Example 174 / P. 247

We know, $a = \frac{dv}{dt}$

$$\Rightarrow dv = a\,dt$$

$$\Rightarrow \int dv = \int a\,dt$$

$$= \int 4t\,dt$$

$$\therefore v = 4 \cdot \frac{t^2}{2} + c$$

$$= 2t^2 + c \quad \text{———— (i)}$$

Given, At $t=0$, $v = 10$ fps

Substituting in eq^n (i)

$$\cancel{\Rightarrow} \quad 10 = 0 + c$$

$$\Rightarrow c = 10$$

Therefore, the eq^n for velocity is

$$v = 2t^2 + 10 \quad \text{———— (ii)}$$

at $t = 4$ sec., $v = 2 \cdot 4^2 + 10 = \boxed{42 \text{ fps}}$ Ans.

Again $v = \frac{ds}{dt} \Rightarrow ds = v\,dt$

$$\int ds = \int v\,dt$$

$$= \int (2t^2 + 10)\,dt$$

$$\therefore S = 2 \cdot \frac{t^3}{3} + 10t + c_1$$

at $t=0$, $S=0 \Rightarrow c_1 = 0$

$$\therefore S = \frac{2}{3}t^3 + 10t$$

Now, at $t = 4$ sec.

$$S = \frac{2}{3} \times 4^3 + 10 \times 4 = \boxed{82.7 \text{ ft}}$$ Ans.

Note: Definite Integral has been used in the book; an alternative way

Ex. 186/P. 258

(a)

$\omega_0 = 0$, counterclock rotation

$\theta = 0.1t^3 - 0.3t^2 + 0.8t$

$t = 6\,s.$

(a) $\theta_6 = ?$ (b) $\omega_6 = ?$ (c) $\alpha_6 = ?$

Sol^n

(a) $\theta_6 = 0.1 \times 6^3 - 0.3 \times 6^2 + 0.8 \times 6 = \boxed{15.6\ s.}$

(b) $\omega = \dfrac{d\theta}{dt} = \dfrac{d}{dt}\left(0.1t^3 - 0.3t^2 + 0.8t\right) = 0.3t^2 - 0.6t + 0.8$

$\therefore \omega_6 = 0.3 \times 6^2 - 0.6 \times 6 + 0.8 = \boxed{8\ rad/s.}$

(c) $\alpha = \dfrac{d\omega}{dt} = \dfrac{d}{dt}\left(0.3t^2 - 0.6t + 0.8\right) = 0.6t - 0.6$

$\therefore \alpha_6 = 0.6 \times 6 - 0.6 = \boxed{3\ rad/s^2}$

Ex. 188/PP. 259-261

Given, $\omega_0 = 300 \text{ rpm} = \frac{300 \times 2\pi}{60} \text{ rad/s.} = 31{\cdot}4 \text{ rad/s.}$

$\alpha = -2 \text{ rad/s}^2$

(a) $t = ?$ for $\omega = 0$

(b) $\omega = ?$ for $t = 10$ s.

(c) No. of revolutions in first 10 s. ie $\theta = ?$

(d) $\theta = ?$ to stop ie $\omega = 0$

(e) No. of revolutions from $t = 10$ s. to $\omega = 0$

Sol^n

(a) $\omega = \omega_0 + \alpha t$

$\Rightarrow 0 = 31{\cdot}4 - 2 \times t$

$\therefore t = \boxed{15{\cdot}7 \text{ s.}}$

(b) $\omega = \omega_0 + \alpha t$

$= 31{\cdot}4 - 2 \times 10$

$= 11{\cdot}4$ rad/s.

$= \frac{11{\cdot}4 \times 60}{2\pi}$ rpm

$\simeq \boxed{109 \text{ rpm}}$

(c) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= 31{\cdot}4 \times 10 + \frac{1}{2} \times (-2) \times 10^2$

$= 214$ rad

$= \frac{214}{2\pi}$ rev.

$= \boxed{34{\cdot}06 \text{ rev.}}$

(d) $\omega^2 = \omega_0^2 + 2\alpha\theta$

$\Rightarrow 0 = (31{\cdot}4)^2 + 2(-2) \cdot \theta$

$\therefore \theta = \boxed{246{\cdot}49 \text{ rad.}}$

(e) from (d) $\rightarrow$ to stop, $\theta = 246{\cdot}49$ rad

from (c) $\rightarrow$ at $t = 10$ s, $\theta = 214$ rad

$\therefore$ From $t = 10$ s. to stop, $\theta = 246{\cdot}49 - 214$

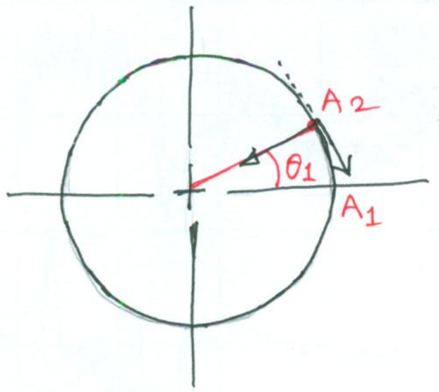
$= 32{\cdot}49$ rad

$= \frac{32{\cdot}49}{2\pi}$ rev.

$= \boxed{5{\cdot}2 \text{ rev.}}$

~~#325~~'

Ex. 191/ PP. 203–204



Total accl^n of a point, that was initially at A_1, after 10 s. = ?

$A_2 \leftarrow$ position of the point after 10 s.

From ex. 188(c), for $t = 10$ s.

$$\theta = 214 \text{ rad} = 34.06 \text{ rev.}$$

$$\therefore \theta_1 = 0.06 \times 360^\circ = 21.6^\circ$$

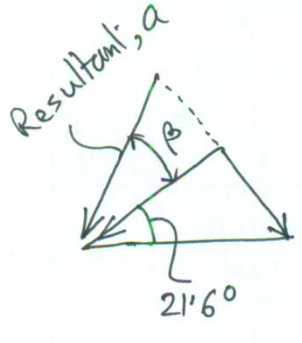
$$r = \frac{30''}{2} = 1.25 \text{ ft}$$

$$\therefore a_t = r\alpha = 1.25 \times 2 = 2.5 \text{ fps}^2 \text{ (clockwise}$$

$$a_n = \frac{v^2}{r} = r\omega^2 = v\omega$$

~~from~~ ex. 118 (b), $\omega = 11.4$ rad/s.

$$\therefore a_n = r\omega^2 = 1.25 \times 11.4^2 = 162.45 \text{ fps}^2 \text{ (towards center)}$$



