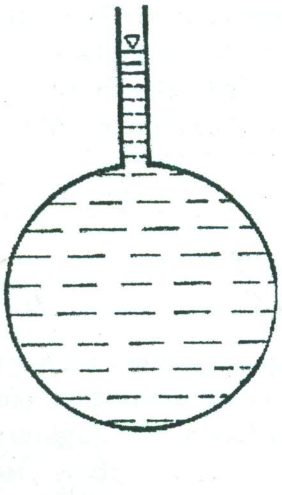
786

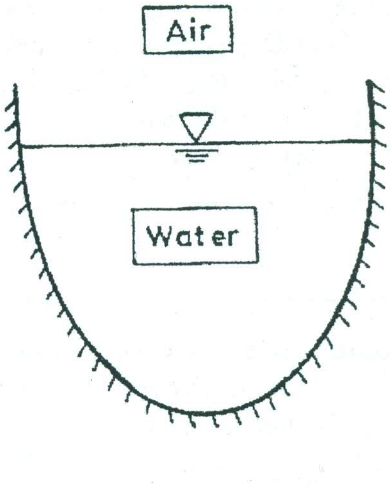
Lecture note on

OPEN CHANNEL FLOW

(for undergraduate students)

MD. ABDUL HALIM





DEPARTMENT OF WATER RESOURCES ENGINEERING
BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY

July 2008

Lecture note on

OPEN CHANNEL FLOW

(for undergraduate students)

MD. ABDUL HALIM
Professor
Department of Water Resources Engineering
Bangladesh University of Engineering and Technology
Dhaka - 1000

July 2008

This lecture note has been compiled for the third year students of the Department of Civil Engineering and the Department of Water Resources Engineering, Bangladesh University of Engineering and Technology (BUET), Dhaka. There is a 4-credit theory course (WRE 301 Open Channel Flow) for them in a term of 14 weeks duration. The contents of this course are given on the back cover page.

The use of materials from various books and technical publications under reference is gratefully acknowledged. During the compilation of this lecture note, the typing, drafting, photocopying and other facilities of the Department of Water Resources Engineering, BUET, have been used. The assistance provided by the numerous individuals is also gratefully acknowledged.

CONTENTS

LIST OF SYMBOLS

The main symbols used in this lecture note are included in the following list. When other symbols have been used, they are defined in the text.

Symbol	Meaning	Unit(s)
A	cross-sectional area of flow	m^2
b	bottom width	m
B	top or free surface width	m
c	celerity of wave	m/s
C	Chezy coefficient	$m^{1/2}/s$
C_c	coefficient of contraction	-
C_d	coefficient of discharge	-
d	depth of flow section	m
	grain diameter	m
d_0	diameter of circular section	m
d_{50}, d_{75}	grain diameters	m
D	hydraulic depth	m
E	specific energy	m
f	Darcy-Weisbach friction factor	-
f_s	Lacey silt factor	-
F	specific force	m^3
F_b	freeboard	m
F_f	friction or resistance force	N
F_g	gravity force	N
F_p	pressure force	N
Fr	Froude number	-
g	acceleration due to gravity	m/s^2
h	vertical depth	m
h_c	critical depth	m
h_e	eddy loss	m
h_f	frictional head loss	m
h_L	total head loss	m
h_n	normal depth	m
h_t	tailwater depth	m
H	total energy or head	m
k_s	Nikuradse equivalent sand grain roughness	m
K	conveyance	m^3/s
L	channel length	m
	length of flow profile	m
	length of weir	m
L_j	length of hydraulic jump	m
M	hydraulic exponent for critical flow computation	-
n	Manning roughness coefficient	$s/m^{1/3}$
N	hydraulic exponent for uniform flow computation	-
p	intensity of pressure	N/m^2
P	wetted perimeter	m

Symbol	Meaning	Unit(s)
q	discharge per unit width	$m^3/s/m$
q_*	transverse or lateral inflow	$m^3/s/m$
Q	discharge	m^3/s
R	hydraulic radius	m
Re	Reynolds number	-
s	side slope (sH:1V)	-
s_s	specific gravity of sediment	-
S	submergence factor	-
S_c	critical slope	-
S_f	friction slope	-
S_n	normal slope	-
S_w	water surface slope	-
S_0	bottom slope	-
t	time	s
u	point velocity in the x-direction	m/s
u_0	velocity at free surface	m/s
u_*	shear or friction velocity	m/s
U	cross-sectional mean velocity	m/s
$\overline{U}$	depth-averaged velocity	m/s
W	weight	N
x	longitudinal coordinate	m
z	vertical coordinate	m
$\overline{z}$	centroidal distance from the free surface	m
z_b	datum or elevation head	m
z_0	roughness height	m
z_w	stage	m
Z	section factor for critical flow	$m^{5/2}$
α	energy coefficient	-
β	momentum coefficient	-
γ	specific weight	N/m^3
δ_v	thickness of viscous or laminar sublayer	m
ω	angle subtended by top width at centre of a circular section	deg or rad
ϕ	side slope angle	deg
ψ	angle of repose	deg
θ	bottom slope angle	deg
κ	von Karman constant	-
μ	dynamic viscosity	$N.s/m^2$
ν	kinematic viscosity	m^2/s
ρ	mass density of fluid	kg/m^3
ρ_w	mass density of water	kg/m^3
ρ_s	mass density of sediment	kg/m^3
τ	shear stress	N/m^2
τ_c	critical shear stress	N/m^2
τ_o	average shear stress	N/m^2

Chapter 1

BASIC CONCEPTS OF OPEN CHANNEL FLOW

1.1 INTRODUCTION

The flow of water in a conduit may be either open channel flow or pipe flow. In pipe flow the flowing water is completely enclosed by solid boundary and flow occurs under pressure. In open channel flow (Fig.1.1) the flowing water is not completely enclosed by solid boundary and flow occurs with a free surface. A free surface is subjected to atmospheric pressure. Therefore, open channel flow may be defined as the flow of water in a conduit with a free surface. Open channel flow is also known as the *free surface flow*. *

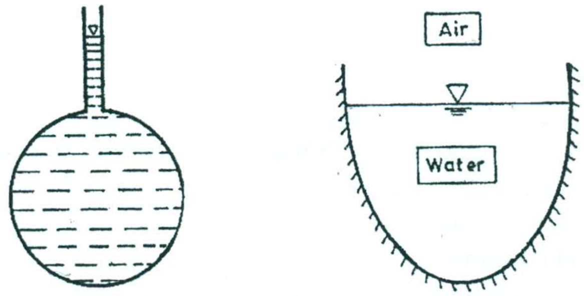


Fig. 1.1 Pipe flow and open channel flow

Flows in rivers and canals are some of the many familiar examples of open channel flow. Flow of water in a closed conduit, e.g. in an underground sewer or in a culvert, may be open channel flow if the flow occurs with a free surface (Fig.1.2). The flow of groundwater with a free surface is also an example of open channel flow.

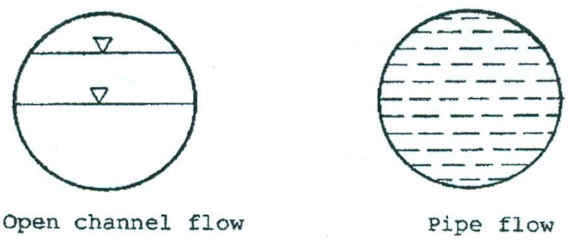


Fig.1.2 Flow in an underground sewer

Open channel flow occurs under the action of gravity and at atmospheric pressure. * Basically all open channels have a bottom slope and flow occurs downstream along the slope. The component of the gravity force or the weight of water along the slope acts as the driving force. Obviously, for open channel flow to occur, the total energy at an upstream section must be greater than the total energy at a downstream section. *

In this lecture note, unless otherwise stated, we will follow the SI systems of units. We will also follow a Cartesian coordinate system in which the x-axis is along the channel bottom, the z-axis is vertically upward and the y-axis is the lateral direction (Fig.1.3). The mean direction of flow is taken to be parallel to the channel bottom and along the x-axis.

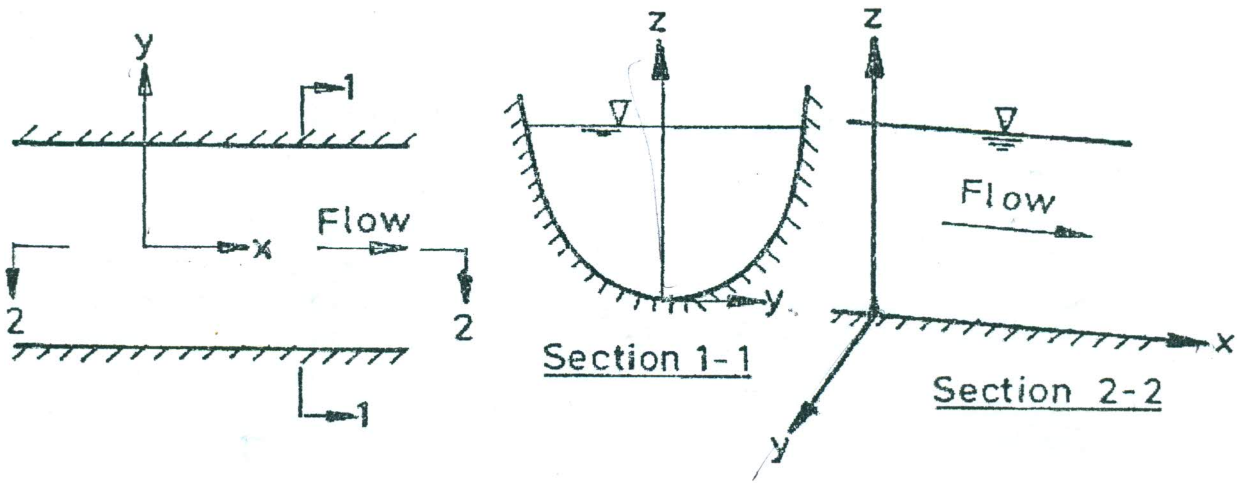


Fig.1.3 The coordinate system

1.2 KINDS OF OPEN CHANNEL

An open channel is a conduit in which water flows with a free surface. Open channels are classified on different criteria as follows.

(a) Natural and Artificial Channels

Natural open channels include all channels that exist naturally on the earth, e.g. rivers and tidal estuaries. They are generally very irregular in shape.

Artificial open channels are the channels developed by men, e.g. irrigation canals, laboratory flumes, spillway chutes, drops, culverts, roadside gutters etc. They are usually designed with regular geometric shapes.

(b) Prismatic and Non-prismatic Channels

A channel with unvarying cross-section and constant bottom slope is called a prismatic channel; otherwise it is non-prismatic. The artificial channels are usually prismatic and the natural channels are generally non-prismatic.

(c) Rigid and Mobile Boundary Channels

A channel with immovable bed and sides is known as a rigid boundary channel, e.g. lined canals, sewers and non-erodible unlined canals. If the channel boundary is composed of loose sedimentary particles moving under the action of flowing water, the channel is called a mobile boundary channel. An alluvial channel is a mobile boundary channel transporting the same type of material as that comprising the channel perimeter.

(d) Small and Large Slope Channels

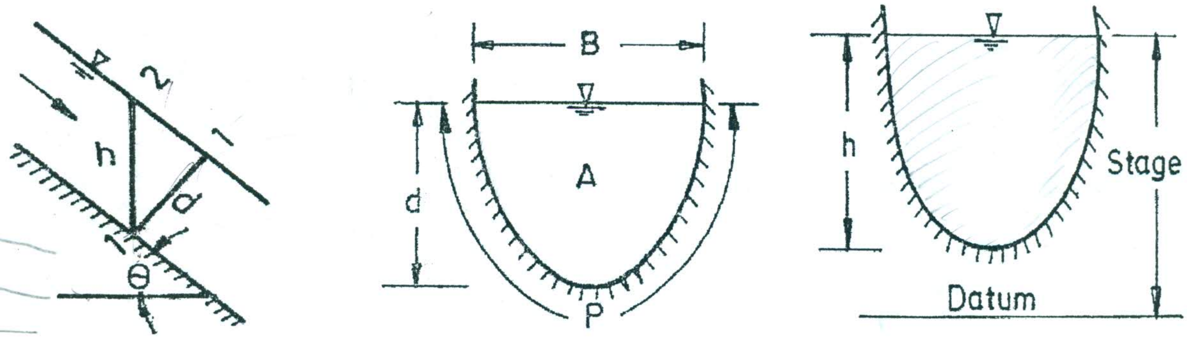
An open channel having a bottom slope greater than 1 in 10 is called a channel of large slope; otherwise it is a channel of small slope (Chow, 1959). The slopes of ordinary channels, natural or artificial, are far less than 1 in 10. However, some artificial channels like drops and chutes have slopes far more than 1 in 10.

1.3 CHANNEL GEOMETRY AND SECTION ELEMENTS

(a) Prismatic Channels

The rectangle, trapezoid, triangle, parabola and circle are the most commonly used shapes of prismatic or regular or uniform channels. Table 1.1 furnishes the formulas for the cross-sectional area A, the wetted perimeter P and the top width B for these channel shapes.

The cross-section of a channel taken normal to the direction of flow is called a *channel section* (Fig. 1.4). A *vertical channel section* is the vertical section passing through the bottom or lowest point of a channel section.



Channel section and vertical channel section

Channel section (section 1-1)

Vertical channel section (section 1-2)

Fig.1.4 Channel section and vertical channel section

The properties of a channel section which are determined by the geometric shape of the channel and the depth of flow are known as the *geometric elements of a channel section.* They are defined below.

Depth of flow h and depth of flow section d: The depth of flow is the vertical distance from the lowest point of a channel section to the water surface. The depth of flow section is the depth of flow normal to the direction of flow. The relation between h and d is

$$d = h\cos\theta \qquad (1.1)$$

where θ is the angle made by the channel bottom with the horizontal.

Stage: The stage is the elevation of the water surface relative to a horizontal datum and may be positive or negative.

Flow area A: The flow area is the cross-sectional area of the flow normal to the direction of flow.

Wetted perimeter P: The wetted perimeter is the length of the interface between water and channel boundary.

Top width B: The top width is the width of a channel section at the water surface.

Hydraulic radius R: The hydraulic radius is the ratio of the flow area to the wetted perimeter, i. e.

$$R = A/P \qquad (1.2)$$

Hydraulic depth D: The hydraulic depth is the ratio of the flow area to the top width, i.e.

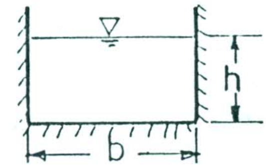
$$D = A/B \qquad (1.3)$$

Obviously, the geometric elements A, P, R etc. of an open channel flow section depend on the depth of flow (Table 1.1). As a result, the solution of an open channel flow problem usually becomes difficult when the depth of flow is unknown and may require a trial-and-error procedure.

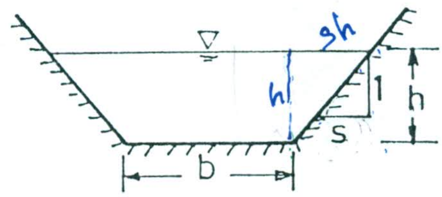
Table 1.1 Geometric elements of some channel sections

1. Rectangle

$$A = bh$$
$$P = b + 2h$$
$$B = b$$



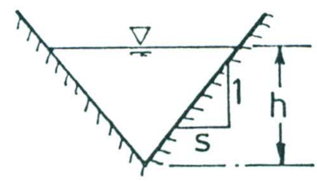
2. Trapezoid



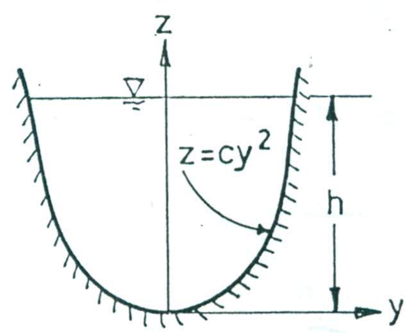
$$A = (b + sh)h$$
$$P = b + 2\sqrt{1+s^2}\,h$$
$$B = b + 2sh$$

3. Triangle

$$A = sh^2$$
$$P = 2\sqrt{1+s^2}\,h$$
$$B = 2sh$$



4. Parabola (perimeter equation $z = cy^2$)



$$A = \frac{2}{3}Bh = \frac{4h^{3/2}}{3\sqrt{c}}$$

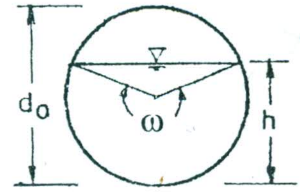
$$P = \frac{B}{2}\left[\sqrt{1+\left(\frac{4h}{B}\right)^2} + \frac{B}{4h}\ln\left(\frac{4h}{B} + \sqrt{1+\left(\frac{4h}{B}\right)^2}\right)\right]$$

$$\approx B + \frac{8h^2}{3B}\left(when\ \ 0 < 4h/B \leq 1\right)$$

$$B = \frac{3A}{2h} = \frac{2h^{1/2}}{\sqrt{c}}$$

5. Circle

$$h = d_0[1-\cos(\omega/2)]/2$$
$$\omega = 2\cos^{-1}(1-2h/d_0)$$
$$A = (\omega - \sin\omega)d_0^{\,2}/8$$
$$P = \omega d_0/2$$
$$B = d_0\sin(\omega/2)$$



Wide Channel

When the width of a rectangular channel is very large compared to the depth, i.e. $b >> h$ (generally if $b \geq 10h$), the sides of the channel have practically no influence on the velocity distribution in the central region. Such a channel is known as a wide channel (Chow, 1959). Many rivers in alluvial plains are treated as a wide channel. For a wide channel

$$R = \frac{A}{P} = \frac{bh}{b+2h} \approx \frac{bh}{b} \approx h \tag{1.4}$$

and the discharge is usually expressed per unit width of the channel which is designated by q (= Q/b, where Q is the total discharge and b is the bottom width) and the units of q are $m^3/s/m$ or simply m^2/s.

(b) Non-prismatic Channels

Non-prismatic or irregular or non-uniform channels like rivers are often encountered in practice and in such cases it is necessary to compute area, top width, wetted perimeter etc. of the channel section. The stream-gauging procedure used by the U.S. Geological Survey involves dividing the total cross-sectional area into (N-1) pockets or segments or vertical strips by N number of successive verticals as shown in Fig. 1.5. The depth of flow at each vertical is measured and is taken to be the depth of the associated segment. The segment cross-sectional area extends laterally to half the distance between the preceding and the following verticals according to the *mid-section rule*. The total cross-sectional area is computed as

$$A = \sum_{i=1}^{N-1} \Delta A_i \tag{1.5}$$

where ΔA_i = area of the ith segment
= depth at the ith segment × (1/2 width to the left + 1/2 width to the right)
= $0.5h_i (W_i + W_{i+1})$

The top width is computed as

$$B = \sum_{i=1}^{N} W_i \tag{1.6}$$

where W_i is the width between the two adjacent verticals.

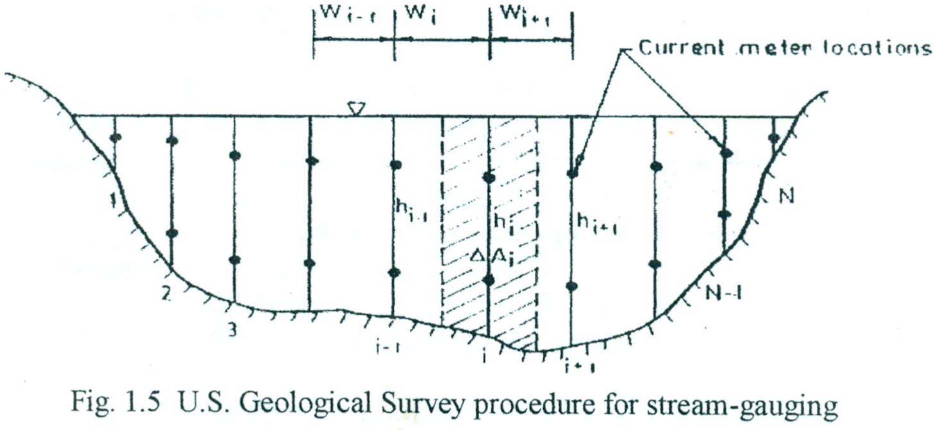


Fig. 1.5 U.S. Geological Survey procedure for stream-gauging

The wetted perimeter P is calculated by the expression

$$P = \sum_{i=2}^{N} \Delta P_i \tag{1.7}$$

where

$$\Delta P_i = \sqrt{W_i^2 + (h_i - h_{i-1})^2} \tag{1.8}$$

and it is assumed that the river bed is linear between the two adjoining verticals.

09

1.4 TYPES OF OPEN CHANNEL FLOW with figure.

Open channel flow is classified on the basis of different criteria as follows.

(a) Steady and Unsteady Flows (time is the criterion)

Flow in an open channel is said to be steady if the depth, mean velocity and discharge at a channel section do not change with time. The flow is unsteady if these quantities at a channel section change with time. Mathematically, for steady flow

$$\partial h / \partial t = \partial U / \partial t = \partial Q / \partial t = 0 \qquad \textit{for fixed } x$$

True steady flow is rare in nature. Flow in a channel may be steady only when the discharge in the channel is constant, i.e. for steady flow not only $\partial Q/\partial t = 0$, but also $\partial Q/\partial x = 0$ (Art.2.2). The flow of water in a straight prismatic channel with a constant discharge (e.g. in a laboratory flume in which a constant discharge is circulated) and the dry-season flow of a river when no rainfall occurs may be considered as steady flow. Flood flows in rivers and tidal flows in estuaries are the familiar examples of unsteady flow.

(b) Uniform and Varied Flows (space is the criterion)

Flow in an open channel is said to be uniform if the depth, mean velocity and discharge do not change along the length of the channel at a given instant of time. When these quantities change along the length of the channel at any instant, the flow is termed as varied or non-uniform. Mathematically, for uniform flow

$$\partial h / \partial x = \partial U / \partial x = \partial Q / \partial x = 0 \qquad \textit{for fixed } t$$

In uniform flow, the channel bottom, the free surface and the energy grade line are parallel to one another, i.e. their slopes are equal.

True uniform flow is rare in nature. The flow of water in a straight prismatic channel (e.g. in a laboratory flume) with a constant discharge and a constant velocity and without any structure like weir or sluice gate may be considered as uniform flow.

* Uniform flow can be steady only. The unsteady uniform flow requires that the water surface fluctuates from time to time remaining parallel to the channel bottom. This is a practically impossible condition and hence "uniform flow" is used to mean "steady uniform flow".

Varied or non-uniform flow is further classified into (i) gradually varied flow, (ii) rapidly varied flow, and (iii) spatially varied flow.

Gradually Varied Flow

If the depth of flow and the mean velocity change gradually along the length of the channel ($\partial h/\partial x \approx 0$, $\partial U/\partial x \approx 0$), the flow is gradually varied. It may again be steady or unsteady. For steady gradually varied flow, $\partial h/\partial t = \partial U/\partial t = \partial Q/\partial t = \partial Q/\partial x = 0$, $\partial h/\partial x \approx 0$, $\partial U/\partial x \approx 0$. For unsteady gradually varied flow, $\partial h/\partial t \neq 0$, $\partial U/\partial t \neq 0$, $\partial Q/\partial t \neq 0$, $\partial Q/\partial x \neq 0$, $\partial h/\partial x \approx 0$, $\partial U/\partial x \approx 0$. The flow upstream of a dam in a river or upstream of a sluice gate in a canal is an example of steady gradually varied flow. The flood flow in a river and the tidal flow in an estuary are two familiar examples of unsteady gradually varied flow. Friction plays an important role in gradually varied flows.

Rapidly Varied Flow

The flow is rapidly varied if the depth of flow and the mean velocity change abruptly over a comparatively short distance ($\partial h/\partial x >> 0$, $\partial U/\partial x >> 0$. Rapidly varied flow is also known as a *local phenomenon*. It may be steady or unsteady. For steady rapidly varied flow, $\partial h/\partial t = \partial U/\partial t = \partial Q/\partial t = \partial Q/\partial x = 0$, $\partial h/\partial x >> 0$, $\partial U/\partial x >> 0$. For unsteady rapidly varied flow, $\partial h/\partial t \neq 0$, $\partial U/\partial t \neq 0$, $\partial Q/\partial t \neq 0$, $\partial Q/\partial x \neq 0$, $\partial h/\partial x >> 0$, $\partial U/\partial x >> 0$. Hydraulic jumps and hydraulic drops are two familiar examples of steady rapidly varied flow and surges in canals and tidal bores are examples of unsteady rapidly varied flow. In a hydraulic jump, the depth changes abruptly from a lower value to a higher value. In a hydraulic drop, the depth changes abruptly from a higher value to a lower value. A surge is a moving wave front that occurs whenever there is an abrupt change in the discharge or depth of flow or both, i.e. during the sudden closure of a sluice gate. The tidal bore is a moving wave front by which the tide invades a river. Friction can usually be neglected in rapidly varied flows.

Spatially Varied Flow

A flow in which the discharge varies along the length of the channel resulting from the transverse addition or withdrawal of water so that $\partial Q/\partial x \neq 0$ is known as a spatially varied or *discontinuous flow*. Obviously, it is a non-uniform flow and may be either steady or unsteady. For steady spatially varied flow, $\partial h/\partial t = \partial U/\partial t = \partial Q/\partial t = 0$, $\partial Q/\partial x \neq 0$, $\partial h/\partial x \neq 0$, $\partial U/\partial x \neq 0$. For unsteady spatially varied flow, $\partial h/\partial t \neq 0$, $\partial U/\partial t \neq 0$, $\partial Q/\partial t \neq 0$, $\partial Q/\partial x \neq 0$, $\partial h/\partial x \neq 0$, $\partial U/\partial x \neq 0$. The flow over a bottom rack is an example of steady spatially varied flow. The production of surface run-off due to rainfall, known as *overland flow*, and the flow over a roadside gutter are examples of unsteady spatially varied flow.

The transverse inflow or outflow, designated by q*, is usually expressed per unit length of the channel and the units of q* are $m^3/s/m$ or simply m^2/s. The inflow is taken to be positive and the outflow is taken to be negative.

The classification of open channel flow is summarized below and the various types of flow are sketched in Fig. 1.6.

```
                                   huQ
                                              ┌──► Uniform
                                              ├──► Gradually varied  ─┐
                    ┌──► Steady    ──────►    ├──► Rapidly varied    ─┴► hU
Open                │                         └──► Spatially varied  ──► Q
channel ───────────►│
flow                │                         ┌──► Gradually varied
                    └──► Unsteady  ──────►    ├──► Rapidly varied
                                              └──► Spatially varied

              (With respect to time)       (With respect to space)
```

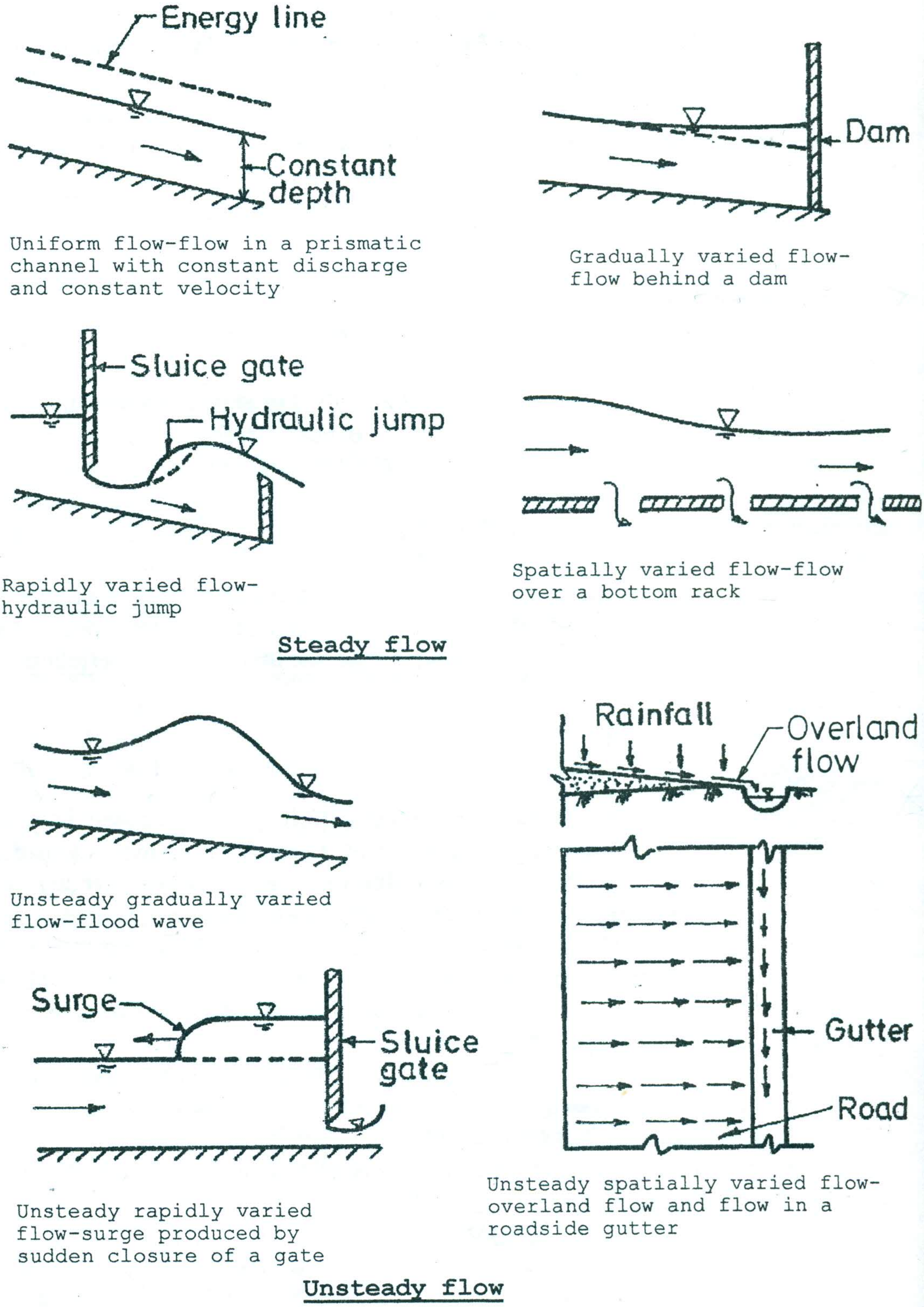


Fig.1.6 Various types of open channel flow

1.5 EFFECTS OF VISCOSITY AND GRAVITY

(a) Effect of Viscosity

The effect of the viscous forces relative to the inertial forces on open channel flow is expressed by the *Reynolds number*, which may be written as

$$\mathrm{Re} = \frac{Inertial\ \ forces}{Viscous\ \ forces} = \frac{Mass * acceleration}{Viscous\ \ shear\ stress * area} = \frac{\rho L^3\left(L/T^2\right)}{\tau.A}$$

$$= \frac{\rho L^3\left(L/T^2\right)}{\mu(du/dz)L^2} = \frac{\rho L^3\left(L/T^2\right)}{\mu(L/T/L)L^2} = \frac{\rho L^2}{\mu T} = \frac{(L/T)L}{\mu/\rho} = \frac{UL}{\nu} \qquad (1.9)$$

where U is a characteristic or representative velocity, taken as the mean velocity of flow, L is a characteristic length and ν is the kinematic viscosity of water. For water at 20^0C, $\nu = 10^{-6}\,\mathrm{m^2/s}$. This value of ν is normally used.

In open channel flow, the characteristic length commonly used is the hydraulic radius R (= A/P), so that

$$\mathrm{Re} = \frac{UR}{\nu} \qquad (1.10)$$

definition

Then, when

i) Re < 500, the flow is *laminar*,

ii) Re > 12,500, the flow is *turbulent*, and

iii) 500 ≤ Re ≤ 12,500, the flow is *transitional*.

In laminar flow the viscous forces are very strong relative to the inertial forces and dominate the flow. When the flow is laminar, the water particles appear to move in definite smooth paths or laminas with only a molecular exchange of momentum. In turbulent flow the viscous forces are very weak relative to the inertial forces and the inertial forces dominate the flow. The water particles move in irregular paths in a random fashion with extensive mixing due to generated eddies. Between the laminar and the turbulent states, there is a transitional state in which the flow changes from laminar to turbulent or vice versa.

By injecting a fine stream of dye, it is possible to identify whether a flow is laminar, transitional or turbulent. The stream of dye looks like a thread and does not mix with water in laminar flow, becomes wavy and irregular in transitional flow, and mixes with water and disappears in turbulent flow.

In laminar flow the head loss due to friction varies as U, whereas in turbulent flow the head loss due to friction varies as U^n, where n ranges from 1.7 (for relatively smooth surfaces) to 2.0 (for fully rough surfaces).

Most open channel flows including those in rivers and canals are turbulent. The Reynolds number of most open channel flows like those in rivers and canals is high, of the order of 10^6. For example, for river flow, U ≈ 1 m/s and R ≈ 5 m, so that Re ≈ 5×10^6. This indicates that the viscous forces are very weak relative to the inertial forces. Therefore, the Reynolds number does not play a significant role in determining the state or behavior of open channel flows.

Laminar open channel flow occurs rarely. The overland flow and the flow of groundwater with a free surface are two examples of laminar open channel flow.

(b) Effect of Gravity

The effect of the gravity forces relative to the inertial forces on open channel flow is determined by the *Froude number*, which may be written as

$$\text{Fr} = \sqrt{\frac{\textit{Inertial forces}}{\textit{Gravity forces}}} = \sqrt{\frac{\textit{Mass} * \textit{inertial acceleration}}{\textit{Mass} * \textit{gravitational acceleration}}}$$

$$= \sqrt{\frac{\textit{Inertial acceleration}}{\textit{Gravitational acceleration}}} = \sqrt{\frac{L/T^2}{g}} = \sqrt{\frac{(L/T)^2}{gL}} = \sqrt{\frac{U^2}{gL}} = \frac{U}{\sqrt{gL}} \qquad (1.11)$$

where U is a characteristic velocity, taken as the mean velocity of flow, L is a characteristic length and g (= 9.81 m/s^2) is the acceleration due to gravity. In open channel flow, the characteristic length commonly used is the hydraulic depth D (= A/B), so that

$$\text{Fr} = \frac{U}{\sqrt{gD}} \qquad (1.12)$$

Then, if

i) $Fr = 1, U = \sqrt{gD}$, the flow is *critical,*

ii) $Fr < 1, U < \sqrt{gD}$, the flow is *subcritical,* and

iii) $Fr > 1, U > \sqrt{gD}$, the flow is *supercritical.*

When the flow is critical, the inertial and gravitational forces are equal. When the flow is subcritical, the gravitational forces are dominant and when the flow is supercritical, the inertial forces are dominant.

The flow in most rivers and canals and upstream of a sluice gate is subcritical. As an example, for river flow, U $\approx$ 1 m/s and D $\approx$ 5 m and hence Fr $\approx$ 0.14. Supercritical flow normally occurs downstream of a sluice gate and at the feet of drops and spillways. Flow upstream of a hydraulic jump is supercritical and downstream of a jump is subcritical.

Thus, the Froude number of open channel flows varies over a wide range covering both subcritical and supercritical flows and the state or behavior of open channel flow is primarily governed by the gravity forces relative to the inertial forces. Therefore, the Froude number is the most important parameter to indicate the state or behavior of open channel flows.

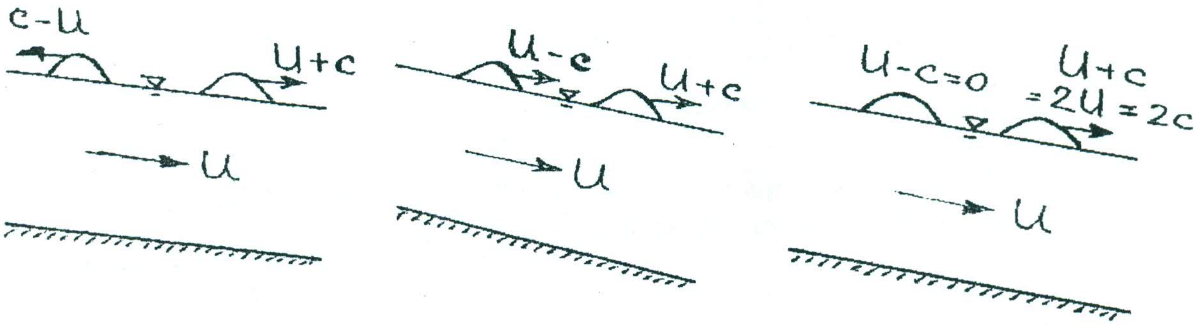
Equation (1.12) states that at the critical state of flow (Fr = 1), the flow velocity is equal to $\sqrt{gD}$. It can be shown that $\sqrt{gD}$ is the velocity (or celerity) of an elementary or small-amplitude wave on the surface of still water (see Appendix 1.1). The Froude number can then be defined as the ratio between the flow velocity U and the wave celerity c, i.e.

$$Fr = \frac{U}{c} \qquad (1.13)$$

Obviously, then

i) for subcritical flow, Fr < 1 and U < c,
ii) for critical flow, Fr = 1 and U = c, and
iii) for supercritical flow, Fr > 1 and U > c.

In an open channel, a small-amplitude wave can be easily produced by gently throwing a small object in water. In subcritical flow, one wave front propagates upstream at a velocity (c-U) and the other wave front propagates downstream at a velocity (U + c) (Fig.1.7a). In supercritical flow, both the wave fronts propagate downstream at velocities (U - c) and (U + c), respectively (Fig. 1.7b). In critical flow, one wave front remains stationary and the other moves downstream at a velocity (U + c) (Fig. 1.7c). Thus, by observing an elementary wave whether it propagates upstream against the flow, remains stationary or propagates downstream can be used as a criterion for physically identifying subcritical, critical or supercritical flow.



Fr < 1, U < c
(a) Subcritical flow

Fr > 1, U > c
(b) Supercritical flow

Fr = 1, U = c
(c) Critical flow

Fig. 1.7 Propagation of wave in subcritical, supercritical and critical flows

The fact that a small disturbance such as an elementary wave can propagate upstream against the flow in subcritical flow, but not in supercritical flow, has important practical significance. In subcritical flow, conditions upstream are affected by downstream conditions. Therefore, subcritical flow is controlled from downstream. In supercritical flow, conditions upstream are not affected by downstream conditions, and hence, supercritical flow is controlled from upstream.

Combined Effect of Viscosity and Gravity: State of Flow

A combined effect of viscosity and gravity may produce any of the following four states of flow in an open channel based on the numerical values of the Froude and the Reynolds numbers:

i) Subcritical laminar $Fr < 1, Re < 500$
ii) Supercritical laminar $Fr > 1, Re < 500$
iii) Subcritical turbulent $Fr < 1, Re > 12{,}500$
iv) Supercritical turbulent $Fr > 1, Re > 12{,}500$

The first two flow states, subcritical laminar and supercritical laminar, are very rare. The flow in most rivers and canals is subcritical turbulent and that at the feet of drops and spillways is supercritical turbulent.

Example 1.1

A trapezoidal channel has a bottom width of 6 m and side slopes of 2:1. Compute the discharge and determine the state of flow in this channel if the depth of flow is 1.5 m and the mean velocity of flow is 2.30 m/s. If elementary waves are created in this channel, determine the speed of the wave fronts upstream and downstream.

Solution Trapezoidal channel, b = 6 m, s = 2, h = 1.5 m, U = 2.30 m/s

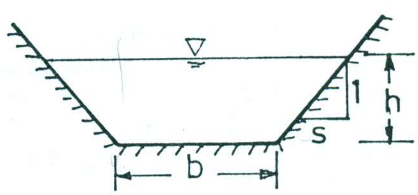
$$A = (b+sh)h = (6+2\times1.5)\times 1.5 = 13.5\ m^2$$

$$P = b + 2\sqrt{1+s^2}\,h = 6 + 2\ \sqrt{1+2^2}\times 1.5 = 12.71\ m$$

$$B = b + 2sh = 6 + 2\times2\times1.5 = 12\ m$$

$$R = A/P = 13.5/12.71 = 1.06\ m$$

$$D = A/B = 13.5/12 = 1.13\ m$$



$$\therefore Q = AU = 13.5\times2.30 = 31.05\ m^3/s$$

$$Re = \frac{UR}{V} = \frac{2.30\times1.06}{10^{-6}} = 2.44\times10^6 > 12{,}500$$

$$Fr = \frac{U}{\sqrt{gD}} = \frac{2.30}{\sqrt{9.81 \times 1.13}} = 0.69 < 1$$

Hence, the flow is subcritical turbulent.

Now, c = $\sqrt{gD} = \sqrt{9.81 \times 1.13} = 3.33\ m/s$

∴ Speed of the wave fronts upstream = c – U = 3.33 – 2.30 = 1.03 m/s
and, speed of the wave fronts downstream = c + U = 3.33 + 2.30 = 5.63 m/s

Example 1.2

A circular channel 2.75 m in diameter carries a discharge of 6.55 m^3/s at a depth of 1.1 m. Determine the state of flow.

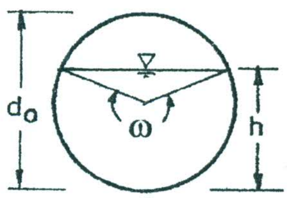
Solution Circular channel, $d_0 = 2.75$ m, $Q = 6.55\ m^3/s$, $h = 1.1$ m

$$\omega = 2\cos^{-1}(1 - 2h/d_0) = 2\cos^{-1}(1 - 2 \times 1.1/2.75) = 2.74\ rad$$

$$A = (\omega - \sin\omega)d_o^2/8 = (2.74 - Sin2.74)2.75^2/8 = 2.22\ m^2$$

$$P = \omega d_o/2 = 2.74 \times 2.75/2 = 3.77\ m$$

$$B = d_o \sin(\omega/2) = 2.75 \times \sin(2.74/2) = 2.69\ m$$



$$R = A/P = 2.22/3.77 = 0.59\ m$$

$$D = A/B = 2.22/2.69 = 0.82\ m$$

$$U = Q/A = 6.55/2.22 = 2.95\ m/s$$

$$\text{Re} = \frac{UR}{\nu} = \frac{2.95 \times 0.59}{10^{-6}} = 1.74 \times 10^6 > 12{,}500$$

$$Fr = \frac{U}{\sqrt{gD}} = \frac{2.95}{\sqrt{9.81 \times 0.82}} = 1.04 > 1$$

Hence, the flow is supercritical turbulent.

1.6 VELOCITY DISTRIBUTION

Velocity Distribution in a Channel Section

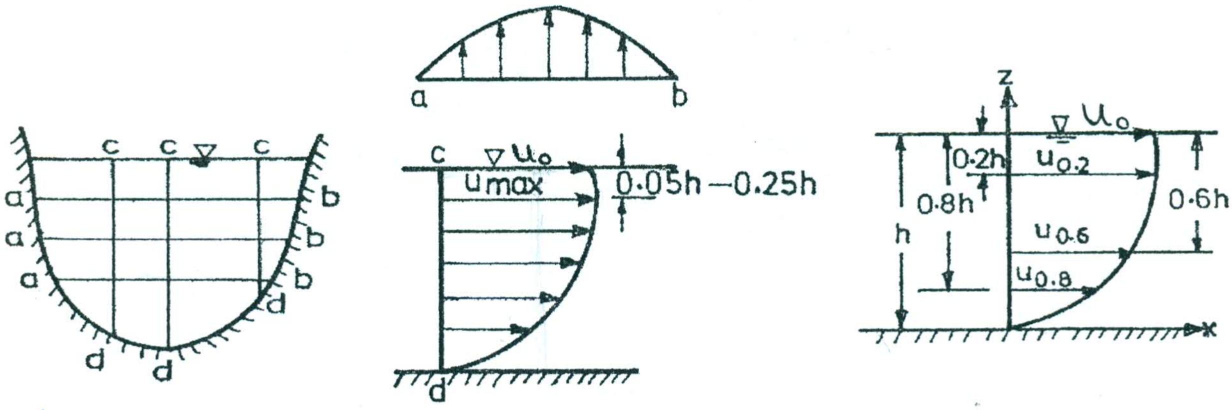
Owing to the presence of the free surface and the friction over the channel bed and banks, the velocities are not uniformly distributed in an open channel flow section. The velocity is zero at the solid boundary and gradually increases with distance from the boundary (Fig.1.8a,b). The velocity distribution in a channel section also depends on the channel geometry, the roughness of the channel, the presence of bends etc. The measured maximum velocity usually occurs below the free surface at a distance of 0.05 to 0.25 of the depth and is about 10% to 30% higher than the cross-sectional mean velocity.

In turbulent flow, the variation of velocity along a vertical can be approximated by a *logarithmic* or *power law*. In this case, the average of the velocities at 0.2 and 0.8 of the depth or the velocity at 0.6 of the depth below the water surface (Fig.1.8c) is approximately equal to the mean velocity in the vertical.

Measurement of Velocity

The velocity of flow in an open channel can be measured with a current meter. It is the standard practice of the U.S. Geological Survey to determine the average velocity in a vertical $\overline{U}$ by measuring the velocities at 0.2 and 0.8 of the depth below the free surface when the depth is more than 0.61 m (2 ft), or at 0.6 of the depth when the depth is less than 0.61 m, i.e.

$$\overline{U} \approx \frac{u_{0.2} + u_{0.8}}{2} \approx u_{0.6} \tag{1.14}$$



(a) Channel section

(b) Velocity variation along a-b and c-d

(c) Logarithmic velocity distribution along a vertical

Fig.1.8 Velocity distribution

The discharge of a stream is computed from the measurement of velocity and area and the method is known as the *area-velocity method.* With reference to Fig.1.5, the total discharge is computed as

$$Q = \sum_{i=1}^{N-1} \Delta Q_i \tag{1.15}$$

where ΔQ_i = discharge in the ith segment
= cross-sectional area of the ith segment × average velocity at the ith vertical
= $\Delta A_i . \overline{U}_i$

The mean velocity in the river section is equal to the discharge divided by the area, i.e.

$$U = \frac{Q}{A} \tag{1.16}$$

The surface velocity u_0 is related to the average velocity in a vertical $\overline{U}$ by

$$\overline{U} = k u_0 \tag{1.17}$$

where k is a coefficient whose value ranges between 0.80 and 0.95 depending on the channel section. The surface velocity can be determined by float tracking or other surface velocity measuring devices.

Example 1.3

The data collected during the stream-gauging operation at a certain river section are given in Table 1.2. Compute the discharge and the mean velocity for the entire section.

Solution

The measurements made 2 m and 15 m from the left bank involve velocity at 0.6 depth which represents the mean velocity in the vertical. Other measurements involve velocities at 0.2 and 0.8 depths and the mean velocity is obtained by averaging the velocities at 0.2 and 0.8 depths. The width associated with each measurement extends halfway between the adjacent

Table 1.2 Computation of discharge from stream-gauging

Distance from left bank (m)	Total Depth (m)	Meter depth (m)	Velocity (m/s)	Width (m)	Area (m^2)	Mean velocity (m/s)	Discharge (m^3/s)
0	0						
2	1.00	0.60	0.54	2.00	2.00	0.54	1.08
4	3.50	2.80	0.98				
		0.70	1.62	2.00	7.00	1.30	9.10
6	5.20	4.16	1.35				
		1.04	1.60	2.50	13.00	1.48	19.18
9	6.30	5.04	1.36				
		1.26	1.81	2.50	15.75	1.59	24.96
11	4.40	3.52	1.51				
		0.88	1.72	2.00	8.80	1.62	14.21
13	2.20	1.32	1.16	2.00	4.40	1.16	5.10
15	0.80	0.48	0.64	2.00	1.60	0.64	1.02
17	0						
Total:					52.55		74.65

segments. So, for example, for the measurement taken 6 m from the left bank, the width is [(6-4)/2+(9-6)/2] = 2.5 m. The corresponding area ΔA is $(5.20\times2.50) = 13.00$ m^2 and the corresponding discharge ΔQ is $(13.00\times1.48) = 19.18$ m^3/s. The areas and discharges associated with other measurements are computed similarly and are shown in Table 1.2 and summed to yield total cross-sectional area $A = 52.55$ m^2 and total discharge $Q = 74.65$ m^3/s. Therefore, the mean velocity for the entire section = 74.65/52.55 = 1.42 m/s.

1.7 VELOCITY DISTRIBUTION COEFFICIENTS

The flow in a straight prismatic channel is in fact three-dimensional, i.e. the flow properties like the velocity and the pressure vary in the longitudinal, lateral and vertical directions. However, the variations of the flow parameters in the lateral and vertical directions are usually small compared to those in the longitudinal direction. Consequently, a majority of open channel flow problems are analyzed by considering that the flow is one-dimensional, i.e. we consider the cross-sectional mean values of the flow parameters that vary from section to section only.

In the one-dimensional method of flow analysis, the discharge and the mean velocity of flow in a channel section are computed as follows. Let u be the velocity over an elementary area ΔA of the channel cross-section (Fig. 1.9). Then, the discharge past a channel section is given by

$$Q = \int_0^A u\,dA \qquad (1.18)$$

and the mean velocity of flow U for the entire cross-section is given by

$$U = \frac{Q}{A} = \frac{1}{A}\int_0^A u\,dA \qquad (1.19)$$

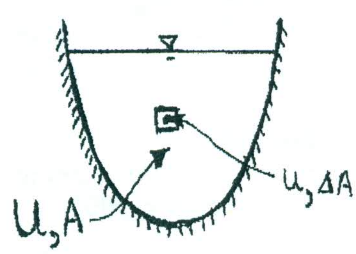


Fig.1.9 Definition sketch

In the one-dimensional method of flow analysis, owing to non-uniform velocity distribution in a channel section, the kinetic energy and the momentum of flow computed from the cross-sectional mean velocity are generally less than their actual values. To get the actual kinetic energy of flow, the kinetic energy based on the mean velocity is multiplied by the coefficient α, known as the *kinetic energy coefficient* or the *Coriolis coefficient*. Similarly, to get the actual momentum, the momentum based on the mean velocity is multiplied by the coefficient β, known as the *momentum coefficient* or the *Boussinesq coefficient*. The energy and momentum coefficients α and β together are known as the *velocity distribution coefficients*.

The kinetic energy of flow passing ΔA per unit time is equal to

$$\frac{1}{2} \times \rho u \Delta A \times u^2 = \frac{\rho}{2} u^3 \Delta A$$

where ρ is the mass density of ~~of~~ water. Therefore, the total kinetic energy of flow passing the channel section is equal to

$$\frac{\rho}{2} \int_0^A u^3 dA$$

where A is the total area of the cross-section. The total kinetic energy based on the mean velocity U and corrected for the non-uniform distribution of velocity is

$$\alpha \frac{\rho}{2} U^3 A$$

Equating the above two quantities and reducing

$$\alpha = \frac{\int_0^A u^3 dA}{U^3 A} \approx \frac{\Sigma u^3 \Delta A}{U^3 A} \qquad (1.20)$$

$\alpha = \frac{\int u^3 dA}{U^3 A}$ $\quad \beta = \frac{\int u^2 dA}{U^2 A}$

The momentum of flow passing ΔA per unit time is $\rho u \Delta A \times u = \rho u^2 \Delta A$. Therefore, the total momentum of flow passing the channel section per unit time is equal to

$$\rho \int_0^A u^2 dA$$

The total momentum based on the mean velocity U and corrected for the non-uniform distribution of velocity is $\beta \rho U^2 A$ so that

$$\beta = \frac{\int_0^A u^2 dA}{U^2 A} \approx \frac{\Sigma u^2 \Delta A}{U^2 A} \qquad (1.21)$$

Equations (1.20) and (1.21) can be integrated if the velocity u is known as an algebraic function of h. When such a function does not exist, the results of actual measurement may be used to evaluate α and β graphically or by evaluating the numerators of Eqs.(1.20) and (1.21) numerically.

The energy and momentum coefficients are always positive and never less than unity. For uniform velocity distribution in the channel section, $\alpha = \beta = 1$. In all other cases, $\alpha > \beta > 1$ and the further the velocity distribution departs from uniform, the greater the coefficients become.

The effect of turbulence is to make the flow more uniform over the channel section. Therefore, the values of α and β are higher for laminar flow than for turbulent flow.

Experimental evidence (e.g. Watts et al., 1967) suggest that when the channels are straight and prismatic and the flow is turbulent and uniform or gradually varied, the two velocity distribution coefficients do not normally exceed 1.10 and 1.04, respectively, and one can assume $\alpha = \beta = 1$. However, in channels of complex cross-section, upstream from weirs, in the vicinity of obstructions, near pronounced irregularities in alignment or when the flow is concentrated in one part of the section, values of α and β may even be greater than 2 and 1.35, respectively.

Although the numerical values of α and β may vary over a wide range depending on the velocity distribution, the ratio $(\alpha\text{-}1)/(\beta\text{-}1)$ tends to vary only slightly, in the range 2.8 to 3.0 (Henderson, 1966).

Example 1.4

In a wide channel the velocity varies along a vertical as $u = 1 + 3z/h$, where h is the total depth and u is the velocity at a distance z from the channel bottom. (i) Compute the discharge per unit width, (ii) determine the state of flow, and (iii) compute the velocity distribution coefficients α and β and the ratio $(\alpha\text{-}1)/(\beta\text{-}1)$, if $h = 5$ m.

Solution

For a wide channel we can consider a unit width of the channel and replace the area A in Eq.(1.19) by the flow depth h. Then, the cross-sectional mean velocity U is becomes the depth-averaged velocity $\overline{U}$. Therefore,

$$\overline{U} = \frac{1}{A}\int_0^A u dA = \frac{1}{h}\int_0^h u dz = \frac{1}{h}\int_0^h (1+\frac{3z}{h})dz = \frac{1}{h}[h+\frac{3h^2}{2h}] = 1+\frac{3}{2} = 2.50\ m/s$$

Alternatively, since the velocity varies linearly from 1 m/s at the channel bottom to 4 m/s at the free surface, the depth-averaged velocity is obtained using the trapezoidal rule as

$$\overline{U} = \frac{1+4}{2} = 2.50\ m/s$$

(i) Then, the discharge per unit width is

$$q = \overline{U}h = 2.50 \times 5 = 12.50\ m^3/s/m \quad \text{or} \quad m^2/s$$

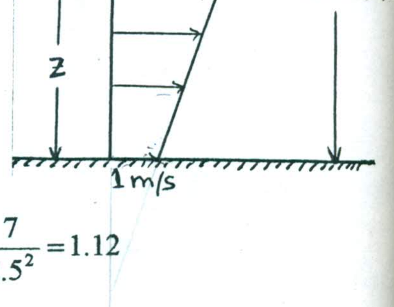
(ii) The Reynolds and Froude numbers Re and Fr are then

$$Re = \frac{\overline{U}R}{\nu} = \frac{\overline{U}h}{\nu} = \frac{2.50\times 5}{10^{-6}} = 12.5\times 10^6 > 12{,}500 \quad \text{(for a wide channel, } R \approx h)$$

$$Fr = \frac{\overline{U}}{\sqrt{gD}} = \frac{\overline{U}}{\sqrt{gh}} = \frac{2.50}{\sqrt{9.81\times 5}} = 0.36 < 1.00 \quad \text{(for a wide channel, } D = h)$$

Hence, the flow is subcritical turbulent.

(iii) The energy and momentum coefficients α and β are then



$$\alpha = \frac{1}{\overline{U}^3 A}\int_0^A u^3 dA = \frac{1}{\overline{U}^3 h}\int_0^h u^3 dz = \frac{1}{\overline{U}^3 h}\int_0^h (1+\frac{3z}{h})^3 dz$$

$$= \frac{1}{\overline{U}^3 h}\int_0^h (1+\frac{9z}{h}+\frac{27z^2}{h^2}+\frac{27z^3}{h^3})dz$$

$$= \frac{1}{\overline{U}^3 h}(h+4.5h+9h+6.75h) = \frac{21.25}{\overline{U}^3} = \frac{21.25}{2.5^3} = 1.36$$

$$\beta = \frac{1}{\overline{U}^2 A}\int_0^A u^2 dA = \frac{1}{\overline{U}^2 h}\int_0^h u^2 dz = \frac{1}{\overline{U}^2 h}\int_0^h (1+\frac{3z}{h})^2 dz$$

$$= \frac{1}{\overline{U}^2 h}\int_0^h (1+\frac{6z}{h}+\frac{9z^2}{h^2})dz = \frac{1}{\overline{U}^2 h}(h+3h+3h) = \frac{7}{\overline{U}^2} = \frac{7}{2.5^2} = 1.12$$

$$\therefore (\alpha-1)/(\beta-1) = 0.36/0.12 = 3$$

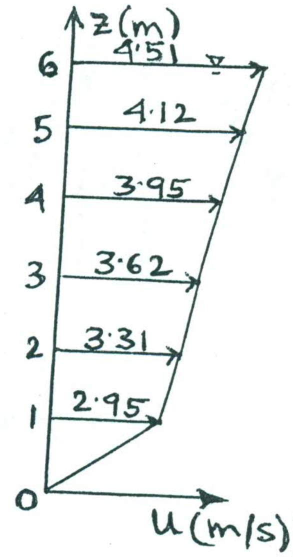
Note that when u is expressed as a function of z/h, the numerical values of $\overline{U}$, α and β become independent of depth of flow.

Example 1.5

Using the trapezoidal rule of numerical integration, compute the discharge per unit width, the mean velocity and the numerical values of α and β for the following velocity measurements (u is the velocity at a distance z from the channel bottom) along a vertical in a wide channel, when the total depth is 6 m.

z (m)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
u (m/s)	0.0	2.95	3.31	3.62	3.95	4.12	4.51

Solution



$$q = \int_0^h u dz = \Sigma u \Delta z = \Delta z[\frac{0+4.51}{2} + 2.95 + 3.31 + 3.62 + 3.95 + 4.12]$$

$$= 1.0 \times 20.21 = 20.21\ m^2/s \quad (\because \Delta z = 1.0 m)$$

$$\overline{U} = \frac{q}{h} = \frac{20.21}{6.0} = 3.37\ m/s$$

$$\alpha = \frac{\int_0^h u^3 dz}{\overline{U}^3 h} = \frac{\Sigma u^3 \Delta z}{\overline{U}^3 h}$$

$$= \frac{1.0}{3.37^3 \times 6}[\frac{0^3 + 4.51^3}{2} + 2.95^3 + 3.31^3 + 3.62^3 + 3.95^3 + 4.12^3] = 1.25$$

$$\beta = \frac{\int_0^h u^2 dz}{\overline{U}^2 h} = \frac{\Sigma u^2 \Delta z}{\overline{U}^2 h}$$

$$= \frac{1.0}{3.37^2 \times 6}[\frac{0^2 + 4.51^2}{2} + 2.95^2 + 3.31^2 + 3.62^2 + 3.95^2 + 4.12^2] = 1.11$$

1.8 PRESSURE DISTRIBUTION

Hydrostatic Pressure Distribution

Let us consider a vertical column of water of height h and cross-sectional area ΔA (Fig. 1.10). Let p be the intensity of pressure or unit pressure (force/unit area) at the bottom of the water column. Then,

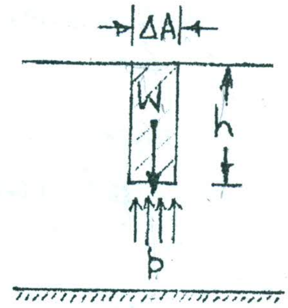


Fig.1.10 Definition sketch

$$p = \frac{Force}{Area} = \frac{Weight\ of\ water\ column}{Area} = \frac{\gamma h \Delta A}{\Delta A} = \gamma h = \rho g h \qquad (1.22)$$

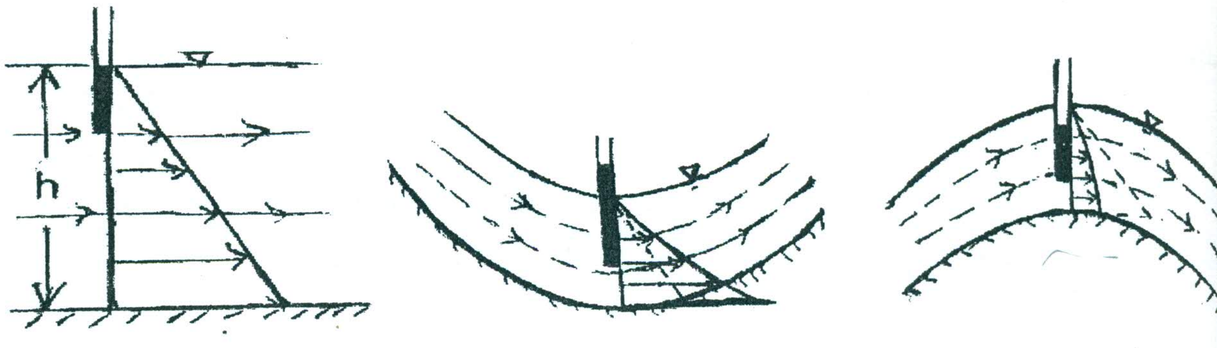
Equation (1.22) indicates that the pressure at any point is directly proportional to the depth of the point below the free surface. This is known as the *hydrostatic distribution of pressure* and h is the *hydrostatic pressure head.*

The hydrostatic law of pressure distribution is valid for parallel flow in a horizontal or small slope channel. When the curvature of the streamlines is small so that the water particles have no acceleration normal to the direction of flow, the flow is known as *parallel flow* (Fig.1.11a). Uniform flow is practically parallel flow. Gradually varied flow may also be regarded as parallel flow since the curvature of the streamlines is small and negligible. Hence, the hydrostatic law of pressure distribution holds exactly for uniform flow and approximately for gradually varied flow.

Pressure Distribution in Curvilinear Flow

When the curvature of the streamlines is considerable, the flow is known as *curvilinear* flow. Such situations may occur when the bottom of the channel is curved, at sluice gates and at free overfalls. In such cases, the acceleration normal to the direction of flow is not negligible and the pressure distribution is not hydrostatic. Curvilinear flows may either be *concave* or *convex*. In concave flow (Fig.1.11b), the centrifugal forces resulting from streamline curvature combine with the gravitational forces and the pressure is more than hydrostatic. In convex flow (Fig.1.11c), the centrifugal forces act against the gravitational forces and the pressure is less than hydrostatic.

diagram



$p = \gamma h$ $\quad$ $p > \gamma h$ $\quad$ $p < \gamma h$

(a) Parallel flow (b) Concave flow (c) Convex flow

Fig.1.11 Pressure distribution in parallel, concave and convex flows

Let us consider the forces acting in the vertical direction on a water column of height h and cross-sectional area ΔA (Fig.1.12). Then,

Mass of the water column = $\rho h \Delta A$

If r is the radius of curvature of the streamline and u is the flow velocity at the point under consideration, then

$$\text{Centrifugal acceleration} = \frac{u^2}{r}$$

and

$$\text{Centrifugal force} = \rho h \Delta A \times \frac{u^2}{r}$$

The intensity of pressure as a result of the centrifugal force is

$$p_c = \frac{Force}{Area} = \frac{\rho h \Delta A}{\Delta A} \times \frac{u^2}{r} = \rho h \frac{u^2}{r} \qquad (1.23)$$

and the pressure head

$$h_c = \frac{p_c}{\gamma} = \frac{\rho h}{\rho g}\frac{u^2}{r} = \frac{1}{g} h \frac{u^2}{r} \qquad (1.24)$$

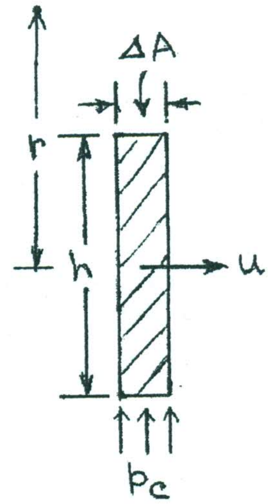


Fig.1.12 Definition sketch

For practical purposes, the velocity u is replaced by the cross-sectional mean velocity U.

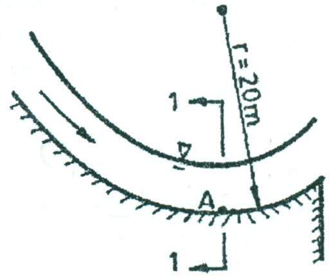
The total pressure head acting at the bottom of the water column is the algebraic sum of the pressure due to the centrifugal action and the weight of the liquid column, i.e.

$$\text{Total pressure head} = h \pm h_c = h \pm \frac{1}{g} h \frac{U^2}{r} = h(1 \pm \frac{1}{g}\frac{U^2}{r}) \qquad (1.25)$$

where the plus and minus signs are used with concave and convex flows, respectively.

Example 1.6

A spillway flip bucket has a radius of curvature of 20 m. If the flow depth at section 1-1 is 3 m and the discharge per unit width is 66 m^2/s, compute the pressure at A.



Solution $r = 20$ m, $q = 66$ m^2/s, $h = 3$ m

$$U = \frac{q}{h} = \frac{66}{3} = 22\ m/s$$

$$p = \gamma h\left(1 + \frac{U^2}{gr}\right) = \rho g h\left(1 + \frac{U^2}{gr}\right)$$

$$= 1000 \times 9.81 \times 3 \times \left(1 + \frac{22^2}{9.81 \times 20}\right) = 102030\ N/m^2 = 102.03\ kN/m^2$$

Effect of Slope on Pressure Distribution

The pressure distribution departs from hydrostatic if the longitudinal slope of the channel is large. Consider a water column of height h and cross-sectional area ΔA (Fig.1.13). The pressure at B in this case balances the component of the weight of the element AB normal to the bed.

