V.T. Chow

Chapter 1

Open Channel Flow & Its Classifications

Open Channel Flow:

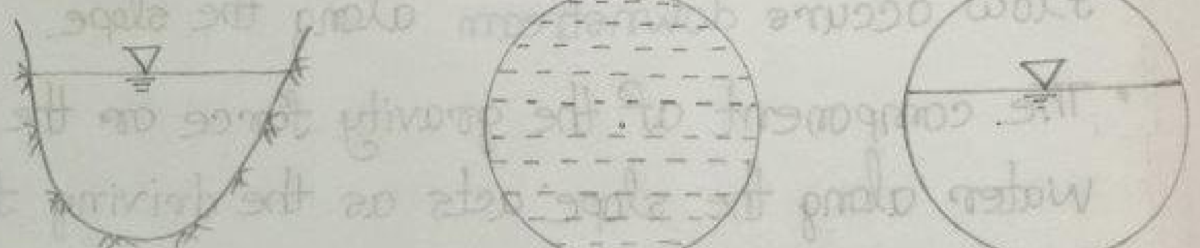
- The flow of water in a conduit may be either open channel flow or pipe flow.
- Open channel flow is defined as the flow of water in a conduit with a free surface subjected to atmospheric pressure.
- Open channel flow occurs under the action of gravity and at atmospheric pressure.
- Basically all open channels have a bottom slope and flow occurs downstream along the slope.
- The component of the gravity force or the weight of water along the slope acts as the driving force.
- For open channel flow to occur, the total energy at an upstream section must be greater than the total energy at a downstream section.
- Open channel flow is also known as the free surface flow.

Examples :

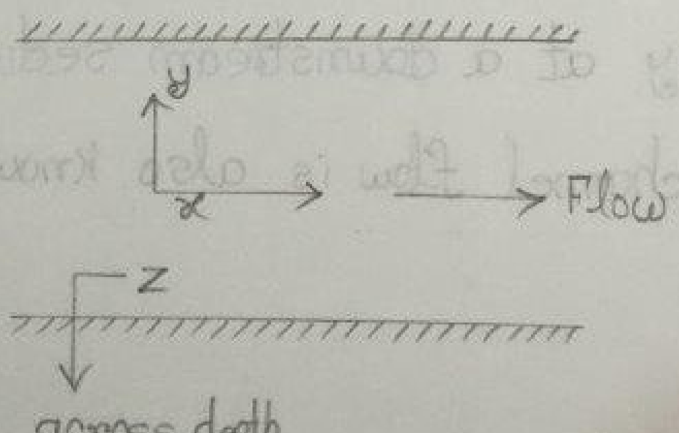
(i) Flows in rivers and canals are some of the many familiar examples of open channel flow.

(ii) Flow of water in a closed conduit, e.g. in an underground sewer, culvert or storm sewer may be open channel flow if the flow occurs with a free surface.

(iii) The flow of groundwater with a free surface is also an example of open channel flow.



Open channel Flow　　Pipe Flow　　Open channel Flow



Co-ordinate System

Kinds of Open Channel: Open channels are classified on different criteria as follows:

According to its origin a channel may be either natural or artificial.

① Natural Channel: Natural open channels include all channels that exist naturally on the earth, e.g. rivers, and tidal estuarie. They are generally very irregular in shape.

② Artifical Channel: Artificial open channels are the channels developed by men or human efforts e.g. irrigation canals, laboratory flumes, spillway chutes, drops, culverts, roadside gutters etc. They are usually designed with regular geometric shapes.

According to channel geometry a channel may be either prismatic or non-prismatic.

③ Prismatic Channel: A channel with unvarying cross-section and constant bottom slope is called a prismatic channel. Artificial channels are usually prismatic.

④ Non-prismatic Channel: A channel with varying cross-section and varying bottom slope is called a non-prismatic channel. Natural channels are generally non-prismatic.

⑤ Rigid Boundary Channel: A channel with immovable bed and sides is known as a rigid boundary channel, e.g. lined canals, sewers and non-erodible unlined canals.

⑥ Mobile Boundary Channel: When the channel boundary is composed of loose sedimentary particles moving under the action of flowing water, the channel is called a mobile boundary channel. An alluvial channel is a mobile boundary channel transporting the same type of material as that comprising the channel perimeter.

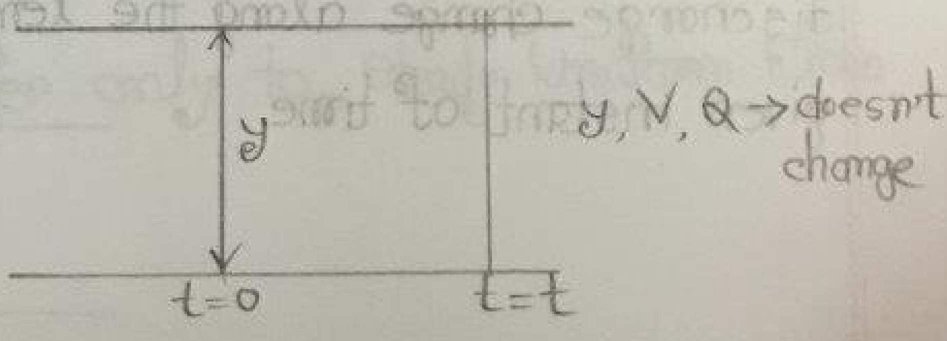
⑦ Large Slope Channel: An open channel having a bottom slope greater than 1 in 10 is called a channel of large slope. Some artificial channels like drops and chutes have slopes far more than 1 in 10.

(8) Small Slope channel: An open channel having a bottom slope less than 1 in 10 is called a channel of small slope. The slopes of ordinary channels, natural or artificial, are far less than 1 in 10.

Types of Open Channel Flow: The following classification is made according to the change in flow depth with respect to time and space.

Steady Flow & Unsteady Flow (Time as the Criterion)

- Flow in an open channel is said to be steady if the depth of flow, mean velocity & discharge at a channel section do not change or if it can be assumed to be constant during the time interval under consideration.
- The flow is unsteady if the depth changes with time.

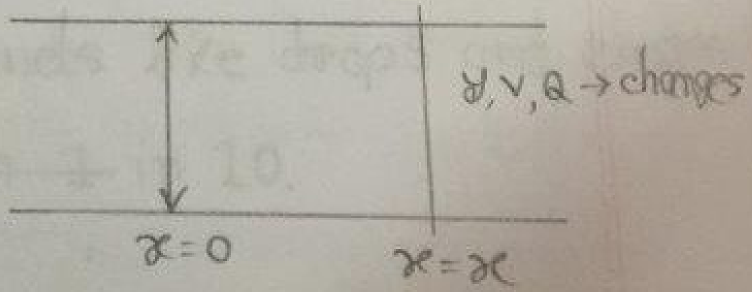


Example :

- The flow of water in a straight prismatic channel with a constant discharge (e.g. in a laboratory flume in which a constant discharge is calculated) and the dry season flow of a river when no rainfall occurs may be considered as steady flow.
- Flood flows in rivers and tidal flows in estuaries are the familiar examples of unsteady flow.

Uniform Flow & Varied Flow (Space as the criterion)

- Flow in an open channel is said to be uniform if the depth of flow, mean velocity & discharge do not change along the length of the channel at a given instant of time.
- Flow is varied if the depth of flow, mean velocity & discharge change along the length of the channel at a given instant of time.



• In uniform flow, the channel bottom, the free surface & the energy grade line are parallel to one another; i.e. their slopes are equal.

Classificiation of Uniform Flow : A uniform flow may be steady or unsteady, depending on whether or not the depth changes with time.

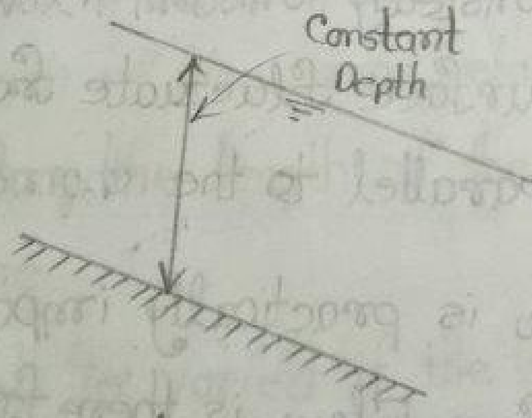
① Steady Uniform Flow is the fundamental type of flow treated in open channel hydraulics. The depth of flow does not change during the time interval under consideration.

② The establishment of Unsteady Uniform Flow would require that the water surface fluctuate from time to time while remaining parallel to the channel bottom.

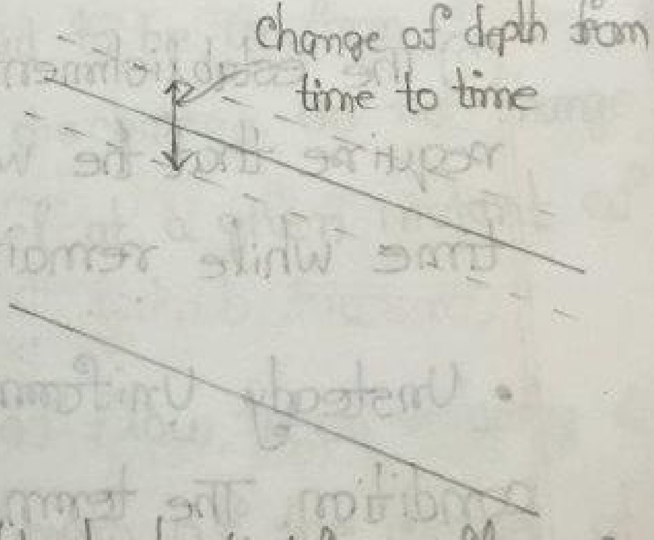
• Unsteady Uniform Flow is practically impossible condition. The term Uniform Flow is, therefore used hereafter to refer only to Steady Uniform Flow.

Classification of Varied Flow: Varied flow may be further classified as either rapidly or gradually varied

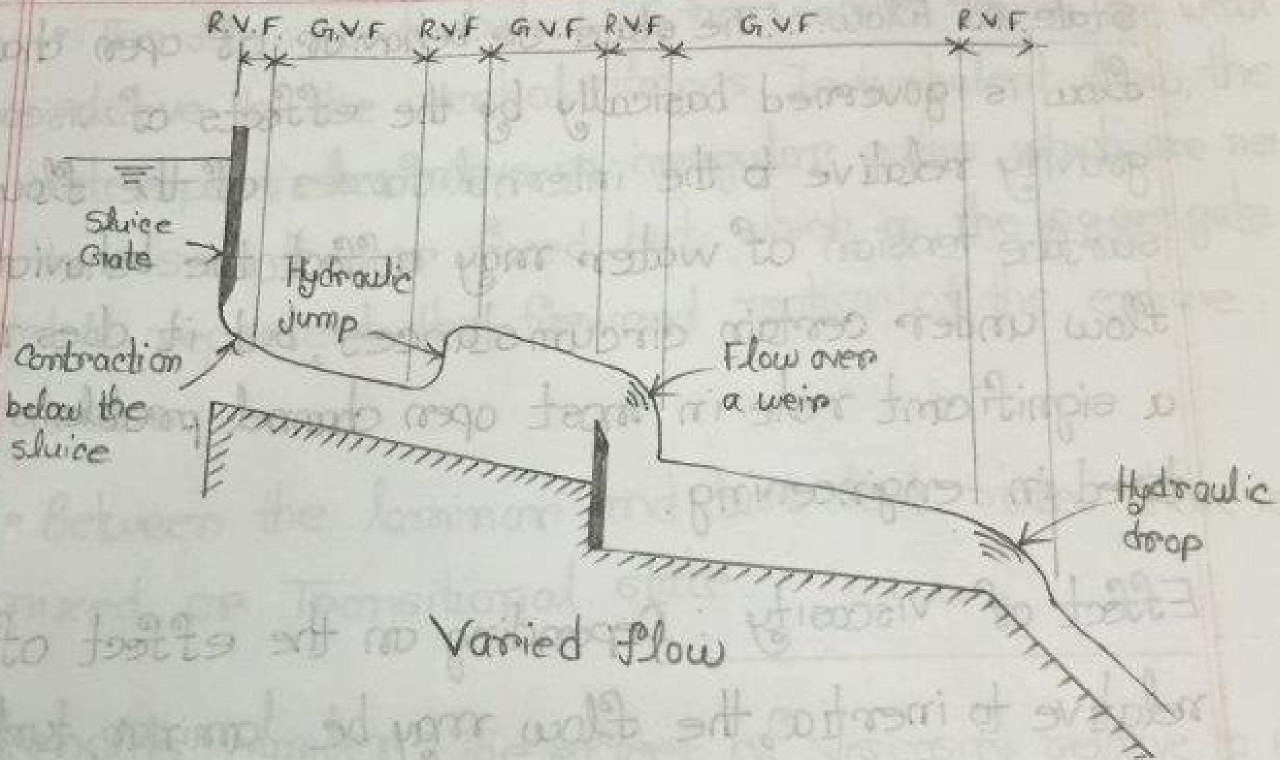
- The flow is rapidly varied if the depth changes abruptly over a comparatively short distance.
- Otherwise, the flow is gradually varied if the depth changes gradually along the length of the channel.
- A rapidly varied flow is also known as a local phenomenon; examples are the hydraulic jump and the hydraulic drop.



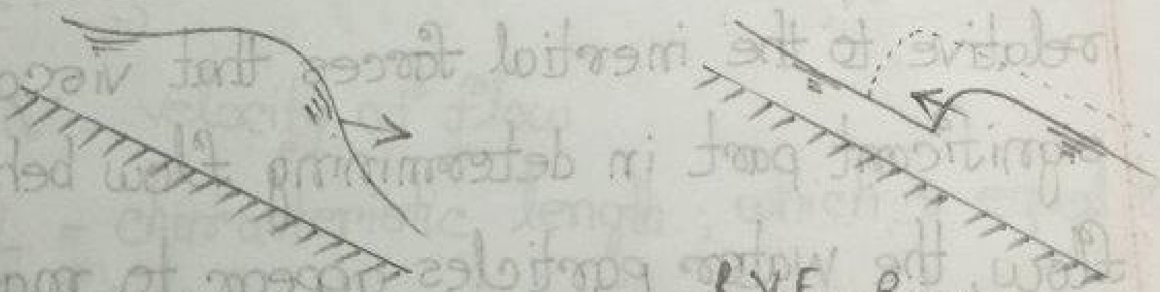
Uniform flow - Flow in a laboratory channel



Unsteady Uniform flow - Rare



Varied flow



G.V.F. - Flood wave

R.V.F. - Bore

Unsteady flow

State of Flow : The state or behavior of open channel flow is governed basically by the effects of viscosity & gravity relative to the internal forces of the flow. The surface tension of water may affect the behavior of flow under certain circumstances, but it does not play a significant role in most open channel problems encountered in engineering.

Effect of Viscosity : Depending on the effect of viscosity relative to inertia, the flow may be laminar, turbulent or transitional.

