Numarical Methods

CE 205

Ansari Sir

1) Numarical Solution of Algebric ($y = ax+b$) and Transcendental Eq^n ($y = ae^x+b$).
2) Solution of system of linear equation
3) Curve Fitting by least squares
4) Finite difference
5) Interpolation
6) Numarical Differentiation
7) Computer application to CE problem
8) Numarical Integration
9) Numarical Solution of Differential Eq^n

Numarical method এর কাজ হল Approximate value বের করা, exact value না।

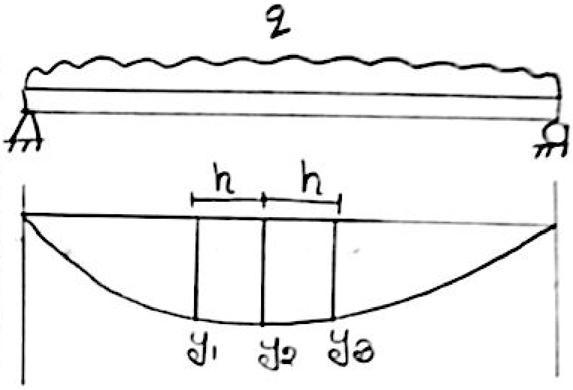
$x_1, x_2, \ldots\ldots x_n$ $\quad |x_{n+1} - x_n| \leqslant \quad \epsilon = 0.001$ Pricition-Hand

$= 0.00001$ Computer-Precition

Reff. Book:

1. Numarical Methods By Goel
2. Introductory methods of Numarical Analysis - Sastry
3. Numarical Mathematical Analysis - scarborough
4. Numarical Methods - Jain

Numarical Differentiation:



$$\theta = \frac{y_2 - y_1}{h} = \frac{dy}{dx} = \text{slope}$$

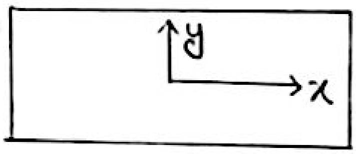
$$M = \frac{\theta_1 - \theta_0}{h}$$

$$V = \frac{M_1 - M_0}{h}$$

$$q = \frac{V_1 - V_0}{h}$$

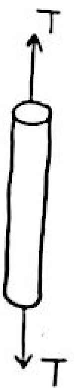
$$M = EI \frac{D^2 y}{Dx^2}$$

FEM (Finite Element Method):

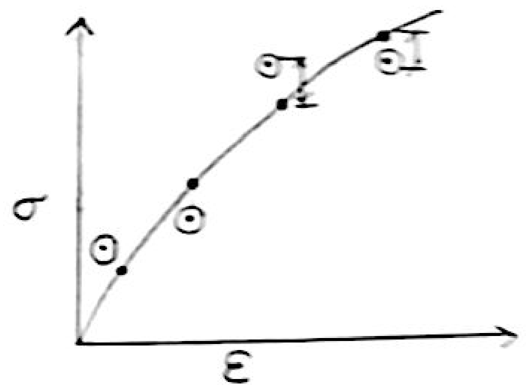


Beam

ETAB দিয়ে Building এর কোথায় কত load আছে সেটা calculate করা যায়।



ε	σ (psi)
1	100
২	150
৩	২00
4	২50



Graphical Interpolation (G.I)

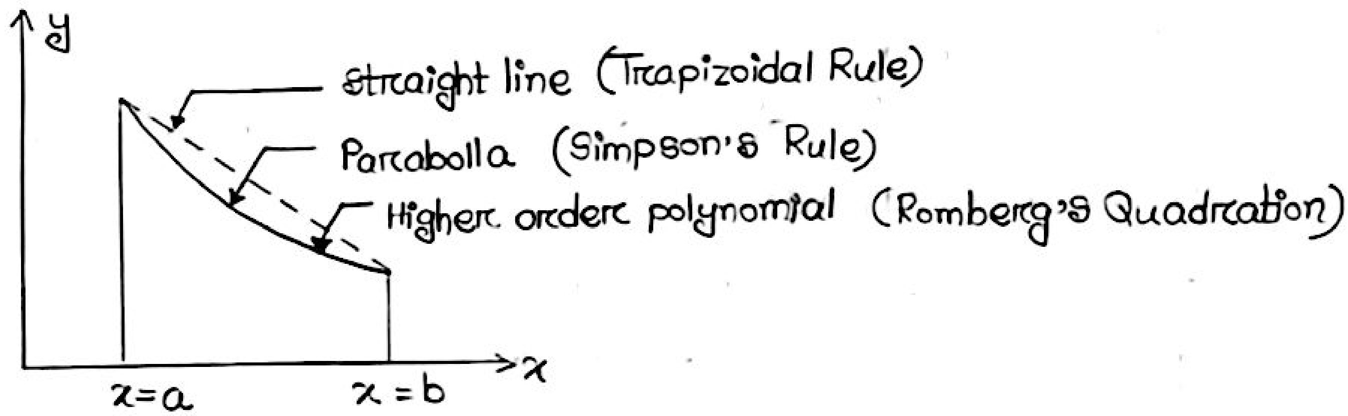
Lab related data হলে Graphical Interpolation use করা হয়।

Curve যখন প্রত্যেকটি বিন্দুগামী হয়, তখন সেই method কে interpolation বলা হয়।

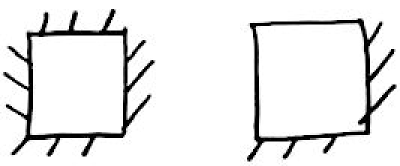
যখন Curve প্রত্যেকটি Data point দিয়ে pass করে না, nearest value দিয়ে pass করে (Best Fitted curve আঁকার সময়) তখন তাকে Curve Fitting বলা হয়। Experimental Data আর graph এর plot কৃত curve এর value এর মধ্যে যে Difference সেটা Least Square এর মাধ্যমে বের করা হয়।

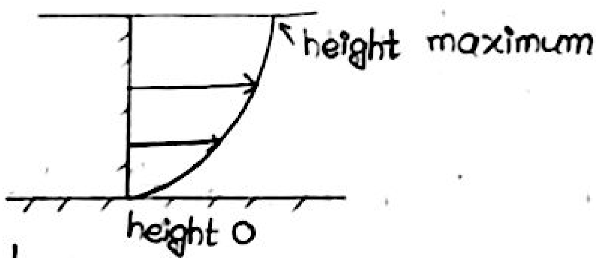
Numarical Integration:

$$I = \int \frac{1}{x} dx \quad ; \quad I = \int \frac{1}{x^2 \sin x + \log x}$$



Boundary Condition:





Numarical Method এর Procedure:

Input Data

↓

The Algorithm

↓

Output

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Solution of System of Linear Equation

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots\cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots\cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots\cdots + a_{mn}x_n = b_m$$

$\boxed{m=n}$

Linear eqn solve করবার দুটি Method.

1) Direct Method:

Yields the solution after an amount of computation that is known in advance.

a) Methods of Matrices / Determinants

b) Methods of Successive Elemination

i) Gauss - Elemination

ii) Gauss - Jordan / Modified Gauss

iii) Crout's Method

iv) Factorization

2) Indirect Method / Itarative Method:

Which starts from an approximation to the true solution and the amount of computation depends on the accuracy needed.

a) Jacobi Method

b) Gauss - Seidal Method

c) Relaxation method

Cramer's Rule:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$x_j = \frac{B_j}{D} \qquad B_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}$$

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Numarical Method:

কোন Formula use না করে math prob. solve করা হয়। prob. কে অনেক ভাগে ভাগ করা। ফলে বড়ো prob. টা solve করতে সময় লাগে।

$F_R = C_d v^2$
(Resistant force of air)

$W = mg$ ↑⊕ ↓⊖

$$mg - C_d v^2 = m\frac{dv}{dt}$$

$$\boxed{\frac{dv}{dt} = g - \frac{C_d}{m}v^2} \quad \text{DE} \cdots ①$$

Performing Integration

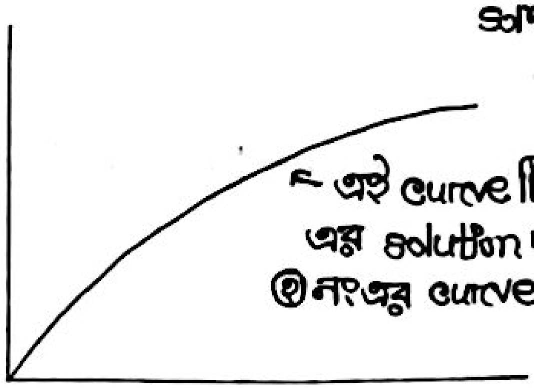
$$v(t) = \sqrt{\frac{gm}{C_d}}\tanh\left(\sqrt{\frac{gC_d}{m}}\,t\right) \cdots ②$$

soln of ①

*solution কে graph এর একটি curve আকারে চিন্তা করা যায়।

② নং soln কে,

Analytic soln বা True/exact soln বলা হয়।



← এই curve টি ① নং এর solution। কারণ এটা ② নং এর curve.

Numarical Soln:

Diff: এর (সংজ্ঞা হতে)

$$\frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} \quad \ldots (1)$$

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2 \quad \ldots (2)$$

Substituting (1) and (2),

$$v(t_{i+1}) = v(t_i) + \left[g - \frac{c_d}{m} \{v(t_i)\}^2 \right] (t_{i+1} - t_i)$$

at $t = 0$ $v(t=0) = 0$

$$v(t=2) = \underbrace{v(t=0)}_{0} + \left(9.81 - \frac{c_d}{m} \underbrace{v(t=0)^2}_{0}\right)(2-0)$$
$$= \ldots$$

$$v(t=4) = v(t=2) \ldots = \ldots$$

$$v(t=6) = v(t=4) + \ldots = \ldots$$

* Numarical ও Analytic solution এর মধ্যকার পার্থক্যকে Numarical error বলা হয়।

2, 4, 6, 7 No. Topic Tanvir Sir পড়াবেন।

<u>Book</u>

Numercical Methods for Engg. — Chapra and Canale

Ansari Sir

Method of Chio for evaluating Determinants:

$$D = \begin{vmatrix} a_1 & a_2 & \cdots & a_m \\ b_1 & b_2 & \cdots & b_m \\ c_1 & c_2 & \cdots & c_m \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix}_{m\times m}$$

$\boxed{a_1 \neq 0}$

$$D = \frac{1}{a_1^{m-1}} \begin{vmatrix} a_1 & a_2a_1 & a_3a_1 & a_ma_1 \\ b_1 & b_2a_1 & \cdots & b_ma_1 \\ c_1 & c_2a_1 & \cdots & c_ma_1 \end{vmatrix}$$

$$= \frac{1}{a_1^{m-1}} \begin{vmatrix} a_1 & 0 & 0 & 0 \\ b_1 & & \boxed{\quad} & \\ c_1 & & & \end{vmatrix}$$

$$= \frac{1}{a_1^{m-2}} \begin{vmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{vmatrix}_{(m-1)\times(m-1)}$$

Gauss Elimination Method:

1. Upper triangle matrix
2. Diagonal element
3. Back substitution
4. Pivatal Condensation

} Short note

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

upper triangle matrix

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

0 0 0

Diagonal element

$$a_{33}x_3 = b_3$$

$$\boxed{\begin{array}{l} x_3 = ? \\ x_2 = ? \\ x_1 = ? \end{array}}$$

Gauss elimination এ last element টা আগে বের হয়। এটাই Back Substitution.

* a_{11} element কখনওই 0 হতে পারবে না।
* প্রথমেই check (scan) করে দেখতে হবে যাতে

  $a_{11} > a_{21} > a_{31}$ হয় এবং $a_{11} \neq 0$.
* যেই procedure এর মাধ্যমে scan করা হয় এবং সবচেয়ে highest value কে a_{11} এ বসানো হয় → Pivatal Condensation
* Check করা না হলে P.C. hang করতে পারে।

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This is the most commonly used method for solving simultaneous linear equation. In this method square co-efficient matrix is first reduced to an upper triangular matrix and then the unknowns are evaluated by back substitution starting from the reduced last eqn.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2$$
$$\vdots \qquad \cdots\cdots (1)$$
$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n = b_n$$

Multiplying the 1st eqn by $\left(\frac{a_{i1}}{a_{11}}\right)$ and subtracting from ith eqn $(i = 2, 3, \cdots n)$ $\frac{a_{21}}{a_{11}} \Big| \frac{a_{31}}{a_{11}} \Big| \frac{a_{n1}}{a_{11}}$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{22}x_2 + \cdots + a_{2n}'x_n = b_2' \qquad \cdots\cdots (2)$$
$$\vdots$$
$$a_{n2}x_2 + \cdots + a_{nn}'x_n = b_n'$$

where,
$$a_{ij}' = a_{ij} - \left(\frac{a_{i1}}{a_{11}}\right) a_{1j}$$
$$b_i' = b_i - \left(\frac{a_{i1}}{a_{11}}\right) b_1$$
$$\forall\; i, j = 2, 3, 4, \cdots n$$

Now multiplying the 2nd eq^n of (2) by $\left(\frac{a_{i2}'}{a_{22}'}\right)$ and subtracting from ith eq^n. $(i = 3, 4, \cdots n)$.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + \cdots a_{1n}x_n = b_1$$
$$a_{22}'x_2 + a_{23}'x_3 + a_{24}'x_4 + \cdots + a_{2n}'x_n = b_2'$$
$$a_{33}^2 x_3 + a_{34}^2 x_4 + \cdots + a_{3n}^2 x_n = b_3^2$$
$$\vdots$$
$$a_{n3}^2 x_3 + a_{n4}^2 x_4 + \cdots + a_{nn}^2 x_n = b_n^2$$

where,
$$\boxed{\begin{aligned} a_{ij}^2 &= a_{ij}' - \left(\frac{a_{i2}'}{a_{22}'}\right) a_{2j}' \\ b_i^2 &= b_i' - \left(\frac{a_{i2}'}{a_{22}'}\right) b_2' \\ i,j &= 3, 4, 5, \ldots n \end{aligned}}$$

After $(n-1)$ steps,

$$a_{11}x_1 + a_{12}x_2 + \cdots \quad + a_{1n}x_n = b_1$$
$$a_{22}'x_2 + \cdots \quad + a_{2n}'x_n = b_2'$$
$$\cdots \quad \cdots$$
$$\vdots$$
$$+ a_{nn}^{n-1} x_n = b_n^{n-1} \Longrightarrow$$

where,
$$\boxed{\begin{aligned} a_{ij}^K &= a_{ij}^{K-1} - \left(\frac{a_{iK}^{K-1}}{a_{KK}^{K-1}}\right) a_{Kj}^{K-1} \\ b_i^K &= b_i^{K-1} - \left(\frac{a_{ik}^{K-1}}{a_{KK}^{K-1}}\right) b_k^{K-1} \\ K &= 1, 2, 3, \ldots (n-1) \\ i,j &= (K+1), (K+2), \ldots n \end{aligned}}$$

K = for steps

upper triangular matrix

$$\Longrightarrow x_n = b_n^{\,n-1} / a_{nn}^{\,n-1}$$

$$x_i = \frac{1}{a_{ii}^{(i-1)}} \left[b_i^{(i-1)} - \sum_{j=i+1}^{n} a_{ij}^{(i-1)} x_j \right]$$ Back Substitution

$i = n, n-1, n-2, \ldots\ldots$

→ এই ৩টি eqn use করে computer এর মাধ্যমে $n \times n$ matrix এর soln পাওয়া যায়।

Ansari Sir [5th class এ C.T]

Problem

Solve the following problem using Gauss-Elimination method:

$$2x_1 + 3x_2 - x_3 = 5$$
$$4x_1 + 4x_2 - 3x_3 = 3$$
$$3x_1 - 4.5x_2 + 1.5x_3 = -1.5$$

Pivotal Condensation:

$$4x_1 + 4x_2 - 3x_3 = 3$$
$$2x_1 + 3x_2 - x_3 = 5$$
$$3x_1 - 4.5x_2 + 1.5x_3 = -1.5$$

$$\begin{bmatrix} 4 & 4 & -3 \\ 2 & 3 & -1 \\ 3 & -4.5 & 1.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ -1.5 \end{bmatrix}$$

$2 - \frac{1}{2} 4 \qquad 3 - \frac{3}{4} 4$

Factor $= \frac{1}{2}$

$R_2' = R_2 - \frac{1}{2} R_1$

Factor =

$R_3' = R_3 - \frac{3}{4} R_1$

$-7.5 - \frac{-7.5}{4} 4$

$R_2' = R_2 - \quad R_1$

$R_3'' = R_3' - \left(\frac{1}{7.5}\right) R_2'$

$$\begin{bmatrix} \cancel{4} & \cancel{4} & \cancel{-3} \\ 0 & 1 & 0.5 \\ 0 & & \end{bmatrix}$$

