Chapter XV

FORCE SYSTEMS THAT PRODUCE RECTILINEAR MOTION

218. Introduction. We are now prepared to return to the consideration of the effects of forces on particles and bodies. In this study, we shall be dealing with external unbalanced force systems which invariably cause changes in the motion of the bodies on which they act. This branch of study is called ***kinetics.*** In this chapter, we shall deal with coplanar force systems (or systems which can be reduced to or considered as coplanar systems) for which the resultant is a *force*. In these systems of forces, there is *no resultant couple.* Renewing this study of forces, we recall that a free body carefully sketched with the external force vectors in their proper relationships is a most effective aid to clear thinking.

219. Sir Isaac Newton. Even the briefest mention of Newton (1642-1727) and his contributions to science is too long for a footnote. Hence, a few words here about the man from whose laws (§ 220) the science of analytic mechanics may be derived.

Many people credit Newton (born on Christmas in the year of Galileo's death) with being the greatest scientist of all times. It would certainly be difficult to prove any one else greater. He was born at Woolsthorpe, England, of parents who were farmers of moderate means. His mechanical talent and his lack of interest in farming became apparent early. Among his grammar school achievements were a water clock and sun dials. Within two years after receiving his degree from Trinity College, Cambridge, he discovered the binomial theorem, took the first steps toward the invention of calculus, started experiments on color, and had begun to speculate on gravitation. His achievements were so varied and numerous that only a few will be mentioned: the reflecting telescope, the composite nature of sunlight, a science of optics, the invention of a thermometer, invention of fluxions (forerunner of modern calculus), and, of course the most monumental, the law of universal gravitation.

The idea that the sun and not the earth was the center of our universe was generally accepted at the time of Newton, and some of Newton's scientific predecessors had speculated upon the notion of universal gravitation, but no proof had been devised. Newton's first calculation in 1666 to establish the concept of universal gravitation was based on an erroneous estimate of the size of the earth. Had not M. Picard made a more accurate estimate of the earth's size in the nick of time, there is no telling how much the development of the science of mechanics might have been delayed. Having learned of Picard's measurement in 1682, Newton returned to this problem with renewed hope. The calculation was a comparatively simple one. It showed that the acceleration governing the falling of a stone also prevented the moon from moving in a straight line. Moving in a closed orbit, the moon has a normal or centrifugal (§ 236) acceleration toward the earth because of the attractive force between masses. With the calculation partially completed, he saw that his theory was going to be verified. He became so jittery that a friend had to complete the calculation for him.

Newton was active in public life in many ways. In 1695 he was made Warden of the Mint of England and he received a knighthood in 1705. He laid no claim to unusual sagacity and possessed none of those eccentricities sometimes found in genius, unless absent mindedness be classed as eccentric. He attributed his successes to application and patient thought. He was very religious and very tolerant of everything except intolerance.

220. Newton's Laws of Motion. As we have learned, Galileo established the kinematic relations of displacement, velocity, acceleration, and time for bodies moving under the influence of a constant force. It was evident from his work that a force was necessary to cause a body to change its motion, but it was not until some one hundred years later that Newton formulated the laws which related the force to the motion. ***Newton's laws*** may be stated as follows:

I. Every particle remains in a state of rest or moves with a constant velocity in a straight line unless an unbalanced force acts on it. (We recognized this statement as defining the condition of equilibrium ($R = O$), a special case.)

II. The acceleration of a particle is directly proportional to the resultant force acting on it and inversely proportional to its mass, and the sense of the acceleration is the same as that of the resultant force. (This is the basic condition of kinetics.)

III. To every action, there is an equal and opposite reaction.

Considering the first law, we see that a resultant force is necessary to cause a body to change its speed or to make it move in other than a straight line. This property of a body or particle which causes it to resist a change in its motion is called its ***inertia.*** Thus, if an unbalanced (resultant) force acts upon a particle, it may be that only the speed of the particle will be changed (if the line of action of the resultant force is in the direction of motion), or only the direction of the velocity may be changed, *or* the velocity may be changed both in magnitude and in direction.

The second law, which gives quantitative expression to the first law, may be expressed in mathematical form. Keeping in mind that a "particle" is a hypothetical something which has weight and mass but occupies no space, let R represent the resultant force, a the acceleration, and m the mass of a particle; then Newton's second law says

$$\text{(a)} \qquad a \propto \frac{R}{m} \qquad \text{or} \qquad a = C\frac{R}{m},$$

where C is a constant of proportionality.

In statics, the action and reaction of the third law refers, for example, to the action of a body A on a body B and the equal and opposite reaction (force) of the body B on the body A. In kinetics, the reaction may be not only in the form of an equal and opposite force, but also in the form of a change in the motion of the body or particle being acted upon.

221. Units. Mechanics is a rational subject. In any equation of mechanics the units must balance; that is, if one side of the equation is in pounds, the other side must be in pounds also. Thus, if we write Newton's second law in the form $R = C'ma$, where C' is the reciprocal of the constant C in equation (**a**), the unit of R must be the same as the unit of $C'ma$. As a matter of fact, we may drop the constant C (or C') altogether, and write the equation in the form

$$\text{(b)} \qquad R = ma,$$

in which case, the units of any two of the quantities R, m, and a must define the units of the other quantity. From equation (**b**), we may say that a unit force is that force which gives to a unit mass a unit acceleration. In fact, such a relation of units exists in the centimeter-gram-second system, where a unit of mass is a gram, a unit of acceleration is a centimeter per second-second, and the corresponding unit of force is a *dyne*. This system of units, which is used for all engineering and scientific work in continental Europe, is commonly used in scientific work and in electrical measurements in this country.

However, the English system of units is not so simple. Some groups, the engineers among them, generally take the unit of force as a pound and the unit of acceleration as feet per second-second. With these two units defined, the unit of mass must be pound-second-second per foot, or a slug (§ 156), if the constant C is to be unity. The scientists often use the unit of mass as a pound, in which event, the unit of force must be

$$R = ma \qquad \frac{\text{lb-ft.}}{\text{sec.}^2}, \qquad \text{[Not used in this text.]}$$

a unit called a ***poundal.*** We shall of course continue to use the pound as the unit of force.

Suppose that a single force R_1 acts on a particle of mass m and produces an acceleration of a_1, in accordance with Newton's second law. If another single force R_2 acting on the *same* particle, produces an acceleration a_2, we may write (from Newton's second law)

$$\frac{R_1}{R_2} = \frac{a_1}{a_2},$$

where R_1 and R_2 are any two resultant forces. We know that if only the force of gravity acts on a particle, its acceleration is the acceleration of gravity, g. The force of gravity W is the weight of the particle. Thus, we know (or can easily measure) the value of one force W and the value of the corresponding acceleration g. Then with any force R producing an acceleration a, we get the proportion

$$\text{(c)} \qquad \frac{R}{a} = \frac{W}{g} \qquad \text{or} \qquad R = \frac{W}{g}a.$$

If a consistent system of units is used, so that, in equation (**a**), $C = 1$, we may compare equations (**c**) and (**b**) and conclude that if the pound is the unit of force, the mass is

$$\text{(d)} \qquad m = \frac{W}{g} \qquad \frac{\text{lb.}}{\text{ft./sec.}^2} = \frac{\text{lb-sec.}^2}{\text{ft.}} \text{ (slug)}.$$

Thus, the equation that we shall use in solving problems of motion involving force, mass, and acceleration is

$$\text{(43)} \qquad R = ma = \frac{W}{g}a,$$

where the resultant force R and the acceleration a, *both vector quantities'* have the *same sense*. If the resultant force is constant, the acceleration is constant. If the resultant force varies, the acceleration varies.*

222. Component Forces and Accelerations. In the solution of problems, we often find it convenient to choose certain coordinate axes. If neither of these axes is in the direction of the resultant acceleration, the relations between the component accelerations and the component forces are

$$\text{(e)} \qquad \Sigma F_x = \frac{W}{g}a_x,$$

$$\text{(f)} \qquad \Sigma F_y = \frac{W}{g}a_y,$$

where

$$[(\Sigma F_x)^2 + (\Sigma F_y)^2]^{1/2} = R \qquad \text{and} \qquad (a_x^2 + a_y^2)^{1/2} = a.$$

*A somewhat longer, but still brief, discussion of units is found in Appendix B.

If it happens, for example, that $a_x = a$, the absolute acceleration, then $a_y = 0$ and $\Sigma F_y = 0$. In the analysis of the motion of a particle in a plane, there are therefore two independent conditions (the force system is a concurrent system). These conditions are expressed by equations (**e**) and (**f**), where either a_x or a_y may be zero. The values of ΣF_x and ΣF_y are found just as explained in the chapters on statics from a free body of the member being studied. Not only should the student use the principles of the free body, but he should also be prepared to use the kinematic relations previously developed, because many problems in kinetics cannot be solved without the application of the principles of kinematics.

223. Example. A body A weighing 20 lb. is resting on a 45° incline for which $f = 0.2$ (kinetic friction). A horizontal force $Q = 10$ lb. acts on the body as shown in Fig. 512. If the body starts from rest, what is its velocity after 5 sec.?

