Numerical Analysis :-

Numerical Analysis involves the development and evaluation of methods for computing numerical reasults from given numerical data.

Algorithm :- A algorithm for a particular problem is a step by step procedure that produces a solution in a finite number of steps.

Numerical Method :-

It is an algorithm for solving a problem whose solution consists of one or more numerical values. Values obtained this way are called numerical solution.

Since numerical methods are needed if the exact answer is known, as before using a numerical method one must

i) Examine the problem

ii) Determine how close to the exact value numerical solution is required.

$$a_{11}x_1 + a_{12}x_2 + \cdots \cdots \cdots a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots \cdots \cdots a_{2n}x_n = b_2$$

$$\vdots \qquad \qquad \vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots \cdots + a_{mn}x_n = b_n$$

Methods :-

(I) **Direct Method :-** yields the solution after an amount of computation that is known in advance.

(II) **Indirect Method / Iterative Method :-**

which starts from an approximation to the exact solution and the amount of computation depends on the accuracy needed.

Direct Methods :-

(A) Methods of Matrices and Determinants

(B) Methods of successive elimination

i) Gauss - Elimination

ii) Modified Gauss or Gauss-Jordan Method

iii) Crout's Method

iv) Triangularization or Factorization

Indirect Method :-

1) Jacobi Method
2) Gauss Seidel
3) Relaxation Method

Cramer's Rule

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$\boxed{D \neq 0}$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$x_j = \frac{B_j}{D};$$

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$x_1 = \frac{B_1}{D}$$

$$x_2 = \frac{B_2}{D}$$

$$x_3 = \frac{B_3}{D}$$

$$B_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}$$

$$B_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}$$

$$B_3 = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{23} & b_3 \end{vmatrix}$$

Methods of Chio for calculating Determinant :-

Let us consider a determinant of nth order

$$D = \begin{vmatrix} a_1 & a_2 & a_3 \cdots & \cdots & a_m \\ b_1 & b_2 & b_3 \cdots & \cdots & b_m \\ c_1 & c_2 & c_3 \cdots & \cdots & c_m \\ \cdots & \cdots & \cdots & & \cdots \end{vmatrix}$$

steps:-

~~Conditions~~ :- (1) $a_1 \neq 0$ if $a_1 = 0$ then row should be exchanged

(1) The elements of 2nd, 3rd --- mth columns are multiplied by a_1 and hence

$$D = \frac{1}{a_1^{m-1}} \begin{vmatrix} a_1 & a_2 a_1 & a_3 a_1 \cdots \\ b_1 & b_2 a_1 & b_3 a_1 \cdots \\ c_1 & c_1 a_1 & c_3 a_1 \cdots \\ \vdots & & \end{vmatrix}$$

(iii) Now substracting $a_2, a_3 \cdots a_m$ times the first column from 2nd, 3rd --- mth column

$$D = \frac{1}{a_1^{m-1}} \begin{vmatrix} a_1 & 0 & 0 & 0 & 0 \\ b_1 & a_1 b_2 - a_2 b_1 & \cdots & & \cdots \\ c_1 & a_1 c_2 - a_2 c_1 & \cdots & & \cdots \\ & \vdots & & & \end{vmatrix}$$

$= 1$

$(m-1)\times(m-1)$

$$= \frac{1}{a_1^{m-2}} \begin{vmatrix} a_1 b_1 - a_2 b_2 & - - - \\ a_1 c_2 - a_2 c_1 & - - - \\ - - - & - \\ - - - & \end{vmatrix}$$

$(m-1)(m-1)$

Gauss - Elimination Method

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

Diagonal element

upper triangular matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

0

$$\boxed{x_3 = \frac{b_3^{2}}{a_{33}^{2}}} \rightarrow x_2 = \cdots \qquad x_3 = \cdots$$

