CE 103
SURVEYING

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Room No. 631, Civil Building

- One Class per week
- One Class Test
- Assignments
- If you have questions, you are welcome to have discussions after class hours

Acknowledgement

Book by Punmia
Oklahoma State University Website
Internet Sources (as stated)

Topics to be covered:

- Problems of Height & Distance
- Computation of Area & Volume
- Astronomical Surveying

Reference Material:

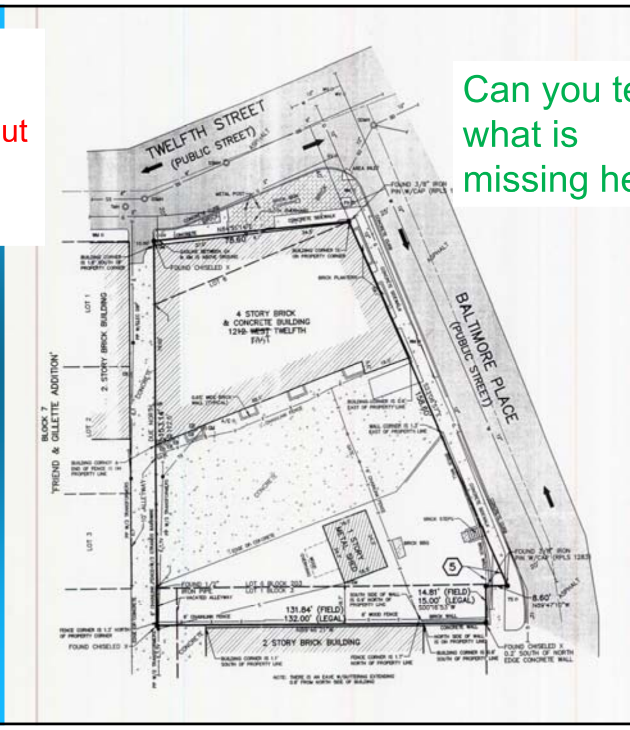
- Class Lecture
- Class Handouts
- Books by Punmia, Shahjahan & Aziz
- Assignments

Introduction to Surveying

The art and science of measuring and locating different features on, above and below the surface of the earth is known as Surveying

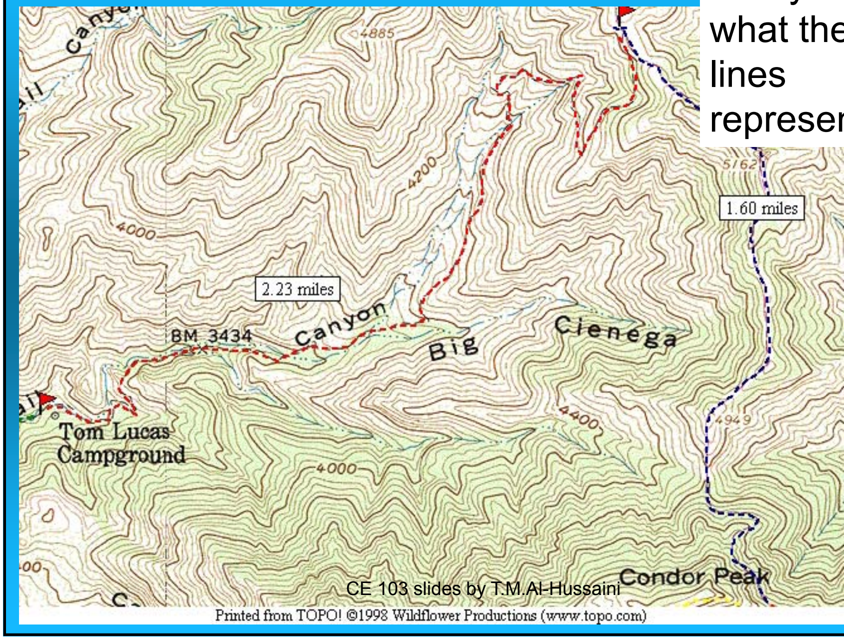
Surveying Measurements lead to final output

MAP



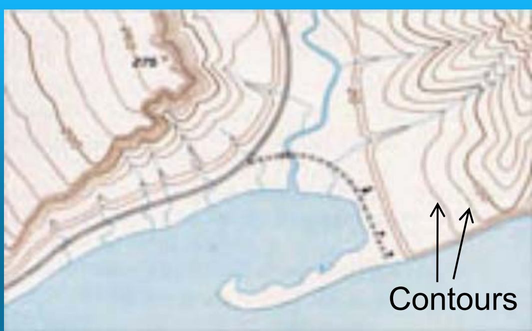
Can you tell what is missing here?

Contour Map



Can you tell what these lines represent?

Contour Map showing elevations



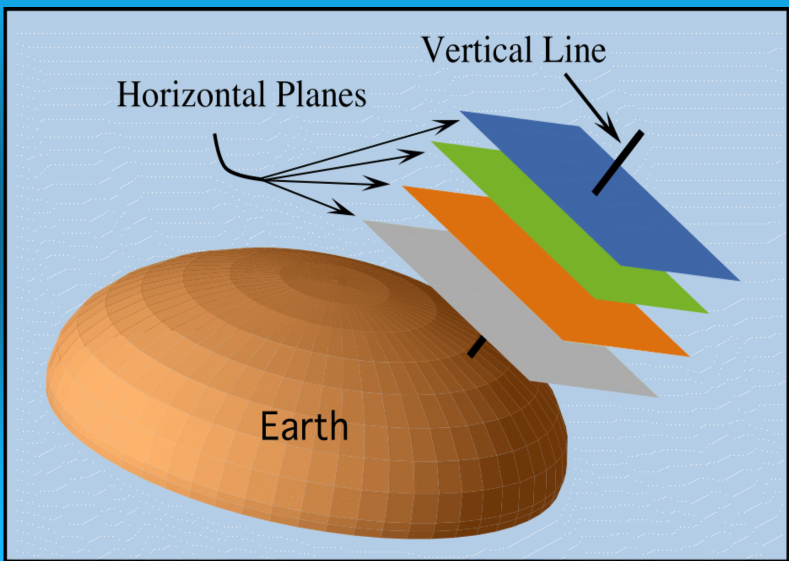
- Contour Map (contours are lines representing same elevation)

- 3D View

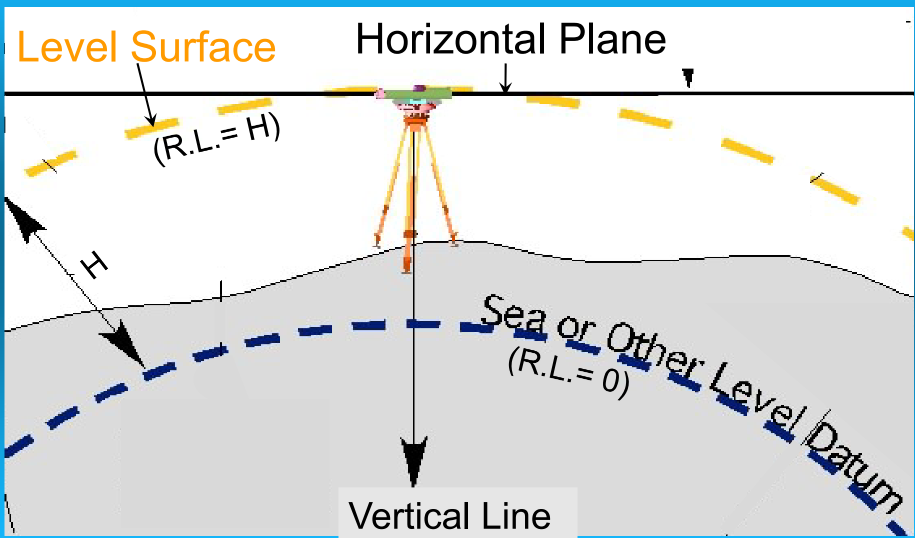
Elevation of a Point and its Determination

Vertical Line represents the direction of gravity.

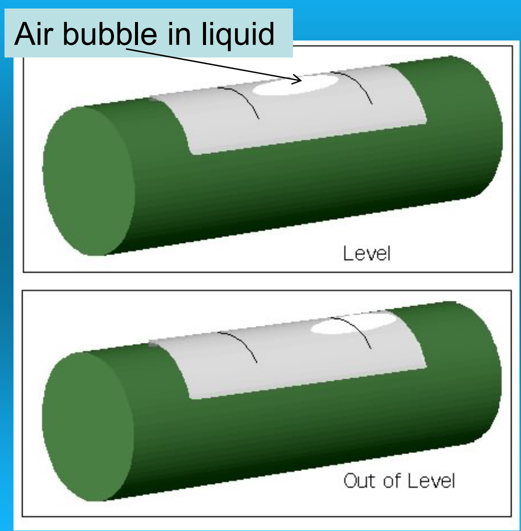
Horizontal Plane is perpendicular to vertical line.



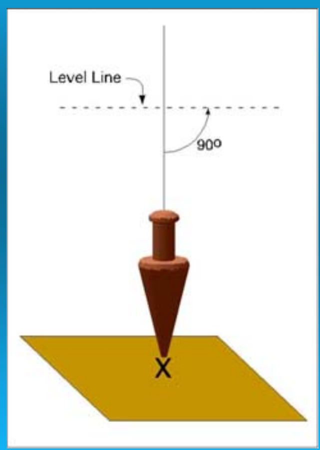
Horizontal Plane and Level Surface are almost same for short distances



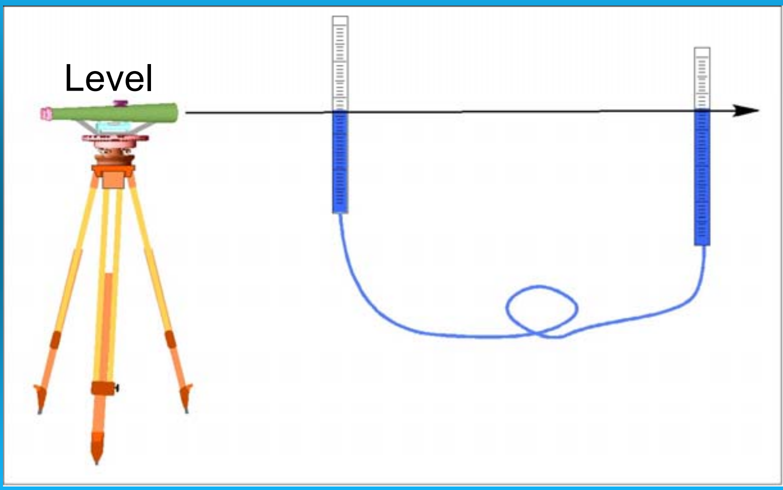
Level Tube indicates level surface



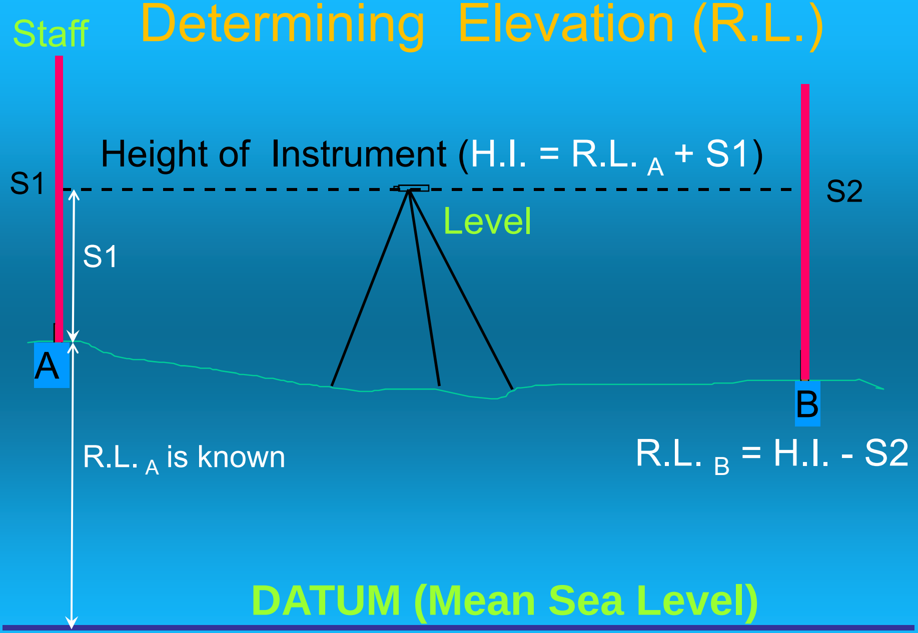
Plumb Bob shows vertical direction



Line of Sight of Level is horizontal



Determining Elevation (R.L.)



