Subject: ANALYTIC MECHANICS (ABU SIDDIQUE SIR)

Date: 24.02.2013

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Problem: 1 Find out the resultant of the forces from figure. 1.

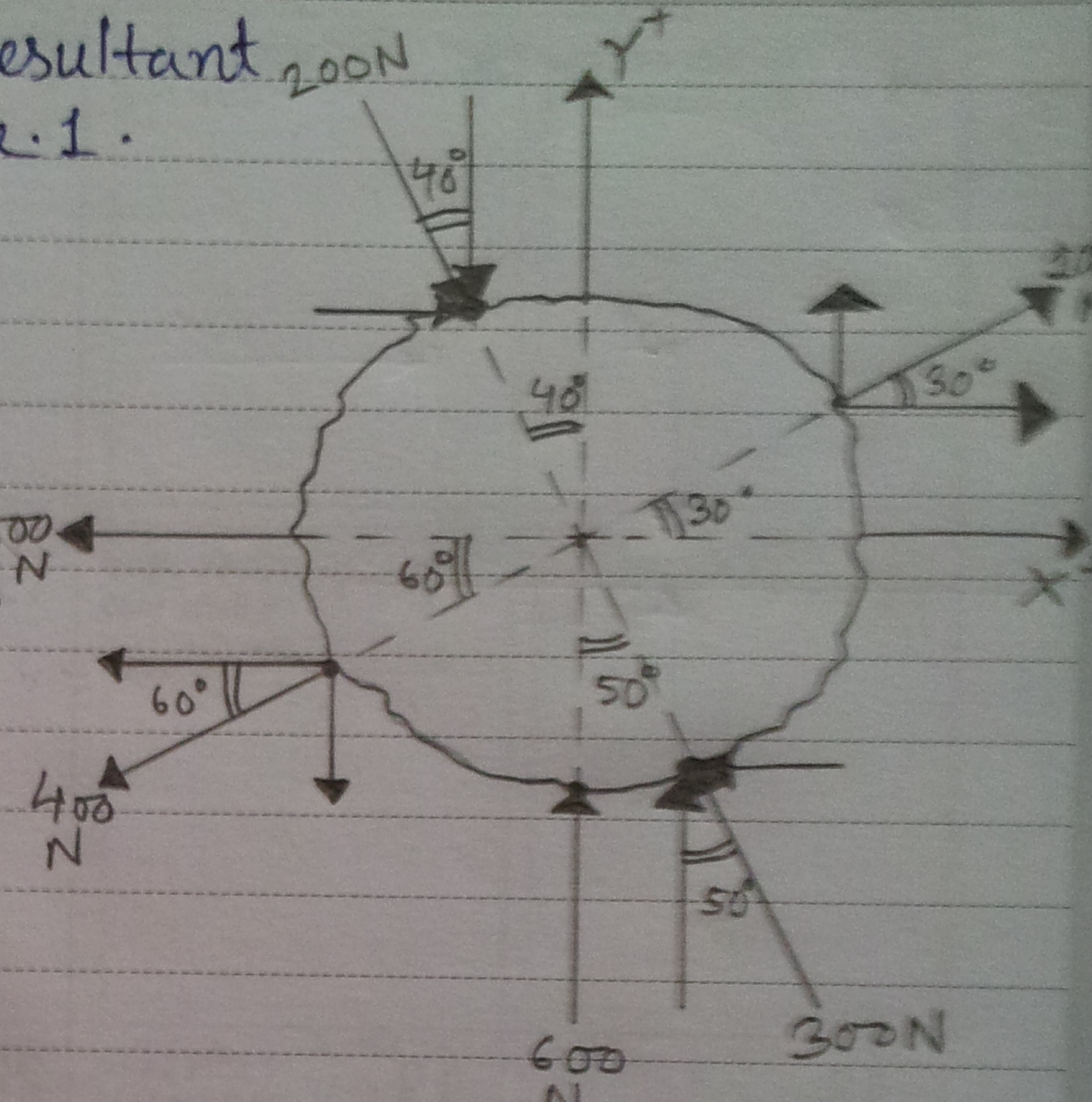


Figure : 1

Sol: $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$\Sigma F_x = 100\cos 30° + 200\cos 50° + 600\cos 90° - 500\cos 0° - 400\cos 60° - 300\cos 40°$

$= -714.65\ N\ (\leftarrow)$

$\Sigma F_y = 100\sin 30° - 200\sin 50° - 500\sin 0° - 400\sin 60° + 600\sin 90° + 300\sin 40°$

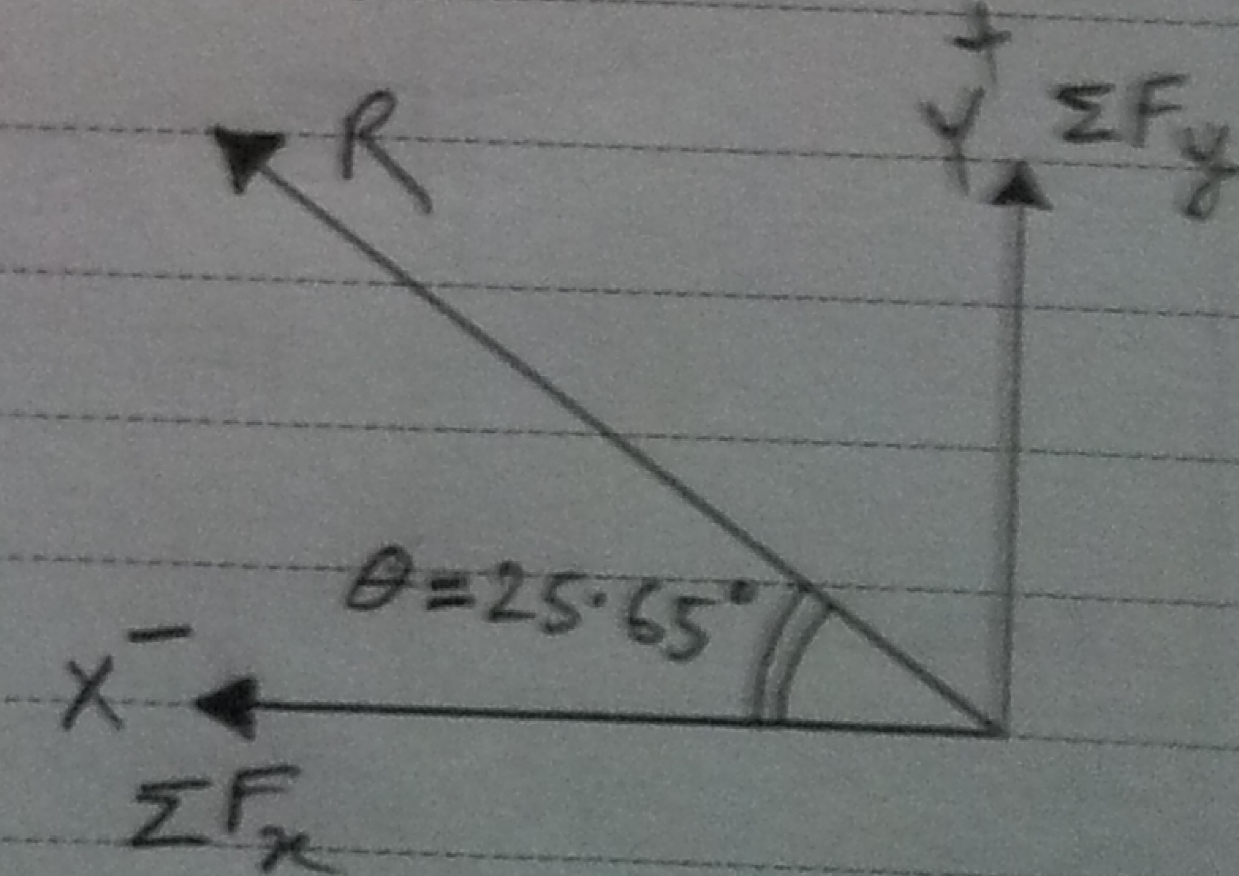
$= 343.22\ N\ (\uparrow)$

$\therefore R = \sqrt{(714.65)^2 + (343.22)^2}\ N = 792.795\ N$

and $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

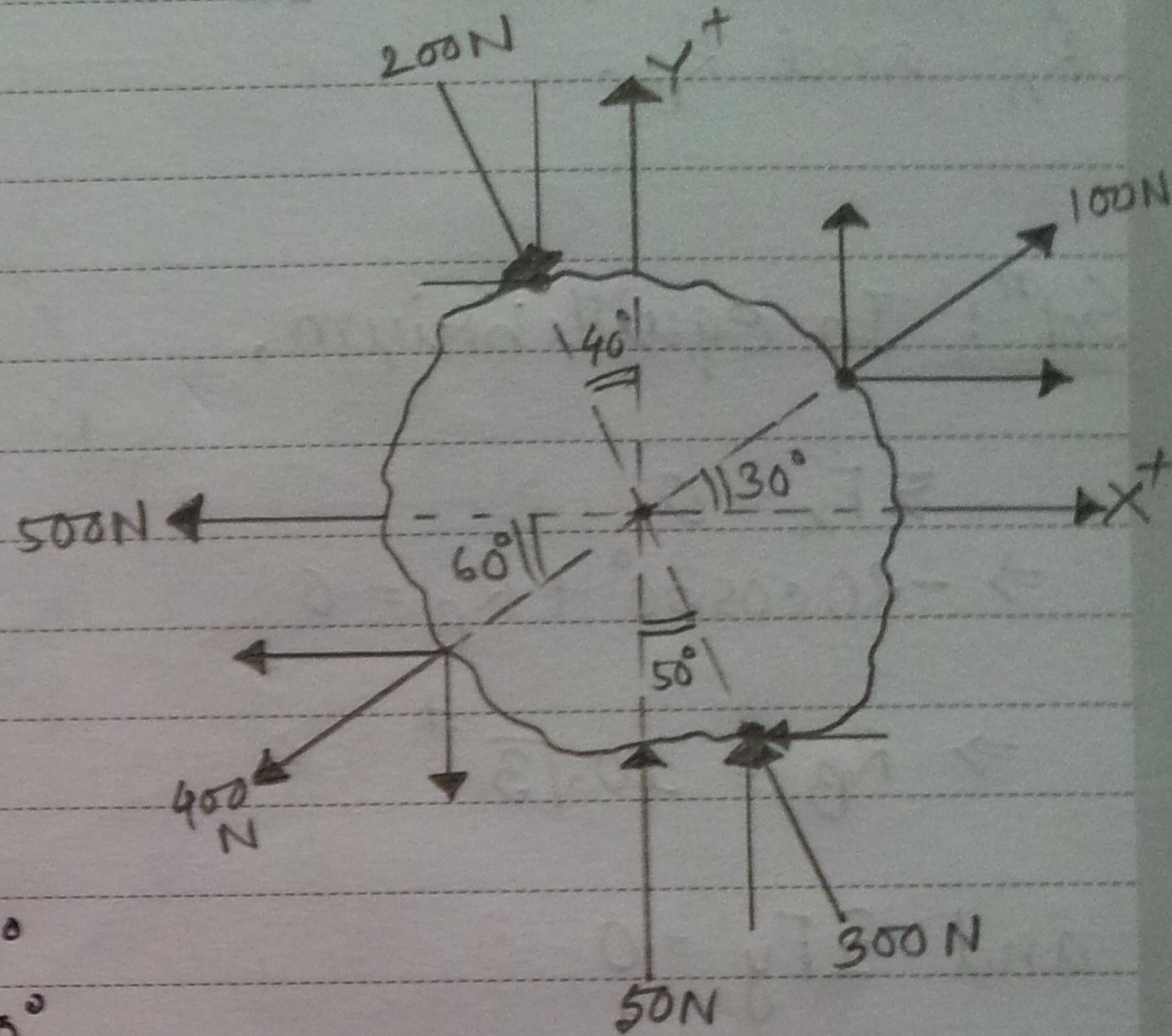
$= \tan^{-1}\left(\frac{343.22}{714.65}\right)$

$= 25.65°$.



Ans: $R = 792.795\ N$ and $\theta = 25.65°$.

Problem: 2 Find out the resultant of the forces from figure. 1.



Soln: $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$\Sigma F_x = 100\cos 30^\circ + 200\cos 50^\circ$
$\quad - 500\cos 0^\circ - 400\cos 60^\circ$
$\quad + 50\cos 90^\circ - 300\cos 40^\circ$

$= -714.65$ N $(\leftarrow)$

$\Sigma F_y = 100\cos 60^\circ - 200\cos 40^\circ$
$\quad - 500\cos 90^\circ - 400\cos 30^\circ$
$\quad + 50\cos 0^\circ + 300\cos 50^\circ$

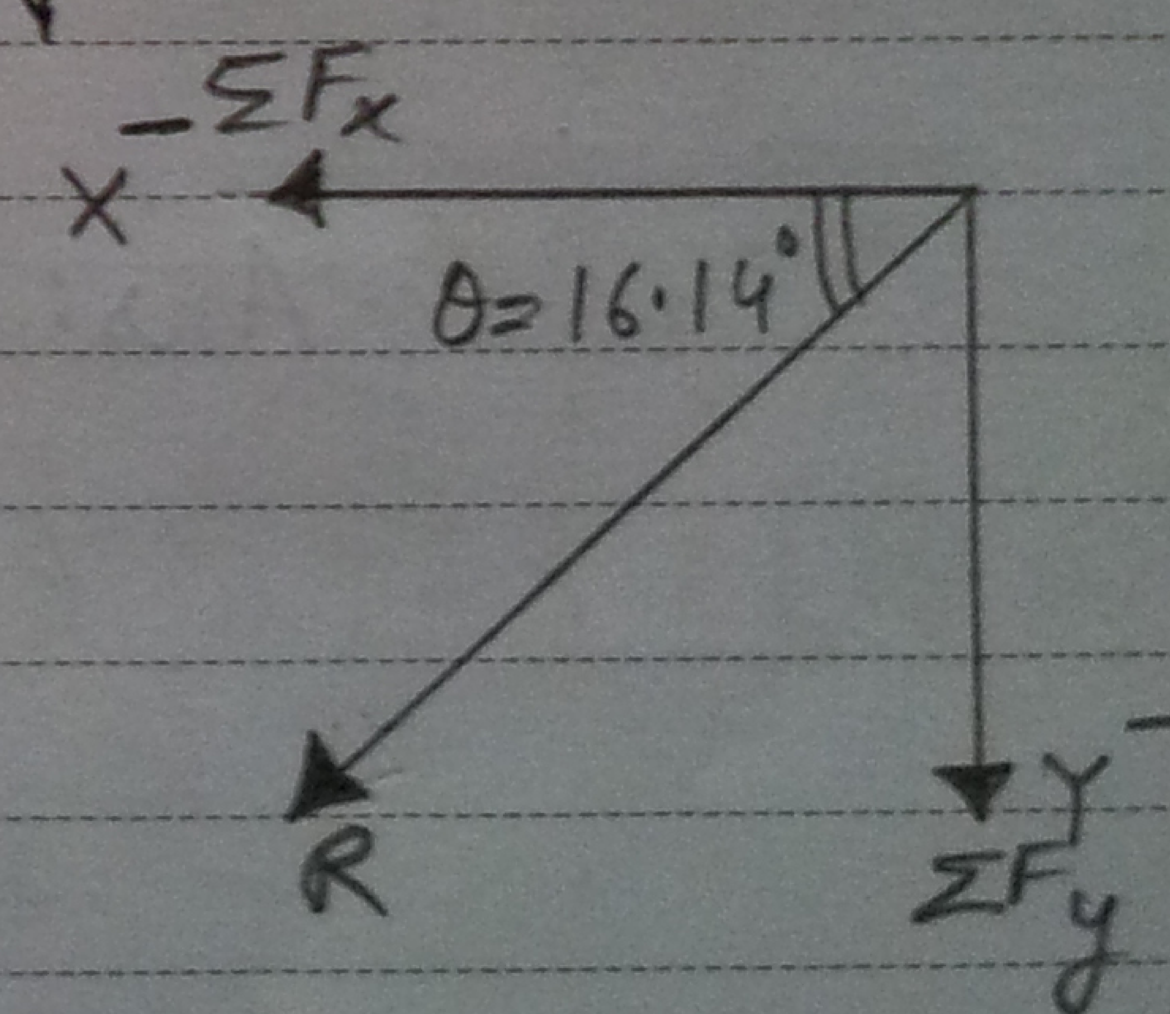
$= -206.78$ N $(\downarrow)$

$\therefore R = \sqrt{(714.65)^2 + (206.78)^2}$ N $= 743.96$ N

and $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$.

$= \tan^{-1}\left(\frac{206.78}{714.65}\right)$

$= 16.14^\circ$



Ans: $R = 743.96$ N and $\theta = 16.14^\circ$.

Problem: 3 Figure-1 is in equillibrium state. Find out R_A and R_B.

