26/04/15

Numerical Method :-

$$P(x) = f(x_1) + \frac{\Delta f(x_1)}{1!h}(x-x_1) + \frac{\Delta^2 f(x_1)}{1!h}(x-x_1) + \frac{\Delta^2 f(x_1)}{2!h^2}(x-x_1)(x_1-x_2)$$

Given values

$x_1 - f(x_1)$

$x_2 - f(x_2)$

$x_3 - f(x_3)$

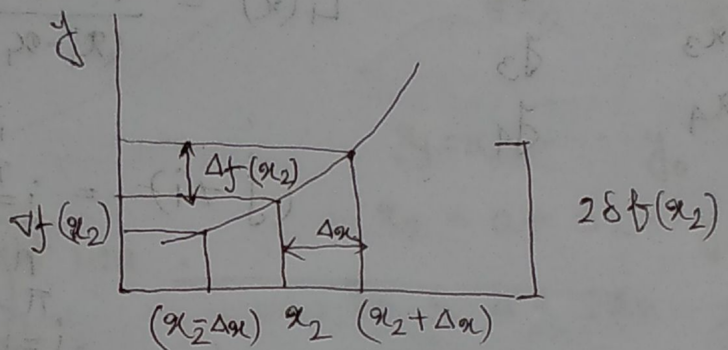
$x_n - f(x_n)$

Finite difference

Δ = Forward difference

∇ = Backward "

δ = Central difference



$$\Delta f(x_1) = f(x_2) - f(x_1)$$

$$\Delta^2 f(x) = 2c_0 f(x_3) - 2c_1 f(x_2) - 2c_2 f(x)$$

~~Finite~~ **Difference Table :-**

		D_1	D_2	$D_3/D^3 f$
x_1	$f(x_1)$			
		$f(x_2)-f(x_1)$		
x_2	$f(x_2)$		$f(x_3)-2f(x_2)+f(x_1)$	
		$f(x_3)-f(x_2)$		$f(x_4)-3f(x_3)+3f(x_3)-f(x_1)$
x_3	$f(x_3)$		$f(x_4)-2f(x_3)+f(x_2)$	
		$f(x_4)-f(x_3)$		$\Delta^3 f(x_1)$
x_4	$f(x_4)$			

Δ^4 D_4 Δ^5 D_5

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~~Logarithm~~ **Lagragian Interpolation :-**

$x_2 - x_1 \neq x_3 - x_2$

$P(x) = L_1(x) y_1 + L_2(x) y_2 + L_3(x) y_3$

x	y
x_1	y_1
x_2	y_2
x_3	y_3
x_4	y_4

$= \sum_{i=1}^{n} L_i(x) y_1$

$$L_i(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_i-x_1)(x_i-x_2)\cdots(x_i-x_n)}$$

$$(j \neq i) \quad = \frac{\prod_{j=1}^{n}(x-x_j)}{\prod_{j=1}^{n}(x_i-x_j)}$$

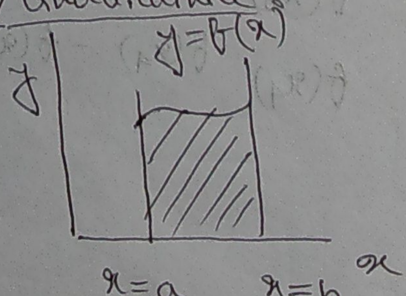
$$L_1(x) = \frac{(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_2)(x_1-x_3)\cdots(x_1-x_1)}$$

$x = 1505$

Interpolation Polimorninial - ও বের করতেহবে,
CT - 70% math
30% Theory

Numerical Integration / Quadrature :-

$$I = \int_a^b f(x)\,dx$$



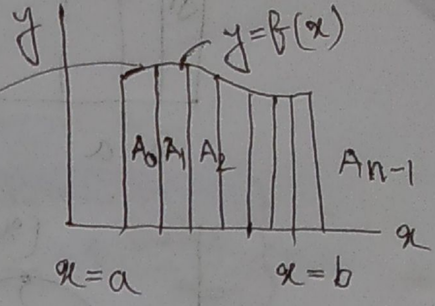
Assumption :- a) a and b are finite
b) $f(x)$ is continuous between limits

<u>Functions</u> functions of one variable

a) Trapizoidal and Simpson's rule

b) Gauss Quadrature

Function of two variables



$$\text{Volume} = \int_{a_1}^{b_1}\int_{a_2}^{b_2} F(x,y)\,dy\,dx$$

$x_1 = a \cdots\cdots y_0$

$x_2 = a + \Delta x \cdots y_1$

<u>Problem</u> This are straight line.

