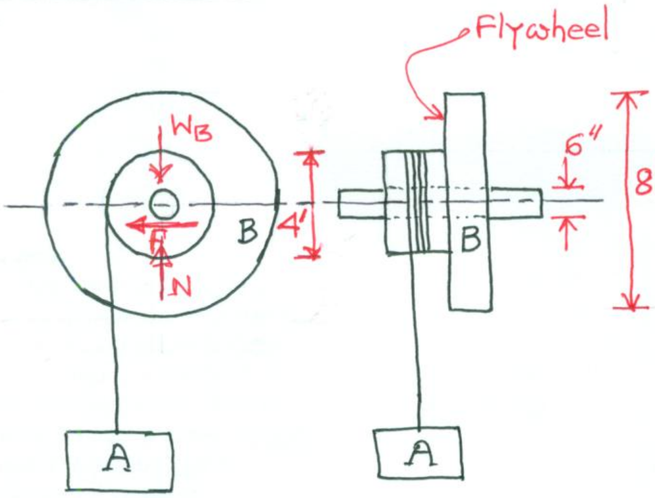
Ex. 278/PP. 398-399



$W_B = 1288$ lb

$\bar{k}_B = 2.5$ ft

$S_A = 80$ ft ↓

$\omega_1 = 10$ rpm $= \frac{10\times 2\pi}{60}$ rad/s $= 1.047$ rad/s

$\omega_2 = 120$ rpm $= \frac{120\times 2\pi}{60}$ rad/s $= 12.566$ rad/s

$F = 70$ lb

$W_A = ?$

Sol^n

U_{net} = Work done by body A to move down − Work done against friction

$= W_A \cdot S_A - F \cdot r\theta$

$= W_A \times 80 - 70 \times \frac{3}{12} \times 40$

$= 80 W_A - 700$ ft-lb

$r = \frac{3}{12}$ ft, $\theta = \frac{S_A}{2} = \frac{80}{2} = 40$ rad

$\Delta KE = \frac{1}{2}\cdot\frac{W_A}{g}(v_2^2 - v_1^2) + \frac{1}{2} I_0(\omega_2^2 - \omega_1^2)$

$= \frac{1}{2}\times\frac{W_A}{32.2}\times(25.13^2 - 2.09^2) + \frac{1}{2}\times 250\times(12.566^2 - 1.047^2)$

$= 9.738 W_A + 19601$ ft-lb

$v_1 = r_A\omega_1 = 2\times 1.047 = 2.09$ fps

$v_2 = r_A\omega_2 = 2\times 12.566 = 25.13$ fps

$I_0 = m_B \bar{k}_B^2 = \frac{1288}{32.2}\times 2.5^2$ slug-ft^2 $= 250$ slug-ft^2

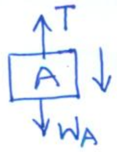
According to the principles of work and kinetic energy

$U_{net} = \Delta KE$

$\Rightarrow 80 W_A - 700 = 9.738 W_A + 19601$

$\therefore W_A = \boxed{288.94 \text{ lb}}$ Ans.

Alternative sol^n: This problem can also be solved by considering freebody of component parts.



Applying 'work done = ΔKE' for body A

$(W_A - T)\times S_A = \frac{W_A}{2g}(v_1^2 - v_2^2)$ —— (1)

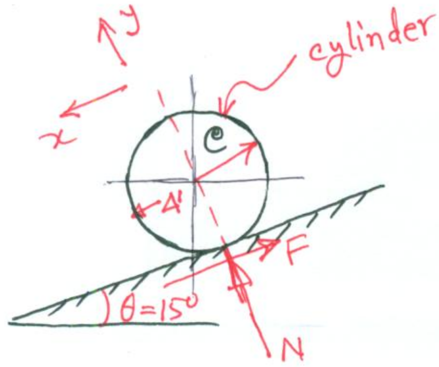
Applying 'work done = ΔKE' for rotating part B

$M_0\theta = \frac{I_0}{2}(\omega_2^2 - \omega_1^2)$

$\Rightarrow T\times 2 - F\times\frac{3}{12} = \frac{I_0}{2}(\omega_2^2 - \omega_1^2)$ —— (2)

In eq^{n}s (1) & (2) there are two unknowns T & W_A, so can be solved.

Ex. 281/P. 402



$W_c = 966$ lb , rolling down
$\theta = 15^\circ$
$v_1 = 0$, $v_2 = ?$
$S_c = 50$ ft

Note: N does no work – no movement in y direction
F does no work – the cylinder is rolling (Art. 280/P. 401)

Sol^n

$$U_{net} = W_c \sin\theta \times S_c = 966 \times \sin 15^\circ \times 50 = 12500{'}96 \text{ ft-lb}$$

$$\Delta KE = \frac{W_c}{2g}\left(v_2^2 - v_1^2\right) + \frac{\bar{I}_c}{2}\left(\omega_2^2 - \omega_1^2\right)$$

Taking c.g. as the reference point

here, $\omega_1 = \frac{v_1}{r_c} = 0$, $\omega_2 = \frac{v_2}{r_c} = \frac{v_2}{2}$

$$\bar{I}_c = \frac{m_c r_c^2}{2} = \frac{966}{32{'}2} \times \frac{2^2}{2} = 60 \text{ slug-ft}^2$$

$$\therefore \Delta KE = \frac{966}{2 \times 32{'}2} \times \left(v_2^2 - 0\right) + \frac{60}{2} \times \left(\frac{v_2^2}{4} - 0\right)$$

$$= 22{'}5\, v_2^2$$

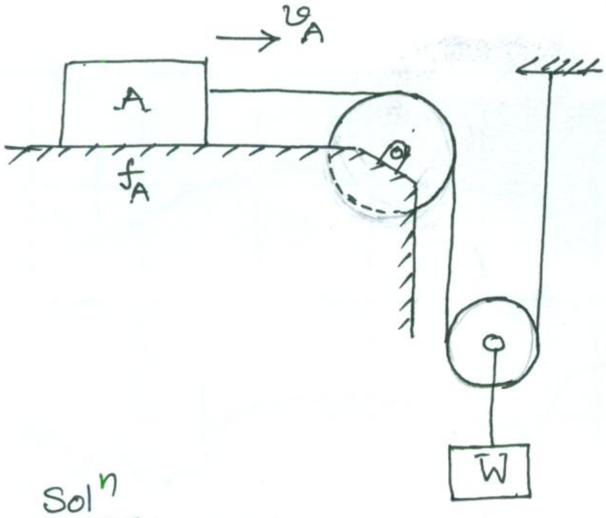
Using $U_{net} = \Delta KE$

$$12500{'}96 = 22{'}5\, v_2^2$$

$$\therefore v_2 = \boxed{23{'}57 \text{ fps.}}$$ Ans.

* In the book, the problem is solved in a different way considering the instantaneous center as the reference point.

#1136/P.327



$W_A = 966$ lb

$f_A = 1/3$

$v_{A1} = 10$ fps

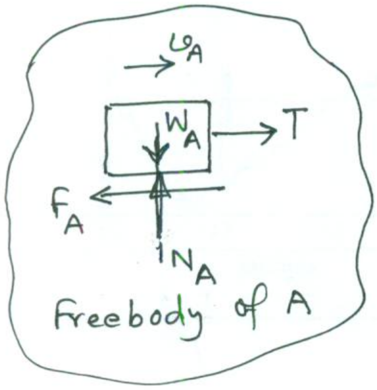
$v_{A2} = 35$ fps } in 25 s.

(a) $W = ?$

(b) S_W during 25 s. = ?

(c) Tension in cable = ?

Sol^n



$$v_{A2} = v_{A1} + a_A t$$

$$\Rightarrow 35 = 10 + a_A \times 25$$

$$\therefore a_A = 1 \text{ fps}^2$$

$$a_W = \frac{a_A}{2} = 0.5 \text{ fps}^2$$

From the freebody of A

$\Sigma F_V = 0 \uparrow +ve$

$$N_A - W_A = 0$$

$$\therefore N_A = W_A = 966 \text{ lb}$$

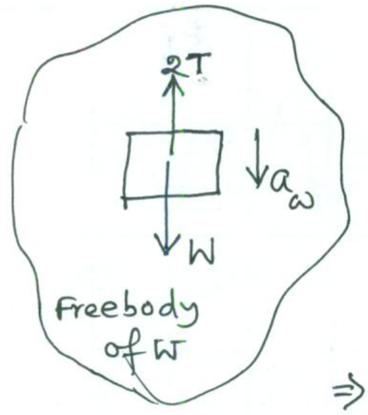
$$\therefore F_A = N_A \cdot f_A = 966 \times \frac{1}{3} = 322 \text{ lb}$$

$\Sigma F_H = 0 \rightarrow +ve$

$$T - F = \frac{W_A}{g} \cdot a_A$$

$$\Rightarrow T - 322 = \frac{966}{32.2} \times 1$$

$$\therefore T = \boxed{352 \text{ lb}}$$ Ans.



From the freebody of the weight W

Taking $\Sigma F_V = ma \downarrow +ve$

$$W - 2T = \frac{W}{g} \cdot a_W$$

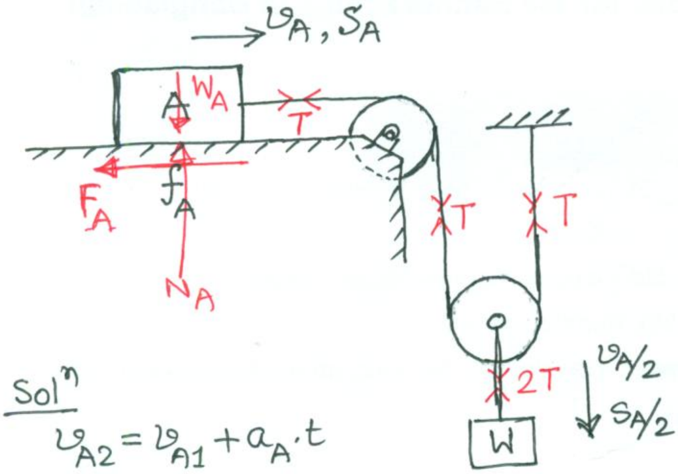
$$\Rightarrow W - 2 \times 352 = \frac{W}{32.2} \times 0.5$$

$$\therefore W = \boxed{715.12 \text{ lb}}$$ Ans.

$$S_A = v_{A1} t + \frac{1}{2} a_A t^2 = 10 \times 25 + \frac{1}{2} \times 1 \times 25^2 = 562.5 \text{ ft}$$

$$S_W = \frac{S_A}{2} = \frac{562.5}{2} = \boxed{281.25 \text{ ft}}$$ Ans.

#1136/P.327 (Using Energy Principle)



$W_A = 966$ lb

$f_A = \frac{1}{3}$

$v_{A1} = 10$ fps, $v_{A2} = 35$ fps; $\Delta t = 25$ s.

(a) $W = ?$
(b) $S_W = ?$
(c) $T = ?$

Weightless cable, weightless and frictionless pulleys.

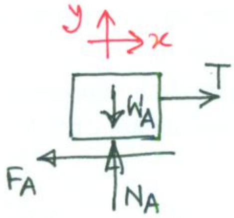
Sol^n

$$v_{A2} = v_{A1} + a_A \cdot t$$

$$\Rightarrow 35 = 10 + a_A \cdot 25$$

$$\therefore a_A = 1 \text{ fps}^2 \qquad \therefore S_A = v_{A1}t + \frac{1}{2}a_A t^2 = 10\times 25 + \frac{1}{2}\times 1\times 25^2 = 562.5 \text{ ft}$$

$$\therefore S_W = \frac{S_A}{2} = \frac{562.5}{2} = \boxed{281.25 \text{ ft}} \text{ Ans.}$$



From the freebody of A, $\Sigma F_y = 0 \uparrow +$ve gives

$$N_A - W_A = 0 \quad \therefore N_A = W_A = 966 \text{ lb}$$

$$\therefore F_A = N_A \cdot f_A = 966 \times \frac{1}{3} = 322 \text{ lb}$$

From the freebody of the whole system

$$\Delta KE = \frac{1}{2}\frac{W_A}{g}\left(v_{A2}^2 - v_{A1}^2\right) + \frac{1}{2}\cdot\frac{W}{g}\left(\frac{v_{A2}^2}{4} - \frac{v_{A1}^2}{4}\right)$$

$$= \frac{1}{2}\times\frac{966}{32.2}\left(35^2 - 10^2\right) + \frac{1}{2}\cdot\frac{W}{32.2}\left(\frac{35^2}{4} - \frac{10^2}{4}\right)$$

$$= 16875 + 4.37W$$

$$U_{net} = -F_A \cdot S_A + W \cdot S_W$$

$$= -322\times 562.5 + W\times 281.25$$

$$= -181125 + 281.25W$$

Now $U_{net} = \Delta KE$

$$\Rightarrow -181125 + 281.25W = 16875 + 4.37W$$

$$\Rightarrow 276.88W = 198000$$

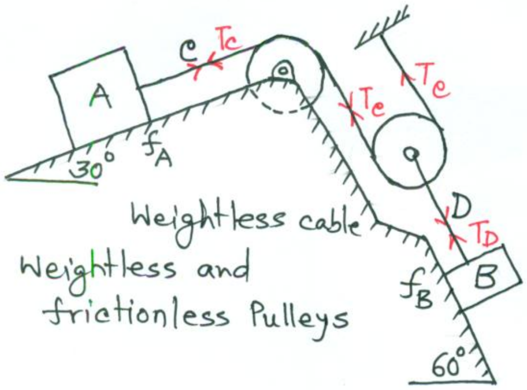
$$\therefore W = \boxed{715.11 \text{ lb}} \text{ Ans}$$

From the freebody of A, taking $\Sigma F_x = ma \rightarrow +$ve

$$T - F_A = \frac{W_A}{g}\cdot a_A$$

$$\Rightarrow T - 322 = \frac{966}{32.1}\times 1 \qquad \therefore T = \boxed{352 \text{ lb}} \text{ Ans}$$

1138/P. 328



$W_A = 200$ lb

$W_B = 100$ lb

$f_A = 1/4$

$f_B = 1/3$

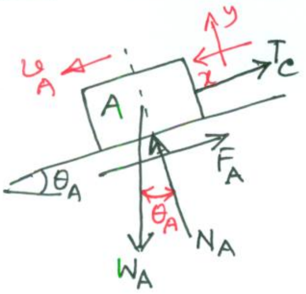
$S_A = ?$ for $t = 30$ s., $v_0 = 0$

Direction ?

$T_c = ?$ and $T_D = ?$

Note: Direct^n of motion not given. It may be assumed and finally obtained from the sign of certain determined quantities. Here it may also be judged/perceived in advance through logic.

Sol^n Let's assume that A moves down the plane



From the freebody of A

$\Sigma F_y = 0$, +ve y direct^n +ve

$$\Rightarrow N_A - W_A \cos\theta_A = 0$$

$$\therefore N_A = 200 \cos 30^\circ = 173{\cdot}2 \text{ lb}$$

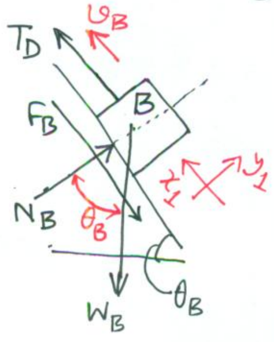
$$F_A = f_A N_A = \frac{1}{4} \times 173{\cdot}2 = 43{\cdot}3 \text{ lb}$$

Considering $\Sigma F_x = ma$, +ve x direct^n +ve

$$\Rightarrow W_A \sin\theta_A - F_A - T_c = \frac{W_A}{g} \cdot a_A$$

$$\Rightarrow 200 \sin 30^\circ - 43{\cdot}3 - T_c = \frac{200}{32{\cdot}2} \times a_A$$

$$\therefore T_c + 6{\cdot}21\, a_A = 56{\cdot}7 \quad \text{——— (1)}$$



From the freebody of B

$\Sigma F_{y_1} = 0$

$$\Rightarrow N_B - W_B \cos\theta_B = 0$$

$$\therefore N_B = 100 \cos 60^\circ = 50 \text{ lb}$$

$$F_B = f_B N_B = \frac{1}{3} \times 50 = 16{\cdot}67 \text{ lb}$$

contd...

#1138/P.328 contd..