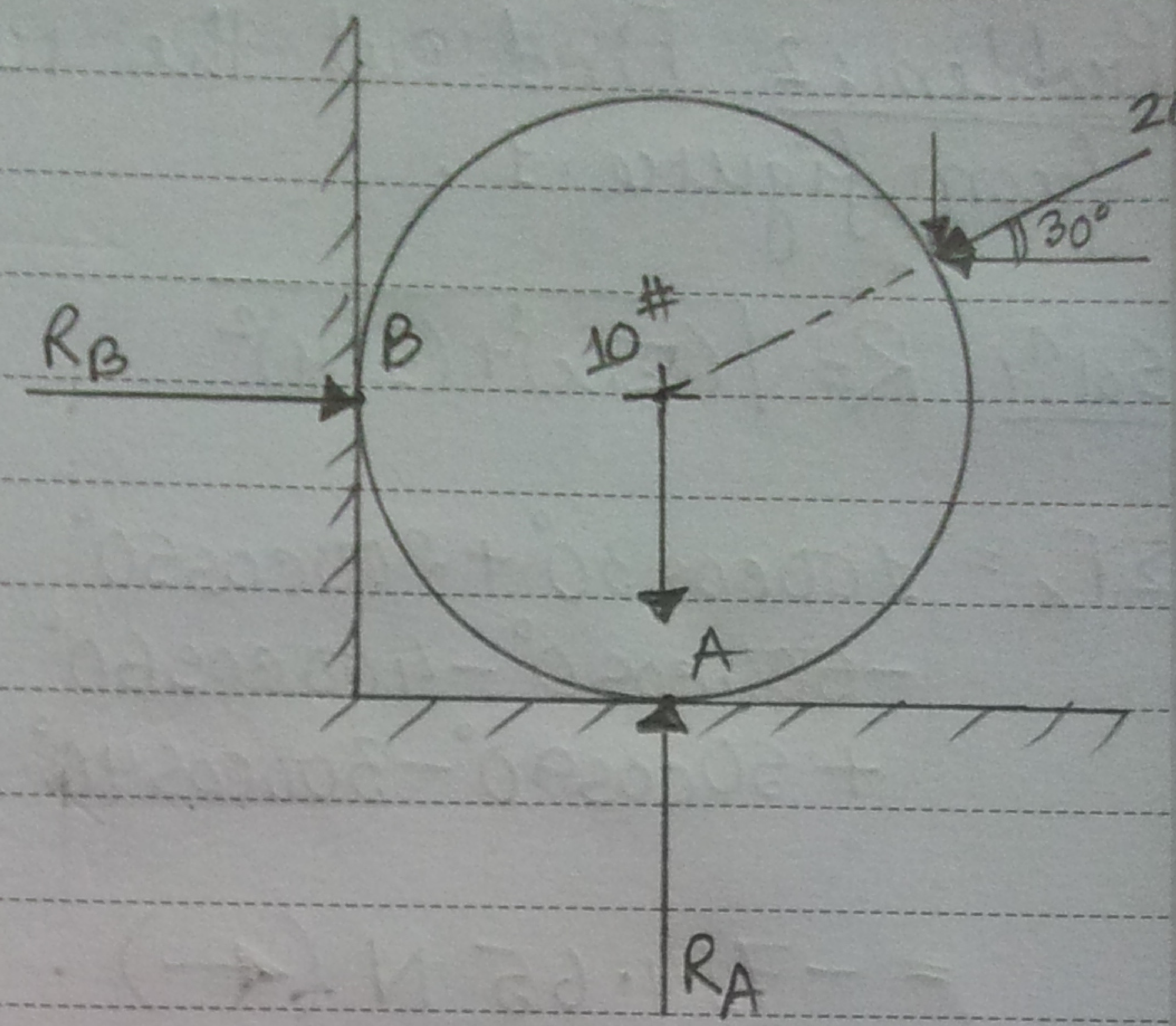


Figure - 1

Sol^n: In equillibrium,

$\Sigma F_x = 0$

$\Rightarrow -20\cos 30° + R_B = 0$

$\Rightarrow R_B = 10\sqrt{3}$ #

and $\Sigma F_y = 0$

$\Rightarrow -20\cos 60° + R_A \cancel{= 0} - 10 = 0$

$\Rightarrow R_A = 20$ #

Ans: $R_A = 20$ # ; $R_B = 10\sqrt{3}$ #.

Problem: 4 Find out R_A and R_B from figure: 1.

Sol^n:

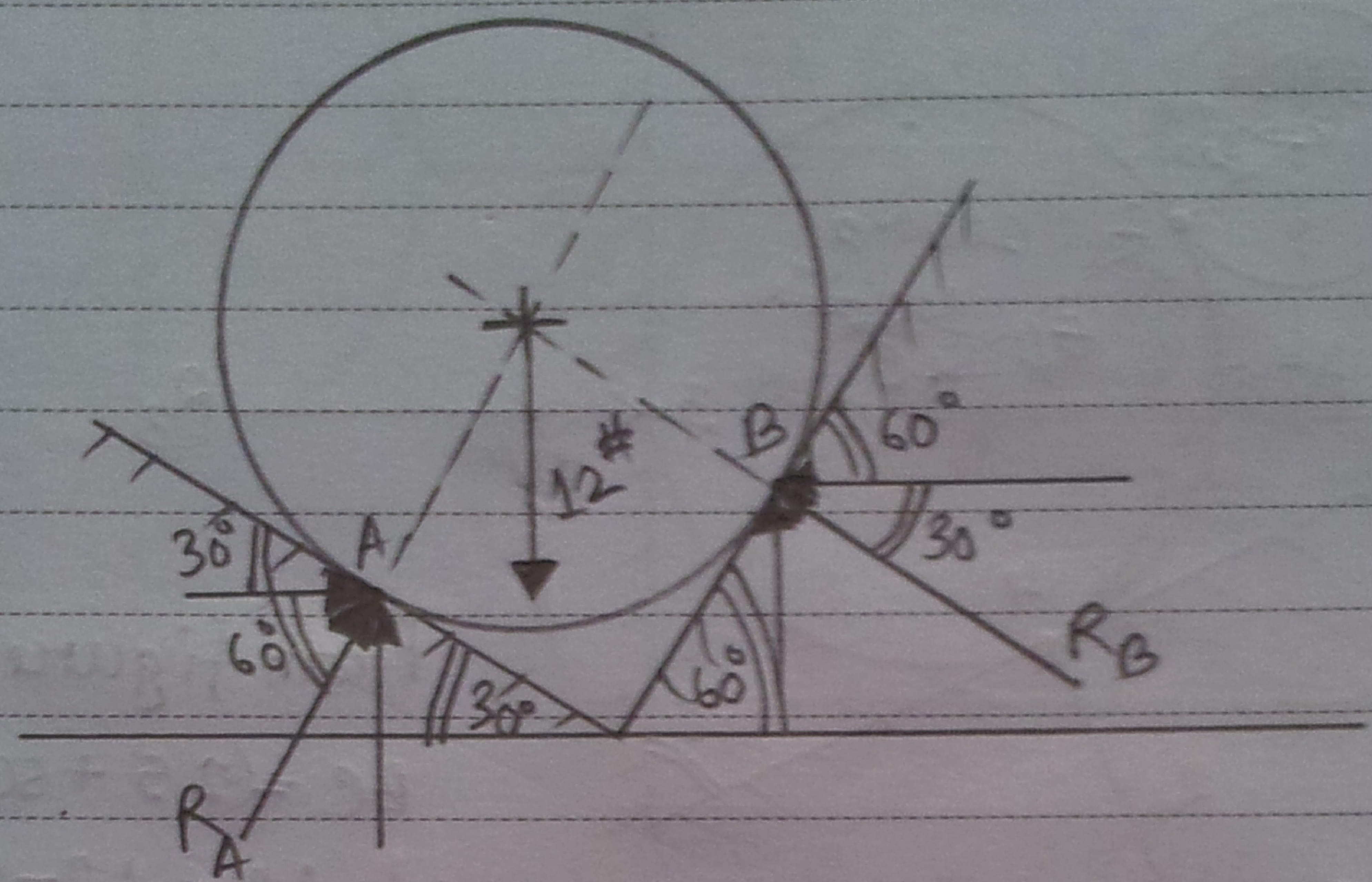


Figure: 1

In equillibrium,

$\Sigma F_x = 0$

$\Rightarrow R_A \cos 60° - R_B \cos 30° = 0$

$\Rightarrow \frac{1}{2} \cos 60° - \frac{\sqrt{3}}{2} R_B = 0$

$\Rightarrow \frac{1}{2} R_A = \frac{\sqrt{3}}{2} R_B$

$\therefore R_A = \sqrt{3} R_B \cdots$ (i)

$\therefore R_A = 6\sqrt{3}$ #

and $\Sigma F_y = 0$

$\Rightarrow -12 + R_A \cos 30° + R_B \cos 60° = 0$

$\Rightarrow -12 + \frac{\sqrt{3}}{2} R_A + \frac{1}{2} R_B = 0$

$\Rightarrow \frac{3}{2} R_B + \frac{1}{2} R_B = 12$; [by (i)]

$\Rightarrow R_B = 6$ #

Ans: $R_A = 6\sqrt{3}$ # and $R_B = 6$ #.

Problem:5 Find out the unknown forces from the figure.

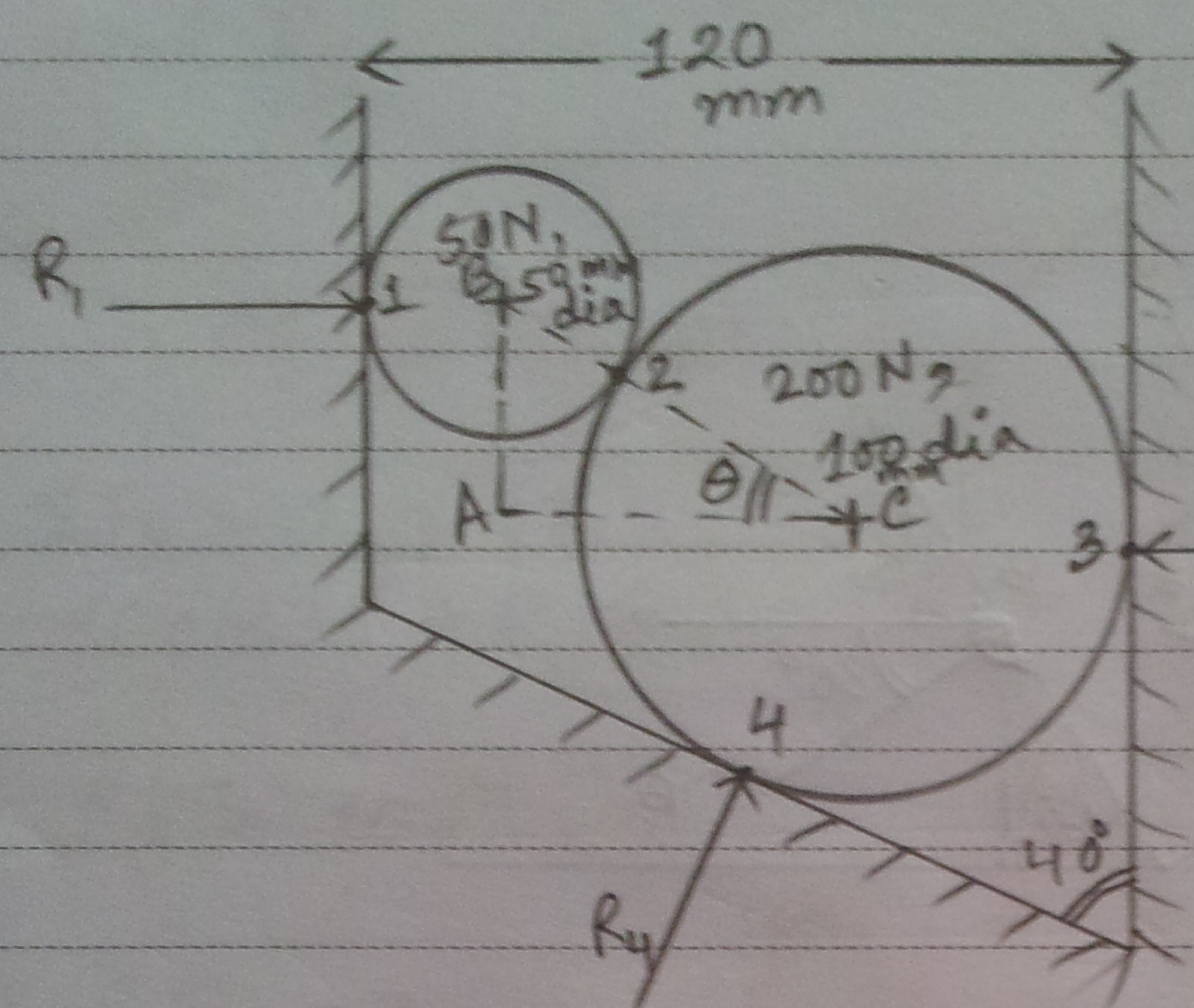


Figure :1

From figure :1,

$BC = (25+50)\,mm = 75\,mm$

and $AC = 12-(50+25)\,mm$

$= 45\,mm$

$\therefore \theta = \cos^{-1}\left(\frac{45}{75}\right) \Rightarrow \theta = 53.13^\circ$

Sol^n:

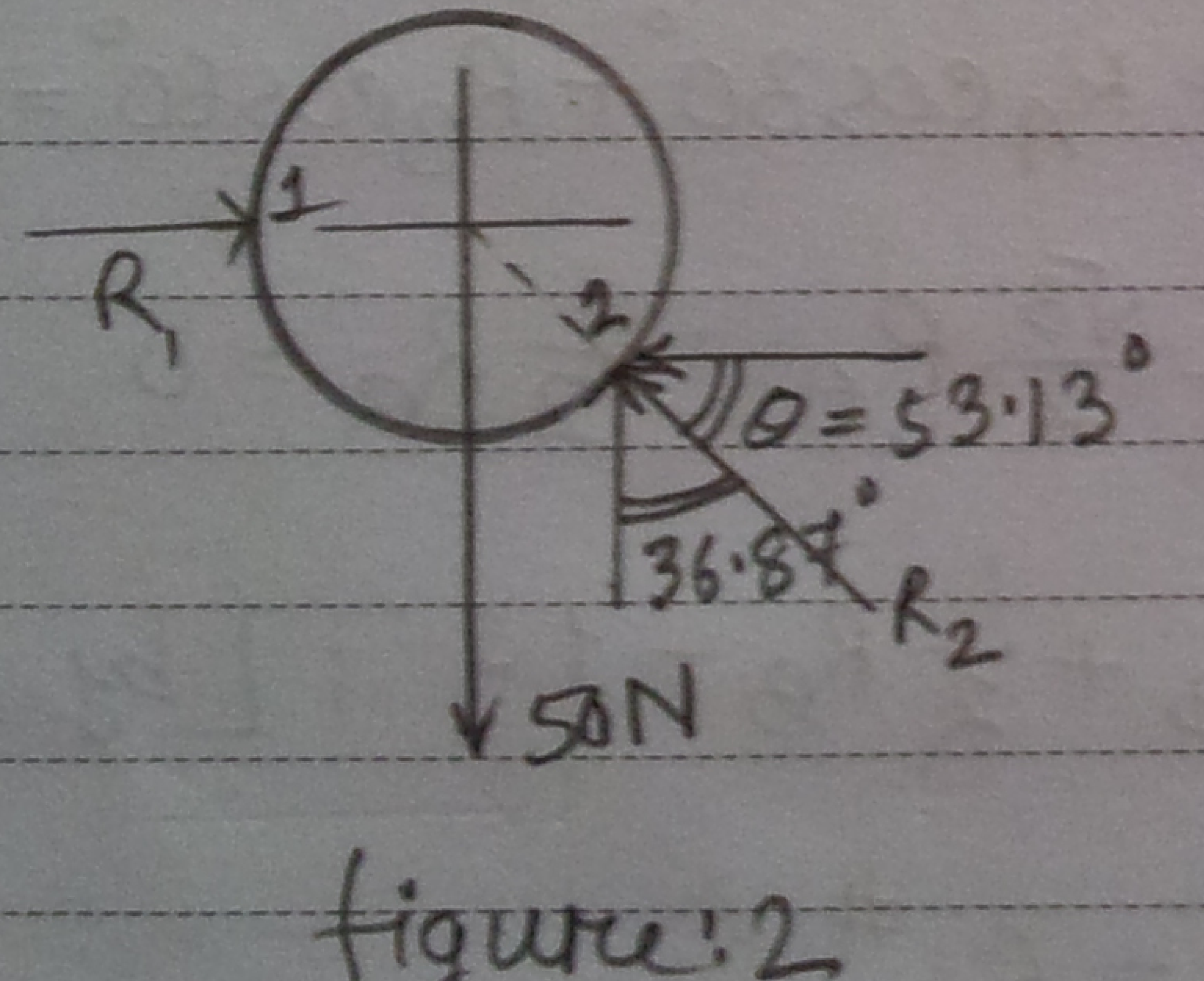


figure:2

From figure-2,

$\Sigma F_x = -R_2 \cos 53.13^\circ + R_1 = 0$

$\Rightarrow R_1 = R_2 \cos 53.13^\circ \ldots\ldots (i)$

and $\Sigma F_y = R_2 \cos 36.87^\circ - 50 = 0$

$\Rightarrow R_2 = 62.5\,N$

$(i) \rightarrow R_1 = 62.5 \times \cos 53.13^\circ\,N$

$= 37.5\,N$

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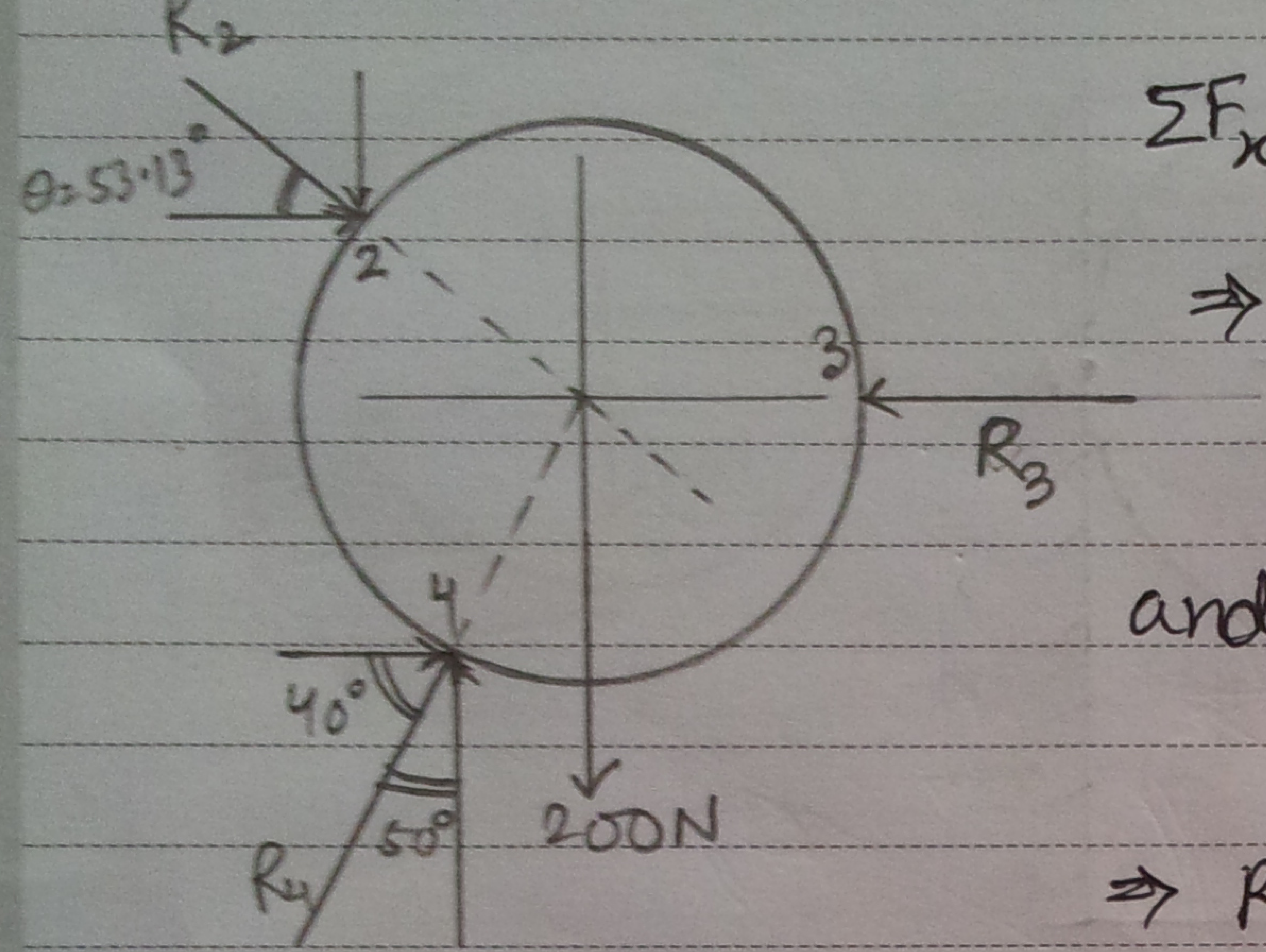

Figure: 3

From figure-3,

$$\Sigma F_x = -R_3 + R_2 \cos 53.13° + R_4 \cos 40° = 0$$

$$\Rightarrow R_4 \cos 40° + R_2 \cos 53.13° = R_3 \quad \ldots (ii)$$

and $$\Sigma F_y = -R_2 \cos 36.87° + R_4 \cos 50° - 200 = 0$$

$$\Rightarrow R_4 = \frac{200 + 62.5 \cos 36.87°}{\cos 50°} \text{ N}$$

$$\therefore R_4 = 388.93 \text{ N}$$

$$(ii) \rightarrow R_3 = 388.93 \cos 40° + 62.5 \cos 53.13°$$

$$\Rightarrow R_3 = 335.44 \text{ N}$$

Ans: $R_1 = 37.5$ N ; $R_2 = 62.5$ N
$R_3 = 335.44$ N ; $R_4 = 388.93$ N.

