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BUILDING CODE REQUIREMENTS FOR STRUCTURAL CONCRETE (ACI 318-02) AND COMMENTARY (ACI 318R-02)

REPORTED BY ACI COMMITTEE 318

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BUILDING CODE REQUIREMENTS FOR STRUCTURAL CONCRETE (ACI 318-02) AND COMMENTARY (ACI 318R-02)

REPORTED BY ACI COMMITTEE 318

The code portion of this document covers the proper design and construction of buildings of structural concrete. The code has been written in such form that it may be adopted by reference in a general building code and earlier editions have been widely used in this manner.

Among the subjects covered are: drawings and specifications; inspection; materials; durability requirements; concrete quality, mixing, and placing; formwork; embedded pipes; construction joints; reinforcement details; analysis and design; strength and serviceability; flexural and axial loads; shear and torsion; development and splices of reinforcement; slab systems; walls; footings; precast concrete; composite flexural members; prestressed concrete; shells and folded plate members; strength evaluation of existing structures; special provisions for seismic design; structural plain concrete; strut-and-tie modeling in Appendix A; alternative design provisions in Appendix B; alternative load and strength-reduction factors in Appendix C; and anchoring to concrete in Appendix D.

The quality and testing of materials used in construction are covered by reference to the appropriate ASTM standard specifications. Welding of reinforcement is covered by reference to the appropriate ANSI/AWS standard.

Because the ACI Building Code is written as a legal document so that it may be adopted by reference in a general building code, it cannot present background details or suggestions for carrying out its requirements or intent. It is the function of this commentary to fill this need.

The commentary discusses some of the considerations of the committee in developing the code with emphasis given to the explanation of new or revised provisions that may be unfamiliar to code users.

References to much of the research data referred to in preparing the code are cited for the user desiring to study individual questions in greater detail. Other documents that provide suggestions for carrying out the requirements of the code are also cited.

Keywords: admixtures; aggregates; anchorage (structural); beam-column frame; beams (supports); **building codes;** cements; cold weather construction; columns (supports); combined stress; composite construction (concrete and steel); composite construction (concrete to concrete); compressive strength; **concrete construction; concretes;** concrete slabs; construction joints; continuity (structural); contraction joints; cover; curing; deep beams; deflections; drawings; earthquake resistant structures; embedded service ducts; flexural strength; floors; folded plates; footings; formwork (construction); frames; hot weather construction; inspection; isolation joints; joints (junctions); joists; lightweight concretes; loads (forces); load tests (structural); materials; mixing; mix proportioning; modulus of elasticity; moments; pipe columns; pipes (tubing); placing; plain concrete; precast concrete; prestressed concrete; prestressing steels; quality control; **reinforced concrete**; reinforcing steels; roofs; serviceability; shear strength; shearwalls; shells (structural forms); spans; specifications; splicing; strength; strength analysis; stresses; **structural analysis; structural concrete; structural design;** structural integrity; T-beams; torsion; walls; water; welded wire fabric.

ACI 318-02 was adopted as a standard of the American Concrete Institute November 1, 2001 to supersede ACI 318-99 in accordance with the Institute's standardization procedure.

A complete metric companion to ACI 318/318R has been developed, 318M/318RM; therefore no metric equivalents are included in this document.

ACI Committee Reports, Guides, Standard Practices, and Commentaries are intended for guidance in planning, designing, executing, and inspecting construction. This Commentary is intended for the use of individuals who are competent to evaluate the significance and limitations of its content and recommendations and who will accept responsibility for the application of the material it contains. The American Concrete Institute disclaims any and all responsibility for the stated principles. The Institute shall not be liable for any loss or damage arising therefrom. Reference to this commentary shall not be made in contract documents. If items found in this Commentary are desired by the Architect/Engineer to be a part of the contract documents, they shall be restated in mandatory language for incorporation by the Architect/Engineer.

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The 2002 ACI Building Code and Commentary are presented in a side-by-side column format, with code text placed in the left column and the corresponding commentary text aligned in the right column. To further distinguish the Code from the Commentary, the Code has been printed in Helvetica, the same type face in which this paragraph is set. Vertical lines in the margins indicate changes from ACI 318-99, including nontechnical changes such as a new section or equation number.

This paragraph is set in Times Roman, and all portions of the text exclusive to the Commentary are printed in this type face. Commentary section numbers are preceded by an "R" to further distinguish them from Code section numbers. Vertical lines in the margins indicate changes from ACI 318R-99, including nontechnical changes such as a new section or equation number.

INTRODUCTION

This commentary discusses some of the considerations of Committee 318 in developing the provisions contained in "Building Code Requirements for Structural Concrete (ACI 318-02)," hereinafter called the code or the 2002 code. Emphasis is given to the explanation of new or revised provisions that may be unfamiliar to code users. In addition, comments are included for some items contained in previous editions of the code to make the present commentary independent of the commentary for ACI 318-99. Comments on specific provisions are made under the corresponding chapter and section numbers of the code.

The commentary is not intended to provide a complete historical background concerning the development of the ACI Building Code,* nor is it intended to provide a detailed résumé of the studies and research data reviewed by the committee in formulating the provisions of the code. However, references to some of the research data are provided for those who wish to study the background material in depth.

As the name implies, "Building Code Requirements for Structural Concrete (ACI 318-02)" is meant to be used as part of a legally adopted building code and as such must differ in form and substance from documents that provide detailed specifications, recommended practice, complete design procedures, or design aids.

The code is intended to cover all buildings of the usual types, both large and small. Requirements more stringent than the code provisions may be desirable for unusual construction. The code and commentary cannot replace sound engineering knowledge, experience, and judgement.

A building code states only the minimum requirements necessary to provide for public health and safety. The code is based on this principle. For any structure, the owner or the structural designer may require the quality of materials and construction to be higher than the minimum requirements necessary to protect the public as stated in the code. However, lower standards are not permitted.

The commentary directs attention to other documents that provide suggestions for carrying out the requirements and intent of the code. However, those documents and the commentary are not a part of the code.

The code has no legal status unless it is adopted by the governments bodies having the police power to regulate building design and construction. Where the code has not been adopted, it may serve as a reference to good practice even though it has no legal status.

The code provides a means of establishing minimum standards for acceptance of designs and construction by a legally appointed building official or his designated representatives. The code and commentary are not intended for use in settling disputes between the owner, engineer, architect, contractor, or their agents, subcontractors, material suppliers, or testing agencies. Therefore, the code cannot define the contract responsibility of each of the parties in usual construction. General references requiring compliance with the code in the job specifications should be avoided since the contractor is rarely in a position to accept responsibility for design details or construction requirements that depend on a detailed knowledge of the design. Generally, the drawings, specifications and contract documents should contain all of the necessary requirements to ensure compliance with the code. In part, this can be accomplished by reference to specific code sections in the job specifications. Other ACI publications, such as "Specifications for Structural Concrete (ACI 301)" are written specifically for use as contract documents for construction.

It is desirable to have testing and certification programs for the individual parties involved with the execution of work performed in accordance with this code. Available for this purpose are the plant certification programs of the Precast/Prestressed Concrete Institute, the Post-Tensioning Institute and the National Ready Mixed Concrete Association; the personnel certification programs of the American Concrete Institute and the Post-Tensioning Institute; and the Concrete Reinforcing Steel Institute's Voluntary Certification Program for Fusion-Bonded Epoxy Coating Applicator Plants. In addition, "Standard Specification for Agencies Engaged in the Testing and/or Inspection of Materials Used in Construction" (ASTM E 329-00b) specifies performance requirements for inspection and testing agencies.

* For a history of the ACI Building Code see Kerekes, Frank, and Reid, Harold B., Jr., "Fifty Years of Development in Building Code Requirements for Reinforced Concrete," ACI JOURNAL, *Proceedings* V. 50, No. 6, Feb. 1954, p. 441. For a discussion of code philosophy, see Siess, Chester P., "Research, Building Codes, and Engineering Practice," ACI JOURNAL, *Proceedings* V. 56, No. 5, May 1960, p. 1105.

Design reference materials illustrating applications of the code requirements may be found in the following documents. The design aids listed may be obtained from the sponsoring organization.

Design aids:

"ACI Design Handbook," ACI Committee 340, Publication SP-17(97), American Concrete Institute, Farmington Hills, Mich., 1997, 482 pp. (Provides tables and charts for design of eccentrically loaded columns by the Strength Design Method. Provides design aids for use in the engineering design and analysis of reinforced concrete slab systems carrying loads by two-way action. Design aids are also provided for the selection of slab thickness and for reinforcement required to control deformation and assure adequate shear and flexural strengths.)

"ACI Detailing Manual—1994," ACI Committee 315, Publication SP-66(94), American Concrete Institute, Farmington Hills, Mich., 1994, 244 pp. (Includes the standard, ACI 315-92, and report, ACI 315R-94. Provides recommended methods and standards for preparing engineering drawings, typical details, and drawings placing reinforcing steel in reinforced concrete structures. Separate sections define responsibilities of both engineer and reinforcing bar detailer.)

"Guide to Durable Concrete (ACI 201.2R-92)," ACI Committee 201, American Concrete Institute, Farmington Hills, Mich., 1992, 41 pp. (Describes specific types of concrete deterioration. It contains a discussion of the mechanisms involved in deterioration and the recommended requirements for individual components of the concrete, quality considerations for concrete mixtures, construction procedures, and influences of the exposure environment. Section R4.4.1 discusses the difference in chloride-ion limits between ACI 201.2R-92 and the code.)

"Guide for the Design of Durable Parking Structures (362.1R-97)," ACI Committee 362, American Concrete Institute, Farmington Hills, Mich., 1997, 40 pp. (Summarizes practical information regarding design of parking structures for durability. It also includes information about design issues related to parking structure construction and maintenance.)

"CRSI Handbook," Concrete Reinforcing Steel Institute, Schaumburg, Ill., 8th Edition, 1996, 960 pp. (Provides tabulated designs for structural elements and slab systems. Design examples are provided to show the basis of and use of the load tables. Tabulated designs are given for beams; square, round and rectangular columns; one-way slabs; and one-way joist construction. The design tables for two-way slab systems include flat plates, flat slabs and waffle slabs. The chapters on foundations provide design tables for square footings, pile caps, drilled piers (caissons) and cantilevered retaining walls. Other design aids are presented for crack control; and development of reinforcement and lap splices.)

"Reinforcement Anchorages and Splices," Concrete Reinforcing Steel Institute, Schaumberg, Ill., 4th Edition, 1997, 100 pp. (Provides accepted practices in splicing reinforcement. The use of lap splices, mechanical splices, and welded splices are described. Design data are presented for development and lap splicing of reinforcement.)

"Structural Welded Wire Reinforcement Manual of Standard Practice," Wire Reinforcement Institute, Findlay, Ohio, 4th Edition, Apr. 1992, 31 pp. (Describes wire fabric material, gives nomenclature and wire size and weight tables. Lists specifications and properties and manufacturing limitations. Book has latest code requirements as code affects welded wire. Also gives development length and splice length tables. Manual contains customary units and soft metric units.)

"Structural Welded Wire Reinforcement Detailing Manual," Wire Reinforcement Institute, Findlay, Ohio, 1994, 252 pp. (Updated with current technical fact sheets inserted.) The manual, in addition to including ACI 318 provisions and design aids, also includes: detailing guidance on welded wire reinforcement in one-way and two-way slabs; precast/prestressed concrete components; columns and beams; cast-in-place walls; and slabs-on-ground. In addition, there are tables to compare areas and spacings of high-strength welded wire with conventional reinforcing.

"Strength Design of Reinforced Concrete Columns," Portland Cement Association, Skokie, Ill., 1978, 48 pp. (Provides design tables of column strength in terms of load in kips versus moment in ft-kips for concrete strength of 5000 psi and Grade 60 reinforcement. Design examples are included. Note that the PCA design tables do not include the strength reduction factor f in the tabulated values; M_u/f and P_u/f must be used when designing with this aid.

"PCI Design Handbook—Precast and Prestressed Concrete," Precast/Prestressed Concrete Institute, Chicago, 5th Edition, 1999, 630 pp. (Provides load tables for common industry products, and procedures for design and analysis of precast and prestressed elements and structures composed of these elements. Provides design aids and examples.)

"Design and Typical Details of Connections for Precast and Prestressed Concrete," Precast/Prestressed Concrete Institute, Chicago, 2nd Edition, 1988, 270 pp. (Updates available information on design of connections for both structural and architectural products, and presents a full spectrum of typical details. Provides design aids and examples.)

"PTI Post-Tensioning Manual," Post-Tensioning Institute, Phoenix, 5th Edition, 1990, 406 pp. (Provides comprehensive coverage of post-tensioning systems, specifications, and design aid construction concepts.)

"PTI Design of Post-Tensioned Slabs," Post-Tensioning Institute, Phoenix, 2nd Edition, Apr. 1984, 56 pp. (Illustrates application of the code requirements for design of one-way and two-way post-tensioned slabs. Detailed design examples are presented.)

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PART 1 — GENERAL

CHAPTER 1 — GENERAL REQUIREMENTS

CODE

1.1 — Scope

1.1.1 — This code provides minimum requirements for design and construction of structural concrete elements of any structure erected under requirements of the legally adopted general building code of which this code forms a part. In areas without a legally adopted building code, this code defines minimum acceptable standards of design and construction practice.

For structural concrete, the specified compressive strength shall not be less than 2500 psi. No maximum specified compressive strength shall apply unless restricted by a specific code provision.

COMMENTARY

R1.1 — Scope

The American Concrete Institute **"Building Code Requirements for Structural Concrete (ACI 318-02),"** referred to as the code, provides minimum requirements for structural concrete design or construction.

The 2002 edition of the code revised the previous standard **"Building Code Requirements for Structural Concrete (ACI 318-99)."** This standard includes in one document the rules for all concrete used for structural purposes including both plain and reinforced concrete. The term "structural concrete" is used to refer to all plain or reinforced concrete used for structural purposes. This covers the spectrum of structural applications of concrete from nonreinforced concrete to concrete containing nonprestressed reinforcement, prestressing steel, or composite steel shapes, pipe, or tubing. Requirements for structural plain concrete are in Chapter 22.

Prestressed concrete is included under the definition of reinforced concrete. Provisions of the code apply to prestressed concrete except for those that are stated to apply specifically to nonprestressed concrete.

Chapter 21 of the code contains special provisions for design and detailing of earthquake resistant structures. See 1.1.8.

In the 1999 code and earlier editions, Appendix A contained provisions for an alternate method of design for nonprestressed reinforced concrete members using service loads (without load factors) and permissible service load stresses. The Alternate Design Method was intended to give results that were slightly more conservative than designs by the Strength Design Method of the code. The Alternate Design Method of the 1999 code may be used in place of applicable sections of the 2002 code.

Appendix A of the code contains provisions for the design of regions near geometrical discontinuities, or abrupt changes in loadings.

Appendix B of the 2002 code contains provisions for reinforcement limits based on $\mathbf{0.75}r_b$, determination of the strength reduction factor f, and moment redistribution that have been in the code for many years, including the 1999 code. The provisions are applicable to reinforced and prestressed concrete members. Designs made using the provisions of Appendix B are equally acceptable as those based on the body of the code, provided the provisions of Appendix B are used in their entirety.

Appendix C of the code allows the use of the factored load combinations given in Chapter 9 of the 1999 code.

CODE

1.1.2 — This code supplements the general building code and shall govern in all matters pertaining to design and construction of structural concrete, except wherever this code is in conflict with requirements in the legally adopted general building code.

1.1.3 — This code shall govern in all matters pertaining to design, construction, and material properties wherever this code is in conflict with requirements contained in other standards referenced in this code.

1.1.4 — For special structures, such as arches, tanks, reservoirs, bins and silos, blast-resistant structures, and chimneys, provisions of this code shall govern where applicable. See also 22.1.2.

COMMENTARY

Appendix D contains provisions for anchoring to concrete.

R1.1.2 — The American Concrete Institute recommends that the code be adopted in its entirety; however, it is recognized that when the code is made a part of a legally adopted general building code, the general building code may modify provisions of this code.

R1.1.4 — Some special structures involve unique design and construction problems that are not covered by the code. However, many code provisions, such as the concrete quality and design principles, are applicable for these structures. Detailed recommendations for design and construction of some special structures are given in the following ACI publications:

"Standard Practice for the Design and Construction of Reinforced Concrete Chimneys" reported by ACI Committee 307.[1.1] (Gives material, construction, and design requirements for circular cast-in-place reinforced chimneys. It sets forth minimum loadings for the design of reinforced concrete chimneys and contains methods for determining the stresses in the concrete and reinforcement required as a result of these loadings.)

"Standard Practice for Design and Construction of Concrete Silos and Stacking Tubes for Storing Granular Materials" reported by ACI Committee 313.[1.2] (Gives material, design, and construction requirements for reinforced concrete bins, silos, and bunkers and stave silos for storing granular materials. It includes recommended design and construction criteria based on experimental and analytical studies plus worldwide experience in silo design and construction.)

"Environmental Engineering Concrete Structures" reported by ACI Committee 350.[1.3] (Gives material, design and construction recommendations for concrete tanks, reservoirs, and other structures commonly used in water and waste treatment works where dense, impermeable concrete with high resistance to chemical attack is required. Special emphasis is placed on a structural design that minimizes the possibility of cracking and accommodates vibrating equipment and other special loads. Proportioning of concrete, placement, curing and protection against chemicals are also described. Design and spacing of joints receive special attention.)

"Code Requirements for Nuclear Safety Related Concrete Structures" reported by ACI Committee 349.[1.4] (Provides minimum requirements for design and construction of concrete structures that form part of a nuclear power plant and have nuclear safety related functions. The code does not cover concrete reactor vessels and concrete containment structures which are covered by ACI 359.)

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"Code for Concrete Reactor Vessels and Containments" reported by ACI-ASME Committee 359.[1.5] (Provides requirements for the design, construction, and use of concrete reactor vessels and concrete containment structures for nuclear power plants.)

1.1.5 — This code does not govern design and installation of portions of concrete piles, drilled piers, and caissons embedded in ground except for structures in regions of high seismic risk or assigned to high seismic performance or design categories. See 21.10.4 for requirements for concrete piles, drilled piers, and caissons in structures in regions of high seismic risk or assigned to high seismic performance or design categories.

R1.1.5 — The design and installation of piling fully embedded in the ground is regulated by the general building code. For portions of piling in air or water, or in soil not capable of providing adequate lateral restraint throughout the piling length to prevent buckling, the design provisions of this code govern where applicable.

Recommendations for concrete piles are given in detail in **"Recommendations for Design, Manufacture, and Installation of Concrete Piles"** reported by ACI Committee 543.[1.6] (Provides recommendations for the design and use of most types of concrete piles for many kinds of construction.)

Recommendations for drilled piers are given in detail in **"Design and Construction of Drilled Piers"** reported by ACI Committee 336.[1.7] (Provides recommendations for design and construction of foundation piers 2-1/2 ft in diameter or larger made by excavating a hole in the soil and then filling it with concrete.)

Detailed recommendations for precast prestressed concrete piles are given in **"Recommended Practice for Design, Manufacture, and Installation of Prestressed Concrete Piling"** prepared by the PCI Committee on Prestressed Concrete Piling.[1.8]

1.1.6 — This code does not govern design and construction of soil-supported slabs, unless the slab transmits vertical loads or lateral forces from other portions of the structure to the soil.

1.1.7 — Concrete on steel form deck

R1.1.7 — Concrete on steel form deck

In steel framed structures, it is common practice to cast concrete floor slabs on stay-in-place steel form deck. In all cases, the deck serves as the form and may, in some cases, serve an additional structural function.

1.1.7.1 — Design and construction of structural concrete slabs cast on stay-in-place, noncomposite steel form deck are governed by this code.

R1.1.7.1 — In its most basic application, the steel form deck serves as a form, and the concrete serves a structural function and, therefore, are to be designed to carry all superimposed loads.

1.1.7.2 — This code does not govern the design of structural concrete slabs cast on stay-in-place, composite steel form deck. Concrete used in the construction of such slabs shall be governed by Parts 1, 2, and 3 of this code, where applicable.

R1.1.7.2 — Another type of steel form deck commonly used develops composite action between the concrete and steel deck. In this type of construction, the steel deck serves as the positive moment reinforcement. The design of composite slabs on steel deck is regulated by **"Standard for the Structural Design of Composite Slabs"** (ANSI/ASCE 3).[1.9] However, ANSI/ASCE 3 references the appropriate portions of ACI 318 for the design and construction of the concrete portion of the composite assembly. Guidelines for the construction of composite steel deck slabs are given in

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"Standard Practice for the Construction and Inspection of Composite Slabs" (ANSI/ASCE 9).[1.10]

1.1.8 — Special provisions for earthquake resistance

R1.1.8 — Special provisions for earthquake resistance

Special provisions for seismic design were first introduced in Appendix A of the 1971 code and were continued without revision in the 1977 code. These provisions were originally intended to apply only to reinforced concrete structures located in regions of highest seismicity.

The special provisions were extensively revised in the 1983 code to include new requirements for certain earthquake-resisting systems located in regions of moderate seismicity. In the 1989 code, the special provisions were moved to Chapter 21.

1.1.8.1 — In regions of low seismic risk, or for structures assigned to low seismic performance or design categories, provisions of Chapter 21 shall not apply.

R1.1.8.1 — For structures located in regions of low seismic risk, or for structures assigned to low seismic performance or design categories, no special design or detailing is required; the general requirements of the main body of the code apply for proportioning and detailing of reinforced concrete structures. It is the intent of Committee 318 that concrete structures proportioned by the main body of the code will provide a level of toughness adequate for low earthquake intensity.

1.1.8.2 — In regions of moderate or high seismic risk, or for structures assigned to intermediate or high seismic performance or design categories, provisions of Chapter 21 shall be satisfied. See 21.2.1.

R1.1.8.2 — For structures in regions of moderate seismic risk, or for structures assigned to intermediate seismic performance or design categories, reinforced concrete moment frames proportioned to resist seismic effects require special reinforcement details, as specified in 21.12. The special details apply only to beams, columns, and slabs to which the earthquake-induced forces have been assigned in design. The special reinforcement details will serve to provide a suitable level of inelastic behavior if the frame is subjected to an earthquake of such intensity as to require it to perform inelastically. There are no Chapter 21 requirements for cast-in-place structural walls provided to resist seismic effects, or for other structural components that are not part of the lateral-force-resisting system of structures in regions of moderate seismic risk, or assigned to intermediate seismic performance or design categories. For precast wall panels designed to resist forces induced by earthquake motions, special requirements are specified in 21.13 for connections between panels or between panels and the foundation. Cast-in-place structural walls proportioned to meet provisions of Chapters 1 through 18 and Chapter 22 are considered to have sufficient toughness at anticipated drift levels for these structures.

For structures located in regions of high seismic risk, or for structures assigned to high seismic performance or design categories, all building components that are part of the lateral-force-resisting system, including foundations (except plain concrete foundations as allowed by 22.10.1), should satisfy requirements of 21.2 through 21.10. In addition, frame members that are not assumed in the design to be part of the lateral-force-resisting system should comply with 21.11. The special proportioning and detailing require-

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1.1.8.3 — The seismic risk level of a region, or seismic performance or design category of a structure, shall be regulated by the legally adopted general building code of which this code forms a part, or determined by local authority.

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ments of Chapter 21 are intended to provide a monolithic reinforced concrete or precast concrete structure with adequate "toughness" to respond inelastically under severe earthquake motions. See also R21.2.1.

R1.1.8.3 — Seismic risk levels (Seismic Zone Maps) and seismic performance or design categories are under the jurisdiction of a general building code rather than ACI 318. Changes in terminology were made to the 1999 edition of the code to make it compatible with the latest editions of model building codes in use in the United States. For example, the phrase "seismic performance or design categories" was introduced. Over the past decade, the manner in which seismic risk levels have been expressed in United States building codes has changed. Previously they have been represented in terms of seismic zones. Recent editions of the "BOCA National Building Code" (NBC)[1.11] and "Standard Building Code" (SBC),[1.12] which are based on the 1991 NEHRP,[1.13] have expressed risk not only as a function of expected intensity of ground shaking on solid rock, but also on the nature of the occupancy and use of the structure. These two items are considered in assigning the structure to a Seismic Performance Category (SPC), which in turn is used to trigger different levels of detailing requirements for the structure. The 2000 "International Building Code" (IBC)[1.14] also uses the two criteria of the NBC and SBC and also considers the effects of soil amplification on the ground motion when assigning seismic risk. Under the IBC, each structure is assigned a Seismic Design Category (SDC). Among its several uses, it triggers different levels of detailing requirements. Table R1.1.8.3 correlates low, moderate/intermediate, and high seismic risk, which has been the terminology used in this code for several editions, to the various methods of assigning risk in use in the U.S. under the various model building codes, the ASCE 7 standard, and the NEHRP Recommended Provisions.

In the absence of a general building code that addresses earthquake loads and seismic zoning, it is the intent of Committee 318 that the local authorities (engineers, geologists, and building code officials) should decide on proper need and proper application of the special provisions for seismic design. Seismic ground-motion maps or zoning maps, such as recommended in References 1.10, 1.15, and 1.16, are suitable for correlating seismic risk.

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TABLE R1.1.8.3—CORRELATION BETWEEN SEISMIC-RELATED TERMINOLOGY IN MODEL CODES

Code, standard, or resource document and edition	Level of seismic risk or assigned seismic performance or design categories as defined in the code section		
	Low (21.2.1.2)	Moderate/ intermediate (21.2.1.3)	High (21.2.1.4)
International Building Code 2000; NEHRP 1997	SDC[1] A, B	SDC C	SDC D, E, F
BOCA National Building Code 1993, 1996, 1999; Standard Building Code 1994, 1997, 1999; ASCE 7-93, 7-95, 7-98; NEHRP 1991, 1994	SPC[2] A, B	SPC C	SPC D, E
Uniform Building Code 1991, 1994, 1997	Seismic Zone 0, 1	Seismic Zone 2	Seismic Zone 3, 4

[1]SDC = *Seismic Design Category* as defined in code, standard, or resource document.
[2]SPC = *Seismic Performance Category* as defined in code, standard, or resource document.

1.2 — Drawings and specifications

1.2.1 — Copies of design drawings, typical details, and specifications for all structural concrete construction shall bear the seal of a registered engineer or architect. These drawings, details, and specifications shall show:

(a) Name and date of issue of code and supplement to which design conforms;

(b) Live load and other loads used in design;

(c) Specified compressive strength of concrete at stated ages or stages of construction for which each part of structure is designed;

(d) Specified strength or grade of reinforcement;

(e) Size and location of all structural elements, reinforcement, and anchors;

(f) Provision for dimensional changes resulting from creep, shrinkage, and temperature;

(g) Magnitude and location of prestressing forces;

(h) Anchorage length of reinforcement and location and length of lap splices;

(i) Type and location of mechanical and welded splices of reinforcement;

(j) Details and location of all contraction or isolation joints specified for plain concrete in Chapter 22;

(k) Minimum concrete compressive strength at time of post-tensioning;

R1.2 — Drawings and specifications

R1.2.1 — The provisions for preparation of design drawings and specifications are, in general, consistent with those of most general building codes and are intended as supplements.

The code lists some of the more important items of information that should be included in the design drawings, details, or specifications. The code does not imply an all-inclusive list, and additional items may be required by the building official.

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(l) Stressing sequence for post-tensioning tendons;

(m) Statement if slab on grade is designed as a structural diaphragm, see 21.10.3.4.

1.2.2 — Calculations pertinent to design shall be filed with the drawings when required by the building official. Analyses and designs using computer programs shall be permitted provided design assumptions, user input, and computer-generated output are submitted. Model analysis shall be permitted to supplement calculations.

1.2.3 — Building official means the officer or other designated authority charged with the administration and enforcement of this code, or his duly authorized representative.

1.3 — Inspection

1.3.1 — Concrete construction shall be inspected as required by the legally adopted general building code. In the absence of such inspection requirements, concrete construction shall be inspected throughout the various

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R1.2.2 — Documented computer output is acceptable in lieu of manual calculations. The extent of input and output information required will vary, according to the specific requirements of individual building officials. However, when a computer program has been used by the designer, only skeleton data should normally be required. This should consist of sufficient input and output data and other information to allow the building official to perform a detailed review and make comparisons using another program or manual calculations. Input data should be identified as to member designation, applied loads, and span lengths. The related output data should include member designation and the shears, moments, and reactions at key points in the span. For column design, it is desirable to include moment magnification factors in the output where applicable.

The code permits model analysis to be used to supplement structural analysis and design calculations. Documentation of the model analysis should be provided with the related calculations. Model analysis should be performed by an engineer or architect having experience in this technique.

R1.2.3 — Building official is the term used by many general building codes to identify the person charged with administration and enforcement of the provisions of the building code. However, such terms as building commissioner or building inspector are variations of the title, and the term building official as used in this code is intended to include those variations as well as others that are used in the same sense.

R1.3 — Inspection

The quality of concrete structures depends largely on workmanship in construction. The best of materials and design practices will not be effective unless the construction is performed well. Inspection is necessary to confirm that the construction is in accordance with the design drawings and project specifications. Proper performance of the structure depends on construction that accurately represents the design and meets code requirements, within the tolerances allowed. Qualification of inspectors can be obtained from a certification program such as the certification program for Reinforced Concrete Inspector sponsored by ACI, International Conference of Building Officials (ICBO), Building Officials and Code Administrators International (BOCA), and Southern Building Code Congress International (SBCCI).

R1.3.1 — Inspection of construction by or under the supervision of the registered design professional responsible for the design should be considered because the person in charge of the design is usually the best qualified to determine if con-

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work stages by or under the supervision of a registered design professional or by a qualified inspector.

1.3.2 — The inspector shall require compliance with design drawings and specifications. Unless specified otherwise in the legally adopted general building code, inspection records shall include:

(a) Quality and proportions of concrete materials and strength of concrete;

(b) Construction and removal of forms and reshoring;

(c) Placing of reinforcement and anchors;

(d) Mixing, placing, and curing of concrete;

(e) Sequence of erection and connection of precast members;

(f) Tensioning of tendons;

(g) Any significant construction loadings on com-

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struction is in conformance with construction documents. When such an arrangement is not feasible, inspection of construction through other registered design professionals or through separate inspection organizations with demonstrated capability for performing the inspection may be used.

Qualified inspectors should establish their qualification by becoming certified to inspect and record the results of concrete construction, including preplacement, placement, and postplacement operations through the Reinforced Concrete Special Inspector program sponsored by ACI, ICBO, BOCA, and SBCCI or equivalent.

When inspection is done independently of the registered design professional responsible for the design, it is recommended that the registered design professional responsible for the design be employed at least to oversee inspection and observe the work to see that the design requirements are properly executed.

In some jurisdictions, legislation has established special registration or licensing procedures for persons performing certain inspection functions. A check should be made in the general building code or with the building official to ascertain if any such requirements exist within a specific jurisdiction.

Inspection reports should be promptly distributed to the owner, registered design professional responsible for the design, contractor, appropriate subcontractors, appropriate suppliers, and the building official to allow timely identification of compliance or the need for corrective action.

Inspection responsibility and the degree of inspection required should be set forth in the contracts between the owner, architect, engineer, contractor, and inspector. Adequate fees should be provided consistent with the work and equipment necessary to properly perform the inspection.

R1.3.2 — By inspection, the code does not mean that the inspector should supervise the construction. Rather it means that the one employed for inspection should visit the project with the frequency necessary to observe the various stages of work and ascertain that it is being done in compliance with contract documents and code requirements. The frequency should be at least enough to provide general knowledge of each operation, whether this is several times a day or once in several days.

Inspection in no way relieves the contractor from his obligation to follow the plans and specifications and to provide the designated quality and quantity of materials and workmanship for all job stages. The inspector should be present as frequently as he or she deems necessary to judge whether the quality and quantity of the work complies with the contract documents; to counsel on possible ways of obtaining the desired results; to see that the general system proposed for formwork appears proper (though it remains the contractor's responsibility to design and build adequate forms and to leave them in place until it is safe to

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pleted floors, members, or walls;

(h) General progress of work.

1.3.3 — When the ambient temperature falls below 40 F or rises above 95 F, a record shall be kept of concrete temperatures and of protection given to concrete during placement and curing.

1.3.4 — Records of inspection required in 1.3.2 and 1.3.3 shall be preserved by the inspecting engineer or architect for 2 years after completion of the project.

1.3.5 — For special moment frames resisting seismic loads in regions of high seismic risk, or in structures assigned to high seismic performance or design categories, continuous inspection of the placement of the reinforcement and concrete shall be made by a qualified inspector. The inspector shall be under the supervision of the engineer responsible for the structural design or under the supervision of an engineer with demonstrated capability for supervising inspection of special moment frames resisting seismic loads in regions of high seismic risk, or in structures assigned to high seismic performance or design categories.

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remove them); to see that reinforcement is properly installed; to see that concrete is of the correct quality, properly placed, and cured; and to see that tests for quality control are being made as specified.

The code prescribes minimum requirements for inspection of all structures within its scope. It is not a construction specification and any user of the code may require higher standards of inspection than cited in the legal code if additional requirements are necessary.

Recommended procedures for organization and conduct of concrete inspection are given in detail in **"Guide for Concrete Inspection,"** reported by ACI Committee 311.[1.17] (Sets forth procedures relating to concrete construction to serve as a guide to owners, architects, and engineers in planning an inspection program.)

Detailed methods of inspecting concrete construction are given in "**ACI Manual of Concrete Inspection"** (SP-2) reported by ACI Committee 311.[1.18] (Describes methods of inspecting concrete construction that are generally accepted as good practice. Intended as a supplement to specifications and as a guide in matters not covered by specifications.)

R1.3.3 — The term ambient temperature means the temperature of the environment to which the concrete is directly exposed. Concrete temperature as used in this section may be taken as the air temperature near the surface of the concrete; however, during mixing and placing it is practical to measure the temperature of the mixture.

R1.3.4 — A record of inspection in the form of a job diary is required in case questions subsequently arise concerning the performance or safety of the structure or members. Photographs documenting job progress may also be desirable.

Records of inspection should be preserved for at least 2 years after the completion of the project. The completion of the project is the date at which the owner accepts the project, or when a certificate of occupancy is issued, whichever date is later. The general building code or other legal requirements may require a longer preservation of such records.

R1.3.5 — The purpose of this section is to ensure that the special detailing required in special moment frames is properly executed through inspection by personnel who are qualified to do this work. Qualifications of inspectors should be acceptable to the jurisdiction enforcing the general building code.

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1.4 — Approval of special systems of design or construction

Sponsors of any system of design or construction within the scope of this code, the adequacy of which has been shown by successful use or by analysis or test, but which does not conform to or is not covered by this code, shall have the right to present the data on which their design is based to the building official or to a board of examiners appointed by the building official. This board shall be composed of competent engineers and shall have authority to investigate the data so submitted, to require tests, and to formulate rules governing design and construction of such systems to meet the intent of this code. These rules when approved by the building official and promulgated shall be of the same force and effect as the provisions of this code.

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R1.4 — Approval of special systems of design or construction

New methods of design, new materials, and new uses of materials should undergo a period of development before being specifically covered in a code. Hence, good systems or components might be excluded from use by implication if means were not available to obtain acceptance.

For special systems considered under this section, specific tests, load factors, deflection limits, and other pertinent requirements should be set by the board of examiners, and should be consistent with the intent of the code.

The provisions of this section do not apply to model tests used to supplement calculations under 1.2.2 or to strength evaluation of existing structures under Chapter 20.

CHAPTER 2 — DEFINITIONS

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2.1 — The following terms are defined for general use in this code. Specialized definitions appear in individual chapters.

Admixture — Material other than water, aggregate, or hydraulic cement, used as an ingredient of concrete and added to concrete before or during its mixing to modify its properties.

Aggregate — Granular material, such as sand, gravel, crushed stone, and iron blast-furnace slag, used with a cementing medium to form a hydraulic cement concrete or mortar.

Aggregate, lightweight — Aggregate with a dry, loose weight of 70 lb/ft^3 or less.

Anchorage device — In post-tensioning, the hardware used for transferring a post-tensioning force from the prestressing steel to the concrete.

Anchorage zone — In post-tensioned members, the portion of the member through which the concentrated prestressing force is transferred to the concrete and distributed more uniformly across the section. Its extent is equal to the largest dimension of the cross section. For anchorage devices located away from the end of a member, the anchorage zone includes the disturbed regions ahead of and behind the anchorage devices.

Basic monostrand anchorage device — Anchorage device used with any single strand or a single 5/8 in. or smaller diameter bar that satisfies 18.21.1 and the anchorage device requirements of ACI 423.6, "Specification for Unbonded Single-Strand Tendons."

Basic multistrand anchorage device — Anchorage device used with multiple strands, bars, or wires, or with single bars larger than 5/8 in. diameter, that satisfies 18.21.1 and the bearing stress and minimum plate stiffness requirements of AASHTO Bridge Specifications, Division I, Articles 9.21.7.2.2 through 9.21.7.2.4.

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R2.1 — For consistent application of the code, it is necessary that terms be defined where they have particular meanings in the code. The definitions given are for use in application of this code only and do not always correspond to ordinary usage. A glossary of most used terms relating to cement manufacturing, concrete design and construction, and research in concrete is contained in **"Cement and Concrete Terminology"** reported by ACI Committee 116.[2.1]

Anchorage device — Most anchorage devices for post-tensioning are standard manufactured devices available from commercial sources. In some cases, designers or constructors develop "special" details or assemblages that combine various wedges and wedge plates for anchoring prestressing steel with specialty end plates or diaphragms. These informal designations as standard anchorage devices or special anchorage devices have no direct relation to the ACI Building Code and AASHTO "Standard Specifications for Highway Bridges" classification of anchorage devices as Basic Anchorage Devices or Special Anchorage Devices.

Anchorage zone — The terminology "ahead of" and "behind" the anchorage device is illustrated in Fig. R18.13.1(b).

Basic anchorage devices are those devices that are so proportioned that they can be checked analytically for compliance with bearing stress and stiffness requirements without having to undergo the acceptance-testing program required of special anchorage devices.

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Bonded tendon — Tendon in which prestressing steel is bonded to concrete either directly or through grouting.

Building official — See 1.2.3.

Cementitious materials — Materials as specified in Chapter 3, which have cementing value when used in concrete either by themselves, such as portland cement, blended hydraulic cements, and expansive cement, or such materials in combination with fly ash, other raw or calcined natural pozzolans, silica fume, and/or ground granulated blast-furnace slag.

Column — Member with a ratio of height-to-least lateral dimension exceeding 3 used primarily to support axial compressive load.

Composite concrete flexural members — Concrete flexural members of precast or cast-in-place concrete elements, or both, constructed in separate placements but so interconnected that all elements respond to loads as a unit.

Compression-controlled section — A cross section in which the net tensile strain in the extreme tension steel at nominal strength is less than or equal to the compression-controlled strain limit.

Compression-controlled strain limit — The net tensile strain at balanced strain conditions. See 10.3.3.

Concrete — Mixture of portland cement or any other hydraulic cement, fine aggregate, coarse aggregate, and water, with or without admixtures.

Concrete, specified compressive strength of, (f_c') — Compressive strength of concrete used in design and evaluated in accordance with provisions of Chapter 5, expressed in pounds per square inch (psi). Whenever the quantity f_c' is under a radical sign, square root of numerical value only is intended, and result has units of pounds per square inch (psi).

Concrete, structural lightweight — Concrete containing lightweight aggregate that conforms to 3.3 and has an air-dry unit weight as determined by "Test Method for Unit Weight of Structural Lightweight Concrete" (ASTM C 567), not exceeding 115 lb/ft^3. In this code, a lightweight concrete without natural sand is termed "all-lightweight concrete" and lightweight concrete in which all of the fine aggregate consists of normal weight sand is termed "sand-lightweight concrete."

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Column — The term compression member is used in the code to define any member in which the primary stress is longitudinal compression. Such a member need not be vertical but may have any orientation in space. Bearing walls, columns, and pedestals qualify as compression members under this definition.

The differentiation between columns and walls in the code is based on the principal use rather than on arbitrary relationships of height and cross-sectional dimensions. The code, however, permits walls to be designed using the principles stated for column design (see 14.4), as well as by the empirical method (see 14.5).

While a wall always encloses or separates spaces, it may also be used to resist horizontal or vertical forces or bending. For example, a retaining wall or a basement wall also supports various combinations of loads.

A column is normally used as a main vertical member carrying axial loads combined with bending and shear. It may, however, form a small part of an enclosure or separation.

Concrete, structural lightweight — By code definition, sand-lightweight concrete is structural lightweight concrete with all of the fine aggregate replaced by sand. This definition may not be in agreement with usage by some material suppliers or contractors where the majority, but not all, of the lightweight fines are replaced by sand. For proper application of the code provisions, the replacement limits should be stated, with interpolation when partial sand replacement is used.

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Contraction joint — Formed, sawed, or tooled groove in a concrete structure to create a weakened plane and regulate the location of cracking resulting from the dimensional change of different parts of the structure.

Curvature friction — Friction resulting from bends or curves in the specified prestressing tendon profile.

Deformed reinforcement — Deformed reinforcing bars, bar mats, deformed wire, welded plain wire fabric, and welded deformed wire fabric conforming to 3.5.3.

Development length — Length of embedded reinforcement required to develop the design strength of reinforcement at a critical section. See 9.3.3.

Duct — A conduit (plain or corrugated) to accommodate prestressing steel for post-tensioned installation. Requirements for post-tensioning ducts are given in Section 18.17.

Effective depth of section (d) — Distance measured from extreme compression fiber to centroid of tension reinforcement.

Effective prestress — Stress remaining in prestressing steel after all losses have occurred.

Embedment length — Length of embedded reinforcement provided beyond a critical section.

Extreme tension steel — The reinforcement (prestressed or nonprestressed) that is the farthest from the extreme compression fiber.

Isolation joint — A separation between adjoining parts of a concrete structure, usually a vertical plane, at a designed location such as to interfere least with performance of the structure, yet such as to allow relative movement in three directions and avoid formation of cracks elsewhere in the concrete and through which all or part of the bonded reinforcement is interrupted.

Jacking force — In prestressed concrete, temporary force exerted by device that introduces tension into prestressing steel.

Load, dead — Dead weight supported by a member, as defined by general building code of which this code forms a part (without load factors).

Load, factored — Load, multiplied by appropriate load factors, used to proportion members by the strength design method of this code. See 8.1.1 and 9.2.

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Deformed reinforcement — Deformed reinforcement is defined as that meeting the deformed bar specifications of 3.5.3.1, or the specifications of 3.5.3.3, 3.5.3.4, 3.5.3.5, or 3.5.3.6. No other bar or fabric qualifies. This definition permits accurate statement of anchorage lengths. Bars or wire not meeting the deformation requirements or fabric not meeting the spacing requirements are "plain reinforcement," for code purposes, and may be used only for spirals.

Loads — A number of definitions for loads are given as the code contains requirements that are to be met at various load levels. The terms dead load and live load refer to the unfactored loads (service loads) specified or defined by the general building code. Service loads (loads without load factors) are to be used where specified in the code to proportion or

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Load, live — Live load specified by general building code of which this code forms a part (without load factors).

Load, service — Load specified by general building code of which this code forms a part (without load factors).

Modulus of elasticity — Ratio of normal stress to corresponding strain for tensile or compressive stresses below proportional limit of material. See 8.5.

Moment frame — Frame in which members and joints resist forces through flexure, shear, and axial force. Moment frames shall be catergorized as follows:

Intermediate moment frame — A cast-in-place frame complying with the requirements of 21.2.2.3 and 21.12 in addition to the requirements for ordinary moment frames.

Ordinary moment frame — A cast-in-place or precast concrete frame complying with the requirements of Chapters 1 through 18.

Special moment frame — A cast-in-place frame complying with the requirements of 21.2 through 21.5, or a precast frame complying with the requirements of 21.2 through 21.6. In addition, the requirements for ordinary moment frames shall be satisfied.

Net tensile strain — The tensile strain at nominal strength exclusive of strains due to effective prestress, creep, shrinkage, and temperature.

Pedestal — Upright compression member with a ratio of unsupported height to average least lateral dimension not exceeding 3.

Plain concrete — Structural concrete with no reinforcement or with less reinforcement than the minimum amount specified for reinforced concrete.

Plain reinforcement — Reinforcement that does not conform to definition of deformed reinforcement. See 3.5.4.

Post-tensioning — Method of prestressing in which prestressing steel is tensioned after concrete has hardened.

Precast concrete — Structural concrete element cast elsewhere than its final position in the structure.

Prestressed concrete — Structural concrete in which internal stresses have been introduced to reduce potential tensile stresses in concrete resulting from loads.

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investigate members for adequate serviceability, as in 9.5, Control of Deflections. Loads used to proportion a member for adequate strength are defined as factored loads. Factored loads are service loads multiplied by the appropriate load factors specified in 9.2 for required strength. The term design loads, as used in the 1971 code edition to refer to loads multiplied by the appropriate load factors, was discontinued in the 1977 code to avoid confusion with the design load terminology used in general building codes to denote service loads, or posted loads in buildings. The factored load terminology, first adopted in the 1977 code, clarifies when the load factors are applied to a particular load, moment, or shear value as used in the code provisions.

Prestressed concrete — Reinforced concrete is defined to include prestressed concrete. Although the behavior of a prestressed member with unbonded tendons may vary from that of members with continuously bonded tendons, bonded

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Prestressing steel — High-strength steel element such as wire, bar, or strand, or a bundle of such elements, used to impart prestress forces to concrete.

Pretensioning — Method of prestressing in which prestressing steel is tensioned before concrete is placed.

Reinforced concrete — Structural concrete reinforced with no less than the minimum amounts of prestressing steel or nonprestressed reinforcement specified in Chapters 1 through 21 and Appendices A through C.

Reinforcement — Material that conforms to 3.5, excluding prestressing steel unless specifically included.

Registered design professional — An individual who is registered or licensed to practice the respective design profession as defined by the statutory requirements of the professional registration laws of the state or jurisdiction in which the project is to be constructed.

Reshores — Shores placed snugly under a concrete slab or other structural member after the original forms and shores have been removed from a larger area, thus requiring the new slab or structural member to deflect and support its own weight and existing construction loads applied prior to the installation of the reshores.

Sheathing — A material encasing prestressing steel to prevent bonding of the prestressing steel with the surrounding concrete, to provide corrosion protection, and to contain the corrosion inhibiting coating.

Shores — Vertical or inclined support members designed to carry the weight of the formwork, concrete, and construction loads above.

Span length — See 8.7.

Special anchorage device — Anchorage device that satisfies 18.15.1 and the standardized acceptance tests of AASHTO "Standard Specifications for Highway Bridges," Division II, Article 10.3.2.3.

Spiral reinforcement — Continuously wound reinforcement in the form of a cylindrical helix.

Splitting tensile strength (f_{ct}) — Tensile strength of concrete determined in accordance with ASTM C 496 as described in "Specification for Lightweight Aggregates for Structural Concrete" (ASTM C 330). See 5.1.4.

COMMENTARY

and unbonded prestressed concrete are combined with conventionally reinforced concrete under the generic term "reinforced concrete." Provisions common to both prestressed and conventionally reinforced concrete are integrated to avoid overlapping and conflicting provisions.

Sheathing — Typically, sheathing is a continuous, seamless, high-density polyethylene material extruded directly on the coated prestressing steel.

Special anchorage devices are any devices (monostrand or multistrand) that do not meet the relevant PTI or AASHTO bearing stress and, where applicable, stiffness requirements. Most commercially marketed multibearing surface anchorage devices are Special Anchorage Devices. As provided in 18.15.1, such devices can be used only when they have been shown experimentally to be in compliance with the AASHTO requirements. This demonstration of compliance will ordinarily be furnished by the device manufacturer.

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Stirrup — Reinforcement used to resist shear and torsion stresses in a structural member; typically bars, wires, or welded wire fabric (plain or deformed) either single leg or bent into L, U, or rectangular shapes and located perpendicular to or at an angle to longitudinal reinforcement. (The term "stirrups" is usually applied to lateral reinforcement in flexural members and the term ties to those in compression members.) See also *Tie*.

Strength, design — Nominal strength multiplied by a strength reduction factor f. See 9.3.

Strength, nominal — Strength of a member or cross section calculated in accordance with provisions and assumptions of the strength design method of this code before application of any strength reduction factors. See 9.3.1.

Strength, required — Strength of a member or cross section required to resist factored loads or related internal moments and forces in such combinations as are stipulated in this code. See 9.1.1.

Stress — Intensity of force per unit area.

Structural concrete — All concrete used for structural purposes including plain and reinforced concrete.

Structural walls — Walls proportioned to resist combinations of shears, moments, and axial forces induced by earthquake motions. A shearwall is a structural wall. Structural walls shall be categorized as follows:

Intermediate precast structural wall — A wall complying with all applicable requirements of Chapters 1 through 18 in addition to 21.13.

Ordinary reinforced concrete structural wall — A wall complying with the requirements of Chapters 1 through 18.

Ordinary structural plain concrete wall — A wall complying with the requirements of Chapter 22.

Special precast structural wall — A precast wall complying with the requirements of 21.8. In addition, the requirements of ordinary reinforced concrete structural walls and the requirements of 21.2 shall be satisfied.

Special reinforced concrete structural wall — A cast-in-place wall complying with the requirements of 21.2 and 21.7 in addition to the requirements for ordinary reinforced concrete structural walls.

COMMENTARY

Strength, nominal — Strength of a member or cross section calculated using standard assumptions and strength equations, and nominal (specified) values of material strengths and dimensions is referred to as "nominal strength." The subscript n is used to denote the nominal strengths; nominal axial load strength P_n, nominal moment strength M_n, and nominal shear strength V_n. "Design strength" or usable strength of a member or cross section is the nominal strength reduced by the strength reduction factor f.

The required axial load, moment, and shear strengths used to proportion members are referred to either as factored axial loads, factored moments, and factored shears, or required axial loads, moments, and shears. The factored load effects are calculated from the applied factored loads and forces in such load combinations as are stipulated in the code (see 9.2).

The subscript u is used only to denote the required strengths; required axial load strength P_u, required moment strength M_u, and required shear strength V_u, calculated from the applied factored loads and forces.

The basic requirement for strength design may be expressed as follows:

Design strength ≥ Required strength

$$f P_n \geq P_u$$

$$f M_n \geq M_u$$

$$f V_n \geq V_u$$

For additional discussion on the concepts and nomenclature for strength design see commentary Chapter 9.

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COMMENTARY

Tendon — In pretensioned applications, the tendon is the prestressing steel. In post-tensioned applications, the tendon is a complete assembly consisting of anchorages, prestressing steel, and sheathing with coating for unbonded applications or ducts with grout for bonded applications.

Tension-controlled section — A cross section in which the net tensile strain in the extreme tension steel at nominal strength is greater than or equal to 0.005.

Tie — Loop of reinforcing bar or wire enclosing longitudinal reinforcement. A continuously wound bar or wire in the form of a circle, rectangle, or other polygon shape without re-entrant corners is acceptable. See also *Stirrup*.

Transfer — Act of transferring stress in prestressing steel from jacks or pretensioning bed to concrete member.

Unbonded Tendon — Tendon in which the prestressing steel is prevented from bonding to the concrete and is free to move relative to the concrete. The prestressing force is permanently transferred to the concrete at the tendon ends by the anchorages only.

Wall — Member, usually vertical, used to enclose or separate spaces.

Wobble friction — In prestressed concrete, friction caused by unintended deviation of prestressing sheath or duct from its specified profile.

Yield strength — Specified minimum yield strength or yield point of reinforcement. Yield strength or yield point shall be determined in tension according to applicable ASTM standards as modified by 3.5 of this code.

CODE

COMMENTARY

Notes

PART 2 — STANDARDS FOR TESTS AND MATERIALS

CHAPTER 3 — MATERIALS

CODE

3.0 — Notation

f_y = specified yield strength of nonprestressed reinforcement, psi

3.1 — Tests of materials

3.1.1 — The building official shall have the right to order testing of any materials used in concrete construction to determine if materials are of quality specified.

3.1.2 — Tests of materials and of concrete shall be made in accordance with standards listed in 3.8.

3.1.3 — A complete record of tests of materials and of concrete shall be retained by the inspector for 2 years after completion of the project, and made available for inspection during the progress of the work.

3.2 — Cements

3.2.1 — Cement shall conform to one of the following specifications:

(a) "Specification for Portland Cement" (ASTM C 150);

(b) "Specification for Blended Hydraulic Cements" (ASTM C 595), excluding Types S and SA which are not intended as principal cementing constituents of structural concrete;

(c) "Specification for Expansive Hydraulic Cement" (ASTM C 845);

(d) "Performance Specification for Hydraulic Cement" (ASTM C 1157).

3.2.2 — Cement used in the work shall correspond to that on which selection of concrete proportions was based. See 5.2.

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R3.0 — Notation

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

R3.1 — Tests of materials

R3.1.3 — The record of tests of materials and of concrete should be retained for at least 2 years after completion of the project. Completion of the project is the date at which the owner accepts the project or when the certificate of occupancy is issued, whichever date is later. Local legal requirements may require longer retention of such records.

R3.2 — Cements

R3.2.2 — Depending on the circumstances, the provision of 3.2.2 may require only the same type of cement or may require cement from the identical source. The latter would be the case if the standard deviation[3.1] of strength tests used in establishing the required strength margin was based on a cement from a particular source. If the standard deviation

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COMMENTARY

was based on tests involving a given type of cement obtained from several sources, the former interpretation would apply.

3.3 — Aggregates

3.3.1 — Concrete aggregates shall conform to one of the following specifications:

(a) "Specification for Concrete Aggregates" (ASTM C 33);

(b) "Specification for Lightweight Aggregates for Structural Concrete" (ASTM C 330).

Exception: Aggregates that have been shown by special test or actual service to produce concrete of adequate strength and durability and approved by the building official.

R3.3 — Aggregates

R3.3.1 — Aggregates conforming to the ASTM specifications are not always economically available and, in some instances, noncomplying materials have a long history of satisfactory performance. Such nonconforming materials are permitted with special approval when acceptable evidence of satisfactory performance is provided. Satisfactory performance in the past, however, does not guarantee good performance under other conditions and in other localities. Whenever possible, aggregates conforming to the designated specifications should be used.

3.3.2 — Nominal maximum size of coarse aggregate shall be not larger than:

(a) 1/5 the narrowest dimension between sides of forms, nor

(b) 1/3 the depth of slabs, nor

(c) 3/4 the minimum clear spacing between individual reinforcing bars or wires, bundles of bars, individual tendons, bundled tendons, or ducts.

These limitations shall not apply if, in the judgment of the engineer, workability and methods of consolidation are such that concrete can be placed without honeycombs or voids.

R3.3.2 — The size limitations on aggregates are provided to ensure proper encasement of reinforcement and to minimize honeycombing. Note that the limitations on maximum size of the aggregate may be waived if, in the judgment of the engineer, the workability and methods of consolidation of the concrete are such that the concrete can be placed without honeycombs or voids.

3.4 — Water

3.4.1 — Water used in mixing concrete shall be clean and free from injurious amounts of oils, acids, alkalis, salts, organic materials, or other substances deleterious to concrete or reinforcement.

3.4.2 — Mixing water for prestressed concrete or for concrete that will contain aluminum embedments, including that portion of mixing water contributed in the form of free moisture on aggregates, shall not contain deleterious amounts of chloride ion. See 4.4.1.

3.4.3 — Nonpotable water shall not be used in concrete unless the following are satisfied:

3.4.3.1 — Selection of concrete proportions shall be based on concrete mixes using water from the same source.

R3.4 — Water

R3.4.1 — Almost any natural water that is drinkable (potable) and has no pronounced taste or odor is satisfactory as mixing water for making concrete. Impurities in mixing water, when excessive, may affect not only setting time, concrete strength, and volume stability (length change), but may also cause efflorescence or corrosion of reinforcement. Where possible, water with high concentrations of dissolved solids should be avoided.

Salts or other deleterious substances contributed from the aggregate or admixtures are additive to the amount which might be contained in the mixing water. These additional amounts are to be considered in evaluating the acceptability of the total impurities that may be deleterious to concrete or steel.

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3.4.3.2 — Mortar test cubes made with nonpotable mixing water shall have 7-day and 28-day strengths equal to at least 90 percent of strengths of similar specimens made with potable water. Strength test comparison shall be made on mortars, identical except for the mixing water, prepared and tested in accordance with "Test Method for Compressive Strength of Hydraulic Cement Mortars (Using 2-in. or 50-mm Cube Specimens)" (ASTM C 109).

3.5 — Steel reinforcement

3.5.1 — Reinforcement shall be deformed reinforcement, except that plain reinforcement shall be permitted for spirals or prestressing steel; and reinforcement consisting of structural steel, steel pipe, or steel tubing shall be permitted as specified in this code.

3.5.2 — Welding of reinforcing bars shall conform to "Structural Welding Code — Reinforcing Steel," ANSI/AWS D1.4 of the American Welding Society. Type and location of welded splices and other required welding of reinforcing bars shall be indicated on the design drawings or in the project specifications. ASTM reinforcing bar specifications, except for ASTM A 706, shall be supplemented to require a report of material properties necessary to conform to the requirements in ANSI/AWS D1.4.

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R3.5 — Steel reinforcement

R3.5.1 — Materials permitted for use as reinforcement are specified. Other metal elements, such as inserts, anchor bolts, or plain bars for dowels at isolation or contraction joints, are not normally considered to be reinforcement under the provisions of this code.

R3.5.2 — When welding of reinforcing bars is required, the weldability of the steel and compatible welding procedures need to be considered. The provisions in ANSI/AWS D1.4 Welding Code cover aspects of welding reinforcing bars, including criteria to qualify welding procedures.

Weldability of the steel is based on its chemical composition or carbon equivalent (CE). The Welding Code establishes preheat and interpass temperatures for a range of carbon equivalents and reinforcing bar sizes. Carbon equivalent is calculated from the chemical composition of the reinforcing bars. The Welding Code has two expressions for calculating carbon equivalent. A relatively short expression, considering only the elements carbon and manganese, is to be used for bars other than ASTM A 706 material. A more comprehensive expression is given for ASTM A 706 bars. The CE formula in the Welding Code for A 706 bars is identical to the CE formula in the ASTM A 706 specification.

The engineer should realize that the chemical analysis, for bars other than A 706, required to calculate the carbon equivalent is not routinely provided by the producer of the reinforcing bars. For welding reinforcing bars other than A 706 bars, the design drawings or project specifications should specifically require results of the chemical analysis to be furnished.

The ASTM A 706 specification covers low-alloy steel reinforcing bars intended for applications requiring controlled tensile properties or welding. Weldability is accomplished in the A 706 specification by limits or controls on chemical composition and on carbon equivalent.[3.2] The producer is required by the A 706 specification to report the chemical composition and carbon equivalent.

The ANSI/AWS D1.4 Welding Code requires the contractor to prepare written welding procedure specifications conforming to the requirements of the Welding Code.

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3.5.3 — Deformed reinforcement

3.5.3.1 — Deformed reinforcing bars shall conform to one of the following specifications:

(a) "Specification for Deformed and Plain Billet-Steel Bars for Concrete Reinforcement" (ASTM A 615);

(b) "Specification for Low-Alloy Steel Deformed and Plain Bars for Concrete Reinforcement" (ASTM A 706);

(c) "Specification for Rail-Steel and Axle-Steel Deformed Bars for Concrete Reinforcement" (ASTM A 996). Bars from rail-steel shall be Type R.

COMMENTARY

Appendix A of the Welding Code contains a suggested form that shows the information required for such a specification for each joint welding procedure.

Often it is necessary to weld to existing reinforcing bars in a structure when no mill test report of the existing reinforcement is available. This condition is particularly common in alterations or building expansions. ANSI/AWS D1.4 states for such bars that a chemical analysis may be performed on representative bars. If the chemical composition is not known or obtained, the Welding Code requires a minimum preheat. For bars other than A 706 material, the minimum preheat required is 300 F for bars No. 6 or smaller, and 400 F for No. 7 bars or larger. The required preheat for all sizes of A 706 is to be the temperature given in the Welding Code's table for minimum preheat corresponding to the range of CE "over 45 percent to 55 percent." Welding of the particular bars should be performed in accordance with ANSI/AWS D 1.4. It should also be determined if additional precautions are in order, based on other considerations such as stress level in the bars, consequences of failure, and heat damage to existing concrete due to welding operations.

Welding of wire to wire, and of wire or welded wire fabric to reinforcing bars or structural steel elements is not covered by ANSI/AWS D1.4. If welding of this type is required on a project, the engineer should specify requirements or performance criteria for this welding. If cold drawn wires are to be welded, the welding procedures should address the potential loss of yield strength and ductility achieved by the cold working process (during manufacture) when such wires are heated by welding. Machine and resistance welding as used in the manufacture of welded wire fabrics is covered by ASTM A 185 and A 497 and is not part of this concern.

R3.5.3 — Deformed reinforcement

R3.5.3.1 — ASTM A 615 covers deformed billet-steel reinforcing bars that are currently the most widely used type of steel bar in reinforced concrete construction in the United States. The specification requires that the bars be marked with the letter *S* for type of steel.

ASTM A 706 covers low-alloy steel deformed bars intended for applications where controlled tensile properties, restrictions on chemical composition to enhance weldability, or both, are required. The specification requires that the bars be marked with the letter *W* for type of steel.

Deformed bars produced to meet both ASTM A 615 and A 706 are required to be marked with the letters *S* and *W* for type of steel.

Rail-steel reinforcing bars used with this code are required to conform to ASTM A 996 including the provisions for Type R bars, and marked with the letter *R* for type of steel.

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3.5.3.2 — Deformed reinforcing bars with a specified yield strength f_y exceeding 60,000 psi shall be permitted, provided f_y shall be the stress corresponding to a strain of 0.35 percent and the bars otherwise conform to one of the ASTM specifications listed in 3.5.3.1. See 9.4.

3.5.3.3 — Bar mats for concrete reinforcement shall conform to "Specification for Fabricated Deformed Steel Bar Mats for Concrete Reinforcement" (ASTM A 184). Reinforcing bars used in bar mats shall conform to one of the specifications listed in 3.5.3.1.

3.5.3.4 — Deformed wire for concrete reinforcement shall conform to "Specification for Steel Wire, Deformed, for Concrete Reinforcement" (ASTM A 496), except that wire shall not be smaller than size D4 and for wire with a specified yield strength f_y exceeding 60,000 psi, f_y shall be the stress corresponding to a strain of 0.35 percent if the yield strength specified in the design exceeds 60,000 psi.

3.5.3.5 — Welded plain wire fabric for concrete reinforcement shall conform to "Specification for Steel Welded Wire Fabric, Plain, for Concrete Reinforcement" (ASTM A 185), except that for wire with a specified yield strength f_y exceeding 60,000 psi, f_y shall be the stress corresponding to a strain of 0.35 percent if the yield strength specified in the design exceeds 60,000 psi. Welded intersections shall not be spaced farther apart than 12 in. in direction of calculated stress, except for wire fabric used as stirrups in accordance with 12.13.2.

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Type R bars are required to meet more restrictive provisions for bend tests.

R3.5.3.2 — ASTM A 615 includes provisions for Grade 75 bars in sizes No. 6 through 18.

The 0.35 percent strain limit is necessary to ensure that the assumption of an elasto-plastic stress-strain curve in 10.2.4 will not lead to unconservative values of the member strength.

The 0.35 strain requirement is not applied to reinforcing bars having yield strengths of 60,000 psi or less. For steels having strengths of 40,000 psi, as were once used extensively, the assumption of an elasto-plastic stress-strain curve is well justified by extensive test data. For higher strength steels, up to 60,000 psi, the stress-strain curve may or may not be elasto-plastic as assumed in 10.2.4, depending on the properties of the steel and the manufacturing process. However, when the stress-strain curve is not elasto-plastic, there is limited experimental evidence to suggest that the actual steel stress at ultimate strength may not be enough less than the specified yield strength to warrant the additional effort of testing to the more restrictive criterion applicable to steels having f_y greater than 60,000 psi. In such cases, the f-factor can be expected to account for the strength deficiency.

R3.5.3.5 — Welded plain wire fabric should be made of wire conforming to "Specification for Steel Wire, Plain, for Concrete Reinforcement" (ASTM A 82). ASTM A 82 has a minimum yield strength of 70,000 psi. The code has assigned a yield strength value of 60,000 psi, but makes provision for the use of higher yield strengths provided the stress corresponds to a strain of 0.35 percent.

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3.5.3.6 — Welded deformed wire fabric for concrete reinforcement shall conform to "Specification for Steel Welded Wire Fabric, Deformed, for Concrete Reinforcement" (ASTM A 497), except that for wire with a specified yield strength f_y exceeding 60,000 psi, f_y shall be the stress corresponding to a strain of 0.35 percent if the yield strength specified in the design exceeds 60,000 psi. Welded intersections shall not be spaced farther apart than 16 in. in direction of calculated stress, except for wire fabric used as stirrups in accordance with 12.13.2.

3.5.3.7 — Galvanized reinforcing bars shall comply with "Specification for Zinc-Coated (Galvanized) Steel Bars for Concrete Reinforcement" (ASTM A 767). Epoxy-coated reinforcing bars shall comply with "Specification for Epoxy-Coated Reinforcing Steel Bars" (ASTM A 775) or with "Specification for Epoxy-Coated Prefabricated Steel Reinforcing Bars" (ASTM A 934). Bars to be galvanized or epoxy-coated shall conform to one of the specifications listed in 3.5.3.1.

3.5.3.8 — Epoxy-coated wires and welded wire fabric shall comply with "Specification for Epoxy-Coated Steel Wire and Welded Wire Fabric for Reinforcement" (ASTM A 884). Wires to be epoxy-coated shall conform to 3.5.3.4 and welded wire fabric to be epoxy-coated shall conform to 3.5.3.5 or 3.5.3.6.

3.5.4 — Plain reinforcement

3.5.4.1 — Plain bars for spiral reinforcement shall conform to the specification listed in 3.5.3.1(a) or (b).

3.5.4.2 — Plain wire for spiral reinforcement shall conform to "Specification for Steel Wire, Plain, for Concrete Reinforcement" (ASTM A 82), except that for wire with a specified yield strength f_y exceeding 60,000 psi, f_y shall be the stress corresponding to a strain of 0.35 percent if the yield strength specified in the design exceeds 60,000 psi.

3.5.5 — Prestressing steel

3.5.5.1 — Steel for prestressing shall conform to one of the following specifications:

(a) Wire conforming to "Specification for Uncoated Stress-Relieved Steel Wire for Prestressed Concrete" (ASTM A 421);

(b) Low-relaxation wire conforming to "Specification for Uncoated Stress-Relieved Steel Wire for Prestressed Concrete" including Supplement "Low-Relaxation Wire" (ASTM A 421);

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R3.5.3.6 — Welded deformed wire fabric should be made of wire conforming to "Specification for Steel Wire, Deformed, for Concrete Reinforcement" (ASTM A 496). ASTM A 496 has a minimum yield strength of 70,000 psi. The code has assigned a yield strength value of 60,000 psi, but makes provision for the use of higher yield strengths provided the stress corresponds to a strain of 0.35 percent.

R3.5.3.7 — Galvanized reinforcing bars (A 767) and epoxy-coated reinforcing bars (A 775) were added to the 1983 code, and epoxy-coated prefabricated reinforcing bars (A 934) were added to the 1995 code recognizing their usage, especially for conditions where corrosion resistance of reinforcement is of particular concern. They have typically been used in parking decks, bridge decks, and other highly corrosive environments.

R3.5.4 — Plain reinforcement

Plain bars and plain wire are permitted only for spiral reinforcement (either as lateral reinforcement for compression members, for torsion members, or for confining reinforcement for splices).

R3.5.5 — Prestressing steel

R3.5.5.1 — Because low-relaxation prestressing steel is addressed in a supplement to ASTM A 421, which applies only when low-relaxation material is specified, the appropriate ASTM reference is listed as a separate entity.

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(c) Strand conforming to “Specification for Steel Strand, Uncoated Seven-Wire for Prestressed Concrete” (ASTM A 416);

(d) Bar conforming to “Specification for Uncoated High-Strength Steel Bars for Prestressing Concrete” (ASTM A 722).

3.5.5.2 — Wire, strands, and bars not specifically listed in ASTM A 421, A 416, or A 722 are allowed provided they conform to minimum requirements of these specifications and do not have properties that make them less satisfactory than those listed in ASTM A 421, A 416, or A 722.

3.5.6 — Structural steel, steel pipe, or tubing

3.5.6.1 — Structural steel used with reinforcing bars in composite compression members meeting requirements of 10.16.7 or 10.16.8 shall conform to one of the following specifications:

(a) “Specification for Carbon Structural Steel” (ASTM A 36);

(b) “Specification for High-Strength Low-Alloy Structural Steel” (ASTM A 242);

(c) “Specification for High-Strength Low-Alloy Columbium-Vanadium Structural Steel” (ASTM A 572);

(d) “Specification for High-Strength Low-Alloy Structural Steel with 50 ksi (345 MPa) Minimum Yield Point to 4 in. (100 mm) Thick” (ASTM A 588).

3.5.6.2 — Steel pipe or tubing for composite compression members composed of a steel encased concrete core meeting requirements of 10.16.6 shall conform to one of the following specifications:

(a) Grade B of “Specification for Pipe, Steel, Black and Hot-Dipped, Zinc-Coated Welded and Seamless” (ASTM A 53);

(b) “Specification for Cold-Formed Welded and Seamless Carbon Steel Structural Tubing in Rounds and Shapes” (ASTM A 500);

(c) “Specification for Hot-Formed Welded and Seamless Carbon Steel Structural Tubing” (ASTM A 501).

3.6 — Admixtures

3.6.1 — Admixtures to be used in concrete shall be subject to prior approval by the engineer.

COMMENTARY

R3.6 — Admixtures

CODE

3.6.2 — An admixture shall be shown capable of maintaining essentially the same composition and performance throughout the work as the product used in establishing concrete proportions in accordance with 5.2.

3.6.3 — Calcium chloride or admixtures containing chloride from other than impurities from admixture ingredients shall not be used in prestressed concrete, in concrete containing embedded aluminum, or in concrete cast against stay-in-place galvanized steel forms. See 4.3.2 and 4.4.1.

3.6.4 — Air-entraining admixtures shall conform to "Specification for Air-Entraining Admixtures for Concrete" (ASTM C 260).

3.6.5 — Water-reducing admixtures, retarding admixtures, accelerating admixtures, water-reducing and retarding admixtures, and water-reducing and accelerating admixtures shall conform to "Specification for Chemical Admixtures for Concrete" (ASTM C 494) or "Specification for Chemical Admixtures for Use in Producing Flowing Concrete" (ASTM C 1017).

3.6.6 — Fly ash or other pozzolans used as admixtures shall conform to "Specification for Fly Ash and Raw or Calcined Natural Pozzolan for Use as a Mineral Admixture in Portland Cement Concrete" (ASTM C 618).

3.6.7 — Ground granulated blast-furnace slag used as an admixture shall conform to "Specification for Ground Granulated Blast-Furnace Slag for Use in Concrete and Mortars" (ASTM C 989).

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R3.6.3 — Admixtures containing any chloride, other than impurities from admixture ingredients, should not be used in prestressed concrete or in concrete with aluminum embedments. Concentrations of chloride ion may produce corrosion of embedded aluminum (e.g., conduit), especially if the aluminum is in contact with embedded steel and the concrete is in a humid environment. Serious corrosion of galvanized steel sheet and galvanized steel stay-in-place forms occurs, especially in humid environments or where drying is inhibited by the thickness of the concrete or coatings or impermeable coverings. See 4.4.1 for specific limits on chloride ion concentration in concrete.

R3.6.7 — Ground granulated blast-furnace slag conforming to ASTM C 989 is used as an admixture in concrete in much the same way as fly ash. Generally, it should be used with portland cements conforming to ASTM C 150, and only rarely would it be appropriate to use ASTM C 989 slag with an ASTM C 595 blended cement that already contains a pozzolan or slag. Such use with ASTM C 595 cements might be considered for massive concrete placements where slow strength gain can be tolerated and where low heat of hydration is of particular importance. ASTM C 989 includes appendices which discuss effects of ground granulated blast-furnace slag on concrete strength, sulfate resistance, and alkali-aggregate reaction.

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3.6.8 — Admixtures used in concrete containing C 845 expansive cements shall be compatible with the cement and produce no deleterious effects.

3.6.9 — Silica fume used as an admixture shall conform to ASTM C 1240.

3.7 — Storage of materials

3.7.1 — Cementitious materials and aggregates shall be stored in such manner as to prevent deterioration or intrusion of foreign matter.

3.7.2 — Any material that has deteriorated or has been contaminated shall not be used for concrete.

3.8 — Referenced standards

3.8.1 — Standards of the American Society for Testing and Materials referred to in this code are listed below with their serial designations, including year of adoption or revision, and are declared to be part of this code as if fully set forth herein:

A 36/ A 36M-00a	Standard Specification for Carbon Structural Steel
A 53/ A 53M-99b	Standard Specification for Pipe, Steel, Black and Hot-Dipped, Zinc-Coated, Welded and Seamless
A 82-97a	Standard Specification for Steel Wire, Plain, for Concrete Reinforcement
A 108-99	Standard Specification for Steel Bars, Carbon, Cold-Finished, Standard Quality
A 184/ A 184M-96	Standard Specification for Fabricated Deformed Steel Bar Mats for Concrete Reinforcement
A 185-97	Standard Specification for Steel Welded Wire Fabric, Plain, for Concrete Reinforcement
A 242/ A 242M-00a	Standard Specification for High-Strength Low-Alloy Structural Steel
A 307-97	Standard Specification for Carbon Steel Bolts and Studs, 60,000 psi Tensile Strength
A 416/ A 416M-99	Standard Specification for Steel Strand, Uncoated Seven-Wire for Prestressed Concrete

COMMENTARY

R3.6.8 — The use of admixtures in concrete containing C 845 expansive cements has reduced levels of expansion or increased shrinkage values. See ACI 223.[3.3]

R3.8 — Referenced standards

The ASTM standard specifications listed are the latest editions at the time these code provisions were adopted. Since these specifications are revised frequently, generally in minor details only, the user of the code should check directly with the sponsoring organization if it is desired to reference the latest edition. However, such a procedure obligates the user of the specification to evaluate if any changes in the later edition are significant in the use of the specification.

Standard specifications or other material to be legally adopted by reference into a building code should refer to a specific document. This can be done by simply using the complete serial designation since the first part indicates the subject and the second part the year of adoption. All standard documents referenced in this code are listed in 3.8, with the title and complete serial designation. In other sections of the code, the designations do not include the date so that all may be kept up-to-date by simply revising 3.8.

ASTM standards are available from ASTM, 100 Barr Harbor Drive, West Conshohocken, Pa., 19428.

CODE

COMMENTARY

A 421/ A 421M-98a	Standard Specification for Uncoated Stress-Relieved Steel Wire for Prestressed Concrete
A 496-97a	Standard Specification for Steel Wire, Deformed, for Concrete Reinforcement
A 497-99	Standard Specification for Steel Welded Wire Fabric, Deformed, for Concrete Reinforcement
A 500-99	Standard Specification for Cold-Formed Welded and Seamless Carbon Steel Structural Tubing in Rounds and Shapes
A 501-99	Standard Specification for Hot-Formed Welded and Seamless Carbon Steel Structural Tubing
A 572/ A 572M-00	Standard Specification for High-Strength Low-Alloy Columbium-Vanadium Structural Steel
A 588/ A 588M-00	Standard Specification for High-Strength Low-Alloy Structural Steel with 50 ksi (345 MPa) Minimum Yield Point to 4 in. (100 mm) Thick
A 615/ A 615M-00	Standard Specification for Deformed and Plain Billet-Steel Bars for Concrete Reinforcement
A 706/ A 706M-00	Standard Specification for Low-Alloy Steel Deformed and Plain Bars for Concrete Reinforcement
A 722/ A 722M-98	Standard Specification for Uncoated High-Strength Steel Bars for Prestressing Concrete
A 767/ A 767M-00b	Standard Specification for Zinc-Coated (Galvanized) Steel Bars for Concrete Reinforcement
A 775/ A 775M-00	Standard Specification for Epoxy-Coated Steel Reinforcing Bars
A 884/ A 884M-99	Standard Specification for Epoxy-Coated Steel Wire and Welded Wire Fabric for Reinforcement
A 934/ A 934M-00	Standard Specification for Epoxy-Coated Prefabricated Steel Reinforcing Bars
A 996/ A 996M-00	Standard Specification for Rail-Steel and Axle-Steel Deformed Bars for Concrete Reinforcement

Type R rail-steel bars are considered a mandatory requirement whenever ASTM A 996 is referenced in the code.

CODE

COMMENTARY

C 31/ C 31M-98	Standard Practice for Making and Curing Concrete Test Specimens in the Field
C 33-99a$^{\varepsilon 1}$	Standard Specification for Concrete Aggregates
C 39/ C 39M-99	Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens
C 42/ C 42M-99	Standard Test Method for Obtaining and Testing Drilled Cores and Sawed Beams of Concrete
C 94/ C 94M-00	Standard Specification of Ready-Mixed Concrete
C 109/ C 109M-99	Standard Test Method for Compressive Strength of Hydraulic Cement Mortars (Using 2-in. or 50-mm Cube Specimens)
C 144-99	Standard Specification for Aggregate for Masonry Mortar
C 150-99a	Standard Specification for Portland Cement
C 172-99	Standard Practice for Sampling Freshly-Mixed Concrete
C 192/ C 192M-98	Standard Practice for Making and Curing Concrete Test Specimens in the Laboratory
C 260-00	Standard Specification for Air-Entraining Admixtures for Concrete
C 330-99	Standard Specification for Lightweight Aggregates for Structural Concrete
C 494/ C 494M-99a	Standard Specification for Chemical Admixtures for Concrete
C 496-96	Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens
C 567-99a	Standard Test Method for Density of Structural Lightweight Concrete
C 595-00	Standard Specification for Blended Hydraulic Cements
C 618-99	Standard Specification for Coal Fly Ash and Raw or Calcined Natural Pozzolan for Use as a Mineral Admixture in Concrete

CODE

C 685-98a	Standard Specification for Concrete Made by Volumetric Batching and Continuous Mixing
C 845-96	Standard Specification for Expansive Hydraulic Cement
C 989-99	Standard Specification for Ground Granulated Blast-Furnace Slag for Use in Concrete and Mortars
C 1017/ C 1017M-98	Standard Specification for Chemical Admixtures for Use in Producing Flowing Concrete
C 1157-00	Standard Performance Specification for Hydraulic Cement
C 1218/ C 1218M-99	Standard Test Method for Water-Soluble Chloride in Mortar and Concrete
C 1240-00$^{\varepsilon 1}$	Standard Specification for Use of Silica Fume as a Mineral Admixture in Hydraulic-Cement Concrete, Mortar, and Grout

3.8.2 — "Structural Welding Code—Reinforcing Steel" (ANSI/AWS D1.4-98) of the American Welding Society is declared to be part of this code as if fully set forth herein.