Resultant accl^n, $a = \sqrt{a_t^2 + a_n^2}$

$$= \sqrt{2.5^2 + 162.45^2}$$

$$= \boxed{162.5 \text{ fps}^2}$$

$$\beta = \tan^{-1}\frac{a_t}{a_n} = \tan^{-1}\frac{2.5}{162.45} = 0.88^\circ$$

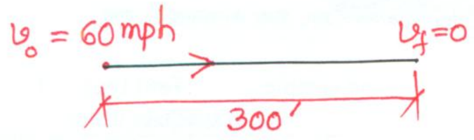
$\therefore$ Direction of total accl^n with horizontal $= \theta_1 + \beta$

$$= 21.6 + 0.88$$

$$= \boxed{22.48^\circ}$$

#896/P. 275

situation 1/ Test



$$60 \text{ mph} = \frac{60 \times 1760 \times 3}{60 \times 60} \text{ fps}$$

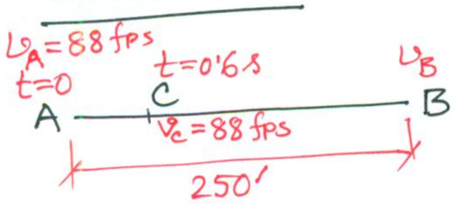
$$= 88 \text{ fps}$$

$$v_f^2 = v_0^2 + 2as$$

$$\Rightarrow \quad 0 = 88^2 + 2 \times a \times 300$$

$$\therefore \; a = -12.91 \text{ fps}^2$$

situation 2



$$AC = v_A t$$

$$= 88 \times 0.6$$

$$= 52.8 \text{ ft}$$

$$CB = AB - AC = 250 - 52.8 = 197.2 \text{ ft}$$

For motion from C to B

$$v_B^2 = v_C^2 + 2a \cdot s_{CB}$$

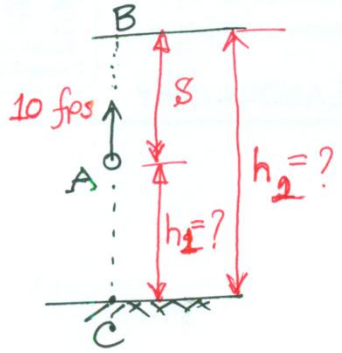
$$= 88^2 + 2 \times (-12.91) \times 197.2$$

$$\therefore \; v_B = \boxed{51.5 \text{ fps}} \text{ Ans}$$

$$= 51.5 \times 60 \text{ fpm} = \boxed{3090 \text{ fpm}} \text{ Ans.}$$

$$= \frac{51.5 \times 60 \times 60}{1760 \times 3} \text{ mph} = \boxed{35.1 \text{ mph}} \text{ Ans.}$$

901/ P. 276



For both baloon & sandbag,

$$v_0 = -10 \text{ fps (downward +ve)}$$

For the upward motion of the sandbag upto max^m altitude

$$v_f = v_0 + g t_1$$

$$\Rightarrow 0 = -10 + 32.2 \times t_1$$

$$\therefore t_1 = \frac{10}{32.2} = 0.31 \text{ s.}$$

$$S = v_0 t_1 + \frac{1}{2} g t_1^2$$

$$= -10 \times 0.31 + \frac{1}{2} \times 32.2 \times 0.31^2$$

$$= -1.55 \text{ ft}$$ (−ve sign implies upward displacement)

For the sandbag to drop from the max^m altitude to ground

$$t_2 = 4 - 0.31 = 3.69 \text{ s.}$$

$$h_2 = \frac{1}{2} g t_2^2 = \frac{1}{2} \times 32.2 \times 3.69^2 = \boxed{219.22 \text{ ft}}$$ Ans.

$$h_1 = h_2 - S = 219.22 - 1.55 = \boxed{217.67 \text{ ft.}}$$ Ans.

Alternatively

Considering motion A→C }

$$h_1 = v_0 t + \frac{1}{2} g t^2 = -10 \times 4 + \frac{1}{2} \times 32.2 \times 4^2 = \boxed{217.67 \text{ ft}}$$ Ans.

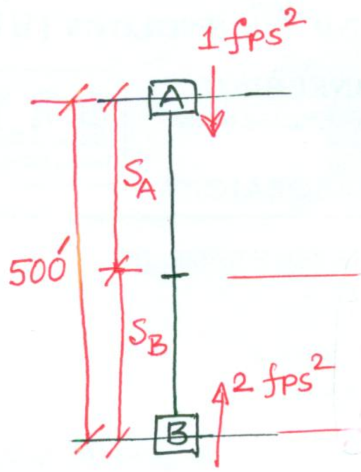
A → B $$v_f^2 = v_0^2 + 2gS$$

$$\Rightarrow 0 = (-10)^2 + 2 \times 32.2 \times S$$

$$\therefore S = \boxed{-1.55 \text{ ft}}$$

$$\therefore h_2 = h_1 + S = 217.67 + 1.55 = \boxed{219.22 \text{ ft.}}$$ Ans

903/P. 276



Given :

A & B start simultaneously

$v_A = 0$, $v_B = 0$

$a_A = 1 \text{ fps}^2 \downarrow$

$a_B = 2 \text{ fps}^2 \uparrow$

$S_A = ?$ $S_B = ?$

Sol^n

Let the two elevators be opposite to each other after t sec. ~~and by this time elev. B travels a distance x.~~

Let's take downward direct^n as +ve

For motion of elev. A, $S_A = \frac{1}{2} a_A t^2$

$= \frac{1}{2} \times 1 \times t^2$

$= 0.5 t^2$

For motion of elev. B, $-S_B = \frac{1}{2} a_B t^2$

$\Rightarrow -S_B = \frac{1}{2} \times (-2) t^2$

$\therefore S_B = t^2$

Now $S_A + S_B = 500$

$\Rightarrow 0.5t^2 + t^2 = 500$

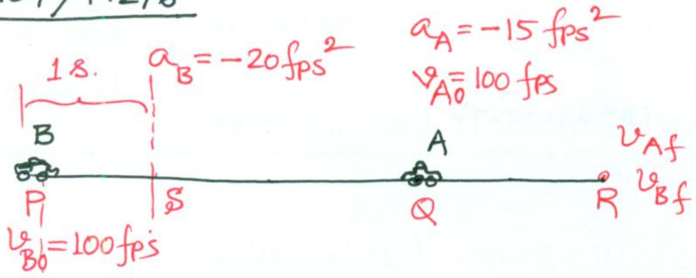
$\Rightarrow 1.5 t^2 = 500$

$\therefore t = 18.26 \text{ s}$

$\therefore S_A = 0.5 \times 18.26^2 = \boxed{166.7 \text{ ft}}$ Ans.

& $S_B = 18.26^2 = \boxed{333.4 \text{ ft.}}$ Ans.

907/P.276



For bumper to bumper touch

$S_{PQ} = ?$

$t = ?$

$v_{Af} = ?$

$v_{Bf} = ?$

Let the points P & Q in the above figure be the initial positions of the automobiles A & B respectively and let ~~R~~ S be the point where B's brake is applied.

For vehicle A (motion from Q to R)

$$v_{Af} = v_{A0} + a_A t = 100 + (-15)t = 100 - 15t \qquad (1)$$

For vehicle B (motion from S to R)

$$v_{Bf} = v_{B0} + a_B (t-1) = 100 + (-20)(t-1) = 120 - 20t \qquad (2)$$

For bumper-to-bumper condition to occur

$$v_{Af} = v_{Bf}$$

$$\Rightarrow 100 - 15t = 120 - 20t$$

$$\Rightarrow 5t = 20$$

$$\therefore t = \boxed{4 \text{ sec.}} \text{ Ans.}$$

$\therefore$ From eqn (1), $\boxed{v_{Af} = 100 - 15 \times 4 = 40 \text{ fps} = v_{Bf}}$ Ans.

