1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Design Example

Failure Investigation of A Prestressed Concrete Bridge Girder

Objective: To investigate the failure of a prestressed girder in accordance with **ACI 318-02**.

Problem: A highway overpass consists of 3 parallel continuous prestressed concrete beams. The length of the overpass structure is 292.8 ft, with a width of 47 ft (Fig. 1 and 2). Each prestressed beam had 5 strands of prestressing steel. There were 22 wires in each strand and each wire had a diameter of 0.6 in. The end of each prestressed beam was supported by a corbel, which was inclined at an angle with respect to the bearing plate (Fig. 3, 4, and 5).

Construction proceeded as planned: the beams were cast-in-place, and after the concrete hardened, they were post-tensioned. Minutes after the prestressing operation, 4 out of the 6 corbels broke (Fig. 3). The State Transportation Authority decided to determine the responsible parties involved in this failure case.

Task: You are hired to be the expert witness on the case. The following information were established:

(a) The reaction force (R) at each end of the beam right before the collapse was estimated at 275 kips.
(b) The horizontal restraint offered by the bearing (i.e., the Teflon disk) is negligible.
(c) Normal weight concrete was used with the compressive strength of f_c' = 5000 psi.
(d) Yield stress for normal reinforcement was f_y = 60 ksi.

Using the above information and the attached drawings, you are asked to assess and testify on the following questions:

(1) Was the design (Fig. 6) adequate in accordance with ACI code requirements?
(2) It was reported that the elastic shortening of the beam due to the initial prestressing was 0.9 in (Fig. 7). Check the design adequacy for this situation.
(3) It is postulated that the workmen might have placed the Teflon disk in the wrong position initially. Together with the elastic shortening due to prestressing, the final position of the Teflon disk was as shown in Fig. 8. Check the design again using the ACI code.
(4) Based on the above information, give your opinion as to the cause(s) of the collapse. It was argued that if instead of having the corbels, the prestressed beams were cast into the piers as a whole unit (i.e., fixed ends), and then the failure would not have occurred. Do you foresee any problems with this design?

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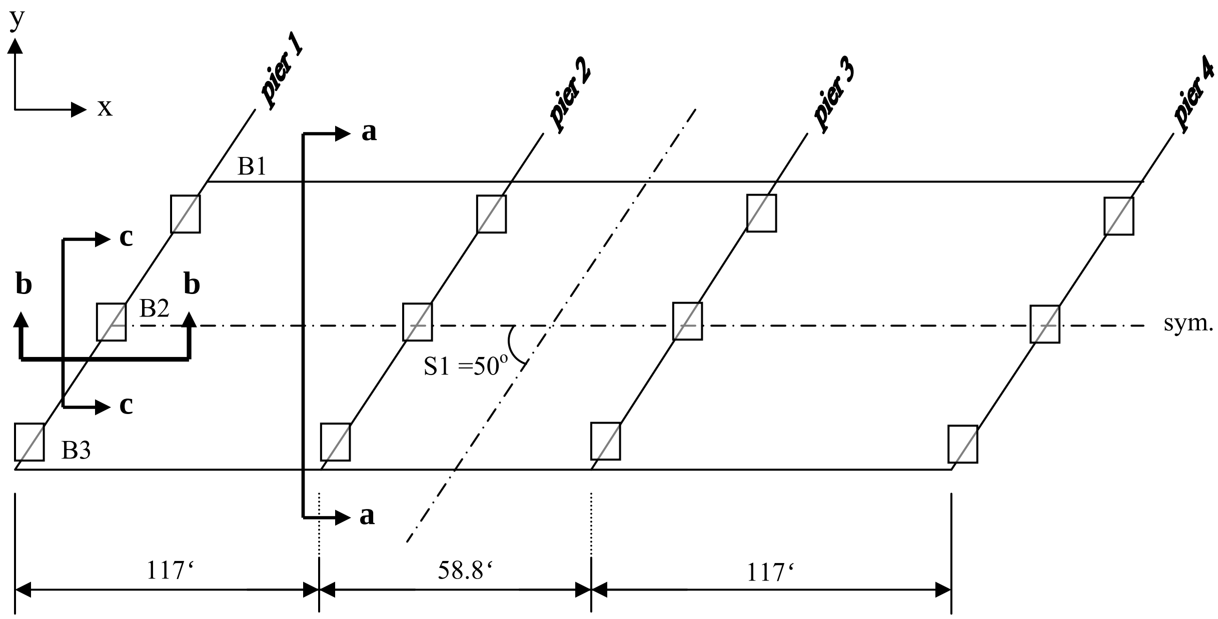


Figure 1. Plan view of overpass

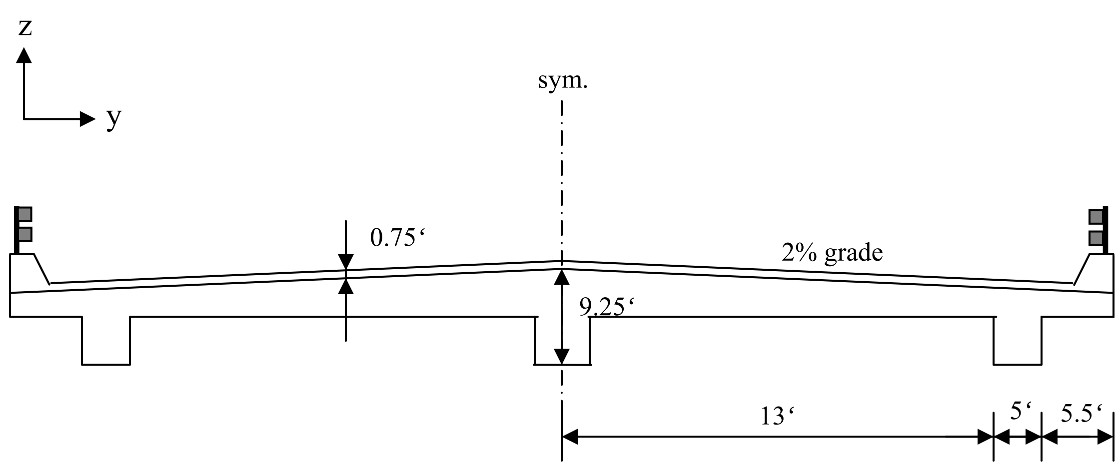


Figure 2. Cross section of overpass (section **a-a**)

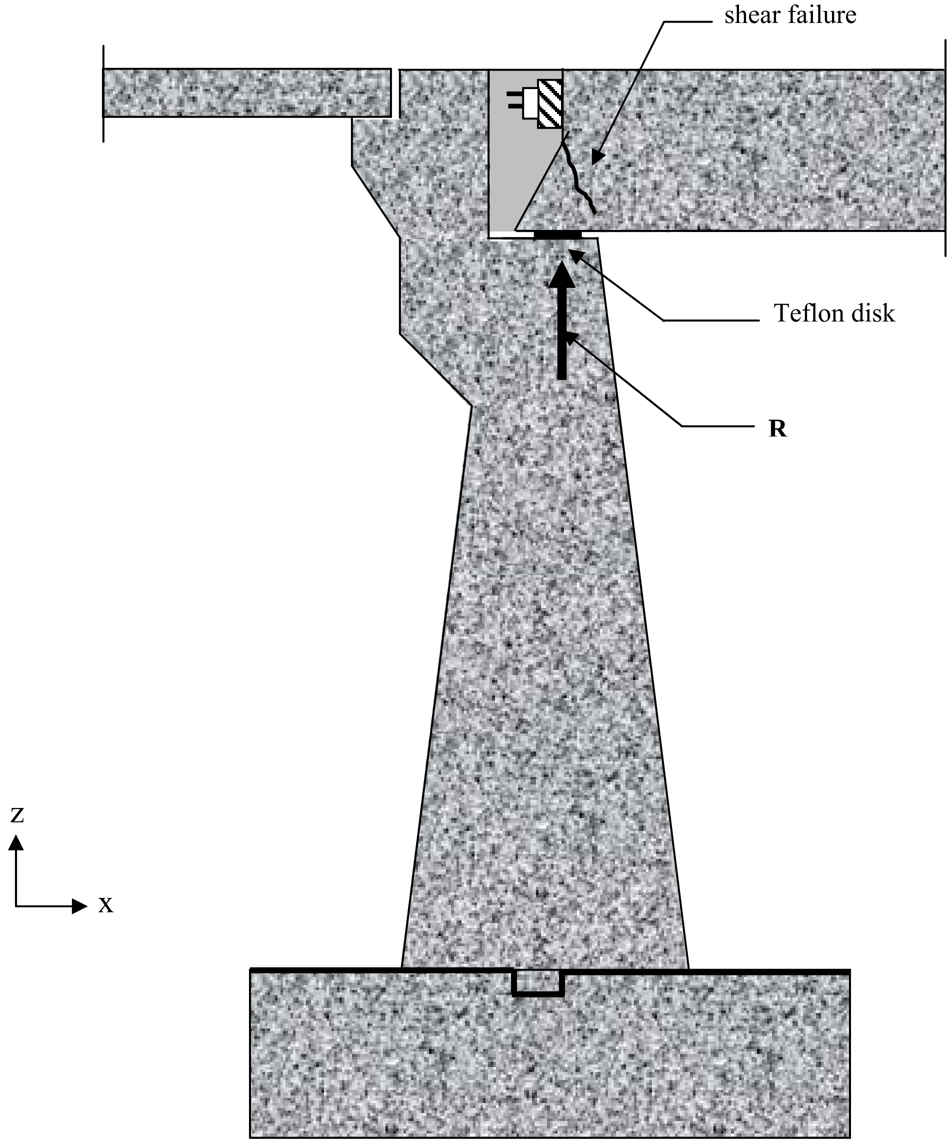


Figure 3. Location of failure (section **b-b**)

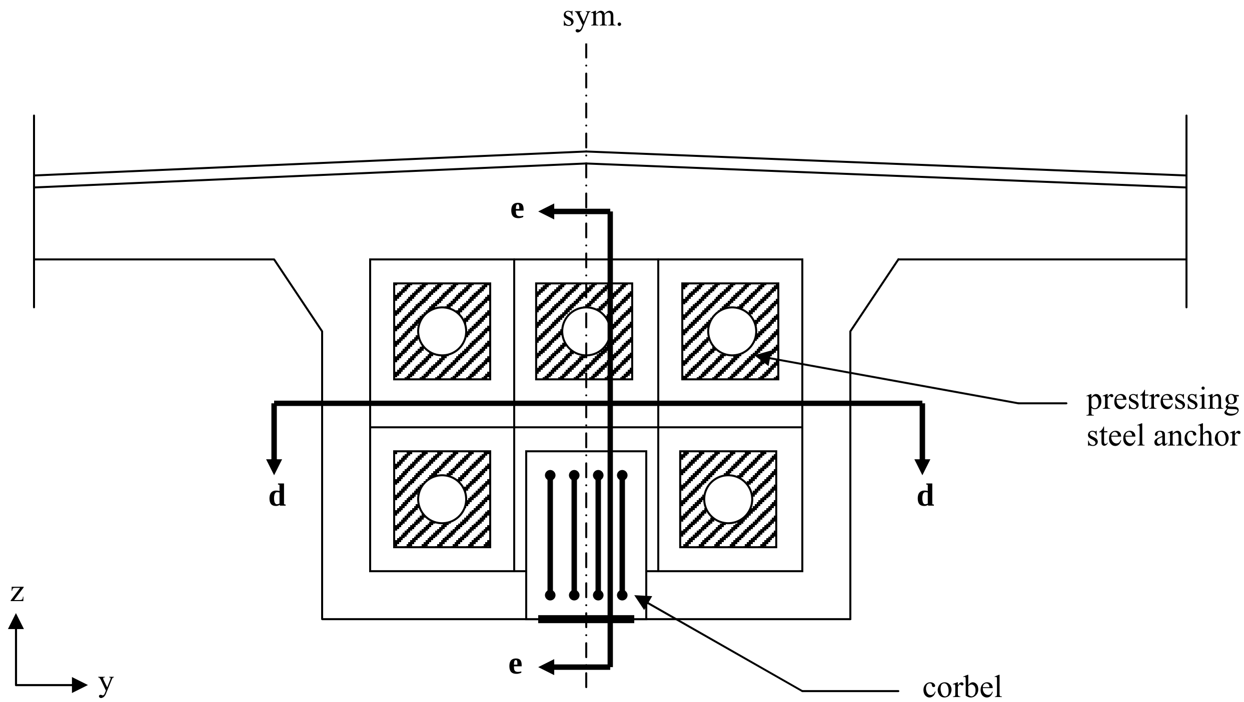


Figure 4. End zone detail for prestressed beam (section **c-c**)

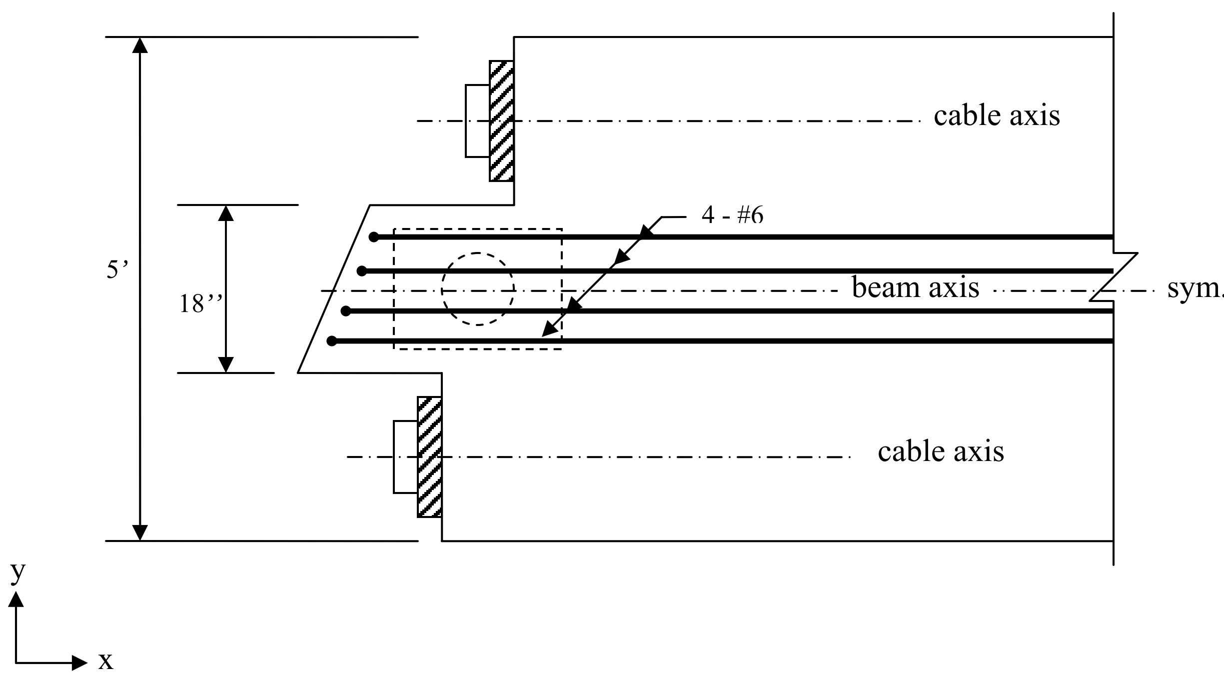


Figure 5. Plan view of end zone (section **d-d**)

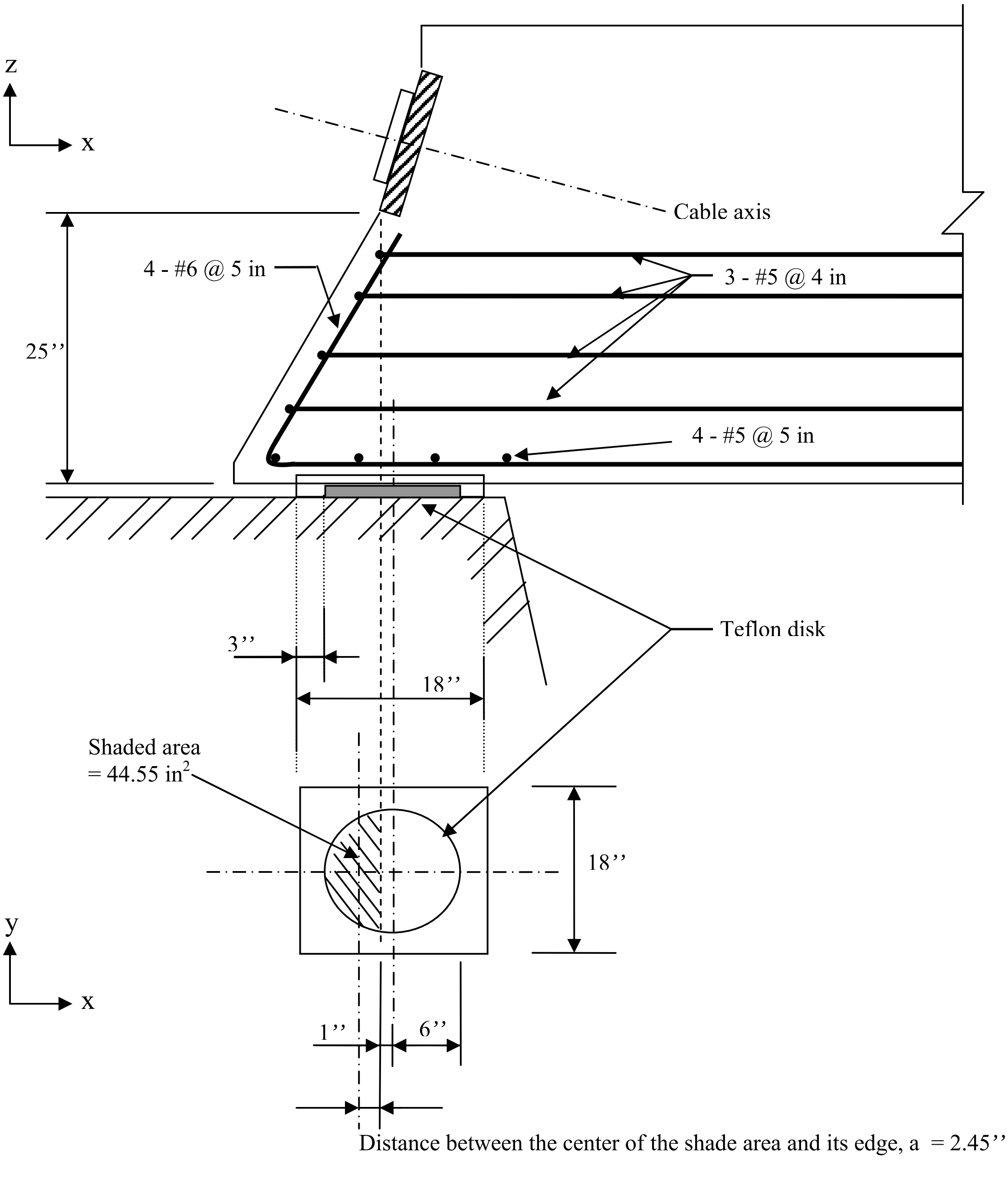


Figure 6. Elevation view of corbel (section **e-e**)
(Design Drawing)

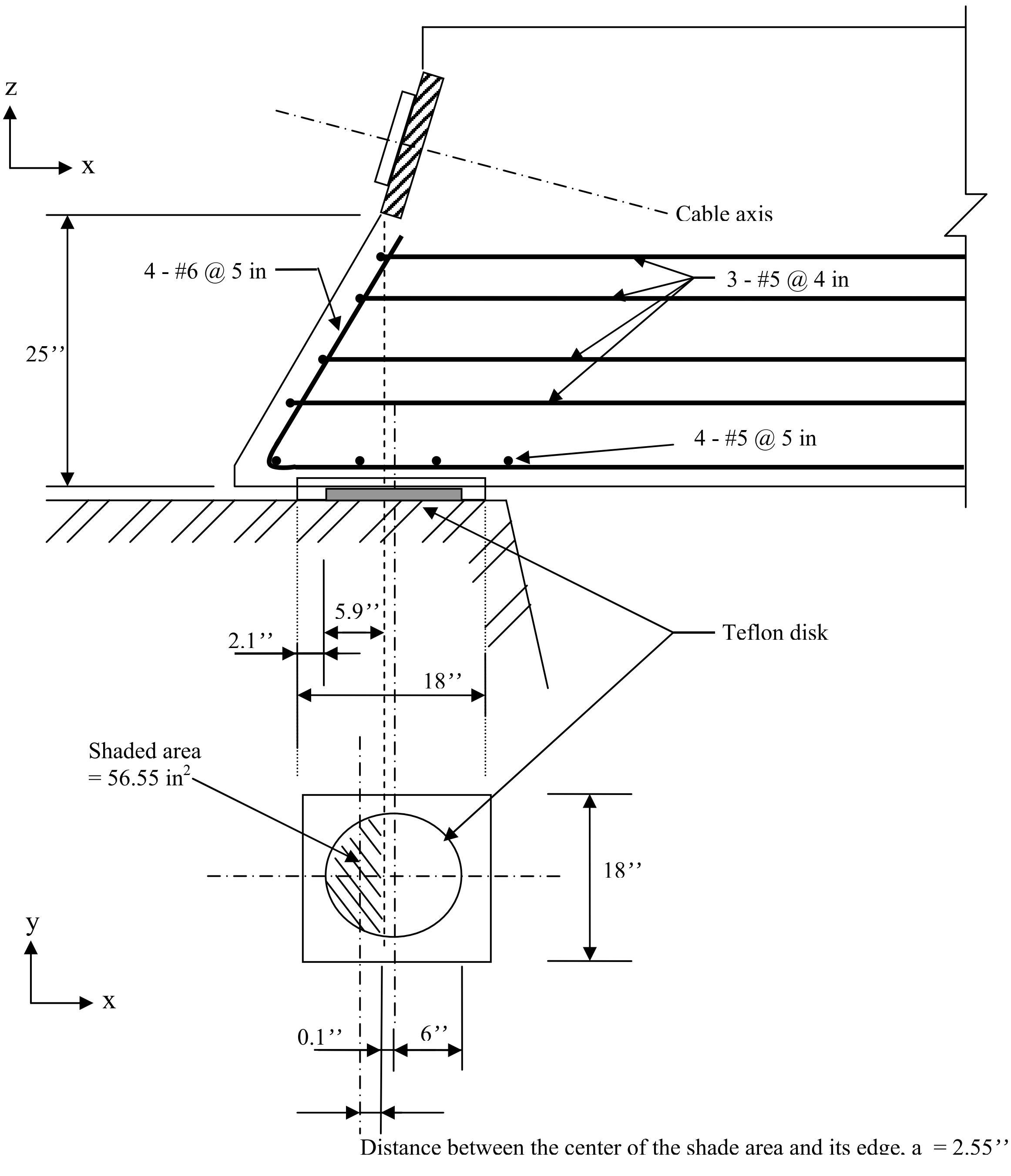


Figure 7. Elevation view of corbel (section **e-e**)
(after initial prestressing, elastic shortening at each end of the beam, ΔL)

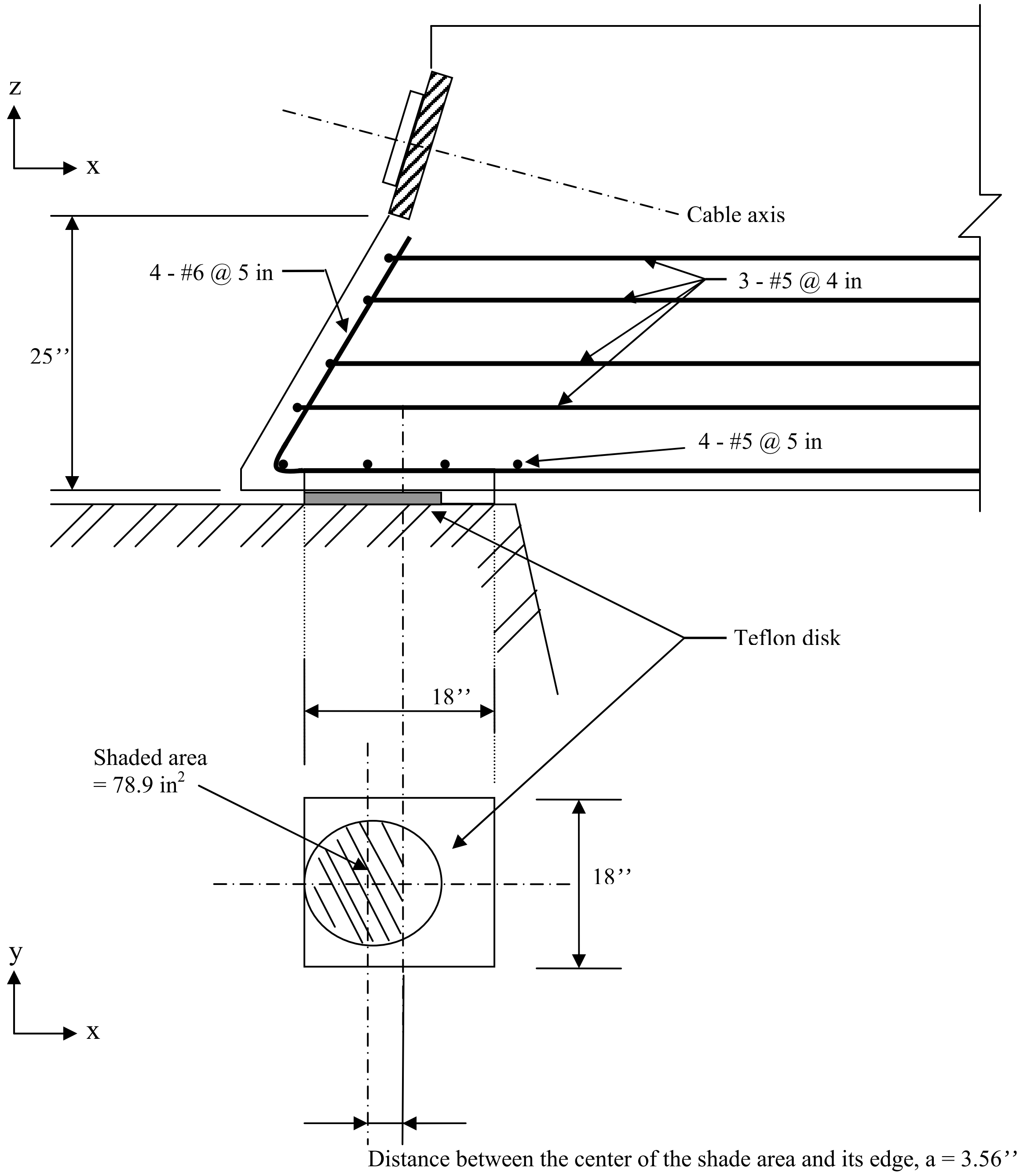


Figure 8. Elevation view of corbel (section **e-e**)
(Postulate failure configuration)

(I) Engineering Drawing: **ACI**

Load on the corbel

– reaction	$R = 275$ kips	
– factored reaction	$R_u = 1.4\,R = 385$ kips	
– area of Teflon disk	$A = \frac{\pi}{4}\cdot(12)^2 = 113.1\ \text{in}^2$	
– uniform stress on the Teflon disk	$\sigma_u = \frac{R_u}{A} = 3.4$ ksi	
– shaded area	$A' = 44.55\ \text{in}^2$	
– shear force	$V_u = \sigma_u \cdot A' = 151.5$ kips	
– tension	$N_{uc} = 0$ kips	**11.9.3.4**
– moment	$M_u = V_u \cdot a$	**11.9.3.2**

$$= 151.5 \times 2.45$$

$$= 371.2 \text{ kips-in}$$

Corbel dimension

$h = 25$ in

$d = 25 - 2 = 23$ in

$b_w = 18$ in

$$\frac{a}{d} = \frac{2.45}{23} = 0.107 < 1.0 \ \text{(O.K.)}$$

and $N_{uc} < V_u$ **11.9.1**

$d_1 = \frac{5}{10}\cdot 23 = 11.5$ in $\geq 0.5d$ (O.K.) **11.9.2**

Shear design

$V_n = \frac{V_u}{\phi} = \frac{151.5}{0.75} = 202$ kips **11.9.3.1**

since $\max V_n = 0.2\cdot f_c' \cdot b_w \cdot d$ **11.9.3.2.1**

$$= 0.2\cdot 5\cdot 18\cdot 23 = 414 \text{ kips}$$

$$\max V_n = 800\cdot b_w \cdot d$$

$$= 800\cdot 18\cdot 23 = 331.2 \text{ kips (governs)}$$

$V_n = 202 < 331.2$ kips (O.K.)

$$\mu = 1.4 \cdot \lambda = 1.4$$ **11.7.4.3**

$$V_n = A_{vf} \cdot f_y \cdot \mu$$ **11.9.3.2**

$$A_{vf} = \frac{202}{60 \cdot 1.4} = 2.4 \text{ in}^2$$ **11.7.4.1**

Flexural design

$$M_u = \phi \cdot 0.85 \cdot f_c' \cdot b_w \cdot x \cdot \left(d - \frac{x}{2} \right)$$ **10.2.10**

$$371.2 = 0.9 \cdot 0.85 \cdot 5 \cdot 18 \cdot x \cdot \left(23 - \frac{x}{2} \right)$$

setting $x = 0.3$

$$A_f = 0.85 \cdot f_c' \cdot b_w \cdot x$$
$$= 0.383 \text{ in}^2$$

Tension design

Since $N_{uc} = 0$ **11.9.3.4**

$A_n = 0$

Primary tension reinforcement

$$A_s = A_f + A_n = 0.383 \text{ in}^2$$ **11.9.3.5**

or $\frac{2}{3} A_{vf} + A_n = 1.41 \text{ in}^2$ (governs)

From the design, there are 4 - #6 bars provided.

$$(A_s)_{provided} = 4 \cdot 0.44 \text{ in}^2$$
$$= 1.76 \text{ in}^2 > 1.41 \text{ in}^2 \text{ (O.K.)}$$

Ties

$$A_n \geq 0.5 \cdot (A_s - A_n)$$ **11.9.4**
$$\geq 0.5 \cdot (1.41 - 0)$$
$$\geq 0.71 \text{ in}^2$$

From the design, over $\frac{2}{3}d = 15.33$ in , there are 3 - #5 bars provided.

$(A_n)_{provided} = 3 \times 2 \times 0.31$

$= 1.86 \text{ in}^2 > 0.71 \text{ in}^2$ (O.K.)

Reinforcement ratio

$$\rho = \frac{A_s}{b \cdot d} = \frac{1.76}{18 \cdot 23} = 0.0043 \qquad \textbf{11.9.5}$$

$$0.04 \cdot \frac{f_c'}{f_y} = 0.04 \cdot \frac{5}{60} = 0.0033 < 0.0043 \text{ (O.K.)}$$

⇒ The engineering design in Fig. 6 is adequate in accordance with ACI code 318-02.

(II) With elastic shortening

Similarly, we have

$A' = 56.55 \text{ in}^2$

$V_u = 3.4 \cdot 56.55 = 192.3$ kips

$N_{uc} = 0$

$M_u = V_u \cdot a$

$= 192.3 \cdot 2.55$

$= 490.8$ kips-in

Corbel dimension

$$\frac{a}{d} = \frac{2.55}{23} = 0.111 < 0.5 \text{ (O.K.)}$$

$$d_1 = \frac{4.1}{10} \cdot 23 = 9.43 < \frac{d}{2} \text{ (N.G.)}$$

Shear design

$$V_n = \frac{V_u}{\phi} = \frac{192.3}{0.75} = 257.6 \text{ kips} < 331.2 \text{ kips (O.K.)}$$

11.9.3.2

$$A_{vf} = \frac{257.6}{60 \times 1.4} = 3.07 \text{ in}^2$$

Flexural design

$$M_n = \phi f_c' bx\left(d - \frac{x}{2}\right)$$

$$490.8 = 0.9\times0.85\times5\times18\times x\times\left(23-\frac{x}{2}\right)$$

$\Rightarrow$ x = 0.316 in

$\Rightarrow$ $A_f = 0.40\ in^2$

Primary tension reinforcement

$$A_s = A_f + A_n = 0.40\ in^2$$

or $$A_s = \frac{2}{3}A_{vf} + A_n = 2.04\ in^2\ \text{(governs)}$$

Since $(A_s)_{provided} = 1.76\ in^2 < 2.04\ in^2$ (N.G.)

$(A_n)_{provided} = 1.86\ in^2 > \frac{A_s}{2} = \frac{2.04}{2} = 1.02\ in^2$

(O.K.)

$\Rightarrow$ With elastic shortening of 0.9 in, the given design is not adequate.

(III) With shortening and misplaced Teflon disk

Similarly, A’ = 78.9 in^2

$V_u = 3.4\times78.9 = 268.3$ kips

$N_{uc} = 0$

$M_u = V_u\times a$

$= 268.3\times3.56$

= 955 kips-in

Corbel dimension

$$\frac{a}{d} = \frac{3.56}{23} = 0.155 < 1\ \text{(O.K.)}$$

$$d_1 = \frac{2}{10}\times23 = 4.6\ in < \frac{d}{2}\ \text{(N.G.)}$$

Shear design

$$V_n = \frac{V_u}{\phi} = \frac{268.3}{0.75} = 357.7\ kips > 331.2\ kips\ \text{(N.G.)}$$

$$A_{vf} = \frac{357.7}{60\times1.4} = 4.26\ in^2\ \text{(O.K.)}$$ **11.9.3.2**

Flexural design

$$M_n = \phi f_c{}' bx\left(d - \frac{x}{2}\right)$$

$$955 = 0.9 \times 0.85 \times 5 \times 18 \times x \times \left(23 - \frac{x}{2}\right)$$

$$\Rightarrow x = 0.62 \text{ in}$$

$$\Rightarrow A_f = 0.79 \text{ in}^2$$

<u>Primary tension reinforcement</u>

$$A_s = A_f + A_n = 0.79 \text{ in}^2$$

or $$A_s = \frac{2}{3}A_{vf} + A_n = 2.84 \text{ in}^2 \text{ (governs)}$$

Since $(A_s)_{provided} = 1.76 \text{ in}^2 < 2.84 \text{ in}^2$ (N.G.)

$$(A_n)_{provided} = 1.86 \text{ in}^2 > \frac{A_s}{2} = \frac{2.84}{2} = 1.42 \text{ in}^2 \quad \text{(O.K.)}$$

$\Rightarrow$ With misplacement of Teflon disk and elastic shortening, the given design is not adequate.

(IV) There is a good chance that the failure was due to poor design or inadequate considerations on the part of engineer. Even if the Teflon disk was correctly placed, with the elastic shortening of beam and live loads, the bridge does not have much of a chance of surviving. Misplacement of the Teflon disk greatly increased the risk of failure since no information on the site supervision on the pat of the engineer was given. A probable cause of the failure could then be attributed to both the engineer and the contractor.

If the beam is cast monolithically into the pier, problems that might arise are

- Secondary stresses induced die to creep, shrinkage and elastic shortening;
- Thermal stresses created due to differential temperature effect.

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Design Example

Shear and Torsion

Objective: To examine the adequacy of given cross section based on shear and torsion capacities.

Problem: At a section of a beam, the internal forces are V_u = 45 kips, M_u = 300 kips-ft, and T_u = 120 kips-ft. The material strengths are f'_c = 4 ksi and f_y = 60 ksi. Assume that the distance from beam faces to the center of stirrups is 2 in and d is 21 in.

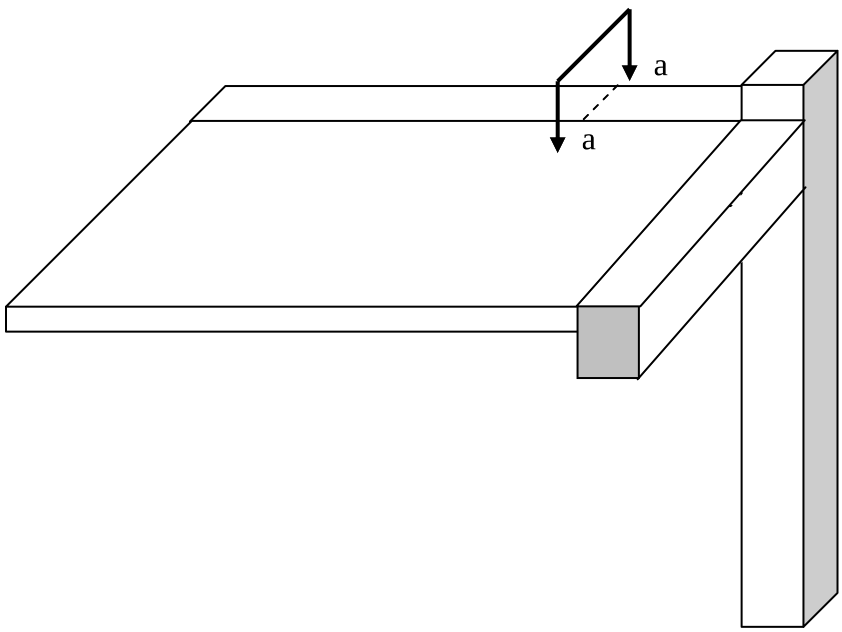


Figure 1. Plan view of overpass

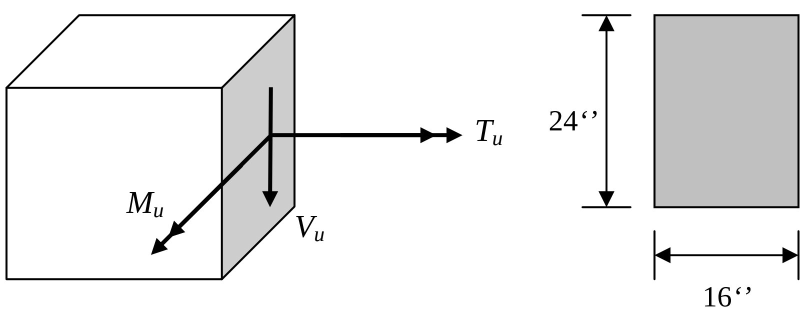


Figure 2. Cross section a-a

<u>Task</u>: Answer or accomplish the following questions and tasks based on given assumption.

- Will this given cross section (16 in wide and 24 in deep) be adequate for shear and torsion requirements? If not, what width is required?
- Assume that the depth is to be held at 24 in.
- Select the required torsion, shear, and bending reinforcement for the minimum required width (to nearest inch).
- Summarize reinforcement on a sketch of the cross section.

