

## Syllabus

1. Concept of Stress and Strain.
2. Constitutive Relationship.
3. Deformation due to tension, compression & temperature change.
4. Beam statics: Reactions, Axial force, Shear force, bending moments, dia.
5. Axial force, shear force & bending moments diagram — using method of section and summation approach.
6. Elastic Analysis of circular shaft, solid non-circular and thin walled tubular members subjected to torsion.
7. Flexural and Shear Stresses in Beams, Shear centre.
8. Thin pressure vessels.

## Lec-2

## Mechanics of Solids

Sat  
16.12.13

### Purpose:

To design the component, parts of structure in order to resist the actual / Probable forces.

### Examples:

- 1) Wall thickness of pressure vessels, such as - Water tank, Gas cylinder.
- 2) Adequate floor size for intended use.
- 3) Size of machine shaft for torque.
- 4) Wing of an Aeroplane to withstand Aerodynamic forces in flight, Take off, Landing.
- 5) Deflection  $\rightarrow$  must not exceed certain level.
- 6) Failure through Elastic Instability (Collapse through buckling due to slenderness under compressive load).

\* We do not know the Actual load. So all calculation is done by probable load.

\* National building code.

### Scope:

In Engineering practice all above will be satisfied with minimum expenditure of a given material. Besides, weight of the package may determine success of mission.

### Goal:

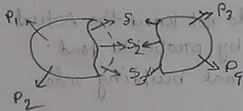
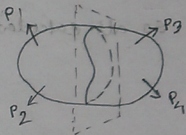
Mechanics of Solids involves Analytical Methods for determining 3 important condition -

1. Strength.
2. Stiffness.
3. Stability.

## Important contributors:

- Indian → Galileo → Early 17<sup>th</sup> century
- French → Coloumb  
Poisson  
Navier  
ST. Venant  
Cauchy } → 19<sup>th</sup> century
- English → R. Hooke  
Thom. Young } → Late 17<sup>th</sup> century
- German → Otto Mohr → 19<sup>th</sup> century.

## Method of section:



Section of a body

## Basic approach:

1. Isolate a member.  
Draw free body diagram of a whole member.
2. Determine reaction using equations of statics or boundary condition.  
For indeterminate structures statics is supplemented with kinematic condition.

3. Pass a section perpendicular to the member axis at the location of interest.

axis  $\rightarrow$  Locs of the CG of the cross section of is the axis. Axis is parallel to the direction of the length of the member.

Isolate any one part of the segment with all forces.

4. Determine these internal forces using equation; condition of the isolated segment.

\* Beam length  $\rightarrow$  দিক হ্রাসক ফিক শাকলে deflection হবনি-  
রলে section বাড়াতে হবে

Lec-3

### Beam Statics

Subique Sunday

17.11.13

**Beam:** In many structural and machine design, members must resist forces applied laterally or transversely to their axes. Such members are called Beams. The main members of supporting floors of buildings are beams.

**N&B** - column - Axial loaded member. Load transmits along the axis.

Moment  $\perp$  Force  $\times$  Perpendicular direction.

Mechanics  $\rightarrow$  deals with forces

Solid "  $\rightarrow$  deals with forces of solid materials.

Statics  $\rightarrow$  3 equations

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

$$\sum_x M = 0$$

$$\sum_y M = 0$$

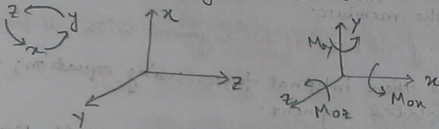
$$\sum_z M = 0$$

} Forces are equilibrium

Statistic law for 3 dimensional body.  
 Moment along the z axis.

All most 2 dimension.

We always use right hand rule.

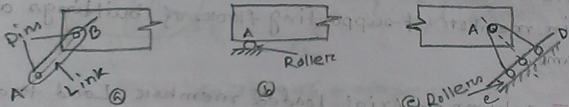


### Diagrammatic Conventions for support:

#### Types of supports:

Three types of supports are recognised for planar structures.

1. Link support - It is capable of resisting a force in only one specific line of action.



In fig (a) can resist a force only in the direction of line AB. In the fig (b) can resist only a vertical force whereas the rollers in fig (c) can resist only a force that acts perpendicular to the plane CD.

A roller support is capable of resisting a force in either direction along the line of action of the reaction.

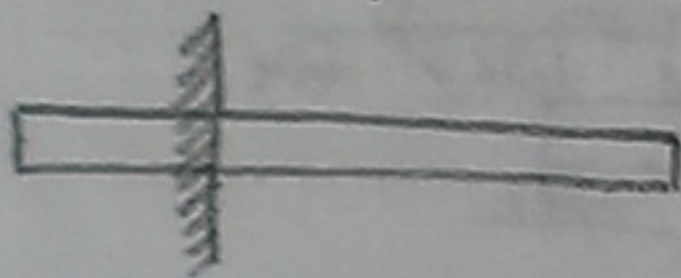
## 2. Pin support

A pinned support is capable of resisting a force acting in any direction of the plane. Hence, in general the reaction component at such a support may have two components. One is horizontal and the other is vertical direction.



## 3. Fixed support

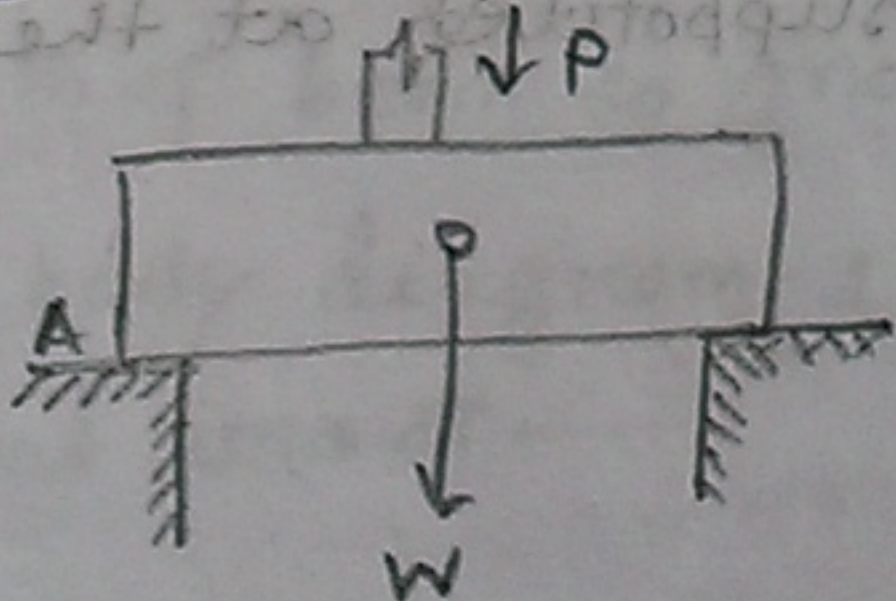
This type of support is able to resist a force in any direction and is also capable of resisting a moment or a couple. A system of three forces can exist at such a support, two component of force and moment.



## Diagrammatic convention for loading

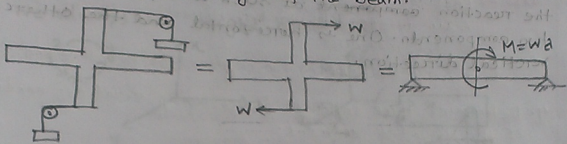
### 1. Concentrated forces

A force is applied to a beam through a post, a hanger, or a bolted detail. Such arrangements apply the force over a very limited portion of the beam and are idealized for the purposes of beam analysis as concentrated forces.



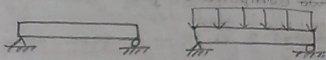
## 2. Uniformly distributed load:

The load is usually expressed as force per unit length of the beam.



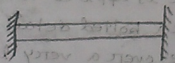
## Classification of Beam:

1. Simple supported beam: If the supports are at the ends and are either pin or rollers.

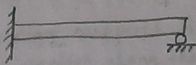


2. Fixed ended beam: The beams if the ends have

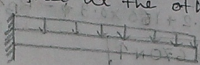
fixed supports.



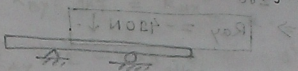
3. Restrained beam: The fixed at one end and completely free at the simply supported at the other.



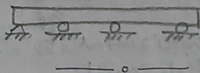
4. Cantilever beam: A beam fixed at one end and completely free at the other end.



5. Overhanging beam: If the beam projects beyond a support.



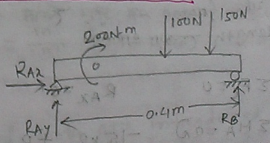
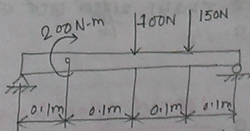
6. Continuous beam: If intermediate supports are provided for a physically continuous member acting as a beam.



### Examples

### Calculation of Beam Reactions

1.



Depth of beam is ignored

Free body diagram -  
support reaction

Reaction at pin support  
At pin, reaction passes through  
the centroid of the cross  
section.

$$\sum F_x = 0$$

$$R_{Ax} = 0$$

$$\sum M_A = 0$$

$$200 + 100 \times 0.2 + 150 \times 0.3 - R_B \times 0.4 = 0$$

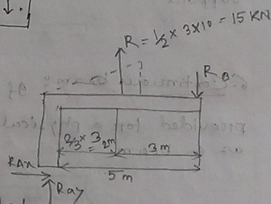
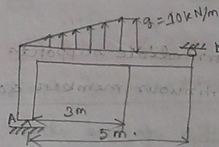
$$\therefore R_B = 670 \text{ N } \uparrow$$

$$\sum M_B = 0$$

$$200 - 100 \times 0.2 - 150 \times 0.3 + R_{Ay} \times 0.4 = 0$$

$$\Rightarrow R_{Ay} = -400 \text{ N } \downarrow$$

2.



\* All frame has a horizontal part and a vertical part.

\* The intensity of force is is varied.

\* Centroid is not at the center.

\* The frame is not a uniform force length of course.

$$\sum F_x = 0$$

$$R_{Ax} = 0$$

$$\sum M_A = 0$$

$$-15 \times 2 + R_B \times 5 = 0$$

$$\Rightarrow R_B = 6 \text{ kN } \downarrow$$

$$\sum M_B = 0$$

$$15 \times 3 + R_{Ay} \times 5 = 0$$

$$\Rightarrow R_{Ay} = -9 \text{ kN } \downarrow$$



$$\sum F_x = 0$$

$$R_{Ax} = 0$$

$$\sum M_A = 0$$

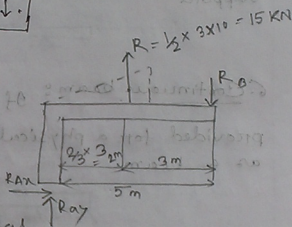
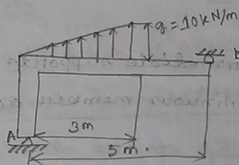
$$200 + 100 \times 0.2 + 150 \times 0.3 - R_B \times 0.4 = 0$$

$$\therefore R_B = 670 \text{ N } \uparrow$$

$$\sum M_B = 0$$

$$200 - 100 \times 0.2 - 150 \times 0.3 + R_{Ay} \times 0.4 = 0$$

$$\Rightarrow R_{Ay} = -490 \text{ N } \downarrow$$



\* All frame has a horizontal part and a vertical part.

\* The intensity of force is varies.

\* Centroid of the frame is at the center.

\* In the frame at the top force is zero at the bottom of course length is 5m.

$$\sum F_x = 0 \quad R_{Ax} = 0$$

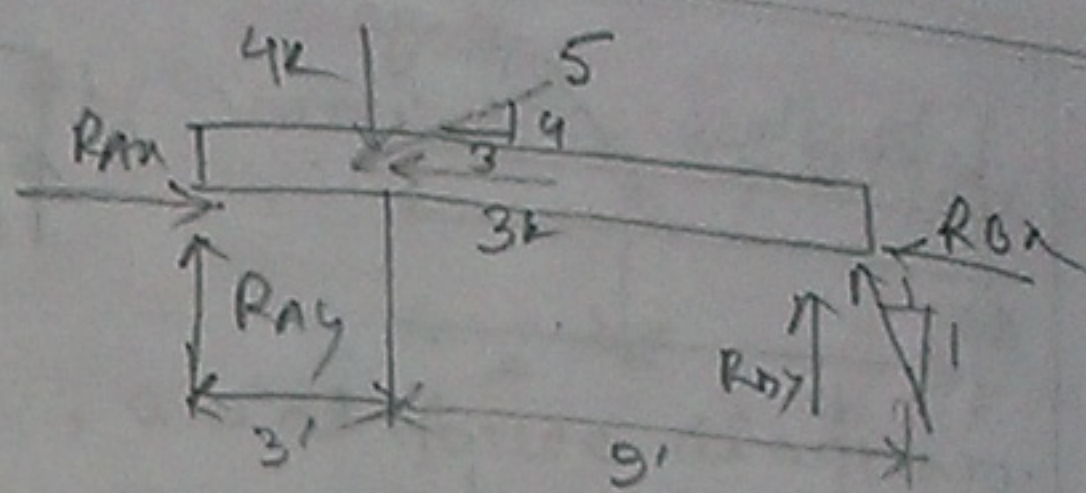
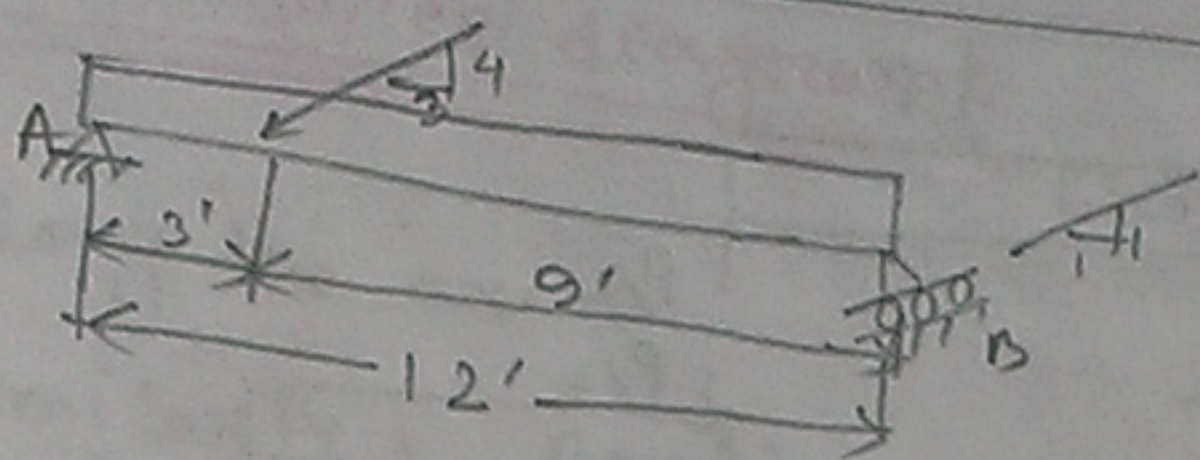
$$\sum M_A = 0 \Rightarrow -15 \times 2 + R_B \times 5 = 0$$

$$\Rightarrow R_B = 6 \text{ kN } \downarrow$$

$$\sum M_B = 0 \Rightarrow 15 \times 3 + R_{Ay} \times 5 = 0$$

$$\Rightarrow R_{Ay} = -9 \text{ kN } \downarrow$$

3.



$$\sum M_A = 0 \Rightarrow 4 \times 3 - R_{By} \times 12 = 0$$

$$\therefore R_{By} = 1k \uparrow = R_{Bx}$$

$$\sum M_B = 0 \Rightarrow R_{Ay} \times 12 - 4 \times 9 = 0$$

$$\therefore R_{Ay} = 3k \uparrow$$

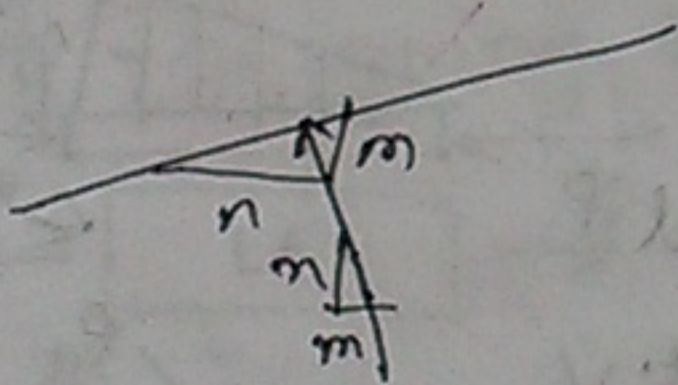
$$\sum F_x = 0 \rightarrow R_{Ax} - 3 - 1 = 0$$

$$\therefore R_{Ax} = 4k$$

$$R_A = \sqrt{4^2 + 3^2} = 5k$$

$$R_B = \sqrt{1^2 + 1^2} = \sqrt{2}k$$

$$R_{Bx} = R_{By}$$



$$\frac{R_{Bx}}{m} = \frac{R_{By}}{n}$$

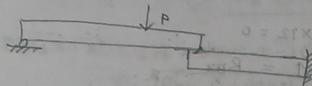
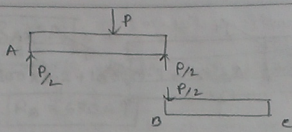
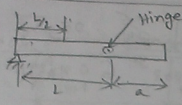
$$\therefore R_{Bx} = \frac{m}{n} R_{By}$$

Occasionally hinges:

Pinned joints are introduced into

Beam.

A hinge can transmit only horizontal and vertical forces no moment.

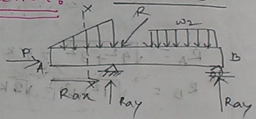


So these hinge points are locations for subdividing structures into parts.

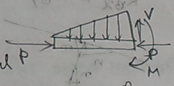
Application of the method of section:

Saturday  
23.11.13

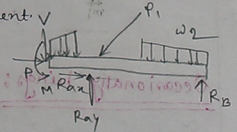
- A free body diagram.
- Compute reactions.
- Apply method of section
- Simple it statically Determinate structure.



At a section in general a horizontal force  $P$  vertical force  $V$  and moment  $M$  are necessary to maintain equilibrium —



- $P \rightarrow$  axial force
- $V \rightarrow$  Shear force
- $M \rightarrow$  Bending moment at the section



Application of method of section:

$\tau$  axis against  $t$  or moment is called twisted moment

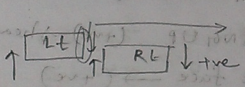
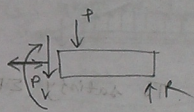
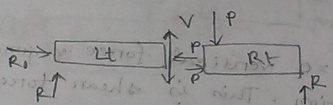
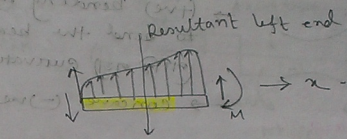
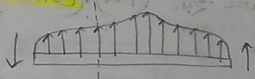
Shear force diagram:

Show the variation of shear force along the axis.

along the axis.

Similarly, Bending moment, axial force diagram.

Shear in Beams:



Rt face of left segment  
Rt face, force विरुद्ध दिशा  
-21773 (+ve)

Left → going to up (+ve)

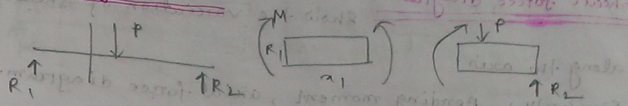
P ← Lt → P P का स्थिति का (+ve)

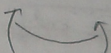

N ↑ ← → आस दिशा 2(+ve)

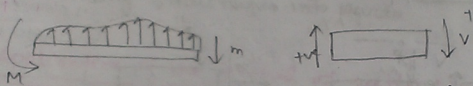
tension → (+ve)

compression → (-ve)

## Bending moment:



 → (+ve) bending moment which tries to bend the beam in such a way that (upward curvature) **(+ve) (convex)**  
 → **Concave (-ve)**



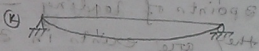
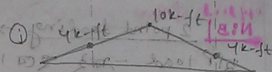
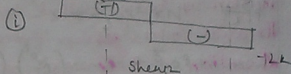
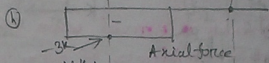
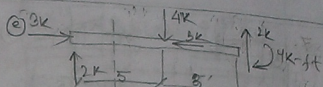
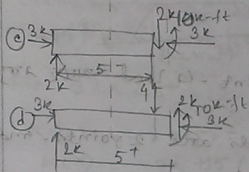
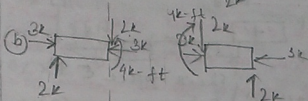
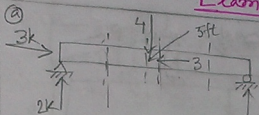
To satisfy  $\sum F_y = 0$  vertical force  $w'$  generally must exist at a section. This is shear force.

Left side → segment GOING UP (+ve) shear.

An upward force on left face → (+ve)

\* Calculation  $\sum F_y = 0$  sign convention  $\sum F_x = 0$

## Examples



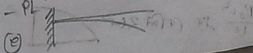
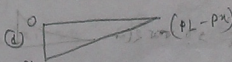
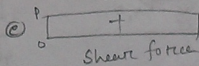
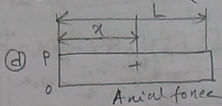
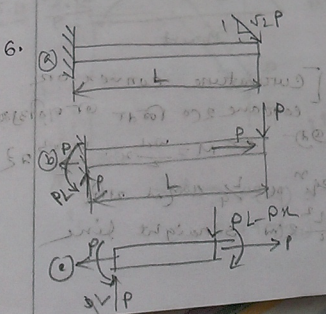
Solve:

$$0 < x < 5 \quad V = +2k$$

$$5 < x < 10 \quad V = -2k$$

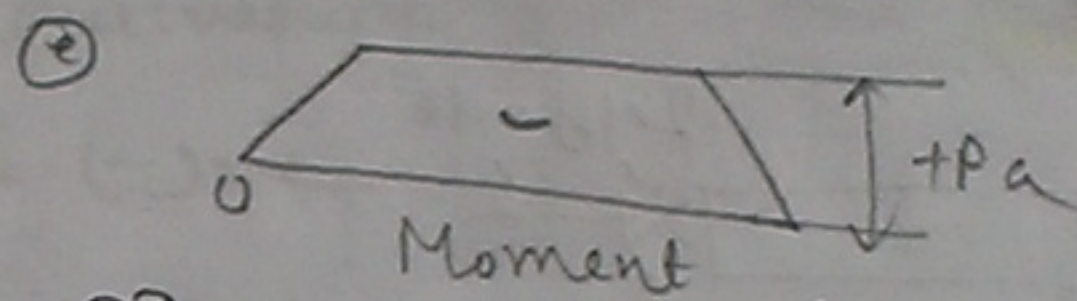
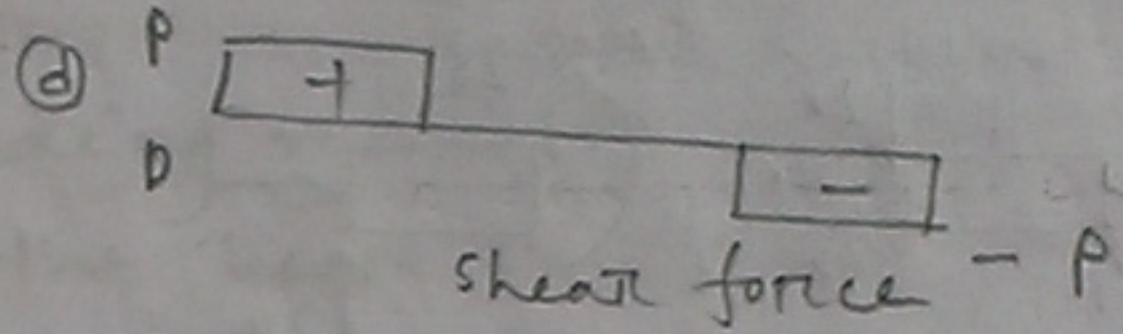
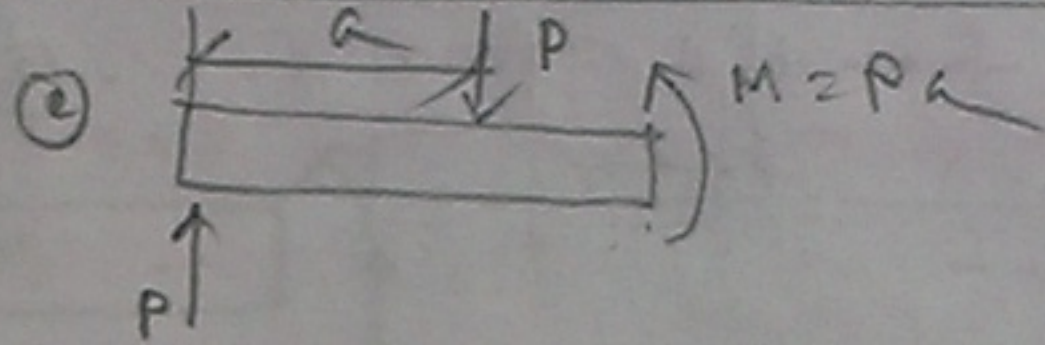
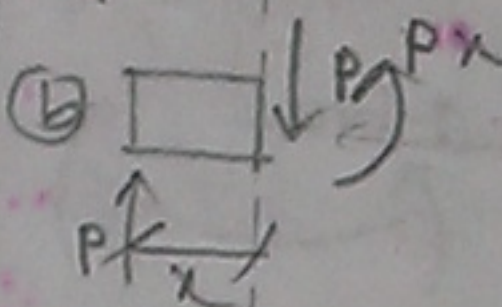
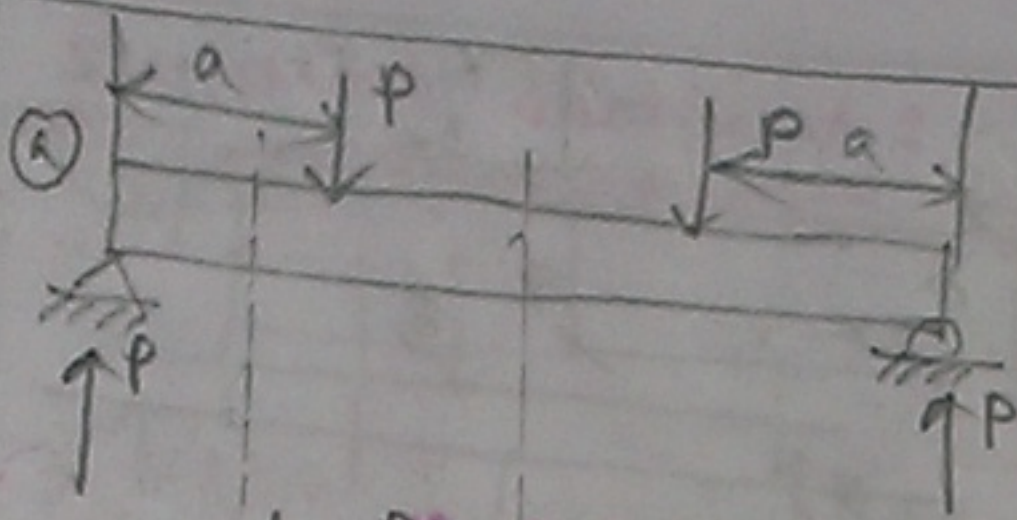
$$0 \leq x \leq 5 \quad M = 2x \text{ k-ft}$$

$$5 \leq x \leq 10 \quad M = 2x - 4(x-5) = 2x - 4x + 20 = (20 - 2x) \text{ k-ft}$$



Examples:

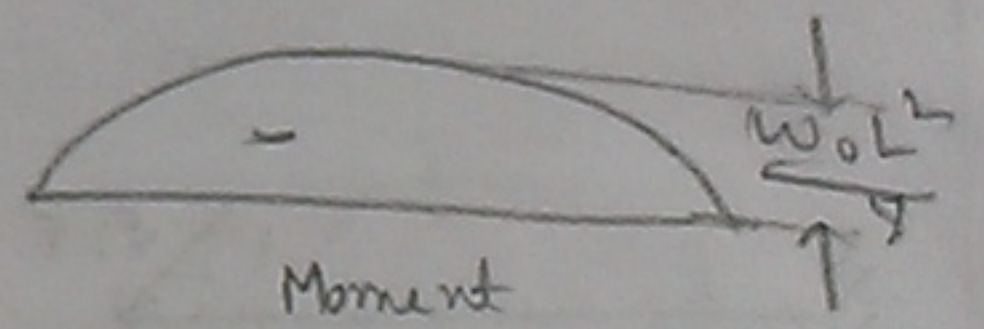
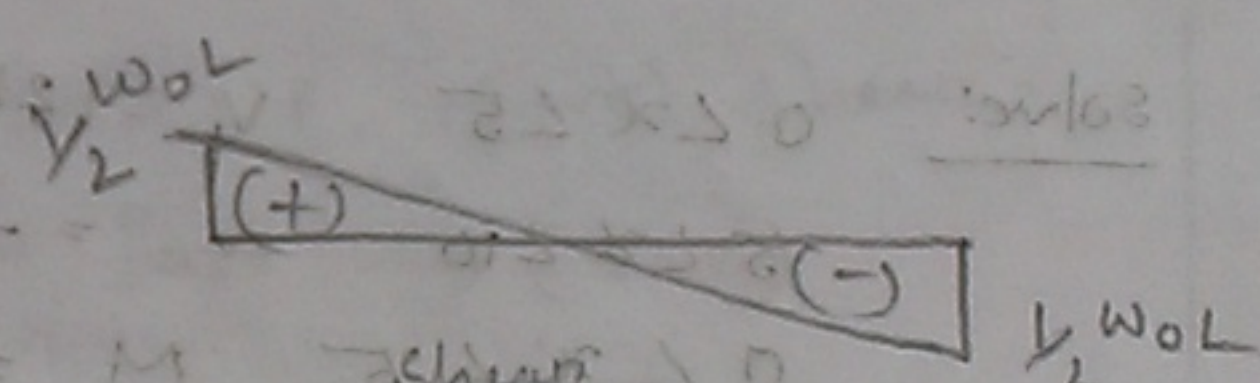
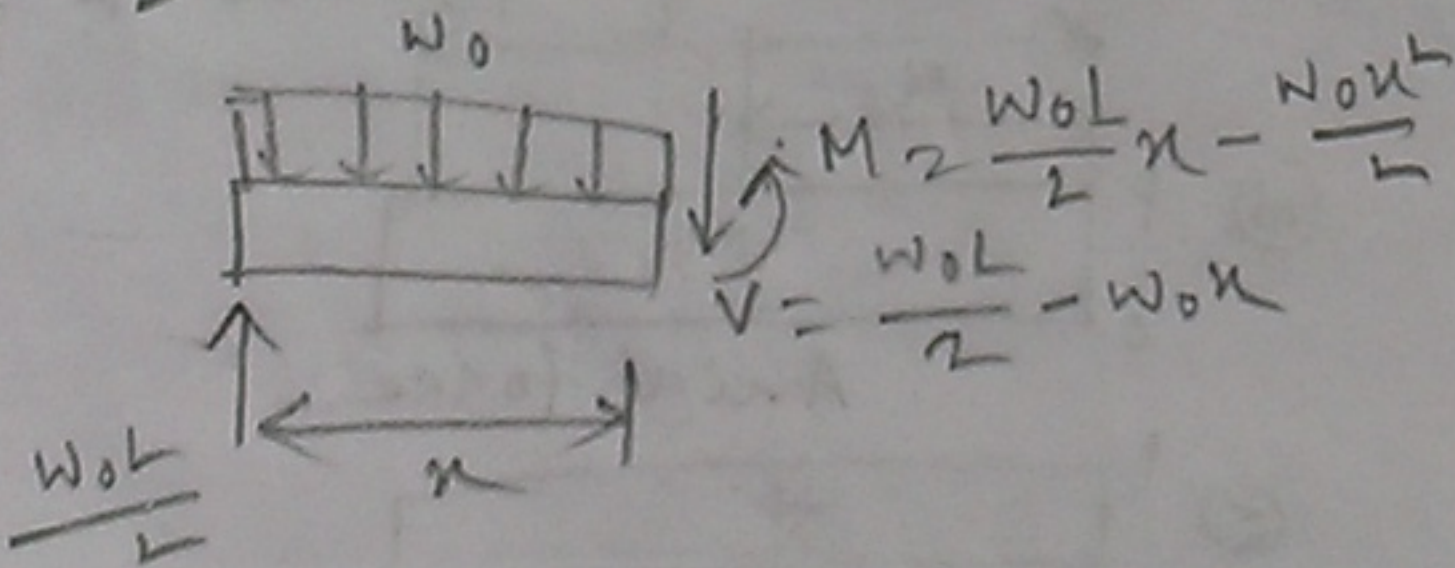
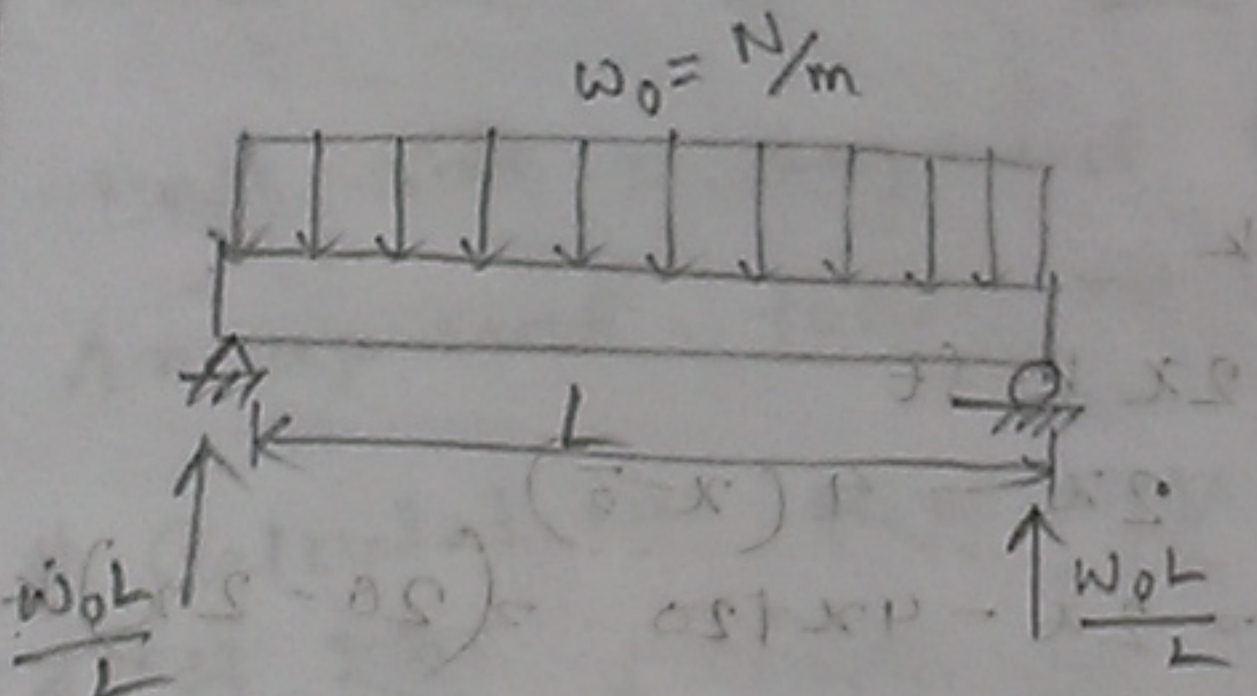
7.