Back substitution

(যেহেতু x_3 সবার আগে বের হবে,

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23} \\ 0 & 0 & a_{33}' \end{bmatrix}$$

$$a_{21}' = a_{21} - \frac{a_{21}}{a_{11}} \times a_{11} = 0$$

$$a_{22}' = a_{22} - \frac{a_{21}}{a_{11}} \times a_{12}$$

$$a_{23}' = a_{23} - \frac{a_{21}}{a_{11}} \times a_{13}$$

if $a_{11} = 0$ $\quad \frac{a_{21}}{a_{11}} = \infty$

eg $\begin{bmatrix} 0 & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

pivotal condensation

সবচেয়ে Max value কে a_{11} এ বসাতে হবে

Gauss Elimination Method

Gauss elimination is the most common method for solving simultaneous linear equation. In this method, the square co-efficient matrix is first reduced to an upper triangular matrix and then the unknown are evaluated by back substitution starting from last reduced equation.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots \quad \cdots \quad \cdots + a_{1n}x_n = b_1$$

$$a_{12}x_2 + a_{22}x_2 + a_{23}x_3 + \cdots \quad \cdots \quad \cdots + a_{2n}x_n = b_2$$

$$\vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots \quad \cdots \quad + a_{nn}x_n = b_n$$

Steps:- (1) Multiplying eqⁿ (1) a_{i1}/a_{11} and substracting it from the ith equation $(i = 2, 3, \cdots n)$

$$a_{11}x_1 + a_{21}x_2 + \cdots \quad \cdots \quad \cdots + a_{1n}x_n = b_1$$

$$a_{22}'x_2 + \cdots \quad \cdots + a^2_{2n}x_n = b_2'$$

$$a_{32}'x_3 + \cdots \quad \cdots + a^2_{3n}x_n = b_3'$$

$$a_{42}'x_2 + \cdots \quad \cdots$$

$$=$$

where $a_{ij}' = a_{ij} - \frac{a_{i1}}{a_{11}} a_{ij}'$

$$b_i' = b_i - \frac{a_{i1}}{a_{11}} b_1$$

$$i, j = 2, 3, \cdots n$$

2. Now multiplying 2nd equation by $\frac{a_{i2}}{a_{22}'}$ and substracting it from ith equation ($i = 3, 4, \cdots n$)

$$a_{11}x_1 + a_{12}x_2 + \cdots \cdots + a_{1n}x_n = b_1$$

$$a_{22}'x_2 + \cdots \cdots + a_{2n}'x_n = b_2'$$

$$+ a_{33}^2 x_3 + \cdots + a_{3n}^2 x_n = b_3^2$$

$$\cancel{a_{nn}^{n-1} x_n = b_n^{n-1}}$$

$$a_{43}^2 x_3 + \cdots \cdots = b_4^2$$

where $a_{ij}^2 = a_{ij}' - \frac{a_{i2}'}{a_{22}'} a_{2j}'$

$b_i = b_i - \frac{a_{i2}'}{a_{22}'} b_2'$

$[i, j = 3, 4, \cdots n]$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1n} \\ 0 & a_{22}' & a_{23}' & & \cdots & a_{2n} \\ 0 & 0 & a_{33}^2 & & & a_{3n}^2 \\ 0 & 0 & 0 & & & a_{nn}^{n-1} \end{bmatrix}$$

After (n-1) steps,

$$a_{ij}^{K} = a_{ij}^{K-1} - \frac{a_{iK}^{K-1}}{a_{KK}^{K-1}} a_{Kj}^{K-1}$$

$$b_i^{K} = b_i^{K-1} - \frac{a_{iK}^{K-1}}{a_{KK}^{K-1}} b_K^{(K-1)}$$

$K = 1, 2, 3 \cdots (n-1)$

$i, j = (K+1), (K+2) \cdots n$

$$\Rightarrow a_{nn}^{n-1} x_n = b_n^{n-1}$$

$$x_n = b_n^{n-1} / a_{nn}^{n-1}$$

$x_{n-1} =$

$x_{n-2} =$

$\vdots$

$x_1 =$

$$x_i = \frac{1}{a_{ii}^{(i-1)}} \left[b_i^{(i-1)} - \sum_{j=i+1}^{n} a_{ij}^{(i-1)} x_j \right]$$

$i = n, n-1, n-2, \ldots$

back substitution

Cramer's Rule.

