Chapter XVIII

WORK, KINETIC ENERGY, POWER

265. Introduction. Where bodies are undergoing an acceleration, there are three possible approaches to the force analyses: first, the method of force, mass, and acceleration as suggested by Newton's second law, $\Sigma F = ma$, which is presented in the three preceding chapters; second, the principle of work and kinetic energy, which is discussed in this chapter; and third, the principle of impulse and momentum, the subject for discussion of the next chapter. Some problems may be analyzed with virtually equal ease by either of the three principles or methods. Some problems are much more easily solved by one of the three principles than by either of the other two. Hence the study of the material of this chapter not only teaches the concepts of work, kinetic energy, and power, but also gives us an additional powerful tool that will be useful in the analyses of many diverse problems in engineering.

266. Work. ***Work*** is a form of energy. In thermodynamics, where the concept of work is explored more fully, work is spoken of as energy in transition. The meaning of this phrase may be discussed from several viewpoints, but for present purposes, we might interpret it to mean that, in order for work to be done, a body must move. That is, a force must undergo a displacement. In this sense, it is a transitory form of energy. No matter how large a force may be acting on a rigid body, if the body does not move, no work is done.

The technical definition of work is in terms of force and displacement. If a force acts on a *particle* which undergoes a displacement ds, the work done by the force is *the product of the displacement and the component of the force in the direction of the displacement.* In the general case, should the particle undergo curvilinear motion with a varying force, we may write the work U as

$$U = \int F\,ds, \tag{48}$$

where the component (F) in the direction of the displacement is considered to be constant for an infinitesimal displacement ds. If the force F is variable, we must of course be able to express it in terms of s in order to integrate (48).

It happens that there are many instances in engineering where the force F is constant, or so nearly constant that no serious error is involved in assuming that it is so. In this event, the integral is readily evaluated. For example, let a constant force F act on the rigid body of weight W (Fig. 616), and let the body move in a straight line through a distance s. In this illustration, the force acts in the direction of the line of motion; hence, the work done by a single constant force F is

(a) $$U = F\int_0^s ds = Fs,$$

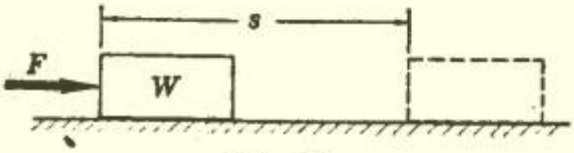

Fig. 616.

where s is the distance moved by the *point of application* in the direction of the constant force. For rectilinear motion, the displacement is equal to the distance moved.

If the force F does not act in the line of motion, as in Fig. 617, we must use the component in the direction of the displacement. This component is $F \cos \theta$, where θ is the angle between the force F and the line of motion. If this component force is constant, we find

(b) $$U = F \cos \theta \int_0^s ds = (F \cos \theta)s.$$

The work of a force acting on a particle which moves in a straight line or which moves through an infinitesimal displacement ds on a curved path may also be defined as the magnitude of the force *times* the component of the displacement of its point of application in the direction of the *force*. Thus, in Fig. 617, the component of the displacement in the direction of the force is $ab = s \cos \theta$, which gives

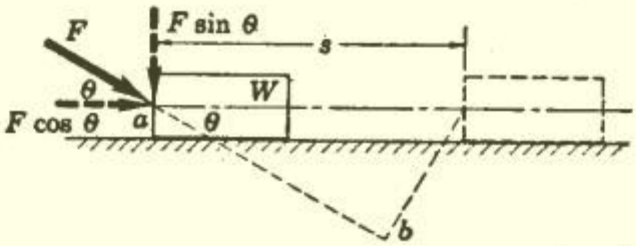

Fig. 617. Component F cos θ does work. Component F sin θ does no work.

$$U = F(s \cos \theta),$$

which agrees with the previous result. This component $s \cos \theta$ of the displacement s is called the *effective displacement*. Observe that the fundamental condition for a force to do work is that its point of application must have a displacement in the direction of some component of the force. In Fig. 617, the component $F \sin \theta$, which acts *perpendicular* to the line of motion, does no work because there is no effective displacement of its point of application in the sense of this component.

267. Positive and Negative Work. Since work is a scalar quantity, it is *not* represented by a vector. It may be considered to be either positive or negative, as may be convenient. Usually in mechanics, the work of a force which has a component in the *same sense* as the displacement is taken as *positive*. Such a force tends to increase the speed of a particle or body. For this scheme of signs, work done *on* the body is positive. If the force has a component which tends to reduce the speed of the body on which it acts, its work is taken as negative.

Forces which have the same sense as the velocity and which therefore tend to accelerate a body or to keep a body in motion (even though it is slowing down) are motivating forces. Forces which act in the opposite sense to the velocity are resisting forces. If the foregoing convention of signs is adopted, motivating forces produce positive work and resisting forces produce negative work. At any rate, it is still advisable to be careful of signs and *to make complete free bodies.*

268. Work of a System of Forces Acting on a Rigid Body. The work of a system of forces acting on a rigid body is equal to the *algebraic sum of the works of the individual forces.* For example, the body A in Fig. 618 is moving up the inclined plane under the action of the constant forces shown. The work of the force Q during a displacement S up the plane is

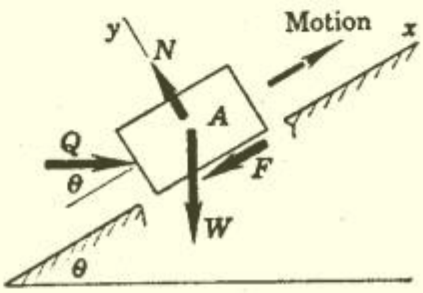

Fig. 618.

$$U_Q = +(Q \cos \theta)s;$$

the work of the frictional force F is

$$U_F = -Fs;$$

the work of the force of gravity W is

$$U_W = -(W \sin \theta)s;$$

and the work of the normal plane reaction N is zero,

$$U_N = 0,$$

since its sense is normal to the direction of motion. Thus, the resultant or *net* work U on the rigid body A is the algebraic sum of the above work quantities, or

(c) $$U = (Q \cos \theta)s - Fs - (W \sin \theta)s,$$

where s is the straight-line displacement of A for which the work is desired. This equation may be written in the form

(d) $$U = (Q \cos \theta - F - W \sin \theta)s,$$

where the part in parentheses is seen to be the resultant force R acting on the body, which is ΣF_x, inasmuch as $\Sigma F_y = 0$. Thus, we may say that for a *particle* or for a rigid *body in rectilinear translation* (where the force system is concurrent), the *net* work done is the work done by the *resultant force.* The point of view expressed by equation (**d**) is particularly convenient for bodies *in rectilinear motion* since we can simply make a complete free body, as we have already learned to do, and then sum the forces in the direction of motion to get the resultant R. The product of R (if it is constant) and the displacement s is the *net* work,

$$\text{Net work} = U = Rs$$
[RECTILINEAR TRANSLATION]

a relation which holds for any rigid body *in rectilinear translation.* While this procedure gives the *net* work, we nevertheless are often interested in the work of some single force, such as the work of friction or the work of gravity.

269. Principle of Work and Kinetic Energy. Let a *particle* of mass dm be acted upon by a resultant force R. Then from Newton's laws, we have $R = (dm)a$. In this expression, setting $a = v\,dv/ds$, we find

$$\text{(e)} \qquad \int R\,ds = \int (dm)v\,dv.$$

In case the mass dm of the particle is constant, which is the usual case in engineering, we find from (**e**), by integrating between any two speeds v_1 and v_2,

$$\text{(49)} \qquad \int R\,ds = \frac{(dm)v_2^2}{2} - \frac{(dm)v_1^2}{2} = \frac{(dW)v_2^2}{2g} - \frac{(dW)v_1^2}{2g},$$

where $dm = dW/g$, dW being the weight of the particle. The left-hand side of (49), $\int R\,ds$, is recognized as the *net* work done on the particle. The expression $(dW)v^2/(2g)$ is called the ***kinetic energy*** KE of a particle. Therefore, the right-hand side of (49) is the *change* of kinetic energy ΔKE. Thus, equation (49) is a mathematical statement of a very useful principle, namely,

> The *net* work done on a particle is equal to the change of kinetic energy of the particle.

In symbol form,

$$\text{(f)} \qquad U_{\text{net}} = \int R\,ds = \Delta KE = KE_2 - KE_1.$$

The symbol ΔKE is always interpreted as the value at the second point minus that at the first point. This principle is a special case of the *law of conservation of energy*; it holds true only if the net work is not affected by temperature changes of the particle which would change the particle's internal energy, by chemical actions, or by electrical energy. Although (**f**) holds only if all energy changes except the net work and the change of kinetic energy are negligible or zero, this principle is very useful to the engineer.

270. Kinetic Energy of a Rigid Body in Translation. If a rigid body A, Fig. 619, is in translation, all particles dm_1, dm_2, etc., have the same velocity v. Therefore the kinetic energy of the body is

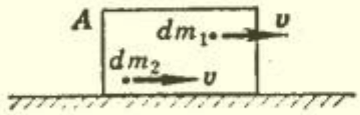

Fig. 619. Kinetic Energy —Translation.

$$KE = \int \frac{(dm)v^2}{2} = \frac{v^2}{2}\int dm = \frac{mv^2}{2} = \frac{Wv^2}{2g},$$

where $m = W/g$ is the mass of the whole body. The energy $Wv^2/(2g)$ is the kinetic energy that a body possesses at a particular speed v. If the speed is v_1, the kinetic energy is $Wv_1^2/(2g)$; if the speed is v_2, then $KE_2 = Wv_2^2/(2g)$; etc.

Since the force system on a body in translation necessarily reduces to a single resultant force,* the *net* work done on the body is $\int R\,ds$, or Rs, if R is constant. Hence, the principle of work and kinetic energy applied to a rigid body in *translation* yields [see equation (f)]

$$U_{\text{net}} = \Delta KE = KE_2 - KE_1,$$
$$\int R\,ds = \frac{Wv_2^2}{2g} - \frac{Wv_1^2}{2g}.$$

[TRANSLATION]

271. Units of Work. The unit of work is evidently a force unit times a space unit. For the type of work in which we use the principles of this chapter, the unit is usually a foot-pound (ft-lb.), although many other units, such as an inch-pound, a mile-ton, a dyne-centimeter, could be used. The work as obtained in foot-pounds may be converted to any other energy unit. The units of some convenient conversion factors are as follows:

$$778\,\frac{\text{ft-lb.}}{\text{Btu}}, \qquad 1.34\,\frac{\text{hp-hr.}}{\text{kw-hr.}},$$
$$33{,}000\,\frac{\text{ft-lb.}}{\text{hp-min.}}, \qquad 0.746\,\frac{\text{kw-hr.}}{\text{hp-hr.}},$$
$$550\,\frac{\text{ft-lb.}}{\text{hp-sec.}} \qquad 2{,}650{,}000\,\frac{\text{ft-lb.}}{\text{kw-hr.}},$$
$$1{,}980{,}000\,\frac{\text{ft-lb.}}{\text{hp-hr.}}, \qquad 3413\,\frac{\text{Btu}}{\text{kw-hr.}},$$
$$2545\,\frac{\text{Btu}}{\text{hp-hr.}}, \qquad 10^{10}\,\frac{\text{ergs}}{\text{kw-sec.}},$$

where Btu stands for British thermal unit, hp for horsepower, hr. for hour, kw for kilowatt. When a large amount of work is involved, as in the instance of power-generating equipment, the horsepower-hour (hp-hr.) and kilowatt-hour (kw-hr.), which are large units of work, are preferred. In dealing with heat and its conversion, we often prefer the Btu.

Since work and kinetic energy are equivalent, the unit of kinetic energy

*R may be zero.

must match that of the corresponding work. Ordinarily, the units of the various terms in $Wv^2/(2g)$ are

$$\frac{Wv^2}{2g},\qquad \frac{(\text{lb.})(\text{ft./sec.})^2}{\text{ft./(sec.)}^2} = \text{ft-lb.}$$

272. Example. A freight train consisting of 60 cars, each weighing 70 tons, starts up a 1.5% grade with an initial speed of 15 mph. The constant drawbar pull (i.e., the force that the locomotive exerts on the train) is 97 tons and the train resistance (including rolling resistance, axial friction, and air resistance) is 15 lb. per ton of weight. At the top of the constant 1.5% grade, the speed is 30 mph. (a) How long is the grade? Neglect the kinetic energy of rotation of the wheels. (b) Express the work of the draw-bar pull in terms of hp-hr. (c) What is the work done against gravity?

SOLUTION. (a) Let Fig. 620 (where θ is exaggerated for clearness) represent a free body of the train: the draw-bar pull is (97)(2000) lb., the total resistance is (60)(70)(15) lb., and the total weight is W = (60)(70)(2000) lb. Since the body is in translation only, the net work is $Rs = (\Sigma F_x)s$. Now using the principle $U_{net} = \Delta KE$, and the fact that 15 mph = 22 fps, we have

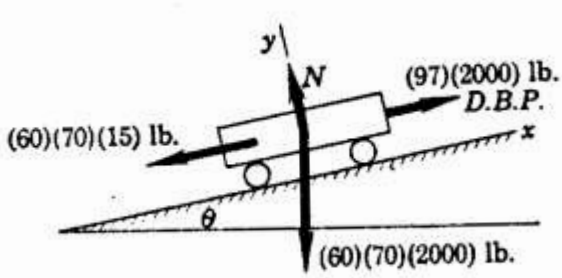

Fig. 620.

$$[(97)(2000) - (60)(70)(15) - (60)(70)(2000)\sin\theta]s = \frac{(60)(70)(2000)}{2g}(44^2 - 22^2),$$

where $\sin\theta = 0.015$.* Solving for s from this equation, we get

$$s = 37{,}900 \text{ ft.} = 7.17 \text{ mi.}$$

(b) The work of the draw-bar pull is

$$U = (97)(2000)(37{,}900) = 7{,}352{,}600{,}000 \text{ ft.-lb.}$$

$$U = \frac{7{,}352{,}600{,}000 \text{ ft-lb.}}{1{,}980{,}000 \text{ ft-lb./(hp-hr.)}} = 3710 \text{ hp-hr.}$$

(c) The work done against gravity is (component of W parallel to the plane times distance)

$$\frac{(60)(70)(2000)(\sin\theta)(37{,}900) \text{ ft-lb.}}{1{,}980{,}000 \text{ ft-lb./hp-hr.}} = 2410 \text{ hp-hr.}$$

There is an *increase* in both the kinetic energy and the potential energy (§ 273) of the train.

273. Potential Energy. The work done against gravity in part (c) of the preceding example is also called the ***change of potential energy.*** The change of potential energy is an increase when the body is elevated; it is a decrease when the body is lowered.

If a *particle* of weight W has an elevation h above some datum plane, it is said to have *potential energy* with reference to the chosen datum, the amount

*A 1.5% grade means a 1.5 ft. rise in 100 ft. measured on the level. Strictly $\tan\theta$ = 0.015, but for small angles $\sin\theta$ is virtually equal to $\tan\theta$.

being Wh, where h is the *vertical* distance through which the particle would travel in passing from its original position to the datum level. In particular, this energy is the ***potential energy of elevation*** and is due to the configuration of two bodies, the earth and the particle whose weight is W.

The potential energy of elevation of a *finite* body with respect to some datum is work that would be done by the force of gravity (W) on the body as its center of gravity passes from one elevation to the elevation of the datum. To prove this statement, let a body, rigid or otherwise (Fig. 621), be composed of particles of weights W_1, W_2, etc., at elevations of y_1, y_2, etc., above some chosen datum. The potential energy PE of the body is the sum of the potential energies of all its particles, that is,

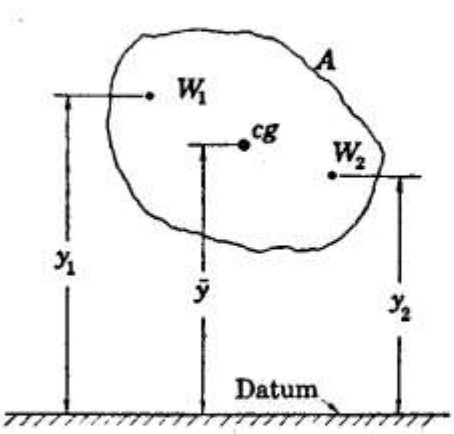

Fig. 621. Potential Energy.

$$PE = W_1y_1 + W_2y_2 + \cdots.$$

But recalling (§ 109) that

$$W_1y_1 + W_2y_2 + \cdots = W\bar{y},$$

we see that the potential energy of a finite body above a particular datum is

$$\text{(g)} \qquad PE = W\bar{y}.$$

Observe that the change of potential energy is independent of the path taken by a body to reach its new position. The path may be curved in any manner, yet the change of potential energy is always $W\bar{y}$, where $\bar{y}$ is the vertical distance moved by the body's center of gravity. If the displacement of the body has a component upward, the body gains potential energy (as does the train of § 272); if the body's displacement has a component downward, the body loses potential energy.*

The energy that is stored in a body by virtue of the configuration of its particles is sometimes called the potential energy of the body. For example, a compressed spring has stored energy because of the internal stresses induced in the material of the spring by the compression. This stored energy is given up when the spring is released; in this sense the energy is *potential*. Similarly, the high-pressure steam entering a steam turbine has stored energy due to the configuration of its molecules; hence the energy is *potential* in form.** Summarizing, we note that kinetic energy is that energy which a body possesses by virtue of its motion, whereas potential energy is energy

*Torricelli (1608-1647) knew that a system of bodies would move under the action of gravity only if the center of gravity of the bodies descended. It was Torricelli who invented the first (liquid) barometer and measured the pressure of the atmosphere.