[Design Procedures]

1. Check maximum torsion capacity **(ACI)**

For the given section: Section/(Equation)

$$\sum x^2 y = (16)^2 (24) = 6144 \text{ in}^3$$

$$C_T = \frac{b_w d}{\sum x^2 y} = \frac{(16)(21)}{6144} = 0.05469 \text{ in}^{-1}$$

$$T_c = \frac{0.8\sqrt{f_c'} \cdot \sum x^2 y}{\sqrt{1 + \left(\frac{0.4V_u}{C_T T_u}\right)^2}} = \frac{0.8\sqrt{4000} \cdot (6144)}{\sqrt{1 + \left(\frac{0.4 \times 45}{0.05469 \times 120 \times 12}\right)^2}} = 318.88 \times 10^3 \text{ lbs-in}$$

Since $(T_s)_{\max} = 4T_c$ **11.6.1 &**

$\Rightarrow (T_u)_{\max} = \phi(T_c + T_s) = \phi(T_c + 4T_c) = 4.25T_c$ ($\phi = 0.85$ for torsion) **11.6.2.2**

$\Rightarrow (T_u)_{\max} = 1355 \times 10^3$ lbs-in

However, $(T_u)_{\text{actual}} = 120 \text{ kips-ft} = 1440 \times 10^3$ lbs-in

$(T_u)_{\text{actual}} > (T_u)_{\max} \Rightarrow$ <u>Section is NOT adequate.</u>

2. Selection of cross section dimensions

Let h = 24 in = constant.

$$(T_u)_{\text{actual}} < 4.25T_c \Rightarrow 1440 \times 10^3 \text{ lbs-in} < 4.25\frac{0.8\sqrt{4000} \cdot x^2 (24)}{1.02579}$$

Assume C_T to be about the same $\Rightarrow$ same dimension.

$\Rightarrow x > 16.92$ in

Try x =17 in,

$\sum x^2 y = (17)^2 (24) = 6936 \text{ in}^3, \; C_T = 0.05147 \text{ in}^{-1}.$

$$T_c = \frac{0.8\sqrt{4000}\cdot(6936)}{\sqrt{1+\left(\frac{0.4\times 45}{0.05147\times 120\times 12}\right)^2}} = 341\times 10^3 \text{ lbs-in}$$

$\Rightarrow (T_u)_{max} = 4.25T_c = 1449\times 10^3 \text{ lbs-in} > T_u$ **(O.K.)**

$\therefore$ Section $17'' \times 24''$ is OK. (Although predicting heavy reinforcement.)

3. Selection of stirrups

$$T_s = \frac{T_u}{\phi} - T_c = \left(\frac{1440}{0.85} - 341\right)\times 10^3 = 1353\times 10^3 \text{ lbs-in}$$

$x_1 = 17 - 2(2) = 13 \text{ in}$

$y_1 = 24 - 2(2) = 20 \text{ in}$

$$\alpha_T = 0.66 + 0.33\frac{y_1}{x_1} = 1.168$$

$$\frac{A_t}{s} = \frac{T_s}{\alpha_T x_1 y_1 f_y} = \frac{1353\times 10^3}{1.168(13)(20)(60\times 10^3)} = 0.0743 \text{ in}^2\text{/in}$$

$$\left(\frac{A_t}{s}\right)_{min} = \frac{25b_w}{f_{yv}} = \frac{(25)(17)}{60000} = 0.00708 \text{ in}^2\text{/in}$$ **(O.K.)** **11.6.5.3**

$$V_c = \frac{2\sqrt{f_c'}b_w d}{\sqrt{1+\left(2.5C_T\frac{T_u}{V_u}\right)^2}} = \frac{2\sqrt{4000}(17)(21)}{\sqrt{1+\left(2.5(0.05147)\frac{120\times 12}{45}\right)^2}} = 10657 \text{ lbs}$$

$$V_s = \frac{V_u}{\phi} - V_c = \left(\frac{45}{0.85} - 10.66\right)\times 10^3 = 42.28\times 10^3 \text{ lbs}$$

$$\frac{A_v}{s} = \frac{V_s}{f_y d} = \frac{42.28\times 10^3}{(60\times 10^3)21} = 0.0336 \text{ in}^2\text{/in}$$ **(11-15)**

Since $\left(\frac{A}{s}\right)_{stirrup} = \frac{A_t}{s} + \frac{1}{2}\frac{A_v}{s} = 0.0743 + \frac{0.0336}{2} = 0.0911 \text{ in}^2\text{/in}$

$$s_{\max} = \frac{x_1 + y_1}{4} = \frac{13+20}{4} = 8.25 \text{ in}$$

$4\sqrt{f_c^{'}}b_w d = 4\sqrt{4000}(17)(21) = 90314$ lbs > V_s **(Not applicable)** **11.5.4.3**

$s_{\max} = \frac{d}{2} = \frac{21}{2} = 10.5$ in **11.5.4.1**

$\phi\left(0.5\sqrt{f_c^{'}}\sum x^2 y\right) = 0.85(0.5)\sqrt{4000}(6936) = 186\times10^3$ lbs-in < T_u **(Applicable)**

$\left(A_v + 2A_t\right)_{\min} = \frac{50 b_w s}{f_y} = \frac{50(17)s}{60\times10^3} = 0.0142s$ **(11-23)**

$$\Rightarrow \left(A_{\text{stirrup}}\right)_{\min} = \frac{0.0142s}{2} = 0.0071s \text{ in}^2$$

Thus, choose A_{stirrup} and *s* to satisfy:

(i) $s > \frac{A}{0.0911}$ in;

(ii) $s < 8.25$ in;

(iii) $A > 0.0071s$ in^2.

Try #5 stirrups @ 3.5in, $A = 0.31$ in^2.

$s_{\min} = \frac{A}{0.0911} = 3.4$ in **(O.K.)**

$s_{\max} = 8.25$ in **(O.K.)**

$A_{\min} = 0.0071s = 0.0071(3.5) = 0.0249$ in^2 **(O.K.)**

$\Rightarrow$ #5 stirrups @ 3.5in are O.K.

4. Selection of longitudinal reinforcement

$$A_l = 2A_t\left(\frac{x_1 + y_1}{s}\right) = 2\frac{A_t}{s}(x_1 + y_1) = 2(0.0743)(13+20) = 4.9 \text{ in}^2$$

$A_{l,\min} = \frac{5\sqrt{f_c^{'}}A_{cp}}{f_{yl}} - \frac{A_t}{s} p_h \frac{f_{yv}}{f_{yl}}$ **(11-24)**

A_{cp} is the area enclosed by outside perimeter of concrete cross section, in^2, and p_h is the perimeter of centerline of outermost closed transverse torsional reinforcement, in.

Assuming $f_{yv} = f_{yl}$ (f_{yv} : yield strength of stirrups/ f_{yl} : yield strength of longitudinal steels) and substituting all known numbers ($A_{cp} = 17\times 24 = 408\ \text{in}^2,\ p_h = 2(17-4+24-4) = 66\ \text{in}$) provides $A_{l,\min} < 0$, therefore (11-24) is disregarded.

Use $\dfrac{50 b_w s}{f_y} < 2A_t$ or $\dfrac{50 b_w}{2 f_y} < \dfrac{A_t}{s}$ to find $2A_t$. **(11-23)**

$$\Rightarrow \frac{50 b_w}{2 f_y} = \frac{50(17)}{2(60\times 10^3)} = 0.0071 < \frac{A_t}{s} = 0.0743$$

Substitute for $2A_t$ in **(11-23)**.

$$A_l = \left[\frac{400x}{f_y}\left(\frac{T_u}{T_u + \dfrac{V_u}{3C_T}}\right) - 2\frac{A_t}{s}\right](x_1 + y_1)$$

$$= \left[\frac{400(17)}{60\times 10^3}\left(\frac{1440\times 10^3}{1440\times 10^3 + \dfrac{45\times 10^3}{3\times 0.05147}}\right) - 2(0.0743)\right](33) < 0$$

This leads to a negative value $\rightarrow$ disregard it!

$\Rightarrow$ <u>$A_l = 4.9\ \text{in}^2$ for torsion</u>

$$M_u = \phi\left(0.85 f_c^{'} ab\right)\left(d - \frac{a}{2}\right) \qquad (1)$$

$$A_s = \frac{0.85 f_c^{'} ab}{f_y} \qquad (2)$$

Eq.(1) provides

$$300\times 12 = 0.9(0.85)(4)a(17)\left(21 - \frac{a}{2}\right) \Rightarrow 26.01a^2 - 1092.42a + 3600 = 0$$

$$\Rightarrow a = 3.605\ \text{in}$$

Eq.(2) provides $A_s = 3.47\ \text{in}^2 \rightarrow \rho = \dfrac{3.47}{(17)(21)} = 0.0097$

$A_{\min} = \dfrac{200}{f_y} = 0.0033\ \text{in}^2 < A_s = 3.47\ \text{in}^2$ **(O.K.)**

In balanced state,

$$C_{\text{balanced}} = \frac{b(0.003)}{0.003+\varepsilon_y} = 12.6\text{ in} \Rightarrow a_{\text{balanced}} = 0.85C_{\text{balanced}} = 10.71\text{ in}$$

$$(A_s)_{\text{balanced}} = (A_s)_b = \frac{0.85(4)(10.71)(17)}{60} = 10.32\ \text{in}^2$$

$$\Rightarrow \rho_b = \frac{10.32}{(17)(21)} = 0.0289$$

$$\rho_{\max} = 0.75\rho_b = 0.0217 \qquad \textbf{(O.K.)}$$

$\Rightarrow$ $A_s = 3.47\ \text{in}^2$ for flexure

5. Summary

Use

4 #10 @ the bottom (3.47 in^2 for M, 1.61 in^2 for T)

2 #6 @ intermediate level (1.50 in^2 for T)

3 #6 @ the top (1.79 in^2 for T, 0.46 in^2 excess)

We have an excess of steel. In the worse case, we have a moment M_u without T_u. The 5.08in^2 of steel at the bottom can all be considered for flexural tensile reinforcement purpose. In that case,

$$\rho = \frac{5.08}{(17)(21)} = 0.0142 < 0.75\rho_b \text{ (Check for flexural capacity as singly-reinforced section)}$$

which is still O.K.

The results are illustrated in Fig. 3:

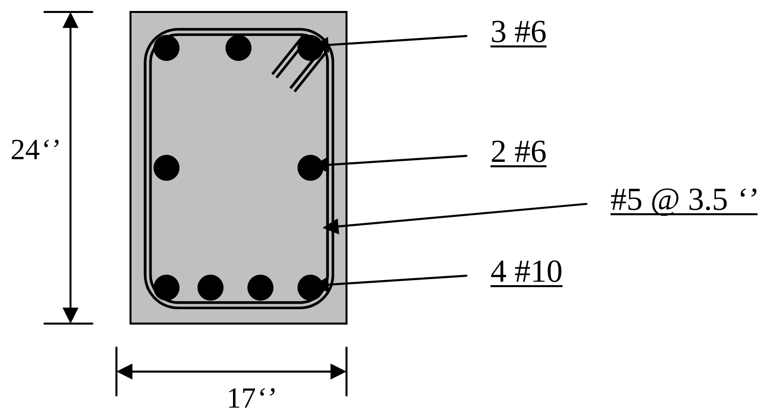


Figure 3. Configuration of the cross section

Comments on the final design:

1. Different design configurations are possible, in general. Various combinations of different sizes of steel bars can achieve same reinforcement ratio. However, relevant designs are made typically considering the convenience of construction and the spacing between any two steel bars (the concrete between two steel bars will crash undesirably if the spacing between them is not enough).
2. Considering the constructability, four corner positions are usually required to deploy longitudinal bars to fix stirrups.
3. It is preferable to use same size of steel bars on each cross section for economic reason unless it is not possible to achieve the requirement of reinforcement.

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Design Example

Analysis of Rectangular Slabs using Yield Line Theory

Objective: To investigate the ultimate load of a rectangular slab supported by four fixed edges.

Problem: A reinforced concrete slab (shown in Fig. 1) is supported by four fixed edges. It has a uniform thickness of 8 in., resulting in effective depths in the long direction of 7 in. and in the short direction of 6.5 in. Bottom reinforcement consists of #4 bars at 15 in. centers in each direction and top reinforcement consists of #4 bars at 12 in. in each direction. Material strengths are

Concrete

Uniaxial compressive strength: $f_c^{'} = 4000$ psi;

Steel

Yield stress: $f_y = 60$ ksi.

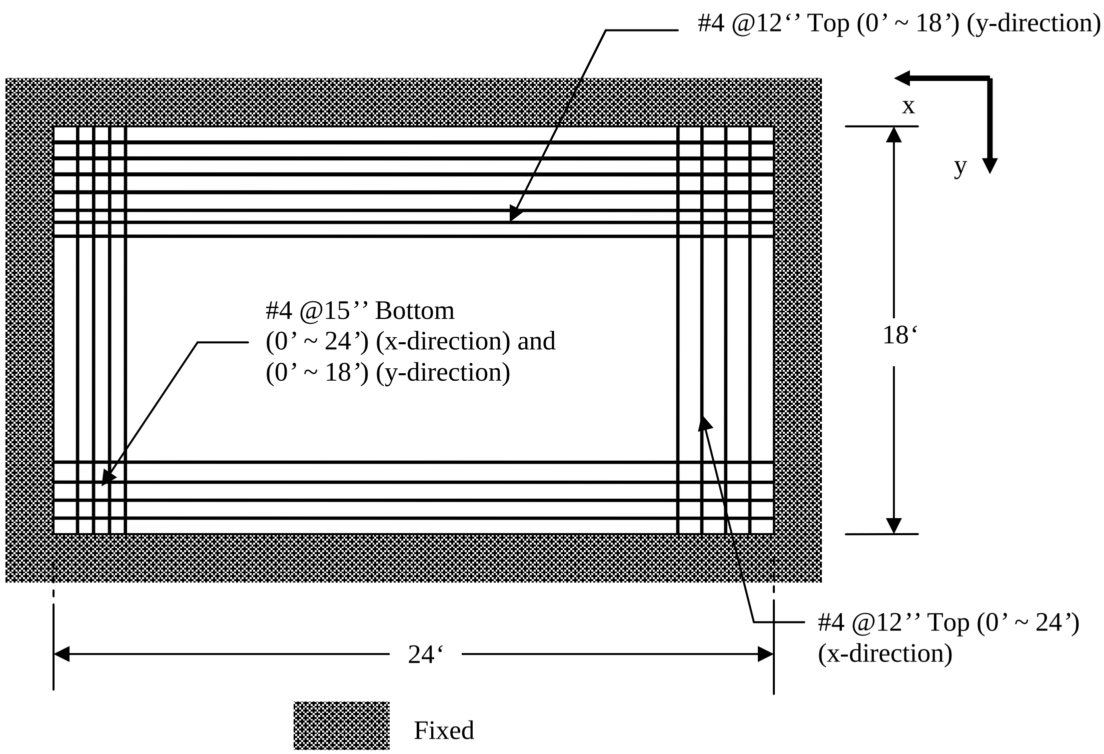


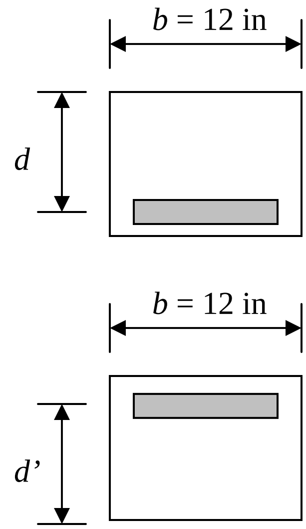
Fig. 1 Reinforced concrete slab and its dimensions

Task: Using the yield line theory method, determine the ultimate load w_u that can be carried by the slab.

[Design Procedures]

Given the information about the slab, shown in Fig. 1, below:

Thickness of the slab: 8 in,
Effective depth in x direction: 7 in,
Effective depth in y direction: 6.5 in,
Notice that *d* = *d'* for each direction.



Reinforcements in both directions:

X direction

Top: #4@12 in (0'~24')
Bottom: #4@15 in (0'~24')

Y direction

Top: #4@12 in (0'~18')
Bottom: #4@15 in (0'~18')

Material strengths:

Concrete

Uniaxial compressive strength: $f_c' = 4000$ psi,

Steel

Yield stress: $f_y = 60$ ksi.

1. Calulation of the moments per unit length in both directions

(1) X direction ($d = d' = 7$ in)

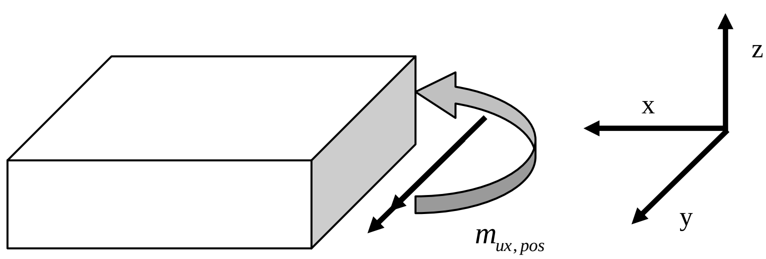


Fig. 2 Positive bending moment in X direction

We have
#4@15 in. for positive (bottom, 0'~18') reinforcement (Fig. 2),
#4@12 in. for negative (top, 0'~18') reinforcement (Fig. 3),

Unit length moments are calculated below.

$\underline{m_{ux,pos}}$:

$$A_s = \frac{12\text{ in}}{15\text{ in}} \cdot 0.2\text{ in}^2 = 0.16\text{ in}^2$$

$\sum C = \sum T$ provides

$$\Rightarrow 0.85 f_c' ab = A_s f_y \Rightarrow a = \frac{f_y}{0.85 f_c' b} A_s = 1.4706 \cdot 0.16 = 0.235 \text{ in}$$

$$m_{ux,pos} = \phi A_s f_y \left(d - \frac{a}{2} \right) = 0.9 \cdot 0.16 \cdot 60 \cdot \left(7 - \frac{0.235}{2} \right) = 59.46 \text{ kips-in} = \underline{4.95 \text{ kips-ft}}$$

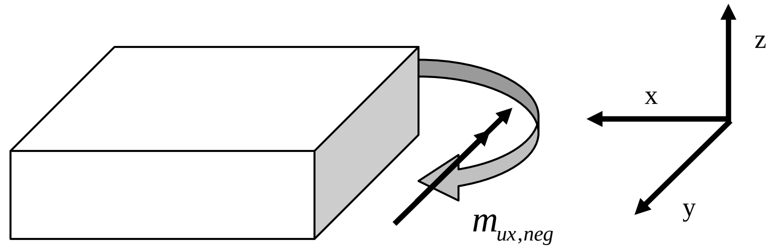


Fig. 3 Negative bending moment in X direction

$\underline{m_{ux,neg}}$ (0'~18'):

$$A_s' = \frac{12 \text{ in}}{12 \text{ in}} \cdot 0.2 \text{ in}^2 = 0.2 \text{ in}^2$$

$\sum C = \sum T$ provides

$$\Rightarrow 0.85 f_c' a' b = A_s' f_y \Rightarrow a' = 1.4706 \cdot 0.2 = 0.294 \text{ in}$$

$$m_{ux,neg} = \phi A_s' f_y \left(d' - \frac{a'}{2} \right) = 0.9 \cdot 0.2 \cdot 60 \cdot \left(7 - \frac{0.294}{2} \right) = 74.01 \text{ kips-in} = \underline{6.17 \text{ kips-ft}}$$

(2) Y direction ($d = d' = 6.5$ in)

We have #4@15 in. for positive (bottom, 0'~24') reinforcement (Fig. 4) and #4@12in. for negative (top, 0'~24') reinforcement (Fig. 5). Calculate the positive and negative moments per unit length respectively.

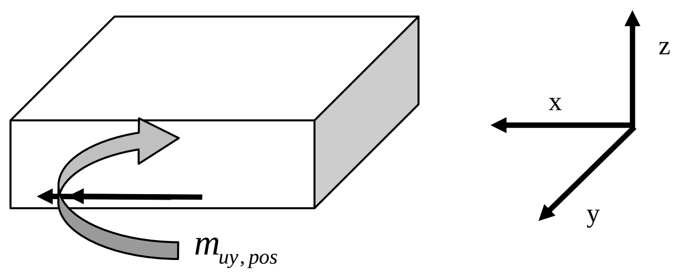


Fig. 4 Positive bending moment in Y direction

$\underline{m_{uy,pos}}$:

$$A_s = \frac{12 \text{ in}}{15 \text{ in}} \cdot 0.2 \text{ in}^2 = 0.16 \text{ in}^2$$

$\sum C = \sum T$ provides

$$\Rightarrow 0.85 f_c^{'} ab = A_s f_y \Rightarrow a = \frac{f_y}{0.85 f_c^{'} b} A_s = \frac{60}{0.85 \cdot 4 \cdot 12} \cdot 0.16 = 1.4706 \cdot 0.16 = 0.235 \text{ in}$$

$$m_{uy,pos} = \phi A_s f_y \left(d - \frac{a}{2} \right) = 0.9 \cdot 0.16 \cdot 60 \cdot \left(6.5 - \frac{0.235}{2} \right) = 55.14 \text{ kips-in} = \underline{4.6 \text{ kips-ft}}$$

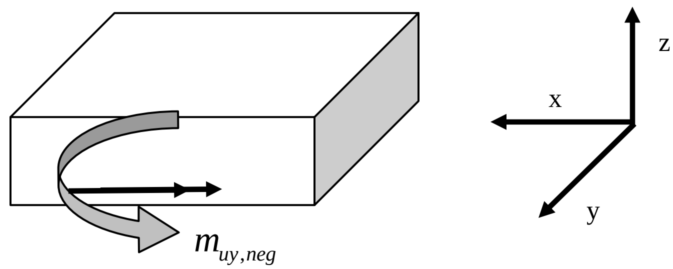


Fig. 5 Negative bending moment in X direction

$\underline{m_{uy,neg}:}$

$$A_s^{'} = \frac{12 \text{ in}}{12 \text{ in}} \cdot 0.2 \text{ in}^2 = 0.2 \text{ in}^2$$

$\sum C = \sum T$ provides

$$\Rightarrow 0.85 f_c^{'} a^{'} b = A_s^{'} f_y \Rightarrow a^{'} = 1.4706 \cdot 0.2 = 0.294 \text{ in}$$

$$m_{uy,neg} = \phi A_s^{'} f_y \left(d^{'} - \frac{a^{'}}{2} \right) = 0.9 \cdot 0.2 \cdot 60 \cdot \left(6.5 - \frac{0.294}{2} \right) = 68.61 \text{ kips-in} = \underline{5.71 \text{ kips-ft}}$$

2. Failure mode and the ultimate load of the slab

(1) One possible mode is postulated for the slab. Its geometry and associated length and angle definitions are provided in Fig. 6.

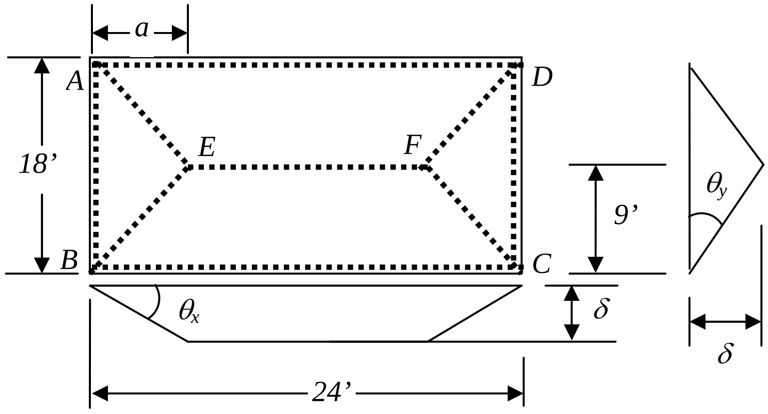


Fig. 6 Postulated failure mode and the associated length and angle definitions

$$\Rightarrow \theta_x = \frac{1}{a},\ \theta_y = \frac{1}{9}$$

Internal work is computed as

Segment	θ_x	θ_y	$m_y \theta_x l_y$	$m_x \theta_y l_x$
AB, CD	$1/a$	0	0	$6.17 \cdot \frac{1}{a} \cdot 18$
AD, BC	0	1/9	0	$5.71 \cdot \frac{1}{9} \cdot 24$
AE, BE, CF, DF	$1/a$	1/9	$5.71^* \cdot \frac{1}{a} \cdot 9$	$4.95^* \cdot \frac{1}{9} \cdot a$
EF	0	2/9	0	$4.6 \cdot \frac{2}{9} \cdot (24 - 2a)$

[*: Use 5.71 and 4.95 kips-in to be conservative although the moment varies along these yield lines.]

$$\sum W_{\text{int}} = 2\left[\frac{111.06}{a} + 15.23\right] + 4\left[\frac{51.39}{a} + 0.55a\right] + 24.48 - 2.04a$$

$$= \frac{427.68}{a} + 54.94 + 0.16a$$

External work is computed as

Segment	Area	δ	$w \cdot A \cdot \delta$
ABE, CDF	$\frac{18 \cdot a}{2}$	1/3	$3wa$
BCFE, ADFE	$9a$; $(24 - 2a) \cdot 9 = 216 - 18a$	1/3; 1/2	$3wa + 108w - 9wa = 108w - 6wa$

$$\sum W_{\text{ext}} = 2[3wa + 108w - 6wa] = 216w - 6wa = w(216 - a)$$

$$\because \sum W_{\text{int}} = \sum W_{\text{ext}}$$

$$\therefore \Rightarrow \frac{427.68}{a} + 54.94 + 0.16a = w(216 - 6a)$$

$$\Rightarrow w = \frac{427.68 + 54.94a + 0.16a^2}{216a - 6a^2}$$

For minimum w, $\frac{dw}{da} = 0$ and $\frac{d^2 w}{da^2} > 0$

$\frac{dw}{da} = 0$ provides

$$\frac{dw}{da} = \frac{\left[\left(54.94 + 0.32a\right)\left(216a - 6a^2\right) - \left(427.68 + 54.94a + 0.16a^2\right)\cdot\left(216 - 12a\right)\right]}{\left(216a - 6a^2\right)^2} = 0$$

It yields to

$$11867a - 329.64a^2 + 69.12a^2 - 1.92a^3 - 92378.88 + 5132.16a - 11867a + 659.28a^2$$
$$-34.56a^2 + 1.92a^3 = 0$$

$$\Rightarrow 364.2a^2 + 5132.16a - 92378.88 = 0$$
$$\Rightarrow a^2 + 14.09a - 253.65 = 0$$

and a = 10.37 in

$$\because w = \frac{427.68 + 54.94a + 0.16a^2}{216a - 6a^2}$$

$$\therefore w_u = 0.636 \frac{\text{kips}}{\text{ft}^2}$$

Therefore the ultimate load this rectangular slab can carry is 0.636 kips/ft^2.

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Outline 1

Introduction / Design Criteria for Reinforced Concrete Structures

- Structural design
  - Definition of design:

    Determination of the general shape and all specific dimensions of a particular structure so that it will perform the function for which it is created and will safely withstand the influences which will act on it throughout its useful life.

    → Principles of mechanics, structural analysis, behavioral knowledge in structures and materials.

    → Engineering experience and intuition.

    → (a) Function, (b) strength with safety requirements will vary for structures.

    → Influences and structural response:

    Influences → Structure → Structural response

Influences	Structural response
Loads	Failure (strength)
Temperature fluctuations	Failure mode
Foundation settlements	Deformations
Time effects	Cracking
Corrosion	Stresses
Earthquakes	Motion
Other environmental effects	

  - Structural mechanics:

    A tool that permits one to predict the response (with a required level of accuracy, and a good degree of certainty) of a structure to defined influences.

- Role of the designer (engineer) of a structure

Design criteria for concrete

- Two schools of thoughts
  1. Base strength predictions on nonlinear theory using actual σ-ε relation
     - 1897 – M.R. von Thullie (flexural theory)
     - 1899 – W. Ritter (parabolic stress distribution theory]
  2. Straight-line theory (elastic)
     - 1900 – E. Coignet and N. de Tedesco (the straight-line (elastic) theory of concrete behavior)

- Working Stress Design (WSD) – Elastic theory
  1. Assess loads (service loads) (Building Code Requirements)
  2. Use linear elastic analysis techniques to obtain the resulting internal forces (load effects): bending, axial force, shear, torsion

  At service loads: $\sigma_{\max} \leq \sigma_{\text{all}}$

  e.g. $\sigma_{\text{all}}^{c} = 0.45 f_c^{'}$ compression in bending

  $\sigma_{\text{all}}^{s} = 0.50 f_y$ flexure

- Ultimate Strength Design (USD)
  - The members are designed taking inelastic strain into account to reach ultimate strength when an ultimate load is applied to the structure.
  - The load effects at the ultimate load may be found by
    (a) assuming a linear-elastic behavior
    (b) taking into account the nonlinear redistribution of actions.
  - Sectional design is based on ultimate load conditions.
  - Some reasons for the trend towards USD are
    (a) Efficient distribution of stresses

(b) Allows a more rational selection of the load factors

(c) Allows designer to assess the ductility of the structure in the post-elastic range

- Limit State Design
  - Serviceability limit state:
    Deformation, fatigue, ductility.
  - Ultimate limit state:
    Strength, plastic collapse, brittle fracture, instability, etc.
  - It has been recognized that the design approach for reinforced concrete (RC) ideally should combine the best features of ultimate strength and working stress designs:
    (a) strength at ultimate load
    (b) deflections at service load
    (c) crack widths at service load

- ACI (American Concrete Institute) Code emphasizes:
  (a) strength provisions
  (b) serviceability provisions (deflections, crack widths)
  (c) ductility provisions (stress redistribution, ductile failure)

Design factors

- 1956 – A.L.L. Baker (simplified method of safety factor determination)
- 1971 – ACI Code (*load factors* and *capacity (strength, resistance) reduction factors*)
- 2002 – ACI 318 Building Code
- Design loads (U) are factored to ensure the safety and reliability of structural performance.
- Structural capacities (ϕ) of concrete material are reduced to account for inaccuracies in construction and variations in properties.

- Safety
  - Semi-probabilistic design is achieved by introducing the use of load factors, γ_l, and capacity reduction factors, ϕ.
  - Load factors – ACI 318 Building Code
    - Load combinations

      U = 1.4(D + F)

      U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r or S or R)

      U = 1.2D + 1.6(L_r or S or R) + (1.0L or 0.8W)

      U = 1.2D + 1.6W + 0.5L + 1.0(L_r or S or R)

      U = 1.2D + 1.0E + 1.0L + 0.2S

      U = 0.9D + 1.6W + 1.6H

      U = 0.9D + 1.0E + 1.6H

      where D = dead load; F = lateral fluid pressure; T = self-straining force (creep, shrinkage, and temperature effects); L = live load; H = load due to the weight and lateral pressure of soil and water in soil; L_r = roof load; S = snow load; R = rain load; W = wind load; E = earthquake load.
    - ACI 318-02 also provides exceptions to the values in above expressions.

  - Capacity reduction factors – ACI 318 Building Code
    - Members subject to structural actions and their associated reduction factor (ϕ)

      Beam or slab in bending or flexure: 0.9

      Columns with ties: 0.65

      Columns with spirals: 0.70

      Columns carrying very small axial loads: 0.65~0.9 for tie stirrups and 0.7~0.9 for spiral stirrups.

      Beam in shear and torsion: 0.75

- Relation between resistance capacity and load effects

  $\phi R_n \geq \sum_{i=1}^{m} \gamma_i l_i$ → resistance ≥ sum of load effects

  For a structure loaded by dead and live loads the overall safety factor is

  $$s = \frac{1.2D + 1.6L}{D + L} \cdot \frac{1}{\phi}$$

Making of concrete

- Cements
  - Portland cements
  - Non-portland cements
- Aggregates – Coarse and fine
- Water
- Chemical admixtures
  - Accelerating admixtures
  - Air-entraining admixtures
  - Water-reducing and set-controlling admixtures
  - Finely divided admixtures
  - Polymers (for polymer-modified concrete)
  - Superplasticizers
  - Silica-fume admixture (for high-strength concrete)
  - Corrosion inhibitors

Raw material components of cement

- Lime (CaO)
- Silica (SiO_2)
- Alumina (Al_2O_3)

Properties of portland cement components

Component	Rate of reaction	Heat liberated	Ultimate cementing value
Tricalcium silicate, C_3S	Medium	Medium	Good
Dicalcium silicate, C_2S	Slow	Small	Good
Tricalcium aluminate, C_3A	Fast	Large	Poor
Tetracalium aluminoferrate, C_4AF	Slow	Small	Poor

- Types of portland cements
  - Type I: All-purpose cement
  - Type II: Comparatively low heat liberation; used in large structures
  - Type III: High strength in 3 days
  - Type IV: Used in mass concrete dams
  - Type V: Used in sewers and structure exposed to sulfates

- Mixture design methods of concrete
  - ACI method of mixture design for normal strength concrete
  - Portland Cement Association (PCA) method of mixture design

- Quality tests on concrete
  - Workability
  - Air content
  - Compressive strength of hardened concrete
  - Flexural strength of plain concrete beams
  - Tensile strength from splitting tests

- Advantages and disadvantages of concrete
  - Advantages
    - Ability to be cast
    - Economical

    - Durable
    - Fire resistant
    - Energy efficient
    - On-site fabrication
    - Aesthetic properties
  - Disadvantages
    - Low tensile strength
    - Low ductility
    - Volume instability
    - Low strength-to-weight ratio

- Properties of steel reinforcement
  - Young's modulus, E_s
  - Yield strength, f_y
  - Ultimate strength, f_u
  - Steel grade
  - Geometrical properties (diameter, surface treatment)

- Types of reinforced concrete structural systems
  - Beam-column systems
  - Slab and shell systems
  - Wall systems
  - Foundation systems

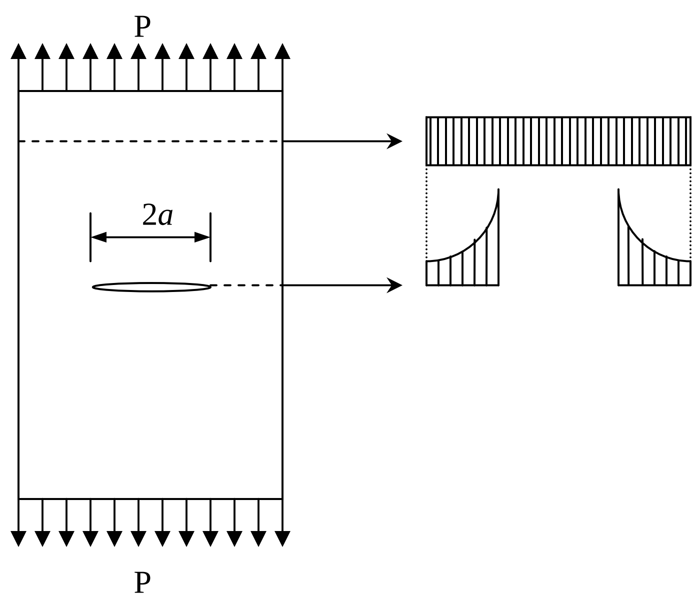
1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Outline 4

Fracture Concepts

- Fracture mechanics:
  - Failure of concrete structures typically involves crack propagation and growth of large cracking zones before the maximum load is reached. Fracture mechanics, for design of concrete structures, has been introduced for a realistic prediction of crack stability.
  - Some reasons for introducing fracture mechanics into the design of concrete structures:
    1. Energy required for crack formation,
    2. The need to achieve objectivity of finite element solutions,
    3. Lack of yield plateau,
    4. The need to rationally predict ductility and energy absorption capability, and
    5. The effect of structure size on the nominal strength, ductility, and energy absorption capability.

  - Fracture problem



- The focus of the problems in fracture mechanics is to study the stress distribution around the crack and the propagation of the crack.

→ Cracks are prescribed geometrically in the material.

→ Energy-based crack propagation (failure) criterion is usually applied in the approach.

→ The method allows investigation of the following phenomena during the propagation of cracks:

1. Strain-softening due to distributed cracking,
2. Localization of cracking into large fractures prior to failure, and
3. Bridging stresses at the fracture front.

→ Traditionally, the size effect has been explained by statistical theories, in terms of the randomness of strength distribution.

Weibull's statistical theory (Weibull's weakest link statistics)

- Weibull-type theory of failure

  The probability of failure of a structure under load P and the mean normal stress at failure are:

$$\text{Prob}(P) = 1 - \exp\left\{-\int_V \left[\frac{\sigma(P,x)}{\sigma_0}\right]^m \frac{dV}{V_r}\right\}$$

$$\bar{\sigma}_N = \frac{\bar{P}}{bd} = \frac{1}{bd}\int_0^\infty \left[1 - \text{Prob}(P)\right] dP$$

where $\bar{\sigma}_N$ = mean normal stress,

$\bar{P}$ = mean load,

V = volume of structure,

V_r = representative volume of material,

m = Weibull modulus of the material,

σ_0 = scaling parameter, and

$\sigma(P,x)$ = a function representing the stress caused by load P at point x.

→ The Weibull-type statistical explanations of the size effect cannot be applied to concrete structures because:

1. It ignores the size effect caused by the redistribution of stress prior to failure, and
2. It ignores the consequent energy release from the structure.

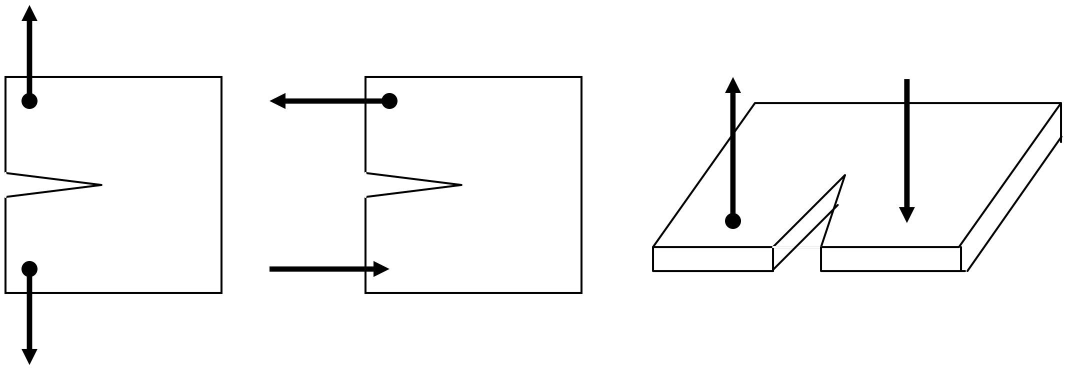
- Assumptions of the brittle failure of concrete structures
  1. The propagation of a fracture or crack band requires an approximately constant energy supply per unit area of fracture plane.
  2. The potential energy released by the structure due to the propagation of the fracture or crack band is a function of both the fracture length and the size of the fracture process zone at the fracture front.
  3. The failure modes of geometrically similar structures of different sizes are also geometrically similar.
  4. The structure does not fail at crack initiation.