Solution. In this type of problem, the body may be considered as a particle. First, we note that in order to obtain v_A, we must know the acceleration of A. Second, we expect that a may be found from the equation $R = ma$. Therefore, draw a free body. We are sure that W acts vertically downward, that N acts normal to the plane. The direction of Q is specified. Since we are not sure in what direction the body will move, we are not sure in what sense to point the frictional force. However, the component of W down the plane is (20)(0.707) = 14.14 lb., which tends to cause the body to move downward. The component of Q up the plane is (10)(0.707) = 7.07 lb. Hence, *if* any motion occurs, it will be down the plane and therefore F points upward. Noting that the motion is entirely parallel to the plane and that consequently $\Sigma F_y = 0$, we find

$$\Sigma F_y = N - Q \sin 45° - W \cos 45° = 0,$$

from which

$$N = (10)(0.707) + (20)(0.707) = 21.21 \text{ lb.}$$

Therefore, if the bodies move, $F = fN = (0.2)(21.21) = 4.242$ lb. Using $\Sigma F_x = ma$, we get (upward direction along the plane is positive)

$$\Sigma F_x = Q \cos 45° + F - W \sin 45° = \frac{W}{g} a$$

$$= 7.07 + 4.24 - 14.14 = \frac{20}{g} a,$$

from which $a = -4.56$ fps², a constant acceleration as long as the forces remain constant. Substituting a in the sum ΣF_x with a *positive* sign is tantamount to assuming that the acceleration is *up* the plane. The negative sign for the answer shows that it is down the plane. With this acceleration, the velocity after 5 sec. is

$$v = at = (-4.56)(5) = -22.8 \text{ fps}$$

down the plane.

224. Example. This example is the same as that in § 223 except that $Q = 25$ lb.

Solution. Summing forces in the y direction and solving for N as before, Fig. 512, we get

$$N = (25)(0.707) + (20)(0.707) = 31.8 \text{ lb.}$$

The maximum frictional force for $f = 0.2$ is

$$F_{\text{kinetic}} = (0.2)(31.8) = 6.36 \text{ lb.}$$

If there is any doubt as to whether motion will occur, a sum of the forces parallel to the plane, omitting the frictional force, compared to the frictional force, will tell. With the upward direction as positive, we find

$$Q \cos 45 - W \cos 45 = (25)(0.707) - (20)(0.707) = 3.5 \text{ lb.}$$

This is the net motivating force. Since it (3.5 lb.) is less than the kinetic friction (6.36 lb.) to be overcome, motion does *not* occur.

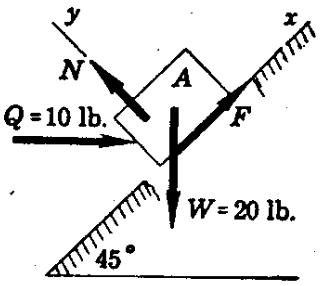

Fig. 512.

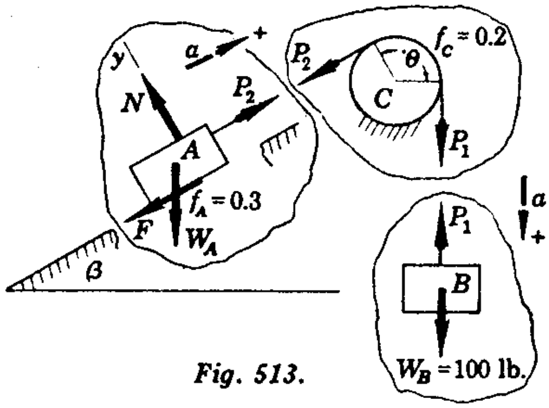

Fig. 513.

225. Example. A body A weighing $W_A = 50$ lb. is on a plane inclined at an angle of $\beta = 30°$ (Fig. 513). The coefficient of friction on this plane is $f_A = 0.3$. A cable attached to this body passes over a stationary member C, for which the coefficient of friction is $f_C = 0.2$. From the other end of the cable is suspended a body B weighing $W_B = 100$ lb. What are the tensions P_1 and P_2 in the cable and what is the speed of the bodies after they move 20 ft. from rest? The weight of the cable is negligible.

Solution. Evidently, if motion occurs, the body B moves downward, pulling the body A up the plane. To solve this problem, break up the system into three free bodies. Starting with the free body of A, we get

$$\Sigma F_y = N - 50 \cos 30° = 0 \qquad \text{or} \qquad N = 43.3 \text{ lb.}$$

Therefore, $F = fN = (0.3)(43.3) = 12.99$ lb. Taking the direction of motion as positive and summing the forces parallel to the plane, we find

$$\Sigma F_x = P_2 - F - W_A \sin 30° = \frac{W_A}{g} a$$

$$= P_2 - 12.99 - 25 = \frac{50}{g} a,$$

(g) $$\Sigma F_x = P_2 - 37.99 = \frac{50}{g} a.$$

In this equation, there are two unknowns. Hence another equation is needed for a solution. The relation between P_1 and P_2 as the cable slides across the member C is given by equation (20), p. 112, from which we find ($\theta = 120° = 2.09$ rad.)

(h) $$P_1 = P_2 e^{f\theta} = P_2 e^{(0.2)(2.09)} = 1.52 P_2.$$

Now there are three unknowns, P_1, P_2, and a, so that still another equation is needed. This equation is obtained from a sum of the vertical forces ΣV on the free body of B.

The positive direction, which has been taken in the *direction of motion*, is now *downward.*

(i) $$\Sigma V = 100 - P_1 = \frac{100}{g} a.$$

Observing that the acceleration of B is the same as that of A, provided the cable is inextensible, we may write $a_A = a_B = a$. To solve the equations (g), (h), and (i), we substitute the value of P_1 from (h) into (i), and obtain

(j) $$100 - 1.52P_2 = \frac{100}{g} a.$$

Equations (g) and (j) may be solved simultaneously for P_2 and a. Doing so, we find

$$P_2 = 50 \text{ lb.} \qquad \text{and} \qquad a = 7.73 \text{ fps}^2.$$

The value of $P_1 = 76$ lb. may be found from either (h) or (i). Since the forces are constant, the acceleration is constant and the speed of the bodies after they have moved 20 ft. is found from the equation

$$v^2 = 2as = (2)(7.73)(20) = 309.2,$$

whence $v = 17.6$ fps.

226. Motion of the Center of Gravity of a Rigid Body. Consider a body of mass m (Fig. 514) in plane motion. If this body is composed of particles whose masses are $dm_1, dm_2, \cdots dm_n$, then

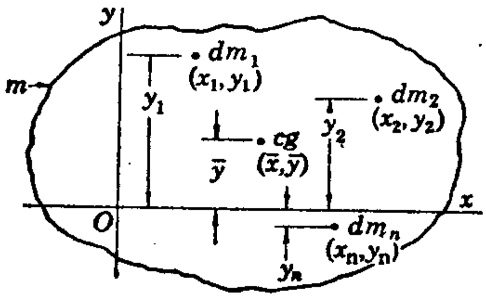

Fig. 514.

$$m = dm_1 + dm_2 + \cdots dm_n = \Sigma dm.$$

No matter what the nature of the motion of a particular particle may be at a certain instant, the resultant force dR on the particle is equal to its mass times its acceleration, that is,

$$dR = (dm)a$$

in accordance with Newton's second law. This force $dR = (dm)a$ is called the ***effective force*** for the particle. Referring the motion to any *fixed x* and y axes, Fig. 514, and using components in these directions we may say for any particle dm in general, that

$$dR_x = (dm)a_x \qquad \text{and} \qquad dR_y = (dm)a_y,$$

where dR_x is the x component of the effective force dR on the particle and a_x is the x component of the acceleration a of the particle, and where the y subscript refers to similar components in the y direction. Considering the x direction only for the moment, we may write for the individual particles

$$dR_{x1} = dm_1 a_{x1}, \qquad dR_{x2} = dm_2 a_{x2}, \qquad dR_{xn} = dm_n a_{xn}.$$

Then by addition, we find

$$dR_{x1} + dR_{x2} + \cdots + dR_{xn} = dm_1 a_{x1} + dm_2 a_{x2} + \cdots dm_n a_{xn}.$$

The sum of the terms on the left side of this equation is seen to be the sum of the x components of the resultant forces for all the particles of the body; that is, this side of the equation may be represented symbolically by $\Sigma\, dR_x$. But the sum of these components for *all* the particles is the x component R_x of the resultant force on the body, $\Sigma dR_x = R_x$. Thus, we have

(k) $$R_x = dm_1a_{x1} + dm_2a_{x2} + \cdots + dm_na_{xn}.$$

Now the x coordinate of the center of mass of the body is defined by the relation (§ 110)

$$m\bar{x} = dm_1x_1 + dm_2x_2 + \cdots + dm_nx_n.$$

Successive differentiations of this equation with respect to time give

$$m\frac{d\bar{x}}{dt} = dm_1\frac{dx_1}{dt} + dm_2\frac{dx_2}{dt} + \cdots + dm_n\frac{dx_n}{dt},$$

$$m\frac{d^2\bar{x}}{dt^2} = dm_1\frac{d^2x_1}{dt^2} + dm_2\frac{d^2x_2}{dt^2} + \cdots + dm_n\frac{d^2x_n}{dt^2}$$

