STRESS-STRAIN BEHAVIOUR OF DIFFERENT MATERIALS IN ISOTHERMAL CONDITION

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CLASS ATTENDANCE

Stress and Strain

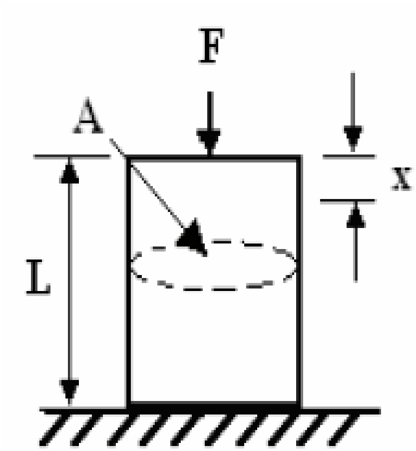
1. DIRECT STRESS σ

When a force is applied to an elastic body, the body deforms.

The way in which the body deforms depends upon the type of force applied to it.

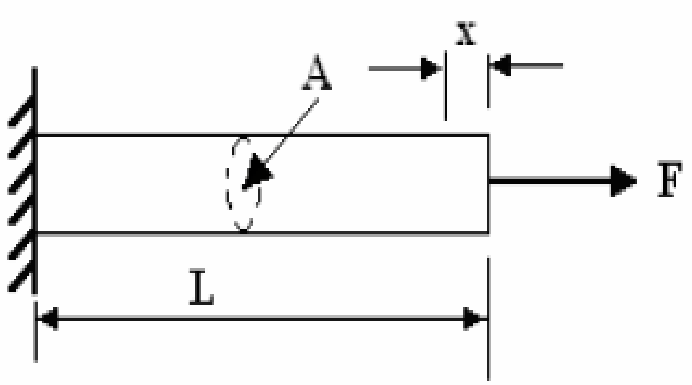
A compression force makes the body shorter.

Figure 1



A tensile force makes the body longer.

Figure 2



Tensile and compressive forces are called **DIRECT FORCES.**

Stress is the force per unit area upon which it acts.

Stress = σ = Force/Area N/m^2 or Pascals.

Stress and Strain

2. DIRECT STRAIN ε

In each case, a force F produces a deformation x. In engineering we usually change this force into stress and the deformation into strain and we define these as follows.

Strain is the deformation per unit of the original length

Strain = ε = x/L

The symbol ε is called EPSILON

Strain has no units since it is a ratio of length to length. Most engineering materials do not stretch very much before they become damaged so strain values are very small figures. It is quite normal to change small numbers in to the exponent for of 10^{-6}. Engineers use the abbreviation με (micro strain) to denote this multiple.

For example a strain of 0.000068 could be written as 68×10^{-6} but engineers would write 68 με.

WORKED EXAMPLE No.1

A metal wire is 2.5 mm diameter and 2 m long. A force of 12 N is applied to it and it stretches 0.3 mm. Assume the material is elastic. Determine the following.

i. The stress in the wire σ.
ii. The strain in the wire ε.

SOLUTION

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 2.5^2}{4} = 4.909 \text{ mm}^2$$

$$\sigma = \frac{F}{A} = \frac{12}{4.909} = 2.44 \text{ N/mm}^2$$

Answer (i) is hence 2.44 MPa

$$\varepsilon = \frac{x}{L} = \frac{0.3 \text{ mm}}{2000} = 0.00015 \text{ or } 150\ \mu\varepsilon$$

Stress and Strain

3. MODULUS OF ELASTICITY E

Elastic materials always spring back into shape when released. They also obey HOOKE'S LAW. This is the law of a spring which states that deformation is directly proportional to the force. F/x = stiffness = k N/m

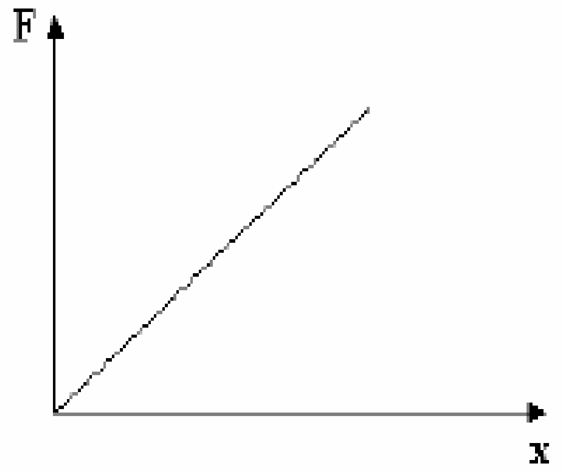


Figure 3

The stiffness is different for different materials and different sizes of the material. We may eliminate the size by using stress and strain instead of force and deformation as follows.

If F and x refer to direct stress and strain then

$F = \sigma A \quad x = \varepsilon L$ hence $\frac{F}{x} = \frac{\sigma A}{\varepsilon L}$ and $\frac{FL}{Ax} = \frac{\sigma}{\varepsilon}$

The stiffness is now in terms of stress and strain only and this constant is called the **MODULUS of ELASTICITY and it has a symbol E.**

$$E = \frac{FL}{Ax} = \frac{\sigma}{\varepsilon}$$

A graph of stress against strain will be a straight line with a gradient of E. The units of E are the same as the units of stress.

Stress and Strain

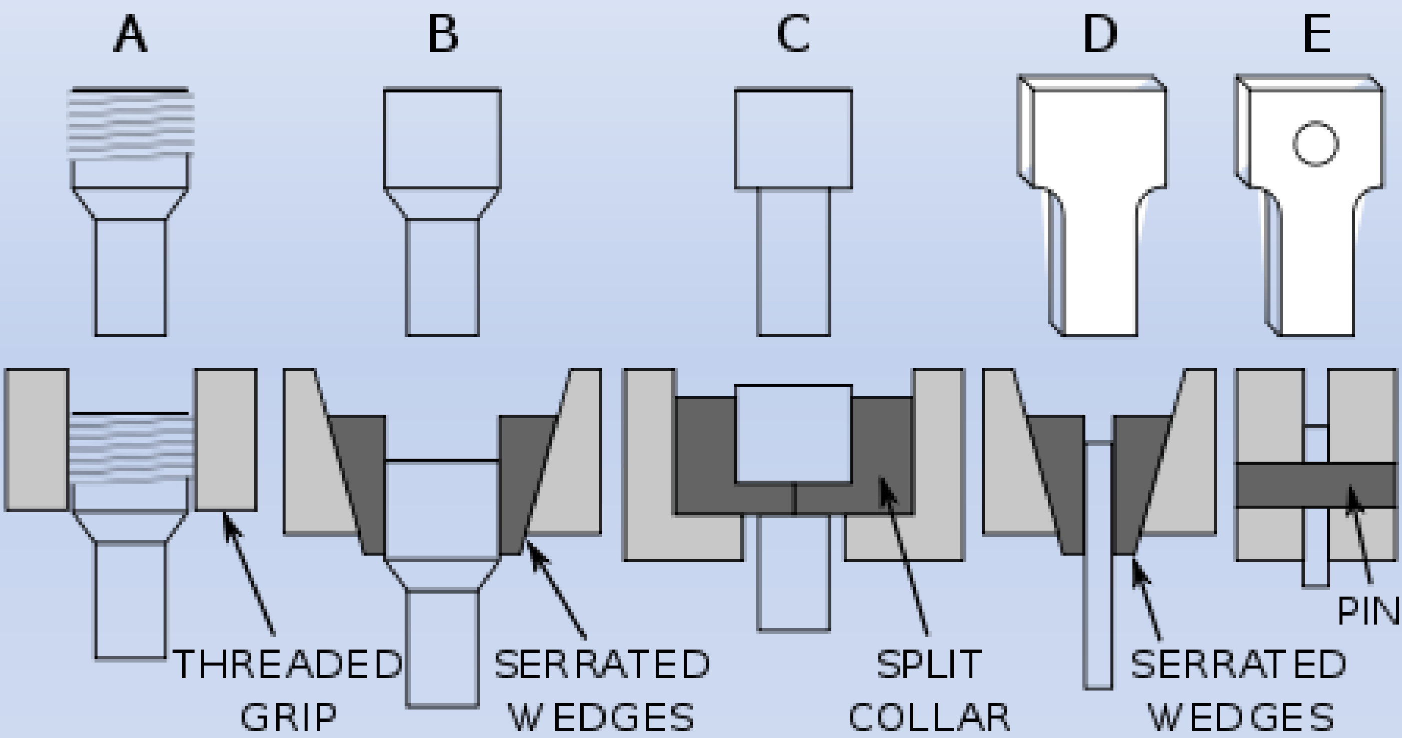
4. ULTIMATE TENSILE STRESS

If a material is stretched until it breaks, the tensile stress has reached the absolute limit and this stress level is called the ultimate tensile stress. Values for different materials may be found in various sources such as the web site Matweb.