N.B | 2 point loading  $\rightarrow$  2nd point - 1st load  $\rightarrow$  2nd load  $\rightarrow$  3rd point

3 points of loading  $\rightarrow$  loads are 2 points, and they are exists in 3rd point.

8.



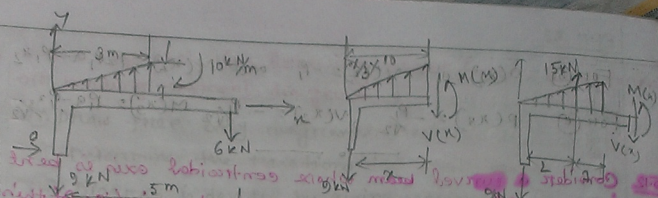
[Curvature convex or concave 2nd order or 3rd order]

eg<sup>n</sup>  $M = \frac{w_0L}{2}x - \frac{w_0x^2}{2}$  2<sup>nd</sup> order straight line

$\frac{P_0 L^2}{16}$  2<sup>nd</sup> order 2(m)

$\frac{P_0 L^2}{16}$  1<sup>st</sup> order 2(m)

5.9

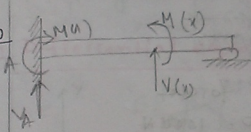


$V(x) = -9 + \frac{1}{2}x^2 \left(\frac{10}{3} \times 10\right) = -9 + \frac{5}{3}x^2 \text{ kN}$   
 $M(x) = -9x + \frac{1}{2}x \left(\frac{10}{3} \times 10\right) \left(\frac{2}{3}\right) = -9x + \frac{5}{9}x^3 \text{ kN-m}$

For  $3L \times L5$

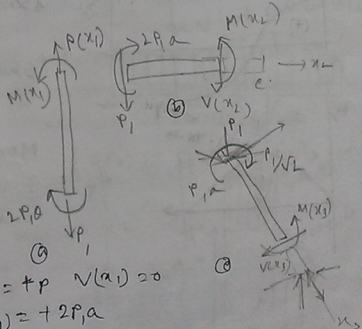
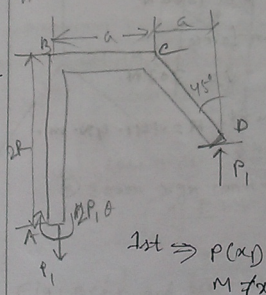
$V(x) = -9 + 15 = 6 \text{ kN}$   
 $M(x) = -9x + 15(x-2) = 6x - 30 \text{ kN-m}$

5.10



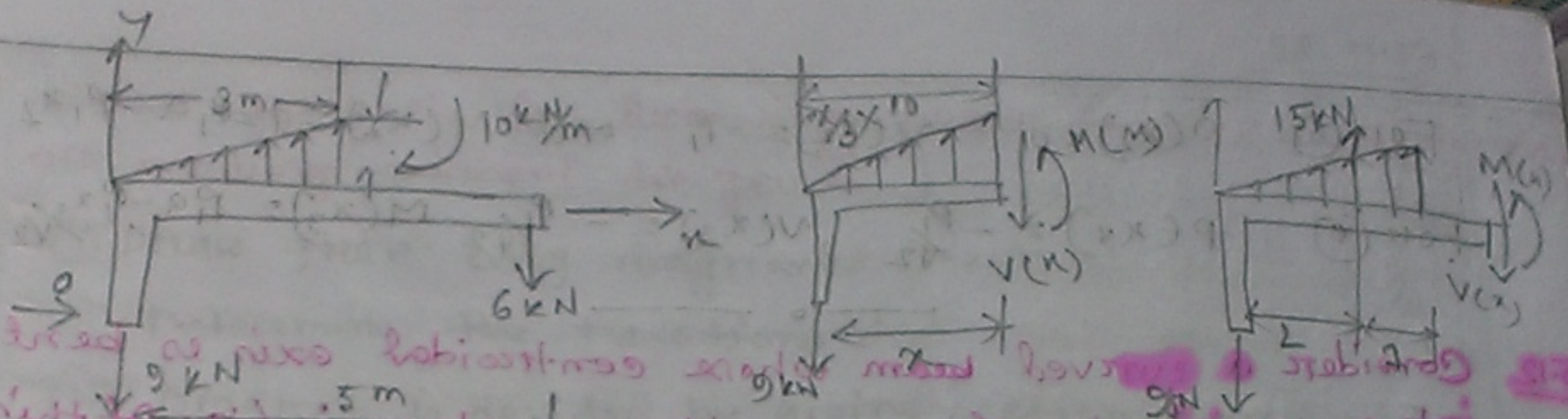
$V = V_A - w_0 x$   
 $M = M_A + V_A x - \frac{w_0 x^2}{2}$

5.11



1st  $\Rightarrow P(x_1) = +P$   $V(x_1) = 0$   
 $M(x_1) = +2P_1 a$

5-9

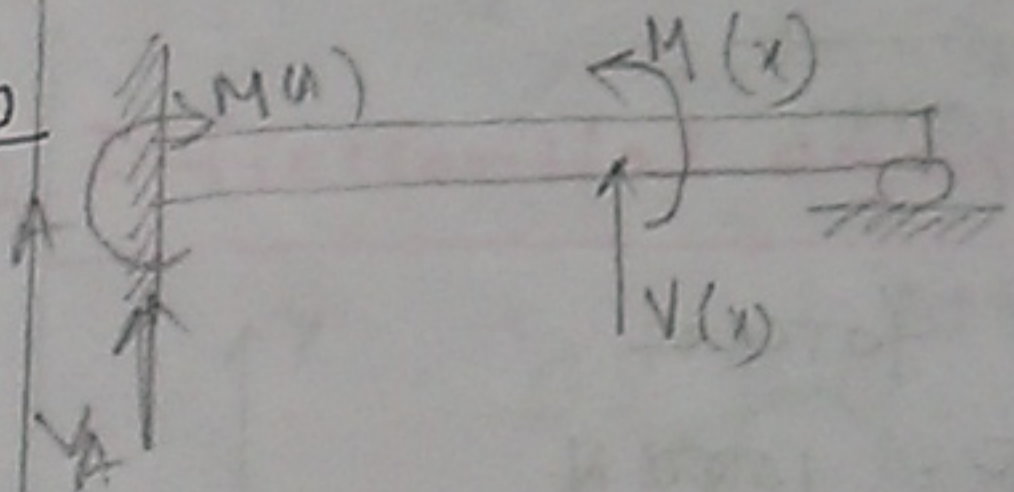


$V(x) = -9 + \frac{1}{2}x \left( \frac{2}{3} \times 10 \right) = -9 + \frac{5}{3}x^2 \text{ kN}$   
 $M(x) = -9x + \frac{1}{2}x \left( \frac{2}{3} \times 10 \right) \left( \frac{2}{3} \right) = -9x + \frac{5}{9}x^3 \text{ kN-m}$

For  $3 \times 5 \times 5$

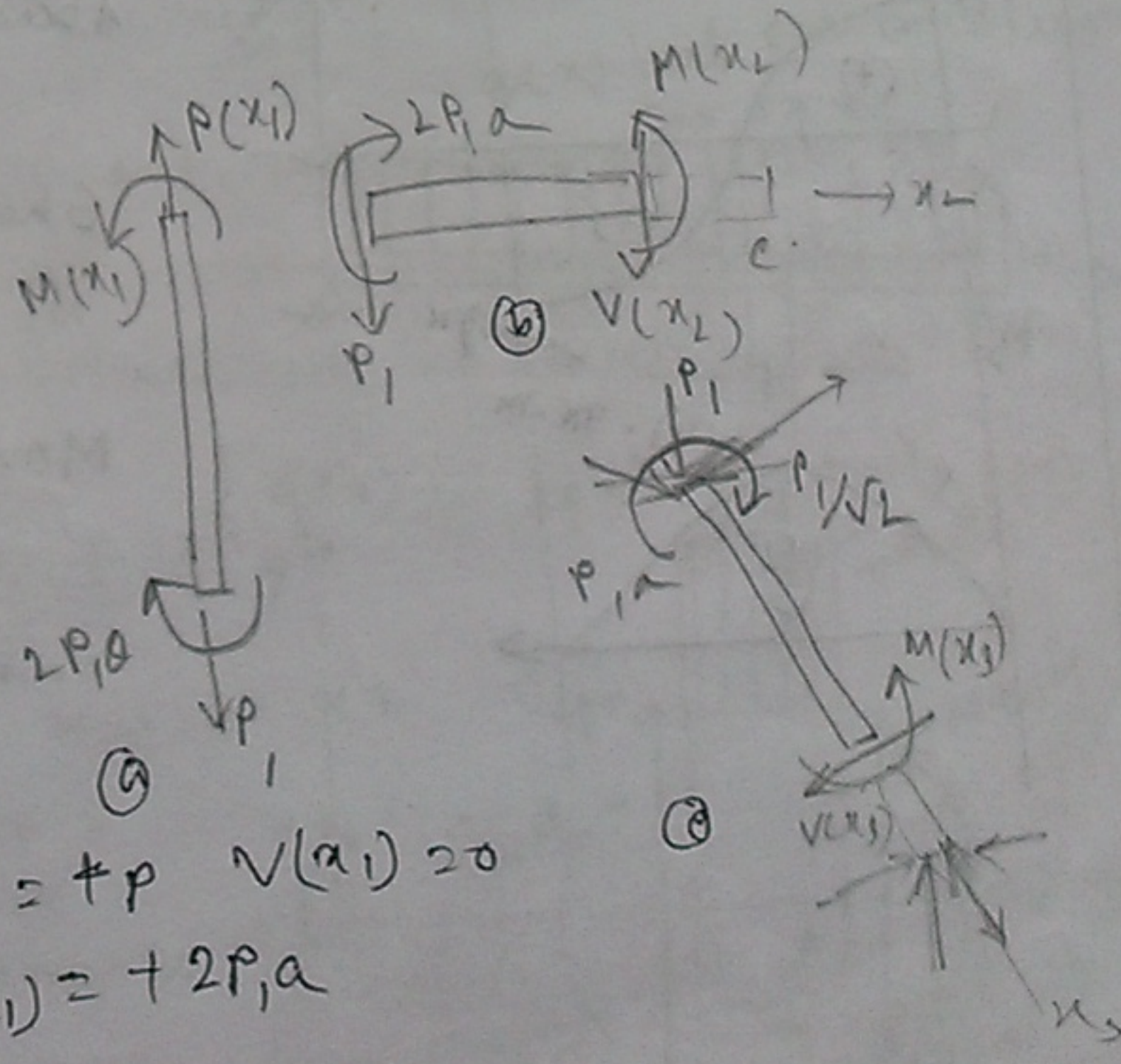
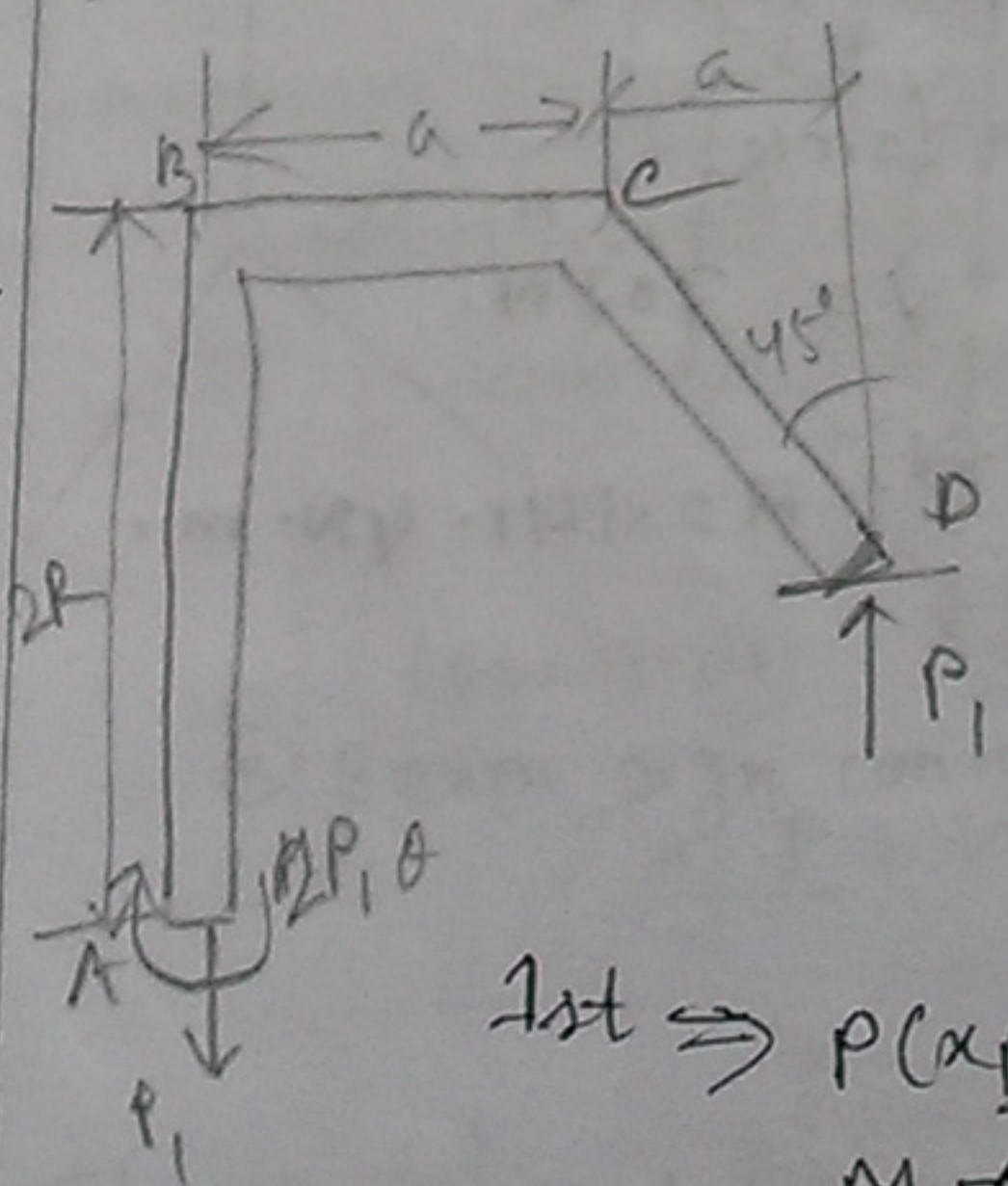
$V(x) = -9 + 15 = 6 \text{ kN}$   
 $M(x) = -9x + 15(x-2) = 6x - 30 \text{ kN-m}$

5-10



$V = V_A - w_0 x$   
 $M = M_A + V_A x - \frac{w_0 x^2}{2}$

5-11

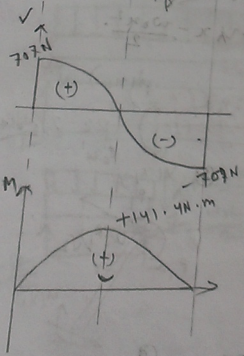
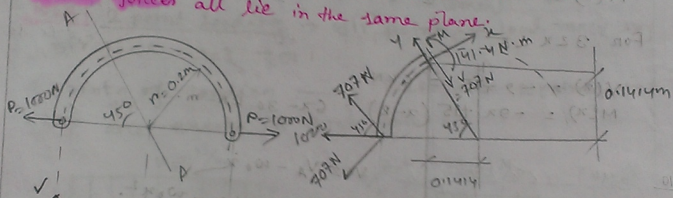


1st  $\Rightarrow P(x_1) = +P$   $V(x_1) = 0$   
 $M(x_1) = +2P_1 a$

For (b)  $P(x_2) = 0$   $V(x_2) = -P_1$  and  $M(x_2) = +2P_1a - P_1x_2$

For (c)  $P(x_3) = -\frac{P_1}{\sqrt{2}}$ ,  $V(x_3) = -\frac{P_1}{\sqrt{2}}$   $M(x_3) = P_1a - P_1x_3/\sqrt{2}$

5.12 Consider a curved beam whose centroidal axis is bent into a semicircle of 0.2 m radius, as shown in fig. If this member is being pulled by the 1000-N forces shown, find the axial force, shear force and the bending moment at section A-A  $\alpha = 45^\circ$ . The centroidal axis and the applied forces all lie in the same plane.



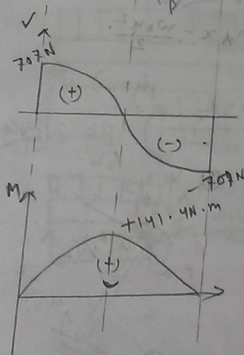
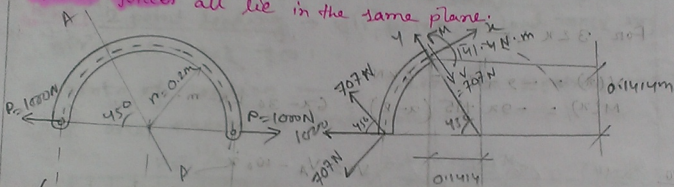
Axial force  
 $P = 1000 \text{ N}$   
 Shear force,  
 $V = 707 \text{ N}$   
 Moment,  $M = 141.4 \text{ N.m}$

For (b)  $P(x_2) = 0$   $V(x_2) = -P_1$  and  $M(x_2) = +2P_1a - P_1x_2$

For (c)  $P(x_3) = -\frac{P_1}{\sqrt{2}}$   $V(x_3) = -\frac{P_1}{\sqrt{2}}$   $M(x_3) = P_1a - P_1x_3/\sqrt{2}$

5.12

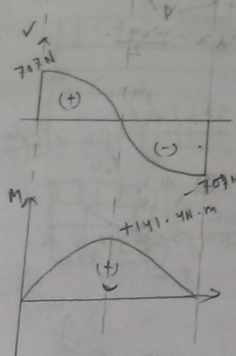
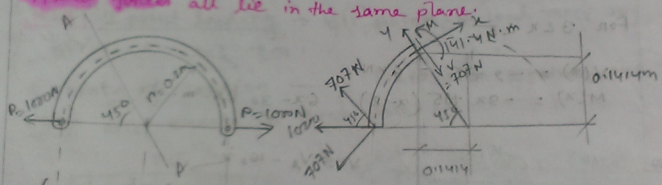
Consider a curved beam whose centroidal axis is bent into a semicircle of 0.2 m radius, as shown in fig. If this member is being pulled by the 1000-N forces shown, find the axial force, shear force and the bending moment at section A-A  $\alpha = 45^\circ$ . The centroidal axis and the applied forces all lie in the same plane.



Axial force  
 $P = 1000 \text{ N}$   
 Shear force,  
 $V = 707 \text{ N}$   
 Moment,  $M = 141.4 \text{ N.m.}$

For (b)  $P(x_2) = 0$ ,  $V(x_2) = -P_1$  and  $M(x_2) = +2P_1 a - P_1 x_2$   
 For (c)  $P(x_3) = -\frac{P_1}{\sqrt{2}}$ ,  $V(x_3) = -\frac{P_1}{\sqrt{2}}$ ,  $M(x_3) = P_1 a - P_1 x_3 \frac{1}{\sqrt{2}}$

5.12 Consider a curved beam whose centroidal axis is bent into a semicircle of 0.2 m radius, as shown in fig. If this member is being pulled by the 1000-N forces shown, find the axial force, shear force and the bending moment at section A-A at  $\theta = 45^\circ$ . The centroidal axis and the applied forces all lie in the same plane.

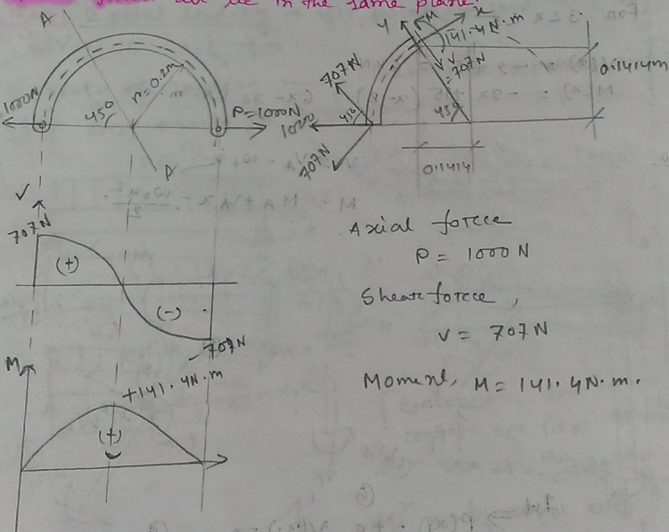


Axial force  
 $P = 1000 \text{ N}$   
 Shear force,  
 $V = 707 \text{ N}$   
 Moment,  $M = 141.4 \text{ N}\cdot\text{m}$ .

For (b)  $P(x_2) = 0$ ,  $V(x_2) = -P$ , and  $M(x_2) = +2P_1a - P_1x_2$

For (c)  $P(x_3) = -\frac{P_1}{\sqrt{2}}$ ,  $V(x_3) = -\frac{P_1}{\sqrt{2}}$ ,  $M(x_3) = P_1a - P_1^2 \frac{x_3}{\sqrt{2}}$

Consider a curved beam whose centroidal axis is bent into a semicircle of 0.2 m radius, as shown in fig. If this member is being pulled by the 1000-N forces shown, find the axial force, shear force and the bending moment at section A-A  $\alpha = 45^\circ$ . The centroidal axis and the applied forces all lie in the same plane.



Axial force  
 $P = 1000 \text{ N}$

Shear force,  
 $V = 707 \text{ N}$

Moment,  $M = 141.4 \text{ N.m.}$

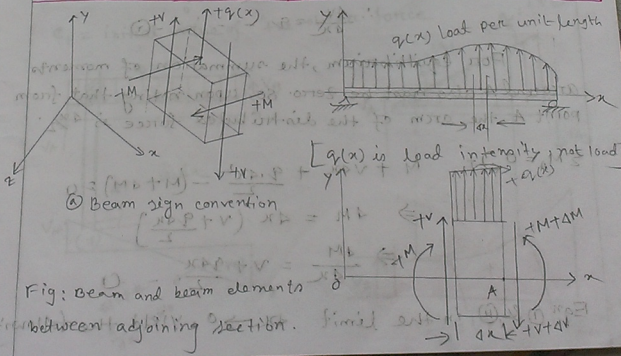
**Basic approach for drawing shear force, axial force or bending moment diagram.**

- (i) Draw free body diagram.
- (ii) Determine the reactions.
- (iii) Diagram is divided by giving section mark and work is done by taking one of the section.
- (iv)  $\sum F_x = 0$ ,  $\sum F_y = 0$  &  $\sum M_z = 0$ .

**N.B** In the case of Beam it has no relation with the Axial force and Bending moment but relation has for the other case.

Otherwise it has relation with the shear force & Bending moment. The

**Differential equations of equilibrium for a beam element**



**Beam sign convention**

Fig: Beam and beam elements between adjoining sections.

Consider a beam element  $\Delta x$  long, isolated by two adjoining sections taken perpendicular to its axis. In figure such an element is shown as a free-body diagram. All the forces shown acting on this element have positive sense. The positive sense of the distributed external force  $q$  is taken to coincide with the direction of positive  $y$  axis. As the shear force & the moment may each change from one section to the next note that on the right side of the element these quantities are, respectively designated  $V + \Delta V$  and  $M + \Delta M$ .

From the condition for equilibrium of vertical forces, one obtains -

$$\sum F_y = 0 \rightarrow V + q\Delta x - (V + \Delta V) = 0$$

$$\Rightarrow \Delta V = q \Delta x$$

$$\therefore \frac{\Delta V}{\Delta x} = q \quad \text{--- (1)}$$

For equilibrium, the summation of moments around A also must be zero. So, upon noting that from point A the arm of the distributed force is  $\Delta x/2$ :

$$\sum M_A = 0 \curvearrowright + M + V\Delta x + \frac{q \cdot \Delta x^2}{2} - (M + \Delta M) = 0$$

$$\Rightarrow \Delta M = \Delta x \left( V + \frac{q\Delta x}{2} \right)$$

$$\Rightarrow \frac{\Delta M}{\Delta x} = V + \frac{q\Delta x}{2} \quad \text{--- (2)}$$

Eqn (1) & (2) in the limit  $\Delta x \rightarrow 0$ , yield the following

two basic differential equations — integration: Example 11.1

$$\frac{dv}{dx} = q \quad \text{--- (ii)} \quad \& \quad \frac{dM}{dx} = v \quad \text{--- (iv)}$$

From (iii) & (iv)

$$\frac{d}{dx} \left( \frac{dM}{dx} \right) = q \quad \Rightarrow \quad \frac{d^2M}{dx^2} = q.$$

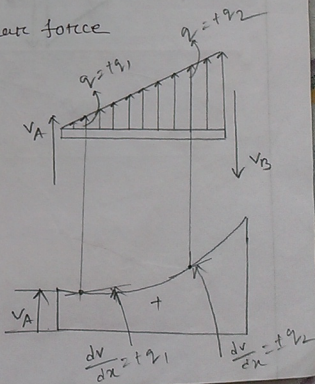
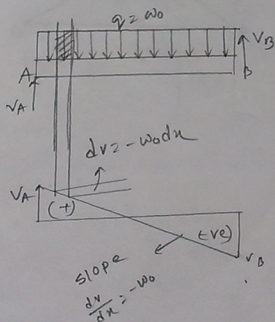
This differential equation can be used for determining reactions of statically determinate beam from the boundary conditions, whether these eqn are very convenient for construction of shear & moment diagram.

Shear diagrams by integration of the load:

From the equation 3 —

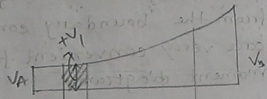
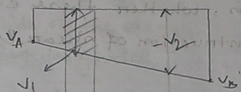
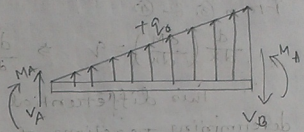
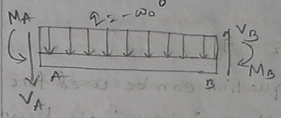
$$v = \int q dx + C_1$$

$C_1$  = initial value of the shear force

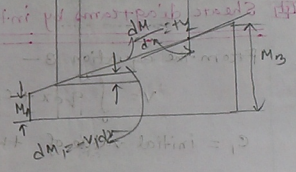
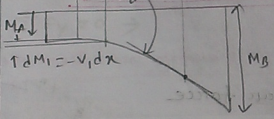


Moment diagrams by integration of the shears:

$$M = \int_0^x V dx + C_2 \quad \text{--- (1)}$$



∴ dM/dx = -V is the integration of the shears



## Shear Force and Bending moment 12-01-14

Example:

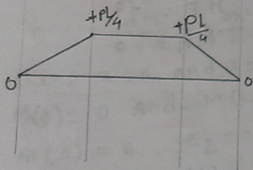
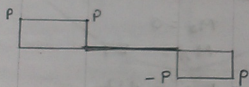
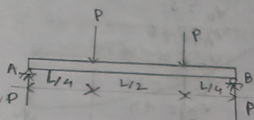
Formula:

$$V = \int P dx + c_1$$

$$M = \int V dx + c_2$$

$$\frac{dV}{dx} = P$$

$$\frac{dM}{dx} = V$$



$$V_{0+} = +P$$

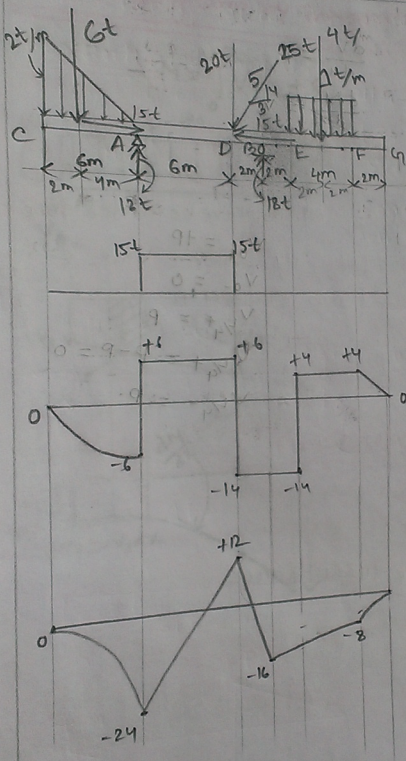
$$V_{0-} = 0$$

$$V_{L/4+} = P$$

$$V_{L/4-} = P - P = 0$$

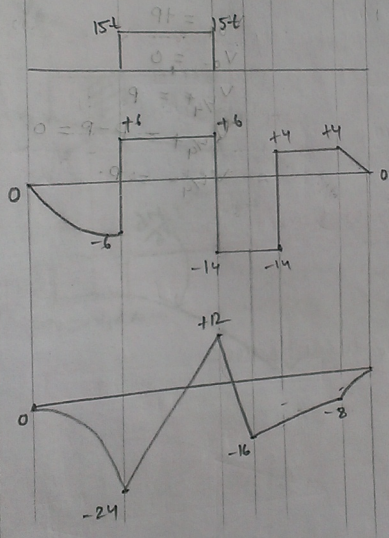
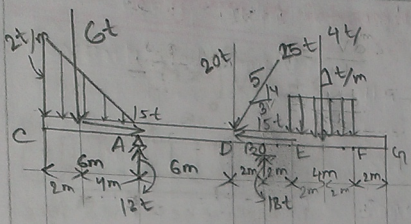
$$V_{L-} = P$$

Shear Force and Bending Moment



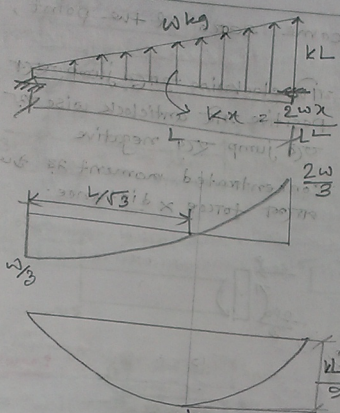
$R_{Ax} = 15t$   
 $M_A = 0$   
 $-24 + 120 - R_B \times 8 + 4 \times 12 = 0$   
 $R_B = 18t$   
 $V_0 = 0$   
 $V_{A+} = +6$   
 $V_{A-} = -6$   
 $V_D = -14$   
 $V_{E+} = +4$   
 $V_{E-} = 0$   
 $M_0 = 0$   
 $M_A = -24$   
 $M_D = +12$   
 $M_E = -16$   
 $M_F = -8$   
 $M_G = 0$

18.01.21 Shear force and bending moment



$R_{Ax} = 15t$   
 $M_A = 0$   
 $-24 + 120 - R_B \times 6 + 4 \times 12 = 0$   
 $R_B = 18t$   
 $V_0 = 0$   
 $V_{A+} = +6$   
 $V_{A-} = -6$   
 $V_D = -14$   
 $V_{E+} = +4$   
 $V_{E-} = 0$   
 $M_0 = 0$   
 $M_A = -24$   
 $M_D = +12$   
 $M_E = -16$   
 $M_F = -8$   
 $M_G = 0$

13.01.14



$\frac{1}{2} \cdot kL \cdot L = w$   
 $\Rightarrow k = \frac{2w}{L}$   
 $\frac{dM}{dx} = V$      $\frac{dV}{dx} = p$   
 $\frac{d^2M}{dx^2} = p$  (load intensity)  
 $x=0$  to  $x=L$  continuous load  
 \* Math - A unit change  
 का 2 unit का, 2 unit  
 का (यही) 2 unit  
 का 2 का।

