

**Pile Foundation in Cohesionless Soil**  
**Lecture -03**

## 9.6 ULTIMATE BEARING CAPACITY IN COHESIONLESS SOILS

### 9.6.2 Calculation of Net Point Bearing Capacity

The ultimate bearing capacity  $Q_u$  in cohesionless soils as per Eq. (9.9) is

$$Q_u = Q_b + Q_f$$

$$Q_u = q_o N_q A_b + \bar{q}_o \bar{K}_s \tan \delta A_s \quad (9.9a)$$

or

$$Q_u = q_b A_b + f_s A_s \quad (9.9b)$$

Eq. (9.9b) implies that both the point resistance  $q_b$  and the skin resistance  $f_s$  are functions of the effective overburden pressure  $q_o$  in cohesionless soils and increases linearly with the depth of embedment,  $L$ , of the pile. However, extensive research works carried out by Vesic (1967) has revealed that the base and frictional resistances remain almost constant beyond a certain depth of embedment which is a function of  $\phi$ . This phenomenon was attributed to arching by Vesic. One conclusion from the investigation of Vesic is that in cohesionless soils, the bearing capacity factor,  $N_q$ , is not a constant depending on  $\phi$  only but also on the ratio  $L/d$  (where  $L$  = length of embedment of pile,  $d$  = diameter or width of pile). In a similar way, the frictional resistance,  $f_s$ , increases with  $L/d$  ratio and remains constant beyond a particular depth. Let  $L_c$  the depth, which may be called here as the critical depth, beyond which both  $q_b$  and  $f_s$ , remain constant. Experiments of Vesic have indicated that  $L_c$ , is a function of  $\phi$ . The  $L_c/d$  ratio as a function of  $\phi$  may be expressed as follows.

For  $28^\circ < \phi < 36.5^\circ$

$$L_c / d = 5 + 0.24(\phi^\circ - 28^\circ) \quad (9.16a)$$

For  $36.5^\circ < \phi < 42^\circ$

$$L_c / d = 7 + 2.35 (\phi^\circ - 36.5^\circ) \quad (9.16b)$$

The above expressions have been developed based on the curve given by Poulos (1980) giving the relationship between  $L_c/d$  and  $\phi^\circ$ .

The Eqs. (9.16) indicate

$$L_c / d = 5 \text{ at } \phi = 28^\circ,$$

$$L_c/d = 7 \text{ at } \phi = 36.5^\circ,$$

$$L_c/d = 20 \text{ at } \phi = 42^\circ.$$

The  $\phi$  values to be used for getting  $L_c/d$  are as follows (Poulos, 1980)

$$\text{for driven piles : } \phi = 0.75 \phi_1 + 10^\circ, \quad (9.17a)$$

$$\text{for bored piles : } \phi = \phi_1 - 3^\circ, \quad (9.17b)$$

where,  $\phi_1$  = angle of internal friction prior to the installation of pile.

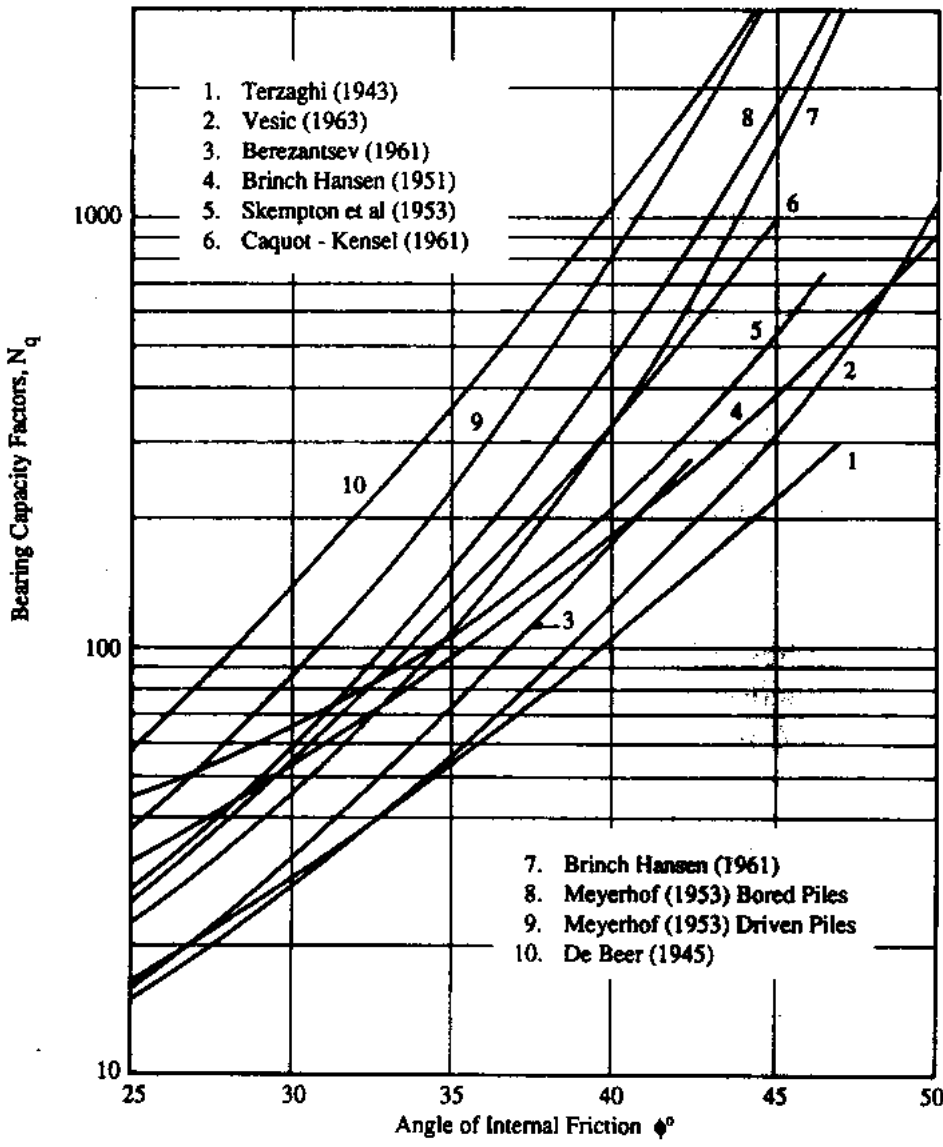
The theoretical,  $N_q$ , factor in Eq. (9.9a) is a function of  $\phi$ . There is a great variation in the values of  $N_q$  derived by different investigators as shown in Fig. (9.4a). Comparison of observed base resistances of piles by Nordlund (1963) and Vesic (1964) have shown (Tomlinson, 1986) that  $N_q$  values established by Berezhantsev *et al* (1961) which take into account the depth to width ratio of the pile, most nearly conform to practical criteria of pile failure. Berezhantsev's values of  $N_q$  as adopted by Tomlinson (1986) are given in Fig. 9.4b.

