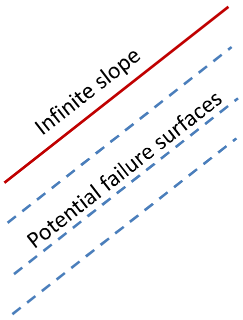
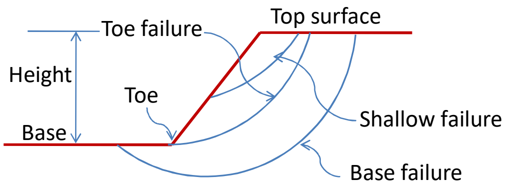


FINITE SLOPES

A finite slope is one with a base and a top surface, the height being limited.



Book: Principles of Geotechnical Engineering, B M Das, 4th edn.

Failure surfaces may consist of a single/multiple plane/curved surfaces depending on the type and thickness of soil layers.

13.5 ANALYSIS OF FINITE SLOPES WITH PLANE FAILURE SURFACES (CULMANN'S METHOD)

Culmann's analysis: based on assumption that failure occurs along a plane

$$W = \tfrac{1}{2}(H)(\overline{BC})(1)(\gamma) = \tfrac{1}{2}H(H\cot\theta - H\cot\beta)\gamma = \frac{1}{2}\gamma H^2\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right] \quad (13.29)$$

The normal and tangential components of W with respect to the plane AC are

$$N_a = \text{normal component} = W\cos\theta = \frac{1}{2}\gamma H^2\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\cos\theta \quad (13.30)$$

N_a, T_a : acting no plane AC

$$T_a = \text{tangential component} = W\sin\theta = \frac{1}{2}\gamma H^2\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\sin\theta \quad (13.31)$$

The average normal stress and the average shear stress on the plane AC are, respectively,

$$\sigma = \frac{N_a}{(\overline{AC})(1)} = \frac{N_a}{\left(\frac{H}{\sin\theta}\right)} = \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\cos\theta\sin\theta \quad (13.32)$$

and

$$\tau = \frac{T_a}{(\overline{AC})(1)} = \frac{T_a}{\left(\frac{H}{\sin\theta}\right)} = \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\sin^2\theta \quad (13.33)$$

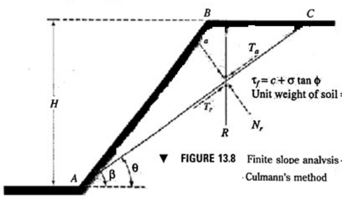


FIGURE 13.8 Finite slope analysis — Culmann's method

H – height of the slope
β - angle of the slope with horizontal
W – weight of wedge ABC
AC – arbitrary failure plane
θ – angle of the failure plane with horizontal

The average resistive shearing stress developed along the plane AC may also be expressed as

$$\tau_d = c_d + \sigma\tan\phi_d = c_d + \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\cos\theta\sin\theta\tan\phi_d \quad (13.34)$$

Taking $\tau = \tau_d$ and substituting σ from eqn. 13.32 and rearranging

$$c_d = \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)(\sin\theta - \cos\theta\tan\phi_d)}{\sin\beta}\right] \quad (13.36)$$

$$c_d = \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)(\sin\theta - \cos\theta\tan\phi_d)}{\sin\beta}\right] \qquad (13.36)$$

The expression in Eq. (13.36) is derived for the trial failure plane *AC*.

To determine the plane for which developed cohesion will be maximum (for a given ϕ_d)

$$\frac{\partial c_d}{\partial\theta} = 0 \qquad (13.37)$$

Because γ, H, and β are constants in Eq. (13.36), we have

$$\frac{\partial}{\partial\theta}[\sin(\beta-\theta)(\sin\theta - \cos\theta\tan\phi_d)] = 0 \qquad (13.38)$$

Solution of Eq. (13.38) gives the critical value of θ, or $\theta_{cr} = \frac{\beta+\phi_d}{2}$ (13.39)

Substitution of the value of $\theta = \theta_{cr}$ into Eq. (13.36) yields

$$c_d = \frac{\gamma H}{4}\left[\frac{1-\cos(\beta-\phi_d)}{\sin\beta\cos\phi_d}\right] \qquad (13.40)$$

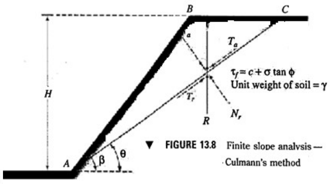
The preceding equation can also be written as

$$\frac{c_d}{\gamma H} = m = \frac{1-\cos(\beta-\phi_d)}{4\sin\beta\cos\phi_d} \qquad (13.41)$$ where m = stability number.

On the basis of Eq. (13.41), the values of $1/m$ for various values of β and ϕ_d are given in Table 13.1.