Surveying Operations

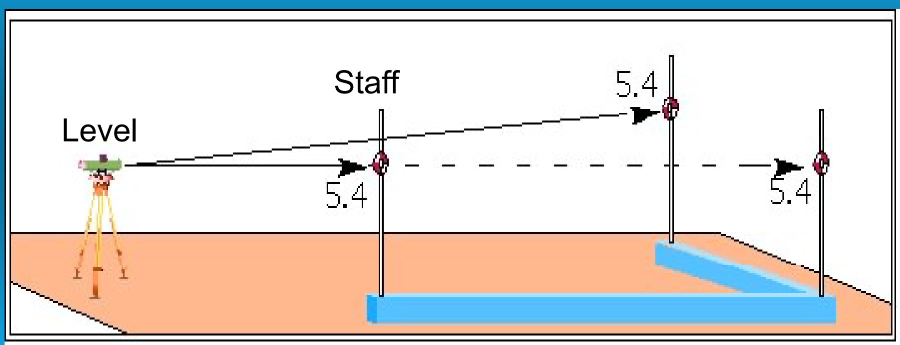
Surveying Operations

- Control Survey: Establish control points for horizontal and vertical control of surveying.
- Boundary Survey: Determine length and direction of land boundaries and locate them on the ground.
- Topographic Survey: Produce topographic map showing configuration of the terrain.
- Hydrographic Survey: Survey of water bodies.
- Mining Survey: Survey related to mining operations.

Surveying Operations (contd.)

- Construction Survey: Survey to lay out, locate and monitor construction works.
- Route Survey: Surveys for locating and constructing highways, railroads, canals, transmission lines, and pipelines.
- Photogrammetric Survey: Utilize aerial photographs to produce maps.
- Astronomical Survey: Survey related to observation of celestial bodies in the sky.

Use of level for levelling the formwork before placing concrete.



Modern Surveying Equipment

Digital Level

Digital Theodolite

Surveyor taking measurements at construction site with level

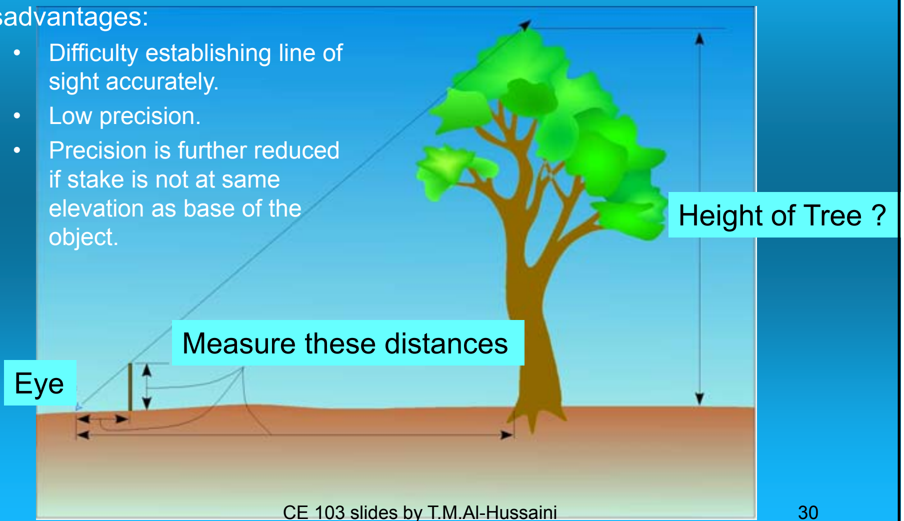
Surveyor measuring distances, elevations and directions on construction site by theodolite

Problems of Height and Distance

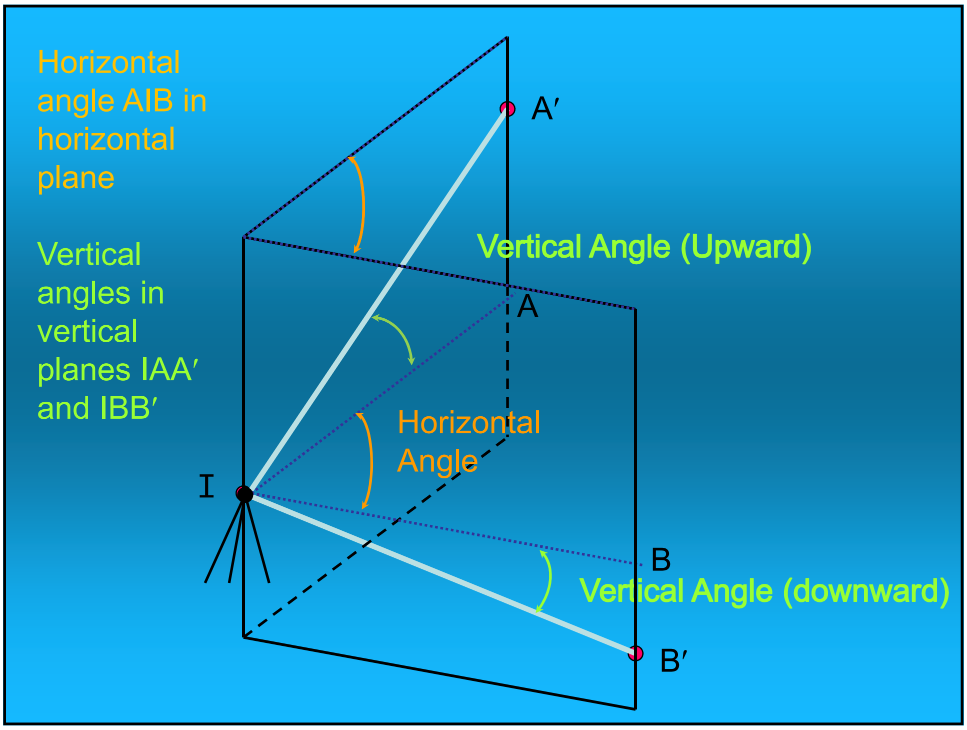
A Simple Low-Tech Method to Determine Height of a High Object

Disadvantages:

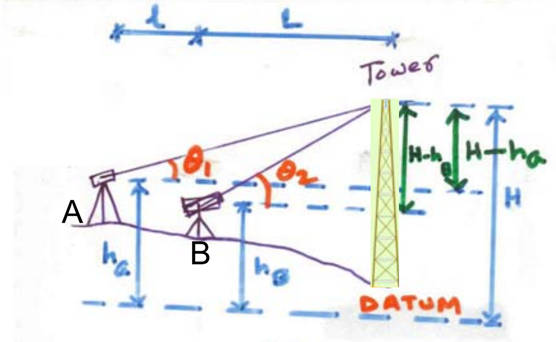
- Difficulty establishing line of sight accurately.
- Low precision.
- Precision is further reduced if stake is not at same elevation as base of the object.



CE 103 slides by T.M.Al-Hussaini 30



Problem 1: Determine R.L. of Tower Top and Distance of Tower



A & B are Instrument (Theodolite) Stations

All lie in same vertical plane.

θ_1 And θ_2 are vertical angles.

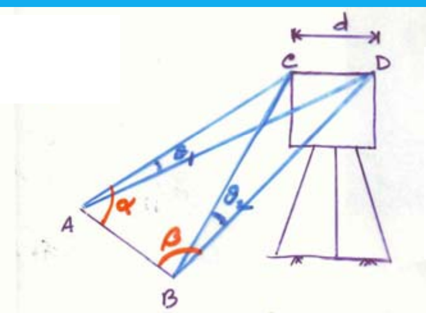
θ_1,θ_2 h_a, h_B, ℓ are known.

$$\tan\theta_1 = \frac{H-h_a}{L+\ell} \quad \text{--- (1)}$$

$$\tan\theta_2 = \frac{H-h_B}{L} \quad \text{--- (2)}$$

Solve (1) & (2) to determine H & L.

Problem 2: Determine Tank Diameter



A & B are Instrument (Theodolite) Stations

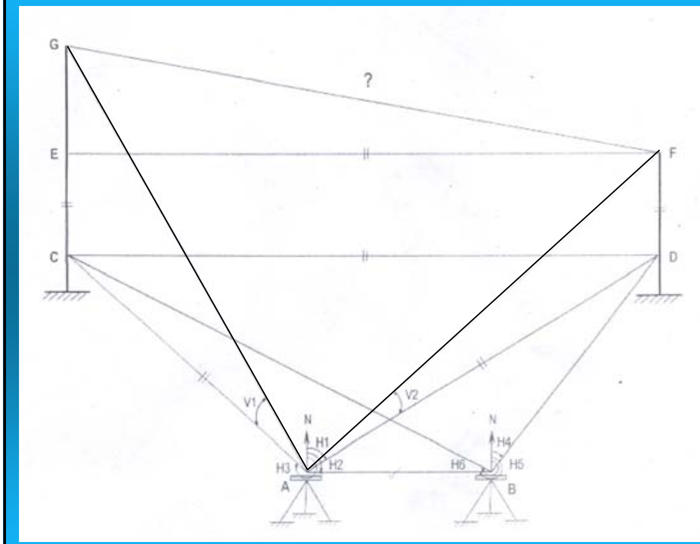
Assume A,B,C,D lie in same plane

$$\Delta ABC \Rightarrow \quad AC = \frac{AB\sin(\beta-\theta_2)}{\sin\angle ACB}$$

$$\Delta ABD \Rightarrow \quad AD = \frac{AB\sin\beta}{\sin\angle ADB}$$

$$\Delta ACD \Rightarrow \quad CD^2 = AC^2 + AD^2 - 2.AC.AD.\cos\theta_1$$

Aerial Distance in 3D Space



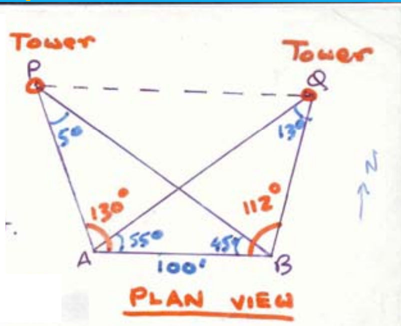
- Instrument Stations A & B
- Points A,B,C,D in same horizontal plane.
- Determine horizontal distance EF using AB & horizontal Angles H1, H2, H3, H4 etc.
- Determine vertical distance GE using vertical angles V1, V2

Problem 3: Determine Aerial Distance between Top of Towers at P & Q

A & B are Theodolite Stations, 100 ft apart

R.L. of Instrument at A & B are 25.2 & 20.4 ft.

Vertical angles from A to tower top at P & Q are 30° & 25°.



Sol^n: First determine horizontal distance PQ.

$\Delta APB \Rightarrow \frac{AP}{\sin 45°} = \frac{AB}{\sin 5°}$

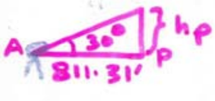
$\therefore AP = 100 \times \frac{\sin 45°}{\sin 5°} = 811.31'$

ΔABQ, $AQ = 100 \times \frac{\sin 112°}{\sin 13°} = 412.17'$

Problem 3 (Contd.)

