13.10 ORDINARY METHOD OF SLICES

AC is an arc of a circle

The soil above the trial failure surface is divided into several vertical slices. The width of each slice need not be the same. Considering a unit length perpendicular to the cross section shown, the forces that act on a typical slice (*n*th slice) are shown in Figure 13.20b.

assumption
pore water pressure is zero.
the resultants of P_n and T_n are equal in magnitude to the resultants of P_{n+1} and T_{n+1}, and that their lines of action coincide.

W_n - weight of the slice.

N_r and T_r - normal and tangential components of the reaction, R, respectively.

P_n and P_{n+1} - normal forces that act on the sides of the slice

T_n and T_{n+1} - shearing forces that act on the sides of the slice

For equilibrium consideration, $N_r = W_n \cos \alpha_n$

The resisting shear force can be expressed as

$$T_r = \tau_d(\Delta L_n) = \frac{\tau_f(\Delta L_n)}{F_s} = \frac{1}{F_s}[c + \sigma \tan \phi]\Delta L_n \qquad (13.55)$$

The normal stress, σ, in Eq. (13.55) is equal to $\dfrac{N_r}{\Delta L_n} = \dfrac{W_n \cos \alpha_n}{\Delta L_n}$

For equilibrium of the trial wedge *ABC*, the moment of the driving force about *O* equals the moment of the resisting force about *O*,

or
$$\sum_{n=1}^{n=p} W_n r \sin \alpha_n = \sum_{n=1}^{n=p} \frac{1}{F_s}\left(c + \frac{W_n \cos \alpha_n}{\Delta L_n} \tan \phi\right)(\Delta L_n)(r)$$

or
$$F_s = \frac{\sum_{n=1}^{n=p} (c\Delta L_n + W_n \cos \alpha_n \tan \phi)}{\sum_{n=1}^{n=p} W_n \sin \alpha_n} \qquad (13.56)$$

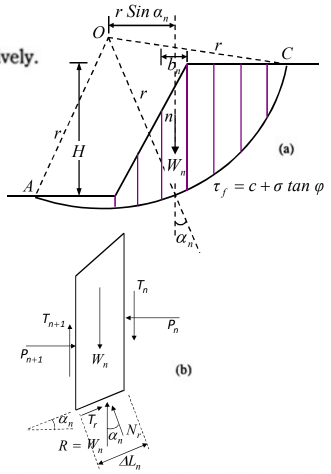


FIGURE 13.20 Stability analysis by ordinary method of slices: (a) trial failure surface; (b) forces acting on *n*th slice

The ordinary method of slices can also be used for drained conditions/ effective stress analysis

$$N_r = W_n \cos \alpha_n - u_n \Delta L_n$$

$$\sigma' = \frac{W_n \cos \alpha_n - u_n \Delta L_n}{\Delta L_n}$$

$$T_r = \frac{1}{F_s}[c' + \sigma' \tan\varphi']\Delta L_n$$

$$= \frac{1}{F_s}\left[c' + \frac{W_n \cos \alpha_n - u_n \Delta L_n}{\Delta L_n}\tan\varphi'\right]\Delta L_n$$

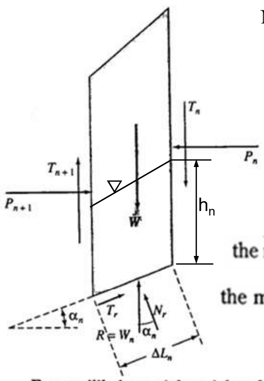
Compare with no seepage case

For equilibrium consideration,

$N_r = W_n \cos \alpha_n$

The resisting shear force can be expressed as

$$T_r = \tau_d(\Delta L_n) = \frac{\tau_f(\Delta L_n)}{F_s} = \frac{1}{F_s}[c + \sigma \tan \phi]\Delta L_n \qquad (13.55)$$



the moment of the driving force about *O*
$$M_d = \sum_{n=1}^{n=p} W_n r \sin\alpha_n$$

the moment of the resisting force about *O*,
$$M_r = \frac{1}{F_s}\left[c' + \frac{W_n \cos\alpha_n - u_n \Delta L_n}{\Delta L_n}\tan\varphi'\right]\Delta L_n . r$$

For equilibrium of the trial wedge *ABC*, the moment of the driving force about *O* equals the moment of the resisting force about *O*, or

$$\sum_{n=1}^{n=p} W_n r \sin\alpha_n = \sum_{n=1}^{n=p} \frac{1}{F_s}\left[c' + \frac{W_n \cos\alpha_n - u_n \Delta L_n}{\Delta L_n}\tan\varphi'\right]\Delta L_n . r \Rightarrow F_s = \frac{c' \Delta L_n + (W_n \cos\alpha_n - u_n \Delta L_n)\tan\varphi'}{\sum_{n=1}^{n=p} W_n \sin\alpha_n}$$

[*Note:* ΔL_n in Eq. (13.56) is approximately equal to $(b_n)/(\cos \alpha_n)$, where b_n = the width of the *n*th slice.]

