Pressure Containers; Riveted and Welded Joints

4-1. Introduction. Two topics are discussed in this chapter. First, a method is demonstrated for finding the forces caused by fluid pressure in a closed container or vessel. After the force is found the unit stress in the wall of the vessel is found by using $S = P/A$.

Second, the riveted, bolted, and welded joints used in making tanks, boilers, pipe, and many types of structures are described, and the stresses in these joints are considered.

The two topics are closely associated because it is necessary to determine the rupturing forces in a boiler, for instance, in order to determine the kind of riveted joints or welded joints needed to join the plates without exceeding allowable stresses.

STRESSES CAUSED BY INTERNAL PRESSURE

4-2. Rupturing Forces in Pressure Containers. The pressure of a liquid or of any confined gas acts normally to the surface of the container in which the pressure exists. This normal pressure sets up stresses in the walls of the container and tends to rupture them. The design of such a container, or the investigation of the stresses set up in one by a given unit pressure, includes two distinct steps: first, the determination of the force which tends to rupture the container along the surface or surfaces where rupture is most likely to occur; second, the determination of the stresses which result from the action of this force.

As an example consider the pressure container shown in Fig. 4-1*a*. Suppose that the unit stress on the section AB is wanted. This unit stress may be found by considering the part of the container above the plane AB as a body in equilibrium, shown in Fig. 4-1*b*. The upward force F is the resultant in a direction *perpendicular* to the plane AB of all the force exerted on the interior surface by the fluid pressure. The downward forces shown represent forces due to the tensile stress in the wall of the container and the sum of these downward forces equals SA, where S is the unit tensile stress and A is the area of the cross-section of the wall cut by the plane. Evidently $S = F/A$.

A method of determining the force F perpendicular to the plane will now be given. In Fig. 4-2 is shown the part of a pressure container above a horizontal plane through the walls of the container. In this figure one-half of the part above the plane has been removed in order to make some of the interior surface visible

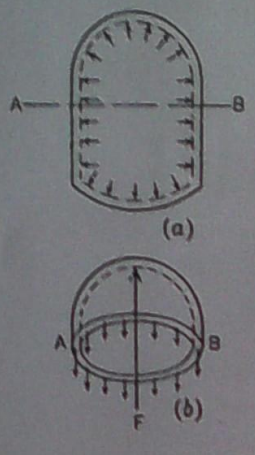
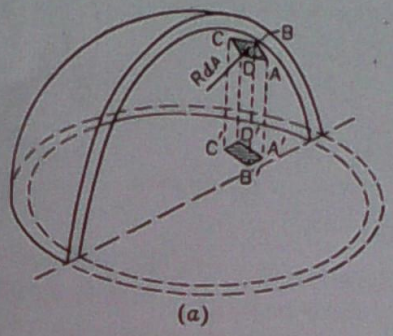


FIG. 4-1



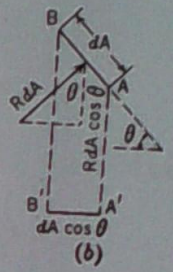


FIG. 4-2

A small rectangular area $ABCD$ on the interior surface is shown. Let the angle between the surface at this point and the horizontal plane be θ, let the area of $ABCD$ be dA sq in., and let the unit pressure of the fluid in the container be R psi. The force P exerted by the fluid on this area equals RdA and is normal to the surface, and therefore makes an angle θ with the vertical (Fig. 4-2*b*). Hence the vertical component of this force is $RdA \cos \theta$.

Now, if perpendiculars to the horizontal plane are dropped from A, B, C, and D, it will be seen that the area $A'B'C'D'$ equals $dA \cos \theta$. Hence the vertical component of P, which is $RdA \cos \theta$, is seen to be equal to R times the projection of the area dA on the horizontal plane. The total vertical force F is the sum of the vertical components of the forces on all the elementary areas comprising the interior surface above the plane. The foregoing reasoning shows that F equals the area of the part of the horizontal plane within the inner surface of the wall multiplied by R. This method of finding F may be stated as a general proposition thus:

The force F tending to rupture a pressure container along any intersecting plane is equal to the area of the part of the plane included within the interior surface multiplied by the unit pressure of the fluid.