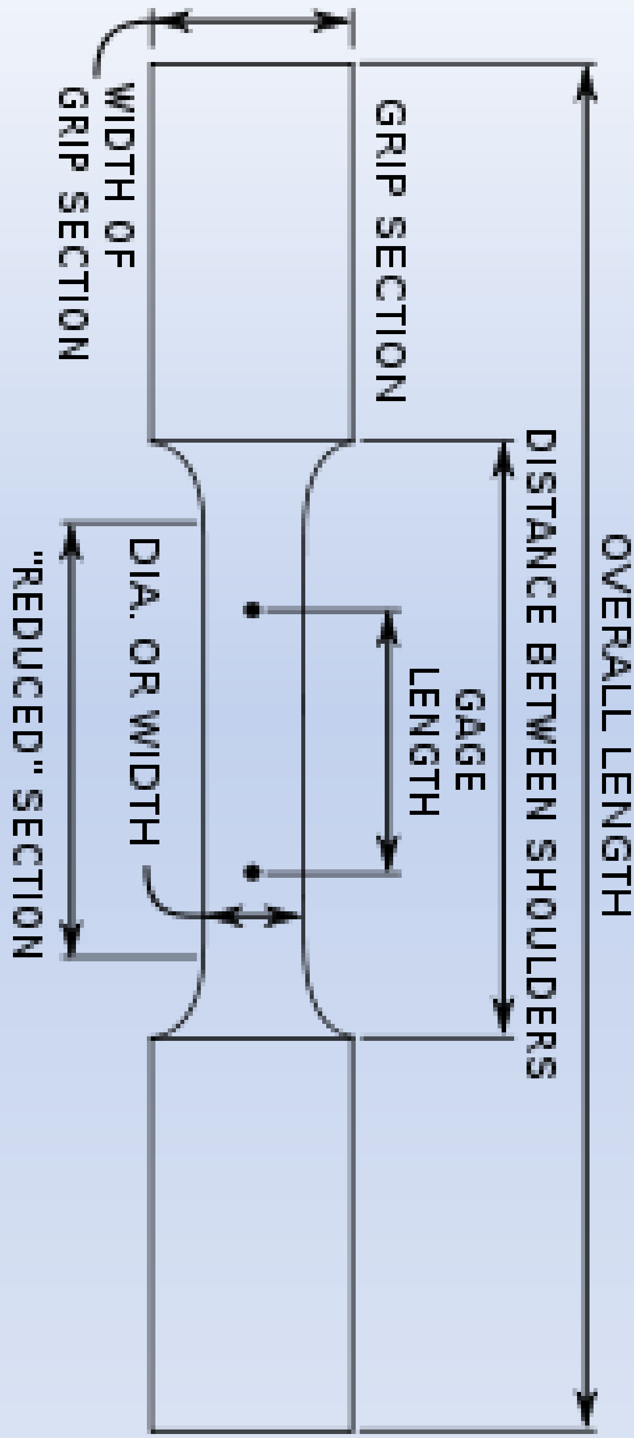
Universal Testing Machine

Closer Look

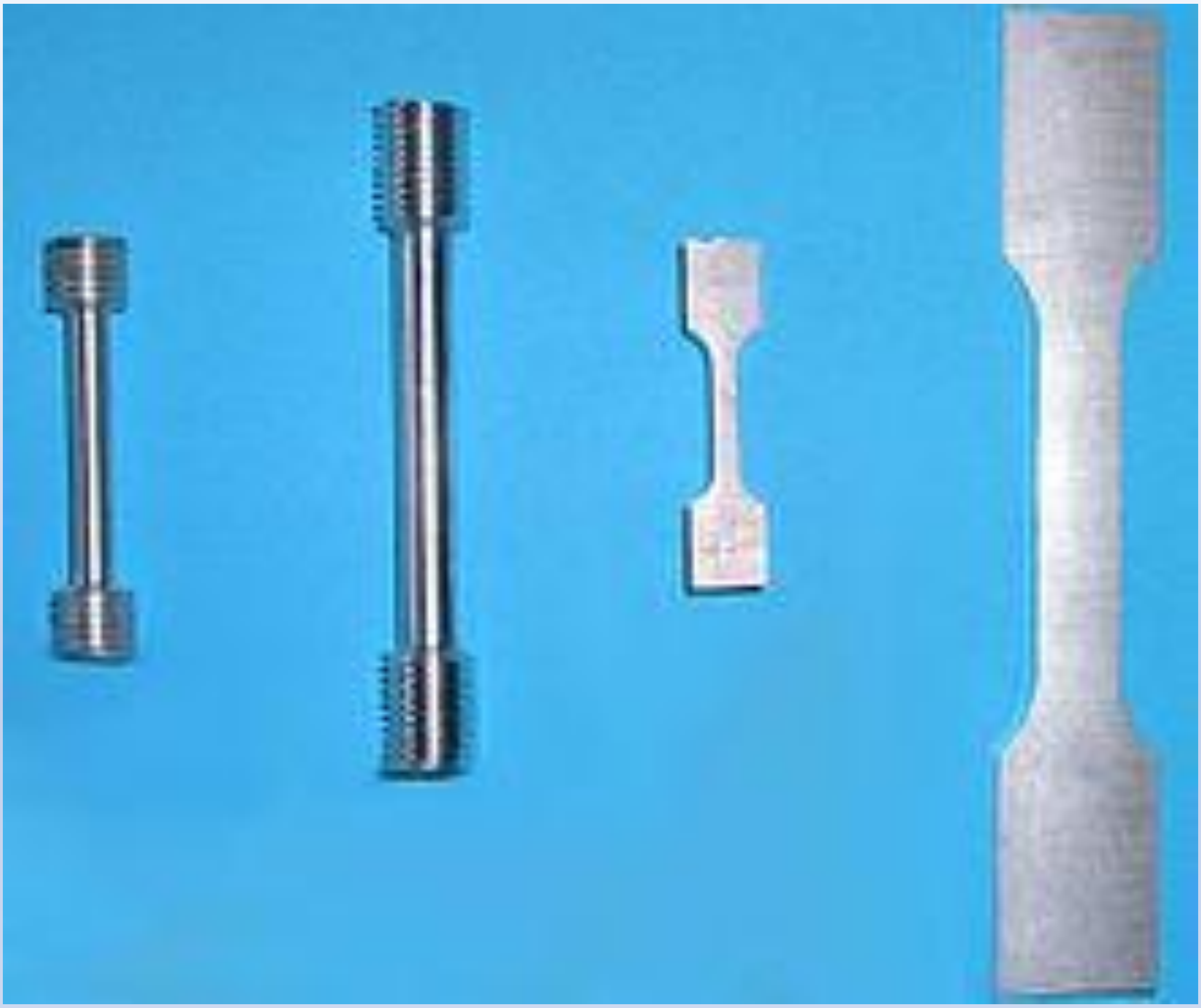
Different Types of Grips



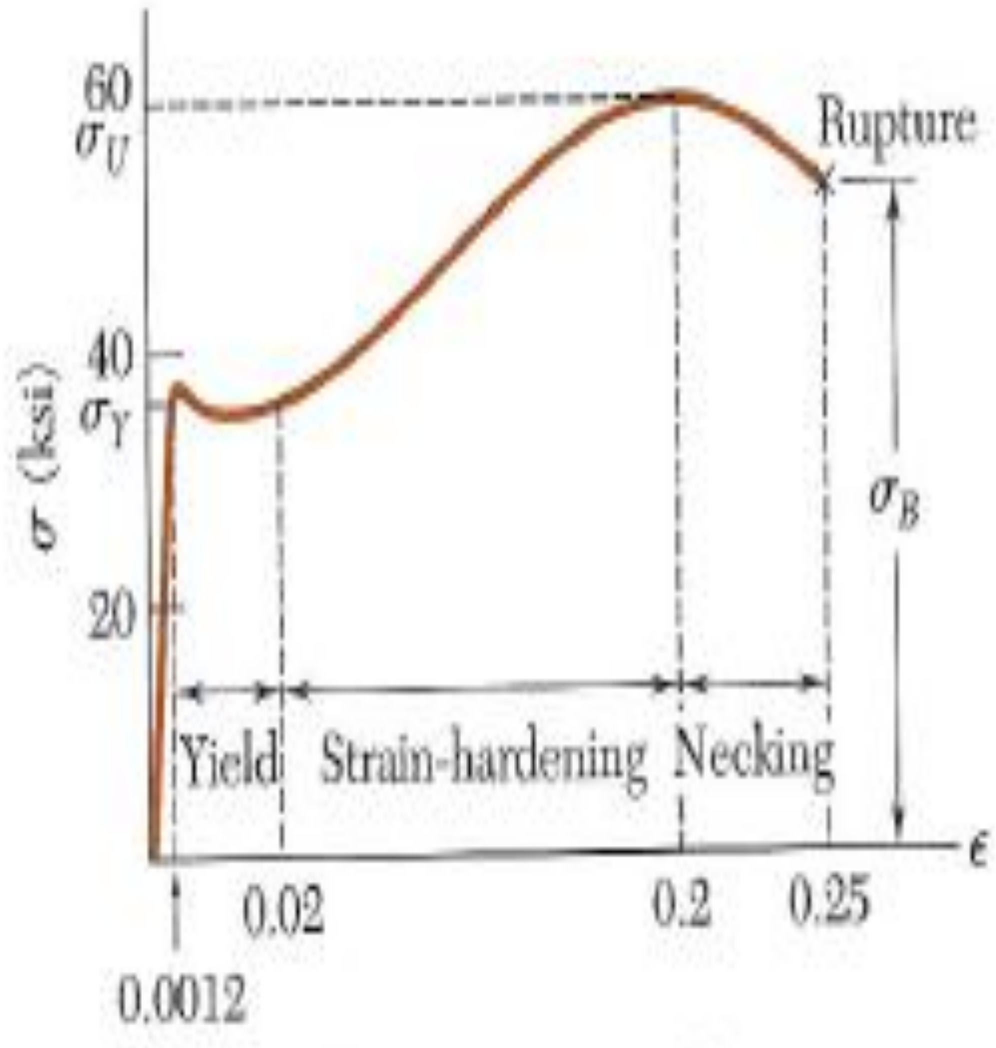
Definition of Gage Length



Test Specimens

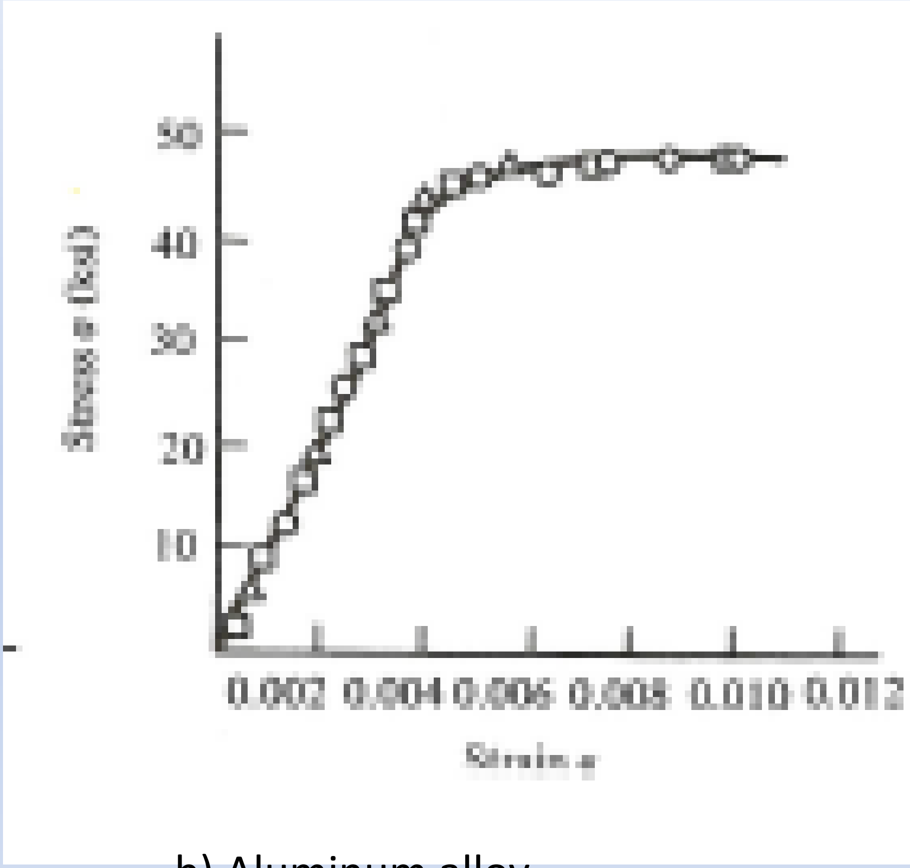


Stress-Strain Curve



(a) Low-carbon steel

Stress-Strain Curve



b) Aluminum alloy

Stress and Strain

WORKED EXAMPLE No.2

A steel tensile test specimen has a cross sectional area of 100 mm^2 and a gauge length of 50 mm, the gradient of the elastic section is 410 x 10^3 N/mm. Determine the modulus of elasticity.

SOLUTION

The gradient gives the ratio F/A = and this may be used to find E.

$$E = \frac{\sigma}{\varepsilon} = \frac{F}{x} x \frac{L}{A} = 410 x 10^3 x \frac{50}{100} = 205\,000 \text{ N/mm}^2 \text{ or } 205\,000 \text{ MPa or } 205 \text{ GPa}$$

WORKED EXAMPLE No.3

A Steel column is 3 m long and 0.4 m diameter. It carries a load of 50 MN. Given that the modulus of elasticity is 200 GPa, calculate the compressive stress and strain and determine how much the column is compressed.

SOLUTION

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 0.4^2}{4} = 0.126\,\text{m}^2$$

$$\sigma = \frac{F}{A} = \frac{50 \times 10^6}{0.126} = 397.9 \times 10^6\,\text{Pa}$$

$$E = \frac{\sigma}{\varepsilon} \quad \text{so} \quad \varepsilon = \frac{\sigma}{E} = \frac{397.9 \times 10^6}{200 \times 10^9} = 0.001989$$

$$\varepsilon = \frac{x}{L} \quad \text{so} \quad x = \varepsilon L = 0.001989 \times 3000\,\text{mm} = 5.97\,\text{mm}$$

Stress and Strain

5. SHEAR STRESS τ

Shear force is a force applied sideways on to the material (transversely loaded). This occurs typically:

when a pair of shears cuts a material

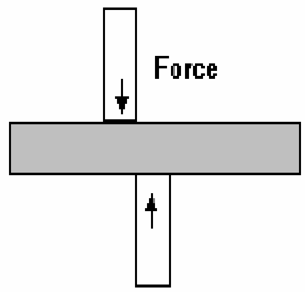


Figure 4

when a material is punched

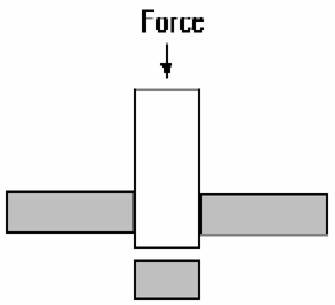


Figure 5

when a beam has a transverse load.

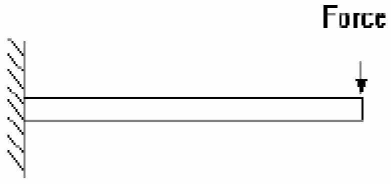


Figure 6

Stress and Strain

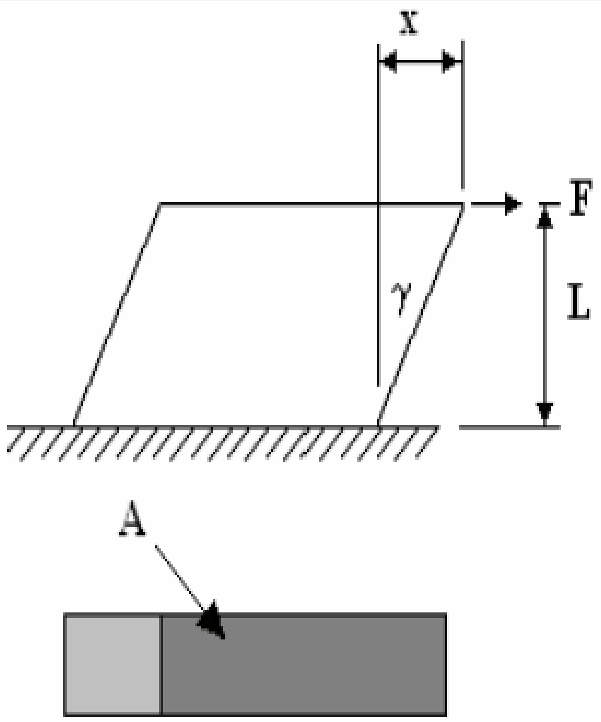


Figure 9

6. SHEAR STRAIN γ

The force causes the material to deform as shown. The shear strain is defined as the ratio of the distance deformed to the height x/L.

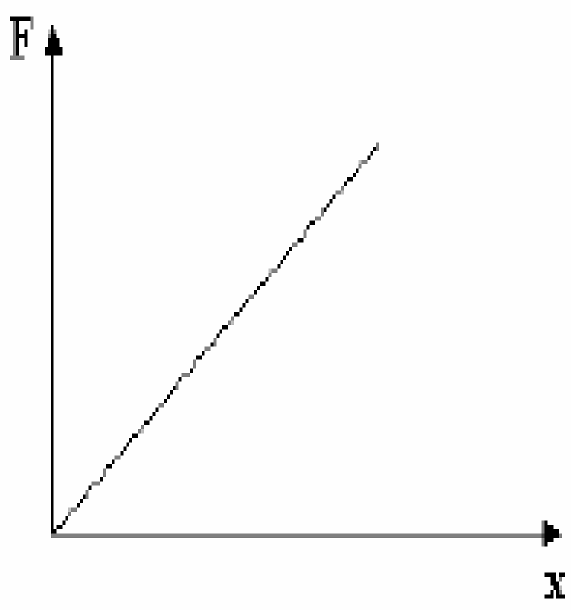
The end face rotates through an angle γ. Since this is a very small angle, it is accurate to say the distance x is the length of an arc of radius L and angle γ so that

$$\gamma = x/L$$

It follows that γ is the shear strain. The symbol γ is called Gamma.

7. MODULUS OF RIGIDITY G

If we were to conduct an experiment and measure x for various values of F, we would find that if the material is elastic, it behave like a spring and so long as we do not damage the material by using too big a force, the graph of F and x is a straight line as shown.



The gradient of the graph is constant so F/x = constant and this is the spring stiffness of the block in N/m.

If we divide F by the area A and x by the height L, the relationship is still a constant and we get

$$\frac{F}{A} \div \frac{x}{L} = \frac{FL}{Ax} = \text{constant}$$

Figure 10

But F/A = τ and x/L = γ so $\frac{F}{A} \div \frac{x}{L} = \frac{FL}{Ax} = \frac{\tau}{\gamma} = \text{constant}$

This constant will have a special value for each elastic material and is called the Modulus of Rigidity with symbol G.

$$\frac{\tau}{\gamma} = G$$

Stress and Strain

8. ULTIMATE SHEAR STRESS

If a material is sheared beyond a certain limit it becomes permanently distorted and does not spring all the way back to its original shape. The elastic limit has been exceeded. If the material is stressed to the limit so that it parts into two(e.g. a guillotine or punch), the ultimate limit has been reached. The ultimate shear stress is τ_u and this value is used to calculate the force needed by shears and punches.

WORKED EXAMPLE No.4

Calculate the force needed to guillotine a sheet of metal 5 mm thick and 0.8 m wide given that the ultimate shear stress is 50 MPa.

SOLUTION

The area to be cut is a rectangle 800 mm x 5 mm

$A = 800 \times 5 = 4000 \text{ mm}^2$ The ultimate shear stress is 50 N/mm^2

$$\tau = \frac{F}{A} \quad \text{so} \quad F = \tau \times A = 50 \times 4000 = 200\,000 \text{ N or } 200 \text{ kN}$$

WORKED EXAMPLE No.5

Calculate the force needed to punch a hole 30 mm diameter in a sheet of metal 3 mm thick given that the ultimate shear stress is 60 MPa.

SOLUTION

The area to be cut is the circumference x thickness = $\pi d \times t$

$A = \pi \times 30 \times 3 = 282.7\ \text{mm}^2$ The ultimate shear stress is 60 N/mm^2

$$\tau = \frac{F}{A} \quad \text{so} \quad F = \tau \times A = 60 \times 282.7 = 16965\ \text{N or } 16.965\ \text{kN}$$

WORKED EXAMPLE No.6

Calculate the force needed to shear a pin 8 mm diameter given that the ultimate shear stress is 60 MPa.

