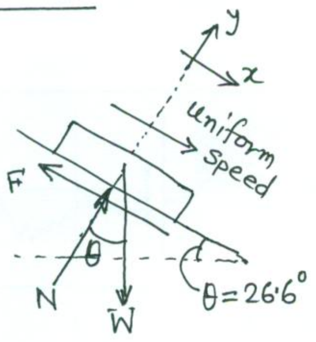
1106. A serious student wishes to perform an experiment in determining the speed of a book. Using a blackboard eraser, he coats one side of the book with chalk dust, so that the book will mark a trail as it slides. Then he places the book on a long table whose inclination is varied until the book will slide down at a uniform speed. This angle of inclination is observed to be 26.6° with the horizontal. Now with the table top horizontal, he throws the book onto the table, and measures a 7.5 ft. trail left by the book as it slid to rest. Calculate the horizontal speed with which the book struck the table. *Ans.* 15.5 fps.

#1106/P.325



Since speed is constant, $a=0$

$\Sigma F_y = 0$ gives

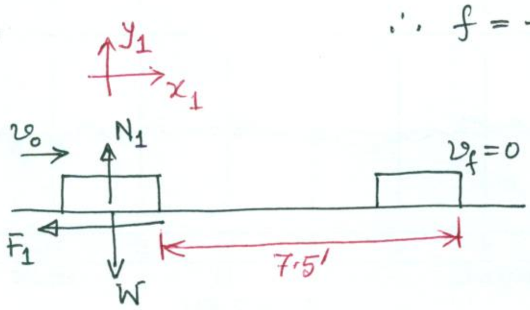
$$N = W\cos\theta$$

Now $\Sigma F_x = 0$ gives, (+x as +ve directⁿ)

$$W\sin\theta - N\cdot f = 0$$

$$\Rightarrow W\sin\theta - W\cos\theta\cdot f = 0$$

$$\therefore f = \frac{\sin\theta}{\cos\theta} = \tan\theta = \tan 26.6^\circ = 0.5$$



$\Sigma F_{y_1} = 0$ gives

$$N_1 - W = 0 \quad \text{i.e. } N_1 = W$$

$$\therefore F_1 = N_1\cdot f = W\times 0.5 = 0.5W$$

Now taking $\Sigma F_{x1} = ma$, $\rightarrow$ +ve

$$-F_1 = \frac{W}{g}\cdot a$$

$$\Rightarrow -0.5W = \frac{W}{g}a$$

$$\therefore a = -0.5\times 32.2 = -16.1 \text{ fps}^2$$

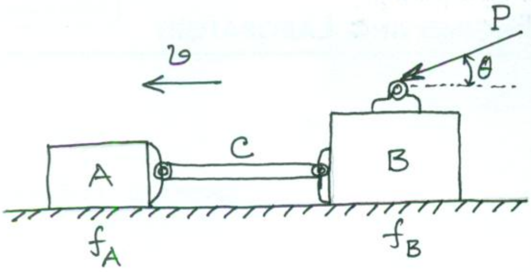
$$v_f^2 = v_0^2 + 2as$$

$$\Rightarrow 0 = v_0^2 + 2\times(-16.1)\times 7.5$$

$$\therefore v_0 = \boxed{15.54 \text{ fps.}}$$ Ans.

1108. Two bodies A and B, connected by a rod C, Fig. 518, have an initial speed of 6 fps and move 300 ft. in 30 sec. Let $W_A = 966$ lb., $W_B = 1288$ lb., $f_A = 0.04$, $f_B = 0.15$, and $\theta = 15°$. For constant acceleration, determine the force P and the final velocity. *Ans.* 270 lb., 14 fps.

#1108/P.325



$v_0 = 6$ fps $W_A = 966$ lb
$S = 300$ ft $W_B = 1288$ lb
$t = 30$ s. $f_A = 0.04$
a = const. $f_B = 0.15$
$\theta = 15°$
$P = ?$ $v_f = ?$

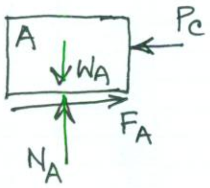
Sol^n

$$S = v_0 t + \frac{1}{2} a t^2$$

$$\Rightarrow 300 = 6 \times 30 + \frac{1}{2} \times a \times 30^2$$

$$\therefore a = 0.267 \text{ fps}^2$$

$$\therefore v_f = v_0 + at = 6 + 0.267 \times 30 = \boxed{14 \text{ fps}}$$ Ans.



Freebody diagram of A

From freebody diagram of A

Taking $\Sigma F_y = 0$ ↑+ve

$$N_A - W_A = 0$$

$$\Rightarrow N_A = W_A = 966 \text{ lb}$$

$$\therefore F_A = N_A \cdot f_A = 966 \times 0.4 = 38.64 \text{ lb}$$

Taking $\Sigma F_H = ma$ ←+ve

$$P_C - F_A = \frac{W_A}{g} \cdot a$$

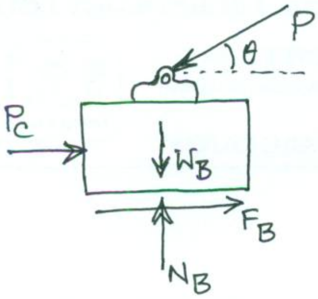
$$\Rightarrow P_C - 38.64 = \frac{966}{32.2} \times 0.267$$

$$\therefore P_C = 46.65 \text{ lb}$$

Note: It is advantageous to consider the direction of motion as +ve.

Note: Inertial force is not included in the freebody of A, so we have used $\Sigma F = ma$. If inertial force is included we shall use $\Sigma F = 0$.

#1108/P.325 ....Contd.



Free body diagram of B

From freebody diagram of B

Taking $\Sigma F_V = 0 \uparrow +ve$

$$-P\sin\theta - W_B + N_B = 0$$

$$\therefore N_B = W_B + P\sin\theta$$

$$= 1288 + P\sin 15^\circ$$

$$F_B = N_B \cdot f_B$$

$$= (1288 + P\sin 15^\circ) \times 0.15$$

$$= 193.2 + 0.039P$$

Now taking $\Sigma F_H = ma \xleftarrow{\quad} +ve$

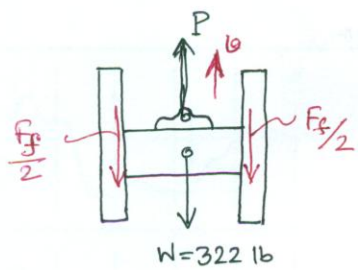
$$P\cos\theta - P_C - F_B = \frac{W_B}{g} \cdot a$$

$$\Rightarrow P\cos 15^\circ - 46.65 - (193.2 + 0.039P) = \frac{1288}{32.2} \times 0.267$$

$$\Rightarrow \quad 0.927P = 250.53$$

$$\therefore P = \boxed{270.3 \text{ lb}}$$ Ans.

#1110/P.325



Initially at rest
$S = 30$ ft
$a =$ const.
$t = 2.5$ s
$F_f = 200$ lb

$P = ?$

Sol^n

$$S = \frac{1}{2} a t^2$$

$$\therefore a = \frac{2S}{t^2} = \frac{2 \times 30}{2.5^2} = 9.6 \text{ fps}^2$$

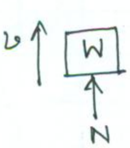
$$\Sigma F_y = ma \uparrow +ve$$

$$\Rightarrow P - W - F_f = \frac{W}{g} \cdot a$$

$$\Rightarrow P - 322 - 200 = \frac{322}{32.2} \times 9.6$$

$$\therefore P = \boxed{618 \text{ lb}} \text{ Ans.}$$

#1111/P.326



$W = 100$ lb
$a = 3$ fps^2
Load supported by the man = ?

Sol^n Load supported = Reaction on the shoulder = N

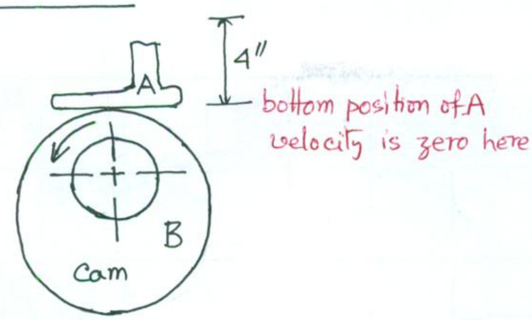
Taking $\Sigma F_v = ma \uparrow +ve$

$$N - W = \frac{W}{g} \cdot a$$

$$\Rightarrow N - 100 = \frac{100}{32.2} \times 3$$

$$\therefore N = \boxed{109.32 \text{ lb}} \text{ Ans.}$$

1112/P. 326



$W_A = 6$ lb

$S_A = 4$ in.