Also from the freebody of B

$\Sigma F_{x1} = 0$ gives

$$T_D - F_B - W_B \sin\theta_B = \frac{W_B}{g} \cdot a_B$$

$$\Rightarrow T_D - 16{\cdot}67 - 100 \sin 60^\circ = \frac{100}{32{\cdot}2} a_B$$

$$\therefore T_D - 3{\cdot}11 a_B = 103{\cdot}27 \quad\text{——— (2)}$$

Now $T_D = 2T_C$

also $a_A = 2a_B$ Note: A moves twice the distance moved by B

$\therefore$ From eq^n (2)

$$2T_C - 3{\cdot}11 \times \frac{a_A}{2} = 103{\cdot}27$$

i.e. $T_C = 0{\cdot}78 a_A + 51{\cdot}64$ ——— (3)

(3) into (1)

$$0{\cdot}78 a_A + 51{\cdot}64 + 6{\cdot}21 a_A = 56{\cdot}7$$

$\therefore a_A = 0{\cdot}724$ fps^2; Since a_A is +ve, assumed direct^n of motion is correct, i.e. A moves down

From eq^n (3) $T_C = 0{\cdot}78 \times 0{\cdot}724 + 51{\cdot}64 = \boxed{52{\cdot}2 \text{ lbs}}$

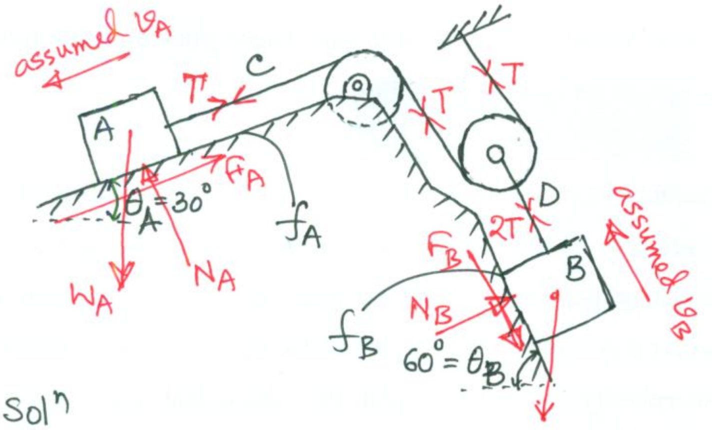
$$T_D = 2T_C = 2 \times 52{\cdot}2 = \boxed{104{\cdot}4 \text{ lb}}$$

$$S_A = v_0 t + \frac{1}{2} a_A t^2$$

$$= 0 + \frac{1}{2} \times 0{\cdot}724 \times 30^2$$

$$= \boxed{325{\cdot}8 \text{ ft}}$$

1138/P.328 (Using Energy Principle)

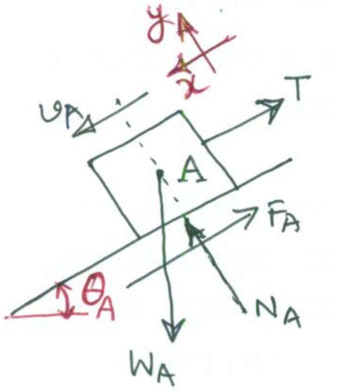


$W_A = 200$ lb
$W_B = 100$ lb
$f_A = \frac{1}{4}$, $f_B = \frac{1}{3}$
$S_A = ?$ in 30 sec./direct^n ?
$T_C = ?$
$T_D = ?$

Initially at rest

Sol^n

Let u assume that body A moves downward.



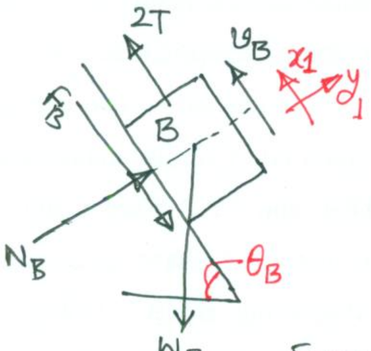
From the freebody of A taking $\Sigma F_y = 0$ +ve y direct^n as +ve

$$N_A - W_A \cos\theta_A = 0$$

$$\Rightarrow N_A - 200 \cos 30^\circ = 0$$

$$\therefore N_A = 173.2 \text{ lb}$$

$$\therefore F_A = N_A \cdot f_A = 173.2 \times \frac{1}{4} = 43.3 \text{ lb}$$



From the freebody of B, $\Sigma F_{y_1} = 0$ gives

$$N_B - W_B \cos\theta_B = 0$$

$$\therefore N_B = 100 \times \cos 60^\circ = 50 \text{ lb}$$

$$\therefore F_B = N_B \cdot f_B = 50 \times \frac{1}{3} = 16.67 \text{ lb}$$

From the freebody of the entire system

$$U_{net} = (W_A \sin\theta_A - F_A) \cdot S_A - (W_B \sin\theta_B + F_B) \cdot S_B$$

$$= (200 \sin 30^\circ - 43.3) S_A - (100 \sin 60^\circ + 16.67) \cdot \frac{S_A}{2}$$

[Substituting $S_B = \frac{S_A}{2}$]

$$= 56.7\, S_A - 51.64\, S_A$$

$$= 5.06\, S_A$$

$$\Delta KE = \frac{1}{2} \frac{W_A}{g} v_A^2 + \frac{1}{2} \cdot \frac{W_B}{g} \cdot v_B^2 \quad \left[\because v_B = \frac{v_A}{2}\right]$$

$$= \frac{1}{2} \cdot \frac{200}{32.2} v_A^2 + \frac{1}{2} \cdot \frac{100}{32.2} \frac{v_A^2}{4}$$

$$= 3.49\, v_A^2$$

$$= 3.49 \times 2 a_A S_A \quad \left[\because v_A^2 = 2 a_A S_A\right]$$

$$= 6.98\, a_A S_A$$

contd....

#1138/P.328 Contd...

Using Energy Principle

Now $U_{net} = \Delta KE$

$$\Rightarrow 5.06\, S_A = 6.98\, a_A\, S_A$$

$$\therefore a_A = 0.725 \text{ fps}^2,$$ Since a_A is +ve, our assumed direction of motion is correct.

$$S_A = \frac{1}{2} a_A t^2 = \frac{1}{2} \times 0.725 \times 30^2 = \boxed{326.25 \text{ ft}}$$ Ans.

From the freebody of A, $\Sigma F_x = m_A a_A$

$$\Rightarrow W_A \sin\theta_A - T - F_A = \frac{W_A}{g} \cdot a_A$$

$$\Rightarrow 200 \sin 30^\circ - T - 43.3 = \frac{200}{32.2} \times 0.725$$

$$\therefore T = 52.2 \text{ lb}$$

$$\text{ie } T_C = \boxed{52.2 \text{ lb}}$$ Ans.

$$T_D = 2T_C = 2 \times 52.2 = \boxed{104.4 \text{ lb}}$$ Ans

#1144/P. 329

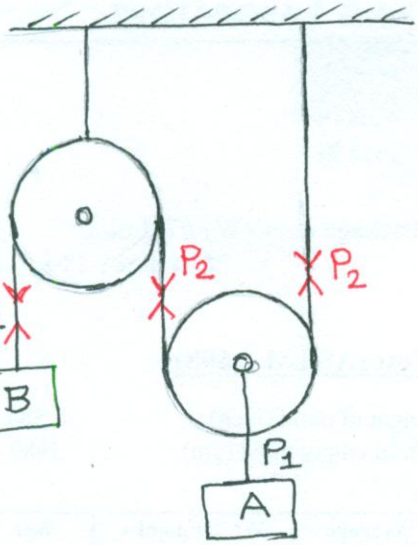
$W_A = 120$ lb

$W_B = 80$ lb

Chord and pulleys are weightless & frictionless

$a_A = ?$ $a_B = ?$

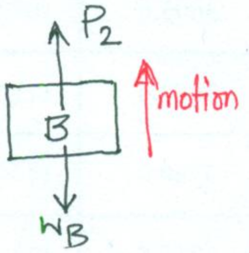
$P_1 = ?$ $P_2 = ?$



sol^n

Since $W_A > W_B$, let's assume that A moves downward and B moves upward.

Here $P_1 = 2P_2$ and $a_B = 2a_A$



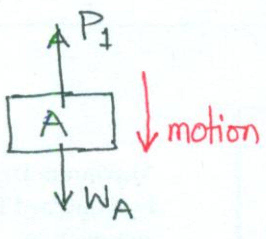
From the freebody of B,

$\Sigma F_y = m_B a_B$ $\uparrow +ve$

$\Rightarrow P_2 - W_B = \frac{W_B}{g} \cdot a_B$

$\Rightarrow \frac{P_1}{2} - 80 = \frac{80}{32 \cdot 2} \times 2a_A$

$\therefore P_1 = 9 \cdot 94\, a_A + 160$ —— (1)



From the freebody of A

$\Sigma F_y = m_A a_A$ $\downarrow +ve$

$\Rightarrow W_A - P_1 = \frac{W_A}{g} \cdot a_A$

$\Rightarrow 120 - P_1 = \frac{120}{32 \cdot 2} \times a_A$

$\Rightarrow 120 - 9 \cdot 94\, a_A - 160 = 3 \cdot 73\, a_A$ [substituting P_1 from (1)]

$\therefore a_A = \boxed{-2 \cdot 926 \text{ fps}^2}$, –ve sign implies that A moves upward

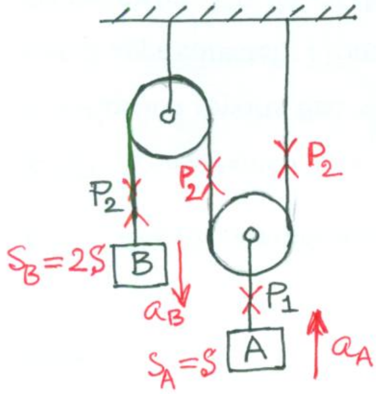
$\therefore a_B = 2a_A = 2 \times 2 \cdot 926 = \boxed{5 \cdot 852 \text{ fps}^2}$

From eq^n (1) $P_1 = 9 \cdot 94 \times (-2 \cdot 962) + 160 = \boxed{130 \cdot 92 \text{ lbs}}$

and $P_2 = P_1/2 = \boxed{65 \cdot 46 \text{ lb}}$

Note: Since these bodies are moving from rest, their velocity and accl^n are in the same sense/direction.

#1144/P. 329 (Using Energy Principles)



$W_A = 120$ lb $\qquad W_B = 80$ lb

$a_A = ?$ $\qquad a_B = ?$

$P_1 = ?$ $\qquad P_2 = ?$

Sol^n

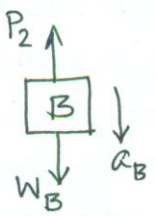
From observation it is understood that A moves upward and B moves downward.

For the whole system

$$U_{net} = W_B \cdot S_B - W_A \cdot S_A$$
$$= 80 \times 2S - 120 \times S$$
$$= 40S$$

$$\Delta KE = \frac{1}{2}\frac{W_A}{g} v_A^2 + \frac{1}{2}\frac{W_B}{g} \cdot v_B^2$$
$$= \frac{1}{2} \cdot \frac{120}{32.2} v_A^2 + \frac{1}{2} \cdot \frac{80}{32.2} v_B^2$$
$$= 1.86 v_A^2 + 1.24 \times (2v_A)^2 \quad [\text{Substituting } v_B = 2v_A]$$
$$= 6.82 v_A^2$$
$$= 6.82 \times 2 a_A S \quad [\because v_A^2 = 2a_A S]$$
$$= 13.64 a_A S$$

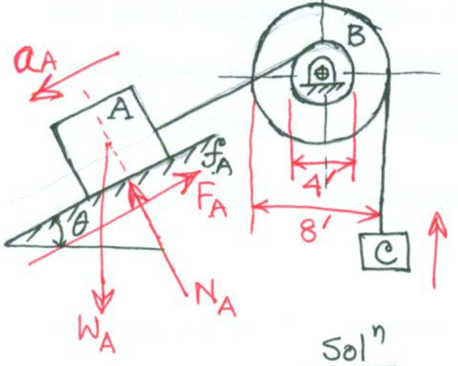
$$U_{net} = \Delta KE$$
$$\Rightarrow 40S = 13.64 a_A S$$
$$\therefore a_A = \boxed{2.93 \text{ fps}^2} \text{ Ans.}$$
$$a_B = 2a_A = 2 \times 2.93 = \boxed{5.86 \text{ fps}^2} \text{ Ans.}$$



From the freebody of B, $\Sigma F_y = ma \downarrow +ve$ gives

$$W_B - P_2 = \frac{W_B}{g} \cdot a_B$$
$$\Rightarrow 80 - P_2 = \frac{80}{32.2} \times 5.86$$
$$\therefore P_2 = \boxed{65.44 \text{ lb}} \text{ Ans.}$$
$$\therefore P_1 = 2P_2 = 2 \times 65.44 = \boxed{130.88 \text{ lb}} \text{ Ans.}$$

1292/P.379



$W_B = 6440$ lb $\qquad W_A = 12880$ lb

$\bar{k}_B = 3$ ft $\qquad f_A = \frac{1}{4}$

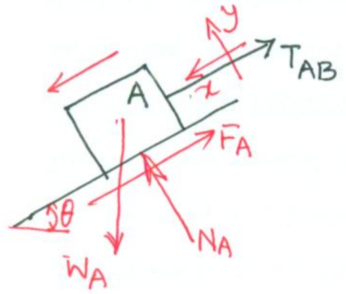
$\theta = 30^\circ$

$S_A = 100$ ft , $t = 40$ sec. , $v_{0A} = 0$

Const. accel^n of A down the plane.

(a) $W_C = ?$ (b) $T_{AB} = ?$ (c) $T_{BC} = ?$

Sol^n



From the freebody of A, taking $\Sigma F_y = 0$

$$N_A - W_A \cos\theta = 0$$

$$\therefore N_A = 12880 \times \cos 30^\circ = 11154.41 \text{ lb}$$

$$\therefore F_A = N_A \cdot f_A = 11154.41 \times \frac{1}{4} = 2788.60 \text{ lb}$$

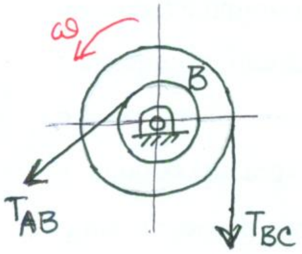
$$S_A = \frac{1}{2} a_A t^2 \Rightarrow 100 = \frac{1}{2} \times a_A \times 40^2 \Rightarrow a_A = 0.125 \text{ fps}^2$$

$$\Sigma F_x = ma$$

$$\Rightarrow W_A \sin\theta - T_{AB} - F_A = \frac{W_A}{g} \cdot a_A$$

$$\Rightarrow 12880 \sin 30^\circ - T_{AB} - 2788.6 = \frac{12880}{32.2} \times 0.125$$

$$\therefore T_{AB} = \boxed{3601.4 \text{ lb}} \text{ Ans.}$$



From the freebody of B, considering $\Sigma M = \bar{I}_B \alpha_B$ ↺+ve

$$T_{AB} \times 2 - T_{BC} \times 4 = \bar{I}_B \alpha_B$$

$$\Rightarrow T_{AB} \times 2 - T_{BC} \times 4 = \bar{k}_B^2 \times \frac{W_B}{g} \times \frac{a_A}{r_A}$$

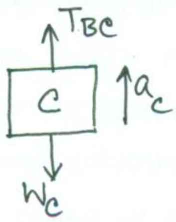
$$\Rightarrow 3601.4 \times 2 - T_{BC} \times 4 = 3^2 \times \frac{6440}{32.2} \times \frac{0.125}{2}$$

$\bar{I}_B = \bar{k}_B^2 m_B = \bar{k}_B^2 \times \frac{W_B}{g}$

$\alpha_B = \frac{a_A}{r_A}$

$$\therefore T_{BC} = \boxed{1772.58 \text{ lb}} \text{ Ans.}$$

$$\omega = \frac{v_A}{r_A} = \frac{v_c}{r_c} \Rightarrow \frac{a_A t}{r_A} = \frac{a_c t}{r_c} \Rightarrow a_c = \frac{r_c}{r_a} \times a_A = \frac{4}{2} \times 0.125 = 0.25 \text{ fps}^2$$



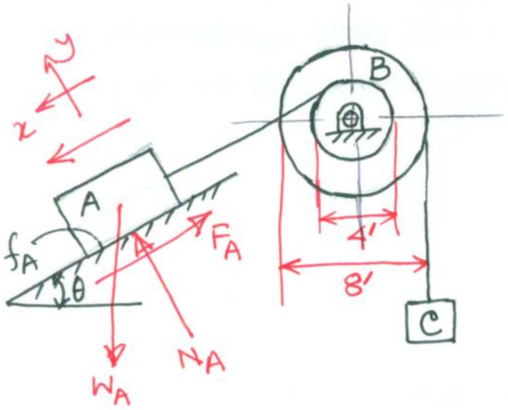
From the freebody of C, $\Sigma F_Y = m_c a_c$ ↑+ve gives

$$T_{BC} - W_c = \frac{W_c}{g} \times a_c$$

$$\Rightarrow 1772.58 - W_c = \frac{W_c}{32.2} \times 0.25$$

$$\therefore W_c = \boxed{1758.9 \text{ lb}} \text{ Ans.}$$

1292/ P. 379 (Using Energy Principle)



Sol^n From the freebody of the entire system

$$U_{net} = (W_A \sin\theta - F_A) \cdot S_A - W_C \cdot S_C$$

$$= (12880 \sin 30^\circ - 2788.6) \times 100 - W_C \times 200$$

$$= 365140 - 200\, W_C \quad (\text{lb-ft})$$

$$S_C = \theta_B \cdot r_C = \frac{S_A}{r_A} \times r_C = \frac{100}{2} \times 4 = 200 \text{ ft}$$

From freebody of B, $\Sigma F_y = 0$ gives $N_A = W_A \cos\theta$

$$\therefore F_A = N_A \cdot f_A = W_A \cos\theta \cdot f_A = 12880 \times \cos 30^\circ \times \frac{1}{4} = 2788.6 \text{ lb}$$

$$\Delta KE = \frac{1}{2}\frac{W_A}{g} v_A^2 + \frac{1}{2}\frac{W_C}{g} v_C^2 + \frac{1}{2}\bar{I}_B \omega_B^2$$

$$S_A = \frac{1}{2} a_A t^2 \Rightarrow 100 = \frac{1}{2} \times a_A \times 40^2 \quad \therefore a_A = 0.125 \text{ fps}^2$$

$$v_A = a_A t = 0.125 \times 40 = 5 \text{ fps}$$

$$v_C = r_C \omega_B = r_C \times \frac{v_A}{r_A} = 4 \times \frac{5}{2} = 10 \text{ fps}$$

$$\bar{I}_B = \bar{k}_B^2 m_B = \bar{k}_B^2 \cdot \frac{W_B}{g} = 3^2 \times \frac{6440}{32.2} = 1800 \text{ slug-ft}^2$$

$$\omega_B = \frac{v_A}{r_A} = \frac{5}{2} = 2.5 \text{ rad/s}$$

substituting the above quantities,

$$\Delta KE = \frac{1}{2} \times \frac{12880}{32.2} \times 5^2 + \frac{1}{2} \times \frac{W_C}{32.2} \times 10^2 + \frac{1}{2} \times 1800 \times 2.5^2$$

$$= 5562.5 + 1.553\, W_C \quad (\text{lb-ft})$$

Now $U_{net} = \Delta KE$

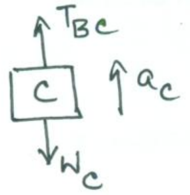
$$\Rightarrow 365140 - 200\, W_C = 5562.5 + 1.553\, W_C$$

$$\therefore W_C = 1784 \text{ lb}$$

contd....