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Problem: 6 Find out the unknown forces from the figure.

Soln:

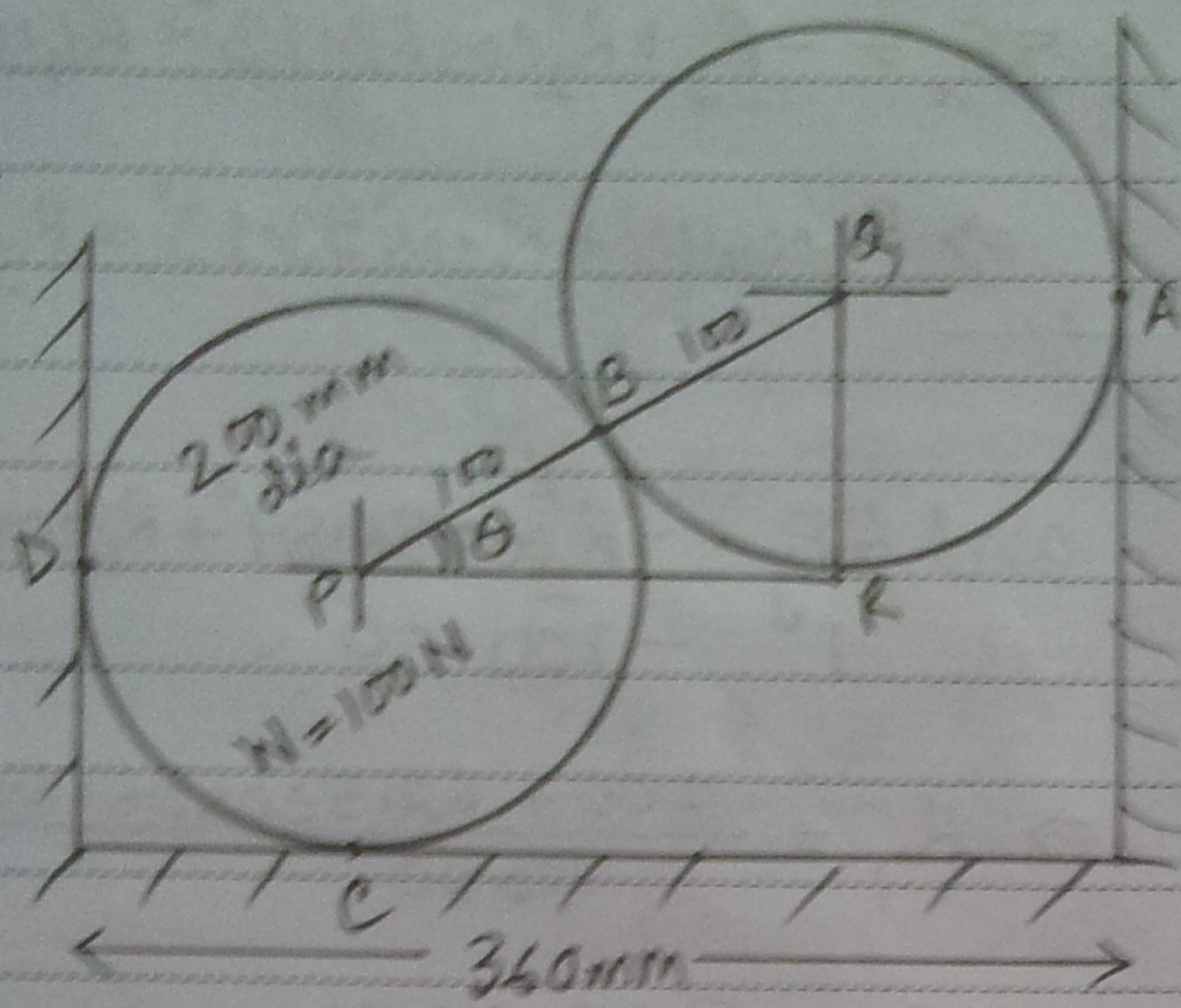

Figure: 1

From figure-1, $PQ = (100+100)\ mm = 200\ mm$

$PR = 360-(100+100)\ mm = 160\ mm$

$\therefore \theta = \cos^{-1}\left(\frac{160}{200}\right) = 36.87^\circ$

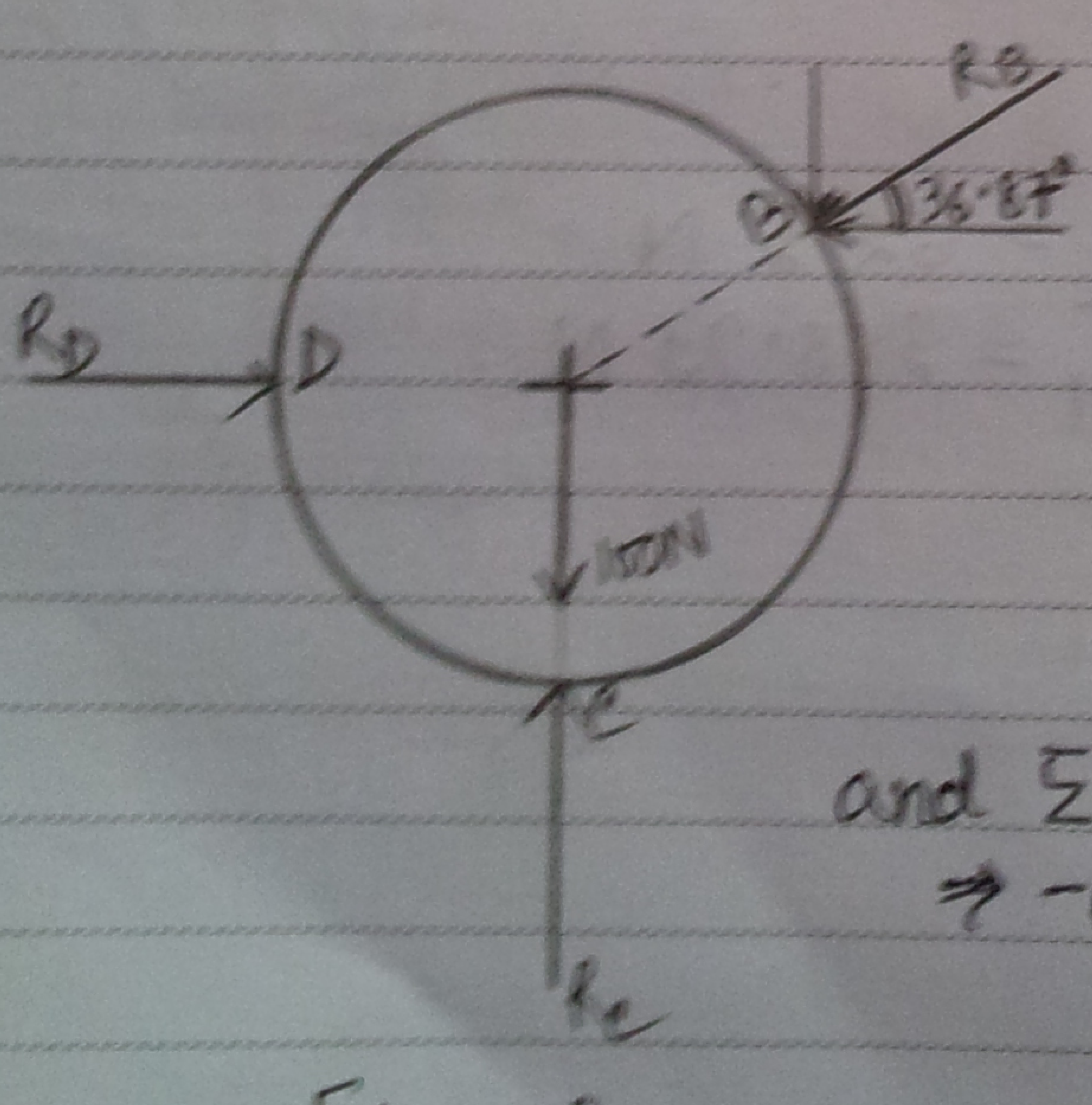

Figure: 2

In equillibrium from figure-2,

$\Sigma F_x = 0$

$\Rightarrow -R_B \cos 36.87^\circ + R_D = 0$

$\Rightarrow R_D = R_B \cos 36.87^\circ$ --- (i)

and $\Sigma F_y = 0$

$\Rightarrow -R_B \sin 36.87^\circ + R_C - 100 = 0$ --- (ii)

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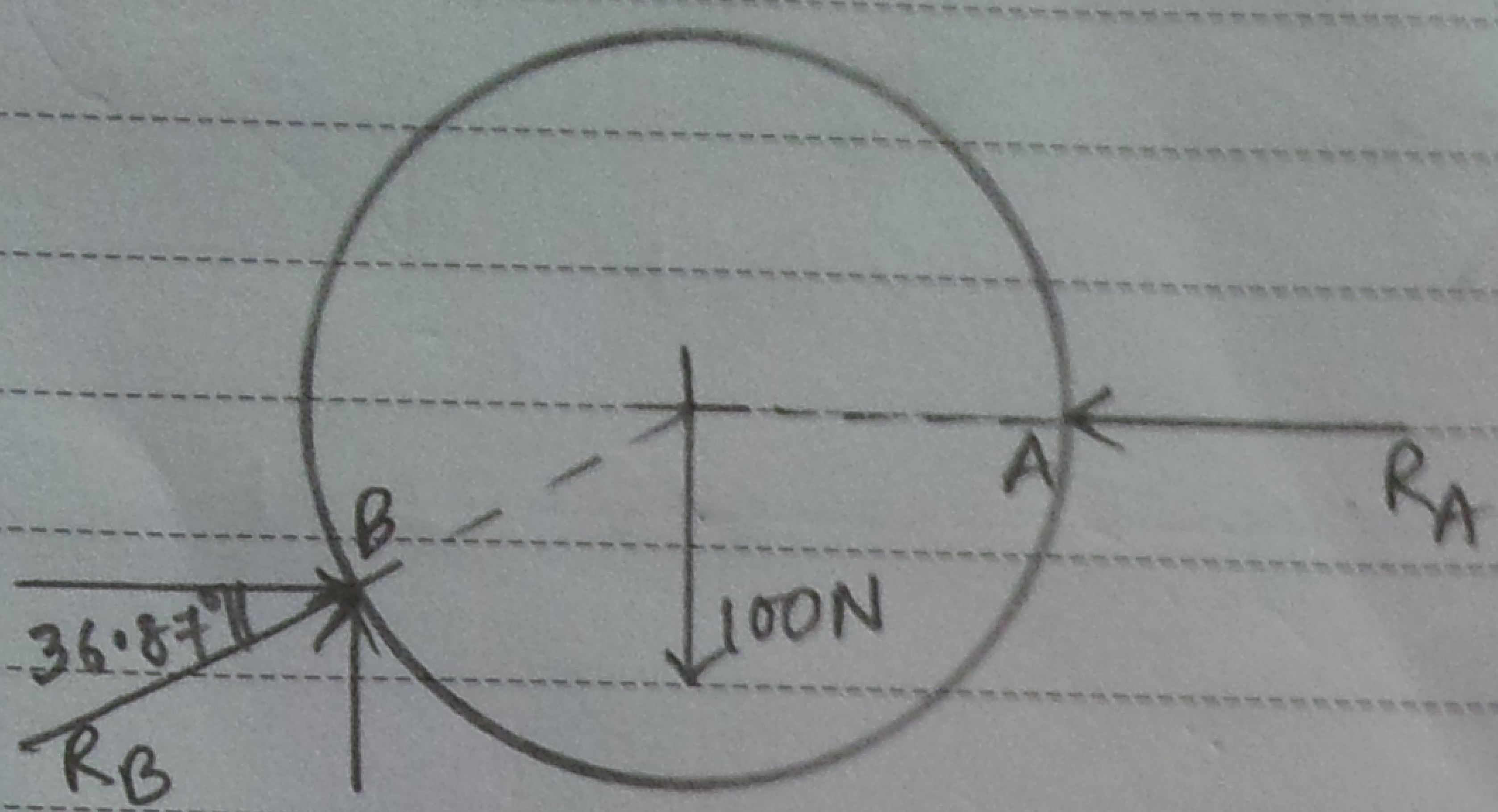


Figure-3

From figure-3 in equillibrium,

$\Sigma F_x = 0$

$\Rightarrow -R_A + R_B \cos 36.87^\circ = 0$

$\Rightarrow R_A = R_B \cos 36.87^\circ$ --- (iii)

and $\Sigma F_y = 0$

$\Rightarrow -100 + R_B \sin 36.87^\circ = 0$

$\Rightarrow R_B = 166.67$ N

$\therefore$ (iii) $\rightarrow R_A = 166.67 \times \cos 36.87^\circ$ N $= 133.33$ N

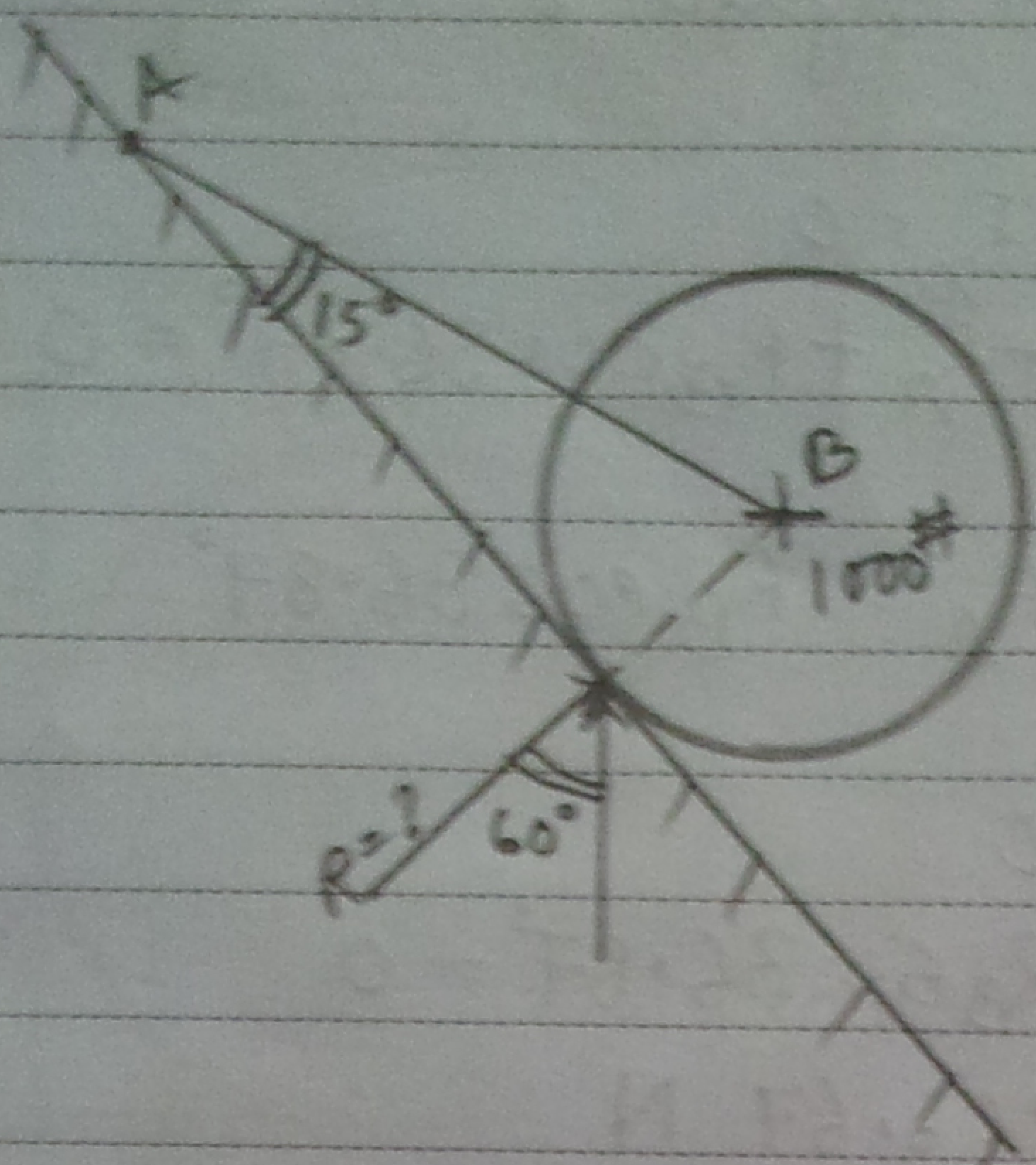
(i) $\rightarrow R_D = 166.67 \times \cos 36.87^\circ$ N $= 133.33$ N

(ii) $\rightarrow R_c = (100 + 166.67 \times \sin 36.87^\circ)$ N $= 200$ N

Ans: $R_A = 133.33$N ; $R_B = 200$ N

$R_C = 166.67$N ; $R_D = 133.33$ N

Problem: 7 From the figure, find R=?