Class Work :-

$$x_1 + 2x_2 - x_3 = 2$$

$$3x_1 + 6x_2 + x_3 = 1$$

$$3x_1 + 3x_2 + 2x_3 = 3$$

Solve this using Cramer's Rule

$$D = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 6 & 1 \\ 3 & 3 & 2 \end{vmatrix}$$

$$= 1\overset{(12-3)}{\underset{9}{\cancel{(18-3)}}} + 2(3-6) - 1(9-18)$$

$$= \cancel{18} - 6 + 9$$

$$= 12$$

$$D_1 = \begin{vmatrix} 2 & 2 & -1 \\ 1 & 6 & 1 \\ 3 & 3 & 2 \end{vmatrix}$$

$$= 2(12-3) - 2(2-3) - 1(3-18)$$

$$= (2\times9) - 2\times(-1) - 1(-15)$$

$$= 18 + 2 + 15$$

$$= 35$$

$x_1 = \frac{B_1}{D} = \frac{35}{12} = \boxed{2.9 = x_1}$

$$B_2 = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 1 \\ 3 & 3 & 2 \end{vmatrix}$$

$= 1(2-3) - 2(6-3) - 1(9-3)$

$= 1(-1) - 2(3) - 1(6)$

$= -1 - 6 - 6$

$= -13$

$x_2 = \frac{B_2}{D} = \frac{-13}{12} = \boxed{1.083 = x_2}$

$$B_3 = \begin{vmatrix} 1 & 2 & 2 \\ 3 & 6 & 1 \\ 3 & 3 & 3 \end{vmatrix}$$

$= 1(18-3) - 2(9-3) + 2(9-18)$

$= 15 - (2 \times 6) + 2(-9)$

$= 15 - 12 - 18$

$= 15$

$x_3 = \frac{B_3}{D} = \frac{15}{12} = \boxed{1.25 = x_3}$

class work-2

Solve the following problem using Gauss-Elimination Technique

$2x_1 + 3x_2 - x_3 = 5$

$4x_1 + 4x_2 - 3x_2 = 3$

$3x_1 - 4.5x_2 + 1.5x_3 = -1.5$

~~Augm~~

$$\begin{pmatrix} 2 & 3 & -1 \\ 4 & 4 & -3 \\ 3 & -4.5 & 1.5 \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 5 \\ 3 \\ -1.5 \end{bmatrix}$$

pivotal condensation

$$\begin{pmatrix} 4 & 4 & -3 \\ 2 & 3 & -1 \\ 3 & -4.5 & 1.5 \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 3 \\ 5 \\ -1.5 \end{bmatrix}$$

$$\begin{pmatrix} 4 & 4 & -3 \\ 0 & 1 & .5 \\ 0 & -7.5 & 3.75 \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 3 \\ 3.5 \\ -3.75 \end{bmatrix} \quad \begin{aligned} R_2' &= R_2 - \frac{R_1}{2} \\ R_3' &= R_3 - \frac{R_1}{4} 3 \end{aligned}$$

Pivotal condensation

$$\begin{pmatrix} 4 & 4 & -3 & 3 \\ 0 & -7.5 & 3.75 & -3.75 \\ 0 & 1 & .5 & 3.5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 2.33 & .33 & 0 & 0 \\ 0 & 1 & -1.44 & -.17 & .06 & 0 \\ 0 & 2 & -5.65 & -1.65 & 0 & 1 \end{pmatrix} \quad R_2' =$$

$$\begin{pmatrix} 1 & 0 & -.55 & -0.01 & .12 & 0 \\ 0 & 1 & -1.44 & -.17 & .06 & 0 \\ 0 & 0 & -2.77 & -1.31 & -.12 & 1 \end{pmatrix} \quad \begin{matrix} R_1' = R_2 \\ R_3 = R \end{matrix}$$

$$= \begin{pmatrix} 1 & 0 & -.55 & -.01 & .12 & 0 \\ 0 & 1 & -1.44 & -.17 & .06 & 0 \\ 0 & 0 & 1 & .47 & .04 & -.36 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & .24 & .14 & -.02 \\ 0 & 1 & 0 & .51 & .12 & -.52 \\ 0 & 0 & 1 & .47 & .04 & -.36 \end{pmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} .24 & .14 & -.02 \\ .51 & .12 & -.52 \\ .47 & .04 & -.36 \end{bmatrix}$$