3.8.3 — Section 2.3 Combining Factored Loads Using Strength Design of "Minimum Design Loads for Buildings and Other Structures" (ASCE 7-98) is declared to be part of this code as if fully set forth herein, for the purpose cited in 9.2.4.

3.8.4 — "Specification for Unbonded Single Strand Tendons (ACI 423.6-01) and Commentary (423.6R-01)" is declared to be part of this code as if fully set forth herein.

3.8.5 — Articles 9.21.7.2 and 9.21.7.3 of Division I and Article 10.3.2.3 of Division II of AASHTO "Standard Specification for Highway Bridges" (AASHTO 16th Edition, 1996) are declared to be a part of this code as if fully set forth herein.

3.8.6 — "Evaluating the Performance of Post-Installed Mechanical Anchors in Concrete (ACI 355.2-01)" is declared to be part of this code as if fully set forth herein, for the purpose cited in Appendix D.

COMMENTARY

R3.8.3 — ASCE 7 is available from ASCE Book Orders, Box 79404, Baltimore, Md., 21279-0404.

R3.8.5 — The 1996 16th Edition of the AASHTO "Standard Specification for Highway Bridges" is available from AASHTO, 444 North Capitol Street, N.W., Suite 249, Washington, D.C., 20001.

R3.8.6 — Parallel to development of the ACI 318-02 provisions for anchoring to concrete, ACI 355 developed a test method to define the level of performance required for post-installed anchors. This test method, ACI 355.2, contains requirements for the testing and evaluation of post-installed anchors for both cracked and uncracked concrete applications.

CODE

3.8.7 — "Structural Welding Code—Steel (AWS D 1.1:2000)" of the American Welding Society is declared to be part of this code as if fully set forth herein.

3.8.8 — "Acceptance Criteria for Moment Frames Based on Structural Testing (ACI T1.1-01)," is declared to be part of this code as if fully set forth herein.

COMMENTARY

CODE

COMMENTARY

Notes

PART 3 — CONSTRUCTION REQUIREMENTS

CHAPTER 4 — DURABILITY REQUIREMENTS

CODE

4.0 — Notation

f_c' = specified compressive strength of concrete, psi

COMMENTARY

Chapters 4 and 5 of earlier editions of the code were reformatted in 1989 to emphasize the importance of considering durability requirements before the designer selects f_c' and cover over the reinforcing steel.

Maximum water-cementitious materials ratios of 0.40 to 0.50 that may be required for concretes exposed to freezing and thawing, sulfate soils or waters, or for preventing corrosion of reinforcement will typically be equivalent to requiring an f_c' of 5000 to 4000 psi, respectively. Generally, the required average concrete strengths, f_{cr}', will be 500 to 700 psi higher than the specified compressive strength, f_c'. Since it is difficult to accurately determine the water-cementitious materials ratio of concrete during production, the f_c' specified should be reasonably consistent with the water-cementitious materials ratio required for durability. Selection of an f_c' that is consistent with the water-cementitious materials ratio selected for durability will help ensure that the required water-cementitious materials ratio is actually obtained in the field. Because the usual emphasis on inspection is for strength, test results substantially higher than the specified strength may lead to a lack of concern for quality and production of concrete that exceeds the maximum water-cementitious materials ratio. Thus an f_c' of 3000 psi and a maximum water-cementitious materials ratio of 0.45 should not be specified for a parking structure, if the structure will be exposed to deicing salts.

The code does not include provisions for especially severe exposures, such as acids or high temperatures, and is not concerned with aesthetic considerations such as surface finishes. These items are beyond the scope of the code and should be covered specifically in the project specifications. Concrete ingredients and proportions are to be selected to meet the minimum requirements stated in the code and the additional requirements of the contract documents.

4.1 — Water-cementitious materials ratio

4.1.1 — The water-cementitious materials ratios specified in Tables 4.2.2 and 4.3.1 shall be calculated using the weight of cement meeting ASTM C 150, C 595, C 845, or C 1157 plus the weight of fly ash and other pozzolans meeting ASTM C 618, slag meeting ASTM C 989, and silica fume meeting ASTM C 1240, if any, except that when concrete is exposed to deicing chemicals, 4.2.3 further limits the amount of fly ash, pozzolans, silica fume, slag, or the combination of these materials.

R4.1 — Water-cementitious materials ratio

R4.1.1 — For concrete exposed to deicing chemicals the quantity of fly ash, other pozzolans, silica fume, slag, or blended cements used in the concrete is subject to the percentage limits in 4.2.3. Further, in 4.3 for sulfate exposures,[4.1] the pozzolan should be Class F by ASTM C 618, or have been tested by ASTM C 1012[4.2] or determined by service record to improve sulfate resistance.

CODE

4.2 — Freezing and thawing exposures

4.2.1 — Normalweight and lightweight concrete exposed to freezing and thawing or deicing chemicals shall be air-entrained with air content indicated in Table 4.2.1. Tolerance on air content as delivered shall be ± 1.5 percent. For specified compressive strength f'_c greater than 5000 psi, reduction of air content indicated in Table 4.2.1 by 1.0 percent shall be permitted.

TABLE 4.2.1—TOTAL AIR CONTENT FOR FROST-RESISTANT CONCRETE

Nominal maximum aggregate size, in.*	Air content, percent	
	Severe exposure	Moderate exposure
3/8	7.5	6
1/2	7	5.5
3/4	6	5
1	6	4.5
1-1/2	5.5	4.5
2†	5	4
3†	4.5	3.5

* See ASTM C 33 for tolerance on oversize for various nominal maximum size designations.

† These air contents apply to total mix, as for the preceding aggregate sizes. When testing these concretes, however, aggregate larger than 1-1/2 in. is removed by handpicking or sieving and air content is determined on the minus 1-1/2 in. fraction of mix (tolerance on air content as delivered applies to this value.). Air content of total mix is computed from value determined on the minus 1-1/2 in. fraction.

4.2.2 — Concrete that will be subject to the exposures given in Table 4.2.2 shall conform to the corresponding maximum water-cementitious materials ratios and minimum specified concrete compressive strength requirements of that table. In addition, concrete that will be exposed to deicing chemicals shall conform to the limitations of 4.2.3.

TABLE 4.2.2—REQUIREMENTS FOR SPECIAL EXPOSURE CONDITIONS

Exposure condition	Maximum water-cementitious materials ratio*, by weight, normalweight aggregate concrete	Minimum f'_c, normalweight and lightweight aggregate concrete, psi*
Concrete intended to have low permeability when exposed to water	0.50	4000
Concrete exposed to freezing and thawing in a moist condition or to deicing chemicals	0.45	4500
For corrosion protection of reinforcement in concrete exposed to chlorides from deicing chemicals, salt, salt water, brackish water, seawater, or spray from these sources.	0.40	5000

* When both Table 4.3.1 and Table 4.2.2 are considered, the lowest applicable maximum *w/cm* and highest applicable minimum f'_c shall be used.

COMMENTARY

R4.2 — Freezing and thawing exposures

R4.2.1 — A table of required air contents for frost-resistant concrete is included in the code, based on **"Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete"** (ACI 211.1).[4.3] Values are provided for both severe and moderate exposures depending on the exposure to moisture or deicing salts. Entrained air will not protect concrete containing coarse aggregates that undergo disruptive volume changes when frozen in a saturated condition. In Table 4.2.1, a severe exposure is where the concrete in a cold climate may be in almost continuous contact with moisture prior to freezing, or where deicing salts are used. Examples are pavements, bridge decks, sidewalks, parking garages, and water tanks. A moderate exposure is where the concrete in a cold climate will be only occasionally exposed to moisture prior to freezing, and where no deicing salts are used. Examples are certain exterior walls, beams, girders, and slabs not in direct contact with soil. Section 4.2.1 permits 1 percent lower air content for concrete with f'_c greater than 5000 psi. Such high-strength concretes will have lower water-cementitious materials ratios and porosity and, therefore, improved frost resistance.

R4.2.2 — Maximum water-cementitious materials ratios are not specified for lightweight aggregate concrete because determination of the absorption of these aggregates is uncertain, making calculation of the water-cementitious materials ratio uncertain. The use of a minimum specified strength will ensure the use of a high-quality cement paste. For normalweight aggregate concrete, use of both minimum strength and maximum water-cementitious materials ratio provide additional assurance that this objective is met.

CODE

4.2.3 — For concrete exposed to deicing chemicals, the maximum weight of fly ash, other pozzolans, silica fume, or slag that is included in the concrete shall not exceed the percentages of the total weight of cementitious materials given in Table 4.2.3.

TABLE 4.2.3—REQUIREMENTS FOR CONCRETE EXPOSED TO DEICING CHEMICALS

Cementitious materials	Maximum percent of total cementitious materials by weight*
Fly ash or other pozzolans conforming to ASTM C 618	25
Slag conforming to ASTM C 989	50
Silica fume conforming to ASTM C 1240	10
Total of fly ash or other pozzolans, slag, and silica fume	50†
Total of fly ash or other pozzolans and silica fume	35†

* The total cementitious material also includes ASTM C 150, C 595, C 845, and C 1157 cement.
The maximum percentages above shall include:
(a) Fly ash or other pozzolans present in Type IP or I(PM) blended cement, ASTM C 595, or ASTM C 1157;
(b) Slag used in the manufacture of a IS or I(SM) blended cement, ASTM C 595, or ASTM C 1157;
(c) Silica fume, ASTM C 1240, present in a blended cement.
† Fly ash or other pozzolans and silica fume shall constitute no more than 25 and 10 percent, respectively, of the total weight of the cementitious materials.

COMMENTARY

R4.2.3 — Section 4.2.3 and Table 4.2.3 establish limitations on the amount of fly ash, other pozzolans, silica fume, and slag that can be included in concrete exposed to deicing chemicals.[4.4-4.6] Recent research has demonstrated that the use of fly ash, slag, and silica fume produce concrete with a finer pore structure and, therefore, lower permeability.[4.7-4.9]

4.3 — Sulfate exposures

4.3.1 — Concrete to be exposed to sulfate-containing solutions or soils shall conform to requirements of Table 4.3.1 or shall be concrete made with a cement that provides sulfate resistance and that has a maximum water-cementitious materials ratio and minimum compressive strength from Table 4.3.1.

R4.3 — Sulfate exposures

R4.3.1 — Concrete exposed to injurious concentrations of sulfates from soil and water should be made with a sulfate-resisting cement. Table 4.3.1 lists the appropriate types of cement and the maximum water-cementitious materials ratios and minimum strengths for various exposure conditions. In selecting a cement for sulfate resistance, the principal consideration is its tricalcium aluminate (C_3A) content. For moderate exposures, Type II cement is limited to a maximum C_3A content of 8.0 percent under ASTM C 150. The blended cements under ASTM C 595 with the MS designation are appropriate for use in moderate sulfate exposures. The appropriate types under ASTM C 595 are IP(MS), IS(MS), I(PM)(MS), and I(SM)(MS). For severe exposures, Type V cement with a maximum C_3A content of 5 percent is specified. In certain areas, the C_3A content of other avail-

TABLE 4.3.1—REQUIREMENTS FOR CONCRETE EXPOSED TO SULFATE-CONTAINING SOLUTIONS

Sulfate exposure	Water soluble sulfate (SO_4) in soil, percent by weight	Sulfate (SO_4) in water, ppm	Cement type	Maximum water-cementitious materials ratio, by weight, normalweight aggregate concrete*	Minimum f'_c, normalweight and lightweight aggregate concrete, psi*
Negligible	$0.00 \le SO_4 < 0.10$	$0 \le SO_4 < 150$	—	—	—
Moderate†	$0.10 \le SO_4 < 0.20$	$150 \le SO_4 < 1500$	II, IP(MS), IS(MS), P(MS), I(PM)(MS), I(SM)(MS)	0.50	4000
Severe	$0.20 \le SO_4 \le 2.00$	$1500 \le SO_4 \le 10{,}000$	V	0.45	4500
Very severe	$SO_4 > 2.00$	$SO_4 > 10{,}000$	V plus pozzolan‡	0.45	4500

* When both Table 4.3.1 and Table 4.2.2 are considered, the lowest applicable maximum *w/cm* and highest applicable minimum f'_c shall be used.
† Seawater.
‡ Pozzolan that has been determined by test or service record to improve sulfate resistance when used in concrete containing Type V cement.

CODE

4.3.2 — Calcium chloride as an admixture shall not be used in concrete to be exposed to severe or very severe sulfate-containing solutions, as defined in Table 4.3.1.

4.4 — Corrosion protection of reinforcement

4.4.1 — For corrosion protection of reinforcement in concrete, maximum water soluble chloride ion concentrations in hardened concrete at ages from 28 to 42 days contributed from the ingredients including water, aggregates, cementitious materials, and admixtures shall not exceed the limits of Table 4.4.1. When testing is performed to determine water soluble chloride ion content, test procedures shall conform to ASTM C 1218.

COMMENTARY

able types such as Type III or Type I may be less than 8 or 5 percent and are usable in moderate or severe sulfate exposures. Note that sulfate-resisting cement will not increase resistance to some chemically aggressive solutions, for example ammonium nitrate. The project specifications should cover all special cases.

Using fly ash (ASTM C 618, Class F) also has been shown to improve the sulfate resistance of concrete.[4.9] Certain Type IP cements made by blending Class F pozzolan with portland cement having a C_3A content greater than 8 percent can provide sulfate resistance for moderate exposures.

A note to Table 4.3.1 lists seawater as moderate exposure, even though it generally contains more than 1500 ppm SO_4. In seawater exposures, other types of cement with C_3A up to 10 percent may be used if the maximum water-cementitious materials ratio is reduced to 0.40.

ASTM test method C 1012[4.2] can be used to evaluate the sulfate resistance of mixtures using combinations of cementitious materials.

In addition to the proper selection of cement, other requirements for durable concrete exposed to concentrations of sulfate are essential, such as, low water-cementitious materials ratio, strength, adequate air entrainment, low slump, adequate consolidation, uniformity, adequate cover of reinforcement, and sufficient moist curing to develop the potential properties of the concrete.

R4.4 — Corrosion protection of reinforcement

R4.4.1 — Additional information on the effects of chlorides on the corrosion of reinforcing steel is given in **"Guide to Durable Concrete"** reported by ACI Committee 201[4.10] and **"Corrosion of Metals in Concrete"** reported by ACI Committee 222.[4.11] Test procedures should conform to those given in ASTM C 1218. An initial evaluation may be obtained by testing individual concrete ingredients for total chloride ion content. If total chloride ion content, calculated on the basis of concrete proportions, exceeds those permitted in Table 4.4.1, it may be necessary to test samples of the hardened concrete for water-soluble chloride ion content described in the ACI 201 guide. Some of the total chloride ions present in the ingredients will either be insoluble or will react with the cement during hydration and become insoluble under the test procedures described in ASTM C 1218.

CODE

TABLE 4.4.1—MAXIMUM CHLORIDE ION CONTENT FOR CORROSION PROTECTION OF REINFORCEMENT

Type of member	Maximum water soluble chloride ion (Cl^-) in concrete, percent by weight of cement
Prestressed concrete	0.06
Reinforced concrete exposed to chloride in service	0.15
Reinforced concrete that will be dry or protected from moisture in service	1.00
Other reinforced concrete construction	0.30

4.4.2 — If concrete with reinforcement will be exposed to chlorides from deicing chemicals, salt, salt water, brackish water, seawater, or spray from these sources, requirements of Table 4.2.2 for water-cementitious materials ratio and concrete strength, and the minimum concrete cover requirements of 7.7 shall be satisfied. See 18.16 for unbonded tendons.

COMMENTARY

When concretes are tested for soluble chloride ion content the tests should be made at an age of 28 to 42 days. The limits in Table 4.4.1 are to be applied to chlorides contributed from the concrete ingredients, not those from the environment surrounding the concrete.

The chloride ion limits in Table 4.4.1 differ from those recommended in ACI 201.2R[4.10] and ACI 222R.[4.11] For reinforced concrete that will be dry in service, a limit of 1 percent has been included to control total soluble chlorides. Table 4.4.1 includes limits of 0.15 and 0.30 percent for reinforced concrete that will be exposed to chlorides or will be damp in service, respectively. These limits compare to 0.10 and 0.15 recommended in ACI 201.2R.[4.10] ACI 222R[4.11] recommends limits of 0.08 and 0.20 percent by weight of cement for chlorides in prestressed and reinforced concrete, respectively, based on tests for acid soluble chlorides, not the test for water soluble chlorides required here.

When epoxy or zinc-coated bars are used, the limits in Table 4.4.1 may be more restrictive than necessary.

R4.4.2 — When concretes are exposed to external sources of chlorides, the water-cementitious materials ratio and specified compressive strength f'_c of 4.2.2 are the minimum requirements that are to be considered. The designer should evaluate conditions in structures where chlorides may be applied, in parking structures where chlorides may be tracked in by vehicles, or in structures near seawater. Epoxy- or zinc-coated bars or cover greater than the minimum required in 7.7 may be desirable. Use of slag meeting ASTM C 989 or fly ash meeting ASTM C 618 and increased levels of specified strength provide increased protection. Use of silica fume meeting ASTM C 1240 with an appropriate high-range water reducer, ASTM C 494, Types F and G, or ASTM C 1017 can also provide additional protection.[4.12] The use of ASTM C 1202[4.13] to test concrete mixtures proposed for use will provide additional information on the performance of the mixtures.

CODE

COMMENTARY

Notes

CHAPTER 5 — CONCRETE QUALITY, MIXING, AND PLACING

CODE

5.0 — Notation

f_c' = specified compressive strength of concrete, psi
f_{cr}' = required average compressive strength of concrete used as the basis for selection of concrete proportions, psi
f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi
s = standard deviation, psi

COMMENTARY

The requirements for proportioning concrete mixtures are based on the philosophy that concrete should provide both adequate durability (Chapter 4) and strength. The criteria for acceptance of concrete are based on the philosophy that the code is intended primarily to protect the safety of the public. Chapter 5 describes procedures by which concrete of adequate strength can be obtained, and provides procedures for checking the quality of the concrete during and after its placement in the work.

Chapter 5 also prescribes minimum criteria for mixing and placing concrete.

The provisions of 5.2, 5.3, and 5.4, together with Chapter 4, establish required mixture proportions. The basis for determining the adequacy of concrete strength is in 5.6.

5.1 — General

5.1.1 — Concrete shall be proportioned to provide an average compressive strength as prescribed in 5.3.2 and shall satisfy the durability criteria of Chapter 4. Concrete shall be produced to minimize the frequency of strengths below f_c', as prescribed in 5.6.3.3. For concrete designed and constructed in accordance with the code, f_c' shall not be less than 2500 psi.

5.1.2 — Requirements for f_c' shall be based on tests of cylinders made and tested as prescribed in 5.6.3.

5.1.3 — Unless otherwise specified, f_c' shall be based on 28-day tests. If other than 28 days, test age for f_c' shall be as indicated in design drawings or specifications.

5.1.4 — Where design criteria in 9.5.2.3, 11.2, and 12.2.4 provide for use of a splitting tensile strength value of concrete, laboratory tests shall be made in accordance with "Specification for Lightweight Aggregates for Structural Concrete" (ASTM C 330) to establish value of f_{ct} corresponding to specified value of f_c'.

5.1.5 — Splitting tensile strength tests shall not be used as a basis for field acceptance of concrete.

R5.1 — General

R5.1.1 — The basic premises governing the designation and evaluation of concrete strength are presented. It is emphasized that the average strength of concrete produced should always exceed the specified value of f_c' used in the structural design calculations. This is based on probabilistic concepts, and is intended to ensure that adequate concrete strength will be developed in the structure. The durability requirements prescribed in Chapter 4 are to be satisfied in addition to attaining the average concrete strength in accordance with 5.3.2.

R5.1.4 — Sections 9.5.2.3 (modulus of rupture), 11.2 (concrete shear strength) and 12.2.4 (development of reinforcement) require modification in the design criteria for the use of lightweight aggregate concrete. Two alternative modification procedures are provided. One alternative is based on laboratory tests to determine the relationship between splitting tensile strength f_{ct} and specified compressive strength f_c' for the lightweight concrete. For a lightweight aggregate from a given source, it is intended that appropriate values of f_{ct} be obtained in advance of design.

R5.1.5 — Tests for splitting tensile strength of concrete (as required by 5.1.4) are not intended for control of, or acceptance of, the strength of concrete in the field. Indirect control will

CODE

COMMENTARY

be maintained through the normal compressive strength test requirements provided by 5.6.

5.2 — Selection of concrete proportions

R5.2 — Selection of concrete proportions

Recommendations for selecting proportions for concrete are given in detail in **"Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete"** (ACI 211.1).[5.1] (Provides two methods for selecting and adjusting proportions for normalweight concrete: the estimated weight and absolute volume methods. Example calculations are shown for both methods. Proportioning of heavyweight concrete by the absolute volume method is presented in an appendix.)

Recommendations for lightweight concrete are given in **"Standard Practice for Selecting Proportions for Structural Lightweight Concrete"** (ACI 211.2).[5.2] (Provides a method of proportioning and adjusting structural grade concrete containing lightweight aggregates.)

5.2.1 — Proportions of materials for concrete shall be established to provide:

(a) Workability and consistency to permit concrete to be worked readily into forms and around reinforcement under conditions of placement to be employed, without segregation or excessive bleeding;

(b) Resistance to special exposures as required by Chapter 4;

(c) Conformance with strength test requirements of 5.6.

R5.2.1 — The selected water-cementitious materials ratio should be low enough, or in the case of lightweight concrete the compressive strength high enough to satisfy both the strength criteria (see 5.3 or 5.4) and the special exposure requirements (Chapter 4). The code does not include provisions for especially severe exposures, such as acids or high temperatures, and is not concerned with aesthetic considerations such as surface finishes. These items are beyond the scope of the code and should be covered specifically in the project specifications. Concrete ingredients and proportions are to be selected to meet the minimum requirements stated in the code and the additional requirements of the contract documents.

5.2.2 — Where different materials are to be used for different portions of proposed work, each combination shall be evaluated.

5.2.3 — Concrete proportions shall be established in accordance with 5.3 or, alternatively, 5.4, and shall meet applicable requirements of Chapter 4.

R5.2.3 — The code emphasizes the use of field experience or laboratory trial mixtures (see 5.3) as the preferred method for selecting concrete mixture proportions.

5.3 — Proportioning on the basis of field experience or trial mixtures, or both

R5.3 — Proportioning on the basis of field experience or trial mixtures, or both

In selecting a suitable concrete mixture there are three basic steps. The first is the determination of the standard deviation. The second is the determination of the required average strength. The third is the selection of mixture proportions required to produce that average strength, either by conventional trial mixture procedures or by a suitable experience record. Fig. R5.3 is a flow chart outlining the mixture selection and documentation procedure.

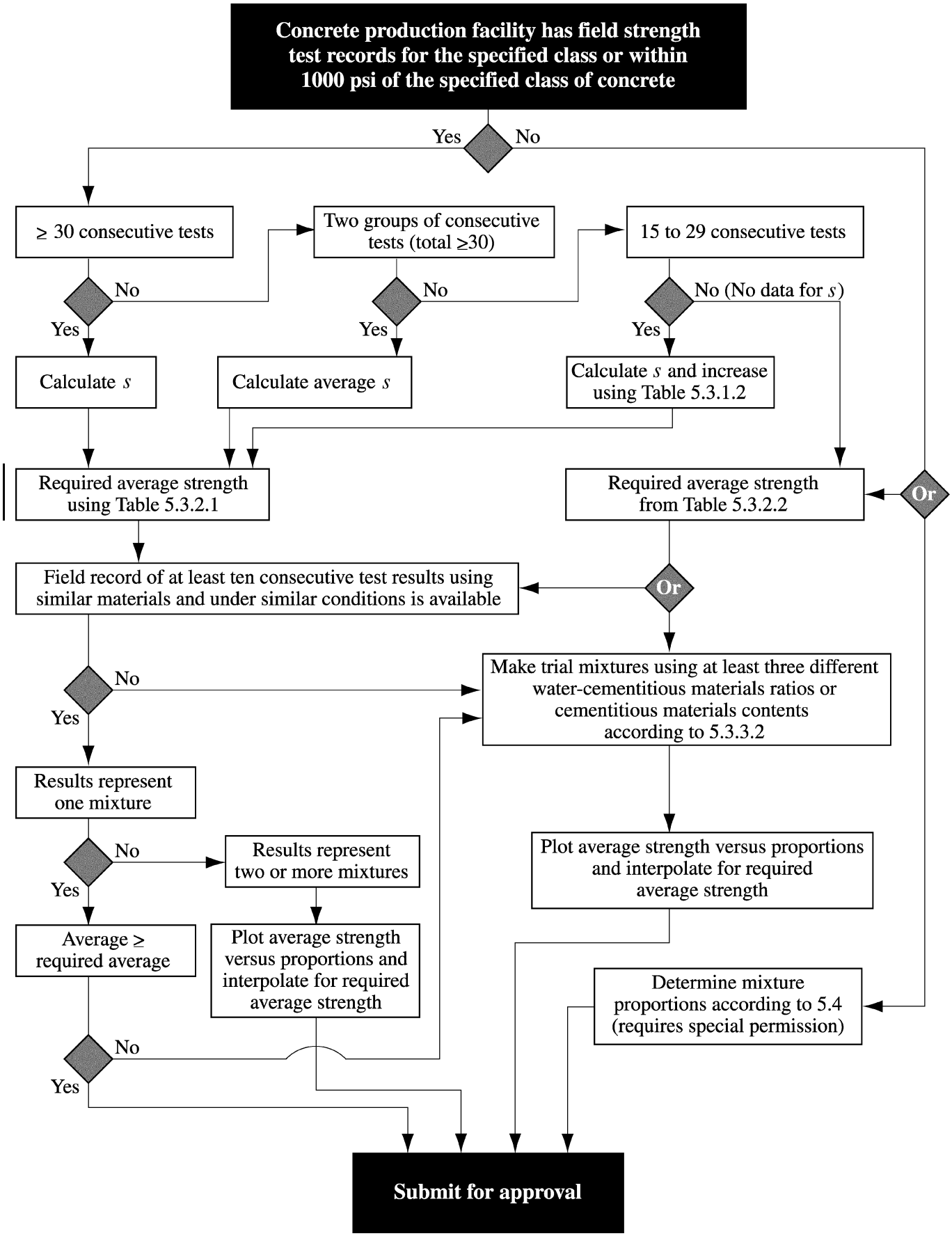


Fig. R5.3—Flow chart for selection and documentation of concrete proportions

CODE

5.3.1 — Standard deviation

5.3.1.1 — Where a concrete production facility has test records, a standard deviation shall be established. Test records from which a standard deviation is calculated:

(a) Shall represent materials, quality control procedures, and conditions similar to those expected and changes in materials and proportions within the test records shall not have been more restricted than those for proposed work;

(b) Shall represent concrete produced to meet a specified strength or strengths f'_c within 1000 psi of that specified for proposed work;

(c) Shall consist of at least 30 consecutive tests or two groups of consecutive tests totaling at least 30 tests as defined in 5.6.2.4, except as provided in 5.3.1.2.

5.3.1.2 — Where a concrete production facility does not have test records meeting requirements of 5.3.1.1, but does have a record based on 15 to 29 consecutive tests, a standard deviation shall be established as the product of the calculated standard deviation and modification factor of Table 5.3.1.2. To be acceptable, test record shall meet requirements (a) and (b) of 5.3.1.1, and represent only a single record of consecutive tests that span a period of not less than 45 calendar days.

TABLE 5.3.1.2—MODIFICATION FACTOR FOR STANDARD DEVIATION WHEN LESS THAN 30 TESTS ARE AVAILABLE

No. of tests*	Modification factor for standard deviation†
Less than 15	Use table 5.3.2.2
15	1.16
20	1.08
25	1.03
30 or more	1.00

* Interpolate for intermediate numbers of tests.

† Modified standard deviation to be used to determine required average strength f'_{cr} from 5.3.2.1.

COMMENTARY

The mixture selected should yield an average strength appreciably higher than the specified strength f'_c. The degree of mixture over design depends on the variability of the test results.

R5.3.1 — Standard deviation

When a concrete production facility has a suitable record of 30 consecutive tests of similar materials and conditions expected, the standard deviation is calculated from those results in accordance with the following formula:

$$s = \left[\frac{\Sigma(X_i - \bar{X})^2}{(n-1)}\right]^{1/2}$$

where:

s = standard deviation, psi
X_i = individual strength tests as defined in 5.6.2.4
$\bar{X}$ = average of n strength test results
n = number of consecutive strength tests

The standard deviation is used to determine the average strength required in 5.3.2.1.

If two test records are used to obtain at least 30 tests, the standard deviation used shall be the statistical average of the values calculated from each test record in accordance with the following formula:

$$\bar{s} = \left[\frac{(n_1-1)(s_1)^2 + (n_2-1)(s_2)^2}{(n_1+n_2-2)}\right]^{1/2}$$

where:

$\bar{s}$ = statistical average standard deviation where two test records are used to estimate the standard deviation
s_1, s_2 = standard deviations calculated from two test records, 1 and 2, respectively
n_1, n_2 = number of tests in each test record, respectively

If less than 30, but at least 15 tests are available, the calculated standard deviation is increased by the factor given in Table 5.3.1.2. This procedure results in a more conservative (increased) required average strength. The factors in Table 5.3.1.2 are based on the sampling distribution of the standard deviation and provide protection (equivalent to that from a record of 30 tests) against the possibility that the smaller sample underestimates the true or universe population standard deviation.

The standard deviation used in the calculation of required average strength should be developed under conditions "similar to those expected" [see 5.3.1.1(a)]. This requirement is important to ensure acceptable concrete.

CODE

5.3.2 — Required average strength

5.3.2.1 — Required average compressive strength f'_{cr} used as the basis for selection of concrete proportions shall be determined from Table 5.3.2.1 using the standard deviation calculated in accordance with 5.3.1.1 or 5.3.1.2.

TABLE 5.3.2.1—REQUIRED AVERAGE COMPRESSIVE STRENGTH WHEN DATA ARE AVAILABLE TO ESTABLISH A STANDARD DEVIATION

Specified compressive strength, f'_c, psi	Required average compressive strength, f'_{cr}, psi
$f'_c \le 5000$	Use the larger value computed from Eq. (5-1) and (5-2) $f'_{cr} = f'_c + 1.34s$ (5-1) $f'_{cr} = f'_c + 2.33s - 500$ (5-2)
Over 5000	Use the larger value computed from Eq. (5-1) and (5-3) $f'_{cr} = f'_c + 1.34s$ (5-1) $f'_{cr} = 0.90f'_c + 2.33s$ (5-3)

COMMENTARY

Concrete for background tests to determine standard deviation is considered to be "similar" to that required if made with the same general types of ingredients under no more restrictive conditions of control over material quality and production methods than on the proposed work, and if its specified strength does not deviate more than 1000 psi from the f'_c required [see 5.3.1.1(b)]. A change in the type of concrete or a major increase in the strength level may increase the standard deviation. Such a situation might occur with a change in type of aggregate (i.e., from natural aggregate to lightweight aggregate or vice versa) or a change from non-air-entrained concrete to air-entrained concrete. Also, there may be an increase in standard deviation when the average strength level is raised by a significant amount, although the increment of increase in standard deviation should be somewhat less than directly proportional to the strength increase. When there is reasonable doubt, any estimated standard deviation used to calculate the required average strength should always be on the conservative (high) side.

Note that the code uses the standard deviation in pounds per square inch instead of the coefficient of variation in percent. The latter is equal to the former expressed as a percent of the average strength.

Even when the average strength and standard deviation are of the levels assumed, there will be occasional tests that fail to meet the acceptance criteria prescribed in 5.6.3.3 (perhaps 1 test in 100).

R5.3.2 — Required average strength

R5.3.2.1 — Once the standard deviation has been determined, the required average compressive strength is obtained from the larger value computed from Eq. (5-1) and (5-2) for f'_c of 5000 psi or less, or the larger value computed from Eq. (5-1) and (5-3) for f'_c over 5000 psi. Equation (5-1) is based on a probability of 1-in-100 that the average of three consecutive tests may be below the specified compressive strength f'_c. Equation (5-2) is based on a similar probability that an individual test may be more than 500 psi below the specified compressive strength f'_c. Equation (5-3) is based on the same 1-in-100 probability that an individual test may be less than $0.90f'_c$. These equations assume that the standard deviation used is equal to the population value appropriate for an infinite or very large number of tests. For this reason, use of standard deviations estimated from records of 100 or more tests is desirable. When 30 tests are available, the probability of failure will likely be somewhat greater than 1-in-100. The additional refinements required to achieve the 1-in-100 probability are not considered necessary, because of the uncertainty inherent in assuming that conditions operating when the test record was accumulated will be similar to conditions when the concrete will be produced.

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5.3.2.2 — When a concrete production facility does not have field strength test records for calculation of standard deviation meeting requirements of 5.3.1.1 or 5.3.1.2, required average strength f_{cr}' shall be determined from Table 5.3.2.2 and documentation of average strength shall be in accordance with requirements of 5.3.3.

TABLE 5.3.2.2—REQUIRED AVERAGE COMPRESSIVE STRENGTH WHEN DATA ARE NOT AVAILABLE TO ESTABLISH A STANDARD DEVIATION

Specified compressive strength, f_c', psi	Required average compressive strength, f_{cr}', psi
Less than 3000	$f_c' + 1000$
3000 to 5000	$f_c' + 1200$
Over 5000	$1.10f_c' + 700$

5.3.3 — Documentation of average strength

Documentation that proposed concrete proportions will produce an average compressive strength equal to or greater than required average compressive strength (see 5.3.2) shall consist of a field strength test record, several strength test records, or trial mixtures.

5.3.3.1 — When test records are used to demonstrate that proposed concrete proportions will produce the required average strength f_{cr}' (see 5.3.2), such records shall represent materials and conditions similar to those expected. Changes in materials, conditions, and proportions within the test records shall not have been more restricted than those for proposed work. For the purpose of documenting average strength potential, test records consisting of less than 30 but not less than 10 consecutive tests are accept-

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R.5.3.3 — Documentation of average strength

Once the required average strength f_{cr}' is known, the next step is to select mixture proportions that will produce an average strength at least as great as the required average strength, and also meet special exposure requirements of Chapter 4. The documentation may consist of a strength test record, several strength test records, or suitable laboratory or field trial mixtures. Generally, if a test record is used, it will be the same one that was used for computation of the standard deviation. However, if this test record shows either lower or higher average strength than the required average strength, different proportions may be necessary or desirable. In such instances, the average from a record of as few as 10 tests may be used, or the proportions may be established by interpolation between the strengths and proportions of two such records of consecutive tests. All test records for establishing proportions necessary to produce the average strength are to meet the requirements of 5.3.3.1 for "similar materials and conditions."

For strengths over 5000 psi where the average strength documentation is based on laboratory trial mixtures, it may be appropriate to increase f_{cr}' calculated in Table 5.3.2.2 to allow for a reduction in strength from laboratory trials to actual concrete production.

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able provided test records encompass a period of time not less than 45 days. Required concrete proportions shall be permitted to be established by interpolation between the strengths and proportions of two or more test records, each of which meets other requirements of this section.

5.3.3.2 — When an acceptable record of field test results is not available, concrete proportions established from trial mixtures meeting the following restrictions shall be permitted:

(a) Combination of materials shall be those for proposed work;

(b) Trial mixtures having proportions and consistencies required for proposed work shall be made using at least three different water-cementitious materials ratios or cementitious materials contents that will produce a range of strengths encompassing the required average strength f'_{cr};

(c) Trial mixtures shall be designed to produce a slump within ± 0.75 in. of maximum permitted, and for air-entrained concrete, within ± 0.5 percent of maximum allowable air content;

(d) For each water-cementitious materials ratio or cementitious materials content, at least three test cylinders for each test age shall be made and cured in accordance with "Method of Making and Curing Concrete Test Specimens in the Laboratory" (ASTM C 192). Cylinders shall be tested at 28 days or at test age designated for determination of f'_c;

(e) From results of cylinder tests a curve shall be plotted showing the relationship between water-cementitious materials ratio or cementitious materials content and compressive strength at designated test age;

(f) Maximum water-cementitious materials ratio or minimum cementitious materials content for concrete to be used in proposed work shall be that shown by the curve to produce the average strength required by 5.3.2, unless a lower water-cementitious materials ratio or higher strength is required by Chapter 4.

5.4 — Proportioning without field experience or trial mixtures

5.4.1 — If data required by 5.3 are not available, concrete proportions shall be based upon other experience or information, if approved by the registered design professional. The required average compres-

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R5.4 — Proportioning without field experience or trial mixtures

R5.4.1 — When no prior experience (5.3.3.1) or trial mixture data (5.3.3.2) meeting the requirements of these sections is available, other experience may be used only when special permission is given. Because combinations of differ-

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sive strength f'_{cr} of concrete produced with materials similar to those proposed for use shall be at least 1200 psi greater than the specified compressive strength f'_c. This alternative shall not be used for specified compressive strengths greater than 5000 psi.

5.4.2 — Concrete proportioned by this section shall conform to the durability requirements of Chapter 4 and to compressive strength test criteria of 5.6.

5.5 — Average strength reduction

As data become available during construction, it shall be permitted to reduce the amount by which f'_{cr} must exceed the specified value of f'_c, provided:

(a) Thirty or more test results are available and average of test results exceeds that required by 5.3.2.1, using a standard deviation calculated in accordance with 5.3.1.1; or

(b) Fifteen to 29 test results are available and average of test results exceeds that required by 5.3.2.1 using a standard deviation calculated in accordance with 5.3.1.2; and

(c) Special exposure requirements of Chapter 4 are met.

5.6 — Evaluation and acceptance of concrete

5.6.1 — Concrete shall be tested in accordance with the requirements of 5.6.2 through 5.6.5. Qualified field testing technicians shall perform tests on fresh concrete at the job site, prepare specimens required for curing under field conditions, prepare specimens required for testing in the laboratory, and record the temperature of the fresh concrete when preparing specimens for strength tests. Qualified laboratory technicians shall perform all required laboratory tests.

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ent ingredients may vary considerably in strength level, this procedure is not permitted for f'_c greater than 5000 psi and the required average strength should exceed f'_c by 1200 psi. The purpose of this provision is to allow work to continue when there is an unexpected interruption in concrete supply and there is not sufficient time for tests and evaluation or in small structures where the cost of trial mixture data is not justified.

R5.6 — Evaluation and acceptance of concrete

Once the mixture proportions have been selected and the job started, the criteria for evaluation and acceptance of the concrete can be obtained from 5.6.

An effort has been made in the code to provide a clear-cut basis for judging the acceptability of the concrete, as well as to indicate a course of action to be followed when the results of strength tests are not satisfactory.

R5.6.1 — Laboratory and field technicians can establish qualifications by becoming certified through certification programs. Field technicians in charge of sampling concrete; testing for slump, unit weight, yield, air content, and temperature; and making and curing test specimens should be certified in accordance with the requirements of ACI Concrete Field Testing Technician—Grade 1 Certification Program, or the requirements of ASTM C 1077,[5.3] or an equivalent program. Concrete testing laboratory personnel should be certified in accordance with the requirements of ACI Concrete Laboratory Testing Technician, Concrete Strength Testing Technician, or the requirements of ASTM C 1077.

Testing reports should be promptly distributed to the owner, registered design professional responsible for the design, contractor, appropriate subcontractors, appropriate suppliers, and building official to allow timely identification of either compliance or the need for corrective action.

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5.6.2 — Frequency of testing

5.6.2.1 — Samples for strength tests of each class of concrete placed each day shall be taken not less than once a day, nor less than once for each 150 yd^3 of concrete, nor less than once for each 5000 ft^2 of surface area for slabs or walls.

5.6.2.2 — On a given project, if total volume of concrete is such that frequency of testing required by 5.6.2.1 would provide less than five strength tests for a given class of concrete, tests shall be made from at least five randomly selected batches or from each batch if fewer than five batches are used.

5.6.2.3 — When total quantity of a given class of concrete is less than 50 yd^3, strength tests are not required when evidence of satisfactory strength is submitted to and approved by the building official.

5.6.2.4 — A strength test shall be the average of the strengths of two cylinders made from the same sample of concrete and tested at 28 days or at test age designated for determination of f_c'.

5.6.3 — Laboratory-cured specimens

5.6.3.1 — Samples for strength tests shall be taken in accordance with "Method of Sampling Freshly Mixed Concrete" (ASTM C 172).

5.6.3.2 — Cylinders for strength tests shall be molded and laboratory-cured in accordance with "Practice for Making and Curing Concrete Test Specimens in the Field" (ASTM C 31) and tested in accordance with "Test Method for Compressive Strength of Cylindrical Concrete Specimens" (ASTM C 39).

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R5.6.2 — Frequency of testing

R5.6.2.1 — The following three criteria establish the required minimum sampling frequency for each class of concrete:

(a) Once each day a given class is placed, nor less than

(b) Once for each 150 yd^3 of each class placed each day, nor less than

(c) Once for each 5000 ft^2 of slab or wall surface area placed each day.

In calculating surface area, only one side of the slab or wall should be considered. Criteria (c) will require more frequent sampling than once for each 150 yd^3 placed if the average wall or slab thickness is less than 9-3/4 in.

R5.6.2.2 — Samples for strength tests are to be taken on a strictly random basis if they are to measure properly the acceptability of the concrete. To be representative, the choice of times of sampling, or the batches of concrete to be sampled, are to be made on the basis of chance alone, within the period of placement. Batches should not be sampled on the basis of appearance, convenience, or other possibly biased criteria, because the statistical analyses will lose their validity. Not more than one test (average of two cylinders made from a sample, 5.6.2.4) should be taken from a single batch, and water may not be added to the concrete after the sample is taken.

ASTM D 3665[5.4] describes procedures for random selection of the batches to be tested.

R5.6.3 — Laboratory-cured specimens

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5.6.3.3 — Strength level of an individual class of concrete shall be considered satisfactory if both of the following requirements are met:

(a) Every arithmetic average of any three consecutive strength tests equals or exceeds f'_c;

(b) No individual strength test (average of two cylinders) falls below f'_c by more than 500 psi when f'_c is 5000 psi or less; or by more than **0.10**f'_c when f'_c is more than 5000 psi.

5.6.3.4 — If either of the requirements of 5.6.3.3 is not met, steps shall be taken to increase the average of subsequent strength test results. Requirements of 5.6.5 shall be observed if requirement of 5.6.3.3(b) is not met.

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R5.6.3.3 — A single set of criteria is given for acceptability of strength and is applicable to all concrete used in structures designed in accordance with the code, regardless of design method used. The concrete strength is considered to be satisfactory as long as averages of any three consecutive strength tests remain above the specified f'_c and no individual strength test falls below the specified f'_c by more than 500 psi if f'_c is 5000 psi or less, or falls below f'_c by more than 10 percent if f'_c is over 5000 psi. Evaluation and acceptance of the concrete can be judged immediately as test results are received during the course of the work. Strength tests failing to meet these criteria will occur occasionally (probably about once in 100 tests) even though concrete strength and uniformity are satisfactory. Allowance should be made for such statistically expected variations in deciding whether the strength level being produced is adequate.

R5.6.3.4 — When concrete fails to meet either of the strength requirements of 5.6.3.3, steps should be taken to increase the average of the concrete test results. If sufficient concrete has been produced to accumulate at least 15 tests, these should be used to establish a new target average strength as described in 5.3.

If fewer than 15 tests have been made on the class of concrete in question, the new target strength level should be at least as great as the average level used in the initial selection of proportions. If the average of the available tests made on the project equals or exceeds the level used in the initial selection of proportions, a further increase in average level is required.

The steps taken to increase the average level of test results will depend on the particular circumstances, but could include one or more of the following:

(a) An increase in cementitious materials content;

(b) Changes in mixture proportions;

(c) Reductions in or better control of levels of slump supplied;

(d) A reduction in delivery time;

(e) Closer control of air content;

(f) An improvement in the quality of the testing, including strict compliance with standard test procedures.

Such changes in operating and testing procedures, or changes in cementitious materials content, or slump should not require a formal resubmission under the procedures of 5.3; however, important changes in sources of cement, aggregates, or admixtures should be accompanied by evidence that the average strength level will be improved.

Laboratories testing cylinders or cores to determine compliance with these requirements should be accredited or inspected for conformance to the requirement of ASTM C 1077[5.3] by a recognized agency such as the American Associ-

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ation for Laboratory Accreditation (A2LA), AASHTO Materials Reference Laboratory (AMRL), National Voluntary Laboratory Accreditation Program (NVLAP), Cement and Concrete Reference Laboratory (CCRL), or their equivalent.

5.6.4 — Field-cured specimens

5.6.4.1 — If required by the building official, results of strength tests of cylinders cured under field conditions shall be provided.

5.6.4.2 — Field-cured cylinders shall be cured under field conditions in accordance with "Practice for Making and Curing Concrete Test Specimens in the Field" (ASTM C 31).

5.6.4.3 — Field-cured test cylinders shall be molded at the same time and from the same samples as laboratory-cured test cylinders.

5.6.4.4 — Procedures for protecting and curing concrete shall be improved when strength of field-cured cylinders at test age designated for determination of f'_c is less than 85 percent of that of companion laboratory-cured cylinders. The 85 percent limitation shall not apply if field-cured strength exceeds f'_c by more than 500 psi.

R5.6.4 — Field-cured specimens

R5.6.4.1 — Strength tests of cylinders cured under field conditions may be required to check the adequacy of curing and protection of concrete in the structure.

R5.6.4.4 — Positive guidance is provided in the code concerning the interpretation of tests of field-cured cylinders. Research has shown that cylinders protected and cured to simulate good field practice should test not less than about 85 percent of standard laboratory moist-cured cylinders. This percentage has been set as a rational basis for judging the adequacy of field curing. The comparison is made between the actual measured strengths of companion job-cured and laboratory-cured cylinders, not between job-cured cylinders and the specified value of f'_c. However, results for the job-cured cylinders are considered satisfactory if the job-cured cylinders exceed the specified f'_c by more than 500 psi, even though they fail to reach 85 percent of the strength of companion laboratory-cured cylinders.

5.6.5 — Investigation of low-strength test results

5.6.5.1 — If any strength test (see 5.6.2.4) of laboratory-cured cylinders falls below specified value of f'_c by more than the values given in 5.6.3.3(b) or if tests of field-cured cylinders indicate deficiencies in protection and curing (see 5.6.4.4), steps shall be taken to assure that load-carrying capacity of the structure is not jeopardized.

5.6.5.2 — If the likelihood of low-strength concrete is confirmed and calculations indicate that load-carrying capacity is significantly reduced, tests of cores drilled from the area in question in accordance with "Method of Obtaining and Testing Drilled Cores and Sawed Beams of Concrete" (ASTM C 42) shall be permitted. In such cases, three cores shall be taken for each strength test that falls below the values given in 5.6.3.3(b).

R5.6.5 — Investigation of low-strength test results

Instructions are provided concerning the procedure to be followed when strength tests have failed to meet the specified acceptance criteria. For obvious reasons, these instructions cannot be dogmatic. The building official should apply judgment as to the significance of low test results and whether they indicate need for concern. If further investigation is deemed necessary, such investigation may include nondestructive tests, or in extreme cases, strength tests of cores taken from the structure.

Nondestructive tests of the concrete in place, such as by probe penetration, impact hammer, ultrasonic pulse velocity or pull out may be useful in determining whether or not a portion of the structure actually contains low-strength concrete. Such tests are of value primarily for comparisons within the same job rather than as quantitative measures of strength. For cores, if required, conservatively safe acceptance criteria are provided that should ensure structural ade-

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5.6.5.3 — Cores shall be prepared for transport and storage by wiping drilling water from their surfaces and placing the cores in watertight bags or containers immediately after drilling. Cores shall be tested no earlier than 48 h and not later than 7 days after coring unless approved by the registered design professional.

5.6.5.4 — Concrete in an area represented by core tests shall be considered structurally adequate if the average of three cores is equal to at least 85 percent of f'_c and if no single core is less than 75 percent of f'_c. Additional testing of cores extracted from locations represented by erratic core strength results shall be permitted.

5.6.5.5 — If criteria of 5.6.5.4 are not met and if the structural adequacy remains in doubt, the responsible authority shall be permitted to order a strength evaluation in accordance with Chapter 20 for the questionable portion of the structure, or take other appropriate action.

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quacy for virtually any type of construction.[5.5-5.8] Lower strength may, of course, be tolerated under many circumstances, but this again becomes a matter of judgment on the part of the building official and design engineer. When the core tests fail to provide assurance of structural adequacy, it may be practical, particularly in the case of floor or roof systems, for the building official to require a load test (Chapter 20). Short of load tests, if time and conditions permit, an effort may be made to improve the strength of the concrete in place by supplemental wet curing. Effectiveness of such a treatment should be verified by further strength evaluation using procedures previously discussed.

A core obtained through the use of a water-cooled bit results in a moisture gradient between the exterior and interior of the core being created during drilling. This adversely affects the core's compressive strength.[5.9] The restriction on the commencement of core testing provides a minimum time for the moisture gradient to dissipate.

Core tests having an average of 85 percent of the specified strength are realistic. To expect core tests to be equal to f'_c is not realistic, since differences in the size of specimens, conditions of obtaining samples, and procedures for curing, do not permit equal values to be obtained.

The code, as stated, concerns itself with assuring structural safety, and the instructions in 5.6 are aimed at that objective. It is not the function of the code to assign responsibility for strength deficiencies, whether or not they are such as to require corrective measures.

Under the requirements of this section, cores taken to confirm structural adequacy will usually be taken at ages later than those specified for determination of f'_c.

5.7 — Preparation of equipment and place of deposit

5.7.1 — Preparation before concrete placement shall include the following:

(a) All equipment for mixing and transporting concrete shall be clean;

(b) All debris and ice shall be removed from spaces to be occupied by concrete;

(c) Forms shall be properly coated;

(d) Masonry filler units that will be in contact with concrete shall be well drenched;

(e) Reinforcement shall be thoroughly clean of ice or other deleterious coatings;

R5.7 — Preparation of equipment and place of deposit

Recommendations for mixing, handling and transporting, and placing concrete are given in detail in **"Guide for Measuring, Mixing, Transporting, and Placing Concrete"** reported by ACI Committee 304.[5.10] (Presents methods and procedures for control, handling and storage of materials, measurement, batching tolerances, mixing, methods of placing, transporting, and forms.)

Attention is directed to the need for using clean equipment and for cleaning forms and reinforcement thoroughly before beginning to deposit concrete. In particular, sawdust, nails, wood pieces, and other debris that may collect inside the forms should be removed. Reinforcement should be thoroughly cleaned of ice, dirt, loose rust, mill scale, or other coatings. Water should be removed from the forms.

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(f) Water shall be removed from place of deposit before concrete is placed unless a tremie is to be used or unless otherwise permitted by the building official;

(g) All laitance and other unsound material shall be removed before additional concrete is placed against hardened concrete.

5.8 — Mixing

5.8.1 — All concrete shall be mixed until there is a uniform distribution of materials and shall be discharged completely before mixer is recharged.

5.8.2 — Ready-mixed concrete shall be mixed and delivered in accordance with requirements of "Specification for Ready-Mixed Concrete" (ASTM C 94) or "Specification for Concrete Made by Volumetric Batching and Continuous Mixing" (ASTM C 685).

5.8.3 — Job-mixed concrete shall be mixed in accordance with the following:

(a) Mixing shall be done in a batch mixer of approved type;

(b) Mixer shall be rotated at a speed recommended by the manufacturer;

(c) Mixing shall be continued for at least 1-1/2 minutes after all materials are in the drum, unless a shorter time is shown to be satisfactory by the mixing uniformity tests of "Specification for Ready-Mixed Concrete" (ASTM C 94);

(d) Materials handling, batching, and mixing shall conform to applicable provisions of "Specification for Ready-Mixed Concrete" (ASTM C 94);

(e) A detailed record shall be kept to identify:

(1) number of batches produced;

(2) proportions of materials used;

(3) approximate location of final deposit in structure;

(4) time and date of mixing and placing.

5.9 — Conveying

5.9.1 — Concrete shall be conveyed from mixer to place of final deposit by methods that will prevent separation or loss of materials.

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R5.8 — Mixing

Concrete of uniform and satisfactory quality requires the materials to be thoroughly mixed until uniform in appearance and all ingredients are distributed. Samples taken from different portions of a batch should have essentially the same unit weight, air content, slump, and coarse aggregate content. Test methods for uniformity of mixing are given in ASTM C 94. The necessary time of mixing will depend on many factors including batch size, stiffness of the batch, size and grading of the aggregate, and the efficiency of the mixer. Excessively long mixing times should be avoided to guard against grinding of the aggregates.

R5.9 — Conveying

Each step in the handling and transporting of concrete needs to be controlled to maintain uniformity within a batch and from batch to batch. It is essential to avoid segregation of the coarse aggregate from the mortar or of water from the other ingredients.

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5.9.2 — Conveying equipment shall be capable of providing a supply of concrete at site of placement without separation of ingredients and without interruptions sufficient to permit loss of plasticity between successive increments.

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The code requires the equipment for handling and transporting concrete to be capable of supplying concrete to the place of deposit continuously and reliably under all conditions and for all methods of placement. The provisions of 5.9 apply to all placement methods, including pumps, belt conveyors, pneumatic systems, wheelbarrows, buggies, crane buckets, and tremies.

Serious loss in strength can result when concrete is pumped through pipe made of aluminum or aluminum alloy.[5.11] Hydrogen gas generated by the reaction between the cement alkalies and the aluminum eroded from the interior of the pipe surface has been shown to cause strength reduction as much as 50 percent. Hence, equipment made of aluminum or aluminum alloys should not be used for pump lines, tremies, or chutes other than short chutes such as those used to convey concrete from a truck mixer.

5.10 — Depositing

5.10.1 — Concrete shall be deposited as nearly as practical in its final position to avoid segregation due to rehandling or flowing.

5.10.2 — Concreting shall be carried on at such a rate that concrete is at all times plastic and flows readily into spaces between reinforcement.

5.10.3 — Concrete that has partially hardened or been contaminated by foreign materials shall not be deposited in the structure.

5.10.4 — Retempered concrete or concrete that has been remixed after initial set shall not be used unless approved by the engineer.

5.10.5 — After concreting is started, it shall be carried on as a continuous operation until placing of a panel or section, as defined by its boundaries or predetermined joints, is completed except as permitted or prohibited by 6.4.

5.10.6 — Top surfaces of vertically formed lifts shall be generally level.

5.10.7 — When construction joints are required, joints shall be made in accordance with 6.4.

5.10.8 — All concrete shall be thoroughly consolidated by suitable means during placement and shall be thoroughly worked around reinforcement and embedded fixtures and into corners of forms.

R5.10 — Depositing

Rehandling concrete can cause segregation of the materials. Hence the code cautions against this practice. Retempering of partially set concrete with the addition of water should not be permitted, unless authorized. This does not preclude the practice (recognized in ASTM C 94) of adding water to mixed concrete to bring it up to the specified slump range so long as prescribed limits on the maximum mixing time and water-cementitious materials ratio are not violated.

Section 5.10.4 of the 1971 code contained a requirement that “where conditions make consolidation difficult or where reinforcement is congested, batches of mortar containing the same proportions of cement, sand, and water as used in the concrete, shall first be deposited in the forms to a depth of at least 1 in.” That requirement was deleted from the 1977 code since the conditions for which it was applicable could not be defined precisely enough to justify its inclusion as a code requirement. The practice, however, has merit and should be incorporated in job specifications where appropriate, with the specific enforcement the responsibility of the job inspector. The use of mortar batches aids in preventing honeycomb and poor bonding of the concrete with the reinforcement. The mortar should be placed immediately before depositing the concrete and should be plastic (neither stiff nor fluid) when the concrete is placed.

Recommendations for consolidation of concrete are given in detail in **“Guide for Consolidation of Concrete”** reported by ACI Committee 309.[5.12] (Presents current information on the mechanism of consolidation and gives recommendations on equipment characteristics and procedures for various classes of concrete.)

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5.11 — Curing

5.11.1 — Concrete (other than high-early-strength) shall be maintained above 50 F and in a moist condition for at least the first 7 days after placement, except when cured in accordance with 5.11.3.

5.11.2 — High-early-strength concrete shall be maintained above 50 F and in a moist condition for at least the first 3 days, except when cured in accordance with 5.11.3.

5.11.3 — Accelerated curing

5.11.3.1 — Curing by high-pressure steam, steam at atmospheric pressure, heat and moisture, or other accepted processes, shall be permitted to accelerate strength gain and reduce time of curing.

5.11.3.2 — Accelerated curing shall provide a compressive strength of the concrete at the load stage considered at least equal to required design strength at that load stage.

5.11.3.3 — Curing process shall be such as to produce concrete with a durability at least equivalent to the curing method of 5.11.1 or 5.11.2.

5.11.4 — When required by the engineer or architect, supplementary strength tests in accordance with 5.6.4 shall be performed to assure that curing is satisfactory.

COMMENTARY

R5.11 — Curing

Recommendations for curing concrete are given in detail in **"Standard Practice for Curing Concrete"** reported by ACI Committee 308.[5.13] (Presents basic principles of proper curing and describes the various methods, procedures, and materials for curing of concrete.)

R5.11.3 — Accelerated curing

The provisions of this section apply whenever an accelerated curing method is used, whether for precast or cast-in-place elements. The compressive strength of steam-cured concrete is not as high as that of similar concrete continuously cured under moist conditions at moderate temperatures. Also the modulus of elasticity E_c of steam-cured speci-mens may vary from that of specimens moist-cured at normal temperatures. When steam curing is used, it is advisable to base the concrete mixture proportions on steam-cured test cylinders.

Accelerated curing procedures require careful attention to obtain uniform and satisfactory results. Preventing moisture loss during the curing is essential.

R5.11.4 — In addition to requiring a minimum curing temperature and time for normal- and high-early-strength concrete, the code provides a specific criterion in 5.6.4 for judging the adequacy of field curing. At the test age for which the strength is specified (usually 28 days), field-cured cylinders should produce strength not less than 85 percent of that of the standard, laboratory-cured cylinders. For a reasonably valid comparison to be made, field-cured cylinders and companion laboratory-cured cylinders should come from the same sample. Field-cured cylinders should be cured under conditions identical to those of the structure. If the structure is protected from the elements, the cylinder should be protected.

Cylinders related to members not directly exposed to weather should be cured adjacent to those members and provided with the same degree of protection and method of curing. The field cylinders should not be treated more favorably than the elements they represent. (See 5.6.4 for additional information.) If the field-cured cylinders do not provide satisfactory strength by this comparison, measures should be taken to improve the curing. If the tests indicate a possible serious deficiency in strength of concrete in the structure, core tests may be required, with or without supplemental wet curing, to check the structural adequacy, as provided in 5.6.5.

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5.12 — Cold weather requirements

5.12.1 — Adequate equipment shall be provided for heating concrete materials and protecting concrete during freezing or near-freezing weather.

5.12.2 — All concrete materials and all reinforcement, forms, fillers, and ground with which concrete is to come in contact shall be free from frost.

5.12.3 — Frozen materials or materials containing ice shall not be used.

5.13 — Hot weather requirements

During hot weather, proper attention shall be given to ingredients, production methods, handling, placing, protection, and curing to prevent excessive concrete temperatures or water evaporation that could impair required strength or serviceability of the member or structure.

COMMENTARY

R5.12 — Cold weather requirements

Recommendations for cold weather concreting are given in detail in **"Cold Weather Concreting"** reported by ACI Committee 306.[5.14] (Presents requirements and methods for producing satisfactory concrete during cold weather.)

R5.13 — Hot weather requirements

Recommendations for hot weather concreting are given in detail in **"Hot Weather Concreting"** reported by ACI Committee 305.[5.15] (Defines the hot weather factors that effect concrete properties and construction practices and recommends measures to eliminate or minimize the undesirable effects.)

CHAPTER 6 — FORMWORK, EMBEDDED PIPES, AND CONSTRUCTION JOINTS

CODE

6.1 — Design of formwork

6.1.1 — Forms shall result in a final structure that conforms to shapes, lines, and dimensions of the members as required by the design drawings and specifications.

6.1.2 — Forms shall be substantial and sufficiently tight to prevent leakage of mortar.

6.1.3 — Forms shall be properly braced or tied together to maintain position and shape.

6.1.4 — Forms and their supports shall be designed so as not to damage previously placed structure.

6.1.5 — Design of formwork shall include consideration of the following factors:

(a) Rate and method of placing concrete;

(b) Construction loads, including vertical, horizontal, and impact loads;

(c) Special form requirements for construction of shells, folded plates, domes, architectural concrete, or similar types of elements.

6.1.6 — Forms for prestressed concrete members shall be designed and constructed to permit movement of the member without damage during application of prestressing force.

6.2 — Removal of forms, shores, and reshoring

6.2.1 — Removal of forms

Forms shall be removed in such a manner as not to impair safety and serviceability of the structure. Concrete exposed by form removal shall have sufficient strength not to be damaged by removal operation.

6.2.2 — Removal of shores and reshoring

The provisions of 6.2.2.1 through 6.2.2.3 shall apply to slabs and beams except where cast on the ground.

6.2.2.1 — Before starting construction, the contractor shall develop a procedure and schedule for removal of shores and installation of reshores and for calculating the loads transferred to the structure during the process.

COMMENTARY

R6.1 — Design of formwork

Only minimum performance requirements for formwork, necessary to provide for public health and safety, are prescribed in Chapter 6. Formwork for concrete, including proper design, construction, and removal, demands sound judgment and planning to achieve adequate forms that are both economical and safe. Detailed information on formwork for concrete is given in: **"Guide to Formwork for Concrete,"** reported by Committee 347.[6.1] (Provides recommendations for design, construction, and materials for formwork, forms for special structures, and formwork for special methods of construction. Directed primarily to contractors, the suggested criteria will aid engineers and architects in preparing job specifications for the contractors.)

Formwork for Concrete[6.2] prepared under the direction of ACI Committee 347. (A how-to-do-it handbook for contractors, engineers, and architects following the guidelines established in ACI 347R. Planning, building, and using formwork are discussed, including tables, diagrams, and formulas for form design loads.)

R6.2 — Removal of forms, shores, and reshoring

In determining the time for removal of forms, consideration should be given to the construction loads and to the possibilities of deflections.[6.3] The construction loads are frequently at least as great as the specified live loads. At early ages, a structure may be adequate to support the applied loads but may deflect sufficiently to cause permanent damage.

Evaluation of concrete strength during construction may be demonstrated by field-cured test cylinders or other procedures approved by the building official such as:

(a) Tests of cast-in-place cylinders in accordance with "Standard Test Method for Compressive Strength of Concrete Cylinders Cast-in-Place in Cylindrical Molds" (ASTM C 873[6.4]). (This method is limited to use in slabs where the depth of concrete is from 5 to 12 in.);

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(a) The structural analysis and concrete strength data used in planning and implementing form removal and shoring shall be furnished by the contractor to the building official when so requested;

(b) No construction loads shall be supported on, nor any shoring removed from, any part of the structure under construction except when that portion of the structure in combination with remaining forming and shoring system has sufficient strength to support safely its weight and loads placed thereon;

(c) Sufficient strength shall be demonstrated by structural analysis considering proposed loads, strength of forming and shoring system, and concrete strength data. Concrete strength data shall be based on tests of field-cured cylinders or, when approved by the building official, on other procedures to evaluate concrete strength.

6.2.2.2 — No construction loads exceeding the combination of superimposed dead load plus specified live load shall be supported on any unshored portion of the structure under construction, unless analysis indicates adequate strength to support such additional loads.

6.2.2.3 — Form supports for prestressed concrete members shall not be removed until sufficient prestressing has been applied to enable prestressed members to carry their dead load and anticipated construction loads.

COMMENTARY

(b) Penetration resistance in accordance with "Standard Test Method for Penetration Resistance of Hardened Concrete" (ASTM C 803[6.5]);

(c) Pullout strength in accordance with "Standard Test Method for Pullout Strength of Hardened Concrete" (ASTM C 900[6.6]);

(d) Maturity factor measurements and correlation in accordance with ASTM C 1074.[6.7]

Procedures (b), (c), and (d) require sufficient data, using job materials, to demonstrate correlation of measurements on the structure with compressive strength of molded cylinders or drilled cores.

Where the structure is adequately supported on shores, the side forms of beams, girders, columns, walls, and similar vertical forms may generally be removed after 12 h of cumulative curing time, provided the side forms support no loads other than the lateral pressure of the plastic concrete. Cumulative curing time represents the sum of time intervals, not necessarily consecutive, during which the temperature of the air surrounding the concrete is above 50 F. The 12-h cumulative curing time is based on regular cements and ordinary conditions; the use of special cements or unusual conditions may require adjustment of the given limits. For example, concrete made with Type II or V (ASTM C 150) or ASTM C 595 cements, concrete containing retarding admixtures, and concrete to which ice was added during mixing (to lower the temperature of fresh concrete) may not have sufficient strength in 12 h and should be investigated before removal of formwork.

The removal of formwork for multistory construction should be a part of a planned procedure considering the temporary support of the whole structure as well as that of each individual member. Such a procedure should be worked out prior to construction and should be based on a structural analysis taking into account the following items, as a minimum:

(a) The structural system that exists at the various stages of construction and the construction loads corresponding to those stages;

(b) The strength of the concrete at the various ages during construction;

(c) The influence of deformations of the structure and shoring system on the distribution of dead loads and construction loads during the various stages of construction;

(d) The strength and spacing of shores or shoring systems used, as well as the method of shoring, bracing, shore removal, and reshoring including the minimum time

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intervals between the various operations;

(e) Any other loading or condition that affects the safety or serviceability of the structure during construction.

For multistory construction, the strength of the concrete during the various stages of construction should be substantiated by field-cured test specimens or other approved methods.

6.3 — Conduits and pipes embedded in concrete

R6.3 — Conduits and pipes embedded in concrete

6.3.1 — Conduits, pipes, and sleeves of any material not harmful to concrete and within limitations of 6.3 shall be permitted to be embedded in concrete with approval of the engineer, provided they are not considered to replace structurally the displaced concrete, except as provided in 6.3.6.

R6.3.1 — Conduits, pipes, and sleeves not harmful to concrete can be embedded within the concrete, but the work should be done in such a manner that the structure will not be endangered. Empirical rules are given in 6.3 for safe installations under common conditions; for other than common conditions, special designs should be made. Many general building codes have adopted ANSI/ASME piping codes B 31.1 for power piping[6.8] and B 31.3 for chemical and petroleum piping.[6.9] The specifier should be sure that the appropriate piping codes are used in the design and testing of the system. The contractor should not be permitted to install conduits, pipes, ducts, or sleeves that are not shown on the plans or not approved by the engineer or architect.

For the integrity of the structure, it is important that all conduit and pipe fittings within the concrete be carefully assembled as shown on the plans or called for in the job specifications.

6.3.2 — Conduits and pipes of aluminum shall not be embedded in structural concrete unless effectively coated or covered to prevent aluminum-concrete reaction or electrolytic action between aluminum and steel.

R6.3.2 — The code prohibits the use of aluminum in structural concrete unless it is effectively coated or covered. Aluminum reacts with concrete and, in the presence of chloride ions, may also react electrolytically with steel, causing cracking and/or spalling of the concrete. Aluminum electrical conduits present a special problem since stray electric current accelerates the adverse reaction.

6.3.3 — Conduits, pipes, and sleeves passing through a slab, wall, or beam shall not impair significantly the strength of the construction.

6.3.4 — Conduits and pipes, with their fittings, embedded within a column shall not displace more than 4 percent of the area of cross section on which strength is calculated or which is required for fire protection.

6.3.5 — Except when drawings for conduits and pipes are approved by the structural engineer, conduits and pipes embedded within a slab, wall, or beam (other than those merely passing through) shall satisfy 6.3.5.1 through 6.3.5.3.

6.3.5.1 — They shall not be larger in outside dimension than 1/3 the overall thickness of slab, wall, or beam in which they are embedded.

CODE

6.3.5.2 — They shall not be spaced closer than 3 diameters or widths on center.

6.3.5.3 — They shall not impair significantly the strength of the construction.

6.3.6 — Conduits, pipes, and sleeves shall be permitted to be considered as replacing structurally in compression the displaced concrete provided in 6.3.6.1 through 6.3.6.3.

6.3.6.1 — They are not exposed to rusting or other deterioration.

6.3.6.2 — They are of uncoated or galvanized iron or steel not thinner than standard Schedule 40 steel pipe.

6.3.6.3 — They have a nominal inside diameter not over 2 in. and are spaced not less than 3 diameters on centers.

6.3.7 — Pipes and fittings shall be designed to resist effects of the material, pressure, and temperature to which they will be subjected.

6.3.8 — No liquid, gas, or vapor, except water not exceeding 90 F nor 50 psi pressure, shall be placed in the pipes until the concrete has attained its design strength.

6.3.9 — In solid slabs, piping, unless it is for radiant heating or snow melting, shall be placed between top and bottom reinforcement.

6.3.10 — Concrete cover for pipes, conduits, and fittings shall not be less than 1-1/2 in. for concrete exposed to earth or weather, nor less than 3/4 in. for concrete not exposed to weather or in contact with ground.

6.3.11 — Reinforcement with an area not less than 0.002 times area of concrete section shall be provided normal to piping.

6.3.12 — Piping and conduit shall be so fabricated and installed that cutting, bending, or displacement of reinforcement from its proper location will not be required.

6.4 — Construction joints

6.4.1 — Surface of concrete construction joints shall be cleaned and laitance removed.

COMMENTARY

R6.3.7 — The 1983 code limited the maximum pressure in embedded pipe to 200 psi, which was considered too restrictive. Nevertheless, the effects of such pressures and the expansion of embedded pipe should be considered in the design of the concrete member.

R6.4 — Construction joints

For the integrity of the structure, it is important that all construction joints be defined in construction documents and constructed as required. Any deviations should be approved by the engineer or architect.

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6.4.2 — Immediately before new concrete is placed, all construction joints shall be wetted and standing water removed.

6.4.3 — Construction joints shall be so made and located as not to impair the strength of the structure. Provision shall be made for transfer of shear and other forces through construction joints. See 11.7.9.

6.4.4 — Construction joints in floors shall be located within the middle third of spans of slabs, beams, and girders. Joints in girders shall be offset a minimum distance of two times the width of intersecting beams.

6.4.5 — Beams, girders, or slabs supported by columns or walls shall not be cast or erected until concrete in the vertical support members is no longer plastic.

6.4.6 — Beams, girders, haunches, drop panels, and capitals shall be placed monolithically as part of a slab system, unless otherwise shown in design drawings or specifications.

COMMENTARY

R6.4.2 — The requirements of the 1977 code for the use of neat cement on vertical joints have been removed, since it is rarely practical and can be detrimental where deep forms and steel congestion prevent proper access. Often wet blasting and other procedures are more appropriate. Because the code sets only minimum standards, the engineer may have to specify special procedures if conditions warrant. The degree to which mortar batches are needed at the start of concrete placement depend on concrete proportions, congestion of steel, vibrator access, and other factors.

R6.4.3 — Construction joints should be located where they will cause the least weakness in the structure. When shear due to gravity load is not significant, as is usually the case in the middle of the span of flexural members, a simple vertical joint may be adequate. Lateral force design may require special design treatment of construction joints. Shear keys, intermittent shear keys, diagonal dowels, or the shear transfer method of 11.7 may be used whenever a force transfer is required.

R6.4.5 — Delay in placing concrete in members supported by columns and walls is necessary to prevent cracking at the interface of the slab and supporting member caused by bleeding and settlement of plastic concrete in the supporting member.

R6.4.6 — Separate placement of slabs and beams, haunches, and similar elements is permitted when shown on the drawings and where provision has been made to transfer forces as required in 6.4.3.

CODE

COMMENTARY

Notes

CHAPTER 7 — DETAILS OF REINFORCEMENT

CODE

7.0 — Notation

d = distance from extreme compression fiber to centroid of tension reinforcement, in.
d_b = nominal diameter of bar, wire, or prestressing strand, in.
f'_{ci} = compressive strength of concrete at time of initial prestress, psi
f_y = specified yield strength of nonprestressed reinforcement, psi
ℓ_d = development length, in. See Chapter 12

7.1 — Standard hooks

The term standard hook as used in this code shall mean one of the following:

7.1.1 — 180-deg bend plus $4d_b$ extension, but not less than 2-1/2 in. at free end of bar.

7.1.2 — 90-deg bend plus $12d_b$ extension at free end of bar.

7.1.3 — For stirrup and tie hooks

(a) No. 5 bar and smaller, 90-deg bend plus $6d_b$ extension at free end of bar; or

(b) No. 6, No. 7, and No. 8 bar, 90-deg bend plus $12d_b$ extension at free end of bar; or

(c) No. 8 bar and smaller, 135-deg bend plus $6d_b$ extension at free end of bar.

7.1.4 — Seismic hooks as defined in 21.1.

7.2 — Minimum bend diameters

7.2.1 — Diameter of bend measured on the inside of the bar, other than for stirrups and ties in sizes No. 3 through No. 5, shall not be less than the values in Table 7.2.

COMMENTARY

Recommended methods and standards for preparing design drawings, typical details, and drawings for the fabrication and placing of reinforcing steel in reinforced concrete structures are given in the ***ACI Detailing Manual***, reported by ACI Committee 315.[7.1]

All provisions in the code relating to bar, wire, or strand diameter (and area) are based on the nominal dimensions of the reinforcement as given in the appropriate ASTM specification. Nominal dimensions are equivalent to those of a circular area having the same weight per foot as the ASTM designated bar, wire, or strand sizes. Cross-sectional area of reinforcement is based on nominal dimensions.

R7.1 — Standard hooks

R7.1.3 — Standard stirrup and tie hooks are limited to No. 8 bars and smaller, and the 90-deg hook with $6d_b$ extension is further limited to No. 5 bars and smaller, in both cases as the result of research showing that larger bar sizes with 90-deg hooks and $6d_b$ extensions tend to pop out under high load.

R7.2 — Minimum bend diameters

Standard bends in reinforcing bars are described in terms of the inside diameter of bend since this is easier to measure than the radius of bend. The primary factors affecting the minimum bend diameter are feasibility of bending without breakage and avoidance of crushing the concrete inside the bend.

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7.2.2 — Inside diameter of bend for stirrups and ties shall not be less than $4d_b$ for No. 5 bar and smaller. For bars larger than No. 5, diameter of bend shall be in accordance with Table 7.2.

7.2.3 — Inside diameter of bend in welded wire fabric (plain or deformed) for stirrups and ties shall not be less than $4d_b$ for deformed wire larger than D6 and $2d_b$ for all other wires. Bends with inside diameter of less than $8d_b$ shall not be less than $4d_b$ from nearest welded intersection.

TABLE 7.2—MINIMUM DIAMETERS OF BEND

Bar size	Minimum diameter
No. 3 through No. 8	$6d_b$
No. 9, No. 10, and No. 11	$8d_b$
No. 14 and No. 18	$10d_b$

7.3 — Bending

7.3.1 — All reinforcement shall be bent cold, unless otherwise permitted by the engineer.

7.3.2 — Reinforcement partially embedded in concrete shall not be field bent, except as shown on the design drawings or permitted by the engineer.

COMMENTARY

R7.2.2 — The minimum $4d_b$ bend for the bar sizes commonly used for stirrups and ties is based on accepted industry practice in the United States. Use of a stirrup bar size not greater than No. 5 for either the 90-deg or 135-deg standard stirrup hook will permit multiple bending on standard stirrup bending equipment.

R7.2.3 — Welded wire fabric, of plain or deformed wire, can be used for stirrups and ties. The wire at welded intersections does not have the same uniform ductility and bendability as in areas which were not heated. These effects of the welding temperature are usually dissipated in a distance of approximately four wire diameters. Minimum bend diameters permitted are in most cases the same as those required in the ASTM bend tests for wire material (ASTM A 82 and A 496).

R7.3 — Bending

R7.3.1 — The engineer may be the design engineer or architect or the engineer or architect employed by the owner to perform inspection. For unusual bends with inside diameters less than ASTM bend test requirements, special fabrication may be required.

R7.3.2 — Construction conditions may make it necessary to bend bars that have been embedded in concrete. Such field bending should not be done without authorization of the engineer. The engineer should determine whether the bars should be bent cold or if heating should be used. Bends should be gradual and should be straightened as required.

Tests[7.2,7.3] have shown that A 615 Grade 40 and Grade 60 reinforcing bars can be cold bent and straightened up to 90 deg at or near the minimum diameter specified in 7.2. If cracking or breakage is encountered, heating to a maximum temperature of 1500 F may avoid this condition for the remainder of the bars. Bars that fracture during bending or straightening can be spliced outside the bend region.

Heating should be performed in a manner that will avoid damage to the concrete. If the bend area is within approximately 6 in. of the concrete, some protective insulation may need to be applied. Heating of the bar should be controlled by temperature-indicating crayons or other suitable means. The heated bars should not be artificially cooled (with water or forced air) until after cooling to at least 600 F.

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7.4 — Surface conditions of reinforcement

7.4.1—At the time concrete is placed, reinforcement shall be free from mud, oil, or other nonmetallic coatings that decrease bond. Epoxy coating of steel reinforcement in accordance with standards referenced in 3.5.3.7 and 3.5.3.8 shall be permitted.

7.4.2 — Except for prestressing steel, steel reinforcement with rust, mill scale, or a combination of both shall be considered satisfactory, provided the minimum dimensions (including height of deformations) and weight of a hand-wire-brushed test specimen comply with applicable ASTM specifications referenced in 3.5.

7.4.3 — Prestressing steel shall be clean and free of oil, dirt, scale, pitting and excessive rust. A light coating of rust shall be permitted.

7.5 — Placing reinforcement

7.5.1 — Reinforcement, including tendons, and post-tensioning ducts shall be accurately placed and adequately supported before concrete is placed, and shall be secured against displacement within tolerances permitted in 7.5.2.

7.5.2 — Unless otherwise specified by the registered design professional, reinforcement, including tendons, and post-tensioning ducts shall be placed within the tolerances in 7.5.2.1 and 7.5.2.2.

COMMENTARY

R7.4 — Surface conditions of reinforcement

Specific limits on rust are based on tests,[7.4] plus a review of earlier tests and recommendations. Reference 7.4 provides guidance with regard to the effects of rust and mill scale on bond characteristics of deformed reinforcing bars. Research has shown that a normal amount of rust increases bond. Normal rough handling generally removes rust that is loose enough to injure the bond between the concrete and reinforcement.