- The flow is laminar if the viscous forces are so strong relative to the inertial forces that viscosity plays a significant part in determining flow behavior. In laminar flow, the water particles appear to move in definite smooth paths, or streamlines, and infinitesimally thin layers of fluid seem to slide over adjacent layers.

- The flow is turbulent if the viscous forces are weak relative to the inertial forces. In turbulent flow, the water particles move in irregular paths which are neither smooth nor fixed but which in the aggregate still represent the forward motion of the entire stream.

- Between the laminar and turbulent states there is a mixed, or Transitional state.

Reynolds Number: The effect of viscosity relative to inertia can be represented by the Reynolds number, defined as

$$R_e = \frac{VL}{\nu}$$

where, V = velocity of flow

L = characteristic length ; which is equal to the hydraulic radius R of a conduit

ν = kinematic viscosity of water.

$$Re = \frac{\text{Inertial Force}}{\text{Viscous Force}}$$

$$= \frac{F_I}{F_V}$$

$$= \frac{ma}{\mu\left(\frac{du}{dy}\right)A}$$

$$= \frac{\rho L^3 \cdot \frac{L}{t^2}}{\mu\left(\frac{V}{L}\right)L^2}$$

$$= \frac{\rho \frac{L^4}{t^2}}{\mu VL}$$

$$= \frac{\rho\left(\frac{L}{t}\right)^2 L^2}{\mu VL}$$

$$= \frac{\rho V^2 L^2}{\mu VL}$$

$$= \frac{VL}{\frac{\mu}{\rho}}$$

$$= \frac{VL}{\nu}$$

Note: The hydraulic radius R is taken as the characteristic length, since the diameter of a pipe is four times its hydraulic radius.

Laminar Flow : $Re < 500$

Transitional Flow : $500 \leq Re \leq 12500$

Turbulent Flow : $Re > 12500$

Stanton Diagram: The laminar, turbulent & transitional states of open-channel flow can be expressed by a diagram that shows a relation between the Reynolds number and the friction factor of the Darcy-Weisbach formula. Such a diagram, generally known as the Stanton Diagram, has been developed for flow in pipes.

According to Darcy-Weisbach formula,

$$h_f = f \cdot \frac{L}{d_o} \cdot \frac{V^2}{2g}$$

where, h_f = frictional loss for flow in the pipe

f = friction factor

L = length of the pipe

d_o = diameter of the pipe

V = velocity of flow

g = acceleration due to gravity

Since, $d_o = 4R$ & the energy gradient $S = \frac{h_f}{L}$

$$f = \frac{h_f}{L} \cdot d_o \cdot \frac{2g}{V^2}$$

$$= S \cdot 4R \cdot \frac{2g}{V^2}$$

$$= \frac{8gRS}{V^2}$$

Effect of Gravity: The effect of gravity upon the state of flow is represented by a ratio of inertial forces to gravity forces.

The ratio is given by the Froude Number,

$$F = \frac{V}{\sqrt{gL}}$$

where, V = mean velocity of flow

g = acceleration due to gravity

L = characteristic length

In open channel flow, the characteristic length L is made equal to the hydraulic depth D.

$$\text{Froude Number, } F = \frac{\text{Inertial Forces}}{\text{Gravity Forces}}$$

$$= \sqrt{\frac{F_I}{F_G}}$$

$$= \sqrt{\frac{ma}{mg}}$$

$$= \sqrt{\frac{\rho L^3 \cdot \frac{L}{t^2}}{\rho L^3 \cdot g}}$$

$$= \sqrt{\frac{\rho \left(\frac{L}{t}\right)^2 L^2}{\rho L^3 g}}$$

$$\Rightarrow F = \sqrt{\frac{\rho V^2 L^2}{\rho L^3 g}}$$

$$= \sqrt{\frac{V^2}{gL}}$$

$$= \frac{V}{\sqrt{gL}}$$

- When F is equal to unity, $V = \sqrt{gD}$ and the flow is said to be in a "critical state."
- If F is less than unity, or $V < \sqrt{gD}$, the flow is sub-critical. In this state, the role played by gravity forces is more pronounced, so the flow has low velocity and is often described as tranquil and streaming.
- If F is greater than unity, or $V > \sqrt{gD}$, the flow is super-critical. In this state, the inertial forces become dominant; so the flow has a high velocity and is usually described as rapid, shooting and torrential.

Depth of flow at critical condition $= y_c$

at sub-critical condition $= y_{sub} > y_c$

at super-critical condition $= y_{sup} < y_c$

$Q = AV$, so depth ↑ velocity ↓ & vice-versa.

Celerity: In the mechanics of water waves, the critical velocity $\sqrt{gD}$ is identified as the celerity of the small gravity waves that occur in shallow water in channels as a result of any momentary change in the local depth of the water.

- The wave velocity remains stagnant at critical condition.
- A gravity wave can be propagated upstream in water of sub-critical flow but not in water of super-critical flow, since the celerity is greater than the velocity of flow in the former case and less in the latter.

Critical condition : wave velocity stagnant

Super-critical condition : wave velocity can't overcome water velocity

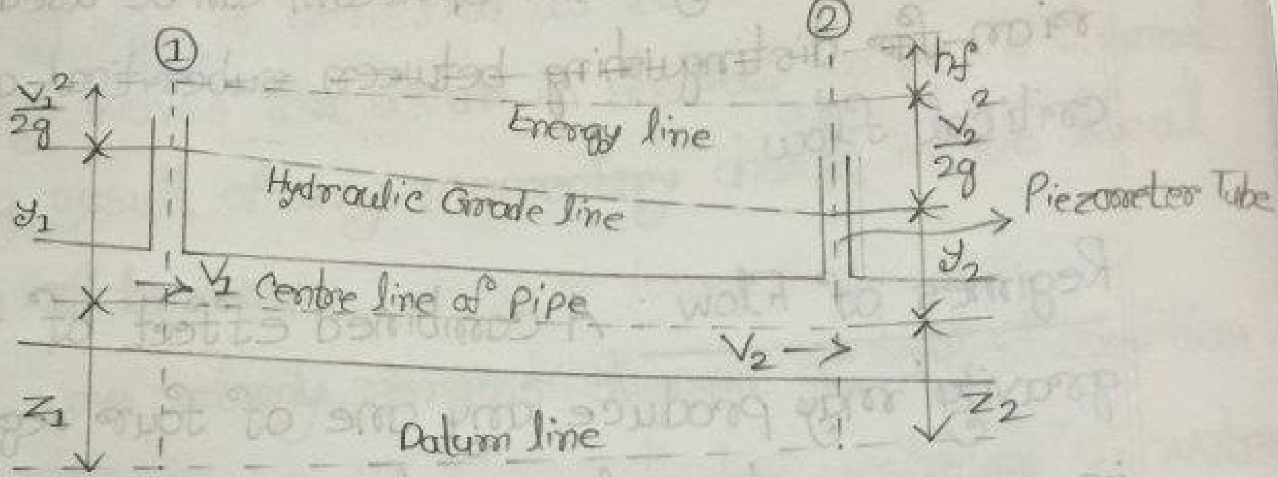
Sub-critical condition : wave front moves toward upstream

• Therefore, the possibility or impossibility of propagating a gravity wave upstream can be used as a criterion for distinguishing between subcritical and supercritical flow.

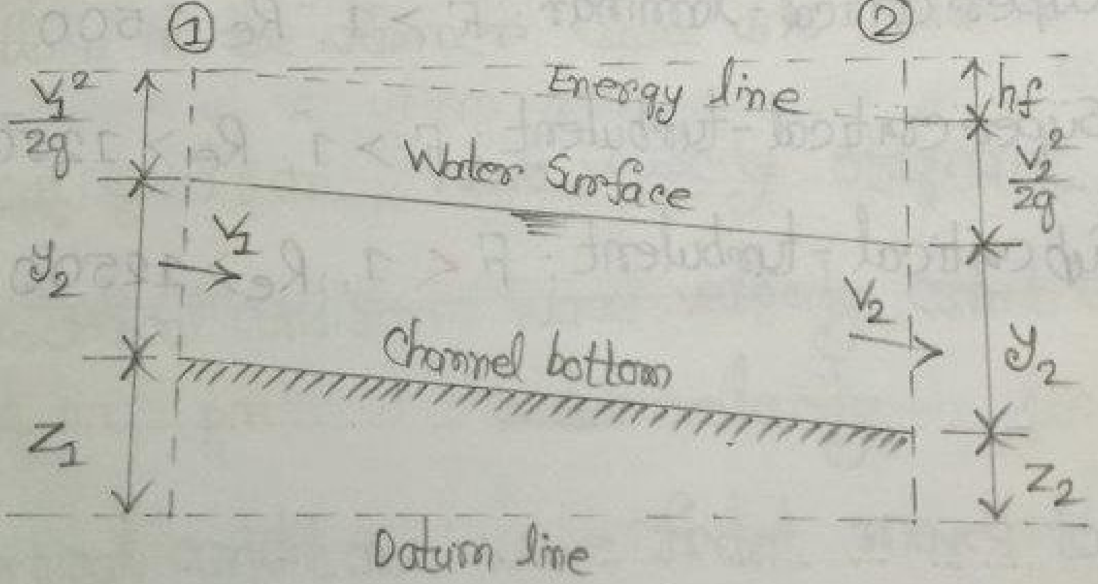
Regimes of Flow : A combined effect of viscosity & gravity may produce any one of four regimes of flow in an open channel, namely,

1. Subcritical - laminar : $F < 1$, $R_e < 500$
2. Supercritical - laminar : $F > 1$, $R_e < 500$
3. Supercritical - turbulent : $F > 1$, $R_e > 12500$
4. Subcritical - turbulent : $F < 1$, $R_e > 12500$

Comparison Between Pipe Flow & Open Channel Flow:



Pipe Flow



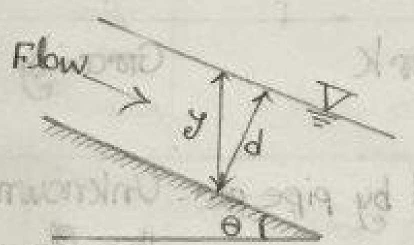
Open Channel Flow

Criteria	Pipe Flow	Open Channel Flow
1. Flow driven by	Pressure work.	Gravity (i.e. potential energy)
2. Flow cross-section	Known (fixed by pipe geometry).	Unknown in advance because the flow depth is unknown beforehand.
3. Characteristic flow parameters	Velocity deduced from continuity equation.	Flow depth and velocity deduced by solving simultaneously the continuity and momentum equations.
4. Free surface	Pipe flow has none, since the water must fill the conduit.	Open channel flow must have a free surface.
5. Specific boundary conditions	Pipe flow, being confined in a closed conduit, exerts no direct atmospheric pressure but hydraulic pressure only.	Atmospheric pressure at the flow free surface.
6. Hydraulic Grade Line	Refers to piezometric height.	Refers to water surface.
7. Physical Conditions	Varies less.	Varies much more widely.
8. Friction Coefficient	Less uncertain.	Greater uncertainty prevails.

Chapter 2 (V.T. Chow)

"Open Channels and Their Properties"

Geometric Properties of Channel:



1. Channel section : Cross-section of a channel taken normal to the direction of flow.

2. Vertical channel section : It is the vertical section passing through the lowest or bottom point of the channel section.

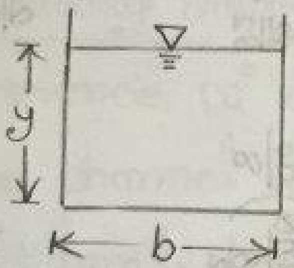
3. Depth of flow (y): It is the vertical distance of the lowest point of a channel section from the free surface.

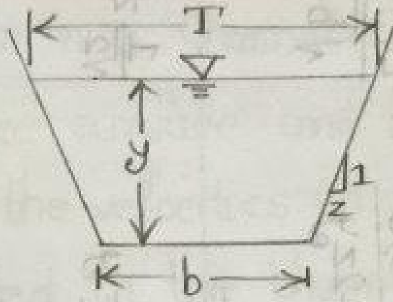
4. Depth of flow section (d): It is the depth of flow normal to the direction of flow, or the height of the channel section containing the water.

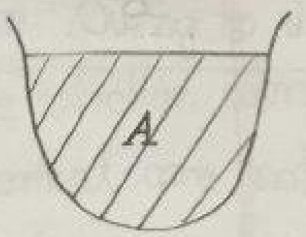
$$y = \frac{d}{\cos\theta}$$

5. Stage : The stage is the elevation or vertical distance of the free surface above a datum.

If the lowest point of the channel section is chosen as the datum, the stage is identical with the depth of flow.







$A = by$

$A = by + \sqrt{1+z^2}\, y$; z = side slope

6. Top width (T) : Width of the channel section at the free surface.

7. Water Area (A) : The cross-sectional area of the flow normal to the direction of flow.

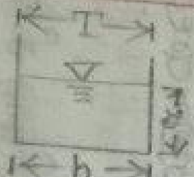
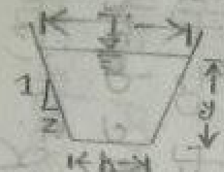
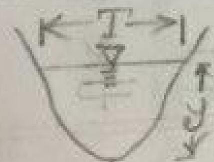
8. Wetted Perimeter (P) : It is the length of the line of intersection of the channel wetted surface with a cross-sectional plane normal to the direction of flow.