It may be seen from Fig. 9.4b that there is a rapid increase in  $N_q$  for high values of  $\phi$ , giving thereby high values of base resistance. *As a general rule (Tomlinson, 1986), the allowable working load on an isolated pile driven to virtual refusal, using normal driving equipment, in a dense sand or gravel consisting of predominantly of quartz particles, is given by the allowable load on the pile considered as a structural member rather than by consideration of failure of the supporting soil, or if the permissible working stress on the material of the pile is not exceeded, then the pile will not fail.*

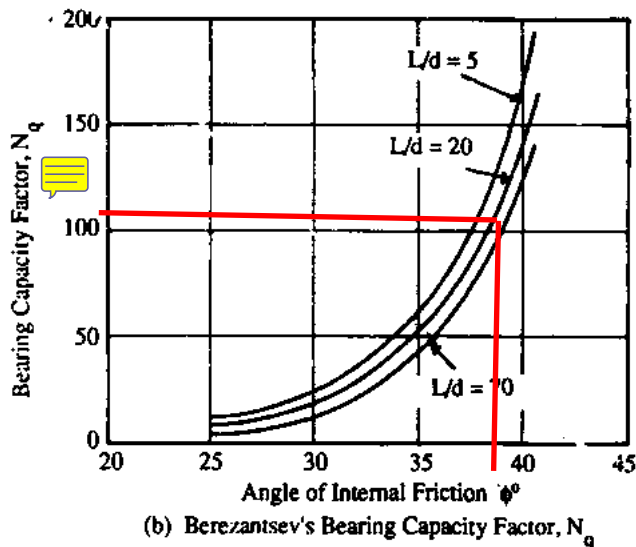
As per Tomlinson, the maximum base resistance  $q_b$  is normally limited to  $11000 \text{ kN/m}^2$  ( $1100 \text{ t/m}^2$ ) whatever might be the penetration depth of the pile.

**$L_c/d$  usually varies in the range of 5 to 20.**

**$L_c/d$  is smaller for coarse sand (i.e.larger  $\phi$ )**



(a) Bearing Capacity Factors for circular deep foundations



(b) Berezantsev's Bearing Capacity Factor,  $N_q$

Fig. 9.4 Bearing capacity factors by various investigators for deep foundations.

## 9.7 THE ULTIMATE SKIN RESISTANCE OF SINGLE PILE IN COHESIONLESS SOILS

### 9.7.1 Skin Resistance (Straight shaft)

The ultimate skin resistance in homogeneous soil as per Eq (9.9) is expressed as

$$Q_f = A_s \bar{q}_o \bar{K}_s \tan \delta . \quad (9.18a)$$

In case  $\bar{q}_o$ ,  $\bar{K}_s$  and  $\delta$  vary with respect to depth, Eq. (9.18a) may be expressed as

$$Q_f = \int_0^L P \bar{q}_o \bar{K}_s \tan \delta dz , \quad (9.18b)$$

where,  $\bar{q}_o$ ,  $\bar{K}_s$  and  $\delta$  refer to thickness  $dz$  of each layer and  $P$  is the perimeter of the pile.

As explained under Section 9.6, the effective overburden pressure does not increase linearly with depth and reaches a constant value beyond a particular depth  $L_c$  called as the critical depth which is a function of  $\phi$ . it is therefore natural to expect the skin resistance  $f_s$  also to remain constant beyond depth  $L_c$ .

Eq. (9.16) can be used for determining the critical length  $L_c$  for any given set of values of  $\phi$  and  $d$ .  $Q_f$  can be calculated from Eq. (9.18) if  $\bar{K}_s$  and  $\delta$  are known.

The values of  $\bar{K}_s$  and  $\delta$  vary not only with the relative density and pile material but also with the method of installation of pile.

Broms (1966) has related the values of  $\bar{K}_s$  and  $\delta$  to the effective angle of internal friction  $\phi$  of cohesionless soils for various pile materials and relative densities ( $D_r$ ) as shown in Table 9.1. The values are applicable to driven piles.