$4x_1 + 4x_2 - 3x_3 = 3$

$0 + x_2 + 0.5x_3 = 3.5$

$-7.5x_2 + 3.75x_3 = -3.75$

Pivotal condensation:

$4x_1 + 4x_2 - 3x_3 = 3$

$-7.5x_2 + 3.75x_3 = -3.75$

$x_2 + 0.5x_3 = 3.5$

$4x_1 + 4x_2 - 3x_3 = 3$

$-7.5x_2 + 3.75x_3 = -3.75$

$\boxed{x_3 = 3}$

$\boxed{x_2 = 2}$

$\boxed{x_1 = 1}$

* Gauss - Jordan Elimination Method:

$a_{11} x_1 + \underset{}{\overset{0}{a_{12}}} x_2 + \overset{0}{a_{13}} x_3 = b_1$

$\underset{}{0}\, a_{21} x_1 + a_{22} x_2 + a_{23} x_3 = b_2$

$\underset{0}{a_{31}} x_1 + \underset{=0}{a_{32}} x_2 + a_{33} x_3 = b_3$

$=1$

Direct solution পাওয়া যায়।

$[A]\{x\} = [B]$

$\{x\} = [A]^{-1} [B]$

Inverse of a matrix :

$$\left(\begin{array}{cccccc} a_{11} & a_{12} & a_{13} & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 1 \end{array}\right)$$

$$\Downarrow \text{ Gauss Jordan}$$

$$\left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \; \underbrace{\begin{array}{ccc} c_1 & c_2 & c_3 \\ c_4 & c_5 & c_6 \\ c_7 & c_8 & c_9 \end{array}}_{A^{-1}}\right)$$

Gauss-Jordan Method :

$$\begin{bmatrix} 4 & 4 & -3 & 1 & 0 & 0 \\ 2 & 3 & -1 & 0 & 1 & 0 \\ 3 & -4.5 & 1.5 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1' = R_1\left(\tfrac{1}{4}\right) \begin{bmatrix} 1 & 1 & -\frac{3}{4} & \frac{1}{4} & 0 & 0 \\ 2 & 3 & -1 & 0 & 1 & 0 \\ 3 & -4.5 & 1.5 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} R_2' = R_{21}(-2) \\ R_3' = R_{31}(-3) \end{array} \begin{bmatrix} 1 & 1 & -\frac{3}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 1 & 0 \\ 0 & -7.5 & 3.75 & -\frac{3}{4} & 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} R_1' = R_1 - R_2 \\ R_3' = R_{32}(7.5) \end{array} \begin{bmatrix} 1 & 0 & -\frac{5}{4} & 0.75 & -1 & 0 \\ 0 & 1 & 0.5 & -0.5 & 1 & 0 \\ 0 & 0 & 7.5 & -4.5 & 7.5 & 1 \end{bmatrix}$$

$$R_3' = R_3\left(\frac{1}{7.5}\right) \quad \begin{bmatrix} 1 & 0 & -1.25 & 0.75 & -1 & 0 \\ 0 & 1 & 0.5 & -0.5 & 1 & 0 \\ 0 & 0 & 1 & -0.6 & 1 & 0.1333 \end{bmatrix}$$

$$R_2' = R_{23}\left(-\tfrac{1}{2}\right)$$

$$R_1' = R_{13}(1.25) \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0.25 & 0.1666 \\ 0 & 1 & 0 & -0.2 & 0.5 & -0.0666 \\ 0 & 0 & 1 & -0.6 & 1 & 0.1333 \end{bmatrix}$$

$$[A]^{-1} = \begin{bmatrix} 0 & 0.25 & 0.1666 \\ -0.2 & 0.5 & -0.0666 \\ -0.6 & 1 & 0.1333 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 3 \\ 5 \\ -1.5 \end{bmatrix}$$

$$\therefore [A]^{-1}[B] = \begin{bmatrix} 0\times 3 + 0.25\times 5 + (0.1666)(-1.5) \\ -0.2\times 3 + 0.5\times 5 + 0.0666\times 1.5 \\ -0.6\times 3 + 1\times 5 - 1.5\times 0.1333 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore x_1 = 1,\ x_2 = 2,\ x_3 = 3$$

Tanvir Sir

Roots of Equation

*Find the roots of $xe^x = 1$

▫ Bracketing method:

1) Bisection / half interval search
2) Method of false position

1) 1st guess, $x_r = \frac{x_l + x_u}{2}$

(1) Find the roots of $xe^x = 1$ using $x_l = 0$. $x_u = 1$.
using bisection method:

$f(x) = xe^x - 1$

$f(0) = -1$ $f(1) = 1.718$

$x_r = \frac{0+1}{2} = 0.5$ 1st guess

ε_a = approximate error

$= \left| \frac{x_{r\,new} - x_{r\,old}}{x_{r\,new}} \right| \times 100\%$

Q. which one is better?

Regular falsi method is better. because we can approach to the same value performing less number of iterations. ***

1st guess: $x_r = \frac{0+1}{2} = 0.5$ $f(0) = -1$, $f(1) = 1.718$

Iter	x_l	x_u	x_r	$f(x_r)$	$\varepsilon_a =$
1	0	1	0.5	-ve	—
2	0.5	1	$\frac{0.5+1}{2} = 0.75$	+ve	$\frac{0.75-0.5}{0.75} \times 100\%$ $= 33\%$
3	0.5	0.75	0.625	-ve	20%
⋮	⋮	⋮	⋮	⋮	11.11%
10	0.5664	0.5684	0.5674	⋮	

$\Delta x = 0.002$, $\frac{\Delta x}{2} = 0.001$ Ans → not exact soln

* যখন x_l বা x_u change করাতে x_r এর মান খুব বেশি change হচ্ছে না, তখন x_r এর সেই মানটিই root ধরে নিব।

Bracketing Method: Bisection:

$$n = \log_2 \left(\frac{\Delta x^0}{E_{a,d}} \right)$$

Δx^0 → initial guess range $= 1 - 0 = 1$

n → No. of iteration

$E_{a,d}$ → desired level of error bound $= \pm \frac{\Delta x}{2}$ at the n-th iteration

$$= \frac{\log (\Delta x^0 / E_{a,d})}{\log 2}$$

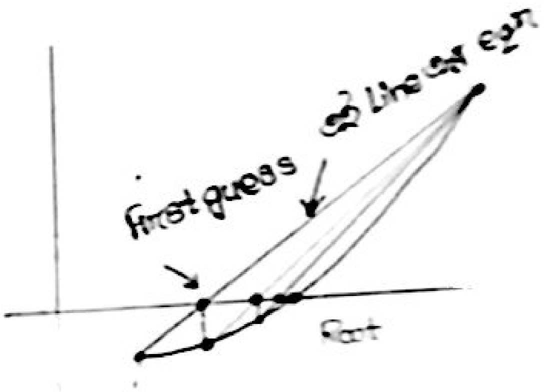
$$= \frac{\log (1/10^{-3})}{\log 2}$$

$= 9.96$

≈ 10

Regular Falsi:

$$\boxed{\frac{x - x_l}{x_l - x_u} = \frac{y - f(x_l)}{f(x_l) - f(x_u)}}$$



For soln, $\dfrac{x_r - x_l}{x_l - x_u} = \dfrac{0 - f(x_l)}{f(x_l) - f(x_u)}$

*Prob: Determine the real roots of $f(x) = -25 + 82x - 90x^2 + 44x^3 - 8x^4 + 0.7x^5$

use $x_u = 1$, $x_l = 0.5$; Perform 4 iterations with Regular Falsi method.

$x_r = x_u - \dfrac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$

Iter	x_l	x_u	$f(x_l)$	$f(x_u)$	x_r	$f(x_r)$	ε_a
1	0.5	1	-1.478	3.70	0.642	0.918	—
2	0.5	0.642	-1.478	0.918	0.588	0.137	9.3%
3	0.5	0.588	-1.478	0.137	0.58	0.018	1.39%
4	0.5	0.58	-1.478	0.018	0.579		0.162

$\boxed{\text{ans}}$

After 4 iteration the ans is 0.579.

* * *

Regular f	Bisection	
9·3%	20%	
1·29%	11%	Ea
0·169%	5·26%	

converges quickly

So, Regular falsi method is better as it converges quickly.

* But, for higher power of x, (function having high power of variables), the shape of the curve → suddenly change হয়ে যায় (suddenly curve turn হয়) এই ধরনের curve এর ক্ষেত্রে, Regular falsi method এ অনেকগুলো iteration করতে হয়, root এর কাছাকাছি পৌঁছাতে। সেই ক্ষেত্রে, bisection method অনেক তাড়াতাড়ি root এ পৌঁছায়।

তাই কোন method এ iteration কম বা বেশি লাগে সেটা নির্ভর করে function এর nature এর উপর।

* Bracketing method use ২ assumption. root lies between the assumption.

* But in open method, there is only 1 assumption. the root may lie either left or right side of the assumption.

Ansari Sir

Crout's Method:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} a_{11}' & a_{12}' & a_{13}' & b_1' \\ a_{21}' & a_{22}' & a_{23}' & b_2' \\ a_{31}' & a_{32}' & a_{33}' & b_3' \end{pmatrix}$$

Soln

$x_3 = b_3'$

$x_2 = b_2' - a_{23}' x_3$

$x_1 = b_1' - a_{13}' x_3 - a_{12}' x_2$

Steps

1) $a_{11}' = a_{11}$
   $a_{21}' = a_{21}$
   $a_{31}' = a_{31}$

2) $a_{12}' = \dfrac{a_{12}}{a_{11}'}$

   $a_{13}' = \dfrac{a_{13}}{a_{11}'}$

   $b_1' = \dfrac{b_1}{a_{11}'}$

3) $a_{22}' = a_{22} - a_{12}' \times a_{21}'$

   $a_{32}' = a_{32} - a_{12}' \times a_{31}'$

4) $a_{23}' = \dfrac{a_{23} - a_{13}' \times a_{21}'}{a_{22}'}$

   $b_2' = \dfrac{b_2 - b_1' \times a_{21}'}{a_{22}'}$

5) $a_{33}' = a_{33} - a_{13}' \times a_{31}' - a_{23}' \times a_{32}'$

6) $b_3' = \dfrac{b_3 - b_1' \times a_{31}' - b_2' \times a_{32}'}{a_{33}'}$

WCT

Method of Interpolation:

*numerical এ 4 decimal পর্যন্ত value নিতে হয় → by default

1. Graphical interpolation
2) Linear "
3) Polynomial "
4) Gregory-Newton "
5) Finite Difference

যখন x এর value গুলো equally spaced থাকে।

6) Lagrangian Interpolation } যখন equally spaced থাকে না।

Mathematical functions are often described in tabular form. That is, for prescribed values of $x_1, x_2, \cdots x_n$ of the independent variable x, corresponding values of $f(x_1), f(x_2) \cdots f(x_n)$ are provided. The logarithmic and trigonometric functions are examples of functions which are provided in tabular form. The process of passing curve through the given points in order to estimate functional values of $f(x)$ for values of 'x' not explicitly shown in the table is called interpolation.

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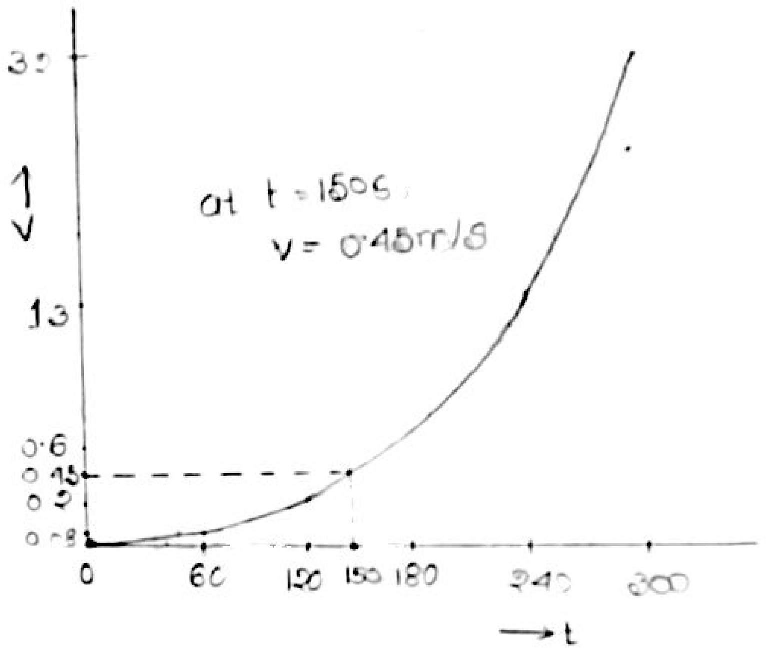
+ 6 টা data দিয়ে pass করালে 5th order polynomial হবে।

Problem

at t = 150s, speed = ?

time (s)	speed (m/s)
0	0
60	0.0824
120	0.2747
180	0.6502
240	1.3851
300	3.2229



Assuming linear:

$$y = a + bx$$

$(120,\ 0.2747)$

$(180,\ 0.6502)$

$\cancel{120 = a + bx}$

$b\,120 + a = 0.2747$

$180b + a = 0.6502$

$\therefore a = -0.4763$

$b = 6.26 \times 10^{-3}$

$\therefore y = -0.4763 + 6.26 \times 10^{-3} x$

$$\boxed{\text{at } x = 150,\ y = 0.4627 \text{ m/s}}$$

2nd order polynomial (parabola):

$[y = a + bx + cx^2]$

$0.0824 = a + b \cdot 60 + c \cdot (60)^2$

$0.2747 = a + b \cdot 120 + c \cdot (120)^2$

$0.6502 = a + b \cdot 180 + c\,(180)^2$

$\cancel{1.3851 = a + b \cdot 240 + c(240)^2}$

$$\Rightarrow \begin{pmatrix} 1 & 60 & 60^2 \\ 1 & 120 & 120^2 \\ 1 & 180 & 180^2 \end{pmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{bmatrix} 0.0824 \\ 0.2747 \\ 0.6502 \end{bmatrix}$$

⇒ factor form এ লিখলে, [এভাবে লিখলে matrix লাগে না, easy হয় calculation]

$$y = a + b(x - x_1) + c(x - x_1)(x - x_2)$$

$\therefore$ ~~0.0824 = a + b(x - 60) + c~~

$$y = a + b(x - 120) + c(x - 120)(x - 180)$$ [আমাদের nearest value 150 এর 120 ও 180, তাই $\underset{x_1}{120}$ $\underset{x_2}{180}$]

এখন, $x = 60$,

$$y = 0.0824 = a + (60 - 120).b + c(60 - 120).(60 - 180)$$

$x = 120$, $y = 0.2747 = a + 0 + 0$

$\therefore a = 0.2747$

$x = 180$, $y = 0.6502 = a + b(180 - 120)$

$\therefore b = \cancel{6.31 \times 10^{-3}}\ 6.2583 \times 10^{-3}$

So, $c = \cancel{2.58 \times 10^{-5}}\ 2.5444 \times 10^{-5}$

$\therefore y = 0.2747 + \overset{6.2583 \times 10^{-3}}{\cancel{6.31 \times 10^{-3}}}(x - 120) + \overset{2.5444 \times 10^{-5}}{\cancel{2.58 \times 10^{-5}}}(x - 120)(x - 180)$

at $x = 150$, $y =$ ~~0.441 m/s~~ ~~0.4407 m/s~~ 0.4395 m/s

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Tanvir Sir

open Method - (Method of iteration)

Find the roots of $f(x) = e^{-x} - x$ using method of iteration.