Solⁿ:

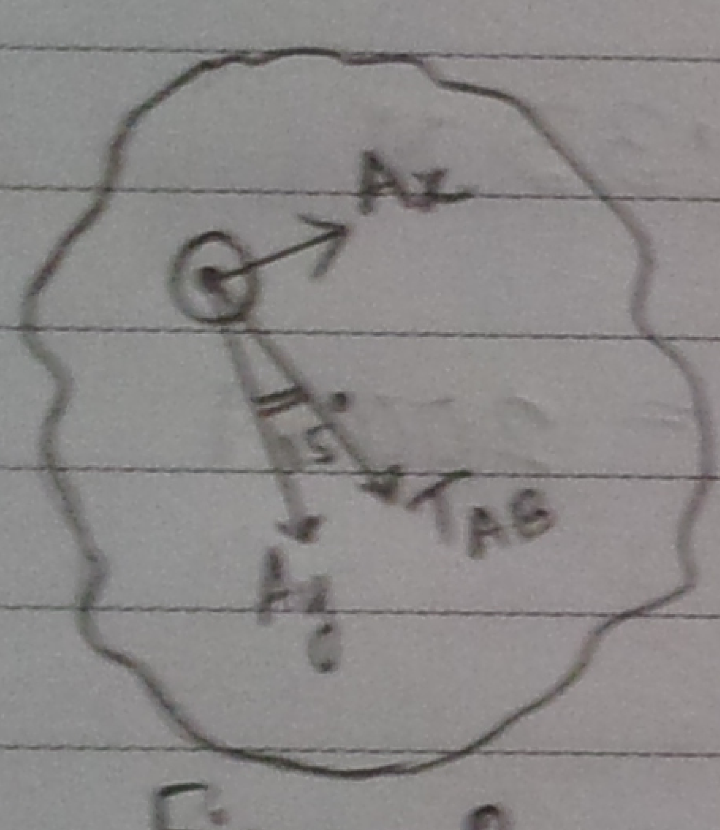


Figure: 2

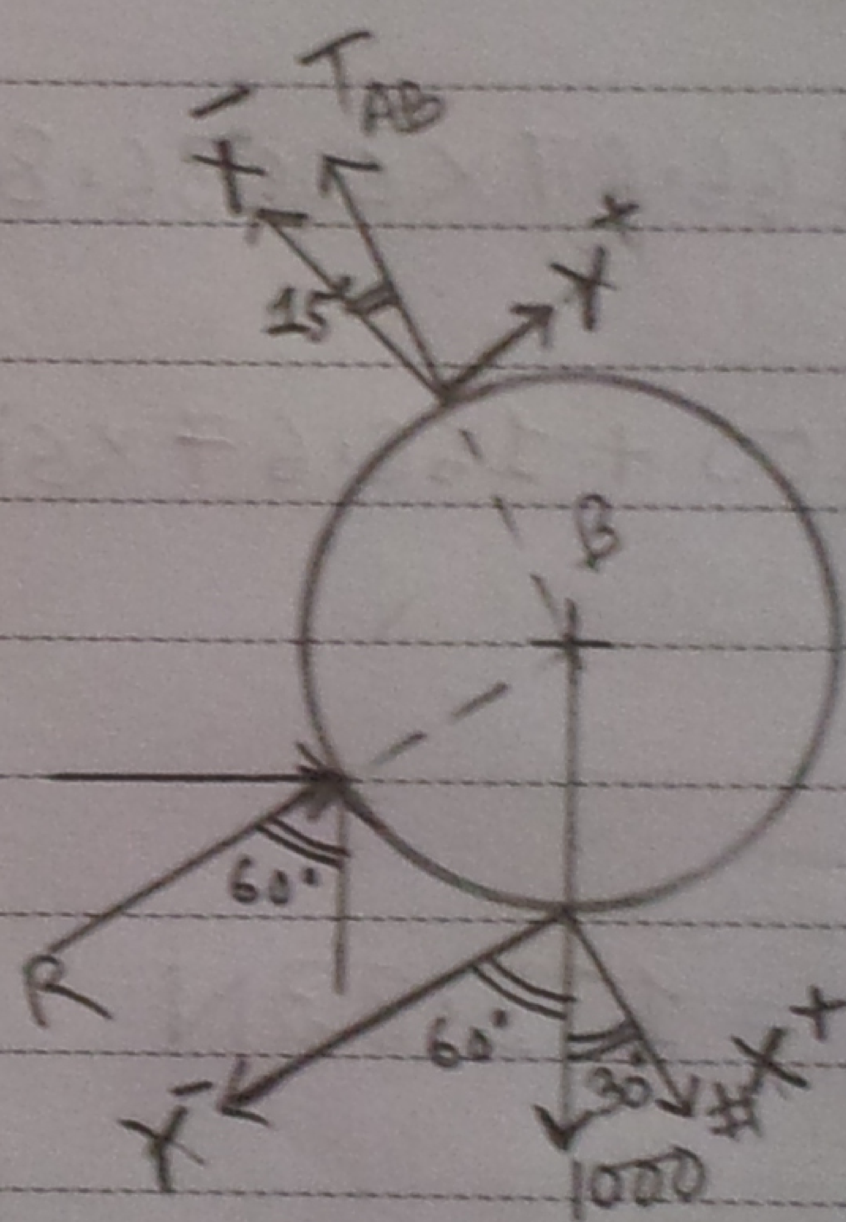


Figure: 1

From figure-1:
In equillibrium,

$\Sigma F_x = -T\cos 15° + 1000\cos 30° = 0$

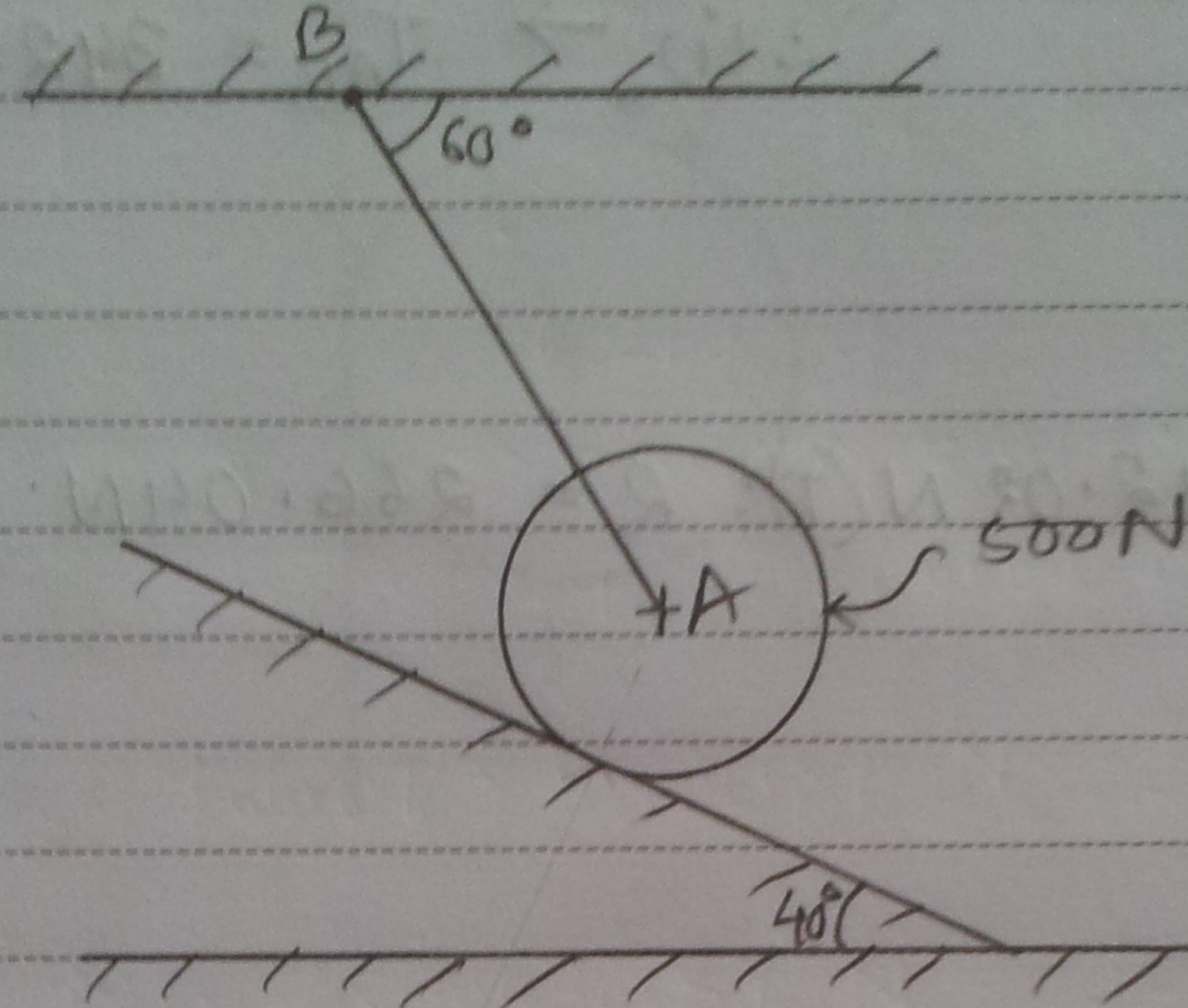
$\Rightarrow T = 896.58\,\#\,(T)$

$\Sigma F_y = T\cos 75° + R - 1000\cos 60° = 0$

$\Rightarrow R = 267.95\,\#$

Ans: $R = 267.95\,\#$.

Problem : 8



Sol$^{\underline{n}}$:

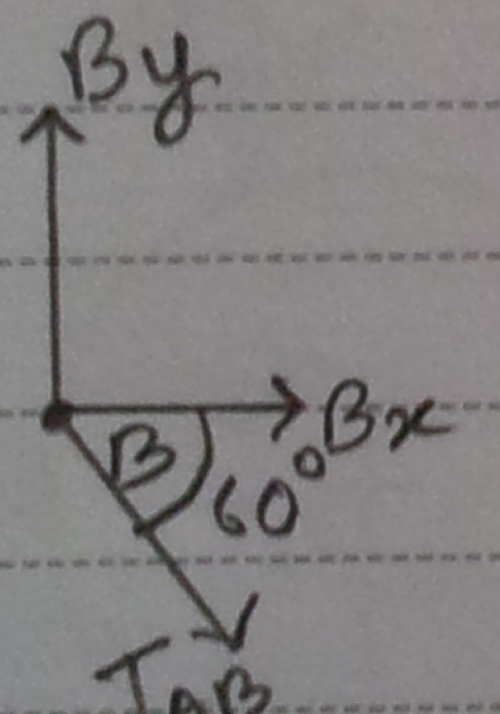


fig-1

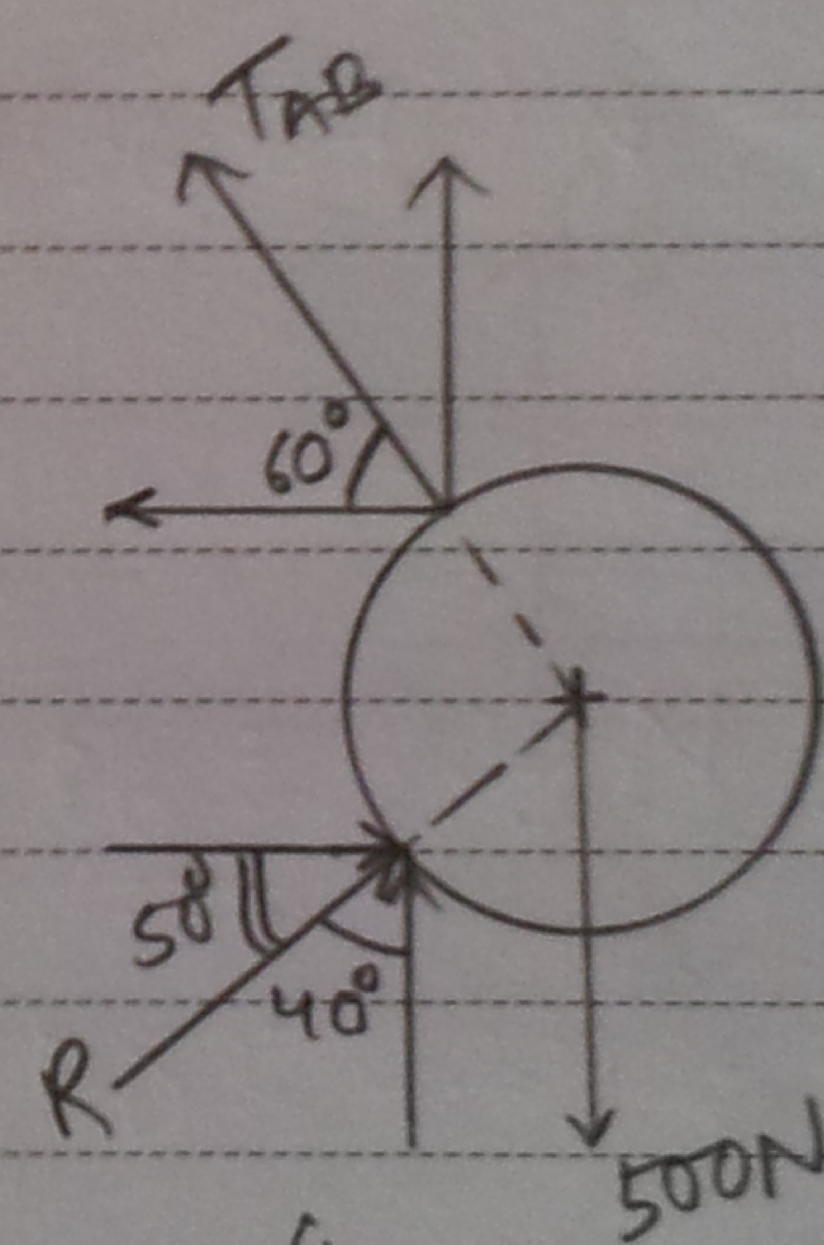


fig-2

For figure-1,

$$\Sigma F_x = B_x + T_{AB}\cos 60^\circ = 0$$

$$\Rightarrow B_x = -342.02 \times \cos 60^\circ \text{ N}$$

$$\therefore B_x = 171.01 \text{ N } (\rightarrow)$$

From fig-2

$$\Sigma F_x = -T_{AB}\cos 60^\circ + R\cos 50^\circ = 0$$

$$\Rightarrow \frac{1}{2} T_{AB} = R\cos 50^\circ$$

$$\Rightarrow T_{AB} = 2R\cos 50^\circ \; \cdots \; (i)$$

& $\Sigma F_y = B_y - T_{AB}\cos 30^\circ = 0$

$\Rightarrow B_y = 342.02 \times \frac{\sqrt{3}}{2}$ N

$\therefore B_y = 296.198$ N $(\uparrow)$

& $\Sigma F_y = T_{AB}\cos 30^\circ + R\cos 40^\circ - 500 = 0$

$\Rightarrow 2R\cos 50^\circ \times \frac{\sqrt{3}}{2} + R\cos 40^\circ = 500$

$\Rightarrow R = 266.04$ N

$\therefore$ (i) $\rightarrow$ $T_{AB} = 342.02$ N (T)

Ans: $T_{AB} = 342.02$ N (T); $R = 266.04$N

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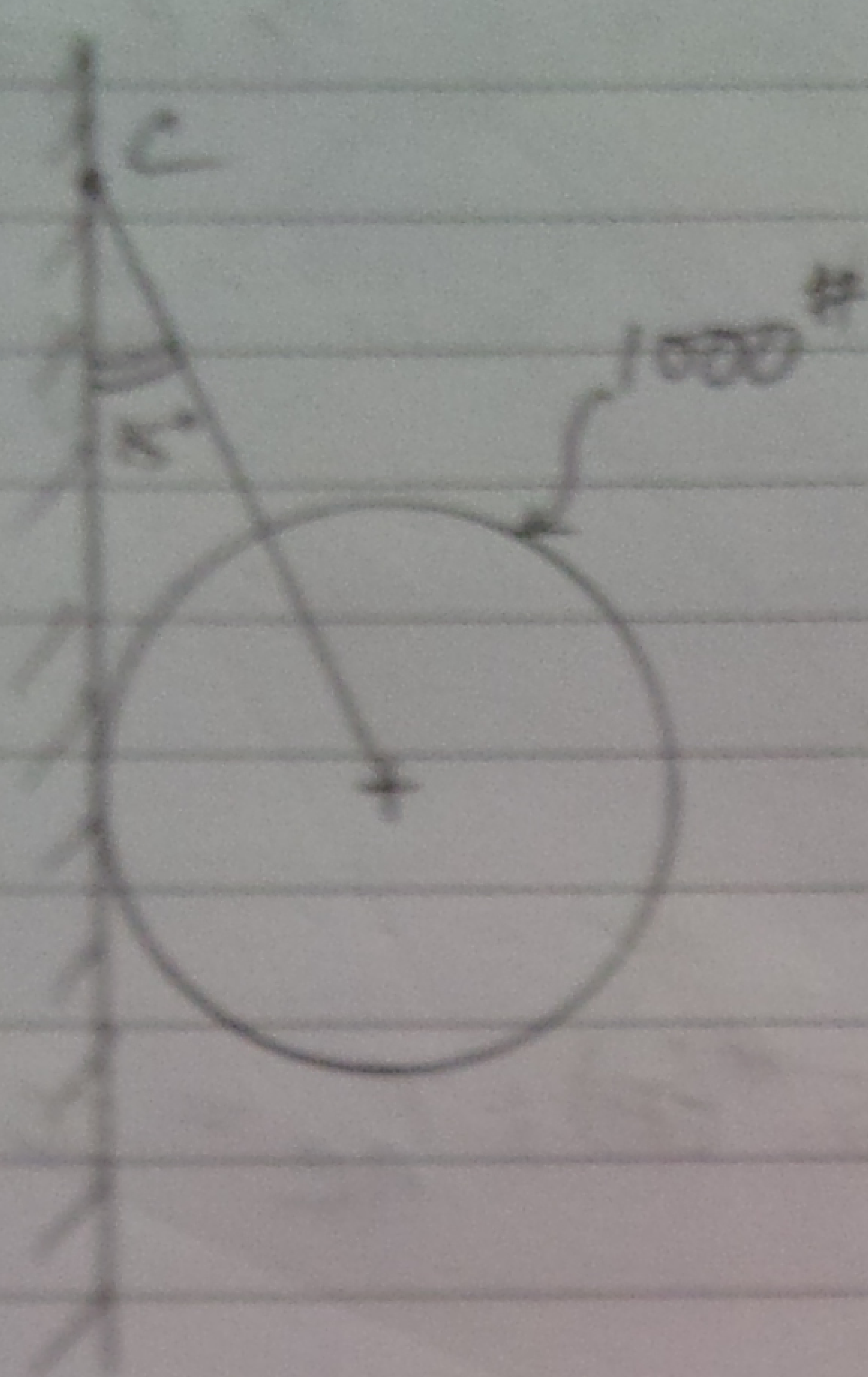
Problem: 9