Again for vehicle A

$$S_{QR} = v_{A0} t + \frac{1}{2} a_A t^2 = 100 \times 4 + \frac{1}{2} \times (-15) \times 4^2 = 280 \text{ ft}$$

and for vehicle B

$$S_{SR} = v_{B0}(t-1) + \frac{1}{2} a_B (t-1)^2 = 100 \times (4-1) + \frac{1}{2} \times (-20) \times (4-1)^2$$

$$= 210 \text{ ft}$$

$\therefore$ Required distance, $S_{PQ} = S_{PS} + (S_{SR} - S_{QR})$

$$= 100 \times 1 + (210 - 280)$$

$$= \boxed{30 \text{ ft}} \text{ Ans.}$$

#916/P. 276

$$a = 5t^{1/3}$$

at $t=0$, $v_0 = 6$ fps and $s_0 = 0$

$s = ?$ betw^n $t = 4$ s. and $t = 10$ s.

Sol^n

$$a = \frac{dv}{dt}$$

$$\Rightarrow dv = a\,dt$$

$$\Rightarrow \int_{v_0}^{v_f} dv = \int_0^t 5t^{1/3}\,dt$$

$$\Rightarrow v_f - v_0 = 5 \times \frac{3}{4} t^{4/3}$$

$$\therefore \quad v = \frac{15}{4} t^{4/3} + 6$$

Again $\quad v = \frac{ds}{dt}$

$$\Rightarrow ds = v\,dt$$

$$\Rightarrow \int_0^s ds = \int_4^{10} \left(\frac{15}{4} t^{4/3} + 6\right) dt$$

$$\therefore S = \frac{15}{4} \times \frac{3}{7} \left[t^{7/3}\right]_4^{10} + 6\left[t\right]_4^{10}$$

$$= 305.42 + 36$$

$$= \boxed{341.42 \text{ ft}}$$

Ans.

921/P.277

$s = (t^2 - 2t + 9)^{1/2}$ ft ; t in sec.

$v_3 = ?$ $a_3 = ?$ at $t = 3$ s.

Avg. accl^n in 3rd sec. = ?

$$v = \frac{ds}{dt} = \frac{d}{dt}(t^2-2t+9)^{1/2} = \frac{1}{2}(t^2-2t+9).(2t-2) = \frac{t-1}{(t^2-2t+9)^{1/2}}$$

$$a = \frac{dv}{dt}$$

$$= \frac{d}{dt}\left[\frac{t-1}{(t^2-2t+9)^{1/2}}\right]$$

$$= \frac{(t^2-2t+9)^{1/2}\cdot 1 - (t-1)\cdot\frac{1}{2}(t^2-2t+9)^{-1/2}\cdot(2t-2)}{t^2-2t+9}$$

$$= \frac{(t^2-2t+9)^{1/2} - (t-1)^2(t^2-2t+9)^{-1/2}}{t^2-2t+9}$$

$$= (t^2-2t+9)^{-1/2} - (t-1)^2(t^2-2t+9)^{-3/2}$$

at $t = 3$ sec.

$$v = \frac{3-1}{(3^2-2\times3+9)^{1/2}} = \boxed{0{\cdot}577 \text{ fps}}$$ Ans.

$$a = (3^2-2\times3+9)^{-1/2} - (3-1)^2(3^2-2\times3+9)^{-3/2} = \boxed{0{\cdot}192 \text{ fps}^2}$$ Ans.

at $t = 2$ sec.

$$a = (2^2-2\times2+9)^{-1/2} - (2-1)^2(2^2-2\times2+9)^{-3/2} = 0{\cdot}296 \text{ fps}^2$$

$\therefore$ Arithmatic avg. accl^n during the 3rd second

$$= \frac{0{\cdot}296 + 0{\cdot}192}{2}$$

$$= \boxed{0{\cdot}244 \text{ fps}^2}$$ Ans.

922 / P. 277

$a = 3t - 12$ fps^2

at $t = 0$, $v_0 = 15$ fps in the same sense as initial acclⁿ

$v = ?$ and $s = ?$ at $t = 3$ s.

Solⁿ

at $t=0$, $a = -12$ fps^2

$\therefore v_0 = -15$ fps

Now $a = \dfrac{dv}{dt}$

$\Rightarrow dv = a\,dt$

$\Rightarrow \displaystyle\int_{-15}^{v} dv = \int_0^t (3t - 12)\,dt$

$\Rightarrow v + 15 = \frac{3}{2}t^2 - 12t$

$\therefore v = \frac{3}{2}t^2 - 12t - 15$

at $t = 3$ s, $v = \frac{3}{2}\times 3^2 - 12\times 3 - 15 = \boxed{-37.5 \text{ fps}}$

–ve sign implies that directⁿ of velocity is same as that of initial acclⁿ.

Ans.

Again $v = \dfrac{ds}{dt}$

$\Rightarrow ds = v\,dt = \left(\frac{3}{2}t^2 - 12t - 15\right)dt$

$\Rightarrow \displaystyle\int_0^s ds = \int_0^t \left(\frac{3}{2}t^2 - 12t - 15\right)dt$

$\therefore S = \frac{1}{2}t^3 - 6t^2 - 15t$

at $t = 3$ s., $S = \frac{1}{2}\times 3^3 - 6\times 3^2 - 15\times 3 = \boxed{-85.5 \text{ ft}}$

–ve sign means opposite to the +ve co-ordinate directⁿ

Ans.

939/P.278

Here, $\omega_0 = 1200 \text{ rpm} = \frac{1200}{60} \times 2\pi = 40\pi \text{ rad/s}$

$\omega_f = 0$, $\alpha = 100 \text{ rpm}^2$

(a) no. of revolution = ?

$t = ?$

(b) $\omega_f = 300$ rpm , $\omega_0 = 1200$ rpm

$\theta = 7500$ rev.

$\alpha = ?$ in rad/s^2

Sol^n

(a) $\omega_f = \omega_0 - \alpha t$

$\Rightarrow 0 = 1200 - 100t$

$\therefore t = \frac{1200}{100} = 12 \text{ min}$

$$\theta = \omega_0 t - \frac{1}{2}\alpha t^2$$

$$= 1200 \times 12 - \frac{1}{2} \times 100 \times 12^2$$

$$= 7200 \text{ rev.}$$

(b) $\omega_f^2 = \omega_0^2 - 2\alpha\theta$

$\Rightarrow 300^2 = 1200^2 - 2\alpha \times 7500$

$$\therefore \alpha = \frac{1}{2 \times 7500}\left(1200^2 - 300^2\right)$$

$$= 90 \text{ rpm}^2$$

$$= \frac{90 \times 2\pi}{60 \times 60} \text{ rad/s}^2$$

$$= 0.157 \text{ rad/s}^2$$

943/P.279

$\theta = t - 0{\cdot}1t^2$ rad , t in sec.

(a) $\theta = ?$, $\omega = ?$, $\alpha = ?$ after $t = 3$ sec.

(b) Same as (a) after 10 sec

(c) Max^m displacement during the first 10 sec.