ΔPAQ, $PQ^2 = AP^2 + AQ^2 - 2 \cdot AP \cdot AQ \cdot \cos\angle PAQ$

$\therefore PQ = 809.33'$



$h_p = 811.31 \tan 30° = 468.41'$

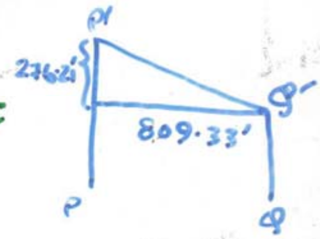
$h_q = 192.20'$

∴ Top of tower at P is 276.21' higher than top of tower at Q

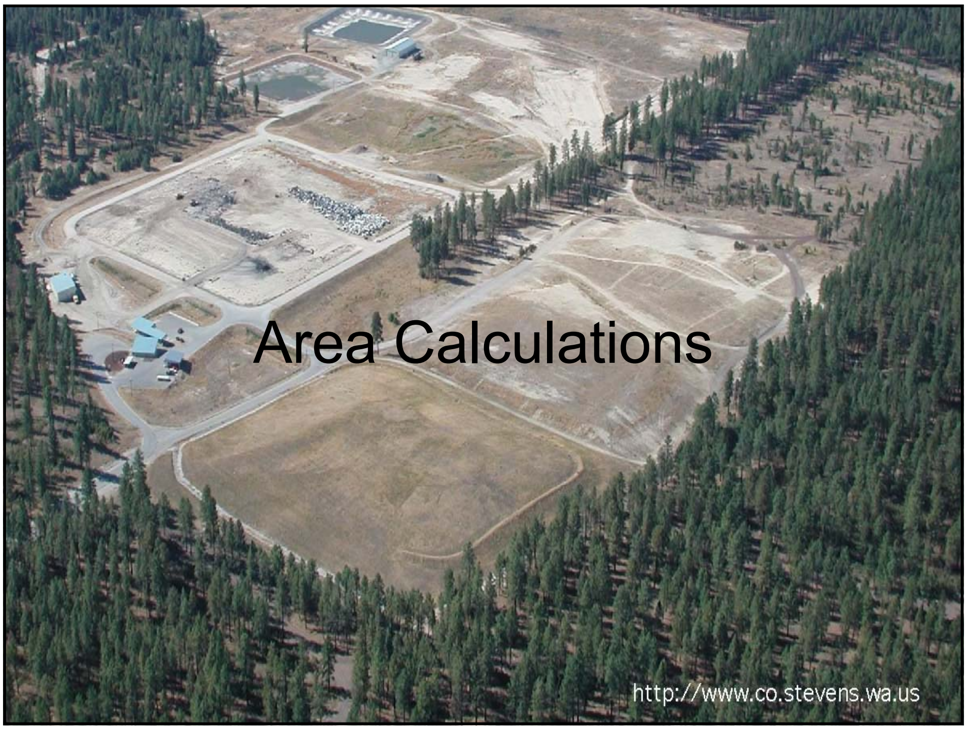
Aerial distance,

$P'Q' = \sqrt{(809.33)^2 + (276.21)^2}$

$= 855.16$ ft.

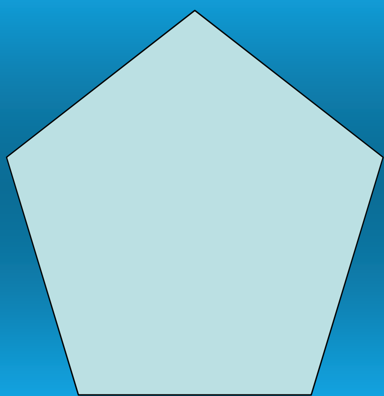


Computation of Area



STRAIGHT BOUNDARY

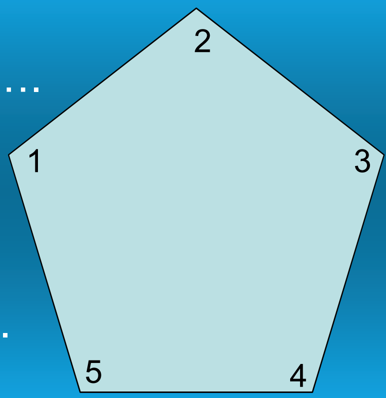
Area of Polygon = ?
You have only tape and can measure only length.



STRAIGHT BOUNDARY (Co-ordinate method)

Area of Polygon =

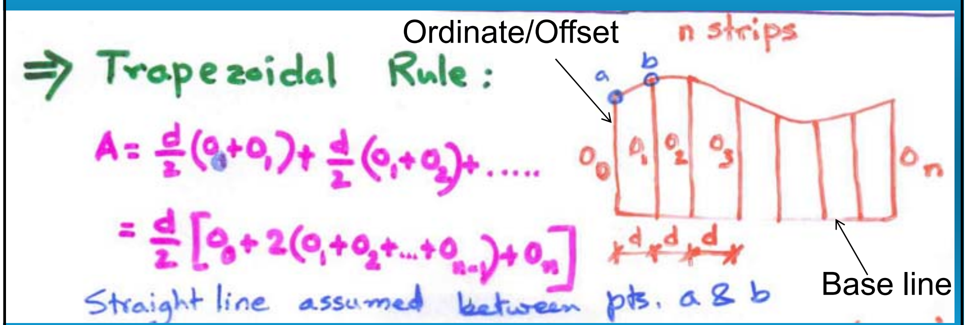
$\frac{1}{2}*[x_1(y_2-y_5) + x_2(y_3-y_1) + \ldots + x_5(y_1-y_4)]$



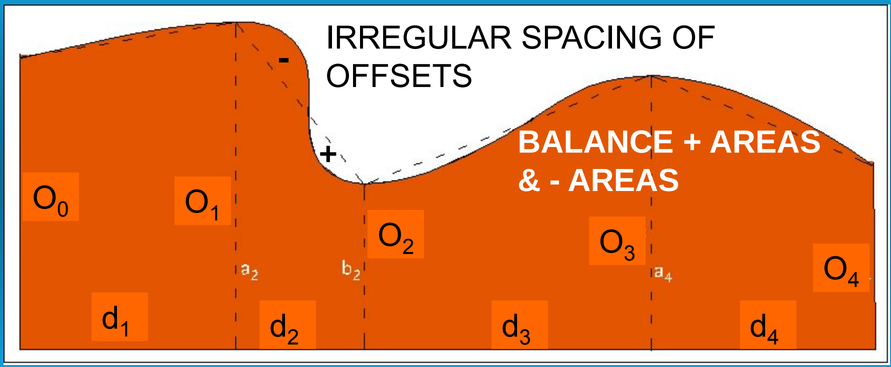
Assignment:

Derive above expression.

CURVED BOUNDARY (TRAPEZOIDAL RULE)



CURVED BOUNDARY
(TRAPEZOIDAL RULE)



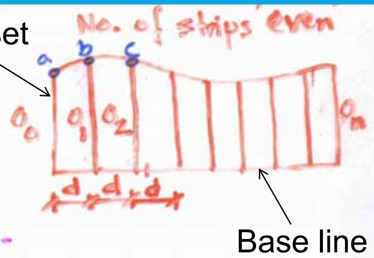
$$\text{Area} = d_1(O_0+O_1)/2 + d_2(O_1+O_2)/2 + \ldots\ldots.$$

CURVED BOUNDARY
(SIMPSON'S RULE)

⇒ Simpson's Rule :

$$A = \frac{2d}{6}(O_0 + 4O_1 + O_2) + \frac{2d}{6}(O_2 + 4O_3 + O_4) + \ldots$$

$$= \frac{d}{3}\left[O_0 + 4(O_1 + O_3 + \ldots\ldots) + 2(O_2 + O_4 + \ldots\ldots) + O_n\right]$$



Parabola assumed through points a, b, c.

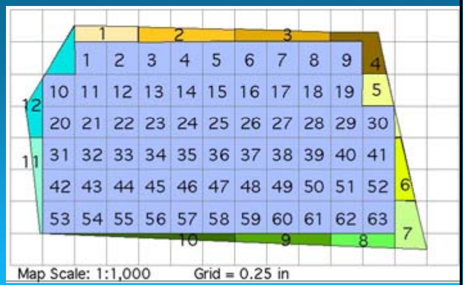
COMPUTATION OF AREA FROM MAPS

(1) Division of Area into simple geometric figures (triangle, rectangle, parallelogram, trapezium, circle, etc.)

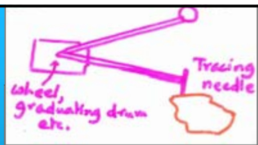
Balance + Areas and – Areas if necessary.

(2) Subdivision into Squares

Area = No. of squares (including fraction) x Area of each square



(3) Mechanical Method: Use of Planimeter



- A Planimeter is a device the determines area by tracing the boundary on a map with a tracing needle.
- Accuracy very good
- Peforms mechanical integration.
- Two types:
  - Mechanical
  - Electronic

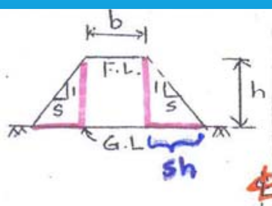
(4) Modern Method: Use of Digitizer

- This method requires a machine called a digitizer, which obtains co-ordinates from map and saves it as digital data in computer.
- The operator moves a special mouse or pen around the map and activates the mouse at each desired location.
- Computer records x - y coordinates of each location.

CROSS-SECTIONAL AREA

1. Level Section:

- F.L. & G.L. are both horizontal.
- Area $= h(b + sh)$



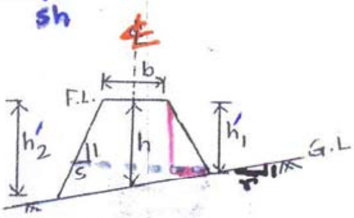
2. Two-Level Section:

- Ground surface is inclined.

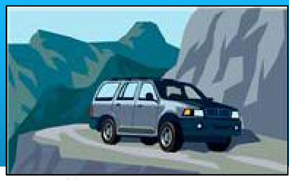
$$\text{Area} = h_1'(b + sh_1') + \frac{1}{2}(b + 2sh_1')(h_2' - h_1')$$

$$= \frac{1}{2}b(h_1' + h_2') + sh_1'h_2'$$

$$= \left[\frac{r^2bh + s(0.5b)^2 + r^2sh^2}{r^2 - s^2}\right] \quad \text{where, } h_1' = \frac{rh - 0.5b}{r + s} \;\; \& \;\; h_2' = \frac{rh + 0.5b}{r - s}$$



CROSS-SECTIONAL AREA
(In Hilly Terrain)



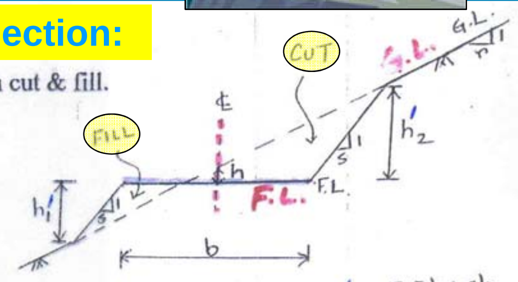
3. Side-Hill Two-Level Section:

- Two-level section consisting of both cut & fill.
- Area of Fill

$$= \frac{1}{2} h_1' \left(\frac{b}{2} - rh \right) = \frac{1}{2} \left\{ \frac{(0.5b - rh)^2}{r - s} \right\}$$

- Area of Cut

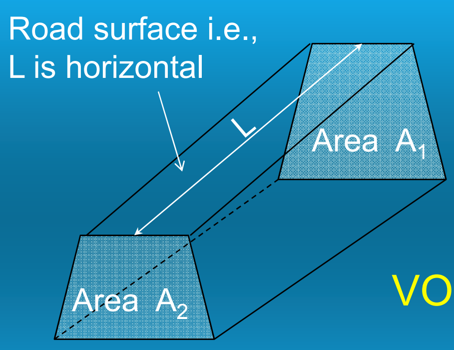
$$= \frac{1}{2} h_2' \left(\frac{b}{2} + rh \right) = \frac{1}{2} \left\{ \frac{(0.5b + rh)^2}{r - s} \right\}$$



$$h_1' = \frac{0.5b - rh}{r - s} \quad \& \quad h_2' = \frac{0.5b + rh}{r - s}$$

Computation of Volume

VOLUME OF ROAD EMBANKMENT



Cross-sections A1 & A2 are vertical

VOLUME = ?

