Book: Principles of Geotechnical Engineering, B M Das, 4th edn.

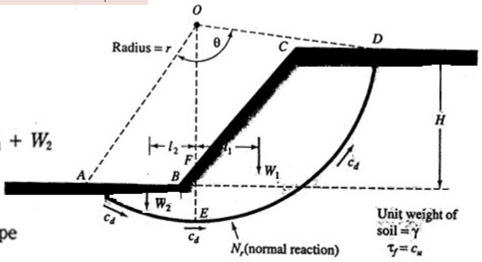
Finite Slope : Circular Failure Surface

This analysis also falls into "Total stress" analysis category

13.8 MASS PROCEDURE – SLOPES IN HOMOGENEOUS CLAY SOIL WITH $\phi = 0$

In undrained condition, $\phi = 0$

Referring to Fig.13.11
- ABCD – Slope in Homogeneous soil
- Undrained condition
- Shear strength is constant with depth, $\tau_f = c_u$
- AED – Trial failure surface, center O, radius r



▼ FIGURE 13.11 Stability analysis of slope in homogeneous clay soil ($\phi = 0$)

weight of the soil above the curve *AED* as $W = W_1 + W_2$

where $W_1 = (\text{Area of } FCDEF)(\gamma)$

and $W_2 = (\text{Area of } ABFEA)(\gamma)$

The moment of the driving force about *O* to cause slope instability is

$$M_d = W_1 l_1 - W_2 l_2 \quad (13.44)$$

where l_1 and l_2 are the moment arms.

If c_d is the cohesion along the potential surface of sliding, then moment of the resisting forces about *O* is

$$M_R = c_d(\widehat{AED})(1)(r) = c_d r^2 \theta \quad (13.45)$$

For equilibrium, $M_R = M_d$; thus,

$$c_d r^2 \theta = W_1 l_1 - W_2 l_2$$

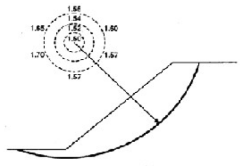
or

$$c_d = \frac{W_1 l_1 - W_2 l_2}{r^2 \theta} \quad (13.46)$$

The factor of safety against sliding may now be found:

$$F_s = \frac{\tau_f}{c_d} = \frac{c_u}{c_d} \quad (13.47)$$

The failure surface AED was chosen arbitrarily. For the critical surface (for which F_s is minimum i.e. c_d is maximum one must analyse a number of trial surface



Location of centre of critical circle by contours of factor of safety

Stability problems of this type have been solved analytically by Fellenius (1927) and Taylor (1937). For the case of *critical circles*, the developed cohesion can be expressed by the relationship

$$c_d = \gamma H m \quad \text{or} \quad \frac{c_d}{\gamma H} = m \quad (13.48)$$

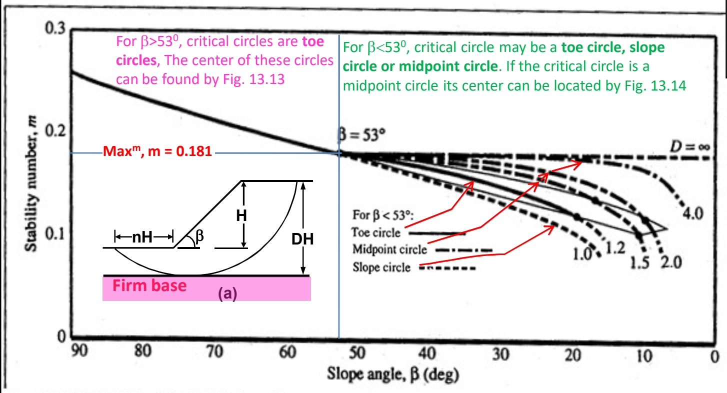
the term *m* is nondimensional and is referred to as the *stability number*. Fig.13.12

For $F_s = 1$

$H = H_{cr}$ and $c_d = c_u$

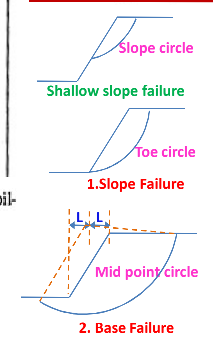
$$H_{cr} = \frac{c_u}{\gamma m} \quad (13.49)$$

Terzaghi used the term $\gamma H / c_d$, the reciprocal of *m*, and called it the *stability factor*.