#1292/P.379 contd..

Using Energy Principle



$$\omega_B = \frac{v_A}{r_A} = \frac{v_c}{r_c}$$

$$\Rightarrow \frac{a_A t}{r_A} = \frac{a_c t}{r_c}$$

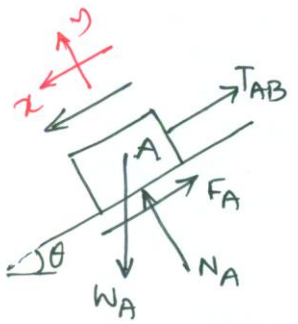
$$\therefore a_c = a_A \times \frac{r_c}{r_A} = 0.125 \times \frac{4}{2} = 0.25 \text{ fps}^2$$

$$\Sigma F_V = m_c a_c \uparrow +ve$$

$$\Rightarrow T_{BC} - W_c = \frac{W_c}{g} \times a_c$$

$$\Rightarrow T_{BC} - 1784 = \frac{1784}{32.2} \times 0.25$$

$$\therefore T_{BC} = 1797.85 \text{ lb.}$$



$$\Sigma F_x = m_A a_A$$

$$\Rightarrow W_A \sin\theta - T_{AB} - F_A = \frac{W_A}{g} \cdot a_A$$

$$\Rightarrow 12880 \sin 30^\circ - T_{AB} - 2788.6 = \frac{12880}{32.2} \times 0.125$$

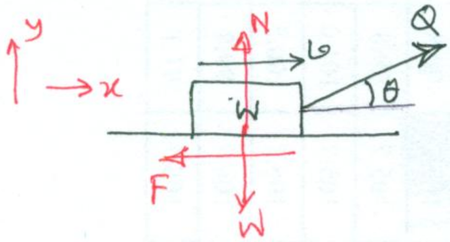
$$\therefore T_{AB} = 3601.4 \text{ lb.}$$

Note: After obtaining W_c, ~~F_{BC}~~ from $U_{net} = \Delta KE$, T_{BC} can be obtained from the free body of C or. T_{AB} can be obtained from the freebody of A and then the other T ~~i.e~~ can be obtained from the freebody of B using $\Sigma M_B = \bar{I}_B \alpha_B$.

$$\Sigma M_B = \bar{I}_B \alpha_B \curvearrowleft +ve$$

$$\Rightarrow T_{AB} \times 2 - T_{BC} \times 4 = \bar{I}_B \times \frac{a_A}{r_A}$$

#1381/P.416



$W = 100$ lb

$Q = 50$ lb

$f = \frac{1}{4}$, $\theta = 30^\circ$

$v_0 = 28$ fps, $s = 20$ ft.

Resultant force = ?

Net work done = ?

Sol^n

$\Sigma F_y = 0 \uparrow +ve$

$\Rightarrow N + Q \sin\theta = W$

$\therefore N = W - Q\sin\theta = 100 - 50 \sin 30^\circ = 75$

$\therefore F = N \cdot f = 75 \times \frac{1}{4} = 18.75$ lb

Resultant force, $R = Q \cos\theta - F$

$= 50 \cos 30^\circ - 18.75$

$= 24.55$ lb

Net Work done $= R \cdot S = 24.55 \times 20$ ft-lb

$= 491$ ft-lb.

Work done by horizontal component of $Q = +Q \cos\theta \times S$

$= +50 \cos 30^\circ \times 20$

$= +866$ ft-lb

Work done by frictional resistance, $F = -F \times S$

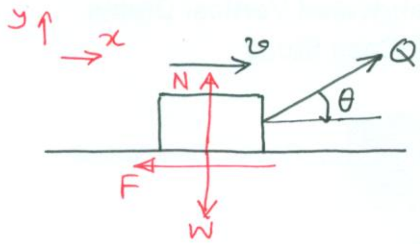
$= -18.75 \times 20$

$= -375$ lb

$\therefore$ Net work done $= +866 - 375$ ft-lb

$= 491$ ft-lb

1382/P.416



$W = 100$ lb

$Q = 10$ lb

$f = \frac{1}{4}$, $\theta = 30^\circ$

$v_0 = 28$ fps, $S = 20$ ft.

Net work done = ?

What does -ve work mean?

Sol^n

$\Sigma F_y = 0 \uparrow +ve$

$\Rightarrow N + Q \sin\theta - W = 0$

$\therefore N = -10 \sin 30^\circ + 100$

$= +95$ lb

$\therefore F = N \cdot f = 95 \times \frac{1}{4} = 23.75$ lb

Resultant force, $R = Q \cos\theta - F$

$= 10 \cos 30^\circ - 23.75$

$= -15.09$ lb

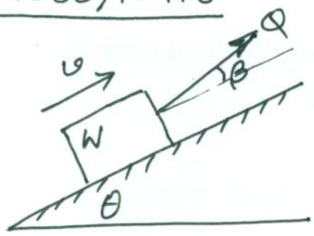
Work done $= R \cdot S$

$= -15.09 \times 20$

$= -301.79$ lb

-ve work means that the direction of resultant force and displacement are opposite. It also means that the resultant force is acting opposite to the velocity.

#1383/P.416



$W = 100$ lb $\theta = 30^\circ$

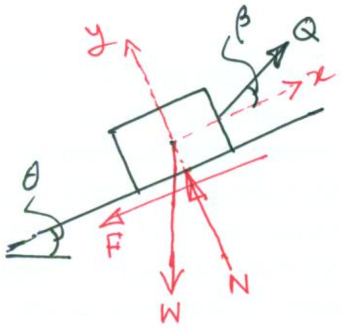
$Q = 120$ lb $\beta = 15^\circ$

$f = \frac{1}{4}$ $S = 8'$ up the plane

Net work done = ?

velocity increases ? or decreases ?

Sol^n



$\Sigma F_Y = 0$, +ve y ~~as~~ direction +ve

$\Rightarrow N + Q \sin\beta - W \cos\theta = 0$

$\Rightarrow N + 120 \sin 15^\circ - 100 \cos 30^\circ = 0$

$\therefore N = 55.54$ lb

$\therefore F = 55.54 \times \frac{1}{4} = 13.89$ lb

$$R_x = Q\cos\beta - F - W\sin\theta$$
$$= 120\cos 15^\circ - 13.89 - 100 \times \sin 30^\circ$$
$$= 52.02 \text{ (towards +ve } x\text{)}$$

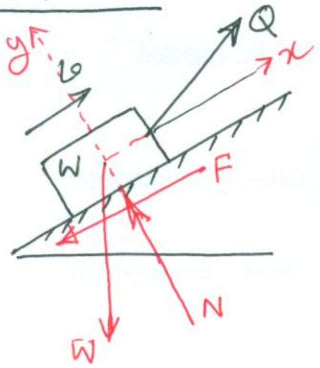
$$R_Y = 0$$

$$R = \sqrt{R_x^2 + R_Y^2} = 52.02 \text{ lb (towards +ve } x\text{)}$$

Net work done $= 52.02 \times 8 = 416.16$ ft-lb

Since the work done is +ve, the velocity will increase.

#1386/P.417



$Q = 46\ \text{lb}$ $\quad \beta = 0 \quad \theta = 15^\circ$

$f = 0.3 \quad U_{net} = -200\ \text{ft-lb}$

$S = 12\ \text{ft}$ (to right)

$W = ?$

Sol^n $\quad \Sigma F_y = 0$, +ve y as +ve

$\Rightarrow Q \sin\beta + N - W = 0$

$\Rightarrow 46 \sin 0^\circ + N - W = 0$

$\therefore N = W$

$F = N \cdot f = W \times 0.3 = 0.3W$

Resultant force $= Q\cos\beta - F - W\sin\theta$

$= 46\cos 0^\circ - 0.3W - W\sin 15^\circ$

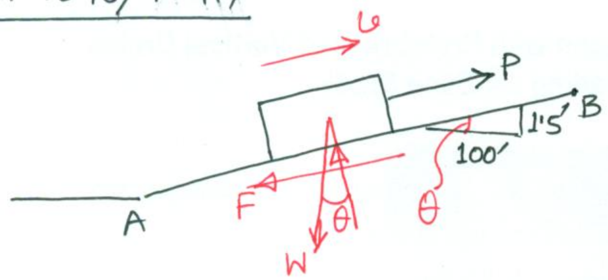
$= 46 - 0.56W$, in the +ve x direct^n

$\therefore (46 - 0.56W) \times 12 = -200$

$\Rightarrow 46 - 0.56W = -\frac{200}{12}$

$\therefore W = \boxed{111.9\ \text{lb}}$ Ans.

#1390/P.417



$W = 3000 \text{ ton} = 3000 \times 2000 \text{ lb}$

$F = 14 \times 3000 \text{ lb} = 42000 \text{ lb}$

$P = 135000 \text{ lb}$

$v_B = 80 \text{ fps}$

$S_{AB} = 1 \text{ mile} = 1760 \times 3 \text{ ft}$

$v_A = ?$

Sol^n

For the train to move from A to B

$$U_{net} = (P - F - W \sin\theta) \times S_{AB}$$

$$= \left(135000 - 42000 - 3000 \times 2000 \times \frac{1.5}{\sqrt{1.5^2 + 100^2}}\right) \times 1760 \times 3 \text{ ft-lb}$$

$$= 15893450.98 \text{ ft-lb}$$

$$\Delta KE = \frac{1}{2} \cdot \frac{W}{g}\left(v_B^2 - v_A^2\right)$$

$$= \frac{1}{2} \times \frac{3000 \times 2000}{32.2}\left(80^2 - v_A^2\right)$$

$$= 93167.7 \times \left(80^2 - v_A^2\right)$$

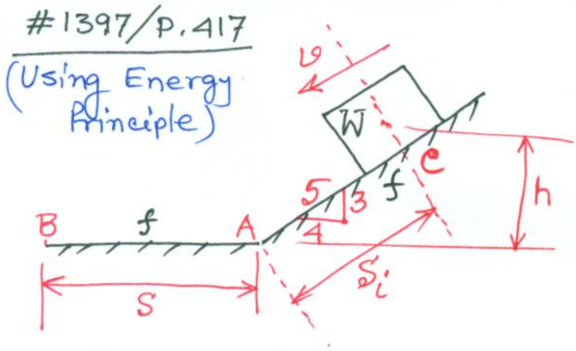
Now $U_{net} = \Delta KE$

$$\Rightarrow 15893450.98 = 93167.7\left(80^2 - v_A^2\right)$$

$$\therefore v_A = \boxed{78.93 \text{ fps.}}$$ Ans.

#1397/p.417

(Using Energy Principle)



Given

$f = \frac{1}{3}$

$h = 10$ ft

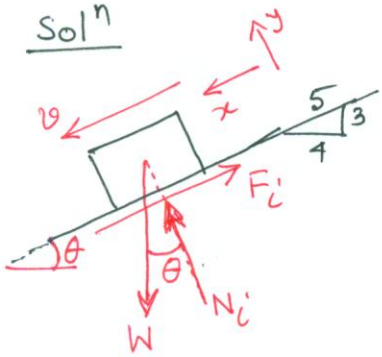
$S = 18$ ft

$v_B = 0$

Determine

v_C

Sol^n



$\Sigma F_y = 0$, +ve y as +ve (box on slope)

$\Rightarrow N_i - W\cos\theta = 0$

$\therefore N_i = W \times \frac{4}{5} = \frac{4}{5}W$

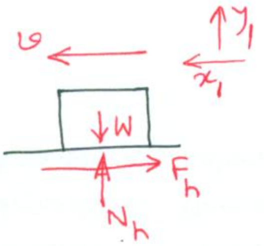
$\therefore F_i = N_i \cdot f = \frac{4}{5}W \times \frac{1}{3} = \frac{4}{15}W$

$\frac{S_i}{h} = \frac{5}{3} \quad \therefore S_i = \frac{5}{3} \times 10 = \frac{50}{3}$ ft

$\therefore$ Work done from C to A $= (W.\sin\theta - F_i) \times S_i$

$= \left(W \times \frac{3}{5} - \frac{4}{15}W\right) \times \frac{50}{3}$

$= \frac{50}{9}W$



When the box is on horizontal plane

$\Sigma F_{y_1} = 0$ gives, $N_h = W$

$\therefore F_h = N_h \cdot f = W \times \frac{1}{3} = \frac{W}{3}$

Work done from A to B $= -F.S = -\frac{W}{3} \times 18 = -6W$

[Here work done is -ve, because force and displacement are opposite]

$U_{net} = \frac{50}{9}W - 6W = -\frac{4}{9}W$

$\Delta KE = \frac{1}{2} \cdot \frac{W}{g}\left(v_B^2 - v_C^2\right) = \frac{1}{2} \times \frac{W}{32.2} \times \left(0 - v_C^2\right) = -\frac{Wv_C^2}{64.4}$

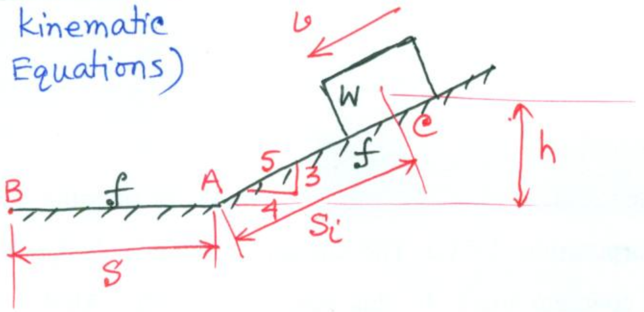
Now $U_{net} = \Delta KE$

$\Rightarrow -\frac{4}{9}W = -\frac{Wv_C^2}{64.4}$

$\therefore v_C = \boxed{5.35 \text{ fps}}$ Ans.

#1397/P.417

(Using kinematic Equations)



$f = \frac{1}{3}$ for both plane

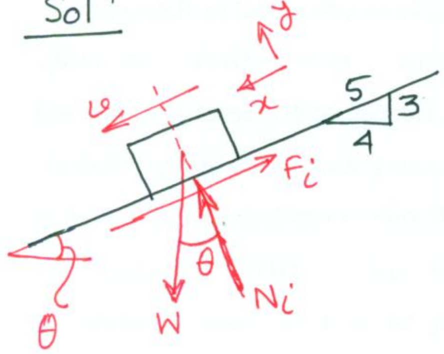
$h = 10$ ft

$S = 18$ ft

$v_B = 0$

$v_c = ?$

Sol^n



For motion from c to A

$\Sigma F_y = 0$, +ve y as +ve

$\Rightarrow N_i - W\cos\theta = 0$

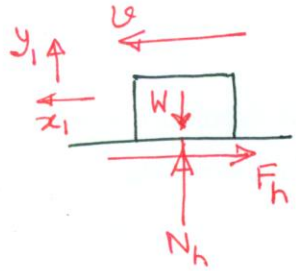
$\therefore N_i = W \times \frac{4}{5} = 0.8W$

$\Sigma F_x = ma$, +ve x as +ve

$\Rightarrow W\sin\theta - F_i = \frac{W}{g} \cdot a_i$

$\Rightarrow W \times \frac{3}{5} - 0.8W = \frac{W}{32.2} a_i$

$\therefore a_i = 10.73 \text{ fps}^2$



For motion from A to B

$\Sigma F_{Y_1} = 0$, +ve y_1 as +ve

$\Rightarrow N_h - W = 0 \quad \therefore N_h = W$

$\therefore F_h = N_h \cdot f = W \times \frac{1}{3} = \frac{W}{3}$

$\Sigma F_{x_1} = ma_h \leftarrow$ +ve

$\Rightarrow -F_h = \frac{W}{g} \cdot a_h$

$\Rightarrow -\frac{W}{3} = \frac{W}{32.2} \times a_h$

$\therefore a_h = -10.73 \text{ fps}^2$

$v_B^2 = v_A^2 + 2a_h \cdot S$

$\Rightarrow 0 = v_A^2 + 2 \times (-10.73) \times 18$

$\therefore v_A = 19.65$ fps

$\frac{S_i}{h} = \frac{5}{3} \quad \therefore S_i = \frac{5}{3} \times h = \frac{5}{3} \times 10 = \frac{50}{3}$ ft

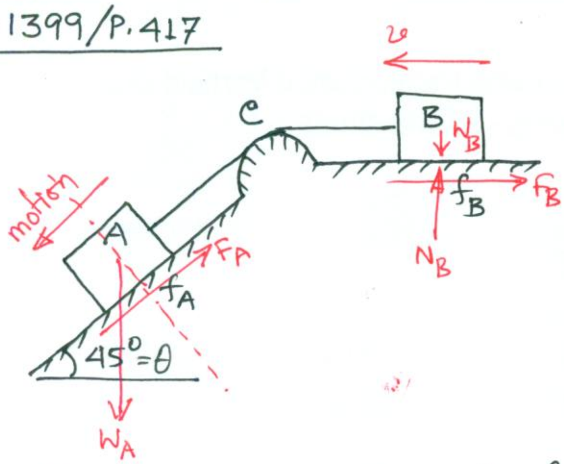
On inclined plane $(c \rightarrow A)$

$v_A^2 = v_c^2 + 2a_i S_i$

$\Rightarrow 19.65^2 = v_c^2 + 2 \times 10.73 \times \frac{50}{3}$

$\therefore v_c = \boxed{5.33 \text{ fps}}$ Ans.