TABLE 9.1  
Values of  $\bar{K}_s$  and  $\delta$

Pile material	$\delta$	Values of $\bar{K}_s$	
		Low $D_r$	High $D_r$
Steel	20°	0.5	1.0
Concrete	3/4 $\phi$	1.0	2.0
Wood	2/3 $\phi$	1.5	4.0

As per the present state of knowledge, the maximum skin friction is limited to 110 kN/m<sup>2</sup> (Tomlinson, 1986).

## 9.16 EXAMPLES

**Example 9.1** A concrete pile of square section  $35 \times 35$  cms is driven into a layered sandy strata upto a depth of 13 m. The average SPT values ( $N$ ), unit weight of soil and the other particulars are as given in Fig. 9.20. Determine the allowable load  $Q_a$  by the conventional method. Assume an overall factor of safety 3.0.

**Solution:**

Determine  $\phi$  from Table 3.6 for the given SPT values  $N$ .

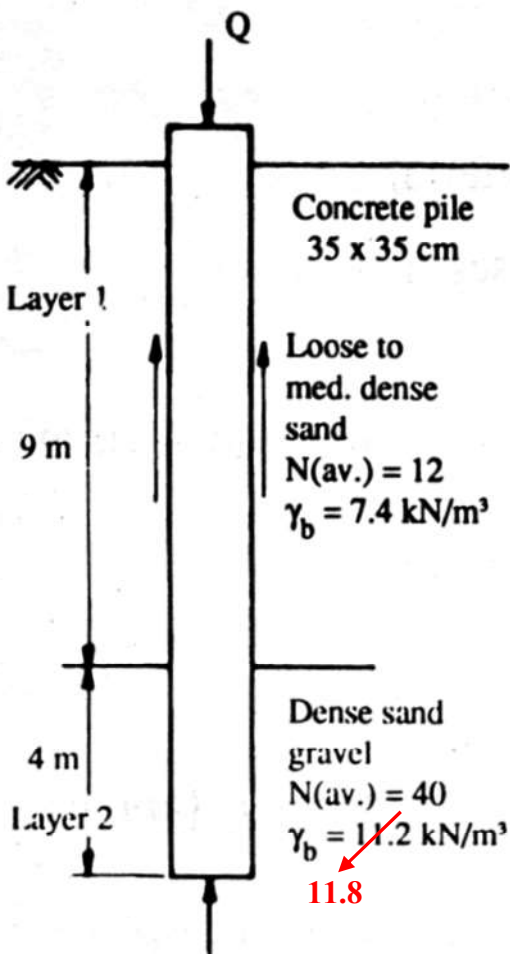
For  $N = 12$ ,  $\phi \approx 31^\circ$ .

For  $N = 40$ ,  $\phi \approx 38.5^\circ$ .

**Point load  $Q_b$**

$$Q_b = q_b N_q, \quad q_b = q'_0 N_q,$$

where,  $q'_0$  = effective overburden pressure =  $9 \times 7.4 + 4 \times 11.8$   
 $= 113.8 \text{ kN/m}^2$ .



**TABLE 3.6**  
***N* and  $\phi$  Related to Relative Density**

<i>N</i>	<i>Compactness</i>	<i>Relative Density, <math>D_r</math> %</i>	$\phi^\circ$
0-4	Very loose	0-15	<28
4-10	Loose	15-35	28-30
10-30	Medium	35-65	30-36
30-50	Dense	65-85	36-41
>50	Very Dense	>85	>41

Before using Table 3.6 the observed *N* value has to be corrected for standard energy, dilatancy and overburden pressure.

Use Fig. 9.4b for determining  $N_q$ .

For  $L/d = \frac{13}{0.35} = 37$ ,  $\phi = 38^\circ.5$ , the value of  $N_q = 115$ .

$$q_b = 113.8 \times 115 = 12,632 \text{ kN/m}^2.$$

As per Tomlinson, Section 9.6 the maximum base resistance allowed = 11,000 kN/m<sup>2</sup>. Therefore,

$$q_b = 11,000 \text{ kN/m}^2,$$

$$Q_b = 11,000 \times 0.35 \times 0.35 = 1347 \text{ kN}.$$

**Friction Load,  $Q_f$**

$$Q_f = Q_{f1} + Q_{f2},$$

where,  $Q_{f1}$  = friction Load for layer 1 =  $A_1 \bar{q}_1 \bar{K}_1 \tan \delta_1$ ,

$Q_{f2}$  = friction Load for layer 2 =  $A_2 \bar{q}_2 \bar{K}_2 \tan \delta_2$ .

**Layer 1**

$$A_1 = 4 \times 0.35 \times 9 = 12.6 \text{ sq.m},$$

$$\bar{q}_1 = \frac{1}{2} \times 9 \times 7.4 = 33.3 \text{ kN/m}^2.$$

From Table 9.1,  $\delta_1 = 0.75 \times 31 = 23.25^\circ$ ,  $\bar{K}_1 \approx 1.25$ ,