Now, weight of the water column

$$W = \gamma d \Delta A$$

The component of W normal to the direction of flow (along AB) $= W\cos\theta = \gamma d \Delta A \cos\theta$

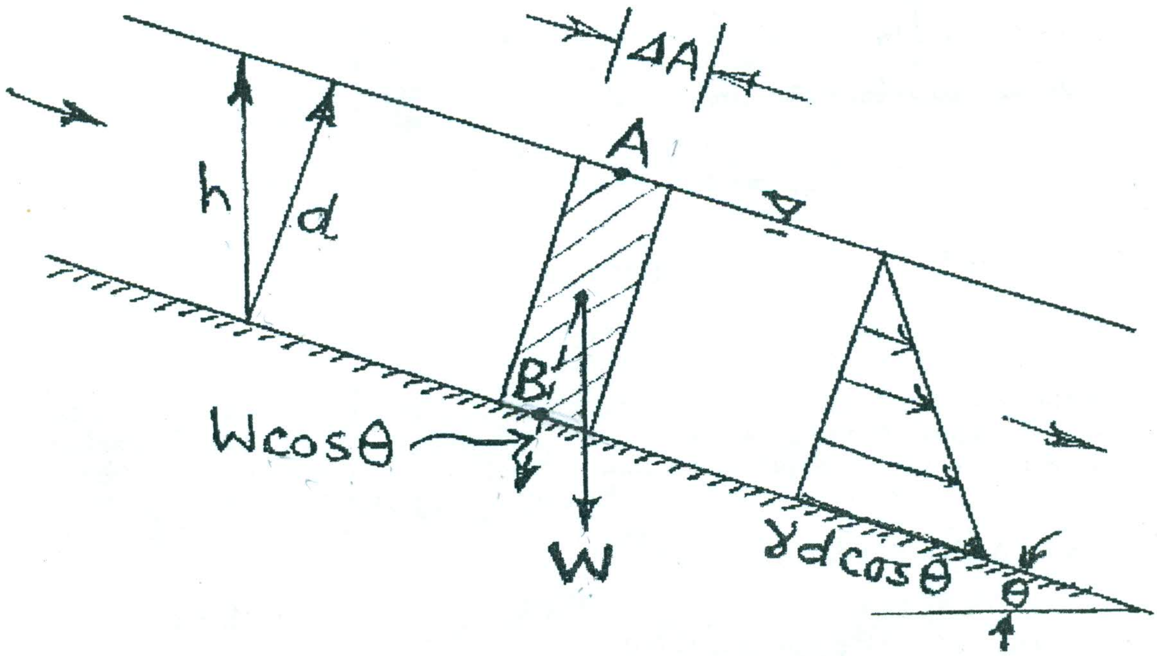


Fig.1.13 Pressure distribution in a channel of large slope

Hence, the intensity of pressure at the bottom of the water column

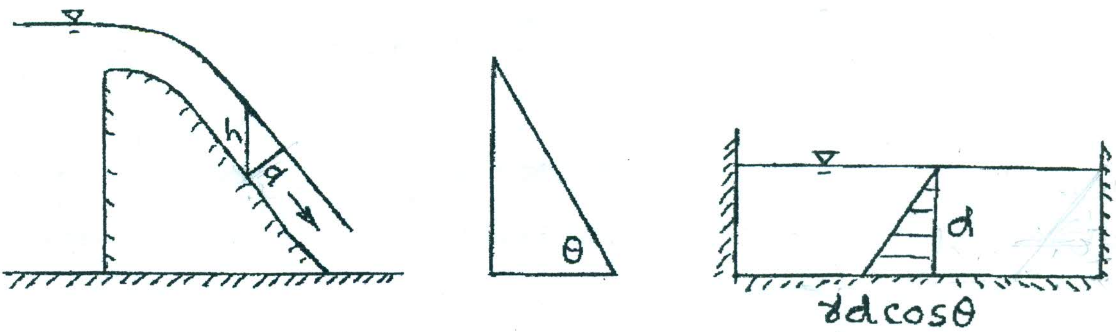
$$p = \frac{Force}{Area} = \frac{\gamma d \Delta A \cos\theta}{\Delta A} = \gamma d \cos\theta = \gamma h \cos^2\theta \quad (\because d = h\cos\theta) \qquad (1.26)$$

$\therefore$ Pressure head, $\dfrac{p}{\gamma} = d\cos\theta = h\cos^2\theta$ (1.27)

Thus, for channels of large slope, the pressure head at any vertical depth is equal to the depth multiplied by a correction factor $\cos^2\theta$. For a small slope channel, $\theta < 6^\circ$ and $S_0 < 0.1$, the use of Eq. (1.22) instead of Eq. (1.27) involves an error less than about 1% and can safely be ignored.

Example 1.7

Prove that the shear force and the overturning moment on the side walls of a steep rectangular channel are $(1/2)\gamma h^2 \cos^3\theta$ and $(1/6)\gamma h^3 \cos^4\theta$, respectively, where h is the depth of flow and θ is the bottom slope of the channel.



Solution $p/\gamma = d\cos\theta$ $\therefore\ p = \gamma d \cos$

$$Shear\ force, F = \frac{1}{2} \times \gamma d\cos\theta \times d \quad (per\ unit\ width)$$

$F = \frac{1}{2} \times p \times d$

$$= \frac{1}{2}\gamma d^2\cos\theta = \frac{1}{2}\gamma h^2\cos^3\theta \quad (\therefore d = h\cos\theta)$$

Length

$$Overturning\ moment = F \times arm = F \times d/3$$

$$= \frac{1}{2}\gamma d^2\cos\theta \times \frac{d}{3} = \frac{1}{6}\gamma d^3\cos\theta = \frac{1}{6}\gamma h^3\cos^4\theta$$

PROBLEMS

(Take ν for water = 10^{-6} m^2/s and g = 9.81 m/s^2)

1.1 (a) The depth and mean velocity upstream and downstream of a vertical sluice gate in a horizontal rectangular channel are 4 m and 1 m and 2 m/s and 8 m/s, respectively. The width of the channel is 6 m. Determine the state of flow both upstream and downstream of the gate.

(b) Consider the following data for the Padma (Ganges) river at the Baruria station in Faridpur on the 2nd July, 1989: A = 33,500 m^2, Q = 56,200 m^3/s and B = 3820 m. Compute the state of flow. Assume that the river is wide.

1.2 Water flows in an open channel at a depth of 1 m and a mean velocity of 3 m/s. Compute the discharge and determine the state of flow if the channel is

i) wide
ii) rectangular with b = 6 m,
iii) trapezoidal with b = 6 m and s = 2,
iv) triangular with s = 2,
v) parabolic with B = 4 m, and
vi) circular whose diameter is 2.5 m.

If elementary waves are created in these channels, determine the speeds of the wave fronts upstream and/or downstream.

1.3(a) The average depth of water in a wide river connected to sea is 5 m. Determine the time taken by a tidal wave to travel from the river mouth to 30 km upstream (i) when there is no flow in the river, and (ii) when the average flow velocity in the river is 1 m/s?

(b) Waves of small amplitude are created at the center of a circular-shaped pond of radius 50 m. The waves are found to reach the edge of the pond in 10 s. Estimate the approximate volume of water in the pond assuming that the depth of water in the pond is same everywhere.

1.4(a) In a wide river the velocity varies linearly along a vertical from 0.10 m/s at the bottom to 2.10 m/s at the surface. (i) Compute the discharge per unit width, and (ii) determine the state of flow, if the depth of flow is 4 m.

(b) In a wide river the velocity varies along a vertical as $u = 1 + 2z/h$, where h is the total depth and u is the velocity at a distance z from the channel bottom. The river is 5 m deep. (i) Compute the discharge per unit width, and (ii) determine the state of flow.

1.5(a) The velocity of flow is zero over one-third of the cross-section of a channel and uniform over the rest of the cross-section. Compute the numerical values of the velocity distribution coefficients α and β.

(b) The velocity is zero along the lower 20% of a vertical in a wide channel and uniform along the rest of the vertical. Compute the numerical values of the velocity distribution coefficients α and β.

1.6(a) For laminar flow the velocity distribution along a vertical can be approximated by

$$u = u_o \sin \frac{\pi z}{2h}$$

where u is the velocity at a distance z from the channel bottom, h is the depth of flow and u_o is the velocity at the free surface. Compute the velocity distribution coefficients α and β and the ratio $(\alpha-1)/(\beta-1)$.

(b) For turbulent flow the velocity distribution along a vertical can be approximated by $u \propto z^n$, when $n = 1/7$ (Prandtl's one-seventh power law). Determine the velocity distribution coefficients α and β and the ratio $(\alpha-1)/(\beta-1)$ in terms of n and for $n = 1/7$. Compare the numerical values of α and β with those obtained for laminar flow in Problem 1.6(a).

1.7 Compute the values of the velocity distribution coefficients α and β and the ratio $(\alpha-1)/(\beta-1)$ for the following velocity distributions along a vertical in a wide channel when the depth of flow in the channel is (a) 5 m, and (b) 10 m.

i) $u = 2z/h$ ii) $u = 2 + 2z/h$
iii) $u = 4(z/h)^{1/2}$ iv) $u = 1 + 2(z/h)^{1/2}$

1.8(a) Solve Example 1.5 using the Simpson's rules of numerical integration.

(b) Figure 1.14 shows the velocity distribution downstream of a sluice gate under submerged condition. Using the trapezoidal rule of numerical integration, compute the discharge per unit width, the mean velocity of flow and the numerical values of α and β.

1.9(a) A steep rectangular chute has a slope of 1H:3V. Compute the pressure at the bed of the chute if the vertical depth of water flowing over the chute is 1 m. Also, compute the force and the overturning moment on its side walls.

(b) While computing the shear force and the overturning moment on the side walls of a steep spillway chute, an engineer assumed that the water pressure varies linearly from zero at the free surface to ρgh at the bed of the chute, where h is the depth measured vertically. Are the computed results correct? If not, compute (i) the correct values of the shear force and the overturning moment, and (ii) the % error. The chute has an inclination of 1H:3V and h = 1 m.

(c) A high-head overflow spillway is shown in Fig.1.15. The flip bucket at the toe of the spillway acts to change the direction of flow from the slope of the spillway face to the horizontal and to discharge the flow into the air. If $r_1 = r_2 = 20$ m, $h_1 = h_2 = h_3 = h_4 = 1$ m and the discharge over the spillway is 6.5 $m^3/s/m$, determine the intensities of pressure at points 1, 2, 3 and 4.

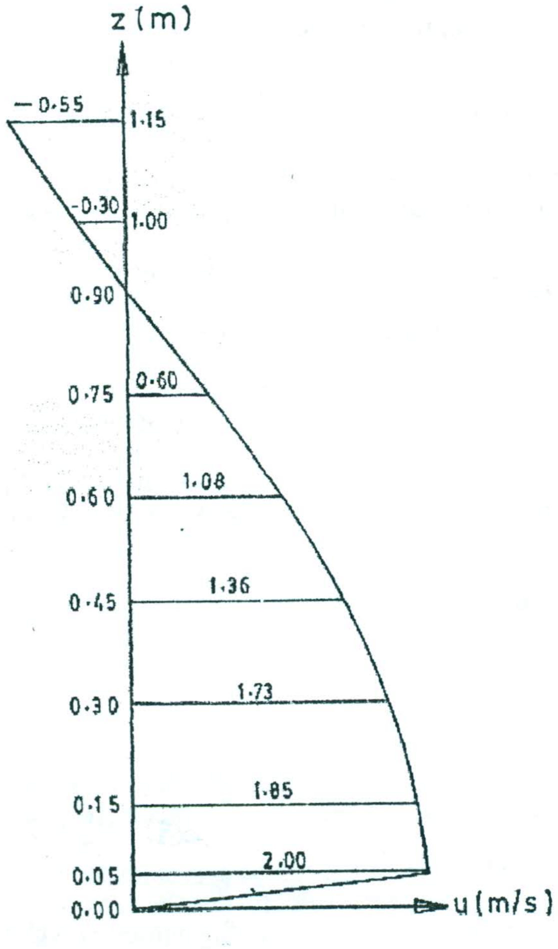


Fig. 1.14

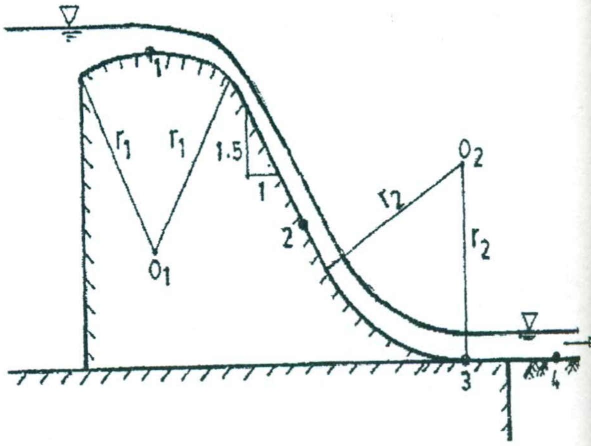


Fig. 1.15

Appendix 1.1

CELERITY OF A SMALL - AMPLITUDE WAVE

The celerity of a small-amplitude wave can be obtained considering the movement of a solitary wave of height Δh travelling to the right with celerity c in a channel as shown in Fig.1.16a. The cross-sectional areas of the channel corresponding to depths h and h + Δh are A and A + ΔA, respectively. The situation defined by Fig.1.16a is obviously unsteady and cannot be analyzed by elementary techniques. However, if the wave form does not change during its travel, the situation may be rendered into one of steady flow by applying a velocity of magnitude c in the direction opposite to that of wave travel as shown in Fig 1.16b.

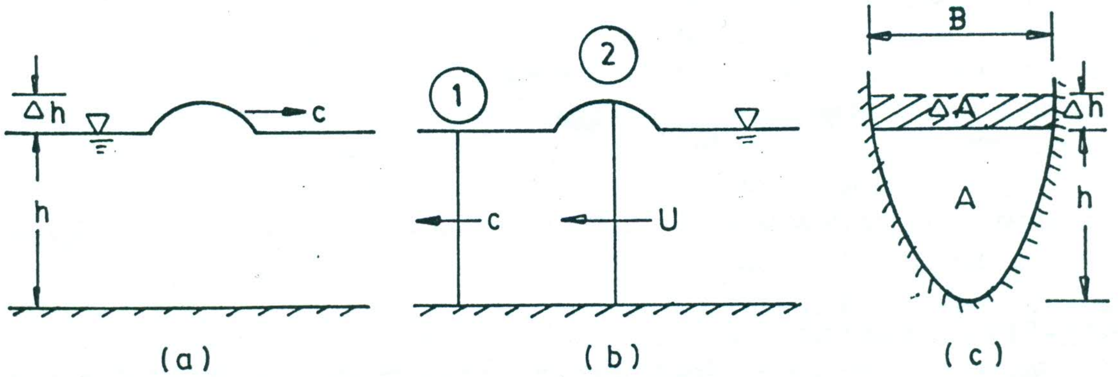


Fig.1.16 Propagation of a wave: (a) unsteady flow, (b) steady flow appeared to an observer moving with the wave crest, and (c) section through the wave crest

Now, application of the equation of continuity for steady flow (derived in Art.2.2) between sections 1 and 2 yields

$$cA = U(A + \Delta A) \tag{1.28}$$

which on simplification gives

$$U = c\left(1 - \frac{\Delta A}{A}\right) \tag{1.29}$$

Assuming a horizontal bed and neglecting friction, application of the energy equation (presented in Art.2.3) between sections 1 and 2 yields

$$h + \frac{c^2}{2g} = h + \Delta h + \frac{c^2}{2g}\left(1 - \frac{\Delta A}{A}\right)^2 \tag{1.30}$$

With reference to Fig.1.16c, the elementary water area ΔA near the free surface is equal to $B\Delta h$. Since the hydraulic depth D = A/B, solving Eq.(1.30) for c and simplifying, one obtains

$$c = \sqrt{gD} \tag{1.31}$$

For a rectangular channel, D = h and the celerity of a wave of small amplitude is given by

$$c = \sqrt{gh} \tag{1.32}$$

Chapter 2

GOVERNING EQUATIONS FOR STEADY ONE-DIMENSIONAL FLOW

2.1 GENERAL

Water movement is the key process in open channel flow. The three basic equations to describe water movement are the continuity, the energy and the momentum equations based on the principles of conservation of mass, energy and momentum, respectively.

As stated earlier, the flow in an open channel is, in fact, three-dimensional, i.e. the flow properties like the velocity and the pressure vary in the longitudinal, lateral and vertical directions. However, if the channel is very wide, the flow properties will approximately be invariant in the lateral direction. Such a flow may be treated as a two-dimensional flow. Since the variations of the flow parameters in the lateral and vertical directions are small compared to those in the longitudinal direction, a majority of open channel flow problems are analyzed by considering that the flow is one-dimensional, i.e. the variations of the flow properties only in the longitudinal direction are taken into consideration. Thus, the flow parameters are cross-sectional mean values that do not vary within the cross-section, but vary from section to section.

2.2 CONTINUITY EQUATION

The continuity equation is developed from the general principle of conservation of mass. According to this principle, for steady flow there cannot be any change of storage of mass within the control volume, i.e. flow must be continuous. If the flow is one-dimensional and if there is no transverse inflow or outflow (Fig.2.1a), then according to the above principle, the mass flow rate (mass per unit time) past various flow sections must be the same, i.e.

$$\rho_1 Q_1 = \rho_2 Q_2 = \ldots.. \tag{2.1}$$

where ρ is the mass density of water and Q is the discharge (volume flow rate) and the subscripts designate different channel sections.

Since the discharge Q is defined as

$$Q = AU \tag{2.2}$$

and water is practically incompressible ($\rho_1 = \rho_2$), Eq.(2.1) takes the form

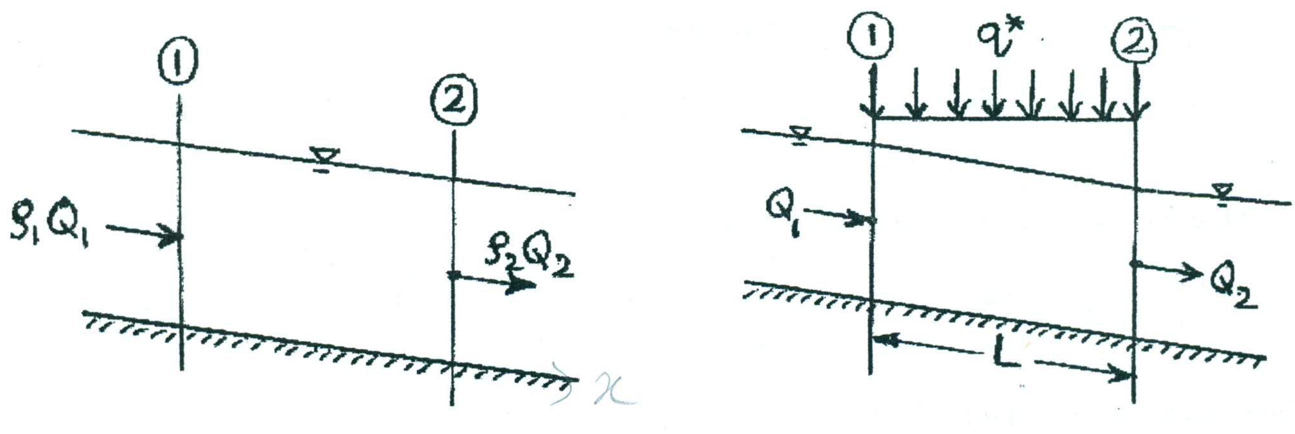
$$Q_1 = Q_2 = \ldots\ldots \qquad or \qquad U_1 A_1 = U_2 A_2 = \ldots\ldots \tag{2.3}$$

where A is the cross-sectional area and U is the mean velocity of flow.

Equation (2.3) is the usual form of the continuity equation for steady one-dimensional open channel flow of an incompressible fluid without transverse inflow or outflow. It indicates that in steady flow not only $\partial Q/\partial t = 0$, but also $\partial Q/\partial x = 0$, i.e. the discharge in the channel is constant.

If there is a transverse inflow of water at the rate of q^* per unit length (Fig.2.1b), flow is spatially varied or discontinuous (Art. 1.4), and the discharge Q_2 at section 2 at a distance L from section 1 where the discharge is Q_1 is given by

$$Q_2 = Q_1 + q^* L \tag{2.4}$$



(a) Without transverse inflow (b) With transverse inflow

Fig.2.1 Definition sketch for the continuity equation

2.3 ENERGY EQUATION

Let us consider a fluid element of cross-sectional area ΔA and length Δs in a steady non-viscous (frictionless) flow (Fig. 2.2). Then there can be only two kinds of impressed force in any chosen direction s; that due to pressure gradient, i.e. $-(\partial p/\partial s)\Delta s\Delta A$ and that due to the weight of the element, i.e. $(\gamma\Delta s\Delta A\sin\theta)$ or $-(\gamma\Delta s\Delta A)\,\partial z/\partial s$, where z is the vertical height above datum and γ is the specific weight of the fluid. Let ρ be the mass density of the fluid and a_s be the acceleration in the s direction. Then. since in any direction, force = mass × acceleration, we have

$$-\frac{\partial p}{\partial s}\Delta s\Delta A-\gamma\Delta s\Delta A\frac{\partial z}{\partial s}=(\rho\Delta s\Delta A)a_s$$

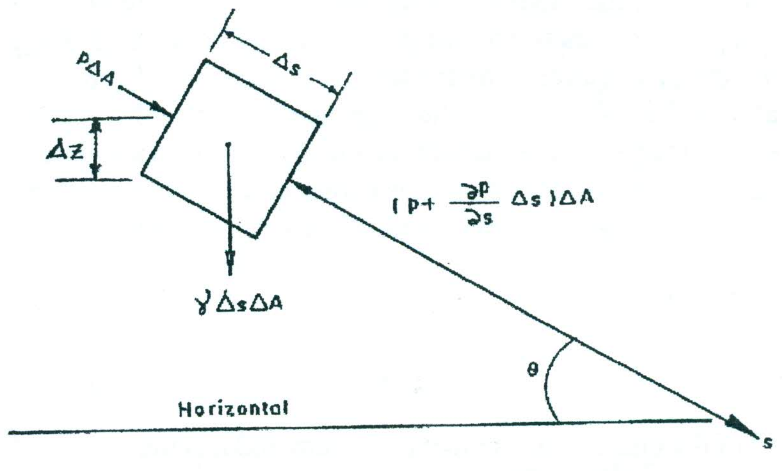


Fig. 2.2 Forces on a fluid element

or

$$\frac{\partial}{\partial s}(p+\gamma z)+\rho a_s=0 \qquad (2.5)$$

If P.E. increases, fluid is decelerated.

which is the *Euler equation.*

Now, from the theory of partial differentiation, we can write

$$\frac{du}{dt} = a_s = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial s} \tag{2.6}$$

When the flow is steady, $\partial u/\partial t = 0$, and Eq. (2.5) becomes

$$\frac{\partial}{\partial s}(p + \gamma z) + \rho u\frac{\partial u}{\partial s} = 0 \tag{2.7}$$

If pressure decreases, velocity increases,

which can be integrated directly to yield

$$p + \gamma z + \frac{1}{2}\rho u^2 = \text{constant}$$

or

$$H = z + \frac{p}{\gamma} + \frac{u^2}{2g} = \text{constant} \tag{2.8}$$

which is the *Bernoulli equation.* The Bernoulli expression ($z + p/\gamma + u^2/2g$) in general varies from one streamline to another but remains constant along a streamline in steady non-viscous (frictionless) flow.

To apply Eq.(2.8) to open channel flow, we replace $u^2/2g$ by $\alpha U^2/2g$, where U is the mean velocity of flow in the channel section and α is the energy coefficient. The velocity (or kinetic energy) head $\alpha U^2/2g$ is then the same for all points in the channel cross-section. Also, the sum ($z + p/\gamma$) is the piezometric head and defines the elevation of the hydraulic grade line above the datum. Since at any point the pressure head p/γ simply equals the depth h of that point below the free surface when the pressure distribution is hydrostatic, the sum ($z + p/\gamma$) must, therefore, be equal to the height of the water surface above the datum, whatever may be the position of the point in the cross-section. It follows that the free surface is the hydraulic grade line for all points in the cross-section provided the pressure distribution is hydrostatic. For convenience, ($z + p/\gamma$) is replaced by ($z_b + h$) and the total energy at an open channel flow section is expressed as

$$H = z_b + h + \alpha\frac{U^2}{2g} \tag{2.9}$$

where z_b is the height of the channel bed above the datum and h is the depth of flow. Obviously, the energy head H remains the same for all points in the cross-section, provided the pressure distribution is hydrostatic.

For a channel of large slope, Eq. (2.9) becomes

$$H = z_b + h\cos^2\theta + \alpha\frac{U^2}{2g} \tag{2.10}$$

According to the principle of conservation of energy, the total energy at the upstream section 1 must be equal to the total energy at the downstream section 2 plus the frictional loss of energy h_f between the two sections (Fig. 2.3), i.e.

$$H_1 = H_2 + h_f$$

or

$$z_{b1} + h_1 + \alpha_1 \frac{U_1^2}{2g} = z_{b2} + h_2 + \alpha_2 \frac{U_2^2}{2g} + h_f \tag{2.11}$$

For a channel of large slope, it becomes

$$z_{b1} + h_1 \cos^2\theta + \alpha_1 \frac{U_1^2}{2g} = z_{b2} + h_2 \cos^2\theta + \alpha_2 \frac{U_2^2}{2g} + h_f \tag{2.12}$$

Either of these two equations is known as the *energy equation*.

When $\alpha_1 = \alpha_2 = 1$ and $h_f = 0$, Eq.(2.11) becomes

$$z_{b1} + h_1 + \frac{U_1^2}{2g} = z_{b2} + h_2 + \frac{U_2^2}{2g} \tag{2.13}$$

This is the well-known *Bernoulli energy equation*.

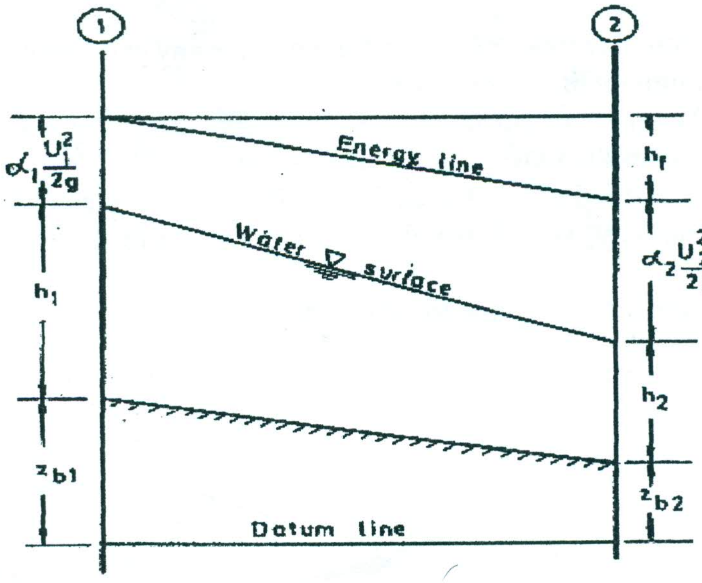


Fig. 2.3 Definition sketch for the energy equation when $\cos\theta \approx 1$

Each term in the energy equation represents energy in m-Newton per Newton. The expression of energy in this form is particularly convenient for dealing with problems in open channel flow.

For the flow of a real fluid like water, the total energy decreases in the downstream direction due to energy or head loss. As a result, in an open channel flow the energy grade line always slopes downward.

The frictional energy or head loss term h_f in the energy equation denotes the energy that is transformed from available mechanical (kinetic plus potential) energy into heat energy. This energy is not recoverable, but it slightly raises the temperature of water, the channel and the surroundings.

Normally, the total head loss h_L can be considered to be made up of frictional head loss h_f and the eddy loss h_e, i.e.

$$h_L = h_f + h_e \tag{2.14}$$

The eddy loss h_e results from flow contractions and expansions and may be appreciable in non-prismatic channels. It is generally taken to be proportional to the absolute magnitude of the change in the velocity head between the two sections, or

$$h_e = k\left|\alpha_1 \frac{U_1^2}{2g} - \alpha_2 \frac{U_2^2}{2g}\right| \tag{2.15}$$

where k is a coefficient which is assumed to range between 0 and 0.1 for gradual contractions, between 0 and 0.2 for gradual expansions and to have a value of 0.5 for abrupt expansions or contractions. The eddy loss is usually neglected in prismatic channels.

The loss of energy may also be due to other reasons, like the presence of bends, flow past submerged bodies etc. and has to be included in the energy equation when it is encountered.

2.4 MOMENTUM EQUATION

The momentum equation is based on Newton's second law of motion which states that the algebraic sum of all the external forces acting on a fluid mass in any particular direction is equal to the time rate of change of momentum in that direction.

Let us consider the control volume bounded by sections 1 and 2 (Fig. 2.4). The various forces acting on the control volume in the longitudinal direction are:

i) the resultant hydrostatic pressure forces F_{p1} and F_{p2} at the two end sections,

ii) the force due to gravity, $W\sin\theta$, which is the component of the weight of water in the longitudinal direction, and

iii) the external frictional force F_f due to boundary friction acting on the surface of contact between water and the channel.

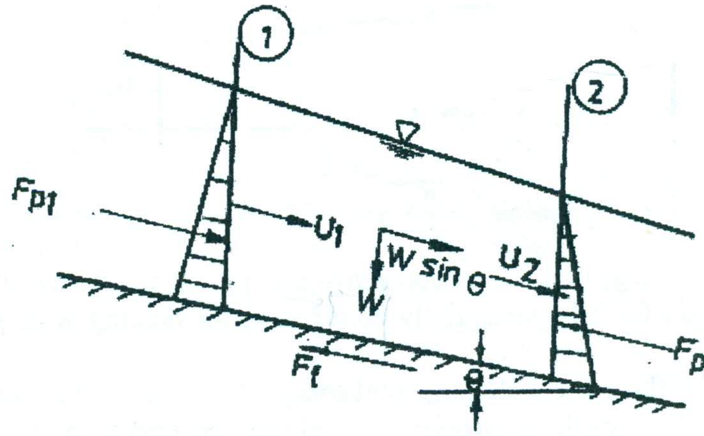


Fig.2.4 Definition sketch for the momentum equation

The momentum of flow passing a channel section per unit time = $\beta\rho QU$, where β is the momentum coefficient. Then, applying the Newton's second law of motion, we can write

$$\rho Q(\beta_2 U_2 - \beta_1 U_1) = F_{p1} - F_{p2} + W \sin\theta - F_f \tag{2.16}$$

which is the momentum equation for one-dimensional steady flow for the flow situation shown in Fig. 2.4.

Note that the terms in the momentum equation have the units of force, i.e. Newton or kg-m/s^2.

If any other external forces are present, they have to be included in the momentum equation. The momentum equation is a particularly useful tool in analyzing rapidly varied expanding flow like a hydraulic jump where the energy losses are significant. The momentum equation is very useful in estimating the forces on different hydraulic structures.

2.5 APPLICABILITY OF THE EQUATIONS

For steady one-dimensional open channel flow we have three basic equations that can be used to compute three unknown quantities. However, in many flow problems, usually it is required to compute two unknown quantities and we need only two equations. For example, it may be required to compute flow depth h and flow velocity U at a downstream section in a channel when the flow conditions at an upstream section are known. The equation of continuity is invariably used as it is the simplest of the three equations. Then, the choice remains whether we will use the energy or the momentum equation. Since these equations are applied between two channel sections, the condition in the channel between the two sections usually determines whether the energy or the momentum equation is to be used.

The energy equation contains a term h_L of internal energy losses. So this equation can be used initially only when this energy loss term h_L is small and negligible. On the other hand, the momentum equation contains an external friction force F_f. So this equation can be used initially only when F_f is small and negligible and, in addition, if any other external force is not present.

As an example, let us consider the flow under a sluice gate as shown in Fig. 2.5. Flow under a sluice gate is an example of converging flow in which energy losses between sections 1 and 2 are usually small and negligible. The flow depth h_2 and the flow velocity U_2 at section 2 can be determined from the known flow conditions at section 1 and using the continuity and energy equations. Initially the momentum equation cannot be used for this situation because of the force on the sluice gate that is unknown and not negligible, although the external friction force F_f is small and negligible. However, once h_2 and U_2 are determined, the momentum equation can be used to compute the force on the sluice gate.

For the hydraulic jump downstream of the sluice gate, the energy equation cannot be initially used because of the significant energy loss h_L involved in the jump. However, the momentum equation can be used without difficulty since the jump takes place in a short distance and the friction force F_f is small and negligible. Therefore, the continuity and momentum equations are used to compute the flow depth h_3 and the flow velocity U_3. The energy equation can then be used to compute the unknown energy loss h_L.

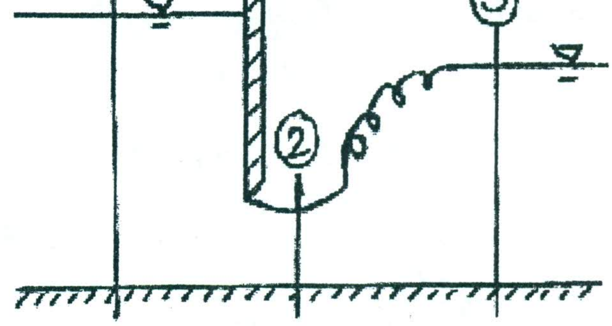


Fig.2.5 Flow beneath a sluice gate

For steady uniform and gradually varied flows, the energy and the momentum equations yield the same results and we can use either of these two equations. However, the energy equation is preferred because it is easy to understand and use than the momentum equation since energy is a scalar quantity and momentum is a vector quantity.

Example 2.1 Derivation of the Law of Torricelli

If a constant level is maintained in a vessel with atmospheric pressure at the water surface and at the discharge point (Fig. 2.6), then application of the Bernoulli equation between points 1 and 2 yields

$$H+0+\frac{U_1^2}{2g}=0+0+\frac{U_2^2}{2g} \tag{2.17}$$

For a wide vessel, $U_1 << U_2$. Hence, from Eq.(2.17), writing U for U_2, it follows that

$$U=\sqrt{2gH} \tag{2.18}$$

which is the Law of Torricelli.

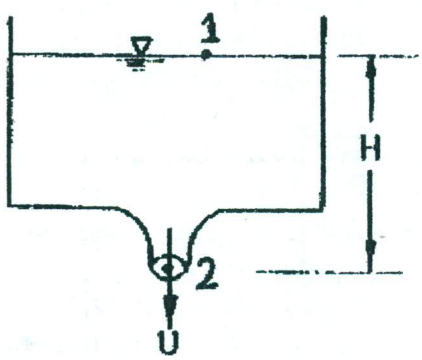


Fig. 2.6

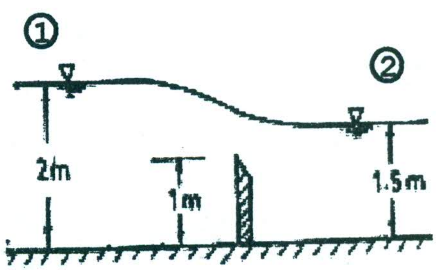


Fig. 2.7

Example 2.2

Figure 2.7 shows a sharp-crested weir in a rectangular channel. If the discharge per unit width of the weir is 4 m^2/s, estimate the energy loss due to the weir and force on the weir plate for the submerged flow condition as shown.

Solution Let the force exerted by the weir plate on water is F. Then, assuming unit width

$$U_1=\frac{q}{h_1}=\frac{4}{2}=2\ m/s$$

$$U_2=\frac{q}{h_2}=\frac{4}{1.5}=2.67\ m/s$$

Applying the energy equation between sections 1 and 2, we obtain

$$h_1+\frac{U_1^2}{2g}=h_2+\frac{U_2^2}{2g}+h_f$$

$$\therefore h_f = (h_1 - h_2) + (\frac{U_1^2}{2g} - \frac{U_2^2}{2g}) = (2.00 - 1.50) + \frac{(2^2 - 2.67^2)}{2 \times 9.81}$$

$$= 0.50 - 0.16 = 0.34 \ m \ of \ water$$

Applying the momentum equation between sections 1 and 2 and assuming unit width, we obtain

$$\rho q(U_2 - U_1) = \frac{1}{2}\gamma h_1^2 - \frac{1}{2}\gamma h_2^2 - F$$

$$\therefore F = \frac{1}{2}\gamma(h_1^2 - h_2^2) - \rho q(U_2 - U_1)$$

$$= \frac{1}{2} \times 1000 \times 9.81 \times (2^2 - 1.5^2) - 1000 \times 4 \times (2.67 - 2.0)$$

$$= 8583.75 - 2666.67 = 5917.08 \, N = 5.92 \, kN$$

The force on the weir plate is equal and opposite to F.

PROBLEMS

2.1 When a Pitot tube is placed in an open channel (Fig. 2.8), the water rises in the tube to a height H. Applying the Bernoulli equation between points 1 and 2, show that the velocity of stream upstream of the tube is

$$u = \sqrt{2gH}$$

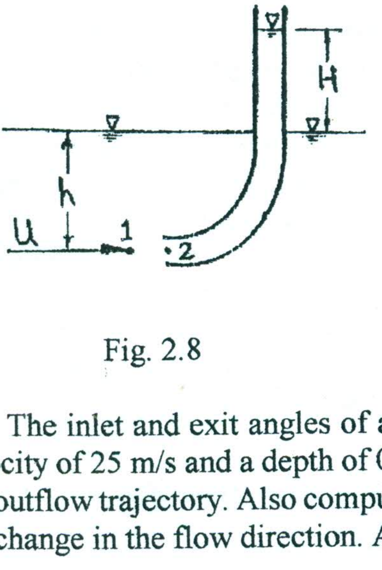


Fig. 2.8

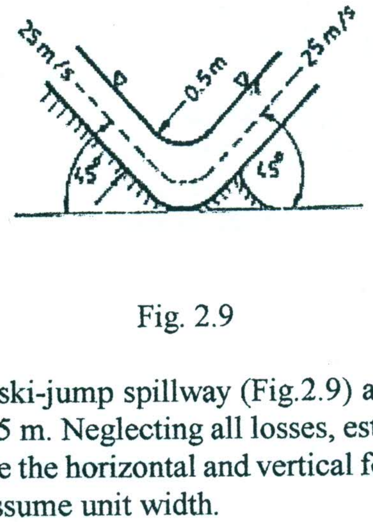


Fig. 2.9

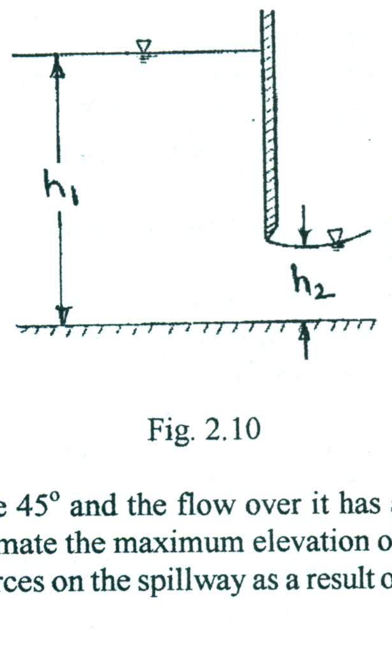


Fig. 2.10

2.2 The inlet and exit angles of a ski-jump spillway (Fig.2.9) are 45° and the flow over it has a velocity of 25 m/s and a depth of 0.5 m. Neglecting all losses, estimate the maximum elevation of the outflow trajectory. Also compute the horizontal and vertical forces on the spillway as a result of the change in the flow direction. Assume unit width.

2.3(a) Show that the force on a vertical sluice gate in a horizontal rectangular channel (Fig.2.10) is given by

$$F = \frac{1}{2}\gamma \frac{(h_1 - h_2)^3}{h_1 + h_2}$$

where γ is the specific weight of water.

(b) The depths of flow a short distance upstream and at the vena contracta downstream of a vertical sluice gate in a horizontal rectangular channel are 4 m and 1 m, respectively. The width of the channel is 6 m.

i) Neglecting energy losses and taking $\alpha_1 = \alpha_2 = 1$, compute the discharge under the gate.

ii) Compute the force on the sluice gate and compare it with that obtained by assuming hydrostatic pressure distribution. Assume that the coefficient of contraction, $C_c = 0.61$.

2.4 A bridge across a river has its piers placed symmetrically at the rate of 30 m center to center. Upstream of the bridge the water depth is 10 m and the velocity is 4 m/s. When the flow has gone far enough downstream to even out again after the disturbance caused by the piers, the depth is 9 m. Compute the thrust on each pier. Neglect the bed slope and the bed friction.

****

Chapter 3

SPECIFIC ENERGY AND CRITICAL FLOW

3.1 SPECIFIC ENERGY

Specific energy (E) at a channel section may be defined as the energy measured with respect to the channel bottom. Thus, from Eq. (2.10) with $z_b = 0$

$$E = h\cos^2\theta + a\frac{U^2}{2g} \tag{3.1}$$

and for a channel of small slope and $\alpha = 1$

$$E = h + \frac{U^2}{2g} \tag{3.2}$$

which indicates that the specific energy is the sum of the depth of flow and the velocity head. Since U = Q/A, Eq. (3.2) may also be written as

$$E = h + \frac{Q^2}{2gA^2} \tag{3.3}$$

which shows that the specific energy depends on the channel section, the depth of flow h and the discharge Q.

The concept of specific energy introduced by Bakhmeteff in 1912 is very useful in the analysis of many open channel flow problems.

Specific Energy Curve

The variation of specific energy with depth for a given section and a constant discharge using Eq. (3.3) is shown in Fig. 3.1. The resulting curve, which is known as the specific energy curve or E-h curve, has two limbs CA and CB. As $h \to 0$, $U^2/2g \to \infty$, $E \to \infty$ and the limb CA approaches the E axis asymptotically toward the right. As $h \to \infty$, $U^2/2g \to 0$, $E \to h$ and the limb CB approaches the line OP whose equation is E = h.

For all points on the specific energy curve excepting point C, there are two values of h for a given value of E, for instance, the lower depth h_1 and the higher depth h_2. These are known as the *alternate depths*. At point C on the specific energy curve, the specific energy is minimum. The state of flow represented by point C is obtained by taking the first derivative of E with respect to h from Eq.(3.3) holding Q constant, i.e.

$$\frac{dE}{dh} = 1 + \frac{Q^2}{2g}\frac{d}{dh}(A^{-2}) = 1 + \frac{Q^2}{2g}(-2)A^{-3}\frac{dA}{dh} = 1 - \frac{Q^2}{gA^3}\frac{dA}{dh} = 1 - \frac{U^2}{gA}\frac{dA}{dh}$$

The elementary water area near the free surface is dA = Bdh, so that $dA/dh = B$. Since $D = A/B$ and $Fr = U/\sqrt{gD}$, we obtain

$$\frac{dE}{dh} = 1 - \frac{U^2 B}{gA} = 1 - \frac{U^2}{gA/B} = 1 - \frac{U^2}{gD} = 1 - Fr^2 \tag{3.4}$$

Now, for minimum specific energy, dE/dh = 0, and hence $1 - Fr^2 = 0$, i.e. Fr = 1, which is the criterion for critical flow. Thus, at the critical state of flow, the specific energy is minimum for a given discharge.

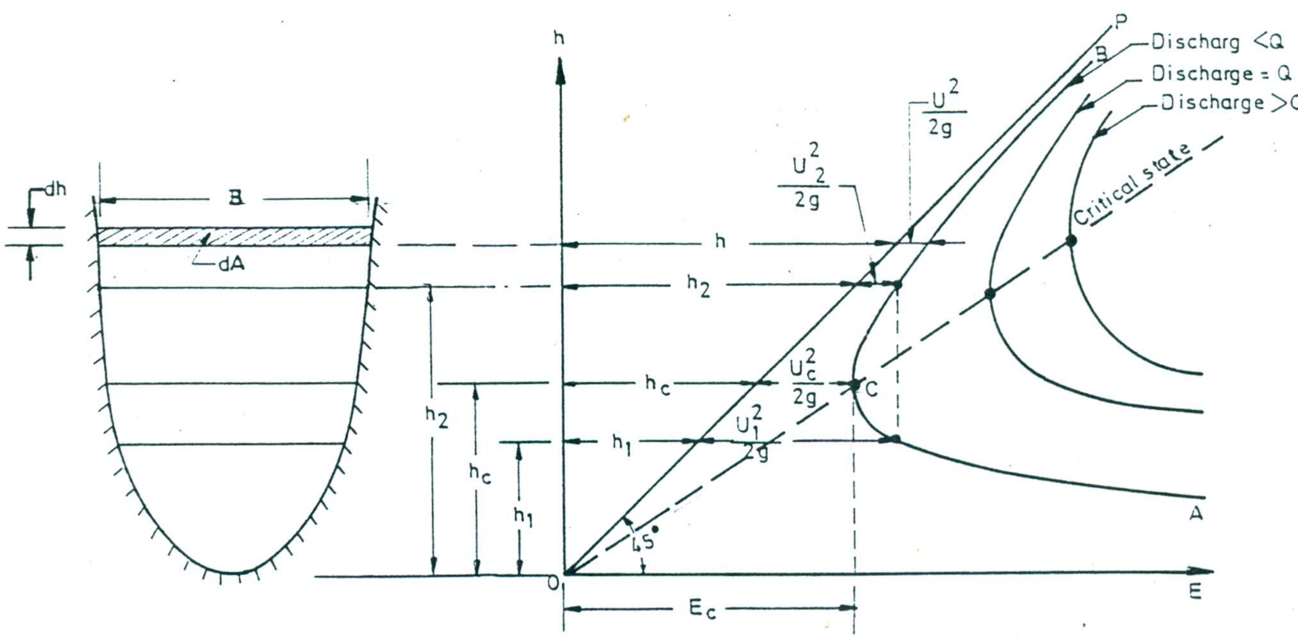


Fig. 3.1 Specific energy curve

Obviously, when the depth of flow is greater than the critical depth, the velocity of flow is less than the critical velocity for the given discharge and the flow is subcritical. When the depth of flow is less than the critical depth, the velocity of flow is greater than the critical velocity and the flow is supercritical. Hence, h_1 is the depth of supercritical flow and h_2 is the depth of subcritical flow and the limbs CB and CA represent subcritical and supercritical flows, respectively. Also, Eq. (3.4) demonstrates that, for a given discharge, the specific energy in a channel section increases with an increase in depth when the flow is subcritical and decreases with an increase in depth when the flow is supercritical. Thus, dE/dh is positive on limb CB, zero at point C and negative on limb CA.

For a channel of large slope and $\alpha \neq 1$, it can be shown that the condition for the critical state of flow is given by

$$Fr = \frac{U}{\sqrt{(gD\cos\theta)/\alpha}} = 1 \qquad (3.5)$$

In this case, the line OP makes an angle $\psi\,(>45^0)$ with the E axis given by

$$\psi = \tan^{-1}\sec^2\theta \qquad (3.6)$$

where θ is the angle made by the channel bottom with the horizontal.

When the discharge changes the specific energy also changes. As shown in Fig. 3.1, the specific energy curves for the flow rates greater and less than Q lie respectively to the right and left of the curve for Q indicating that the specific energy increases with an increase in discharge and vice versa.

In an open channel flow the energy grade line always slopes downward and the available energy is decreased. The specific energy, however, remains constant for uniform flow and can either increase or decrease along the channel in varied flow.

Near the critical state the E-h curve is almost vertical (Fig. 3.1) and a small change in E results in a large change in h. As a result, flow at or near the critical condition is unstable and there will be an wavy water surface. Hence, it is undesirable to design channels at or near the critical state.

Discharge-Depth Curve

So far the critical flow condition has been derived by considering the variation of E with h for a given Q. It is also of practical interest to study the variation of Q with h for a given E. Equation (3.3) may be written as

$$Q^2 = 2gA^2(E-h) \qquad (3.7)$$

Clearly, the variation of Q with h will be of the general form shown in Fig. 3.2. When either h = 0 or h = E, Q = 0 and there will be a maximum value of Q for some value of h between 0 and E which may be obtained as follows. Differentiating Eq. (3.7) with respect to h and using Eq.(3.2) one obtains

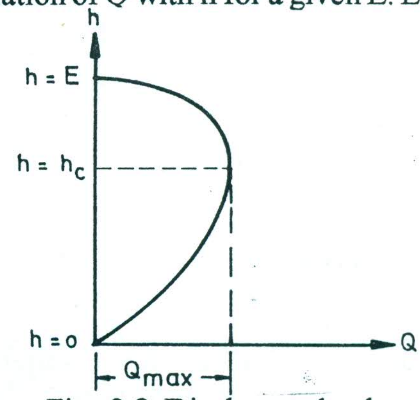


Fig. 3.2 Discharge-depth curve for a given specific energy

$$2Q\frac{dQ}{dh} = 2g[A^2(-1)+(E-h)\times 2A\frac{dA}{dh}] = 2g[-A^2 + \frac{U^2}{2g}\times 2AB]$$

$$=-2gA^2[1-\frac{U^2}{gA/B}] = -2gA^2[1-\frac{U^2}{gD}] = -2gA^2(1-Fr^2) \qquad (3.8)$$

For maximum discharge, dQ/dh = 0, and we obtain Fr = 1, which is the criterion for critical flow. Thus, at the critical state of flow, the discharge is maximum for a given specific energy. When the specific energy increases, the discharge also increases and vice versa.

3.2 CRITICAL FLOW

The critical flow has been defined as the flow for which the numerical value of the Froude number is equal to unity. Using Eq.(1.12), it can be shown that at the critical state of flow

$$\frac{U_c^2}{2g} = \frac{D_c}{2} \qquad (3.9)$$

and when α ≠ 1

U Fr

$$a\frac{U_c^2}{2g} = \frac{D_c}{2} \qquad (3.10)$$

i.e., the velocity head is equal to one-half of the hydraulic depth.

For a channel of large slope and α ≠ 1, the condition for the critical state of flow is given, using Eq.(3.5), by

$$a\frac{U_c^2}{2g} = \frac{D_c \cos\theta}{2} \qquad (3.11)$$

For a rectangular channel, $D_c = h_c$. Hence

$$\alpha\frac{U_c^2}{2g} = \frac{D_c}{2} = \frac{h_c}{2}$$

$$\therefore\ E_c = h_c + \alpha\frac{U_c^2}{2g} = h_c + \frac{h_c}{2} = 1.5h_c \tag{3.12}$$

Section Factor for Critical Flow Computation

The product of the flow area and the square root of the hydraulic depth is known as the section factor in connection with critical flow, denoted by Z, i.e.

$$Z = A\sqrt{D} \tag{3.13}$$

It can be computed when the channel section and the depth of flow h are given.

When the flow is critical, the product of the flow area and the square root of the hydraulic depth is known as the section factor for critical flow computation, denoted by Z_c, i.e.

$$Z_c = A_c\sqrt{D_c} \tag{3.14}$$

Substituting U = Q/A in Eq. (3.10) and using Eq.(3.14), it can be shown that

$$\alpha\frac{Q^2}{2gA_c^2} = \frac{D_c}{2} \quad or,\ A_c^2D_c = \frac{Q^2}{g/\alpha} \quad or,\ \ Z_c = \frac{Q}{\sqrt{g/\alpha}} \tag{3.15}$$

Thus, Z_c can be computed using Eq.(3.14) when the channel section and the critical depth h_c are given, or alternatively, using Eq.(3.15) when the discharge Q and the energy coefficient α are given. Equations (3.14) and (3.15) are very useful in the analysis of critical flow.

Using Eqs.(1.12), (3.13) and (3.14), we obtain

$$Fr^2 = \frac{\alpha U^2}{gD} = \frac{\alpha Q^2}{gA^2D} = \frac{\alpha Q^2/g}{A^2D} = \frac{Z_c^2}{Z^2} \tag{3.16}$$

Hydraulic Exponent for Critical Flow Computation

The section factor Z is a function of the depth of flow for a given channel section. It is convenient to express Z in the form

$$Z^2 = C_1h^M \tag{3.17}$$

where C_1 is a coefficient and M is an exponent which is known as the hydraulic exponent for critical flow computation.

Taking logarithm of both sides of Eq.(3.17) and then differentiating with respect to h, one obtains

$$\frac{d(\ln Z)}{dh} = \frac{M}{2h} \tag{3.18}$$

Also, substituting D = A/B, taking logarithm of both sides of Eq. (3.13) and then differentiating with respect to h, we get

$$\frac{d(\ln Z)}{dh}=\frac{3}{2}\frac{1}{A}\frac{dA}{dh}-\frac{1}{2}\frac{1}{B}\frac{dB}{dh}=\frac{3B}{2A}-\frac{1}{2B}\frac{dB}{dh}=\frac{1}{2A}(3B-D\frac{dB}{dh}) \tag{3.19}$$

Equating the right sides of Eqs. (3.18) and (3.19) and solving for M, we obtain

$$M=\frac{h}{A}(3B-D\frac{dB}{dh}) \tag{3.20}$$

which is the general equation for the hydraulic exponent for critical flow computation M and indicates that M is a function of the channel section and the depth of flow. The values of M for different channel sections are given in Table 3.1.

Table 3.1 Values of M for different channel sections

Channel section	M
1. Rectangular	**3**
2. Triangular	**5**
3. Parabolic	**4**
4. Trapezoidal	$\frac{3[1+2s(h/b)]^2-2s(h/b)[1+s(h/b)]}{[1+2s(h/b)][1+s(h/b)]}$
5. Circular	$(1-\cos\omega/2)\frac{12\sin\omega/2}{\omega-\sin\omega}-\frac{\cos\omega/2}{\sin^2\omega/2}$

Example 3.1

Determine the numerical value of the hydraulic exponent for critical flow computation M for a rectangular channel.

Solution For a rectangular channel, A = bh, B = b, D = A/B = h, dB/dh = 0

$$\therefore\ M=\frac{h}{A}(3B-D\frac{dB}{dh})=\frac{h}{bh}(3b-h\times 0)=3$$

Example 3.2

Compute the hydraulic exponent for critical flow computation M for a trapezoidal channel with b = 6.1 m, s = 2 and h = 2 m.

Solution h/b = 2/6.1 = 0.328

$$M=\frac{3[1+2s(h/b)]^2-2s(h/b)[1+s(h/b)]}{[1+2s(h/b)][1+s(h/b)]}$$

$$=\frac{3(1+2\times2\times0.328)^2-2\times2\times0.328\times(1+2\times0.328)}{(1+2\times2\times0.328)(1+2\times0.328)}=3.62$$

Alternative solution A = (6.1 + 2×2)×2 = 20.2 m^2, B = 6.1 + 2×2×2 = 14.1 m

D = A/B = 20.2/14.1 = 1.43 m, dB/dh = 2s = 2×2 = 4

$$\therefore\ M = \frac{h}{A}(3B - D\frac{dB}{dh}) = \frac{2}{20.2}(3\times14.1 - 1.43\times4) = 3.62$$

Computation of Critical Depth

Analytical Method

The critical depth is an important parameter in the analysis of open channel flow. It may be computed when the channel section, the energy coefficient α and the discharge Q are given. For wide, rectangular, triangular and parabolic channels, the following analytical expressions for the critical depth can be easily obtained.

Wide channel

$$h_c = \sqrt[3]{\frac{\alpha q^2}{g}} \qquad (3.21)$$

$y_c = \left(\frac{\alpha q^2}{g}\right)^{1/3}$

Rectangular channel

$$h_c = \sqrt[3]{\frac{\alpha Q^2}{g b^2}} \qquad (3.22)$$

Triangular channel

$$h_c = \sqrt[5]{\frac{2\alpha Q^2}{g s^2}} \qquad (3.23)$$

Parabolic channel (z = cy^2)

$$h_c = \sqrt[4]{\frac{27\alpha c Q^2}{32 g}} \qquad (3.24)$$

Example 3.3

Compute the critical depth and velocity in a (i) wide rectangular channel with q = 3 m^2/s, (ii) rectangular channel with b = 6 m and Q = 20 m^3/s, (iii) triangular channel with s = 2 and Q = 10 m^3/s, and (iv) parabolic channel whose profile is given by y^2 = 4z with Q = 20 m^3/s. In all cases assume α = 1.12.

Solution α = 1.12

(i) Wide channel q = 3 m^2/s

$$h_c = \sqrt[3]{\frac{\alpha q^2}{g}} = \sqrt[3]{\frac{1.12\times3^2}{9.81}} = 1.01\ m$$

$$U_c = \frac{q}{h_c} = \frac{3}{1.01} = 2.97\ m/s$$

(ii) Rectangular channel, b = 6 m, Q = 20 m^3/s

$$h_c = \sqrt[3]{\frac{\alpha Q^2}{gb^2}} = \sqrt[3]{\frac{1.12 \times 20^2}{9.81 \times 6^2}} = 1.08\ m$$

Then,

$$A_c = bh_c = 6 \times 1.08 = 6.50\, m^2$$

$$U_c = \frac{Q}{A_c} = \frac{20}{6.50} = 3.08\ m/s$$

(iii) Triangular channel, s = 2, Q = 10 m^3/s

$$h_c = \sqrt[5]{\frac{2\alpha Q^2}{gs^2}} = \sqrt[5]{\frac{2 \times 1.12 \times 10^2}{9.81 \times 2^2}} = 1.42\ m$$

Then,

$$A_c = sh_c^{\ 2} = 2 \times 1.42^2 = 4.01\ m^2$$

$$U_c = \frac{Q}{A_c} = \frac{10}{4.01} = 2.49\ m/s$$

(iv) Parabolic channel, $y^2 = 4z$, $z = 0.25y^2$, $c = 0.25$, Q = 20 m^2/s

$$h_c = \sqrt[4]{\frac{27\alpha c Q^2}{32g}} = \sqrt[4]{\frac{27 \times 1.12 \times 0.25 \times 20^2}{32 \times 9.81}} = 1.76\ m$$

Then,

$$A_c = \frac{4h_c^{\ 3/2}}{3\sqrt{c}} = \frac{4 \times 1.76^{3/2}}{3 \times \sqrt{0.25}} = 6.24\ m^2$$

$$U_c = \frac{Q}{A_c} = \frac{20}{6.24} = 3.21\ m/s$$

Trial-and-Error Method

For other simple geometric channel sections, like the trapezoidal and circular channel sections, the critical depth can be conveniently obtained by the trial-and-error solution of Eq. (3.15).

Example 3.4

For a trapezoidal channel with b = 6 m and s = 2, compute the critical depth and velocity if Q = 50 m^3/s. Take $\alpha = 1$.

Solution Trapezoidal channel, b = 6 m, s = 2, Q = 50 m^3/s, $\alpha = 1$

$$Z_c = \frac{Q}{\sqrt{g/\alpha}} = \frac{50}{\sqrt{9.81/1}} = 15.964$$

Now, assume several values of h and compute the section factor $Z = A\sqrt{D}$ until the computed value of Z is very close to 15.964.

h (m)	A (m²)	B (m)	D (m)	Z=A√D	Remarks
1.00	8.000	10.00	0.800	7.155	h small
2.00	20.000	14.00	1.429	23.905	h large
1.60	14.720	12.40	1.187	16.038	h closest
1.59	14.596	12.36	1.181	15.862	

Hence, the critical depth, $h_c = 1.60$ m and the critical velocity

$$U_c = \frac{Q}{A_c} = \frac{50}{14.720} = 3.40\ m/s$$

Example 3.5

A circular channel 2 m in diameter carries a discharge of 4 m ^{3}/s. Compute the critical depth and velocity. Take $\alpha = 1.10$.

Solution Circular section, $d_0 = 2$ m, $Q = 4\ m^3/s$, $\alpha = 1.10$

$$Z_c = \frac{Q}{\sqrt{g/a}} = \frac{4}{\sqrt{9.81/1.10}} = 1.339$$

ω (rad)	A (m²)	B (m)	D (m)	Z=A√D
1	0.079	0.959	0.083	0.023
2	0.545	1.683	0.324	0.310
3	1.429	1.995	0.716	1.210
4	2.378	1.819	1.308	2.719
3.10	1.529	2.000	0.765	1.337
3.11	1.539	2.000	0.770	1.350

Hence, $\omega = 3.10$ rad and the critical depth

$$h_c = \frac{d_0}{2}(1 - \cos\frac{\omega}{2}) = \frac{2}{2}(1 - \cos\frac{3.10}{2}) = 0.98\ m$$

and, the critical velocity

$$U_c = \frac{Q}{A_c} = \frac{4}{1.529} = 2.62\ m/s$$

Numerical Methods

A number of numerical methods are available for solving non-linear algebraic equations involving a single variable, e.g. the method of bisection, the method of iteration, the method of false position, the secant method, the Newton-Raphson method etc. (Churchhouse, 1981). These methods can be conveniently used to compute the critical depth in trapezoidal and circular channel sections. The application of the method of bisection and the Newton-Raphson method for computing critical depth is considered here.

Bisection method: The bisection method is very convenient to solve an algebraic equation $f(x) = 0$ which contains only one root. Suppose we want to compute the critical depth in a channel for a given section, discharge Q and energy coefficient α. Then the function

$$f(h) = 1 - Fr^2 = 1 - (Z_c/Z)^2 = 1 - \frac{\alpha Q^2 B}{gA^3} \tag{3.25}$$

must be satisfied by some positive depth greater than say h_{min} and less than say h_{max}. The critical depth is taken equal to $(h_{min} + h_{max})/2$ and f(h) is determined. If f(h) is positive, then the root is less than (h_{min} + h_{max})/2 and the upper limit is taken as (h_{min} + h_{max})/2. On the other hand, if f(h) is negative, then the lower limit is taken as (h_{min} + h_{max})/2. The procedure is repeated till the desired accuracy is attained.

Example 3.6

For a trapezoidal channel with b = 6 m and s = 2, compute the critical depth by the method of bisection if Q = 14 m^3/s and α = 1.

Solution A = (6+2h)h, B = 6+4h, $\alpha Q^2/g$ = 19.98

$$f(h)=1-\frac{\alpha Q^2 B}{gA^3}=1-\frac{19.98(6+4h)}{[(6+2h)h]^3}$$

Initially the values of h_{min} and h_{max} are taken as 0 and 10 m, respectively. The computation is carried out as follows.

h_{min}	h_{max}	h=(h_{min}+ h_{max})/2	f(h)	Root lies between
0	10	5	0.999	0 and 5
0	5	2.5	0.984	0 and 2.5
0	2.5	1.25	0.8168	0 and 1.25
0	1.25	0.625	- 0.8254	0.625 and 1.25
0.625	1.25	0.9375	0.5159	0.625 and 0.9375
0.625	0.9375	0.7813	0.1160	0.625 and 0.7813
0.625	0.7813	0.7031	- 0.2468	0.7031 and 0.7813
0.7031	0.7813	0.7422	- 0.0455	0.7422 and 0.7813
0.7422	0.7813	0.7617	0.0396	0.7422 and 0.7617
0.7422	0.7617	0.7519	- 0.0018	0.7520 and 0.7617
0.7520	0.7617	0.7568	0.0192	0.7520 and 0.7568
0.7520	0.7568	0.7544	0.0088	0.7520 and 0.7544

Hence, the critical depth, h_c = 0.75 m

In general, 12 to 15 iterations, depending on the values of h_{min} and h_{max} taken, reduce to interval within which the root is correct up to 0.01 m.

Newton - Raphson Method: The Newton-Raphson method is particularly convenient for solving an algebraic equation which is easily differentiable and when the value of the desired root is known approximately. Let y = f(x) be the equation, with x = x_n an approximation of the root. Then a better approximation x_{n+1} to the root is obtained using the equation

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)} \tag{3.26}$$

Suppose we want to compute the critical depth in a channel for a given section, Q and α. Obviously when h = h_c

$$1-Fr^2=1-(Z/Z_c)^2=0 \qquad or, \qquad 1-\frac{\alpha Q^2 B}{gA^3}=0 \qquad or, \qquad A^3-\frac{\alpha Q^2 B}{g}=0$$

If we now assume

$$f(h)=A^3-\frac{\alpha Q^2 B}{g} \tag{3.27}$$

then

$$f'(h)=3A^2B-\frac{\alpha Q^2}{g}\frac{dB}{dh} \tag{3.28}$$

For a given section, $f(h)$ and $f'(h)$ depend on the depth of flow only and can be easily obtained.

Example 3.7

For a trapezoidal channel with b = 6 m and s = 2, compute the critical depth by the Newton-Raphson method if Q = 14 m^3/s and $\alpha = 1$.