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$$\begin{pmatrix} 4 & 4 & -3 & 3 \\ 0 & -7.5 & 3.75 & -3.75 \\ 0 & 1 & .5 & 3.5 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 4 & -3 & 3 \\ 0 & -7.5 & 3.75 & -3.75 \\ 0 & 0 & 1 & \cancel{4}\,3 \end{pmatrix} \qquad R_3' = R_3 + \frac{R_2}{7.5}$$

$$= \begin{pmatrix} 4 & 4 & -3 & 3 \\ 0 & 1 & -.5 & .5 \\ 0 & 0 & 1 & 3 \end{pmatrix} \qquad R_2' = \left(R/7.5\right)$$

$$4x_1 + 4x_2 + (-3x_3) = 3$$

$$x_2 - .5x_3 = .5$$

$$\boxed{x_3 = \cancel{4}\,3}$$

$$x_2 = .5 + (.5 \times 3) = \cancel{2.5} \qquad \boxed{2 = x_2}$$

$$4x_1 = 3 + 3(4) - 4(2.5)$$

$$=$$

$$x_1 = 5/4$$

$$4x_1 = 3 + (3 \times 3) - 4(2)$$

$$\boxed{x_1 = 1}$$

Classwork - 3 Find the inverse of a Matrix using Gauss Jordan Method

$3x_1 - 6x_2 + 7x_3 = 3$

$9x_1 \quad - 5x_3 = 3$

$5x_1 - 8x_2 + 6x_3 = -4$

$$\begin{pmatrix} 3 & -6 & 7 & 1 & 0 & 0 \\ 9 & 0 & -5 & 0 & 1 & 0 \\ 5 & -8 & 6 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -2 & 2.33 & .33 & 0 & 0 \\ 9 & 0 & -5 & 0 & 1 & 0 \\ 5 & -8 & 6 & 0 & 0 & 1 \end{pmatrix} \quad R_1' = R_1/3$$

$$= \begin{pmatrix} 1 & -2 & 2.33 & .33 & 0 & 0 \\ 0 & 18 & 25.95 & -2.97 & 1 & 0 \\ 0 & 2 & -5.65 & -1.65 & 0 & 1 \end{pmatrix} \quad \begin{matrix} R_2' = R_2 - 9R \\ R_3' = R_3 - 5R \end{matrix}$$

=

$$\left(\begin{array}{cccccc} 1 & -2 & 2.33 & .33 & 0 & 0 \\ 0 & 1 & -1.44 & -.17 & .06 & 0 \\ 0 & 2 & -5.65 & -1.65 & 0 & 1 \end{array}\right) \quad R_2' = R_2/18$$

$$\left(\begin{array}{cccccc} 1 & 0 & -.55 & -0.01 & .12 & 0 \\ 0 & 1 & -1.44 & -.17 & .06 & 0 \\ 0 & 0 & -2.77 & -1.31 & -.12 & 1 \end{array}\right) \quad \begin{array}{l} R_1' = R_1 + 2R_2 \\ R_3 = R_3 - 2R_2 \end{array}$$

$$= \left(\begin{array}{cccccc} 1 & 0 & -.55 & -.01 & .12 & 0 \\ 0 & 1 & -1.44 & -.17 & .06 & 0 \\ 0 & 0 & .1 & .47 & .04 & -.36 \end{array}\right)$$

$$\left(\begin{array}{cccccc} 1 & 0 & 0 & .24 & .14 & -.02 \\ 0 & 1 & 0 & .51 & .12 & -.52 \\ 0 & 0 & 1 & .47 & .04 & -.36 \end{array}\right) \quad \begin{array}{l} R_2' \\ = R_3 \times 1.44 \\ + R_2 \end{array}$$

$$\therefore A^{-1} = \begin{bmatrix} .24 & .14 & -.02 \\ .51 & .12 & -.52 \\ .47 & .04 & -.36 \end{bmatrix}$$