Sol^n:

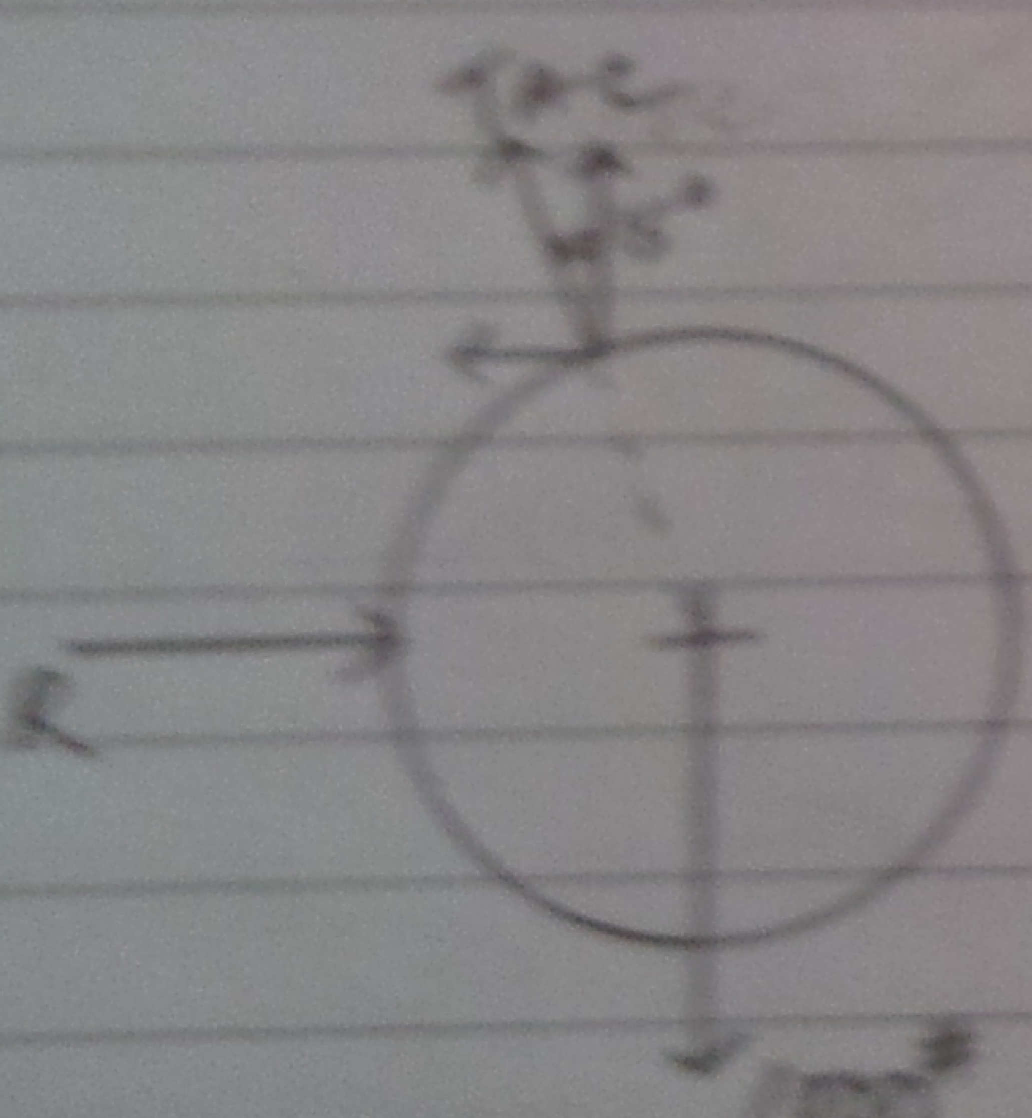


fig-1

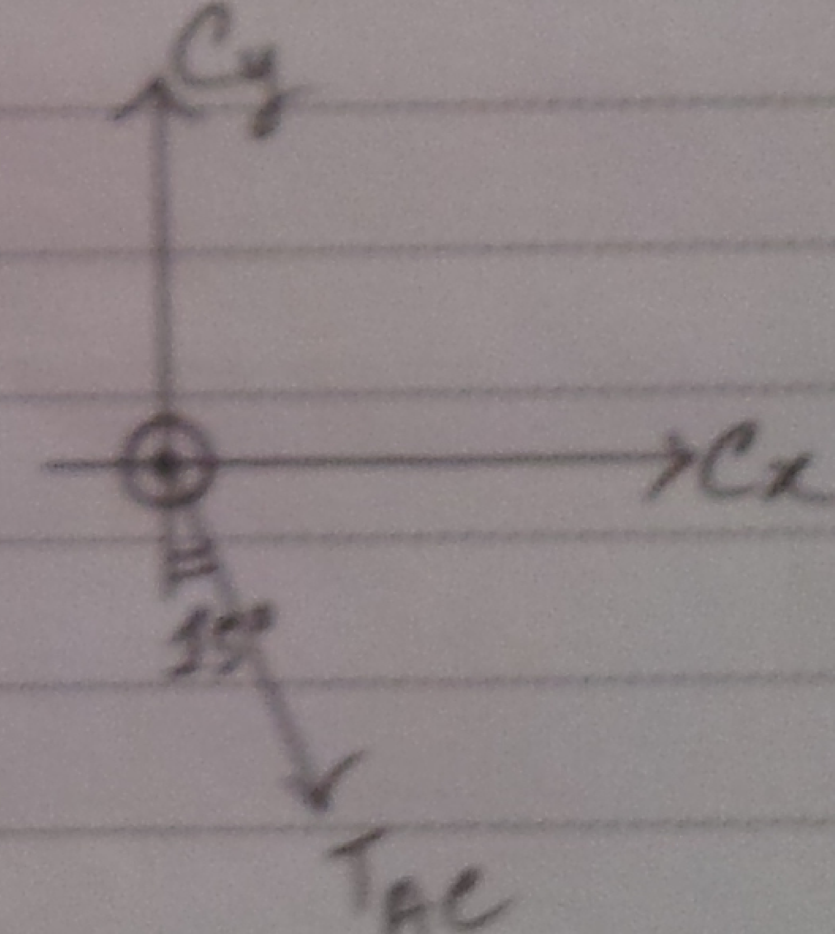


fig-2

From figure-1,

$$\Sigma F_x = R - T_{AC}\cos 75^\circ = 0$$

$$\Rightarrow R = T_{AC}\cos 75^\circ \text{ --- (i)}$$

From figure-2,

$$\Sigma F_x = C_x + T_{AC}\cos 75^\circ = 0$$

$$\Rightarrow C_x = -1035.28^{\#} \times \cos 75^\circ$$

$$\therefore C_x = 267.95^{\#} (\leftarrow)$$

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& $\Sigma F_y = T_{AC} \cos 15^\circ - 1000 = 0$

$\Rightarrow T_{AC} = 1035.28^{\#}$ (T)

(i) $\longrightarrow R = 1035.28^{\#} \times \cos 75^\circ$

$\Rightarrow R = 267.95^{\#}$

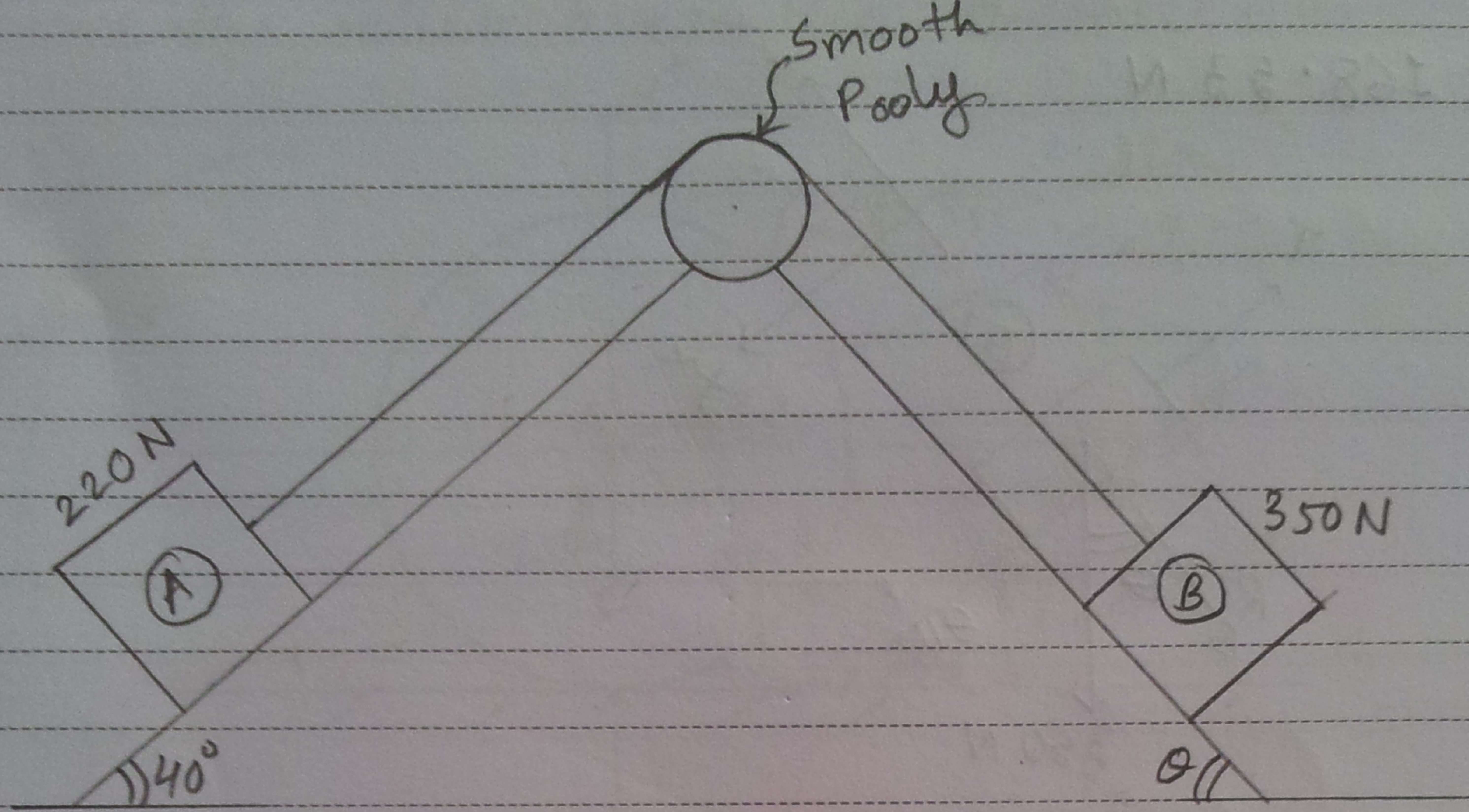
& $\Sigma F_y = C_y - T_{AC} \cos 15^\circ = 0$

$\Rightarrow C_y = 1035.28^{\#} \times \cos 15^\circ$

$\therefore C_y = 1000.004^{\#}$ $(\uparrow)$

Ans: $R = 267.95^{\#}$ ~~(←)~~; $T_{AC} = 1035.28^{\#}$ (T)

Problem: 10



Sol^n:

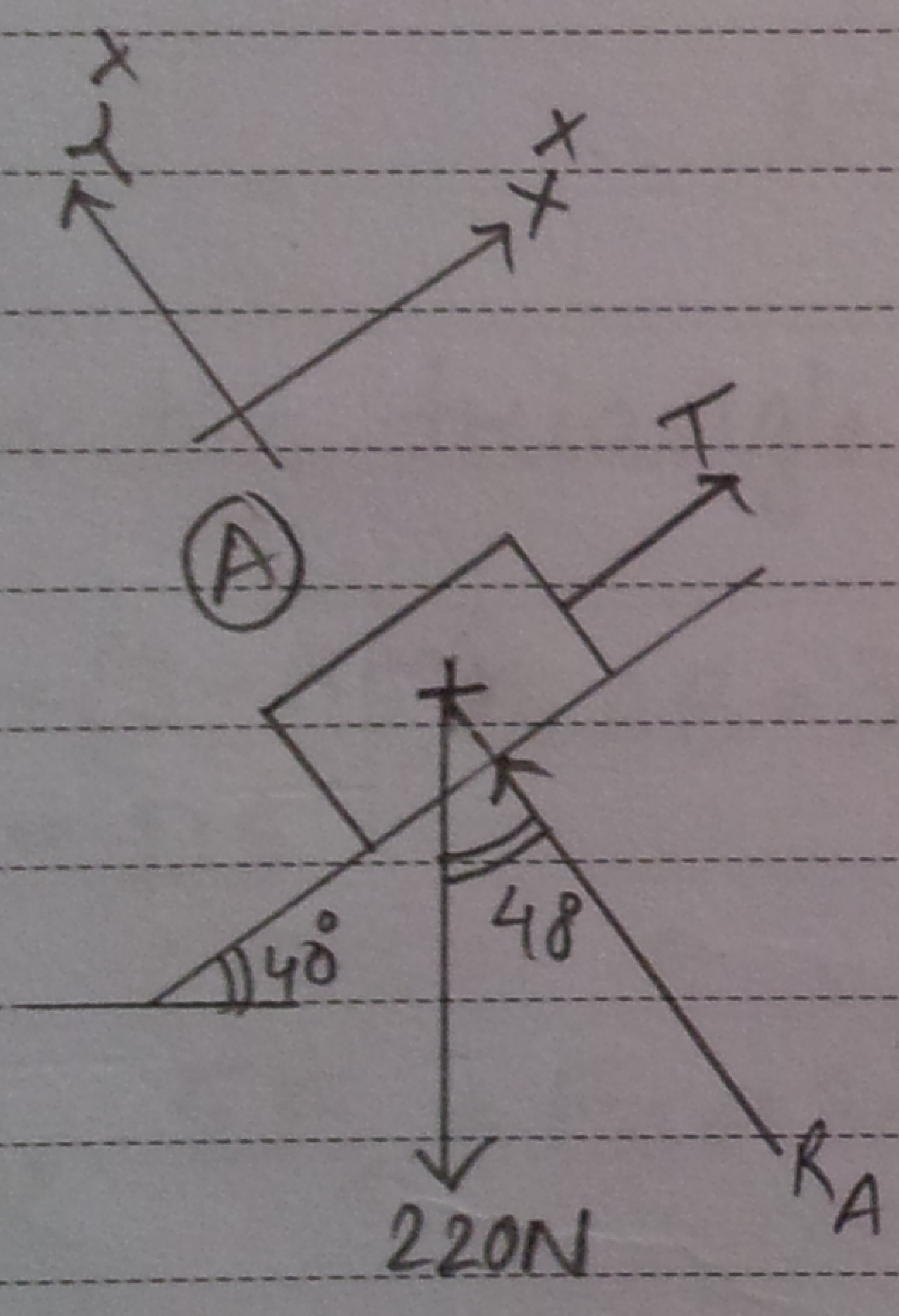


fig-1

From figure-1,

$$\Sigma F_x = T - R_A \cos 90^\circ = 0 - 220 \cos 50^\circ = 0$$

$$\Rightarrow T = 22 \cos 50^\circ$$

$$\Rightarrow T = 141 \cdot 4\ N$$

& $\Sigma F_y = R_A - 220\cos 40^\circ = 0$

$\Rightarrow R_A = 168.53\ N$

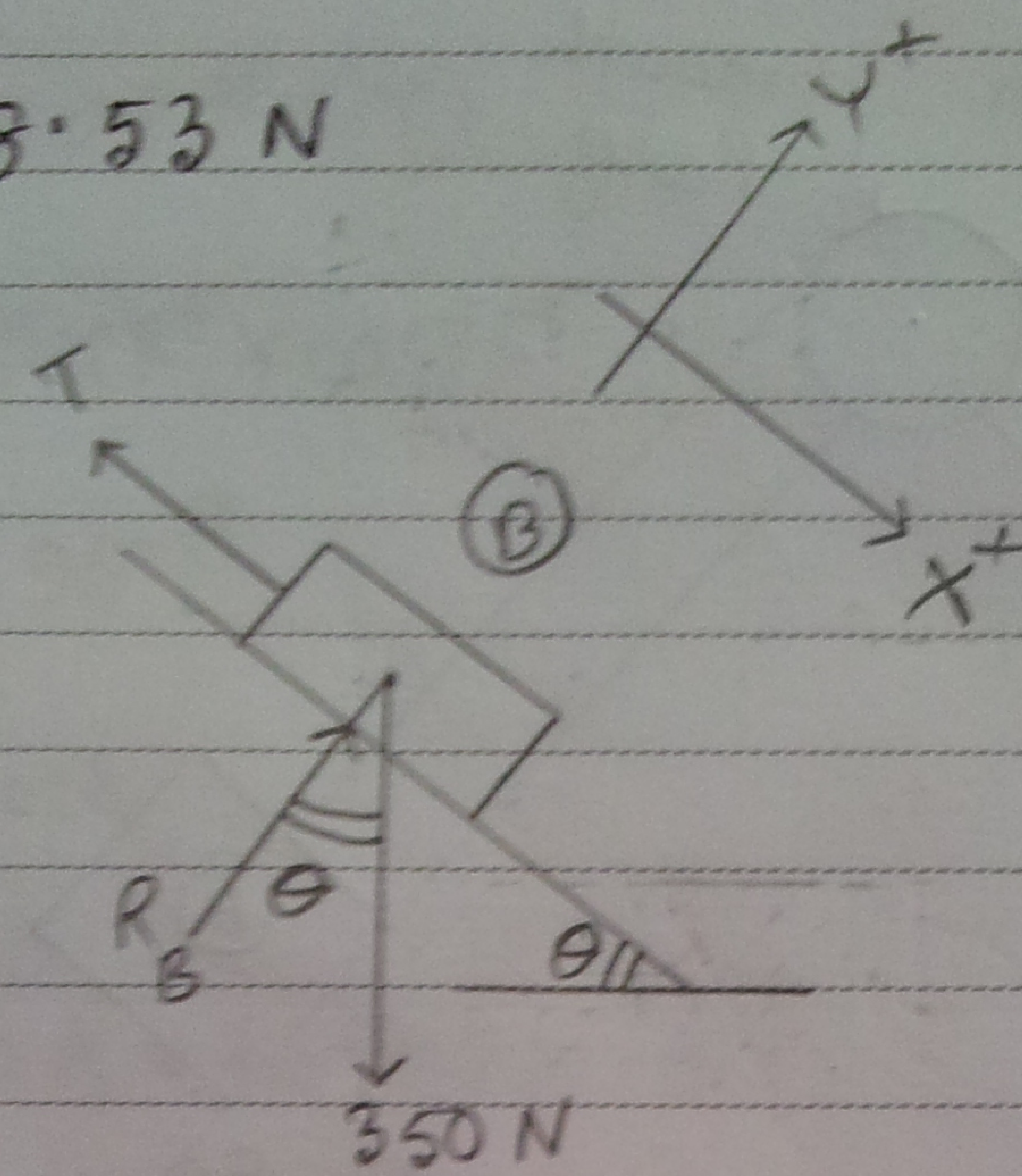


fig-2

From figure-2,

$\Sigma F_x = -T + 350\sin\theta = 0$

$\Rightarrow \theta = \sin^{-1}\left(\frac{141.4}{350}\right)$

$\therefore \theta = 23.83^\circ$

& $\Sigma F_y = R_B - 350\cos\theta = 0$

$\Rightarrow R_B = 350 \times \cos(23.83)$

$\therefore R_B = 320.16\ N$

Ans: $R_A = 168.53 N$; $R_B = 320.16\ N$; $\theta = 23.83^\circ$

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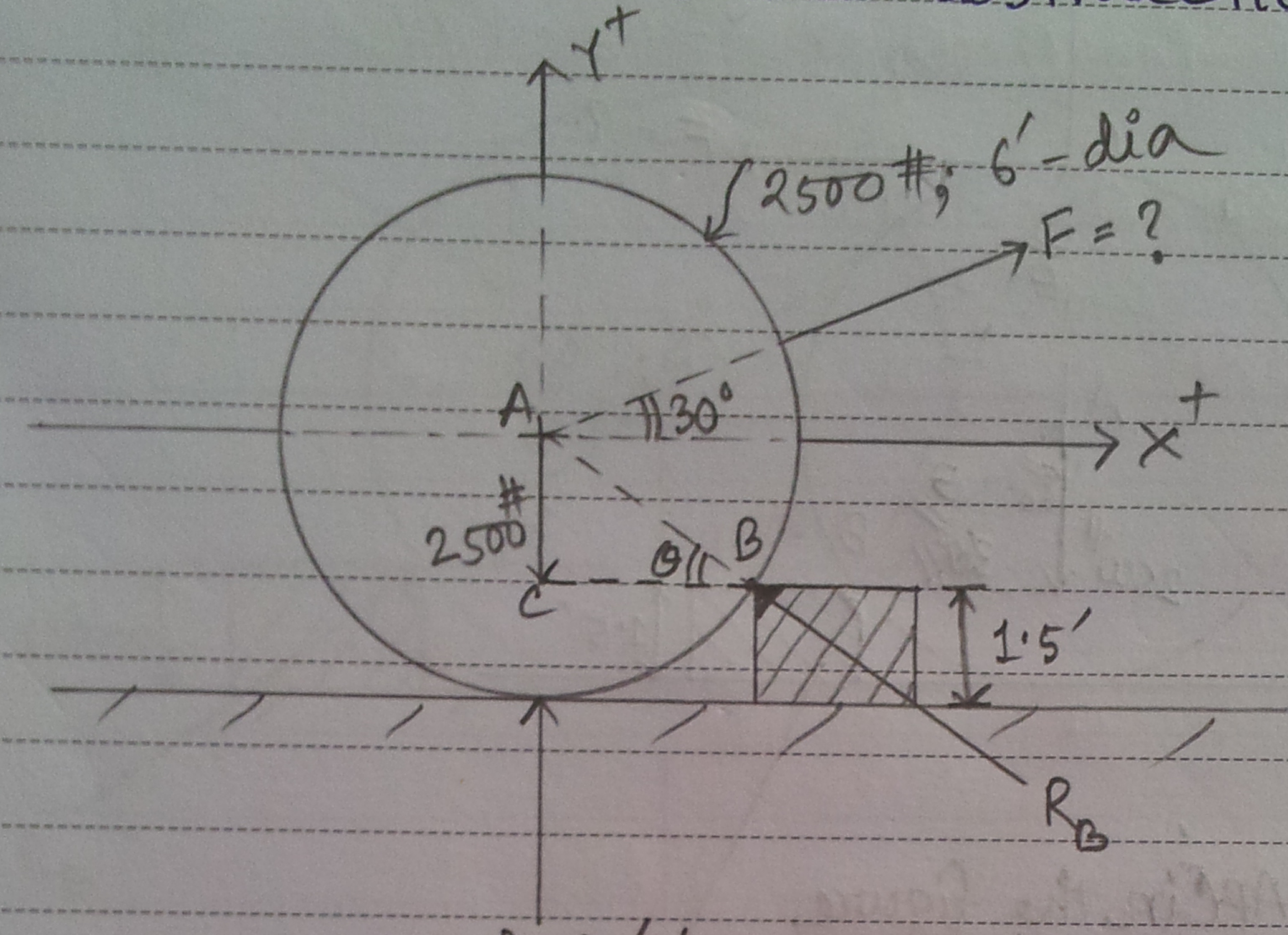
Problem: 11 Calculate minimum force required to move the roller over the obstruction.

(i)



R = 0 (for overcoming the obstacle)

From the triangle ABC in the figure,

$AC = (3 - 1.5)' \; ; \; AB = 3' \quad \therefore \theta = \sin^{-1}\left(\frac{1.5}{3}\right)$

$= 1.5' \qquad \Rightarrow \theta = 30^\circ$

Now, $\Sigma F_x = F\cos 30^\circ - R_B \cos 30^\circ = 0$

$\Rightarrow F = R_B \quad \text{---(i)}$

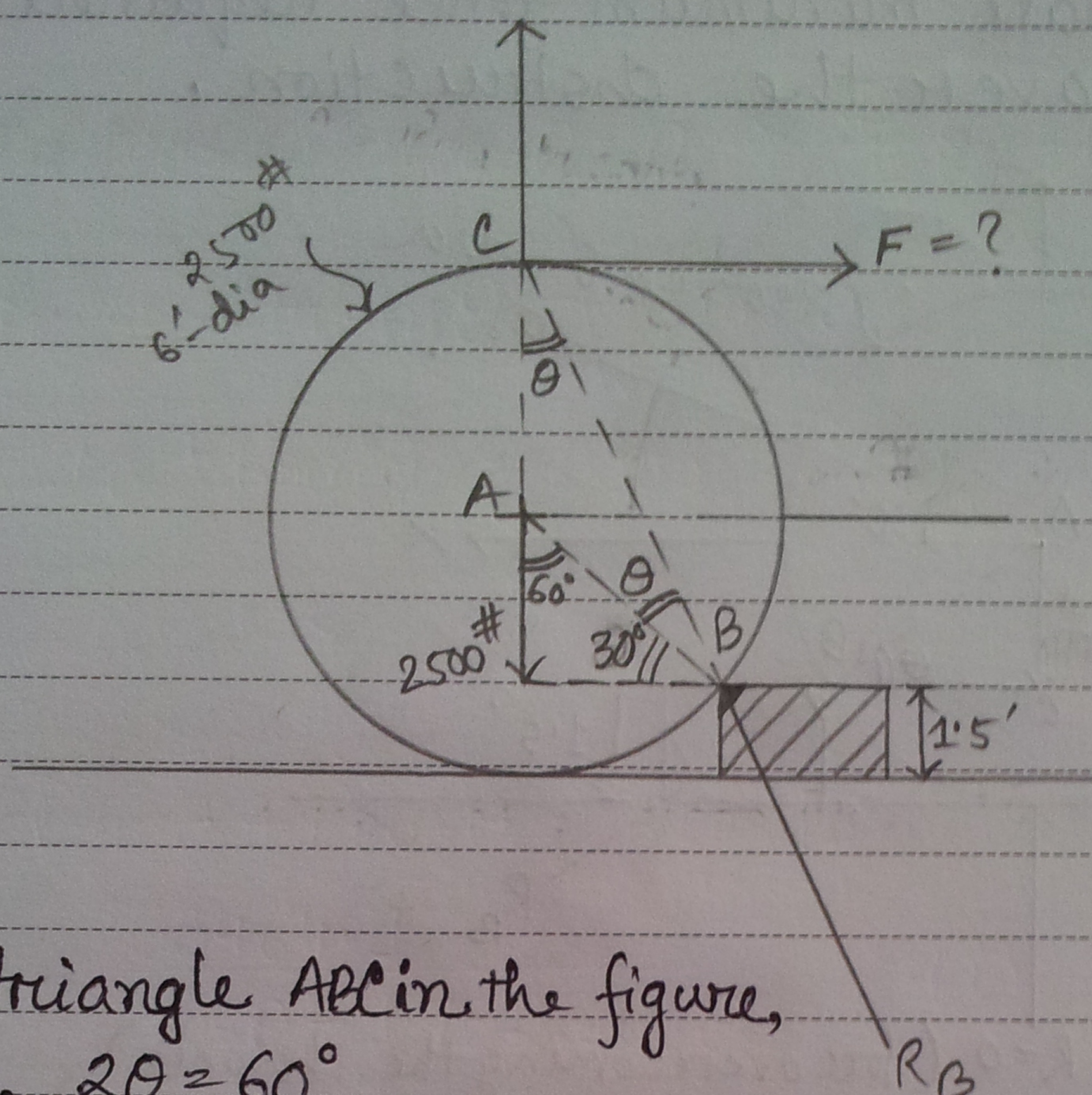
& $\Sigma F_y = R_B \cos 60^\circ + F\cos 60^\circ - 2500 = 0$

$\Rightarrow 2F\cos 60^\circ = 2500 \; ; \quad [\because F = R_B]$

$\Rightarrow F = 2500^{\#}$

Ans: $F = 2500^{\#}$.

(ii)



From triangle ABC in the figure,
we get, $2\theta = 60^\circ$
$\Rightarrow \theta = 30^\circ$

Now, $\Sigma F_x = F - R_B \sin\theta = 0$

$\Rightarrow F = R_B \sin 30^\circ$ ---(i)

& $\Sigma F_y = R_B \cos 30^\circ - 2500 = 0$

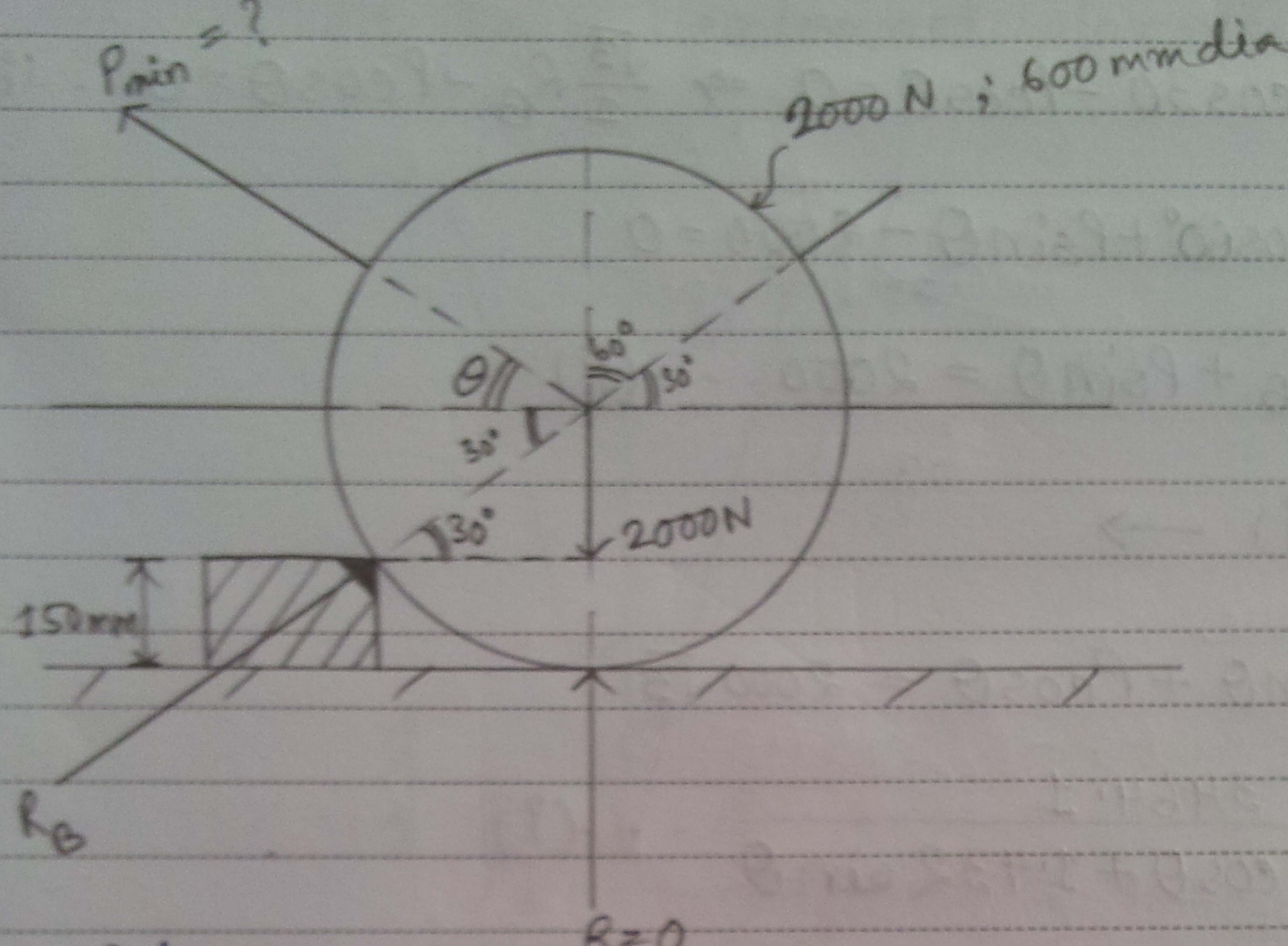
$\Rightarrow R_B = 2886.75\#$

(i) $\rightarrow$ $F = 2886.75 \times \sin 30^\circ$

$\Rightarrow F = 1443.38\#$

Ans: $F = 1443.38\#$

Problem: 12



(Process-1)(Lami)

Here, $\dfrac{P}{\sin(90^\circ+30^\circ)} = \dfrac{R_B}{\sin(90^\circ+\theta)} = \dfrac{2000}{\sin(90^\circ-\theta+60^\circ)}$

$\Rightarrow \dfrac{P}{\sin 150^\circ} = \dfrac{R}{\cos\theta} = \dfrac{2000}{\sin(150-\theta)}$

If P is minimum, $\sin(150-\theta)$ will be maximum.

$\therefore \sin(150-\theta) = 1$

$\Rightarrow 150 - \theta = 90^\circ$

$\therefore \theta = 60^\circ$

$\therefore P_{min} = 2000 \sin 120^\circ = 1732.05\,N$

Ans: 1732.05 N.

(Process-2)

$\Sigma F_x = R_B \cos 30^\circ - P\cos\theta = 0 \Rightarrow \frac{\sqrt{3}}{2} R_B - P\cos\theta = 0$ --(i)

$\Sigma F_y = R_B \cos 60^\circ + P\sin\theta - 2000 = 0$

$\Rightarrow \frac{1}{2} R_B + P\sin\theta = 2000$ ---(ii)

(ii) $\times \sqrt{3}$ − (i) ⟶

$\sqrt{3} P\sin\theta + P\cos\theta = 2000\sqrt{3}$

$\Rightarrow P = \dfrac{3464.1}{\cos\theta + 1.732\sin\theta}$ ---(i)

P will be minimum when $(\cos\theta + 1.732\sin\theta)$ is maximum.