9. Hydraulic Radius (R) : Ratio of the water area to its wetted perimeter, $R = \frac{A}{P}$

10. Hydraulic Depth (D) : Ratio of the water area to the top width, $D = \frac{A}{T}$

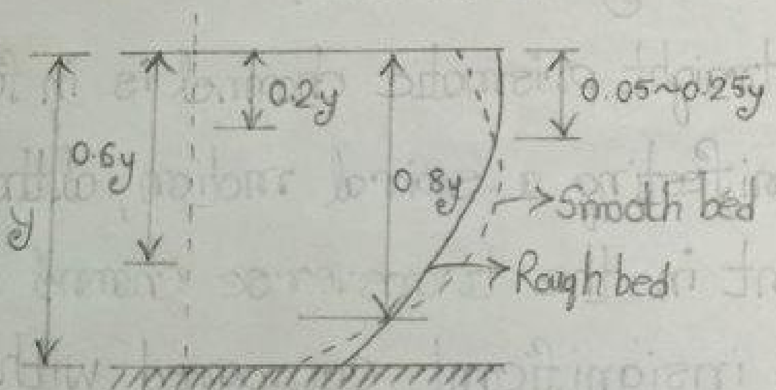
11. Section Factor (Z) : Product of the water area and the square root of the hydraulic depth,

$$Z = A\sqrt{D} = A\sqrt{\frac{A}{T}} = AR^{2/3} \text{ (for uniform flow)}$$

Section	Area A	Wetted Perimeter P	Hydraulic Radius R	Top width T	Hydraulic Depth D	Section Factor Z
 Rectangle	by	$b+2y$	$\frac{by}{b+2y}$	b	y	$by^{3/2}$
 Trapezoid	$(b+zy)y$	$b+2y\sqrt{1+z^2}$	$\frac{(b+zy)y}{b+2y\sqrt{1+z^2}}$	$b+2zy$	$\frac{(b+zy)y}{b+2zy}$	$\frac{[(b+zy)y]^{3/2}}{\sqrt{b+2zy}}$
 Triangle	zy^2	$2y\sqrt{1+z^2}$	$\frac{zy}{2\sqrt{1+z^2}}$	$2zy$	$\frac{1}{2}y$	$\frac{1}{\sqrt{2}}zy^{5/2}$
 Circle	$\frac{1}{8}(\theta-\sin\theta)d_0^2$ Note: $\theta=2\cos^{-1}(1-\frac{2y}{d_0})$	$\frac{1}{2}\theta d_0$	$\frac{1}{4}(1-\frac{\sin\theta}{\theta})d_0$	$(\sin\frac{\theta}{2})d_0$ or, $2\sqrt{y(d_0-y)}$	$(\frac{\theta-\sin\theta}{\sin\frac{\theta}{2}})\frac{d_0}{8}$	$\frac{\sqrt{2}}{32}\frac{(\theta-\sin\theta)^{3/2}}{(\sin\frac{\theta}{2})^{1/2}}\cdot d_0^{5/2}$
 Parabola	$\frac{2}{3}Ty$ or, $\frac{4y^{3/2}}{3\sqrt{c}}$	$T+\frac{8y^2}{3T}$	$\frac{2T^2y}{3T^2+8y^2}$	$\frac{3}{2}\cdot\frac{A}{y}$	$\frac{2}{3}y$	$\frac{2}{9}\sqrt{6}\,Ty^{3/2}$

Velocity Distribution in a Channel Section: Owing to the presence of a free surface and to the friction along the channel wall, the velocities in a channel are not uniformly distributed in the channel section.

The measured maximum velocity in ordinary channels usually appears to occur below the free surface at a distance of 0.05 to 0.25 of the depth; the closer to the banks, the deeper is the maximum.



The velocity distribution in a channel section depends on some factors such as,

① Unusual shape of the section

② Roughness of the channel

③ Presence of bends

- In a broad, rapid and shallow stream or in a very smooth channel, the maximum velocity may often be found at the free surface.
- The roughness of the channel will cause the curvature of the "vertical-velocity-distribution" curve to increase.
- On a bend the velocity increases greatly at the convex side, owing to the centrifugal action of the flow.
- Contrary to the usual belief, a surface wind has very little effect on velocity distribution.
- The flow in a straight prismatic channel is in fact three-dimensional, manifesting a spiral motion, although the velocity component in the transverse channel section is usually small & insignificant compared with the longitudinal velocity component.
- In short laboratory flumes, a small disturbance at the entrance, which is usually unavoidable, is sufficient to cause the zone of highest water level to shift to one side, thus giving rise to a single spiral motion.

- In a long and uniform reach remote from the entrance, a double spiral motion will occur to permit equalization of shear stresses on both sides of the channel.
- In practical considerations, it is quite safe to ignore the spiral motion in straight prismatic channels. Spiral flow in curved channels, however, is an important phenomenon to be considered in design.

Wide Open Channel: Observations in very wide open channels have shown that the velocity distribution in the central region of the section is essentially the same as it would be in a rectangular channel of infinite width.

- In other words, under this condition, the sides of the channel have practically no influence on the velocity distribution in the central region, and the flow in the central region can therefore be regarded as two-dimensional in hydraulic analyses.

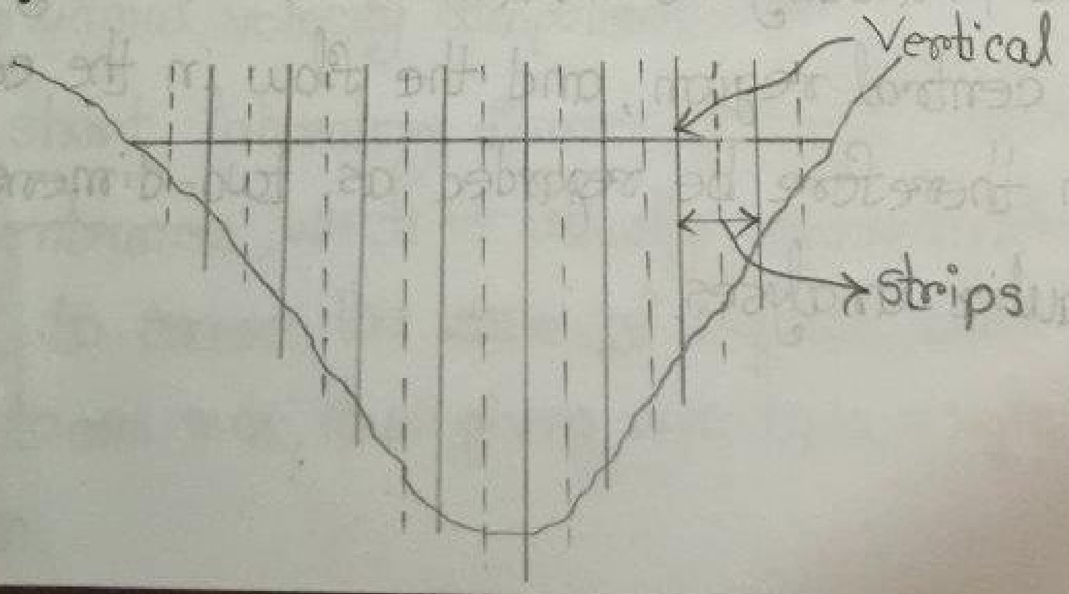
- This central region exists in rectangular channels only when the width is greater than 5 to 10 times the depth of flow, depending on the condition of surface roughness.

$$b > 5 \sim 10\,y$$

- Thus, a wide open channel can safely be defined as a rectangular channel whose width is greater than 10 times the depth of flow.

$$b > 10\,y$$

Measurement of Velocity : The velocity of flow in an open channel can be measured with a current meter. According to the stream-gaging procedure of the U.S. Geological Survey, the channel cross-section is divided



into vertical strips by a number of successive verticals, and mean velocities in verticals are determined by measuring the velocity

Case-1 : at 0.6 of the depth in each vertical when the depth of flow $y < 0.61\,m$; $V_{avg} = V_{0.6y}$

Case-2 : average of the velocities at 0.2 and 0.8 of the depth when the depth of flow $y > 0.61\,m$;

$$V_{avg} = \frac{V_{0.2y} + V_{0.8y}}{2}$$

- The average of the mean velocities in any two adjacent verticals multiplied by the area between the verticals gives the discharge through this vertical strip of the cross-section.
- The sum of discharges through all strips is the total discharge.
- The mean velocity of the whole section is, therefore, equal to the total discharge divided by the whole area.

$$V_{mean} = \frac{\Sigma Q_i}{\Sigma A_i}$$

Propeller Type Current Meter :

No. of revolution = N

Time = t

Revolution per second,

$$n = \frac{N}{t}$$

Point velocity, $v = an + b$

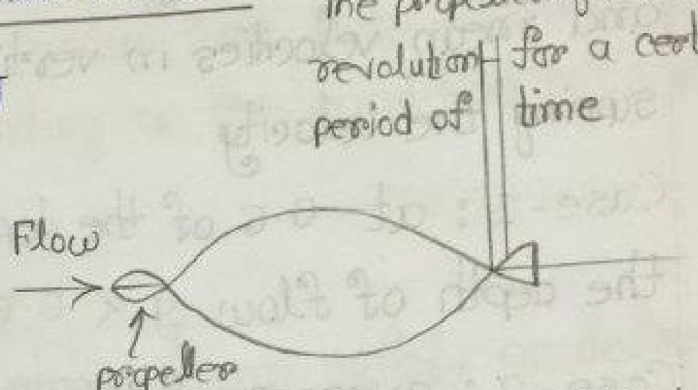


Figure : Current Meter

where, a & b are the coefficients of current meter.

Velocity Distribution Coefficients :

- As a result of nonuniform distribution of velocities over a channel section, the velocity head of an open-channel flow is generally greater than the value computed according to the expression "$\frac{V^2}{2g}$", where V is the mean velocity.
- When the energy principle is used in computation, the true velocity head may be expressed as $\alpha \frac{V^2}{2g}$, where α is known as the energy co-

efficient or Coriolis coefficient.

- Experimental data indicate that the value of α varies from about 1.03 to 1.36 for fairly straight prismatic channels.
- The value is generally higher for small channels and lower for large streams of considerable depth.

- The non-uniform distribution of velocities also affects the computation of momentum in open-channel flow.
- From the principle of mechanics, the momentum of the fluid passing through a channel section per unit time is expressed by $\frac{\beta w Q V}{g}$,

where, β = momentum coefficient or Boussinesq coefficient

w = unit weight of water

Q = discharge

V = mean velocity

- The value of β for fairly straight prismatic channels varies approximately from 1.01 to 1.12.

Salient Features of α & β :

- The energy & momentum coefficients are always positive and never less than unity.
- For uniform velocity distribution in the channel section, $\alpha = \beta = 1$. In all other cases, $\alpha > \beta > 1$ and further the velocity distribution departs from uniform, the greater the coefficients become.
- The effect of turbulence is to make the flow more uniform over the channel section.
- Therefore, the values of α & β are higher for laminar flow than for turbulent flow.
- Although the numerical values of α & β may vary over a wide range depending on the velocity distribution, the ratio $\frac{\alpha - 1}{\beta - 1}$ tends to vary only slightly, in the range 2.8 to 3.0.

Determination of Velocity Distribution Coefficient :

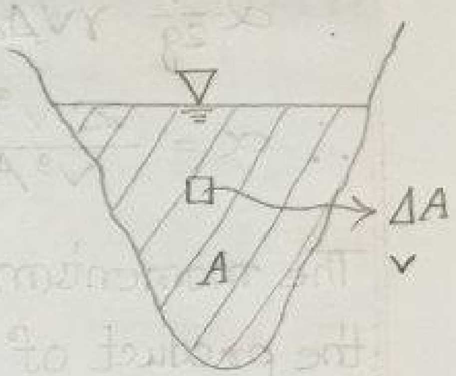
Let,

A = whole water area

ΔA = elementary water area

v = velocity

γ = unit weight of water



The weight of water passing ΔA per unit time, with a velocity v,

$$\gamma v \Delta A$$

$\xleftrightarrow{\quad v \quad}$ in unit time, crosses v length

The kinetic energy of water passing ΔA per unit time, is the product of the weight & the velocity head,

$$\gamma v \Delta A \cdot \frac{v^2}{2g} = \frac{1}{2} \cdot \frac{\gamma v \Delta A}{g} \cdot v^2$$

Total kinetic energy for the whole water area,

$$\Sigma \frac{\gamma v^3 \Delta A}{2g} \quad \ldots\ldots \quad (1)$$

Now, taking the whole area A, the mean velocity V and the corrected velocity head for the whole area as, $\alpha \frac{V^2}{2g}$ the total kinetic energy is $\alpha \frac{V^2}{2g} \cdot \gamma V A$

Equating this quantity with $\Sigma \frac{\gamma v^3 \Delta A}{2g}$ we get,

$$\alpha \cdot \frac{V^2}{2g} \cdot \gamma V A = \Sigma \frac{\gamma v^3 \Delta A}{2g}$$

$$\therefore \alpha = \frac{\Sigma v^3 \Delta A}{V^3 A} \quad \ldots\ldots\ldots \quad (2)$$

The momentum of water passing ΔA per unit time is the product of the mass, $\frac{\gamma v \Delta A}{g}$ and the velocity v, or $\frac{\gamma v \Delta A}{g} \cdot v$

The total momentum is $\Sigma \frac{\gamma v^2 \Delta A}{g}$.

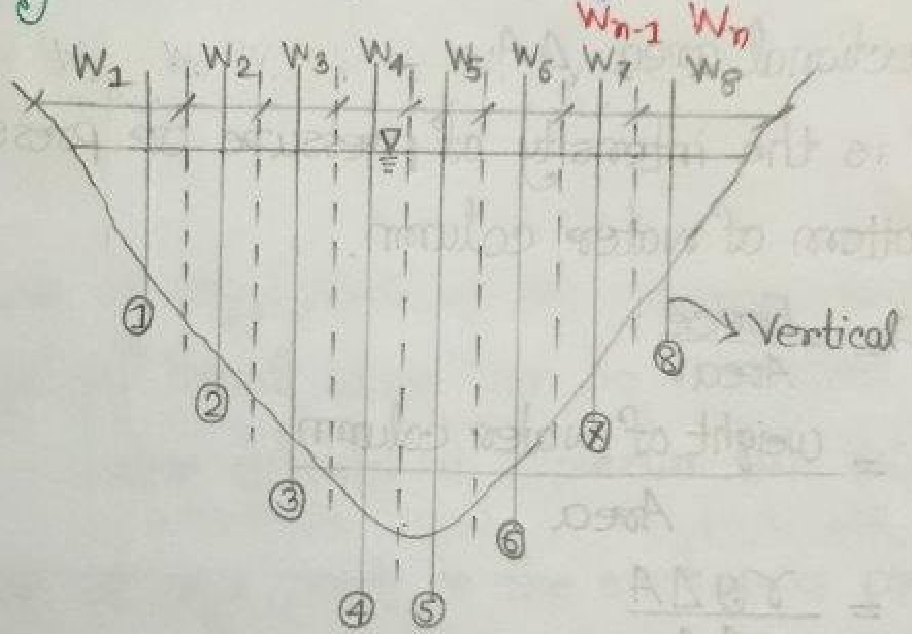
Equating this quantity with the corrected momentum for the whole area, or $\frac{\beta \gamma A V^2}{g}$, and reducing,

$$\Sigma \frac{\gamma v^2 \Delta A}{g} = \frac{\beta \gamma V^2 A}{g}$$

$$\therefore \beta = \frac{\Sigma v^2 \Delta A}{V^2 A} \quad \ldots\ldots\ldots \quad (3)$$

Velocity Measurement :

Area-Velocity Method : For Discharge Measurement



First Segment :

$$\overline{W_1} = \frac{\left(W_1 + \frac{W_2}{2}\right)^2}{2W_1}$$

Last Segment :

$$\overline{W_n} = \frac{\left(W_n + \frac{W_{n-1}}{2}\right)^2}{2W_n}$$

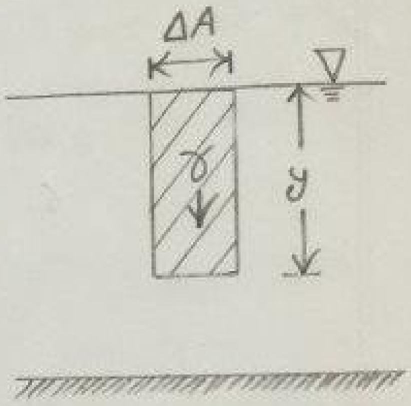
Intermediate Segment :

$$\overline{W_i} = \frac{W_i + W_{i+1}}{2}$$

Hydrostatic Pressure Distribution :

Let, consider a vertical column of height y and cross-sectional area ΔA.