$x_3 = a + 2\Delta x \cdots y_2$

যত বেশী Segment তত বেশী Accurate

$x_4 = a + 3\Delta x \cdots y_3$

$$A_0 = \frac{1}{2}(y_0 + y_1)\Delta x$$

$$A_1 = \frac{1}{2}(y_1 + y_2)\Delta x$$

$$A_2 = \frac{1}{2}(y_2 + y_3)\Delta x$$

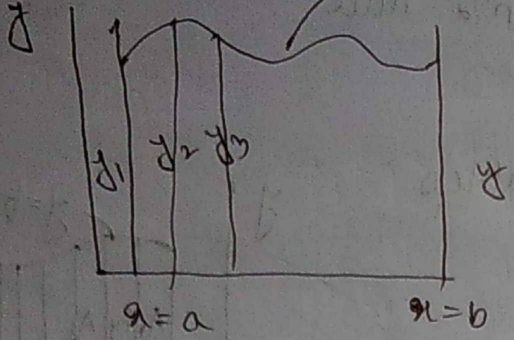
$$A_{n-1} = \frac{1}{2}(y_{n-1} + y_n)\Delta x$$

$$A = \frac{\Delta x}{2}(y_0 + y_n + 2(y_1 + y_2 + \cdots$$

Derivation important :-

Simpson's Rule

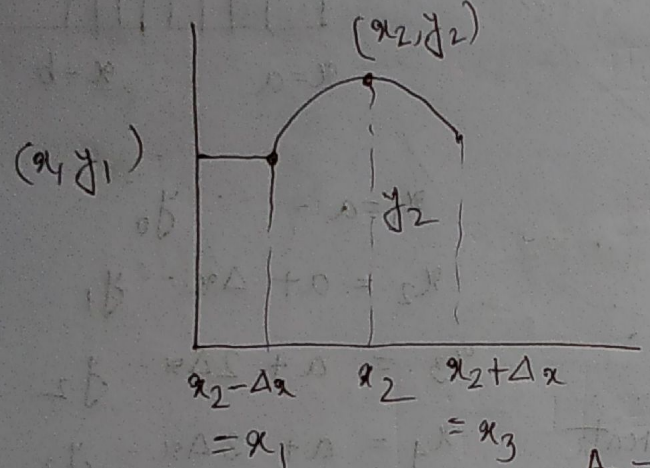
$y = a_1x^2 + a_2x + a$



$y_1 = a_1(x_2-\Delta x)^2 + a_2(x_2-\Delta x) + a_3$

$y_2 = a_1x_2^2 + a_2x_2 + a_3$

$y_3 = a_1(x_2+\Delta x)$



$$A_0 = \frac{\Delta x}{3}\left[y_1 + 4y_2 + y_3\right]$$

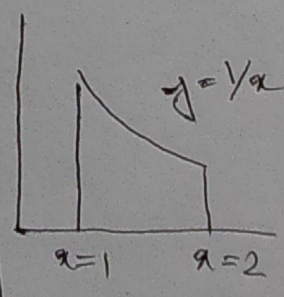
$$A_0 = \int_{x_1}^{x_3} (a_1x^2 + a_2x + a_3)\,dx$$

$$= \left[\frac{a_1x^3}{3} + \frac{a_2x^2}{2} + a_3x\right]_{x_2-\Delta x}^{x_2+\Delta x} \quad \text{---(I)}$$

Problem $\int_1^2 \frac{1}{x}dx$ Trapizoidal

1, 2, 4, 8, 16

n	Δx	A
1	$\Delta = \frac{2-1}{1} = 1$	$\frac{1}{2}[1 + .05] = .75$
2	$\Delta x = \frac{2-1}{2} = .5$	$\frac{.5}{2}[1 + 2\times .6666 + .5] = .7082$