α_n may be either positive or negative. The value of α_n is positive when the slope of the arc is in the same quadrant as the ground slope.

To find the minimum factor of safety — that is, the factor of safety for the critical circle — one must make several trials by changing the center of the trial circle.

Compare with the expression for no seepage/water pressure

$$F_s = \frac{\sum_{n=1}^{n=p} (c\Delta L_n + W_n \cos \alpha_n \tan \phi)}{\sum_{n=1}^{n=p} W_n \sin \alpha_n} \qquad (13.56)$$

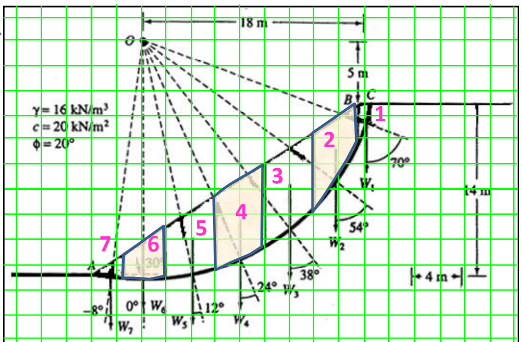
▼ **EXAMPLE 13.9**

For the slope shown in Figure 13.22, find the factor of safety against sliding for the trial slip surface AC. Use the ordinary method of slices.

Solution The sliding wedge is divided into seven slices.

$$F_s = \frac{\sum_{n=1}^{n=p}(c\Delta L_n + W_n \cos\alpha_n \tan\phi)}{\sum_{n=1}^{n=p} W_n \sin\alpha_n} \quad (13.56)$$

$$F_s = \frac{c\sum_{n=1}^{n=p}\Delta L_n + \tan\phi\sum_{n=1}^{n=p} W_n\cos\alpha_n}{\sum_{n=1}^{n=p} W_n\sin\alpha_n}$$



▼ **FIGURE 13.22** Stability analysis of a slope by ordinary method of slices

Slice No. (1)	Width, b_n m (2)	Avg. ht., h_n, m, (3)	Wt., W, kN/m (4)	α_n (deg), (5)	ΔL_n, m (6)	$W_n \sin\alpha_n$ (7)	$W_n \cos\alpha_n$ (8)
1	1	1.4	22.4	70	2.92	21.05	7.66
2	4	4.6	294.4	54	6.80	238.17	173.04
3	4	6.8	435.2	38	5.08	267.94	342.94
4	4	6.8	435.2	24	4.38	177.01	397.57
5	4	6.1	390.4	12	4.09	81.17	381.87
6	4	4.2	268.8	0	4.00	0.00	268.80
7	3.2	1.3	66.6	-8	3.23	-9.26	65.91
Σ					30.50	776.08	1637.80

$$F_s = \frac{20 \times 30.50 + \tan 20 \times 1637.80}{776.08}$$

$$= 1.55$$

13.11 BISHOP'S SIMPLIFIED METHOD OF SLICES

Probably most widely used method

Ordinary method of slices is too conservative

In this method, the effect of forces on the sides of each slice are accounted for to some degree.

The forces that act on the nth slice have been drawn in Figure 13.23a. Now, let $P_n - P_{n+1} = \Delta P$ and $T_n - T_{n+1} = \Delta T$.

So, we can write

$$T_r = N_r(\tan\phi_d) + c_d\Delta L_n = N_r\left(\frac{\tan\phi}{F_s}\right) + \frac{c\Delta L_n}{F_s} \quad (13.57)$$

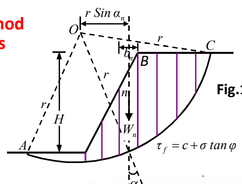


Fig.13.20a

Figure 13.23b shows the force polygon for equilibrium of the nth slice. Summing the forces in the vertical direction gives

$$W_n + \Delta T = N_r\cos\alpha_n + \left[\frac{N_r\tan\phi}{F_s} + \frac{c\Delta L_n}{F_s}\right]\sin\alpha_n$$

or
$$N_r = \frac{W_n + \Delta T - \frac{c\Delta L_n}{F_s}\sin\alpha_n}{\cos\alpha_n + \frac{\tan\phi\sin\alpha_n}{F_s}} \quad (13.58)$$