$\theta_B = 75^\circ = \frac{\pi}{180} \times 75 = \frac{5\pi}{12}$ rad

$\omega_B = 120$ rpm

$= \frac{120 \times 2\pi}{60}$ rad/s

$= 4\pi$ rad/s

$F = ?$ Width of cam $= ?$

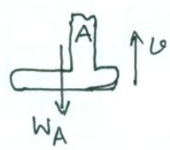
$$\theta_B = \omega_B \cdot t$$

$$\therefore t = \frac{\theta_B}{\omega_B} = \frac{5\pi}{12} \times \frac{1}{4\pi} = 0.104 \text{ s.}$$

Now $S_A = v_{A0}\, t + \frac{1}{2} a_A \cdot t^2$

$$\Rightarrow \frac{4}{12} = 0 \times t + \frac{1}{2} \times a_A \times (0.104)^2$$

$$\therefore a = 61.64 \text{ fps}^2$$



Force on the follower,

$$F = m_A \cdot a_A$$

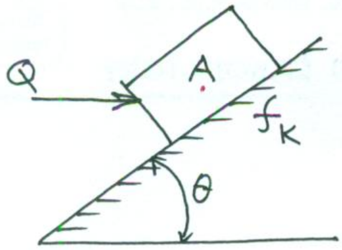
$$= \frac{6}{32.2} \times 61.64$$

$$= \boxed{11.45 \text{ lb}}$$ Ans.

Permissible load $= 100$ lb/in.

$$\therefore \text{Reqd. width} = \frac{11.45}{100} \text{ in} = \boxed{0.1145 \text{ in.}}$$ Ans.

1114/P. 326



$W_A = 50$ lb

$\theta = 30^\circ$

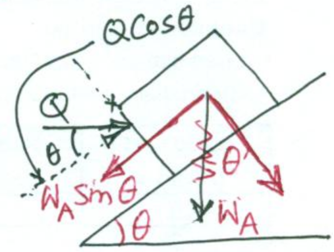
$f_K = 0.1$

$Q = 40$ lb

$v_0 = 0$

$v_f = ?$ after $t = 4$ s.

Note: Direction of motion is not stated. But we need it to apply Newton's 2nd law. We can assume a direction and then interpret the results (from sign). In this problem we can find it beforehand.

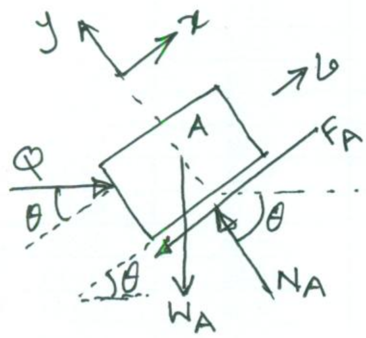


Sol^n

$$Q \cos\theta = 40 \cos 30^\circ = 34.64 \text{ lb}$$

$$W_A \sin\theta = 50 \sin 30^\circ = 25 \text{ lb}$$

Since $Q\cos\theta > W_A \sin\theta$, the body moves upward



From the freebody diagram of A

Taking $\Sigma F_y = 0 \uparrow +ve$

$$-Q\sin\theta - W_A \cos\theta + N_A = 0$$

$$\Rightarrow -40 \sin 30^\circ - 50 \cos 30^\circ + N_A = 0$$

$$\therefore N_A = 63.30 \text{ lb}$$

$$\therefore F_A = N_A . f_k = 63.3 \times 0.1 = 6.33 \text{ lb}$$

Now applying $\Sigma F_x = ma$, +ve x direct^n +ve

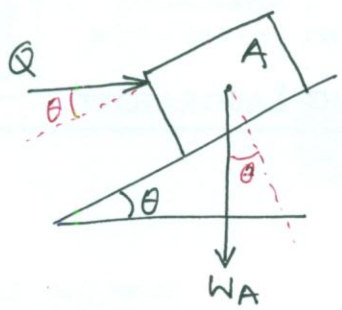
$$Q\cos\theta - W_A \sin\theta - F_A = \frac{W_A}{g} . a$$

$$\Rightarrow 40 \cos 30^\circ - 50 \sin 30^\circ - 6.33 = \frac{50}{32.2} \times a$$

$$\therefore a = 2.132 \text{ fps}^2$$

$$v_f = v_0 + at = 0 + 2.132 \times 4 = \boxed{8.53 \text{ fps}}$$ Ans.

 # 1115 / P. 326



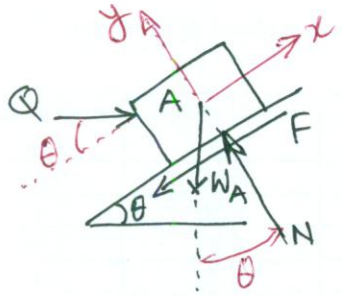
$Q = 40$ lb
$W_A = 50$ lb
$\theta = 30^\circ$
$f_k = 0.1$

$v_0 = 0$
(a) $f_s = ?$
(b) $v = ?$ for $S = 10$ ft

Sol^n

$Q\cos\theta = 40\cos 30^\circ = 34.64$ lb
$W_A\sin\theta = 50\sin 30^\circ = 25$ lb

$\because$ $Q\cos\theta > W_A\sin\theta$, motion, if occurs will be upward



From the freebody of A

$\Sigma F_y = 0$, $+y$ direct^n as +ve

$N - W_A\cos\theta - Q\sin\theta = 0$

$\Rightarrow N - 50\cos 30^\circ - 40\sin 30^\circ = 0$

$\therefore N = 63.3$ lb

For impending motion $\Sigma F_x = 0$, taking $+x$ direct^n +ve

$Q\cos\theta - F - W_A\sin\theta = 0$

$\Rightarrow 40\cos 30^\circ - 63.3 \times f_s - 50\sin 30^\circ = 0$

$\therefore f_s = 0.152$

Note: If motion occurs then $\Sigma F = ma$

When the body is in motion, $\Sigma F_x = ma$

$\therefore Q\cos\theta - F - W_A\sin\theta = \frac{W_A}{g}\cdot a$

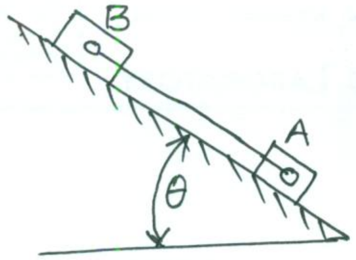
$\Rightarrow 40\cos 30^\circ - 0.1 \times 63.3 - 50\sin 30^\circ = \frac{50}{32.2} \times a$

$\therefore a = 2.13$ fps^2

Now $v^2 = v_0^2 + 2aS = 0 + 2 \times 2.13 \times 10$

$\therefore v = \boxed{6.52 \text{ fps}}$

1125/P. 326



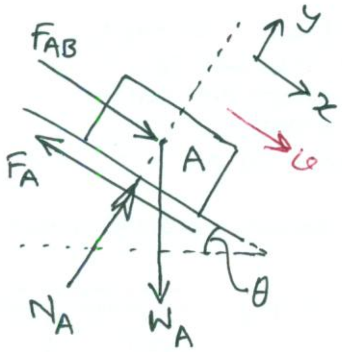
Motion down the plane

$W_A = W_B = 10$ lb

$\theta = 30°$, $a = -1.61$ fps^2

$F_{AB} = 1$ lb (comp.)

$f_A = ?$ $f_B = ?$



From the freebody of A.