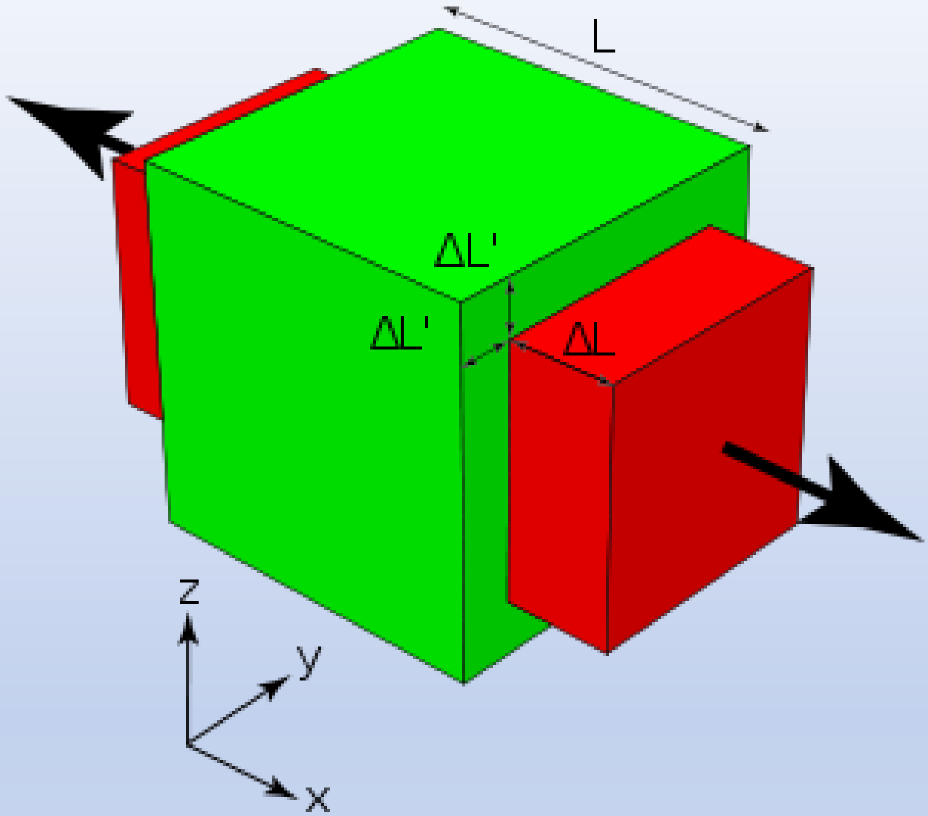
SOLUTION

The area to be sheared is the circular area $A = \frac{\pi d}{4}$

$A = \frac{\pi \times 8^2}{4} = 50.26\text{mm}^2$ The ultimate shear stress is 60 N/mm^2

$$\tau = \frac{F}{A} \quad \text{so} \quad F = \tau \times A = 60 \times 50.26 = 3016\ \text{N or } 3.016\ \text{kN}$$

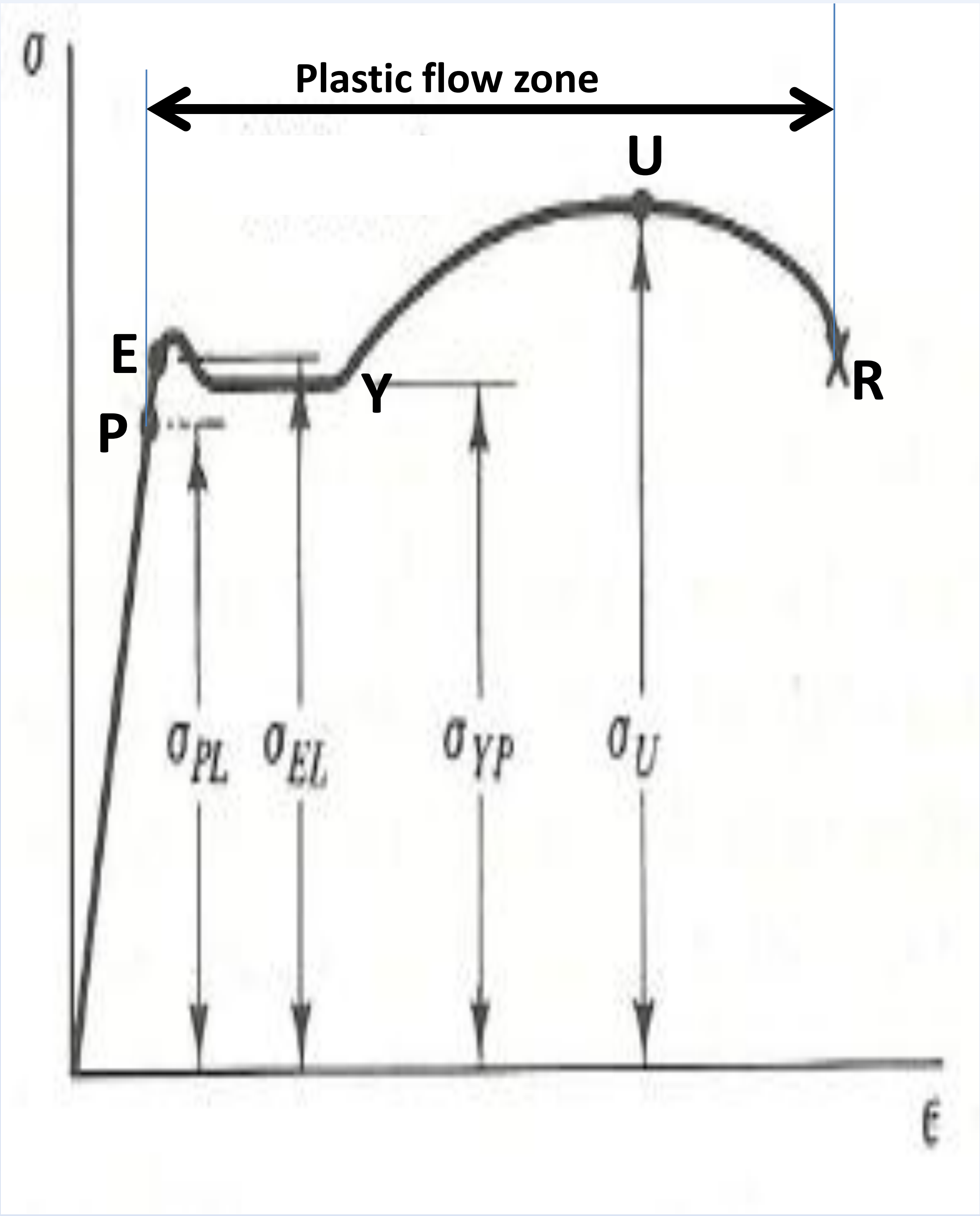
9. Poisson's Ratio



$$\nu = -\frac{d\varepsilon_{\mathrm{trans}}}{d\varepsilon_{\mathrm{axial}}} = -\frac{d\varepsilon_{\mathrm{y}}}{d\varepsilon_{\mathrm{x}}} = -\frac{d\varepsilon_{\mathrm{z}}}{d\varepsilon_{\mathrm{x}}}$$

Rubber: 0.5
Glass: 0.18-0.30
Concrete: 0.2
Steel: 0.27-0.30

10. Definitions Related to Stress-Strain Curves



10. Definitions Related to Stress-Strain Curves

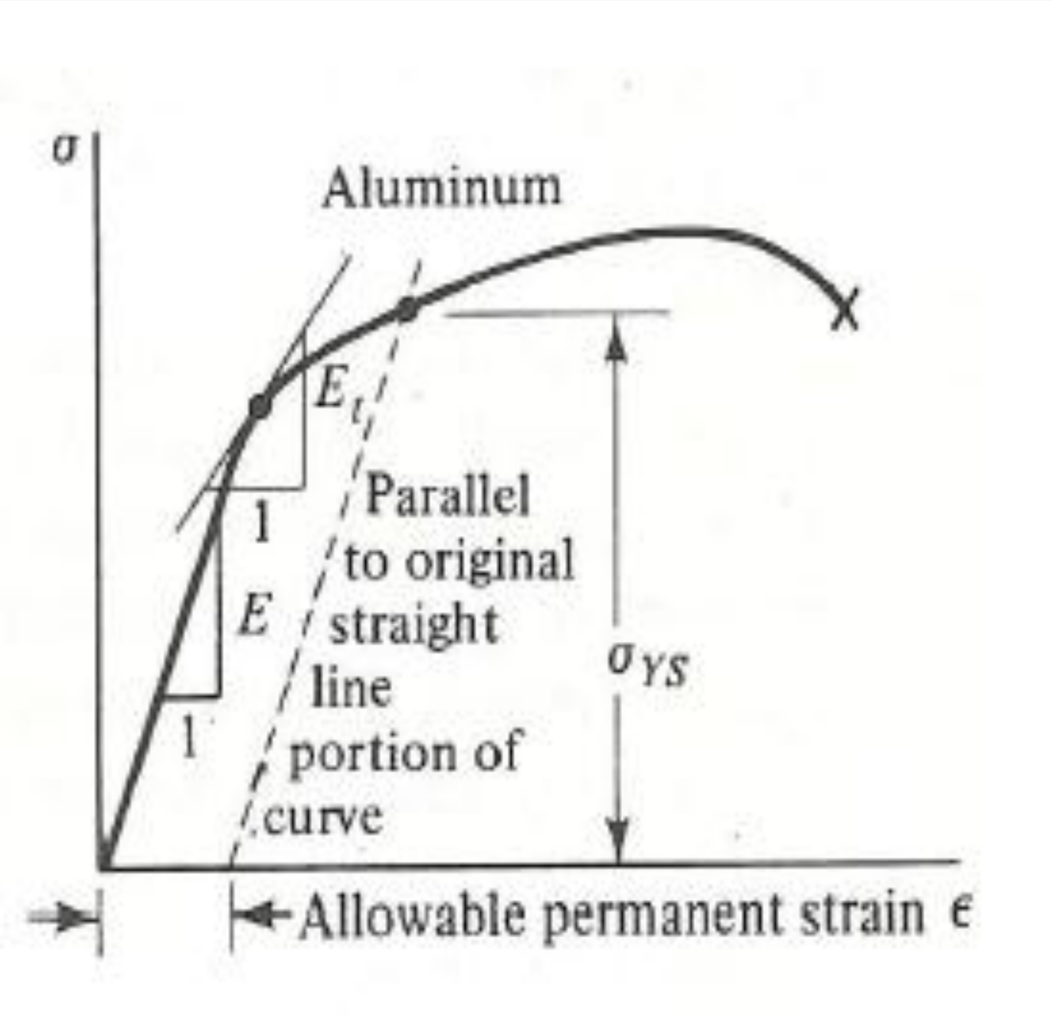
$\sigma_{PL} \Rightarrow$ **Proportional Limit** - Stress above which stress is not longer proportional to strain.

$\sigma_{EL} \Rightarrow$ **Elastic Limit** - The maximum stress that can be applied without resulting in permanent deformation when unloaded.

$\sigma_{YP} \Rightarrow$ **Yield Point** - Stress at which there are large increases in strain with little or no increase in stress. Among common structural materials, only steel exhibits this type of response.

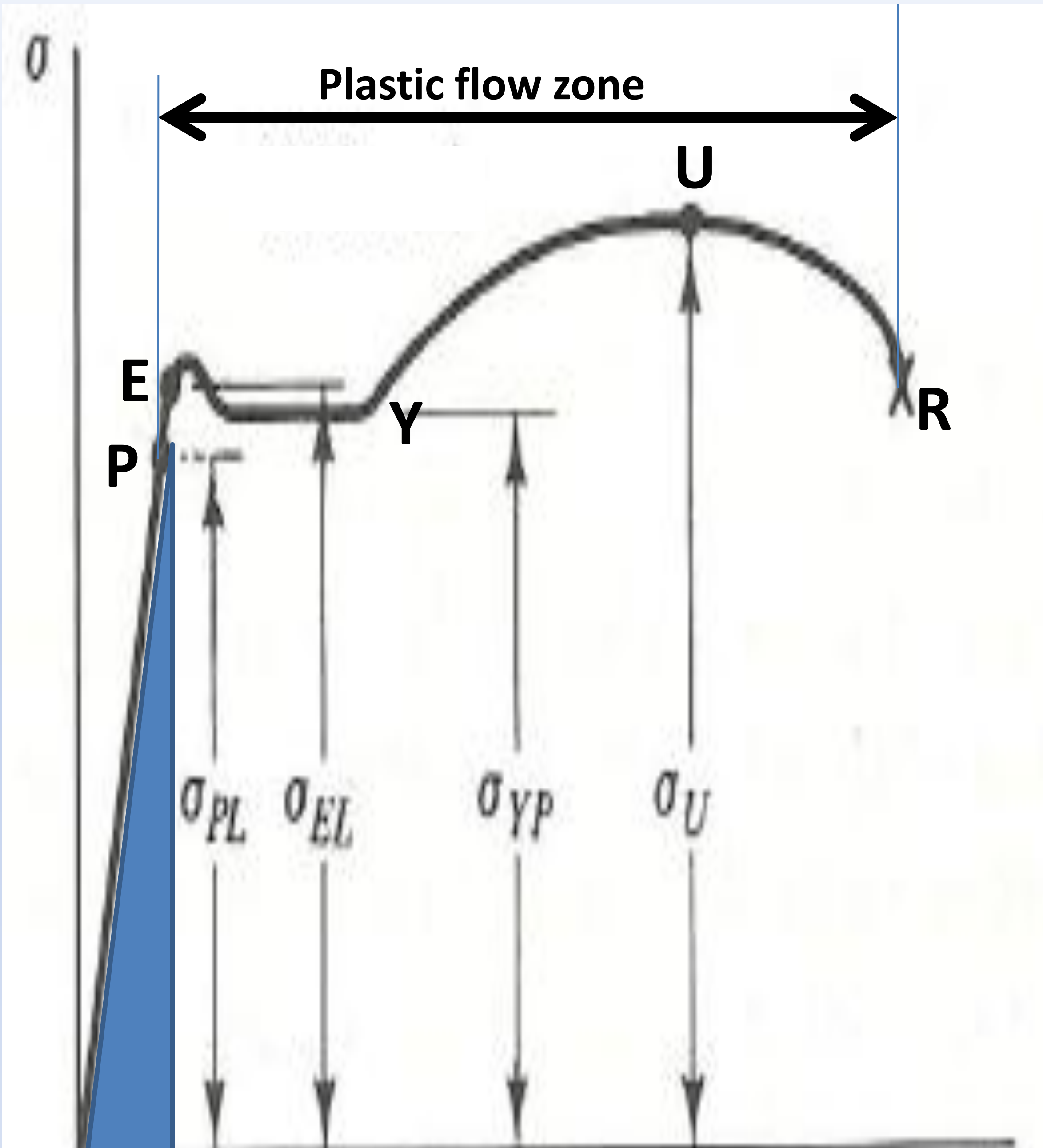
$\sigma_{U} \Rightarrow$ **Ultimate Strength** - The maximum stress the material can withstand (based on the original area).

