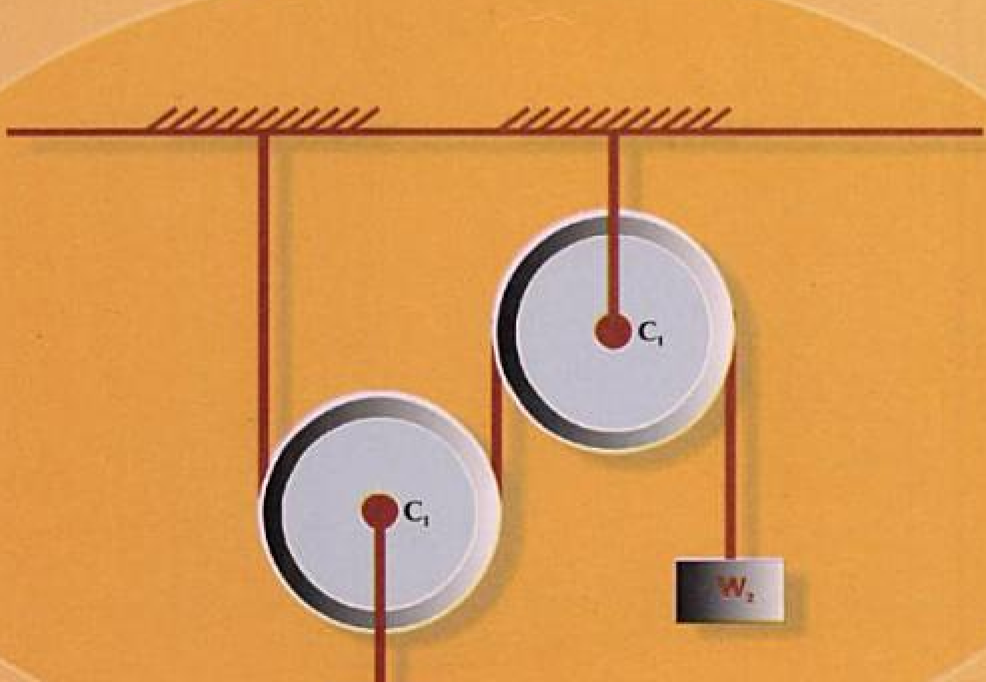
Revised Second Edition

PROBLEMS AND SOLUTIONS IN ENGINEERING MECHANICS

S.S. Bhavikatti
A. Vittal Hegde



NEW AGE INTERNATIONAL PUBLISHERS

First Edition : 2002
Second Edition : 2005

NEW AGE INTERNATIONAL (P) LIMITED, PUBLISHERS
4835/24, Ansari Road, Daryaganj,
New Delhi - 110 002
Visit us at : www.newagepublishers.com

Offices at :
Bangalore, Chennai, Cochin, Guwahati, Hyderabad, Jalandhar,
Kolkata, Lucknow, Mumbai and Ranchi

Rs. 125.00

ISBN : 81-224-1601-2

Published by New Age International (P) Ltd.,
4835/24, Ansari Road, Daryaganj, New Delhi-110 002 and
typesetter Innovative, Delhi
printed in India at Pack Printers, New Delhi

Contents

Contents

CHAPTER 1

Coplanar Concurrent Force Systems

GENERAL INFORMATION

COPLANAR CONCURRENT FORCE SYSTEMS

The various forces acting on a body constitute a system of forces. If all the forces in the system lie in a single plane, it is called as *Coplanar Force System*. If the lines of action of all the forces in the system pass through a single point, it is called a *Concurrent Force System.*

Resultant of Concurrent Forces

It is possible to find a single force, which would have the same effect as that of a number of forces acting on a given body. Such a single force is called the *Resultant Force.*

Composition of Forces

The process of finding the resultant of a force system is called *Composition of Forces*.

DETERMINATION OF RESULTANT

(a) Graphical Method

(*i*) *The Parallelogram Law of Forces:* This law states that if two forces acting simultaneously on a body at a point are represented in magnitude and direction by two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces. In Fig. 1.1, the resultant of forces F_1 and F_2 is R.

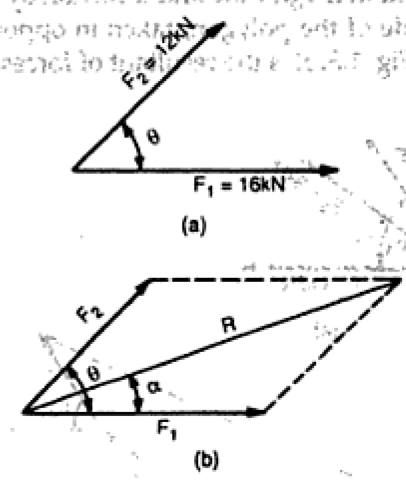


Fig. 1.1

(*ii*) *The Triangle Law of Forces:* This law states that if two forces acting simultaneously on a body are represented by the sides of a triangle taken in order, then their resultant is represented by the closing side of the triangle, taken in the opposite order. In Fig. 1.2, forces F_1 and F_2 are represented in magnitude (to suitable scale) and direction by *AB* and *BC*. According to this law, the closing line of the triangle *ABC* i.e., *AC* represent the resultant *R*.

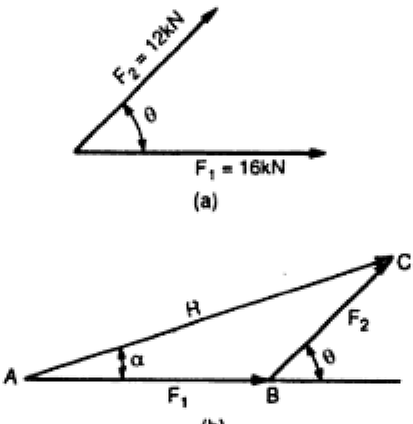


Fig. 1.2

(iii) The Polygon Law of Forces: This law states that if a number of concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a Polygon, taken in order, then the resultant is represented in magnitude and direction by the closing side of the polygon, taken in opposite order. In Fig. 1.3, R is the resultant of forces F_1, F_2 and F_3.

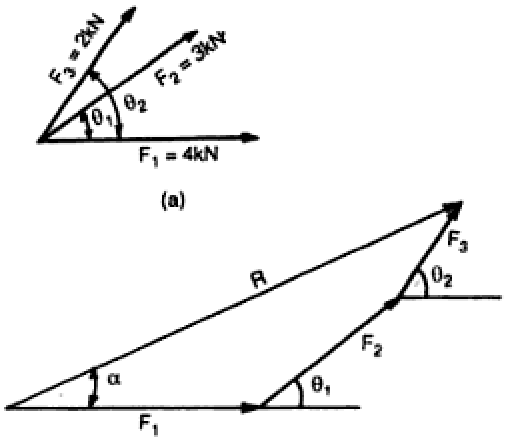


Fig. 1.3

(b) Resultant by Analytical Method

(*i*) If only two forces are acting as shown in Fig. 1.2,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} \tag{1.1}$$

and $$\alpha = \tan^{-1}\frac{F_2\sin\theta}{F_1 + F_2\cos\theta} \tag{1.2}$$

(*ii*) *Resultant by the Method of Resolution:*

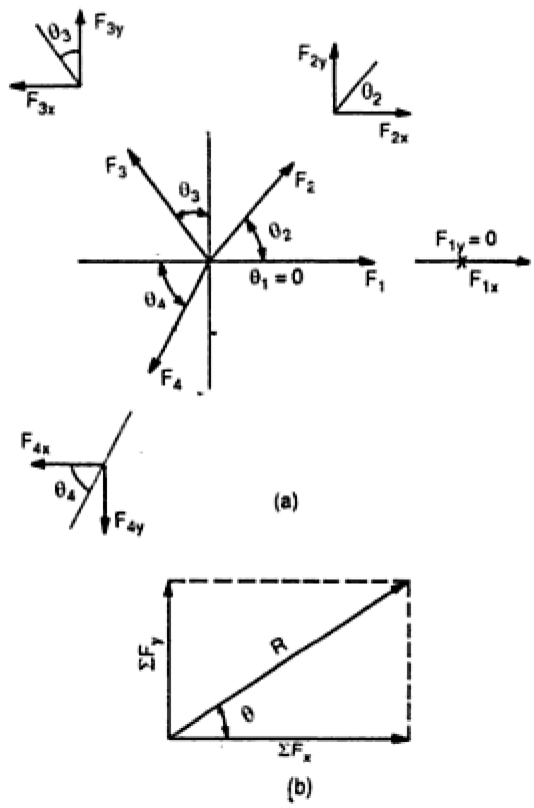


Fig. 1.4

Referring to Fig. 1.4,

$$\Sigma F_x = F_{1x} + F_{2x} + F_{3x} + F_{4x} \tag{1.3}$$

$$\Sigma F_y = F_{1y} + F_{2y} + F_{3y} + F_{4y} \tag{1.4}$$

$$\therefore \quad P = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \tag{1.5}$$

and $$\theta = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x} \tag{1.6}$$

(c) Equilibrium of a Body

A body is said to be in equilibrium, when it is at rest or continues to be in steady linear motion. Mathematically, it means, resultant R of the system of forces acting on the body is zero.

$$\left.\begin{aligned}\Sigma F_x &= 0\\ \Sigma F_y &= 0\end{aligned}\right\} \tag{1.7}$$

EQUILIBRIANT OF A FORCE SYSTEM

It is that single force which is equal and opposite to the resultant of the given force system.

Lami's Theorem

This theorem states that, if a body is in equilibrium under the action of three forces, each force is proportional to the sine of the angle between the other two forces.

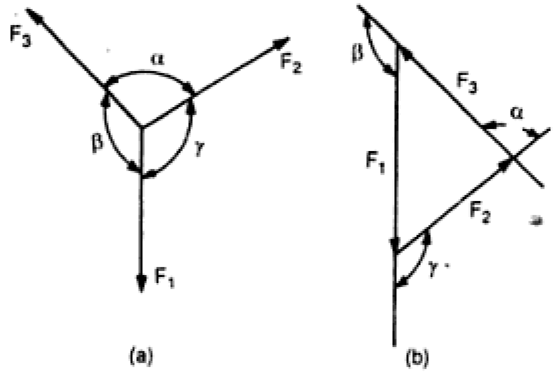


Fig. 1.5

For the system of forces shown in Fig. 1.5,

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma} \qquad (1.8)$$

(d) Free Body Diagram

It is the diagram of a body, in which the body under consideration is freed from all the contact surfaces, and all the forces acting on it, including the reactions at contact surfaces are indicated.

(e) Principle of Transmissibility of Forces

This principle states that the state of rest or of uniform motion of a rigid boay is unaltered, if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the rigid body along the line of action of the force.

SOLVED PROBLEMS

(Following sign convention is used in solving the problems: rightward and upward forces positive, leftward and downward forces negative.)

1. The body on the incline in Fig. 1.6(*a*) is subjected to the vertical and horizontal forces as shown. Find the component of each force along *x*-*y* axes oriented parallel and perpendicular to the incline.

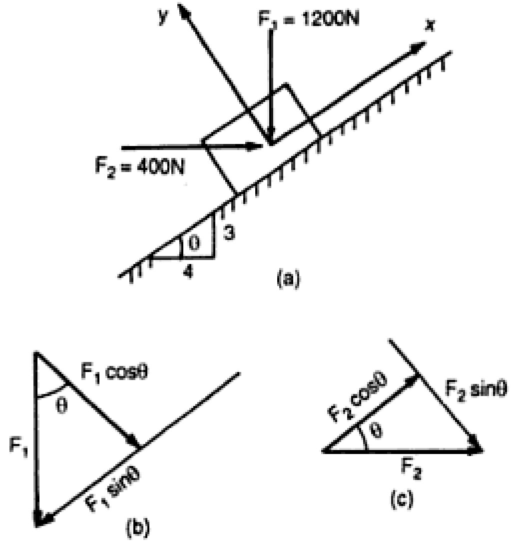


Fig. 1.6

Solution:

$\tan\theta = \frac{3}{4} \qquad \therefore\ \theta = 36.87°$

Component of F_1 are F_x (along *x*-direction) and F_y (along *y*-direction) and may be easily found by moving from the tail of the force F_1 to its head in the desired direction [Refer Fig. 1.6(*b*)].

$$F_{1x} = -F_1 \sin\theta = -1200 \sin 36.87° = -720 \text{ N. (Ans.)}$$

$$F_{1y} = -F_1 \cos\theta = -1200 \cos 36.87° = -960 \text{ N. (Ans.)}$$

Component of F_2 are [(Refer Fig. 1.6(*c*)]

$$F_{2x} = F_2 \cos\theta = 400 \cos 36.87° = 320 \text{ N (Ans.)}$$

$$F_{2y} = -F_2 \sin\theta = -400 \sin 36.87° = -240 \text{ N (Ans.)}$$

2. Determine the *x* and *y* components of each of the forces shown in Fig. 1.7(*a*).

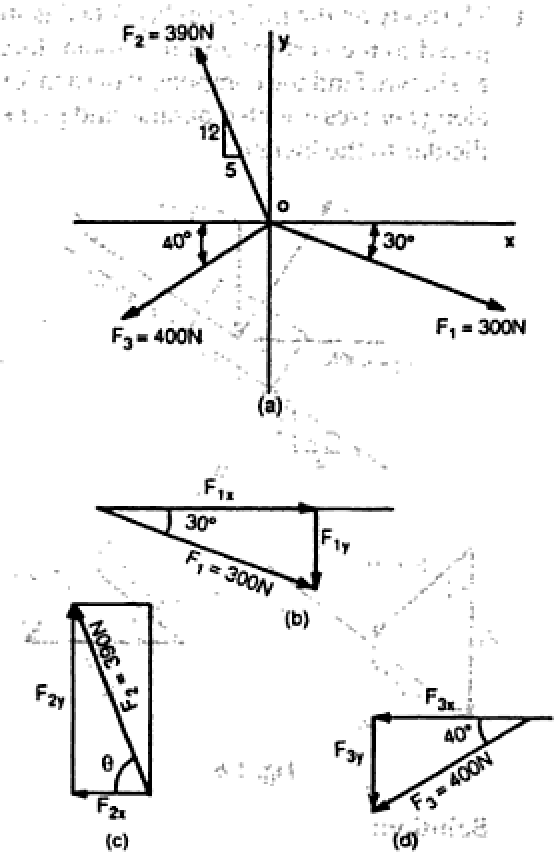


Fig. 1.7

Solution:

$F_{1x} = 300 \cos 30° = 259.81$ N **(Ans.)**

$F_{1y} = -300 \sin 30° = -150$ N **(Ans.)**

$\tan\theta = \dfrac{12}{5} \qquad \therefore\ \theta = 67.38°$

$F_{2x} = -F_2 \cos 67.38 = -390 \cos 67.38$
$= -150$ N **(Ans.)**

$F_{2y} = F_2 \sin 67.38 = 390 \sin 67.38$
$= 360$ N **(Ans.)**

$F_{3x} = -F_3 \cos 40° = -400 \cos 40°$
$= -306.42$ N **(Ans.)**

$F_{3y} = -F_2 \sin 40° = -400 \sin 40°$
$= -257.12$ N **(Ans.)**

3. Find the resultant of the two forces shown in Fig. 1.8(*a*).

Solution: From Fig. 1.8(*a*),

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{20^2 + 60^2 + 2 \times 20 \times 60 \times \cos 25°}$$

$= 78.58$ N **(Ans.)**

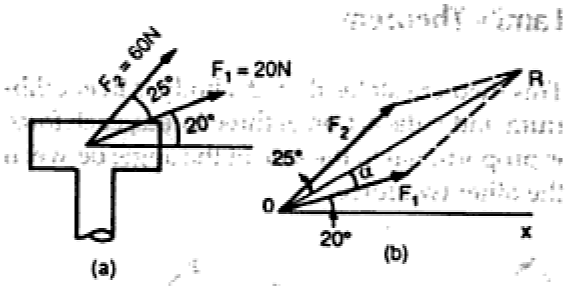


Fig. 1.8

$$\alpha = \tan^{-1} \frac{F_1 \sin\theta}{F_1 + F_2 \cos\theta}$$

$$= \tan^{-1} \frac{20 \sin 25°}{20 + 60 \cos 25°} = 0.1136$$

$= 6.48°$ **(Ans.)**

$\therefore$ Inclination with x-axis $= 20 + 6.48°$
$= 26.48°$ **(Ans.)**

4. Find the resultant of the force system shown in Fig. 1.9(*a*).

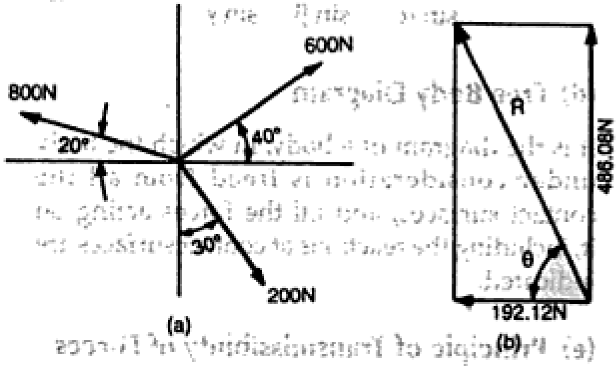


Fig. 1.9

Solution: Resolving each force in x and y directions, and adding algebraically, we get,

$\Sigma F_x = 600 \cos 40° - 800 \cos 20° + 200 \sin 30°$
$= -192.12$ N (←)

$\Sigma F_y = 600 \sin 40° + 800 \sin 20° - 200 \cos 30°$
$= 486.08$ N (↑)

$$\therefore R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{192.12^2 + 486.08^2} = 522.67 \text{ N} \textbf{ (Ans.)}$$

$$\theta = \tan^{-1} \frac{\Sigma F_y}{\Sigma F_x} = \tan^{-1} \frac{486.08}{192.12}$$

$= 68.43°$ as shown in Fig. 1.9(*b*). **(Ans.)**

5. Find the resultant of the force system acting on the hook shown in Fig. 1.10(*a*).

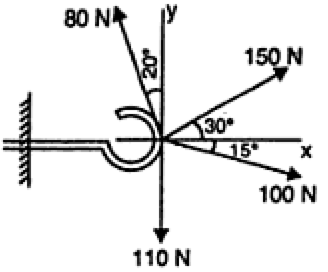


Fig. 1.10(*a*)

Solution: Resolving each force in x and y directions and adding algebraically,

$\Sigma F_x = 150 \cos 30° + 100 \cos 15° - 80 \sin 20°$
$= 199.13 \text{ N} (\rightarrow)$

$\Sigma F_y = 150 \sin 30° - 100 \sin 15° - 110 + 80 \cos 20°$
$= 14.29\text{N} (\uparrow)$

$\therefore R = \sqrt{192.13^2 + 14.29^2} = 199.64$ N **(Ans.)**

$\theta = \tan^{-1} \dfrac{14.29}{199.64}$
$= 4.09°$ as shown in Fig. 1.10(*b*).

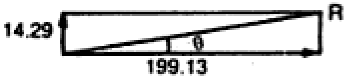


Fig. 1.10(*b*)

6. A system of forces acting on a body resting on an incline plane is as shown in Fig. 1.11. Determine the resultant force, if $\theta = 30°$, $w = 1000$ N, $N = 866.03$ N, $F = 200$ N and $T = 1200$ N.

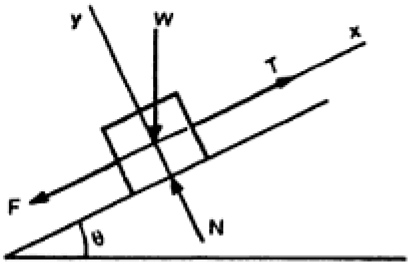


Fig. 1.11

Solution:

$\Sigma F_x = T - F - W \sin \theta$
$= 1200 - 200 - 1000 \sin 30°$
$= 500$ N

$\Sigma F_y = N - W \cos 30°$
$= 866.03 - 1000 \cos 30°$
$= 0$ N

Hence, the resultant force is 500 N acting up the plane.

7. A disabled ship is pulled by means of two tugboats as shown in Fig. 1.12(*a*). If the resultant of the two forces exerted by the ropes is a 300 N force parallel to the axis of the ship, find:
   (*a*) Force exerted by each of the tugboats knowing $\alpha = 30°$.
   (*b*) The value of α such that the force of tugboat 2 is minimum, while that of tugboat 1 acts in the same direction. Find the corresponding force to be exerted by tug- boat 1 also.

Solution:

(*a*) Let T_1 and T_2 be the tensile forces in the two ropes as shown in Fig. 1.12(*a*) with $\alpha = 30°$. According to law of triangle of forces, the system will be as shown in Fig. 1.12(*b*). Applying sine rule to the triangle of forces,

$$\frac{T_1}{\sin 30°} = \frac{T_2}{\sin 20°} = \frac{300}{\sin 130°}$$

$$\therefore \quad T_1 = \frac{300 \sin 30°}{\sin 130°} = 195.81 \text{ N } \textbf{(Ans.)}$$

$$T_2 = \frac{300 \sin 20°}{\sin 130°} = 133.94 \text{ N } \textbf{(Ans.)}$$

(*b*) From Fig. 1.12(*c*), it is clear that T_2 is least, when it is at right angles to T_1. Hence,

$\alpha = 70°$ **(Ans.)**

Applying sine rule to the triangle of forces,

$$\frac{T_1}{\sin 70°} = \frac{T_2}{\sin 20°} = \frac{300}{\sin 90°}$$

$$\therefore \quad T_1 = 300 \frac{\sin 70°}{\sin 90°} = 281.91 \text{ N } \textbf{(Ans.)}$$

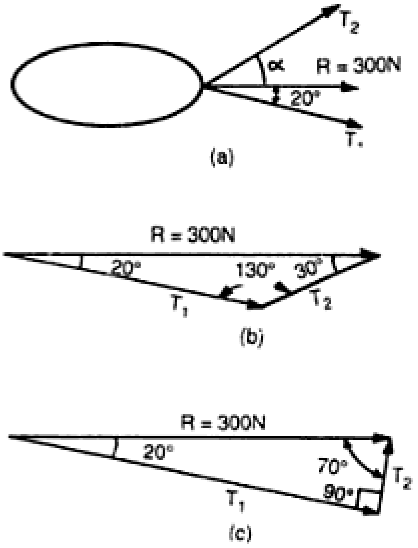


Fig. 1.12

$$T_2 = 300\ \frac{\sin 20^\circ}{\sin 90^\circ} = 102.61\ \text{N (Ans.)}$$

8. An automobile which is disabled is pulled by two ropes as shown in Fig. 1.13(*a*). Find the force P and resultant R such that R is directed as shown in the figure.