use an initial guess, $x_0 = 0$

$$x_{i+1} = e^{-x_i}$$

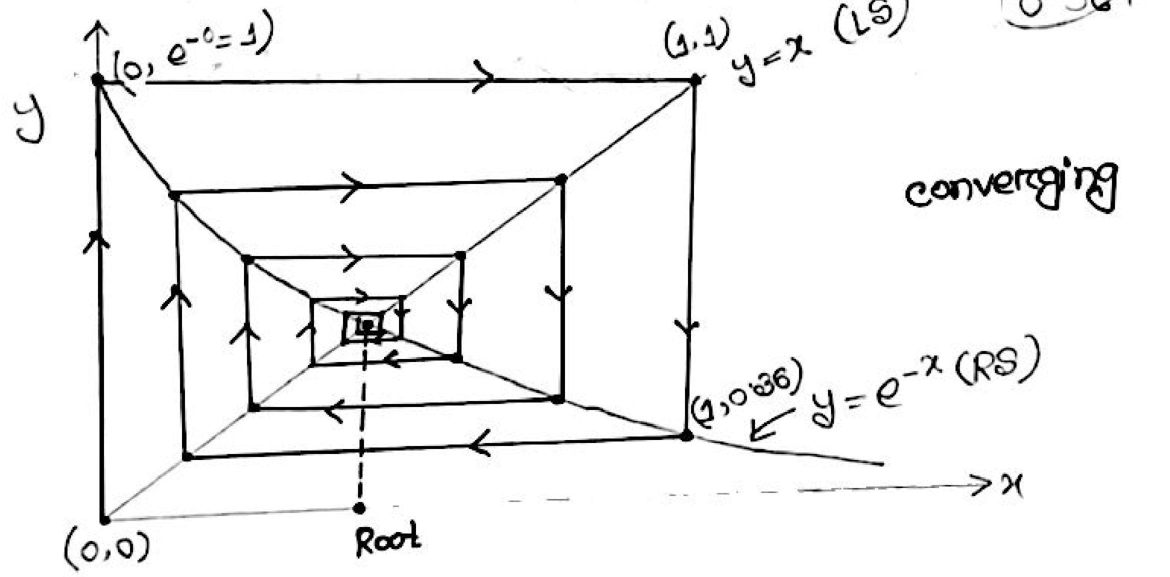
$$f(x) = e^{-x} - x = 0 \Rightarrow \underbrace{x}_{LS} = \underbrace{e^{-x}}_{RS}$$

[L.S সবসময় x হতে হবে]

Iter.	x_i	x_{i+1}	ε_{at}
0.	0	$e^{-0} = 1$	$\frac{1-0}{1} \times 100\% = 100\%$
1	1	$e^{-1} = 0.36$	76.3%
2	0.36	0.69	35%
3	0.69	0.5	22%
4	0.5	0.606	11.8%
5	0.606	0.545	6.89%

So, যখন L.S. ও R.S প্রায় (x_i ও x_{i+1}) সমান হয়ে যাবে। ε_a দেখলেও বুঝা যাবে। যখন $\varepsilon_a \approx 0.1$ or 0.2%, তখন বুঝব root এর কাছাকাছি চলে এসেছে।

0.567 -



* R.S → $g(x)$, $|g'(x)| < 1$ → convergence criteria

Find the square root of 5 using method of iteration.

$$x = \sqrt{5}$$
$$\Rightarrow x^2 = 5$$
$$\therefore f(x) = x^2 - 5 = 0$$

× Method-1 (Diverging)

$x = \frac{5}{x} \Rightarrow x^2 - 5 = 0$

$x_0 = 1$, $x_1 = \frac{5}{1} = 5$, $x_2 = \frac{5}{5} = 1$, $x_3 = 5$, $x_4 = 1$, $x_5 = 5$

× Method-2 (Div) $x = x^2 + x - 5$

$x_0 = 0$, $x_1 = -5$, $x_2 = 15$, $x_3 = 235$, $x_4 = 55455$

✓ Method-3 (conv) $x = \frac{5}{2x} + \frac{x}{2}$

$x = \frac{5 + x^2}{2x}$

$x_0 = 1$, $x_1 = 3$, $x_2 = 2.33$, $x_3 = 2.23$

আগে $|g'(x)|$ check করে নিব:

↓

x = initial guess

at least for 2 or 3 successive iteration $|g'(x)|$ should be < 1.

$2x^2 = 5 + x^2$

$x = \frac{5}{2x} + \frac{x}{2}$

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Newton-Rapshon Method:

* Find the roots of $f(x) = x^3 - 2x - 5$ using Newton Raphson method. use an initial guess, $x_0 = 2$

$$\boxed{x_{i+1} = x_i - \frac{x_i^3 - 2x_i - 5}{3x_i^2 - 2} \quad \frac{f(x_i)}{f'(x_i)}}$$

itr	x_i	x_{i+1}	ε_a
1	2	2.1	
2	2.1	2.09	
3	2.09	2.09	

* For $f(x) = x\sin x + \cos x = 0$

$$x_{i+1} = x_i - \frac{x_i \sin x_i + \cos x_i}{x_i \cos x_i} \quad \frac{f(x_i)}{f'(x_i)}$$

* $\cos(x)$ x = এর value radian এ হতে হবে।

inflection point:
The point at which a curve suddenly changes it's curvature.

Problems:

* inflection point ← এইসকল ক্ষেত্রে False method useful

*

Tanvir Sir

Q. Find the roots of $f(x) = e^{-x} - x$ using secant method. use initial guesses 0 and 1.

$f(x) = e^{-x} - x$

$$x_{i+1} = x_i \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)} \quad f'(x_i)$$

$\begin{cases} x_{-1} = 0 \\ x_0 = 1 \end{cases}$ initial guess

converging ↓

iter	x_{i-1}	x_i	$f(x_{i-1})$	$f(x_i)$	x_{i+1}
1	0	1	1	−0.632	0.6127
* 2	1*	0.6127	−0.632	−0.07	0.5638
* 3	*0.6127	0.5638	−0.07	0.005	0.567

[∵ 0 ও 1 এর মধ্যে 1 is closer to 0.6127, তাই 1 রাখা হয়েছে, 0 বাদ দেয়া হয়েছে]

[3rd iter এর ক্ষেত্রে, 1 ও 0.6127 এর মধ্যে 0.6127 is closer to 0.5638. তাই 1 বাদ দেয়া হয়েছে]

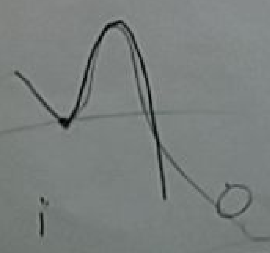
* open method performs generally better than bracketing method.

math করতে হবে না বুঝার জন্য

- Modified newton raphson method :
- Roots of polynomials :
- Muller method:
- convergence & divergence

Q. what is the basic principal of these. Explain with picture.

V.V. Imp → picture clearly আঁকতে হবে। picture এর ও উপর marking হবে।



0.6 1 $\frac{d}{dx}\left[\frac{f(x)}{f'(x)^2}\right]$

Errors in Numerical Method

Round off error :

Truncation error :

$f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$

Predict x_{i+1} using $x_i = 0$ and step size of $h = 1$

Taylor Series

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + f''(x_i)\frac{h^2}{2!} + f'''(x_i)\frac{h^3}{3!} + \cdots$$

$f(0) = 1.2$

$f(1) = ?$ Approximated

$f(1) = 0.2$ [True value . $f(x)$এ বসিয়ে পেয়েছি]

$$f(1) = f(0) + f'(0)\cdot 1 + f''(0)\frac{1^2}{2!} + f'''(0)\frac{1^3}{3!}$$

⇒ Zero-order approximation :

$f(x_{i+1}) \cong f(x_i)$

$f(1) \cong f(0) = 1.2$

Error $= 1.2 -$ true value

$= 1.2 - 0.2 = 1$

$f'(x) = -0.4x^3 - 0.45x^2 - x - 0.25$

$f'(0) = -0.25$

⇒ 1st order approx.

$f(1) = f(0) + f'(0)\cdot 1 = \overset{1.2 - 0.25}{\cdots} = 0.95$

Error $= 0.2 - 0.95 = -0.75$

$|error| = 0.75$

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⇒ 2nd order app:

$$f(1) = f(0) + f'(0)\cdot 1 + f''(0)\cdot \frac{1^2}{2!}$$

$$|error| = 0.25$$

* এই ক্ষেত্রে যত বেশি পদ নেয়া হচ্ছে, value ততই true value এর কাছাকাছি যাচ্ছে।

* কিন্তু f(x) যদি sin বা cos এর হয়, তবে যতই expand করি (Taylor series) খুব একটা সুবিধাজনক হবে না।

* $\overline{f(x_{i+1}) = f(x_i) + f'(x_i)}\,h + f''(x_i)\frac{h^2}{2!} + \cdots$

for 1st order approx. we use ⌐¬ this part. So, the remaining part becomes truncation error.

round off error = limitation of calculator (ignore করা যায় না)

truncation error = taylor series থেকে ইচ্ছাকৃত বাদ দেয়া part (ignore করা যায়)

Total error of numerical method

curve fitting → next class

Ansari Sir

Parabolla

$y = a_1 + a_2x + a_3x^2$

Modified · $y = a_1 + a_2(x - x_1) + a_3(x - x_1)(x - x_2)$

nth order polynomial, $y = a_1 + a_2x + a_3x^2 + \dots + a_{n+1}x^2$

$= a_1 + a_2(x - x_1) + a_3(x - x_1)(x - x_2) + \dots +$
$a_{n+1}(x - x_1)(x - x_n)$.

Gregory- Newton Interpolation:

x	$f(x)$
x_1	$f(x_1)$
x_2	$f(x_2)$
$\vdots$	
x_{n+1}	$f(x_{n+1})$

Data গুলোকে increasing order এ সাজাতে হবে।

* polynomial সবগুলো point দিয়ে pass করে।

① $x_{n+1} > x_n$

$f(x_1) = a_1$
$f(x_2) = a_1 + a_2(x_2 - x_1)$
$f(x_3) = a_1 + a_2(x_3 - x_1) + a_3(x_3 - x_1)(x_3 - x_2)$
$\vdots$

② $x_2 - x_1 = x_3 - x_2$
$= x_{n+1} - x_n$
$= h$

$f(x_1 + h) = a_1 + a_2(h)$
$f(x_1 + 2h) = a_1 + a_2(2h) + a_3(2h)(h)$
$f(x_1 + nh) =$

$x_2 = x_1 + h$
$x_3 = x_1 + 2h$

so, $x_{n+1} = x_1 + nh$

$$a_1 = f(x_1)$$

$$a_2 = \frac{f(x_1+h) - f(x_1)}{\lfloor 1\, h} = \frac{\Delta f(x_1)}{\lfloor 1\, h}$$

$$a_3 = \frac{\overset{2c_0}{f(x_1+2h)} - \overset{2c_1}{2f(x_1+h)} + \overset{2c_2}{f(x_1)}}{\lfloor 2\, h^2} = \frac{\Delta^2 f(x_1)}{\lfloor 2\, h^2}$$

Backward (∇)
Central (δ)
Forward (Δ)

$\vdots$

$$a_{n+1} = \frac{\Delta^n f(x_1)}{\lfloor n\, h^n}$$

$+ nc_0 - nc_1 + nc_2$ ——.... nc_n আকারে যাবে।

$$y = f(x_1) + \frac{\Delta f(x_1)}{\lfloor 1\, h}(x - x_1) + \frac{\Delta^2 f(x_1)}{\lfloor 2\, h^2}(x-x_1)(x-x_2) + \cdots$$
$$+ \frac{\Delta^n f(x_1)}{\lfloor n\, h^n}(x-x_1)(x-x_2)\cdots(x-x_n)$$

Let, $x - x_1 = uh$
$x - x_2 = (u-1)h$
$x - x_3 = (u-2)h$

modified form

$$y = f(x_1) + \frac{\Delta f(x_1)}{\lfloor 1\, h} uh + \frac{\Delta^2 f(x_1)}{\lfloor 2\, h^2}(uh)(u-1)h + \cdots$$

এটা valid for increasing order

$$= f(x_1) + \frac{\Delta f(x_1)}{\lfloor 1} u + \frac{\Delta^2 f(x_1)}{\lfloor 2} u(u-1) + \cdots\cdots$$

এ উপরোক্ত method টি Forward (Δ) system এ বলা হয়েছে, কারণ increasing order এ সাজানো হয়েছে।

Tanveer Sir

□ Truncation error:

Problem 2:

sin x =

Truncation error in numerical differenciation:

approximation - means it is almost equal to, but not the exact value.

Numerical Differentiation

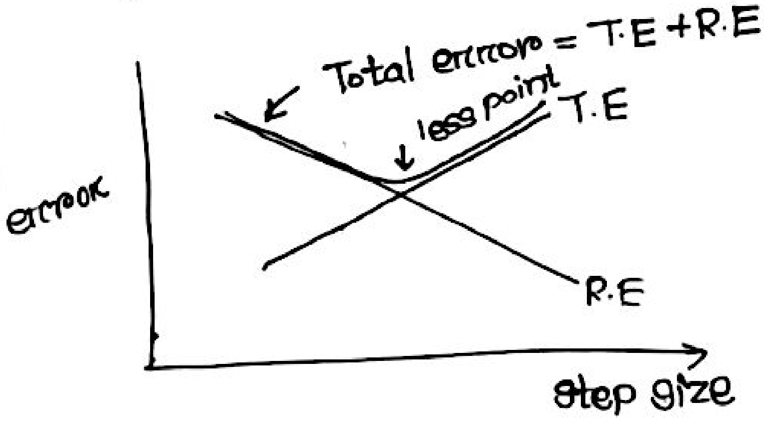
Centered → gives the least amount of truncation error. It is better than the other.

Problem

higher the step size → larger will be truncation error.

$h \uparrow \quad O(h^2) \uparrow\uparrow$, so T.E $\uparrow\uparrow$

2



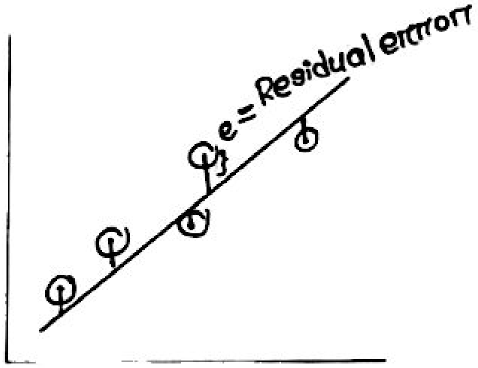
round off error,

$h \uparrow$ round off error $\downarrow$

truncation error $\uparrow$

Q. What is the relation between step size, truncation and round off error and why the curve is like that?

Curve Fitting



Data point দেয়া থাকবে।
straight line এর equation
লিখতে হবে।

Problem:

Fit a straight line to the following data:

x	y	xy	x^2
1	0.5	0.5	1
2	2.5	5	4
3	2	6	9
4	4	16	16
5	3.5		
6	6		
7	5.5		
$\Sigma x = 28$	$\Sigma y = 24$	$\Sigma xy = 119.5$	$\Sigma x^2 = 140$

$\bar{x} = \frac{28}{7} = 4 \quad \bar{y} = \frac{24}{7} = 3.428$

* cont.

$a_1 = \frac{7(119.5) - 28 \times 24}{7(140)^2 - (28)^2}$

$= 0.839$

$a_0 = 3.428 - 0.839 \times 4$

$= 0.0714$

$\boxed{y = 0.0714 + 0.839x}$

Least square Regression:

$$\left.\begin{aligned}\frac{\partial S_r}{\partial a_0} &= 0 \\ \frac{\partial S_r}{\partial a_1} &= 0\end{aligned}\right\} \text{minimizing}$$

$a_0, a_1 =$ unknown
$n =$ no. of data point

Improvement by linear regression:

$\bar{y} =$ always the worst possible straight line

$$r^2 = \frac{S_t - S_r}{S_t}$$

1)

2) $S_t = S_r$, so, $r^2 = \frac{0}{S_t} = 0$

$r^2 > 0.5$ is a good fit curve

r^2 close to zero $\Rightarrow$ ~~wor~~ mean value

r^2 close to 1, best fit curve

cont.

$(y_i - \bar{y})^2$	$(y_i - a_0 - a_1 x)^2$	
8.57	0.1687	
⋮	0.5625	
	⋮	
$\Sigma S_t = 22.71$	$S_r = 2.99$	

$$r^2 = \frac{S_t - S_r}{S_t} = \frac{22.71 - 2.99}{22.71} = 0.87$$

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Tanveer Sir

Q. Fit the model $y = ae^{bx}$ with the following data:

x	y	$\ln y = Y$	xY	x^2	$(Y-\bar{Y})^2$		$\{Y-(a_0+a_1X)\}^2$
2	4.078	1.405	2.81	4			
4	11.084	2.405	9.62	16		$X = b \cdot \ln x$	
6	30.128	3.405	20.43	36		$Y = \ln y$	
8	81.897	4.405	35.24	64		$XY -$	
10	222.62	5.405	50.05	100			
					$\Sigma = S_t$		$\Sigma = S_r$

$\therefore r^2 = \dfrac{S_t - S_r}{S_t}$

[converted variable এর জন্য r^2 বের করব]

$y = ae^{bx}$

$\Rightarrow \ln y = \ln a + bx \ln e$

$\Rightarrow \underbrace{\ln y}_{Y} = \underbrace{\ln a}_{a_0} + \underbrace{b}_{a_1}\underbrace{x}_{X}$

$Y = a_0 + a_1 X$

$\Sigma x = 30$

$\Sigma Y = 17.025$

$\Sigma x^2 = 220$

$\Sigma xY = 122.150$

$b = a_1 = \dots\dots = 0.5$ (applying formula)

$\ln a = a_0 = \dots\dots = 0.405$ (App. for)

$\therefore a = e^{0.405} = 1.499$

$\boxed{y = 1.499\, e^{0.5x}}$ (Ans)

$x, y \rightarrow$ original variable

$X, Y \rightarrow$ converted "

$a_1 = 0.6149$ $\quad r^2 = 0.9980$

$a_0 = -0.0683$

$\downarrow$

$$\underbrace{\frac{1}{y}}_{Y} = \underbrace{\frac{1}{\alpha_3}}_{a_0} + \underbrace{\frac{\beta_3}{\alpha_3}}_{a_1} \cdot \underbrace{\frac{1}{x}}_{X}$$

Q. $y = ax^b$

$$\underbrace{\log y}_{Y} = \underbrace{\log a}_{a_0} + \underbrace{b}_{a_1} \underbrace{\log x}_{X}$$

$$\boxed{Y = a_0 + a_1 X}$$

আগের Math এর Dataset use করলে, MATLAB

$a_1 = 2.4216 = b$

$a_0 = -0.2570 \quad \therefore a = 0.5533$

$\therefore y = 0.5533\, x^{2.4216} \quad r^2 = 0.9473$

Linearization:

Polynomial regression:

Q. $y_i = a_0 + a_1 x_i + a_2 x_i^2 + e$ $\qquad e =$ residual error

$$\therefore \underbrace{\sum e^2}_{} = \sum (y_i - a_0 - a_1 x_i - a_2 x_i^2)^2 = S_r$$

This has to be minimized.