$\therefore \frac{d}{d\theta}(\cos\theta + 1.732\sin\theta) = 0$

$\Rightarrow 1.732\cos\theta - \sin\theta = 0$

$\Rightarrow \tan\theta = 1.732$

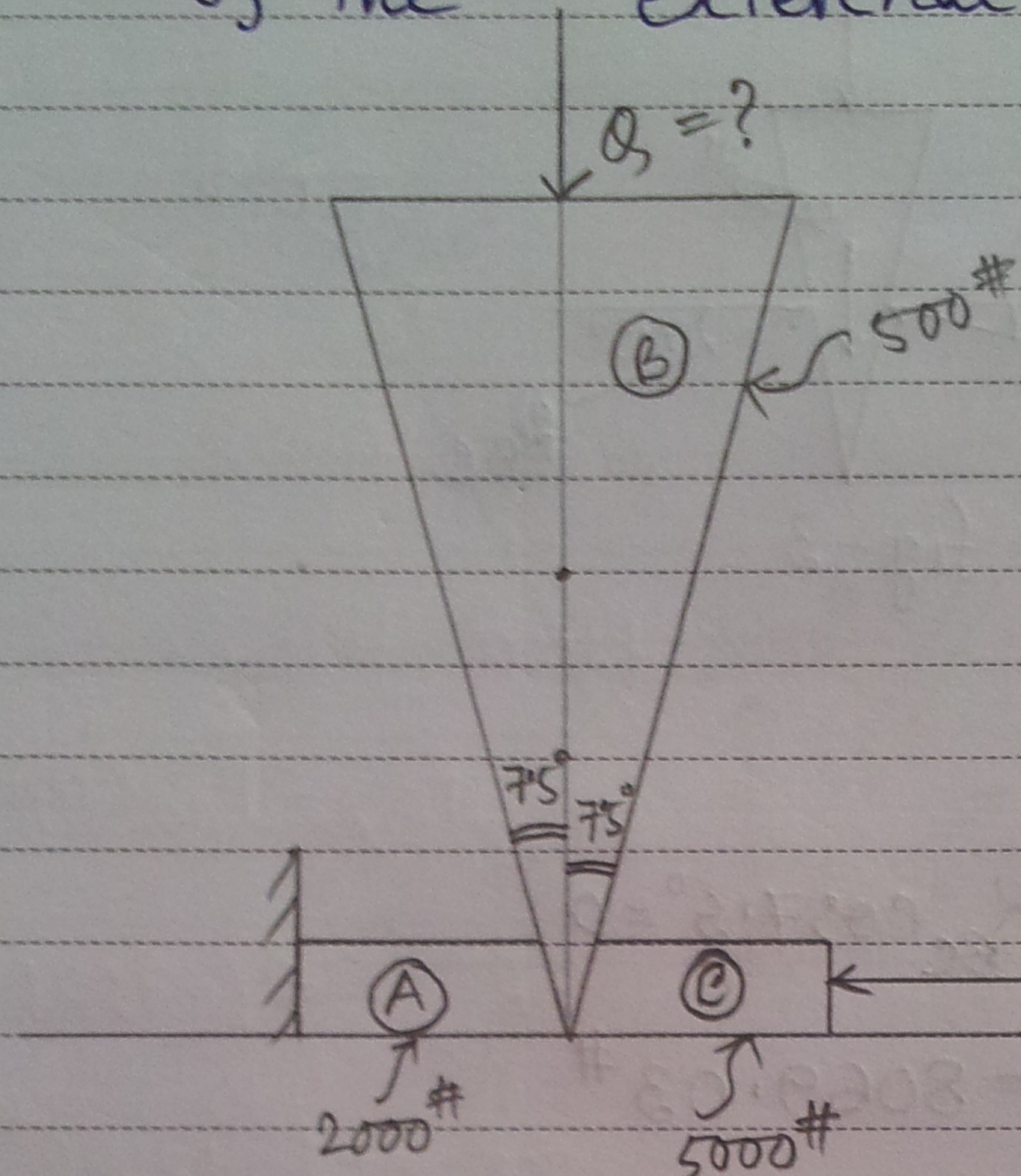
$\therefore \theta = 60^\circ$

$\therefore$ From eq. (i) $\rightarrow P_{min} = \dfrac{3464.1}{\cos 60^\circ + 1.732\sin 60^\circ}$

$\Rightarrow P_{min} = 1732.05$ N

Ans: $P_{min} = 1732.05$ N

Problem-13 (Fairies-89) What is the value of Q and reactions of the external planes?



Solⁿ:

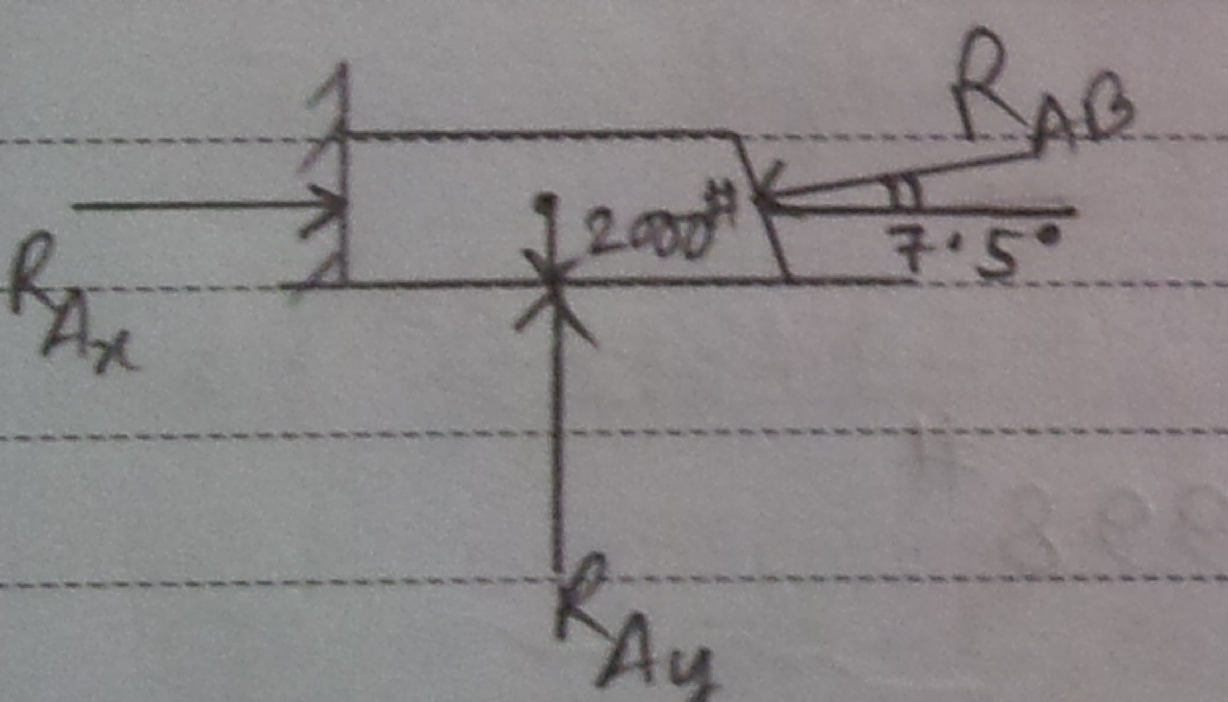


fig-1

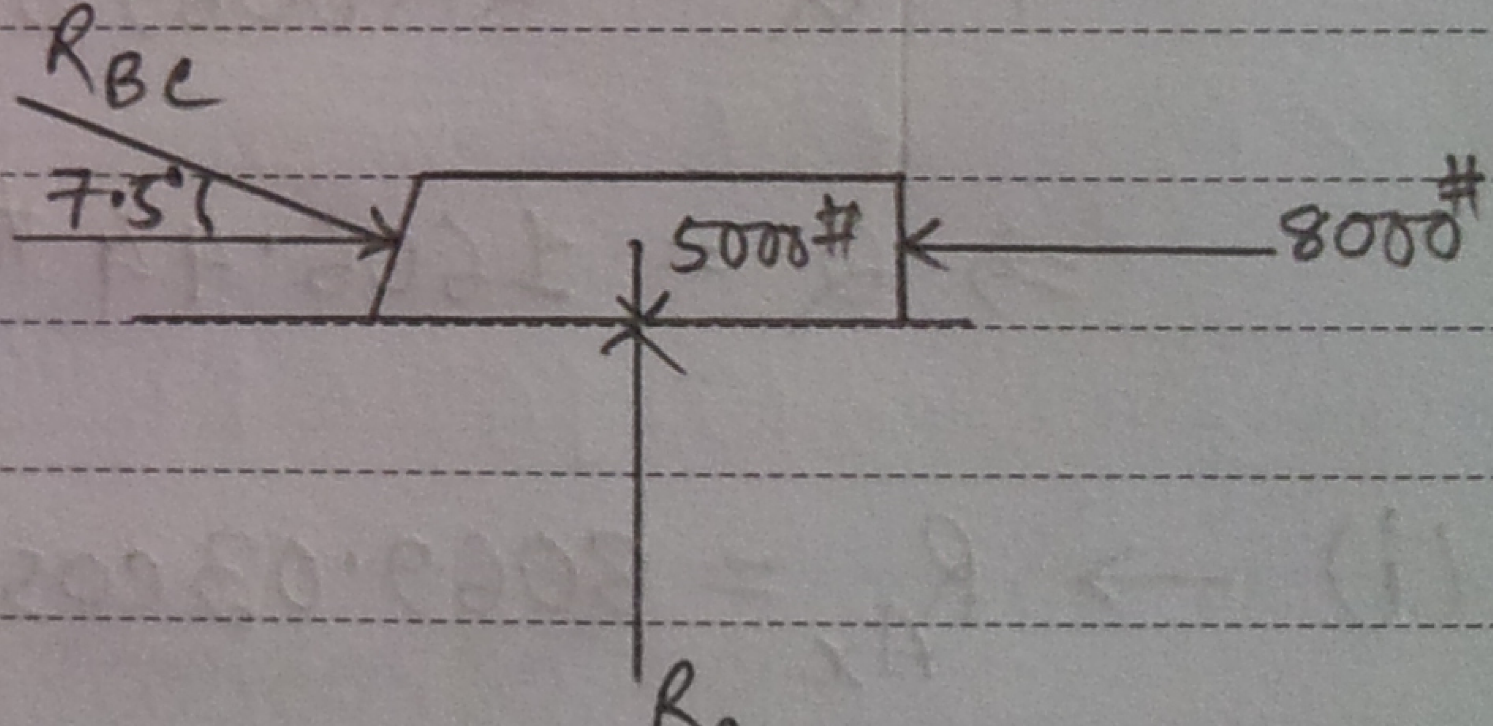


fig-2

From figure-1

$\Sigma F_x = -R_{AB}\cos 7.5^\circ + R_{A_x} = 0$

$\Rightarrow R_{A_x} = R_{AB}\cos 7.5^\circ \quad \ldots(i)$

& $\Sigma F_y = R_{Ay} - R_{AB}\cos 82.5^\circ - 2000 = 0$

$\Rightarrow R_{Ay} = R_{AB}\cos 82.5^\circ + 2000 \quad \ldots(ii)$

From figure-2

$\Sigma F_x = R_{BC}\cos 7.5^\circ - 8000 = 0$

$\Rightarrow R_{BC} = 8069.03\#$

& $\Sigma F_y = R_c - R_{BC}\cos 82.5^\circ - 5000 = 0$

$\Rightarrow R_c = 6053.22\#$

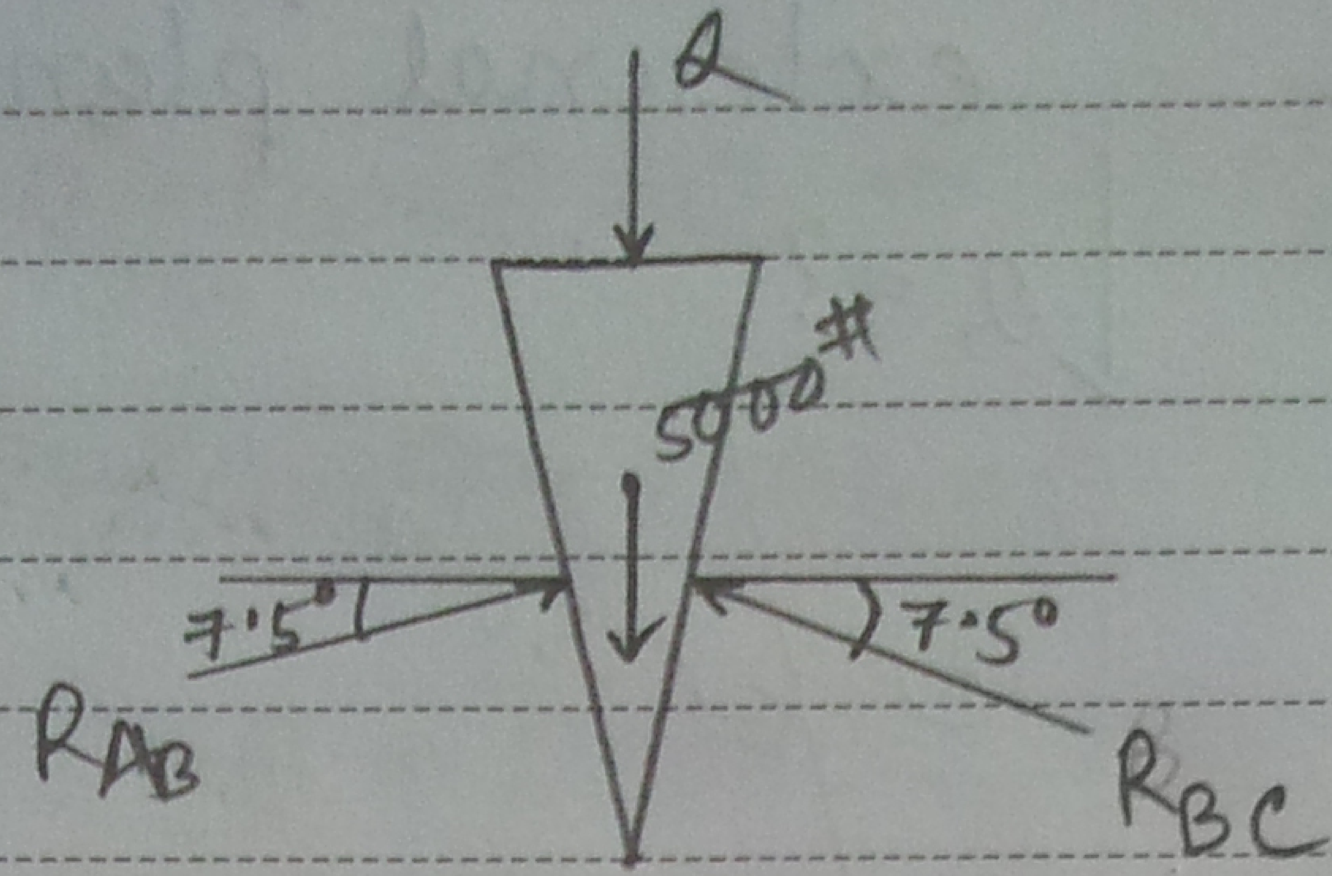


fig-3

From figure-3,

$$\Sigma F_x = R_{AB}\cos 7.5^\circ - R_{BC}\cos 7.5^\circ = 0$$

$$\Rightarrow R_{AB} = R_{BC} = 8069.03^{\#}$$

$$\& \ \Sigma F_y = -Q - 5000 + R_{AB}\cos 82.5^\circ + R_{BC}\cos 82.5^\circ$$

$$\Rightarrow Q = 2 \times 8069.03 \times \cos 82.5^\circ - 5000$$

$$\Rightarrow Q = 1606.44^{\#}$$

(i) $\rightarrow R_{Ax} = 8069.03\cos 7.5^\circ = 7999.998^{\#}$

(ii) $\rightarrow R_{Ay} = 8069.03\cos 82.5^\circ + 2000 = 3053.22^{\#}$

Ans: $Q = 1606.44^{\#}$; $R_{Ax} = 7999.998^{\#}$;
$R_{Ay} = 3053.22^{\#}$; $R_C = 6053.22^{\#}$

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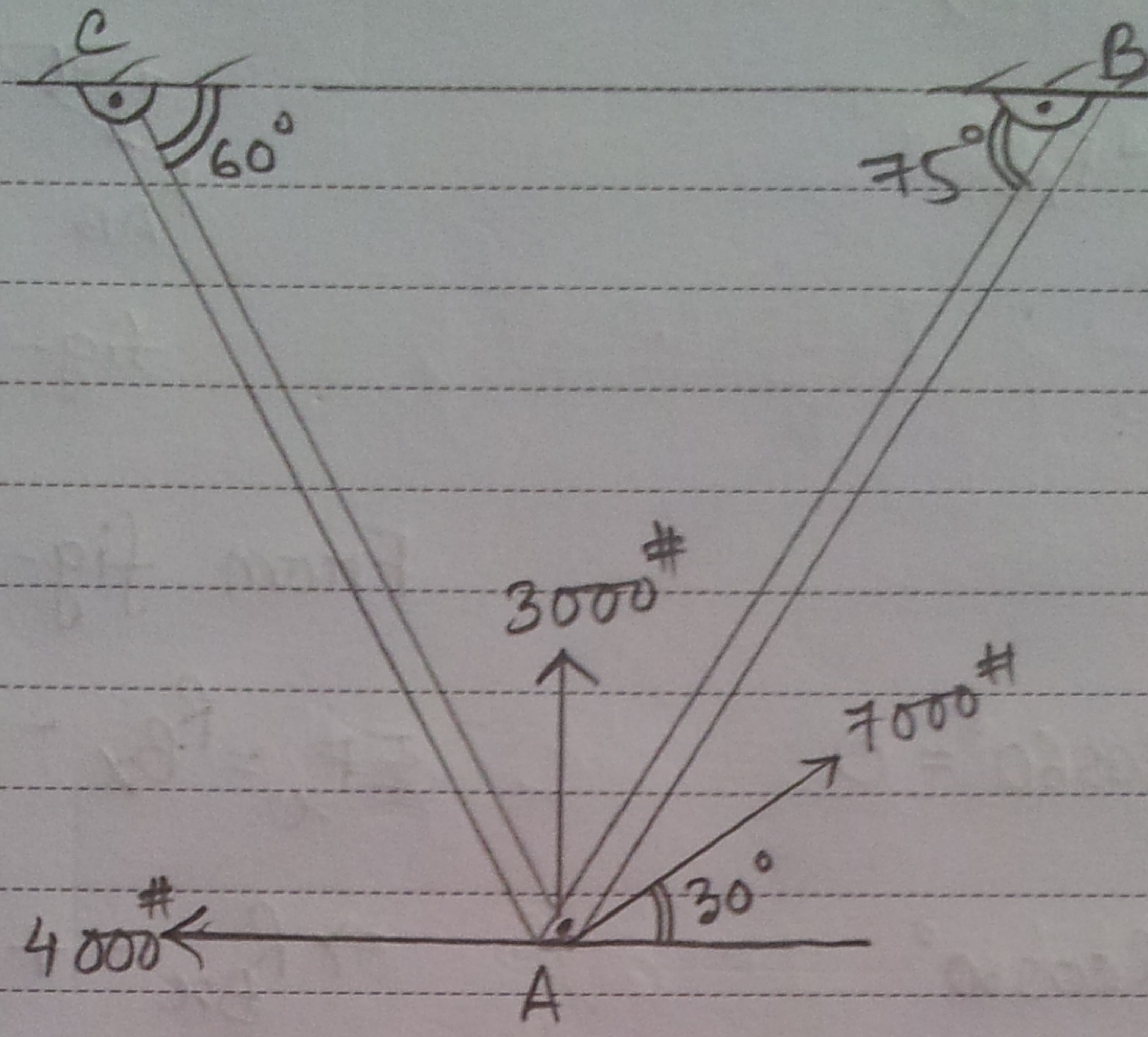
Problem : 14