Sol^n

(a) $\theta = t - 0{\cdot}1t^2$

$\therefore$ after 3 sec. $\theta = 3 - 0{\cdot}1 \times 3^2 = \boxed{2{\cdot}1 \text{ rad}}$ Ans.

$$\omega = \frac{d\theta}{dt} = 1 - 0{\cdot}2t$$

$\therefore \omega_{3\,sec.} = 1 - 0{\cdot}2 \times 3 = \boxed{0{\cdot}4 \text{ rad/s}}$ Ans.

$\alpha = \dfrac{d\omega}{dt} = \boxed{-0{\cdot}2 \text{ rad/s}^2}$ Ans.

(b) after 10 sec., $\theta = 10 - 0{\cdot}1 \times 10^2 = \boxed{0 \text{ rad}}$ Ans.

$\omega = 1 - 0{\cdot}2 \times 10 = \boxed{-1 \text{ rad/s}}$ Ans.

$\alpha = \boxed{-0{\cdot}2 \text{ rad/s}^2}$ Ans.

(c) for maximum displacement

$$\frac{d\theta}{dt} = 0$$

$$\Rightarrow 1 - 0{\cdot}2t = 0$$

$$\therefore t = \frac{1}{0{\cdot}2} = 5 \text{ s}$$

$\therefore \theta_{max} = 5 - 0{\cdot}1 \times 5^2 = \boxed{2{\cdot}5 \text{ rad}}$ Ans.

948/P.279

$\alpha = 2\theta$

at $t=0$, $\omega = \omega_0$ and $\theta = 0$

Find the general expression for ω.

Sol^n

We know $\quad \alpha \, d\theta = \omega \, d\omega$

$$\Rightarrow \quad 2\theta \, d\theta = \omega \, d\omega$$

$$\Rightarrow \quad 2\int_{\theta_0=0}^{\theta} \theta \, d\theta = \int_{\omega_0}^{\omega} \omega \, d\omega$$

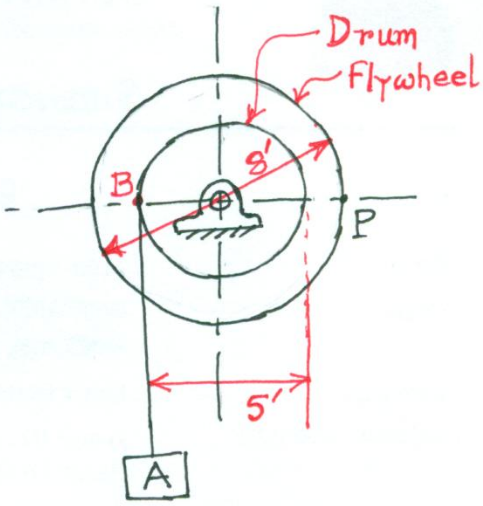
$$\Rightarrow \quad \left[\theta^2\right]_0^{\theta} = \frac{1}{2}\left[\omega^2\right]_{\omega_0}^{\omega}$$

$$\Rightarrow \quad \theta^2 = \frac{1}{2}\left(\omega^2 - \omega_0^2\right)$$

$$\Rightarrow \quad \omega^2 = \omega_0^2 + 2\theta^2$$

$$\therefore \quad \omega = \pm\left(\omega_0^2 + 2\theta^2\right)^{1/2}$$

#958/P.280



Here, $v_A = \text{Const.} = 10 \text{ fps} \downarrow$

(a) $\omega_p = ?$ and $v_p = ?$ at $t = 3$ s.

(b) $a_{np} = ?$ and $a_{tp} = ?$

Solution

(a) $v_B = v_A = 10 \text{ fps}$

$$v_B = r_B \omega$$

$$v_p = r_p \omega$$

$$\therefore \frac{v_p}{v_B} = \frac{r_p}{r_B} \Rightarrow v_p = \frac{r_p}{r_B} \cdot v_B = \frac{4}{2.5} \times 10 = \boxed{16 \text{ fps}}$$

$$\omega = \omega_p = \frac{v_p}{r_p} = \frac{16}{4} = \boxed{4 \text{ rad/s}}$$

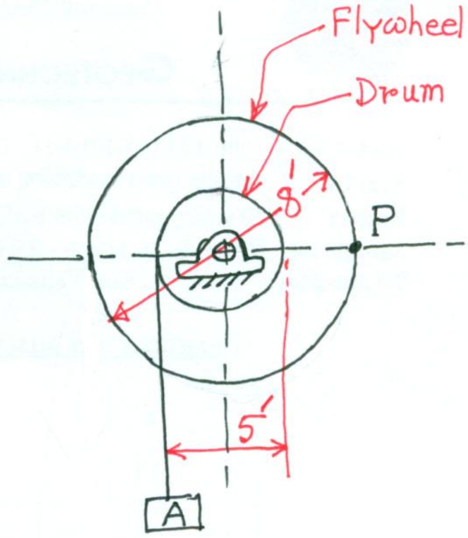
(b) $$a_{np} = r_p \omega_p^2 = 4 \times 4^2 = \boxed{64 \text{ fps}^2}$$

$$a_{tp} = r_p \alpha_p = r_p \cdot \frac{d\omega_p}{dt} = \boxed{0}$$ [Because $\omega_p = \text{Constant}$]

960/P.280

$a_A = \frac{g}{2} - \frac{t}{12}$

$v_0 = 10$ fps



(a) $\omega_{P3} = ?$ $\quad v_{P3} = ?$ $\Big\}$ when $t = 3$ sec.

(b) $a_{nP3} = ?$ $\quad a_{tP3} = ?$

Sol^n

$$dv_A = a_A\, dt$$

$$\Rightarrow \int_{v_0}^{v_{At}} dv_A = \int_0^t \left(\frac{g}{2} - \frac{t}{12}\right) dt$$

$$\Rightarrow \left[v_A\right]_{v_0}^{v_{At}} = \left[\frac{gt}{2} - \frac{t^2}{24}\right]_0^t$$

$$\Rightarrow v_{At} - v_0 = \frac{gt}{2} - \frac{t^2}{24}$$

$$\therefore v_{At} = v_0 + \frac{gt}{2} - \frac{t^2}{24} = 10 + \frac{gt}{2} - \frac{t^2}{24} \quad \text{——— (1)}$$

at $t = 3$ s., $v_{A3} = 10 + \frac{32.2 \times 3}{2} - \frac{3^2}{24} = 57.93$ fps.