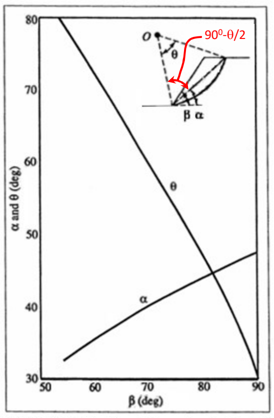


▼ FIGURE 13.12 (a) Definition of parameters for midpoint circle type of failure; (b) plot of stability number against slope angle (redrawn from Terzaghi and Peck, 1967)

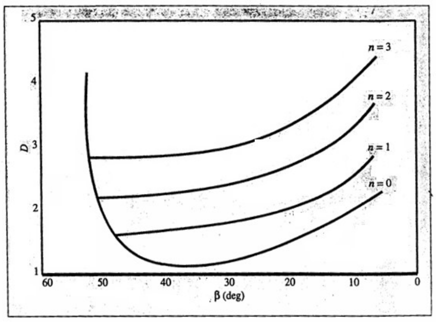
Figure 13.12 is valid for slopes of saturated clay and is applicable to only undrained conditions ($\phi = 0$).



Fellenius (1927) also investigated the case of critical toe circles for slopes with $\beta < 53°$. The location of the center of these circles can be determined with the use of Figure 13.15 and Table 13.2. Note that these critical toe circles are not necessarily the most critical circles that exist.



▼ FIGURE 13.13 Location of the center of critical circles for $\beta > 53°$

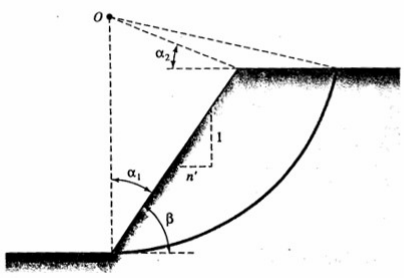


▼ FIGURE 13.14 Location of midpoint circles (after Terzaghi and Peck, 1967)

TABLE 13.2 Location of the Center of Critical Toe Circles ($\beta < 53°$)

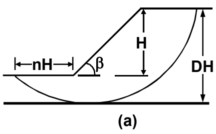
n'	β (deg)	α_1 (deg)	α_2 (deg)
1.0	45	28	37
1.5	33.68	26	35
2.0	26.57	25	35
3.0	18.43	25	35
5.0	11.32	25	37

Note: For notations of n', β, α_1, and α_2, see Figure 13.15.



▼ FIGURE 13.15 Location of the center of critical toe circles for $\beta < 53°$

▼ **EXAMPLE 13.4**



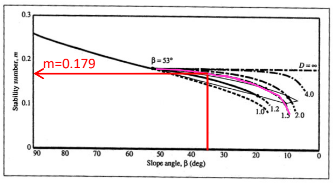
A saturated clay embankment has a height (H) of 30 ft. A rock layer is located at a depth of 45 ft measured from the top of the embankment. Given that the slope angle (β) = 35°, c_u = 1000 lb/ft², and γ = 120 lb/ft³, determine

a. Factor of safety against sliding, F_s
b. Nature of the critical failure circle

Solution

a. Given: $\beta = 35°$ and $H = 30$ ft. So, from Eq. (13.50),

$$D = \frac{45}{30} = 1.5$$



Referring to Figure 13.12, we find that $m = 0.179$.

From Eq. (13.48), $m = 0.179 = \dfrac{c_d}{\gamma H}$

⇨ $c_d = (0.179)(120)(30) = 644.4 \text{ lb/ft}^2$

From Eq. (13.47), $F_s = \dfrac{c_u}{c_d} = \dfrac{1000}{644.4} = \mathbf{1.55}$

b. Referring to Figure 13.12, we can see that the critical circle is a **mid point circle.**

▼ EXAMPLE 13.5

A cut slope is to be made in a soft clay with its sides rising at an angle of 75° to the horizontal (Figure 13.16). Given: $c_u = 650 \text{ lb/ft}^2$ and $\gamma = 110 \text{ lb/ft}^3$.