$$Q_{f1} = 12.6 \times 33.2 \times 1.25 \tan 23.25^\circ = 225 \text{ kN}.$$

**Layer 2**

$$A_2 = 4 \times 0.35 \times 4 = 5.6 \text{ sq.m},$$

$$\bar{q}_2 = 9 \times 7.4 + 2 \times 11.2 = 89 \text{ kN/m}^2.$$

From Table 9.1,  $\delta_2 = 0.75 \times 38.5 = 28.9^\circ$ ,  $\bar{k}_2 = 2$ ,

$$Q_{f2} = 5.6 \times 89 \times 2 \times \tan 28.9^\circ = 550 \text{ kN}.$$

$$Q_f = 225 + 550 = 775 \text{ kN}.$$

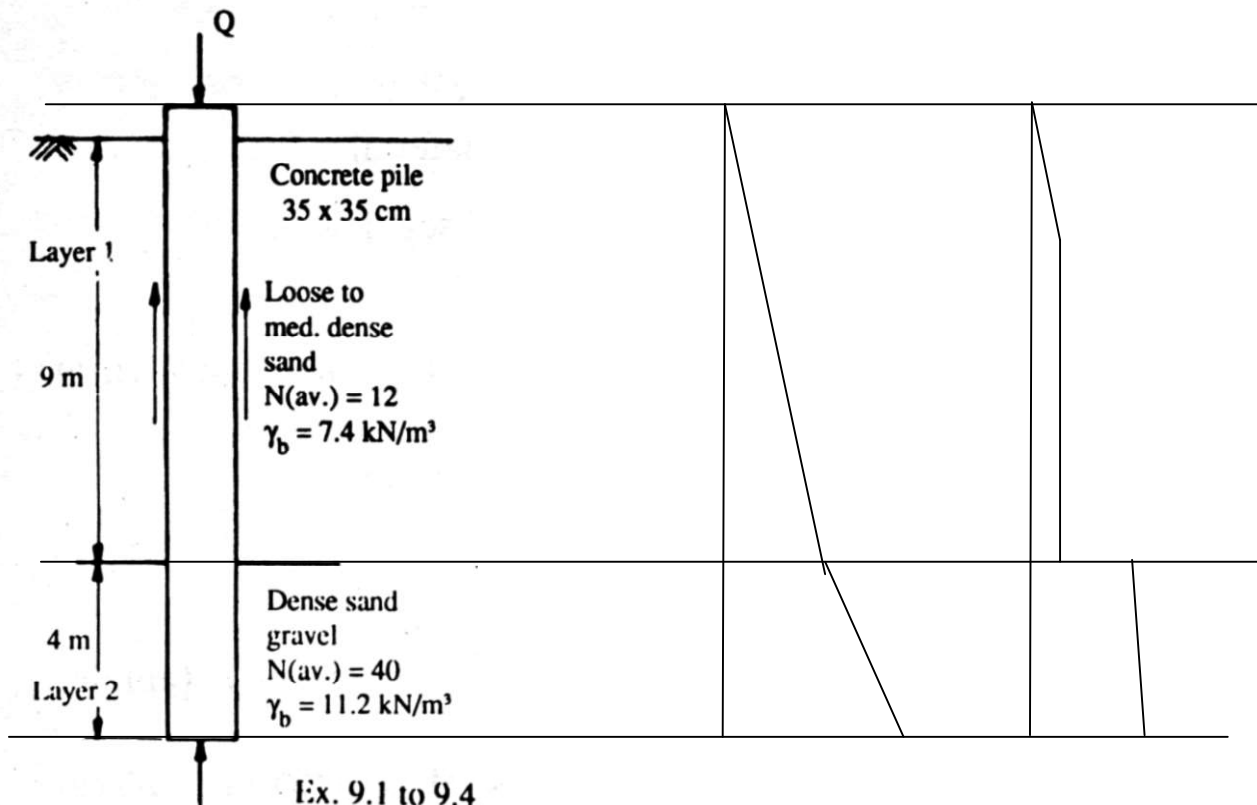
**Ultimate load bearing capacity**

$$Q_u = Q_b + Q_f = 1347 + 775 = 2122 \text{ kN}.$$

**Allowable Load**

$$Q_a = \frac{Q_u}{3} = \frac{2122}{3} = 707 \text{ kN or say } 700 \text{ kN}.$$

**Example 9.2** By making use of critical depth concept as explained in Section 9.6, determine the allowable load for the problem in Ex. 9.1 and Fig. 9.20. Use  $F_s = 2.5$ .



Ex. 9.1 to 9.4

**Fig. 9.20**

**Solution:**

As per the concept of critical depth the unit frictional resistance  $f_s$  and the base resistance  $q_b$  remains constant beyond a depth, called as the critical depth  $L_c$  which is a function of  $\phi$ .

**Critical depth for Layer 1**

Use Eq. (9.16)

$$\text{For } \phi = 31^\circ, L_c/d = 5 + 0.24(\phi^\circ - 28^\circ) = 5 + 0.24(31 - 28^\circ) = 5.72,$$

$$L_c = 0.35 \times 5.72 = 2.0 \text{ m.}$$

Effective overburden pressure at 2.0 m depth

$$q'_o = 2 \times 7.4 = 14.8 \text{ kN/m}^2,$$

$q'_o$  is supposed to remain constant from 2.0 to 9.0 m depth, and hence  $f_s$  remains constant.

### Critical depth for Layer 2

No method is suggested for computing critical depth for a layered system. In this case the effective overburden pressure at the middle of pile penetration in layer 2 may be calculated as follows

$$q'_2 = q'_o = 14.8 + \frac{1}{2} \times 4 \times 11.2 = 37.2 \text{ kN/m}^2.$$

### Friction Load

The total friction load  $Q_f$  is

$$Q_f = Q_{f1} + Q_{f2} + Q_{f3}$$

$$= \frac{1}{2} q'_1 \bar{K}_1 \tan \delta_1 A_1 + q'_1 \bar{K}_1 \tan \delta_1 A'_1 + q'_2 \bar{K}_2 \tan \delta_2 A_2,$$

where,  $q'_1 = 14.8 \text{ kN/m}^2$ ,  $\delta_1 = 23.25^\circ$ ,  $\bar{K}_1 = 1.2$ ,