**However, the steam also has energy in the form of the kinetic energy of its moving molecules.

that a body has by virtue of its position relative to the earth or by virtue of the configuration of its molecules.

274. Example. In a hydroelectric plant, the elevation of the water surface above the plant is $y_1 = 400$ ft. above a certain reference level (Fig. 622). The elevation of the water below the plant above this same reference level is $y_2 = 100$ ft. How much work may be done by 100,000 cu. ft. of water passing through the plant if all of the processes are frictionless and if the turbine blades are 100% efficient?

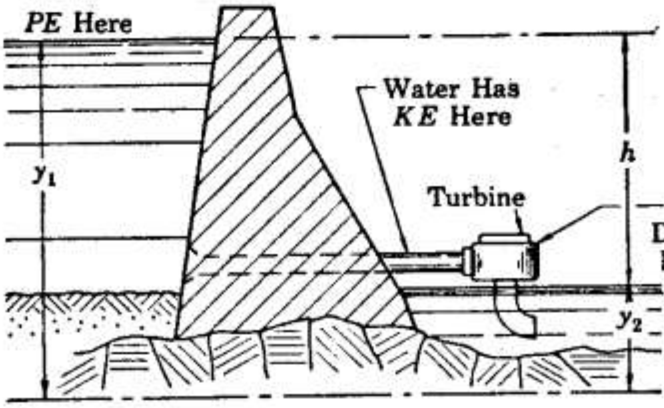

Fig. 622. **The loss of potential energy is numerically equal to the gain of kinetic energy, friction neglected. In the ideal case assumed, the gain of kinetic energy is converted into work in the turbine; so the work is equal to the change of potential energy. If one were thinking in terms of the law of conservation of energy, the energy relationship would be $\Delta PE + \Delta KE = 0$; that is, up *to* the turbine, no energy enters or leaves the system, so that the net change must be zero.**

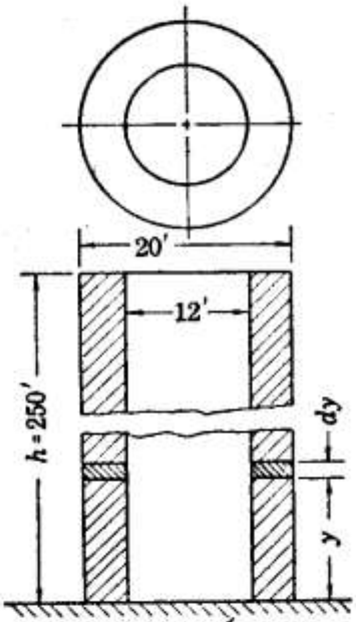

Fig. 623.

Solution. For the conditions specified, all of the change of potential energy is converted into work. Hence we have

$$U = Wh = (100{,}000)(62.4)(400 - 100)$$
$$= 1{,}872{,}000{,}000 \text{ ft-lb.}$$

275. Example. What is the work done against gravity in lifting the material to its place during the building of a masonry chimney which is 250 ft. high, 20 ft. OD, and 12 ft. ID? The masonry weighs 100 lb. per cu. ft. (Fig. 623).

Solution. Choose a differential volume of thickness dy at a height of y from the ground. This differential volume weighs

$$dW = w\,dV = (100)\left(\frac{\pi}{4}\right)(20^2 - 12^2)\,dy = 20{,}150\,dy.$$

The work done against gravity on this differential weight is its increase in potential energy. From the ground level, this is $(20{,}150\,dy)(y)$ and the total increase in potential energy is

$$PE = 20{,}150 \int_0^{250} y\,dy = \frac{(20{,}150)(250)^2}{2} = 628{,}000{,}000 \text{ ft-lb.}$$

This result may be obtained more directly by noting that the increase in potential energy of the material in the chimney, as measured from ground level, is $W\bar{y}$, where W is its total weight and $\bar{y}$ is the elevation of its center of gravity. Thus we find

$$PE = wV\frac{h}{2} = (100)\left(\frac{\pi}{4}\right)(20^2 - 12^2)(250)\left(\frac{250}{2}\right) = 628{,}000{,}000 \text{ ft-lb.}$$

276. Work of a Couple. The two equal forces F, Fig. 624, form a couple with a moment arm $2r$. If this couple is rotated through a small angle $d\theta$, *each* force F acts through a displacement $ds = r\,d\theta$ and therefore does the work $F\,ds = Fr\,d\theta$. Therefore, the total work for both forces F is

$$\text{(h)} \qquad U = \int 2F\,ds = \int F2r\,d\theta = \int M\,d\theta,$$

where $M = F(2r)$, the moment of the couple. If M in (h) is *constant*, we find

$$\text{(i)} \qquad U = M\int_0^\theta d\theta = M\theta,$$

the work done by a constant couple, where θ is in radians and M is generally in foot-pounds to give U in foot-pounds.* If M in (h) is not constant, the integration may be made when M is expressed as a function of θ.

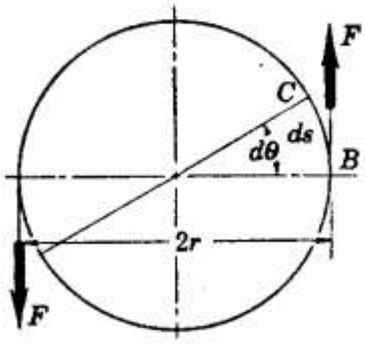

Fig. 624. Work of a Couple.

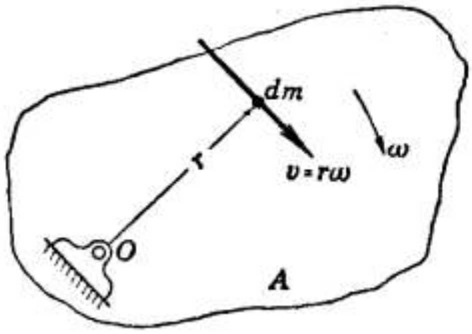

Fig. 625. Kinetic Energy—Rotation.

277. Kinetic Energy of a Rotating Body. A body A (Fig. 625) is rotating about a fixed center O. A particle dm of this body has kinetic energy whose value is $(dm)v^2/2$ at some instant, where v is the speed of the particle (§ 269). For a rotating body, the speed of a particle is $v = r\omega$, where ω is the same for all particles of the body and therefore is a constant that can be taken out of an integral. Since the sum of the kinetic energies of all its particles is the total kinetic energy of a rotating body, we have

$$KE = \int\frac{(dm)v^2}{2} = \int\frac{(dm)\,(r\omega)^2}{2} = \frac{\omega^2}{2}\int r^2\,dm = \frac{I_o\omega^2}{2},$$

*The reader should observe that the units of a couple and the units of work may be the same, foot-pounds, for example. Moreover, each one may be found from a force times a distance. Yet the conceptions of a couple (or any moment) and of work are entirely different. Be certain that each idea is clearly in mind in order to avoid mistakes due to confusion.

Moment of inertia of mass/p.227/Ch.XII

Cylinder, $\bar{I} = \frac{1}{2}mr^2$ about geometric axis

Sphere, $\bar{I} = \frac{2}{5}mr^2$ about a diameter

$I = \bar{I} + md^2$ about any other parallel axes or

$$KE = \frac{I_o\omega^2}{2}, \tag{50}$$

[ROTATION]

where we recall that $\int r^2\,dm = I_o$, the moment of inertia of the body about the axis O, the center of rotation in this case. Thus, the kinetic energy of a rotating body is $I_o\omega^2/2$ (foot-pounds in this book) for any angular velocity ω. If the angular velocity is ω_1, the kinetic energy is $I_o\omega_1^2/2$; etc., where the reference point O is the center of rotation.

As in the case of a body in rectilinear translation, the *net* work done on a rotating body is equal to its change of kinetic energy. To prove this, we recall that the resultant couple acting on a rotating body will be equal to the sum of the moments of all the forces on the body about the center of rotation. Let this resultant be represented by M_o. We have learned (§ 244) that $M_o = I_o\alpha$. Using $\alpha = \omega\,d\omega/d\theta$, we find

$$\int M_o\,d\theta = \int I_o\omega\,d\omega. \tag{j}$$

For a particular body with a fixed center of rotation, I_o is constant, so that the integration of the right-hand side of the equation (j) between the limits of any two angular velocities ω_1 and ω_2 gives

$$\int M_o\,d\theta = \frac{I_o\omega_2^2}{2} - \frac{I_o\omega_1^2}{2}\ \text{ft-lb.}, \tag{51}$$

which accords with equation (f), $U_{net} = \Delta KE$. If the moment M_o is constant, the left-hand integral in (51) becomes

$$\int M_o\,d\theta = M_o\int d\theta = M_o\theta,$$

where M_o is the same as we have previously designated ΣM_o, the sum of the moments of all the external forces with respect to the center of rotation O. In equation (51), we recognize the left-hand side as the *net* work (§ 276) and the right-hand side as the *change* of kinetic energy (ΔKE). It is implicit in (51) that each couple acting on the rotating body, if there is more than one couple, has the same angular displacement $d\theta$; otherwise, the left-hand side of (51) must be found from a sum of the works of the individual couples, $\int M_1\,d\theta_1 + \int M_2\,d\theta_2 + \cdots$.

If you care to satisfy yourself concerning the units of $I_o\omega^2/2$, recall that a radian is a ratio of like quantities and therefore has no units (see § 351). It follows that the unit for ω is 1/sec. Then the units of $I\omega^2 = (W/g)k^2\omega^2$ are found from

$$\frac{W}{g}k^2\omega^2 \rightarrow \frac{(\text{lb.})(\text{ft.}^2)(1/\text{sec.}^2)}{(\text{ft./sec.}^2)} = \text{ft-lb.}$$

278. Example. A weight A is supported from a cable which is wound about a 4-ft. drum (Fig. 626). An 8-ft. flywheel turns with the drum. The total weight of the rotating parts is 1288 lb. and the radius of gyration is 2.5 ft. While A travels 80 ft. vertically downward, the speed of the rotating parts changes from 10 rpm to 120 rpm. The frictional force in the bearings acting tangentially to the 6-in. shaft is $F = 70$ lb. What is the weight W of A?

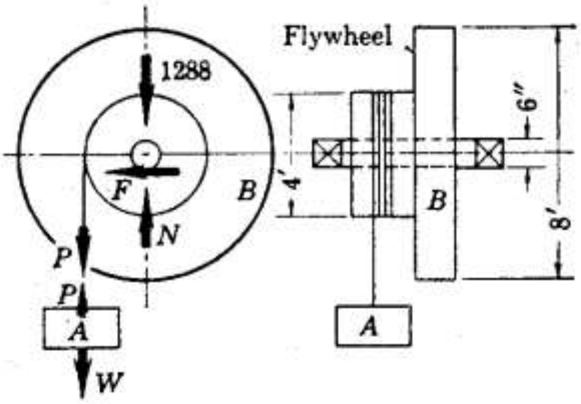

Fig. 626.

SOLUTION. Two bodies are involved, one in rotation, one in translation. The initial and final angular speeds of the flywheel B are

$$\omega_1 = \frac{2\pi n}{60} = \frac{2\pi 10}{60} = 1.047 \text{ rad. per sec.},$$

$$\omega_2 = \frac{2\pi n}{60} = \frac{2\pi 120}{60} = 12.58 \text{ rad. per sec.}$$

The corresponding linear velocities of the weight A are

$$v_1 = r\omega = (2)(1.047) = 2.09 \text{ fps} \qquad \text{and} \qquad v_2 = (2)(12.58) = 25.2 \text{ fps}.$$

Considering the free body of A and letting the downward direction be positive, we have

$$U_{net} = \Delta KE$$

$$(W - P)s = \frac{Wv_2^2}{2g} - \frac{Wv_1^2}{2g},$$

$$(W - P)80 = \frac{W}{2g}(25.2^2 - 2.09^2) = 9.79W,$$

from which

(**k**) $$W = 1.138\ P.$$

While the weight A moves 80 ft., the rotating parts turn through an angle of

$$\theta = \frac{s}{r} = \frac{80}{2} = 40 \text{ rad.}$$

From the free body of the rotating parts, we get the sum of the moments about O as

$$M_o = (P)(2) - (F)(0.25)$$
$$= 2\ P - (70)(0.25) = 2P - 17.5.$$

Now using the principle of equation (51) with M_o constant and using for I_o the value

$$I_o = mk_o^2 = \left(\frac{1288}{g}\right)(2.5)^2 = 250 \text{ slug-ft}^2.,$$

we find

$$\int M_o d\theta = M_o\theta = \frac{I_o}{2}(\omega_2^2 - \omega_1^2)$$

$$(2P - 17.5)40 = \frac{250}{2}(12.58^2 - 1.047^2),$$

from which $P = 254$ lb. Using this value of P in (**k**), we get $W = 289$ lb.

ALTERNATE SOLUTION. In setting up the energy relations for a series of connected bodies, it is not always necessary to take each member of the series as a free body. For example, in this case, imagine that the cable is whole, so that the force P is solely an internal force. Now we have implicitly assumed in this chapter that the *internal* forces do no net work, that is, that they are in equilibrium among themselves (§ 226). For example, the work of one force P downward is cancelled by the work of the other force P upward.

In applying the principle

$$U_{net} = \Delta KE = KE_2 - KE_1$$

to a series of connected bodies, the safest procedure is usually to find the work done by each force (or couple) separately and to consider separately also the changes of kinetic energy of each body in the system. Thus, neither the normal reaction N at the bearings nor the weight 1288 lb. of the rotating parts does work. The work done by W for s = 80 ft. is

$$U_W = +Ws = +80W.$$

The work done by the frictional force F is

$$U_F = -M\theta = -Fr\,\theta = -(70)(0.25)(40) = -700 \text{ ft-lb.},$$

where the minus sign indicates that this work tends to retard the bodies. Therefore the net work is

$$U_{net} = 80W - 700 \text{ ft-lb.}$$

The change in kinetic energy of A is

$$\Delta KE_A = \frac{Wv_2^2}{2g} - \frac{Wv_1^2}{2g} = \frac{W}{2g}(25.2^2 - 2.09^2) = 9.79\,W \text{ ft-lb.}$$

The change in kinetic energy of the rotating parts is

$$\Delta KE_B = \frac{I_o\omega_2^2}{2} - \frac{I_o\omega_1^2}{2} = \frac{250}{2}(12.58^2 - 1.047^2) = 19{,}600 \text{ ft-lb.}$$

Equating the *net* work to the total change of kinetic energy of all the connected parts, we have

$$80W - 700 = 9.79W + 19{,}600 \text{ ft-lb.},$$

from which W = 289 lb. If the internal force P in the cable were now desired, it would be necessary to use a free body of either A or B. This plan of considering a group of connected bodies as a free body often results in some simplification of a solution, but it requires care in being sure to include *all* energy quantities.

279. Bodies in Plane Motion. Let the rigid body A (Fig. 627) be in random plane motion. The path followed by any point is in general a plane curved path. Choose any convenient reference point, say the point O. While point O is moving in a curved path, we shall assume that at the instant under consideration, it is moving in a horizontal direction, as shown. Any differential mass dm, located at some point B, will have an absolute velocity

$$v_B = v_o \nrightarrow v_{B/o},$$

by the principle of relative velocities (§ 204). The velocity of B relative to O is necessarily perpendicular to the radius $BO = r$, since the body is rigid (§ 208). Hence $v_{B/o} = r\omega$, where ω is the angular velocity of A. The

$v_{B/o}$ = Velocity of B relative to point O

algebraic expression for v_B is obtained by applying the law of cosines to the vector triangle shown at B; thus

$$v_B^2 = v_o^2 + v_{B/o}^2 - 2v_o v_{B/o}\cos(180 - \theta)$$
$$= v_o^2 + r^2\omega^2 + 2v_o r\omega\cos\theta.$$

Since the kinetic energy of the particle at B, by definition, is $(dm)v_B^2/2$, the total kinetic energy of the body A is

$$KE = \int \frac{dm}{2}(v_o^2 + r^2\omega^2 + 2v_o r\omega\cos\theta)$$
$$= \frac{v_o^2}{2}\int dm + \frac{\omega^2}{2}\int r^2\,dm + v_o\omega\int (r\cos\theta)dm.$$

We see that (Fig. 627)

$$\int (r\cos\theta)dm = \int y\,dm = m\bar{y},$$

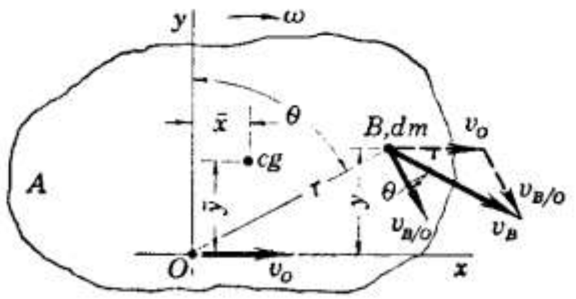

Fig. 627. Kinetic Energy — Plane Motion. **Point O is *not* a center of rotation; it is a random reference point whose velocity is known.**

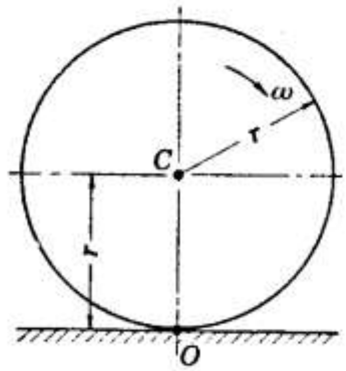

Fig. 628.

and we recognize that $\int r^2 dm = I_o$. From the foregoing expression, we then get the kinetic energy of a body in any kind of plane motion as

(1) $$KE = \frac{mv_o^2}{2} + \frac{I_o\omega^2}{2} + v_o\omega m\bar{y}.$$

Equation (1) gives the kinetic energy of a body in plane motion when the reference point O, whose velocity is known, is chosen at random with the x axis through O in the direction v_o. However, it is expedient in most engineering situations to choose one of the following two reference points; because they simplify the solutions.