$\Sigma F_y = 0$, +ve y direct^n as +ve

$N_A - W_A \cos\theta = 0$

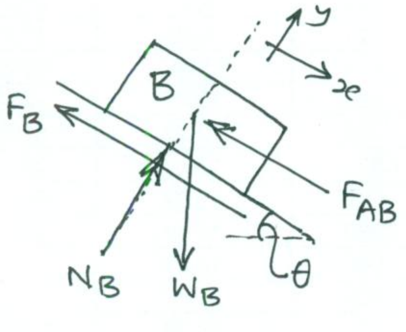
$\therefore N_A = 10 \cos 30° = 8.66$ lb

Again taking $\Sigma F_x = ma$, +x direct^n as +ve

$$F_{AB} - F_A + W_A \sin\theta = \frac{W_A}{g} a$$

$$\Rightarrow 1 - 8.66 \times f_A + 10 \sin 30° = \frac{10}{32.2} \times (-1.61)$$

$$\therefore f_A = \boxed{0.75} \text{ Ans.}$$



From the freebody of B

$\Sigma F_y = 0$, +y as +ve

$N_B - W_B \cos\theta = 0$

$\therefore N_B = 10 \cos 30° = 8.66$ lb

$\Sigma F_x = ma$, +x as +ve

$$-F_{AB} - F_B + W_B \sin\theta = \frac{W_B}{g} \cdot a$$

$$\Rightarrow -1 - 8.66 \times f_B + 10 \sin 30° = \frac{10}{32.2} \times (-1.61)$$

$$\therefore f_B = \boxed{0.52} \text{ Ans.}$$

#1126/P. 326

Train moving up 1% grade

Locomotive pull, $P = 60{,}000$ lb

Resistance, $F = 15$ lb/ton

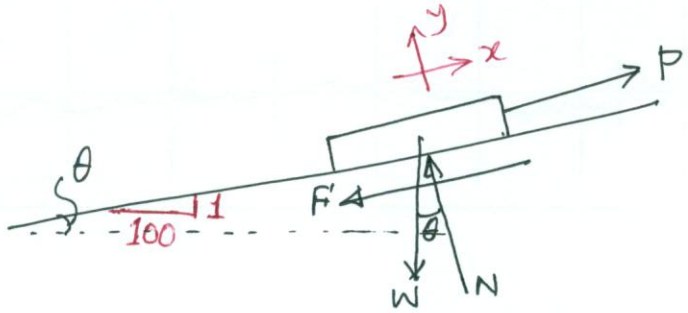
$v_0 = 5 \text{ mph} = \dfrac{5 \times 1760 \times 3}{60 \times 60} \text{ fps} = 7.33 \text{ fps}$

$v_f = 30 \text{ mph} = \dfrac{30 \times 1760 \times 3}{60 \times 60} \text{ fps} = 44 \text{ fps}$

$S = 8 \text{ miles} = 8 \times 1760 \times 3 \text{ ft}$

Weight of train, $W = ?$

Sol^n



$$v_f^2 = v_0^2 + 2aS$$

$$\Rightarrow 44^2 = 7.33^2 + 2 \times a \times (8 \times 1760 \times 3)$$

$$\therefore a = 0.0222 \text{ fps}^2$$

Now Taking $\Sigma F_x = ma$, +ve x direction +ve

$$P - W\sin\theta - F = \frac{W}{g} \cdot a$$

$$\Rightarrow 60000 - W \times \frac{1}{\sqrt{100^2 + 1^2}} - \frac{W}{2000} \times 15 = \frac{W}{32.2} \times 0.0222$$

$$\Rightarrow W = \left[\frac{0.0222}{32.2} + \frac{15}{2000} + \frac{1}{\sqrt{10001}} \right. = 60000$$

$$\therefore W = 3298707.71 \text{ lb}$$

$$= 1649.36 \text{ ton}$$

#1126/P.326

Train moving up 1% grade

Locomotive pull, $P = 60{,}000$ lb

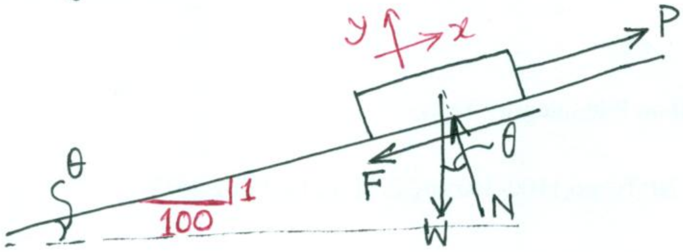
Resistance, $F = 15$ lb/ton

$v_0 = 5$ mph , $v_f = 30$ mph

$S = 8$ miles

Weight of train, $W = ?$

Sol^n



$$v_f^2 = v_0^2 + 2aS$$

$$\Rightarrow 30^2 = 5^2 + 2 \times a \times 8$$

$$\therefore a = 54.69 \text{ mph}^2 = \frac{54.69 \times 1760 \times 3}{(60 \times 60)^2} \text{fps}^2 = 0.0222 \text{ fps}^2$$

Applying $\Sigma F_x = ma$, +ve x direct^n +ve

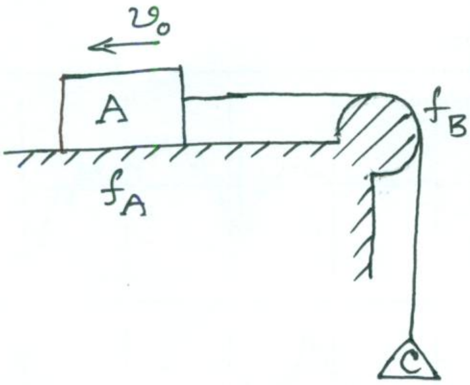
$$P - W \sin\theta - F = \frac{W}{g} a \quad \text{[Consider W in ton]}$$

$$\Rightarrow \frac{60000}{2000} - W \times \frac{1}{\sqrt{1^2 + 100^2}} - W \times 15 = \frac{W}{32.2} \times 0.0222$$

$$\Rightarrow W\left[\frac{0.0222}{32.2} + \frac{15}{2000} + \frac{1}{\sqrt{10001}}\right] = \frac{60{,}000}{2000}$$

$$\therefore W = \boxed{1649.36 \text{ ton}}$$ Ans.

1131/P.327



$f_A = \frac{1}{3}$, $f_B = 0$

$W_A = 64.4$ lb , $W_C = 96.6$ lb

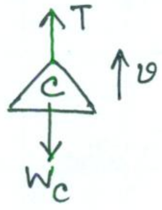
$v_0 = 30$ fps (to left)

$S = 10$ ft

$t = ?$

$\sum F = ma$

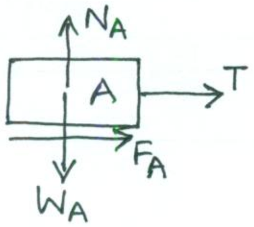
Sol^n



From freebody of C, taking ~~$\Sigma F_y = 0$~~ $\uparrow$ +ve

$$T - 96.6 = \frac{96.6}{32.2} a$$

$$\text{i.e. } T = 96.6 + 3a \quad \text{--- (1)}$$



Taking $\Sigma F_v = 0$, $\uparrow$ +ve from freebody of A

$$N_A - 64.4 = 0 \quad \text{i.e. } N_A = 64.4 \text{ lb}$$

$$\Sigma F_H = ma \leftarrow +\text{ve}$$

$$\Rightarrow -T - F_A = \frac{W_A}{g} \cdot a$$

$$\Rightarrow -T - 64.4 \times \frac{1}{3} = \frac{64.4}{32.2} \times a$$

$$\therefore T = -21.47 - 2a \quad \text{--- (2)}$$

From (1) and (2)

$$96.6 + 3a = -21.47 - 2a$$

$$\Rightarrow 5a = -118.07$$

$$\therefore a = -23.61 \text{ fps}^2$$ (−ve sign implies that velocity is decreasing)

Now, $S = v_0 t + \frac{1}{2} a t^2$

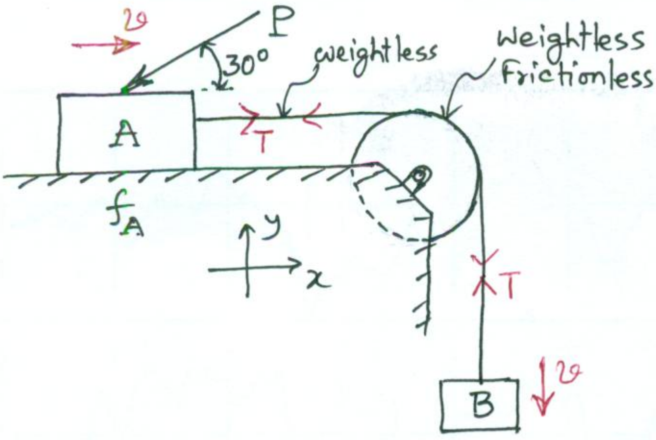
$$\Rightarrow 10 = 30t + \frac{1}{2}(-23.61) \times t^2$$

$$\Rightarrow 11.805 t^2 - 30t + 10 = 0$$

$$\therefore t = \frac{30 \pm \sqrt{30^2 - 4 \times 11.805 \times 10}}{2 \times 11.805} = \frac{30 \pm 20.68}{2 \times 11.805} \text{ s.}$$