$\frac{\partial S_r}{\partial a_0} = \quad \dots \quad = 0 \qquad a_0 = ?$

$\frac{\partial S_r}{\partial a_1} = \quad \dots \quad = 0 \qquad a_1 = ?$

$\frac{\partial S_r}{\partial a_2} = \quad \dots \quad = 0 \qquad a_2 = ?$

$$\begin{bmatrix} n & \sum x & \sum x^2 \\ \sum x & \sum x^2 & \sum x^3 \\ \sum x^2 & \sum x^3 & \sum x^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum xy \\ \sum x^2 y \end{bmatrix}$$

Q. Fit a parrabolla to the following data points:

x	y	x^2	x^3	x^4	xy	x^2y
0	2.1					
1	7.7					
2	13.6					
3	27.2					
4	40.9					
5	61.1					

$\sum x = 15$ $\quad\quad \sum y = 152.6$

$\sum x^2 = 55$ $\quad\quad \sum xy = 5856$

$\sum x^3 = 225$ $\quad\quad \sum x^4 = 979$

$n = 6$ $\quad\quad \sum x^2 y = 2488.8$

$$\begin{bmatrix} 6 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 152.6 \\ 585.6 \\ 2488.8 \end{bmatrix}$$

$a_0 = 2.47$

$a_1 = 2.35$

$a_2 = 1.86$

$\boxed{y = 2.47 + 2.35x + 1.86x^2}$ (Ans)

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Multiple Linear Regression:

$$y = a_0 + a_1x_1 + a_2x_2 \quad [\text{eq}^n \text{ of plane surface}]$$

$x_1, x_2 \rightarrow$ independent variable
$y \rightarrow$ dependent "

$$\frac{\partial S_r}{\partial a_0} = \cdots = 0$$

$$\frac{\partial S_r}{\partial a_1} = \cdots = 0$$

$$\frac{\partial S_r}{\partial a_2} = \cdots = 0$$

$$\begin{bmatrix} n & \Sigma x_1 & \Sigma x_2 \\ \Sigma x_1 & \Sigma x_1^2 & \Sigma x_1x_2 \\ \Sigma x_2 & \Sigma x_1x_2 & \Sigma x_2^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \Sigma y \\ \Sigma x_1y \\ \Sigma x_2y \end{bmatrix}$$

Q. Fit the following data to $y = a_0 + a_1x_1 + a_2x_2$

x_1	x_2	y
0	0	5
2	1	10
2.5	2	9
1	3	0
4	6	3
7	2	27

$$\begin{bmatrix} 6 & 16.5 & 14 \\ 16.5 & 76.25 & 48 \\ 14 & 48 & 54 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 54 \\ 243.5 \\ 100 \end{bmatrix}$$

solving $\Rightarrow$

$a_0 = 5, a_1 = 4, a_2 = -3$

$$\boxed{\therefore y = 5 + 4x_1 - 3x_2}$$

C.T → finding roots (next class)

Linear regression – Single axis

Multiple Linear " → Multiple axis

Q. $\underbrace{y}_{Y} = a_0 + a_1 \underbrace{\cos(\omega x)}_{X_1} + a_2 \underbrace{\sin(\omega x)}_{X_2}$ | x vs y data set

$$\boxed{Y = a_0 + a_1 X_1 + a_2 X_2}$$ solve using multiple linear regression

Q. solve, $\boxed{Z = a_0 \cdot x^{a_1} y^{a_2}}$

$$\underbrace{\log Z}_{Y} = \underbrace{\log a_0}_{A_0} + a_1 \underbrace{\log x}_{X_1} + a_2 \underbrace{\log y}_{X_2}$$

$$\boxed{Y = A_0 + a_1 X_1 + a_2 X_2}$$ solve using multiple linear regression

Tanvir Sir

Ordinary Differential Equation

ODE : 2 types : (Difference)

1) Initial Value Problem

only single point is given.

(I.V.P)

2) Boundary Value Problem

2 different values are given (boundary).

So, the solution should lie between this two given value.

(B.V.P)

* The soln methodology of I.V.P is different from B.V.P.
* Methods used for I.V.P can't be used in B.V.P.

Initial Value Problem :

Techniques :

- Euler
- Heun
- Midpoint
- Fourth - order Runge Kutta

1) Euler's Method:

Problem $\frac{dy}{dx} = -2x^3 + 12x^2 - 20x + 8.5$

Given, $y(0) = 1$

Solve from $x = 0$ to $x = 4.0$ using Euler's method with step size. $h = 0.5$.

Soln $y(0) = 1$, $y(0.5) = ?$, $y(1) = ?$, $y(1.5) = ?$, $y(2) = ?$ ..... $y(4.0) = ?$

$$\boxed{y_{i+1} = y_i + \text{slope} \times h}$$

$$y(0.5) = y(0) + \left.\frac{dy}{dx}\right|_{x=0} \times 0.5$$

$$= 1 + 8.5 \times 0.5$$

$$= 5.25$$

$$y(1) = y(0.5) + \left.\frac{dy}{dx}\right|_{x=1} \times 0.5$$

$$= 5.25 + \left(-\frac{3}{2}\right) \times 0.5$$

$$= 5.875$$

$y(1.5) = \cdots$

Analytical soln:

$$\frac{dy}{dx} = -2x^3 + 12x^2 - 20x + 8.5$$

$$\therefore y = -2\frac{x^4}{4} + 12\frac{x^3}{3} - 20\frac{x^2}{2} + 8.5x + C$$

at $x = 0$, $y = 1$, so, $C = 1$

$$\boxed{\therefore y = -\frac{x^4}{2} + 4x^3 - 10x^2 + 8.5x + 1}$$ Analytical soln

x	y_{Euler}	y_{True}	E_t	L.TE %
0	1	1	—	—
0.5	5.25	3.21	−63.1%	−63.1%
1	5.87	3	−95.8%	−28%
1.5	5.12	2.21	131%	−1.41%
2	4.5	2	−125%	20.5%
2.5	4.75	2.71	−74.7%	17.3%
3	5.87	4	46.9%	4 %
3.5	7.12	4.71	−51%	−11.3%
4	7 sum	3	−133.3%	−53 %

$$\text{True Error} = E_t = \frac{y_{True} - y_{Euler}}{y_{True}} \times 100\%$$

[y_{Euler} ও y_{True} এর মধ্যে Huge error আসার কারণ হল Local Truncation Error.]

Error
- Truncation error
  1) Local
  2) Propagated
- Round off

* Local Truncation error Taylor Series থেকে বের করা যায়।

$$y_{i+1} = y_i + y_i' h + y_i'' \frac{h^2}{2!} + \cdots \quad \text{(Taylor Series)}$$

$$\Rightarrow \underbrace{\boxed{y_{i+1} = y_i + \frac{dy}{dx} h}}_{\text{Euler's Method}} + \underbrace{y_i'' \frac{h^2}{2!} + \cdots\cdots}_{\text{[error]}}$$

[error] because it was ignored. And this error is being accumulated in each step.

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* How to estimate Local Truncation Error :

$y_i'' = -6x^2 + 24x - 20$

$y_i''' = -12x + 24$

$y_i^{IV} = -12$

$y_i^{V} = 0$

$y_i^{VI} = 0$

....

$\therefore$ Local Truncation Error $= y_i'' \frac{h^2}{2!} + y_i''' \frac{h^3}{3!} + y_i^{IV} \frac{h^4}{4!}$

$= -2.5 + 0.5 - 0.03125$

$= -2.03$

$\therefore$ L.T.E % $= \frac{-2.03}{3.21} \times 100\% = -63.1\%$

$\downarrow$

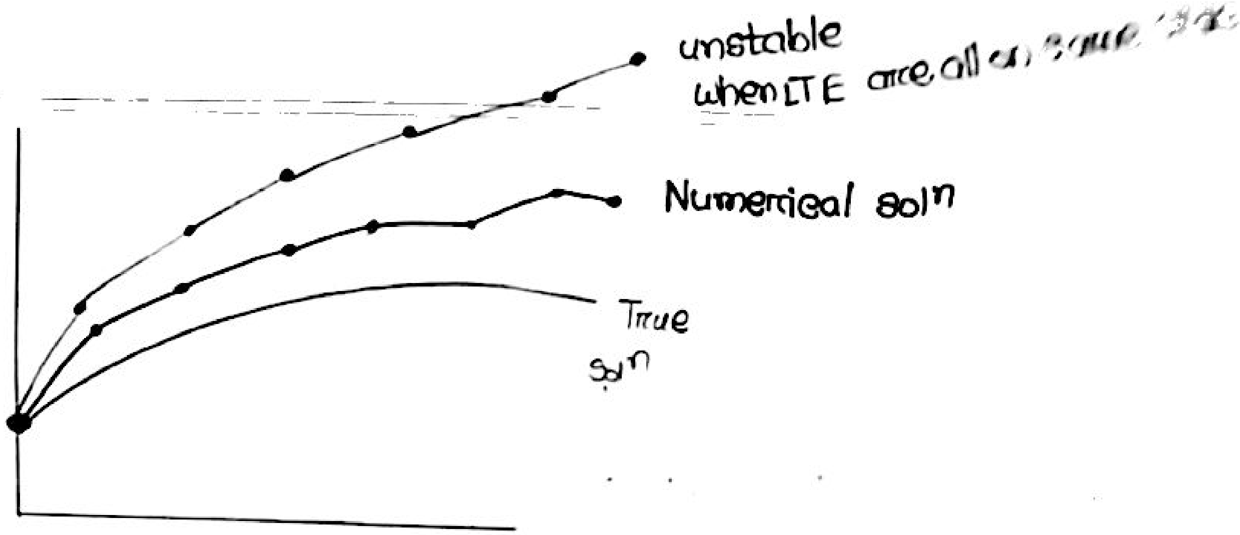
at $x = 0$

□ The effect of Local truncation Error on global error :

1) Change in sign of LTE reduces global error.
(compensation effect)

2) If LTE all are the same sign numerical solution diverges further → becomes unstable

* অনেক DE এর True soln পাওয়া যায় না। তখন y_{True} ও E_t পাওয়া যায় না।



* LTE indicates the nature of global error. and we should try to minimize LTE all time.

To minimize LTE :

1) Euler method না use করে অন্য method use করব।
২) Euler method apply করা হলে h কমাতে হবে। h ↓; LTE ↓↓

Q What are the drawbacks of Euler's Method ?

→ It provides a huge no. of Truncation error.

→ And to minimize LTE, h should be reduced, so no. of calculation would be increased

*** LTE% এমন হতে হবে যেন sign changing থাকে ওদের মধ্যে।

Euler's method এ → error is very large,
reducing step size we can increase accuracy

Tanvir Sir

Ordinary Differential Equation

□ Solve $\frac{dy}{dx} = \frac{2y}{x}$ with $y(1)=2$, from $x=1$ to 2, use $h=0.25$

Apply Heun's Method: (average slope)

$y(1.25) = ?$ 3.1
$y(1.5) = ?$ 4.44
$y(1.75) = ?$ 6.03
$y(2) = ?$ 7.86
Ans

$$y' = \frac{2y}{x}$$

$$m(1) = \frac{2y}{x}\Big|_{\substack{x=1\\y=2}} = \frac{2\times 2}{1} = 4$$

Euler $y_e(1.25) = y(1.0) + 4\times 0.25 = 3$

$$m(2) = \frac{2y}{x}\Big|_{\substack{x=1.25\\y_e=3}}$$

$$= \frac{2\times 3}{1.25} = 4.8$$

$$m = \frac{m_1+m_2}{2} = \frac{4+4.8}{2} = 4.4$$

$$y(1.25) = y(1.0) + 4.4\times \underbrace{0.25}_{h} = 3.1$$

y_e = euler's estimated y

y = real y

*যখন $y(1.5)$ = ধু বের করতে যাব তখন $y(1.25)=3.1$ use করব।

*যখন $y(1.75)$ " " " $y(1.5)=4.44$ use করব।

x	y_{Euler}	y_{Heun}	y_{True}
1.0	2.00	2.00	2.00
1.25	3	3.1	3.125
1.5	4.2	4.44	4.5
1.75	5.6	6.03	6.125
2	7.2	7.86	8

So, Heun's method performs better.

□ Midpoint method: (slope at the mid point) (better than Heun's method)

$y' = \frac{2y}{x}$ = slope

Slope at midpoint, $y' = y'(1.125) = y'\left(1 + \frac{0.25}{2}\right)$

$y_e(1.125) = y(1.0) + \left.\frac{dy}{dx}\right|_{\substack{x=1 \\ y=2}} \times \underbrace{0.125}_{\frac{h}{2}} = 2.5$

$y'(1.125) = \left.\frac{2y}{x}\right|_{\substack{x=1.125 \\ y=2.5}}$

$= \frac{2 \times 2.5}{1.125} = 4.44$

$\therefore y(1.25) = y(1.0) + 4.44 \times 0.25 = 3.11$

using this

Similarly, $y(1.5) = 4.47$

$y(1.75) =$

$y(2.0) =$

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□ 4th order Runge-Kutta Method: (Best method for solving ODE)

<u>Prob</u> $\frac{dy}{dx} = x^2+y^2$, $y(0)=0$

Estimate $y(0.4)$ with $h=0.2$. Use 4th order R-K method.

$$\begin{cases} K_1 = f(t_i , y_i) \\ K_2 = f(t_i + \frac{1}{2}h , y_i + \frac{1}{2}K_1 h) \\ K_3 = f(t_i + \frac{1}{2}h , y_i + \frac{1}{2}K_2 h) \\ K_4 = f(t_i + h , y_i + K_3 h) \\ y_{i+1} = y_i + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)h \end{cases}$$

$f(x,y) = x^2+y^2$

$y(0.2) = ?$, $y(0.4) = ?$

$K_1 = f(0,0) = 0^2+0^2 = 0$

$K_2 = f\left(0+\frac{0.2}{2} , 0+\frac{1}{2}\times 0\times 0.2\right)$

$= f(0.1,0)$

$= 0.1^2+0^2$

$= 0.01$

$K_3 = f\left(0+\frac{0.2}{2} , 0+\frac{1}{2}\times 0.01\times 0.2\right)$

$= f(0.1 , 0.001)$

$= (0.1)^2+(0.001)^2$

$= 0.01$

$K_4 = f(0+0.2 , 0+0.01\times 0.2)$

$= f(0.2 , 2\times 10^{-3})$

$= (0.2)^2+(2\times 10^{-3})^2$

$= 0.04$

$\therefore y(0.2) = y(0) + \frac{1}{6}(0 + 2\times 0.01 + 2\times 0.01 + 0.04)\times 0.2$

$\boxed{y(0.2) = 0.002667}$ (Ans)

similarly,

$\boxed{y(0.4) = 0.02136}$ (Ans)

এটার জন্য $y(0.2) = 0.002667$ use করব।

Ansari Sir

□ Gregory- Newton Interpolation: (Derivation)

$$\rightarrow P(x) = f(x_1) + \frac{\Delta f(x_1)}{\lfloor 1\, h}(x-x_1) + \frac{\Delta^2 f(x_1)}{\lfloor 2\, h^2}(x-x_1)(x-x_2) + \dots + \frac{\Delta^n f(x_1)}{\lfloor n\, h^n}(x-x_1)\dots(x-x_n)$$

Let, $x - x_1 = hu$

$\therefore u = \frac{x-x_1}{h}$

$$\rightarrow P(u) = f(x_1) + \frac{\Delta f(x_1)}{\lfloor 1}u + \frac{\Delta^2 f(x_1)}{\lfloor 2}u(u-1) + \dots$$