$$\omega_{P3} = \frac{v_{A3}}{r_A} = \frac{57.93}{2.5} = \boxed{23.17 \text{ rad/s}} \text{ Ans.}$$

$$v_{P3} = r_P\, \omega_{P3} = 4 \times 23.17 = \boxed{92.68 \text{ fps}} \text{ Ans.}$$

$$a_{nP3} = r_P\, \omega_{P3}^2 = 4 \times \cancel{92.68}\; 23.17^2 = \boxed{2147.40 \text{ fps}^2} \text{ Ans.}$$

$$a_{tP3} = r_P\, \alpha_{P3}$$

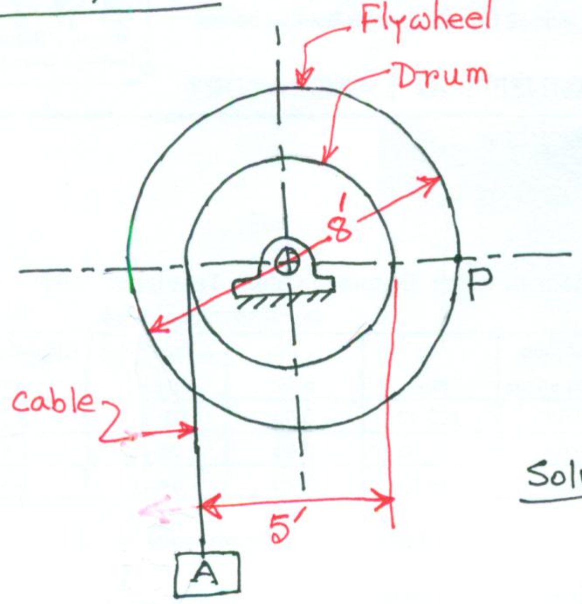
$$\alpha_{Pt} = \frac{d\omega_{Pt}}{dt} = \frac{d}{dt}\left[\omega_{At}\right] = \frac{d}{dt}\left[\frac{v_{At}}{r_A}\right] = \frac{1}{r_A}\frac{d}{dt}\left(10 + \frac{gt}{2} - \frac{t^2}{24}\right)$$

$$= \frac{1}{r_A} \times \left(\frac{g}{2} - \frac{t}{12}\right)$$

$$\therefore \alpha_{P3} = \frac{1}{2.5}\left(\frac{32.2}{2} - \frac{3}{12}\right) = 6.34 \text{ rad/s}^2$$

$$\therefore a_{tP3} = r_P \cdot \alpha_{P3} = 4 \times 6.34 = \boxed{25.36 \text{ fps}^2} \text{ Ans.}$$

#960/P. 280



$a_A = \frac{g}{2} - \frac{t}{12}$

$v_o = 10$ fps

Determine:

(a) $\omega_{P3} = ?$ $v_{P3} = ?$
(b) $a_{nP3} = ?$ $a_{tP3} = ?$
when $t = 3$ s.

Solution

$$dv_A = a_A\,dt$$

$$\Rightarrow \int_{v_o}^{v_{At}} dv_A = \int_0^t \left(\frac{g}{2} - \frac{t}{12}\right) dt$$

$$\Rightarrow \Big[v_A\Big]_{v_o}^{v_{At}} = \left[\frac{gt}{2} - \frac{t^2}{24}\right]_0^t$$

$$\Rightarrow v_{At} - v_o = \frac{gt}{2} - \frac{t^2}{24}$$

$$\therefore v_{At} = v_o + \frac{gt}{2} - \frac{t^2}{24}$$

$$= 10 + \frac{gt}{2} - \frac{t^2}{24} \qquad (1)$$

$$\omega_{Pt} = \omega_{At} = \frac{v_{At}}{r_A} = \frac{1}{2.5}\left(10 + \frac{gt}{2} - \frac{t^2}{24}\right) \qquad (2)$$

$$\therefore \omega_{P3} = \frac{1}{2.5}\left(10 + \frac{32.2 \times 3}{2} - \frac{3^2}{24}\right) = \boxed{23.17 \text{ rad/s}} \quad \text{Ans.}$$

$$v_{Pt} = r_{Pt}\,\omega_{Pt} = 4 \times \frac{1}{2.5}\left(10 - \frac{gt}{2} - \frac{t^2}{24}\right) \qquad (3)$$

$$\therefore v_{P3} = \frac{4}{2.5}\left(10 + \frac{32.2 \times 3}{2} - \frac{3^2}{24}\right) = \boxed{92.68 \text{ fps}} \quad \text{Ans.}$$

$$a_{nPt} = r_P\,\omega_{Pt}^2 = 4 \times \frac{1}{2.5^2}\left(10 + \frac{gt}{2} - \frac{t^2}{24}\right)^2 \qquad (4)$$

$$\therefore a_{nP3} = \frac{4}{2.5^2}\left(10 + \frac{32.2 \times 3}{2} - \frac{3^2}{24}\right)^2 = \boxed{2147.39 \text{ fps}^2} \quad \text{Ans}$$

$$a_{tPt} = r_P\,\alpha_{Pt} = r_P \cdot \frac{d\omega_{Pt}}{dt} = 4 \times \frac{d}{dt}\left[\frac{1}{2.5}\left(10 + \frac{gt}{2} - \frac{t^2}{24}\right)\right]$$

$$= \frac{4}{2.5}\left(\frac{g}{2} - \frac{t}{12}\right) \qquad (5)$$

$$\therefore a_{tP3} = \frac{4}{2.5}\left(\frac{32.2}{2} - \frac{3}{12}\right) = \boxed{25.36 \text{ fps}^2} \quad \text{Ans.}$$

962/P.280

Here, $\alpha = 0{\cdot}1t$

$\omega_0 = 5$ rad/s

$t = 10$ s

$r = 6$ inch

$a_t = ?$ $a_n = ?$

Sol^n

at $t = 10$ s, $\alpha = 0{\cdot}1 \times 10 = 1$ rad/s^2

$$\therefore a_t = r\alpha = 6 \times 1 = \boxed{6 \text{ in/s}^2}$$ Ans.

$$\alpha = \frac{d\omega}{dt}$$

$$\Rightarrow d\omega = \alpha\, dt$$

$$\Rightarrow \int_5^{\omega_f} d\omega = \int_0^{10} 0{\cdot}1 t\, dt$$

$$\Rightarrow \left[\omega\right]_5^{\omega_f} = \frac{0{\cdot}1}{2}\left[t^2\right]_0^{10}$$

$$\Rightarrow \omega_f - 5 = 0{\cdot}05\left(10^2 - 0\right)$$

$$\therefore \omega_f = 10 \text{ rad/s}$$

$$a_n = r\omega_f^2 = 6 \times 5^2 = \boxed{150 \text{ in/s}^2}$$ Ans.

963/P.280

Here, $\alpha = -4\theta^{1/2}$ $\qquad a_{nf} = ?$

$\omega_0 = 50$ rad/s $\qquad a_{tf} = ?$

$\theta_f = 7$ rev $= 7 \times 2\pi = 14\pi$ rad $~~14\pi rad~~$

$r = 18$ in.

Sol^n

We know, $\alpha \, d\theta = \omega \, d\omega$

$$\Rightarrow -4\theta^{1/2} d\theta = \omega \, d\omega$$

$$\Rightarrow -4\int_0^{14\pi} \theta^{1/2} d\theta = \int_{50}^{\omega_f} \omega \, d\omega$$

$$\Rightarrow -4\left[\theta^{3/2} \cdot \frac{2}{3}\right]_0^{14\pi} = \left[\frac{\omega^2}{2}\right]_{50}^{\omega_f}$$

$$\Rightarrow -\frac{8}{3}(14\pi)^{3/2} = \frac{1}{2}\left(\omega_f^2 - 50^2\right)$$

$$\Rightarrow \omega_f^2 = -\frac{16}{3}(14\pi)^{3/2} + 50^2$$

$$\therefore \omega_f = 30{\cdot}75 \text{ rad/s.}$$

$$\alpha_f = -4\theta_f^{1/2} = -4 \times (14\pi)^{1/2} = -26{\cdot}52 \text{ rad/s}^2$$

$$a_{tf} = r\alpha_f = 18 \times (-26{\cdot}52) \text{ in/s}^2 = \boxed{-39{\cdot}78 \text{ fps}^2}$$

–ve means velocity is reducing

Ans.

$$a_{nf} = r\omega_f^2 = 18 \times 30{\cdot}75^2 \text{ in/s}^2 = \boxed{1418{\cdot}34 \text{ fps}^2}$$

Ans.