$$= \boxed{2.15 \text{ s.}, \; 0.39 \text{ s.}} \quad \text{Ans.}$$

#1135/P. 327



$W_A = 600$ lb

$W_B = 225$ lb

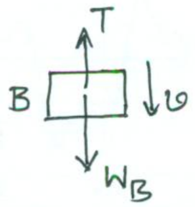
$f_A = 1/3$

$v_0 = 20$ fps , $v_f = 0$

$S = 30$ ft

$P = ?$ $T = ?$

Sol^n



Considering the motion of body B

$$v_f^2 = v_0^2 + 2aS$$

$$\Rightarrow 0 = 20^2 + 2 \times a \times 30$$

$$\therefore a = -6.67 \text{ fps}^2$$

Now from the freebody of B, taking $\Sigma F_y = ma \downarrow +ve$

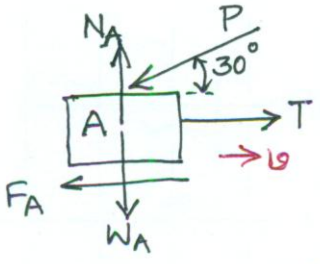
$$W_B - T = \frac{W_B}{g} \cdot a$$

$$\Rightarrow 225 - T = \frac{225}{32.2} \times (-6.67)$$

$$\boxed{\therefore T = 271.61 \text{ lb}}$$ Ans.

Note: In the eq^n $R = \frac{W}{g} a$ the resultant force R and accl^n a are of same sense.

–ve because retardat^n



For body A, taking $\Sigma F_y = 0$, $\uparrow +ve$

$$-P \sin 30^\circ - W_A + N_A = 0$$

$$\therefore N_A = P \sin 30^\circ + 600$$

Again taking $\Sigma F_x = ma \rightarrow +ve$

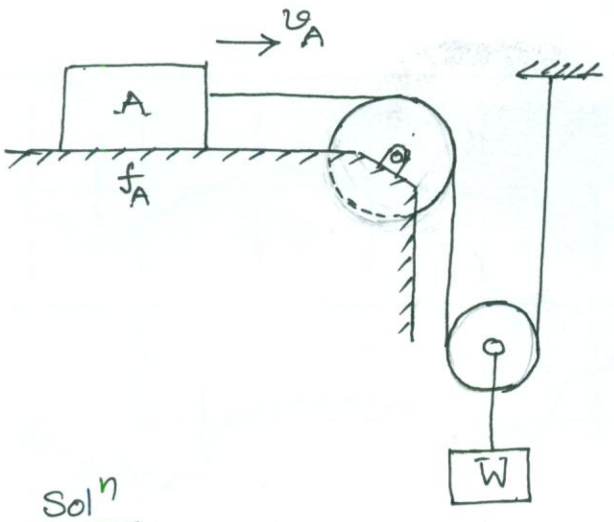
$$-P \cos 30^\circ - F_A + T = \frac{W_A}{g} \cdot a$$

$$\Rightarrow -P \cos 30^\circ - (P \sin 30^\circ + 600) \times \frac{1}{3} + 271.61 = \frac{600}{32.2} \times (-6.67)$$

$$\Rightarrow 1.032P = 195.89$$

$$\boxed{\therefore P = 189.82 \text{ lb}}$$ Ans.

#1136/P.327



$W_A = 966$ lb

$f_A = 1/3$

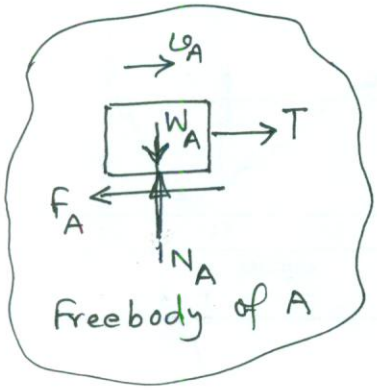
$\left.\begin{array}{l} v_{A1} = 10 \text{ fps} \\ v_{A2} = 35 \text{ fps} \end{array}\right\}$ in 25 s.

(a) $W = ?$

(b) S_W during 25 s. = ?

(c) Tension in cable = ?

Sol^n



$v_{A2} = v_{A1} + a_A t$

$\Rightarrow 35 = 10 + a_A \times 25$

$\therefore a_A = 1 \text{ fps}^2$

$a_W = \dfrac{a_A}{2} = 0{\cdot}5 \text{ fps}^2$

From the freebody of A

$\Sigma F_V = 0 \uparrow +ve$

$N_A - W_A = 0$

$\therefore N_A = W_A = 966$ lb

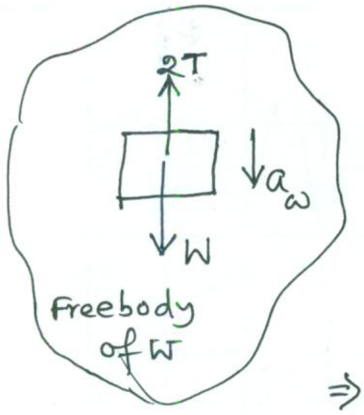
$\therefore F_A = N_A \cdot f_A = 966 \times \frac{1}{3}$
$= 322$ lb

$\Sigma F_H = 0 \rightarrow +ve$

$T - F = \dfrac{W_A}{g} \cdot a_A$

$\Rightarrow T - 322 = \dfrac{966}{32.2} \times 1$

$\therefore T = \boxed{352 \text{ lb}}$ Ans.



From the freebody of the weight W

Taking $\Sigma F_V = ma \downarrow +ve$

$W - 2T = \dfrac{W}{g} \cdot a_W$

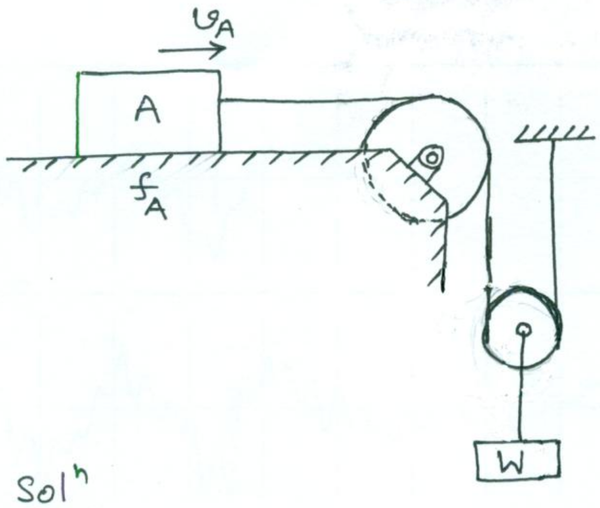
$\Rightarrow W - 2 \times 352 = \dfrac{W}{32{\cdot}2} \times 0{\cdot}5$

$\therefore W = \boxed{715{\cdot}12 \text{ lb}}$ Ans.

$S_A = v_{A1} t + \frac{1}{2} a_A t^2 = 10 \times 25 + \frac{1}{2} \times 1 \times 25^2 = 562{\cdot}5$ ft

$S_W = \dfrac{S_A}{2} = \dfrac{562{\cdot}5}{2} = \boxed{281{\cdot}25 \text{ ft}}$ Ans.

1137/P.327



$W_A = 966$ lb

$f_A = 1/3$

$\left.\begin{array}{l} v_{A1} = 60 \text{ fps} \\ v_{A2} = 10 \text{ fps} \end{array}\right\}$ in 25 s.

(a) $W = ?$

(b) $S_W = ?$

(c) cable tension = ?

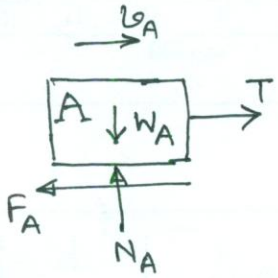
Sol^n

Considering motion of body A

$v_{A2} = v_{A1} + a_A t \Rightarrow 10 = 60 + a_A \times 25$ $\quad \therefore a_A = -2 \text{ fps}^2$

−ve sign means retardat^n

$\therefore a_w = \dfrac{a_A}{2} = -1 \text{ fps}^2$ (direct^n ↓, −ve implies retardat^n)



For body A taking $\Sigma F_V = 0$

$N_A = W_A = 966$ lb

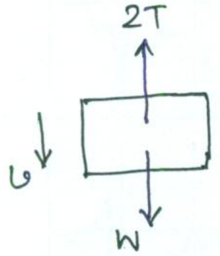
$\therefore F_A = N_A \cdot f_A = 966 \times \frac{1}{3} = 322$ lb.