এই দুটোর যে কোন একটা form use করে math করা যাবে।

Math

To estimate speed at $t = 150s$.

	t(s)		Speed (m/s)
x_1	0	$f(x_1)$	0
x_1+h	60	$f(x_1+h)$	0.0824
x_1+2h	120	$f(x_1+2h)$	0.2747
x_1+3h	180	$f(x_1+3h)$	0.6502
x_1+4h	240	$f(x_1+4h)$	1.3851
x_1+5h	300	$f(x_1+5h)$	3.2229

Data assending order এ সাজাতে হবে।

Graphical — 0.45 m/s

Straight line → 0.4624 m/s

Parabolla → 0.4395 m/s

5th order polinomial →

$$\therefore p(u) = f(x_1) + \frac{\Delta f(x_1)}{\lfloor 1}u + \frac{\Delta^2 f(x_1)}{\lfloor 2}u(u-1) + \frac{\Delta^3 f(x_1)}{\lfloor 3}u(u-1)(u-2)$$
$$+ \frac{\Delta^4 f(x_1)}{\lfloor 4}u(u-1)(u-2)(u-3) + \frac{\Delta^5 f(x_1)}{\lfloor 5}u(u-1)(u-2)(u-3)(u-4)$$

$$\Delta f(x_1) = f(x_1+h) - f(x_1)$$
$$\Delta^2 f(x_1) = {}^2C_0\, f(x_1+2h) - {}^2C_1\, f(x_1+h) + {}^2C_2\, f(x_1)$$
$$\Delta^3 f(x_1) = {}^3C_0\, f(x_1+3h) - {}^3C_1\, f(x_1+2h) + {}^3C_2\, f(x_1+h) - {}^3C_3\, f(x_1)$$
$$\Delta^4 f(x_1) = {}^4C_0\, f(x_1+4h) - {}^4C_1\, f(x_1+3h) + {}^4C_2\, f(x_1+2h) - {}^4C_3\, f(x_1+h) + {}^4C_4\, f(x_1)$$
$$\Delta^5 f(x_1) = {}^5C_0\, f(x_1+5h) - {}^5C_1\, f(x_1+4h) + {}^5C_2\, f(x_1+3h) - {}^5C_3\, f(x_1+2h) + {}^5C_4\, f(x_1+h) - {}^5C_5\, f(x_1)$$

$\therefore \Delta f(x_1) = 0.0824$

$\Delta^2 f(x_1) = 0.1099$

$\Delta^3 f(x_1) = 0.0733$

$\Delta^4 f(x_1) = 0.1029$

$\Delta^5 f(x_1) = 0.4635$

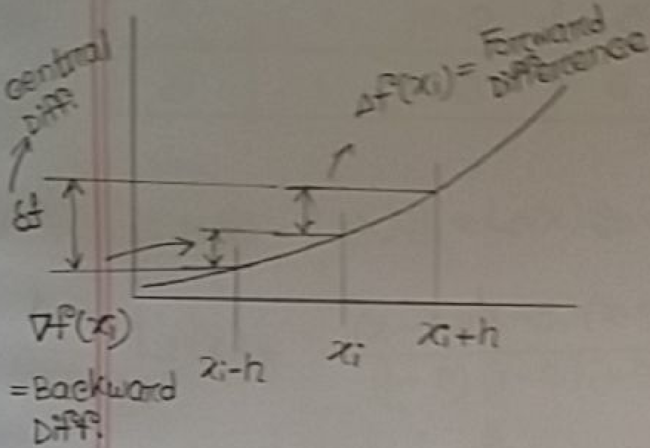
$$u = \frac{-0+150}{60} = 2.5$$

$h = \text{step size} = 120-60 = 60$
$= 180-120 = 60.$

$$\therefore p(u) = 0 + \frac{0.0824}{\lfloor 1} \times 2.5 + \frac{0.1099}{\lfloor 2}\, 2.5 \times (2.5-1) + \frac{0.0733}{\lfloor 3} \times 2.5$$
$$\times 1.5 \times 0.5 + \frac{0.1029}{\lfloor 4} \times 2.5 \times 1.5 \times 0.5 \times (2.5-3)$$
$$+ \frac{0.4635}{\lfloor 5} \times 2.5 \times 1.5 \times 0.5 \times (-0.5) \times (-1.5)$$
$$= 0.43638$$

Difference → Difference of ordinate

Finite Difference:



A finite difference (or simply difference) of the function $f(x)$ is the value of the function at one point x_1 minus value at a 2nd point x_2.

Algebraically, $f(x_1) - f(x_2)$.

Difference Table

x	$f(x)$	D1 $\Delta' f(x_i)$	D2 $\Delta^2 f(x_i)$
x_i	$f(x_i)$	$f(x_i+h) - f(x_i)$	$f(x_i+2h) - 2f(x_i+h) + f(x_i)$
x_i+h	$f(x_i+h)$	$f(x_i+2h) - f(x_i+h)$	$f(x_i+3h) - 2f(x_i+2h) + f(x_i+h)$
x_i+2h	$f(x_i+2h)$	$f(x_i+3h) - f(x_i+2h)$	$f(x_i+4h) - 2f(x_i+3h) + f(x_i+2h)$
x_i+3h	$f(x_i+3h)$	$f(x_i+4h) - f(x_i+3h)$	
x_i+4h	$f(x_i+4h)$		

$\Delta^3 f(x_i)$

D3

$f(x_i+3h) - 3f(x_i+2h) + 3f(x_i+h) - f(x_i)$

$\Delta^4 f(x_i)$

D4

□ Unequal spacing এর জন্য: (Lagrange Interpolation)
also for equal spacing

$$P(x) = L_1(x)y_1 + L_2(x)y_2 + L_3(x)y_3 + \cdots\cdots$$

i	x_i	y_i	
1	1	3	
2	2	4	
3	5	6	
4	9	10	

$$L_i(x) = \frac{\prod_{j=1}^{n+1} (x - x_j)}{\prod_{j=1}^{n+1} (x_i - x_j)} \quad j \neq i$$

Problem

i	1	2	3	4
x_i	1	2	5	9
y_i	1	3	6	10

Find
a) E_q^n of the polynomial, $p(x)$.
(b) $f(6) = ?$

$$\therefore p(x) = L_1(x)y_1 + L_2(x)y_2 + L_3(x)y_3 + L_4(x)y_4$$

$$L_1(x) = \frac{(x-2)(x-5)(x-9)}{(1-2)(1-5)(1-9)}$$

$$L_1(x=6) = ? \quad \frac{3}{8}$$

$$L_2(x) = \frac{(x-1)(x-5)(x-9)}{(2-1)(2-5)(2-9)}$$

$$\therefore L_2(x=6) = \frac{-5}{7}$$

$$\cancel{L_3(x=6)} \quad L_3(x) = \frac{(x-1)(x-2)(x-9)}{(5-1)(5-2)(5-9)}$$

$$\therefore L_3(x=6) = \frac{5}{4}$$

$$L_4(x) = \frac{(x-1)(x-2)(x-5)}{(9-1)(9-2)(9-5)}$$

$$\therefore L_4(x=6) = \frac{5}{56}$$

$$\therefore p(x) = f(6) = \frac{3}{8} \times 1 + \frac{-5}{7} \times 3 + \frac{5}{4} \times 6 + \frac{5}{56} \times 10$$

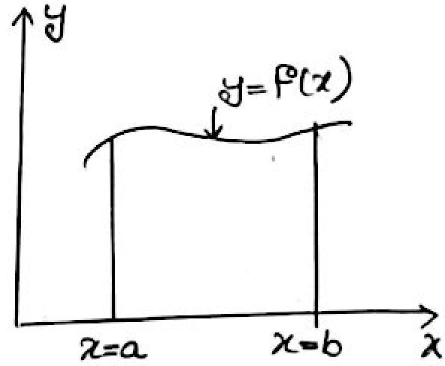
$$\boxed{f(6) = 6.625} \quad (b)$$

Ansari Sir

Numerical Integration / Quadrature

It is a technique of estimating the numerical value of a definite integral:

$$I = \int_a^b f(x)dx$$



Integration করার assumption:

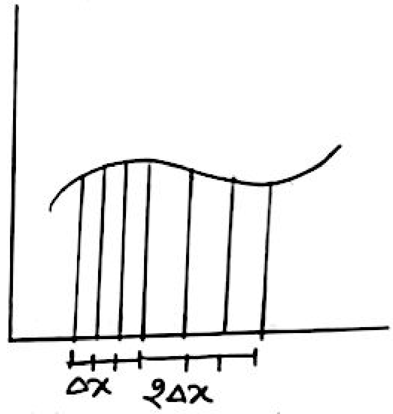
1) Both the limits are finite.

2) f(x) is continuous over the closed interval a and b.

Values of function:

(1) at equidistant values of x

(2) at non equidistant " " "



Functions → of one variable [area]

(a) Trapezoidal / Simpson's Rule / Romberg's Quadrature

(b) Gauss- Quadrature

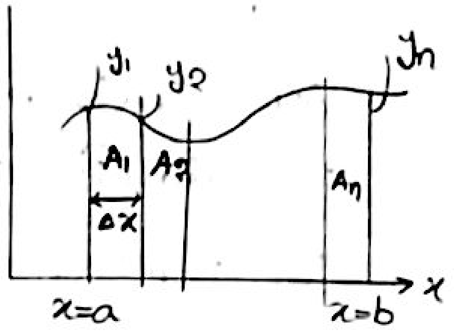
Functions of two variable [Volm]

(a) integration over a rectangular domain

(b) " " " " non rectangular "

$$\therefore V = \int_{y_1}^{y_2}\int_{x_1}^{x_2} F(x,y)\, dy\, dx$$

Trapezoidal Rule: (single area পাওয়া যায়)



$$\Delta x = \frac{b-a}{n}$$ [n = যত সংখ্যক segment এ ভাগ করেছি]

$$A = A_1 + A_2 + \cdots + A_n$$

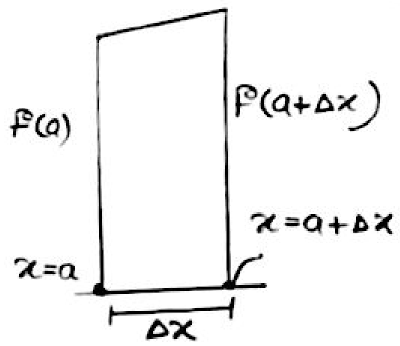
$$A_1 = \frac{\Delta x}{2}(y_1 + y_2)$$

$$A_2 = \frac{\Delta x}{2}(y_2 + y_3)$$

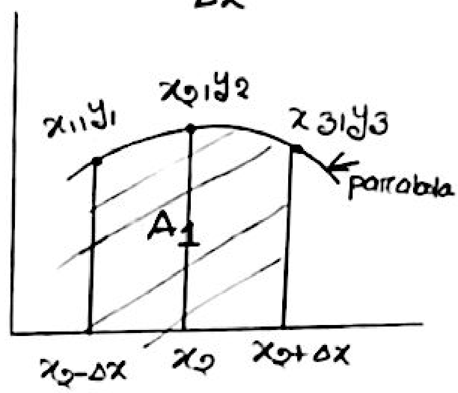
$$A_3 = \frac{\Delta x}{2}(y_3 + y_4)$$

$$\vdots$$

$$A = \frac{\Delta x}{2}\left[y_1 + y_n + 2(y_2 + y_3 + \cdots)\right]$$



Simpson's Rule: (pair of area পাওয়া যায়)



parabola, $y = a_1x^2 + a_2x + a_3$

$$A_1 = \int_{x_1}^{x_3} y\,dx = \int_{x_2-\Delta x}^{x_2+\Delta x} (a_1x^2 + a_2x + a_3)\,dx$$

$$y_1 = a_1(x_2 - \Delta x)^2 + a_2(x_2 - \Delta x) + a_3$$

$$y_2 = a_1x^2 + a_2x_2 + a_3$$

$$y_3 = a_1(x_2 + \Delta x)^2 + a_2(x_2 + \Delta x) + a_3$$

$$\therefore A_1 = \frac{\Delta x}{3}\left[y_1 + 4y_2 + y_3\right]$$

$$A_2 = \frac{\Delta x}{3}\left[y_3 + 4y_4 + y_5\right]$$

$$A_n = \frac{\Delta x}{3}\left[y_{n-2} + 4y_{n-1} + y_n\right]$$

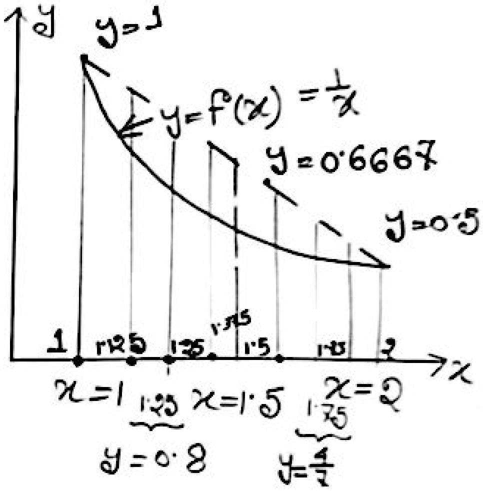
$$A = \frac{\Delta x}{3}\left[y_1 + y_n + 4(y_2 + y_4 + y_6 + \cdots) + 2(y_3 + y_5 + y_7 + \cdots)\right]$$

Problem

Approximate the following integral using the trapezoidal rule with 1, 2, 4, 8 and 16 equal subdivision.

$\underbrace{1,\ 2, 4, 8 \text{ and } 16}_{n \text{ এর value}}$ $\int_1^2 \frac{1}{x}dx = 0.6931$

For 1 division



n	Δx	A_{Trap}	A_{simp}
1	1	$\frac{1}{2}(1+0.5) = 0.75$	—
2	0.5	$\frac{0.5}{2}[1+0.5+2\times 0.6667]$ $= 0.70835$	$\frac{0.5}{3}[1+0.5+4\times 0.6667]$ $= 0.6945$
4	0.25	$\frac{0.25}{2}[1+0.5+2(0.8+\frac{4}{7}+0.6667)]$ $= 0.6971$	$\frac{0.25}{3}[1+0.5+4(0.8+\frac{4}{7})+2\times 0.6667]$ $= 0.6933$
8	0.125	$\frac{0.125}{2}[1+0.5+$ $2($	
16	0.0625		

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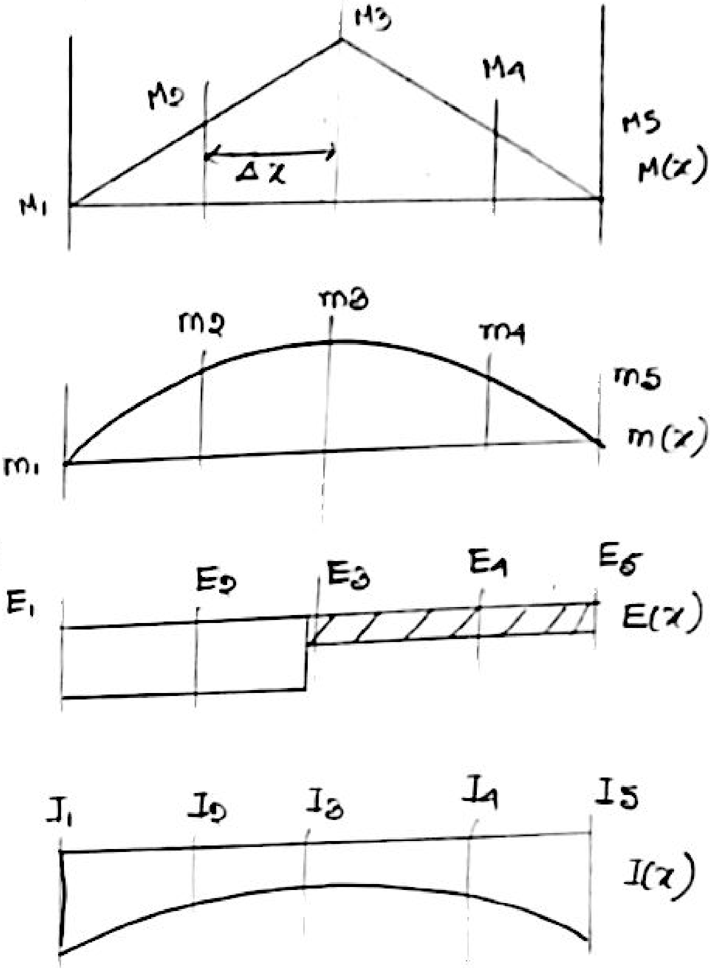
Product of Integrals:

$$\int_a^b \frac{M\,m}{EI}\,dx \cong$$

$$\frac{\Delta x}{3}\left[\frac{M_1 m_1}{E_1 I_1} + \frac{M_5 m_5}{E_5 I_5} + 4\left(\frac{M_2 m_2}{E_2 I_2} + \frac{M_4 m_4}{E_4 I_4}\right) + 2\,\frac{M_3 m_3}{E_3 I_3}\right]$$