Solution $A = (6+2h)h$, $B = 6+4h$, $dB/dh = 2s = 2\times2 = 4$

$$\therefore\ f(h) = A^3 - \frac{\alpha Q^2 B}{g} = [(6+2h)h]^3 - \frac{1\times14^2\times(6+4h)}{9.81}$$

$$= [(6+2h)h]^3 - 19.9796(6+4h)$$

and, $$f'(h) = 3A^2B - \frac{\alpha Q^2}{g}\frac{dB}{dh} = 3[(6+2h)h]^2(6+4h) - \frac{1\times14^2\times4}{9.81}$$

$$= 24(3+2h)[(3+h)h]^2 - 79.918$$

The computation of critical depth is carried out as follows.

f	$f(h)$	$f'(h)$	$\Delta h = -\dfrac{f(h)}{f'(h)}$	$h = h + \Delta h$
1.000	312.204	1840.082	-0.170	0.830
0.830	70.776	1050.264	-0.067	0.773
0.773	8.488	815.536	-0.010	0.753
0.753	0.496	783.752	-0.001	0.752

Hence, the critical depth, $h_c = 0.75$ m

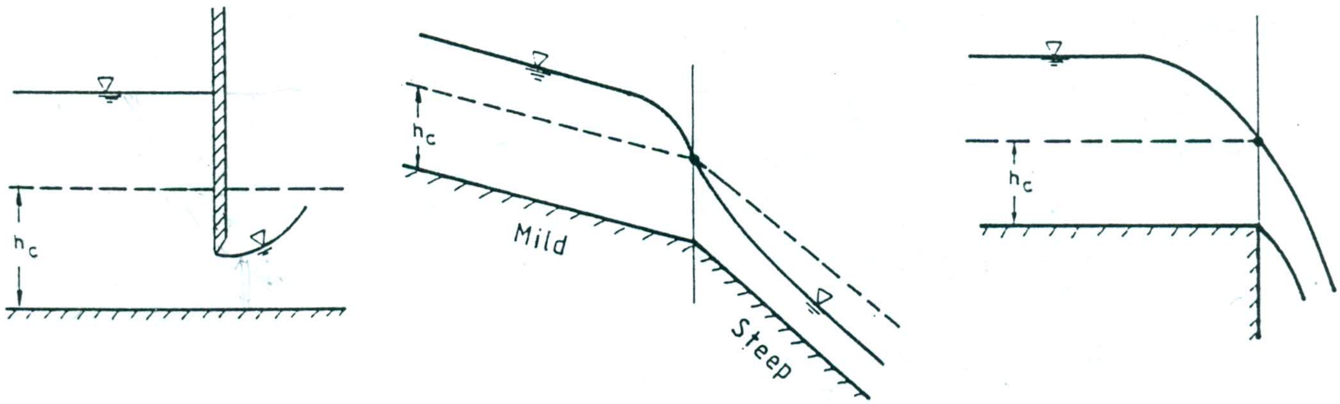
The Newton-Raphson method is particularly suitable for computing the critical depth in an open channel. Normally it is necessary to repeat the procedure 3 to 4 times to obtain a value of the root correct up to 0.01 m..

Critical Flow and Control

The concept of control is of paramount importance in the study of open channel flow and is closely related to the concept of critical flow. Any feature which establishes a definite relationship between the depth (or the stage) and the discharge is a control. From this definition, it follows that for any feature which acts as a control the discharge can be computed once the depth of flow (or the stage) is known and vice versa.

The location of a control in a channel is governed by the state of flow in the channel. In has been shown in Art. 1.5 that a small disturbance (e.g. an elementary wave) can travel upstream in subcritical flow and can only travel downstream in supercritical flow. That is to say, subcritical flow is affected by conditions downstream and supercritical flow is affected by conditions upstream. Accordingly, subcritical flow is subjected to downstream control and supercritical flow is subjected to upstream control. In other words, the flow upstream of a control must be subcritical and that downstream of a control must be supercritical.

Control sections occur at entrances and exits to channels and at changes in channel slopes, under certain conditions. A gate in a channel can be a control for both the upstream and downstream flows. Three control sections are illustrated in Fig 3.3. In Fig. 3.3(a), a sluice gate in a horizontal channel provides control for both upstream and downstream from it. In Fig. 3.3(b) a change in channel slope from mild to steep causes the flow to change from subcritical to supercritical and a control section exists at or near the break in bottom slope. In Fig. 3.3(c) the free overfall at the end of a horizontal channel causes the flow to change from subcritical to supercritical and a control section occurs behind the brink.



(a) (b) (c)

Fig. 3.3 Control sections

A control (or a control section) can be conveniently used for flow measurement since the discharge is readily obtained once the depth of flow (or the stage) is known or measured. On the other hand, for a channel section that is not a control section, the discharge is obtained by the area-velocity method (Art.1.6) which is lengthy and involves elaborate measurements of cross-section and velocities. It is for this reason that a control section is always a suitable site for a gauging station for developing the discharge rating curves.

A critical flow section is a control section, because it establishes a definite relationship between the depth and the discharge, represented by Eq. (3.15), which is independent of channel roughness and other uncontrolled circumstances. As a result, a critical flow section can be conveniently used for flow measurement (Art. 3.4). The control sections in Figs. 3.3(b) and (c) are actually the critical flow sections.

3.3 TRANSITION PROBLEM

A transition may be defined as a change either in the direction or slope or cross-section (i.e. change in bed level and/or width) of the channel that produces a change, either temporary or permanent, in the state of the flow in the channel. The concepts of specific energy and critical flow are extremely useful for solving the transition problems. In this section, two simple transitions, viz. changes in the bed level and the channel width in rectangular channels are considered. The principles are also equally applicable to channels of any shape and other types of transitions and their combinations.

Channel with a Change in Bed Level

Example 3.8

Water flows at a velocity of 1 m/s and a depth of 1.50 m in a long rectangular channel 3 m wide. Compute (a) the height of a smooth upward step in the channel bed to produce critical flow, and (b) the depth and the change in water level produced by (i) a smooth upward step of 0.45 m, (ii) a smooth upward step of 0.80 m, and (iii) a smooth downward step of 0.45 m, assuming that the discharge in the channel does not change. In all cases, neglect energy losses and take $\alpha = 1$.

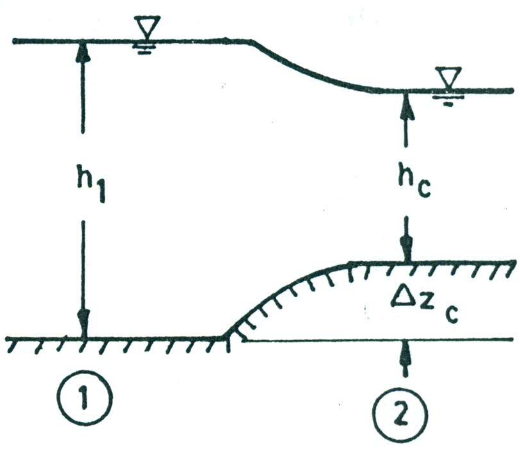
Solution Rectangular section, $b = 3$ m, $h_f = 0$, $\alpha = 1$,

a) Taking section 1 upstream of the step and section 2 over the step, we obtain

$$Q = A_1U_1 = bh_1U_1 = 3\times 1.50\times 1 = 4.5 \text{ m}^3/\text{s}$$

$$E_1 = h_1 + \frac{U_1^{\ 2}}{2g} = 1.50 + \frac{1^2}{2\times 9.81} = 1.55\,m$$

$$Fr_1 = \frac{U_1}{\sqrt{gD_1}} = \frac{U_1}{\sqrt{gh_1}} = \frac{1}{\sqrt{9.81\times 1.5}} = 0.26 < 1$$



Hence, the upstream flow is subcritical. Now,

$$h_c = \sqrt[3]{\frac{Q^2}{gb^2}} = \sqrt[3]{\frac{4.5^2}{9.81\times 3^2}} = 0.61m$$

$$U_c = \frac{Q}{A_c} = \frac{Q}{bh_c} = \frac{4.5}{3\times 0.61} = 2.45\ m/s$$

$$E_c = h_c + \frac{U_c^{\ 2}}{2g} = 0.61 + \frac{2.45^2}{2\times 9.81} = 0.92\ m$$

or, alternatively, for a rectangular channel

$$E_c = 1.5h_c = 1.5\times 0.61 = 0.92 \text{ m}$$

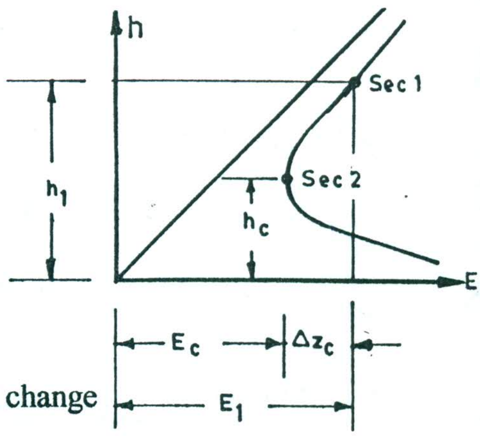
Applying the energy equation between sections 1 and 2 and taking the original channel bed as datum, we get

$$h_1 + \frac{U_1^{\ 2}}{2g} = h_c + \frac{U_c^{\ 2}}{2g} + \Delta z_c$$

or $\quad E_1 = E_c + \Delta z_c$

$\therefore \quad \Delta z_c = E_1 - E_c = 1.55 - 0.92 = 0.63$ m

$\therefore$ *Drop in water level* $= h_1 - h_c - \Delta z_c = 1.50 - 0.61 - 0.63 = 0.26\,m$



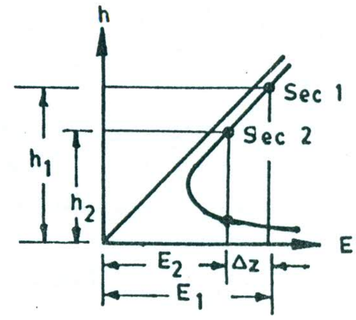
b)(i) In this case, $\Delta z = 0.45$ m. Since the upstream conditions do not change

$$Q = 4.5 \text{ m}^3/\text{s and } E_1 = 1.55 \text{ m}$$

The specific energy at section 2 is equal to

$$E_2 = E_1 - \Delta z = 1.55 - 0.45 = 1.10 \text{ m}$$

$$\therefore\ 1.10 = h_2 + \frac{Q^2}{2gb^2h_2^{\ 2}} = h_2 + \frac{4.5^2}{2\times 9.81\times 3^2\times h_2^{\ 2}} = h_2 + \frac{0.115}{h_2^{\ 2}}$$

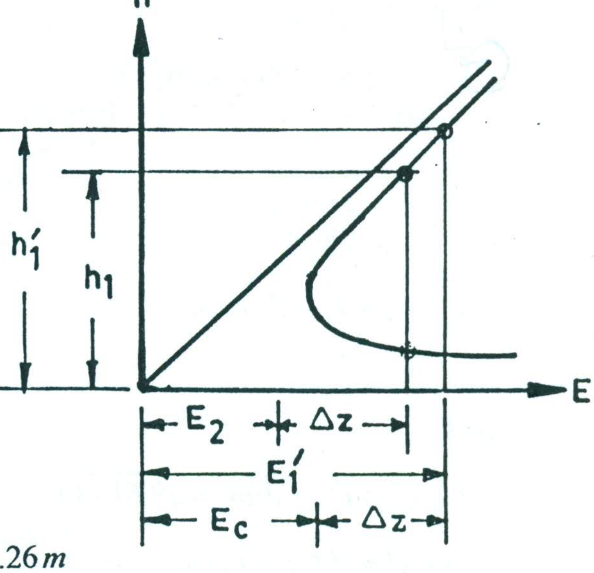


Solving for h_2 by trial, we get $h_2 = 0.98$ m or 0.41 m. Since the upstream flow is subcritical ($Fr_1 = 0.26$), the only subcritical value of 0.98 m is possible.

$\therefore h_2 = 0.98$ m

$\therefore$ *Drop in water level* $= h_1 - h_2 - \Delta z = 1.50 - 0.98 - 0.45 = 0.07\,m$

ii) In this case, $\Delta z = 0.80$ m. $\therefore E_2 = E_1 - \Delta z = 1.55 - 0.80 = 0.75$ m, which is less than $E_c = 0.92$ m. Hence, either the discharge in the channel has to reduce or the upstream specific energy has to increase. Since the discharge in the channel does not change, hence the upstream specific energy has to increase from E_1 to E_1'. However, the flow over the step will remain critical corresponding to $Q = 4.5\ m^3/s$.



$$\therefore h_2 = h_c = 0.61\ m \qquad E_2 = E_c = 0.92\ m$$

$$E_1' = E_c + \Delta z = 0.92 + 0.80 = 1.72\ m$$

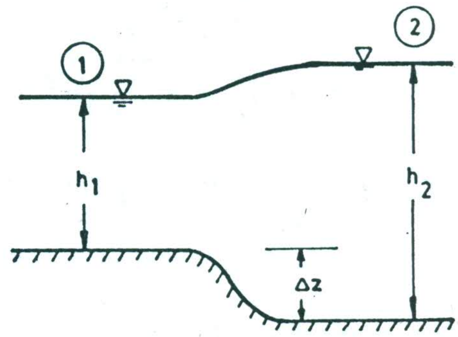
$$\therefore\ 1.72 = h_1' + \frac{Q^2}{2gb^2 h_1'^2} = h_1' + \frac{4.5^2}{2\times 9.81\times 3^2 \times h_1'^2} = h_1' + \frac{0.115}{h_1'^2}$$

Solving for h_1', we get

$$h_1' = 1.67\,m$$

$\therefore$ *Drop in water level* $= h_1' - h_c - \Delta z = 1.67 - 0.61 - 0.80 = 0.26\,m$

iii) Applying the energy equation between sections 1 and 2, we get



$$h_1 + \frac{U_1^2}{2g} + \Delta z = h_2 + \frac{U_2^2}{2g}$$

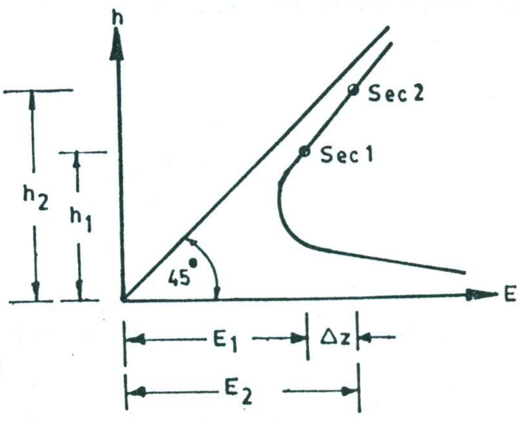
or $\qquad E_1 + \Delta z = E_2$

or, $\qquad E_2 = E_1 + \Delta z = 1.55 + 0.45 = 2.00$ m

$$\therefore\ 2.00 = h_2 + \frac{Q^2}{2gb^2 h_2^2} = h_2 + \frac{4.5^2}{2\times 9.81\times 3^2 \times h_2^2} = h_2 + \frac{0.115}{h_2^2}$$

Solving for h_2 by trial, we get

$h_2 = 1.97$ m



$\therefore$ *Rise in water level* $= h_2 - h_1 - \Delta z = 1.97 - 1.50 - 0.45 = 0.02\ m$

This example demonstrates that a critical state of flow can be produced in a channel by raising the channel bed, but not by lowering the channel bed. The amount by which the channel bed is to be raised is given by $\Delta z \geq \Delta z_c$

Channel with a Change in Width

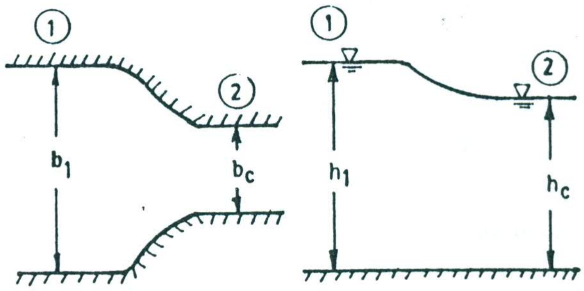
Example 3.9

Water flows at a velocity of 1 m/s and a depth of 1.50 m in a long rectangular channel 3 m wide. Compute (a) the contraction in width of the channel for producing critical flow, and (b) the depth and the change in water level produced by (i) a smooth contraction in width to 2 m, (ii) a smooth contraction in width to 1 m, and (iii) a smooth expansion in width to 4 m, assuming that the discharge in the channel does not change. In all cases, neglect energy losses and take $\alpha = 1$.

Solution

a) Taking section 1 upstream of the contraction and section 2 over the contraction, we obtain Q = 4.5 m^3/s and $E_1 = 1.55$ m. Then applying the energy equation between sections 1 and 2, we get

$$h_1 + \frac{U_1^{\,2}}{2g} = h_c + \frac{U_c^{\,2}}{2g}$$



or, $\quad E_1 = E_c = 1.55\ m$

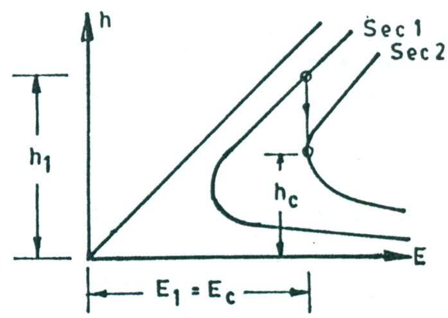
$$\therefore h_c = (2/3)E_c = 1.03\ m$$

and

$$U_c = \sqrt{gD_c} = \sqrt{gh_c} = \sqrt{9.81 \times 1.03} = 3.19\ m$$

Since $Q = A_c U_c = b_c h_c U_c$, we get

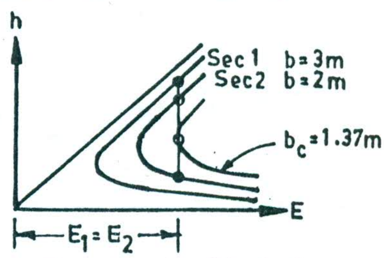
$$b_c = \frac{Q}{h_c U_c} = \frac{4.5}{1.03 \times 3.19} = 1.37\,m$$



Hence, the width required to produce critical flow, $b_c = 1.37\,m$.

$$\therefore \ \textit{Drop in water level} = h_1 - h_c = 1.50 - 1.03 = 0.47\ m$$

b)(i) Since $b_2 > b_c$, the upstream conditions do not change. Then applying the energy equation between sections 1 and 2



$$E_2 = E_1 = 1.55 = h_2 + \frac{Q^2}{2gb_2^{\,2}h_2^{\,2}} = h_2 + \frac{4.5^2}{2 \times 9.81 \times 2^2 \times h_2^{\,2}}$$

$$or, \qquad 1.55 = h_2 + \frac{0.258}{h_2^{\,2}}$$

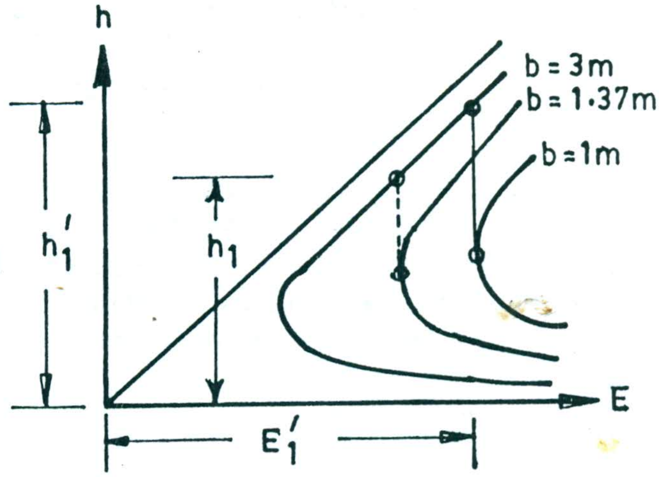
Solving by trial, we get $h_2 = 1.42$ m or 0.49 m. Since the upstream flow is subcritical, the only subcritical value of 1.42 m is possible, i.e.

$h_2 = 1.42$ m.

$$\therefore \ \textit{Drop in water level} = h_1 - h_2 = 1.50 - 1.42 = 0.08\ m$$

(ii) Since $b_2 < b_c$, the flow will not be possible with the given upstream conditions. Since the discharge in the channel does not change, the upstream specific energy has to increase and flow over the constriction will be critical corresponding to $b_2 = 1$ m. Therefore,

$$h_c = \sqrt[3]{\frac{Q^2}{g{b_2}^2}} = \sqrt[3]{\frac{4.5^2}{9.81\times 1^2}} = 1.27\ m$$



$$E_1' = E_c = 1.5h_c = 1.5\times 1.27 = 1.91\ m$$

$$or,\ h_1' + \frac{Q^2}{2g{b_1}^2{h_1'}^2} = h_1' + \frac{4.5^2}{2\times 9.81\times 3^2\times h_1} = 1.91$$

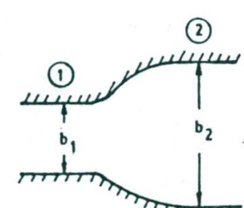
$$or,\ h_1' + \frac{0.1147}{{h_1'}^2} = 1.91$$

Solving by trial, we get

$$h_1' = 1.88\ m$$

$$\therefore\ Drop\ in\ water\ level = h_1' - h_c = 1.88 - 1.27 = 0.61\ m$$

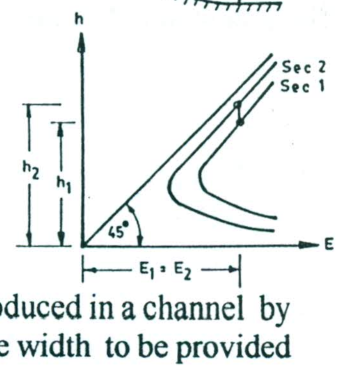
iii) Applying the energy equation between sections 1 and 2 , we get



$$E_2 = E_1 = 1.55\ m$$

$$\therefore\ 1.55 = h_2 + \frac{Q^2}{2g{b_2}^2{h_2}^2} = h_2 + \frac{4.5^2}{2\times 9.81\times 4^2\times {h_2}^2} = h_2 + \frac{0.0645}{{h_2}^2}$$

Solving by trial, we get



$h_2 = 1.52$ m

$$\therefore\ Rise\ in\ water\ level = h_2 - h_1 = 1.52 - 1.50 = 0.02\ m$$

This example demonstrates that a critical state of flow can be produced in a channel by reducing the channel width, but not by increasing the channel width. The width to be provided is given by $b_2 \le b_c$.

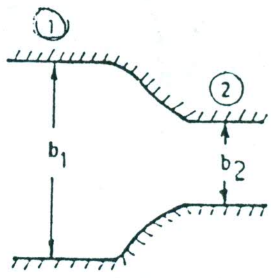
Combination of Transitions

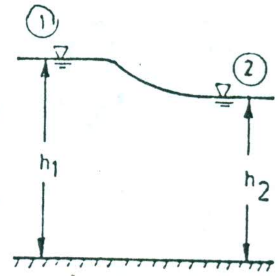
Example 3.10

Water flows in a 6 m wide rectangular channel at a depth of 2 m and a velocity of 2 m/s. The channel is contracted to a width of 3 m. How much the channel bottom is to be simultaneously raised or lowered for the flow to be possible as specified? Neglect energy losses and take $\alpha = 1$.

Solution

Designating as sections 1 and 2 the sections upstream of the constriction and over the constriction, respectively, one obtains





$$Q = A_1 U_1 = b_1 h_1 U_1 = 6 \times 2 \times 2 = 24\ m^3/s$$

$$E_1 = h_1 + \frac{U_1^2}{2g} = 2 + \frac{2^2}{2 \times 9.81} = 2.20\ m^3/s$$

$$Fr_1 = \frac{U_1}{\sqrt{gD_1}} = \frac{U_1}{\sqrt{gh_1}} = \frac{2}{\sqrt{9.81 \times 2}} = 0.45$$

Therefore, the upstream flow is subcritical.

Considering only the contraction, the downstream specific energy is

$$E_2 = E_1 = 2.20\ m$$

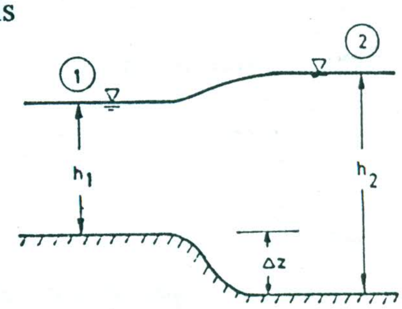
The critical depth corresponding to $b_2 = 3$ m at section 2 is

$$h_{c2} = \sqrt[3]{\frac{Q^2}{g b_2^{\ 2}}} = \sqrt[3]{\frac{24^2}{9.81 \times 3^2}} = 1.87\ m$$

and the minimum specific energy at section 2 is

$$E_{c2} = 1.5 h_{c2} = 1.5 \times 1.87 = 2.80\ m$$

Since $E_2 < E_{c2}$, it is not possible to maintain the flow as specified without lowering the channel bed. If the flow at section 2 occurs at the critical state, then the lowering of the channel bottom required is minimum. Therefore, the required condition is



$$E_2 = E_{C2} = E_1 + \Delta z$$

$$\therefore \quad \Delta z = E_{C2} - E_1 = 2.80 - 2.20 = 0.60\ m$$

Hence, the channel bottom is to be lowered by 0.60 m.

The following points may be noted regarding the effects of transitions considered above:

i) When $\Delta z < \Delta z_c$ or $b_1 > b_2 > b_c$, the transition produces a temporary change in flow depth, but no change in the state of flow.

ii) When $\Delta z = \Delta z_c$ or $b_2 = b_c$, critical flow occurs over the transition, but the upstream flow is not affected.

iii) When $\Delta z > \Delta z_c$ or $b_2 < b_c$, the transition becomes a control. In this case, the flow over the transition becomes critical and the upstream flow is affected.

iv) A critical state of flow can be produced in a channel either by raising the channel bed or reducing the channel width, but not by lowering the channel bed or increasing the channel width.

3.4 FLOW MEASUREMENT

It has been mentioned in Art.3.2 that a definite relationship between depth (or stage) and discharge exists at a control section. Such a definite depth-discharge relationship offers a theoretical basis for the measurement of discharge in open channels. Since a critical flow section is a control section, it can be used for flow measurement. Based on the principle of critical flow, various devices for flow measurement have been developed. In such devices, the critical depth is produced either by the construction of a hump or step on the channel bed, such as a broad-crested weir, or by reducing the channel width, such as a critical flow flume.

Broad-Crested Weir

If the channel bed is raised by an amount Δz such that $\Delta z \geq \Delta z_c$ over a length sufficient enough to develop parallel flow over the hump, the flow over the hump will be critical (Art.3.3). Such a device is called a broad-crested weir and provides an excellent means of measuring the discharge in open channels.

Consider a rectangular broad-crested weir as shown in Fig. 3.4. Using Eq.(3.22), the discharge over the weir with $\alpha = 1$ is given by

$$Q = \sqrt{g}\, b h_c^{1.5} = 3.13 b h_c^{1.5} \tag{3.29}$$

where b is the width of the channel.

The usual difficulty in using Eq.(3.29) for computing discharge in an open channel lies in locating the critical flow section and measuring the critical depth accurately. This difficulty is, however, avoided by measuring the depth of flow upstream of the weir where the flow is not affected by the presence of the weir. With reference to Fig.3.4, neglecting the velocity of approach and the frictional losses and applying the energy equation between the upstream section and the critical flow section, one obtains

$$h_1 = h_c + \frac{U_c^2}{2g} = h_c + \frac{h_c}{2} = \frac{3}{2} h_c \tag{3.30}$$

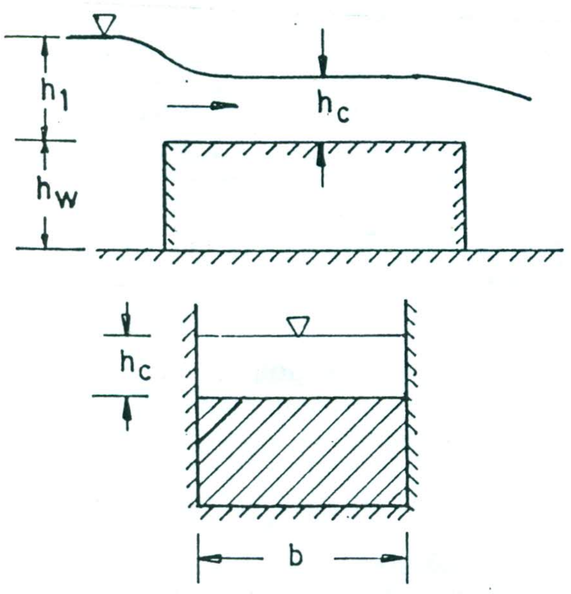


Fig. 3.4 Flow over a rectangular broad-crested weir

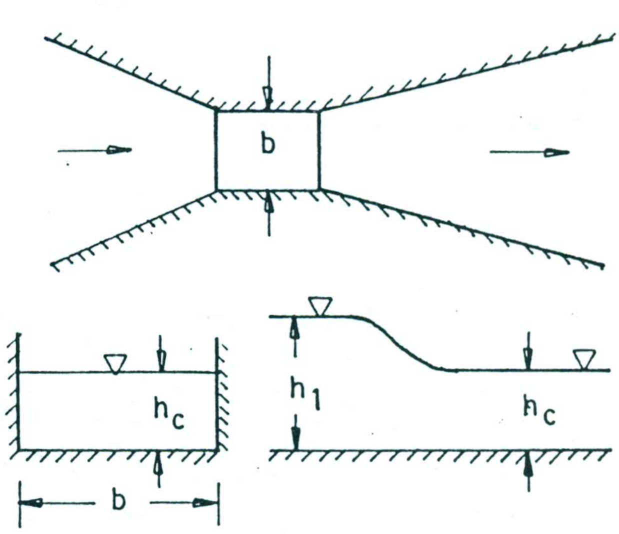


Fig. 3.5 Flow over a rectangular Venturi flume

Using Eq.(3.30), Eq.(3.29) becomes

$$Q = (2/3)^{1.5}\, b\sqrt{g}\, h_1^{1.5} = 1.705 b h_1^{1.5} \tag{3.31}$$

If it is desired to include the velocity of approach, then Eq.(3.30) becomes

$$h_1 + \frac{U_1^2}{2g} = h_c + \frac{U_c^2}{2g} = h_c + \frac{h_c}{2} = \frac{3}{2} h_c \tag{3.32}$$

and the discharge equation, Eq.(3.31), becomes

$$Q = 1.705 b \left(h_1 + \frac{U_1^2}{2g} \right)^{1.5} \tag{3.33}$$

Example 3.11

A broad-crested weir is built in a rectangular channel of width 2 m. The height of the weir crest above the channel bed is 1.20 m and the head over the weir is 0.80 m. Calculate the discharge.

Solution $b = 2$ m, $h_w = 1.20$ m, $h_1 = 0.80$ m

Initially we neglect the velocity of approach. Then, the discharge is obtained using Eq.(3.31) as

$$Q = 1.705 b h_1^{1.5} = 1.705 \times 2 \times 0.80^{1.50} = 2.440 \ m^3/s$$ Ans.

A more accurate discharge can be obtained by using Eq.(3.33) as follows.

Assumed Q	A_1	U_1	$U_1^2/2g$	$h_1+U_1^2/2g$	Computed Q
2.440	4	0.6100	0.0190	0.8190	2.527
2.527	4	0.6318	0.0203	0.8203	2.534
2.534	4	0.6334	0.0204	0.8204	2.534
2.534	4	0.6335	0.0205	0.8205	2.534

Hence, the discharge, $Q = 2.534$ m^3/s

Critical Flow Flumes

A broad-crested weir has the disadvantage of having a dead water region upstream in which silt and debris can accumulate. This difficulty can be overcome by the use of a critical flow flume, in which the occurrence of critical flow is forced by a contraction in channel width, followed by a short length of supercritical flow and a hydraulic jump. Obviously, the width at the contracted section or throat (Fig. 3.5) must be equal to or less than the width required for producing critical flow, i. e. $b \leq b_c$ (Art.3.3).

The critical flow flume, also known as the *Venturi flume*, has been designed in various forms. The discharge through the flume is given by Eq. (3.29), or when it is difficult to locate the critical flow section and measure the critical depth accurately, by Eq. (3.31) or (3.33), where b is now the throat width and h_c is the critical depth at the throat (Fig. 3.5).

PROBLEMS

3.1 A rectangular channel has a bottom width of 6 m. (i) Construct the specific energy curve for $Q = 15\ m^3/s$ and determine the critical depth and the minimum value of the specific energy. (ii) Construct the discharge-depth curve for E = 3 m and determine the critical depth and the maximum value of the discharge.

3.2 Compute the numerical values of the hydraulic exponent for critical flow computation M for h = 1 m in a (i) trapezoidal channel with b = 6m, s = 2 and $Q = 20\ m^3/s$, and (ii) circular channel with $d_0 = 3$ m.

3.3 Prove the following equations when the flow is critical in a rectangular channel:

$$i)\ h_c = \sqrt[3]{\frac{\alpha Q^2}{gb^2}} = \sqrt[3]{\frac{\alpha q^2}{g}} \qquad ii)\ U_c = \sqrt{\frac{gh_c}{\alpha}} = \sqrt[3]{\frac{gQ}{\alpha b}}$$

$$iii)\ E_c = 1.5h_c \qquad iv)\ Q = \sqrt{g}\, b\, h_c^{\ 1.5} = 3.132 b h_c^{\ 1.5} = 0.544\sqrt{g}\, bE_c^{\ 1,5}$$

3.4 Compute the critical depth and velocity in a (i) wide rectangular channel with $q = 4\ m^2/s$, (ii) rectangular channel with b = 6 m and $Q = 35\ m^3/s$, (iii) triangular channel with s = 1 and Q = 5 m^3/s, and (iv) parabolic channel whose profile is given by $y^2 = 5z$ with $Q = 25\ m^3/s$. In all cases assume $\alpha = 1.12$.

3.5 (a) Compute the critical depth and velocity in a trapezoidal channel with b = 6m, s = 2 and Q = 30 m^3/s by (i) the trial-and-error, (ii) the bisection, and (iii) the Newton-Raphson methods, if $\alpha = 1.12$.

(b) Compute the critical depth and velocity in a circular channel with $d_0 = 3$ m and $Q = 5\ m^3/s$ by the trial-and-error method, if (i) $\alpha = 1$, and (ii) $\alpha = 1.12$.

3.6 (a) Prove that the section of a channel at which the flow is critical at all depths can be expressed by

$$B^2G^3 = Q^2 / 8g$$

where B is the top width, G is the vertical distance between the energy line and the water surface, Q is the discharge and g is the acceleration due to gravity.

(b) A rectangular channel section is to take a certain discharge Q at the critical state and at the same time the wetted perimeter is to be a minimum. Show that the width of the channel must be equal to 4/3 times the depth.

3.7(a) Show that the relation between the alternate depths h_1 and h_2 for a rectangular channel is given by

$$\frac{2h_1^{\ 2}h_2^{\ 2}}{h_1 + h_2} = h_c^{\ 3}$$

where h_c is the critical depth.

(b) The alternate depths in a rectangular channel 6 m wide are 2 m and 1 m. Compute the discharge, the specific energy and the critical depth.

3.8 Compute the maximum discharge that may be carried by a channel for a specific energy of 2.0 m when the channel is (i) rectangular with b = 6 m, (ii) triangular with s = 1, (iii) parabolic whose profile is given by $y^2 = 4z$, (iv) trapezoidal with b = 6 m and s = 2, and (v) circular with $d_0 = 3$ m.

3.9 The depth upstream of a vertical sluice gate in a rectangular channel is 2 m and the discharge under the gate is 30.67 m^3/s. The channel is 6 m wide. Compute the downstream depth.

3.10 Prove that the minimum height of a hump that will produce critical flow in a rectangular channel is given by

$$\Delta z_c = h_1(1 + 0.50 Fr_1^{\,2} - 1.50 Fr_1^{\,0.67})$$

where h_1 is the upstream depth and Fr_1 is the Froude number corresponding to h_1.

3.11 Water is flowing at a velocity of 2 m/s and a depth of 2.5 m in a long rectangular channel 6 m wide. Compute (a) the height of a smooth upward step in the channel bed to produce critical flow, and (b) the depth and the change in water level produced by (i) a smooth upward step of 0.40 m, (ii) a smooth upward step of 0.80 m, and (iii) a smooth downward step of 0.40 m. In all cases, neglect energy losses and take $\alpha = 1$.

3.12 Water is flowing at a velocity of 2 m/s and a depth of 2.5 m in a long rectangular channel 6 m wide. Compute (a) the contraction in width of the channel for producing critical flow, and (b) the depth and the change in water level produced by (i) a smooth contraction in width to 5 m, (ii) a smooth contraction in width to 3 m, and (iii) a smooth expansion in width to 8 m. In all cases neglect energy losses and take $\alpha = 1$.

3.13 The upstream conditions are as in Prob. 3.11 and there is a smooth upward step of 0.80 m in the channel bed. What expansion or contraction in width must simultaneously take place to produce critical flow in the channel?

3.14 The upstream conditions are as in Prob. 3.11 and the width of the channel is reduced to 5 m. How much the channel bottom is to be raised or lowered to produce critical flow in the channel?

3.15 A bridge is to be constructed across a 10 km wide river carrying a discharge of 1,00,000 m^3/s at a depth of 10 m. If it is intended to provide the minimum length of the bridge by reducing the river width, what would be the minimum river width without affecting the upstream flow? Neglect energy losses and assume $\alpha = 1$.

3.16(a) A broad-crested weir is built in a rectangular channel of width 1 m. The height of the weir crest above the channel bed is 0.60 m and the head over the weir is 0.40 m. Calculate the discharge.

(b) Compute the discharge through a Venturi flume having a throat width of 0.30 m when the upstream depth is 0.50 m.

****

Chapter 4

UNIFORM FLOW

4.1 INTRODUCTION

When uniform flow occurs in a channel, (1) the discharge, the velocity and the depth remain constant along the length of the channel, and (2) the energy line, the water surface and the channel bottom are parallel, i.e. $S_f = S_w = S_0$. In uniform flow, water is neither accelerated nor retarded and the net external force on water is zero.

The flow in a long straight prismatic channel under normal condition, i.e. when there is no inflow or outflow or no transition or control structures like sluice gates, weirs, dams etc., tends to be uniform.

Uniform flow is considered to be steady only, since unsteady uniform flow is not practically possible. True uniform flow does not normally occur in natural channels, because changes in the cross-section along the length of the channel induce non-uniform flow conditions. Still, the concept of uniform flow is central to the understanding and solution of most problems in open channel hydraulics. In fact, the resistance relations developed for uniform flow are used to predict channel resistance in gradually varied flows, both steady and unsteady.

4.2 ESTABLISHMENT OF UNIFORM FLOW

The condition for the establishment of uniform flow in an open channel can be determined considering the momentum equation, Eq.(2.16). For uniform flow, $U_1 = U_2$, $F_{p1} = F_{p2}$ and $\beta_1 = \beta_2$. Hence, Eq.(2.16) reduces to

$$\underline{\underline{W \sin\theta = F_f}} \quad \rho Q(\beta_2 U_2 - \beta_1 U_1) = F_{P_1} - F_{P_2} + W\sin\theta - F_f. \tag{4.1}$$

which indicates that when uniform flow occurs in a channel, the active component of the gravity force causing the flow is equal to the total force of friction or resistance.

The above condition implies that (i) flow cannot be uniform in a horizontal channel for which $\theta = 0$, and for uniform flow to occur, the channel must have a slope in the downstream direction, (ii) flow cannot be uniform in an adverse slope channel in which both $W\sin\theta$ and F_f act in the same direction, which is opposite to the direction of flow, and (iii) flow cannot be uniform in a frictionless channel for which $F_f = 0$, and uniform flow of an ideal fluid is impossible, since an ideal fluid has no friction.

The condition for the establishment of uniform flow, $W\sin\theta = F_f$, can be used to explain why the flow in a long straight prismatic channel under normal condition tends to be uniform. Suppose that at some location of a channel $W\sin\theta > F_f$ and flow is non-uniform. As the flow proceeds downstream, the flow is accelerated and the flow velocity increases. Since $F_f \propto U^2$, the friction or resistance force also increases and a balance between $W\sin\theta$ and F_f tends to reach and the flow tends to be uniform. On the other hand, if $W\sin\theta < F_f$ at some location of a channel, the flow is retarded and the flow velocity decreases. Hence, the friction or resistance force also decreases and a balance between $W\sin\theta$ and F_f tends to reach and the flow tends to be uniform as the flow proceeds downstream. Thus, uniform flow seems to be self-adjusting and any departure from the condition $W\sin\theta = F_f$ tends to reestablish this condition.

4.3 VELOCITY DISTRIBUTION

Shear Stress and Friction Velocity

When water flows in a channel, the pull of water produces a force that acts on the channel bed in the direction of flow. This force is known as the shear or tractive or drag force and is equal to the friction or resistance force F_f. If the average value of this force per unit wetted area, which is known as the shear stress, is denoted by τ_0, then

$$F_f = \tau_0 P L \tag{4.2}$$

where P is the wetted perimeter and L is the length of the channel. Therefore, from Eq.(4.1)

$$W \sin\theta = F_f = \tau_0 PL \tag{4.3}$$

When the angle of bottom slope θ is small, $\sin\theta \approx \tan\theta$. Also, $\tan\theta = S_0$ and when the flow is uniform, $S_0 = S_f$. Therefore, the active component of the gravity force $= W\sin\theta = \gamma AL\sin\theta \approx \gamma AL\tan\theta = \gamma ALS_0$, where γ is the specific weight of water, A is the cross-sectional area and S_0 is the channel bottom slope. Therefore,

$$\gamma ALS_0 = \tau_0 PL \tag{4.4}$$

or

$$\tau_0 = \gamma \frac{A}{P} S_0 = \gamma RS_0 = \rho gRS_0 \tag{4.5}$$

For a wide channel, $R \approx h$, hence Eq. (4.5) becomes

$$\tau_0 = \gamma hS_0 = \rho ghS_0 \tag{4.6}$$

The quantity $\sqrt{\tau_0 / \rho}$ has the dimensions of velocity and the shear stress τ_0 is expressed as

$$\tau_0 = \rho u^{*2} \tag{4.7}$$

or

$$u^* = \sqrt{\tau_0 / \rho} \tag{4.8}$$

where u* is known as the shear or friction or drag velocity. It does not represent a velocity which is physically real. However, it is used as the velocity scale in the study of velocity distribution in open channels.

Using Eqs.(4.5) and (4.8) it can be shown that

$$u^* = \sqrt{gRS_0} \tag{4.9}$$

and when the channel is wide

$$u^* = \sqrt{ghS_0} \tag{4.10}$$

Laminar or Viscous Sublayer

Even in a turbulent flow, there is a very thin layer near the boundary in which the flow is laminar and is known as the laminar or viscous sublayer. The thickness of this layer is given by

$$\delta_v = \frac{11.6\nu}{u^*} \tag{4.11}$$

Smooth and Rough Boundaries

As stated earlier, the velocity distribution across a channel section is not uniform owing to the presence of boundary (wall) roughness. The effect of boundary roughness on the velocity distribution in turbulent flow was first investigated by Nikuradse who introduced the concept of *equivalent sand grain roughness* (k_s) as standard for all other types of roughness elements. The ratio k_s/R of the roughness height to the hydraulic radius is known as the *relative roughness*. The boundary surfaces are classified based on the following criteria:

1. Hydraulically smooth boundary

$$\frac{u^* k_s}{\nu} \le 5 \tag{4.12}$$

The roughness elements are well-covered by the viscous sublayer ($k_s < \delta_v$) and do not affect the velocity distribution outside the sublayer. The velocity distribution depends on the viscosity of water.

2. Hydraulically rough boundary

$$\frac{u^* k_s}{\nu} \ge 70 \tag{4.13}$$

The roughness elements project through the viscous sublayer ($k_s > \delta_v$) and the velocity distribution outside the sublayer is affected by the surface roughness. The viscosity of water has no effect on the velocity distribution.

3. Transition boundary

$$5 < \frac{u^* k_s}{\nu} < 70 \tag{4.14}$$

The velocity distribution is affected both by the viscosity of water and the bottom roughness.

Velocity Distribution in Turbulent Flow

Along a Vertical

The velocity distribution along a vertical in a wide channel in turbulent flow is given by

$$\frac{u_z}{u^*} = \frac{1}{\kappa} \ln \frac{z}{z_0} \tag{4.15}$$

where u_z is the velocity at a distance z from the channel bottom, κ (= 0.4) is the *von Karman constant* and z_0 is the zero velocity level, i.e. u = 0 at z = z_0. Equation (4.15) is commonly known as the *Prandtl-von Karman universal velocity distribution law.*

Experimental evidence suggests that the logarithmic velocity profile is a good approximation for the full depth of the flow. The values of z_0 for different boundaries are as follows:

1. Hydraulically smooth surface ($u^*k_s/\nu \leq 5$)

$$z_o = 0.11\frac{\nu}{u^*} \tag{4.16}$$

2. Hydraulically rough surface ($u^*k_s/\nu \geq 70$)

$$z_o = 0.033k_S \tag{4.17}$$

3. Transition regime ($5 < u^*k_s/\nu < 70$)

$$z_0 = 0.11\frac{\nu}{u^*} + 0.033k_S \tag{4.18}$$

Depth-Averaged Velocity

For logarithmic velocity distribution, Eq.(4.15), Vanoni (1941) showed that the flow velocity, measured at 0.632h from the free surface, is equal to the depth-averaged velocity in the vertical. Also, it can be shown that the velocity at 0.6h depth from the free surface or the average of the velocities at 0.2h and 0.8h depths from the free surface, when h is the total depth of flow, is approximately equal to the average velocity in the vertical.

Cross-Sectional Mean Velocity

The cross-sectional mean velocity for turbulent flow in open channels are given by the following equations:

1. Hydraulically smooth surface ($u^*k_s/v \leq 5$)

$$\frac{U}{u^*} = 5.75\log\left(\frac{3.64u^*R}{\nu}\right) \tag{4.19}$$

2. Hydraulically rough surface ($u^*k_s/v \geq 70$)

$$\frac{U}{u^*} = 5.75\log\left(\frac{12.2R}{k_s}\right) \tag{4.20}$$

3. Transition regime ($5 < u^*k_s/v < 70$)

$$\frac{U}{u^*} = 5.75\log\left(\frac{12.2R}{k_s + 3.35\nu/u^*}\right) \tag{4.21}$$

Example 4.1

A rectangular channel is 6 m wide and laid on a slope of 0.25%. The channel is made of concrete ($k_s = 2$ mm) and carries water at a depth of 0.50 m. Compute the mean velocity of flow.

Solution $k_s = 2 \text{ mm} = 0.002 \text{ m}$

$$S_0 = 0.25/100 = 0.0025$$

$$R = A/P = (6\times0.50)/(6 + 2\times0.50) = 0.4286 \text{ m}$$

$$u^* = \sqrt{gRS_0} = \sqrt{9.81\times0.4286\times0.0025} = 0.1025 \ m/s$$

$$\frac{k_s u^*}{\nu} = \frac{0.002\times0.1025}{10^{-6}} = 205 > 70$$

Hence, the boundary is hydraulically rough and the mean velocity of flow is obtained by Eq.(4.20), i.e.

$$\frac{U}{u^*} = 5.75\log\frac{12.2R}{k_s} = 5.75\log\frac{12.2\times0.4286}{0.002} = 19.65$$

$$\therefore\ U = 19.65\times0.1025 = 2.014 \ m/s$$

4.4 UNIFORM FLOW FORMULAS

Chezy Formula

The Chezy formula can be found mathematically from two assumptions. The first assumption states that, in steady uniform flow the active component of the gravity force causing the flow must be equal to the total force of friction or resistance, as indicated by Eq.(4.1). When the channel slope is small, the active component of the gravity force $= W\sin\theta = \gamma AL\sin\theta \approx \gamma AL\tan\theta = \gamma ALS_0 = \gamma ALS_f$.

The second assumption states that, in turbulent flow the resistance force per unit wetted area varies as the square of the mean velocity. The total wetted area is the product of the wetted perimeter P and the length of the channel L. Hence, the total force of resistance is given by

$$F_f = kPLU^2 \tag{4.22}$$

where k is a constant of proportionality. Hence, Eq.(4.1) gives

$$\gamma ALS_f = kPLU^2 \tag{4.23}$$

Equation (4.23) can be rearranged to yield

$$U = CR^{1/2}S_f^{1/2} \tag{4.24}$$

where $\sqrt{\gamma/k}$ is written as one constant C.

Equation (4.24) is probably the first steady uniform flow formula developed by the French engineer Antoine Chezy in 1769. The resistance factor C is referred to as the *Chezy's C.*

The Chezy formula can be used in any systems of units. The dimensions of Chezy's C in SI units are $m^{1/2}/s$ and $C/\sqrt{g}$ is dimensionless. The numerical value of Chezy's C varies with the systems of units. For the rivers of Bangladesh, the numerical value of C varies from 30 $m^{1/2}/s$ to 80 $m^{1/2}/s$ and the mean value of C may be taken as 50 $m^{1/2}/s$.

The Chezy formula is applicable for steady uniform and nearly uniform flows. It is widely used in Europe.

Darcy-Weisbach Formula

The Darcy-Weisbach formula, first presented by Julius Weisbach in 1845 and primarily developed for pipe flow, is given by

$$h_f = f\frac{L}{d_0}\frac{U^2}{2g} \qquad (4.25)$$

$\Rightarrow h_f/L \cdot 1/f \cdot 8gR = U^2$

$\Rightarrow U = S_f^{1/2} R^{1/2} \sqrt{8g/f}$

where h_f is the frictional loss, f is the friction factor, L is the length of the pipe, d_0 is the diameter of the pipe, U is the mean velocity of flow and g is the acceleration due to gravity. Since $d_0 = 4R$ and the energy gradient $S_f = h_f/L$, the above formula may be written as

$S_0 \rightarrow$ slope of water surface

$$U = \sqrt{\frac{8g}{f}}R^{1/2}S_f^{1/2} \qquad (4.26)$$

$S_f \rightarrow$ energy grad slope

This formula is same in all the systems of units and may be applied to uniform and nearly uniform flows in open channels. The friction factor f is dimensionless and its numerical value remains same in all the systems of units.

Manning Formula

In 1889 the Irish engineer Robert Manning presented a formula for steady uniform flow in open channels. This formula is completely empirical in nature. In SI and English units this formula is given by

$$U = \frac{1}{n}R^{2/3}S_f^{1/2} \qquad (4.27)$$

and

$$U = \frac{1.486}{n}R^{2/3}S_f^{1/2} \qquad (4.28)$$

respectively, where U is the mean velocity, n ($s/m^{1/3}$ or $sec/ft^{1/3}$) is the Manning's roughness coefficient, specifically known as *Manning's n*, R is the hydraulic radius and S_f is the slope of the energy line.

The numerical value of n is the same in all the systems of units, but the coefficient in the Manning formula is different in different systems of units. For example, when the Manning formula in SI units (Eq.4.27) is converted to English units (Eq. 4.28), it has to be multiplied by the factor $(3.28)^{1/3} = 1.486$, as shown below.

$$n \; in \; s/m^{1/3} = \frac{n}{3.28^{1/3}} \; in \; \sec/ft^{1/3} \quad \text{(since 1 m = 3.28 ft)}$$

$$\therefore U = \frac{1}{n} R^{2/3} S_f^{\;1/2} \quad in \;\; SI \;\; units$$

$$= \frac{1}{n/3.28^{1/3}} R^{2/3} S_f^{\;1/2} \quad in \;\; English \;\; units$$

$$= \frac{3.28^{1/3}}{n} R^{2/3} S_f^{\;1/2} \quad in \;\; English \;\; units$$

$$= \frac{1.486}{n} R^{2/3} S_f^{\;1/2} \quad in \;\; English \;\; units$$

The Manning formula has been verified by many laboratory and field measurements and found to give satisfactory results. Therefore, it has been the most widely used of all the uniform flow formulas for open channel flow computation. However, in applying the Manning formula the main difficulty lies in the determination of the roughness coefficient n, for there is no exact method of selecting the value of n. Even at the present stage of knowledge, in addition to the information and the methods available for selecting n, the veteran engineers have to exercise sound engineering judgement and experience in selecting the proper value of n.

Typical values of Manning's roughness coefficient n for some open channel surfaces are presented in Table 4.1.

Table 4.1 Typical values of Manning's roughness coefficient n for some open channel surfaces

Surface	Value of n	Surface	Value of n
Glass	0.010	Plastic	0.010
Cement	0.011	Concrete	0.013
Wood	0.015	Earth canals	0.025
Rivers	0.025	Flood plains	0.040

The Manning formula is valid for fully rough turbulent flow for values of R/k_s up to about 1500.

Relationship Between Chezy's C, Darcy-Weichbach Friction Factor f and Manning's n

Using Eqs.(4.24), (4.26) and (4.27), we obtain the following relationships between C and n, C and f, and n and f in SI units:

$$C = \frac{1}{n} R^{\frac{1}{6}} \tag{4.29}$$

$$\frac{C}{\sqrt{g}} = \sqrt{\frac{8}{f}} \tag{4.30}$$

$$n = R^{\frac{1}{6}} \sqrt{\frac{f}{8g}} \tag{4.31}$$

Also, combination of Eqs.(4.9),(4.24) and (4.30) yields with $S_0 = S_f$

$$\frac{U}{u*} = \frac{C}{\sqrt{g}} = \sqrt{\frac{8}{f}} \tag{4.32}$$

which indicates that $C/\sqrt{g}$ is dimensionless.

Estimation of Chezy's C

White-Colebrook Formula: For hydraulically smooth flow ($u^*k_s/\nu \leq 5$), the Chezy's C is estimated from

$$C = 18\log\left(\frac{3.64u^*R}{\nu}\right) = 18\log\left(\frac{11.4\mathrm{Re}}{C}\right) \tag{4.33}$$

in which Re (= UR/ν) is the Reynolds number.

For hydraulically rough flow ($u^*k_s/\nu \geq 70$), C is obtained from

$$C = 18\log\left(\frac{12.2R}{k_s}\right) \tag{4.34}$$

and for the transition boundary, C is obtained from

$$\frac{U}{u*} = \frac{C}{\sqrt{g}} = 5.75\log\left(\frac{12.2R}{k_s + 3.35\nu/u*}\right) \tag{4.35}$$

Estimation of Manning's n

Strickler fomula: The most simple and the best-known of the methods used for estimating Manning's n is the empirical formula presented by the Swiss engineer Strickler in 1923. This formula is based on data from (i) streams with beds consisting of coarse material and free from bed undulations, and (ii) fixed bed channels with grains pasted to the bottom and sides. The formula originally proposed by Strickler is

$$n = \frac{d_{50}^{\ 1/6}}{21.1} = 0.047d_{50}^{\ 1/6} \tag{4.36}$$

where d_{50} is the median diameter or the diameter of the bed material in meters such that 50 percent of the material by weight is smaller.

The Strickler formula has two major advantages: (i) it relates n to the size of the grains which can be measured easily, and (ii) since d_{50} is raised to 1/6th power, an error in estimating its value has a corresponding less effect on the computed value of n.

Factors Affecting Manning's n

The value of n is highly variable and depends on a number of factors, which are to some extent interdependent. The factors that exert the greatest influence upon Manning's n in both natural and artificial channels are briefly described below.

i) **Roughness of the surface**: The value of Manning's n depends on the roughness of the surface which in turn depends on the size and shape of the grains of the material forming the channel perimeter. In general, fine-grained soils (e.g. clay, silt and sand) result in a low value of n and coarse-grained soils (e.g. gravels and boulders) result in a high value of n.

ii) **Vegetation**: The presence of vegetation in a channel retards the flow and increases n depending on the height, density, distribution and type of vegetation. Owing to the seasonal growth of aquatic plants, the value of n may increase in the growing season and diminish in the dormant season.

iii) **Channel irregularity**: Channel irregularities include sand bars, depressions, holes, humps, etc. and increase the value of n.

iv) **Channel alignment**: The value of n is low for straight channels and high for curved channels and increases with the curvature of the channel.

v) **Silting and scouring**: In general, silting converts an irregular channel into a regular one and decreases n, whereas scouring does the reverse and increases n.

vi) **Obstruction**: The presence of obstructions like logs, bridge piers, boats, ships, launches, steamers etc. tends to increase n depending on the size, shape, number and distribution of the obstructions.

vii) **Stage and discharge**: The value of n generally decreases with increase in stage and discharge. However, the value of n may be high when the flood plains in a river are submerged at high stages.

viii) **Suspended material and bed load**: The suspended material and bed load cause an increase in Manning's n because additional energy is required to move the sediment.

Example 4.2

An open channel lined with concrete (d_{50} = 1.5 mm) is laid on a slope of 0.1%. The channel is trapezoidal with b = 6 m and s = 2. Compute the uniform flow discharge in the channel if the depth of flow is 2 m. Also compute the numerical values of Chezy's C and friction factor f.

Solution $S_f = S_0 = 0.1\% = 0.1/100 = 0.001$, $d_{50} = 1.5 \text{ mm} = 1.5/1000 \text{ m} = 0.0015 \text{ m}$

$$n = 0.047 d_{50}^{1/6} = 0.047 \times 0.0015^{1/6} = 0.016$$

$$A = (b + sh)h = (6+2\times2)\times2 = 20 \text{ m}^2$$

$$P = b + 2\sqrt{1+s^2}\,h = 6 + 2\sqrt{1+2^2} \times 2 = 14.94\, m$$

$$R = A/P = 1.34 \text{ m}$$

$$Q = (1/n)AR^{2/3}S_0^{1/2} = (1/0.016)\times20\times1.34^{2/3}\times0.001^{1/2}$$

$$= 48.29 \text{ m}^3/\text{s}$$

$$C = (1/n)R^{1/6} = (1/0.0159)\times1.338^{1/6} = 66.01 \text{ m}^{1/2}/\text{s}$$

$$f = 8g/C^2 = 8\times9.81/66.01^2 = 0.018$$

When the flow is uniform, then using the Manning formula (Eq.4.38), it can be shown that

$$Q_n = K_n \sqrt{S_n} \tag{4.42}$$

where

$$K_n = \frac{1}{n} A_n R_n^{2/3} \tag{4.43}$$

is the *conveyance for uniform flow*. It can be computed using Eq.(4.42) when Q_n and S_n are given or using Eq.(4.43) from the given section, n and the normal depth h_n.

In terms of the Chezy formula, the conveyance is given by

$$K = CAR^{1/2} \tag{4.44}$$

and using Eq.(4.37), we obtain

$$Q_n = K_n \sqrt{S_n} \tag{4.45}$$

where

$$K_n = CA_n R_n^{1/2} \tag{4.46}$$

is the conveyance for uniform flow.

For unit longitudinal slope of the channel, i.e. when $S_n = 1$, Eq.(4.42) or (4.45) shows that $Q_n = K_n$, i.e. the conveyance is numerically equal to the discharge. Also, for a given longitudinal slope, $Q_n \propto K_n$. Thus, the conveyance is a measure of the carrying capacity of the channel.

Hydraulic Exponent for Uniform Flow Computation

The conveyance K is a function of the depth of flow for a given channel section and roughness and it is convenient to express K in the form

$$K^2 = C_2 h^N \tag{4.47}$$

where C_2 is a coefficient and N is an exponent which is known as the hydraulic exponent for uniform flow computation.

Assume that n and N are independent of h. Taking logarithm of both sides of Eq.(4.47) and then differentiating with respect to h, one obtains

$$\frac{d(\ln K)}{dh} = \frac{N}{2h} \tag{4.48}$$

Also, using R = A/P in Eq.(4.41), taking logarithm of both sides of Eq.(4.41), then differentiating with respect to h and using dA/dh = B, we get

$$\frac{d(\ln K)}{dh} = \frac{1}{3A}\left(5B - 2R\frac{dP}{dh}\right) \tag{4.49}$$

Equating the right sides of Eqs. (4.48) and (4.49) and solving for N, we obtain

$$N = \frac{2h}{3A}\left(5B - 2R\frac{dP}{dh}\right) \tag{4.50}$$

which is the general equation for the hydraulic exponent N when the conveyance is expressed in terms of the Manning's formula.

When the conveyance is expressed in terms of the Chezy formula, Eq.(4.44), it can be shown in a similar way that

$$N = \frac{h}{A}\left(3B - R\frac{dP}{dh}\right) \tag{4.51}$$

Equations (4.50) and (4.51) indicate that the numerical value N depends on the channel shape and the depth of flow. It also depends on whether the conveyance is expressed in terms of the Manning or the Chezy formula. The values of N for different channel sections are given in Table 4.2.

Table 4.2 Values of N for different channel sections

Channel section	Conveyance computed by Manning equation	Conveyance computed by Chezy equation
1. Wide (b>>h)	3.33	3.00
2. Triangle	5.33	5.00
3. Rectangle	$\frac{2}{3}\left[5 - \frac{4(h/b)}{1+2(h/b)}\right]$	$3 - \frac{2(h/b)}{1+2(h/b)}$
4. Trapezoid	$\frac{2}{3}\left[5\frac{1+2s(h/b)}{1+s(h/b)} - 4\frac{\sqrt{1+s^2}(h/b)}{1+2\sqrt{1+s^2}(h/b)}\right]$	$3\frac{1+2s(h/b)}{1+s(h/b)} - \frac{2\sqrt{1+s^2}(h/b)}{1+2\sqrt{1+s^2}(h/b)}$
5. Circle	$\frac{16h}{3d_o}\left(\frac{5\sin\omega/2}{\omega-\sin\omega} - \frac{1}{\omega\sin\omega/2}\right)$	$\frac{4h}{d_o}\left(\frac{6\sin\omega/2}{\omega-\sin\omega} - \frac{1}{\omega\sin\omega/2}\right)$
6. Parabola (perimeter equation $z = cy^2$)	$3 + \frac{4}{3+2ch}$	$3 + \frac{3}{3+2ch}$

Example 4.3

Derive the expression for the hydraulic exponent for uniform flow computation N for a rectangular channel based on the Manning formula. Then compute the numerical values of N for (i) wide, and (ii) narrow channels.

Solution For a rectangular channel, $A = bh$, $B = b$, $P = b + 2h$, $R = A/P$, $dP/dh = 2$

$$N = \frac{2h}{3A}\left(5B - 2R\frac{dP}{dh}\right) = \frac{2h}{3bh}\left(5b - 2\times\frac{bh}{b+2h}\times 2\right)$$

$$= \frac{2}{3}\left(5 - \frac{4h}{b+2h}\right) = \frac{2}{3}\left[5 - \frac{4(h/b)}{1+2(h/b)}\right]$$

This is the expression for N for a rectangular channel based on the Manning formula.

(i) For a wide channel, $h/b \approx 0$. Hence,

$$N = \frac{2}{3}\left[5 - \frac{4(h/b)}{1+2(h/b)}\right] = \frac{2}{3}\left[5 - \frac{4\times 0}{1+2\times 0}\right] = \frac{10}{3} = 3.33$$

(ii) For a narrow channel, $h/b \to \infty$. Hence,

$$N = \frac{2}{3}\left[5 - \frac{4(h/b)}{1+2(h/b)}\right] = \frac{2}{3}\left[5 - \frac{4}{1/(h/b)+2}\right] = \frac{2}{3}\left[5 - \frac{4}{0+2}\right] = 2$$

Example 4.4

Compute the hydraulic exponent for uniform flow computation N of a trapezoidal channel with b = 6.1 m, s = 2 and h = 2 m based on the Manning formula.

Solution

$$\frac{h}{b} = \frac{2}{6.1} = 0.328$$

$$N = \frac{2}{3}\times 5\times\frac{1+2s(h/b)}{1+s(h/b)} - \frac{2}{3}\times 4\times\frac{\sqrt{1+s^2}\,(h/b)}{1+2\sqrt{1+s^2}\,(h/b)}$$

$$= \frac{2}{3}\times 5\times\frac{1+2\times 2\times 0.328}{1+2\times 0.328} - \frac{2}{3}\times 4\times\frac{\sqrt{1+2^2}\times 0.328}{1+2\sqrt{1+2^2}\times 0.328}$$

$$= 4.653 - 0.793 = 3.860$$

Alternative solution $A = (6.1+2\times 2)\times 2 = 20.2\ \text{m}^2$, $P = 6.1 + \sqrt{1+2^2}\times 2 = 15.044$ m, $R = A/P =$ $20.2/15.04 = 1.342$ m, $B = 6.1 + 2\times 2\times 2 = 14.1$ m, $dP/dh = 2\sqrt{1+s^2} = 2\sqrt{5}$

$$\therefore\ N = \frac{2h}{3A}\left(5B - 2R\frac{dP}{dh}\right) = \frac{2\times 2}{3\times 20.2}(5\times 14.1 - 2\times 1.342\times 2\sqrt{5}) = 3.860$$

105. Subramanya.

Computation of Normal Depth

Analytical Method

The normal depth is an important parameter in the analysis of open channel flow. It may be computed using the Manning or the Chezy formula when the channel section, the discharge Q, the bottom slope S_0 and the Manning's n or the Chezy's C are given. For wide and triangular channels, the following analytical (explicit) expressions for the normal depth can be easily obtained.

a) Using the Manning formula

i. Wide channel

$$h_n = \left(\frac{nq}{\sqrt{S_o}}\right)^{3/5} \tag{4.52}$$

ii. Triangular channel

$$h_n = \frac{2^{1/4}(1+s^2)^{1/8}}{s^{5/8}}\left(\frac{nQ}{\sqrt{S_o}}\right)^{3/8} \tag{4.53}$$

b) Using the Chezy formula

i. Wide channel

$$h_n = \left(\frac{q}{C\sqrt{S_o}}\right)^{2/3} \tag{4.54}$$

ii. Triangular channel

$$h_n = \frac{2^{1/5}(1+s^2)^{1/10}}{s^{3/5}}\left(\frac{Q}{C\sqrt{S_o}}\right)^{2/5} \tag{4.55}$$

Example 4.5

A wide channel with $S_0 = 0.0025$ carries a discharge of 3 m^2/s. Compute the normal depth and velocity (i) using the Manning formula when $n = 0.020$, and (ii) using the Chezy formula when $C = 45\ m^{1/2}/s$.