P is the intensity of pressure or pressure $\left(\frac{\text{force}}{\text{area}}\right)$ at the bottom of water column.



$$P = \frac{\text{Force}}{\text{Area}}$$

$$= \frac{\text{weight of water column}}{\text{Area}}$$

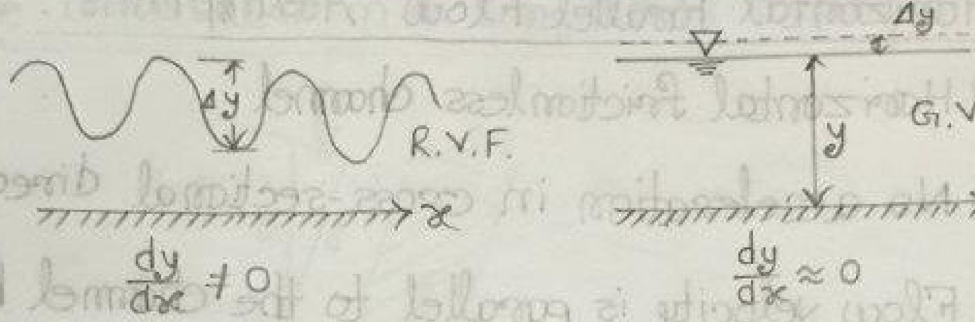
$$= \frac{\gamma y \Delta A}{\Delta A}$$

$$= \gamma y$$

$$= \rho g y$$

Pressure Distribution in a Channel Section :

- It is valid for "gradually varied flow" where depth changes with respect to channel height so mildly that the streamlines have neither appreciable curvature nor divergence but not for "rapidly varied flow".



- The pressure at any point on the cross-section of flow in a channel of "small slope" can be measured by the height of a water column in a piezometric tube.

- The pressure at any point on the section is proportional to the depth at a point below the surface and equal to hydrostatic pressure corresponding to its depth, $P = \rho g h = \gamma h$.

This is known as Hydrostatic Law of Pressure.

- Hydrostatic law is valid for parallel flows where stream-lines are parallel or not substantially curvature. In such cases, the flow has no acceleration components in the plane of cross-section.

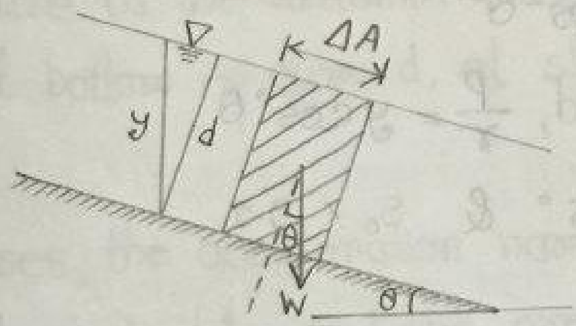
Horizontal Parallel Flow Assumptions:

1. Horizontal frictionless channel
2. No acceleration in cross-sectional direction
3. Flow velocity is parallel to the channel bottom
4. Streamlines are parallel
5. Vertical component of the resultant force acting on the column of liquid is zero.

Effect of Slope on Pressure Distribution on Parallel Flow in a Sloping Channel:

1. Sloping frictionless channel
2. No acceleration in cross-sectional direction
3. Flow velocity is parallel to the channel bottom
4. Streamlines are parallel

Pressure Distribution in a Longitudinal Sloping (θ) channel:



Consider a water column of height y and cross-sectional area ΔA.

The pressure at the point on the channel bottom balances the component of the weight of the element normal to the bed.

Weight of the column normal to the direction of flow

$$= W\cos\theta = \gamma d \Delta A \cos\theta$$

The intensity of pressure at the bottom of water column,

$$p = \frac{\text{Force}}{\text{Area}} = \frac{\gamma d \Delta A \cos\theta}{\Delta A} = \gamma d \cos\theta$$

Again, $y = \dfrac{d}{\cos\theta}$

$$\Rightarrow d = y\cos\theta$$

$\therefore P = \gamma y \cos\theta \cdot \cos\theta$

$\Rightarrow P = \gamma y \cos^2\theta$

Pressure Head, $\frac{P}{\gamma} = y\cos^2\theta$

when, $\theta < 6°$ & $S_0 < 0.1$

then, $\frac{P}{\gamma} = y$

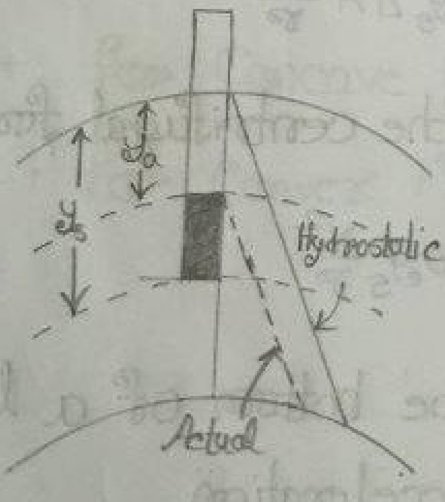
when, $\theta > 6°$ & $S_0 > 0.1$

then $\frac{P}{\gamma} = y\cos^2\theta$

where, $\cos^2\theta$ is the correction factor of pressure head.

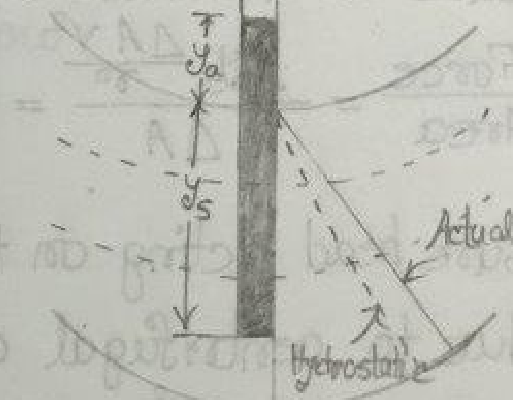
Pressure Distribution in Curvilinear Flow:

- The curvature of the streamlines is considerable. e.g. the channel bottom is curved, at sluice gate and at free overfalls.
- In such cases, the acceleration normal to the direction of flow is not negligible and the pressure distribution is not hydrostatic.
- Curvilinear flows may be concave/convex.



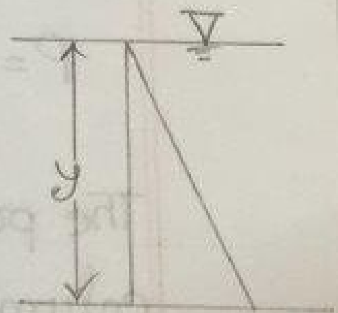
$P < \gamma y$

Convex Flow



$P > \gamma y$

Concave Flow



$P = \gamma h = \gamma y$

Let us consider the forces acting in the vertical direction on a column of a liquid with cross-sectional area ΔA.

Mass of liquid column $= \rho y_s \Delta A$ . . . . . ①

If r is the radius of curvature and v is the flow velocity of the point under consideration,

Centrifugal acceleration $= \frac{v^2}{r}$

Centrifugal force $= \frac{mv^2}{r} = \rho y_s \Delta A \frac{v^2}{r}$

The pressure as a result of the centrifugal force is,

$$P = \frac{\text{Force}}{\text{Area}} = \frac{\rho y_5 \frac{\Delta A\, v^2}{r}}{\Delta A} = \rho y_5 \frac{v^2}{r}$$

The pressure head acting on the bottom of a liquid column due to centrifugal acceleration,

$$y_a = \frac{P}{\gamma} = \frac{\rho y_5 \frac{v^2}{r}}{\gamma} = \frac{\rho y_5 \frac{v^2}{r}}{\rho g}$$

$$\therefore\ y_a = \frac{1}{g} y_5 \frac{v^2}{r}$$

The total pressure head acting on the bottom is the algebraic sum of pressure due to centrifugal action & the weight of the liquid column.

$\therefore$ Total Pressure Head $= y_s \pm y_a$

$$= y_s \pm \frac{1}{g} y_s \frac{v^2}{r}$$

$$= y_s \left(1 \pm \frac{1}{g} \cdot \frac{v^2}{r}\right)$$

where, $\frac{1}{g}\frac{v^2}{r}$ is the correction for curvilinear flow.

'+' for Concave Flow

'−' for Convex Flow

Open Channel Flow By Halim

Chapter 1

Solution of Problems

C.W. **Example 1.1** : A trapezoidal channel has a bottom width of 6m and side slopes of 2 : 1. Compute the discharge & determine the state of flow in this channel if the depth of flow is 1.5 m and the mean velocity of flow is 2.30 m/s. If elementary waves are created in this channel, determine the speed of wave fronts upstream and downstream.

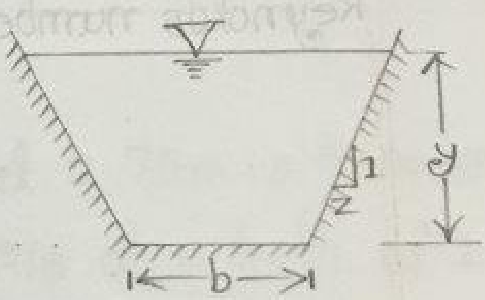
Sol^n: Given, for a Trapezoidal channel,

Bottom width, $b = 6m$

Side slope, $z = 2$

Depth of flow, $y = 1.5m$

Velocity, $v = 2.30 \, m/s$



Cross-sectional area, $A = (b + zy)\, y$

$$= (6 + 2 \times 1.5) \times 1.5$$

$$= 13.5 \, m^2$$

Wetted perimeter, $P = b + 2\sqrt{1+z^2} \cdot y$

$$= 6 + 2 \times \sqrt{1 + (2)^2} \cdot 1.5$$

$$= 12.71 \, m$$

Top width, $T = b + 2zy$

$= 6 + 2 \times 2 \times 1.5$

$= 12 m$

Hydraulic radius, $R = \frac{A}{P} = \frac{13.5}{12.71} = 1.06 m$

Hydraulic depth, $D = \frac{A}{T} = \frac{13.5}{12} = 1.125 m$

$\therefore$ Discharge, $Q = VA = (2.30 \times 13.5) = 31.05 m^3/s$

(Ans:)

Reynolds number, $R_e = \frac{VR}{\nu}$ $[\nu = 1 \times 10^{-6} m^2/s]$

$= \frac{2.3 \times 1.06}{10^{-6}}$

$= 2.438 \times 10^6 > 12,500$

Froude number, $F_r = \frac{V}{\sqrt{gD}}$

$= \frac{2.30}{\sqrt{9.81 \times 1.125}}$

$= 0.69 < 1.0$

Hence, the flow is Sub-critical - Turbulent.

(Ans:)

Now, Celerity, $c = \sqrt{gD} = \sqrt{9.81 \times 1.125} = 3.32$ m/s

$\therefore$ Speed of wave fronts upstream $= c - V$

$= 3.32 - 2.30$

$= 1.02$ m/s (Ans:)

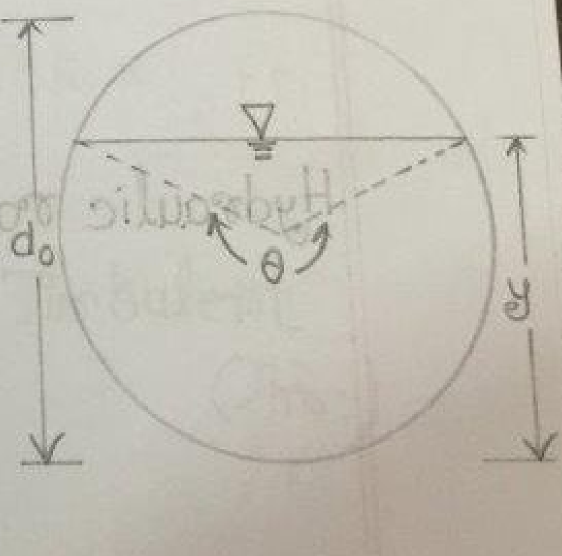
& Speed of wave fronts downstream $= c + V$

$= 3.32 + 2.30$

$= 5.62$ m/s (Ans:)

Example 1.2 : A circular channel 2.75m in diameter carries a discharge of 6.55 m^3/s at a depth of 1.1m. Determine the state of flow.

Solⁿ: Given, for a circular channel,



Diameter, $d_0 = 2.75$ m

Discharge, $Q = 6.55$ m^3/s

Depth of flow, $y = 1.1$ m

Angle, $\theta = 2\cos^{-1}\left(1 - \frac{2y}{d_0}\right)$ [θ in radian]

$= 2\cos^{-1}\left(1 - \frac{2\times 1.1}{2.75}\right)$

$= 156.9261°$ or, 2.74 radian

Area, $A = \frac{1}{8}(\theta - \sin\theta)d_0^2$

$= \frac{1}{8}(2.74 - \sin 2.74)\times(2.75)^2$

$= 2.22\ m^2$

Wetted perimeter, $P = \frac{1}{2}\theta d_0$

$= \frac{1}{2}\times 2.74 \times 2.75$

$= 3.77\ m$

Top width, $T = \left(\sin\frac{\theta}{2}\right) d_0$

$= \sin\left(\frac{2.74}{2}\right)\times 2.75$

$= 2.69\ m$

Hydraulic radius, $R = \frac{A}{P}$

$= \frac{2.22}{3.77}$

$= 0.59\ m$

Hydraulic depth, $D = \frac{A}{T}$

$= \frac{2.22}{2.69}$

$= 0.83\ m$

Velocity, $V = \frac{Q}{A}$

$= \frac{6.55}{2.22}$

$= 2.95\ m/s$

Reynolds number, $R_e = \frac{VR}{\nu}$

$= \frac{2.95 \times 0.59}{10^{-6}}$ $[\nu = 10^{-6}\ m^2/s]$

$= 1.7405 \times 10^{6} > 12,500$

Froude number, $F_r = \frac{V}{\sqrt{gD}}$

$= \frac{2.95}{\sqrt{9.81 \times 0.83}}$

$= 1.03 > 1.0$

Hence, the flow is Super-critical - Turbulent

(Ans:)

C.W. Example 1.3: The data collected during the stream-gauging operation at a certain river section are given in the table. Compute the discharge and the mean velocity for the entire section.