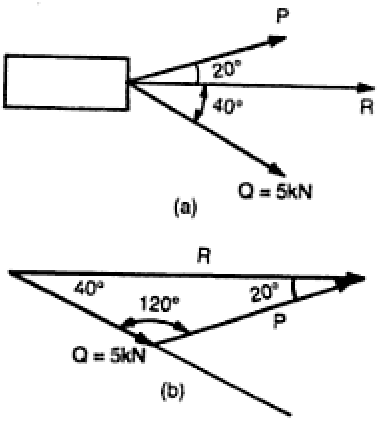


Fig. 1.13

Solution: From Fig. 1.13(*b*),

$$\frac{R}{\sin 120^\circ} = \frac{5}{\sin 20^\circ} = \frac{P}{\sin 40^\circ}$$

$\therefore$ $R = 12.66$ kN **(Ans.)**

$P = 9.40$ kN **(Ans.)**

9. A Collar, which may slide on a vertical rod, is subjected to three forces as shown in Fig. 1.14. The direction of the force F may be varied. If possible, determine the direction of the force F, so that resultant of the three forces is horizontal, knowing that the magnitude of F is equal to (*a*) 2400 N, (*b*) 1400 N.

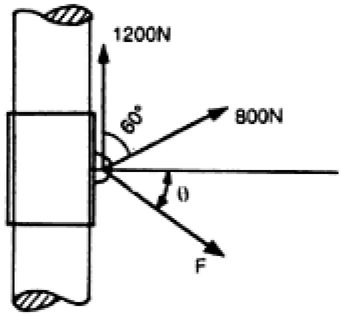


Fig. 1.14

Solution:

Case (*a*): $F = 2400$ N.

Since the resultant should be horizontal,

$$\Sigma F_y = 0$$

i.e., $1200 + 800\cos 60^\circ - F\sin\theta = 0$

$$\therefore \quad \sin\theta = \frac{1600}{2400}$$

$\therefore$ $\theta = 41.81^\circ$ **(Ans.)**

Case (*b*): $F = 1400$ N

$$\Sigma F_y = 0$$

i.e., $1200 + 800\cos 60^\circ - 1400\sin\theta = 0$

$$\therefore \quad \sin\theta = \frac{1600}{1400} = 1.143 > 1$$

which is not possible as sine of an angle can not be greater than unity. Hence, it is not possible to have the resultant of the three forces in horizontal direction with $F = 1400$ N only. The force F has to be greater than or equal to 1600 N.

10. Determine the angle α and the magnitude of force Q such that the resultant of the three forces on the pole is vertically downward and of magnitude 12 kN. (Ref. Fig. 1.15)

Solution: Since the resultant force should be along the vertical, the horizontal, summation of the component of three forces should be zero.

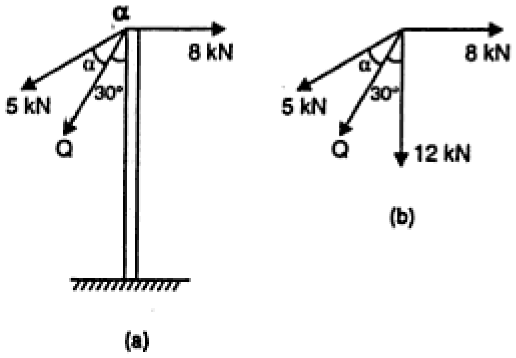


Fig. 1.15

i.e., $\Sigma F_x = 0 = -5 \sin (30 + \alpha)$
$-Q \sin 30° + 8$ ...(*i*)
$\Sigma F_y = -12 = -5 \cos (30 + \alpha) - Q \cos 30°$

i.e., $Q = \dfrac{12 - 5\cos(30+\alpha)}{\cos 30°}$ ...(*ii*)

Substituting this value of Q in (*i*),

$0 = -5 \sin (\alpha + 30) + 5 \tan 30°$
$\cos (\alpha + 30) + 1.0718$

Solving by trial and error,

When $\alpha = 10°$, RHS = 0.0698
$\alpha = 10.5°$, RHS = 0.0197
$\alpha = 10.7°$, RHS = $-0.00015 \approx 0$

$\therefore$ $\alpha = 10.7°$ **(Ans.)**

$$Q = \frac{12 - 5 \cos (30 + 10.7)}{\cos 30°}$$
= 9.479 kN **(Ans.)**

11. Find the resultant of the force system shown in Fig. 1.16.

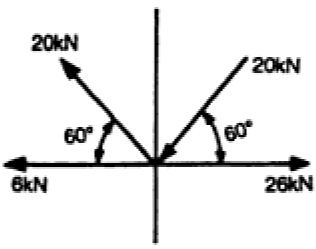


Fig. 1.16

$\Sigma F_x = 20 \sin 60° - 20 \sin 60° = 0$
$\Sigma F_y = 26 - 20 \cos 60° - 20 \cos 60° - 6 = 0$
$\therefore$ Resultant force is zero. **(Ans.)**

12. Two forces acting on a body are 1500 N and 1000 N as shown in Fig. 1.17(*a*). Determine the third force *F* such that the resultant of all the three forces is 1000 *N* directed at 45° to the *x*-axis.

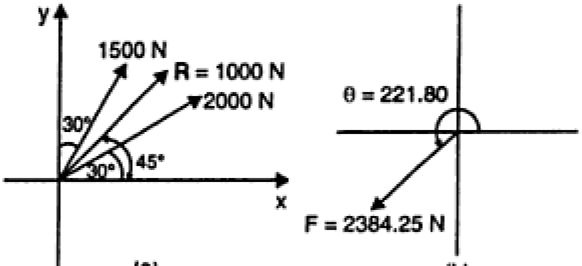


Fig. 1.17

Solution: Let the third force be *F* making an angle θ with *x*-axis.

$R \cos \alpha = \Sigma F_x$

$\therefore$ $1000 \cos 45° = 1500 \sin 30° + 2000 \cos 30°$
$+ F \cos \theta$

$F \cos \theta = -1774.94$ ...(*i*)

$R \sin \alpha = \Sigma F_y$

$1000 \sin 45° = 1500 \cos 30° + 2000 \sin 30°$
$+ F \sin \theta$

or $F \sin \theta = -1591.93$ ...(*ii*)

Squaring and adding (*i*) and (*ii*),

$(F \sin \theta)^2 + (F \cos \theta)^2 = (-1774.94)^2$
$+ (-1591.93)^2$

i.e., $F^2 = (-1774.94)^2 + (-1591.93)^2$
$F = 2384.25$ N **(Ans.)**

Dividing (*ii*) by (*i*),

$$\tan \theta = \frac{-1591.93}{-1774.94} = 0.897$$

$\therefore$ $\theta = 41.89°$ or $(180 + 41.89°)$

Since both *x* and *y* components are to be –ve

$\theta = 180 + 41.89° = 221.89°$ as shown in Fig. 1.17 (*b*). **(Ans.)**

13. Three forces acting at a point are shown in Fig. 1.18. The direction of the 300 N forces may vary, but the angle between them is always 40°. Determine α for which the resultant of the three forces is directed parallel to the plane b-b.

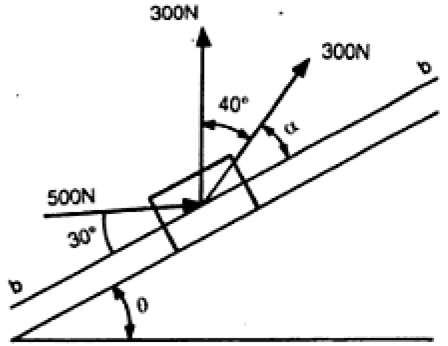


Fig. 1.18

Solution: Since the resultant has to be parallel to the plane b-b, in the ⊥r direction to it (y-direction), the resultant = 0.

i.e., $\Sigma F_y = 0$

$-500 \sin 30° + 300 \sin \alpha + 300 \sin (\alpha + 40) = 0$

$300 [\sin \alpha + \sin (\alpha + 40°)] = 250$

$\sin \alpha + \sin (\alpha + 40°) = 0.833$

i.e.,

$$2 \sin\left(\frac{\alpha + \alpha + 40}{2}\right) \cos\left(\frac{\alpha + 40 - \alpha}{2}\right) = 0.833$$

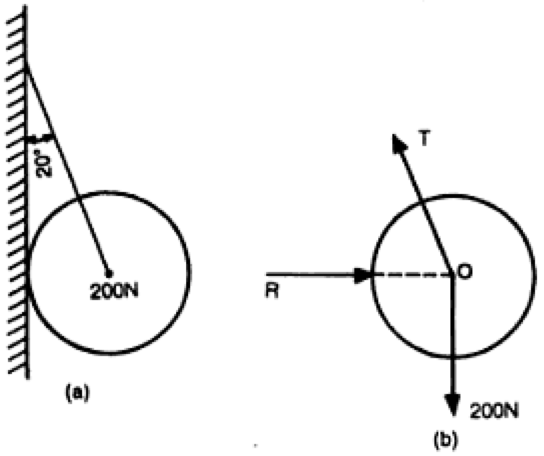
$2 \sin (\alpha + 20) \cos 20° = 0.833$

$\sin (\alpha + 20) = 0.4434$

$\alpha + 20 = 26.32°$

or $\alpha = 6.32°$ **(Ans.)**

14. A sphere weighing 200 N is tied to a smooth wall by a string as shown in Fig. 1.19(*a*). Find the tension T in the string and reaction R of the wall.



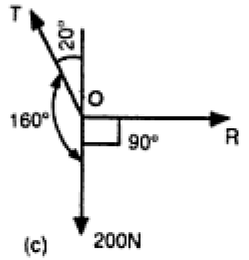


Fig. 1.19

Solution: The free body diagram of the sphere is as shown in Fig. 1.19(*b*). These concurrent forces acting away from the point 'o' are shown in Fig. 1.19(*c*), after using law of Transmissibility of forces. Applying Lami's theorem,

$$\frac{T}{\sin 90°} = \frac{R}{\sin(180 - 20)} = \frac{200}{\sin(90 + 20)}$$

$T = 212.84$ N **(Ans.)**

$$R = 200 \frac{\sin 160°}{\sin 110°} = 72.79 \text{ N } \textbf{(Ans.)}$$

It may be solved using the equations of equilibrium also.

$\Sigma F_x = 0$ gives $T \cos 20° - 200 = 0$

$$T = \frac{200}{\cos 20°} = 212.84 \text{ N } \textbf{(Ans.)}$$

$\Sigma F_y = 0$ gives $-T \sin 20° + R = 0$

or $R = T \sin 20° = 72.79$ N **(Ans.)**

15. Determine the horizontal force P to be applied to a block of weight 2500 N to hold it in position on a smooth inclined plane *AB*, which makes an angle of 30° with the horizontal [Ref. Fig. 1.20(*a*)].

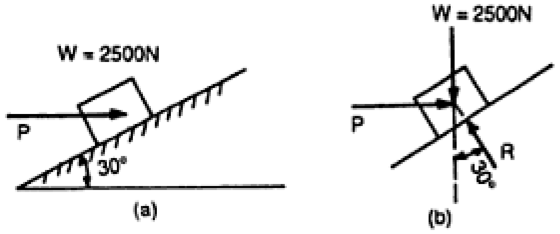


Fig. 1.20

Solution: The free body diagram of the block is shown in Fig. 1.20(*b*). Since, there are only three forces acting on the body, which keep it in equilibrium, Lami's theorem may be used.

$$\frac{P}{\sin(180° - 30°)} = \frac{R}{\sin 90°} = \frac{2500}{\sin(90° + 30°)}$$

$\therefore$ $P = 1443.38$ N (Ans.)

$R = 2886.75$ N (Ans.)

16. Two smooth spheres each of radius 150 mm and weight 250 N rest in a horizontal channel having vertical walls, the distance between which is 560 mm. Find the reaction at the points of contact *A*, *B*, *C* and *D* as shown in Fig. 1.21(*a*).

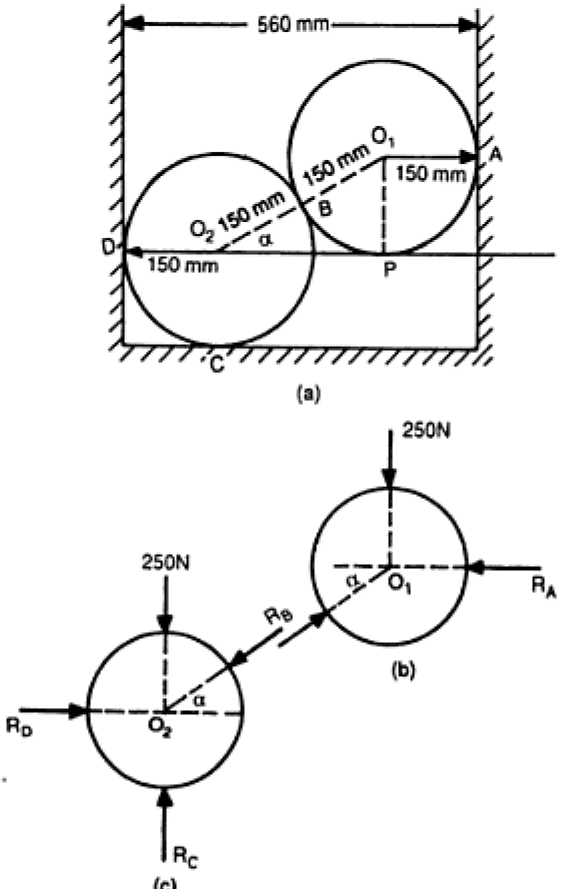


Fig. 1.21

Solution: Referring to Fig. 1.21(*a*), angle α is given by

$$\cos\alpha = \frac{O_2P}{O_2O_1} = \frac{560 - 150 - 150}{150 + 150} = 0.867$$

$\therefore \quad \alpha = 29.92°$

Considering *FBD* of sphere with centre O_1 [Ref. Fig. 1.21(*b*)],

$$\frac{R_A}{\sin(90 + \alpha)} = \frac{R_B}{\sin 90°} = \frac{250}{\sin(180 - \alpha)}$$

Since $\alpha = 29.92°$,

$$\frac{R_A}{\sin 119.92°} = \frac{R_B}{\sin 90°} = \frac{250}{\sin 150.08°}$$

$\therefore$ $R_A = 434.32$ N (Ans.)

$R_B = 501.11$ N (Ans.)

Considering *FBD* of sphere with centre O_2 (Fig. 1.21*c*),

$\Sigma F_x = 0 \Rightarrow R_D - R_B\cos(29.92°) = 0$

$R_D = 501.11\cos 29.92$

$= 432.32$ N (Ans.)

$\Sigma F_y = 0 \Rightarrow R_C - 250 - R_B\sin(29.92°) = 0$

$R_C - 250 - 501.11\sin 29.92 = 0$

or $R_C = 500$ N (Ans.)

17. A cord *ACB* 8 m long is attached at two points *A* and *B* to two vertical walls 5 m apart as shown in Fig. 1.22(*a*). A pulley *C* of negligible radius carries a suspended load of 100 N and is free to roll without friction along the chord. Determine the position of equilibrium, as defined by the distance *x*, that the pulley will assume and also the tensile force in the cord.

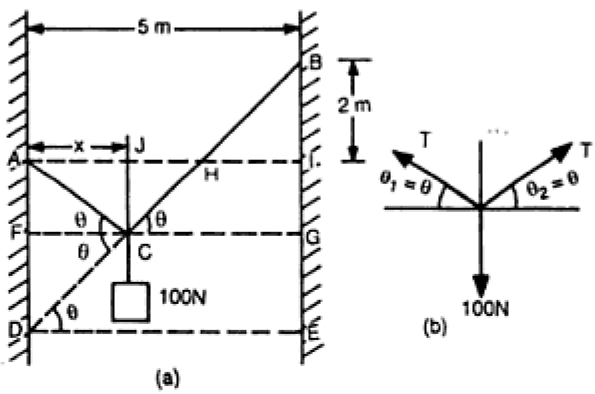


Fig. 1.22

Solution: Since the pulley is frictionless, tensile forces in *CA* and *CB* should be equal. Referring to *FBD* of pulley as shown in Fig. 1.22(*b*),

$\Sigma F_x = 0 \Rightarrow T\cos\theta_1 = T\cos\theta_2$

or $\theta_1 = \theta_2$; say $= \theta$

Referring to Fig. 1.22(*a*),

$\Delta CFD = \Delta CFA$

$\therefore \quad CD = CA$

$BD = BC + CD = BC + CA$

= length of chord = 8 m

DE = 5 m, the distance between the walls.

$\therefore \cos\theta = \frac{5}{8}$ or $\theta = 51.32°$

$\therefore \quad BE = 8\sin\theta = 8\sin 51.32 = 6.245$ m

ΔBHI is ||| to ΔBDE

$$\therefore \quad HI = \frac{BI}{BE}DE = \frac{2}{6.245}\times 5 = 1.60 \text{ m}$$

$\therefore \quad AH = 5 - HI = 5 - 1.6 = 3.4$ m

Since $\Delta ACJ = \Delta HCJ$,

$AJ = JH = x$

But $AJ + JH = 3.4$

or $2x = 3.4$

$x = 1.7$ **(Ans.)**

Now at *C*,

$$\Sigma F_y = 0 \Rightarrow 2T\sin\theta = 100$$

$$T = \frac{100}{2\sin 51.32°}$$

$= 64.05$ N **(Ans.)**

18. A roller of radius r = 500 mm and weight 4000 N is to be pulled over a curb of height 250 mm by a horizontal force *P* applied to the end of a string, wound tightly around the circumference of the roller. Find the magnitude of force *P* required to start the roller move over the curb. [Ref. Fig. 1.23(*a*)].

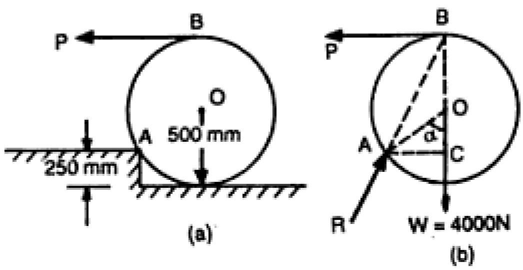


Fig. 1.23

Solution: Since the body is in equilibrium under the action of only three forces, namely self-weight, applied force *P* and reaction at curb, they should be concurrent as shown in Fig. 1.23(*b*).

From the figure,

$$\cos\alpha = \frac{OC}{AO} = \frac{250}{500} = 0.5$$

$\therefore \quad \alpha = 60°$

From ΔAOB, $\angle OAB = \angle OBA$

since $OA = OB$

But $\angle OAB + \angle OBA = \alpha$

$2\angle OBA = 60$

$\therefore \ \angle OBA = 30°$

$\Sigma F_y = 0 \Rightarrow R\cos 30° = 4000$

$\therefore \quad R = 4618.8$ N **(Ans.)**

$\Sigma F_x = 0 \Rightarrow R\sin 30° - P = 0$

$\therefore \quad P = R\sin 30° = 2309.4$ N **(Ans.)**

19. In the above problem what is the least pull *P*, through the centre of the wheel to just turn the roller over the curb?

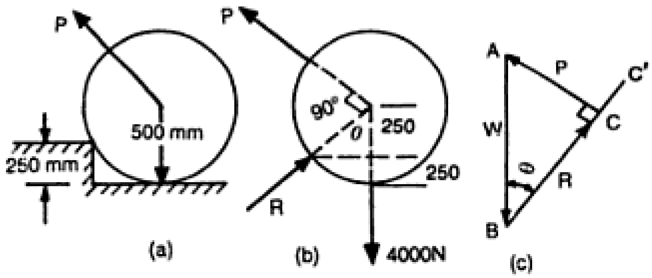


Fig. 1.24

Solution: For this case the reaction *R* should pass through the center, since the other two forces pass through it. Referring to Fig. 1.24 (*b*), $\cos\theta = \frac{250}{500} = 0.5$. $\therefore \theta = 60°$, Referring to Fig. 1.24(*c*), if *AB* represents graphically the self-weight, the direction of *R* is along *BC′*. Since the body is in equilibrium, the figure drawn, representing the forces one after the other must close. Hence for *P* to be minimum, *AC* should be perpendicular to *BC′*. From ΔABC,

$P = CA = AB\sin\theta = 4000\cos 60° = 3464.1$ N **(Ans.)**

20. In the Fig. 1.25(*a*), find the forces in the bars *AB* and *AC*. Neglect size of the pulley, which is frictionless.

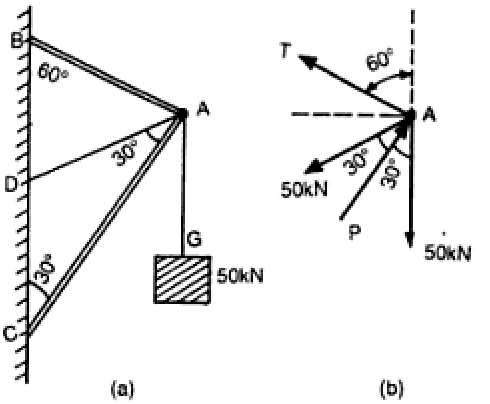


Fig. 1.25

Solution: Since pulley is frictionless,
Tension in segment AD = tension in segment AG = 50 kN.

Free body diagram of pulley at A is shown in Fig. 1.25(*b*).

$\Sigma F_x = 0$

$P \sin 30° - T \sin 60° - 50 \sin 60° = 0$

$0.5\,P - 0.866\,T = 43.30$ ...(*i*)

$\Sigma F_y = 0$

$P \cos 30° + T \cos 60° - 50 - 50 \cos 60° = 0$

$0.866\,P + 0.5\,T = 75$ ...(*ii*)

Multiplying equation (2) by $\frac{0.5}{0.866}$, we get

$0.5\,P + 0.2887\,T = 43.30$ ...(*iii*)

Subtracting eqn. (1) from eqn. (3), we get

$T = 0$ **(Ans.)**

$\therefore \quad P = \frac{43.3}{0.5} = 86.6$ kN **(Ans.)**

21. An electric light fixture weighing 200 N is supported as shown in Fig. 1.26(*a*). Determine the tensile forces in the wires *BA* and *BC*.

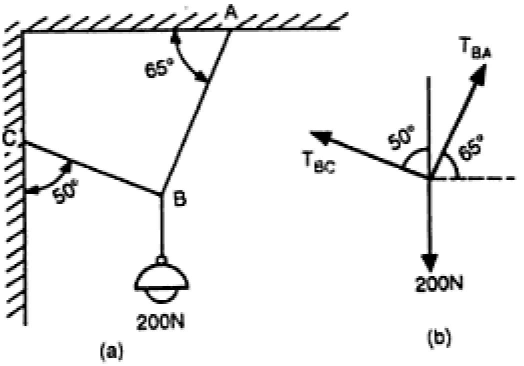


Fig. 1.26

Solution: Free body diagram is shown in Fig. 1.26(*b*). Applying Lami's theorem,

$$\frac{T_{BC}}{\sin(90+65)} = \frac{T_{BA}}{\sin(180-50)} = \frac{200}{\sin[(90-65)+50]}$$

$T_{BC} = 87.5$ N **(Ans.)**

$T_{BA} = 158.6$ N **(Ans.)**

22. A ball weighing 100 N is at rest in a right angle trough as shown in Fig. 1.27(*a*). Determine the forces exerted on the sides of the trough at *D* and *E*. Assume all surfaces to be smooth.