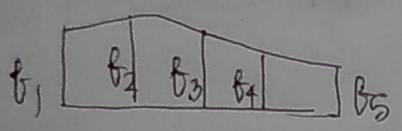


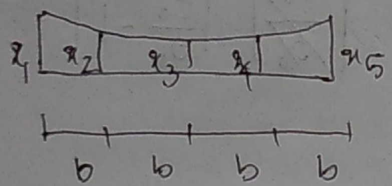
Always Numerical এ রাখতে হবে

Solve this using Simpsons Rule $= \cdot 6937$

<u>product of integrals</u>

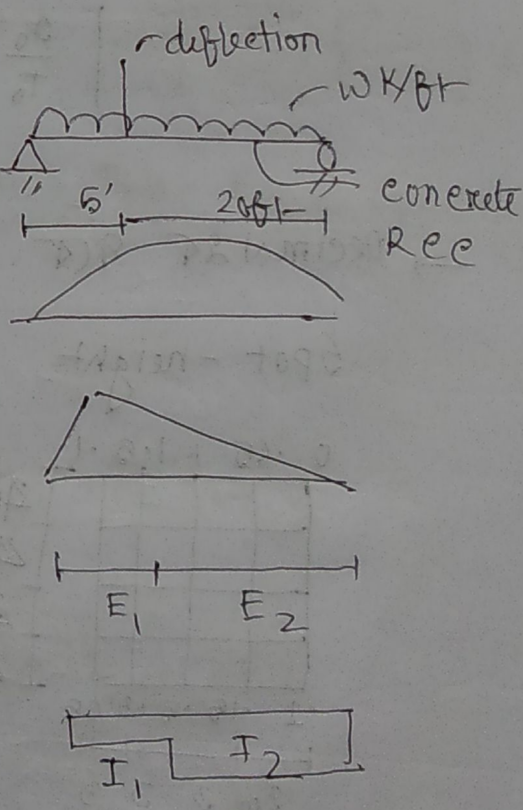
$$\int_a^b \frac{f_1(x)\, f_2(x)}{f_3(x)}\, dx \simeq \frac{b}{3}\left[\frac{f_1 y_1}{x_1} + 4\left\{\frac{f_2 y_2}{x_2} + \frac{f_4 f_4}{x_4}\right\} + 2\frac{f_3 y_3}{x_3} + \frac{f_5 y_5}{x_5}\right]$$





<u>problem</u>

$$\Delta_a = \int \frac{Mm}{EI}\, dx$$



10/5/15

Romberg's Table $I = \int_1^2 \frac{1}{x}\,dx$ $\quad \varepsilon = .0001$

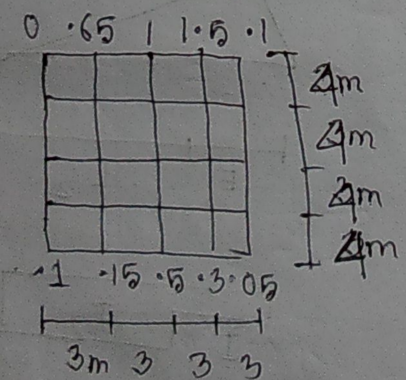
	Trapizoidal	Simpson's	C	D	E
1	T_0	$\rightarrow S_0 = \frac{4T_1 - T_0}{4-1}$	$- C_0 = \frac{4^2 S_1 - S_0}{4^2-1}$	$\rightarrow D_0 = \frac{4^3 C_1 - C_0}{4^3-1}$	$E_0 = \frac{4^4 D_1 - D_0}{4^4-1}$
2	T_1	$\rightarrow S_1 = \frac{4T_2 - T_1}{4-1}$	$- C_1 = \frac{4^2 S_1 - S_1}{4^2-1}$	$D_1 = 4^3 C_2 - C_1$	
4	T_2	$\rightarrow S_2 = \frac{4T_3 - T_2}{4-1}$	$- C_2 = \frac{4^2 S_3 - S_2}{4-1}$		
8	T_3	$- S_3 = \frac{4T_4 - T_3}{4-1} -$			