Distance from left bank (m)	Total Depth (m)	Meter Depth (m)	Velocity (m/s)	Width (m)	Area (m^2)	Mean velocity (m/s)	Discharge (m^3/s)
0	0						
2	1.00	0.60	0.54	2.25	2.25	0.54	1.215
4	3.50	2.80 0.70	0.98 1.62	2.00	7.00	1.30	9.10
6	5.20	4.16 1.04	1.35 1.60	2.50	13.00	1.475	19.175
9	6.30	5.04 1.26	1.36 1.81	2.50	15.75	1.585	24.96
11	4.40	3.52 0.88	1.51 1.72	2.00	8.80	1.615	14.212
13	2.20	1.32	1.16	2.00	4.40	1.16	5.104
15	0.80	0.48	0.64	2.25	1.80	0.64	1.152
17	0						
Total					53.00		74.918

$\underline{\text{Sol}^n}$: For the section at a distance 2m from left bank,

Width, $\overline{W_1} = \dfrac{\left(W_1 + \frac{W_2}{2}\right)^2}{2W_1} = \dfrac{\left(2 + \frac{4-2}{2}\right)^2}{2 \times 2} = 2.25\ m$

Area, A = Total Depth $\times$ Width

$= 1.00 \times 2.25$

$= 2.25\ m^2$

Mean Velocity, $V_{mean} = V_{0.6y} = 0.54\ m/s$

Discharge, $Q = V_{mean} \cdot A = (0.54 \times 2.25) = 1.215\ m^3/s$

For the section at a distance 9m from left bank,

Width, $\overline{W_4} = \dfrac{W_4 + W_5}{2} = \dfrac{(9-6) + (11-9)}{2} = 2.50\ m$

Area, A = Total Depth $\times$ Width

$= 6.30 \times 2.50$

$= 15.75\ m^2$

Mean velocity, $V_{mean} = \dfrac{V_{0.2y} + V_{0.8y}}{2} = \dfrac{1.81 + 1.36}{2} = 1.585\ m/s$

Discharge, $Q = V_{mean} \cdot A = (1.585 \times 15.75) = 24.96\ m^3/s$

For the section at a distance 15 m from left bank,

Width, $\overline{W}_7 = \frac{(W_7 + \frac{W_6}{2})^2}{2W_7} = \frac{[(17-15) + \frac{(15-13)}{2}]^2}{2 \times 2} = 2.25\ m$

Area, A = Total Depth × Width

$= 0.80 \times 2.25$

$= 1.80\ m^2$

Mean Velocity, $V_{mean} = V_{0.6y} = 0.64\ m/s$

Discharge, $Q = V_{mean} \cdot A = (0.64 \times 1.80) = 1.152\ m^3/s$

Mean Velocity for entire section,

$$V_{mean} = \frac{\Sigma Q_i}{\Sigma A_i}$$

$$= \frac{74.918}{53.00}$$

$$= 1.4135\ m/s$$ (Ans.)

C.W. Example 1.4 : In a wide channel the velocity varies along a vertical as $v = 1 + \frac{3z}{y}$, where y is the total depth, v is the velocity at a distance z from the channel bottom.

(i) Compute the discharge per unit width

(ii) Determine the state of flow

(iii) Compute the velocity distribution coefficients α and β and the ratio $\frac{(\alpha-1)}{(\beta-1)}$, if $y = 5\,m$.

Sol^n: For a wide channel we can consider a unit width of the channel and replace the area by the flow depth, y. Then, the cross-sectional mean velocity V becomes the depth averaged velocity $\overline{V}$. Therefore,

$$\overline{V} = \frac{1}{A}\int_0^A v\,dA$$

$$= \frac{1}{y}\int_0^y v\,dz$$

$$= \frac{1}{y}\int_0^y \left(1+\frac{3z}{y}\right)dz$$

$$= \frac{1}{y}\left[z + \frac{3z^2}{2y}\right]_0^y$$

$$= \frac{1}{y}\left[y + \frac{3y^2}{2y}\right]$$

$\Rightarrow \bar{V} = 1 + \frac{3}{2}$

$\therefore \bar{V} = 2.5$ m/s

(i) Discharge per unit width, $Q = y\bar{V}$

$\left[\begin{array}{l} \because Q = VA \\ \Rightarrow Q = Vby \\ \Rightarrow \frac{Q}{b} = yV \end{array}\right]$

$\Rightarrow Q = 5 \times 2.5$

$\therefore Q = 12.5 \ m^3/s/m$ (Ans:)

(ii) Reynolds Number, $R_e = \frac{\bar{V}R}{\nu}$

$\left[\begin{array}{l} \text{For wide channel,} \\ R = \frac{A}{P} = \frac{by}{b+2y} \; ; \; b >> y \\ \text{So, } R = \frac{by}{b} \approx \\ D = \frac{A}{T} = \frac{by}{b} = y \end{array}\right]$

$\Rightarrow R_e = \frac{2.5 \times 5}{10^{-6}}$

$\therefore R_e = 12.5 \times 10^6 > 12{,}500$

Froude Number, $F_r = \frac{\bar{V}}{\sqrt{gD}}$

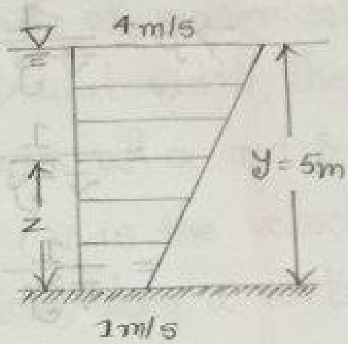
$\Rightarrow F_r = \frac{2.5}{\sqrt{9.81 \times 5}}$

$\therefore F_r = 0.36 < 1$

Hence, the flow is Sub-critical - Turbulent

(Ans:)

(iii) Energy coefficient,



$$\alpha = \frac{\int v^3 \Delta A}{\bar{V}^3 A}$$

$$= \frac{1}{\bar{V}^3 y} \int_0^y v^3 dz$$

$$= \frac{1}{\bar{V}^3 y} \int_0^y \left(1 + \frac{3z}{y}\right)^3 dz$$

$$= \frac{1}{\bar{V}^3 y} \int_0^y \left[1 + 3 \cdot \frac{3z}{y} + 3 \cdot \frac{9z^2}{y^2} + \frac{27z^3}{y^3}\right] dz$$

$$= \frac{1}{\bar{V}^3 y} \left[z + \frac{9z^2}{2y} + \frac{27z^3}{3y^2} + \frac{27z^4}{4y^3}\right]_0^y$$

$$= \frac{1}{\bar{V}^3 y} \left[y + \frac{9y^2}{2y} + \frac{27y^3}{3y^2} + \frac{27y^4}{4y^3}\right]$$

$$= \frac{1}{(2.5)^3} \left[1 + 4.5 + 9 + 6.75\right]$$

$$= 1.36$$ (Ans:)

Momentum coefficient,

$$\beta = \frac{\int v^2 \Delta A}{\bar{V} A}$$

$$= \frac{1}{\bar{V}^2 y} \int_0^y v^2 dz$$

$$= \frac{1}{\bar{V}^2 y} \int_0^y \left(1 + \frac{3z}{y}\right)^2 dz$$

$$\Rightarrow \beta = \frac{1}{\bar{V}^2 y}\int_0^y \left(1 + 2\cdot\frac{3z}{y} + \frac{9z^2}{y^2}\right) dz$$

$$= \frac{1}{\bar{V}^2 y}\left[z + \frac{6z^2}{2y} + \frac{9z^3}{3y^2}\right]_0^y$$

$$= \frac{1}{\bar{V}^2 y}\left[y + \frac{3y^2}{y} + \frac{3y^3}{y^2}\right]$$

$$= \frac{1}{(2.5)^2}\left[1 + 3 + 3\right]$$

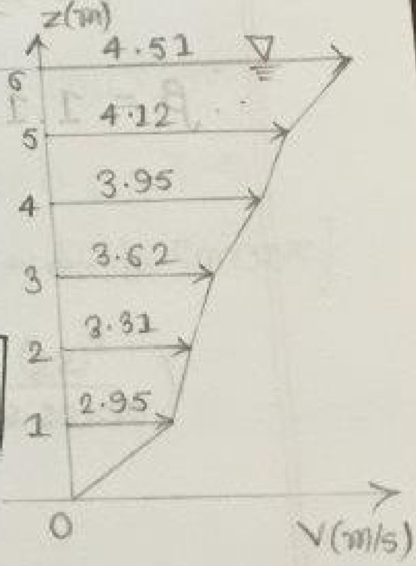
$$= 1.12$$ (Ans:)

The ratio of $\frac{\alpha - 1}{\beta - 1} = \frac{1.36 - 1}{1.12 - 1} = 3$ (Ans:)

Note: When v is expressed as a function of $\frac{z}{y}$, the numerical values of $\bar{V}$, α, β become independent of depth of flow.

C.W. **Example 1.5:** Using the trapezoidal rule of numerical integration, compute the discharge per unit width, the mean velocity and the numerical values of α and β for the following velocity measurements (v is the velocity at a distance z from the channel bottom) along a vertical in a wide channel, when the total depth is 6 m.

z(m)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
v (m/s)	0.0	2.95	3.31	3.62	3.95	4.12	4.51



Sol^n: Discharge per unit width,

$$q = \int_0^y v\,dz$$

$$= \sum v \Delta z$$

$$= \Delta z \left[\frac{0}{2} + 2.95 + 3.31 + 3.62 + 3.95 + 4.12 + \frac{4.51}{2}\right]$$

$$= 1.0 \times 20.205 \quad [\because \Delta z = 1.0\,m]$$

$$= 20.205\ m^3/s/m$$

(Ans:)

Mean velocity, $\bar{V} = \frac{q}{y} = \frac{20.205}{6} = 3.3675\ m/s$ (Ans:)

Energy coefficient, $\alpha = \dfrac{\int v^3 dz}{\bar{V}^3 y} = \dfrac{\Sigma v^3 \Delta z}{\bar{V}^3 y}$

$$\Rightarrow \alpha = \frac{1.0}{(3.3675)^3 \times 6} \times \left[\frac{0^3}{2} + (2.95)^3 + (3.31)^3 + (3.62)^3 + (3.95)^3 + (4.12)^3 + \frac{(4.51)^3}{2}\right]$$

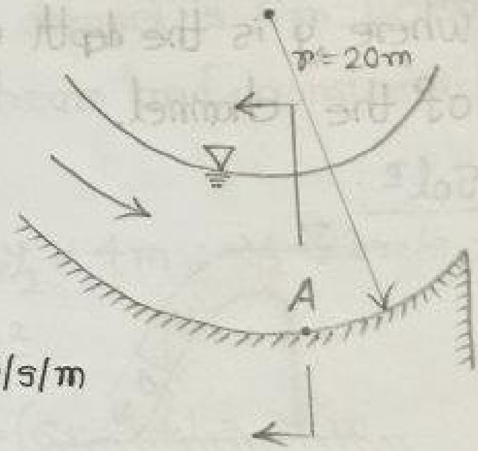
$\Rightarrow \alpha = 1.25$ (Ans:)

Momentum coefficient, $\beta = \dfrac{\int v^2 dz}{\bar{V}^2 y} = \dfrac{\Sigma v^2 \Delta z}{\bar{V}^2 y}$

$$\Rightarrow \beta = \frac{1.0}{(3.3675)^2 \times 6} \times \left[\frac{0^2}{2} + (2.95)^2 + (3.31)^2 + (3.62)^2 + (3.95)^2 + (4.12)^2 + \frac{(4.51)^2}{2}\right]$$

$\therefore \beta = 1.11$ (Ans:)

Example 1.6 : A spillway flip bucket has a radius of curvature of 20m. If the flow depth at section 1-1 is 3m and the discharge per unit width is 66 $m^3/s/m$, compute the pressure at A.



Sol^n: Given,

Radius of curvature, $r = 20m$

Depth of flow, $y = 3m$

Discharge per unit width, $q = 66 \, m^3/s/m$

Now, Mean velocity, $V = \frac{q}{y}$

$$= \left(\frac{66}{3}\right) = 22 \, m/s$$

Pressure at A, $p = \rho g y \left(1 + \frac{V^2}{gr}\right)$ ['+' due to Concave]

$$= 1000 \times 9.81 \times 3 \times \left(1 + \frac{(22)^2}{9.81 \times 20}\right)$$

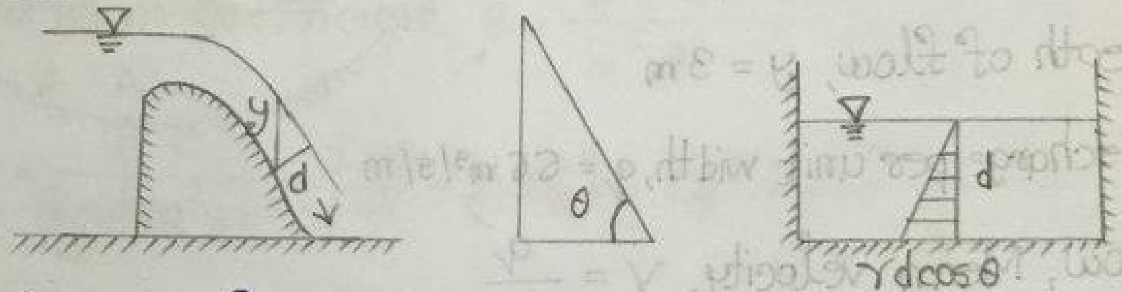
$$= 102030 \, N/m^2$$

$$= 102.30 \, kN/m^2$$

(Ans:)

Example 1.7: Prove that the shear force and the overturning moment on the side walls of a steep rectangular channel are $\frac{1}{2}\gamma y^2 \cos^3\theta$ and $\frac{1}{6}\gamma y^3 \cos^4\theta$, respectively, where y is the depth of flow and θ is the bottom slope of the channel.

Sol^n:



We know, $\frac{P}{\gamma} = d\cos\theta$ for a Sloping Channel.

$\Rightarrow P = \gamma d \cos\theta$

Shear Force, $F = \frac{1}{2} \times \gamma d \cos\theta \times d$ [per unit width]

$= \frac{1}{2}\gamma d^2 \cos\theta$

$= \frac{1}{2}\gamma y^2 \cos^3\theta$ $[\because d = y\cos\theta]$
(Proved)

Overturning moment $= F \times \text{Arm} = F \times \frac{d}{3}$ [from channel bottom]

$= \frac{1}{2}\gamma d^2 \cos\theta \times \frac{d}{3}$

$= \frac{1}{6}\gamma d^3 \cos\theta$

$= \frac{1}{6}\gamma y^3 \cos^4\theta$ $[\because d = y\cos\theta]$
(Proved)

Problem 1.1 (a): The depth and mean velocity upstream and downstream of a vertical sluice gate in a horizontal rectangular channel are 4m and 1m and 2m/s and 8m/s respectively. The width of the channel is 6m. Determine the state of flow both upstream and downstream of the gate.

Sol^n: Upstream of the gate, $y_1 = 4m$; $V_1 = 2m/s$; $b = 6m$

Area, $A_1 = by_1 = (6 \times 4) = 24 m^2$

Wetted Perimeter, $P_1 = b + 2y_1 = (6 + 2\times4) = 14 m$

Top Width, $T_1 = b = 6m$

Hydraulic Radius, $R_1 = \frac{A_1}{P_1} = \frac{24}{14} = 1.71 m$

Hydraulic Depth, $D_1 = \frac{A_1}{T_1} = \frac{24}{6} = 8 m$

Reynolds Number, $R_e = \frac{V_1 R_1}{\nu} = \frac{2\times1.71}{10^{-6}} = 3.42\times10^6 > 12{,}500$

Froude Number, $F_r = \frac{V_1}{\sqrt{gD_1}} = \frac{2}{\sqrt{9.81\times8}} = 0.23 < 1.0$

Hence, the flow is Sub-critical - Turbulent

(Ans.)