Solution: Referring to *FBD* of the ball shown in Fig. 1.27(*b*),

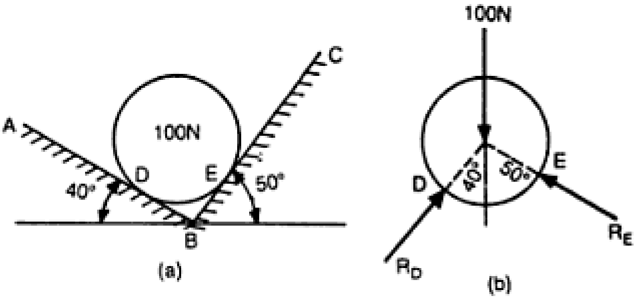


Fig. 1.27

$$\frac{R_D}{\sin(180-50)} = \frac{R_E}{\sin(180-40)} = \frac{100}{\sin(40+50)}$$

$\therefore \quad R_D = 76.6$ N **(Ans.)**

$R_E = 64.3$ N **(Ans.)**

23. Determine the forces developed in the members of the system shown in Fig. 1.28(*a*), when a load $P = 2000$ N acts. Neglect self-weight of the members and assume ideal hinge at *A* and a perfectly flexible string *BC*.

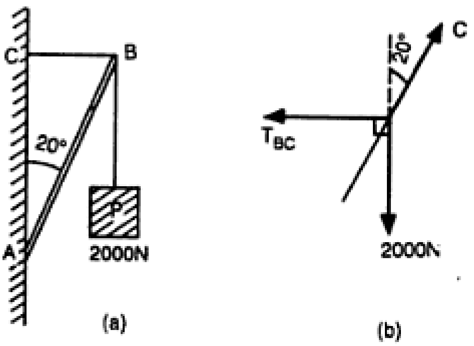


Fig. 1.28

Solution: Referring to *FBD* of *B*, shown in Fig. 1.28(*b*),

$$\frac{T_{BC}}{\sin(180-20)} = \frac{C}{\sin(90)} = \frac{2000}{\sin(90+20)}$$

$\therefore$ $T_{BC} = 727.9$ N **(Ans.)**

$C = 2128.4$ N **(Ans.)**

24. A right circular roller of weight 5000 N rests on a smooth inclined plane and is held in position by a chord *AC* as shown in Fig. 1.29(*a*). Find the tension in the chord and reaction at *B*, if there is a horizontal force *P* = 1000 N acting at *C*.

Solution: Free body diagram of the ball is shown in Fig. 1.29(*b*),

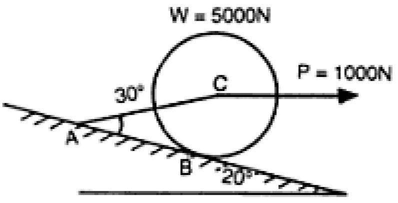
Noting that tension in *AC* makes 30 – 20 = 10° angle with horizontal,

$\Sigma F_x = 0 \Rightarrow R_B \sin 20° + 1000 - T_{CA} \cos 10° = 0$

i.e., $0.342\, R_B - 0.985\, T_{CA} = -1000$ ...(*i*)

$\Sigma F_y = 0 \Rightarrow R_B \cos 20° - T_{CA} \sin 10° = 5000$

$0.9397\, R_B - 0.1736 T_{CA} = 5000$ ...(*ii*)



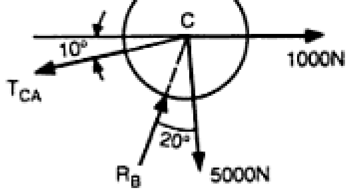


Fig. 1.29

Multiplying eqn. (2) by $\frac{0.985}{0.1736}$, we get

$5.33\, R_B - 0.985\, T_{CA} = 28361.9$ ...(*iii*)

From eqn. (*i*) and (*iii*), we get

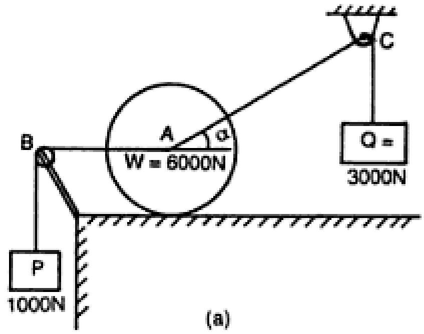
$4.98834\, R_B = 29361.9$

or $R_B = 5886.1$ N **(Ans.)**

and $T_{CA} = 3058.9$ N **(Ans.)**

25. A ball of weight *W* = 6000 N rests upon a smooth horizontal plane and has attached to its centre two strings *AB* and *AC*, which pass over frictionless pulleys at *B* and *C* and carry loads *P* = 1000 N and *Q* = 3000 N respectively as shown in Fig. 1.30(*a*). If the string *AB* is horizontal, find the angle α shown in the figure, when the ball is in a position of equilibrium. Find also the pressure *R* between the ball and the plane.

Solution: Consider *FBD* of the ball, which is as shown in Fig. 1.30(*b*),



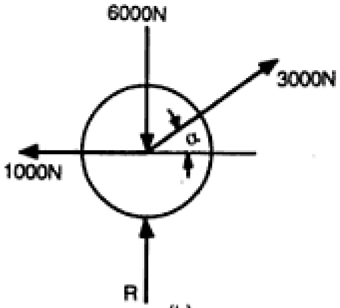


Fig. 1.30

$\Sigma F_x = 0 \Rightarrow 3000 \cos \alpha = 1000$

$\therefore$ $\alpha = 70.53°$ **(Ans.)**

$\Sigma F_y = 0 \Rightarrow R - 6000 + 3000 \sin \alpha = 0$

$\therefore$ $R = 3171.6$ N **(Ans.)**

26. In Fig. 1.31, *P* = 500 N and *Q* = 1000 N, are suspended in a vertical plane by strings *AD*, *AB* and *AC*. Find the tensions induced in each of these strings.

Solution: Tension in string *AD*

$= Q = 1000 \text{ N} = T_{AD}$ **(Ans.)**

Now consider the *FBD* of joint *A* [Ref. Fig. 1.31(*b*)]

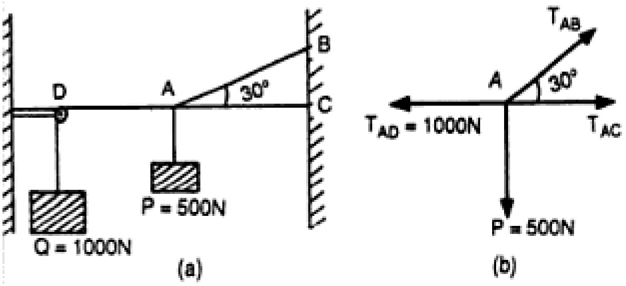


Fig. 1.31

$$\Sigma F_y = 0 \Rightarrow T_{AB} \sin 30° = 500$$

$$\therefore \quad T_{AB} = 1000 \text{ N (Ans.)}$$

$$\Sigma F_x = 0 \Rightarrow T_{AB} \cos 30° + T_{AC} - T_{AD} = 0$$

$$1000 \cos 30° + T_{AC} - 1000 = 0$$

$$T_{AC} = 133.97 \text{ N (Ans.)}$$

27. A system of connected flexible cables shown in Fig. 1.32(*a*) is supporting two loads 400 N and 500 N at points *B* and *D*. Determine tensions in various segments of the cable.

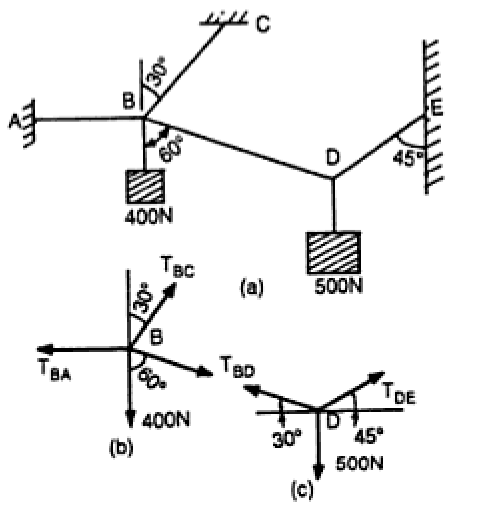


Fig. 1.32

Solution: Referring to *FBD* of joint *D* [Ref. Fig. 1.32(*c*)],

$$\frac{T_{BD}}{\sin(90+45)} = \frac{T_{DE}}{\sin(90+30)} = \frac{500}{\sin(180-30-45)}$$

$$\therefore \quad T_{BD} = 366.02 \text{ N (Ans.)}$$

$$T_{DE} = 448.29 \text{ N (Ans.)}$$

Considering *FBD* of joint *B*,

$$\Sigma F_x = 0 \Rightarrow T_{BD} \sin 60 + T_{BC} \sin 30° - T_{BA} = 0$$

$$0.866\, T_{BD} + 0.5\, T_{BC} - T_{BA} = 0$$

$$\text{i.e., } T_{BA} - 0.5\, T_{BC} = 0.866 \times 366.02 = 316.98 \qquad \ldots(i)$$

$$\Sigma F_y = 0 \Rightarrow T_{BC} \cos 30 - 400 - T_{BD} \cos 60 = 0$$

$$0.866\, T_{BC} = 400 + 366.02 \cos 60°$$

$$\therefore \quad T_{BC} = 673.2 \text{ N (Ans.)}$$

From eqn (*i*)

$$T_{AB} = 316.98 + 0.5 \times 673.2 = 653.58 \text{ N (Ans.)}$$

28. Two equal loads are supported by a flexible cable *ACDB* as shown in Fig. 1.33(*a*). Determine tensile force developed in portion *AC*, *CD* and *DB* respectively, if the span $l = 12$ m and sag $h = 1.5$ m. Neglect weight of the cable.

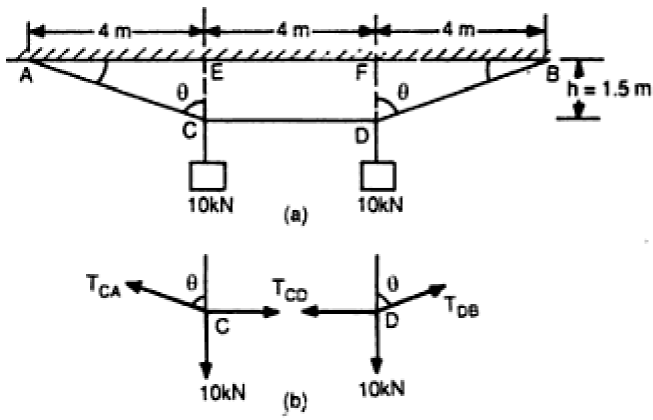


Fig. 1.33

Solution: From ΔAEC and ΔBFD,

$$\tan\theta = \frac{4}{1.5} \quad \therefore \theta = 69.44°$$

$\therefore$ From *FBD* of point *C* [Fig. 1.33(*b*)],

$$\Sigma F_y = 0 \Rightarrow$$

$$T_{CA} \cos\theta = 10$$

$$T_{CA} = \frac{10}{\cos\theta} = \frac{10}{\cos 69.44}$$

$$= 28.48 \text{ kN (Ans.)}$$

$$\Sigma F_x = 0 \Rightarrow$$

$$T_{CD} = T_{CA} \sin\theta = T_{CA} \sin 69.44°$$

$$\therefore \quad T_{CD} = 26.67 \text{ kN (Ans.)}$$

From *FBD* of point *D* [Fig. 1.33(*c*)],

$T_{DB} \cos 69.44° = 10$

$\therefore \quad T_{DB} = 28.48$ kN **(Ans.)**

29. On the string *ACEDB* are hung three equal weights, placed symmetrically with respect to the vertical line through the mid point *E* as shown in Fig. 1.34(*a*). Determine angle β, if the other angles are as shown in the figure.

Solution: From *FBD* of point E [Fig. 1.34(*c*)],

$\Sigma F_x = 0 \Rightarrow T_{CE} = T_{ED}$

$\Sigma F_y = 0 \Rightarrow T_{CE} \sin\beta + T_{ED} \sin\beta = W$

i.e., $2\,T_{CE} \sin\beta = W$

or $T_{CE} = \dfrac{W}{2\sin\beta}$ ...(*i*)

From *FBD* of point *C* [Fig. 1.33(*b*)],

$\Sigma F_x = 0 \Rightarrow T_{AC} \cos 60° = T_{CE} \cos\beta$

or $T_{AC} = \dfrac{W}{2\sin\beta} \cdot \cos\beta \dfrac{1}{\cos 60°}$

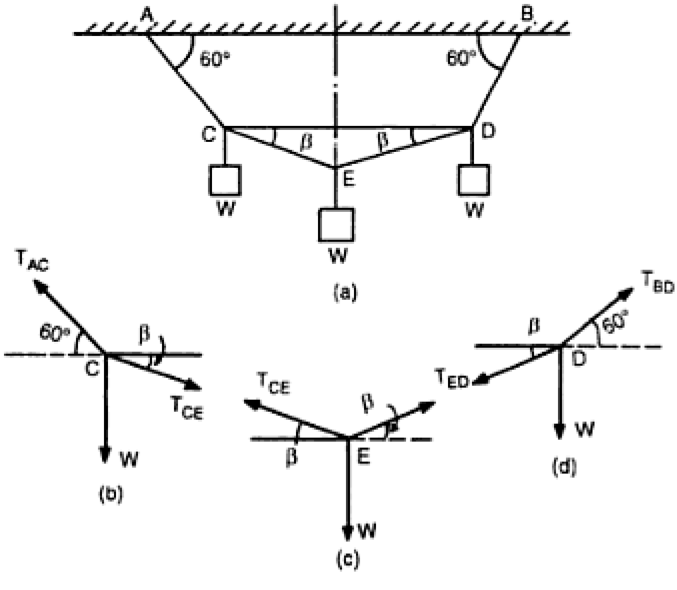


Fig. 1.34

$= \dfrac{W \cot\beta}{2\cos 60°}$ ...(*ii*)

$\Sigma F_y = 0 \Rightarrow$

$T_{AC} \sin 60° - T_{CE} \sin\beta - W = 0$

Substituting the values of T_{CE} and T_{AC} from eqns. (*i*) and (*ii*),

$\dfrac{W}{2} \cot\beta \tan 60° - \dfrac{W}{2} - W = 0$

or $\cot\beta \tan 60° = 3$

$\cot\beta = \sqrt{3}$, or $\tan\beta = \dfrac{1}{\sqrt{3}}$

$\therefore \quad \beta = 30°$ **(Ans.)**

30. A wire is fixed at two points *A* and *D* as shown in Fig. 1.35 (*a*). Two weights 10 kN and 30 kN are supported at *B* and *C* respectively. When equilibrium is reached it is found that inclination of *AB* is 20° and that of *CD* is 50° to the vertical. Determine the tension in the segments *AB*, *BC* and *CD* of the wire and also the inclination of *BC* to the vertical.

Solution: Consider *FBD* of point *B* [Ref. Fig. 1.35(*b*)],

$\Sigma F_x = 0 \Rightarrow T_{AB} \sin 20° = T_{BC} \cos\theta$

$T_{AB} = 2.9238\, T_{BC} \cos\theta$ ...(*i*)

$\Sigma F_y = 0 \Rightarrow T_{AB} \cos 20° - T_{BC} \sin\theta = 10$

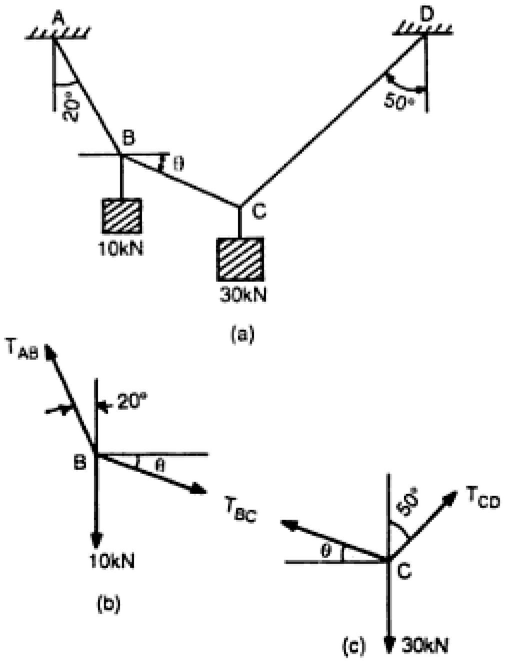


Fig. 1.35

i.e.,

$T_{BC}\,(2.9238 \cos 20 \cos\theta - \sin\theta) = 10$ ...(*ii*)

Consider *FBD* of point C [Fig. 1.35(*c*)],

$\Sigma F_x = 0 \Rightarrow T_{BC} \cos\theta = T_{CD} \sin 50°$

$T_{CD} = 1.3054\, T_{BC} \cos\theta$ ...(*iii*)

$\Sigma F_y = 0 \Rightarrow T_{BC} \sin\theta + T_{CD} \cos 50° = 30$

$T_{BC}[\sin\theta + 1.3054 \cos\theta \cos 50°] = 30$

$T_{BC}[\sin\theta + 0.8391 \cos\theta] = 30$ ...(*iv*)

Dividing eqn. (2) by eqn. (4), we get

$$\frac{2.7475\cos\theta - \sin\theta}{\sin\theta + 0.8391\cos\theta} = \frac{1}{3}$$

i.e., $8.2425 \cos\theta - 3 \sin\theta$

$= \sin\theta + 0.8391 \cos\theta$

or $4 \sin\theta = 7.4034 \cos\theta$

$\tan\theta = 1.85085$

or $\theta = 61.62°$ **(Ans.)**

∴ From eqn. (*ii*), $T_{BC} = 23.46$ kN **(Ans.)**

From eqn. (*i*), $T_{AB} = 32.61$ kN **(Ans.)**

From eqn. (*iii*), $T_{CD} = 14.56$ kN **(Ans.)**

31. A rope *AB* 6 m long is connected at two points *A* and *B* at the same level 4.5 m apart. A load of 2000 N is suspended from point *C* on the rope at 2 m from *A*, as shown in Fig. 1.36(*a*). What load connected at point *D* on the rope, 1.5 m from *B*, will be necessary to keep *CD* horizontal?

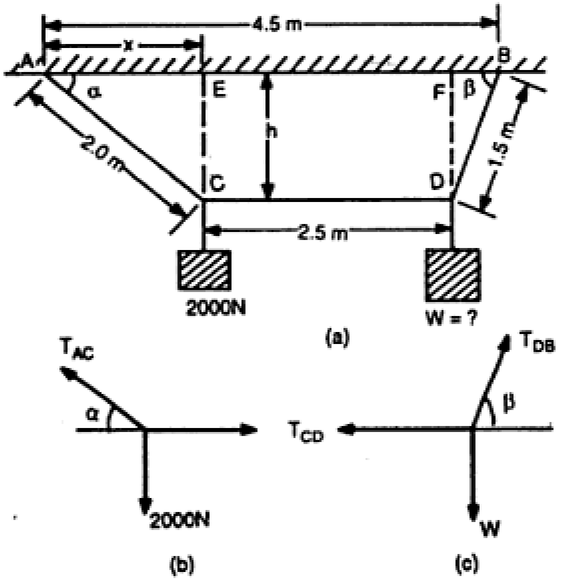


Fig. 1.36

Solution: Referring to Fig. 1.36(*a*),

Let $AE = x$ and seg $EC = h$

From ΔAEC, $x^2 + h^2 = 4$ ...(*i*)

From ΔBDF, $BF^2 + h^2 = 1.5^2$

i.e., $(4.5 - 2.5 - x)^2 + h^2 = 2.25$

i.e., $4 - 4x + x^2 + h^2 = 2.25$

i.e., $x^2 - 4x + h^2 = -1.75$ ...(*ii*)

From eqn. (*i*) and (*ii*)

$-4x + 4 = -1.75$

$$\therefore \quad x = \frac{5.75}{4} = 1.4375 \text{ m}$$

$$h = \sqrt{4 - 1.4375^2} = 1.3905$$

$$\alpha = \tan^{-1}\frac{1.3905}{1.4375} = 44.05°$$

$$\tan\beta = \frac{h}{BF} = \frac{1.3905}{4.5 - 2.5 - 1.4375}$$

∴ $\beta = 68.039°$

Considering equilibrium of joint *C*, we get

$T_{AC} \sin 44.05° = 2000$

∴ $T_{AC} = 2871.3$ N

$T_{CD} = T_{AC} \cos 44.05$

$= 2871.3 \cos 44.05$

$= 2063.7$ N

Considering the equilibrium of joint *D*,

$T_{DB} \cos 68.039 = T_{CD} = 2063.9$

$T_{DB} = 5518.3$ N

and $W = T_{DB} \sin 68.039°$

$= 5117.9$ N **(Ans.)**

32. In the Fig. 1.37 find the tensile force in each of the two guy wires *BF* and *DG*, if the load $Q = 2500$ N, $L = 6$ m and $d = 0.30$ m.

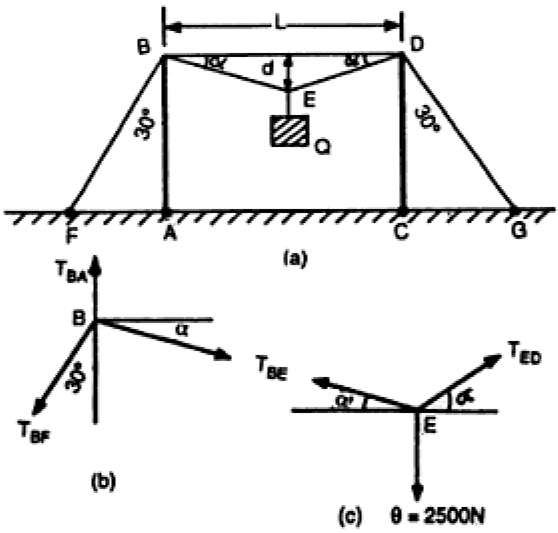


Fig. 1.37

Solution:

From ΔBED, $\tan\alpha = \dfrac{d}{L/2} = \dfrac{0.3}{3} = 0.1$

$\therefore \quad \alpha = 5.7106°$

Equilibrium of joint E gives,

$$T_{BE} = T_{ED}$$

and $T_{BE}\sin\alpha + T_{ED}\sin\alpha = 2500$

$$T_{BE} = \frac{1}{2}\frac{2500}{\sin 5.7106}$$

Since $\quad T_{BE} = T_{ED}$ and $\alpha = 5.7106$

$\therefore \quad T_{BE} = 12562.3$ N

From the equilibrium of joint B,

$$T_{BF}\sin 30° = T_{BE}\cos\alpha$$

$$\therefore \quad T_{BF} = \frac{12562.3\cos 5.7106°}{\sin 30°}$$

$= 2500$ N **(Ans.)**

$T_{BF} = T_{DG}$ by symmetry. **(Ans.)**

33. Three bars hinged at A and D and pinned at B and C, as shown in Fig. 1.38, form a four linked mechanism. Determine the value of P that will prevent movement of bars.