Again, taking $\Sigma F_H = ma$, → +ve for body A

$$T - F_A = \frac{W_A}{g} \cdot a_A$$

$$\Rightarrow T - 322 = \frac{966}{32.2} \times (-2)$$

$$\therefore T = \boxed{262 \text{ lb}}$$



For body B, Taking $\Sigma F_v = ma$ ↓ +ve

$$W - 2T = \frac{W}{g} \cdot a_\omega$$

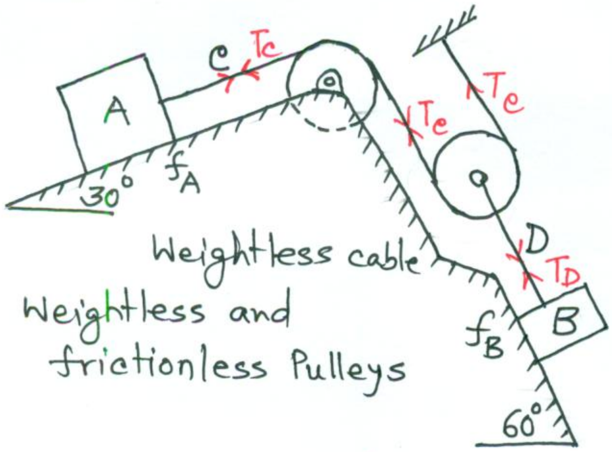
$$\Rightarrow W - 2 \times 262 = \frac{W}{32.2} \times (-1)$$

$$\therefore W = \boxed{508.22 \text{ lb}}$$

$$S_A = v_{A1} t + \frac{1}{2} a_A t^2 = 60 \times 25 + \frac{1}{2} \times (-2) \times 25^2 = 875 \text{ ft.}$$

$$\therefore S_W = \frac{875}{2} = \boxed{437.5 \text{ ft.}}$$

1138/P. 328



$W_A = 200$ lb

$W_B = 100$ lb

$f_A = 1/4$

$f_B = 1/3$

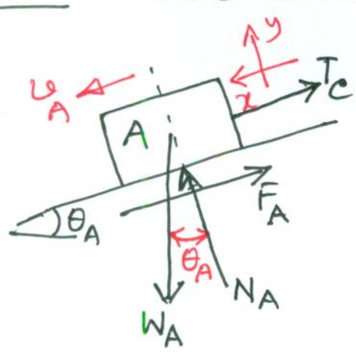
$S_A = ?$ for $t = 30$ s., $v_0 = 0$

Direction ?

$T_c = ?$ and $T_D = ?$

Note: Direct^n of motion not given. It may be assumed and finally obtained from the sign of certain determined quantities. Here it may also be judged/perceived in advance through logic.

Sol^n Let's assume that A moves down the plane



From the freebody of A

$\Sigma F_y = 0$, +ve y direct^n +ve

$$\Rightarrow N_A - W_A \cos\theta_A = 0$$

$$\therefore N_A = 200 \cos 30^0 = 173.2 \text{ lb}$$

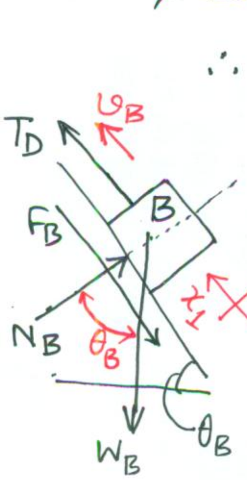
$$F_A = f_A N_A = \frac{1}{4} \times 173.2 = 43.3 \text{ lb}$$

Considering $\Sigma F_x = ma$, +ve x direct^n +ve

$$\Rightarrow W_A \sin\theta_A - F_A - T_c = \frac{W_A}{g} \cdot a_A$$

$$\Rightarrow 200 \sin 30^0 - 43.3 - T_c = \frac{200}{32.2} \times a_A$$

$$\therefore T_c + 6.21\, a_A = 56.7 \quad \text{——— (1)}$$



From the freebody of B

$\Sigma F_{y_1} = 0$

$$\Rightarrow N_B - W_B \cos\theta_B = 0$$

$$\therefore N_B = 100 \cos 60^0 = 50 \text{ lb}$$

$$F_B = f_B N_B = \frac{1}{3} \times 50 = 16.67 \text{ lb}$$

contd...

#1138/P.328 contd..

Also from the freebody of B

$\Sigma F_{x1} = 0$ gives

$$T_D - F_B - W_B \sin\theta_B = \frac{W_B}{g} \cdot a_B$$

$$\Rightarrow T_D - 16{\cdot}67 - 100 \sin 60^0 = \frac{100}{32{\cdot}2} a_B$$

$$\therefore T_D - 3{\cdot}11\, a_B = 103{\cdot}27 \quad \text{——— (2)}$$

Now $T_D = 2T_C$

also $a_A = 2a_B$ Note: A moves twice the distance moved by B

$\therefore$ From eq^n (2)

$$2T_C - 3{\cdot}11 \times \frac{a_A}{2} = 103{\cdot}27$$

i.e. $T_C = 0{\cdot}78\, a_A + 51{\cdot}64$ ——— (3)

(3) into (1)

$$0{\cdot}78\, a_A + 51{\cdot}64 + 6{\cdot}21\, a_A = 56{\cdot}7$$

$\therefore a_A = 0{\cdot}724$ fps^2; Since a_A is +ve, assumed direct^n of motion is correct, i.e. A moves down

From eq^n (3) $T_C = 0{\cdot}78 \times 0{\cdot}724 + 51{\cdot}64 = \boxed{52{\cdot}2 \text{ lbs}}$

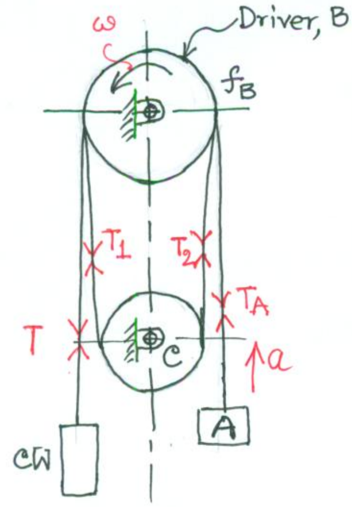
$$T_D = 2T_C = 2 \times 52{\cdot}2 = \boxed{104{\cdot}4 \text{ lb}}$$

$$S_A = v_0 t + \frac{1}{2} a_A t^2$$

$$= 0 + \frac{1}{2} \times 0{\cdot}724 \times 30^2$$

$$= \boxed{325{\cdot}8 \text{ ft}}$$

1142/P.328



$W_A = 5000$ lb

$a_A = 3$ fps^2

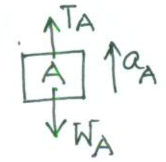
$f_B = 0{\cdot}1$

CW = ?

Neglect inertia effects.

Note: Here min^m CW is asked for. If CW is less than this min^m, body A will drop down, irrespective of whether sheave B rotates ↷ or ↶. In other words, for min^m CW, body A will be in a ~~sta~~ impending state to drop down. It indicates $T_A > T_1$

Sol^n

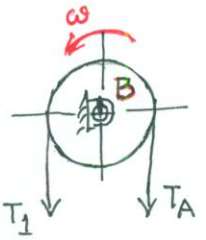


From the freebody of A

$$\Sigma F = ma \uparrow +ve$$

$$\Rightarrow T_A - W_A = \frac{W_A}{g} \cdot a_A$$

$$\therefore T_A = 5000 + \frac{5000}{32{\cdot}2} \times 3 = 5465{\cdot}84 \text{ lb}$$



From body B, since $T_A > T_1$

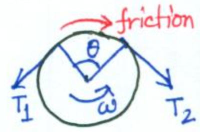
$$T_A = T_1 e^{f_B \theta}$$

$$\therefore T_1 = \frac{T_A}{e^{f_B\theta}} = \frac{5465{\cdot}84}{e^{0{\cdot}1 \times \pi}} = 3992{\cdot}26 \text{ lb}$$

$$= T_2$$

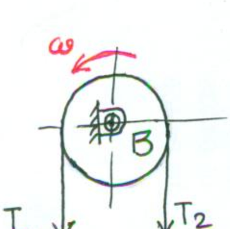
[∵ Sheave C is frictionless]