10. Definitions Related to Stress-Strain Curves



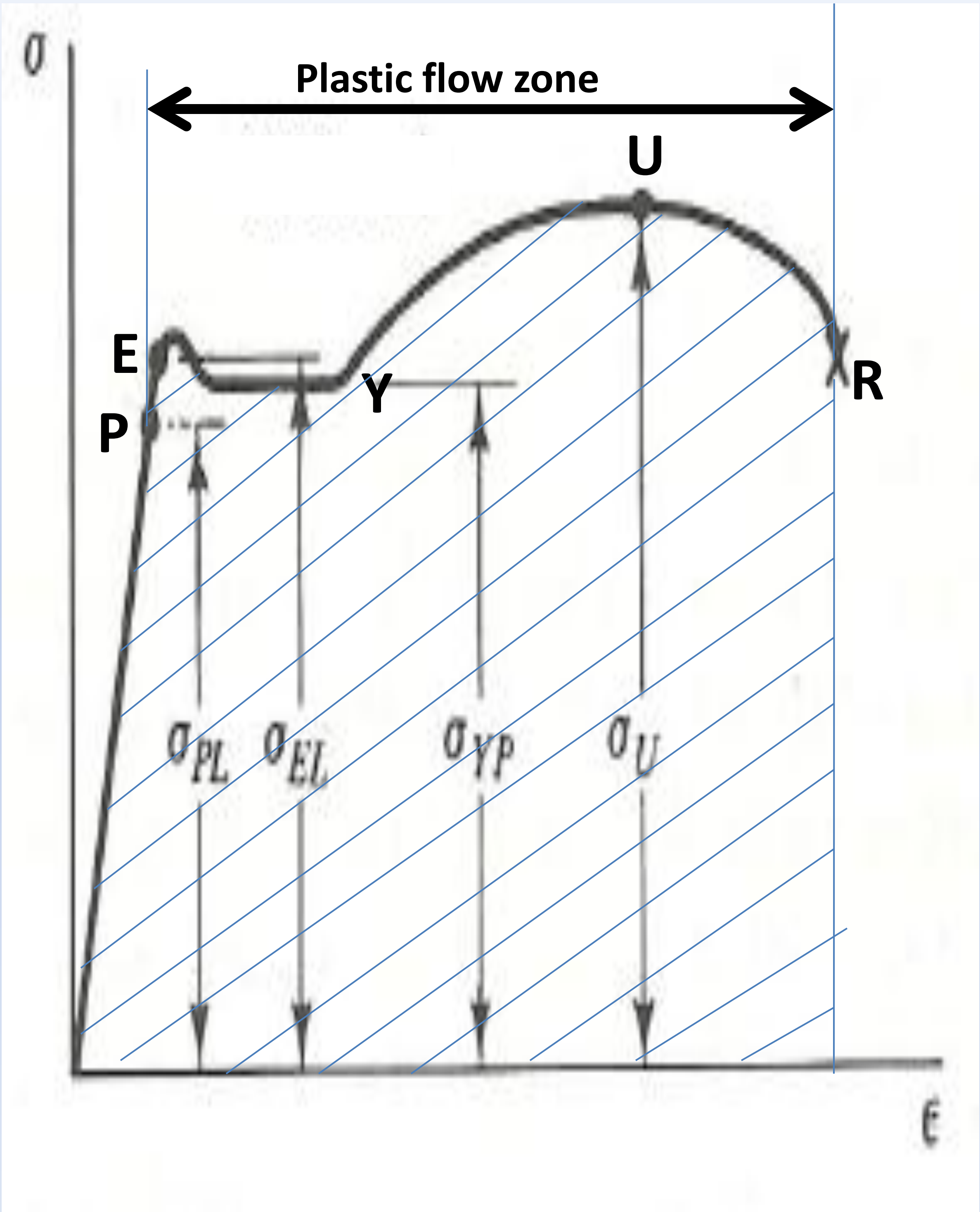
$\sigma_{YS} \Rightarrow$ **Yield Strength** - The maximum stress that can be applied without exceeding a specified value of permanent strain (typically .2% = .002 in/in).

10. Definitions Related to Stress-Strain Curves



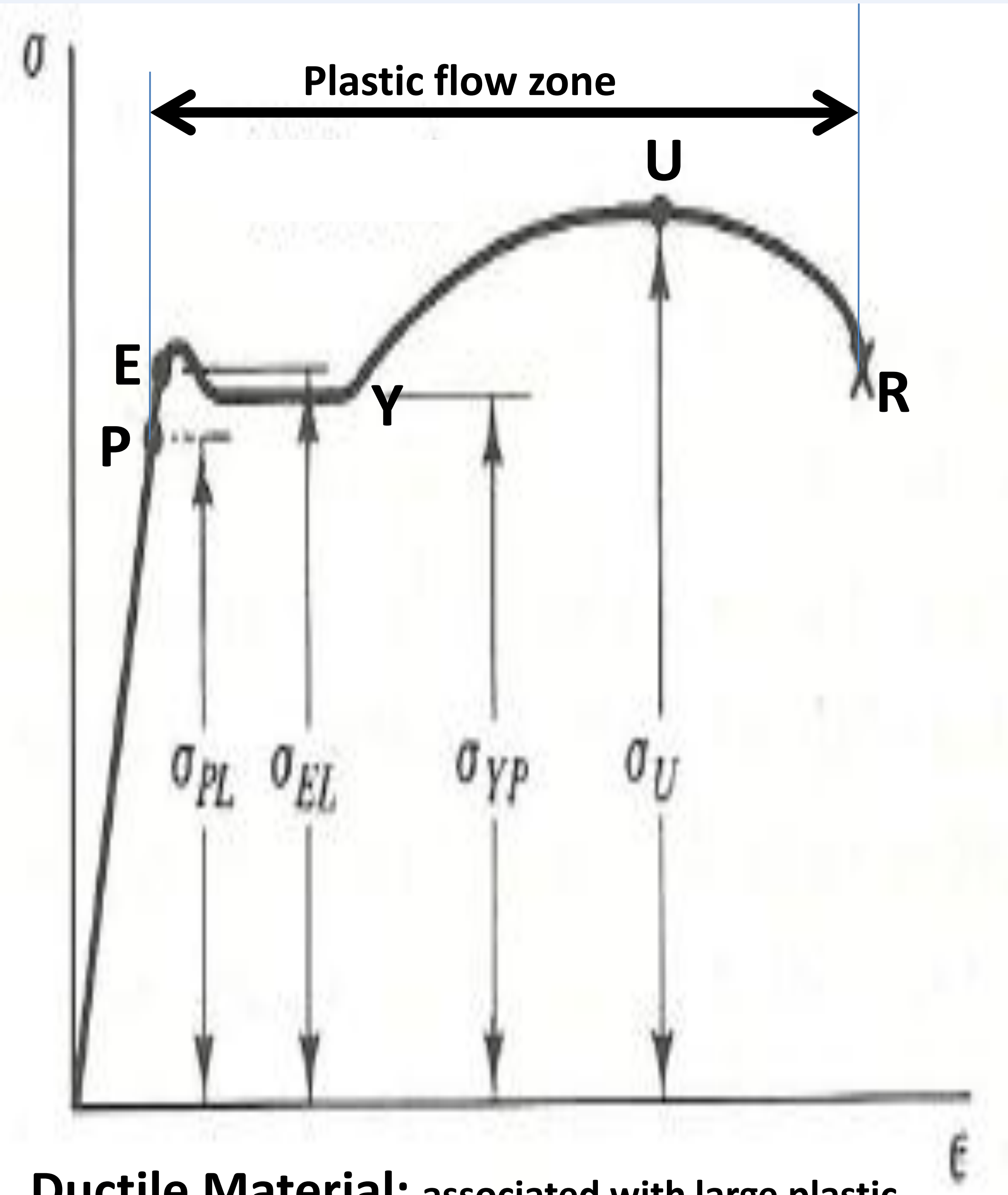
Modulus of Resilience: ability to absorb energy without causing any permanent deformation

10. Definitions Related to Stress-Strain Curves



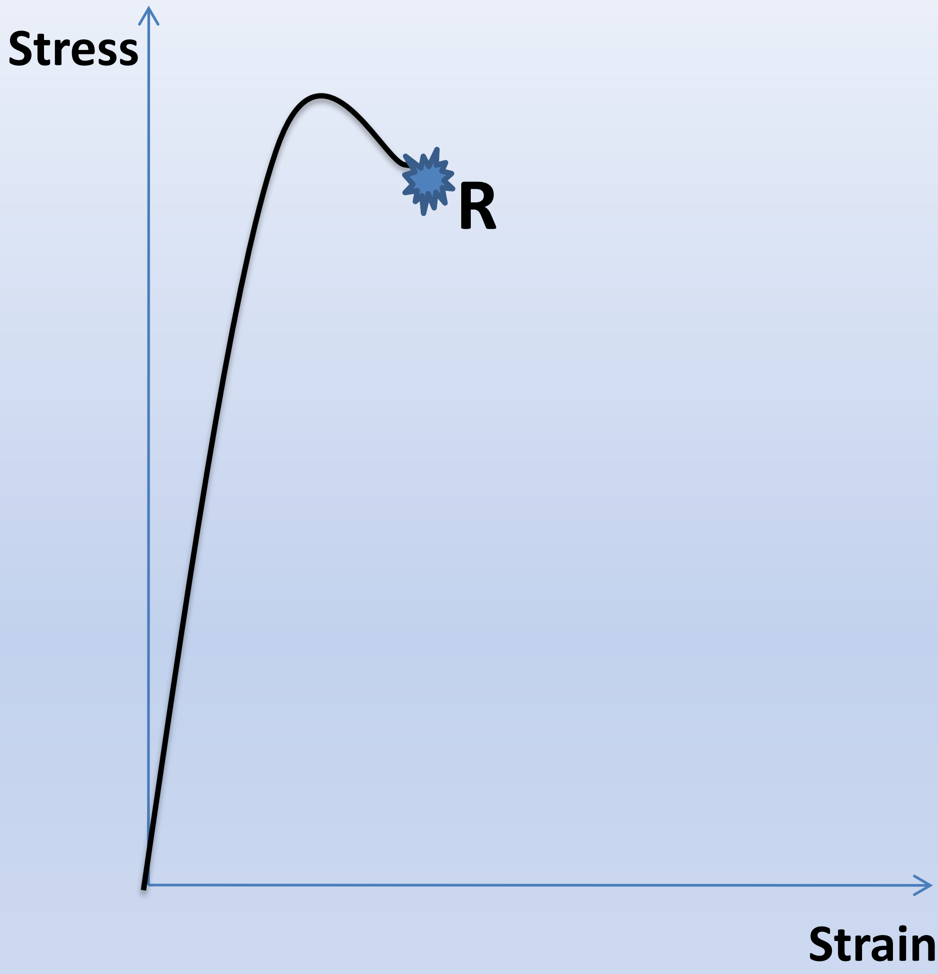
Modulus of Toughness: ability to absorb energy without causing it to break

11. Ductile and Brittle Material



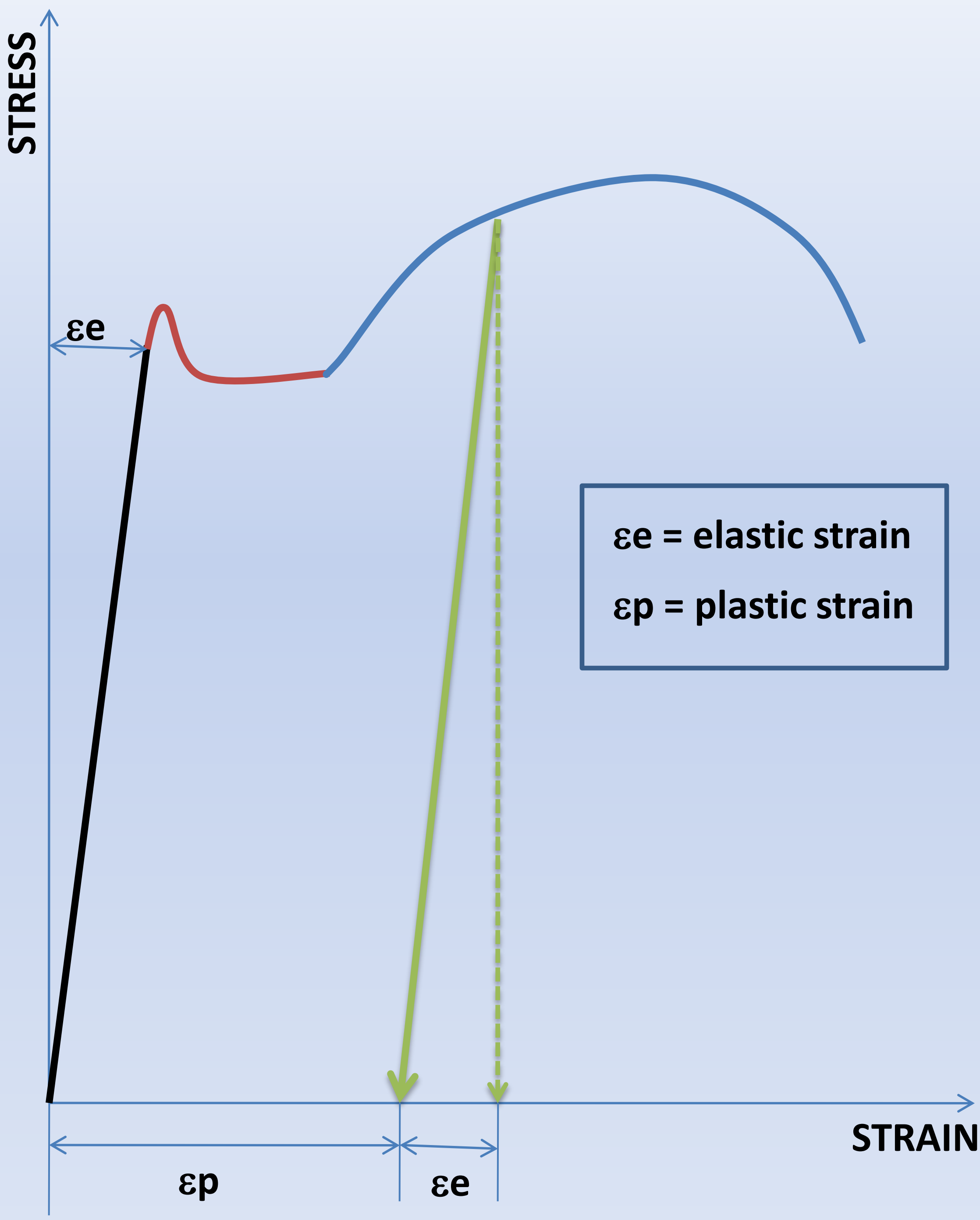
Ductile Material: **associated with large plastic deformation before failure**

11. Ductile and Brittle Material



Brittle Material: **does not show any large plastic deformation before failure**

Typical Stress-Strain Behaviour of Steel



Load-Strain Behaviour with Time (Elastic Material)