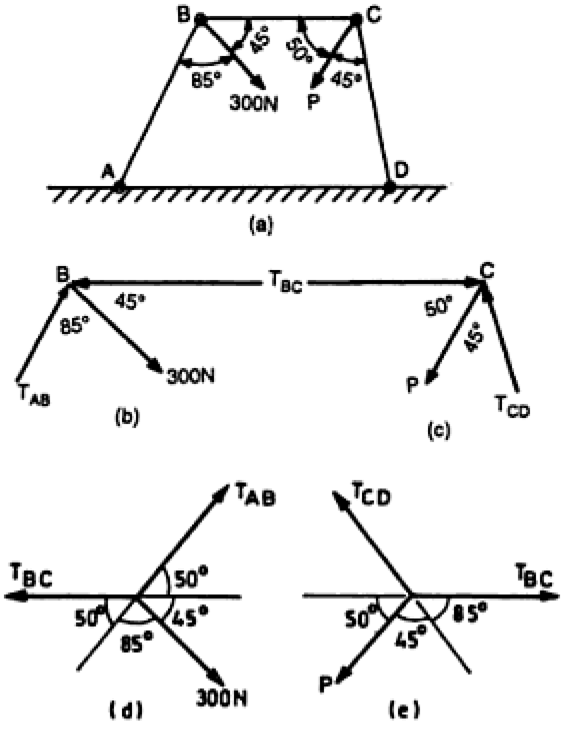


Fig. 1.38

Solution: Referring to *FBD* of joint B [Fig. 1.38(*d*)],

$$\frac{T_{BC}}{\sin(180-85)°} = \frac{300}{\sin(180-50)}$$

$$T_{BC} = 390.1 \text{ N}$$

Referring to *FBD* of joint C [Fig. 1.38(*i*)]

$$\frac{P}{\sin(180-85)°} = \frac{T_{BC}}{\sin(180-45)}$$

$P = 549.6$ N **(Ans.)**

34. A 500 N cylinder is supported by the frame *ABC*, which is hinged at A, and rests against wall *AD*. Determine the reactions at contact surfaces A, B, C and D. [Ref. Fig. 1.39(*a*)].

Solution: Referring to *FBD* of Cylinder [Fig. 1.39(*b*)],

$R_C = 500$ N **(Ans.)**

$R_B = R_D$

Referring to *FBD* of rigid frame [Fig. 1.39(*c*)],

$$\tan\alpha = \frac{300}{100}$$

$\therefore \quad \alpha = 71.565°$

$R_A \sin\alpha = R_C = 500$

$$\therefore \quad R_A = \frac{500}{\sin 71.565°} = 527.0 \text{ N} \quad \textbf{(Ans.)}$$

and $\quad R_B = R_A\cos\alpha = 527\cos 71.565°$

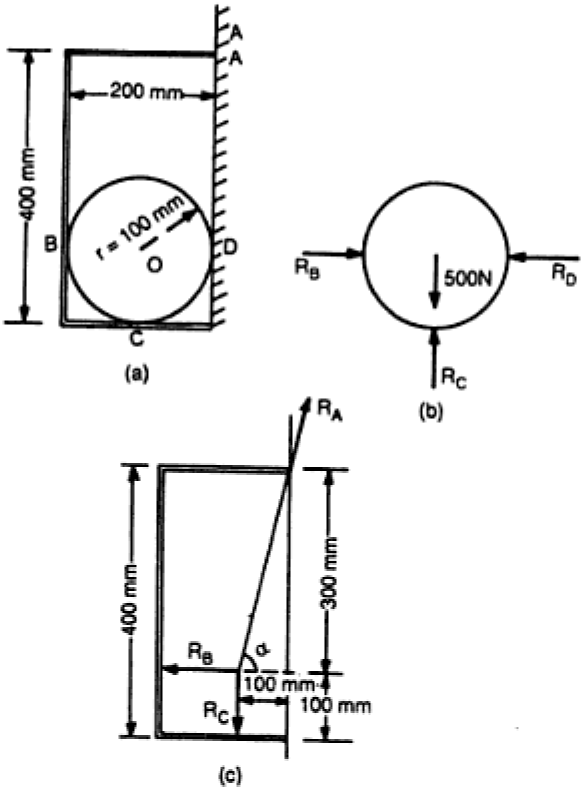


Fig. 1.39

$= 166.67$ N **(Ans.)**

$\therefore \quad R_D = R_B = 166.67$ N **(Ans.)**

35. Two smooth spheres, each of radius 150 mm and weighing 250 N rest in a horizontal channel having vertical walls, the distance between the walls being 560 mm. Find the reactions at the points of contacts *A, B, C* and *D* as shown in Fig. 1.40.

Solution: Referring to Fig. 1.40(*a*), angle α is given by

$$\cos\alpha = \frac{560-150-150}{O_1O_2} = \frac{260}{150+150} = 0.867$$

or $\alpha = 29.93°$.

Considering the equilibrium of sphere with centre at O_1 [Ref. Fig. 1.40(*c*)],

$\Sigma F_y = 0 \Rightarrow R_B \sin\alpha = 250$

i.e., $R_B = 501.1$ N **(Ans.)**

and $\Sigma F_x = 0 \Rightarrow R_A = R_B \cos\alpha$

$= 501.1 \cos 29.93°$

$= 432.3$ N **(Ans.)**

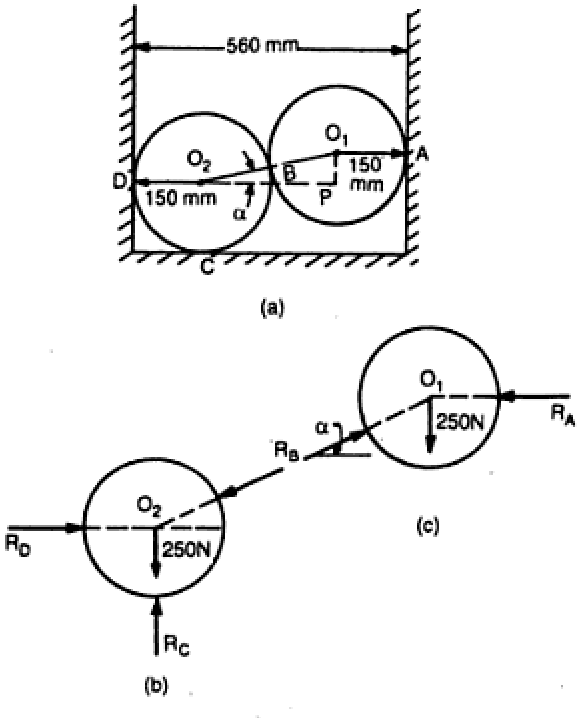


Fig. 1.40

Now, considering the equilibrium of sphere with centre at O_2 [Ref. Fig. 1.40 (*b*)],

$\Sigma F_y = 0 \Rightarrow R_C - 250 - R_B \sin\alpha = 0$

$R_C = 250 + 501.1 \sin 29.93°$

$= 500$ N **(Ans.)**

$\Sigma F_x = 0 \Rightarrow R_D - R_B \cos\alpha = 0$

$\therefore \quad R_D = 501.1 \cos 29.93°$

$= 434.29$ N **(Ans.)**

36. Two identical rollers, each weighing 120 N are placed in a trough as shown in Fig. 1.41(*a*). Assuming all surfaces of contact are smooth, find the reactions developed at the contact surfaces.

Solution: Referring to *FBD* of first roller [Fig. 1.41(*c*)],

Σ forces normal to the plane = 0, gives

$R_A - 120 \cos 40° = 0$

$\therefore R_A = 91.94$ N **(Ans.)**

Σ forces parallel to plane = 0, gives

$R_B - 120 \cos 50° = 0 \therefore R_B = 77.1$ N **(Ans.)**

Referring to the equilibrium of second roller, [Ref. Fig. 1.41(*b*)],

$\Sigma F_y = 0 \Rightarrow R_C \cos 40° - 120 - R_B \sin 40° = 0$

$$R_C = \frac{120 + 77.1 \sin 40°}{\cos 40°} = 221.4 \text{ N } \textbf{(Ans.)}$$

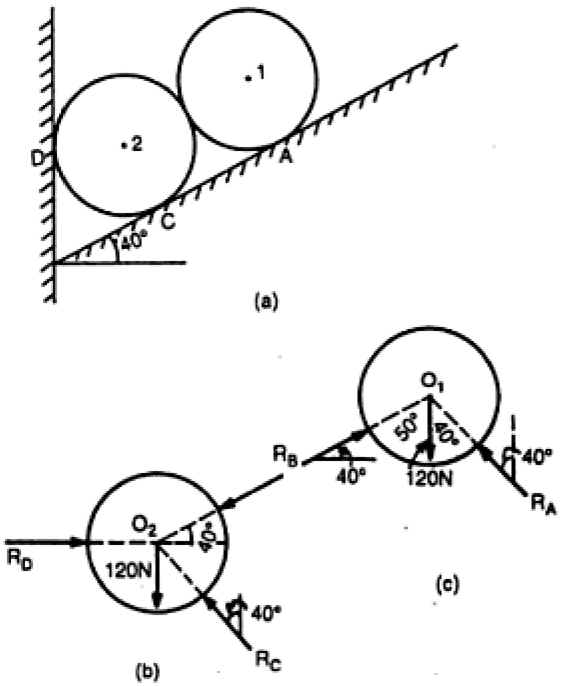


Fig. 1.41

$\Sigma F_x = 0 \Rightarrow R_D - R_C \sin 40° - R_B \cos 40° = 0$

$R_D = 221.4 \sin 40° + 77.1 \cos 40°$

$= 201.4$ **(Ans.)**

37. Two cylinders are placed in a trough as shown in Fig. 1.42(*a*). Neglecting friction, find the reactions at all contact surfaces, given.

Diameter of first cylinder = 120 mm
Diameter of second cylinder = 60 mm
Weight of first cylinder = 250 N
Weight of second cylinder = 100 N

Solution: Referring to Fig. 1.42(*a*),

$$\cos\alpha = \frac{140-30-60}{60+30} = \frac{50}{90}$$

$\therefore \quad \alpha = 56.25°$

Considering equilibrium of second cylinder [Ref. 1.42(*b*)]

$\Sigma F_y = 0 \Rightarrow R_2 \sin 56.25° = 100$

$\therefore \quad R_2 = 120.3$ N **(Ans.)**

$\Sigma F_x = 0 \Rightarrow R_1 - R_2 \cos\alpha = 0$

$\therefore \quad R_1 = 120.3 \cos 56.25° = 66.82$ N **(Ans.)**

Considering equilibrium of first cylinder [Ref. Fig. 1.42(*c*)],

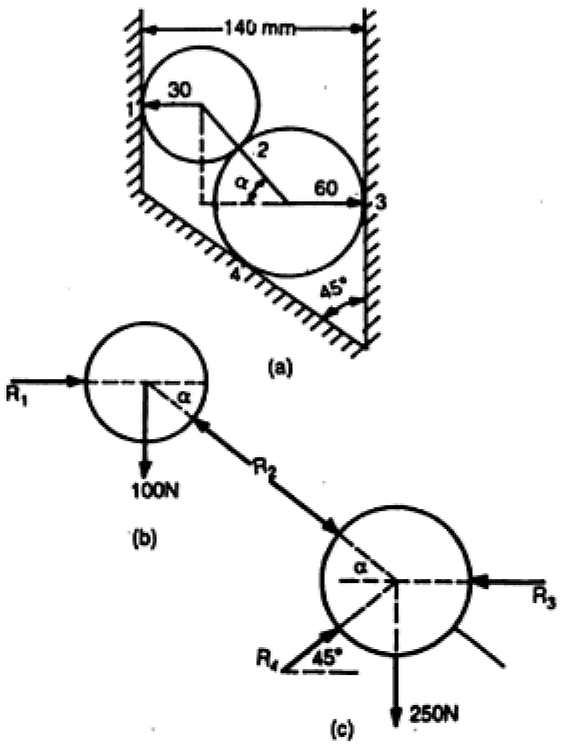


Fig. 1.42

$\Sigma F_y = 0 \Rightarrow R_4 \sin 45° - 250 - R_2 \sin\alpha = 0$

$$R_4 = \frac{250 + 120.3 \sin 56.25°}{\sin 45°} = 495 \text{ N (Ans.)}$$

$\Sigma F_x = 0 \Rightarrow R_4 \cos 45° + R_2 \cos\alpha - R_3 = 0$

$\therefore R_3 = 495 \cos 45° + 120.3 \cos 56.25°$

$= 416.9$ N **(Ans.)**

38. Cylinder *A* and *B* weighing 5000 N and 2000 N rest on smooth incline planes as shown in Fig. 1.43. Neglecting the weight of connecting bar and assuming smooth pin connections, find the force *P* to be applied such that the system is in the equilibrium.

Solution: Applying Lami's theorem to the equilibrium condition of cylinder *A* [Ref. Fig. 1.43(*b*)].

$$\frac{C}{\sin 60°} = \frac{5000}{\sin(60+90-20)}$$

$\therefore \quad C = 5652.6$ N

Considering the equilibrium of cylinder *B* [Fig. 1.42(*c*)].

$\Sigma F_x = 0 \Rightarrow -P \cos 45° - R_2 \sin 30°$

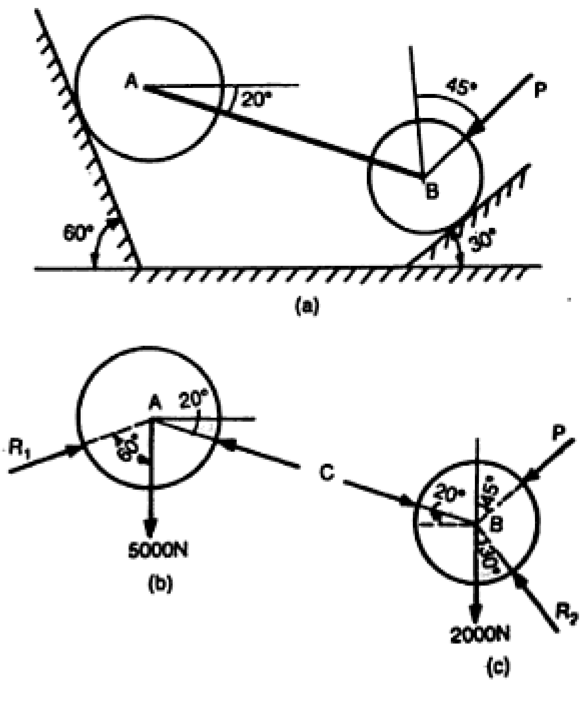


Fig. 1.43

$+ 5652. \cos 20° = 0$

$$\frac{P}{\sqrt{2}} + 0.5 R_2 = 5311.7 \qquad ...(i)$$

$$\Sigma F_y = 0 \Rightarrow -P\cos 45° + R_2\cos 30° - 2000 - 5652.6\sin 20° = 0$$

$$\frac{-P}{\sqrt{2}} + 0.866\,R_2 = 3933.3 \qquad ...(ii)$$

From eqns. (*i*) and (*ii*),

$$1.366\,R_2 = 5311.7 + 3933.3$$

$$R_2 = 6767.9 \text{ N}$$

$$\therefore \quad P = (5311.7 - 0.5 \times 6767.9)\sqrt{2} = 2726.25 \text{ N } \textbf{(Ans.)}$$

39. The weights and radii of the three cylinders piled in a rectangular ditch as shown in Fig. 1.44 are as given below:

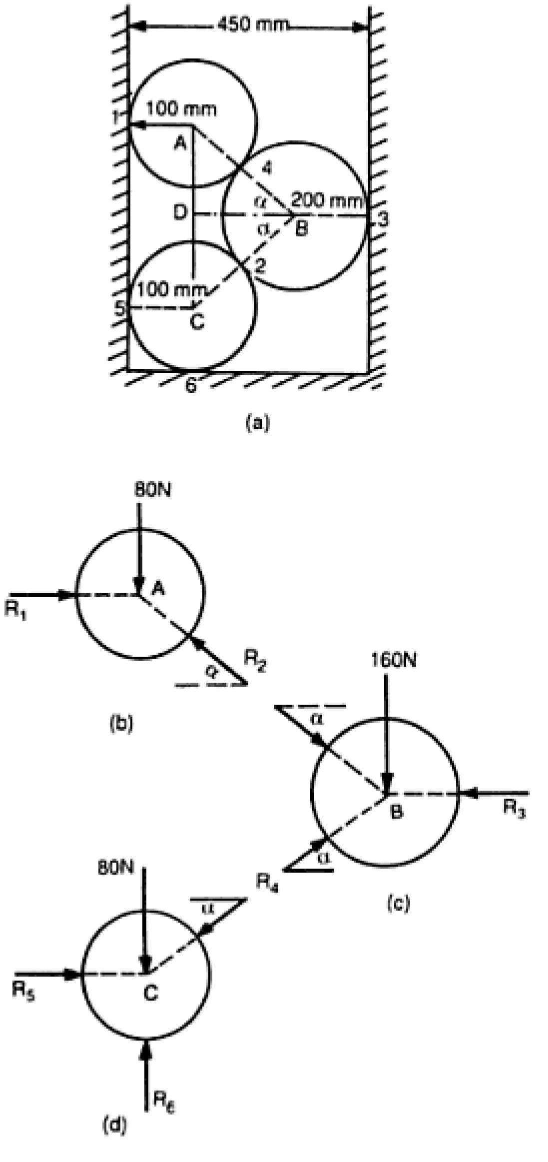


Fig. 1.44

Cylinder	*Weight*	*Radius*
A	80 N	100 mm
B	160 N	200 mm
C	80 N	100 mm

Assuming all contact surfaces to be smooth, determine the reactions acting on cylinder C

Solution: Referring to the Fig. 1.44(*a*),

$$\cos\alpha = \frac{BD}{BA} = \frac{450-100-200}{200+100} = 0.5$$

$$\therefore \quad \alpha = 60°$$

From the equilibrium condition for cylinder *A* [Fig. 1.44(*b*)],

$$\Sigma F_y = 0 \Rightarrow R_2\sin\alpha = 80$$

$$\therefore \quad R_2 = \frac{80}{\sin 60°} = 92.4 \text{ N}$$

Referring to the *FBD* of cylinder *B* [Fig. 1.44(*c*)],

$$\Sigma F_y = 0 \Rightarrow R_4\sin\alpha - 160 - R_2\sin\alpha = 0$$

$$\therefore \quad R_4 = \frac{160 + 92.4\sin 60°}{\sin 60°} = 277.1 \text{ N } \textbf{(Ans.)}$$

Referring to the *FBD* of cylinder *C* [Fig. 1.44(*d*)],

$$\Sigma F_x = 0 \Rightarrow R_5 - R_4\cos\alpha = 0$$

$$R_5 = 277.1\cos 60° = 138.6 \text{ N } \textbf{(Ans.)}$$

$$\Sigma F_y = 0 \Rightarrow R_6 - 80 - R_4\sin\alpha = 0$$

$$R_6 = 80 + 277.1\sin 60° = 320 \text{ N } \textbf{(Ans.)}$$

40. The spheres *A*, *B* and *C* weighing 200 N, 400 N and 200 N respectively and having radii 400 mm, 600 mm and 400 mm respectively are placed in a trench as shown in Fig. 1.45(*a*). Treating all contact surfaces as smooth, determine the reactions developed.

Solution: Referring to the Fig. 1.45(*a*),

$$\sin\alpha = \frac{BD}{AB} = \frac{600-400}{400+600} = 0.2$$

$$\therefore \quad \alpha = 11.537°$$

Referring to *FBD* of sphere *A* [Fig. 1.45(*b*)],

$$R_2\cos\alpha = 200$$

$$\therefore \quad R_2 = \frac{200}{\cos 11.537°} = 204.1 \text{ N } \textbf{(Ans.)}$$

and $R_1 - R_2\sin\alpha = 0$

$$\therefore \quad R_1 = 40.8 \text{ N } \textbf{(Ans.)}$$

Referring to the *FBD* of sphere *C* [Fig. 1.45(*c*)],

Σ forces $\parallel$ to inclined plane = 0

$\Rightarrow$ $R_4 \cos\alpha - 200 \cos 45° = 0$

$R_4 = 144.3$ N **(Ans.)**

$\Sigma F_x = 0 \Rightarrow R_4 \cos(45 - \alpha) - R_3 \cos 45 = 0$

$R_3 = 170.3$ N **(Ans.)**

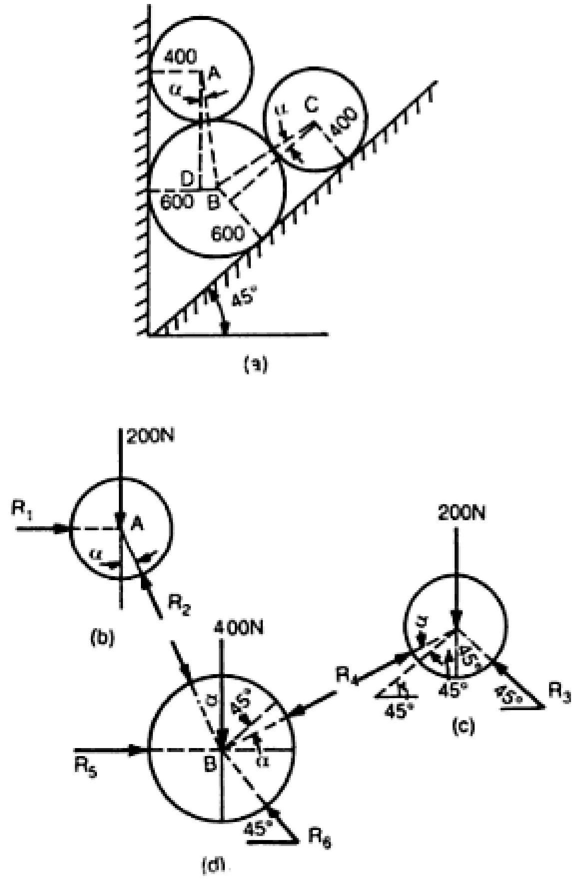


Fig. 1.45

Referring to *FBD* of cylinder *B* [Fig. 1.45(*d*)],

$\Sigma F_y = 0 \Rightarrow R_6 \sin 45 - 400 - R_2 \cos\alpha$
$- R_4 \cos(45 + \alpha) = 0$

$R_6 \sin 45° = 400 + 204.1 \cos 11.537°$
$+ 144.3 \cos 56.537°$

$\therefore$ $R_6 = 961.0$ N **(Ans.)**

$\Sigma F_x = 0 \Rightarrow R_5 - R_2 \sin\alpha - R_4 \sin(45 + \alpha) - R_6 \cos 45° = 0$

$\therefore$ $R_5 = 204.1 \sin 11.537 + 144.3 \sin 56.537 + 961.0 \cos 45°$
$= 840.7$ N **(Ans.)**

41. Three spheres are piled in a trench as shown in Fig. 1.46(*a*). Self weight and radii of the cylinders are as given below:

Spheres	*Weight*	*Radius*
A	2 kN	400 mm
B	2 kN	400 mm
C	4 kN	600 mm

Treating all contact surfaces as smooth, determine the reactions developed at the contact surfaces *P*, *Q*, *R* and *S*. Given: centre to centre distance between sphere *A* and *B* is 500 mm.