$$A_1 = 2 \times 4 \times 0.35 = 2.8 \text{ sq.m},$$

$$A'_1 = (9 - 2) \times 4 \times 0.35 = 9.8 \text{ sq.m},$$

$$q'_2 = 37.2 \text{ kN/m}^2, \bar{K}_2 = 2.0, \delta_2 = 28.9^\circ,$$

$$A_2 = 4 \times 4 \times 0.35 = 5.6 \text{ sq.m},$$

$$Q_f = \frac{1}{2} \times 14.8 \times 1.2 \times \tan 23.25^\circ \times 2.8 \\ + 14.8 \times 1.2 \times \tan 23.25^\circ \times 9.8 + 37.2 \\ \times 2.0 \tan 28.9^\circ \times 5.6$$

or  $Q_f = 10.7 + 74.8 + 230 = 315 \text{ kN}.$

### Point load $Q_b$

$$Q_b = q_b A_b = q'_o N_q A_b = (14.8 + 4 \times 11.8) \times 115 \times 0.35^2 = 873 \text{ kN}.$$

Total load  $Q_u = 315 + 873 = 1188 \text{ kN}.$

$$\text{Allowable load, } Q_a = \frac{1188}{2.5} = 475 \text{ kN}.$$

**Note:** There is no way of checking the veracity of the two methods except by load tests. The value for  $Q_a$  obtained by this method is about 68 per cent of the conventional method.

**Pile Foundation in Cohesionless Soil**  
**Lecture -04**

$$Q_f = A_s \bar{q}_o \bar{K}_s \tan \delta \quad (9.18a)$$

**$\beta$ -Method** (Applicable to Cohesionless soil)

Eq. (9.18b) may also be written as

$$Q_f = \int_0^L P \bar{q}_o \beta dz \quad (9.18c)$$

where,  $\beta = \bar{K}_s \tan \delta$ .

Poulos (1980) has given a curve giving the relationship between  $\beta$  and  $\phi^\circ$  which is applicable for driven piles and to all types of material surfaces. According to Poulos there is not sufficient evidence to show that  $\beta$  would vary with the pile material. The relationship between  $\beta$  and  $\phi$  is given in Fig. 9.5a. For bored piles, Poulos recommends the relationship given by Meyerhof (1976) between  $\phi$  and  $\beta$  (Fig. 9.5b).

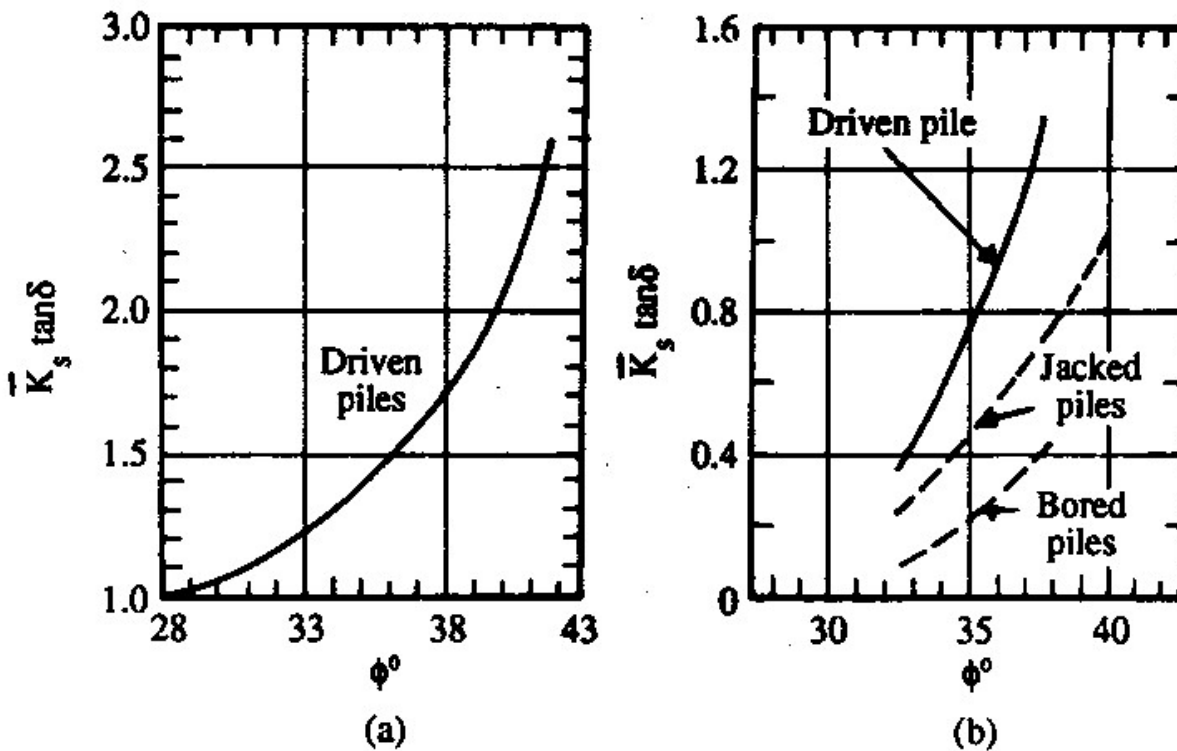


Fig. 9.5 Values of  $K_s \tan \delta$  in sand as per (a) Poulos, 1980 (b) Meyerhof, 1976.

Meyerhof has obtained the curve by assuming  $\delta = 0.75 \phi$ . Poulos is also of the opinion that the curves given by Meyerhof (1976) for driven piles are considerably smaller as compared to the one given in Fig. 9.5a. The angles to be used for getting  $\bar{K}_s \tan \delta$  in Fig 9.5a are the same as that given by the equations (9.17a) and (9.17b) for driven and bored piles respectively.