The above reasoning applies to a vessel of any shape. Consequently for any vessel, such as those in Fig. 4-3, the resultant force exerted by the pressure on the part on one side of a plane a–a equals the unit pressure times the area of the part of the plane included within the inner surface of the vessel.

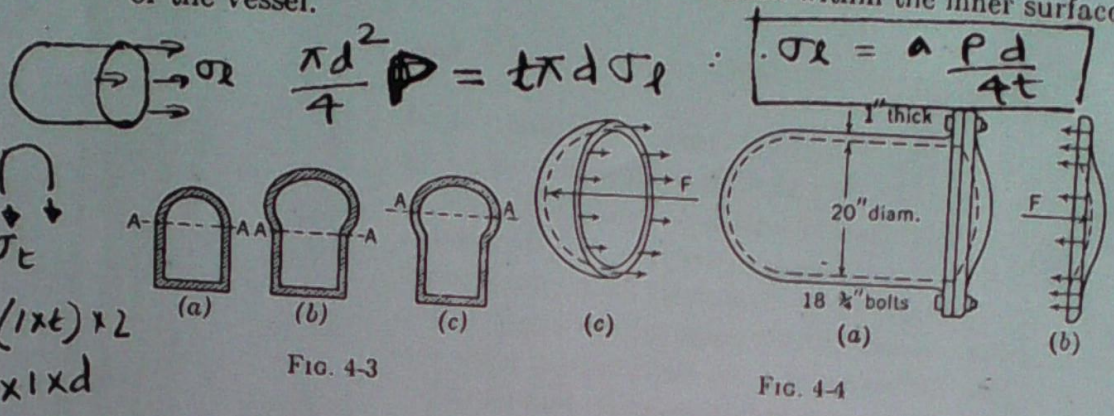


FIG. 4-3

FIG. 4-4

4-3. Unit Stresses in Pressure Vessels. Examples showing applications of the principles of Art. 4-2 will now be given:

Example 1. A cast-iron cylinder with a bolted cover is shown in Fig. 4-4*a*. The cover is attached with 18 bolts, $\frac{3}{4}$ in. in diameter and equally spaced. Calculate the maximum unit tensile stress in the bolts when the pressure of the gas in the cylinder is 80 psi.

Solution: The cover is shown as a body in equilibrium in Fig. 4-4*b*. The force $F = 100\pi \times 80 = 25{,}200$ lb. The area of one $\frac{3}{4}$-in. bolt at the root of the thread is found in Appendix C to be 0.302 sq in. Hence $S = F/A = 25{,}200/18 \times 0.302 = 4{,}630$ psi.

Example 2. Calculate the unit stress that occurs on any cross-section of the cylinder shown in Fig. 4-4*a*. The unit stress on a transverse section perpendicular to the longitudinal axis of the cylinder is called "longitudinal stress."

Solution: Pass an imaginary transverse plane A–A cutting the cylinder into two parts and show one of the two parts as a body in equilibrium, as in Fig. 4-4*c*. The forces due to the tensile unit stress on the cut section of the wall of the cylinder must balance the force F. The area of the cut section of the wall is equal to the thickness of the wall times the mean circumference. Hence

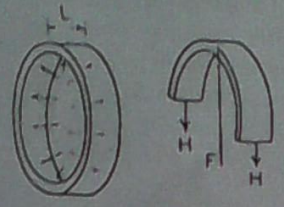
$$S = \frac{F}{A} = \frac{100\pi \times 80}{21\pi \times 1}$$

$$S = 8{,}000/21 = 381 \text{ psi}$$

Note that this stress would also be the unit stress in a sphere with an inside diameter of 20 in. and walls 1 in. thick. The method of solution for the sphere would differ only in that the transverse plane would cut the sphere into two equal parts, one of which would be taken as a body in equilibrium. The force F is the same as for the cylinder.