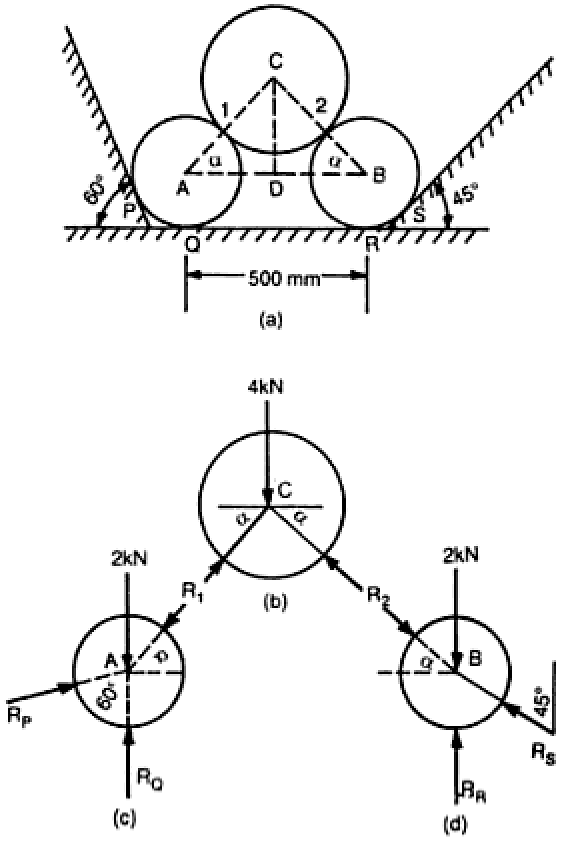


Fig. 1.46

Solution: From triangle *ABC* in Fig. 1.46(*a*),

$$\cos\alpha = \frac{AD}{AC} = \frac{250}{400 + 600} = 0.25$$

$\therefore$ $\alpha = 75.522°$

Referring to *FBD* of sphere C [Fig. 1.46(*b*)],

$\Sigma F_x = 0 \Rightarrow R_1 \cos\alpha = R_2 \cos\alpha$

i.e., $R_1 = R_2$

$$\Sigma F_y = 0 \Rightarrow R_1 \sin\alpha + R_2 \sin\alpha = 4$$

$$2R_1 \sin\alpha = 4$$

$$R_1 = \frac{4}{2\sin 75.522} = 2.066 \text{ kN} = R_2$$

Referring to *FBD* of sphere *A* [Fig. 1.46(*c*)],

$$\Sigma F_x = 0 \Rightarrow R_p \sin 60° - R_1 \cos\alpha = 0$$

$$R_p = \frac{2.066 \cos 75.522°}{\sin 60°}$$

$$= 0.596 \text{ kN (Ans.)}$$

$$\Sigma F_y = 0 \Rightarrow R_Q - R_1 \sin\alpha - 2 - R_p \cos 60° = 0$$

$$R_Q = 2.066 \sin 75.522 + 2 + 0.596 \cos 60°$$

$$= 4.298 \text{ kN (Ans.)}$$

Referring to *FBD* of sphere *B* [Fig. 1.46(*d*)],

$$\Sigma F_x = 0 \Rightarrow R_2 \cos\alpha - R_s \sin 45° = 0$$

$$R_s = \frac{2.066 \cos 75.522°}{\sin 45°} = 0.730 \text{ kN} \quad \textbf{(Ans.)}$$

$$\Sigma F_y = 0 \Rightarrow R_R + R_S \cos 45° - R_2 \sin\alpha - 2 = 0$$

$$R_R = -0.730 \cos 45° + 2.066 \sin 75.522 + 2$$

$$= 1.484 \text{ kN (Ans.)}$$

CHAPTER 2

Coplanar Non-concurrent Force Systems

GENERAL INFORMATION

COPLANAR NON-CONCURRENT FORCE SYSTEM

If all the forces in a system lie in the same plane and the lines of action of all the forces do not pass through a single point, the system is said to be *Coplanar Non-concurrent Force System.*

Moment of a Force

Moment of force about a point is the measure of its rotational effect. It is defined as the product of the magnitude of the force and the perpendicular distance of the point from the line of action of the force. The point about which the moment is considered is called *'moment centre'* and the perpendicular distance of the point from the line of action of the force is called *'moment arm'*. In Fig. 2.1 *'O'* is moment centre and *d* is moment arm.

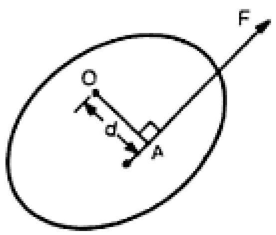


Fig. 2.1

If a point lies on the line of action of a force the moment of the force about that point is zero. Moment of a force has got both magnitude and sense. The sense can be clockwise or anti-clockwise. Commonly used units of moment are Nmm, kNmm, Nm etc.

Varignon's Theorem

This theorem is also known as *'Principle of moments'*. The theorem states that:
"The algebraic sum of moments of a system of coplanar forces about a moment centre in their plane is equal to the moment of the resultant force about the same moment centre.

Couple

A couple consists of two parallel forces equal in magnitude and opposite in direction and separated by a definite distance. The sum of forces forming a couple in any direction is zero, which means that the *translatory effect of the couple is zero. Moment of the couple about any point is same.* The effect of the couple is unchanged if

⇒ The couple is rotated through an angle.
⇒ The couple is shifted to any other position.
⇒ The couple is replaced by another pair of forces whose rotational effect is the same.

Solution: The resultant force of the system is obtained by algebraic summation of the forces, since the forces are all parallel.

$$R = \Sigma F_y = +10 - 50 - 40$$
$$= -80 \text{ kN} = 80 \text{ kN} (\downarrow) \text{ (Ans.)}$$

This acts at a distance of d, where

$$d = \frac{+(50 \times 4) + (40 \times 6)}{80}$$
$$= 5.5 \text{ m from } O \text{ (Ans.)}$$

Now, if an additional force of 20 kN acts along the bar A to B, the two forces should be added vectorially to obtain the resultant action.

$$R = \sqrt{80^2 + 20^2}$$
$$= 82.46 \text{ N (Ans.)}$$
$$\alpha = \tan^{-1}\left(\frac{80}{20}\right)$$
$$= 75.96° \text{ (Ans.)}$$

7. Find the resultant of the coplanar parallel forces acting on the truss shown in Fig. 2.11.

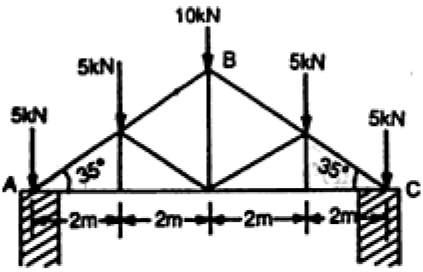


Fig. 2.11

Solution: The forces are all parallel and are in same direction. Hence they may be added arithmetically to get the resultant R.

$$R = -5 - 5 - 10 - 5 - 5$$
$$= -30 \text{ kN} = 30 \text{ kN} (\downarrow) \text{ (Ans.)}$$

To determine the position of R, take moments of all forces about A,

$$R \times d = 30 \times d = (5 \times 0) + (5 \times 2) + (10 \times 4) + (5 \times 6) + (5 \times 8) = 120$$
$$d = \frac{120}{30}$$
$$= 4 \text{ m (Ans.)}$$

$\therefore$ R acts vertically downwards 4 m from point A, and its magnitude is 30 kN.

8. A pulley of 1 m diameter is subjected to 2 kN and 3 kN forces at A and B respectively as shown in Fig. 2.12. Its own weight of 1 kN acts through the centre O. Determine resultant force and its line of action with respect to AOB.

Solution: The 2 kN and 3 kN forces acting at A and B are to be transferred to O by an equivalent force couple system. Hence their would be a force of (2 + 3) = 5 kN acting

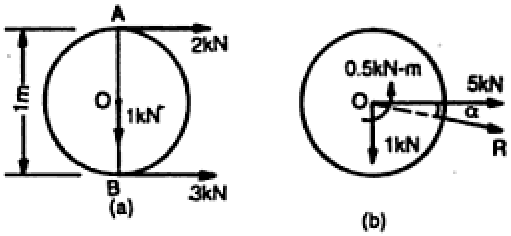


Fig. 2.12

rightwards. At 'O' with couples of moment + 1 kNm and −1.5 kNm.

Now vertically adding the self weight of 1 kN and the 5 kN force at O,

$$R = \sqrt{5^2 + 1^2}$$
$$= \sqrt{26} = 5.1 \text{ kN (Ans.)}$$
$$\alpha = \tan^{-1}\left(\frac{1}{5}\right) \text{ (Angle with horizontal)}$$
$$= 11.31° \text{ (Ans.)}$$

Net moment at centre O = −0.5 kNm **(Ans.)**

9. A 2 × 4 m plate is subjected to a system of two coplanar forces as shown in Fig. 2.13. Determine the equivalent action at 'O', which may replace the force system.

Solution: Resolving 3 kN and 6 kN forces in their x and y components as shown in Fig. 2.13(b).

Moment at 'O' due to a force of 3 sin 60 = 3 sin 60 × 2 = 5.196 kNm

Moment at 'O' due to 3 cos 60

$$= 3 \cos 60 \times 1 = +1.5 \text{ kNm}$$

Moment at 'O' due to 6 cos 45

$$= -6 \cos 45 \times 1 = -4.24 \text{ kNm}$$

Moment at 'O' due to 6 sin 45

$$= -6 \sin 45 \times 2 = -8.49 \text{ kNm}$$

∴ Total moment at 'O'
$= -6.03 = 6.03$ kN-m, Anticlockwise **(Ans.)**
Now adding forces vectorially using the method of components,

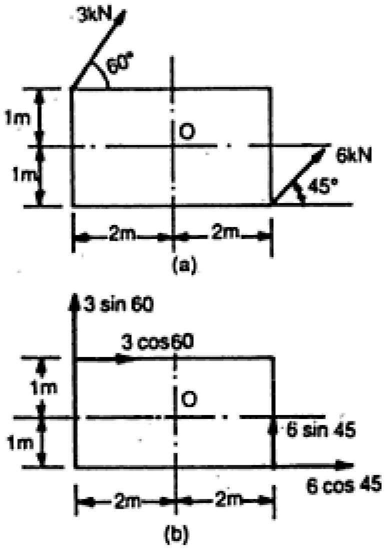


Fig. 2.13

$\sum F_x = 3\cos 60 + 6\cos 45 = 1.5 + 4.24$
$= 5.74$ kN
$\sum F_y = 3\sin 60 + 6\sin 45 = 2.6 + 4.24$
$= 6.84$ kN

$$R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2}$$

$$= \sqrt{5.74^2 + 6.84^2}$$

$= 8.93$ kN **(Ans.)**

$$\alpha = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right) = 50° \text{ (Ans.)}$$

10. A dam is subjected to 3 forces: 50 kN force on the upstream vertical face *AB*, 40 kN force on the downstream inclined face and it's own weight of 140 kN as shown in Fig. 2.14. Determine the single equivalent force and locate it's point of intersection with the base *AD*, assuming all the forces to lie in the same plane.

Solution:

$\sum F_x = +50 - 40\sin 60 = +15.36$ kN
$\sum F_y = -140 - 40\cos 60 = -160$ kN

$$R = \sqrt{\sum F_x^2 + \sum F_y^2} = 160.74 \text{ kN (Ans.)}$$

α = Angle with horizontal

$$= \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right)$$

$$= \tan^{-1}\left(\frac{160}{15.36}\right)$$

$\alpha = 84.51°$ As shown in Fig. 2.14 **(Ans.)**

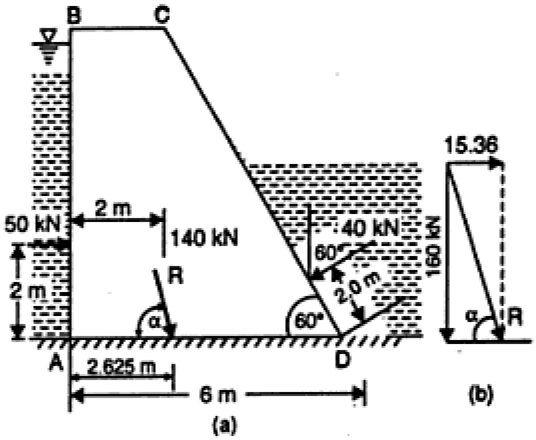


Fig. 2.14

To locate position of the resultant w.r.t. point *A*, along the base width

$\sum M_A = +(50 \times 2) + (140 \times 2) - (40\sin 60) \times 2\sin 60 + (40\cos 60) \times (6 - 2\cos 60)$
$= +100 + 280 - 60 + (100)$
$= 420$ kNm

$$\therefore \quad d = \frac{\sum M_A}{\sum F_y}$$

$$= \frac{420}{160}$$

$= 2.625$ m from point *A*. **(Ans.)**

11. Determine the resultant of the forces acting on a bell-crank lever as shown in the Fig. 2.15.

Solution:

$\sum F_x = +150\cos 60 + 300$
$= +375$ N

$\sum F_y = -150\sin 60 - 100$
$= -229.9$ N

$$R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} = 439.9 \text{ N. (Ans.)}$$

$$\alpha = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right)$$

$$= \tan^{-1}\left(\frac{229.9}{375}\right)$$

= 31.511° (With horizontal) as shown in Fig. 2.15(*b*). (Ans.)

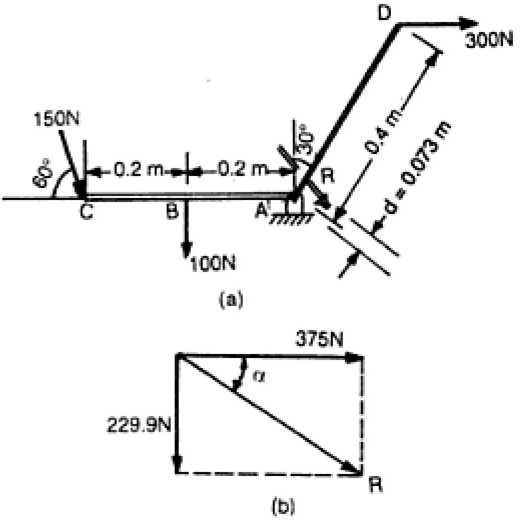


Fig. 2.15

Taking moments about point A,

$$\Sigma M_A = -(150 \times \sin 60) \times 0.4 + (300 \times 0.4)\cos 30 - 100 \times 0.2 = 31.96 \text{ Nm}$$

Equating it to the moment of resultant force, we get

$$Rd = 31.96$$

$$\therefore \quad d = \frac{31.96}{439.9} = 0.073 \text{ m (Ans.)}$$

12. A bracket is subjected to a coplanar force system as shown in Fig. 2.16(*a*). Determine the magnitude and the line of action of the single resultant of the system.

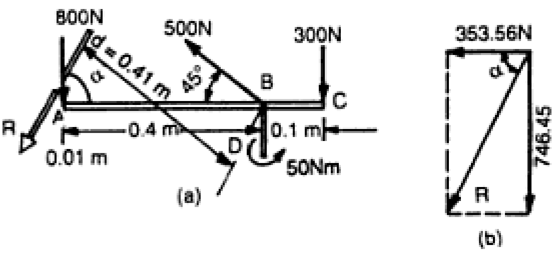


Fig. 2.16

Solution:

$$\Sigma F_x = -500 \cos 45$$
$$= -353.56 \text{ N}$$

$$\Sigma F_y = -800 - 300 + 500 \sin 45$$
$$= -746.45 \text{ N}$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$= \sqrt{(-353.56)^2 + (746.45)^2}$$
$$= 825.95 \text{ N (Ans.)}$$

$$\alpha = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right) = \tan^{-1}\left(\frac{746.45}{353.56}\right)$$

= 64.66° with horizontal as shown in Fig. 2.16(*b*). (Ans.)

Taking moment about point B,

$$-(800 \times 0.4) + (300 \times 0.1) - 50 = \Sigma M_B$$
$$\Sigma M_B = -340 \text{ (anticlockwise)}$$

Equating it to the moment of resultant,

$$825.95\, d = 340$$

$$\therefore \quad d = 0.41 \text{ m from } B.$$

= 0.41 m as shown in Fig. 2.16 (*a*) (Ans.)

13. Compute the resultant of the force system shown in the Fig. 2.17.

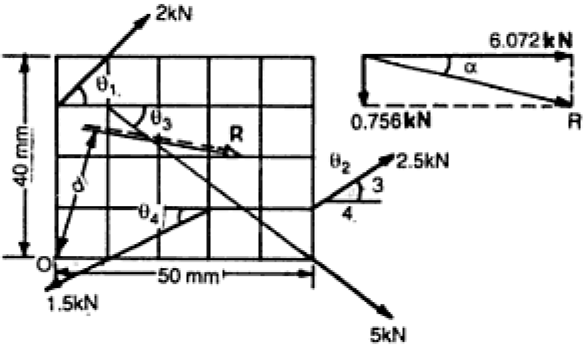


Fig. 2.17

Solution:

$$\theta_1 = \tan^{-1}\frac{10}{10} = 45°$$

$$\theta_2 = \tan^{-1}\frac{3}{4} = 36.87°$$

$$\theta_3 = \tan^{-1}\frac{30}{40} = 36.87°$$

$$\theta_4 = \tan^{-1}\frac{1}{2} = 26.56°$$

$$\Sigma F_x = 2\cos 45 + 2.5\cos 36.87 + 5\cos 36.87 - 1.5\cos 26.56$$
$$= +6.072 \text{ kN } (\rightarrow)$$

Solution:

$$\Sigma F_x = +100 - 130\cos 30 - 150\cos 60$$
$$= -87.58 \text{ N}$$
$$\Sigma F_y = -100 + 150\sin 60 - 130\sin 30$$
$$= -35.09 \text{ N}$$

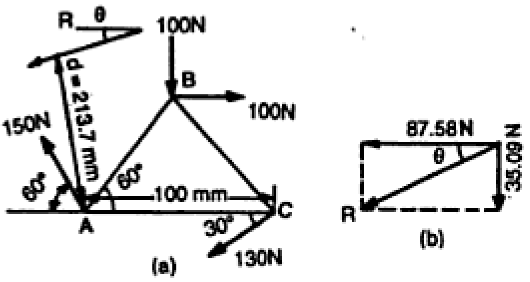


Fig. 2.22

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$= \sqrt{(87.58)^2 + (35.09)^2}$$
$$= 94.35 \text{ N} \quad \textbf{(Ans.)}$$
$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{35.09}{87.58}\right)$$
$$= 21.83° \text{ [as shown in Fig. 2.22(b)]}$$

Taking moment of all forces about *A*,

$$\Sigma M_A = +100 \times 100\cos 60° + 100 \times 100\sin 60° + 130 \times \sin 30 \times 100$$
$$= 20160.3 \text{ Nmm}$$
$$\Sigma M_A = R.d$$
$$20160.3 = 94.35\, d$$
$$\therefore \quad d = 213.7 \text{ mm from } A. \quad \textbf{(Ans.)}$$

19. Determine and locate the resultant *R* of the two forces and one couple acting on the *I* beam shown in Fig. 2.23.

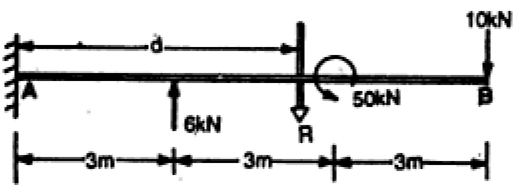


Fig. 2.23

Solution:

$$\Sigma F_y = -10 + 6$$
$$= -4 \text{ kN}$$
$$\Sigma F_x = 0$$
$$\therefore \quad R = -4 \text{ kN}$$
$$= 4 \text{ kN } (\downarrow)$$
$$\Sigma M_A = +10 \times 9 - 6 \times 3 - 50$$
$$= +22 \text{ kNm}$$
$$\therefore \quad Rd = \Sigma M_A$$
$$4(d) = 22$$
$$d = 5.5 \text{ m from } A.$$

[as shown in Fig. 2.23] **(Ans.)**

20. Determine the resultant *R* of three forces and two couples shown in Fig. 2.24. Find the co-ordinate *x* of the point on the *x* axis through which *R* passes.

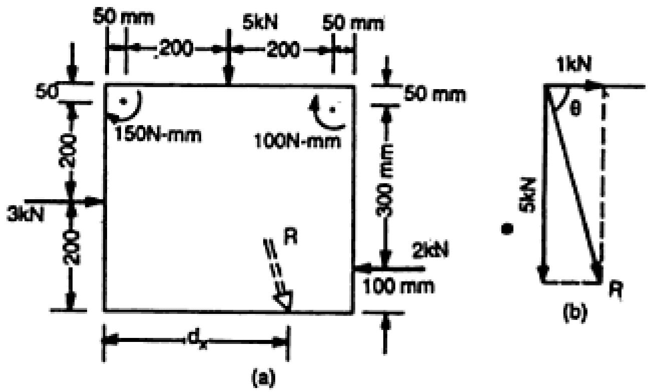


Fig. 2.24

Solution:

$$\Sigma F_x = 3 - 2$$
$$= 1 \text{ kN}$$
$$\Sigma F_y = -5 \text{ kN}$$
$$R = \sqrt{1^2 + 5^2}$$
$$= \sqrt{26} = 5.1 \text{ kN.} \quad \textbf{(Ans.)}$$
$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$
$$= \tan^{-1}\left(\frac{5}{1}\right)$$
$$= 78.69° \quad \textbf{(Ans.)}$$

Let d_x be the horizontal distance from *A* through which the resultant acts.