#1399/P.417



$W_A = 1000$ lb

$f_A = 0.15$

$f_B = 0.6$

$v_0 = 20$ fps (to left)

$S = 160$ ft

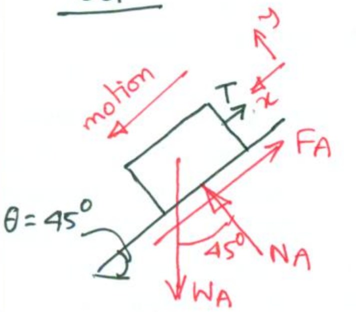
$v_f = 10$ fps

(a) $W_B = ?$

(b) $T_{BC} = T_{AC} = T = ?$

(c) $\Delta PE = ?$

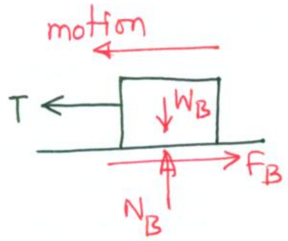
Sol^n



From the freebody of A, $\Sigma F_y = 0$ gives

$N_A - W_A \cos\theta = 0 \quad \therefore N_A = 1000 \cos 45^0 = 707.11$ lb

$\therefore F_A = N_A \cdot f_A = 707.11 \times 0.15 = 106.07$ lb



From the freebody of B, taking $\Sigma F_v = 0 \uparrow$ +ve

$N_B - W_B = 0 \quad \therefore N_B = W_B$

$F_B = N_B \cdot f_B = W_B \times 0.6 = 0.6 W_B$

Now for the entire system

$$U_{net} = (W_A \sin\theta - F_A) \times S - F_B \times S$$

$$= (1000 \sin 45^0 - 106.07) \times 160 - 0.6 W_B \times 160$$

$$= 96165.88 - 96 W_B$$

$$\Delta KE = \frac{1}{2} \cdot \frac{W_A}{g}(v_f^2 - v_0^2) + \frac{1}{2} \cdot \frac{W_B}{g}(v_f^2 - v_0^2)$$

$$= \frac{1}{2} \times \frac{1000}{32.2} \times (10^2 - 20^2) + \frac{1}{2} \times \frac{W_B}{32.2} \times (10^2 - 20^2)$$

$$= -4658.39 - 4.66 W_B$$

$$U_{net} = \Delta KE$$

$$\Rightarrow 96165.88 - 96 W_B = -4658.39 - 4.66 W_B$$

$$\therefore W_B = \boxed{1103.8 \text{ lb}}$$

contd.....

#1399/P.417 contd.--

Using Energy Principle

From the freebody of B

$$U_{net} = -F_B \times S + T \times S$$
$$= -0.6 W_B \times 160 + T \times 160$$
$$= -0.6 \times 1103.8 \times 160 + T \times 160$$
$$= -105964.8 + 160T \quad \text{lb-ft}$$

$$\Delta KE = \frac{1}{2} \cdot \frac{W_B}{g} \left(v_f^2 - v_0^2 \right)$$
$$= \frac{1}{2} \times \frac{1103.8}{32.2} \times \left(10^2 - 20^2 \right)$$
$$= -5141.9 \quad \text{lb-ft}$$

$$U_{net} = \Delta KE$$

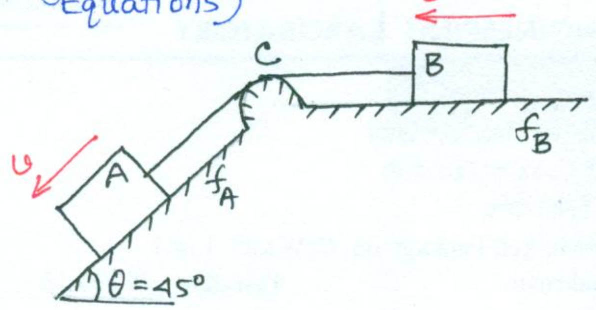
$$\Rightarrow -105964.8 + 160T = -5141.9$$
$$\therefore T = \boxed{630.14 \text{ lb}}$$ Ans.

$$\Delta PE = -W_A \times S \times \sin\theta$$
$$= -1000 \times 160 \sin 45^\circ$$
$$= \boxed{-113137.09 \text{ lb-ft}}$$

-ve sign means decrease in PE. Ans.

1399/P. 417

(Using Kinematic Equations)



$W_A = 1000$ lb

$f_A = 0.45$

$f_B = 0.6$

$v_0 = 20$ fps (to left)

$S = 160$ ft

$v_f = 10$ fps

(a) $W_B = ?$

(b) $T_{BC} = T_{AC} = T = ?$

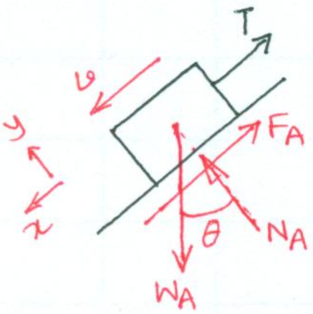
(c) $\Delta PE = ?$

Sol^n

$$v_f^2 = v_0^2 + 2aS$$

$$\Rightarrow 10^2 = 20^2 + 2 \times a \times 160$$

$$\therefore a = -0.938 \text{ fps}^2$$



From the freebody of A, $\Sigma F_y = 0$ gives

$$N_A = W_A \cos\theta = 1000 \cos 45^\circ = 707.11 \text{ lb}$$

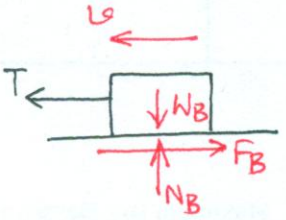
$$\therefore F_A = N_A \cdot f_A = 707.11 \times 0.15 = 106.07 \text{ lb}$$

Now $\Sigma F_x = m_A a$, down the plane +ve

$$W_A \sin\theta - T - F_A = \frac{W_A}{g} \cdot a$$

$$\Rightarrow 1000 \sin 45^\circ - T - 106.07 = \frac{1000}{32.2} \times (-0.938)$$

$$\therefore T = \boxed{630.17 \text{ lb}} \text{ Ans.}$$



From the freebody of B

$$\Sigma F_h = m_B \cdot a \quad \leftarrow +ve$$

$$\Rightarrow T - F_B = \frac{W_B}{g} \cdot a$$

$$\Rightarrow 630.17 - 0.6 W_B = \frac{W_B}{32.2} \times (-0.938)$$

$$\Rightarrow 0.57 W_B = 630.17$$

$$\therefore W_B = \boxed{1103.8 \text{ lb}} \text{ Ans.}$$

$\Sigma F_V = 0$ gives

$$N_B = W_B$$

$$F_B = N_B \cdot f_B = W_B \times 0.6 = 0.6 W_B$$

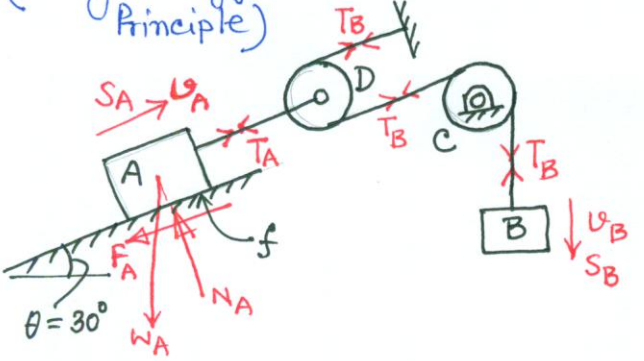
$$\Delta PE = -W_A \times S \cdot \sin\theta$$

$$= -1000 \times 160 \sin 45^\circ$$

$= \boxed{-113137.09 \text{ lb-ft}}$, −ve sign means decrease in PE. Ans.

1402/P. 418

(Using Energy Principle)



$W_A = 1000$ lb

$f = \frac{1}{3}$

$S_A = 60$ ft

$t = 12$ s.

$v_{0A} = 0$

(a) $W_B = ?$

(b) $T_A = ?$ $T_B = ?$

(c) $\Delta PE_A = ?$

$\Delta PE_B = ?$

Pulleys C & D are weightless & frictionless.

Sol^n For the entire system, considering $U_{net} = \Delta KE$

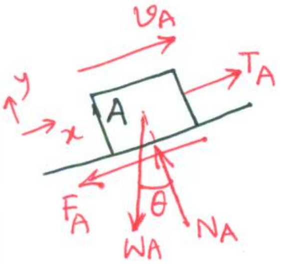
$$-F_A \cdot S_A - W_A \sin\theta \cdot S_A + W_B \cdot S_B = \frac{1}{2} \cdot \frac{W_A}{g} v_A^2 + \frac{1}{2} \cdot \frac{W_B}{g} \cdot v_B^2 \quad \text{—— (1)}$$

$$S_A = v_{0A} t + \frac{1}{2} a_A t^2 \Rightarrow 60 = 0 + \frac{1}{2} \times a_A \times 12^2 \quad \therefore a_A = \frac{5}{6} \text{ fps}^2$$

$$v_A = v_{0A} + a_A t = 0 + \frac{5}{6} \times 12 = 10 \text{ fps}$$

$$v_B = 2 v_A = 2 \times 10 = 20 \text{ fps}$$

$$S_B = 2 S_A = 2 \times 60 = 120 \text{ ft}$$



From the freebody of A, $\Sigma F_y = 0$ +ve y direct^n +ve

$$N_A - W_A \cos\theta = 0 \quad \Rightarrow N_A = 1000 \cos 30^0 = 866 \text{ lb}$$

$$\therefore F_A = N_A \cdot f_A = 866 \times \frac{1}{3} = 288.68 \text{ lb}$$

Now from eq^n (1)

$$-288.68 \times 60 - 1000 \sin 30^0 \times 60 + W_B \times 120 = \frac{1}{2} \times \frac{1000}{32.2} \times 10^2 + \frac{1}{2} \times \frac{W_B}{32.2} \times 20^2$$

$$\Rightarrow 113.78 \, W_B = 48873.6$$

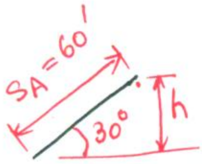
$$\therefore W_B = \boxed{429.54 \text{ lb}}$$

$\Sigma F_x = m_A a_A$, +ve x direct^n as +ve

$$\Rightarrow T_A - F_A - W_A \sin\theta = \frac{W_A}{g} \cdot a_A$$

$$\Rightarrow T_A - 288.68 - 1000 \sin 30^0 = \frac{1000}{32.2} \times \frac{5}{6}$$

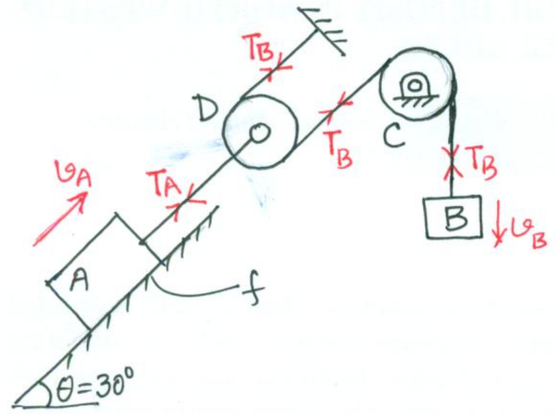
$$\therefore T_A = \boxed{814.55 \text{ lb.}} \qquad T_B = T_A/2 = \frac{814.55}{2} = \boxed{407.28 \text{ lb}}$$



$$\Delta PE_A = +\frac{W_A}{g} \cdot g \cdot h = W_A \cdot h = 1000 \times 60 \sin 30^0 = \boxed{30000 \text{ lb, increase}}$$

$$\Delta PE_B = -\frac{W_B}{g} \cdot g \cdot S_B = -W_B S_B = -429.54 \times 120 = \boxed{-51545 \text{ lb, decrease}}$$

1402/P. 418



$W_A = 1000$ lb

$f = \frac{1}{3}$

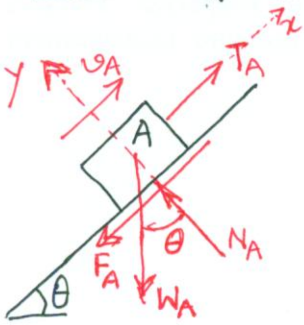
(a) $S_A = 60$ ft, $v_{0A} = 0$ } $W_B = ?$ at $t = 12$ s.

(b) $T_A = ?$ $T_B = ?$

(c) $\Delta PE_A = ?$ $\Delta PE_B = ?$

Pulleys C & D are weightless and frictionless.

Sol^n $\quad S_A = v_{0A} t + \frac{1}{2} a_A t^2 \Rightarrow 60 = 0 + \frac{1}{2} \times a_A \times 12^2 \quad \therefore a_A = 0.833 \text{ fps}^2$



From the freebody of A

$\Sigma F_y = 0$, $+y$ direct^n as +ve

$\Rightarrow N_A - W_A \cos\theta = 0$

$\therefore N_A = 1000 \cos 30^\circ = 866.03$ lb

$\Sigma F_x = m_A a_A$, up the plane +ve

$\Rightarrow T_A - W_A \sin\theta - F_A = \frac{W_A}{g} \times a_A$

$\Rightarrow T_A - 1000 \sin 30^\circ - 866.03 \times \frac{1}{3} = \frac{1000}{32.2} \times 0.833$

$\therefore T_A = \boxed{814.55 \text{ lb}}$ Ans.

From observation of the whole system

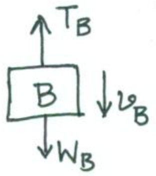
$a_B = 2 a_A$ [Also notice → $v_B = 2 v_A$, $S_B = 2 S_A$]

$= 2 \times 0.833$

$= 1.666 \text{ fps}^2$

$T_A = 2 T_B$

$\therefore T_B = T_A/2 = \frac{814.55}{2} = \boxed{407.28 \text{ lb}}$ Ans



From the freebody of B, taking $\Sigma F_v = m_B a_B \downarrow$ +ve

$W_B - T_B = \frac{W_B}{g} \cdot a_B$

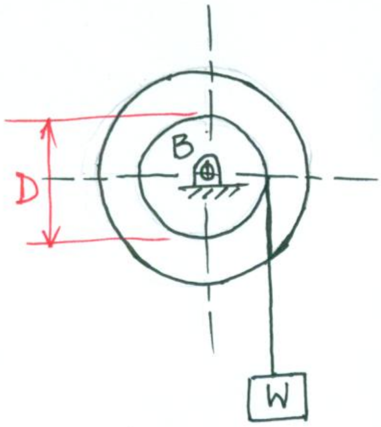
$\Rightarrow W_B - 407.28 = \frac{W_B}{32.2} \times 1.666$

$\therefore W_B = \boxed{429.5 \text{ lb}}$ Ans.

$\Delta PE_A = + \frac{W_A}{g} \cdot g \cdot h = W_A h = 1000 \times 60 \sin 30^\circ = \boxed{30000 \text{ lb increase}}$

$\Delta PE_B = - \frac{W_B}{g} \cdot g \cdot S_B = - W_B \cdot S_B = -429.5 \times (2 \times 60) = \boxed{-51540 \text{ lb decrease}}$

1414/P.418



$W_B = 2576$ lb

$\bar{k}_B = 14$ in.

$D = 32$ in.

$\omega_0 = 20$ rpm

$\omega_f = 40$ rpm

$S_W = 40$ ft ↓

$W = ?$

Friction negligible.

Sol^n

$$\Delta KE = \frac{W}{2g}\left(v_f^2 - v_0^2\right) + \frac{\bar{I}_B}{2}\left(\omega_f^2 - \omega_0^2\right)$$

Here, $\omega_0 = \dfrac{20 \times 2\pi}{60}$ rad/s $= 2{\cdot}09$ rad/s.

$\omega_f = \dfrac{40 \times 2\pi}{60}$ rad/s $= 4{\cdot}19$ rad/s.