Only moment equilibrium is satisfied, force equilibrium is not satisfied

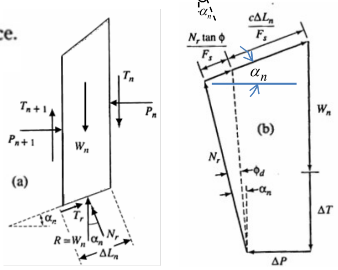


FIGURE 13.23 (a) forces acting on the nth slice; (b) force polygon for equilibrium

For equilibrium of the wedge ABC (Figure 13.20a), taking the moment about O gives

$$\sum_{n=1}^{n=p} W_n r\sin\alpha_n = \sum_{n=1}^{n=p} T_r r \quad (13.59)$$

where
$$T_r = \frac{1}{F_s}(c + \sigma\tan\phi)\Delta L_n = \frac{1}{F_s}(c\Delta L_n + N_r\tan\phi) \quad (13.60)$$

Substitution of Eqs. (13.58) and (13.60) into Eq. (13.59) gives
$$F_s = \frac{\sum_{n=1}^{n=p}(cb_n + W_n\tan\phi + \Delta T\tan\phi)\frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n\sin\alpha_n} \quad (13.61)$$

where
$$m_{\alpha(n)} = \cos\alpha_n + \frac{\tan\phi\sin\alpha_n}{F_s} \quad (13.62)$$

For simplicity, if we let $\Delta T = 0$, Eq. (13.61) becomes
$$F_s = \frac{\sum_{n=1}^{n=p}(cb_n + W_n\tan\phi)\frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n\sin\alpha_n} \quad (13.63)$$

F_s exists on both sides, so trial and error procedure is required to find F_s.

A number of failure surfaces are to be analysed to find critical surface and minimum F_s.

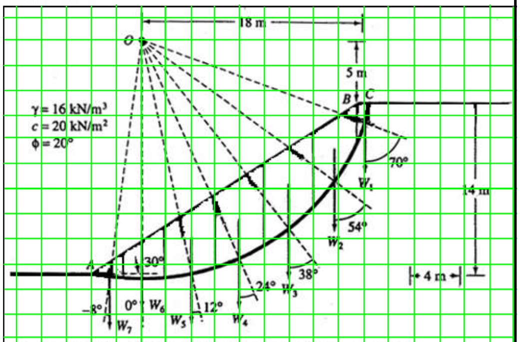
▼ EXAMPLE 13.9

For the slope shown in Figure 13.22, find the factor of safety against sliding for the trial slip surface *AC*. Use the ordinary method of slices.

Solution The sliding wedge is divided into seven slices.

$$F_s = \frac{\sum_{n=1}^{n=p} (cb_n + W_n \tan\phi)\frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n \sin\alpha_n} \qquad (13.63)$$

where $m_{\alpha(n)} = \cos\alpha_n + \frac{\tan\phi \sin\alpha_n}{F_s}$ (13.62)



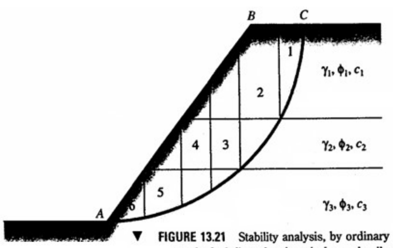
▼ FIGURE 13.22 Stability analysis of a slope by ordinary method of slices

γ, kN/m^3 =	16
ϕ, deg =	20
c, kN/m^2 =	20

F_s =	1.58

Slice No. (1)	Width of slice b_n, m (2)	Avg. ht. of slice, h_n, m, (3)	W_n, kN/m (4)	α_n (deg), (5)	$m_{\alpha(n)}$, (6)	$W_n/m_{\alpha(n)}$, (7)	$W_n\sin\alpha_n$ (8)
1	1	1.4	22.4	70	0.56	40.11	21.0
2	4	4.6	294.4	54	0.77	380.29	238.2
3	4	6.8	435.2	38	0.93	468.04	267.9
4	4	6.8	435.2	24	1.01	432.07	177.0
5	4	6.1	390.4	12	1.03	380.49	81.2
6	4	4.2	268.8	0	1.00	268.80	0.0
7	3.2	1.3	66.6	-8	0.96	69.46	-9.3
Σ	24.2					2039.26	776.1

Fs = 1.58



▼ FIGURE 13.21 Stability analysis, by ordinary method of slices, for slope in layered soils

Method of slices can applied to layered soil
c, ϕ, γ will not be same for all slices; γ may vary within a slice