COMPUTATION OF VOLUME USING CROSS-SECTIONS

Basic Assumptions:

- Cross-sections are parallel
- Surfaces bounding the volume are plane
- Consider volume as prismoid

Two Approaches:

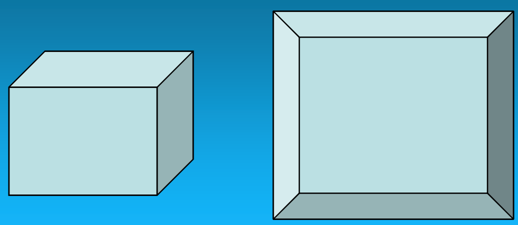
- Apply **Prismoidal rule**
- Apply Trapezoidal rule and then apply correction. Why??

PRISMOID

A solid, bounded by two parallel plane ends having the form of polygons, joined by longitudinal faces which are plane surfaces.

The longitudinal faces will be plane if the sides of the end polygons are parallel.

The dimensions of middle section midway between end sections is average of dimensions of end sections



Imagine different types of prismoids

VOLUME BY PRISMOIDAL RULE

Volume of earth between two successive cross-sections A_1 & A_2 at distance L apart, is considered as prismoid.

Volume of prismoid, $V_p = L/6\ [A_1+4A_m+A_2]$
where A_m is cross-sectional area midway between A_1 & A_2.

Note that although linear dimensions of mid-section is average of corresponding dimensions of end areas, in general, $A_m \neq (A_1+A_2)/2$

This formula is exact when the solid consists of prisms, wedges and pyramids only.

VOLUME BY TRAPEZOIDAL RULE

Volume of earth between two successive cross-sections A_1 & A_2 at distance L apart is calculated as: $V_T = L/2\,[A_1+A_2]$

This formula agrees with prismoidal rule only if $A_m = (A_1+A_2)/2$

This formula is exact when the solid consists of prisms and wedges only. It cannot calculate the volume of pyramid correctly.

If trapezoidal rule is used, prismoidal correction must be applied.

Prism volume = height x base area
Wedge volume = 1/2 x base area x height
Pyramid volume = 1/3 x base area x height

PRISMOIDAL CORRECTION (Level Section)

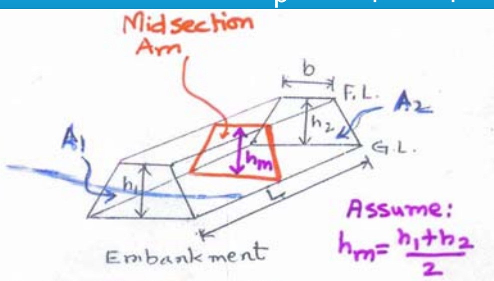
Prismoidal Correction is defined as $C_p = V_T - V_P$

1. Level Sections:

$$V_P = \frac{L}{6}[A_1 + 4A_m + A_2] = L\left[b\left(\frac{h_1+h_2}{2}\right) + \frac{s}{3}\left(h_1^2 + h_2^2 + h_1 h_2\right)\right]$$

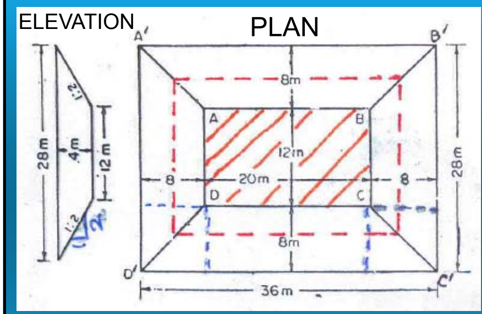
$$V_T = \frac{L}{2}[A_1 + A_2] = L\left[b\left(\frac{h_1+h_2}{2}\right) + \frac{s}{2}\left(h_1^2 + h_2^2\right)\right]$$

$$C_p = V_T - V_P = \frac{Ls}{6}[h_1 - h_2]^2$$



$$A_1 = (b + sh_1)h_1$$
$$A_2 = (b + sh_2)h_2$$
$$A_m = (b + sh_m)h_m = b\left(\frac{h_1+h_2}{2}\right) + s\left(\frac{h_1+h_2}{2}\right)^2$$

Problem 1: Determine Volume of Pond by Prismoidal & Trapezoidal Rule



Top and bottom planes of pond are horizontal & parallel, its volume may be considered as prismoid.

A_{bottom} = 20 x 12 = 240 m^2
A_{top} = 28 x 36 = 1008 m^2
V_T = 4/2 [240+1008]= 2496 m^3

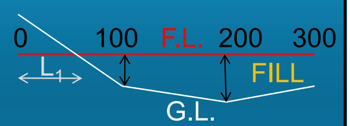
A_{mid} = 28 x 20 = 560 m^2
V_P = 4/6 [240+4x560+1008]
= 2325 m^3

Note: $A_{mid} \neq (A_{bottom} + A_{top})/2$
$V_T > V_P$

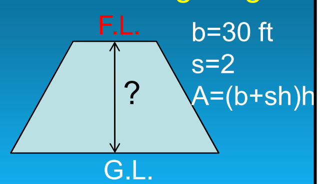
Problem 2: Determine Volume of Earthwork for Road Embankment

F.L. = 15 ft. ➡ (road top) Width at F.L.= 30 ft. Side Slope=2H:1V
Ground is level across embankment width ➡ Level Section
G.L. of centreline at Chainage 0, 100, 200, 300 ft is 18, 13, 12, 13 ft respectively.

Ch. (ft)	G.L. (ft)	F.L. (ft)	h (ft)	A (sft)	Remarks
0	18	15	3		CUT
L_1	15	15			
100	13	15	2		FILL
200	12	15			FILL
300	13	15			FILL



Elevation along length



Cross-Section at Ch.100

What is the Cross-Section at Ch.0 like?

Problem 2 (Contd)

- ❑ The earthwork consists of both Cut and Fill. These two types of volume should be calculated separately.
- ❑ Consider Volume between two consective sections as Prismoids. Can you tell how many Prismoids in this problem?
- ❑ For each Prismoid, calculate volume by Trapezoidal Rule first, then apply Prismoidal Correction.

Volume (CUT) between Ch.0 & Ch.L_1

$L = L_1 =$

$h_1 = 3$ ft, Area $A_1 = (30+2x3)3 = 108\ ft^2$

$h_2 =$, Area $A_2 =$

Volume V_T (cut) $= L\,(A_1+A_2)/2 =$

Prismoidal correction C_p (cut) $= L_1 s(h_1 - h_2)^2/6 =$

Volume V_p (cut) $= V_T - C_p =$

Problem 2 (Contd)

Volume (FILL) between Ch.L_1 & Ch.100

$L = 100 - L_1$

$h_1 =$, Area $A_1 =$

$h_2 = 2$ ft, Area $A_2 =$

Volume $V_T = L\,(A_1+A_2)/2 =$

Prismoidal correction $C_p = Ls(h_1 - h_2)^2/6 =$

Volume V_p (fill) $= V_T - C_p =$

Volume (FILL) between Ch.100 & Ch.300

$V_T =$

$C_p =$

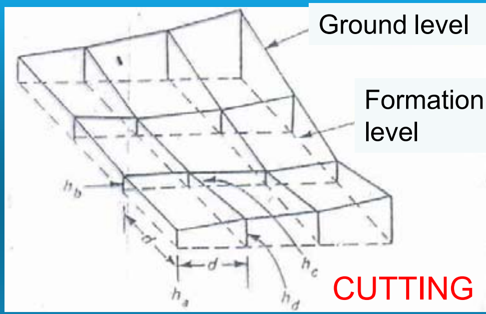
V_P (fill) $=$

Assignment:

Apply Prismoidal Rule directly to compute volume and compare.

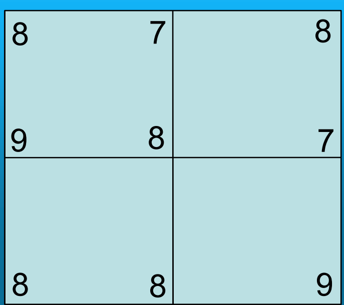
Volume from Spot levels

- ❑ Area is divided into several squares, rectangles or triangles
- ❑ Basic assumption: A plane surface passes through the corners of each square or triangle. Volume within each square/triangle can then be considered as a prism.



- ❑ Volume of square prism = Plan Area x Average depth at four corners [Note that depth must be perpendicular to the Area]
- ❑ This method is convenient in determining volume of excavation or filling in an area, where existing ground level is raised or lowered.

Problem 3: Determine Volume of Land-Fill



PLAN (R.L. shown)

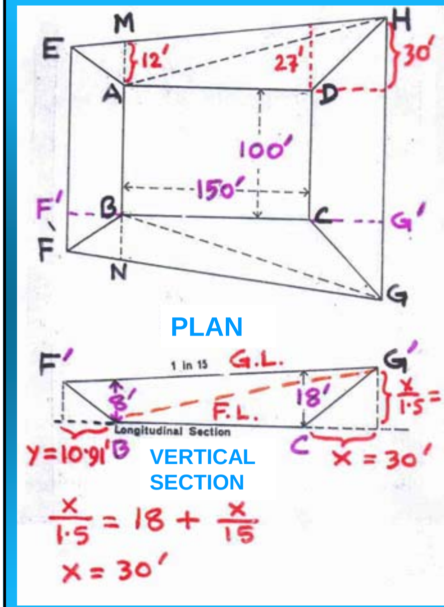
Assignment:
Consider triangular prisms instead of square prisms. Is there any difference in Volume? Explain why?

Existing G.L. (m) of Grid Points are shown in figure. Each square is 5 m x 5 m. This land is to be raised to an elevation (R.L.) of 10 m.

Consider Earthfill to consist of 4 Square Prisms. Vertical height of prism at four corners is obtained from difference between G.L. and F.L. The cross-sectional area of the prism is the plan (horizontal) area which is perpendicular to the height.

Volume of Earthfill = ¼ x 5 x 5 [2+1+3+2+3+2+2+3+1+2+2+2+2+2+3+1] = 206.25 m^3

Problem 4: Determine Volume of Excavation by Prisms



<u>Given Data:</u>

Ground surface is sloping in one direction. So, top plane (EFGH) and bottom plane (ABCD) of pond are not parallel. Hence prismoidal rule cannot be applied.

Depth of pond = 8 ft (above AB)
Depth of pond = 18 ft (above CD)

Ground slope = 1V:15H along AD or BC but horizontal along BA or CD. Side slope of pond = 1V:1.5H. Bottom dimensions 150 ft x 100 ft

Problem 4 (Contd.)

While bottom ABCD dimensions are known, top surface EFGH dimensions need to be calculated. Determine horizontal distances GH=160 ft, EF=121.82 ft, F″G″=190.91 ft

The total pond volume is considered to consist of two triangular prisms EFGHAB on top of ABCDGH. The edges (height) of these prisms are horizontal, so the cross-sectional area must be vertical (consider vertical plane BCG′F′). For Prism EFGHAB, triangular area BG′F′ and for prism ABCDGH, triangular area BCG′ is the required cross-sections.

Volume of prism ABCDGH = ½ x 150 x 20 [100+100+160]/3
Volume of prism EFGHAB = ½ x 8 x 190.91 [100+160+121.82]/3
Total volume of pond = 277190 ft^3.