1. The *center of gravity* of the body, in which case $\bar{y} = 0$, the last term drops out, and equation (1) becomes

(52) $$KE = \frac{m\bar{v}^2}{2} + \frac{\bar{I}\omega^2}{2},$$

[PLANE MOTION]

where $\bar{v}$ is the velocity of the center of gravity (or centroid) and $\bar{I}$ is the moment of inertia of the body about a gravity axis perpendicular to the plane of motion.

Sec fig 1145 p 271

2. The *instantaneous center* of the body, in which event $v_o = 0$, the first and last terms drop out, and equation (1) becomes

See page 271 for more on Instantaneous center

$$KE = \frac{I_o\omega^2}{2}, \tag{53}$$

[PLANE MOTION]

where I_o is the moment of inertia about an axis normal to the plane of motion through the instantaneous center (point of zero velocity). (Note that O is no longer a random point.)

For example (Fig. 628), the kinetic energy of a rolling cylinder is, from (52),

$$KE = \frac{m\bar{v}^2}{2} + \frac{\bar{I}\omega^2}{2} = \frac{m\bar{v}^2}{2} + \frac{mr^2\omega^2}{4} = \frac{3m\bar{v}^2}{4}$$

where $\bar{v} = r\omega$ is the velocity of the cg when there is no slipping and where $\bar{I} = mr^2/2$.

To use (53), we find

$$I_o = \bar{I} + mr^2 = \frac{mr^2}{2} + mr^2 = \frac{3mr^2}{2},$$

and then the kinetic energy is ($\bar{v} = r\omega$)

$$KE = \frac{1}{2}I_o\omega^2 = \frac{1}{2}\left(\frac{3mr^2}{2}\right)\omega^2 = \frac{3m\bar{v}^2}{4},$$

the same answer as obtained from (52). The instantaneous center of a *rolling* body is its point of contact with the ground (Fig. 628). Equation (52) suggests that plane motion may be thought of as a translatory motion and a rotational motion about the center of gravity; that is, the total kinetic energy is the kinetic energy of translation $mv^2/2$ plus the kinetic energy of rotation $\bar{I}\omega^2/2$.

The principle of work and kinetic energy, $U_{net} = \Delta KE$, being a special case of the law of the conservation of energy, holds for bodies in any kind of plane motion.

280. Frictional Force in Plane Rolling. When a body slides, the frictional force undergoes displacement and therefore does work. When a body rolls without slipping, there is

> no displacement of the point of contact between the rolling body and the ground;

hence the frictional force which induces pure rolling does *no* work. Hesitate long enough here to "see" in your own mind that the frictional force does not "move through a distance" if the body does not slide.

There is a rolling resistance (§ 77) against which work must be done in order to maintain motion. In a careful analysis, this resistance should be estimated; but in many cases, the work corresponding to this resistance is relatively small and is often neglected, or lumped together with other frictional resistance.

In general, in problems involving a combination of translation and rotation, it will be safer to *find the work* of each *individual* force *which does work* and then *add algebraically these work quantities.*

281. Example. A 4-ft. cylinder, which weighs 966 lb., rolls down a 15° incline from rest. What is its speed after it has rolled 50 ft.?

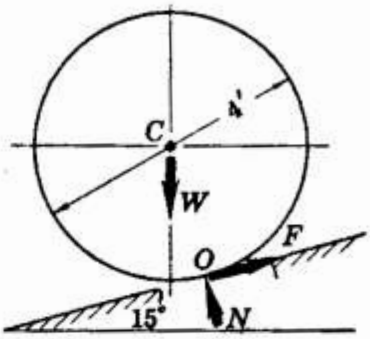

***Fig. 629.* For pure rolling, *F* does no work.**

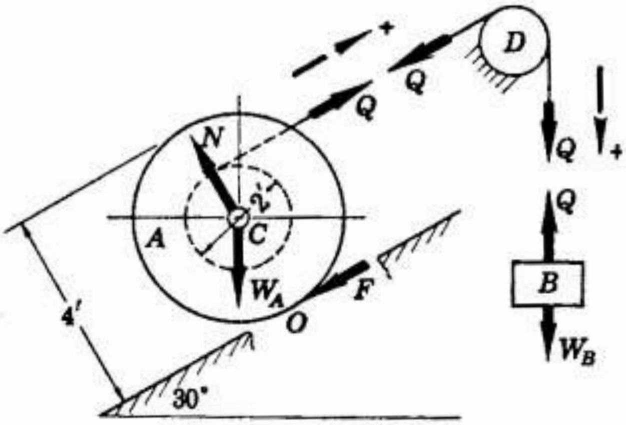

Fig. 630.

Solution. The forces acting on the cylinder are W, N, and F, as shown in Fig. 629. The force N does no work because it acts normal to the direction of motion. The force F does no work because the cylinder is rolling. Hence the only work quantity involves that component of W which is parallel to the incline. Using $U_{net} = \Delta KE$ and equation (52), we have

$$\textbf{(m)} \qquad (W \sin 15°)(50) = \frac{W}{2g}(\bar{v}_2^2 - \bar{v}_1^2) + \frac{\bar{I}}{2}(\omega_2^2 - \omega_1^2).$$

In this example, $v_1 = 0$, $\omega_1 = 0$, $\omega_2 = v_2/2$, and

$$\bar{I} = \frac{mr^2}{2} = \frac{(966)(4)}{(32.2)(2)} = 60 \text{ slug-ft.}^2$$

Using these values in (**m**), we find

$$(966)(0.259)(50) = \frac{966}{64.4}\bar{v}_2^2 + \left(\frac{60}{2}\right)\frac{\bar{v}_2^2}{4},$$

from which $\bar{v}_2 = 23.6$ fps. Observe that the work done by gravity on the cylinder is equal to its loss of potential energy.

Alternate Solution. Using the instantaneous center O (Fig. 629) as the reference point, and applying equation (53), we find

$$I_o = \frac{mr^2}{2} + mr^2 = \frac{(966)(4)}{2g} + \frac{(966)(4)}{g} = 180 \text{ slug-ft.}^2$$

$$(W \sin 15°)(50) = \frac{I_o\omega_2}{2} = \left(\frac{180}{2}\right)\frac{\bar{v}_2^2}{4},$$

from which $\bar{v}_2 = 23.6$ fps, as before.

282. Example. A 4-ft. cylinder A, Fig. 630, has a central 2-ft. groove about which is wound a weightless inextensible cord. This cord passes parallel to the 30°

incline and over a *smooth* post D, thence vertically downward to a body B which weighs $W_B = 300$ lb. The cylinder weighs $W_A = 500$ lb. and its radius of gyration about its axis C is $k_c = \bar{k} = 1$ ft. The frictional force is sufficient to cause the cylinder to roll. (a) If the system is released, in what direction does motion occur? (b) Determine the tension in the cord and the velocity of the cylinder after B moves 15 ft. from rest. (c) What is the acceleration of the body B?

Solution. (a) The direction in which the cylinder moves depends upon the relative values of the clockwise moment of Q about O and of the counterclockwise moment of W_A about O. If the cylinder is held at rest by some force not shown, the force in the cord is $Q = W_B = 300$ lb. and the clockwise moment is $(300)(3) = 900$ ft-lb. The counterclockwise moment is $W_A r \sin 30° = (500)(1) = 500$ ft-lb. Since the clockwise moment is greater than the counterclockwise moment, the cylinder will turn clockwise (or move up the incline) when only the forces shown are acting.

(b) Let $\bar{v}_A$ represent the speed of the center of gravity C of A. Since rolling occurs, the instantaneous center is at the point of contact O, Fig. 630. Inasmuch as the velocities of points in A are proportional to their distances from the instantaneous center, the velocity v_Q of a point on the cord is found from

$$\frac{v_Q}{\bar{v}_A} = \frac{3}{2} \qquad \text{or} \qquad v_Q = \frac{3}{2}\bar{v}_A.$$

And of course the speed of B is the same as that of a point on the cord; hence $v_B = (3/2)\bar{v}_A$.

For the free body A, Fig. 630, we may write ($U_{net} = \Delta KE$)

$$\text{(n)} \qquad (Q)(15) - (W_A \sin 30°)(10) = \frac{W_A\bar{v}_A^2}{2g} + \frac{\bar{I}\omega_A^2}{2},$$

where the force Q acts through a distance of 15 ft. while the component $W_A \sin 30°$ acts through 10 ft.; also

$$\bar{I} = m\bar{k}^2 = \frac{500}{g}, \qquad \omega_A = \frac{\bar{v}_A}{2}.$$

Observe particularly that the distances "moved by the forces" on A are not the same and that therefore the work quantities have to be found separately for each force. Substituting known values into (n), we have

$$\text{(o)} \qquad 15Q - 2500 = \frac{(500)(\bar{v}_A^2)}{2g} + \frac{(500)(\bar{v}_A^2)}{8g} = \frac{312.5\bar{v}_A^2}{g}.$$

For the free body B, we find ($W_B = 300$)

$$\text{(p)} \qquad (300 - Q)(15) = \frac{W_B v_B^2}{2g} = \frac{(300)(3/2)^2\bar{v}_A^2}{2g} = \frac{337.5\bar{v}_A^2}{g}.$$

Adding equations (o) and (p), we get

$$2000 = \frac{650\bar{v}_A^2}{g},$$

from which $\bar{v}_A = 9.95$ fps. Using this value of $\bar{v}_A$ in equation (o) [or (p)] we find

$$Q = \frac{2500 + 961}{15} = 230.5 \text{ lb.}$$

(c) Using the kinematic relation $v^2 = 2as$, obtained from $v\,dv = a\,ds$, we find that the acceleration of the axis of the cylinder is

$$\bar{a}_A = \frac{\bar{v}_A^2}{2s} = \frac{99.2}{(2)(10)} = 4.96 \text{ fps}^2.$$

Since the acceleration of B is to the acceleration of the axis of A as the ratio of the instantaneous radii (3/2), we have $a_B = (3/2)(4.96) = 7.44$ fps².

Alternate Solution. (b) A direct solution for the velocity of A (or B) may be made by considering the two bodies together as a free body. In this event, the net work is the algebraic sum of the works of the forces W_A and W_B only (Fig. 631). (Note that Q is now an internal force.) This net work is equal to the total change of kinetic energy of both A and B. Thus

$$U_{net} = \Delta KE$$

$$(W_B)(15) - (W_A \sin 30°)(10) = \frac{W_B v_B^2}{2g} + \frac{W_A \bar{v}_A^2}{2g} + \frac{\bar{I}\omega_A^2}{2}.$$

Using $v_B = (3/2)\bar{v}_A$ and the values of the other terms as found above, we get $\bar{v}_A = 9.95$ fps, as before. The tension in the cord may now be found from a free body of either A or B, and the other quantities called for in the problem are found as in the first solution.

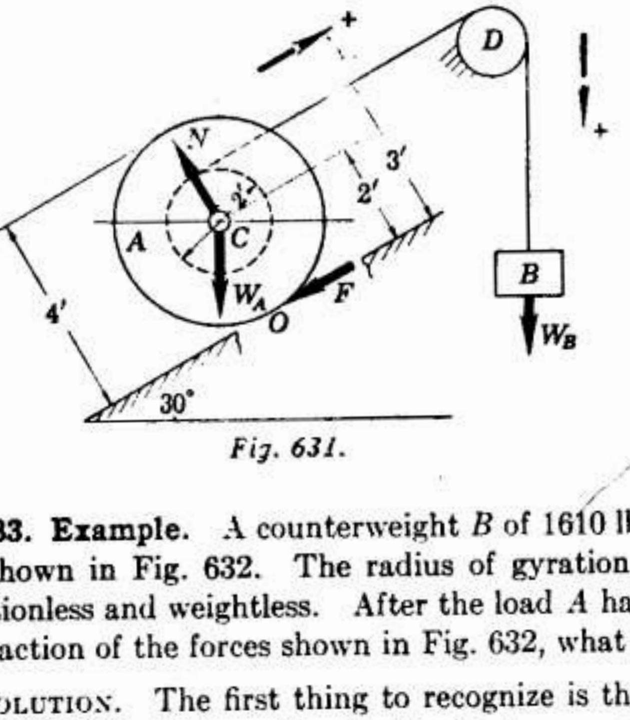

Fig. 631.

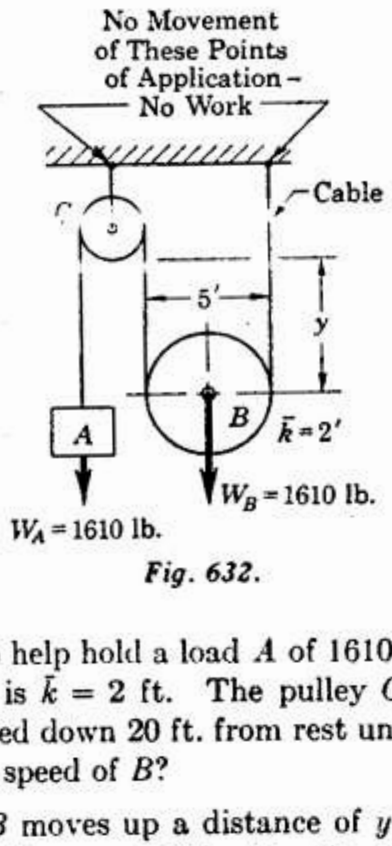

Fig. 632.

283. Example. A counterweight B of 1610 lb. is to help hold a load A of 1610 lb. as shown in Fig. 632. The radius of gyration of B is $\bar{k} = 2$ ft. The pulley C is frictionless and weightless. After the load A has moved down 20 ft. from rest under the action of the forces shown in Fig. 632, what is the speed of B?

Solution. The first thing to recognize is that if B moves up a distance of y, A moves down a distance $2y$; that is, if A moves 20 ft., B moves 10 ft. See Fig. 632. It follows also that the velocity of A is twice that of B, $v_A = 2v_B$. Let the work done by the weight A be positive; then that by B is negative. Using equation (52) for the kinetic energy of B, we get

$$U_{net} = \frac{W_A v_A^2}{2g} + \frac{W_B \bar{v}_B^2}{2g} + \frac{\bar{I}_B \omega^2}{2},$$

$$20W_A - 10W_B = \frac{W_A(4\bar{v}_B^2)}{2g} + \frac{W_B \bar{v}_B^2}{2g} + \frac{m_B \bar{k}^2(\bar{v}_B/2.5)^2}{2},$$

$$(20)(1610) - (10)(1610) = \frac{(1610)(4\bar{v}_B^2)}{2g} + \frac{1610\bar{v}_B^2}{2g} + \frac{(1610)(4)\bar{v}_B^2}{(2g)(6.25)},$$

from which $\bar{v}_B = 10.7$ fps.

284. Variable Forces. As previously mentioned, a force doing work may not be constant. The most common example of a variable force in engineering is that in which the force varies directly as the displacement, as in a spring (§ 231). If the scale of the spring in Fig. 633 is K lb. per ft. (§ 231), the magnitude of F after the spring has been gradually compressed a distance of y ft. is $F = Ky$. During a further infinitesimal displacement dy, the force Ky is virtually constant, so that the work done in compressing (or in stretching) a spring an infinitesimal amount is $dU = F\,dy = Ky\,dy$, and the total work of deforming a spring an amount of s ft. from its free length is

$$U = K\int_0^s y\,dy = \frac{Ks^2}{2}\text{ ft-lb.}$$

When the work done by a variable force involves a change of kinetic energy, the principle $U_{net} = \Delta KE$ may be used.

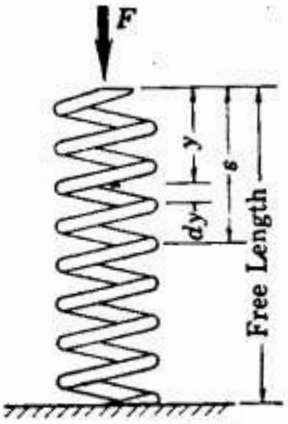

Fig. 633. Spring.

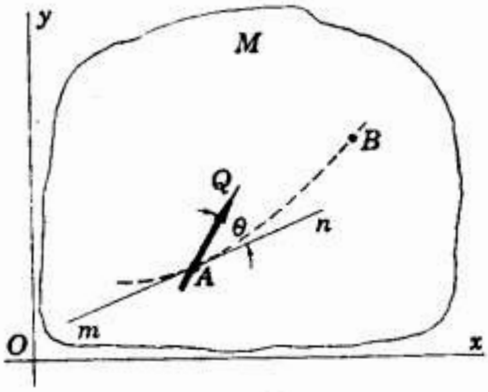

Fig. 634.