Example 3. Calculate the circumferential stress or "hoop tension" in the cast-iron cylinder of Example 1.

Solution: Imagine two transverse planes a short distance L apart and perpendicular to the axis of the cylinder. Between these two planes is a ring or "hoop" shown in Fig. 4-5*a*. Against the inner surface of this ring are normal forces due to the gas pressure in the cylinder. Next the hoop is cut into two equal parts by a horizontal plane, and the upper half is shown in Fig. 4-5*b*. This is a body in equilibrium with the vertical force F (the resultant of the gas pressure on the inner surface) and two forces H, due to the circumferential stress in the wall of the cylinder. Equilibrium requires that



(*a*) (*b*)

Fig. 4-5

$$2H = F$$

whence

$$2 \times 1 \times L \times S = 20L \times 80$$

from which

$$S = 1{,}600/2 = 800 \text{ psi}$$

Note that the longitudinal unit stress calculated in Example 2 is somewhat less than half the circumferential stress. For cylinders with thinner walls in proportion to the inside diameter, the longitudinal stress is more nearly equal to half the circumferential stress. The statement is often made that in "thin cylinders" the longitudinal stress is one-half the circumferential stress, and this is nearly true. For instance, if the wall thickness is one-fiftieth of the inside diameter, the ratio of longitudinal stress to circumferential stress is 0.49.

A formula for longitudinal stress in a thin-walled cylinder will now be derived. Let Fig. 4-4*c* be the end of a thin-walled cylinder with inside diameter of D in. and wall thickness of t in. The area of the metal cut by the cross-section equals the mean circumference times the thickness, or $\pi (D + t) t$; but, if t is small in comparison to D, a close approximation will be πDt. Equating the force F due to the internal pressure of R psi with the unit stress in the shell times the cut area,

$$(\pi/4)D^2R = \pi DtS \qquad \sigma_l = \frac{PD}{4t}$$

whence

$$S = RD/4t$$

Note that this formula also gives a value for the stress in a thin spherical shell with the same degree of approximation.

A formula for the "hoop tension" or circumferential stress in a thin-walled cylinder is derived as follows. Let Fig. 4-5*a* represent a ring cut from a thin-walled tank with internal fluid pressure of R psi. Let L be 1 in. Now consider half this ring, as shown in Fig. 4-5*b*, as a body in equilibrium. If t is the thickness of the shell, the force H equals $S \times 1 \times t = St$. The force $F = RD$. Equating the upward and downward forces, $2St = RD$, from which

$$S = RD/2t \qquad \sigma_t = \frac{PD}{2t} = 2 \cdot \frac{PD}{4t} = 2\sigma_l$$

Note that this is exactly twice the value of the longitudinal stress as given by the approximate formula derived just above.

Since the plane dividing the ring into two half rings (Fig. 4-5*b*) may cut the ring at any two opposite points, it follows that the total tension is the same at all cross-sections of the ring. If for any reason the area of the ring is not the same for all cross-sections, the maximum unit stress will occur at the cross-sections where the area is least.

Tanks and pipes are sometimes made of wooden staves held together by hoops. The construction is somewhat like that of a wooden barrel. The total stress on one hoop may be found much as the force H is found. The maximum unit stress is then determined by dividing the total stress by the minimum cross-section of the hoop. The distance L between the two transverse planes should be taken equal to the distance between the hoops. This same method applies to other types of fastenings which occur at intervals along a pipe or tank.

PROBLEMS

4-1. A fire extinguisher has a copper tank holding 2.5 gal. The inside diameter is 7 in., and the thickness of the shell of the tank is 0.12 in. The extinguisher was tested with a water pressure of 350 psi. What stress did this pressure cause in the shell of the tank? *Ans.* $S = 10{,}200$ psi.

4-2. A "blind flange" or cover is used to close the end of a 14-in. (outside diameter, see Table VIII) steam line which is subjected to a pressure of 600 psi at a temperature of 750°F. The "American standard" for this service requires that the flange be held on with twenty $1\frac{1}{8}$-in. alloy steel bolts. (*a*) What is the maximum stress in each bolt? (*b*) On this same pipe and flange a hydraulic (non-shock) pressure of 1,100 psi at ordinary temperature is permitted. What is the maximum bolt stress?