Solve:  $\frac{d^2M}{dx^2} = p = +kx = \frac{2w}{L}x$

$\frac{dM}{dx} = \frac{kx^2}{2} + c_1$  and  $M = \frac{kx^3}{6} + c_2x + c_3$

when,  $x=0$  and  $x=L$  then

$M(0) = 0$  and  $M(L) = 0$

$M(0) = 0$      $c_3 = 0$

Similarly  $M(L) = 0$

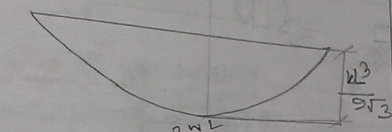
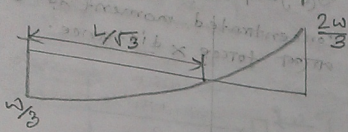
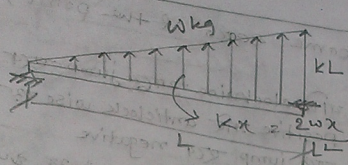
$\frac{kL^3}{6} + c_1L = 0$

$\Rightarrow c_1 = -\frac{kL^2}{6} = -\frac{2}{3}w$

$V = \frac{dM}{dx} = \frac{kx^2}{2} - \frac{kL^2}{6} = \frac{wx^2}{L^2} - \frac{w}{3}$

$M = \frac{kx^3}{6} - \frac{kL^2x}{6} = \frac{wx^3}{3L^2} - \frac{wx}{3}$

13.01.14



$$\frac{1}{2} \cdot kL \cdot L = w$$

$$\Rightarrow k = \frac{2w}{L}$$

$$\frac{dM}{dx} = V \quad \frac{dV}{dx} = -p$$

$$\frac{d^2M}{dx^2} = -p \text{ (load intensity)}$$

$x=0$  and  $x=L$  end of continuous load उत्तर  
 \* Math-A unit change  
 का मतलब है, (x) unit  
 A (यही उत्तर (x) unit  
 का मतलब है 2 का।

Solve:  $\frac{d^2M}{dx^2} = -p = -kx = -\frac{2w}{L}x$

$$\frac{dM}{dx} = -\frac{kx^2}{2} + c_1 \quad \text{and} \quad M = -\frac{kx^3}{6} + c_1x + c_2$$

when,  $x=0$  and  $x=L$  then

$$M(0) = 0 \quad \text{and} \quad M(L) = 0$$

$$M(0) = 0 \quad c_2 = 0$$

Similarly  $M(L) = 0$

$$\frac{KL^3}{6} + c_1L = 0$$

$$\Rightarrow c_1 = -\frac{KL^2}{6} = -\frac{2}{3}wL$$

$$V = \frac{dM}{dx} = -\frac{kx^2}{2} - \frac{KL^2}{6} = -\frac{wx^2}{L^2} - \frac{w}{3}$$

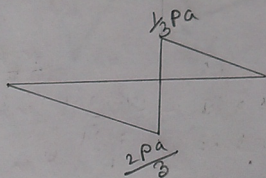
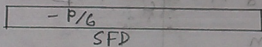
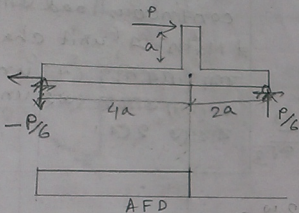
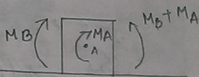
$$M = \frac{kx^3}{6} - \frac{KL^2x}{6} = \frac{wx^3}{3L^2} - \frac{wx}{3}$$

The shear force become zero at the point,

Concentrated moment:

clockwise jump, हल positive  
 anticlockwise jump, हल negative

Concentrated moment is equal to force  $\times$  distance.



$$x \frac{w}{l} = x^2 + c = 9 = \frac{11^2 - b^2}{1 - b^2}$$

$$11^2 - b^2 = 9(1 - b^2)$$

$$121 - b^2 = 9 - 9b^2$$

$$112 = -8b^2$$

$$b^2 = -14$$

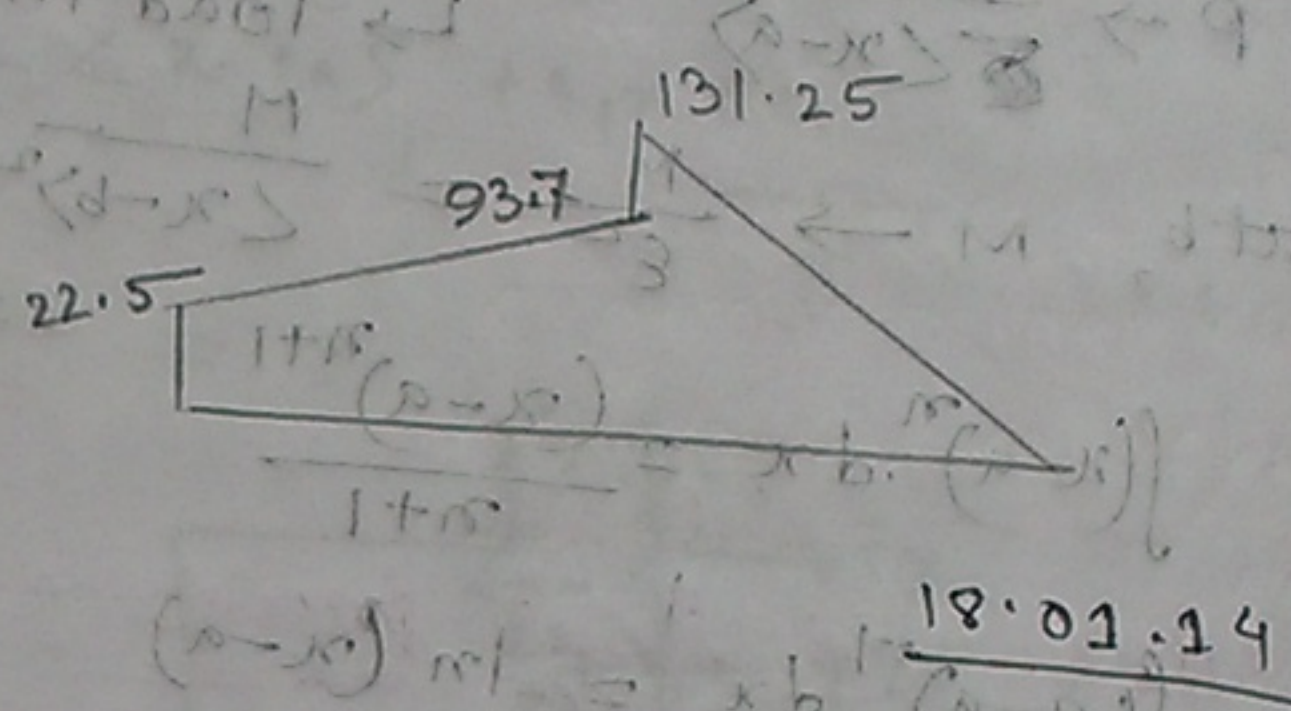
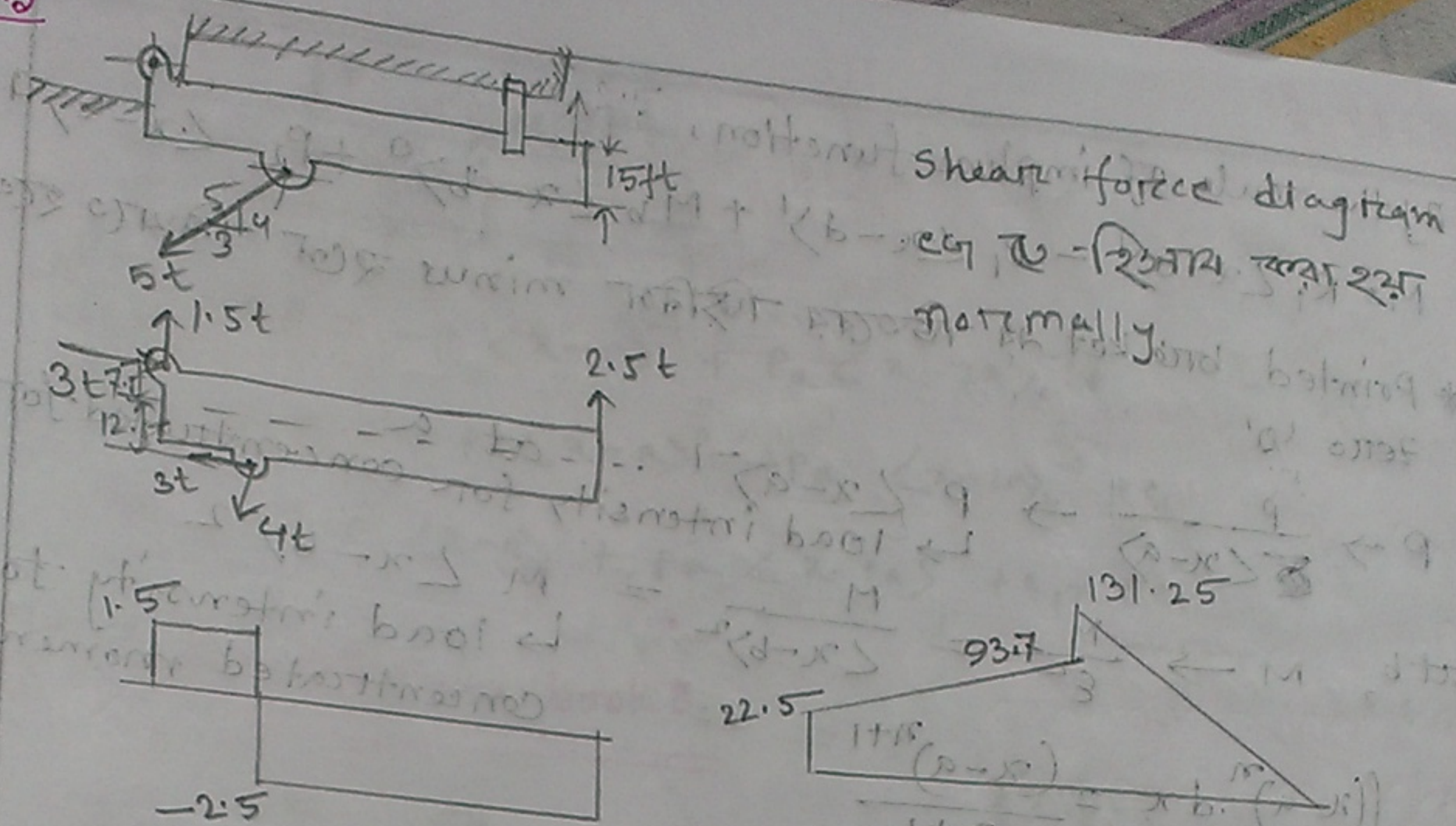
$$b = \sqrt{-14}$$

$$M = -\frac{P}{6} \times 4a = -\frac{2Pa}{3}$$

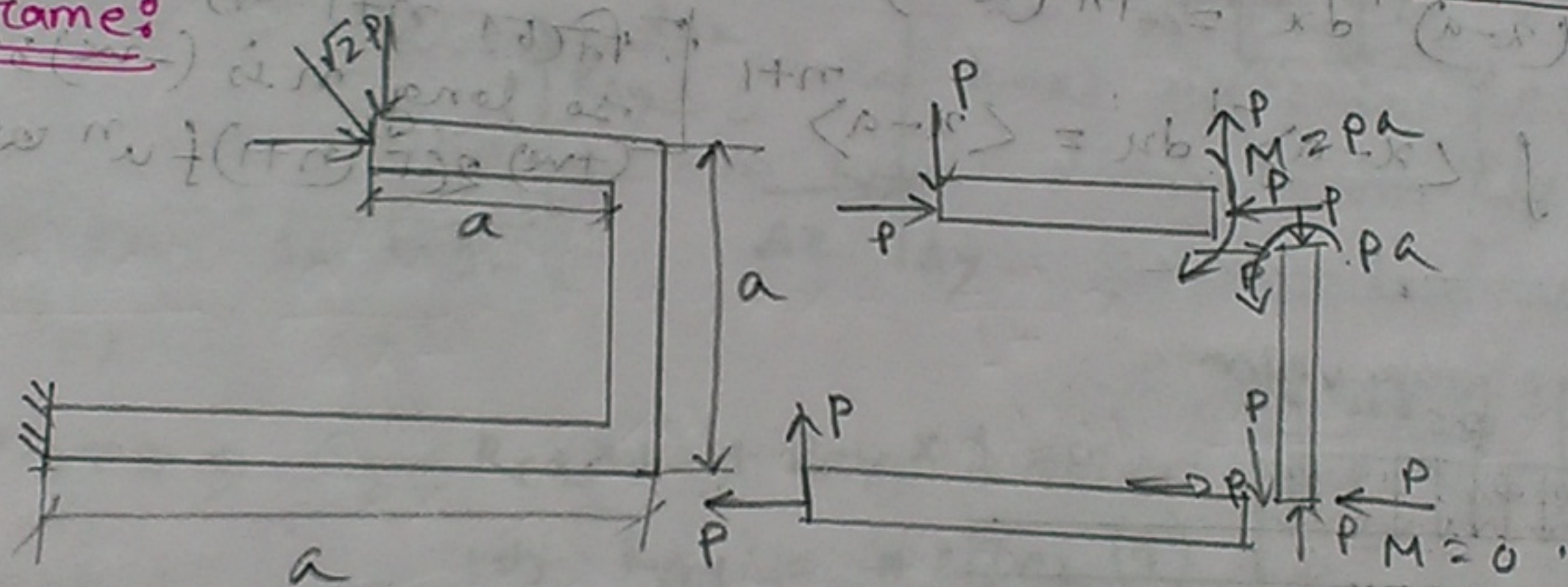
$$M_{4a} = \frac{P}{6} \times 2a = \frac{Pa}{3}$$

$M = \int v dx$  valid upto concentrated moment and beyond but not there.

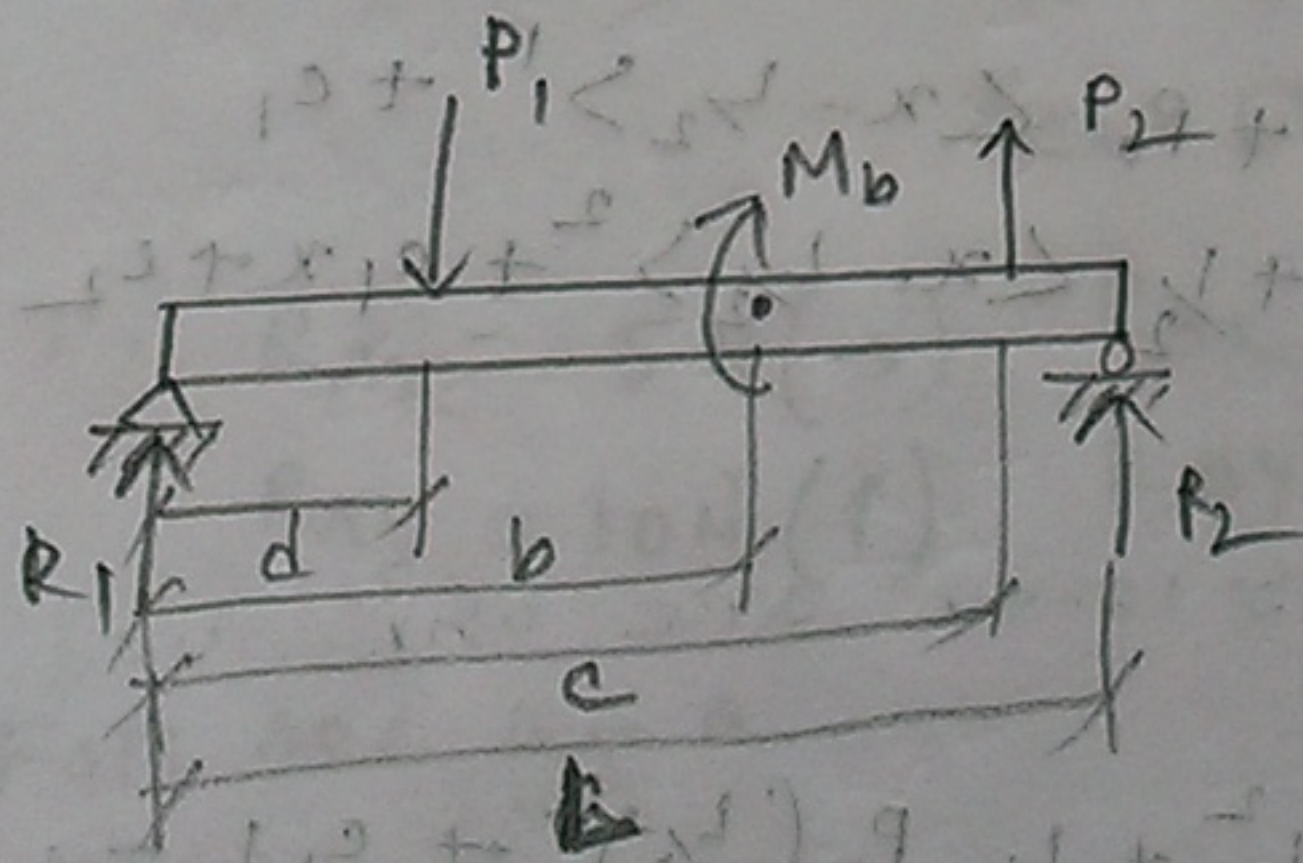
2.13



III Frame:



IV Singularity function / Impulse function:



$$M = R_1 x \quad 0 \leq x < d$$

$$M = R_1 x - P_1(x-d) \quad d \leq x < b$$

$$M = R_1 x - P_1(x-d) + M_b \quad b \leq x < c$$

$$M = R_1 x - P_1(x-d) + M_b + P_2(x-c) \quad c \leq x \leq L$$

By the rule of impulse function,

$$M = R_1 \langle x-0 \rangle^1 - P_1 \langle x-d \rangle^1 + M_b \langle x-b \rangle^0 + P_2 \langle x-c \rangle^1$$

\* Pointed bracket का डिफरेंस जितना minus इतना लिखा जा सकता है zero को

$$P \rightarrow \frac{P}{\epsilon \langle x-a \rangle} \rightarrow P \langle x-a \rangle^{-1} \quad \text{at } a$$

↳ load intensity for concentrated force

at b,  $M \rightarrow \frac{M}{\epsilon^2} = \frac{M}{\langle x-b \rangle^2} = M \langle x-b \rangle^{-2}$

↳ load intensity for concentrated moment

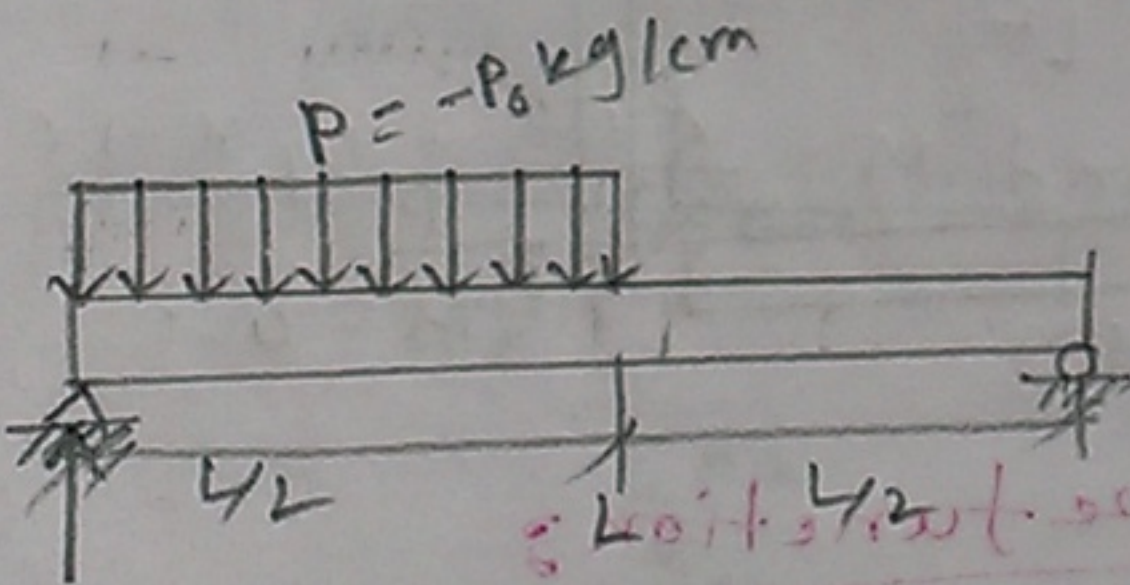
$$\int (x-a)^n dx = \frac{(x-a)^{n+1}}{n+1}$$

$$\int (x-a)^{-1} dx = \ln(x-a)$$

But  $\int \langle x-a \rangle^{-1} dx = \langle x-a \rangle^{n+1}$

त्रिकोण n+1 अंशों में है तो  
 so long n is (-ve); n is  
 (+ve) इतना (n+1) f u^n आता है

Ex



: Location of impulse function

$$\frac{d^2M}{dx^2} + p = -p_0 \langle x-0 \rangle^0 + p_0 \langle x-L/2 \rangle^0$$

$$\frac{dM}{dx} = -V = -p_0 \langle x-0 \rangle^1 + p_0 \langle x-L/2 \rangle^1 + c_1$$

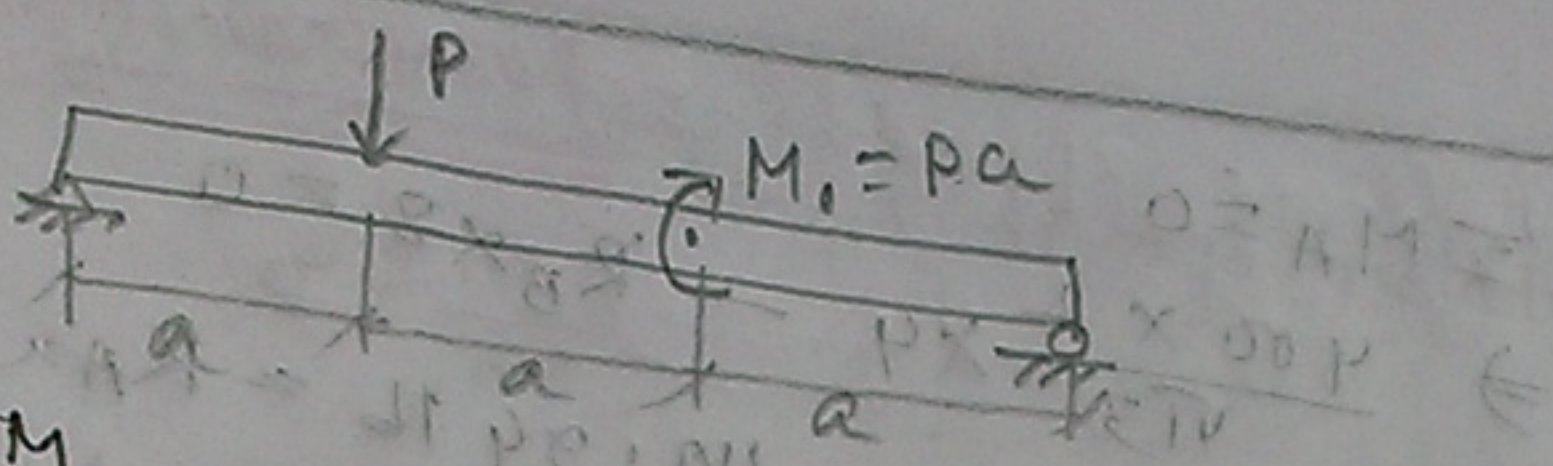
$$M(x) = -\frac{1}{2} p_0 \langle x-0 \rangle^2 + \frac{1}{2} p_0 \langle x-L/2 \rangle^2 + c_1 x + c_2$$

By boundary condition

①  $M(0) = 0$ , yields  $c_2 = 0$

②  $M(L) = 0$ , yields  $-\frac{1}{2} p_0 L^2 + \frac{1}{2} p_0 (L/2)^2 + c_1 L = 0$

$\Rightarrow c_1 = \frac{3}{8} p_0 L$

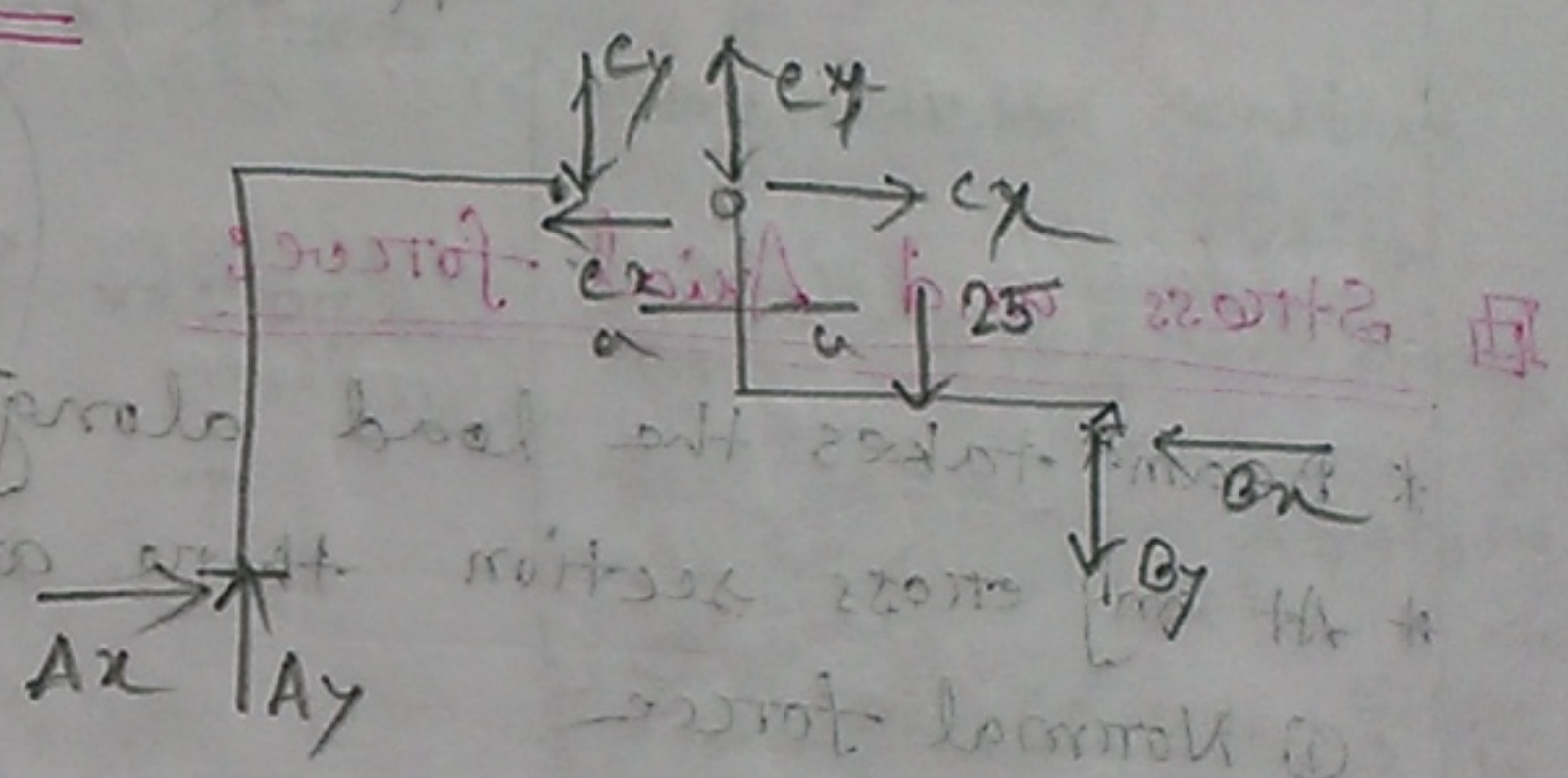
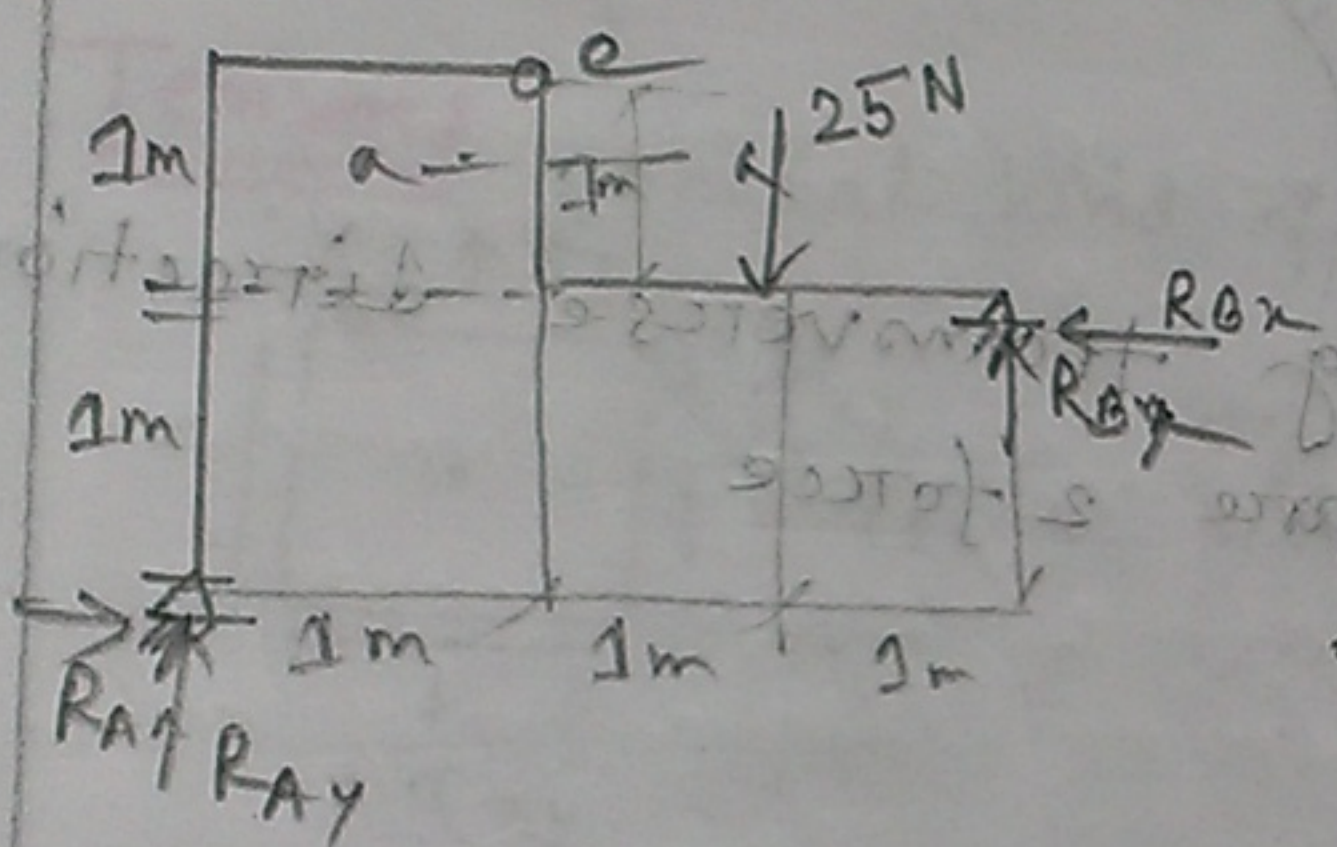


$$\frac{d^2M}{dx^2} = +P = -P \langle x-a \rangle^{-1} + Pa \langle x-2a \rangle^{-2}$$

$$\frac{dM}{dx} = -V = -P \langle x-a \rangle^0 + Pa \langle x-2a \rangle^{-1} + c_1$$

