

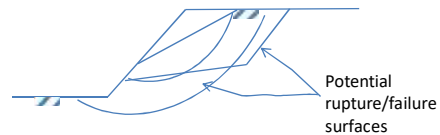
Book: Principles of Geotechnical Engineering, B M Das, 4th edn.

Slope

An exposed ground surface that stands at an angle with the horizontal is called an *unrestrained slope*. The slope can be natural or man-made. If the ground surface is not horizontal, a component of gravity will tend to move the soil downward. If the component of gravity is large enough, slope failure can occur.

Slope Stability Analysis

In many cases, civil engineers are expected to make calculations to check the safety of natural slopes, slopes of excavations, and compacted embankments. This check involves determining and comparing the shear stress developed along the most likely rupture surface with the shear strength of the soil. This process is called *slope stability analysis*. The most likely rupture surface is the critical plane that has the minimum factor of safety.



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Macro stability of slope

The material composing a slope has a natural tendency to slide under the influence of gravitational and other forces (e.g. seismic activity) which is resisted by the shearing resistance of the material. **Instability occurs when the shearing resistance is not enough to counterbalance the forces tending to cause movement along any surface within a slope.** A slope that has been stable for many years may suddenly fail due to one or several of the following main causes.

- (a) external disturbances in the form of cutting / dredging / scour near toe.
- (b) seismic activity
- (c) increase in pore water pressure within a slope (may be due to rise in water level or exceptional rainfall)
- (d) progressive decrease in shear strength of the slope material
- (e) progressive change in the stress field within a slope (may be due to subsidence of foundation soil, piping or internal erosion, movement of soil particles)

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Phenomena Governing Stability of Slope

Stable earth slopes, both natural and man-made, are of great importance to mankind. Although many landslides occur in slopes because of **natural influences, human activities such as undercutting, piling earth on unstable slopes, or raising the ground water level** by constructing reservoirs, **are important causes of landslides**. Unfavourable ground water and seepage conditions are among the most frequent. **Water lowers stability and contributes to slope failures** in the following ways:

- By reducing or eliminating cohesive strength
- By producing pore water pressure which reduce effective stresses, thereby lowering effective strength
- By producing horizontally inclined seepage forces which increase the overturning moments and possibility of failure
- By lubricating failure planes after occurring small initial movement
- By supplying an excess of fluid that becomes trapped in soil pores during earthquakes or other severe shocks, leading to liquefaction failures.

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Factor of Safety

As engineer we need to ensure the safety of a slope through determining the factor of safety. The factor of safety is defined as

$$F_s = \frac{\tau_f}{\tau_d}$$

Where

F_s = Factor of safety with respect to strength

τ_f = Average shear strength of the soil

τ_d = average shear stress developed along the potential failure surface

The shear strength of a soil consists of two components, cohesion and friction, and may be written as

$$\tau_f = c + \sigma \tan \phi \quad (13.2)$$

where c = cohesion

ϕ = angle of friction

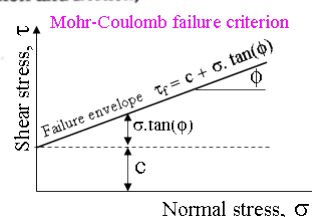
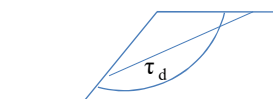
σ = normal stress on the potential failure surface

In a similar manner, we can write

$$\tau_d = c_d + \sigma \tan \phi_d \quad (13.3)$$

where c_d and ϕ_d are, respectively, the cohesion and the angle of friction that develop along the potential failure surface. Substituting Eqs. (13.2) and (13.3) into Eq. (13.1), we get

$$F_s = \frac{c + \sigma \tan \phi}{c_d + \sigma \tan \phi_d} \quad (13.4)$$



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Now we can introduce some other aspects of the factor of safety — that is, the factor of safety with respect to cohesion, F_c , and the factor of safety with respect to friction, F_ϕ . They are defined as follows:

$$F_c = \frac{c}{c_d} \quad (13.5)$$

and

$$F_\phi = \frac{\tan \phi}{\tan \phi_d} \quad (13.6)$$

When we compare Eqs. (13.4) through (13.6), we can see that when F_c becomes equal to F_ϕ , it gives the factor of safety with respect to strength. Or, if

$$\frac{c}{c_d} = \frac{\tan \phi}{\tan \phi_d} \quad \text{we can write} \quad F_s = F_c = F_\phi \quad (13.7)$$

When F_s is equal to 1, the slope is in a state of impending failure. Generally, a value of 1.5 for the factor of safety with respect to strength is acceptable for the design of a stable slope.