(l) $$m\frac{d^2\bar{x}}{dt^2} = dm_1a_{x1} + dm_2a_{x2} + \cdots + dm_na_{xn},$$

where we recognize that, for example, $d^2x_1/dt^2 = a_{x1}$, the x component of the acceleration of particle 1. The acceleration $d^2\bar{x}/dt^2$ is seen to be the x component of the acceleration of the center of mass of the body, which we may designate by $\bar{a}_x$. We also see that the right-hand sides of (**k**) and (**l**) are identical. Therefore, the left-hand sides are equal; that is,

(m) $$R_x = m\frac{d^2\bar{x}}{dt^2} = m\bar{a}_x.$$

In a like manner, using the y components, we find

(n) $$R_y = m\frac{d^2\bar{y}}{dt^2} = m\bar{a}_y.$$

Then, in terms of the resultant force R and the resultant acceleration $\bar{a}$ of the mass center, we get

(o) $$R = m\bar{a}.$$

It has been shown that, for any body in plane motion, the resultant force on it *is equal to the mass of the body times the acceleration of its center of mass* (or its center of gravity). The resultant force on a *particle* of a body may be either an internal force alone, or it may be the resultant of an external and an internal force. If the particle is in the interior of the body, it is acted upon only by the adjacent particles and the actions of adjacent particles are considered as *internal* forces. If the body is rigid, that is, if the relative positions of the particles remain fixed, these internal forces remain in equilibrium among themselves. Some of the particles on the surface of the body are subjected to *external* forces, which may or may not be in equilibrium. Thus, the resultant of the effective forces acting on all the particles of a rigid body is simply the resultant of the *external* force system. This principle is called ***D'Alembert's principle.**** Therefore, we find R in equation (o), or R_x or R_y, in the usual manner from a free-body diagram showing the *external* forces. It is important to remember that the acceleration in equation (o) is the acceleration of the *center of mass* and that the vectors for R and $\bar{a}$ have the same sense.

227. Location of the Resultant—Body in Rectilinear Translation. It is often convenient to know the location, relative to the body, of the line of action of the resultant. Taking moments of the x components of the effective forces $dR_x = (dm)a_x$ on the particles of Fig. 514 about the xz plane, where the z axis is perpendicular to the page, we have

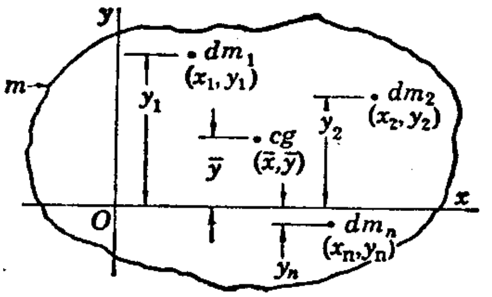

Fig. 514. **Repeated.**

$$(dm_1a_{x1})y_1 + (dm_2a_{x2})y_2 + \cdots + (dm_na_{xn})y_n.$$

For the special case of *rectilinear translation,* we know that $a_{x1} = a_{x2} = a_{xn} = a_x$, since all particles have the same acceleration. The preceding expression then becomes

$$a_x(dm_1y_1 + dm_2y_2 + \cdots + dm_ny_n) = a_x\Sigma(dm)y.$$

We recognize that $\Sigma(dm)y = m\bar{y}$; hence

$$\text{(p)} \qquad a_x\Sigma(dm)y = a_xm\bar{y} = R_x\bar{y},$$

where $a_xm = R_x$, by equation (**m**). Similarly, by taking moments about the yz plane, we may write

$$\text{(q)} \qquad (dm_1a_{y1})x_1 + (dm_2a_{y2})x_2 + \cdots + (dm_na_{yn})x_n = R_y\bar{x}.$$

Since the components R_x and R_y intersect on the line of action of the resultant R, equations (**p**) and (**q**) show that the x and y coordinates of a point on the line of action of the resultant are the same as the coordinates of the center of mass; therefore, the line of action of the *resultant force* on a body in *translation* passes through the body's *center of mass* (cg). Note carefully that this statement holds for a *body in translation* (no angular motion).

*Jean le Rond D'Alembert (1717-1783), a Paris-born mathematician and philosopher, studied law and medicine, but his principal contributions to posterity were in mathematics, especially integral calculus. His book, *Traité de Dynamique,* is important not only for the presentation of the principle which goes by his name, but also for its use of the calculus in the field of mechanics. Not having had calculus as a tool, Newton relied on geometry in his *Principia.* D'Alembert was a contemporary and good friend of Voltaire. He was an excellent musician. It should be mentioned that the job of developing analytic mechanics (the mathematical approach) was practically completed by Lagrange (1736-1813) only a few years after the death of D'Alembert.

228. Inertia Force. If the right-hand term in the equation $R = m\bar{a}$ is transposed to the left-hand side, we have

$$R - m\bar{a} = 0. \tag{r}$$

From this equation, we see that *if* a force equal to $m\bar{a}$, collinear and opposite in sense to the resultant R, is added to a free body, the result is a system of forces in equilibrium (since ΣF in any direction would then be equal to zero). This statement applies when the resultant is not a couple, a case we shall discuss later.

The resultant force is called the effective force; hence, this opposite force $m\bar{a}$ is called a ***reversed effective force*** (REF) or an ***inertia force.*** The inertia force is a ***dynamic reaction*** and it should not be included in a free-body diagram unless the method of solving a problem, as explained in the next article, calls for its addition. If desired, the components of the inertia force, $m\bar{a}_x$ and $m\bar{a}_y$, may be used, though this procedure is seldom advantangeous for bodies in rectilinear motion.

The line of action of the reversed effective force (inertia force) for a body in *translation* passes through the center of gravity (cg). Also notice the statement that the inertia force is a dynamic reaction (Newton's third law) and that it is not a *force* in the same sense as the other external forces. Its significance will become clearer as we proceed through succeeding chapters.

229. Methods of Solving Problems. From the preceding discussion we conclude that either of two plans may be used in solving problems. One plan is to make a free body of only the actual external forces; then the conditions which apply are

$$\Sigma F_x = m\bar{a}_x, \qquad \Sigma F_y = m\bar{a}_y, \qquad \Sigma M_{cg} = 0, \tag{s}$$

[INERTIA FORCE NOT IN FREE BODY]

where we note that the sum of the moments ($\Sigma M_{cg} = 0$) is taken about the cg. The cg is used as a center of moments because it is known to be on the line of action of the resultant. Other points may be used, but we must remember that the $\Sigma M \neq 0$ for an unbalanced system of forces except when the *center of moments is on the line of action of the resultant.*

The other plan for solving problems is to add the inertia force $m\bar{a}$ to the free body and then use the conditions of equilibrium.

$$\Sigma F_x = 0, \qquad \Sigma F_y = 0, \qquad \Sigma M = 0, \tag{t}$$

[INERTIA FORCE INCLUDED IN FREE BODY]

where the x and y directions are any two convenient directions and where ΣM represents the sum of the moments about *any* convenient point. However, the line of action of the inertia force must be properly located; through the *center of mass* (for translation) and its sense is ***opposite*** to that of the acceleration (inertia force = dynamic reaction).

230. Example. A ladle of molten metal is supported by a carriage whose wheels A and B are 12 ft. apart (Fig. 515). The total weight of the ladle and carriage is 1000 lb., the center of gravity of which is 8 ft. below the track. A force $P = 150$ lb. is exerted horizontally at a distance $d = 4$ ft. below the surface of the track. If the frictional force at each wheel is 0.05 of the corresponding normal force, find the acceleration, and find the normal and frictional forces at the wheels. Assume that the ladle and the carriage are rigidly connected.

First Solution. From the free body of Fig. 515, which shows all of the *external* forces, we sum forces in the y direction where the acceleration is zero and get

$$\Sigma F_y = N_A + N_B - 1000 = 0.$$

A sum in the x direction gives

$$\Sigma F_x = P - F_A - F_B = m\bar{a}$$
$$= 150 - F_A - F_B = \frac{1000\bar{a}}{g}.$$

The relations between the frictional and normal forces are

$$F_A = fN_A = 0.05N_A, \qquad F_B = fN_B = 0.05N_B.$$

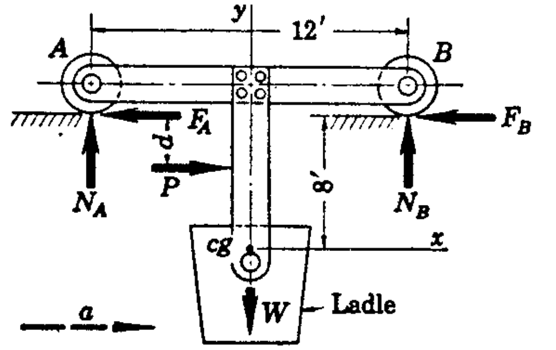

Fig. 515.