Art 71/P.111–114 Fairies & Chambers



$T_1 > T_2$

$T_1 = T_2 e^{f\theta}$

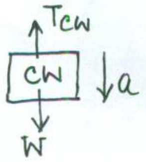


$T_2 > T_{cw}$

Again from body B

$$T_2 = T_{cw} e^{f_B\theta}$$

$$\therefore T_{cw} = \frac{T_2}{e^{f_B\theta}} = \frac{3992{\cdot}26}{e^{0{\cdot}1\pi}} = 2915{\cdot}95 \text{ lb}$$



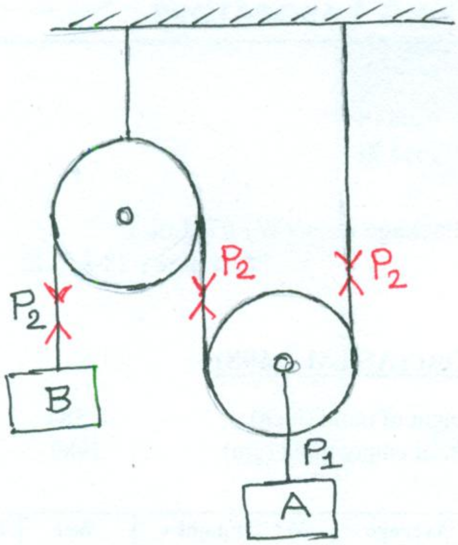
From freebody of counter weight,

$$\Sigma F = ma \downarrow +ve$$

$$\Rightarrow W - T_{cw} = \frac{W}{g} \cdot a \Rightarrow W\left(1 - \frac{a}{g}\right) = T_W$$

$$\therefore W = 2915{\cdot}95 \Big/ \left(1 - \frac{3}{32{\cdot}2}\right) = \boxed{3214{\cdot}9 \text{ lb}}$$

#1144/P. 329



$W_A = 120$ lb

$W_B = 80$ lb

Chord and pulleys are weightless & frictionless

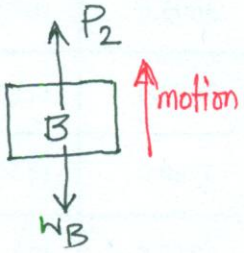
$a_A = ?$ $a_B = ?$

$P_1 = ?$ $P_2 = ?$

Sol^n

Since $W_A > W_B$, let's assume that A moves downward and B moves upward.

Here $P_1 = 2P_2$ and $a_B = 2a_A$



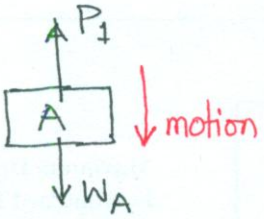
From the freebody of B,

$\Sigma F_y = m_B a_B$ ↑ +ve

$\Rightarrow P_2 - W_B = \frac{W_B}{g} \cdot a_B$

$\Rightarrow \frac{P_1}{2} - 80 = \frac{80}{32 \cdot 2} \times 2a_A$

$\therefore P_1 = 9.94\, a_A + 160$ —— (1)



From the freebody of A

$\Sigma F_y = m_A a_A$ ↓ +ve

$\Rightarrow W_A - P_1 = \frac{W_A}{g} \cdot a_A$

$\Rightarrow 120 - P_1 = \frac{120}{32 \cdot 2} \times a_A$

$\Rightarrow 120 - 9.94\, a_A - 160 = 3.73\, a_A$ [substituting P_1 from (1)]

$\therefore a_A = \boxed{-2.926 \text{ fps}^2}$, −ve sign implies that A moves upward

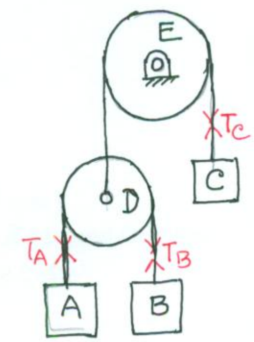
$\therefore a_B = 2a_A = 2 \times 2.926 = \boxed{5.852 \text{ fps}^2}$

From eq^n (1) $P_1 = 9.94 \times (-2.962) + 160 = \boxed{130.92 \text{ lbs}}$

and $P_2 = P_1/2 = \boxed{65.46 \text{ lb}}$

Note: Since these bodies are moving from rest, their velocity and accl^n are in the same sense/direction.

#1146/P.329



$W_A = 1$ slug

$W_B = 2$ slug

$W_C = 4$ slug

$a_A = ?$
$v_A = ?$
$S_A = ?$
} absolute values at $t = 2\,s$.

Sheaves D & E weightless

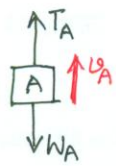
Chords are weightless & flexible

Released from at rest cond^n.

Sol^n

$W_A + W_B < W_C$, so C will move down & sheave D will move up

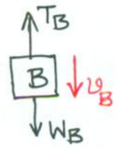
But $W_B > W_A$, so B moves down and A moves up relative to B.



From free body A

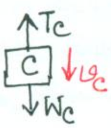
$\Sigma F_y = ma$ gives ($\uparrow$+ve)

$T_A - W_A = m_A \cdot a_A$ ——— (1)



From freebody of B, ($\downarrow$ +ve)

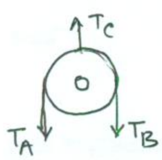
$W_B - T_B = m_B \cdot a_B$ ——— (2)



From freebody of C, ($\downarrow$ +ve)

$W_C - T_C = m_C \cdot a_C$ ——— (3)

Since the sheaves are frictionless and chords are weightless & frictionless



$T_A = T_B$ ——— (4)

$T_C = T_A + T_B = 2T_A$ ——— (5)

Displacements of A & B are same (though opposite) and their accl^n should be same (also opposite)

$\therefore\ a_A - a_C = a_B + a_C$

$\Rightarrow\ 2a_C = a_A - a_B$ ——— (6)

Substituting T_C from (5) and a_C from (6) into eq^n (3)

$W_C - 2T_A = \frac{m_C}{2}(a_A - a_B)$ ——— (7)

Substituting T_B from (4) in eq^n (2)

$W_B - T_A = m_B a_B$ ——— (8)

Contd....

#1146/P.329 contd...

① + ⑧ gives

$$W_B - W_A = m_A a_A + m_B a_B$$

$$\Rightarrow (2-1)\times 32.2 = 1 \cdot a_A + 2 \cdot a_B$$

$$\therefore \quad a_A + 2a_B = 32.2 \qquad \text{———— (9)}$$

2 × ① + ⑦ gives

$$W_c - W_A = 2m_A a_A + \frac{m_c}{2}(a_A - a_B)$$

$$\Rightarrow (4-2)\times 32.2 = 2\times 1 \times a_A + \frac{4}{2}(a_A - a_B)$$

$$\Rightarrow \quad 64.4 = 2a_A + 2a_A - 2a_B$$

$$\Rightarrow \quad 2a_A - a_B = 32.2 \qquad \text{———— (10)}$$

$$2\times(9) - (10) \Rightarrow 5a_B = 32.2$$

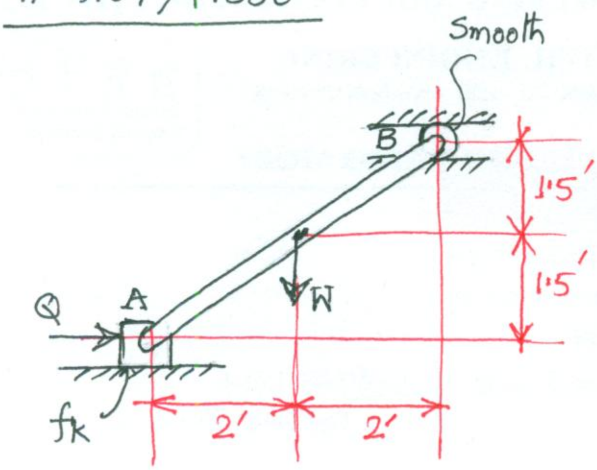
$$\therefore a_B = 6.44 \text{ fps}^2$$

$$\therefore \text{from (9)} \quad a_A = 32.2 - 2\times 6.44 = \boxed{19.32 \text{ fps}^2}$$

$$\text{Now } v_{A2} = v_{0A} + a_A t = 0 + 19.32\times 2 = \boxed{38.64 \text{ fps}}$$

$$S_{A2} = v_0 t + \frac{1}{2}a_A t^2 = 0 + \frac{1}{2}\times 19.32\times 2^2 = \boxed{38.64 \text{ ft}}$$