$$M = -P \langle x-a \rangle + Pa \langle x-2a \rangle + c_1 x + c_2$$

Exercises of Popov book 8



$$\sum M_A = 0 \Rightarrow -R_{Cx} \times 2 + R_{Cy} \times 1 = 0$$

$$\Rightarrow R_{Ay} = 2R_{Cx} \quad (1)$$

$$\sum M_B = 0 \Rightarrow R_{Cy} \times 1 + R_{Cx} \times 1 - 25 \times 1 = 0$$

$$\Rightarrow 4R_{Cx} + R_{Cx} = 25$$

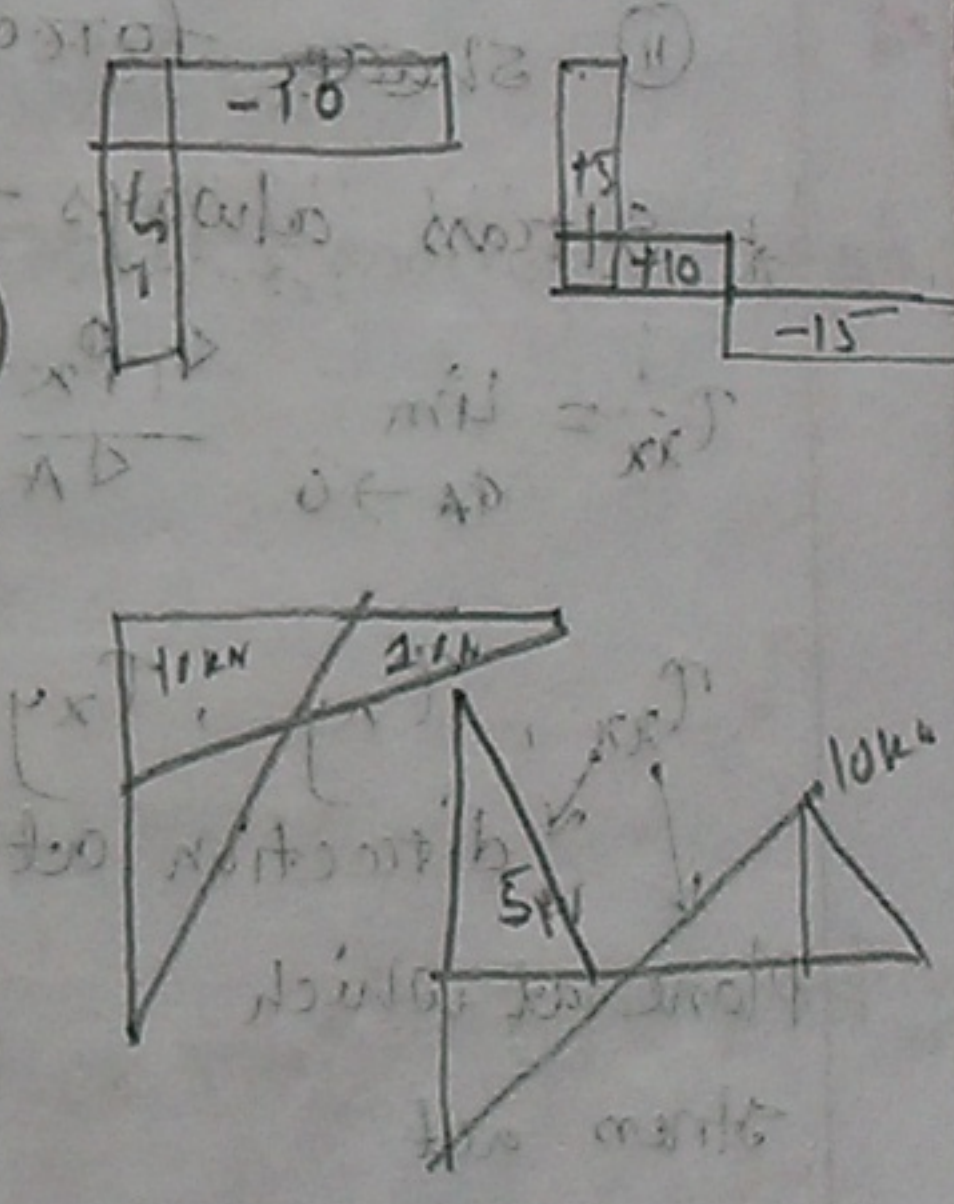
$$R_{Cx} = 5N (\rightarrow)$$

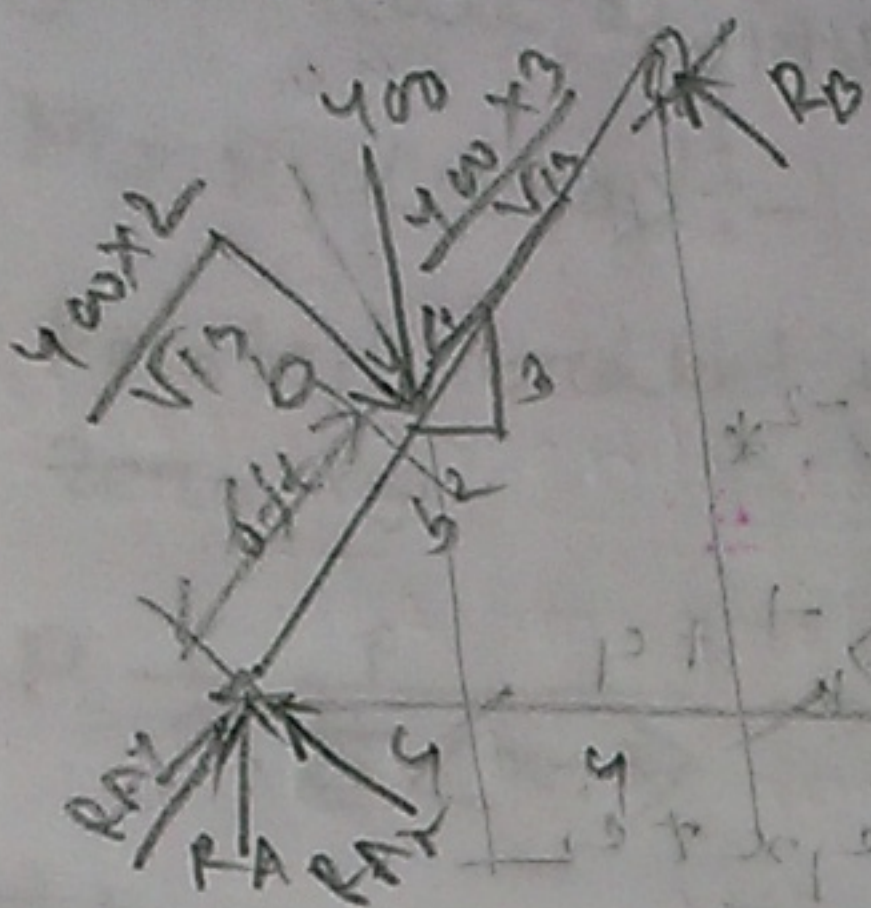
$$\therefore R_{Ax} = 5N (\rightarrow) \quad R_{Bx} = 5N (\leftarrow)$$

$$R_{Ay} = 10N (\uparrow) \quad R_{By} = 10N (\uparrow)$$

For see a-a

- P = 10 N
- V = 5 N
- M = 2.5 N.m





$$\sum M_A = 0$$

$$\Rightarrow \frac{400 \times 2}{\sqrt{3}} \times 4 - R_B \times 8 = 0$$

$$\Rightarrow R_B = 110.94 \text{ lb} = R_{Ax}$$

$$R_{Ay} = 332.82 \text{ lb}$$

Force see b-b

$$P = 332.82 \text{ lb}$$

$$V = 110.94 \text{ lb}$$

$$M = 110.94 \text{ lb} \times 6 \text{ ft} = 665.64 \text{ lb/ft}$$

## Stress and Axial force:

\* Beam takes the load along transverse direction

\* At any cross section there are 2 force

(i) Normal force

(ii) shear force

\* Stress always relates to a point:

$$\tau_{xx} = \lim_{\Delta A \rightarrow 0} \frac{\Delta P_x}{\Delta A}; \quad \tau_{xy} = \lim_{\Delta A \rightarrow 0} \frac{\Delta P_y}{\Delta A}; \quad \tau_{xz} = \lim_{\Delta A \rightarrow 0} \frac{\Delta P_z}{\Delta A}$$

$\tau_{xx}, \tau_{xy}, \tau_{xz}$

direction at which stress act.

Plane at which stress act

\* Any cross section total 9 loads are observed

(i) Normal stress  $\rightarrow 3$

(ii) shear stress  $\rightarrow 6$

25.01.14

Stress tensor:

Stress always relates to a point.

$$\sigma = \frac{dP}{dA}$$

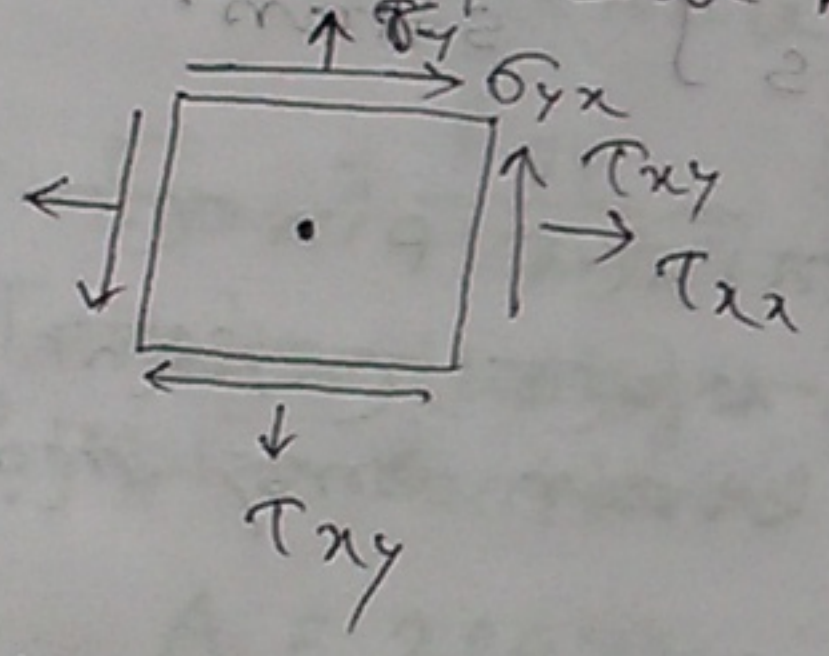
There are 9 types of stress at a point

$$\begin{matrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{matrix} = \begin{matrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{matrix}$$

$\tau_{xx} = \sigma_x, \tau_{yy} = \sigma_y, \tau_{zz} = \sigma_z$   
 $\tau_{ij} \rightarrow$  stress at a point  
 $\sigma_x, \sigma_y \text{ \& } \sigma_z = \text{Normal stress.}$

Tensor:

Special kind of vector.



Concrete unit wt.  $25 \text{ kN/m}^3$   
 $\gamma_{con} = 150 \text{ lb/ft}^3 \Rightarrow 2400 \text{ kg/m}^3$   
 $\gamma_{steel} = 490 \text{ lb/ft}^3 \Rightarrow 7850 \text{ kg/m}^3$   
 $\gamma_{water} = 62.4 \text{ lb/ft}^3 \Rightarrow 1000 \text{ kg/m}^3$

Normal stress - a moment  $2(\tau_{xy} \cdot z)$ . They passing through the centroid.

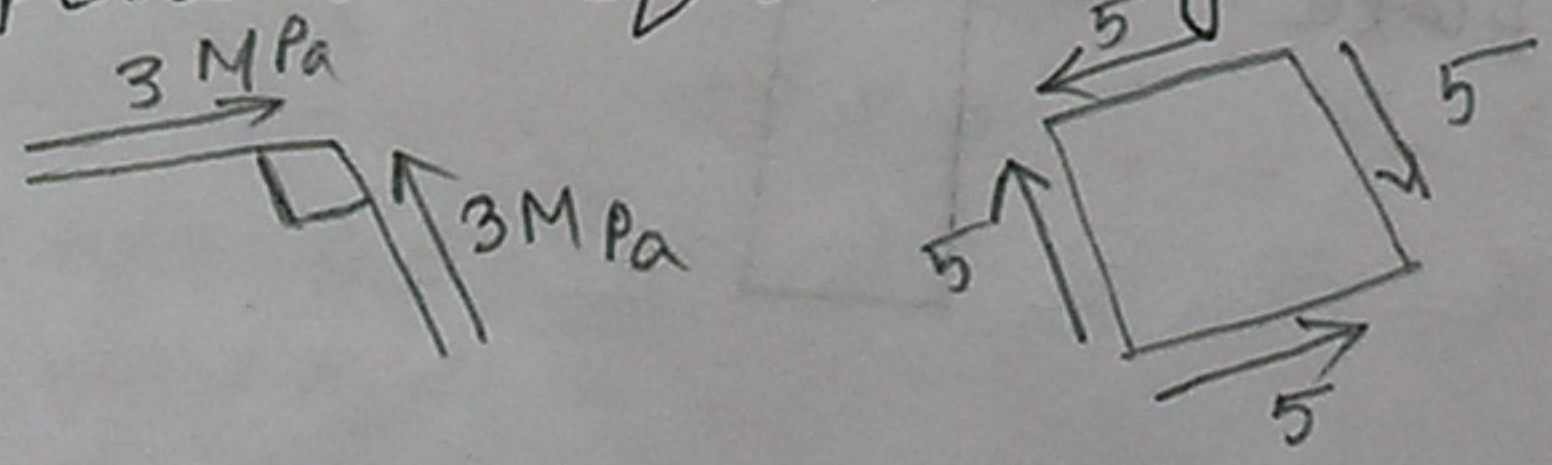
$$(\tau_{xy} \cdot dy \cdot dz) dx - (\tau_{yx} \cdot dx \cdot dz) dy = 0$$

$$\tau_{xy} = \tau_{yx}$$

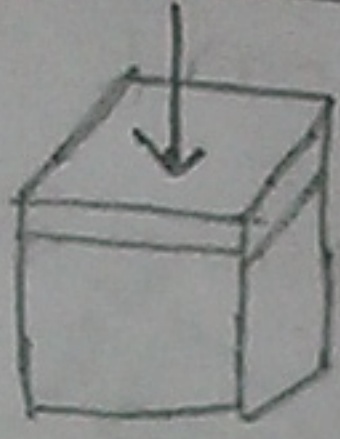
\* Complementary:

shearing stresses of two mutually perpendicular

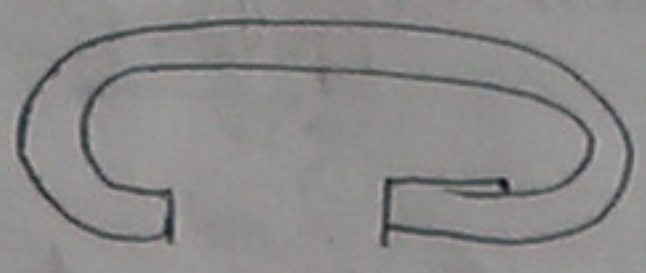
plane are equal in magnitude.



Axial load & Normal stress



point force applied on surface  
 $\frac{P}{A}$  stress



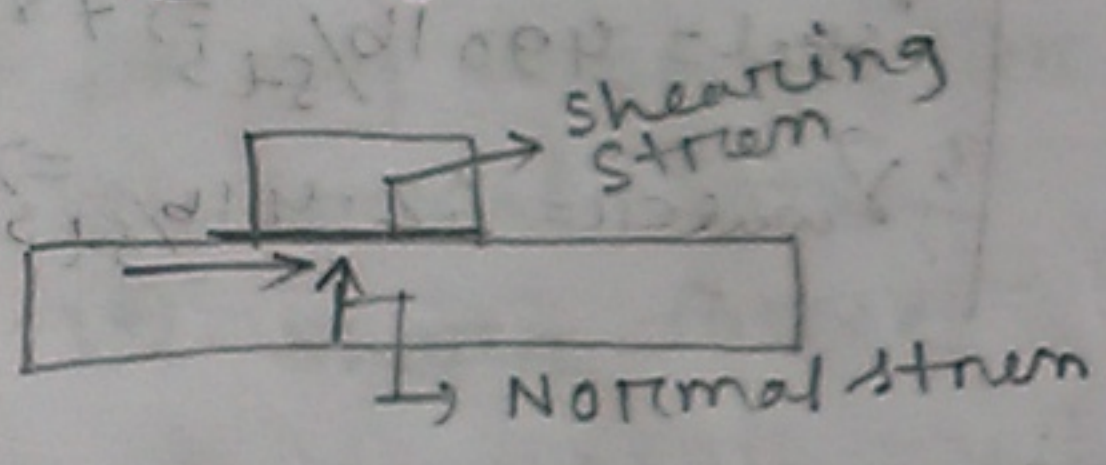
Stress concentration:

The magnitude of stress on the force applied plane point is more than  $\frac{P}{A}$ .

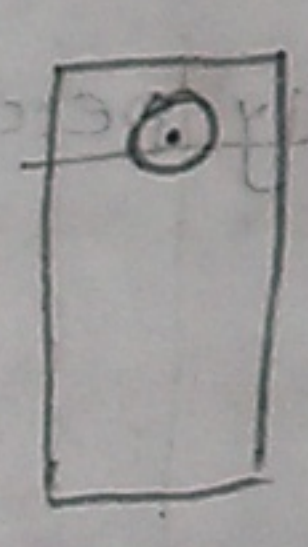
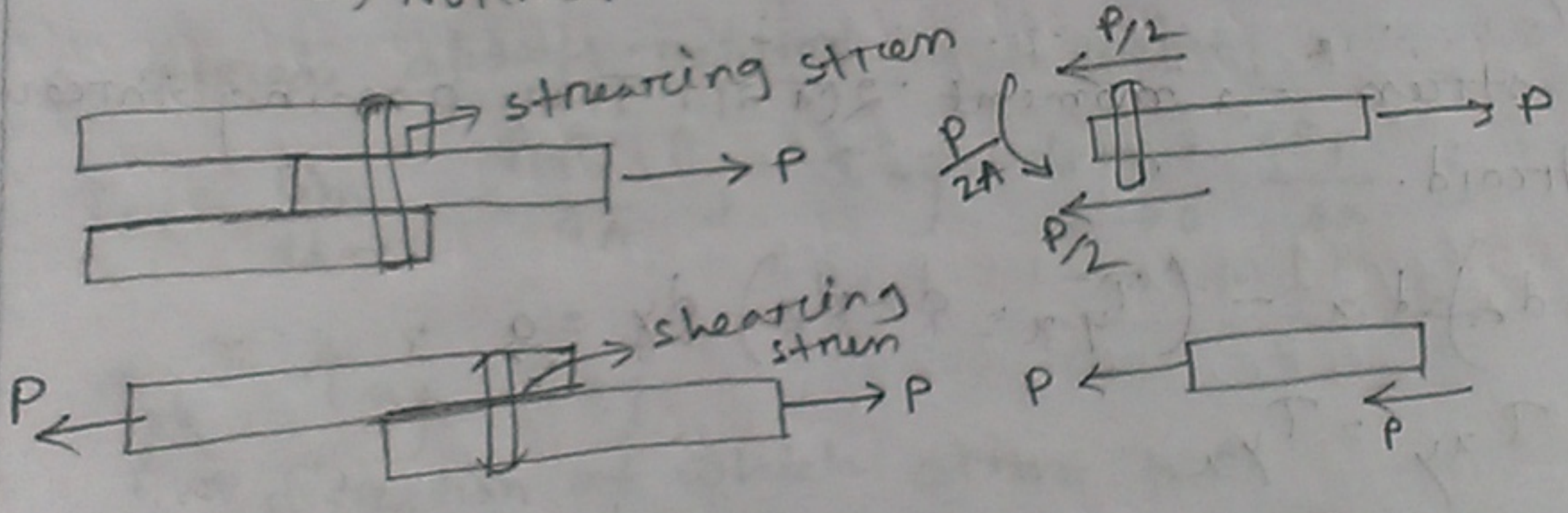
Bearing stress:

The stress of 2 dissimilar surface is called bearing stress. They are only normal stress.

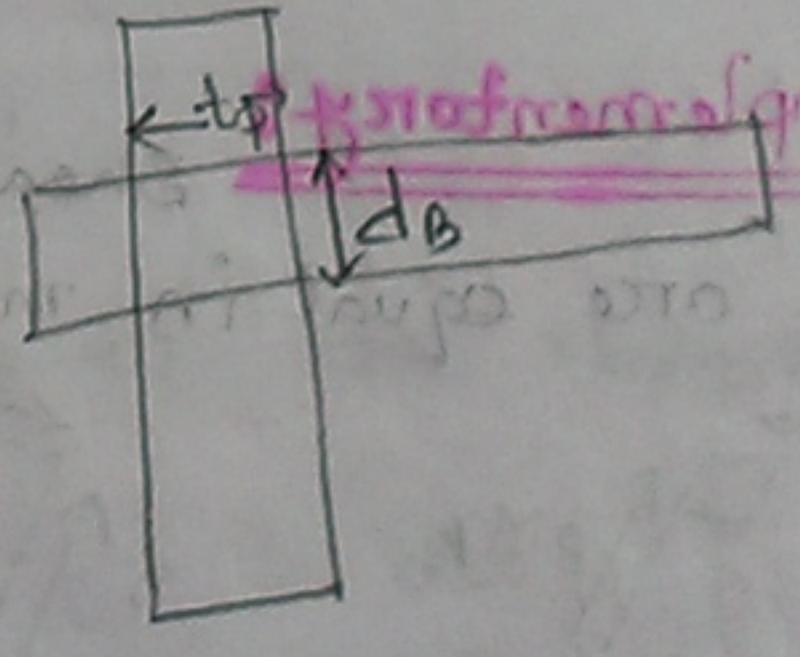
(column + footing) surface  $\rightarrow$  B.S } Normal stress  
 (footing + soil) surface  $\rightarrow$  B.S }



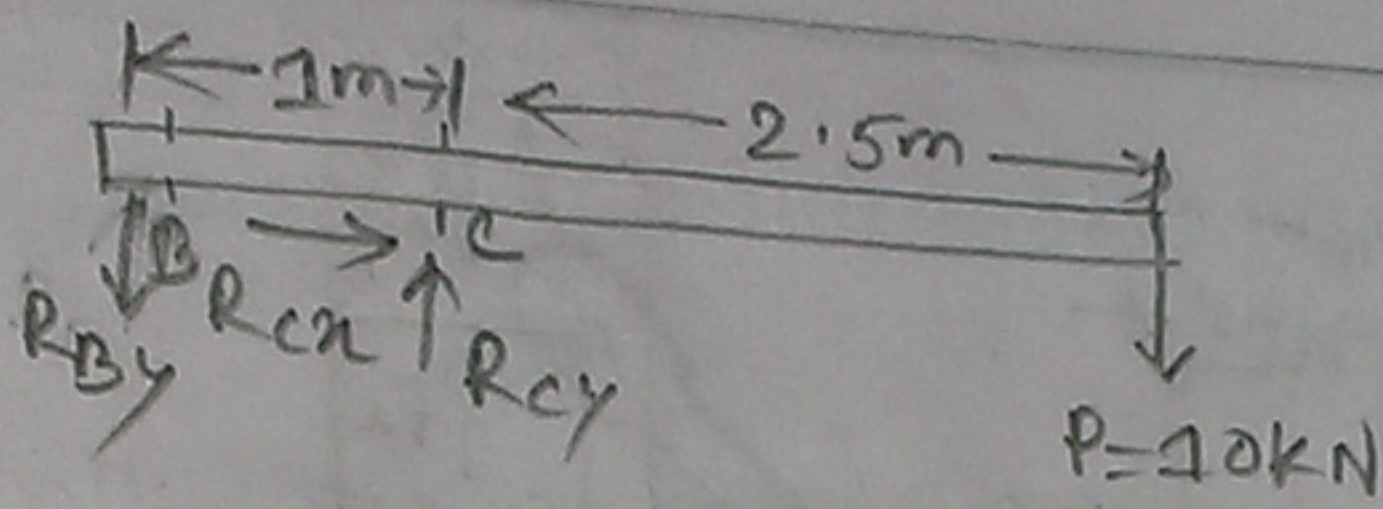
board plate pin point - a - b - c - d - e - f - g - h - i - j - k - l - m - n - o - p - q - r - s - t - u - v - w - x - y - z - aa - ab - ac - ad - ae - af - ag - ah - ai - aj - ak - al - am - an - ao - ap - aq - ar - as - at - au - av - aw - ax - ay - az - ba - bb - bc - bd - be - bf - bg - bh - bi - bj - bk - bl - bm - bn - bo - bp - bq - br - bs - bt - bu - bv - bw - bx - by - bz - ca - cb - cc - cd - ce - cf - cg - ch - ci - cj - ck - cl - cm - cn - co - cp - cq - cr - cs - ct - cu - cv - cw - cx - cy - cz - da - db - dc - dd - de - df - dg - dh - di - dj - dk - dl - dm - dn - do - dp - dq - dr - ds - dt - du - dv - dw - dx - dy - dz - ea - eb - ec - ed - ee - ef - eg - eh - ei - ej - ek - el - em - en - eo - ep - eq - er - es - et - eu - ev - ew - ex - ey - ez - fa - fb - fc - fd - fe - ff - fg - fh - fi - fj - fk - fl - fm - fn - fo - fp - fq - fr - fs - ft - fu - fv - fw - fx - fy - fz - ga - gb - gc - gd - ge - gf - gg - gh - gi - gj - gk - gl - gm - gn - go - gp - gq - gr - gs - gt - gu - gv - gw - gx - gy - gz - ha - hb - hc - hd - he - hf - hg - hh - hi - hj - hk - hl - hm - hn - ho - hp - hq - hr - hs - ht - hu - hv - hw - hx - hy - hz - ia - ib - ic - id - ie - if - ig - ih - ii - ij - ik - il - im - in - io - ip - iq - ir - is - it - iu - iv - iw - ix - iy - iz - ja - jb - jc - jd - je - jf - jg - jh - ji - jj - jk - jl - jm - jn - jo - jp - jq - jr - js - jt - ju - jv - jw - jx - jy - jz - ka - kb - kc - kd - ke - kf - kg - kh - ki - kj - kk - kl - km - kn - ko - kp - kq - kr - ks - kt - ku - kv - kw - kx - ky - kz - la - lb - lc - ld - le - lf - lg - lh - li - lj - lk - ll - lm - ln - lo - lp - lq - lr - ls - lt - lu - lv - lw - lx - ly - lz - ma - mb - mc - md - me - mf - mg - mh - mi - mj - mk - ml - mm - mn - mo - mp - mq - mr - ms - mt - mu - mv - mw - mx - my - mz - na - nb - nc - nd - ne - nf - ng - nh - ni - nj - nk - nl - nm - nn - no - np - nq - nr - ns - nt - nu - nv - nw - nx - ny - nz - oa - ob - oc - od - oe - of - og - oh - oi - oj - ok - ol - om - on - oo - op - oq - or - os - ot - ou - ov - ow - ox - oy - oz - pa - pb - pc - pd - pe - pf - pg - ph - pi - pj - pk - pl - pm - pn - po - pp - pq - pr - ps - pt - pu - pv - pw - px - py - pz - qa - qb - qc - qd - qe - qf - qg - qh - qi - qj - qk - ql - qm - qn - qo - qp - qq - qr - qs - qt - qu - qv - qw - qx - qy - qz - ra - rb - rc - rd - re - rf - rg - rh - ri - rj - rk - rl - rm - rn - ro - rp - rq - rr - rs - rt - ru - rv - rw - rx - ry - rz - sa - sb - sc - sd - se - sf - sg - sh - si - sj - sk - sl - sm - sn - so - sp - sq - sr - ss - st - su - sv - sw - sx - sy - sz - ta - tb - tc - td - te - tf - tg - th - ti - tj - tk - tl - tm - tn - to - tp - tq - tr - ts - tt - tu - tv - tw - tx - ty - tz - ua - ub - uc - ud - ue - uf - ug - uh - ui - uj - uk - ul - um - un - uo - up - uq - ur - us - ut - uu - uv - uw - ux - uy - uz - va - vb - vc - vd - ve - vf - vg - vh - vi - vj - vk - vl - vm - vn - vo - vp - vq - vr - vs - vt - vu - vv - vw - vx - vy - vz - wa - wb - wc - wd - we - wf - wg - wh - wi - wj - wk - wl - wm - wn - wo - wp - wq - wr - ws - wt - wu - wv - ww - wx - wy - wz - xa - xb - xc - xd - xe - xf - xg - xh - xi - xj - xk - xl - xm - xn - xo - xp - xq - xr - xs - xt - xu - xv - xw - xx - xy - xz - ya - yb - yc - yd - ye - yf - yg - yh - yi - yj - yk - yl - ym - yn - yo - yp - yq - yr - ys - yt - yu - yv - yw - yx - yy - yz - za - zb - zc - zd - ze - zf - zg - zh - zi - zj - zk - zl - zm - zn - zo - zp - zq - zr - zs - zt - zu - zv - zw - zx - zy - zz



$A = d \cdot t$   
 diameter of the board  
 thickness of plate



01.02.14



$$\sum F_x = 0 \Rightarrow R_{cx} = 0$$

$$\sum M_B = 0 \Rightarrow 10 \times 3.5 - R_{cy} \times 1 = 0$$

$$\therefore R_{cy} = 35 \text{ kN} \uparrow$$

$$\sum M_C = 0 \Rightarrow R_{By} \times 1 + 10 \times 2.5 = 0$$

$$\therefore R_{By} = -25 \text{ kN} \downarrow$$

cross-sectional area of BD

$$A = \pi \times 10^2 = 314 \text{ mm}^2$$

This is not the min area of bolt.

$$\text{So, } A_{\text{net}} = \frac{\pi \times 16^2}{4} = 201 \text{ mm}^2$$

Normal tensile stress of Bolt BD

$$\sigma_{\text{max}} = \frac{R_{By}}{2A} = \frac{25 \times 10^3}{2 \times 201} = 62 \text{ MPa} = 62 \text{ N/mm}^2$$

Tensile strength of Bolt BD,  $\sigma = \frac{25 \times 10^3}{2 \times 314} = 39.8 \text{ MPa}$

Cross-sectional area of C

$$A = 200 \times 200 = 4 \times 10^4 \text{ mm}^2$$

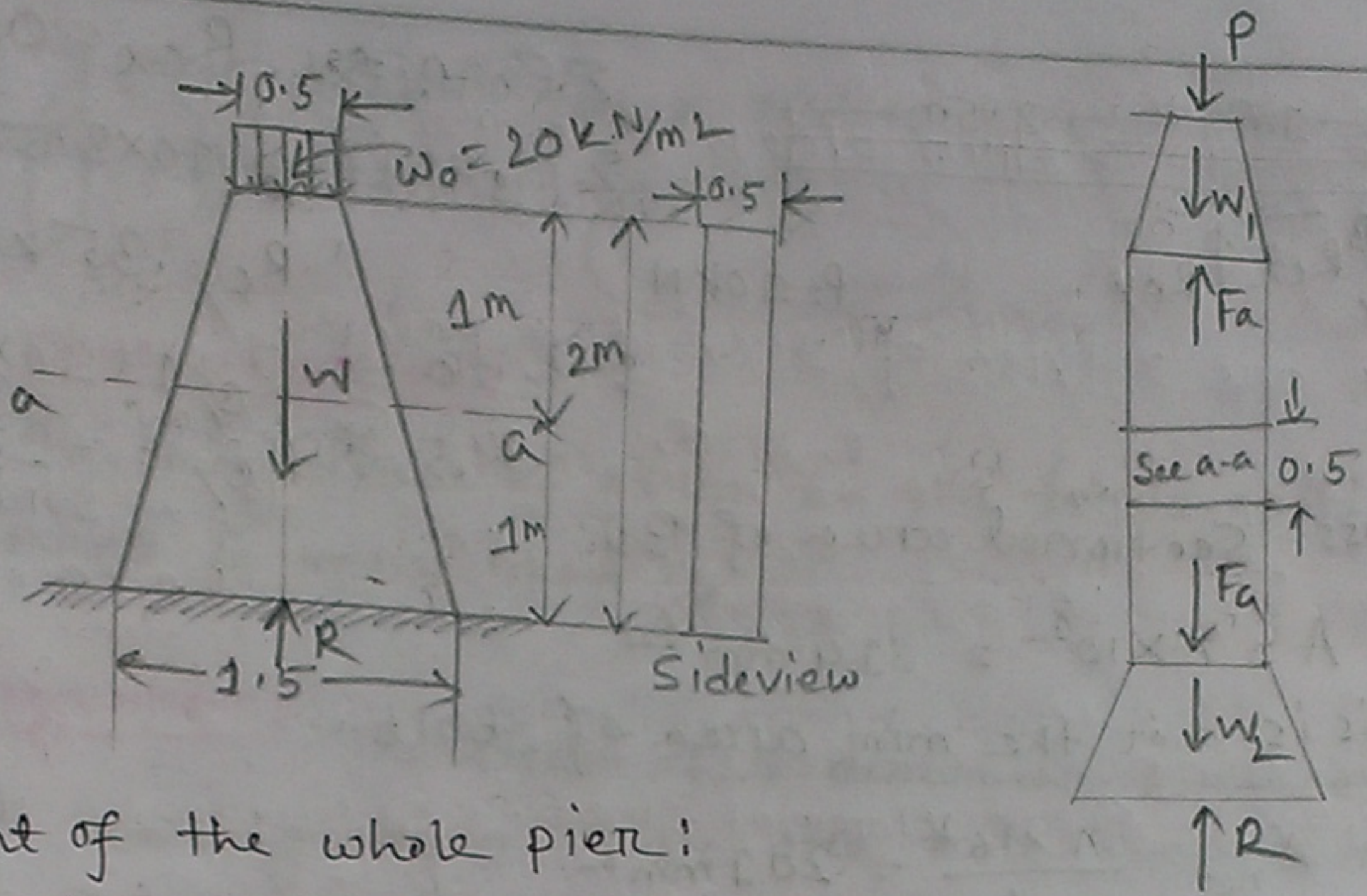
$\therefore$  Bearing stress of C

$$\sigma_b = \frac{R_{cy}}{A} = \frac{35 \times 10^3}{4 \times 10^4} = 0.875 \text{ MPa} = 0.875 \text{ N/mm}^2$$

This calculation can also be represented as

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & +39.8 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ MPa}$$

1.2



Weight of the whole pier:

$$W = \left[ \frac{(0.5 + 1.5)}{2} \right] \times 0.5 \times 2 \times 25 = 25 \text{ kN.}$$

Total applied force,

$$P = 20 \times 0.5 \times \frac{1}{2} = 20 \text{ kN.}$$

Then,  $\sum F_y = 0 \Rightarrow R - W - P = 0 \Rightarrow R = W + P = 25 + 20 = 45 \text{ kN}$