Downstream of the gate : $y_2 = 1m$; $V_2 = 8 m/s$; $b = 6m$

Area, $A_2 = by_2 = (6 \times 1) = 6 m^2$

Wetted Perimeter, $P_2 = b + 2y_2 = (6 + 2\times1) = 8m$

Top Width, $T_2 = b = 6m$

Hydraulic Radius, $R_2 = \frac{A_2}{P_2} = \frac{6}{8} = 0.75 m$

Hydraulic Depth, $D_2 = \frac{A_2}{T_2} = \frac{6}{6} = 1.0 m$

Reynolds Number, $R_e = \frac{V_2 R_2}{\nu} = \frac{8 \times 0.75}{10^{-6}} = 6\times10^6 > 12,500$

Froude Number, $F_r = \frac{V_2}{\sqrt{g D_2}} = \frac{8}{\sqrt{9.81\times1}} = 2.55 > 1.0$

Hence, the flow is Super-critical-Turbulent.

(Ans:)

Problem 1.1 (b): Consider the following data for the Padma (Ganges) river at the Baruria station in Faridpur on the 2nd July, 1989 : $A = 33,500 \ m^2$, $Q = 56,200 \ m^3/s$, and $B = 3820 \ m$. Compute the state of flow. Assume that the river is wide.

Sol^n: Depth of flow, $y = \frac{A}{b} = \frac{A}{B}$ [For wide river, $b = B$ Top width]

$$\Rightarrow y = \frac{33500}{3820} = 8.77 m$$

For wide river, Hydraulic Radius, $R = y = 8.77 m$

& Hydraulic Depth, $D = y = 8.77 m$

Mean velocity, $V = \frac{Q}{A} = \frac{56200}{33500} = 1.68 \ m/s$

Reynolds Number, $R_e = \frac{VR}{\nu} = \frac{1.68 \times 8.77}{10^{-6}} = 14.7336 \times 10^6$

$> 12,500$

Froude Number, $F_r = \frac{V}{\sqrt{gD}} = \frac{1.68}{\sqrt{9.81 \times 8.77}} = 0.18 < 1.0$

Hence, the flow is Sub-critical - Turbulent.

(Ans.)

Problem 1.2 : Water flows in an open channel at a depth of 1m and a mean velocity of 3m/s. Compute the discharge and determine the state of flow if the channel is, i) wide

ii) rectangular with $b = 6m$

iii) trapezoidal with $b = 6m$ and $z = 2$

iv) triangular with $z = 2$

v) parabolic with $T = 4m$

vi) circular whose diameter is 2.5m

If elementary waves are created in these channels, determine the speeds of the wave fronts upstream and/or downstream.

Sol^n : (i) Given, $y = 1m$; $V = 3m/s$

For wide channel, Discharge per unit width,

$$q = yV = (1 \times 3) = 3\ m^3/s/m \quad \text{(Ans:)}$$

Hydraulic Radius, $R = y = 1m$

Hydraulic Depth, $D = y = 1m$

Reynolds Number, $R_e = \frac{VR}{\nu} = \frac{3\times1}{10^{-6}} = 3\times10^6 > 12{,}500$

Froude Number, $F_r = \frac{V}{\sqrt{gD}} = \frac{3}{\sqrt{9.81\times1}} = 0.96 < 1.0$

Hence, the flow is Subcritical - Turbulent (Ans:)

Celerity, $c = \sqrt{gD} = \sqrt{9.81\times1} = 3.13$ m/s

Speed of wave front upstream $= c - V = (3.13 - 3) = 0.13$ m/s ; u/s (Ans:)

Speed of wave fronts downstream $= c + V = (3.13 + 3) = 6.13$ m/s; d/s (Ans:)

(ii) Given, $y = 1$ m, $V = 3$ m/s, $b = 6$ m

Area, $A = by = (6\times1) = 6\ m^2$

Discharge, $Q = VA = (3\times6) = 18\ m^3/s$ (Ans:)

Wetted Perimeter, $P = b + 2y = (6 + 2\times1) = 8$ m

Top width, $T = b = 6$ m

Hydraulic Radius, $R = \frac{A}{P} = \frac{6}{8} = 0.75$ m

Hydraulic Depth, $D = \frac{A}{T} = \frac{6}{6} = 1$ m

Reynolds Number, $R_e = \frac{VR}{\nu} = \frac{3 \times 0.75}{10^{-6}} = 2.25 \times 10^6 > 12,500$

Froude Number, $F_r = \frac{V}{\sqrt{gD}} = \frac{3}{\sqrt{9.81 \times 1}} = 0.96 < 1.0$

Hence, the state of flow is Subcritical-Turbulent. (Ans:)

Celerity, $c = \sqrt{gD} = \sqrt{9.81 \times 1} = 3.13$ m/s

Speed of wavefronts upstream $= c - V = (3.13 - 3) = 0.13$ m/s; u/s (Ans:)

Speed of wavefronts downstream $= c + V = (3.13 + 3) = 6.13$ m/s; d/s (Ans:)

(iii) Given, $y = 1$ m; $V = 3$ m/s, $b = 6$ m; $z = 2$

Area, $A = (b + zy)\,y = (6 + 2 \times 1) \times 1 = 8\ m^2$

Discharge, $Q = VA = (3 \times 8) = 24\ m^3/s$ (Ans:)

Wetted Perimeter, $P = b + 2\sqrt{1+z^2}\,y = \left[6 + \sqrt{1+(2)^2} \times 1\right] = 8.24$ m

Top Width, $T = b + 2zy = 6 + (2 \times 2 \times 1) = 10$ m

Hydraulic Radius, $R = \frac{A}{P} = \frac{8}{8.24} = 0.97$ m

Hydraulic Depth, $D = \frac{A}{T} = \frac{8}{10} = 0.8$ m

Reynolds Number, $R_e = \frac{VR}{\nu} = \frac{3 \times 0.97}{10^{-6}} = 2.91 \times 10^6 > 12,500$

Froude Number, $F_r = \frac{V}{\sqrt{gD}} = \frac{3}{\sqrt{9.81 \times 0.8}} = 1.07 > 1.0$

Hence, the flow is Supercritical-Turbulent. (Ans:)

Celerity, $c = \sqrt{gD} = \sqrt{9.81 \times 0.8} = 2.80$ m/s

Speed of wavefronts upstream $= c - V = (2.8 - 3) = -0.2$ m/s ; d/s (Ans:)

Speed of wavefronts downstream $= c + V = (2.8 + 3) = 5.8$ m/s ; d/s (Ans:)

(iv) Given, $y = 1$ m ; $V = 3$ m/s ; $z = 2$

Area, $A = zy^2 = 2 \times (1)^2 = 2\ m^2$

Discharge, $Q = VA = (3 \times 2) = 6\ m^3/s$ (Ans:)

Wetted Perimeter, $P = 2\sqrt{1+z^2}\, y = 2 \times \sqrt{1+(2)^2} \times 1 = 4.47$ m

Top Width, $T = 2zy = (2 \times 2 \times 1) = 4$ m

Hydraulic Radius, $R = \frac{A}{P} = \frac{2}{4.47} = 0.45$ m

Hydraulic Depth, $D = \frac{A}{T} = \frac{2}{4} = 0.5$ m

Reynolds Number, $R_e = \frac{VR}{\nu} = \frac{3 \times 0.45}{10^{-6}} = 1.35 \times 10^{6} > 12,500$

Froude Number, $F_r = \frac{V}{\sqrt{gD}} = \frac{3}{\sqrt{9.81 \times 0.5}} = 1.35 > 1.0$

Hence, the flow is Supercritical - Turbulent. (Ans:)

Celerity, $c = \sqrt{gD} = \sqrt{9.81 \times 0.5} = 2.21$ m/s

Speed of wavefronts upstream $= c - V = (2.21 - 3) = 0.79$ m/s ; d/s (Ans:)

Speed of wavefronts downstream $= c + V = (2.21 + 3) = 5.21$ m/s ; d/s (Ans:)

(V) Given, $y = 1$m ; $V = 3$m/s ; $T = 4$m

Area, $A = \frac{2}{3} Ty = \left(\frac{2}{3} \times 4 \times 1\right) = 2.67 \text{m}^2$

Discharge, $Q = VA = \left(3 \times \frac{8}{3}\right) = 8 \text{ m}^3/\text{s}$ (Ans:)

Wetted Perimeter, $P = T + \frac{8}{3} \cdot \frac{y^2}{T} = 4 + \frac{8}{3} \cdot \frac{(1)^2}{4} = 4.67$ m

Top Width, $T = 4$m

Hydraulic Radius, $R = \frac{A}{P} = \frac{\frac{8}{3}}{\frac{14}{3}} = 0.57$ m

Hydraulic Depth, $D = \frac{A}{T} = \frac{\frac{8}{3}}{4} = 0.67$m

Reynolds Number, $R_e = \frac{VR}{\nu} = \frac{3\times0.57}{10^{-6}} = 1.71\times10^{6} > 12,500$

Froude Number, $F_r = \frac{V}{\sqrt{gD}} = \frac{3}{\sqrt{9.81\times0.67}} = 1.17 > 1.0$

Hence, the flow is Supercritical-Turbulent. (Ans:)

Celerity, $c = \sqrt{gD} = \sqrt{9.81\times0.67} = 2.56$ m/s

Speed of wavefronts upstream $= c - V = (2.56-3.0) = -0.44$ m/s; d/s (Ans:)

Speed of wavefronts downstream $= c + V = (2.56+3) = 5.56$ m/s; d/s (Ans:)

(vi) Given, $y = 1$ m; $V = 3$ m/s; $d_0 = 2.5$ m

Angle, $\theta = 2\cos^{-1}\left(1-\frac{2y}{d_0}\right) = 2\cos^{-1}\left(1-\frac{2\times1}{2.5}\right) = 156.93°$ or $= 2.74$ radian

Area, $A = \frac{1}{8}(\theta - \sin\theta)d_0^2$

$= \frac{1}{8}(2.74 - \sin 2.74)(2.5)^2$ [θ in radian]

$= 1.84\ m^2$

Wetted Perimeter, $P = \frac{1}{2}\theta d_0 = \frac{1}{2}\times2.74\times2.5 = 3.425$ m

Top Width, $T = \left(\sin\frac{\theta}{2}\right)d_0 = \left(\sin\frac{2.74}{2}\right)\times2.5 = 2.45$ m

Hydraulic Radius, $R = \frac{A}{P} = \frac{1.84}{3.425} = 0.54m$

Hydraulic Depth, $D = \frac{A}{T} = \frac{1.84}{2.45} = 0.75\ m$

Reynolds Number, $R_e = \frac{VR}{\nu} = \frac{3 \times 0.54}{10^{-6}} = 1.62 \times 10^6 > 12,500$

Froude Number, $F_r = \frac{V}{\sqrt{gD}} = \frac{3}{\sqrt{9.81 \times 0.75}} = 1.11 > 1.0$

Hence, the flow is Super critical-Turbulent. (Ans:)

Celerity, $c = \sqrt{gD} = \sqrt{9.81 \times 0.75} = 2.71\ m/s$

Speed of wave fronts upstream $= c - V = (2.71 - 3) = -0.29\ m/s$; d/s

(Ans:)

Speed of wave fronts downstream $= c + V = (2.71 + 3) = 5.71\ m/s$; d/s

(Ans:)

Problem 1.3(a): The average depth of water in a wide river connected to sea is 5m. Determine the time taken by a tidal wave to travel from the river mouth to 30km upstream,

(i) when there is no flow in the river, and

(ii) when the average flow velocity in the river is 1 m/s?

Sol^n: Given, Average depth of water, $y = 5m$

For wide river, Hydraulic depth, $D = y = 5m$

Celerity, $c = \sqrt{gD} = \sqrt{9.81 \times 5} = 7.0$ m/s

(i) As there is no flow in the river, $v = 0$ m/s

Speed of wavefronts upstream $= c - V = (7-0) = 7$ m/s ; u/s

Time required, $t = \frac{s}{(c-V)} = \frac{30 \times 10^3 m}{7 m/s} = 4286$ sec (Ans:)

(ii) Average flow velocity in the river, $v = 1$ m/s

Speed of wavefronts upstream $= c - V = (7-1) = 6$ m/s ; u/s

Time required, $t = \frac{s}{(c-V)} = \frac{30 \times 10^3}{6} = 5000$ sec

(Ans:)

Problem 1.3(b): Waves of small amplitude are created at the center of a circular-shaped pond of radius 50m. The waves are found to reach the edge of the pond in 10s. Estimate the approximate volume of water in the pond assuming that the depth of water in the pond is same every where.

Sol^n: Given, $r = 50\,m$; $t = 10s$

Velocity of wave, $c = \frac{s}{t} = \frac{r}{t} = \frac{50}{10} = 5\,m/s$

We know, Celerity, $c = \sqrt{gD}$

$\Rightarrow 5 = \sqrt{9.81 \times D}$

$\therefore D = 2.5484\,m$

Assuming wide channel, Depth of flow, $y = D = 2.5484\,m$

So, Volume of water $= \pi r^2 y$

$= 3.1416 \times (50)^2 \times 2.5484$

$= 20015\,m^3$

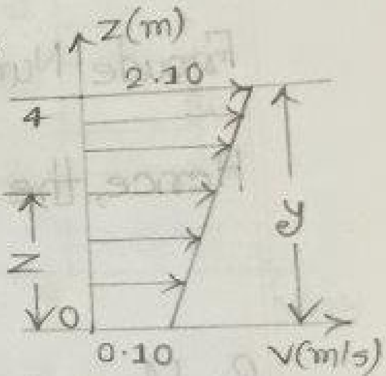
(Ans:)

Problem 1.4(a) : In a wide river the velocity varies linearly along a vertical from 0.10 m/s at the bottom to 2.10 m/s at the surface.

(i) Compute the discharge per unit width, and

(ii) Determine the state of flow, if depth of flow is 4 m.