Then

$$\Sigma F_y\, d_x = \Sigma M_A = 5 \times 250 + 3 \times 200 - 2 \times 100 + 150 + 100 = 1900$$
$$d_x = \frac{1900}{5} = 380 \text{ mm} \quad \textbf{(Ans.)}$$

21. What force and moment is transmitted to the supporting wall at *A*? (Refer Fig. 2.25)

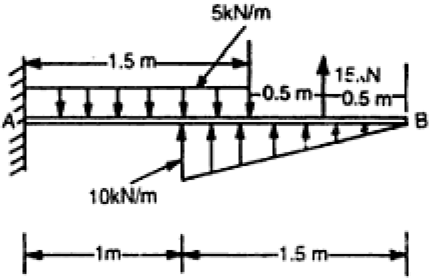


Fig. 2.25

Solution: $\Sigma F_x = 0$

$$\Sigma F_y = -5 \times 1.5 + 15 + \frac{1}{2} \times 1.5 \times 10$$
$$= 15 \text{ kN}$$

$$\Sigma M_A = 1.5 \times 5 \times 0.75 - 15 \times 2 - \frac{1}{2} \times 1.5 \times 10 \times (2.5 - 1.0)$$
$$= -35.625 \text{ kNm}$$

∴ A force of 15 kN (vertical) (↑) is transmitted to the wall along with an anticlockwise moment of 35.625 kNm.

22. A jet plane of mass 30 Mg is climbing at a 15° angle with constant velocity. If the drag force *D* is = 10 kN, find the required thrust, *T*, lift force *L*, and force *P* acting on the rear stabilizers.

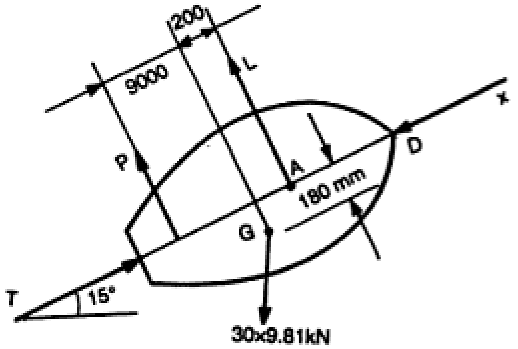


Fig. 2.26

Solution: Since the jet climbs at a constant velocity,

$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma M = 0$

Note: 1 Mg = 1000 kg = 9.81 × 1000 N = 9.81 kN.

$\Sigma F_x = 0 \Rightarrow +T - D - (30 \times 9.81) \sin 15$ ...(*i*)

∴ $T = 86.17$ kN **(Ans.)**

$\Sigma F_y = 0 \Rightarrow P + L - 30 \times 9.81 \cos 15 = 0$

$P + L = (30 \times 9.81) \cos 15$
$= 284.27$ kN

$\Sigma M_A = 0 \Rightarrow (P \times 9200) + (30 \times 9.81) \sin 15 \times 180 - (30 \times 9.81) \cos 15 \times 200 = 0$

∴ $P = 4.69$ kN **(Ans.)**

∴ $L = 284.27 - P$
$= 284.27 - 4.69$
$L = 279.58$ kN **(Ans.)**

23. Determine the magnitude, direction and line of action of the resultant of the given set of coplanar forces acting on a planar structure shown in Fig. 2.27.

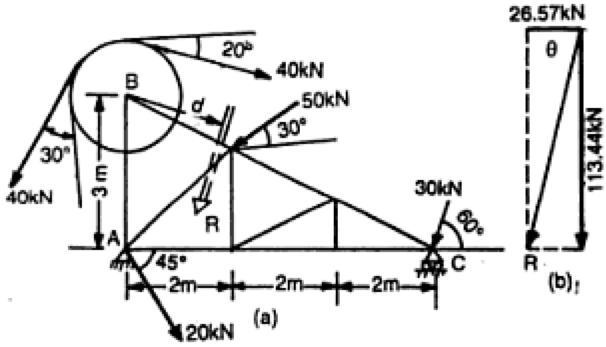


Fig. 2.27

Solution:

$$\Sigma F_x = -40 \sin 30 + 40 \cos 20 - 50 \cos 30 - 30 \cos 60 + 20 \cos 45$$
$$= -26.57 \text{ kN}$$

$$\Sigma F_y = -40 \cos 30 - 40 \sin 20 - 50 \sin 30 - 30 \sin 60 - 20 \sin 45$$
$$= -113.44 \text{ kN}$$

$$\therefore R = \sqrt{(-26.57)^2 + (-113.44)^2}$$
$$= 116.51 \text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{113.44}{26.57}\right)$$

= 76.82° (as shown in Fig. 2.27) **(Ans.)**

Replacing the two 40 kN forces by their equivalents at *B* and noticing that the moment by the two forces cancel each other,

$$M_B = 50 \cos 30\left[3 - \frac{4}{6}3\right] + 50 \sin 30 \times 2$$

$$+ 30 \cos 60 \times 3 + 30 \sin 60° \times 6$$

$$- 20 \cos 45° \times 3$$

$$= 251.76 \text{ kNm}$$

$$\therefore R.d = 251.76$$

$$\text{or } d = \frac{251.76}{116.51} = 2.16 \text{ m (Ans.)}$$

EQUILIBRIUM OF COPLANAR NON-CONCURRENT FORCE SYSTEM

A body acted upon by a system of coplanar Non-concurrent forces is said to be in static equilibrium, when it does not have any translatory or rotatory motion in any direction. According to Newton's second law of motion, it means, the resultant force acting on the body should be zero. i.e., Algebraic sum of components of all the forces in any two mutual perpendicular directions should be zero,

i.e.,

$$\Sigma F_x = 0 \quad ...(2.6)$$

$$\Sigma F_y = 0 \quad ...(2.7)$$

and the algebraic sum of moments of all forces acting on the body about any point in the plane of the body should be zero.

$$\Sigma M_A = 0 \quad ...(2.8)$$

Reactions at Supports of Beams

A beam may be defined as a structural element which has one dimension (length) considerably larger compared to the other two directions i.e., breadth and depth, and is supported at few points. It is usually loaded in vertical direction. Due to applied loads reactions develop at supports. The system of forces consisting of applied loads and reactions keep the beam in equilibrium.

Following are different types of supports (See Fig. 2.28)

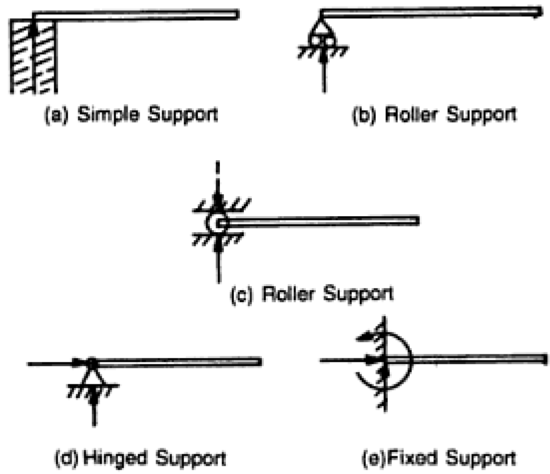


Fig. 2.28

Following are the types of beams (See Fig. 2.29)

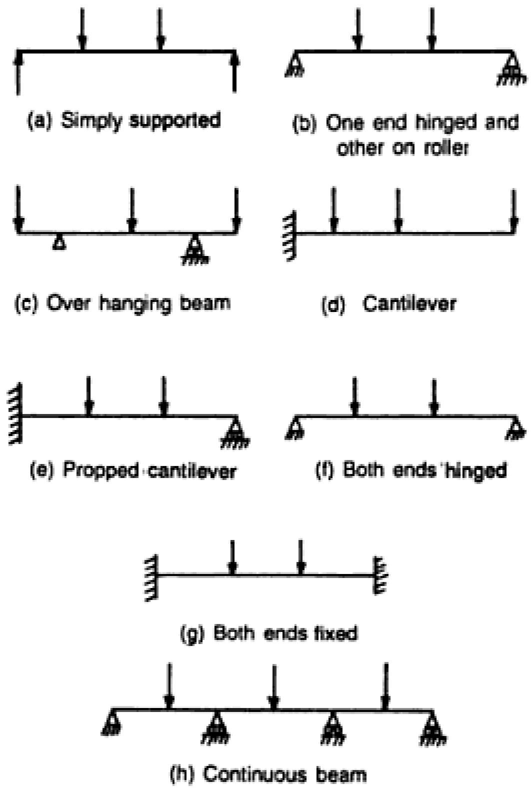


Fig. 2.29

Types of Loading

Beams are subjected to different types of loadings like concentrated or point loads, uniformly distributed loads (UDL), uniformly varying loads, external moments and general loading. See Fig. 2.30.

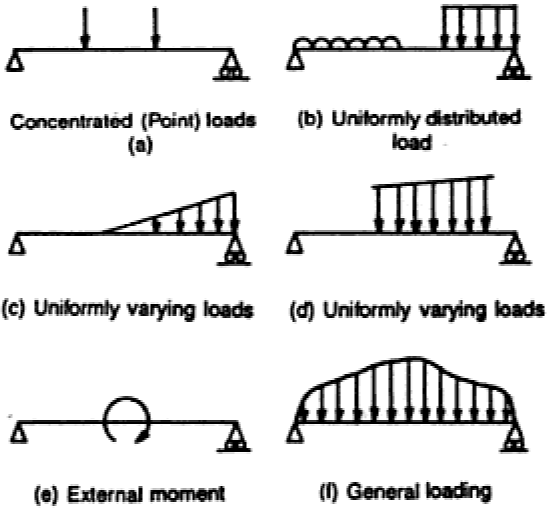


Fig. 2.30

SOLVED PROBLEMS

24. In the Fig. 2.31(*a*) calculate the tension T in the nail and magnitude of the force exerted by the hammer head at A on the block. Assume no slipping at A.

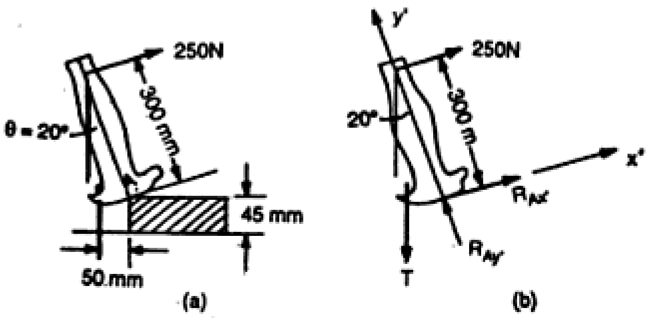


Fig. 2.31

Solution: *FBD* of hammer is as shown in Fig. 2.31(*b*).

Equating moment of 250 N force about A to that of force T in the nail about same point, (i.e., considering moment equilibrium condition at A).

$$250 \times 300 = T \times 50$$

$$T = 1500 \text{ N} \quad \textbf{(Ans.)}$$

Since the hammer is in equilibrium, (point A is a hinge)

$$\Sigma F_{x'} = 0 = -1500 \sin 20 + 250 + R_{Ax'}$$

$$R_{Ax'} = 263 \text{ N}$$

$$\Sigma F_{y'} = 0 = -1500 \cos 20 + R_{Ay'}$$

$$\therefore \quad R_{Ay'} = 1409.54 \text{ N}$$

$$R_A = \sqrt{(R_{Ax'})^2 + (R_{Ay'})^2}$$

$$= \sqrt{263^2 + 1409.54^2} \text{ N}$$

$$= 1433.87 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{R_{Ay'}}{R_{Ax'}}\right) = \tan^{-1}\left(\frac{1409.54}{263}\right)$$

$$= 79.43° \quad \textbf{(Ans.)}$$

25. Calculate the tension T in the cable that supports the body weighing 1000 N with the pulley arrangement shown in Fig. 2.32. Pulleys are free to rotate about their bearing, and masses of all parts are small compared with that of the loads. Find the magnitude of the total force on the bearing of pulley C.

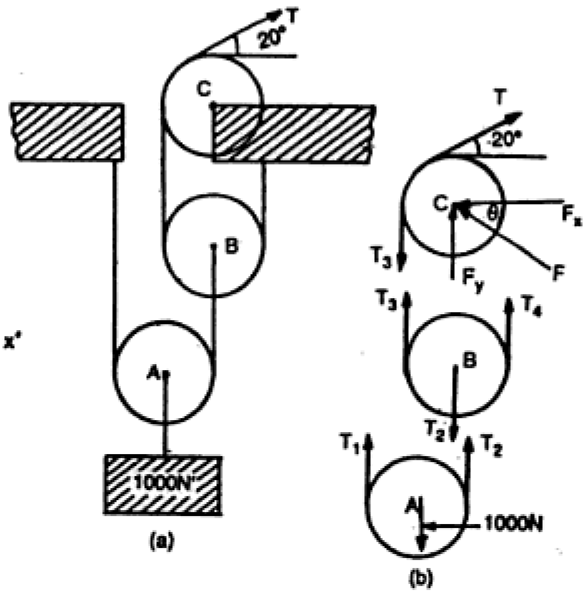


Fig. 2.32

Solution: Consider the pulley A first [See Fig. 2.32(*b*)] when the method of joints fail to proceed for want of a joint, with only two unknown member forces.

Combination of the above two methods also may be used advantageously.

In the above methods, once a member force is found, mark the direction of the force near the joint and then mark the joint force at the other end of the member in the opposite direction. Since the forces marked are the forces of the members on the joint, if marking of joint forces are as shown in Fig. 3.1, the members *AB*, *BC*, *CD* and *BE* are in compression while the forces in members *AE*, *ED* and *EC* are tensile forces. Thus, if a member force is towards the joint, it is compressive force and if it is away from the joint, it is tensile force.

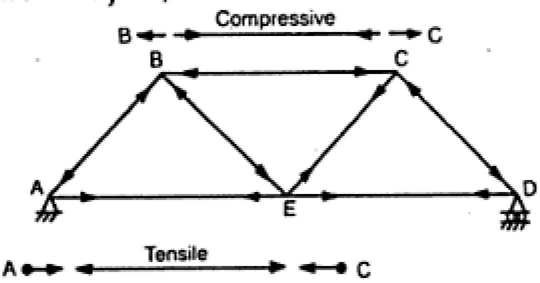


Fig. 3.1

SOLVED PROBLEMS

1. Compute the forces in all the members of the cantilever truss shown in Fig. 3.2(*a*) and indicate the forces on a sketch of the truss along with their nature.

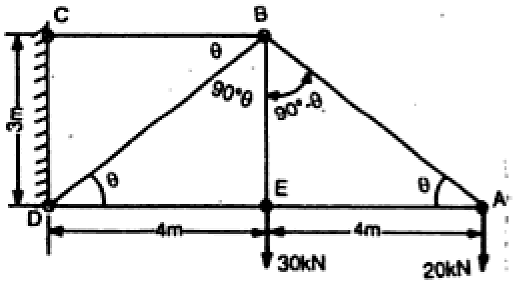


Fig. 3.2 (a)

Solution:

$$\tan\theta = \frac{3}{4}$$

$\therefore$ $\sin\theta = 0.6$ and $\cos\theta = 0.8$

Since at joint *A*, there are only two unknown forces, we may start from joint *A* itself. Assume directions for F_{AB} and F_{AE} as shown in Fig. 3.2(*b*).

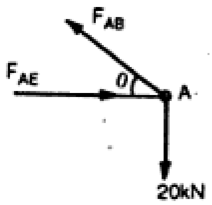


Fig. 3.2 (b)

$$\Sigma F_y = 0 \Rightarrow F_{AB}\sin\theta = 20$$

$$F_{AB} = \frac{20}{0.6} = 33.33 \text{ kN } (\textit{Tensile})$$

$$\Sigma F_x = 0 \Rightarrow F_{AE} - F_{AB}\cos\theta = 0$$

$$F_{AE} = 33.33 \times 0.8$$

$$= 26.67 \text{ kN } (\textit{Comp.})$$

Since F_{AB} and F_{AE} have + signs, the directions assumed for the forces in the members *AB* and *AE* are correct.

Now consider joint E: [Ref. Fig. 3.2(*c*)]

$$\Sigma F_y = 0 \Rightarrow F_{BE} = 30 \text{ kN } (\textit{Tensile})$$

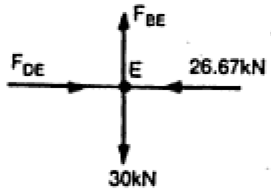


Fig. 3.2 (c)

$$\Sigma F_x = 0 \Rightarrow F_{DE} = 26.67 \text{ kN } (\textit{Comp.})$$

Now consider joint B: FBD of *B* is as shown in Fig. 3.2(*d*).

$$\Sigma F_y = 0 \Rightarrow -33.33\cos(90-\theta) - 30 + F_{BD}\cos(90-\theta) = 0$$

$$-33.33\sin\theta - 30 + F_{BD}\sin\theta = 0$$

$$F_{DB} = \frac{33.33 \times 0.6 + 30}{0.6} = 83.33 \text{ kN } (\textit{Comp.})$$

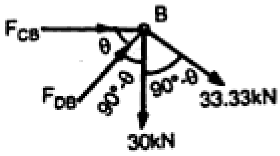


Fig. 3.2 (d)

$\Sigma F_x = 0 \Rightarrow F_{CB} + F_{DB} \cos\theta + 33.33 \sin.$
$(90 - \theta) = 0$

$F_{CB} + 83.33 \times 0.8 + 33.33 \times 0.8 = 0$

$F_{CB} = -93.33$ kN

Since F_{CB} is negative, its direction is to be reversed. Hence F_{CB} is tensile force. Member forces are shown in Fig. 3.2(*e*) in their correct senses.

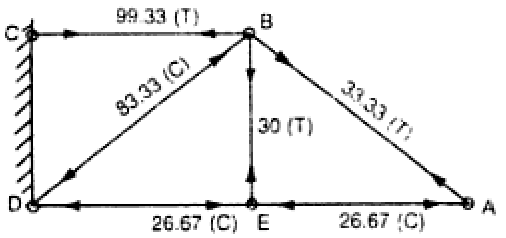


Fig. 3.2(e)

2. For the cantilever truss shown in Fig. 3.3(*a*) compute the forces in all the members. Compute the reactions at the supports also.

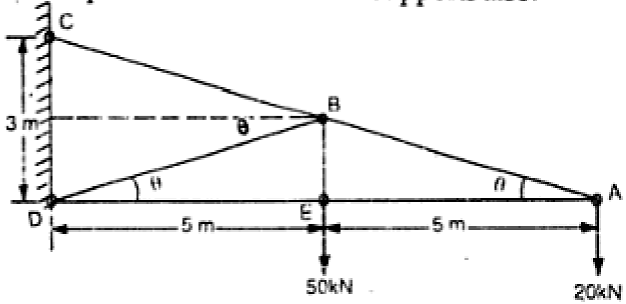


Fig. 3.3(a)

Solution:

$$\theta = \tan^{-1}\frac{1.5}{5} = 16.69°$$

Consider the joint A: [Ref. Fig. 3.3(*b*)]

$\Sigma F_y = 0 \Rightarrow F_{AB} \sin 16.69 - 20 = 0$

$\therefore\ F_{AB} = 69.64$ kN (*Tensile*)

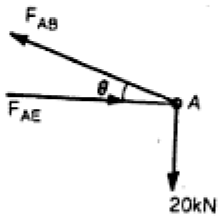


Fig. 3.3(b)

$\Sigma F_x = 0 \Rightarrow F_{AE} - F_{AB} \cos 16.69 = 0$

$\therefore\ F_{AE} = 66.71$ kN (*Comp.*)

Now consider joint E: [Ref. Fig. 3.3(*c*)]

$\Sigma F_y = 0 \Rightarrow F_{BE} = 50$ kN (*Tensile*)

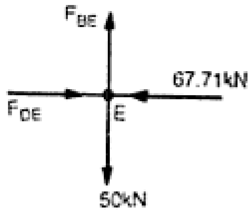


Fig. 3.3(c)

$\Sigma F_x = 0 \Rightarrow F_{DE} = 66.71$ kN (*Comp.*)

From FBD of joint B: [Ref. Fig. 3.3(*d*)],

$\Sigma F_y = 0 \Rightarrow F_{BC} \sin 16.69 - F_{BD} \sin 16.69$
$- 50 - 69.64 \sin 16.69 = 0$

i.e., $F_{BC} - F_{BD} = 243.74$ ...(*i*)

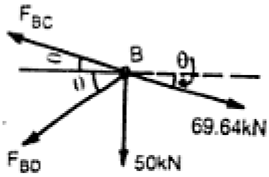


Fig. 3.3(d)

$\Sigma F_x = 0 \Rightarrow -F_{BC} \cos 16.69 - F_{BD} \cos 16.69$
$+ 69.64 \cos 16.69 = 0$

i.e., $F_{BC} + F_{BD} = 69.64$ ...(*ii*)

Solving equations (*i*) and (*ii*),

$F_{BC} = 156.69$ kN (*Tensile*)

$F_{BD} = -87.05$ kN

Since F_{BD} is negative its direction is to be reversed [Fig. 3.3(*d*)],

$\therefore\ F_{BD} = 87.05$ kN (*Comp.*)

Now consider joint D: [Ref. Fig. 3.3(e)],

Let V_D and H_D be vertical and horizontal components of reactions at *D*.