Using the upper part of sec a-a.

$$W_1 = \frac{(0.5 + 1) \times 0.5 \times 1 \times 25}{2} = 9.4 \text{ kN}$$

The applied force  $P = 5$ .

$$\therefore \sum F_y = 0 \Rightarrow F_a = W_1 + P = 5 + 9.4 = 14.4 \text{ kN}$$

$\therefore$  The normal stress,

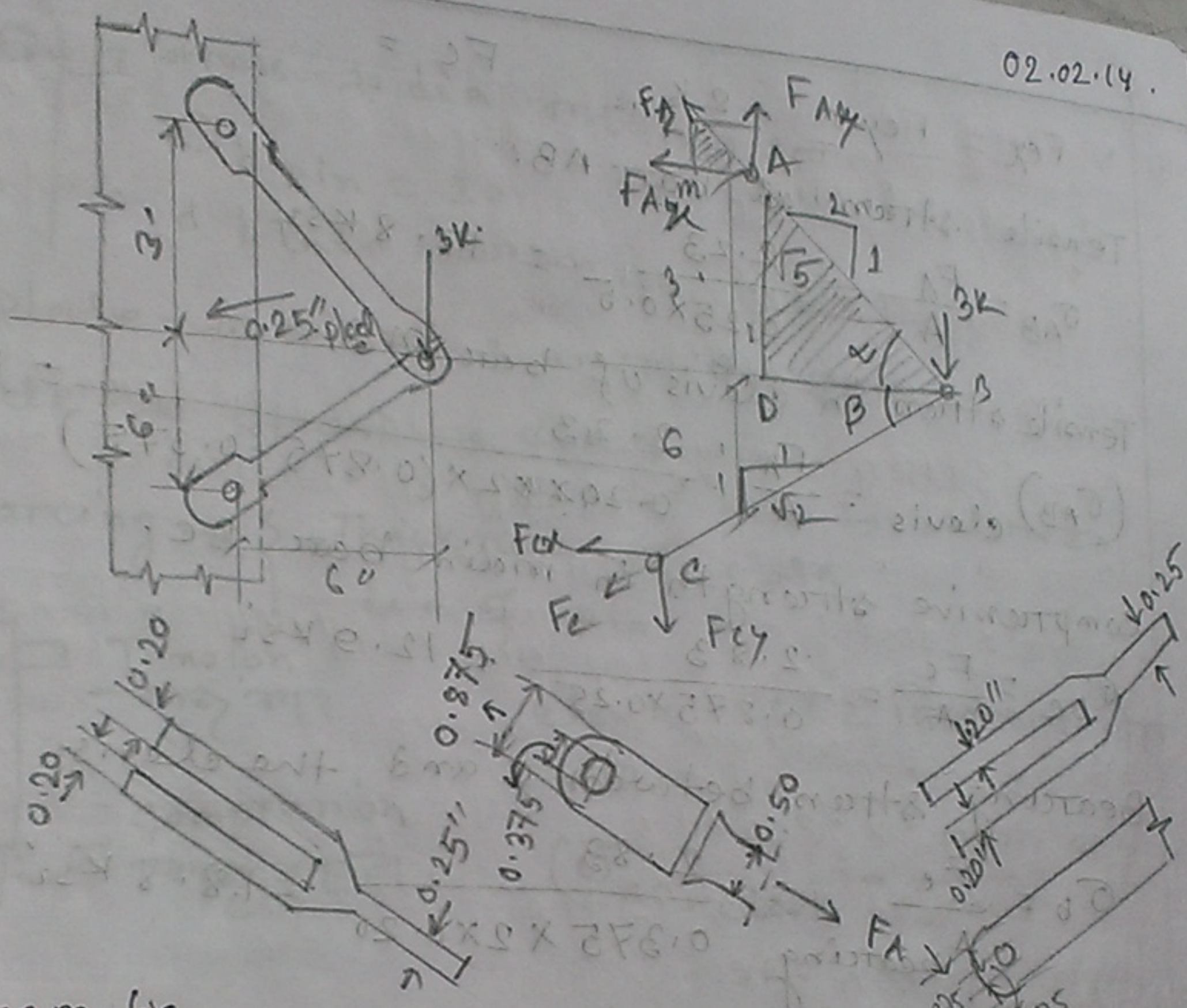
$$\sigma_{a-a} = \frac{F_a}{A} = \frac{14.4}{0.5 \times 1} = 28.8 \text{ kN/m}^2$$

Similarly the lower part of the sec.

$$W_2 = \frac{(1 + 1.5) \times 1 \times 1 \times 25}{2} = 15.6 \text{ kN.}$$

$$\sum F_y = 0 \Rightarrow R = F_a + W_2 \Rightarrow F_a = R - W_2 = 45 - 15.6 = 29.4 \text{ kN.}$$

02.02.14



From fig.  $\triangle AKM$  &  $\triangle ADM$  are similar

$$\frac{F_A}{F_{Ax}} = \frac{AB}{DB} \Rightarrow F_A = \frac{AB}{DB} \times F_{Ax}$$

$$\frac{F_{Ay}}{F_{Ax}} = \frac{AD}{DB} \Rightarrow F_{Ay} = \frac{AD}{DB} F_{Ax} = F_{Ax}$$

So from the members AB & BC:

$$F_A = \frac{\sqrt{5}}{2} F_{Ax} \quad \& \quad F_{Ay} = F_{Ax}/2$$

$$\sum M_C = 0 \Rightarrow 3 \times 6 - F_{Ax} \times 9 = 0 \Rightarrow F_{Ax} = +2k$$

$$\sum M_A = 0 \Rightarrow 3 \times 6 + F_{Cx} \times 9 = 0 \Rightarrow F_{Cx} = -2k$$

$$\therefore F_{Ay} = +1k$$

$$F_A = \frac{\sqrt{5}}{2} \times 2 = 2.23k$$

$$\therefore F_{cx} = F_{cy} = -2k, \quad F_c =$$

Tensile stress at bar AB.

$$\sigma_{AB} = \frac{F_A}{A} = \frac{2.23}{0.25 \times 0.5} = 17.8 \text{ ksi}$$

Tensile stress in clevis of bar AB.

$$(\sigma_{AB})_{\text{clevis}} = \frac{F_A}{A} = \frac{2.23}{0.20 \times 2 \times (0.875 - 0.375)} = 11.2 \text{ ksi}$$

Compressive strength in main bar BC.

$$\sigma_{bc} = \frac{F_c}{A} = \frac{2.83}{0.875 \times 0.25} = 12.9 \text{ ksi}$$

Bearing stress between c and the clevis

$$\sigma_b = \frac{F_c}{A_{\text{bearing}}} = \frac{2.83}{0.375 \times 2 \times 0.20} = 18.8 \text{ ksi}$$

Bearing between the main plate

$$\sigma_b = \frac{F_c}{A} = \frac{2.83}{0.375 \times 0.25} = 30.2 \text{ ksi}$$

Double shear in pin c.

$$\tau = \frac{F_c}{A} = \frac{2.83}{2A \left( \frac{0.375}{2} \right)^2} = 12.9 \text{ ksi}$$



Stress - 20 dia वरुन दिएको २००।

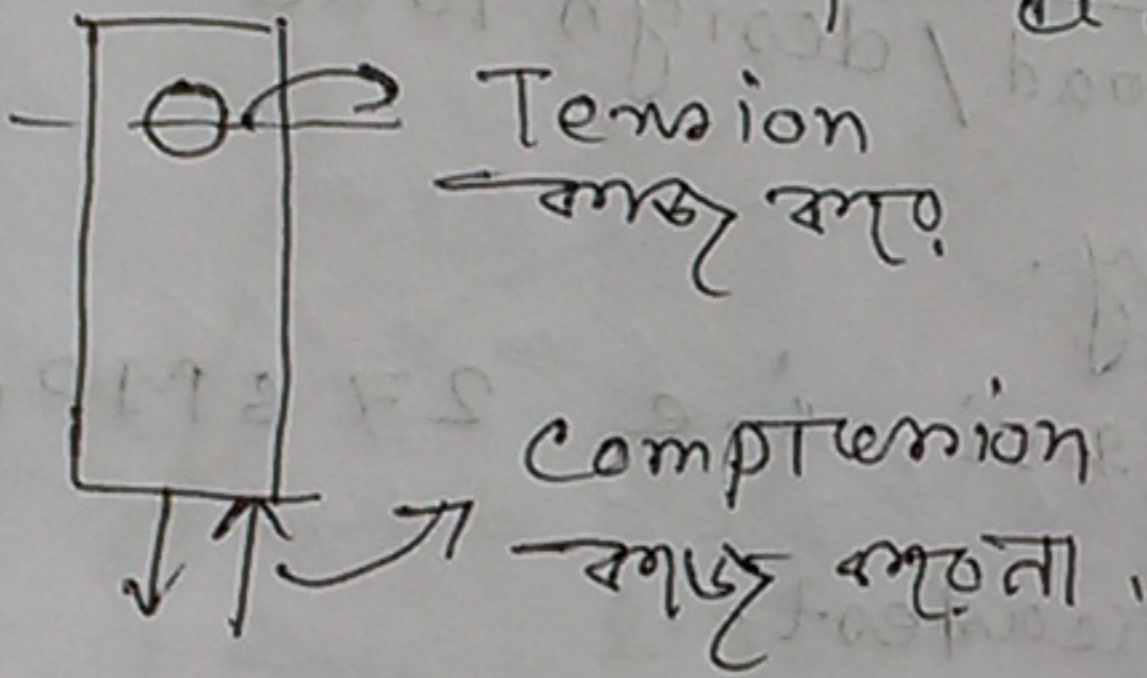
$$d_{\text{pin}} = 20$$

$$d_{\text{plate}} = 22 \text{ mm}$$

Plate - 20 dia 2-3 mm वरुन दिएको २००।

Plate - 20 dia 2-3 mm वरुन दिएको २००।

bearing  $\rightarrow$  } Thickness of plate  
 dia of pin.  $\rightarrow$  2-3 mm वरुन दिएको २००।



Stress  $\rightarrow$  २०० २ दिने-लग दिएको २००।

$$(\sigma_{AB})_{\text{stress}} = \frac{F_A}{A_{\text{net}}}$$

Universal testing machine.  $\rightarrow$  material-20 properties.

Design of axially loaded members & pins

$$\frac{U}{A} = \sigma \leq \frac{U}{A} = 20$$

$$\frac{F}{A} = \sigma \leq \frac{F}{A} = 20$$

03.02.14.

## Working stress:

Repeated loading  $\rightarrow$  fatigue test.

$\hookrightarrow$  material do not behave in same way.

stress reverse ২(ল)

SN  $\rightarrow$  stress-number  $\rightarrow$  infinity time stress  
দেওয়া পর tolerance ability.

Factor of safety  $\rightarrow$   $\frac{\text{ultimate load}}{\text{working load / design load}}$

f.s = 1  $\rightarrow$  margin of safety.

yield stress  $\rightarrow$  2800 kg/cm<sup>2</sup>, 40 ksi i.e. 276 MPa

yield stress is the greatest.

High carbon steel.

40 grade - ২ ৬০ এর বেশি.

90,000 এর উপর ২(৭) ultimate stress.

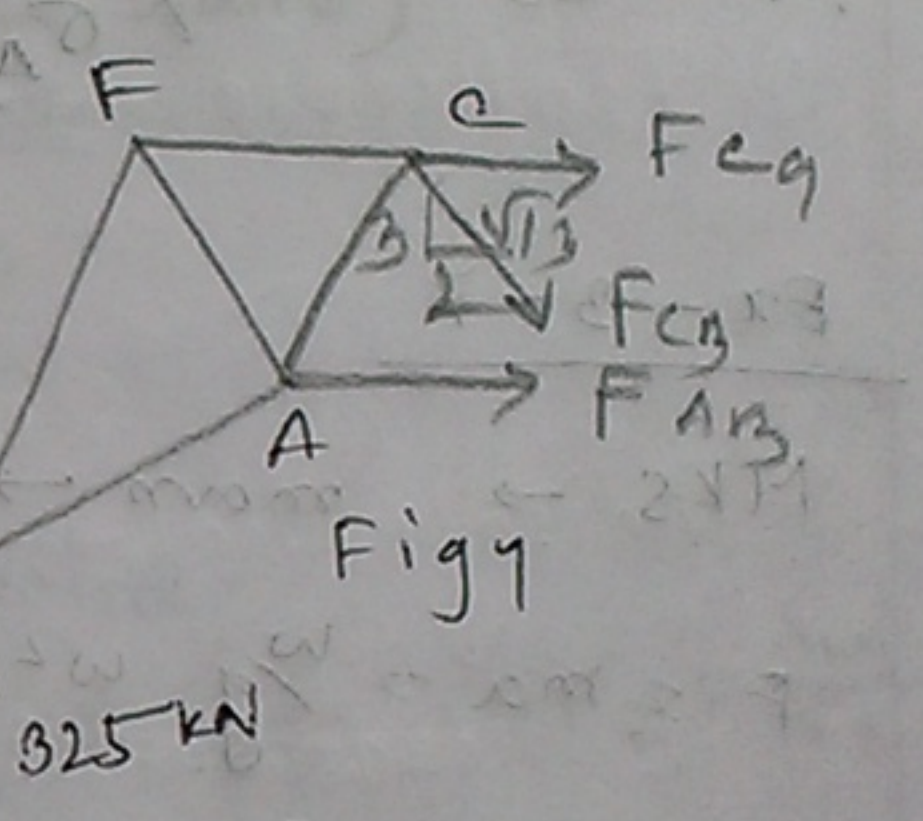
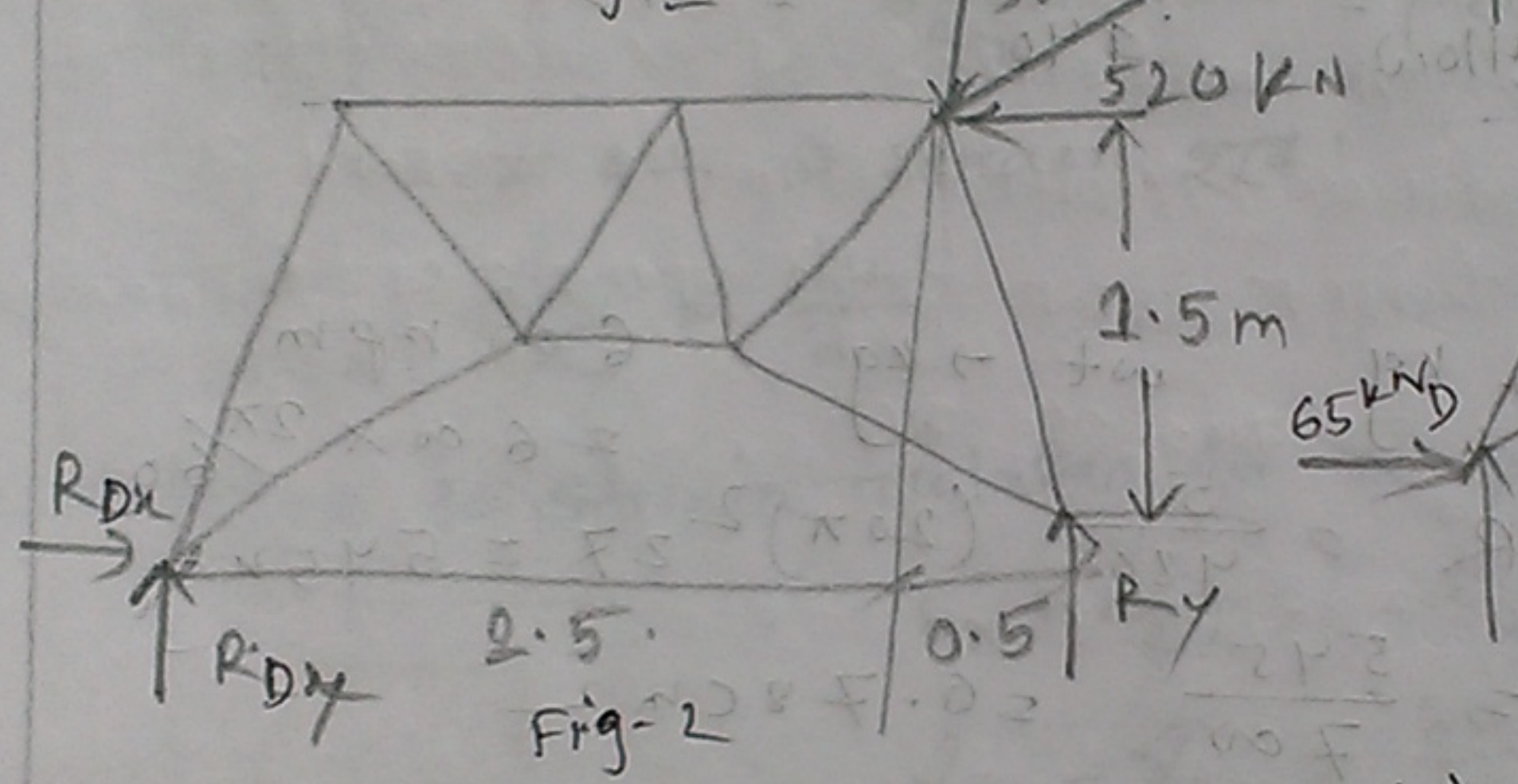
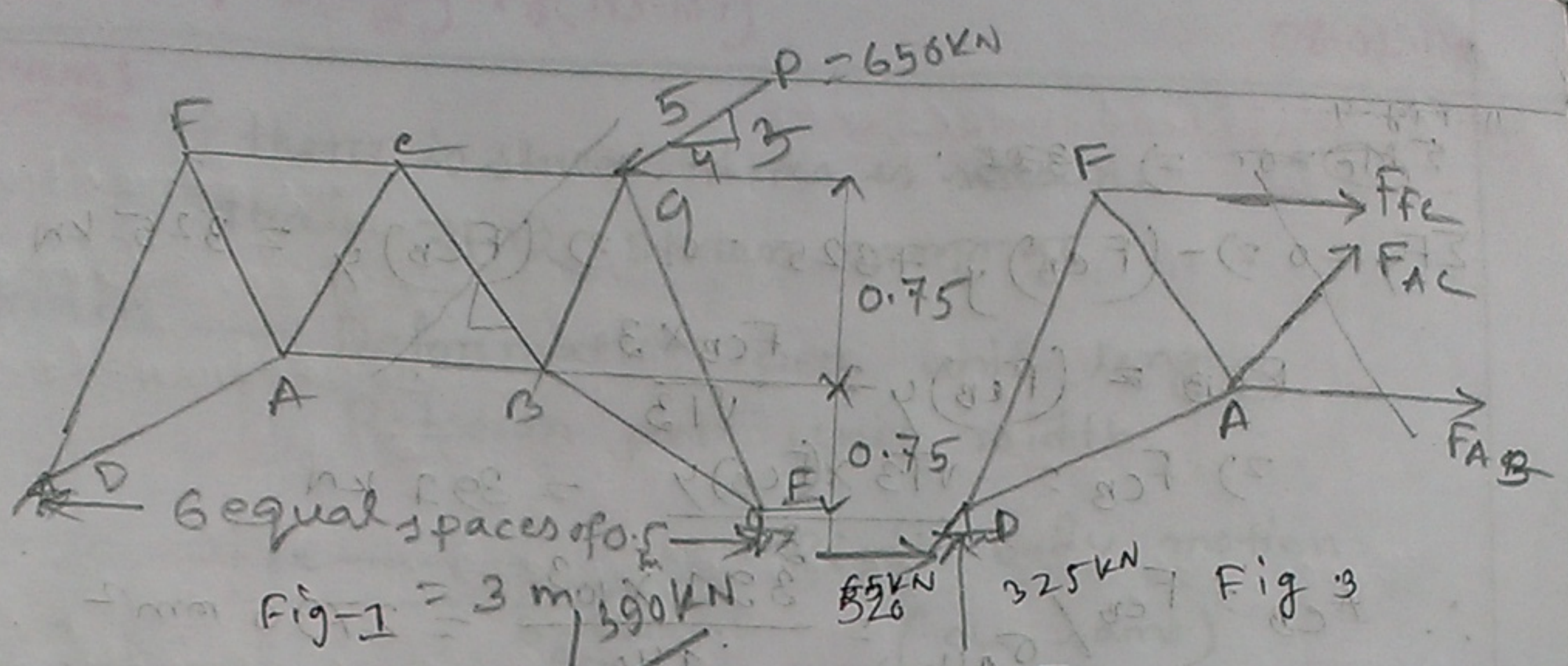
## Design of axially loaded members & pins:

$$F_{OS} = \frac{U}{A} \Rightarrow A = \frac{U}{F_{OS}}$$

$$F = \sigma A \Rightarrow A = \frac{F}{\sigma_{allow}}$$

$$0.33 \text{ cm}^2 \Rightarrow 0.11 \times 0.3 \Rightarrow 0.7 \times 0.5 = 0.35$$

1.5



$\sum M_A = 0$  + Force the whole dia -

$\sum F_x = 0 \Rightarrow R_{Dx} = 520 \text{ kN}$

$\sum M_D = 0 \Rightarrow -R_{Ey} \times 3 + 390 \times 2.5 - 520 \times 1.5 = 0$

$\therefore R_{Ey} = 1065 \text{ kN}$

$\sum M_E = 0 \Rightarrow R_{Dy} \times 3 - 390 \times 0.5 - 520 \times 1.5 = 0$

$\therefore R_{Dy} = 325 \text{ kN}$

From fig-3,

$\sum M_A = 0 \Rightarrow 325 \times 0.5 - 65 \times 0.75 + F_{Ac} \times 0.75 = 0$

$\therefore F_{Ac} = +86.7 \text{ kN}$

$A_{FC} = \frac{F_{FC}}{\sigma_{Allow}} = \frac{86.7 \times 10^3}{140} = 620 \text{ mm}^2$

Fig-4

$$\sum M_0 = 0 \Rightarrow 325 \cdot$$

$$\sum F_y = 0 \Rightarrow -(F_{CB})_y + 325 = 0 \Rightarrow (F_{CB})_y = 325 \text{ kN}$$

$$F_{CB} = (F_{CB})_y = \frac{F_{CB} \times 3}{\sqrt{13}}$$

$$\Rightarrow F_{CB} = \frac{\sqrt{13} \times (F_{CB})_y}{3} = 391 \text{ kN}$$

$$\therefore A_{CB} = \frac{F_{CB}}{\sigma_{allow}} = \frac{391 \times 10^3}{140} = 2790 \text{ mm}^2$$

Ex-3-7

MKS  $\rightarrow$  mm  $\rightarrow$  kg wt  $\rightarrow$  kg 600 rpm

$$F = ma = \frac{w}{g} \omega^2 R = \frac{5}{980} (20\pi)^2 \cdot 27 = 545 \text{ kg}$$

$$A_{net} = \frac{F}{\sigma_{allow}} = \frac{545}{700} = 0.78 \text{ cm}^2$$

$$F_1 = \int_0^L (m \cdot dr) \omega^2 r = m_1 \omega^2 \frac{r^2}{2} \quad m_1 = \frac{\sigma}{g} \cdot d$$

dia bar  $\rightarrow$  mm  $\rightarrow$  cm  $\rightarrow$  dia of rod

additional pull  $\rightarrow$  rod self weight

$$P_1 = \int_0^L (m \cdot dr) \omega^2 r$$

$$\checkmark = (\text{unit wt}) \times A_{net}$$

= unit weight

$$A_{FC} = \frac{F_{FC}}{\sigma_{allow}} = \frac{86.5 \times 10^3}{140} = 618 \text{ mm}^2$$

अभ्यास - Chap-3 (part B) - Pg (143-151)

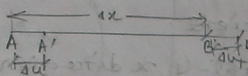
08.02.14

Strain:

If there is stress, there is strain.

Strain  $\propto$  Stress

Strain  $\rightarrow$  Deformation per unit length  
Rotation per unit width.

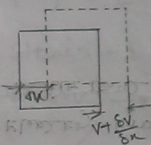


Rigid body motion  
(du same)

$du \neq dv \Rightarrow$  Strain  $\propto$

$$\lim_{dx \rightarrow 0} \frac{du}{dx} = \frac{du}{dx}$$

\* Stress & Strain relates to a point.



$$\epsilon_x = \frac{\partial u}{\partial x}, \quad \epsilon_y = \frac{\partial v}{\partial y}, \quad \epsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \gamma_{yx} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$\gamma_{xz} = \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

$$\gamma_{yz} = \gamma_{zy} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$

Total angular strain.

\* Mechanical strain

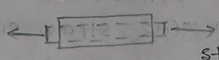
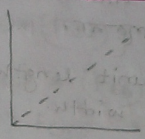
$$\Rightarrow \epsilon_{xy} = \epsilon_{yx} = \frac{\gamma_{xy}}{2}$$

$$\epsilon_{yz} = \epsilon_{zy} = \frac{\gamma_{yz}}{2}$$

$$\epsilon_{zx} = \epsilon_{xz} = \frac{\gamma_{zx}}{2}$$

$$\epsilon_{zx} = \epsilon_{xz} = \frac{\gamma_{zx}}{2}$$

Generalized Hooke's law:



x direction of stress due to y or z direction - stress  $\frac{2}{2(v)}$

There are 36 co-efficient.

- ① Isotropic  $\rightarrow$  For material - x direction - stress, y direction - stress  $\frac{2}{2(v)}$  (Identical)
- ② Anisotropic  $\rightarrow$  is responsible to a strain

- ① Homogeneous  $\rightarrow$  Stress-strain relation
- ② Heterogeneous  $\rightarrow$  Stress-strain relation

$$\{\epsilon\}_{ij} = [A_{ij}] \{\tau\}_{ij} \rightarrow 36 \text{ independent co-efficient.}$$

$$\hat{\epsilon}_{ij} = A_{ij} \cdot \hat{\tau}_{ij} \rightarrow 3 \text{ co-efficient for}$$

1st 3 0 (isotropic material) last 3  $\rightarrow$  0 (isotropic material)

$$\epsilon_{xx} = \frac{\tau_{xx}}{E}$$

$$\epsilon_{yy} = -\nu \frac{\sigma_x}{E} \text{ ; } \nu \text{ is poisson ratio}$$

$$A_{11} = \frac{1}{E} \quad A_{12} = -\frac{\nu}{E} \quad A_{13} = \frac{1}{E}$$

V1.50.00

(10-10) 9-5-19

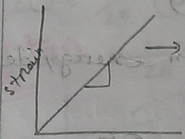
$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{zx} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2G} \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

$$\{\epsilon\} = [A] \{\sigma\}$$

$$\{\sigma\} = [A^{-1}] \{\epsilon\} ; [A^{-1}] \Rightarrow \text{constitutive matrix}$$

\* Strain - 2 x c.m. दिव स्ट्रेन calculate करे 200.

Determine  $\rightarrow$  Constitutive matrix for normal strain



linear slope

$$E = \frac{\sigma}{\epsilon} \quad \epsilon = 1 \text{ (at } E = \sigma)$$

Poisson ratio

$$\nu = \frac{|\epsilon_x|}{\epsilon_x} \rightarrow \text{lateral direction}$$

$$\tau_{xy} = G \gamma_{xy} ; G = \frac{E}{2(1+\nu)} \text{ [memorize]}$$

E & u test दिव G करे करे करे

2" (25mm) त्रिज्या नात्रिले diameter करे करे करे करे

$$\epsilon_1 = \text{transverse direction} \quad G = \frac{E}{2.67} \times 10^4 = 28 \times 10^4 \text{ kg/cm}^2$$

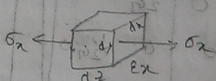
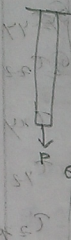
Energy

capacity to do work.

$W = F \cdot s$

Strain energy  $\rightarrow$  strain for load

Unit point  $\rightarrow$  load  $\rightarrow$  force



Deformation,  $\epsilon_x dx$

$\epsilon_x = \frac{du}{dx}$

$\sigma_x =$  final end product

$\therefore du = \epsilon_x dx$

$F = dy dz \int \frac{1}{2} (\sigma_x \cdot dy dz) \epsilon_x dx$

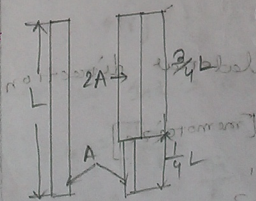
$U = \int \frac{1}{2} \sigma_x \epsilon_x dv = \frac{1}{2} \sigma_x \epsilon_x (dx dy dz)$

$\therefore \epsilon_x = \frac{\sigma_x}{E}$ ;  $dv = \frac{1}{2} \times \text{strain} \times \text{volume}$

$= \int \frac{\sigma_x^2}{2E} dv$  (strain as form)

$U_0 = \frac{dU}{dv} = \frac{1}{2} \sigma_x \epsilon_x = \frac{\sigma_x^2}{2E}$  (strain energy density)

Ex



$U = \int \frac{\sigma_x^2}{2E} dv = \frac{\sigma_1^2}{2E} \int dv = \frac{\sigma_1^2}{2E} (AL)$

A = cross sectional area

L = length.

Handwritten notes and scribbles at the bottom of the page, including some calculations and diagrams.

$$U_2 = \int_V \frac{\sigma^2}{2E} dv = \frac{\sigma_2^2}{2E} \int_{\text{lower part}} dv + \frac{(\sigma_2/2)^2}{2E} \int_{\text{upper part}} dv$$

$$= \frac{\sigma_2^2}{2E} \left( \frac{AL}{4} \right) + \frac{(\sigma_2/2)^2}{2E} \left( 2A \cdot \frac{3L}{4} \right) = \frac{\sigma_2^2 AL}{8E} + \frac{3\sigma_2^2 AL}{16E}$$

$$= \frac{2\sigma_2^2 AL + 3\sigma_2^2 AL}{16E} = \frac{5\sigma_2^2 AL}{16E} = \frac{\sigma_2^2}{2E} \left( \frac{5}{8} AL \right)$$

If  $U_1 = U_2$

$$\frac{\sigma_1^2}{2E} (AL) = \frac{\sigma_2^2}{2E} \left( \frac{5}{8} AL \right) \therefore \sigma_2 = 1.265 \sigma_1$$

Hence for the same energy load, the stress in the reinforced bar is 26.5% higher than in the plain bar.

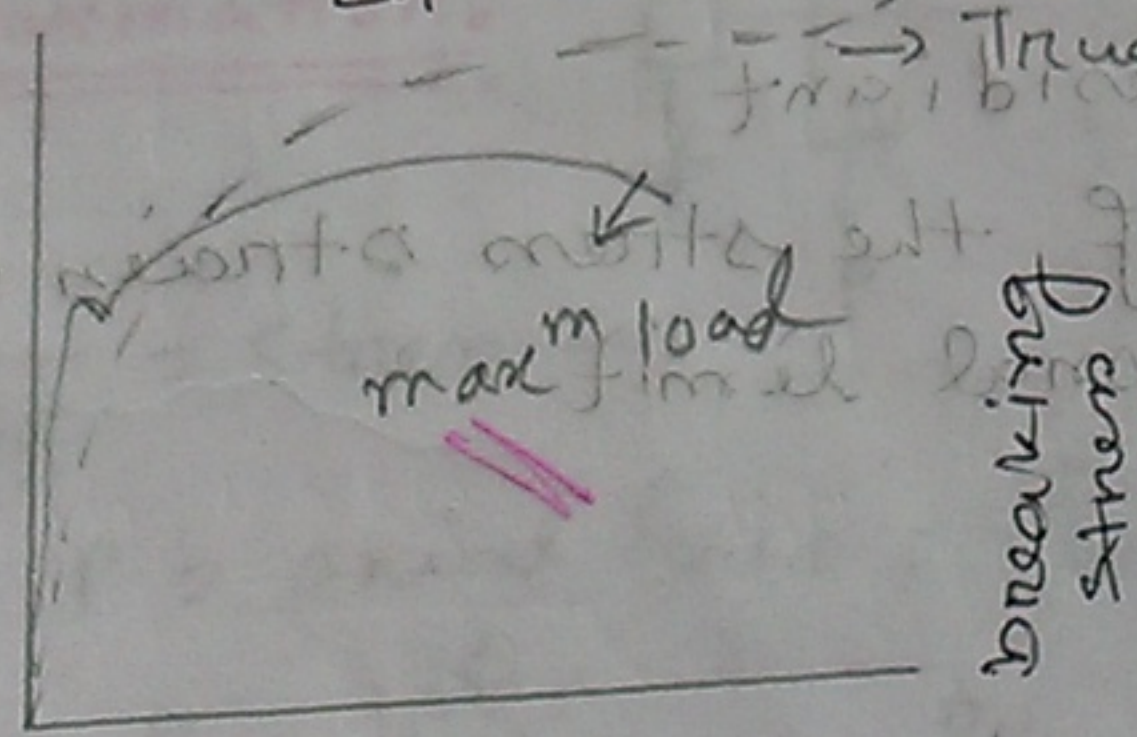
$$U_{\text{shear}} = \int_V \frac{1}{2} \tau_{xy} \gamma_{xy} dv = \int_V \frac{1}{2} \frac{\tau_{xy}^2}{G} dv$$

$$U = \int_V \left[ \frac{1}{2} \sigma_x \epsilon_x + \frac{1}{2} \sigma_y \epsilon_y + \frac{1}{2} \sigma_z \epsilon_z + \frac{1}{2} \tau_{xy} \gamma_{xy} + \frac{1}{2} \tau_{yz} \gamma_{yz} + \frac{1}{2} \tau_{zx} \gamma_{zx} \right] dv$$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E} \sigma_y - \frac{\nu}{E} \sigma_z$$

$$U = \frac{1}{2E} (\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - \frac{\nu}{E} (\sigma_x \sigma_y + \sigma_x \sigma_z + \sigma_y \sigma_z)$$

LHP-2  
Pg-62-75



Flow of the material at constant stress  $\rightarrow$  yield.