Crouts Method

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{pmatrix}$$

Step 1

$a_{11}' = a_{11}$

$a_{21}' = a_{21}$

$a_{31}' = a_{31}$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{pmatrix}$$

$\Downarrow$

$$\begin{pmatrix} a_{11}' & a_{12}' & a_{13}' & b_1' \\ a_{21}' & a_{22}' & a_{23}' & b_2' \\ a_{31}' & a_{32}' & a_{33}' & b_3' \end{pmatrix}$$

Step 2

$a_{12}' = \dfrac{a_{12}}{a_{11}'}$

$a_{13}' = \dfrac{a_{13}}{a_{11}'}$

$b_1' = \dfrac{b_1}{a_{11}'}$

Step 3

$a_{22}' = a_{22} - a_{12}' a_{21}'$

$a_{32}' = a_{32} - a_{12}' a_{31}'$

Step 4 :-

$a_{23}' = \dfrac{a_{23} - a_{13}' a_{21}'}{a_{22}}$

$b_2 = \dfrac{b_2 - b_1' a_{21}'}{a_{22}'}$

Step 5

$a_{33}' = a_{33} - a_{13}' a_{31}' - a_{23}' a_{32}'$

Step 6

$b_3' = \dfrac{b_3 - b_1' a_{31}' - b_2' a_{32}'}{a_{33}'}$

\# $x_1 + 2x_2 + 3x_3 = 1$

$3x_1 + x_2 + x_3 = 0$

$2x_1 + x_2 + x_3 = 0$

$$= \begin{vmatrix} 1 & 2 & 3 & 1 \\ 3 & 1 & 1 & 0 \\ 2 & 1 & 1 & 0 \end{vmatrix}$$

① $a_{11}' = 1$

$a_{21}' = 3$

$a_{31}' = 2$

② $a_{12}' = 2/1 = 2$

$a_{13}' = 3/1 = 3$

$b_1' = \frac{1}{1} = 1$

③ $a_{22}' = 1 - (2\times3) = -5$

$a_{32}' = 1 - (2\times2) = -3$

④ $a_{23}' = \frac{1-(3\times3)}{-5} = 8/5$

$b_2' = \frac{0-3}{-5} = -3/5$

Step 5

$a_{33}' = 1 - (3\times2) - (-3)(8/5)$

$= 1 - 6 - \cancel{6}$

$= -1/5$

$b_3' = \frac{0 - (1\times2) - (-3/5)(-3)}{-1/5}$

$= 1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 8/5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3/5 \\ 1 \end{bmatrix}$$

$$x_3 = 1$$

$$x_2 + \frac{8}{5} \times x_3 = 3/5$$

$$\Rightarrow x_2 = 3/5 - \frac{8}{5}(1)$$

$$= -1$$

$$x_1 + 2x_2 + 3x_3 = 1$$

$$\Rightarrow x_1 + (-2) + 3 = 1$$

$$\Rightarrow \boxed{x_1 = 0}$$

$$\boxed{x_2 = -1}$$

$$\boxed{x_3 = 1}$$

23-03-15

Interpolation :-

Mathematical function are often described in tabular form. That is for prescribed values of $x_1, x_2 \cdots x_n$ corresponding values of $f(x_1), f(x_2) \ldots$ are provided e.g sine, cosine, logarithmic. The process of passing curve through the given point in order to determine functional values $f(x)$ for values of 'x' not explicity shown in the table is called interpolation

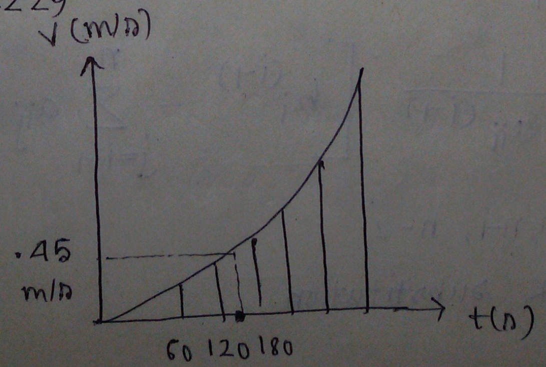
problem :-

Time (s)	Speed (m/s)
0	0
60	0.0824
120	0.2747
180	0.6502
240	1.3851
300	3.2229

$t = 150$

$v = ?$

Solution :- 1) Graphical



2) Linear interpolation [Round off করা যাবেনা]