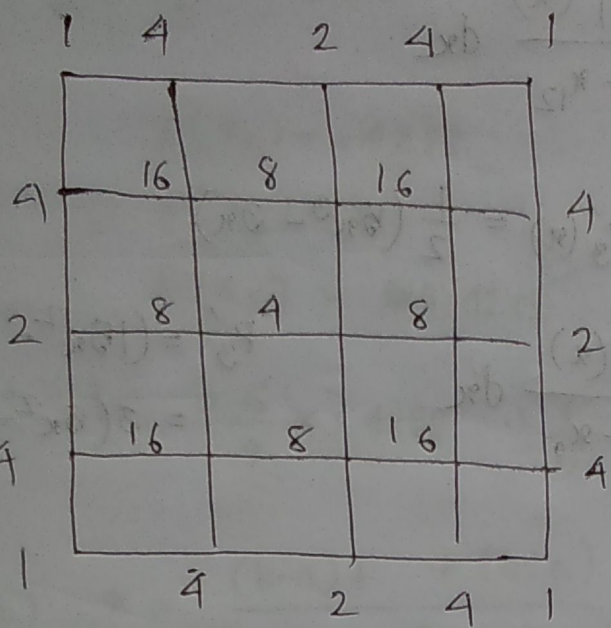
$\left| \frac{S_0}{T_0} - 1 \right| = ?$ $\qquad \left| \cancel{\frac{C_0}{S_0}} \right| \left| \frac{C_0}{S_0} - 1 \right|$

→ Decimal এর পরে 4 Digit পর্যন্ত

Spot - height   Slope - থাকলে How to calculate Area?

ছাদ যদি slope হয়





$$\text{Volume} = \frac{h_1}{3} + \frac{h_{12}}{3}$$

$$\text{Volume} = \frac{h_1}{3} + \frac{h_2}{3}\left[1\times\{\quad\}\right] + 2\{\cdots\} + 4\{\quad\} + 8$$

$\rightarrow$ Main condition for simpson's Rule - Equal Spacing

Gauss Quadrature

$$\int_{-1}^{1} f(x)\,dx \simeq w_0 f(x_0) + w_1 f(x_1) + w_2 f(x_2)$$

বলা না থাকলে $n = 2$

$$P_n(x) = \frac{1}{2^n \lfloor n} \frac{d^n}{dx^n}(x^2-1)^n$$

n	x	w
2	0 ±.07746	5/9 5/9, 5/9
3	±0.34 ±0.8611	0.6521 .3479
4		

$P_0(x) = 1$

$P_1(x) = x$

$P_2(x) = \frac{1}{2}(3x^2-1)$

$P_3(x) = \frac{1}{2}(5x^3 - 3x)$

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$$w_k = \frac{1}{P_{n+1}(x_k)} \int_{-1}^{1} \frac{P_{n+1}(x)}{x - x_{12}} dx$$

$n = 2 \quad P_{n+1}(x) = P_3'(x) = \frac{1}{2}(5x^3 - 3x)$

$$w_0 = \frac{1}{P_3'(x_0)} \int_{-1}^{1} \frac{P_3(x)}{x - x_0} dx$$

$P_3' = (15x^2 - 3)$

$= 3(5x^2 - 1)$

$= 5/9$

$w_1 = 8/9$

$w_2 = 5/9$

problem (a)

a) $\int_{-1}^{1} x^2 \cos x \, dx$

b) $\int_{2}^{3} x^2 \cos x \, dx$

$$P(x) = \frac{1}{2^n \lfloor n} \frac{d^n}{dx^n} (x^2 \cos x)^n$$

$f(x) = x^2 \cos x$ radian এ বসাতে হবে

$x = -\sqrt{3/5}$

$f(x_0) = ?$ $\quad w_0 = 5/9$

$x_1 = 0 \quad f(x_1) = 0$

$x_2 = \pm\sqrt{3/5}$ $f(x_2) = ?$ $w_2 = 5/9$

$f(x_0) = .4288$

$f(x_1) = 0$

$f(x_2) = 0.4288$

Radian এ রাখলে

.4765

$$\int_{-1}^{1} f(x)\,dx = \frac{5}{9} \times .4258 + \frac{5}{9} \times .4288 = .4764$$

b) $$x = \frac{(b-a)t + (b+a)}{2}$$

$a = 2$

$b = 3$

$$dx = \frac{b-a}{2}\,dt = \frac{1}{2}\,dt$$

$$x = \frac{(t+5)}{2}$$

t = transformation function

$$\int_{-1}^{1} \frac{1}{2}\left(\frac{t+5}{2}\right)^2 \cos\left(\frac{t+5}{2}\right) dt$$