If the force varies in direction as well as in magnitude, the general problem is to set up for the work an integral that can be evaluated. In Fig. 634, a force Q acts on the body M at point A, which moves in a path AB. The line of action of Q makes an angle θ with the tangent mn to the curve at A. If the point of application of the force Q moves along the curve a distance ds, the work done is

$$Q\cos\theta\,ds,$$

inasmuch as the component $Q\sin\theta$ does no work. If Q, θ, and s all vary, the integral may be evaluated if any two of the three variables are expressed in terms of the third, for example, if θ and Q are in terms of s; or if all three variables can be expressed in terms of another variable, such as x and y. In this connection, we might recall that

(q) $$ds = [(dx)^2 + (dy)^2]^{1/2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2} dx.$$

285. Example. A 90-lb. body A, Fig. 635, rests upon a 30° incline where $f = 0.2$. It is in contact with a spring which has been compressed 10 in. and whose scale is $K = 30$ lb. per in. The lower end of the spring is attached to a fixed wall, and at the instant the spring reaches its free length, it ceases to act upon the body A. (a) What is the speed of the body at the instant the spring reaches its free length? (b) How far up the incline does the body go before coming to rest?

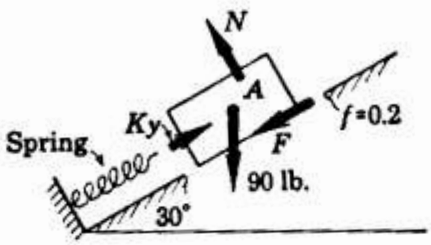

Fig. 635.

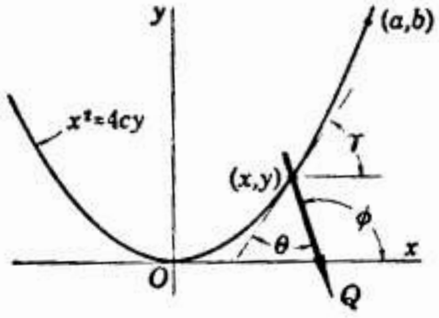

Fig. 636.

Solution. (a) A free body of A is shown in Fig. 635. The work of the spring on A is the same as the work of compressing the spring 10 in. = 10/12 ft. Assuming that this work is positive, we have

$$U_1 = K\int_0^{10/12} y\,dy = (30)(12)\left(\frac{10}{12}\right)^2\left(\frac{1}{2}\right) = 125 \text{ ft-lb.},$$

where (30)(12) is K, the scale of the spring, converted to pounds per foot. The normal force is obtained from a sum normal to the plane; we find

$$N = (90)(\cos 30) = 77.94 \text{ lb.},$$

from which

$$F = fN = (0.2)(77.94) = 15.59 \text{ lb.}$$

The work of the frictional force through the 10-in. displacement is

$$U_2 = -(15.59)\left(\frac{10}{12}\right) = -13 \text{ ft-lb.},$$

which is taken as negative because it opposes the work of the spring. The work against gravity or change of potential energy is

$$U_3 = -(90)(\sin 30)\left(\frac{10}{12}\right) = -37.5 \text{ ft-lb.}$$

The net work is then the algebraic sum of the individual work quantities, or

$$U_{net} = U_1 + U_2 + U_3 = 125 - 13 - 37.5$$
$$= 74.5 \text{ ft-lb.}$$

Using the relation $U_{net} = \Delta KE = mv^2/2$, we have

$$74.5 = \frac{90v^2}{2g},$$

from which $v = 7.3$ fps, the speed of A when the spring regains its free length.

(b) As the body A comes to rest, its kinetic energy (= 74.5 ft-lb.) is used up by work against friction and gravity. If the displacement is s ft. during this action, we have

$$(90 \sin 30° + 15.59)s = 74.5,$$

from which $s = 1.23$ ft. $= 14.75$ in. from the end of the spring in its free length or $14.75 + 10 = 24.75$ in. (nearly) from its initial position.

One of the lessons of this example is that when a force acts on a body for only part of its motion, the analysis must be made separately for those intervals involving the same force systems. Another illustration of this idea is when a body moves from a plane with one slope onto a plane with another slope, as when a car goes downhill and then moves onto a stretch of level road. The force system on the car while it is going downhill is not the same as it is while the car is on the level.

286. Example. A particle moves along a parabolic curve, $x^2 = 4cy$, from the origin to a point defined by the coordinates (a,b). It is acted upon by a variable force Q which is defined by the components $Q_x = mx$ and $Q_y = nx^2$, where m and n are constants. What is the work done by Q?

SOLUTION. Considering the particle in any position, defined by the coordinates (x,y), Fig. 636, we see that

$$U = \int Q \cos\theta \, ds,$$

In this expression, the value of Q is

$$Q = (Q_x^2 + Q_y^2)^{1/2} = (m^2x^2 + n^2x^4)^{1/2} = x(m^2 + n^2x^2)^{1/2},$$

We now need to use equation (**q**) *of* § 284 to find ds in terms of x. Differentiating the equation of the curve $x^2 = 4cy$, we get

$$2x\,dx = 4c\,dy, \qquad \text{or} \qquad \frac{dy}{dx} = \frac{2x}{4c} = \frac{x}{2c}.$$

Then,

$$ds = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2} dx = \left(1 + \frac{x^2}{4c^2}\right)^{1/2} dx = \frac{1}{2c}(4c^2 + x^2)^{1/2}\,dx.$$

To find θ as a function of x, we observe in Fig. 636 that $\theta = \phi - \gamma$. Also

$$\tan\gamma = \frac{dy}{dx} = \frac{x}{(2\,c)} \qquad \text{and}$$

$$\tan\phi = -\frac{Q_y}{Q_x} = -\frac{nx^2}{mx} = -\frac{nx}{m}.$$

Substituting these values in

$$\tan\theta = \tan(\phi - \gamma) = \frac{\tan\phi - \tan\gamma}{1 + \tan\phi\tan\gamma},$$

we find

$$\tan\theta = \frac{x(2cn + m)}{nx^2 - 2cm}.$$

If this expression is $\tan\theta$, the value of $\cos\theta$ is

$$\cos\theta = \frac{nx^2 - 2cm}{(4c^2 + x^2)^{1/2}(m^2 + n^2x^2)^{1/2}}.$$

Putting these values of Q, ds, and $\cos\theta$ into the integral for the work, we find

$$U = \frac{1}{2c}\int_0^a (nx^2 - 2cm)(x\,dx) = \frac{a^2}{8c}(a^2n - 4cm),$$

where the unit is ft-lb. if Q is expressed in pounds and the distances in feet.

287. Graphical Representation of Work. Suppose we plot a curve in which the ordinate is the force F and the abscissa is displacement, Fig. 637. Now consider a differential area between this curve and the x axis. Its width is dx, its height is F, and its area is $F\,dx$. The total area under the curve between any two values x_1 and x_2 is

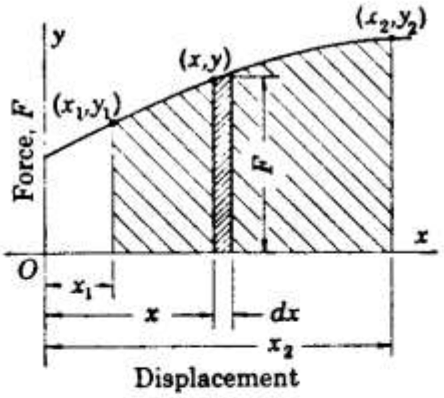

Fig. 637.

(1) $$\text{Area} = \int_{x_1}^{x_2} F\,dx.$$

However, we recognize $\int F\,dx$ as the expression for the work done by the force F. Hence, we conclude that the area under a curve whose coordinates are displacement and force represents the work done by the force. Engineers find this knowledge very useful in a number of circumstances. In making tests on various kinds of reciprocating engines, we often obtain what is called an *indicator diagram* or *indicator card* (Fig. 638), which is a record of the variation of the pressure p in the cylinder. Since the area A of the piston over which the pressure acts is constant, the indicator diagram is also a record of the variation of the force $F(= pA)$ on the piston.

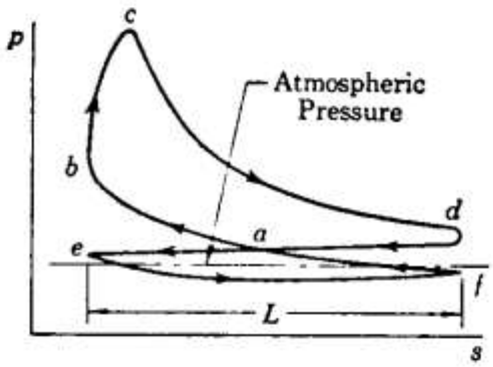

Fig. 638. Indicator Card. **This is a typical indicator card taken from a slow-speed internal combustion engine. The fuel begins to burn at *b*, and hot gases expand from *c* to *d*, doing work. In this diagram, the area *abcda* represents work done *by* the gases, and the area *aefa* represents work done *on* the gases (usually considered to be *negative* work in thermodynamics). The negative work occurs because the piston must push out the exhaust gases and draw in a new charge. The net work of the engine is represented by area *abcda* minus area *aefa*. Indicator cards and their uses are covered in some detail in heat power courses.**

The scale of the area is the product of the scale of the ordinate and the scale of the abscissa. Thus, if the ordinate scale is 1 in. = 300 lb. and the abscissa scale is 1 in. = 1/2 ft., then one square inch of area represents $(300)(0.5) = 150$ ft-lb., the scale of the area. After the area under a curve or the area enclosed *within* a force-displacement diagram has been found, this area in square inches times the *scale* of the area is the work done.

If the ordinate is pressure in pounds per square inch instead of force, the area of the diagram represents the work per square inch of *piston area*. Let 1 in. = 50 psi (ordinate) and 1 in. = 0.5 ft. (abscissa). Then the scale of the area is (50 psi)(0.5) = 25 ft-lb. per sq. in. of piston area, which means that each square inch of diagram area represents 25 ft-lb. of work for each square inch of piston area. Thus, suppose that the area of the diagram is 2 sq. in. and that the area of the piston is 150 sq. in. The total work is then (25)(2)(150) = 7500 ft-lb. as represented by the diagram.

In finding the area under a curve or within a closed curve, we may sometimes integrate for it when the curve or curves involved are such that we can readily express F as a function of x. If this procedure is not feasible, the area may be found by using a planimeter or by using some approximate rule, such as Simpson's rule.

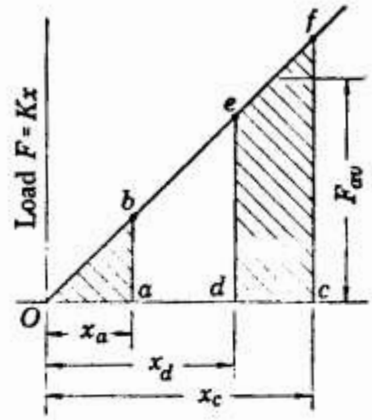

Fig. 639. Work of a Spring.

Since $F = Kx$ for a spring, the F–x curve is a straight line whose slope is K, the scale of the spring. The curve O–f, Fig. 639, represents the force-deflection curve for a spring, so that any area under this curve represents work done on or by the spring. For example, the energy stored in a spring when it is compressed a distance x_a from its free length is represented by the triangular area Oab. Since the ordinate $ab = Kx_a$, we have

$$\text{(s)} \qquad U = \text{Area } Oab = \frac{1}{2}(x_a)(Kx_a) = \frac{Kx_a^2}{2},$$

which agrees with the result obtained by integration. Observe in Fig. 639 that if the spring is compressed or stretched from a deflection of x_d to a deflection x_c, the work done on the spring *during this particular deflection* is represented by the area $defc$, which is seen to be $F_{av}(x_c - x_d)$, where F_{av} is the average force $[F_{av} = (Kx_c + Kx_d)/2]$, or

$$U_{d-c} = \frac{K(x_c + x_d)}{2}(x_c - x_d) = \frac{Kx_c^2}{2} - \frac{Kx_d^2}{2}.$$

Interpreting this expression in terms of equation (s), we see that the work done in compressing the spring from d to c is represented by

$$\text{Area } Ocf \text{ minus Area } Ode = \text{Area } defc;$$

that is, this work (d to c) is the total work done on the spring (Ocf) minus the work that had already been done (Ode).

288. Example. The expansion of steam in a steam engine is sometimes assumed to follow the law $px = C$; that is, the pressure in the cylinder times the displacement of the piston is a constant. This law is represented by the curve of Fig. 640, where we see that the pressure falls from p_c at a displacement x_a to p_d at a displacement of x_b. Let $x_a = 1$ ft., $p_c = 100$ psi, $x_b = 3$ ft. (= the stroke of the piston), and let the diameter of the piston be 20 in. For the assumption $px = C$, what is the work done by the steam on the piston?

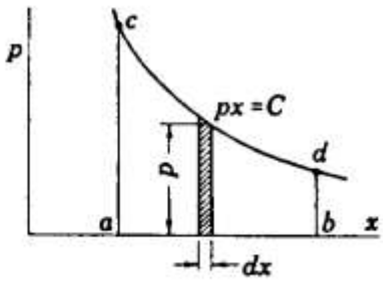

Fig. 640.

SOLUTION. The area under the curve cd is $\int p\,dx$, where $p = C/x$. Using this value of p, we find

$$\int p\,dx = C\int_{x_a}^{x_b}\frac{dx}{x} = C\log_e\frac{x_b}{x_a}.$$

For the data given, $C = p_c x_a = (100)(144)(1) = 14{,}400$, where the 144 converts pounds per square inch to pounds per square foot. Hence the work *per square foot* of piston area is

$$C\log_e\frac{x_b}{x_a} = (14{,}400)\log_e 3 = 15{,}800 \text{ ft-lb. per sq. ft.}$$

The area of the piston is $\pi D^2/4 = \pi(20/12)^2/4$ sq. ft.; hence the total work performed by the expansion of the steam from c to b is

$$U = \frac{\pi}{4}\left(\frac{20}{12}\right)^2(15{,}800) = 34{,}400 \text{ ft-lb.}$$

289. Power. *Power* is the time rate of doing work. A gasoline engine or a steam turbine, for example, is capable of doing repeatedly a particular amount of work during each unit of time. Thus the amount of work performed by a prime mover depends at least in part on how long it operates. To say that an engine does, for example, 100,000 ft-lb. of work, tells nothing of its size or capabilities, since the work may have been performed in a second or in a week. Power units therefore invariably involve some time unit. The most common unit of power in English-speaking countries is the horsepower. If an engine delivers one horsepower, it is, by definition, doing work at the rate of 33,000 ft-lb. per min.

For electrical machinery and in countries using the c.g.s. system of units, the power is often expressed in kilowatts (equal to 1000 watts), where a kilowatt is the same as 44,250 ft-lb. per min. Since

$$\text{Power} = \frac{\text{Work}}{\text{Time}}, \qquad \text{Work} = (\text{Power})(\text{Time}).$$

If therefore we multiply a power unit by time, we obtain a work unit. As pointed out previously (§ 271), the work units of horsepower-hour or kilowatt-hour are convenient for large work quantities. Suppose the work done is 1000 hp-hr. This work may be done by a 1000-hp engine running for one hour, by a 500-hp engine running for two hours, or by a 3000-hp engine running for 20 minutes (1/3 hr.). At any rate, the 1000 hp-hr. of work is equivalent to (see § 271).

$$(1000\text{ hp-hr.})\left(1{,}980{,}000\,\frac{\text{ft-lb.}}{\text{hp-hr.}}\right) = 1{,}980{,}000{,}000\text{ ft-lb.},$$

or

$$(1000\text{ hp-hr.})\left(2545\,\frac{\text{Btu}}{\text{hp-hr.}}\right) = 2{,}545{,}000\text{ Btu.}$$

290. Conversion Factors. Some horsepower equivalents are shown in the following conversions:

$$(\text{hp})\left(33{,}000\,\frac{\text{ft-lb.}}{\text{min-hp}}\right) = \frac{\text{ft-lb.}}{\text{min.}}, \qquad (\text{hp})\left(550\,\frac{\text{ft-lb.}}{\text{sec-hp}}\right) = \frac{\text{ft-lb.}}{\text{sec.}},$$

$$(\text{hp})\left(2545\,\frac{\text{Btu}}{\text{hr-hp}}\right) = \frac{\text{Btu}}{\text{hr.}}.$$

In the generation of electricity, the kilowatt is the common unit of power.

$$(\text{hp})\left(0.746\,\frac{\text{kw}}{\text{hp}}\right) = \text{kw.}$$

See also § 271 for other conversion equivalents.

291. Equations for Horsepower. When the need arises, the engineer should, of course, be able to find horsepower from its definition if the work done in a unit of time can be readily calculated. Hence the reader should look upon the discussion in this article as simply pointing to a method of reasoning.