- Maximum stress while the crack is propagating

$$\sigma = \sqrt{\frac{2E\gamma}{\pi a}}$$

where E = Young's modulus, $2a$ = characteristic length of the crack, γ = surface energy. r is the function of material property and crack geometry.

- Cracking modes:



Mode I: Opening mode | Mode II: In-plane mode | Mode III: Tearing mode

Linear elastic fracture mechanics (LEFM)

- Assumptions:
  1. All of the fracture process happens at the crack tip, and
  2. The entire volume of the body remains elastic.

  → Under these assumptions, crack propagation and structural failure can be investigated by methods of linear elasticity.

- Stress singularity

  In a sufficiently close neighborhood of the sharp crack tip, the stress components are the same regardless of the shape of the body and the manner of loading.

$$\sigma_{ij}^{I} = \frac{K_I f_{ij}^{I}(\theta)}{\sqrt{2\pi r}}, \quad \sigma_{ij}^{II} = \frac{K_{II} f_{ij}^{II}(\theta)}{\sqrt{2\pi r}}, \quad \sigma_{ij}^{III} = \frac{K_{III} f_{ij}^{III}(\theta)}{\sqrt{2\pi r}}$$

  where *I*, *II*, *III* refer to the elementary modes,

  θ = the polar angle,

  K_I, K_{II}, K_{III} = stress intensity factors, and

  f_{ij}^{I}, f_{ij}^{II}, f_{ij}^{III} functions are the same regardless of the body geometry and the manner of loading.

- Energy criterion

  As the crack tip propagates, energy flows into the crack tip where it is dissipated by the fracture process. The energy flow is characterized by the energy release rate:

$$Gb = -\frac{\partial \Pi(a)}{\partial a} \cong -\frac{1}{\Delta a}\left[\Pi\left(a+\frac{\Delta a}{2}\right) - \Pi\left(a-\frac{\Delta a}{2}\right)\right]$$

  where Π = U-W = potential energy of the structure,

  U = strain energy of the structure as a function of the crack length *a*, and

  W = work of loads.

- Critical energy release rate

  Define G = energy release rate and G_c = critical energy release rate,

  → the crack is stable if the calculated G < G_c,

  → the crack is propagating if the calculated G > G_c, and

  → the crack is in meta stable equilibrium if the calculated G = G_c.

  Suppose a structure with an interior crack (existing crack) with thickness b, the energy released by crack growth is calculated as

$$Gb\Delta_a = P\Delta_x - \Delta U_e$$

  where ΔU_e = change in elastic energy due to crack growth Δ_a.

  The equation can be rewritten as

$$Gb = P\frac{dx}{da} - \frac{dU_e}{da}$$

  Introduce the compliance function

$$c = \frac{x}{P} \Rightarrow x = cP$$

  The strain energy U_e is given by

$$U_e = \frac{cP^2}{2}$$

thus,

$$Gb = P\frac{d(cP)}{da} - \frac{d}{da}\left(\frac{cP^2}{2}\right)$$

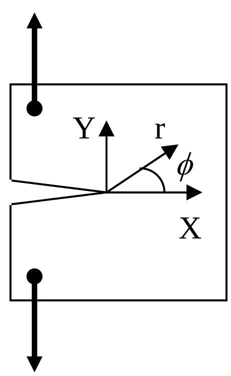
$$\Rightarrow Gb = P\frac{d\left(cP\right)}{da} - \frac{1}{2}\frac{d\left(cP^2\right)}{da}$$

$$\Rightarrow Gb = P^2\frac{dc}{da} - \frac{1}{2}P^2\frac{dc}{da} = \frac{1}{2}P^2\frac{dc}{da}$$

$$\Rightarrow G = \frac{P^2}{2b}\frac{dc}{da}$$

- G_c corresponds to that at fracture. The method is known as "critical energy release rate" method.

o Stress intensity



$$\sigma_x = \frac{K_1}{\sqrt{2\pi r}}\cos\left(\frac{\phi}{2}\right)\left[1-\sin\left(\frac{\phi}{2}\right)\sin\left(\frac{3\phi}{2}\right)\right]$$

$$\sigma_y = \frac{K_1}{\sqrt{2\pi r}}\cos\left(\frac{\phi}{2}\right)\left[1+\sin\left(\frac{\phi}{2}\right)\sin\left(\frac{3\phi}{2}\right)\right]$$

$$\tau_{xy} = \frac{K_1}{\sqrt{2\pi r}}\cos\left(\frac{\phi}{2}\right)\left[\sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right)\cos\left(\frac{3\phi}{2}\right)\right]$$

where K_1 is stress intensity factor for Mode I. Dimensional analysis shows that $K_1 = \sigma\sqrt{a}f\left(g\right) \rightarrow$ σ = applied stress, a = characteristic length $f\left(g\right)$= a function representing geometrical properties.

- Calculation of G_i

For Model I, $G_I = \frac{K_I^2}{E^'}$.

For Model II, $G_{II} = \frac{K_{II}^2}{E^'}$.

For Model III, $G_{III} = \frac{K_{III}^2}{\mu}$.

where $E^'$ = E = Young's modulus, and μ = elastic shear modulus. For the case of plane strain,

$$E^' = \frac{E}{1-\nu^2}$$

where ν = Poisson's ratio.

→ For general loading, the total energy release rate is

$$G = G_I + G_{II} + G_{III}$$

- Calculation of K_I

The stress intensity factor K_I can be calculated from σ_{ij} (or τ_{ij}) expressions or from $K_I = \sigma\sqrt{a} f(g)$. Also,

$$K_I = \frac{P}{bd}\sqrt{\pi a} f(\alpha) = \frac{P}{bd}\sqrt{d}\varphi(\alpha), \ \alpha = \frac{a}{d}$$

where α = the relative crack length,

d = characteristic structure dimension, and

$\varphi(\alpha) = f(\alpha)\sqrt{\pi a}$ = a nondimensional function.

→ For geometrically similar structures of different sizes, the stress intensity factor is proportional to the square root of the size, and the energy release rate is proportional to the size of the structure.

→ The condition of mode I crack propagation is as follow:

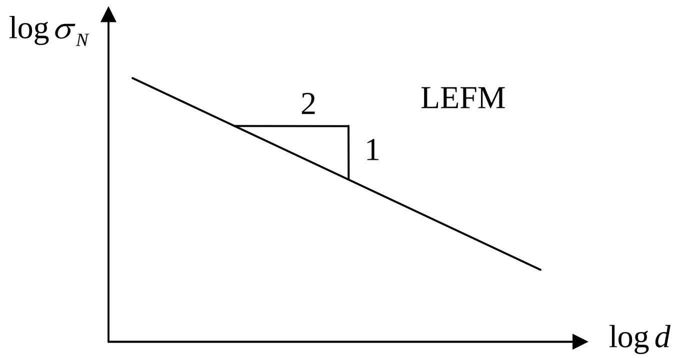
$$K_I = K_{Ic}$$

where $K_{Ic} = G_f E^'$ = critical value of K_I = fracture toughness

→ The nominal stress at failure is

$$\sigma_N = \frac{K_{Ic}}{\sqrt{\pi a}\, f(\alpha)} \Rightarrow \log \sigma_N = -\frac{1}{2}\log d + const.$$

→ The size effect plot according to linear elastic fracture mechanics is an inclined straight line of slope of -1/2.



- Fracture process zone
  - In concrete, microcracks develop ahead of crack tip creating a "fracture process zone". Characteristics of this zone is of fundamental importance in the development of nonlinear fracture mechanics of concrete. Experimental methods are continuously developed.
  - In the HSC the tensile strength can be 2-5 times greater than NSC. However, the increase in fracture energy or elastic modulus is not much. Consequently, HSC may be brittle.

- Applicability of LEFM:
  - In fracture mechanics, the fracture process zone must be having some finite size. The size is characterized by material properties, such as the maximum aggregate size d_a.
  - The length and effective width of the fracture process zone of concrete in three-point bend specimens are roughly $12 d_a$ and 3 d_a, respectively.

- LEFM is applicable when the length of the fracture process zone is much smaller than the cross section dimension of the structure, which is not satisfied for most of the concrete structures.
- Brittleness → Brittleness is the function of fracture energy.
- Nonhomogeneity of cracked concrete

Fictitious crack model

- Developed by Hillerborg to capture the complex nature of concrete in tension.
- Up to the peak, the change in length due to crack propagation is assumed to be a linear function of the strain.

  $\Delta L = L\varepsilon$

  After peak a localized fracture develops. There is a softening behavior inside the fracture zone. This is called strain localization.

  However, in the region near crack tip (localization of deformation), this assumption is not valid. The modified form for ΔL is thus:

  $\Delta L = L\varepsilon + w$

  where w = the term representing the influence of localization in fracture zone.
- Therefore, two relationships are needed to characterize the mechanical behavior:

  1. $\sigma - \varepsilon$ relationship for region outside the fracture zone.
  2. $\sigma - w$ relationship for the fracture zone.

     $G_f = \int_0^\infty \sigma(w)\, dw$

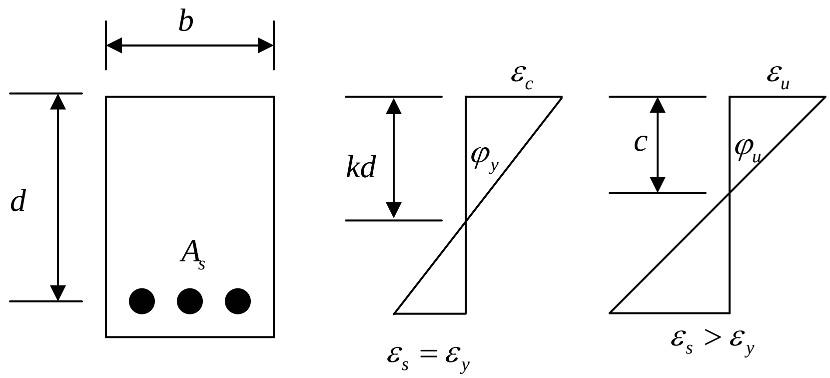
     This is determined experimentally using notched specimens.

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Outline 6

Ductility and Deflections

- Ductility
  - Toughness, deformability, energy absorption capacity



  - For beams failing in flexure the ductility ratio can be defined as the ratio of the curvature at ultimate moment to the curvature at yield.
    - Yield condition: $\varphi_y = \dfrac{f_y}{E_s}\dfrac{1}{d(1-k)}$
    - Ultimate condition: $\varphi_u = \dfrac{\varepsilon_u}{c} = \dfrac{\varepsilon_u \beta_1}{a}$

    → Ductility = $\dfrac{\varphi_u}{\varphi_y} = \dfrac{\varepsilon_u}{f_y/E_s}\dfrac{d(1-k)}{a/\beta_1}$

    → The ratio gives a measure of the curvature ductility of the cross section.

  - Ductility characterizes the deformation capacity of members (structures) after yielding, or their ability to dissipate energy.
  - In general, ductility is a structural property which is governed by fracture and depends on structure size.

- The defined ductility (ratio) shown above does not give information about effects of a moderate number of cycles nor about the shape of the descending branch in moment-rotation curves.

- For doubly reinforced cross section,

$$k = \left[\left(\rho + \rho' \right)^2 n^2 + 2 \left(\rho + \frac{\rho' d'}{d} \right) n \right]^{1/2}$$

$$M_y = A_s f_y jd$$

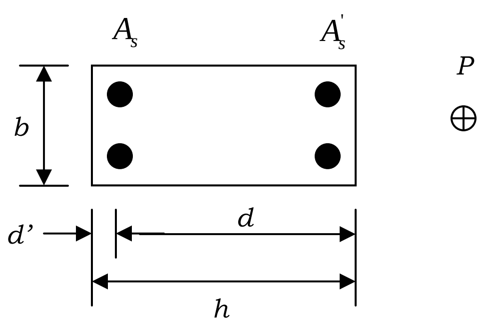
$$a = \frac{A_s f_y - A_s' f_y}{0.85 f_c' b}$$

and the curvature ductility ratio factor is given

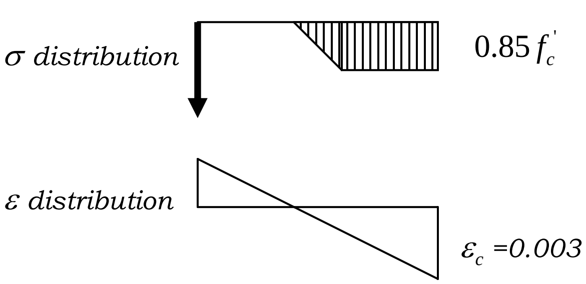
$$\frac{\varphi_u}{\varphi_y} = \frac{0.85 \beta_1 E_s \varepsilon_c f_c'}{f_y^{\,2} \left(\rho - \rho' \right)} \left\{ 1 + \left(\rho + \rho' \right) n - \left[\left(\rho + \rho' \right)^2 n^2 + 2 \left(\rho + \frac{\rho' d'}{d} \right) n \right]^{1/2} \right\}$$

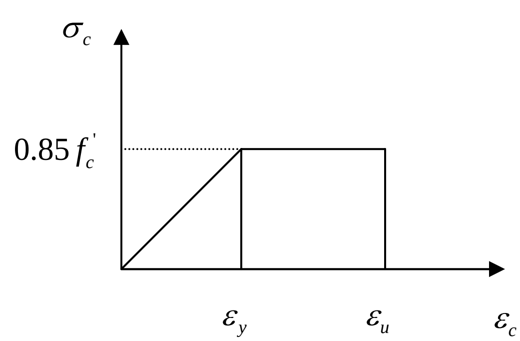
- Effect of axial load on flexural ductility

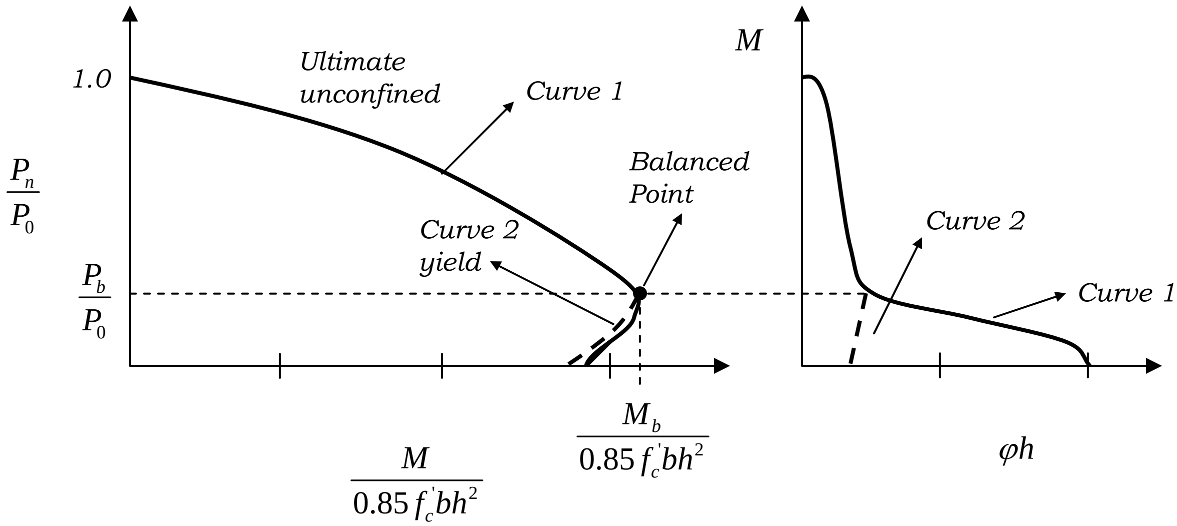
Ductility of unconfined column sections:

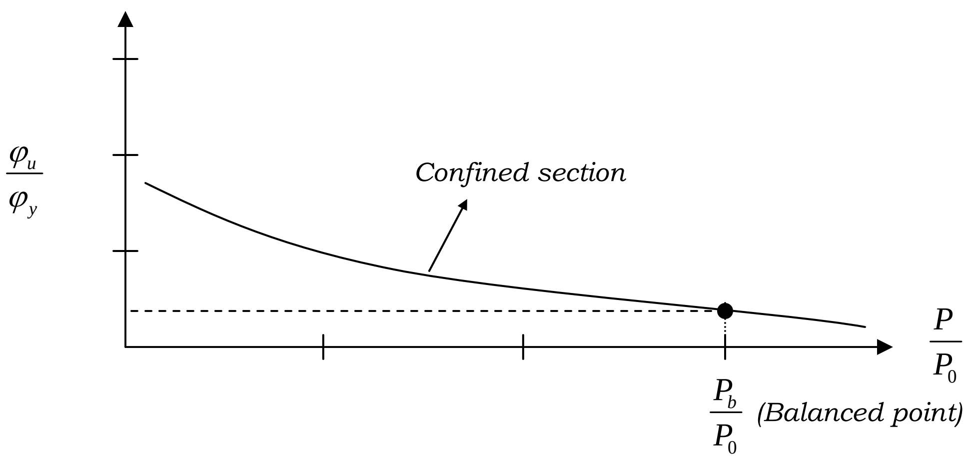


$A_s = A_s'$
Given A_s, f_c', f_y









Curve 1: Combinations of P and M and φh that cause the column to reach ε_u (ultimate concrete strain) without confinement

Curve 2: Combinations of P and M and φh at which the tension steel first reaches the yield strength.

→ The difference between Curve 1 and 2 indicates the amount of inelastic bending deformation (energy absorption), which occurs once the yielding starts.
(P_0 = pure axial strength.)

→ Ductility is reduced by the presence of axial load.

Because of the brittle behavior of unconfined columns it is recommended that the ends of the columns in frames in earthquake regions be confined by closely spaced transverse reinforcement when, generally, $P > 0.4P_0$.

- Effect of confinement
  - If the compression zone is confined by ties, hoops, and spirals, ductility will be improved.
  - Additional confinement due to loading and support conditions.

- Displacement ductility factor

  A measure of the ductility of a structure may be defined by the displacement ductility factor.

$$\mu = \frac{\Delta_u}{\Delta_y}$$

  where Δ_u = the lateral deflection at the end of the post-elastic range, and Δ_y = the lateral deflection when yield is first reached.

- For column members, factors affecting the ductility, other than those related to confinement, are:
  - rate of loading
  - concrete strength
  - bar diameter
  - content of longitudinal steel
  - yield strength of the transverse steel

Deflections

- Types of deflection
  1. Immediate deflections (short-term)

– Deflections that occur at once upon application of load.

– Time-independent

– Elastic-plastic

2. Long-term deflections

– Deflections that occur due to time-dependent behavior of materials, mainly creep and shrinkage.

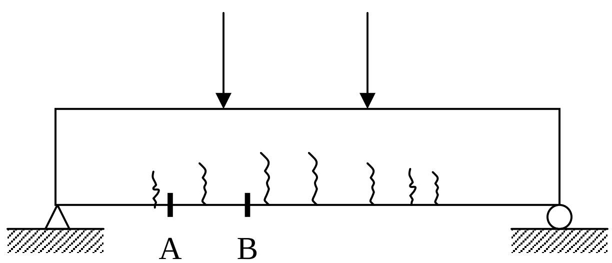
– Creep (under sustained loading)

– Shrinkage (independent of loading)

Short-term deflection

- Effective moment of inertia

Consider a beam structure subjected to bending deflection,



The deflection angle between A and B is

$$\theta = \int_A^B \varphi dx$$

where φ = curvature

Due the existence of cracks, the effective moment of inertia will be calculated as

$$I_e = \left(\frac{M_{cr}}{M_{\max}}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{\max}}\right)^3\right] I_{cr}$$

where $M_{\max}$ = max moment capacity of the cross section,

M_{cr} = moment capacity at first cracking,

I_g = moment of inertia of gross section, and

I_{cr} = moment of inertia of cracked section.

It is known that

$$M_{cr} = \frac{f_{cr} I_g}{y}$$

where f_{cr} = maximum stress at cracking,

y = maximum distance from N.A. to the outer-most fibre on the cross section.

- Long-term deflection
  - The member when loaded undergoes an immediate deflection. The additional deflections are caused due to creep and shrinkage. The rate of these additional deflections decreases by time.

  - Effect of creep on the flexure behavior of concrete member
    - Concrete creep results in a shortening of the compressed part of the concrete cross section, hence causes additional curvature and stress redistribution in the section.
    - Effective modulus of elasticity considering creep:

      For stress less than $0.5 f_c^{'}$ $(\sigma < f_c^{'})$:

$$\varepsilon_{creep} = C_t \frac{f_c}{E_c}$$

where E_c = modulus of elasticity at the instant of loading.

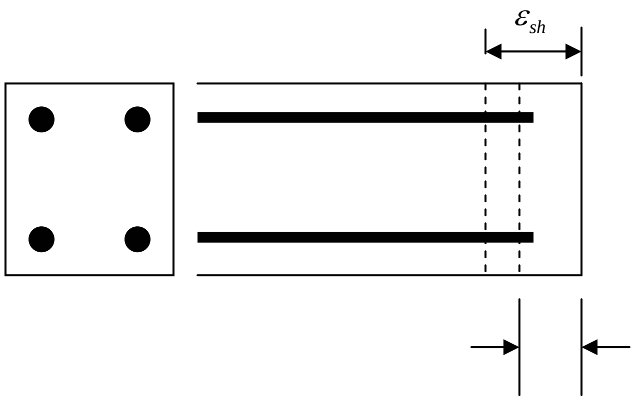
$$\varepsilon_{total} = \frac{f_c}{E_c} + C_t \frac{f_c}{E_c} = \frac{f_c}{E_c}(1 + C_t)$$

The effective modulus including creep is

$$E_{eff} = \frac{f_c}{\varepsilon_{total}} = \frac{E_c}{1 + C_t}$$

  - Effect of shrinkage on the flexure behavior of concrete member

Concrete sections reinforced symmetrically causes uniform stress distribution. In unsymmetrical sections a non-uniform stress distribution, and hence, a curvature is resulted.



Concrete will tend to shrink, but cannot due to the steel restraint, hence concrete undergoes tensile stress and steel reinforcement will be in compression.

- Effective concrete stress considering creep and shrinkage:

  $f_c = \left(\varepsilon_{sh} - \varepsilon_{sh}^{a}\right) E_{eff}$ (tensile stress in concrete)

  $f_s = \varepsilon_{sh}^{a} E_s$ (compressive stress in steel)

  $f_s A_s = f_c A_c$ (equilibrium)

  $$\Rightarrow \varepsilon_{sh}^{a} = \frac{f_s}{E_s} = \frac{f_c}{E_S}\frac{A_c}{A_s}$$

  $$\Rightarrow f_c = \left(\varepsilon_{sh} - \frac{f_c}{E_s}\frac{A_c}{A_s}\right)\frac{E_c}{1+C_t} = \left(\varepsilon_{sh} - \frac{f_c}{E_s}\frac{A_c}{A_s}\right)E_{c,effective}$$

  where A_c = area of concrete,

  ε_{sh} = free shrinkage of concrete

  $\varepsilon_{sh}{}^{a}$ = actual shrinkage of concrete

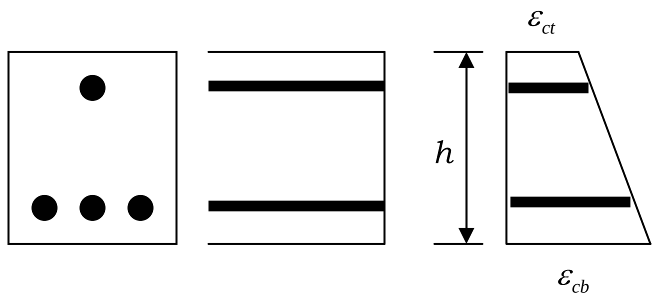
  A_s = area of steel,

  E_s = Young's modulus of steel, and

E_c = Young's modulus of concrete.

This tensile stress, f_c, may exceed the tensile strength of concrete causing cracking.

→ Note that for symmetrical reinforcement the curvature is zero, due to shrinkage. Otherwise,



$$\phi_{sh} = \frac{\varepsilon_{cb} - \varepsilon_{ct}}{h}$$

- Simplified approach in computing long-term deflections:
  → Due to complexities, for traditional applications simplified methods are used.
  → Computer based analysis can be performed with effective E modulus to predict long-term deflection, taking into account the load history.

Control of deflections

- Excessive deflections can lead to cracking of supported walls and partitions, ill-fitting doors and windows, poor roof drainage, misalignment of sensitive machinery and equipment, or visually offensive sag, and interfere with service conditions.
- The need for deflection control
  1. Sensory acceptability
  2. Serviceability of the structure
  3. Effect on nonstructural elements

4. Effect on structural elements

- Control strategies of deflections
  1. Limiting span/thickness ratios
     Deflections are controlled by setting suitable upper limits on the span-depth ratio of members.
  2. Limiting computed deflections
     Deflections are controlled by calculating predicted deflections and comparing those with specific limitations that may be imposed by codes or by special requirements.

  → In any case, maximum allowable deflections are constrained by structural and functional limitations.

- Some permissible deflections by ACI Code

  Minimum thickness of beams:

Type of support	**Minimum thickness**
Simply supported	$\frac{l}{16}$
One end continuous	$\frac{l}{18.5}$
Both ends continuous	$\frac{l}{21}$
Cantilever	$\frac{l}{8}$

PS: Clear span l in inches.

- Additional long-term deflection multipliers

$$\lambda = \frac{\xi}{1+50\rho'}$$

where ξ = a time-dependent coefficient characterizing material properties, and $\rho' = \frac{A_s'}{bd}$.

→ Long-term deflection $\Delta_{total} = \lambda \Delta_{immediate}$

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Outline 7

Shear Failures, Shear Transfer, and Shear Design

- Structural behavior
  - Structural members are subjected to shear forces, generally, in combination with flexure, axial force, and sometimes with torsion.
  - Shear failures are brittle failures primarily because shear resistance in R/C relies on tensile as well as the compressive strength of concrete. Although cracking introduces complications it is still convenient to use classical concepts in analyzing concrete beams under shear failure. Such concepts indicate that shear failure is related to diagonal tensile behavior in concrete. R/C beam must be safe against premature failure due to diagonal tension.

- Failure modes due to shear in beams
  - Diagonal tension failure – sudden
  - Shear-compression failure – gradual
  - Shear-bond failure

  → In general, the design for shear is based on consideration of diagonal (inclined) tension failure.

- Failure of R/C by inclined cracking
  - Inclined cracking load

    $V_c = V_{cc} + V_{ci} + V_d$

    where V_{cc} = shear transferred through the uncracked portion of the concrete,

V_{ci} = vertical component of the aggregate interlocking force in the cracked portion of the concrete, and

V_d = shear force carried through the dowel action of the longitudinal steel.

- Shear strength of the beam without transverse reinforcement is based on the interactive effect of shear stress $v = K_1\left(\frac{V}{bd}\right)$ and flexural stress $f_x = K_2\left(\frac{M}{bd^2}\right)$ leading to dependence on the ratio of $\frac{v}{f} = K \cdot \frac{Vd}{M}$.

Basis of design

- Total ultimate shear force V_u

  $V_u \le \phi V_n$

  where ϕ = the strength reduction factor for shear, and

  V_n = nominal shear strength.

- Nominal shear strength

  $V_n = V_c + V_s$

  where V_c = inclined cracking load of concrete,

  V_s = shear carried by transverse reinforcement.

- Shear strength expression of concrete given by ACI:

$$V_c = \left(1.9\sqrt{f_c^{'}} + 2500\frac{\rho_w V_u d}{M_u}\right)b_w d \le 3.5\sqrt{f_c^{'}}b_w d$$

  where $f_c^{'}$ = compressive strength of the concrete, in psi,

  $\rho_w = \frac{A_s}{b_w d}$ = longitudinal tensile steel ratio,

  b_w = the effective width of the beam, in inches

M_u = total bending moment of the beam, $\frac{V_u d}{M_u} \leq 1$, in lbs-in,

d = the effective depth of the beam, in inches, and

s = spacing of stirrups, in inches.

An alternative simpler equation is

$$V_c = 2\sqrt{f_c^{'}} b_w d$$

- The required shear strength to be provided by the steel (vertical web reinforcement):

$$V_s = \frac{(V_u - \phi V_c)}{\phi} = \frac{V_u}{\phi} - V_c = \frac{A_v f_y d}{s}$$

where f_y = yield strength of the steel and

s = spacing of stirrups.

When stirrups with inclination θ are used, the contribution of steel to shear strength becomes:

$$V_s = \frac{A_v f_y d}{s}(\sin\theta + \cos\theta)$$

- Contribution of axial forces
- Minimum web reinforcement
- Shear transfer
  - Shear in concrete can cause inclined cracking across a member. It is also possible that shear stresses may cause a sliding type of failure along a well-defined plane. Because of previous load history, external tension, shrinkage, etc., a crack may have formed along such a plane even before shear is applied. Upon application of shear forces we have the problem of quantifying shear stress transferred across the cracked sections.
  - General shear transfer mechanisms are

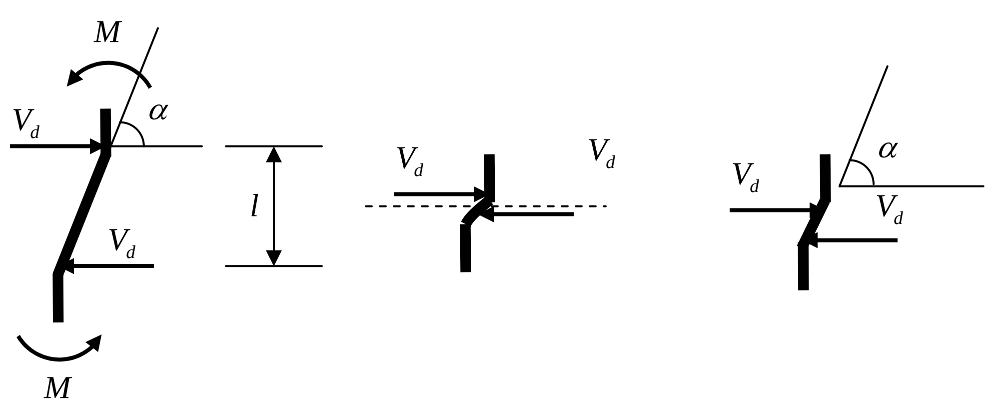
    1. Through still uncracked concrete

2. Direct thrust
3. Dowel action
4. Aggregate interlock

→ Reinforcement provides clamping action.

1. Transfer of shear through intact concrete such as the compression region in a beam
2. Direct thrust
   - Models of shear transfer:
     1. Arch analogy: Beam example
     2. Truss analogy (strut-and-tie action): Corbel example

     → Failure mechanisms of corbels:
       - Flexural-tension failure
       - Diagonal splitting failure
       - Diagonal cracks and shear force failure
       - Splitting along flexural reinforcement failure
       - Local cracking at support
       - Local splitting due to cracking
3. Dowel action
   - Three mechanisms of dowel action:



Flexural

$$V_d = \frac{2M}{l}$$

V_d shear

$$V_d = \frac{A_s f_y}{\sqrt{2}}$$

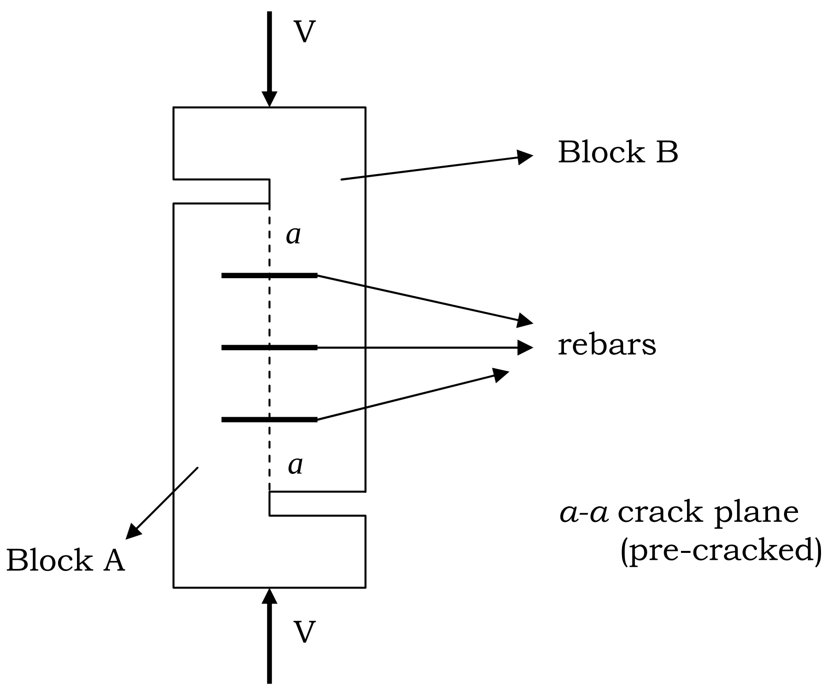
Kinking

$$V_d = A_s f_y \cos\alpha$$

- Shear transfer through dowel action is approximately 25~30% of the shear resisted by the interface shear mechanism.
- Note that the shear yield stress may be determined from von Mises yield function:

$$\tau_y = \frac{A_s f_y}{\sqrt{3}}$$

4. Interface shear transfer: Aggregate interlock + Dowel action
   - Simple friction behavior



   - Assume that the movement of Block A is restraint. Upon application of V Block B moves downward and tends to go to right-opening of crack. Crack plane is in compression. Dowel is in tension.
   - Shear force due to simple friction and dowel

     $V_f = \mu A_s f_y$

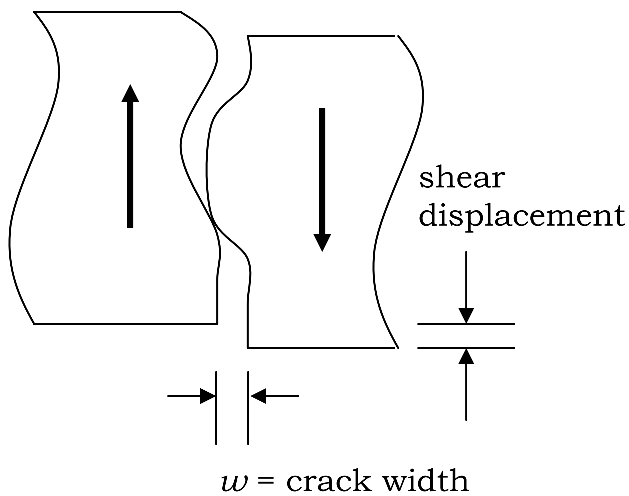
     where μ = friction coefficient.

- Total shear capacity

$$V = V_d + V_f = \frac{A_s f_y}{\sqrt{3}} + \mu A_s f_y = A_s f_y \left(\frac{1}{\sqrt{3}} + \mu \right)$$

→ $A_s = \dfrac{V}{f_y \left(\frac{1}{\sqrt{3}} + \mu \right)}$ = required area of the steel

- Modeling of aggregate interlock and shear modulus of cracked concrete in R/C elements



Crack surface

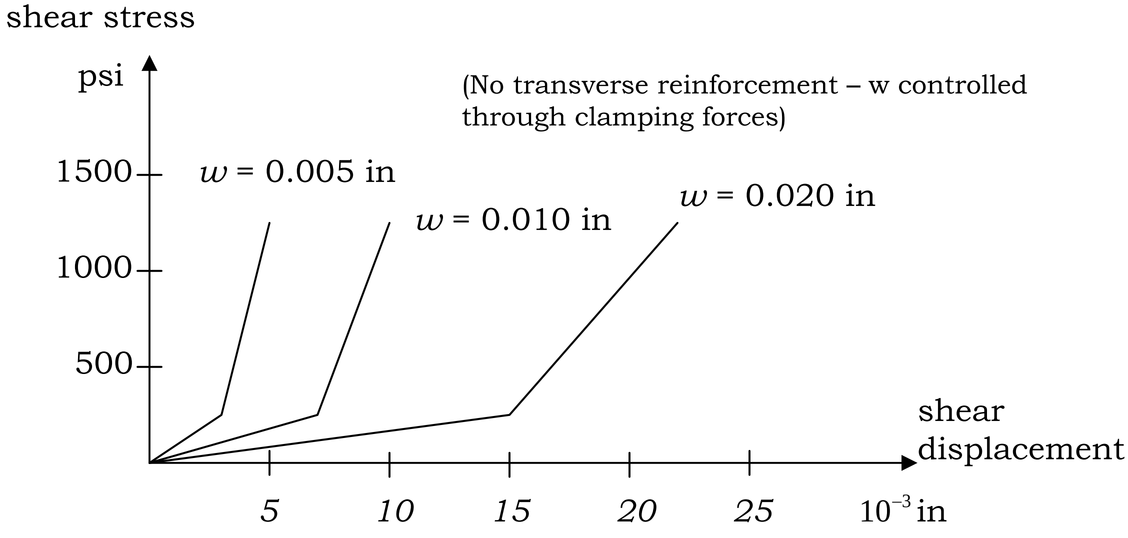
→ Sufficient shear displacement should take place before interlock occurs.

→ Crack width will increase with increased shear displacement.