$v = a + bt$

$\Rightarrow .2747 = a + b(120)$

$\Rightarrow .6502 = a + b(180)$

$a = -0.4763$

$b = 6.258 \times 10^{-3}$

$v = -0.4763 + 6.258 \times 10^{-3} (150)$

$v = 0.4624 \ ms^{-1}$

(3) nth order polynomial :-

(n+1) of points

$P(x) = a_1 + a_2 x + a_3 x^2 + \cdots \quad \cdots + a_{n+1} x^n$

i) Parabola: $P(x) = a_1 + a_2 x + a_3 x^2$

$.824 = a_1 + a_2(60) + a_3(60)^2$

$.2747 = a_1 + a_2(120) + a_3(120)^2$

$.6502 = a_1 + a_2(180) + a_3(180)^2$

$a_1 = .0733$

$a_2 = -1.375 \times 10^{-3}$

$a_3 = 2.54 \times 10^{-5}$

$$\begin{pmatrix} 1 & 60 & 60^2 \\ 1 & 120 & 120^2 \\ 1 & 180 & 180^2 \end{pmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \begin{Bmatrix} 6.0824 \\ 0.2747 \\ 0.6502 \end{Bmatrix}$$

$v = .0733 - 1.375 \times 10^{-3} (150) + 2.54 \times 10^{-5} (150)^2$

$v = .43855 \ ms^{-1}$

ii) A new form

$$P(x) = a_1 + a_2(x-x_1) + a_3(x-x_1)(x-x_2)$$

$$\cancel{P(x) = a_1 + a_2(x-120) + a_3(x-x_1)(}$$

$$P(x) = a_1 + a_2(x-120) + a_3(x-120)(x-180)$$

Passing 180,

$$\cdot 6502 = a_1 + a_2(60)$$

Passing 120,

$$\cdot 2747 = a_1$$

Passing 60,

$$0\cdot 0824 = a_1 + a_2(-60) + a_3(-60)(-120)$$

4) <u>Gregory – Newton</u> :-

x		$f(x)$
x_1	———	$f(x_1)$
x_2	———	$f(x_2)$
x_3	———	$f(x_3)$
$\vdots$		$\vdots$
x_{n+1}		$f(x_{n+1})$

$$x_{i+1} > x_i$$

$x_{n+1} - x_n = x_2 - x_1 = h$ এই Condition fulfil করলে

$$P(x) = a_1 + a_2(x-x_1) + a_3(x-x_1)(x-x_2) + \cdots + a_{n+1}(x-x_1)(x-x_2)$$

$f(x_1) = a_1$

$f(x_2) = a_1 + a_2(x_2 - x_1)$

$f(x_3) = a_1 + a_2(x_3 - x_1) + a_3(x_3 - x_1)(x_3 - x_2)$

$x_2 = x_1 + h$

$x_3 = x_1 + 2h$

$\vdots$

$x_{n+1} = x_1 + hn$

$a_1 = f(x_1), c_0$

$$a_2 = \frac{f(x_1+h) - f(x_1)}{1 \cdot h} = \frac{\Delta f(x_1)}{\lfloor 1 \, h}$$

$$a_3 = \frac{f(x_1+2h) - 2f(x_1+h) + f(x_1)}{2 \cdot h^2}$$

$$= \frac{\Delta^2 f(x_1)}{\lfloor 2 h^2}$$

$$a_{n+1} = \frac{\Delta^n f(x_1)}{\lfloor n \, h^n}$$

$$P(x) = f(x_1) + \frac{\Delta f(x_1)}{\lfloor 1 \cdot h}(x - x_1) + \frac{\Delta^2 f(x_1)}{\lfloor 2 \, h^2}(x - x_1)(x - x_2) + \ldots$$