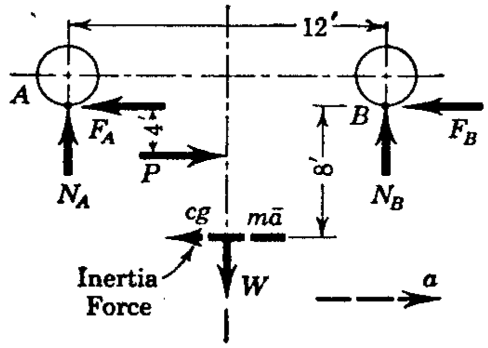

Fig. 516.

Taking moments about the cg, (we recall that the center of moments must be on the line of action of R if $\Sigma M = 0$), we find

$$\Sigma M_{cg} = 6N_A - 6N_B - 8F_A - 8F_B + 4P = 0$$
$$= 6N_A - 6N_B - 8F_A - 8F_B + 600 = 0.$$

We now have five equations and five unknowns. The solution of these equations for the unknowns yields (the student should make this solution)

$N_B = 517$ lb., $F_B = 25.85$ lb., $N_A = 483$ lb., $F_A = 24.15$ lb., $\bar{a} = 3.22$ fps^2.

Second Solution. In this solution, the inertia force $m\bar{a} = 1000\ \bar{a}/g$ is added to the free body (Fig. 516). By this act, we place the body in a simulated equilibrium, so that the sum of the forces in any direction and the sum of the moments about *any* point is each equal to zero. Summing forces in the vertical direction, we get

$$\Sigma F_y = N_A + N_B - 1000 = 0,$$

as before. A sum in the horizontal direction (Fig. 516) gives

$$\Sigma F_x = P - F_A - F_B - \frac{W\bar{a}}{g} = 0$$
$$= 150 - F_A - F_B - \frac{1000\bar{a}}{g} = 0.$$

The relations between the frictional and normal forces are as before,

$$F_A = fN_A = 0.05N_A, \qquad F_B = fN_B = 0.05N_B.$$

Now choosing point A as the center of moments, we find

$$\Sigma M_A = 4P + 12N_B - 6W - 8\frac{W\bar{a}}{g} = 0$$
$$= 600 + 12N_B - 6000 - \frac{8000\bar{a}}{g} = 0.$$

Again, we have five unknowns and five equations, from which we should find the same values of the unknowns as we did in the first solution. The student should be sure to do all the mathematical work in making each of these solutions. It is worth noting that, since $f = 0.05$ for both points A and B,

$$F_A + F_B = fN_A + fN_B = f(N_A + N_B)$$
$$= (0.05)(1000) = 50 \text{ lb.}$$

This value of $F_A + F_B$ in the equation for ΣF_x yields $\bar{a}$. The other details are left to the student.

There are times when one or the other of the preceding methods is advantageous in solving problems. The reader is advised to learn both methods; however, at the discretion of the instructor, he may concentrate on one method to the exclusion of the other where time is short.

231. Variable Forces. There are a number of instances where the effective force is not constant, one of the most common being the force exerted by a spring. The action of helical and leaf springs usually follows Hooke's law* with reasonable accuracy, at least as long as the material of the spring is not stressed beyond a certain point (called the elastic limit). As applied to springs, Hooke's law states that the force exerted by a spring is directly

*Robert Hooke (1635-1703), a brilliant contemporary of Newton, born on the Isle of Wight, was the son of a minister. Being a sickly youth and unable to attend school reguarly, he was left much to his own devices. There was soon no doubt but that he had a marvelous mechanical aptitude, an interesting and usually disagreeable personality, and also a prodigious mind. He mastered Euclid's six books of geometry in one week. Probably because of an inferiority complex, he drove himself hard. He was a great originator, but lacked that characteristic (which Newton had) of carrying through his ideas to a logical conclusion. While attending Christ Church, Oxford, he aided Boyle in his experiments with air. Among his prolific endeavors were: a study of the role of air in respiration and combustion; a statement that indicated his belief in universal gravitation (about which he quarreled with Newton); the relation between changes in barometric pressure and weather; the determination of the frequency of vibrations for each musical tone; origination of the short wave theory of light; first application of the spiral spring to time pieces (the center of a violent dispute with Huygens who had independently done the same thing); pointed out the rotation of Jupiter. He thought of many things first, but usually left his notions unproved so that eventually someone else got the credit.

Physically unattractive, somewhat deformed, always in ill health, and irascible, Hooke was involved in many unpleasant controversies with his contemporaries. His discovery of Hooke's law was first written in cryptic form in 1660 as follows: c e i i i n o s s s t t u u, which rearranges to *ut tensio sic vis*, free translation of which is "force is proportional to elongation." The cryptic form was for the purpose of keeping his discovery a secret so that he might profit from it by a blanket patent, which he never got. He lived in near poverty, but left a good sum of money at his death in a locked chest.

proportional to the deflection of the spring. Let s represent the deformation of a spring; then the corresponding force

$$F \propto s \qquad \text{or} \qquad F = Ks,$$

where K is a constant for a particular spring. This constant K has various names—the ***spring constant,*** the ***rate*** of the spring, the ***modulus*** of the spring, and most common, the ***scale*** of the spring. Since $K = F/s$, the unit of K is seen to be *force unit per length unit.* We ordinarily state the scale of a spring in pounds per inch. However, to be consistent in the units, we generally convert the inch unit to a foot. Thus, a scale of 50 lb. per in. is equivalent to $(12)(50) = 600$ lb. per ft.; that is, 50 lb. will compress the spring 1 in., and 600 lb. will compress it 12 in. or 1 ft. (if such a compression were possible within the elastic limit of the material). The ***free length*** (Fig. 517) of a spring is its length when no load is imposed on it.

In general, if a force is variable, it is shown on the free body in terms of some function of s, t, v, etc. Then, the sum of the forces is made in the usual manner and equated to ma ($\Sigma F = ma$). Then we may be able to use one of the equations $a = v\,dv/ds$, $a = dv/dt$, or $a = d^2s/dt^2$ and integrate for some desired result.

232. Example. A body ($W = 75$ lb.) is resting on a *smooth* plane which inclines $\theta = 30°$ with the horizontal. It is held in position by being attached to a spring whose weight is negligible and whose scale is $K = 50$ lb. per in. If the body is pulled down the plane from its *equilibrium position* for a distance of $r = 2$ in. and then released, determine (a) its velocity when it has returned to the equilibrium position, (b) the acceleration at the instant it is released, (c) the time of vibration.

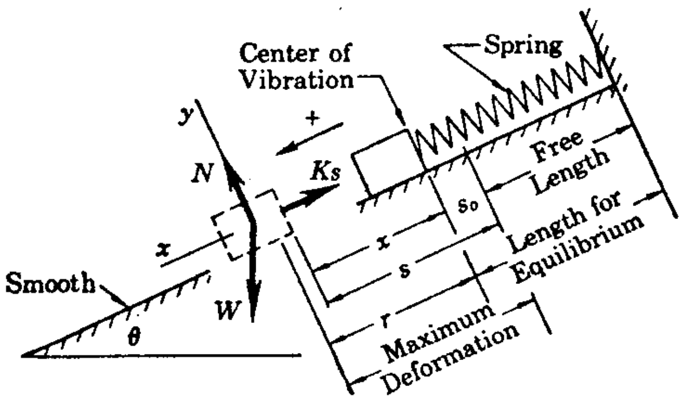

Fig. 517.

SOLUTION. (a) The situation described in the problem is pictured in a general way in Fig. 517. If the body is at rest, the extension of the spring is s_o. In this equilibrium position, a sum of the forces parallel to the plane, placed equal to zero, shows that

$$Ks_o = W \sin \theta.$$

Now let the body be pulled down a distance r from the equilibrium position and released. It will vibrate with an amplitude $r = 2$ in. When the body is x distance from the equilibrium position, its total deflection is s. At this instant the force up the plane is

$$Ks = K(s_o + x) = Ks_o + Kx = W \sin \theta + Kx.$$

Taking as positive the direction down the plane (as assumed in the foregoing displacements s_o and x), we get

$$\Sigma F_x = W \sin \theta - Ks = W \sin \theta - W \sin \theta - Kx = \frac{W}{g} a,$$

or

(u) $$a = -\frac{Kg}{W}x$$

This equation gives the acceleration a at any displacement x. Compare with equation (42), p. 269. Substituting the foregoing value of a in $v\,dv = a\,dx$, we have

$$\int_0^v v\,dv = -\frac{Kg}{W}\int_{1/6}^0 x\,dx = -\frac{Kg}{W}\left[\frac{x^2}{2}\right]_{1/6}^0 = -\frac{Kg}{W}\left(-\frac{1}{72}\right),$$

where the limits are from zero velocity at the instant of release to the desired velocity v, and from a displacement of $x = r = 2$ in. $= 1/6$ ft. to zero displacement (the position of equilibrium). Integration gives

$$\frac{v^2}{2} = -\frac{Kg}{W}\left(-\frac{1}{72}\right) = \frac{(600)(32.2)}{(75)(72)},$$

from which $v = 2.67$ fps for $W = 75$ lb. and $K = 600$ lb. per ft.