- A fundamental theory was developed at M.I.T.
- At contact points:

  Frictional resistance due to general roughness of a crack in concrete,

  Additional frictional resistance due to local roughness. (also involves cyclic shear effects)



- Calculation of the deflection due to shear-slip
- Equilibrium + Compatibility + Deformation

$$\delta = \frac{\alpha}{1 + K_D \frac{\beta}{K_N}} w_0 + \frac{1}{\frac{K_N}{\beta} + K_D} V$$

where δ = shear-slip deflection,

α = a coefficient representing gaps produced between asperities,

w_0 = initial crack displacement,

K_D , K_N = coefficients relating to dowel and normal stiffnesses,

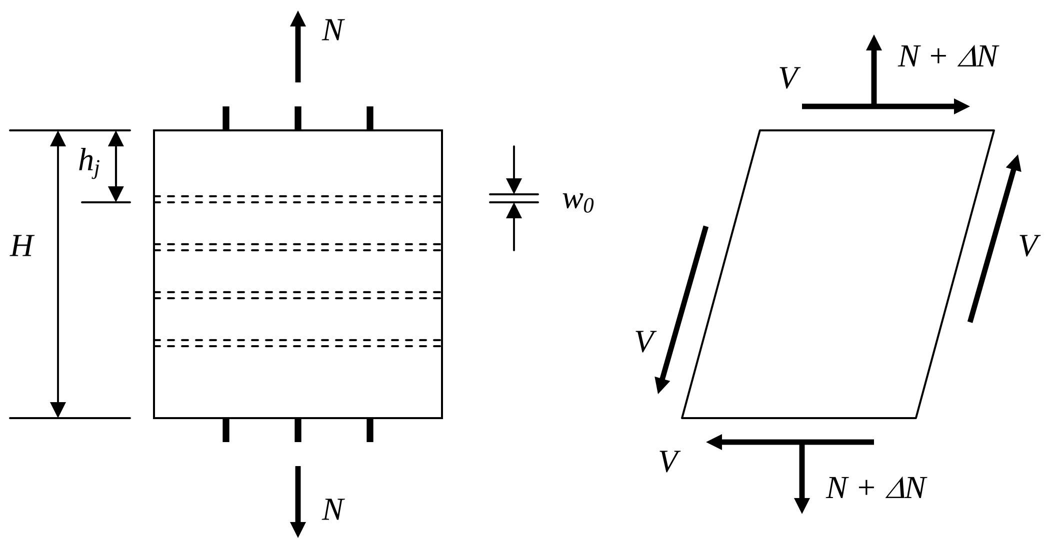
β = a coefficient representing frictional effects at contact points, and

V = applied shear load.

→ α increases with the number of loading cycles.

→ β decreases with the number of loading cycles.

- Overall cracked panel shear modulus G_{cr}



  - Overall effective shear modulus is calculated by

$$G_{cr} = \left[\frac{1}{h\left(\frac{K_N}{\hat{\beta}} + K_D\right)} + \frac{1}{G_e} \right]^{-1}$$

where h = distance between two cracks,

$\hat{\beta}$ = a coefficient obtained from regression analysis, and

G_e = elastic shear modulus.

- Examples of structural applications where inclusion of shear transfer mechanism is important to reduce the required transverse reinforcement for constructability and efficiency
  - Nuclear containment structures (R/C, P/C, hybrid systems)
  - Offshore concrete gravity structures
  - Shear walls

- Design Example – Failure investigation of a prestressed concrete bridge girder

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Outline 9

Beam Column Joints

- Importance of joint behavior
  - Weak link theory
  - Deterioration mechanisms
  - Detailing

- Monolithic beam-column joints
  - In the design with the philosophy of limit states it is seen that joints are often weakest links in a structural system.
  - The knowledge of joint behavior and of existing detailing practice is in need of much improvement.
  - Joint behavior is especially critical for structures subject to earthquake effects.
  - The shear forces developed as a result of such an excitation should be safely transferred through joints. The R/C system should be designed as a “ductile system”.

- Design of joints
  - Joint types

    Type I – Static loading

    → strength important

    → ductility secondary

    Type II – Earthquake and blast loading

    → ductility + strength

    → inelastic range of deformation

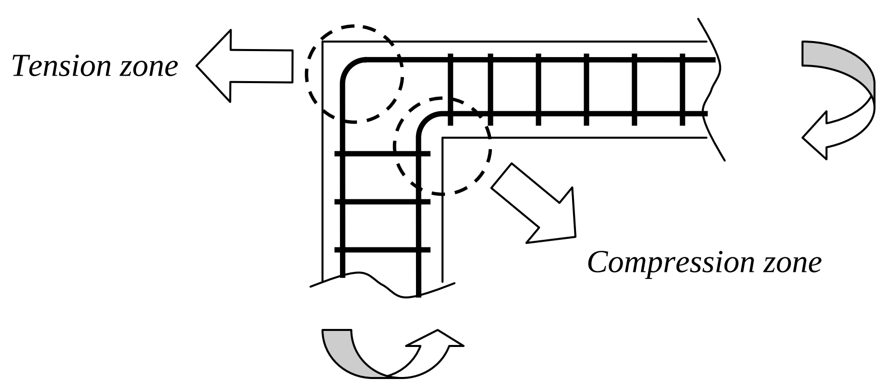
→ stress reversal

- Joints should exhibit a service load performance equal to that of the members it joins.
- Joints should possess strength at least equal to that of the members it joins (sometimes several times more).
- Philosophy: Members fail first, then joints.
  → The joint strength and behavior should not govern the strength of the structure.
- Detailing and constructability.

Behavior of joints

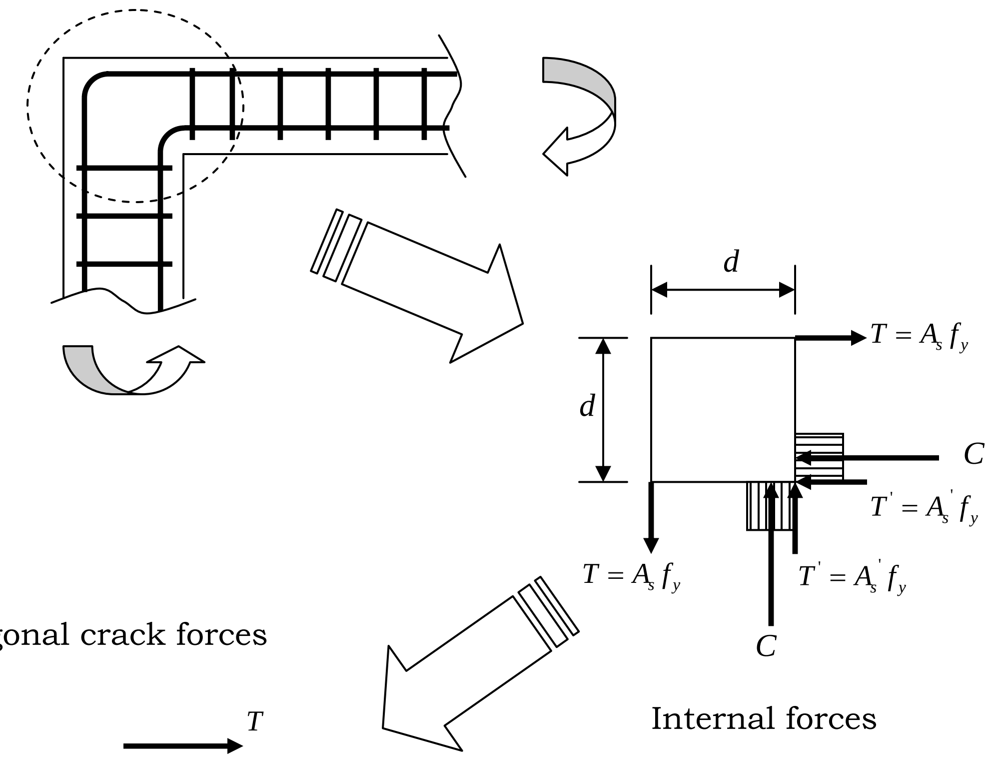
- Knee joint
  - Typical example of a portal frame. The internal forces generated at such a knee joint may cause failure with the joint before the strength of the beam or column.
  - Even if the members meet at an angle, continuity in behavior is necessary.

- Corner joints under closing loads
  - Biaxial compression: $\varepsilon_u > 0.003$



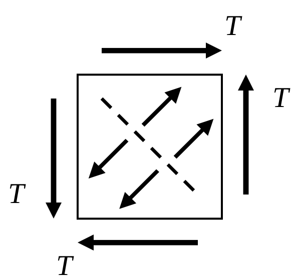
  - Full strength of the bars can be developed if there is no bond failure.

- Joint core



Diagonal crack forces

Internal forces



$$f_t' = \frac{T}{bd} = \frac{A_s f_y}{bd} = \rho f_y \cong 6\sqrt{f_c'}$$

The joint strength:

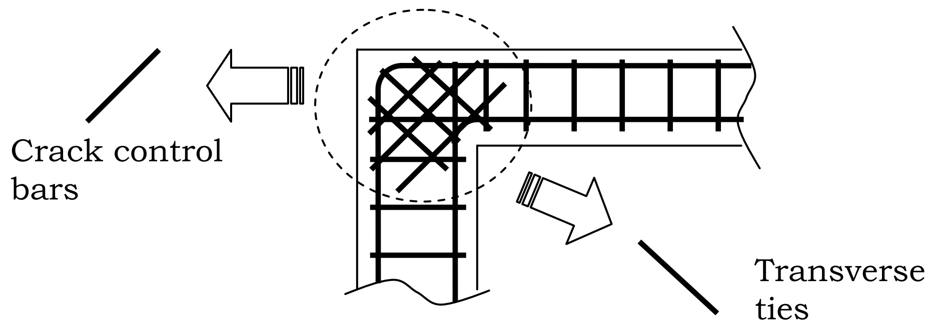
$$f_t' > \rho f_y \rightarrow \rho \le \frac{f_t'}{f_y} \cong \frac{6\sqrt{f_c'}}{f_y}$$

- Factors influencing joint strength
  1. Tension steel is continuous around the corner (i.e., not lapped within the joint).
  2. The tension bars are bent to a sufficient radius to prevent bearing or splitting failure under the bars.

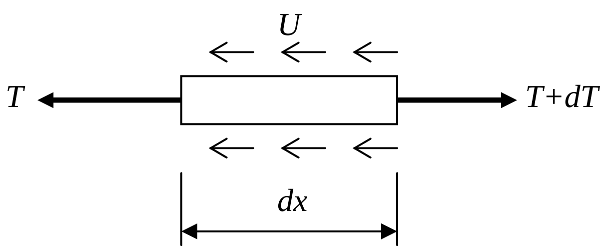
3. The amount of reinforcement is limited to

$$\rho \le \frac{6\sqrt{f_c^{'}}}{f_y}$$

4. Relative size will affect strength and detailing for practical reasons.



5. Bond force

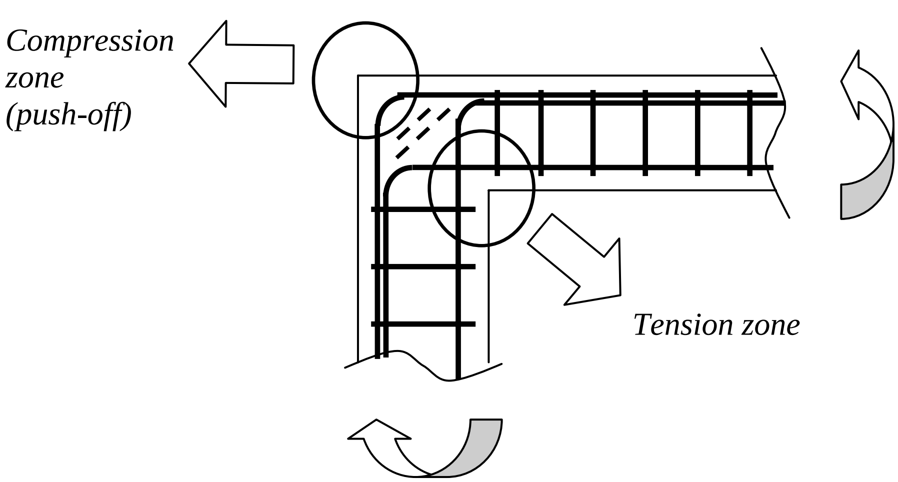


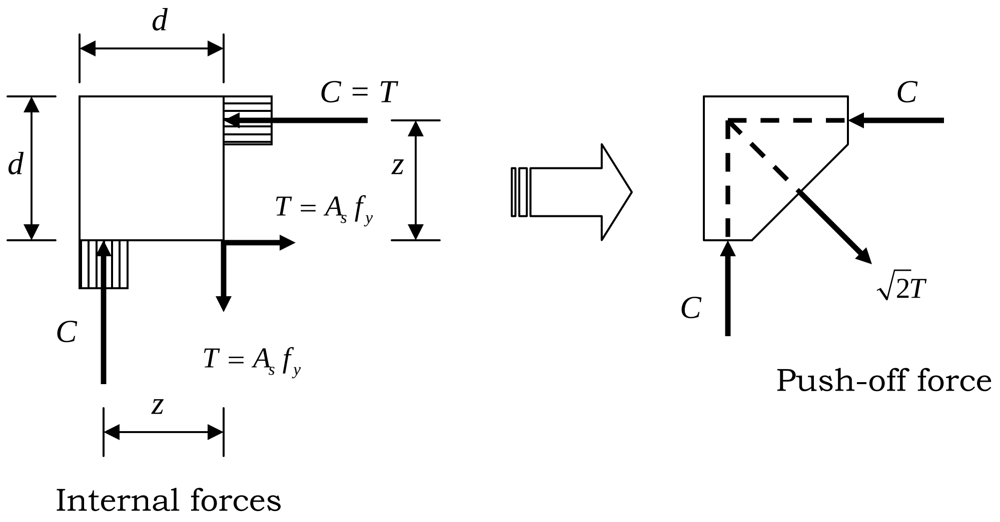
$$\Rightarrow U = \frac{dT}{dx}$$

6. Full bond strength needs to be developed to transfer shear forces into the concrete core.

- Corner joints under opening loads

→ When subjected to opening moments the joint effects are more severe.





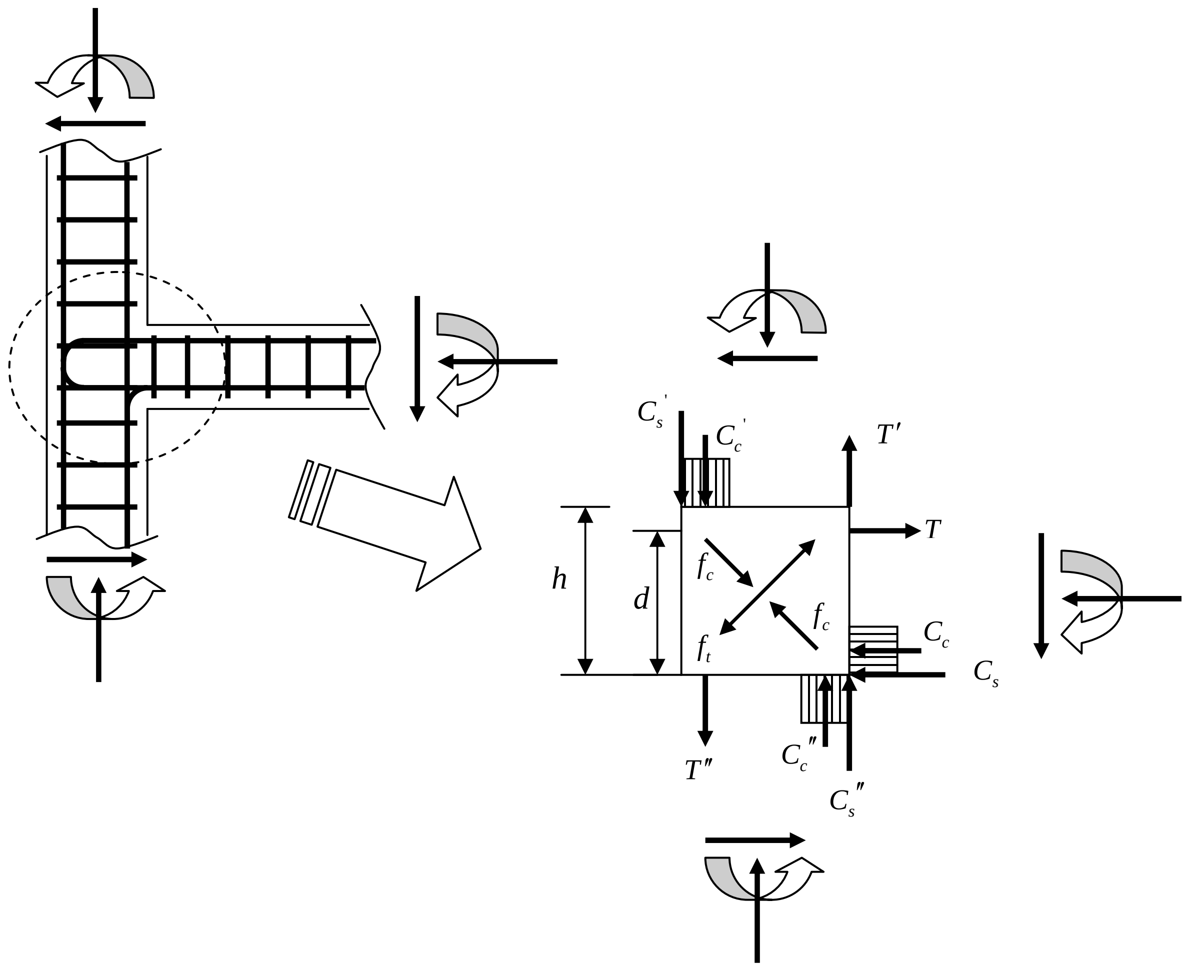
- Behavior under seismic loading
  → Concrete with joint cracks due to cycling.
  → Degradation of bond strength.
  → Flexural bars should be anchored carefully.
  → No benefit should be expected from axial loads.
  → Rely on ties within the joint.
  → Effects from both opening and closing should be considered.
  → An orthogonal mesh of reinforcing bars would be efficient.

o Corner joints under cyclic loads

→ When subjected to cyclic loading (opening moment), one should consider the interaction between tension and compression zones.

o Exterior joints

- Exterior joints of multistory plane frames
  → Issues:
  a. Bond performance as affected by the state of the concrete around anchorage.
  b. Transmission of compression and shearing forces though the joint when the joint core cracks



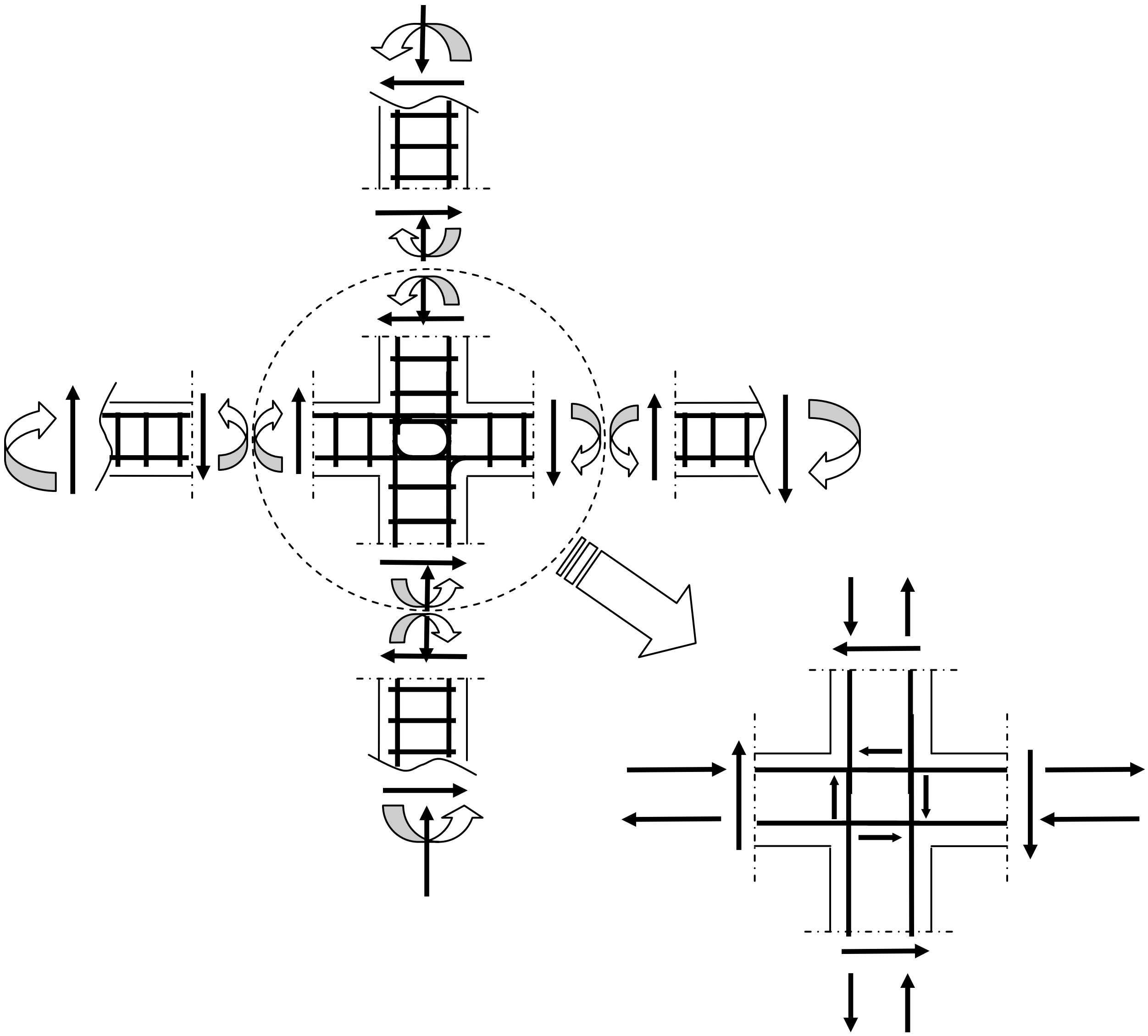
- Also consider load reversals. This is critical for seismic effects.
- Top beam bars

  → Subject to transverse tension

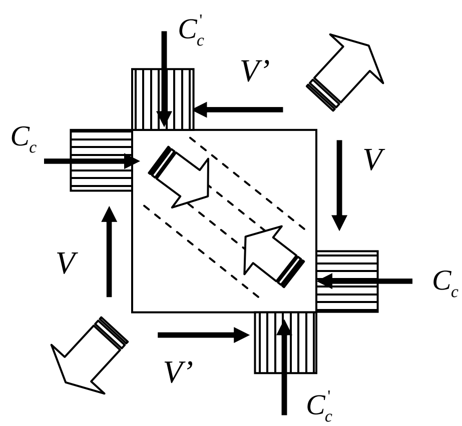
  → The anchorage condition of the reinforcement steel
- Bottom beam bars

  → Subject to transverse compression
- Outer column bars are subjected to severe stress conditions.
- Transmission of shearing and compression forces by diagonal strut across the joint

- Interior joints

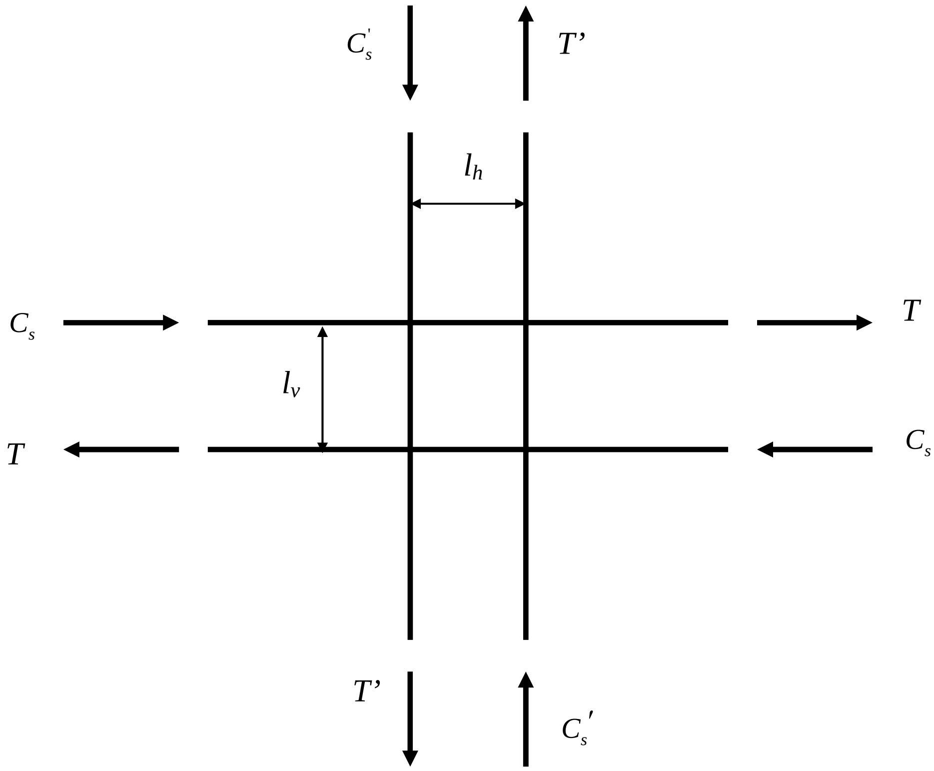


- Concrete



→ $V_c = C_c - V'$ = shear force transferred through concrete

- Steel

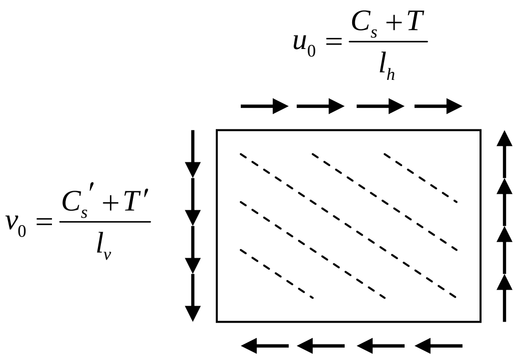


→ $V_s = C_s + T$ = shear force transferred through steel

→ $u_0 = \frac{C_s + T}{l_h}$ and $v_0 = \frac{C_s' + T'}{l_v}$

- Combined behavior:

  Shear transfer by bond



$V_j = V_c + V_s$

- Reduction in compressive strength due to biaxiality in concrete and deterioration of bond due to load cycling are of importance in joint integrity.
- Effect of axial force
- Effect of confinement

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Outline 12

Reinforced Concrete Thin Shell Structures

- Thin shell
  - Definition – A thin shell is a curved slab whose thickness h is small compared with its other dimensions and compared with its principal radius of curvature.
  - Middle surface

    The surface that bisects the shell is called the middle surface. It specifies the form of this surface and the thickness h at every point.
  - Analysis of thin shells consists the following steps:
    - Establish equilibrium of a differential element cut from the shell
    - Achieve strain compatibility so that each element remains continuous with each adjacent element after deformation.
  - Stress resultants and stress couples

- Shell theories
  - The Kirchhoff-Love theory – The first-approximation of shells
    - Assumptions:

      (1) The shell thickness is negligibly small in comparison with the least radius of curvature of the shell middle surface.

      (2) Strains and displacements that arise within the shells are small.

      (3) Straight lines that are normal to the middle surface prior to deformation remain straight and normal to the middle surface during deformation, and experience no change in length.

(*Analogous to Navier's hypothesis for beams – Bernoulli-Euler theory for beams*)

(4) The direct stress acting in the direction normal to the shell middle surface is negligible.

- Results of the assumptions:
  - Normal directions to the reference surface remain *straight* and *normal* to the deformed reference surface.
  - The hypothesis precludes any transverse-shear strain, i.e., no change in the right angle between the normal and any line in the surface.
  - It is strictly applicable to *thin* shells.
  - It is not descriptive of the behavior near localized loads or junctions. (A*ssumption (4) is not valid in the vicinity of concentrated transverse loads.*)

- The Flügge-Byrne theory – The second-approximation of shells
  - Assumptions:
    - It adopts only assumption (2).
    - It is referred to as "higher-order approximations" of the Kirchhoff-Love assumptions

Classification of shells:

- Classified by governing equation of geometry:
  - Paraboloid of revolution
  - Hyperboloid of revolution
  - Circular cylinder
  - Elliptic paraboloid
  - Hyperbolic paraboloid
  - Circular cone

- Geometrical analysis of shells:
  - Orthogonal curvilinear coordinates

    Consider the position vector

    $$r(\alpha,\beta) = f_1(\alpha,\beta)\mathbf{u} + f_2(\alpha,\beta)\mathbf{v} + f_3(\alpha,\beta)\mathbf{w}$$

    where $f_1(\alpha,\beta)$, $f_2(\alpha,\beta)$, and $f_3(\alpha,\beta)$ are continuous, single-valued functions. The surface is determined by α and β uniquely. α and β are called curvilinear coordinates. **u**, **v**, and **w** are unit vectors in the Cartesian coordinate system.

    Orthogonality is addressed as

    $\frac{\partial r}{\partial \alpha}\cdot\frac{\partial r}{\partial \beta} = 0$ → Inner product is zero.

    The distance between (α,β) and $(\alpha+d\alpha,\beta+d\beta)$ is

    $$ds = \frac{\partial r}{\partial \alpha}d\alpha + \frac{\partial r}{\partial \beta}d\beta$$

    The scalar product of ds with itself is

    $$ds\cdot ds = \left(\frac{\partial r}{\partial \alpha}\cdot\frac{\partial r}{\partial \alpha}\right)d\alpha^2 + \left(\frac{\partial r}{\partial \beta}\cdot\frac{\partial r}{\partial \beta}\right)d\beta^2 = A^2 d\alpha^2 + B^2 d\beta^2 = ds^2$$

    where A and B are called Lame's parameters or measure numbers. The formula shown above is the first quadratic form of the theory of surfaces.
  - Principal radii of curvature
  - Gaussian curvature

    $$K = \frac{1}{r_x}\cdot\frac{1}{r_y}$$

    where r_x and r_y are the principal radii of curvature.

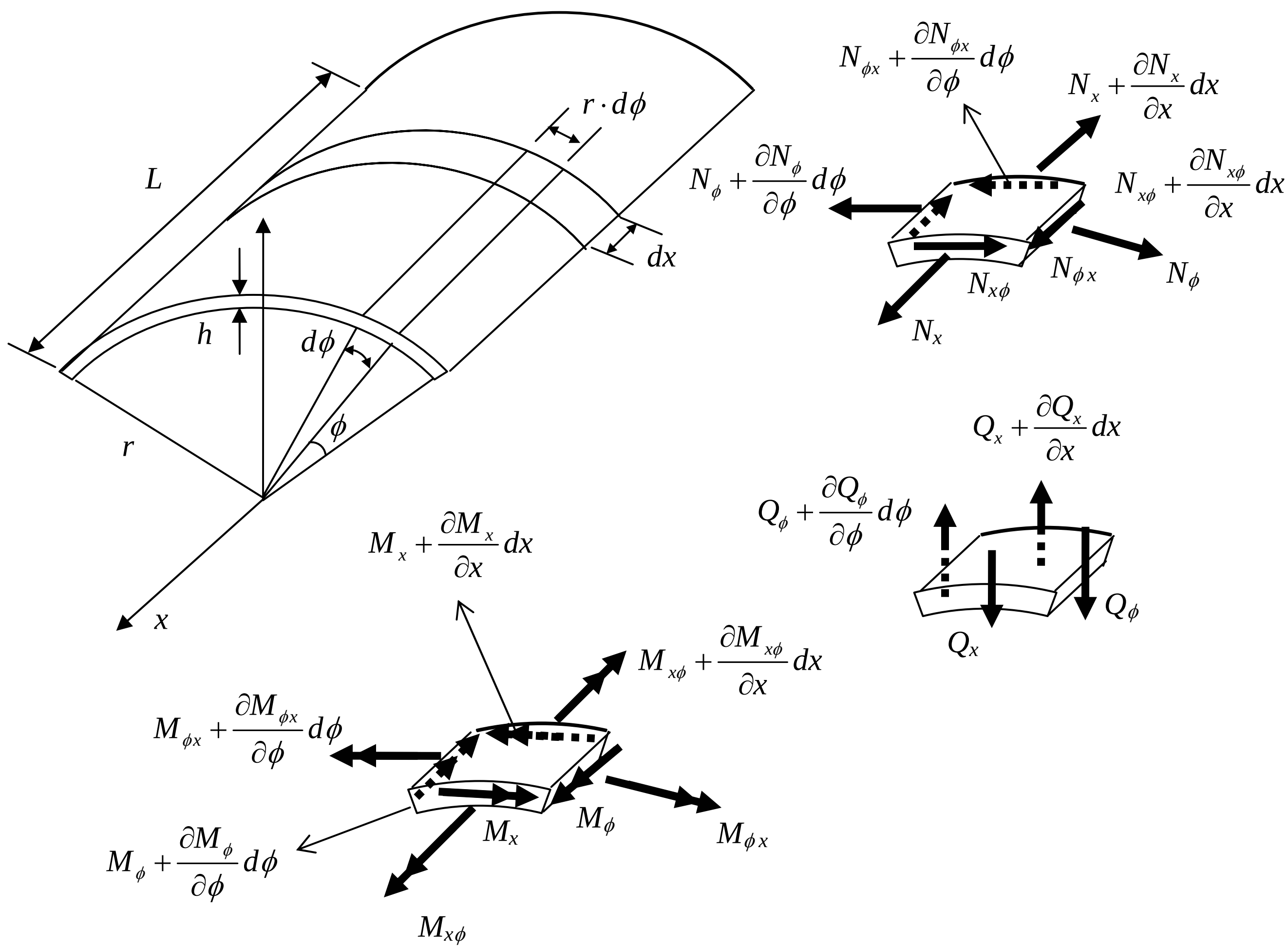
    – $K > 0$: Synclastic shells, i.e., spherical domes and elliptic paraboloids.

    – $K = 0$ (either r_x or r_y is zero): Single-curvature shells, i.e., cylinders and cones.

– $K < 0$: Anticlastic shells, i.e., hyperbolic paraboloids and hyperbolas of revolution.

- Analysis of cylindrical shell:
  - o Discussion of behavior of singly curved vs. doubly curved shells
  - o Definition:

    A cylindrical shell may be defined as a curved slab which has been cut form a full cylinder. The slab is bounded by two straight "longitudinal" edges parallel to the axis of the cylinder and by two curved transverse edges in planes perpendicular to the axis; the slab is curved in only one direction. The cylindrical shell is circular when the curvature is constant.
  - o Effects of shell edges on the load carrying behavior
  - o Stress resultants and stress couples of cylindrical shells



- 10 unknowns: $(N_x, N_\phi, N_{x\phi}, N_{\phi x})$, (Q_x, Q_ϕ), $(M_x, M_\phi, M_{x\phi}, M_{\phi x})$.

  → The problem is statically indeterminate.

- In most R/C cylindrical shells, M_x will be small and Q_x will be small.

  → $M_{x\phi}$, $M_{\phi x}$ are negligible.

  → 6 unknowns $(N_x, N_\phi, N_{x\phi}, N_{\phi x})$, (Q_ϕ, M_ϕ).

- Line loads: Forces applied along the free edge.

- Classification:

  - Long shells → $\frac{L}{r} \geq 2.5$

    Line loads produce significant magnitudes of M_ϕ and Q_ϕ, membrane forces become insignificant. Stresses can be estimated using the beam theory.

  - Intermediate shells → $0.5 \leq \frac{L}{r} < 2.5$

  - Short shells → $\frac{L}{r} < 0.5$

    The line loads produce internal forces generally in the region near the longitudinal edge. Greater part of the shell behaves with membrane values.

- Beam theory for long cylindrical shells

  - For long shells the stresses can be estimated closely by beam theory. The shell is considered as a beam of a curved cross section between end supports.

  - Assumption: Relative displacements within each transverse cross section are negligible.

- The extension of the beam method of analysis to multiple shells

- Membrane theory
  - For a certain class of shells which the stress couples are an order of magnitude smaller than the extensional and in-plane shear stress resultants, the transverse shear stress resultants are similarly small and may be neglected in the force equilibrium.
  - The assumption is valid only if at least one radius of curvature is finite. (Flat plates are excluded from resisting transverse loading in this manner.)
  - The shell may achieve force equilibrium through the action of in-plane forces alone. Hence, the state of stress in the shell is completely determined by equations of equilibrium, i.e., the shell is statically determinate.
  - The boundary conditions must provide for those shell edge forces which are computed from the equations of equilibrium. The boundary conditions must also permit those shell edge displacements (translations and rotations) which are computed from the forces found by the membrane theory.

- Approximation methods of analysis
  - Energy method
    - Strain energy and potential energy
  - Rayleigh-Ritz methods
  - Galerkin method

Nonlinear analysis using doubly-curved isoparametric thin shell R/C elements

- Assumptions:
  - The actual reinforcing bars are represented by equivalent anisotropic steel layers by making appropriate adjustments to the elastic-plastic incremental strain-to-stress transformation matrix.

These layers carry uniaxial stress only in the same direction as the actual bars. Dowel action is neglected.

- Strain compatibility between steel and concrete is maintained.

- Constitutive law for isotropic materials

  In matrix form: $\{\sigma\} = [C]\{\varepsilon\}$

- Stress resultant-strain, and stress couple-curvature relations

  In matrix form: $\{N\} = [D]\{\varepsilon\}$

  → More complex material properties can be represented through a modification of $[C]$ and $[D]$.

- Incremental stress-strain relation

  $\{d\sigma\} = [D]\{d\varepsilon\}$

  where $\{d\sigma\}$ = increment of total stress vector,

  $[D]$ = the elastic-plastic incremental strain-to-stress transformation matrix, and

  $\{d\varepsilon\}$ = increment of total strain vector.

- Generalized stress-strain relation for full concrete elements

$$\begin{Bmatrix} dN \\ dM \end{Bmatrix} = \int_{-H/2}^{H/2} \begin{bmatrix} [M] & [M]h \\ [M]h & [M]h^2 \end{bmatrix} \begin{Bmatrix} d\varepsilon_0 \\ dk \end{Bmatrix} dh$$

  where dN = increment of direct stress resultant,

  dM = increment of bending stress resultant,

  $d\varepsilon_0$ = mid-wall component of strain increment,

  dk = bending component of the strain increment, and

  H = the wall thickness.

- Numerical integration in finite element analysis

- At each integration point of the element the shell wall is divided into a number of stations with constant intervals, T, through the thickness.
- Using $[D]$, the matrix $[M]$ is evaluated at the start of each increment and for each of the stations.

- Incremental stress-strain relation for steel layer element with anisotropic properties consistent with uniaxial condition

  $d\sigma_i = (R_i D_i) d\varepsilon_i$

  where $R_i = \frac{t_i}{T}$ and t_i = the portion of the equivalent steel area contributed to the i th integration station.