965/P. 281

Here, $a_t = (s+6)^{1/2}$, circular path, $r = 600$ ft

at $t=0$, $v_0 = 0$

$t = t_f$, $s = 100$ ft

Find ⇒ a_{nf} and a_{tf}

Sol^n

$$a_{tf} = (100+6)^{1/2} = \boxed{10{\cdot}29 \text{ fps}^2}$$

For rectilinear motion, $a\,ds = v\,dv$

Here, $a_t\,ds = v\,dv$

$$\Rightarrow (s+6)^{1/2}\,ds = v\,dv$$

$$\Rightarrow \int_0^{100} (s+6)^{1/2}\,ds = \int_0^{v_f} v\,dv$$

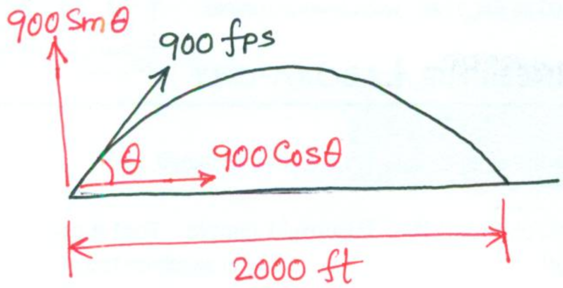
$$\Rightarrow \frac{2}{3}\left[(s+6)^{3/2}\right]_0^{100} = \frac{1}{2}\left[v^2\right]_0^{v_f}$$

$$\Rightarrow v_f^2 = \frac{4}{3}(100+6)^{3/2}$$

$$\therefore v_f = 38{\cdot}15 \text{ fps}$$

$$\therefore a_{nf} = \frac{v_f^2}{r} = \frac{38{\cdot}15^2}{600} = \boxed{2{\cdot}43 \text{ fps}^2}$$

977/P. 281



$\theta = ?$

Considering horizontal motion

$$2000 = (900 \cos\theta) \cdot t \quad \text{—— (1)}$$

Considering vertical motion (downward +ve)

$$0 = -(900 \sin\theta) \cdot t + \frac{1}{2} g t^2 \quad \text{—— (2)}$$

From eq^n (1) $\quad 900 \cos\theta = \frac{2000}{t} \quad$ —— (3)

from eq^n (2) $\quad 900 \sin\theta = 16{\cdot}1\, t \quad$ —— (4)

Now taking square of both sides of eq^n (3) & (4) and adding —

$$900^2 = \frac{2000^2}{t^2} + 16{\cdot}1^2 t^2$$

$$\Rightarrow 259{\cdot}21\, t^4 + 2000^2 - 900^2 t^2 = 0$$

$$\Rightarrow t^4 - 3124{\cdot}88\, t^2 + 15431{\cdot}5 = 0$$

$$\Rightarrow t^2 = \frac{3124{\cdot}88 \pm \sqrt{3124{\cdot}88^2 - 4 \times 15431{\cdot}5}}{2}$$

$$= \frac{3124{\cdot}88 \pm 3114{\cdot}99}{2}$$

$$= 3119{\cdot}94 \text{ s}, \; 4{\cdot}95 \text{ s}$$

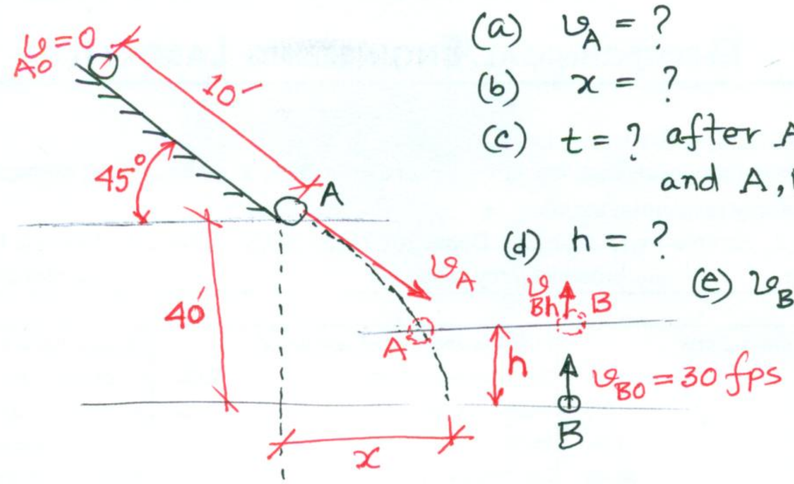
$$\therefore t = 55{\cdot}86 \text{ s}, \; 2{\cdot}22 \text{ s}$$

From eq^n (4) $\quad \theta = \sin^{-1}\left(\frac{16{\cdot}1\, t}{900}\right)$

when $t = 55{\cdot}86$ s, $\quad \theta = \boxed{87{\cdot}82^0}$ Ans

$t = 2{\cdot}22$ s, $\quad \theta = \boxed{2{\cdot}28^0}$ Ans.

981/P. 281



(a) $v_A = ?$

(b) $x = ?$

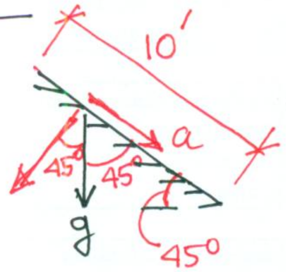
(c) $t = ?$ after A leaves the plane and A, B are at same level

(d) $h = ?$

(e) $v_{Bh} = ?$

Sol^n

(a)



Accl^n along the plane, $a = g \cos 45^\circ$

$= 32.2 \times \cos 45^\circ$

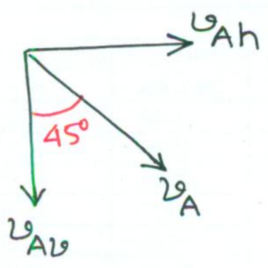
$= 22.77 \text{ fps}^2$

Now $v_A^2 = 2as$

$= 2 \times 22.77 \times 10$

$\boxed{\therefore v_A = 21.34 \text{ fps}}$ Ans.

(b) Let t_1 be the time taken by body A to drop from end of the plane to the ground.



$v_{Ah} = v_{Av} = v_A \cos 45^\circ = 21.34 \cos 45^\circ$

$= 15.09 \text{ fps}$

For vertical component of the motion

$40 = v_{Av} \cdot t_1 + \frac{1}{2} g t_1^2$

$\Rightarrow 40 = 15.09\, t_1 + \frac{1}{2} \times 32.2 \times t_1^2$

$\Rightarrow 16.1\, t_1^2 + 15.09\, t_1 - 40 = 0$

$\therefore t_1 = \dfrac{-15.09 \pm \sqrt{15.09^2 + 4 \times 16.1 \times 40}}{2 \times 16.1}$

$= \dfrac{-15.09 \pm 52.95}{32.2}$

$= 1.176$ sec. (taking +ve value only)

$\therefore x = v_{Ah} \cdot t_1 = 15.09 \times 1.176 = \boxed{17.75 \text{ ft}}$ Ans.