(b) At the instant of release, the displacement is $x = r = 2$ in. $= 1/6$ ft. Hence the acceleration is

$$a = -\frac{Kgx}{W} = -\frac{(600)(32.2)(\frac{1}{6})}{75} = -42.9 \text{ fps}^2.$$

(c) By comparing equation (u) of this example with equation (42), p. 269, we see that the motion of this body is simple harmonic, where

$$C = \omega^2 = Kg/W.$$

Thus, the equations for simple harmonic motion may be used to solve parts (a) and (b), and the student should check the preceding results by the harmonic equations. From

(v) $$\omega^2 = \frac{Kg}{W} = \frac{(600)(32.2)}{75} = 258,$$

we find $\omega = 16.03$ rad. per sec. Then by equation (z), p. 270, we have the period as

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{16.03} = 0.391 \text{ sec.},$$

the time for the body to return to the position from which it was released. On a frictionless plane, this vibration, called an ***undamped free vibration,*** will continue indefinitely. With friction, the body will soon come to rest. Such a vibration is called a ***damped free vibration.***

We may derive various equations, if we desire, for a body vibrating on an incline as in Fig. 517. For instance, from the first equation in this example, we find

$$W = \frac{Ks_o}{\sin\theta}.$$

Using this in the expression for ω^2, we get

$$\omega^2 = \frac{Kg}{W} = \frac{g\sin\theta}{s_o}.$$

The frequency ϕ is the reciprocal of the period,

$$\phi = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\left(\frac{g\sin\theta}{s_o}\right)^{1/2},$$

where $g \sin \theta$ is observed to be the component of the gravity acceleration vector parallel to the plane. However, the reader is urged to solve problems at this stage by making a free body and reasoning his way through as illustrated in the foregoing example.

Referring to equation (v), we see that the period (and the frequency) depend on the weight W and the scale K. In the days of the automobile without shock absorbers, it was always noticeable how much easier riding the car was when loaded than with only the driver in it. The heavier load reduced the frequency of vibration. If the scale K of the spring is increased, the spring is "stiffer" and the frequency is increased. In an automobile, increasing K makes the ride more jarring (same road conditions). A "soft" spring (smaller K) will result in your bouncing farther but not so jarringly. It might be said that the ride you get in a modern automobile is the result of a compromise between stiffness (K) and the action of shock absorbers, based largely on experience. See Chapter XX for additional material on vibrations.

233. Closure. This chapter has dealt with the application of Newton's second law only to bodies in translation. Whenever possible, determine the direction of motion (the sense of the velocity vector) and choose this direction as positive. Then in making sums, remember that acceleration, as well as force, is a vector quantity and that, therefore, we must be vigilant about giving the proper sign to a. If the direction of motion is taken as positive, a is negative for a deceleration. If the numerical value of a is known, put it into the equations with its correct sign or the solution will be wrong. If nothing is known about a, it may be put into the equations with a positive sign, equivalent to assuming a positive acceleration; then if the answer is negative, we know we chose the wrong direction for a.

By all means, always make a *complete* free body before beginning the algebraic solution, and somewhere on this sketch include a vector representing the acceleration—to help you to remember about its sign. In the process of doing this, consider whether you want to include the inertia force ma (reversed effective force, abbreviated REF) in the free body and use

$$\Sigma F = 0, \qquad \text{[Free body should match this equation, include REF.]}$$

where ma is included in ΣF; or to omit the inertia force and use

$$\Sigma F = ma, \qquad \text{[Free body should match this equation, omit REF.]}$$

where ma is not included in ΣF. If the inertia force is included, make it a dotted vector to distinguish it from the real forces. If you must take moments, there is often some advantage in including the inertia force on the free body because this allows you to take moments about *any* convenient point. If moments are taken without the inertia force as part of the free body, the center of moments must be on the line of action of the resultant; such a point for a body in translation is the cg.

Problems

NOTE. *Always show complete free bodies in each solution.*

HORIZONTAL OR VERTICAL MOTION OF PARTICLES, *a* CONSTANT

1101. The weight of the reciprocating parts of the steam engine of § 213 is 322 lb. For the conditions of this example, the acceleration of the piston was found to be $a_c = 63$ fps² (p. 297). What is the resultant force on the reciprocating parts at this instant?

1102. A body slides down a 60° incline for which $f = 1/4$. (a) If it starts from rest, how long does it take to slide 60 ft.? (b) What should be the inclination of the plane if the body slides down with constant speed? *Ans.* (a) 2.24 sec.; (b) 14.03°.

1103. A 644-lb. loaded sled is towed behind a truck by a rope that is inclined upward from the sled at a 20° angle with the horizontal. If $f = 1/5$ for the sled and if the tension in the rope is 200 lb., find (a) the acceleration and (b) the time required for the speed to change from 12 fps to 40 fps.

1104. A 3000-lb. automobile is to be brought to rest with a constant from 60 mph in a distance of 160 ft. What is the total frictional force between the tires and the ground? *Ans.* 2250 lb.

1105. A book is thrown across the room and found to slide 17 ft. as it comes to rest. If the kinetic coefficient of friction is 0.2, compute the horizontal speed with which the book is thrown.

1106. A serious student wishes to perform an experiment in determining the speed of a book. Using a blackboard eraser, he coats one side of the book with chalk dust, so that the book will mark a trail as it slides. Then he places the book on a long table whose inclination is varied until the book will slide down at a uniform speed. This angle of inclination is observed to be 26.6° with the horizontal. Now with the table top horizontal, he throws the book onto the table, and measures a 7.5 ft. trail left by the book as it slid to rest. Calculate the horizontal speed with which the book struck the table. *Ans.* 15.5 fps.

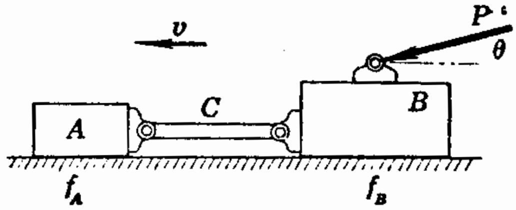

Fig. 518. Problems 1107, 1108.

1107. Two bodies A and B, connected by a rod C, Fig. 518, have an initial speed of 30 fps. A force of $P = 150$ lb. acts at $\theta = 0°$. Let $W_A = 322$ lb., $W_B = 644$ lb., $f_A = 0.04$, and $f_B = 0.15$. Determine the force in the rod C and the velocity of the bodies after 15 sec.

1108. Two bodies A and B, connected by a rod C, Fig. 518, have an initial speed of 6 fps and move 300 ft. in 30 sec. Let $W_A = 966$ lb., $W_B = 1288$ lb., $f_A = 0.04$, $f_B = 0.15$, and $\theta = 15°$. For constant acceleration, determine the force P and the final velocity. *Ans.* 270 lb., 14 fps.

1109. A hydrogen-filled balloon of weight W is falling vertically downward with a constant acceleration a. What amount of ballast Q must be thrown out in order to give the balloon an equal upward acceleration a? Neglect air resistance.

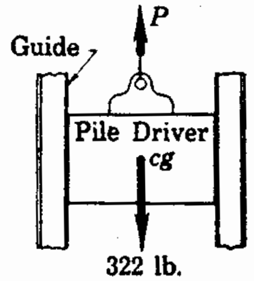

Fig. 519. Problem 1110.

1110. A pile-driver hammer, Fig. 519, weighing 322 lb., is to be raised 30 ft. with a constant acceleration in 2.5 sec. The total friction in the guides is constant at 200 lb. What is the required pull P in the cable? *Ans.* 618 lb.

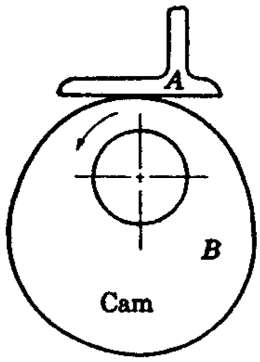

Fig. 520. Problems 1112, 1113.

1111. A man in an elevator has a weight of 100 lb. on his shoulders. If the elevator moves upward with an acceleration of 3 fps², what load does he support?

1112. The reciprocating follower A, Fig. 520, which together with its attached parts weighs 6 lb., is moved upward by the cam B a distance of 4 in. during a 75° turn of the cam with constant acceleration. If the cam turns at a constant speed of 120 rpm, what is the force between the cam and the follower? If the permissible load is 100 lb. per in. of face of the cam, how wide should the face be, if the computed force only is considered? *Ans.* 11.45 lb., 0.1145 in.

1113. The same as **1112** except that the speed of the cam is 400 rpm.

PARTICLES ON AN INCLINED PLANE, a CONSTANT

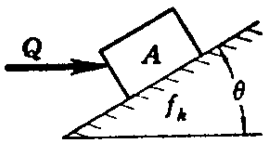

Fig. 521. Problems 1114-1117.