$v = r\omega$

$$v_0 = \frac{16}{12} \times 2{\cdot}09 = 2{\cdot}787 \text{ fps}$$

$$v_f = \frac{16}{12} \times 4{\cdot}19 = 5{\cdot}587 \text{ fps}$$

$$\bar{I}_B = \bar{k}_B^2 m_B = \left(\frac{14}{12}\right)^2 \times \frac{2576}{32{\cdot}2} = 108{\cdot}89 \text{ slug-ft}^2$$

$$\therefore \Delta KE = \frac{W}{2 \times 32{\cdot}2}\left(5{\cdot}587^2 - 2{\cdot}787^2\right) + \frac{108{\cdot}89}{2}\left(4{\cdot}19^2 - 2{\cdot}09^2\right)$$

$$= 0{\cdot}364\,W + 718{\cdot}02 \text{ ft-lb}$$

$$U_{net} = W \cdot S_W = W \times 40 = 40\,W \text{ ft-lb}$$

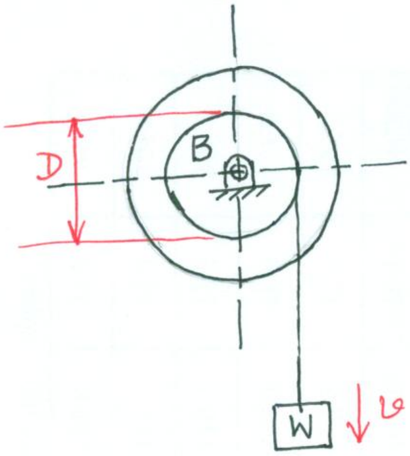
Now, according to the principle of work and kinetic energy

$$U_{net} = \Delta KE$$

$$\Rightarrow 40\,W = 0{\cdot}364\,W + 718{\cdot}02$$

$$\therefore W = \boxed{18{\cdot}12 \text{ lb}}$$ Ans.

#1416/P.419



$W_B = 200$ lb

$D = 2$ ft

$W = 32.2$ lb

Neglect friction and mass of cable

$v_{oW} = 0$, $S = 20$ ft, $t = 4$ sec.

(a) Tension in cable, $T = ?$

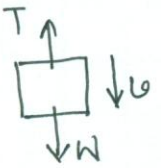
(b) Radius of gyration of B, $\bar{k}_B = ?$

Sol^n

$$S = v_{oW} t + \frac{1}{2} a t^2$$

$$\Rightarrow 20 = 0 \times 4 + \frac{1}{2} \times a \times 4^2$$

$$\therefore a = 2.5 \text{ fps}^2$$



From the freebody of the weight W, taking $\Sigma F_y = 0 \downarrow +$

$$W - T = \frac{W}{g} \cdot a$$

$$\Rightarrow 32.2 - T = \frac{32.2}{32.2} \times 2.5$$

$$\therefore T = \boxed{29.7 \text{ lb}} \text{ Ans.}$$

Change in kinetic energy of the whole system,

$$\Delta KE = \Delta KE_B + \Delta KE_W$$

$$= \frac{1}{2} \bar{I}_B \left(\omega_B^2 - \omega_{oB}^2\right) + \frac{1}{2} \frac{W}{g} \left(v_W^2 - v_{oW}^2\right)$$

$$= \frac{1}{2} \times \frac{200}{32.2} \times \bar{k}_B^2 \times 10^2 + \frac{1}{2} \times \frac{32.2}{32.2} \times 10^2$$

$$= 310.56\, \bar{k}_B^2 + 50$$

$v_W = at = 2.5 \times 4 = 10$ fps

$\omega_B = \frac{v_W}{r} = \frac{10}{1} = 10$ rad/s.

$\omega_{oB} = \frac{v_{oW}}{r} = \frac{0}{1} = 0$

$\bar{I}_B = m_B \bar{k}_B^2 = \frac{W_B}{g} \bar{k}_B^2$

$= \frac{200}{32.2} \bar{k}_B^2$

Net work done by the system, $U_{net} = W \cdot S = 32.2 \times 20 = 644$ lb

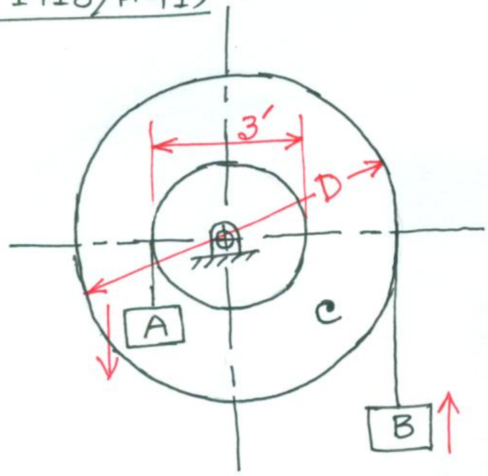
$$U_{net} = \Delta KE$$

$$\Rightarrow 644 = 310.56\, \bar{k}_B^2 + 50$$

$$\therefore \bar{k}_B = \boxed{1.38 \text{ ft}} \text{ Ans.}$$

Note: For an explanation of U_{net} see example 278/ alternative sol^n/ pp. 398 - 399

1418/P. 419



$W_A = 500$ lb (a) $S_B = 10$ ft $v_{A0} = 0$

$W_B = 150$ lb $\omega_{c0} = 0$ $v_{B0} = 0$

$D = 9$ ft $v_{Af} = ?$ $v_{Bf} = ?$

$W_c = 600$ lb (b) $a_A = ?$ $a_B = ?$ $\alpha_c = ?$

$\bar{K}_c = 3$ ft. (c) $\Delta PE = ?$

Sol^n ↶ moment produced by $A = 500 \times 1.5 = 750$ ft-lb

↷ " " " $B = 150 \times 4.5 = 675$ ft-lb

Since ↶ moment is larger, A moves down & B moves up.

$$\Delta KE = \frac{W_A}{2g}\left(v_{Af}^2 - v_{A0}^2\right) + \frac{W_B}{2g}\left(v_{Bf}^2 - v_{B0}^2\right) + \frac{\bar{I}_c}{2}\left(\omega_{cf}^2 - \omega_{c0}^2\right)$$

$$\frac{v_{Af}}{v_{Bf}} = \frac{r_A}{r_B} = \frac{1.5}{4.5} = \frac{1}{3} \Rightarrow v_{Bf} = 3v_{Af}$$

$$\omega_{cf} = \frac{v_{Bf}}{r_B} = \frac{3v_{Af}}{4.5} = \frac{2v_{Af}}{3}$$

$$\bar{I}_c = \bar{k}_c^2 m_c = 3^2 \times \frac{600}{32.2} = 167.7 \text{ slug-ft}^2$$

$$\therefore \Delta KE = \frac{500}{2\times 32.2} v_{Af}^2 + \frac{150}{2\times 32.2} \times 9 v_{Af}^2 + \frac{167.7}{2} \times \left(\frac{2}{3}v_{Af}\right)^2$$

$$= 66.37\, v_{Af}^2$$

$$U_{net} = W_A \times S_A - W_B \times S_B$$

$$= 500 \times 3.33 - 150 \times 10$$

$$= 165 \text{ lb-ft}$$

$S_B = 10$ ft

$$\theta = \frac{S_B}{r_B} = \frac{S_A}{r_A}$$

$$\therefore S_A = S_B \times \frac{r_B}{r_A}$$

$$= 10 \times \frac{1.5}{4.5}$$

$$= 3.33 \text{ ft.}$$

According to the principles of work an energy, $U_{net} = \Delta KE$

$$\Rightarrow 165 = 66.37\, v_{Af}^2$$

$$\therefore v_{Af} = \boxed{1.58 \text{ fps}} \text{ Ans.}$$

$$\therefore v_{Bf} = 3 \times 1.58 = \boxed{4.74 \text{ fps.}} \text{ Ans.}$$

Contd....

#1418/P. 419 Contd....

(b) $v_{Af}^2 = 2a_A S_A$

$\Rightarrow 1.58^2 = 2 \times a_A \times 3.33$

$\therefore a_A = \boxed{0.375 \text{ fps}^2}$ Ans.

$v_{Bf}^2 = 2a_B S_B$

$\Rightarrow 4.74^2 = 2a_B \times 10$

$\therefore a_B = \boxed{1.123 \text{ fps}^2}$ Ans.

$\omega_{cf} = \dfrac{2v_{Af}}{3} = \dfrac{2}{3} \times 1.58 = 1.059 \text{ rad/sec.}$

$\omega_{cf}^2 = \omega_{co}^2 + 2\alpha_c \theta$ | $\theta = \dfrac{S_B}{r_B} = \dfrac{10}{4.5} = 2.22 \text{ rad.}$

$\Rightarrow 1.059^2 = 0 + 2 \times \alpha_c \times 2.22$

$\therefore \alpha_c = \boxed{0.252 \text{ rad/sec}^2}$ Ans.

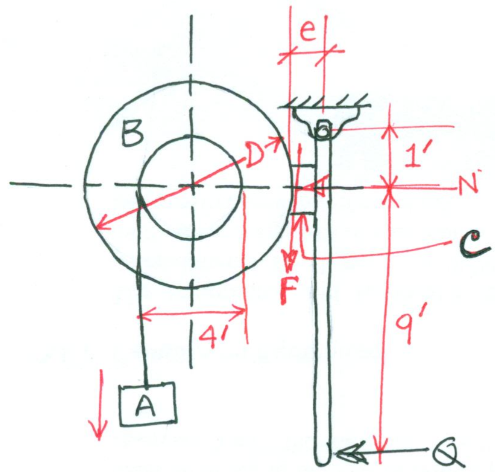
(c) Change in potential energy,

$\Delta PE = W_B S_B - W_A S_A$

$= 150 \times 10 - 500 \times 3.33$

$= \boxed{-165 \text{ lb-ft, -ve sign implies decrease}}$ Ans.

1420/P. 419



$W_B = 1288$ lb

$\bar{k}_B = 2.5$ ft

$\omega_{BO} = 120$ rpm $= \frac{120 \times 2\pi}{60}$

$= 4\pi$ rad/sec.

$S_A = 80$ ft

$v_{Af} = 0$

$W_A = 278$ lb

$Q = ?$

$D = 8'$

$f = \frac{1}{3}$

$e = 0$

Sol^n

Let S_B be the distance that a point on the outer circle moved while A moved 80 ft downward.

$\therefore$ Angle traversed, $\theta = \frac{S_A}{r_A} = \frac{S_B}{r_B}$

$\Rightarrow \frac{80}{2} = \frac{S_B}{4}$

$\therefore S_B = 160$ ft

For the entire system,

$U_{net} = W_A S_A - F \cdot S_B = 278 \times 80 - F \times 160 = 22240 - 160F$

$\Delta KE = \frac{1}{2} \cdot \frac{W_A}{g} \left(v_{Af}^2 - v_{AO}^2 \right) + \frac{1}{2} \bar{I}_B \left(\omega_{Bf}^2 - \omega_{BO}^2 \right)$

$= \frac{1}{2} \times \frac{278}{32.2} \left(0 - 64\pi^2 \right) + \frac{1}{2} \times 250 \left(0 - 16\pi^2 \right)$

$= -2726.71 - 19739.21$

$= -22465.92$ lb-ft

$v_{AO} = r_A \, \omega_{BO}$

$= 2 \times 4\pi$

$= 8\pi$ fps

$\bar{I}_B = m_B \bar{k}_B^2$

$= \frac{1288}{32.2} \times 2.5^2$

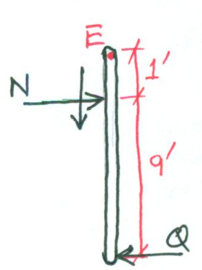
$= 250$ slug-ft^2

According to the principle of work and kinetic energy

$U_{net} = \Delta KE$

$\Rightarrow 22240 - 160F = -22465.92$

i.e. $F = 279.41$ lb



$N = \frac{F}{f} = \frac{279.41}{1/3} = 838.24$ lb

Taking $\sum M_E = 0$ ↻+ve

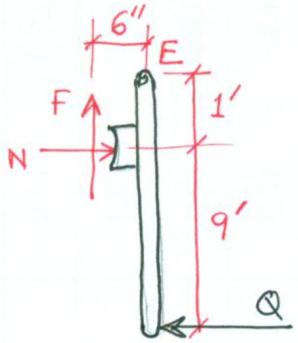
$Q \times 10 - N \times 1 = 0$

$\Rightarrow Q \times 10 - 838.24 \times 1 = 0$

$\therefore Q = \boxed{83.82 \text{ lb}}$

#1421/P.419

Similar to # 1420 find F = 279.41 lb



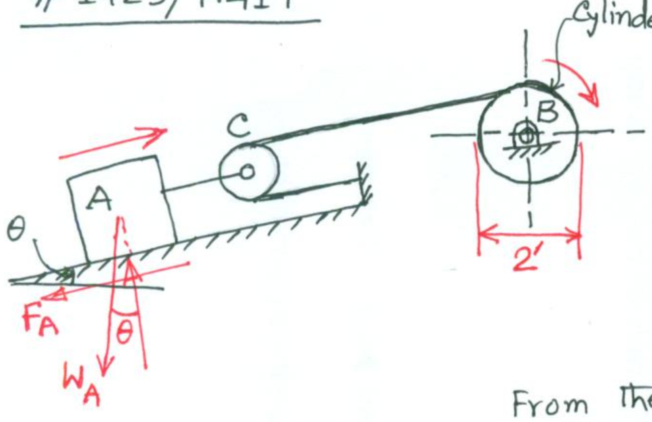
$\sum M_E = 0$ ↷ +ve

$\Rightarrow Q \times 10 + F \times \frac{6}{12} - N \times 1 = 0$

$\Rightarrow Q \times 10 + 279.41 \times \frac{6}{12} - 838.24 \times 1 = 0$

$\therefore Q = \boxed{69.85 \text{ lb.}}$ Ans.

#1423/P.419



$W_A = 128.8$ lb $\quad S_A = 20$ ft (up the plane)

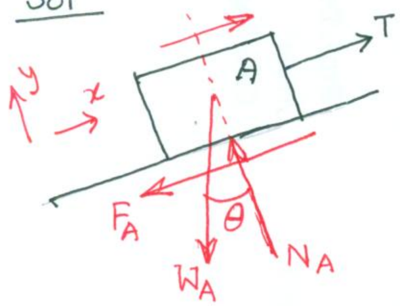
$\theta = 15^\circ$

$f = 0.15 \quad W_B = ?$

$\omega_{B0} = 40$ rpm

$\omega_{Bf} = 0$

Sol^n



From the freebody of A, taking $\Sigma F_y = 0$, +y direct^n as +ve

$$N_A - W_A \cos\theta = 0$$

$$\therefore N_A = W_A \cos\theta = 128.8 \times \cos 15^\circ = 124.41 \text{ lb}$$

$$F_A = N_A \times f_A = 124.41 \times 0.15 = 18.66 \text{ lb}$$

For the entire system,

$$U_{net} = -W_A \sin\theta \times S_A - F_A \times S_A$$

$$= -128.8 \sin 15^\circ \times 20 - 18.66 \times 20$$

$$= -1039.92 \text{ lb-ft}$$

$$\Delta KE = \frac{1}{2} m_A \left(v_{Af}^2 - v_{A0}^2\right) + \frac{1}{2} \bar{I}_B \left(\omega_{Bf}^2 - \omega_{B0}^2\right)$$

$$= \frac{1}{2} \times \frac{128.8}{32.2} \times \left(0 - 2.09^2\right) + \frac{1}{2} \times \frac{W_B}{64.4} \times \left(0 - 4.19^2\right)$$

$$= -8.74 - 0.136\, W_B$$

[Note: For a cylinder, $\bar{I} = \frac{m r^2}{2}$, Art. 160, P.230
Art. 167, P.236]

$$\omega_{B0} = 40 \text{ rpm} = \frac{40}{60} \times 2\pi = 4.19 \text{ rad/s.}$$

$$v_{A0} = \frac{1}{2} r_B\, \omega_{B0} = \frac{1}{2} \times 1 \times 4.19 = 2.09 \text{ fps}$$

$$v_{Af} = \frac{1}{2} r_B\, \omega_{Bf} = \frac{1}{2} \times 1 \times 0 = 0$$

$$\bar{I}_B = \frac{m_B r_B^2}{2} = \frac{W_B \times 1^2}{2 \times 32.2} = \frac{W_B}{64.4} \text{ slug-ft}^2$$

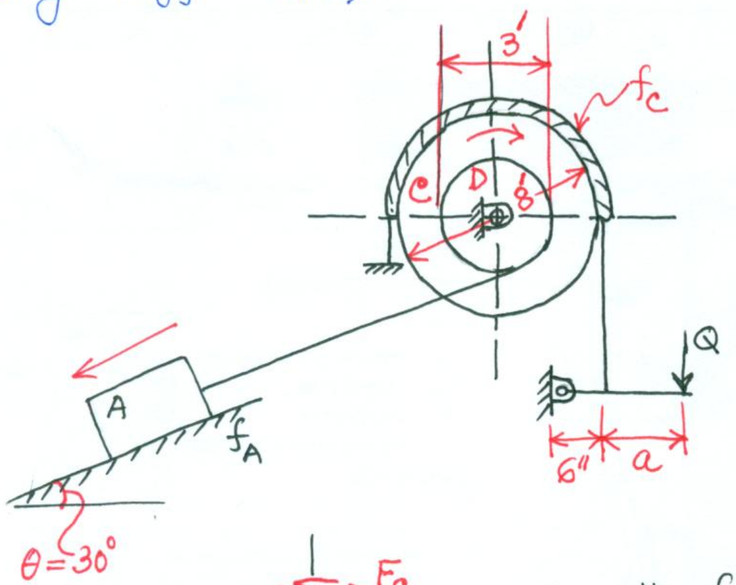
Now, $U_{net} = \Delta KE$

$$\Rightarrow -1039.92 = -8.74 - 0.136\, W_B$$

$$\therefore W_B = \boxed{7582.2 \text{ lb}}$$ Ans.