(c) $t \rightarrow$ time elapsed after A leaves the plane when both A & B are at the same elevation.

for body A $\rightarrow$ $40 - h = 15{\cdot}09\,t + \frac{1}{2} g t^2$ —— ①

for body B $\rightarrow$ $-h = -30t + \frac{1}{2} g t^2$ —— ②

Taking downward +ve

① − ② gives, $40 = 45{\cdot}09\,t$

$\boxed{\therefore t = 0{\cdot}89 \text{ sec.}}$ Ans.

(d) Substituting $t = 0{\cdot}89$ s. in eq^n ②

$h = 30 \times 0{\cdot}89 - \frac{1}{2} \times 32{\cdot}2 \times 0{\cdot}89^2$

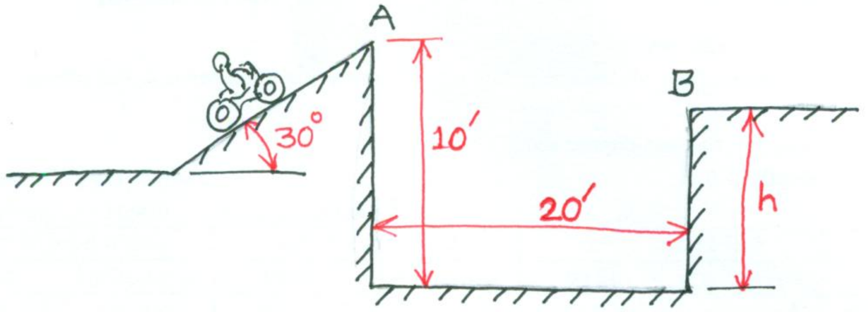
$= \boxed{13{\cdot}95 \text{ ft}}$ Ans.

(e) $v_{Bh} = v_{B0} + gt$

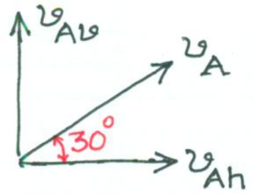
$= -30 + 32{\cdot}2 \times 0{\cdot}89$

$= \boxed{-1{\cdot}34 \text{ fps (−ve means upward)}}$ Ans.

983/P. 282



$$v_A = 75 \text{ mph} = \frac{75 \times 3 \times 1760}{60 \times 60} \text{ fps.} = 110 \text{ fps.}$$



$$v_{Ah} = v_A \cos 30^\circ = 95.26 \text{ fps}$$

$$v_{Av} = v_A \sin 30^\circ = 55 \text{ fps}$$

For horizontal motion, $s = v_{Ah} \cdot t$

$$\Rightarrow 20 = 95.26 \times t$$

$$\therefore t = 0.21 \text{ s.}$$

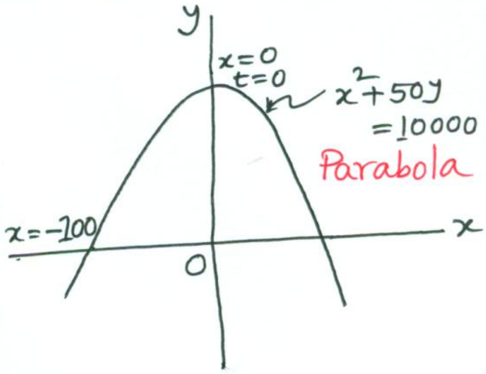
Considering downward direction +ve, for vertical motion

$$10 - h = v_{Av} t + \frac{1}{2} g t^2$$

$$\Rightarrow 10 - h = (-55) \times 0.21 + \frac{1}{2} \times 32.2 \times 0.21^2$$

$$\therefore h = \boxed{20.84 \text{ ft.}}$$ Ans.

987/P. 282



$$x^2 + 50y = 10{,}000$$

Differentiating w.r.t. 't'

$$2x\frac{dx}{dt} + 50\frac{dy}{dt} = 0$$

$$\Rightarrow 2x\,v_x + 50\,v_y = 0 \quad \text{———— (i)}$$

Again differentiating w.r.t. 't'

$$2v_x\frac{dx}{dt} + 50\frac{dv_y}{dt} = 0$$

[Notice that v_x is mentioned as constant]

$$\Rightarrow 2v_x^2 + 50\,a_y = 0$$

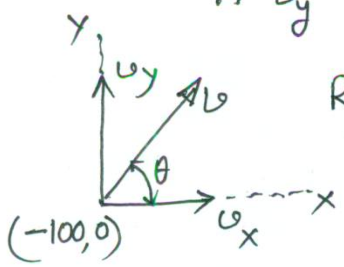
$$\Rightarrow 2v_x^2 + 50\times(-g) = 0$$

$$\therefore v_x = \sqrt{\frac{50g}{2}} = \sqrt{\frac{50\times 32.2}{2}} = 28.37 \text{ fps}$$

Now from eq^n (i), at point $(-100, 0)$

$$2\times(-100)\times 28.37 + 50\,v_y = 0$$

$$\therefore v_y = 113.48 \text{ fps}$$



Resultant velocity at $(-100, 0)$

$$= \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{28.37^2 + 113.48^2}$$

$$= \boxed{116.97 \text{ fps}}$$ Ans

$$\tan\theta = \frac{v_y}{v_x} \Rightarrow \theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{113.48}{28.37} = \boxed{75.96^\circ}$$ Ans.

#988/P.282

Given, tangential velocity = Constant = 12 fps

i.e. $v_x^2 + v_y^2 = 12^2$ ——— ①

Now $y = e^x$

$$\Rightarrow \frac{dy}{dt} = e^x \cdot \frac{dx}{dt}$$

$$\Rightarrow v_y = y \cdot v_x \qquad \text{——— ②}$$

From (1) and (2)

$$v_x^2 + y^2 v_x^2 = 144$$

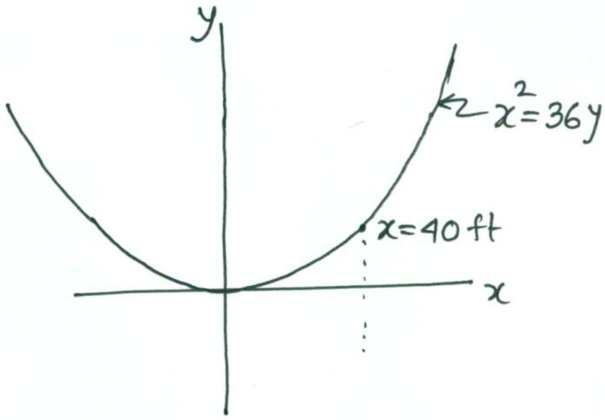
$$\Rightarrow v_x^2 (1 + y^2) = 144$$

$$\therefore v_x = \frac{12}{\sqrt{1+y^2}}$$

at $y = 10$ ft, $v_x = \dfrac{12}{\sqrt{1+10^2}} = \boxed{1{\cdot}194 \text{ fps}}$ Ans.

and from (2), $v_y = 10 \times 1{\cdot}194 = \boxed{11{\cdot}94 \text{ fps}}$ Ans.