- The generalized stress-strain relation at station *i*:

  $$\begin{Bmatrix} dN \\ dM \end{Bmatrix} = \int_{-H/2}^{H/2} \begin{bmatrix} R_i D_i & R_i D_i h \\ R_i D_i h & R_i D_i h^2 \end{bmatrix} \begin{Bmatrix} d\varepsilon_0 \\ dk \end{Bmatrix} dh$$

  For integration stations with no reinforcement contribution, the R_i multiplier will be zero.

Design of cylindrical concrete shells

- ASCE method – Solution is based on superposition.
- Steps of the method:
  1. Assume that the surface loads are transmitted to the supports by direct stress only. This is the membrane solution, equivalent to assuming the structures to be statically determinate. This assumption leads to displacements and reactions along the long edges which are inconsistent with the actual boundary condition.

     → Surface loads – The membrane analysis of a cylindrical shell subjected to surface loads involves only successive differentiations and interpolations of the radial and tangential components of the surface load. This is always possible for continuous loads. For most conditions the surface loads on the shell are uniform.

→ Representation of loads using Fourier series

2. Apply line loads along the long edges. The stresses from line loads are superimposed with those from membrane solution.

   → Line loads along the longitudinal edges

   → The effect of edge line loads on the shell

- Definitions
  - The *primary system* is obtained by reducing the general theory to the membrane theory which reduces to three equations with three unknowns, and hence to a statically determinate system.
  - The *errors* correspond to the incompatible edge effects.
  - The *corrections* correspond to unit edge effects derived from a bending theory.
  - *Compatibility* is obtained by determining the size of the corrections required to remove the errors in the membrane theory.

- Edge beams
  - Vertical beams
    - Usually employed for long shells, where the principal structural action is *longitudinal bending*.
  - Horizontal beams
    - Typically used with short shells, where the principal structural action is *transverse arching*.

- Edge effects
  - $K > 0$: The edge effects tend to damp rapidly and are usually confined to a narrow zone at the edge. Thus in these shells the membrane theory will often be valid throughout the entire shell except just at the boundaries.

– $K = 0$ (either r_x or r_y is zero): The edge effects are damped out but tend to extend further into the shell than for shells of positive curvature.
– $K < 0$: The damping is markedly less than for the others. The boundary effects tend to become significant over large portions of the shell.

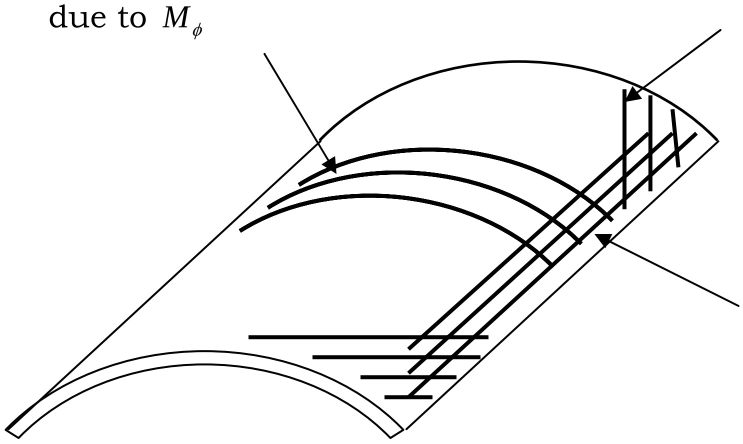
- Reinforcement patterns of a simply supported single shell

  I: Longitudinal reinforcement in the edge beam

  II: Transverse membrane and bending reinforcement

  III: Shear reinforcement

  IV: Negative moment bending reinforcement near the diaphragms



❑ Design considerations

- Analysis is based on homogeneous isotropic material assumption. Principal stresses are computed from the values for the in-plane stress resultants, and stress trajectories are drawn; reinforcement is placed to carry tensile stresses.

- Transverse reinforcement must be provided to resist bending moment M_ϕ. Transverse shear Q_ϕ and thrust N_ϕ are usually small and do not influence design.
- Other Design Issues
  - Boundary conditions; edge beam compatibility and design
  - Instability – Buckling
  - Minimum reinforcement requirements

- Design of containment structures (thick shell effects)

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Outline 13

Non-destructive Testing for Concrete Structures

Demand for the development of non-destructive testing (NDT) techniques for concrete structures has increased with the growing concern about the deteriorating condition of the World's infrastructure. Efficient and accurate imaging techniques are needed for a reliable evaluation of safety and serviceability of concrete structures. Although, presently, imaging is routinely used in various fields, implementation of these technologies in NDT of civil engineering systems, especially of concrete structures, offers many challenges and requires additional development due to the composite nature of the concrete material and the complexities of reinforced or prestressed concrete systems.

This lecture introduces the basic principles of various imaging techniques associated with several NDT methods applicable to concrete structures. The techniques considered are radiography, radioactive computerized tomography, infrared thermography, radar imaging and acoustic imaging. Special considerations regarding the applicability and accuracy of these techniques for the condition assessment of concrete structures are discussed, and examples of imaging applications are given.

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Outline 14

Earthquake Risk Assessment and Hazard Mitigation

Seismic risk assessment and hazard mitigation for urban infrastructures located in seismic regions is a challenge faced by many countries around the world, especially those with infrastructures known for their variability in seismic resistance and quality of construction. Two recent major earthquakes that hit the densely populated urban areas in Northwest Turkey resulted in a large-scale destruction and loss of life. Scientific studies indicate a high probability of occurrence of another severe earthquake along the North Anatolian Fault in this region is quite high in the next thirty years. This situation presents a serious threat to the large building stock and their occupants, lifelines, and critical facilities, which are primarily reinforced concrete structures. Limited time and funds do not allow for a detailed evaluation of the entire inventory of structures according to seismic codes. Thus, there is an urgent need for a systemic strategy that will allow for a reliable assessment of the seismic hazard risk of existing structures through an effective and economical methodology. Prioritization of these structures according to their hazard risk, and implementation of the necessary mitigation measures are required. In this lecture we present seismic resistance issues associated with reinforced concrete structures, and the methodologies and advances in large-scale seismic risk evaluation and hazard reduction are presented.

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Outline 15

High-rise Concrete Buildings

High-rise buildings are a distinct part of urban life around the world. Scarcity of land in the urban settings, increasing demand for business and residential space, economic growth, technological advancements, and innovations in structural systems and materials have lead to an impressive evolution of these systems. Desire for aesthetics in urban design and the concept of city skyline, cultural significance and prestige, and human aspiration to build higher are also among the forces behind the demand for high-rise buildings. During the initial rapid development phase, the race to the sky was pushed from the 10 story Home Insurance Building of Chicago built in 1885 to the 102 story Empire State Building in 1931, in less than 50 years. Following a hiatus of over thirty years, developments in high-rise buildings were revived once again in 1960s, reaching new heights by leaving the shear frame systems behind and recognizing a hierarchy of new structural forms such as interacting systems, partial tubular systems, tubular systems, and hybrid systems. Recent developments in the area of high performance materials have further fueled this evolution. The tallest buildings of the 21st century are in excess of 1500 ft (457 m) high, and the dreams of mile-high building and sky cities may be hindered by issues other than technical feasibility. Increase in height of buildings leads to enhanced challenges including structural safety, control of deflections, human comfort, fire safety, exit strategies, crowd control, and security. Owing to the advances in load prediction methods and structural analysis tools, specialty design process, and quality construction practice, high-rise buildings are among the safest structures against wind and seismic loads. In addition, efficient structural systems coupled with passive and active damping/control systems help reduce structural vibrations to prevent discomfort. However, accidental and deliberate extreme loadings such as impact and blast effects may jeopardize structural stability especially when coupled with fire. These effects bring the issues of fire safety, security and exit concerns. Among several classes of specialty structures, high-rise buildings generally allow easy public access and their symbolism of cultural and economical dominance makes them well-defined targets. Failure of a high-rise building is often of a colossal type having a profound social and economical impact. Thus, especially in the wake of the collapse of World Trade Center (WTC) towers, there will be a considerable increase in the performance demand for high-rise buildings, particularly against progressive failure mechanisms. Structural systems that maintain stability for extended periods under fire by means of redundancy, use of fire resistant materials, and introduction of efficient egress routes are essential elements of an effective emergency exit strategy. This lecture reviews evolutionary development aspects of high-rise building systems with emphasis on concrete tall buildings and the recent increase of the role of high performance concrete in developing innovative high rise building solutions. Forward-looking strategies on fire safety and egress systems will also be discussed.

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Outline 2

Micro-cracking of Concrete / Behavior under Multiaxial Loading

- Strength properties of concrete
  - Concrete is a complex material consisting of coarse aggregate, sand, cement gel, unhydrated cement particles, capillary and gel pores, pore water, air voids etc.
  - Concrete is multi-phased at several levels:
    - Micro-level: multi-element
    - Meso-level: multi-phase
    - Macro-level: homogeneous
  - A mortar-aggregate system is considered to be at meso-level.
  - Cracking stages:

    1. Bond cracks at $\frac{\sigma}{f_c^{'}} \cong 0.3$
    2. Mortar cracks at $\frac{\sigma}{f_c^{'}} \cong 0.75 - 0.80$
    3. Unstable crack propagation
    4. Failure

    where $f_c^{'}$ = reference strength, which is usually obtained from uniaxial testing (compression) of standard size cylinders.

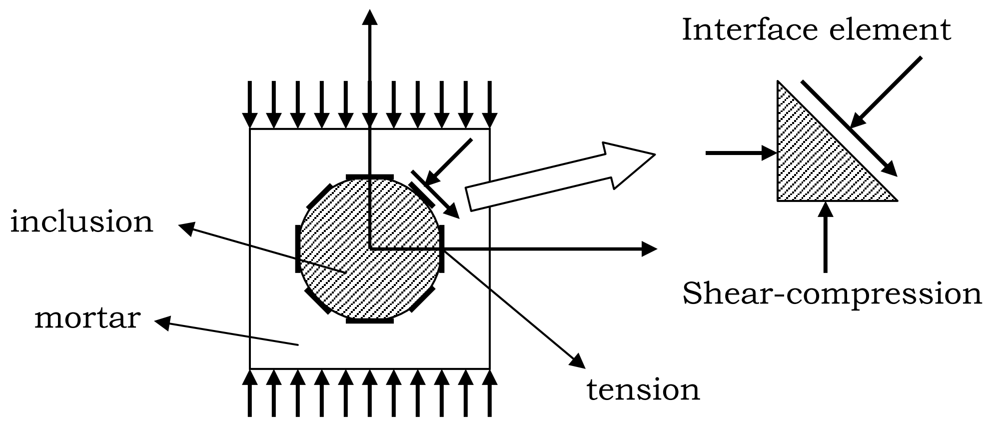
- Stress-strain behavior of concrete
  - Stress-strain $(\sigma - \varepsilon)$ relation is used to determine material property, such as the modulus of elasticity (Young's modulus), E, strength, and failure strain as a basis for analysis and design purposes.

- For concrete, since it is mainly used in compression, the modulus of elasticity, E_c , is determined from compressive stress-strain curves that are generally obtained from uniaxial compressive tests.

Deflection and failure behavior as affected by microcracking

- Model study:

  Deformation and fracture of a concrete model:



- Interface elements are used to study the influence of matrix-inclusion that is bond effect and particle-interaction phenomena in modeling.
- Precise analysis should account for the influence of nonhomogeneity, models of interfacial behavior, and include recognition of interface debonding and resulting progressive fracture.
- Sophisticated analysis shows that bond failure initiates due to compression-shear and that the failure behavior is controlled by microcracking due to direct tensile stresses and splitting of concrete.

- Two debonding modes:
  1. Compression-shear (including pure shear) failure at interface
  2. Tensile-shear (including pure tension) debonding

Compression-shear strength

- The compression-shear strength of concrete is based on:
  1. Chemical bond which is an attraction between the mortar and aggregate constituents.
  2. Mechanical bond which results from the interlocking of the mortar and the aggregate in the irregularities of the aggregate surfaces.

Uniaxial compression

- Inelastic volume increase → "dilatancy"
- The behavior is related to microcracking.

Uniaxial tension

- $f_t^{'} \cong 4\sqrt{f_c^{'}}$

Bending test

- Bending test

  Modulus of rupture: $f_r^{'} \cong 7.5\sqrt{f_c^{'}}$

Strain rate effects

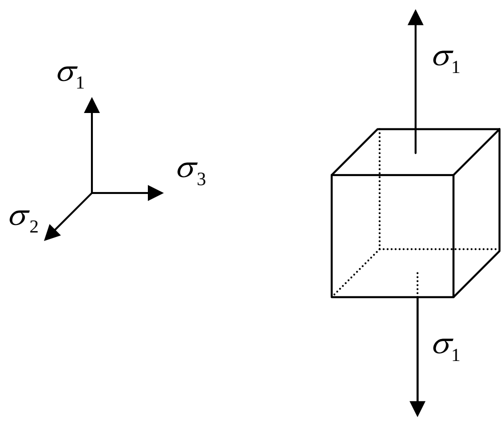
Stress-strain under cyclic loading

Behavior of concrete in combined stress

- In many structural situations concrete is subjected to multiaxial state of stress.
  - Beams: tension, compression + shear
  - Slabs: compression + shear
  - Thin shells: biaxial stresses
  - Thick shells (containments etc.): triaxial stresses

→ Any combined stress state can be reduced to three normal stresses acting on three mutually perpendicular planes. These three normal stresses are the principal stresses, and the shear stresses acting on these planes are zero (eigenvalue problem). Thus, uniaxial, biaxial, and triaxial stress states are considered.

- Uniaxial loading



- σ_1, σ_2, σ_3 are principal stresses.
- σ_2, σ_3 = 0 → unaxial condition,

  $\sigma_1 < 0$ → uniaxial compression,

  $\sigma_1 > 0$ → uniaxial tension.
- The strength of concrete depends on the type of mixture, the properties of aggregate, and the time and quality of curing. Concrete is neither homogeneous nor isotropic. The non-isotropic feature of concrete makes its compressive strength different from its tensile strength.

- Uniaxial compressive strength

  The compressive strength, $f_c^{'}$, is obtained on standard 6 in.-by-12 in. cylinders cured under standard laboratory conditions and tested at a specified loading rate at 28 days of age.

Normal strength concrete is considered to have a compressive strength range from 3000 psi to 6000 psi.

(Discussion in this outline pertains to the behavior of normal strength concrete.)

ACI Code suggests:

$E_c = 33 w_c^{1.5}\sqrt{f_c'}$ for 90 < w_c < 155 lb/ft³

where w_c = the density of concrete. For normal weight concrete,

$E_c = 57000\sqrt{f_c'}$ psi

- "Dilatancy": Inelastic volume increase.

  → It is related to microcracking and brings a complex deformation and failure behavior including dependency on hydrostatic pressure in failure.

- Uniaxial tensile strength

  Cylinder splitting test – Tensile splitting strength, f_t'.

- Flexural test

  ACI Code suggests, for normal weight concrete,

  $f_r = 7.5\sqrt{f_c'}$ (modulus of rupture)

  and

  $f_r = 1.09 f_{ct} \le 7.5\sqrt{f_c'}$

  for lightweight concrete, where f_{ct} = splitting tensile strength.

- Short-time response in compressive loading tests

  From the uniaxially compressive testing results, the short-time response curves consist of an initial relatively straight-line portion and a curve-to-level portion that contains the maximum stress. The curve descends after the maximum stress is reached. Different ways of testing result in different characteristics of the after-peak behavior.

Maximum strain for normal density concrete generally ranges from 0.002 to 0.003 and from 0.003 to 0.0035 for lightweight concrete.

In general, the modulus of elasticity calculated from the initial straight portion of the stress-strain curve is larger when the strength of the concrete is higher.

- Testing conditions, such as specimen boundary condition and loading rate, influence stress-strain curve characteristics.

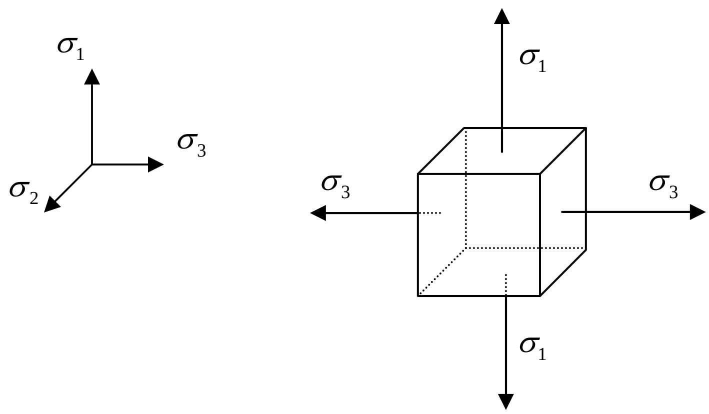
- Long-time response in compressive loading tests
  (a) The influence of creep deformation → concrete deforms under sustained load.
  (b) The influence of fatigue → the fatigue strength of concrete depends not only on the static strength of concrete but also on moisture condition, age, and rate and range of loading.

- Several observations on the stress-strain curves of concrete of various strengths by the Portland Cement Association (PCA):
  (a) The lower the strength of concrete, the higher the failure strain.
  (b) The length of the initial relatively linear portion increases with the increase in the compressive strength of concrete.
  (c) There is a marked reduction in ductility with increased strength.

- Biaxial stress state
  - Effect of microcracking
  - Biaxial stress state is considered with principal stresses acting only in two directions.
  - When concrete is compressed in biaxial stress state:
    1. The observed compressive strength increases.

2. The tensile ductility is greater than that under unaxial compression. For compression-tension region ductility decreases with the increase in tension.
3. Elastic limit is shifted up.
4. As the failure point is approached, an increase in volume occurs as the compressive stress continues to increase. In biaxial compression volumetric strains increase.
5. Failure modes depend on various stress combinations.

- Biaxial loading



- $\sigma_2 = 0$ → biaxial condition,

  $\sigma_1 < 0$ and $\sigma_3 < 0$ → biaxial compression,

  $\sigma_1 > 0$ and $\sigma_3 > 0$ → biaxial tension,

  $\sigma_1 < 0$ and $\sigma_3 > 0$ or $\sigma_1 > 0$ and $\sigma_3 < 0$ → shear.
- Various forms of analytical expression of the strength of concrete under combined stress (biaxial stress) have been proposed.
- The biaxial strength of concrete in compression is higher than its uniaxial strength.
- The biaxial strength of concrete in tension is lower than its uniaxial tension strength.

- Constitutive equation
  - The constitutive equation for an isotropic elastic material under biaxial stresses:

$$\begin{bmatrix} \sigma_{\bar{y}} \\ \sigma_{\bar{x}} \\ \sigma_{\overline{xy}} \end{bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & G \end{bmatrix} \begin{bmatrix} \varepsilon_{\bar{y}} \\ \varepsilon_{\bar{x}} \\ \varepsilon_{\overline{xy}} \end{bmatrix} \text{ or } \bar{\sigma} = \bar{C}\bar{\varepsilon}$$

where $\sigma_{\bar{y}}$, $\sigma_{\bar{x}}$, $\sigma_{\overline{xy}}$ = stresses in the local coordinate systems,

$\varepsilon_{\bar{y}}$, $\varepsilon_{\bar{x}}$, $\varepsilon_{\overline{xy}}$ = strains in the local coordinate systems,

E = modulus of elasticity,

G = shear modulus,

ν = Poisson's ratio, and

$\bar{C}$ = the biaxial constitutive matrix of an isotropic, continuous, and elastic material.

  - The constitutive matrix accounting for the compression-shear cracks

(a) In the local coordinate systems:

$$\bar{C}_m = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Note that actually the shear modulus is a non-zero value, αG, to reflect the effect of shear transfer across cracked surfaces (aggregate interlock, dowel action etc.) ($\alpha < 1$). See outline on shear transfer.

(b) In the global coordinate systems:

$$C_m = \frac{E}{1-\nu^2}\begin{bmatrix} \cos^4\phi + \sin^4\phi + 2\nu\sin^2\phi\cos^2\phi & & \text{sym.} \\ \nu\left(\cos^4\phi + \sin^4\phi\right) + 2\sin^2\phi\cos^2\phi & \cos^4\phi + \sin^4\phi + 2\nu\sin^2\phi\cos^2\phi & \\ \left(\frac{1-\nu}{2}\right)\sin 2\phi\cos 2\phi & \left(\frac{\nu-1}{2}\right)\sin 2\phi\cos 2\phi & 2\sin^2\phi\cos^2\phi\left(1-\nu\right) \end{bmatrix}$$

- The constitutive matrix accounting for the tensile-shear cracks in the local coordinate systems:

  (a) In the local coordinate systems:

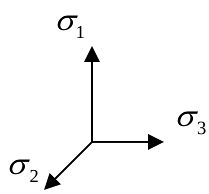
$$\bar{C}_m = \frac{E}{1-\nu^2}\begin{bmatrix} 1-\nu^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

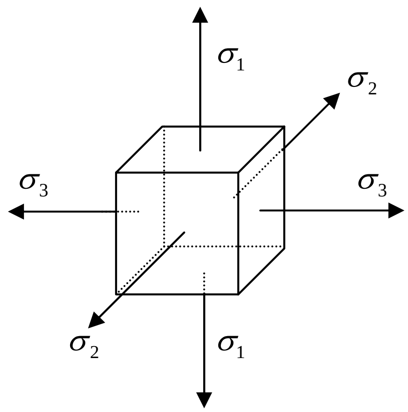
  (b) In the global coordinate systems:

$$C_m = \frac{E}{1-\nu^2}\begin{bmatrix} \left(1-\nu^2\right)\sin^4\phi & & \text{sym.} \\ \left(1-\nu^2\right)\sin^2\phi\cos^2\phi & \left(1-\nu^2\right)\cos^4\phi & \\ \left(\nu^2-1\right)\sin^3\phi\cos\phi & \left(\nu^2-1\right)\sin\phi\cos^3\phi & \left(1-\nu^2\right)\sin^2\phi\cos^2\phi \end{bmatrix}$$

- Deformational and fractural behavior of the analytical model under uniaxial compressive stress is affected by debonding at the interface between the matrix and the inclusions.
- Fracture occurs by progressive cracking, starting at the interface and resulting in separation between the two constituents.
- Ultimate failure is obtained by progression of the interfacial cracks into the matrix, forming crack patterns parallel to the loading direction.
- Bond failure at the interface, resulting in local deformation accumulation, affects the overall nonlinear deformational behavior.

- Triaxial loading





- In normal strength concrete fracture occurs by progressive cracking, starting at the interface and resulting in separation between the two constituents.
- σ_1, σ_2, $\sigma_3 \neq 0$ → triaxial condition,

  $\sigma_1 < 0$ and $\sigma_2 < 0$ and $\sigma_3 < 0$ → triaxial compression,

  $\sigma_1 > 0$ and $\sigma_2 > 0$ and $\sigma_3 > 0$ → triaxial tension,

  $\{\sigma_1 < 0$ and $\sigma_2 > 0$ and $\sigma_3 > 0\}$ or $\{\sigma_1 > 0$ and $\sigma_2 < 0$ and $\sigma_3 < 0\}$ or

  $\{\sigma_1 > 0$ and $\sigma_2 < 0$ and $\sigma_3 > 0\}$ or $\{\sigma_1 < 0$ and $\sigma_2 > 0$ and $\sigma_3 < 0\}$ or

  $\{\sigma_1 > 0$ and $\sigma_2 > 0$ and $\sigma_3 < 0\}$ or $\{\sigma_1 < 0$ and $\sigma_2 < 0$ and $\sigma_3 > 0\}$

  → triaxial shear.
- Nilson (1997) reported several observations regarding the triaxial strength of concrete:

  "*1. In a state of equal triaxial compression, concrete strength may be an order of magnitude larger than the uniaxial compressive strength.*

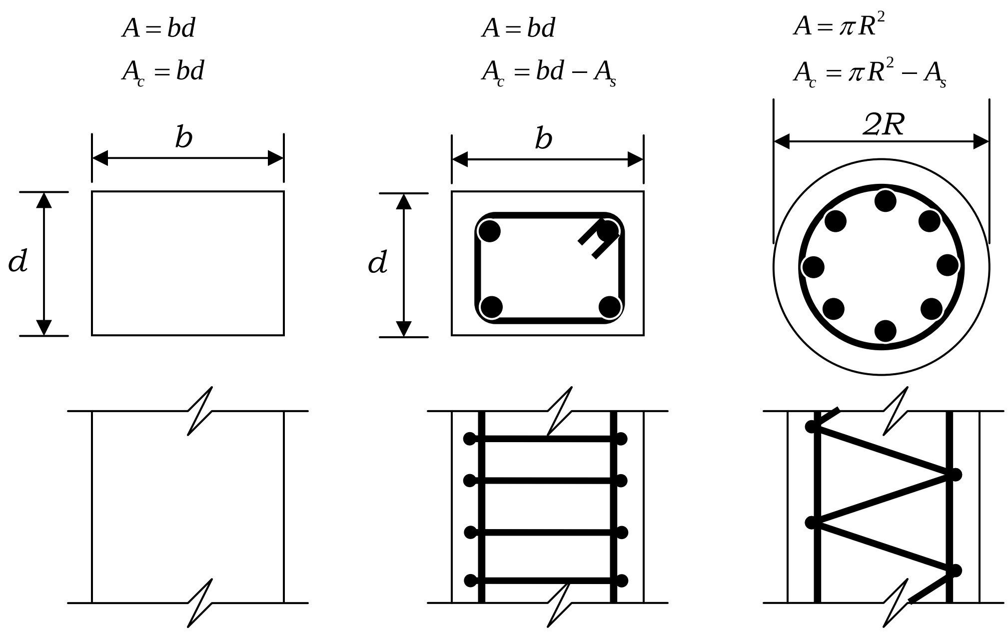
  2. For equal biaxial compression combined with a smaller value of compression in the third direction, a strength increase greater than 20 percent can be expected.

  3. For stress states including compression combined with tension in at least one other direction, the intermediate principal stress is of *little consequence, and the compressive strength can be predicted safely…”*

- Failure of concrete in triaxial stress state
  1. The failure surface in triaxial stress state is described by a hydro-axis and deviatoric sections. Intersection of the failure surface with $\sigma_3 = 0$ plane gives $\sigma_1 - \sigma_2$ biaxial failure curve. Other representations involve failure description in deviatoric and hydrostatic planes.
  2. Under high confining stress the possibility of bond cracking is reduced; and failure mode shifts from cleavage to crushing of the cement paste.
  3. Concrete is a brittle material which fails through brittle cleavage (splitting) at the interfaces and in mortar except for high triaxial compression where shear slippage occurs resulting in a ductile behavior.

- The strength of concrete under combined stress cannot be determined analytically, in a precise manner, mainly due to the following reasons:
  1. The existence of cracks in concrete makes it become non-homogeneous and non-isotropic. Traditional approach using continuum mechanics cannot be directly applied to develop the analytical expression of strength of concrete in this circumstance. Currently the prediction of concrete strength under combined stress relies on experimental results.
  2. In many situations it is not possible to determine all the acting stresses and their directions in concrete structures.

- Concrete confinement by transverse and longitudinal reinforcement
  - Different confinement scenarios that may be used in practice are illustrated below:



Plain concrete | Longitudinal + transverse reinforcement with ties (hoop) | Helical reinforcement (spiral + longitudinal)

  - Smaller spacings result in more effective confinement.
  - Closely spaced spirals may result near uniform pressure.
  - Tests have shown that transverse reinforcement can considerably improve the stress-strain characteristics at high strains.

- In practice, concrete may be confined by transverse reinforcement (in combination with the longitudinal reinforcement); Closely spaced steel spirals or hoops.
- Concrete becomes confined when at stresses approaching the uniaxial strength, the transverse strains become very high and concrete bears out against the transverse reinforcement.

- Effect of different confinement scenarios on axial capacity

  Total axial capacity:

  $P_0 = 0.85 f_c^{'} A_c + A_s f_y$

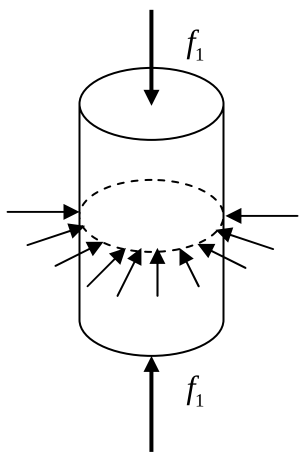
  - The factor 0.85 is used for vertically cast members for the effects of:

    — bleeding in long columns (strength variation along the axis)

    — eccentricity effects

    — size effects

  - The effect of the spiral is comparable to the specimens subjected to lateral pressure f_2 in which case



e.g. for f_2 = 1000 psi, $f_c^{'}$ = 3000 psi, $f_1 \cong$ 7000 psi.

If h = core diameter (out-to-out) and A_n = core area, the above result implies that the contribution of the lateral pressure to the ultimate load = $4.1 f_2 A_n$.

  - Define A_b = area of spiral and s = pitch, then the volumetric steel percentage is

$$\rho_s = \frac{\pi h A_b}{\pi h^2 s/4} = \frac{\text{volume of spiral}}{\text{total volume of concrete in } s} = \frac{4A_b}{hs}$$

  - The lateral confining pressure may be expressed approximately in terms of the tension in spiral reinforcement.

$$2A_s f_s = f_2 hs$$

$$f_2 = \frac{2A_s f_s}{hs} = \frac{\rho_s}{2} f_s \quad (f_s = f_y \text{ in tension at failure})$$

Since the contribution of the lateral pressure to the ultimate axial load = $4.1 f_2 A_n$, it turns out that

$$4.1 f_2 A_n = 4.1 \frac{f_s}{2} \rho_s A_n = 2.05 \rho_s f_s A_n$$

Validity of this equation has been verified by the tests where the coefficient ranged from 1.7 to 2.9.

- When P_0 is reached, the shell of concrete outside the core will fail and the load capacity of the column will reduce. But due to confinement no buckling will occur in the longitudinal steel.

- ACI recommends that the contribution of the transverse reinforcement to the strength should be at least equal to the contribution of the concrete cover.

$$0.85 f_c^{'} (A_c - A_n) = 2.05 \rho_s f_s A_n \cong 2 \rho_s f_s A_n$$

$$\rho > \rho_s = 0.425 \left(\frac{A_c}{A_n} - 1 \right) \frac{f_c^{'}}{f_y}$$

- Other confinement mechanism
  - Confinement by steel shells and FRP sheets. Discuss various models.

- Failure theories for concrete under multiaxial loading and constitutive relations are summarized in Outline 3.

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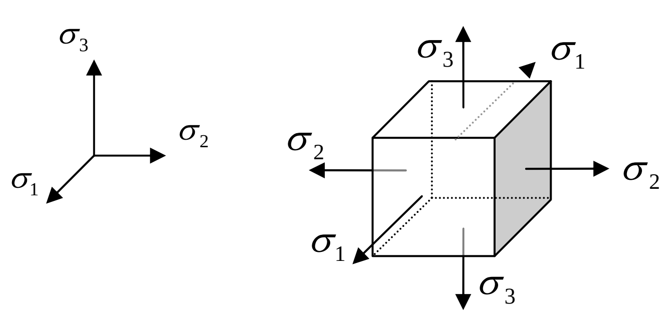
Outline 3

Failure Theories and Concrete Plasticity

- Failure of concrete
  - Concrete is a brittle material which fails through brittle cleavage (splitting) at the interfaces and in mortar except for high triaxial compression where shear slippage occurs resulting in a ductile behavior. Failure occurs by tensile splitting with the fractured surface orthogonal to the direction of the maximum tensile stress or strain.

- Prediction of multiaxial behavior
  - In general the material properties are known from simple tests such as uniaxial loadings giving $f_c^{'}$ and f_t . Prediction involves strength calculation in multiaxial situations given the data from the uniaxial tests.
  - In the field of concrete research attempts have been made to apply some of the classical failure theories to concrete. These theories were altered to overcome some disadvantages or otherwise improve their agreement with the phenomenological behavior of concrete. New failure theories were therefore formed.

- Principal stresses:



- Some classical failure theories
  - Maximum principal stress theory
  - Maximum principal strain theory
  - Maximum shear stress theory
  - Internal friction theory
  - Maximum strain energy theory
  - Distortion energy theory
  - Fracture mechanics based theories – stress intensity, toughness – fracture energy release.

  → These introduce either limitations or contradictions when applied to concrete. Modifications to concrete have resulted:
  - Internal friction-maximum stress theory
  - Octahedral shear-normal stress theory
  - Newman's two-part criterion
  - Local deformation theories, etc.

  → Extensive research has been conducted to develop better theories: Elastic-plastic, plastic-fracturing, endochronic, bounding surface etc. approaches.

- Maximum principal stress theory (elastic behavior)
  - $\sigma_1 > \sigma_2 > \sigma_3$
  - Failure occurs when:

    $\sigma_1^{\max} = \sigma_t$ (Tensile strength)

    $\sigma_3^{\max} = \sigma_c = f_c^{'}$ (Compressive strength)

    → It does not reflect splitting nature of failure.

- Maximum principal strain theory (elastic behavior)
  - Failure occurs when:

    $\varepsilon^{\max} = \varepsilon_{\text{limit}} = \varepsilon_t$

- Maximum shear stress theory
  - $\sigma_1 > \sigma_2 > \sigma_3$
  - Failure occurs when:

    $\sigma_1 - \sigma_3 + \lambda(\sigma_1 + \sigma_3) = 2\sigma_s$

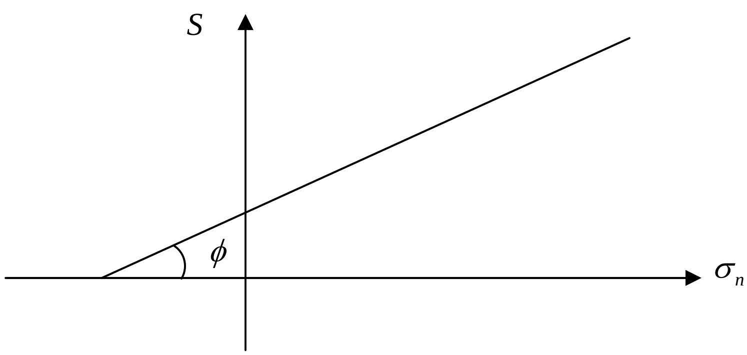
    where $\sigma_1 - \sigma_3$ = shear stress,

    $\lambda(\sigma_1 + \sigma_3)$ = portion of the volumetric stress,

    σ_s = a critical shear stress value (e.g. under pure shear)
  - For metals, $\lambda \cong 0$. For brittle materials, $\lambda \neq 0$.
  - For $\lambda = 0$, the failure criterion becomes

    $\frac{\sigma_1 - \sigma_3}{2} = \sigma_s$
  - The theory gives equal uniaxial tensile and compressive strengths. It is also independent of intermediate stress σ_2. (pressure sensitivity)

- Internal friction theory



  - Consider the effect of normal stress on shear strength:

    $S = K - \tan\phi\sigma_n$

    where S = shear strength,

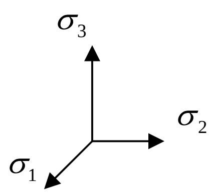
    K = cohesive strength,

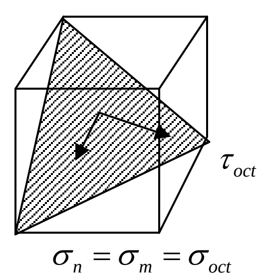
    ϕ = angle of internal friction, and

    σ_n = normal stress.
  - Compression increases S and tension decreases S.

- Mohr's theory (generalization of internal friction theory)
  - $S = \tau = f(\sigma)$
  - σ_2 has no effect on strength.
  - $f(\sigma)$ is the envelop of all the circles corresponding to the various states of stress at which failure takes place.

- Octahedral shear and normal stress theory





  - $\sigma_1 > \sigma_2 > \sigma_3$
  - Failure occurs when the octahedral stress exceeds a limiting value.

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

In uniaxial tension and compression, $\tau_{oct} = \frac{\sqrt{2}}{3}\sigma_i$, i = 1, 2, 3

  - The failure criterion provided by octahedral shear stress theory:

$$\tau_{oct} = \tau_{\text{limit}} \Rightarrow (\tau_{\text{limit}})_{tension} = (\tau_{\text{limit}})_{compression}$$

    - This gives the same ultimate strength for uniaxial tension and compression. → It is not valid for concrete.
    - Inclusion of $\sigma_{oct} = \sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ improves the prediction.

  - Bresler, Pister tests

$$\frac{\tau_a}{f_c'} = k_1 + k_2\left(\frac{\sigma_a}{f_c'}\right) + k_3\left(\frac{\sigma_a}{f_c'}\right)^2$$

where $\tau_a = \frac{1}{\sqrt{15}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$ and

$$\sigma_a = \sigma_{oct} = \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)$$

- Invariant formulation
  - A failure criterion should be based upon an invariant function of the state of stress, i.e., independent of the choice of the coordinate systems.
  - Stress invariants

    $I_1 = \sigma_1+\sigma_2+\sigma_3$, (more suitable for applying to concrete)

    $I_2 = \sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1$,

    $I_3 = \sigma_1\sigma_2\sigma_3$

    where σ_1, σ_2, σ_3 are principal stresses.

- General stress state representation
  - $\begin{bmatrix}\sigma_x & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \sigma_y & \tau_{yz}\\ \tau_{zx} & \tau_{zy} & \sigma_z\end{bmatrix}$ or $\begin{bmatrix}\sigma_{11} & \sigma_{12} & \sigma_{13}\\ \sigma_{21} & \sigma_{22} & \sigma_{23}\\ \sigma_{31} & \sigma_{32} & \sigma_{33}\end{bmatrix} = \sigma_{ij}$, $\sigma_{ij} = \sigma_{ji}$ $\forall\, i, j = 1, 2, 3$

    → $\sigma_{11} = \sigma_1$, $\sigma_{22} = \sigma_2$, $\sigma_{33} = \sigma_3$
  - $\sigma_m = \frac{1}{3}(\sigma_{11}+\sigma_{22}+\sigma_{33}) = \frac{1}{3}\sigma_{ii} = \frac{1}{3}I_1$
  - Average normal stress: $\sigma_a = \frac{1}{3}I_1$
  - Average shear stress: $\tau_a = \sqrt{\frac{2}{15}}\left[I_1^2 - 3I_2\right]^{1/2}$

- Deviatoric stress

  - $S_{ij} = \sigma_{ij} - \sigma_m \delta_{ij}$, δ_{ij} = Kronecker's delta $\left(\delta_{ij} = \begin{cases} 1, & \forall i = j \\ 0, & \forall i \neq j \end{cases} \right)$

    $S_{11} = \sigma_{11} - \sigma_m$ $\quad S_{22} = \sigma_{22} - \sigma_m$ $\quad S_{33} = \sigma_{33} - \sigma_m$

    $S_{12} = \sigma_{12}$, $\quad S_{21} = \sigma_{21}$, $\quad S_{31} = \sigma_{31}$

    $S_{13} = \sigma_{13}$ $\quad S_{23} = \sigma_{23}$ $\quad S_{32} = \sigma_{32}$

    where S_{11}, S_{22}, S_{33} are principal stresses.