1114. In Fig. 521, body A weighs 50 lb., $\theta = 30°$, $f_k = 0.1$, and force $Q = 40$ lb. If the body starts from rest, what is its velocity after 4 sec.? *Ans.* 8.5 fps.

1115. In Fig. 521, body A weighs 50 lb., $\theta = 30°$, $f_k = 0.1$, and $Q = 40$ lb. (a) If the block is initially at rest, what is the maximum value of the coefficient of static friction if motion is to occur? (b) What is the velocity of the body A after it has traveled 10 ft.?

Ans. (a) 0.1517, (b) 6.43 fps.

1116. In Fig. 521, $\theta = 40°$, $f_k = 1/3$, and force Q is removed while the body A is at rest. Find the velocity of body A after 5 sec. Find the limiting value of f_s if A is to move ($Q = 0$).

1117. In Fig. 521, body A weighs 32.2 lb., $f_k = 1/4$, and force $Q = 30$ lb. Find the angle θ for which the body will have an acceleration of 3 fps² up the plane.

1118. This problem is the same as the example of § 225 except that $\beta = 60°$.

1119. A toboggan slides 100 yd. from rest down a 20% grade in 15 sec. Find the average coefficient of friction.

Ans. 0.115.

1120. A toboggan slides 100 yd. from rest down a slope in 10 sec. Find the angle the runway makes with the horizontal f the coefficient of friction is 0.1.

1121. In Fig. 522, the bodies A and B are in motion down the plane. The rigid bar AB is of negligible weight. If $W_A = 10$ lb., $W_B = 20$ lb., $f_A = 0.2$, $f_B = 1/3$, and $\theta = 30°$, find the acceleration of the bodies and the force in bar AB.

Ans. 8.04 fps², 0.77 lb. (T).

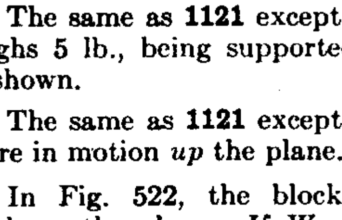

Fig. 522. Problems 1121-1125.

1122. The same as **1121** except that bar AB weighs 5 lb., being supported by the pins as shown.

1123. The same as **1121** except that the bodies are in motion *up* the plane.

1124. In Fig. 522, the blocks are in motion down the plane. If $W_A = W_B = 16.1$ lb., $\theta = 30°$, $f_A = 1/3$, and $f_B = 1/4$, find the acceleration of the blocks and the force in the weightless bar AB.

Ans. 7.97 fps², 0.58 lb. (C).

1125. In Fig. 522, the blocks A and B are in motion down the plane. If $W_A = W_B = 10$ lb., $\theta = 30°$, $a = -1.61$ fps², and the compressive force in the weightless rod AB is 1 lb., find the coefficients of friction f_A and f_B. *Ans.* 0.75, 0.52.

1126. While moving up a 1% grade, a freight locomotive exerts a constant drawbar pull of 60,000 lb. The train resistance is 15 lb. per ton. (The train resistance consists of the friction in the bearings, the rolling resistance of the wheels, air resistance, etc., and is generally estimated as a force of so many pounds opposed to the direction of motion.) If the speed of the train changes from 5 mph to 30 mph in a distance of 8 miles, what is the weight of the train? *Ans.* 1650 tons.

1127. A 1000-ton train has a resistance (see note in **1126**) of 10 lb. per ton and a drawbar pull of 135,200 lb. produced by a Virginian steam locomotive. To reach the top of a 1-mile long, 1½% grade with a speed of 20 fps, (a) find the speed with which the train must "hit" the grade. *Ans.* 126 fps.

1128. The same as **1127** except that the train is double-headed with 2 Virginian locomotives, each producing the same drawbar pull.

TWO DIRECTIONS OF MOTION (PARTICLES)

1129. A freight car starts down a 1.5% grade from rest under the action of gravity. The constant frictional resistance to motion is 10 lb. per ton of weight. After 5000 ft., it passes onto level track where the resistance remains the same. How far does it go before it stops. There are no cows or other obstructions on the track. *Ans.* 15,000 ft.

1130. The same as **1129** except that the grade is 1%.

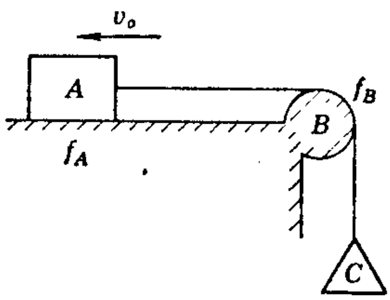

Fig. 523. Problems 1131-1133.

1131. In Fig. 523, bodies A and C are connected by a weightless flexible cord over a smooth surface B ($f_B = 0$). The coefficient of friction $f_A = 1/3$. If $W_A = 64.4$ lb., $W_C = 96.6$ lb., and the initial velocity of A is 30 fps toward the left, find the time in seconds for A to travel 10 ft. *Ans.* 0.39 sec.

1132. The same as **1131** except that $f_B = 0.2$.

1133. With the initial conditons described in problem **1131**, find the displacement of A after 6 sec. *Ans.* +149 ft.

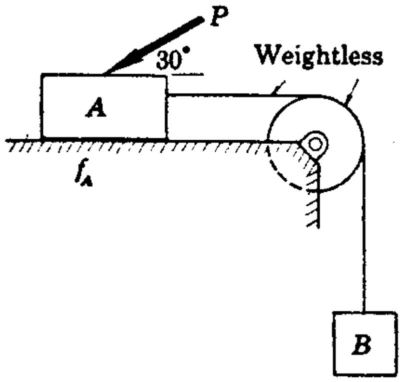

Fig. 524. Problems 1134, 1135.

1134. In Fig. 524, $P = 0$, $W_A = 600$ lb., $W_B = 225$ lb., and $f_A = 1/3$. Neglecting the weight of the cable and pulley and the friction at the pulley, compute (a) the distance A moves in 20 sec., (b) the tension in the cable, and (c) the speed of the bodies after 20 sec. The bodies start from rest.

1135. The bodies A and B, Fig. 524, are moving toward the right at 20 fps. Let $W_A = 600$ lb., $W_B = 225$ lb., $f_A = 1/3$, and neglect the weight of the cable and pulley and the friction at the pulley. What constant force P will bring the bodies to rest in a distance of 30 ft.? What is the tension in the cable? *Ans.* 189.4 lb., 271.6 lb.

INVOLVING PULLEYS (PARTICLES)

1136. In Fig. 525, let $W_A = 966$ lb. and $f_A = 1/3$. The speed of A changes from $v_{A1} = 10$ fps to $v_{A2} = 35$ fps during 25 sec. Determine (a) the weight W, (b) the distance moved by W during 25 sec., and (c) the tension in the cable.

1137. The same as **1136** except that $v_{A1} = 60$ fps and $v_{A2} = 10$ fps.

Ans. (a) 508 lb.; (b) 437.5 ft.; (c) 262 lb.

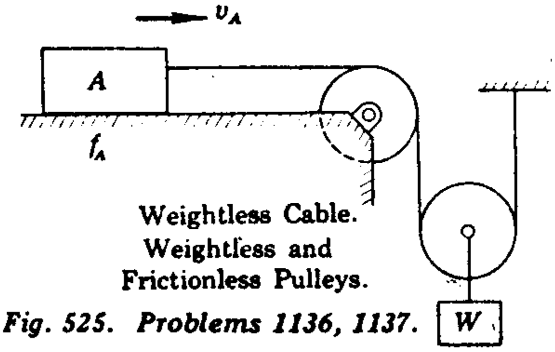

Fig. 525. Problems 1136, 1137.

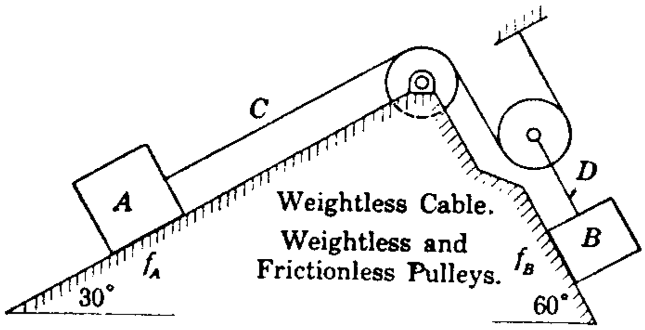

Fig. 526. Problems 1138, 1139.

1138. In Fig. 526, $W_A = 200$ lb., $W_B = 100$ lb., $f_A = 1/4$, and $f_B = 1/3$. How far and in what direction does A travel from rest during 30 sec.? What is the tension in the cable C? in the cable D?

Ans. 326 ft., 52.2 lb., 104.4 lb.

1139. The same as **1138** except that $W_A = 30$ lb.

1141. Let the weight of the bodies in Fig. 528 be represented by W_A and W_B. Derive an expression for their acceleration and the equation giving the tension in the weightless cord. Let $W_A > W_B$ and neglect the inertia and friction of the pulley.