Suppose that a constant force of F lb. does work, its point of application moving through a distance s ft. in the direction of F. If the distance s is covered in *one* minute, the speed of the point of application of F is $v_m = s/1$ fpm and the work done is Fv_m ft-lb. per min. Similarly, if the speed of the point of application of F is v_s fps, the work is at the rate of Fv_s ft-lb. per sec.; hence

$$hp = \frac{Fv_m}{33{,}000} = \frac{Fv_s}{550}. \tag{t}$$

We recall that work done on a rotating body is represented by $M\theta$ ft-lb., where M is the constant torque on the body in foot-pounds and θ is the angle in radians through which the torque is applied (§ 276). If this angle θ is turned through in one second or one minute, the corresponding angular velocities are ω_s rad. per sec. and ω_m rad. per min.; hence

$M\omega_s$ is work in foot-pounds per second and
$M\omega_m$ is work in foot-pounds per minute and

$$hp = \frac{M\omega_s}{550} = \frac{M\omega_m}{33{,}000}. \tag{u}$$

The work done on a rotating body may also be determined from a tangential force. Suppose a constant driving force of F lb. always acts tangent to a circular path in a rotating body at a constant radius of r ft., Fig. 641. The work done in *one revolution* is (Force)(Distance) $= F(2\pi r)$ ft-lb. If the body makes n_m revolutions per minute, the work done in *one minute* is $F(2\pi r)n_m$ ft-lb. per min.; or if it makes n_s revolutions per second, the work is $F(2\pi r)n_s$ ft-lb. per sec.; thus

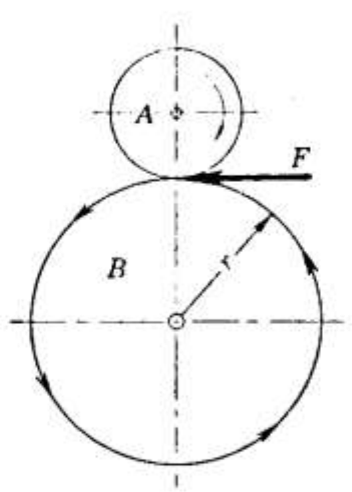

Fig. 641. For a practical view, consider that A and B are two gears, with gear A driving gear B, and exerting a constant force F while B rotates.

$$\text{(v)} \qquad hp = \frac{2\pi r n_m F}{33{,}000} = \frac{2\pi r n_s F}{550} = \frac{M(2\pi n_m)}{33{,}000},$$

where $M = Fr$ is the torque in foot-pounds.

In a hydroelectric power plant, some of the potential energy of water in a reservoir is converted into kinetic energy by the fall of the water through pipes to a hydraulic turbine. Then part of the kinetic energy of the water as it enters the turbine is converted into work done on the turbine shaft, which is finally converted into electrical energy. The amount of water which reaches the turbine is generally measured in cubic feet per some unit of time or in pounds per unit of time. Suppose that V cu. ft. per min. (cfm) of water arrives at a turbine. If the water weighs 62.4 lb. per cu. ft., the weight of water is $62.4V$ lb. per min. After falling a distance of h ft., the water has a kinetic energy equal to the loss of potential energy, in the ideal case where there are no frictional losses. Thus, with a loss of potential energy of $62.4Vh$ ft-lb. per min., the horsepower developed in the ideal turbine is

$$hp = \frac{62.4Vh}{33{,}000}.$$

We not only use the horsepower and other power units to express the rate at which work is done by an engine which *generates* power, but we also often use such units to express the rate at which work is lost; for example, in a brake (see § 294 and see Fig. 642).

292. Efficiency. The ***mechanical efficiency*** is a term used to express the losses that occur in machines due to friction between parts which have relative motion. The single word *efficiency* has a variety of technical meanings and is generally defined by its context, or by such phrases as *indicated thermal efficiency*. Detailed knowledge of thermal efficiencies is gained from works on thermodynamics. In this book, we shall sometimes use the simple term *efficiency* e, defined by the equation

$$\text{(w)} \qquad e = \frac{\text{Output}}{\text{Input}} \text{ [output and input in energy units]},$$

an expression that can be interpreted to apply to most of the various kinds of efficiencies. For example, in a geared hoist, the amount of work done by the operator (or electric motor or other engine driving the hoist) not only raises the load, but also overcomes the frictional losses in the bearings supporting the gears and sheaves and the losses in the meshing gear teeth. That is, the input work of the operator is greater than the useful work done, the difference being the frictional work. The work of the operator on the driving shaft of the hoist *divided into* the work done on the body being moved is termed the efficiency of the hoist.

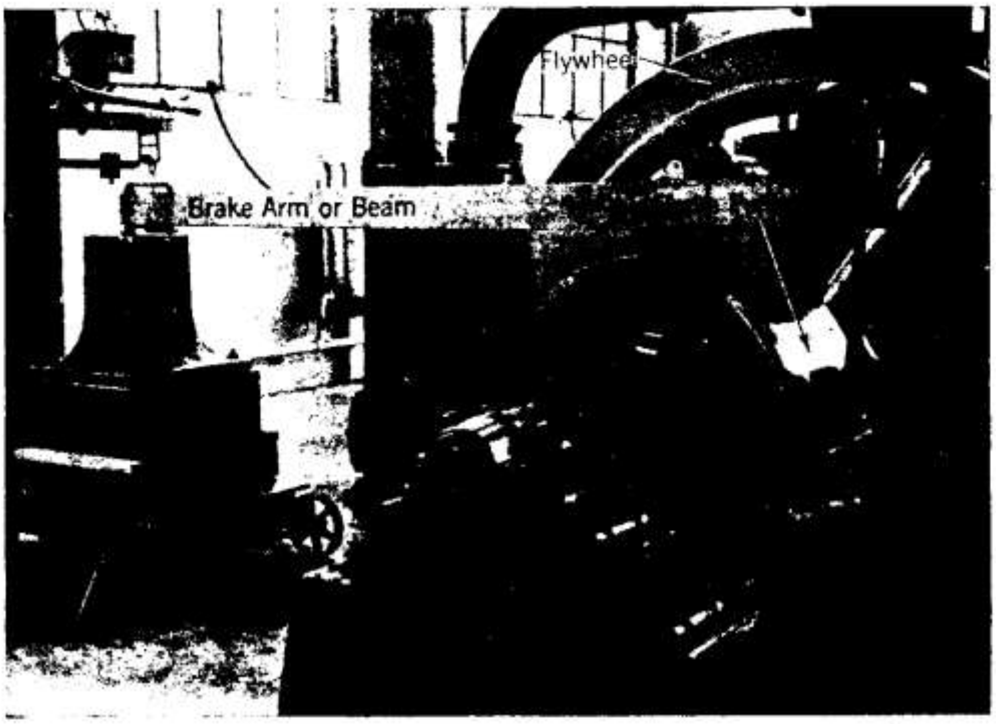

Fig. 642. ***Prony Brake.*** **A typical setup of a prony brake for measuring the brake horsepower of a small oil engine. Observe the clamping wheel in the upper right-hand corner which clamps the brake to the brake drum; and, on the left side of the illustration, observe the scales which weigh the force exerted at the end of the brake arm when the engine is running.**

The mechanical efficiency of a steam engine is defined as the work delivered by the shaft (called the brake work) *divided by* the work done by the steam in the cylinder (called the indicated work). The ratio of the brake horsepower (*bhp*) and the indicated horsepower (*ihp*) gives the same result; thus, for a steam engine, the mechanical efficiency is

$$e = \frac{\text{Brake work}}{\text{Indicated work}} = \frac{bhp}{ihp}.$$

293. Example. An automobile engine is delivering 100 hp from its crankshaft. The efficiency of the transmission, differential, etc., including wheel bearings, is 80%. The car is going 60 mph and the wheels are 30 in. in diameter. What is the tractive force exerted between the wheels and road?

SOLUTION. The output (to the wheels) is found from

$$e = \frac{\text{Output}}{\text{Input}} = 0.80 = \frac{\text{Output}}{100},$$

or the output is 80 hp. At 60 mph = 88 fps, the tractive force F does work

$$U = Fv_s = 88F \text{ ft-lb. per sec.}$$

The horsepower corresponding to this work per second is $88F/550$, which is equal to 80 hp as found above; that is,

$$80 = \frac{88F}{550},$$

from which $F = 500$ lb.

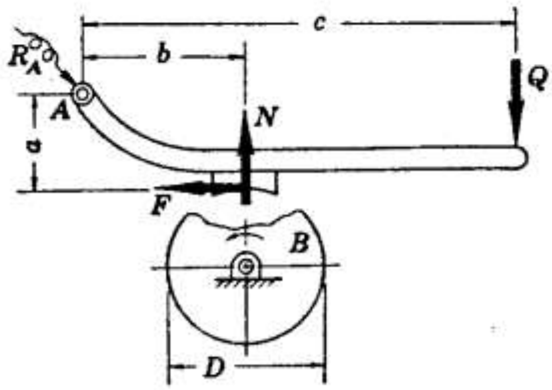

Fig. 643.

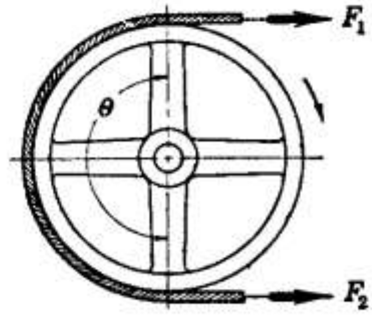

Fig. 644.

294. Example. A single-block brake (Fig. 643) is used to lower a load at a constant speed with the 30-in. brake-drum B which turns at 60 rpm. If the force $Q = 120$ lb., $a = 4$ in., $b = 20$ in., $c = 50$ in., and the coefficient of kinetic friction $f = 1/3$, what is the frictional horsepower (fhp)?

SOLUTION. Observing that $N = F/f = 3F$ and taking moments about the fixed pivot A in the free body of the lever, Fig. 643, we find

$$\begin{aligned}\Sigma M_A &= Fa - Nb + Qc = 0\\ &= (F)(4) - (3F)(20) + (120)(50) = 0,\end{aligned}$$

from which, $F = 107.1$ lb. The peripheral speed of the drum is

$$v_m = \pi Dn = \pi\left(\frac{30}{12}\right)(60) = 471 \text{ fpm.}$$

The frictional loss in horsepower is

$$fhp = \frac{Fv_m}{33{,}000} = \frac{(107.1)(471)}{33{,}000} = 1.53 \text{ hp.}$$

The brake either must be able to rid itself of heat at this rate (1.53 × 2545 Btu per hr.) without overheating, or it must be used intermittently. Brakes are usually rated in terms of the braking torque; in this case, the braking torque exerted is

$$M_f = (107.1)(15) = 1610 \text{ in-lb.} = 134 \text{ ft-lb.}$$

295. Example. A belt is transmitting 300 hp at a belt speed of 2400 fpm. If the coefficient of friction between the belt and the pulley is $f = 0.4$ and if the angle of contact is $\theta = 180°$, what is the value of the tight tension F_1? Consider that the belt is on the point of slipping and that the speed of the belt is low enough that the centrifugal force of the belt may be neglected with little error.

SOLUTION. The net driving force on the pulley is $F_1 - F_2$ (Fig. 644), the value of which is found from the horsepower equation $hp = Fv_m/33{,}000$. This gives

$$F = F_1 - F_2 = \frac{33{,}000\,hp}{v_m} = \frac{(33{,}000)(300)}{2400} = 4125 \text{ lb.} \tag{x}$$

Since the centrifugal force is negligible and slipping is imminent, we may use the relation $F_1/F_2 = e^{f\theta}$ (§ 71), and find

$$F_2 = \frac{F_1}{e^{f\theta}} = \frac{F_1}{e^{(0.4)(\pi)}} = \frac{F_1}{3.51} = 0.285F_1,$$

where π radians are equivalent to 180° and where it is understood that the belt is in limiting friction on the pulley. Using this value of F_2 in equation (x), we get

$$F_1 - 0.285F_1 = 4125,$$

from which the tight tension is $F_1 = 5780$ lb.

296. Closure. In applying the principle $U_{net} = \Delta KE$, the beginner must be particularly careful (1) that he has accounted for the work of each and every force in a system in obtaining the net work, (2) that he has correctly determined "the distance through which *each* force acts" in doing work, and (3) that he has included *all* changes of kinetic energies. In finding the change of kinetic energy of a body in plane motion, we may conveniently use either the center of gravity as a reference point, in which case

$$KE = \frac{W\bar{v}^2}{2g} + \frac{\bar{I}\omega^2}{2}, \tag{52}$$

Or we may use the *instantaneous center*, in which case

$$KE = \frac{I_o\omega^2}{2}, \tag{53}$$

where the subscript o refers to the axis of instantaneous rotation. If a wheel slips on a surface, the instantaneous center is *not* at the point of contact.

It is convenient to memorize that the terms $mv^2/2$ and $I\omega^2/2$ represent kinetic energy. Then it is a matter of understanding their application. We might write

$$U_{net} = \Delta\left(\frac{mv^2}{2}\right) + \Delta\left(\frac{I\omega^2}{2}\right) \tag{y}$$

and apply this to all situations regarding work and kinetic energy. If work is done on a system of bodies, this work is equal to the change of kinetic energy of the system and the change of kinetic energy may be computed separately for each body in the system. If a body is in translation ($\omega = 0$), equation (y) reduces to

$$U_{net} = \Delta\left(\frac{mv^2}{2}\right)$$

[TRANSLATION]

as found in § 270. If a body is rotating about a fixed center, the term $\Delta(mv^2/2)$ for translation drops out of (y) to give

$$U_{net} = \Delta\left(\frac{I\omega^2}{2}\right)$$

[ROTATION]

which agrees with (51) of § 277 when I is taken with respect to the center of rotation. Then for general plane motion, the right-hand side of (y) is taken in accordance with (52). Also, we need to know that $U = \int F\,ds$ and $U = \int M\,d\theta$, from which, if F and M are constant, we get $U = Fs$ and $U = M\theta$.

Our modern concept of work (and energy) evolved rather slowly. As typical in the early stages of the development of any science, there were vocabulary difficulties. Galileo variously called the quantity *wv momentum, impulse*, or *energy*. Descartes called *wv* the *quantity of motion*. Newton, with the conception of mass, called *mv* the *quantity of motion* (now called *momentum*). Leibnitz called mv^2 *vis viva* or *living force*. Coriolis decided to call $mv^2/2$ *vis viva*. Later, to avoid confusion, Belanger proposed the term *living power* for $mv^2/2$, which is now called *kinetic energy*. Coriolis did use the name *work* for *Fs*. Newton used the word *force* in its modern sense. The writer has not learned just when the word *power* took on its present day meaning, but students (and others) still have difficulty in keeping clear conceptions in their minds of such terms as *force* and *moment, work* and *power*.

Problems

NOTE. *The following problems are to be solved using work and energy principles.*

TRANSLATION

1381. In Fig. 645, $W = 100$ lb., $Q = 50$ lb., $\theta = 30°$, $f = 1/4$, and the body is moving with an initial velocity of 28 fps. For a displacement of 20 ft. to the right, find the resultant force on the body and the *net* work done. Check your answer by adding algebraically the work done by each force. *Ans.* 24.5 lb., 491 ft-lb.

1382. The same as **1381** except that $Q = 10$ lb. What does a negative answer for work mean?

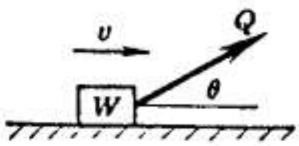

Fig. 645. Problems 1381, 1382.

1383. In Fig. 646, $W = 100$ lb., $\theta = 30°$, $Q = 120$ lb., $\beta = 15°$, $f = 1/4$, and the body is displaced 8 ft. up the incline. Find the resultant force on W and the *net* work done. Check your answer by adding algebraically the work done by each force. Does the velocity increase or decrease?

Ans. 52 lb., 416 ft-lb.; increases.

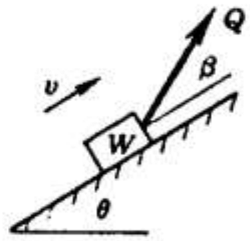

Fig. 646. Problems 1383-1387.

1384. The same as **1383** except that $\beta = 0$.

1385. The same as **1383** except that $W = 200$ lb.

1386. In Fig. 646, $Q = 46$ lb., $\beta = 0$, $\theta = 15°$, $f = 0.3$, and the net work is -200 ft-lb. as the body is displaced 12 ft. to the right. Find the weight of the body.

Ans. 114 lb.

1387. The same as **1386** except that the net work done is $+200$ ft-lb.

1388. A 900-lb. body lies on a 30° incline for which the kinetic coefficient of friction is $f = 0.1$. What is the least force that will keep the body in motion up the plane? What is the work done by this force in a distance of 250 ft.? With this force acting, what is the change in velocity of the body? *Ans.* 528 lb., 132,000 ft-lb., no change in v.

1389. The same as **1388** except that $f = 0.25$.

1390. A 3000-ton train has a resistance of 14 lb. per ton and a drawbar pull of 135,000 lb. produced by a Virginian steam locomotive. With what minimum speed must the train "hit" the foot of a 1-mile long. 1.5% grade to clear the top with a speed of 80 fps? Would you suggest a smaller locomotive or the use of two of these locomotives to attain the objective of 80 fps?

1391. An ore car, weighing 18 tons, is loaded with 50 tons of ore. The total resistance to motion of the car is represented by a force of 500 lb. parallel to the track. If the car is on a level track, what constant drawbar pull is necessary to change its speed from 5 fps to 15 fps in a distance of 200 ft.? The kinetic energy of rotation of the wheels is to be neglected. *Ans.* 2610 lb.

1392. The same as **1391** except that the car is being moved up a 2% grade.

1393. The same as **1391** except that the car is being moved down a 2% grade.

1394. A freight car becomes uncoupled from a train which is moving up a 1.5% grade at a speed of 10 fps. It is acted upon by a constant resistance to motion of 10 lb. per ton of weight. (a) What is its speed in mph when it reaches a point 1-mile down grade from the point at which it became uncoupled? (b) If the car moves onto a level track after this 1 mile is traveled, how far does it go on the level before it comes to rest? Its motion is unobstructed except for friction.