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Methods of slope stability analysis can be grouped into two categories

- (1) Limit equilibrium methods
- (2) Stress analysis methods - e.g. Finite element analysis

In this course we shall discuss only Limit equilibrium methods

The approach for Limit equilibrium method

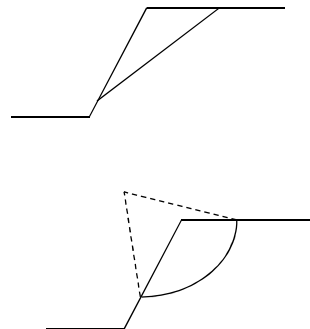
A failure surface is assumed (plane or curve)

For a plane failure surface,

imposed shear stress = resisting shear stress

For circular failure surface,

the disturbing moment = resisting moment



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Semi infinite – unbounded in one direction

Infinite Slope

An 'infinite slope' is one which represents the boundary surface of a semi-infinite soil mass inclined to the horizontal. In practice, if the height of the slope is very large, one may consider it as an infinite one.

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13.2 STABILITY OF INFINITE SLOPES WITHOUT SEEPAGE

$T_i = T_a = W \sin \beta = \gamma L H \sin \beta$

$\tau = \frac{T_a}{\text{Area}} = \frac{\gamma L H \sin \beta}{L / \cos \beta} = \gamma H \sin \beta \cos \beta$

$\sigma = \frac{N_a}{\text{Area}} = \frac{W \cos \beta}{L / \cos \beta} = \frac{\gamma L H \cos^2 \beta}{L} = \gamma H \cos^2 \beta$

Shear stress on the plane can also be written as
 $\tau_d = c_d + \sigma \tan \phi_d$ *$c_d, \phi_d \rightarrow$ mobilized parameters*

From definition of Factor of safety
 $c_d = \frac{c}{F_s}$ and $\tan \phi_d = \frac{\tan \phi}{F_s}$ *$c, \phi \rightarrow$ ultimate / peak strength parameters*

Taking $\tau = \tau_d$, substituting $\sigma, c_d, \tan \phi_d$ and re-arranging

$$F_s = \frac{c}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi}{\tan \beta} \quad (13.15)$$

For granular soils, $c = 0$, and the factor of safety, F_s , becomes equal to $(\tan \phi) / (\tan \beta)$. This indicates that in an infinite slope in sand, the value of F_s is independent of the height, H , and the slope is stable as long as $\beta < \phi$.

If a soil possesses cohesion and friction, the depth of the plane along which critical equilibrium occurs may be determined by substituting $F_s = 1$ and $H = H_c$ into Eq. (13.15). Thus,

$$H_c = \frac{c}{\gamma \cos^2 \beta (\tan \beta - \tan \phi)} \quad (13.16)$$

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13.3 STABILITY OF INFINITE SLOPES WITH SEEPAGE

$$T_r = T_a = W \sin \beta = \gamma_{sat} L H \sin \beta$$

T_a = in-plane component of W
 T_r = in-plane reaction force
 τ = shear stress
 σ = normal stress

$$\tau = \frac{T_r}{\text{Area}} = \frac{\gamma L H \sin \beta}{L / \cos \beta} = \gamma H \sin \beta \cos \beta$$

$$\sigma = \frac{N_a}{\text{Area}} = \frac{W \cos \beta}{L / \cos \beta} = \frac{\gamma_{sat} L H \cos^2 \beta}{L} = \gamma_{sat} H \cos^2 \beta$$

Shear stress on the plane can also be written as

$$\tau_d = c_d + \sigma' \tan \phi_d \quad c_d, \phi_d \rightarrow \text{mobilized parameters}$$

Now, $\sigma' = \sigma - u$ Where, $u = h \gamma_w = H \gamma_w \cos^2 \beta$

From definition of Factor of safety

$$c_d = \frac{c}{F_s} \quad \text{and} \quad \tan \phi_d = \frac{\tan \phi}{F_s}$$