$\epsilon_{\text{creep}}$ : strain at constant load

convention stress-strain

deformation (stress u 20 gnt)

Breaking stress:  $\frac{\text{max load}}{\text{area}}$  will be highest.

Yield stress

Ultimate stress

চক্রে মধ্যম অক্ষর রাখা দিলে (cup and cone) দিলে cone.

so this is called cup & cone structure.

ductile

Brittle → absence of ductility

concrete → Brittle

যদি কঠিন stress হওয়ার কথা হয় তবে কঠিন দেওয়া হয়  
যাও concrete - crash করার সময় warning দেয়।

Proportional limit:

যেখানে proportionality শেষ হয় তার

(i) Yield stress

(ii) Proportional limit

(iii) Elastic limit

↓  
use করা ২য়

↓  
Stress constant

↓  
Elongation বাড়বে

↓  
determine করা difficult

↓  
proportional limit এর  
সময়

\* 3 এর same

$\frac{1}{2} \sigma_p \epsilon_p$  → strain energy which can be modulus of elastic resiliant

→ is the area of the stress strain diagram

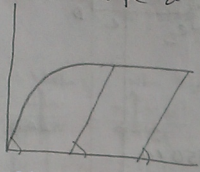
up to proportional limit

conversion of stress-strain curve  
proportional stress  
modulus of elasticity

Ch 2 (93, 95)

10.02.19

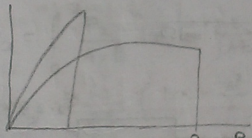
Yield point  $\rightarrow$  elastic limit  
 non-linear elastic material  
 initial slope & final slope is same



cold working  
 strain hardening  
 $\Delta$  in deformation more load

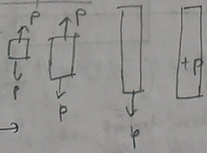
- (i) Stronger
- (ii) Tougher  $\rightarrow$  full area under curve deformation  
 $\downarrow$   
 above more energy

$A > B$   
 modulus of elasticity  $> M \cdot E \rightarrow$  strength ratio



weak but tough

Deformation:



$\frac{P}{AE} = \text{stress} \rightarrow$

$P =$  Axial force = internal force

$= \frac{P}{AE} \int_0^L du = \frac{PL}{AE}$  ;  $P \rightarrow$  axial force  
 $\downarrow$   
 internal force

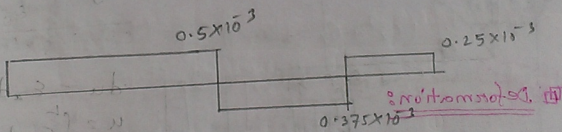
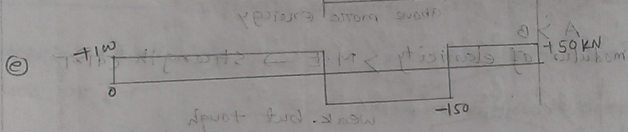
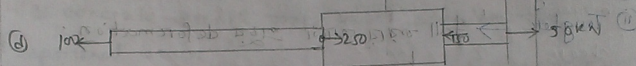
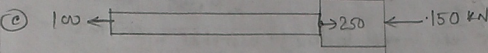
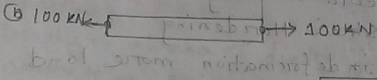
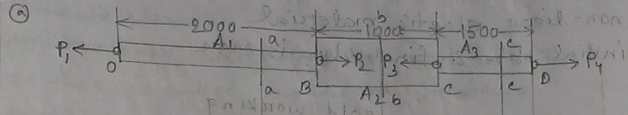
$du = \epsilon_x dx$   $U =$  strain energy  
 $u = \int_0^L \epsilon_x dx$   
 $= \int_0^L \frac{\sigma_x}{E} = \frac{P}{AE} dx$   
 $= \int_0^L \frac{P}{AE} \cdot dx$

Pl. 50.01

Mech-ehp-2 - Pg 76

(29.8.2021)

limit elastic ← finding blott



$$\frac{P}{AE} = \frac{P}{AE}$$

$$\frac{P}{AE} = \frac{P}{AE}$$

$$\frac{P}{AE} = \frac{P}{AE}$$

$$\frac{P}{AE} = \frac{P}{AE}$$

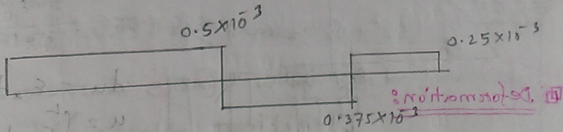
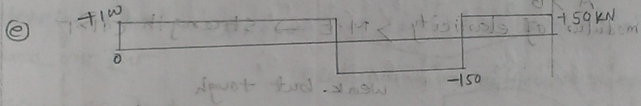
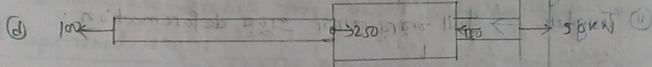
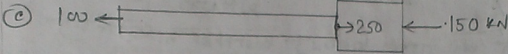
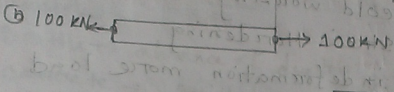
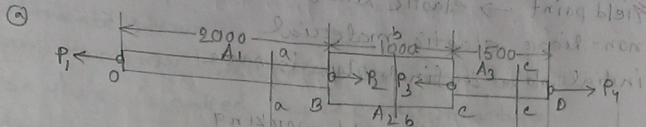
$$\frac{P}{AE} = \frac{P}{AE}$$

$$\frac{P}{AE} = \frac{P}{AE}$$

11.05.01

Math-chp-2 - Pg 76

(20.00.2021)



Deformation

$$\frac{P}{AE} = \text{stress}$$

$$P = \text{Axial force} = \text{stress} \times A = \sigma \times A$$

$$\frac{P}{AE} = \frac{\sigma}{E} \times E = \epsilon$$

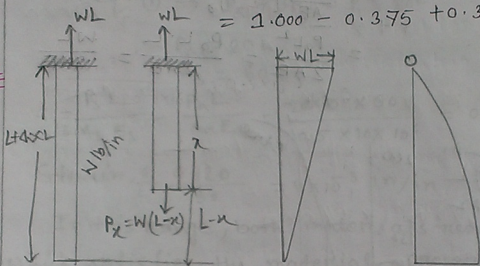
$$\frac{P}{AE} = \frac{\sigma}{E} \times E = \epsilon$$

internal force  $\rightarrow$  force force  $\rightarrow$  force force

$$\Delta = \sum \frac{P_i L_i}{A_i E} = \frac{P_{OB} L_{OB}}{A_{OB} E} + \frac{P_{BC} L_{BC}}{A_{BC} E} + \frac{P_{CD} L_{CD}}{A_{CD} E}$$

$$= \frac{100 \times 10^3 \times 2000}{1000 \times 200 \times 10^3} - \frac{150 \times 10^3 \times 1000}{2000 \times 200 \times 10^3} + \frac{50 \times 10^3 \times 1500}{1000 \times 200 \times 10^3}$$

$$= 1.000 - 0.375 + 0.375 = 1.000 \text{ mm}$$



$$\Delta x = \int_0^x \frac{P_x}{A_x E} dx = \int_0^x \frac{1}{AE} w(L-x) dx$$

$$= \frac{w}{AE} \left( Lx - \frac{x^2}{2} \right)$$

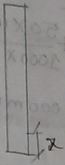
The deflection of B is,

$$\Delta = \Delta(L) = \frac{w}{AE} \left( L^2 - \frac{L^2}{2} \right) = \frac{wL^2}{2AE} = \frac{WL}{2AE}$$

Where  $w = wL$  is the total weight of the bar.

$$\Delta = \frac{PL}{AE} + \frac{WL}{2AE} = \frac{[P + w/2] L}{AE}$$

Ex



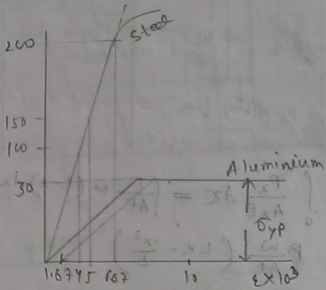
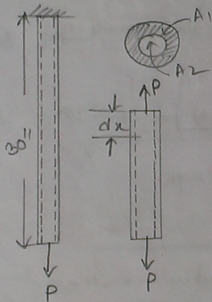
Per unit length weight  $= P_0$

$$u = \int_0^L \frac{P_0 x}{AE} \cdot dx = \frac{P_0}{AE} \int_0^L x dx$$

$$= \frac{P_0}{AE} \left[ \frac{x^2}{2} \right]_0^L$$

$$= \frac{P_0 L^2}{2AE} = \frac{P_0 \cdot L \cdot L}{2AE} = \frac{WL}{2AE}$$

Ex



From equilibrium:  $P_a + P_s = P$ , or  $P_2 = P - P_1$  — (1)

From compatibility:  $\delta_a = \delta_s$  or  $\epsilon_a = \epsilon_s$

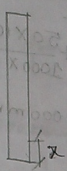
From material properties:

$$\epsilon_a = \sigma_a / E_a \quad \text{and} \quad \epsilon_s = \sigma_s / E_s$$

$$\sigma_a = \frac{P_a}{A_a}, \quad \sigma_s = \frac{P_s}{A_s}$$

$$E_s = 30 \times 10^6 \text{ psi} \quad \text{and} \quad E_a = 10 \times 10^6 \text{ psi}$$

Ex



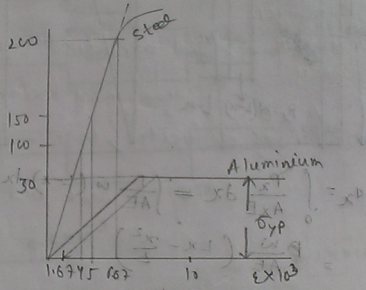
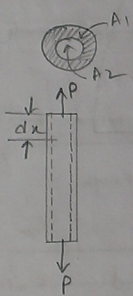
Perc unit length weight =  $P_0$

$$u = \int_0^L \frac{P_0 x}{AE} \cdot dx = \frac{P_0}{AE} \int_0^L x dx$$

$$= \frac{P_0}{AE} \left[ \frac{x^2}{2} \right]_0^L$$

$$= \frac{P_0 L^2}{2AE} = \frac{P_0 L \cdot L}{2AE} = \frac{WL}{2AE}$$

Ex



From equilibrium:  $P_a + P_s = P_1$  or  $P_2$  — (1)

From compatibility:  $\Delta_a = \Delta_s$  or  $\epsilon_a = \epsilon_s$

From material properties:

$$\epsilon_a = \frac{\sigma_a}{E_a} \quad \text{and} \quad \epsilon_s = \frac{\sigma_s}{E_s}$$

$$\sigma_a = \frac{P_a}{A_a}, \quad \sigma_s = \frac{P_s}{A_s}$$

$$E_s = 30 \times 10^6 \text{ psi} \quad \text{and} \quad E_a = 10 \times 10^6 \text{ psi}$$

$$P. \quad \epsilon_a = \epsilon_s = \frac{\sigma_a}{E_a} = \frac{\sigma_s}{E_s} = \frac{P_a}{A_a E_a} = \frac{P_s}{A_s E_s}$$

$$P_s = \left( \frac{A_s E_s}{A_a E_a} \right) P_a = 3 P_a$$

$$\text{From eqn (1)} \Rightarrow P_a + 3P_a = P_1 = 80 \text{ k}$$

$$\therefore P_a = 20 \text{ k}$$

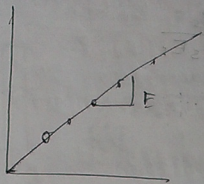
$$P_s = 60 \text{ k}$$

$$\Delta = \frac{P_s L}{A_s E_s} = \frac{P_a L}{A_a E_a} = \frac{20 \times 10^3 \times 30}{0.15 \times 10 \times 10^6} = 0.120 \text{ in}$$

$$\text{Strain, } \frac{\Delta}{L} = \frac{0.120}{30} = 4 \times 10^{-3} \text{ in/in}$$

In this range, both materials respond elastically which satisfies the material-property assumption made at the beginning of this solution. In fact

08 02.14



$\Delta l / l = P / EA$  for fixed length,  
 $E = \frac{Pl}{A \Delta l}$

$14.5 \rightarrow \frac{1450}{\pi/4 \times (5.6)^2} \rightarrow \text{stress}$

cor. strain  $\rightarrow 0.000782$  cm per cm

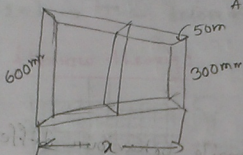
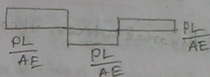
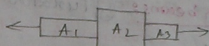
$\frac{588}{0.000782} = 75 \times 10^4 \text{ kg/cm}^2$

$G = 29 \times 10^4 \text{ kg/cm}^2$

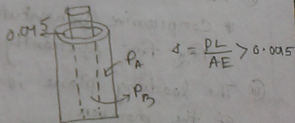
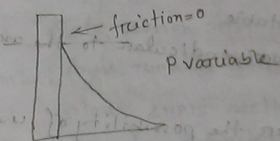
In this range, the material property assumption made at the beginning of this solution is that

22-02-14

$$U = \int \frac{P_x}{A_x E} dx$$

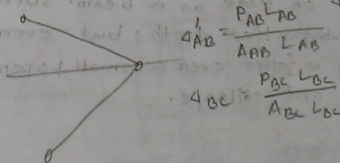


P & E constant  
A variable → integration required



$$P_A + P_B = 6000$$

$$\Delta_B = \Delta_A + 0.0015$$

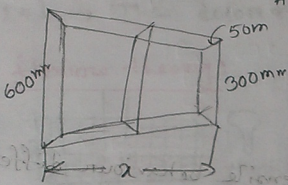
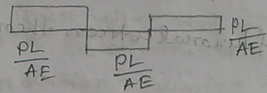
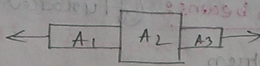


$$\Delta_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} L_{AB}}$$

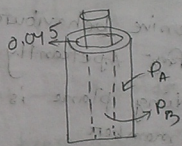
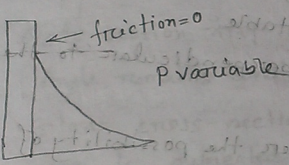
$$\Delta_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} L_{BC}}$$

22.02.14

$$U = \int \frac{P_x}{A_x E} dx$$



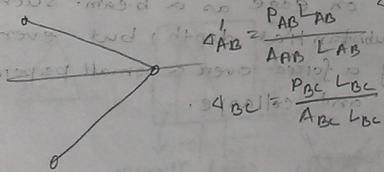
P & E constant  
A variable → integration



$$\Delta = \frac{PL}{AE} > 0.005$$

$$P_A + P_B = 6000$$

$$\Delta_B = \Delta_A + 0.0015$$



$$\Delta_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} E}$$

$$\Delta_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} E}$$

## Bending stress in beams: (flexural stress)

$$\frac{P}{A} \text{ average stress}$$

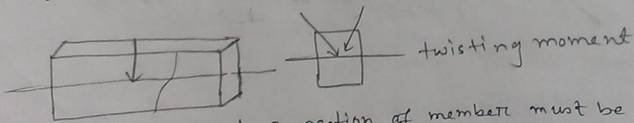
Navier  $\rightarrow$  flexural stress theory

$\downarrow$

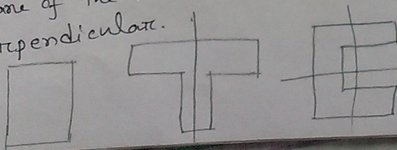
Navier hypothesis.

## Limitation:

- (i) static analysis
- (ii) member must be stable
  - \* Compressive behaviour, tensile behaviour different
  - $f_{comp} \neq f_{tens}$  Asymmetrically stable
- (iii) The loading plane is perpendicular to the axis of the member
- (iv) An ex. example consider the possibility of using a sheet of paper on edge as a beam. Such a beam is as a substantial depth, but even if it is used to carry a force over a small paper it will buckle sidewise and collapse.



Plane of the load & section of member must be perpendicular.



(v) Symmetric either major or minor axis.

### Basic kinematic Assumption:

1st - reduce the internally statically indeterminate problem to a determinate

2nd  $\rightarrow$  The deformation causing strain to relate

### flexure theory:

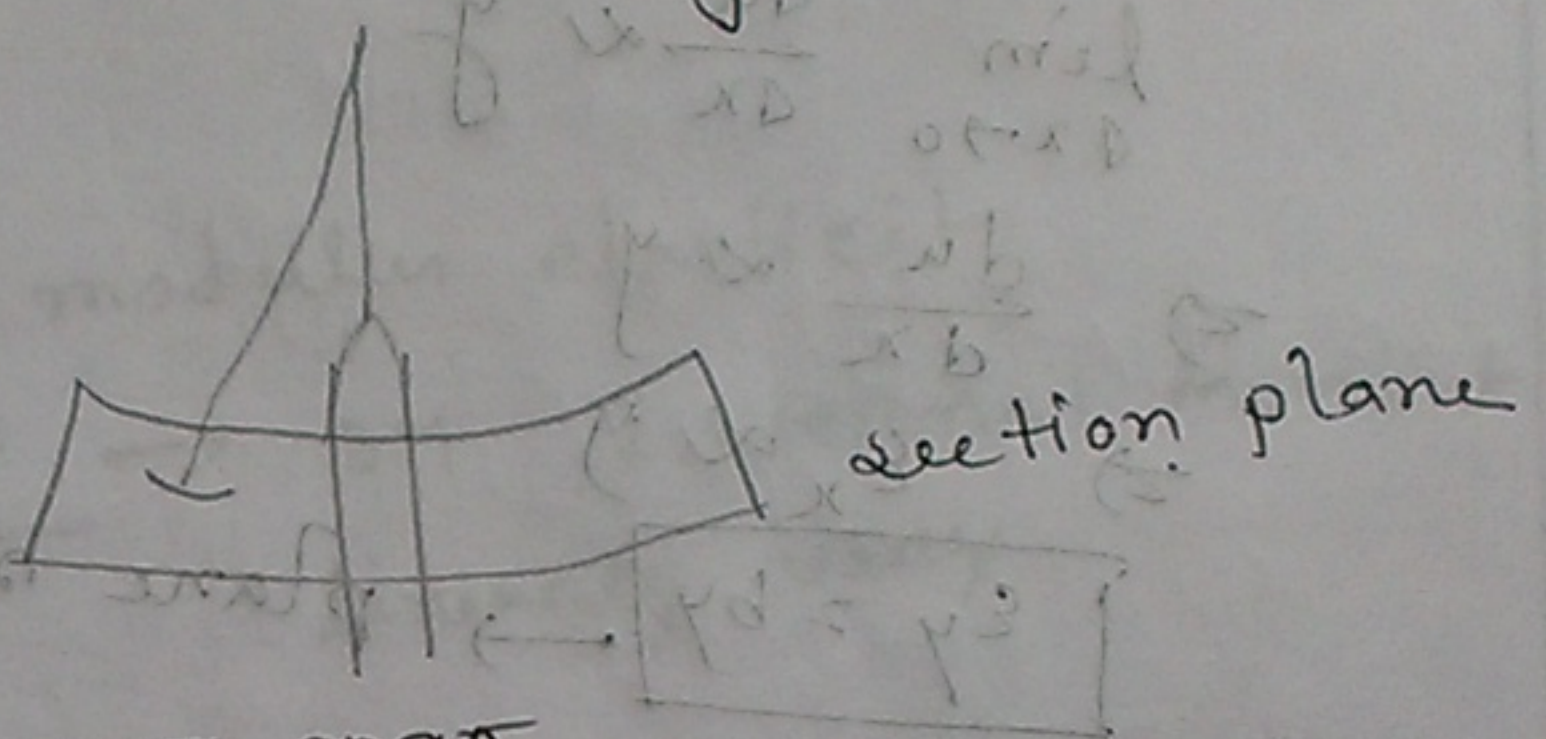
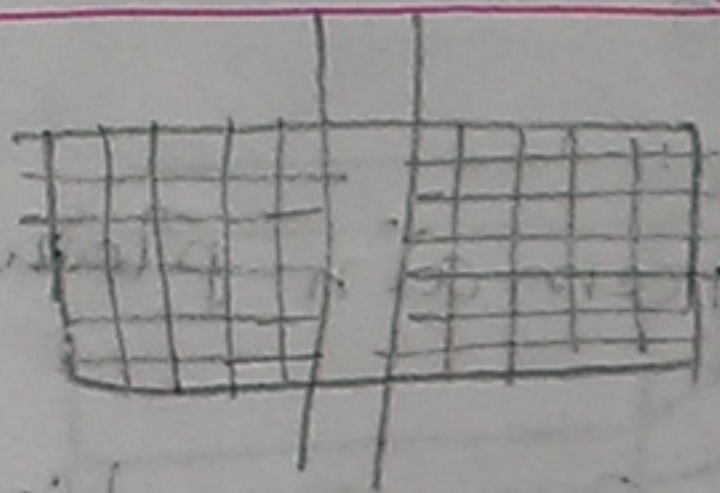
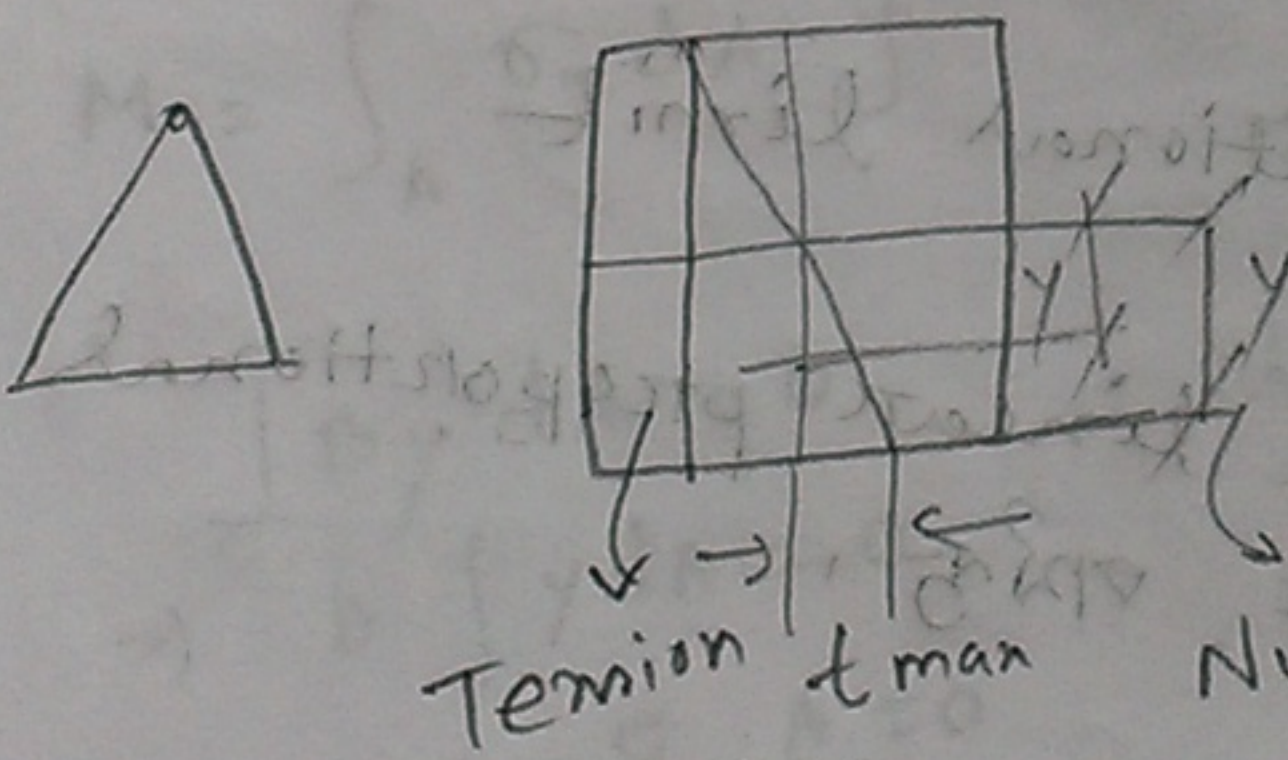


plate section  $\rightarrow$  level  $\rightarrow$  axis

Angle remains constant  $\rightarrow$  Navier's hypothesis

plane cross section beam remain plane after

the beam is subjected to bending.



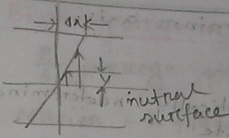
Tension  $t_{max}$

Neutral surface  $\rightarrow$  neutral surface

neutral surface  $\leftrightarrow$  cross sectional plane.

intersection is line

This line is called  $\rightarrow$  neutral axis



$u \propto y$   
 deformation of the distance  
 from the neutral surface

$$\lim_{dx \rightarrow 0} \frac{du}{dx} \propto y$$

$$\Rightarrow \frac{du}{dx} \propto y$$

$$\Rightarrow \epsilon_x \propto y$$

$$\epsilon_y = by$$

plane section remain plane

linearly vary  $\sigma_x$   $\rightarrow$  otherwise plane section  
 otherwise  $\sigma_x$

Navier's hypothesis is true after breaking

$$\sigma_x = E \epsilon_x$$

$\rightarrow$  Hooke's law

True up to proportional limit

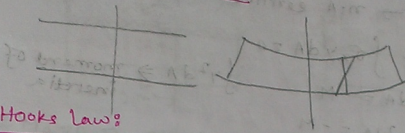
$$\sigma_x = by$$

Stress linear proportional  
 limit only

— 0 —

## Navier's Hypothesis

23.02.14



$$u = c \cdot y$$

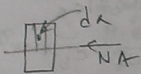
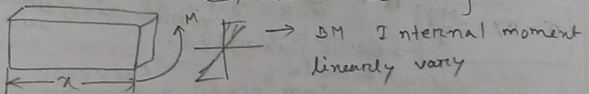
$$\epsilon_x = b \cdot y$$

mathematical

## Hooke's Law

$$\sigma_x = E \cdot \epsilon_x$$

$$E = \frac{\sigma}{\epsilon} \rightarrow \text{modulus of elasticity}$$



$$\text{Force} = \sigma_x dA$$

$$dM = \sigma_x dA \cdot y$$

Net Horizontal force = 0

$$M = \int_A \sigma_x dA \cdot y$$

$$dF = \sigma_x dA$$

$$\sum F_x = 0 \Rightarrow \int \sigma_x dA = 0$$

$$\int E \cdot \epsilon_x dA = 0$$

$$\Rightarrow E \int y dA = 0$$

$$\therefore E \cdot \bar{y} \cdot A = 0$$

$E$  &  $A$  not equal to 0.

$\int y dA \Rightarrow$  moment of area

$\int y dA = \int A \bar{y} = \text{centroidal axis}$   
distance from reference line.



$$\bar{y} = 0$$

$\rightarrow$  That means neutral Axis passing through the centroid.

17.10.20

For elastic beam  $\rightarrow$  n.A centroidal axis

$$\sum M = 0 \Rightarrow M + \int \sigma xy dA = 0$$

$$\int \sigma y^2 dA = -M$$

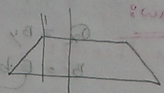
$\int y^2 dA \Rightarrow$  moment of inertia

$$\Rightarrow B \int y^2 dA = -M$$

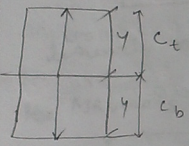
$$B \cdot I = -M$$

$$\sigma = By = -\frac{My}{I}$$

$\Rightarrow$  elastic flexural formula.

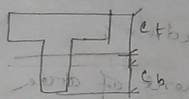


neutral axis  $\Rightarrow$  centroidal axis

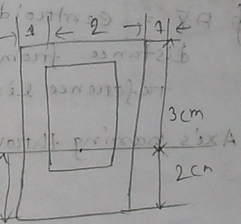


$$\sigma_{max} = \frac{Mc}{I}; c_t = e + \text{or } c_b$$

For rectangle section,  $c_t = c_b$



Top fibre —  $y(+)$   $\rightarrow$  max<sup>m</sup> compression  
 Bottom " —  $y(-)$   $\rightarrow$  max<sup>m</sup> tension



$$\bar{y} = \frac{\sum A_i y_i}{\sum A} = \frac{20 \times 3 - 6 \times 3.5}{10} = 2.83 \text{ cm}$$

$$\therefore I = \frac{bh^3}{12} - \frac{b_1 h_1^3}{12} = \frac{4 \times 6^3}{12} + 24(0.17)^2 - \left( \frac{2 \times 3^3}{12} - 6 \cdot v^2 \right)$$

$$= 65.5 \text{ cm}^4$$

11.10.20

For inelastic beam  $\rightarrow$  n.A centroid दिए गए त्रि

$$\sum M = 0 \Rightarrow M + \int \sigma_x y dA = 0$$

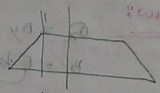
$$\int \sigma_y y^2 dA = -M \quad \int y^2 dA \Rightarrow \text{moment of inertia}$$

$$\Rightarrow B \int y^2 dA = -M$$

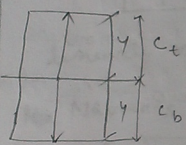
$$B \cdot I = -M$$

$$\sigma = B y = \frac{-M y}{I}$$

$\Rightarrow$  elastic flexural formula.

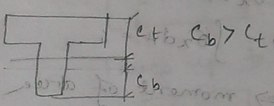


neutral axis  $\Rightarrow$  centroidal axis



$$\sigma_{max} = \frac{M c}{I} ; c_t = c_b$$

For rectangle section,  $c_t = c_b$

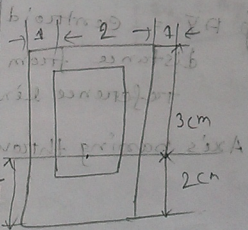


$y(t)$   $\rightarrow$  (at  $M(t)$ , max<sup>m</sup> compression

Top fibre  $- y(t)$

Bottom "  $- y(t)$   $\rightarrow$  max<sup>m</sup> tension

(अक्षात् हरे पर है केन्द्रात् बाह्ये शत)



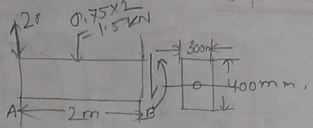
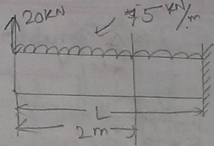
$$\bar{y} = \frac{\sum A_i y_i}{\sum A} = \frac{20 \times 3 - 6 \times 3.5}{10} = 2.83 \text{ cm}$$

$$\therefore I = \frac{bh^3}{12} - \frac{b_1 h_1^3}{12} = 2.83 \text{ cm}$$

$$= \left[ \frac{4 \times 6^3}{12} + 24(0.17)^2 \right] - \left( \frac{2 \times 3^3}{12} - 6.6^2 \right)$$

$$= 65.5 \text{ cm}^3$$

Example:



$$\sum M_A = 0 \Rightarrow 20 \times L - 1.5 \times L = M = 0$$

$$\therefore M = 38.5 \text{ kN}\cdot\text{m}$$

$$\sum F_y = 0 \Rightarrow 20 - 1.5 - B = 0$$

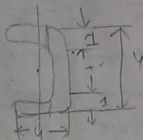
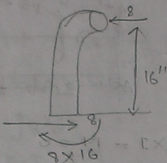
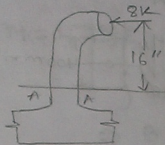
$$\therefore B = 18.5 \text{ kN}$$

$$I = \frac{300 \times 400^3}{12} = 1.6 \times 10^9 \text{ mm}^4$$

$$\sigma_{\max} = \frac{M \cdot c}{I} = \frac{38.5 \times 10^6 \times 200 \text{ mm}}{1.6 \times 10^9 \text{ mm}^4} = \pm 4.81 \text{ MPa}$$

29/02/2014

Example:



$$\bar{y} = \frac{4 \times 1 \times 0.5 + 2 \times 3 \times 1 \times 2.5}{4 \times 1 + 2 \times 3 \times 1} = 1.7$$

$$I = \frac{4 \times 1^3}{12} + [4 \times 1 \times (1.7 - 0.5)^2] + \frac{2 \times 1 \times 3^3}{12} + [2 \times 1 \times 3 \times (2.5 - 1.7)^2]$$

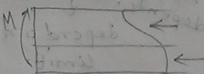
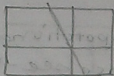
$$= 14.43 \text{ in}^4$$

$$\sigma_{\max} = \frac{M c}{I} = \frac{8 \times 16 \times 2.3}{14.43} = 20.4 \text{ ksi (compression)}$$

$$\sigma_{\max} = \frac{M c}{I} = \frac{8 \times 16 \times 1.7}{14.43} = 15.1 \text{ ksi (tension)}$$

## Inelastic bending of beams:

Procedure same, eqn



Stress diagram.