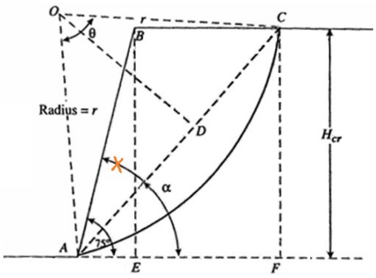
a. Determine the maximum depth up to which the excavation can be carried out.
b. Find the radius, r, of the critical circle when the factor of safety is equal to 1 [part (a)].
c. Find the distance $\overline{BC}$.

Analyze for undrained condition by - circular failure surface / Fellenius method/ Taylor method

Solution

a. Because the slope angle (β) = 75° > 53°, the critical circle is a toe circle. From Figure 13.12, for $\beta = 75°$, the stability number (m) = 0.219. From Eq. (13.49),

$$H_{cr} = \frac{c_u}{\gamma m} = \frac{650}{110 \times 0.219} = \mathbf{26.98 \text{ ft}}$$



▼ FIGURE 13.16 Cut slope in soft clay

b. From Figure 13.16, $r = \dfrac{\overline{DC}}{\sin\dfrac{\theta}{2}}$

But,

$$\overline{DC} = \frac{\overline{AC}}{2} = \frac{\left(\dfrac{H_{cr}}{\sin\alpha}\right)}{2}$$

So $r = \dfrac{H_{cr}}{2\sin\alpha\sin\dfrac{\theta}{2}}$

From Figure 13.13, for $\beta = 75°$, $\alpha = 41.8°$ and $\theta = 51.8°$. Substituting these values into the equation for r, we get

$$r = \frac{26.98}{2(\sin 41.8)(\sin 25.9)} = \mathbf{46.34 \text{ ft}}$$

c. $\overline{BC} = \overline{EF} = \overline{AF} - \overline{AE}$

$$= H_{cr}(\cot\alpha - \cot 75)$$

$$= 26.98(\cot 41.8 - \cot 75) = \mathbf{22.95 \text{ ft}}$$

Do yourself

▼ EXAMPLE 13.6

If the cut described in Example 13.5 is made to a depth of only 10 ft, what is the factor of safety of the slope against sliding?

Solution The stability number corresponding to $\beta = 75°$ is 0.219. From Eq. (13.48),

$$m = \frac{c_d}{\gamma H}$$

So,

$$0.219 = \frac{c_d}{(110)(10)}$$

or

$$c_d = (0.219)(110)(10) = 240.9 \text{ lb/ft}^2$$

From Eqs. (13.1) and (13.47),

$$F_s = \frac{\tau_f}{\tau_d} = \frac{c_u}{c_d} = \frac{650}{240.9} = \mathbf{2.7}$$ ▲

Do yourself

▼ EXAMPLE 13.7

A cut slope was excavated in a saturated clay. The slope made an angle of 40° with the horizontal. Slope failure occurred when the cut reached a depth of 6.1 m. Previous soil explorations showed that a rock layer was located at a depth of 9.15 m below the ground surface. Assuming an undrained condition and that $\gamma_{sat} = 17.29\ \text{kN/m}^3$,

a. Determine the undrained cohesion of the clay (use Figure 13.12).
b. What was the nature of the critical circle?
c. With reference to the toe of the slope, at what distance did the surface of sliding intersect the bottom of the excavation?