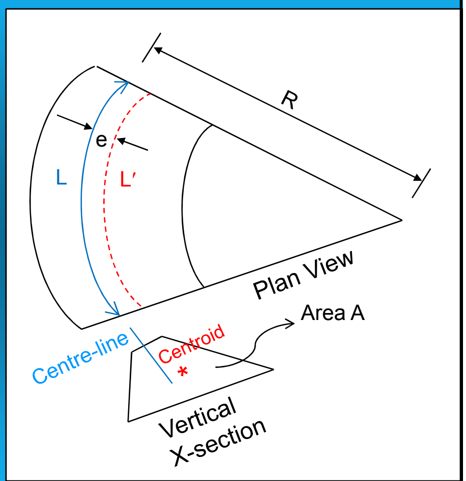
Curvature Correction for Volume

- Assumption in Prismoidal formula: cross-sections are parallel, this is true for straight embankment or canal.
- When the centre-line of an embankment is a circular curve, cross-sections are along radial lines, hence cross-sections are no longer parallel.
- To account for the effect of curvature, a curvature correction is derived using Pappus theorem which states *"Volume of solid formed by rotating a plane figure about an axis is equal to the area of the figure A multiplied by the length of path L traced by the centroid of the area"*.

- Let us consider an embankment with uniform cross-sectional area A curved with a radius of curvature R. L is length along centre-line between two cross-sections. Let us determine the volume of this curved embankment using Pappus' theorem.

Curvature Correction (contd.)

The centroid of the cross-section (as for example two-level section) is at a horizontal distance e from the centre line toward the centre of curvature (see figure).



Uncorrected Volume = AL
Radius of path of centroid = R-e
Length of path of centroid, L′ = (R-e)L/R
Volume (using Pappus theorem) = A(R-e)L/R
Curvature correction to volume = A(R-e)L/R - AL = -AeL/R

Curvature Correction (contd.)

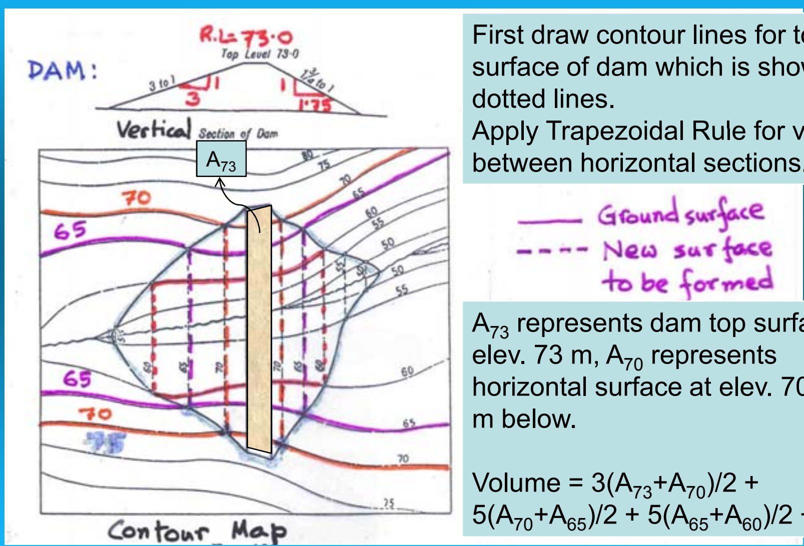
- If centroid of cross-section lies on centre-line (such as level section), no curvature correction is needed.
- If centroid of cross-section and centre of curvature lies on same side of centre-line the curvature correction is negative and vice-versa.

- In reality, the cross-sectional area A varies, then either of the two following approaches may be followed:

(a) Determine A and e for each cross-section and use the mean value $0.5(A_1e_1+A_2e_2)$ in place of Ae in the expression for volume correction $\pm AeL/R$. Calculate volume using prismoidal formula as if cross-sections are parallel and then apply volume correction.

(b) Correct each cross-sectional area A for eccentricity e. Calculate $A_1(1\pm e_1/R)$ instead of A_1, and $A_2(1\pm e_2/R)$ instead of A_2, and use these corrected areas in the prismoidal formula.

Problem 5: Determine Volume of Earth-Fill to Construct a Dam



First draw contour lines for top surface of dam which is shown by dotted lines.
Apply Trapezoidal Rule for volume between horizontal sections.

A_{73} represents dam top surface at elev. 73 m, A_{70} represents horizontal surface at elev. 70 m., 3 m below.

Volume = $3(A_{73}+A_{70})/2 + 5(A_{70}+A_{65})/2 + 5(A_{65}+A_{60})/2 + \ldots$

Let us start thinking about the earth, the sun, the stars and their relative motion

Astronomical Surveying

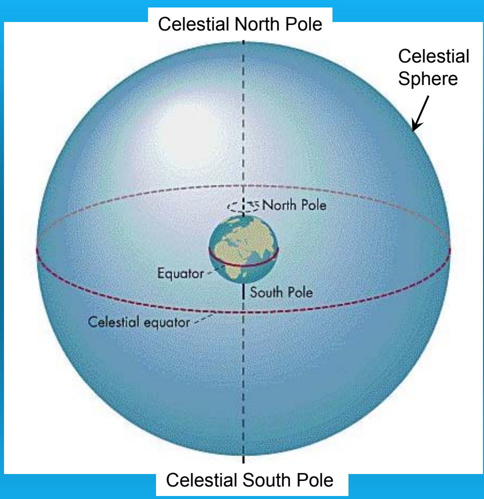
Astronomical Surveying

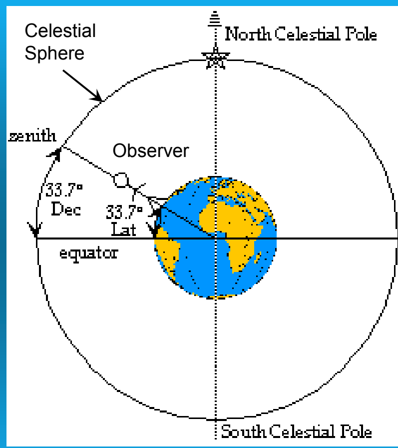
Branch of surveying that deals with the observation of celestial bodies (sun, stars) for determining the absolute location of any point or the absolute location and direction of any line on the surface of earth and also for detrmining time.

Celestial Sphere

Imaginary sphere on which the distant stars appear to lie with the earth centre as its centre. The concept of celestial sphere is useful, because the surveyor is concerned with the angular position of the stars only and not with the actual distances.

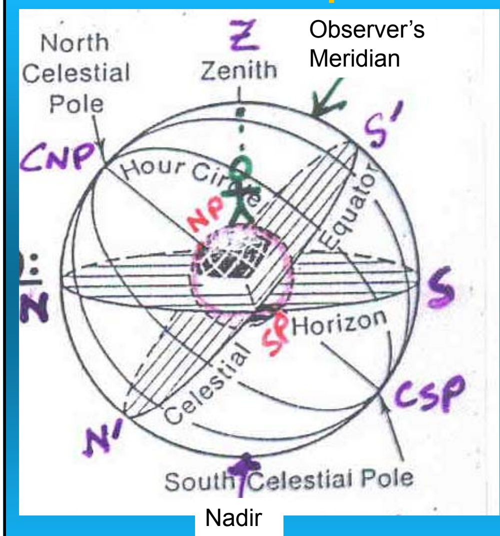
Celestial Sphere with Earth at centre





Latitude = Angle between zenith & equator

Celestial Sphere



Observer in the Northern hemisphere, Celestial North Pole (CNP) lies between North Point (N) and Zenith (Z)

What happens when observer is in Southern hemisphere? Draw your figure to find out.

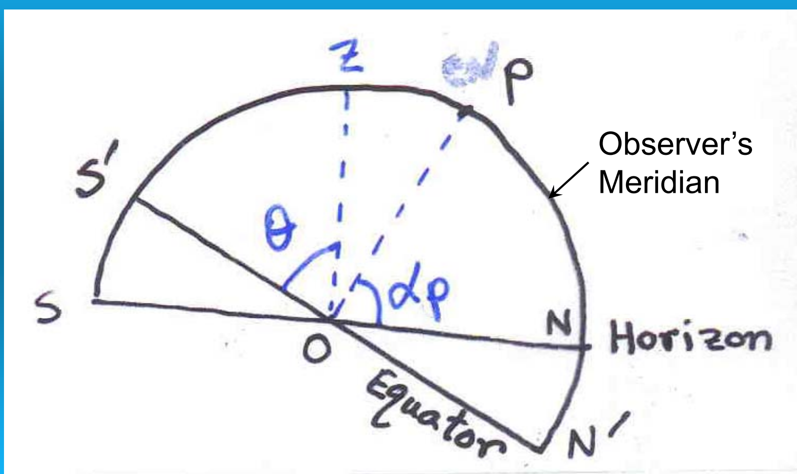
Observer's Meridian

Great Circle passing through zenith (Z), celestial north pole (CNP), celestial south pole (CSP), north point (N) & south point (S).

Great circle may be conceived as a plane passing through the centre.

Observer's meridian represents the vertical plane at the location of observer in the north-south direction

Prove: Altitude of Pole (α_p) = Latitude of Observer's position (θ)



CO-ORDINATE SYSTEMS

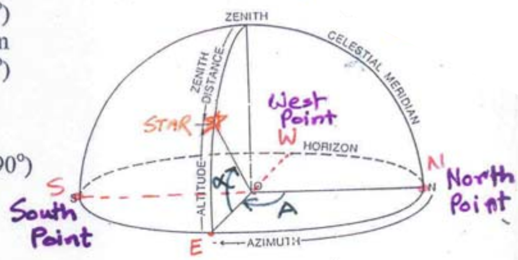
1. Horizon System (Altitude & azimuth System)

Azimuth (A):

- Angle measured in horizontal plane.
- In Northern hemisphere, measured from North eastward or westward (0° to 180°)
- In Southern hemisphere, measured from South eastward or westward (0° to 180°)

Altitude (∝):

- Angle measured in vertical plane.
- Measured from horizon upward (0° to 90°) Altitude of Zenith is 90°.
- Zenith Distance, z = 90° - ∝



- It is easy to visualize these coordinates in the sky. These coordinates can be directly measured with simple instruments.
- *Disadvantage:* These coordinates are dependent on the observer's position.
- Azimuths and altitudes of celestial bodies are continuously changing with time because of apparent daily motion of the celestial sphere.

2. Independent Equatorial System

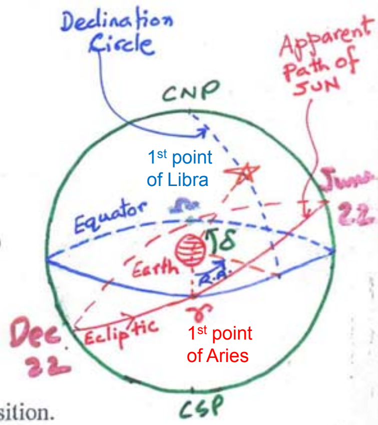
2. INDEPENDENT EQUATORIAL SYSTEM:

Right Ascension (R.A.):

- Angle (range: 0° to 360°) measured in equatorial plane, eastward from 1st Point of Aries (♈), the point where the sun's centre crosses the celestial equator moving from south to north of equator.

