L-2/T-2/CE **Date : 18/12/2012**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2010-2011

Sub : **CE 205** (Numerical Methods)

Full Marks: 140 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Water is flowing in a trapezoidal channel at a rate of Q = 20 m^3/s. The critical depth y for such a channel must satisfy the equation **(20)**

$$1-\frac{Q^2}{gA_c^3}B=0$$

where g = 9.81 m/s^2, A_c = cross sectional area $(m^2)=3y+\frac{y^2}{2}$, B = width of the channel at surface $(m)=3+y$

(i) Solve for the critical depth using both bisection method and regula falsi method. In both methods, use initial guesses of $x_L=0.5$ and $x_u=2.5$, and iterate until the approximate error falls below 1% or the number of iterations exceed 10.

(ii) Which method performs better for the given stopping criteria and why?

(b) Give a graphical depiction of convergence and divergence of simple fixed-point iteration. What is the general condition for convergence of simple fixed-point iteration? $(3\frac{1}{3})$

2. (a) Use least-squares regression to fit a saturation-growth-rate equation $\left(y=a\frac{x}{b+x}\right)$ for the given data: $(13\frac{1}{3})$

x	5	10	15	20	25	30	35	40	45	50
y	17	24	31	33	37	37	40	40	42	41

(b) Find to four places of decimal, the smallest root of the equation $e^{-x}=\sin x$ using Newton-Raphson method. **(10)**

3. (a) Solve the following equation numerically over the interval from x = 0 to 1. $(15\frac{1}{3})$

$$\frac{dy}{dx}=(1+2x)\sqrt{y} \qquad y(0)=1$$

Use the classical fourth order Runge-Kutta method with h = 0.5. Also compare the results with the analytical solution.

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(b) Write down the set of linear equations (in matrix form) resulting from the finite difference approximation of the following boundary value problem: (Use $\Delta x = 1$) **(8)**

$$\frac{d^2T}{dx^2} = 0.15\,T = 0 \qquad \begin{cases} T(0) = 240 \\ T(10) = 150 \end{cases}$$

4. (a) Why do we need to use numerical computing techniques to solve differential equations? **(3)**

(b) What is an initial value problem? How is it different from a boundary value problem? Given examples. **(5)**

(c) "Heun's method is an improved version of Euler's method" — Comment. **(3)**

(d) The differential equation governing the displacement of a body attached to a spring over time(t) is given by **(12⅓)**

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 0$$

Find the displacement y at time t = 1.5 given that y(0) = 2 and y′(0) = –4. Use Heun's method with a stepsize of 0.5.

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Write short notes on: **(10)**

   (i) Pivotal Condensation and Back Substitution

   (ii) Matrix Inversion using Gauss-Jordan method.

(b) S~~a~~et a polynomial equation passing through the points provided in the following table and use it to find the interpolated value for x = 3.0. **(13⅓)**

x	3.2	2.7	1.0	4.8	5.6
$f(x)$	22.0	17.8	14.2	38.3	51.7

6. (a) Derive the general expression of $I = \int_a^b f(x)\,dx$, using Simpson's Rule. **(8)**

(b) Use the following data to get the integral between the limits of x = 1.6 and x = 3.8 using Romberg's Quadrature. **(15⅓)**

x	$f(x)$	x	$f(x)$	x	$f(x)$
1.6	4.953	2.4	11.023	3.2	24.533
1.8	6.050	2.6	13.464	3.4	29.964
2.0	7.389	2.8	16.445	3.6	36.598
2.2	9.025	3.0	20.086	3.8	44.701

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7. (a) Derive the final expression of Gregory-Newton Interpolation formula. **($11\frac{1}{3}$)**

(b) Solve the following system using Crout's method: **(12)**

$$\begin{aligned} 2x_1 + x_2 - x_3 &= 2 \\ 3x_1 - x_2 + 2x_3 &= 4 \\ 2x_1 - 2x_2 + x_3 &= 6 \end{aligned}$$

8. (a) Evaluate $I = \int_0^{\pi/2} \sin x dx$ for n = 3 using Gauss Quadrature technique. **($10\frac{1}{3}$)**

(b) Experimentally observed values of deflections of a beam are shown in the following figure. Estimate slope, bending moment and shear force at all points. **(13)**

Given E = 30×10^6 psi; I = 1500 in^4.