Solution

a. We know that

$$D = \frac{9.15}{6.1} = 1.5$$

$$\gamma_{sat} = 17.29\ \text{kN/m}^3$$

$$H_{cr} = \frac{c_u}{\gamma m} \qquad \text{[Eq. (13.49)]}$$

From Figure 13.12, for $\beta = 40°$ and $D = 1.5$, $m = 0.175$. So,

$$c_u = (H_{cr})(\gamma)(m) = (6.1)(17.29)(0.175) = \mathbf{18.5\ kN/m^2}$$

b. From Figure 13.12, we find that the critical circle is a **midpoint circle.**

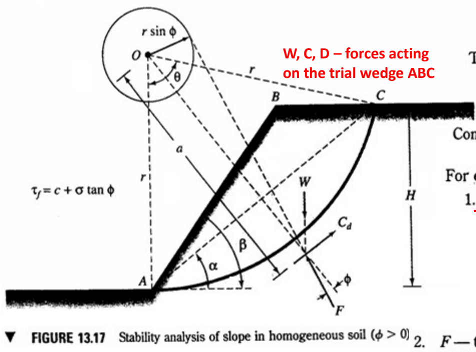
c. From Figure 13.14, for $D = 1.5$ and $\beta = 40°$, $n = 0.9$. So,

$$\text{Distance} = (n)(H_{cr}) = (0.9)(6.1) = \mathbf{5.49\ m}$$ ▲

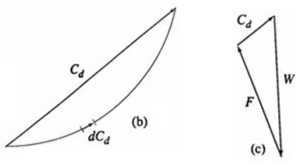
13.9 MASS PROCEDURE — SLOPES IN HOMOGENEOUS SOIL WITH $\phi > 0$

Assumptions: Soil is homogeneous
Pore pressure is zero

W, C, D – forces acting on the trial wedge ABC



▼ FIGURE 13.17 Stability analysis of slope in homogeneous soil ($\phi > 0$)



The shear strength of the soil is given by

$$\tau_f = c + \sigma \tan \phi$$

$\overset{\frown}{AC}$ is a trial circular arc that passes through the toe of the slope, and O is the center of the circle.

Considering a unit length perpendicular to the section of the slope,

Weight of soil wedge $ABC = W =$ (Area of ABC)(γ)

For equilibrium, the following other forces are acting on the wedge:

1. C_d — resultant of the cohesive force along cord $\overline{AC}$.

$$C_d = c_d(\overline{AC}) \qquad (13.51)$$

C_d acts in a direction parallel to the cord $\overline{AC}$ and at a distance a from the center of the circle O such that

$$C_d(a) = c_d(\overset{\frown}{AC})r \quad \text{or} \quad a = \frac{c_d(\overset{\frown}{AC})r}{C_d} = \frac{\overset{\frown}{AC}}{\overline{AC}}r \qquad (13.52)$$

2. F — the resultant of the normal and frictional forces along the surface of sliding. For equilibrium, the line of action of F will pass through the point of intersection of the line of action of W and C_d.

Now, if we assume that full friction is mobilized ($\phi_d = \phi$ or $F_\phi = 1$), the line of action of F will make an angle of ϕ with a normal to the arc and will thus be a tangent to a circle with its center at O and having a radius of $r \sin \phi$. This circle is called the *friction circle*.

Because the directions of W, C_d, and F are known and the magnitude of W is known, The magnitude of C_d can be determined from the force polygon.

So, the cohesion per unit area developed can be found: $c_d = \dfrac{C_d}{\overline{AC}}$

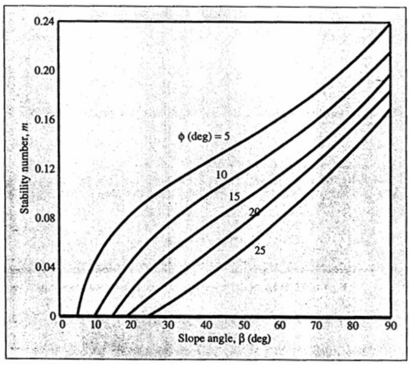
Determination of the magnitude of c_d described previously is based on a trial surface of sliding. Several trials must be made to obtain the most critical sliding surface, along which the developed cohesion is a maximum. Thus, we can express the maximum cohesion developed along the critical surface as

$$c_d = \gamma H[f(\alpha, \beta, \theta, \phi)] \qquad (13.53)$$

For critical equilibrium — that is, $F_c = F_\phi = F_s = 1$ — we can substitute $H = H_{cr}$ and $c_d = c$ into Eq. (13.53):

$$c = \gamma H_{cr}[f(\alpha, \beta, \theta, \phi)] \quad \text{or} \quad \frac{c}{\gamma H_{cr}} = f(\alpha, \beta, \theta, \phi) = m \qquad (13.54)$$

where m = stability number. The values of m for various values of ϕ and β are given in Figure 13.18. Example 13.8 illustrates the use of this chart.