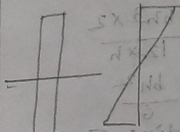
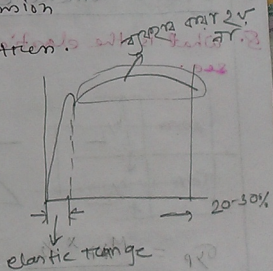
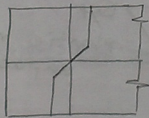
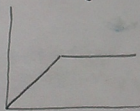
$$\sum F_x = 0 \Rightarrow \int_A \sigma_x dA = 0$$

$$\sum M_x = 0 \Rightarrow \int_A \sigma_x y dA = M$$

Neutral axis centroid  $\Rightarrow$   $\bar{y} = 0$

Total compression  $\neq$  total tension

Bending moment  $\rightarrow$  Normal stress.

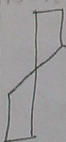


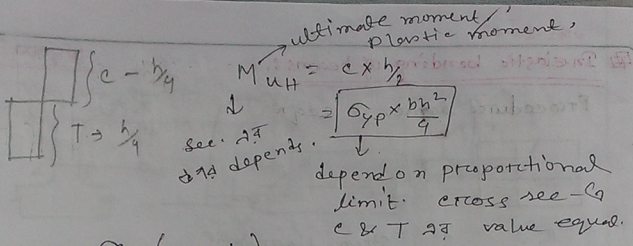
$$\sigma = \frac{M y}{I}$$

\* Stress proportional limit  $\Rightarrow$   $\sigma_y$   
 max<sup>m</sup> value  $\sigma_y$

min<sup>m</sup> length  $\Rightarrow$   $\sigma_y$   
 don't know

elastic curve



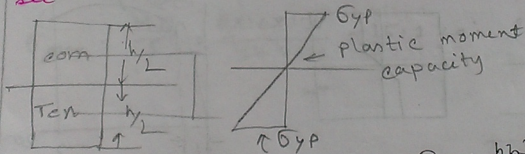


$c = T = \left( \frac{\sigma_y b h/2}{\text{stress area}} \right) = \text{max force generated (fail)}$   
 Total force

$m_p = \text{plastic moment}$

Elastic moment capacity:

8. What is the elastic moment capacity of a rectangular sec.



$$\sigma_{yp} = \frac{M_{yp} \times h/2}{I}$$

$$M_{yp} = \sigma_{yp} \times \frac{bh^3 \times 2}{12 \times h} = \sigma_{yp} \times \frac{bh^2}{6}$$

Residual stress  $\rightarrow$  without exceeding Top fibers & bottom fibre.

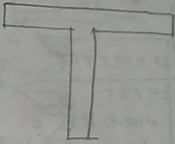
$$\frac{M_p}{M_{yp}} = 1.5 = \frac{6}{4}$$

Elastic limit  $\rightarrow$  50%  $\rightarrow$   $\frac{6}{4}$

07.07.2013

\* Shape factor of a rectangular beam is  $\boxed{1.5}$

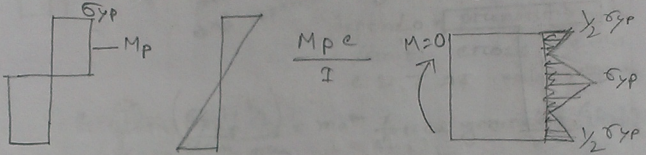
Q. What is the shape factor of T beam?



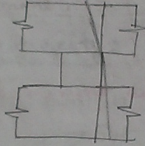
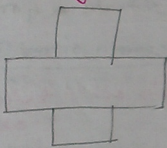
01.03.2013

load  $\rightarrow$  moment = 0

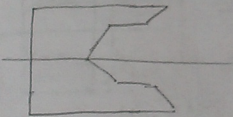
- \* Moment 0  $\rightarrow$  (बिना) internal stress (होना) चाहिए,
- \* बिना  $\rightarrow$  100% hooks law follow  $\rightarrow$   $\sigma = \epsilon E$



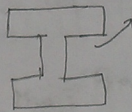
Bending strain of composite beam:



Strain linear  $\rightarrow$   
 Timber  $\epsilon_E$   $\rightarrow$   $\sigma = \epsilon E$   
 steel  $\rightarrow$   $\sigma = \epsilon E$



दोनों steel

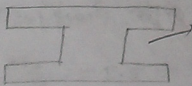


depth  $\rightarrow$   $\eta$   $\rightarrow$   $2\eta$

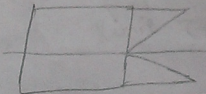
$$\eta = \frac{E_s}{E_w} = \frac{E_1}{E_2} \text{ ratio}$$

$$\Rightarrow \eta = \frac{E_2}{E_1}$$

Strain diagram.  
 steel  $\rightarrow$   $\sigma = \epsilon E$

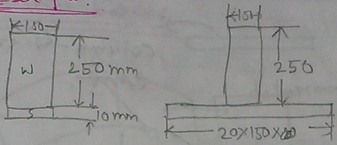


depth  $\rightarrow$   $2\eta$   
 height  $\rightarrow$   $2\eta$



Modular ratio  
 $\rightarrow$  ratio of  $\epsilon_E$  of two materials.

Example:



$$n = \frac{E_s}{E_w} = \frac{200}{10} = 20$$

$$\bar{y} = \frac{150 \times 250 \times 125 + (20 \times 150 \times 10 \times 255)}{150 \times 250 + 20 \times 150 \times 10}$$

$$= 183 \text{ mm}$$

$$I_{\bar{y}} = \frac{150 \times 250^3}{12} + [150 \times 250 \times (183 - 125)^2] + \frac{20 \times 150 \times 10^3}{12} + [20 \times 150 \times 10 \times (255 - 183)^2]$$

$$= 478 \times 10^6 \text{ mm}^4$$

Q 2

$$(\sigma_w)_{\max} = \frac{M c}{I}$$

$$= \frac{30 \times 10^6 \times 183}{478 \times 10^6}$$

$$= 11.5 \text{ MPa}$$

$$(\sigma_s)_{\max} = n \sigma_w$$

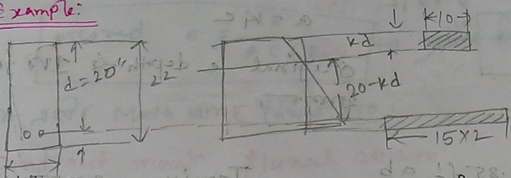
$$= 20 \times 11.5$$

$$= 230 \text{ MPa}$$

$$= \frac{20 \times 30 \times 10^6}{478 \times 10^6}$$

$$= 96.7 \text{ MPa}$$

Example:



$$10 \times kd \times \frac{kd}{2} = 15 \times 2 \times (20 - kd) \Rightarrow 5kd^2 = 600 - 30kd$$

$$\Rightarrow 5kd^2 + 30kd - 600 = 0 \Rightarrow kd = 8.36$$

$$\therefore 20 - kd = 11.64$$

$$I = \frac{10 \times 8.36^3}{12} + \left[ 10 \times 8.36 \times \left( \frac{8.36}{2} \right)^2 \right] + 0 + 30 \times (11.64)^2 = 6020 \text{ in}^4$$

$$(\sigma_c)_{\max} = \frac{50,000 \times 12 \times 8.56}{6020} = 833 \text{ psi}$$

$$\sigma_s = 15 \times \frac{50,000 \times 12 \times 11.64}{6020} = 17400 \text{ psi}$$

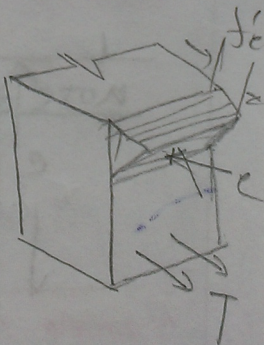
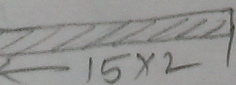
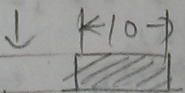
$$10 \approx 20.$$

Q 2

5)

$$\begin{aligned}(\sigma_w)_{\max} &= \frac{Mc}{I} \\ &= \frac{30 \times 10^6 \times 183}{478 \times 10^6} \\ &= 11.5 \text{ MPa}\end{aligned}$$

$$\begin{aligned}(\sigma_s)_{\max} &= n \sigma_w \\ &= 20 \times 11.5 \\ &= \frac{20 \times 30 \times 10^6}{486 \times 10^6} \\ &= 96.7 \text{ MPa}.\end{aligned}$$

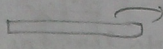
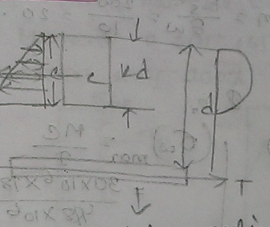


$$5kd^2 = 600 - 30kd$$

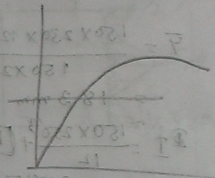
$$kd = 8.36$$

02.03.2019

Example:



column-tension joint



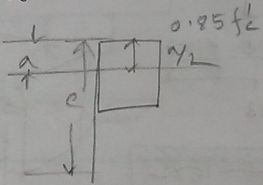
Modular ratio

\*  $j d \Rightarrow$  depth of reinforcement.

Ultimate moment capacity:

Linear  $\rightarrow$  depth of  $n \cdot A = k d$

Non linear  $\rightarrow$  " " " =  $c$



$$a = k c$$

original e depth -> 1/8 fraction

Example 3

$$e = 0.85 f'_d a b$$

$$A_{st} = 0.85 f'_c a b$$

$$M_u = A_{st} f_y (d - \frac{a}{2})$$

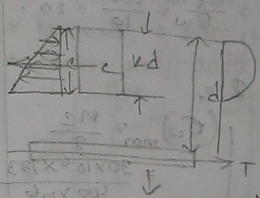
Tension = compression

$\sigma_y =$  yield stress

$$12 \times 10^6 \times 100 \times 100 = 1200 \times 10^6$$

02.03.2019

Example:



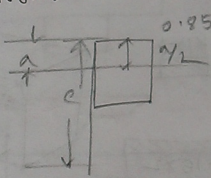
\* Modular ratio

\*  $jd \Rightarrow$  depth of reinforcement.

Ultimate moment capacity:

Linear  $\rightarrow$  depth of  $n \cdot A = kd$

Non linear  $\rightarrow$   $n \cdot A = c$



$a = kc$

original  $c$  depth -> IR fracture

$e = 0.85 f_c' ab$

$A_s f_y = 0.85 f_c' ab$

$M_u = A_s f_y (d - \frac{a}{2})$

Tension = compression

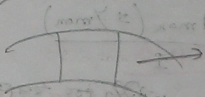
$\sigma_y = \text{yield stress}$

$$M_u = A_s f_y (d - \frac{a}{2}) = 12 \times 50000 \times 11.04 \times (12 - \frac{10.8}{2}) = 833424 \text{ Nmm}$$

curved beams:



deformation linearly vary  $\sigma(x)$



original depth is not equal strain doesn't proportional.

Straight beam

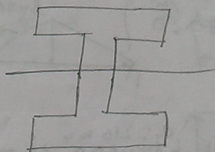
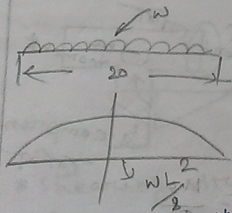
deformation linear strain-strain linear.

curved beam: deformation linear strain strain non linear.

proved  $\sigma = \frac{My}{Ac(R-y)}$  →

\* 2nd Math come (page 270)

Find out max<sup>m</sup> flexural stress.



$$\sigma = \frac{My}{I}$$

$$I = \frac{bh^3}{12} - \frac{b_1h_1^3}{12}$$

\* Bending moment diagram  $\frac{wL^2}{8}$

$$\frac{wL^2}{8} = \frac{10 \times 20^2}{8} = 25 \text{ kNm}$$

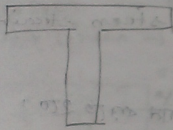
$$\sigma = \frac{M_{\max} \gamma_{\max}}{I}$$

\* I see  $\rightarrow \gamma_{\max}$  2 side  $\rightarrow$  equal.

$$\pm \text{ sign } \rightarrow \sigma = \frac{M_{\max} (\pm \gamma_{\max})}{I}$$

\* Tensile & compressive stress

$\pm =$   $(-)$  top  
 $(+)$  bottom

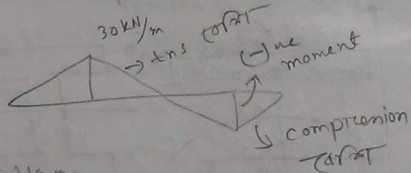
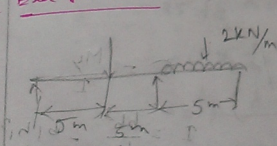


Neutral axis, unsymmetric  $\rightarrow$  tension & compression indicate

$$\sigma_{\text{com}} = \frac{M c_1}{I}$$

$$\sigma_{\text{ten}} = \frac{M c_2}{I}$$

### Example-2



$$\sigma = \frac{M y}{I}$$

$$\frac{wL^2}{8} = \frac{10 \times 20^2}{8} = 25 \text{ kNm}$$

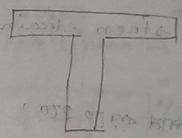
$$\sigma = \frac{M_{\max} \gamma_{\max}}{I}$$

\* I see  $\pm \gamma_{\max}$  2 side  $\rightarrow$  equal.

$$\pm \text{ sign } \rightarrow \sigma = \frac{M_{\max} (\pm \gamma_{\max})}{I}$$

\* Tensile & compressive stress

$\pm$  = (-) top position  
(+) bottom

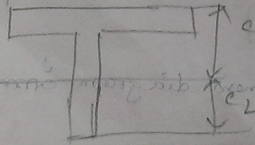
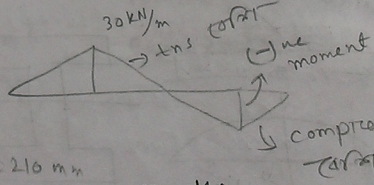
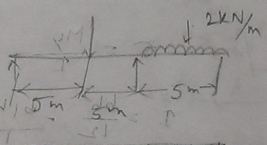


Neutral axis, unsymmetric  $\rightarrow$  tension or compression indicate

$$\sigma_{\text{com}} = \frac{M c_1}{y}$$

$$\sigma_{\text{ten}} = \frac{M c_2}{y}$$

Example-2



$$\sigma = \frac{M y}{I}$$

distance  $r_{12}$  multiple  $r_{12} @ 2r_{12}$

30x210  
25x390

4 stress  $r_{12} @ 2r_{12}$

$\sigma_{top}$ ,  $\sigma_{bottom}$  at + 30kN/m

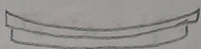
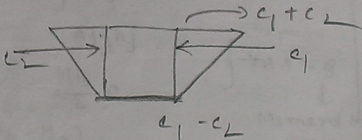
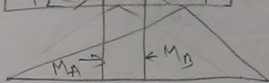
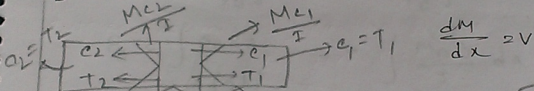
$\sigma_{top}$ ,  $\sigma_{bottom}$  at -25 kN.

## chapter-7

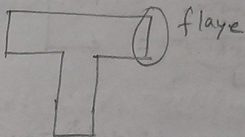
### shearing stress: (flexural shear)

all shear flexural stress associated with

Direct shear stress.



Bending moment vary  $r_{12} @ 2r_{12}$

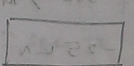


$$\int_{y_1}^{y_2} \sigma_z dz$$

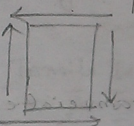
\* Shearing stress is occurred due to flexural stress.

$F_A \neq F_B \rightarrow$  but can be calculated by integration,

$c_1 - c_2 = 20 \Rightarrow$  difference of flexural stress.



shearing stress does not exist long horizontal.



$\tau_{xy} = \tau_{yx}$  (shearing stress) =  $\tau_{yx}$  (flexural stress)

$v = \frac{Mx}{I}$

Bending moment vary with  $x$

$\left. \begin{matrix} \tau \\ \sigma \end{matrix} \right\} \begin{matrix} \tau \\ \sigma \end{matrix}$

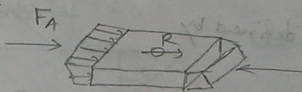
shearing stress is occurred due to flexure.

### Example - 8.6

Consider a rectangular beam of two materials bonded together as shown in fig 8.14 (a). The upper is 150

Shear flow:

8.03.14



$$\sigma = -\frac{My}{I}$$

$$F_A = \int_{A_{fghj}} \sigma_L dA = - \int_{A_{fghj}} \frac{M_A \cdot y}{I} dA = -M$$

$$F_B = \int_{A_{fghj}} \sigma_R dA = - \int_{A_{fghj}} \frac{M_B \cdot y}{I} dA$$

$$= -\frac{M_B}{I} \int y dA$$

$\int y dA = \bar{y}$   
 $\downarrow$   
 Moment of area

$$F_B - F_A = df = -\frac{(M_B - M_A)}{I} \cdot \bar{y}$$

$$\rightarrow = -\left(\frac{dM}{dx}\right) \bar{y} \quad \leftarrow \text{shear force}$$

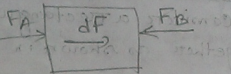
shear flow: resembles flow of water

$$q = \frac{V\bar{y}}{I}$$

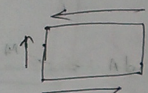
$$\tau = \frac{Q}{A} = \frac{VQ}{Ib}$$

horizontal plane

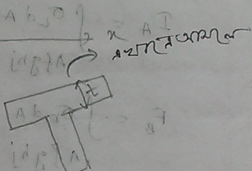
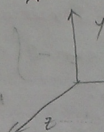
A plane is defined by normal



horizontal plane  
← b →

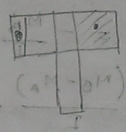
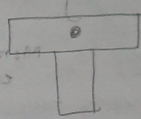


$$\tau_{xy} = \tau_{yx}$$



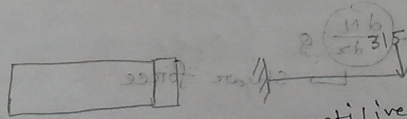
Example:

$\tau = \frac{VQ}{Ib}$



50mm

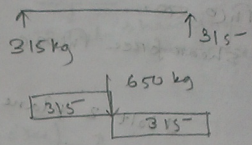
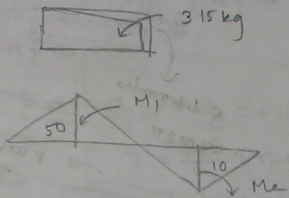
200mm  
mild steel beam



$$\tau = \frac{116}{200} \times 315 \text{ kg}$$

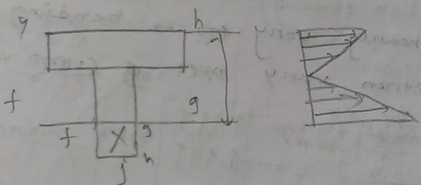
cantilever beam shear force is constant.

Two cases possible:



beam में loading diagram → पोपव टारन

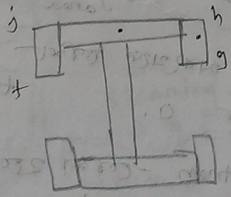
$F_A - F_B$  ३ अक्षात रूने वगैरे किने एपेता



Neutral axis में टारनता २० side २ तिते २०।

unit → careful  $q \rightarrow \text{kg/cm}^2$

4.13



leg screw spacing  
वर्गल sheat  
force diagram

From symmetry

१ वर्गल को (w) spacing



$$\frac{\int \tau_{xy} dA}{A} = \frac{V}{2I} \int_{-h/2}^{h/2} \left[ \frac{h}{2} \right]^2 = \frac{V}{2I} \cdot \frac{h^3}{4} = \frac{V}{2I} \cdot \frac{h^3}{4}$$

↓  
Shear force

↓  
Total shearing stress force

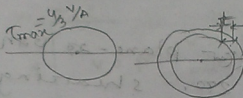
$\tau_{xy} \text{ max } 2 \text{ @ } y_1 = 0.$

$$\frac{Vh^2}{8I} \Rightarrow I = \frac{bh^3}{12} \text{ (rect. sec)}$$

$$\tau_{\text{max}} = \frac{3V}{2A} \quad | \quad 1.5 \frac{V}{A}$$

Average shearing stress (2 or 3 or 4)

Assignment:



$$\tau_{\text{max}} = \frac{2V}{A}$$

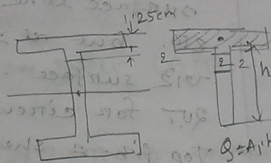
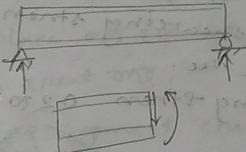
$$\tau_{\text{max}} = \frac{3}{2} \frac{V}{A}$$

2 for tree. see

\* prove @ 2 to

$$\tau_{\text{max}} = \frac{4}{3} \frac{V}{A} \text{ and } \tau_{\text{max}} = \frac{2V}{A}$$

Example:



$$\tau = \frac{VQ}{It}$$

top fb - 2 stress

2 to cor of 20.

$\tau_{\text{max}} \Rightarrow \text{N.A.}$

shearing stress

$$\tau = \frac{V}{I}$$

$$Q = A_1 h_1$$

2-2 flange stress

$$Q = A_1 h_1 + A_2 h_2$$

$$I = I_{\text{flange}} + I_{\text{web}}$$

$$\frac{\int \tau_{xy} dA}{A} = \frac{V}{2I} \int_{-h/2}^{h/2} \left[ \frac{h}{2} \right]^2 = \frac{V}{A}$$

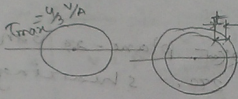
↓
↓  
 Shear force Total shearing stress force

$\tau_{xy} \text{ max at } y_1 = 0.$

$\frac{Vh^2}{8I} \Rightarrow I = \frac{bh^3}{12}$  (rec. sec)

$\tau_{\text{max}} = \frac{3V}{2A} \quad | \quad 1.5 \frac{V}{A}$       Average shearing stress (2/3 or 3/2)

Assignment:



$\tau_{\text{max}} = 2 \frac{V}{A}$

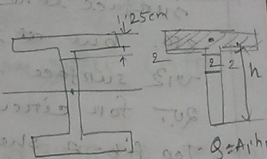
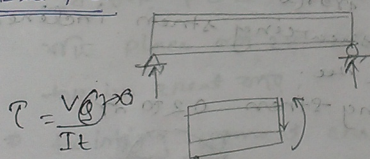
$\tau_{\text{max}} = \frac{3}{2} \frac{V}{A}$

2 for rec. sec

\* prove for 2 to

$\tau_{\text{max}} = \frac{4}{3} \frac{V}{A}$  and  $\tau_{\text{min}} = \frac{2V}{A}$

Example:



top fl - 2 stress

2 to coz of 20.1

$\tau_{\text{max}} \Rightarrow N.A. \Rightarrow$

shearing stress

$\tau = \frac{VQ}{I}$

$Q = A_1 h_1 + A_2 h_2$   
 $\tau = \frac{VQ}{I}$  Area

$$\int_A \tau_{xy} dA = \frac{V}{2I} \int_{-h/2}^{h/2} \left[ \frac{h}{2} \right]^2 = \frac{V}{2} \cdot \frac{h^2}{2} = \frac{Vh^2}{4}$$

↓  
Shear force

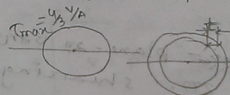
↓  
Total shearing stress force

$\tau_{xy} \text{ max}$  at  $y_1 = 0$ .

$$\frac{Vh^2}{8I} \Rightarrow I = \frac{bh^3}{12} \text{ (rect. sec)}$$

$$\tau_{\text{max}} = \frac{3V}{2A} \quad \left| \quad 1.5 \frac{V}{A} \right. \quad \text{Average shearing stress (rect. cross)}$$

Assignment:



$$\tau_{\text{max}} = \frac{2V}{A}$$

$$\tau_{\text{max}} = \frac{3}{2} \frac{V}{A}$$

2 for tree. sec



\* prove for 2 to  $\tau_{\text{max}} = \frac{4}{3} \frac{V}{A}$  and  $\tau_{\text{max}} = \frac{9V}{A}$

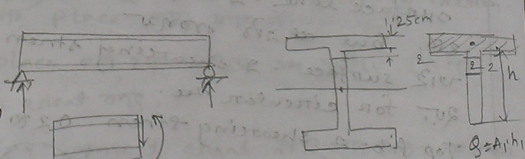
Example:

$$\tau = \frac{VQ}{It}$$

top fl - 2 stem

2 to cor  $\phi 20.1$

$\tau_{\text{max}} \Rightarrow N.A. \rightarrow$

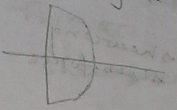


shearing stress

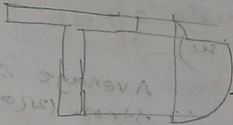
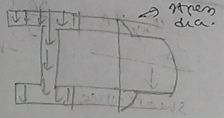
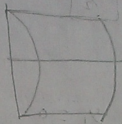
$$\tau = \frac{V}{I}$$

$Q = A_1 h_1 + A_2 h_2$   
 $\tau = \frac{V}{I} Q$

$\tau$  diagram



$\tau$  diagram



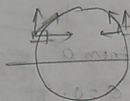
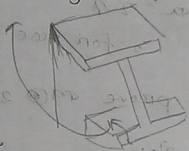
shearing stress down ward

shearing stress diagram

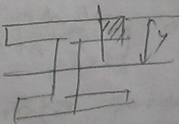
Limitations:

① plane section  $\tau$  or plane - is  $\tau$  and shearing stress along  $\tau$ , shearing stress alone  $\tau$  and  $\tau$ .

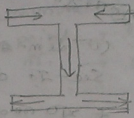
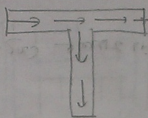
② spherical  $\tau$  or opposite surface line - is  $\tau$  and  $\tau$ , but at  $\tau$  or  $\tau$  or  $\tau$  surface - is shearing stress inclined  $\tau$ , for circular sec. Top fib - is shearing stress  $\tau$  and  $\tau$ .



Plane section:



shearing stress 2 direction horizontal & vertical flange - is  $\tau$  or  $\tau$  or  $\tau$  opposite direction.



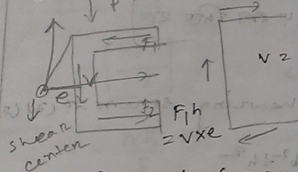
Shear flow  $\rightarrow$  per unit section area shearing stress flow area  $\rightarrow$   $v = \frac{VQ}{I}$

shearing flow area  $\rightarrow$   $v = \frac{VQ}{I}$

Vertical load  $\rightarrow$   $V$

Vertical, horizontal load  $\rightarrow$   $V$  and  $H$

channel section  $\rightarrow$   $V$



$v$  is the summation of all vertical load.

$\sum H = P$  balance  $\rightarrow$   $\sum H = 0$

$\sum \text{horizontal force} = 0$

\* horizontal force equal but moment create  $\rightarrow$  vertical force  $\rightarrow$  section  $\rightarrow$  place  $\rightarrow$   $\rightarrow$

place  $\rightarrow$   $\rightarrow$  see  $\rightarrow$  (vertical)  $\rightarrow$  torsion create  $\rightarrow$

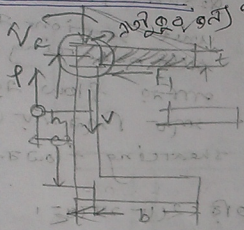
\* twisting moment create  $\rightarrow$  section  $\rightarrow$  vertical force  $\rightarrow$   $\rightarrow$

$\rightarrow$  vertical force  $\rightarrow$   $\rightarrow$

$\rightarrow$  vertical shear center  $\rightarrow$   $\rightarrow$

\* torsion  $\rightarrow$   $\rightarrow$

shear centre:



(U) dim - 1 (height 2mm) cur r2  
 See - 2 dim.  
 + 2D small.

$$q = \frac{VQ}{I} = \frac{V(bxt) \cdot \frac{h}{2}}{I}$$

$$= \frac{Vbt h}{2I}$$

$$P = \frac{Vbh}{2I} \left[ P \cdot 2 \text{ (top) } + \text{ (bottom) } \right]$$

$$F_1 = \frac{1}{2} q \cdot b = \frac{Vb^2 t h}{4I}$$

$F_1 = \frac{1}{2} \times q \cdot (b \times t) \rightarrow$  shearing stress at (a)

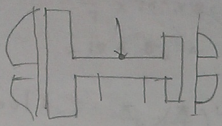
$$\text{Moment} = F_1 \times h = \frac{Vb^2 t h^2}{4I}$$

$$P \cdot e = F_1 h$$

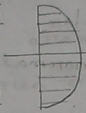
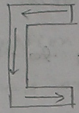
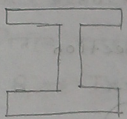
$$e = \frac{F_1 \cdot h}{P} = \frac{Vb^2 t h^2}{4I} \cdot \frac{4I}{Vbh} = \frac{b^2 t h}{3I}$$

11-03-2014

10.03.2014

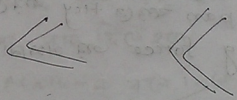


Unsymmetric beam 2<sup>nd</sup> order  
 point - 1 load form or shear  
 $R_1 \rightarrow$  Shear centre.



$v_2 - h = p \cdot e$   
 $I = I_1 + I_2$

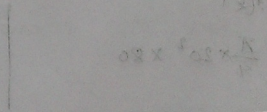
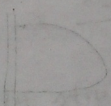
$$\frac{h I_2}{I}$$



$$e = \frac{h I_2}{I}$$

$$f = \frac{h I_1}{I} \quad E \cdot m_3$$

Parting section  $\rightarrow$  Canal section.



15.09.14

Thin pressure vessels:

Longitudinal tension

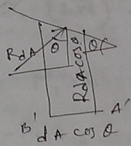
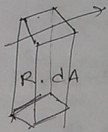
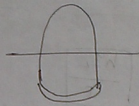
Riveted and welded joints:

Pressure - shear stress - tension relation

pressure perpendicular

- \* Pressure acts normal to the section
- \* Rupturing force:

Force per unit area  
bursting force on skin



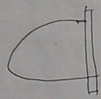
Force x projected area

Bursting force =  $R (\text{press}) \times A$  area enclosed of the rupturing plane  
 Rupturing force = Area of the skin x stress.

Example: 1

bursting area  $a = \pi r^2$

circle of force area



$$\frac{\pi}{4} \times 20^2 \times 80$$

71 bolt stress =  $\sigma$   
 $F = \sigma \times A$

$$BF = \frac{\pi}{4} \times 20^2 \times 80$$

$$= 181600 \text{ mm}^3 = \frac{\pi}{4} \times 20^2 \times 80$$

$$\text{bolt} = \frac{\pi}{4} \times \left(\frac{B}{4}\right)^2 \times L$$

R = reduction factor

\* Minimum cross section area is

R → Thread - 2 area is

Thread length: 20%, 25%, use max 20

\* Thread is length max 20

→ Math-1 value 20

area max → percentage max value

\* 18000 + 6400 = Tension

↑

→ Tension (max value)

pre tension

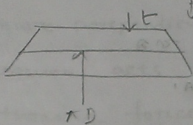
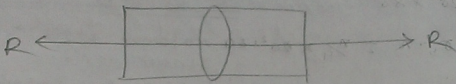
[ stress leak max ]

→ Tension

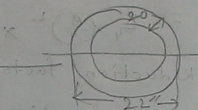
\* Pre compressed is value less (max value)

$$s/2 \frac{F}{A} = \frac{252000}{18} \times 0.5 = 4600 \text{ PSI}$$

Normally stress is value is 20 (force)



$$S = \frac{F}{A} = \frac{1000 \times 80}{22 \times \pi \times 1}$$



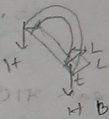
Actual area

$$= \frac{\pi}{4} (22^2 - 20^2)$$

$$\approx \pi D' t$$

Transverse direction - 2 burst area -

pressure intensity  $\times$  area  
 = Rupturing force.



$$H \cdot BF = 2H = \sigma_h \cdot L \cdot t \quad [\text{both sides}]$$

$$\Rightarrow 80 \times 20 \times 2 = 20 h \cdot L \cdot t$$

$$\Rightarrow \sigma_h = 800 \text{ psi}$$

$$\sigma_l = 381 \text{ psi}$$

} transverse stress  
 more than 2 times

(যদিও গ্যাসের চাপ উচ্চ হলেও) transverse direction.  
 develop stress

$\sigma_L$  cylinder  $\rightarrow$  Thin wall  
ENR CO  $\rightarrow$  cylinder  $\rightarrow$  ENR

General formula for cylindrical section:

Rupturing plane in longitudinal direction

$$BF = R \cdot \frac{\pi D^2}{4}$$

$$RF = \pi (D+t) \cdot L \cdot \sigma_L$$

$\rightarrow$  inside dia

$$= \frac{\pi R D^2}{4}$$

$$\therefore \sigma_L = \frac{RD^2}{4(D+t) \cdot t} \rightarrow \text{longitudinal direction}$$

$$D \gg t$$

$$\therefore \sigma_L = \frac{RD}{4t} = \frac{80 \times 20}{4 \times 5} = \underline{\underline{400 \text{ psi}}}$$

Approximate.