$\Sigma F_y = 0 \Rightarrow V_D - 87.05 \sin 16.69 = 0$

$\therefore\ V_D = 25$ kN

$\Sigma F_x = 0 \Rightarrow H_D - 67.71 - 87.05 \cos 16.69 = 0$

$\therefore\ H_D = 151.1$ kN

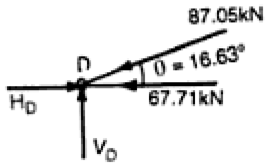


Fig. 3.3(e)

Consider joint C: It is in equilibrium under the action of only two forces i.e., force in member *BC* and the reaction at C. Hence they should be collinear. [Ref. Fig. 3.3*f*]

i.e., $R_C = F_{BC} = 156.69$ kN

Member forces and the reactions are shown in Fig. 3.3(*g*)

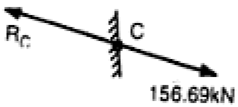


Fig. 3.3(f)

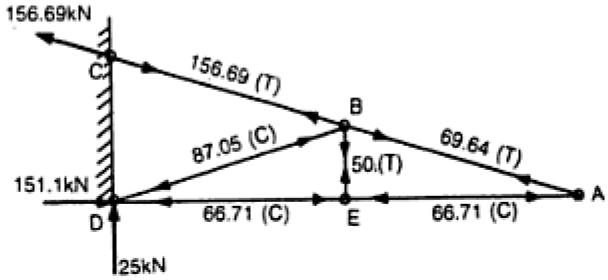


Fig. 3.3(g)

3. For the cantilever truss shown in Fig. 3.4(*a*), determine the member forces.

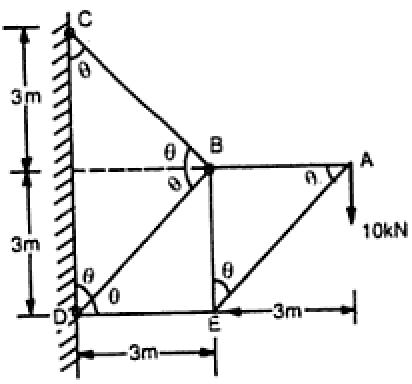


Fig. 3.4(a)

Solution:

$$\theta = \tan^{-1}\frac{3}{3} = 45°$$

Consider equilibrium of joint A: [Ref. Fig. 3.4(*b*)]

$$\Sigma F_y = 0 \Rightarrow F_{AE} \sin 45 - 10 = 0$$

$\therefore$ $F_{AE} = 14.14$ kN (*Comp.*)

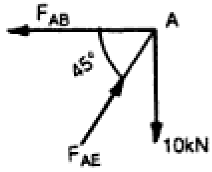


Fig. 3.4(b)

$$\Sigma F_x = 0 \Rightarrow -F_{AB} + 14.14 \cos 45° = 0$$

$\therefore$ $F_{AB} = 10$ kN (*Tensile*)

Consider equilibrium of joint E: [Ref. Fig. 3.4(*c*)]

$$\Sigma F_y = 0 \Rightarrow F_{BE} - 14.14 \cos 45 = 0$$

$\therefore$ $F_{BE} = 10$ kN (*Tensile*)

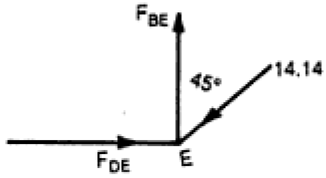


Fig. 3.4(c)

$$\Sigma F_x = 0 \Rightarrow F_{DE} - 14.14 \sin 45° = 0$$

$\therefore$ $F_{DE} = 10$ kN (*Comp.*)

Consider equilibrium of joint B: [Ref. Fig. 3.4(*d*)]

$$\Sigma F_x = 0 \Rightarrow -F_{BC} \cos 45 - F_{BD} \cos 45 + 10 = 0$$

i.e., $F_{BC} + F_{BD} = 14.14$ kN ...(*i*)

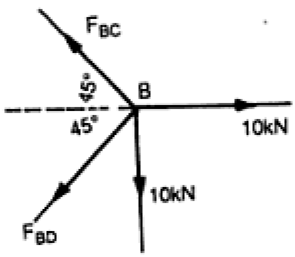


Fig. 3.4(d)

$\Sigma F_y = 0 \Rightarrow F_{BC} \sin 45 - F_{BD} \sin 45 - 10 = 0$

i.e., $F_{BC} - F_{BD} = 14.14$ kN ...(*ii*)

From (*i*) and (*ii*),

$F_{BC} = 14.14$ kN (*Tensile*)

$F_{BD} = 0$

Member forces are indicated in Fig. 3.4(*e*).

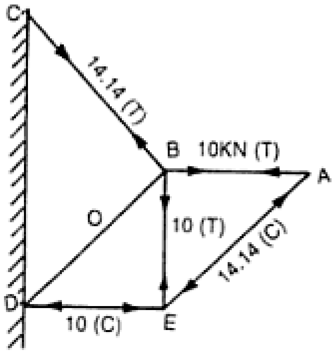


Fig. 3.4(e)

4. Compute the member forces in the cantilever truss shown in Fig. 3.5(*a*).

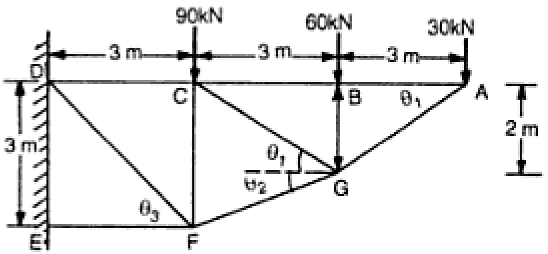


Fig. 3.5(a)

Solution:

$$\theta_1 = \tan^{-1}\frac{2}{3} = 33.69°$$

$$\theta_2 = \tan^{-1}\frac{1}{3} = 18.43°$$

$$\theta_3 = \tan^{-1}\frac{3}{3} = 45°$$

Consider the equilibrium of joint A: [Ref. Fig. 3.5(*b*)],

$\Sigma F_y = 0 \Rightarrow F_{AG} \sin 33.69 - 30 = 0$

$\therefore$ $F_{AG} = 54.08$ kN (*Comp.*)

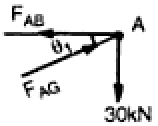


Fig. 3.5(b)

$\Sigma F_x = 0 \Rightarrow -F_{AB} + 54.08 \cos 33.69 = 0$

$\therefore$ $F_{AB} = 45$ kN (*Tensile*)

Consider joint B: [Ref. Fig. 3.5(*c*)]

$\Sigma F_y = 0 \Rightarrow F_{BG} = 60$ kN (*Comp.*)

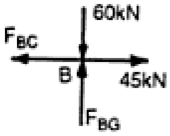


Fig. 3.5(c)

$\Sigma F_x = 0 \Rightarrow F_{BC} = 45$ kN (*Tensile*)

Consider joint G: [Ref. Fig. 3.5(*d*)]

$\Sigma F_y = 0 \Rightarrow -60 - F_{CG} \sin 33.69 + F_{FG} \sin 18.43 - 54.085 \sin 33.69 = 0$

i.e., $-F_{CG} + 0.57 F_{FG} = 162.23$ ...(*i*)

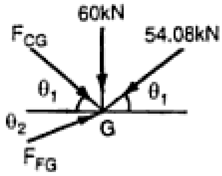


Fig. 3.5(d)

$\Sigma F_x = 0 \Rightarrow F_{CG} \cos 33.69 + F_{FG} \cos 18.43 - 54.085 \sin 33.69 = 0$

i.e., $F_{CG} + 1.14 F_{FG} = 54.08$ ...(*ii*)

Solving equations (*i*) and (*ii*)

$$F_{FG} = \frac{162.23 + 54.08}{0.57 + 1.14} = 126.5 \text{ kN (Comp.)}$$

$\therefore$ $F_{CG} = 54.08 - 1.14 \times 126.5 = -90.14$ kN

i.e., F_{CG} direction is opposite to assumed direction.

$\therefore$ $F_{CG} = 90.14$ kN (*Tensile*)

Consider joint C: [Ref. Fig. 3.5(*e*)]

$\Sigma F_y = 0 \Rightarrow F_{CF} - 90 - 90.14 \sin 33.69° = 0$

$\therefore$ $F_{CF} = 140$ kN (*Comp.*)

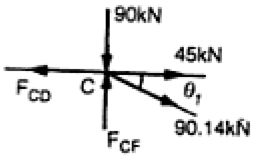


Fig. 3.5 (e)

$\Sigma F_x = 0 \Rightarrow -F_{CD} + 45 + 90.14 \cos 33.69° = 0$

$\therefore F_{CD} = 120$ kN (*Tensile*)

Consider joint F: [Ref. Fig. 3.5(*f*)]

$\Sigma F_y = 0 \Rightarrow F_{DF} \sin 45° - 140 - 126.5 \sin 18.43° = 0$

$\therefore \; F_{DF} = +254.55$ kN (*Tensile*)

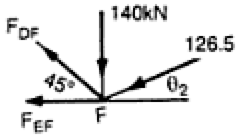


Fig. 3.5 (f)

$\Sigma F_x = 0 \Rightarrow -F_{EF} - F_{DF} \cos 45° - 126.5 \cos 18.43° = 0$

$\therefore \; F_{EF} = -300$ kN

The direction of F_{EF} is to be reversed.

$\therefore \; F_{EF} = 300$ kN (*Comp.*)

Fig. 3.5(*g*) shows member forces with their senses.

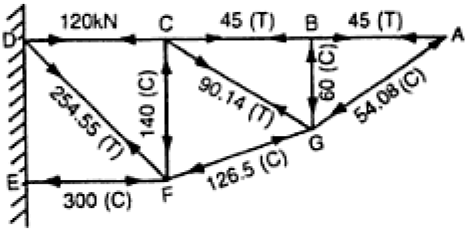


Fig. 3.5(g)

5. Determine the forces in the members of truss shown in Fig. 3.6(*a*).

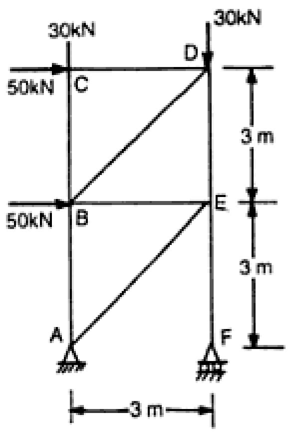


Fig. 3.6(a)

Solution:

Consider FBD of joint C: [Ref. Fig. 3.6(*b*)],

$\Sigma F_y = 0 \; \Rightarrow \; F_{BC} = 30$ kN (*Comp.*)

$\Sigma F_x = 0 \; \Rightarrow \; F_{CD} = 50$ kN (*Comp.*)

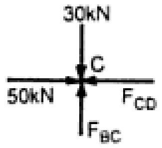


Fig. 3.6 (b)

Consider Joint D: [Ref. Fig. 3.6(*c*)], Noting all inclined members are at 45° to horizontal/vertical.

$\Sigma F_x = 0 \; \Rightarrow \; -F_{BD} \cos 45° + 50 = 0$

or $\quad F_{BD} = 70.71$ kN (*Tensile*)

$\Sigma F_y = 0 \; \Rightarrow \; F_{DE} - 30 - 70.71 \sin 45° = 0$

$\therefore \quad F_{DE} = 80$ kN (*Comp.*)

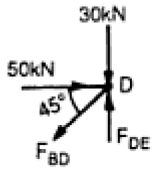


Fig. 3.6(c)

Consider joint B: [Ref. Fig. 3.6(*d*)],

$\Sigma F_x = 0 \; \Rightarrow \; -F_{BE} + 50 + 70.71 \cos 45° = 0$

$\therefore \; F_{BE} = 100$ kN (*Comp.*)

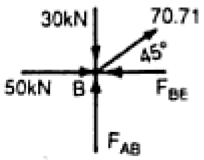


Fig. 3.6(d)

$\Sigma F_y = 0 \; \Rightarrow \; F_{AB} - 30 + 70.71 \sin 45° = 0$

$\therefore \; F_{AB} = -20$ kN (*Comp.*)

$\therefore$ Direction of F_{AB} is opposite to the assumed direction.

$\therefore \; F_{AB} = 20$ kN (*Comp.*)

Consider FBD of joint E: [Ref. Fig. 3.6(*e*)],

$\Sigma F_x = 0 \; \Rightarrow \; -F_{AE} \cos 45° + 100 = 0$

$\therefore \; F_{AE} = 141.42$ kN (*Tensile*)

$\therefore \quad F_{BK} = \dfrac{-100+140}{2} = 20 \text{ kN } (Comp.)$

and $F_{BC} = 140 - 20 = 120$ kN (*Comp.*)

Consider joint K: [Ref. Fig. 3.9(*f*)],

$$\Sigma F_y = 0 \Rightarrow F_{CK} - 20 \sin 30° = 0$$

$\therefore \quad F_{CK} = 10$ kN

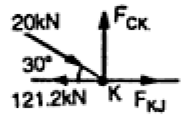


Fig. 3.9(f)

$$\Sigma F_x = 0 \Rightarrow 20 \cos 30° - 121.2 + F_{KJ} = 0$$

$\therefore \quad F_{KJ} = 103.9$ kN (*Tensile*)

Consider joint C: [Ref. Fig. 3.9(*g*)],

$$\tan\theta = \frac{CK}{KJ} = \frac{8 \tan 30}{4} = 1.1547$$

$\therefore \quad \theta = 49.11°$

$$\Sigma F_y = 0 \Rightarrow -20 - 10 + 120 \sin 30° - F_{CD} \sin 30° + F_{CJ} \sin 49.11° = 0$$

$$F_{CD} + 1.512\, F_{CJ} = -60 \qquad ...(iii)$$

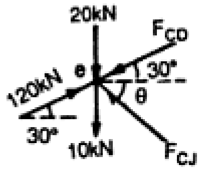


Fig. 3.9(g)

$$\Sigma F_x = 0 \Rightarrow 120 \cos 30° - F_{CD} \cos 30° - F_{CJ} \cos 49.11° = 0$$

$$F_{CD} + 0.756\, F_{CJ} = 120 \qquad ...(iv)$$

From equations (*iii*) and (*iv*),

$$F_{CJ} = \frac{-60+120}{1.512+0.756} = 26.46 \text{ kN } (Comp.)$$

and $F_{CD} = 120 - 0.756 \times 26.46$
$= 100$ kN (*Comp.*)

Now Consider joint D: [Ref. Fig. 3.9(*h*)],

Due to symmetry

$$F_{DE} = F_{CD} = 100 \text{ kN } (Comp.)$$

$$\Sigma F_y = 0 \Rightarrow 100 \cos 60° + 100 \cos 60° - 60 + F_{DJ} = 0$$

$\therefore \quad F_{DJ} = -40$ kN

$\therefore$ Direction is to be reversed.

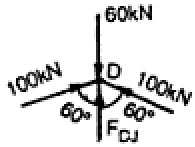


Fig. 3.9(h)

$\therefore F_{DJ} = 40$ kN (*Tensile*)

Member forces are shown on Fig. 3.9(*b*).

9. Compute the member forces in all the members of truss shown in Fig. 3.10(*a*).

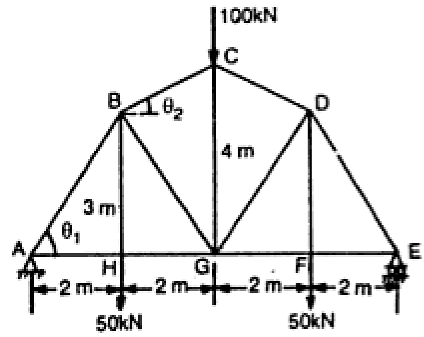


Fig. 3.10(a)

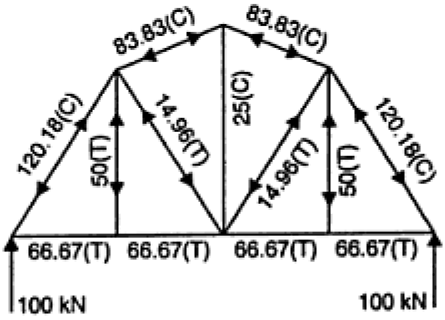


Fig. 3.10(b)

Solution: Since the geometry and loading are symmetrical, each of the reaction,

$$V_A = V_E = \frac{1}{2} \times \text{total load} = \frac{1}{2} \times (100 + 50 + 50)$$

$= 100$ kN, as shown in Fig. 3.10(*b*).

Now consider the FBD of joint A: [Ref. Fig. 3.10(*c*)],

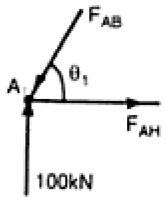


Fig. 3.10(c)

$$\theta_1 = \tan^{-1}\frac{3}{2} = 56.31°$$

$\Sigma F_y = 0 \Rightarrow 100 - F_{AB}\sin 56.31° = 0$

$\therefore\ F_{AB} = 120.18$ kN (*Comp.*)

$\Sigma F_x = 0 \Rightarrow F_{AH} - 120.18\cos 56.31° = 0$

$\therefore\ F_{AH} = 66.67$ kN (*Tensile*)

Consider joint H: [Ref. Fig. 3.10(*d*)],

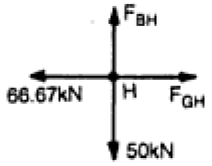


Fig. 3.10(d)

$\Sigma F_y = 0 \Rightarrow F_{BH} = 50$ kN (*Tensile*)

$\Sigma F_x = 0 \Rightarrow F_{GH} = F_{AH} = 66.67$ kN (*Tensile*)

Consider joint B: [Ref. Fig. 3.10(*e*)],

$$\theta_2 = \tan^{-1}\frac{4-3}{2} = 26.57°$$

$\Sigma F_y = 0 \Rightarrow -50 + 120.18\sin 56.31°$
$+ F_{BG}\sin 56.31° - F_{BC}\sin 26.57° = 0$

$\therefore\quad F_{BG} - 0.538\,F_{BC} = -60.09$ ...(*i*)

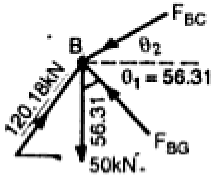


Fig. 3.10(e)

$\Sigma F_x = 0 \Rightarrow 120.18\cos 56.31°$
$- F_{BG}\cos 56.31°$
$- F_{BC}\cos 26.57° = 0$

$\therefore\ F_{BG} + 1.612\,F_{BC} = 120.18$ ...(*ii*)

From equations (*i*) and (*ii*),

$$F_{BC} = \frac{120.18 + 60.09}{1.612 + 0.538} = 83.83 \text{ kN } (Comp.)$$

and

$F_{BG} = 120.18 - 1.612 \times 83.83 = -14.96$

Direction of F_{BG} is to be reversed.

Hence $F_{BG} = 14.96$ kN (*Tensile*)

Consider joint C: [Ref. Fig. 3.10(*f*)],

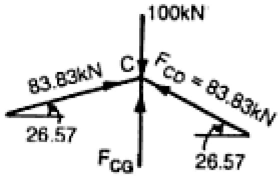


Fig. 3.10(f)

Due to symmetry,

$F_{CD} = F_{BC} = 83.83$ kN (*Comp.*)

$\Sigma F_y = 0 \Rightarrow 83.83\sin 26.57° + 83.83\sin 26.57° - 100 + F_{CG} = 0$

$\therefore\ F_{CG} = 25$ kN (*Comp.*)

Using symmetry, forces in the other half are noted down and are indicated in Fig. 3.10(*b*).

10. In the truss shown in Fig. 3.11(*a*), compute the forces in all the members.

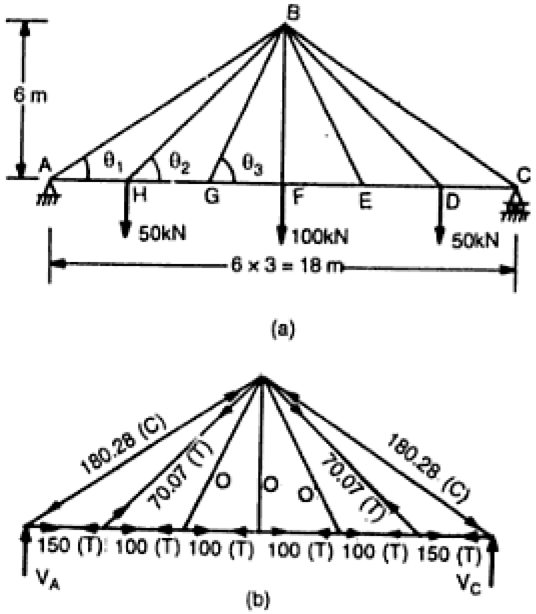


Fig. 3.11(a & b)

Solution:

$$V_A = V_C = \frac{50+100+50}{2} = 100 \text{ kN}$$

Consider the equilibrium of Joint A: [Ref. Fig. 3.11(*c*)],

$$\theta_1 = \tan^{-1}\frac{6}{9} = 33.69°$$

$$\Sigma F_y = 0 \Rightarrow 100 - F_{AB}\sin 33.69° = 0$$

$\therefore$ $F_{AB} = 180.28$ kN (*Comp.*)

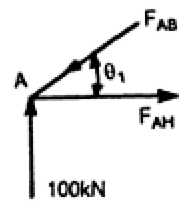


Fig. 3.11(c)

$$\Sigma F_x = 0 \Rightarrow F_{AH} - 180.28\cos 33.69° = 0$$

$\therefore F_{AH} = 150$ kN (*Tensile*)

Consider joint H: [Ref. Fig. 3.11(*d*)],

$$\theta_2 = \tan^{-1}\frac{6}{6} = 45°$$

$$\Sigma F_y = 0 \Rightarrow -50 + F_{BH}\sin 45° = 0$$

$\therefore$ $F_{BH} = 70.07$ kN (*Tensile*)

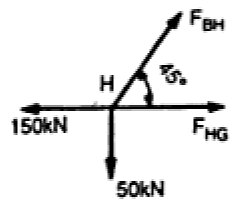


Fig. 3.11(d)

$$\Sigma F_x = 0 \Rightarrow -150 + 70.07\cos 45° + F_{HG} = 0$$

$\therefore$ $F_{HG} = 100$ kN (*Tensile*)

Consider joint G: [Ref. Fig. 3.11(*e*)],

$$\Sigma F_y = 0 \Rightarrow F_{BG} = 0$$

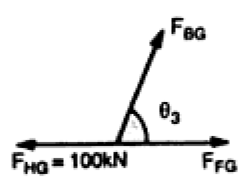


Fig. 3.11(e)

$$\Sigma F_x = 0 \Rightarrow F_{FG} = F_{HG} = 100 \text{ kN (Tensile)}$$

Consider joint F: [Ref. Fig. 3.11(*f*)],

$$\Sigma F_y = 0 \Rightarrow F_{BF} = 0$$

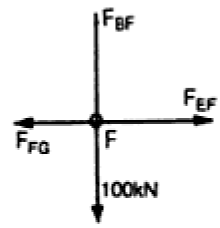
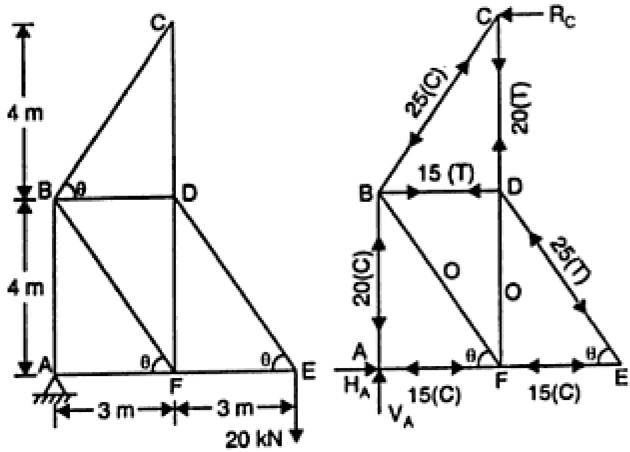


Fig. 3.11(f)

Using symmetry forces in all other members are noted down as shown in Fig. 3.11(*b*)

11. Analyse the truss shown in Fig. 3.12(*a*) and indicate member forces on the sketch of the truss.



(a) (*b*)

Fig. 3.12(a & b)

Solution:

$$\tan\theta = \frac{4}{3}$$

$\therefore$ $\sin\theta = 0.8$ and $\cos\theta = 0.6$ [3:4:5 rule]

Consider joint E: [Ref. Fig. 3.12(*c*)],

$$\Sigma F_y = 0 \Rightarrow F_{DE}\sin\theta = 20$$

$$\therefore \quad F_{DE} = 20 \times \frac{1}{0.8} = 25 \text{ kN (Tensile)}$$

$$\Sigma F_x = 0 \Rightarrow F_{EF} - F_{DE}\cos\theta = 0.$$

$\therefore$ $F_{EF} = 25 \times 0.6 = 15$ kN (*Comp.*)

Now consider the *FBD* of whole truss [Ref. Fig. 3.12(*b*)], to determine the reactions, since without them we cannot proceed with the method of joints.