$\nabla f(x_1)$ = Backward

$\delta f(x_1)$ = central

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26/04/15

Numerical Method :-

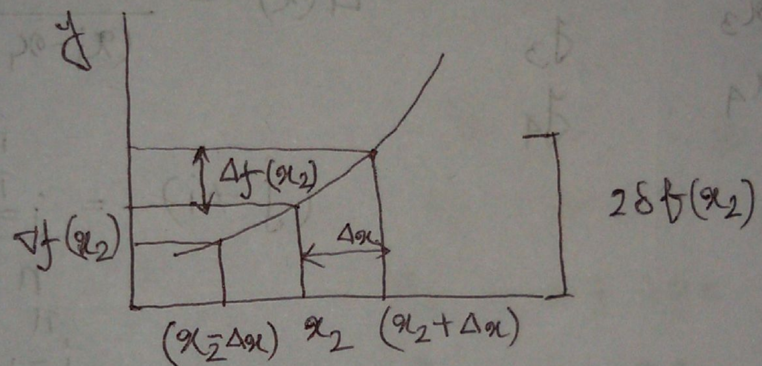
$$P(x) = f(x_1) + \frac{\Delta f(x_1)}{\lfloor 1\, h}(x-x_1) + \frac{\Delta^2 f(x_1)}{\lfloor 1\, h}(x-x_1) + \frac{\Delta^2 f(x_1)}{\lfloor 2\, h^2}(x-x_1)(x-x_2)$$

Given values

$x_1 - f(x_1)$
$x_2 - f(x_2)$
$x_3 - f(x_3)$
$\downarrow$
$x_n - f(x_n)$

Finite difference

Δ = Forward difference
∇ = Backward "
δ = Central difference



$\Delta f(x_1) = f(x_2) - f(x_1)$

$\Delta^2 f(x) = 2c_0 f(x_3) - 2c_1 f(x_2) - 2c_2 f(x)$

~~Finite~~ **Difference Table :-**

		D_1	D_2	$D_3/D^3 f$
x_1	$f(x_1)$	$f(x_2)-f(x_1)$	$f(x_3)-2f(x_2)+f(x_1)$	$f(x_4)-3f(x_3)+3f(x_3)-f(x_1)$
x_2	$f(x_2)$	$f(x_3)-f(x_2)$	$f(x_4)-2f(x_3)+f(x_2)$	$\Delta^3 f(x_1)$
x_3	$f(x_3)$	$f(x_4)-f(x_3)$		
x_4	$f(x_4)$			

Δ^4 D_4 Δ^5 D_5

~~Logarithm~~ Lagragian Interpolation :-

$x_2 - x_1 \neq x_3 - x_2$

$P(x) = L_1(x)y_1 + L_2(x)y_2 + L_3(x)y_3$

x	y
x_1	y_1
x_2	y_2
x_3	y_3
x_4	y_4

$= \sum_{i=1}^{n} L_i(x) y_1$

$$L_i(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_i-x_1)(x_1-x_2)\cdots(x_1-x_n)}$$

$$(j \neq i) = \frac{\prod_{j=1}^{n}(x-x_j)}{\prod_{j=1}^{n}(x_i-x_j)}$$

$$L_1(x) = \frac{(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_2)(x_1-x_3)\cdots(x_1-x_1)}$$

$x = 1505$

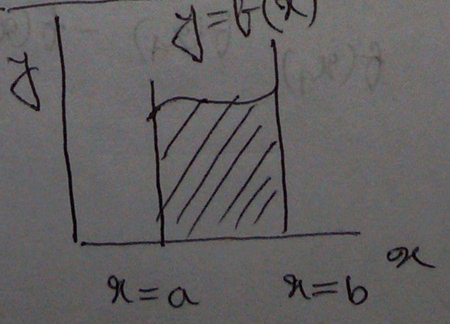
Interpolation Polimorainial - ৩ বের করতে হবে।

CT - 70% math

30% theory

Numerical Integration / Quadrature :-

$$I = \int_a^b f(x)\,dx$$



Assumption :- a) a and b are finite

b) f(x) is continuous between limits