#1424/P.420

(Using Energy Principle)



$W_{CD} = 2576$ lb

$\bar{k}_{CD} = 3$ ft

$W_A = 2000$ lb

$f_A = 1/3$

$v_{A0} = 40$ fps, $v_{Af} = 10$ fps

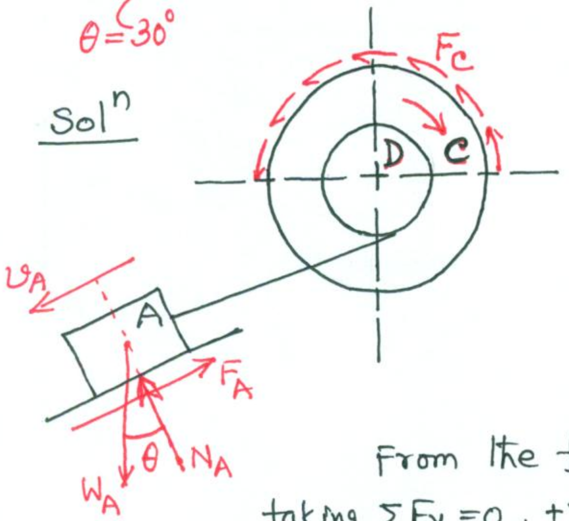
$S_A = 170$ ft

$f_c = 0.25$

$a = 30''$

$Q = ?$

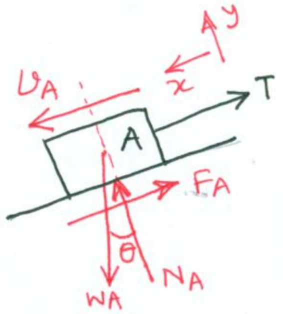
Sol^n



From the freebody shown on left

$$U_{net} = (W_A \sin\theta - F_A) \times S_A - F_c \times S_c \qquad (1)$$

$$\Delta KE = \frac{1}{2}\cdot\frac{W_A}{g}\left(v_{Af}^2 - v_{A0}^2\right) + \frac{1}{2}\bar{I}_{CD}\left(\omega_{CDf}^2 - \omega_{CD0}^2\right) \qquad (2)$$



From the freebody of A

taking $\Sigma F_y = 0$, +ve y direct^n as +ve

$$N_A - W_A \cos\theta = 0$$

$$\therefore N_A = 2000 \cos 30^\circ = 1732.05 \text{ lb}$$

$$\therefore F_A = N_A \cdot f_A = 1732.05 \times \frac{1}{3} = 577.35 \text{ lb}$$

Considering the circular displacement of points on C & D and on the same radial line —

$$\text{rotation, } \theta = \frac{S_D}{r_D} = \frac{S_c}{r_c}$$

$$\Rightarrow \frac{S_A}{1.5} = \frac{S_c}{4}$$

$$\therefore S_c = S_A \times \frac{4}{1.5} = 170 \times \frac{4}{1.5} = 453.33 \text{ ft}$$

Now substituting known quantities in eq^n (1)

$$U_{net} = (2000 \sin 30^\circ - 577.35) \times 170 - F_c \times 453.33$$

$$= 71850.5 - 453.33 F_c \qquad (3)$$

Contd---

#1424/P.420 contd--

$$\bar{I}_{CD} = m_{CD}\bar{k}_{CD}^2 = \frac{W_{CD}}{g}\bar{k}_{CD}^2 = \frac{2576}{32.2}\times 3^2 = 720 \text{ slug-ft}^2$$

$$\omega_{CDf} = \frac{v_{Af}}{r_D} = \frac{10}{1.5} = \quad \text{rad/s}$$

$$\omega_{CD0} = \frac{v_{A0}}{r_D} = \frac{40}{1.5} = 26.67 \text{ rad/s}$$

Now from eq^n (2)

$$\Delta KE = \frac{1}{2}\cdot\frac{2000}{32.2}\times\left(10^2 - 40^2\right) + \frac{1}{2}\times 720\times\left(6.67^2 - 26.67^2\right)$$

$$= -46583.85 - 240048$$

$$= -286631.85 \text{ ft-lb} \quad \text{——— (4)}$$

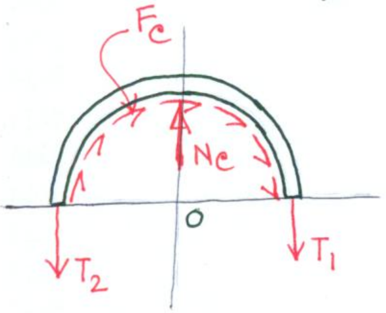
According to principles of work and energy

$$U_{net} = \Delta KE$$

$\therefore$ Equating (3) & (4)

$$71850.5 - 453.33F_c = -286631.85$$

$$\therefore F_c = 790.78 \text{ lb}$$



From the freebody of the brake band

$$T_2 = T_1 e^{f_c\theta_1} = T_1 e^{0.25\times\pi} = 2.19\,T_1$$

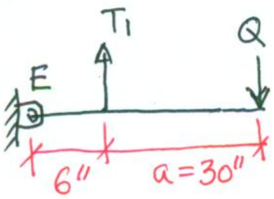
Taking $\Sigma M_o = 0$ ↻ +ve

$$T_1\times 4 + F_c\times 4 - T_2\times 4 = 0$$

$$\Rightarrow T_1 + F_c - T_2 = 0$$

$$\Rightarrow T_1 + 790.78 - 2.19T_1 = 0$$

$$\therefore T_1 = 664.52 \text{ lb}$$



From the freebody of brake lever considering $\Sigma M_E = 0$ ↻ +ve

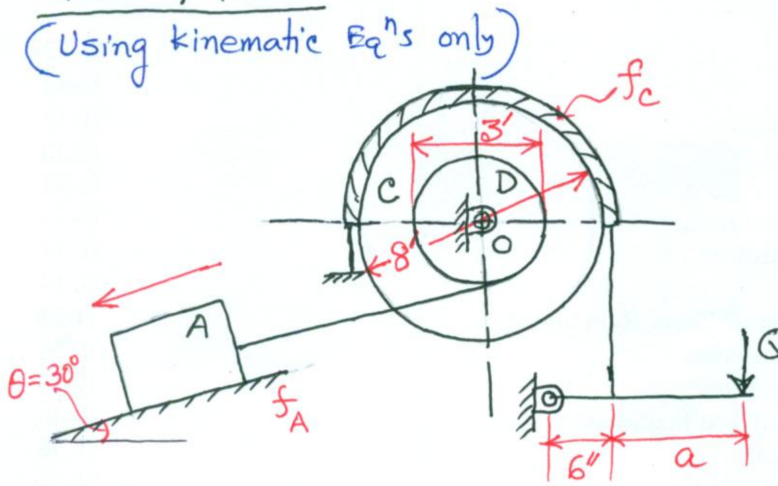
$$-T_1\times 6 + Q\times 36 = 0$$

$$\Rightarrow -664.52\times 6 + Q\times 36 = 0$$

$$\therefore Q = \boxed{110.75 \text{ lb}} \text{ Ans.}$$

#1424/ P. 420

(Using kinematic Eq^{n}s only)



$W_{CD} = 2576$ lb

$\bar{k}_{CD} = 3$ ft

$W_A = 2000$ lb

$f_A = \frac{1}{3}$

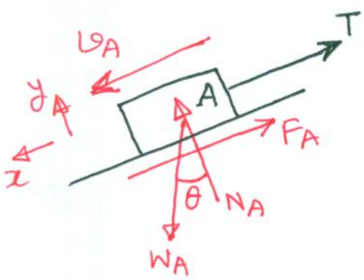
$v_{A0} = 40$ fps, $v_{Af} = 10$ fps

$S_A = 170$ ft

$a = 30$ inch

$Q = ?$

Sol^n



From the freebody of A, taking $\Sigma F_Y = 0$, +ve y direct^n as +ve

$$N_A - W_A \sin\theta = 0$$

$$\Rightarrow N_A - 2000 \sin 30^\circ = 0$$

$$\therefore N_A = 1732.05 \text{ lb}$$

$$\therefore F_A = N_A \cdot f_A = 1732.05 \times \frac{1}{3} = 577.35 \text{ lb}$$

$$v_{Af}^2 = v_{A0}^2 + 2 a_A S_A$$

$$\Rightarrow 10^2 = 40^2 + 2 a_A \times 170$$

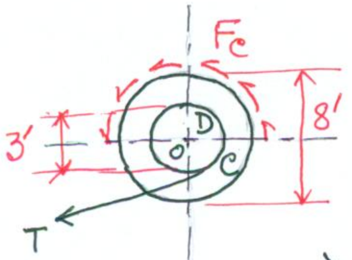
$$\therefore a = -4.41 \text{ fps}^2$$

$\Sigma F_x = ma$, down the plane +ve

$$\Rightarrow W_A \sin\theta - F_A - T = \frac{W_A}{g} \cdot a_A$$

$$\Rightarrow 2000 \sin 30^\circ - 577.35 - T = \frac{2000}{32.2} \times (-4.41)$$

$$\therefore T = 696.56 \text{ lb}$$



From the freebody of drum D and brake-wheel C,

$\Sigma M_o = \bar{I}\alpha$ gives ↻+ve

$$T \times 1.5 - F_c \times 4 = m_{CD} \bar{k}_{CD}^2 \alpha$$

$$\Rightarrow 696.56 \times 1.5 - F_c \times 4 = \frac{2576}{32.2} \times 3^2 \times (-2.94)$$

$$\therefore F_c = 790.41 \text{ lb.}$$

$$\omega_{CDf}^2 = \omega_{CD0}^2 + 2\alpha\theta$$

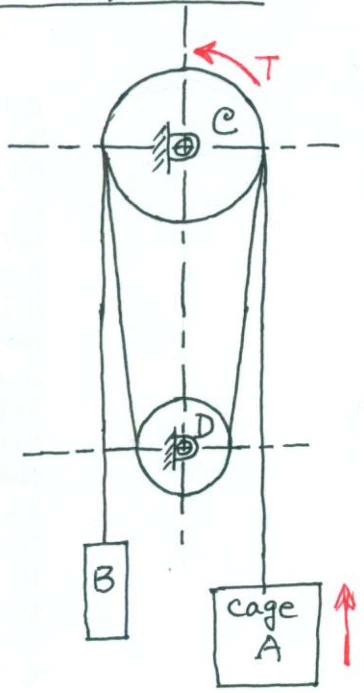
$$\Rightarrow \left(\frac{v_{Af}}{r_D}\right)^2 = \left(\frac{v_{A0}}{r_D}\right)^2 + 2\alpha \cdot \frac{S_A}{r_D}$$

$$\Rightarrow \left(\frac{10}{1.5}\right)^2 = \left(\frac{40}{1.5}\right)^2 + 2\alpha \cdot \frac{170}{1.5}$$

$$\therefore \alpha = -2.94 \text{ rad/s}^2$$

The rest is same as has been done in the solution using energy principle.

1426/P. 420



$W_A = 6000$ lb

$W_B = 5000$ lb

$D_C = D_D = 30'' = 2.5$ ft

$\bar{I}_C = \bar{I}_D = 5$ slug-ft^2

$v_{A0} = 0$, $v_{Af} = 10$ fps

$S_A = 10$ ft ↑

$S_B = 10$ ft ↓

Torque, T = ?

$\Delta PE = ?$

Sol^n

$$\theta_C = \frac{S_A}{r_C} = \frac{10}{2.5/2} = 8 \text{ rad}$$

$$\omega_C = \frac{v_{Af}}{r_C} = \frac{10}{2.5/2} = 8 \text{ rad/s}$$

$$\omega_D = \omega_C = 8 \text{ rad/s}$$

$$v_{Bf} = v_{Af} = 10 \text{ fps}$$

$$U_{net} = -W_A \times S_A + W_B \times S_B + T \times \theta_C$$

$$= -6000 \times 10 + 5000 \times 10 + T \times 8$$

$$= -10000 + 8T$$

$$\Delta KE = \frac{1}{2} \cdot \frac{W_A}{g} v_{Af}^2 + \frac{1}{2} \cdot \frac{W_B}{g} \cdot v_{Bf}^2 + \frac{1}{2} \bar{I}_C \omega_C^2 + \frac{1}{2} \bar{I}_D \omega_D^2$$

$$= \frac{1}{2} \times \frac{6000}{32.2} \times 10^2 + \frac{1}{2} \times \frac{5000}{32.2} \times 10^2 + \frac{1}{2} \times 5 \times 8^2 + \frac{1}{2} \times 5 \times 8^2$$

$$= 9316.77 + 7763.98 + 320$$

$$= 17400.75 \text{ lb-ft}$$

According to principles of work and kinetic energy

$$U_{net} = \Delta KE$$

$$\Rightarrow -10000 + 8T = 17400.75$$

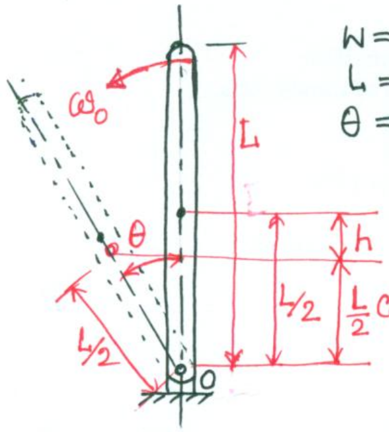
$$\therefore T = \boxed{3425.09 \text{ lb-ft}} \text{ Ans.}$$

$$\Delta PE = W_A \times S_A - W_B \times S_B$$

$$= 6000 \times 10 - 5000 \times 10$$

$$= \boxed{10000 \text{ lb-ft}} \text{ Ans.}$$

1430/P.420



$W = 16.1$ lb
$L = 4$ ft
$\theta = 120^\circ$ from rest
$\bar{v} = ?$
KE = ?
$\Delta PE = ?$

Note:

For a slender rod moment of inertia of mass about an axis through O & perpendicular to the plane of paper is – $I_0 = \frac{mL^2}{3}$

about centroidal axis, $\bar{I} = \frac{mL^2}{12}$

Example 164/PP. 233-234.

Sol^n

$$h = \frac{L}{2} - \frac{L}{2}\cos\theta = \frac{L}{2}(1-\cos\theta)$$

$$U_{net} = W \times h = W \times \frac{L}{2}(1-\cos\theta) = 16.1 \times \frac{4}{2}(1-\cos 120^\circ) = 48.3 \text{ lb-ft}$$

$$I_0 = \frac{mL^2}{3} = \frac{W}{g}\cdot\frac{L^2}{3} = \frac{16.1}{32.2}\times\frac{4^2}{3} = 2.67 \text{ slug-ft}^2$$

$$\Delta KE = \frac{1}{2} I_0 \omega^2 = \frac{1}{2}\times 2.67 \times \omega^2$$

According to principles of work and kinetic energy

$$U_{net} = \Delta KE$$

$$\Rightarrow 48.3 = \frac{1}{2}\times 2.67\times\omega^2$$

$$\therefore \omega = 6.01 \text{ rad/s.}$$

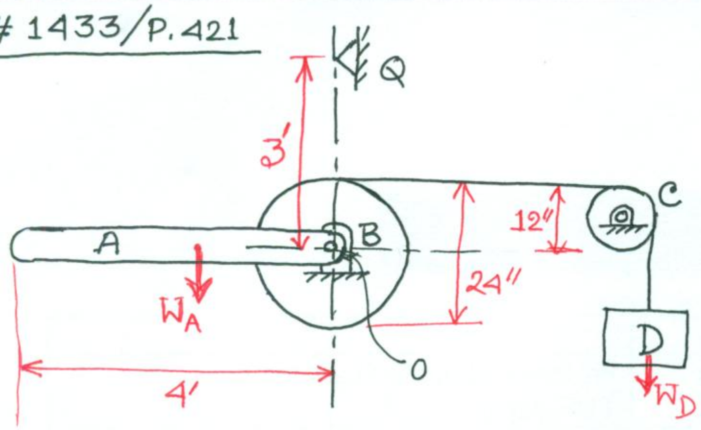
$$\bar{v} = r\omega = \frac{4}{2}\times 6.01 = \boxed{12.02 \text{ fps}}$$

$$\Delta KE = \frac{1}{2}\times 2.67\times 6.01^2 = \boxed{48.2 \text{ ft-lb}}$$

$$\Delta PE = -W\times h = -U_{net} = \boxed{-48.3 \text{ ft-lb}}$$

–ve means decrease in PE

#1433/P.421



$W_A = 48.3$ lb

$\bar{W}_B = 128.8$ lb

$\bar{k}_B = 10$ inch

$W_D = 644$ lb

Start from rest.

$\theta_B = \theta_A = \frac{\pi}{2}$ rad.