1142. In Fig. 529, an elevator A is supported by a cable which passes over a

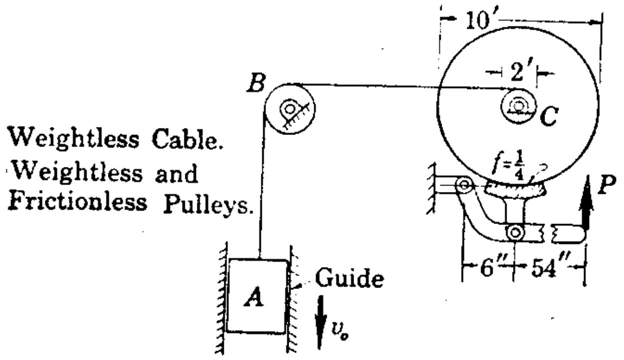

Fig. 527. Problem 1140.

1140. A mine cage A, Fig. 527, weighs 9660 lb. and has a speed of $v_o = 30$ fps downward. The total friction in the guides is constant at 100 lb. The cable supporting the cage wraps around a 2-ft. drum which is connected to a 10-ft. brake wheel. Between the brake shoe and the wheel, $f = 1/4$. Consider the mass of the wheel and drum to be negligible. If the cage comes to rest in a distance of 60 ft., compute the tension in the cable and the force P on the braking lever.

Ans. 11,810 lb., 945 lb.

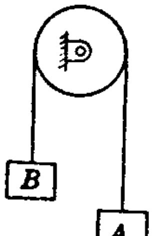

Fig. 528. Problem 1141.

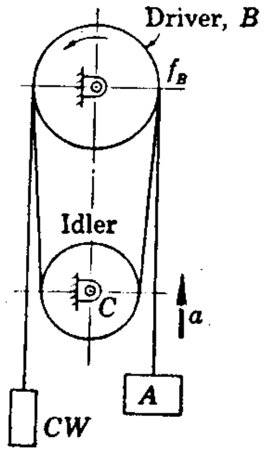

Fig. 529. Problems 1142, 1143.

driving sheave B, thence half around an *idling* sheave, thence over the driving sheave again, and down to a counterweight CW. In this type of drive, the motor turns the sheave B, which, by virtue of the friction between the cable and the sheave, causes the elevator to move up or down. Let $W_A = 5000$ lb., $a_A = 3$ fps², and let the coefficient of friction of the cable over sheave B be $f_B = 0.1$. Neglecting the inertia effects of the sheaves and assuming that sheaves B and C are the same size, determine the necessary minimum weight of the counterweight CW if the cable is not to slip on the driving sheave.

Ans. 3220 lb.

1143. In the traction drive described in **1142**, Fig. 529, let $W_A = 7500$, $f_B = 0.1$, and let the weight of the counterweight be 6000 lb. What maximum upward acceleration may be given the elevator A without slipping of the cable on the driving sheave?

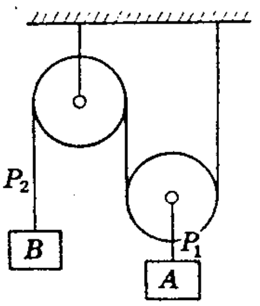

Fig. 530. Problems 1144, 1145.

1144. In Fig. 530, $W_A = 120$ lb. and $W_B = 80$ lb. Consider the cord and pulleys weightless, and neglect friction. Determine the acceleration of each body and the tensions in the cords P_1 and P_2.

Ans. $a_A = 2.93$ fps², 131 lb., 65.5 lb.

1145. The same as **1144** except that $W_A = 240$ lb.

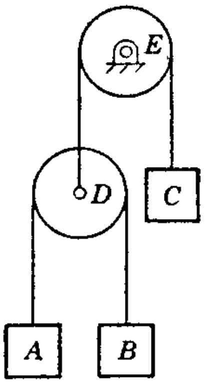

Fig. 531. Problems 1146, 1147.

1146. In Fig. 531, the masses of A, B, and C are, respectively, 1 slug, 2 slugs, and 4 slugs. The cords are weightless and flexible. Sheaves D and E are considered free and weightless. In 2 sec. after all elements are simultaneously released from rest, find the absolute acceleration, velocity, and displacement of body A.

Ans. 19.25 fps², 38.5 fps, 38.5 ft.

1147. The same as **1146** except that the mass of C is 2 slugs.

RIGID BODIES

NOTE. *Unless the instructor specifies otherwise, make two solutions of the following problems, one without a reversed effective force* (REF) *on the free body and one with it. Make two free bodies. This procedure will make clear the advantages of each method.*

1148. Body A, Fig. 532, is accelerated from rest to 40 fps in 20 ft. Attached to it is the 50-lb. vertical homogeneous bar B that is pivoted at pin C. Find the forces at C and D, (a) not using the REF, (b) using the REF.

Ans. $C = 52.4$ lb. at 72.7°, $D = 46.6$ lb.

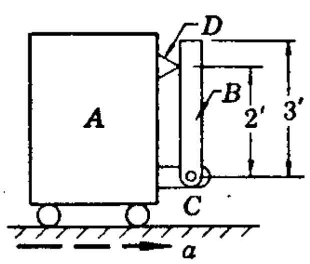

Fig. 532. Problem 1148.

1149. This problem is the same as the example of § 230 except that the line of action of P is a distance $d = 10$ ft. below the track. Solve (a) without REF, (b) with REF.

1150. A 128.8-lb. garage door is supported on frictionless rollers as shown in Fig. 533. For a force $F = 20$ lb. applied $y = 3$ ft. from the bottom of the door, what are the reactions at A and B? Solve (a) without REF, (b) with REF.

Ans. 66.9 lb., 61.9 lb.

1151. A 128.8-lb. garage door is supported on frictionless rollers as shown in Fig. 533. (a) What force F applied at a distance $y = 2$ ft. would result in a zero reaction at roller B? (b) What force F applied at a distance $y = 6$ ft. would result

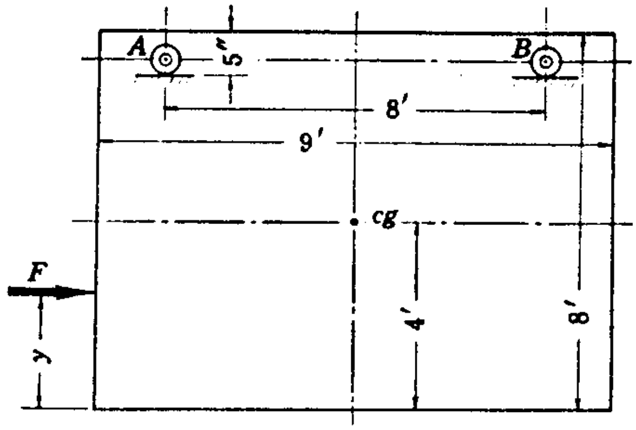

Fig. 533. Problems 1150, 1151.

in a zero reaction at roller A? Solve (a) without REF, (b) with REF.

1152. An overhead crane in a foundry moves a 20-ton ladle, which is suspended from cables. *From* the point of tangency of the cables on the crane drum *to* the center of gravity of the ladle is 30 ft. In a distance of 60 ft., the speed of the crane steadily increases from 5 to 15 fps. What is the total pull in the cables and the inclination of the cables with the vertical?

Ans. 40,100 lb., 2.96°.

1153. A truck, with a gross weight of 6440 lb., has a wheel base of 12 ft. Its center of gravity is 4 ft. above the pavement and 5 ft. ahead of the rear axle. If the truck is brought to rest in a straight line from a speed of 70 fps in a distance of 90 ft., (a) what are the vertical reactions at the front and rear wheels, and (b) what is the total frictional force between tires and pavement?

1154. The truck described in **1153** is going down a 10% grade at a speed of 50 mph. The coefficient of kinetic friction between the wheels and the road is 0.4. If the driver locks all four wheels, determine the normal reactions on the front and rear wheels. The truck moves at all times in a straight path. *Ans.* 3520 lb., 2885 lb.

1155. A driver of a 3000-lb. automobile brakes his car so that each wheel is about to slip on the pavement, where $f = 0.8$. The wheel base of the car is 120 in., and its center of gravity is 2 ft. above the ground and 6 ft. back of the front axles. If the car is going 60 mph when the brakes are applied, how far does it go before it comes to rest? What are the frictional and normal forces at the front and at the rear wheels?

Ans. 150 ft., $F_f = 1344$ lb., $F_r = 1056$ lb.

1156. The same as **1155** except that the car is on a gravel road, where $f = 1/4$.

1157. In Fig. 534, the weight of the bar is $W = 100$ lb. Neglect the size and weight of the wheel and of the block. If the reaction at B is 10 lb. upward, and $f_k = 0.2$, find $\bar{a}$ and Q (a) without REF, and (b) with REF. *Ans.* 34.3 fps², 124.7 lb.

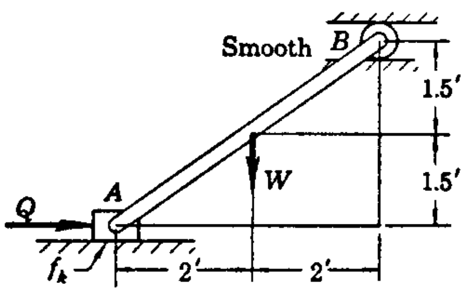

Fig. 534. Problems 1157-1160.