R7.4.3 — Guidance for evaluating the degree of rusting on strand is given in Reference 7.5.

R7.5 — Placing reinforcement

R7.5.1 — Reinforcement, including tendons, and post-tensioning ducts should be adequately supported in the forms to prevent displacement by concrete placement or workers. Beam stirrups should be supported on the bottom form of the beam by positive supports such as continuous longitudinal beam bolsters. If only the longitudinal beam bottom reinforcement is supported, construction traffic can dislodge the stirrups as well as any prestressing tendons tied to the stirrups.

R7.5.2 — Generally accepted practice, as reflected in **"Standard Specifications for Tolerances for Concrete Construction and Materials,"** reported by ACI Committee 117.[7.6] has established tolerances on total depth (formwork or finish) and fabrication of truss bent reinforcing bars and closed ties, stirrups, and spirals. The engineer should specify more restrictive tolerances than those permitted by the code when necessary to minimize the accumulation of tolerances resulting in excessive reduction in effective depth or cover.

More restrictive tolerances have been placed on minimum clear distance to formed soffits because of its importance for durability and fire protection, and because bars are usually supported in such a manner that the specified tolerance is practical.

More restrictive tolerances than those required by the code may be desirable for prestressed concrete to achieve camber control within limits acceptable to the designer or owner. In such cases, the engineer should specify the necessary tolerances. Recommendations are given in Reference 7.7.

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7.5.2.1 — Tolerance for depth ***d***, and minimum concrete cover in flexural members, walls and compression members shall be as follows:

	Tolerance on ***d***	Tolerance on minimum concrete cover
d ≤ 8 in.	±3/8 in.	–3/8 in.
d > 8 in.	±1/2 in.	–1/2 in.

except that tolerance for the clear distance to formed soffits shall be minus 1/4 in. and tolerance for cover shall not exceed minus 1/3 the minimum concrete cover required in the design drawings or specifications.

7.5.2.2 — Tolerance for longitudinal location of bends and ends of reinforcement shall be ± 2 in., except the tolerance shall be ± 1/2 in. at the discontinuous ends of brackets and corbels, and ± 1 in. at the discontinuous ends of other members. The tolerance for minimum concrete cover of 7.5.2.1 shall also apply at discontinuous ends of members.

7.5.3 — Welded wire fabric (with wire size not greater than W5 or D5) used in slabs not exceeding 10 ft in span shall be permitted to be curved from a point near the top of slab over the support to a point near the bottom of slab at midspan, provided such reinforcement is either continuous over, or securely anchored at support.

7.5.4 — Welding of crossing bars shall not be permitted for assembly of reinforcement unless authorized by the engineer.

7.6 — Spacing limits for reinforcement

7.6.1 — The minimum clear spacing between parallel bars in a layer shall be d_b, but not less than 1 in. See also 3.3.2.

7.6.2 — Where parallel reinforcement is placed in two or more layers, bars in the upper layers shall be placed directly above bars in the bottom layer with clear distance between layers not less than 1 in.

7.6.3 — In spirally reinforced or tied reinforced compression members, clear distance between longitudinal bars shall be not less than $1.5d_b$ nor less than 1-1/2 in. See also 3.3.2.

7.6.4 — Clear distance limitation between bars shall apply also to the clear distance between a contact lap splice and adjacent splices or bars.

COMMENTARY

R7.5.2.1 — The code specifies a tolerance on depth ***d***, an essential component of strength of the member. Because reinforcing steel is placed with respect to edges of members and formwork surfaces, the depth ***d*** is not always conveniently measured in the field. Engineers should specify tolerances for bar placement, cover, and member size. See ACI 117.[7.6]

R7.5.4 — "Tack" welding (welding crossing bars) can seriously weaken a bar at the point welded by creating a metallurgical notch effect. This operation can be performed safely only when the material welded and welding operations are under continuous competent control, as in the manufacture of welded wire fabric.

R7.6 — Spacing limits for reinforcement

Although the minimum bar spacings are unchanged in this code, the development lengths given in Chapter 12 became a function of the bar spacings since the 1989 code. As a result, it may be desirable to use larger than minimum bar spacings in some cases. The minimum limits were originally established to permit concrete to flow readily into spaces between bars and between bars and forms without honeycomb, and to ensure against concentration of bars on a line that may cause shear or shrinkage cracking. Use of nominal bar diameter to define minimum spacing permits a uniform criterion for all bar sizes.

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7.6.5 — In walls and slabs other than concrete joist construction, primary flexural reinforcement shall not be spaced farther apart than three times the wall or slab thickness, nor farther apart than 18 in.

7.6.6 — Bundled bars

7.6.6.1 — Groups of parallel reinforcing bars bundled in contact to act as a unit shall be limited to four in any one bundle.

7.6.6.2 — Bundled bars shall be enclosed within stirrups or ties.

7.6.6.3 — Bars larger than No. 11 shall not be bundled in beams.

7.6.6.4 — Individual bars within a bundle terminated within the span of flexural members shall terminate at different points with at least $40d_b$ stagger.

7.6.6.5 — Where spacing limitations and minimum concrete cover are based on bar diameter d_b, a unit of bundled bars shall be treated as a single bar of a diameter derived from the equivalent total area.

7.6.7 — Tendons and ducts

7.6.7.1 — Center-to-center spacing of pretensioning tendons at each end of a member shall be not less than $4d_b$ for strands, or $5d_b$ for wire, except that if concrete strength at transfer of prestress, f_{ci}, is 4000 psi or more, minimum center to center spacing of strands shall be 1-3/4 in. for strands of 1/2 in. nominal diameter or smaller and 2 in. for strands of 0.6 in. nominal diameter. See also 3.3.2. Closer vertical spacing and bundling of tendons shall be permitted in the middle portion of a span.

7.6.7.2 — Bundling of post-tensioning ducts shall be permitted if shown that concrete can be satisfactorily placed and if provision is made to prevent the prestressing steel, when tensioned, from breaking through the duct.

COMMENTARY

R7.6.6 — Bundled bars

Bond research[7.8] showed that bar cutoffs within bundles should be staggered. Bundled bars should be tied, wired, or otherwise fastened together to ensure remaining in position whether vertical or horizontal.

A limitation that bars larger than No. 11 not be bundled in beams or girders is a practical limit for application to building size members. (The "**Standard Specifications for Highway Bridges**"[7.9] permits two-bar bundles for No. 14 and No. 18 bars in bridge girders.) Conformance to the crack control requirements of 10.6 will effectively preclude bundling of bars larger than No. 11 as tensile reinforcement. The code phrasing "bundled in contact to act as a unit," is intended to preclude bundling more than two bars in the same plane. Typical bundle shapes are triangular, square, or L-shaped patterns for three- or four-bar bundles. As a practical caution, bundles more than one bar deep in the plane of bending should not be hooked or bent as a unit. Where end hooks are required, it is preferable to stagger the individual bar hooks within a bundle.

R7.6.7 — Tendons and ducts

7.6.7.1 — The allowed decreased spacing in this section for transfer strengths of 4000 psi or greater is based on Reference 7.10, 7.11.

R7.6.7.2 — When ducts for prestressing steel in a beam are arranged closely together vertically, provision should be made to prevent the prestressing steel from breaking through the duct when tensioned. Horizontal disposition of ducts should allow proper placement of concrete. A clear spacing of one and one-third times the size of the coarse aggregate, but not less than 1 in., has proven satisfactory. Where concentration of tendons or ducts tends to create a weakened plane in the concrete cover, reinforcement should be provided to control cracking.

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7.7 — Concrete protection for reinforcement

7.7.1 — Cast-in-place concrete (nonprestressed)

The following minimum concrete cover shall be provided for reinforcement, but shall not be less than required by 7.7.5 and 7.7.7:

	Minimum cover, in.
(a) Concrete cast against and permanently exposed to earth	3
(b) Concrete exposed to earth or weather:	
No. 6 through No. 18 bars	2
No. 5 bar, W31 or D31 wire, and smaller	1-1/2
(c) Concrete not exposed to weather or in contact with ground:	
Slabs, walls, joists:	
No. 14 and No. 18 bars	1-1/2
No. 11 bar and smaller	3/4
Beams, columns:	
Primary reinforcement, ties, stirrups, spirals	1-1/2
Shells, folded plate members:	
No. 6 bar and larger	3/4
No. 5 bar, W31 or D31 wire, and smaller	1/2

7.7.2 — Cast-in-place concrete (prestressed)

The following minimum concrete cover shall be provided for prestressed and nonprestressed reinforcement, ducts, and end fittings, but shall not be less than required by 7.7.5, 7.7.5.1, and 7.7.7:

	Minimum cover, in.
(a) Concrete cast against and permanently exposed to earth	3
(b) Concrete exposed to earth or weather:	
Wall panels, slabs, joists	1
Other members	1-1/2
(c) Concrete not exposed to weather or in contact with ground:	
Slabs, walls, joists	3/4
Beams, columns:	
Primary reinforcement	1-1/2
Ties, stirrups, spirals	1

COMMENTARY

R7.7 — Concrete protection for reinforcement

Concrete cover as protection of reinforcement against weather and other effects is measured from the concrete surface to the outermost surface of the steel to which the cover requirement applies. Where minimum cover is prescribed for a class of structural member, it is measured to the outer edge of stirrups, ties, or spirals if transverse reinforcement encloses main bars; to the outermost layer of bars if more than one layer is used without stirrups or ties; or to the metal end fitting or duct on post-tensioned prestressing steel.

The condition "concrete surfaces exposed to earth or weather" refers to direct exposure to moisture changes and not just to temperature changes. Slab or thin shell soffits are not usually considered directly exposed unless subject to alternate wetting and drying, including that due to condensation conditions or direct leakage from exposed top surface, run off, or similar effects.

Alternative methods of protecting the reinforcement from weather may be provided if they are equivalent to the additional concrete cover required by the code. When approved by the building official under the provisions of 1.4, reinforcement with alternative protection from the weather may have concrete cover not less than the cover required for reinforcement not exposed to weather.

The development length given in Chapter 12 is now a function of the bar cover. As a result, it may be desirable to use larger than minimum cover in some cases.

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Shells, folded plate members:
No. 5 bar, W31 or D31 wire, and smaller 3/8
Other reinforcement .. d_b but not less than 3/4

7.7.3 — Precast concrete (manufactured under plant control conditions)

The following minimum concrete cover shall be provided for prestressed and nonprestressed reinforcement, ducts, and end fittings, but shall not be less than required by 7.7.5, 7.7.5.1, and 7.7.7:

	Minimum cover, in.
(a) Concrete exposed to earth or weather:	
Wall panels:	
No. 14 and No. 18 bars, prestressing tendons larger than 1-1/2 in. diameter	1-1/2
No. 11 bar and smaller, prestressing tendons 1-1/2 in. diameter and smaller, W31 and D31 wire and smaller	3/4
Other members:	
No. 14 and No. 18 bars, prestressing tendons larger than 1-1/2 in. diameter	2
No. 6 through No. 11 bars, prestressing tendons larger than 5/8 in. diameter through 1-1/2 in. diameter	1-1/2
No. 5 bar and smaller, prestressing tendons 5/8 in. diameter and smaller, W31 and D31 wire, and smaller	1-1/4
(b) Concrete not exposed to weather or in contact with ground:	
Slabs, walls, joists:	
No. 14 and No. 18 bars, prestressing tendons larger than 1-1/2 in. diameter	1-1/4
Prestressing tendons 1-1/2 in. diameter and smaller	3/4
No. 11 bar and smaller, W31 or D31 wire, and smaller	5/8
Beams, columns:	
Primary reinforcement ..... d_b but not less than 5/8 and need not exceed 1-1/2	
Ties, stirrups, spirals	3/8
Shells, folded plate members:	
Prestressing tendons	3/4
No. 6 bar and larger	5/8
No. 5 bar and smaller, W31 or D31 wire, and smaller	3/8

COMMENTARY

R7.7.3 — Precast concrete (manufactured under plant control conditions)

The lesser cover thicknesses for precast construction reflect the greater convenience of control for proportioning, placing, and curing inherent in precasting. The term "manufactured under plant control conditions" does not specifically imply that precast members should be manufactured in a plant. Structural elements precast at the job site will also qualify under this section if the control of form dimensions, placing of reinforcement, quality control of concrete, and curing procedure are equal to that normally expected in a plant.

Concrete cover to pretensioned strand as described in this section is intended to provide minimum protection against weather and other effects. Such cover may not be sufficient to transfer or develop the stress in the strand, and it may be necessary to increase the cover accordingly.

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7.7.4 — Bundled bars

For bundled bars, minimum concrete cover shall be equal to the equivalent diameter of the bundle, but need not be greater than 2 in.; except for concrete cast against and permanently exposed to earth, where minimum cover shall be 3 in.

7.7.5— Corrosive environments

In corrosive environments or other severe exposure conditions, amount of concrete protection shall be suitably increased, and denseness and nonporosity of protecting concrete shall be considered, or other protection shall be provided.

7.7.5.1 — For prestressed concrete members exposed to corrosive environments or other severe exposure conditions, and which are classified as Class T or C in 18.3.3, minimum cover to the prestressed reinforcement shall be increased 50 percent. This requirement shall be permitted to be waived if the precompressed tensile zone is not in tension under sustained loads.

7.7.6 — Future extensions

Exposed reinforcement, inserts, and plates intended for bonding with future extensions shall be protected from corrosion.

7.7.7 — Fire protection

When the general building code (of which this code forms a part) requires a thickness of cover for fire protection greater than the minimum concrete cover specified in 7.7, such greater thicknesses shall be used.

7.8 — Special reinforcement details for columns

7.8.1 — Offset bars

Offset bent longitudinal bars shall conform to the following:

7.8.1.1 — Slope of inclined portion of an offset bar with axis of column shall not exceed 1 in 6.

7.8.1.2 — Portions of bar above and below an offset shall be parallel to axis of column.

COMMENTARY

R7.7.5 — Corrosive environments

Where concrete will be exposed to external sources of chlorides in service, such as deicing salts, brackish water, seawater, or spray from these sources, concrete should be proportioned to satisfy the special exposure requirements of Chapter 4. These include minimum air content, maximum water-cementitious materials ratio, minimum strength for normal weight and lightweight concrete, maximum chloride ion in concrete, and cement type. Additionally, for corrosion protection, a minimum concrete cover for reinforcement of 2 in. for walls and slabs and 2-1/2 in. for other members is recommended. For precast concrete manufactured under plant control conditions, a minimum cover of 1-1/2 and 2 in., respectively, is recommended.

R7.7.5.1 — Corrosive environments are defined in sections 4.4.2 and R4.4.2. Additional information on corrosion in parking structures is given in ACI 362.1R-97,[7.12] "Design of Parking Structures," pp. 21-26.

R7.8 — Special reinforcement details for columns

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7.8.1.3 — Horizontal support at offset bends shall be provided by lateral ties, spirals, or parts of the floor construction. Horizontal support provided shall be designed to resist 1-1/2 times the horizontal component of the computed force in the inclined portion of an offset bar. Lateral ties or spirals, if used, shall be placed not more than 6 in. from points of bend.

7.8.1.4 — Offset bars shall be bent before placement in the forms. See 7.3.

7.8.1.5 — Where a column face is offset 3 in. or greater, longitudinal bars shall not be offset bent. Separate dowels, lap spliced with the longitudinal bars adjacent to the offset column faces, shall be provided. Lap splices shall conform to 12.17.

7.8.2 — Steel cores

Load transfer in structural steel cores of composite compression members shall be provided by the following:

7.8.2.1 — Ends of structural steel cores shall be accurately finished to bear at end bearing splices, with positive provision for alignment of one core above the other in concentric contact.

7.8.2.2 — At end bearing splices, bearing shall be considered effective to transfer not more than 50 percent of the total compressive stress in the steel core.

7.8.2.3 — Transfer of stress between column base and footing shall be designed in accordance with 15.8.

7.8.2.4 — Base of structural steel section shall be designed to transfer the total load from the entire composite member to the footing; or, the base shall be designed to transfer the load from the steel core only, provided ample concrete section is available for transfer of the portion of the total load carried by the reinforced concrete section to the footing by compression in the concrete and by reinforcement.

7.9 — Connections

7.9.1 — At connections of principal framing elements (such as beams and columns), enclosure shall be provided for splices of continuing reinforcement and for anchorage of reinforcement terminating in such connections.

7.9.2 — Enclosure at connections shall consist of external concrete or internal closed ties, spirals, or stirrups.

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R7.8.2 — Steel cores

The 50 percent limit on transfer of compressive load by end bearing on ends of structural steel cores is intended to provide some tensile capacity at such splices (up to 50 percent), since the remainder of the total compressive stress in the steel core are to be transmitted by dowels, splice plates, welds, etc. This provision should ensure that splices in composite compression members meet essentially the same tensile capacity as required for conventionally reinforced concrete compression members.

R7.9 — Connections

Confinement is essential at connections to ensure that the flexural capacity of the members can be developed without deterioration of the joint under repeated loadings.[7.13,7.14]

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7.10 — Lateral reinforcement for compression members

7.10.1 — Lateral reinforcement for compression members shall conform to the provisions of 7.10.4 and 7.10.5 and, where shear or torsion reinforcement is required, shall also conform to provisions of Chapter 11.

7.10.2 — Lateral reinforcement requirements for composite compression members shall conform to 10.16. Lateral reinforcement requirements for tendons shall conform to 18.11.

7.10.3 — It shall be permitted to waive the lateral reinforcement requirements of 7.10, 10.16, and 18.11 where tests and structural analysis show adequate strength and feasibility of construction.

7.10.4 — Spirals

Spiral reinforcement for compression members shall conform to 10.9.3 and to the following:

7.10.4.1 — Spirals shall consist of evenly spaced continuous bar or wire of such size and so assembled to permit handling and placing without distortion from designed dimensions.

7.10.4.2 — For cast-in-place construction, size of spirals shall not be less than 3/8 in. diameter.

7.10.4.3 — Clear spacing between spirals shall not exceed 3 in., nor be less than 1 in. See also 3.3.2.

7.10.4.4 — Anchorage of spiral reinforcement shall be provided by 1-1/2 extra turns of spiral bar or wire at each end of a spiral unit.

7.10.4.5 — Spiral reinforcement shall be spliced, if needed, by any one of the following methods:

(a) Lap splices not less than the larger of 12 in. and the length indicated in one of (1) through (5) below:

(1) deformed uncoated bar or wire..... $48d_b$
(2) plain uncoated bar or wire ..... $72d_b$
(3) epoxy-coated deformed bar or wire... $72d_b$
(4) plain uncoated bar or wire with a standard stirrup or tie hook in accordance with 7.1.3 at ends of lapped spiral reinforcement.

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R7.10 — Lateral reinforcement for compression members

R7.10.3 — Precast columns with cover less than 1-1/2 in., prestressed columns without longitudinal bars, columns smaller than minimum dimensions prescribed in earlier code editions, columns of concrete with small size coarse aggregate, wall-like columns, and other special cases may require special designs for lateral reinforcement. Plain or deformed wire, W4, D4, or larger, may be used for ties or spirals. If such special columns are considered as spiral columns for load strength in design, the ratio of spiral reinforcement r_s is to conform to 10.9.3.

R7.10.4 — Spirals

For practical considerations in cast-in-place construction, the minimum diameter of spiral reinforcement is 3/8 in. (3/8"f, No. 3 bar, or W11 or D11 wire). This is the smallest size that can be used in a column with 1-1/2 in. or more cover and having concrete strengths of 3000 psi or more if the minimum clear spacing for placing concrete is to be maintained.

Standard spiral sizes are 3/8, 1/2, and 5/8 in. diameter for hot rolled or cold drawn material, plain or deformed.

The code allows spirals to be terminated at the level of lowest horizontal reinforcement framing into the column. However, if one or more sides of the column are not enclosed by beams or brackets, ties are required from the termination of the spiral to the bottom of the slab or drop panel. If beams or brackets enclose all sides of the column but are of different depths, the ties should extend from the spiral to the level of the horizontal reinforcement of the shallowest beam or bracket framing into the column. These additional ties are to enclose the longitudinal column reinforcement and the portion of bars from beams bent into the column for anchorage. See also 7.9.

Spirals should be held firmly in place, at proper pitch and alignment, to prevent displacement during concrete placement. The code has traditionally required spacers to hold the fabricated spiral cage in place but was changed in 1989 to allow alternate methods of installation. When spacers are used, the following may be used for guidance: For spiral bar or wire smaller than 5/8 in. diameter, a minimum of two spacers should

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The hooks shall be embedded within the core confined by the spiral reinforcement ..... $48d_b$

(5) epoxy-coated deformed bar or wire with a standard stirrup or tie hook in accordance with 7.1.3 at ends of lapped spiral reinforcement. The hooks shall be embedded within the core confined by the spiral reinforcement..... $48d_b$

(b) Full mechanical or welded splices in accordance with 12.14.3.

7.10.4.6 — Spirals shall extend from top of footing or slab in any story to level of lowest horizontal reinforcement in members supported above.

7.10.4.7 — Where beams or brackets do not frame into all sides of a column, ties shall extend above termination of spiral to bottom of slab or drop panel.

7.10.4.8 — In columns with capitals, spirals shall extend to a level at which the diameter or width of capital is two times that of the column.

7.10.4.9 — Spirals shall be held firmly in place and true to line.

7.10.5 — Ties

Tie reinforcement for compression members shall conform to the following:

7.10.5.1 — All nonprestressed bars shall be enclosed by lateral ties, at least No. 3 in size for longitudinal bars No. 10 or smaller, and at least No. 4 in size for No. 11, No. 14, No. 18, and bundled longitudinal bars. Deformed wire or welded wire fabric of equivalent area shall be permitted.

7.10.5.2 — Vertical spacing of ties shall not exceed 16 longitudinal bar diameters, 48 tie bar or wire diameters, or least dimension of the compression member.

7.10.5.3 — Ties shall be arranged such that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie with an included angle of not more than 135 deg and no bar shall be farther than 6 in. clear on each side along the tie from such a laterally supported bar. Where longitudinal bars are located around the perimeter of a circle, a complete circular tie shall be permitted.

7.10.5.4 — Ties shall be located vertically not more than one-half a tie spacing above the top of footing or slab in any story, and shall be spaced as provided

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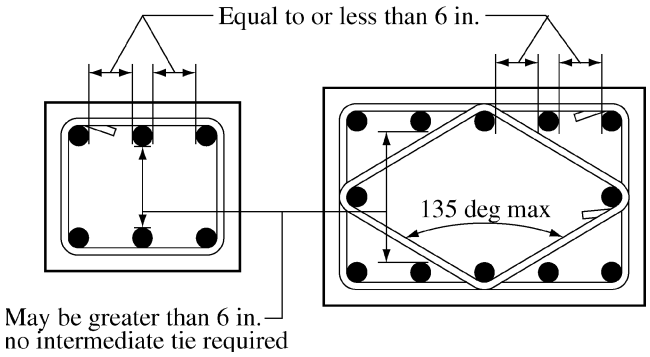


Fig. R7.10.5—Sketch to clarify measurements between laterally supported column bars

be used for spirals less than 20 in. in diameter, three spacers for spirals 20 to 30 in. in diameter, and four spacers for spirals greater than 30 in. in diameter. For spiral bar or wire 5/8 in. diameter or larger, a minimum of three spacers should be used for spirals 24 in. or less in diameter, and four spacers for spirals greater than 24 in. in diameter. The project specifications or subcontract agreements should be clearly written to cover the supply of spacers or field tying of the spiral reinforcement. In the 1999 code, splice requirements were modified for epoxy-coated and plain spirals and to allow mechanical splices.

R7.10.5 — Ties

All longitudinal bars in compression should be enclosed within lateral ties. Where longitudinal bars are arranged in a circular pattern, only one circular tie per specified spacing is required. This requirement can be satisfied by a continuous circular tie (helix) at larger pitch than required for spirals under 10.9.3, the maximum pitch being equal to the required tie spacing (see also 7.10.4.3).

The 1956 code required "lateral support equivalent to that provided by a 90-deg corner of a tie," for every vertical bar. Tie requirements were liberalized in 1963 by increasing the permissible included angle from 90 to 135 deg and exempting bars that are located within 6 in. clear on each side along the tie from adequately tied bars (see Fig. R7.10.5). Limited tests[7.15] on full-size, axially-loaded, tied columns containing full-length bars (without splices) showed no appreciable difference between ultimate strengths of columns with full tie requirements and no ties at all.

Since spliced bars and bundled bars were not included in the tests of Reference 7.15, is prudent to provide a set of ties at each end of lap spliced bars, above and below end-bearing splices, and at minimum spacings immediately below sloping regions of offset bent bars.

Standard tie hooks are intended for use with deformed bars only, and should be staggered where possible. See also 7.9.

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herein to not more than one-half a tie spacing below the lowest horizontal reinforcement in slab or drop panel above.

7.10.5.5 — Where beams or brackets frame from four directions into a column, termination of ties not more than 3 in. below lowest reinforcement in shallowest of such beams or brackets shall be permitted.

7.10.5.6 — Where anchor bolts are placed in the top of columns or pedestals, the bolts shall be enclosed by lateral reinforcement that also surrounds at least four vertical bars of the column or pedestal. The lateral reinforcement shall be distributed within 5 in. of the top of the column or pedestal, and shall consist of at least two No. 4 or three No. 3 bars.

7.11 — Lateral reinforcement for flexural members

7.11.1 — Compression reinforcement in beams shall be enclosed by ties or stirrups satisfying the size and spacing limitations in 7.10.5 or by welded wire fabric of equivalent area. Such ties or stirrups shall be provided throughout the distance where compression reinforcement is required.

7.11.2 — Lateral reinforcement for flexural framing members subject to stress reversals or to torsion at supports shall consist of closed ties, closed stirrups, or spirals extending around the flexural reinforcement.

7.11.3 — Closed ties or stirrups shall be formed in one piece by overlapping standard stirrup or tie end hooks around a longitudinal bar, or formed in one or two pieces lap spliced with a Class B splice (lap of $\mathbf{1.3}\ell_d$) or anchored in accordance with 12.13.

7.12 — Shrinkage and temperature reinforcement

7.12.1 — Reinforcement for shrinkage and temperature stresses normal to flexural reinforcement shall be provided in structural slabs where the flexural reinforcement extends in one direction only.

7.12.1.1 — Shrinkage and temperature reinforcement shall be provided in accordance with either 7.12.2 or 7.12.3.

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Continuously wound bars or wires can be used as ties provided their pitch and area are at least equivalent to the area and spacing of separate ties. Anchorage at the end of a continuously wound bar or wire should be by a standard hook as for separate bars or by one additional turn of the tie pattern. A circular continuously wound bar or wire is considered a spiral if it conforms to 7.10.4, otherwise it is considered a tie.

R7.10.5.5 — With the 1983 code, the wording of this section was modified to clarify that ties may be terminated only when elements frame into all four sides of square and rectangular columns; for round or polygonal columns, such elements frame into the column from four directions.

R7.10.5.6 — Provisions for confinement of anchor bolts that are placed in the top of columns or pedestals were added in the 2002 code. Confinement improves load transfer from the anchor bolts to the column or pier for situations where the concrete cracks in the vicinity of the bolts. Such cracking can occur due to unanticipated forces caused by temperature, restrained shrinkage, and similar effects.

R7.11 — Lateral reinforcement for flexural members

R7.11.1 — Compression reinforcement in beams and girders should be enclosed to prevent buckling; similar requirements for such enclosure have remained essentially unchanged through several editions of the code, except for minor clarification.

R7.12 — Shrinkage and temperature reinforcement

R7.12.1 — Shrinkage and temperature reinforcement is required at right angles to the principal reinforcement to minimize cracking and to tie the structure together to ensure its acting as assumed in the design. The provisions of this section are intended for structural slabs only; they are not intended for soil-supported slabs on grade.

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7.12.1.2 — Where shrinkage and temperature movements are significantly restrained, the requirements of 8.2.4 and 9.2.3 shall be considered.

7.12.2 — Deformed reinforcement conforming to 3.5.3 used for shrinkage and temperature reinforcement shall be provided in accordance with the following:

7.12.2.1 — Area of shrinkage and temperature reinforcement shall provide at least the following ratios of reinforcement area to gross concrete area, but not less than 0.0014:

(a) Slabs where Grade 40 or 50 deformed bars are used 0.0020

(b) Slabs where Grade 60 deformed bars or welded wire fabric (plain or deformed) are used..... 0.0018

(c) Slabs where reinforcement with yield stress exceeding 60,000 psi measured at a yield strain of 0.35 percent is used ..... $\frac{0.0018 \times 60{,}000}{f_y}$

7.12.2.2 — Shrinkage and temperature reinforcement shall be spaced not farther apart than five times the slab thickness, nor farther apart than 18 in.

7.12.2.3 — At all sections where required, reinforcement for shrinkage and temperature stresses shall develop the specified yield strength f_y in tension in accordance with Chapter 12.

7.12.3 — Prestressing steel conforming to 3.5.5 used for shrinkage and temperature reinforcement shall be provided in accordance with the following:

7.12.3.1 — Tendons shall be proportioned to provide a minimum average compressive stress of 100 psi on gross concrete area using effective prestress, after losses, in accordance with 18.6.

7.12.3.2 — Spacing of tendons shall not exceed 6 ft.

7.12.3.3 — When spacing of tendons exceeds 54 in., additional bonded shrinkage and temperature reinforcement conforming to 7.12.2 shall be provided

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R7.12.1.2 — The area of shrinkage and temperature reinforcement required by 7.12 has been satisfactory where shrinkage and temperature movements are permitted to occur. For cases where structural walls or large columns provide significant restraint to shrinkage and temperature movements, it may be necessary to increase the amount of reinforcement normal to the flexural reinforcement in 7.12.1.2 (see Reference 7.16). Top and bottom reinforcement are both effective in controlling cracks. Control strips during the construction period, which permit initial shrinkage to occur without causing an increase in stresses, are also effective in reducing cracks caused by restraint.

R7.12.2 — The amounts specified given for deformed bars and welded wire fabric are empirical but have been used satisfactorily for many years. Splices and end anchorages of shrinkage and temperature reinforcement are to be designed for the full specified yield strength in accordance with 12.1, 12.15, 12.18, and 12.19.

R7.12.3 — Prestressed reinforcement requirements have been selected to provide an effective force on the slab approximately equal to the yield strength force for nonprestressed shrinkage and temperature reinforcement. This amount of prestressing, 100 psi on the gross concrete area, has been successfully used on a large number of projects. When the spacing of tendons used for shrinkage and temperature reinforcement exceeds 54 in., additional bonded reinforcement is required at slab edges where the prestressing forces are applied in order to adequately reinforce the area between the slab edge and the point where compressive stresses behind individual anchorages have spread sufficiently such that the slab is uniformly in compression.

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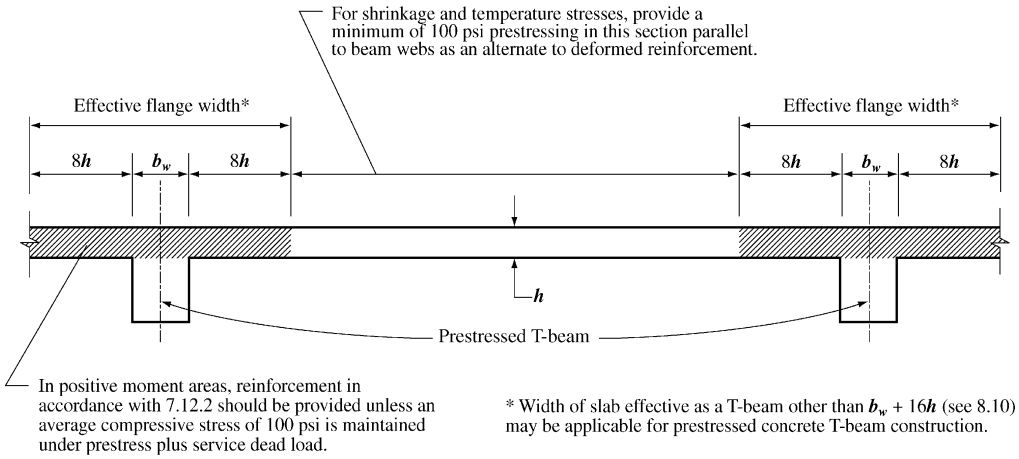


Fig. R7.12.3—Prestressing used for shrinkage and temperature

between the tendons at slab edges extending from the slab edge for a distance equal to the tendon spacing.

Application of the provisions of 7.12.3 to monolithic cast-in-place post-tensioned beam and slab construction is illustrated in Fig. R7.12.3.

Tendons used for shrinkage and temperature reinforcement should be positioned vertically in the slab as close as practicable to the center of the slab. In cases where the shrinkage and temperature tendons are used for supporting the principal tendons, variations from the slab centroid are permissible; however, the resultant of the shrinkage and temperature tendons should not fall outside the kern area of the slab.

The designer should evaluate the effects of slab shortening to ensure proper action. In most cases, the low level of prestressing recommended should not cause difficulties in a properly detailed structure. Special attention may be required where thermal effects become significant.

7.13 — Requirements for structural integrity

R7.13 — Requirements for structural integrity

Experience has shown that the overall integrity of a structure can be substantially enhanced by minor changes in detailing of reinforcement. It is the intent of this section of the code to improve the redundancy and ductility in structures so that in the event of damage to a major supporting element or an abnormal loading event, the resulting damage may be confined to a relatively small area and the structure will have a better chance to maintain overall stability.

7.13.1 — In the detailing of reinforcement and connections, members of a structure shall be effectively tied together to improve integrity of the overall structure.

7.13.2 — For cast-in-place construction, the following shall constitute minimum requirements:

7.13.2.1 — In joist construction, at least one bottom bar shall be continuous or shall be spliced with a Class A tension splice or a mechanical or welded splice satisfying 12.14.3 and at noncontinuous supports shall be terminated with a standard hook.

7.13.2.2 — Beams along the perimeter of the structure shall have continuous reinforcement consisting of:

R7.13.2 — With damage to a support, top reinforcement that is continuous over the support, but not confined by stirrups, will tend to tear out of the concrete and will not provide the catenary action needed to bridge the damaged support. By making a portion of the bottom reinforcement continuous, catenary action can be provided.

Requiring continuous top and bottom reinforcement in perimeter or spandrel beams provides a continuous tie around the structure. It is not the intent to require a tensile tie of continuous reinforcement of constant size around the

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(a) at least one-sixth of the tension reinforcement required for negative moment at the support, but not less than two bars; and

(b) at least one-quarter of the tension reinforcement required for positive moment at midspan, but not less than two bars.

7.13.2.3 — Where splices are needed to provide the required continuity, the top reinforcement shall be spliced at or near midspan and bottom reinforcement shall be spliced at or near the support. Splices shall be Class A tension splices or mechanical or welded splices satisfying 12.14.3. The continuous reinforcement required in 7.13.2.2(a) and 7.13.2.2(b) shall be enclosed by the corners of U-stirrups having not less than 135-deg hooks around the continuous top bars, or by one-piece closed stirrups with not less than 135-deg hooks around one of the continuous top bars. Stirrups need not be extended through any joints.

7.13.2.4 — In other than perimeter beams, when stirrups as defined in 7.13.2.3 are not provided, at least one-quarter of the positive moment reinforcement required at midspan, but not less than two bars, shall be continuous or shall be spliced over or near the support with a Class A tension splice or a mechanical or welded splice satisfying 12.14.3, and at noncontinuous supports shall be terminated with a standard hook.

7.13.2.5 — For two-way slab construction, see 13.3.8.5.

7.13.3 — For precast concrete construction, tension ties shall be provided in the transverse, longitudinal, and vertical directions and around the perimeter of the structure to effectively tie elements together. The provisions of 16.5 shall apply.

7.13.4 — For lift-slab construction, see 13.3.8.6 and 18.12.6.

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entire perimeter of a structure, but simply to require that one half of the top flexural reinforcement required to extend past the point of inflection by 12.12.3 be further extended and spliced at or near midspan. Similarly, the bottom reinforcement required to extend into the support by 12.11.1 should be made continuous or spliced with bottom reinforcement from the adjacent span. If the depth of a continuous beam changes at a support, the bottom reinforcement in the deeper member should be terminated with a standard hook and bottom reinforcement in the shallower member should be extended into and fully developed in the deeper member.

In the 2002 code, provisions were added to permit the use of mechanical or welded splices for splicing reinforcement, and the detailing requirements for the longitudinal reinforcement and stirrups in beams were revised. Section 7.13.2 was revised in 2002 to require U-stirrups with not less than 135-deg hooks around the continuous bars, or one-piece close stirrups, because a crosstie forming the top of a two-piece closed stirrup is ineffective in preventing the top continuous bars from tearing out of the top of the beam.

R7.13.3 — The code requires tension ties for precast concrete buildings of all heights. Details should provide connections to resist applied loads. Connection details that rely solely on friction caused by gravity forces are not permitted.

Connection details should be arranged so as to minimize the potential for cracking due to restrained creep, shrinkage and temperature movements. For information on connections and detailing requirements, see Reference 7.17.

Reference 7.18 recommends minimum tie requirements for precast concrete bearing wall buildings.

CODE

COMMENTARY

Notes

PART 4 — GENERAL REQUIREMENTS

CHAPTER 8 — ANALYSIS AND DESIGN — GENERAL CONSIDERATIONS

CODE

8.0 — Notation

A_s = area of nonprestressed tension reinforcement, in.^{2}
A_s' = area of compression reinforcement, in.^{2}
b = width of compression face of member, in.
d = distance from extreme compression fiber to centroid of tension reinforcement, in.
E_c = modulus of elasticity of concrete, psi. See 8.5.1
E_s = modulus of elasticity of reinforcement, psi. See 8.5.2 and 8.5.3
f_c' = specified compressive strength of concrete, psi
f_y = specified yield strength of nonprestressed reinforcement, psi
ℓ_n = clear span for positive moment or shear and average of adjacent clear spans for negative moment
V_c = nominal shear strength provided by concrete
w_c = unit weight of concrete, lb/ft^3
w_u = factored load per unit length of beam or per unit area of slab
b_1 = factor defined in 10.2.7.3
e_t = net tensile strain in extreme tension steel at nominal strength
r = ratio of nonprestressed tension reinforcement
= A_s/bd
r' = ratio of nonprestressed compression reinforcement
= A_s'/bd
r_b = reinforcement ratio producing balanced strain conditions. See 10.3.2
f = strength reduction factor. See 9.3

8.1 — Design methods

8.1.1 — In design of structural concrete, members shall be proportioned for adequate strength in accordance with provisions of this code, using load factors and strength reduction factors f specified in Chapter 9.

8.1.2 — Design of reinforced concrete using the provisions of Appendix B, Alternative Provisions for Reinforced and Prestressed Concrete Flexural and Compression Members, shall be permitted.

COMMENTARY

R8.0 — Notation

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

The definition of net tensile strain in 2.1 excludes strains due to effective prestress, creep, shrinkage, and temperature.

R8.1 — Design methods

R8.1.1 — The strength design method requires service loads or related internal moments and forces to be increased by specified load factors (required strength) and computed nominal strengths to be reduced by specified strength reduction factors f (design strength).

R8.1.2 — Designs in accordance with Appendix B are equally acceptable, provided the provisions of Appendix B are used in their entirety.

An appendix may be judged not to be an official part of a legal document unless specifically adopted. Therefore, spe-

CODE

8.1.3 — Anchors within the scope of Appendix D, Anchoring to Concrete, installed in concrete to transfer loads between connected elements shall be designed using Appendix D.

8.2 — Loading

8.2.1 — Design provisions of this code are based on the assumption that structures shall be designed to resist all applicable loads.

8.2.2 — Service loads shall be in accordance with the general building code of which this code forms a part, with such live load reductions as are permitted in the general building code.

8.2.3 — In design for wind and earthquake loads, integral structural parts shall be designed to resist the total lateral loads.

8.2.4 — Consideration shall be given to effects of forces due to prestressing, crane loads, vibration, impact, shrinkage, temperature changes, creep, expansion of shrinkage-compensating concrete, and unequal settlement of supports.

COMMENTARY

cific reference is made to Appendix B in the main body of the code, to make it a legal part of the code.

R8.1.3 — The code has included specific provisions for anchoring to concrete for the first time in the 2002 edition. As has been done in the past with a number of new sections and chapters, the new material has been presented as an appendix.

An appendix may be judged not to be an official part of a legal document unless specifically adopted. Therefore, specific reference is made to Appendix D in the main part of the code of make it a legal part of the code.

R8.2 — Loading

The provisions in the code are for live, wind, and earthquake loads such as those recommended in **"Minimum Design Loads for Buildings and Other Structures,"** (ASCE 7), of the American Society of Civil Engineers (ASCE) (formerly ANSI A58.1). If the service loads specified by the general building code (of which ACI 318 forms a part) differ from those of ASCE 7, the general building code governs. However, if the nature of the loads contained in a general building code differs considerably from ASCE 7 loads, some provisions of this code may need modification to reflect the difference.

Roofs should be designed with sufficient slope or camber to ensure adequate drainage accounting for any long-term deflection of the roof due to the dead loads, or the loads should be increased to account for all likely accumulations of water. If deflection of roof members may result in ponding of water accompanied by increased deflection and additional ponding, the design should ensure that this process is self-limiting.

R8.2.3 — Any reinforced concrete wall that is monolithic with other structural elements is considered to be an "integral part." Partition walls may or may not be integral structural parts. If partition walls may be removed, the primary lateral load resisting system should provide all of the required resistance without contribution of the removable partition. However, the effects of all partition walls attached to the structure should be considered in the analysis of the structure because they may lead to increased design forces in some or all elements. Special provisions for seismic design are given in Chapter 21.

R8.2.4 — Information is accumulating on the magnitudes of these various effects, especially the effects of column creep and shrinkage in tall structures,[8.1] and on procedures for including the forces resulting from these effects in design.

CODE

8.3 — Methods of analysis

8.3.1 — All members of frames or continuous construction shall be designed for the maximum effects of factored loads as determined by the theory of elastic analysis, except as modified according to 8.4. It shall be permitted to simplify design by using the assumptions specified in 8.6 through 8.9.

8.3.2 — Except for prestressed concrete, approximate methods of frame analysis shall be permitted for buildings of usual types of construction, spans, and story heights.

8.3.3 — As an alternate to frame analysis, the following approximate moments and shears shall be permitted for design of continuous beams and one-way slabs (slabs reinforced to resist flexural stresses in only one direction), provided:

(a) There are two or more spans;

(b) Spans are approximately equal, with the larger of two adjacent spans not greater than the shorter by more than 20 percent;

(c) Loads are uniformly distributed;

(d) Unit live load does not exceed three times unit dead load; and

(e) Members are prismatic.

Positive moment	
End spans	
Discontinuous end unrestrained	$w_u \ell_n^2/11$
Discontinuous end integral with support	$w_u \ell_n^2/14$
Interior spans	$w_u \ell_n^2/16$
Negative moments at exterior face of first interior support	
Two spans	$w_u \ell_n^2/9$
More than two spans	$w_u \ell_n^2/10$
Negative moment at other faces of interior supports	$w_u \ell_n^2/11$

COMMENTARY

R8.3 — Methods of analysis

R8.3.1 — Factored loads are service loads multiplied by appropriate load factors. For the strength design method, elastic analysis is used to obtain moments, shears, and reactions.

R8.3.3 — The approximate moments and shears give reasonably conservative values for the stated conditions if the flexural members are part of a frame or continuous construction. Because the load patterns that produce critical values for moments in columns of frames differ from those for maximum negative moments in beams, column moments should be evaluated separately.

CODE

Negative moment at face of all supports for

Slabs with spans not exceeding 10 ft; and beams where ratio of sum of column stiffnesses to beam stiffness exceeds eight at each end of the span ..... $w_u \ell_n^2/12$

Negative moment at interior face of exterior support for members built integrally with supports

Where support is spandrel beam .... $w_u \ell_n^2/24$

Where support is a column..... $w_u \ell_n^2/16$

Shear in end members at face of first interior support ..... $1.15\, w_u \ell_n /2$

Shear at face of all other supports ..... $w_u \ell_n /2$

8.3.4 — Strut-and-tie models shall be permitted to be used in the design of structural concrete. See Appendix A.

COMMENTARY

R8.3.4 — The strut-and-tie model in Appendix A is based on the assumption that portions of concrete structures can be analyzed and designed using hypothetical pin-jointed trusses consisting of struts and ties connected at nodes. This design method can be used in the design of regions where the basic assumptions of flexure theory are not applicable, such as regions near force discontinuities arising from concentrated forces or reactions, and regions near geometric discontinuities, such as abrupt changes in cross section.

8.4 — Redistribution of negative moments in continuous flexural members

8.4.1 — Except where approximate values for moments are used, it shall be permitted to increase or decrease negative moments calculated by elastic theory at supports of continuous flexural members for any assumed loading arrangement by not more than $1000\varepsilon_t$ percent, with a maximum of 20 percent.

8.4.2 — The modified negative moments shall be used for calculating moments at sections within the spans.

8.4.3 — Redistribution of negative moments shall be made only when ε_t is equal to or greater than 0.0075 at the section at which moment is reduced.

R8.4 — Redistribution of negative moments in continuous flexural members

Moment redistribution is dependent on adequate ductility in plastic hinge regions. These plastic hinge regions develop at points of maximum moment and cause a shift in the elastic moment diagram. The usual result is a reduction in the values of negative moments in the plastic hinge region and an increase in the values of positive moments from those computed by elastic analysis. Because negative moments are determined for one loading arrangement and positive moments for another, each section has a reserve capacity that is not fully utilized for any one loading condition. The plastic hinges permit the utilization of the full capacity of more cross sections of a flexural member at ultimate loads.

Using conservative values of limiting concrete strains and lengths of plastic hinges derived from extensive tests, flexural members with small rotation capacity were analyzed for moment redistribution up to 20 percent, depending on the reinforcement ratio. The results were found to be conservative (see Fig. R8.4). Studies by Cohn[8.2] and Mattock[8.3] support this conclusion and indicate that cracking and

CODE

COMMENTARY

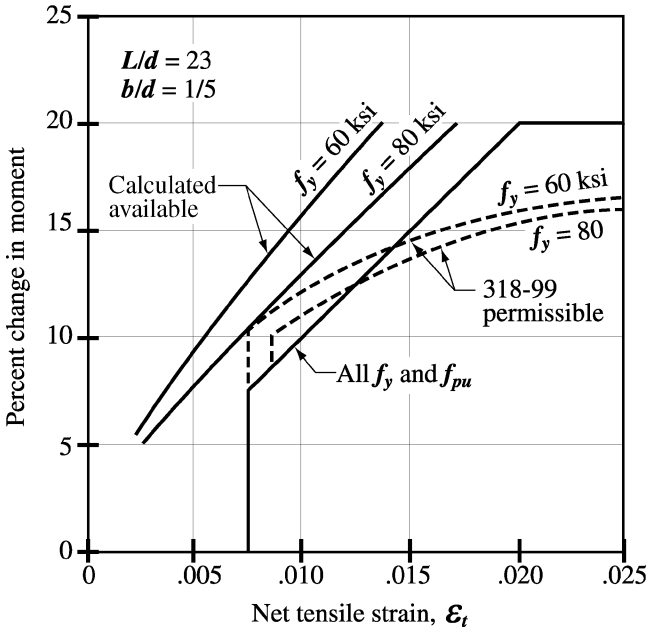


Fig. R8.4—Permissible moment redistribution for minimum rotation capacity

deflection of beams designed for moment redistribution are not significantly greater at service loads than for beams designed by the elastic theory distribution of moments. Also, these studies indicated that adequate rotation capacity for the moment redistribution allowed by the code is available if the members satisfy the code requirements.

Moment redistribution may not be used for slab systems designed by the Direct Design Method (see 13.6.1.7).

In previous codes, Section 8.4 specified the permissible redistribution percentage in terms of reinforcement indices. The 2002 code specifies the permissible redistribution percentage in terms of the net tensile strain ε_t. See Reference 8.4 for a comparison of these moment redistribution provisions.

8.5 — Modulus of elasticity

8.5.1 — Modulus of elasticity E_c for concrete shall be permitted to be taken as $w_c^{1.5}\,33\sqrt{f'_c}$ (in psi) for values of w_c between 90 and 155 lb/ft.^{3} For normal weight concrete, E_c shall be permitted to be taken as $57{,}000\sqrt{f'_c}$.

8.5.2 — Modulus of elasticity E_s for nonprestressed reinforcement shall be permitted to be taken as 29,000,000 psi.

8.5.3 — Modulus of elasticity E_s for prestressing steel shall be determined by tests or supplied by the manufacturer.

R8.5 — Modulus of elasticity

R8.5.1 — Studies leading to the expression for modulus of elasticity of concrete in 8.5.1 are summarized in Reference 8.5 where E_c was defined as the slope of the line drawn from a stress of zero to a compressive stress of $0.45f'_c$. The modulus for concrete is sensitive to the modulus of the aggregate and may differ from the specified value. Measured values range typically from 120 to 80 percent of the specified value. Methods for determining Young's modulus for concrete are described in Reference 8.6.

CODE

8.6 — Stiffness

8.6.1 — Use of any set of reasonable assumptions shall be permitted for computing relative flexural and torsional stiffnesses of columns, walls, floors, and roof systems. The assumptions adopted shall be consistent throughout analysis.

8.6.2 — Effect of haunches shall be considered both in determining moments and in design of members.

8.7 — Span length

8.7.1 — Span length of members not built integrally with supports shall be considered as the clear span plus the depth of the member, but need not exceed distance between centers of supports.

8.7.2 — In analysis of frames or continuous construction for determination of moments, span length shall be taken as the distance center-to-center of supports.

8.7.3 — For beams built integrally with supports, design on the basis of moments at faces of support shall be permitted.

8.7.4 — It shall be permitted to analyze solid or ribbed slabs built integrally with supports, with clear spans not more than 10 ft, as continuous slabs on knife edge supports with spans equal to the clear spans of the slab and width of beams otherwise neglected.

COMMENTARY

R8.6 — Stiffness

R8.6.1 — Ideally, the member stiffnesses ***EI*** and ***GJ*** should reflect the degree of cracking and inelastic action that has occurred along each member before yielding. However, the complexities involved in selecting different stiffnesses for all members of a frame would make frame analyses inefficient in design offices. Simpler assumptions are required to define flexural and torsional stiffnesses.

For braced frames, relative values of stiffness are important. Two usual assumptions are to use gross ***EI*** values for all members or, to use half the gross ***EI*** of the beam stem for beams and the gross ***EI*** for the columns.

For frames that are free to sway, a realistic estimate of ***EI*** is desirable and should be used if second-order analyses are carried out. Guidance for the choice of ***EI*** for this case is given in R10.11.1.

Two conditions determine whether it is necessary to consider torsional stiffness in the analysis of a given structure: (1) the relative magnitude of the torsional and flexural stiffnesses, and (2) whether torsion is required for equilibrium of the structure (equilibrium torsion) or is due to members twisting to maintain deformation compatibility (compatibility torsion). In the case of compatibility torsion, the torsional stiffness may be neglected. For cases involving equilibrium torsion, torsional stiffness should be considered.

R8.6.2 — Stiffness and fixed-end moment coefficients for haunched members may be obtained from Reference 8.7.

R8.7 — Span length

Beam moments calculated at support centers may be reduced to the moments at support faces for design of beams. Reference 8.8 provides an acceptable method of reducing moments at support centers to those at support faces.

CODE

8.8 — Columns

8.8.1 — Columns shall be designed to resist the axial forces from factored loads on all floors or roof and the maximum moment from factored loads on a single adjacent span of the floor or roof under consideration. Loading condition giving the maximum ratio of moment to axial load shall also be considered.

8.8.2 — In frames or continuous construction, consideration shall be given to the effect of unbalanced floor or roof loads on both exterior and interior columns and of eccentric loading due to other causes.

8.8.3 — In computing gravity load moments in columns, it shall be permitted to assume far ends of columns built integrally with the structure to be fixed.

8.8.4 — Resistance to moments at any floor or roof level shall be provided by distributing the moment between columns immediately above and below the given floor in proportion to the relative column stiffnesses and conditions of restraint.

8.9 — Arrangement of live load

8.9.1 — It shall be permitted to assume that:

(a) The live load is applied only to the floor or roof under consideration;

(b) The far ends of columns built integrally with the structure are considered to be fixed.

8.9.2 — It shall be permitted to assume that the arrangement of live load is limited to combinations of:

(a) Factored dead load on all spans with full factored live load on two adjacent spans;

(b) Factored dead load on all spans with full factored live load on alternate spans.

8.10 — T-beam construction

8.10.1 — In T-beam construction, the flange and web shall be built integrally or otherwise effectively bonded together.

COMMENTARY

R8.8 — Columns

Section 8.8 has been developed with the intent of making certain that the most demanding combinations of axial load and moments be identified for design.

Section 8.8.4 has been included to make certain that moments in columns are recognized in the design if the girders have been proportioned using 8.3.3. The moment in 8.8.4 refers to the difference between the moments in a given vertical plane, exerted at column centerline by members framing into that column.

R8.9 — Arrangement of live load

For determining column, wall, and beam moments and shears caused by gravity loads, the code permits the use of a model limited to the beams in the level considered and the columns above and below that level. Far ends of columns are to be considered as fixed for the purpose of analysis under gravity loads. This assumption does not apply to lateral load analysis. However in analysis for lateral loads, simplified methods (such as the portal method) may be used to obtain the moments, shears, and reactions for structures that are symmetrical and satisfy the assumptions used for such simplified methods. For unsymmetrical and high-rise structures, rigorous methods recognizing all structural displacements should be used.

The engineer is expected to establish the most demanding sets of design forces by investigating the effects of live load placed in various critical patterns.

Most approximate methods of analysis neglect effects of deflections on geometry and axial flexibility. Therefore, beam and column moments may have to be amplified for column slenderness in accordance with 10.11, 10.12, and 10.13.

R8.10 — T-beam construction

This section contains provisions identical to those of previous codes for limiting dimensions related to stiffness and flexural calculations. Special provisions related to T-beams and other flanged members are stated in 11.6.1 with regard to torsion.

CODE

8.10.2 — Width of slab effective as a T-beam flange shall not exceed one-quarter of the span length of the beam, and the effective overhanging flange width on each side of the web shall not exceed:

(a) eight times the slab thickness;

(b) one-half the clear distance to the next web.

8.10.3 — For beams with a slab on one side only, the effective overhanging flange width shall not exceed:

(a) one-twelfth the span length of the beam;

(b) six times the slab thickness;

(c) one-half the clear distance to the next web.

8.10.4 — Isolated beams, in which the T-shape is used to provide a flange for additional compression area, shall have a flange thickness not less than one-half the width of web and an effective flange width not more than four times the width of web.

8.10.5 — Where primary flexural reinforcement in a slab that is considered as a T-beam flange (excluding joist construction) is parallel to the beam, reinforcement perpendicular to the beam shall be provided in the top of the slab in accordance with the following:

8.10.5.1 — Transverse reinforcement shall be designed to carry the factored load on the overhanging slab width assumed to act as a cantilever. For isolated beams, the full width of overhanging flange shall be considered. For other T-beams, only the effective overhanging slab width need be considered.

8.10.5.2 — Transverse reinforcement shall be spaced not farther apart than five times the slab thickness, nor farther apart than 18 in.

8.11 — Joist construction

8.11.1 — Joist construction consists of a monolithic combination of regularly spaced ribs and a top slab arranged to span in one direction or two orthogonal directions.

8.11.2 — Ribs shall be not less than 4 in. in width, and shall have a depth of not more than 3-1/2 times the minimum width of rib.

8.11.3 — Clear spacing between ribs shall not exceed 30 in.

COMMENTARY

R8.11 — Joist construction

The size and spacing limitations for concrete joist construction meeting the limitations of 8.11.1 through 8.11.3 are based on successful performance in the past.

R8.11.3 — A limit on the maximum spacing of ribs is required because of the special provisions permitting higher shear strengths and less concrete protection for the reinforcement for these relatively small, repetitive members.

CODE

8.11.4 — Joist construction not meeting the limitations of 8.11.1 through 8.11.3 shall be designed as slabs and beams.

8.11.5 — When permanent burned clay or concrete tile fillers of material having a unit compressive strength at least equal to that of the specified strength of concrete in the joists are used:

8.11.5.1 — For shear and negative moment strength computations, it shall be permitted to include the vertical shells of fillers in contact with the ribs. Other portions of fillers shall not be included in strength computations.

8.11.5.2 — Slab thickness over permanent fillers shall be not less than one-twelfth the clear distance between ribs, nor less than 1-1/2 in.

8.11.5.3 — In one-way joists, reinforcement normal to the ribs shall be provided in the slab as required by 7.12.

8.11.6 — When removable forms or fillers not complying with 8.11.5 are used:

8.11.6.1 — Slab thickness shall be not less than one-twelfth the clear distance between ribs, nor less than 2 in.

8.11.6.2 — Reinforcement normal to the ribs shall be provided in the slab as required for flexure, considering load concentrations, if any, but not less than required by 7.12.

8.11.7 — Where conduits or pipes as permitted by 6.3 are embedded within the slab, slab thickness shall be at least 1 in. greater than the total overall depth of the conduits or pipes at any point. Conduits or pipes shall not impair significantly the strength of the construction.

8.11.8 — For joist construction, contribution of concrete to shear strength V_c shall be permitted to be 10 percent more than that specified in Chapter 11. It shall be permitted to increase shear strength using shear reinforcement or by widening the ends of ribs.

8.12 — Separate floor finish

8.12.1 — A floor finish shall not be included as part of a structural member unless placed monolithically with the floor slab or designed in accordance with requirements of Chapter 17.

8.12.2 — It shall be permitted to consider all concrete floor finishes as part of required cover or total thickness for nonstructural considerations.

COMMENTARY

R8.11.8 — The increase in shear strength permitted by 8.11.8 is justified on the basis of: (1) satisfactory performance of joist construction with higher shear strengths, designed under previous codes, which allowed comparable shear stresses, and (2) redistribution of local overloads to adjacent joists.

R8.12 — Separate floor finish

The code does not specify an additional thickness for wearing surfaces subjected to unusual conditions of wear. The need for added thickness for unusual wear is left to the discretion of the designer.

As in previous editions of the code, a floor finish may be considered for strength purposes only if it is cast monolithically with the slab. Permission is given to include a separate

CODE

COMMENTARY

finish in the structural thickness if composite action is provided for in accordance with Chapter 17.

All floor finishes may be considered for nonstructural purposes such as cover for reinforcement, fire protection, etc. Provisions should be made, however, to ensure that the finish will not spall off, thus causing decreased cover. Furthermore, development of reinforcement considerations requires minimum monolithic concrete cover according to 7.7.

CHAPTER 9 — STRENGTH AND SERVICEABILITY REQUIREMENTS

CODE

9.0 — Notation

A_g = gross area of section, in.^{2}
A_s' = area of compression reinforcement, in.^{2}
b = width of compression face of member, in.
c = distance from extreme compression fiber to neutral axis, in.
d = distance from extreme compression fiber to centroid of tension reinforcement, in.
d' = distance from extreme compression fiber to centroid of compression reinforcement, in.
d_s = distance from extreme tension fiber to centroid of tension reinforcement, in.
d_t = distance from extreme compression fiber to extreme tension steel, in.
D = dead loads, or related internal moments and forces
E = load effects of seismic forces, or related internal moments and forces
E_c = modulus of elasticity of concrete, psi. See 8.5.1
f_c' = specified compressive strength of concrete, psi
$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi
f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi
f_r = modulus of rupture of concrete, psi
f_y = specified yield strength of nonprestressed reinforcement, psi
F = loads due to weight and pressures of fluids with well-defined densities and controllable maximum heights, or related internal moments and forces
h = overall thickness of member, in.
H = loads due to weight and pressure of soil, water in soil, or other materials, or related internal moments and forces
I_{cr} = moment of inertia of cracked section transformed to concrete, in.^{4}
I_e = effective moment of inertia for computation of deflection, in.^{4}
I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in.^{4}
ℓ = span length of beam or one-way slab, as defined in 8.7; clear projection of cantilever, in.
ℓ_n = length of clear span in long direction of two-way construction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases, in.

COMMENTARY

R9. 0 — Notation

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

CODE

L = live loads, or related internal moments and forces

L_r = roof live load, or related internal moments and forces

M_a = maximum moment in member at stage deflection is computed, in.-lb

M_{cr} = cracking moment, in.-lb. See 9.5.2.3

P_b = nominal axial load strength at balanced strain conditions, lb. See 10.3.2

P_n = nominal axial load strength at given eccentricity, lb

R = rain load, or related internal moments and forces

S = snow load, or related internal moments and forces

T = cumulative effect of temperature, creep, shrinkage, differential settlement, and shrinkage-compensating concrete

U = required strength to resist factored loads or related internal moments and forces

W = wind load, or related internal moments and forces

w_c = weight of concrete, lb/ft^3

y_t = distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension, in.

a = ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels (if any) on each side of beam. See Chapter 13

a_m = average value of α for all beams on edges of a panel

b = ratio of clear spans in long to short direction of two-way slabs

e_t = net tensile strain in extreme tension steel at nominal strength

l = multiplier for additional long-term deflection as defined in 9.5.2.5

x = time-dependent factor for sustained load. See 9.5.2.5

r = ratio of nonprestressed tension reinforcement, A_s/bd

$r¢$ = reinforcement ratio for nonprestressed compression reinforcement, A_s'/bd

r_b = reinforcement ratio producing balanced strain conditions. See 10.3.2

f = strength reduction factor. See 9.3

9.1 — General

9.1.1 — Structures and structural members shall be designed to have design strengths at all sections at least equal to the required strengths calculated for the factored loads and forces in such combinations as are stipulated in this code.

COMMENTARY

The definition of net tensile strain in 2.1 excludes strains due to effective prestress, creep, shrinkage, and temperature.

R9.1 — General

In the 2002 code, the load factor combinations and strength reduction factors of the 1999 code were revised and moved to Appendix C. The 1999 combinations have been replaced with those of ASCE 7-98.[9.1] The strength reduction factors were replaced with those of the 1999 Appendix C, except that the factor for flexure was increased.

CODE

9.1.2 — Members also shall meet all other requirements of this code to ensure adequate performance at service load levels.

9.1.3 — Design of structures and structural members using the load factor combinations and strength reduction factors of Appendix C shall be permitted. Use of load factor combinations from this chapter in conjunction with strength reduction factors of Appendix C shall not be permitted.

COMMENTARY

The changes were made to further unify the design profession on one set of load factors and combinations, and to facilitate the proportioning of concrete building structures that include members of materials other than concrete. When used with the strength reduction factors in 9.3, the designs for gravity loads will be comparable to those obtained using the strength reduction and load factors of the 1999 and earlier codes. For combinations with lateral loads, some designs will be different, but the results of either set of load factors are considered acceptable.

Chapter 9 defines the basic strength and serviceability conditions for proportioning structural concrete members.

The basic requirement for strength design may be expressed as follows:

$$\text{Design Strength} \geq \text{Required Strength}$$

$$f\,(\text{Nominal Strength}) \geq U$$

In the strength design procedure, the margin of safety is provided by multiplying the service load by a load factor and the nominal strength by a strength reduction factor.

9.2 — Required strength

R9.2 — Required strength

9.2.1 — Required strength U shall be at least equal to the effects of factored loads in Eq. (9-1) through (9-7). The effect of one or more loads not acting simultaneously shall be investigated.

$$U = 1.4(D + F) \quad (9\text{-}1)$$

$$U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R) \quad (9\text{-}2)$$

$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.8W) \quad (9\text{-}3)$$

$$U = 1.2D + 1.6W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R) \quad (9\text{-}4)$$

$$U = 1.2D + 1.0E + 1.0L + 0.2S \quad (9\text{-}5)$$

$$U = 0.9D + 1.6W + 1.6H \quad (9\text{-}6)$$

$$U = 0.9D + 1.0E + 1.6H \quad (9\text{-}7)$$

except as follows:

(a) The load factor on L in Eq. (9-3) to (9-5) shall be permitted to be reduced to 0.5 except for garages, areas occupied as places of public assembly, and all areas where the live load L is greater than 100 lb/ft^2.

The required strength U is expressed in terms of factored loads, or related internal moments and forces. Factored loads are the loads specified in the general building code multiplied by appropriate load factors.

The factor assigned to each load is influenced by the degree of accuracy to which the load effect usually can be calculated and the variation that might be expected in the load during the lifetime of the structure. Dead loads, because they are more accurately determined and less variable, are assigned a lower load factor than live loads. Load factors also account for variability in the structural analysis used to compute moments and shears.

The code gives load factors for specific combinations of loads. In assigning factors to combinations of loading, some consideration is given to the probability of simultaneous occurrence. While most of the usual combinations of loadings are included, the designer should not assume that all cases are covered.

Due regard is to be given to sign in determining U for combinations of loadings, as one type of loading may produce effects of opposite sense to that produced by another type. The load combinations with $0.9D$ are specifically included for the case where a higher dead load reduces the effects of other loads. The loading case may also be critical for tension-controlled column sections. In such a case, a reduction in axial load and an increase in moment may result in a critical load combination.

CODE

(b) Where wind load ***W*** has not been reduced by a directionality factor, it shall be permitted to use **1.3*W*** in place of **1.6*W*** in Eq. (9-4) and (9-6).

(c) Where earthquake load ***E*** is based on service-level seismic forces, **1.4*E*** shall be used in place of **1.0*E*** in Eq. (9-5) and (9-7).

(d) The load factor on ***H*** shall be set equal to zero in Eq. (9-6) and (9-7) if the structural action due to ***H*** counteracts that due to ***W*** or ***E***. Where lateral earth pressure provides resistance to structural actions from other forces, it shall not be included in ***H*** but shall be included in the design resistance.

9.2.2 — If resistance to impact effects is taken into account in design, such effects shall be included with live load ***L***.

9.2.3 — Estimations of differential settlement, creep, shrinkage, expansion of shrinkage-compensating concrete, or temperature change shall be based on a realistic assessment of such effects occurring in service.

9.2.4 — For a structure in a flood zone, the flood load and load combinations of ASCE 7 shall be used.

9.2.5 — For post-tensioned anchorage zone design, a load factor of 1.2 shall be applied to the maximum prestressing steel jacking force.

COMMENTARY

Consideration should be given to various combinations of loading to determine the most critical design condition. This is particularly true when strength is dependent on more than one load effect, such as strength for combined flexure and axial load or shear strength in members with axial load.

If special circumstances require greater reliance on the strength of particular members than encountered in usual practice, some reduction in the stipulated strength reduction factors f or increase in the stipulated load factors ***U*** may be appropriate for such members.

The wind load equation in ASCE 7-98[9.1] and IBC 2000[9.2] includes a factor for wind directionality that is equal to 0.85 for buildings. The corresponding load factor for wind in the load combination equations was increased accordingly (1.3/0.85 = 1.53 rounded up to 1.6). The code allows use of the previous wind load factor of 1.3 when the design wind load is obtained from other sources that do not include the wind directionality factor.

Model building codes and design load references have converted earthquake forces to strength level, and reduced the earthquake load factor to 1.0 (ASCE 7-93[9.3]; BOCA/NBC 93[9.4]; SBC 94[9.5]; UBC 97[9.6]; and IBC 2000[9.2]). The code requires use of the previous load factor for earthquake loads, approximately 1.4, when service-level earthquake forces from earlier editions of these references are used.

R9.2.2 — If the live load is applied rapidly, as may be the case for parking structures, loading docks, warehouse floors, elevator shafts, etc., impact effects should be considered. In all equations, substitute (***L*** + impact) for ***L*** when impact should be considered.

R9.2.3 — The designer should consider the effects of differential settlement, creep, shrinkage, temperature, and shrinkage-compensating concrete. The term realistic assessment is used to indicate that the most probable values rather than the upper bound values of the variables should be used.

R9.2.4 — Areas subject to flooding are defined by flood hazard maps, usually maintained by local governmental jurisdictions.

R9.2.5 — The load factor of 1.2 applied to the maximum tendon jacking force results in a design load of about 113 percent of the specified prestressing steel yield strength but not more than 96 percent of the nominal ultimate strength of the prestressing steel. This compares well with the maximum attainable jacking force, which is limited by the anchor efficiency factor.

CODE

9.3 — Design strength

9.3.1 — Design strength provided by a member, its connections to other members, and its cross sections, in terms of flexure, axial load, shear, and torsion, shall be taken as the nominal strength calculated in accordance with requirements and assumptions of this code, multiplied by the strength reduction factors ϕ in 9.3.2, 9.3.4, and 9.3.5.

9.3.2 — Strength reduction factor ϕ shall be as follows:

9.3.2.1 — Tension-controlled sections as defined in 10.3.4 ..... 0.90
(See also 9.3.2.7)

9.3.2.2 — Compression-controlled sections, as defined in 10.3.3:

(a) Members with spiral reinforcement conforming to 10.9.3..... 0.70

(b) Other reinforced members 0.65

For sections in which the net tensile strain in the extreme tension steel at nominal strength is between the limits for compression-controlled and tension-controlled sections, ϕ shall be permitted to be linearly increased from that for compression-controlled sections to 0.90 as the net tensile strain in the extreme tension steel at nominal strength increases from the compression-controlled strain limit to 0.005.

Alternatively, when Appendix B is used, for members in which f_y does not exceed 60,000 psi, with symmetric reinforcement, and with $(h - d' - d_s)/h$ not less than 0.70, ϕ shall be permitted to be increased linearly to 0.90 as ϕP_n decreases from $0.10 f'_c A_g$ to zero. For

COMMENTARY

R9.3 — Design strength

R9.3.1 — The design strength of a member refers to the nominal strength calculated in accordance with the requirements stipulated in this code multiplied by a strength reduction factor ϕ, which is always less than one.

The purposes of the strength reduction factor ϕ are (1) to allow for the probability of under-strength members due to variations in material strengths and dimensions, (2) to allow for inaccuracies in the design equations, (3) to reflect the degree of ductility and required reliability of the member under the load effects being considered, and (4) to reflect the importance of the member in the structure.[9.7,9.8]

In the 2002 code, the strength reduction factors were adjusted to be compatible with the ASCE 7-98[9.1] load combinations, which were the basis for the required factored load combinations in model building codes at that time. These factors are essentially the same as those published in Appendix C of the 1995 edition, except the factor for flexure/tension controlled limits is increased from 0.80 to 0.90. This change is based on past[9.7] and current reliability analyses,[9.9] statistical study of material properties, as well as the opinion of the committee that the historical performance of concrete structures supports $\phi = 0.90$.

R9.3.2.1 — In applying 9.3.2.1 and 9.3.2.2, the axial tensions and compressions to be considered are those caused by external forces. Effects of prestressing forces are not included.

R9.3.2.2 — Before the 2002 edition, the code specified the magnitude of the ϕ-factor for cases of axial load or flexure, or both, in terms of the type of loading. For these cases, the ϕ-factor is now determined by the strain conditions at a cross section, at nominal strength.

A lower ϕ-factor is used for compression-controlled sections than is used for tension-controlled sections because compression-controlled sections have less ductility, are more sensitive to variations in concrete strength, and generally occur in members that support larger loaded areas than members with tension-controlled sections. Members with spiral reinforcement are assigned a higher ϕ than tied columns since they have greater ductility or toughness.

For sections subjected to axial load with flexure, design strengths are determined by multiplying both P_n and M_n by the appropriate single value of ϕ. Compression-controlled and tension-controlled sections are defined in 10.3.3 and 10.3.4 as those that have net tensile strain in the extreme tension steel at nominal strength less than or equal to the compression-controlled strain limit, and equal to or greater than 0.005, respectively. For sections with net tensile strain

CODE

COMMENTARY

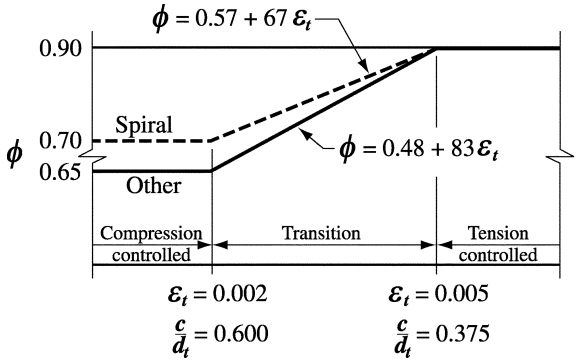


Fig. R9.3.2—Variation of f with net tensile e_t and c/d$_t$ for Grade 60 reinforcement and for prestressing steel.

other reinforced members, f shall be permitted to be increased linearly to 0.90 as fP_n decreases from $0.10f'_cA_g$ or fP_b, whichever is smaller, to zero.

e_t in the extreme tension steel at nominal strength between the above limits, the value of f may be determined by linear interpolation, as shown in Fig. R9.3.2. The concept of net tensile strain e_t is discussed in R10.3.3.

Since the compressive strain in the concrete at nominal strength is assumed in 10.2.3 to be 0.003, the net tensile strain limits for compression-controlled members may also be stated in terms of the ratio c/d_t, where c is the depth of the neutral axis at nominal strength, and d_t is the distance from the extreme compression fiber to the extreme tension steel. The c/d_t limits for compression-controlled and tension-controlled sections are 0.6 and 0.375, respectively. The 0.6 limit applies to sections reinforced with Grade 60 steel and to prestressed sections. Fig. R9.3.2 also gives equations for f as a function of c/d_t.

The net tensile strain limit for tension-controlled sections may also be stated in terms of the r/r_b as defined in the 1999 and earlier editions of the code. The net tensile strain limit of 0.005 corresponds to a r/r_b ratio of 0.63 for rectangular sections with Grade 60 reinforcement. For a comparison of these provisions with the 1999 code Section 9.3, see Reference 9.10.

9.3.2.3 — Shear and torsion 0.75

9.3.2.4 — Bearing on concrete (except for post-tensioned anchorage zones and strut-and-tie models)..... 0.65

9.3.2.5 — Post-tensioned anchorage zones 0.85

R9.3.2.5 — The f-factor of 0.85 reflects the wide scatter of results of experimental anchorage zone studies. Since 18.13.4.2 limits the nominal compressive strength of unconfined concrete in the general zone to $0.7f'_{ci}$, the effective design strength for unconfined concrete is $0.85 \times 0.7f'_{ci} \approx 0.6f'_{ci}$.

9.3.2.6 — Strut-and-tie models (Appendix A), and struts, ties, nodal zones, and bearing areas in such models 0.75

R9.3.2.6 — The f factor used in strut-and-tie models is taken equal to the f factor for shear. The value of f for strut-and-tie models is applied to struts, ties, and bearing areas in such models.

CODE

9.3.2.7 — Flexure sections without axial load in pretensioned members where strand embedment is less than the development length as provided in 12.9.1.1 ..... 0.75

9.3.3 — Development lengths specified in Chapter 12 do not require a ϕ-factor.

9.3.4 — In structures that rely on special moment resisting frames or special reinforced concrete structural walls to resist earthquake effects, the strength reduction factors ϕ shall be modified as given in (a) through (c):

(a) The strength reduction factor for shear shall be 0.60 for any structural member that is designed to resist earthquake effects if its nominal shear strength is less than the shear corresponding to the development of the nominal flexural strength of the member. The nominal flexural strength shall be determined considering the most critical factored axial loads and including earthquake effects;

(b) The strength reduction factor for shear in diaphragms shall not exceed the minimum strength reduction factor for shear used for the vertical components of the primary lateral-force-resisting system;

(c) The strength reduction factor for shear in joints and diagonally reinforced coupling beams shall be 0.85.

9.3.5 — Strength reduction factor ϕ for flexure, compression, shear, and bearing of structural plain concrete in Chapter 22 shall be 0.55.

9.4 — Design strength for reinforcement

Designs shall not be based on a yield strength of reinforcement f_y in excess of 80,000 psi, except for prestressing steel.

COMMENTARY

R9.3.2.7 — If a critical section occurs in a region where strand is not fully developed, failure may be by bond slip. Such a failure resembles a brittle shear failure; hence, the requirements for a reduced ϕ.

R9.3.4 — Strength reduction factors in 9.3.4 are intended to compensate for uncertainties in estimation of strength of structural members in buildings. They are based primarily on experience with constant or steadily increasing applied load. For construction in regions of high seismic risk, some of the strength reduction factors have been modified in 9.3.4 to account for the effects of displacement reversals into the nonlinear range of response on strength.

Section 9.3.4(a) refers to brittle members such as low-rise walls, portions of walls between openings, or diaphragms that are impractical to reinforce to raise their nominal shear strength above nominal flexural strength for the pertinent loading conditions.

Short structural walls were the primary vertical elements of the lateral-force-resisting system in many of the parking structures that sustained damage during the 1994 Northridge earthquake. Section 9.3.4(b) requires the shear strength reduction factor for diaphragms to be 0.60 if the shear strength reduction factor for the walls is 0.60.

R9.3.5 — The strength reduction factor ϕ for structural plain concrete design is the same for all strength conditions. Since both flexural tension strength and shear strength for plain concrete depend on the tensile strength characteristics of the concrete, with no reserve strength or ductility possible due to the absence of reinforcement, equal strength reduction factors for both bending and shear are considered appropriate.

R9.4 — Design strength for reinforcement

In addition to the upper limit of 80,000 psi for yield strength of nonprestressed reinforcement, there are limitations on yield strength in other sections of the code.

In 11.5.2, 11.6.3.4, and 11.7.6, the maximum f_y that may be used in design for shear and torsion reinforcement is 60,000 psi, except that f_y up to 80,000 psi may be used for shear reinforcement meeting the requirements of ASTM A 497.

In 19.3.2 and 21.2.5, the maximum specified f_y is 60,000 psi in shells, folded plates, and structures governed by the special seismic provisions of Chapter 21.

The deflection provisions of 9.5 and the limitations on distribution of flexural reinforcement of 10.6 become increasingly critical as f_y increases.

CODE

9.5 — Control of deflections

9.5.1 — Reinforced concrete members subjected to flexure shall be designed to have adequate stiffness to limit deflections or any deformations that adversely affect strength or serviceability of a structure.

9.5.2 — One-way construction (nonprestressed)

9.5.2.1 — Minimum thickness stipulated in Table 9.5(a) shall apply for one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections, unless computation of deflection indicates a lesser thickness can be used without adverse effects.

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE COMPUTED

	Minimum thickness, h			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Notes:
1) Span length ℓ is in inches.
2) Values given shall be used directly for members with normalweight concrete (w_c = 145 lb/ft^3) and Grade 60 reinforcement. For other conditions, the values shall be modified as follows:
a) For structural lightweight concrete having unit weight in the range 90-120 lb/ft^3, the values shall be multiplied by ($1.65 - 0.005w_c$) but not less than 1.09, where w_c is the unit weight in lb/ft^3.
b) For f_y other than 60,000 psi, the values shall be multiplied by ($0.4 + f_y/100{,}000$).