### 9.6.1 Effect of Pile Installation on the Value of the Angle of friction $\phi$

When a pile is driven into loose sand its density is increased (Meyerhof, 1959), and the horizontal extent of the compacted zone has a width of about 6 to 8 times the pile diameter. However, in dense sand, pile driving decreases the relative density because of the dilatancy of the sand and the loosened sand along the shaft has a width of about 5 times the pile diameter (Kerisel, 1961). On the basis of field and model test results, Kishida (1967) proposed that the angle of internal friction decreases linearly from a maximum value of  $\phi_2$  at the pile tip to a low value of  $\phi_1$  at a distance of  $3.5d$  from the tip where  $d$  is the diameter of the pile,  $\phi_1$  is the angle of friction before the installation of the pile and  $\phi_2$  after the installation as shown in Fig. 9.3d. Based on the field data, the relationship between  $\phi_1$  and  $\phi_2$  in sands may be written as

$$\phi_2 = \frac{\phi_1 + 40}{2} \quad (9.13)$$

An angle of  $\phi_1 = \phi_2 = 40^\circ$  in Eq. (9.13) means no change of relative density due to pile driving. Value of  $\phi_1$  are obtained from the *in-situ* penetration tests (with no correction due to overburden pressure) by using the relationships established between  $\phi$  and SPT or CPT values (Chapter 3). Kishida (1967) has suggested the following relationship between  $\phi$  and SPT value  $N$  as

$$\phi^\circ = \sqrt{2ON} + 15^\circ \quad (9.14)$$

However, Tomlinson (1986) is of the opinion that it is unwise to use higher values for  $\phi$  due to driving of pile. His argument is that the sand may not get compacted always, as for example, when piles are driven into loose sand, the resistance is so low and little compaction is given to the soil. He suggests, therefore, that the value of  $\phi$ , used for the design should represent the *in-situ* condition only that existed before driving.

With regards to driven and *cast-in-situ* piles, there is no suggestion by any investigator as to what value of  $\phi$  should be used for calculating the base resistance. However, it is safer to assume the *in-situ*  $\phi$  value for computing the base resistance.

With regards to bored and *cast-in-situ* piles, the soil gets loosened during boring. Tomlinson (1986) therefore, suggests that the  $\phi$  value

for calculating both the base and skin resistance should represent the loose state. However, Poulos (1980) suggests that for bored piles, the value of  $\phi$  may be taken as

$$\phi = \phi_1 - 3 \quad (9.15)$$

where,  $\phi_1$  = angle of internal friction prior to installation of pile.

**Example 9.4** Compute the safe load on pile given in Ex. 9.1 and Fig. 9.20 by making use of  $\phi$  modified due to driving by conventional method. Use a factor of safety of 3.

**Solution:**

Due to driving of piles into cohesionless soils, the soil around the pile is supposed to get densified and thereby the value of  $\phi$  gets modified. As per Eq. 9.17a,

$$\phi = 0.75 \phi_1 + 10^\circ,$$

where,  $\phi$  = modified value,

$\phi_1$  = value of  $\phi$  prior to driving.

Here we consider  $\phi_1$  as the values obtained from relationship established between  $N$  and  $\phi$ .

From Ex. 9.1. we have for  $N = 12$ ,  $\phi_1 = 31^\circ$ ,

$$N = 40, \phi_1 = 38^\circ.5.$$

Now for  $N = 12$ ,  $\phi = 0.75 \times 31 + 10 = 33^\circ.25$ .

$$\begin{aligned} N = 40, \phi &= 0.75 \times 38.5 + 10 = 28.9 + 10 \\ &= 38.9^\circ. \end{aligned}$$

From Table 9.1 for  $\phi = 33^\circ.25$ ,  $\delta = 3/4 \times 33.25 = 25^\circ$ ,  $\bar{K}_s = 1.5$ .

For  $\phi = 38^\circ.9$ ,  $\delta = 3/4 \times 38^\circ.9 = 29^\circ.18$ ,  $\bar{K}_s = 2.0$ .

**Skin Load**

Layer 1 
$$\begin{aligned} Q_{f1} &= \bar{q}_1 \bar{K}_1 \tan \delta_1 A_1 \\ &= 1/2 \times 9 \times 7.4 \times 1.5 \tan 25^\circ \times 9 \times 4 \times .35 \\ &= 293 \text{ kN.} \end{aligned}$$

Layer 2 
$$\begin{aligned} Q_{f2} &= q_2 \bar{K}_1 \tan \delta_2 A_2 \\ &= (9 \times 7.4 + 2 \times 11.2) \times 2.0 \tan 29^\circ.18 \times 4 \times 4 \times 0.35 \\ &= 556 \text{ kN.} \end{aligned}$$

$$Q_f = 293 + 256 = 849 \text{ KN.}$$

**Base Load  $Q_b$**

$$\begin{aligned} Q_b &= q_b A_b = q'_o N_q A_b. \\ q'_o &= 9 \times 7.4 + 4 \times 11.2 = 111.4 \text{ kN/m}^2. \end{aligned}$$

From Fig. 9.4b, for  $\phi \approx 39^\circ$ ,  $N_q = 125$ , for  $L/d = 37$ .

$$Q_b = 111.4 \times 125 \times 0.35^2 = 1706 \text{ kN.}$$

Total Load 
$$Q_u = 849 + 1706 = 2555 \text{ kN.}$$

Allowable load, 
$$Q_a = \frac{2555}{3} = 852 \text{ kN or say } 850 \text{ kN.}$$

*Note:* Tomlinson (1986) does not recommend the use of modified values for  $\phi$ . Example 9.1 is solved without modification of  $\phi$ . The value of  $Q_a$  as from Ex. 9.1 is about 82 per cent of the value in this example.