এটা apply করে আমরা deflection at any point বের করতে পারি।



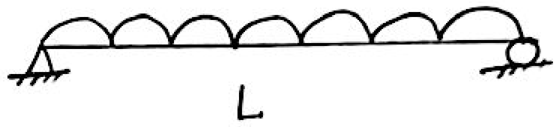
Ansari Sir

$$\Delta = \int \frac{Mm}{EI} dx$$

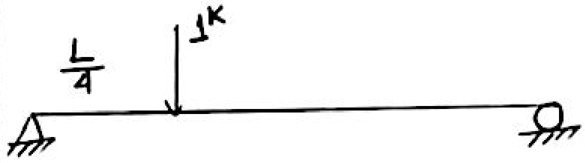
m = mising element

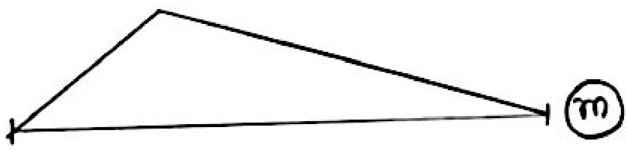
M → diagram থেকে বের করব

E, I → given



Deflection at L/4





Romberg's Quadrature:

Division	Trapezoidal	Simpson's		
1	T^0	$S^0 = \frac{4T_1 - T_0}{4-1}$	$C_0 = \frac{4^2 S_1 - S_0}{4^2-1}$	$D^0 = \frac{4^3 C_1 - C_0}{4^3-1}$
2	T^1	$S' = \frac{4T_2 - T_1}{4-1}$	$C' = \frac{4^2 S_2 - S_1}{4^2-1}$	$D' = \frac{4^3 C_2 - C_1}{4^3-1}$
4	T^2	$S^2 = \frac{4T_3 - T_2}{4-1}$	$C^2 = \frac{4^2 S_3 - S_2}{4^2-1}$	$E^0 = \frac{4^4 D_1 - D_0}{4^4-1}$
8	T^3	$S^3 = \frac{4T_4 - T_3}{4-1}$		
16	T^4			

Criteria

$$\cancel{\left|\frac{C_0}{S_0}\right|} \quad \left|\frac{C_0}{S_0} - 1\right| < \epsilon = 0.0001$$

$$\left|\frac{D^0}{C_0} - 1\right| = ?$$

প্রত্যেক level এ check করতে হবে. $\int_1^2 \frac{1}{x}\,dx$

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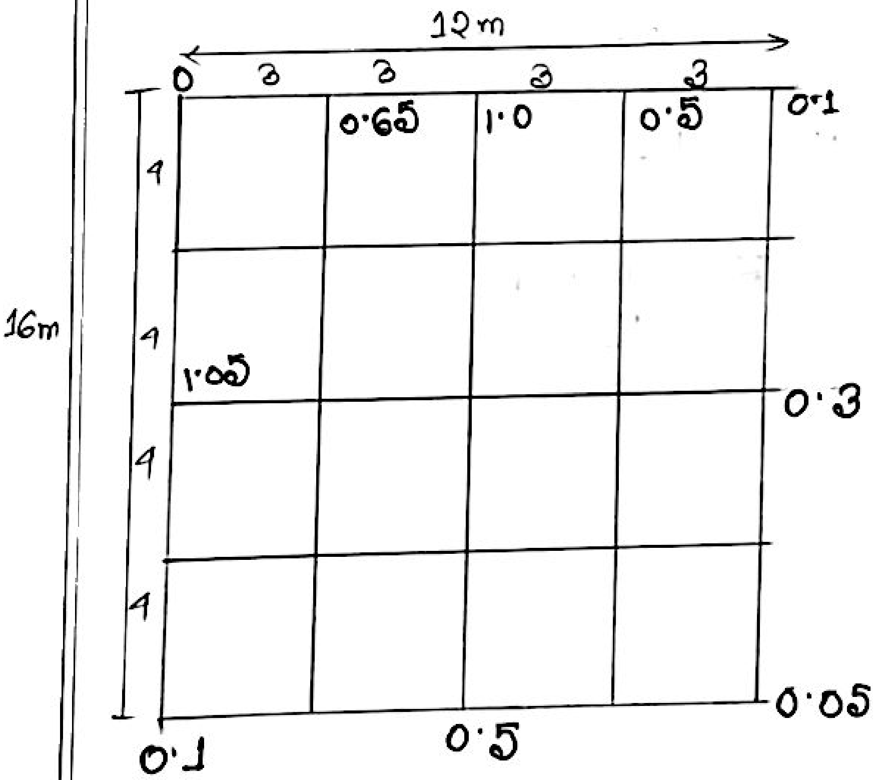
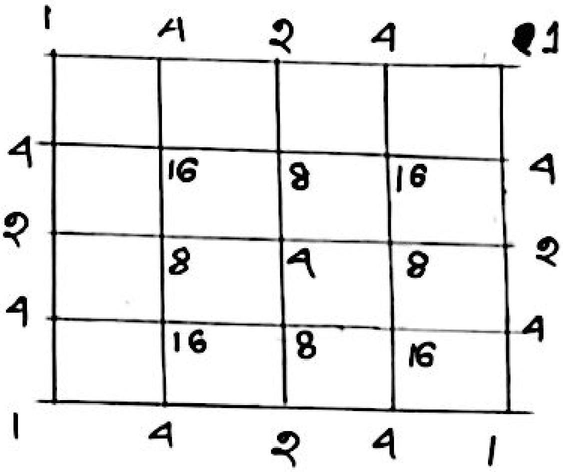
Division	T	S
1	$0.75\,T^0$	$S_0 = \frac{4T_1 - T_0}{4-1} = \frac{4 \times 0.7083 - 0.75}{4-1} = 0.6944$
2	$0.7083\,T^1$	
4	$0.6970\,T^2$	$S^1 = \frac{4T_2 - T_1}{4-1} = 0.6932$
8	$0.6941\,T^3$	$S^2 = \frac{4T_3 - T_2}{4-1} = 0.69313$
16	$0.6934\,T^4$	
		$S^3 = \frac{4T_4 - T_3}{4-1} = 0.69316$

$$\left| \frac{S_0}{T_0} - 1 \right|$$

Integration over a rectangular domain :



$$A = \frac{\Delta x}{3} \left[1y_1 + 4y_2 + 2y_3 + 4y_4 + 1y_5 \right]$$



$$\text{Volume} = \frac{3}{3} \times \frac{4}{3} \times \Big[1 \times \{0 + 0.1 + 0.05 + 0.1\} + 2\{1 + 0.3 + 0.5 + 1.05\}$$
$$+ 4\{\underset{\substack{\downarrow \\ \text{height}}}{\ldots\ldots}\} + 8\{\ldots\} + 16\{\ldots\}\Big]$$

$$= 267.7 \text{ m}^3$$

Gauss Quadrature:

$$\int_{-1}^{1} f(x)\,dx \cong w_0 f(x_0) + w_1 f(x_1) + w_2 f(x_2) + \cdots + \cdots$$

$w \rightarrow$ weighting co-efficients

$x \rightarrow$ associated points

Legender's polynomial:

$$P_n(x) = \frac{1}{2^n \lfloor n} \frac{d^n}{dx^n} (x^2 - 1)^n$$

$$P_0(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$
$$P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

$$W_k = \frac{1}{P'_{n+1}(x_k)} \int_{-1}^{1} \frac{P_{n+1}(x)}{x - x_k}\, dx$$

কোন কিছু না বললে by default $\boxed{n=2}$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x) = 0 \Rightarrow P_3'(x) = \frac{1}{2}(15x^2 - 3)$$

$$\therefore \frac{5x}{2}\left(x^2 - \frac{3}{5}\right) = 0$$

$$\leftarrow x_0 = -\sqrt{3/5}$$

$$x_1 = 0$$

$$x_2 = +\sqrt{\frac{3}{5}}$$

$$w_0 = \frac{1}{\frac{1}{2}\left\{15 \times \frac{3}{5} - 3\right\}} \int \frac{\frac{5x}{2}\left(x^2 - \frac{3}{5}\right) dx}{x - \left(-\sqrt{3/5}\right)}$$

$$= \frac{5}{9}$$

$w_1 = 8/9$ [$x_1 = 0$ বসিয়ে]

$w_2 = \frac{5}{9}$ [$x_2 = \sqrt{\frac{3}{5}}$ বসিয়ে]

Table Weighing co-efficients (w_k) and associated points (x_k) for the Gaussian Quadrature :

n	x_k	w_k
২	0 ± ৭৭46	8/৯ 5/৯
৩	± 0·৩4 ± 0·8611	0·65২1 0·34৭৯
4	0 ± 0·5৩85 ± 0·৯06২	0·5689 0·4786 0·২369

যদি বলে derive কর, তবে আগের page এর মত proof করতে হবে। যদি বলে value বের কর. তবে table থেকে direct value use করব।

Tanvir Sir

System of ODE: (1 independt, 2 dependent variable)

Given, $\dfrac{dy_1}{dx} = x + y_1 + y_2 \cdots (1) \quad y_1(0) = 1$

$$\frac{dy_2}{dx} = 1 + y_1 + y_2 \cdots\cdots (2) \quad y_2(0) = -1$$

Q. Estimate $y_1(0.1)$ and $y_2(0.1)$ using Heun's method. $(h = 0.1)$

Soln

$$M_1(1) = \left.\frac{dy_1}{dx}\right|_{\substack{x=0 \\ y_1=1 \\ y_2=-1}} = x + y_1 + y_2 = 0 + 1 - 1 = 0 \text{ slope}$$

$$M_1(2) = 1 + y_1 + y_2 = 1 + 1 - 1 = 1$$

$$M_2(1) = \underbrace{x + y_1 + y_2}_{\text{using euler}} = 0.1 + 1 - 0.9 = 0.2$$

$$M_2(2) = \cancel{x+y} \; 1 + y_1 + y_2 = 1 + 1 - 0.9 = 1.1$$

$$M(1) = \frac{M_1(1) + M_2(1)}{2} = \frac{0 + 0.2}{2} = 0.1$$

$$M(2) = \frac{M_1(2) + M_2(2)}{2} = \frac{1 + 1.1}{2} = 1.05$$

$$y_1(0.1) = y_1(0) + \text{avg. slope} \times 0.1 = 1 + 0.1 \times 0.1 = \boxed{1.01} \text{ (Ans)}$$

$$y_2(0.1) = y_2(0) + \text{avg. solope} \times 0.1 = -1 + 1.05 \times 0.1$$
$$= \boxed{-0.895} \text{ (Ans)}$$

Using Euler's Method :

* ৩ বা 4 variable এর জন্যও Math আসতে পারে।

y_1 at $x = 0.1$

$y_1(0.1) = y_1(0) + \text{slope} \times h$

$\Rightarrow y_1(0.1) = 1 + 0 \times 0.1 = 1$

y_2 at $x = 0.1$

$y_2(0.1) = y_2(0) + \text{slope} \times h$

$= -1 + 1 \times 0.1$

$= -0.9$

* Higher order ODE

Problem:

Given $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 6x$; $y(0) = 0$, $y'(0) = 1$

solve for $y(0.2)$ using Heun's method. use $h = 0.2$

Assume, $y = y_1$, $\frac{dy}{dx} = y_2$

$\therefore y_1(0) = 0$
$y_2(0) = 1$

$\frac{dy_1}{dx} = y_2$ .... ①

$\frac{dy_2}{dx} = 6x + 3y_1 - 2y_2$ .... ②

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Soln

$M_1(1) = 1$

$M_1(2) = 6x - 3y_1 - 2y_2 = \cdots\cdots = -2$

$M_2(1) = y_2|_{x=0} \underbrace{+ 0.2 \times (-2)} = 0.6$

$y_2 = y_2(0) + \text{slope} \times 0.2$

y_2 এর জন্য slope use করব।

যেহেতু y_2 এর জন্য কোন কিছু evaluate করা হচ্ছে

$$M_2(2) = \underbrace{6 \times 0.2}_{x=0.2} + 3\underbrace{(0 + 0.2 \times 1)}_{y_1 \text{ evaluated at } x=0.2} - \underbrace{2 \times (1 + (-2) \times 0.2)}_{y_2 \text{ evaluated at } x=0.2}$$

$= 0.6$

$M(1) = \frac{1+0.6}{2} = 0.8$

$M(2) = \frac{-2+0.6}{2} = -0.7$

$y_1(0.2) = y_1(0) + 0.8 \times 0.2 = \boxed{0.16}$ (Ans)

$y_2(0.2) = y_2(0) + (-0.7) \times 0.2 = 0.86$

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Problem $\frac{d^3y}{dx^3} + 6x = 5$

$y = y_1$

$\frac{dy}{dx} = y_2$

$\frac{d^2y}{dx^2} = y_3$

Ansari Sir

V. Imp

Problem: Solve the following using Gauss Quadrature for $n = 2$:

(a) $\int_{-1}^{1} x^2 \cos x \, dx$

(b) $\int_{0}^{3} x^2 \cos x \, dx$

কোন কিছু না বলা থাকলে $n = 2$ by default.

Limit অবশ্যই -1 থেকে $+1$ হতে হবে। নইলে Gauss Quad. apply হবে না।

$$\int_{-1}^{1} f(x)dx \equiv \underbrace{f(x_0)w_0 + f(x_1)w_1 + f(x_2)w_2}_{\text{For this problem}} + \underbrace{f(x_3)w_3}_{\text{for } n=3}$$

x	w
$-\sqrt{3}/5$	$5/9$
0	$8/9$
$+\sqrt{3}/5$	$5/9$

$f(x) = x^2 \cos x$

$$\therefore \int_{-1}^{1} x^2 \cos x = \cancel{0.06 + 0 + 0.06 = 0.1254}$$

$$= \left(-\frac{\sqrt{3}}{\sqrt{5}}\right)^2 \cos\left(-\frac{\sqrt{3}}{\sqrt{5}}\right) \times \frac{5}{9} + 0 + \left(\frac{\sqrt{3}}{\sqrt{5}}\right)^2 \cos\left(\frac{\sqrt{3}}{\sqrt{5}}\right) \frac{5}{9}$$

$$= 0.476$$

(b) $x = \dfrac{(b-a)t + (b+a)}{2} = \dfrac{3}{2}(t+1)$

$\dfrac{dx}{dt} = \left(\dfrac{b-a}{2}\right) = \dfrac{3-0}{2} = \dfrac{3}{2}$

$[b = 3, a = 0]$

$x = 0, \; t = -1$

$x = 3, \; t = 1$

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$$\therefore (b) = \int_0^3 x^2 \cos x \, dx$$

$$= \int_{-1}^{1} \left\{\frac{3}{2}(t+1)\right\}^2 \cos\left\{\frac{3}{2}(t+1)\right\} \cdot \frac{3}{2} dt$$

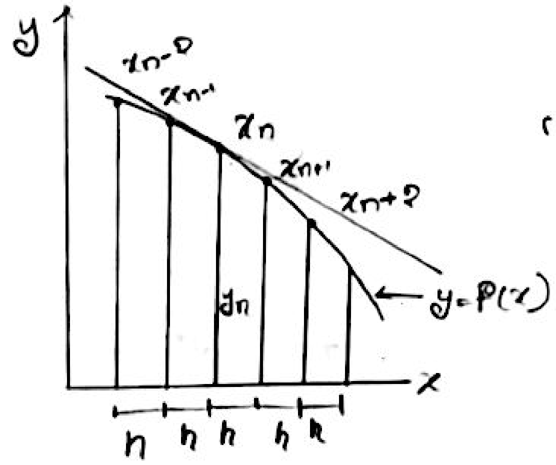
$$= \frac{9}{4} \cdot \frac{3}{2} \int_{-1}^{1} (t+1)^2 \cos\left(\frac{3t}{2}+\frac{3}{2}\right) dt$$

$$\therefore f(x) = f(t) = (t+1)^2 \cos\left(\frac{3t}{2}+\frac{3}{2}\right)$$

$$\therefore \frac{27}{8}\int_{-1}^{1} (t+1)^2 \cos\left\{\frac{3}{2}(t+1)\right\} dt \simeq \left[w_0 f(t_0) + w_1 f(t_1) + w_2 f(t_2)\right] \frac{27}{8}$$

$$= -4.986$$

Numerical Differentiation:



$$\theta|_n = \frac{dy}{dx} \xrightarrow{} Dy = \frac{y_{n+1} - y_n}{h}$$

$$'M' \longrightarrow D^2y = \frac{d}{dx}(\theta) = \frac{\theta_{n+1} - \theta_n}{h}$$

$$D^3y \rightarrow V = \frac{\frac{y_{n+2} - y_{n+1}}{h} - \frac{y_{n+1} - y_n}{h}}{h}$$

$$D^4y \rightarrow q$$

$$= \frac{y_{n+2} - 2y_{n+1} + y_n}{h^2}$$

$$M = -EI D^2y$$

Mathematical Molecules:

(A) Backward Difference:

	y_n	y_{n-1}	y_{n-2}	y_{n-3}	y_{n-4}
hD	1	-1			
h^2D^2	1	-2	1		
h^3D^3	1	-3	3	1	
h^4D^4	1	-4	6	-4	1

(B) Forward Difference:

	y_n	y_{n+1}	y_{n+2}	y_{n+3}	y_{n+4}
hD	-1	1			
h^2D^2	1	-2	1		
h^3D^3	-1	3	-3	1	
h^4D^4	1	-4	6	-4	1

(C) Central Difference:

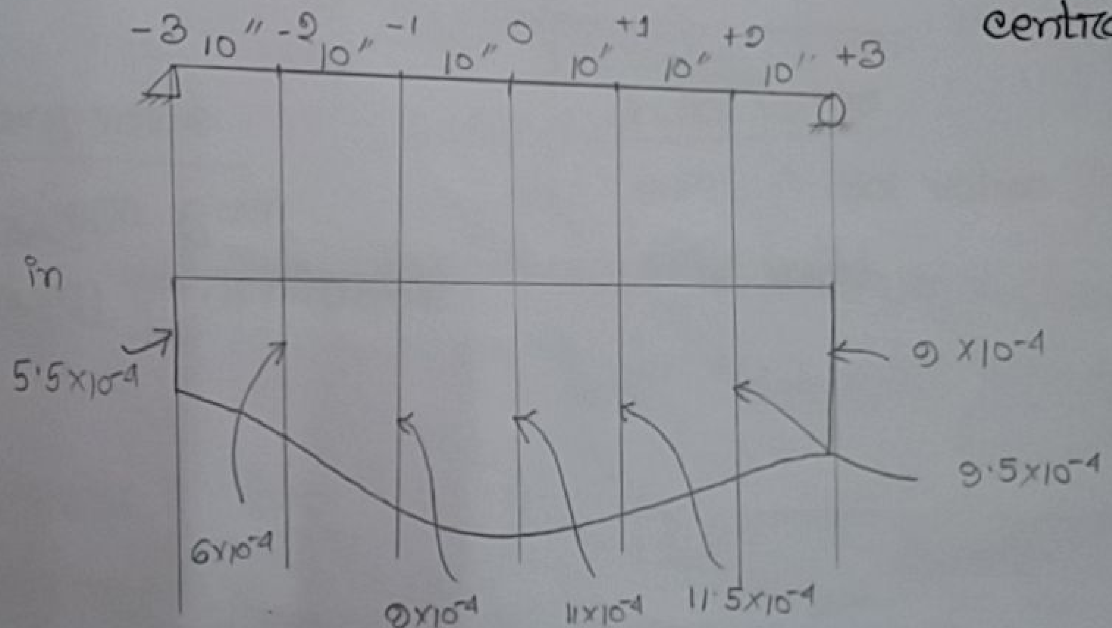
	y_{n-2}	y_{n-1}	y_n	y_{n+1}	y_{n+2}
$2hD$		-1	0	1	
h^2D^2		1	-2	1	
$2h^3D^3$	-1	2	0	-2	1
h^4D^4	1	-4	6	-4	1

Problem:

Experimentally observed values of deflection of a beam is shown below.

Given: $E = 30\times10^6$ psi and $I = 1000\ in^4$. Estimate bending moments at $\boxed{+1,\ 0 \text{ and } -1}$

কোন কিছু না বললে central টা ভাল, use করা



$$D^2y = \frac{1}{h^2}\left[y_{n+1} - 2y_n + y_{n-1}\right]$$

যদি (+3) ও (-3) point এ Moment চাই, তবে central use করা যাবে না।

$$(+1) = \frac{1}{10^2}\left[y_2 - 2y_1 + y_0\right] = -2.5\times10^{-6}$$

$$(0) = \frac{1}{10^2}\left[y_1 - 2y_0 + y_{-1}\right] = -1.5\times10^{-6}$$

+3 এর জন্য Backward
-3 " " Forward use করব।

$$(-1) = \frac{1}{10^2}\left[y_0 - 2y_{-1} + y_{-2}\right] = -1\times10^{-6}$$

$$(+1)\quad M = -EID^2y = -30\times10^6\times1000\times(-2.5\times10^{-6}) = 75000\ lb\text{-}in$$

$$(0)\quad M = -30\times10^6\times1000\times(-1.5\times10^{-6}) = 45000\ lb\text{-}in$$

$$(-1)\quad M = -30\times10^6\times1000\times(-1\times10^{-6}) = 30000\ lb\text{-}in$$

For (+3) $D^2y = \frac{1}{h^2}[y_n - 2y_{n-1} + y_{n-2}]$

$= \frac{1}{10^2}[y_3 - 2y_2 + y_1] =$

For (−3) $D^2y = \frac{1}{10^2}[y_{-1} - 2y_{-2} + y_{-3}]$

Boundary value Problem: (Finite Di

Prob

Given the equation $\frac{d^2y}{dx^2} = e^{x^2}$ with $y(0)=0$,
$y(1) = 0$

estimate the values of $y(x)$ at $x = 0.25$, 0.5, 0.75

boundary value

Sol^n শুধুমাত্র given boundary তে applicable

initial value

Sol^n initial value থেকে ∞ পর্যন্ত valid.

Tanvir Sir

Method of Finite Difference:

$$\frac{dy}{dx} \cong \frac{y_{i+1} - y_{i-1}}{2\Delta x}$$

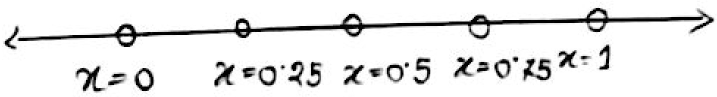
$$\frac{d^2y}{dx^2} \cong \frac{y_{i+1} - 2y_i + y_{i-1}}{(\Delta x)^2}$$

Prob

Given the equation $\frac{d^2y}{dx^2} = e^{x^2}$ with $y(0) = 0$, $y(1) = 0$.
Estimate the values of $y(x)$ at $x = 0.25, 0.5, 0.75$.

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{(\Delta x)^2} = e^{x^2}$$

This problem is bounded within 0 and 1.



$i = 1, x = 0.25$

$$\therefore \frac{y_{i+1} - 2y_i + y_{i-1}}{(0.25)^2} = e^{x^2}$$

$i = 1$, $x = 0.25$

$\Rightarrow y_2 - 2y_1 + \cancel{y_0} \xrightarrow{} 0 = 0.665$ ...... (1)

$i = 2, x = 0.5$

$$y_3 - 2y_2 + y_1 = e^{(0.5)^2}$$

$$\Rightarrow y_3 - 2y_2 + y_1 = 0.0803 \cdots\cdots (২)$$

Similarly, $\cancel{y_4}^{0} - 2y_3 + y_2 = 0.1097 \cdots\cdots (3)$

$$\left.\begin{aligned} y_1 &= -0.1175 = y(0.25) \\ y_2 &= -0.1684 = y(0.5) \\ y_3 &= -0.1391 = y(0.75) \end{aligned}\right\} \text{(Ans)}$$

* যদি Δx অনেক ছোট হয়, the amount of work increases.

$\Delta x \uparrow\uparrow$, large amount of calculation

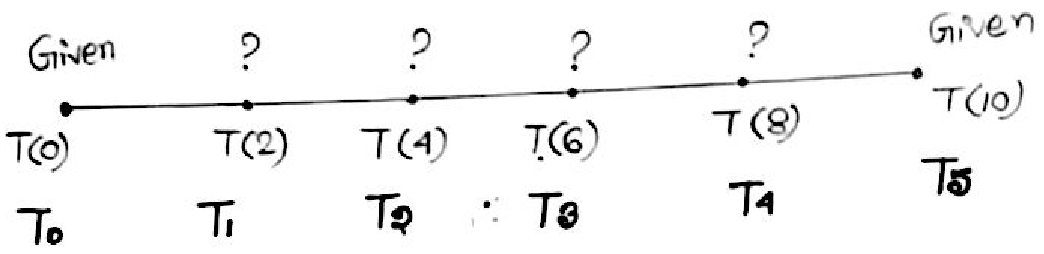
$\downarrow$

so, we have to use computer

Δx ছোট হলে. eq^n এর সংখ্যা বেড়ে যায়।

Prob

Solve $\frac{d^2T}{dx^2} - 0.01\,(T-20) = 0$, $T(0) = 40$, $T(10) = 200$, $\Delta x = 2$



$$\frac{T_{i+1} - 2T_i + T_{i-1}}{2^2} - 0.01\,(T_i - 20) = 0$$

$$\Rightarrow \boxed{-T_{i-1} + 2.04\,T_i + T_{i+1} = 0.8}$$ Finite D. approximation

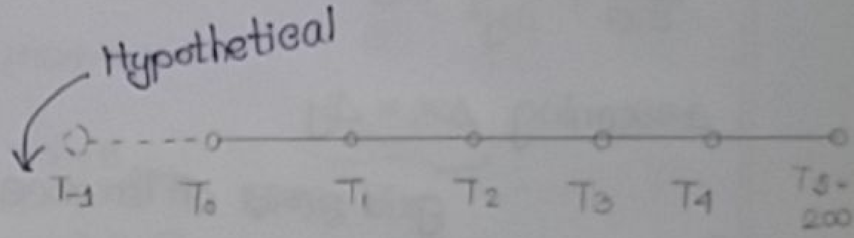
$$\begin{bmatrix} 2.04 & -1 & 0 & 0 \\ -1 & 2.04 & -1 & 0 \\ 0 & -1 & 2.04 & -1 \\ 0 & 0 & -1 & 2.04 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 40.8 \\ 0.8 \\ 0.8 \\ 200.8 \end{bmatrix}$$

$T = [65.97 \quad 93.78 \quad 124.53 \quad 159.48]$ $\boxed{\text{sol}^n}$

এখানে $i = 1, 2, 3 \ldots$ বসিয়ে

Prob

$$\left.\frac{dT}{dx}\right|_{x=0} = 5 \quad T(10) = 200 \quad \Delta x = 2 \quad \longrightarrow \text{for } \frac{d^2T}{dx^2} - 0.01(T-20) = 0$$



at $i=1$ $\quad \dfrac{T_1 - T_{-1}}{2\Delta x} = 5$

$\Rightarrow T_{-1} = T_1 - 20$

replace this with $T_1 - 20$

$i=0,\ x=0$ $\quad -T_{-1} + 2.04\,T_0 - T_1 = 0.8$

$\Rightarrow 2.04\,T_0 - 2T_1 = -19.2 \quad \cdots\cdots \quad (1)$

unknown

$$\begin{bmatrix} 2.04 & -2 & 0 & 0 & 0 \\ -1 & 2.04 & -1 & 0 & 0 \\ 0 & -1 & 2.04 & -1 & 0 \\ 0 & 0 & -1 & 2.04 & -1 \\ 0 & 0 & 0 & -1 & 2.04 \end{bmatrix} \begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} -19.2 \\ 0.8 \\ 0.8 \\ 0.8 \\ 200.8 \end{bmatrix}$$

যদি problem এ 8-9 টি unknown থাকে তবে শুধু Matrix বানাতে হবে

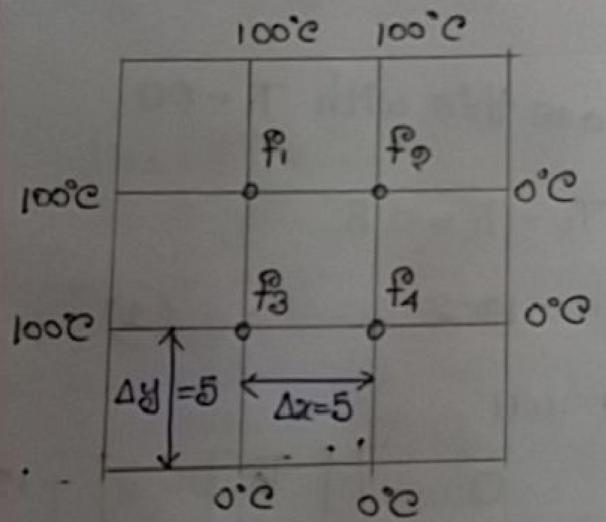
যদি 4-5 " " " " solve করতে হবে।

PDE

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

assuming $\underbrace{\Delta x = \Delta y}_{\text{গ্রিড গুলোর difference equal}}$

Prob



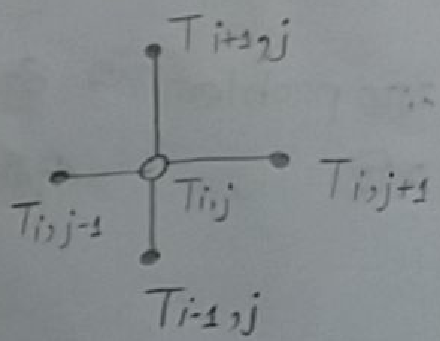
Consider a steel plate of size 15 cm × 15 cm.
What are the steady state temperature at the intermediate points
$f_1 = ?$ $f_2 = ?$ $f_3 = ?$ $f_4 = ?$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

$$T_{i,j+1} + T_{i,j-1} + T_{i+1,j} + T_{i-1,j} - 4T_{i,j} = 0$$

$i \rightarrow$ row (x axis)

$j \rightarrow$ column (y axis)



at point 1. $f_2 + f_3 - 4f_1 + 100 + 100 = 0$ .... (1)

point 2. $f_1 + f_4 - 4f_2 + 100 + 0 = 0$ .... (2)

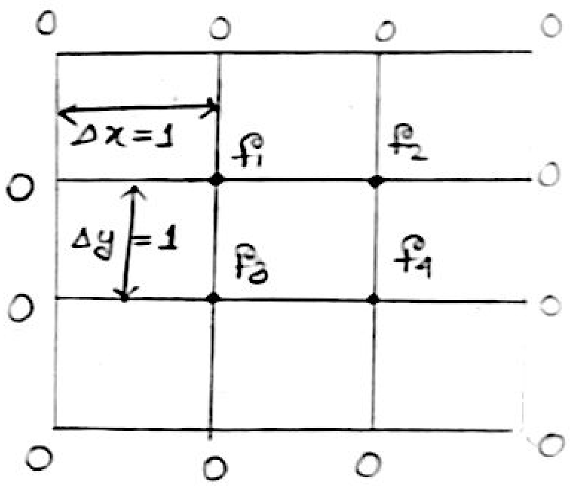
point 3. $f_1 + f_4 - 4f_3 + 100 + 0 = 0$ ..... (3)

point 4. $f_2 + f_3 - 4f_4 + 0 + 0 = 0$ .... (4)

solving (1), (2), (3), (4),

$$\begin{bmatrix} f_1 = 75 & f_3 = 50 \\ f_2 = 50 & f_4 = 25 \end{bmatrix} \text{(Ans)}$$

Problem



$$\underbrace{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 2x^2y^2}$$

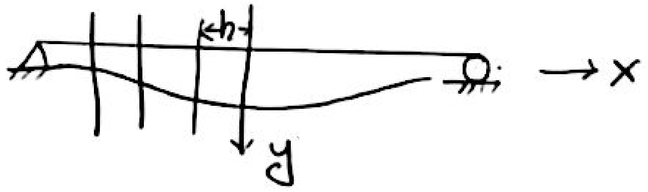
$\downarrow$

Solution

$$\begin{bmatrix} f_1 = -\frac{22}{4} \\ f_2 = -\frac{13}{4} \\ f_3 = -\frac{43}{4} \\ f_4 = -\frac{22}{4} \end{bmatrix} \text{(Ans)}$$

Ansari Sir

BM estimation for a slab using partial differentiation:



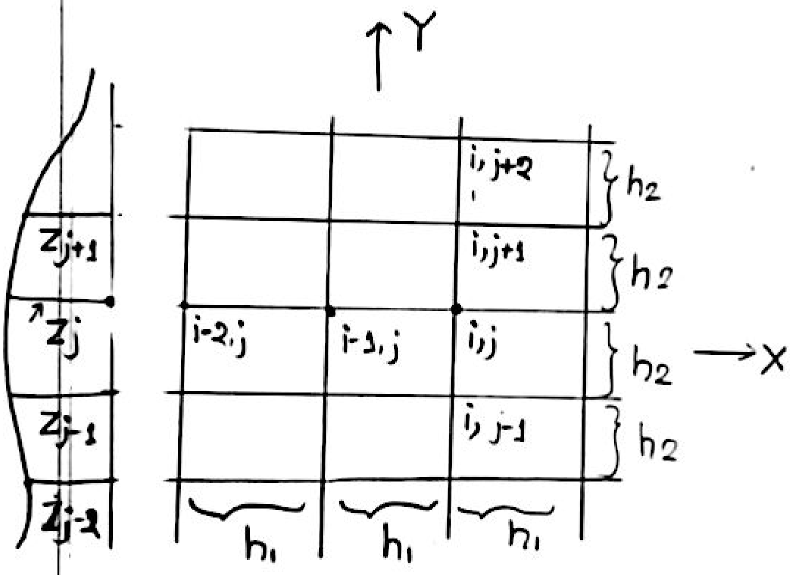
$$y = f(x)$$

$$\theta = \frac{dy}{dx}$$

$$M = -EI\frac{d^2y}{dx^2}$$

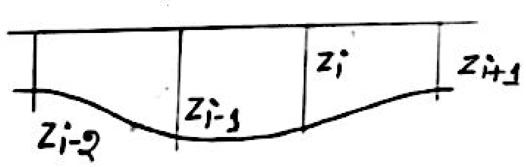
$$Z = f(x,y)$$

deflection



$$\frac{\partial z}{\partial x} = \frac{1}{2h_1}\left[Z_{i+1} - Z_{i-1}\right]$$

$$\frac{\partial z}{\partial y} = \frac{1}{2h_2}\left[Z_{j+1} - Z_{j-1}\right]$$



BM for Plate / Slab:

$$M_x = -D\left[\frac{\partial^2 z}{\partial x^2} + \nu\frac{\partial^2 z}{\partial y^2}\right]$$

$$M_y = -D\left[\nu\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right]$$

D = plate stiffness

$$= \frac{Et^3}{12(1-\nu^2)}$$

ν = poission's ratio $\qquad$ $t = 4'' \sim 6''$

$= 0.15 - 0.35$ $\qquad$ = plate thickness

$\boxed{D = 3\times10^8 \text{ in-lb}}$ $\qquad$ $E = 30\times10^6$ psi

$$\left.\frac{\partial^2 z}{\partial x^2}\right|_i = \frac{1}{h_1^2}\left[z_{i+1} - 2z_i + z_{i-1}\right]$$

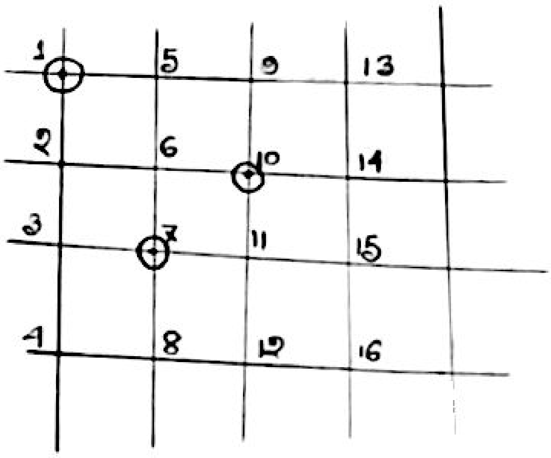
$$\left.\frac{\partial^2 z}{\partial y^2}\right|_j = \frac{1}{h_2^2}\left[z_{j+1} - 2z_j + z_{j-1}\right]$$

Problem

The deflection at various points on a normally loaded plate are shown below. Estimate the BM at #1, #7 and #10 points. Given: $D = 3\times10^8$ lb-in, $\nu = 0.15$, $h = 10''$

Pt	Deflection ($\times10^{-4}$ in)
1	3
2	4
3	4.5
4	3.7
5	3.2
6	4.2
7	4.7
8	4.0

Pt	Deflection ($\times10^{-4}$ in)
9	3.8
10	4.5
11	4.8
12	4.3
13	3.6
14	4.2
15	4.5
16	4



#1

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{10^2}\left[Z_9 - 2Z_5 + Z_1\right] = \frac{1}{100}\left[3.8 - 2\times 3.2 + 3\right]\times 10^{-4}$$
$$= \cancel{4.7}\ 4\times 10^{-7}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{1}{10^2}\left[Z_3 - 2Z_2 + Z_1\right] = \frac{1}{100}\left(4.5 - 2\times 4 + 3\right)\times 10^{-4}$$
$$= -5\times 10^{-7}$$

$$\therefore M_x = -3\times 10^8\left[4\times 10^{-7} + 0.15\left(-5\times 10^{-7}\right)\right] = -97.5 \text{ lb-in}$$

$$M_y = -3\times 10^8\left[0.15\left(4\times 10^{-7}\right) + \left(-5\times 10^{-7}\right)\right] = 132 \text{ lb-in}$$

#10

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{10^2}\left[Z_6 - 2Z_{10} + Z_{14}\right] = \frac{1}{10^2}\left[4.2 - 2\times 4.5 + 4.2\right]\times 10^{-4}$$
$$= -6\times 10^{-7}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{1}{10^2}\left[Z_9 - 2Z_{10} + Z_{11}\right] = \frac{1}{10^2}\left(3.8 - 2\times 4.5 + 4.8\right)\times 10^{-4}$$
$$= -4\times 10^{-7}$$

$$M_x = -3\times10^8\left[-6\times10^{-7} + 0.15\left(-4\times10^{-7}\right)\right] = 198 \text{ in-lb}$$

$$M_y = -3\times10^8\left[0.15\left(-6\times10^{-7}\right) - 4\times10^{-7}\right] = 147 \text{ in-lb}$$

Tanvir Sir

PDE

Method Of Finite Difference

Last class: $\underbrace{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}}_{x,y \text{ space dimension}} = 0$ (Elliptic Eqⁿ)

Parabolic PDE Eqⁿ:

$$K \underbrace{\frac{\partial^2 T}{\partial x^2}}_{\substack{\text{space} \\ \text{dimension}}} = \underbrace{\frac{\partial T}{\partial t}}_{\substack{\text{time} \\ \text{dimension}}}$$

$$K \cdot \frac{T_{i+1}^{l} - 2T_i^{l} + T_{i-1}^{l}}{(\Delta x)^2} = \frac{T_i^{l+1} - T_i^{l}}{\Delta t}$$

$$\Rightarrow \boxed{T_i^{l+1} = T_i^{l} + \lambda\left(T_{i+1}^{l} - 2T_i^{l} + T_{i-1}^{l}\right)}$$

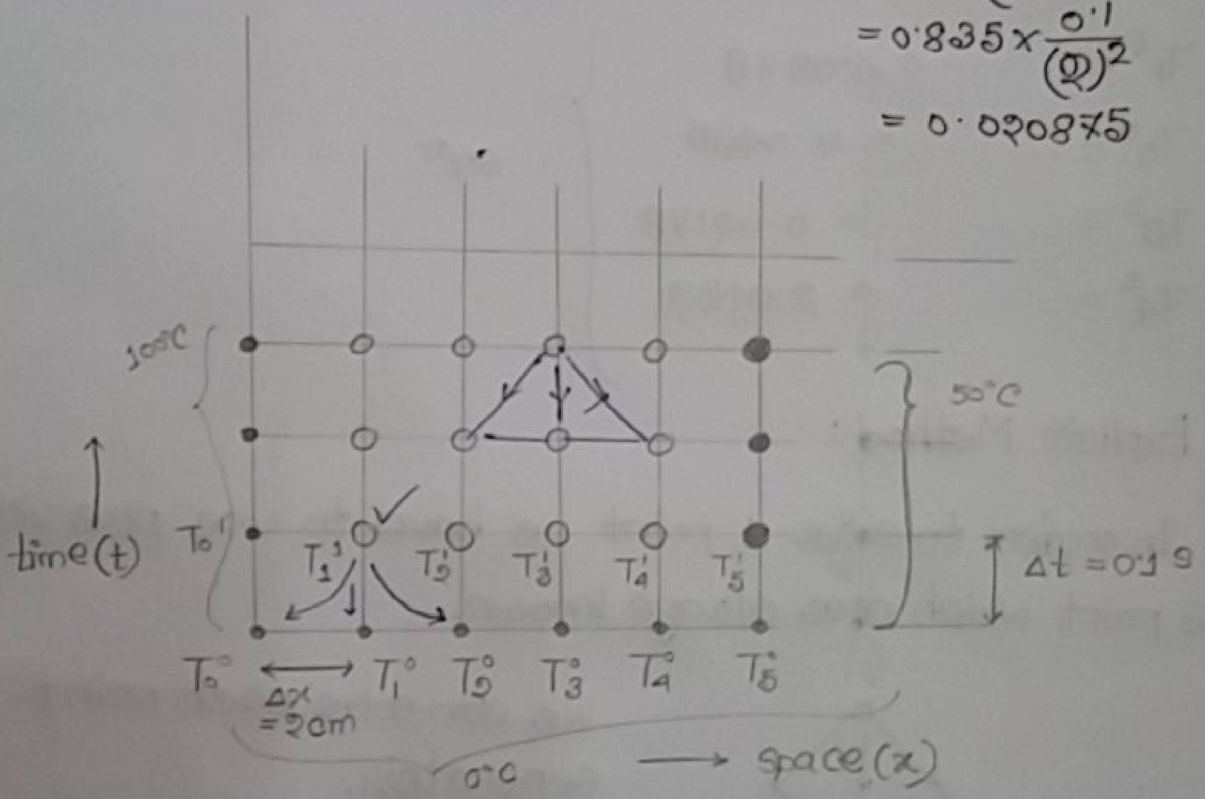
where, $\lambda = K \frac{\Delta t}{(\Delta x)^2}$ (given)

$$\underbrace{T_{i,j}}_{\text{space}}^{\,l \leftarrow \text{Time}}$$

* যখন Time dimension এ derivative নিব, তখন space constant রাখব। and vice versa.

Problem : Explicit method

$$\lambda = K \frac{\Delta t}{(\Delta x)^2} = 0.835 \times \frac{0.1}{(2)^2} = 0.020875$$



@ $t = 0.1$ s and $x = 2$

$$T_1^1 = T_1^0 + \lambda\left(T_2^0 - 2T_1^0 + T_0^0\right) \quad [\, i = 1,\ l = 0 \,]$$

$$= 0 + 0.020875(0 - 2\times 0 + 100)$$

$$= 2.0875$$

@ $x = 4$cm $(i = 2, l = 0)$

$$T_2' = 0 + 0.020875\,[0 - 2\times 0 + 0] = 0$$

<u>$x = 6$ cm</u>

$$T_3' = \ldots\ldots = 0$$

<u>$x = 8$ cm</u>

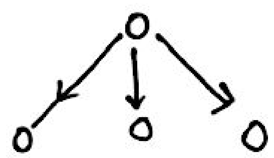
$$T_4' = 0 + 0.020875\,[50 - 2\times 0 + 0] = 1.0438$$

Solving for $t=0.2$ sec. using the sol^n of $t=0.1s$,

$$\left.\begin{aligned} T_1^2 &= \ldots\ldots = 4.0878 \\ T_2^2 &= \ldots\ldots = 0.0435 \\ T_3^2 &= \ldots\ldots = 0.02178 \\ T_4^2 &= \ldots = 2.0439 \end{aligned}\right\} sol^n$$

Explicit Method:

In order to solve 1 point we have to use previous 3 point which are always known.



We can solve each point seperately.

Implicit Method: (not included in syllabus)

we can't solve each point seperately. we have to solve system of linear eq^n in this method.

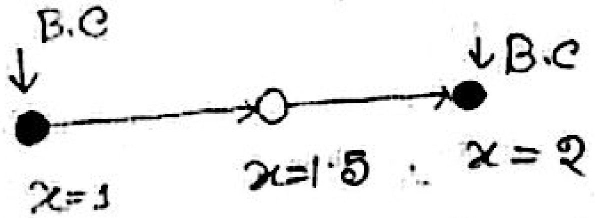
Tanvir Sir

Boundary value problem:

Solve the diff. eqn $\frac{d^2y}{dx^2} = 6x$, within the interval $(1, 2)$
use $h = 0.5$.

$\underbrace{y(1) = 2,\ y(2) = 9}_{\text{boundary condition}}$

Soln



1st method (Higher order এর জন্য এটা use হয়েছে)

$x = 1.5,\ y = ?$ .... (1)

assume $\frac{dy}{dx} = z,\ y(1) = 2$

$\frac{dz}{dx} = 6x,\ \underbrace{z(1) = y'(1) = ?}_{\text{assume} = 2}$ .... (2)

Iter-1

$M_1(1) = 2$ $\qquad M_2(1) = 5$

$M_1(2) = 6$ $\qquad M_2(2) = 9$

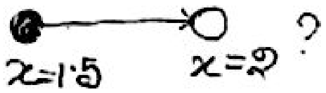
$M(1) = \frac{2+5}{2} = 3.5$

$M(2) = \frac{6+9}{2} = 7.5$

$y(1.5) = y(1) + 3.5 \times 0.5 = 3.75$

$z(1.5) = z(1) + 7.5 \times 0.5 = 5.75$

Iter-2



$M_1(1) = 5.75$ $M_2(1) = 10.25$

$M_1(2) = 9$ $M_2(2) = 12$

$M(1) = 8$ $M(2) = 10.5$

$y(2.0) = \ldots\ldots = \boxed{7.75} < 9$

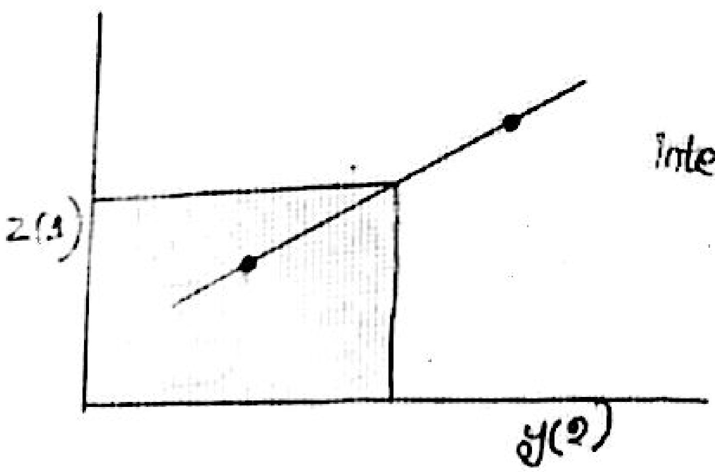
$z(2) = \ldots$

So, the assumption was not correct.

Again, Let, $z(1) = 4$,

Doing the same thing we get, $y(2.0) = \ldots = \boxed{9.75} > 9$

So, this assumption is also not correct.



Interpolate করে

$M_1 \left\{ Z(1) = 2 \longrightarrow y(2) = 7.75 \right\} B_1$

$M_2 \left\{ Z(1) = 4 \longrightarrow y(2) = 9.75 \right\} B_2$

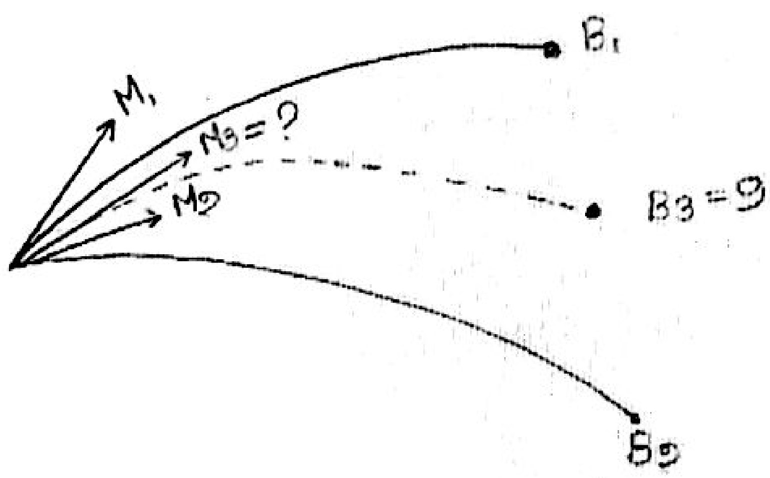
$M_3 \left\{ Z(1) = ? \longleftarrow y(2) = 9 \right\} B_3$

$$\frac{M_3 - M_2}{B_3 - B_2} = \frac{M_2 - M_1}{B_2 - B_1}$$

~~If B3~~ putting values, we get, $\boxed{M_3 = 3.25}$

Now using $z(1) = 3.25$ to solve the eqn

$$\left.\begin{aligned} y(1) &= 2 \\ y(1.5) &= 4.375 \\ y(2) &= 9 \end{aligned}\right] \text{solution}$$



"shooting method"

Exam এ
বিভিন্ন value assume করে নিয়ে value বের করতে দিবে।