Ans. (a) 40 mph; (b) 2.04 mi.

1395. It is desired to design a steel chute to be used to deliver packing cases to a shipping platform from a second floor which is 12 ft. above. These cases are expected to be given an initial speed of about 2 fps down the chute and is is desired that they shall move down the chute, then out on the level for a distance of about 10 ft., at which point they are to come to rest under the action of friction alone. The coefficient of friction between the chute and cases is expected to be about $f = 0.3$. Neglecting the effect of a short curved path between the incline and the horizontal, determine the constant slope of the incline.

Ans. 21.65°

1396. The same as **1395** except that $f = 0.5$.

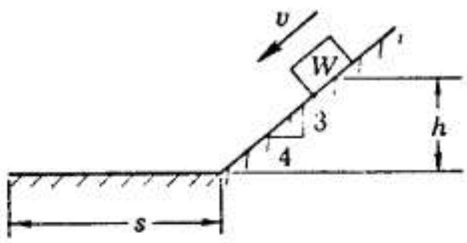

Fig. 647. Problems 1397, 1398.

1397. In Fig. 647, $f = 1/3$ for both planes, $h = 10$ ft., and the box comes to rest when $s = 18$ ft. Find the initial velocity.

Ans. 5.35 fps.

1398. In Fig. 647, $W = 64.4$ lb., $f = 1/3$, the initial velocity is $v_o = 10$ fps, and the box comes to rest when $s = 18$ ft. Find the initial height h, and the kinetic energy at the foot of the incline.

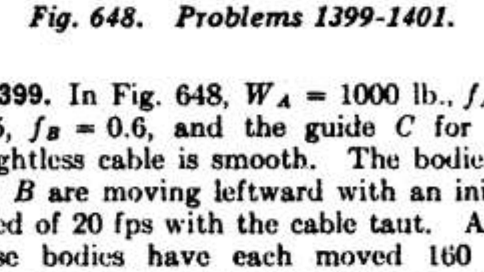

Fig. 648. Problems 1399-1401.

1399. In Fig. 648, $W_A = 1000$ lb., $f_A = 0.15$, $f_B = 0.6$, and the guide C for the weightless cable is smooth. The bodies A and B are moving leftward with an initial speed of 20 fps with the cable taut. After these bodies have each moved 160 ft., neither one changing its direction, their speed is 10 fps. (a) What is the weight W_B of the body B? (b) What are the tensions in the cables AC and BC? Use only energy methods in solving this part.

(c) What is the change of potential energy of the system?
Ans. (a) 1100 lb.; (b) 627 lb.; (c) −113,000 ft-lb.

1400. The same as **1399** except that $f_B = 0.4$.

1401. The same as **1399** except that the guide C is not smooth. The value of f at C is $f_c = 0.1$.

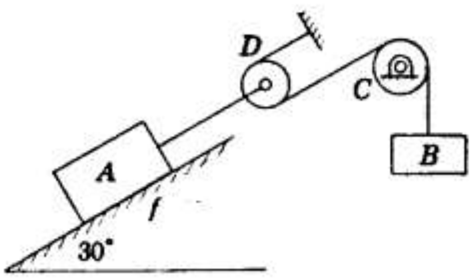

Fig. 649. Problems 1402, 1403.

1402. In Fig. 649, $W_A = 1000$ lb., $f = 1/3$, and the pulleys C and D are to be considered frictionless and weightless. (a) If A moves 60 ft. from rest up the incline in 12 sec., what is the weight W_B of the body B? (b) What are the forces on the cables attached to A and B? (c) What is the change of potential energy of A? Of B?
Ans. (a) 430 lb.; (b) 815 lb., 407.5 lb.; (c) 30,000 ft-lb., −51,500 ft-lb.

1403. The same as **1402** except that the displacement of A is 48 ft. *down* the incline.

1404. A book is thrown horizontally at a height of 4 ft. above a floor. It travels 20 ft. horizontally before striking the floor, and then slides 50 ft. along the floor. (a) What happened to the initial potential energy? (b) What happened to the initial kinetic energy? (c) Find the coefficient of kinetic friction between the book and the floor. State the assumptions under which your solution is valid.

1405. A water tank in the shape of a hemispherical shell, 96 ft. in diameter, is to be filled with water from a lake whose surface is 120 ft. below the top surface of the tank. The intake is at the bottom of the tank and the connecting pipe is 12 in. in diameter. Assume that there is no loss due to friction and that the water weighs 62.4 lb. per cu. ft. What is the work done in filling the tank? *Ans.* 745 hp-hr.

1406. A water cistern is in the form of a frustum of a cone with the smaller base, 14 ft. in diameter, below. The diameter of the larger base is 30 ft. and the depth of the tank is 20 ft. The water in the full cistern is to be pumped to a level 30 ft. above the bottom of the cistern. The friction head is equivalent to 8 ft. of lift. What is the work done in horsepower-hours?

ROTATION ABOUT CENTER OF GRAVITY

1407. The driver of a car turns a 17-in. steering wheel through 90°. Each hand exerts a constant tangential force of 10 lb. at the perimeter of the wheel. What is the work done by the hands? *Ans.* 267 in-lb.

1408. To speed up a flywheel, a constant net torque of 1200 in-lb. is applied through 8 revolutions. What net work is done?

1409. A screw jack with a 1.5-in. square thread is used to raise a load of 2000 lb. through a distance of 6 in. The mean diameter of the screw is 1.35 in., there are 3 single threads per inch, and the coefficient of friction is $f = 0.1$. What torque must be applied to the screw? How much work is done? See § 74.
Ans. 244 in-lb., 2290 ft-lb.

1410. The same as **1409** except that $f = 0.15$.

1411. A 500-lb. flywheel with a radius of gyration of 18 in. must be speeded up from 148 rpm to 150 rpm in 0.7 of a turn. What constant torque must be applied?

1412. The rotor of a steam turbine weighs 2400 lb. and has a radius of gyration of 15 in. It is supported in bearings 10 in. in diameter for which the coefficient of friction is $f = 0.007$. The steam is shut off while the turbine is rotating at 1800 rpm. If there is no resistance to rotation except the frictional forces in the bearings, how many turns does the rotor make before coming to rest? How long does it take? (Actually, the friction of a connected generator, the fluid friction of fanning the steam in the turbine casing, etc., would result in a much shorter time.)
Ans. 47,100 rev., 52.4 min.

1413. The same as **1412** except that $k = 18$ in.

1414. The weight of the rotating drum assembly B, Fig. 650, is 2576 lb. and its radius of gyration with respect to the axis of rotation is 14 in. The weight W is suspended from a cable which wraps about the $D = 32$ in. diameter. While W moves downward through a distance of 40 ft., the speed of the drum is increased from 20 rpm to 40 rpm. If the frictional effects are negligible, what is the weight W?

Ans. 18 lb.

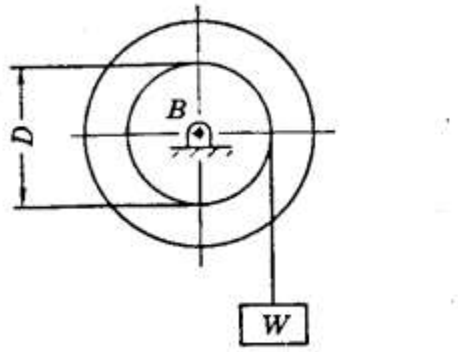

Fig. 650. Problems 1414-1417.

1415. A 193.2-lb. body W, Fig. 650, is suspended from a cable which wraps about a D = 32-in. drum B. The drum weighs 2576 lb. and has a radius of gyration of 14 in. with respect to its axis. (a) After W has moved 20 ft. from rest, what is the angular velocity of the drum? (b) What is the acceleration of W?

1416. In Fig. 650, the rotating assembly weighs 200 lb., D = 2 ft., and the weight W = 32.2 lb. Neglect friction and the mass of the cable. If W is released from rest and descends 20 ft. in 4 sec., find (a) the tension in the cable and (b) the radius of gyration of the rotating assembly.

Ans. (a) 29.7 lb.; (b) 1.38 ft.

1417. The same as **1416** except that the rotating assembly weighs 161 lb.

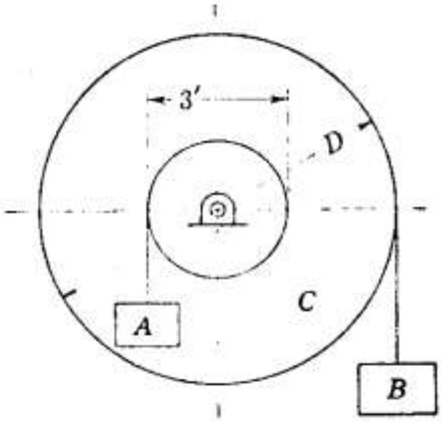

Fig. 651. Problems 1418, 1419.

1418. In Fig. 651, the bodies A and B weigh W_A = 500 lb., W_B = 150 lb., and D = 9 ft. The rotating part C weighs 600 lb. and has a radius of gyration of 3 ft. with respect to its axis. (a) After B has moved 10 ft. from rest, what is the speed of A and of B? (b) What is the acceleration of A and of B, and the angular acceleration of C? (c) What is the change of potential energy of the system?

Ans. (a) 1.59 fps, 4.77 fps; (b) 0.379 fps², 1.132 fps², 0.252 rad. per sec.²; (c) − 167 ft-lb.

1419. The same as **1418** except that W_B = 400 lb.

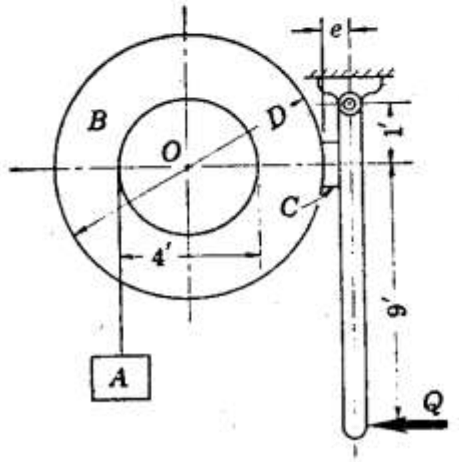

Fig. 652. Problems 1420-1422.

1420. In Fig. 652, the rotating elements B, which weigh 1288 lb. and have a radius of gyration of $\bar{k}$ = 2.5 ft., are turning 120 rpm. While it moves 80 ft. downward, the 278-lb. weight A is brought to rest by the constant frictional force at the brake shoe C, where f = 1/3. The shape of the brake arm is such that e = 0. What is the value of the force Q applying the brake?

Ans. 84 lb.

1421. The same as **1420** except that the weight A is moving upward when the brake is applied.

1422. The same as **1420** except that e = 6 in. *Ans.* 69.8 lb.

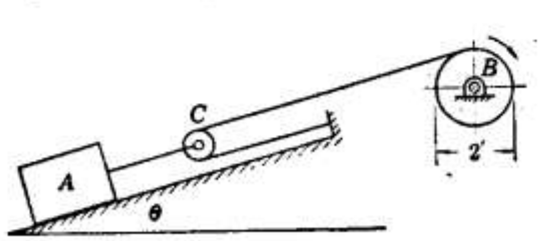

Fig. 653. Problem 1423.

1423. A 128.8-lb. body A, Fig. 653, is on a θ = 15° incline where f = 0.15. At a certain instant, the solid cast-iron cylinder B is rotating at 40 rpm and the block A is being moved up the incline by virtue of the cable connection shown. The cable CB wraps around the 2-ft. cylinder B. After A

moves 20 ft. up the incline, it comes to rest. What is the weight of the cylinder B? Neglect the axial friction for B and C and the mass of the pulley C. See problem **1293**.

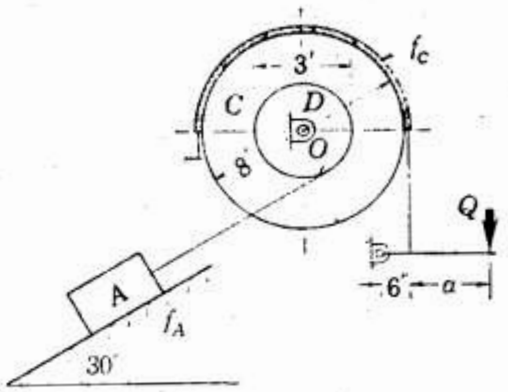

Fig. 654. Problems 1424, 1425.

1424. The drum D, the brake wheel C, and their supporting shaft, Fig. 654, weigh 2576 lb. and have a radius of gyration of $\bar{k} = 3$ ft. The 2000-lb. body A is moving down the incline, where $f_A = 1.3$, at a speed of 40 fps. The flexible brake band subtends 180° on the brake wheel, where the coefficient of friction is $f_C = 0.25$. When the brake is applied by a constant force Q, the speed of A decreases to 10 fps while it moves 170 ft. What is the magnitude of Q when $a = 30$ in.? The weight of the cable is negligible. *Ans.* 111 lb.

1425. The same as **1424** except that the initial speed of A is 80 fps.

1426. Figure 655 represents diagrammatically a traction drive for an elevator, in which A is a 6000-lb. cage and B is a 5000-lb. counterweight. The cable from the cage passes over the driving sheave C, thence around the idler sheave D, back around C, and thence to the counterweight. The

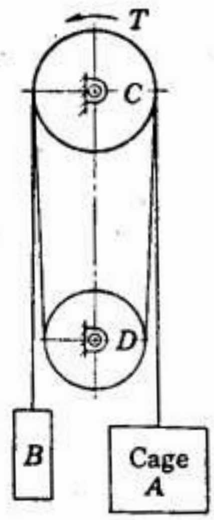

Fig. 655. Problems 1426, 1427.

power is delivered to the axis of the driving sheave C. Each sheave is 30 in. in diameter and each has a moment of inertia of 5 slug-ft.² about its axis. If the elevator attains an upward speed of 10 fps in a distance of 10 ft. with constant acceleration, what torque must be applied to the axis of C? What is the change of potential energy of the system?

Ans. 3420 ft-lb., +10,000 ft-lb.

1427. The same as **1426** except that the elevator attains a speed of 5 fps.

1428. Steam leaves a nozzle at a speed of 2000 fps and enters the blades of a turbine. The rate of discharge is 0.5 lb. per sec. The rotor carrying the blades rotates at 1800 rpm. If all of the kinetic energy of the steam is converted into work on the shaft of the turbine, what is the torque exerted on the shaft? *Ans.* 165 ft-lb.

1429. The same as **1428** except that the velocity of the steam leaving the turbine is 500 fps. Other losses are to be neglected.

ROTATION NOT ABOUT CENTER OF GRAVITY

1430. A slender rod, which weighs 16.1 lb. and is $L = 4$ ft. long, is pivoted at one end. It rotates from rest under the action of gravity only, starting from a vertical position, Fig. 656. (a) What is the speed of its center of gravity after it has turned through $\theta = 120°$? (b) What is its kinetic energy at this instant? (c) What is its change of potential energy?

Ans. (a) 12.04 fps; (b) 48.3 ft-lb.; (c) 48.3 ft-lb.

1431. A homogeneous slender rod of length L is pivoted at one end in a vertical position, Fig. 656. In this position, its angular velocity is $\omega_o = 0$. Derive an ex-

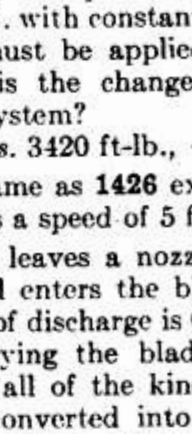

Fig. 656. Problems 1430-1432.

pression for $\dot{\theta}_2$ when $\theta = 120°$, rotation being forced by gravity alone.

Ans. $\dot{\theta}_2 = 3(2gL)^{1/2}/4$.

1432. The same as **1431** except that $\omega_o = 4$ rad. per sec.

1433. In Fig. 657, the uniform bar A weighs $W_A = 48.3$ lb., the drum B weighs 128.8 lb. and has a radius of gyration $\bar{k} =$ 10 in., the sheave C and the cable are considered weightless, the body D weighs 644 lb., and all friction is neglected. If bar A is released from rest in the horizontal position shown, (a) how much kinetic energy does the system have when A strikes the stop Q? (b) What is the velocity of D when impact occurs?

1434. The same as **1433** except that D weighs 322 lb.

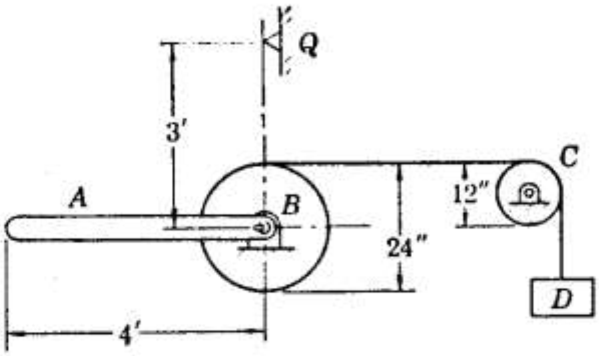

Fig. 657. Problems 1433, 1434.

ROTATION AND TRANSLATION

1435. A solid homogeneous cylinder, 16 in. in diameter and weighing 322 lb., rolls on a rough horizontal plane under the action of a constant horizontal force $Q =$ 100 lb., which acts through the center of gravity. If the initial speed of its cg is 5 fps, what is its speed after it has moved 20 ft.? What is the acceleration of the cg?