COMMENTARY

R9.5 — Control of deflections

R9.5.1 — The provisions of 9.5 are concerned only with deflections or deformations that may occur at service load levels. When long-term deflections are computed, only the dead load and that portion of the live load that is sustained need be considered.

Two methods are given for controlling deflections.[9.11] For nonprestressed beams and one-way slabs, and for composite members, provision of a minimum overall thickness as required by Table 9.5(a) will satisfy the requirements of the code for members not supporting or attached to partitions or other construction likely to be damaged by large deflections. For nonprestressed two-way construction, minimum thickness as required by 9.5.3.1, 9.5.3.2, and 9.5.3.3 will satisfy the requirements of the code.

For nonprestressed members that do not meet these minimum thickness requirements, or that support or are attached to partitions or other construction likely to be damaged by large deflections, and for all prestressed concrete flexural members, deflections should be calculated by the procedures described or referred to in the appropriate sections of the code, and are limited to the values in Table 9.5(b).

R9.5.2 — One-way construction (nonprestressed)

R9.5.2.1 — The minimum thicknesses of Table 9.5(a) apply for nonprestressed beams and one-way slabs (see 9.5.2), and for composite members (see 9.5.5). These minimum thicknesses apply only to members not supporting or attached to partitions and other construction likely to be damaged by deflection.

Values of minimum thickness should be modified if other than normalweight concrete and Grade 60 reinforcement are used. The notes beneath the table are essential to its use for reinforced concrete members constructed with structural lightweight concrete or with reinforcement having a yield strength other than 60,000 psi. If both of these conditions exist, the corrections in footnotes (a) and (b) should both be applied.

The modification for lightweight concrete in footnote (a) is based on studies of the results and discussions in Reference 9.12. No correction is given for concretes weighing between 120 and 145 lb/ft^3 because the correction term would be close to unity in this range.

The modification for yield strength in footnote (b) is approximate but should yield conservative results for the type of members considered in the table, for typical reinforcement ratios, and for values of f_y between 40,000 and 80,000 psi.

CODE

9.5.2.2 — Where deflections are to be computed, deflections that occur immediately on application of load shall be computed by usual methods or formulas for elastic deflections, considering effects of cracking and reinforcement on member stiffness.

9.5.2.3 — Unless stiffness values are obtained by a more comprehensive analysis, immediate deflection shall be computed with the modulus of elasticity E_c for concrete as specified in 8.5.1 (normalweight or lightweight concrete) and with the effective moment of inertia as follows, but not greater than I_g.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \qquad (9\text{-}8)$$

where:

$$M_{cr} = \frac{f_r I_g}{y_t} \qquad (9\text{-}9)$$

and for normalweight concrete,

$$f_r = 7.5\sqrt{f'_c} \qquad (9\text{-}10)$$

When lightweight aggregate concrete is used, one of the following modifications shall apply:

(a) When f_{ct} is specified and concrete is proportioned in accordance with 5.2, f_r shall be modified by substituting $f_{ct}/6.7$ for $\sqrt{f'_c}$, but the value of $f_{ct}/6.7$ shall not exceed $\sqrt{f'_c}$;

(b) When f_{ct} is not specified, f_r shall be multiplied by 0.75 for all-lightweight concrete, and 0.85 for sand-lightweight concrete. Linear interpolation shall be permitted if partial sand replacement is used.

9.5.2.4 — For continuous members, effective moment of inertia shall be permitted to be taken as the average of values obtained from Eq. (9-8) for the critical positive and negative moment sections. For prismatic members, effective moment of inertia shall be permitted to be taken as the value obtained from Eq. (9-8) at midspan for simple and continuous spans, and at support for cantilevers.

9.5.2.5 — Unless values are obtained by a more comprehensive analysis, additional long-term deflection resulting from creep and shrinkage of flexural members (normalweight or lightweight concrete) shall be determined by multiplying the immediate deflection caused

COMMENTARY

R9.5.2.2 — For calculation of immediate deflections of uncracked prismatic members, the usual methods or formulas for elastic deflections may be used with a constant value of $E_c I_g$ along the length of the member. However, if the member is cracked at one or more sections, or if its depth varies along the span, a more exact calculation becomes necessary.

R9.5.2.3 — The effective moment of inertia procedure described in the code and developed in Reference 9.13 was selected as being sufficiently accurate for use to control deflections.[9.14-9.16] The effective I_e was developed to provide a transition between the upper and lower bounds of I_g and I_{cr} as a function of the ratio M_{cr}/M_a. For most cases I_e will be less than I_g.

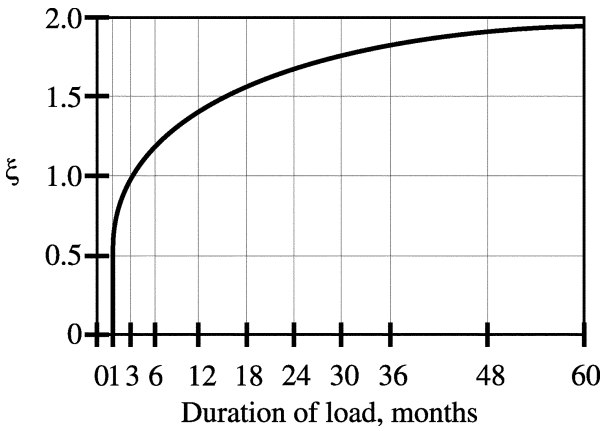


Fig. R9.5.2.5—Multipliers for long-term deflections

R9.5.2.4 — For continuous members, the code procedure suggests a simple averaging of I_e values for the positive and negative moment sections. The use of the midspan section properties for continuous prismatic members is considered satisfactory in approximate calculations primarily because the midspan rigidity (including the effect of cracking) has the dominant effect on deflections, as shown by ACI Committee 435[9.17,9.18] and SP-43.[9.11]

R9.5.2.5 — Shrinkage and creep due to sustained loads cause additional long-term deflections over and above those which occur when loads are first placed on the structure. Such deflections are influenced by temperature, humidity, curing conditions, age at time of loading, quantity of com-

CODE

by the sustained load considered, by the factor

$$\lambda = \frac{\xi}{1 + 50\rho'} \quad (9\text{-}11)$$

where ρ' shall be the value at midspan for simple and continuous spans, and at support for cantilevers. It shall be permitted to assume the time-dependent factor ξ for sustained loads to be equal to:

5 years or more	2.0
12 months	1.4
6 months	1.2
3 months	1.0

9.5.2.6 — Deflection computed in accordance with 9.5.2.2 through 9.5.2.5 shall not exceed limits stipulated in Table 9.5(b).

9.5.3 — Two-way construction (nonprestressed)

9.5.3.1 — Section 9.5.3 shall govern the minimum thickness of slabs or other two-way construction designed in accordance with the provisions of Chapter 13 and conforming with the requirements of 13.6.1.2. The thickness of slabs without interior beams spanning between the supports on all sides shall satisfy the requirements of 9.5.3.2 or 9.5.3.4. The thickness of slabs with beams spanning between the supports on all sides shall satisfy requirements of 9.5.3.3 or 9.5.3.4.

9.5.3.2 — For slabs without interior beams spanning between the supports and having a ratio of long to short span not greater than 2, the minimum thickness shall be in accordance with the provisions of Table 9.5(c) and shall not be less than the following values:

(a) Slabs without drop panels as defined in 13.3.7.1 and 13.3.7.2	5 in.

COMMENTARY

pression reinforcement, and magnitude of the sustained load. The expression given in this section is considered satisfactory for use with the code procedures for the calculation of immediate deflections, and with the limits given in Table 9.5(b). The deflection computed in accordance with this section is the additional long-term deflection due to the dead load and that portion of the live load that will be sustained for a sufficient period to cause significant time-dependent deflections.

Eq. (9-11) was developed in Reference 9.19. In Eq. (9-11) the multiplier on ξ accounts for the effect of compression reinforcement in reducing long-term deflections. $\xi = 2.0$ represents a nominal time-dependent factor for 5 years duration of loading. The curve in Fig. R9.5.2.5 may be used to estimate values of ξ for loading periods less than five years.

If it is desired to consider creep and shrinkage separately, approximate equations provided in References 9.13, 9.14, 9.19, and 9.20 may be used.

R9.5.2.6 — It should be noted that the limitations given in this table relate only to supported or attached nonstructural elements. For those structures in which structural members are likely to be affected by deflection or deformation of members to which they are attached in such a manner as to affect adversely the strength of the structure, these deflections and the resulting forces should be considered explicitly in the analysis and design of the structures as required by 9.5.1. (See Reference 9.16.)

Where long-term deflections are computed, the portion of the deflection before attachment of the nonstructural elements may be deducted. In making this correction use may be made of the curve in Fig. R9.5.2.5 for members of usual sizes and shapes.

R9.5.3 — Two-way construction (nonprestressed)

R9.5.3.2 — The minimum thicknesses in Table 9.5(c) are those that have been developed through the years. Slabs conforming to those limits have not resulted in systematic problems related to stiffness for short- and long-term loads. These limits apply to only the domain of previous experience in loads, environment, materials, boundary conditions, and spans.

CODE

COMMENTARY

TABLE 9.5(b) — MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\ell/180$*
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\ell/360$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)†	$\ell/480$‡
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\ell/240$§

* Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

† Long-term deflection shall be determined in accordance with 9.5.2.5 or 9.5.4.3, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

‡ Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

§ Limit shall not be greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

TABLE 9.5(c)—MINIMUM THICKNESS OF SLABS WITHOUT INTERIOR BEAMS

	Without drop panels†			With drop panels†		
	Exterior panels		Interior panels	Exterior panels		Interior panels
Yield strength, f_y, psi*	Without edge beams	With edge beams‡		Without edge beams	With edge beams‡	
40,000	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{40}$	$\frac{\ell_n}{40}$
60,000	$\frac{\ell_n}{30}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$
75,000	$\frac{\ell_n}{28}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{34}$	$\frac{\ell_n}{34}$

* For values of reinforcement yield strength between the values given in the table, minimum thickness shall be determined by linear interpolation.

† Drop panel is defined in 13.3.7.1 and 13.3.7.2.

‡ Slabs with beams between columns along exterior edges. The value of α for the edge beam shall not be less than 0.8.

(b) Slabs with drop panels as defined in 13.3.7.1 and 13.3.7.2 ..... 4 in.

9.5.3.3 — For slabs with beams spanning between the supports on all sides, the minimum thickness shall be as follows:

(a) For α_m equal to or less than 0.2, the provisions of 9.5.3.2 shall apply;

(b) For α_m greater than 0.2 but not greater than 2.0, the thickness shall not be less than

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{200{,}000}\right)}{36 + 5\beta(\alpha_m - 0.2)} \qquad (9\text{-}12)$$

and not less than 5 in.;

(c) For α_m greater than 2.0, the thickness shall not be less than

R9.5.3.3 — For panels having a ratio of long to short span greater than 2, the use of Eq. (9-12) and (9-13), which express the minimum thickness as a fraction of the long span, may give unreasonable results. For such panels, the rules applying to one-way construction in 9.5.2 should be used.

The requirement in 9.5.3.3(a) for α_m equal to 0.2 made it possible to eliminate Eq. (9-13) of the 1989 code. That equation gave values essentially the same as those in Table 9.5(c), as does Eq. (9-12) at a value of α_m equal to 0.2.

CODE

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{200{,}000}\right)}{36 + 9\beta} \quad (9\text{-}13)$$

and not less than 3.5 in.;

(d) At discontinuous edges, an edge beam shall be provided with a stiffness ratio α not less than 0.80 or the minimum thickness required by Eq. (9-12) or (9-13) shall be increased by at least 10 percent in the panel with a discontinuous edge.

9.5.3.4 — Slab thickness less than the minimum thickness required by 9.5.3.1, 9.5.3.2, and 9.5.3.3 shall be permitted to be used if shown by computation that the deflection will not exceed the limits stipulated in Table 9.5(b). Deflections shall be computed taking into account size and shape of the panel, conditions of support, and nature of restraints at the panel edges. The modulus of elasticity of concrete E_c shall be as specified in 8.5.1. The effective moment of inertia shall be that given by Eq. (9-8); other values shall be permitted to be used if they result in computed deflections in reasonable agreement with results of comprehensive tests. Additional long-term deflection shall be computed in accordance with 9.5.2.5.

9.5.4 — Prestressed concrete construction

9.5.4.1 — For flexural members designed in accordance with provisions of Chapter 18, immediate deflection shall be computed by usual methods or formulas for elastic deflections, and the moment of inertia of the gross concrete section shall be permitted to be used for Class U flexural members, as defined in 18.3.3.

9.5.4.2 — For Class C and Class T flexural members, as defined in 18.3.3, deflection calculations shall be based on a cracked transformed section analysis. It shall be permitted to base computations on a bilinear moment-deflection relationship, or an effective moment of inertia as defined by Eq. (9-8).

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R9.5.3.4 — The calculation of deflections for slabs is complicated even if linear elastic behavior can be assumed. For immediate deflections, the values of E_c and I_e specified in 9.5.2.3 may be used.[9.16] However, other procedures and other values of the stiffness EI may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests.

Since available data on long-term deflections of slabs are too limited to justify more elaborate procedures, the additional long-term deflection for two-way construction is required to be computed using the multipliers given in 9.5.2.5.

R9.5.4 — Prestressed concrete construction

The code requires deflections for all prestressed concrete flexural members to be computed and compared with the allowable values in Table 9.5(b).

R9.5.4.1 — Immediate deflections of Class U prestressed concrete members may be calculated by the usual methods or formulas for elastic deflections using the moment of inertia of the gross (uncracked) concrete section and the modulus of elasticity for concrete specified in 8.5.1.

R9.5.4.2 — Class C and Class T prestressed flexural members are defined in 18.3.3. Reference 9.21 gives information on deflection calculations using a bilinear moment-deflection relationship and using an effective moment of inertia. Reference 9.22 gives additional information on deflection of cracked prestressed concrete members.

Reference 9.23 shows that the I_e method can be used to compute deflections of Class T prestressed members loaded above the cracking load. For this case, the cracking moment should take into account the effect of prestress. A method for predicting the effect of nonprestressed tension steel in reducing creep camber is also given in Reference 9.23, with approximate forms given in References 9.16 and 9.24.

CODE

9.5.4.3 — Additional long-term deflection of prestressed concrete members shall be computed taking into account stresses in concrete and steel under sustained load and including effects of creep and shrinkage of concrete and relaxation of steel.

9.5.4.4 — Deflection computed in accordance with 9.5.4.1 or 9.5.4.2, and 9.5.4.3 shall not exceed limits stipulated in Table 9.5(b).

9.5.5 — Composite construction

9.5.5.1 — Shored construction

If composite flexural members are supported during construction so that, after removal of temporary supports, dead load is resisted by the full composite section, it shall be permitted to consider the composite member equivalent to a monolithically cast member for computation of deflection. For nonprestressed members, the portion of the member in compression shall determine whether values in Table 9.5(a) for normal-weight or lightweight concrete shall apply. If deflection is computed, account shall be taken of curvatures resulting from differential shrinkage of precast and cast-in-place components, and of axial creep effects in a prestressed concrete member.

COMMENTARY

R9.5.4.3 — Calculation of long-term deflections of prestressed concrete flexural members is complicated. The calculations should consider not only the increased deflections due to flexural stresses, but also the additional long-term deflections resulting from time-dependent shortening of the flexural member.

Prestressed concrete members shorten more with time than similar nonprestressed members due to the precompression in the slab or beam which causes axial creep. This creep together with concrete shrinkage results in significant shortening of the flexural members that continues for several years after construction and should be considered in design. The shortening tends to reduce the tension in the prestressing steel, reducing the precompression in the member and thereby causing increased long-term deflections.

Another factor that can influence long-term deflections of prestressed flexural members is adjacent concrete or masonry that is nonprestressed in the direction of the prestressed member. This can be a slab nonprestressed in the beam direction adjacent to a prestressed beam or a nonprestressed slab system. As the prestressed member tends to shrink and creep more than the adjacent nonprestressed concrete, the structure will tend to reach a compatibility of the shortening effects. This results in a reduction of the precompression in the prestressed member as the adjacent concrete absorbs the compression. This reduction in precompression of the prestressed member can occur over a period of years and will result in additional long-term deflections and in increase tensile stresses in the prestressed member.

Any suitable method for calculating long-term deflections of prestressed members may be used, provided all effects are considered. Guidance may be found in References 9.16, 9.23, 9.25, and 9.26.

R9.5.5 — Composite construction

Since few tests have been made to study the immediate and long-term deflections of composite members, the rules given in 9.5.5.1 and 9.5.5.2 are based on the judgment of ACI Committee 318 and on experience.

If any portion of a composite member is prestressed or if the member is prestressed after the components have been cast, the provisions of 9.5.4 apply and deflections are to be calculated. For nonprestressed composite members, deflections need to be calculated and compared with the limiting values in Table 9.5(b) only when the thickness of the member or the precast part of the member is less than the minimum thickness given in Table 9.5(a). In unshored construction the thickness of concern depends on whether the deflection before or after the attainment of effective composite action

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9.5.5.2 — Unshored construction

If the thickness of a nonprestressed precast flexural member meets the requirements of Table 9.5(a), deflection need not be computed. If the thickness of a nonprestressed composite member meets the requirements of Table 9.5(a), it is not required to compute deflection occurring after the member becomes composite, but the long-term deflection of the precast member shall be investigated for magnitude and duration of load prior to beginning of effective composite action.

9.5.5.3 — Deflection computed in accordance with 9.5.5.1 or 9.5.5.2 shall not exceed limits stipulated in Table 9.5(b).

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is being considered. (In Chapter 17, it is stated that distinction need not be made between shored and unshored members. This refers to strength calculations, not to deflections.)

CHAPTER 10 — FLEXURE AND AXIAL LOADS

CODE

10.0 — Notation

a = depth of equivalent rectangular stress block as defined in 10.2.7.1, in.

A_b = area of an individual horizontal bar or wire, in.^{2}

A_c = area of core of spirally reinforced compression member measured to outside diameter of spiral, in.^{2}

A_g = gross area of section, in.^{2}

A_s = area of nonprestressed tension reinforcement, in.^{2}

$A_{s,min}$ = minimum amount of flexural reinforcement, in.^{2} See 10.5

A_{st} = total area of longitudinal reinforcement, (bars or steel shapes), in.^{2}

A_t = area of structural steel shape, pipe, or tubing in a composite section, in.^{2}

A_1 = loaded area, in.^{2}

A_2 = the area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal, in.^{2}

b = width of compression face of member, in.

b_w = web width, in.

c = distance from extreme compression fiber to neutral axis, in.

c_c = clear cover from the nearest surface in tension to the surface of the flexural tension reinforcement, in.

C_m = a factor relating actual moment diagram to an equivalent uniform moment diagram

d = distance from extreme compression fiber to centroid of tension reinforcement, in.

E_c = modulus of elasticity of concrete, psi. See 8.5.1

E_s = modulus of elasticity of reinforcement, psi. See 8.5.2 or 8.5.3

EI = flexural stiffness of compression member. See Eq. (10-11) and Eq. (10-12), in.^{2}-lb

f_c' = specified compressive strength of concrete, psi

f_s = calculated stress in reinforcement at service loads, ksi

f_y = specified yield strength of nonprestressed reinforcement, psi

h = overall thickness of member, in.

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R10.0 — Notation

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

d_t = distance from extreme compression fiber to extreme tension steel, in.

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I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in.^{4}

I_{se} = moment of inertia of reinforcement about centroidal axis of member cross section, in.^{4}

I_t = moment of inertia of structural steel shape, pipe, or tubing about centroidal axis of composite member cross section, in.^{4}

k = effective length factor for compression members

ℓ_c = length of compression member in a frame, measured from center to center of the joints in the frame, in.

ℓ_u = unsupported length of compression member, in.

M_c = factored moment to be used for design of compression member, in.-lb

M_s = moment due to loads causing appreciable sway, in.-lb

M_u = factored moment at section, in.-lb

M_1 = smaller factored end moment on a compression member, positive if member is bent in single curvature, negative if bent in double curvature, in.-lb

M_{1ns} = factored end moment on a compression member at the end at which M_1 acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis, in.-lb

M_{1s} = factored end moment on compression member at the end at which M_1 acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis, in.-lb

M_2 = larger factored end moment on compression member, always positive, in.-lb

$M_{2,\,min}$ = minimum value of M_2, in.-lb

M_{2ns} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis, in.-lb

M_{2s} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis, in.-lb

P_b = nominal axial load strength at balanced strain conditions. See 10.3.2, lb

P_c = critical load. See Eq. (10-10), lb

P_n = nominal axial load strength at given eccentricity, lb

P_o = nominal axial load strength at zero eccentricity, lb

P_u = factored axial load at given eccentricity, lb $\leq \phi P_n$

Q = stability index for a story. See 10.11.4

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r = radius of gyration of cross section of a compression member, in.

s = center-to-center spacing of flexural tension reinforcement nearest to the extreme tension face, in. (where there is only one bar or wire nearest to the extreme tension face, s is the width of the extreme tension face.)

s_{sk} = spacing of skin reinforcement, in.

V_u = factored horizontal shear in a story, lb

b_1 = factor defined in 10.2.7.3

b_d = (a) for nonsway frames, b_d is the ratio of the maximum factored axial sustained load to the maximum factored axial load associated with the same load combination;
(b) for sway frames, except as required in (c) of this definition, b_d is the ratio of the maximum factored sustained shear within a story to the maximum factored shear in that story;
(c) for stability checks of sway frames carried out in accordance with 10.13.6, b_d is the ratio of the maximum factored sustained axial load to the maximum factored axial load

d_{ns} = moment magnification factor for frames braced against sidesway, to reflect effects of member curvature between ends of compression member

d_s = moment magnification factor for frames not braced against sidesway, to reflect lateral drift resulting from lateral and gravity loads

D_o = relative lateral deflection between the top and bottom of a story due to V_u, computed using a first-order elastic frame analysis and stiffness values satisfying 10.11.1, in.

e_t = net tensile strain in extreme tension steel at nominal strength

r = ratio of nonprestressed tension reinforcement
= A_s/bd

r_b = reinforcement ratio producing balanced strain conditions. See 10.3.2

r_s = ratio of volume of spiral reinforcement to total volume of core (out-to-out of spirals) of a spirally reinforced compression member

f = strength reduction factor. See 9.3

f_K = stiffness reduction factor. See R10.12.3

10.1 — Scope

Provisions of Chapter 10 shall apply for design of members subject to flexure or axial loads or to combined flexure and axial loads.

10.2 — Design assumptions

10.2.1 — Strength design of members for flexure and axial loads shall be based on assumptions given in 10.2.2 through 10.2.7, and on satisfaction of applicable conditions of equilibrium and compatibility of strains.

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The definition of net tensile strain in 2.1 excludes strains due to effective prestress, creep, shrinkage, and temperature.

R10.2 — Design assumptions

R10.2.1 — The strength of a member computed by the strength design method of the code requires that two basic conditions be satisfied: (1) static equilibrium, and (2) compatibility of strains. Equilibrium between the compressive

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and tensile forces acting on the cross section at nominal strength should be satisfied. Compatibility between the stress and strain for the concrete and the reinforcement at nominal strength conditions should also be established within the design assumptions allowed by 10.2.

10.2.2 — Strain in reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis, except that, for deep beams as defined in 10.7.1, an analysis that considers a nonlinear distribution of strain shall be used. Alternatively, it shall be permitted to use a strut-and-tie model. See 10.7, 11.8, and Appendix A.

R10.2.2—Many tests have confirmed that the distribution of strain is essentially linear across a reinforced concrete cross section, even near ultimate strength.

The strain in both reinforcement and in concrete is assumed to be directly proportional to the distance from the neutral axis. This assumption is of primary importance in design for determining the strain and corresponding stress in the reinforcement.

10.2.3 — Maximum usable strain at extreme concrete compression fiber shall be assumed equal to 0.003.

R10.2.3 — The maximum concrete compressive strain at crushing of the concrete has been observed in tests of various kinds to vary from 0.003 to higher than 0.008 under special conditions. However, the strain at which ultimate moments are developed is usually about 0.003 to 0.004 for members of normal proportions and materials.

10.2.4 — Stress in reinforcement below specified yield strength f_y for grade of reinforcement used shall be taken as E_s times steel strain. For strains greater than that corresponding to f_y, stress in reinforcement shall be considered independent of strain and equal to f_y.

R10.2.4 — For deformed reinforcement, it is reasonably accurate to assume that the stress in reinforcement is proportional to strain below the yield strength f_y. The increase in strength due to the effect of strain hardening of the reinforcement is neglected for strength computations. In strength computations, the force developed in tensile or compressive reinforcement is computed as,

when $e_s < e_y$ (yield strain)

$$A_s f_s = A_s E_s e_s$$

when $e_s \geq e_y$

$$A_s f_s = A_s f_y$$

where e_s is the value from the strain diagram at the location of the reinforcement. For design, the modulus of elasticity of steel reinforcement E_s may be taken as 29,000,000 psi (see 8.5.2).

10.2.5 — Tensile strength of concrete shall be neglected in axial and flexural calculations of reinforced concrete, except when meeting requirements of 18.4.

R10.2.5 — The tensile strength of concrete in flexure (modulus of rupture) is a more variable property than the compressive strength and is about 10 to 15 percent of the compressive strength. Tensile strength of concrete in flexure is neglected in strength design. For members with normal percentages of reinforcement, this assumption is in good agreement with tests. For very small percentages of reinforcement, neglect of the tensile strength at ultimate is usually correct.

The strength of concrete in tension, however, is important in cracking and deflection considerations at service loads.

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10.2.6 — The relationship between concrete compressive stress distribution and concrete strain shall be assumed to be rectangular, trapezoidal, parabolic, or any other shape that results in prediction of strength in substantial agreement with results of comprehensive tests.

10.2.7 — Requirements of 10.2.6 are satisfied by an equivalent rectangular concrete stress distribution defined by the following:

10.2.7.1 — Concrete stress of $0.85f_c'$ shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross section and a straight line located parallel to the neutral axis at a distance $a = b_1 c$ from the fiber of maximum compressive strain.

10.2.7.2 — Distance c from the fiber of maximum strain to the neutral axis shall be measured in a direction perpendicular to that axis.

10.2.7.3 — Factor b_1 shall be taken as 0.85 for concrete strengths f_c' up to and including 4000 psi. For strengths above 4000 psi, b_1 shall be reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi, but b_1 shall not be taken less than 0.65.

10.3 — General principles and requirements

10.3.1 — Design of cross sections subject to flexure or axial loads, or to combined flexure and axial loads, shall be based on stress and strain compatibility using assumptions in 10.2.

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R10.2.6 — This assumption recognizes the inelastic stress distribution of concrete at high stress. As maximum stress is approached, the stress-strain relationship for concrete is not a straight line but some form of a curve (stress is not proportional to strain). The general shape of a stress-strain curve is primarily a function of concrete strength and consists of a rising curve from zero to a maximum at a compressive strain between 0.0015 and 0.002 followed by a descending curve to an ultimate strain (crushing of the concrete) from 0.003 to higher than 0.008. As discussed under R10.2.3, the code sets the maximum usable strain at 0.003 for design.

The actual distribution of concrete compressive stress is complex and usually not known explicitly. Research has shown that the important properties of the concrete stress distribution can be approximated closely using any one of several different assumptions as to the form of stress distribution. The code permits any particular stress distribution to be assumed in design if shown to result in predictions of ultimate strength in reasonable agreement with the results of comprehensive tests. Many stress distributions have been proposed. The three most common are the parabola, trapezoid, and rectangle.

R10.2.7 — For design, the code allows the use of an equivalent rectangular compressive stress distribution (stress block) to replace the more exact concrete stress distribution. In the equivalent rectangular stress block, an average stress of $0.85f_c'$ is used with a rectangle of depth $a = b_1 c$. The b_1 of 0.85 for concrete with $f_c' \leq 4000$ psi and 0.05 less for each 1000 psi of f_c' in excess of 4000 was determined experimentally.

In the 1976 supplement to the 1971 code, a lower limit of b_1 equal to 0.65 was adopted for concrete strengths greater than 8000 psi. Research data from tests with high strength concretes[10.1,10.2] supported the equivalent rectangular stress block for concrete strengths exceeding 8000 psi, with a b_1 equal to 0.65. Use of the equivalent rectangular stress distribution specified in the 1971 code, with no lower limit on b_1, resulted in inconsistent designs for high strength concrete for members subject to combined flexure and axial load.

The equivalent rectangular stress distribution does not represent the actual stress distribution in the compression zone at ultimate, but does provide essentially the same results as those obtained in tests.[10.3]

R10.3 — General principles and requirements

R10.3.1 — Design strength equations for members subject to flexure or combined flexure and axial load are derived in the paper, "Rectangular Concrete Stress Distribution in Ultimate Strength Design."[10.3] Reference 10.3 and previous editions of this commentary also give the derivations of strength equations for cross sections other than rectangular.

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10.3.2 — Balanced strain conditions exist at a cross section when tension reinforcement reaches the strain corresponding to its specified yield strength f_y just as concrete in compression reaches its assumed ultimate strain of 0.003.

10.3.3 — Sections are compression-controlled when the net tensile strain in the extreme tension steel is equal to or less than the compression-controlled strain limit at the time the concrete in compression reaches its assumed strain limit of 0.003. The compression-controlled strain limit is the net tensile strain in the reinforcement at balanced strain conditions. For Grade 60 reinforcement, and for all prestressed reinforcement, it shall be permitted to set the compression-controlled strain limit equal to 0.002.

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R10.3.2 — A balanced strain condition exists at a cross section when the maximum strain at the extreme compression fiber just reaches 0.003 simultaneously with the first yield strain f_y/E_s in the tension reinforcement. The reinforcement ratio r_b, which produces balanced conditions under flexure, depends on the shape of the cross section and the location of the reinforcement.

R10.3.3 — The nominal flexural strength of a member is reached when the strain in the extreme compression fiber reaches the assumed strain limit 0.003. The net tensile strain e_t is the tensile strain in the extreme tension steel at nominal strength, exclusive of strains due to prestress, creep, shrinkage, and temperature. The net tensile strain in the extreme tension steel is determined from a linear strain distribution at nominal strength, shown in Fig. R10.3.3, using similar triangles.

When the net tensile strain in the extreme tension steel is sufficiently large (equal to or greater than 0.005), the section is defined as tension-controlled where ample warning of failure with excessive deflection and cracking may be expected. When the net tensile strain in the extreme tension steel is small (less than or equal to the compression-controlled strain limit), a brittle failure condition may be expected, with little warning of impending failure. Flexural members are usually tension-controlled, whereas compression members are usually compression-controlled. Some sections, such as those with small axial load and large bending moment, will have net tensile strain in the extreme tension steel between the above limits. These sections are in a transition region between compression- and tension-controlled sections. Section 9.3.2 specifies the appropriate strength reduction factors for tension-controlled and compression-controlled sections, and for intermediate cases in the transition region.

Before the development of these provisions, the limiting tensile strain for flexural members was not stated, but was implicit in the maximum tension reinforcement ratio that was given as a fraction of r_b, which was dependent on the yield

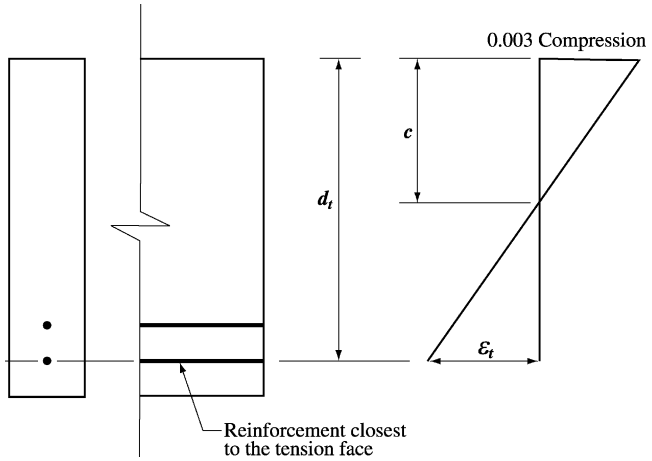


Fig. R10.3.3— Strain distribution and net tensile strain.

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strength of the reinforcement. The net tensile strain limit of 0.005 for tension-controlled sections was chosen to be a single value that applies to all types of steel (prestressed and nonprestressed) permitted by this code.

Unless unusual amounts of ductility are required, the 0.005 limit will provide ductile behavior for most designs. One condition where greater ductile behavior is required is in design for redistribution of moments in continuous members and frames. Section 8.4 permits redistribution of negative moments. Since moment redistribution is dependent on adequate ductility in hinge regions, moment redistribution is limited to sections that have a net tensile strain of at least 0.0075.

For beams with compression reinforcement, or T-beams, the effects of compression reinforcement and flanges are automatically accounted for in the computation of net tensile strain ε_t.

10.3.4 — Sections are tension-controlled when the net tensile strain in the extreme tension steel is equal to or greater than 0.005 just as the concrete in compression reaches its assumed strain limit of 0.003. Sections with net tensile strain in the extreme tension steel between the compression-controlled strain limit and 0.005 constitute a transition region between compression-controlled and tension-controlled sections.

10.3.5 — For nonprestressed flexural members and nonprestressed members with axial load less than $0.10 f'_c A_g$, the net tensile strain ε_t at nominal strength shall not be less than 0.004.

10.3.5.1 — Use of compression reinforcement shall be permitted in conjunction with additional tension reinforcement to increase the strength of flexural members.

R10.3.5 — The effect of this limitation is to restrict the reinforcement ratio in nonprestressed beams to about the same ratio as in editions of the code prior to 2002. The reinforcement limit of $0.75\rho_b$ results in a net tensile strain at nominal strength of 0.00376. The proposed limit of 0.004 is slightly more conservative. This limitation does not apply to prestressed members.

10.3.6 — Design axial load strength ϕP_n of compression members shall not be taken greater than the following:

10.3.6.1 — For nonprestressed members with spiral reinforcement conforming to 7.10.4 or composite members conforming to 10.16:

$$\phi P_{n,max} = 0.85\phi [0.85 f'_c (A_g - A_{st}) + f_y A_{st}] \quad (10\text{-}1)$$

10.3.6.2—For nonprestressed members with tie reinforcement conforming to 7.10.5:

$$\phi P_{n,max} = 0.80\phi [0.85 f'_c (A_g - A_{st}) + f_y A_{st}] \quad (10\text{-}2)$$

R10.3.6 and R10.3.7 — The minimum design eccentricities included in the 1963 and 1971 codes were deleted from the 1977 code except for consideration of slenderness effects in compression members with small or zero computed end moments (see 10.12.3.2). The specified minimum eccentricities were originally intended to serve as a means of reducing the axial load design strength of a section in pure compression to account for accidental eccentricities not considered in the analysis that may exist in a compression member, and to recognize that concrete strength may be less than f'_c under sustained high loads. The primary purpose of the minimum eccentricity requirement was to limit the maximum design axial load strength of a compression member. This is now accomplished directly in 10.3.6 by

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10.3.6.3 — For prestressed members, design axial load strength ϕP_n shall not be taken greater than 0.85 (for members with spiral reinforcement) or 0.80 (for members with tie reinforcement) of the design axial load strength at zero eccentricity ϕP_o.

10.3.7 — Members subject to compressive axial load shall be designed for the maximum moment that can accompany the axial load. The factored axial load P_u at given eccentricity shall not exceed that given in 10.3.6. The maximum factored moment M_u shall be magnified for slenderness effects in accordance with 10.10.

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limiting the design axial load strength of a section in pure compression to 85 or 80 percent of the nominal strength. These percentage values approximate the axial load strengths at e/h ratios of 0.05 and 0.10, specified in the earlier codes for the spirally reinforced and tied members, respectively. The same axial load limitation applies to both cast-in-place and precast compression members. Design aids and computer programs based on the minimum eccentricity requirement of the 1963 and 1971 codes are equally applicable.

For prestressed members, the design axial load strength in pure compression is computed by the strength design methods of Chapter 10, including the effect of the prestressing force.

Compression member end moments should be considered in the design of adjacent flexural members. In nonsway frames, the effects of magnifying the end moments need not be considered in the design of the adjacent beams. In sway frames, the magnified end moments should be considered in designing the flexural members, as required in 10.13.7.

Corner and other columns exposed to known moments about each axis simultaneously should be designed for biaxial bending and axial load. Satisfactory methods are available in the *ACI Design Handbook*[10.4] and the *CRSI Handbook*.[10.5] The reciprocal load method[10.6] and the load contour method[10.7] are the methods used in those two handbooks. Research[10.8,10.9] indicates that using the equivalent rectangular stress block provisions of 10.2.7 produces satisfactory strength estimates for doubly symmetric sections. A simple and somewhat conservative estimate of nominal strength P_{ni} can be obtained from the reciprocal load relationship[10.6]

$$\frac{1}{P_{ni}} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_o}$$

where:

P_{ni} = nominal axial load strength at given eccentricity along both axes
P_o = nominal axial load strength at zero eccentricity
P_{nx} = nominal axial load strength at given eccentricity along *x*-axis
P_{ny} = nominal axial load strength at given eccentricity along *y*-axis

This relationship is most suitable when values P_{nx} and P_{ny} are greater than the balanced axial force P_b for the particular axis.

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10.4 — Distance between lateral supports of flexural members

10.4.1 — Spacing of lateral supports for a beam shall not exceed 50 times the least width ***b*** of compression flange or face.

10.4.2 — Effects of lateral eccentricity of load shall be taken into account in determining spacing of lateral supports.

10.5 — Minimum reinforcement of flexural members

10.5.1 — At every section of a flexural member where tensile reinforcement is required by analysis, except as provided in 10.5.2, 10.5.3, and 10.5.4, the area A_s provided shall not be less than that given by

$$A_{s,min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \qquad (10\text{-}3)$$

and not less than $200 b_w d/f_y$.

10.5.2 — For statically determinate members with a flange in tension, the area $A_{s,min}$ shall be equal to or greater than the value given by Eq. (10-3) with b_w replaced by either $2b_w$ or the width of the flange, whichever is smaller.

10.5.3 — The requirements of 10.5.1 and 10.5.2 need not be applied if at every section the area of tensile reinforcement provided is at least one-third greater than that required by analysis.

10.5.4 — For structural slabs and footings of uniform thickness the minimum area of tensile reinforcement in the direction of the span shall be the same as that

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R10.4 — Distance between lateral supports of flexural members

Tests[10.10,10.11] have shown that laterally unbraced reinforced concrete beams of any reasonable dimensions, even when very deep and narrow, will not fail prematurely by lateral buckling provided the beams are loaded without lateral eccentricity that causes torsion.

Laterally unbraced beams are frequently loaded off center (lateral eccentricity) or with slight inclination. Stresses and deformations set up by such loading become detrimental for narrow, deep beams, the more so as the unsupported length increases. Lateral supports spaced closer than **50*b*** may be required by loading conditions.

R10.5 — Minimum reinforcement of flexural members

The provision for a minimum amount of reinforcement applies to flexural members, which for architectural or other reasons, are larger in cross section than required for strength. With a very small amount of tensile reinforcement, the computed moment strength as a reinforced concrete section using cracked section analysis becomes less than that of the corresponding unreinforced concrete section computed from its modulus of rupture. Failure in such a case can be sudden.

To prevent such a failure, a minimum amount of tensile reinforcement is required by 10.5.1 in both positive and negative moment regions. When concrete strength higher than about 5000 psi is used, the $200/f_y$ value previously prescribed may not be sufficient. Equation (10-3) gives the same amount of reinforcement as $200 b_{wd}/f_y$ when f'_c equals 4440 psi. When the flange of a section is in tension, the amount of tensile reinforcement needed to make the strength of the reinforced section equal that of the unreinforced section is about twice that for a rectangular section or that of a flanged section with the flange in compression. A higher amount of minimum tensile reinforcement is particularly necessary in cantilevers and other statically determinate members where there is no possibility for redistribution of moments.

R10.5.3 — The minimum reinforcement required by Eq. (10-3) is to be provided wherever reinforcement is needed, except where such reinforcement is at least one-third greater than that required by analysis. This exception provides sufficient additional reinforcement in large members where the amount required by 10.5.1 or 10.5.2 would be excessive.

R10.5.4 — The minimum reinforcement required for slabs should be equal to the same amount as that required by 7.12 for shrinkage and temperature reinforcement.

CODE

required by 7.12. Maximum spacing of this reinforcement shall not exceed three times the thickness, nor 18 in.

COMMENTARY

Soil-supported slabs such as slabs on grade are not considered to be structural slabs in the context of this section, unless they transmit vertical loads from other parts of the structure to the soil. Reinforcement, if any, in soil-supported slabs should be proportioned with due consideration of all design forces. Mat foundations and other slabs that help support the structure vertically should meet the requirements of this section.

In reevaluating the overall treatment of 10.5, the maximum spacing for reinforcement in structural slabs (including footings) was reduced from the **5*h*** for temperature and shrinkage reinforcement to the compromise value of **3*h***, which is somewhat larger than the **2*h*** limit of 13.3.2 for two-way slab systems.

10.6 — Distribution of flexural reinforcement in beams and one-way slabs

R10.6 — Distribution of flexural reinforcement in beams and one-way slabs

10.6.1 — This section prescribes rules for distribution of flexural reinforcement to control flexural cracking in beams and in one-way slabs (slabs reinforced to resist flexural stresses in only one direction).

R10.6.1 — Many structures designed by working stress methods and with low steel stress served their intended functions with very limited flexural cracking. When high strength reinforcing steels are used at high service load stresses, however, visible cracks should be expected, and steps should be taken in detailing of the reinforcement to control cracking. For protection of reinforcement against corrosion, and for aesthetic reasons, many fine hairline cracks are preferable to a few wide cracks.

Control of cracking is particularly important when reinforcement with a yield strength in excess of 40,000 psi is used. Current good detailing practices will usually lead to adequate crack control even when reinforcement of 60,000 psi yield is used.

Extensive laboratory work[10.12-10.14] involving deformed bars has confirmed that crack width at service loads is proportional to steel stress. The significant variables reflecting steel detailing were found to be thickness of concrete cover and the area of concrete in the zone of maximum tension surrounding each individual reinforcing bar.

Crack width is inherently subject to wide scatter even in careful laboratory work and is influenced by shrinkage and other time-dependent effects. The better crack control is obtained when the steel reinforcement is well distributed over the zone of maximum concrete tension.

10.6.2 — Distribution of flexural reinforcement in two-way slabs shall be as required by 13.3.

10.6.3 — Flexural tension reinforcement shall be well distributed within maximum flexural tension zones of a member cross section as required by 10.6.4.

R10.6.3 — Several bars at moderate spacing are much more effective in controlling cracking than one or two larger bars of equivalent area.

CODE

10.6.4 — The spacing s of reinforcement closest to a surface in tension shall not exceed that given by

$$s = \frac{540}{f_s} - 2.5c_c \quad (10\text{-}4)$$

but not greater than $12(36/f_s)$.

Calculated stress f_s (in ksi) in reinforcement at service load shall be computed as the unfactored moment divided by the product of steel area and internal moment arm. It shall be permitted to take f_s as 60 percent of specified yield strength.

10.6.5 — Provisions of 10.6.4 are not sufficient for structures subject to very aggressive exposure or designed to be watertight. For such structures, special investigations and precautions are required.

10.6.6 — Where flanges of T-beam construction are in tension, part of the flexural tension reinforcement shall be distributed over an effective flange width as defined in 8.10, or a width equal to one-tenth the span, whichever is smaller. If the effective flange width exceeds one-tenth the span, some longitudinal reinforcement shall be provided in the outer portions of the flange.

10.6.7 — If the effective depth d of a beam or joist exceeds 36 in., longitudinal skin reinforcement shall be uniformly distributed along both side faces of the member for a distance $d/2$ nearest the flexural tension reinforcement. The spacing s_{sk} between longitudinal bars or wires of the skin reinforcement shall not exceed the least of $d/6$, 12 in., and $1000A_b/(d - 30)$. It shall be permitted to include such reinforcement in strength computations if a strain compatibility analysis is made to determine stress in the individual bars or wires. The total area of longitudinal skin reinforcement in both faces need not exceed one-half of the required flexural tensile reinforcement.

COMMENTARY

R10.6.4 — This section replaces the z factor requirements of the 1995 and previous code editions. The maximum bar spacing is now specified directly.[10.15,10.16,10.17] For the usual case of beams with Grade 60 reinforcement and 2 in. clear cover to the main reinforcement, with f_s = 36 ksi, the maximum bar spacing is 10 in.

Crack widths in structures are highly variable. In previous codes, provisions were given for distribution of reinforcement that were based on empirical equations using a calculated maximum crack width of 0.016 in. The current provisions for spacing are intended to limit surface cracks to a width that is generally acceptable in practice but may vary widely in a given structure.

The role of cracks in the corrosion of reinforcement is controversial. Research[10.18,10.19] shows that corrosion is not clearly correlated with surface crack widths in the range normally found with reinforcement stresses at service load levels. For this reason, the former distinction between interior and exterior exposure has been eliminated.

R10.6.5 — Although a number of studies have been conducted, clear experimental evidence is not available regarding the crack width beyond which a corrosion danger exists. Exposure tests indicate that concrete quality, adequate compaction, and ample concrete cover may be of greater importance for corrosion protection than crack width at the concrete surface.

R10.6.6 — In major T-beams, distribution of the negative reinforcement for control of cracking should take into account two considerations: (1) wide spacing of the reinforcement across the full effective width of flange may cause some wide cracks to form in the slab near the web and, (2) close spacing near the web leaves the outer regions of the flange unprotected. The one-tenth limitation is to guard against too wide a spacing, with some additional reinforcement required to protect the outer portions of the flange.

R10.6.7 — For relatively deep flexural members, some reinforcement should be placed near the vertical faces of the tension zone to control cracking in the web.[10.16] (See Fig. R10.6.7.) Without such auxiliary steel, the width of the cracks in the web may exceed the crack widths at the level of the flexural tension reinforcement.

Where the provisions for deep beams, walls, or precast panels require more steel, those provisions (along with their spacing requirements) will govern.

CODE COMMENTARY

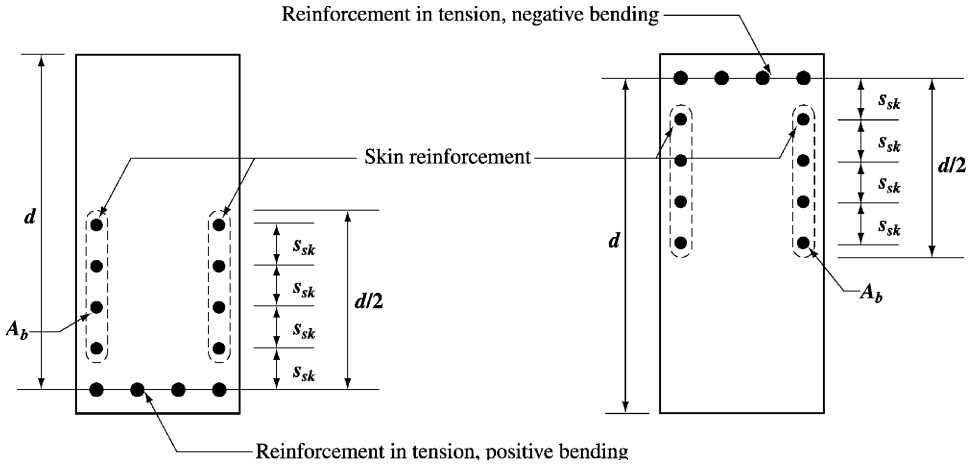


*Fig. R10.6.7—Skin reinforcement for beams and joists with **d** > 36 in.*

10.7 — Deep beams

10.7.1 — Deep beams are members loaded on one face and supported on the opposite face so that compression struts can develop between the loads and the supports, and have either:

(a) clear spans, ℓ_n, equal to or less than four times the overall member depth; or

(b) regions loaded with concentrated loads within twice the member depth from the face of the support.

Deep beams shall be designed either taking into account nonlinear distribution of strain, or by Appendix A. (See also 11.8.1 and 12.10.6.) Lateral buckling shall be considered.

10.7.2 — Shear strength of deep beams shall be in accordance with 11.8.

10.7.3 — Minimum flexural tension reinforcement shall conform to 10.5.

10.7.4 — Minimum horizontal and vertical reinforcement in the side faces of deep beams shall satisfy either A.3.3 or 11.8.4 and 11.8.5.

R10.7 — Deep beams

The span-to-depth ratios used to define deep beams in the 1999 and earlier codes were based on papers published in 1946 and 1953. The definitions of deep beams given in 10.7.1 and 11.8.1 of these earlier codes were different from each other and different from the 2002 code definition that is based on D-region behavior (see Appendix A). The definitions of deep beams in Sections 10.7.1 and 11.8.1 of the 2002 code are consistent with each other.

The 2002 code does not contain detailed requirements for designing deep beams for flexure except that nonlinearity of strain distribution and lateral buckling is to be considered. Suggestions for the design of deep beams for flexure are given in References 10.20, 10.21, and 10.22.

10.8 — Design dimensions for compression members

10.8.1 — Isolated compression member with multiple spirals

Outer limits of the effective cross section of a compression member with two or more interlocking spirals shall be taken at a distance outside the extreme limits of the spirals equal to the minimum concrete cover required by 7.7.

R10.8 — Design dimensions for compression members

With the 1971 code, minimum sizes for compression members were eliminated to allow wider utilization of reinforced concrete compression members in smaller size and lightly loaded structures, such as low-rise residential and light office buildings. The engineer should recognize the need for careful workmanship, as well as the increased significance of shrinkage stresses with small sections.

CODE

10.8.2 — Compression member built monolithically with wall

Outer limits of the effective cross section of a spirally reinforced or tied reinforced compression member built monolithically with a concrete wall or pier shall be taken not greater than 1-1/2 in. outside the spiral or tie reinforcement.

10.8.3 — Equivalent circular compression member

As an alternative to using the full gross area for design of a compression member with a square, octagonal, or other shaped cross section, it shall be permitted to use a circular section with a diameter equal to the least lateral dimension of the actual shape. Gross area considered, required percentage of reinforcement, and design strength shall be based on that circular section.

10.8.4 — Limits of section

For a compression member with a cross section larger than required by considerations of loading, it shall be permitted to base the minimum reinforcement and strength on a reduced effective area A_g not less than one-half the total area. This provision shall not apply in regions of high seismic risk.

10.9 — Limits for reinforcement of compression members

10.9.1 — Area of longitudinal reinforcement for noncomposite compression members shall be not less than 0.01 nor more than 0.08 times gross area A_g of section.

COMMENTARY

R10.8.2, R10.8.3, and R10.8.4 — For column design,[10.23] the code provisions for quantity of reinforcement, both vertical and spiral, are based on the gross column area and core area, and the design strength of the column is based on the gross area of the column section. In some cases, however, the gross area is larger than necessary to carry the factored load. The basis of 10.8.2, 10.8.3, and 10.8.4 is that it is satisfactory to design a column of sufficient size to carry the factored load and then simply add concrete around the designed section without increasing the reinforcement to meet the minimum percentages required by 10.9.1. The additional concrete should not be considered as carrying load; however, the effects of the additional concrete on member stiffness should be included in the structural analysis. The effects of the additional concrete also should be considered in design of the other parts of the structure that interact with the oversize member.

R10.9 — Limits for reinforcement of compression members

R10.9.1 — This section prescribes the limits on the amount of longitudinal reinforcement for noncomposite compression members. If the use of high reinforcement ratios would involve practical difficulties in the placing of concrete, a lower percentage and hence a larger column, or higher strength concrete or reinforcement (see R9.4) should be considered. The percentage of reinforcement in columns should usually not exceed 4 percent if the column bars are required to be lap spliced.

Minimum reinforcement — Since the design methods for columns incorporate separate terms for the load carried by concrete and by reinforcement, it is necessary to specify some minimum amount of reinforcement to ensure that only reinforced concrete columns are designed by these procedures. Reinforcement is necessary to provide resistance to bending, which may exist whether or not computations show that bending exists, and to reduce the effects of creep and shrinkage of the concrete under sustained compressive stresses. Tests have shown that creep and shrinkage tend to transfer load from the concrete to the reinforcement, with a consequent increase in stress in the reinforcement, and that this increase is greater as the ratio of reinforcement decreases. Unless a lower limit is placed on this ratio, the stress in the reinforcement may increase to the yield level

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COMMENTARY

under sustained service loads. This phenomenon was emphasized in the report of ACI Committee 105[10.24] and minimum reinforcement ratios of 0.01 and 0.005 were recommended for spiral and tied columns, respectively. However, in all editions of the code since 1936, the minimum ratio has been 0.01 for both types of laterally reinforced columns.

Maximum reinforcement — Extensive tests of the ACI column investigation[10.24] included reinforcement ratios no greater than 0.06. Although other tests with as much as 17 percent reinforcement in the form of bars produced results similar to those obtained previously, it is necessary to note that the loads in these tests were applied through bearing plates on the ends of the columns and the problem of transferring a proportional amount of the load to the bars was thus minimized or avoided. Maximum ratios of 0.08 and 0.03 were recommended by ACI Committee 105[10.24] for spiral and tied columns, respectively. In the 1936 code, these limits were made 0.08 and 0.04, respectively. In the 1956 code, the limit for tied columns with bending was raised to 0.08. Since the 1963 code, it has been required that bending be considered in the design of all columns, and the maximum ratio of 0.08 has been applied to both types of columns. This limit can be considered a practical maximum for reinforcement in terms of economy and requirements for placing.

10.9.2 — Minimum number of longitudinal bars in compression members shall be 4 for bars within rectangular or circular ties, 3 for bars within triangular ties, and 6 for bars enclosed by spirals conforming to 10.9.3.

R10.9.2 — For compression members, a minimum of four longitudinal bars are required when bars are enclosed by rectangular or circular ties. For other shapes, one bar should be provided at each apex or corner and proper lateral reinforcement provided. For example, tied triangular columns require three longitudinal bars, one at each apex of the triangular ties. For bars enclosed by spirals, six bars are required.

When the number of bars in a circular arrangement is less than eight, the orientation of the bars will affect the moment strength of eccentrically loaded columns and should be considered in design.

10.9.3 — Ratio of spiral reinforcement ρ_s shall be not less than the value given by

$$\rho_s = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_y} \qquad (10\text{-}5)$$

where f_y is the specified yield strength of spiral reinforcement but not more than 60,000 psi.

R10.9.3 — The effect of spiral reinforcement in increasing the load-carrying strength of the concrete within the core is not realized until the column has been subjected to a load and deformation sufficient to cause the concrete shell outside the core to spall off. The amount of spiral reinforcement required by Eq. (10-5) is intended to provide additional load-carrying strength for concentrically loaded columns equal to or slightly greater than the strength lost when the shell spalls off. This principle was recommended by ACI Committee 105[10.24] and has been a part of the code since 1963. The derivation of Eq. (10-5) is given in the ACI Committee 105 report. Tests and experience show that columns containing the amount of spiral reinforcement required by this section exhibit considerable toughness and ductility.

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10.10 — Slenderness effects in compression members

10.10.1 — Except as allowed in 10.10.2, the design of compression members, restraining beams, and other supporting members shall be based on the factored forces and moments from a second-order analysis considering material nonlinearity and cracking, as well as the effects of member curvature and lateral drift, duration of the loads, shrinkage and creep, and interaction with the supporting foundation. The dimensions of each member cross section used in the analysis shall be within 10 percent of the dimensions of the members shown on the design drawings or the analysis shall be repeated. The analysis procedure shall have been shown to result in prediction of strength in substantial agreement with the results of comprehensive tests of columns in statically indeterminate reinforced concrete structures.

10.10.2 — As an alternate to the procedure prescribed in 10.10.1, it shall be permitted to base the design of compression members, restraining beams, and other supporting members on axial forces and moments from the analyses described in 10.11.

10.11 — Magnified moments — General

COMMENTARY

R10.10 — Slenderness effects in compression members

Provisions for slenderness effects in compression members and frames were revised in the 1995 code to better recognize the use of second-order analyses and to improve the arrangement of the provisions dealing with sway (unbraced) and nonsway (braced) frames.[10.25] The use of a refined nonlinear second-order analysis is permitted in 10.10.1. Sections 10.11, 10.12, and 10.13 present an approximate design method based on the traditional moment magnifier method. For sway frames, the magnified sway moment $\delta_s M_s$ may be calculated using a second-order elastic analysis, by an approximation to such an analysis, or by the traditional sway moment magnifier.

R10.10.1 — Two limits are placed on the use of the refined second-order analysis. First, the structure that is analyzed should have members similar to those in the final structure. If the members in the final structure have cross-sectional dimensions more than 10 percent different from those assumed in the analysis, new member properties should be computed and the analysis repeated. Second, the refined second-order analysis procedure should have been shown to predict ultimate loads within 15 percent of those reported in tests of indeterminate reinforced concrete structures. At the very least, the comparison should include tests of columns in planar nonsway frames, sway frames, and frames with varying column stiffnesses. To allow for variability in the actual member properties and in the analysis, the member properties used in analysis should be multiplied by a stiffness reduction factor f_K less than one. For consistency with the second-order analysis in 10.13.4.1, the stiffness reduction factor f_K can be taken as 0.80. The concept of a stiffness reduction factor f_K is discussed in R10.12.3.

R10.10.2 — As an alternate to the refined second-order analysis of 10.10.1, design may be based on elastic analyses and the moment magnifier approach.[10.26,10.27] For sway frames the magnified sway moments may be calculated using a second-order elastic analysis based on realistic stiffness values. See R10.13.4.1.

R10.11 — Magnified moments — General

This section describes an approximate design procedure that uses the moment magnifier concept to account for slenderness effects. Moments computed using an ordinary first-order frame analysis are multiplied by a moment magnifier that is a function of the factored axial load P_u and the critical buckling load P_c for the column. Nonsway and sway frames are treated separately in 10.12 and 10.13. Provisions applicable to both nonsway and sway columns are given in 10.11. A first-order frame analysis is an elastic analysis that does not include the internal force effects resulting from deflections.

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10.11.1 — The factored axial forces P_u, the factored moments M_1 and M_2 at the ends of the column, and, where required, the relative lateral story deflections D_o shall be computed using an elastic first-order frame analysis with the section properties determined taking into account the influence of axial loads, the presence of cracked regions along the length of the member, and effects of duration of the loads. Alternatively, it shall be permitted to use the following properties for the members in the structure:

(a) Modulus of elasticity ..... E_c from 8.5.1

(b) Moments of inertia

Beams	$0.35I_g$
Columns	$0.70I_g$
Walls—Uncracked	$0.70I_g$
—Cracked	$0.35I_g$
Flat plates and flat slabs	$0.25I_g$

(c) Area ..... $1.0A_g$

The moments of inertia shall be divided by $(1 + b_d)$

(d) When sustained lateral loads act; or

(e) For stability checks made in accordance with 10.13.6.

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R10.11.1 — The stiffnesses EI used in an elastic analysis used for strength design should represent the stiffnesses of the members immediately prior to failure. This is particularly true for a second-order analysis that should predict the lateral deflections at loads approaching ultimate. The EI values should not be based totally on the moment-curvature relationship for the most highly loaded section along the length of each member. Instead, they should correspond to the moment-end rotation relationship for a complete member.

The alternative values of E_c, I_g, and A_g given in 10.11.1 have been chosen from the results of frame tests and analyses and include an allowance for the variability of the computed deflections. The modulus of elasticity E_c is based on the specified concrete strength while the sway deflections are a function of the average concrete strength, which is higher. The moments of inertia were taken as 0.875 times those in Reference 10.28. These two effects result in an overestimation of the second-order deflections in the order of 20 to 25 percent, corresponding to an implicit stiffness reduction factor f_K of 0.80 to 0.85 on the stability calculation. The concept of a stiffness reduction factor f_K is discussed in R10.12.3

The moment of inertia of T-beams should be based on the effective flange width defined in 8.10. It is generally sufficiently accurate to take I_g of a T-beam as two times the I_g for the web, $2(b_w h^3/12)$.

If the factored moments and shears from an analysis based on the moment of inertia of a wall taken equal to $0.70I_g$ indicate that the wall will crack in flexure, based on the modulus of rupture, the analysis should be repeated with $I = 0.35I_g$ in those stories where cracking is predicted at factored loads.

The alternative values of the moments of inertia given in 10.11.1 were derived for nonprestressed members. For prestressed members, the moments of inertia may differ from the values in 10.11.1 depending on the amount, location, and type of the reinforcement and the degree of cracking prior to ultimate. The stiffness values for prestressed concrete members should include an allowance for the variability of the stiffnesses.

Sections 10.11 through 10.13 provide requirements for strength and assume frame analyses will be carried out using factored loads. Analyses of deflections, vibrations, and building periods are needed at various service (unfactored) load levels[10.29,10.30] to determine the serviceability of the structure and to estimate the wind forces in wind tunnel laboratories. The seismic base shear is also based on the service load periods of vibration. The magnified service loads and deflections by a second-order analysis should also be computed using service loads. The moments of inertia of the structural members in the service load analyses should, therefore, be representative of the degree of cracking at the

CODE

10.11.2 — It shall be permitted to take the radius of gyration r equal to 0.30 times the overall dimension in the direction stability is being considered for rectangular compression members and 0.25 times the diameter for circular compression members. For other shapes, it shall be permitted to compute the radius of gyration for the gross concrete section.

10.11.3 — Unsupported length of compression members

10.11.3.1 — The unsupported length ℓ_u of a compression member shall be taken as the clear distance between floor slabs, beams, or other members capable of providing lateral support in the direction being considered.

10.11.3.2 — Where column capitals or haunches are present, the unsupported length shall be measured to the lower extremity of the capital or haunch in the plane considered.

10.11.4 — Columns and stories in structures shall be designated as nonsway or sway columns or stories. The design of columns in nonsway frames or stories shall be based on 10.12. The design of columns in sway frames or stories shall be based on 10.13.

10.11.4.1 — It shall be permitted to assume a column in a structure is nonsway if the increase in column end moments due to second-order effects does not exceed 5 percent of the first-order end moments.

10.11.4.2 — It also shall be permitted to assume a story within a structure is nonsway if:

$$Q = \frac{\Sigma P_u \Delta_o}{V_u \ell_c} \qquad (10\text{-}6)$$

is less than or equal to 0.05, where ΣP_u and V_u are the total vertical load and the story shear, respectively, in the story in question and Δ_o is the first-order relative deflection between the top and bottom of that story due to V_u.

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various service load levels investigated. Unless a more accurate estimate of the degree of cracking at design service load level is available, it is satisfactory to use **1/0.70 = 1.43** times the moments of inertia given in 10.11.1 for service load analyses.

Item (d) in 10.11.1 refers to the unusual case of sustained lateral loads. Such a case might exist, for example, if there were permanent lateral loads resulting from unequal earth pressures on two sides of a building.

R10.11.4 — The moment magnifier design method requires the designer to distinguish between nonsway frames, which are designed according to 10.12, and sway frames, which are designed according to 10.13. Frequently this can be done by inspection by comparing the total lateral stiffness of the columns in a story to that of the bracing elements. A compression member may be assumed nonsway by inspection if it is located in a story in which the bracing elements (shearwalls, shear trusses, or other types of lateral bracing) have such substantial lateral stiffness to resist the lateral deflections of the story that any resulting lateral deflection is not large enough to affect the column strength substantially. If not readily apparent by inspection, 10.11.4.1 and 10.11.4.2 give two possible ways of doing this. In 10.11.4.1, a story in a frame is said to be nonsway if the increase in the lateral load moments resulting from $P\Delta$ effects does not exceed 5 percent of the first-order moments.[10.28] Section 10.11.4.2 gives an alternative method of determining this based on the stability index for a story Q. In computing Q, ΣP_u should correspond to the lateral loading case for which ΣP_u is greatest. A frame may contain both nonsway and sway stories. This test would not be suitable if V_u is zero.

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10.11.5 — Where an individual compression member in the frame has a slenderness $k\ell_u/r$ of more than 100, 10.10.1 shall be used to compute the forces and moments in the frame.

10.11.6 — For compression members subject to bending about both principal axes, the moment about each axis shall be magnified separately based on the conditions of restraint corresponding to that axis.

10.12 — Magnified moments — Nonsway frames

10.12.1 — For compression members in nonsway frames, the effective length factor k shall be taken as 1.0, unless analysis shows that a lower value is justified. The calculation of k shall be based on the E and I values used in 10.11.1.

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If the lateral load deflections of the frame have been computed using service loads and the service load moments of inertia given in 10.11.1, it is permissible to compute Q in Eq. (10-6) using 1.2 times the sum of the service gravity loads, the service load story shear, and 1.43 times the first-order service load story deflections.

R10.11.5 — An upper limit is imposed on the slenderness ratio of columns designed by the moment magnifier method of 10.11 to 10.13. No similar limit is imposed if design is carried out according to 10.10.1. The limit of $k\ell_u/r = 100$ represents the upper range of actual tests of slender compression members in frames.

R10.11.6 — When biaxial bending occurs in a compression member, the computed moments about each principal axes should be magnified. The magnification factors d are computed considering the buckling load P_c about each axis separately based on the appropriate effective length $k\ell_u$ and the stiffness EI. If the buckling capacities are different about the two axes, different magnification factors will result.

R10.12 — Magnified moments — Nonsway frames

R10.12.1 — The moment magnifier equations were derived for hinged end columns and should be modified to account for the effect of end restraints. This is done by using an effective length $k\ell_u$ in the computation of P_c.

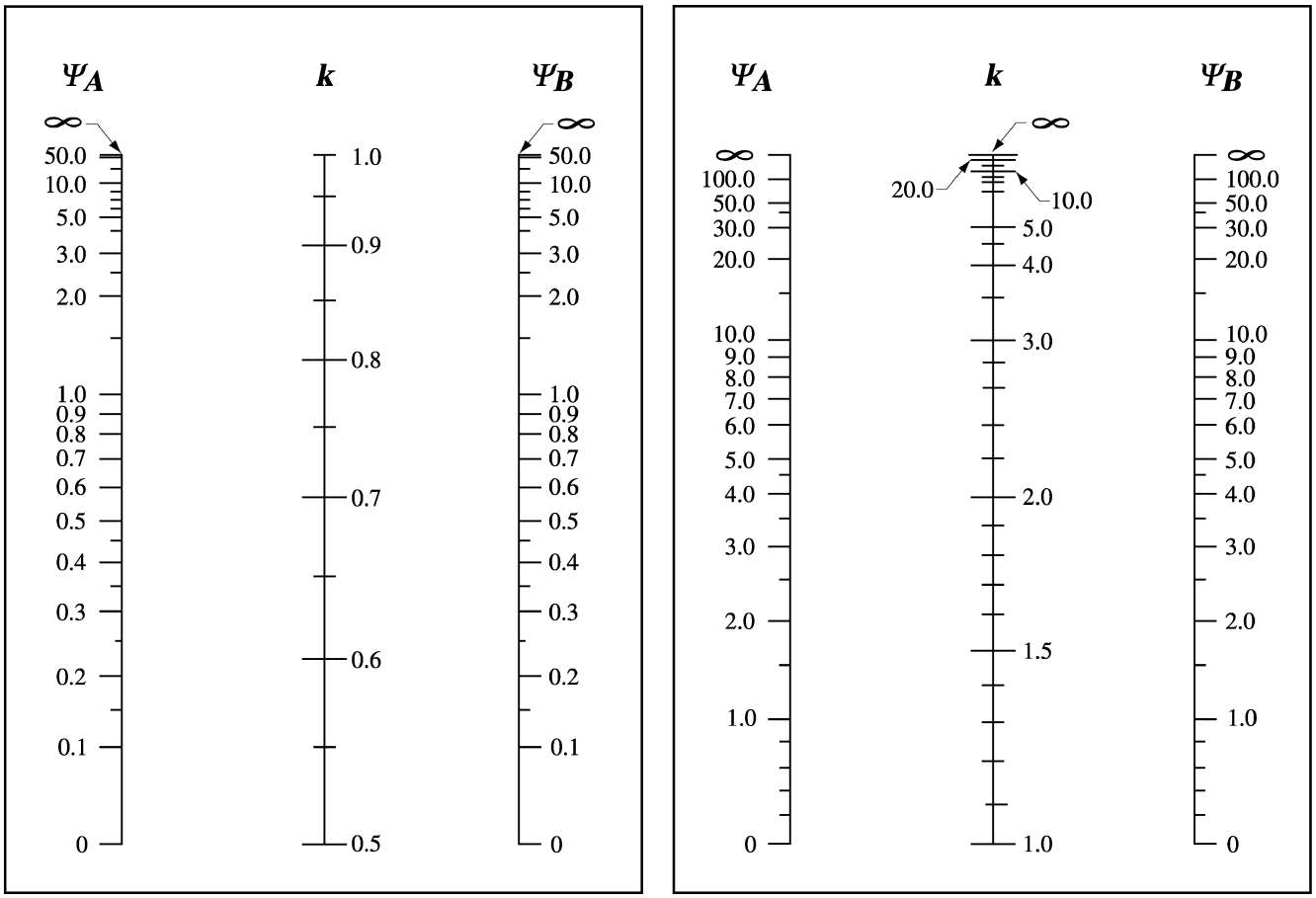
The primary design aid to estimate the effective length factor k is the Jackson and Moreland Alignment Charts (Fig. R10.12.1), which allow a graphical determination of k for a column of constant cross section in a multibay frame.[10.31,10.32]

The effective length is a function of the relative stiffness at each end of the compression member. Studies have indicated that the effects of varying beam and column reinforcement percentages and beam cracking should be considered in determining the relative end stiffnesses. In determining y for use in evaluating the effective length factor k, the rigidity of the flexural members may be calculated on the basis of $0.35I_g$ for flexural members to account for the effect of cracking and reinforcement on relative stiffness, and $0.70I_g$ for compression members.

The simplified equations (A-E), listed below for computing the effective length factors for nonsway and sway members, may be used. Eq. (A), (B), and (E) are taken from the 1972 British Standard Code of Practice.[10.33,10.34] Eq. (C) and (D) for sway members were developed in Reference 10.32.

For compression members in a nonsway frame, an upper bound to the effective length factor may be taken as the smaller of the following two expressions:

COMMENTARY



(a) Nonsway Frames

(b) Sway Frames

Ψ = ratio of $\Sigma(EI/\ell_c)$ of compression members to $\Sigma(EI/\ell)$ of flexural members in a plane at one end of a compression member

ℓ = span length of flexural member measured center to center of joints

Fig. R10.12.1—Effective length factors, k

$$k = 0.7 + 0.05\,(y_A + y_B) \leq 1.0 \quad \text{(A)}$$

$$k = 0.85 + 0.05\,y_{min} \leq 1.0 \quad \text{(B)}$$

where y_A and y_B are the values of y at the two ends of the column and y_{min} is the smaller of the two values.

For compression members in a sway frame, restrained at both ends, the effective length factor may be taken as:

For $y_m < 2$

$$k = \frac{20 - y_m}{20}\sqrt{1 + y_m} \quad \text{(C)}$$

For $y_m \geq 2$

$$k = 0.9\sqrt{1 + y_m} \quad \text{(D)}$$

where y_m is the average of the y-values at the two ends of the compression member.

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10.12.2 — In nonsway frames it shall be permitted to ignore slenderness effects for compression members that satisfy:

$$\frac{k\ell_u}{r} \le 34 - 12(M_1/M_2) \qquad (10\text{-}7)$$

where the term $[34 - 12M_1/M_2]$ shall not be taken greater than 40. The term M_1/M_2 is positive if the member is bent in single curvature, and negative if the member is bent in double curvature.

10.12.3 — Compression members shall be designed for the factored axial load P_u and the moment amplified for the effects of member curvature M_c as follows:

$$M_c = \delta_{ns} M_2 \qquad (10\text{-}8)$$

where

$$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \ge 1.0 \qquad (10\text{-}9)$$

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \qquad (10\text{-}10)$$

EI shall be taken as

$$EI = \frac{(0.2E_c I_g + E_s I_{se})}{1 + \beta_d} \qquad (10\text{-}11)$$

or

$$EI = \frac{0.4E_c I_g}{1 + \beta_d} \qquad (10\text{-}12)$$

COMMENTARY

For compression members in a sway frame, hinged at one end, the effective length factor may be taken as:

$$k = 2.0 + 0.3\psi \qquad (E)$$

where ψ is the value at the restrained end.

The use of the charts in Fig. R10.12.1, or the equations in this section, may be considered as satisfying the requirements of the code to justify k less than 1.0.

R10.12.2 — Eq. (10-7) is derived from Eq. (10-9) assuming that a 5 percent increase in moments due to slenderness is acceptable.[10.26] The derivation did not include ϕ in the calculation of the moment magnifier. As a first approximation, k may be taken equal to 1.0 in Eq. (10-7).

R10.12.3 — The ϕ-factors used in the design of slender columns represent two different sources of variability. First, the stiffness reduction ϕ-factors in the magnifier equations in the 1989 and earlier codes were intended to account for the variability in the stiffness EI and the moment magnification analysis. Second, the variability of the strength of the cross section is accounted for by strength reduction ϕ-factors of 0.70 for tied columns and 0.75 for spiral columns. Studies reported in Reference 10.35 indicate that the stiffness reduction factor ϕ_K, and the cross-sectional strength reduction ϕ-factors do not have the same values, contrary to the assumption in the 1989 and earlier codes. These studies suggest the stiffness reduction factor ϕ_K for an isolated column should be 0.75 for both tied and spiral columns. The 0.75 factors in Eq. (10-9) and (10-18) are stiffness reduction factors ϕ_K and replace the ϕ-factors in these equations in the 1989 and earlier codes. This has been done to avoid confusion between a stiffness reduction factor ϕ_K in Eq. (10-9) and (10-18), and the cross-sectional strength reduction ϕ-factors.

In defining the critical load, the main problem is the choice of a stiffness EI that reasonably approximates the variations in stiffness due to cracking, creep, and the nonlinearity of the concrete stress-strain curve. Eq. (10-11) was derived for small eccentricity ratios and high levels of axial load where the slenderness effects are most pronounced.

Creep due to sustained load will increase the lateral deflections of a column and hence the moment magnification. This is approximated for design by reducing the stiffness EI used to compute P_c and hence δ_{ns} by dividing EI by $(1 + \beta_d)$. Both the concrete and steel terms in Eq. (10-11) are divided by $(1 + \beta_d)$. This reflects the premature yielding of steel in columns subjected to sustained load.

CODE

10.12.3.1 — For members without transverse loads between supports, C_m shall be taken as

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4 \quad (10\text{-}13)$$

where M_1/M_2 is positive if the column is bent in single curvature. For members with transverse loads between supports, C_m shall be taken as 1.0.

10.12.3.2 — The factored moment M_2 in Eq. (10-8) shall not be taken less than

$$M_{2,min} = P_u(0.6 + 0.03h) \quad (10\text{-}14)$$

about each axis separately, where 0.6 and h are in inches. For members for which $M_{2,min}$ exceeds M_2, the value of C_m in Eq. (10-13) shall either be taken equal to 1.0, or shall be based on the ratio of the computed end moments M_1 and M_2.

10.13 — Magnified moments — Sway frames

COMMENTARY

Either Eq. (10-11) or (10-12) may be used to compute EI. Eq. (10-12) is a simplified approximation to Eq. (10-11). It is less accurate than Eq. (10-11).[10.36] Eq. (10-12) may be simplified further by assuming $\beta_d = 0.6$. When this is done Eq. (10-12) becomes

$$EI = 0.25E_cI_g \quad (F)$$

The term β_d is defined differently for nonsway and sway frames. See 10.0. For nonsway frames, β_d is the ratio of the maximum factored axial sustained load to the maximum factored axial load.

R10.12.3.1 — The factor C_m is an equivalent moment correction factor. The derivation of the moment magnifier assumes that the maximum moment is at or near midheight of the column. If the maximum moment occurs at one end of the column, design should be based on an equivalent uniform moment C_mM_2 that would lead to the same maximum moment when magnified.[10.26]

In the case of compression members that are subjected to transverse loading between supports, it is possible that the maximum moment will occur at a section away from the end of the member. If this occurs, the value of the largest calculated moment occurring anywhere along the member should be used for the value of M_2 in Eq. (10-8). In accordance with the last sentence of 10.12.3.1, C_m is to be taken as 1.0 for this case.

R10.12.3.2 — In the code, slenderness is accounted for by magnifying the column end moments. If the factored column moments are very small or zero, the design of slender columns should be based on the minimum eccentricity given in this section. It is not intended that the minimum eccentricity be applied about both axes simultaneously.

The factored column end moments from the structural analysis are used in Eq. (10-13) in determining the ratio M_1/M_2 for the column when the design should be based on minimum eccentricity. This eliminates what would otherwise be a discontinuity between columns with computed eccentricities less than the minimum eccentricity and columns with computed eccentricities equal to or greater than the minimum eccentricity.

R10.13 — Magnified moments — Sway frames

The design of sway frames for slenderness was revised in the 1995 code. The revised procedure consists of three steps:

(1) The magnified sway moments δ_sM_s are computed. This should be done in one of three ways. First, a second-order elastic frame analysis may be used (10.13.4.1).

CODE

COMMENTARY

Second, an approximation to such analysis (10.13.4.2) may be used. The third option is to use the sway magnifier d_s from previous editions of the code (10.13.4.3);

(2) The magnified sway moments $d_s M_s$ are added to the unmagnified nonsway moment M_{ns} at each end of each column (10.13.3). The nonsway moments may be computed using a first-order elastic analysis;

(3) If the column is slender and heavily loaded, it is checked to see whether the moments at points between the ends of the column exceed those at the ends of the column. As specified in 10.13.5 this is done using the nonsway frame magnifier d_{ns} with P_c computed assuming $k = 1.0$ or less.

10.13.1 — For compression members not braced against sidesway, the effective length factor k shall be determined using E and I values in accordance with 10.11.1 and shall not be less than 1.0.

R10.13.1 — See R10.12.1.

10.13.2 — For compression members not braced against sidesway, it shall be permitted to neglect the effects of slenderness when $k\ell_u/r$ is less than 22.

10.13.3 — The moments M_1 and M_2 at the ends of an individual compression member shall be taken as

$$M_1 = M_{1ns} + d_s M_{1s} \quad (10\text{-}15)$$

$$M_2 = M_{2ns} + d_s M_{2s} \quad (10\text{-}16)$$

where $d_s M_{1s}$ and $d_s M_{2s}$ shall be computed according to 10.13.4.

R10.13.3 — The analysis described in this section deals only with plane frames subjected to loads causing deflections in that plane. If torsional displacements are significant, a three-dimensional second-order analysis should be used.

10.13.4 — Calculation of $d_s M_s$

10.13.4.1 — The magnified sway moments $d_s M_s$ shall be taken as the column end moments calculated using a second-order elastic analysis based on the member stiffnesses given in 10.11.1.