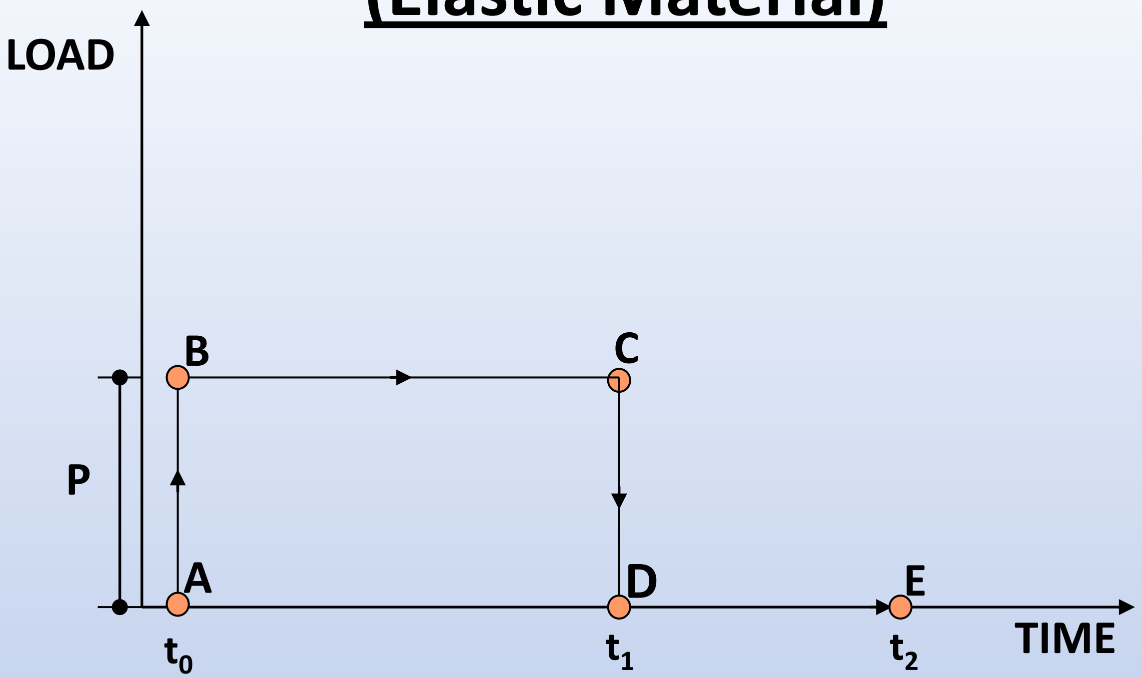


Figure . Loading-unloading sequence for identifying strain components

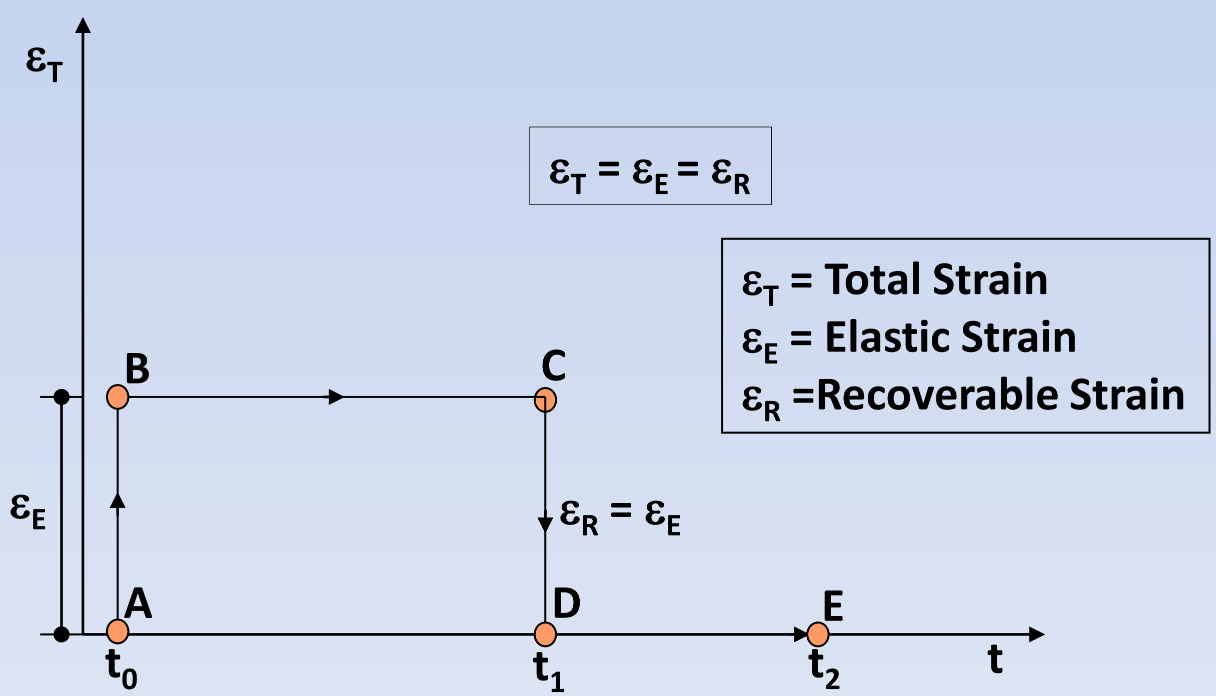


Figure . Strain components of an elastic material

Characteristics of Elastic Strain

. Elastic strain develops in a material immediately upon application of a load.

. The amount of elastic strain that develops upon application of a load can be completely recovered if the load is removed.

Load-Strain Behaviour with Time (Plastic Material)

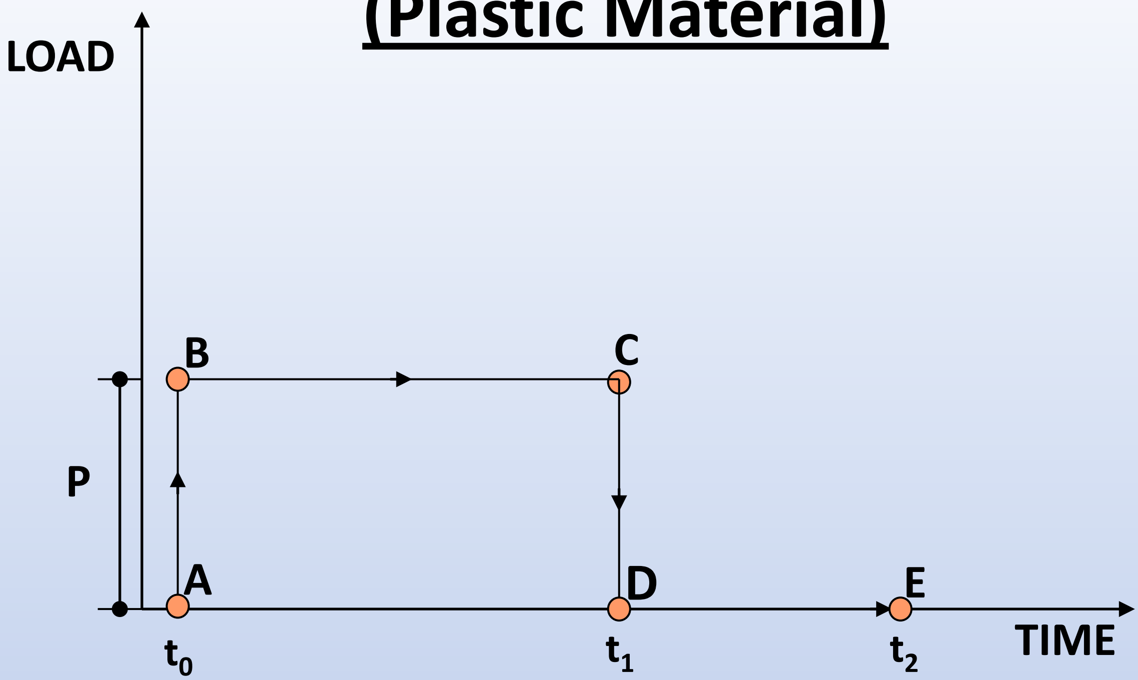


Figure . Loading-unloading sequence

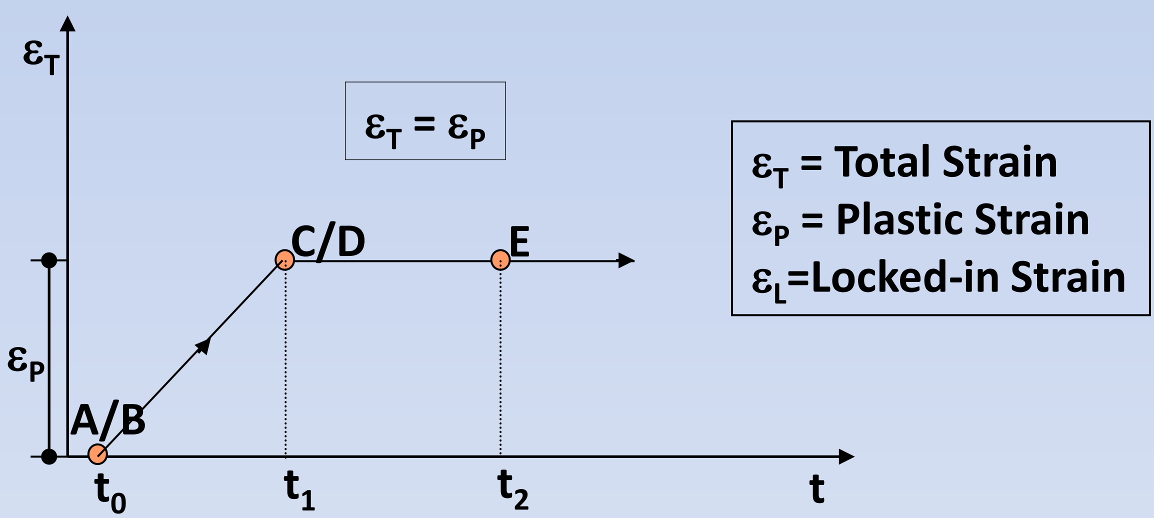


Figure. Strain components of plastic materials

Characteristics of Plastic Strain

. Plastic strain develops in a material with time under a sustained load.

. The amount of plastic strain that develops upon application of a sustained load can never be recovered even if the load is fully removed.

Load-Strain Behaviour with Time (Elasto-Plastic Material)

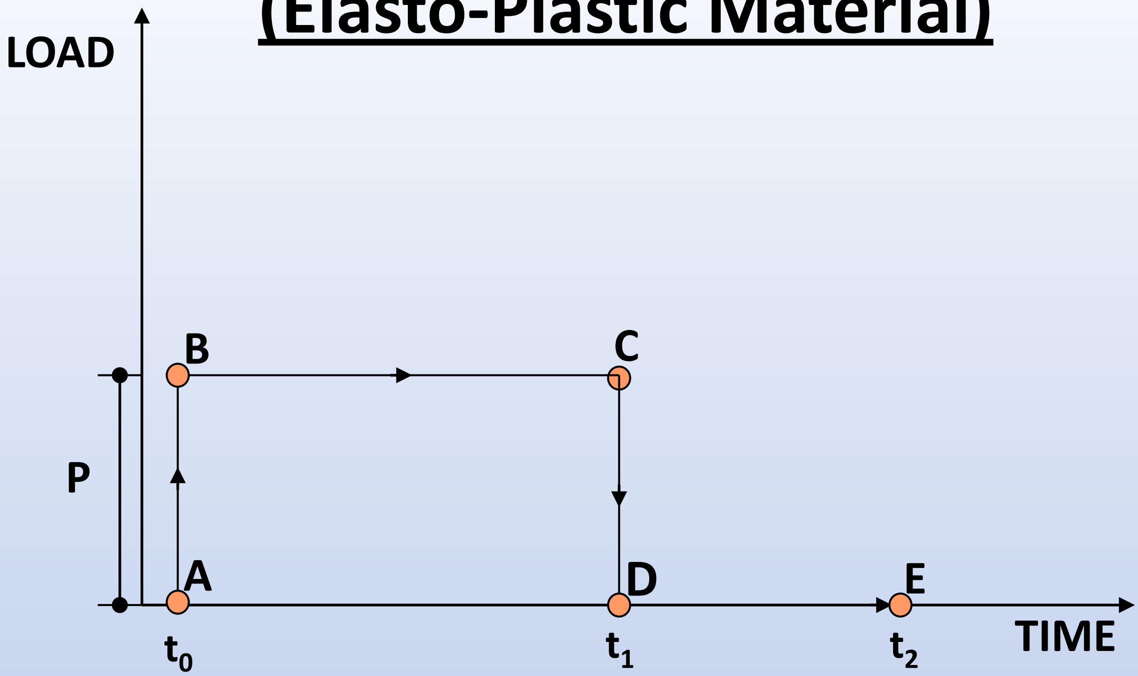


Figure . Loading-unloading sequence

$\varepsilon_T = \varepsilon_E + \varepsilon_P$
$= \varepsilon_R + \varepsilon_L$

ε_T = Total Strain
ε_E = Elastic Strain
ε_P = Plastic Strain
ε_R =Recoverable Strain
ε_L = Locked-in Strain

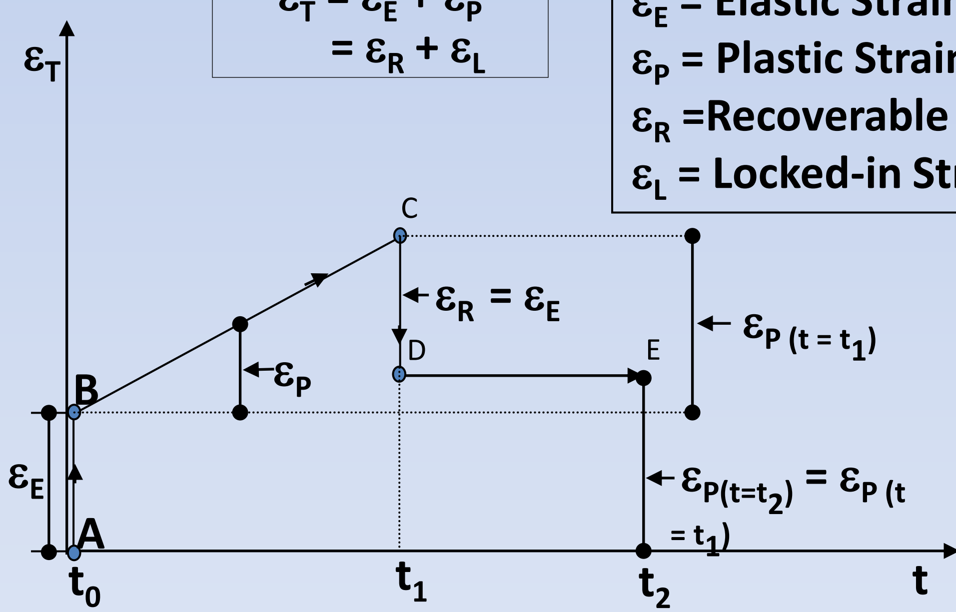


Figure . Strain components of elasto-plastic materials

Load-Strain Behaviour with Time (Elasto-Visco-Plastic Material)