Ans. 17.1 fps. 6.67 fps².

1436. The same as **1435** except that the rolling body is a solid homogeneous sphere.

1437. A solid homogeneous cylinder, 24 in. in diameter and weighing 483 lb., rolls down a 30° rough incline under the action of gravity. If it starts from rest, how far has it moved when the speed of its center of gravity is 25 fps? *Ans.* 29.1 ft.

1438. The same as **1437** except that the rolling body is a cylindrical thin shell.

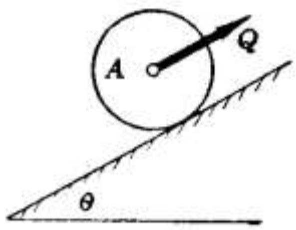

Fig. 658. Problems 1439-1441.

1439. The body A, Fig. 658, is a 161-lb. cylinder, 12 in. in diameter. It is being rolled up the incline, where $\theta = 30°$, by a constant force $Q = 96.5$ lb. (a) What is the speed of its center of gravity after a displacement of 15 ft. from rest? (b) What is its angular acceleration? (c) What is the frictional force between the plane and the cylinder? What coefficient of friction is necessary for rolling?

Ans. (a) 8 fps; (b) 4.26 rad. per sec.²; (c) 5.35 lb., 0.0384.

1440. The same as **1439** except that $Q = 224.5$ lb.

1441. The same as **1439** except that the rolling body A is a sphere.

Ans. (a) 8.3 fps; (b) 4.6 rad. per sec.²; (c) 4.6 lb., 0.033.

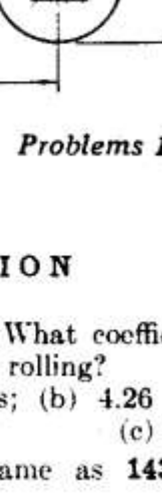

Fig. 659. Problems 1442, 1443, 1444.

1442. In Fig. 659, the body A is a solid homogeneous cylinder with a weightless cord wrapped about its midsection. One end of the cord is attached to a fixed surface at B. If the cylinder is released from rest in the position shown and moves vertically downward, what is the speed of its cg after a displacement of 15 ft.? The axis remains in a horizontal position. What is its change of potential energy?

1443. The same as **1442** except that A is a solid homogeneous sphere.

Ans. 26.2 fps, $-15W$.

1444. The same as **1442** except that A is a hollow cylinder with a thin shell.

1445. A 322-lb. grooved cylinder, Fig. 660, is rolled toward the right by a constant

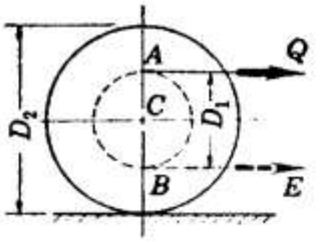

Fig. 660. Problems 1445, 1446.

force Q until the displacement of its cg is 54 ft. The force Q acts on a weightless cord along the line AQ which wraps about a groove whose diameter is $D_1 = 18$ in. The diameter of the cylinder is $D_2 = 36$ in. and its moment of inertia is $\bar{I} = 10$ slug-ft.^{2} If the motion starts from rest and if the final speed is 45 fps, what is the magnitude of Q? There is no force along the line BE.

Ans. 181 lb.

1446. The same as **1445** except that the force Q acts along the line BE and the cord wraps around the groove in a clockwise direction. There is no force along the line AQ.

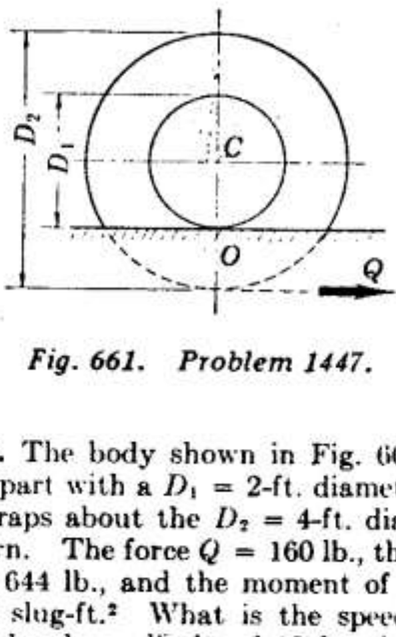

Fig. 661. Problem 1447.

1447. The body shown in Fig. 661 rolls on the part with a $D_1 = 2$-ft. diameter. A cord wraps about the $D_2 = 4$-ft. diameter, as shown. The force $Q = 160$ lb., the body weighs 644 lb., and the moment of inertia $\bar{I} = 12$ slug-ft.^{2} What is the speed of C after C has been displaced 10 ft., the body having started from rest? *Ans.* 10 fps.

1448. A disk A, Fig. 662, has a weightless cord wrapped about its midsection. This cord passes over a frictionless and weightless sheave C, and thence downward to a 50-lb. weight B. Let $W_A = 80$ lb., $\theta = 30°$, $\bar{I}_A = 4$ slug-ft.^{2}, and let the displacement of B be 20 ft. (a) If the system starts from rest, determine the final speed of the cg of A and the acceleration of B. (b) What is the tension in the cord?

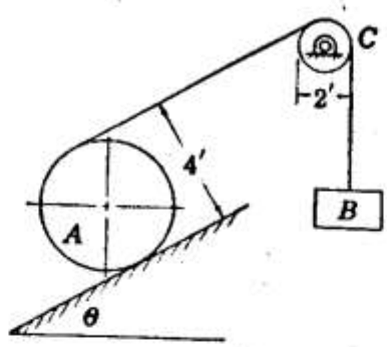

Fig. 662. Problems 1448, 1449.

1449. The same as **1448** except that the sheave C has a moment of inertia $\bar{I}_C = 0.3$ slug-ft.^{2}

Ans. (a) 10.5 fps, 11 fps^2; (b) 32.9 lb.

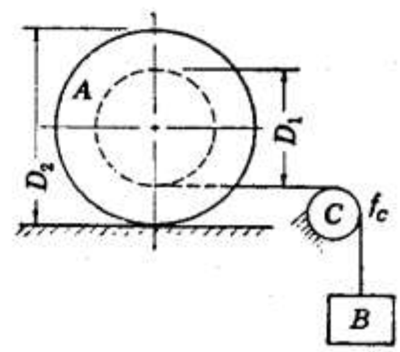

Fig. 663. Problems 1450, 1451.

1450. In Fig. 663, the grooved cylinder A weighs 200 lb. and has a moment of inertia of $\bar{I}_A = 6$ slug-ft.^{2} Let $D_1 = 2$ ft., $D_2 = 3$ ft., $W_B = 32.2$ lb., and $f_c = 0$ (that is, the fixed peg C is smooth). (a) Determine the speed of the cg of A and the acceleration of B after B has moved downward through 20 ft. (b) What is the tension in the cord? (c) How would the acceleration of the cg of A vary as the diameter D_1 increases, other conditions remaining the same?

Ans. (a) 11.95 fps, 0.397 fps^2; (b) 31.8 lb.; (c) inversely as D_1.

1451. The same as **1450** except that the peg C is not smooth and $f_c = 0.3$.

1452. A 2-ft. diameter sphere whose mass is 5 slugs travels 10 ft. from rest down a 100% grade for which the coefficient of friction is 0.3. Find the work of friction and the final kinetic energy. Did the sphere roll or roll and slide?

Ans. $KE_2 = 797$ ft-lb.; rolls and slides.

1453. The same as **1452** except that the grade is 10%.

SPRINGS—VARIABLE FORCES

1454. A force of 100 lb. is required to compress a spring 4 in. Sketch a force-displacement diagram and use it in finding (a) the scale (or modulus) of the spring, (b) the work and maximum force required to compress the spring a total of 6 in., and (c) the potential energy stored in the spring when it is compressed 4 in.
Ans. (a) 25 lb. per in.; (b) 450 in-lb., 150 lb.; (c) 200 in-lb.

1455. A loaded freight car, weighing 80,000 lb. and moving on a horizontal track at 3 mph, strikes a nest of springs on a bumper post. The car's brakes are applied so that the total resistance to motion is constant at 300 lb. per ton of weight. If the springs are compressed 2 in. in bringing the car to rest, what is the combined scale of the springs? How far back does the car move on the rebound? Neglect the kinetic energy of rotation of the car's wheels and the effect of the mass of the springs.

1456. A 100-lb. body falls 24 in. from rest and strikes the free end of a helical spring whose scale is 30 lb. per in. and whose axis is vertical. Find (a) the maximum compression of the spring, (b) the maximum kinetic energy of the falling body, and (c) the velocity of the body when the spring is compressed 12 in. Neglect friction and the mass of the spring. Sketch a velocity-displacement curve for the motion of the body from the moment it contacts the spring until the maximum compression of the spring is reached.
Ans. (a) 16.43 in.; (b) 214 ft-lb.; (c) 8.8 fps.

1457. A helical spring whose scale is 1000 lb. per in. is compressed 4 in. with its axis in a vertical position. A body weighing 20 lb. is placed on the compressed spring, which is then released. Neglecting air resistance and assuming that the spring acts on the 20-lb. body only until it regains its free length, determine the kinetic energy of the 20-lb. body at the instant the spring has regained its free length, and determine the height to which the body rises as measured from its original position.
Ans. 660 ft-lb., 400 in.

1458. A 40-lb. body falls freely and strikes a spring whose scale is 5000 lb. per in. If the spring is compressed 6 in., through what total vertical distance does the 40-lb. body move before it is brought to rest? What is the maximum kinetic energy of the body during this period?
Ans. 187.5 ft., 7480 ft-lb.

1459. The same as **1458** except that the scale of the spring is 40 lb. per in.

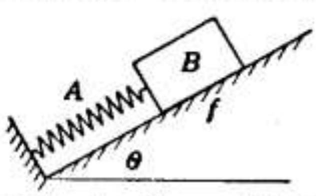

Fig. 664. Problems 1460-1462.

1460. In Fig. 664, a coiled spring A, whose scale is 30 lb. per in., is compressed 10 in. against a fixed surface with a 90-lb. body B at the free end. The coefficient of friction between B and the plane is $f = 0.2$, and $\theta = 30°$. When it is released, the spring acts on B until the free length is reached. How far up the incline from the point of release does B go? During this movement, what is B's maximum kinetic energy? *Ans.* 1.23 ft., 114.8 ft-lb.

1461. A 100-lb. body B slides from rest down a $\theta = 45°$ incline, where $f = 0.2$, for a distance of 10 ft. At this instant, it strikes a spring A, Fig. 664, which is compressed 3 in. before B comes to rest. Determine (a) the scale of spring, (b) the maximum kinetic energy of the body during this period, and (c) the distance the block will go up the plane on the rebound. For uniformity, let zero displacement be at the upper end of the free spring.
Ans. (a) 1547 lb. per in.; (b) 565.6 ft-lb., (c) 6.59 ft.

1462. The same as **1461** except that $\theta = 30°$.

1463. A coil spring has a scale of 70 lb. per in. From an initial deflection of 2 in., it is compressed 1.5 in. more. Integrate for the addional work done on the spring, and make a drawing to show an area which represents this work. *Ans.* 288 in-lb.

1464. From an initial deflection of 2 in., a coiled tension spring is stretched an additional 3 in. The work necessary for this action is 60,000 ft-lb. Determine the scale of the spring by integration and check by using a force-deflection diagram.

1465. The work done upon a coiled spring in deflecting it 0.7 in. from an initial deflection of 0.3 in. is 500 in-lb. If an additional 1500 in-lb. of work is done upon the spring, determine the final total deflection and the magnitude of the final force.
Ans. 1.93 in., 2120 lb.

1466. A gun weighing 200,000 lb. has an initial speed of recoil of 10 fps. The recoil is resisted by a nest of springs whose scale is 50,000 lb. per in. What is the distance of recoil? The gun moves in a horizontal plane.

1467. A 900-lb. body moves on a horizontal plane, where $f = 0.25$, under the action of a horizontal force $Q = 3x^2 + 250$ lb., where x is the displacement in feet. If the body starts from rest, determine the net work and the speed of the body after a displacement of 10 ft.

Ans. 1250 ft-lb., 9.45 fps.

1468. A 500-lb. body moves up a 30° incline, where $f = 0.2$, under the action of a force $Q = 2x^2 + 4x + 400$ lb. which is directed upward and parallel to the plane. The variable x is the displacement in feet parallel to the plane. If the body starts from rest, determine the net work and the speed of the body after a displacement of 20 ft. *Ans.* 7400 ft-lb., 30.9 fps.

1469. The same as **1468** except that the 500-lb. body is moving down the incline and the force Q is directed downward.

1470. A particle moves along the parabolic curve $x^2 = 16y$ from the origin to the point where $x = 4$ ft. It is acted upon by a resultant force, $Q_x = 2x$ lb. ($Q_y = 0$.) What work is done? (Solve by the methods § 286. Do you see a simpler solution?)

Ans. 16 ft-lb.

1471. The force on a pneumatic device varies according to $Fs^{0.85} = C$. If a force of 100 lb. is required when $s_1 = 1$ in., find the force required when the displacement is $s_2 = 4$ in. How much work is done from s_1 to s_2? *Ans.* 30.8 lb., 154 in-lb.

1472. The same as **1471** except that the relation between F and s is $Fs^{1/3} = C$.

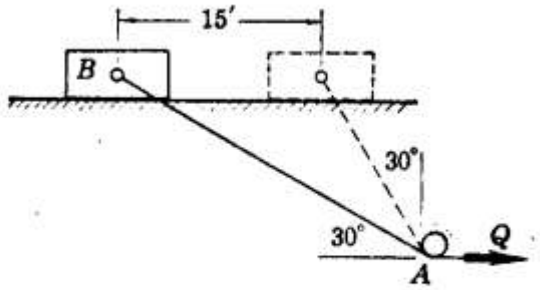

Fig. 665. Problems 1473, 1474.

1473. A constant force $Q = 30$ lb. acts on the cord shown in Fig. 665. The cord passes over a smooth peg A of negligible diameter and is attached to the body B. Determine, by two methods, one of which uses the calculus, the work done on B by Q while B is displaced a distance of 15 ft.

1474. The same as **1473** except that Q is variable and $Q = 10s + 10$, where s is the displacement in feet of the point of application of Q. A solution by one method only is required. *Ans.* 715 ft-lb.

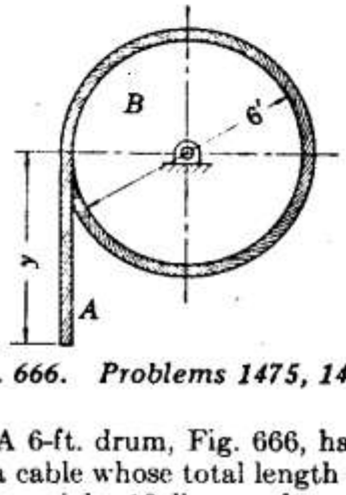

Fig. 666. Problems 1475, 1476.

1475. A 6-ft. drum, Fig. 666, has wound about it a cable whose total length is 100 ft. The cable weighs 10 lb. per ft. and 10 ft. of it hang from the drum when rotation starts. The shaft and drum together weigh 6440 lb. and have a radius of gyration of 2.5 ft. Assume that the bearing friction is negligible, that all coils of the cable have a mean diameter of 6 ft., and that the cable does not slip on the drum. What is the angular velocity of the drum after the 90 ft. of cable have run off?

Ans. 8.03 rad. per sec.

1476. The same as **1475** except that the cable weighs 4 lb. per ft.

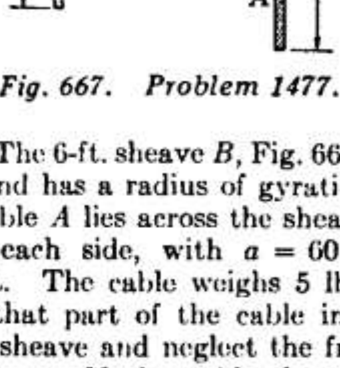

Fig. 667. Problem 1477.

1477. The 6-ft. sheave B, Fig. 667, weighs 644 lb. and has a radius of gyration of 2.5 ft. A cable A lies across the sheave, overhanging each side, with $a = 60$ ft. and $b = 40$ ft. The cable weighs 5 lb. per ft. Neglect that part of the cable in contact with the sheave and neglect the friction in the bearings. If the cable does not slip on the sheave and starts from rest, what is the angular velocity of the sheave after the length a has increased from 60 ft. to 100 ft. *Ans.* 9.51 rad. per sec.

1478. The force-displacement diagram for a punch while it is punching a hole in a metal plate is similar to the solid curve $OABC$, Fig. 668. It is seen that the work area under the curve is represented roughly by the triangular area ODC, at least closely enough for some engineering purposes. The maximum force necessary to punch a 3/4-in. hole in a 1/2-in. thick, soft steel plate is about $F_{max} = 50{,}000$ lb. (a) What is the work done? (b) It is frequently assumed that the flywheel supplies all this energy. If the flywheel has a moment of inertia of 34 slug-ft.^{2} and is turning at 150 rpm at the beginning of the punching, what is its angular velocity just as the punching is completed? Assume that the frictional losses and the power supplied by the driving motor during this short time element are negligible.