1157/P.330



$W = 100$ lb , $f_k = 0{\cdot}2$

React^n at B = 10 lb (↑)

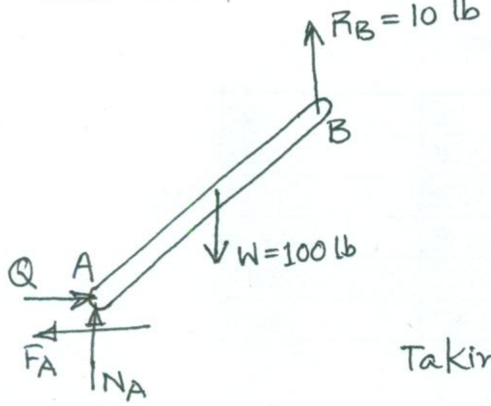
$\bar{a} = ?$ $\bar{a} \rightarrow$ resultant accl^n of mass center

$Q = ?$

Solve (a) without REF

(b) with REF

Sol^n without REF



From the freebody of bar AB

$\Sigma F_v = 0$ ↑+ve gives

$$10 - 100 + N_A = 0$$

$$\therefore N_A = 90 \text{ lb}$$

$$\therefore F_A = N_A \cdot f_k = 90 \times 0{\cdot}2 = 18 \text{ lb}$$

Taking $\Sigma M_{c.g} = 0$ ↻+ve

We can take $\sum M_B=0$ as well

$$-Q \times 1{\cdot}5 + F_A \times 1{\cdot}5 + N_A \times 2 - R_B \times 2 = 0$$

$$\Rightarrow -Q \times 1{\cdot}5 + 18 \times 1{\cdot}5 + 90 \times 2 - 10 \times 2 = 0$$

$$\therefore Q = \boxed{124{\cdot}67 \text{ lb}}$$

$\Sigma F_h = m\bar{a}$ → +ve

$$\Rightarrow Q - F_A = \frac{W}{g} \times \bar{a}$$

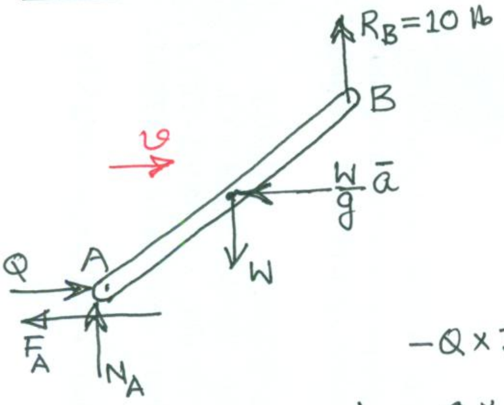
$$\Rightarrow 124{\cdot}67 - 18 = \frac{100}{32{\cdot}2} \times \bar{a}$$

$$\therefore \bar{a} = \boxed{34{\cdot}35 \text{ fps}^2}$$

contd...

#1157/P.330 Contd.

Sol^n with REF



$N_A = 90$ lb
$F_A = 18$ lb
[Similar as done for sol^n without REF]

$$\Sigma M_B = 0 \quad \curvearrowright +ve$$

$$-Q \times 3 - W \times 2 + F_A \times 3 + N_A \times 4 + \frac{W}{g}\bar{a} \times 1.5 = 0$$

$$\Rightarrow -Q \times 3 - 100 \times 2 + 18 \times 3 + 90 \times 4 + \frac{100}{32.2}\bar{a} \times 1.5 = 0$$

$$\Rightarrow 3Q - 4.66\,\bar{a} = 214$$

$$\therefore Q = 71.33 + 1.55\,\bar{a} \quad \text{——— (1)}$$

$$\Sigma F_h = 0 \quad \rightarrow +ve$$

$$\Rightarrow Q - F_A - \frac{W}{g}.\bar{a} = 0$$

$$\Rightarrow 71.33 + 1.55\,\bar{a} - 18 - \frac{100}{32.2} \times \bar{a} = 0$$

[Substituting Q from (1)]

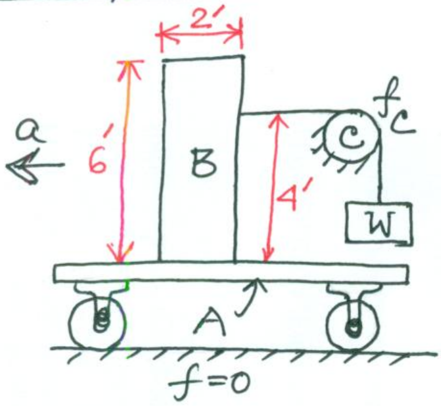
$$\Rightarrow 1.55\,\bar{a} = 53.33$$

$$\therefore \bar{a} = \boxed{34.41 \text{ fps}^2}$$

Now from eq^n (1)

$$Q = 71.33 + 1.55 \times 34.41 = \boxed{124.67 \text{ lb}}$$

#1161/P. 330



$W_B = 322$ lb

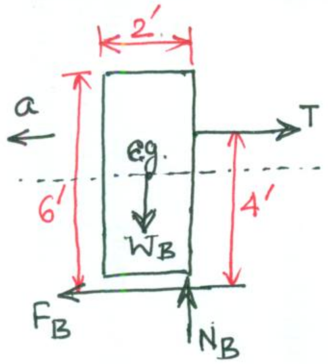
$f_C = 0$

$a = 3.22$ fps^2 ←

(a) W = ? for B turning over
Solve with & without REF

(b) $f_B = 0.1$ will B turn over or slide?

(a) Sol^n without REF



Note: The direction of frictional resistance is opposite to the relative displacement/motion.

~~ΣF_Y~~

Considering $\Sigma M_{cg} = 0$ ↻ +ve, from the freebody of B

$$T \times 1 + F_B \times 3 - N_B \times 1 = 0 \quad (1)$$

$\Sigma F_y = 0$ ↑ +ve gives

$$N_B - W_B = 0 \Rightarrow N_B = W_B = 322 \text{ lb}$$

Now from eq^n (1)

$$T + F_B \times 3 - 322 = 0$$

$$\therefore T + 3F_B = 322 \quad (2)$$

Again $\Sigma F_h = ma$ ← +ve gives

$$-T + F_B = \frac{W_B}{g} \cdot a$$

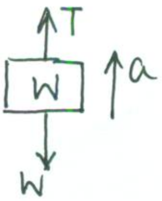
$$\therefore -T + F_B = \frac{322}{32.2} \times 3.22 = 32.2 \quad (3)$$

Adding eq^{n}s (2) & (3) we obtain

$$4F_B = 354.2$$

$$\therefore F_B = 88.55 \text{ lb}$$

From eq^n (2), $T = 322 - 3 \times 88.55 = 56.35$ lb



From the freebody of the weight, W

$\Sigma F_y = 0$ ↑ +ve gives

$$T - W = \frac{W}{g} \cdot a$$

$$\Rightarrow 56.35 - W = \frac{W}{32.2} \times 3.22$$

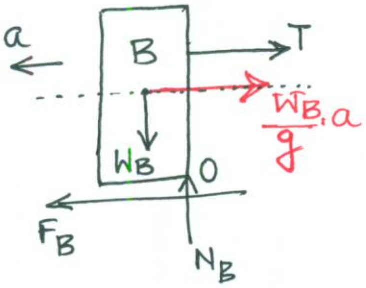
$$\Rightarrow 1.1W = 56.35$$

$$\therefore W = 51.23 \text{ lb}$$

contd.....

\# 1161/P.330 contd....