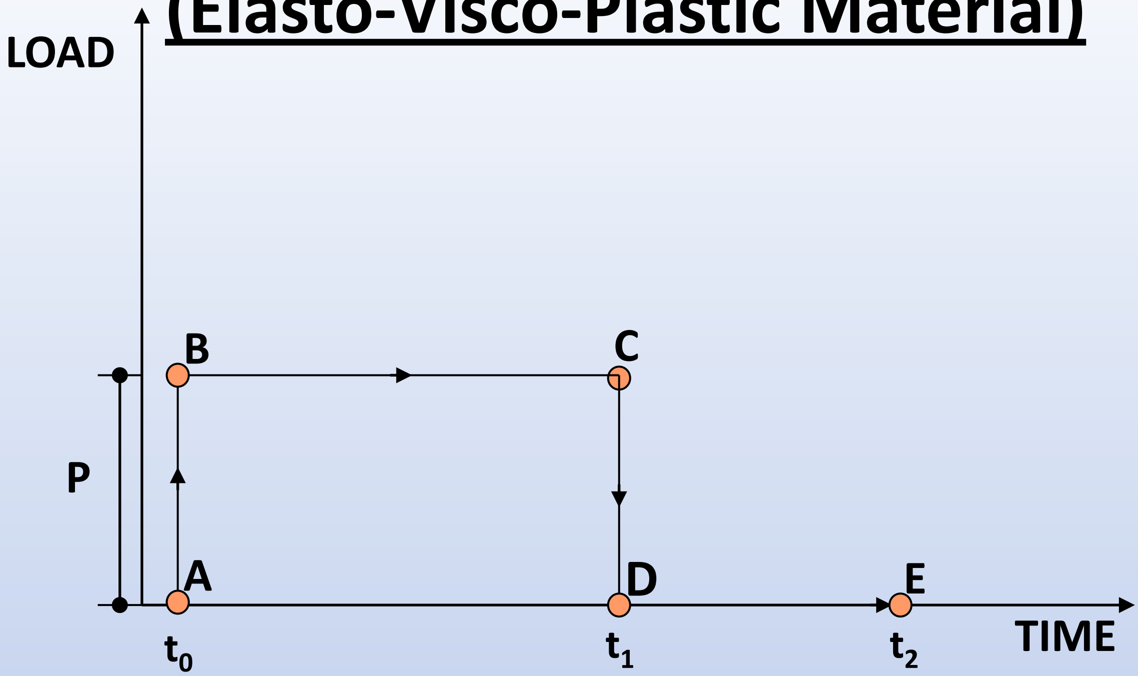


Figure . Loading-unloading sequence

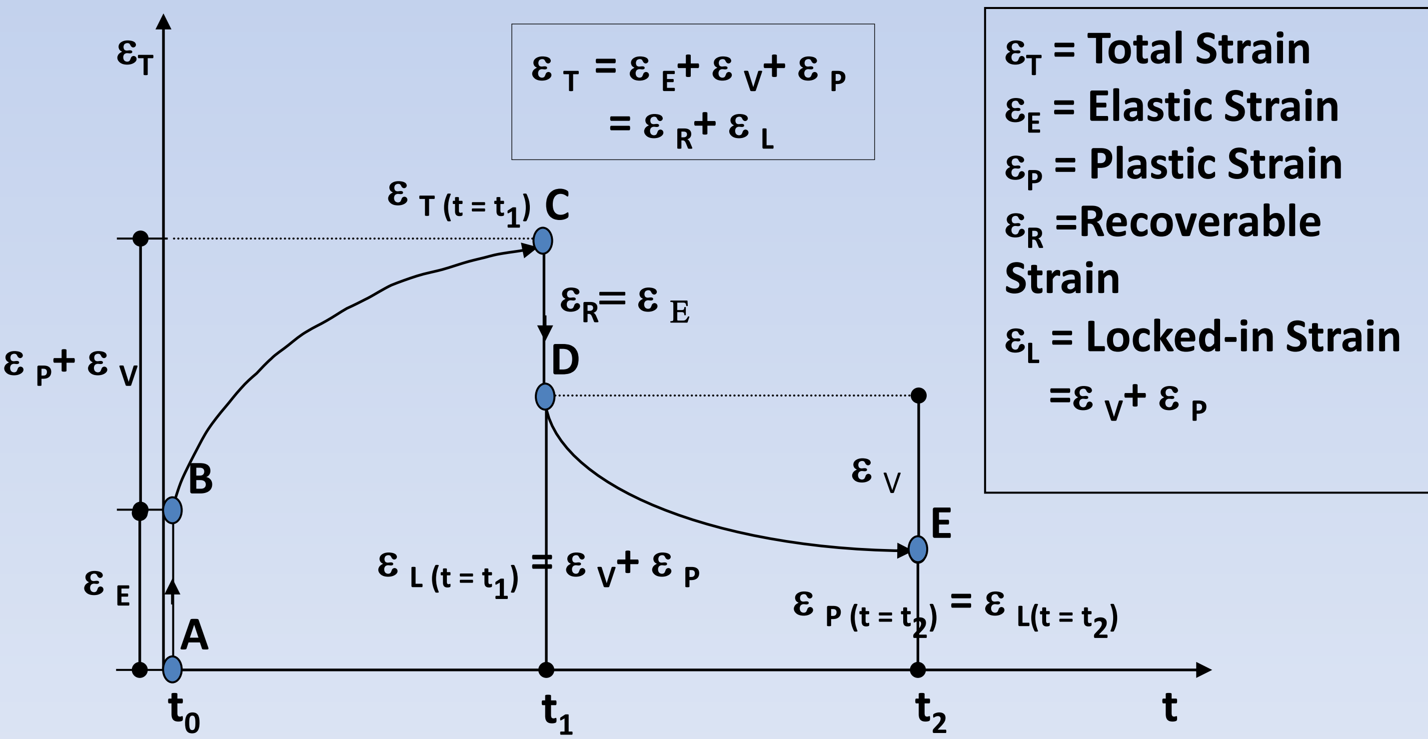


Figure. Strain components of elasto-visco-plastic materials

Characteristics of Viscous Strain

. Viscous strain develops in a material with time under a sustained load.

. The amount of viscous strain that develops upon application of a sustained load does not recover immediately upon removal of the load.

- This strain gets recovered at a certain period of time after removal of the load.