4-3. Specifications of the American Water Works Association provide that a 30-in.-diameter Class A (wall thickness 1.15 in.) cast-iron pipe must withstand a hydrostatic pressure of 200 psi. What circumferential unit stress does this pressure cause?

4-4. The inside diameter of a wood-stave pipe is 66 in. (Fig. 4-6). Hoops are steel rods 1 in. in diameter spaced 6 in. center to center. The ends of each rod are threaded so that the hoop can be tightened by turning a nut. What is the maximum unit stress in the hoop when the water pressure in the pipe is 50 psi? *Ans.* $S = 17{,}900$ psi.

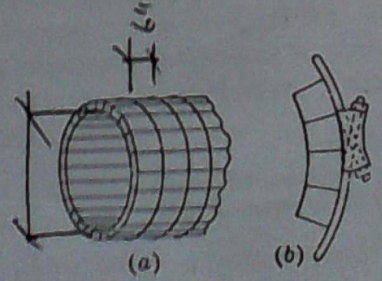


FIG. 4-6

4-5. A rectangular tank is 20 in. square by 40 in. long (interior dimensions). It is cast in two sections, as shown in Fig. 4-7. The two halves are bolted together with ten $1\frac{1}{2}$-in. bolts having an ultimate strength of 65,000 psi. Find the factor of

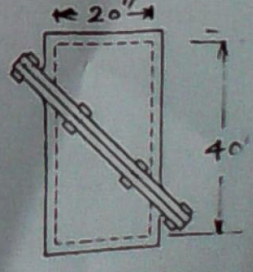


FIG. 4-7

pressure in the tank is 210 psi. Is there any shearing

4-6. An air chamber for a pump is shown in Fig. 4-8. For a pressure of 240 psi calculate the number of $\frac{7}{8}$-in.-diameter bolts required at A and also at B. Stress is not to exceed 6,000 psi. *Ans.* 10 bolts at A.

4-7. A small cylinder subject to high pressure is to be made by brazing a plug p into each end of a steel tube (inside diameter d of 2.00 in.) as shown in Fig. 4-9. Through one of the plugs there is an accurately drilled hole 0.60 in. in diameter. The pressure is produced by filling the tube with a liquid and forcing in the 0.60-in.-diameter plunger with a load F. (*a*) What pressure in pounds per square inch is allowable in the liquid if the allowable shearing stress in the brazing is 1,500 psi? (*b*) What is the load F to produce this pressure? (*c*) The tube is made of metal 0.20 in. thick. What is the maximum unit stress in the tube when this load is applied? (*d*) What is the longitudinal unit stress in the tube?

Ans. (*a*) R = 3,600 psi.

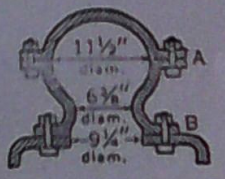


FIG. 4-8

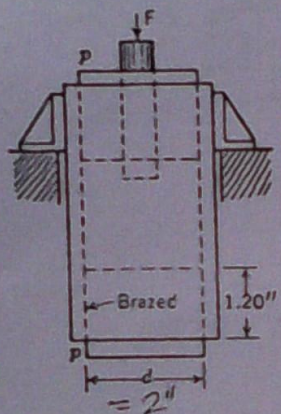


FIG. 4-9

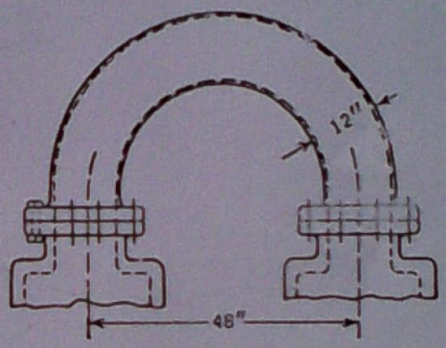


FIG. 4-10

4-8. A bent steam pipe connecting a valve chamber and a cylinder is shown in Fig. 4-10. The pipe is 12 in. in outside diameter and 11 in. in inside diameter. Steam pressure is 450 psi. Determine the total number of bolts required to attach the bent pipe to the two chambers. Bolts are $1\frac{1}{4}$ in. in diameter, and allowable stress in the bolts is 5,000 psi.