Solution

(i) Using the Manning formula

$$h_n = \left(\frac{nq}{\sqrt{S_o}}\right)^{3/5} = \left(\frac{0.020x3}{\sqrt{0.0025}}\right)^{3/5} = 1.12\ m$$

$$U_n = \frac{q}{h_n} = \frac{3}{1.12} = 2.69\ m/s$$

(ii) Using the Chezy formula

$$h_n = \left(\frac{q}{C\sqrt{S_0}}\right)^{2/3} = \left(\frac{3}{45\times\sqrt{0.0025}}\right)^{2/3} = 1.21\ m$$

$$U_n = \frac{q}{h_n} = \frac{3}{1.21} = 2.48\ m/s$$

Example 4.6

For a triangular channel with side slopes of 2:1, a longitudinal slope of 0.0016 and n = 0.015, determine the normal depth if Q = 10 m^3/s.

Solution $s = 2,\ S_0 = 0.0016,\ n = 0.015,\ Q = 10\ m^3/s$

$$h_n = \frac{2^{1/4}\left(1+s^2\right)^{1/8}}{s^{5/8}}\left(\frac{nQ}{\sqrt{S_o}}\right)^{3/8} = \frac{2^{1/4}\left(1+2^2\right)^{1/8}}{2^{5/8}}\left(\frac{0.015\times 10}{\sqrt{0.0016}}\right)^{3/8} = 1.55\ m$$

$$A = sh^2 = 2\times 1.55^2 = 4.79\ m^2$$

$$U_n = \frac{Q}{A_n} = \frac{10}{4.79} = 2.09\ m/s$$

Alternative solution

$$AR^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.015\times 10}{\sqrt{0.0016}} = 3.75$$

$$\therefore\ sh^2\left(\frac{sh}{2\sqrt{1+s^2}}\right)^{2/3} = 3.75$$

$$or,\ \ 2h_n^{\ 2}\left(\frac{2h_n}{2\sqrt{1+2^2}}\right)^{2/3} = 3.75$$

which gives $h_n^{8/3} = 3.206$

$$\therefore\ h_n = 1.55\ m$$

Then, $A = sh^2 = 2\times 1.55^2 = 4.79\ m^2$

$$U_n = \frac{Q}{A_n} = \frac{10}{4.79} = 2.09\ m/s$$

Trial-and-Error Method

For other simple geometric channel sections, like the rectangular, trapezoidal, circular and parabolic sections, the computation of normal depth can be conveniently carried out by the trial-and-error solution of Eq. (4.39).

Example 4.7

For a rectangular channel with b = 6.0 m, n = 0.025 and S_0 = 0.0025, compute the normal depth and velocity if Q = 20 m^3/s.

Solution

$$A_n R_n^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.025 \times 20}{\sqrt{0.0025}} = 10.000$$

Now, assume several values of h and compute the section factor $AR^{2/3}$ until the computed value of $AR^{2/3}$ is close to 10.000.

h(m)	bh A(m^2)	b+2h P(m)	A/P R(m)	$AR^{2/3}$	Remarks
1.00	6.000	8.000	0.750	4.952	h too small
2.00	12.000	10.000	1.200	13.551	h too large
1.60	9.600	9.200	1.043	9.876	
1.62	9.720	9.240	1.052	10.054	
1.61	9.660	9.220	1.048	9.965	h closest

Hence, the normal depth, h_n = 1.61 m and the normal velocity

$$U_n = \frac{Q}{A_n} = \frac{20}{9.66} = 2.07\ m/s$$

Example 4.8

For a trapezoidal channel with b = 6 m, s = 2, n = 0.025 and S_0 = 0.001, compute the normal depth and velocity if Q = 14 m^3/s.

Solution

$$A_n R_n^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.025 \times 14}{\sqrt{0.001}} = 11.068$$

h(m)	A(m^2)	P(m)	R(m)	$AR^{2/3}$
1.00	8.000	10.472	0.764	6.684
2.00	20.000	14.944	1.338	24.288
1.30	11.180	11.814	0.946	10.776
1.31	11.292	11.858	0.952	10.929
1.32	11.405	11.903	0.958	11.084

Hence, the normal depth, h_n = 1.32 m and the normal velocity

$$U_n = \frac{Q}{A_n} = \frac{14}{11.405} = 1.23\ m/s$$

Example 4.9

Compute the normal depth and velocity in a parabolic channel with Q = 20 m^3/s, n = 0.025 and S_0 = 0.0025 when the profile of the channel is given by $y^2 = 4z$.

Solution

$$A_n R_n^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.025 \times 20}{\sqrt{0.0025}} = 10.000$$

Since $y^2 = 4z$, we have $z = 0.25y^2$ so that c = 0.25. Also, note that $4h/B = 2h^{1/2}\sqrt{c} = 2h^{1/2}\sqrt{0.25} = h^{1/2}$.

h(m)	B(m)	A(m^2)	P(m)	R(m)	AR$^{2/3}$
1.00	4.000	2.667	4.591	0.581	1.856
2.00	5.657	7.542	7.192	1.049	7.785
3.00	6.928	13.856	9.562	1.449	17.743
2.26	6.013	9.060	7.822	1.158	9.993
2.27	6.027	9.120	7.846	1.162	10.008

Hence, the normal depth, h_n = 2.26 m and the normal velocity

$$U_n = \frac{Q}{A_n} = \frac{20}{9.060} = 2.21\ m/s$$

Example 4.10

A circular channel 2 m in diameter is laid on a slope of 0.001 and carries a discharge of 4 m^3/s. Compute the normal depth and velocity when n = 0.013.

Solution

$$A_n R_n^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.013 \times 4}{\sqrt{0.001}} = 1.644$$

$\omega(rad)$	A(m^2)	P(m)	R(m)	AR$^{2/3}$
1	0.079	1	0.079	0.015
2	0.545	2	0.273	0.229
3	1.429	3	0.476	0.872
4	2.378	4	0.595	1.682
3.94	2.328	3.94	0.591	1.639
3.95	2.337	3.95	0.592	1.647

Hence, ω_n = 3.95 rad and the normal depth

$$h_n = \frac{d_o}{2}\left(1 - \cos\frac{\omega_n}{2}\right) = \frac{2}{2}\left(1 - \cos\frac{3.95}{2}\right) = 1.39\ m$$

and, the normal velocity

$$U_n = \frac{Q}{A_n} = \frac{4}{2.337} = 1.71\ m/s$$

Numerical Methods

The numerical methods, used for solving nonlinear algebraic equations involving a single variable, e.g. the method of bisection, the method of iteration, the method of false position, the secant method, the Newton-Raphson method etc., as stated in Art.3.2, can be conveniently used to compute the normal depth for rectangular, trapezoidal, circular and parabolic channel sections. The computation of normal depth using the bisection and the Newton-Raphson methods is considered here.

Bisection method: Suppose that we want to compute the normal depth in a channel for a given section, discharge Q, roughness coefficient n, bottom slope S_0. Then the function

$$f(h) = AR^{2/3} - A_n R_n^{2/3} = AR^{2/3} - \frac{nQ}{\sqrt{S_0}} \qquad (4.56)$$

must be satisfied by some positive depth greater than say h_{min} and less than say h_{max}. The normal depth is taken equal to $(h_{min} + h_{max})/2$ and f(h) is determined. If f(h) is positive, then the root is less than $(h_{min} + h_{max})/2$ and the upper limit is taken as $(h_{min} + h_{max})/2$. On the other hand, if f(h) is negative, then the lower limit is taken as $(h_{min} + h_{max})/2$. The procedure is repeated till the desired accuracy is attained.

Example 4.11

For a trapezoidal channel with b = 6 m, s = 2, n = 0.025 and S_0 = 0.001, compute the normal depth by the method of bisection if Q = 14 m^3/s.

Solution $\qquad A = (6+2h)h \qquad P = 6+4.472h \qquad R = A/P$

$$f(h) = AR^{2/3} - \frac{nQ}{\sqrt{S_0}} = \frac{A^{5/3}}{P^{2/3}} - \frac{nQ}{\sqrt{S_0}} = \frac{[(6+2h)h]^{5/3}}{(6+4.472h)^{2/3}} - 11.068$$

Initially the values of h_{min} and h_{max} are taken as 0 and 10 m, respectively. The computation is carried out as follows.

h_{min}	h_{max}	$h=(h_{min}+h_{max})/2$	f(h)	Root lies between
0	10	5	148.647	0 and 5
0	5	2.5	26.562	0 and 2.5
0	2.5	1.25	-1.041	1.25 and 2.5
1.25	2.50	1.875	10.381	1.25 and 1.875
1.25	1.875	1.5625	4.105	1.25 and 1.5625
1.25	1.5625	1.4063	1.394	1.25 and 1.4063
1.25	1.4063	1.3281	0.143	1.25 and 1.3281
1.25	1.3281	1.2891	-0.458	1.2891 and 1.3281
1.2891	1.3281	1.3086	-9.357	1.3086 and 1.3281
1.3086	1.3281	1.3184	-0.009	1.3184 and 1.3281

Hence, the normal depth, h_n = 1.32 m

Newton-Raphson method: Suppose we want to compute the normal depth in a channel for given section, Q, n and S_0. Obviously, when $h = h_n$

$$AR^{2/3} - \frac{nQ}{\sqrt{S_0}} = 0 \qquad or, \quad A^{5/3} - \frac{nQ}{\sqrt{S_0}}P^{2/3} = 0 \qquad \text{(since R = A/P)}$$

If we now assume

$$f(h) = A^{5/3} - \frac{nQ}{\sqrt{S_0}}P^{2/3} \tag{4.57}$$

then

$$f'(h) = \frac{5}{3}A^{2/3}\frac{dA}{dh} - \frac{nQ}{\sqrt{S_0}} \times \frac{2}{3}P^{-1/3}\frac{dP}{dh} = \frac{5}{3}A^{2/3}B - \frac{2nQ}{3\sqrt{S_0}}P^{-1/3}\frac{dP}{dh} \quad (\therefore \frac{dA}{dh} = B) \tag{4.58}$$

For a given channel section, $f(h)$ and $f'(h)$ depend on the depth of flow only and hence can be easily evaluated.

Example 4.12

For a trapezoidal channel with b = 6 m, s = 2, n = 0.025 and $S_0 = 0.001$, compute the normal depth by the Newton-Raphson method if Q = 14 m^3/s.

Solution $\quad A = (6+2h)h \qquad P = 6 + 2\sqrt{5}h \qquad and \qquad B = 6+4h$

$$\frac{nQ}{\sqrt{S_0}} = \frac{0.025 \times 14}{\sqrt{0.001}} = 11.068$$

$$f(h) = [(6+2h)h]^{5/3} - 11.068(6+2\sqrt{5}h)^{2/3} = 3.175[(3+h)h]^{5/3} - 17.569(4+\sqrt{5}h)^{2/3}$$

$$f'(h) = 5.291(3+2h)[(3+h)h]^{2/3} - 26.191(3+\sqrt{5}h)^{-1/3}$$

The computation of normal depth is carried out as follows.

h	$f(h)$	$f'(h)$	$\Delta h = -\frac{f(h)}{f'(h)}$	$h = h + \Delta h$
1.000	-20.979	51.579	0.407	1.407
1.407	7.493	89.555	- 0.083	1.324
1.324	0.406	81.215	- 0.005	1.319
1.319	0.003	79.094	- 0.000	1.319

Hence, the normal depth, h_n = 1.32 m

Normal Depth for a Conduit with Gradually Closing Top

In some channel sections the top width either remains constant or increases with flow depth. The section factor $AR^{2/3}$ for these channel sections increases with an increase in the depth of flow and there is only one normal depth for a given discharge. The rectangular, triangular, trapezoidal and parabolic sections fall in this category. However, for channel sections having a gradually closing top, the section factor at first increases with depth and decreases with depth when the full depth is approached. Consequently, it is possible to have two normal depths for the same value of the section factor and, therefore, for the same discharge. The circular section falls in this later category.

In a circular section when the depth is greater than about $0.82d_0$, it is possible to have two different normal depths for the same discharge and a small disturbance in the water surface may cause the water surface to seek alternate normal depths, thus contributing to the instability of the water surface. Therefore, it is advisable in the design of a circular section to restrict the depth to a value less than or equal to $0.80d_0$.

4.6 COMPUTATION OF NORMAL AND CRITICAL SLOPES

The normal slope (S_n) is the longitudinal slope of the channel that is required to maintain uniform flow in the channel. When the Manning formula is used

$$S_n = \frac{n^2 U_n^{\ 2}}{R_n^{\ 4/3}} = \frac{n^2 Q_n^{\ 2}}{A_n^{\ 2} R_n^{\ 4/3}} \tag{4.59}$$

or, when the Chezy formula is used

$$S_n = \frac{U_n^{\ 2}}{C^2 R_n} = \frac{Q_n^{\ 2}}{C^2 A_n^{\ 2} R_n} \tag{4.60}$$

Equations (4.59) and (4.60) indicate that the normal slope depends on the channel section, the discharge, the depth and the channel roughness. Thus, when the channel section, Q, n or C and h_n are given, the normal slope can be obtained using Eq.(4.59) or (4.60).

The critical slope (S_c) is the longitudinal slope of the channel for which the flow in the channel is both uniform and critical, i.e. uniform flow occurs in a critical state and $S_n = S_c$, $U_n = U_c$ and $h_n = h_c$. When the channel section, n or C and h or Q are given, the critical slope can be determined using the Manning formula as

$$S_c = \frac{n^2 U^2}{R^{4/3}} = \frac{n^2 Q^2}{A^2 R^{4/3}} \tag{4.61}$$

or, using the Chezy formula as

$$S_c = \frac{U^2}{C^2 R} = \frac{Q^2}{C^2 A^2 R} \tag{4.62}$$

When Q is given, the normal depth h_n, which is also equal to the critical depth h_c, is first computed using the critical condition and then the critical slope is computed using Eq.(4.61) or (4.62). On the other hand, when h_n ($= h_c$) is given, the mean velocity U or the discharge Q is first determined using the critical condition and then the critical slope is computed using Eq.(4.61) or (4.62).

Example 4.13

A rectangular channel has a bottom width of 6 m, $\alpha = 1.12$ and $n = 0.020$. (i) For $h_n = 1$ m and $Q = 11$ m^3/s, determine the normal slope. (ii) Determine the critical slope for $Q = 11$ m^3/s. (iii) Determine the critical slope for $h_n = 1$ m.

Solution Rectangular channel, $b = 6$ m, $\alpha = 1.12$, $n = 0.020$

(i) $h_n = 1$ m $Q = 11$ m^3/s

$A = bh = 6\times 1 = 6$ m^2, $P = b + 2h = 8$ m, $R = A/P = 0.75$ m

$$\therefore S_n = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.020\times 11}{6\times 0.75^{2/3}}\right)^2 = 0.0020$$

(ii) $Q = 11$ m^3/s

$$h_c = \sqrt[3]{\frac{\alpha Q^2}{gb^2}} = \sqrt[3]{\frac{1.12\times 11^2}{9.81\times 6^2}} = 0.73\ m$$

$$\therefore h_n = h_c = 0.73\ m$$

$A = bh = 6\times 0.73 = 4.36$ m^2, $P = b + 2h = 6 + 2\times 0.73 = 7.45$ m, $R = A/P = 0.58$ m

$$\therefore S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.020\times 11}{4.36\times 0.58^{2/3}}\right)^2 = 0.0053$$

(iii) $h_c = h_n = 1$ m

$A = bh = 6\times 1 = 6$ m^2, $P = b + 2h = 6 + 2\times 1 = 8$ m, $R = A/P = 0.75$ m

$$\therefore U_n = U_c = \sqrt{gD_c/\alpha} = \sqrt{gh_c/\alpha} = \sqrt{9.81\times 1/1.12} = 2.96\ m/s$$

$$\therefore Q = AU_c = 6\times 2.96 = 17.76\ m^3/s$$

$$or,\ Q = \sqrt{g/\alpha}\, bh_c^{1.5} = \sqrt{9.81/1.12}\times 6\times 1^{1..5} = 17.76\ m^3/s$$

$$\therefore S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.020\times 17.76}{6\times 0.75^{2/3}}\right)^2 = 0.0051$$

4.7 CHANNEL SECTION WITH COMPOSITE ROUGHNESS

The roughness may vary along the perimeter of a channel as shown in Fig. 4.1(a). Such a channel section is known as a channel section with composite roughness. A good example of such a section is provided by a rectangular flume built with a wooden bottom and glass walls having different n-values for the bottom and the walls.

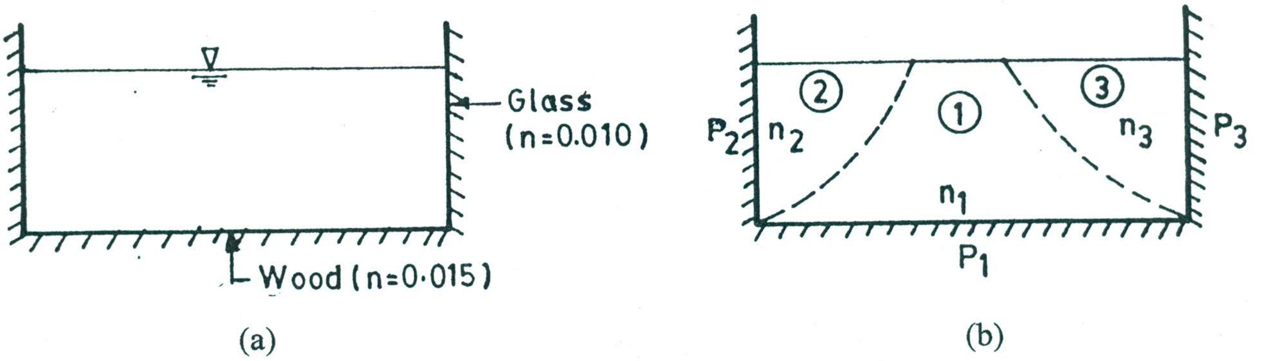


Fig. 4.1 Channel section with composite roughness

In applying the Manning or the Chezy formula to compute flow in such a section, it is necessary to compute an equivalent n-value for the entire perimeter. Consider a channel in which the flow area is divided into 3 parts, as shown in Fig.4.1(b), of which the wetted perimeters P_1, P_2 and P_3 and the corresponding coefficients of roughness n_1, n_2 and n_3 are known. Following Horton (1933), it is assumed that each part of the area has the same mean velocity that is also equal to the mean velocity of the whole section, i.e.

$$U_1 = U_2 = U_3 = U \tag{4.63}$$

On the basis of this assumption, the equivalent n-value for the entire section is obtained as follows. Since the energy slope is the same for all the sections, Eqs. (4.27) and (4.63) give

$$\frac{1}{n_1} R_1^{2/3} = \frac{1}{n_2} R_2^{2/3} = \frac{1}{n_3} R_3^{2/3} = \frac{1}{n} R^{2/3} \tag{4.64}$$

or

$$\frac{1}{n_1}\left(\frac{A_1}{P_1}\right)^{2/3} = \frac{1}{n_2}\left(\frac{A_2}{P_2}\right)^{2/3} = \frac{1}{n_3}\left(\frac{A_3}{P_3}\right)^{2/3} = \frac{1}{n}\left(\frac{A}{P}\right)^{2/3} \tag{4.65}$$

so that

$$A_1 = \left(\frac{n_1}{n}\right)^{3/2} \left(\frac{P_1}{P}\right) A \tag{4.66a}$$

$$A_2 = \left(\frac{n_2}{n}\right)^{3/2} \left(\frac{P_2}{P}\right) A \tag{4.66b}$$

$$A_3 = \left(\frac{n_3}{n}\right)^{3/2} \left(\frac{P_3}{P}\right) A \tag{4.66c}$$

Using the above three equations in

$$A = A_1 + A_2 + A_3$$

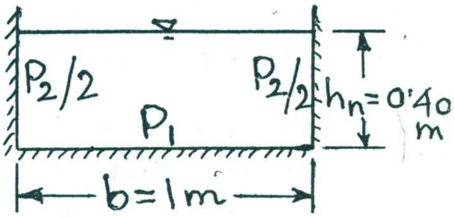
simplifying and rearranging, we get

$$n = \left(\frac{P_1 n_1^{3/2} + P_2 n_2^{3/2} + P_3 n_3^{3/2}}{P} \right)^{2/3} \tag{4.67}$$

which can be used to compute the equivalent n-value for the entire section.

Example 4.14

The sides of a laboratory flume are made of glass (n = 0.010) and the bottom is made of wood (n = 0.015). The flume is rectangular with b = 1 m and is laid on a slope of 0.001. Compute the discharge in the flume if h_n = 0.4 m.



Solution Designating the perimeter of the bottom as P_1 and the combined perimeter of the two sides as P_2, we have $P_1 = 1$ m, $P_2 = 2\times0.4 = 0.8$ m, $P = P_1 + P_2 = 1.8$ m, $n_1 = 0.015$, $n_2 = 0.010$. Then,

$$A = 1\times0.4 = 0.4 \text{ m}^2, \quad R = A/P = 0.4/1.8 = 0.222 \text{ m}$$

$$n = \left(\frac{P_1 n_1^{3/2} + P_2 n_2^{3/2}}{P} \right)^{2/3} = \left(\frac{1\times0.015^{3/2} + 0.8\times0.010^{3/2}}{1.8} \right)^{2/3} = 0.013$$

$$Q = \frac{1}{n} A R^{2/3} S_0^{1/2} = \frac{1}{0.013} \times 0.4 \times 0.222^{2/3} \times 0.001^{1/2} = 0.36 \ m^3/s$$

4.8 COMPOUND CROSS-SECTION

The cross-section of a channel may be composed of several distinct subsections as shown in Fig.4.2. Such a channel section is known as a compound section. For example, an alluvial river subjected to seasonal floods generally consists of a main channel and two side channels. The side channels are usually shallower and rougher than the main channel. So the mean velocity in the main channel is greater than the mean velocity in the side channels.

In dealing with a compound cross-section, the subsections (the main and the side channels) are first separated by drawing vertical lines at the banks (shown as dotted lines in Fig.4.2). It is assumed that (i) the water surface and energy line are horizontal across the cross-section, and (ii) the vertical separation lines are the lines of zero shear and, therefore, do not contribute to the wetted perimeters of the subsections.

The discharges in the subsections are computed by applying the Manning or the Chezy formula separately to each subsection, i.e.

$$Q_1 = \frac{1}{n_1} A_1 R_1^{2/3} S_0^{1/2} \tag{4.68a}$$

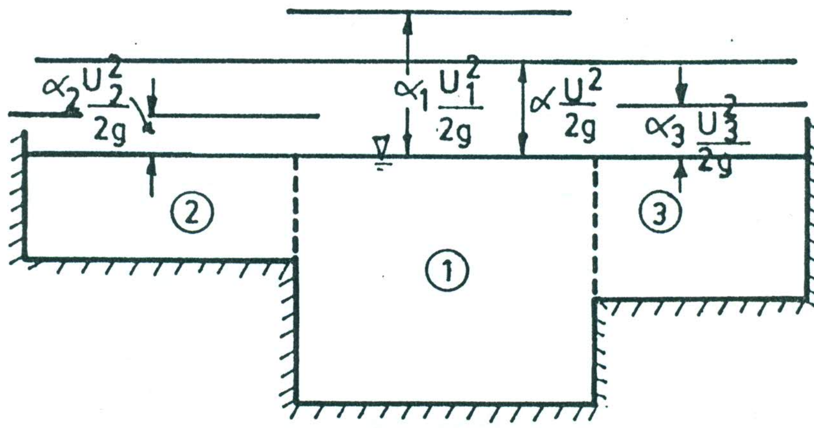


Fig. 4.2 Compound cross-section

$$Q_2 = \frac{1}{n_2} A_2 R_2^{2/3} S_0^{1/2} \tag{4.68b}$$

$$Q_3 = \frac{1}{n_3} A_3 R_3^{2/3} S_0^{1/2} \tag{4.68c}$$

The total discharge for the entire section is equal to the sum of these discharges, i.e.

$$Q = Q_1 + Q_2 + Q_3 \tag{4.69}$$

The mean velocity for the entire section is equal to the total discharge divided by the total area, i.e.

$$U = Q/A \tag{4.70}$$

where the total area

$$A = A_1 + A_2 + A_3 \tag{4.71}$$

The equivalent n-value for the entire section can be computed using the Manning formula for the entire section as

$$n = AR^{2/3} S_0^{1/2} / Q \tag{4.72}$$

or, using the equation

$$n = \frac{AR^{2/3}}{K_1 + K_2 + K_3} = \frac{AR^{2/3}}{K} \tag{4.73}$$

where K_1, K_2 and K_3 are the conveyances of the individual subsections and K ($= K_1 + K_2 + K_3$) is the conveyance for the entire section.

The energy and momentum coefficients for the entire section are obtained using the equations

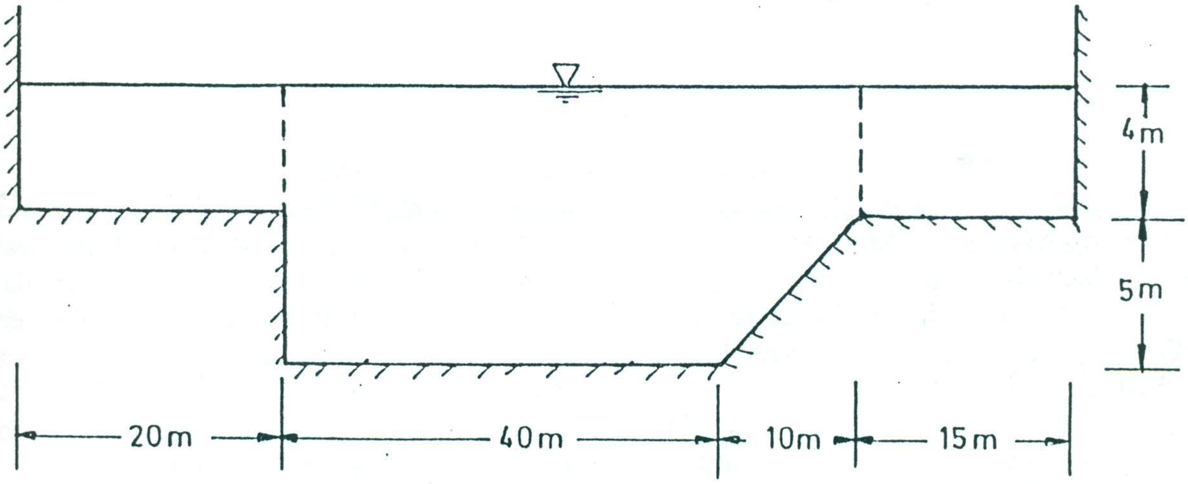
$$\alpha = \frac{\alpha_1 K_1^3 / A_1^2 + \alpha_2 K_2^3 / A_2^2 + \alpha_3 K_3^3 / A_3^2}{K^3 / A^2} \tag{4.74}$$

$$\beta = \frac{\beta_1 K_1^2 / A_1 + \beta_2 K_2^2 / A_2 + \beta_3 K_3^2 / A_3}{K^2 / A} \tag{4.75}$$

where α_1, α_2 and α_3 are the energy coefficients and β_1, β_2 and β_3 are the momentum coefficients for the individual subsections, respectively.

Example 4.15

A channel consists of a main section and two side sections as shown in the figure. Compute the total discharge and the mean velocity of flow for the entire section when n = 0.025 for the main section, n = 0.035 for the side sections and $S_0 = 0.0001$. Also compute the numerical values of n, α and β for the entire section assuming that α = 1.12 and β = 1.04 for the main and side sections.



Solution

The main and the two side (left and right) sections are separated by drawing vertical (dotted) lines as shown. The computed values of A, P, R, Q and K for the three subsections and A, P, Q and K for the entire section are shown below.

Section	A	P	R	Q	K
Main	425.0	56.18	7.565	655.12	65512
Left	80.0	24.00	3.333	51.00	5100
Right	60.0	19.00	3.158	36.90	3690
Σ	565.0	99.18		743.02	74302

Hence, the total discharge, Q = 743.02 m^3/s

The mean velocity U, roughness coefficient n, energy coefficient α and momentum coefficient β for the entire section are obtained as follows.

$$U = \frac{Q}{A} = \frac{743.02}{565.0} = 1.315\ m/s$$

$$n = AR^{2/3}S_0^{\ 1/2}/Q = 565.0 \times (565.0/99.18)^{2/3} \times 0.0001^{1/2}/743.02 = 0.024$$

$$\alpha = \frac{\alpha_1 K_1^{\ 3}/A_1^{\ 2} + \alpha_2 K_2^{\ 3}/A_2^{\ 2} + \alpha_3 K_3^{\ 3}/A_3^{\ 2}}{K^3/A^2}$$
$$= \frac{1.12 \times 65512^3/425^2 + 1.12 \times 5100^3/80^2 + 1.12 \times 3690^3/60^2}{74302^3/565^2} = 1.236$$

$$\beta = \frac{\beta_1 K_1^{\ 2}/A_1 + \beta_2 K_2^{\ 2}/A_2 + \beta_3 K_3^{\ 2}/A_3}{K^2/A}$$
$$= \frac{1.04 \times 65512^2/425 + 1.04 \times 5100^2/80 + 1.04 \times 3690^2/60}{74302^2/565} = 1.090$$

4.9 COMPUTATION OF FLOOD DISCHARGE BY SLOPE-AREA METHOD

The slope-area method is an indirect method of estimating the flood discharge in a river from past records of stages at different sections using the Manning or the Chezy formula. Flood flows in natural channels are usually unsteady and varied. The use of a steady uniform flow formula for computing flood discharge is acceptable only when the changes in flood stage and discharge are sufficiently gradual and the data available are not sufficient to justify the use of a more sophisticated method.

The selection of a suitable channel reach is probably the most important aspect of the slope-area method. The following points must be considered in selecting the channel reach.

i) Reliable and good quality highwater marks must be available in the selected reach.

ii) A straight and uniform reach is preferred and a gradually contracting reach should be chosen over an expanding reach if there is a choice. The change in conveyance in the reach should be less than 30%.

iii) The length of the reach L should be at least 75 times the average depth of flow, the fall of water surface F should be equal to or greater than the velocity head and the fall should at least be equal to 0.15 m.

With reference to Fig. 4.3, applying the energy equation between sections 1 and 2, we get

$$F + \alpha_1 \frac{U_1^{\ 2}}{2g} = \alpha_2 \frac{U_2^{\ 2}}{2g} + h_f + h_e \tag{4.76}$$

or

$$h_f = F + (\alpha_1 \frac{U_1^{\ 2}}{2g} - \alpha_2 \frac{U_2^{\ 2}}{2g}) - h_e \tag{4.77}$$

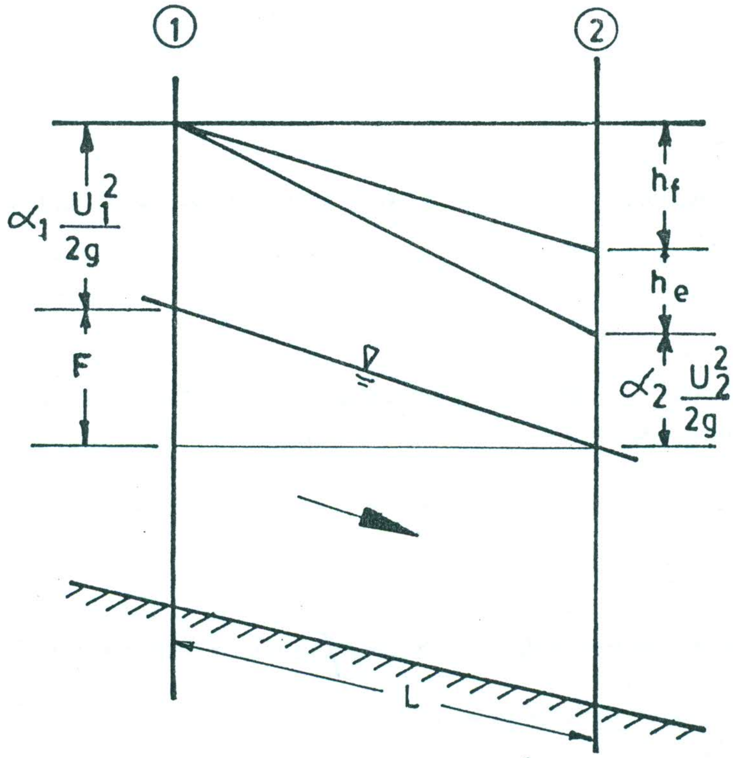


Fig. 4.3 Definition sketch

where h_e is the eddy loss, given by

$$h_e = k\left|\alpha_1 \frac{U_1^2}{2g} - \alpha_2 \frac{U_2^2}{2g}\right| \tag{4.78}$$

and the coefficient k is assumed to range between 0 and 0.1 for gradual contractions, between 0 and 0.2 for gradual expansions and to have a value of 0.5 for abrupt expansions or contractions.

The following data are required for the slope-area method: (i) the cross-sectional areas A_1 and A_2 of the upstream and downstream sections of the selected reach, (ii) the wetted perimeters P_1 and P_2, (iii) the Manning roughness coefficients n_1 and n_2, (iv) the energy coefficients α_1 and α_2, (v) the length of the reach L, and (vi) the fall of the water surface F between the two sections. In addition, the eddy-loss coefficient k is to be given if the eddy loss is to be included.

The computation of flood discharge using this method involves the following steps:

i) Compute the conveyances K_1 and K_2 for the two end sections.

ii) Compute the geometric mean conveyance for the reach, i.e.

$$K = \sqrt{K_1 K_2} \tag{4.79}$$

iii) Since the discharge Q is not known initially, as a first approximation assume that $h_f = F$ and hence

$$S_f = \frac{h_f}{L} = \frac{F}{L} \tag{4.80}$$

iv) The first approximation of the discharge (which is also the uniform flow discharge) is then computed using the equation

$$Q = K\sqrt{S_f} \tag{4.81}$$

v) A more accurate value of the energy slope is now obtained using the equation

$$S_f = \frac{h_f}{L} \tag{4.82}$$

where h_f is given by Eq.(4.77). The corresponding discharge is then computed by Eq.(4.81) using the revised slope given by Eq.(4.82).

vi) Repeat step (v) until the assumed and the computed discharges agree.

Example 4.16

Compute the flood discharge through a river ~~a~~ reach of 850 m using the following data:

$A_1 = 10350\ m^2$, $P_1 = 2035$ m, $n_1 = 0.030$, $\alpha_1 = 1.15$

$A_2 = 9275\ m^2$, $P_2 = 1965$ m, $n_2 = 0.030$, $\alpha_2 = 1.18$

The fall of water surface in the reach is 0.76 m. Neglect eddy loss.

Solution L = 850 m, F = 0.76 m, $h_e = 0$

$$K_1 = \frac{1}{n_1} A_1 R_1^{2/3} = \frac{1}{0.030} \times 10350 \times (10350/2035)^{2/3} = 1020320$$

$$K_2 = \frac{1}{n_2} A_2 R_2^{2/3} = \frac{1}{0.030} \times 9275 \times (9275/1965)^{2/3} = 869950$$

$$K = \sqrt{K_1 K_2} = \sqrt{1020320 \times 869950} = 942140$$

Approxi-mation	Assumed Q(m^3/s)	F (m)	$\alpha_1 \frac{U_1^2}{2g}$(m)	$\alpha_2 \frac{U_2^2}{2g}$(m)	h_f (m)	S_f ($\times 10^{-4}$)	Computed Q(m^3/s)
1st	-	0.76	-	-	0.7600	8.94	28170
2nd	28170	"	0.4343	0.5548	0.6395	7.52	25840
3rd	25840	"	0.3653	0.4668	0.6585	7.75	26220
4th	26220	"	0.3762	0.4806	0.6556	7.73	26170
5th	26170	"	0.3747	0.4788	0.6559	7.72	26170

$\therefore$ The flood discharge, Q = 26,170 m^3/s

PROBLEMS

4.1 Assuming that the velocity distribution along a vertical in an open channel is logarithmic, compute the position of the mean velocity below the free surface. Also show that (i) the velocity at 0.6 depth, and (ii) the average of the velocities at 0.2 and 0.8 depths are approximately equal to the mean velocity in a vertical.

4.2(a) A trapezoidal channel has a bottom width of 6.0 m, side slopes of 1.5H:1V, a depth of flow of 2.0 m, n = 0.025 and $S_0 = 0.0001$. Assuming that the flow is uniform, (i) compute Q, (ii) compute C, f, τ_0 and u*, and (iii) compute k_s, determine whether the channel boundary is smooth or rough and state if the Manning formula is applicable for computing flow in this channel. Assume that the velocity distribution is logarithmic.

(b) Consider the following data for the Padma (Ganges) river at the Baruria station in Faridpur on the 2nd July, 1989: A = 33,500 m^2, Q = 56,200 m^3/s and B = 3820 m. Assuming that the flow is uniform, (i) compute n, C, f, u* and τ_0, and (ii) determine whether the channel boundary is smooth or rough taking the velocity distribution as logarithmic. Assume that the river is wide. Longitudinal slope of the river is 4 cm/km.

4.3(a) Show that for a wide rough channel with logarithmic velocity distribution in the vertical, the Manning's roughness coefficient n may be expressed by

$$n = \frac{(r-1)h^{1/6}}{5.57(r+0.95)}$$

where r (= $u_{0.2}/u_{0.8}$)is the ratio between the measured velocities at two-tenths and eight-tenths of depth.

(b) The velocities at 0.2 and 0.8 of the depth along a vertical in a wide river are 1.25 m/s and 1 m/s, respectively. (i) Compute the numerical value of Manning's n, and (ii) determine the variation of velocity in the vertical, if the river is 10 m deep.

4.4(a) Using the Manning formula and taking h = 1 m, compute the hydraulic exponent for uniform flow computation N for a

i. rectangular channel with b = 6 m,
ii. trapezoidal channel with b = 6 m and s = 2,
iii. parabolic channel whose profile is given by $y^2 = 4z$, and
iv. circular channel whose diameter is 2 m.

(b) Solve Problem 4.4(a) using the Chezy formula.

4.5(a) A wide channel with n = 0.025 and S_0 = 0.0025 carries a discharge of 3 m^2/s. Compute the normal depth and velocity.

(b) A wide channel with S_0 = 0.006 and C = 50 $m^{1/2}$/s carries a discharge of 4 m^2/s. Compute the normal depth and velocity.

4.6(a) A triangular channel with side slopes 1:1 is laid on a slope of 0.001. If n = 0.015 and h_n = 1 m, compute the discharge.

(b) A triangular channel with s = 1, n = 0.025 and S_0 = 0.0025 carries a discharge of 5 m^3/s. Compute the normal depth and velocity.

4.7 Water flows at a velocity of 1 m/s in an open channel under uniform flow condition. The longitudinal slope of the channel is 0.0016 and n = 0.020. Compute the depth of flow when the channel is

i) rectangular with b = 6 m,
ii) trapezoidal with b = 6 m and s = 2,
iii) triangular with s = 1,
iv) parabolic whose profile is given by $y^2 = 4z$, and
v) circular whose diameter is 2 m.

4.8 Uniform flow occurs in an open channel with h_n = 1 m, S_0 = 0.0001 and n = 0.015. Compute the discharge if the channel is

i) rectangular with b = 6 m,
ii) trapezoidal with b = 6 m and s = 1,
iii) triangular with s = 1.5,
iv) parabolic whose profile is given by $y^2 = 4z$, and
v) circular whose diameter is 1.5 m.

4.9(a) A rectangular channel having n = 0.025 and S_0 = 0.0001 carries a discharge of 6 m^3/s at a normal depth of 1.5 m. Compute the bottom width.

(b) A trapezoidal channel having side slopes of 1.5H:1V, n = 0.020 and S_0 = 0.0002 carries a discharge of 25 m^3/s at a normal depth of 2 m. Compute the bottom width.

4.10 Compute the normal depth and velocity in a

i. rectangular channel with b = 8 m and Q = 22 m^3/s,
ii. trapezoidal channel with b = 6 m, s = 2 and Q = 30 m^3/s,
iii. parabolic channel whose profile is given by $y^2 = 5z$ and Q = 15 m^3/s, and
iv. ~~a~~ circular channel whose diameter is 2 m and Q = 3 m^3/s.

In all cases, take n = 0.025 and S_0 = 0.0025.

4.11(a) A trapezoidal channel has a bottom width of 6 m, side slopes of 1.5:1, α = 1 and n = 0.025. (i) Determine the normal slope at a normal depth of 1 m when the discharge is 20 m^3/s. (ii) Determine the critical slope when the discharge is 20 m^3/s. (iii) Determine the critical slope at the normal depth of 1 m.

(b) Solve Problem 4.11(a) when α = 1.12.

4.12 When the Manning formula is used, show the critical slope at a given normal depth h_n may be expressed by

$$S_c = \frac{gn^2 D_n}{R_n^{4/3}}$$

and that this slope for a wide channel is

$$S_c = \frac{gn^2}{h_n^{1/3}} = \frac{n^2 g^{10/9}}{q^{2/9}}$$

where q is the discharge per unit width.

4.13 A channel consists of a main section and two side sections as shown in Fig.4.4. Compute the total discharge, the mean velocity of flow and the Manning's n for the entire section when $n = 0.025$ for the main channel, $n = 0.045$ for the side channels and $S_0 = 0.0002$. Also, compute the numerical values α and β for the entire section assuming that $\alpha = \beta = 1.00$ for the main and the side sections.

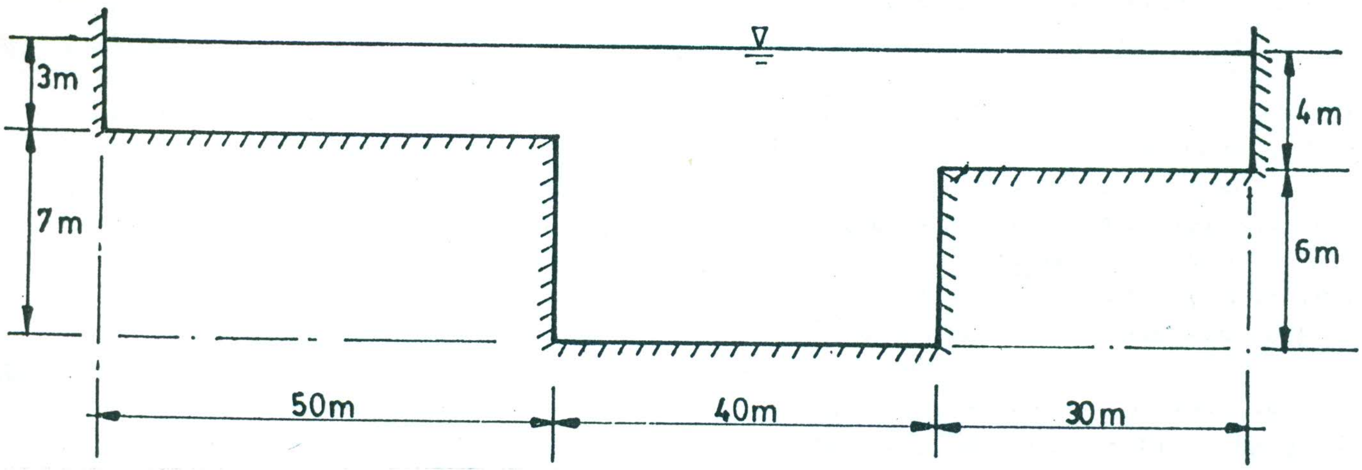


Fig. 4.4 for Problem 4.13

4.14 An unlined irrigation canal ($n = 0.025$) is trapezoidal and has a bottom width of 6 m, side slopes of 1:1 and a depth of flow of 2 m. The longitudinal slope of the canal is 0.0005. Compute the discharge carried by the canal under uniform flow condition. It is proposed to line the canal with concrete having $n = 0.013$. Compute the discharge that would be carried by the canal when (i) only the sides are lined, (ii) only the bottom is lined, and (iii) both the bottom and the sides are lined.

4.15 A rectangular testing channel is 0.60 m wide and is laid on a slope of 0.1%. When the channel bed and walls were made smooth by neat cement, the measured normal depth of flow was 0.40 m for a discharge of 0.23 m^3/s. The same channel was then roughened by cemented sand grains and the measured normal depth was 0.35 m for a discharge of 0.12 m^3/s. Determine the discharge for a normal depth of 0.45 m if the bed is roughened and the walls are made smooth.

4.16 Compute the flood discharge through a river reach 1000 m long having a fall in water surface of 0.85 m. Neglect the eddy loss. Use the following data:

Section	A (m^2)	P (m)	n	α
Upstream	12,000	2,150	0.030	1.15
Downstream	10,500	2,050	0.030	1.18

****

Chapter 5

DESIGN OF CHANNELS

definitions of all

5.1 INTRODUCTION

The hydraulic design of a channel, like an irrigation canal or a drainage channel, involves the determination of the cross-sectional dimensions of the channel, e.g. the bottom width b, the side slope s, if any, the depth of flow h etc., to convey the required discharge with the available difference in head. Since a channel may be lined or unlined, and may or may not carry sediment, we consider the design of three types of channels:(i) rigid-boundary or non-erodible channels carrying clear water with little or no sediment, (ii) mobile-boundary or erodible channels carrying clear water which scour but do not silt, and (iii) alluvial or mobile boundary channels carrying sediment which both scour and silt.

The shape of the cross-section to be used is generally decided by the discharge and the engineering properties of the material forming the channel body. Normally, a trapezoidal section is used when the discharge is large. For small discharges, triangular sections are used. Rectangular cross-sections are also used when the discharge is small or in special situations, such as rock cuts, steep chutes and cross-drainage works.

The type of materials forming the channel body determines the roughness characteristics and the side slope of the channel. The longitudinal slope of the channel generally depends on the topography of the land, the energy head required for the flow of water, and in many cases, on the purpose of the channel.

In the design of a channel, it is desirable to maintain subcritical flow in the channel having a Froude number range of 0.3 to 0.4. When the Froude number is high and the flow approaches the critical state, the water surface becomes unstable and wavy and large disturbances are expected at bends and obstructions

The *freeboard* is the vertical distance between the top of the channel and the water surface at the design condition. It is provided to prevent the water level from overtopping the sides of the channel due to its fluctuation caused by wind, tide, superelevation at bends, hydraulic jump etc. Freeboards varying from 5% to 30% of the depth of flow are commonly used in design. As a rough estimate of freeboard in unlined channels, the United States Bureau of Reclamation (USBR) suggested the formula

$$F_b = \sqrt{ch} \tag{5.1}$$

where F_b is the freeboard in ft, h is the depth of flow in ft and c is a factor varying from 1.5 for $Q < 20$ ft^3/sec (0.57 m^3/s) to 2.5 for $Q > 300$ ft^3/sec (8.52 m^3/s).

The current practice of providing freeboard in India is as follows (Ranga Raju,1993):

Discharge, Q(m^3/s)	< 0.75	0.75 - 1.50	1.50 – 85.0	> 85.0
Freeboard, F_b(m)	0.45	0.60	0.75	0.90

In case of lined channels, the top of the lining is generally located half the total freeboard above the water surface.

5.2 RIGID-BOUNDARY OR NON-ERODIBLE CHANNELS

Most of the lined channels and the built-up unlined channels fall into this category. A uniform flow formula, like the Manning or the Chezy formula, is used to compute the section dimensions of the channel by maintaining a velocity which will not cause sedimentation of the particles in suspension on the channel boundary.

For lined channels, the materials used include concrete, stone or brick masonry, steel, cast iron, timber, glass, plastic, geotextile etc. The choice of a material depends mainly on the availability and cost of the material and the purpose and the method of construction. The provision of lining in a channel (i) permits the water to flow at high velocities, (ii) decreases seepage and percolation losses, (iii) reduces the costs of operation and maintenance, and (iv) ensures the stability of the channel section.

The *minimum permissible velocity* or the *non-silting velocity* is the lowest mean velocity of flow that will prevent sedimentation and vegetative growth. The exact value of this velocity cannot be easily determined. In general, an average velocity of 2 to 3 ft/sec (0.61 to 0.91 m/s) will prevent sedimentation when the silt load of the flow is low (Chow, 1959). A velocity of 2.5 ft/sec (0.75 m/s) is usually sufficient to prevent the growth of vegetation.

The *maximum permissible velocity* or the *non-erodible velocity* is the greatest mean velocity that will not cause erosion of the channel body. The maximum permissible velocity is not usually the criterion in the design of non-erodible channels. However, when the flow velocities are very high, there is a tendency for the rapidly flowing water to lift the lining blocks and push them out of position. For brick and concrete tile linings, the mean velocity of flow is restricted to 2 m/s to avoid any danger to the lining materials.

Best Hydraulic Section

A channel section that conveys the maximum discharge for a given area is known as the best hydraulic section. Since $Q \infty AR^{2/3}$ and $R = A/P$, the best hydraulic section gives minimum wetted perimeter P and maximum hydraulic radius R for a given area A. It is to be noted that a minimum wetted perimeter also gives minimum amount of lining.

Among all possible open channel cross-sections, the best hydraulic section is a semicircle ($h = d_0/2$). Also, for any channel section of a given geometric shape, there is a relationship between the various geometric elements to form the best hydraulic section such that a semicircle can be inscribed in it. Thus, the best hydraulic rectangular section is one-half of a square ($B = 2h$), the best hydraulic triangular section is one-half of a square ($s = 1$), the best hydraulic trapezoidal section is one-half of a regular hexagon ($s = 1/\sqrt{3}$) and for the best hydraulic parabolic section the top width is equal to $2\sqrt{2}$ times the depth of flow ($B = 2\sqrt{2}h$). Table 5.1 lists the geometric elements of some best hydraulic sections.

Table 5.1 Geometric elements of some best hydraulic sections

Cross-section	A	P	R	B	D	S
Rectangle (half of a square)	$2h^2$	$4h$	$h/2$	$2h$	h	
Triangle (half of a square)	h^2	$2\sqrt{2}h$	$\sqrt{2}h/4$	$2h$	$h/2$	1
Trapezoid (half of a hexagon)	$\sqrt{3}h^2$	$2\sqrt{3}h$	$h/2$	$4\sqrt{3}h/3$	$3h/4$	$1/\sqrt{3}$
Circle (semi-circle)	$\pi h^2/2$	πh	$h/2$	$2h$	$\pi h/4$	
Parabola ($B=2\sqrt{2}h$)	$4\sqrt{2}h^2/3$	$8\sqrt{2}h/3$	$h/2$	$2\sqrt{2}h$	$2h/3$	

Problem → Q for a rectangular section.

Question: $v = 2.5 m/sec$, $Q = ?$ if $y = 1.5 m$ [assume ~~secti~~ best hydraulic rectangular section]

$b = 2y$

Example 5.1

Show that the best hydraulic rectangular section is one-half of a square. Also, determine the geometric elements of best hydraulic rectangular section.

Solution For a rectangular section, A = bh $\therefore$ b = A/h

$$\therefore P = b + 2h = \frac{A}{h} + 2h$$

Considering A to be constant and differentiating P with respect to h, we get

$$\frac{dP}{dh} = -\frac{A}{h^2} + 2$$

For P to be minimum, dP/dh = 0. Hence we get

$$-\frac{A}{h^2} + 2 = 0 \quad or, \ \frac{bh}{h^2} = 2$$

or, $b = 2h$

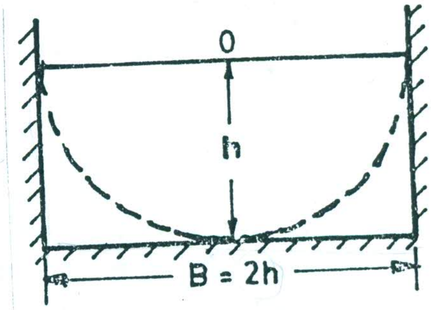


Fig.5.1 Best hydraulic rectangular section

This means that the best hydraulic rectangular section is one-half of a square, as shown in Fig. 5.1. A semi-circle with O as center and h as radius can be inscribed in the best hydraulic rectangular section. Also, for the best hydraulic rectangular section, we get

$$A = bh = 2h^2 \qquad P = b + 2h = 4h \qquad R = A/P = 2h^2/4h = h/2$$

$$B = b = 2h \qquad D = A/B = 2h^2/2h = h$$

as given in Table 5.1.

Example 5.2

Show that the best hydraulic trapezoidal channel section is one-half of a regular hexagon.

Solution For a trapezoidal section, $A = (b + sh)h$ (i)

$$or, \ b = \frac{A}{h} - sh$$

$$\therefore P = b + 2\sqrt{1+s^2}\,h \qquad \text{(ii)}$$

$$= \frac{A}{h} - sh + 2\sqrt{1+s^2}\,h$$

$$or, \ P = \frac{A}{h} + (2\sqrt{1+s^2} - s)h \qquad \text{(iii)}$$

Considering A and s to be constant and differentiating P with respect to h, we get

$$\frac{dP}{dh} = -\frac{A}{h^2} + 2\sqrt{1+s^2} - s$$

For the minimum value of P, dP/dh = 0. Hence, we get

$$A = (2\sqrt{1+s^2} - s)h^2 \qquad \text{(iv)}$$

Combining (i) and (iv), we get

$$(b + sh)h = (2\sqrt{1+s^2} - s)h^2$$

$$or, \ b = 2(\sqrt{1+s^2} - s)h \qquad \text{(v)}$$

Substituting (v) into (ii), we get

$$P = 2(\sqrt{1+s^2} - s)h + 2\sqrt{1+s^2}\,h = 2h(2\sqrt{1+s^2} - s) \qquad \text{(vi)}$$

Dividing (iv) by (vi), we get

$$R = A/P = h/2 \qquad \text{(vii)}$$

i.e. for the best hydraulic trapezoidal section, the hydraulic radius R is one-half of the depth of flow h, irrespective of the side slope.

From (vi), we have

$$P^2 = 4h^2(2\sqrt{1+s^2} - s)^2 \qquad \text{(viii)}$$

and from (iv), we have

$$h^2 = \frac{A}{2\sqrt{1+s^2} - s} \qquad \text{(ix)}$$

Using (ix) in (viii), we get

$$P^2 = \frac{4A(2\sqrt{1+s^2} - s)^2}{2\sqrt{1+s^2} - s} = 4A(2\sqrt{1+s^2} - s) \qquad \text{(x)}$$

Considering A to be constant and differentiating P with respect to s, we get

$$2P\frac{dP}{ds} = 4A\left(\frac{2\times\frac{1}{2}\times 2s}{\sqrt{1+s^2}} - 1\right) = 4A\left(\frac{2s}{\sqrt{1+s^2}} - 1\right)$$

For minimum P, dP/ds = 0. Hence

$$\frac{2s}{\sqrt{1+s^2}} - 1 = 0 \quad or,\ 2s = \sqrt{1+s^2} \quad or,\ 4s^2 = 1 + s^2 \quad or,\ 3s^2 = 1$$

$$\therefore\ s = \frac{1}{\sqrt{3}} \qquad \text{(xi)}$$

i.e. $\tan\phi = \frac{1}{s} = \sqrt{3} = \tan 60^0$

$$\therefore\ \phi = 60^0 \qquad \text{(xii)}$$

This means that the section is one-half of a regular hexagon, as shown in Fig. 5.2.

Now, from Fig. 5.2

$$OA = OB = OP\sin\phi = \frac{1}{2}PQ\sin\phi$$

$$= \frac{PQ}{2\sqrt{1+s^2}} = (b+2sh)\times\frac{1}{2\sqrt{1+s^2}}$$

$$= 2\sqrt{1+s^2}\times h\times\frac{1}{2\sqrt{1+s^2}} \quad \text{(using (v))}$$

$$= h$$

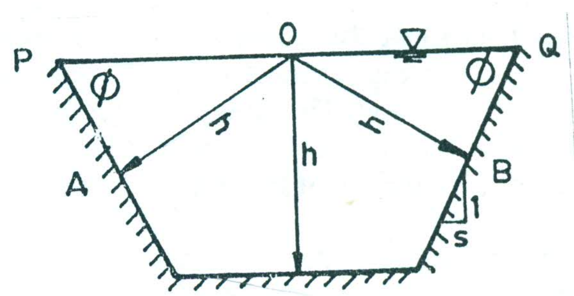


Fig.5.2 Best hydraulic trapezoidal section

Thus, a semi-circle with O as center and h as radius can be inscribed in the best hydraulic trapezoidal section.

Procedure for Designing a Channel with Best Hydraulic Section

When the design discharge Q, the Manning's roughness coefficient n and the bottom slope S_0 are known or determined, the design of rigid boundary or non-erodible channels based on the concept of best hydraulic section involve the following steps:

1. Compute the section factor for uniform flow computation $AR^{2/3}$ using

$$AR^{2/3} = \frac{nQ}{\sqrt{S_0}} \tag{5.2}$$

2. Substitute the expressions for A and R from Table 5.1 in Eq. (5.2) and solve directly for the depth.

3. Check for the minimum permissible velocity for siltation and/or for vegetation.

4. Check for the Froude number of the flow.

5. Add a proper freeboard to the depth of the channel section.

Example 5.3

A trapezoidal channel carrying 20 m^3/s is built with non-erodible bed having a slope of 1 in 1000 and n = 0.025. Design the channel by the concept of best hydraulic section.

Solution Trapezoidal channel, Q = 20 m^3/s, S_0 = 1 in 1000 = 0.001, n = 0.025

$$AR^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.025 \times 20}{\sqrt{0.001}} = 15.81$$

From Table 5.1, A = $\sqrt{3}h^2$ and R = 0.5h. Hence

$$\sqrt{3}h^2 \times (0.5h)^{2/3} = 15.81$$

which gives

$$h^{8/3} = 14.49$$

$$\therefore \; h = 2.73 \text{ m}$$

The side slope of the best hydraulic trapezoidal section is given by s = $1/\sqrt{3}$ = 0.577 so that b = 2h($\sqrt{1+s^2} - s$) = 4.12 m, A = $\sqrt{3}h^2$ = 12.86 m^2, P = $2\sqrt{3}h$ = 9.44 m, B = $4\sqrt{3}h/3$ = 6.30 m, R = 0.5h = 1.36 m and D = 3h/4 = 2.05 m. Also, U = Q/A = 1.56 m/s, which is greater than the minimum permissible velocity. The Froude number, Fr = U/$\sqrt{gD}$ = 0.33, which seems satisfactory. Adding a freeboard of 0.77 m, the total depth becomes 3.5 m.

Practical Rigid-Boundary Channel Sections

From the practical point of view, the best hydraulic section is not necessarily the most economic section. This is because (i) the area to be excavated to achieve the area for the best hydraulic section may be significantly larger, (ii) the type of material in the channel body may not permit the adoption of the slope required by the best hydraulic section, (iii) the cost of excavation depends not only on the amount of material which is removed, but also on the ease of access of the site and the cost of depositing the material removed, and (iv) the sharp corners in a cross-section, which are virtually the zones of stagnation, may lead to deposition if the water carries silt.

In view of the above factors, the best hydraulic sections need to be modified in practice. The side slopes of a channel depend mainly on the kind of material (Table 5.2). Usually the slopes are steeper in cutting than in filling. For lined channels, the side slopes usually vary from 1:1 to 2:1 and roughly correspond to the angle of repose of the natural soil (Fig. 5.6). The angle of repose is the angle made by a heap of soil under natural condition with the horizontal.

Also, the triangular and trapezoidal channel sections are provided with rounded corners instead of sharp corners, as shown in Fig. 5.3 (Ranga Raju, 1993). For the triangular section

$$A = h^2(\phi + \cot\phi) \tag{5.3}$$

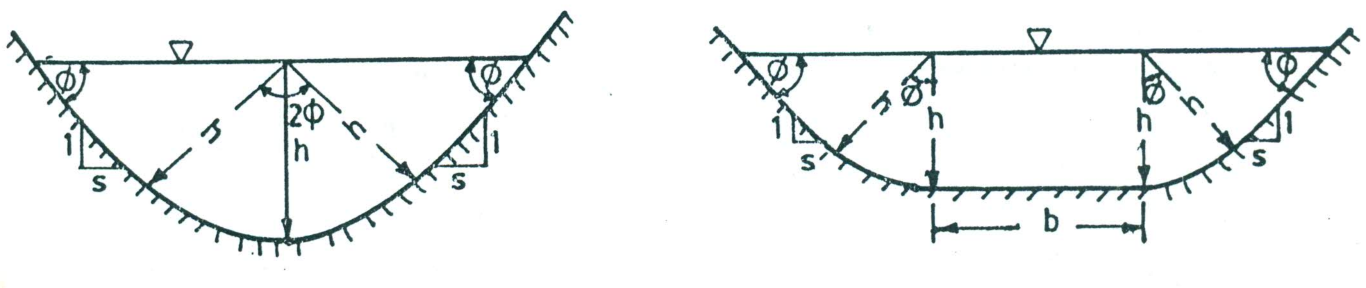
$$P = 2h(\phi + \cot\phi) \tag{5.4}$$

and for the trapezoidal section

$$A = bh + h^2(\phi + \cot\phi) \tag{5.5}$$

$$P = b + 2h(\phi + \cot\phi) \tag{5.6}$$

For given values of Q, n, s and S_0, the depth of flow in a triangular channel can be determined directly using the Manning formula. However, the determination of the depth of flow and the bottom width of a trapezoidal channel is based on a limiting velocity. The limiting or the maximum permissible velocity is generally taken as 2 m/s) (Ranga Raju, 1993).



(a) (b)

Fig. 5.3 Lined channel sections for (a) $Q < 55$ m^3/s, and (b) $Q > 55$ m^3/s (Ranga Raju, 1993)

Table 5.2 Suitable side slopes for channels built in various kinds of materials (Chow, 1959)

Material	Side slope
Rock	Nearly vertical
Muck and peat soils	0.25:1
Stiff clay or earth with concrete lining	0.5:1 to 1:1
Earth with stone lining or earth for large channels	1:1
Firm clay or earth for small ditches	1.5:1
Loose sandy earth	2:1
Sandy loam or porous clay	3:1

The USBR developed experience curves (Chow, 1959) showing the relations between discharge Q, bottom width b and depth of flow h which can be used as a guide in selecting proper section dimensions for lined trapezoidal channels without rounding the corners. Some of the bottom widths and depths extracted from those curves are given in Table 5.3.

Table 5.3 Bottom width and depth of lined trapezoidal channels (U.S. Bureau of Reclamation) (Chow, 1959)

Q(m^3/s)	1	5	10	20	50	100
b(m)	1.13	1.83	2.20	2.74	4.25	8.23
h(m)	0.61	1.52	2.07	2.50	3.23	4.21

Example 5.4

A channel lined with concrete is to be laid on a slope of 1 in 3600. The side slope of the channel is to be maintained at 1:1 and the lining is expected to give n = 0.013. Determine the section dimensions if (a) Q = 35 m^3/s, and (b) Q = 100 m^3/s and the limiting or maximum permissible velocity is 2 m/s.

Solution (a) Since Q < 55 m^3/s, use a triangular section with rounded corner as shown in Fig. 5.3(a). Here, s = $\cot\phi$ = 1 and ϕ = 0.785 radian. Then,

$$A = h^2(\phi + \cot\phi) = 1.785h^2$$
$$P = 2h(\phi + \cot\phi) = 3.571h$$
$$R = A/P = 0.5h$$

Now, using the Manning formula, we get

$$35 = (1/0.013) \times 1.785h^2 \times (0.5h)^{2/3} \times (1/3600)^{1/2}$$

from which we obtain

$$h^{8/3} = 24.28$$

or, $h = 3.307$ m

(b) Since Q > 55 m^3/s, use a trapezoidal section with rounded corners as shown in Fig. 5.3(b). Here, s = $\cot\phi$ = 1 and ϕ = 0.785 radian. As the maximum permissible velocity is 2 m/s, so

$$A = 100/2 = 50.0 \text{ m}^2$$

Again, from the Manning formula

$$100 = (1/0.013) \times 50 \times R^{2/3} \times (1/3600)^{1/2}$$

which gives

$$R = 1.95 \text{ m}$$

Since R = A/P, we have

$$P = A/R = 50.0/1.95 = 25.66 \ m$$

So, we have the equations

$$A = bh + h^2(\phi + \cot\phi) = bh + 1.785h^2 = 50.00$$
$$P = b + 2h(\phi + \cot\phi) = b + 3.571h = 25.66$$

Eliminating b between the above two equations, we get the quadratic equation

$$h^2 - 14.37h + 28.00 = 0$$

which gives h = 2.32 m and 12.04 m. Using these two values of h, we get b = 17.36 m and –17.34 m. Since b cannot be negative, we accept

h = 2.32 m and b = 17.36 m.

Example 5.5

A trapezoidal channel lined with concrete (n = 0.013) and laid on a slope of 1 in 3600 carries a discharge of 100 m^3/s. Determine the section dimensions of the channel (a) taking b = 6 m and side slopes of 1:1, (b) for the best hydraulic section when the side slope is 1:1, and (c) when the side slope is 1:1 and the bottom width as given in Table 5.3.

Solution Trapezoidal channel, n = 0.013, S_0 = 1/3600, Q = 100 m^3/s

(a) $$AR^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.013 \times 100}{\sqrt{1/3600}} = 78.00$$

With b = 6 m and s = 1, the depth of flow h can be determined by trial as follows.

h (m)	A (m^2)	P (m)	R (m)	AR$^{2/3}$
2.00	16.00	11.66	1.37	19.76
4.00	40.00	17.31	2.31	69.90
4.24	43.42	17.99	2.41	78.11
4.25	43.56	18.02	2.42	78.45

Hence, h = 4.24 m.