## 9.9 BEARING CAPACITY OF PILES IN GRANULAR SOILS BASED ON SPT VALUE

Meyerhof (1956) suggests the following equations for single piles in granular soils based on SPT values.

For displacement piles

Also see S K Garg page 490

$$Q_u = 400NA_b + 2\bar{N}A_s, \quad (9.25a)$$

for H-Piles

$$Q_u = 400NA_b + \bar{N}A_s, \quad (9.25b)$$

for bored piles

$$Q_u = 133NA_b + 0.67\bar{N}A_s, \quad (9.25c)$$

where,  $Q_u$  = ultimate total load in kN,

$N$  = average SPT value below pile tip,

$\bar{N}$  = Average SPT value along the pile shaft,

$A_b$  = base area of pile in  $m^2$ ,

$A_s$  = Shaft surface area in  $m^2$ .

A minimum factor of safety of 4 is recommended. The allowable load,  $Q_a$  is

$$Q_a = \frac{Q_u}{4}. \quad (9.25d)$$

**Example 9.3** Using the SPT Values  $N$ , determine the allowable load for the pile given in Ex. 9.1 and Fig. 9.20. Use a factor of safety 4.0.

**Solution:**

Use Eq. (9.25a).

$$Q_u = 400NA_b + 2\bar{N}A_s = Q_b + Q_f.$$

For Layer 1,  $2\bar{N}A_s = 2 \times 12 \times 9 \times 4 \times 0.35 = 302$  kN.

For Layer 2,  $2\bar{N}A_s = 2 \times 40 \times 4 \times 4 \times 0.35 = 112$  kN.

$$Q_f = 414 \text{ kN.}$$

$$Q_b = 400 NA_b = 400 \times 40 \times 0.35^2 = 1960 \text{ kN.}$$

$$Q_f + Q_b = 2374 \text{ kN.}$$

$$Q_a = \frac{2374}{4} = 594 \text{ kN.}$$

*Note:* The value of  $Q_a$  obtained by this method is about 85 per cent of the conventional method.

### 9.13 PILE BEARING CAPACITY FROM DYNAMIC PILE DRIVING FORMULAE

The resistance offered by a soil to penetration of pile during driving gives an indication of its bearing capacity. Qualitatively speaking, a pile which meets greater resistance during driving is capable of carrying a greater load. A number of dynamic formulae have been developed which equates the pile capacity in terms of driving energy.

The basis for all these formulae is the simple energy relationship which may be stated by either of the following equations. (Fig. 9.17).

$$Wh = Q_u s,$$

$$Q_u = Wh/s,$$

(9.47)

in which,  $W$  = weight of driving hammer,

$h$  = height of fall of hammer,

$Wh$  = energy of hammer blow,

$Q_u$  = ultimate resistance to penetration,

$s$  = pile penetration under the one hammer blow,

$Q_u s$  = resisting energy of pile.

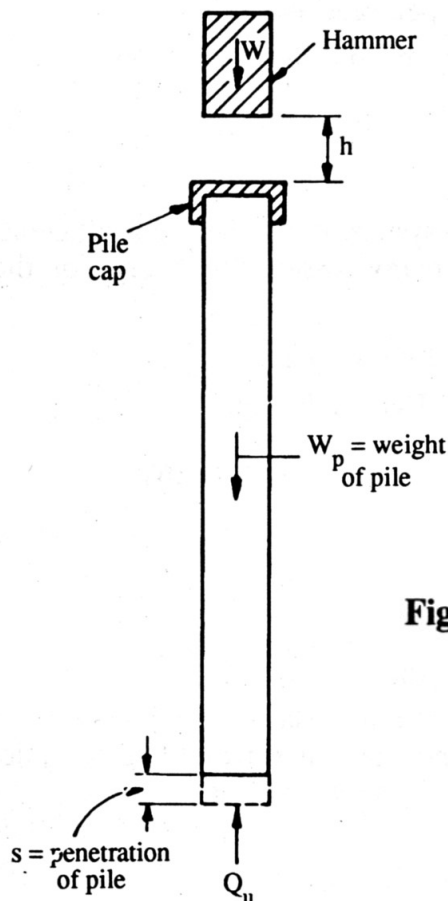


Fig. 9.17 Basic Energy Relationship

### Hiley Formula

Eq. (9.47) holds good only if the system is 100 per cent efficient. Since the driving of a pile involves many losses, the energy of the system may be written as

$$\text{Energy input} = \text{Energy used} + \text{Energy losses}$$

or  $\text{Energy used} = \text{Energy input} - \text{Energy losses}.$

The expression for the various energy terms used are

1. Energy used =  $Q_u s,$
2. Energy input =  $\eta_h Wh,$

where,  $\eta_h$  is the efficiency of the hammer.

3. The energy losses are due to the following :

- (i) The energy loss  $E_1$  due to the elastic compressions of pile cap, pile material and the soil surrounding the pile. The expression for  $E_1$  may be written as

$$E_1 = \frac{1}{2} Q_u (c_1 + c_2 + c_3) = Q_u C,$$

where,  $c_1$  = elastic compression of pile cap,  
 $c_2$  = elastic compression of pile,  
 $c_3$  = elastic compression of soil.