Response of Diff. Materials

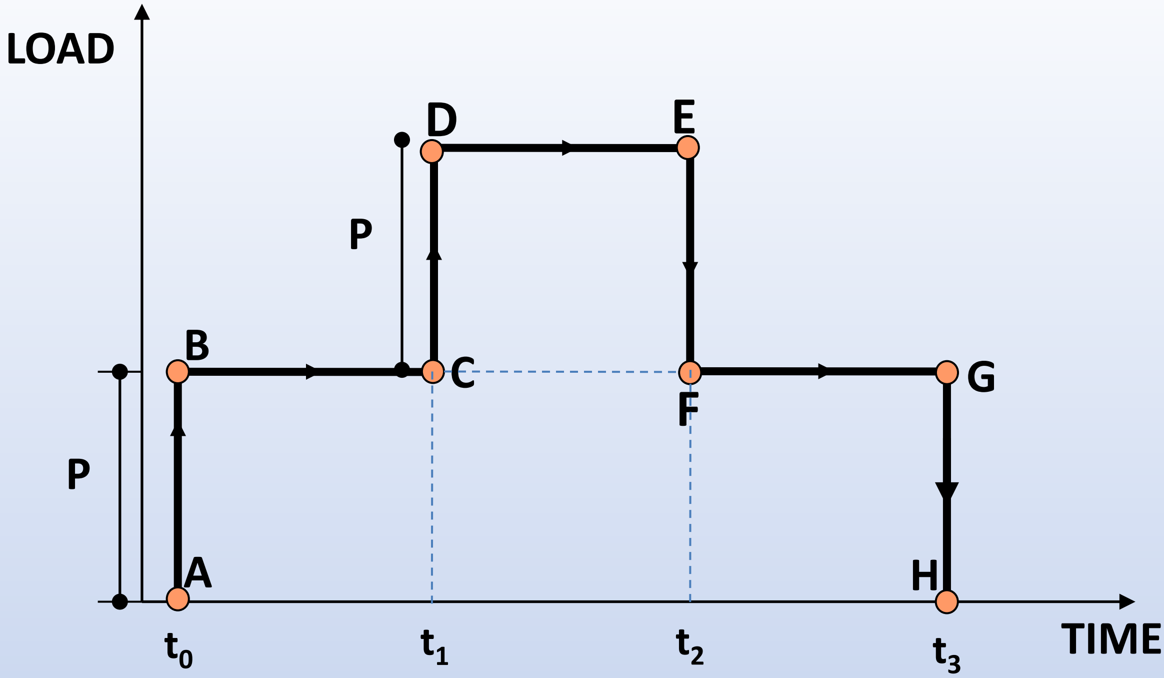


Figure . Loading-unloading sequence

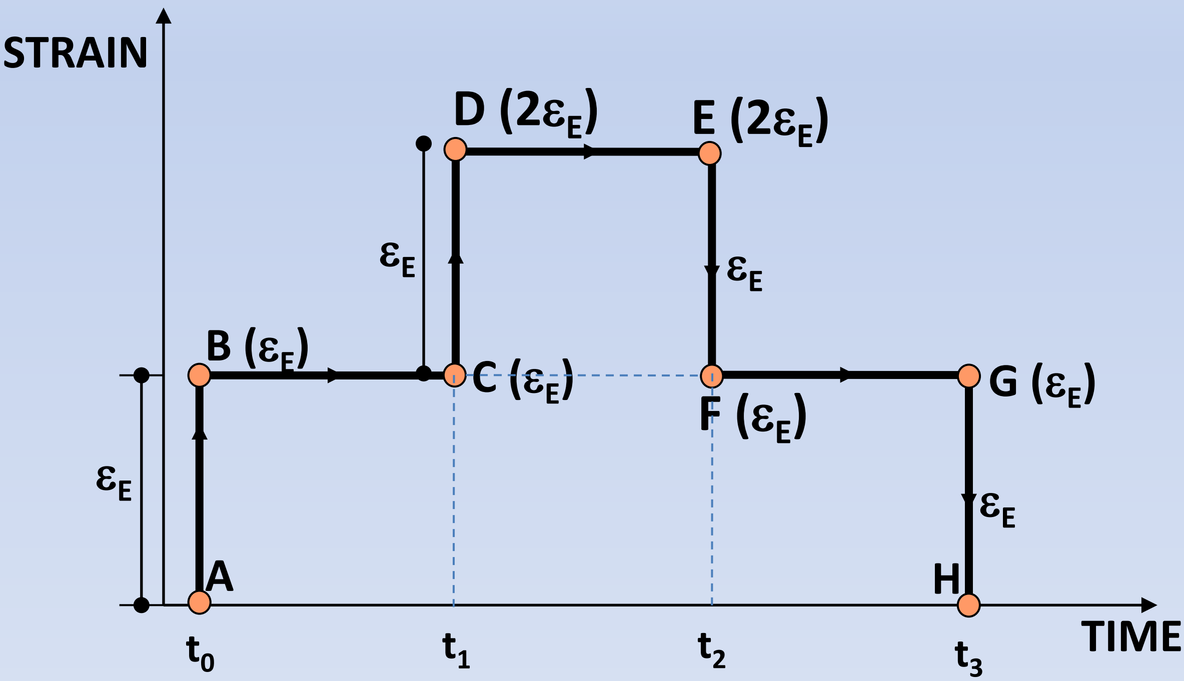


Figure . Strain response of an elastic material to the above loading regime

Response of Diff. Materials

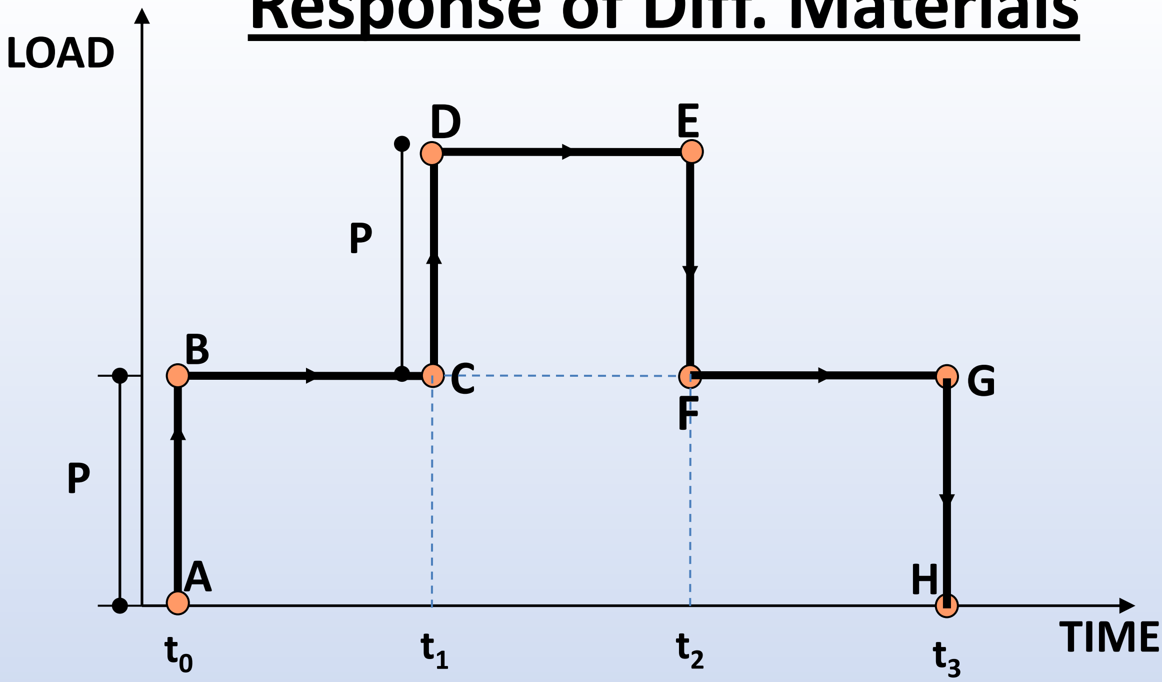


Figure . Loading-unloading sequence

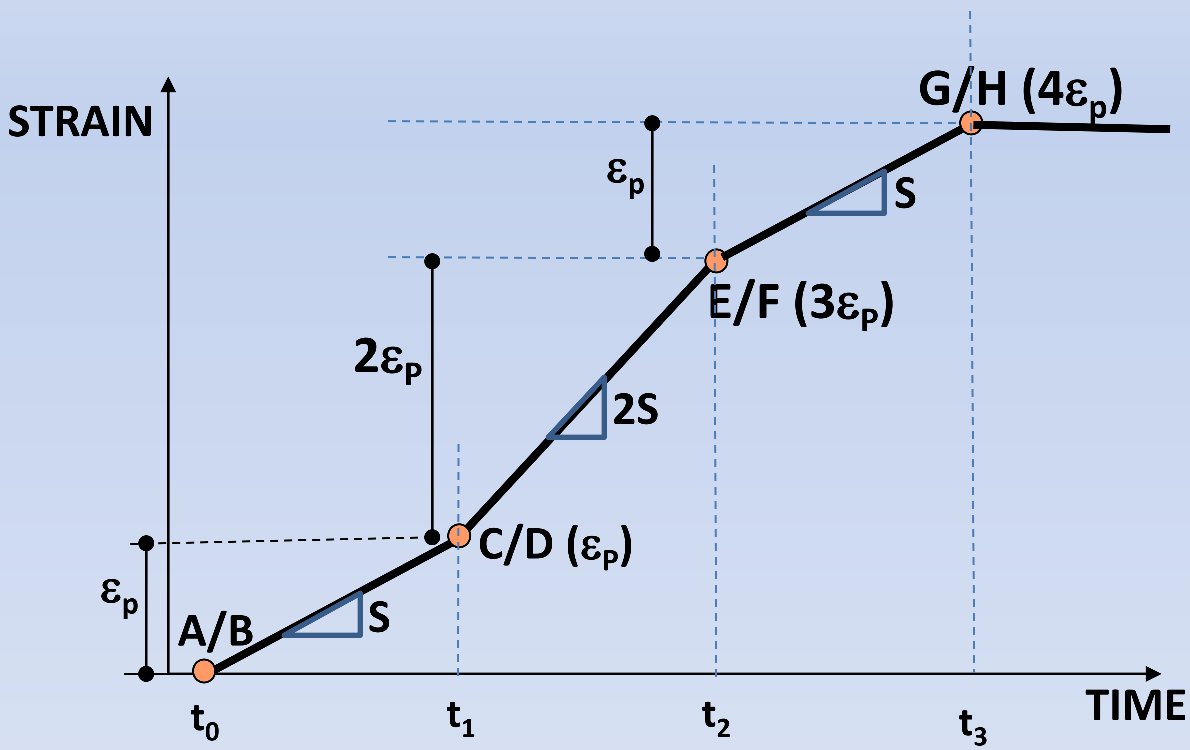


Figure . Strain response of a plastic material to the above loading regime

Response of Diff. Materials

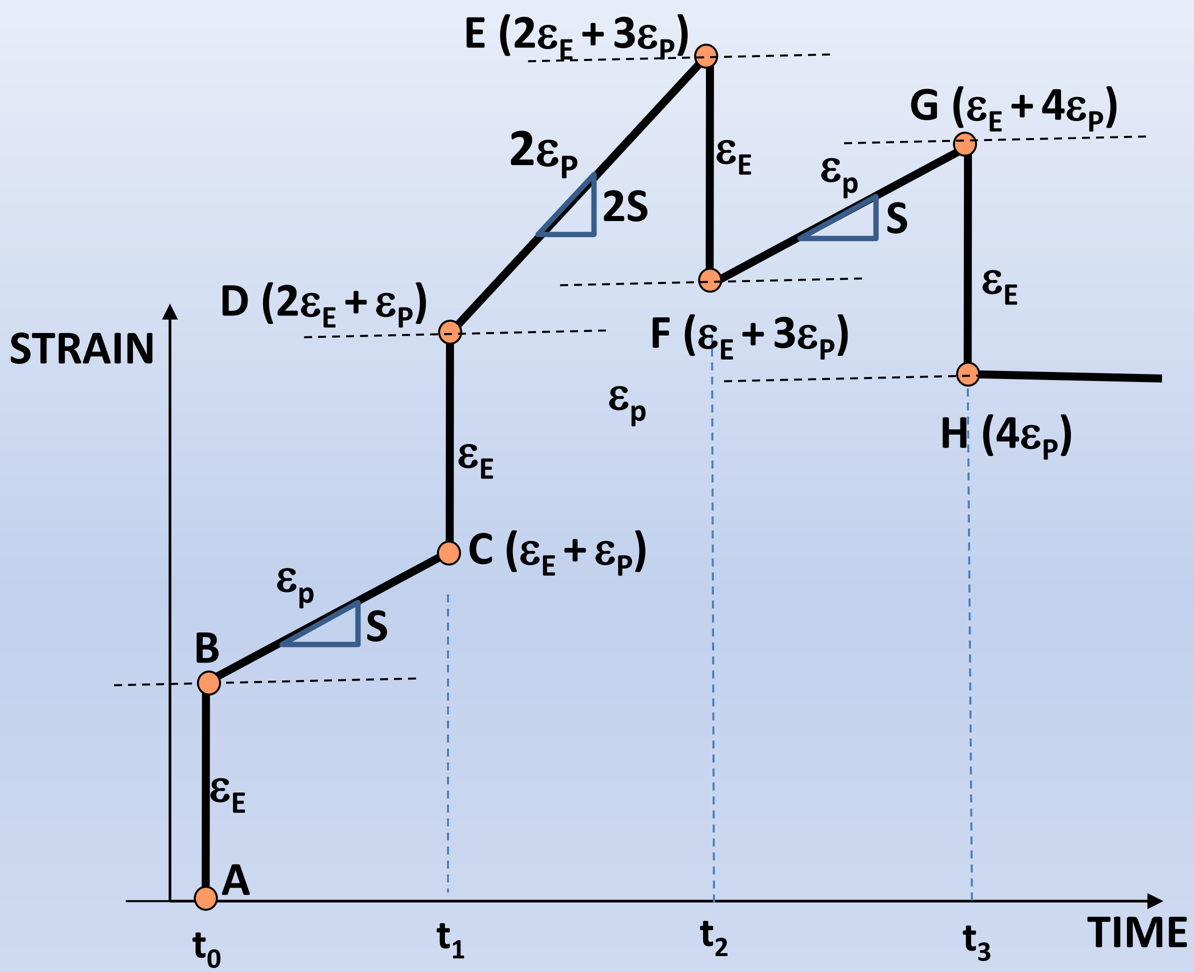
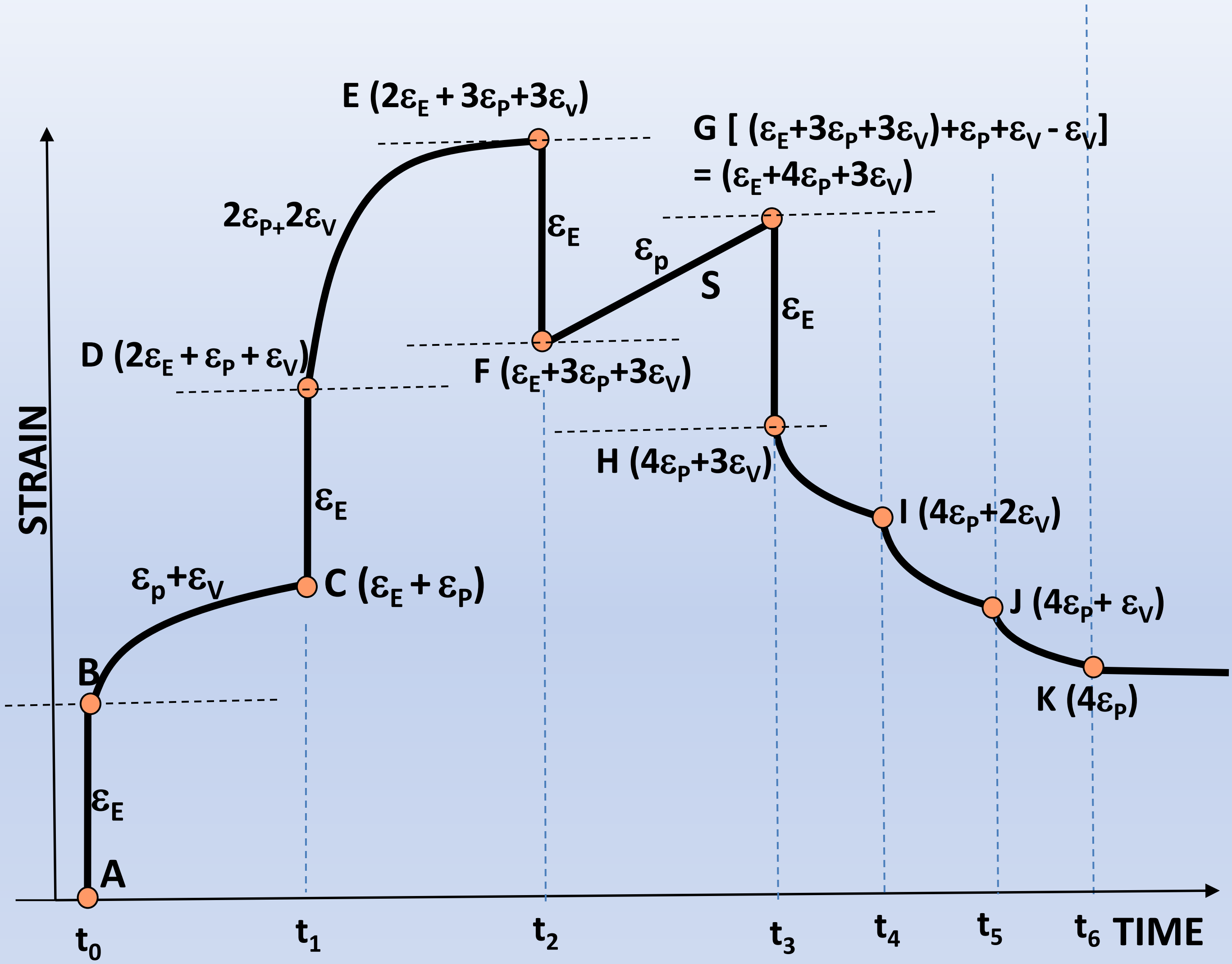


Figure . Strain response of an elasto-plastic material to the above loading regime

Response of Diff. Materials



NOTE: $\Delta t = t_1 - t_0 = t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = t_5 - t_4 = t_6 - t_5$

Figure . Strain response of an elasto-visco-plastic material to the above loading regime