▼ **FIGURE 13.18** Plot of stability number with slope angle; $\phi > 0$ (after Taylor, 1937)

▼ **EXAMPLE 13.8**

A slope with $\beta = 45°$ is to be constructed with a soil that has effective stress parameters of $\phi = 20°$ and $c = 23.95$ kN/m². The unit weight of the compacted soil will be 18.87 kN/m³.

a. Find the critical height of the slope.
b. If the height of the slope is 10 m, determine the factor of safety with respect to strength.

Solution

a. From Eq. (13.54), $m = \frac{c}{\gamma H_{cr}}$

From Figure 13.18, for $\beta = 45°$ and $\phi = 20°$, $m = 0.06$. So, $H_{cr} = \frac{c}{\gamma m} = \frac{23.95}{(18.87)(0.06)} = \mathbf{21.15\ m}$

b. If we assume that full friction is mobilized, then, referring to Figure 13.18 (for $\beta = 45°$ and $\phi_d = \phi = 20°$), we have

$$m = 0.06 = \frac{c_d}{\gamma H} \quad \text{or} \quad c_d = (0.06)(18.87)(10) = 11.32 \text{ kN/m}^2$$

Thus, from Eq. (13.6), $F_\phi = \frac{\tan \phi}{\tan \phi_d} = \frac{\tan 20}{\tan 20} = 1$

and, from Eq. (13.5), $F_c = \frac{c}{c_d} = \frac{23.95}{11.32} = 2.12$

Because $F_c \neq F_\phi$, this is not the factor of safety with respect to strength.

Now we can make another trial. Let the developed angle of friction, ϕ_d, be equal to 15°. For $\beta = 45°$ and the friction angle equal to 15°, we find

$$m = 0.085 = \frac{c_d}{\gamma H} \quad \text{(Figure 13.18)} \quad \text{or} \quad c_d = (0.085)(18.87)(10) = 16.04 \text{ kN/m}^2$$

For this trial,

$$F_\phi = \frac{\tan \phi}{\tan \phi_d} = \frac{\tan 20}{\tan 15} = 1.36$$

and

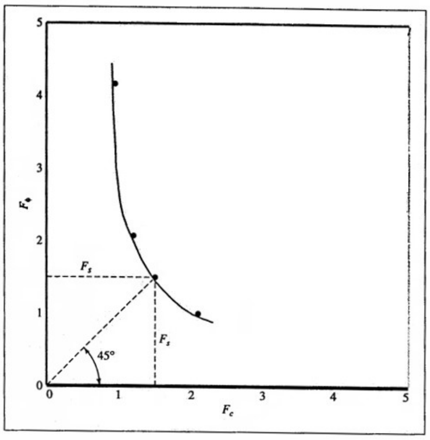
$$F_c = \frac{c}{c_d} = \frac{23.95}{16.04} = 1.49$$

Similar calculations of F_ϕ and F_c for various assumed values of ϕ_d can be made. They are given in the following table:

ϕ_d	$\tan \phi_d$	F_ϕ	m	c_d (kN/m²)	F_c
20	0.364	1.0	0.06	11.32	2.12
15	0.268	1.36	0.085	16.04	1.49
10	0.176	2.07	0.11	20.75	1.15
5	0.0875	4.16	0.136	25.66	0.93

The values of F_ϕ are plotted against their corresponding Figure 13.19, from which we find

$F_c = F_\phi = F_s =$ **1.45**



▼ FIGURE 13.19 Determination of F_s by plotting values of F_c versus values of F_ϕ