- (ii) The energy loss  $E_2$  is due to the interaction of the pile hammer system (impact of two bodies). The expression for  $E_2$  may be written as

$$E_2 = WhW_p \frac{1 - C_r^2}{W + W_p},$$

where,  $W_p$  = weight of pile,  
 $C_r$  = coefficient of restitution.

$C_r$  is the ratio of the velocity before and after collision of two bodies. Value ranges from 0 to 1. It indicates how much kinetic energy remains after collision.

Substituting the various expressions in the energy equation and simplifying, we have

$$Q_u = \frac{\eta_h Wh}{s + C} \cdot \frac{1 + C_r^2 R}{1 + R}, \quad (9.48) \quad \text{where, } R = \frac{W_p}{W}.$$

Eq. (9.48) is called as Hiley formula. The allowable load  $Q_a$  may be obtained by dividing  $Q_u$  by a suitable factor of safety.

If the pile tip rests on rock or relatively impenetrable material, Eq. (9.48) is not valid. Chellis suggests for this condition that the use of  $W_p/2$  instead of  $W_p$  may be more correct. The various coefficients used in the Eq. (9.48) are as given below:

1. *Elastic Compression  $c_1$  of cap and pile head.*

<i>Pile Material</i>	<i>Range of Driving Stress kg/cm<sup>2</sup></i>	<i>Range of <math>c_1</math></i>
Precast concrete pile with packing inside cap	30 – 150	0.12 – 0.50
Timber pile without cap	30 – 150	0.05 – 0.20
Steel H-pile	30 – 150	0.04 – 0.16

2. *Elastic compression,  $c_2$  of pile.*

This may be computed using the equation

$$c_2 = \frac{Q_u L}{AE},$$

where,  
 $L$  = embedded length of pile,  
 $A$  = average cross-sectional area of pile,  
 $E$  = Young's modulus.

3. *Elastic Compression,  $c_3$  of soil.*

The average value of  $c_3$  may be taken as 0.1 (The value ranges from 0.0 for hard soil to 0.2 for resilient soils).

4. *Pile-hammer efficiency  $\eta_h$*

Hammer Type	$\eta_h$
Drop	1.00
Single acting	0.75 – 0.85
Double acting	0.85
Diesel	1.00

5. *Coefficients of restitution  $C_r$*

Material	$C_r$
Wood pile	0.25
Compact wood cushion on steel pile	0.32
C.I. Hammer on concrete pile without cap	0.40
C.I. Hammer on steel pipe without cushion	0.55

**Engineering News Formula**

The general form of Engineering News formula for the allowable load  $Q_a$  may be obtained from Eq. (9.48) by putting.

$\eta_h = 1$  and  $C_r = 1$  and a factor of safety equal to 6. The formula as proposed by A.M. Wellington, editor of the Engineering News, in 1886, is

$$Q_a = \frac{Wh}{6(s + C)}, \quad (9.49)$$

wherein,  $Q_a$  = allowable load in kg,

$W$  = weight of hammer in kg,

$h$  = height of fall of hammer in cms,

$s$  = final penetration in cms per blow (which is termed as *set*). The *set* is taken as the average penetration per blow for the last 5 blows of a drop hammer or 20 blows of a steam hammer,

$C$  = empirical constant

= 2.5 for a drop hammer,

= 0.25 for single and double acting hammers.

The equations for the various types of hammers may, therefore be written as:

## 1. Drop hammer

$$Q_a = \frac{Wh}{6(s + 2.5)}. \quad (9.50)$$

## 2. Single-acting hammer

$$Q_a = \frac{Wh}{6(s + 0.25)}. \quad (9.51)$$

## 3. Double-acting hammer

$$Q_a = \frac{(W + ap)}{6(s + 0.25)}, \quad (9.52)$$

$a$  = effective area of piston in sq. cm,

$p$  = mean effective steam pressure in kg/cm<sup>2</sup>.

### *Comments on the use of Dynamic Formulae*

1. The detailed investigations carried out by Vesic on deep foundations in granular soils indicate that the Engineering News Formula applicable to drop hammers, Eq. (9.50), gives pile loads as low as 44 % of the actual loads. In order to obtain better agreement between the one computed and observed loads, Vesic suggests the following values for the coefficient  $C$  in Eq. (9.49).

For Steel pipe piles,  $C = 1$  cm.

For precast concrete piles  $C = 1.5$  cms.

2. The tests carried out by Vesic in granular soils indicate that Hiley's formula does not give consistent results. The values computed from Eq. (9.48) are sometimes higher and sometimes lower than the observed values.
3. Dynamic formulae in general have limited value in pile foundation work mainly because the dynamic resistance of soil does not represent the static resistance, and because often the results obtained from the use of dynamic equations are of questionable dependability. However, engineers prefer to use Engineering News formula because of its simplicity.
4. Dynamic formulae could be used with more confidence in freely draining materials such as coarse sand. If the pile is driven to saturated loose fine sand and silt, there is every possibility of development of liquefaction which reduces the bearing capacity of the pile.
5. Dynamic formulae are not recommended for computing allowable loads of piles driven into cohesive soils. In cohesive soils, the resistance to driving increases through the sudden increase in stress in porewater and decreases because of the decreased value of the internal friction between soil and pile because of porewater. These two oppositely directed forces do not lend themselves to analytical treatment and as such the dynamic penetration resistance to pile driving has no relationship to static bearing capacity.

There is another effect of pile driving in cohesive soils. During driving the soil gets remoulded and the shear strength of the soil gets reduced considerably. Though there will be a regaining of shear strength after a lapse of some days after the driving operation, this will not be reflected in the resistance value obtained from the dynamic formulae.