Sol^n: Velocity at a distance z from bottom,



$$V = 0.10 + \left(\frac{2.10 - 0.10}{4}\right) z$$

$$= 0.10 + 0.5 z$$

(i) Discharge per unit width,

$$q = \int_0^y v\,dz$$

$$= \int_0^4 (0.10 + 0.5z)\,dz$$

$$= \left[0.10z + \frac{0.5z^2}{2}\right]_0^4$$

$$= \left(0.10 \times 4 + \frac{(4)^2}{4}\right)$$

$$= 4.4 \text{ m}^3/\text{s}/\text{m} \quad \text{(Ans:)}$$

(ii) For wide river, Mean velocity, $V = \frac{q}{y} = \frac{4.4}{4} = 1.1 \ m/s$

Hydraulic Radius, $R = y = 4m$

Hydraulic Depth, $D = y = 4m$

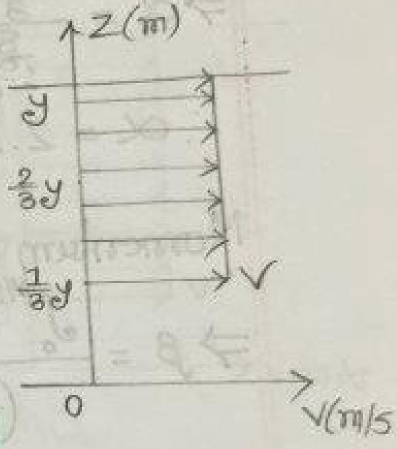
Reynolds Number, $R_e = \frac{VR}{\nu} = \frac{1.1 \times 4}{10^{-6}} = 4.4 \times 10^6 > 12,500$

Froude Number, $F_r = \frac{V}{\sqrt{gD}} = \frac{1.1}{\sqrt{9.81 \times 4}} = 0.18 < 1.0$

Hence, the flow is subcritical-Turbulent. (Ans.)

Problem 1.4 (b) : Same as Example 1.4

Problem 1.5(a): The velocity of flow is zero over one-third of the cross-section of a channel and uniform over the rest of the cross-section. Compute the numerical values of the velocity distribution coefficients α and β.



Soln: Let, depth of flow $= y$

Uniform velocity $= V$

Discharge per unit width,

$$q = \int_0^y V\,dz$$

$$= \int_0^{1/3 y} 0\,dz + \int_{\frac{1}{3}y}^{y} V\,dz \quad [V = \text{constant}]$$

$$= V[z]_{\frac{y}{3}}^{y}$$

$$= V\left[y - \frac{y}{3}\right]$$

$$= \frac{2}{3}Vy$$

Mean velocity, $V_{mean} = \dfrac{q}{y} = \dfrac{\frac{2}{3}Vy}{y} = \dfrac{2}{3}V$

Energy Coefficient, $\alpha = \dfrac{\int_0^y V^3 dz}{V_{mean}^3\, y}$

$$= \frac{\int_0^{\frac{y}{3}} 0\,dz + \int_{\frac{y}{3}}^{y} V^3 dz}{\left(\frac{2}{3}V\right)^3 y}$$

$$\Rightarrow \alpha = \frac{v^3 [z]_{\frac{y}{3}}^{y}}{\frac{8}{27} v^3 y} \quad [\because v = \text{constant}]$$

$$\Rightarrow \alpha = \frac{v^3 [y - \frac{y}{3}]}{\frac{8}{27} v^3 y}$$

$$\Rightarrow \alpha = \frac{\frac{2}{3} v^3 y}{\frac{8}{27} v^3 y}$$

$\therefore \alpha = 2.25$ (Ans:)

Momentum Coefficient, $\beta = \frac{\int_0^y v^2 dz}{V_{mean}^2 y}$

$$\Rightarrow \beta = \frac{\int_0^{y/3} 0 dz + \int_{y/3}^{y} v^2 dz}{(\frac{2}{3} v)^2 y}$$

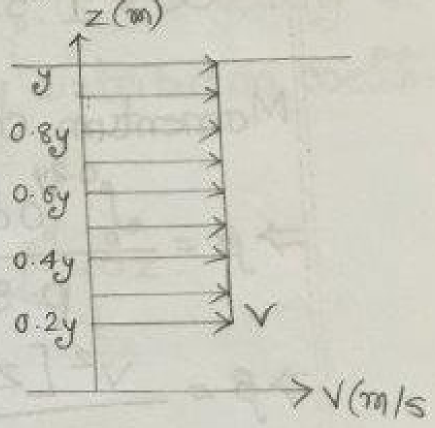
$$\Rightarrow \beta = \frac{v^2 [z]_{y/3}^{y}}{\frac{4}{9} v^2 y}$$

$$\Rightarrow \beta = \frac{v^2 [y - \frac{y}{3}]}{\frac{4}{9} v^2 y}$$

$$\Rightarrow \beta = \frac{\frac{2}{3} v^2 y}{\frac{4}{9} v^2 y}$$

$\therefore \beta = 1.5$ (Ans:)

Problem 1.5(b): The velocity is zero along the lower 20% of a vertical in a wide channel and uniform along the rest of the vertical. Compute the numerical values of the velocity distribution coefficients α and β.



Sol^n: Let, Depth of flow = y

Uniform velocity = V

Discharge per unit width,

$$q = \int_0^y V\,dz$$

$$= \int_0^{0.2y} 0\,dz + \int_{0.2y}^{y} V\,dz$$

$$= V[y - 0.2y]$$

$$= 0.8\,Vy$$

Mean velocity, $\bar{V} = \frac{q}{y} = \frac{0.8Vy}{y} = 0.8V$

Energy coefficient, $\alpha = \frac{\int_0^y V^3 dz}{\bar{V}^3 y}$

$$\Rightarrow \alpha = \frac{\int_0^{0.2y} 0^3 dz + \int_{0.2y}^{y} V^3 dz}{(0.8V)^3 y}$$

$$\Rightarrow \alpha = \frac{V^3 [z]_{0.2y}^{y}}{0.512\,V^3 y}$$

$$\Rightarrow \alpha = \frac{v^3 [y - 0.2y]}{0.512 v^3 y}$$

$$\Rightarrow \alpha = \frac{v^3 \times 0.8y}{0.512 v^3 y}$$

$$\therefore \alpha = 1.5625 \text{ (Ans:)}$$

Momentum coefficient, $\beta = \frac{\int_0^y v^2 dz}{\bar{V}^2 y}$

$$\Rightarrow \beta = \frac{\int_0^{0.2y} 0 dz + \int_{0.2y}^{y} v^2 dz}{(0.8v)^2 y}$$

$$\Rightarrow \beta = \frac{v^2 [z]_{0.2y}^{y}}{0.64 v^2 y}$$

$$\Rightarrow \beta = \frac{v^2 [y - 0.2y]}{0.64 v^2 y}$$

$$\Rightarrow \beta = \frac{v^2 \times 0.8y}{0.64 v^2 y}$$

$$\therefore \beta = 1.25 \text{ (Ans:)}$$

Problem 1.6(a): For laminar flow the velocity distribution along a vertical can be approximated by, $v = v_0 \sin\frac{\pi z}{2y}$, where, v is the velocity at a distance z from the channel bottom, y is the depth of flow and v_0 is the velocity at the free surface. Compute the velocity distribution coefficients α and β and the ratio $\frac{\alpha - 1}{\beta - 1}$.

Sol^n: Discharge per unit width, $q = \int_0^y v\,dz$

$$\Rightarrow q = \int_0^y v_0 \sin\frac{\pi z}{2y}\,dz$$

$$\Rightarrow q = \frac{2y}{\pi} \cdot v_0 \left[-\cos\frac{\pi z}{2y}\right]_0^y$$

$$\Rightarrow q = \frac{2y v_0}{\pi}\left[-\cos\frac{\pi}{2} + \cos 0\right]$$

$$\therefore q = \frac{2y v_0}{\pi}$$

Mean velocity, $\bar{v} = \frac{q}{y} = \frac{\frac{2y v_0}{\pi}}{y} = \frac{2 v_0}{\pi}$

Energy coefficient, $\alpha = \frac{\int_0^y v^3\,dz}{\bar{v}^3 y}$

$$\Rightarrow \alpha = \frac{\int_0^y v_0^3 \sin^3\left(\frac{\pi z}{2y}\right) dz}{\left(\frac{2 v_0}{\pi}\right)^3 y}$$

$4\sin^3 x = 3\sin x - \sin 3x$
$2\sin^2 x = 1 - \cos 2x$ } Formula

$$\Rightarrow \alpha = \frac{\pi^3}{8y} \times \frac{1}{4}\int_0^y \left[3\sin\left(\frac{\pi z}{2y}\right) - \sin\left(\frac{3\pi z}{2y}\right)\right] dz$$

$$\Rightarrow \alpha = \frac{\pi^3}{32y}\left[3\cdot\frac{2y}{\pi}\cos\left(\frac{\pi z}{2y}\right) + \frac{2y}{3\pi}\cos\left(\frac{3\pi z}{2y}\right)\right]_0^y$$

$$\Rightarrow \alpha = \frac{\pi^3}{32y}\left[-\frac{6y}{\pi}\cos\left(\frac{\pi}{2}\right) + \frac{2y}{3\pi}\cos\left(\frac{3\pi}{2}\right) + \frac{6y}{\pi}\cos(0) - \frac{2y}{3\pi}\cos(0)\right]$$

$$\Rightarrow \alpha = \frac{\pi^3}{32y} \times \left[\frac{6y}{\pi} - \frac{2y}{3\pi}\right]$$

$$\therefore \alpha = \frac{\pi^2}{6} = 1.645 \text{ (Ans:)}$$

Momentum coefficient, $\beta = \dfrac{\int_0^y v^2 dz}{\bar{V}^2 y}$

$$\Rightarrow \beta = \frac{\int_0^y v_0^2 \sin^2\left(\frac{\pi z}{2y}\right) dz}{\left(\frac{2v_0}{\pi}\right)^2 y}$$

$$\Rightarrow \beta = \frac{\pi^2}{4y} \times \frac{1}{2}\int_0^y \left[1 - \cos\left(\frac{2\pi z}{2y}\right)\right] dz$$

$$\Rightarrow \beta = \frac{\pi^2}{8y}\left[z - \frac{y}{\pi}\sin\left(\frac{\pi z}{y}\right)\right]_0^y$$

$$\Rightarrow \beta = \frac{\pi^2}{8y}\left[y - \frac{y}{\pi}\sin(\pi) - 0 + \frac{y}{\pi}\sin(0)\right]$$

$$\Rightarrow \beta = \frac{\pi^2}{8y}\left[y - 0 - 0 + 0\right]$$

$\therefore \beta = \frac{\pi^2}{8} = 1.234$ (Ans.)

The ratio, $\frac{\alpha - 1}{\beta - 1} = \frac{1.645 - 1}{1.234 - 1} = 2.76$ (Ans.)

Problem 1.6 (b): For turbulent flow, the velocity distribution along a vertical can be approximated by $v \propto z^n$, when $n = \frac{1}{7}$ (Prandtl's one-seventh power law). Determine the velocity distribution coefficients α and β and the ratio $\frac{\alpha - 1}{\beta - 1}$ in terms of n and for $n = \frac{1}{7}$. Compare the numerical values of α and β with those obtained for laminar flow in Problem 1.6(a).

Sol$^{\underline{n}}$: Given, $v \propto z^n$

$\therefore v = v_0 z^n$; where v_0 is the velocity at the free surface

Discharge per unit width, $q = \int_0^y v\,dz$

$$\Rightarrow q = \int_0^y v_0 z^n dz$$

$$\Rightarrow q = \left[\frac{v_0 z^{n+1}}{n+1}\right]_0^y$$

$$\Rightarrow q = \frac{v_0 y^{n+1}}{n+1}$$

Mean velocity, $\bar{V} = \frac{q}{y} = \frac{V_0 y^{n+1}}{(n+1)y} = \frac{V_0 y^n}{n+1}$

Energy coefficient, $\alpha = \frac{\int_0^y v^3 dz}{\bar{V}^3 y}$

$$\Rightarrow \alpha = \frac{\int_0^y V_0^3 z^{3n} dz}{\frac{V_0^3 y^{3n}}{(n+1)^3} \cdot y}$$

$$\Rightarrow \alpha = \frac{(n+1)^3}{y^{3n+1}} \left[\frac{z^{3n+1}}{3n+1}\right]_0^y$$

$$\Rightarrow \alpha = \frac{(n+1)^3}{y^{3n+1}} \times \frac{y^{3n+1}}{(3n+1)}$$

$\therefore \alpha = \frac{(n+1)^3}{3n+1}$ for $n = \frac{1}{7}$; $\alpha = 1.045$ (Ans.)

(Ans.)

Momentum coefficient, $\beta = \frac{\int_0^y v^2 dz}{\bar{V}^2 y}$

$$\Rightarrow \beta = \frac{\int_0^y V_0^2 z^{2n} dz}{\frac{V_0^2 y^{2n}}{(n+1)^2} \cdot y}$$

$$\Rightarrow \beta = \frac{(n+1)^2}{y^{2n+1}} \left[\frac{z^{2n+1}}{2n+1}\right]_0^y$$

$$\Rightarrow \beta = \frac{(n+1)^2}{y^{2n+1}} \times \frac{y^{2n+1}}{(2n+1)}$$

$\therefore \beta = \frac{(n+1)^2}{2n+1}$ (Ans:) for $n = \frac{1}{7}$; $\beta = 1.016$ (Ans:)

The ratio, $\frac{\alpha - 1}{\beta - 1} = \frac{1.045 - 1}{1.016 - 1} = 2.8125$ (Ans:)

The numerical values of α & β has become less as the flow turns into turbulent from laminar.