(b) For the best hydraulic trapezoidal section, R = 0.5h. Since s = 1, we have

$$A=(2\sqrt{1+s^2}-s)h^2=(2\sqrt{2}-1)h^2=1.828h^2$$

Hence

$$AR^{2/3}=(1.828h^2)(0.5h)^{2/3}=78.00$$

which gives

$$h^{8/3}=67.73$$

or, $h = 4.86$ m

Then, $b=2h(\sqrt{1+s^2}-s)=4.03\ m$

(c) From Table 5.3, for a discharge of 100 m^3/s, the bottom width b to be used is 8.23 m. Substituting the expressions for A and R for a trapezoidal section from Table 1.1, one obtains

$$AR^{2/3}=\frac{[(b+sh)h]^{5/3}}{(b+2\sqrt{1+s^2}h)^{2/3}}=78.00$$

Using s = 1 and b = 8.23 m and solving the above equation for h, we obtain h = 3.70 m.

5.3 ERODIBLE CHANNELS WHICH SCOUR BUT DO NOT SILT: SHEAR OR TRACTIVE FORCE MOTHOD

The erodible channels with coarse non-cohesive materials on the boundary and carrying either clear water or water with fine sediment in suspension which will not deposit are designed by the method developed by Lane (1955) of USBR. The method is based on the *threshold* or *incipient* or impen-
motion condition of the soil particles, which denotes the limiting condition at which the soil particles ding
just begin to move. In this method it is assumed that a channel scours when the shear stress developed on the channel boundary exceeds the critical shear stress value. The average shear stress on the boundary of an open channel at which the soil particles just begin to move is called the *critical shear stress* τ_c. Since the threshold or incipient motion condition of the particles on the channel boundary (or over a part of the boundary which is more susceptible to scour) is considered, the design is economical.

As stated in Art. 4.3, in uniform flow the average shear stress on the channel boundary is given by

$$\tau_0=\gamma RS_0=\rho gRS_0 \qquad (5.7)$$

and when the channel is wide

$$\tau_0=\gamma hS_0 \qquad (5.8)$$

The shear stress, however, is not uniformly distributed along the wetted perimeter when the channel is not wide. For a trapezoidal section, the maximum shear stress on the bottom is close to γhS_0 and on the sides close to $0.76\gamma hS_0$, as shown in Fig.5.4 (Lane, 1955).

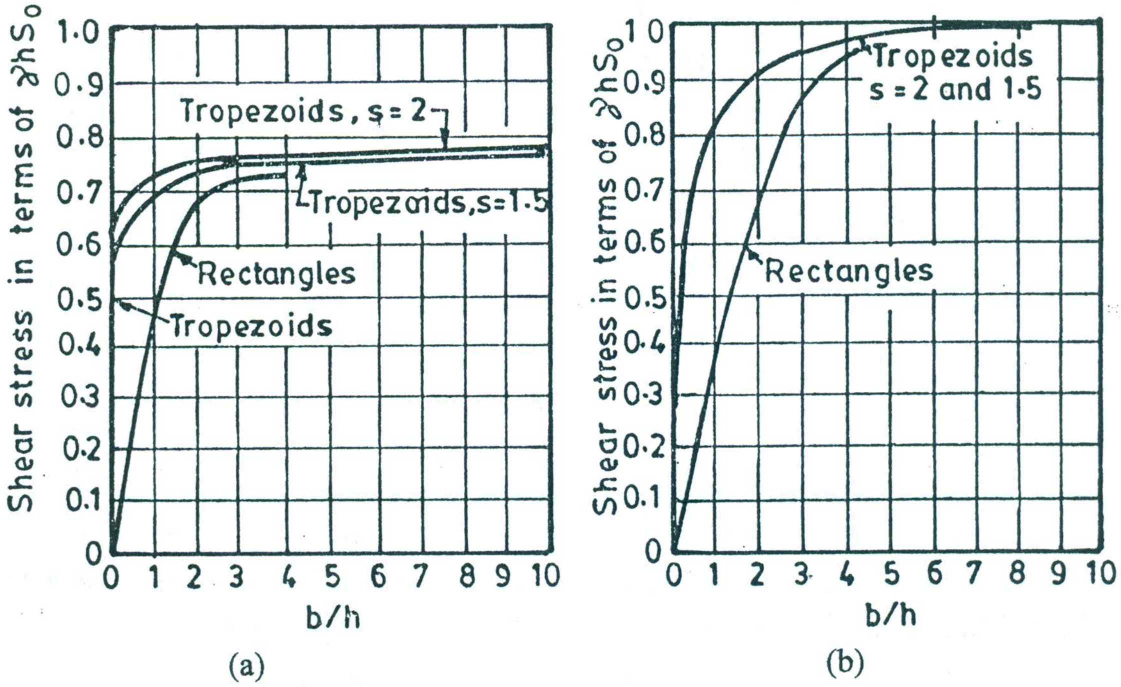


Fig. 5. 4 Maximum shear stresses on (a) sides and (b) bottom of trapezoidal channels

Shear Stress Ratio

On a soil particle resting on the sloping side of a trapezoidal channel in which water is flowing (Fig. 5.5), two forces, viz. the tractive or shear force $a\tau_s$ and the gravity force component $W_s\sin\varphi$, tend to move the soil particle. Since the two forces act on the soil particle at right angles to each other, the resultant force on the soil particle is

$$\sqrt{W_s^2 \sin^2 \phi + a^2 \tau_s^2}$$

where a is the effective area of the soil particle, τ_s is the shear stress on the sloping side and W_s is the submerged weight of the particle. The force resisting the movement of the soil particle is equal to $W_s\cos\varphi\tan\psi$, where ψ is the *angle of repose* and $\tan\psi$ is the *coefficient of friction*. For the incipient motion condition of the soil particle

$$W_s \cos\phi \tan\psi = \sqrt{W_s^2 \sin^2 \phi + a^2 \tau_s^2} \tag{5.9}$$

which gives

$$\tau_s = \frac{W_s}{a} \cos\phi \tan\psi \sqrt{1 - \frac{\tan^2 \phi}{\tan^2 \psi}} \tag{5.10}$$

Similarly, for the incipient motion condition of a soil particle on the level or horizontal bottom, Eq.(5.10) gives with $\phi = 0$

$$\tau_b = \frac{W_s}{a} \tan\psi \tag{5.11}$$

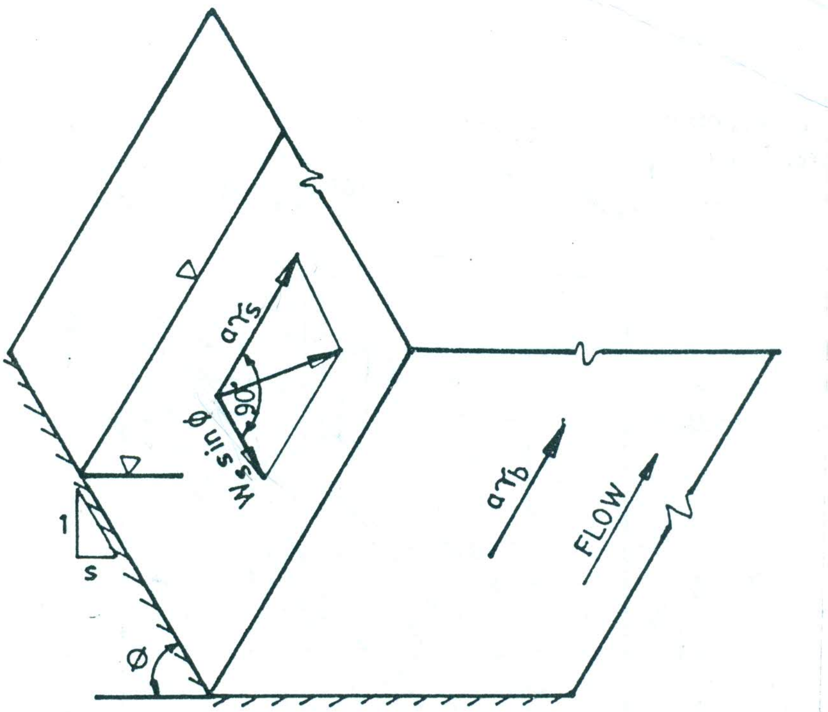


Fig.5.5 Forces acting on a soil particle resting on the side and the bottom of a trapezoidal channel

where τ_b is the shear stress on the level bottom. From Eqs.(5.10) and (5.11), the shear stress ratio, which is the ratio between the shear stress on the sloping side and that on the level bottom of a trapezoidal section, is given by

$$K = \frac{\tau_s}{\tau_b} = \cos\phi\sqrt{1-\frac{\tan^2\phi}{\tan^2\psi}} = \sqrt{\cos^2\phi - \frac{\sin^2\phi}{\tan^2\psi}} = \sqrt{1-\sin^2\phi - \frac{\sin^2\phi}{\tan^2\psi}}$$

$$= \sqrt{1-\sin^2\phi\left(1+\frac{1}{\tan^2\psi}\right)} = \sqrt{1-\sin^2\phi\left(\frac{\sin^2\psi+\cos^2\psi}{\sin^2\psi}\right)}$$

Or, $$K = \sqrt{1-\frac{\sin^2\phi}{\sin^2\psi}} \tag{5.12}$$

For cohesive and fine non-cohesive materials, the gravity force component causing the particle to roll down the side slope is much smaller than the cohesive forces and can safely be neglected. Therefore, the angle of repose need to be considered only for coarse non-cohesive materials. In general, the angle of repose increases with both size and angularity of the material. Figure 5.6 shows the curves prepared by USBR for the angle of repose for non-cohesive material larger than 0.2 inch (5 mm) in diameter. In this figure, the particle size is the d_{75} size which is the particle size than which 75% of the material by weight is smaller (25% is larger).

The USBR recommends a value of the permissible shear stress on level bottom for coarse non-cohesive materials in pounds per square foot equal to $0.40d_{75}$, where d_{75} is in inches.

Shields in 1936 first gave a semi-empirical approach to define the threshold of movement and his results are most widely used today. His results can be stated in terms of two dimensionless parameters

$$Re^* = \frac{u^* d}{\nu} \tag{5.13}$$

and

$$\tau_c^* = \frac{\tau_c}{\gamma(s_s-1)d} = \frac{u^{*2}}{g(s_s-1)d} \tag{5.14}$$

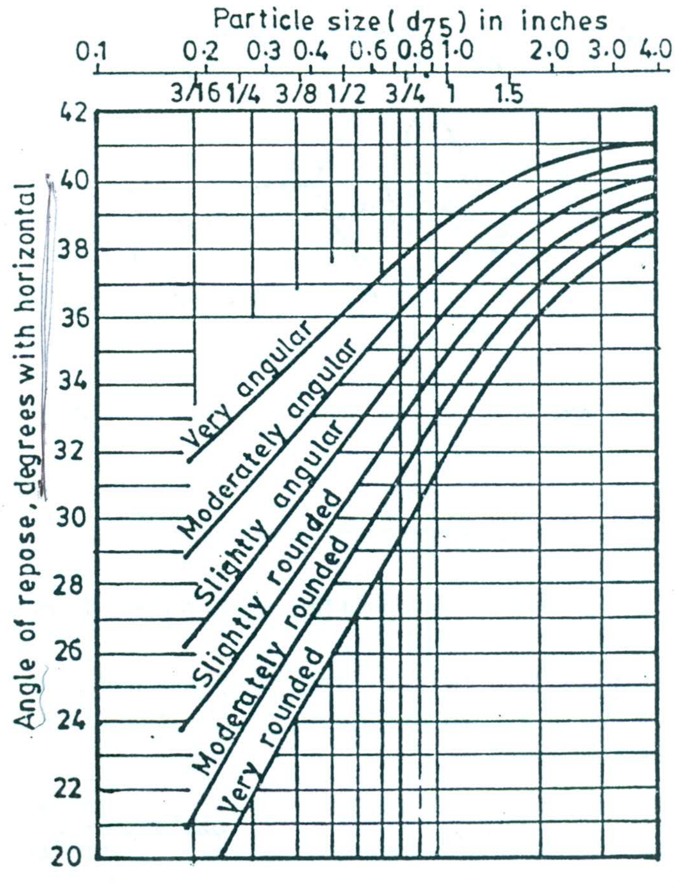


Fig.5.6 Angle of repose of non-cohesive materials (Lane, 1955)

when s_s is the specific gravity of the sediment and d is the size of the sediment particle. The parameter Re^* is known as the *particle Reynolds number*.

The Shields curve, plotted in Fig.5.7 (French, 1986), delineates the threshold of movement, i.e. the regime above the curve represents a moving bed and that below the curve represents a rigid bed. The first part of the curve represents laminar flow, the middle part represents a transition region and the last part represents a region of fully developed turbulence. The boundary is rough at large values of Re^* and when Re^* exceeds a value of about 400, τ_c* remains constant at 0.056.

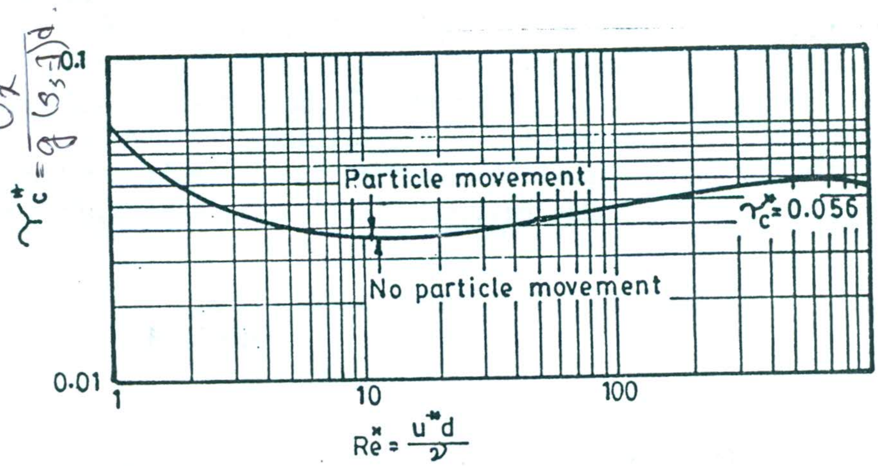


Fig.5.7 Shields curve for incipient motion condition

The Shields curve is based on a limited amount of field data. The relationship proposed by Yalin and Karahan (1979) based on considerably more data is also shown in Fig. 5.8 and is more reliable. As a result, it is recommended that the relationship of Yalin and Karahan be used to define τ_c. In the Yalin-Karahan relationship, when Re^* exceeds a value of about 70, τ_c^* remains constant at 0.045.

The permissible shear stress on level bottom τ_b may be taken to be equal to or slightly less (to provide a factor of safety) than the critical shear stress τ_c and can be obtained using the Yalin-Karahan curve. However, the computation of τ_c using Fig. 5.8 requires a trial procedure. Garde and Ranga Raju (1977) presented a modified form of the Yalin-Karahan curve as shown in Fig.5.9 which gives the value of τ_c directly without involving any trial.

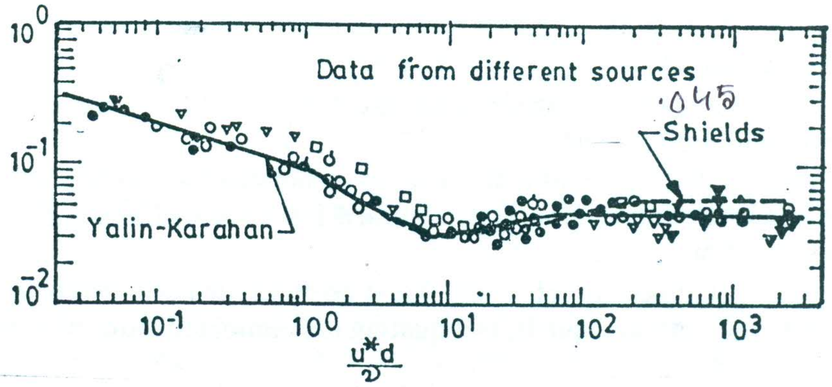


Fig.5.8 Yalin-Karahan curve for incipient motion condition

Since sinuous channels are apt to scour more easily than straight channels, the permissible shear stresses for sinuous channels are reduced approximately, as suggested by Lane (1955), by 10% for slightly sinuous channels (sinuosity 1.5-2.0), 25% for moderately sinuous channels (sinuosity 2.0-2.5) and 40% for very sinuous channels (sinuosity > 2.5). The sinuosity is the ratio between the channel (curved) length and the valley (straight) length of a curved channel.

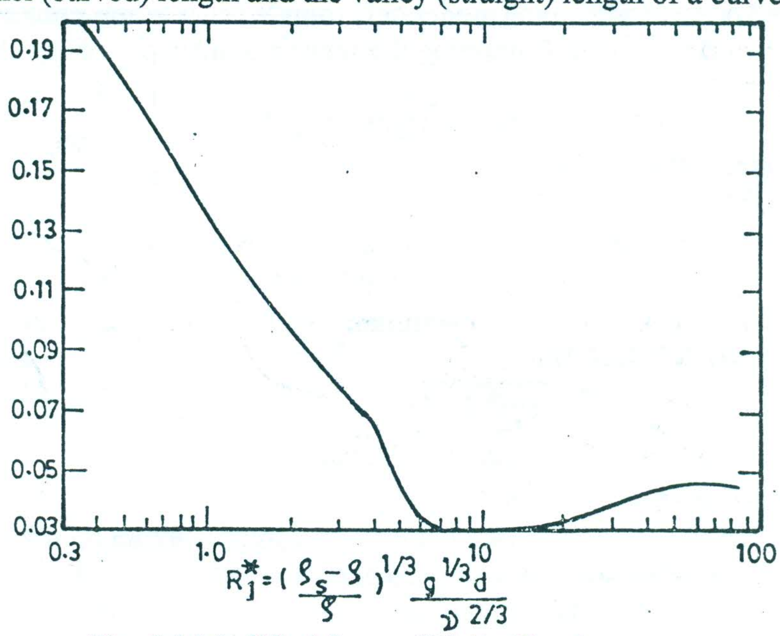


Fig. 5.9 Modified form of Yalin-Karahan curve

From Eq.(5.12), it is apparent that the side slope angle ϕ of a trapezoidal channel must be less than the angle of repose ψ, otherwise K becomes imaginary. Also, a rectangular section cannot be designed by this method, because for a rectangular section $\phi = 90^0$ and K becomes imaginary.

Procedure for Designing a Trapezoidal Channel

For a trapezoidal channel, the shear stress on the sloping sides is usually less than that on the bottom. Hence, the side force is generally the limiting factor in the channel design. The section dimensions should therefore be determined for the side force and checked for the bottom force.

When the design discharge Q, the bottom slope S_0, the relevant engineering properties of the soil and the Manning's n are known or determined, the design of a trapezoidal channel section in coarse non-cohesive materials involves the following steps:

1. Estimate the angle of repose ψ for the perimeter material (Fig.5.6).
2. Assume s and b/h_n. Compute ϕ (= $\cot^{-1}$s).
3. Determine the maximum shear stress developed on sides in terms of h_n (Fig.5.4(a)).
4. Calculate K (Eq.(5.12)).
5. Determine τ_b, the permissible shear stress on bottom ($\tau_b = 0.40 d_{75}$ when τ_b is in pounds per square foot and d_{75} is in inches, or $\tau_b = \tau_c$ obtained from the Yalin-Karahan curve,Fig.5.9) and correct for sinuosity, if any.
6. Compute τ_s, the permissible shear stress on sides, using $\tau_s = K\tau_b$.
7. Compute the normal depth h_n by equating the computed value of τ_s in step 6 to the shear stress obtained in step 3.
8. Compute b.
9. Compute Q and compare this with the design Q.
10. Repeat steps 2 to 9 until the computed Q is close to the design Q.
11. Compare the actual shear stress on bottom τ_b (Fig.5.4(b)) with the permissible value obtained in step 5.

Example 5.6

A trapezoidal channel is to be laid on a slope of 1 in 1000 and carry a discharge of 20 m^3/s. It is to be excavated in earth containing moderately rounded coarse non-cohesive particles with d_{50} = 2 cm, d_{75} = 2.5 cm and n = 0.025. Determine the section dimensions of the channel.

Solution $S_0 = 1/1000 = 0.001$

i) Design by the method of Lane

$d_{75} = 2.5/2.54 = 0.98$ in

Then, from Fig.5.6, $\psi = 33^0$

Assume s = 2 and b/h_n = 4. Then, $\phi = \cot^{-1} 2 = \tan^{-1} ½ = 26.56^0$

Then from Fig.5.4(a), the maximum shear stress on sides is $0.75\gamma h_n S_0$ = $0.75 \times 9810 \times h_n \times 0.001 = 7.36 h_n$ N/m^2.

$$K = \frac{\tau_s}{\tau_b} = \sqrt{1 - \frac{\sin^2 \phi}{\sin^2 \psi}} = \sqrt{1 - \frac{\sin^2 26.56^0}{\sin^2 33^0}} = 0.57$$

The permissible shear stress on bottom is

$\tau_b = 0.4 \times 0.98 = 0.39$ lb/ft^2 = 18.77 N/m^2 ($\therefore$ 1 lb/ft^2 = 47.86 N/m^2)

Then the permissible shear stress on sides is

$\tau_s = K\tau_b = 0.57 \times 18.77 = 10.72$ N/m^2

For a state of impending motion of the particles on the sides

$7.36 h_n = 10.72$ $\therefore h_n = 1.47$ m

$\therefore$ b = 1.47×4 = 5.83 m

For this trapezoidal section, A = 12.73 m^2, P = 12.34 m and R = 1.03 m. Hence

$$Q=(1/0.025)\times 12.73\times 1.03^{2/3}\times 0.001^{1/2}=16.44\ m^3/s$$

which is less than the design discharge of 20 m^3/s.

Further computations are done in tabular form as shown below for different values of b/h$_n$.

b/h$_n$	h$_n$ (m)	b (m)	A (m^2)	P (m)	R (m)	Q (m^3/s)
4.5	1.47	6.62	14.05	13.19	1.065	18.53
5.0	1.47	7.35	15.13	13.92	1.086	20.22
4.9	1.47	7.20	14.91	13.78	1.082	19.88

Then, for s = 2 and b/h$_n$ = 5.0, the section dimensions are h$_n$ = 1.47 m, b = 7.35 m, A = 15.13 m^2, P = 13.92 m and R = 1.086 m, and Q = 20.22 m^3/s which is very close to 20 m^3/s.

With s = 2 and b/h$_n$ = 5.0, the shear stress on bottom (Fig.5.4(b)) is $0.97\gamma h_n S_0$ = 0.97×9810×1.42×0.001 = 13.51 N/m^2 which is less than 18.77 N/m^2, the permissible shear stress on bottom. Hence the design is acceptable.

Obviously, alternative section dimensions may be obtained by taking other values of s.

ii) Design by using modified Yalin-Karahan curve

The permissible shear stress on bottom τ_b taken to be equal to τ_c obtained from Fig. 5.9. The median size d_{50} is taken to be equal to the size of the sediment particle. Then,

$$R_1{*}=\left(\frac{\rho_s-\rho}{\rho}\right)^{1/3}\times\frac{g^{1/3}d_{50}}{\nu^{2/3}}=\left(\frac{2650-1000}{1000}\right)^{1/3}\times\frac{9.81^{1/3}\times 0.02}{(10^{-6})^{2/3}}=505.92$$

The corresponding value of $\tau_c{*}$ from Fig. 5.9 is 0.045.

$$\therefore \tau_b=\tau_c=\tau_c{*}\gamma(s_s-1)d_{50}=0.045\times 9810\times(2.65-1)\times 0.02=14.57\ N/m^2$$

Then $\tau_s = K\tau_b$ = 0.57×14.57 = 8.30 N/m^2

$\therefore$ 7.36h$_n$ = 8.30

$\therefore$ h$_n$ = 1.13 m

$\therefore$ b = 1.13×4 = 4.52 m

For this trapezoidal section, A = 7.66 m^2, P = 9.57 m and R = 0.80 m. Hence

$$Q=(1/0.025)\times 7.66\times 0.80^{2/3}\times 0.001^{1/2}=8.35\ m^3/s$$

which is far less than the design discharge of 20 m^3/s.

Further computations are done in tabular form as shown below for different values of b/h$_n$.

b/h$_n$	h$_n$ (m)	b (m)	A (m^2)	P (m)	R (m)	Q (m^3/s)
8.0	1.13	9.04	12.77	14.03	0.906	15.12
10.0	1.13	11.30	15.32	16.35	0.096	18.56
11.0	1.13	12.43	16.60	17.43	0.949	20.23
10.9	1.13	12.32	16.47	17.37	0.948	20.10

Then, for s = 2 and b/h$_n$ = 10.9, h$_n$ = 1.13 m, b = 12.32 m, A = 16.47 m^2, P = 17.37 m, R = 0.948 m and Q = 20.10 m^3/s which is very close to 20 m^3/s.

With s = 2 and b/h$_n$ = 10.9, the maximum unit tractive force on bottom (Fig.5.4(b)) is $\gamma h_n S_0$ = 9810×1.13×0.001 = 11.08 N/m^2 which is less than 14.57 N/m^2, the permissible tractive force on bottom. Hence the design is acceptable.

5.4 ALLUVIAL CHANNELS: REGIME APPROACH

An alluvial channel has been defined as a channel transporting the same type of material as that comprising the channel perimeter. Such a channel can be stable only when sediment inflow into channel is equal to sediment outflow, i.e. the channel cross-section and bottom slope do not change due to erosion and deposition. A channel is said to be in a regime when it has adjusted its shape and slope to an equilibrium condition.

The two commonly adopted methods for the design of stable alluvial channels are the shear or tractive force approach, considered earlier, and the regime approach. The shear force approach is more rational, as it makes use of the laws governing sediment transport and resistance laws. The regime theory is purely empirical and has been developed using data of stable canals in India and Pakistan carrying sediment load generally less than 500 ppm by weight.

The first regime formula developed by Kennedy (1895) is given by

$$U_0 = 0.546h^{0.64} \tag{5.15}$$

where U_0 is the non-silting and non-scouring mean velocity and h is the depth of flow. The main limitation of the Kennedy equation is that it does not specify a stable width, thereby making an infinite number of width-to-depth ratios possible. However, experience shows that stability is possible only if the width does not vary over a wide range. Lindley (1919) recognized this fact and introduced a relation between non-silting and non-scouring velocity and the bed width. Later on, Lacey (1930,1946) carried out extensive investigations on the design of stable channels in alluvium using data of stable canals in the Indo-Gangetic plains and put forward his new theory. He differentiated between two regime conditions: (i) initial regime, and (ii) final regime. A channel under initial or false regime is not a channel in regime, although it appears to be in regime as the~~ir~~re is no silting or scouring, and the regime theory is not applicable to them. According to Lacey, an artificially constructed channel having a certain fixed section and certain fixed slope can be in true or final regime if the discharge is constant, flow is uniform, the silt grade and the silt charge are constant and the channel flows through incoherent alluvium of the same type as is transported without changing its cross-section and slope. Lacey's regime theory is applicable only to channels which are in final regime.

There

*

The various equations proposed by Lacey for the design of stable channels in alluvium are

$$P = 4.75\sqrt{Q} \tag{5.16}$$

$$R = 0.47(Q / f_s)^{1/3} \tag{5.17}$$

and

$$S_0 = \frac{f_s^{5/3}}{3340Q^{1/6}} \tag{5.18}$$

with

$$f_s = 1.76\sqrt{d} \tag{5.19}$$

where P is the wetted perimeter in m, R is the hydraulic radius in m, Q is the discharge in m^3/s, d is the average particle size in mm and f_s is the *silt factor* which takes into account the effect of grain size of the material forming the channel. Combination of Eqs.(5.16) to (5.19) results in the following resistance formula similar to the Manning formula:

$$U = 10.8R^{2/3}S_0^{1/2} \tag{5.20}$$

The alluvial channels are usually provided with trapezoidal sections having side slopes equal to or less than the angle of repose of the perimeter material. But due to deposition of fine sediments, the final side slopes attained by the channels are much steeper. Hence, it is customary to assume a side slope of 1/2:1 (i.e. s = 1/2) for the design of alluvial channels.

Example 5.7

Design a stable alluvial channel using the Lacey's theory. The channel is to carry 10 m^3/s through 1 mm sand.

Solution $Q = 10\ m^3/s \quad d = 1\ mm$

$$f_s = 1.76\sqrt{d} = 1.76\sqrt{1} = 1.76$$

$$S_0 = \frac{f_s^{5/3}}{3340Q^{1/6}} = \frac{1.76^{5/3}}{3340 \times 10^{1/6}} = 5.233 \times 10^{-4}$$

$$R = 0.47(Q/f_s)^{1/3} = 0.47 \times (10/1.76)^{1/3} = 0.8387\ m$$

$$P = 4.75\sqrt{Q} = 4.75\sqrt{10} = 15.02\ m$$

so that

$$A = PR = 15.02 \times 0.8387 = 12.60\ m^2$$

Assuming that the side slope is 1/2H:1V so that s = 0.5, we obtain

$$P = 15.02 = b + 2\sqrt{1+0.5^2} \times h = b + 2.236h$$

$$A = 12.60 = (b+0.5h)h = bh + 0.50h^2$$

Eliminating b between the above two equations, we get the quadratic equation

$$h^2 - 8.652h + 7.258 = 0$$

which gives h = 0.941 m and 7.711 m. Using these two values of h, we get b = 15.02 – 2.236h = 12.916 m and – 2.222 m. Since b cannot be negative, we accept

h = 0.941 m and b = 12.916 m.

PROBLEMS

5.1 (a) Show that the best hydraulic circular section is a semi-circle.

(b) Show that the best hydraulic triangular section is one-half of a square.

(c) Show that for the best hydraulic parabolic section the top width is equal to $2\sqrt{2}$ times the depth of flow.

5.2 (a) The cross-sectional area of a channel is 40 m^2. Calculate the wetted perimeter and the hydraulic radius of the best hydraulic section if the channel is (i) rectangular, (ii) triangular, (iii) trapezoidal, (iv) circular, and (v) parabolic. Which section has the minimum wetted perimeter?

(b) Show that for a given area A, the best hydraulic rectangular and triangular sections have the same wetted perimeter.

5.3 Compute the wetted perimeter of the best hydraulic section for a lined channel to carry a discharge of 15 m^3/s with n = 0.013 and S_0 = 0.001 if the section is (i) rectangular, (ii) triangular, (iii) trapezoidal, (iv) circular, and (v) parabolic. Which section has the minimum wetted perimeter?

5.4 (a) Design the best hydraulic trapezoidal section to carry a discharge of 20 m^3/s on a slope of 1 in 2500 if s = 1 and n = 0.012.

(b) Determine the bottom width of the best hydraulic trapezoidal section to carry a discharge of 10 m^3/s if s = 2 and n = 0.015 in which the depth of flow is to be restricted to 1 m. Also, determine the bottom slope of the channel.

5.5 A lined channel (n = 0.015) is to be laid on a slope of 1 in 2000. The side slope of the channel is to be maintained at 1.5:1. (i) Determine the depth of flow of a triangular section with rounded corner to carry a discharge of 40 m^3/s. (ii) Determine the dimensions of a trapezoidal section with rounded corners to carry a discharge of 80 m^3/s when the maximum permissible velocity is 2 m/s.

5.6 An irrigation canal has to carry a discharge of 30 m^3/s through a coarse non-cohesive material having d_{50} = 2.5 cm, d_{75} = 3 cm and n = 0.025. The angle of repose of the perimeter material is 32^0. The canal is to be trapezoidal in shape having s = 2 and laid on a slope of 1 in 1000. Compute the bottom width and the depth of flow (i) using the method of Lane, and (ii) using the modified Yalin-Karahan diagram.

5.7 Using the Lacey method, design a stable alluvial channel when d = 1.5 mm and Q = 25 m^3/s.

****

Chapter 6

GRADUALLY VARIED FLOW

6.1 INTRODUCTION

In a long straight prismatic channel in which no transition or control structure is present, the flow tends to be uniform. In practice, however, it becomes necessary to change the channel section or bottom slope and use transition and control structures like sluice gates, weirs etc. in the channel as a result of which the flow in the channel usually becomes varied or non-uniform between two uniform states of flow. Varied flow may be either gradually varied or rapidly varied.

In gradually varied flow the water depth and flow velocity vary gradually along the channel length ($\partial h/\partial x \approx 0$, $\partial U/\partial x \approx 0$). The streamlines are practically parallel so that there is no appreciable acceleration component normal to the direction of flow and the pressure distribution over the channel section is hydrostatic. Since in gradually varied flow the depth of flow changes gradually, to produce a significant change in depth, long channel lengths are usually involved in the analysis of gradually varied flow. Consequently, the frictional losses, which are proportional to the channel length, play a dominating role in determining the flow characteristics and must be included. Flow behind a dam and flow upstream of a sluice gate or weir are examples of gradually varied flow.

The analysis of gradually varied flow involves the assumption that the friction losses in gradually varied flow are not significantly different from those in uniform flow. By virtue of this assumption, the friction slope in gradually varied flow is computed using a uniform flow formula, i.e.

$$S_f = \frac{U^2}{C^2 R} = \frac{Q^2}{C^2 A^2 R} \qquad (6.1)$$

when the Chezy formula is used, and

$$S_f = \frac{n^2 U^2}{R^{4/3}} = \frac{n^2 Q^2}{A^2 R^{4/3}} \qquad (6.2)$$

when the Manning formula is used.

6.2 GOVERNING EQUATIONS

At any channel section, the total energy is given by (Fig. 6.1)

$$H = z_b + h + \alpha \frac{U^2}{2g} \qquad (6.3)$$

where z_b is the elevation of the channel bottom above a horizontal datum, h is the depth of flow, U is the mean velocity of flow and α is the energy coefficient.

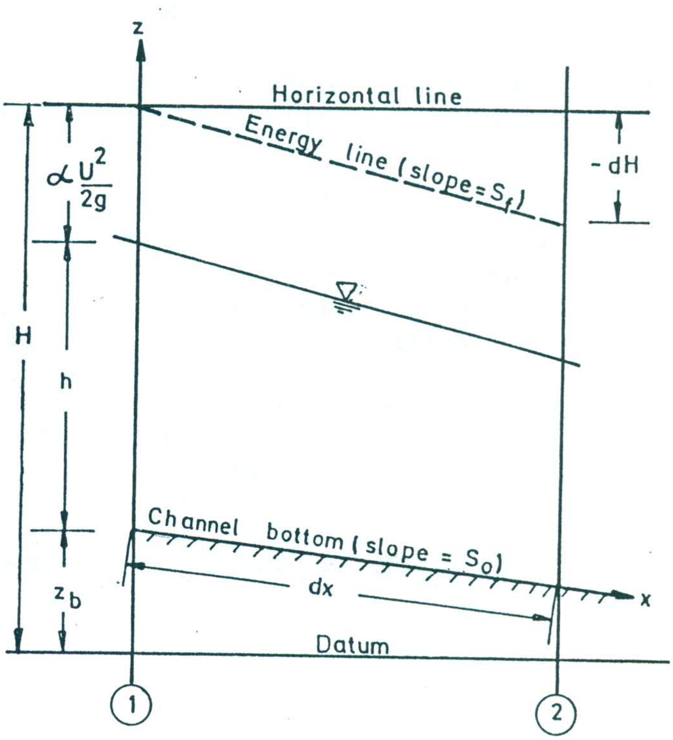


Fig. 6.1 Derivation of the gradually varied flow equation

Differentiating Eq.(6.3) with respect to x yields

$$\frac{dH}{dx}=\frac{dz_b}{dx}+\frac{dh}{dx}+\frac{d}{dx}\left(\alpha\frac{U^2}{2g}\right) \tag{6.4}$$

The term dH/dx represents the slope of the energy line. It is usual to consider the slope of the energy line which descends in the direction of flow as positive. Since the total energy of flow decreases (as x increases) in the direction of flow, it follows that

$$\frac{dH}{dx}=-S_f \tag{6.5}$$

The term dz_b/dx represents the bottom slope of the channel. It is usual to consider the bottom slope that falls in the direction of flow as positive. Since the change in the bottom elevation dz_b is negative when the channel bottom descends in the direction of flow, we have

$$\frac{dz_b}{dx}=-S_0 \tag{6.6}$$

For a specified flow rate and channel section

$$\frac{d}{dx}(\alpha\frac{U^2}{2g})=\frac{d}{dh}(\alpha\frac{Q^2}{2gA^2})\frac{dh}{dx}=\frac{\alpha Q^2}{2g}\frac{d}{dh}(A^{-2})\frac{dh}{dx}=-\frac{\alpha Q^2}{gA^3}\frac{dA}{dh}\frac{dh}{dx}=-\frac{\alpha U^2}{gD}\frac{dh}{dx}=-Fr^2\frac{dh}{dx} \tag{6.7}$$

where dA/dh = B and A/B = D have been used.

Using Eqs. (6.5) to (6.7), Eq. (6.4) becomes

$$\frac{dh}{dx}=\frac{S_0-S_f}{1-Fr^2} \tag{6.8}$$

Equation (6.8) is the basic differential equation of steady gradually varied flow and is also known as the *dynamic equation of (steady) gradually varied flow*. It represents the slope of the water surface with respect to the channel bottom and gives the variation of h in the x direction.

The water surface curve or profile represents a *backwater curve* when the depth of flow increases in the direction of flow (dh/dx > 0) and a *drawdown curve* when the depth of flow decreases in the direction of flow (dh/dx < 0). When dh/dx = 0, the water surface is parallel to the channel bottom and the flow is uniform.

Other useful forms of Eq. (6.8) can also be obtained. The section factor and the section factor for critical flow computation can be expressed, respectively, as (Art. 3.2)

$$Z=A\sqrt{D} \tag{6.9}$$

and

$$Z_c=\frac{Q}{\sqrt{g/\alpha}} \tag{6.10}$$

so that

$$\frac{Z_c^{\,2}}{Z^2}=\frac{\alpha Q^2}{gA^2D}=\frac{U^2}{gD/\alpha}=Fr^2 \tag{6.11}$$

where Z represents simply the numerical value of $A\sqrt{D}$ to be computed for Q at the actual depth h of the gradually varied flow and Z_c is the section factor to be computed for Q as if the flow in the channel is critical.

The slope of the channel bottom and the slope of the energy line can be expressed, respectively, as (Art. 4.5)

$$S_0 = \frac{Q^2}{K_n^{\,2}} \tag{6.12}$$

and

$$S_f = \frac{Q^2}{K^2} \tag{6.13}$$

so that

$$\frac{S_f}{S_0} = \frac{K_n^{\,2}}{K^2} \tag{6.14}$$

where K is the numerical value of the conveyance to be computed for Q at the actual depth h of the gradually varied flow and K_n is the conveyance to be computed for Q as if the flow in the channel is uniform.

Substitution of Eqs. (6.11) and (6.14) into Eq.(6.8) yields

$$\frac{dh}{dx} = S_0 \frac{1-(K_n/K)^2}{1-(Z_c/Z)^2} \tag{6.15}$$

Now, for a given channel section, the section factor and the conveyance are functions of the depth of flow, i.e.

$$Z^2 = C_1\, h^M \qquad Z_c^{\,2} = C_1 h_c^{\,M} \tag{6.16}$$

$$K^2 = C_2\, h^N \qquad K_n^{\,2} = C_2 h_n^{\,N} \tag{6.17}$$

where C_1 and C_2 are the coefficients and M and N are the hydraulic exponents for critical flow computation and uniform flow computation, respectively. With the above expressions, Eq.(6.15) becomes

$$\frac{dh}{dx} = S_0 \frac{1-(h_n/h)^N}{1-(h_c/h)^M} \tag{6.18}$$

For a wide rectangular channel, M = 3, and when the Chezy formula is used to compute S_f, N = 3, and hence

$$\frac{dh}{dx} = S_0 \frac{1-(h_n/h)^3}{1-(h_c/h)^3} \tag{6.19}$$

Equation (6.19) is known as the *Belanger equation*. When the Manning formula is used to compute S_f, N = 10/3, and therefore

$$\frac{dh}{dx} = S_0 \frac{1-(h_n/h)^{10/3}}{1-(h_c/h)^3} \tag{6.20}$$

Equations (6.19) and (6.20) can be used to compute gradually varied flow in wide rectangular channels.

The differential equation of gradually varied flow can also be expressed in terms of the specific energy E. Since

$$E = h + \alpha \frac{U^2}{2g} \quad and \quad H = z_b + h + \alpha \frac{U^2}{2g} = z_b + E$$

therefore

$$\frac{dE}{dx} = \frac{d}{dx}(H - z_b) = \frac{dH}{dx} - \frac{dz_b}{dx} = -S_f - (-S_0) = S_0 - S_f \qquad (6.21)$$

6.3 CHARACTERISTICS AND CLASSIFICATION OF FLOW PROFILES

Types of Bottom Slopes

The channel bottom slopes are conveniently classified as *sustaining* or *positive slope* ($S_o > 0$) and *non-sustaining slope* ($S_0 \leq 0$). A positive slope is the slope for which the channel bottom falls in the direction of flow. It may be

i) *mild* ($S_o < S_c$, $h_n > h_c$)
ii) *critical* ($S_o = S_c$, $h_n = h_c$), and
iii) *steep* ($S_o > S_c$, $h_n < h_c$).

Uniform flow can occur in positive slope channels only. In a mild slope channel the uniform flow is subcritical, in a critical slope channel the uniform flow is critical and in a steep slope channel the uniform flow is supercritical.

A non-sustaining slope is the slope for which the channel bottom does not fall in the direction of flow. It may be

i) *horizontal* ($S_o = 0$), and
ii) *adverse* ($S_o < 0$)

A horizontal slope is a zero slope. An adverse slope is a negative slope for which the channel bottom rises in the direction of flow.

The symbols M, C, S, H and A are respectively used to designate the **M**ild, **C**ritical, **S**teep, **H**orizontal and **A**dverse slopes.

Types of Flow Profiles

For the given discharge and channel conditions, the normal depth line (NDL) and the critical depth line (CDL) divide the space above the channel into three zones:

Zone 1 Space above upper line ($h > h_n$, $h > h_c$)
Zone 2 Space between two lines ($h_n > h > h_c$ or $h_c > h > h_n$)
Zone 3 Space between channel bed and lower line ($h < h_n$, $h < h_c$)

The three zones are designated by 1 to 3 starting from the top. The flow profiles are classified according to the channel slope and the zone in which the flow profile lies. The name of a flow profile includes the symbol used for the channel slope followed by the zone number. Thus, the name of the flow profile which lies in Zone 1 of a Mild slope channel is M1. It will be shown later in this section that the horizontal and the adverse slope channels have only two zones and the flow profiles H1 and A1 are physically not possible. Thus, the flow profiles may be classified into thirteen different types designated as M1, M2, M3, C1, C2, C3, S1, S2, S3, H2, H3, A2 and A3. Of the thirteen flow profiles, twelve are for gradually varied flow and one, C2, is for uniform flow. The general characteristics of these flow profiles are given in Table 6.1 and their shapes are shown in Figs. 6.2. Some examples of flow profiles are given in Fig. 6.3.

Table 6.1 Types of flow profiles in prismatic channels

Channel Slope	Zone	Designatio	Relation of h to h_n and h_c	Type of curve	Type of flow
Horizontal $S_0 = 0$	1	-	-	-	-
	2	H2	$h > h_c$	Drawdown	Subcritical
	3	H3	$h_c > h$	Backwater	Supercritical
Mild $0 < S_0 < S_c$ $h_n > h_c$	1	M1	$h > h_n > h_c$	Backwater	Subcritical
	2	M2	$h_n > h > h_c$	Drawdown	Subcritical
	3	M3	$h_n > h_c > h$	Backwater	Supercritical
Critical $S_0 = S_c > 0$ $h_n = h_c$	1	C1	$h > h_c = h_n$	Backwater	Subcritical
	2	C2	$h_c = h = h_n$	Parallel to channel bottom	Uniform critical
	3	C3	$h_c = h_n > h$	Backwater	Supercritical
Steep $S_0 > S_c > 0$ $h_n < h_c$	1	S1	$h > h_c > h_n$	Backwater	Subcritical
	2	S2	$h_c > h > h_n$	Drawdown	Supercritical
	3	S3	$h_c > h_n > h$	Backwater	Supercritical
Adverse $S_0 < 0$	1	-	-	-	-
	2	A2	$h > h_c$	Drawdown	Subcritical
	3	A3	$h_c > h$	Backwater	Supercritical

Behavior of Flow Profiles at Specific Depths

Let us now consider the theoretical behavior of the flow profiles at some specific depths.

i) When $h \to h_n$, Eq.(6.18) shows that $dh/dx \to 0$, i.e. the flow profile approaches the normal depth line tangentially.

ii) When $h \to h_c$, Eq.(6.18) shows that $dh/dx \to \infty$, i.e. the flow profile becomes vertical in crossing the critical depth line. This indicates a hydraulic jump if the depth changes suddenly from a lower value to a higher value or a hydraulic drop if the depth changes abruptly from a higher value to a lower value. In both cases the flow becomes rapidly varied and the theory of gradually varied flow does not apply.

iii) When $h \to \infty$, Eq.(6.18) shows that $dh/dx \to S_o$, i.e. the flow profile tends to be horizontal. Note that the slope of the channel bottom S_0 is determined with respect to the horizontal line, whereas the slope of the flow profile dh/dx is determined with respect to the channel bottom. Therefore, the flow profile, whose slope is $dh/dx = S_0$ with respect to the channel bottom, must be horizontal.

iv) As $h \to 0$, Eq.(6.18) shows that $dh/dx \to \infty/\infty$. However, using Eqs.(6.19) and (6.20), it can be shown that when $h = 0$ and the channel is wide

$$\frac{dh}{dx} = S_0 \left(\frac{h_n}{h_c}\right)^3 \qquad (6.22)$$

when the Chezy formula is used, and

$$\frac{dh}{dx} = \infty \qquad (6.23)$$

when the Manning formula is used. Thus, the theoretical behavior of the flow profile at or near $h = 0$ depends on the type of uniform flow formula used in the computation. However, this result is not of much practical importance since zero depth does never occur.

General Procedure for Sketching Qualitative Flow Profiles

The general procedure for sketching the qualitative flow profiles in a channel is as follows:

1. Draw the profile of the channel. Plot the critical depth line (CDL) and normal depth line (NDL), if any.

2. For the zone in which the profile lies, determine the relation of the depth h to h_c and h_n, if any. For example, for Zone 1 of a mild slope channel, $h > h_n > h_c$.

3. Name the profile considering the channel slope and the zone in which it lies. For example, the name of the profile which lies in Zone 2 of a steep slope channel is S2.

4. Determine the sign of dh/dx from the signs of the numerator and denominator of the right hand side of Eq.(6.18). The numerator is positive if $h > h_n$ and negative if $h < h_n$. Similarly, the denominator is positive if $h > h_c$ and negative if $h < h_c$.

5. Determine whether the profile is a backwater curve or a drawdown curve. The profile is a backwater curve if $dh/dx > 0$ and a drawdown curve if $dh/dx < 0$.

6. Consider the conditions of the profile at its upstream and downstream ends. These conditions help to determine the actual shape of the profile, i.e. whether the profile is concave or convex.

7. Sketch the qualitative flow profile.

8. Determine whether the flow is subcritical or supercritical. Flow is subcritical if $h > h_c$ and supercritical if $h < h_c$.

Flow Profiles in Mild Slope Channels ($S_o > 0$ and $h_n > h_c$)

Using Eq. (6.18), the sign of dh/dx in each zone can be determined as follows:

$$i)\ Zone\ 1:\ h > h_n > h_c,\ \frac{dh}{dx} = +\frac{+}{+} = +,\ i.e.\ \frac{dh}{dx} > 0$$

$$ii)\ Zone\ 2:\ h_n > h > h_c,\ \frac{dh}{dx} = +\frac{-}{+} = -,\ i.e.\ \frac{dh}{dx} < 0$$

$$iii)\ Zone\ 3:\ h_n > h_c > h,\ \frac{dh}{dx} = +\frac{-}{-} = +,\ i.e.\ \frac{dh}{dx} > 0$$

The water surface profile in Zone 1, designated as M1, is a backwater curve and represents subcritical flow. At the upstream boundary ($h \to h_n$, $dh/dx \to 0$), the profile is tangential to the normal depth line and at the downstream boundary ($h \to \infty$, $dh/dx \to S_o$), the profile asymptotically approaches a horizontal line. It may be noted that the water surface in an M1 profile falls in the downstream direction and approaches its horizontal asymptote from above. The M1 profile occurs behind a dam, upstream of a weir or sluice gate in a mild slope channel, when a long mild slope channel ends in a reservoir to a depth greater than the normal depth and when a mild channel is followed by a milder channel. The M1 profiles may be very long compared to other flow profiles. In rivers and canals, the M1 profiles may extend considerable distance before merging with the normal depth. The M1 profile represents the most common flow profile and it is the most important flow profile from the practical point of view, since the slope of most rivers and canals is mild.

The M2 drawdown curve in Zone 2 is tangential to the normal depth line at its upstream boundary ($h \rightarrow h_n$, $dh/dx \rightarrow 0$) and normal to the critical depth line ($h \rightarrow h_c$, $dh/dx \rightarrow \infty$), indicating a hydraulic drop, at its downstream boundary. This type of profile can occur at a free overfall, when a mild slope is followed by a steeper mild or critical or steep slope, when a mild slope channel ends in a reservoir to a depth less than the normal depth and at the upstream side of a sudden enlargement of a channel section.

The M3 backwater profile in Zone 3 starts theoretically from the channel bottom at its upstream end and terminates in a hydraulic jump at its upstream boundary ($h \rightarrow h_c$, $dh/dx \rightarrow \infty$). The M3 profile occurs downstream of a sluice gate in a mild slope channel and when a supercritical flow enters a mild slope channel. The M3 profiles are relatively shorter than M1 and M2 profiles.

Flow Profiles in Steep Slope Channels ($S_o > 0$ and $h_n < h_c$)

The S1 backwater profile begins with a hydraulic jump at the upstream boundary and tends to be horizontal at the downstream boundary. In the S1 profile the water surface rises in the downstream direction and approaches its horizontal asymptote from below. The S1 profile occurs behind a dam or upstream of a weir built in a steep channel and in a steep channel ending in a reservoir to a depth more than the critical depth.

The S2 drawdown curve starts from the critical depth line with a vertical slope at its upstream end and is tangential to the normal depth line at its downstream end. It is usually very short and acts like a transition between a hydraulic drop and uniform flow. This type of profile can occur downstream of an enlargement of a channel section and also downstream of a transition of slope from mild to steep or steep to steeper.

The S3 backwater profile starts from the channel bottom and approaches the normal depth line tangentially. It may occur below a sluice gate on a steep slope or at a transition between steep slope and milder steep slope.

Flow Profiles in Critical Slope Channels ($S_o > 0$ and $h_n = h_c$)

In a critical slope channel the NDL and the CDL coincide since $h_n = h_c$. Therefore, Zone 2 and the C2 profile which satisfy the condition $h_n = h = h_c$ also coincide with the NDL and the CDL. The C2 profile thus represents uniform critical flow and may occur in a long prismatic critical slope channel. Since it represents uniform flow, it is not considered as a profile of gradually varied flow. The C1 backwater profile in Zone 1 starts from the $h_n = h_c$ line and tends to be horizontal downstream. The C3 backwater profile in Zone 3 starts from the channel bottom and meets the $h_n = h_c$ line at its downstream end. Using the condition $h_n = h_c$, Eq.(6.19) gives

$$\frac{dh}{dx} = S_0$$

i.e. the C1 and C3 profiles in a wide channel are exactly horizontal. For channels which are not wide, generally $M \approx N$. So, using the condition $h_c = h_n$, Eq.(6.18) becomes

$$\frac{dh}{dx} \approx S_0$$

indicating that the C1 and C3 profiles are approximately horizontal. The C1 profile may occur upstream of a sluice gate on a critical slope or when a critical slope is followed by a mild or horizontal or adverse slope or it may connect a supercritical flow with a reservoir pool on a critical slope. The C3 profile may occur downstream of a sluice gate in a critical slope channel or at the transition between steep and critical slopes. The critical slope profiles are very rare.

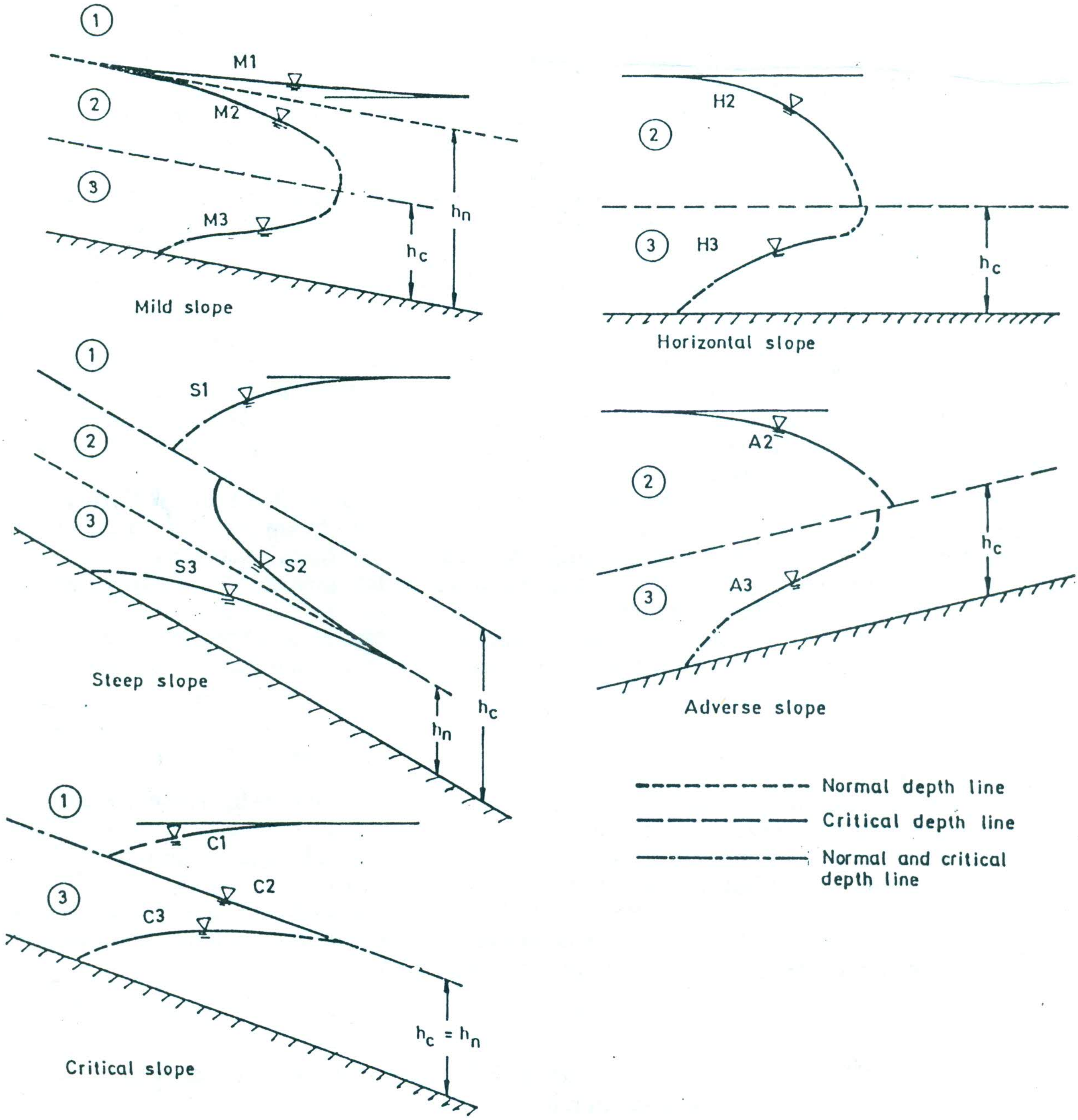


Fig.6.2 Classification of flow profiles of gradually varied flow

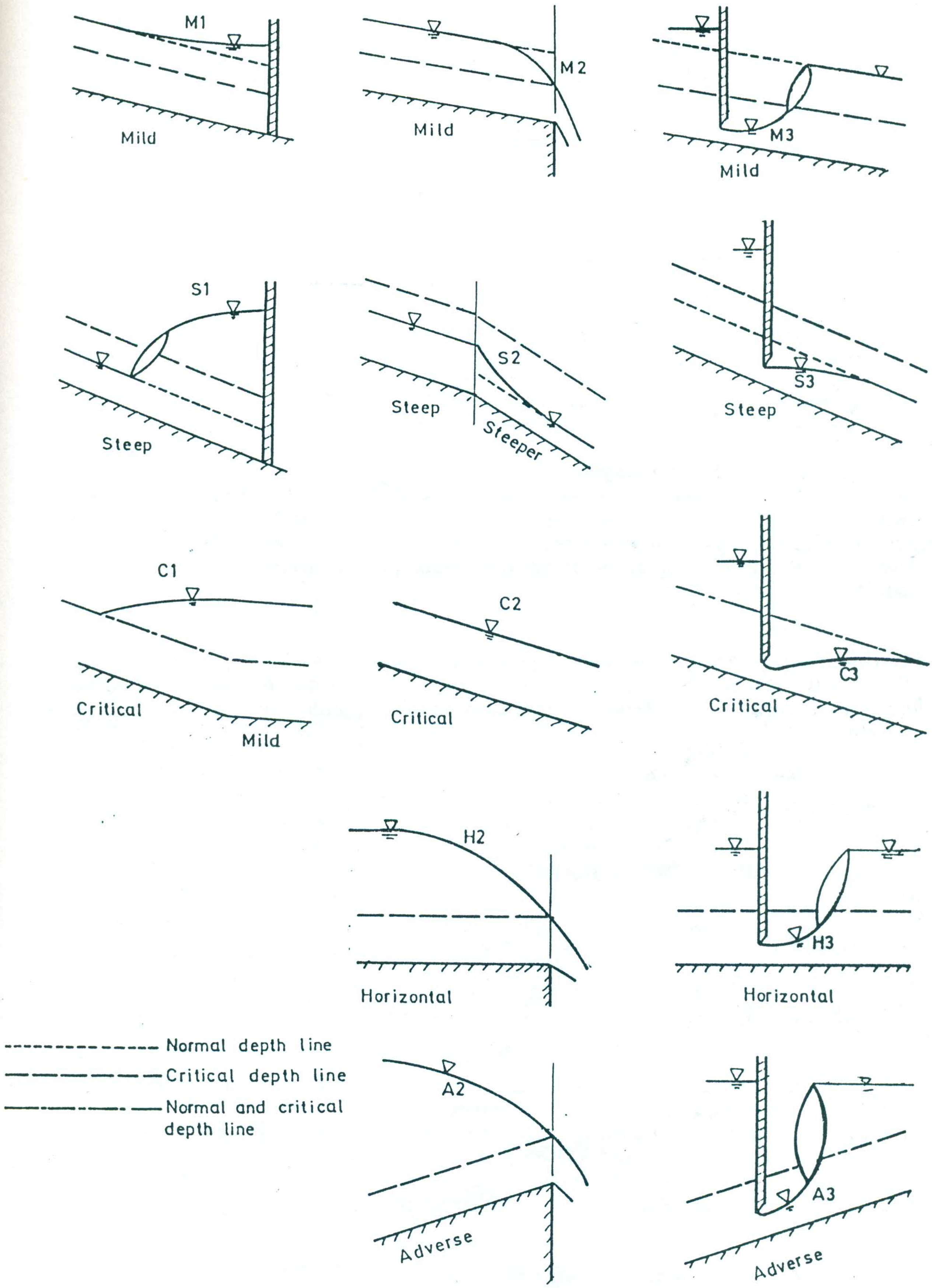


Fig. 6.3 Examples of flow profiles

Flow Profiles in Horizontal Channels ($S_o = 0$ and $h_n = \infty$)

For a horizontal slope ($S_o = 0$), Eq.(6.12) gives $K_n = \infty$ or $h_n = \infty$, and, therefore, Zone 1 and an H1 profile satisfying the condition $h > h_n > h_c$ are not physically possible. With $S_o = 0$, combination of Eqs. (6.8), (6.11), (6.13) and (6.16) gives

$$\frac{dh}{dx} = \frac{-S_f}{1-Fr^2} = \frac{-(Q/K)^2}{1-(h_c/h)^M} \tag{6.24}$$

so that the sign of dh/dx is obtained as follows:

$$i)\ Zone\ 2: h > h_c,\ \frac{dh}{dx} = \frac{-}{+} = -,\ i.e.\ \frac{dh}{dx} < 0$$

$$ii)\ Zone\ 3: h < h_c,\ \frac{dh}{dx} = \frac{-}{-} = +,\ i.e.\ \frac{dh}{dx} > 0$$

The H2 drawdown profile has a horizontal asymptote at its upstream end ($h \to \infty$) and ends in a hydraulic drop at its downstream end ($h \to h_c$, $dh/dx \to \infty$). It may occur on a horizontal slope upstream of a free overfall. The H3 backwater profile, which is similar to the M3 profile, is obtained downstream of sluice gates and spillways on a horizontal slope. The horizontal slope profiles may be considered to be the limiting cases of the mild slope profiles when the channel becomes horizontal.

Flow Profiles in Adverse Slope Channels ($S_o < 0$ and h_n imaginary)

For an adverse slope ($S_o < 0$), Eq. (6.12) indicates that K_n^2 is negative and h_n is imaginary. Therefore, Zone 1 and an A1 profile are not physically possible. Combining Eqs. (6.8), (6.11) and (6.16), we obtain

$$\frac{dh}{dx} = \frac{S_0 - S_f}{1-Fr^2} = \frac{-|S_0 + S_f|}{1-(h_c/h)^M} \tag{6.25}$$

and, hence, the sign of dh/dx is obtained as follows:

$$i)\ Zone\ 2: h > h_c,\ \frac{dh}{dx} = \frac{-}{+} = -,\ i.e.\ \frac{dh}{dx} < 0$$

$$ii)\ Zone\ 3: h < h_c,\ \frac{dh}{dx} = \frac{-}{-} = +,\ i.e.\ \frac{dh}{dx} > 0$$

The A2 and A3 profiles are similar to H2 and H3 profiles and are very rare. Only short lengths of adverse slope profiles may be expected to occur in practice.

It is evident from Table 6.1 and Figs. 6.2 and 6.3 that the profiles in Zone 1 (i.e. M1, S1 and C1) and Zone 3 (i.e. M3, S3, C3, H3 and A3) are backwater curves and those in Zone 2 (i.e. M2, S2, H2 and A2) are drawdown curves. All the profiles in Zones 1 and 2, excepting the S2 profile, represent subcritical flow and those in Zone 3 and the S2 profile represent supercritical flow.

6.4 FLOW PROFILES IN SERIAL ARRANGEMENT OF CHANNELS

When two or more prismatic channels of the same cross-section but with different bottom slopes are combined and carry the same discharge, the following procedure for the analysis of flow profile is to be adopted:

1. Draw the channel profile. Plot the CDL and the NDL, if any, in each channel.
2. Locate all possible control sections at which the depth is known .
3. Starting from the known depth, which may be the normal depth or the depth at a control section, draw the possible flow profiles in the channels.

The following points must be noted in connection with the flow profiles in a number of channels:

1. The critical depth h_c will be the same for all the channels, since it does not depend on the channel bottom slope S_0.

2. The normal depth h_n will be different in different channels. Since h_n is inversely related to the channel bottom slope S_0, the normal depth h_n for a channel will be higher if S_0 is lower and vice versa.

3. Flow upstream of a control must be subcritical and downstream of a control must be supercritical. The control itself locates the subcritical flow profile upstream of it and the supercritical flow profile downstream of it. In fact, the gradually varied flow profile(s) are the results of interaction between the flow and the control(s).

4. When the flow changes from subcritical to supercritical, a hydraulic drop usually forms. On the other hand, when the flow changes from supercritical to subcritical, a hydraulic jump usually forms.

5. Under normal condition, the flow in a long straight prismatic channel having positive slope is taken to be uniform. Therefore, the flow beyond the influence of a control or transition in a mild, critical or steep slope channel will be uniform, i.e. at the normal depth h_n.

6. Under normal condition, the flow in a long horizontal or adverse slope channel is subcritical. Therefore, the flow beyond the influence of a control or transition in a horizontal or adverse slope channel will be in Zone 2, i.e. the flow profile will be H2 in a horizontal channel and A2 in an adverse slope channel.

Specific Examples

Mild slope channel followed by steep slope channel

Let us sketch the qualitative flow profiles in a mild slope channel followed by a steep slope channel (Fig. 6.4). Obviously, the flow is uniform both upstream and downstream of the transition point as shown. The five possible flow profiles, numbered from 1 to 5, over the transition in slope are shown in the figure. For the flow profiles marked 1, the flow is uniform in the entire mild slope channel and the flow passes from the uniform flow condition in the upstream channel to the uniform condition in the downstream channel through an S1 profile in Zone 1 and an S2 profile in Zone 2 of the steep channel. Obviously, this is an impossible case because an S1 profile must be a backwater curve, not a drawdown curve, as shown in Fig. 6.4. In a similar way, the flow profiles marked 5, consisting of an M2 profile in Zone 2 and an M3 profile in Zone 3 of the upstream mild slope channel, are also impossible since the profile M3 must be a backwater curve. Proceeding in this way we come to the conclusion that the flow profiles marked 3, consisting of an M2 drawdown profile in Zone 2 of the upstream mild slope channel and an S2 drawdown profile in Zone 2 of the downstream steep slope channel, are the only acceptable flow profiles. Obviously, the flow changes from subcritical in the upstream mild slope channel to supercritical in the downstream steep slope channel which is possible only through a hydraulic drop.