The maximum height of the slope for which critical equilibrium occurs can be obtained by substituting $c_d = c$ and $\phi_d = \phi$ into Eq. (13.40). Thus,

$$H_{cr} = \frac{4c}{\gamma}\left[\frac{\sin\beta\cos\phi}{1-\cos(\beta-\phi)}\right] \qquad (13.42)$$



▼ **FIGURE 13.8** Finite slope analysis—Culmann's method

▼ **TABLE 13.1** Stability Numbers Based on Culmann's Analysis [Eq. (13.41)]

Slope angle, β (deg)	ϕ_d (deg)	$1/m$	Slope angle, β (deg)	ϕ_d (deg)	$1/m$
10	0	45.72	50	20	21.49
	5	181.84		25	29.64
15	0	30.38		30	44.00
	5	67.89	60	0	6.93
	10	267.93		5	8.09
20	0	22.69		10	9.55
	5	40.00		15	11.42
	10	88.68		20	13.91
	15	347.27		25	17.36
25	0	18.04		30	22.39
	5	27.92	70	0	5.71
	10	48.86		5	6.49
	15	107.48		10	7.40
	20	417.45		15	8.51
30	0	14.93		20	9.89
	5	21.27		25	11.63
	10	32.66		30	13.91
	15	56.70	80	0	4.77
	20	123.71		5	5.29
	25	476.34		10	5.90
40	0	10.99		15	6.59
	5	14.16		20	7.40
	10	18.90		25	8.37
	15	26.51		30	9.55
	20	40.06	90	0	4.00
	25	68.39		5	4.37
	30	146.57		10	4.77
50	0	8.58		15	5.21
	5	10.42		20	5.71
	10	12.89		25	6.28
	15	16.37		30	6.93

▼ **EXAMPLE 13.3**

A cut is to be made in a soil that has $\gamma = 105$ lb/ft³, $c = 600$ lb/ft², and $\phi = 15°$. The side of the cut slope will make an angle of 45° with the horizontal. What should be the depth of the cut slope that will have a factor of safety, F_s, of 3?

Solution We are given that $\phi = 15°$ and $c = 600$ lb/ft². If $F_s = 3$, then, from Eq. (13.7), F_c and F_ϕ should both be equal to 3. From Eq. (13.5),

$$F_c = \frac{c}{c_d} \qquad \text{or} \qquad c_d = \frac{c}{F_c} = \frac{c}{F_s} = \frac{600}{3} = 200\ \text{lb/ft}^2$$

Similarly, from Eq. (13.6),

$$F_\phi = \frac{\tan\phi}{\tan\phi_d} \quad ==> \quad \tan\phi_d = \frac{\tan\phi}{F_\phi} = \frac{\tan\phi}{F_s} = \frac{\tan 15}{3}$$

or $\phi_d = \tan^{-1}\left[\frac{\tan 15}{3}\right] = 5.1°$

Substituting the preceding values of c_d and ϕ_d into Eq. (13.40) gives

$$H = \frac{4c_d}{\gamma}\left[\frac{\sin\beta\cos\phi_d}{1-\cos(\beta-\phi_d)}\right] = \frac{4\times 200}{105}\left[\frac{\sin 45\cos 5.1}{1-\cos(45-5.1)}\right]$$

$= \mathbf{23\ ft}$

13.6 SLOPES WITH WATER IN THE TENSILE CRACK

In many cases, tensile cracks develop at the top of a slope. The cracks are filled with water, as shown in Figure 13.9.

Let z_c = depth of crack

z_1 = depth of water in the crack

We can consider a linear distribution of water pressure along trial failure surface AB (zero at A and $z_c\gamma_w$ at B)

So, the forces per unit length acting on the trial wedge *ABCD* are as follows:

1. W — total weight of the wedge
2. U_1 — force due to water in the crack
3. U_2 — force due to pore water pressure along *AB*

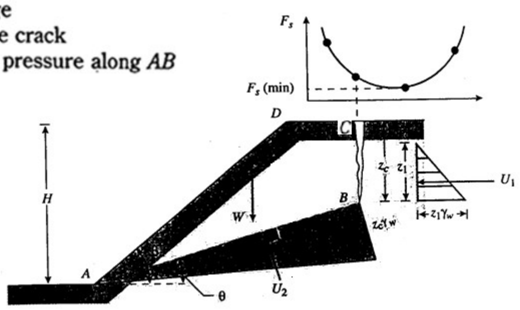
However,

$$U_1 = \frac{1}{2}\gamma_w z_1^2$$

$$U_2 = \frac{1}{2}\gamma_w z_c X$$

where $X = AB = (H - z_c)\operatorname{cosec}\theta$. So,

$$U_2 = \frac{1}{2}\gamma_w z_c (H - z_c)\operatorname{cosec}\theta$$



▼ **FIGURE 13.9** Slope with water in tensile crack

The sum of the components (F) of these forces parallel to AB will tend to slide the wedge, or

$$F = W\sin\theta + U_1\cos\theta$$

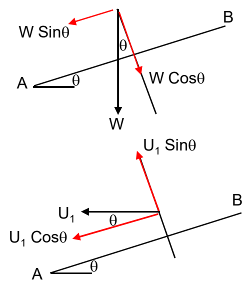
The maximum resisting force that can be developed along AB is

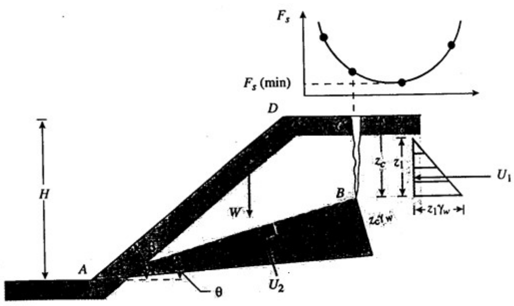
$$R = cX + (W\cos\theta - U_1\sin\theta - U_2)\tan\phi$$

The factor of safety with respect to strength can then be given as

$$F_s = \frac{R}{F} = \frac{cX + (W\cos\theta - U_1\sin\theta - U_2)\tan\phi}{W\sin\theta + U_1\cos\theta} \qquad (13.43)$$

The magnitude of F_s for various trial wedges can be calculated by varying the value of θ. The minimum value of F_s is the factor of safety of the slope with respect to strength.





▼ **FIGURE 13.9** Slope with water in tensile crack