R10.13.4 — Calculation of $d_s M_s$

R10.13.4.1 — A second-order analysis is a frame analysis that includes the internal force effects resulting from deflections. When a second-order elastic analysis is used to compute $d_s M_s$, the deflections should be representative of the stage immediately prior to the ultimate load. For this reason the reduced $E_c I_g$ values given in 10.11.1 should be used in the second-order analysis.

The term b_d is defined differently for nonsway and sway frames. See 10.0. Sway deflections due to short-term loads such as wind or earthquake are a function of the short-term stiffness of the columns following a period of sustained gravity load. For this case the definition of b_d in 10.0 gives $b_d = 0$. In the unusual case of a sway frame where the lateral loads are sustained, b_d will not be zero. This might occur if a building on a sloping site is subjected to earth pressure on one side but not on the other.

CODE

10.13.4.2 — Alternatively, it shall be permitted to calculate $d_s M_s$ as

$$d_s M_s = \frac{M_s}{1 - Q} \geq M_s \quad (10\text{-}17)$$

If d_s calculated in this way exceeds 1.5, $d_s M_s$ shall be calculated using 10.13.4.1 or 10.13.4.3.

10.13.4.3 — Alternatively, it shall be permitted to calculate the magnified sway moment $d_s M_s$ as

$$d_s M_s = \frac{M_s}{1 - \frac{\Sigma P_u}{0.75 \Sigma P_c}} \geq M_s \quad (10\text{-}18)$$

COMMENTARY

In a second-order analysis the axial loads in all columns that are not part of the lateral load resisting elements and depend on these elements for stability should be included.

In the 1989 and earlier codes, the moment magnifier equations for d_b and d_s included a stiffness reduction factor f_K to cover the variability in the stability calculation. The second-order analysis method is based on the values of E and I from 10.11.1. These lead to a 20 to 25 percent overestimation of the lateral deflections that corresponds to a stiffness reduction factor f_K between 0.80 and 0.85 on the $P\Delta$ moments. No additional f-factor is needed in the stability calculation. Once the moments are established, selection of the cross sections of the columns involves the strength reduction factors f from 9.3.2.2.

R10.13.4.2 — The iterative $P\Delta$ analysis for second-order moments can be represented by an infinite series. The solution of this series is given by Eq. (10-17).[10.28] Reference 10.37 shows that Eq. (10-17) closely predicts the second-order moments in a sway frame until d_s exceeds 1.5.

The $P\Delta$ moment diagrams for deflected columns are curved, with Δ related to the deflected shape of the columns. Eq. (10-17) and most commercially available second-order frame analyses have been derived assuming that the $P\Delta$ moments result from equal and opposite forces of $P\Delta/\ell_c$ applied at the bottom and top of the story. These forces give a straight line $P\Delta$ moment diagram. The curved $P\Delta$ moment diagrams lead to lateral displacements in the order of 15 percent larger than those from the straight line $P\Delta$ moment diagrams. This effect can be included in Eq. (10-17) by writing the denominator as $(1 - 1.15Q)$ rather than $(1 - Q)$. The 1.15 factor has been left out of Eq. (10-17) to maintain consistency with available computer programs.

If deflections have been calculated using service loads, Q in Eq. (10-17) should be calculated in the manner explained in R10.11.4.

In the 1989 and earlier codes, the moment magnifier equations for d_b and d_s included a stiffness reduction factor f_K to cover the variability in the stability calculation. The Q factor analysis is based on deflections calculated using the values of E_c and I_g from 10.11.1, which include the equivalent of a stiffness reduction factor f_K as explained in R10.13.4.1. As a result, no additional f-factor is needed in the stability calculation. Once the moments are established using Eq. (10-17), selection of the cross sections of the columns involves the strength reduction factors f from 9.3.2.2.

R10.13.4.3 — To check the effects of story stability, d_s is computed as an averaged value for the entire story based on use of $\Sigma P_u / \Sigma P_c$. This reflects the interaction of all sway resisting columns in the story in the $P\Delta$ effects since the lateral deflection of all columns in the story should be equal in the absence of torsional displacements about a vertical axis.

CODE

where ΣP_u is the summation for all the vertical loads in a story and ΣP_c is the summation for all sway resisting columns in a story. P_c is calculated using Eq. (10-10) using k from 10.13.1 and EI from Eq. (10-11) or Eq. (10-12).

10.13.5 — If an individual compression member has

$$\frac{\ell_u}{r} > \frac{35}{\sqrt{\frac{P_u}{f'_c A_g}}} \qquad (10\text{-}19)$$

it shall be designed for the factored axial load P_u and the moment M_c calculated using 10.12.3 in which M_1 and M_2 are computed in accordance with 10.13.3, b_d as defined for the load combination under consideration, and k as defined in 10.12.1.

10.13.6 — In addition to load cases involving lateral loads, the strength and stability of the structure as a whole under factored gravity loads shall be considered.

(a) When $d_s M_s$ is computed from 10.13.4.1, the ratio of second-order lateral deflections to first-order lateral deflections for 1.4 dead load and 1.7 live load plus lateral load applied to the structure shall not exceed 2.5;

(b) When $d_s M_s$ is computed according to 10.13.4.2, the value of Q computed using ΣP_u for 1.4 dead load plus 1.7 live load shall not exceed 0.60;

(c) When $d_s M_s$ is computed from 10.13.4.3, d_s computed using ΣP_u and ΣP_c corresponding to the factored dead and live loads shall be positive and shall not exceed 2.5.

COMMENTARY

In addition, it is possible that a particularly slender individual column in a sway frame could have substantial midheight deflections even if adequately braced against lateral end deflections by other columns in the story. Such a column will have ℓ_u/r greater than the value given in Eq. (10-19) and should be checked using 10.13.5.

If the lateral load deflections involve a significant torsional displacement, the moment magnification in the columns farthest from the center of twist may be underestimated by the moment magnifier procedure. In such cases, a three-dimensional second-order analysis should be considered.

The 0.75 in the denominator of Eq. (10-18) is a stiffness reduction factor f_K as explained in R10.12.3.

In the calculation of EI, b_d will normally be zero for a sway frame because the lateral loads are generally of short duration. (See R10.13.4.1).

R10.13.5 — The unmagnified nonsway moments at the ends of the columns are added to the magnified sway moments at the same points. Generally, one of the resulting end moments is the maximum moment in the column. However, for slender columns with high axial loads the point of maximum moment may be between the ends of the column so that the end moments are no longer the maximum moments. If ℓ_u/r is less than the value given by Eq. (10-19) the maximum moment at any point along the height of such a column will be less than 1.05 times the maximum end moment. When ℓ_u/r exceeds the value given by Eq. (10-19), the maximum moment will occur at a point between the ends of the column and will exceed the maximum end moment by more than 5 percent.[10.25] In such a case the maximum moment is calculated by magnifying the end moments using Eq. (10-8).

R10.13.6 — The possibility of sidesway instability under gravity loads alone should be investigated. When using second-order analyses to compile $d_s M_s$ (10.13.4.1), the frame should be analyzed twice for the case of factored gravity loads plus a lateral load applied to the frame. This load may be the lateral load used in design or it may be a single lateral load applied to the top of the frame. The first analysis should be a first-order analysis; the second analysis should be a second-order analysis. The deflection from the second-order analysis should not exceed 2.5 times the deflection from the first-order analysis. If one story is much more flexible than the others, the deflection ratio should be computed in that story. The lateral load should be large enough to give deflections of a magnitude that can be compared accurately. In unsymmetrical frames that deflect laterally under gravity loads alone, the lateral load should act in the direction for which it will increase the lateral deflections.

CODE

In cases (a), (b), and (c) above, b_d shall be taken as the ratio of the maximum factored sustained axial load to the maximum factored axial load.

10.13.7 — In sway frames, flexural members shall be designed for the total magnified end moments of the compression members at the joint.

10.14 — Axially loaded members supporting slab system

Axially loaded members supporting a slab system included within the scope of 13.1 shall be designed as provided in Chapter 10 and in accordance with the additional requirements of Chapter 13.

10.15 — Transmission of column loads through floor system

When the specified compressive strength of concrete in a column is greater than 1.4 times that specified for a floor system, transmission of load through the floor system shall be provided by 10.15.1, 10.15.2, or 10.15.3.

COMMENTARY

When using 10.13.4.2 to compute $d_s M_s$, the value of Q evaluated using factored gravity loads should not exceed 0.60. This is equivalent to $d_s = 2.5$. The values of V_u and D_o used to compute Q can result from assuming any real or arbitrary set of lateral loads provided that V_u and D_o are both from the same loading. If Q as computed in 10.11.4.2 is 0.2 or less, the stability check in 10.13.6 is satisfied.

When $d_s M_s$ is computed using Eq. (10-18), an upper limit of 2.5 is placed on d_s. For higher d_s values, the frame will be very susceptible to variations in EI and foundation rotations. If d_s exceeds 2.5, the frame should be stiffened to reduce d_s. SP_u shall include the axial load in all columns and walls including columns that are not part of the lateral load resisting system. The value $d_s = 2.5$ is a very high magnifier. It has been chosen to offset the conservatism inherent in the moment magnifier procedure.

For nonsway frames, b_d is the ratio of the maximum factored axial sustained load to the maximum factored axial load.

R10.13.7 — The strength of a sway frame is governed by the stability of the columns and by the degree of end restraint provided by the beams in the frame. If plastic hinges form in the restraining beam, the structure approaches a failure mechanism and its axial load capacity is drastically reduced. Section 10.13.7 provides that the designer make certain that the restraining flexural members have the capacity to resist the magnified column moments.

R10.15 — Transmission of column loads through floor system

The requirements of this section are based on a paper on the effect of floor concrete strength on column strength.[10.38] The provisions mean that when the column concrete strength does not exceed the floor concrete strength by more than 40 percent, no special precautions need be taken. For higher column concrete strengths, methods in 10.15.1 or 10.15.2 should be used for corner or edge columns. Methods in 10.15.1, 10.15.2, or 10.15.3 should be used for interior columns with adequate restraint on all four sides.

CODE

10.15.1 — Concrete of strength specified for the column shall be placed in the floor at the column location. Top surface of the column concrete shall extend 2 ft into the slab from face of column. Column concrete shall be well integrated with floor concrete, and shall be placed in accordance with 6.4.5 and 6.4.6.

10.15.2 — Strength of a column through a floor system shall be based on the lower value of concrete strength with vertical dowels and spirals as required.

10.15.3 — For columns laterally supported on four sides by beams of approximately equal depth or by slabs, it shall be permitted to base strength of the column on an assumed concrete strength in the column joint equal to 75 percent of column concrete strength plus 35 percent of floor concrete strength. In the application of 10.15.3, the ratio of column concrete strength to slab concrete strength shall not be taken greater than 2.5 for design.

10.16 — Composite compression members

10.16.1 — Composite compression members shall include all such members reinforced longitudinally with structural steel shapes, pipe, or tubing with or without longitudinal bars.

10.16.2 — Strength of a composite member shall be computed for the same limiting conditions applicable to ordinary reinforced concrete members.

COMMENTARY

R10.15.1 — Application of the concrete placement procedure described in 10.15.1 requires the placing of two different concrete mixtures in the floor system. The lower strength mixture should be placed while the higher strength concrete is still plastic and should be adequately vibrated to ensure the concretes are well integrated. This requires careful coordination of the concrete deliveries and the possible use of retarders. In some cases, additional inspection services will be required when this procedure is used. It is important that the higher strength concrete in the floor in the region of the column be placed before the lower strength concrete in the remainder of the floor to prevent accidental placing of the low strength concrete in the column area. It is the designer's responsibility to indicate on the drawings where the high and low strength concretes are to be placed.

With the 1983 code, the amount of column concrete to be placed within the floor is expressed as a simple 2-ft extension from face of the column. Since the concrete placement requirement should be carried out in the field, it is now expressed in a way that is directly evident to workers. The new requirement will also locate the interface between column and floor concrete farther out into the floor, away from regions of very high shear.

R10.15.3 — Research[10.39] has shown that heavily loaded slabs do not provide as much confinement as lightly loaded slabs when ratios of column concrete strength to slab concrete strength exceed about 2.5. Consequently, a limit is placed on the concrete strength ratio assumed in design.

R10.16 — Composite compression members

R10.16.1 — Composite columns are defined without reference to classifications of combination, composite, or concrete-filled pipe column. Reference to other metals used for reinforcement has been omitted because they are seldom used in concrete construction.

R10.16.2 — The same rules used for computing the load-moment interaction strength for reinforced concrete sections can be applied to composite sections. Interaction charts for concrete-filled tubing would have a form identical to those of ACI SP-7[10.40] and the *ACI Design Handbook*[10.32] but with g slightly greater than 1.0.

CODE

10.16.3 — Any axial load strength assigned to concrete of a composite member shall be transferred to the concrete by members or brackets in direct bearing on the composite member concrete.

10.16.4 — All axial load strength not assigned to concrete of a composite member shall be developed by direct connection to the structural steel shape, pipe, or tube.

10.16.5 — For evaluation of slenderness effects, radius of gyration of a composite section shall be not greater than the value given by

$$r = \sqrt{\frac{(E_c I_g / 5) + E_s I_t}{(E_c A_g / 5) + E_s A_t}} \qquad (10\text{-}20)$$

and, as an alternative to a more accurate calculation, EI in Eq. (10-10) shall be taken either as Eq. (10-11) or

$$EI = \frac{(E_c I_g / 5)}{1 + \beta_d} + E_s I_t \qquad (10\text{-}21)$$

10.16.6 — Structural steel encased concrete core

10.16.6.1 — For a composite member with a concrete core encased by structural steel, the thickness of the steel encasement shall be not less than

$b\sqrt{\frac{f_y}{3E_s}}$ for each face of width b

nor

$h\sqrt{\frac{f_y}{8E_s}}$ for circular sections of diameter h

10.16.6.2 — Longitudinal bars located within the encased concrete core shall be permitted to be used in computing A_t and I_t.

10.16.7 — Spiral reinforcement around structural steel core

A composite member with spirally reinforced concrete around a structural steel core shall conform to 10.16.7.1 through 10.16.7.5.

10.16.7.1 — Specified compressive strength of concrete f'_c shall not be less than that given in 1.1.1.

10.16.7.2 — Design yield strength of structural steel core shall be the specified minimum yield strength for the grade of structural steel used but not to exceed 50,000 psi.

COMMENTARY

R10.16.3 and R10.16.4 — Direct bearing or direct connection for transfer of forces between steel and concrete can be developed through lugs, plates, or reinforcing bars welded to the structural shape or tubing before the concrete is cast. Flexural compressive stress need not be considered a part of direct compression load to be developed by bearing. A concrete encasement around a structural steel shape may stiffen the shape, but it would not necessarily increase its strength.

R10.16.5 — Eq. (10-20) is given because the rules of 10.11.2 for estimating the radius of gyration are overly conservative for concrete filled tubing and are not applicable for members with enclosed structural shapes.

In reinforced concrete columns subject to sustained loads, creep transfers some of the load from the concrete to the steel, increasing the steel stresses. In the case of lightly reinforced columns, this load transfer may cause the compression steel to yield prematurely, resulting in a loss in the effective EI. Accordingly, both the concrete and steel terms in Eq. (10-11) are reduced to account for creep. For heavily reinforced columns or for composite columns in which the pipe or structural shape makes up a large percentage of the cross section, the load transfer due to creep is not significant. Accordingly, Eq. (10-21) was revised in the 1980 code supplement so that only the EI of the concrete is reduced for sustained load effects.

R10.16.6 — Structural steel encased concrete core

Steel encased concrete sections should have a metal wall thickness large enough to attain longitudinal yield stress before buckling outward.

R10.16.7 — Spiral reinforcement around structural steel core

Concrete that is laterally confined by a spiral has increased load-carrying strength, and the size of the spiral required can be regulated on the basis of the strength of the concrete outside the spiral the same reasoning that applies for columns reinforced only with longitudinal bars. The radial pressure provided by the spiral ensures interaction between concrete, reinforcing bars, and steel core such that longitudinal bars will both stiffen and strengthen the cross section.

CODE

10.16.7.3 — Spiral reinforcement shall conform to 10.9.3.

10.16.7.4 — Longitudinal bars located within the spiral shall be not less than 0.01 nor more than 0.08 times net area of concrete section.

10.16.7.5 — Longitudinal bars located within the spiral shall be permitted to be used in computing A_t and I_t.

10.16.8 — Tie reinforcement around structural steel core

A composite member with laterally tied concrete around a structural steel core shall conform to 10.16.8.1 through 10.16.8.8.

10.16.8.1 — Specified compressive strength of concrete f_c' shall not be less than that given in 1.1.1.

10.16.8.2 — Design yield strength of structural steel core shall be the specified minimum yield strength for the grade of structural steel used but not to exceed 50,000 psi.

10.16.8.3 — Lateral ties shall extend completely around the structural steel core.

10.16.8.4 — Lateral ties shall have a diameter not less than 0.02 times the greatest side dimension of composite member, except that ties shall not be smaller than No. 3 and are not required to be larger than No. 5. Welded wire fabric of equivalent area shall be permitted.

10.16.8.5 — Vertical spacing of lateral ties shall not exceed 16 longitudinal bar diameters, 48 tie bar diameters, or 0.5 times the least side dimension of the composite member.

10.16.8.6 — Longitudinal bars located within the ties shall be not less than 0.01 nor more than 0.08 times net area of concrete section.

10.16.8.7 — A longitudinal bar shall be located at every corner of a rectangular cross section, with other longitudinal bars spaced not farther apart than one-half the least side dimension of the composite member.

10.16.8.8 — Longitudinal bars located within the ties shall be permitted to be used in computing A_t for strength but not in computing I_t for evaluation of slenderness effects.

COMMENTARY

R10.16.8 — Tie reinforcement around structural steel core

Concrete that is laterally confined by tie bars is likely to be rather thin along at least one face of a steel core section. Therefore, complete interaction between the core, the concrete, and any longitudinal reinforcement should not be assumed. Concrete will probably separate from smooth faces of the steel core. To maintain the concrete around the structural steel core, it is reasonable to require more lateral ties than needed for ordinary reinforced concrete columns. Because of probable separation at high strains between the steel core and the concrete, longitudinal bars will be ineffective in stiffening cross sections even though they would be useful in sustaining compression forces. The yield strength of the steel core should be limited to that which exists at strains below those that can be sustained without spalling of the concrete. It has been assumed that axially compressed concrete will not spall at strains less than 0.0018. The yield strength of $0.0018 \times 29{,}000{,}000$, or 52,000 psi, represents an upper limit of the useful maximum steel stress.

CODE

COMMENTARY

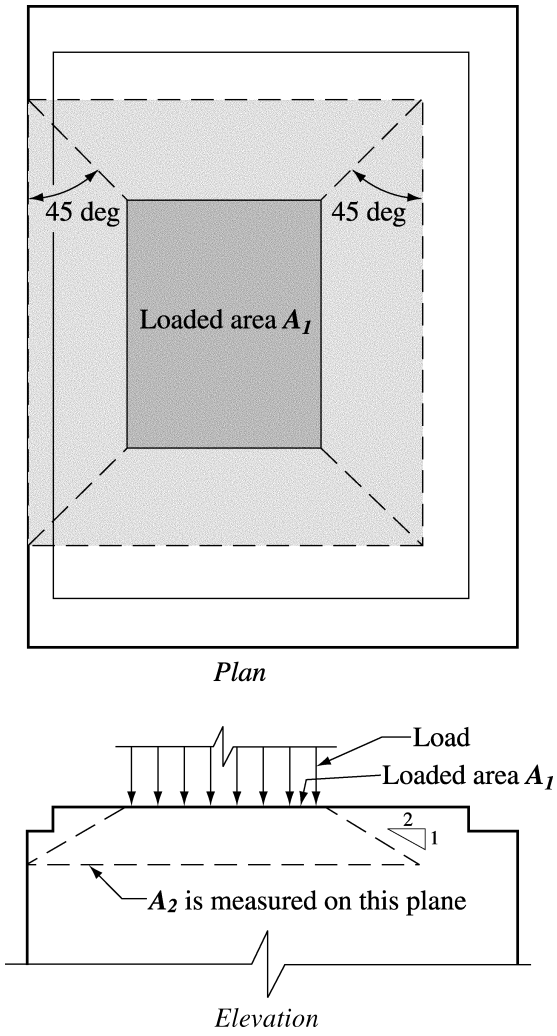


Fig. R10.17—Application of frustum to find A_2 in stepped or sloped supports

10.17 — Bearing strength

10.17.1 — Design bearing strength of concrete shall not exceed ϕ **(0.85$f_c'A_1$)**, except when the supporting surface is wider on all sides than the loaded area, then the design bearing strength of the loaded area shall be permitted to be multiplied by $\sqrt{A_2/A_1}$ but not more than 2.

R10.17 — Bearing strength

R10.17.1 — This section deals with bearing strength of concrete supports. The permissible bearing stress of **0.85f_c'** is based on tests reported in Reference 10.41. (See also 15.8).

When the supporting area is wider than the loaded area on all sides, the surrounding concrete confines the bearing area, resulting in an increase in bearing strength. No minimum depth is given for a supporting member. The minimum depth of support will be controlled by the shear requirements of 11.12.

When the top of the support is sloped or stepped, advantage may still be taken of the condition that the supporting member is larger than the loaded area, provided the supporting member does not slope at too great an angle. Fig. R10.17 illustrates the application of the frustum to find A_2. The frustum should not be confused with the path by which a load spreads out as it travels downward through the support.

CODE

COMMENTARY

Such a load path would have steeper sides. However, the frustum described has somewhat flat side slopes to ensure that there is concrete immediately surrounding the zone of high stress at the bearing. A_1 is the loaded area but not greater than the bearing plate or bearing cross-sectional area.

10.17.2 — Section 10.17 does not apply to post-tensioning anchorages.

R10.17.2 — Post-tensioning anchorages are usually laterally reinforced, in accordance with 18.13.

CHAPTER 11 — SHEAR AND TORSION

CODE

COMMENTARY

This chapter includes shear and torsion provisions for both nonprestressed and prestressed concrete members. The shear-friction concept (11.7) is particularly applicable to design of reinforcement details in precast structures. Special provisions are included for deep flexural members (11.8), brackets and corbels (11.9), and shearwalls (11.10). Shear provisions for slabs and footings are given in 11.12.

11.0 — Notation

R11.0 — Notation

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

a = shear span, distance between concentrated load and face of support, in.

A_c = area of concrete section resisting shear transfer, in.^{2}

A_{cp} = area enclosed by outside perimeter of concrete cross section, in.^{2} See 11.6.1

A_f = area of reinforcement in bracket or corbel resisting factored moment, $[V_u a + N_{uc} (h - d)]$, in.^{2}

A_g = gross area of section, in.^{2} For a hollow section, A_g is the area of the concrete only and does not include the area of the void(s). See 11.6.1.

A_h = area of shear reinforcement parallel to flexural tension reinforcement, in.^{2}

A_ℓ = total area of longitudinal reinforcement to resist torsion, in.^{2}

A_n = area of reinforcement in bracket or corbel resisting tensile force N_{uc}, in.^{2}

A_o = gross area enclosed by shear flow path, in.^{2}

A_{oh} = area enclosed by centerline of the outermost closed transverse torsional reinforcement, in.^{2}

A_{ps} = area of prestressed reinforcement in tension zone, in.^{2}

A_s = area of nonprestressed tension reinforcement, in.^{2}

A_t = area of one leg of a closed stirrup resisting torsion within a distance s, in.^{2}

A_v = area of shear reinforcement within a distance s, or area of shear reinforcement perpendicular to flexural tension reinforcement within a distance s for deep flexural members, in.^{2}

A_{vf} = area of shear-friction reinforcement, in.^{2}

A_{vh} = area of shear reinforcement parallel to flexural tension reinforcement within a distance s_2, in.^{2}

b = width of compression face of member, in.

b_o = perimeter of critical section for slabs and footings, in.

b_t = width of that part of cross section containing the closed stirrups resisting torsion, in.

CODE

b_w = web width, or diameter of circular section, in.

b_1 = width of the critical section defined in 11.12.1.2 measured in the direction of the span for which moments are determined, in.

b_2 = width of the critical section defined in 11.12.1.2 measured in the direction perpendicular to b_1, in.

c_1 = size of rectangular or equivalent rectangular column, capital, or bracket measured in the direction of the span for which moments are being determined, in.

c_2 = size of rectangular or equivalent rectangular column, capital, or bracket measured transverse to the direction of the span for which moments are being determined, in.

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, but need not be less than $0.80h$ for circular sections and prestressed members, in.

f'_c = specified compressive strength of concrete, psi

$\sqrt{f'_c}$ = square root of specified compressive strength of concrete, psi

f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi

f_d = stress due to unfactored dead load, at extreme fiber of section where tensile stress is caused by externally applied loads, psi

f_{pc} = compressive stress in concrete (after allowance for all prestress losses) at centroid of cross section resisting externally applied loads or at junction of web and flange when the centroid lies within the flange, psi. (In a composite member, f_{pc} is resultant compressive stress at centroid of composite section, or at junction of web and flange when the centroid lies within the flange, due to both prestress and moments resisted by precast member acting alone)

f_{pe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads, psi

f_{pu} = specified tensile strength of prestressing steel, psi

f_y = specified yield strength of nonprestressed reinforcement, psi

f_{yh} = specified yield strength of circular tie, hoop, or spiral reinforcement, psi

f_{yv} = yield strength of closed transverse torsional reinforcement, psi

$f_{y\ell}$ = yield strength of longitudinal torsional reinforcement, psi

h = overall thickness of member, in.

h_v = total depth of shearhead cross section, in.

COMMENTARY

Tests[11.1] have indicated that the average shear stress over the full effective section also may be applicable for circular sections. Note the special definition of d for such sections.

Although the value of d may vary along the span of a prestressed beam, studies[11.2] showed that, for prestressed concrete members, d need not be taken less than $0.8h$. The beams considered had some straight tendons or reinforcing bars at the bottom of the section and had stirrups that enclosed that steel.

CODE

COMMENTARY

h_w = total height of wall from base to top, in.
I = moment of inertia of section resisting externally applied factored loads, in.[4]
ℓ_n = clear span measured face-to-face of supports, in.
ℓ_v = length of shearhead arm from centroid of concentrated load or reaction, in.
ℓ_w = horizontal length of wall, in.
M_{cr} = moment causing flexural cracking at section due to externally applied loads. See 11.4.2.1
M_m = modified moment, in.-lb
M_{max} = maximum factored moment at section due to externally applied loads, in.-lb
M_p = required plastic moment strength of shearhead cross section, in.-lb
M_u = factored moment at section, in.-lb
M_v = moment resistance contributed by shearhead reinforcement, in.-lb
N_u = factored axial load normal to cross section occurring simultaneously with V_u *or* T_u; to be taken as positive for compression, lb
N_{uc} = factored tensile force applied at top of bracket or corbel acting simultaneously with V_u, to be taken as positive for tension, lb
p_{cp} = outside perimeter of the concrete cross section, in. See 11.6.1
p_h = perimeter of centerline of outermost closed transverse torsional reinforcement, in.
s = spacing of shear or torsion reinforcement measured in a direction parallel to longitudinal reinforcement, in.
s_1 = spacing of vertical reinforcement in wall, in.
s_2 = spacing of shear or torsion reinforcement measured in a direction perpendicular to longitudinal reinforcement—or spacing of horizontal reinforcement in wall, in.
t = thickness of a wall of a hollow section, in.
T_n = nominal torsional moment strength, in.-lb
T_u = factored torsional moment at section, in.-lb
V_c = nominal shear strength provided by concrete, lb
V_{ci} = nominal shear strength provided by concrete when diagonal cracking results from combined shear and moment, lb
V_{cw} = nominal shear strength provided by concrete when diagonal cracking results from excessive principal tensile stress in web, lb
V_d = shear force at section due to unfactored dead load, lb
V_i = factored shear force at section due to externally applied loads occurring simultaneously with M_{max}, lb
V_n = nominal shear strength, lb
V_p = vertical component of effective prestress force at section, lb
V_s = nominal shear strength provided by shear reinforcement, lb
V_u = factored shear force at section, lb
v_n = nominal shear stress, psi. See 11.12.6.2

CODE

y_t = distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension, in.

a = angle between inclined stirrups and longitudinal axis of member

a_f = angle between shear-friction reinforcement and shear plane

a_s = constant used to compute V_c in slabs and footings

a_v = ratio of flexural stiffness of shearhead arm to that of the surrounding composite slab section. See 11.12.4.5

b_c = ratio of long side to short side of concentrated load or reaction area

b_p = constant used to compute V_c in prestressed slabs

g_f = fraction of unbalanced moment transferred by flexure at slab-column connections. See 13.5.3.2

g_v = fraction of unbalanced moment transferred by eccentricity of shear at slab-column connections. See 11.12.6.1

= $1 - g_f$

h = number of identical arms of shearhead

q = angle of compression diagonals in truss analogy for torsion

l = correction factor related to unit weight of concrete

m = coefficient of friction. See 11.7.4.3

r = ratio of nonprestressed tension reinforcement

= A_s/bd

r_h = ratio of horizontal shear reinforcement area to gross concrete area of vertical section

r_n = ratio of vertical shear reinforcement area to gross concrete area of horizontal section

r_w = $A_s/b_w d$

f = strength reduction factor. See 9.3

11.1 — Shear strength

11.1.1 — Except for members designed in accordance with Appendix A, design of cross sections subject to shear shall be based on:

$$f V_n \geq V_u \quad (11\text{-}1)$$

where V_u is the factored shear force at the section considered and V_n is nominal shear strength computed by:

$$V_n = V_c + V_s \quad (11\text{-}2)$$

where V_c is nominal shear strength provided by concrete in accordance with 11.3, 11.4, or 11.12, and V_s is nominal shear strength provided by shear reinforcement in accordance with 11.5.6, 11.10.9, or 11.12.

COMMENTARY

R11.1 — Shear strength

The shear strength is based on an average shear stress on the full effective cross section $b_w d$. In a member without shear reinforcement, shear is assumed to be carried by the concrete web. In a member with shear reinforcement, a portion of the shear strength is assumed to be provided by the concrete and the remainder by the shear reinforcement.

The shear strength provided by concrete V_c is assumed to be the same for beams with and without shear reinforcement and is taken as the shear causing significant inclined cracking. These assumptions are discussed in References 11.1, 11.2, and 11.3.

Appendix A allows the use of strut-and-tie models in the shear design of disturbed regions. The traditional shear design procedures, which ignore D-regions, are acceptable in shear spans that include B-regions.

CODE

11.1.1.1 — In determining shear strength V_n, the effect of any openings in members shall be considered.

11.1.1.2 — In determining shear strength V_c, whenever applicable, effects of axial tension due to creep and shrinkage in restrained members shall be considered and effects of inclined flexural compression in variable depth members shall be permitted to be included.

11.1.2 — The values of $\sqrt{f'_c}$ used in this chapter shall not exceed 100 psi except as allowed in 11.1.2.1.

11.1.2.1 — Values of $\sqrt{f'_c}$ greater than 100 psi shall be permitted in computing V_c, V_{ci}, and V_{cw} for reinforced or prestressed concrete beams and concrete joist construction having minimum web reinforcement in accordance with 11.5.5.3, 11.5.5.4, or 11.6.5.2.

11.1.3 — Computation of maximum factored shear force V_u at supports in accordance with 11.1.3.1 or 11.1.3.2 shall be permitted when all of the following conditions are satisfied:

(a) Support reaction, in direction of applied shear, introduces compression into the end regions of member;

(b) Loads are applied at or near the top of the member;

(c) No concentrated load occurs between face of support and location of critical section defined in 11.1.3.1 or 11.1.3.2.

COMMENTARY

R11.1.1.1 — Openings in the web of a member can reduce its shear strength. The effects of openings are discussed in Section 4.7 of Reference 11.1 and in References 11.4 and 11.5.

R11.1.1.2 — In a member of variable depth, the internal shear at any section is increased or decreased by the vertical component of the inclined flexural stresses. Computation methods are outlined in various textbooks and in the 1940 Joint Committee Report.[11.6]

R11.1.2 — Because of a lack of test data and practical experience with concretes having compressive strengths greater than 10,000 psi, the 1989 edition of the code imposed a maximum value of 100 psi on $\sqrt{f'_c}$ for use in the calculation of shear strength of concrete beams, joists, and slabs. Exceptions to this limit were permitted in beams and joists when the transverse reinforcement satisfied an increased value for the minimum amount of web reinforcement. There are limited test data on the two-way shear strength of high-strength concrete slabs. Until more experience is obtained for two-way slabs built with concretes that have strengths greater than 10,000 psi, it is prudent to limit $\sqrt{f'_c}$ to 100 psi for the calculation of shear strength.

R11.1.2.1 — Based on the test results in References 11.7, 11.8, 11.9, 11.10, and 11.11, an increase in the minimum amount of transverse reinforcement is required for high-strength concrete. These tests indicated a reduction in the reserve shear strength as f'_c increased in beams reinforced with the specified minimum amount of transverse reinforcement, which is equivalent to an effective shear stress of 50 psi. A provision introduced in the 1989 edition of the code required an increase in the minimum amount of transverse reinforcement for concrete strengths between 10,000 and 15,000 psi. This provision, which led to a sudden increase in the minimum amount of transverse reinforcement at a compressive strength of 10,000 psi, has been replaced by a gradual increase in the minimum A_v as f'_c increases, as given by Eq. (11-13).

CODE

COMMENTARY

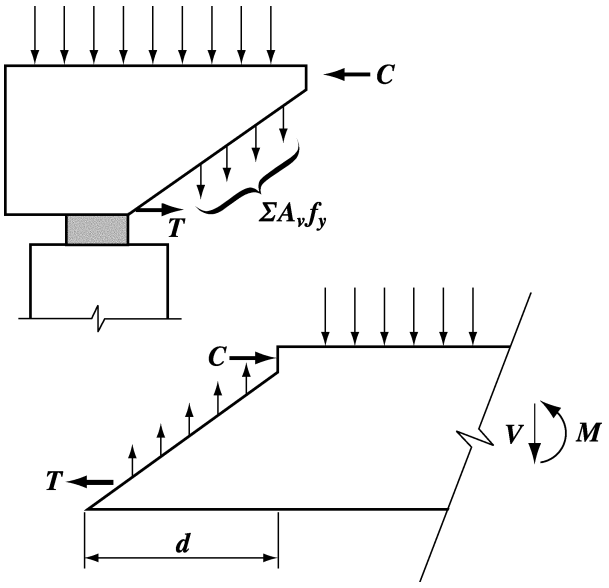


Fig. R11.1.3.1(a)—Free body diagrams of the end of a beam

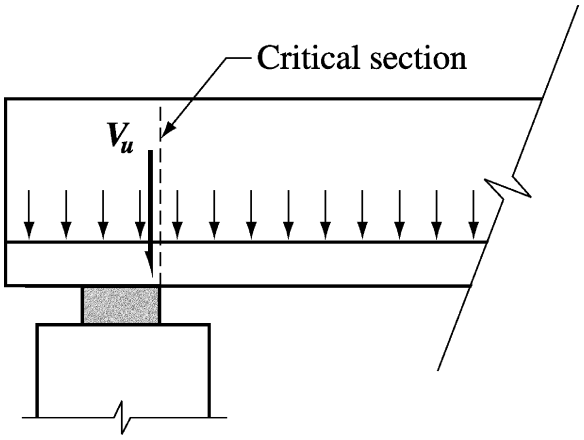


Fig. R11.1.3.1(b)—Location of critical section for shear in a member loaded near bottom

11.1.3.1 — For nonprestressed members, sections located less than a distance ***d*** from face of support shall be permitted to be designed for the same shear V_u as that computed at a distance ***d***.

R11.1.3.1 — The closest inclined crack to the support of the beam in Fig. R11.1.3.1(a) will extend upwards from the face of the support reaching the compression zone about ***d*** from the face of the support. If loads are applied to the top of this beam, the stirrups across this crack are stressed by loads acting on the lower freebody in Fig. R11.1.3.1(a). The loads applied to the beam between the face of the column and the point ***d*** away from the face are transferred directly to the support by compression in the web above the crack. Accordingly, the code permits design for a maximum factored shear force V_u at a distance ***d*** from the support for nonprestressed members, and at a distance ***h*/2** for prestressed members. Two things are emphasized: first, stirrups are required across the potential crack designed for the shear at ***d*** from the support, and second, a tension force exists in the longitudinal reinforcement at the face of the support.

CODE

COMMENTARY

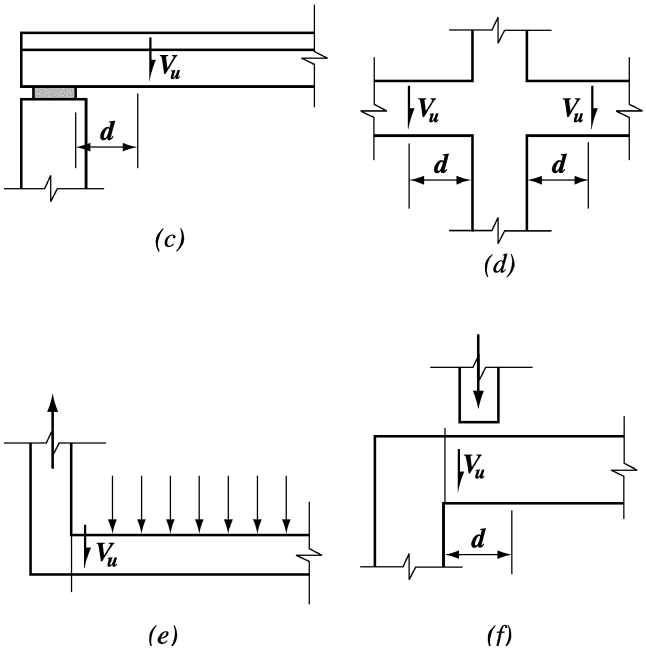


Fig. R11.1.3.1(c), (d), (e), (f)—Typical support conditions for locating factored shear force V_u

In Fig. R11.1.3.1(b), loads are shown acting near the bottom of a beam. In this case, the critical section is taken at the face of the support. Loads acting near the support should be transferred across the inclined crack extending upward from the support face. The shear force acting on the critical section should include all loads applied below the potential inclined crack.

Typical support conditions where the shear force at a distance ***d*** from the support may be used include: (1) members supported by bearing at the bottom of the member, such as shown in Fig. R11.1.3.1(c); and (2) members framing monolithically into another member as illustrated in Fig. R11.1.3.1(d).

Support conditions where this provision should not be applied include: (1) Members framing into a supporting member in tension, such as shown in Fig. R11.1.3.1(e). For this case, the critical section for shear should be taken at the face of the support. Shear within the connection should also be investigated and special corner reinforcement should be provided. (2) Members for which loads are not applied at or near the top of the member. This is the condition referred to in Fig. 11.1.3.1(b). For such cases the critical section is taken at the face of the support. Loads acting near the support should be transferred across the inclined crack extending upward from the support face. The shear force acting on the critical section should include all loads applied below the potential inclined crack. (3) Members loaded such that the shear at sections between the support and a distance ***d*** from the support differs radically from the shear at distance ***d***. This commonly occurs in brackets and in beams where a concentrated load is located close to the support, as shown in Fig. R11.1.3.1(f) or in footings supported on piles. In this case the shear at the face of the support should be used.

CODE

11.1.3.2 — For prestressed members, sections located less than a distance ***h*/2** from face of support shall be permitted to be designed for the same shear V_u as that computed at a distance ***h*/2**.

11.1.4 — For deep beams, brackets and corbels, walls, and slabs and footings, the special provisions of 11.8 through 11.12 shall apply.

11.2 — Lightweight concrete

11.2.1 — Provisions for shear and torsion strength apply to normalweight concrete. When lightweight aggregate concrete is used, one of the following modifications shall apply to $\sqrt{f_c'}$ throughout Chapter 11, except 11.5.4.3, 11.5.6.9, 11.6.3.1, 11.12.3.2, and 11.12.4.8

11.2.1.1 — When f_{ct} is specified and concrete is proportioned in accordance with 5.2, $f_{ct}/6.7$ shall be substituted for $\sqrt{f_c'}$, but the value of $f_{ct}/6.7$ shall not exceed $\sqrt{f_c'}$.

11.2.1.2 — When f_{ct} is not specified, all values of $\sqrt{f_c'}$ shall be multiplied by 0.75 for all-lightweight concrete and 0.85 for sand-lightweight concrete. Linear interpolation shall be permitted when partial sand replacement is used.

11.3 — Shear strength provided by concrete for nonprestressed members

11.3.1 — Shear strength V_c shall be computed by provisions of 11.3.1.1 through 11.3.1.3, unless a more detailed calculation is made in accordance with 11.3.2.

11.3.1.1 — For members subject to shear and flexure only,

$$V_c = 2\sqrt{f_c'}\, b_w d \qquad (11\text{-}3)$$

11.3.1.2 — For members subject to axial compression,

$$V_c = 2\left(1 + \frac{N_u}{2000 A_g}\right)\sqrt{f_c'}\, b_w d \qquad (11\text{-}4)$$

Quantity N_u/A_g shall be expressed in psi.

11.3.1.3 — For members subject to significant axial tension, shear reinforcement shall be designed to carry total shear unless a more detailed analysis is made using 11.3.2.3.

COMMENTARY

R11.1.3.2 — Because ***d*** frequently varies in prestressed members, the location of the critical section has arbitrarily been taken as ***h*/2** from the face of the support.

R11.2 — Lightweight concrete

Two alternative procedures are provided to modify the provisions for shear and torsion when lightweight aggregate concrete is used. The lightweight concrete modification applies only to the terms containing $\sqrt{f_c'}$ in the equations of Chapter 11.

R11.2.1.1 — The first alternative bases the modification on laboratory tests to determine the relationship between splitting tensile strength f_{ct} and the compressive strength f_c' for the lightweight concrete being used. For normalweight concrete, the splitting tensile strength f_{ct} is approximately equal to $6.7\sqrt{f_c'}$.[11.10,11.11]

R11.2.1.2 — The second alternative bases the modification on the assumption that the tensile strength of lightweight concrete is a fixed fraction of the tensile strength of normalweight concrete.[11.12] The multipliers are based on data from tests[11.13] on many types of structural lightweight aggregate concrete.

R11.3 — Shear strength provided by concrete for nonprestressed members

R11.3.1.1 — See R11.3.2.1.

R11.3.1.2 and R11.3.1.3 — See R11.3.2.2.

CODE

11.3.2 — Shear strength V_c shall be permitted to be computed by the more detailed calculation of 11.3.2.1 through 11.3.2.3.

11.3.2.1 — For members subject to shear and flexure only,

$$V_c = \left(1.9\sqrt{f'_c} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d \qquad (11\text{-}5)$$

but not greater than $3.5\sqrt{f'_c}\, b_w d$. Quantity $V_u d/M_u$ shall not be taken greater than 1.0 in computing V_c by Eq. (11-5), where M_u is factored moment occurring simultaneously with V_u at section considered.

11.3.2.2 — For members subject to axial compression, it shall be permitted to compute V_c using Eq. (11-5) with M_m substituted for M_u and $V_u d/M_u$ not then limited to 1.0, where

$$M_m = M_u - N_u \frac{(4h - d)}{8} \qquad (11\text{-}6)$$

However, V_c shall not be taken greater than

$$V_c = 3.5\sqrt{f'_c} b_w d \sqrt{1 + \frac{N_u}{500 A_g}} \qquad (11\text{-}7)$$

Quantity N_u/A_g shall be expressed in psi. When M_m as computed by Eq. (11-6) is negative, V_c shall be computed by Eq. (11-7).

COMMENTARY

R11.3.2.1 — Eq. (11-5) is the basic expression for shear strength of members without shear reinforcement.[11.3] Designers should recognize that the three variables in Eq. (11-5), $\sqrt{f'_c}$ (as a measure of concrete tensile strength), ρ_w, and $V_u d/M_u$, are known to affect shear strength, although some research data[11.1,11.14] indicate that Eq. (11-5) overestimates the influence of f'_c and underestimates the influence of ρ_w and $V_u d/M_u$. Further information[11.15] has indicated that shear strength decreases as the overall depth of the member increases.

The minimum value of M_u equal to $V_u d$ in Eq. (11-5) is to limit V_c near points of inflection.

For most designs, it is convenient to assume that the second term of Eq. (11-5) equals $0.1\sqrt{f'_c}$ and use V_c equal to $2\sqrt{f'_c}\, b_w d$ as permitted in 11.3.1.1.

R11.3.2.2 — Eq. (11-6) and (11-7), for members subject to axial compression in addition to shear and flexure are derived in the ACI-ASCE Committee 326 report.[11.3] As N_u is increased, the value of V_c computed from Eq. (11-5) and (11-6) will exceed the upper limit given by Eq. (11-7) before the value of M_m given by Eq. (11-6) becomes negative. The value of V_c obtained from Eq. (11-5) has no physical significance if a negative value of M_m is substituted. For this condition, Eq. (11-7) or Eq. (11-4) should be used to calculate V_c. Values of V_c for members subject to shear and axial load are illustrated in Fig. R11.3.2.2. The background for these equations is discussed and comparisons are made with test data in Reference 11.2.

Because of the complexity of Eq. (11-5) and (11-6), an alternative design provision, Eq. (11-4), is permitted.

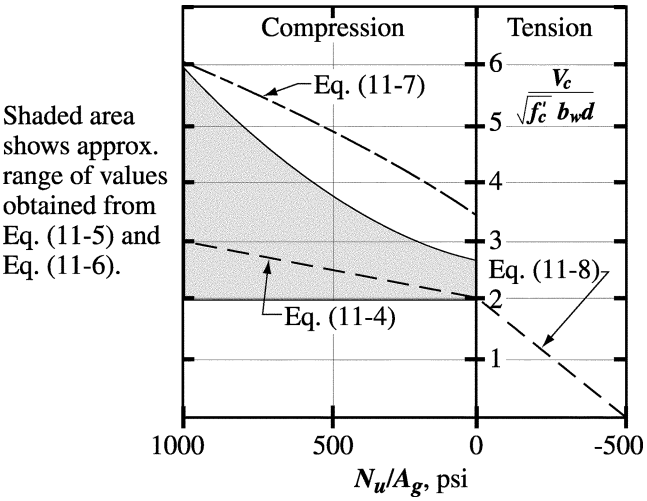


Fig. R11.3.2.2—Comparison of shear strength equations for members subject to axial load

CODE

11.3.2.3 — For members subject to significant axial tension,

$$V_c = 2\left(1 + \frac{N_u}{500A_g}\right)\sqrt{f_c'}\,b_w d \qquad (11\text{-}8)$$

but not less than zero, where N_u is negative for tension. Quantity N_u/A_g shall be expressed in psi.

11.3.3 — For circular members, the area used to compute V_c shall be taken as the product of the diameter and effective depth of the concrete section. It shall be permitted to take the effective depth as 0.8 times the diameter of the concrete section.

11.4 — Shear strength provided by concrete for prestressed members

11.4.1 — For members with effective prestress force not less than 40 percent of the tensile strength of flexural reinforcement, unless a more detailed calculation is made in accordance with 11.4.2,

$$V_c = \left(0.6\sqrt{f_c'} + 700\frac{V_u d}{M_u}\right)b_w d \qquad (11\text{-}9)$$

but V_c need not be taken less than $2\sqrt{f_c'}\,b_w d$ nor shall V_c be taken greater than $5\sqrt{f_c'}\,b_w d$ nor the value given in 11.4.3 or 11.4.4. The quantity $V_u d/M_u$ shall not be taken greater than 1.0, where M_u is factored moment occurring simultaneously with V_u at the section considered. When applying Eq. (11-9), d in the term $V_u d/M_u$ shall be the distance from extreme compression fiber to centroid of prestressed reinforcement.

COMMENTARY

R11.3.2.3 — Eq. (11-8) may be used to compute V_c for members subject to significant axial tension. Shear reinforcement may then be designed for $V_n - V_c$. The term significant is used to recognize that a designer must use judgment in deciding whether axial tension needs to be considered. Low levels of axial tension often occur due to volume changes, but are not important in structures with adequate expansion joints and minimum reinforcement. It may be desirable to design shear reinforcement to carry total shear if there is uncertainty about the magnitude of axial tension.

R11.3.3 — Shear tests of members with circular sections indicate that the effective area can be taken as the gross area of the section or as an equivalent rectangular area.[11.1, 11.16, 11.17]

R11.4 — Shear strength provided by concrete for prestressed members

R11.4.1 — Eq. (11-9) offers a simple means of computing V_c for prestressed concrete beams.[11.2] It may be applied to beams having prestressed reinforcement only, or to members reinforced with a combination of prestressed reinforcement and nonprestressed deformed bars. Eq. (11-9) is most applicable to members subject to uniform loading and may give conservative results when applied to composite girders for bridges.

In applying Eq. (11-9) to simply supported members subject to uniform loads $V_u d/M_u$ can be expressed as:

$$\frac{V_u d}{M_u} = \frac{d(\ell - 2x)}{x(\ell - x)}$$

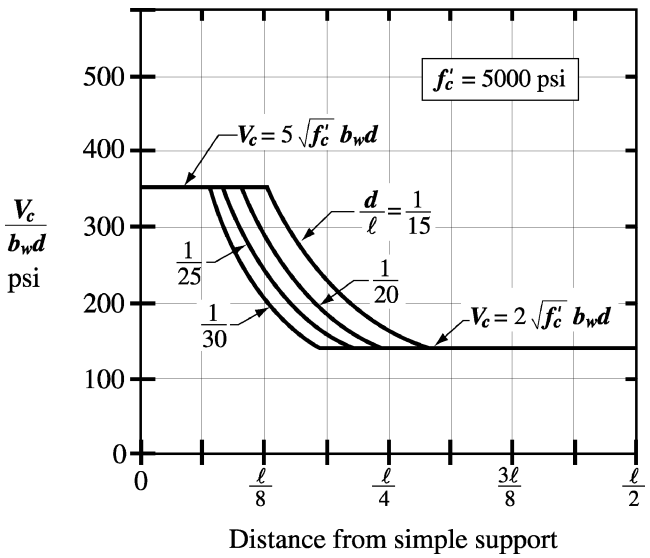


Fig. R11.4.1—Application of Eq. (11-9) to uniformly loaded prestressed members

CODE

11.4.2 — Shear strength V_c shall be permitted to be computed in accordance with 11.4.2.1 and 11.4.2.2, where V_c shall be the lesser of V_{ci} or V_{cw}.

11.4.2.1—Shear strength V_{ci} shall be computed by

$$V_{ci} = 0.6\sqrt{f'_c}\,b_w d + V_d + \frac{V_i M_{cr}}{M_{max}} \quad (11\text{-}10)$$

but V_{ci} need not be taken less than $1.7\sqrt{f'_c}\,b_w d$, where

$$M_{cr} = (I/y_t)(6\sqrt{f'_c} + f_{pe} - f_d) \quad (11\text{-}11)$$

and values of M_{max} and V_i shall be computed from the load combination causing maximum moment to occur at the section.

11.4.2.2—Shear strength V_{cw} shall be computed by

$$V_{cw} = (3.5\sqrt{f'_c} + 0.3f_{pc})b_w d + V_p \quad (11\text{-}12)$$

Alternatively, V_{cw} shall be computed as the shear force corresponding to dead load plus live load that results in a principal tensile stress of $4\sqrt{f'_c}$ at the centroidal axis of member, or at the intersection of flange and web when the centroidal axis is in the flange. In composite members, the principal tensile stress shall be computed using the cross section that resists live load.

11.4.2.3 — In Eq. (11-10) and (11-12), d shall be the distance from extreme compression fiber to centroid of prestressed reinforcement or $0.8h$, whichever is greater.

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where ℓ is the span length and x is the distance from the section being investigated to the support. For concrete with f'_c equal to 5000 psi, V_c from 11.4.1 varies as shown in Fig. R11.4.1. Design aids based on this equation are given in Reference 11.18.

R11.4.2 — Two types of inclined cracking occur in concrete beams: web-shear cracking and flexure-shear cracking. These two types of inclined cracking are illustrated in Fig. R11.4.2.

Web-shear cracking begins from an interior point in a member when the principal tensile stresses exceed the tensile strength of the concrete. Flexure-shear cracking is initiated by flexural cracking. When flexural cracking occurs, the shear stresses in the concrete above the crack are increased. The flexure-shear crack develops when the combined shear and tensile stress exceeds the tensile strength of the concrete.

Eq. (11-10) and (11-12) may be used to determine the shear forces causing flexure-shear and web-shear cracking, respectively. The shear strength provided by the concrete V_c is assumed equal to the lesser of V_{ci} and V_{cw}. The derivations of Eq. (11-10) and (11-12) are summarized in Reference 11.19.

In deriving Eq. (11-10) it was assumed that V_{ci} is the sum of the shear required to cause a flexural crack at the point in question given by:

$$V = \frac{V_i M_{cr}}{M_{max}}$$

plus an additional increment of shear required to change the flexural crack to a flexure-shear crack. The externally applied factored loads, from which V_i and M_{max} are determined, include superimposed dead load, earth pressure, and live load. In computing M_{cr} for substitution into Eq. (11-10), I and y_t are the properties of the section resisting the externally applied loads.

For a composite member, where part of the dead load is resisted by only a part of the section, appropriate section properties should be used to compute f_d. The shear due to

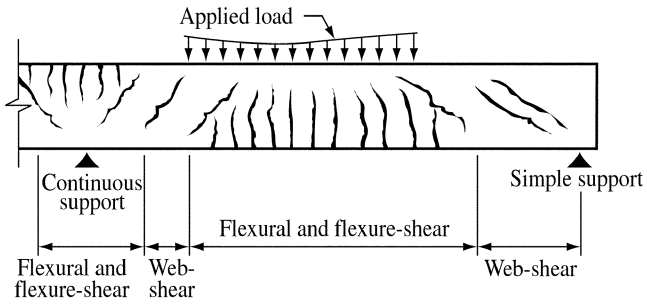


Fig. R11.4.2—Types of cracking in concrete beams

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dead loads, V_d and that due to other loads V_i are separated in this case. V_d is then the total shear force due to unfactored dead load acting on that part of the section carrying the dead loads acting prior to composite action plus the unfactored superimposed dead load acting on the composite member. The terms V_i and M_{max} may be taken as:

$$V_i = V_u - V_d$$

$$M_{max} = M_u - M_d$$

where V_u and M_u are the factored shear and moment due to the total factored loads, and M_d is the moment due to unfactored dead load (the moment corresponding to f_d).

For noncomposite, uniformly loaded beams, the total cross section resists all the shear and the live and dead load shear force diagrams are similar. In this case Eq. (11-10) reduces to:

$$V_{ci} = 0.6\sqrt{f'_c}\,b_w d + \frac{V_u M_{ct}}{M_u}$$

where:

$$M_{ct} = (I/y_t)(6\sqrt{f'_c} + f_{pe})$$

The symbol M_{ct} in the two preceding equations represents the total moment, including dead load, required to cause cracking at the extreme fiber in tension. This is not the same as M_{cr} in code Eq. (11-10) where the cracking moment is that due to all loads except the dead load. In Eq. (11-10) the dead load shear is added as a separate term.

M_u is the factored moment on the beam at the section under consideration, and V_u is the factored shear force occurring simultaneously with M_u. Since the same section properties apply to both dead and live load stresses, there is no need to compute dead load stresses and shears separately. The cracking moment M_{ct} reflects the total stress change from effective prestress to a tension of $6\sqrt{f'_c}$, assumed to cause flexural cracking.

Eq. (11-12) is based on the assumption that web-shear cracking occurs due to the shear causing a principal tensile stress of approximately $4\sqrt{f'_c}$ at the centroidal axis of the cross section. V_p is calculated from the effective prestress force without load factors.

11.4.3 — In a pretensioned member in which the section at a distance $h/2$ from face of support is closer to the end of member than the transfer length of the prestressing steel, the reduced prestress shall be considered when computing V_{cw}. This value of V_{cw} shall also

R11.4.3 and R11.4.4 — The effect of the reduced prestress near the ends of pretensioned beams on the shear strength should be taken into account. Section 11.4.3 relates to the shear strength at sections within the transfer length of prestressing steel when bonding of prestressing steel extends to the end of the member.

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be taken as the maximum limit for Eq. (11-9). The prestress force shall be assumed to vary linearly from zero at end of the prestressing steel, to a maximum at a distance from end of the prestressing steel equal to the transfer length, assumed to be 50 diameters for strand and 100 diameters for single wire.

11.4.4—In a pretensioned member where bonding of some tendons does not extend to the end of member, a reduced prestress shall be considered when computing V_c in accordance with 11.4.1 or 11.4.2. The value of V_{cw} calculated using the reduced prestress shall also be taken as the maximum limit for Eq. (11-9). The prestress force due to tendons for which bonding does not extend to the end of member shall be assumed to vary linearly from zero at the point at which bonding commences to a maximum at a distance from this point equal to the transfer length, assumed to be 50 diameters for strand and 100 diameters for single wire.

11.5 — Shear strength provided by shear reinforcement

11.5.1 — Types of shear reinforcement

11.5.1.1 — Shear reinforcement consisting of the following shall be permitted:

(a) Stirrups perpendicular to axis of member;

(b) Welded wire fabric with wires located perpendicular to axis of member;

(c) Spirals, circular ties, or hoops.

11.5.1.2 — For nonprestressed members, shear reinforcement shall be permitted to also consist of:

(a) Stirrups making an angle of 45 deg or more with longitudinal tension reinforcement;

(b) Longitudinal reinforcement with bent portion making an angle of 30 deg or more with the longitudinal tension reinforcement;

(c) Combinations of stirrups and bent longitudinal reinforcement.

11.5.2 — Design yield strength of shear reinforcement shall not exceed 60,000 psi, except that the design yield strength of welded deformed wire fabric shall not exceed 80,000 psi.

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Section 11.4.4 relates to the shear strength at sections within the length over which some of the prestressing steel is not bonded to the concrete, or within the transfer length of the prestressing steel for which bonding does not extend to the end of the beam.

R11.5 — Shear strength provided by shear reinforcement

R11.5.2 — Limiting the design yield strength of shear reinforcement to 60,000 psi provides a control on diagonal crack width. In the 1995 code, the limitation on design yield strength of 60,000 psi for shear reinforcement was raised to 80,000 psi for welded deformed wire fabric. Research[11.20,11.21,11.22] has indicated that the performance of higher strength steels as shear reinforcement has been

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11.5.3 — Stirrups and other bars or wires used as shear reinforcement shall extend to a distance d from extreme compression fiber and shall be anchored at both ends according to 12.13 to develop the design yield strength of reinforcement.

11.5.4 — Spacing limits for shear reinforcement

11.5.4.1 — Spacing of shear reinforcement placed perpendicular to axis of member shall not exceed $d/2$ in nonprestressed members or $0.75h$ in prestressed members, nor 24 in.

11.5.4.2 — Inclined stirrups and bent longitudinal reinforcement shall be so spaced that every 45 deg line, extending toward the reaction from mid-depth of member $d/2$ to longitudinal tension reinforcement, shall be crossed by at least one line of shear reinforcement.

11.5.4.3 — When V_s exceeds $4\sqrt{f'_c}\,b_w d$, maximum spacings given in 11.5.4.1 and 11.5.4.2 shall be reduced by one-half.

11.5.5 — Minimum shear reinforcement

11.5.5.1 — A minimum area of shear reinforcement shall be provided in all reinforced concrete flexural members (prestressed and nonprestressed) where factored shear force V_u exceeds one-half the shear strength provided by concrete ϕV_c, except:

(a) Slabs and footings;

(b) Concrete joist construction defined by 8.11;

(c) Beams with total depth not greater than 10 in., 2.5 times thickness of flange, or 0.5 the width of web, whichever is greatest.

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satisfactory. In particular, full-scale beam tests described in Reference 11.21 indicated that the widths of inclined shear cracks at service load levels were less for beams reinforced with smaller diameter deformed welded wire fabric cages designed on the basis of a yield strength of 75 ksi than beams reinforced with deformed Grade 60 stirrups.

R11.5.3 — It is essential that shear (and torsion) reinforcement be adequately anchored at both ends to be fully effective on either side of any potential inclined crack. This generally requires a hook or bend at the end of the reinforcement as provided by 12.13.

R11.5.5 — Minimum shear reinforcement

R11.5.5.1 — Shear reinforcement restrains the growth of inclined cracking. Ductility is increased and a warning of failure is provided. In an unreinforced web, the sudden formation of inclined cracking might lead directly to failure without warning. Such reinforcement is of great value if a member is subjected to an unexpected tensile force or an overload. Accordingly, a minimum area of shear reinforcement not less than that given by Eq. (11-13) or (11-14) is required wherever the total factored shear force V_u is greater than one-half the shear strength provided by concrete ϕV_c. Slabs, footings and joists are excluded from the minimum shear reinforcement requirement because there is a possibility of load sharing between weak and strong areas. However, research results[11.23] have shown that deep, lightly reinforced one-way slabs, particularly if constructed with high-strength concrete, may fail at shear loads less than V_c, calculated from Eq. (11-3).

Even when the total factored shear strength V_u is less than one-half of the shear strength provided by the concrete ϕV_c, the use of some web reinforcement is recommended in all thin-web post-tensioned prestressed concrete members (joists, waffle slabs, beams, and T-beams) to reinforce against tensile forces in webs resulting from local deviations from the design tendon profile, and to provide a means of

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11.5.5.2 — Minimum shear reinforcement requirements of 11.5.5.1 shall be permitted to be waived if shown by test that required nominal flexural and shear strengths can be developed when shear reinforcement is omitted. Such tests shall simulate effects of differential settlement, creep, shrinkage, and temperature change, based on a realistic assessment of such effects occurring in service.

11.5.5.3 — Where shear reinforcement is required by 11.5.5.1 or for strength and where 11.6.1 allows torsion to be neglected, the minimum area of shear reinforcement for prestressed (except as provided in 11.5.5.4) and nonprestressed members shall be computed by

$$A_v = 0.75\sqrt{f'_c}\frac{b_w s}{f_y} \qquad (11\text{-}13)$$

but shall not be less than $(50 b_w s)/f_y$ where b_w and s are in inches.

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supporting the tendons in the design profile during construction. If sufficient support is not provided, lateral wobble and local deviations from the smooth parabolic tendon profile assumed in design may result during placement of the concrete. In such cases, the deviations in the tendons tend to straighten out when the tendons are stressed. This process may impose large tensile stresses in webs, and severe cracking may develop if no web reinforcement is provided. Unintended curvature of the tendons, and the resulting tensile stresses in webs, may be minimized by securely tying tendons to stirrups that are rigidly held in place by other elements of the reinforcing cage and held down in the forms. The maximum spacing of stirrups used for this purpose should not exceed the smaller of $1.5h$ or 4 ft. When applicable, the shear reinforcement provisions of 11.5.4 and 11.5.5 will require closer stirrup spacings.

For repeated loading of flexural members, the possibility of inclined diagonal tension cracks forming at stresses appreciably smaller than under static loading should be taken into account in the design. In these instances, it would be prudent to use at least the minimum shear reinforcement expressed by Eq. (11-13) or (11-14), even though tests or calculations based on static loads show that shear reinforcement is not required.

R11.5.5.2 — When a member is tested to demonstrate that its shear and flexural strengths are adequate, the actual member dimensions and material strengths are known. The strength used as a basis for comparison should therefore be that corresponding to a strength reduction factor of unity **(ϕ = 1.0)**, i.e. the required nominal strength V_n and M_n. This ensures that if the actual material strengths in the field were less than specified, or the member dimensions were in error such as to result in a reduced member strength, a satisfactory margin of safety will be retained.

R11.5.5.3 — Previous versions of the code have required a minimum area of transverse reinforcement that is independent of concrete strength. Tests[11.9] have indicated the need to increase the minimum area of shear reinforcement as concrete strength increases to prevent sudden shear failures when inclined cracking occurs. Equation (11-13) provides for a gradual increase in the minimum area of transverse reinforcement, while maintaining the previous minimum value.

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11.5.5.4 — For prestressed members with an effective prestress force not less than 40 percent of the tensile strength of the flexural reinforcement, the area of shear reinforcement shall not be less than the smaller A_v from Eq. (11-13) or (11-14)

$$A_v = \frac{A_{ps} f_{pu} s}{80 f_y d} \sqrt{\frac{d}{b_w}} \qquad (11\text{-}14)$$

11.5.6 — Design of shear reinforcement

11.5.6.1 — Where factored shear force V_u exceeds shear strength ϕV_c, shear reinforcement shall be provided to satisfy Eq. (11-1) and (11-2), where shear strength V_s shall be computed in accordance with 11.5.6.2 through 11.5.6.9.

11.5.6.2 — When shear reinforcement perpendicular to axis of member is used,

$$V_s = \frac{A_v f_y d}{s} \qquad (11\text{-}15)$$

where A_v is the area of shear reinforcement within a distance s.

11.5.6.3 — When circular ties, hoops, or spirals are used as shear reinforcement, V_s shall be computed using Eq. (11-15) where d shall be taken as the effective depth defined in 11.3.3. A_v shall be taken as two times the area of the bar in a circular tie, hoop, or spiral at a spacing s, and f_{yh} is the specified yield strength of circular tie, hoop, or spiral reinforcement.

11.5.6.4 — When inclined stirrups are used as shear reinforcement,

$$V_s = \frac{A_v f_y (\sin\alpha + \cos\alpha) d}{s} \qquad (11\text{-}16)$$

11.5.6.5 — When shear reinforcement consists of a single bar or a single group of parallel bars, all bent up at the same distance from the support,

$$V_s = A_v f_y \sin\alpha \qquad (11\text{-}17)$$

but not greater than $3\sqrt{f_c'}\, b_w d$.

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R11.5.5.4 — Tests[11.24] of prestressed beams with minimum web reinforcement based on Eq. (11-13) and (11-14) indicated that the smaller A_v from these two equations was sufficient to develop ductile behavior.

Eq. (11-14) may be used only for prestressed members meeting the minimum prestress force requirements given in 11.5.5.4. This equation is discussed in Reference 11.24.

R11.5.6 — Design of shear reinforcement

Design of shear reinforcement is based on a modified truss analogy. The truss analogy assumes that the total shear is carried by shear reinforcement. However, considerable research on both nonprestressed and prestressed members has indicated that shear reinforcement needs to be designed to carry only the shear exceeding that which causes inclined cracking, provided the diagonal members in the truss are assumed to be inclined at 45 deg.

Eq. (11-15), (11-16), and (11-17) are presented in terms of shear strength V_s attributed to the shear reinforcement. When shear reinforcement perpendicular to axis of member is used, the required area of shear reinforcement A_v and its spacing s are computed by

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_y d}$$

Research[11.25,11.26] has shown that shear behavior of wide beams with substantial flexural reinforcement is improved if the transverse spacing of stirrup legs across the section is reduced.

R11.5.6.3 — Although the transverse reinforcement in a circular section may not consist of straight legs, tests indicate that Eq. (11-15) is conservative if d is taken as defined in 11.3.3.[11.16, 11.17]

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11.5.6.6 — When shear reinforcement consists of a series of parallel bent-up bars or groups of parallel bent-up bars at different distances from the support, shear strength V_s shall be computed by Eq. (11-16).

11.5.6.7 — Only the center three-fourths of the inclined portion of any longitudinal bent bar shall be considered effective for shear reinforcement.

11.5.6.8 — Where more than one type of shear reinforcement is used to reinforce the same portion of a member, shear strength V_s shall be computed as the sum of the V_s values computed for the various types.

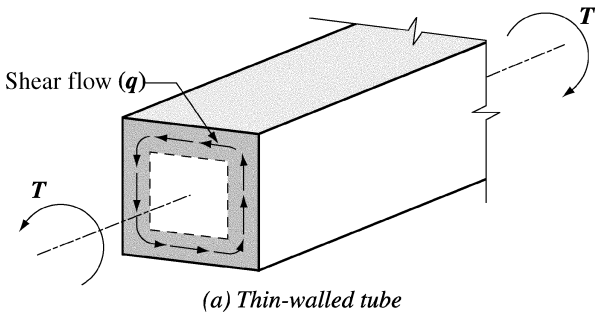
11.5.6.9 — Shear strength V_s shall not be taken greater than $8\sqrt{f'_c}\, b_w d$.

11.6 — Design for torsion

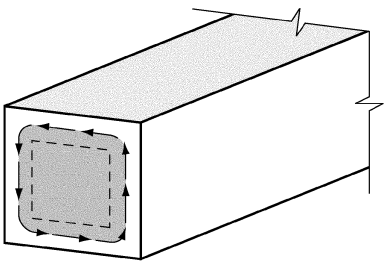
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R11.6 — Design for torsion

The design for torsion is based on a thin-walled tube, space truss analogy. A beam subjected to torsion is idealized as a thin-walled tube with the core concrete cross section in a solid beam neglected as shown in Fig. R11.6(a). Once a reinforced concrete beam has cracked in torsion, its torsional resistance is provided primarily by closed stirrups and longitudinal bars located near the surface of the member. In the thin-walled tube analogy the resistance is assumed to be provided by the outer skin of the cross section roughly centered on the closed stirrups. Both hollow and solid sections are idealized as thin-walled tubes both before and after cracking.



(a) Thin-walled tube



(b) Area enclosed by shear flow path

Fig. R11.6—(a) Thin-walled tube; (b) area enclosed by shear flow path

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11.6.1 — Threshold torsion

It shall be permitted to neglect torsion effects when the factored torsional moment T_u is less than:

(a) For nonprestressed members:

$$\phi\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

(b) For prestressed members:

$$\phi\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\sqrt{f'_c}}}$$

(c) For nonprestressed members subjected to an axial tensile or compressive force:

$$\phi\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\sqrt{f'_c}}}$$

For members cast monolithically with a slab, the overhanging flange width used in computing A_{cp} and p_{cp} shall conform to 13.2.4. For a hollow section, A_g shall be used in place of A_{cp} in 11.6.1, and the outer boundaries of the section shall conform to 13.2.4.

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In a closed thin-walled tube, the product of the shear stress t and the wall thickness t at any point in the perimeter is known as the shear flow, $q = t\,t$. The shear flow q due to torsion acts as shown in Fig. R11.6(a) and is constant at all points around the perimeter of the tube. The path along which it acts extends around the tube at midthickness of the walls of the tube. At any point along the perimeter of the tube the shear stress due to torsion is $t = T/(2A_o t)$ where A_o is the gross area enclosed by the shear flow path, shown shaded in Fig. R11.6(b), and t is the thickness of the wall at the point where t is being computed. The shear flow follows the midthickness of the walls of the tube and A_o is the area enclosed by the path of the shearflow. For a hollow member with continuous walls, A_o includes the area of the hole.

In the 1995 code, the elliptical interaction between the shear carried by the concrete, V_c, and the torsion carried by the concrete was eliminated. V_c remains constant at the value it has when there is no torsion, and the torsion carried by the concrete is always taken as zero.

The design procedure is derived and compared with test results in References 11.27 and 11.28.

R11.6.1 — Threshold torsion

Torques that do not exceed approximately one-quarter of the cracking torque T_{cr} will not cause a structurally significant reduction in either the flexural or shear strength and can be ignored. The cracking torsion under pure torsion T_{cr} is derived by replacing the actual section with an equivalent thin-walled tube with a wall thickness t prior to cracking of $0.75A_{cp}/p_{cp}$ and an area enclosed by the wall centerline A_o equal to $2A_{cp}/3$. Cracking is assumed to occur when the principal tensile stress reaches $4\sqrt{f'_c}$. In a nonprestressed beam loaded with torsion alone, the principal tensile stress is equal to the torsional shear stress, $t = T/(2A_o t)$. Thus, cracking occurs when t reaches $4\sqrt{f'_c}$, giving the cracking torque T_{cr} as:

$$T_{cr} = 4\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

For solid members, the interaction between the cracking torsion and the inclined cracking shear is approximately circular or elliptical. For such a relationship, a torque of $0.25T_{cr}$, as used in 11.6.1, corresponds to a reduction of 3% in the inclined cracking shear. This reduction in the inclined cracking shear was considered negligible. The stress at cracking $4\sqrt{f'_c}$ has purposely been taken as a lower bound value.

For prestressed members, the torsional cracking load is increased by the prestress. A Mohr's Circle analysis based on average stresses indicates the torque required to cause a principal tensile stress equal to $4\sqrt{f'_c}$ is $\sqrt{1+f_{pc}/(4\sqrt{f'_c})}$ times the corresponding torque in a nonprestressed beam. A similar modification is made in part (c) of 11.6.1 for members subjected to axial load and torsion.

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11.6.1.1 — For isolated members with flanges and for members cast monolithically with a slab, the overhanging flange width used in computed A_{cp} and p_{cp} shall conform to 13.2.4, except that the overhanging flanges shall be neglected in cases where the parameter A_{cp}^2/p_{cp} calculated for a beam with flanges is less than that computed for the same beam ignoring the flanges.

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For torsion, a hollow member is defined as having one or more longitudinal voids, such as a single-cell or multiple-cell box girder. Small longitudinal voids, such as ungrouted post-tensioning ducts that result in A_g/A_{cp} greater than or equal to **0.95**, can be ignored when computing the threshold torque in 11.6.1. The interaction between torsional cracking and shear cracking for hollow sections is assumed to vary from the elliptical relationship for members with small voids, to a straight-line relationship for thin-walled sections with large voids. For a straight-line interaction, a torque of $0.25T_{cr}$ would cause a reduction in the inclined cracking shear of about 25%. This reduction was judged to be excessive.

In the 2002 code, two changes were made to modify 11.6.1 to apply to hollow sections. First, the minimum torque limits from the 1999 code were multiplied by (A_g/A_{cp}) because tests of solid and hollow beams[11.29] indicate that the cracking torque of a hollow section is approximately (A_g/A_{cp}) times the cracking torque of a solid section with the same outside dimensions. The second change was to multiply the cracking torque by (A_g/A_{cp}) a second time to reflect the transition from the circular interaction between the inclined cracking loads in shear and torsion for solid members, to the approximately linear interaction for thin-walled hollow sections.

11.6.2 — Calculation of factored torsional moment T_u

R11.6.2 — Calculation of factored torsional moment T_u

11.6.2.1 — If the factored torsional moment T_u in a member is required to maintain equilibrium and exceeds the minimum value given in 11.6.1, the member shall be designed to carry that torsional moment in accordance with 11.6.3 through 11.6.6.

11.6.2.2 — In a statically indeterminate structure where reduction of the torsional moment in a member can occur due to redistribution of internal forces upon cracking, the maximum factored torsional moment T_u shall be permitted to be reduced to the values given in (a), (b), or (c), as applicable:

(a) For nonprestressed members, at the sections described in 11.6.2.4:

$$\phi 4\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

(b) For prestressed members, at the sections described in 11.6.2.5:

$$\phi 4\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\sqrt{f'_c}}}$$

R11.6.2.1 and R11.6.2.2 — In designing for torsion in reinforced concrete structures, two conditions may be identified:[11.30,11.31]

(a) The torsional moment cannot be reduced by redistribution of internal forces (11.6.2.1). This is referred to as equilibrium torsion, since the torsional moment is required for the structure to be in equilibrium.

For this condition, illustrated in Fig. R11.6.2.1, torsion reinforcement designed according to 11.6.3 through 11.6.6 must be provided to resist the total design torsional moments.

(b) The torsional moment can be reduced by redistribution of internal forces after cracking (11.6.2.2) if the torsion arises from the member twisting to maintain compatibility of deformations. This type of torsion is referred to as compatibility torsion.

For this condition, illustrated in Fig. R11.6.2.2, the torsional stiffness before cracking corresponds to that of the uncracked section according to St. Venant's theory. At torsional cracking, however, a large twist occurs under an essentially constant torque, resulting in a large redistribution of forces in the structure.[11.30,11.31] The cracking torque under combined shear, flexure, and torsion corresponds to a principal tensile stress somewhat less than the $4\sqrt{f'_c}$ quoted in R11.6.1.

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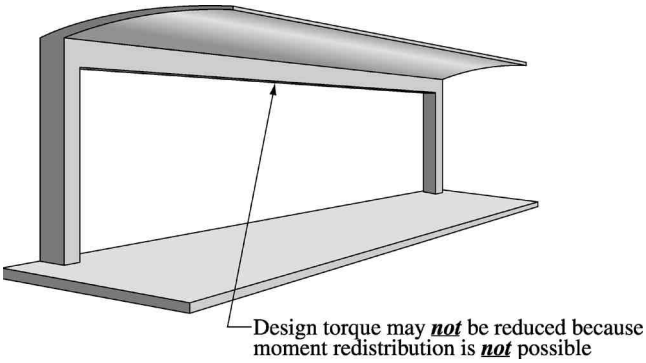


Fig. R11.6.2.1—Design torque may not be reduced (11.6.2.1)

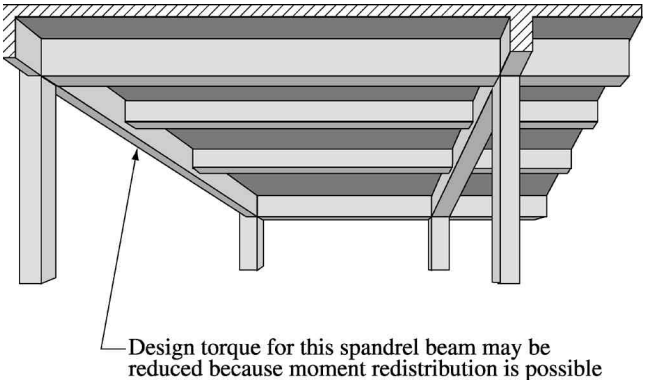


Fig. R11.6.2.2—Design torque may be reduced (11.6.2.2)

(c) For nonprestressed members subjected to an axial tensile or compressive force:

$$f\,4\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\sqrt{f'_c}}}$$

In (a), (b), or (c), the correspondingly redistributed bending moments and shears in the adjoining members shall be used in the design of these members. For hollow sections, A_{cp} shall not be replaced with A_g in 11.6.2.2.

When the torsional moment exceeds the cracking torque, a maximum factored torsional moment equal to the cracking torque may be assumed to occur at the critical sections near the faces of the supports. This limit has been established to control the width of torsional cracks. The replacement of A_{cp} with A_g, as in the calculation of the threshold torque for hollow sections in 11.6.1, is not applied here. Thus, the torque after redistribution is larger and hence more conservative.

Section 11.6.2.2 applies to typical and regular framing conditions. With layouts that impose significant torsional rotations within a limited length of the member, such as a heavy torque loading located close to a stiff column, or a column that rotates in the reverse directions because of other loading, a more exact analysis is advisable.