Sol^n with REF



From freebody of B

$\Sigma F_y = 0 \uparrow +ve \Rightarrow N_B - W_B = 0$

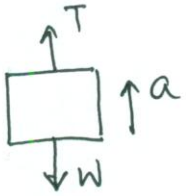
$\therefore N_B = W_B = 322$ lb

Taking $\Sigma M_O = 0 \curvearrowright +ve$

$T \times 4 + \frac{W_B}{g} \times a \times 3 - W_B \times 1 = 0$

$\Rightarrow T \times 4 + \frac{322}{32.2} \times 3.22 \times 3 - 322 = 0$

$\therefore T = 56.35$ lb



From freebody of weight W, $\Sigma F_y = 0 \uparrow +ve$

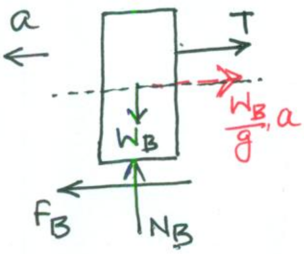
$T - W = \frac{W}{g} \times a$

$\Rightarrow 56.35 - W = \frac{W}{32.2} \times 3.22$

$\Rightarrow 1.1\, W = 56.35$

$\therefore W = 51.23$ lb

(b) Case : Sliding



$\Sigma F_y = 0 \uparrow +ve$

$\Rightarrow N_B - W_B = 0$

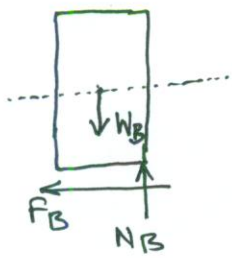
$\therefore N_B = W_B = 322$ lb

$F_B = N_B \times f_B$
$= 322 \times 0.1$
$= 32.2$ lb

Inertia force $= \frac{W_B}{g} a = \frac{322}{32.2} \times 3.22 = 32.2$ lb

Since inertia force = Limiting frictional resistance, sliding is incipient.

Case : Overturning



From $\Sigma F_y = 0$, $N_B = W_B = 322$ lb

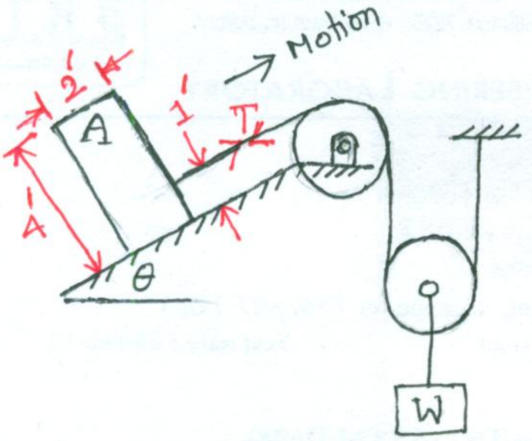
$\Sigma F_h = ma$, $F_B = \frac{W_B}{g} \times a = \frac{322}{32.2} \times 3.22 = 32.2$ lb

$\Sigma M_{c.g.}$ $\curvearrowright +ve$ $= F_B \times 3 - N_B \times 1 = 32.2 \times 3 - 322 \times 1$

$= -225.4$ ft-lb, which is anti-clockwise ie stabilizing

The box would slide.

1165/P. 331



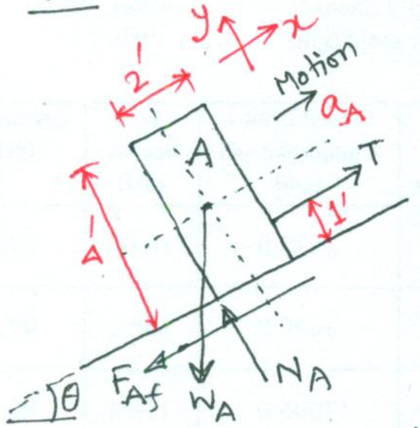
$W_A = 1200$ lb

$\theta = 30^\circ$

$f_A = 0.3$

(a) $W = ?$
(b) $T = ?$ } body A is on the point of turning over

Sol^n



From the freebody of A considering $\Sigma F_y = 0$, +ve y direct^n as +ve

$$N_A - W_A \cos 30^\circ = 0$$

$$\therefore N_A = 1200 \times \cos 30^\circ = 1039.2 \text{ lb}$$

$$\therefore F_{Af} = N_A \cdot f_A = 1039.2 \times 0.3 = 311.77 \text{ lb}$$

Again for body A, $\Sigma M_{cg.} = 0$ ↻ +ve

$$-T \times 1 + N_A \times 1 + F_{Af} \times 2 = 0$$

$$\Rightarrow -T + 1039.2 \times 1 + 311.77 \times 2 = 0$$

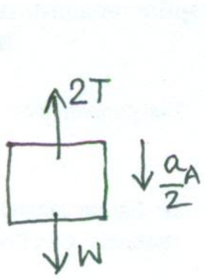
$$\therefore T = \boxed{1662.74 \text{ lb}} \text{ Ans.}$$

For body A, considering $\Sigma F_x = ma$, +ve x direct^n as +ve

$$T - W_A \sin\theta - F_{Af} = \frac{W_A}{g} \cdot a_A$$

$$\Rightarrow 1662.74 - 1200 \sin 30^\circ - 311.77 = \frac{1200}{32.2} \times a_A$$

$$\therefore a_A = 20.15 \text{ fps}^2$$



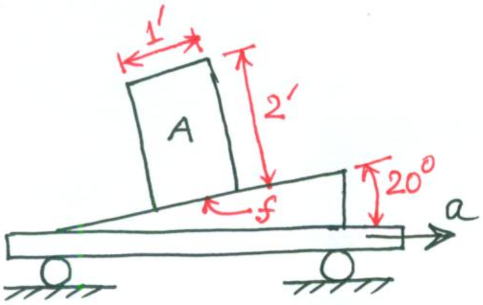
From the freebody of W, $\Sigma F = ma$ ↓ +ve gives

$$W - 2T = \frac{W}{g} \cdot \frac{a_A}{2}$$

$$\Rightarrow W - 2 \times 1662.74 = \frac{W}{32.2} \times \frac{20.15}{2}$$

$$\therefore W = \boxed{4839.79 \text{ lb}} \text{ Ans.}$$

1167/P.331



$f = 0.4$

$W_A = 200$ lb

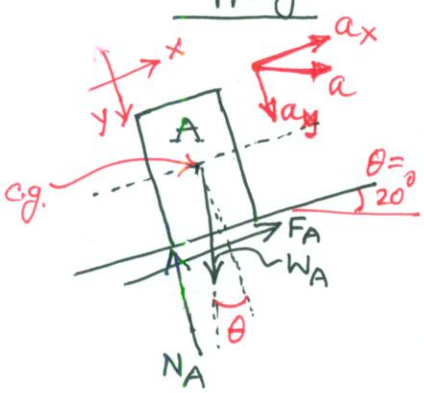
a gradually increased

(a) tip over or slide ?

(b) $a_{max} = ?$

Sol^n

Tipping



$\Sigma F_Y = 0$, +ve y as +ve

$\Rightarrow -W_A \cos\theta + N_A = 0$

$\Rightarrow -200 \times \cos 20^\circ + N_A = 0$

$\therefore N_A = 187.94$ lb

$\Sigma M_{c.g.} = 0$ ↻ +ve

$\Rightarrow N_A \times 0.5 - F_A \times 1 = 0$

$\therefore F_A = N_A \times 0.5 = 187.94 \times 0.5 = 93.97$ lb

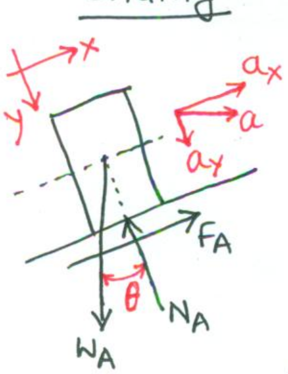
$$\Sigma F_x = m a_x$$

$$\Rightarrow F_A - W_A \sin\theta = \frac{W_A}{g} \cdot a_x$$

$$\Rightarrow 93.97 - 200 \sin 20^\circ = \frac{200}{32.2} \times a \cos 20^\circ$$

$$\therefore a = 4.38 \text{ fps}^2$$

Sliding



$\Sigma F_y = m a_y$, +ve y as +ve

$\Rightarrow W_A \cos\theta - N_A = \frac{W_A}{g} . a_y$

$\Rightarrow 200 \cos 20^\circ - N_A = \frac{200}{32.2} a_y$

$\therefore N_A = 187.94 - 2.12a$

$F_A = N_A \cdot f_A$

$= (187.94 - 2.12a) \times 0.4$

$= 75.18 - 0.848a$

$\Sigma F_x = m a_x$, +ve x as +ve

$$\Rightarrow F_A - W_A \sin\theta = \frac{W_A}{g} \cdot a_x$$

$$\Rightarrow 75.18 - 0.848a - 200 \sin 20^\circ = \frac{200}{32.2} \times a \cos 20^\circ$$

$$\Rightarrow 6.685\, a = 6.776$$

$$\therefore a = 1.014 \text{ fps}^2 < a \text{ for tipping.}$$

$\boxed{\text{Therefore, the body will slide and } a_{max} = 1.014 \text{ fps}^2}$