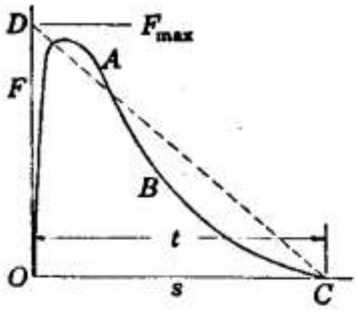

Fig. 668. Problems 1478, 1479.

1479. The work of punching a 1/2-in. hole in a 1/4-in. steel plate may be found as described in **1478**. The maximum force for this hole is approximately $F_{max} = 18{,}000$ lb. The flywheel of this machine, turning at 200 rpm. is to supply the energy for punching the hole. Its speed should not fall below 170 rpm during punching. (a) What is the minimum permissible moment of inertia of the flywheel? Frictional losses are negligible. (b) Assuming that the moment of inertia of the hub and spokes are negligible, that the mean diameter of the rim of the flywheel is 30 in., and that the width of the rim is 4 in., determine the minimum thickness of the cast-iron rim.

Ans. (a) 3.05 slug-ft.^{2}; (b) approx. 5/8 in.

POWER

1480. If a 3000-lb. automobile coasts at a constant speed of 30 mph down a 0.5% grade, what is the total resistance to motion as measured by a force parallel to the grade? How much is this resistance expressed in kilowatt units of power?

Ans. 15 lb., 0.895 kw.

1481. How long does it take a 200-lb. man to run up a stairway and increase his elevation 60 ft. if his average net power output during this period is 0.2 kw? (In a similar manner, a student may determine his individual power output for a short period.)

1482. (a) What is the maximum speed at which a 10-hp motor can lift a 1000-lb. elevator? (b) If friction consumes 25% of the motor's power, what is the maximum speed?

Ans. (a) 5.5 fps; (b) 4.125 fps.

1483. An 1800-rpm motor has an output of 2 hp. It is 85% efficient. Find (a) the torque on its shaft and (b) the cost to operate it 30 min. at 2 cents per kw-hr.

1484. What is the greatest head against which a 20-hp motor can pump 1000 gpm of water that weighs 62.1 lb. per cu. ft. The overall efficiency of the operation is 80%. *Ans.* 63.6 ft.

1485. A 12-in. gear is transmitting 36 hp at 100 rpm. What is the driving force on the gear teeth?

1486. A 3220-lb. automobile on a level road utilizes 60 hp to travel at a constant 60 mph and 75 hp to travel at 75 mph. Find the acceleration (a) at the instant when the car is going 60 mph and 75 hp is applied and (b) at the instant when the car is going 75 mph and 60 hp is applied. (Do not assume that 1 hp per mph is a constant relationship. It may be true for two points by coincidence.)

Ans. (a) $+0.93$ fps^2; (b) -0.75 fps^2.

1487. An autombile engine develops a maximum of 100 hp at 3600 rpm. The 30-in. diameter rear wheels turn 1/4 as fast as the engine, and 75% of the engine power is delivered to the wheels. For the maximum power, find (a) the torque on the rear wheels, (b) the driving force between the wheels and the road, and (c) the least weight that may be on the rear wheels if they are not slipping when $f = 0.2$.

Ans. (a) 5250 in-lb.; (b) 350 lb.; (c) 1750 lb.

1488. A 4000-lb. automobile with a fluid torque-converter transmission might conceivably have its engine so throttled that it continuously delivers 100 hp to the rear wheels. Observe that the driving force is variable and assume that the wheels do not slip. (a) Find the speed after the car has traveled 500 ft., measured from the instant when its speed was 30 mph. (b) How much net work is done in this distance?

Ans. (a) 90.8 fps; (b) 391,000 ft-lb.

1489. The same as **1488** except that the initial speed of the car is 45 mph.

1490. A locomotive exerts a constant drawbar pull of 50,000 lb. on a freight train whose gross weight is 2000 tons. The total train resistance is 15 lb. per ton. This train starts up a 1% grade with a speed of 60 mph. (a) If the grade is 2 miles long, what is the train's speed in mph when it reaches the top? What is the maximum horsepower developed at the drawbar during this period? What is the drawbar horsepower at the instant that the train reaches the top of the grade? (b) If the grade is 4 miles long, what is the speed at the top?
Ans. (a) 44.9 mph, 8000 hp, 5980 hp; (b) 21 mph.

1491. In problem **1428**, what is the horsepower being developed by this ideal turbine?

Ans. 56.5 hp.

1492. What maximum horsepower must be delivered by an electric motor which operates a mine hoist, when the total load is 22 tons and when the hoist is uniformly accelerated upward from 5 fps to 20 fps in 10 sec.? Assume that the guides are frictionless.

1493. A hoist with its load weighs 40,000 lb. It attains a speed of 10 fps in a distance of 10 ft. with constant acceleration. The constant friction in the guides is 200 lb. What is the maximum horsepower delivered by the driving motor during this period? What horsepower does the motor develop when the elevator is on the point of starting?

1494. A motor truck with its load weighs 8.05 tons and is to be accelerated on a level highway from 10 mph to 45 mph in a distance of 2000 ft. The resistance to motion is 500 lb. If the acceleration is constant throughout this period, (a) what maximum horsepower must be delivered to the rear wheels; (b) what is the delivered horsepower when the speed is 10 mph; (c) what is the tractive force for the 36-in. driving wheels?

Ans. (a) 122 hp; (b) 27.2 hp; (c) 1018 lb.

1495. If a hoisting engine is to be able to lift a 50-ton girder through 400 ft. in 5 min. at constant speed, what horsepower must it deliver, friction negligible?

1496. The rotating parts of an electric generator weigh 14 tons and have a radius of gyration of 7 ft. These parts are brought up to a speed of 600 rpm in 30 sec. with uniform acceleration. What maximum horsepower is necessary, if the friction is neglected? *Ans.* 10,190 hp.

1497. The same as **1496** except that a constant total frictional force of 200 lb. acts at the surface of the 6-in. journals.

1498. What average horsepower is needed in filling a water reservoir 100 ft. long, 100 ft. wide, and 8 ft. deep in 4 hr.? The water is pumped against a constant head of 30 ft. Neglect the friction.

Ans. 18.9 hp.

1499. At a point where the cross section of a river is 900 sq. ft. in area, the average speed of flow is 3 mph. In this vicinity a fall of 30 ft. is available. If the efficiency of the hydraulic turbines is 80%, what horsepower could be obtained from the total flow of the river?

1500. The ideal pumping engine in problem **1405** develops 100 hp. How long will it take to fill the hemispherical tank?

Ans. 7.45 hr.

1501. The work done to operate an actual screw jack in raising a 2000-lb. load through 6 in. is 2300 ft-lb. What is the efficiency of the jack? See problem **1409**.

1502. The same as **1501** except that the work done is 3000 ft-lb. *Ans.* 33.3%.

1503. The General Electric Company rates the 2400-lb., J-47 turbo-jet engine at a static thrust of 5000 lb. for take-off and at 10,000 hp at 750 mph. (a) What is the power output when the plane is on the point of starting down the runway? (b) What is the thrust at 750 mph? (c) If the thrust is constant at all speeds, what is the horsepower developed at 375 mph? See Fig. 678.

Ans. (a) 0; (b) 5000 lb.; (c) 5000 hp.

1504. An Air Force F-86 fighter plane weighs 12,000 lb. loaded. If its jet engine can produce a constant thrust of 5000 lb. and if this thrust causes the speed to change from 500 mph to 600 mph, find (a) the net work done, (b) the final power output in horsepower, and (c) the altitude that might have been gained if the pilot had chosen to increase the potential energy instead of the kinetic energy of the plane. (These data do not reflect the true performance of the plane.)
Ans. (a) 4.42×10^7 ft-lb.; (b) 8000 hp; (c) 3680 ft.

1505. An Air Force F-86 fighter plane weighs 12,000 lb. loaded and climbs at 5700 fpm, maximum. (a) Find the average "lifting horsepower" needed. (b) If the "lifting horsepower" were applied to produce horizontal acceleration when the plane's speed is 480 mph, what would be the instantaneous horizontal unbalanced force on the plane? (c) What instantaneous acceleration of the plane would be produced by this unbalanced force?

1506. A B-36 bomber with a bomb load weighs 320,000 lb. From a cruising speed in level flight of 300 mph, the pilot maneuvers the plane to gain 500 ft. of altitude in a horizontal distance of 2 miles, and then the plane levels out at a speed of 240 mph. Find (a) the increase of potential energy, (b) the loss of kinetic energy. (c) From a consideration of (a) and (b) decide whether the pilot increased or decreased the power output of the engines during this maneuver. *Ans.* (a) 1.6×10^8 ft-lb.; (b) 3.46×10^8 ft-lb.; (c) decrease.

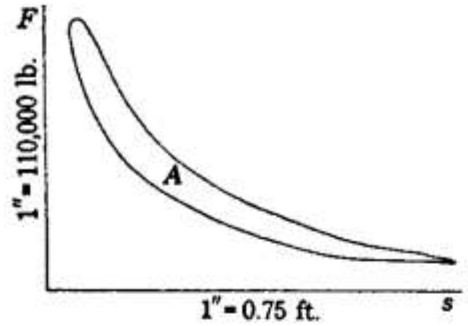

Fig. 669. Problems 1507, 1508.

1507. Figure 669 is an indicator diagram for a two-stroke-cycle diesel engine. The enclosed area A represents to scale the work done in a cylinder during each revolution of the engine. Suppose that the area A is 0.55 sq. in., that the force scale is 1 in. = 110,000 lb., and that the displacement scale is 1 in. = 0.75 ft. (a) What work is done per revolution in each cylinder? (b) If the engine has 3 cylinders and turns at 200 rpm, what horsepower is the engine developing?

1508. The same as **1507** except that the area of the diagram A is 0.86 sq. in.

Ans. (a) 71,000 ft-lb.; (b) 1290 hp.

1509. The diagram *abcde* of Fig. 670 shows what is termed a conventional diagram for a steam engine. It is something of an idealized picture of the variation of the force on a piston at different piston positions. The actual work of the engine is somewhat less than that represented by the enclosed area. Suppose that the magnitude of the force at the point a is F_a = 10,000 lb. and that F_c = 1600 lb. (a) Determine the work

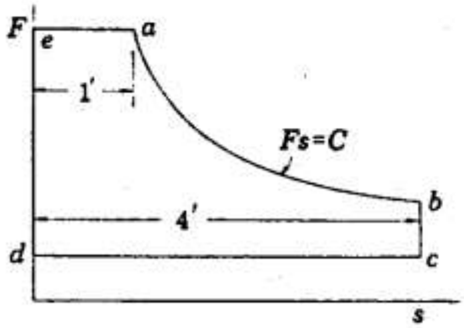

Fig. 670. Problems 1509, 1510.

represented by this diagram. (b) If two such diagrams are completed during each revolution of a double-acting engine, what horsepower corresponds to an angular speed of 120 rpm?

Ans. (a) 17,480 ft-lb.; (b) 127 hp.

1510. The same as **1509** except that F_a = 25,000 lb. and F_c = 5000 lb.

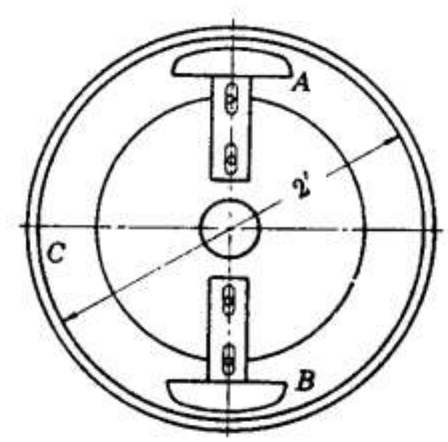

Fig. 671. Problem 1511.

1511. In Fig. 671, C is a rim which is connected to turn with a shaft, which we shall call shaft 2. The weights A and B through a connection by pins in slots are arranged to turn with a shaft 1, which is collinear with shaft 2. As shaft 1 speeds up, centrifugal force causes the weights A and B to move outward and contact the rim C. Faster speed finally results in a frictional force large enough to turn C and shaft 2. This figure represents a clutch, the object of which is to pick up the load gradually with little shock to shaft 2. The contact surfaces of the weights are faced with asbestos for which f = 0.4 and the weights weigh 32.2 lb. each. The center of gravity of each of the weights A and B is 9 in. from the axis of the shafts. If n = 300 rpm, what horsepower may be transmitted through this clutch? *Ans.* 33.8 hp.

1512. In the brake shown in Fig. 672, assume that the pressure is uniformly dis-

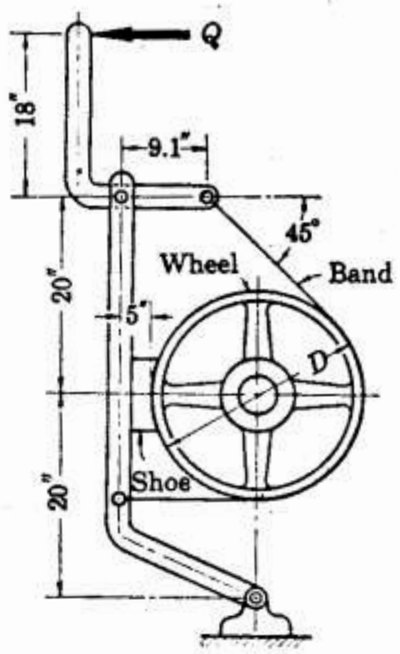

Fig. 672. Problems 1512, 1513.

tributed on the brake shoe. The force $Q = 100$ lb., $f = 0.35$ for both shoe and band, and $D = 20$ in. (a) If the brake wheel turns counterclockwise, what braking torque is exerted? (b) If the speed of rotation is 200 rpm, what power is dissipated as heat? *Ans.* (a) 389 ft-lb.; (b) 14.8 hp.

1513. The same as **1512** except that the wheel turns clockwise.

1514. An ore car A, Fig. 673, weighing 50,000 lb. with its load is moving down a 10° incline with a speed of 10 fps. The resistance to motion is 15 lb. per ton. A cable from A wraps about a drum whose diameter is $D_1 = 3$ ft. Attached to the drum is an 8-ft. ($= D_2$) brake wheel, for which $f = 0.3$. The car A is to be brought to rest in a displacement of 50 ft. with uniform deceleration. What force Q must be applied? What is the maximum rate at which power is absorbed by the brake? Express as fhp (frictional horsepower).

Ans. 697 lb., 179 hp.

1515. Figure 674 is a diagrammatic representation of a prony brake, which is a device to measure the output of engines. A brake is clamped to the flywheel of an engine, but it is prevented from rotating with the flywheel by connecting the brake arm to a scale which is itself fixed. All the power of the engine is absorbed by friction at the brake blocks. Observing that the moment of the net scale reading (the force at the end of the brake arm) about the center of the flywheel must be equal to the moment of the frictional forces at the brake about the same point, we can easily compute the "brake" horsepower of an engine. Suppose that the brake arm is $a = 60$ in. long, the scale reading is 40 lb., and the flywheel turns 300 rpm. What is the brake horsepower? See Fig. 642, p. 413.

Ans. 11.4 hp.

1516. In Fig. 674, the brake arm is $a =$ 3 ft. long, the scale reading is 250 lb., and the angular speed of the flywheel (and engine) is 275 rpm. What is the brake horsepower? See problem **1515**.

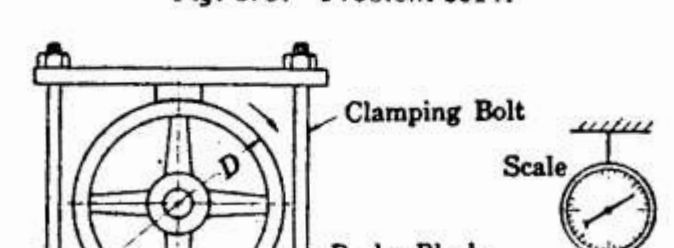

Fig. 673. Problem 1514.

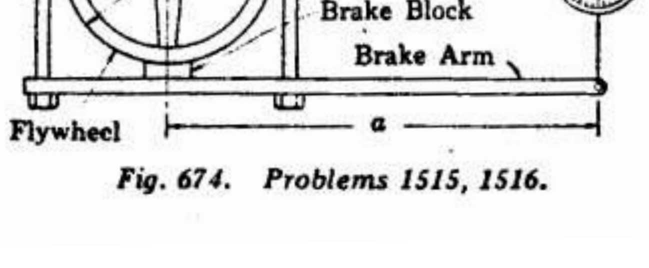

Fig. 674. Problems 1515, 1516.

1517. For checking the horsepower output of a small engine, a band brake was rigged up, as shown diagrammatically in Fig. 675. The brake drum A was attached to the shaft of the engine. One end of the band was connected to a scale and the other end supported a weight of $W = 50$ lb. If the scale reading was 200 lb. and the angular speed of the engine was 300 rpm, determine the brake horsepower.

Ans. 4.28 hp.

1518-1530. These numbers may be used for other problems.

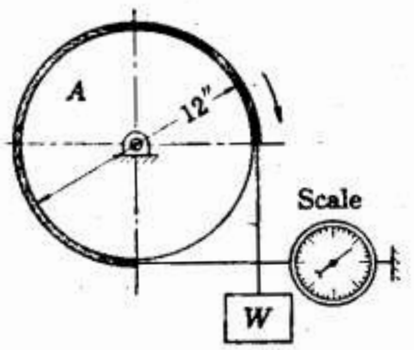

Fig. 675. Problem 1517.