

Full Marks : 140

Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION - A

There are FOUR questions in this Section. Answer any THREE.

1. (a) Deduce the general quadrature formula and hence derive Simpson's $\frac{1}{3}$ rd rule for numerical integration. (10 $\frac{1}{3}$)

- (b) Solve the following system of equations by relaxation method. (13)

$$3x_1 + 9x_2 - 2x_3 = 11$$

$$4x_1 + 2x_2 + 13x_3 = 24$$

$$4x_1 - 4x_2 + 3x_3 = -8$$

Consider a tolerance of 0.01 for consecutive trials of all the variables.

2. (a) State various Elimination and Iterative approaches for the solution of systems of linear equations. Discuss, with examples, the various possibilities of solutions of systems of linear equations. (10 $\frac{1}{3}$)

- (b) Evaluate the following integral by trapezoidal rule considering four equal intervals and hence refine the solution by Romberg's method. (13)

$$\int_0^1 \frac{dx}{1+x^2}$$

3. (a) Describe a step by step procedure for solving a system of linear equation by Crout's method. (10 $\frac{1}{3}$)

- (b) Monthly faculty salary in three departments of an institute is given below. Assuming that the salary for a particular category of faculty is same in all the departments, calculate the salary of each category of faculty by formulating linear equations. (13)

| Departments | Number of Faculties | | | Total Salary Permonth (Tk. In Lacs) |
|-------------|---------------------|--------------|----------|--|
| | Professor | Asstt. Prof. | Lecturer | |
| A | 2 | 2 | 7 | 1.50 |
| B | 3 | 1 | 1 | 1.00 |
| C | 1 | 4 | 1 | 0.50 |

Use Gauss-Seidel method and a tolerance of 0.001 for all the variables.

ContdP/2

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4. (a) The following Table gives the angular displacements, θ (radians) at different intervals of time, t (seconds).

(10 $\frac{1}{3}$)

| | | | | | |
|----------|-------|-------|-------|-------|-------|
| θ | 0.052 | 0.105 | 0.168 | 0.242 | 0.327 |
| t | 0 | 0.02 | 0.04 | 0.06 | 0.08 |

Calculate the angular velocity at the instant $t = 0.06$.

- (b) The population of a town in decennial census were as under. Estimate the population for the year 1944.

(13)

| | | | | | |
|---------------------------|------|------|------|------|------|
| Year | 1921 | 1931 | 1941 | 1951 | 1961 |
| Population (in Thousands) | 46 | 66 | 81 | 93 | 100 |

Use Gauss's Central difference formula. Given that-

$$y_p = y_0 + G_1 \Delta y_0 + G_2 \Delta^2 y_0 + G_3 \Delta^3 y_{-1} + G_4 \Delta^4 y_{-2} + \dots$$

$$G_1 = P, \quad G_2 = \frac{P(P-1)}{2}, \quad G_3 = \frac{(P+1)P(P-1)}{3}, \quad G_4 = \frac{(P+1)P(P-1)(P-2)}{4}, \text{ etc.}$$

SECTION - B

There are SEVEN questions in this Section. Answer any FIVE.

5. (a) Use the Newton-Raphson method to estimate the root of the equation $e^{-x} = x$ such that the root is correct up to 4 decimals. Use $x = 0$ as your initial guess. (10)
- (b) How many additional iterations will be required if you want the function to be correct up to 10 decimals. (4)
6. (a) Find the real root of the equation $x \log_{10} x - 1.5 = 0$ such that the function value is correct up to 5 decimals using the method of false position. (10)
- (b) How does the method of the false position differ from bisection method? (4)
7. (a) Find the cubic root of 10 using numerical methods. Use a starting value of 2 and show five iterations. (7)
- (b) Find one root of the equation $x^2 - 5x + 6 = 0$ using fixed point iteration. Use a starting value of 1 and show at least three iterations. (7)
8. (a) An experiment is performed to define the relationship between applied stress and the time to fracture for a stainless steel specimen. Eight different values of stress are applied and the resulting data are as follows:

| | | | | | | | | |
|----------------------------------|----|----|----|----|----|----|----|----|
| Applied stress, kg/mm^2 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
| Fracture time, h | 40 | 30 | 25 | 40 | 18 | 20 | 22 | 15 |

- (a) Fit a straight line to the data with least square method. (12)
- (b) Use the best fit equation to predict the fracture time for an applied stress of 33 kg/mm^2 . (2)

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9. (a) Determine the constants a and b by the least squares method such that $y = ae^{bx}$ fits the following data:

(10)

| | | | | |
|---|--------|--------|---------|--------|
| x | 1.0 | 1.2 | 1.4 | 1.6 |
| y | 40.170 | 73.196 | 133.372 | 243.02 |

- (b) Write the normal equations for the case where the first two observations are assigned weights equal to 0.5 and 0.75 respectively (no need to find the constants, only formulation and writing down the normal equations will be fine).

(4)

| | | | | |
|---|--------|--------|---------|--------|
| x | 1.0 | 1.2 | 1.4 | 1.6 |
| y | 40.170 | 73.196 | 133.372 | 243.02 |
| w | 0.5 | 0.75 | 1 | 1 |

- 10 (a) Solve $y' = 2x^2 - y$, $y(0) = -1$ by the 4th order Runge-Kutta Method and determine $y(3)$. Use step-size equal to 1.

(12)

- (b) How does Runge-Kutta method differ from Multi-step Methods?

(2)

11. (a) Solve $y' = \sin(x) + y$, $y(0) = 2$ by the Modified Euler's method to get $y(0.3)$. Use a step-size equal to 0.1.

(12)

- (b) Why do we use adaptive/non-equal step-size in solving ordinary differential equations?

(2)

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আমি বিশ্বাস করি মুহম্মদের (স) মতো ব্যক্তির নিকট যদি আধুনিক বিশ্বের একনায়কতন্ত্র অর্পণ করা হতো তবে এর সমস্যাগুলো তিনি এমনভাবে সফলতার সাথে সমাধান করতেন যা বহু প্রতিক্ষীত শান্তি ও সুখ আনয়ন করতো। আমি ভবিষ্যতবাণী করছি যে মুহাম্মদের ধর্মবিশ্বাস আগামীদিনের ইউরোপের কাছে গ্রহণযোগ্য হবে, যা ইতিমধ্যে বর্তমান ইউরোপে গ্রহণযোগ্যতা পেতে আরম্ভ করেছে।

Sir George Bernard Shaw in 'The Genuine Islam,' Vol. 1, No. 8, 1936.