  - Discussion of physical meaning of deviatoric and hydrostatic stresses.

- Deviatoric stress invariants

  - $J_1 = S_{ij} = S_{11} + S_{22} + S_{33}$

    $$J_2 = \frac{1}{2} S_{ij} S_{ji} = \frac{1}{2}\left(S_{11}{}^2 + S_{22}{}^2 + S_{33}{}^2 \right)$$

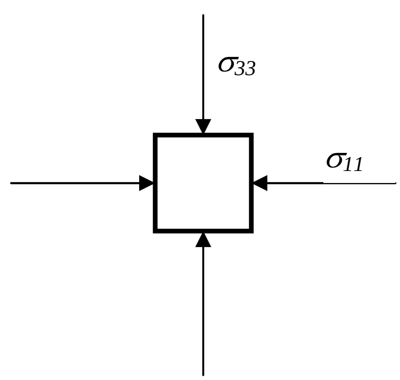
    $$J_3 = \frac{1}{3} S_{ij} S_{jk} S_{ki} = \frac{1}{3}\left(S_{11}{}^3 + S_{22}{}^3 + S_{33}{}^3 \right)$$

- Biaxial loading $(\sigma_{22} = 0)$

  - $S_{11} = \frac{1}{3}(2\sigma_{11} - \sigma_{33})$

    $S_{22} = -\frac{1}{3}(2\sigma_{11} + \sigma_{33})$

    $S_{33} = \frac{1}{3}(2\sigma_{33} - \sigma_{11})$



  - $I_1 = \sigma_{11} + \sigma_{33}$

    $$J_2 = \frac{1}{2} \cdot \frac{1}{9}\left[(2\sigma_{11} - \sigma_{33})^2 + (\sigma_{11} + \sigma_{33})^2 + (2\sigma_{33} - \sigma_{11})^2 \right]$$

    → In general, stress invariants I_1, J_2 are used to characterize the behavior of concrete structures.

- Invariant formulation of the concrete failure

- $F\left(I_1, J_2\right)=0$
- Failure criteria –

  $3J_2+\sigma_3 I_1+\frac{I_1^{\,2}}{5}=\frac{\sigma_c^{\,2}}{9}$ and

  $\frac{K^2}{3}J_2-\frac{K^2}{36}I_1 \pm \frac{1}{2}I_1+\frac{1}{3}A_u I_1=\tau_u^{\,2}$, where A_u, τ_u are material constants.

❑ Multiaxial failure criterion

- Principal stresses based

  $F\left(\sigma_1, \sigma_2, \sigma_3\right)=0$

- Stress invariants based

  $F\left(I_1, J_2, J_3\right)=0$

- One model considering the effect of all the three stress invariants and possessing the observed features of the failure surface such as smoothness, symmetry, convexity, and curved meridians is provided.

$$f\left(\sigma_m, \tau_m, \theta\right)=\frac{1}{\bar{r}\left(\sigma_m, \theta\right)}\frac{\tau_m}{f_c^{'}}-1=0$$

where $\tau_m^2=\frac{3}{5}\tau_{oct}^2=\frac{2}{5}J_2$,

$$\sigma_m=\frac{1}{3}I_1,$$

$$\bar{r}\left(\sigma_m, \theta\right)=\frac{1}{\sqrt{5}f_c^{'}}r\left(\sigma_m, \theta\right)$$

$$r\left(\sigma_m, \theta\right)=\frac{2r_c\left(r_c^2-r_t^2\right)\cos\theta+r_c\left(2r_t-r_c\right)\sqrt{4\left(r_c^2-r_t^2\right)\cos^2\theta+5r_t^2-4r_t r_c}}{4\left(r_c^2-r_t^2\right)\cos^2\theta+\left(r_c-2r_t\right)^2}$$

$$\cos\theta=\frac{2\sigma_1-\sigma_2-\sigma_3}{\sqrt{2\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]}}$$

$$\frac{r_t}{\sqrt{5}f_c^{'}}=a_0+a_1\frac{\sigma_m}{f_c^{'}}+a_2\left(\frac{\sigma_m}{f_c^{'}}\right)^2 \text{ at } \theta=0^\circ$$

$$\frac{r_c}{\sqrt{5} f_c^{'}} = b_0 + b_1 \frac{\sigma_m}{f_c^{'}} + b_2 \left(\frac{\sigma_m}{f_c^{'}} \right)^2 \text{ at } \theta = 60^\circ$$

- In the tension-biaxial compression zone, the tensile strength is

$$\sigma_{3c} = f_t^{'} \left[1 - \frac{2}{3} \frac{\sigma_1}{1.5 f_c^{'}} \right] \left[1 - \frac{2}{3} \frac{\sigma_2}{f_c^{'}} \right]$$

- In the triaxial tension zone, the failure is defined as

$$\sigma_{ic} = f_t^{'} \quad \forall i = 1, 2, 3$$

- In the compression-biaxial tension zone, the failure is defined as

$$\sigma_{1c} = f_c^{'}$$

$$\sigma_{2c} = f_t^{'} \left[1 - \frac{2}{3} \frac{\sigma_1}{f_c^{'}} \right] = \sigma_{3c}$$

Damage model

- Incremental damage

$$dK = \frac{d\gamma_0^p}{F(I_1, \theta)}$$

where γ_0^p = plastic component of octahedral shear strain,

I_1 = volumetric stress invariant, and

$F(\sigma_{ij}, K_{max}) = 0$ → bounding surface.

- $D = \frac{r}{R}$

where r = current stress vector (distance), and R = distance to bounding surface.

When D = 1, the material is assumed to have failed.

- Constitutive modeling of concrete
  - Approaches for defining stress-strain behavior of concrete:
    - Linear and nonlinear elasticity theories
    - Elastic perfectly plastic models
    - Elastic strain hardening plasticity models
    - Plastic damage (fracturing)-type models
    - Endochronic theory of inelasticity

  - Isotropic stress model
    - The stress-strain law

      $$\sigma_{oct} = 3K_S\left(\varepsilon_{oct}\right)\varepsilon_{oct}$$

      $$\tau_{oct} = G_S\left(\gamma_{oct}\right)\gamma_{oct}$$

      where σ_{oct} = octahedral normal stress,

      ε_{oct} = octahedral normal strain,

      K_S = secant bulk modulus,

      τ_{oct} = octahedral shear stress,

      γ_{oct} = octahedral shear strain, and

      G_S = secant shear modulus.

    - The nondimensional secant bulk and shear moduli are approximated by

      $$\frac{K_S}{K_0} = ab^{-\varepsilon_{oct}/c} + d$$

      $$\frac{G_S}{G_0} = pq^{-\gamma_{oct}/r} - s\gamma_{oct} + t$$

    - The tangent bulk and shear moduli are

      $$\frac{K_T}{K_0} = a\left[1 - \frac{(\ln b)\,\varepsilon_{oct}}{c}\right]b^{-\varepsilon_{oct}/c} + d$$

$$\frac{G_T}{G_0} = p\left[1 - \frac{(\ln q)\gamma_{oct}}{c}\right] q^{-\gamma_{oct}/r} + t$$

- The elastic material stiffness matrix:

$$D = \begin{bmatrix} K+4/3G & K-2/3G & K-2/3G & 0 & 0 & 0 \\ & K+4/3G & K-2/3G & 0 & 0 & 0 \\ & & K+4/3G & 0 & 0 & 0 \\ & & & G & 0 & 0 \\ & & & & G & 0 \\ & sym. & & & & G \end{bmatrix}$$

(Young's modulus: $E = \frac{9KG}{3K+G}$, Poisson's ratio: $\nu = \frac{3K-2G}{2(3K+G)}$)

- Isotropic strain model
  - Nonlinear isotropic elastic model
  - The nondimensional secant bulk and shear moduli:

$$\frac{K_S}{K_0} = \frac{1}{1+0.52\left(\sigma_{oct}/f_c^{'}\right)^{1.09}}$$

$$\frac{G_S}{G_0} = \frac{2}{1+3.57\left(\tau_{oct}/f_c^{'}\right)^{1.7}}$$

  - The tangent bulk and shear moduli:

$$\frac{K_T}{K_0} = \frac{1}{1+1.08\left(\sigma_{oct}/f_c^{'}\right)^{1.09}}$$

$$\frac{G_T}{G_0} = \frac{2}{1+9.63\left(\tau_{oct}/f_c^{'}\right)^{1.7}}$$

- Orthotropic model
  - The concept of equivalent uniaxial strains
  - The constitutive law in terms of the material stiffness tensor D_{ijkl}

$$d\sigma_{ij} = D_{ijkl} d\varepsilon_{kl}$$

where $d\sigma_{ij}$ = the tensor of incremental stresses, and

$d\varepsilon_{ij}$ = the tensor of incremental strains.

- The matrix of the tangent stiffness tensor:

$$D = \begin{bmatrix} (1-\nu^2)E_1 & \nu(1+\nu)\sqrt{E_1E_2} & \nu(1+\nu)\sqrt{E_1E_3} & 0 & 0 & 0 \\ & (1-\nu^2)E_2 & \nu(1+\nu)\sqrt{E_2E_3} & 0 & 0 & 0 \\ & & (1-\nu^2)E_3 & 0 & 0 & 0 \\ & & & \phi G_{12} & 0 & 0 \\ & & & & \phi G_{23} & 0 \\ sym. & & & & & \phi G_{31} \end{bmatrix}$$

where E_1, E_2, E_3 = tangent Young's moduli in directions 1, 2, and 3,

$\phi = 1 - 3\nu^2 - 2\nu^3$

G_{12}, G_{23}, G_{31} = incremental shear moduli for planes parallel to coordinates 1-2, 2-3, and 3-1. $G_{12} = \alpha\sqrt{E_1E_2}$, $G_{23} = \alpha\sqrt{E_2E_3}$, $G_{31} = \alpha\sqrt{E_3E_1}$. For uncracked concrete, $\alpha = \frac{1}{2}(1+\nu)$.

- The equivalent uniaxial strain ε_{iu}

$$\varepsilon_{iu} = \frac{\varepsilon_i}{1 - \nu\dfrac{\sigma_j + \sigma_k}{\sigma_i}}$$

where ε_i = principal strain in direction *i*.

- Elastic-hardening plasticity model
  - It is developed for short-term monotonic compressive loading of concrete.
  - The constitutive relationships feature such characteristics of concrete deformational behavior as inelastic dilatancy and frictional effect and inelastic shear caused by hydrostatic pressure (hydrostatic pressure sensitivity).

- Following the incremental theory of plasticity, the total strain increments are

  $\{d\varepsilon\} = \{d\varepsilon^e\} + \{d\varepsilon^p\}$ or $d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p$

  where $d\varepsilon_{ij} = C_{ijkl}^e d\sigma_{kl}$

  $C_{ijkl}^e = \frac{1}{2G}\delta_{ik}\delta_{jl} + \left(\frac{1}{9K} - \frac{1}{6G}\right)\delta_{ij}\delta_{kl}$ = the elastic compliance tensor

- The equation of plastic flow provides

  $d\varepsilon_{ij}^e = \frac{1}{R}\left[\frac{\partial F}{\partial \sigma_{kl}} d\sigma_{kl}\right]\left(\xi_{ij} - \frac{1}{3}\xi_{ii}\right)$ (as the incremental form of elastic strain)

  $d\varepsilon_{ij}^p = \frac{1}{R}\left[\frac{\partial F}{\partial \sigma_{kl}} d\sigma_{kl}\right]\xi_{ij}$ (as the incremental form of plastic strain)

  where R and ξ_{ij} depend on the loading history, and

  F = yield function.

o Plasticity based model

- $f = 3\sqrt{3J_2 + \bar{\sigma} I_1 + \frac{I_1^2}{5}} = \bar{\sigma}$, where $\bar{\sigma}$ = equivalent stress.

  $d\bar{\sigma} = H' \cdot d\bar{\varepsilon}^P$

  $\frac{\partial f}{\partial \sigma}\{d\sigma\} + \frac{\partial f}{\partial \bar{\sigma}} d\bar{\sigma} = d\bar{\sigma}$

  $\Rightarrow \frac{\partial f}{\partial \sigma}\{d\sigma\} + \frac{3I_1}{2\bar{\sigma}} d\bar{\sigma} = d\bar{\sigma}$

  $\Rightarrow \frac{\partial f}{\partial \sigma}\{d\sigma\} = \left(1 - \frac{3I_1}{2\bar{\sigma}}\right) \cdot H' \cdot d\bar{\varepsilon}^P$

o Hardening law (Flow rule) –

$\{d\varepsilon^P\} = d\bar{\varepsilon}^P \cdot \left\{\frac{\partial f}{\partial \sigma}\right\}$

→ If the shape of the curve is assumed to expand uniformly in all directions, the flow rule is referred to as the isotropic hardening law.

- Stress increment –

  $\{d\sigma\} = [C]\cdot\{d\varepsilon^e\}$

  $\{d\varepsilon\} = \{d\varepsilon^e\} + \{d\varepsilon^p\}$ = incremental total strain

  $[C]$ = elastic strain to stress transformation matrix

  $\{d\varepsilon^e\} = \{d\varepsilon\} - \{d\varepsilon^p\}$ = elastic strain increment

  $\{d\varepsilon^p\}$ = plastic strain increment

  therefore,

  $$\{d\sigma\} = \left\{[C] - \frac{[C]\cdot\left\{\frac{\partial f}{\partial\sigma}\right\}\cdot\left[\frac{\partial f}{\partial\sigma}\right]\cdot[C]}{\left(1-\frac{3I_1}{2\bar{\sigma}}\right)\cdot H' + \left[\frac{\partial f}{\partial\sigma}\right]\cdot[C]\cdot\left\{\frac{\partial f}{\partial\sigma}\right\}}\right\}\cdot\{d\varepsilon\}$$

  Note that for perfect plasticity, $H' = 0$, this formulation, leading to a non-singular $[C]$, does not cause any numerical difficulty.

Nonlinear analysis of reinforced concrete

- Early studies, by necessity, concentrated on the behavior of isolated elements such as beams, columns, joints, etc. As facilities developed and computing capability expanded the scope broadened to include entire systems such as slabs and beams, coupled shear walls, folded plates, and shells complete with supporting beams for example. This broadening stems both from
  - the desire for better understanding of the behavior of the complete system with the possibility of therefore achieving better structural efficiency, and

- the need because of increased complexity of the problems requiring solution coupled with more severe demands being placed on the structure.

- Factors which complicate the analysis of reinforcement concrete structures
  - Analytical procedures which may accurately determine stress and deformation states in reinforced concrete members and structures are complicated due to many factors:
    1. Non-linear stress-strain relation
    2. Progressive cracking
    3. Consideration of steel reinforcement
    4. Creep and shrinkage (time-dependent behavior)
    5. Special problems (shear transfer, cyclic loading)

    → The development of finite element method permits realistic evaluation of internal stresses and displacements on which the limit requirements may be based for improved structural efficiency. Furthermore, such refined analytical solutions help in understanding and interpreting the observed behavior of structural elements from experiments.

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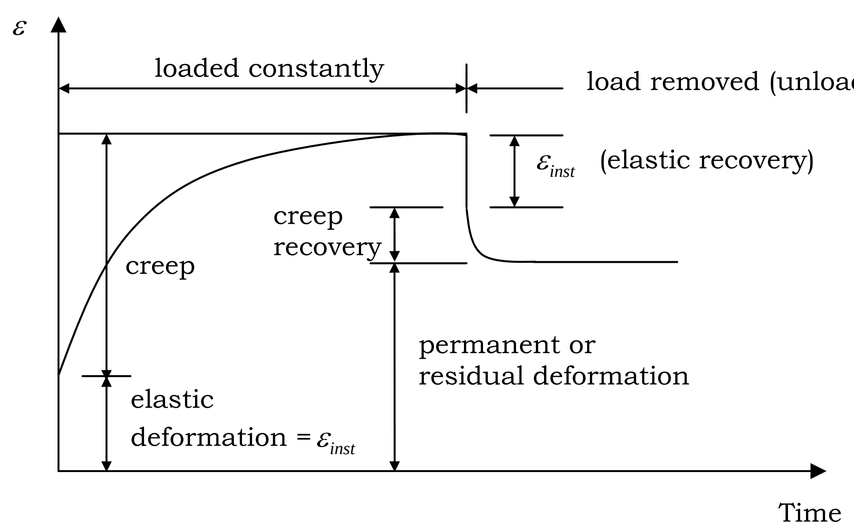
Outline 5

Creep and Shrinkage Deformation

- Creep of concrete
  - Concrete under stress undergoes a gradual increase of strain with time. The final creep strain may be several times as large as the initial elastic strain.
  - Creep is the property of materials by which they continue deforming over considerable length of time under sustained stress.
  - Relaxation is the loss of stress by time with constant strain.
  - In concrete, creep deformations are generally larger than elastic deformation and thus creep represents an important factor affecting the deformation behavior.

  - Concrete under constant axial compressive stress



    - Experiments indicate that for working stress range – i.e. stresses not exceeding 0.5 $f_c^{'}$ – creep strains are directly proportional to stress that is a linear relationship between σ and ε_{cr}.

- Microcracking effect on creep in high stresses.

Mechanisms of concrete creep

- Two phenomena are involved:
  1. Time dependent deformation that occurs when concrete is loaded in a sealed condition so that moisture cannot escape. → basic creep
  2. Creep of the material when moisture exchange is permitted. → drying creep
- Basic creep is primarily influenced by the material properties only, while drying creep and shrinkage also depend on the environment and the size of the specimen.
- The real situation might be the combination of the two phenomena, sometimes, one being the dominating factor.
- Creep deformation contains three regions:
  1. Primary creep → initial increase in deformation
  2. Secondary creep → relatively a steady deformation region
  3. Tertiary creep → leads to creep fracture

Specific creep

- $\varepsilon_{sp} = \dfrac{\varepsilon_c}{\sigma}$, $\sigma < 0.5f_c^{'}$

  where ε_c = a function of time.

  → Stress levels above $0.8f_c^{'}$ creep produces failure in time.
- Relationship between $f_c^{'}$ and ε_{sp}:

$f_c^{'}$ (psi)	ε_{sp} (10^{-6} per psi)	Final strain (10^{-6} per psi)
3000	1.0	3.1
4000	0.80	2.9
6000	0.55	2.4
8000	0.40	2.0

- Creep coefficient

  - $C_t = \dfrac{\varepsilon_c}{\varepsilon_{inst}}$

    where ε_{inst} = instantaneous (initial) strain.

- Factors influencing creep:
  - Internal factors (composition)
    - Aggregate (concentration + stiffness) ↗ ⇨ Creep ↘
    - Water/cement ratio ↗ ⇨ Creep ↗
    - Aggregate permeability ↗ ⇨ Creep ↗
    - Aggregate creep ↗ ⇨ Creep ↗
    - Aggregate stiffness ↗ ⇨ Creep ↘
    - Aggregate grading and distribution
    - Cement
  - External factors (environment, time history)
    - Size
    - Shape
    - Cross-section ↗ ⇨ Creep ↘
    - Environmental factors (ambient humidity, temperature)
    - Stress intensity ↗ ⇨ Creep ↗
    - Time (age of loading) → the loading history is important to the total deformation (strain).

      → age of loading ↗ ⇨ Creep ↘

- Mathematical modeling of creep:
  - Strain decomposition

    The total strain of concrete may be decomposed as

$$\varepsilon = \varepsilon_\sigma + \varepsilon^0 = \left(\varepsilon_E + \varepsilon_C\right) + \varepsilon^0 = \left(\varepsilon_E + \varepsilon_C\right) + \left(\varepsilon_S + \varepsilon_T\right) = \varepsilon_E + \varepsilon''$$

where ε_σ = stress-induced strain,

ε_E = reversible strain,

ε_C = creep strain,

ε^0 = stress-independent inelastic strain,

ε_S = shrinkage strain,

ε_T = thermal expansion, and

$\varepsilon^{''}$ = inelastic strain.

- Incremental formulation of stress-strain relation

  $\{d\sigma\} = [D]\{d\varepsilon\}$

  where $\{d\sigma\}$ = change in stress tensor,

  $[D]$ = constitutive law of the material, and

  $\{d\varepsilon\}$ = change in strain tensor.

- First-order and higher orders formulations of creep and shrinkage deformation:
  - First-order formulations: The incremental elastic stiffness matrix changes from one time step to the next as proportional to the elastic modulus. Applicable to homogeneous materials.
  - Higher order formulations: The incremental elastic stiffness matrix is different and not proportional to the elastic modulus from one time step to another. Applicable to non-homogeneous materials.

- Incremental quasi-elastic stress-strain law

  $\Delta\varepsilon = J\Delta\sigma + \Delta\varepsilon^0$

  where $\Delta\varepsilon$ = a column matrix consisting of the strain increments,

  J = compliance (square) matrix (function),

  $\Delta\sigma$ = a column matrix consisting of the stress increments, and

$\Delta\varepsilon^0$ = a column matrix consisting of the inelastic strain increments.

- Linear methods for the calculation of creep strain
  - Effective modulus method

    Total stress-strain relation:

$$\varepsilon(t) = D\frac{1}{E_{eff}(t,t^{'})}\sigma(t)+\varepsilon^0(t)$$

where t = current time,

$t^{'}$ = the time at which the instantaneous elastic modulus is characterized,

$$D = \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ & 1 & -\nu & 0 & 0 & 0 \\ & & 1 & 0 & 0 & 0 \\ & & & 1+\nu & 0 & 0 \\ & & & & 1+\nu & 0 \\ sym. & & & & & 1+\nu \end{bmatrix}$$ for the case of isotropy,

$E_{eff}(t,t^{'}) = \dfrac{1}{J(t,t^{'})} = \dfrac{E(t^{'})}{1+\phi(t,t^{'})}$ = effective modulus of elasticity,

$\varepsilon^0(t)$ = stress-independent inelastic strain,

$J(t,t^{'}) = \dfrac{1}{E(t^{'})}+C(t,t^{'}) = \dfrac{1+\phi(t,t^{'})}{E(t^{'})}$ = compliance function (the creep function) representing the strain (elastic + creep) at time t,

$E(t^{'})$ = modulus of elasticity characterizing the instantaneous deformation at time $t^{'}$,

$\phi(t,t^{'}) = E(t^{'})J(t,t^{'})-1$ = creep coefficient representing the ratio of the creep deformation to the initial elastic deformation, and

$C(t,t') = \frac{\phi(t,t')}{E(t')} = \frac{\varepsilon_c}{\varepsilon_{inst}} \frac{1}{E(t')} = \frac{\varepsilon_c}{\sigma}$ = specific creep function ($C(t,t')$ = ε_{sp}).

- Incremental strain

  The method is applied incrementally from t_0 to t if the structural system is changed after t_0.

$$\Delta\varepsilon(t) = D\left[\frac{1}{E_{eff}(t,t')}\Delta\sigma(t) + \frac{\sigma(t_0)}{E(t_0)}\phi(t,t_0)\right] + \Delta\varepsilon^0(t)$$

  where $\Delta\sigma(t)$ = incremental form of the stress at time *t*,

  $\Delta\varepsilon^0(t)$ = incremental form of the stress-independent inelastic strain, and

  t_0 = time of first loading.

→ Effective modulus method is exact when the stress is constant from t' to t.

o Linear variation method

- Assuming the mechanical strain $\varepsilon - \varepsilon^0$ is zero up to time t_0 and jumps to the value of *a*, apply linear variation of strain from t_0 to t and obtain the following expression:

$$\varepsilon(t) - \varepsilon^0(t) = a + c\phi(t,t_0)$$

  The stress for $t_0 > t$ is

$$\sigma(t) = (a-c)R(t,t_0') + cE(t_0)$$

  where *a*, *c* = constants.

One can then obtain by algebraic manipulation the basic incremental stress-strain relation of the age-adjusted effective modulus.

- Age-adjusted effective modulus method

  Effective elastic modulus with a correction of the creep coefficient:

  $$E''(t,t_0)=\frac{E(t_0)}{1+\chi\phi(t,t_0)}$$

  where χ = the relaxation coefficient (aging coefficient, age-adjusted effective modulus) = 0.5~1.0.

  The quasi-elastic incremental stress-strain relation of the age-adjusted effective modulus:

  $$\Delta\varepsilon(t)=\frac{1}{E''(t,t_0)}D\Delta\sigma(t)+\Delta\varepsilon''(t)$$

  $$\Delta\varepsilon''(t)=\frac{\phi(t,t_0)}{E''(t,t_0)}D\sigma(t_0)+\Delta\varepsilon^0(t)$$

  where $\Delta\varepsilon(t)=\varepsilon(t)-\varepsilon(t_0)$.

  We assume isotropic material with a constant Poisson's ratio, which is approximately true for concrete.

Practical considerations of creep prediction

- Considering the specific strain function expressed as

  $$\phi(t,\tau)=\frac{1}{E(\tau)}+C(t,\tau)$$

  That is strain per unit stress at time t for the stress applied at age τ.

- The specific creep function

  $$C(t,\tau)=F(\tau)\,f(t-\tau)$$

- Forms of the aging function $F(\tau)$ and $f(t-\tau)$:
  - $F(\tau)$

- Power law: $F(\tau) = a + b\tau^{-c}$
- Exponential law: $F(\tau) = a + be^{-c\tau}$

- $f(t-\tau)$
  - Power law: $f(t-\tau) = a(t-\tau)^m$
  - Logarithmic expression: $f(t-\tau) = a + \log\left[1+(t-\tau)\right]$, $\quad \dfrac{\sigma}{f_c^{'}} < 0.4$

    where $(t\text{-}\tau)$ is measured in days.

→ Experimental data is used to obtain coefficients.

- Prediction of creep deformation:
  - Perform short-term experimental results → Develop short-term creep curves to be used in long-term prediction.
  - When experimental data is not available, the design has to rely on a relevant code. Various code recommendations have been quite controversial.
  - ACI 209 Committee defines the creep coefficient $\varphi(t,t_0)$ as

$$\varphi(t,t_0) = \frac{(t-t_0)^{0.6}}{10+(t-t_0)^{0.6}}\varphi(\infty,t_0)$$

where $(t-t_0)$ = time since application of load,

$\varphi(\infty,t_0) = \alpha K_1K_2K_3K_4K_5K_6$ = ultimate creep,

K_1 and K_2 relate to curing process, K_3 relates to the thickness of member, K_4, K_5, and K_6 relate to concrete composition. (α = 2.35)

  - Principle of superposition

For analysis and design purposes the time dependent linear relation between stress and creep strain can be written in the following way:

$$\varepsilon_{cr}(t) = f_0 \varepsilon_{sp}(t,\tau) + \sum_{i=\tau}^{t} \left[\Delta F(\tau_i) \varepsilon_{sp}(t,\tau_i) \right]$$

where f_0 = initial stress in concrete at the time τ of first loading,

$\varepsilon_{sp}(t,\tau)$ = specific creep strain at time t for concrete loaded at time τ,

$\Delta F(\tau_i)$ = additional stress increment or decrement applied at time $\tau < \tau_i < t$,

$\varepsilon_{sp}(t,\tau_i)$ = specific creep strain at time t for concrete at age $\tau_i > t$.

- Effect of stress on creep is predicted by the superposition method.
- Creep strains produced in concrete at any time t by an increment of stress applied at any time τ are independent of the effects of any other stress increment applied earlier or later than time τ.
- This method of analysis predicts creep recovery and generates stress-strain curves of a shape similar to experimental results. It is assumed that the concrete creeps in tension at the same rate as it creeps in compression.
- Creep under variable stress can be obtained by superimposing appropriate creep curves introduced for corresponding changes in stress at different time intervals. This is true if creep is proportional to applied stress.

Creep under different states of stress:

- Creep takes place in tension in the same manner as in compression. The magnitude in tension is much higher.
- Creep occurs under torsional loading.
- Under unaxial compression, creep occurs not only in the axial but also in the normal directions.

- Under multiaxial loading creep occurs in all directions and affected by stresses in other normal stresses.

- Non-linear factors in the evaluation of creep strain
  - The influences of
    - Temperature dependence
    - Creep in combined stress
    - Shrinkage
  - Humidity and temperature variation

    → Cross coupling of creep with shrinkage or thermal expansion.
  - Cracking or strain-softening
  - Cyclic loading
  - High stress and multiaxial viscoplasticity

- Examples of complex structures with significant effect of creep on deformation and failure behavior
  - Thermal effect (pressure vessels, process vessels)
  - Segmental concrete bridge deformation (construction and service stages)

- Shrinkage deformation
  - Shrinkage is basically a volume change of the element which is considered to occur independently of externally imposed stresses. This volume change can also be negative and is then called swelling. Thin sections are particularly susceptible to drying shrinkage and therefore must contain a certain minimum quantity of steel. In restrained concrete shrinkage results in cracking, before any loading. The minimum reinforcement is provided to control this cracking.
  - Causes of shrinkage:
    - Loss of water on drying

- Volume change on carbonation

o On average ultimate shrinkage $\varepsilon_{sh,u} = 8 \times 10^{-4}$.

o ACI 209 Committee suggests:

$$\varepsilon_{sh} = \varepsilon_{sh,u} S_t S_h S_{th} S_s S_f S_e S_c$$

where $\varepsilon_{sh,u}$ = ultimate factor of shrinkage,

S_t = factor of time,

S_h = factor of humidity,

S_{th} = factor of thickness,

S_s = factor of slump,

S_f = factor of fineness,

S_e = factor of air content, and

S_c = factor of cement property.

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Outline 8

Biaxial Bending

- R/C columns under biaxial bending
  - The problem of columns under biaxial bending is nonlinear and the number of unknowns is large. For any defined column the problem may be expressed as:

    $\left(P, e_x, e_y\right) = f\left(c, \theta, \varepsilon_c\right)$

    where f = a nonlinear function of the variables that can be derived from the equilibrium equations and geometry of any given column section and the stress-strain curves of the materials,

    P = axial load,

    e_x, e_y = eccentricities measured parallel to x and y axes, respectively,

    θ = inclination of neutral axis,

    c = depth of neutral axis measured from extreme compressive fiber, and

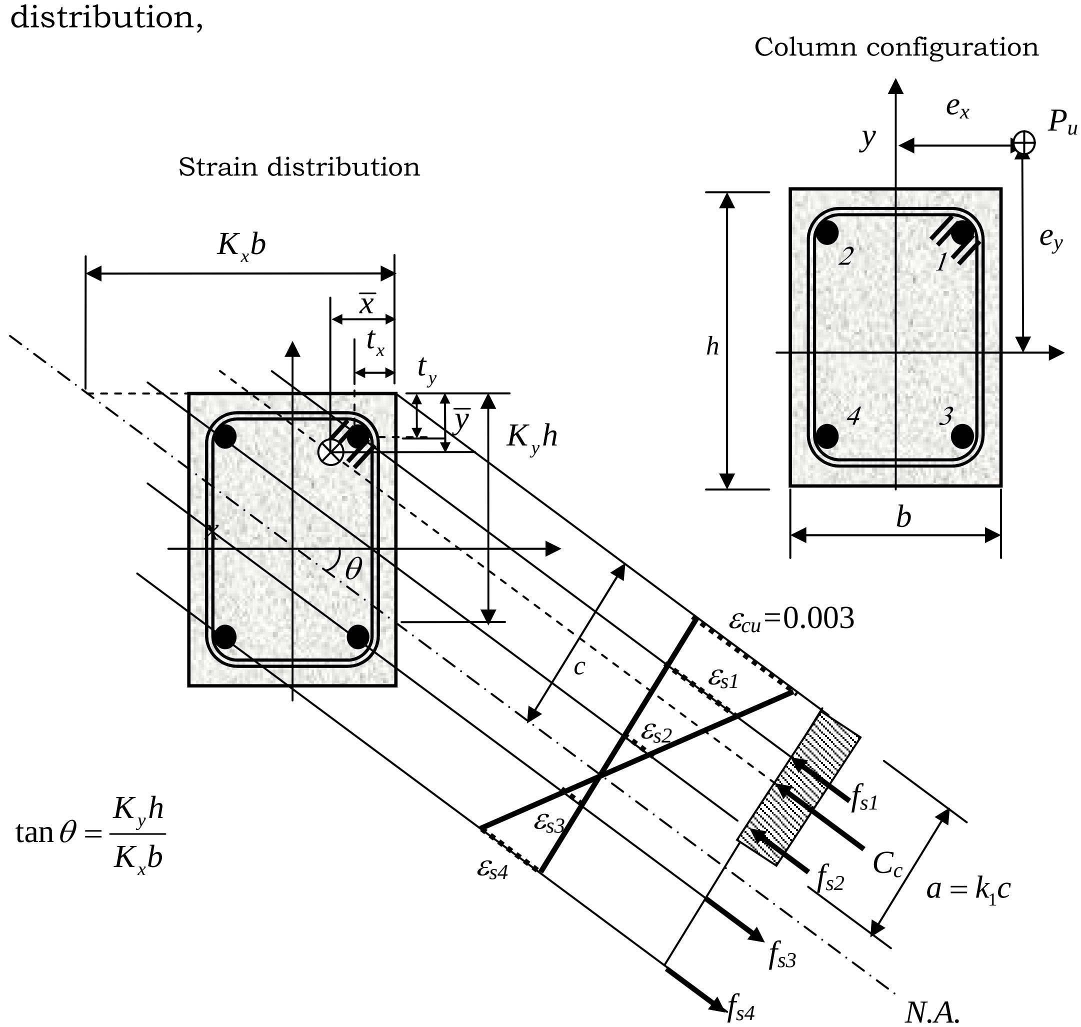
    ε_c = failure strain of concrete in compression.

  - A number of approximate methods which are based on simplifying assumptions have been developed. However, for certain situations, the simplifying assumptions may lead to inaccurate results, and the use of presently available design charts is often limited.
  - The use of computers to solve this problem with improved accuracy has been based upon iterative analysis of trial sections until a satisfactory result is achieved.

- Apart from these considerations, the design process should not be limited to a purely numerical evaluation of the loads, stresses, and strains involved; basic design issues, such as seismic requirements, architectural preferences, availability of material types and sizes, economy, and constructability must be taken into account.

Basic concept of analysis

- The analysis is based on the strain compatibility and equilibrium equations for the column section.
- For a given neutral axis position, the strains, stresses, and forces in the steel can be found. The resultant force in the concrete depends on the shape of the stress block.
- Consider the following configuration of column section and its strain distribution,



Strains in steel bars:

$$\varepsilon_{s1} = 0.003\left(1 - \frac{t_x}{K_x b} - \frac{t_y}{K_y h}\right)$$

$$\varepsilon_{s2} = 0.003\left(1 - \frac{b - t_x}{K_x b} - \frac{t_y}{K_y h}\right)$$

$$\varepsilon_{s3} = \frac{0.003}{c}\left\{\left[h\left(1 - K_y\right) - t_y\right]\cos\theta + t_y \sin\theta\right\}$$

$$\varepsilon_{s4} = \frac{0.003}{c}\left\{\frac{K_y h}{K_x b}\left[b\left(1 - K_x\right) - t_x\right] + h - t_y\right\}\cos\left[\tan^{-1}\left(\frac{K_y h}{K_x b}\right)\right]$$

- Determination of the stress of steel bars

  For $\varepsilon_{si} < \varepsilon_y$, $f_{si} = E_s \varepsilon_{si}$.

  For $\varepsilon_{si} \geq \varepsilon_y$, $f_{si} = f_y$.

- Equilibrium conditions

  Forces in steel bars:

$$S_1 = f_{s1} A_{s1} = \left(\varepsilon_{s1} E_s\right) A_{s1}$$

$$S_2 = f_{s2} A_{s2} = \left(\varepsilon_{s2} E_s\right) A_{s2}$$

$$S_3 = f_{s3} A_{s3} = \left(\varepsilon_{s3} E_s\right) A_{s3}$$

$$S_4 = f_{s4} A_{s4} = \left(\varepsilon_{s4} E_s\right) A_{s4}$$

  Force equilibrium:

$$\sum F = 0;\ C_c + S_1 + S_2 - S_3 - S_4 = 0$$

  Conditions of moment equilibrium are expressed in x and y directions.

$$M_{ux} = P_u e_y = C_c\left(\frac{h}{2} - \bar{y}\right) + \left(S_1 + S_2\right)\left(\frac{h}{2} - t_y\right) + \left(S_3 + S_4\right)\left(\frac{h}{2} - t_y\right)$$

$$M_{uy} = P_u e_x = C_c\left(\frac{b}{2} - \bar{x}\right) + S_1\left(\frac{b}{2} - t_x\right) - S_2\left(\frac{b}{2} - t_x\right) - S_3\left(\frac{b}{2} - t_x\right) + S_4\left(\frac{b}{2} - t_x\right)$$

Analysis and design requires trials and iterations to find the inclination and depth of the neutral axis satisfying the equilibrium equations.

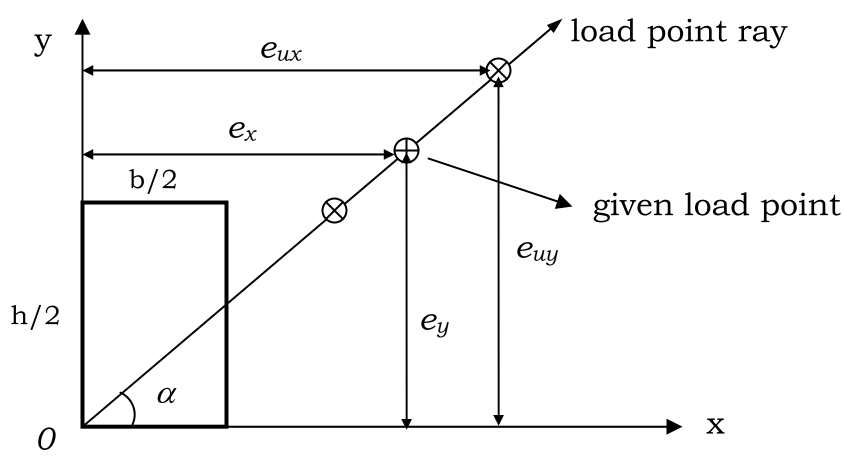
- Failure interaction curve
  - The relationship expressed in

    $\left(P, e_x, e_y\right) = f\left(c, \theta, \varepsilon_c\right)$

    describes the three dimensional failure surface (failure interaction curve) if the concrete strain is taken to be ε_u (usually 0.003). Any combination of neutral axis depth c and inclination θ will give a unique triplet of P_u, e_{ux}, and e_{uy} corresponding to a point on this failure surface.