Rupturing plane in transverse

$$BF = RDL$$

$$RF = 2t = 2\sigma_h L \cdot t$$

$$\Rightarrow 2\sigma_h L \cdot t = RDL$$

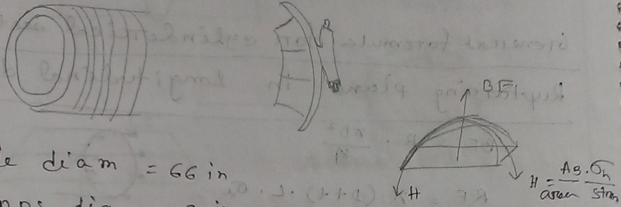
$$\therefore \sigma_h = \frac{RD}{2t}$$

# Example

16.03.2014

Bursting force = Resisting force

4-4



Inside diam = 66 in

Hoops dia = 2 in

Pressure of water 50 psi

bursting force transverse direction.

$$BF = R \cdot D \cdot L = 50 \times 66 \times 6 = 19800 \text{ lb}$$

$$R.F = 2H = 2 \times \sigma_h \times A_h =$$

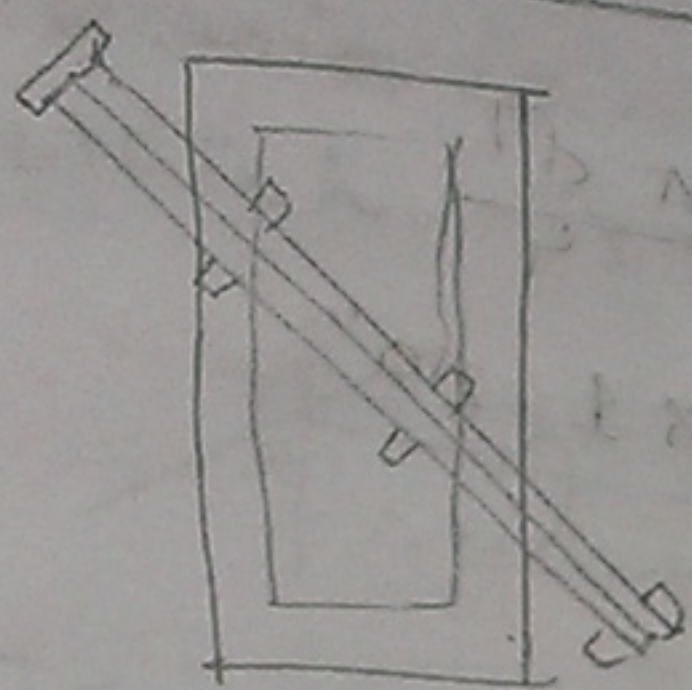
No thread,  $= 0.785 \text{ in}^2$  [2 in of 5/8]

Thread  $= 0.602 \text{ in}^2$

$\therefore$  considering thread  $= 2 \cdot \sigma_h \times A_h$

$$= 2 \times \sigma_h \times 0.602 = 50 \times 66 \times 6$$

$$\sigma_h = 16445 \text{ psi}$$



interior dimension

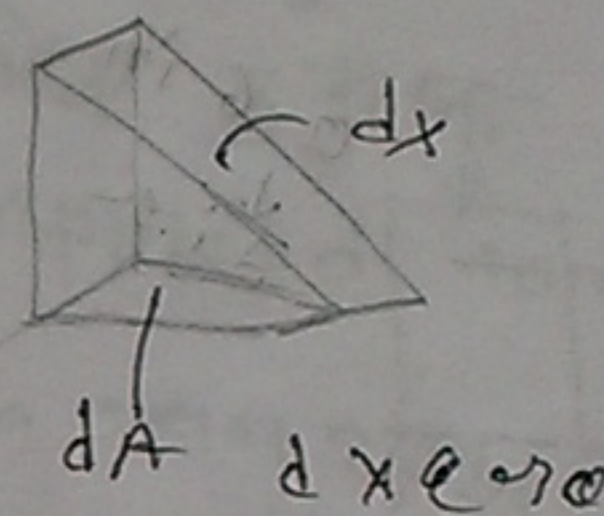
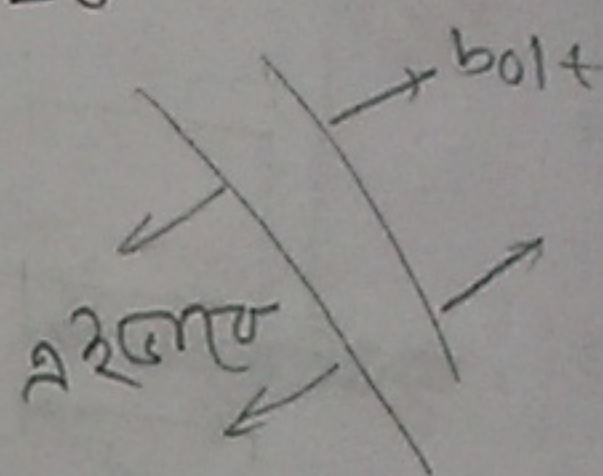
Area  $\text{int} = 20 \text{ square-in.}$

$20 \text{ in} \times 20 \text{ in} = 20 \times 20$

Normally EN 61  $\rightarrow 300 \text{ + psi}$

\* Area = 40 square in

Bursting force



$B F = \frac{210}{\text{pressure}} \times (20 \times 20) \cos 45^\circ$

$R F = A_b \cdot \sigma_b \times 10$

$A_b = (1.5)^2 \times 0.707 = 1.76$

$= 10 \times 1.76 \times \sigma_b$

$10 \times 1.76 \times \sigma_b = \frac{210 \times 400}{0.707}$

$10 \times 1.76 \cdot$

$\therefore \sigma_b = 6064 \text{ psi}$

Factor of safety =  $\frac{65000}{6064} = 11$

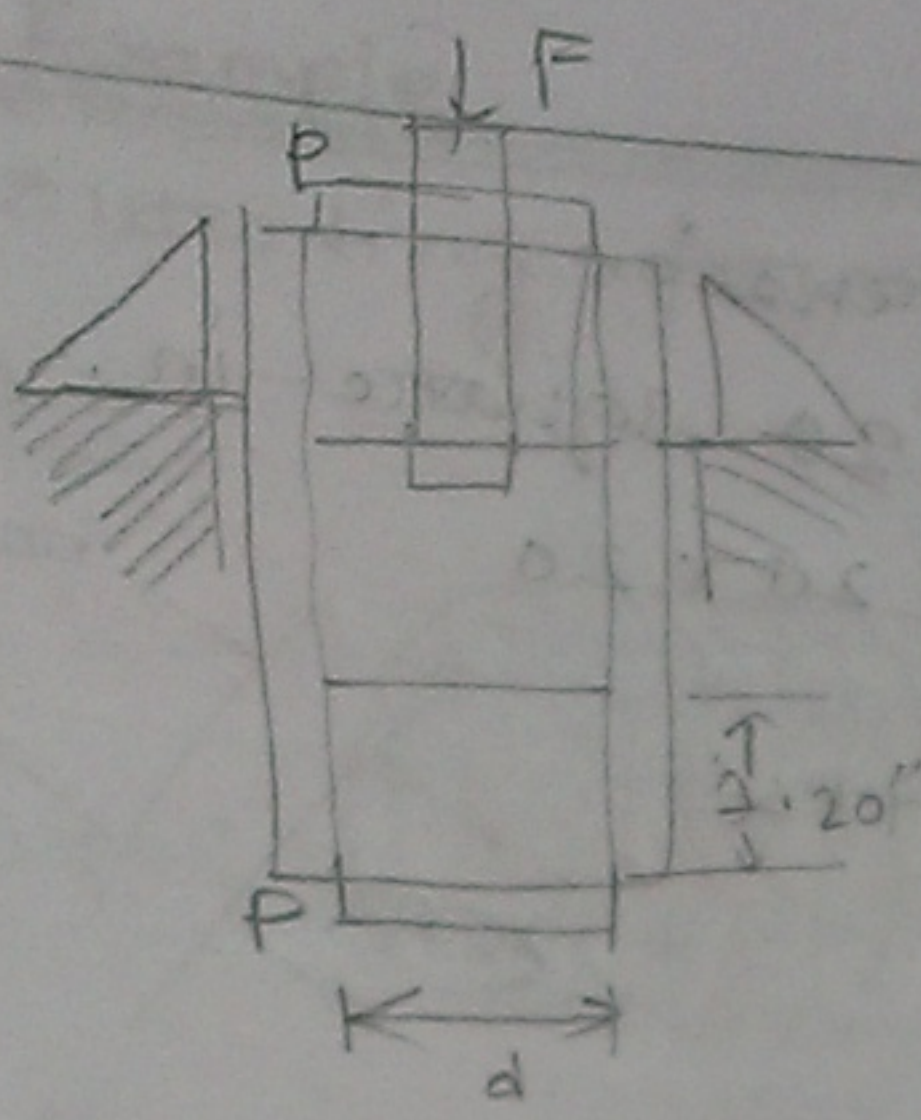
For pressure container  
f.F = 8, 10, 12

Factor of safety  $\rightarrow$   $\frac{\text{ultimate strength}}{\text{working stress}}$

bolt vertical  $\rightarrow$  ultimate strength

bolt inclined  $\rightarrow$  ultimate strength

4.7

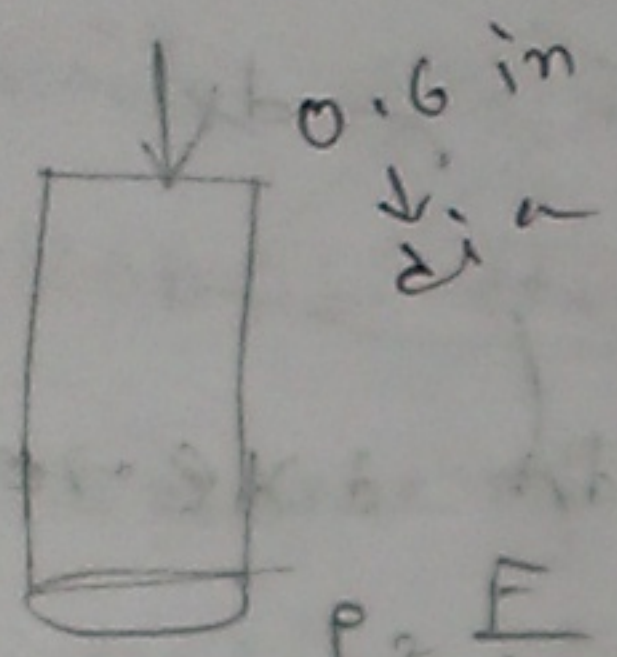


Perimeter  $= \frac{2\pi d^2}{4}$

Area  $= \frac{\pi d^2}{4} \times 1.20$

$R = \frac{F}{\pi d t}$

$B.F. = R \times \frac{\pi d^2}{4}$



Per shearing stress 1500 psi

Find out R.

allowable  $\rightarrow$  plung etc angle

$F = \text{pressure} \times \text{area}$

Andro bytle

Bolt

shearing stress 2000 psi bolt vertically

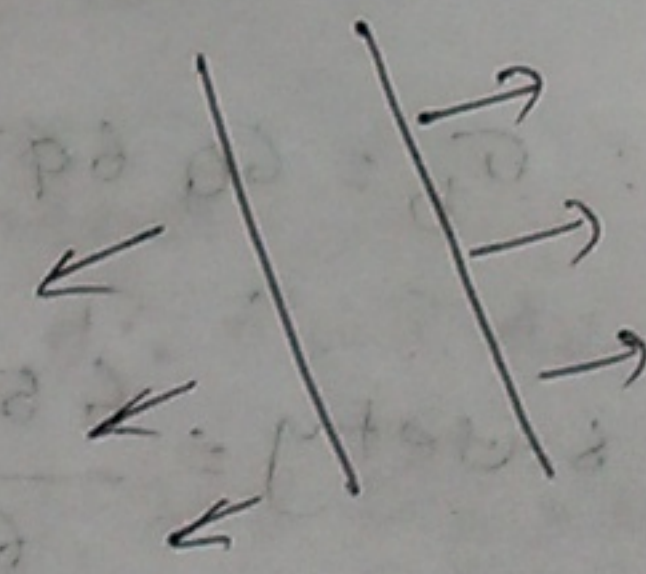
shearing stress

Rupture force

Brazing - welding

बrazing

inside dia 2"  
0.6"

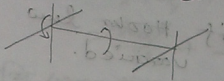


brazing etc

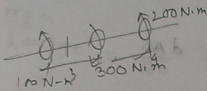
# Torsion

22.03.2014

Torsion is one kind of moment which is act on the axis.



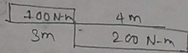
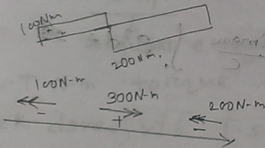
Torsion का लक्ष्य twisting diagram खींचना है।



Machine फिर खूना ला-

- applied torque

1 motor - 3 machine - torsion का लक्ष्य खींचना है।

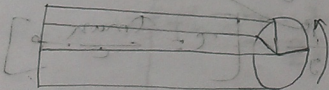


अक्ष  $x$  axis में  
संकेत positive  $x$  axis में  
दिशा में धनात्मक (+) है।

## Basic assumption :

physical movement outside

होना है।

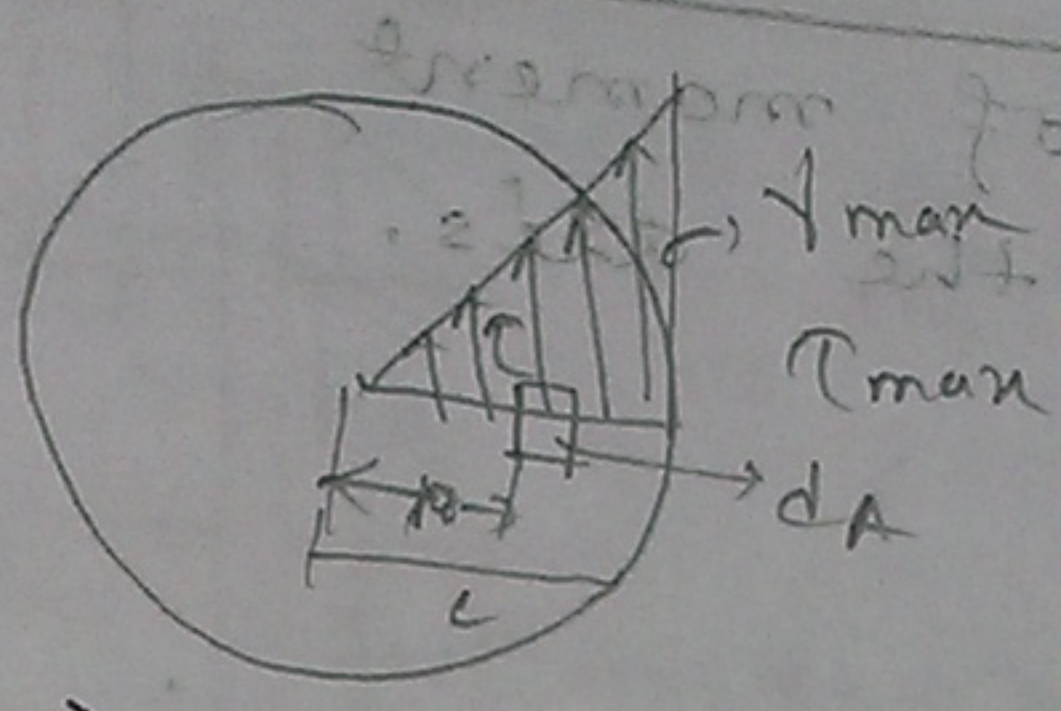


Angular direction linearly vary नहीं है।

Radial distance

plane sections remain plane.

$$\frac{\tau}{r} = \frac{G \theta}{L}$$



if Hook's law varied.  
 $\tau_{max} = \frac{T_{max}}{c}$   
 $\tau_{max} = E_{max}$

Shearing stress  $\tau$   $dA$

$$\int_A \tau dA \cdot p$$

$$\frac{\tau_{max}}{c} \cdot p \Rightarrow T = \int_A \frac{\tau_{max}}{c} \cdot p \cdot p dA$$

$$T = \frac{\tau_{max}}{c} \int p^2 dA$$

polar

$$\therefore T = \frac{\tau_{max}}{c} J$$

$$\tau_{max} = \frac{T \cdot c}{J}$$

$$\tau = \frac{T \cdot p}{J} \quad \left[ \tau = \frac{\tau_{max}}{c} \cdot p \right]$$

$p =$  radial distance

max<sup>m</sup> shearing stress  $\tau_{max} = \frac{T \cdot c}{J}$

$$\tau_{max} = \frac{M \cdot c}{I}$$

$$\tau_{max} = \frac{16 T}{32 d^3}$$

plane sections remain plane.

Polar moment of Inertia,  $I_p = \frac{\pi d^4}{64}$

$$J = I_p = I_x + I_y$$

$$= \frac{\pi d^4}{32} + \frac{\pi d^4}{32} = \pi$$

वेग (वेग)  $\rightarrow$  torque  
 वेग  $\rightarrow$  वेग

$N$  - rpm

$$f - \text{hertz} = \frac{N}{60}$$

$$1 \text{ rev} \rightarrow 2\pi$$

$$f \text{ rev} \rightarrow 2\pi f$$

Angular change

वेग  $T$  Nm torque use  $2\pi$

Work done,  $W \propto f \cdot s$

$$\propto T \cdot 2\pi f = P \text{ (power)}$$

(HP)  
(KW)

$$1 \text{ HP} \times 746 = T \cdot 2\pi f \text{ N.m}$$

$$\Rightarrow T = \frac{\text{HP} \times 746}{2\pi f}$$

$$= \frac{119 \text{ HP}}{f}$$

$$\text{In KW} \Rightarrow T = \frac{159 \text{ KW}}{f}$$

In FPS

$$T = \frac{63000 \text{ HP}}{N} \cdot \text{in-lb}$$

$$1 \text{ HP} = 550 \text{ ft-lb}$$

6-5

Select solid shaft  $P = 150 \text{ kW}$

$$\tau_{\text{max}} = 70 \text{ MPa}$$

$$f_1 = 0.5 \text{ Hz}$$

$$f_2 = 300 \text{ Hz}$$

Solve:

$$T_1 = \frac{150 \times 150}{0.5} \text{ N-m} = 45000 \text{ N-m}$$

$$T_2 = \frac{150 \times 150}{300} \text{ N-m} = 75 \text{ N-m}$$

$$\tau = \frac{Tc}{J} \Rightarrow \frac{T}{\tau} = \frac{J}{c}$$

$$\frac{\pi d^3}{16} = \frac{T}{\tau}$$

$$d^3 = \frac{16T}{\pi \tau}$$

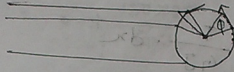
$$d = \sqrt[3]{\frac{16T}{\pi \tau}}$$

$$d_1 = 180 \text{ mm}$$

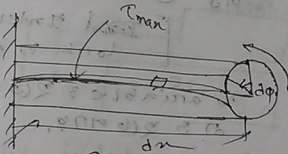
$$d_2 = 28 \text{ mm}$$

Handwritten notes and scribbles at the bottom of the page.

23.03.2019



Perpendicular plane  $\rightarrow$  shearing stress is at parallel to grain + to grain  $\rightarrow$  shear stress



$\tau_{max}$  surface - as  
centre  $\rightarrow \tau_{max} = 0$

$$AA' = \tau_{max} \cdot dx$$

$$AA' = c \cdot d\phi \quad \left[ \text{centre } \tau_{max} \text{ is } 0 \right]$$

$$c \cdot d\phi = \tau_{max} dx$$

$$\frac{d\phi}{dx} = \frac{\tau_{max}}{c}$$

$$\tau_{max} = G \tau_{max}$$

$$\Rightarrow \tau_{max} = \frac{T_{max}}{G}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{1}{G}$$

$$\textcircled{1} \Rightarrow \frac{d\phi}{dx} = \frac{\tau_{max}}{Gc}$$

$$= \frac{Tc}{J \cdot Gc} = \frac{T}{GJ}$$

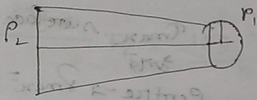
$$\int d\theta = \int \frac{T}{GJ} \cdot da$$

$$\phi_{AB} = \int_A^B d\theta = \int \frac{T}{GJ} \cdot dx$$

$$= \frac{T}{GJ} \int dx$$

$$= \frac{Ta}{GJ} \quad \left[ \text{when } T, G, J \text{ are constant} \right]$$

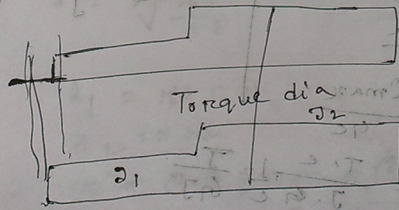
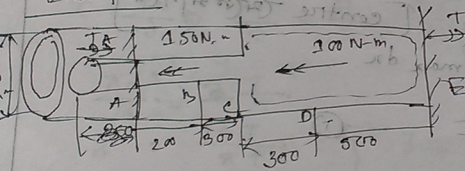
Internal torque is A-B



Variables 3 2 3 2 3

2 3 2 3 2 3

Example:



$$J_1 = \frac{\pi (50^4 - 25^4)}{32}$$

$$J_2 = \frac{\pi (32^4)}{32}$$

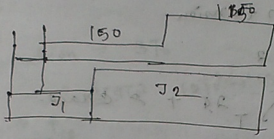
$$\phi_{AE} = \phi_{AB} + \phi_{BC} + \phi_{CD} + \phi_{DE}$$

$$= 0 + \frac{150 \times 200}{GJ_1} + \frac{150 \times 1000 \times 300}{G \cdot J_2} + \frac{1150 \times 500}{G \cdot J}$$

$$\phi = \frac{TL}{GJ}$$

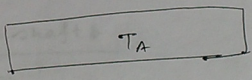
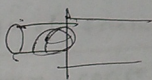
$G = 280 \text{ GPa}$   
 $= 80,000 \text{ N/mm}^2$

$$= 23.3 \times 10^3 \text{ rad}$$



Statically indeterminate:

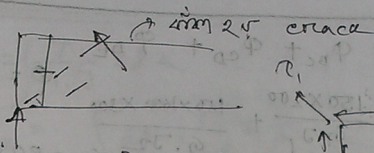
$$T_A + T_B = 150 + 1000$$



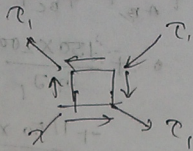
$$\phi'_{AB} = T_A \odot$$

Bear & Johnston

Wanpage



$$\frac{VQ}{It}$$

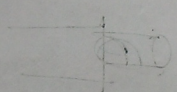
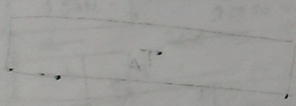


fail on shear force  
- at crack

concrete - weak in tension

corner - a beam - a

Crack at  $T$   $\tau + \sigma$  crack



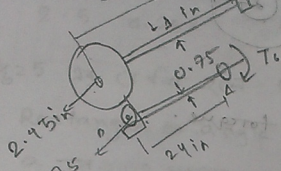
$$V_{AT} = \frac{1}{2} A \phi$$

Beam & column

Welded

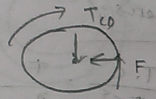
29.03.2019

Ex-1 AB shaft  $\rightarrow$  (ହାଉ) gear  
 CD "  $\rightarrow$  ବଡ଼ gear



Rotation  $\rightarrow$  370

45 - 1° ଝୁରାଇ (ହାଉ)  $\rightarrow$  ଓ ଝୁରାଇ ଭେଲି ସେଗେ ଧୁରାଏ



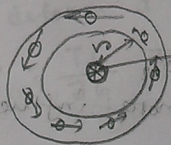
8000 PSI ରୁ 56116 ମିଲି 2ଟି

Design of transmission shafts

\* Stress concentration

cross section area ~~adversely~~ change 20

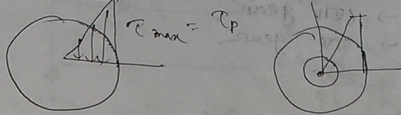
Stress ବଢ଼ାଇ ଦେଇ ଥାଏ



collar

$$N \cdot (F \cdot b) = T$$

Elasto plastic



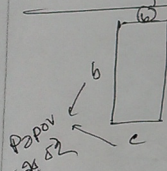
$$M_p = 1.5 M_y$$

$T_p = \frac{4}{3} T_y$  = plastic torque.

Residual stress:

Net moment = 0

Torsion of Noncircular Members:



$$\tau = \frac{Tc}{J} = \frac{T \cdot c}{\frac{\pi c^4}{2}} = \frac{T}{\frac{\pi}{2} c^3}$$

$$= \frac{T}{k \cdot c^3}$$

$$\tau_{max} = \frac{T}{c_1 a b^2}$$

Popov  $\rightarrow \tau = \frac{T}{\alpha_1 b c^2}$  [square root of width dimension]

$$\phi = \frac{TL}{c_2 a b^3 G}$$

Popov  $\rightarrow \phi = \frac{TL}{\beta b c^3 G}$

\* small influence size  $\rightarrow$  more influenced

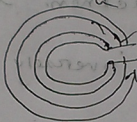
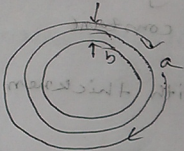
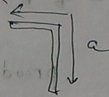
$\frac{a}{b}$	$e_1$	$e_2$
2.0	0.246	0.229
2.5	0.258	0.249

$\frac{a}{b} = 2.5$  ત્રિજ્યા 2cm  $e_1$  &  $e_2 = 0.333$

Rectangle ko angle or capacity name

a સહકારો સમજાવો મોળાવો.

ત્રિજ્યા 2cm.



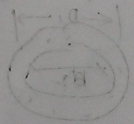
\* compare the stiffness of circular tube.

with

stiffness  $\Rightarrow k$ .  $\rightarrow$  unit length deformation  
જાથ જાથ (કર) force નીકળ

Torque required to  
deform  $1^\circ$ .

$$\phi = \frac{TL}{JG} \Rightarrow \frac{T}{\phi} = \frac{JG}{L}$$



$$\frac{T}{\phi} = \frac{JG}{L}$$

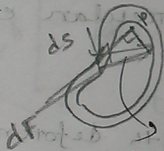
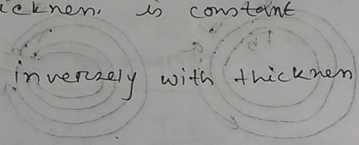
# Thin walled hollow shafts:



$$\tau_A t_A = \tau_B t_B = \tau t = q = \text{shear flow}$$

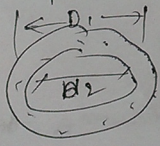
product of shearing stress & thickness is constant

shear stress varies inversely with thickness.



triangle - area  
 $= \frac{1}{2} \times ds \times t$

$$T = 2q(A) \rightarrow \text{area}$$



$$\pi \left[ \frac{D_1^2 - D_2^2}{4} \right]$$

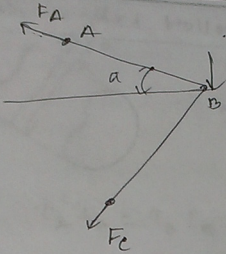
$$\tau = \frac{T}{2t(A)}$$

Angle of twist

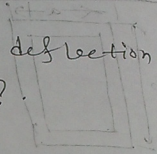
$$\phi = \frac{TL}{4(A)^2 G} \int \frac{ds}{t}$$



Last class



B pt - 1 deflection  
का 20?

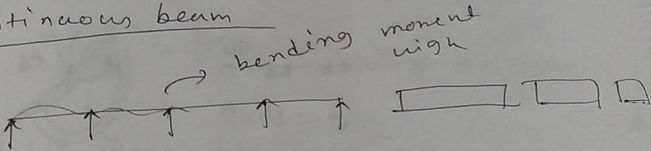


Thermal deflection

$$\Delta = \alpha \cdot \Delta T \cdot L$$

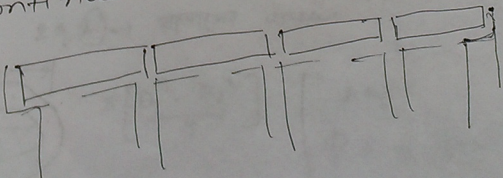
$\Sigma = \alpha \cdot \Delta T$  \* Strain का कुल length मागले वा,

Continuous beam



alluvial land →

continuous bridge का 20 DT 1

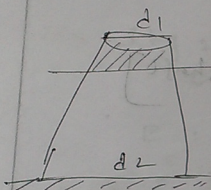


(i) Stress,  $P/A$

(ii) Strain  $\epsilon = \frac{\sigma}{E}$   $\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E} \sigma_y - \frac{\nu}{E} \sigma_z$   
 (Hooke's Law)

$\sigma_x, \sigma_y, \sigma_z$

$U = \int \frac{\sigma}{AE} \frac{P dx}{A'E} dx = \int \frac{\sigma}{E} dx$  [if  $P/A$  is constant]  
 $= \frac{PL}{AE}$ ; special case.



ಗಿಟಿ (ಗಿಟಿ) ಉದಾಹರಣೆ.

$P$  variable,  $A$  variable,

$E$  constant.

\* Axial strain energy,  $U = \int \frac{1}{2} \sigma_x \epsilon_x dv$

$\frac{1}{2}$  = average  $= \int \frac{\sigma_x^2}{2E} dv$

\* Temperature strain

$\epsilon = \alpha \Delta T$

deformation,  $\Delta L = \alpha \Delta T \cdot L$

(iii) AF, BM & SF (from sectional perpendicular)

BM  $\Rightarrow$  loading diagram.

$$\frac{dv}{dx} = M - \frac{d^2v}{dx^2} = V.$$

loading dia

(iv) Flexural stress:  $\sigma = \frac{My}{I}$

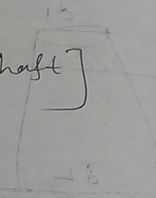
(v) Shearing stress:  $\tau = \frac{VQ}{I}$   
 $v = \frac{VQ}{I}$

(vi) Torsion  $T \rightarrow \tau = \frac{Tc}{J}$  [circular shaft]  
 For rectangular:

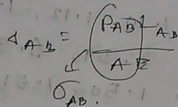
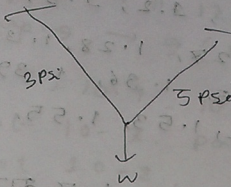
$$\tau = \frac{Tc}{J} = \frac{T}{\alpha bc^2} = \frac{T}{\beta bc^3}$$

$$\tau = \frac{T}{2A}$$

$$\tau = \frac{T}{2A}$$



AF, BM & SF (from section) ...  
 BM  $\rightarrow$  loading dia



Modulus of Elasticity of steel

— at (in) 21 TMS  $200 \text{ GPa}$   
 $29 \times 10^6 \text{ PSI}$

1 kg  $\approx 9.8 \text{ N}$   
 $\approx 10 \text{ N}$

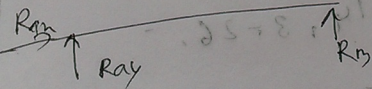
$$E = 200 \text{ GPa}$$

$$= \frac{200,000 \times 100}{10} \text{ N/mm}^2$$

$$= 2 \times 10^6 \text{ kg/cm}^2$$

Aluminium,  $E_{al} = 70 \text{ GPa}$

$E_{br} = 84 \text{ GPa}$



Sitzen - ~~1.1~~ Example - 1.1, 1.2, 1.5,

1.9, 1.10, 1.12, 1.21, 1.27, 1.28, 1.30, 1.32,  
1.33, 1.34, 1.34, 1.35, 1.36, 1.39, 1.44,  
1.47, 1.50, 1.51, 1.52, 1.53, 1.55, 1.57,  
1.58,

1.55  $\Rightarrow$  ACE two force members at

1.52  $\Rightarrow$  DEF force  $\cdot$   $\beta$  members at.

\* 1.3.

page - 31.

Ex: - 1.5, 1.6.

P<sub>1</sub> - 33.

Ex - 3.1, 3.2, 3.4, 3.3, 3.5, 3.7, 3.8

Ant - 3.3 & 3.5 page - 109; fig 3.9.

Ant - 5.6. Ant - 5.9.

page - 70 ex - 2.1,

Ant - 2.8  $\rightarrow$  fig 2.18 + fig 2 - 20.

Prob - 3.12, 3.13, 3.14, 3.26.

Ex 4.8.

Ex - 7.5, 7.6, 7.7, 7.8, 7.9, 7.11, 7.19,  
7.13, 7.15, 7.16, 7.17, 7.60, 7.18,

Ex - 8.4, 8.5, 8.12, 8.13, 8.11,

chap-8  $\rightarrow$  11, 12, 13, 14, 15, 17, 18, 20, 21, 25, 26.  
chap-9  $\rightarrow$  9 (11, 12, 15)

10.2, 10.4, 10.7, 10.8, 10.5,

Art-10-7. page-93 & 139.

Art 10-8.

EX-10-7,

Ch-10 - 2, 3, 4, 6, 8, 9, 10, 13, 15, 18, 20, 21,  
22, 23, 28, 29, 40, 41, 42.

EX-6.1, 6.2, 6.4, 6.5, 6.6, 6.7, 6.9,

Art-6.3, 6.6, 6.14, 6.16,

4.5 Prob.

fig 4.9.