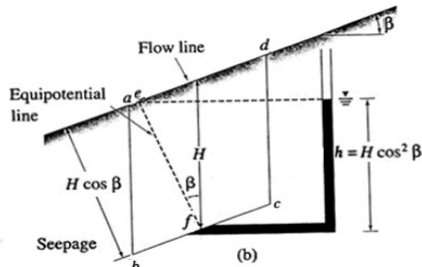
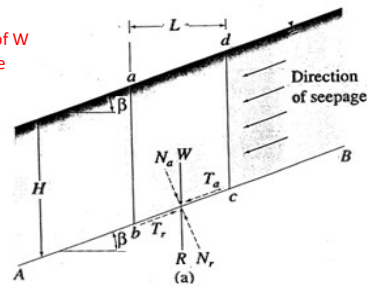
Taking $\tau = \tau_d$, substituting $\sigma, c_d, \tan \phi_d$ and re-arranging

$$F_s = \frac{c}{\gamma_{sat} H \cos^2 \beta \tan \beta} + \frac{\gamma' \tan \phi}{\gamma_{sat} \tan \beta} \quad (13.28)$$

Compare with the expression for the case of no seepage

$$F_s = \frac{c}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi}{\tan \beta}$$

In the first term, in the denominator $\gamma_{sat} > \gamma$; so the first term reduces
 In the second term, the multiplier $\gamma'/\gamma_{sat} < 1$; so the second term also reduces
 Thus it is quite obvious that factor of safety reduces when seepage forces act



EXAMPLE 13.1

An infinite slope is shown in Figure 13.6. The shear strength parameters at the interface of soil and rock are as follows: $c = 18 \text{ kN/m}^2$ and $\phi = 25^\circ$.

- If $H = 8 \text{ m}$ and $\beta = 20^\circ$, find the factor of safety against sliding on the rock surface.
- If $\beta = 30^\circ$, find the height, H , for which $F_s = 1$. (Assume pore water pressure to be zero.)

Solution

- Given that $\rho = 1900 \text{ kg/m}^3$, the unit weight of soil is as follows:

$$\gamma = \rho \cdot g = \frac{1900 \times 9.81}{1000} = 18.64 \text{ kN/m}^3$$

From Eq. (13.15),

$$F_s = \frac{c}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi}{\tan \beta} = \frac{18}{(18.64)(8)(\cos 20^\circ)^2 (\tan 20^\circ)} + \frac{\tan 25^\circ}{\tan 20^\circ} = 0.376 + 1.28 = 1.656$$

- From Eq. (13.16),

$$H_{cr} = \frac{c}{\gamma \cos^2 \beta (\tan \beta - \tan \phi)} = \frac{18}{18.64 \cos^2 30^\circ (\tan 30^\circ - \tan 25^\circ)} = 11.6 \text{ m}$$

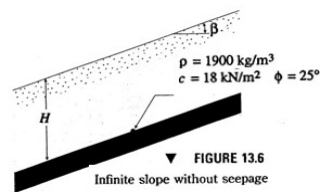


FIGURE 13.6 Infinite slope without seepage

▼ **EXAMPLE 13.2**

Refer to Figure 13.6. If there were seepage through the soil, and the groundwater table coincided with the ground surface, what would be the value of F_s ? Use $H = 8$ m, $\rho_{sat} = 1900$ kg/m³, and $\beta = 20^\circ$.

Solution $\gamma_{sat} = 18.64$ kN/m³ and $\gamma_w = 9.81$ kN/m³. So,

$$\gamma' = \gamma_{sat} - \gamma_w = 18.64 - 9.81 = 8.83 \text{ kN/m}^3$$

From Eq. (13.28),

$$F_s = \frac{c}{\gamma_{sat} H \cos^2 \beta \tan \beta} + \frac{\gamma' \tan \phi}{\gamma_{sat} \tan \beta}$$

$$= \frac{18}{(18.64)(8)(\cos 20^\circ)^2 \tan 20^\circ} + \frac{8.83 \tan 25^\circ}{18.64 \tan 20^\circ}$$

$$= 0.376 + 0.606 = \mathbf{0.98}$$

The value of F_s would be less than 1; hence the slope would be

Comparing Example 13.1 and 13.2 we find that the given slope is stable when dry but will fail when gets saturated.