989/P.282



$$x^2 = 36y$$

Differentiating w.r.t. 't'

$$2x\frac{dx}{dt} = 36\frac{dy}{dt}$$

$$\Rightarrow 2x v_x = 36 v_y$$

$$\Rightarrow x v_x = 18 v_y$$

at $x = 40$ ft, $40 v_x = 18 v_y$

$$\therefore v_x = 0.45 v_y \quad \text{—— (1)}$$

Given, tangential velocity = 12 fps

i.e. $v_x^2 + v_y^2 = 12^2$ —— (2)

Substituting (1) into (2)

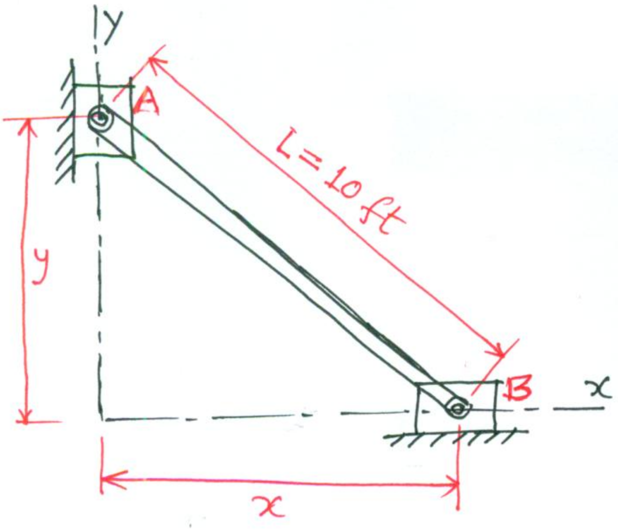
$$(0.45 v_y)^2 + v_y^2 = 12^2$$

$$\therefore v_y = \boxed{10.94 \text{ fps}}$$ Ans.

Now from eq^n (1) $v_x = 0.45 \times 10.94 = \boxed{4.92 \text{ fps}}$

Ans.

990/P.282



Given

$L = 10$ ft

when $x = 8$ ft, $v_B = 20$ fps $\rightarrow$

$a_B = -15$ fps^2 $\leftarrow$

$v_A = ?$ $a_A = ?$

Here, $x^2 + y^2 = 10^2$ —— (1)

Differentiating eq^n (1) w.r.t. 't'

$$2x\frac{dx}{dt} + 2y\cdot\frac{dy}{dt} = 0$$

$$\Rightarrow x v_B + y v_A = 0 \quad \text{—— (2)}$$

When $x = 8$ ft, $v_B = 20$ fps, $y = \sqrt{10^2 - 8^2} = 6$ ft

Substituting in eq^n (2)

$$8 \times 20 + 6 \times v_A = 0$$

$\therefore v_A = \boxed{-26.67 \text{ fps}}$, −ve sign means motion of A is downward

Ans.

Differentiating (2) w.r.t. 't'

$$x\cdot\frac{dv_B}{dt} + v_B\cdot\frac{dx}{dt} + y\frac{dv_A}{dt} + v_A\cdot\frac{dy}{dt} = 0$$

$$\Rightarrow x a_B + v_B^2 + y a_A + v_A^2 = 0$$

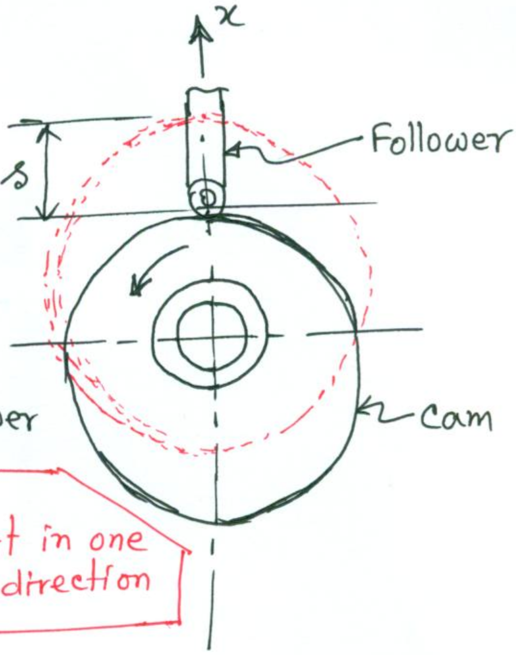
For the ~~mo~~ instant in consideration

$$8\times(-15) + 20^2 + 6\times a_A + (-26.67^2) = 0$$

$\therefore a_A = \boxed{-165.21 \text{ fps}^2}$ −ve means $\downarrow$

Ans.

997/P.283



Given $s = 3$ inch with harmonic motion

Cam speed = 60 rpm

$v_{max} = ?$ $a_{max} = ?$ for follower

Stroke → Complete movement of a reciprocating machine part in one direction

For Harmonic motion

$$x = r\cos\omega t$$

$$\therefore v = \frac{dx}{dt} = -r\omega\sin\omega t$$

$$\therefore a = \frac{dv}{dt} = -r\omega^2\cos\omega t$$

Here, $r = \frac{s}{2} = \frac{3}{2} = 1.5$ inch

$$\omega = 60 \text{ rpm} = \frac{60 \times 2\pi}{60} \text{ rad/s} = 2\pi \text{ rad/s}$$

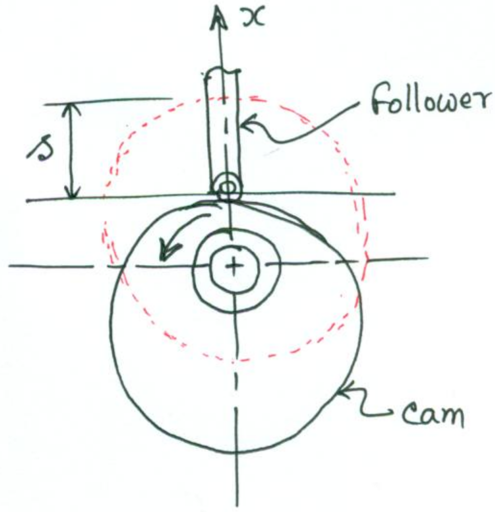
$\therefore v_{max} = r\omega = 1.5 \times 2\pi = 9.42$ in per sec.

$= \boxed{0.79 \text{ fps}}$ Ans.

and $a_{max} = r\omega^2 = 1.5 \times (2\pi)^2 = 59.16$ in per sec.

$= \boxed{4.93 \text{ fps}^2}$ Ans.

998/P.283



Given: $T = 2$ sec.

Harmonic motion

$r = \frac{5}{2} = 2.5$ inch

$v_{max} = ?$ $a_{max} = ?$

Because of Harmonic motion of the follower

$$x = r\cos\omega t$$

$$\therefore v = \frac{dx}{dt} = -r\omega\sin\omega t$$

$$a = \frac{dv}{dt} = -r\omega^2\cos\omega t$$

Here, $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$ rad/s

$$\therefore v_{max} = r\omega = 2.5\times\pi = \boxed{7.85 \text{ inch/s.}}$$ Ans.

$$a_{max} = r\omega^2 = 2.5\times\pi^2 = \boxed{24.67 \text{ inch/s.}}$$ Ans.