Steep slope channel followed by mild slope channel

In Fig. 6.5, the flow profiles in a steep slope channel followed by a mild slope channel are shown. The uniform flow is supercritical in the upstream channel and subcritical in the downstream channel. The change in the flow state from supercritical to subcritical can only occur through a hydraulic jump. The location of the jump depends on the relative magnitudes of the two slopes. If the magnitude of the two slopes are such that the jump is formed in the steep slope channel, an S1 profile is formed in the steep channel and the flow is uniform just from the beginning of the mild slope channel. Now, if the slope of the downstream mild slope channel is gradually increased, the normal depth line for the mild slope channel lowers and the jump gradually moves downstream and finally the jump forms on the mild slope channel. In this situation, the flow is uniform in the entire upstream channel, an M3 profile is formed in the downstream channel and the flow eventually returns to the uniform state after the hydraulic jump in the mild slope channel.

It is to be noted that a hydraulic jump usually occurs when a steep slope channel is followed by either a mild or a horizontal or an adverse slope channel. The jump forms either in the upstream channel or in the downstream channel, but such a situation does never occur that a part of the jump forms in the upstream channel and the rest of the jump forms in the downstream channel.

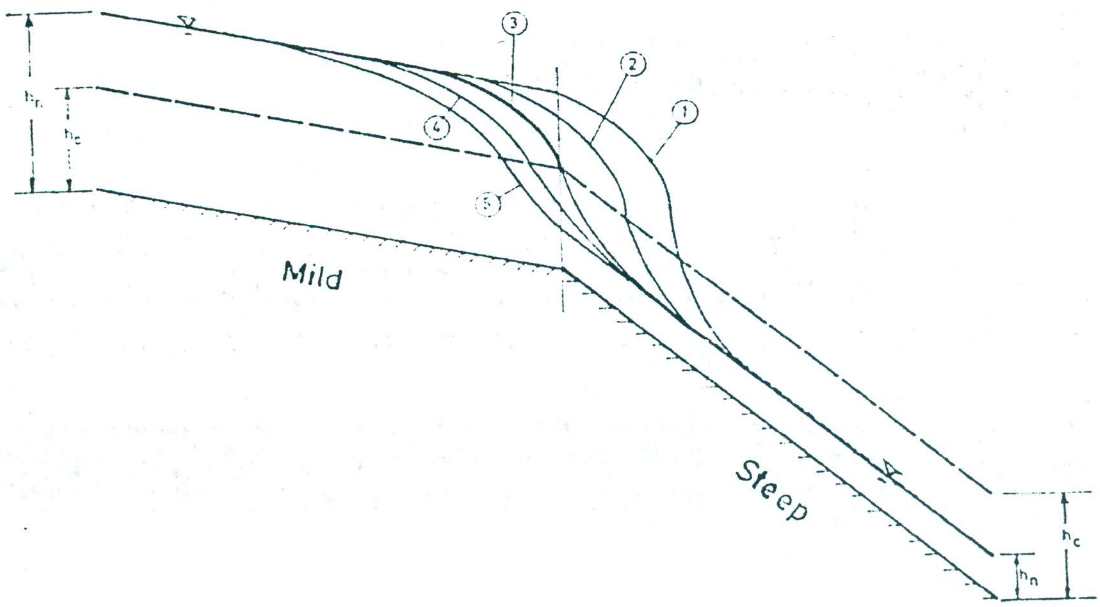


Fig. 6.4 Flow profiles in a mild slope channel followed by a steep slope channel

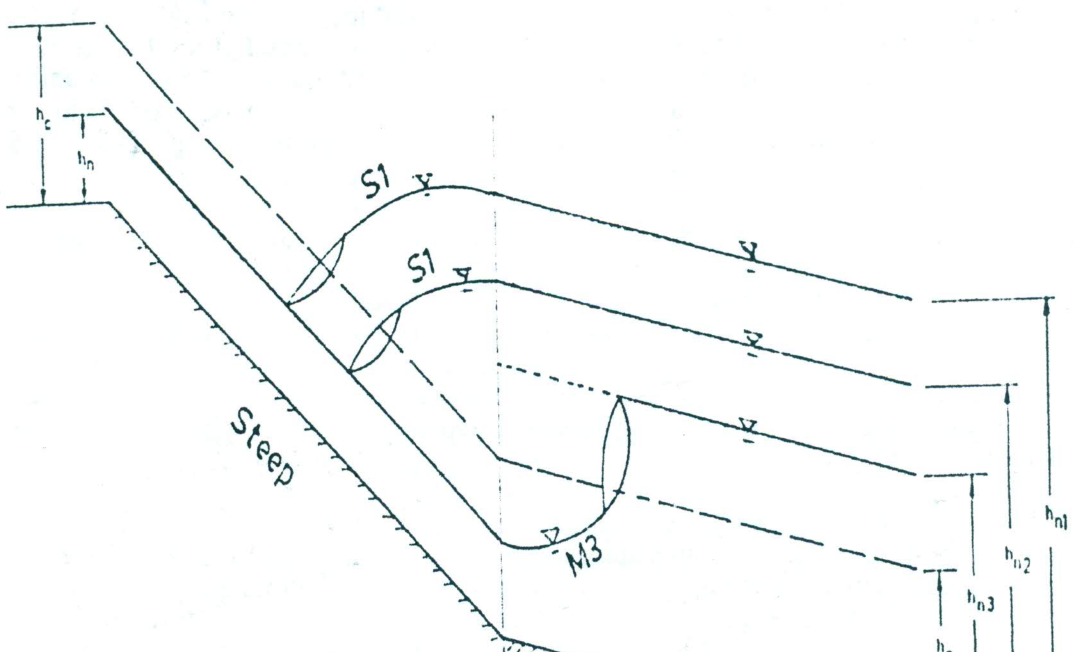


Fig. 6.5 Flow profile in a steep slope channel followed by a mild slope channel

Free overfall at the end of a mild slope channel

Suppose there is a free overfall at the end of a mild slope channel (Fig. 6.6a). The flow is uniform in the channel far upstream of the free overfall. The water surface falls as a result of the free overfall, an M2 profile develops immediately upstream of the free overfall and the critical depth occurs just upstream of the brink.

Different water levels downstream of a mild slope channel

Three positions of the water level h_d downstream of a mild slope channel are as shown in Fig. 6.6(b). Obviously,

i) when $h_d > h_n$, an M1 profile is formed,
ii) when $h_n > h_d > h_c$, an M2 profile is formed, and
iii) when $h_n > h_c > h_d$, the situation is similar to a free overfall and an M2 profile is formed.

Sluice gate in a steep slope channel

There is a vertical sluice gate in a steep slope channel. The flow in the channel far upstream from of the sluice gate is uniform and supercritical. The presence of the sluice gate, which is a control, changes the flow to subcritical with the formation of a hydraulic jump and an S1 profile, as shown in Fig. 6.6(c). An S3 profile representing supercritical flow is formed downstream of the sluice gate through which the water surface joins the NDL downstream.

Overflow weir in a mild slope chanel

The flow in the channel far upstream of the weir is uniform and subcritical. The depth above the weir is approximately equal to the critical depth, i.e. critical section occurs just upstream of the weir. Therefore, an M1 profile is formed upstream of the weir (Fig. 6.6d).

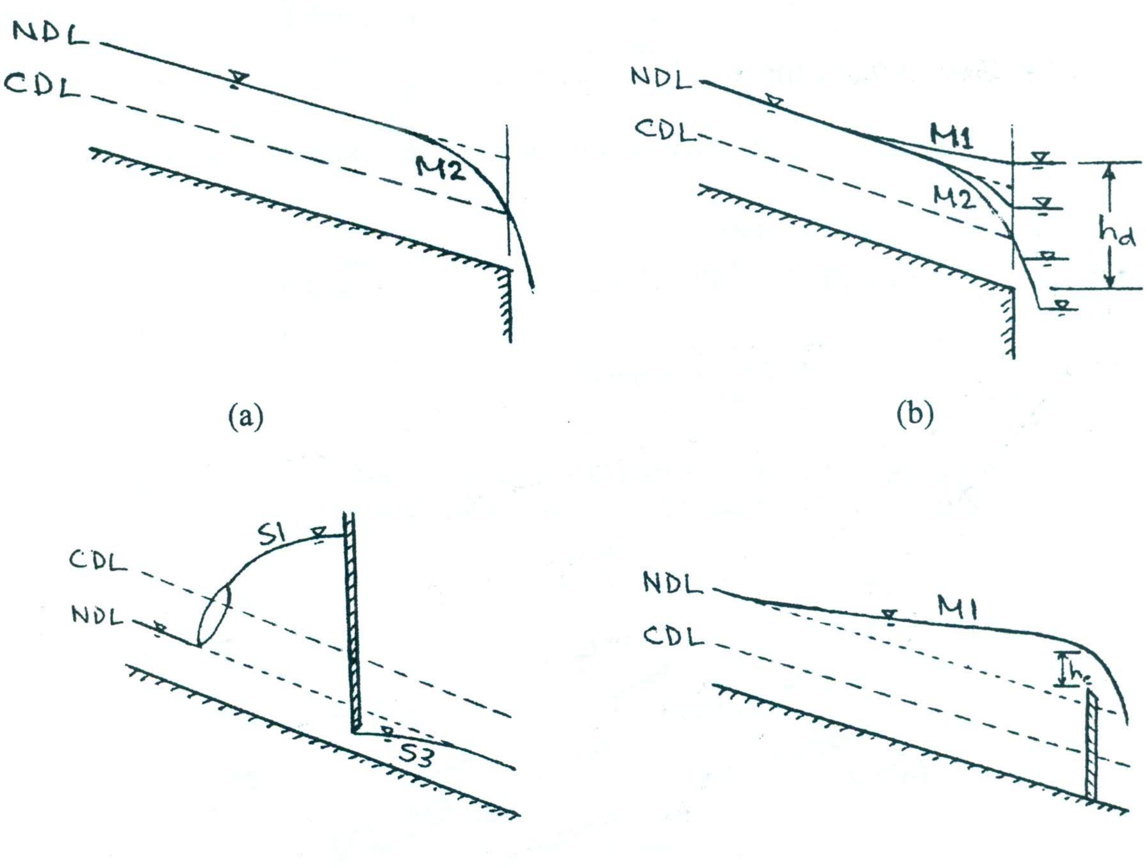


Fig. 6.6(a) Flow profile due to a free overfall at the end of a mild slope channel. (b) Flow profiles for different positions of water level downstream of a mild slope channel.(c) Flow profiles upstream and downstream of a sluice gate in a steep slope channel. (d) Flow profile upstream of an overflow weir in a mild slope channel.

Example 6.1

A trapezoidal channel with b = 6 m, n = 0.025, s = 2 and S_0 = 0.001 carries a discharge of 28 m^3/s. At a certain section A of the channel the depth of flow is 1.30 m. (i) Determine the type of channel slope. (ii) Determine the type of flow profile. (iii) If at another section B, the depth of flow is 1.50 m, state whether section B is located upstream or downstream of section A.

Solution For the given section, n, S_0 and Q, the critical and normal depths are found to be

$$h_c = 1.14 \text{ m} \quad \text{and} \quad h_n = 1.91 \text{ m}$$

Also, the actual depth of flow at section A , h_A = 1.30 m.

(i) Since $h_n > h_c$, the channel slope is mild.

(ii) Since $h_n > h_A > h_c$, the profile is M2.

(iii) Since the M2 profile is a drawdown profile and the depth at section B (= 1.50 m) is more than the depth at section A (= 1.30 m), section B is located upstream of section A.

Example 6.2

A rectangular channel 10 m wide and having α = 1.10 and n = 0.025 has three reaches arranged serially. The bottom slopes of these reaches are 0.0040, 0.0065 and 0.0090, respectively. For a discharge of 35 m^3/s in this channel, sketch the resulting flow profiles. Steep

Solution The critical depth for the given conditions is obtained as

Mild Steeper

$$h_c = \sqrt[3]{\frac{\alpha Q^2}{g b^2}} = \sqrt[3]{\frac{1.10 \times 35^2}{9.81 \times 10^2}} = 1.11\ m$$

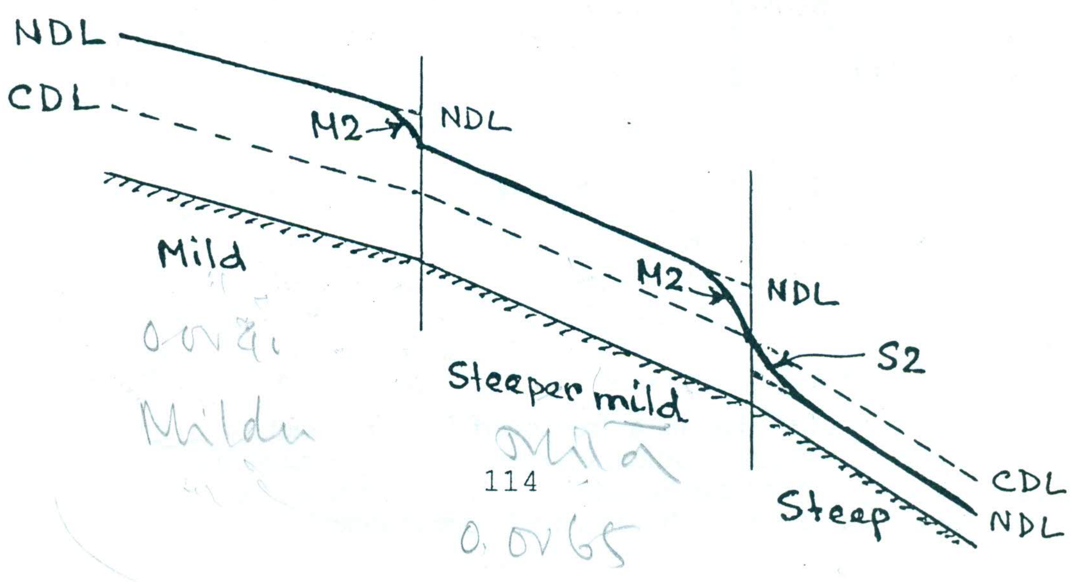
Since critical slope is the slope for which flow in the channel is both uniform and critical, hence

$$h_n = h_c = 1.11 \text{ m}$$

Therefore, A = 10× 1.11 = 11.12 m^2, P = 10 + 2× 1.11 = 12.22 m and R = A/P = 0.91 m, and

$$S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.025 \times 35}{11.12 \times 0.91}\right)^2 = 0.0070$$

Thus, the bottom slopes of the three reaches are mild, steeper mild and steep, respectively. The resulting flow profiles are M2, M2 and S2, as shown in the following figure.



6.5 COMPUTATION OF GRADUALLY VARIED FLOW PROFILES

6.5.1 Introduction

The computation of the gradually varied flow basically involves the integration of the dynamic equation of gradually varied flow, presented earlier. This equation is a non-linear ordinary differential equation of the first order and its solution requires one boundary condition for depth, i. e. the depth at the section where the computation begins must be given. This equation can be easily integrated (i) for a wide channel, and (ii) for a horizontal channel. For other channels, the integration of the gradually varied flow equation has to be performed either graphically or numerically.

The computation of gradually varied flow profile must begin with the known depth of flow at a control and proceed in the direction in which the control operates. Thus, the computation of the subcritical flow profiles must start from the downstream end of the channel reach and proceed upstream, and the computation of supercritical flow profiles must start from the upstream end of the channel reach and proceed downstream.

The profiles M1, M2, S2 and S3 approach the normal depth line asymptotically, i. e. theoretically these profiles extent indefinitely before merging with the normal depth. Such a situation presents difficulties from the computational point of view. As a result, the computation of these flow profiles is usually terminated at a section where the depth of flow is about 5% greater or less than the normal depth.

In computing a flow profile, the following information are generally required:

1. The discharge Q for which the flow profile is desired.
2. The depth of flow or stage at the control section where the computation begins.
3. The channel shape at various channel sections.
4. The bottom slope S_0 of the channel.
5. The energy coefficient α.
6. The Manning's n or Chezy's C.

There are many methods for computing gradually varied flow profiles. However, these methods can be broadly classified into the following two categories:

i) Methods used for computing flow profiles in prismatic channels
ii) Methods used for computing flow profiles in non-prismatic channels.

The methods used for computing flow profiles in prismatic or regular or uniform channels compute a longitudinal distance x for a given h explicitly without involving any trial. The direct step method and the direct integration method fall in this category. On the other hand, the methods used for computing flow profiles in non-prismatic or irregular or non-uniform channels compute h from a given x. In this case, a trial-and-error procedure is necessary. The standard step method falls in this second category.

6.5.2 Computation of Flow Profiles in Prismatic Channels

(a) Direct Integration Method

Direct integration of the gradually varied flow equation for computing flow profiles in a wide channel and in a horizontal channel is simple and considered here. The integration of the gradually varied flow equation for other cases is presented by Chow (1959) and Gill (1976).

The main advantage of the direct integration method over other methods is that the total length of the flow profile may, if desired, be computed using a single step.

(i) Flow Profile in a Wide Channel: Bresse Method

Equation (6.18) can be integrated exactly for a wide rectangular channel with the conveyance expressed in terms of the Chezy equation. For this case, M = N = 3 and

$$\frac{dh}{dx} = S_0 \frac{1-(h_n/h)^3}{1-(h_c/h)^3} \tag{6.26}$$

Putting $u = h/h_n$, so that $du = dh/h_n$ in Eq.(6.26) and rearranging yields

$$dx = \frac{h_n}{S_0}\left[1-\left(1-\frac{h_c^{\ 3}}{h_n^{\ 3}}\right)\frac{1}{1-u^3}\right]du \tag{6.27}$$

which on integration gives

$$x = \frac{h_n}{S_O}\left[u-\left(1-\frac{h_c^{\ 3}}{h_n^{\ 3}}\right)\phi\right]+C_1 \tag{6.28}$$

where ϕ is the *Bresse function* given by

$$\phi = \int_O^u \frac{du}{1-u^3} = \frac{1}{6}\ln\frac{u^2+u+1}{(u-1)^2} - \frac{1}{\sqrt{3}}\tan^{-1}\frac{\sqrt{3}}{2u+1} \tag{6.29}$$

and C_1 is a constant of integration. This integration was first performed by J. A. Ch. Bresse in 1860. A determination of the flow profile by this solution is widely known as the *Bresse method.*

For a wide channel the critical depth h_c is given by

$$h_c = \sqrt[3]{\frac{q^2}{g}} \tag{6.30}$$

and, using the Chezy formula, the normal depth h_n is given by

$$h_n = \sqrt[3]{\frac{q^2}{C^2 S_O}} \tag{6.31}$$

where C is Chezy's C and q is the discharge per unit width, and hence

$$\left(\frac{h_c}{h_n}\right)^3 = \frac{C^2 S_O}{g} \tag{6.32}$$

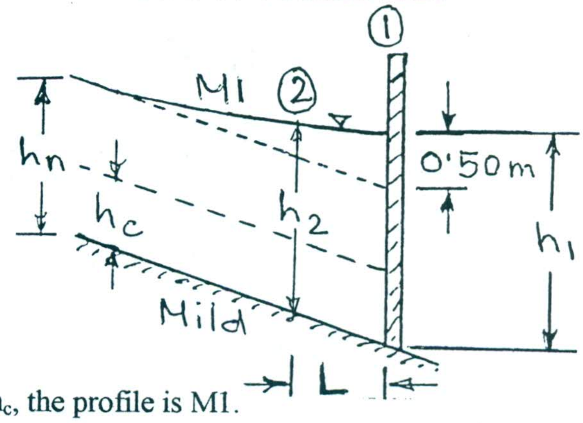
The length of the flow profile between two consecutive sections of depths h_1 and h_2 is

$$L = x_2 - x_1 = \frac{h_n}{S_O}\left[(u_2-u_1)-\left(1-\frac{h_c^{\ 3}}{h_n^{\ 3}}\right)(\phi_2-\phi_1)\right] \tag{6.33}$$

where ϕ_1 and ϕ_2 are the values of ϕ corresponding to $u_1 = h_1/h_n$ and $u_2 = h_2/h_n$, respectively.

Example 6.3

A wide rectangular channel with Chezy's C = 47 $m^{1/2}/s$ and S_0 = 0.0001 carries a discharge of 2 m^2/s. A dam raises the water level by 0.50 m above the normal depth at the dam site. Compute the length of the flow profile between the dam site and the location where the depth is 2.90 m.



Solution Since the channel is wide,

$$h_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{2^2}{9.81}} = 0.742\ m$$

$$h_n = \sqrt[3]{\frac{q^2}{C^2 S_O}} = \sqrt[3]{\frac{2^2}{47.0^2 x 0.0001}} = 2.626\ m$$

Since $h_n > h_c$, the channel slope is mild.

Now, $h_1 = 2.626 + 0.50 = 3.126\ m$, $u_1 = h_1 / h_n = 1.190$,

$h_2 = 2.90\ m$, $u_2 = h_2 / h_n = 1.104$. Since h_1 or $h_2 > h_n > h_c$, the profile is M1.

$$\phi_1 = \frac{1}{6} \ln \frac{u_1^2 + u_1 + 1}{(u_1 - 1)^2} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}}{2u_1 + 1} = 0.7667 - 0.2734 = 0.4933$$

$$\phi_2 = \frac{1}{6} \ln \frac{u_2^2 + u_2 + 1}{(u_2 - 1)^2} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}}{2u_2 + 1} = 0.9545 - 0.2858 = 0.6687$$

Hence, the length of the profile is obtained using Eq.(6.33) as

$$L = \frac{h_n}{S_O}\left[(u_2 - u_1) - \left(1 - \frac{h_c^3}{h_n^3}\right)(\phi_2 - \phi_1)\right]$$

$$= \frac{2.626}{0.0001}\left[(1.104 - 1.190) - \left(1 - \frac{0.742^3}{2.626^3}\right)(0.6687 - 0.4933)\right] = -6760.06\ m$$

(ii) Flow Profile in a Horizontal Channel

For a horizontal channel, $S_0 = 0$ and Eq. (6.8) becomes

$$\frac{dh}{dx} = \frac{-S_f}{1 - Fr^2} = \frac{-(Q/K)^2}{1 - (h_c/h)^M} \tag{6.34}$$

where $Q = K\sqrt{S_f}$. Since the critical slope S_c is the slope that will produce a discharge Q at a normal depth equal to the critical depth h_c, $Q = K_c\sqrt{S_c}$ and hence Eq.(6.34) becomes

$$\frac{dh}{dx} = S_c \frac{-(K_c/K)^2}{1 - (h_c/h)^M} \tag{6.35}$$

Since $K^2 = C_2 h^N$ and $K_c^2 = C_2 h_c^N$, so that $(K_c/K)^2 = (h_c/h)^N$, Eq.(6.35) becomes

$$\frac{dh}{dx} = S_c \frac{-(h_c/h)^N}{1 - (h_c/h)^M} \tag{6.36}$$

Putting $p = h/h_c$ in Eq.(6.36) so that $dp = dh/h_c$ and rearranging, we obtain

$$h_c \frac{dp}{dx} = S_c \frac{-(1/p)^N}{1-(1/p)^M} = S_c \frac{p^M}{p^N(1-p^M)} = S_c \frac{p^{M-N}}{1-p^M} \tag{6.37}$$

or

$$dx = \frac{h_c}{S_c}(p^{N-M} - p^N)dp \tag{6.38}$$

Integrating the above equation, we get

$$x = \frac{h_c}{S_c}\left(\frac{p^{N-M+1}}{N-M+1} - \frac{p^{N+1}}{N+1}\right) + C_1 \tag{6.39}$$

where C_1 is a constant of integration. The length of the flow profile between two consecutive sections of depths h_1 and h_2 is given by

$$L = x_2 - x_1 = \frac{h_c}{S_c}\left[\frac{1}{N-M+1}(p_2^{\;N-M+1} - p_1^{\;N-M+!}) - \frac{1}{N+1}(p_2^{\;N+1} - p_1^{\;N+1})\right] \tag{6.40}$$

When M and N are not constant, then they are to be computed for the average depth $\bar{h} = (h_1 + h_2)/2$ and taken to be constant for the reach.

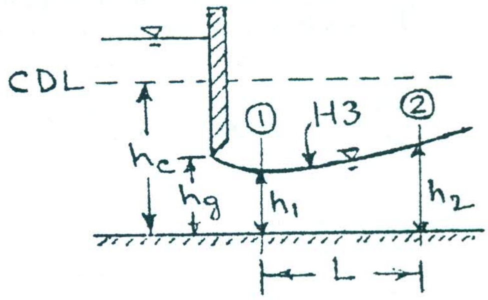
Example 6.4

A vertical sluice gate having a coefficient of contraction, $C_c = 0.61$ and a gate opening, $h_g = 1.00$ m, discharges 25 m^3/s into a horizontal rectangular channel 5 m wide. Compute the length of the flow profile between the vena contracta and the location where the depth is 0.75 m. Take n = 0.015 and $\alpha = 1.12$.

Solution Depth at the vena contracta,

$h_1 = C_c \times h_g = 0.61 \times 1.00 = 0.61$ m

$$h_c = \sqrt[3]{\frac{\alpha Q^2}{gb^2}} = \sqrt[3]{\frac{1.12 \times 25^2}{9.81 \times 5^2}} = 1.419 \; m$$



Since $h_1 < h_c$, an H3 backwater profile is created.

The critical slope is obtained using the Manning formula with $h_n = h_c = 1.419$ m. Then, $A = 5 \times 1.419 = 7.09$ m^2, $P = 5 + 2 \times 1.419 = 7.84$ m, $R = 7.09/7.84 = 0.91$ m so that

$$S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.015 \times 25}{7.09 \times 0.91^{2/3}}\right)^2 = 0.0032$$

The channel is rectangular. Hence, M = 3 and the value of N is computed for the average depth, $\bar{h} = (h_1 + h_2)/2 =$ (0.61+0.75)/2 = 0.68 m so that $\bar{h}/b = 0.68/5 = 0.136$.

$$\therefore N = \frac{2}{3}\left[5 - \frac{4(\bar{h}/b)}{1+2(\bar{h}/b)}\right] = \frac{2}{3}\left[5 - \frac{4 \times 0.136}{1 + 2 \times 0.136}\right] = 3.048$$

Therefore, N – M + 1 = 1.048 and N + 1 = 4.048. Also, $p_1 = h_1/h_c = 0.61/1.419 = 0.4297$ and $p_2 = h_2/h_c = 0.75/1.419 = 0.5284$. Hence, the length of the flow profile is obtained using Eq. (6.40) as

$$L = \frac{h_c}{S_c}\left[\frac{1}{N-M+1}(p_2^{\;N-M+1} - p_1^{\;N-M+!}) - \frac{1}{N+1}(p_2^{\;N+1} - p_1^{\;N+1})\right]$$

$$= \frac{1.419}{0.0032}\left[\left(\frac{0.5284^{1.048} - 0.4297^{1.048}}{1.048}\right) - \left(\frac{0.5284^{4.048} - 0.4297^{4.048}}{4.048}\right)\right] = 37.63 \text{ m}$$

(b) Direct Step Method

In general a step method is characterized by dividing the channel into short reaches (Fig. 6.7) and carrying the computation step by step from one end of the reach to the other. The direct step method is the simplest step method applicable to prismatic channels. It predicts a longitudinal distance x for a given depth h explicitly without involving any trial.

In this method the equation

$$\frac{dE}{dx} = S_0 - S_f \qquad (6.41)$$

is used. In finite difference form this equation can be written as

$$\frac{\Delta E}{\Delta x} = S_0 - S_f \qquad (6.42)$$

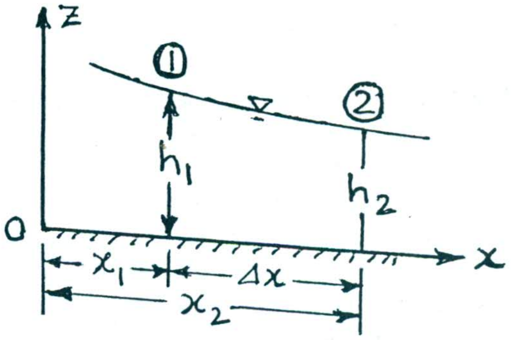


Fig. 6.7 Definition sketch for direct step method

In this equation, all variables with the exception of Δx, are functions of the depth of flow h. So, by selecting the value of h, Eq. (6.42) can be solved for Δx as

$$\Delta x = \frac{\Delta E}{S_0 - \bar{S}_f} = \frac{E_2 - E_1}{S_0 - \bar{S}_f} \qquad (6.43)$$

where $\bar{S}_f$ is mean value of the friction slope over the space interval Δx. Since $\Delta x = x_2 - x_1$ and the distance x_1 of section 1 from the origin is known, the distance x_2 from the origin can be obtained from

$$x_2 = x_1 + \Delta x \qquad (6.44)$$

In uniform flow the friction slope is the same at every section of the channel. But in gradually varied flow the friction slope is different at different channel sections. The following are some of the methods of estimating the average friction slope for a reach.

Arithmetic mean friction slope

$$\bar{S}_f = \frac{S_{f1} + S_{f2}}{2} \qquad (6.45)$$

Geometric mean friction slope

$$\bar{S}_f = \sqrt{S_{f1} \times S_{f2}} \qquad (6.46)$$

Harmonic mean friction slope

$$\bar{S}_f = \frac{2S_{f1}S_{f2}}{S_{f1} + S_{f2}} \qquad (6.47)$$

The above three formulas give almost identical results for most problems. We use Eq. (6.45) because it is the simplest of the three equations.

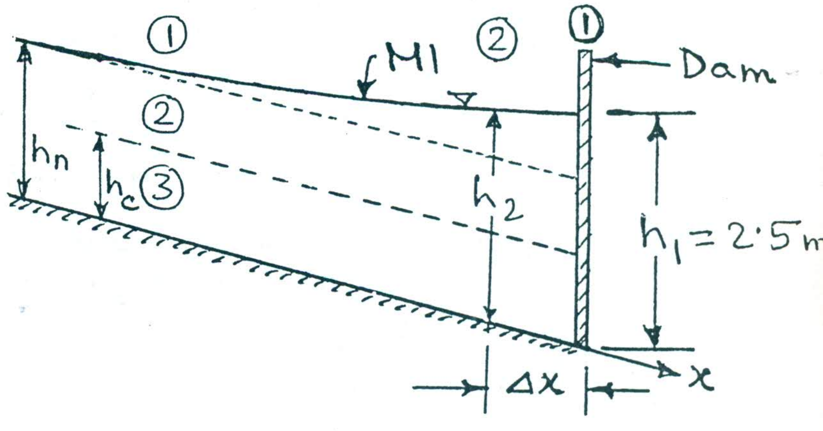
Example 6.5

A trapezoidal channel with b = 6 m and s = 2 is laid on a slope of 0.0025 and carries a discharge of 30 m^3/s. The depth produced by a dam immediately upstream of it is 2.50 m. Compute the resulting flow profile. Take $\alpha = 1.12$ and n = 0.025.

Solution

For the given data, the critical and normal depths are found to be $h_c = 1.23$ m and $h_n = 1.55$ m. Since $h_n > h_c$, the channel slope is mild, and since $h > h_n > h_c$, the profile is M1.

The channel bottom at the section where h = 2.5 m is taken as the origin and the distances measured downstream are taken to be positive. The computation begins at x = 0 and proceeds upstream step by step until the depth is 1.60 m, which is about 3% higher than the normal depth. The computation of Δx using Eq.(6.43) for various values of h are given in Table 6.2 which is almost self-explanatory.



6.5.3 Computation of Flow Profiles in Non-prismatic Channels

Standard Step Method

The direct integration method and the direct step method, considered above, compute a longitudinal distance x for a given depth h in a prismatic channel. These methods cannot be employed to compute flow profiles in non-prismatic channels where it is necessary to compute the depth of flow h or the stage z_w for a given longitudinal distance x.

Using the standard step method, the depth of flow h or the stage z_w for a given x can be computed. This method is, therefore, best suited to the computation of flow profiles in irregular or non-prismatic channels like the natural rivers. In rivers, the channel properties are usually measured only at certain fixed sections, known as *stations*, and it is necessary to calculate the depth h or the stage z_w from the chosen value of x. We must, therefore, use some sort of trial process. In the computation of flow profiles in non-prismatic channels like rivers, the *stage* z_w is most commonly used instead of the depth of flow h.

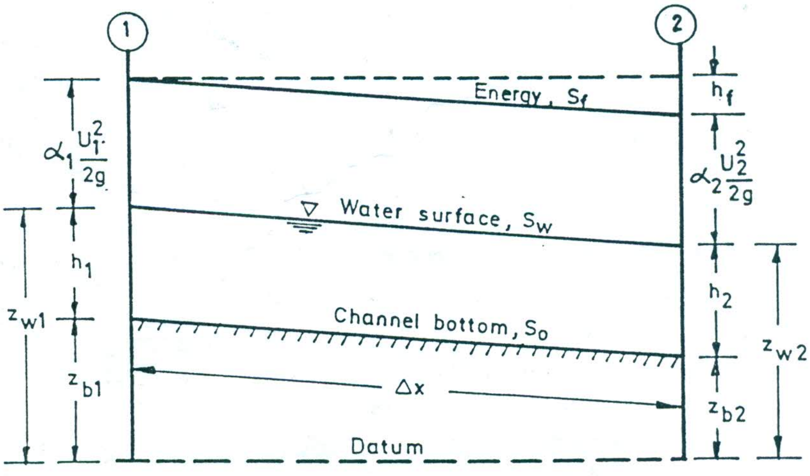


Fig. 6.8 Channel reach definition for standard step method

The application of the energy equation between the two stations shown in Fig.6.8 yields

$$z_{b1} + h_1 + \alpha_1 \frac{U_1^2}{2g} = z_{b2} + h_2 + \alpha_2 \frac{U_2^2}{2g} + h_f + h_e$$

or,
$$z_{w1} + \alpha_1 \frac{U_1^2}{2g} = z_{w2} + \alpha_2 \frac{U_2^2}{2g} + h_f + h_e \tag{6.48}$$

where

$$z_{w1} = z_{b1} + h_1 \tag{6.49}$$

and

$$z_{w2} = z_{b2} + h_2 \tag{6.50}$$

are the stages and z_{b1} and z_{b2} are the elevations of the channel bottom at sections 1 and 2, respectively, h_f is the friction loss in the reach and h_e is the eddy loss occurring in the reach.

The friction loss is given by

$$h_f = \bar{S}_f \Delta x = \frac{1}{2}(S_{f1} + S_{f2})\Delta x \tag{6.51}$$

where S_{f1} and S_{f2} are the friction slopes at sections 1 and 2, respectively.

The eddy losses, which may be appreciable in non-prismatic channels, are generally taken to be proportional to the absolute magnitude of the change in the velocity head in the reach or

$$h_e = k\left|\alpha_1 \frac{U_1^2}{2g} - \alpha_2 \frac{U_2^2}{2g}\right| = k\left|\Delta(\alpha U^2/2g)\right| \tag{6.52}$$

where k is a coefficient which is assumed to range between 0 and 0.1 for gradually converging reaches, between 0 and 0.2 for gradually diverging reaches and to have a value of 0.5 for abrupt expansions or contractions. The eddy loss h_e is assumed to be zero for prismatic channels. In some cases, we increase the Manning's roughness coefficient n and neglect h_e.

The total heads at sections 1 and 2 are given by

$$H_1 = z_{w1} + \alpha_1 \frac{U_1^2}{2g} \tag{6.53}$$

and

$$H_2 = z_{w2} + \alpha_2 \frac{U_2^2}{2g} \tag{6.54}$$

Using Eqs. (6.53) and (6.54), Eq.(6.48) becomes

$$H_2 = H_1 - h_f - h_e \tag{6.55}$$

Equation (6.55) is solved by trial-and-error, i.e. for a given space interval Δx, a value of h_2 (or z_{w2}) is assumed which allows the computation of H_2 by Eq. (6.54), h_f and h_e are then computed and H_2 is estimated by Eq. (6.55). If the two computed values of H_2 agree, then the assumed depth (or stage) at station 2 is correct. If not, the calculations are repeated with an improved trial value of h_2 (or z_{w2}). If the value of h_2 (or z_{w2}) computed by Eq.(6.54) is more than that computed by Eq.(6.55), then h_2 (or z_{w2} has to be reduced for the next trial and vice versa.

The important step in this analysis is the selection of an improved trial value of h_2 or z_{w2}. On the basis of ith trial, the (i+1)th trial value of the depth h_2 (and hence the stage z_{w2}) can be found by the *Newton-Raphson method.* Let $F(h_2)$ be a function such that

$$F(h_2) = H_2 - (H_1 - h_f - h_e) \tag{6.56}$$

Then using Eqs.(6.50),(6.51) and (6.54), one obtains

$$F(h_2) = z_{w2} + \alpha_2 \frac{U_2^{\ 2}}{2g} - H_1 + \frac{1}{2} S_{f1} \Delta x + \frac{1}{2} S_{f2} \Delta x + h_e$$

$$= z_{b2} + h_2 + \alpha_2 \frac{U_2^{\ 2}}{2g} - H_1 + \frac{1}{2} S_{f1} \Delta x + \frac{1}{2} S_{f2} \Delta x + h_e \tag{6.57}$$

Then

$$\frac{dF(h_2)}{dh_2} = \frac{d}{dh_2}(z_{b2} + h_2 + \alpha_2 \frac{U_2^{\ 2}}{2g} - H_1 + \frac{1}{2} S_{f1} \Delta x + \frac{1}{2} S_{f2} \Delta x + h_e) \tag{6.58}$$

Since z_{b2}, H_1 and S_{f1} are already known, their derivatives with respect to h_2 are equal to zero. The derivative dh_e/dh_2 is neglected, since the variation of h_e with respect to h_2 is small (if desired, this can be included). Further, $d(\alpha U^2/2g)/dh = - Fr^2$, where Fr is the Froude number, and using Eqs.(6.13) and (6.17), it can be proved that $dS_f/dh = - NS_f/h$, where N is the hydraulic exponent for uniform flow computation. Therefore, Eq.(6.58) reduces to

$$\frac{dF(h_2)}{dh_2} = 1 - Fr_2^{\ 2} - \frac{N_2 S_{f2} \Delta x}{2h_2} \tag{6.59}$$

Then, according to the Newton-Raphson method, the amount by which the depth h_2 (or stage z_{w2}) must be adjusted is given by

$$\Delta h_2 \ (or\ \Delta z_{w2}) = -\frac{F(h_2)}{dF(h_2)/dh_2} = -\frac{F(h_2)}{1 - Fr_2^{\ 2} - N_2 S_{f2} \Delta x / 2h_2} \tag{6.60}$$

Equation (6.60) reduces the number of trial values, usually to 3 or 4, needed to compute the depth of flow or stage correctly.

For a wide channel, N = 3.00 or 3.33 depending on whether the conveyance is expressed by the Chezy or the Manning formula and $R \approx h$. Hence, for a river, which can be considered wide, one can use the equation

$$\Delta h_2 \ (or\ \Delta z_{w2}) = -\frac{F(h_2)}{dF(h_2)/dh_2} = -\frac{F(h_2)}{1 - Fr_2^{\ 2} - 3S_{f2}\Delta x / 2R_2} \qquad (6.61)$$

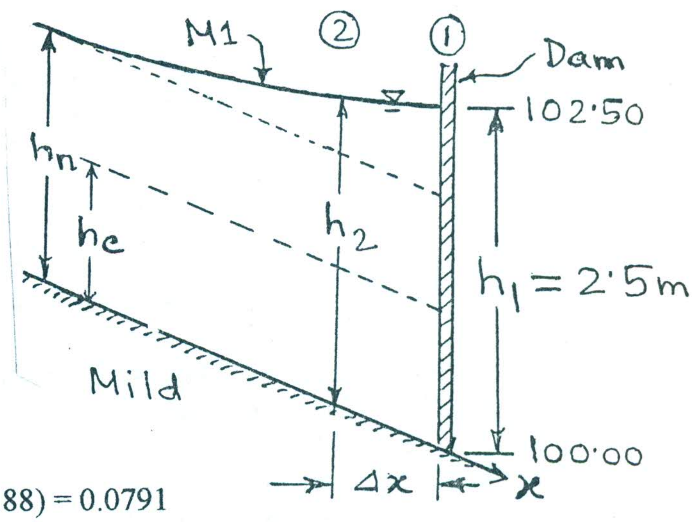
to obtain the next trial value of Δh_2 or Δz_{w2}.

Example 6.6

Considering the channel described in Example 6.5, compute the depths (or stages) at distances of 100 m, 200 m and 300 m upstream from the dam site by the standard step method. The elevation of the channel bottom at the dam site as 100.00 m. Take $h_e = 0$.

Solution

The channel bottom at the dam site is taken as the origin (x = 0) and distances measured downstream are taken to be positive. The computation starts at the dam and proceeds upstream step by step. The computation of depths (or stages) at the three upstream sections are given in Table 6.3.



At section 2 which is situated at a distance of 100 m upstream from the dam site ($\Delta x = -100$ m), the trial value of h_2 is taken as 2.50 m. The elevation of the channel bottom at section 2 is $(100.00 + 100 \times 0.0025)$ m = 100.25 m. The stage at this section is (100.25 + 2.50) m = 102.75 m. The value of H_2 determined by Eq.(6.54) is 102.8179 m and the value of H_2 determined by Eq.(6.55) is 102.6076 m. These two values of H_2 do not agree and hence it is necessary to revise the trial value of h_2 (and z_{w2}). The next trial value of h_2 is obtained as follows:

$F(h_2) = 102.8179 - 102.6076 = 0.2103$

$B_2 = b + 2sh_2 = 6 + 2\times 2 \times 2.5 = 16$ m

$D_2 = A_2/B_2 = 27.50/16 = 1.7188$ m

$Fr_2^{\ 2} = \alpha U_2^{\ 2}/(gD_2) = 1.12 \times 1.091^2/(9.81 \times 1.7188) = 0.0791$

$$N_2 = \frac{2h_2}{3A_2}\left(5B_2 - 2R_2\frac{dP}{dh}\right) = \frac{2\times 2.50}{3\times 27.50}(5\times 16 - 2\times 1.601 \times 2\sqrt{5}) = 3.981$$

$$\Delta h_2 = -\frac{F(h_2)}{1 - Fr_2^{\ 2} - N_2 S_{f2}\Delta x / 2h_2} = -\frac{0.2103}{1 - 0.0791 - 3.981 \times 0.000397 \times (-100)/(2\times 2.50)}$$

$$= -\frac{0.2103}{0.9528} = -0.221$$

Hence, the next trial value of $h_2 = (2.50 - 0.221)$ m = 2.279 m

Note that if the channel is assumed to be wide, then taking N = 3.00, we obtain $\Delta h_2 = -0.2103/0.9447 = -0.223$ and the next trial value of h_2 is 2.277 m, which is practically the same as that obtained when N = 3.981.

6.5.4 Computation of Flow Profiles by Numerical Methods

As stated earlier, the dynamic equation of gradually varied flow is a non-linear first-order ordinary differential equation and requires one boundary condition (i.e. the depth at the starting section) for its solution. There are a group of numerical methods, known as the *Runge-Kutta methods*, which are particularly suitable and commonly used for solving the first-order ordinary differential equations similar to the dynamic equation of gradually varied flow. The Runge-Kutta

Table 6.3 Computation of flow profile for Example 6.5 by standard step method

b = 6 m, s = 2, S_0 = 0.0025, Q = 30 m^3/s, α = 1.12, n = 0.025, h_c = 1.23 m, h_n = 1.55 m

x	h	z_w	A	P	R	$R^{2/3}$	U	$\frac{\alpha U^2}{2g}$	H	S_f	$\bar{S}_f$	Δx	h_f	H
0	2.500	102.500	27.500	17.180	1.601	1.368	1.091	0.0679	102.5679	0.0003972	-	-	-	102.5679
-100	2.500	102.750	27.500	17.180	1.601	1.368	1.091	0.0679	102.8179	0.0003972	0.0003972	-100	-0.0397	102.6076
	2.279	102.529	24.066	16.193	1.486	1.302	1.246	0.0887	102.6180	0.0005727	0.0004850	-100	-0.0485	102.6164
	2.278	102.528	24.041	16.186	1.485	1.302	1.248	0.0889	102.6165	0.0005743	0.0004858	-100	-0.0486	102.6165
-200	2.278	102.778	24.041	16.186	1.485	1.302	1.248	0.0889	102.8665	0.0005743	0.0005743	-100	-0.0574	102.6739
	2.072	102.572	21.022	15.267	1.377	1.238	1.427	0.1163	102.6885	0.0008309	0.0007026	-100	-0.0703	102.6868
	2.070	102.570	20.995	15.259	1.376	1.237	1.429	0.1166	102.6869	0.0008339	0.0007041	-100	-0.0704	102.6869
-300	2.070	102.820	20.995	15.259	1.376	1.237	1.429	0.1166	102.9369	0.0008339	0.0008339	-100	-0.0834	102.7703
	1.889	102.639	18.473	14.449	1.279	1.178	1.624	0.1505	102.7897	0.0011878	0.0010110	-100	-0.1011	102.7880
	1.887	102.637	18.447	14.440	1.277	1.177	1.626	0.1510	102.7882	0.0011925	0.0010132	-100	-0.1013	102.7882

methods of various orders exist. Two of these methods, namely, the Euler method or the first-order Runge-Kutta method and the modified Euler method or the second-order Runge-Kutta method (Churchouse, 1981) are considered here. In using these methods, the channel is divided into short reaches of known space interval Δx. Starting from the known depth at one end of the channel, the depth at the end of each length step Δx is systematically calculated till the other end of the channel is reached.

We can write the dynamic equation of gradually varied flow as

$$\frac{dh}{dx} = f(x,h) \tag{6.62}$$

where

$$f(x,h) = \frac{S_0 - S_f}{1 - Fr^2} = \frac{S_0 - S_f}{1 - \dfrac{\alpha Q^2 B}{g A^3}} = S_0 \frac{1-(h_n/h)^N}{1-(h_c/h)^M} \tag{6.63}$$

and suppose that the water depth h_j at point x_j is known, i.e.

$$h_j(x_j) = h_j \tag{6.64}$$

The Euler Method

In this method the water depth h_{j+1} at the end of a space step Δx is obtained in a single step, i.e.

$$h_{j+1} = h_j + \Delta x \left(\frac{dh}{dx}\right)_j = h_j + \Delta x\, f(x_j, h_j) \tag{6.65}$$

This method is of first-order accuracy, i.e. accuracy $\approx 0(\Delta x)$. It is very slow and to obtain reasonable accuracy, we need to take a smaller value of Δx.

The Modified Euler Method

In this method, the accuracy of the Euler method is improved by using the slope of the curve at the mid-point of the space interval Δx, i.e. at $x_{j+1/2}$, when $x_{j+1/2} = (x_j + x_{j+1})/2$. To do so, we first determine $h_{j+1/2}$ as

$$h_{j+1/2} = h_j + \frac{\Delta x}{2}\left(\frac{dh}{dx}\right)_j \tag{6.66}$$

and then use the equation

$$h_{j+1} = h_j + \Delta x \left(\frac{dh}{dx}\right)_{j+1/2} = h_j + \Delta x\, f(x_{j+1/2}, h_{j+1/2}) \tag{6.67}$$

to determine the unknown depth h_{j+1}.

This method is of second-order accuracy, i.e. accuracy $\approx 0(\Delta x^2)$. Therefore, it is more accurate than the Euler method.

The Euler and the modified Euler methods are direct methods, i.e. they do not involve any iteration.

Example 6.7

Determine the depth of flow 100 m upstream of the dam of Example 6.5 using the Euler and the modified Euler methods.

Solution Trapezoidal channel, b = 6 m, s = 2, S_0 = 0.0025, Q = 30 m^3/s, α = 1.12, n = 0.025

Euler mothod h_1 = 2.50 m, A_1 = 27.50 m^2, P_1 = 17.18 m, R_1 = 1.601 m, $R_1^{2/3}$ = 1.368, S_{f1} = 0.000397, B_1 = 16 m and Δx = 100 m

$$\left(\frac{dh}{dx}\right)_1 = \frac{S_0 - S_{f1}}{1 - \dfrac{\alpha Q^2 B_1}{g A_1{}^3}} = \frac{0.0025 - 0.000379}{1 - \dfrac{1.12 \times 30^2 \times 16}{9.81 \times 27.50^3}} = 2.283 \times 10^{-3}$$

$$h_2 = h_1 + \Delta x \left(\frac{dh}{dx}\right)_1 = 2.50 + (-100) \times 2.283 \times 10^{-3} = 2.2717\, m$$

Modified Euler Method As in the Euler method, $(dh/dx)_1 = 2.283 \times 10^{-3}$.

$$\therefore h_{1\frac{1}{2}} = h_1 + \frac{\Delta x}{2}\left(\frac{dh}{dx}\right)_1 = 2.50 + 0.5 \times (-100) \times 2.283 \times 10^{-3} = 2.3853\ m$$

For this depth, we obtain A = 25.701 m^2, P = 16.67 m, R = 1.54 m, $R^{2/3}$ = 1.3345, S_f = 0.000478 and B = 15.543 m.

$$\therefore \left(\frac{dh}{dx}\right)_{1\frac{1}{2}} = \frac{S_0 - S_f}{1 - \dfrac{\alpha Q^2 B}{g A^3}} = \frac{0.0025 - 0.000478}{1 - \dfrac{1.12 \times 30^2 \times 15.543}{9.81 \times 25.701^3}} = 2.232 \times 10^{-3}$$

$$\therefore h_2 = h_1 + \Delta x \left(\frac{dh}{dx}\right)_{1\frac{1}{2}} = 2.50 + (-100) \times 2.232 \times 10^{-3} = 2.2768\ m$$

PROBLEMS

6.1 Show that the gradually varied flow equation for flow in a rectangular channel of variable width b may be expressed as

$$\frac{dh}{dx} = \frac{S_0 - S_f + (\alpha Q^2 h / g A^3)(db/dx)}{1 - \alpha Q^2 b / g A^3}$$

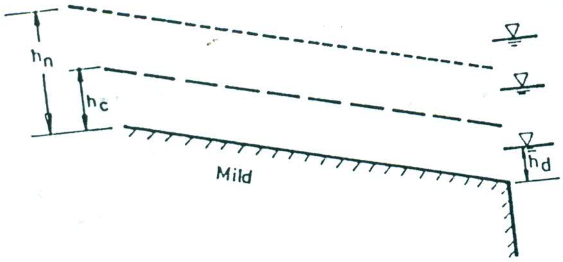
6.2 Prove that the specific energy of the M1, S1, S2 and C1 profiles increases and of the M2, M3, S3, C3, H2, H3, A2 and A3 profiles decreases in the downstream direction.

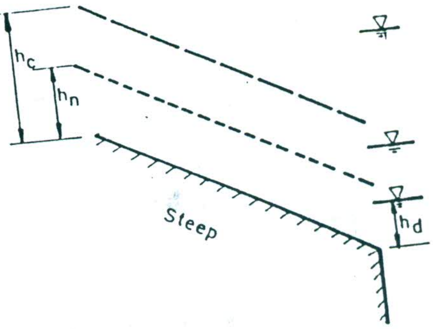
6.3 Sketch the possible flow profiles produced on the upstream and downstream of a sluice gate in a (i) mild, (ii) critical, (iii) steep, (iv) horizontal and (v) adverse slope channels.

6.4 (a) Draw the possible flow profiles when there is a free overfall at the end of a (i) mild, (ii) critical, (iii) steep, (iv) horizontal and (v) adverse slope channels.

(b) There is a free overfall at the end of mild slope channel. Draw all the possible flow profiles for different water levels downstream of the channel.

(c) Same as Prob. 6.4(b), but now the channel is steep.





Problem 6.4(b) Problem 6.4(c)

6.5 Sketch the possible flow profiles in the following combination of slopes:

a) mild-horizontal
b) horizontal-mild
c) steep-horizontal
d) critical-horizontal
e) horizontal-adverse
f) mild-critical
g) adverse-mild
h) critical-steep
i) mild-steeper mild
j) horizontal-critical
k) steep-critical
l) adverse-steep

6.9

6.6 Sketch the possible flow profiles in the following serial arrangement of channels. The flow is from left to right.

a) mild-milder-steep
b) critical-steep-mild
c) steep-mild-milder 3
d) horizontal-mild-critical
e) steep-critical-mild
f) critical-horizontal-steep
g) mild-horizontal-critical-free overfall
h) mild-steep-milder steep-free overfall
i) critical-adverse-horizontal
j) horizontal-adverse-steep-free overfall
k) mild-adverse-horizontal-free overfall 3
l) mild-critical-steep

Mild - milder → M1
Mild - steeper ML → M2
e St - mSl → S3
6.7

6.7 Sketch the possible flow profiles in the channels shown in Fig. 6.9.

6.8(a) Determine the flow profile developed as a result of an increase in surface roughness in a (i) mild slope, and (ii) steep slope channel.

(b) Determine the flow profile developed as a result of a decrease in surface roughness in a (i) mild slope, and (ii) steep slope channel.

6.9 A rectangular channel with b = 6.0 m and n = 0.020 carries a discharge of 24 m^3/s. Identify the flow profiles produced in the channel for the following changes in the bottom slope:

i) $S_0 = 0.0040$ to $S_0 = 0.0090$
ii) $S_0 = 0.0030$ to $S_0 = 0.0050$
iii) $S_0 = 0.0085$ to $S_0 = 0.0000$
iv) $S_0 = 0.0095$ to $S_0 = 0.0075$
v) $S_0 = 0.0000$ to $S_0 = 0.0045$

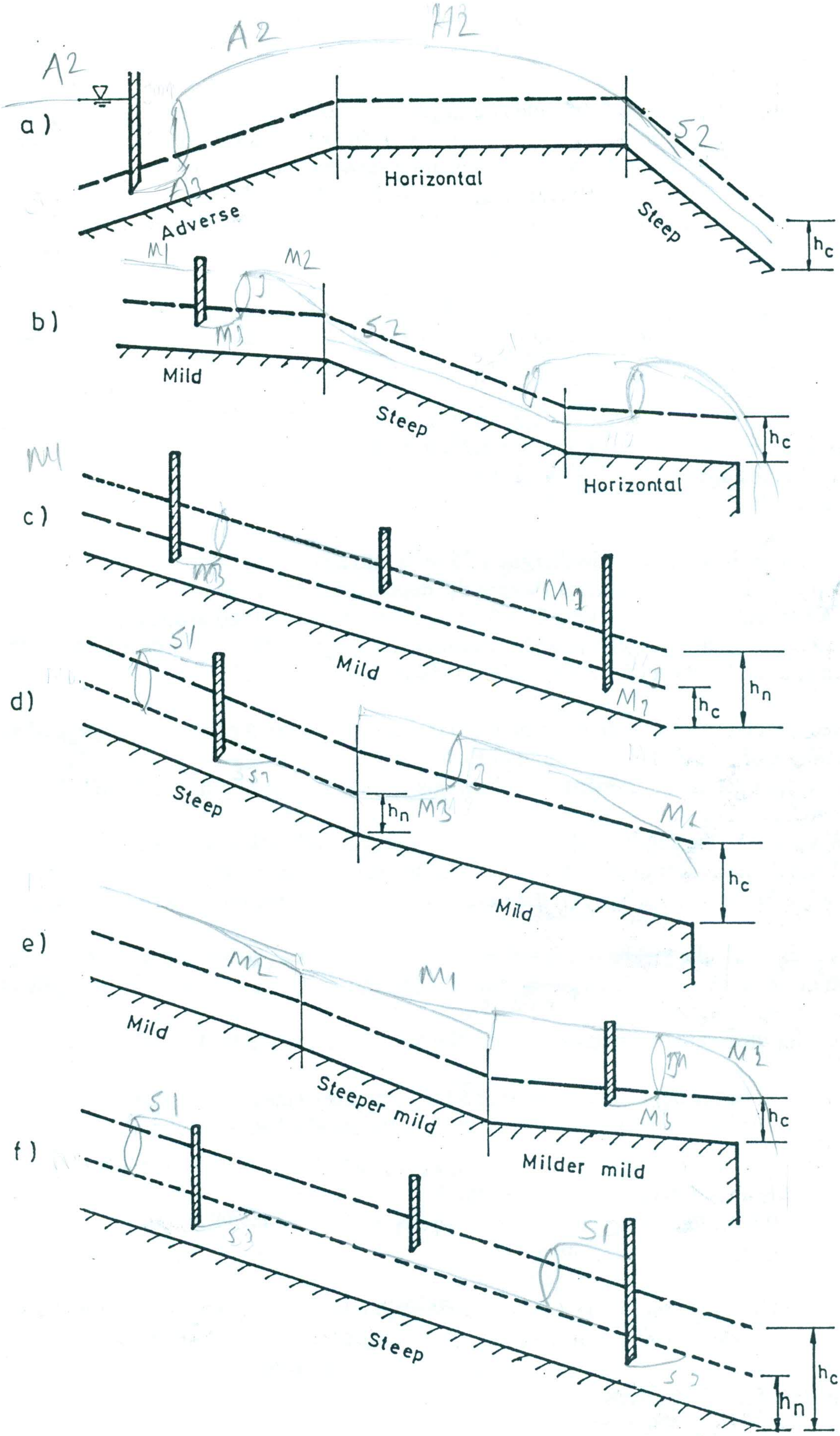


Fig. 6.9 for Problem 6.7

6.10(a) A rectangular channel 6 m wide and having n = 0.025 has three reaches arranged serially. The bottom slopes of the three reaches are 0.0016, 0.0150 and 0.0064, respectively. For a discharge of 20 m^3/s through this channel, sketch the resulting flow profiles.

(b) Same as Problem 6.10(a) except that n = 0.015 for the middle reach.

(c) Same as Problem 6.10(a) except that the Manning roughness coefficient values for the three reaches are 0.020, 0.015 and 0.025, respectively.

6.11(a) A wide rectangular channel with C = 45 $m^{1/2}$/s and S_0 = 0.0001 carries a discharge of 1.8 m^2/s. A weir causes the water level to be raised by 0.50 m above the normal depth. Compute the length of the resulting flow profile between the weir site and the location where the depth is 2.80 m by the Bresse method.

(b) A wide river has an average depth of 5 m, an average slope of 1 in 10,000 and n = 0.025. A dam increases the water depth by 1.0 m. Find out the length of the flow profile created by the dam assuming that the upstream end of the profile is at a depth 10% higher than the average depth. Use the Bresse method.

6.12(a) An overflow spillway discharges 4.25 m^2/s into a horizontal floor. Compute the length of the flow profile between the sections where the depths are 0.35 m and 0.65 m.. Take n = 0.015.

(b) A vertical sluice gate having C_c = 0.61 and gate opening = 0.60 m discharges 27 m^3/s into a horizontal rectangular channel 6 m wide. Compute the length of the flow profile between the vena contracta and the location where the depth is 0.50 m. Take n = 0.013.

6.13 A rectangular channel with b = 6 m, n = 0.025 and S_0 = 0.0025 carries a discharge of 40 m^3/s. At a section A of this channel the depth of flow is 2 m.

(a) How far upstream or downstream from this section will the depth be 2.25 m? Use the direct step method.

(b) What will be the depth at a distance of 50 m upstream of section A? Assume that the elevation of the channel bed at section A is 100.00 m. Use the standard step method.

(c) Solve Problem 6.13(b) by (i) the Euler method, and (ii) the modified Euler method.

6.14 A trapezoidal channel having b = 5 m, s = 2, n = 0.020 and S_0 = 0.002 carries a discharge of 48.67 m^3/s. A dam constructed ~~along~~ across the channel raises the water level to a depth of 5 m immediately upstream of it.

a) How far upstream or downstream from the dam will the depth be 4.90 m? Use the direct step method.

b) What will be the depth at a distance of 50 m upstream of the dam? Assume that the elevation of the channel bed at the dam site is 100.00 m. Use the standard step method.

****

Chapter 7

HYDRAULIC JUMP

7.1 INTRODUCTION

In open channels when a supercritical flow is made to change abruptly to subcritical flow, the result is usually an abrupt rise of the water surface. This feature is known as the hydraulic jump (Fig. 7.1). It usually results when two controls, one from upstream and the other from downstream, operate on the same reach of a channel. The upstream control produces supercritical flow downstream of itself and the downstream control produces subcritical flow upstream of itself and the result is the formation of a hydraulic jump. A hydraulic jump may occur at the foot of a spillway or behind a sluice gate in a mild or horizontal or adverse slope channel, or when a steep slope channel is followed by a mild or horizontal or adverse slope channel.

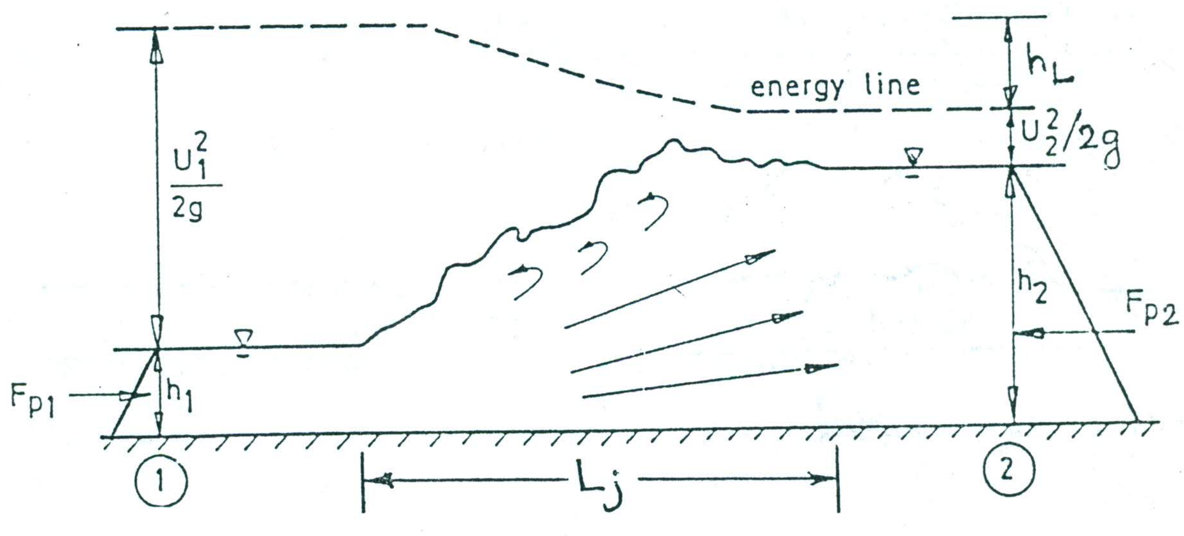


Fig. 7.1 Hydraulic jump

The depth of flow before the jump is known as the *initial depth* and that after the jump as the *conjugate* or *sequent depth*. The strength of a hydraulic jump is determined by the Froude number of the flow before the jump. The hydraulic jump is accompanied by considerable turbulence and energy dissipation.

use of hydraulic jump

Practical applications of the hydraulic jump in the field of open channel flow include (i) the dissipation of kinetic energy in high-velocity flows over weirs, spillways, gates and other hydraulic structures to prevent scouring downstream, (ii) the increase of depth of water in channels for irrigation and water distribution purposes, (iii) the increase of the discharge of a sluice by repelling the tailwater so that it works under free-flow condition, (iv) the reduction of uplift pressure under a structure by increasing weight on its apron, (v) the mixing of chemicals for water purification or wastewater treatment, (vi) the aeration of flows for city water supplies, and (vii) the removal of air pockets from water supply lines.

7.2 JUMPS IN HORIZONTAL RECTANGULAR CHANNELS

Types of Jumps

The hydraulic jumps in horizontal rectangular channels have been most extensively studied and are known as the *classical jumps*. Based on the results of extensive studies, the United States Bureau of Reclamation (USBR) classified the hydraulic jumps in horizontal rectangular channels into the following five categories (Fig.7.2) according to the Froude number Fr_1 of the incoming flow:

1. For $1 < Fr_1 < 1.7$, the water surface shows undulations and the change from initial to sequent depth is small and gradual. This jump is called an *undular jump*. $E_L/E_1 = 0$ (Practically)

2. For $1.7 < Fr_1 < 2.5$, a series of small rollers appear on the jump surface, but the downstream water surface remains smooth. This type of jump is known as a *weak jump*. $F_r = 1.7$, $E_L/E_1 = 5$; $= 2.5$, $E_L/E_1 = 18$

3. For $2.5 < Fr_1 < 4.5$, the incoming jet oscillates between the bed and the bottom of the surface roller. Each oscillation produces a large surface wave of irregular period which may persist for a considerable distance downstream causing unlimited damage to earth banks and riprap. This jump is known as an *oscillating jump*. $E_L/E_1 = 45$ at $F_r = 4.5$

4. For $4.5 < Fr_1 < 9.0$, a steady jump with appreciable energy dissipation and fairly smooth water surface downstream is formed. The action and position of the jump is least sensitive to the tailwater fluctuation. This type of jump is known as a *steady jump*. E_L/E_1 (45 – 70)

5. For $Fr_1 > 9.0$, the jump surface and the water surface downstream become very rough and the high-velocity jet generates waves downstream. The jump is effective since the energy dissipation is high. This jump is known as a *strong jump*.