$\Sigma M_A = 0 \Rightarrow R_C \times 8 - 20 \times 6 = 0 \therefore R_C = 15$ kN
$\Sigma F_y = 0 \Rightarrow V_A = 20$ kN
$\Sigma F_x = 0 \Rightarrow H_A = R_C = 15$ kN

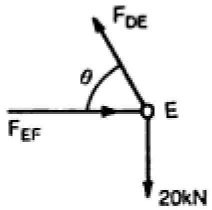


Fig. 3.12(c)

Consider joint A: [Ref. Fig. 3.12(*d*)],

$\Sigma F_y = 0 \Rightarrow F_{AB} = 20$ kN (*Comp.*)

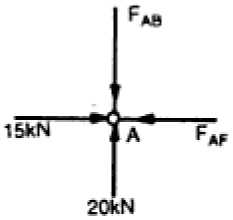


Fig. 3.12(d)

$\Sigma F_x = 0 \Rightarrow 15 - F_{AF} = 0$

or $F_{AF} = 15$ kN (*Comp.*)

Consider joint C: [Ref. Fig. 3.12(*e*)],

$\Sigma F_x = 0 \Rightarrow F_{BC} \cos\theta = 15$

$$F_{BC} = 15 \times \frac{1}{0.6} = 25 \text{ kN (Comp.)}$$

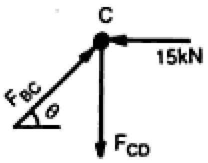


Fig. 3.12(e)

$\Sigma F_y = 0 \Rightarrow F_{BC} \sin\theta - F_{CD} = 0$

$F_{CD} = 25 \times 0.8 = 20$ kN (*Tensile*)

Consider joint B: [Ref. Fig. 3.12(*f*)],

$\Sigma F_y = 0 \Rightarrow F_{BF} \sin\theta + 20 - 25 \times \sin\theta = 0$

$F_{BF} \times 0.8 = 25 \times 0.8 - 20$

or $F_{BF} = 0$

$\Sigma F_x = 0 \Rightarrow F_{BD} - 0 - 25 \cos\theta = 0$

$F_{BD} = 25 \times 0.6 = 15$ kN (*Tensile*)

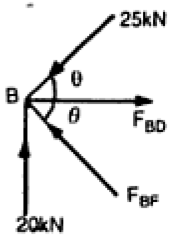


Fig. 3.12(f)

Consider joint F: [Ref. Fig. 3.12(*g*)],

$\Sigma F_y = 0 \Rightarrow F_{DF} = 0$

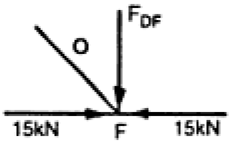


Fig. 3.12(g)

Forces in various members are indicated in Fig. 3.12(*b*).

12. Determine the forces in the members of the frame shown in Fig. 3.13(*a*)

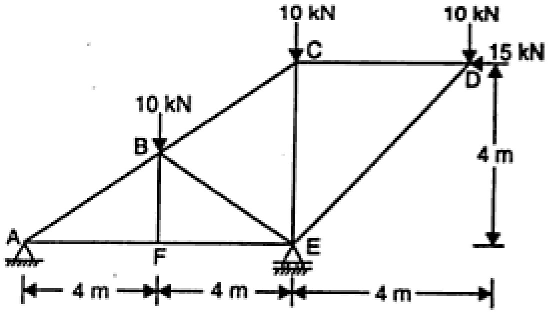


Fig. 3.13(a)

Solution: Consider the *FBD* of the entire truss [Fig. 3.13(*b*)],

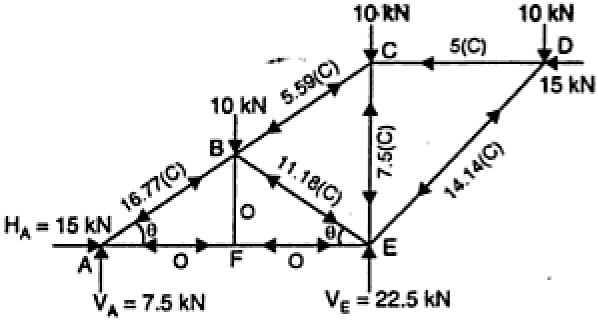


Fig. 13(b)

$\therefore\ F_{CE} \sin 30 \times 3 + F_{CE} \cos 30 \times 1.732 + 10 \times 2.2 = 0$

$$\therefore\ F_{CE} = -\frac{22}{3 \sin 30 + 1.732 \cos 30}$$

$= -7.33$ kN

$\Sigma F_Y = 0 \Rightarrow F_{CD} \sin 30° - F_{CE} \sin 30° - 10 + 10 = 0$

$\therefore\ F_{CD} = F_{CE} = -7.33$ kN

$\Sigma F_X = 0 \Rightarrow F_{AE} + F_{CD} \cos 30° + F_{CE} \cos 30° = 0$

$F_{AE} - 7.33 \cos 30° - 7.33 \cos 30° = 0$

$\therefore\ F_{AE} = 12.70$ kN (*Tensile*)

The directions of forces F_{CD} and F_{CE} are to be reversed.

$\therefore\ F_{CD} = F_{CE} = 7.33$ kN (*Comp.*)

18. Determine the forces in the members *CD*, *CI* and *HI* of the truss shown in Fig. 3.19(*a*).

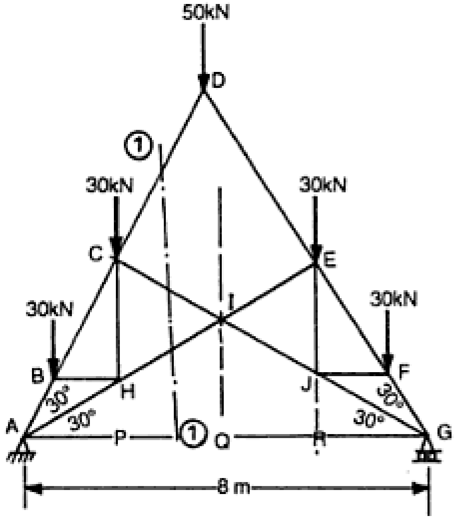


Fig. 3.19(a)

Solution: Since the truss is symmetric with respect to geometry and loading, the reactions are:

$$R_A = R_G = \frac{30 + 30 + 50 + 30 + 30}{2}$$

$= 85$ kN, vertically upwards.

Let *P*, *Q* and *R* be projections of *CH*, *DI* and *EJ* on the horizontal line *AG*. Then,

$CG = 8 \cos 30°$

And $PG = CG \cos 30° = 8 \cos^2 30° = 6$ m

$\therefore\ AP = 8 - 6 = 2$ m

$\therefore$ Due to symmetry $AP = PQ = QR = RG = 2$m

section (1)-(1) is taken to cut the members *CD*, *CI* and *HI* and consider the *FBD* of LHS of the section [Ref. Fig. 3.19(*b*)].

$\Sigma M_A = 0 \Rightarrow F_{CI} \times AC - 30 \times 2 - 30 \times 4 = 0$

$F_{CI} \times 8 \sin 30° = 180$

$\therefore\ F_{CI} = 45$ kN (*Tensile*)

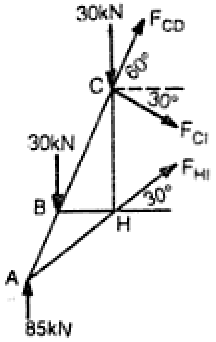


Fig. 3.19(b)

$\Sigma F_Y = 0 \Rightarrow\ F_{CD} \sin 60 - F_{CI} \sin 30 + F_{HI} \sin 30 - 30 - 30 + 85 = 0$

i.e., $F_{CD} + 0.577 F_{HI} = -2.887$ ...(i)

$\Sigma F_X = 0 \Rightarrow F_{CD} \cos 60° + F_{CI} \cos 30° + F_{HI} \cos 30° = 0$

i.e., $F_{CD} + F_{HI} \times 1.732 = -45 \times 1.732$ ...(*ii*)

From (*i*) and (*ii*),

$(0.577 - 1.732)\ F_{HI} = -75.053$

or $F_{HI} = 64.98$ kN (*Tensile*)

$\therefore\ F_{CD} = -2.887 - 0.577 \times 64.9$

$= -40.38$

The direction of F_{CD} is to be reversed.

$\therefore\ F_{CD} = 40.38$ kN (*Comp.*)

19. For the truss shown in Fig. 3.20(*a*), compute the member forces *BG*, *AC* and *HG*.

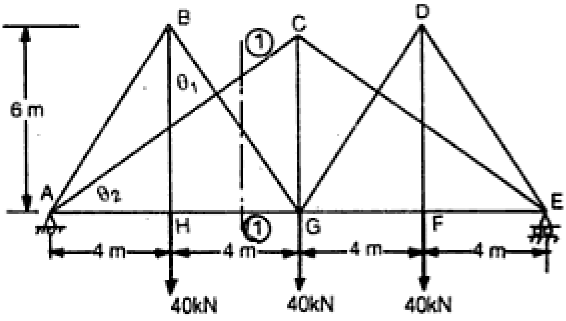


Fig. 3.20(a)

Solution: Due to symmetry

$$R_A = R_B = \frac{1}{2}(40 + 40 + 40)$$
$$= 60 \text{ kN, Vertically up wards.}$$

Take section (1)-(1) and consider LHS portion [Ref. Fig. 3.20(*b*)],

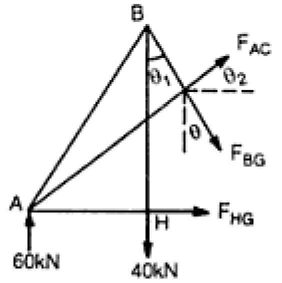


Fig. 3.20(b)

$$\theta_1 = \tan^{-1}\frac{4}{6} = 33.69°$$

$$\Sigma M_A = 0 \Rightarrow F_{BG}\cos 33.69° \times 4 + F_{BG}\sin 33.69° \times 6 + 40 \times 4 = 0$$

$\therefore\ F_{BG} = -24.0$ kN

$$\theta_2 = \tan^{-1}\frac{6}{8} = 36.87°$$

$$\Sigma F_Y = 0 \Rightarrow F_{AC}\sin 36.87° - F_{BG}\cos 33.69° + 60 - 40 = 0$$

$$F_{AC} = \frac{-24\sin 33.69° - 20}{\sin 36.87°} = -6.0$$

$$\Sigma F_X = 0 \Rightarrow F_{HG} + F_{BG}\sin 33.69° + F_{AC}\cos 36.87 = 0$$

$\therefore\ F_{HCo} = -(-24)\sin 33.69 - (-6)\cos 36.87$
$= 18.11$ kN (*Tensile*)

Sign for F_{BC} is to be reversed.

$\therefore\ F_{BG} = 24.0$ kN (*Comp.*)

20. For the truss shown in Fig. 3.21(*a*), determine the forces in the members *BC*, *GC* and *GF*.

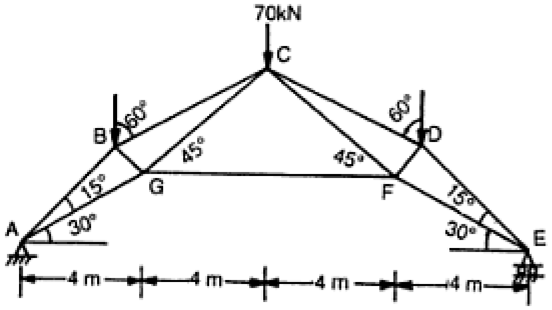


Fig. 3.21(a)

Solution: Due to symmetry

$$R_A = R_E = \frac{1}{2} \times 70 = 35 \text{ kN}$$

Take the section through *BC*, *GC* and *GH* and consider left half portion [Ref. Fig. 3.21(*b*)],

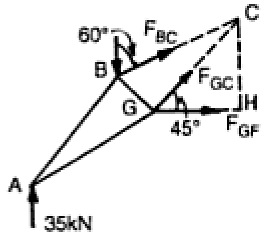


Fig. 3.21(b)

$$\Sigma M_C = 0 \Rightarrow F_{GF} \times CH = 35 \times 8$$
$$F_{GF} \times 4 = 35 \times 8$$

or $F_{GF} = 70$ kN (*Tensile*)

Ref. Fig. 3.21(b)

$$\Sigma F_Y = 0 \Rightarrow F_{GC}\sin 45° + F_{BC}\cos 60° + 35 = 0$$

$$0.707\,F_{GC} + 0.5\,F_{BC} = -35 \qquad ...(i)$$

$$\Sigma F_X = 0 \Rightarrow F_{GC}\cos 45° + F_{BC}\sin 60° + F_{GF} = 0$$

$$0.707\,F_{GC} + 0.866\,F_{BC} = -70 \qquad ...(ii)$$

From equation (*i*) and (*ii*),

$0.366\,F_{BC} = -70 + 35 = -35$

$\therefore\ F_{BC} = -95.62$ kN

From (*i*), $0.707\,F_{GC} + 0.5\,(-95.62) = -35$

$\therefore\ F_{BC} = 18.12$ kN (*Tensile*)

The direction of F_{GC} to be reversed

$\therefore\ F_{GC} = 18.12$ kN (*Comp.*)

21. Determine the forces in the members *DE*, *NE* and *LP* of the French truss shown in Fig. 3.22(*a*).

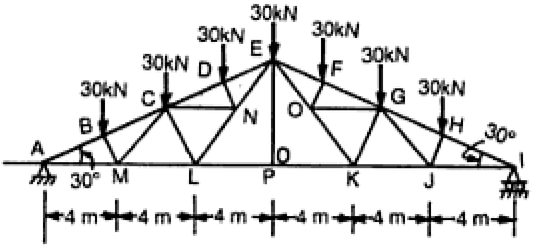


Fig. 3.22(a)

Solution: Due to symmetry

$$R_A = R_I = \frac{1}{2}(30 + 30 + 30 + 30 + 30 + 30 + 30) = 105 \text{ kN, Vertical.}$$

Consider LHS portion of the truss. Its *FBD* is as shown in Fig. 3.22(*b*).

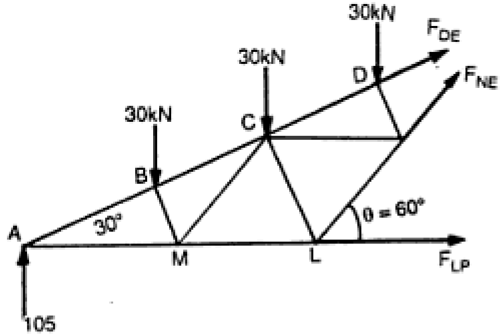


Fig. 3.22(b)

$$A_E = \frac{12}{\cos 30^\circ} = 13.86 \text{ m}$$

$$\therefore \quad AB = BC = CD = DE = \frac{13.86}{4} = 3.46 \text{ m}$$

$$EP = 12 \tan 30^\circ$$

$$\therefore \tan\theta = \frac{EP}{LP} = \frac{12 \tan 30^\circ}{4} \doteq 1.732$$

or $\theta = 60^\circ$

$\Sigma M_A = 0 \Rightarrow F_{NE} \sin 60^\circ \times 8 - 30 \times 3.46 \cos 30^\circ - 30 \times 2 \times 3.46 \cos 30^\circ - 30 \times 3 \times 3.46 \cos 30^\circ = 0$

$\therefore$ $F_{NE} = 77.85$ kN (*Tensile*) **(Ans.)**

$\Sigma F_Y = 0 \Rightarrow F_{NE} \sin 60^\circ + F_{DE} \sin 30^\circ + 105 - 30 - 30 - 30 = 0$

$$F_{DE} = \frac{-77.85 \sin 60^\circ - 15}{\sin 30^\circ} = -164.84 \text{ kN}$$

$\Sigma F_X = 0 \Rightarrow F_{LP} + F_{NE} \cos 60^\circ + F_{DE} \cos 30^\circ = 0$

$F_{LP} = -77.85 \cos 60^\circ - (-164) \cos 30^\circ$
$= 103.83$ kN (*Tensile*) **(Ans.)**

The direction of F_{DE} is to be reversed

$F_{DE} = 164.84$ kN (*Comp.*) **(Ans.)**

22. Determine the forces in the members *BC*, *BK*, *AL*, *ML* of the truss shown in Fig. 3.23.

Solution: Due to symmetry

$$R_N = R_H = \frac{1}{2}(50 + 50 + 50 + 50 + 50)$$

$= 125$ kN

Note: Method of section is not directly Applicable, since section cutting 3 members will not separate the two portions.

Joint N: $\Sigma F_X = 0 \Rightarrow F_{NM} = 0$

Joint M: $\Sigma F_X = 0 \Rightarrow F_{ML} = 0$

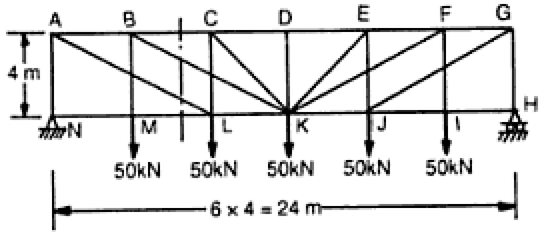


Fig. 3.23(a)

Now consider a section through the members *BC*, *BK* and *AL* [Ref. Fig. 3.23(*b*)],

$$\theta = \tan^{-1}\frac{4}{8} = 26.57^\circ$$

$$\Sigma M_B = 0 \Rightarrow F_{AL} \sin 26.57^\circ \times 4 - 125 \times 4 = 0$$

$$\therefore \quad F_{AL} = \frac{125}{\sin 26.57^\circ} = 279.5 \text{ kN } (\textit{Tensile})$$

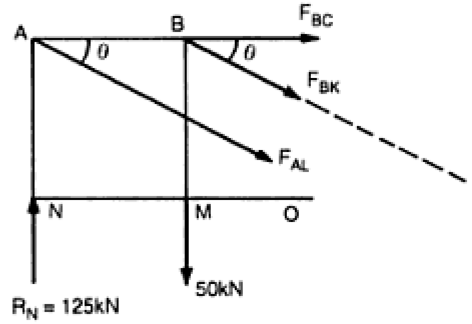


Fig. 3.23(b)

$\Sigma F_Y = 0 \Rightarrow -F_{BK} \sin 26.57^\circ - 279.5 \sin 26.57^\circ + 125 - 50 = 0$

$\therefore$ $F_{BK} = -111.78$ kN

$\Sigma F_X = 0 \Rightarrow F_{BC} + F_{BK} \cos 26.57^\circ + F_{AL} \cos 26.57^\circ = 0$

$F_{BC} = -(-111.78) \cos 26.57^\circ - 279.5 \cos 26.57^\circ$

$\therefore$ $F_{BC} = 100 - 250 = -150$ kN

$\therefore$ The directions of F_{BK} and F_{BC} are to be reserved.

PROBLEMS AND SOLUTIONS IN ENGINEERING MECHANICS

Problem solving is a vital requirement for any aspiring engineer. This book aims to develop this ability in students by explaining the basic principles of mechanics through a series of graded problems and their solutions.

Each chapter begins with a quick discussion of the basic concepts and principles. It then provides several well developed solved examples which illustrate the various dimensions of the concept under discussion. A set of practice problems is also included to encourage the student to test his mastery over the subject.

The book would serve as an excellent text for both Degree and Diploma students of all engineering disciplines. AMIE candidates would also find it most useful.

Dr S. S. Bhavikatti studied at BVB College of Engineering and Technology, Hubli for his BE (Civil) degree and graduated from Karnataka University, Dharwad, sharing first rank with another candidate. The same year he joined Karnataka Regional Engineering College, Surathkal (presently, National Institute of Technology, Karnataka, Surathkal). He secured his M.E. Degree in Structural Engineering in 1967 from University of Roorkee (presently IIT Roorkee) under Technical Teachers Training Programme and Ph.D. degree in 1977 from IIT Delhi under Quality Improvement Programme. He served NITK, Surathkal in different capacities like Head of Civil Engineering Department; Chairman, Central Computer Centre; Chairman, Centre for Continuing Education and Dean (Administration). In November 2001 he joined S.D.M. College of Engineering and Technology, Dharwad as Head of Civil Engineering Department. He is Dean (Academic) also. He has published 45 technical papers in national and international journals and conferences. He has already published 10 books in Civil Engineering. He was member of organizing/scientific committees of international seminars of International Association of Shells and Spatial Structures (IASS) held at Taegue (South Korea) in 1990, Tokyo (Japan) in 1993 and Taipei (Taiwan) in 1997. He is member of several Indian and international professional bodies. He is Chairman of Indian Concrete Institute, Karnataka, Hubli-Dharwad Centre.

A.V. Hegde obtained his B.Tech. degree in Civil Engineering from KREC, Surathkal in 1980. In 1982, he hoined KREC in the department of Applied Mechanics & Hydraulics as Lecturer. He was deputed to IIT, Mumbai for his M.Tech. degree in Offshore Engineering, under QIP of Government of India, in 1984. In 1997 he got his doctorate from Mangalore University working on "Optimum CAD of Breakwaters". Dr. Hegde worked for his Post-Doctoral Programme in the world renowned WL/Delft Hydraulics Lab., the Netherlands, in 1999 and developed an algorithm for the design of Breakwater structures.

He has to his credit about 45 technical papers in the various national and international journals and conferences.

ISBN 81-224-1601-2

9788122416015

NEW AGE INTERNATIONAL (P) LIMITED, PUBLISHERS

New Delhi • Bangalore • Chennai • Cochin • Guwahati • Hyderabad
Jalandhar • Kolkata • Lucknow • Mumbai • Ranchi

www.newagepublishers.com