(a) KE = ? when A strikes Q

(b) $v_D = ?$

Sol^n

For the entire system

$$U_{net} = W_D \times S_D - W_A \times \frac{L_A}{2}$$

$$= 644 \times \frac{\pi}{2} - 48.3 \times \frac{4}{2}$$

$$= 914.99 \text{ ft-lb}$$

$$S_D = r_B \theta_B = 1 \times \frac{\pi}{2} = \frac{\pi}{2} \text{ ft}$$

$$\Delta KE = \frac{1}{2} \cdot \frac{W_D}{g} v_D^2 + \frac{1}{2} I_o \omega_o^2$$

$$= \frac{1}{2} \times \frac{644}{32.2} \times v_D^2 + \frac{1}{2} \times 10.78 \times v_D^2$$

$$= 10 v_D^2 + 5.39 v_D^2$$

$$= 15.39 v_D^2$$

$$I_o = I_{AO} + I_{BO}$$

$$I_{AO} = \frac{1}{3} m_A L_A^2 = \frac{1}{3} \times \frac{48.3}{32.2} \times 4^2$$

$$= 8 \text{ slug-ft}^2$$

$$I_{BO} = m_B k_B^2 = \frac{128.8}{32.2} \times \left(\frac{10}{12}\right)^2$$

$$= 2.78 \text{ slug-ft}^2$$

$$\therefore I_o = 8 + 2.78 = 10.78 \text{ slug-ft}^2$$

$$\omega_o = \frac{v_D}{r_B} = \frac{v_D}{1} = v_D$$

According to principles of work and kinetic energy

$$U_{net} = \Delta KE$$

$$\Rightarrow 914.99 = 15.39 v_D^2$$

$$\therefore v_D = \boxed{7.71 \text{ fps}}$$

K.E. when A strikes Q $= \Delta KE$

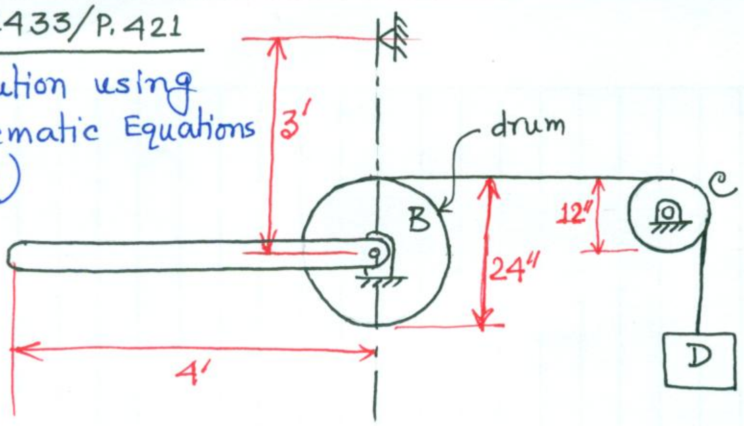
$$= 15.39 v_D^2$$

$$= 15.39 \times 7.71^2$$

$$= \boxed{914.84 \text{ lb-ft}}$$

#1433/P.421

(Solution using Kinematic Equations only)



$W_A = 48.3$ lb

$W_B = 128.8$ lb

$\bar{k}_B = 10$ inch

$W_D = 644$ lb

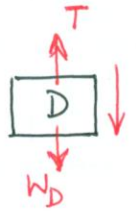
Start from rest

$\theta_B = \theta_A = \frac{\pi}{2}$ rad.

(a) K.E. = ? when A strikes Q

(b) $v_D = ?$

Sol^n



From the freebody of D

$\Sigma F_V = m_D a_D \downarrow +ve$

$\Rightarrow W_D - T = \frac{W_D}{g} \cdot a_D$

$\Rightarrow 644 - T = \frac{644}{32.2} \times \frac{v_D^2}{\pi}$

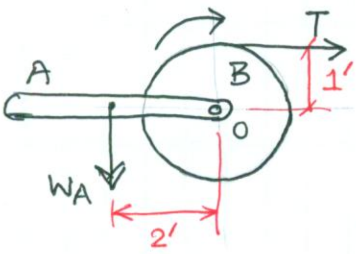
$\Rightarrow 644 - T = 6.37\, v_D^2$ —— (1)

$v_D^2 = 2 a_D S_D$

$S_D = r_B \theta_B = 1 \times \frac{\pi}{2} = \frac{\pi}{2}$ ft

$\therefore v_D^2 = 2 a_D \times \frac{\pi}{2}$

i.e. $a_D = \frac{v_D^2}{\pi}$



From the freebody of drum B

taking $\Sigma M_O = I_O \alpha_{AB}$ ↻ +ve

$\Rightarrow T \times 1 - W_A \times 2 = I_O \alpha_{AB}$ —— (2)

Now $I_O = I_{AO} + I_{BO}$

$= \frac{1}{3} m_A L_A^2 + m_B k_B^2$

$= \frac{1}{3} \times \frac{48.3}{32.2} \times 4^2 + \frac{128.8}{32.2} \times \left(\frac{10}{12}\right)^2$

$= 8 + 2.78$

$= 10.78$ slug-ft^2

$\omega_{ABf} = \frac{v_D}{r_B} = \frac{v_D}{1} = v_D$

$\omega_{ABf}^2 = \omega_{ABO}^2 + 2 \alpha_{AB} \theta_{AB}$

$\Rightarrow v_D^2 = 0 + 2 \cdot \alpha_{AB} \cdot \frac{\pi}{2}$

$\therefore \alpha_{AB} = \frac{v_D^2}{\pi}$

$\therefore$ From eq^n (2) $\quad T \times 1 - 48.3 \times 2 = 10.78 \times \frac{v_D^2}{\pi}$

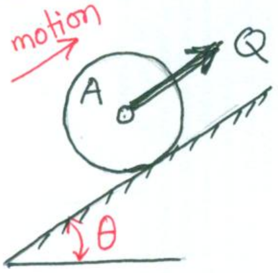
$\Rightarrow T - 96.6 = 3.43\, v_D^2$ —— (3)

Adding eq^{n}s (1) & (3) $\quad 547.4 = 9.8\, v_D^2 \Rightarrow v_D = \boxed{7.47 \text{ fps}}$

$\therefore$ K.E. when A strikes Q $= \Delta KE = \frac{1}{2} \frac{W_D}{g} v_D^2 + \frac{1}{2} I_O \omega_{ABf}^2$

$= \frac{1}{2} \frac{W_D}{g} \cdot v_D^2 + \frac{1}{2} I_O v_D^2 = \frac{1}{2}\left(\frac{644}{32.2} + 10.78\right) \times 7.47^2 = \boxed{858.8 \text{ lb-ft}}$

#1439/p.421



$W_A = 161$ lb $\quad D_A = 12$ inch

$\theta = 30^0 \quad Q = 96.5$ lb

(a) $\bar{v}_{Af} = ? \quad \bar{v}_{A0} = 0 \quad S_A = 15$ ft

(b) $\alpha_A = ?$

(c) $F = ? \quad f_c = ?$

Sol^n

(a) $U_{net} = (Q - W_A \sin\theta) \times S_A = (96.5 - 161 \sin 30^0) \times 15 = 240$ lb-ft

$$\Delta KE = \frac{1}{2} \cdot \frac{W_A}{g} \bar{v}_{Af}^2 + \frac{1}{2} \bar{I}_A \omega_{Af}^2$$

$$= \frac{1}{2} \times \frac{161}{32.2} \times \bar{v}_{Af}^2 + \frac{1}{2} \times 0.625 \times (2\bar{v}_{Af})^2$$

$$= 3.75\, \bar{v}_{Af}^2 \text{ lb-ft.}$$

$$\bar{I}_A = \frac{m_A r_A^2}{2} = \frac{1}{2} \times \frac{161}{32.2} \times \left(\frac{6}{12}\right)^2 = 0.625 \text{ slug-ft}^2$$

$$\omega_{Af} = \frac{v_{Af}}{r_A} = \frac{v_{Af}}{6/12} = 2v_A$$

According to the principles of work and kinetic energy

$U_{net} = \Delta KE \Rightarrow 240 = 3.75\, \bar{v}_{Af}^2 \quad \therefore \bar{v}_{Af} = \boxed{8 \text{ fps}}$

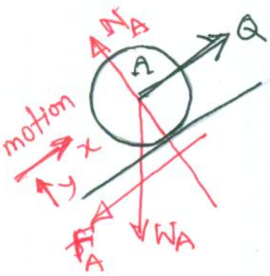
(b) $\omega_{Af} = 2v_{Af} = 2 \times 8 = 16$ rad/s

$\omega_{Af}^2 = 0 + 2\alpha_A \theta_A \qquad \theta_A = \dfrac{S_A}{r_A} = \dfrac{15}{6/12} = 30$ rad/s.

$\Rightarrow 16^2 = 2 \times \alpha_A \times 30$

$\therefore \alpha_A = \boxed{4.27 \text{ rad/s}^2}$

(c) $v_{Af}^2 = v_{A0}^2 + 2aS_A \Rightarrow 8^2 = 0 + 2 \times a \times 15 \quad \therefore a = 2.13$ fps^2



$\Sigma F_x = m_A a$, +ve x direct^n +ve

$\Rightarrow Q - W_A \sin\theta - F_A = \dfrac{W_A}{g} \cdot a$

$\Rightarrow 96.5 - 161 \sin 30^0 - F_A = \dfrac{161}{32.2} \times 2.13$

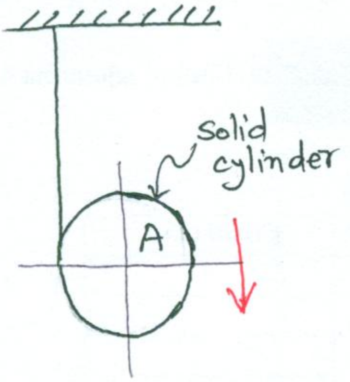
$\therefore F_A = \boxed{5.35 \text{ lb.}}$

$\Sigma F_y = 0$ gives, $N_A - W_A \cos\theta = 0$

$\therefore N_A = 161 \cos 30^0 = 139.43$ lb

$\therefore f = \dfrac{F_A}{N_A} = \dfrac{5.35}{139.43} = \boxed{0.0384}$

1442/p. 421



$v_{A0} = 0$

$S = 15$ ft

$\bar{v}_{Af} = ?$

$\Delta PE = ?$

$$U_{net} = W_A \times 15$$

$$\Delta KE = \frac{1}{2} \cdot \frac{W_A}{g} \cdot \left(\bar{v}_{Af}^2 - \bar{v}_{A0}^2 \right) + \frac{1}{2} \bar{I}_A \left(\omega_{Af}^2 - \omega_{A0}^2 \right)$$

$$\omega_{A0} = \frac{\bar{v}_{A0}}{r_A} = 0 \quad , \quad \omega_{Af} = \frac{\bar{v}_{Af}}{r_A}$$

$$\bar{I}_A = \frac{1}{2} m_A r_A^2 = \frac{1}{2} \cdot \frac{W_A}{g} \cdot r_A^2$$

$$\therefore \Delta KE = \frac{1}{2} \cdot \frac{W_A}{g} \cdot \bar{v}_{Af}^2 + \frac{1}{2} \cdot \frac{W_A}{2g} r_A^2 \cdot \frac{\bar{v}_{Af}^2}{r_A^2}$$

$$= \frac{W_A}{2g} \bar{v}_{Af}^2 + \frac{W_A}{4g} \bar{v}_{Af}^2$$

$$= \frac{3}{4} \cdot \frac{W_A}{g} \cdot \bar{v}_{Af}^2$$

According to the principles of work and kinetic energy

$$U_{net} = \Delta KE$$

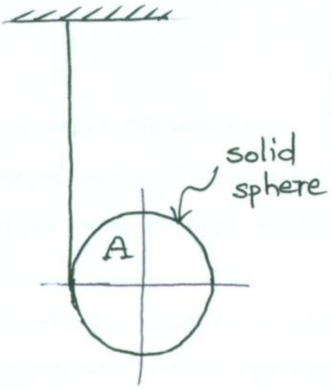
$$\Rightarrow W_A \times 15 = \frac{3}{4} \cdot \frac{W_A}{g} \cdot \bar{v}_{Af}^2$$

$$\therefore \bar{v}_{Af} = \sqrt{\frac{15 \times 4 \times 32.2}{3}} = \boxed{25.38 \text{ fps.}}$$

$$\Delta PE = -W \times 15 = \boxed{-15W}$$

–ve sign means decrease in potential energy

#1443/P.421



$v_{A0} = 0$

$S_A = 15$ ft

$v_{Af} = ?$

$\Delta PE = ?$

Note: Moment of inertia of mass of a sphere about a diameter, $\bar{I} = \frac{2}{5} m r^2$
Example 161/P.231, Art. 167/P.236.

Sol^n

$U_{net} = W_A \times 15$

$$\Delta KE = \frac{W_A}{2g}\left(v_{Af}^2 - v_{A0}^2\right) + \frac{\bar{I}_A}{2}\left(\omega_{Af}^2 - \omega_{A0}^2\right)$$

$$= \frac{W_A}{2g} v_{Af}^2 + \frac{1}{2} \times \frac{2}{5} \cdot \frac{W_A}{g} r_A^2 \times \frac{v_{Af}^2}{r_A^2}$$

$$= \frac{W_A v_{Af}^2}{2g} + \frac{W_A v_{Af}^2}{5g}$$

$$= \frac{7}{10} \cdot \frac{W_A v_{Af}^2}{g}$$

$$\omega_{A0} = \frac{v_{A0}}{r_A} = \frac{0}{r_A} = 0$$

$$\omega_{Af} = \frac{v_{Af}}{r_A}$$

$$\bar{I}_A = \frac{2}{5} m_A r_A^2 = \frac{2}{5} \cdot \frac{W_A}{g} \cdot r_A^2$$

According to principles of work and kinetic energy

$$U_{net} = \Delta KE$$

$$\Rightarrow W_A \times 15 = \frac{7}{10} \cdot \frac{W_A v_{Af}^2}{g}$$

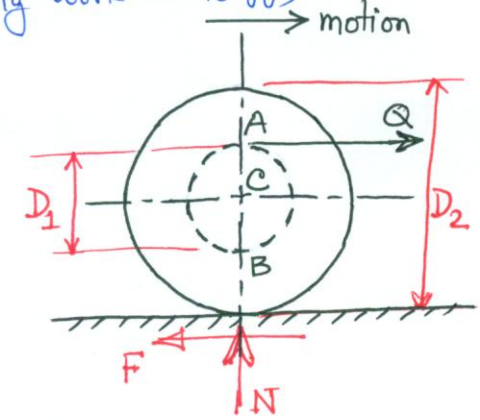
$$\therefore v_{Af} = \left(\frac{15 \times 10 \times 32.2}{7}\right)^{1/2}$$

$$= \boxed{26.26 \text{ fps}}$$

Change in P.E. $= -W_A \times 15 = \boxed{-15 W_A}$
–ve sign means decrease

1445/PP. 421-422

(Using Work & Energy)



Grooved cylinder, rolling

$W = 322$ lb

$S_c = 54$ ft

$D_1 = 18$ inch $= 1.5$ ft

$D_2 = 36$ inch $= 3.0$ ft

$\bar{I} = 10$ slug-ft^2

$\bar{v}_0 = 0$, $\bar{v}_f = 45$ fps

$Q = ?$

Sol^n

As the cylinder is rolling, the friction force F does no work
See Art. 280/P. 401

$$\Delta KE = \frac{1}{2}\frac{W}{g}\bar{v}_f^2 + \frac{1}{2}\bar{I}\omega_f^2$$ [Taking c.g. as the ref. point]

$$= \frac{1}{2} \times \frac{322}{32.2} \times 45^2 + \frac{1}{2} \times 10 \times 30^2 \quad \Big| \quad \omega_f = \frac{\bar{v}_f}{r} = \frac{45}{3/2} = 30 \text{ rad/s.}$$

$$= 14625 \text{ lb-ft}$$

Let S_Q be the displacement of the point of application of Q

$$\therefore \theta = \frac{S_Q}{\frac{D_2}{2} + \frac{D_1}{2}} = \frac{S_c}{\frac{D_2}{2}}$$ [Taking instantaneous center as reference point]

$$\Rightarrow \frac{S_Q}{1.5 + 0.75} = \frac{54}{1.5}$$

$$\therefore S_Q = \frac{54}{1.5} \times 2.25 = 81 \text{ ft}$$

$\therefore$ Net work done, $U_{net} = Q \cdot S_Q = Q \times 81$ lb-ft

According to principles of work and kinetic energy

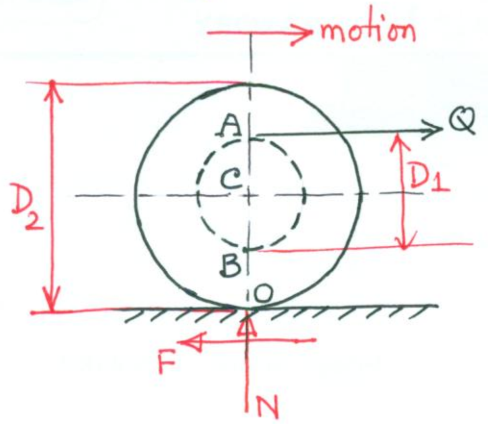
$$U_{net} = \Delta KE$$

$$\Rightarrow 81Q = 14625$$

$$\therefore Q = \boxed{180.6 \text{ lb}}$$ Ans.