When the factored torsional moment from an elastic analysis based on uncracked section properties is between the values in 11.6.1 and the values given in this section, torsion reinforcement should be designed to resist the computed torsional moments.

11.6.2.3 — Unless determined by a more exact analysis, it shall be permitted to take the torsional loading from a slab as uniformly distributed along the member.

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11.6.2.4 — In nonprestressed members, sections located less than a distance ***d*** from the face of a support shall be designed for not less than the torsion T_u computed at a distance ***d***. If a concentrated torque occurs within this distance, the critical section for design shall be at the face of the support.

11.6.2.5 — In prestressed members, sections located less than a distance ***h/2*** from the face of a support shall be designed for not less than the torsion T_u computed at a distance ***h/2***. If a concentrated torque occurs within this distance, the critical section for design shall be at the face of the support.

11.6.3 — Torsional moment strength

11.6.3.1 — The cross-sectional dimensions shall be such that:

(a) For solid sections:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \qquad (11\text{-}18)$$

(b) For hollow sections:

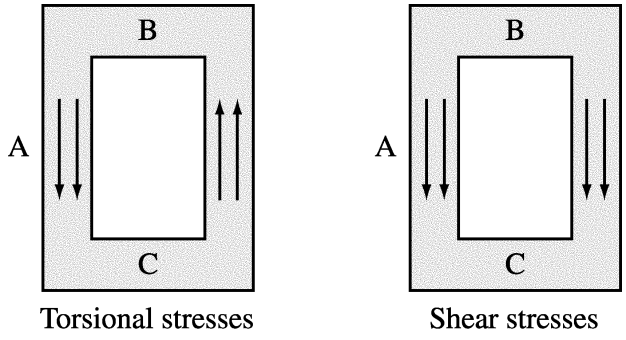
$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right) \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \qquad (11\text{-}19)$$

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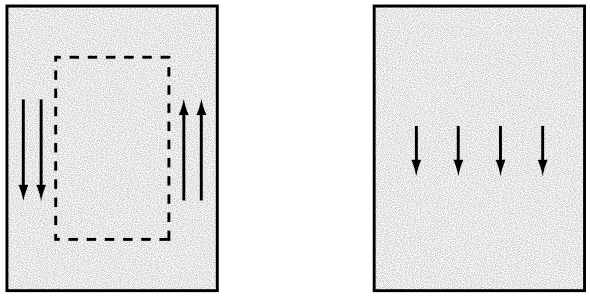
R11.6.2.4 and R11.6.2.5 — It is not uncommon for a beam to frame into one side of a girder near the support of the girder. In such a case a concentrated shear and torque are applied to the girder.

R11.6.3 — Torsional moment strength

R11.6.3.1 —The size of a cross section is limited for two reasons, first to reduce unsightly cracking and second to prevent crushing of the surface concrete due to inclined compressive stresses due to shear and torsion. In Eq. (11-18) and (11-19), the two terms on the left hand side are the shear stresses due to shear and torsion. The sum of these stresses may not exceed the stress causing shear cracking plus $8\sqrt{f'_c}$, similar to the limiting strength given in 11.5.6.9 for shear without torsion. The limit is expressed in terms of V_c to allow its use for nonprestressed or prestressed concrete. It was originally derived on the basis of crack control. It is not necessary to check against crushing of the web since this happens at higher shear stresses.



(a) Hollow section



(b) Solid section

Fig. R11.6.3.1—Addition of torsional and shear stresses

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11.6.3.2—If the wall thickness varies around the perimeter of a hollow section, Eq. (11-19) shall be evaluated at the location where the left-hand side of Eq. (11-19) is a maximum.

11.6.3.3 — If the wall thickness is less than A_{oh}/p_h, the second term in Eq. (11-19) shall be taken as:

$$\left(\frac{T_u}{1.7A_{oh}t}\right)$$

where t is the thickness of the wall of the hollow section at the location where the stresses are being checked.

11.6.3.4 — Design yield strength of nonprestressed torsion reinforcement shall not exceed 60,000 psi.

11.6.3.5 — The reinforcement required for torsion shall be determined from:

$$\phi T_n \geq T_u \qquad (11\text{-}20)$$

11.6.3.6 — The transverse reinforcement for torsion shall be designed using:

$$T_n = \frac{2A_oA_tf_{yv}}{s}\cot\theta \qquad (11\text{-}21)$$

where A_o shall be determined by analysis except that it shall be permitted to take A_o equal to $0.85A_{oh}$; θ shall not be taken smaller than 30 deg nor larger than 60 deg. It shall be permitted to take θ equal to:

(a) 45 deg for nonprestressed members or members with less prestress than in (b); or

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In a hollow section, the shear stresses due to shear and torsion both occur in the walls of the box as shown in Fig. 11.6.3.1(a) and hence are directly additive at point A as given in Eq. (11-19). In a solid section the shear stresses due to torsion act in the "tubular" outside section while the shear stresses due to V_u are spread across the width of the section as shown in Fig. R11.6.3.1(b). For this reason stresses are combined in Eq. (11-18) using the square root of the sum of the squares rather than by direct addition.

R11.6.3.2 — Generally, the maximum will be on the wall where the torsional and shearing stresses are additive [Point A in Fig. R11.6.3.1(a)]. If the top or bottom flanges are thinner than the vertical webs, it may be necessary to evaluate Eq. (11-19) at points B and C in Fig. R11.6.3.1(a). At these points the stresses due to the shear force are usually negligible.

R11.6.3.4 — Limiting the design yield strength of torsion reinforcement to 60,000 psi provides a control on diagonal crack width.

R11.6.3.5 — The factored torsional resistance ϕT_n must equal or exceed the torsion T_u due to the factored loads. In the calculation of T_n, all the torque is assumed to be resisted by stirrups and longitudinal steel with $T_c = 0$. At the same time, the shear resisted by concrete V_c is assumed to be unchanged by the presence of torsion. For beams with V_u greater than about $0.8\phi V_c$ the resulting amount of combined shear and torsional reinforcement is essentially the same as required by the 1989 code. For smaller values of V_u, more shear and torsion reinforcement will be required.

R11.6.3.6 — Eq. (11-21) is based on the space truss analogy shown in Fig. R11.6.3.6(a) with compression diagonals at an angle θ, assuming the concrete carries no tension and the reinforcement yields. After torsional cracking develops, the torsional resistance is provided mainly by closed stirrups, longitudinal bars, and compression diagonals. The concrete outside these stirrups is relatively ineffective. For this reason A_o, the area enclosed by the shear flow path around the perimeter of the tube, is defined after cracking in terms of A_{oh}, the area enclosed by the centerline of the outermost closed hoops. The area A_{oh} is shown in Fig. R11.6.3.6(b) for various cross sections. In an I-, T-, or L-shaped section, A_{oh} is taken as that area enclosed by the outermost legs of interlocking stirrups as shown in Fig. R11.6.3.6(b). The expression for A_o given by Hsu[11.32] may be used if greater accuracy is desired.

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(b) 37.5 deg for prestressed members with an effective prestress force not less than 40 percent of the tensile strength of the longitudinal reinforcement.

11.6.3.7 — The additional longitudinal reinforcement required for torsion shall not be less than:

$$A_\ell = \frac{A_t}{s} p_h \left(\frac{f_{yv}}{f_{y\ell}}\right) \cot^2 \theta \qquad (11\text{-}22)$$

where θ shall be the same value used in Eq. (11-21) and A_t/s shall be taken as the amount computed from Eq. (11-21) not modified in accordance with 11.6.5.2 or 11.6.5.3.

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The shear flow q in the walls of the tube, discussed in R11.6, can be resolved into the shear forces V_1 to V_4 acting in the individual sides of the tube or space truss, as shown in Fig. R11.6.3.6(a).

The angle θ can be obtained by analysis[11.32] or may be taken to be equal to the values given in 11.6.3.6(a) or (b). The same value of θ should be used in both Eq. (11-21) and (11-22). As θ gets smaller, the amount of stirrups required by Eq. (11-21) decreases. At the same time the amount of longitudinal steel required by Eq. (11-22) increases.

R11.6.3.7 — Fig. R11.6.3.6(a) shows the shear forces V_1 to V_4 resulting from the shear flow around the walls of the tube. On a given wall of the tube, the shear flow V_i is resisted by a diagonal compression component, $D_i = V_i/\sin\theta$, in the concrete. An axial tension force, $N_i = V_i(\cot\theta)$, is needed in the longitudinal steel to complete the resolution of V_i.

Fig. R11.6.3.7 shows the diagonal compressive stresses and the axial tension force, N_i, acting on a short segment along one wall of the tube. Because the shear flow due to torsion is constant at all points around the perimeter of the tube, the resultants of D_i and N_i act through the midheight of side i. As a result, half of N_i can be assumed to be resisted by each of the top and bottom chords as shown. Longitudinal reinforcement with a capacity $A_\ell f_{y\ell}$ should be provided to resist the sum of the N_i forces, ΣN_i, acting in all of the walls of the tube.

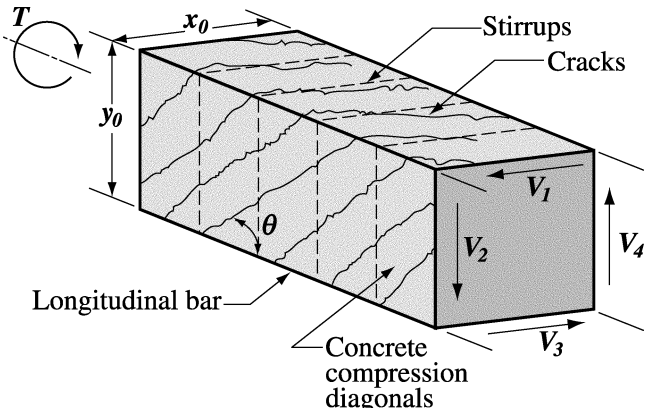
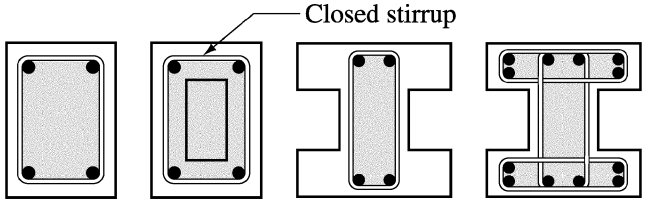


Fig. R11.6.3.6(a)—Space truss analogy



A_{oh} = shaded area

Fig. R11.6.3.6(b)—Definition of A_{oh}

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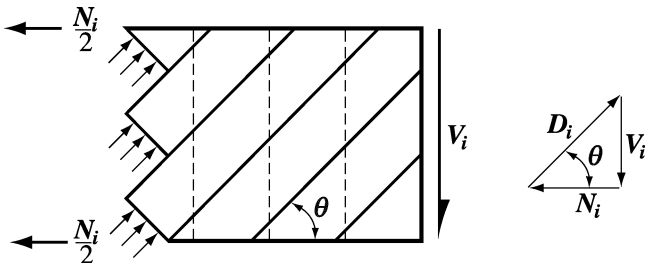


Fig. R11.6.3.7—Resolution of shear force $\mathbf{V_i}$ *into diagonal compression force* $\mathbf{D_i}$ *and axial tension force* $\mathbf{N_i}$ *in one wall of the tube*

In the derivation of Eq. (11-22), axial tension forces are summed along the sides of the area A_o. These sides form a perimeter length, p_o, approximately equal to the length of the line joining the centers of the bars in the corners of the tube. For ease in computation this has been replaced with the perimeter of the closed stirrups, p_h.

Frequently, the maximum allowable stirrup spacing governs the amount of stirrups provided. Furthermore, when combined shear and torsion act, the total stirrup area is the sum of the amounts provided for shear and torsion. To avoid the need to provide excessive amounts of longitudinal reinforcement, 11.6.3.7 states that the A_t/s used in calculating A_ℓ at any given section should be taken as the A_t/s calculated at that section using Eq. (11-21).

11.6.3.8 — Reinforcement required for torsion shall be added to that required for the shear, moment and axial force that act in combination with the torsion. The most restrictive requirements for reinforcement spacing and placement shall be met.

R11.6.3.8 — The stirrup requirements for torsion and shear are added and stirrups are provided to supply at least the total amount required. Since the stirrup area A_v for shear is defined in terms of all the legs of a given stirrup while the stirrup area A_t for torsion is defined in terms of one leg only, the addition of stirrups is carried out as follows:

$$\text{Total}\left(\frac{A_{v+t}}{s}\right) = \frac{A_v}{s} + 2\frac{A_t}{s}$$

If a stirrup group had four legs for shear, only the legs adjacent to the sides of the beam would be included in this summation since the inner legs would be ineffective for torsion.

The longitudinal reinforcement required for torsion is added at each section to the longitudinal reinforcement required for bending moment that acts at the same time as the torsion. The longitudinal reinforcement is then chosen for this sum, but should not be less than the amount required for the maximum bending moment at that section if this exceeds the moment acting at the same time as the torsion. If the maximum bending moment occurs at one section, such as the midspan, while the maximum torsional moment occurs at another, such as the support, the total longitudinal steel required may be less than that obtained by adding the maximum flexural steel plus the maximum torsional steel. In such a case the required longitudinal steel is evaluated at several locations.

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11.6.3.9 — It shall be permitted to reduce the area of longitudinal torsion reinforcement in the flexural compression zone by an amount equal to $M_u / (0.9 d f_{y\ell})$, where M_u is the factored moment acting at the section in combination with T_u, except that the reinforcement provided shall not be less than that required by 11.6.5.3 or 11.6.6.2.

11.6.3.10 — In prestressed beams:

(a) The total longitudinal reinforcement including prestressing steel at each section shall resist the factored bending moment at that section plus an additional concentric longitudinal tensile force equal to $A_\ell f_{y\ell}$, based on the factored torsion at that section;

(b) The spacing of the longitudinal reinforcement including tendons shall satisfy the requirements in 11.6.6.2.

11.6.3.11 — In prestressed beams, it shall be permitted to reduce the area of longitudinal torsional reinforcement on the side of the member in compression due to flexure below that required by 11.6.3.10 in accordance with 11.6.3.9.

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The most restrictive requirements for spacing, cut-off points, and placement for flexural, shear, and torsional steel should be satisfied. The flexural steel should be extended a distance d, but not less than $12 d_b$, past where it is no longer needed for flexure as required in 12.10.3.

R11.6.3.9 — The longitudinal tension due to torsion is offset in part by the compression in the flexural compression zone, allowing a reduction in the longitudinal torsion steel required in the compression zone.

R11.6.3.10 — As explained in R11.6.3.7, torsion causes an axial tension force. In a nonprestressed beam this force is resisted by longitudinal reinforcement having an axial tensile capacity of $A_\ell f_{y\ell}$. This steel is in addition to the flexural reinforcement and is distributed uniformly around the sides of the perimeter so that the resultant of $A_\ell f_{y\ell}$ acts along the axis of the member.

In a prestressed beam, the same technique (providing additional reinforcing bars with capacity $A_\ell f_{y\ell}$) can be followed, or the designer can use any overcapacity of the prestressing steel to resist some of the axial force $A_\ell f_{y\ell}$ as outlined in the next paragraph.

In a prestressed beam, the prestressing steel stress at ultimate at the section of the maximum moment is f_{ps}. At other sections, the prestressing steel stress at ultimate will be between f_{se} and f_{ps}. A portion of the $A_\ell f_{y\ell}$ force acting on the sides of the perimeter where the prestressing steel is located can be resisted by a force $A_{ps} \Delta f_p$ in the prestressing steel, where Δf_p is f_{ps} minus the prestressing steel stress due to flexure at the ultimate load at the section in question. This can be taken as M_u at the section, divided by $(f 0.9 d_p A_{ps})$, but Δf_p should not be more than 60 ksi. Longitudinal reinforcing bars will be required on the other sides of the member to provide the remainder of the $A_\ell f_{y\ell}$ force, or to satisfy the spacing requirements given in 11.6.6.2, or both.

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11.6.4 — Details of torsional reinforcement

11.6.4.1 — Torsion reinforcement shall consist of longitudinal bars or tendons and one or more of the following:

(a) Closed stirrups or closed ties, perpendicular to the axis of the member;

(b) A closed cage of welded wire fabric with transverse wires perpendicular to the axis of the member;

(c) In nonprestressed beams, spiral reinforcement.

11.6.4.2 — Transverse torsional reinforcement shall be anchored by one of the following:

(a) A 135 deg standard hook around a longitudinal bar;

(b) According to 12.13.2.1, 12.13.2.2, or 12.13.2.3 in regions where the concrete surrounding the anchorage is restrained against spalling by a flange or slab or similar member.

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R11.6.4 — Details of torsional reinforcement

R11.6.4.1 — Both longitudinal and closed transverse reinforcement are required to resist the diagonal tension stresses due to torsion. The stirrups must be closed, since inclined cracking due to torsion may occur on all faces of a member.

In the case of sections subjected primarily to torsion, the concrete side cover over the stirrups spalls off at high torques.[11.33] This renders lapped-spliced stirrups ineffective, leading to a premature torsional failure.[11.34] In such cases, closed stirrups should not be made up of pairs of U-stirrups lapping one another.

R11.6.4.2 — When a rectangular beam fails in torsion, the corners of the beam tend to spall off due to the inclined compressive stresses in the concrete diagonals of the space truss changing direction at the corner as shown in Fig. R11.6.4.2(a). In tests,[11.33] closed stirrups anchored by 90 deg hooks failed when this occurred. For this reason, 135 deg hooks are preferable for torsional stirrups in all cases. In regions where this spalling is prevented by an adjacent slab or flange, 11.6.4.2(b) relaxes this and allows 90 deg hooks.

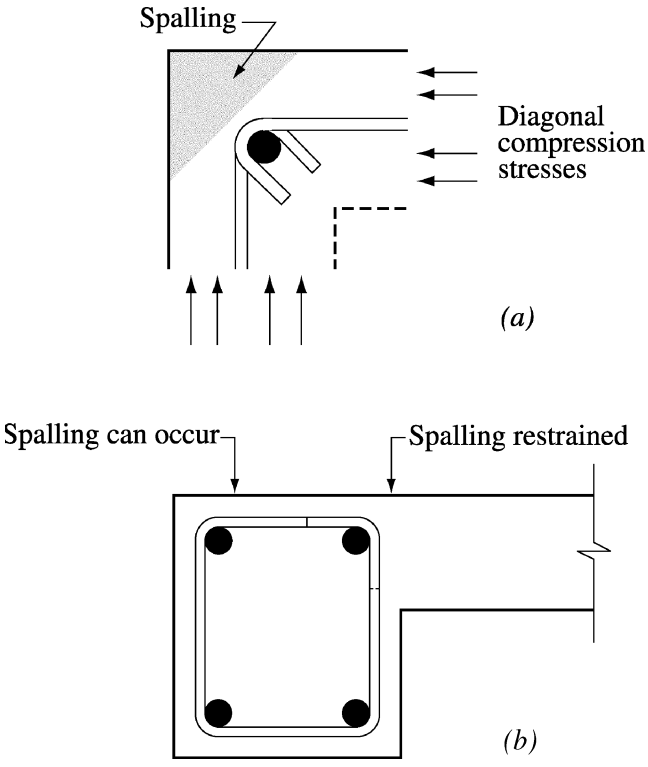


Fig. R11.6.4.2—Spalling of corners of beams loaded in torsion

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11.6.4.3 — Longitudinal torsion reinforcement shall be developed at both ends.

11.6.4.4 — For hollow sections in torsion, the distance from the centerline of the transverse torsional reinforcement to the inside face of the wall of the hollow section shall not be less than $0.5A_{oh}/p_h$.

11.6.5 — Minimum torsion reinforcement

11.6.5.1 — A minimum area of torsion reinforcement shall be provided in all regions where the factored torsional moment T_u exceeds the values specified in 11.6.1.

11.6.5.2 — Where torsional reinforcement is required by 11.6.5.1, the minimum area of transverse closed stirrups shall be computed by:

$$(A_v + 2A_t) = 0.75\sqrt{f'_c}\frac{b_w s}{f_{yv}} \qquad (11\text{-}23)$$

but shall not be less than $(50b_w s)/f_{yv}$.

11.6.5.3 — Where torsional reinforcement is required by 11.6.5.1, the minimum total area of longitudinal torsional reinforcement shall be computed by:

$$A_{\ell,min} = \frac{5\sqrt{f'_c}A_{cp}}{f_{y\ell}} - \left(\frac{A_t}{s}\right)p_h\frac{f_{yv}}{f_{y\ell}} \qquad (11\text{-}24)$$

where A_t/s shall not be taken less than $25b_w/f_{yv}$.

11.6.6 — Spacing of torsion reinforcement

11.6.6.1 — The spacing of transverse torsion reinforcement shall not exceed the smaller of $p_h/8$ or 12 in.

11.6.6.2 — The longitudinal reinforcement required for torsion shall be distributed around the perimeter of the closed stirrups with a maximum spacing of 12 in. The longitudinal bars or tendons shall be inside the stirrups. There shall be at least one longitudinal bar or

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R11.6.4.3 — If high torsion acts near the end of a beam, the longitudinal torsion reinforcement should be adequately anchored. Sufficient development length should be provided outside the inner face of the support to develop the needed tension force in the bars or tendons. In the case of bars, this may require hooks or horizontal U-shaped bars lapped with the longitudinal torsion reinforcement.

R11.6.4.4 — The closed stirrups provided for torsion in a hollow section should be located in the outer half of the wall thickness effective for torsion where the wall thickness can be taken as A_{oh}/p_h.

R11.6.5 — Minimum torsion reinforcement

R11.6.5.1 and R11.6.5.2 — If a member is subject to a factored torsional moment T_u greater than the values specified in 11.6.1, the minimum amount of transverse web reinforcement for combined shear and torsion is $50b_w s/f_{yv}$. The differences in the definition of A_v and the symbol A_t should be noted; A_v is the area of two legs of a closed stirrup while A_t is the area of only one leg of a closed stirrup.

Tests[11.9] of high-strength reinforced concrete beams have indicated the need to increase the minimum area of shear reinforcement to prevent shear failures when inclined cracking occurs. Although there are a limited number of tests of high-strength concrete beams in torsion, the equation for the minimum area of transverse closed stirrups has been changed for consistency with calculations required for minimum shear reinforcement.

R11.6.5.3 — Reinforced concrete beam specimens with less than 1 percent torsional reinforcement by volume have failed in pure torsion at torsional cracking.[11.27] In the 1989 and prior codes, a relationship was presented that required about 1 percent torsional reinforcement in beams loaded in pure torsion and less in beams with combined shear and torsion, as a function of the ratio of shear stresses due to torsion and shear. Eq. (11-24) was simplified by assuming a single value of this reduction factor and results in a volumetric ratio of about 0.5 percent.

R11.6.6 — Spacing of torsion reinforcement

R11.6.6.1 — The spacing of the stirrups is limited to ensure the development of the ultimate torsional strength of the beam, to prevent excessive loss of torsional stiffness after cracking, and to control crack widths. For a square cross section the $p_h/8$ limitation requires stirrups at $d/2$, which corresponds to 11.5.4.1.

R11.6.6.2 — In R11.6.3.7 it was shown that longitudinal reinforcement is needed to resist the sum of the longitudinal tensile forces due to torsion in the walls of the thin-walled tube. Since the force acts along the centroidal axis of the section, the centroid of the additional longitudinal reinforcement

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tendon in each corner of the stirrups. Bars shall have a diameter at least 0.042 times the stirrup spacing, but not less than a No. 3 bar.

11.6.6.3 — Torsion reinforcement shall be provided for a distance of at least $(b_t + d)$ beyond the point theoretically required.

11.7 — Shear-friction

11.7.1 — Provisions of 11.7 are to be applied where it is appropriate to consider shear transfer across a given plane, such as: an existing or potential crack, an interface between dissimilar materials, or an interface between two concretes cast at different times.

11.7.2 — Design of cross sections subject to shear transfer as described in 11.7.1 shall be based on Eq. (11-1), where V_n is calculated in accordance with provisions of 11.7.3 or 11.7.4.

11.7.3 — A crack shall be assumed to occur along the shear plane considered. The required area of shear-friction reinforcement A_{vf} across the shear plane shall be designed using either 11.7.4 or any other shear transfer design methods that result in prediction of strength in substantial agreement with results of comprehensive tests.

11.7.3.1 — Provisions of 11.7.5 through 11.7.10 shall apply for all calculations of shear transfer strength.

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for torsion should approximately coincide with the centroid of the section. The code accomplishes this by requiring the longitudinal torsional reinforcement to be distributed around the perimeter of the closed stirrups. Longitudinal bars or tendons are required in each corner of the stirrups to provide anchorage for the legs of the stirrups. Corner bars have also been found to be very effective in developing torsional strength and in controlling cracks.

R11.6.6.3 — The distance $(b_t + d)$ beyond the point theoretically required for torsional reinforcement is larger than that used for shear and flexural reinforcement because torsional diagonal tension cracks develop in a helical form.

R11.7 — Shear-friction

R11.7.1 — With the exception of 11.7, virtually all provisions regarding shear are intended to prevent diagonal tension failures rather than direct shear transfer failures. The purpose of 11.7 is to provide design methods for conditions where shear transfer should be considered: an interface between concretes cast at different times, an interface between concrete and steel, reinforcement details for precast concrete structures, and other situations where it is considered appropriate to investigate shear transfer across a plane in structural concrete. (See References 11.35 and 11.36.)

R11.7.3 — Although uncracked concrete is relatively strong in direct shear there is always the possibility that a crack will form in an unfavorable location. The shear-friction concept assumes that such a crack will form, and that reinforcement must be provided across the crack to resist relative displacement along it. When shear acts along a crack, one crack face slips relative to the other. If the crack faces are rough and irregular, this slip is accompanied by separation of the crack faces. At ultimate, the separation is sufficient to stress the reinforcement crossing the crack to its yield point. The reinforcement provides a clamping force $A_{vf}f_y$ across the crack faces. The applied shear is then resisted by friction between the crack faces, by resistance to the shearing off of protrusions on the crack faces, and by dowel action of the reinforcement crossing the crack. Successful application of 11.7 depends on proper selection of the location of an assumed crack.[11.18,11.35]

The relationship between shear-transfer strength and the reinforcement crossing the shear plane can be expressed in various ways. Eq. (11-25) and (11-26) of 11.7.4 are based on the shear-friction model. This gives a conservative prediction of shear-transfer strength. Other relationships that give a closer estimate of shear-transfer strength[11.18,11.37,11.38] can be used under the provisions of 11.7.3. For example,

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11.7.4 — Shear-friction design method

11.7.4.1 — When shear-friction reinforcement is perpendicular to the shear plane, shear strength V_n shall be computed by

$$V_n = A_{vf} f_y \mu \quad (11\text{-}25)$$

where μ is coefficient of friction in accordance with 11.7.4.3.

11.7.4.2 — When shear-friction reinforcement is inclined to the shear plane, such that the shear force produces tension in shear-friction reinforcement, shear strength V_n shall be computed by

$$V_n = A_{vf} f_y (\mu \sin \alpha_f + \cos \alpha_f) \quad (11\text{-}26)$$

where α_f is angle between shear-friction reinforcement and shear plane.

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when the shear-friction reinforcement is perpendicular to the shear plane, the shear strength V_n is given by[11.37,11.38]

$$V_n = 0.8 A_{vf} f_y + A_c K_1$$

where A_c is the area of concrete section resisting shear transfer (in.^{2}) and K_1 = **400** psi for normalweight concrete, 200 psi for all-lightweight concrete, and 250 psi for sand-lightweight concrete. These values of K_1 apply to both monolithically cast concrete and to concrete cast against hardened concrete with a rough surface, as defined in 11.7.9.

In this equation, the first term represents the contribution of friction to shear-transfer resistance (0.8 representing the coefficient of friction). The second term represents the sum of the resistance to shearing of protrusions on the crack faces and the dowel action of the reinforcement.

When the shear-friction reinforcement is inclined to the shear plane, such that the shear force produces tension in that reinforcement, the shear strength V_n is given by

$$V_n = A_{vf} f_y (0.8 \sin \alpha_f + \cos \alpha_f) + A_c K_1 \sin^2 \alpha_f$$

where α_f is the angle between the shear-friction reinforcement and the shear plane, (i.e. $0 < \alpha_f < 90$ deg).

When using the modified shear-friction method, the terms $(A_{vf} f_y / A_c)$ or $(A_{vf} f_y \sin \alpha_f / A_c)$ should not be less than 200 psi for the design equations to be valid.

R11.7.4 — Shear-friction design method

R11.7.4.1 — The required area of shear-transfer reinforcement A_{vf} is computed using

$$A_{vf} = \frac{V_u}{\phi f_y \mu}$$

The specified upper limit on shear strength should also be observed.

R11.7.4.2 — When the shear-friction reinforcement is inclined to the shear plane, such that the component of the shear force parallel to the reinforcement tends to produce tension in the reinforcement, as shown in Fig. R11.7.4, part of the shear is resisted by the component parallel to the shear plane of the tension force in the reinforcement.[11.38] Eq. (11-26) should be used only when the shear force component parallel to the reinforcement produces tension in the reinforcement, as shown in Fig. R11.7.4. When α_f is greater than 90 deg, the relative movement of the surfaces tends to compress the bar and Eq. (11-26) is not valid.

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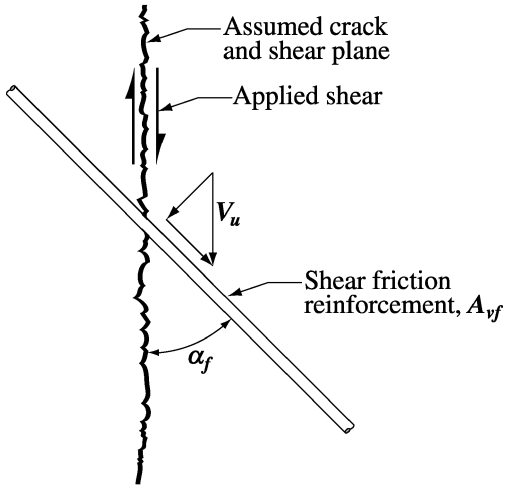


Fig. R11.7.4—Shear-friction reinforcement at an angle to assumed crack

11.7.4.3 — The coefficient of friction m in Eq. (11-25) and Eq. (11-26) shall be

Concrete placed monolithically	**1.4** l
Concrete placed against hardened concrete with surface intentionally roughened as specified in 11.7.9	**1.0** l
Concrete placed against hardened concrete not intentionally roughened	**0.6** l
Concrete anchored to as-rolled structural steel by headed studs or by reinforcing bars (see 11.7.10)	**0.7** l

where l **= 1.0** for normalweight concrete, 0.85 for sand-lightweight concrete and 0.75 for all lightweight concrete. Linear interpolation shall be permitted when partial sand replacement is used.

R11.7.4.3 — In the shear-friction method of calculation, it is assumed that all the shear resistance is due to the friction between the crack faces. It is, therefore, necessary to use artificially high values of the coefficient of friction in the shear-friction equations so that the calculated shear strength will be in reasonable agreement with test results. For concrete cast against hardened concrete not roughened in accordance with 11.7.9, shear resistance is primarily due to dowel action of the reinforcement and tests[11.39] indicate that reduced value of m **= 0.6** l specified for this case is appropriate.

The value of m for concrete placed against as-rolled structural steel relates to the design of connections between precast concrete members, or between structural steel members and structural concrete members. The shear-transfer reinforcement may be either reinforcing bars or headed stud shear connectors; also, field welding to steel plates after casting of concrete is common. The design of shear connectors for composite action of concrete slabs and steel beams is not covered by these provisions, but should be in accordance with Reference 11.40.

11.7.5 — Shear strength V_n shall not be taken greater than $0.2f_c'A_c$ nor $800A_c$ in lb, where A_c is area of concrete section resisting shear transfer.

R11.7.5 — This upper limit on shear strength is specified because Eq. (11-25) and (11-26) become unconservative if V_n has a greater value.

11.7.6 — Design yield strength of shear-friction reinforcement shall not exceed 60,000 psi.

CODE

11.7.7 — Net tension across shear plane shall be resisted by additional reinforcement. Permanent net compression across shear plane shall be permitted to be taken as additive to the force in the shear-friction reinforcement $A_{vf} f_y$ when calculating required A_{vf}.

11.7.8 — Shear-friction reinforcement shall be appropriately placed along the shear plane and shall be anchored to develop the specified yield strength on both sides by embedment, hooks, or welding to special devices.

11.7.9 — For the purpose of 11.7, when concrete is placed against previously hardened concrete, the interface for shear transfer shall be clean and free of laitance. If μ is assumed equal to $\mathbf{1.0}\lambda$, interface shall be roughened to a full amplitude of approximately 1/4 in.

COMMENTARY

R11.7.7 — If a resultant tensile force acts across a shear plane, reinforcement to carry that tension should be provided in addition to that provided for shear transfer. Tension may be caused by restraint of deformations due to temperature change, creep, and shrinkage. Such tensile forces have caused failures, particularly in beam bearings.

When moment acts on a shear plane, the flexural tension stresses and flexural compression stresses are in equilibrium. There is no change in the resultant compression $A_{vf} f_y$ acting across the shear plane and the shear-transfer strength is not changed. It is therefore not necessary to provide additional reinforcement to resist the flexural tension stresses, unless the required flexural tension reinforcement exceeds the amount of shear-transfer reinforcement provided in the flexural tension zone. This has been demonstrated experimentally.[11.41]

It has also been demonstrated experimentally[11.36] that if a resultant compressive force acts across a shear plane, the shear-transfer strength is a function of the sum of the resultant compressive force and the force $A_{vf} f_y$ in the shear-friction reinforcement. In design, advantage should be taken of the existence of a compressive force across the shear plane to reduce the amount of shear-friction reinforcement required, only if it is certain that the compressive force is permanent.

R11.7.8 — If no moment acts across the shear plane, reinforcement should be uniformly distributed along the shear plane to minimize crack widths. If a moment acts across the shear plane, it is desirable to distribute the shear-transfer reinforcement primarily in the flexural tension zone.

Since the shear-friction reinforcement acts in tension, it should have full tensile anchorage on both sides of the shear plane. Further, the shear-friction reinforcement anchorage should engage the primary reinforcement, otherwise a potential crack may pass between the shear-friction reinforcement and the body of the concrete. This requirement applies particularly to welded headed studs used with steel inserts for connections in precast and cast-in-place concrete. Anchorage may be developed by bond, by a welded mechanical anchorage, or by threaded dowels and screw inserts. Space limitations often require a welded mechanical anchorage. For anchorage of headed studs in concrete see Reference 11.18.

CODE

11.7.10 — When shear is transferred between as-rolled steel and concrete using headed studs or welded reinforcing bars, steel shall be clean and free of paint.

11.8 — Deep beams

11.8.1 — The provisions of 11.8 shall apply to members with clear spans, ℓ_n, equal to or less than four times the overall member depth or regions of beams loaded with concentrated loads within twice the member depth from the support that are loaded on one face and supported on the opposite face so that compression struts can develop between the loads and supports. See also 12.10.6.

11.8.2 — Deep beams shall be designed using either nonlinear analysis as permitted in 10.7.1, or Appendix A.

11.8.3 — Shear strength V_n for deep beams shall not exceed $10\sqrt{f'_c}\,b_w d$.

11.8.4 — The area of shear reinforcement perpendicular to the span, A_v, shall not be less than $0.0025 b_w s$, and s shall not exceed $d/5$, nor 12 in.

11.8.5 — The area of shear reinforcement parallel to the span, A_{vh}, shall not be less than $0.0015 b_w s_2$, and s_2 shall not exceed $d/5$, nor 12 in.

11.8.6 — It shall be permitted to provide reinforcement satisfying A.3.3 instead of the minimum horizontal and vertical reinforcement specified in 11.8.4 and 11.8.5.

COMMENTARY

R11.8 — Deep beams

R11.8.1 — The behavior of a deep beam is discussed in References 11.5 and 11.38. For a deep beam supporting gravity loads, this section applies if the loads are applied on the top of the beam and the beam is supported on its bottom face. If the loads are applied through the sides or bottom of such a member, the design for shear should be the same as for ordinary beams.

The longitudinal reinforcement in deep beams should be extended to the supports and adequately anchored by embedment, hooks, or welding to special devices. Bent-up bars are not recommended.

R11.8.2 — Deep beams can be designed using strut-and-tie models, regardless of how they are loaded and supported. Section 10.7.1 allows the use of nonlinear stress fields when proportioning deep beams. Such analyses should consider the effects of cracking on the stress distribution.

R11.8.3 — In the 1999 and earlier codes, a sliding maximum shear strength was specified. A re-examination of the test data suggests that this strength limit was derived from tests in which the beams failed due to crushing of support regions. This possibility is specifically addressed in the design process specified in this code.

R11.8.4 and **R11.8.5** — The relative amounts of horizontal and vertical shear reinforcement have been interchanged from those required in the 1999 and earlier codes because tests[11.42,11.43,11.44] have shown that vertical shear reinforcement is more effective than horizontal shear reinforcement. The maximum spacing of bars has been reduced from 18 in. to 12 in. because this steel is provided to restrain the width of the cracks.

CODE

11.9 — Special provisions for brackets and corbels

11.9.1 — Brackets and corbels with a shear span-to-depth ratio a/d less than 2 shall be permitted to be designed using Appendix A. Design shall be permitted using 11.9.3 and 11.9.4 for brackets and corbels with:

(a) a/d not greater than 1, and

(b) subject to horizontal tensile force for N_{uc} not larger than V_u.

The requirements of 11.9.2, 11.9.3.2.1, 11.9.3.2.2, 11.9.5, 11.9.6, and 11.9.7 shall apply to design of brackets and corbels. Distance d shall be measured at the face of the support.

COMMENTARY

R11.9 — Special provisions for brackets and corbels

Brackets and corbels are cantilevers having shear span-to-depth ratios not greater than unity, which tend to act as simple trusses or deep beams, rather than flexural members designed for shear according to 11.3.

The corbel shown in Fig. R11.9.1 may fail by shearing along the interface between the column and the corbel, by yielding of the tension tie, by crushing or splitting of the compression strut, or by localized bearing or shearing failure under the loading plate. These failure modes are illustrated and are discussed more fully in Reference 11.1. The notation used in 11.9 is illustrated in Fig. R11.9.2.

R11.9.1 — An upper limit of 1.0 for a/d is imposed for design by 11.9.3 and 11.9.4 for two reasons. First, for shear span-to-depth ratios exceeding unity, the diagonal tension cracks are less steeply inclined and the use of horizontal stirrups alone as specified in 11.9.4 is not appropriate. Second, this method of design has only been validated experimentally for a/d of unity or less. An upper limit is provided for N_{uc} because this method of design has only been validated experimentally for N_{uc} less than or equal to V_u, including N_{uc} equal to zero.

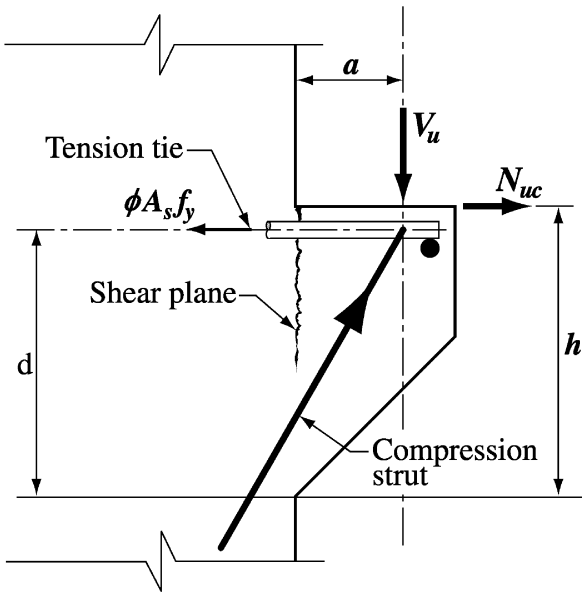


Fig. R11.9.1—Structural action of a corbel

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COMMENTARY

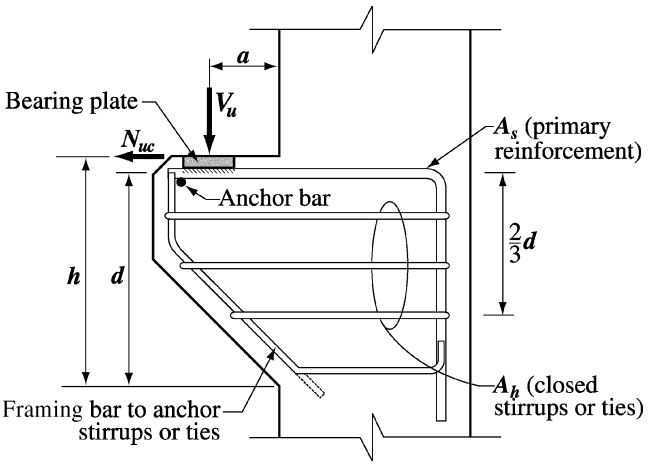


Fig. R11.9.2—Notation used in Section 11.9

11.9.2 — Depth at outside edge of bearing area shall not be less than $0.5d$.

R11.9.2 — A minimum depth is required at the outside edge of the bearing area so that a premature failure will not occur due to a major diagonal tension crack propagating from below the bearing area to the outer sloping face of the corbel or bracket. Failures of this type have been observed[11.45] in corbels having depths at the outside edge of the bearing area less than required in this section of the code.

11.9.3 — Section at face of support shall be designed to resist simultaneously a shear V_u, a moment $[V_u a + N_{uc}(h - d)]$, and a horizontal tensile force N_{uc}.

11.9.3.1 — In all design calculations in accordance with 11.9, strength reduction factor ϕ shall be taken equal to 0.75

R11.9.3.1 — Corbel and bracket behavior is predominantly controlled by shear; therefore, a single value of $\phi = 0.75$ is required for all design conditions.

11.9.3.2 — Design of shear-friction reinforcement A_{vf} to resist shear V_u shall be in accordance with 11.7.

11.9.3.2.1 — For normalweight concrete, shear strength V_n shall not be taken greater than $0.2f_c'b_wd$ nor $800b_wd$, in lb.

11.9.3.2.2 — For all-lightweight or sand-lightweight concrete, shear strength V_n shall not be taken greater than $(0.2 - 0.07a/d)f_c'b_wd$ nor $(800 - 280a/d)b_wd$, in lb.

R11.9.3.2.2 — Tests[11.46] have shown that the maximum shear strength of lightweight concrete corbels or brackets is a function of both f_c' and a/d. No data are available for corbels or brackets made of sand-lightweight concrete. As a result, the same limitations have been placed on both all-lightweight and sand-lightweight brackets and corbels.

11.9.3.3 — Reinforcement A_f to resist moment $[V_u a + N_{uc}(h - d)]$ shall be computed in accordance with 10.2 and 10.3.

R11.9.3.3 — Reinforcement required to resist moment can be calculated using flexural theory. The factored moment is calculated by summing moments about the flexural reinforcement at the face of support.

CODE

11.9.3.4 — Reinforcement A_n to resist tensile force N_{uc} shall be determined from $N_{uc} \le \phi A_n f_y$. Tensile force N_{uc} shall not be taken less than $0.2V_u$ unless special provisions are made to avoid tensile forces. Tensile force N_{uc} shall be regarded as a live load even when tension results from creep, shrinkage, or temperature change.

11.9.3.5 — Area of primary tension reinforcement A_s shall be made equal to the greater of $(A_f + A_n)$ or $(2A_{vf}/3 + A_n)$.

11.9.4 — Closed stirrups or ties parallel to A_s, with a total area A_h not less than $0.5(A_s - A_n)$, shall be uniformly distributed within two-thirds of the effective depth adjacent to A_s.

11.9.5 — Ratio $\rho = A_s/bd$ shall not be less than $0.04(f_c'/f_y)$.

11.9.6 — At front face of bracket or corbel, primary tension reinforcement A_s shall be anchored by one of the following:

(a) By a structural weld to a transverse bar of at least equal size; weld to be designed to develop specified yield strength f_y of A_s bars;

(b) By bending primary tension bars A_s back to form a horizontal loop; or

(c) By some other means of positive anchorage.

COMMENTARY

R11.9.3.4 — Because the magnitude of horizontal forces acting on corbels or brackets cannot usually be determined with great accuracy, it is required that N_{uc} be regarded as a live load.

R11.9.3.5 — Tests[11.46] suggest that the total amount of reinforcement $(A_s + A_h)$ required to cross the face of support should be the greater of:

(a) The sum of A_{vf} calculated according to 11.9.3.2 and A_n calculated according to 11.9.3.4;

(b) The sum of 1.5 times A_f calculated according to 11.9.3.3 and A_n calculated according to 11.9.3.4.

If (a) controls, $A_s = (2A_{vf}/3 + A_n)$ is required as primary tensile reinforcement, and the remaining $A_{vf}/3$ should be provided as closed stirrups parallel to A_s and distributed within $2d/3$, adjacent to A_s. Section 11.9.4 satisfies this by requiring $A_h = 0.5(2A_{vf}/3)$.

If (b) controls, $A_s = (A_f + A_n)$ is required as primary tension reinforcement, and the remaining $A_f/2$ should be provided as closed stirrups parallel to A_s and distributed within $2d/3$, adjacent to A_s. Again 11.9.4 satisfies this requirement.

R11.9.4 — Closed stirrups parallel to the primary tension reinforcement are necessary to prevent a premature diagonal tension failure of the corbel or bracket. The required area of closed stirrups $A_h = 0.5(A_s - A_n)$ automatically yields the appropriate amounts, as discussed in R11.9.3.5 above.

R11.9.5 — A minimum amount of reinforcement is required to prevent the possibility of sudden failure should the bracket or corbel concrete crack under the action of flexural moment and outward tensile force N_{uc}.

R11.9.6 — Because the horizontal component of the inclined concrete compression strut (see Fig. R11.9.1) is transferred to the primary tension reinforcement at the location of the vertical load, the reinforcement A_s is essentially uniformly stressed from the face of the support to the point where the vertical load is applied. It should, therefore, be anchored at its outer end and in the supporting column, so as to be able to develop its yield strength from the face of support to the vertical load. Satisfactory anchorage at the outer end can be obtained by bending the A_s bars in a horizontal loop as specified in (b), or by welding a bar of equal diameter or a suitably sized angle across the ends of the A_s bars. The welds should be designed to develop the yield strength of the reinforcement A_s. The weld detail used successfully in the corbel tests reported in Reference 11.45 is shown in Fig. R11.9.6. The reinforcement A_s should be anchored

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COMMENTARY

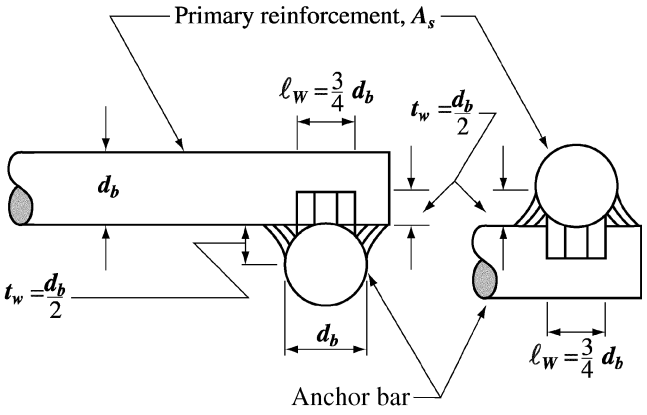


Fig. R11.9.6—Weld details used in tests of Reference 11.45

within the supporting column in accordance with the requirements of Chapter 12. See additional discussion on end anchorage in R12.10.6.

11.9.7 — Bearing area of load on bracket or corbel shall not project beyond straight portion of primary tension bars A_s, nor project beyond interior face of transverse anchor bar (if one is provided).

R11.9.7 — The restriction on the location of the bearing area is necessary to ensure development of the yield strength of the reinforcement A_s near the load. When corbels are designed to resist horizontal forces, the bearing plate should be welded to the tension reinforcement A_s.

11.10 — Special provisions for walls

R11.10 — Special provisions for walls

11.10.1 — Design for shear forces perpendicular to face of wall shall be in accordance with provisions for slabs in 11.12. Design for horizontal in-plane shear forces in a wall shall be in accordance with 11.10.2 through 11.10.9. Alternatively, it shall be permitted to design walls with a height not exceeding two times the length of the wall for horizontal shear forces in accordance with Appendix A and 11.10.9.2 through 11.10.9.5.

R11.10.1 — Shear in the plane of the wall is primarily of importance for shearwalls with a small height-to-length ratio. The design of higher walls, particularly walls with uniformly distributed reinforcement, will probably be controlled by flexural considerations.

11.10.2 — Design of horizontal section for shear in plane of wall shall be based on Eq. (11-1) and (11-2), where shear strength V_c shall be in accordance with 11.10.5 or 11.10.6 and shear strength V_s shall be in accordance with 11.10.9.

11.10.3 — Shear strength V_n at any horizontal section for shear in plane of wall shall not be taken greater than $10\sqrt{f'_c}\,hd$.

R11.10.3 — Although the width-to-depth ratio of shearwalls is less than that for ordinary beams, tests[11.47] on shearwalls with a thickness equal to $\ell_w/25$ have indicated that ultimate shear stresses in excess of $10\sqrt{f'_c}$ can be obtained.

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11.10.4 — For design for horizontal shear forces in plane of wall, d shall be taken equal to $0.8\ell_w$. A larger value of d, equal to the distance from extreme compression fiber to center of force of all reinforcement in tension, shall be permitted to be used when determined by a strain compatibility analysis.

11.10.5 — Unless a more detailed calculation is made in accordance with 11.10.6, shear strength V_c shall not be taken greater than $2\sqrt{f'_c}hd$ for walls subject to N_u in compression, or V_c shall not be taken greater than the value given in 11.3.2.3 for walls subject to N_u in tension.

11.10.6 — Shear strength V_c shall be permitted to be the lesser of the values computed from Eq. (11-29) or (11-30).

$$V_c = 3.3\sqrt{f'_c}hd + \frac{N_u d}{4\ell_w} \quad (11\text{-}29)$$

or

$$V_c = \left[0.6\sqrt{f'_c} + \frac{\ell_w\left(1.25\sqrt{f'_c} + 0.2\frac{N_u}{\ell_w h}\right)}{\frac{M_u}{V_u} - \frac{\ell_w}{2}}\right]hd \quad (11\text{-}30)$$

where N_u is negative for tension. When $(M_u/V_u - \ell_w/2)$ is negative, Eq. (11-30) shall not apply.

11.10.7 — Sections located closer to wall base than a distance $\ell_w/2$ or one-half the wall height, whichever is less, shall be permitted to be designed for the same V_c as that computed at a distance $\ell_w/2$ or one-half the height.

11.10.8 — When factored shear force V_u is less than $\phi V_c/2$, reinforcement shall be provided in accordance with 11.10.9 or in accordance with Chapter 14. When V_u exceeds $\phi V_c/2$, wall reinforcement for resisting shear shall be provided in accordance with 11.10.9.

11.10.9 — Design of shear reinforcement for walls

11.10.9.1 — Where factored shear force V_u exceeds shear strength ϕV_c, horizontal shear reinforcement shall be provided to satisfy Eq. (11-1) and (11-2), where shear strength V_s shall be computed by

$$V_s = \frac{A_v f_y d}{s_2} \quad (11\text{-}31)$$

COMMENTARY

R11.10.5 and R11.10.6 — Eq. (11-29) and (11-30) may be used to determine the inclined cracking strength at any section through a shearwall. Eq. (11-29) corresponds to the occurrence of a principal tensile stress of approximately $4\sqrt{f'_c}$ at the centroid of the shearwall cross section. Eq. (11-30) corresponds approximately to the occurrence of a flexural tensile stress of $6\sqrt{f'_c}$ at a section $\ell_w/2$ above the section being investigated. As the term

$$\left(\frac{M_u}{V_u} - \frac{\ell_w}{2}\right)$$

decreases, Eq. (11-29) will control before this term becomes negative. When this term becomes negative Eq. (11-29) should be used.

R11.10.7 — The values of V_c computed from Eq. (11-29) and (11-30) at a section located a lesser distance of $\ell_w/2$ or $h_w/2$ above the base apply to that and all sections between this section and the base. However, the maximum factored shear force V_u at any section, including the base of the wall, is limited to ϕV_n in accordance with 11.10.3.

R11.10.9 — Design of shear reinforcement for walls

Both horizontal and vertical shear reinforcement are required for all walls. For low walls, test data[11.48] indicate that horizontal shear reinforcement becomes less effective with vertical reinforcement becoming more effective. This change in effectiveness of the horizontal versus vertical reinforcement is recognized in Eq. (11-32); when h_w/ℓ_w is less than 0.5, the amount of vertical reinforcement is equal to the amount of horizontal reinforcement. When h_w/ℓ_w is

CODE

where A_v is area of horizontal shear reinforcement within a distance s_2 and distance d is in accordance with 11.10.4. Vertical shear reinforcement shall be provided in accordance with 11.10.9.4.

11.10.9.2 — Ratio r_h of horizontal shear reinforcement area to gross concrete area of vertical section shall not be less than 0.0025.

11.10.9.3 — Spacing of horizontal shear reinforcement s_2 shall not exceed $\ell_w/5$, $3h$, nor 18 in.

11.10.9.4 — Ratio r_n of vertical shear reinforcement area to gross concrete area of horizontal section shall not be less than

$$r_n = 0.0025 + 0.5\left(2.5 - \frac{h_w}{\ell_w}\right)(r_h - 0.0025) \quad (11\text{-}32)$$

nor 0.0025, but need not be greater than the required horizontal shear reinforcement.

11.10.9.5 — Spacing of vertical shear reinforcement s_1 shall not exceed $\ell_w/3$, $3h$, nor 18 in.

11.11 — Transfer of moments to columns

11.11.1 — When gravity load, wind, earthquake, or other lateral forces cause transfer of moment at connections of framing elements to columns, the shear resulting from moment transfer shall be considered in the design of lateral reinforcement in the columns.

11.11.2 — Except for connections not part of a primary seismic load-resisting system that are restrained on four sides by beams or slabs of approximately equal depth, connections shall have lateral reinforcement not less than that required by Eq. (11-13) within the column for a depth not less than that of the deepest connection of framing elements to the columns. See also 7.9.

11.12 — Special provisions for slabs and footings

11.12.1 — The shear strength of slabs and footings in the vicinity of columns, concentrated loads, or reactions is governed by the more severe of two conditions:

COMMENTARY

greater than 2.5, only a minimum amount of vertical reinforcement is required **($0.0025\, s_1 h$)**.

Eq. (11-31) is presented in terms of shear strength V_s provided by the horizontal shear reinforcement for direct application in Eq. (11-1) and (11-2).

Vertical shear reinforcement also should be provided in accordance with 11.10.9.4 within the spacing limitation of 11.10.9.5.

R11.11 — Transfer of moments to columns

R11.11.1 — Tests[11.49] have shown that the joint region of a beam-to-column connection in the interior of a building does not require shear reinforcement if the joint is confined on four sides by beams of approximately equal depth. However, joints without lateral confinement, such as at the exterior of a building, need shear reinforcement to prevent deterioration due to shear cracking.[11.50]

For regions where strong earthquakes may occur, joints may be required to withstand several reversals of loading that develop the flexural capacity of the adjoining beams. See Chapter 21 for special provisions for seismic design.

R11.12 — Special provisions for slabs and footings

R11.12.1 — Differentiation should be made between a long and narrow slab or footing acting as a beam, and a slab or footing subject to two-way action where failure may occur by punching along a truncated cone or pyramid around a concentrated load or reaction area.

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11.12.1.1 — Beam action where each critical section to be investigated extends in a plane across the entire width. For beam action the slab or footing shall be designed in accordance with 11.1 through 11.5.

11.12.1.2 — Two-way action where each of the critical sections to be investigated shall be located so that its perimeter b_o is a minimum but need not approach closer than $d/2$ to

(a) Edges or corners of columns, concentrated loads, or reaction areas; or

(b) Changes in slab thickness such as edges of capitals or drop panels.

For two-way action the slab or footing shall be designed in accordance with 11.12.2 through 11.12.6.

11.12.1.3 — For square or rectangular columns, concentrated loads, or reaction areas, the critical sections with four straight sides shall be permitted.

11.12.2 — The design of a slab or footing for two-way action is based on Eq. (11-1) and (11-2). V_c shall be computed in accordance with 11.12.2.1, 11.12.2.2, or 11.12.3.1. V_s shall be computed in accordance with 11.12.3. For slabs with shearheads, V_n shall be in accordance with 11.12.4. When moment is transferred between a slab and a column, 11.12.6 shall apply.

11.12.2.1 — For nonprestressed slabs and footings, V_c shall be the smallest of (a), (b), and (c):

(a) $$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f'_c}\,b_o d \qquad (11\text{-}33)$$

where β_c is the ratio of long side to short side of the column, concentrated load or reaction area;

(b) $$V_c = \left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f'_c}\,b_o d \qquad (11\text{-}34)$$

where α_s is 40 for interior columns, 30 for edge columns, 20 for corner columns; and

COMMENTARY

R11.12.1.2 — The critical section for shear in slabs subjected to bending in two directions follows the perimeter at the edge of the loaded area.[11.3] The shear stress acting on this section at factored loads is a function of $\sqrt{f'_c}$ and the ratio of the side dimension of the column to the effective slab depth. A much simpler design equation results by assuming a pseudocritical section located at a distance $d/2$ from the periphery of the concentrated load. When this is done, the shear strength is almost independent of the ratio of column size to slab depth. For rectangular columns, this critical section was defined by straight lines drawn parallel to and at a distance $d/2$ from the edges of the loaded area. Section 11.12.1.3 allows the use of a rectangular critical section.

For slabs of uniform thickness, it is sufficient to check shear on one section. For slabs with changes in thickness, such as the edge of drop panels, it is necessary to check shear at several sections.

For edge columns at points where the slab cantilevers beyond the column, the critical perimeter will either be three-sided or four-sided.

R11.12.2.1 — For square columns, the shear stress due to ultimate loads in slabs subjected to bending in two directions is limited to $4\sqrt{f'_c}$. However, tests[11.51] have indicated that the value of $4\sqrt{f'_c}$ is unconservative when the ratio β_c of the lengths of the long and short sides of a rectangular column or loaded area is larger than 2.0. In such cases, the actual shear stress on the critical section at punching shear failure varies from a maximum of about $4\sqrt{f'_c}$ around the corners of the column or loaded area, down to $2\sqrt{f'_c}$ or less along the long sides between the two end sections. Other tests[11.52] indicate that v_c decreases as the ratio b_o/d increases. Eq. (11-33) and (11-34) were developed to account for these two effects. The words “interior,” “edge,” and “corner columns” in 11.12.2.1(b) refer to critical sections with 4, 3, and 2 sides, respectively.

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(c) $V_c = 4\sqrt{f_c'}\, b_o d$ (11-35)

11.12.2.2 — At columns of two-way prestressed slabs and footings that meet the requirements of 18.9.3

$$V_c = (b_p\sqrt{f_c'} + 0.3f_{pc})b_o d + V_p \quad (11\text{-}36)$$

where b_p is the smaller of 3.5 or $(a_s d/b_o + 1.5)$, a_s is 40 for interior columns, 30 for edge columns, and 20 for corner columns, b_o is perimeter of critical section defined in 11.12.1.2, f_{pc} is the average value of f_{pc} for the two directions, and V_p is the vertical component of all effective prestress forces crossing the critical section. V_c shall be permitted to be computed by Eq. (11-36) if the following are satisfied; otherwise, 11.12.2.1 shall apply:

(a) No portion of the column cross section shall be closer to a discontinuous edge than 4 times the slab thickness;

(b) f_c' in Eq. (11-36) shall not be taken greater than 5,000 psi; and

(c) f_{pc} in each direction shall not be less than 125 psi, nor be taken greater than 500 psi.

COMMENTARY

For shapes other than rectangular, b_c is taken to be the ratio of the longest overall dimension of the effective loaded area to the largest overall perpendicular dimension of the effective loaded area, as illustrated for an L-shaped reaction area in Fig. R11.12.2. The effective loaded area is that area totally enclosing the actual loaded area, for which the perimeter is a minimum.

R11.12.2.2 — For prestressed slabs and footings, a modified form of code Eq. (11-33) and (11-36) is specified for two-way action shear strength. Research[11.53,11.54] indicates that the shear strength of two-way prestressed slabs around interior columns is conservatively predicted by Eq. (11-36). V_c from Eq. (11-36) corresponds to a diagonal tension failure of the concrete initiating at the critical section defined in 11.12.1.2. The mode of failure differs from a punching shear failure of the concrete compression zone around the perimeter of the loaded area predicted by Eq. (11-33). Consequently, the term b_c does not enter into Eq. (11-36). Design values for f_c' and f_{pc} are restricted due to limited test data available for higher values. When computing f_{pc}, loss of prestress due to restraint of the slab by shearwalls and other structural elements should be taken into account.

In a prestressed slab with distributed tendons, the V_p term in Eq. (11-36) contributes only a small amount to the shear strength; therefore, it may be conservatively taken as zero. If V_p is to be included, the tendon profile assumed in the calculations should be noted.

For an exterior column support where the distance from the outside of the column to the edge of the slab is less than four times the slab thickness, the prestress is not fully effective around the total perimeter b_o of the critical section. Shear strength in this case is therefore conservatively taken the same as for a nonprestressed slab.

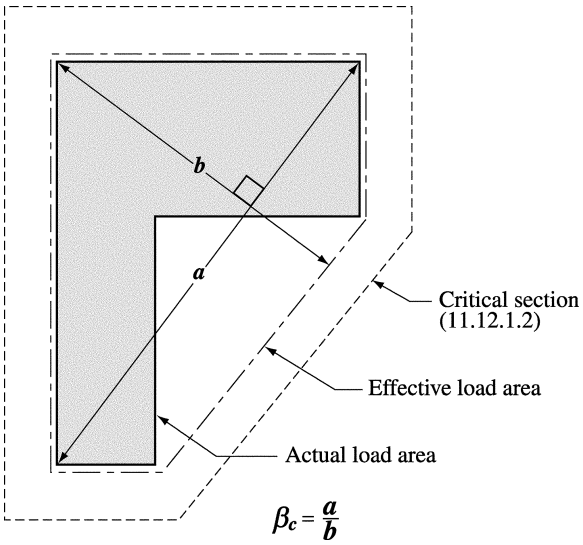


Fig. R11.12.2—Value of b_c for a nonrectangular loaded area

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11.12.3 — Shear reinforcement consisting of bars or wires and single- or multiple-leg stirrups shall be permitted in slabs and footings with an effective depth, d, greater than or equal to 6 in., but not less than 16 times the shear reinforcement bar diameter. Shear reinforcement shall be in accordance with 11.12.3.1 through 11.12.3.4.

11.12.3.1 — V_n shall be computed by Eq. (11-2), where V_c shall not be taken greater than $2\sqrt{f'_c}\,b_o d$, and the strength of shear reinforcement V_s shall be calculated in accordance with 11.5. The area of shear reinforcement A_v used in Eq. (11-15) is the cross-sectional area of all legs of reinforcement on one peripheral line that is geometrically similar to the perimeter of the column section.

11.12.3.2 — V_n shall not be taken greater than $6\sqrt{f'_c}\,b_o d$.

11.12.3.3 — The distance between the column face and the first line of stirrup legs that surround the column shall not exceed $d/2$. The spacing between adjacent stirrup legs in the first line of shear reinforcement shall not exceed $2d$ measured in a direction parallel to the column face. The spacing between successive lines of shear reinforcement that surround the column shall not exceed $d/2$ measured in a direction perpendicular to the column face.

11.12.3.4 — Slab shear reinforcement shall satisfy the anchorage requirements of 12.13 and shall engage the longitudinal flexural reinforcement in the direction being considered.

11.12.4—Shear reinforcement consisting of structural steel I- or channel-shaped sections (shearheads) shall be permitted in slabs. The provisions of 11.12.4.1 through 11.12.4.9 shall apply where shear due to gravity load is transferred at interior column supports. Where moment is transferred to columns, 11.12.6.3 shall apply.

11.12.4.1 — Each shearhead shall consist of steel shapes fabricated by welding with a full penetration weld into identical arms at right angles. Shearhead arms shall not be interrupted within the column section.

11.12.4.2 — A shearhead shall not be deeper than 70 times the web thickness of the steel shape.

11.12.4.3 — The ends of each shearhead arm shall be permitted to be cut at angles not less than 30 deg with the horizontal, provided the plastic moment strength of the remaining tapered section is adequate to resist the shear force attributed to that arm of the shearhead.

COMMENTARY

R11.12.3 — Research[11.55,11.56,11.57,11.58,11.59] has shown that shear reinforcement consisting of properly anchored bars or wires and single- or multiple-leg stirrups, or closed stirrups, can increase the punching shear resistance of slabs. The spacing limits given in 11.12.3.3 correspond to slab shear reinforcement details that have been shown to be effective. Sections 12.13.2 and 12.13.3 give anchorage requirements for stirrup-type shear reinforcement that should also be applied for bars or wires used as slab shear reinforcement. It is essential that this shear reinforcement engage longitudinal reinforcement at both the top and bottom of the slab, as shown for typical details in Fig. R11.12.3(a) to (c). Anchorage of shear reinforcement according to the requirements of 12.13 is difficult in slabs thinner than 10 in. Shear reinforcement consisting of vertical bars mechanically anchored at each end by a plate or head capable of developing the yield strength of the bars has been used successfully.[11.59]

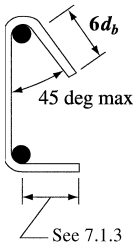
In a slab-column connection for which the moment transfer is negligible, the shear reinforcement should be symmetrical about the centroid of the critical section (Fig. R11.12.3(d)). Spacing limits defined in 11.12.3.3 are also shown in Fig. R11.12.3(d) and (e). At edge columns or for interior connections where moment transfer is significant, closed stirrups are recommended in a pattern as symmetrical as possible. Although the average shear stresses on faces *AD* and *BC* of the exterior column in Fig. R11.12.3(e) are lower than on face *AB*, the closed stirrups extending from faces *AD* and *BC* provide some torsional capacity along the edge of the slab.

R11.12.4—Based on reported test data,[11.60] design procedures are presented for shearhead reinforcement consisting of structural steel shapes. For a column connection transferring moment, the design of shearheads is given in 11.12.6.3.

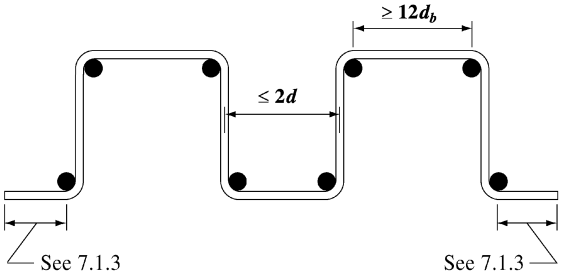
Three basic criteria should be considered in the design of shearhead reinforcement for connections transferring shear due to gravity load. First, a minimum flexural strength should be provided to ensure that the required shear strength of the slab is reached before the flexural strength of the shearhead is exceeded. Second, the shear stress in the slab at the end of the shearhead reinforcement should be limited. Third, after these two requirements are satisfied, the designer can reduce the negative slab reinforcement in proportion to the moment contribution of the shearhead at the design section.

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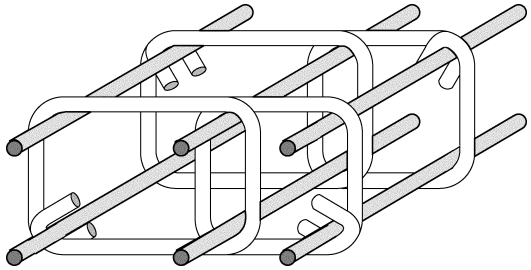
COMMENTARY



(a) single-leg stirrup or bar



(b) multiple-leg stirrup or bar



(c) closed stirrups

Fig. R11.12.3(a)-(c): Single- or multiple-leg stirrup-type slab shear reinforcement.

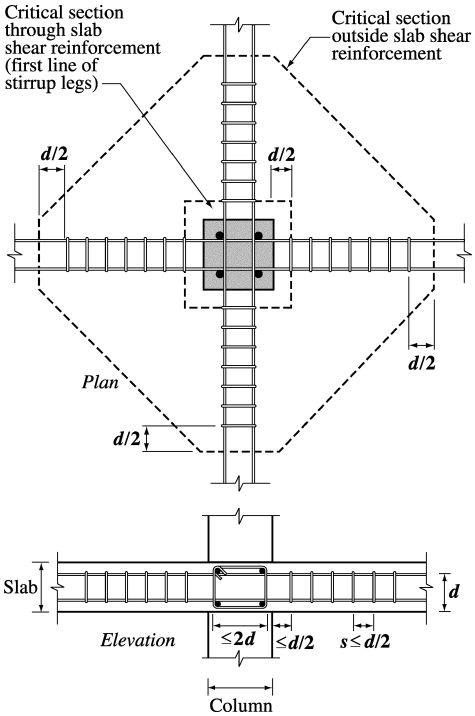


Fig. R11.12.3(d)—Arrangement of stirrup shear reinforcement, interior column.

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COMMENTARY

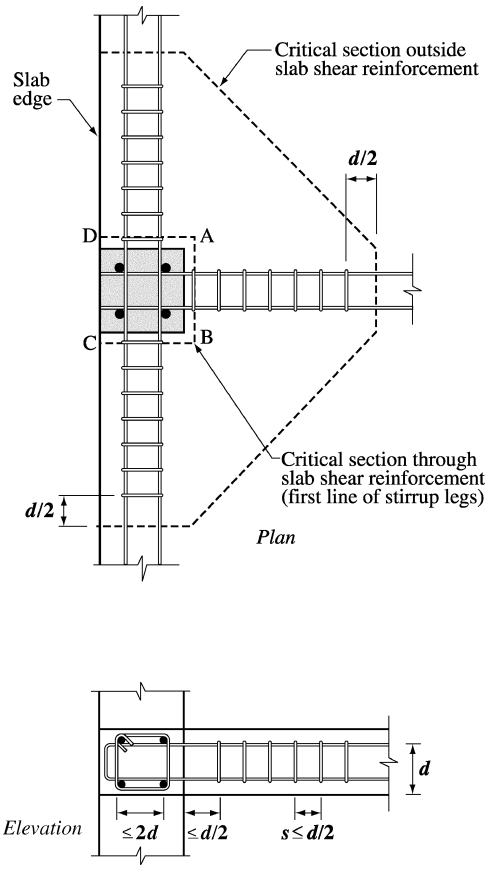


Fig. R11.12.3(e)—Arrangement of stirrup shear reinforcement, edge column

11.12.4.4 — All compression flanges of steel shapes shall be located within **0.3*d*** of compression surface of slab.

11.12.4.5 — The ratio α_v between the flexural stiffness of each shearhead arm and that of the surrounding composite cracked slab section of width $(c_2 + d)$ shall not be less than 0.15.

11.12.4.6 — The plastic moment strength M_p required for each arm of the shearhead shall be computed by

$$M_p = \frac{V_u}{2\phi h}\left[h_v + \alpha_v\left(\ell_v - \frac{c_1}{2}\right)\right] \qquad (11\text{-}37)$$

where ϕ is the strength reduction factor for tension-controlled members, h is the number of arms, and ℓ_v is the minimum length of each shearhead arm required to comply with requirements of 11.12.4.7 and 11.12.4.8.

R11.12.4.5 and R11.12.4.6 — The assumed idealized shear distribution along an arm of a shearhead at an interior column is shown in Fig. R11.12.4.5. The shear along each of the arms is taken as $\alpha_v V_c / h$, where α_v and h are defined in 11.12.4.5 and 11.12.4.6, and V_c is defined in 11.12.2.1. However, the peak shear at the face of the column is taken as the total shear considered per arm $V_u / \phi h$ minus the shear considered carried to the column by the concrete compression zone of the slab. The latter term is expressed as $(V_c / h)(1 - \alpha_v)$, so that it approaches zero for a heavy shearhead and approaches $V_u / \phi h$ when a light shearhead is used. Equation (11-37) then follows from the assumption that ϕV_c is about one-half the factored shear force V_u. In this equation, M_p is the required plastic moment strength of each shearhead arm necessary to ensure that factored shear V_u is attained as the moment strength of the shearhead is reached. The quantity ℓ_v is the length from the center of the column to the point at which the shearhead is no longer required, and the distance $c_1/2$ is one-half the dimension of the column in the direction considered.

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COMMENTARY

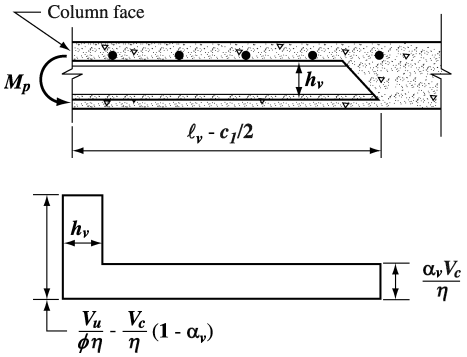


Fig. R11.12.4.5—Idealized shear acting on shearhead

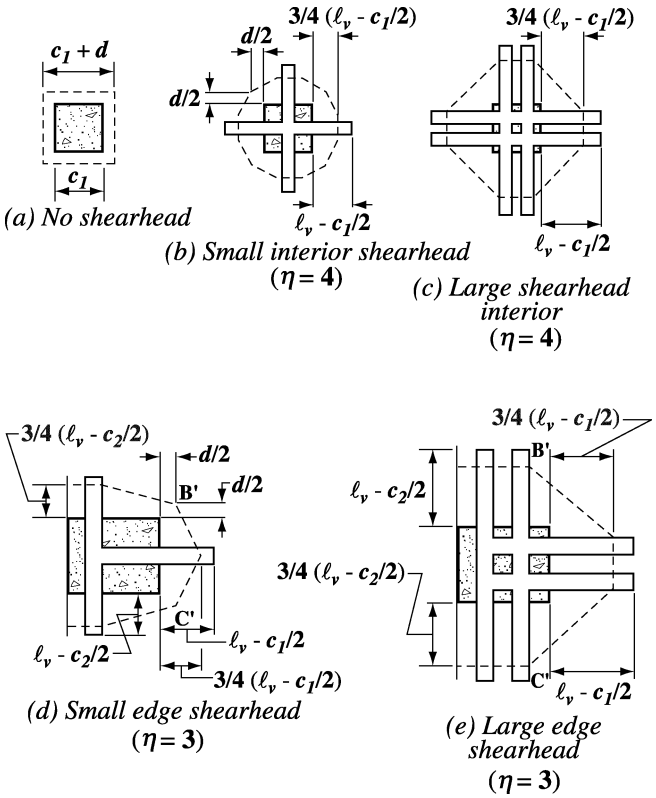


Fig. R11.12.4.7—Location of critical section defined in 11.12.4.7

11.12.4.7 — The critical slab section for shear shall be perpendicular to the plane of the slab and shall cross each shearhead arm at three-quarters the distance $[\ell_v - (c_1/2)]$ from the column face to the end of the shearhead arm. The critical section shall be located so that its perimeter b_o is a minimum, but need not be closer than the perimeter defined in 11.12.1.2(a).

R11.12.4.7 — The test results[11.60] indicated that slabs containing under-reinforcing shearheads failed at a shear stress on a critical section at the end of the shearhead reinforcement less than $4\sqrt{f_c'}$. Although the use of over-reinforcing shearheads brought the shear strength back to about the equivalent of $4\sqrt{f_c'}$, the limited test data suggest that a conservative design is desirable. Therefore, the shear strength is calculated as $4\sqrt{f_c'}$ on an assumed critical section located inside the end of the shearhead reinforcement.

The critical section is taken through the shearhead arms three-fourths of the distance $[\ell_v - (c_1/2)]$ from the face of the column to the end of the shearhead. However, this assumed critical section need not be taken closer than $d/2$ to the column. See Fig. R11.12.4.7.

CODE

11.12.4.8 — V_n shall not be taken greater than $4\sqrt{f'_c}\,b_o d$ on the critical section defined in 11.12.4.7. When shearhead reinforcement is provided, V_n shall not be taken greater than $7\sqrt{f'_c}\,b_o d$ on the critical section defined in 11.12.1.2(a).

11.12.4.9 — The moment resistance M_v contributed to each slab column strip by a shearhead shall not be taken greater than

$$M_v = \frac{\phi\alpha_v V_u}{2h}\left(\ell_v - \frac{c_1}{2}\right) \qquad (11\text{-}38)$$

where ϕ is the strength reduction factor for tension-controlled members, h is the number of arms, and ℓ_v is the length of each shearhead arm actually provided. However, M_v shall not be taken larger than the smaller of:

(a) 30 percent of the total factored moment required for each slab column strip;

(b) The change in column strip moment over the length ℓ_v;

(c) The value of M_p computed by Eq. (11-37).

11.12.4.10 — When unbalanced moments are considered, the shearhead must have adequate anchorage to transmit M_p to the column.

11.12.5 — Openings in slabs

When openings in slabs are located at a distance less than 10 times the slab thickness from a concentrated load or reaction area, or when openings in flat slabs are located within column strips as defined in Chapter 13, the critical slab sections for shear defined in 11.12.1.2 and 11.12.4.7 shall be modified as follows:

11.12.5.1 — For slabs without shearheads, that part of the perimeter of the critical section that is enclosed by straight lines projecting from the centroid of the column, concentrated load, or reaction area and tangent to the boundaries of the openings shall be considered ineffective.

11.12.5.2 — For slabs with shearheads, the ineffective portion of the perimeter shall be one-half of that defined in 11.12.5.1.

COMMENTARY

R11.12.4.9 — If the peak shear at the face of the column is neglected, and ϕV_c is again assumed to be about one-half of V_u, the moment contribution of the shearhead M_v can be conservatively computed from Eq. (11-38), in which ϕ is the factor for flexure.

R11.12.4.10 — See R11.12.6.3.

R11.12.5 — Openings in slabs

Provisions for design of openings in slabs (and footings) were developed in Reference 11.3. The locations of the effective portions of the critical section near typical openings and free edges are shown by the dashed lines in Fig. R11.12.5. Additional research[11.51] has confirmed that these provisions are conservative.