  - Evaluation of column adequacy using a numerical scheme:
    In order to declare the adequacy of the column section to resist a given combination of P, e_x, and e_y only one point on the failure surface need to be computed. Such point satisfies the following condition

    $P_u = P$ and $\dfrac{e_{uy}}{e_{ux}} = \dfrac{e_y}{e_x}$



    - The procedure is summarized as follows:

(a) Find the neutral axis inclination satisfying $\frac{e_{uy}}{e_{ux}} = \frac{e_y}{e_x}$.

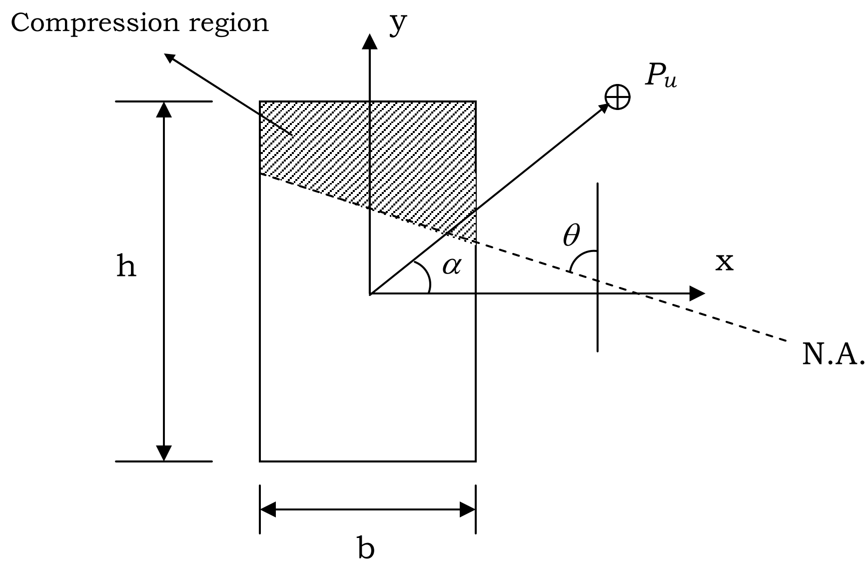
(b) Set the neutral axis depth c equal to the neutral axis depth computed from balanced failure condition for the section.

(c) Compute the value of P_u and update c using a modified secant numerical method until $P_u = P$.

(d) Compute e_{ux} and e_{uy} and compare with e_x and e_y to decide whether the section is adequate or not.

o Approximate method for the determination of the neutral axis inclination θ

For rectangular column sections shown:



Approximately,

$\theta = 90 - y + c - \frac{z}{\sqrt{2}}$ = inclination of N.A. to y axis

where $y = \frac{c}{2} + \sqrt{\frac{c^2}{4} - x^2 + cx}$

$$c = \frac{127}{\frac{h}{b} - 1} = \frac{127b}{h-b} \quad (\text{if } \frac{h}{b} = 1,\ \theta = \alpha)$$

$$x = \alpha + c - \frac{z}{\sqrt{2}}$$

$$z = -13\left(\frac{h}{b} - 1\right)^2 + 39.4\left(\frac{h}{b} - 1\right) + 63.6$$

- Approximate design methods
  - Reciprocal load method:

$$\frac{1}{P_u} = \frac{1}{P_{ux}} + \frac{1}{P_{uy}} - \frac{1}{P_0}$$

where P_u = ultimate load under biaxial loading,

P_{ux} = ultimate load when only e_x is present,

P_{uy} = ultimate load when only e_y is present, and

P_0 = ultimate load when $e_x = e_y = 0$.

  - Load contour method: ($M_{ux} - M_{uy}$ interaction curve in 2D)

For various loads of constant P_u,

$$\left(\frac{M_{ux}}{M_{ux0}}\right)^m + \left(\frac{M_{uy}}{M_{uy0}}\right)^n = 1$$

where $M_{ux} = P_u e_y$, $M_{uy} = P_u e_x$, and M_{ux0}, M_{uy0} are uniaxial flexural strengths about x and y axes for the constant load level considered.

→ Experiments suggest that $m = n = \alpha$ depends on column geometry. Typically $1.15 < \alpha < 1.55$ for most rectangular columns with uniform reinforcement.

→ There is no single value that can be assigned to the exponent to represent the true shape of the load contour in all cases.

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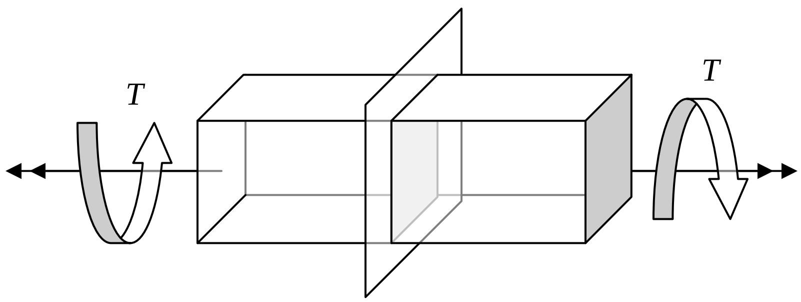
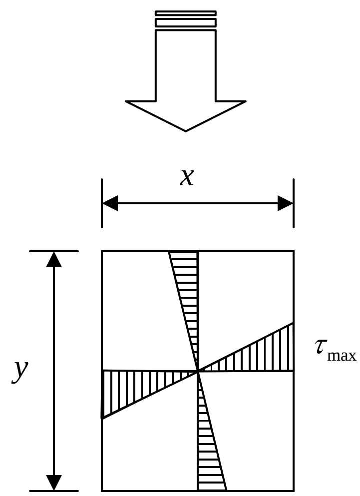
Outline 10

Torsion, Shear, and Flexure

- Torsion
  - Stress distribution on a cross section subject to torsion





x: narrow side

y: wide side

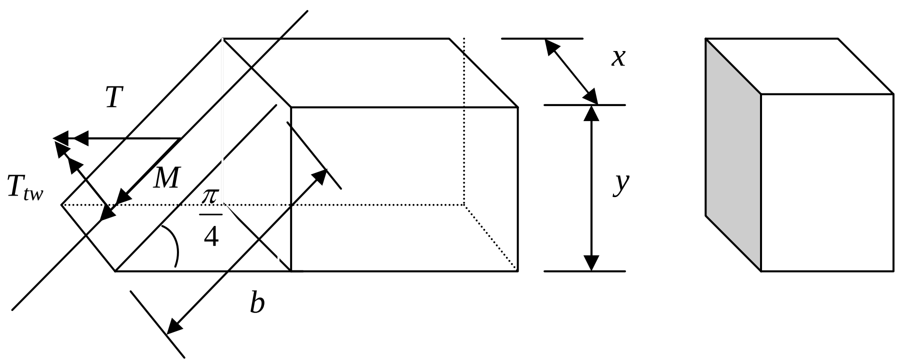
  - Maximum shear stress, τ_{max}

$$\tau_{max} = \eta \frac{T}{x^2 y}$$

where η = shape factor, T = torque, x, y = dimensions of the cross section. The shape factor is different for linear and nonlinear cases.

- Failure mode
  - Torsion failure of plain concrete occurs suddenly with an inclined tension crack in one of the wider faces, then extending into the narrow faces. Concrete crushing occurs in the opposite wider face.

- Torsional strength, T_{up}, of plain concrete
  - Several theories have been presented for computing torsional strength of plain concrete including elastic, plastic, and skew bending theories.
  - Skew bending:



→ T is the applied torque and M, T_{tw}, are the bending and twisting moments, respectively, on the $\frac{\pi}{4}$ plane.

→ $M = \frac{T}{\sqrt{2}}$, $b = \sqrt{2} \cdot y$,

$$S = \frac{\left(\sqrt{2}y\right)x^2}{6} = \frac{x^2 y}{3\sqrt{2}}$$

$$\sigma_t = \frac{M}{S} = \frac{T_{up}/\sqrt{2}}{x^2 y / 3\sqrt{2}} = \frac{3T_{up}}{x^2 y},$$

where T_{up} = ultimate torsion for plain concrete when σ reaches σ_t.

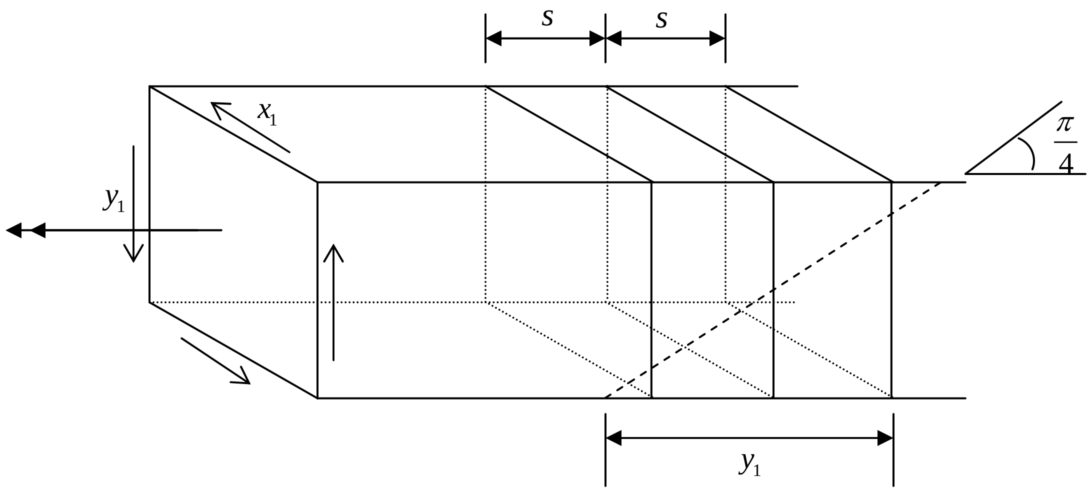
→ $T_{up} = \frac{x^2 y}{3}\sigma_t$

$$\sigma_t = 0.85 f_r \approx 0.85\left(7.5\sqrt{f_c^{'}}\right) = 6\sqrt{f_c^{'}}$$

$$\rightarrow T_{cr} = \frac{x^2 y}{3} \cdot 6\sqrt{f_c^{'}} = 2x^2 y\sqrt{f_c^{'}} = T_{up}$$

- Torsional strength contributed by steel
  - Consider the system consisting of longitudinal and transverse (stirrups) steel:

    (x_1, y_1 are the dimensions of steel frame as shown.)



  - Torsional moment with respect to axis of the vertical stirrups

$$T_{s1} = \left(A_t \alpha_1 f_s\right)\frac{y_1}{s} x_1$$

where A_t = area of one stirrup leg,

f_s = stirrup stress, and

s = stirrup spacing.

  - Torsional moment with respect to axis of the horizontal stirrups

$$T_{s2} = \left(A_t \alpha_2 f_s\right)\frac{x_1}{s} y_1$$

  - Total torsional moment

$$T_s = \frac{A_t f_s}{s} x_1 y_1 \left(\alpha_1 + \alpha_2\right)$$

$$T_s = \frac{A_t f_s}{s} x_1 y_1 \alpha_t$$ (α_t is determined from experiment.)

- Design concept
  - Total ultimate torsion capacity, T_u

    $T_u = T_c + T_s$

    where T_c = torsional capacity contributed by concrete, and

    T_s = torsional capacity contributed by reinforcement.

    $T_c = \beta T_{up} \quad (\beta \approx 0.4)$

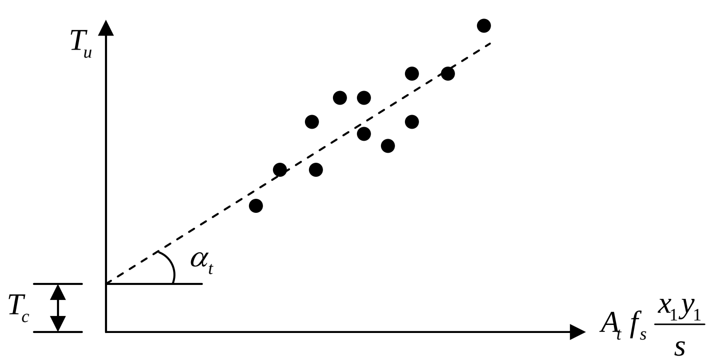
    Thus,

    $T_c = 0.8\sqrt{f_c^{'}}x^2 y$

    The coefficient β represents reduction in torsional strength provided by concrete after cracking. Upon cracking of concrete stress and strain are partially transferred to steel. Stiffness and strength of the system will depend on the amount of transverse and longitudinal reinforcements.

  - The final failure may be in one of the following ways:
    1. Under reinforced → Both transverse and longitudinal steel yield before failure.
    2. Over reinforced → Concrete crushes before yielding of steel.
    3. Partially over (under) reinforced

  - For under reinforced elements, α_t is independent of the steel ratio.



- Code suggestion:

  $$\alpha_t = 0.66 + 0.33\frac{y_1}{x_1} \le 1.50$$

- Role of longitudinal steel

  1. It anchors the stirrups, particularly at corners.
  2. It provides dowel resistance.
  3. It controls crack widening.

- Condition of under reinforcement

  $$A_l \le 2A_t \frac{x_1 + y_1}{s}$$

  where A_l = volume per length of longitudinal steel.

  → Steel yields first.

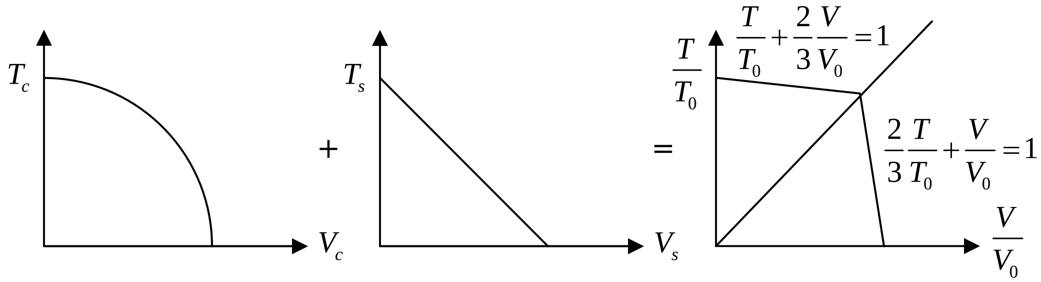
- Torsion combined with flexure

- Torsion combined with shear

  - Generally shear exists simultaneously with bending.

    → The existence of shear will reduce the resisting ability in torsion. Thus, it is necessary to consider the case of torsion combined with shear.

  - For RC beams with transverse reinforcement:
    - Pure torsion: $T_u = T_c + T_s$
    - Pure shear: $V_u = V_c + V_s$



- ACI Code
  - Design for torsion

    → Same interaction as in members without transverse reinforcement.

  - Excess torque

    → Over and above that resisted by concrete, the same amount of reinforcement is provided in members subject to torsion plus shear as would be required for purely torsional members.

    → This torsional reinforcement is added to that required for carrying bending moments and flexural shears.

  - $T_u \leq \phi T_n = \phi\left(T_c + T_s\right)$

    where T_u = factored torque,

    ϕ = capacity reduction factor for torsion = 0.75,

    T_n = nominal strength for torsion,

    T_c = torsional moment carried by concrete, and

    T_s = torsional moment carried by steel.

  - $T_c = \dfrac{T_0}{\sqrt{1+\left(\dfrac{T_0}{V_0}\right)^2\left(\dfrac{V_c}{T_c}\right)^2}}$

    where $T_0 = 0.8\sqrt{f_c^{'}}x^2 y$ = pure torsion and $V_0 = 2\sqrt{f_c^{'}}bd$ = pure shear.

    → $\dfrac{T_0}{V_0} = 0.4\dfrac{x^2 y}{bd} = \dfrac{0.4}{C_T}$ and $C_T = \dfrac{bd}{x^2 y}$

  - Assume $\dfrac{V_c}{T_c} = \dfrac{V_u}{T_u}$, such that

$$\rightarrow T_c = \frac{0.8\sqrt{f_c^{'}}x^2y}{\sqrt{1+\left(\frac{0.4}{C_T}\frac{V_u}{T_u}\right)^2}}, \quad V_c = \frac{2\sqrt{f_c^{'}}bd}{\sqrt{1+\left(2.5C_T\frac{T_u}{V_u}\right)^2}}$$

$$T_s = \frac{\alpha_t A_t f_y}{s}x_1y_1, \quad V_s = \frac{A_v f_y d}{s}$$

$$\rightarrow T_u = \phi\left(T_c + T_s\right) = \phi T_n$$

$$T_s = \frac{T_u - \phi T_c}{\phi}$$

$$\rightarrow A_t = \frac{sT_s}{\alpha_t f_y x_1 y_1} = \frac{\left(T_u - \phi T_c\right)s}{\alpha_t \phi f_y x_1 y_1}$$

- $T_s \le 4T_c$ is required to assure yielding of steel first.
- Minimum spacing of torsional stirrups → $4\left(x_1 + y_1\right)$ or 12 in.

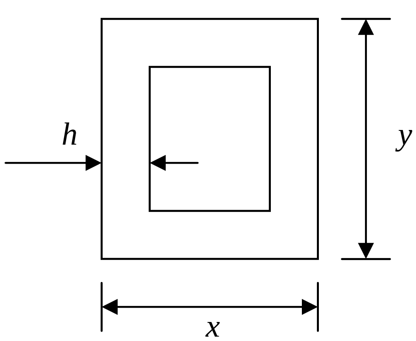
Condition of neglecting torsional effects

- Torsional effects may be neglected if

$$T_u < 0.5\phi\sqrt{f_c^{'}}\sum_{i=1}^{n}\left(x_i^2 y_i\right)$$

where $\sum_{i=1}^{n}\left(x_i^2 y_i\right)$ = sum of the small rectangles for irregular shapes.

Hollow sections



- When $h > \frac{x}{4}$, consider the cross section as solid.
- When $\frac{x}{10} \le h \le \frac{x}{4}$, assume it as solid but multiply $\sum(x^2 y)$ by $\left(4\frac{h}{x}\right)$.
- When $h < \frac{x}{10}$, consider it as a thin-walled section. → Check for instability (local buckling).

- General formulation of post-cracking behavior of flexure, shear, and tension interaction in R/C beams

- Discussion of applications: Concrete guideway systems from monorail and maglev transportation infrastructure.

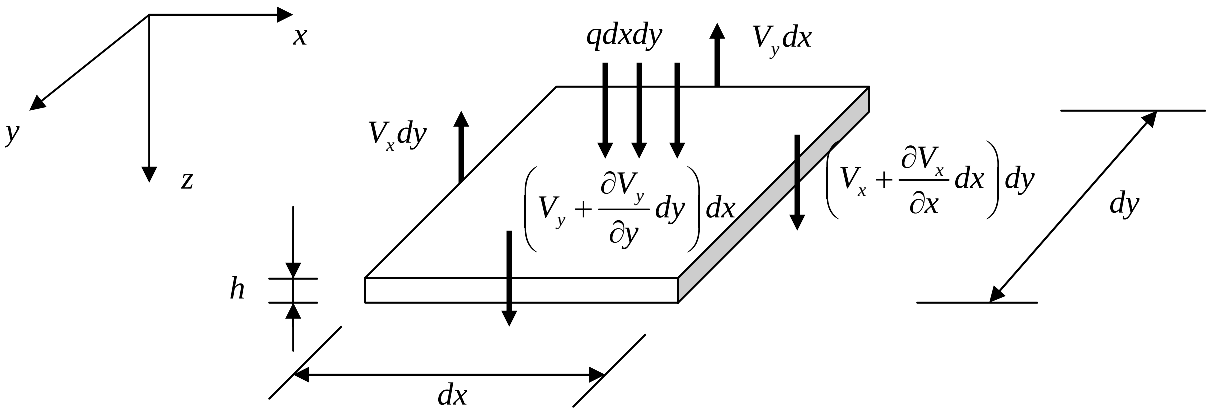
- Design Example – Shear and torsion

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

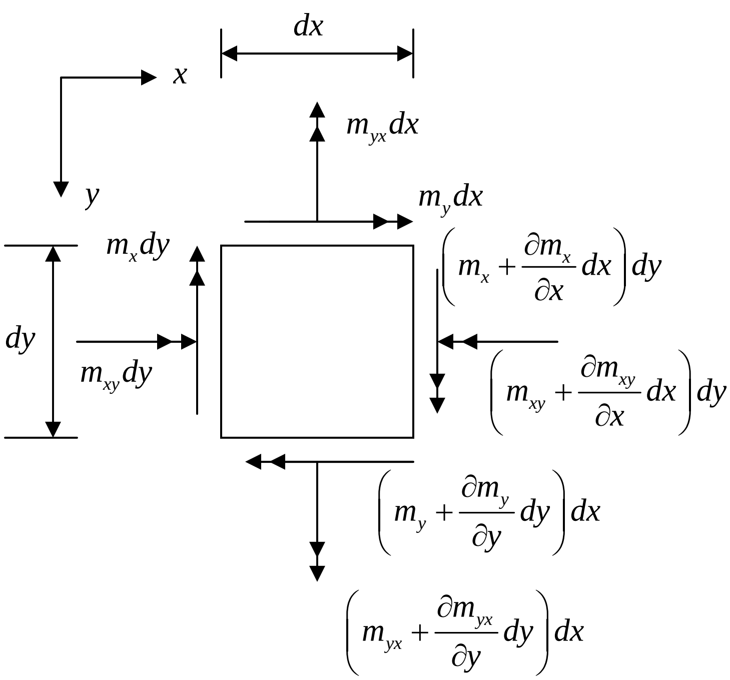
Outline 11

Yield Line Theory for Slabs

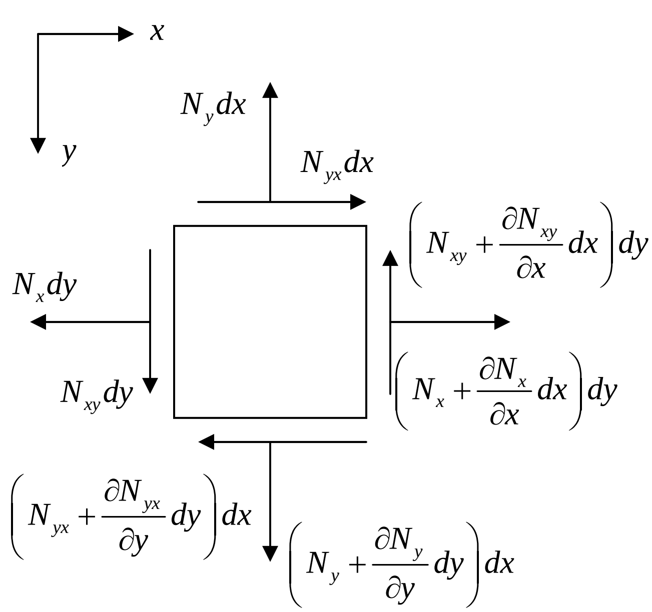
- Loads and load effects



Surface and shear forces



Moments



Membrane forces

- Load effects to be solved: $V_x, V_y, m_x, m_y, m_{xy}, m_{yx}, N_x, N_y, N_{xy}, N_{yx}$

  → Ten unknowns and six equations

  → Indeterminate problem: We need to include stress-strain relation for complete elastic solution.

- The relative importance of the load effects is related to the thickness of the slab. Most reinforced and prestressed concrete floor slabs fall within "medium-thick" class, i.e., plates are
  - thin enough that shear deformations are small, and
  - thick enough that in-plane or membrane forces are small.

Analysis methods:

- Elastic theory
- Elastic-plastic analysis

  – Finite element analysis (FEA)
- Approximate methods of analysis
- Limit analysis – **Yield Line Theory**

  – Lower & upper bound analysis

Elastic theory

- Lagrange's fourth-order PDE governing equation of isotropic plates loaded normal to their plane:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

where

w = deflection of plate in direction of loading at point (x, y).

q = loading imposed on plate per unit area, $q \approx f(x, y)$

D = flexural rigidity of plate, $D = \dfrac{Eh^3}{12(1-\mu^2)}$

E = Young's modulus

h = plate thickness

μ = Poisson's ratio.

- Navier's solution of Lagrange's equation using doubly infinite Fourier series:

  $$w(x,y) = q \cdot C \cdot \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

  where

  a, b = lengths of panel sides

  m, n = integers

  C, A_{mn} = constants – Boundary conditions.

Finite difference (FD) method

- It replaces Lagrange's fourth-order PDE with a series of simultaneous linear algebraic equations for the deflections of a finite number of points on the slab surface. Deflections, moments, and shears are computed.

Finite element (FE) method

- It utilizes discretization of the physical system into elements. Displacement functions are chosen. Exact compatibility and approximate equilibrium considerations are used.

Approximate methods

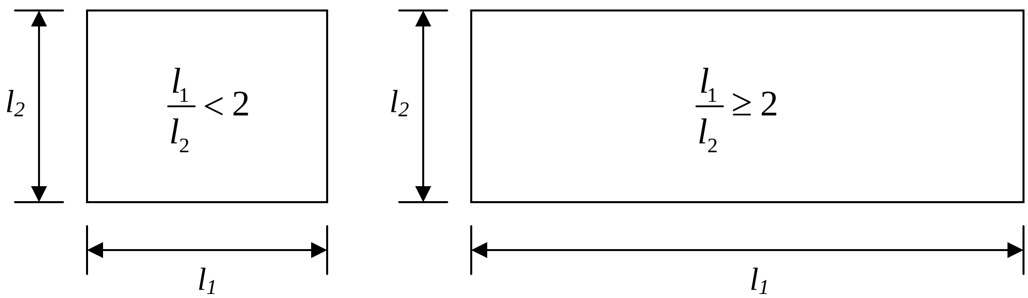
- Direct design method
- Equivalent frame method
- Assignment of moments

Types of slabs

- According to the structural action

— One-way slabs

— Two-way slabs



Two-way slab One-way slab

- According to the support and boundary conditions

❑ Choice of slab type

❑ Limit analysis – Yield Line Theory

- Ductility and Yield Line Theory
- Yield Line Analysis:

  Yield line theory permits prediction of the ultimate load of a slab system by postulating a collapse mechanism which is compatible with the boundary conditions. Slab sections are assumed to be ductile enough to allow plastic rotation to occur at critical section along yield lines.

  1. Postulate a collapse mechanism compatible with the boundary conditions
  2. Moment at plastic hinge lines ≈ Ultimate moment of resistance of the sections
  3. Determine the ultimate load
  4. Redistributions of bending moments are necessary with plastic rotations.

- Moment-curvature relationship

  – Curvature ductility factor: $\frac{\phi_u}{\phi_y}$

  – $\varepsilon_{\mathrm{E}} << \varepsilon_{Plastic}$

  – $M_u \approx$ constant at yield lines.

- Determinate structures → mechanism
- Indeterminate structures – moment redistribution

- Assumptions and guidelines for establishing axes of rotation and yield lines
- Determination of the ultimate load (or moment):
  - Equilibrium method
  - Analysis by Principle of Virtual Work

Isotropic and orthotropic slabs

- Isotropic slabs

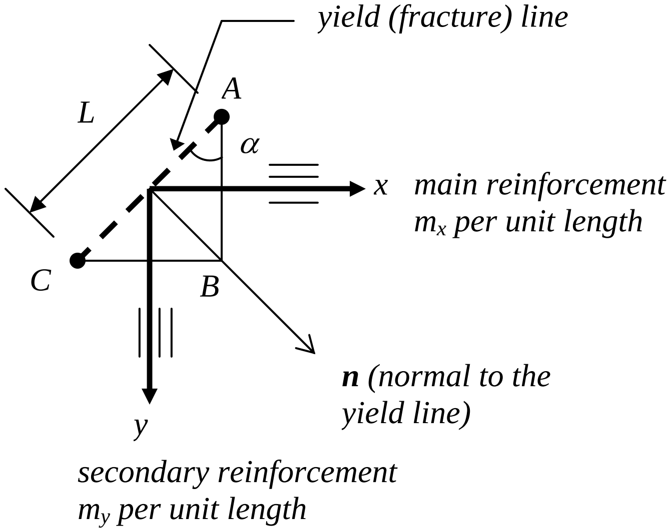
  A slab is said to be isotropically reinforced if it is reinforced identically in orthogonal directions and its ultimate resisting moment is the same in these two directions as it is along any line regardless of its direction.

- Orthotropic slabs

  A slab is said to be orthotropically reinforced if its ultimate strengths are different in two perpendicular directions. In such cases, yield lines will occur across these orthogonal directions.

- Determination of the moment capacity M_u for orthotropic slabs

  Computation for the moment capacity M_u consistent with the yield line given the moment capacities M_x and M_y in the direction of the reinforcing bars:



m_x = ultimate resisting moment per length along the x axis

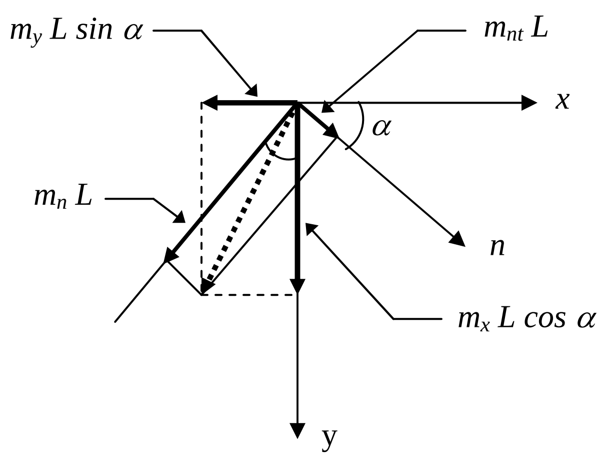
m_y = ultimate resisting moment per length along the y axis

m_n = ultimate resisting moment per length along AC

m_{nt} = ultimate resisting moment per length along normal direction to the yield line (torsion)

- Equilibrium in vector notation

$m_x\,(AB) = m_x\,L\cos\alpha$
$m_y\,(BC) = m_x\,L\sin\alpha$
$m_n\,(AC) = m_n\,L$
$m_{nt}\,(AC) = m_{nt}\,L$



$m_n L = (m_x L\cos\alpha)\cos\alpha +$
$\quad (m_y L\sin\alpha)\sin\alpha$
$m_n = m_x\cos^2\alpha + m_y\sin^2\alpha$
$m_{nt} L = (m_x L\cos\alpha)\sin\alpha -$
$\quad (m_y L\sin\alpha)\cos\alpha$
$m_{nt} = (m_x - m_y)\,L\sin\alpha\cos\alpha$

if $\alpha = 0$ *or* $\pi/2$, *then* $m_{nt} = 0$
if $m_x = m_y = m$, *then*
$m_n = m(\cos^2\alpha + \sin^2\alpha)$, $m_{nt} = 0$
$m_n = m$

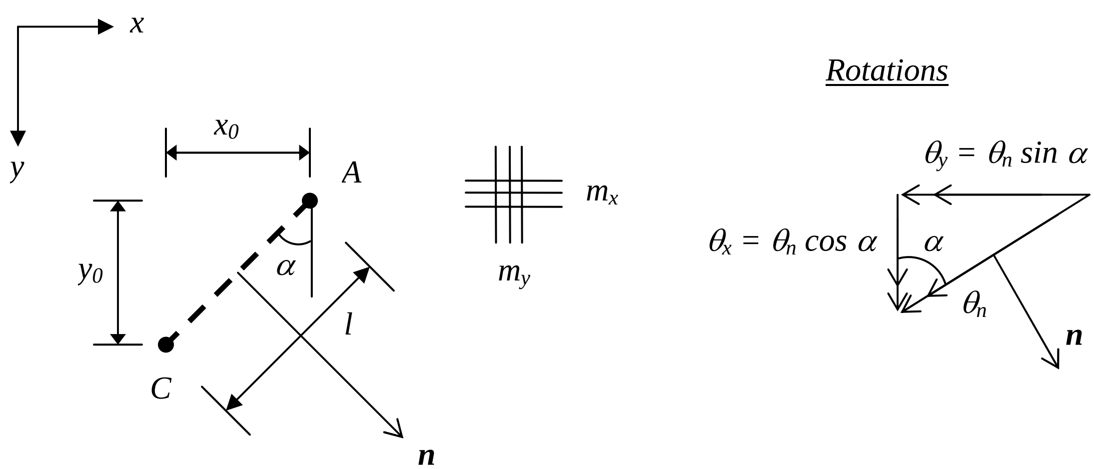
Square yield criterion
(isotropic reinforcement)

if $m_x \neq m_y$, *then*
orthogonally anisotropic or orthotropic

Orthotropic slabs can be reduced to equivalent isotropic cases by modifying the slab dimensions.

In analyzing orthotropic plates it is usually easier to deal separately with the x and y direction components of the internal work done by the ultimate moments: $\sum m_n \theta_n l$

- Components of internal work done:



Equilibrium:

$$\sum m_n \theta_n L = \sum \left(m_x \theta_n \cos\alpha \cdot y_0 + m_y \theta_n \sin\alpha \cdot x_0 \right) = \sum \left(m_x \theta_x \cdot y_0 + m_y \theta_y \cdot x_0 \right)$$

Virtual Work:

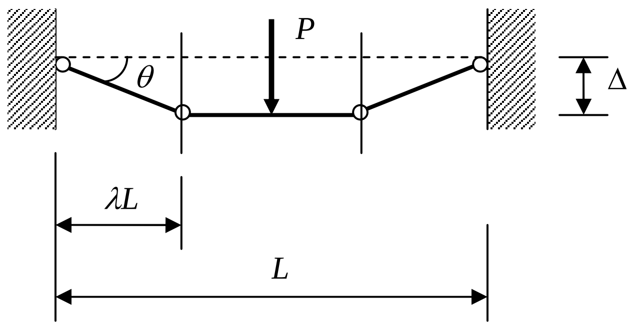
$$\sum W\Delta = \sum m_x \theta_x y_0 + \sum m_y \theta_y x_0$$

Upper-Bound Solution (Energy Approach)

- Energy method, with an initial selection of a collapse mechanism, gives an upper bound solution, i.e., if failure mode (mechanism) is chosen incorrectly (still satisfying boundary condition) the solution for the ultimate load will be unconservative. The method involves:
  - Select a failure (collapse) mechanism which satisfies the displacement boundary conditions everywhere (kinematic admissibility), and which satisfies the yield criterion at the yield line.

- Impose the condition that work done by the external loads must equal the work done by the resisting forces.
- If the correct mechanism is chosen the method leads to the correct value, otherwise, the predicted load is unconservative.
- This is explained with the following example:

  A fixed ended beam has a positive and negative moment capacity of M_u. Assume the following collapse mechanism,



Conservation of energy:

$$P_u\Delta = 4M_u\theta = 4M_u\frac{\Delta}{\lambda L}$$

$$\Rightarrow P_u = \frac{4M_u}{\lambda L}$$

The correct collapse load is found for λ = 0.5, $P_u = \frac{8M_u}{L}$. For any other value of λ < 0.5, P_u is unconservative.

- Comments on yield line theory:

1. In the equilibrium method, equilibrium of each individual segment of the yield pattern under the action of its bending and torsional moments, shear forces and external loads is considered.
2. In the virtual work method, shear force and torsional moment magnitudes and distribution need not be known because they do not work when summed over the whole slab when the yield line pattern is given a small displacement.

- Limitations on yield line theory:

1. Analysis is based on rotation capacity at the yield line, i.e., lightly reinforced slabs.
2. The theory focuses attention on the moment capacity of the slab. It is assumed an earlier failure would not occur due to shear, bond, etc.
3. The theory does not give any information on stresses, deflections, or service load conditions.

- Design Example – Design of rectangular slabs using yield line theory

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Homework #1

Assigned: Thursday, February 12, 2004 **Due:** Tuesday, February 24, 2004

Design of A Concrete Gravity Platform

This problem is about failure criteria for concrete in a variety of stress states. An application considered in this problem is the concrete gravity platform (offshore structure). A brief introduction is first presented followed by questions on the behavior of concrete in this application.

Introduction

A concrete gravity platform is one that is placed on the seabed, and by its own weight, is capable of withstanding the environmental forces (wind and waves) it may be exposed to during its lifetime. Gravity-type concrete platforms are used for oil-drilling purpose in the oil industry. They are used when the soil/rock of the seabed is hard and relevant for supporting them since they rest directly on the ocean floor without pile foundation. Concrete platforms are larger and heavier than steel platforms. Although there are various designs of a concrete gravity platform, the base part is usually made of reinforced concrete and consisted of huge sub sea concrete tanks. These tanks are used for the storage of crude oil. For example, the base of a concrete gravity platform built for Mobil Oil in the North Sea consists of 19 hollow concrete cylinders of 2 ft wall thickness, 66 ft inner diameter, and 164 ft height. The cylindrical concrete storage tanks of the base are built to such a height that when the dock is flooded the base has sufficient freeboard to float on its own buoyancy. The base is then towed out into the deep-water site and the tanks are flooded, causing the base to sink to the seabed.

The platform rests on several (usually three or four) taller, tapered concrete legs (columns), which rest on several of the tanks. These columns can rise up to 300 ft above the top of the tanks. Finally the steel superstructure is placed on top of the completed concrete substructure. Sometimes grout is pumped beneath the bottom of the tanks to provide a firmer foundation. The platform is heavy enough to remain stable without pile foundation when subject to environmental forces.

Questions

1. Consider the base of the concrete gravity platform as a cylinder model. How deep could the open, flooded cylinders go before being crushed by the water pressure? Assume f_c' to be 5000 psi (lb/in^2) and ignore the end conditions of the cylinder and the contribution of reinforced steels. Use constant gravity weight for seawater[1]. Consider the tanks as open-ended tubes to simplify the problem.