1445/ pp. 421 – 422

(Without using energy principles)



Grooved cylinder, rolling

$W = 322$ lb

$S_c = 54$ ft

$D_1 = 18$ in. $= 1.5$ ft

$D_2 = 36$ in. $= 3.0$ ft

$\bar{I} = 10$ slug-ft^2

$\bar{v}_0 = 0$

$\bar{v}_f = 45$ fps

$Q = ?$

Sol^n

$$\bar{v}_f^2 = \bar{v}_0^2 + 2\bar{a}S_c$$

$$\Rightarrow 45^2 = 0 + 2 \times a \times 54$$

$$\therefore a = 18.75 \text{ fps}^2$$

We know, for angular motion, $M = I\alpha$

Considering instantaneous center O as reference point

$$M_o = I_o \alpha \quad \text{—— (1)}$$

Now, $M_o = Q(r_2 + r_1) = Q(1.5 + 0.75) = 2.25Q$

$$I_o = \bar{I} + m r_2^2 = 10 + \frac{322}{32.2} \times 1.5^2 = 32.5 \text{ slug-ft}^2$$

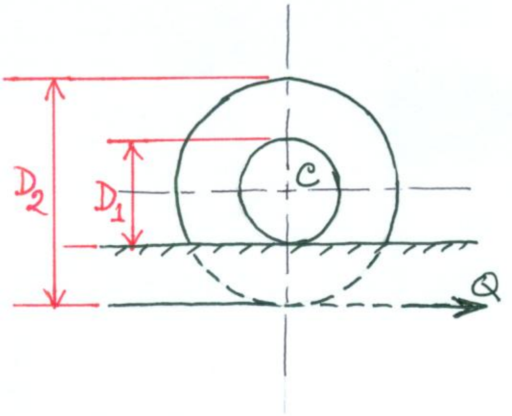
$$\alpha = \frac{a}{r_2} = \frac{18.75}{1.5} = 12.5 \text{ rad/s}^2$$

$\therefore$ From eq^n (1)

$$2.25Q = 32.5 \times 12.5$$

$$\therefore Q = \boxed{180.6 \text{ lb}}$$

1447/p. 422



Rolling

$D_1 = 2$ ft

$D_2 = 4$ ft

$Q = 160$ lb

$W = 644$ lb

$\bar{I} = 12$ slug-ft^2

$v = ?$ when $S = 10$ ft from rest

Sol^n

$$U_{net} = Q.S = 160 \times 10 = 1600 \text{ ft-lb}$$

$$\Delta KE = \frac{1}{2}\frac{W}{g}v^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2} \times \frac{644}{32.2}v^2 + \frac{1}{2} \times 12 \times \left(\frac{v}{1}\right)^2$$

$$= 10v^2 + 6v^2$$

$$= 16v^2$$

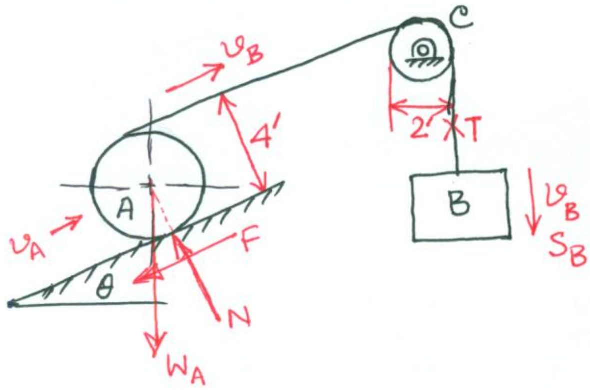
According to principles of work and kinetic energy

$$U_{net} = \Delta KE$$

$$\Rightarrow 1600 = 16v^2$$

$$\therefore v = \boxed{10 \text{ fps.}}$$ Ans

1449/P.422



$W_B = 50$ lb
$W_A = 80$ lb
$\theta = 30^\circ$
$\bar{I}_A = 4$ slug-ft^2
$S_B = 20$ ft
$\bar{I}_c = 0.3$ slug-ft^2
$v_A = ?$ $v_A \rightarrow$ velocity of c.g. of A
$T = ?$ $a_B = ?$

Sol^n

For the entire system,

$$U_{net} = W_B \cdot S_B - W_A \sin\theta \cdot S_A \quad \text{——(1)}$$

$$\Delta KE = \frac{1}{2}\frac{W_B}{g}v_B^2 + \frac{1}{2}\cdot\frac{W_A}{g}v_A^2 + \frac{1}{2}\bar{I}_A\omega_A^2 + \frac{1}{2}\bar{I}_c\omega_c^2 \quad \text{——(2)}$$

Note: Since A is rolling, friction force F does no work. Art. 280/P.401.

Now $\frac{v_B}{v_A} = \frac{4}{2} = 2$

$\therefore v_B = 2v_A$

$\therefore S_B = 2S_A$ ie $S_A = \frac{S_B}{2} = \frac{20}{2} = 10$ ft

$\omega_A = \frac{v_A}{r_A} = \frac{v_A}{2}$

$\omega_c = \frac{v_B}{r_c} = \frac{v_B}{1} = v_B = 2v_A$

Note: Because of rolling, the instantaneous center is the point of contact and velocity of any other point is proportional to the distance from instanteneous center. Art. 198/PP271-273, Art 199/P.274

From eq^n (1), $U_{net} = 50\times 20 - 80\sin 30^\circ \times 10 = 600$ lb-ft

& From eq^n (2), $\Delta KE = \frac{1}{2}\cdot\frac{50}{32.2}(2v_A)^2 + \frac{1}{2}\cdot\frac{80}{32.2}v_A^2 + \frac{1}{2}\cdot 4\cdot\left(\frac{v_A}{2}\right)^2 + \frac{1}{2}\times 0.3\times(2v_A)^2$

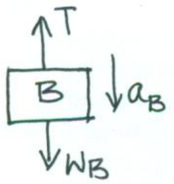
$= 5.448\, v_A^2$ lb-ft

According to the principles of work and kinetic energy

$U_{net} = \Delta KE$

$\Rightarrow 600 = 5.448\, v_A^2$ $\therefore v_A = \boxed{10.5 \text{ fps}}$ Ans.

& $v_B = 2v_A = 2\times 10.5 = 21$ fps.



Considering the motion of body B

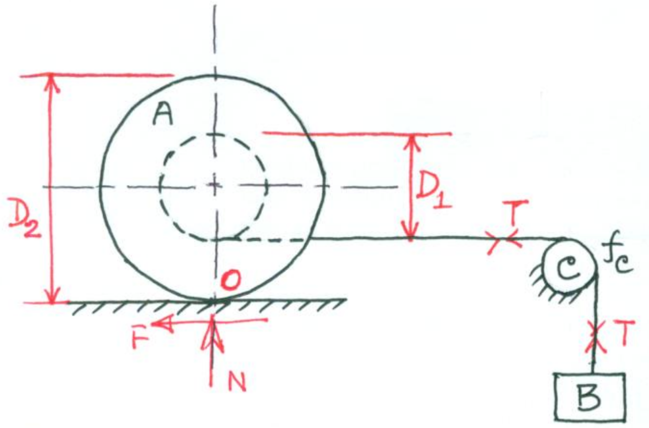
$v_B^2 = 2a_B S_B$ ie $a_B = \frac{v_B^2}{2S_B} = \frac{21^2}{2\times 20} = \boxed{11.03 \text{ fps}^2}$ Ans

Taking $\Sigma F_y = m_B a_B \downarrow +ve$

$W_B - T = \frac{W_B}{g}\cdot a_B$

$\therefore T = W_B - \frac{W_B}{g}\cdot a_B = 50 - \frac{50}{32.2}\times 11.03 = \boxed{32.87 \text{ lb}}$ Ans.

1450/P. 422



$W_A = 200$ lb
$\bar{I}_A = 6$ slug-ft^2
$D_1 = 2$ ft
$D_2 = 3$ ft
$W_B = 32.2$ lb
$f_c = 0$ (i.e. smooth peg)

(a) $\bar{v}_A = ?$ $a_B = ?$ for $s_B = 20$ ft
(b) $T = ?$
(c) How does $\bar{a}_A$ vary with increase of D_1?

Soln

For the entire system,

$$U_{net} = W_B . S_B = 32.2 \times 20 = 644 \text{ lb-ft}$$

$$\Delta KE = \frac{1}{2}\frac{W_B}{g} v_B^2 + \frac{1}{2}\cdot\frac{W_A}{g}\cdot\bar{v}_A^2 + \frac{1}{2}\bar{I}_A\omega_A^2 \quad \text{—— (1)}$$

[Considering c.g. of A as the ref. point]

Note: As B moves downward, A rolls to the left. Because of plane rolling, frictional force F does no work.

Now $\frac{\bar{v}_A}{v_B} = \frac{r_A\,\omega}{r_B\,\omega} = \frac{r_A}{r_B} = \frac{1.5}{0.5} = 3$

$\therefore \bar{v}_A = 3 v_B$ or $v_B = \frac{\bar{v}_A}{3}$

$\omega = \frac{\bar{v}_A}{r_A} = \frac{\bar{v}_A}{1.5}$

$\omega \to$ angular velocity of c.g. of A.
$\bar{v}_A \to$ velocity of the c.g. of A
$v_B \to$ velocity of a point on the chord

Note: v_A and v_B are the velocities relative to point O, therefore ω is also the angular velocity of the c.g. of A about the instantaneous center O.

Therefore, from eqn (1)

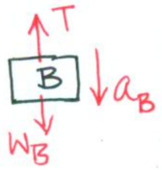
$$\Delta KE = \frac{1}{2}\times\frac{32.2}{32.2}\times\frac{\bar{v}_A^2}{9} + \frac{1}{2}\times\frac{200}{32.2}\times\bar{v}_A^2 + \frac{1}{2}\times 6\times\frac{\bar{v}_A^2}{1.5^2}$$

$$= 4.494\,\bar{v}_A^2$$

According to the principles of work and kinetic energy

$U_{net} = \Delta KE \Rightarrow 644 = 4.494\,\bar{v}_A^2 \quad \therefore \bar{v}_A = \boxed{11.97 \text{ fps}}$

$\therefore v_B = \frac{\bar{v}_A}{3} = \frac{11.97}{3} = 3.99$ fps.



For body B

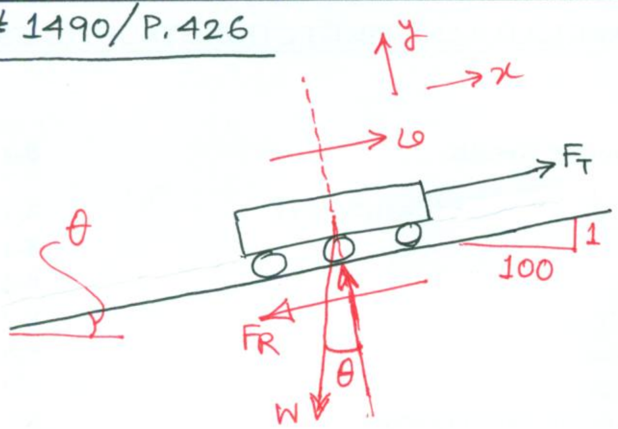
$v_B^2 = 2 a_B . S_B \quad \therefore a_B = \frac{v_B^2}{2S_B} = \frac{3.99^2}{2\times 20} = \boxed{0.398 \text{ fps}^2}$

$\Sigma F_y = m_B a_B \downarrow +ve$

$\Rightarrow W_B - T = \frac{W_B}{g}.a_B$

$\therefore T = W_B - \frac{W_B}{g}.a_B = 32.2 - \frac{32.2}{32.2}\times 0.398 = \boxed{31.8 \text{ lb}}$

1490/P. 426



$F_T = 50{,}000$ lb

$W = 2000$ Ton $= 2000 \times 2000$ lb

$F_R = 15$ lb/Ton

$= 15 \times 2000$ lb

$v_0 = 60$ mph $= \frac{60 \times 1760 \times 3}{60 \times 60}$ fps

$= 88$ fps

(a) for $s = 2$ mile, $v_{top} = ?$

max^m hp of drawbar = ?

hp of drawbar at top = ?

(b) for $s = 4$ mile, $v_{top} = ?$

Sol^n

(a) Considering $\Sigma F_x = m a_x$

$\Rightarrow F_T - F_R - W \sin\theta = \frac{W}{g} \cdot a$

$\Rightarrow 50000 - 15 \times 2000 - 2000 \times 2000 \times \frac{1}{\sqrt{100^2 + 1^2}}$

$= \frac{2000 \times 2000}{32.2} \times a$

$\therefore a = -0.161$ fps^2, $-$ve sign indicates retardation

$v_{top}^2 = v_0^2 + 2aS = 88^2 + 2 \times (-0.161) \times (2 \times 1760 \times 3)$

$\therefore v_{top} = 65.9$ fps $= \frac{65.90 \times 60 \times 60}{1760 \times 3}$ mph $= \boxed{44.93 \text{ mph}}$

Max^m power is delivered when the velocity is maximum.

$\therefore$ Max^m h.p. $= \frac{F_T \cdot v_0}{550} = \frac{50000 \times 88}{550} = \boxed{8000}$ [P. 411/Eq^n for h.p. 1 hp = 550 ft-lb/sec.]

h.p. when the train is at the top

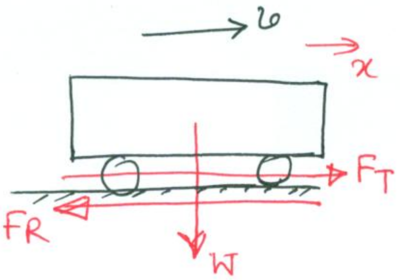
$= \frac{F_T \cdot v_{top}}{550} = \frac{50000 \times 65.9}{550} = \boxed{5991}$

(b) for $S = 4$ mile

$v_{top}^2 = v_0^2 + 2as = 88^2 + 2 \times (-0.161) \times (4 \times 1760 \times 3)$

$\therefore v_{top} = 30.71$ fps $= \frac{30.71 \times 60 \times 60}{1760 \times 3}$ mph $= \boxed{20.94 \text{ mph}}$

1494/P. 426



$W = 8{\cdot}05$ Ton
$v_0 = 10$ mph
$v_f = 45$ mph
$s = 2000$ ft
$F_R = 500$ lb

(a) Max^m h.p. = ?
(b) h.p. for 10 mph = ?
(c) F_T = ?

$F_T \rightarrow$ Tractive force
$F_R \rightarrow$ Resistive force

Sol^n

$$v_f^2 = v_0^2 + 2as$$

here, $v_0 = \dfrac{10 \times 1760 \times 3}{60 \times 60} = 14{\cdot}67$ fps

$v_f = \dfrac{45 \times 1760 \times 3}{60 \times 60} = 66$ fps

$\therefore 66^2 = 14{\cdot}67^2 + 2 \times a \times 2000 \quad \Rightarrow a = 1{\cdot}035 \text{ fps}^2$

$$\Sigma F_x = ma$$

$\Rightarrow F_T - F_R = \dfrac{W}{g} \cdot a$

$\Rightarrow F_T - 500 = \dfrac{8{\cdot}05 \times 2000}{32{\cdot}2} \times 1{\cdot}035$

$\therefore F_T = \boxed{1017{\cdot}5 \text{ lb}}$

Max^m h.p. $= \dfrac{1017{\cdot}5 \times 66}{550} = \boxed{122{\cdot}14}$ $\quad \boxed{1 \text{ hp.} = 550 \text{ ft-lb/s}}$

h.p. for 10 mph $= \dfrac{1017{\cdot}5 \times 14{\cdot}67}{550} = \boxed{27{\cdot}13}$

#1504/P.426

$W = 12000$ lb

$F = 5000$ lb

$v_0 = 500$ mph

$v_f = 600$ mph

(a) $U_{net} = ?$

(b) Output h.p. = ?

(c) Altitude $h = ?$ if potential energy is increased instead of K.E.

Sol^n

$$F = ma = \frac{W}{g} a$$

$$\Rightarrow 5000 = \frac{12000}{32.2} \times a$$

$$\therefore a = 13.42 \text{ fps}^2$$

$$v_0 = 500 \text{ mph} = \frac{500 \times 1760 \times 3}{60 \times 60} \text{ fps} = 733.33 \text{ fps}$$

$$v_f = 600 \text{ mph} = \frac{600 \times 1760 \times 3}{60 \times 60} \text{ fps} = 880 \text{ fps}$$

$$v_f^2 = v_0^2 + 2as$$

$$\Rightarrow 880^2 = 733.33^2 + 2 \times 13.42 \times S$$

$$\therefore S = 8834.24 \text{ ft}$$

(a) $U_{net} = F.S. = 5000 \times 8834.24 \text{ lb-ft} = \boxed{4.42 \times 10^7 \text{ ft-lb}}$ Ans.

(b) Final power output $= \frac{F v_f}{550}$ h.p. $= \frac{5000 \times 880}{550}$ h.p. $= \boxed{8000 \text{ h.p.}}$ Ans

(c) $W.h = \Delta KE = U_{net}$

$$\Rightarrow 12000 \times h = 4.42 \times 10^7$$

$$\therefore h = \boxed{3683.3 \text{ ft}}$$ Ans.