4-9. A water tank made of wood staves has an inside diameter of 12 ft and is 18 ft high. Hoops are flat steel bars 2 in. by $\frac{3}{8}$ in., spaced 10 in. center to center. (*a*) What is the unit stress in a hoop 10 in. above the bottom of the tank when the tank is full? (*b*) If instead of fresh water the tank is to hold brine (specific gravity = 1.20), to what should the hoop spacing be reduced for retention of the same factor of safety?

4-10. A welded steel water pipe used as a "siphon" in the Owyhee reclamation project in eastern Oregon has a diameter of 9 ft and is made of $\frac{13}{16}$-in. plate. After fabrication this pipe was tested under a water pressure of 220 psi. What circumferential stress was developed? *Ans.* S = 14,600 psi.

4-11. The Outardes hydroelectric project in Canada includes what is believed to be the largest wooden-stave pipe so far constructed. (See *Civil Engineering*, December, 1937.) This pipe has an internal diameter of 17 ft 6 in. and operates under a maximum head of 113.0 ft. The staves are held together by 1-in. steel bars, threaded. What maximum tensile unit stress does the water pressure cause in these bars where the head is 100 ft and the hoops are spaced 2.5 in. center to center?

4–4. Thick-Walled Cylinders. The relation between unit pressure and circumferential stress developed for "thin-walled" cylinders does not hold for "thick-walled" cylinders. The reasoning applied to the thin-walled cylinder gives a correct value for the *average* circumferential stress in the wall of any circular cylinder subjected to internal pressure. But if the cylinder wall is thick (in comparison with the internal radius) the *maximum* unit stress is higher than the average. If the wall thickness is 2/10 of the inner radius, the maximum stress is 10 per cent above the average stress. For greater relative thicknesses, the maximum stress increases somewhat more rapidly than the thickness.

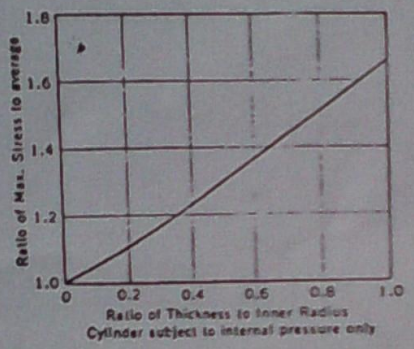


FIG. 4-11

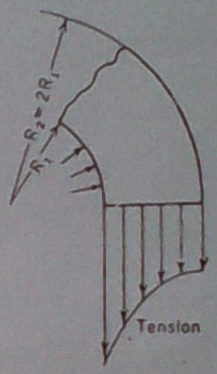


FIG. 4-12

The analysis of stresses in thick-walled cylinders will not be given in this book.[1] However, a curve giving the ratio of the maximum circumferential unit stress to the average unit stress is given in Fig. 4-11. By means of this curve thick-walled cylinders subject to internal static pressure may be designed or investigated. As an example illustrating the variation of circumferential stress, Fig. 4-12 shows the stresses in the wall of a cylinder having a thickness equal to the inner radius. For this case the curve (Fig. 4-11) indicates that the maximum stress is about 1.65 times the average stress obtained by the methods of this chapter. The curve of Fig. 4-11 is so nearly straight that generally satisfactory results can be obtained from a straight line giving a ratio of unity when the thickness is zero and 1.65 when the thickness equals the inner radius.

Example. A chrome-vanadium steel cylinder used in a chemical manufacturing plant has an inside diameter of 24 in. and an outside diameter of 32 in. When the

[1] See Seely and Smith, *Advanced Mechanics of Materials*, 2nd ed., John Wiley & Sons, 1952.