Problem 1.7 (b) (iv) : Compute the values of the velocity distribution coefficients α and β and the ratio $\frac{\alpha - 1}{\beta - 1}$ for the following velocity distribution along a vertical in a wide channel when the depth of flow in the channel is 10 m.

$$v = 1 + 2\left(\frac{z}{y}\right)^{1/2}$$

Sol^n: Mean velocity, $\bar{v} = \frac{1}{A}\int_0^A v\,dA = \frac{1}{y}\int_0^y v\,dz$

$$\Rightarrow \bar{v} = \frac{1}{y}\int_0^y \left[1 + 2\left(\frac{z}{y}\right)^{1/2}\right] dz$$

$$\Rightarrow \bar{v} = \frac{1}{y}\left[z + \frac{2}{\sqrt{y}} \cdot \frac{z^{3/2}}{\frac{3}{2}}\right]_0^y$$

$$\Rightarrow \bar{v} = \frac{1}{y}\left[y + \frac{4}{3\sqrt{y}} \cdot y^{3/2}\right]$$

$$\Rightarrow \bar{v} = \left(1 + \frac{4}{3}\right) = \text{[illegible]} \text{ m/s}$$

Energy coefficient, $\alpha = \dfrac{\int_0^y v^3 dz}{\bar{V}^3 y}$

$$\Rightarrow \alpha = \frac{\int_0^y \left[1+2\left(\frac{z}{y}\right)^{1/2}\right]^3 dz}{\left(\frac{7}{3}\right)^3 y}$$

$$\Rightarrow \alpha = \frac{\int_0^y \left[1+3\cdot(1)^2\cdot 2\left(\frac{z}{y}\right)^{1/2} + 3.1.4\left(\frac{z}{y}\right) + 8\left(\frac{z}{y}\right)^{3/2}\right] dz}{\left(\frac{7}{3}\right)^3 y}$$

$$\Rightarrow \alpha = \frac{\left[z + \frac{6}{\sqrt{y}}\cdot\frac{z^{3/2}}{\frac{3}{2}} + \frac{12 z^2}{2y} + \frac{8}{y^{3/2}}\cdot\frac{z^{5/2}}{5/2}\right]_0^y}{\left(\frac{7}{3}\right)^3 y}$$

$$\Rightarrow \alpha = \frac{y + \frac{4}{\sqrt{y}}\cdot y^{3/2} + \frac{6y^2}{y} + \frac{16 y^{5/2}}{5y^{3/2}}}{\left(\frac{7}{3}\right)^3 y}$$

$$\Rightarrow \alpha = \frac{1+4+6+\frac{16}{5}}{\left(\frac{7}{3}\right)^3}$$

$\therefore \alpha = 1.118$ (Ans:)

Momentum coefficient, $\beta = \dfrac{\int_0^y v^2 dz}{V^2 y}$

$$\Rightarrow \beta = \frac{\int_0^y \left[1+2\left(\frac{z}{y}\right)^{1/2}\right]^2 dz}{\left(\frac{7}{3}\right)^2 y}$$

$$\Rightarrow \beta = \frac{\int_0^y \left[1+2.1.2\left(\frac{z}{y}\right)^{1/2} + 4\left(\frac{z}{y}\right)\right] dz}{\left(\frac{7}{3}\right)^2 y}$$

$$\Rightarrow \beta = \frac{\left[z + \frac{4}{\sqrt{y}} \cdot \frac{z^{3/2}}{3/2} + \frac{4}{y} \cdot \frac{z^2}{2}\right]_0^y}{\left(\frac{7}{3}\right)^2 y}$$

$$\Rightarrow \beta = \frac{y + \frac{8}{3\sqrt{y}} \cdot y^{3/2} + \frac{2y^2}{y}}{\left(\frac{7}{3}\right)^2 y}$$

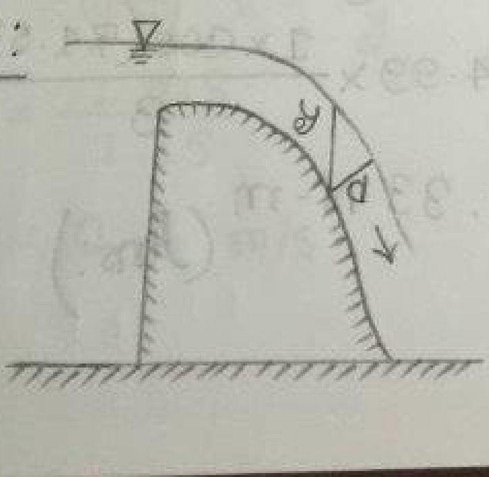
$$\Rightarrow \beta = \frac{1 + \frac{8}{3} + 2}{\left(\frac{7}{3}\right)^2}$$

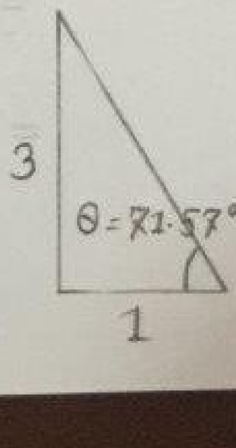
$$\therefore \beta = 1.041 \text{ (Ans:)}$$

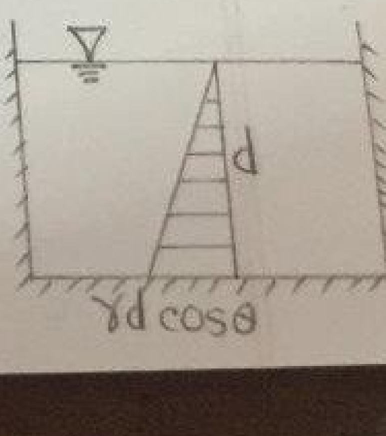
The ratio, $\frac{\alpha - 1}{\beta - 1} = \frac{1.118 - 1}{1.041 - 1} = 2.878$ (Ans:)

Problem 1.9(a): A steep rectangular chute has a slope of 1 H : 3 V. Compute the pressure at the bed of the chute if the vertical depth of water flowing over the chute is 1m. Also, compute the force and the overturning moment on its side walls.

Sol^n:







Bottom slope of channel, $\theta = \tan^{-1}\left(\frac{3}{1}\right) = 71.57°$

Vertical depth of water, $y = 1\,m$

Pressure at the bed, $P = \gamma d \cos\theta$

$\Rightarrow P = \gamma y \cos\theta \cdot \cos\theta$ $\quad [\because d = y\cos\theta]$

$\Rightarrow P = (1000 \times 9.81) \times 1 \times \cos^2(71.57)$

$\therefore P = 980.49\ N/m^2$ (Ans:)

Force on side walls, $F = \frac{1}{2} \times \gamma d \cos\theta \times d$

$\Rightarrow F = \frac{1}{2} \gamma y^2 \cos^2\theta \cdot \cos\theta$

$\Rightarrow F = \frac{1}{2} \times (1000 \times 9.81) \times (1)^2 \times \cos^3(71.57)$

$\therefore F = 154.99\ N$ (Ans:)

Overturning moment = F × Arm [from bottom of channel]

$$= F \times \frac{d}{3}$$

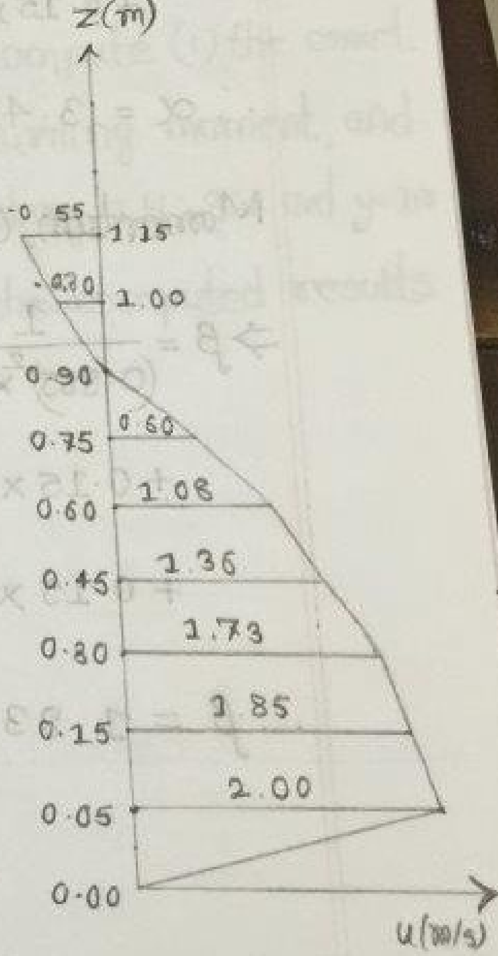
$$= F \times \frac{y\cos\theta}{3}$$

$$= 154.99 \times \frac{1 \times \cos(71.57)}{3}$$

$$= 16.33\ N\text{-}m$$

(Ans:)

Problem 1.8(b): Figure shows the velocity distribution down-stream of a sluice gate under submerged condition. Using the trapezoidal rule of numerical integration, compute the discharge per unit width, the mean velocity of flow and the numerical values of α and β.



Sol^n: Discharge per unit width,

$$q = \int_0^y v\,dz = \Sigma u \Delta z$$

$$\Rightarrow q = 0.05\times\left(\frac{0+2}{2}\right) + 0.10\times\left(\frac{2+1.85}{2}\right)$$

$$+ 0.15\times\left(\frac{1.85}{2}+1.73+1.36+1.08+\frac{0.60}{2}\right)$$

$$+ 0.15\times\left(\frac{0.60+0}{2}\right) + 0.10\times\left(\frac{0-0.30}{2}\right)$$

$$+ 0.15\times\left(\frac{-0.30-0.55}{2}\right)$$

$$\therefore q = 1.018\ m^3/s/m \quad \text{(Ans:)}$$

Here, depth of flow, $y = 1.15\ m$

Mean Velocity, $\bar{V} = \frac{q}{y}$

$$\Rightarrow \bar{V} = \frac{1.018}{1.15}$$

$$\therefore \bar{V} = 0.8\ m/s$$

Energy coefficient, $\alpha = \frac{\int_0^y v^3 dz}{\bar{V}^3 y} = \frac{\sum v^3 \Delta z}{\bar{V}^3 y}$

$$\Rightarrow \alpha = \frac{1}{(0.885)^3 \times 1.15} \times \Big[0.05 \times \frac{(0)^3 + (2)^3}{2} + 0.10 \times \frac{(2)^3 + (1.85)^3}{2}$$

$$+ 0.15 \times \left\{ \frac{(1.85)^3}{2} + (1.73)^3 + (1.36)^3 + (1.08)^3 + \frac{(0.60)^3}{2} \right\}$$

$$+ 0.15 \times \frac{(0.60)^3 + (0)^3}{2} + 0.10 \times \frac{(0)^3 + (-0.30)^3}{2} + 0.15 \times \frac{(-0.30)^3 + (-0.55)^3}{2} \Big]$$

$\therefore \alpha = 3.451$ (Ans:)

Momentum Coefficient, $\beta = \frac{\int_0^y v^2 dz}{\bar{V}^2 y} = \frac{\sum v^2 \Delta z}{\bar{V}^2 y}$

$$\Rightarrow \beta = \frac{1}{(0.885)^2 \times 1.15} \times \Big[0.05 \times \frac{(0)^2 + (2)^2}{2} + 0.10 \times \frac{(2)^2 + (1.85)^2}{2}$$

$$+ 0.15 \times \left\{ \frac{(1.85)^2}{2} + (1.73)^2 + (1.36)^2 + (1.08)^2 + \frac{(0.60)^2}{2} \right\}$$

$$+ 0.15 \times \frac{(0.60)^2 + (0)^2}{2} + 0.10 \times \frac{(0)^2 + (-0.30)^2}{2} + 0.15 \times \frac{(-0.30)^2 + (-0.55)^2}{2}$$

$\therefore \beta = 1.831$ (Ans:)

Problem 1.9 (b): While computing the shear force and the overturning moment on the side walls of a steep spillway chute, an engineer assumed that the water pressure varies linearly from zero at the free surface to $\rho g y$ at the bed of the chute, where y is the depth measured vertically. Are the computed results correct? If not compute (i) the correct values of the shear force and the overturning moment, and (ii) the % error. The chute has an inclination 1H : 3V and $y = 1m$.

Sol^n: Considering hydrostatic pressure, the computed results will not correct. (Ans:)

Using hydrostatic law of pressure,

Shear Force, $F = \frac{1}{2} \times \rho g y \times y$

$$= \frac{1}{2} \times 1000 \times 9.81 \times 1 \times 1$$

$$= 4905 \ N/m^2$$

Overturning moment $= F \times Arm$

$$= F \times \frac{y}{3}$$

$$= 4905 \times \frac{1}{3}$$

$$= 1635 \ N\text{-}m$$

Angle, $\theta = \tan^{-1}\left(\frac{3}{1}\right) = 71.5651^\circ$

(ii) The correct values,

Shear Force, $F = \frac{1}{2} \times \gamma d\cos\theta \times d$

$= \frac{1}{2}\gamma d^2 \cos\theta$

$= \frac{1}{2}\gamma y^2 \cos^3\theta$ $\quad [\because d = y\cos\theta]$

$= \frac{1}{2} \times (1000 \times 9.81) \times (1)^2 \times \cos^3(71.5651)$

$= 155.11\ N/m^2$ (Ans:)

Overturning moment, $= F \times Arm$

$= F \times \frac{d}{3}$

$= F \times \frac{y\cos\theta}{3}$

$= 155.11 \times \frac{1 \times \cos(71.5651)}{3}$

$= 16.35\ N\text{-}m$ (Ans:)

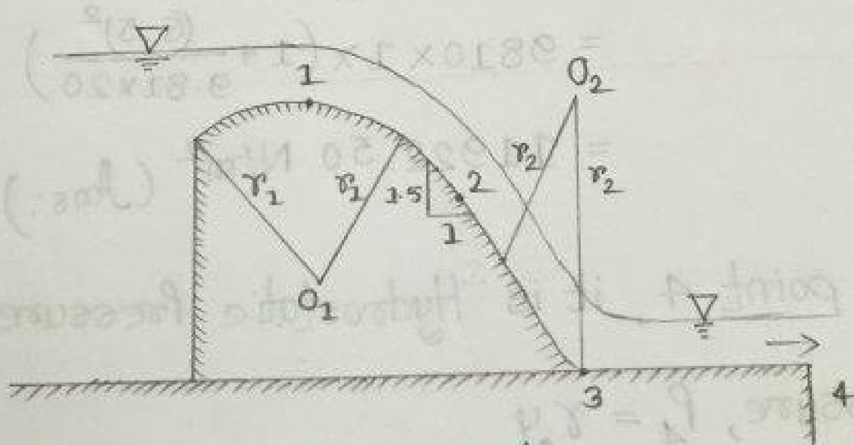
(ii) % Error in Shear Force $= \frac{(4905 - 155.11)}{155.11} \times 100\%$

$= 3063\%$ (Ans:)

% Error in Overturning moment $= \frac{(1635 - 16.35)}{16.35} \times 100\%$

$= 9900\%$ (Ans:)

Problem 1.9(c): A high-head overflow spillway is shown in figure. The flip bucket at the toe of the spillway acts to change the direction of flow from the slope of the spillway face to the horizontal and to discharge the flow into the air. If $r_1 = r_2 = 20\,m$, $y_1 = y_2 = y_3 = y_4 = 1\,m$ and the discharge over the spillway is $6.5\ m^3/s/m$, determine the intensities of pressure at points 1, 2, 3, 4.



Sol^n: At point 1, it is Convex Flow,

Pressure, $P_1 = \gamma y \left(1 - \frac{V^2}{g r}\right)$

$= (1000 \times 9.81) \times 1 \times \left(1 - \frac{(6.5)^2}{9.81 \times 20}\right)$

$= 7697.5\ N/m^2$ (Ans:)

Given,

Discharge, $q = 6.5\ m^3/s/m$

Depth of flow, $y = 1\,m$

Velocity $= \frac{q}{y} = \frac{6.5}{1}$

$= 6.5\ m/s$

Angle, $\theta = \tan^{-1}\left(\frac{1.5}{1}\right)$

$= 56.31$

$r = 20\,m$

At point 2, it is Sloping Flow,

Pressure, $P_2 = \gamma d\cos\theta = \gamma y.\cos^2\theta$ $\left[\because y = \frac{d}{\cos\theta}\right]$

$= 9810 \times (1) \times \cos^2(56.31)$

$= 3018.45\ N/m^2$ (Ans:)

At point 3, it is Concave Flow,

Pressure, $P_3 = \gamma y\left(1 + \frac{V^2}{gr}\right)$

$= 9810 \times 1 \times \left(1 + \frac{(6.5)^2}{9.81 \times 20}\right)$

$= 11922.50\ N/m^2$ (Ans:)

At point 4, it is Hydrostatic Pressure Distribution.

Pressure, $P_4 = \gamma y$

$= 9810 \times 1$

$= 9810\ N/m^2$ (Ans:)