CODE

COMMENTARY

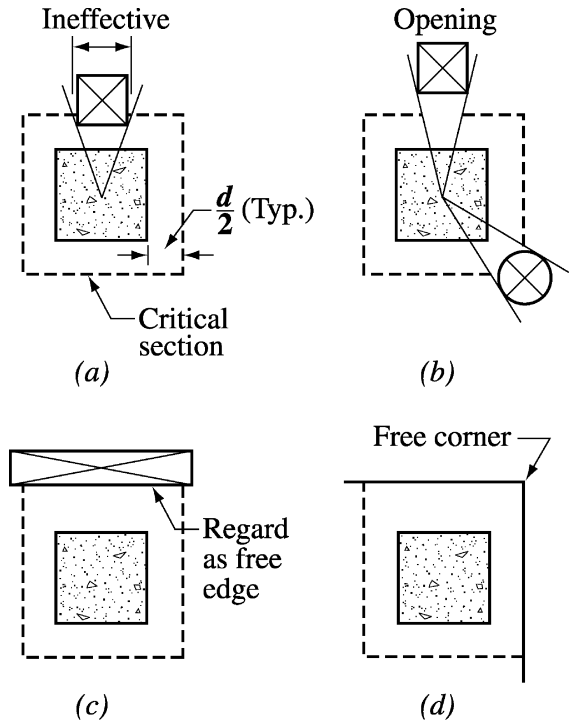


Fig. R11.12.5—Effect of openings and free edges (effective perimeter shown with dashed lines)

11.12.6 — Transfer of moment in slab-column connections

11.12.6.1 — When gravity load, wind, earthquake, or other lateral forces cause transfer of unbalanced moment M_u between a slab and a column, a fraction $g_f M_u$ of the unbalanced moment shall be transferred by flexure in accordance with 13.5.3. The remainder of the unbalanced moment given by $g_v M_u$ shall be considered to be transferred by eccentricity of shear about the centroid of the critical section defined in 11.12.1.2 where

$$g_v = (1 - g_f) \quad (11\text{-}39)$$

11.12.6.2 — The shear stress resulting from moment transfer by eccentricity of shear shall be assumed to vary linearly about the centroid of the critical sections defined in 11.12.1.2. The maximum shear stress due to the factored shear force and moment shall not exceed ϕv_n:

(a) For members without shear reinforcement:

$$\phi v_n = \phi V_c \le \phi (b_o d) \quad (11\text{-}40)$$

R11.12.6 — Transfer of moment in slab-column connections

R11.12.6.1 — In Reference 11.61 it was found that where moment is transferred between a column and a slab, 60 percent of the moment should be considered transferred by flexure across the perimeter of the critical section defined in 11.12.1.2, and 40 percent by eccentricity of the shear about the centroid of the critical section. For rectangular columns, the portion of the moment transferred by flexure increases as the width of the face of the critical section resisting the moment increases, as given by Eq. (13-1).

Most of the data in Reference 11.61 were obtained from tests of square columns, and little information is available for round columns. These can be approximated as square columns. Fig. R13.6.2.5 shows square supports having the same area as some nonrectangular members.

R11.12.6.2 — The stress distribution is assumed as illustrated in Fig. R11.12.6.2 for an interior or exterior column. The perimeter of the critical section, ***ABCD***, is determined in accordance with 11.12.1.2. The factored shear force V_u and unbalanced moment M_u are determined at the centroidal axis ***c-c*** of the critical section. The maximum factored shear stress may be calculated from:

$$v_{u(AB)} = \frac{V_u}{A_c} + \frac{g_v M_u c_{AB}}{J_c}$$

CODE

where V_c is as defined in 11.12.2.1 or 11.12.2.2.

(b) For members with shear reinforcement other than shearheads:

$$\phi V_n = \phi (V_c + V_s) \le \phi (b_o d) \qquad (11\text{-}41)$$

where V_c and V_s are defined in 11.12.3.1. The design shall take into account the variation of shear stress around the column. The shear stress due to factored shear force and moment shall not exceed $2\sqrt{f'_c}$ at the critical section located $d/2$ outside the outermost line of stirrup legs that surround the column.

11.12.6.3 — When shear reinforcement consisting of structural steel I- or channel-shaped sections (shearheads) is provided, the sum of the shear stresses due to vertical load acting on the critical section defined by 11.12.4.7 and the shear stresses resulting from moment transferred by eccentricity of shear about the centroid of the critical section defined in 11.12.1.2(a) and 11.12.1.3 shall not exceed $\phi 4\sqrt{f'_c}$.

COMMENTARY

or

$$v_{u(CD)} = \frac{V_u}{A_c} - \frac{\gamma_v M_u c_{CD}}{J_c}$$

where γ_v is given by Eq. (11-39). For an interior column, A_c and J_c may be calculated by

A_c = area of concrete of assumed critical section
$= 2d(c_1 + c_2 + 2d)$

J_c = property of assumed critical section analogous to polar moment of inertia

$$= \frac{d(c_1 + d)^3}{6} + \frac{(c_1 + d)d^3}{6} + \frac{d(c_2 + d)(c_1 + d)^2}{2}$$

Similar equations may be developed for A_c and J_c for columns located at the edge or corner of a slab.

The fraction of the unbalanced moment between slab and column not transferred by eccentricity of the shear should be transferred by flexure in accordance with 13.5.3. A conservative method assigns the fraction transferred by flexure over an effective slab width defined in 13.5.3.2. Often designers concentrate column strip reinforcement near the column to accommodate this unbalanced moment. Available test data[11.61] seem to indicate that this practice does not increase shear strength but may be desirable to increase the stiffness of the slab-column junction.

Test data[11.62] indicate that the moment transfer capacity of a prestressed slab to column connection can be calculated using the procedures of 11.12.6 and 13.5.3.

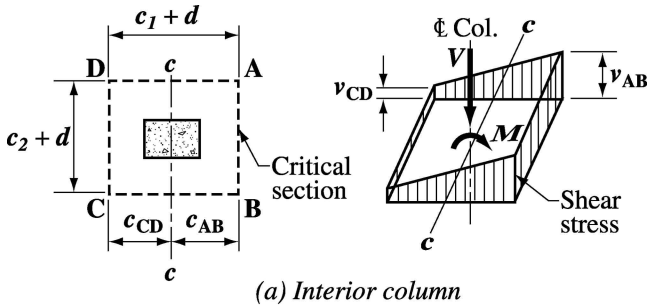
Where shear reinforcement has been used, the critical section beyond the shear reinforcement generally has a polygonal shape (Fig. R11.12.3(d) and (e)). Equations for calculating shear stresses on such sections are given in Reference 11.58.

R11.12.6.3 — Tests[11.63] indicate that the critical sections are defined in 11.12.1.2(a) and 11.12.1.3 and are appropriate for calculations of shear stresses caused by transfer of moments even when shearheads are used. Then, even though the critical sections for direct shear and shear due to moment transfer differ, they coincide or are in close proximity at the column corners where the failures initiate. Because a shearhead attracts most of the shear as it funnels toward the column, it is conservative to take the maximum shear stress as the sum of the two components.

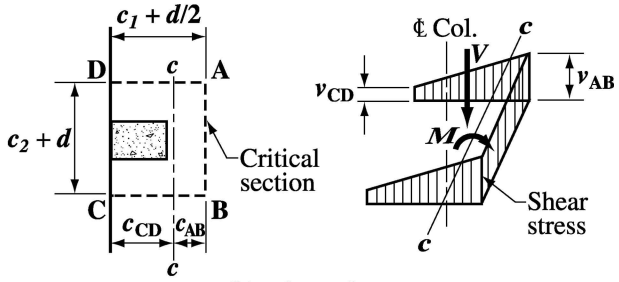
Section 11.12.4.10 requires the moment M_p to be transferred to the column in shearhead connections transferring unbalanced moments. This may be done by bearing within the column or by mechanical anchorage.

CODE

COMMENTARY



(a) Interior column



(b) Edge column

Fig. R11.12.6.2—Assumed distribution of shear stress

CHAPTER 12 — DEVELOPMENT AND SPLICES OF REINFORCEMENT

CODE

12.0 — Notation

a = depth of equivalent rectangular stress block as defined in 10.2.7.1, in.
A_b = area of an individual bar, in.^{2}
A_s = area of nonprestressed tension reinforcement, in.^{2}
A_{tr} = total cross-sectional area of all transverse reinforcement that is within the spacing s and that crosses the potential plane of splitting through the reinforcement being developed, in.^{2}
A_v = area of shear reinforcement within a distance s, in.^{2}
A_w = area of an individual wire to be developed or spliced, in.^{2}
b_w = web width, or diameter of circular section, in.
c = spacing or cover dimension, in. See 12.2.4
d = distance from extreme compression fiber to centroid of tension reinforcement, in.
d_b = nominal diameter of bar, wire, or prestressing strand, in.
f_c' = specified compressive strength of concrete, psi
$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi
f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi
f_{ps} = stress in prestressed reinforcement at nominal strength, ksi
f_{se} = effective stress in prestressed reinforcement (after allowance for all prestress losses), ksi
f_y = specified yield strength of nonprestressed reinforcement, psi
f_{yt} = specified yield strength of transverse reinforcement, psi
h = overall thickness of member, in.
K_{tr} = transverse reinforcement index
= $\frac{A_{tr} f_{yt}}{1500 s n}$ (constant 1500 carries the unit lb/in.^{2})
ℓ_a = additional embedment length at support or at point of inflection, in.
ℓ_d = development length of deformed bars and deformed wire in tension, in.
ℓ_{dc} = development length of deformed bars and deformed wire in compression, in.
ℓ_{dh} = development length of standard hook in tension, measured from critical section to outside end of hook (straight embedment length between critical section and start of hook [point of tangency] plus radius of bend and one bar diameter), in.

COMMENTARY

The development length concept for anchorage of reinforcement was first introduced in the 1971 code, to replace the dual requirements for flexural bond and anchorage bond contained in earlier editions. It is no longer necessary to consider the flexural bond concept, which placed emphasis on the computation of nominal peak bond stresses. Consideration of an average bond resistance over a full development length of the reinforcement is more meaningful, partially because all bond tests consider an average bond resistance over a length of embedment of the reinforcement, and partially because uncalculated extreme variations in local bond stresses exist near flexural cracks.[12.1]

The development length concept is based on the attainable average bond stress over the length of embedment of the reinforcement. Development lengths are required because of the tendency of highly stressed bars to split relatively thin sections of restraining concrete. A single bar embedded in a mass of concrete should not require as great a development length; although a row of bars, even in mass concrete, can create a weakened plane with longitudinal splitting along the plane of the bars.

In application, the development length concept requires minimum lengths or extensions of reinforcement beyond all points of peak stress in the reinforcement. Such peak stresses generally occur at the points in 12.10.2.

The strength reduction factor f is not used in the development length and lap splice equations. An allowance for strength reduction is already included in the expressions for determining development and splice lengths.

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

CODE

ℓ_{hb} = basic development length of standard hook in tension, in.
M_n = nominal moment strength at section, in.-lb
= $A_s f_y (d - a/2)$
n = number of bars or wires being spliced or developed along the plane of splitting
s = maximum center-to-center spacing of transverse reinforcement within ℓ_d, in.
s_w = spacing of wire to be developed or spliced, in.
V_u = factored shear force at section, lb
a = reinforcement location factor. See 12.2.4
b = coating factor. See 12.2.4
b_b = ratio of area of reinforcement cut off to total area of tension reinforcement at section
g = reinforcement size factor. See 12.2.4
l = lightweight aggregate concrete factor. See 12.2.4

12.1 — Development of reinforcement — General

12.1.1 — Calculated tension or compression in reinforcement at each section of structural concrete members shall be developed on each side of that section by embedment length, hook or mechanical device, or a combination thereof. Hooks shall not be used to develop bars in compression.

12.1.2 — The values of $\sqrt{f_c'}$ used in this chapter shall not exceed 100 psi.

12.2 — Development of deformed bars and deformed wire in tension

12.2.1 — Development length ℓ_d, in inches, for deformed bars and deformed wire in tension shall be determined from either 12.2.2 or 12.2.3, but ℓ_d shall not be less than 12 in.

COMMENTARY

R12.1 — Development of reinforcement — General

From a point of peak stress in reinforcement, some length of reinforcement or anchorage is necessary to develop the stress. This development length or anchorage is necessary on both sides of such peak stress points. Often the reinforcement continues for a considerable distance on one side of a critical stress point so that calculations need involve only the other side, for example, the negative moment reinforcement continuing through a support to the middle of the next span.

R12.2 — Development of deformed bars and deformed wire in tension

The general development length equation (Eq. (12-1)) is given in 12.2.3. The equation is based on the expression for development length previously endorsed by Committee 408.[12.2,12.3] In Eq. (12-1), c is a factor that represents the smallest of the side cover, the cover over the bar or wire (in both cases measured to the center of the bar or wire), or one-half the center-to-center spacing of the bars or wires. K_{tr} is a factor that represents the contribution of confining reinforcement across potential splitting planes. a is the traditional reinforcement location factor to reflect the adverse effects of the top reinforcement casting position. b is a coating factor reflecting the effects of epoxy coating. There is a limit on the product ab. g is a reinforcement size factor that reflects the more favorable performance of smaller diameter reinforcement. l is a factor reflecting the lower tensile strength of lightweight concrete and the resulting reduction of the splitting resistance, which increases the development length in lightweight concrete. A limit of 2.5 is placed on the term $(c + K_{tr})/d_b$. When $(c + K_{tr})/d_b$ is less than 2.5, splitting failures are likely to occur. For values above 2.5, a pullout failure is expected and an increase in cover or transverse reinforcement is unlikely to increase the anchorage capacity.

CODE

12.2.2 — For deformed bars or deformed wire, ℓ_d shall be as follows:

	No. 6 and smaller bars and deformed wires	No. 7 and larger bars
Clear spacing of bars being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the code minimum or Clear spacing of bars being developed or spliced not less than $2d_b$ and clear cover not less than d_b	$\left(\frac{f_y \alpha\beta\lambda}{25\sqrt{f_c'}}\right) d_b$	$\left(\frac{f_y \alpha\beta\lambda}{20\sqrt{f_c'}}\right) d_b$
Other cases	$\left(\frac{3 f_y \alpha\beta\lambda}{50\sqrt{f_c'}}\right) d_b$	$\left(\frac{3 f_y \alpha\beta\lambda}{40\sqrt{f_c'}}\right) d_b$

12.2.3 — For deformed bars or deformed wire, ℓ_d shall be:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f_c'}} \frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b}\right)} \right) d_b \qquad (12\text{-}1)$$

in which the term $(c + K_{tr})/d_b$ shall not be taken greater than 2.5.

COMMENTARY

Equation (12-1) allows the designer to see the effect of all variables controlling the development length. The designer is permitted to disregard terms when such omission results in longer and hence, more conservative, development lengths.

The provisions of 12.2.2 and 12.2.3 give a two-tier approach. The user can either calculate ℓ_d based on the actual $(c + K_{tr})/d_b$ (12.2.3) or calculate ℓ_d using 12.2.2, which is based on two preselected values of $(c + K_{tr})/d_b$.

Section 12.2.2 recognizes that many current practical construction cases utilize spacing and cover values along with confining reinforcement, such as stirrups or ties, that result in a value of $(c + K_{tr})/d_b$ of at least 1.5. Examples include a minimum clear cover of d_b along with either minimum clear spacing of $2d_b$, or a combination of minimum clear spacing of d_b and minimum ties or stirrups. For these frequently occurring cases, the development length for larger bars can be taken as $\ell_d = [f_y \alpha\beta\lambda/(20\sqrt{f_c'})]d_b$. In the development of ACI 318-95, a comparison with past provisions and a check of a database of experimental results maintained by ACI Committee 408[12.2] indicated that for No. 6 deformed bars and smaller, as well as for deformed wire, the development lengths could be reduced 20% using $\gamma = 0.80$. This is the basis for the middle column of the table in 12.2.2. With less cover and in the absence of minimum ties or stirrups, the minimum clear spacing limits of 7.6.1 and the minimum concrete cover requirements of 7.7 result in minimum values of c of d_b. Thus, for "other cases," the values are based on using $(c + K_{tr})/d_b = 1.0$ in Eq. (12-1).

The user may easily construct simple, useful expressions. For example, in all structures with normalweight concrete ($\lambda = 1.0$), uncoated reinforcement ($\beta = 1.0$), No. 7 or larger bottom bars ($\alpha = 1.0$) with $f_c' = 4$ ksi and Grade 60 reinforcement, the equations reduce to

$$\ell_d = \frac{(60{,}000)(1.0)(1.0)(1.0)}{20\sqrt{4000}} d_b = 47 d_b$$

or

$$\ell_d = \frac{3(60{,}000)(1.0)(1.0)(1.0)}{40\sqrt{4000}} d_b = 71 d_b$$

Thus, as long as minimum cover of d_b is provided along with a minimum clear spacing of $2d_b$, or a minimum clear cover of d_b and a minimum clear spacing of d_b are provided along with minimum ties or stirrups, a designer knows that $\ell_d = 47d_b$. The penalty for spacing bars closer or providing less cover is the requirement that $\ell_d = 71d_b$.

Many practical combinations of side cover, clear cover, and confining reinforcement can be used with 12.2.3 to produce significantly shorter development lengths than allowed by

CODE

12.2.4 — The factors for use in the expressions for development of deformed bars and deformed wires in tension in Chapter 12 are as follows:

α = reinforcement location factor

Horizontal reinforcement so placed that more than 12 in. of fresh concrete is cast in the member below the development length or splice 1.3

Other reinforcement 1.0

β = coating factor

Epoxy-coated bars or wires with cover less than $3d_b$, or clear spacing less than $6d_b$ 1.5

All other epoxy-coated bars or wires 1.2

Uncoated reinforcement 1.0

However, the product $\alpha\beta$ need not be taken greater than 1.7.

γ = reinforcement size factor

No. 6 and smaller bars and deformed wires 0.8

No. 7 and larger bars 1.0

λ = lightweight aggregate concrete factor

When lightweight aggregate concrete is used 1.3

However, when f_{ct} is specified, λ shall be permitted to be taken as $6.7\sqrt{f_c'}/f_{ct}$ but not less than 1.0

When normalweight concrete is used 1.0

c = spacing or cover dimension, in.

Use the smaller of either the distance from the center of the bar or wire to the nearest concrete surface or one-half the center-to-center spacing of the bars or wires being developed.

K_{tr} = transverse reinforcement index

$$= \frac{A_{tr} f_{yt}}{1500sn}$$

It shall be permitted to use $K_{tr} = 0$ as a design simplification even if transverse reinforcement is present.

12.2.5 — Excess reinforcement

Reduction in development length shall be permitted where reinforcement in a flexural member is in excess of that required by analysis except where anchorage or development for f_y is specifically required or the reinforcement is designed under provisions of 21.2.1.4 ..... (A_s required)/(A_s provided).

COMMENTARY

12.2.2. For example, bars or wires with minimum clear cover not less than $2d_b$ and minimum clear spacing not less than $4d_b$ and without any confining reinforcement would have a $(c + K_{tr})/d_b$ value of 2.5 and would require a development length of only $28d_b$ for the example above.

R12.2.4 — The reinforcement location factor α accounts for position of the reinforcement in freshly placed concrete. The factor was reduced to 1.3 in the 1989 code to reflect research.[12.4,12.5]

The factor λ for lightweight aggregate concrete was made the same for all types of lightweight aggregates in the 1989 code. Research on hooked bar anchorages did not support the variations in previous codes for all-lightweight and sand-lightweight concrete and a single value, 1.3, was selected. Section 12.2.4 allows a lower factor to be used when the splitting tensile strength of the lightweight concrete is specified. See 5.1.4.

Studies[12.6,12.7,12.8] of the anchorage of epoxy-coated bars show that bond strength is reduced because the coating prevents adhesion and friction between the bar and the concrete. The factors reflect the type of anchorage failure likely to occur. When the cover or spacing is small, a splitting failure can occur and the anchorage or bond strength is substantially reduced. If the cover and spacing between bars is large, a splitting failure is precluded and the effect of the epoxy coating on anchorage strength is not as large. Studies[12.9] have shown that although the cover or spacing may be small, the anchorage strength may be increased by adding transverse steel crossing the plane of splitting, and restraining the splitting crack.

Because the bond of epoxy-coated bars is already reduced due to the loss of adhesion between the bar and the concrete, an upper limit of 1.7 is established for the product of the top reinforcement and epoxy-coated reinforcement factors.

Although there is no requirement for transverse reinforcement along the tension development or splice length, recent research[12.10, 12.11] indicates that in concrete with very high compressive strength, brittle anchorage failure occurred in bars with inadequate transverse reinforcement. In splice tests of No. 8 and No. 11 bars in concrete with an f_c' of approximately 15,000 psi, transverse reinforcement improved ductile anchorage behavior.

R12.2.5 — Excess reinforcement

The reduction factor based on area is not to be used in those cases where anchorage development for full f_y is required. For example, the excess reinforcement factor does not apply for development of positive moment reinforcement at supports according to 12.11.2, for development of shrinkage and temperature reinforcement according to 7.12.2.3, or for development of reinforcement provided according to 7.13 and 13.3.8.5.

CODE

12.3 — Development of deformed bars and deformed wire in compression

12.3.1 — Development length ℓ_{dc}, in inches, for deformed bars and deformed wire in compression shall be determined from 12.3.2 and applicable modification factors of 12.3.3, but ℓ_{dc} shall not be less than 8 in.

12.3.2 — For deformed bars and deformed wire, ℓ_{dc} shall be taken as the larger of $(\mathbf{0.02}f_y/\sqrt{f_c'})d_b$ and $(\mathbf{0.0003}f_y)d_b$, where the constant 0.0003 carries the unit of $in.^2/lb$.

12.3.3 — The length ℓ_{dc} in 12.3.2 shall be permitted to be multiplied by the applicable factors for:

(a) Reinforcement in excess of that required by analysis	(A_s required)/(A_s provided)
(b) Reinforcement enclosed within spiral reinforcement not less than 1/4 in. diameter and not more than 4 in. pitch or within No. 4 ties in conformance with 7.10.5 and spaced at not more than 4 in. on center	0.75

12.4 — Development of bundled bars

12.4.1 — Development length of individual bars within a bundle, in tension or compression, shall be that for the individual bar, increased 20 percent for three-bar bundle, and 33 percent for four-bar bundle.

12.4.2 — For determining the appropriate factors in 12.2, a unit of bundled bars shall be treated as a single bar of a diameter derived from the equivalent total area.

COMMENTARY

R12.3 — Development of deformed bars and deformed wire in compression

The weakening effect of flexural tension cracks is not present for bars and wire in compression, and usually end bearing of the bars on the concrete is beneficial. Therefore, shorter development lengths are specified for compression than for tension. The development length may be reduced 25 percent when the reinforcement is enclosed within spirals or ties. A reduction in development length is also permitted if excess reinforcement is provided.

R12.4 — Development of bundled bars

R12.4.1 — An increased development length for individual bars is required when three or four bars are bundled together. The extra extension is needed because the grouping makes it more difficult to mobilize bond resistance from the core between the bars.

The designer should also note 7.6.6.4 relating to the cutoff points of individual bars within a bundle and 12.14.2.2 relating to splices of bundled bars. The increases in development length of 12.4 do apply when computing splice lengths of bundled bars in accordance with 12.14.2.2. The development of bundled bars by a standard hook of the bundle is not covered by the provisions of 12.5.

R12.4.2 — Although splice and development lengths of bundled bars are based on the diameter of individual bars increased by 20 or 33 percent, as appropriate, it is necessary to use an equivalent diameter of the entire bundle derived from the equivalent total area of bars when determining factors in 12.2, which considers cover and clear spacing and represents the tendency of concrete to split.

CODE

12.5 — Development of standard hooks in tension

12.5.1 — Development length ℓ_{dh}, in inches, for deformed bars in tension terminating in a standard hook (see 7.1) shall be determined from 12.5.2 and the applicable modification factors of 12.5.3, but ℓ_{dh} shall not be less than $8d_b$, nor less than 6 in.

12.5.2 — For deformed bars, ℓ_{dh} shall be $(0.02\beta\lambda f_y/\sqrt{f_c'})d_b$ with β taken as 1.2 for epoxy-coated reinforcement, and λ taken as 1.3 for lightweight aggregate concrete. For other cases, β and λ shall be taken as 1.0.

12.5.3 — The length ℓ_{dh} in 12.5.2 shall be permitted to be multiplied by the following applicable factors:

(a) For No. 11 bar and smaller hooks with side cover (normal to plane of hook) not less than 2-1/2 in., and for 90 deg hook with cover on bar extension beyond hook not less than 2 in. 0.7

(b) For 90 deg hooks of No. 11 and smaller bars that are either enclosed within ties or stirrups perpendicular to the bar being developed, spaced not greater than $3d_b$ along the development length ℓ_{dh} of the hook; or enclosed within ties or stirrups parallel to the bar being developed, spaced not greater than $3d_b$ along the length of the tail extension of the hook plus bend 0.8

(c) For 180 deg hooks of No. 11 and smaller bars that are enclosed within ties or stirrups perpendicular to the bar being developed, spaced not greater than $3d_b$ along the development length ℓ_{dh}, of the hook ... 0.8

(d) Where anchorage or development for f_y is not specifically required, reinforcement in excess of that required by analysis ..... (A_s required)/(A_s provided)

In 12.5.3(b) and 12.5.3(c), d_b is the diameter of the hooked bar, and the first tie or stirrup shall enclose the bent portion of the hook, within $2d_b$ of the outside of the bend.

COMMENTARY

R12.5 — Development of standard hooks in tension

The provisions for hooked bar anchorage were extensively revised in the 1983 code. Study of failures of hooked bars indicate that splitting of the concrete cover in the plane of the hook is the primary cause of failure and that splitting originates at the inside of the hook where the local stress concentrations are very high. Thus, hook development is a direct function of bar diameter d_b,which governs the magnitude of compressive stresses on the inside of the hook. Only standard hooks (see 7.1) are considered and the influence of larger radius of bend cannot be evaluated by 12.5.

The hooked bar anchorage provisions give the total hooked bar embedment length as shown in Fig. R12.5. The development length ℓ_{dh} is measured from the critical section to the outside end (or edge) of the hook.

The development length for standard hooks ℓ_{dh} of 12.5.2 can be reduced by all applicable modification factors of 12.5.3. As an example, if the conditions of both 12.5.3(a) and (c) are met, both factors may be applied.

The effects of bar yield strength, excess reinforcement, lightweight concrete, and factors to reflect the resistance to splitting provided from confinement by concrete and transverse ties or stirrups are based on recommendations from References 12.2 and 12.3.

Tests[12.12] indicate that closely spaced ties at or near the bend portion of a hooked bar are most effective in confining the hooked bar. For construction purposes, this is not always practicable. The cases where the modification factor of 12.5.3(b) may be used are illustrated in Fig. R12.5.3(a) and (b). Figure R12.5.3(a) shows placement of ties or stirrups perpendicular to the bar being developed, spaced along the development length, ℓ_{dh}, of the hook. Figure R12.5.3(b) shows placement of ties or stirrups parallel to the bar being developed along the length of the tail extension of the hook plus bend. The latter configuration would be typical in a beam column joint.

The factor for excess reinforcement in 12.5.3(d) applies only where anchorage or development for full f_y is not specifically required. The λ factor for lightweight concrete is a simplification over the procedure in 12.2.3.3 of ACI 318-83 in which the increase varies from 18 to 33 percent, depending on the amount of lightweight aggregate used. Unlike straight bar development, no distinction is made between top bars and other bars; such a distinction is difficult for hooked bars in any case. A minimum value of ℓ_{dh} is specified to prevent failure by direct pullout in cases where a hook may be located very near the critical section. Hooks cannot be considered effective in compression.

CODE

COMMENTARY

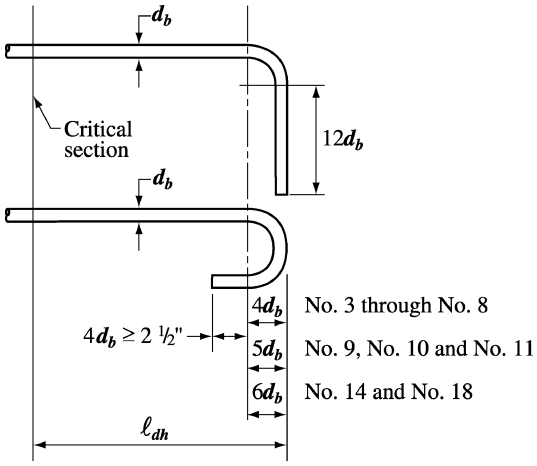


Fig. R12.5—Hooked bar details for development of standard hooks

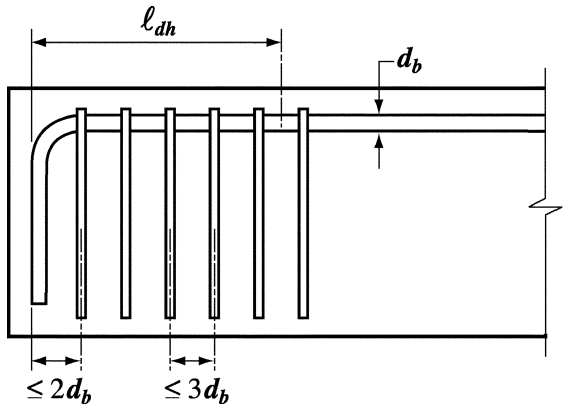


Fig. R12.5.3(a)—Ties or stirrups placed perpendicular to the bar being developed, spaced along the development length ℓ_{dh}.

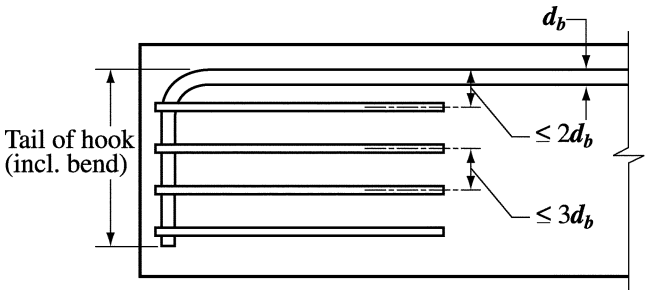


Fig. R12.5.3(b)—Ties or stirrups placed parallel to the bar being developed, spaced along the length of the tail extension of the hook plus bend.

Tests[12.13] indicate that the development length for hooked bars should be increased by 20 percent to account for reduced bond when reinforcement is epoxy coated.

12.5.4 — For bars being developed by a standard hook at discontinuous ends of members with both side cover and top (or bottom) cover over hook less than 2-1/2 in., the hooked bar shall be enclosed within ties or stirrups

R12.5.4 — Bar hooks are especially susceptible to a concrete splitting failure if both side cover (normal to plane of hook) and top or bottom cover (in plane of hook) are small. See Fig. R12.5.4. With minimum confinement provided by

CODE

perpendicular to the bar being developed, spaced not greater than $3d_b$ along the development length ℓ_{dh} of the hook. The first tie or stirrup shall enclose the bent portion of the hook, within $2d_b$ of the outside of the bend, where d_b is the diameter of the hooked bar. For this case, the factors of 12.5.3(b) and (c) shall not apply.

12.5.5 — Hooks shall not be considered effective in developing bars in compression.

12.6 — Mechanical anchorage

12.6.1 — Any mechanical device capable of developing the strength of reinforcement without damage to concrete is allowed as anchorage.

12.6.2 — Test results showing adequacy of such mechanical devices shall be presented to the building official.

12.6.3 — Development of reinforcement shall be permitted to consist of a combination of mechanical anchorage plus additional embedment length of reinforcement between the point of maximum bar stress and the mechanical anchorage.

12.7 — Development of welded deformed wire fabric in tension

12.7.1 — Development length ℓ_d, in inches, of welded deformed wire fabric measured from the point of critical section to the end of wire shall be computed as the product of the development length ℓ_d, from 12.2.2 or 12.2.3 times a wire fabric factor from 12.7.2 or 12.7.3. It shall be permitted to reduce the development length in accordance with 12.2.5 when applicable, but ℓ_d shall not be less than 8 in. except in computation of lap

COMMENTARY

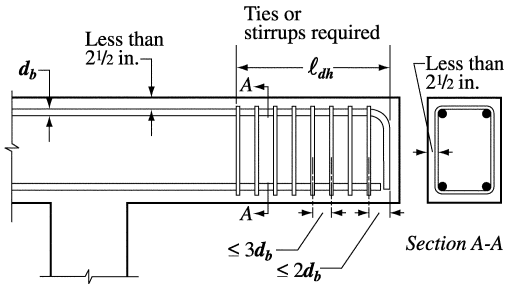


Fig. R12.5.4—Concrete cover per 12.5.4

concrete, additional confinement provided by ties or stirrups is essential, especially if full bar strength should be developed by a hooked bar with such small cover. Cases where hooks may require ties or stirrups for confinement are at ends of simply supported beams, at free end of cantilevers, and at ends of members framing into a joint where members do not extend beyond the joint. In contrast, if calculated bar stress is so low that the hook is not needed for bar anchorage, the ties or stirrups are not necessary. Also, provisions of 12.5.4 do not apply for hooked bars at discontinuous ends of slabs with confinement provided by the slab continuous on both sides normal to the plane of the hook.

R12.5.5 — In compression, hooks are ineffective and may not be used as anchorage.

R12.6 — Mechanical anchorage

R12.6.1 — Mechanical anchorage can be made adequate for strength both for tendons and for bar reinforcement.

R12.6.3 — Total development of a bar consists of the sum of all the parts that contribute to anchorage. When a mechanical anchorage is not capable of developing the required design strength of the reinforcement, additional embedment length of reinforcement should be provided between the mechanical anchorage and the critical section.

R12.7 — Development of welded deformed wire fabric in tension

Fig. R12.7 shows the development requirements for deformed wire fabric with one cross wire within the development length. ASTM A 497 for deformed wire fabric requires the same strength of the weld as required for plain wire fabric (ASTM A 185). Some of the development is assigned to welds and some assigned to the length of deformed wire. The development computations are simplified from earlier code provisions for wire development by

CODE

splices by 12.18. When using the wire fabric factor from 12.7.2, it shall be permitted to use an epoxy-coating factor b of 1.0 for epoxy-coated welded wire fabric in 12.2.2 and 12.2.3.

12.7.2 — For welded deformed wire fabric with at least one cross wire within the development length and not less than 2 in. from the point of the critical section, the wire fabric factor shall be the greater of:

$$\left(\frac{f_y - 35{,}000}{f_y}\right)$$

or

$$\left(\frac{5d_b}{s_w}\right)$$

but need not be greater than 1.

12.7.3 — For welded deformed wire fabric with no cross wires within the development length or with a single cross wire less than 2 in. from the point of the critical section, the wire fabric factor shall be taken as 1, and the development length shall be determined as for deformed wire.

12.7.4—When any plain wires are present in the deformed wire fabric in the direction of the development length, the fabric shall be developed in accordance with 12.8.

12.8—Development of welded plain wire fabric in tension

Yield strength of welded plain wire fabric shall be considered developed by embedment of two cross wires with the closer cross wire not less than 2 in. from the point of the critical section. However, the development length ℓ_d in inches, measured from the point of the critical section to the outermost cross wire shall not be less than

$$0.27\,\frac{A_w}{s_w}\left(\frac{f_y}{\sqrt{f'_c}}\right)l$$

COMMENTARY

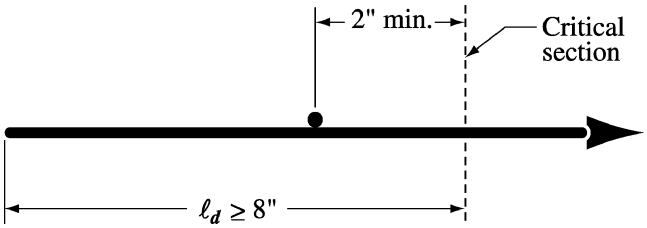


Fig. R12.7—Development of welded deformed wire fabric

assuming that only one cross wire is contained in the development length. The factors in 12.7.2 are applied to the deformed wire development length computed from 12.2, but with an absolute minimum of 8 in. The explicit statement that the mesh multiplier not be taken greater than 1 corrects an oversight in earlier codes. The multipliers were derived using the general relationships between deformed wire mesh and deformed wires in the ℓ_{db} values of the 1983 code.

Tests[12.14] have indicated that epoxy-coated welded wire fabric has essentially the same development and splice strengths as uncoated fabric since the cross wires provide the primary anchorage for the wire. Therefore, an epoxy-coating factor of 1.0 is used for development and splice lengths of epoxy-coated welded wire fabric with cross wires within the splice or development length.

R12.8 — Development of welded plain wire fabric in tension

Fig. R12.8 shows the development requirements for plain wire fabric with development primarily dependent on the location of cross wires. For fabrics made with the smaller wires, an embedment of at least two cross wires 2 in. or more beyond the point of critical section is adequate to develop the full yield strength of the anchored wires. However, for fabrics made with larger closely spaced wires, a longer embedment is required and a minimum development length is provided for these fabrics.

CODE

except that when reinforcement provided is in excess of that required, this length may be reduced in accordance with 12.2.5. ℓ_d shall not be less than 6 in. except in computation of lap splices by 12.19.

COMMENTARY

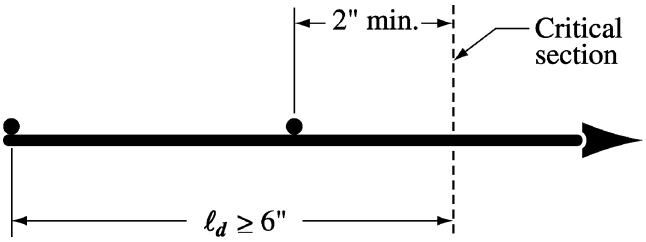


Fig. R12.8—Development of welded plain wire fabric

12.9 — Development of prestressing strand

R12.9 — Development of prestressing strand

12.9.1 — Except as provided in 12.9.1.1, seven-wire strand shall be bonded beyond the critical section for a development length ℓ_d, in inches, not less than

$$\ell_d = \left(\frac{f_{se}}{3}\right)d_b + (f_{ps} - f_{se})d_b \qquad (12\text{-}2)$$

where d_b is strand diameter in inches, and f_{ps} and f_{se} are expressed in kips/in.^{2} The expressions in parenthesis are used as a constant without units.

The development requirements for prestressing strand are intended to provide bond integrity for the strength of the member. The provisions are based on tests performed on normalweight concrete members with a minimum cover of 2 in. These tests may not represent the behavior of strand in low water-cementitious materials ratio, no-slump, concrete. Fabrication methods should ensure consolidation of concrete around the strand with complete contact between the steel and concrete. Extra precautions should be exercised when low water-cementitious materials ratio, no-slump concrete is used.

The first term in Eq. (12-2) represents the transfer length of the strand, that is, the distance over which the strand should be bonded to the concrete to develop the prestress f_{se} in the strand. The second term represents the additional length over which the strand should be bonded so that a stress f_{ps} may develop in the strand at nominal strength of the member.

The bond of strand is a function of a number of factors, including the configuration and surface condition of the steel, the stress in the steel, the depth of concrete beneath the strand, and the method used to transfer the force in the strand to the concrete. For bonded applications, quality assurance procedures should be used to confirm that the strand is capable of adequate bond.[12.15,12.16] The precast concrete manufacturer may rely on certification from the strand manufacturer that the strand has bond characteristics that comply with this section. Strand with a slightly rusted surface can have an appreciably shorter transfer length than clean strand. Gentle release of the strand will permit a shorter transfer length than abruptly cutting the strands.

The provisions of 12.9 do not apply to plain wires or to end-anchored tendons. The length for smooth wire could be expected to be considerably greater due to the absence of mechanical interlock. Flexural bond failure would occur with plain wire when first slip occurred.

12.9.1.1 — Embedment less than the development length shall be permitted at a section of a member provided the design strand stress at that section does not exceed values obtained from the bilinear relationship defined by Eq. (12-2).

R12.9.1.1 — Figure R12.9 shows the relationship between steel stress and the distance over which the strand is bonded to the concrete represented by Eq. (12-2). This idealized variation of strand stress may be used for analyzing sections within the development region.[12.17, 12.18] The expressions for

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COMMENTARY

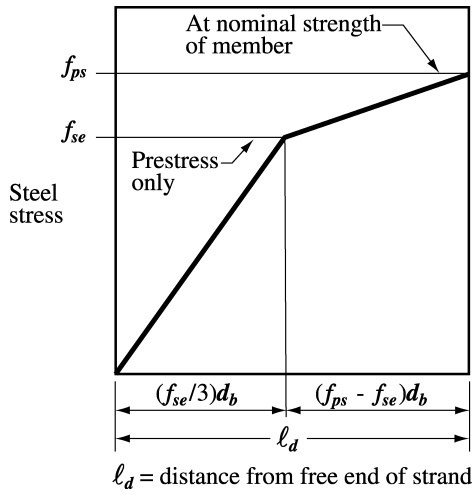


Fig. R12.9—Idealized bilinear relationship between steel stress and distance from the free end of strand.

transfer length, and for the additional bonded length necessary to develop an increase in stress of $(f_{ps} - f_{se})$, are based on tests of members prestressed with clean, 1/4, 3/8, and 1/2 in. diameter strands for which the maximum value of f_{ps} was 275 kips/in.[2] See References 12.19, 12.20, and 12.21.

12.9.2 — Limiting the investigation to cross sections nearest each end of the member that are required to develop full design strength under specified factored loads shall be permitted except where bonding of one or more strands does not extend to the end of the member, or where concentrated loads are applied in the strand development length.

R12.9.2 — Where bonding of one or more strands does not extend to the end of the member, critical sections may be at locations other than where full design strength is required to be developed, and detailed analysis may be required. References 12.17 and 12.18 show a method that may be used in the case of strands with different points of full development. Conservatively, only the strands that are fully developed at a section may be considered effective at that section. If critical sections occur in the transfer region, special considerations may be necessary. Some loading conditions, such as where heavy concentrated loads occur within the strand development length, may cause critical sections to occur away from the section that is required to develop full design strength.

12.9.3 — Where bonding of a strand does not extend to end of member, and design includes tension at service load in precompressed tensile zone as permitted by 18.4.2, development length specified in 12.9.1 shall be doubled.

R12.9.3 — Exploratory tests conducted in 1965[12.19] that study the effect of debonded strand (bond not permitted to extend to the ends of members) on performance of pretensioned girders indicated that the performance of these girders with embedment lengths twice those required by 12.9.1 closely matched the flexural performance of similar pretensioned girders with strand fully bonded to ends of girders. Accordingly, doubled development length is required for strand not bonded through to the end of a member. Subsequent tests[12.22] indicated that in pretensioned members designed for zero tension in the concrete under service load conditions (see 18.4.2), the development length for debonded strands need not be doubled. For analysis of sections with debonded strands at locations where strand is not fully developed, it is usually assumed that both the transfer length and development length are doubled.

CODE

12.10 — Development of flexural reinforcement — General

12.10.1 — Development of tension reinforcement by bending across the web to be anchored or made continuous with reinforcement on the opposite face of member shall be permitted.

12.10.2 — Critical sections for development of reinforcement in flexural members are at points of maximum stress and at points within the span where adjacent reinforcement terminates, or is bent. Provisions of 12.11.3 must be satisfied.

12.10.3 — Reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the effective depth of member or $12d_b$, whichever is greater, except at supports of simple spans and at free end of cantilevers.

COMMENTARY

R12.10 — Development of flexural reinforcement — General

R12.10.2 — Critical sections for a typical continuous beam are indicated with a "c" or an "x" in Fig. R12.10.2. For uniform loading, the positive reinforcement extending into the support is more apt to be governed by the requirements of 12.11.3 rather than by development length measured from a point of maximum moment or bar cutoff.

R12.10.3 — The moment diagrams customarily used in design are approximate; some shifting of the location of maximum moments may occur due to changes in loading, settlement of supports, lateral loads, or other causes. A diagonal tension crack in a flexural member without stirrups may shift the location of the calculated tensile stress approximately a distance d towards a point of zero moment. When stirrups are provided, this effect is less severe, although still present to some extent.

To provide for shifts in the location of maximum moments, the code requires the extension of reinforcement a distance d or $12d_b$ beyond the point at which it is theoretically no longer required to resist flexure, except as noted.

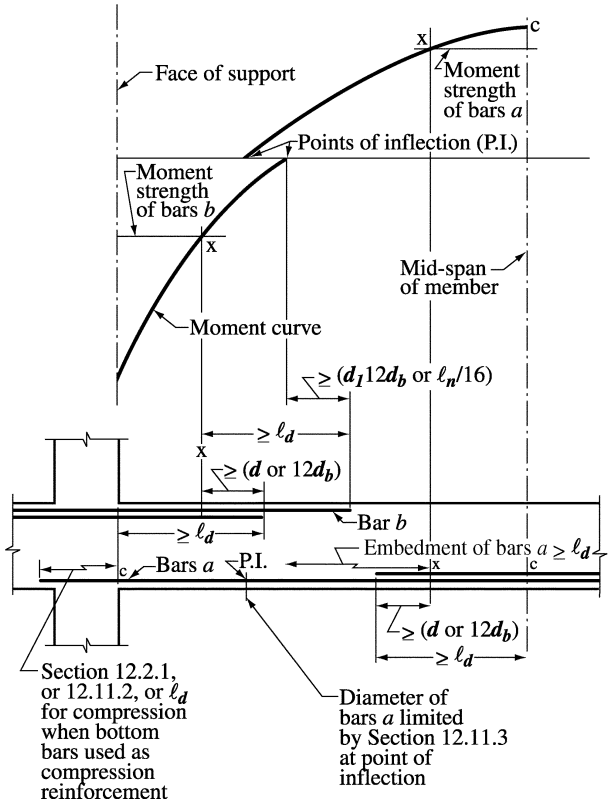


Fig. R12.10.2—Development of flexural reinforcement in a typical continuous beam

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Cutoff points of bars to meet this requirement are illustrated in Fig. R12.10.2.

When bars of different sizes are used, the extension should be in accordance with the diameter of bar being terminated. A bar bent to the far face of a beam and continued there may logically be considered effective, in satisfying this section, to the point where the bar crosses the mid-depth of the member.

12.10.4 — Continuing reinforcement shall have an embedment length not less than the development length ℓ_d beyond the point where bent or terminated tension reinforcement is no longer required to resist flexure.

R12.10.4 — Peak stresses exist in the remaining bars wherever adjacent bars are cut off, or bent, in tension regions. In Fig. R12.10.2 an "x" is used to indicate the peak stress points remaining in continuing bars after part of the bars have been cut off. If bars are cut off as short as the moment diagrams allow, these peak stresses become the full f_y, which requires a full ℓ_d extension as indicated. This extension may exceed the length required for flexure.

12.10.5 — Flexural reinforcement shall not be terminated in a tension zone unless 12.10.5.1, 12.10.5.2, or 12.10.5.3 is satisfied.

12.10.5.1 — Factored shear at the cutoff point does not exceed two-thirds of the design shear strength, ϕV_n.

12.10.5.2 — Stirrup area in excess of that required for shear and torsion is provided along each terminated bar or wire over a distance from the termination point equal to three-fourths the effective depth of member. Excess stirrup area A_v shall be not less than $60 b_w s / f_y$. Spacing s shall not exceed $d / 8 b_b$.

12.10.5.3 — For No. 11 bars and smaller, continuing reinforcement provides double the area required for flexure at the cutoff point and factored shear does not exceed three-fourths the design shear strength, ϕV_n.

R12.10.5 — Reduced shear strength and loss of ductility when bars are cut off in a tension zone, as in Fig. R12.10.2, have been reported. The code does not permit flexural reinforcement to be terminated in a tension zone unless special conditions are satisfied. Flexure cracks tend to open early wherever any reinforcement is terminated in a tension zone. If the steel stress in the continuing reinforcement and the shear strength are each near their limiting values, diagonal tension cracking tends to develop prematurely from these flexure cracks. Diagonal cracks are less likely to form where shear stress is low (see 12.10.5.1). Diagonal cracks can be restrained by closely spaced stirrups (see 12.10.5.2). A lower steel stress reduces the probability of such diagonal cracking (see 12.10.5.3). These requirements are not intended to apply to tension splices which are covered by 12.2, 12.13.5, and the related 12.15.

12.10.6 — Adequate anchorage shall be provided for tension reinforcement in flexural members where reinforcement stress is not directly proportional to moment, such as: sloped, stepped, or tapered footings; brackets; deep flexural members; or members in which tension reinforcement is not parallel to compression face. See 12.11.4 and 12.12.4 for deep flexural members.

R12.10.6 — Brackets, members of variable depth, and other members where steel stress f_s does not decrease linearly in proportion to a decreasing moment require special consideration for proper development of the flexural reinforcement. For the bracket shown in Fig. R12.10.6, the stress at ultimate in the reinforcement is almost constant at approximately f_y from the face of support to the load point. In such a case, development of the flexural reinforcement depends largely on the end anchorage provided at the loaded end. Reference 12.1 suggests a welded cross bar of equal diameter as a means of providing effective end anchorage. An end hook in the vertical plane, with the minimum diameter bend, is not totally effective because an essentially plain concrete corner will exist near loads applied close to the corner. For wide brackets (perpendicular to the plane of the figure) and loads not applied close to the corners, U-shaped bars in a horizontal plane provide effective end hooks.

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COMMENTARY

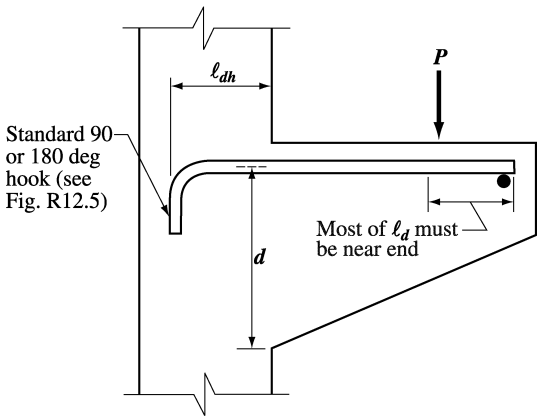


Fig. R12.10.6—Special member largely dependent on end anchorage

12.11 — Development of positive moment reinforcement

R12.11 — Development of positive moment reinforcement

12.11.1 — At least one-third the positive moment reinforcement in simple members and one-fourth the positive moment reinforcement in continuous members shall extend along the same face of member into the support. In beams, such reinforcement shall extend into the support at least 6 in.

R12.11.1 — Positive moment reinforcement is carried into the support to provide for some shifting of the moments due to changes in loading, settlement of supports, and lateral loads.

12.11.2 — When a flexural member is part of a primary lateral load resisting system, positive moment reinforcement required to be extended into the support by 12.11.1 shall be anchored to develop the specified yield strength f_y in tension at the face of support.

R12.11.2 — When a flexural member is part of a primary lateral load resisting system, loads greater than those anticipated in design may cause reversal of moment at supports; some positive reinforcement should be well anchored into the support. This anchorage is required to ensure ductility of response in the event of serious overstress, such as from blast or earthquake. It is not sufficient to use more reinforcement at lower stresses.

12.11.3 — At simple supports and at points of inflection, positive moment tension reinforcement shall be limited to a diameter such that ℓ_d computed for f_y by 12.2 satisfies Eq. (12-3); except, Eq. (12-3) need not be satisfied for reinforcement terminating beyond centerline of simple supports by a standard hook, or a mechanical anchorage at least equivalent to a standard hook.

$$\ell_d \leq \frac{M_n}{V_u} + \ell_a \qquad (12\text{-}3)$$

where:

M_n is nominal moment strength assuming all reinforcement at the section to be stressed to the specified yield strength f_y;

V_u is factored shear force at the section;

ℓ_a at a support shall be the embedment length beyond center of support; or

R12.11.3 — At simple supports and points of inflection such as "P.I." in Fig. R12.10.2, the diameter of the positive reinforcement should be small enough so that computed development length of the bar ℓ_d does not exceed $M_n/V_u + \ell_a$, or under favorable support conditions, $1.3M_n/V_u + \ell_a$. Fig. R12.11.3(a) illustrates the use of the provision.

At the point of inflection the value of ℓ_a should not exceed the actual bar extension used beyond the point of zero moment. The M_n/V_u portion of the available length is a theoretical quantity not generally associated with an obvious maximum stress point. M_n is the nominal strength of the cross section without the ϕ-factor and is not the applied factored moment.

The length M_n/V_u corresponds to the development length for the maximum size bar obtained from the previously used flexural bond equation $S_o = V/ujd$, where u is bond stress, and jd is the moment arm. In the 1971 code, this anchorage requirement was relaxed from previous codes by crediting the

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ℓ_a at a point of inflection shall be limited to the effective depth of member or $12d_b$, whichever is greater.

An increase of 30 percent in the value of M_n/V_u shall be permitted when the ends of reinforcement are confined by a compressive reaction.

12.11.4 — At simple supports of deep beams, positive moment tension reinforcement shall be anchored to develop its specified yield strength, f_y, in tension at the face of the support except that if design is carried out using Appendix A, the positive moment tension reinforcement shall be anchored in accordance with A.4.3. At interior supports of deep beams, positive moment tension reinforcement shall be continuous or be spliced with that of the adjacent spans.

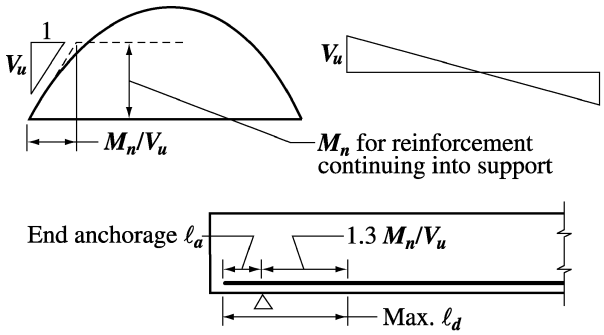
COMMENTARY

available end anchorage length ℓ_a and by including a 30 percent increase for M_n/V_u when the ends of the reinforcement are confined by a compressive reaction.

For example, a bar size is provided at a simple support such that ℓ_d as computed by 12.2 is equal to $0.04 A_b f_y/\sqrt{f_c'}$. The bar size provided is satisfactory only if $0.04 A_b f_y/\sqrt{f_c'}$ does not exceed $1.3M_n/V_u + \ell_a$.

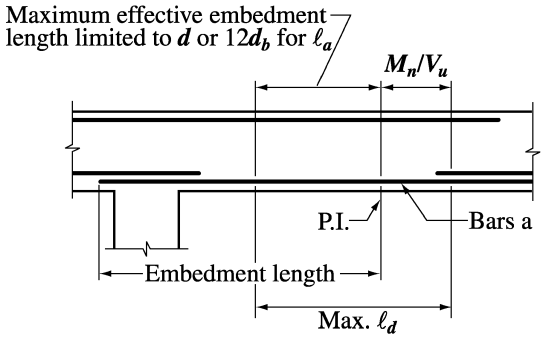
The ℓ_a to be used at points of inflection is limited to the effective depth of the member d or 12 bar diameters $(12d_b)$, whichever is greater. Fig. R12.11.3(b) illustrates this provision at points of inflection. The ℓ_a limitation is added since test data are not available to show that a long end anchorage length will be fully effective in developing a bar that has only a short length between a point of inflection and a point of maximum stress.

R12.11.4 — The use of the strut and tie model for the design of reinforced concrete deep flexural members clarifies that there is significant tension in the reinforcement at the face of the support. This requires the tension reinforcement to be continuous or be developed through and beyond the support.[12.23]



Note: The 1.3 factor is usable only if the reaction confines the ends of the reinforcement.

(a) Maximum size of bar at simple support



(b) Maximum size of bar "a" at point of inflection

Fig. R12.11.3—Concept for determining maximum bar size per 12.11.3

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12.12 — Development of negative moment reinforcement

12.12.1 — Negative moment reinforcement in a continuous, restrained, or cantilever member, or in any member of a rigid frame, shall be anchored in or through the supporting member by embedment length, hooks, or mechanical anchorage.

12.12.2 — Negative moment reinforcement shall have an embedment length into the span as required by 12.1 and 12.10.3.

12.12.3 — At least one-third the total tension reinforcement provided for negative moment at a support shall have an embedment length beyond the point of inflection not less than effective depth of member, $12d_b$, or one-sixteenth the clear span, whichever is greater.

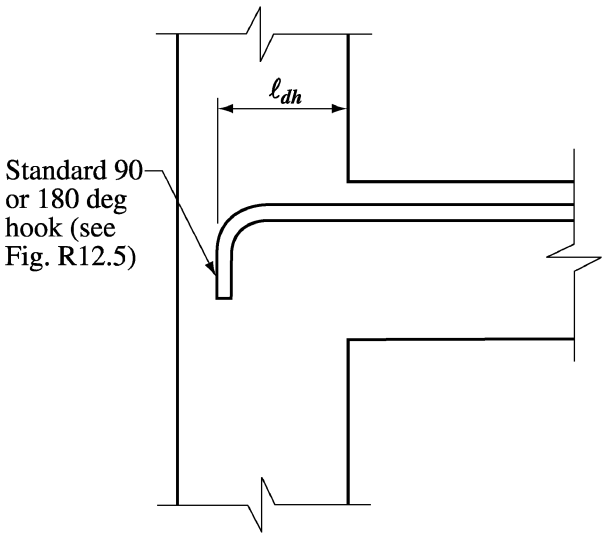
12.12.4 — At interior supports of deep flexural members, negative moment tension reinforcement shall be continuous with that of the adjacent spans.

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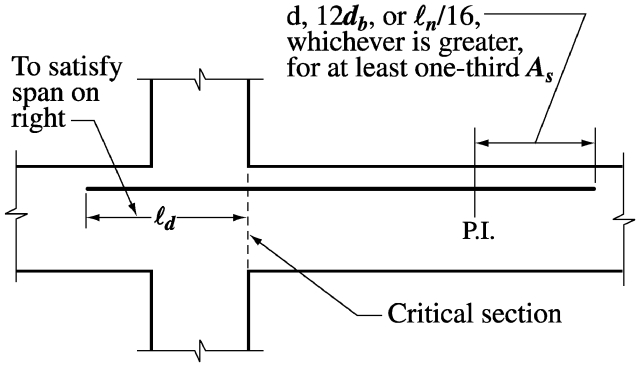
R12.12 — Development of negative moment reinforcement

Fig. R12.12 illustrates two methods of satisfying requirements for anchorage of tension reinforcement beyond the face of support. For anchorage of reinforcement with hooks, see R12.5.

Section 12.12.3 provides for possible shifting of the moment diagram at a point of inflection, as discussed under R12.10.3. This requirement may exceed that of 12.10.3, and the more restrictive of the two provisions governs.



(a) Anchorage into exterior column



Note: Usually such anchorage becomes part of the adjacent beam reinforcement.

(b) Anchorage into adjacent beam

Fig. R12.12—Development of negative moment reinforcement

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12.13 — Development of web reinforcement

R12.13 — Development of web reinforcement

12.13.1 — Web reinforcement shall be as close to the compression and tension surfaces of the member as cover requirements and proximity of other reinforcement permits.

R12.13.1 — Stirrups should be carried as close to the compression face of the member as possible because near ultimate load the flexural tension cracks penetrate deeply.

12.13.2 — Ends of single leg, simple U-, or multiple U-stirrups shall be anchored as required by 12.13.2.1 through 12.13.2.5.

R12.13.2 — The anchorage or development requirements for stirrups composed of bars or deformed wire were changed in the 1989 code to simplify the requirements. The straight anchorage was deleted as this stirrup is difficult to hold in place during concrete placement and the lack of a hook may make the stirrup ineffective as it crosses shear cracks near the end of the stirrup.

12.13.2.1 — For No. 5 bar and D31 wire, and smaller, and for No. 6, No. 7, and No. 8 bars with f_y of 40,000 psi or less, a standard hook around longitudinal reinforcement.

R12.13.2.1 — For a No. 5 bar or smaller, anchorage is provided by a standard stirrup hook, as defined in 7.1.3, hooked around a longitudinal bar. The 1989 code eliminated the need for a calculated straight embedment length in addition to the hook for these small bars, but 12.13.1 requires a full depth stirrup. Likewise, larger stirrups with f_y equal to or less than 40,000 are sufficiently anchored with a standard stirrup hook around the longitudinal reinforcement.

12.13.2.2 — For No. 6, No. 7, and No. 8 stirrups with f_y greater than 40,000 psi, a standard stirrup hook around a longitudinal bar plus an embedment between midheight of the member and the outside end of the hook equal to or greater than $0.014 d_b f_y / \sqrt{f_c'}$.

R12.13.2.2 — Since it is not possible to bend a No. 6, No. 7, or No. 8 stirrup tightly around a longitudinal bar and due to the force in a bar with a design stress greater than 40,000 psi, stirrup anchorage depends on both the value of the hook and whatever development length is provided. A longitudinal bar within a stirrup hook limits the width of any flexural cracks, even in a tensile zone. Since such a stirrup hook cannot fail by splitting parallel to the plane of the hooked bar, the hook strength as utilized in 12.5.2 has been adjusted to reflect cover and confinement around the stirrup hook.

For stirrups with f_y of only 40,000 psi, a standard stirrup hook provides sufficient anchorage and these bars are cov-

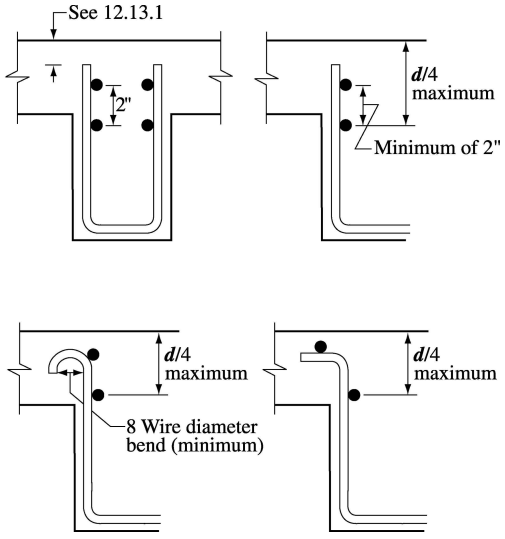


Fig. R12.13.2.3—Anchorage in compression zone of welded plain wire fabric U-stirrups

CODE

12.13.2.3 — For each leg of welded plain wire fabric forming simple U-stirrups, either:

(a) Two longitudinal wires spaced at a 2 in. spacing along the member at the top of the U; or

(b) One longitudinal wire located not more than $d/4$ from the compression face and a second wire closer to the compression face and spaced not less than 2 in. from the first wire. The second wire shall be permitted to be located on the stirrup leg beyond a bend, or on a bend with an inside diameter of bend not less than $8d_b$.

12.13.2.4 — For each end of a single leg stirrup of welded plain or deformed wire fabric, two longitudinal wires at a minimum spacing of 2 in. and with the inner wire at least the greater of $d/4$ or 2 in. from $d/2$. Outer longitudinal wire at tension face shall not be farther from the face than the portion of primary flexural reinforcement closest to the face.

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ered in 12.13.2.1. For bars with higher strength, the embedment should be checked. A 135 deg or 180 deg hook is preferred, but a 90 deg hook may be used provided the free end of the 90 deg hook is extended the full 12 bar diameters as required in 7.1.3.

R12.13.2.3—The requirements for anchorage of welded plain wire fabric stirrups are illustrated in Fig. R12.13.2.3.

R12.13.2.4 — Use of welded wire fabric for shear reinforcement has become commonplace in the precast, prestressed concrete industry. The rationale for acceptance of straight sheets of wire fabric as shear reinforcement is presented in a report by a joint PCI/WRI Ad Hoc Committee on Welded Wire Fabric for Shear Reinforcement.[12.24]

The provisions for anchorage of single leg welded wire fabric in the tension face emphasize the location of the longitudinal wire at the same depth as the primary flexural reinforcement to avoid a splitting problem at the tension steel level. Fig. R12.13.2.4 illustrates the anchorage requirements for single leg, welded wire fabric. For anchorage of single leg, welded wire fabric, the code has permitted hooks and embedment length in the compression and tension faces of members (see 12.13.2.1 and 12.13.2.3), and embedment only in the compression face (see 12.13.2.2). Section 12.13.2.4 provides for anchorage of straight, single leg, welded wire fabric using longitudinal wire anchorage with adequate embedment length in compression and tension faces of members.

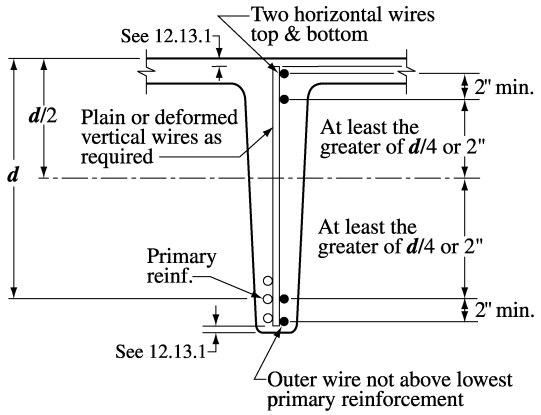


Fig. R12.13.2.4—Anchorage of single leg welded wire fabric shear reinforcement

CODE

12.13.2.5 — In joist construction as defined in 8.11, for No. 4 bar and D20 wire and smaller, a standard hook.

12.13.3 — Between anchored ends, each bend in the continuous portion of a simple U-stirrup or multiple U-stirrup shall enclose a longitudinal bar.

12.13.4 — Longitudinal bars bent to act as shear reinforcement, if extended into a region of tension, shall be continuous with longitudinal reinforcement and, if extended into a region of compression, shall be anchored beyond mid-depth $d/2$ as specified for development length in 12.2 for that part of f_y required to satisfy Eq. (11-17).

12.13.5 — Pairs of U-stirrups or ties so placed as to form a closed unit shall be considered properly spliced when length of laps are $1.3\ell_d$. In members at least 18 in. deep, such splices with $A_b f_y$ not more than 9000 lb per leg shall be considered adequate if stirrup legs extend the full available depth of member.

12.14 — Splices of reinforcement — General

12.14.1 — Splices of reinforcement shall be made only as required or permitted on design drawings, or in specifications, or as authorized by the engineer.

12.14.2 — Lap splices

12.14.2.1 — Lap splices shall not be used for bars larger than No. 11 except as provided in 12.16.2 and 15.8.2.3.

12.14.2.2 — Lap splices of bars in a bundle shall be based on the lap splice length required for individual bars within the bundle, increased in accordance with 12.4. Individual bar splices within a bundle shall not overlap. Entire bundles shall not be lap spliced.

12.14.2.3 — Bars spliced by noncontact lap splices in flexural members shall not be spaced transversely farther apart than one-fifth the required lap splice length, nor 6 in.

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R12.13.2.5 — In joists, a small bar or wire can be anchored by a standard hook not engaging longitudinal reinforcement, allowing a continuously bent bar to form a series of single-leg stirrups in the joist.

R12.13.5 — These requirements for lapping of double U-stirrups to form closed stirrups control over the provisions of 12.15.

R12.14 — Splices of reinforcement — General

Splices should, if possible, be located away from points of maximum tensile stress. The lap splice requirements of 12.15 encourage this practice.

R12.14.2 — Lap splices

R12.14.2.1 — Because of lack of adequate experimental data on lap splices of No. 14 and No. 18 bars in compression and in tension, lap splicing of these bar sizes is prohibited except as permitted in 12.16.2 and 15.8.2.3 for compression lap splices of No. 14 and No. 18 bars with smaller bars.

R12.14.2.2 — The increased length of lap required for bars in bundles is based on the reduction in the exposed perimeter of the bars. Only individual bars are lap spliced along the bundle.

R12.14.2.3 — If individual bars in noncontact lap splices are too widely spaced, an unreinforced section is created. Forcing a potential crack to follow a zigzag line (5 to 1 slope) is considered a minimum precaution. The 6 in. maximum spacing is added because most research available on the lap splicing of deformed bars was conducted with reinforcement within this spacing.

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12.14.3 — Mechanical and welded splices

12.14.3.1 — Mechanical and welded splices shall be permitted.

12.14.3.2 — A full mechanical splice shall develop in tension or compression, as required, at least 125 percent of specified yield strength f_y of the bar.

12.14.3.3 — Except as provided in this code, all welding shall conform to "Structural Welding Code—Reinforcing Steel" (ANSI/AWS D1.4).

12.14.3.4 — A full welded splice shall develop at least 125 percent of the specified yield strength f_y of the bar.

12.14.3.5 — Mechanical or welded splices not meeting requirements of 12.14.3.2 or 12.14.3.4 shall be permitted only for No. 5 bars and smaller and in accordance with 12.15.4.

12.15 — Splices of deformed bars and deformed wire in tension

12.15.1 — Minimum length of lap for tension lap splices shall be as required for Class A or B splice, but not less than 12 in., where:

Class A splice ..... $1.0\ell_d$

Class B splice ..... $1.3\ell_d$

where ℓ_d is the tensile development length for the specified yield strength f_y in accordance with 12.2 without the modification factor of 12.2.5.

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R12.14.3 — Mechanical and welded splices

R12.14.3.2 — The maximum reinforcement stress used in design under the code is the specified yield strength. To ensure sufficient strength in splices so that yielding can be achieved in a member and thus brittle failure avoided, the 25 percent increase above the specified yield strength was selected as both an adequate minimum for safety and a practicable maximum for economy.

R12.14.3.3 — See R3.5.2 for discussion on welding.

R12.14.3.4 — A full welded splice is primarily intended for large bars (No. 6 and larger) in main members. The tensile strength requirement of 125 percent of specified yield strength is intended to provide sound welding that is also adequate for compression. See the discussion on strength in R12.14.3.2. The 1995 code eliminated a requirement that the bars be butted since indirect butt welds are permitted by ANSI/AWS D1.4, although ANSI/AWS D1.4 does indicate that wherever practical, direct butt splices are preferable for No. 7 bars and larger.

R12.14.3.5 — The use of mechanical or welded splices of less strength than 125 percent of specified yield strength is permitted if the minimum design criteria of 12.15.4 are met. Therefore, lap welds of reinforcing bars, either with or without backup material, welds to plate connections, and end-bearing splices are allowed under certain conditions. The 1995 code limited these lower strength welds and connections to No. 5 bars and smaller due to the potentially brittle nature of failure at these welds.

R12.15 — Splices of deformed bars and deformed wire in tension

R12.15.1 — Lap splices in tension are classified as Type A or B, with length of lap a multiple of the tensile development length ℓ_d. The development length ℓ_d used to obtain lap length should be based on f_y because the splice classifications already reflect any excess reinforcement at the splice location; therefore, the factor from 12.2.5 for excess A_s should not be used. When multiple bars located in the same plane are spliced at the same section, the clear spacing is the minimum clear distance between the adjacent splices. For splices in columns with offset bars, Fig. R12.15.1(a) illustrates the clear spacing to be used. For staggered splices, the clear spacing is the minimum distance between adjacent splices [distance x in Fig. R12.15.1(b)].

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COMMENTARY

The 1989 code contained several changes in development length in tension that eliminated many of the concerns regarding tension splices due to closely spaced bars with minimal cover. Thus, the Class C splice was eliminated although development lengths, on which splice lengths are based, have in some cases increased. Committee 318 considered suggestions from many sources, including ACI Committee 408, but has retained a two-level splice length primarily to encourage designers to splice bars at points of minimum stress and to stagger splices to improve behavior of critical details.

12.15.2 — Lap splices of deformed bars and deformed wire in tension shall be Class B splices except that Class A splices are allowed when:

(a) the area of reinforcement provided is at least twice that required by analysis over the entire length of the splice; and

(b) one-half or less of the total reinforcement is spliced within the required lap length.

R12.15.2 — The tension lap splice requirements of 12.15.1 encourage the location of splices away from regions of high tensile stress to locations where the area of steel provided is at least twice that required by analysis. Table R12.15.2 presents the splice requirements in tabular form as presented in earlier code editions.

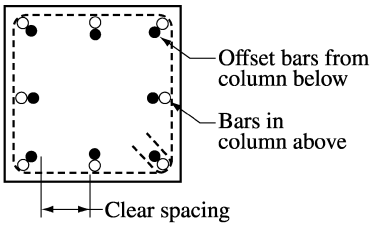
TABLE R12.15.2—TENSION LAP SPLICES

$\frac{A_s \text{ provided}^*}{A_s \text{ required}}$	Maximum percent of A_s spliced within required lap length	
	50	100
Equal to or greater than 2	Class A	Class B
Less than 2	Class B	Class B

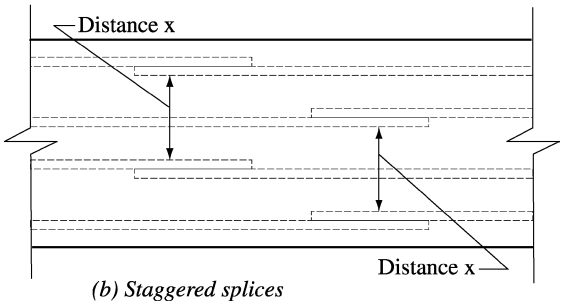
* Ratio of area of reinforcement provided to area of reinforcement required by analysis at splice locations.

12.15.3 — Mechanical or welded splices used where area of reinforcement provided is less than twice that required by analysis shall meet requirements of 12.14.3.2 or 12.14.3.4.

R12.15.3 — A mechanical or welded splice should develop at least 125 percent of the specified yield strength when located in regions of high tensile stress in the reinforcement. Such splices need not be staggered, although such staggering is encouraged where the area of reinforcement provided is less than twice that required by the analysis.



(a) Offset column bars



(b) Staggered splices

Fig. R12.15.1—Clear spacing of spliced bars

CODE

12.15.4 — Mechanical or welded splices not meeting the requirements of 12.14.3.2 or 12.14.3.4 shall be permitted for No. 5 bars and smaller if the requirements of 12.15.4.1 through 12.15.4.3 are met:

12.15.4.1 — Splices shall be staggered at least 24 in.

12.15.4.2 — In computing the tensile forces that can be developed at each section, the spliced reinforcement stress shall be taken as the specified splice strength, but not greater than f_y. The stress in the unspliced reinforcement shall be taken as f_y times the ratio of the shortest length embedded beyond the section to ℓ_d, but not greater than f_y.

12.15.4.3 — The total tensile force that can be developed at each section must be at least twice that required by analysis, and at least 20,000 psi times the total area of reinforcement provided.

12.15.5 — Splices in tension tie members shall be made with a full mechanical or full welded splice in accordance with 12.14.3.2 or 12.14.3.4 and splices in adjacent bars shall be staggered at least 30 in.

12.16 — Splices of deformed bars in compression

COMMENTARY

R12.15.4 — See R12.14.3.5. Section 12.15.4 concerns the situation where mechanical or welded splices of strength less than 125 percent of the specified yield strength of the reinforcement may be used. It provides a relaxation in the splice requirements where the splices are staggered and excess reinforcement area is available. The criterion of twice the computed tensile force is used to cover sections containing partial tensile splices with various percentages of total continuous steel. The usual partial tensile splice is a flare groove weld between bars or bar and structural steel piece.

To detail such welding, the length of weld should be specified. Such welds are rated at the product of total weld length times effective size of groove weld (established by bar size) times allowable stress permitted by **"Structural Welding Code—Reinforcing Steel"** (ANSI/AWS D1.4).

A full mechanical or welded splice conforming to 12.14.3.2 or 12.14.3.4 can be used without the stagger requirement in lieu of the lower strength mechanical or welded splice.

R12.15.5 — A tension tie member, has the following characteristics: member having an axial tensile force sufficient to create tension over the cross section; a level of stress in the reinforcement such that every bar must be fully effective; and limited concrete cover on all sides. Examples of members that may be classified as tension ties are arch ties, hangers carrying load to an overhead supporting structure, and main tension elements in a truss.

In determining if a member should be classified as a tension tie, consideration should be given to the importance, function, proportions, and stress conditions of the member related to the above characteristics. For example, a usual large circular tank, with many bars and with splices well staggered and widely spaced should not be classified as a tension tie member, and Class B splices may be used.

R12.16 — Splices of deformed bars in compression

Bond research has been primarily related to bars in tension. Bond behavior of compression bars is not complicated by the problem of transverse tension cracking and thus compression splices do not require provisions as strict as those specified for tension splices. The minimum lengths for column splices contained originally in the 1956 code have been carried forward in later codes, and extended to compression bars in beams and to higher strength steels. No changes have been made in the provisions for compression splices since the 1971 code.

CODE

12.16.1 — Compression lap splice length shall be $0.0005 f_y d_b$, for f_y of 60,000 psi or less, or $(0.0009 f_y - 24) d_b$ for f_y greater than 60,000 psi, but not less than 12 in. For f_c' less than 3000 psi, length of lap shall be increased by one-third.

12.16.2 — When bars of different size are lap spliced in compression, splice length shall be the larger of either development length of larger bar, or splice length of smaller bar. Lap splices of No. 14 and No. 18 bars to No. 11 and smaller bars shall be permitted.

12.16.3 — Mechanical or welded splices used in compression shall meet requirements of 12.14.3.2 or 12.14.3.4.

12.16.4 — End-bearing splices

12.16.4.1 — In bars required for compression only, transmission of compressive stress by bearing of square cut ends held in concentric contact by a suitable device shall be permitted.

12.16.4.2 — Bar ends shall terminate in flat surfaces within 1.5 deg of a right angle to the axis of the bars and shall be fitted within 3 deg of full bearing after assembly.

12.16.4.3 — End-bearing splices shall be used only in members containing closed ties, closed stirrups, or spirals.

12.17 — Special splice requirements for columns

12.17.1 — Lap splices, mechanical splices, butt-welded splices, and end-bearing splices shall be used with the limitations of 12.17.2 through 12.17.4. A splice shall satisfy requirements for all load combinations for the column.

COMMENTARY

R12.16.1 — Essentially, lap requirements for compression splices have remained the same since the 1963 code.

The 1963 code values were modified in the 1971 code to recognize various degrees of confinement and to permit design with reinforcement up to 80,000 psi yield strength. Tests[12.1,12.25] have shown that splice strengths in compression depend considerably on end bearing and do not increase proportionally in strength when the splice length is doubled. Accordingly, for yield strengths above 60,000 psi, compression lap lengths are significantly increased, except where spiral enclosures are used (as in spiral columns) the where the increase is about 10 percent for an increase in yield strength from 60,000 to 75,000 psi.

R12.16.2 — The lap splice length is to be computed based on the larger of the compression splice length of the smaller bar; or the compression development length of the larger bar. Lap splices are generally prohibited for No. 14 or No. 18 bars; however, for compression only, lap splices are permitted for No. 14 or No. 18 bars to No. 11 or smaller bars.

R12.16.4 — End-bearing splices

R12.16.4.1 — Experience with end-bearing splices has been almost exclusively with vertical bars in columns. If bars are significantly inclined from the vertical, special attention is required to ensure that adequate end-bearing contact can be achieved and maintained.

R12.16.4.2 — These tolerances were added in the 1971 code, representing practice based on tests of full-size members containing No. 18 bars.

R12.16.4.3 — This limitation was added in the 1971 code to ensure a minimum shear resistance in sections containing end-bearing splices.

R12.17 — Special splice requirements for columns

In columns subject to flexure and axial loads, tension stresses may occur on one face of the column for moderate and large eccentricities as shown in Fig. R12.17. When such tensions occur, 12.17 requires tension splices to be used or an adequate tensile resistance to be provided. Furthermore, a minimum tension capacity is required in each face of all columns even where analysis indicates compression only.