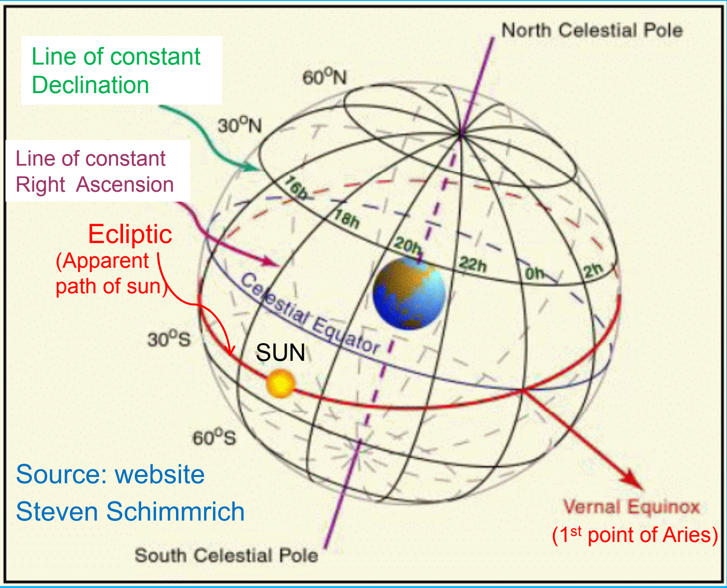
Declination (δ) :

- Angle measured north (N or +ve) or south (S or -ve) of equator in a plane (Plane of Declination Circle) normal to equatorial plane. Range: 0° to 90°
- *Declination of Celestial North Pole is 90° N.*
- Polar Distance, p = 90° - δ

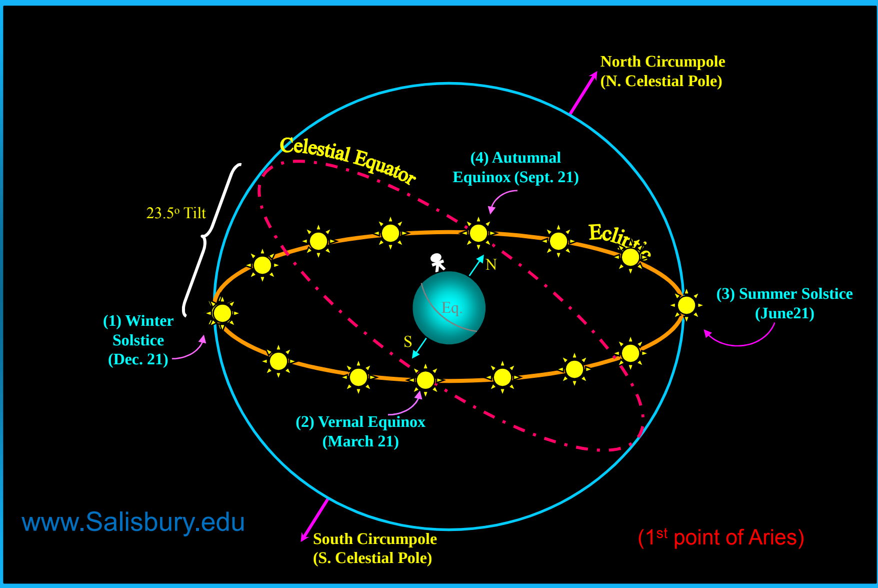


- These coordinates are independent of observer's position.
- Used in star catalogues and maps.
- Declination and right ascension of the distant stars are nearly constant. They may, however, be subjected to small annual changes.
- Declination and right ascension of the sun changes significantly with time.

Independent Equatorial System (contd.)



Solstice & Equinox



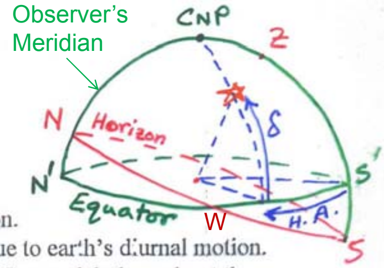
3. Dependent Equatorial System
(Hour Angle and Declination)

3. DEPENDENT EQUATORIAL SYSTEM:

Hour Angle (H.A.):

- Angle measured in equatorial plane, westward from south (Range: 0° to 360°)

Declination **(δ) :** .... *described above*



- The Hour Angle is dependent on observer's position.
- H.A. of a star at a location changes continuously due to earth's diurnal motion.
- H.A. of stars observed daily at convenient times at Greenwich throughout the year is reported in star catalogues.

4. Celestial Latitude and Longitude System

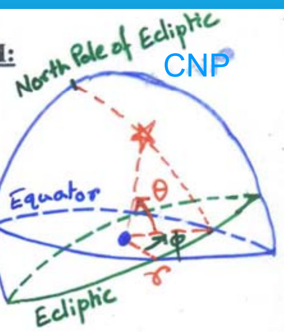
4. CELESTIAL LATITUDE & LONGITUDE SYSTEM:

Celestial longitude (ϕ):

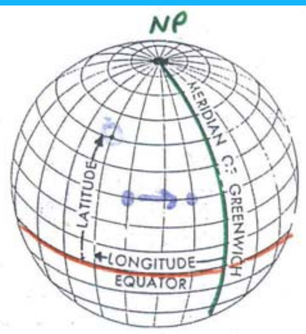
- Angle measured in plane of ecliptic, eastward from 1st Point of Aries (Range: 0° to 360°)

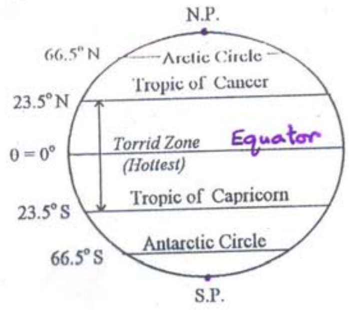
Celestial latitude (θ):

- Angle measured in plane normal to plane of ecliptic. (Range: 0° to 90° N or S of ecliptic)

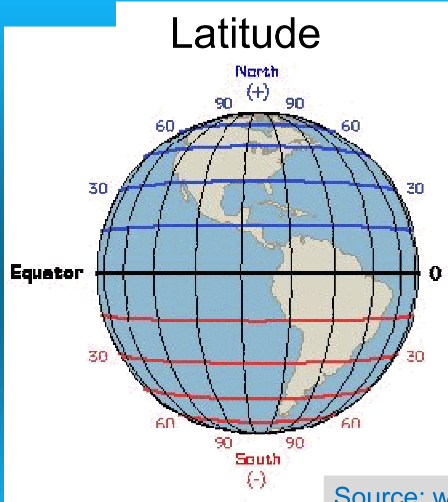


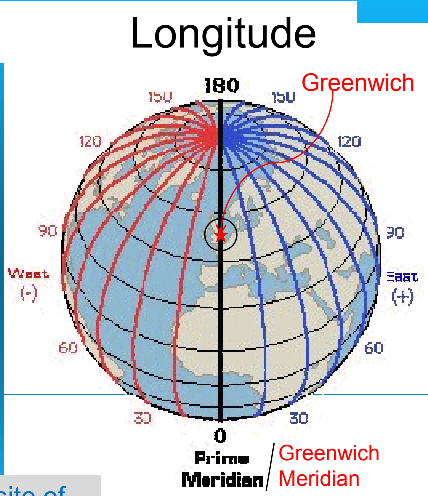
EARTH





Great Circle: Circle formed on the surface of sphere by plane passing through the centre.
Meridian: Half of great circle joining the poles which is perpendicular to the equator
Longitude (ϕ): Angle measured in equatorial plane. Range 0° to 180° east or west of Greenwich Meridian.
Latitude (θ): Angle measured in plane normal to equatorial plane. Range 0° to 90° north or south of equator.





Source: website of Steven Schimmrich

EARTH

Parallel of Latitude: Small circle parallel to plane of equator where latitude is constant. Tropic of cancer, arctic circle etc. are parallel of latitude.

Distance (n.m.) along Parallel of Latitude = Difference in ϕ (min) x cos(θ)
1 nautical mile = 6080 ft (Arc length on great circle creating 1 min angle at centre of earth.

Earth is not a perfect sphere, it is flattened at the poles and bulged at the equator:
Equatorial diameter = 12756.8 km
Polar diameter = 12713.8 km.

Polar axis: Earth is continuously rotating from west to east about an axis joining the poles called the polar axis.

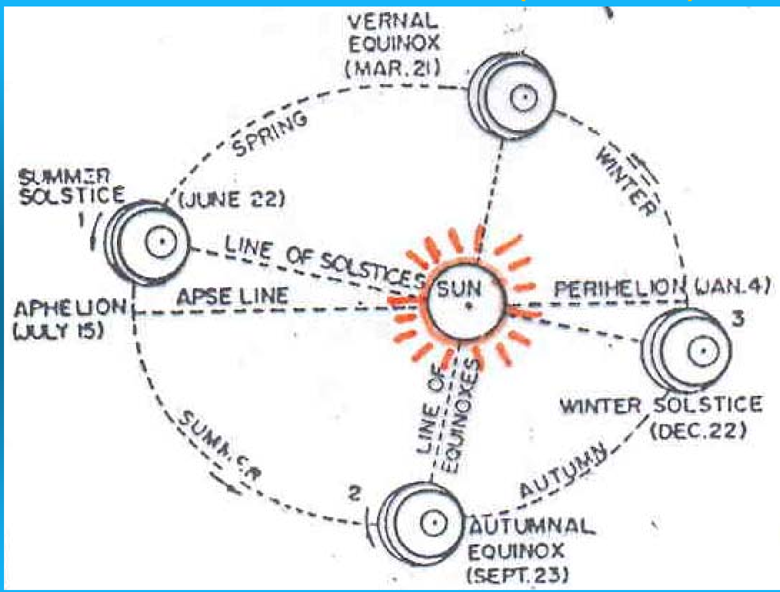
ORBIT OF EARTH

- The earth moves eastward in an elliptic orbit completely around the sun in a year maintaining an angle of 23°27′ between its orbit and the equator. That is the reason, the ecliptic (apparent path of sun around the earth) makes an angle of 23°27′ with the equator.

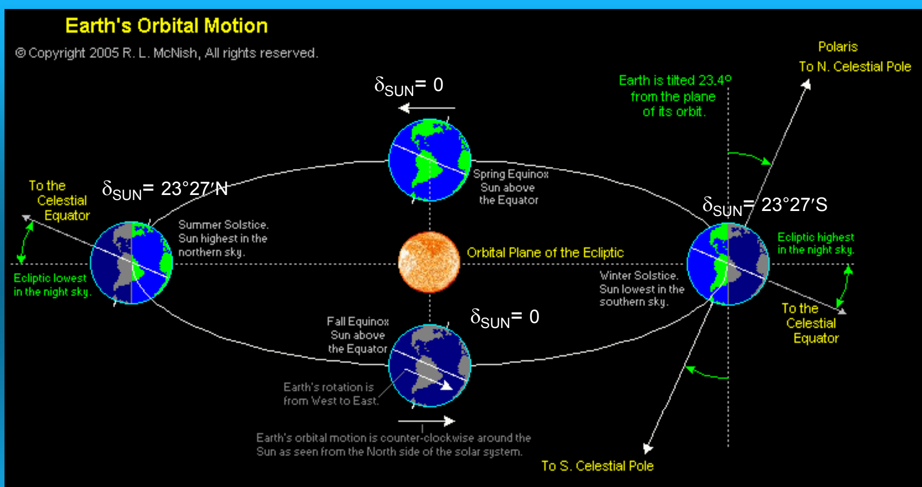
- The axis of the earth remains practically parallel to itself as it moves around the sun. The distance between the earth and sun varies between 147 million km (Jan) to 152 million km (July).

- The declination angle of the sun is therefore continuously changing from 23°27′N (June 22: Summer Solstice) to 23°27′S (Dec.22: Winter Solstice). It is zero on Sept.23 (Autumnal Equinox) and Mar.21 (Vernal Equinox).

ORBIT OF EARTH (Contd.)



ORBIT OF EARTH (Contd.)