$E_L/E_1 > 70$.

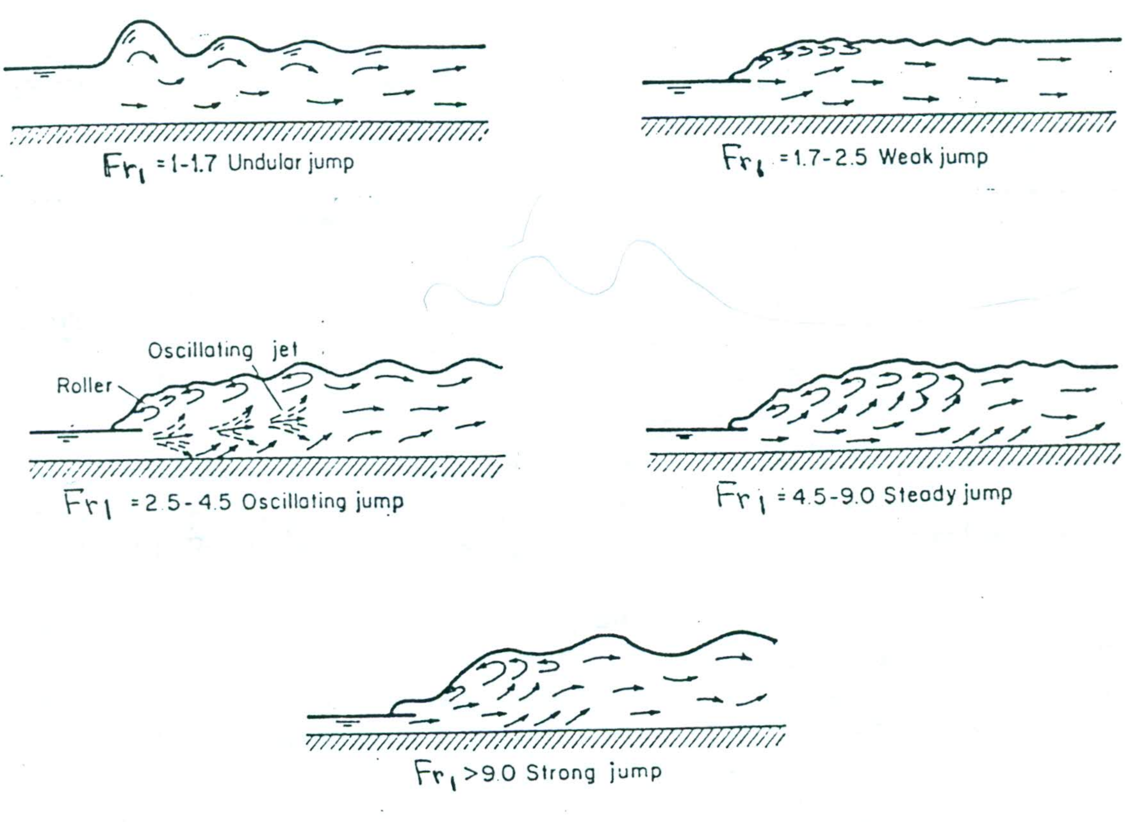


Fig. 7.2 Hydraulic jumps in horizontal rectangular channels

Sequent Depth

Since a hydraulic jump is accompanied by considerable energy dissipation, the energy principle cannot be used initially for its analysis. However, as the jump takes place in a short distance so that the external friction force F_f can generally be neglected, the computation of hydraulic jumps always begins with the momentum equation

$$\rho Q(\beta_2 U_2 - \beta_1 U_1) = F_{p1} - F_{p2} + W \sin\theta - F_f \tag{7.1}$$

Consider a hydraulic jump occurring in a horizontal ($\theta = 0$) rectangular channel. Since the jump takes place in a short reach of the channel, $F_f \approx 0$, and since the channel is prismatic, $\beta_1 = \beta_2 = 1$. The hydrostatic forces F_{P1} and F_{P2} may be expressed as

$$F_{p1} = \gamma \bar{z}_1 A_1 \tag{7.2}$$

and

$$F_{p2} = \gamma \bar{z}_2 A_2 \tag{7.3}$$

where $\bar{z}_1$ and $\bar{z}_2$ are the vertical distances of the centroids of the respective water areas A_1 and A_2 from the free surface. Since $U_1 = Q/A_1$ and $U_2 = Q/A_2$, the momentum equation, Eq.(7.1), may then be expressed as

$$\frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2 \tag{7.4}$$

or

$$F_1 = F_2 \tag{7.5}$$

where

$$F = \frac{Q^2}{gA} + \bar{z}A \tag{7.6}$$

and is known as the *specific force* or *specific momentum*. The term Q^2/gA in Eq. (7.6) represents the momentum of flow passing through the channel section per unit time per unit weight of water and the term $\bar{z}A$ represents the pressure force per unit weight of water. Thus, the specific force represents the force per unit weight of water.

Equation (7.5) indicates that the specific forces before and after a hydraulic jump in a horizontal channel are equal.

Since for a rectangular channel $A_1 = bh_1$, $A_2 = bh_2$, $\bar{z}_1 = h_1/2$ and $\bar{z}_2 = h_2/2$, Eq.(7.4) gives

$$\frac{Q^2}{gb^2} = \frac{h_1 h_2}{2}(h_1 + h_2) \tag{7.7}$$

Since $Q = A_1 U_1 = bh_1 U_1$, Eq. (7.7) may be expressed as

$$\frac{U_1^2}{gh_1} = Fr_1^2 = \frac{1}{2}\frac{h_2}{h_1}\left(\frac{h_2}{h_1} + 1\right) \tag{7.8}$$

and since $Q = A_2U_2 = bh_2U_2$, Eq. (7.7) may also be expressed as

$$\frac{U_2^{\ 2}}{gh_2} = Fr_2^{\ 2} = \frac{1}{2}\frac{h_1}{h_2}\left(\frac{h_1}{h_2}+1\right) \tag{7.9}$$

where Fr_1 $(= U_1/\sqrt{gh_1})$ and Fr_2 $(= U_2/\sqrt{gh_2})$ are the Froude numbers of the flow before and after the jump. Equations (7.8) and (7.9) may be solved to yield

$$\frac{h_2}{h_1} = \frac{1}{2}(\sqrt{1+8Fr_1^{\ 2}} - 1) \tag{7.10}$$

and

$$\frac{h_1}{h_2} = \frac{1}{2}(\sqrt{1+8Fr_2^{\ 2}} - 1) \tag{7.11}$$

respectively.

Equations (7.10) and (7.11) each contains three independent variables and two of them must be known before the third may be computed. Normally the upstream conditions, i.e. h_1 and Fr_1 will be known and the downstream depth h_2 can be determined using Eq. (7.10). It must be realised, however, that the downstream depth h_2 is the result of a control acting further downstream. If the downstream control produces the required depth h_2, then a jump will form.

The depth produced by a downstream control is called the *tailwater depth* h_t. When a jump is formed, $h_2 = h_t$. If the tailwater depth is increased, the jump moves upstream and if the tailwater depth is decreased, the jump moves downstream.

Length of Jump

The length of a hydraulic jump L_j (Fig. 7.1) is the horizontal distance from the front face of the jump to a point immediately downstream from the roller. It is an important design parameter, but it cannot be determined theoretically. For jumps occurring in horizontal rectangular channels, the length of the jump is preferably estimated from the Bradley and Paterka (1957) curve which is a plot of Fr_1 vs. L_j/h_2 (Fig.7.3). This curve has a fairly horizontal portion where $L_j/h_2 \approx 6.0$ to 6.1 in the range of the Froude numbers ($Fr_1 \approx 5$ to 13) yielding the best performance. Silvester (1964) demonstrated that for free hydraulic jumps in horizontal rectangular channels

$$\frac{L_j}{h_1} = 9.75(Fr_1 - 1)^{1.01} \tag{7.12}$$

$L/h_1 = 9.75\,(F_1-1)^{1.01}$

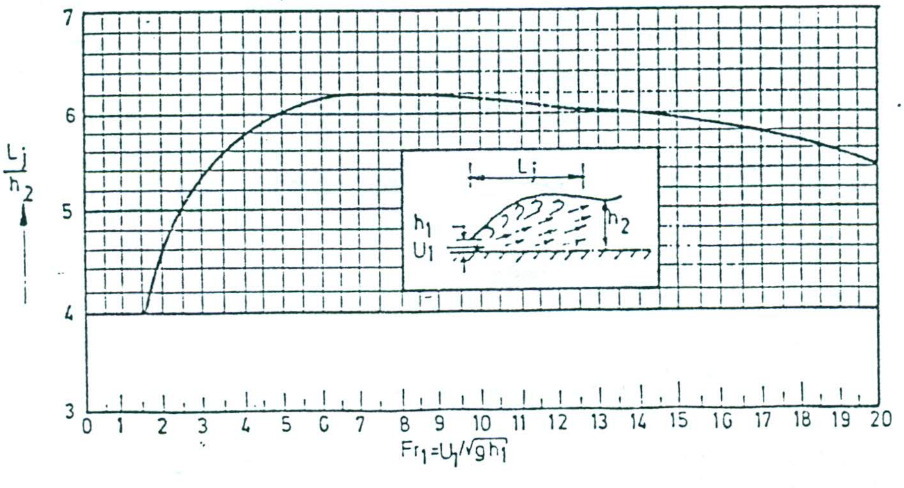


Fig.7.3 Length of hydraulic jump in horizontal rectangular channels

This functional relationship is probably the best because the resulting curve can best be defined by the experimental data.

Location of Jump

The location of a hydraulic jump in a channel can be determined by applying the flow profile equation, Eq.(6.8), and the hydraulic jump equation, Eq.(7.10), together with the length of the jump L_j. Let us consider, for example, a hydraulic jump downstream of a sluice gate in a horizontal rectangular channel. as shown in Fig 7.4. Equation (6.8) may be used to compute the flow profile AB (which is a H3 profile) and the flow profile CD (which is a H2 profile). Now, (i) assume a value of h_1 at point P on AB. (ii) Compute the corresponding sequent depth h_2 by Eq.(7.10). (iii) Find the corresponding point Q on CD at which the depth is h_2. (iv) Measure the horizontal distance between P and Q. (v) Determine the length of the jump L_j corresponding to h_1 and Fr_1 by Eq.(7.12) or Fig. 7.3. (vi) If the horizontal distance between P and Q is equal to L_j, the jump forms at P and ends at Q. (vii) If not, then assume another value of h_1 and repeat steps (i) to (vi) till the jump is located. In this way, the jump may be located in any channel by a trial-and-error procedure as mentioned.

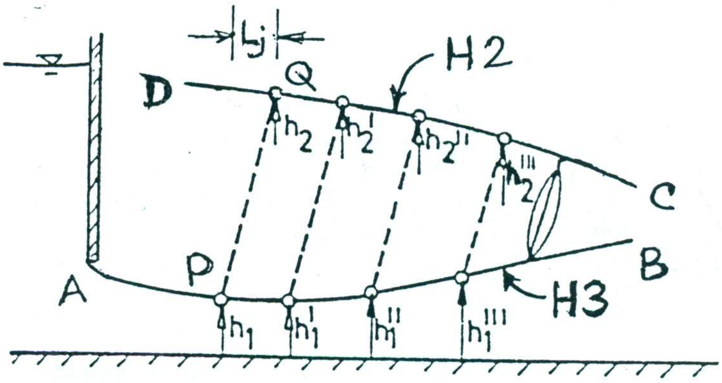


Fig. 7.4 Location of a hydraulic jump

Basic Characteristics

Energy loss: The energy loss involved in a hydraulic jump is the difference between the total energies immediately before and after the jump. It can be determined by applying the energy equation before and after the jump, i.e.

$$h_L = H_1 - H_2 = (z_{b1} + h_1 + \alpha_1 \frac{U_1^2}{2g}) - (z_{b2} + h_2 + \alpha_2 \frac{U_2^2}{2g})$$

For a jump on a horizontal channel ($z_{b1} = z_{b2}$), the energy loss h_L is given by (assuming $\alpha_1 = \alpha_2 = 1$)

$$h_L = H_1 - H_2 = (h_1 + \frac{U_1^2}{2g}) - (h_2 + \frac{U_2^2}{2g}) = E_1 - E_2 \tag{7.13}$$

where E_1 and E_2 are the specific energies before and after the jump. Since $U_1 = Q/A_1$, $U_2 = Q/A_2$, $A_1 = bh_1$ and $A_2 = bh_2$, Eq.(7.13) may be expressed as

$$h_L = (h_1 - h_2) + \frac{Q^2}{2gb^2} \frac{(h_1 + h_2)(h_2 - h_1)}{h_1^2 h_2^2} \tag{7.14}$$

Then, combining Eqs.(7.7) and (7.14), it can be shown that for a hydraulic jump occurring in a horizontal rectangular channel

$$h_L = \frac{(h_2 - h_1)^3}{4h_1h_2} \tag{7.15}$$

The ratio h_L/E_1 (= $1 - E_2/E_1$) is known as the *relative loss.*

Efficiency: The ratio of the specific energy after the jump to that before the jump, E_2/E_1, is known as the efficiency of the jump. It can be shown that

$$\frac{E_1}{h_1} = \frac{h_1 + U_1^2/2g}{h_1} = \frac{1}{2}(2 + Fr_1^2) \tag{7.16}$$

and writing $U_2 = U_1h_1/h_2$ and using Eq.(7.10), it can be shown that

$$\frac{E_2}{h_1} = \frac{h_2 + U_2^2/2g}{h_1} = \frac{1}{2}\left[\frac{(1+8Fr_1^2)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2}\right] \tag{7.17}$$

Now, combining Eqs.(7.16) and (7.17), the efficiency E_2/E_1 is given by

$$\frac{E_2}{E_1} = \frac{(1+8Fr_1^2)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2(2+Fr_1^2)} \tag{7.18}$$

Height of jump: The height of the jump is the difference between the sequent and initial depths, i.e. $h_j = h_2 - h_1$. The ratio h_j/E_1 is known as the *relative height*. It can be shown that

$$\frac{h_j}{h_1} = \frac{1}{2}(\sqrt{1+8Fr_1^2} - 3) \tag{7.19}$$

Then, combination of Eqs.(7.16) and (7.19) gives

$$\frac{h_j}{E_1} = \frac{\sqrt{1+8Fr_1^2} - 3}{2+Fr_1^2} \tag{7.20}$$

Submerged Jumps

A hydraulic jump is formed when the initial and sequent depths satisfy Eq.(7.10). If the downstream depth is increased, the jump moves upstream . Thus, by increasing the downstream depth a jump can be moved upstream. Now if there is an upstream control like a sluice gate (Fig.7.5), the jump cannot move upstream and a submerged jump is formed. The submerged jumps usually form downstream of gates in irrigation systems and the main unknown quantity in this case is the submerged depth h_3.

Applying the momentum principle and the equation of continuity, it can be shown that

$$\frac{h_3}{h_t} = \left[1 + 2Fr_t^2\left(1 - \frac{h_t}{h_g}\right)\right]^{1/2} \tag{7.21}$$

where h_g is the height of the sluice gate opening, h_t is the tailwater depth, h_2 is the subcritical sequent depth of the free jump corresponding to h_g and Fr_t is the Froude number corresponding to h_t.

The length of a submerged hydraulic jump in a horizontal rectangular channel is estimated by the empirical equation

$$L_j / h_2 = 4.9S + 6.1 \tag{7.22}$$

where S is the *submergence factor* given by

$$S = \frac{h_t - h_2}{h_2} \tag{7.23}$$

Equation (7.22) demonstrates that the length of a submerged jump exceeds the length of a free jump by the amount 4.9S.

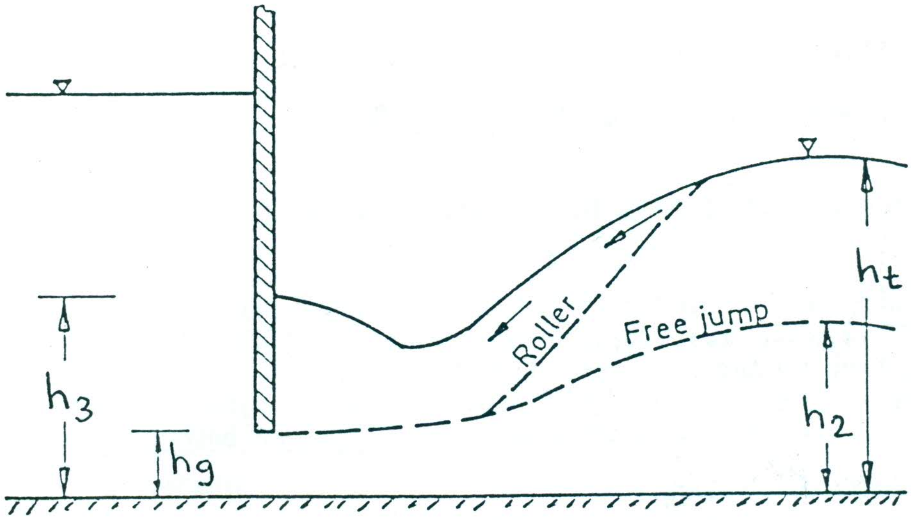


Fig. 7.5 A submerged jump downstream of a sluice gate

Example 7.1

Water flows in a horizontal rectangular channel 6 m wide at a depth of 0.52 m and a velocity of 15.2 m/s. If a hydraulic jump forms in this channel, determine (i) the type of jump, (ii) the downstream depth needed to form the jump, (iii) the horse-power dissipation in the jump, (iv) the efficiency of the jump, (v) the relative height of the jump, and (vi) the length of the jump.

Solution Horizontal rectangular channel, b = 6 m, h_1 = 0.52 m, U_1 = 15.2 m/s
$Q = A_1U_1 = bh_1U_1 = 6\times0.52\times15.2 = 47.42\ m^3/s$

(i) $$Fr_1 = \frac{U_1}{\sqrt{gh_1}} = \frac{15.2}{\sqrt{9.81\times0.52}} = 6.73$$

Hence, the jump is a steady jump.

(ii) $$\frac{h_2}{h_1} = \frac{1}{2}(\sqrt{1+8Fr_1^{\ 2}} - 1) = \frac{1}{2}(\sqrt{1+8\times6.73^2} - 1) = 9.03$$

$$\therefore h_2 = 9.03\times0.52 = 4.70\ m$$

(iii) $$h_L = \frac{(h_2-h_1)^3}{4h_1h_2} = \frac{(4.70-0.52)^3}{4\times4.70\times0.52} = 7.47\ m - \cancel{kg/kg}$$

$$\therefore \textit{Horse power dissipation} = \frac{7.22\rho Q h_L}{550} \quad (\because 1\ \text{m-kg/s}=7.22\ \text{lb-ft/sec, 1 horse power}=550\ \text{lb-ft/sec})$$

$$= \frac{7.22\times1000\times47.42\times7.47}{550} = 4650.4$$

$\gamma Q h$
= kg/m³ 1000×9.81 N/m³
= ~~kgf~~ × 47.42 m³/s × 7.43 m

(iv) $\quad Efficiency, \dfrac{E_2}{E_1} = \dfrac{(1+8Fr_1^{\,2})^{3/2} - 4Fr_1^{\,2} + 1}{8Fr_1^{\,2}(2+Fr_1^{\,2})}$

$$= \frac{(1+8\times 6.73^2)^{3/2} - 4\times 6.73^2 + 1}{8\times 6.73^2 \times (2+6.73^2)} = 0.3937 = 39.37\%$$

(v) $Relative\, height\, of\ jump, \dfrac{h_j}{E_1} = \dfrac{\sqrt{1+8Fr_1^{\,2}} - 3}{2+Fr_1^{\,2}} = \dfrac{\sqrt{1+8\times 6.73^2} - 3}{2+6.73^2} = 0.3396 = 33.96\%$

(vi) Length of jump (using Fig. 7.3)

$$L_j = 6.1h_2 = 6.1\times 4.70 = 28.67 \text{ m}$$

or by the Silvester formula (Eq. 7.12)

$$L_j = 9.75h_1(Fr_1\text{-}1)^{1.01} = 9.75\times 0.52\times (6.73\text{-}1)^{1.01} = 29.56 \text{ m}$$

7.3 JUMPS IN HORIZONTAL NON-RECTANGULAR CHANNELS

Sequent Depth

For hydraulic jumps occurring in horizontal non-rectangular channels, there are no equations analogous to Eqs.(7.10) and (7.11) to compute the sequent depth. For these jumps the sequent depth can be determined from a trial-and- error solution of Eq.(7.5).

The trial-and-error solution of Eq. (7.5) requires the determination of $\bar{z}$ of the flow section. The expression for $\bar{z}$ of different channel sections derived from the basic principles are given in Table 7.1.

Table 7.1 Expression for $\bar{z}$ of different channel sections

Section	$\bar{z}$
1. Rectangle	$h/2$
2. Triangle	$h/3$
3. Trapezoid	$\dfrac{h}{6}\left(\dfrac{3b+2sh}{b+sh}\right)$
4. Parabola	$2h/5$
5. Circle	$\dfrac{2(d_0h-h^2)^{3/2}}{3A} - \dfrac{d_0}{2} + h$

Example 7.2

A horizontal trapezoidal channel with b = 6 m and s = 2 carries a discharge of 120 m^3/s. If the upstream depth of flow is 1 m, compute the downstream depth that will create a hydraulic jump.

Solution Trapezoidal channel, b = 6 m, s = 2, Q = 120 m^3/s, h_1 = 1 m

At the upstream section

$A_1 = (b + sh_1)h_1 = (6 + 2\times 1)\times 1 = 8 \text{ m}^2$

$B_1 = b + 2sh_1 = 6 + 2\times 2\times 1 = 10 \text{ m}$

$D_1 = A_1/B_1 = 8/10 = 0.8 \text{ m}$

$U_1 = Q/A_1 = 120/8 = 15 \text{ m/s}$

$$Fr_1 = U_1/\sqrt{gD_1} = 15/\sqrt{9.81\times 0.8} = 5.35$$

$$\bar{z}_1 = \frac{h_1}{6}\left(\frac{3b+2sh_1}{b+sh_1}\right) = \frac{1}{6}\left(\frac{3\times 6+2\times 2\times 1}{6+2\times 1}\right) = 0.458\ m$$

$$F_1 = \frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{120^2}{9.81\times 8} + 0.458\times 8 = 187.15$$

At the downstream section

$$F_2 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2$$

The condition which must be satisfied to cause a hydraulic jump between sections 1 and 2 is

$$F_1 = F_2$$

The value of h_2 which satisfies this condition is determined by trial-and-error as follows.

h_2	A_2	$\bar{z}_2$	F_2
3.00	36.00	1.250	85.77
4.00	56.00	1.619	116.88
5.00	80.00	1.979	176.68
5.10	82.62	2.015	184.23
5.20	85.28	2.050	192.07
5.13	83.41	2.025	186.54
5.14	83.68	2.029	187.33

Hence, the downstream depth required to produce a hydraulic jump, $h_2 = 5.14$ m.

Energy Loss

For computing energy loss involved in a hydraulic jump in horizontal non-rectangular channels, there are no equations analogous to Eqs.(7.14) and (7.15). In such cases, the general energy equation, Eq.(7.14), is to be solved on a case-by-case basis.

Example 7.3

Compute the relative energy loss that will occur if there is a hydraulic jump in the channel considered in Example 7.2.

Solution From Example 7.2, $h_1 = 1$ m, $U_1 = 15$ m/s, $Fr_1 = 5.35$ and if there is a hydraulic jump, then $h_2 = 5.14$ m and $A_2 = 83.68$ m^2.

$\therefore$ $U_2 = Q/A_2 = 120/83.68 = 1.434$ m/s

$$E_1 = h_1 + \frac{U_1^2}{2g} = 1 + \frac{15^2}{2\times 9.81} = 12.47\ m$$

$$E_2 = h_2 + \frac{U_2^2}{2g} = 5.14 + \frac{1.434^2}{2\times 9.81} = 5.24\ m$$

$$\therefore \text{Relative energy loss}, \frac{h_L}{E_1} = \frac{E_1 - E_2}{E_1} = \frac{12.47 - 5.24}{12.47} = 0.5798 = 57.98\%$$

7.4 JUMPS IN SLOPING CHANNELS

In the analysis of hydraulic jumps in sloping channels, it is essential to consider the weight of water in the jump. In horizontal channels the effect of this weight in negligible.

Hydraulic jumps in sloping channels may occur in various forms as shown in Fig. 7.6. Let h_t be the tailwater depth (i.e. the depth produced by the downstream control), h_1 be the supercritical depth of flow on the slope which is assumed to be constant, h_2 is the sequent depth corresponding to h_1 when the jump occurs on horizontal channel given by Eq.(7.10) and h_2^* is the sequent depth when the jump occurs on a sloping channel. Then,

i) if $h_t \leq h_2$, the jump forms on the horizontal bed and type A jump occurs,

ii) if $h_2^* > h_t > h_2$, the toe of the jump is on the slope and the end on the horizontal bed and a type B jump forms,

iii) if $h_2^* = h_t > h_2$, the end of the jump coincides with the intersection of the sloping and the horizontal beds and a type C jump occurs, and

iv) if $h_t > h_2^* > h_2$, a type D jump occurs completely on the sloping section.

The type E jumps occur on sloping beds which have no break in slope. The rare type F jump forms in adverse slope channels and is normally found in stilling basins below drop structures.

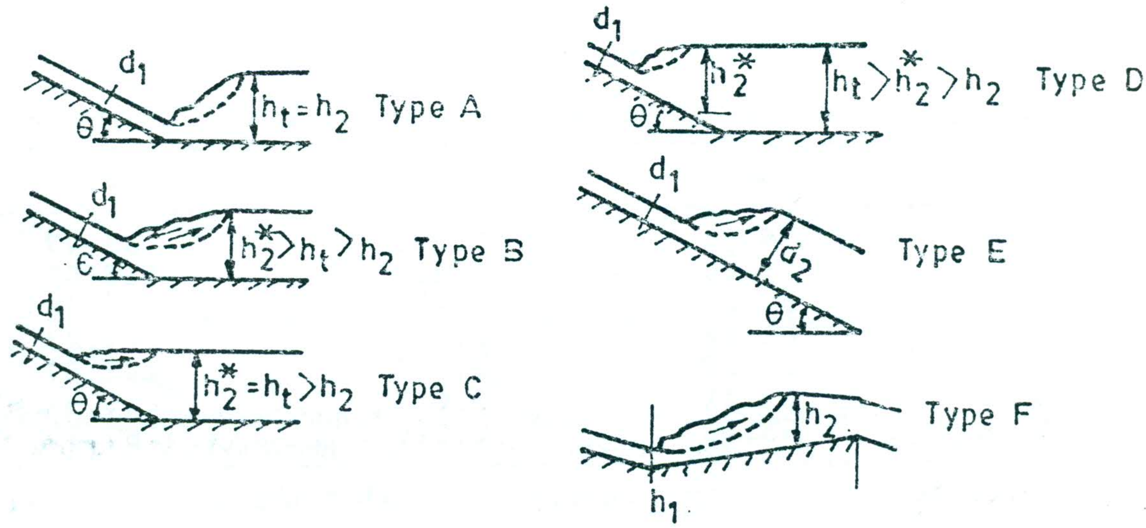


Fig. 7.6 Types of jumps that occur in sloping channels

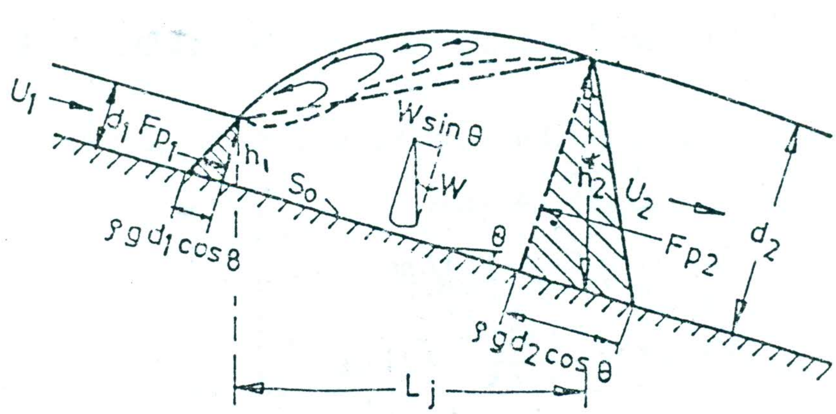


Fig. 7.7 Forces acting on a type E jump

The forces which act on a type E jump in a rectangular channel are shown in Fig. 7.7. Considering unit width of the channel and all forces parallel to the channel bottom, the momentum equation may be written as

$$\rho q(\beta_2 U_2 - \beta_1 U_1) = F_{p1} - F_{p2} + W\sin\theta - F_f \qquad (7.24)$$

Now, $q = U_1 d_1$, $U_2 = U_1 d_1/d_2$, $F_{p1} = 0.5\rho g d_1^2 \cos\theta$, $F_{p2} = 0.5\rho g d_2^2 \cos\theta$, $d_1 = h_1 \cos\theta$, $d_2 = h_2^* \cos\theta$, the friction force F_f is negligible and β_1 and β_2 may be taken as unity. If the jump profile is a straight line, W can be computed easily. The difference between the straight line and the actual profile and the effect of slope may be corrected by a factor Γ. Thus,

$$W = 0.5\Gamma \rho g L_j (d_1 + d_2) \tag{7.25}$$

Substituting Eq.(7.25) in Eq.(7.24) and simplifying, it can be shown that

$$\frac{h_2^*}{h_1} = \frac{d_2}{d_1} = \frac{1}{2}(\sqrt{1+8G^2} - 1) \tag{7.26}$$

where

$$G = \frac{Fr_1}{\sqrt{\cos\theta - \dfrac{\Gamma L_j \sin\theta}{d_2 - d_1}}} \tag{7.27}$$

and

$$Fr_1 = U_1/\sqrt{gd_1} \tag{7.28}$$

One may expect Γ and L_j to be functions of Fr_1 and θ. Hence, G, h_2^*/h_1 and d_2/d_1 are functions of Fr_1 and θ. The term G can be computed using the empirical relationship (Rajaratnam, 1967)

$$G^2 = k_1^2 \, Fr_1^2 \tag{7.29}$$

where

$$k_1 = 10^{0.027\theta} \tag{7.30}$$

and θ is in degrees.

The relative length of jump L_j/h_t as a function of Fr_1 and S_o are presented in Fig 7.8.

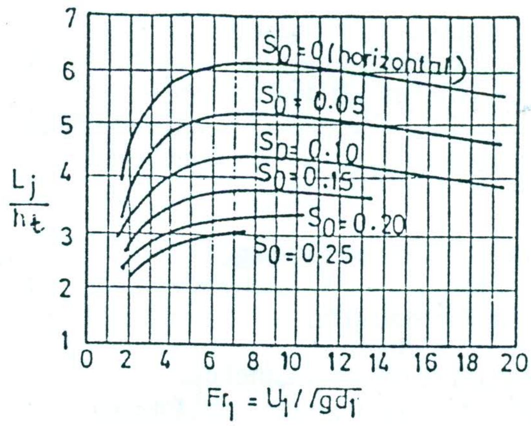


Fig. 7.8 Length of jumps in sloping channels

The solutions for type E jump are believed to be applicable for type C and D jumps. The solutions for type B jumps can be found in French (1986).

Example 7.4

A rectangular channel is 1 m wide and inclined at an angle of 3.5^0 with the horizontal. Determine the type of jump when the discharge is 0.15 m^3/s, the initial depth of flow section (d_1) is 0.02 m and the tailwater depth is 0.70 m. Also compute the energy loss in the jump if the length of the jump is 2 m.

Solution Rectangular channel, $b = 1$ m, $\theta = 3.5^0$, $Q = 0.15$ m^3/s, $d_1 = 0.02$ m, $h_t = 0.70$ m, $L_j = 2$ m

$h_1 = d_1/\cos\theta = 0.02/\cos 3.5^0 = 0.02$ m

$A_1 = bd_1 = 1\times 0.02 = 0.02$ m^2

$U_1 = Q/A_1 = 0.15/0.02 = 7.5$ m/s

$Fr_1 = U_1/\sqrt{gd_1} = 7.5/\sqrt{9.81\times 0.02} = 16.93$

$$h_2 = \frac{h_1}{2}(\sqrt{1+8Fr_1^2}-1) = \frac{0.02}{2}(\sqrt{1+8\times 16.93^2}-1) = 0.467\ m$$

Since $h_t > h_2$, the jump occurs on the sloping channel.

$k_1 = 10^{0.027x3.5} = 1.243$

$G^2 = k_1^2 Fr_1^2 = 442.9$

$$h_2^* = \frac{h_1}{2}(\sqrt{1+8G^2}-1) = \frac{0.02}{2}(\sqrt{1+8\times 442.9}-1) = 0.585\ m$$

Since $h_t > h_2^* > h_2$, the jump occurs entirely on the sloping channel and a D type jump occurs.
Now,

$d_2 = h_2^*\cos\theta = 0.585\times\cos 3.5^0 = 0.584$ m

$A_2 = bd_2 = 1\times 0.584 = 0.584$ m^2

$U_2 = Q/A_2 = 0.15/0.584 = 0.257$ m/s

Hence, applying the energy equation between section 1 before the jump and section 2 after the jump, one obtains

$$z_{b1} + d_1\cos\theta + \frac{U_1^2}{2g} = z_{b2} + d_2\cos\theta + \frac{U_2^2}{2g} + h_L$$

or, $$(z_{b1} - z_{b2}) + d_1\cos\theta + \frac{U_1^2}{2g} = d_2\cos\theta + \frac{U_2^2}{2g} + h_L$$

or, $$L_j\tan\theta + d_1\cos\theta + \frac{U_1^2}{2g} = d_2\cos\theta + \frac{U_2^2}{2g} + h_L$$

or, $$2\times\tan 3.5^0 + 0.02\times\cos 3.5^0 + \frac{7.5^2}{2\times 9.81} = 0.584\times\cos 3.5^0 + \frac{0.257^2}{2\times 9.81} + h_L$$

or, $0.122 + 0.020 + 2.867 = 0.583 + 0.003 + h_L$

or, $h_L = 3.009 - 0.586 = 2.423$ m-kg/kg

7.5 STILLING BASINS

A stilling basin is a short length of paved channel placed at the end of a spillway or any other source of supercritical flow to which a hydraulic jump used for energy dissipation is confined partly or entirely. The aim of the designer is to make a hydraulic jump form within the basin so that flow is converted to subcritical before it reaches the exposed earthen bed downstream, to promote the formation of the jump, to make it stable in one position and to make it as short as possible.

Because of the widespread use of stilling basins, a number of generalized designs have been developed based on model studies, experience and observation of existing stilling basins (Figs. 7.9 and 7.10). The basins thus designed are usually provided with special appurtenances which include chute blocks, sills and baffle piers.

Chute Blocks

The chute blocks are used to form a serrated device at the entrance to the basin. Their function is to furrow the incoming jet and lift a portion of it from the floor. The purpose of the blocks is to shorten the length of the jump, stabilize it and improve its performance.

Sill, Dentated or Solid

The sill, dentated or solid, is usually provided at the end of the basin. The function of this appurtenance is to further reduce the length of the jump and to control scour downstream. In large stilling basins, the sill is usually dentated (also known as the Rehbock sill) to aid in the diffusion of the high velocity jet that may reach the end of the basin.

Baffle Piers or Floor Blocks

Baffle piers are placed at intermediate locations in the basin. Their primary function is to dissipate energy mostly by impact. When the approach velocities are low, baffle piers may be very effective. However, when incoming velocities are high, this type of appurtenance is unsuitable because of the possibility of cavitation. This is why the baffle piers are normally used when the incoming velocity is less than about 16 m/s (50 ft/sec).

Of the many generalized designs of stilling basin, four typical designs are considered below.

USBR Basin II

The USBR Basin II (Fig. 7.9) is recommended for use on large structures e.g. large high-dam and earth-dam spillways, large canal structures etc. when $Fr_1 > 4.5$ and the incoming velocity U_1 exceeds about 18 m/s (60 ft/sec). The length of the basin is reduced by about 33 percent with the provision of chute blocks at the upstream end and a dentated sill at the downstream. This design may be safe and conservative for spillways with fall up to 60 m (200 ft) and for flows up to 46 m^3/s per m of basin width.

The elevation of the basin floor is set to utilize full tailwater depth plus an added factor of safety. The dotted lines in Fig. 7.9(b) are guides based on various ratios of the actual tailwater depth to sequent depth. There is a lower limit of this ratio which is determined by the curve labeled "minimum TW depth". The basin should never be designed for less than the sequent depth and a minimum safety factor of 5% should be added to the sequent depth.

The length of the basin is obtained from Fig.7.9(c). The height, width and spacing of the chute blocks is equal to h_1. A space equal to $0.5h_1$ is provided along each basin wall. The height of the dentated sill is equal to $0.2h_2$ and the width and the spacing of the dentates is approximately $0.15h_2$.

USBR Basin III

USBR Basin III (Fig. 7.10a) is similar to USBR Basin II , but it is used when the incoming velocity U_1 is less than 18 m/s (60 ft/sec). The major difference between the designs of Basin II and Basin III is that in the latter lower velocities allow the installation of baffle piers downstream of the chute blocks. The added resistance offered by the piers allows the use of a shorter basin. With the help of the appurtenances the basin length can be reduced about 60%.

USBR Basin IV

USBR Basin IV (Fig. 7.10b) is designed in conjunction with canal and diversion structures for the special purpose of suppressing the waves at their source generated in oscillating jumps when $2.5 < Fr_1 < 4.5$. This is achieved by intensifying the rollers which appear in the upper portion of the jump. The appurtenances used include a few chute blocks at the entrance and a solid sill at the end.

Saint Anthony Falls (SAF) Stilling Basin

The SAF Basin (Fig. 7.10c), developed at the Saint Anthony Falls Hydraulic Laboratory, University of Minnesota, for the U. S. Soil Conservation Service, is usually used in conjunction with small spillways outlet works and canal structures. It is intended for much the same use as the USBR Basin III, but it is designed for a greater range of upstream Froude numbers, viz. $1.7 < Fr_1 < 17$. The reduction in basin length achieved through the use of appurtenances (chute blocks, baffle piers and a solid end sill) ranges from 70% to 90%. Thus, the SAF Basin is shorter and more economical but, in consequence, has a lower factor of safety than the USBR Basin III.

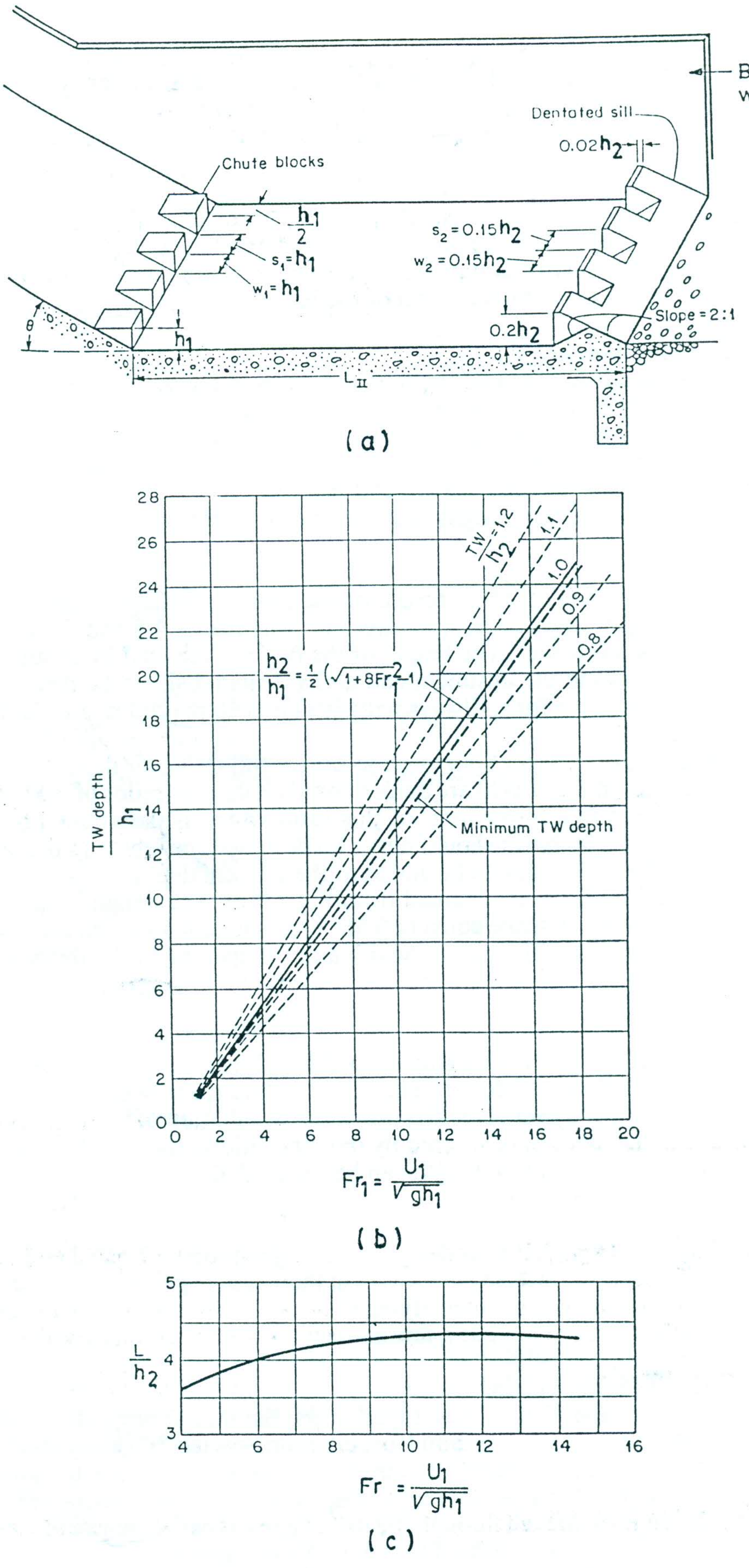
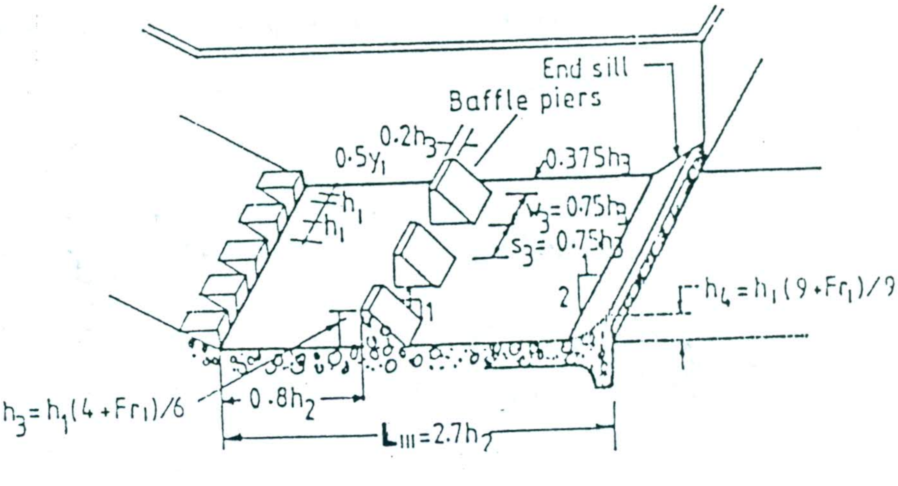
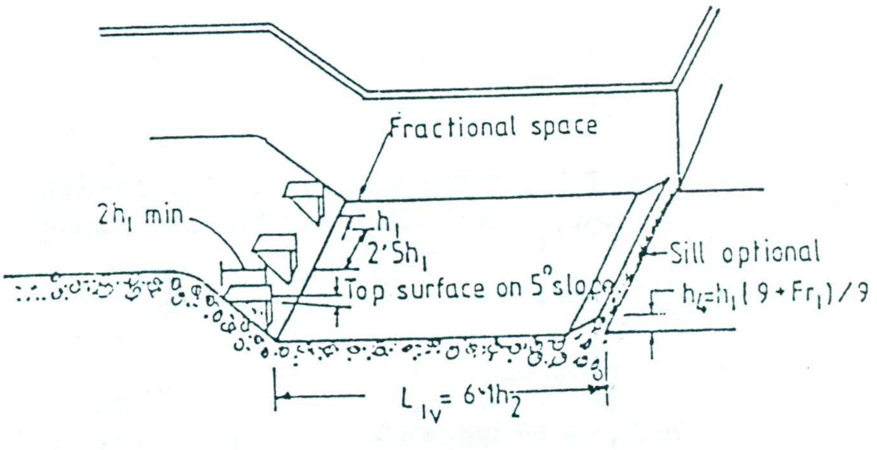


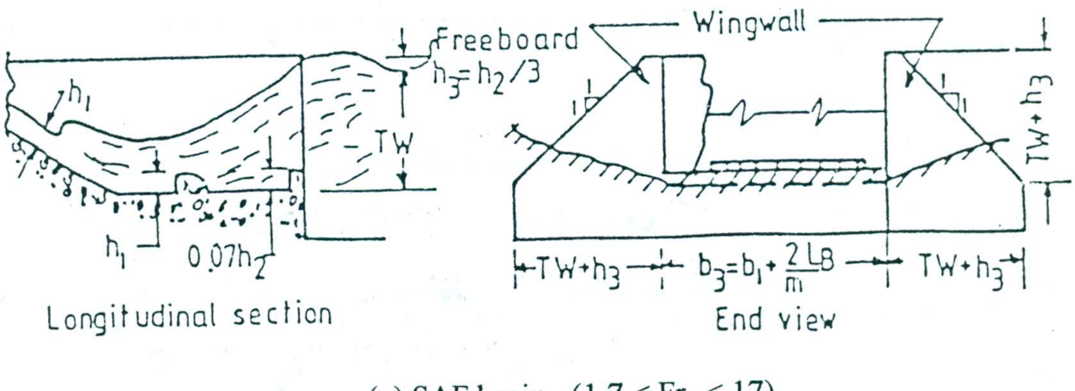
Fig. 7.9 USBR stilling basin II. (a) Recommended proportions, (b) Minimum tailwater depths, and (c) Length of hydraulic jump (after Bradley and Paterka, 1957)



(a) USBR basin III ($Fr_1 \geq 4.5$)



(b) USBR basin IV ($2.5 < Fr_1 < 4.5$)



(c) SAF basin ($1.7 < Fr_1 < 17$)

Fig. 7.10 USBR stilling basin III, USBR stilling basin IV and SAF stilling basin

Example 7.5

Proportion a USBR stilling basin II for an overflow spillway with the following data:

Design discharge	$= 15{,}870\ m^3/s$
TW level	$= 17.26$ m
Basin width	$= 227.1$ m
Elevation of ground	$= 0.00$ m
Velocity at the foot of the spillway	$= 24.70$ m/s

Solution $Q = 15870\ m^3/s$, $h_t = 17.26$ m, $B = 227.1$ m, $U_1 = 24.70$ m/s

$$h_1 = \frac{Q}{BU_1} = \frac{15870}{227.1 \times 24.70} = 2.83\ m$$

$$Fr_1 = \frac{U_1}{\sqrt{gh_1}} = \frac{24.70}{\sqrt{9.81 \times 2.83}} = 4.69 > 4.50$$

$$\frac{h_2}{h_1} = \frac{1}{2}(\sqrt{1+8Fr_1^{\,2}} - 1) = \frac{1}{2}(\sqrt{1+8\times 4.69^2} - 1) = 6.15$$

$$\therefore\ h_2 = 6.15 \times 2.83 = 17.40\ m \quad and \quad h_t = 17.26\,m$$

$$\therefore\ h_2 > h_t$$

So, when the basin floor is set at 0.00 m, the jump moves downstream and more basin length need to be provided. Hence, the floor must lowered. With 5% safety margin

$$h_t = 1.05 h_2 = 1.05 \times 17.40 = 18.27\,m$$

Hence, the floor must be set at elevation (17.26 – 18.27) m = - 1.01 m

From Fig. 7.9(c), for $Fr_1 = 4.69$, $L/h_2 = 3.70$. Hence, the length of the basin

$$L = 3.70 \times 17.40 = 64.40\ m$$

The height, width and spacing of the chute blocks $= h_1 = 2.83$ m

The height of the dentated sill $= 0.2h_2 = 0.2 \times 17.40 = 3.48$ m and the width and spacing of the dentates $= 0.15h_2 = 0.15 \times 17.40 = 2.61$ m

PROBLEMS

7.1 Verify Eqs. (7.10), (7.11), (7.15), (7.18), (7.20), (7.21) and (7.26).

7.2 (a) Compute the sequent depth ratio h_2/h_1, the relative energy loss h_L/E_1 and the relative height of jump h_j/E_1 for $Fr_1 = 1$, 1.7, 2.5, 4.5, 9 and 15 for jumps in horizontal rectangular channels.

(b) Show that for jumps in horizontal rectangular channels, the maximum relative height h_j/E_1 is 0.51 and it occurs at $Fr_1 = 2.77$.

7.3(a) Water flows at a velocity of 6.1 m/s and a depth of 1 m in a horizontal rectangular channel 6.1 m wide. Find (i) the downstream depth necessary to form a hydraulic jump, (ii) the type of jump, (iii) the height of the jump, (iv) the length of the jump, (v) the horsepower dissipation in the jump, and (vi) the efficiency.

(b) The depth and velocity at the foot of an overflow spillway are 0.50 m and 15.50 m, respectively. What tailwater depth is needed to form a hydraulic jump? If a jump is formed, determine the type of jump, the height of jump, the length of jump and the energy loss in the jump as a percentage of the initial energy.

7.4 The values of two variables in connection with a hydraulic jump in a horizontal rectangular channel are given in the following table. Compute the values of other variables in this table.

	h_1 (m)	U_1 (m/s)	h_2 (m)	U_2 (m/s)	q (m^2/s)	Fr_1	Fr_2	h_L (m)
a)	0.25	12.75						
b)	0.25		2.76					
c)			2.76	1.16				
d)		12.75		1.16				
e)						8.14		5.71
f)							0.22	5.71
g)					3.19			5.71

7.5 Water flows at a depth of 1 m in a horizontal trapezoidal channel having a base width 5 m and side slope 1:1 and $Q = 30\ m^3/s$. If a hydraulic jump occurs in this channel, compute the sequent depth and the energy lost in the jump.

7.6 A horizontal triangular channel having $s = 2$ carries a discharge of 20 m^3/s at a depth of 1 m. Compute the downstream depth that will form a hydraulic jump.

7.7 A horizontal parabolic channel contains a discharge of 10 m^3/s at a depth of 0.50 m. The profile of the channel is given by the equation $y^2 = 4z$. If a hydraulic jump occurs in this channel, compute the sequent depth.

7.8(a) A rectangular channel is 1.2 m wide and inclined at an angle of 3° with the horizontal. The channel carries a discharge of 0.14 m^3/s at a vertical depth of 0.02 m. If a hydraulic jump occurs in this channel, compute the sequent depth.

(b) A rectangular channel 6.0 m wide is inclined at an angle of 3.5^0 with the horizontal. Determine the jump type if $Q = 0.75\ m^3/s$, $h_1 = 0.02$ m and $h_t = 0.45$ m.

7.9 A rectangular channel 6 m wide and inclined at an angle of 5^0 with the horizontal carries a discharge of 20 m^3/s. Determine the jump type if the upstream depth (normal to the direction of flow) is (i) 0.20 m, (ii) 0.30 m, and (iii) 0.40 m, when the tailwater depth is 3.20 m.

****

Chapter 8

CHANNEL CONTROLS AND TRANSITIONS

8.1 INTRODUCTION

A control has been defined as a feature, natural or artificial, which establishes a definite relationship between the discharge and the depth of flow. Sluice gates, free overfalls, weirs and spillways are some familiar examples of controls. Many of these structures are used for flow measurement. In addition, these control structures govern the depth of flow upstream in case of subcritical flow and downstream in supercritical flow.

A transition has been defined as a change either in the direction or slope or cross-section of the channel that produces a change, either temporary or permanent, in the state of flow in the channel. Channel bends, humps, depressions, expansions and contractions are typical examples of channel transitions.

A transition producing a permanent change in the state of flow in effect becomes a control. Therefore, all controls are transitions, but all transitions are not necessarily controls under all conditions.

8.2 SLUICE GATE

The vertical sluice gate is one of the most commonly used types of underflow gates used for flow measurement. In its simplest form, it consists of a vertical gate that can be lifted up and down. It is sometimes used to raise the water level and maintain a constant water level in an irrigation canal. It acts as a control provided the height of the gate opening h_g is less than the critical depth of flow h_c in the channel. When this condition is fulfilled, the flows upstream and downstream of the gate are subcritical and supercritical, respectively, and decreasing h_g increases the upstream depth h_1 and decreases the downstream depth h_2 and vice versa (Fig. 8.1). As the water issues out of the sharp edge of the gate, the streamlines converge rapidly till the flowing water attains a minimum depth at the vena contracta approximately at a distance equal to h_g from the plane of the gate. The gate may operate under free or submerged condition depending on the tailwater depth.

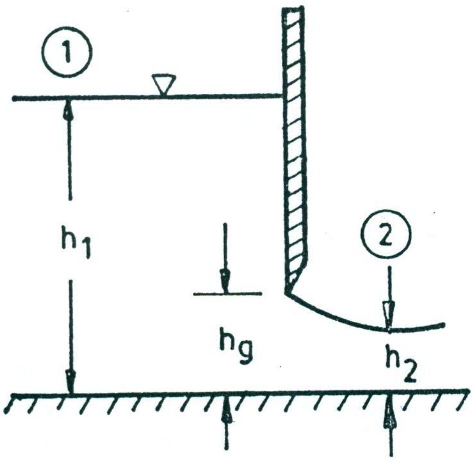
Free Flow

Consider a vertical sluice gate in a horizontal rectangular channel as shown in Fig. 8.1. Flow under a sluice gate is an example of converging flow in which the energy losses between sections 1 and 2 are small and negligible. So, considering free flow and taking the channel bed as datum, we can write

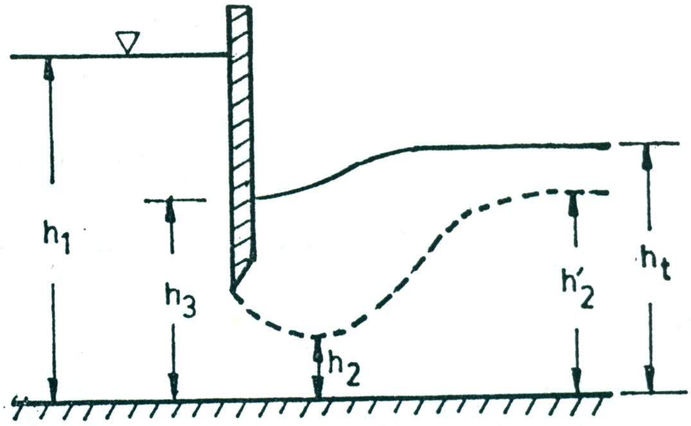
$$h_1 + \frac{U_1^2}{2g} = h_2 + \frac{U_2^2}{2g} \qquad or,\ h_1 + \frac{U_1^2}{2g} = C_c h_g + \frac{U_2^2}{2g} \tag{8.1}$$

where C_c (= h_2/h_g) is the coefficient of contraction. From the continuity equation, we have

$$Q = C_c b h_g U_2 = b h_1 U_1 \tag{8.2}$$



a) Free flow



(b) Submerged flow.

Fig. 8.1 Flow beneath a sluice gate

Combining Eqs. (8.1) and (8.2) and simplifying

$$Q = C_d b h_g \sqrt{2gh_1} \tag{8.3}$$

in which the discharge coefficient C_d is given by

$$C_d = \frac{C_c}{\sqrt{1 + C_c h_g / h_1}} \tag{8.4}$$

Thus, the sluice gate is in effect a control because for a given gate opening there is a definite relationship between Q and the upstream depth h_1 as given by Eq.(8.3).

Submerged Flow

By operating a control located downstream, the tailwater depth h_t can be increased and a hydraulic jump can be formed and advanced upstream. Till the toe of the jump reaches the vena cotracta, the discharge beneath the sluice gate is not affected and the free flow condition exists. Any further increase of h_t causes the jump to be submerged. The effect of submergence is to reduce the discharge in the channel and/or to increase the depth h_1.

Equation (8.3) may also be used for submerged flow. However, for submerged flow

$$C_d = f(h_1/h_g, h_t/h_g) \tag{8.5}$$

Rajaratnam and Subramanya (1967) proposed an alternative method of computing the discharge as

$$Q = C_d b h_g \sqrt{2g\Delta h} \tag{8.6}$$

where

$$\Delta h = h_1 - h_2 = h_1 - C_c h_g \tag{8.7}$$

for free flow, and

$$\Delta h = h_1 - h_3 \tag{8.8}$$

for submerged flow.

In order to compute the discharge under submerged flow condition using Eqs. (8.6) and (8.8), the prediction of h_3 is necessary. Applying the momentum principle and the equation of continuity, it can be shown that in horizontal rectangular channels

$$\frac{h_3}{h_t} = \left[1 + 2Fr_t^2\left(1 - \frac{h_t}{h_g}\right)\right]^{1/2} \tag{8.9}$$

where h_g is the height of the sluice gate opening, h_t is the tailwater depth, h_2 is the subcritical sequent depth of the free jump corresponding to h_g and Fr_t is the Froude number corresponding to h_t. Equation (8.9) can be used to compute h_3. However, computation of h_3 using this equation requires that Q must be known initially. Using the momentum principle, Rajaratnam and Subramanya(1967) gave the equation

$$\frac{h_3}{h_g C_d} = 2\left(1 - \frac{h_g C_d}{h_t}\right) + \sqrt{4\left(1 - \frac{h_g C_d}{h_t}\right)^2 + \left(\frac{h_t}{h_g C_d}\right)^2 - 4\left(\frac{h_1}{h_g C_d} - \frac{h_1}{h_t}\right)} \tag{8.10}$$

which can be used to compute h_3 without the discharge Q. Once h_3 is computed, Eqs.(8.6) and (8.8) may be used to compute the discharge under submerged condition.

Coefficients C_c and C_d

The value of C_c does not vary significantly from 0.61 and is nearly independent of the ratio h_1/h_g. Therefore, a value of 0.61 can be used for C_c both for free and submerged flow conditions.

The analysis of experimental data indicated that C_d for both free and submerged flow conditions is uniquely related to h_g/h_1 as shown in Fig. 8.2. The values of C_d for some values of h_g/h_1 are given in Table 8.1. From Fig. 8.2 and Table 8.1, it may be noted that the variation C_d in the range of h_g/h_1 from 0 to 0.30 is small and we can use a constant value of 0.60 for C_d when $h_g/h_1 \leq$ 0.40.

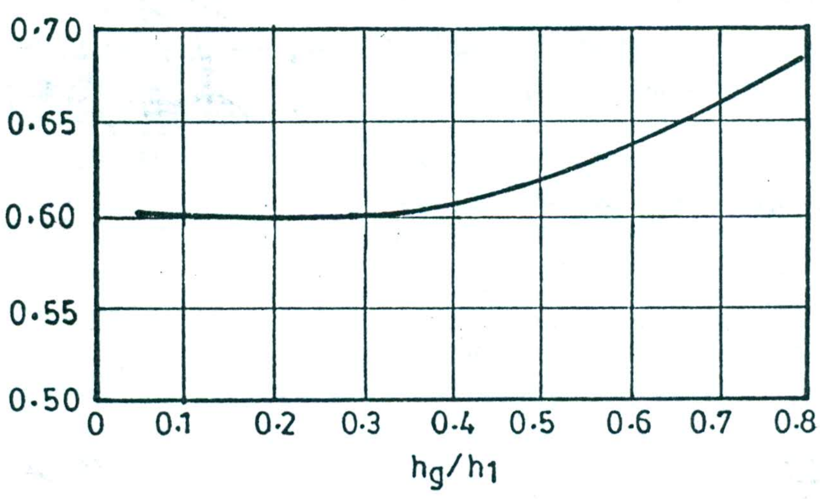


Fig 8.2 Coefficient of discharge for vertical sluice gates

Table 8.1 Values of C_d for some values of h_g/h_1 (Rajaratnam and Subramanya, 1967)

h_g/h_1	0.00	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70
C_d	0.61	0.60	0.60	0.605	0.605	0.607	0.620	0.640	0.660

Example 8.1

Compute the discharge through a vertical sluice gate in a horizontal rectangular channel 6 m wide and having a depth of 1 m at the vena contracta when the upstream head is 4 m, for (i) free flow condition and compare it with the discharge obtained in Problem 2.3, and (ii) submerged condition when the tailwater depth is 3.5 m.

Solution Horizontal rectangular channel, b = 6 m, $h_1 = 4$ m, $h_2 = 1$ m, $h_t = 3.5$ m
Take $C_c = 0.61$ so that $h_g = h_1/C_c = 1/0.61 = 1.64$ m ($< h_c = 1.86$ m)

(i) For free flow condition, using Eq. (8.4), we get

$$C_d = \frac{C_c}{\sqrt{1 + C_c h_g / h_1}} = \frac{0.61}{\sqrt{1 + 0.61 \times 1.64/4}} = 0.546$$

Then, using Eq.(8.3)

$$Q = C_d b h_g \sqrt{2gh_1} = 0.546 \times 6 \times 1.64 \times \sqrt{2 \times 9.81 \times 4} = 47.56\ m^3/s$$

which is exactly the discharge obtained in Problem 2.3 (Q= 47.54 m^3/s).

Again, $h_g/h_1 = 1.64/4 = 0.41$ and from Fig.8.2, $C_d = 0.61$. Hence, using Eq.(8.6), we get

$$Q = C_d b h_g \sqrt{2g\Delta h} = C_d b h_g \sqrt{2g(h_1 - h_2)}$$

$$= 0.61 \times 6 \times 1.64 \times \sqrt{2 \times 9.81 \times (4-1)} = 46.05\ m^3/s$$

which is about 3% lower than, but still considered to be very near to, the discharge obtained in Problem 2.3.

(ii) For submerged flow condition, we have $h_g C_d / h_t = 1.64 \times 0.61/3.5 = 0.286$. Then, using Eq.(8.10), we get

$$\frac{h_3}{h_g C_d} = 2\left(1 - \frac{h_g C_d}{h_t}\right) + \sqrt{4\left(1 - \frac{h_g C_d}{h_t}\right)^2 + \left(\frac{h_t}{h_g C_d}\right)^2 - 4\left(\frac{h_1}{h_g C_d} - \frac{h_1}{h_t}\right)}$$

$$= 2(1 - 0.286) + \sqrt{4(1-0.286)^2 + (\frac{1}{0.286})^2 - 4(\frac{4}{1.64 \times 0.61} - \frac{4}{3.5})} = 3.341$$

$\therefore h_3 = 3.341 \times 1.64 \times 0.61 = 3.342$ m

$$Q = C_d b h_g \sqrt{2g\Delta h} = C_d b h_g \sqrt{2g(h_1 - h_3)}$$

$$= 0.61 \times 6 \times 1.64 \times \sqrt{2 \times 9.81 \times (4 - 3.342)} = 21.57\ m^3/s$$

8.3 FREE OVERFALL

A free overfall occurs when the bottom of a channel is discontinued causing the flow to separate from the channel bed and forming a nappe as shown in Fig. 8.3. The pressure above and below the nappe is atmospheric and the water surface profile of the nappe is a parabola.

The flow downstream of brink or edge of channel is supercritical. Now, if the flow in the channel is subcritical, then there is a section at which the flow is critical. Theoretically, the critical section or the section of minimum specific energy should occur at the brink. However, the determination of critical depth using Eq.(3.9) is based on the assumption of parallel flow and applicable only approximately to gradually varied flow. The flow in the brink is actually curvilinear

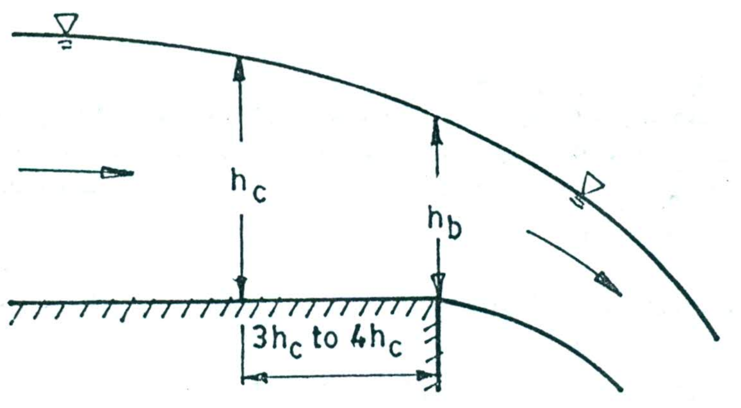


Fig. 8.3 Free overfall at the end of a channel with subcritical flow

having appreciable curvature of the streamlines which causes the pressure distribution at the brink section to depart from hydrostatic. The actual situation is that the brink section is the true section of minimum specific energy, but it is not the critical section as computed by the assumption of parallel flow. The critical flow section occurs a short distance (approximately $3h_c$ to $4h_c$) upstream of the brink and the brink depth is less than the critical depth.