$$= \frac{1}{8}\int_{-1}^{1} (t+5)^2 \cos\left(\frac{t+5}{2}\right) dt$$

x	2	3
t	-1	1

$t = -\sqrt{3/5}$ $w_0 = 8/9$

$t_1 = 0$ $w_1 = 5/9$

$t_2 = \sqrt{3/5}$ $w_2 = 5/9$

$f(t_0) = -1.15$

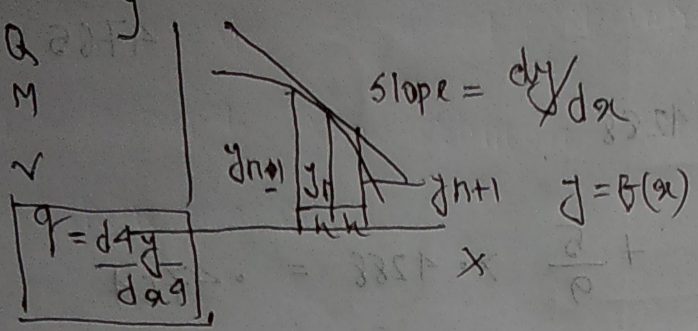
$f(t_1) = 0$

$f(t_2) = -4.03$

$f(a) = -2.87$

25/05/15

Numerical Differenciation



$$\frac{dy}{dx} = \text{slope} = \frac{y_{n+1} - y_n}{h}$$

$$\frac{d}{dx} = D = \frac{y_n - y_{n-1}}{h}$$

$$= \frac{y_{n+1} - y_{n-1}}{2h}$$

$$\frac{d^2y}{dx^2} = \frac{d\theta_n}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{y_{n+2} - 2y_{n+1} + y_n}{h^2}$$

$$D^2y = \frac{\theta_{n+1} - \theta_n}{h} = \frac{\frac{y_{n+2} - y_{n+1}}{h} - \frac{y_{n+1} - y_n}{h}}{h}$$

Backward Difference

	y_n	y_{n-1}	y_{n-2}	y_{n-3}	y_{n-4}
hD	1	-1			
h^2D^2	1	-2	1		
h^3D^3	1	-3	3	-1	
h^4D^4	1	-4	6	-4	1

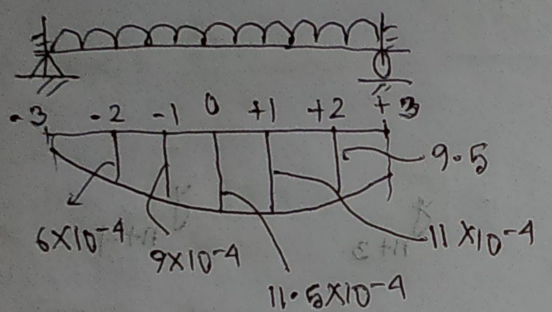
Forward difference

	y_n	y_{n+1}	y_{n+2}	y_{n+3}	y_{n+4}
hD	-1	1			
h^2D^2	1	-2	1		
h^3D^3	-1	3	-3	1	
h^4D^4	1	-4	6	-4	1

Central Difference

	y_{n-2}	y_{n-1}	y_n	y_{n+1}	y_{n+2}
$2hD$		-1	0	1	
h^2D^2		1	-2	1	
$2h^3D^3$	-1	2	0	-2	1
h^4D^4	1	-4	6	-4	1

Problem:- Experimentally observed value of deflections of a beam are shown below. Calculate Bending moment at $-1, 0, +1$ points given $E = 30 \times 10^6$ psi. $I = 1000 \text{ in}^4$. Also estimate slope and shear force at their points.



ছবি আঁকা থাকতেও পারে
না থাকলে এঁকে
নিতে হবে

Plate

$$M_x = -D\left[\frac{\partial^2 z}{\partial x^2} + \gamma \frac{\partial^2 z}{\partial y}\right]$$

$$M_y = -D\left[\frac{\partial^2 z}{\partial y^2} + \gamma \frac{\partial^2 z}{\partial x^2}\right]$$

D = plate

$$= \frac{Et^3}{12(1-\gamma^2)}$$

($\gamma = 0.15 - 0.2$

t = slab thickness

$= 5'' - 6''$

At point 1

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{h^2}(z_1 - 2z_2 + z_3)$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{1}{\ell^2}(z_1 - 2z_6 + z_{11})$$

point	deflection (10^{-4})
1	6.0
2	5.0
16	4.0