2. Scrambling to be the first to tap the oil reservoirs in Siberia, Exxon hires 20 M.I.T. graduate students to design the base of a platform that is to be constructed in a 1700-ft deep fresh water lake[2]. The tanks are to have the same dimensions as the Mobil tanks described above with f_c' to be 5000 psi. Apply appropriate idealizing assumptions and use a biaxial state of stress theory to compute how much platform each tank (tube) can support with a factor of safety (F.S.) of 3. Ignore stability problem (buckling) and consider only the through-the-wall section under the hydrostatic water pressure and the weight of tank and water above it.

3. Once the platform is complete and operational, the tanks are used to store crude oil. The Exxon engineers have told you that the platform weighs 500,000 tons to be supported on three tanks (tubes). Use a triaxial failure theory to compute the required compressive strength of the concrete to support the platform, with a factor of safety of 4. Consider hydrostatic pressure from seawater and the difference in density of fresh water and crude oil. Use 0.85 as the specific gravity[3] for crude oil. Do you think this compressive strength of concrete is manufacturable in Siberia? Please note the failure criterion you used and explain the reason why you use it.
   This question examines the concrete under triaxial stress. There will be three stresses acting on the concrete: σ_1, the pressure in radius direction, σ_2, the pressure in vertical direction, and σ_3, the hoops stress caused by the differences in the density of sea water and the density of crude oil. Use the following definition of hoop stress for convenience,

$$\sigma_{\mathrm{h}} = \frac{p \cdot r}{t}$$

   where σ_{h} = hoop stress,
   p = difference in hydrostatic pressure,
   r = radius of cylinder,
   t = thickness of cylinder wall.

In solving this problem, use your knowledge of concrete behavior in multiaxial loading. Make appropriate assumptions with respect to uniaxial, biaxial, and triaxial failure criteria. Note sources you may have used.

Note:

[1] Specific weight for fresh water at 60F is 62.4 lb/ft^3 (9.8 kN/ m^3), for seawater at 60F is 64 lb/ft^3 (10.1 kN/ m^3),

[2] Fresh water lakes could be as deep as 1932 ft, which is the record in the U.S. (Crater Lake, Oregon) (Fig. 6).

[3] Specific gravity is dimensionless unit defined as the ratio of density of the material to the density a specific temperature.

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Homework #2

Assigned: Thursday, February 26, 2004 **Due:** Tuesday, March 9, 2004

Creep Strain in A Reinforced Concrete Building Column

Introduction

A typical interior column in a 3-story reinforced concrete building is to be investigated, considering the load increments during construction. From recorded dates of casting and form removal, the following load history was determined for the first floor column.

Time (days)	Event
0	Column cast
30	First floor shoring removed, ΔP = 300 kips
60	Second floor cast, ΔP = 200 kips
120	Penthouse and roof case, ΔP = 250 kips

The column is 30x30 inches and contains #8 bars ($A_s = 6.32$ in^2), the modulus of elasticity for the reinforcing steel is $E_s = 29 \times 10^6$ psi, and for the concrete $E_c = 2.9 \times 10^6$ psi. The attached data indicates the specific creep function for the concrete used in this question (See Attachment A).

Use the modified supervision method outlined in class to compute the creep strain effect on the concrete, the reinforcing steel, and the column. The attached summary of a numerical procedure is given as a concept. You may develop your own procedure and interpretation of the problem. (See Attachment B).

Questions

1. Consider only elastic and creep effects. Plot the following information at 30-day intervals.
   1.1) Column strain (stress-strain curve)
   1.2) Stress in the concrete (stress-time curve)
   1.3) Stress in the reinforcing steel (stress-time curve)

   To provide a basis for comparison, show on these plots the elastic stress-strain (σ–ε) history, neglecting creep effects. Continue the analysis for the first 180 days. The column is to be considered concentrically loaded for the purpose of this analysis. Also present your solution in a tabular format.

2. The exterior columns of the building are 10x20 inches. Discuss the effect of creep in this building, and the possible problems that could arise from it. Assume for the purpose of the discussion that the column will only be subjected to axial forces.

Attachment A

Specific creep, $C(t, \tau) \times 10^{-6}$ (in/in/psi)

τ (days) \ t (days)	**15**	**30**	**45**	**60**	**75**	**90**	**105**	**120**	**135**	**150**	**165**	**180**
15	0	0.563	0.657	0.712	0.754	0.787	0.813	0.835	0.852	0.867	0.880	0.891
30		0	0.364	0.424	0.460	0.487	0.508	0.525	0.539	0.550	0.560	0.568
45			0	0.276	0.322	0.349	0.370	0.386	0.399	0.410	0.418	0.426
60				0	0.229	0.267	0.289	0.307	0.320	0.331	0.339	0.347
75					0	0.200	0.233	0.253	0.268	0.280	0.289	0.297
90						0	0.181	0.211	0.229	0.242	0.253	0.261
105							0	0.168	0.196	0.212	0.225	0.253
120								0	0.159	0.185	0.201	0.213
135									0	0.152	0.177	0.192
150										0	0.147	0.171
165											0	0.143
180												0

Attachment B

Numerical procedures using modified supervision method:

$\Delta t_i =$ time step (i = time index) $\tau =$ age of concrete at loading

$\rho' =$ reinforcement ratio, A_s / A_c $t_i =$ age of concrete at step i

$m =$ modular ratio, E_s / E_c

Initial conditions:

$$\varepsilon_{column,-1} = 0, \qquad \sigma_{concrete,-1} = 0, \qquad \sigma_{steel,-1} = 0$$

Changes in free creep strain:

$$\delta\varepsilon_n^{creep} = \sum_{i=0}^{n-1} \Delta\sigma_{concrete,i} \times \left[C\left(t_n, \tau_i\right) - C\left(t_{n-1}, \tau_i\right) \right]$$

Correction strain and stress:

$$\delta\varepsilon_{column,n}^{creep} = \frac{\delta\varepsilon_n^{creep}}{1 + m \cdot \rho'}, \qquad \delta\sigma_{concrete,n}^{creep} = \frac{-\delta\varepsilon_n^{creep} \cdot E_c}{1 + \frac{1}{m \cdot \rho'}}, \qquad \delta\sigma_{steel,n}^{creep} = \frac{-\delta\sigma_{concrete,n}^{creep}}{\rho'}$$

Total change in strain and stress:

$$\Delta\varepsilon_{column,n} = \delta\varepsilon_{column,n}^{creep} + \Delta\varepsilon_{concrete,n}^{load}$$

$$\Delta\sigma_{concrete,n} = \delta\sigma_{concrete,n}^{creep} + \Delta\sigma_{concrete,n}^{load}$$

$$\Delta\sigma_{steel,n} = \delta\sigma_{steel,n}^{creep} + \Delta\sigma_{steel,n}^{load}$$

Current state of strain and stress:

$$\varepsilon_{column,n} = \varepsilon_{column,n-1} + \Delta\varepsilon_{column,n}$$

$$\sigma_{concrete,n} = \sigma_{concrete,n-1} + \Delta\sigma_{concrete,n}$$

$$\sigma_{steel,n} = \sigma_{steel,n-1} + \Delta\sigma_{steel,n}$$

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Homework #3

Assigned: Thursday, March 18, 2004 **Due:** Thursday, April 1, 2004

Biaxial Column Interaction

Introduction

To demonstrate the adequacy of a given column section subjected to a given biaxial loading using the approximate contour method and the theoretical method. A square cross section is as shown in Fig. 1 with the following information:

Concrete

Uniaxial compressive strength: $f_c^{'} = 5000$ psi;

Maximum concrete strain: $\varepsilon_{cu} = 0.003$;

Steel

Yield stress: $f_y = 60$ ksi;

Yield strain: $\varepsilon_{sy} = 0.002$.

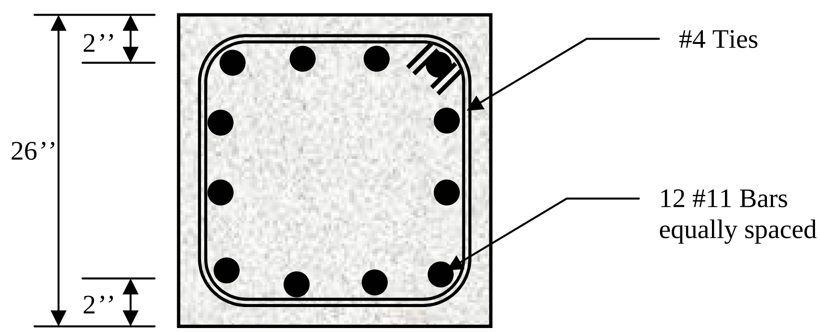


Figure 1. Configuration of the reinforced concrete column section

Questions

1. (Uniaxial Column Interaction Diagram)

For the cross section shown in Fig. 1, construct the axial load-moment interaction diagram. As a minimum calculation, establish the points corresponding to

(a) pure axial load ($P_o^{'}$),

(b) balanced loads ($P_b^{'}$, $M_b^{'}$), and

(c) pure moment ($M_o^{'}$).

You may assume straight lines between these points. Also, plot the axial load ultimate curvature diagram.

2. (Biaxial Column Interaction)

For the same column cross section, assume that the neutral axis is originated at 45 degree to the principal axes, and has a depth equal to 12 inches. At the extreme compression fiber, the compression strain is equal to 0.003. Accomplish the following tasks:

(a) Calculate the axial load and bending moment acting on the section,

(b) Plot the load contour (P vs. M_x vs. M_y) using the Bresler load contour method with α = 1.5, and the uniaxial interaction diagram from part 1.

(c) Show the loading points on the load contour and comment on the adequacy of the section for the loading considered.

- **Note:** Bresler's Load Contour Method

The general nondimensional equaiton for the load contour at constant may be expressed in the form

$$\left(\frac{M_{nx}}{M_{ox}}\right)^{\alpha_1} + \left(\frac{M_{ny}}{M_{oy}}\right)^{\alpha_2} = 1 \qquad (1)$$

where $M_{nx} = P_n e_y$; $M_{ny} = P_n e_x$;

$M_{ox} = M_{nx}$ capacity at axial load P_n when $M_{ny} = 0$ or $(e_x = 0)$;

$M_{oy} = M_{ny}$ capacity at axial load P_n when $M_{nx} = 0$ or $(e_y = 0)$.

Bresler (1960) suggests that it is acceptable to take $\alpha_1 = \alpha_2 = \alpha$; then

$$\left(\frac{M_{nx}}{M_{ox}}\right)^{\alpha} + \left(\frac{M_{ny}}{M_{oy}}\right)^{\alpha} = 1 \qquad (2)$$

which is shown graphically in Fig. 2.

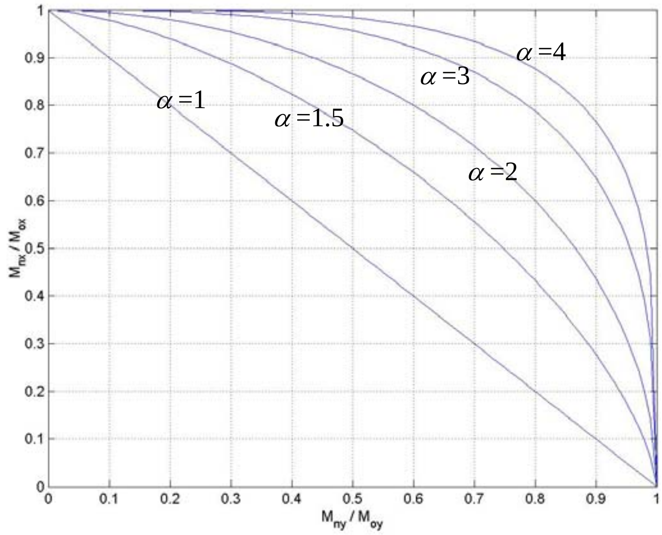


Figure 2. Interaction curves for Eq. (2)

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Homework #4

Assigned: Thursday, April 8, 2004 **Due:** Thursday, April 15, 2004

Shear and Torsion

Introduction

Architectural and clearance requirements call for the use of a transfer girder (shown in Fig. 1) spanning 28 ft between supporting column faces. The girder carries a concentrated load of 25 kips at midspan, applied with eccentricity of 25 in. from the girder centerline (considering load factors and including girder self weight). Dimensions of the member are x = 10 in., y = 22 in., x_1 = 6 in., y_1 = 17 in., and d = 19 in. Assuming supporting columns provide full torsional rigidity. Flexural rigidity at the ends of the span may also be assumed to develop 40 percent of the maximum moment that would be obtained if the girder were simply supported. Material strengths are

Concrete
Uniaxial compressive strength: $f_c' = 5000$ psi;
Steel
Yield stress: $f_y = 40$ ksi;

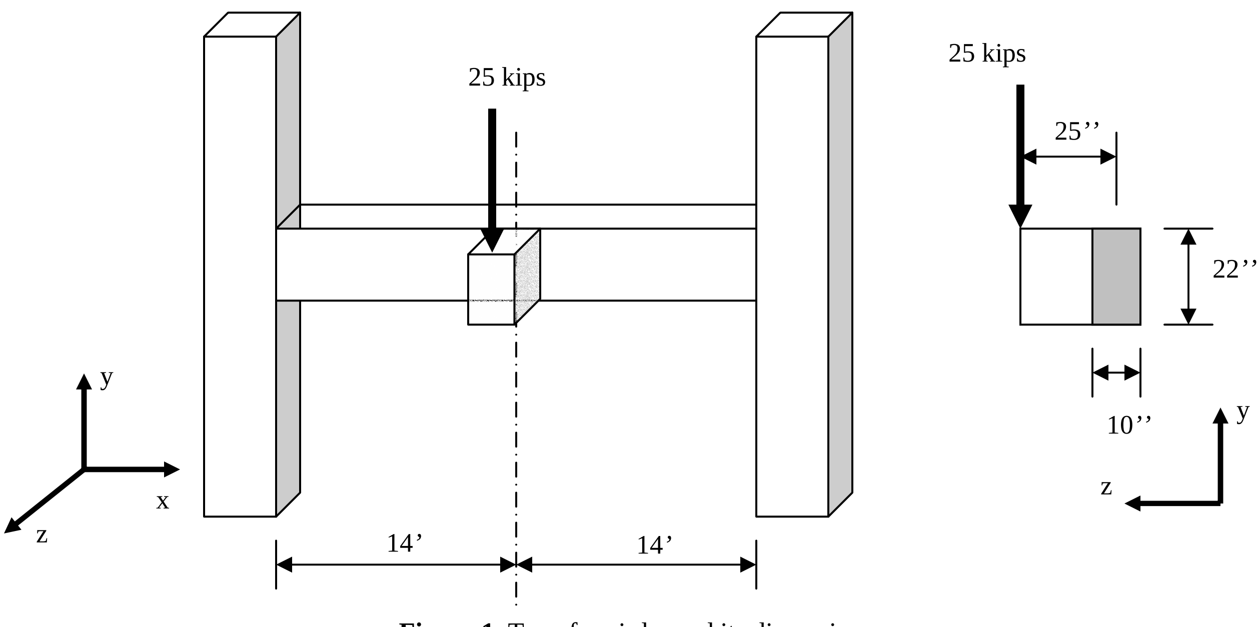


Figure 1. Transfer girder and its dimensions

Question

Design both transverse and longitudinal steel for the girder.

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Homework #5

Assigned: Thursday, April 22, 2004 **Due:** Thursday, April 29, 2004

Yield Line Theory

Introduction

A reinforced concrete slab (shown in Fig. 1) is supported by two fixed edges and one simply supported edge but has no support along one long side. It has a uniform thickness of 8 in., resulting in effective depths in the long direction of 7 in. and in the short direction of 6.5 in. Bottom reinforcement consists of #4 bars at 15 in. centers in each direction, continued to the supports and free edge. Top negative steel in the x direction consists of #4 bars at 12 in. on center, except that in a 2 ft. wide “strong band” parallel and adjacent to the free edge, four #5 bars are used. Top negative steel in the y direction consists of #4 bars at 24 in. on center. Material strengths are

Concrete

Uniaxial compressive strength: $f_c' = 4000$ psi;

Steel

Yield stress: $f_y = 60$ ksi;

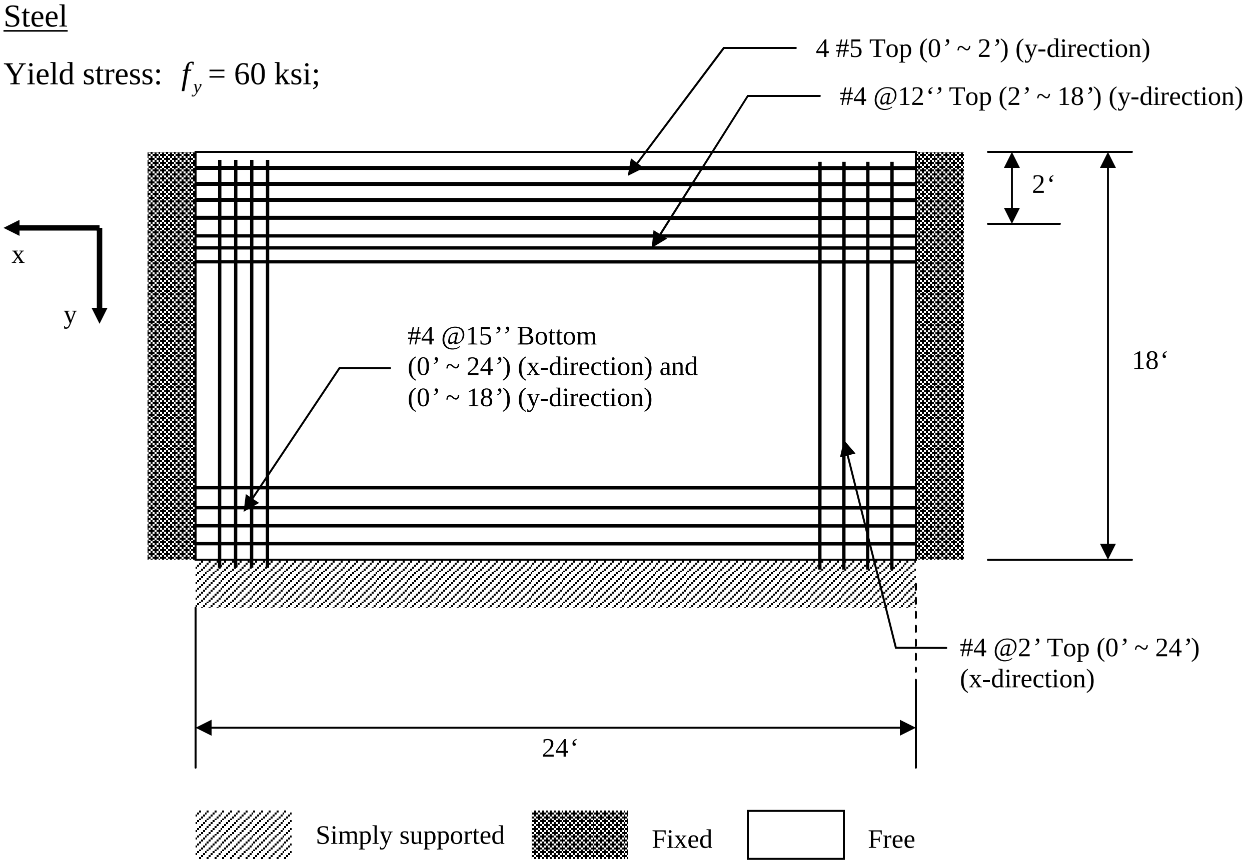


Figure 1. Reinforced concrete slab and its dimensions

Question

Using the yield line theory method, determine the ultimate load w_u that can be carried by the slab.

Note:

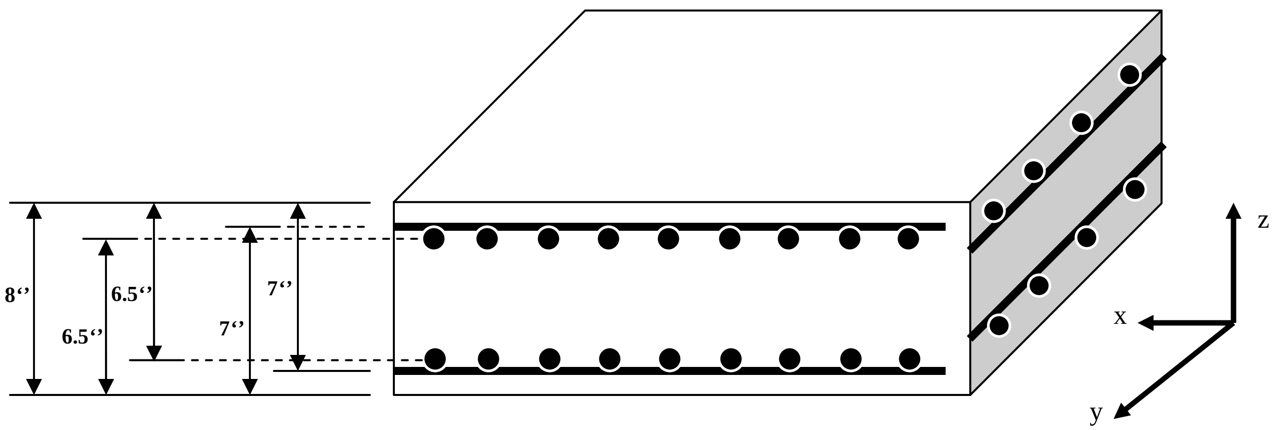


Figure 2. Illustration of the effeective depths in x- and y- directions

1. Fig. 2 is the partial illustration of the reinforced concrete slab in 3D, shown in Fig. 1. Notice that the given effective depths are valid for distances either from the top or the bottom. You will need these values to calculate positive and negative bending moments in the slab.
2. In y-direction, notice that the bending moment capacity of the cross section is NOT constant due to different spacings in two ranges (range 0’~2’ and range 2’~18’). You need to compute both of them for the virtual work, associated with their ranges, done by the slab (you CANNOT take average of them).

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Term Project

Objective

The term project report is intended to encourage you to examine, in some depth, a subject matter in which you are interested. It is also intended to introduce you to the vast body of research and development upon which current design practices are based.

Scope

You are to choose a topic in which you are interested and prepare a project report. A list of some current areas of interest is attached. The subject should then be explored through an examination of the relevant literature. In the report, you should attempt to establish the most current understanding of the basic concepts of physical behavior and/or design concepts related to your specific topic; you should examine the origin of these concepts (including some reference to experimental results where applicable) and their implications to design.

Schedule

There are three documents to be submitted:

1. Project proposal – **February 26** (Thursday)
2. Progress report – **March 30** (Tuesday)
3. Final project report – **May 6** (Thursday)

There will be two progress presentations and one final presentation:

1. Progress presentation I – **March 30** (Tuesday)
2. Progress presentation II – **April 22** (Thursday)
3. Final presentation – **May 6** (Thursday) and **May 11** (Tuesday)

Format

Project Proposal

The project proposal will contain:

1. Title of the research topic
2. Your name and contact information
3. Main text of proposal

Proposal must be typed as singly spaced for text and doubly spaced for section titles. The length of the proposal is limited to **1 page** (Letter size).

Progress Report

The progress report will contain the following components:

1. Title of the research topic
2. Your name and contact information

3. Main text of progress

Progress report must be typed as singly spaced for text and doubly spaced for section titles. The length of the report is limited to **3 pages** (Letter size).

Final Project Report

The final report will consist of the following components:

1. Title Page (1 page)
2. Abstract (1 page).
3. Introduction
4. Main text (sections and subsections)
5. Conclusions
6. Tables and figures
7. List of References

The final report must be typed. Singly spaced for text and doubly spaced for section titles. Length of the text (*Introduction*, main text, *Conclusions*) is limited to **10 pages** (Letter size, not including *Title Page*, *Abstract*, *Tables and Figures*, and *List of References*). You may organize your main text as several main sections with subsections.

Introduction section introduces the topic and its background, defines the objectives of your work and your approach of investigation. The main text explains your methodology of the study and presentation and discussions of the results. *Conclusions* summaries the important findings and structural implications of your investigation. Provide a list of references. All figures, tables, and references should be referred to in the main text.

Papers or books cited as references should be listed by the following format,

Name, “Title”, *Journal title/Book title*, Volume No. (for papers), Issue No. (for papers) Publisher (for books), Pages, Date (month, year).

Example:

Buyukozturk, O., "Failure Behavior of Precracked Concrete Beams Retrofitted with FRP," *ASCE Journal of Composites for Construction*, Vol. 2, No. 3, pp. 138-144, August 1998.

Some Topics of Current Research Interest:

- Behavior of fiber reinforced mortar and concrete materials: Study the effect of fiber materials (steel, polypropylene, carbon), fiber content, strain rate, and cyclic loads on the stress-strain response. Characterization of material behavior under tensile, compressive, or flexural loads. Analysis of fracture and failure.
- Behavior and strength of reinforced and prestressed concrete structures and their components. Case studies of an actual bridge. Fracture and finite element analysis. Innovative bridge design methods.
- Constitutive modeling of concrete for finite element analysis. Critical review and comparison of existing models. In-depth study of a specific model. New model development. Finite element implementation.
- Development of computer software for the design and analysis of reinforced concrete or prestressed concrete members and structures. Knowledge based design systems. Expert interactive design development.
- Development of mix design, study of mechanical behavior and application of high strength concrete.
- Behavior of partially prestressed concrete beams. Behavior of unbonded tendons in prestressed segments. Development of design tools for partial prestressing and unbonded tendons.
- Development of innovative structural design methodologies using conventional and advanced construction materials.
- Ceramics and refractories. Behavior of ceramics and refractories, and their uses in structural applications.
- Development and behavior of high performance concrete materials and their use in innovative structural solutions.
- Fracture of cememtitious materials and design based on fracture analysis. Interface fracture concepts. Size effects. Composite materials and structural systems.
- Design case studies and design evaluation.
- Durability of concrete structures: corrosion and spalling problems. Bridge deck, roadway, and airport pavement deterioration.
- Nondestructive evaluation (NDE) methods used for condition assessment of concrete structures: ultrasonic, acoustic emission, microwave, and other methods.
- Innovative techniques in repair and strengthening of concrete structures.
- Use of fiber reinforced polymer (FRP) in new construction and in strengthening of existing structures.
- Behavior and design of earthquake resistant concrete structures.
- Repair and retrofit of concrete structures.
- Structural health monitoring (SHM) and damage detection problem in concrete structures.

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Exam #1

Date: Tuesday, March 16, 2004 **Time:** 2:30pm~4pm

Note: This is an open-book and open-note exam.
This exam is designed to test your knowledge of the concepts. Be brief and specific in your discussions.

A high-rise building is being designed using an innovative concept for the columns. The columns have no reinforcement bars but are confined by thin steel shells, which also perform a function of formwork for the concrete. High strength concrete in conjunction with conventional steel is being used with the following properties:

Concrete

Uniaxial compressive strength of concrete: $f_c' = 8100\ psi$;

Uniaxial splitting tensile strength of concrete: $f_t' = 7\sqrt{f_c'}\ (psi)$; and

Young's Modulus: $E_c = 40000\sqrt{f_c'} + 1.0 \times 10^6\ (psi)$

Steel

Yield stress: $f_y = 60\ ksi$

Young's modulus: $E_s = 29\text{x}10^6\ psi$

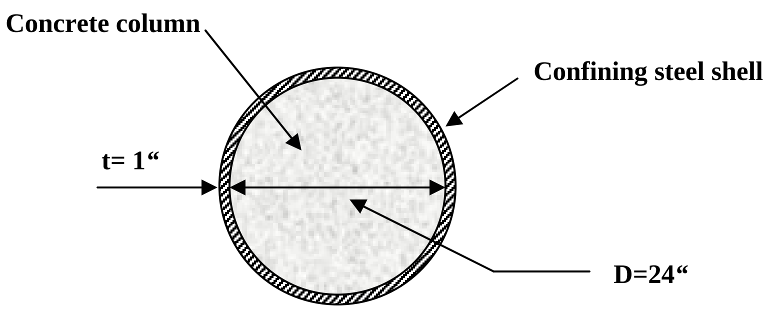


Figure 1. Concrete column confined by thin steel shell

Question 1 – Confinement Effect (30%)

(a) Consider the column section shown in Fig. 1. Assuming that the axial strength of the concrete increases with confinement in a similar way as for the conventional concrete, compute the maximum concentrically applied load that can be carried by the column. You may assume:

- at failure steel yields and concrete strength reaches the peak value;
- the concrete is perfectly bonded to the steel.

(b) Briefly discuss the effect of varying axial load on the confinement.

Question 2 – Behavior in Combined Stress (30%)

(a) Consider that the column section shown in Fig. 1 is subjected a concentrically applied axial load of P = 8,000 kips, and a simultaneous moment. Based on your knowledge of concrete in combined stress, compute the value of the moment that causes first cracking in this beam-column member. For simplification, you may assume:

- Before cracking, there is a transverse confinement level of, for example, 5000 psi and that this confinement is valid in the tension zone of section as well.
- Before cracking, concrete and steel are in elastic regime and that the principle of superposition is valid.

(Hint: For simplicity use the concept of transformed area and transformed moment of inertia for this composite section.)

(b) For a more accurate analysis, briefly discuss the factors influencing the interaction between the confinement and the applied bending moment. Comment on the presence of simultaneously applied axial force.

Question 3 – Creep and Shrinkage (25%)

Consider the column in Fig. 1 under a sustained concentric axial load of P = 8,000 kips.

(a) Compute the final stress value in the concrete after one year. Assume that the creep coefficient C_t = 0.9 for 1 year. Indicate whether the final stress is tensile or compressive and explain.
(b) How would this applied load influence the shrinkage behavior of concrete during that time?

Question 4 – Ductility (15%)

(a) Describe very briefly, with no calculations, ductility for
 (i) a reinforced concrete flexural member;
 (ii) a reinforced concrete beam-column;
 (iii) a reinforced concrete building.

(b) Briefly comment on the importance of ductility for the building in an earthquake region.

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Exam #2

Date: Tuesday, May 4, 2004 **Time:** 2:30pm ~ 4pm

Note: This is an open-book and open-note exam.
This exam is designed to test your knowledge of the concepts. Be brief and specific in your discussions.

Question 1 – Bridge Torsion (45%)

A small simply supported bridge in an industrial plant carrying a conveyor and spanning over 40 ft. has a T cross section as shown in Fig. 1. The bridge is to carry a full service live load of 110 lb/ft^2 over its entire width; when only one half of the width of the bridge is loaded a service live load of 180 lb/ft^2 shall be considered. Neglect the weight of the concrete for brevity. Capacity reduction factor for shear and torsion is $\phi = 0.85$.

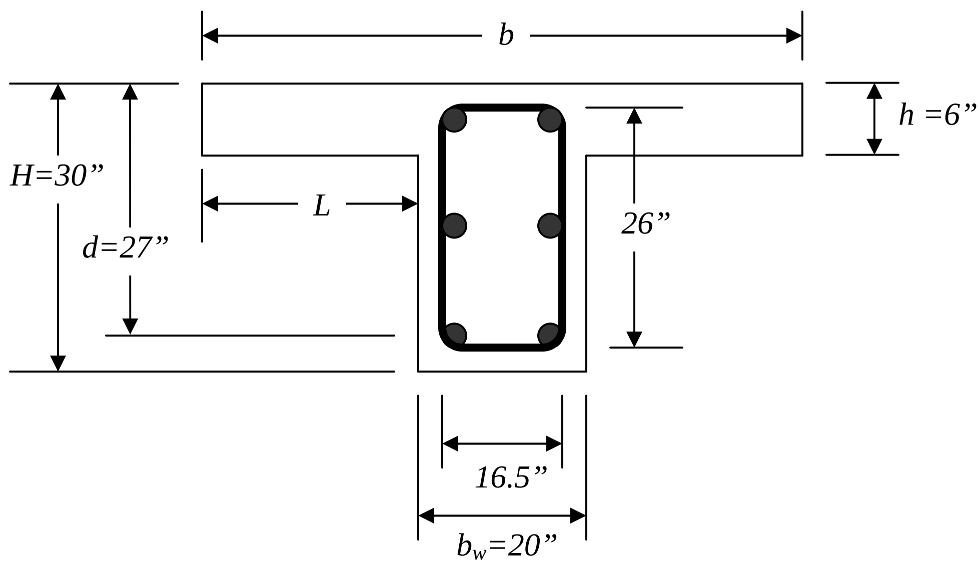


Fig. 1 Dimensions of the T cross section

Material strengths are provided below:

Concrete
Uniaxial compressive strength of concrete: f_c'= 4000 *psi*;

Steel
Yield stress: f_y = 40 *ksi*

The beam has been reinforced for shear and torsion. Combined shear reinforcement (A_v) and torsional reinforcement ($2A_t$) have the following relationship

$$\rho_{v,total} = \frac{A_v + 2A_t}{b_w \cdot s} = 0.01$$

where s = uniform spacing of the reinforcement. Also, we assume $A_v = A_t$.

Compute the maximum allowable width, *b*, of the bridge deck for:

(a) pure shear considering that the bridge is fully loaded (neglect bending effects).
(b) combined shear and torsion. (Describe your methodology and set up the equations for solving b_{max}. If an explicit solution is difficult, describe briefly the procedure for a solution based on numerical iteration.)

Hints:

Check the ultimate shear capacity when the bridge is carrying load over its entire width, and the shear and torsional capacities when the bridge is loaded only on one half of the width.
$V_u \le \phi\left(V_s + V_c\right)$, (Use live load factor of 1.7 in computing V_u)

For pure shear: $V_s = f_y \cdot d \cdot \dfrac{A_{v,total}}{s}$

Question 2 – Yield Line Slab Analysis (35%)

The one-way slab shown in Fig. 2 is uniformly reinforced to provide a positive and negative moment capacity of M_u.

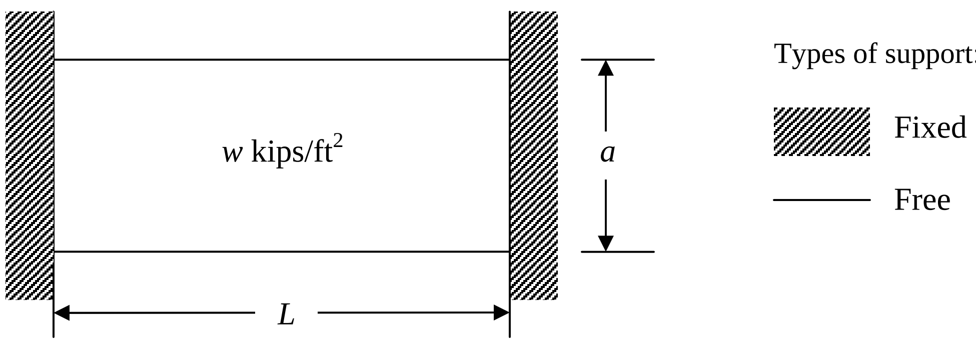


Fig. 2 One-way slab

(a) Sketch the most likely yield line pattern for the slab, and obtain an expression for the ultimate load capacity, w_u, for this slab.
(b) The collapse mechanism shown in Fig. 3 is considered for this slab. Compute the ultimate load capacity, w_u, for this collapse mechanism.

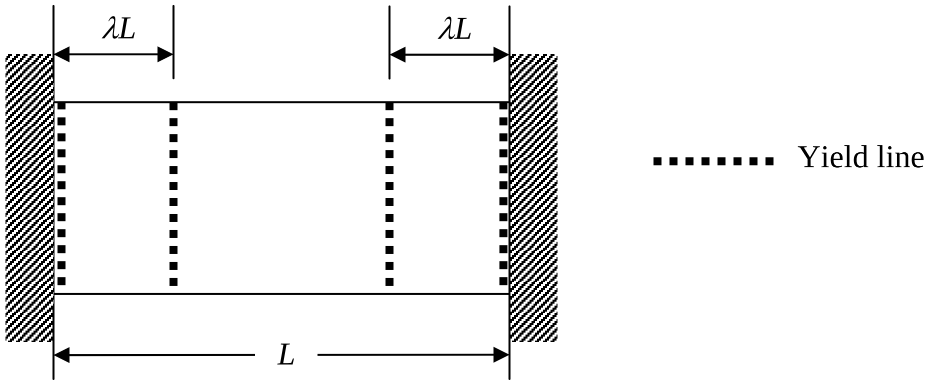


Fig. 3 The collapse mechanism

Compare the values found from (a) and (b). Explain any difference in these values for varied λ. Comment on the safety implication of this solution.

Question 3 – Thin Shell Structures (20%)

Consider the cylindrical shell under uniformly distributed surface load.

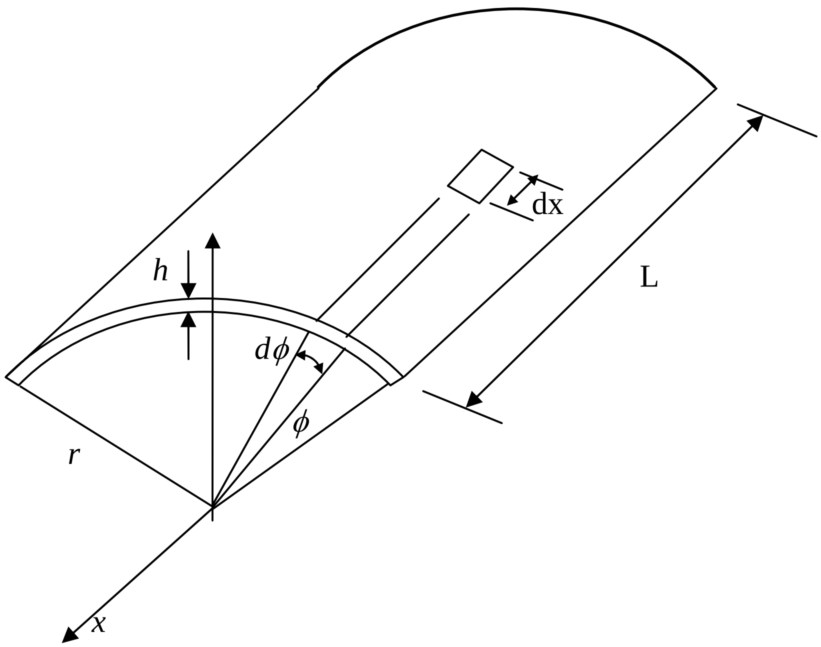


Fig. 4 The cylindrical shell

(a) What are the conditions (or parameters) that approximately classify this shell as i) a long shell; ii) a short shell.

(b) Describe briefly the behavioral difference of the short and long shells and indicate the significant internal effects associated with each of these shell types.

(c) You are asked to provide an approximate solution of internal effects as a basis for preliminary design of this shell. Briefly explain your analysis methodology for each of these shell types.