Rouse (1936) was probably the first to determine experimentally that the value of h_b/h_c for subcritical flow in a horizontal rectangular channel is 0.715, i.e. $h_c = 1.4h_b$. Since then a large number of experimental and semi-theoretical studies have confirmed that the value of h_b/h_c obtained by Rouse (1936) for horizontal rectangular channels is not significantly different from 0.715. For other slopes, the relative brink depth can be expressed as

$$h_b/h_c = f(S_0/S_c, \text{channel shape})$$

Thus, for a given channel shape, the variation of h_b/h_c can be expressed as a unique function of S_0/S_c. The variation of h_b/h_c with channel slope, experimentally determined by Delleur al.(1956) using data on adverse and mild slopes, is shown in Fig. 8.4.

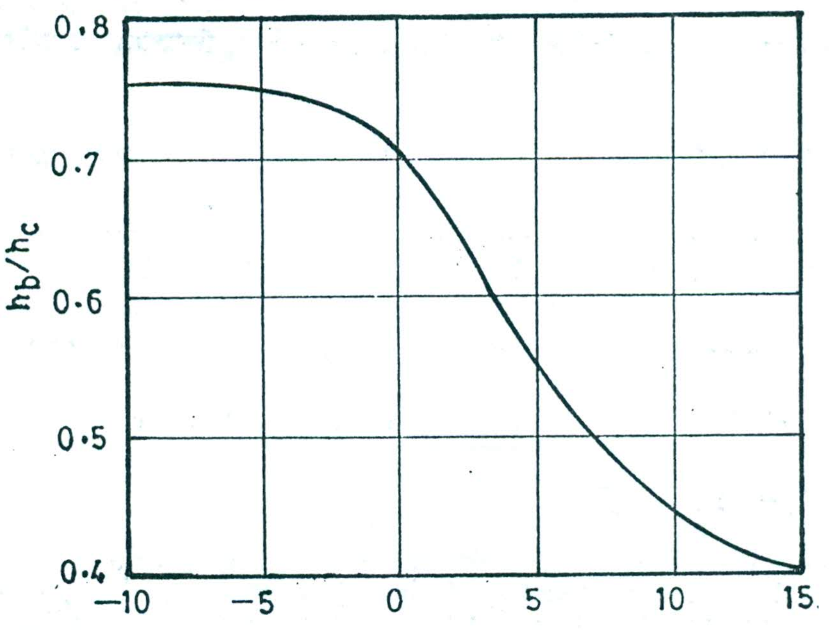


Fig. 8.4 Brink depth in rectangular channels for different slopes

Flow Measurement Using Brink Depth

The unique relationship between h_c and h_b for a given channel shape gives rise to a unique relationship between the brink depth and the discharge for that channel shape. This is of considerable practical interest, because it permits the use of a free overfall for flow measurement. For the purpose of flow measurement, the channel must be level in the lateral direction and preceded by a channel of length not less than $15h_c$. The depth should be measured at the brink section on the centerline of the channel accurately. A channel with horizontal or small slope must be used.

The following discharge formula are to be used when the channel is horizontal or of small slope:

1. Rectangular channel

$$h_b/h_c = 0.715 \qquad Q = 5.18bh_b^{1.5} \tag{8.11}$$

2. Triangular channel:

$$h_b/h_c = 0.795 \qquad Q = 3.93sh_b^{2.5} \tag{8.12}$$

3. Parabolic channel

$$h_b/h_c = 0.772 \qquad Q = 5.72c^{-1/2}h_b^{2} \tag{8.13}$$

The values of h_b/h_c for other slopes can be obtained from Fig. 8.4 and the corresponding discharge can be estimated. However, in this case a trial procedure is needed.

For circular and trapezoidal channels, explicit relationship between the brink depth and the discharge can not be determined. However, in this case by determining h_b, h_c can be estimated and the discharge Q can be estimated. The values of h_b/h_c for free overfalls in circular conduits for different bottom slopes are given in Table 8.2. The value of h_b/h_c for a horizontal circular channel is 0.725.

Table 8.2 Brink depth in circular channels for different slopes

S_0/S_c	-4.00	0.00	2.00	4.00	6.00	8.00
h_b/h_c	0.75	0.725	0.61	0.53	0.491	0.487

For a trapezoidal channel, the general functional relationship is

$$h_b/h_c = f(S_0/S_c, sh_c/b) \tag{8.14}$$

Rajaratnam and Muralidhar (1970) performed experiments on trapezoidal outfalls and analysed the data with those of Diskin (1961) in accordance with Eq.(8.14). The relationship developed by them is shown in Fig. 8.5. For a horizontal trapezoidal channel the ratio h_b/h_c varies from 0.715 to 0.795. The determination of discharge corresponding to a measured brink depth involves a trial procedure.

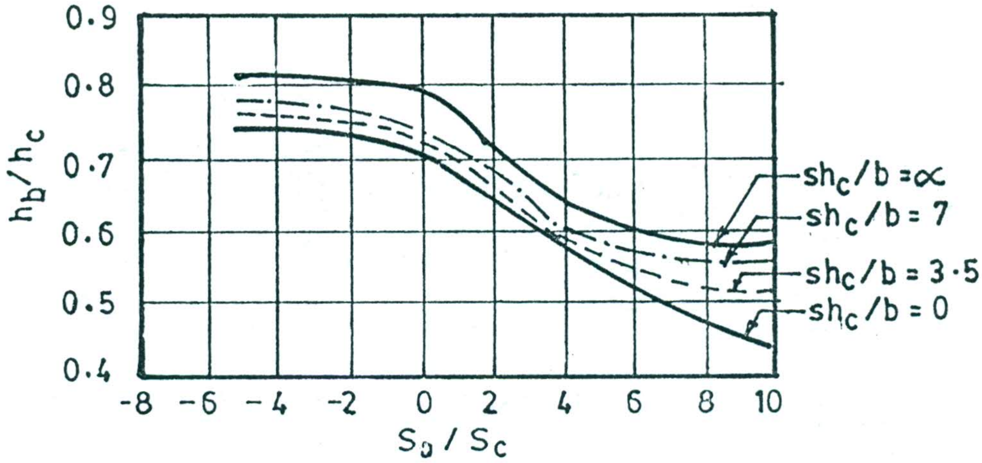


Fig. 8.5 Brink depth in smooth trapezoidal channels for different slopes

Example 8.2

A rectangular channel 6 m wide is made of concrete ($n = 0.013$) and ends in a free overfall. Compute the discharge in the channel if the brink depth is 0.75 m, when (i) the channel is horizontal, and (ii) the channel is laid on a slope of 0.0020.

Solution Rectangular channel, $b = 6$ m, $n = 0.013$, $h_b = 0.75$ m

(i) When the channel is horizontal, using Eq.(8.11), we get

$$Q = 5.18bh_b^{1.5} = 5.18 \times 6 \times 0.75^{1.5} = 20.19 \text{ m}^3/\text{s}$$

or, alternatively, since $h_b/h_c = 0.715$, $h_c = h_b/0.715 = 0.75/0.715 = 1.049$ m, we get

$$Q = \sqrt{g}bh_c^{1.5} = \sqrt{9.81} \times 6 \times 1.049^{1.5} = 20.19 \; m^3/s$$

(ii) When the channel slope is 0.0025, let us take $Q = 20.19$ m^3/s as a first approximation to determine the critical slope. Then, applying the Manning formula

$$Q = \frac{1}{n} AR^{2/3} S_c^{\,1/2}$$

we have

$$20.19 = \frac{1}{0.013} \times (6 \times 1.049) \times \left(\frac{6 \times 1.049}{6 + 2 \times 1.049}\right)^{2/3} \times S_c^{\,1/2}$$

which gives the critical slope as $S_c = 0.0024$ so that

$$S_0/S_c = 0.0020/0.0024 = 0.82$$

Then, from Fig 8.5, we get $h_b/h_c = 0.70$ and $h_c = 0.75/0.70 = 1.08$ m. For this critical depth, $Q = 21.07$ m^3/s and $S_c = 0.00243$ which is the same as we obtained previously.

$$\therefore Q = 21.08 \text{ m}^3/\text{s}$$

Example 8.3

A trapezoidal channel with b = 6 m, s = 1 and n = 0.015 ends in a free overfall. The brink depth is measured and found to be 1.00 m Compute the discharge in the channel if (i) the channel is horizontal, and (ii) the channel has a longitudinal slope of 0.002.

Solution Trapezoidal channel, b = 6 m, s = 1, n = 0.015, h_b = 1.00 m

(i) When the channel is horizontal, the ratio h_b/h_c varies from 0.715 to 0.795. Assume that h_b/h_c = 0.75 so that h_c = 1/0.75 = 1.33 m.

$$\therefore \; sh_c/b = 1 \times 1.33/6 = 0.22$$

Then, from Fig.8.5, $h_b/h_c = 0.73$

$$\therefore \; h_c = 1/0.73 = 1.37 \text{ m and } sh_c/b = 1 \times 1.37/6 = 0.23$$

Therefore, the value of h_c may be taken as 1.37 m.

$$\therefore \; A_c = (6 + 1 \times 1.37) \times 1.37 = 10.10 \text{ m}^2$$

$$B_c = 6 + 2 \times 1 \times 1.37 = 8.74 \text{ m}$$

$$D_c = A_c/B_c = 10.10/8.74 = 1.16 \text{ m}$$

$$U_c = \sqrt{gD_c} = \sqrt{9.81 \times 1.16} = 3.37 \; m/s$$

$$\therefore \; Q_c = A_c U_c = 33.99 \text{ m}^3/\text{s}$$

(ii) When the channel slope is 0.002, assume that h_b/h_c = 0.70 so that h_c = 1/0.70 = 1.428 m.

$$\therefore A_c = (6 + 1 \times 1.428) \times 1.423 = 10.612 \text{ m}$$

$$B_c = 6 + 2 \times 1 \times 1.428 = 8.857 \text{ m}$$

$$P_c = 6 + 2\sqrt{1+1^2} \times 1.428 = 10.041 \; m$$

$$R = A_c/P_c = 10.612/10.041 = 1.057 \text{ m}$$

$$D_c = A_c/B_c = 10.612/8.857 = 1.979 \text{ m}$$

$$U_c = \sqrt{gD_c} = \sqrt{9.81 \times 1.979} = 3.428 \; m/s$$

$$\therefore \; Q_c = A_c U_c = 36.37 \text{ m}^3/\text{s}$$

Now, from the Manning formula

$$Q = \frac{1}{n} A R^{2/3} S_c^{\ 1/2}$$

we have

$$36.37 = \frac{1}{0.015} \times 10.612 \times 1.057^{2/3} \times S_c^{\ 1/2}$$

which gives the critical slope as $S_c = 0.0025$.

$$\therefore\ S_0/S_c = 0.002/0.002456 = 0.814 \ \text{ and } \ sh_c/b = 1 \times 1.423/6 = 0.237$$

From Fig 8.5, $h_b/h_c = 0.70$. Hence, the assumed value of h_b/h_c is correct and

$$Q = 36.37\ \text{m}^3/\text{s}$$

8.4 FLOW OVER A WEIR

A weir is an overflow structure built across a channel whose primary faction is to estimate the discharge in the channel. Weirs may be of different cross-sectional shapes like rectangular, trapezoidal, triangular, circular, parabolic and so on.

Depending on the value of h_1/L, where h_1 is the upstream depth measured above the weir crest and L is the length of the weir crest along the direction of flow (Fig.8.6), the flow over a weir may be classified as follows.

1. $h_1/L < 0.10$: Flow over the weir crest is subcritical and resistance effects are appreciable. Critical flow section occurs at the downstream end of the weir. This type of weir is termed *long-crested* and cannot be used for flow measurement. However, it can be used for flow measurement based on the brink depth as discussed in Art. 8.3 provided the crest is long enough to develop the full drawdown profile.

2. $0.10 \le h_1/L \le 0.35$: A region of parallel flow and the critical flow (control) section occur in the vicinity of the midpoint of the crest. The weir can precisely be defined as broad–crested and is widely used for flow measurement. In general, the coefficient of discharge has a constant value in this rage of h_1/L.

3. $0.35 \le h_1/L \le 1.5$: The water surface is generally curvilinear all over the crest and the control section occurs near the upstream end of the weir. The weir is termed *short-crested or narrow-crested.*

4. $h_1/L > 1.5$: The flow may separate completely at the upstream edge of the weir. The flow above the crest has curved streamlines. The weir is similar to a sharp-crested weir ($h_1/L > 15$) and can be used for flow measurement.

Broad-Crested Weir

It was shown in Chapter 3 that if the height of a hump provided on the channel bed Δz is equal to or greater than a limiting value Δz_c, the flow over the hump will be critical. However, the length of the crest in the direction of flow must be adequate so that parallel flow with hydrostatic pressure distribution develops over the hump. As started earlier, such a situation exists when $0.10 \le h_1/L \le 0.35$ and the hump, in effect, because a broad-crested weir

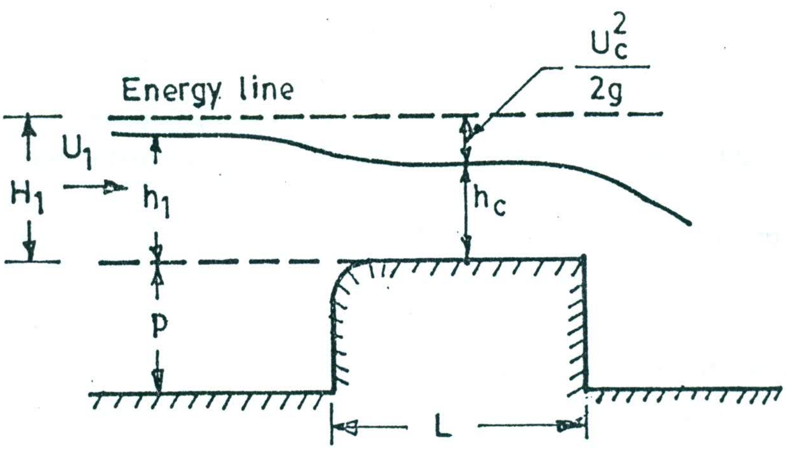


Fig. 8.6 Free flow over a rectangular broad-crested weir

A broad-crested weir may operate under free-flow or submerged condition. Under free-flow condition the discharge does not depend on the depth downstream of the weir. Under submerged condition the discharge depends on the downstream water level. Under free-flow condition the flow over the weir is critical and the discharge is maximum. Such a weir provides an excellent means of measuring the discharge in open channels.

Rectangular broad-crested weir

Free flow: Consider a rectangular broad-crested weir with vertical faces and rounded upstream corner under free-flow condition spanning over the entire width of the channel (Fig. 8.6). Neglecting the frictional losses and applying the energy equation between the upstream section and the critical flow section, one obtains

$$H_1 = h_c + \frac{U_c^{\,2}}{2g} = h_c + \frac{h_c}{2} = \frac{3}{2} h_c \tag{8.15}$$

or

$$h_c = \frac{2}{3} H_1 \tag{8.16}$$

For critical flow condition

$$Fr = U_c / \sqrt{g h_c} = 1$$

and hence

$$U_c = \sqrt{g h_c} \tag{8.17}$$

The discharge over the broad-crested weir is $Q = A_c U_c = b h_c U_c$, or

$$Q = b\sqrt{g}\, h_c^{\,1.5} = 3.132 b\, h_c^{\,1.5} \tag{8.18}$$

where b is the width of the channel.

Equation (8.18) can be used for computing the discharge in an open channel when frictional losses over the weir crest are negligible, the flow over the weir is parallel, i.e. the water pressure over the weir crest is hydrostatic and the upstream velocity of approach is negligible. However, the usual difficulty in using Eq.(8.18) for computing discharge in an open channel lies in locating the critical flow section and measuring the critical depth accurately. These difficulties are, however, avoided by measuring the depth of flow upstream of the weir where the flow is not affected by the presence of the weir.

Combination of Eqs.(8.16) and (8.18) yields

$$Q = \left(\frac{2}{3}\right)^{1.5} b\sqrt{g}H_1^{1.5} = 1.705bH_1^{1.5} \tag{8.19}$$

Introducing a discharge coefficient C_d, the equation for actual discharge over a rectangular broad-crested weir becomes

$$Q = \left(\frac{2}{3}\right)^{1.5} C_d b\sqrt{g}H_1^{1.5} = 1.705C_d bH_1^{1.5} \tag{8.20}$$

The discharge coefficient C_d takes into account the effects of fluid viscosity, surface tension, non-uniform velocity distribution, curvature and friction and usually varies from 0.80 to 1.00. It is a function of the parameters h_1/L and $h_1/(h_1 + p)$. However, for $0.10 < h_1/L < 0.35$ and $h_1/(h_1 + p) < 0.35$ (i.e. $h_1/p < 0.54$), the value of C_d is found to remain constant at 0.848, a value which is termed as the *basic discharge coefficient*. When the above conditions are not met, the basic discharge coefficient must be multiplied by a correction factor f which can be obtained using Fig.8.7 (French, 1986).

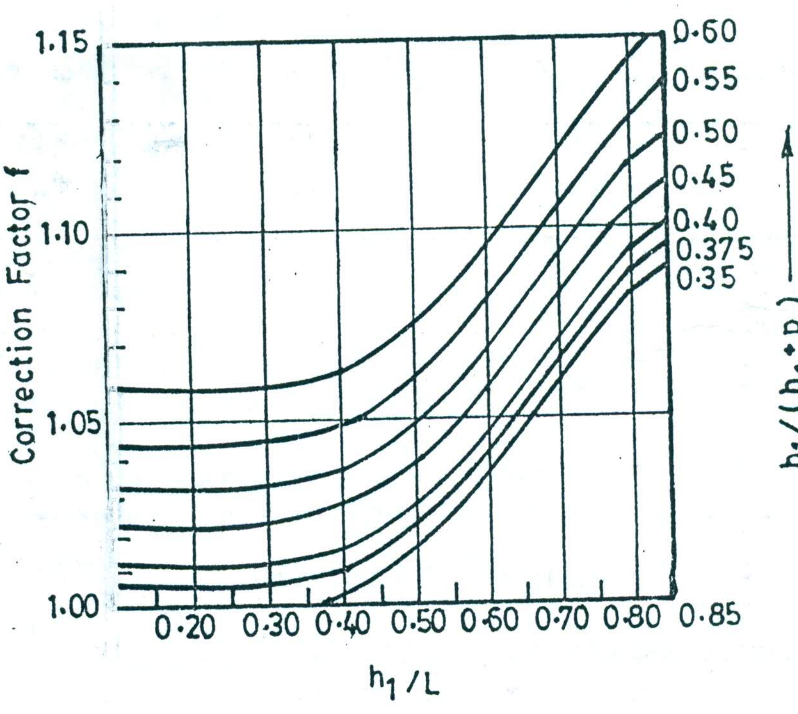


Fig. 8.7 Correction factor f as a function of h_1/L and $h_1/(h_1+p)$ for rectangular broad-crested weir (French, 1986)

Equation (8.20) is rather inconvenient for use as it contains the energy head H_1. If the velocity of approach is small so that $U_1^2/2g << h_1$, one can use the equation

$$Q = \left(\frac{2}{3}\right)^{1.5} C_d b\sqrt{g} h_1^{1.5} = 1.705 C_d b h_1^{1.5} \tag{8.21}$$

for estimating the discharge. However, if the velocity of approach U_1 is appreciable, Eq.(8.20) has to be used, initially neglecting the velocity of approach and then, improving the solution by including the velocity of approach.

Submerged flow: The effect of submergence is to drown the critical depth on the weir crest and make the flow over the weir crest subcritical. Under submerged conditions the discharge over the weir depends on the downstream depth h_2 (Fig.8.8). However, the flow is affected by downstream depth only when h_2/h_1 is greater than about 0.80, and when $h_2/h_1 < 0.80$ the weir discharges freely with no submergence effects.

The discharge formula for a submerged weir may be written as

$$Q = \left(\frac{2}{3}\right)^{1.5} C_d C_s b\sqrt{g} h_1^{1.5} = 1.705 C_d C_s b h_1^{1.5} \tag{8.22}$$

where the coefficient C_s takes into account the effect of submergence and depends on h_2/h_1 and the geometry of the weir. For a rectangular broad-crested weir with vertical upstream and downstream faces, the numerical value of C_s varies from 1.00 at $h_2/h_1 = 0.80$ (no submergence effect) to 0 at $h_2/h_1 = 1$ as given in Table 8.3.

Table 8.3 Variation of C_s with h_2/h_1 for weirs with vertical faces (Ranga Raju, 1993)

h_2/h_1	≤ 0.80	0.85	0.90	0.95	0.98	0.99	1.00
C_s	1.00	0.95	0.82	0.63	0.35	0.20	0

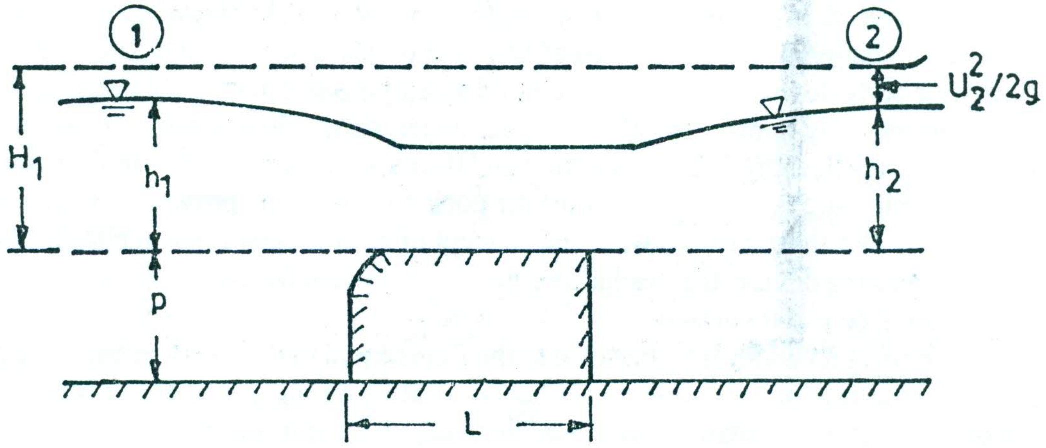


Fig. 8.8 Submerged flow over a rectangular broad-crested weir

Example 8.4

A rectangular broad-crested weir with vertical faces is 1 m high, has crest length of 2 m and span over the entire width of the channel. If the head over the weir is 0.50 m, (i) compute the discharge per unit width. (ii) What would be the discharge if the depth downstream of the weir is 0.45 m?

Solution

(i) In this case, the weir operates under free flow condition. We have, $p = 1$ m, $L = 2$ m and $h_1 =$ 0.50 m.

$$\therefore \frac{h_1}{L} = \frac{0.50}{2} = 0.25 < 0.35$$

$$\frac{h_1}{h_1 + p} = \frac{0.50}{0.50 + 1} = 0.33 < 0.35$$

Hence, the weir is practically a broad-crested weir with $C_d = 0.848$. The computation of discharge is carried out with Eq.(8.20) as follows.

h_1	U_1	$U_1^2/2g$	H_1	q
0.50	-	-	0.50	0.511
0.50	0.3407	0.0059	0.5059	0.520
0.50	0.3468	0.0061	0.5061	0.521
0.50	0.3471	0.0061	0.5061	0.521

Hence, the discharge per unit width, $q = 0.521$ m^2/s

ii) Since $h_2/h_1 = 0.45/0.50 = 0.90 > 0.80$, the weir operates under submerged condition. From Table 8.3, we obtain $C_s =$ 82. Therefore, $q = 0.521 \times 0.82 = 0.427$ m^2/s.

Sharp-Crested Weir

If the length of the weir in the direction of flow is such that $h_1/L > 15$ (Fig.8.9), then the weir is termed sharp-crested. In practice, the crest length provided in a sharp-crested weir is only 1 mm to 2 mm and the downstream end is bevelled at an angle of 45° to 60°. In this case, the flow springs clear of the weir body downstream of the weir and an air pocket is formed beneath the nappe from which air is continuously removed by the overflowing jet. As the flow continues, the pressure in the air pocket falls below the atmospheric pressure and the nappe is depressed. For flow measurement, the atmospheric pressure is maintained in the air pocket through the provision of air vents.

In the case of a sharp-crested weir, the concept of critical flow is not applicable. For this type of discharge measuring device, the discharge equation is derived by assuming that the weir behaves as an orifice with free water surface.

A weir having its width B (transverse to the flow) equal to the width of the channel b, so that only vertical contraction of the nappe takes places, is called a *suppressed or full-width weir*. When the width of the weir is less than the width of the channel so that the nappe contracts both in the vertical and lateral directions, the weir is termed a *contracted weir*.

A sharp-crested weir may operate under free-flow or submerged condition depending on whether the water level downstream of the weir lies below or above the crest.

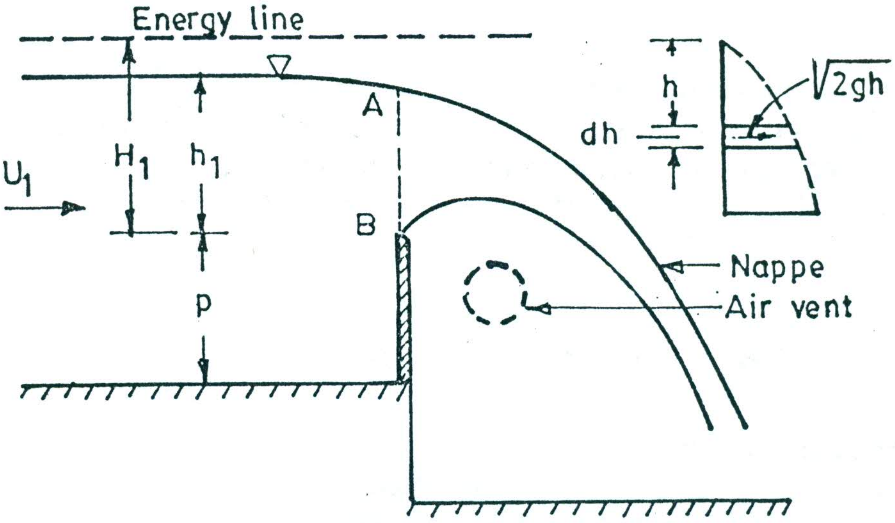


Fig. 8.9 Free flow over a rectangular sharp-crested suppressed weir

Rectangular sharp-crested suppressed weir

Free flow: Consider a rectangular sharp-crested weir spanning the full width b of a rectangular channel as shown in Fig.8.9. It is assumed that the flow does not contract as it passes over the weir and the pressure is atmospheric across the whole section AB. The velocity at any depth h below the energy line is equal to $\sqrt{2gh}$ and the discharge through an elementary strip of thickness dh is given by

$$dQ = b\sqrt{2gh}\,dh \tag{8.23}$$

The total discharge Q is then

$$Q = b\sqrt{2g}\int_{U_1^2/2g}^{h_1+U_1^2/2g}\sqrt{h}\,dh$$

$$= \frac{2}{3}\sqrt{2g}\,b\left[\left(h_1+\frac{U_1^2}{2g}\right)^{1.5}-\left(\frac{U_1^2}{2g}\right)^{1.5}\right] \tag{8.24}$$

$$= \frac{2}{3}\sqrt{2g}\,bh_1^{1.5}\left[\left(1+\frac{U_1^2}{2gh_1}\right)^{1.5}-\left(\frac{U_1^2}{2gh_1}\right)^{1.5}\right] \tag{8.25}$$

where

$$U_1 = \frac{Q}{b(h_1+p)} \tag{8.26}$$

is the approach velocity.

The effect of flow contraction is taken into account by a coefficient of contraction C_c. Then

$$Q = \frac{2}{3}C_c\sqrt{2g}\,bh_1^{1.5}\left[\left(1+\frac{U_1^2}{2gh_1}\right)^{1.5}-\left(\frac{U_1^2}{2gh_1}\right)^{1.5}\right] \tag{8.27}$$

Introducing a discharge coefficient C_d, Eq.(8.27) can be written in a more compact form as

$$Q = \frac{2}{3} C_d \sqrt{2g}\, b h_1^{1.5} \tag{8.28}$$

where

$$C_d = C_c \left[\left(1 + \frac{U_1^2}{2gh_1}\right)^{1.5} - \left(\frac{U_1^2}{2gh_1}\right)^{1.5} \right] \tag{8.29}$$

If the Reynolds number of the flow is sufficiently high and the upstream depth h_1 is at least 0.11 m so that the surface tension and viscosity effects are negligible, then C_d becomes independent of the Reynolds and Weber numbers and depends only on the ratio h_1/p. The variation of C_d for rectangular sharp-crested weirs is given with satisfactory accuracy using the well-known *Rehbock formula*

$$C_d = 0.611 + 0.08\, h_1/p \tag{8.30}$$

which is valid for $h_1/p \leq 5$.

When p becomes very large, C_d becomes equal to 0.611. Since in this case $U_1^2/2gh_1$ becomes negligibly small, Eq.(8.29) shows that C_c also becomes equal to 0.611.

When $p = 0$ so that h_1/p is infinite, the situation becomes a free overfall, considered in Art. 8.3. For a very low weir, in the range $h_1/p > 20$, critical flow occurs just upstream of the weir. Such a weir is termed as a *sill*. In this case

$$h_1 + p = h_c = \left(\frac{Q^2}{gb^2}\right)^{1/3}$$

or

$$Q = \sqrt{g}\, b h_c^{1.5} = \sqrt{g}\, b (h_1 + p)^{1.5} = \frac{2}{3} C_d b \sqrt{2g}\, h_1^{1.5} \tag{8.31}$$

so that

$$C_d = 1.06 \left(1 + \frac{p}{h_1}\right)^{1.5} \tag{8.32}$$

In the range $20 > h_1/p > 5$, the intermediate region between a weir and a sill, the coefficient of discharge C_d is expected to have a smooth transition from Eq. (8.30) to Eq. (8.32).

Submerged flow: The discharge over a broad-crested weir is affected by the tailwater level downstream of the weir if it is above the weir crest. Such a flow is called a submerged flow. Under submerged conditions, the discharge over the weir depends on the submergence ratio h_2/h_1, where h_2 is the water surface elevation measured above the weir crest (Fig. 8.10), and is given by the *Villemonte equation*

$$Q_s = Q \left[1 - \left(\frac{h_2}{h_1}\right)^n \right]^{0.385} \tag{8.33}$$

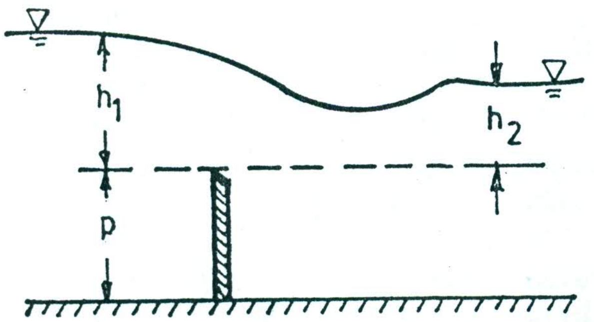


Fig. 8.10 Submerged sharp-crested weir

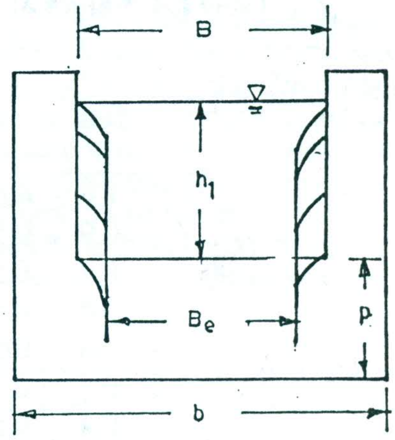


Fig. 8.11 Weir with end contractions

where Q is the free-flow discharge under head h_1 (Eq.(8.28)) and n is the exponent of head in the head-discharge relationship $Q = kh^n$. For rectangular weirs, n = 1.5 and for triangular weirs, n = 2.5.

To ensure free flow over a broad-crested weir, the water level downstream of the weir must be kept a few centimeters below the weir crest.

Rectangular sharp-crested contracted weir

In a contracted weir (Fig.8.11), the effective width of the weir (transverse to the direction of flow) is reduced and is given by the well-known Francis formula

$$B_e = B - 0.1nh_1 \tag{8.34}$$

where B is the width of the weir and n is the number of end contractions. The discharge equation for a contracted weir may be developed in the same way as for a suppressed weir. However, Kindsvater and Carter (1957), based on their extensive experimental investigation, modified the theoretical equation so that it would apply to all rectangular sharp-crested weirs regardless of whether they are suppressed or contracted. The equation for discharge over a sharp-crested weir is

$$Q = \frac{2}{3} C_{de} \sqrt{2g}\, B_e h_{1e}^{\ 1.5} \tag{8.35}$$

where C_{de} is the effective coefficient of discharge and B_e and h_{1e} are the effective width and head of the weir; and

$$C_{de} = K_1 + K_2 h_1/p \tag{8.36a}$$

$$B_e = B + K_b \tag{8.36b}$$

$$h_{1e} = h_1 + K_h \tag{8.36c}$$

where the parameters K_b and K_h represent the combined effects of viscosity and surface tension on the flow. Usually K_h is taken to be equal to 0.001 m. The parameters K_1, K_2 and K_b, which depend on B/b, are given in Table 8.4.

Table 8.4 Values of K_1, K_2 and K_b for broad-crested weirs (Kindsvater and Carter, 1957)

B/b	K_1	K_2	K_b
1.0	0.602	0.0750	-0.0009
0.9	0.599	0.0640	0.0037
0.8	0.597	0.0450	0.0043
0.7	0.595	0.0300	0.0041
0.6	0.593	0.0180	0.0037
0.5	0.592	0.0110	0.0030
0.4	0.591	0.0058	0.0027
0.3	0.590	0.0020	0.0025
0.2	0.589	-0.0018	0.0024
0.1	0.588	-0.0021	0.0024

Example 8.5

A rectangular sharp-crested weir spanning the full width of a rectangular channel 2 m wide is 1 m high. Compute the discharge over the weir under an upstream head of 0.75 m.

Solution Rectangular channel, $b = 2$ m, $p = 1$ m, $h_1 = 0.75$ m

$$C_d = 0.611 + 0.08h_1/p = 0.611 + 0.08 \times 0.75/1 = 0.671$$

$$\therefore Q = \frac{2}{3}C_d\sqrt{2g}bh_1^{1.5} = \frac{2}{3}\times 0.671\times\sqrt{2\times 9.81}\times 2\times 0.75^{1.5} = 2.574\,m^3/s$$

Example 8.6

Compute the discharge over a sharp-crested contracted weir 1 m wide and 1 m high set in a rectangular channel 2 m wide if the head over the weir is 0.75 m.

Solution $b = 2$ m, $B = 1$ m, $p = 1$ m, $h_1 = 0.75$ m

$$\frac{B}{b} = \frac{1}{2} = 0.50$$

$\therefore$ From Table 8.4, $K_b = 0.0030$, $K_1 = 0.592$ and $K_2 = 0.0110$.

$$\therefore\ C_{de} = K_1 + K_2\times\frac{h_1}{p} = 0.592 + 0.0110\times\frac{0.75}{1} = 0.60025$$

$$B_e = B + K_b = 1 + 0.0030 = 1.0030$$

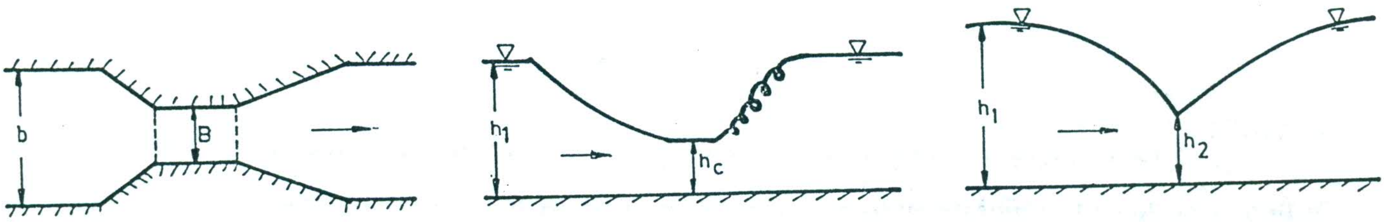
$$h_{1e} = h_1 + 0.001 = 0.75 + 0.001 = 0.751$$

$$\therefore Q = \frac{2}{3}C_{de}\sqrt{2g}B_e h_{1e}^{1.5} = \frac{2}{3}\times 0.60025\times\sqrt{2\times 9.81}\times 1.0030\times 0.751^{1.5} = 1.161\ m^3/s$$

8.5 CRITICAL FLOW FLUMES

It has been stated in Chapter 3 that a critical flow section is a control section and can be used for flow measurement. Based on critical flow, various devices have been used for flow measurement. In these devices, critical flow is produced either by raising the channel bottom, as in a broad-crested weir, or by reducing the channel width, as in a critical flow flume. The use of a broad-crested weir for flow measurement has been considered in Art. 8.3. It has, however, the disadvantage of having a dead water region upstream in which silt and debris can accumulate. This difficulty can be overcome by the use of a critical flow flume, in which the occurrence of critical flow is forced by a contraction in channel width, followed by a short length of supercritical flow and a hydraulic jump. Obviously, the width at the contracted section or throat must be equal to or less than the width required for producing critical flow, i. e. $b \leq b_c$, as shown in Chapter3.

The critical flow flume, also known as the *Venturi flume*, has been designed in various forms. It is usually operated with a free-flow condition having the critical depth at the contracted section or throat and a hydraulic jump in the exit section. Under certain conditions of flow, however, the jump may be submerged.



(a) Plan (b) Free flow (c) Submerged flow

Fig.8. 12 Flow in a rectangular critical flow flume

Let us consider a rectangular flume as shown in Fig. 8.12. For free-flow (Fig. 8.13b) condition, the discharge through the flume is given by the equation

$$Q = B\sqrt{g}\, h_c^{1.5} = 3.132 B h_c^{1.5} \tag{8.37}$$

and when it is difficult to locate the critical flow section and measure the critical depth accurately, by the equation

$$Q = \left(\frac{2}{3}\right)^{1.5} B\sqrt{g} H_1^{1.5} = 1.705 B H_1^{1.5} \tag{8.38}$$

where B is the throat width and h_c is the critical depth at the throat. Equations (8.37) and (8.38) are the same as the Equations (8.18) and (8.19), respectively, used for computing the discharge over a broad-crested weir under free flow condition.

For drowned or submerged condition (Fig. 8.12c), the discharge is given by

$$Q = A_2\sqrt{2g(h_1 - h_2)} / \sqrt{1 - r^2} \tag{8.39}$$

where A_2 is the throat area (= Bh_2), r = A_2/A_1 and A_1 is the upstream area (= bh_1).

The most extensively used critical flow flume is the Parshall flume, developed by R. L. Parshall in 1920 and named after him. Detailed designs for this flume have been developed for a wide range of discharges, and the usual difficulties of locating the critical flow section and measuring the critical depth accurately are readily disposed of by suitable choice of a standard section (not necessarily the critical one) at which the depth is measured. Figure 8.13 shows the

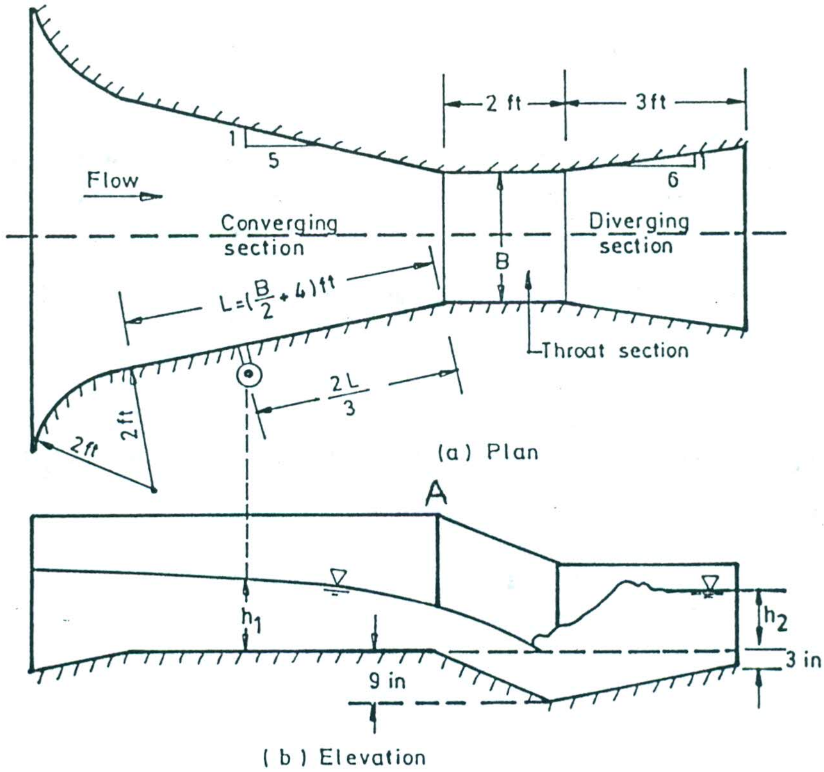


Fig. 8.13 Dimensions of Parshall flume for widths B of 1 ft to 8 ft (Henderson, 1966)

standard design of Parshall flume for throat widths b of 1 ft (0.30 m) to 8 ft (2.67 m). The size of the flume is determined by the throat width in inch or ft. The sidewall convergence and sudden dip in the bed are provided to promote the formation of critical flow near section A followed by a short length of supercritical flow and a hydraulic jump at the exit section. For the above range of throat widths, reliable empirical formula for discharge has been established by calibration and is given by

$$Q = 4Bh_1^{1.522B^{0.026}} \tag{8.40}$$

where B is the throat width and h_1 is the depth measured at the upstream location as shown in Fig. 8.13 and all quantities are in English units.

The above formula is true for values of the submergence ratio, h_b/h_a, up to 0.7. When the submergence ratio exceeds 0.7, the flow becomes submerged. The effect of submergence is to reduce the discharge, i. e. the discharge for submerged flow is less than the free flow discharge. In this case the discharge given by the above formula must be corrected by a negative quantity.

Other designs and the discharge formulae for Parshall flumes covering a range of widths from 3 in to 50 ft are also given by Parshall (see Chow, 1959; French,1986).

Example 8.7

Compute the theoretical discharge through a Venturi flume having a throat width of 0.25 m, (i) when the critical depth measured at the throat is 0.35 m under free-flow condition, and (ii) when the upstream and downstream depths measured are 0.50 m and 0.45 m, respectively, and the channel width is 0.60 m, under submerged condition.

Solution $\quad B = 0.25$ m

(i) Under free-flow condition, the theoretical discharge through the Venturi flume is given by

$$Q = B\sqrt{g}\, h_c^{1.5} = 0.25 \times \sqrt{9.81} \times 0.35^{1.5} = 0.162\ m^3/s$$

(ii) $b = 0.60$ m, $h_1 = 0.50$ m, $h_2 = 0.45$ m, $A_1 = bh_1 = 0.60 \times 0.50 = 0.30$ m^2, $A_2 = Bh_2 = 0.25 \times 0.45 = 0.1125$ m^2. $\therefore$ $r = A_2/A_1 = 0.1125/0.30 = 0.375$. Hence, the theoretical discharge under submerged condition is

$$Q = A_2\sqrt{2g(h_1 - h_2)}/\sqrt{1-r^2} = 0.1125 \times \sqrt{2 \times 9.81 \times (0.50 - 0.45)}/\sqrt{1 - 0.375^2} = 0.103\ m^3/s$$

Example 8.8

Determine the discharge through a 4-ft Parshall flume if the gage reading h_1 is 1.25 m under free-flow condition.

Solution $B = 4$ ft $h_1 = 1.25$ m $= 1.25 \times 3.28 = 4.10$ ft

$$\therefore\ Q = 4Bh_1^{1.522B^{0.026}} = 4 \times 4 \times 4.10^{1.522 \times 4^{0.026}} = 148.254\, ft^3/\sec = 4.202 m^3/s$$

PROBLEMS

8.1 Find the discharge through a vertical sluice gate in a horizontal rectangular channel 6 m wide and having a gate opening of 1 m under an upstream head of 4 m (i) for free flow condition, and (ii) for submerged condition when the tailwater depth is 3.25 m. (iii) Also compute the depth of submergence by Eq.(8.9).

8.2(a) A horizontal rectangular channel carries a discharge of 1.30 m^2/s. There is a vertical sluice gate in the channel. What would be the height of the gate opening to pass the stated flow when the upstream head is 4 m and the gate operates under free flow condition?

(b) Assuming that the flow though the sluice gate in Prob. 8.3(a) occurs under submerged condition and the tailwater depth is 3.2 m, what would be the upstream depth h_1 and the submergence depth h_3 if the discharge in the channel remains the same?

(c) Compute the force on the sluice gate in Problems 8.3(a) and 8.3(b).

8.3 Derive Eqs.(8.11) to (8.13).

8.4 A horizontal channel ends in a free overfall. The brink depth is measured and found to be 0.50 m. Compute the discharge if the channel is

i) rectangular with $b = 6$ m,
ii) triangular with $s = 2$,
iii) parabolic with perimeter equation $y^2 = 4z$,
iv) circular with $d_0 = 2$ m, and
v) trapezoidal with $b = 6$ m and $s = 2$.

8.5. An open channel having a slope of 0.0025 ends in a free overfall. The brink depth is measured and found to be 0.50 m. Compute the discharge if the channel is

i) rectangular with $b = 6$ m,
ii) triangular with $s = 2$,
iii) trapezoid with $b = 6$ m and $s = 2$.

8.6 A broad-crested weir with vertical faces is 1 m high, has a crest length of 2 m and spans the entire width of the channel. If the head over the weir is 0.80 m, compute the discharge per unit width. What would be the discharge per unit width if the depth downstream of the weir is (i) 0.60 m, and (ii) 0.75 m?

8.7 A broad-crested weir with sharp square corner and vertical faces at the upstream and spanning the full width of a rectangular channel has a crest length of 2.50 m. The height of the weir is 1.50 m and the channel is 6 m wide. Calculate the depth of flow upstream of the weir when the discharge is (i) 6 m^3/s, and (ii) 10 m^3/s, assuming free flow.

8.8 A rectangular channel 10 m wide is to carry a discharge of 15 m^3/s. It is desired to place a broad-crested weir across the channel. Compute the height of the weir and its crest length.

8.9 Compute the discharge over a suppressed sharp-crested weir of height 0.50 m and constructed in 2 m wide rectangular channel. The head over the weir is 0.50 m.

8.10 Compute the discharge over a contracted sharp-crested weir 2 m wide and 0.50 m high constructed in a 2.5 m wide rectangular channel. The head over the weir is 0.50 m.

8.11 The depth and discharge in a rectangular channel 2 m wide are 0.75 m and 0.50 m^3/s, respectively. Find the height of a supressed sharp-crested weir that will pass the channel discharge.

8.12 In Problem 8.10, if a contracted sharp-crested weir of width 1.6 m is used, what would be the height of the weir?

8.13 Estimate the discharge over a suppressed sharp-crested weir spanning the full width of a rectangular channel 2 m wide when the depths upstream and downstream of the weir above the weir crest are 1.6 m and 1.3 m, respectively, and the height of the weir is 1m.

8.14 Show from Eqs.(8.29) and (8.32) that $C_c = 0.715$ for the completely free overfall ($p = 0$).

8.15 Compute the theoretical discharge through a Venturi flume having a throat width of 0.65 m, (i) when the critical depth measured at the throat is 0.45 m under free-flow condition, and (ii) when the upstream and downstream depths measured are 0.60 m and 0.50 m, respectively, and the channel width is 0.75 m, under submerged condition.

8.16 Determine the discharge through a 5-ft Parshall flume if the gage reading h_1 is 1.32 m under free-flow condition.

****

ANSWERS TO THE PROBLEMS

Chapter 1

1.1(a) Upstream: subcritical turbulent, Downstream: supercritical turbulent (b) Subcritical turbulent

1.2 (i) 3 m^2/s, subcritical turbulent, 0.13 m/s upstream, 6.13 m/s downstream
(ii) 18 m^3/s, subcritical turbulent, 0.13 m/s upstream, 6.13 m/s downstream
(iii) 24 m^3/s, supercritical turbulent, 0.20 m/s downstrtem, 5.80 m/s downstrem
(iv) 6 m^3/s, supercritical turbulent, 0.79 m/s downstream, 5.21 m/s downstream
(v) 8 m^3/s, supercritical turbulent, 0.44 m/s downstream, 5.56 m/s downstream
(vi) 5.50 m^3/s, supercritical turbulent, 0.29 m/s downstream, 5.71 m/s downstream

1.3(a) (i) 4286 s (ii) 5000 s (b) 20,015 m^3

1.4(a) (i) 4.4 m^2/s (ii) Subcritical turbulent (b) (i) 10 m^2/s (ii) Subcritical turbulent

1.5(a) $\alpha = 2.25$, $\beta = 1.50$ (b) $\alpha = 1.56$, $\beta = 1.25$

1.6(a) $\alpha = \pi^2/6 = 1.645$, $\beta = \pi^2/8 = 1.234$, Ratio = 2.76
(b) $\alpha = (n+1)^3/(3n+1) = 1.045$, $\beta = (n+1)^2/(2n+1) = 1.016$, Ratio = 2.83

1.7 (i) $\alpha = 2.000$, $\beta = 1.330$, Ratio = 3.00 (ii) $\alpha = 1.111$, $\beta = 1.037$, Ratio = 3.00
(iii) $\alpha = 1.350$, $\beta = 1.125$, Ratio = 2.80 (iv) $\alpha = 1.118$, $\beta = 1.041$, Ratio = 2.89

1.8(a) $q = 20.597$ m^2/s, $\overline{U} = 3.433$ m/s, $\alpha = 1.181$, $\beta = 1.078$
(b) $q = 1.018$ m^2/s, $\overline{U} = 0.885$ m/s, $\alpha = 3.450$, $\beta = 1.906$

1.9(a) $p = 980.83$ N/m^2, F = 155.07 N, O.M. = 16.35 m-N (b) Not correct, (i) F = 155.07 N, O.M. = 16.35 m-N (ii) Error for F = +3,063%, Error for O.M. = +9,900%
(c) $p_1 = 7{,}697.50$ N/m^2, $p_2 = 3{,}018.46$ N/m^2, $p_3 = 11{,}922.50$ N/m^2, $p_4 = 9{,}810$ N/m^2

Chapter 2

2.2 15.93 m, $F_x = 0$, $F_z = 441{,}941$ N

2.3 (b) i) $Q = 47.54$ m^3/s ii) $F_{actual} = 159.06$ kN, $F_{hydrostatic} = 163.91$ kN

2.4 Thrust on each pier = 2,268 kN

Chapter 3

3.1 (i) $h_c = 0.85$ m, $E_{min} = E_c = 1.30$ m (ii) $h_c = 2$ m, $Q_{max} = 53.20$ m^3/s

3.2 (i) M = 3.25 (ii) M = 3.74

3.4 (i) $h_c = 1.222$ m, $U_c = 3.372$ m/s (ii) $h_c = 1.083$ m, $U_c = 3.079$ m/s
(iii) $h_c = 1.417$ m, $U_c = 2.490$ m/s (iv) $h_c = 1.863$ m, $U_c = 3.298$ m/s

3.5(a) $h_c = 1.23$ m, $U_c = 2.883$ m/s (by all methods)
(b) (i) $h_c = 0.950$ m, $U_c = 2.600$ m/s (ii) $h_c = 0.977$ m, $U_c = 2.501$ m/s

3.7(b) $Q = 30.688$ m^3/s, E = 2.333 m, $h_c = 1.387$ m

3.8 (i) 28.933 m^3/s (ii) 7.172 m^3/s (iii) 15.344 m^3/s (iv) 42.270 m^3/s (v) 11.13 m^3/s

3.9 1 m

3.11 (a) $\Delta z_c = 0.655$ m
(b) (i) $h_2 = 1.98$ m, Drop in water level = 0.12 m
(ii) $h_1' = 2.67$ m, Drop in water level = 0.51 m
(iii) $h_2 = 2.96$ m, Rise in water level = 0.06 m

3.12 (a) $b_c = 3.96$ m
(b) (i) $h_2 = 2.38$ m, Drop in water level = 0.12 m
(ii) $h_1' = 3.12$ m, Drop in water level = 0.95 m
(iii) $h_2 = 2.60$ m, Rise in water level = 0.10 m

3.13 Expansion in width = 0.70 m 3.14 Bottom is to be raised by 0.39 m

3.15 Minimum river width = 1840 m 3.16(a) 0.448 m^3/s (b) 0.181 m^3/s

Chapter 4

4.1 0.368h from channel bottom (0.632h from free surface)

4.2(a) (i) $Q = 8.838\ m^3/s$ (ii) $C = 42.1\ m^{1/2}/s$, $f = 0.044$, $\tau_0 = 1.334\ N/m^2$, $u^* = 0.0365$ m/s
(iii) $k_s = 0.0759$ m, Hydraulically rough, Manning formula is applicable
(b)(i) $n = 0.016$, $C = 89.57\ m^{1/2}/s$, $f = 0.010$, $u^* = 0.059$ m/s, $\tau_0 = 3.34\ N/m^2$
(ii) $k_s = 0.0012$ m, Hydraulically rough

4.3(b) (i) $n = 0.030$ (ii) $u^* = 0.072$ m/s, $z_0 = 0.00771$ m, $u_z = 0.18 \ln(129.67z)$ when $z \geq z_0$

4.4(a) (i) 3 (ii) 3.598 (iii) 4.143 (iv) 3.395
(b) (i) 2.75 (ii) 3.323 (iii) 3.857 (iv) 3.058

4.5(a) $h_n = 1.275$ m, $U_n = 2.35$ m/s (b) $h_n = 1.0217$ m, $U_n = 3.915$ m/s

4.6(a) $Q = 1.054\ m^3/s$ (b) $h_n = 1.8285$ m, $U_n = 1.4953$ m/s

4.7 (i) 0.40 m (ii) 0.41 m (iii) 1 m (iv) 0.57 m (v) 0.62 m

4.8 (i) 3.301 m^3/s (ii) 4.994 m^3/s (iii) 0.557 m^3/s (iv) 1.238 m^3/s (v) 0.48 m^3/s

4.9 (i) 9.21 m (ii) 10.37 m

4.10 (i) $h_n = 1.36$ m, $U_n = 2.022$ m/s (ii) $h_n = 1.55$ m, $U_n = 2.127$ m/s
(iii) $h_n = 1.84$ m, $U_n = 2.016$ m/s (iv) $h_n = 1.30$ m, $U_n = 1.387$ m/s

4.11(a)(i) $S_n = 0.0062$ (ii) $S_c = 0.0071$ (iii) $S_c = 0.0071$
(b)(i) $S_n = 0.0062$ (ii) $S_c = 0.0072$ (iii) $S_c = 0.0063$

4.13 $Q = 1052.35\ m^3/s$, $U = 1.571$ m/s, $n = 0.026$, $\alpha = 1.621$, $\beta = 1.221$

4.14 17.675 m^3/s (i) 22.49 m^3/s (ii) 22.895 m^3/s (iii) 33.99 m^3/s

4.15 0.21 m^3/s 4.16 30,690 m^3/s

Chapter 5

5.2(a)(i) $P = 17.889$ m, $R = 2.236$ m (ii) $P = 17.889$ m, $R = 2.236$ m
(iii) $P = 16.647$ m, $R = 2.403$ m (iv) $P = 15.853$ m, $R = 2.523$ m
(v) $P = 17.369$ m, $R = 2.303$ m Circular section has the minimum wetted perimeter

5.3(a)(i) $P = 7.256$ m (ii) $P = 7.256$ m (iii) $P = 6.632$ m (iv) $P = 6.239$ m
(v) $P = 6.994$ m Circular section has the minimum wetted perimeter

5.4(a) $h = 2.408$ m, $b = 1.994$ m (b) $b = 0.472$ m, $S_0 = 9.278 \times 10^{-3}$

5.5(i) $h = 3.098$ m (ii) $h = 1.824$ m, $b = 18.124$ m

5.6 (i) $b = 9.67$ m, $h = 1.61$ m (ii) $b = 14.76$ m, $h = 1.30$ m

5.7 $b = 21.150$ m, $h = 1.163$ m

Chapter 6

6.3 (i) M1, M3 (ii) C1, C3 (iii) S1, S3 (iv) H2, H3 (v) A2, A3
6.4(a) (i) M2 (ii) C2 (iii) None (iv) H2 (v) A2
(b) M2 when $h_d < h_n$ and M1 when $h_d > h_n$
(c) Jump and S1 when $h_d > h_c$ and none when $h_d < h_c$
6.5 (a) M1, H2 (b) H2 (c) S1 or H3, H2 (d) C1, H2
(e) H2, A2 (f) M2,C2 (g) A2 (h) C2, S2
(i) M2 (j) H2, C2 (k) C3 (l) A2,S2
6.6 (a) M1, M2, S2 (b) C2, S2, S1 or M3 (c) S1 or M3, M1 (d) H2, M2, C2
(e) C3, C2,C1 (f) C1, H2, S2 (g) M1, H2, C2 (h)M2, S2, S3
(i) C1, A2, H2 (j) H2, A2, S2 (k) M1,A2, H2 (l) M2, C2, S2
6.7 (a) A3, A2, H2, S2 (b) M1, M3, M2, S2, S1 or H3, H2 (c) M1, M3, M1, M3
d) S1, S3, S1 or M3, M2 (e) M2, M1, M1, M3, M2 (f) S1, S3, S1, S3
6.8 (a) (i) M1 (ii) S3 (b)(i) M2 (ii) S2
6.9 (i) M2, S2 (ii) M2 (in upstream channel) (iii) S1 or H3
(iv) S3 (in downstream channel) (v) H2
6.10(a) M2, S2, S1, M3 (b) M2, S2, S1, M3 (c) M2, S2, S1 or M3
6.11(a) – 6,484.22 m (b) – 14,816.25 m 6.12(a) 99.63 m (b) 43.85 m
6.13(a) 60.09 m upstream (b) 2.22 m (c) (i) 2.3396 m (ii) 2.1653 m
6.14(a) 50.41m upstream (b) 4.90m

Chapter 7

7.2(a)

Fr_1	h_2/h_1	h_L/E_1	h_j/E_1	Fr_1	h_2/h_1	h_L/E_1	h_j/E_1
1	1	0	0	1.7	1.96	0.05	0.39
2.5	3.07	0.18	0.50	4.5	5.88	0.44	0.44
9	12.24	0.70	0.27	15	20.71	0.82	0.17

7.3(a)(i) 2.30 m (ii) weak jump (iii) 1.30 m (iv) L_j = 9.66 m using Fig. 7.3 and L_j = 9.28 m by Silvester formula (v) 116.2 (vi) 91.78%
(b) 4.71 m, steady jump, 4.21 m, L_j = 28.7 m using Fig. 7.3, 62.1%
7.4 For the two given variables, values of other variables in the table are obtained.
7.5 1.88 m, 0.12 m 7.6 3.05 m 7.7 2.45 m 7.8(a) 0.438 m (b) Jump type B
7.9(i) Jump type A (ii) Jump type B (iii) Jump type D

Chapter 8

8.1(i) 30.20 m^3/s using Eq.(8.3) and 29.60 m^3/s using Eq.(8.6) (ii) 16.85 m^3/s (iii) 3.07 m
8.2(a) 0.25 m (b) 6.70 m, 2.83 m (c) 67,706.46 N, 180,557.30 N
8.4(i) 10.99 m^3/s (ii) 1.39 m^3/s (iii) 2.86 m^3/s (iv) 2.137 m^3/s (vi) 12.20 m^3/s
8.5(i) 11.85 m^3/s (ii) 1.51 m^3/s (iii) 13.93 m^3/s
8.6 1.093 m^2/s (i) 1.093 m^2/s (ii) 0.732 m^2/s 8.7(i) 0.77 m (ii) 1.07 m
8.8 2.50 m, 3.50 m 8.9 1.443 m^3/s 8.10 1.347 m^3/s 8.11 0.49 m 8.12 0.45 m
8.13 5.318 m^3/s 8.15(i) 0.614 m^3/s (ii) 0.218 m^3/s 8.16 5.80 m^3/s

REFERENCES

Bradley, J.M. and Paterka, A.J., The hydraulic design of stilling basins, J. Hyd. Div., Proc. ASCE, Vol. 83, No. HY5, 1957.

Chow, V.T., Open Channel Hydraulics, McGraw-Hill Book Co. Inc., New York, 1959.

Churchhouse, R.F. (ed.), Handbook of Applied Mathematics, Vol. III: Numerical Methods, John Wiley and Sons, New York, 1981.

Delleur, J.W., Dooge, J.C. I. and Gent, K.W., Influence of slope and roughness on the free overfall, J. Hyd. Div., Proc. ASCE, Vol.82, No.8, Aug., 1956, 30-35.

Diskin, M.H.,The end depth at a drop in trapezoidal channels, J. Hyd. Div., Proc. ASCE, Vol. 87, No.7, 1961, 11-32.

French, R.F., Open Channel Hydraulics, McGraw-Hill Book Co. Inc. New York, 1986.

Garde, R. J. and Ranga Raju, K. G., Mechanics of Sediment Transportation and Alluvial Stream Problems, Wiley Eastern, New Delhi, 1977.

Gill, M.A., Exact solution of gradually varied flow, J. Hyd. Div., Proc. ASCE, Vol. 102, No. HY9, 1976, 1353-1364.

Henderson, F. M., Open Channel Flow, Macmillan Company, New York, 1966.

Horton, R.E., Separate roughness coefficients for channel bottom and sides, Engineering News – Record, Vol. 111, No. 22, 1933, 651 – 653.

Kindsvater, C.E. and Carter, R.W., Discharge characteristics of rectangular thin plate weirs, J. Hyd. Div., Proc. ASCE, Vol. 83, No. HY12, 1957, 1-36.

Kennedy, R.G., The prevention of silting in irrigation canals, Proc. Instn. Civil Engrs., London, Vol. 119, 1895, 281-290.

Lacey, G., Stable channels in alluvium, Proc. Instn. Civil Engrs., London, Vol. 229, 1930, 259-384.

Lacey, G., A general theory of flow in alluvium, Proc. Instn. Civil Engrs., London, Vol. 27, 1946, 16-47.

Lane, E.W., Design of stable channels, Trans. ASCE, Vol.120, 1955, 1234-1260.

Lindley, E. S., Regime channels, Proc. Punjab Engineering Congress, Lahore, India, Vol. 7, 1919. 63-74.

Ranga Raju, K.J., Flow through Open Channels, Tata McGraw Hill Publishing Co. Ltd., New Delhi, 1993.

Rajaratnam, N., Hydraulic jumps, Advances in Hydroscience, Vol. 4, Academic Press, New York, 1967, 197 – 280.

Rajaratnam, N. and Muralidhar, D., The trapezoidal free overfall, J. Hyd. Res., IAHR, Vol. 8, No. 4, 1970, 419-447.

Rajaratnam, N. and Subramanya, K., Flow equation for the sluice gate, J. Irrigation and Drainage Division, Proc. ASCE, Vol. 93 , No. HY9, 1967, 167-186.

Rouse,A., Discharge characteristics of the free overfall, Civil Engineering, Vol.6, No.7, 1936, 257-260.

Silvester, R., Hydraulic jump in all shapes of horizontal channels, J. Hyd. Div., Proc. ASCE, Vol. 90, No. HY1, 1964, 23 – 55.

Subramanya, K., Flow in Open Channels, Tata McGraw-Hill Publishing Company, New Delhi, 1997.

Vanoni, V.A., Velocity distribution in open channels, Civil Engineering, ASCE, Vol.11, No.6, 1941, 356-357.

Watts, F.J., Simons, D. B. and Richardson, E.V., Variation of α and β values in a lined open channel, J. Hyd. Div., Proc. ASCE, Vol. 93, No. HY6, 1967, 217-233.

Yalin, M.S. and Karahan, E., Inception of sediment transport, J. Hyd. Div., Proc. ASCE, Vol. 105, No. HY11, 1979.

WRE 301 OPEN CHANNEL FLOW (4 hours/week)

Open channel flow and its classification. Velocity and pressure distributions. Energy equation, specific energy and transition problems. Critical flow and control. Principles of flow measurement and devices. Concept of uniform flow, Chezy and Manning equations, estimation of resistance coefficients and computation of uniform flow. Momentum equation and specific momentum. Hydraulic jump. Theory and analysis of gradually varied flow. Computation of flow profiles. Design of channels.

REFERENCE BOOKS

V. T. Chow, 1959, Open Channel Hydraulics, McGraw-Hill Book Co. Inc., New York.

F. M. Henderson, 1966, Open Channel Flow, Macmillan Co., New York.

R. H. French , 1986, Open Channel Hydraulics, McGraw- Hill Book Co. Inc., New York.

L. C. Van Rijn, 1990, Principles of fluid flow and surface waves in rivers, estuaries, seas and oceans, Aqua Publications, Amsterdam.

M. H. Chaudhry, 1993, Open Channel Flow, Prentice Hall of India Private Ltd., New Delhi.

K. G. Ranga Raju, 1993, Flow through Open Channels, Tata McGraw-Hill Publishing Co. Ltd., New Delhi.

K. Subramanya, 1997, Flow in Open Channels, Tata McGraw-Hill Publishing Co. Ltd., New Delhi.

S. C. Sutradhar, 2003, Open Channel Hydraulics, EPC Ltd. Office, Dhaka.