Creep Behaviour of Materials

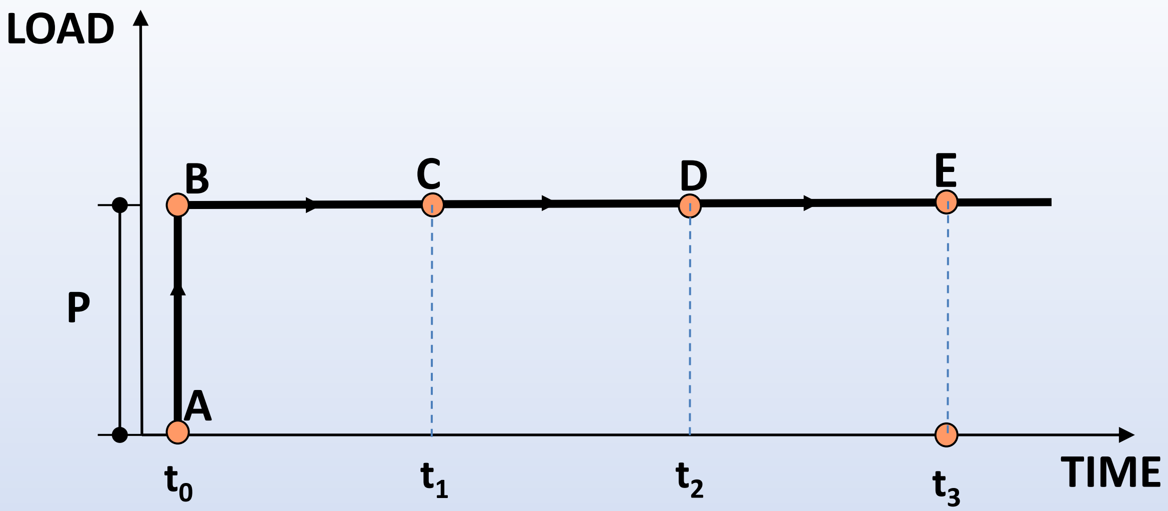


Figure . Loading sequence

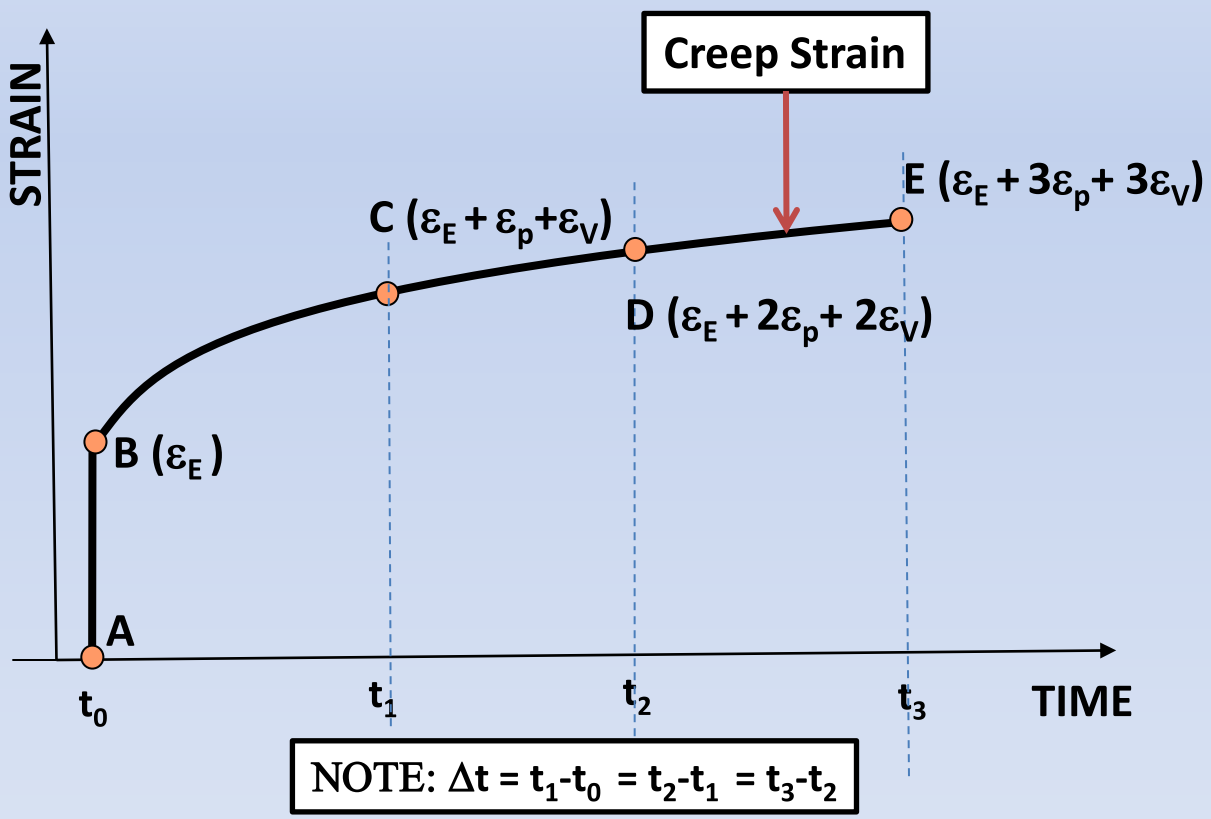


Figure . Creep behaviour of an elasto-visco-plastic material

ISOCHRONOUS LOAD-STRAIN CURVES

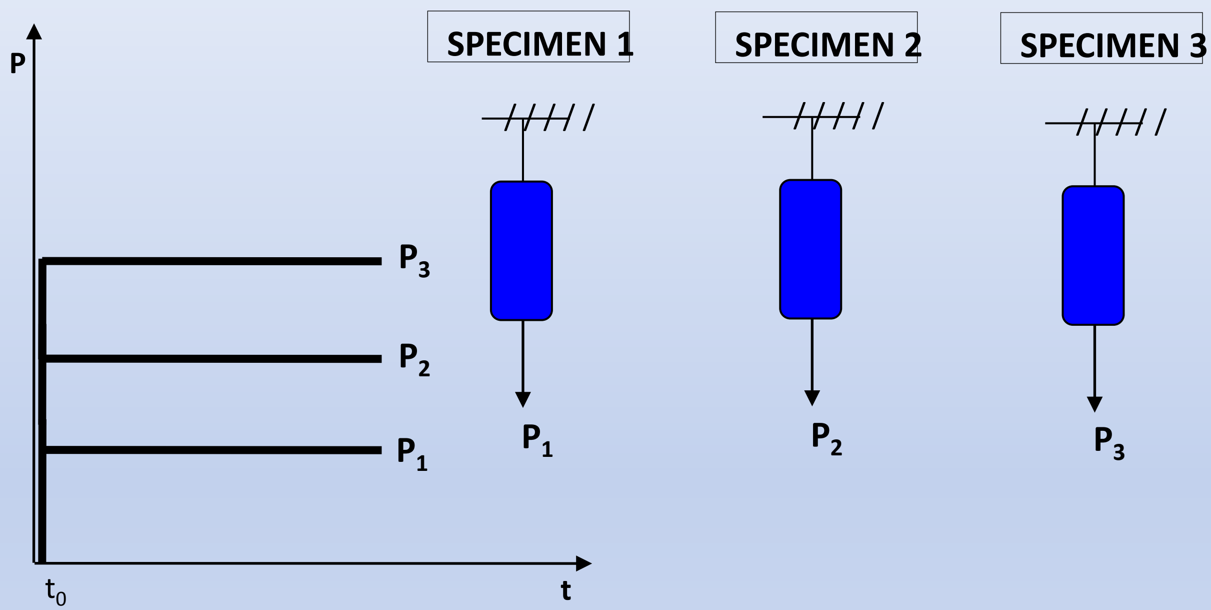


Figure . Series of loading verses time for deriving Isochronous curves

ISOCHRONOUS LOAD-STRAIN CURVES

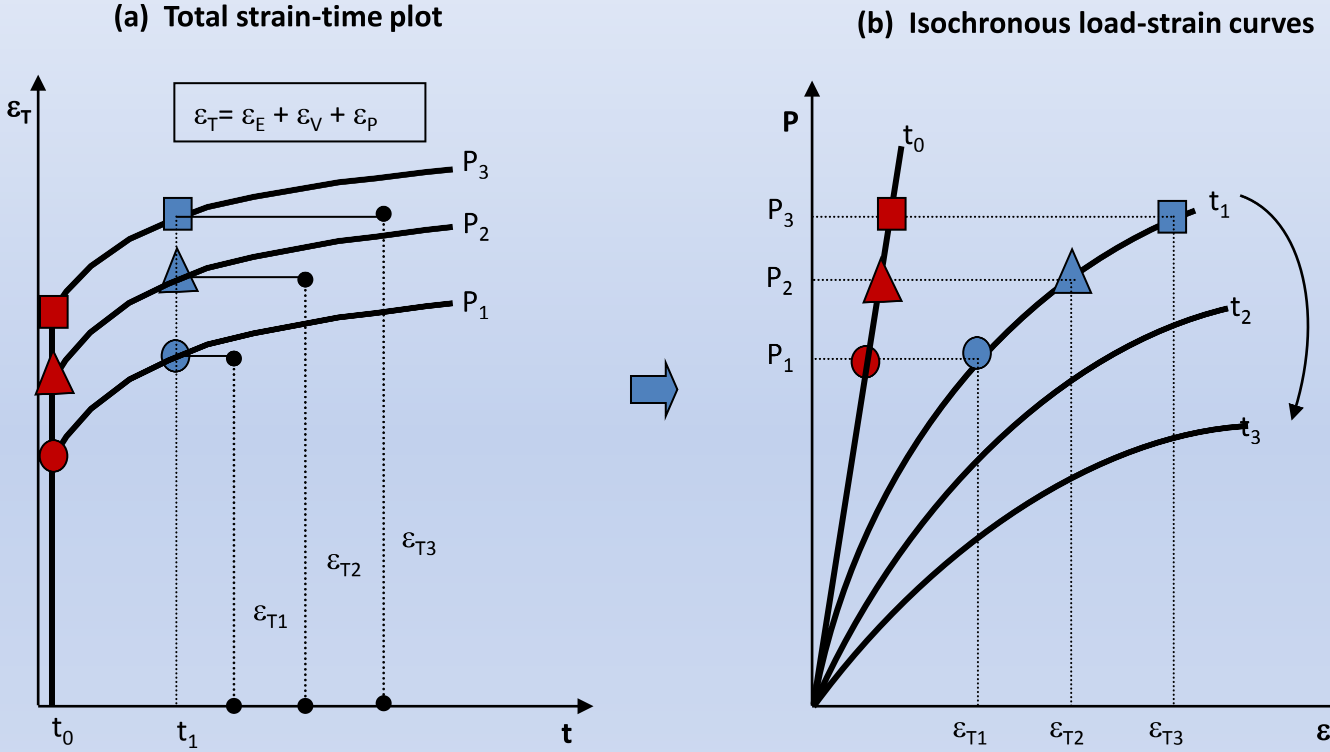
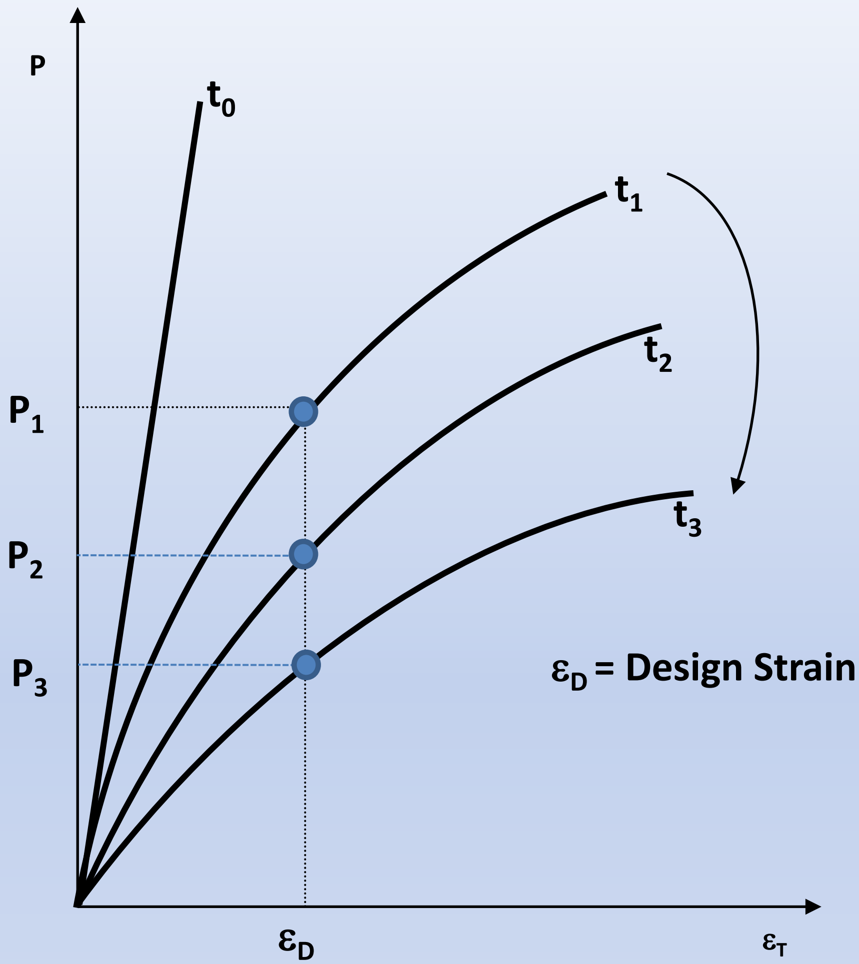


Figure . Deriving Isochronous load-strain curves for elasto-visco-plastic materials

SIGNIFICANCE OF ISOCHRONOUS LOAD-STRAIN CURVES



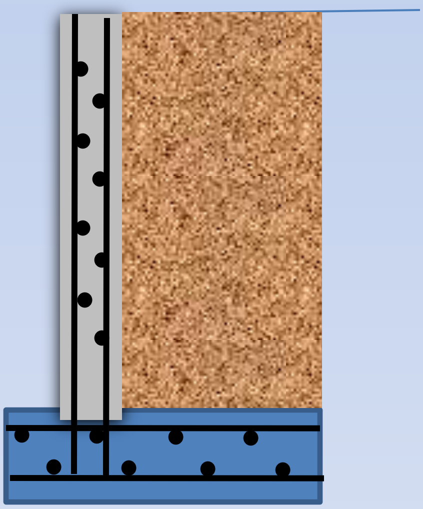
Load carrying capacity of a creeping material reduces with time for a particular design strain

EXAMPLES AND APPLICATIONS OF DIFFERENT MATERIALS

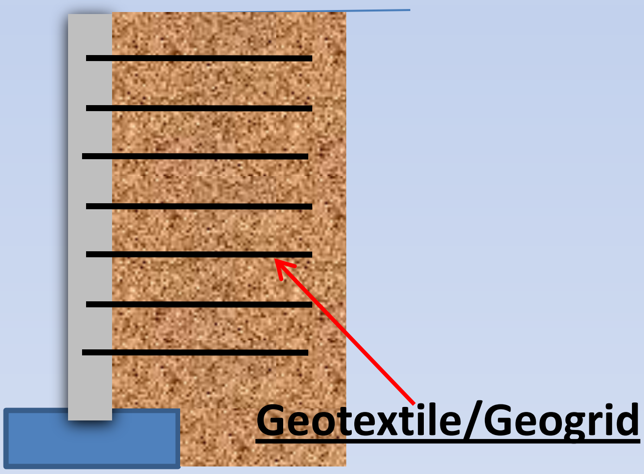
1) Elastic material: Steel up to proportional limit.

2) Elasto-plastic material: Jute geotextile, Bitumen, Steel up to ultimate stress, Sandy soil etc.

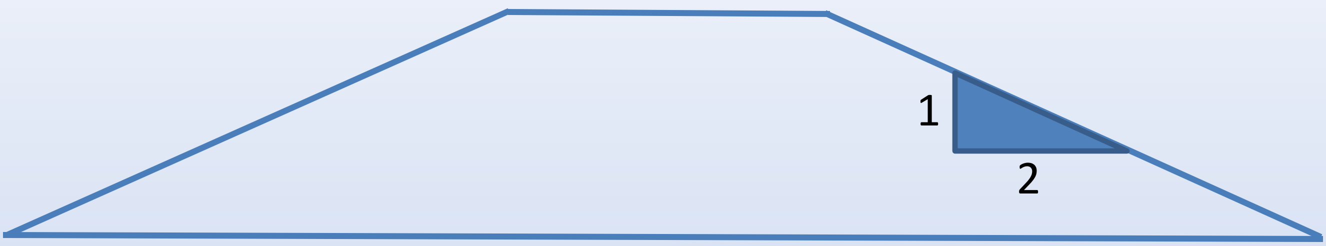
3) Elasto-visco-plastic material: Synthetic geotextile, Synthetic geogrid and saturated clay soil.



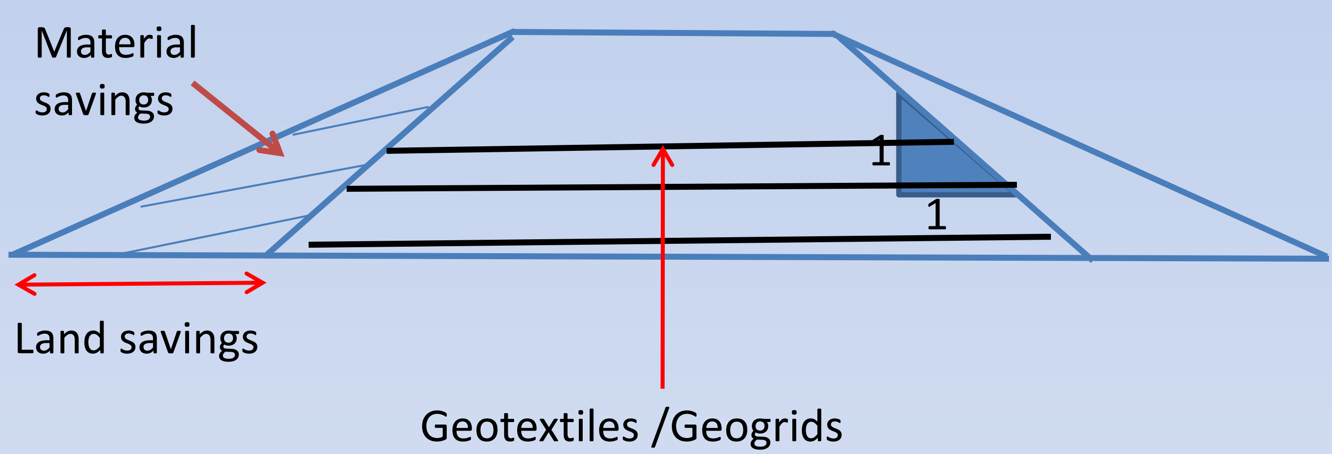
RC Cantilever Wall



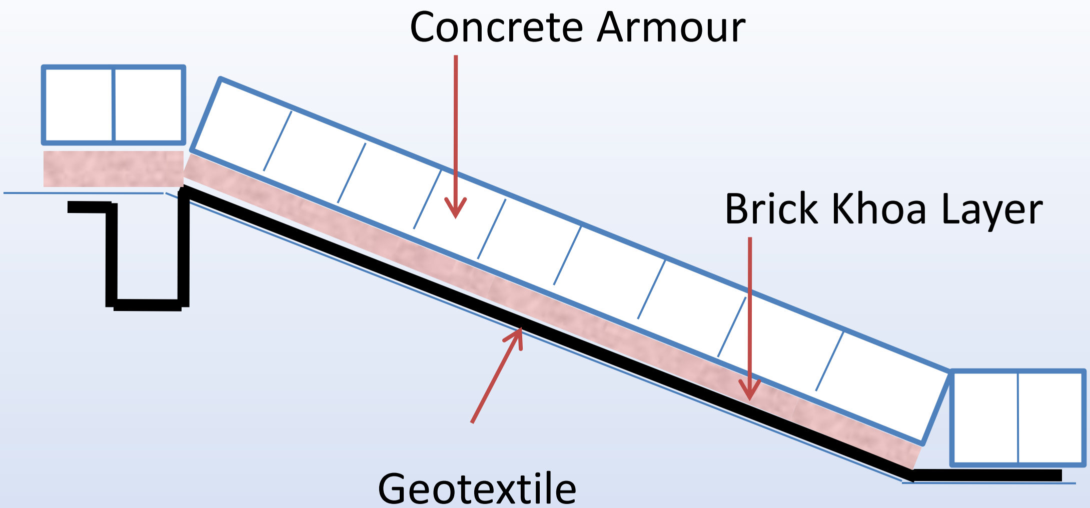
Reinforced Earth Wall



Conventional Earth Embankment



Reinforced Earth Embankment



River Bank Protection Scheme


শ্রমিকরা জুট জিওর উপরে ব্লক স্থাপন করিতেছে।


সমাপ্ত কৃত কাজ দেখা যাইতেছে।