(Fairies 84)



Sol^n:

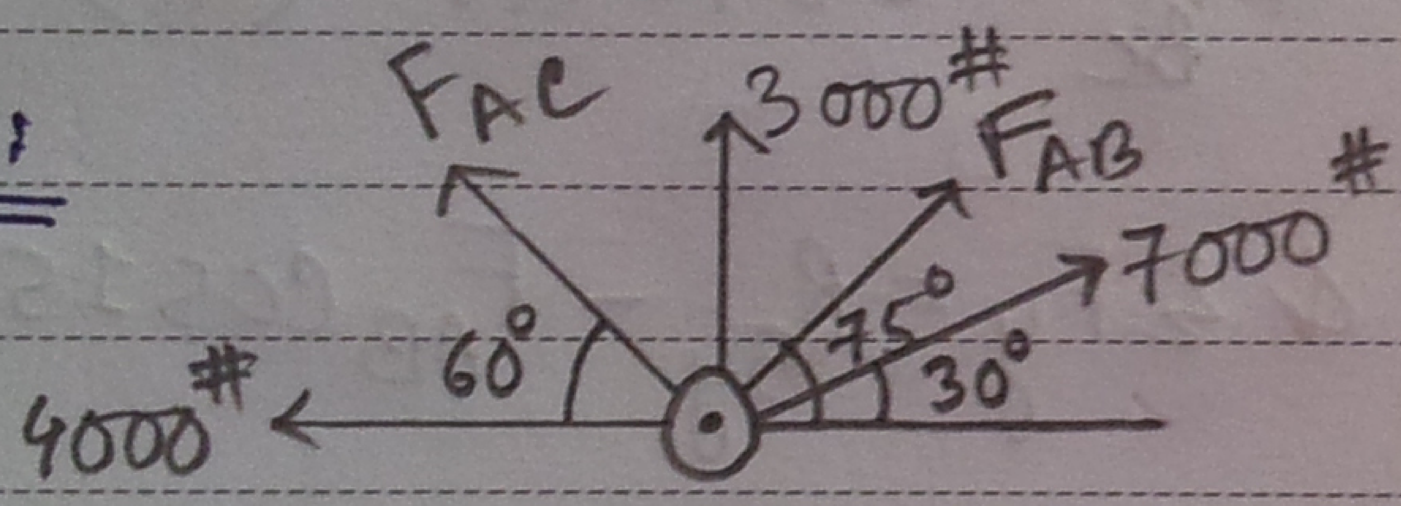


fig-1

From figure-1

$$\Sigma F_x = -4000 - F_{AC}\cos 60^\circ + F_{AB}\cos 75^\circ + 7000\cos 30^\circ = 0$$

$$\Rightarrow 0.26 F_{AB} - 0.5 F_{AC} + 2062.18 = 0 \text{ --- (i)}$$

$$\& \ \Sigma F_y = F_{AC}\cos 30^\circ + 3000 + F_{AB}\cos 15^\circ + 7000\cos 60^\circ = 0$$

$$\Rightarrow 0.97 F_{AB} + 0.87 F_{AC} + 6500 = 0 \text{ --- (ii)}$$

Solving eq. (i) & (ii) $\rightarrow$ $F_{AB} = -7121.8^{\#}_{(c)}$ & $F_{AC} = +437.82^{\#}_{(T)}$

Date :

Sun | Mon | Tue | Wed | Thu | Fri | Sat

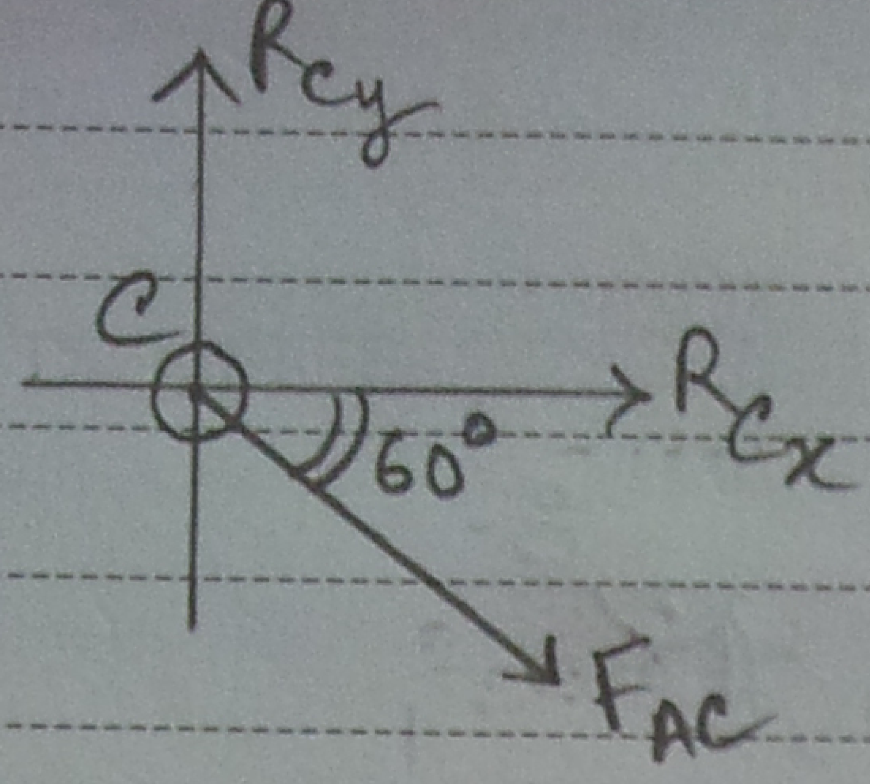


fig-2

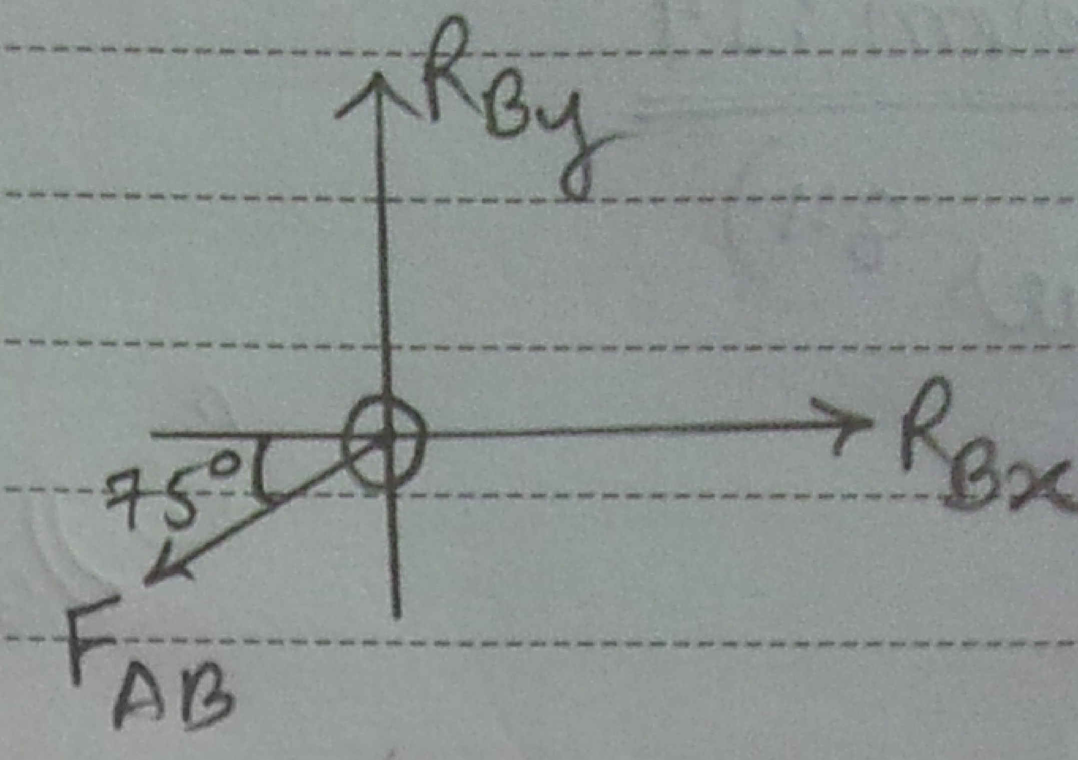


fig-3

From fig-2,

$\Sigma F_x = R_{cx} + F_{Ac} \cos 60° = 0$

$\Rightarrow R_{cx} = -437.82 \cos 60°$

$\therefore R_{cx} = -218.91^{\#} (\leftarrow)$

& $\Sigma F_y = R_{cy} - F_{Ac} \cos 30° = 0$

$\Rightarrow R_{cy} = 437.82 \cos 30°$

$\therefore R_{cy} = 379.16^{\#} (\uparrow)$

From fig-3,

$\Sigma F_x = R_{Bx} - F_{AB} \cos 75° = 0$

$\Rightarrow R_{Bx} = +(-7121.8) \cos 75°$

$\Rightarrow R_{Bx} = -1843.26^{\#} (\leftarrow)$

& $\Sigma F_y = R_{By} - F_{AB} \cos 15° = 0$

$\Rightarrow R_{By} = (-7121.8) \cos 15°$

$\Rightarrow R_{By} = 6879.13^{\#} (\downarrow)$

Ans:

Problem : 15

(Fairies 93)

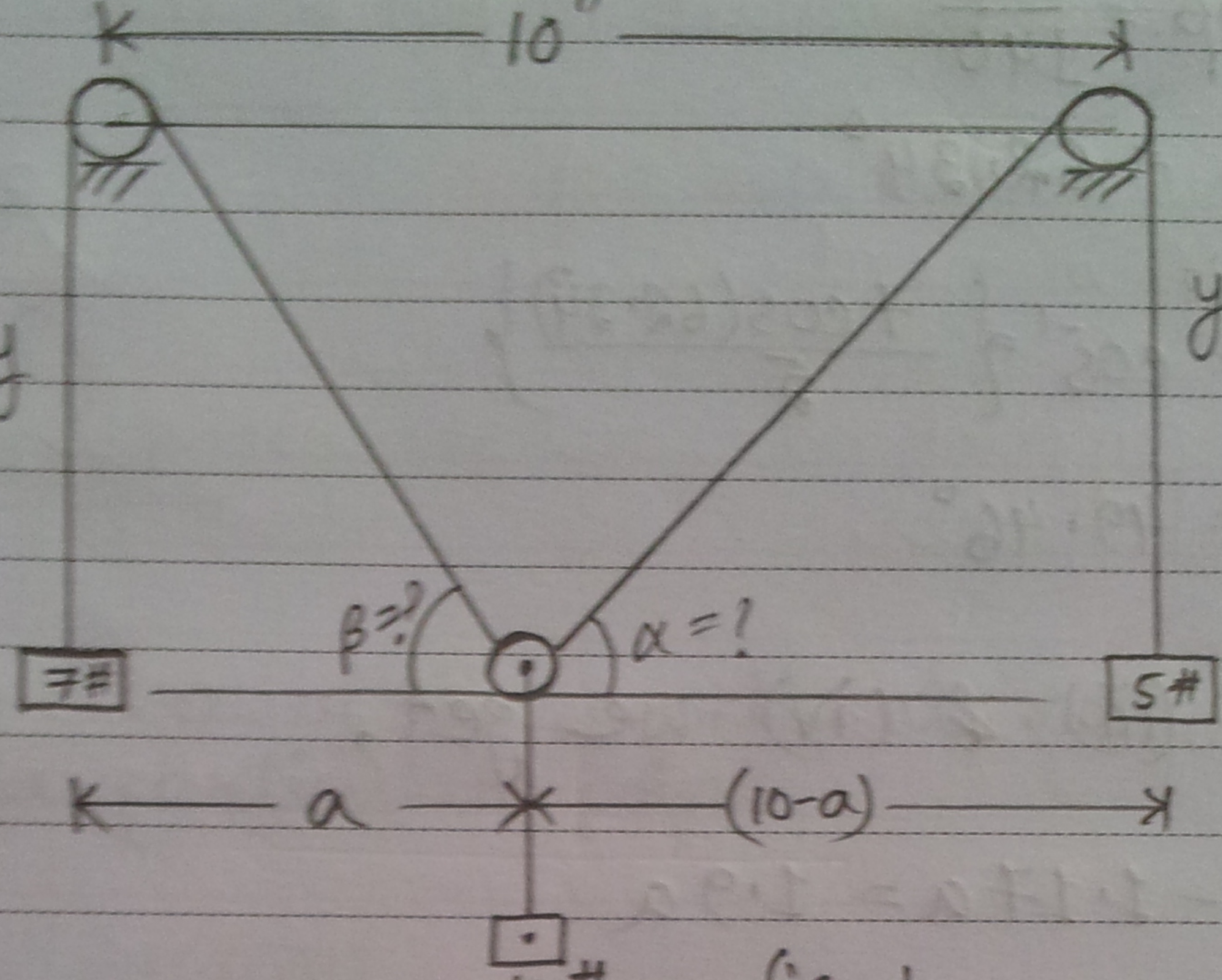


fig-1

Sol^n:

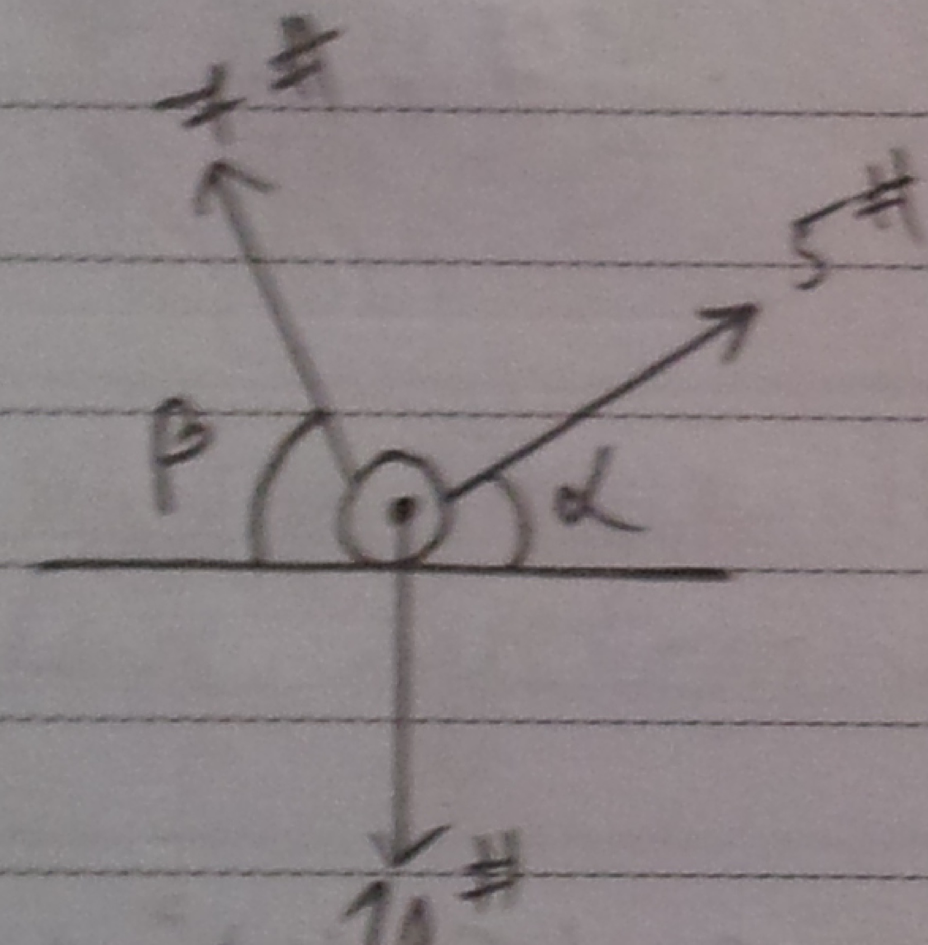


fig-2

From fig-1,

$\tan\alpha = \frac{y}{10-a}$

$\Rightarrow \tan(49.46^\circ) = \frac{y}{10-a}$

$\Rightarrow 1.17 = \frac{y}{10-a}$

$\Rightarrow 11.7 - 1.17a = y$ ...(iii)

Again, $\tan\beta = \frac{y}{a}$

$\Rightarrow y = a\tan(62.34^\circ)$

$\Rightarrow y = 1.9a$ ...(iv)

From fig-2,

$\Sigma F_x = 5\cos\alpha - 7\cos\beta = 0$ ---(i)

$\Rightarrow 5\cos\alpha = 7\cos\beta$

$\& \Sigma F_y = 5\sin\alpha + 7\sin\beta - 10 = 0$ --(ii)

$\Rightarrow 5\sin\alpha = 10 - 7\sin\beta$

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$(i)^2 + (ii)^2 \rightarrow 25 = 49 - 140\sin\beta + 100$

$\Rightarrow \sin\beta = \frac{124}{140}$

$\Rightarrow \beta = 62.34^\circ$

$(i) \rightarrow \alpha = \cos^{-1}\left\{\frac{7\cos(62.34)}{5}\right\}$

$\Rightarrow \alpha = 49.46^\circ$

From eq. (iii) & (iv) we get,

$11.7 - 1.17a = 1.9a$

$\Rightarrow 3.07a = 11.7$

$\Rightarrow a = 3.81''$

Ans: $a = 3.81''$; $\alpha = 49.46^\circ$;

$\beta = 62.34^\circ$.