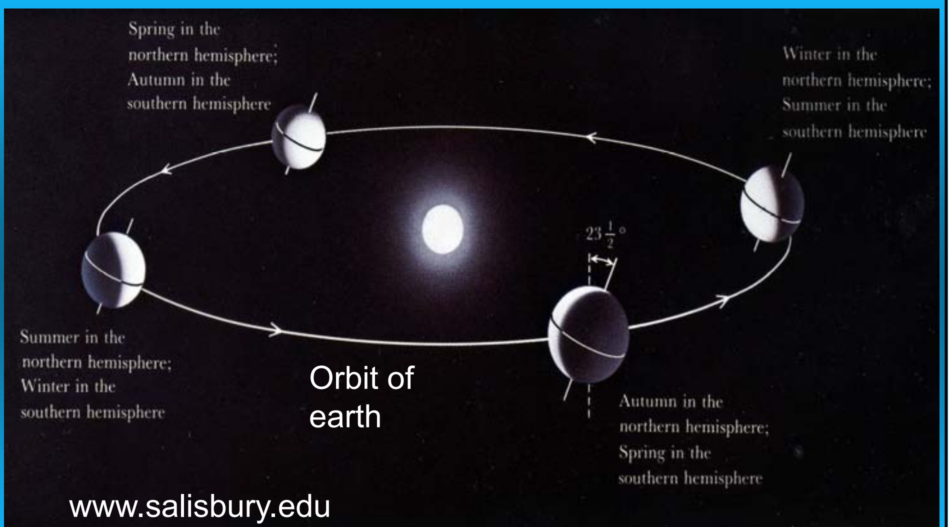


SEASONS: DAY & NIGHT

- ***On June 22*** (Summer Solastice in Northern Hemisphere):
  At Equator: Day = Night =12 hrs. North of Equator: Day > 12 hrs.
  North of Arctic Circle: Day =24 hrs. South of Antarctic Circle: Night =24 hrs.
- It is warmer in summer due to: (i) Days are longer (ii) Intensity of sun energy falling on unit area of earth surface is larger.

On Dec 22 (Winter Solstice in Northern Hemisphere):
It is winter in Northern hemisphere while summer in Southern hemisphere

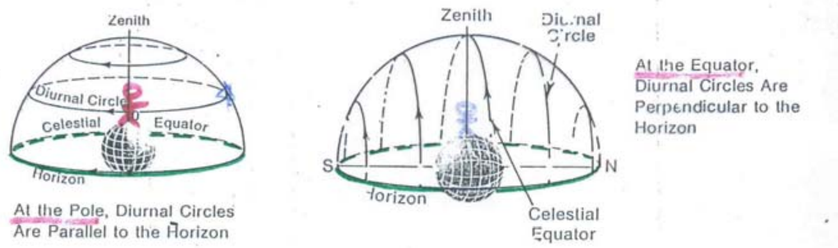
SEASONS: DAY & NIGHT



DIURNAL CIRCLE

Apparent path traced by a star due to daily rotation of earth about its own axis

At Pole | At Equator



- Diurnal circles are parallel to equator and normal to earth's axis of rotation (Polar axis)

TRANSIT OF STAR

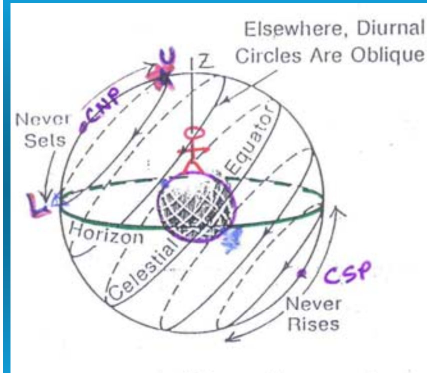
- Each star crosses the observer's meridian twice each day – these two points are known as transit of the star

Star at **Upper Transit** – Altitude maximum
Star at **Lower Transit** – Altitude minimum (cannot be seen if below horizon)

Circumpolar Star: Star that is always above horizon and thus never sets

Required Condition for circumpolar star:
Declination (δ) should be greater than co-latitude ($90°-\theta$) of the place… PROVE

Observer at Northern Hemisphere



Problem 1: 2D Problem on Transit

Altitude of a circumpolar star at upper transit is 80° and south of zenith. At lower transit, it is 10° and north of zenith. Determine latitude (θ) of observer and declination of star (δ).

Draw observer's meridian, start with horizon NS, zenith first, … then transits…then equator N′S′

NL = α_L = 10°

SU = α_U = 80°

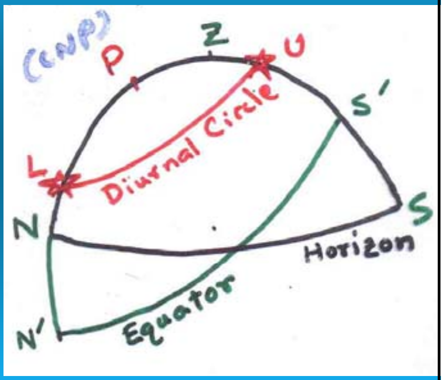
Note that PL = PU (star is always equidistant from pole)

LZ = 90° - 10° = 80°

ZU = 90° - 80° = 10°

LU =

PU = PL =



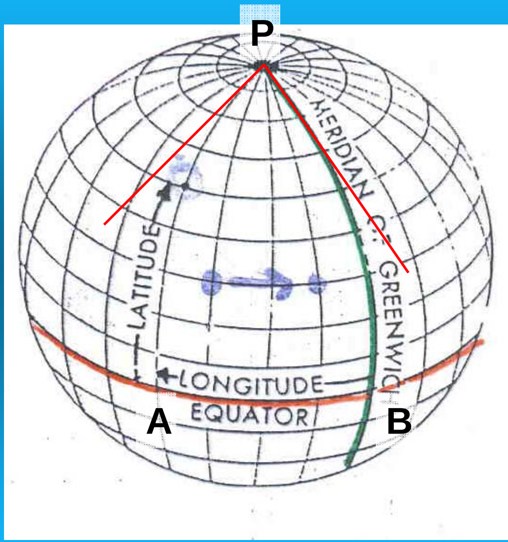
Spherical Trigonometry

Spherical Triangle. Triangle formed on the surface of sphere by parts of three great circles. The spherical triangle ABC has three sides (a,b,c) and three angles (A,B,C) all of them represented by angles.

Spherical side: The three arcs forming the spherical triangle. Each arc is represented by the angle formed by it at the centre.

Spherical angle: The angle between the tangents at each corner point of the spherical triangle

Spherical Triangle



PAB is a Spherical Triangle. The spherical triangle ABC has three sides (p,a,b) and three angles (P,A,B).

Problem 2: Distance Problem on Earth

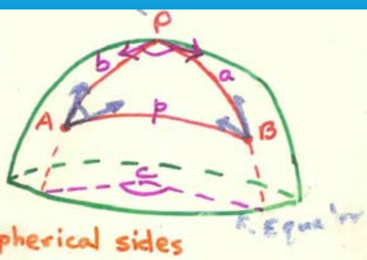
Shortest distance between A (ϕ=120°W, θ=23.5°N) and B (ϕ=70°E, θ=23.5°N).

Shortest distance will be along a great circle AB.

PAB is a spherical triangle

b = Arc AP = $90° - 23.5° = 66.5°$

a = Arc BP = $90° - 23.5° = 66.5°$ } Spherical sides



Spherical angle at P = Difference in longitudes

$P = 360° - (120° + 70°) = 170°$ ($\leq 180°$)

$\cos p = \cos P \sin a \sin b + \cos a \cos b = -0.669$

$\therefore p = 131.99°$

Problem 2 (contd.)

Considering radius of earth = 6370 km

Arc AB = $\frac{131.99}{360} \times 2\pi \times 6370$ = 14674 km. Ans

Direction of AB from A:

$\tan\frac{A+B}{2} = \frac{\cos\frac{a-b}{2}}{\cos\frac{a+b}{2}} \cot\frac{P}{2} = \frac{\cos 0^\circ}{\cos 66.5^\circ} \cot 85^\circ = 0.22$

$\therefore \frac{A+B}{2} = 12.41^\circ$

$\tan\frac{A-B}{2} = \frac{\sin\frac{a-b}{2}}{\sin\frac{a+b}{2}} \cot\frac{P}{2} = 0$

$\therefore A = B = 12.41^\circ$

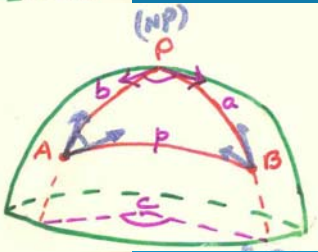
Direction of B from A = N12.41°E

Direction of A from B = N12.41°W

Distance along parallel of latitude:

A→B = (170×60) cos 23.5° = 9354 n.m. = 17339 km

(compare)



Problem 3: Azimuth Problem

Prob. At Long. = 15°W, Lat. = 53°30'N, Altitude (after all corrections) of sun at 2:12 pm is 22°31'48.5". At G.M.N. δ_{SUN} = 3°25'S decreasing at rate of 1'/hr. W.C.B. at observation = 226°50'. Determine (i) Azimuth of sun at observati (ii) Angle between True North & Magnetic North

G.M.T. at time of observation

= 14h12m + 1h = 15h 12m

which is 3.2 h after G.M.N.

$\therefore \delta_{SUN} = 3^\circ 25' - 1' \times 3.2$

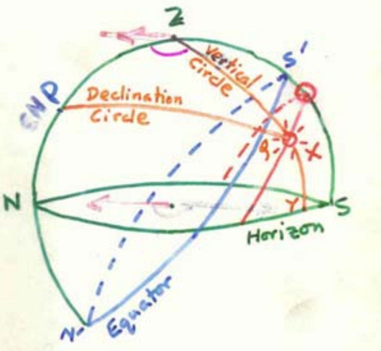
$= 3^\circ 21' 48''$S (QX)

At observation, sun is at X

Given: ZS' = 53°30'

XY = α_{SUN} = 22.53°

Spherical Triangle PZX,



Problem 3: Azimuth Problem (contd.)

Spherical Triangle PZX,

$PZ = 90^\circ - 53.5^\circ = 36.5^\circ$

$ZX = 90^\circ - 22.53^\circ = 67.47^\circ$

$PX = 90^\circ + 3.36^\circ = 93.36^\circ$

$$\cos Z = \frac{\cos PX - \cos PZ \cos ZX}{\sin PZ \cdot \sin ZX} = -0.667$$

$\therefore Z = 131.86^\circ = 131^\circ 51' 27.1''$

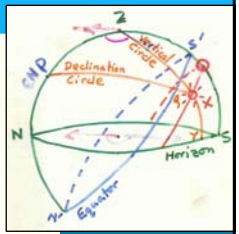
Azimuth of SUN = $131^\circ 51' 27.1''$ west → with respect to true north

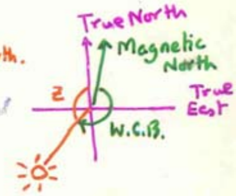
$131^\circ 51' 27.1'' + 226^\circ 50' = 358^\circ 41' 27.1''$ ($< 360^\circ$)

W.C.B. ← measured w.r.t. magnetic north.

∴ Angle Magnetic North makes with True North $= 360^\circ - 358^\circ 41' 27.1''$

$= 1^\circ 18' 32.9''$ E





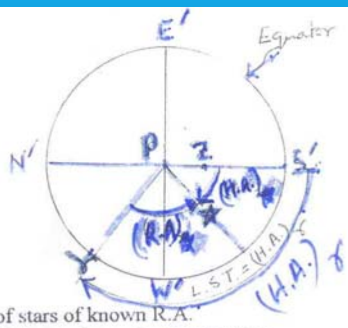
TIME

Measurement of time is based on observations of celestial bodies which appear to revolve round the earth due to earths rotation about its own axis

1. SIDEREAL TIME

- ***Sidereal Day:*** Time interval between two successive upper transits of 1st point of Aries (♈).
- ***Local Sidereal Time*** (L.S.T.) = $(H.A.)_{♈}$ = $(R.A.)_{☆} + (H.A.)_{☆}$

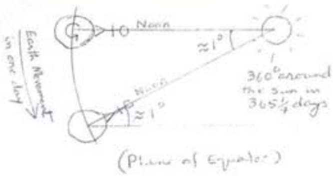
In Northern Hemisphere, upper transit of star will be between CNP and S′(Explain why?)
At upper transit, $(H.A.)_{☆} = 0$
Hence, L.S.T. = $(R.A.)_{☆}$ when star is at its upper transit



- Sidereal time is, thus, determined by observing transits of stars of known R.A.
- Sidereal time is continuously monitored in astronomical laboratories and maintained by sidereal clocks which run about 4 min. faster than ordinary clock in a day.
- This time is used for checking standard time clocks.
- Although sidereal time is suited for certain activities in the observatories, it is not useful for civil use.
  It is convenient to use the sun as the time reference, since our daily affairs are governed by the sun.