1158. The same as **1157** except that $f_k = 0$.

1159. The same as **1157** except let the reaction at B be 10 lb. downward.

1160. In Fig. 534, the weight of the bar is $W = 100$ lb. Let $f_k = 0$ and neglect the size and weight of the wheel and of the block. If $Q = 500$ lb., find $\bar{a}$ and the reaction at B. Solve (a) without REF, (b) with REF.

1161. A 2 x 2 x 6-ft. box B containing cork and weighing 322 lb. is placed on a car A, as shown in Fig. 535. At C is a fixed peg for which $f_C = 0$. The car A is given an acceleration of $a = 3.22$ fps² leftward. (a) What is the weight W if the box is about to turn over? Solve without and with REF. (b) If the coefficient of friction between A and B is $f_B = 0.1$, would the box turn over or slide?

Ans. (a) 51.2 lb.; (b) slide.

1162. The same as **1161** except that $f_C = 0.3$.

and block B are released, how far up the plane does the block go? The spring (considered weightless) acts on the block only for the 2 in. which it was compressed.

Ans. 31.47 ft.

1170. A weight of 50 lb. is suspended from a weightless spring in a vertical position. The scale of the spring is 20 lb. per in. If the weight is pulled down $2\frac{1}{2}$ in. and then released, determine (a) the maximum acceleration, (b) the maximum velocity, and (c) the frequency of the motion.

1171. A coil spring used for the front suspension of an automobile has a scale of 400 lb. per in. If the weight on this spring is 1000 lb., determine the frequency of the vibration when the wheel hits a bump, assuming that no shock absorbers are used.

Ans. 1.98 cycles per sec.

1172. A 10-lb. body falls a distance of 3 ft. and strikes a spring whose scale is $K = 30$ lb. per in. How much is the spring compressed when the body has come to rest?

1173. The problem is the same as the example of § 232 except that $W = 30$ lb.

Ans. (a) 4.24 fps; (b) -107.3 fps^2, (c) 0.247 sec.

1174. A loaded freight car, weighing 70 tons and moving at 6 mph, strikes a spring bumper, consisting of a nest of springs, at a point 4 ft. above the top of the rails. The nest of springs as a group has a scale of 40,000 lb. per in. and may be compressed as much as 12 in. The distance between the front and rear trucks of the car is 30 ft. and the cg of the whole is 6 ft. above the top of the rails. Neglect rolling resistance and determine (a) the maximum compression of the springs, (b) the maximum acceleration, and (c) the maximum vertical reactions at the front and rear trucks.

Ans. (a) 10.04 in.; (b) 92.4 fps^2; (c) 96,784 lb.; 43,216 lb.

1175. The same as **1174** except that the rolling resistance is constant at 15 lb. per ton and is assumed to act at the surface of the rails.

1176. A 322-lb. weight is suspended from a spring whose axis is vertical. It is pulled down a certain distance and released. When the displacement is -2 in., the velocity is $+14.1$ fps. When the displacement is $+3$ in., the velocity is -10.6 fps. Determine the scale of the spring, the period, the maximum acceleration, and maximum velocity of the weight.

Ans. 8000 lb. per ft., 0.209 sec., 450 fps^2, 15 fps.

1177. The resistance to motion of a 3220-lb. boat in still water is approximately $3v$ lb., where v is in fps. If the speed of the boat is 9 fps when the engine is stopped, (a) how far will the boat go and (b) how long will it take before it comes to rest.

Ans. (a) 300 ft.; (b) infinite.

1178. The resistance to motion of a 1610-lb. boat in still water is approximately $3v$ lb., where v is in fps. If the speed of the boat is 6 fps when the engine is stopped, find the speed after it goes 100 ft. and the time required for the displacement.

1179. A 3220-lb. boat is propelled from rest by a constant thrust of 50 lb. The resistance to motion is $3v$, where v is in fps. (a) How long does it take for the boat to attain a speed of 6 mph? (b) For a thrust of 50 lb.. what is the maximum speed of the boat? (c) How far does the boat go in attaining a speed of 6 mph?

Ans. (a) 25 sec.; (b) 16.6 fps.; (c) 125 ft.

1180. A man and parachute weighing 196 lb. reach a speed of 80 fps before the parachute opens. At this instant the parachute opens, and the resistance to falling is $0.81v^2$, where v is in fps. (a) What is the limiting speed with the parachute open? (b) How far does the man move while his speed drops to 20 fps? (c) How long does it take to reach this speed?

1181. A resultant force acting on a 64.4-lb. body in the direction of motion is $F = 4t^2$. The initial velocity of the body is 4 fps. What is its velocity after 2 sec.? What is the corresponding displacement?

Ans. 9.33 fps, 10.67 ft.

1182. A motive force of $F = 4t^2$ acts on a 64.4-lb. body. There is a constant force of 30 lb. acting opposite to the sense of the initial motion. The initial speed is 4 fps. Determine the velocity of the body after (a) 0.2 sec., (b) 2 sec., and (c) 5 sec.

1183. A particle moves in a straight line according to the law $s = t^3 - 75t$ ft., where t is in sec. (a) What is its arithmetic average velocity during the fourth second? (b) When does it come to rest? What is its acceleration at the instant it is at rest? (c) If the rightward direction is positive, in what direction is it moving after 4 sec.? Where is it located at this instant? (d) If the particle weighs 6.44 lb., what is the resultant force on it at 4 sec.?

Ans. (a) -37.5 fps; (b) 5 sec., 30 fps^2; (c) left, -236 ft.; (d) 4.8 lb.

1184. The connection between the crank pin and piston rod shown in Fig. 539 is

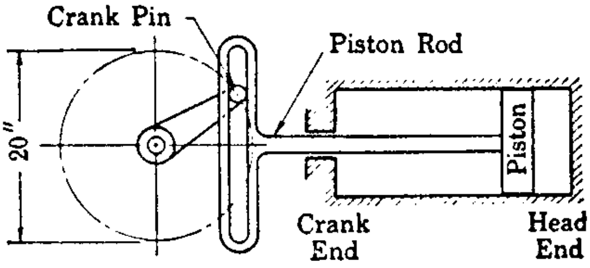

Fig. 539. Problem 1184.

called a Scotch crosshead. This connection is sometimes used in driving pumps. The stroke is 20 in. and the crank turns uniformly at 120 rpm. (a) If the weight of the reciprocating parts is 644 lb., what is the corresponding inertia force when the piston is 4 in. from its head end position? What is the maximum inertia force? (b) If the diameter of the cylinder is 18 in. and the net fluid pressure on the piston is 60 psi for a complete stroke, what is the maximum force on the crank pin?

Ans. (a) 1580 lb., 2630 lb.; (b) 17,870 lb.

1185. A flexible chain weighs 20 lb. per ft. and is 30 ft. long. It is placed on a horizontal surface, for which $f = 1/3$, with 10 ft. of it hanging vertically over the edge. If the chain is released in this position, how long does it take for the remainder of the chain to leave the horizontal surface?

Ans. 2.42 sec.

1186. A flexible chain weighs 10 lb. per ft. and is 20 ft. long. It is placed on a 30° inclined plane, for which $f = 0.4$, with 5-ft. of the chain hanging over the lower end of the incline. If the chain is released in this position, what is its velocity at the instant the chain leaves the incline? How long does it take for the chain to leave the incline?

Ans. 25.7 fps, 1.425 sec.

1187. The maximum horizontal recoil of a 966-lb. gun is to be 1.5 ft. with an initial recoil speed of 12 fps. The reaction is to be handled by compressing air in a cylinder. The volume of air in the cylinder at any point is xA cu. ft., where A is the area of a section of the cylinder and x is the distance from the piston to the cylinder head. At the beginning of the reaction, the value of $x = 2.5$ ft. giving a volume of air of $2.5A$, and the pressure of the ai at this point is $p_1 = 14.7$ psi. At the end of the reaction, the value of x is $2.5 - 1.5 = 1$ ft. During the compression of the air, the relation $pV = C$, a constant, is assumed to be true, where p is the variable pressure of the air. The variable resisting force is then $F = pA$. Determine the constant C in terms of A, and compute the necessary diameter of the recoil cylinder.

Ans. 9.03 in.

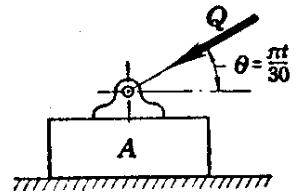

Fig. 540. Problem 1188.

1188. A constant force of $Q = 100$ lb. acts on the 360-lb. body A, Fig. 540, but the angle θ varies according to the law $\theta = \pi t/30$, where θ, measured counterclockwise, is in radians and t is in seconds. Motion of A starts when $t = 0$ and $\theta = 0$. Neglecting friction, find the velocity and displacement of A after 15 sec.

1189-1200. These numbers may be used for other problems.