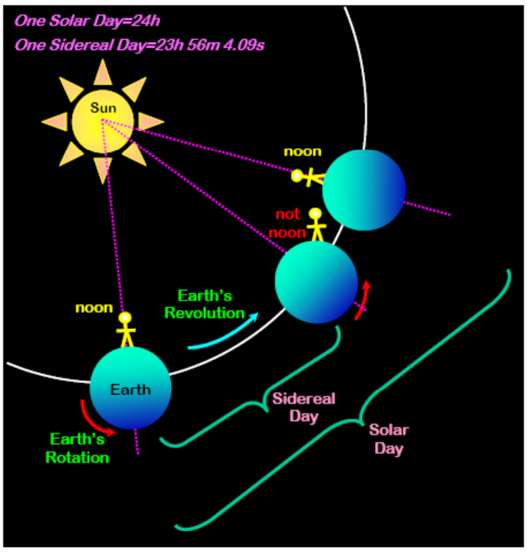
2. APPARENT SOLAR TIME

- ***Apparent Solar Day:*** Time interval between two successive lower transits of the sun.
  A.S.T. = $(H.A.)_{SUN}$ + 12 hr = 0 hr at midnight (lower transit)
  = 12 hr at noon (upper transit)
- Apparent solar time is based on observation of sun's centre and may be measured by SUNDIAL
- Apparent Solar Day > Sidereal Day
  by about 4 minutes.
  Reason: Earth has to rotate an additional angle (close to 1°) from one noon to reach the next day's noon after one complete rotation. This is due to the movement of earth around the sun. (see figure)



- Disadvantage of apparent solar time:
  Apparent solar days are not of constant length due to two reasons:
  (1) Variable speed of earth in its orbit around the sun, depending on distance between earth and sun.
  (2) Inclination of the apparent sun's path (ecliptic) to the equator. The projection of the apparent movement of sun with respect to earth on the plane of equator is consequently non-uniform.
  In other words, the rate of change of right ascension (R.A.) of sun is nonuniform due to the above two reasons.
  Note that the sun does not move, but the earth moves round the sun in a year.
  To the observer on the earth, it is like an apparent motion of sun around the earth.
- The difference in duration of the apparent solar day and the more nearly constant sidereal day depends on the daily change of the sun's right ascension.

Apparent Solar Day vs. Sidereal Day



3. MEAN SOLAR TIME

- ***Mean Solar Day:*** Time interval between two successive lower transits of **mean sun**.
- ***Mean Sun:*** Imaginary sun considered to move at a uniform rate along the equator (i.e. rate of change of R.A. of mean sun is uniform), completing its circuit (orbit) in the same period of time as required by the real sun to complete its apparent circuit.
- Mean Solar Day = Average of all Apparent Solar Days of the year.
- Mean solar days are equal. Hence clock running at a fixed speed may be used. Standard clock time is based on mean solar time.
- ~~A.S.T. = (H.A.) of mean sun + 12 hr~~ = 0 hr (lower transit of mean sun)
  = 12 hr (upper transit of mean sun)
- Note that 0 hr (midnight) and 12 hr (noon) of mean solar time generally do not agree with the midnight and noon of the apparent solar time (real sun).
- A.M.: 0 hr to 12 hr P.M.: 12 hr to 24 hr

STANDARD TIME & E.T.

- **Greenwich Mean Time (G.M.T.)**
  = Mean Solar Time at Greenwich Meridian (Meridian through Greenwich, England)

- **Local Mean Time (L.M.T.)** at another Meridian
  = G.M.T. - West Longitude
  = G.M.T. + East Longitude

Standard Meridian:

A particular meridian selected to represent the same mean time for the whole or part of a country. Such a standard meridian lies an exact multiple of half hour (7.5°) from Greenwich Meridian. One country may have one or more standard meridians. Standard Meridian for Bangladesh is 90°E.

Standard Time:

Mean Solar Time at the Standard Meridian. This time is maintained throughout the whole or part of a country. Standard clocks that we use maintain this time.

EQUATION OF TIME (E.T.):

- E.T. = A.S.T. - M.S.T.
- E.T. is continuously varying and is the result of nonuniform apparent solar day throughout the year.
- E.T. varies from -14m 20s (Feb.12) to + 16m 23 s (Nov.4) and is zero four times in a year.

Nomenclature used for Time

1st Letter:

L=Local, G=Greenwich

2nd Letter:

S=Sidereal, A=Apparent (solar), M=Mean (solar)

3rd Letter:

T=Time, N=Noon, M=Midnight

L.S.T: Local Sidereal Time. (Sidereal time)

L.A.T: Local Apparent Time. (Apparent solar time)

G.M.T: Greenwich Mean Time (Mean solar time)

G.M.M: Greenwich Mean Midnight (Mean solar time)

EQUATION OF TIME (Contd.)

Equation of Time.

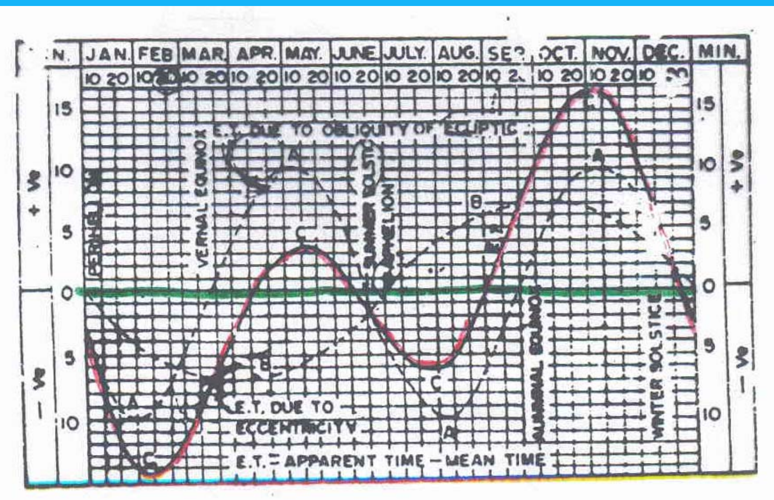


Fig. 1·33. The equation of time : The correction to be added to the mean time to obtain apparent time.

EQUATION OF TIME (Contd.)

(i) Effect of Obliquity of Ecliptic on E.T.

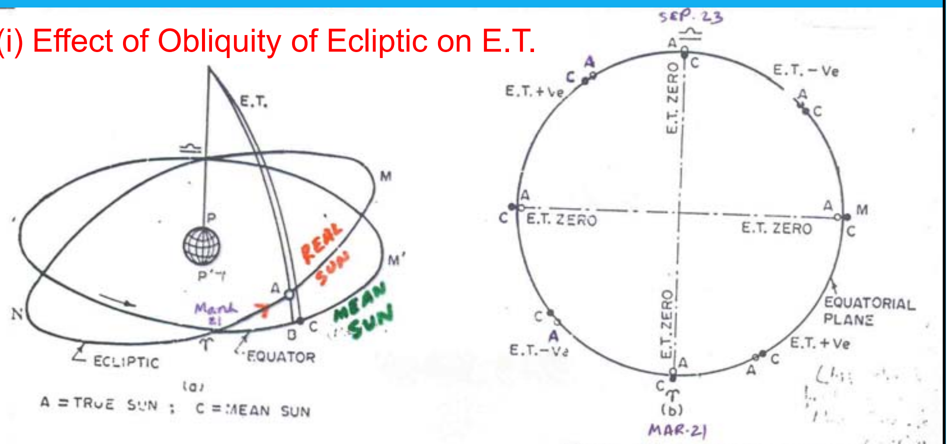


Fig. 1·31. Effect of obliquity of the ecliptic.

EQUATION OF TIME (Contd.)

(ii) Effect of Ellipticity of Orbit on E.T.

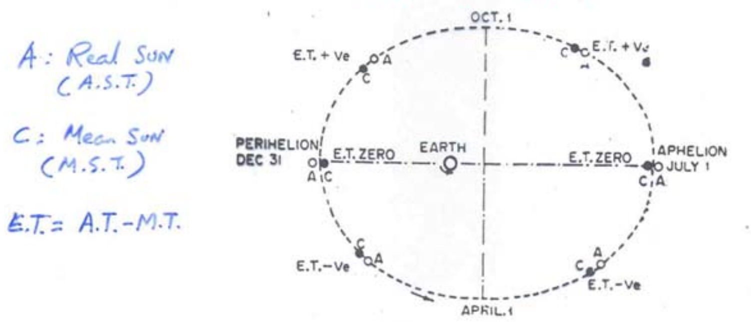


Fig. 1·32. Effect of ellipticity of the orbit.

Problem 4: Equation of Time

L.M.T. = 10h20m30s at ϕ=60°18′E. Equation of time at G.M.N. is 5m4.35s decreasing at a rate of 0.32s/h. L.A.T.=?

60°18′E = 4h 1m 12s E

G.M.T. = L.M.T. − East Long. = 10h20m30s − 4h1m12s = 6h19m18s

Time before G.M.N. (Greenwich Mean Noon) = 12h − 6h19m18s = 5h40m42s = 5.678h

E.T. at G.M.N = 5m 4.35s

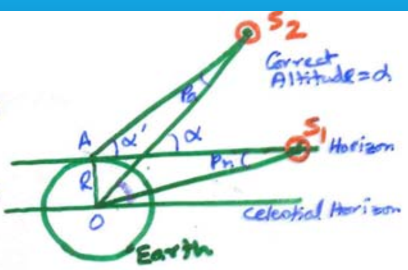
∴ E.T. at instant of observation = 5m4.35s + 0.32×5.678 s
= 5m 6.17s

L.A.T. = L.M.T. + E.T.
= 10h 20m 30s + 5m 6.17s
= 10h 25m 36.17s

Astronomical Corrections (applicable to Altitude readings)

(1) Correction for Parallax

Due to difference in direction of a heavenly body as seen from earth's centre 2 from observation point.



$P_h \approx \sin P_h = \frac{R}{OS_1}$ (small P_h)

ΔOAS_2, $\frac{R}{\sin p_a} = \frac{OS_2}{\sin(90°+\alpha')}$

$p_a \approx \sin p_a = \frac{R}{OS_2}\cos\alpha' = P_h\cos\alpha'$, where average value of $P_h = 8.8''$ (small p_a)

$\alpha = \alpha' + p_a$ (Additive correction)

Parallax error negligible for distant stars, important for sun.

Astronomical Corrections (Contd.) (applicable to Altitude readings)

(2) Correction for Refraction

(due to refraction of light ray in earth's atmosphere)

Refraction depends on:

(i) Temperature (ii) Air density

(iii) Barometric Pressure (iv) Altitude

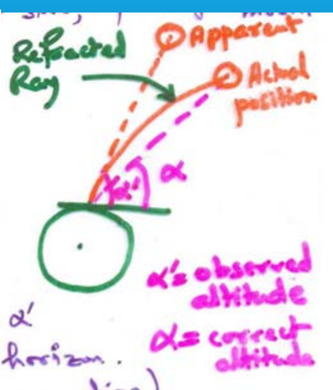
Approx. correction (Pr) = $58'' \cot\alpha'$

at pressure 29.6″ of Hg & 50°F

This correction uncertain for low α'

Correction = 33′ when on horizon.

$\alpha = \alpha' - P_r$ (subtractive correction)



Refraction error applicable for all celestial objects.

Astronomical Corrections (Contd.) (applicable to Altitude readings)

(3) Correction for Semi-Diameter of Sun

Readings are taken at edge of Sun, not at centre of Sun.
For altitude readings are taken at lower edge or upper edge of Sun

(4) Correction for Dip of Horizon (using Sextant)

Altitude readings from ships in the sea are taken by this optical instrument called Sextant with respect to the apparent horizon instead of the horizontal plane. There is a vertical angle between the horizontal plane and the apparent horizon. This instrument is still being used, I recently saw it (probably first time in my life) during a visit to a modern Navy ship.