The 1989 code clarifies this section on the basis that a compressive lap splice has a tension capacity of at least one-

CODE

12.17.2 — Lap splices in columns

12.17.2.1 — Where the bar stress due to factored loads is compressive, lap splices shall conform to 12.16.1, 12.16.2, and, where applicable, to 12.17.2.4 or 12.17.2.5.

12.17.2.2 — Where the bar stress due to factored loads is tensile and does not exceed $\mathbf{0.5}f_y$ in tension, lap splices shall be Class B tension lap splices if more than one-half of the bars are spliced at any section, or Class A tension lap splices if half or fewer of the bars are spliced at any section and alternate lap splices are staggered by ℓ_d.

12.17.2.3 — Where the bar stress due to factored loads is greater than $\mathbf{0.5}f_y$ in tension, lap splices shall be Class B tension lap splices.

COMMENTARY

quarter f_y, which simplifies the calculation requirements in previous codes.

Note that the column splice should satisfy requirements for all load combinations for the column. Frequently, the basic gravity load combination will govern the design of the column itself, but a load combination including wind or seismic loads may induce greater tension in some column bars, and the column splice should be designed for this tension.

R12.17.2 — Lap splices in columns

R12.17.2.1 — The 1989 code was simplified for column bars always in compression on the basis that a compressive lap splice is adequate for sufficient tension to preclude special requirements.

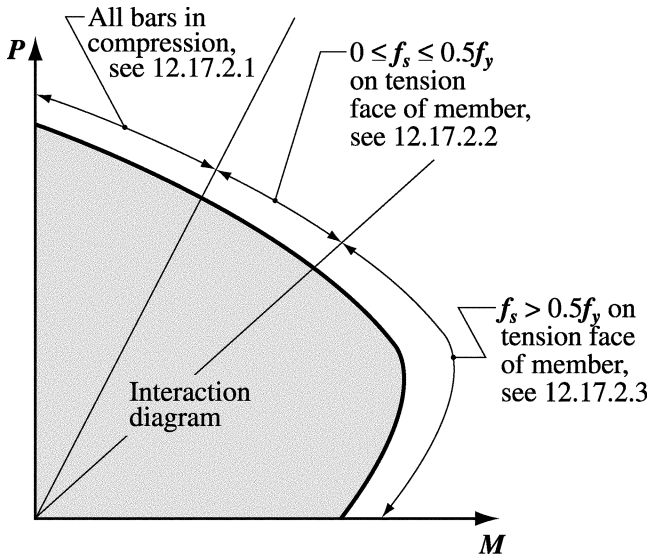


Fig. R12.17—Special splice requirements for columns

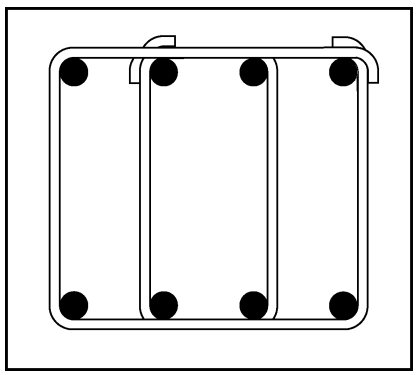


Fig. R.12.17.2—Tie legs which cross the axis of bending are used to compute effective area. In the case shown, four legs are effective

CODE

12.17.2.4 — In tied reinforced compression members, where ties throughout the lap splice length have an effective area not less than $0.0015hs$, lap splice length shall be permitted to be multiplied by 0.83, but lap length shall not be less than 12 in. Tie legs perpendicular to dimension h shall be used in determining effective area.

12.17.2.5 — In spirally reinforced compression members, lap splice length of bars within a spiral shall be permitted to be multiplied by 0.75, but lap length shall not be less than 12 in.

12.17.3 — Mechanical or welded splices in columns

Mechanical or welded splices in columns shall meet the requirements of 12.14.3.2 or 12.14.3.4.

12.17.4 — End-bearing splices in columns

End-bearing splices complying with 12.16.4 shall be permitted to be used for column bars stressed in compression provided the splices are staggered or additional bars are provided at splice locations. The continuing bars in each face of the column shall have a tensile strength, based on the specified yield strength f_y, not less than $0.25f_y$ times the area of the vertical reinforcement in that face.

COMMENTARY

R12.17.2.4 — Reduced lap lengths are allowed when the splice is enclosed throughout its length by minimum ties.

The tie legs perpendicular to each direction are computed separately and the requirement must be satisfied in each direction. This is illustrated in Fig. R12.17.2, where four legs are effective in one direction and two legs in the other direction. This calculation is critical in one direction, which normally can be determined by inspection.

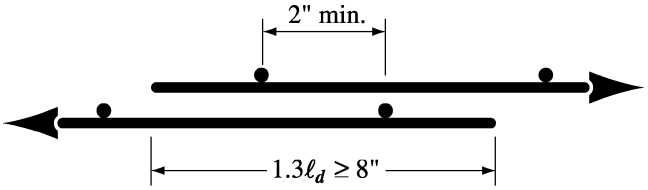
R12.17.2.5 — Compression lap lengths may be reduced when the lap splice is enclosed throughout its length by spirals because of increased splitting resistance. Spirals should meet requirements of 7.10.4 and 10.9.3.

R12.17.3 — Mechanical or welded splices in columns

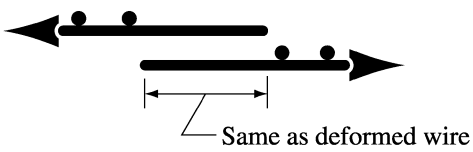
Mechanical or welded splices are allowed for splices in columns but should be designed as a full mechanical splice or a full welded splice developing 125 percent f_y as required by 12.14.3.2 or 12.14.3.4. Splice capacity is traditionally tested in tension and full strength is required to reflect the high compression loads possible in column reinforcement due to creep effects. If a mechanical splice developing less than a full mechanical splice is used, then the splice is required to conform to all requirements of end-bearing splices of 12.16.4 and 12.17.4.

R12.17.4 — End-bearing splices in columns

End-bearing splices used to splice column bars always in compression should have a tension capacity of 25 percent of the yield strength of the steel area on each face of the column, either by staggering the end-bearing splices or by adding additional steel through the splice location. The end-bearing splice should conform to 12.16.4.



(a) Section 12.18.1



(b) Section 12.18.2

Fig. R12.18—Lap splices of deformed fabric

CODE

12.18 — Splices of welded deformed wire fabric in tension

12.18.1 — Minimum length of lap for lap splices of welded deformed wire fabric measured between the ends of each fabric sheet shall be not less than $\mathbf{1.3}\ell_d$ nor 8 in., and the overlap measured between outermost cross wires of each fabric sheet shall be not less than 2 in. ℓ_d shall be the development length for the specified yield strength f_y in accordance with 12.7.

12.18.2 — Lap splices of welded deformed wire fabric, with no cross wires within the lap splice length, shall be determined as for deformed wire.

12.18.3—When any plain wires are present in the deformed wire fabric in the direction of the lap splice or when deformed wire fabric is lap spliced to plain wire fabric, the fabric shall be lap spliced in accordance with 12.19.

12.19 — Splices of welded plain wire fabric in tension

Minimum length of lap for lap splices of welded plain wire fabric shall be in accordance with 12.19.1 and 12.19.2.

12.19.1 — When area of reinforcement provided is less than twice that required by analysis at splice location, length of overlap measured between outermost cross wires of each fabric sheet shall be not less than one spacing of cross wires plus 2 in., nor less than $\mathbf{1.5}\ell_d$, nor 6 in. The development length ℓ_d for the specified yield strength f_y shall be in accordance with 12.8.

12.19.2 — When area of reinforcement provided is at least twice that required by analysis at splice location, length of overlap measured between outermost cross wires of each fabric sheet shall not be less than $\mathbf{1.5}\ell_d$, nor 2 in. ℓ_d shall be the development length for the specified yield strength f_y in accordance with 12.8.

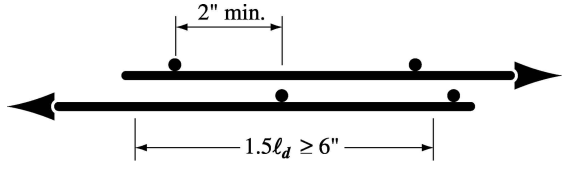
COMMENTARY

R12.18 — Splices of welded deformed wire fabric in tension

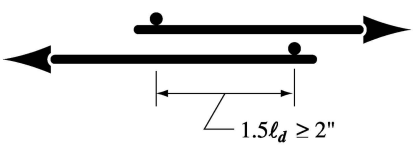
Splice provisions for deformed fabric are based on available tests.[12.26] The requirements were simplified (1976 code supplement) from provisions of the 1971 code by assuming that only one cross wire of each fabric sheet is overlapped and by computing the splice length as $\mathbf{1.3}\ell_d$. The development length ℓ_d is that computed in accordance with the provisions of 12.7 without regard to the 8 in. minimum. The 8 in. applies to the overall splice length. See Fig. R12.18. If no cross wires are within the lap length, the provisions for deformed wire apply.

R12.19 — Splices of welded plain wire fabric in tension

The strength of lap splices of welded plain wire fabric is dependent primarily on the anchorage obtained from the cross wires rather than on the length of wire in the splice. For this reason, the lap is specified in terms of overlap of cross wires rather than in wire diameters or inches. The 2 in. additional lap required is to assure overlapping of the cross wires and to provide space for satisfactory consolidation of the concrete between cross wires. Research[12.27] has shown an increased splice length is required when fabric of large, closely spaced wires is lapped and as a consequence additional splice length requirements are provided for these fabrics, in addition to an absolute minimum of 6 in. The development length ℓ_d is that computed in accordance with the provisions of 12.8 without regard to the 6 in. minimum. Splice requirements are illustrated in Fig. R12.19.



A_s prov./A_s req'd. < 2
(a) Section 12.19.1



A_s prov./A_s req'd. ≥ 2
(b) Section 12.19.2

Fig. R12.19—Lap splices of plain fabric

PART 5 — STRUCTURAL SYSTEMS OR ELEMENTS

CHAPTER 13 — TWO-WAY SLAB SYSTEMS

CODE

13.0 — Notation

b_1 = width of the critical section defined in 11.12.1.2 measured in the direction of the span for which moments are determined, in.

b_2 = width of the critical section defined in 11.12.1.2 measured in the direction perpendicular to b_1, in.

c_1 = size of rectangular or equivalent rectangular column, capital, or bracket measured in the direction of the span for which moments are being determined, in.

c_2 = size of rectangular or equivalent rectangular column, capital, or bracket measured transverse to the direction of the span for which moments are being determined, in.

C = cross-sectional constant to define torsional properties

$$= \sum\left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3}$$

The constant C for T- or L-sections shall be permitted to be evaluated by dividing the section into separate rectangular parts and summing the values of C for each part

E_{cb} = modulus of elasticity of beam concrete, psi

E_{cs} = modulus of elasticity of slab concrete, psi

h = overall thickness of member, in.

I_b = moment of inertia about centroidal axis of gross section of beam as defined in 13.2.4, in.^{4}

I_s = moment of inertia about centroidal axis of gross section of slab, in.^{4}

= $h^3/12$ times width of slab defined in notations a and b_t

K_t = torsional stiffness of torsional member; moment per unit rotation. See R13.7.5.

ℓ_n = length of clear span in direction that moments are being determined, measured face-to-face of supports, in.

ℓ_1 = length of span in direction that moments are being determined, measured center-to-center of supports, in.

ℓ_2 = length of span transverse to ℓ_1, measured center-to-center of supports, in. See also 13.6.2.3 and 13.6.2.4.

M_o = total factored static moment, in.-lb

M_u = factored moment at section, in.-lb

V_c = nominal shear strength provided by concrete, lb. See 11.12.2.1

COMMENTARY

13.0 — Notation

The design methods given in Chapter 13 are based on analysis of the results of an extensive series of tests[13.1-13.7] and the well established performance record of various slab systems. Much of Chapter 13 is concerned with the selection and distribution of flexural reinforcement. The designer is cautioned that the problem related to safety of a slab system is the transmission of load from the slab to the columns by flexure, torsion, and shear. Design criteria for shear and torsion in slabs are given in Chapter 11.

Design aids for use in the engineering analysis and design of two-way slab systems are given in the *ACI Design Handbook*.[13.8] Design aids are provided to simplify application of the direct design and equivalent frame methods of Chapter 13.

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

CODE

V_u = factored shear force at section, lb
w_d = factored dead load per unit area
w_ℓ = factored live load per unit area
w_u = factored load per unit area
x = shorter overall dimension of rectangular part of cross section, in.
y = longer overall dimension of rectangular part of cross section, in.
α = ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels (if any) on each side of the beam

$$= \frac{E_{cb}I_b}{E_{cs}I_s}$$

α_1 = α in direction of ℓ_1
α_2 = α in direction of ℓ_2
β_t = ratio of torsional stiffness of edge beam section to flexural stiffness of a width of slab equal to span length of beam, center-to-center of supports

$$= \frac{E_{cb}C}{2E_{cs}I_s}$$

γ_f = fraction of unbalanced moment transferred by flexure at slab-column connections. See 13.5.3.2
γ_v = fraction of unbalanced moment transferred by eccentricity of shear at slab-column connections
$= 1 - \gamma_f$
ρ = ratio of nonprestressed tension reinforcement
ρ_b = reinforcement ratio producing balanced strain conditions
ϕ = strength reduction factor

13.1 — Scope

13.1.1 — Provisions of Chapter 13 shall apply for design of slab systems reinforced for flexure in more than one direction, with or without beams between supports.

13.1.2 — For a slab system supported by columns or walls, the dimensions c_1 and c_2 and the clear span ℓ_n shall be based on an effective support area defined by the intersection of the bottom surface of the slab, or of the drop panel if there is one, with the largest right circular cone, right pyramid, or tapered wedge whose surfaces are located within the column and the capital or bracket and are oriented no greater than 45 deg to the axis of the column.

13.1.3 — Solid slabs and slabs with recesses or pockets made by permanent or removable fillers between ribs or joists in two directions are included within the scope of Chapter 13.

COMMENTARY

R13.1 — Scope

The fundamental design principles contained in Chapter 13 are applicable to all planar structural systems subjected to transverse loads. Some of the specific design rules, as well as historical precedents, limit the types of structures to which Chapter 13 applies. General characteristics of slab systems that may be designed according to Chapter 13 are described in this section. These systems include flat slabs, flat plates, two-way slabs, and waffle slabs. Slabs with paneled ceilings are two-way wide-band beam systems.

True one-way slabs, slabs reinforced to resist flexural stresses in only one direction, are excluded. Also excluded are soil-supported slabs, such as slabs on grade, that do not transmit vertical loads from other parts of the structure to the soil.

For slabs with beams, the explicit design procedures of Chapter 13 apply only when the beams are located at the edges of the panel and when the beams are supported by

CODE

13.1.4 — Minimum thickness of slabs designed in accordance with Chapter 13 shall be as required by 9.5.3.

COMMENTARY

columns or other essentially nondeflecting supports at the corners of the panel. Two-way slabs with beams in one direction, with both slab and beams supported by girders in the other direction, may be designed under the general requirements of Chapter 13. Such designs should be based upon analysis compatible with the deflected position of the supporting beams and girders.

For slabs supported on walls, the explicit design procedures in this chapter treat the wall as a beam of infinite stiffness; therefore, each wall should support the entire length of an edge of the panel (see 13.2.3). Wall-like columns less than a full panel length can be treated as columns.

13.2 — Definitions

R13.2 — Definitions

13.2.1 — Column strip is a design strip with a width on each side of a column centerline equal to $\mathbf{0.25}\ell_2$ or $\mathbf{0.25}\ell_1$, whichever is less. Column strip includes beams, if any.

13.2.2 — Middle strip is a design strip bounded by two column strips.

13.2.3 — A panel is bounded by column, beam, or wall centerlines on all sides.

R13.2.3 — A panel includes all flexural elements between column centerlines. Thus, the column strip includes the beam, if any.

13.2.4 — For monolithic or fully composite construction, a beam includes that portion of slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness.

R13.2.4 — For monolithic or fully composite construction, the beams include portions of the slab as flanges. Two examples of the rule are provided in Fig. R13.2.4.

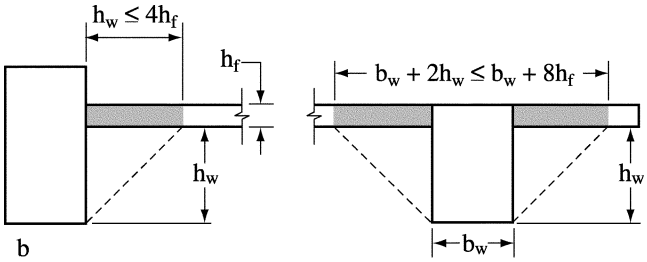


Fig. R13.2.4—Examples of the portion of slab to be included with the beam under 13.2.4

13.3 — Slab reinforcement

R13.3 — Slab reinforcement

13.3.1 — Area of reinforcement in each direction for two-way slab systems shall be determined from moments at critical sections, but shall not be less than required by 7.12.

13.3.2 — Spacing of reinforcement at critical sections shall not exceed two times the slab thickness, except for portions of slab area of cellular or ribbed construction.

R13.3.2 — The requirement that the center-to-center spacing of the reinforcement be not more than two times the slab thickness applies only to the reinforcement in solid slabs,

CODE

In the slab over cellular spaces, reinforcement shall be provided as required by 7.12.

13.3.3 — Positive moment reinforcement perpendicular to a discontinuous edge shall extend to the edge of slab and have embedment, straight or hooked, at least 6 in. in spandrel beams, columns, or walls.

13.3.4 — Negative moment reinforcement perpendicular to a discontinuous edge shall be bent, hooked, or otherwise anchored in spandrel beams, columns, or walls, and shall be developed at face of support according to provisions of Chapter 12.

13.3.5 — Where a slab is not supported by a spandrel beam or wall at a discontinuous edge, or where a slab cantilevers beyond the support, anchorage of reinforcement shall be permitted within the slab.

13.3.6 — In slabs with beams between supports with a value of α greater than 1.0, special top and bottom slab reinforcement shall be provided at exterior corners in accordance with 13.3.6.1 through 13.3.6.4.

13.3.6.1 — The special reinforcement in both top and bottom of slab shall be sufficient to resist a moment per foot of width equal to the maximum positive moment in the slab.

13.3.6.2 — The moment shall be assumed to be about an axis perpendicular to the diagonal from the corner in the top of the slab and about an axis parallel to the diagonal from the corner in the bottom of the slab.

13.3.6.3 — The special reinforcement shall be provided for a distance in each direction from the corner equal to one-fifth the longer span.

13.3.6.4 — The special reinforcement shall be placed in a band parallel to the diagonal in the top of the slab and a band perpendicular to the diagonal in the bottom of the slab. Alternatively, the special reinforcement shall be placed in two layers parallel to the sides of the slab in both the top and bottom of the slab.

13.3.7 — Where a drop panel is used to reduce amount of negative moment reinforcement over the column of a flat slab, size of drop panel shall be in accordance with the 13.3.7.1, 13.3.7.2, and 13.3.7.3.

13.3.7.1 — Drop panel shall extend in each direction from centerline of support a distance not less than one-sixth the span length measured from center-to-center of supports in that direction.

COMMENTARY

and not to reinforcement joists or waffle slabs. This limitation is to ensure slab action, cracking, and provide for the possibility of loads concentrated on small areas of the slab. See also R10.6.

R13.3.3-R13.3.5 — Bending moments in slabs at spandrel beams can be subject to great variation. If spandrel beams are built solidly into walls, the slab approaches complete fixity. Without an integral wall, the slab could approach simply supported, depending on the torsional rigidity of the spandrel beam or slab edge. These requirements provide for unknown conditions that might normally occur in a structure.

CODE

13.3.7.2 — Projection of drop panel below the slab shall be at least one-quarter the slab thickness beyond the drop.

13.3.7.3 — In computing required slab reinforcement, the thickness of the drop panel below the slab shall not be assumed greater than one-quarter the distance from edge of drop panel to edge of column or column capital.

13.3.8 — Details of reinforcement in slabs without beams

13.3.8.1 — In addition to the other requirements of 13.3, reinforcement in slabs without beams shall have minimum extensions as prescribed in Fig. 13.3.8.

13.3.8.2 — Where adjacent spans are unequal, extensions of negative moment reinforcement beyond the face of support as prescribed in Fig. 13.3.8 shall be based on requirements of the longer span.

13.3.8.3 — Bent bars shall be permitted only when depth-span ratio permits use of bends of 45 deg or less.

13.3.8.4 — In frames where two-way slabs act as primary members resisting lateral loads, lengths of reinforcement shall be determined by analysis but shall not be less than those prescribed in Fig. 13.3.8.

13.3.8.5 — All bottom bars or wires within the column strip, in each direction, shall be continuous or spliced with Class A tension splices or with mechanical or welded splices satisfying 12.14.3. Splices shall be located as shown in Fig. 13.3.8. At least two of the column strip bottom bars or wires in each direction shall pass within the column core and shall be anchored at exterior supports.

13.3.8.6 — In slabs with shearheads and in lift-slab construction where it is not practical to pass the bottom bars required by 13.3.8.5 through the column, at least two bonded bottom bars or wires in each direction shall pass through the shearhead or lifting collar as close to the column as practicable and be continuous or spliced with a Class A splice. At exterior columns, the reinforcement shall be anchored at the shearhead or lifting collar.

COMMENTARY

R13.3.8 — Details of reinforcement in slabs without beams

In the 1989 code, bent bars were removed from Fig. 13.3.8. This was done because bent bars are seldom used and are difficult to place properly. Bent bars are permitted, however, if they comply with 13.3.8.3. Refer to 13.4.8 of the 1983 code.

R13.3.8.4 — For moments resulting from combined lateral and gravity loadings, the minimum lengths and extensions of bars in Fig. 13.3.8 may not be sufficient.

R13.3.8.5 — The continuous column strip bottom reinforcement provides the slab some residual ability to span to the adjacent supports should a single support be damaged. The two continuous column strip bottom bars or wires through the column may be termed integrity steel, and are provided to give the slab some residual capacity following a single punching shear failure at a single support.[13.9] In the 2002 code, mechanical and welded splices were explicitly recognized as alternative methods of splicing reinforcement.

R13.3.8.6 — In the 1992 code, this provision was added to require the same integrity steel as for other two-way slabs without beams in case of a punching shear failure at a support.

In some instances, there is sufficient clearance so that the bonded bottom bars can pass under shearheads and through the column. Where clearance under the shearhead is inadequate, the bottom bars should pass through holes in the shearhead arms or within the perimeter of the lifting collar. Shearheads should be kept as low as possible in the slab to increase their effectiveness.

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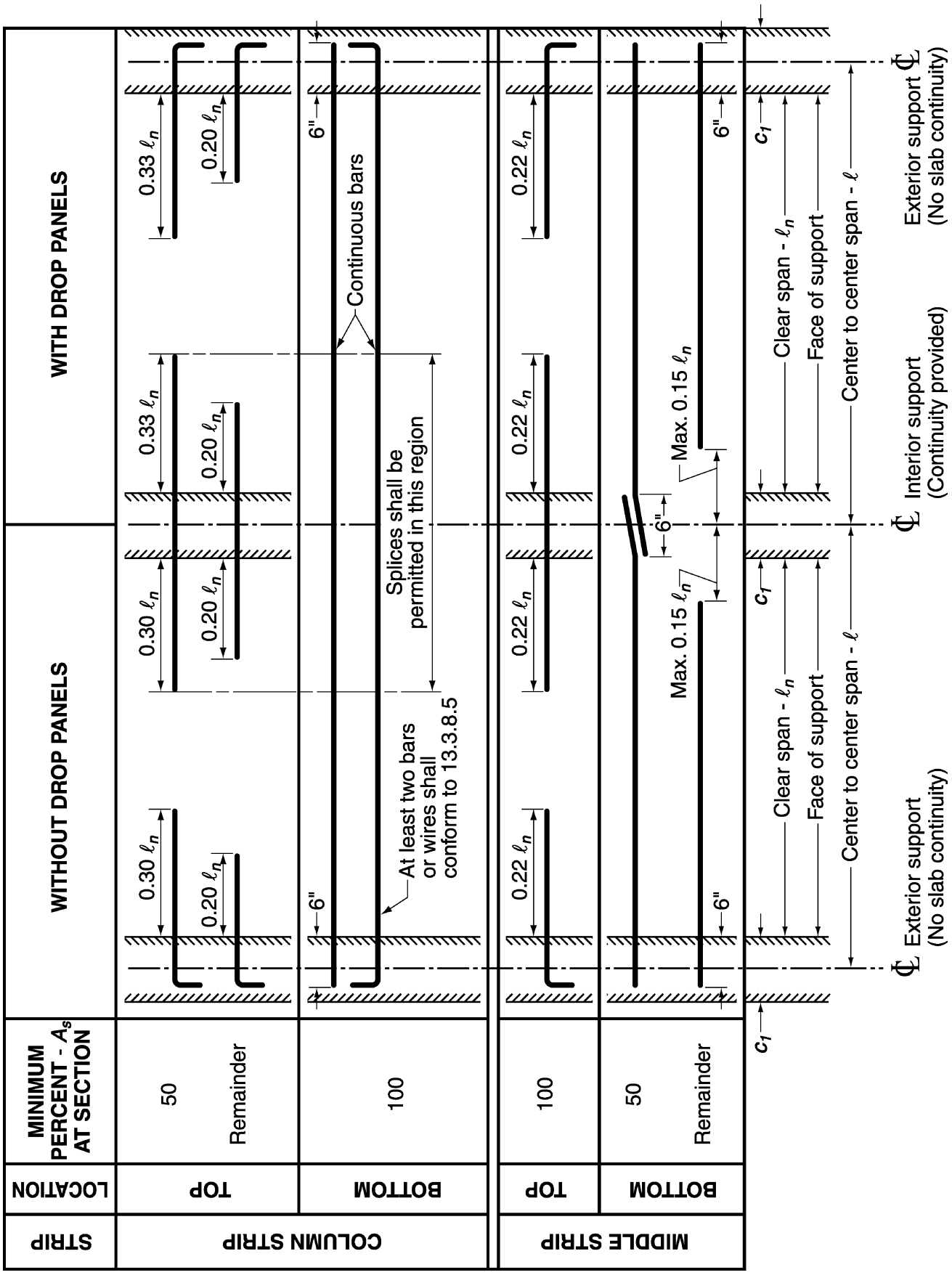


Fig. 13.3.8—Minimum extensions for reinforcement in slabs without beams. (See 12.11.1 for reinforcement extension into supports)

CODE

13.4 — Openings in slab systems

13.4.1 — Openings of any size shall be permitted in slab systems if shown by analysis that the design strength is at least equal to the required strength set forth in 9.2 and 9.3, and that all serviceability conditions, including the limits on deflections, are met.

13.4.2 — As an alternate to special analysis as required by 13.4.1, openings shall be permitted in slab systems without beams only in accordance with 13.4.2.1 through 13.4.2.4.

13.4.2.1 — Openings of any size shall be permitted in the area common to intersecting middle strips, provided total amount of reinforcement required for the panel without the opening is maintained.

13.4.2.2 — In the area common to intersecting column strips, not more than one-eighth the width of column strip in either span shall be interrupted by openings. An amount of reinforcement equivalent to that interrupted by an opening shall be added on the sides of the opening.

13.4.2.3 — In the area common to one column strip and one middle strip, not more than one-quarter of the reinforcement in either strip shall be interrupted by openings. An amount of reinforcement equivalent to that interrupted by an opening shall be added on the sides of the opening.

13.4.2.4 — Shear requirements of 11.12.5 shall be satisfied.

13.5 — Design procedures

13.5.1 — A slab system shall be designed by any procedure satisfying conditions of equilibrium and geometric compatibility, if shown that the design strength at every section is at least equal to the required strength set forth in 9.2 and 9.3, and that all serviceability conditions, including limits on deflections, are met.

COMMENTARY

R13.4 — Openings in slab systems

See R11.12.5.

R13.5 — Design procedures

R13.5.1 — This section permits a designer to base a design directly on fundamental principles of structural mechanics, provided it can be demonstrated explicitly that all safety and serviceability criteria are satisfied. The design of the slab may be achieved through the combined use of classic solutions based on a linearly elastic continuum, numerical solutions based on discrete elements, or yield-line analyses, including, in all cases, evaluation of the stress conditions around the supports in relation to shear and torsion as well as flexure. The designer should consider that the design of a slab system involves more than its analysis, and justify any deviations in physical dimensions of the slab from common practice on the basis of knowledge of the expected loads and the reliability of the calculated stresses and deformations of the structure.

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13.5.1.1 — Design of a slab system for gravity loads, including the slab and beams (if any) between supports and supporting columns or walls forming orthogonal frames, by either the Direct Design Method of 13.6 or the Equivalent Frame Method of 13.7 shall be permitted.

13.5.1.2 — For lateral loads, analysis of frames shall take into account effects of cracking and reinforcement on stiffness of frame members.

13.5.1.3 — Combining the results of the gravity load analysis with the results of the lateral load analysis shall be permitted.

13.5.2 — The slab and beams (if any) between supports shall be proportioned for factored moments prevailing at every section.

COMMENTARY

R13.5.1.1 — For gravity load analysis of two-way slab systems, two analysis methods are given in 13.6 and 13.7. The specific provisions of both design methods are limited in application to orthogonal frames subject to gravity loads only. Both methods apply to two-way slabs with beams as well as to flat slabs and flat plates. In both methods, the distribution of moments to the critical sections of the slab reflects the effects of reduced stiffness of elements due to cracking and support geometry.

R13.5.1.2 — During the life of a structure, construction loads, ordinary occupancy loads, anticipated overloads, and volume changes will cause cracking of slabs. Cracking reduces stiffness of slab members, and increases lateral flexibility when lateral loads act on the structure. Cracking of slabs should be considered in stiffness assumptions so that drift caused by wind or earthquake is not grossly underestimated.

The designer may model the structure for lateral load analysis using any approach that is shown to satisfy equilibrium and geometric compatibility and to be in reasonable agreement with test data.[13.10,13.11] The selected approach should recognize effects of cracking as well as parameters such as ℓ_2/ℓ_1, c_1/ℓ_1, and c_2/c_1. Some of the available approaches are summarized in Reference 13.12, which includes a discussion on the effects of cracking. Acceptable approaches include plate-bending finite-element models, the effective beam width model, and the equivalent frame model. In all cases, framing member stiffnesses should be reduced to account for cracking.

For nonprestressed slabs, it is normally appropriate to reduce slab bending stiffness to between one-half and one-quarter of the uncracked stiffness. For prestressed construction, stiffnesses greater than those of cracked, nonprestressed slabs may be appropriate. When the analysis is used to determine design drifts or moment magnification, lower-bound slab stiffnesses should be assumed. When the analysis is used to study interactions of the slab with other framing elements, such as structural walls, it may be appropriate to consider a range of slab stiffnesses so that the relative importance of the slab on those interactions can be assessed.

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13.5.3 — When gravity load, wind, earthquake, or other lateral forces cause transfer of moment between slab and column, a fraction of the unbalanced moment shall be transferred by flexure in accordance with 13.5.3.2 and 13.5.3.3.

13.5.3.1 — The fraction of unbalanced moment not transferred by flexure shall be transferred by eccentricity of shear in accordance with 11.12.6.

13.5.3.2 — A fraction of the unbalanced moment given by $\gamma_f M_u$ shall be considered to be transferred by flexure within an effective slab width between lines that are one and one-half slab or drop panel thicknesses **(1.5*h*)** outside opposite faces of the column or capital, where M_u is the moment to be transferred and

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} \qquad (13\text{-}1)$$

13.5.3.3 — For unbalanced moments about an axis parallel to the edge at exterior supports, the value of γ_f by Eq. (13-1) shall be permitted to be increased up to 1.0 provided that V_u at an edge support does not exceed $0.75\phi V_c$ or at a corner support does not exceed $0.5\phi V_c$. For unbalanced moments at interior supports, and for unbalanced moments about an axis transverse to the edge at exterior supports, the value of γ_f in Eq. (13-1) shall be permitted to be increased by up to 25 percent provided that V_u at the support does not exceed $0.4\phi V_c$. The reinforcement ratio ρ, within the effective slab width defined in 13.5.3.2, shall not exceed $0.375\rho_b$. No adjustments to γ_f shall be permitted for prestressed slab systems.

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R13.5.3 — This section is concerned primarily with slab systems without beams. Tests and experience have shown that, unless special measures are taken to resist the torsional and shear stresses, all reinforcement resisting that part of the moment to be transferred to the column by flexure should be placed between lines that are one and one-half the slab or drop panel thickness, **1.5*h***, on each side of the column. The calculated shear stresses in the slab around the column are required to conform to the requirements of 11.12.2. See R11.12.1.2 and R11.12.2.1 for more details on application of this section.

R13.5.3.3 — The 1989 code procedures remain unchanged, except that under certain conditions the designer is permitted to adjust the level of moment transferred by shear without revising member sizes. Tests indicate that some flexibility in distribution of unbalanced moments transferred by shear and flexure at both exterior and interior supports is possible. Interior, exterior, and corner supports refer to slab-column connections for which the critical perimeter for rectangular columns has 4, 3, or 2 sides, respectively. Changes in the 1995 code recognized, to some extent, design practices prior to the 1971 code.[13.13]

At exterior supports, for unbalanced moments about an axis parallel to the edge, the portion of moment transferred by eccentricity of shear $\gamma_v M_u$ may be reduced provided that the factored shear at the support (excluding the shear produced by moment transfer) does not exceed 75 percent of the shear capacity ϕV_c as defined in 11.12.2.1 for edge columns or 50 percent for corner columns. Tests[13.14,13.15] indicate that there is no significant interaction between shear and unbalanced moment at the exterior support in such cases. Note that as $\gamma_v M_u$ is decreased, $\gamma_f M_u$ is increased.

Evaluation of tests of interior supports indicate that some flexibility in distributing unbalanced moments transferred by shear and flexure is possible, but with more severe limitations than for exterior supports. For interior supports, the unbalanced moment transferred by flexure is permitted to be increased up to 25 percent provided that the factored shear (excluding the shear caused by the moment transfer) at the interior supports does not exceed 40 percent of the shear capacity ϕV_c as defined in 11.12.2.1.

Tests of slab-column connections indicate that a large degree of ductility is required because the interaction between shear and unbalanced moment is critical. When the factored shear is large, the column-slab joint cannot always develop all of the reinforcement provided in the effective width. The modifications for edge, corner, or interior slab-

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column connections in 13.5.3.3 are permitted only when the reinforcement ratio (within the effective width) required to develop the unbalanced moment $\gamma_f M_u$ does not exceed $0.375\rho_b$. The use of Eq. (13-1) without the modification permitted in 13.5.3.3 will generally indicate overstress conditions on the joint. The provisions of 13.5.3.3 are intended to improve ductile behavior of the column-slab joint. When a reversal of moments occurs at opposite faces of an interior support, both top and bottom reinforcement should be concentrated within the effective width. A ratio of top to bottom reinforcement of about 2 has been observed to be appropriate.

13.5.3.4 — Concentration of reinforcement over the column by closer spacing or additional reinforcement shall be used to resist moment on the effective slab width defined in 13.5.3.2.

13.5.4 — Design for transfer of load from slabs to supporting columns or walls through shear and torsion shall be in accordance with Chapter 11.

13.6 — Direct design method

R13.6 — Direct design method

The direct design method consists of a set of rules for distributing moments to slab and beam sections to satisfy safety requirements and most serviceability requirements simultaneously. Three fundamental steps are involved as follows:

(1) Determination of the total factored static moment (see 13.6.2);

(2) Distribution of the total factored static moment to negative and positive sections (see 13.6.3);

(3) Distribution of the negative and positive factored moments to the column and middle strips and to the beams, if any (see 13.6.4 through 13.6.6). The distribution of moments to column and middle strips is also used in the equivalent frame method (see 13.7).

13.6.1 — Limitations

R13.6.1 — Limitations

Design of slab systems within the limitations of 13.6.1.1 through 13.6.1.8 by the direct design method shall be permitted.

The direct design method was developed from considerations of theoretical procedures for the determination of moments in slabs with and without beams, requirements for simple design and construction procedures, and precedents supplied by performance of slab systems. Consequently, the slab systems to be designed using the direct design method should conform to the limitations in this section.

13.6.1.1 — There shall be a minimum of three continuous spans in each direction.

R13.6.1.1 — The primary reason for the limitation in this section is the magnitude of the negative moments at the interior support in a structure with only two continuous spans. The rules given for the direct design method assume that the slab system at the first interior negative moment section is neither fixed against rotation nor discontinuous.

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13.6.1.2 — Panels shall be rectangular, with a ratio of longer to shorter span center-to-center of supports within a panel not greater than 2.

13.6.1.3 — Successive span lengths center-to-center of supports in each direction shall not differ by more than one-third the longer span.

13.6.1.4 — Offset of columns by a maximum of 10 percent of the span (in direction of offset) from either axis between centerlines of successive columns shall be permitted.

13.6.1.5 — All loads shall be due to gravity only and uniformly distributed over an entire panel. Live load shall not exceed two times dead load.

13.6.1.6 — For a panel with beams between supports on all sides, the relative stiffness of beams in two perpendicular directions

$$\frac{a_1 \ell_2^2}{a_2 \ell_1^2} \qquad (13\text{-}2)$$

shall not be less than 0.2 nor greater than 5.0.

13.6.1.7 — Moment redistribution as permitted by 8.4 shall not be applied for slab systems designed by the Direct Design Method. See 13.6.7.

13.6.1.8 — Variations from the limitations of 13.6.1 shall be permitted if demonstrated by analysis that requirements of 13.5.1 are satisfied.

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R13.6.1.2 — If the ratio of the two spans (long span/short span) of a panel exceeds two, the slab resists the moment in the shorter span essentially as a one-way slab.

R13.6.1.3 — The limitation in this section is related to the possibility of developing negative moments beyond the point where negative moment reinforcement is terminated, as prescribed in Fig. 13.3.8.

R13.6.1.4 — Columns can be offset within specified limits from a regular rectangular array. A cumulative total offset of 20 percent of the span is established as the upper limit.

R13.6.1.5 — The direct design method is based on tests[13.16] for uniform gravity loads and resulting column reactions determined by statics. Lateral loads such as wind or seismic require a frame analysis. Inverted foundation mats designed as two-way slabs (see 15.10) involve application of known column loads. Therefore, even where the soil reaction is assumed to be uniform, a frame analysis should be performed.

In the 1995 code, the limit of applicability of the direct design method for ratios of live load to dead load was reduced from 3 to 2. In most slab systems, the live to dead load ratio will be less than 2 and it will not be necessary to check the effects of pattern loading.

R13.6.1.6 — The elastic distribution of moments will deviate significantly from those assumed in the direct design method unless the requirements for stiffness are satisfied.

R13.6.1.7 — Moment redistribution as permitted by 8.4 is not intended for use where approximate values for bending moments are used. For the direct design method, 10 percent modification is allowed by 13.6.7.

R13.6.1.8 — The designer is permitted to use the direct design method even if the structure does not fit the limitations in this section, provided it can be shown by analysis that the particular limitation does not apply to that structure. For a slab system carrying a nonmovable load (such as a water reservoir in which the load on all panels is expected to be the same), the designer need not satisfy the live load limitation of 13.6.1.5.

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13.6.2 — Total factored static moment for a span

13.6.2.1 — Total factored static moment for a span shall be determined in a strip bounded laterally by centerline of panel on each side of centerline of supports.

13.6.2.2 — Absolute sum of positive and average negative factored moments in each direction shall not be less than

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} \quad (13\text{-}3)$$

13.6.2.3 — Where the transverse span of panels on either side of the centerline of supports varies, ℓ_2 in Eq. (13-3) shall be taken as the average of adjacent transverse spans.

13.6.2.4 — When the span adjacent and parallel to an edge is being considered, the distance from edge to panel centerline shall be substituted for ℓ_2 in Eq. (13-3).

13.6.2.5 — Clear span ℓ_n shall extend from face to face of columns, capitals, brackets, or walls. Value of ℓ_n used in Eq. (13-3) shall not be less than $0.65\ell_1$. Circular or regular polygon shaped supports shall be treated as square supports with the same area.

13.6.3 — Negative and positive factored moments

13.6.3.1 — Negative factored moments shall be located at face of rectangular supports. Circular or regular polygon shaped supports shall be treated as square supports with the same area.

13.6.3.2 — In an interior span, total static moment M_o shall be distributed as follows:

Negative factored moment	0.65
Positive factored moment	0.35

COMMENTARY

R13.6.2 — Total factored static moment for a span

R13.6.2.2 — Eq. (13-3) follows directly from Nichol's derivation[13.17] with the simplifying assumption that the reactions are concentrated along the faces of the support perpendicular to the span considered. In general, the designer will find it expedient to calculate static moments for two adjacent half panels that include a column strip with a half middle strip along each side.

R13.6.2.5 — If a supporting member does not have a rectangular cross section or if the sides of the rectangle are not parallel to the spans, it is to be treated as a square support having the same area, as illustrated in Fig. R13.6.2.5.

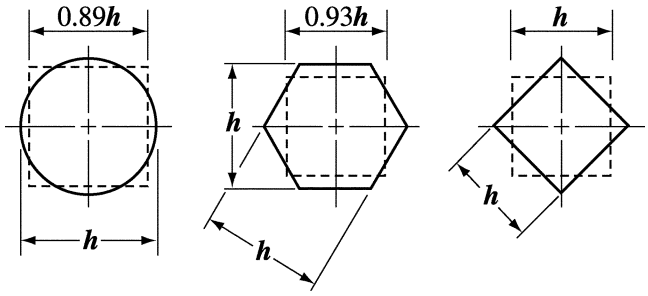


Fig. R13.6.2.5—Examples of equivalent square section for supporting members

R13.6.3 — Negative and positive factored moments

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13.6.3.3 — In an end span, total factored static moment M_o shall be distributed as follows:

	(1)	(2)	(3)	(4)	(5)
	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0	0.16	0.26	0.30	0.65

13.6.3.4 — Negative moment sections shall be designed to resist the larger of the two interior negative factored moments determined for spans framing into a common support unless an analysis is made to distribute the unbalanced moment in accordance with stiffnesses of adjoining elements.

13.6.3.5 — Edge beams or edges of slab shall be proportioned to resist in torsion their share of exterior negative factored moments.

13.6.3.6 — The gravity load moment to be transferred between slab and edge column in accordance with 13.5.3.1 shall be $0.3M_o$.

13.6.4 — Factored moments in column strips

13.6.4.1 — Column strips shall be proportioned to resist the following portions in percent of interior negative factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_1\ell_2/\ell_1) \geq 1.0$	90	75	45

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R13.6.3.3 — The moment coefficients for an end span are based on the equivalent column stiffness expressions from References 13.18, 13.19, and 13.20. The coefficients for an unrestrained edge would be used, for example, if the slab were simply supported on a masonry or concrete wall. Those for a fully restrained edge would apply if the slab were constructed integrally with a concrete wall having a flexural stiffness so large compared to that of the slab that little rotation occurs at the slab-to-wall connection.

For other than unrestrained or fully restrained edges, coefficients in the table were selected to be near the upper bound of the range for positive moments and interior negative moments. As a result, exterior negative moments were usually closer to a lower bound. The exterior negative moment capacity for most slab systems is governed by minimum reinforcement to control cracking. The final coefficients in the table have been adjusted so that the absolute sum of the positive and average moments equal M_o.

For two-way slab systems with beams between supports on all sides (two-way slabs), moment coefficients of column (2) of the table apply. For slab systems without beams between interior supports (flat plates and flat slabs), the moment coefficients of column (3) or (4) apply, without or with an edge (spandrel) beam, respectively.

In the 1977 code, distribution factors defined as a function of the stiffness ratio of the equivalent exterior support were used for proportioning the total static moment M_o in an end span. The approach may be used in place of values in 13.6.3.3.

R13.6.3.4 — The differences in slab moment on either side of a column or other type of support should be accounted for in the design of the support. If an analysis is made to distribute unbalanced moments, flexural stiffness may be obtained on the basis of the gross concrete section of the members involved.

R13.6.3.5 — Moments perpendicular to, and at the edge of, the slab structure should be transmitted to the supporting columns or walls. Torsional stresses caused by the moment assigned to the slab should be investigated.

R13.6.4, R13.6.5, and R13.6.6 — Factored moments in column strips, beams, and middle strips

The rules given for assigning moments to the column strips, beams, and middle strips are based on studies[13.21] of moments in linearly elastic slabs with different beam stiffness tempered by the moment coefficients that have been used successfully.

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Linear interpolations shall be made between values shown.

13.6.4.2 — Column strips shall be proportioned to resist the following portions in percent of exterior negative factored moments:

ℓ_2/ℓ_1		0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$(\alpha_1\ell_2/\ell_1) \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45

Linear interpolations shall be made between values shown.

13.6.4.3 — Where supports consist of columns or walls extending for a distance equal to or greater than three-quarters the span length ℓ_2 used to compute M_o, negative moments shall be considered to be uniformly distributed across ℓ_2.

13.6.4.4 — Column strips shall be proportioned to resist the following portions in percent of positive factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_1\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

13.6.4.5 — For slabs with beams between supports, the slab portion of column strips shall be proportioned to resist that portion of column strip moments not resisted by beams.

13.6.5 — Factored moments in beams

13.6.5.1 — Beams between supports shall be proportioned to resist 85 percent of column strip moments if $\alpha_1\ell_2/\ell_1$ is equal to or greater than 1.0.

13.6.5.2 — For values of $\alpha_1\ell_2/\ell_1$ between 1.0 and zero, proportion of column strip moments resisted by beams shall be obtained by linear interpolation between 85 and zero percent.

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For the purpose of establishing moments in the half column strip adjacent to an edge supported by a wall, ℓ_n in Eq. (13-3) may be assumed equal to ℓ_n of the parallel adjacent column to column span, and the wall may be considered as a beam having a moment of inertia I_b equal to infinity.

R13.6.4.2 — The effect of the torsional stiffness parameter β_t is to assign all of the exterior negative factored moment to the column strip, and none to the middle strip, unless the beam torsional stiffness is high relative to the flexural stiffness of the supported slab. In the definition of β_t, the shear modulus has been taken as $E_{cb}/2$.

Where walls are used as supports along column lines, they can be regarded as very stiff beams with an $\alpha_1\ell_2/\ell_1$, value greater than one. Where the exterior support consists of a wall perpendicular to the direction in which moments are being determined, β_t may be taken as zero if the wall is of masonry without torsional resistance, and β_t may be taken as 2.5 for a concrete wall with great torsional resistance that is monolithic with the slab.

R13.6.5 — Factored moments in beams

Loads assigned directly to beams are in addition to the uniform dead load of the slab; uniform superimposed dead loads, such as the ceiling, floor finish, or assumed equivalent partition loads; and uniform live loads. All of these loads are normally included with w_u in Eq. (13-3). Linear loads applied directly to beams include partition walls over or along beam centerlines and additional dead load of the projecting beam stem. Concentrated loads include posts above or hangers below the beams. For the purpose of assigning directly applied loads, only loads located within the width of the beam stem should be considered as directly

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13.6.5.3 — In addition to moments calculated for uniform loads according to 13.6.2.2, 13.6.5.1, and 13.6.5.2, beams shall be proportioned to resist all moments caused by concentrated or linear loads applied directly to beams, including weight of projecting beam stem above or below the slab.

13.6.6 — Factored moments in middle strips

13.6.6.1 — That portion of negative and positive factored moments not resisted by column strips shall be proportionately assigned to corresponding half middle strips.

13.6.6.2 — Each middle strip shall be proportioned to resist the sum of the moments assigned to its two half middle strips.

13.6.6.3 — A middle strip adjacent to and parallel with a wall-supported edge shall be proportioned to resist twice the moment assigned to the half middle strip corresponding to the first row of interior supports.

13.6.7 — Modification of factored moments

Modification of negative and positive factored moments by 10 percent shall be permitted provided the total static moment for a panel in the direction considered is not less than that required by Eq. (13-3).

13.6.8 — Factored shear in slab systems with beams

13.6.8.1 — Beams with $\alpha_1\ell_2/\ell_1$ equal to or greater than 1.0 shall be proportioned to resist shear caused by factored loads on tributary areas which are bounded by 45 deg lines drawn from the corners of the panels and the centerlines of the adjacent panels parallel to the long sides.

13.6.8.2 — In proportioning of beams with $\alpha_1\ell_2/\ell_1$ less than 1.0 to resist shear, linear interpolation, assuming beams carry no load at $\alpha_1 = 0$, shall be permitted.

13.6.8.3 — In addition to shears calculated according to 13.6.8.1 and 13.6.8.2, beams shall be proportioned to resist shears caused by factored loads applied directly on beams.

13.6.8.4 — Computation of slab shear strength on the assumption that load is distributed to supporting beams in accordance with 13.6.8.1 or 13.6.8.2 shall be permitted. Resistance to total shear occurring on a panel shall be provided.

13.6.8.5 — Shear strength shall satisfy the requirements of Chapter 11.

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applied to the beams. (The effective width of a beam as defined in 13.2.4 is solely for strength and relative stiffness calculations.) Line loads and concentrated loads located on the slab away from the beam stem require special consideration to determine their apportionment to slab and beams.

R13.6.8 — Factored shear in slab systems with beams

The tributary area for computing shear on an interior beam is shown shaded in Fig. R13.6.8. If the stiffness for the beam $\alpha_1\ell_2/\ell_1$ is less than 1.0, the shear on the beam may be obtained by linear interpolation. In such cases, the beams framing into the column will not account for all of the shear force applied on the column. The remaining shear force will produce shear stresses in the slab around the column that should be checked in the same manner as for flat slabs, as required by 13.6.8.4. Sections 13.6.8.1 through 13.6.8.3 do not apply to the calculation of torsional moments on the

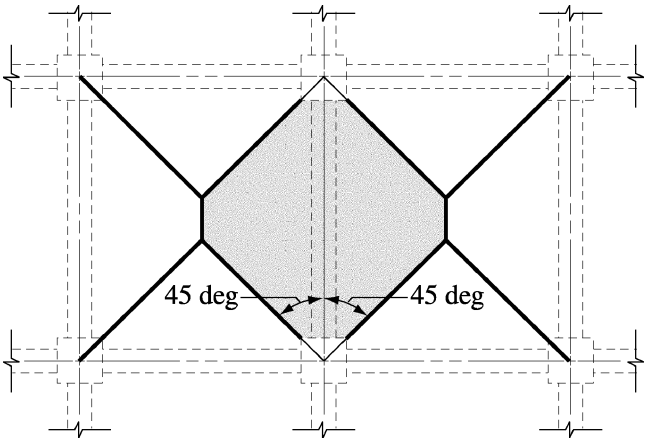


Fig. R13.6.8—Tributary area for shear on an interior beam

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13.6.9 — Factored moments in columns and walls

13.6.9.1 — Columns and walls built integrally with a slab system shall resist moments caused by factored loads on the slab system.

13.6.9.2 — At an interior support, supporting elements above and below the slab shall resist the moment specified by Eq. (13-4) in direct proportion to their stiffnesses unless a general analysis is made.

$$M = 0.07[(w_d + 0.5w_\ell)\ell_2 \ell_n^2 - w_d\ell\ell_2\ell(\ell_n\ell)^2] \quad (13\text{-}4)$$

where $w_d\ell$, $\ell_2\ell$, and $\ell_n\ell$ refer to shorter span.

13.7 — Equivalent frame method

13.7.1 — Design of slab systems by the equivalent frame method shall be based on assumptions given in 13.7.2 through 13.7.6, and all sections of slabs and supporting members shall be proportioned for moments and shears thus obtained.

13.7.1.1 — Where metal column capitals are used, it shall be permitted to take account of their contributions to stiffness and resistance to moment and to shear.

13.7.1.2 — It shall be permitted to neglect the change in length of columns and slabs due to direct stress, and deflections due to shear.

13.7.2 — Equivalent frame

13.7.2.1 — The structure shall be considered to be made up of equivalent frames on column lines taken longitudinally and transversely through the building.

13.7.2.2 — Each frame shall consist of a row of columns or supports and slab-beam strips, bounded laterally by the centerline of panel on each side of the centerline of columns or supports.

13.7.2.3 — Columns or supports shall be assumed to be attached to slab-beam strips by torsional members (see 13.7.5) transverse to the direction of the span for which moments are being determined and extending to bounding lateral panel centerlines on each side of a column.

COMMENTARY

beams. These moments should be based on the calculated flexural moments acting on the sides of the beam.

R13.6.9 — Factored moments in columns and walls

Eq. (13-4) refers to two adjoining spans, with one span longer than the other, and with full dead load plus one-half live load applied on the longer span and only dead load applied on the shorter span.

Design and detailing of the reinforcement transferring the moment from the slab to the edge column is critical to both the performance and the safety of flat slabs or flat plates without edge beams or cantilever slabs. It is important that complete design details be shown on design drawings, such as concentration of reinforcement over the column by closer spacing or additional reinforcement.

R13.7 — Equivalent frame method

The equivalent frame method involves the representation of the three-dimensional slab system by a series of two-dimensional frames that are then analyzed for loads acting in the plane of the frames. The negative and positive moments so determined at the critical design sections of the frame are distributed to the slab sections in accordance with 13.6.4 (column strips), 13.6.5 (beams), and 13.6.6 (middle strips). The equivalent frame method is based on studies reported in References 13.18, 13.19, and 13.20. Many of the details of the equivalent frame method given in the Commentary in the 1989 code were removed in the 1995 code.

R13.7.2 — Equivalent frame

Application of the equivalent frame to a regular structure is illustrated in Fig. R13.7.2. The three-dimensional building is divided into a series of two-dimensional frame bents (equivalent frames) centered on column or support centerlines with each frame extending the full height of the building. The width of each equivalent frame is bounded by the centerlines of the adjacent panels. The complete analysis of a slab system for a building consists of analyzing a series of equivalent (interior and exterior) frames spanning longitudinally and transversely through the building.

The equivalent frame comprises three parts: (1) the horizontal slab strip, including any beams spanning in the direction of the frame, (2) the columns or other vertical supporting members, extending above and below the slab, and (3) the elements of the structure that provide moment transfer between the horizontal and vertical members.

CODE

13.7.2.4 — Frames adjacent and parallel to an edge shall be bounded by that edge and the centerline of adjacent panel.

13.7.2.5 — Analysis of each equivalent frame in its entirety shall be permitted. Alternatively, for gravity loading, a separate analysis of each floor or roof with far ends of columns considered fixed shall be permitted.

13.7.2.6 — Where slab-beams are analyzed separately, determination of moment at a given support assuming that the slab-beam is fixed at any support two panels distant therefrom, shall be permitted, provided the slab continues beyond that point.

COMMENTARY

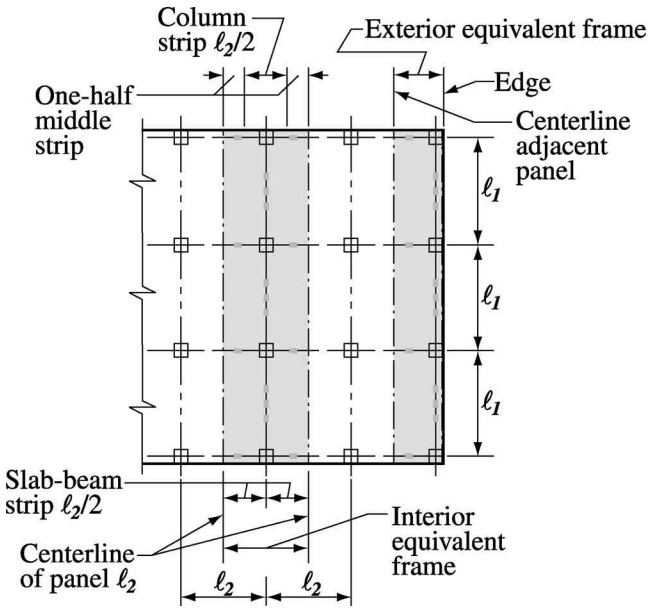


Fig. R13.7.2—Definitions of equivalent frame

13.7.3 — Slab-beams

R13.7.3 — Slab-beams

13.7.3.1—Determination of the moment of inertia of slab-beams at any cross section outside of joints or column capitals using the gross area of concrete shall be permitted.

13.7.3.2 — Variation in moment of inertia along axis of slab-beams shall be taken into account.

13.7.3.3 — Moment of inertia of slab-beams from center of column to face of column, bracket, or capital shall be assumed equal to the moment of inertia of the slab-beam at face of column, bracket, or capital divided by the quantity $(1 - c_2/\ell_2)^2$, where c_2 and ℓ_2 are measured transverse to the direction of the span for which moments are being determined.

R13.7.3.3 — A support is defined as a column, capital, bracket, or wall. A beam is not considered to be a support member for the equivalent frame.

13.7.4 — Columns

R13.7.4 — Columns

13.7.4.1 — Determination of the moment of inertia of columns at any cross section outside of joints or column capitals using the gross area of concrete shall be permitted.

13.7.4.2 — Variation in moment of inertia along axis of columns shall be taken into account.

13.7.4.3 — Moment of inertia of columns from top to bottom of the slab-beam at a joint shall be assumed to be infinite.

Column stiffness is based on the length of the column from mid-depth of slab above to mid-depth of slab below. Column moment of inertia is computed on the basis of its cross section, taking into account the increase in stiffness provided by the capital, if any.

When slab-beams are analyzed separately for gravity loads, the concept of an equivalent column, combining the stiffness of the slab-beam and torsional member into a composite element, is used. The column flexibility is modified to account for the torsional flexibility of the slab-to-column connection that reduces its efficiency for transmission of

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COMMENTARY

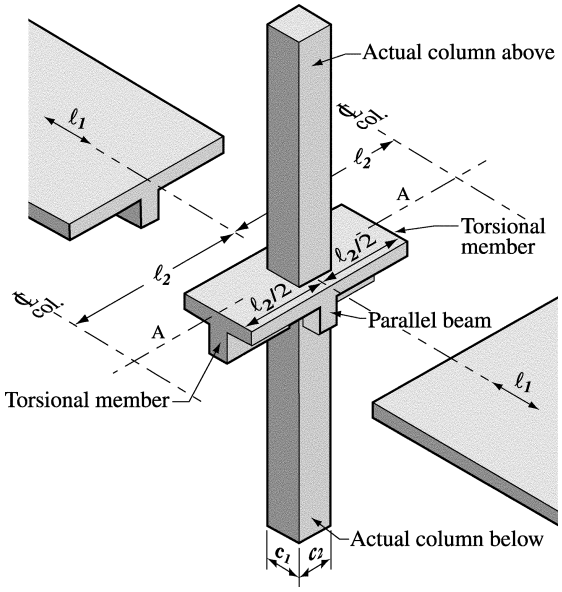


Fig. R13.7.4—Equivalent column (column plus torsional members)

moments. The equivalent column consists of the actual columns above and below the slab-beam, plus attached torsional members on each side of the columns extending to the centerline of the adjacent panels as shown in Fig. R13.7.4.

13.7.5 — Torsional members

13.7.5.1 — Torsional members (see 13.7.2.3) shall be assumed to have a constant cross section throughout their length consisting of the largest of (a), (b), and (c):

(a) A portion of slab having a width equal to that of the column, bracket, or capital in the direction of the span for which moments are being determined;

(b) For monolithic or fully composite construction, the portion of slab specified in (a) plus that part of the transverse beam above and below the slab;

(c) The transverse beam as defined in 13.2.4.

13.7.5.2 — Where beams frame into columns in the direction of the span for which moments are being determined, the torsional stiffness shall be multiplied by the ratio of the moment of inertia of the slab with such a beam to the moment of inertia of the slab without such a beam.

R13.7.5 — Torsional members

Computation of the stiffness of the torsional member requires several simplifying assumptions. If no transverse-beam frames into the column, a portion of the slab equal to the width of the column or capital is assumed to be the torsional member. If a beam frames into the column, T-beam or L-beam action is assumed, with the flanges extending on each side of the beam a distance equal to the projection of the beam above or below the slab but not greater than four times the thickness of the slab. Furthermore, it is assumed that no torsional rotation occurs in the beam over the width of the support.

The member sections to be used for calculating the torsional stiffness are defined in 13.7.5.1. In the 1989 code, Eq. (13-6) specified the stiffness coefficient K_t of the torsional members. The approximate expression for K_t has been moved to the commentary and the expression for the torsional constant (Eq. (13-7) in the 1989 code) is now defined in 13.0.

Studies of three-dimensional analyses of various slab configurations suggest that a reasonable value of the torsional stiffness can be obtained by assuming a moment distribution along the torsional member that varies linearly from a maximum at the center of the column to zero at the middle of the panel. The assumed distribution of unit twisting moment along the column centerline is shown in Fig. R13.7.5.

An approximate expression for the stiffness of the torsional member, based on the results of three-dimensional analyses

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COMMENTARY

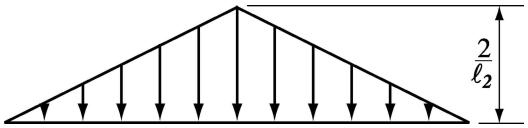


Fig. R13.7.5—Distribution of unit twisting moment along column centerline AA shown in Fig. R13.7.4

of various slab configurations (References 13.18, 13.19, and 13.20) is given below as

$$K_t = S\frac{9E_{cs}C}{\ell_2\left(1-\frac{c_2}{\ell_2}\right)^3}$$

where an expression for C is given in 13.0.

13.7.6 — Arrangement of live load

13.7.6.1 — When the loading pattern is known, the equivalent frame shall be analyzed for that load.

13.7.6.2 — When live load is variable but does not exceed three-quarters of the dead load, or the nature of live load is such that all panels will be loaded simultaneously, it shall be permitted to assume that maximum factored moments occur at all sections with full factored live load on entire slab system.

13.7.6.3 — For loading conditions other than those defined in 13.7.6.2, it shall be permitted to assume that maximum positive factored moment near midspan of a panel occurs with three-quarters of the full factored live load on the panel and on alternate panels; and it shall be permitted to assume that maximum negative factored moment in the slab at a support occurs with three-quarters of the full live load on adjacent panels only.

13.7.6.4 — Factored moments shall be taken not less than those occurring with full factored live load on all panels.

R13.7.6 — Arrangement of live load

The use of only three-quarters of the full factored live load for maximum moment loading patterns is based on the fact that maximum negative and maximum positive live load moments cannot occur simultaneously and that redistribution of maximum moments is thus possible before failure occurs. This procedure, in effect, permits some local overstress under the full factored live load if it is distributed in the prescribed manner, but still ensures that the ultimate capacity of the slab system after redistribution of moment is not less than that required to carry the full factored dead and live loads on all panels.

13.7.7 — Factored moments

13.7.7.1 — At interior supports, the critical section for negative factored moment (in both column and middle strips) shall be taken at face of rectilinear supports, but not farther away than $0.175\ell_1$ from the center of a column.

13.7.7.2 — At exterior supports with brackets or capitals, the critical section for negative factored moment in the span perpendicular to an edge shall be taken at a distance from face of supporting element not greater than one-half the projection of bracket or capital beyond face of supporting element.

R13.7.7 — Factored moments

R13.7.7.1-R13.7.7.3 — These code sections adjust the negative factored moments to the face of the supports. The adjustment is modified at an exterior support to limit reductions in the exterior negative moment. Fig. R13.6.2.5 illustrates several equivalent rectangular supports for use in establishing faces of supports for design with nonrectangular supports.

CODE

13.7.7.3 — Circular or regular polygon shaped supports shall be treated as square supports with the same area for location of critical section for negative design moment.

13.7.7.4 — When slab systems within limitations of 13.6.1 are analyzed by the equivalent frame method, it shall be permitted to reduce the resulting computed moments in such proportion that the absolute sum of the positive and average negative moments used in design need not exceed the value obtained from Eq. (13-3).

13.7.7.5 — Distribution of moments at critical sections across the slab-beam strip of each frame to column strips, beams, and middle strips as provided in 13.6.4, 13.6.5, and 13.6.6 shall be permitted if the requirement of 13.6.1.6 is satisfied.

COMMENTARY

R13.7.7.4 — Previous codes have contained this section. It is based on the principle that if two different methods are prescribed to obtain a particular answer, the code should not require a value greater than the least acceptable value. Due to the long satisfactory experience with designs having total factored static moments not exceeding those given by Eq. (13-3), it is considered that these values are satisfactory for design when applicable limitations are met.

CHAPTER 14 — WALLS

CODE

14.0 — Notation

A_g = gross area of section, in.^{2}
A_s = area of longitudinal tension reinforcement in wall segment, in.^{2}
A_{se} = area of effective longitudinal tension reinforcement in wall segment, in.^{2}, as calculated by Eq. (14-8)
c = distance from extreme compression fiber to neutral axis, in.
d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in.
E_c = modulus of elasticity of concrete, psi
f_c' = specified compressive strength of concrete, psi
f_y = specified yield strength of nonprestressed reinforcement, psi
h = overall thickness of member, in.
I_{cr} = moment of inertia of cracked section transformed to concrete, in.^{4}
I_e = effective moment of inertia for computation of deflection, in.^{4}
k = effective length factor
ℓ_c = vertical distance between supports, in.
ℓ_w = horizontal length of wall, in.
M = maximum unfactored moment due to service loads, including $P\Delta$ effects, in.-lb
M_a = maximum moment in member at stage deflection is computed, in.-lb
M_{cr} = moment causing flexural cracking due to applied lateral and vertical loads, in.-lb
M_n = nominal moment strength at section, in.-lb
M_{sa} = maximum unfactored applied moment due to service loads, not including $P\Delta$ effects, in.-lb
M_u = factored moment at section including $P\Delta$ effects, in.-lb
M_{ua} = moment at the midheight section of the wall due to factored lateral and eccentric vertical loads, in.-lb
n = modular ratio of elasticity, but not less than 6
= E_s/E_c
P_{nw} = nominal axial load strength of wall designed by the empirical method, lb (see 14.5)
P_s = unfactored axial load at the design (midheight) section including effects of self-weight, lb
P_u = factored axial load, lb
D_s = maximum deflection at or near midheight due to service loads, in.
D_u = deflection at midheight of wall due to factored loads, in.

COMMENTARY

R14.0 — Notation

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

CODE

f = strength reduction factor. See 9.3
r = ratio of tension reinforcement
= $A_s/(\ell_w d)$
r_b = reinforcement ratio producing balanced strain conditions

14.1 — Scope

14.1.1 — Provisions of Chapter 14 shall apply for design of walls subjected to axial load, with or without flexure.

14.1.2 — Cantilever retaining walls are designed according to flexural design provisions of Chapter 10 with minimum horizontal reinforcement according to 14.3.3.

14.2 — General

14.2.1 — Walls shall be designed for eccentric loads and any lateral or other loads to which they are subjected.

14.2.2 — Walls subject to axial loads shall be designed in accordance with 14.2, 14.3, and either 14.4, 14.5, or 14.8.

14.2.3 — Design for shear shall be in accordance with 11.10.

14.2.4 — Unless otherwise demonstrated by an analysis, the horizontal length of wall considered as effective for each concentrated load shall not exceed center-to-center distance between loads, nor the bearing width plus four times the wall thickness.

14.2.5 — Compression members built integrally with walls shall conform to 10.8.2.

14.2.6 — Walls shall be anchored to intersecting elements, such as floors and roofs; or to columns, pilasters, buttresses, of intersecting walls; and to footings.

14.2.7 — Quantity of reinforcement and limits of thickness required by 14.3 and 14.5 shall be permitted to be waived where structural analysis shows adequate strength and stability.

14.2.8 — Transfer of force to footing at base of wall shall be in accordance with 15.8.

COMMENTARY

R14.1 — Scope

Chapter 14 applies generally to walls as vertical load carrying members. Cantilever retaining walls are designed according to the flexural design provisions of Chapter 10. Walls designed to resist shear forces, such as shearwalls, should be designed in accordance with Chapter 14 and 11.10 as applicable.

In the 1977 code, walls could be designed according to Chapter 14 or 10.15. In the 1983 code these two were combined in Chapter 14.

R14.2 — General

Walls should be designed to resist all loads to which they are subjected, including eccentric axial loads and lateral forces. Design is to be carried out in accordance with 14.4 unless the wall meets the requirements of 14.5.1.

CODE

14.3 — Minimum reinforcement

14.3.1 — Minimum vertical and horizontal reinforcement shall be in accordance with 14.3.2 and 14.3.3 unless a greater amount is required for shear by 11.10.8 and 11.10.9.

14.3.2 — Minimum ratio of vertical reinforcement area to gross concrete area shall be:

(a) 0.0012 for deformed bars not larger than No. 5 with a specified yield strength not less than 60,000 psi; or

(b) 0.0015 for other deformed bars; or

(c) 0.0012 for welded wire fabric (plain or deformed) not larger than W31 or D31.

14.3.3 — Minimum ratio of horizontal reinforcement area to gross concrete area shall be:

(a) 0.0020 for deformed bars not larger than No. 5 with a specified yield strength not less than 60,000 psi; or

(b) 0.0025 for other deformed bars; or

(c) 0.0020 for welded wire fabric (plain or deformed) not larger than W31 or D31.

14.3.4 — Walls more than 10 in. thick, except basement walls, shall have reinforcement for each direction placed in two layers parallel with faces of wall in accordance with the following:

(a) One layer consisting of not less than one-half and not more than two-thirds of total reinforcement required for each direction shall be placed not less than 2 in. nor more than one-third the thickness of wall from the exterior surface;

(b) The other layer, consisting of the balance of required reinforcement in that direction, shall be placed not less than 3/4 in. nor more than one-third the thickness of wall from the interior surface.

14.3.5 — Vertical and horizontal reinforcement shall not be spaced farther apart than three times the wall thickness, nor farther apart than 18 in.

14.3.6 — Vertical reinforcement need not be enclosed by lateral ties if vertical reinforcement area is not greater than 0.01 times gross concrete area, or where vertical reinforcement is not required as compression reinforcement.

14.3.7 — In addition to the minimum reinforcement required by 14.3.1, not less than two No. 5 bars shall be

COMMENTARY

R14.3 — Minimum reinforcement

The requirements of 14.3 are similar to those in previous codes. These apply to walls designed according to 14.4, 14.5, or 14.8. For walls resisting horizontal shear forces in the plane of the wall, reinforcement designed according to 11.10.9.2 and 11.10.9.4 may exceed the minimum reinforcement in 14.3.

CODE

provided around all window and door openings. Such bars shall be extended to develop the bar beyond the corners of the openings but not less than 24 in.

14.4 — Walls designed as compression members

Except as provided in 14.5, walls subject to axial load or combined flexure and axial load shall be designed as compression members in accordance with provisions of 10.2, 10.3, 10.10, 10.11, 10.12, 10.13, 10.14, 10.17, 14.2, and 14.3.

14.5 — Empirical design method

14.5.1 — Walls of solid rectangular cross section shall be permitted to be designed by the empirical provisions of 14.5 if the resultant of all factored loads is located within the middle third of the overall thickness of the wall and all limits of 14.2, 14.3, and 14.5 are satisfied.

14.5.2 — Design axial load strength ϕP_{nw} of a wall satisfying limitations of 14.5.1 shall be computed by Eq. (14-1) unless designed in accordance with 14.4.

$$\phi P_{nw} = 0.55 \phi f'_c A_g \left[1 - \left(\frac{k\ell_c}{32h}\right)^2\right] \qquad (14\text{-}1)$$

where $\phi = 0.70$ and effective length factor k shall be:

For walls braced top and bottom against lateral translation and

(a) Restrained against rotation at one or both ends (top, bottom, or both)..... 0.8

(b) Unrestrained against rotation at both ends.... 1.0

For walls not braced against lateral translation 2.0

COMMENTARY

R14.5 — Empirical design method

The empirical design method applies only to solid rectangular cross sections. All other shapes should be designed according to 14.4.

Eccentric loads and lateral forces are used to determine the total eccentricity of the factored axial load P_u. When the resultant load for all applicable load combinations falls within the middle third of the wall thickness (eccentricity not greater than $h/6$) at all sections along the length of the undeformed wall, the empirical design method may be used. The design is then carried out considering P_u as the concentric load. The factored axial load P_u should be less than or equal to the design axial load strength ϕP_{nw} computed by Eq. (14-1), $P_u \le \phi P_{nw}$.

With the 1980 code supplement, (Eq. 14-1) was revised to reflect the general range of end conditions encountered in wall designs. The wall strength equation in the 1977 code

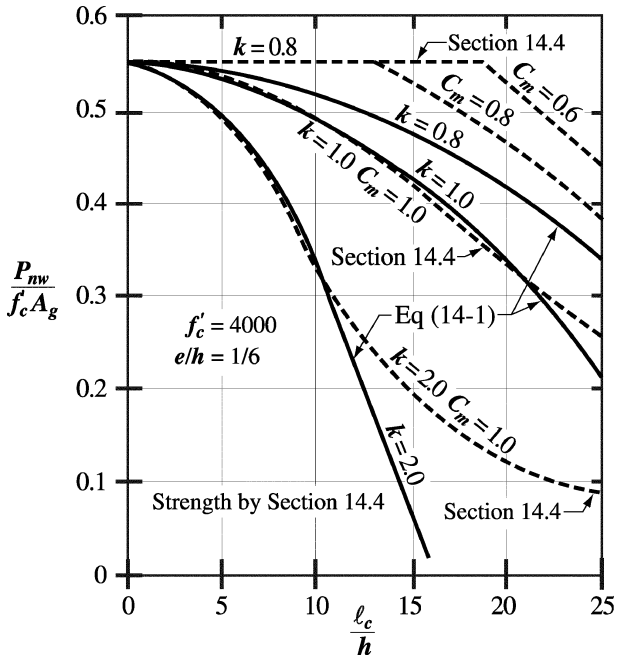


Fig. R14.5—Empirical design of walls, Eq. (14-1) versus 14.4

CODE

14.5.3 — Minimum thickness of walls designed by empirical design method

14.5.3.1 — Thickness of bearing walls shall not be less than 1/25 the supported height or length, whichever is shorter, nor less than 4 in.

14.5.3.2 — Thickness of exterior basement walls and foundation walls shall not be less than 7-1/2 in.

14.6 — Nonbearing walls

14.6.1 — Thickness of nonbearing walls shall not be less than 4 in., nor less than 1/30 the least distance between members that provide lateral support.

14.7 — Walls as grade beams

14.7.1 — Walls designed as grade beams shall have top and bottom reinforcement as required for moment in accordance with provisions of 10.2 through 10.7. Design for shear shall be in accordance with provisions of Chapter 11.

14.7.2 — Portions of grade beam walls exposed above grade shall also meet requirements of 14.3.

14.8 — Alternative design of slender walls

14.8.1 — When flexural tension controls the design of a wall, the requirements of 14.8 are considered to satisfy 10.10.

COMMENTARY

was based on the assumption of a wall with top and bottom fixed against lateral movement, and with moment restraint at one end corresponding to an effective length factor between 0.8 and 0.9. Axial load strength values determined from the original equation were unconservative when compared to test results[14.1] for walls with pinned conditions at both ends, as occurs with some precast and tilt-up applications, or when the top of the wall is not effectively braced against translation, as occurs with free-standing walls or in large structures where significant roof diaphragm deflections occur due to wind and seismic loads. Eq. (14-1) gives the same results as the 1977 code for walls braced against translation and with reasonable base restraint against rotation.[14.2] Values of effective vertical length factors k are given for commonly occurring wall end conditions. The end condition "restrained against rotation" required for a k-factor of 0.8 implies attachment to a member having flexural stiffness EI/ℓ at least as large as that of the wall.

The slenderness portion of Eq. (14-1) results in relatively comparable strengths by either 14.3 or 14.4 for members loaded at the middle third of the thickness with different braced and restrained end conditions. See Fig. R14.5.

R14.5.3 — Minimum thickness of walls designed by empirical design method

The minimum thickness requirements need not be applied to walls designed according to 14.4.

R14.8 — Alternative design of slender walls

Section 14.8 is based on the corresponding requirements in the Uniform Building Code (UBC)[14.3] and experimental research.[14.4]

CODE

14.8.2 — Walls designed by the provisions of 14.8 shall satisfy 14.8.2.1 through 14.8.2.6.

14.8.2.1 — The wall panel shall be designed as a simply supported, axially loaded member subjected to an out-of-plane uniform lateral load, with maximum moments and deflections occurring at midspan.

14.8.2.2 — The cross section shall be constant over the height of the panel.

14.8.2.3 — The reinforcement ratio ρ shall not exceed $0.6\rho_b$.

14.8.2.4 — Reinforcement shall provide a design strength

$$\phi M_n \geq M_{cr} \qquad (14\text{-}2)$$

where M_{cr} shall be obtained using the modulus of rupture given by Eq. (9-10).

14.8.2.5 — Concentrated gravity loads applied to the wall above the design flexural section shall be assumed to be distributed over a width:

(a) Equal to the bearing width, plus a width on each side that increases at a slope of 2 vertical to 1 horizontal down to the design section; but

(b) Not greater than the spacing of the concentrated loads; and

(c) Not extending beyond the edges of the wall panel.

14.8.2.6 — Vertical stress P_u/A_g at the midheight section shall not exceed $0.06f'_c$.

14.8.3 — The design moment strength ϕM_n for combined flexure and axial loads at the midheight cross section shall be

$$\phi M_n \geq M_u \qquad (14\text{-}3)$$

where:

$$M_u = M_{ua} + P_u D_u \qquad (14\text{-}4)$$

M_{ua} is the moment at the midheight section of the wall due to factored loads, and D_u is:

$$D_u = \frac{5M_u\ell_c^2}{(0.75)48E_cI_{cr}} \qquad (14\text{-}5)$$

M_u shall be obtained by iteration of deflections, or by direct calculation using Eq. (14-6).

COMMENTARY

The procedure is presented as an alternative to the requirements of 10.10 for the out-of-plane design of precast wall panels, where the panels are restrained against overturning at the top.

The procedure, as prescribed in the UBC,[14.3] has been converted from working stress to factored load design.

Panels that have windows or other large openings are not considered to have constant cross section over the height of the panel. Such walls are to be designed taking into account the effects of openings.

Many aspects of the design of tilt-up walls and buildings are discussed in References 14.5 and 14.6.

CODE

COMMENTARY

$$M_u = \frac{M_{ua}}{1 - \frac{5P_u\ell_c^2}{(0.75)48E_cI_{cr}}} \quad (14\text{-}6)$$

where:

$$I_{cr} = nA_{se}(d - c)^2 + \frac{\ell_w c^3}{3} \quad (14\text{-}7)$$

and

$$A_{se} = \frac{P_u + A_s f_y}{f_y} \quad (14\text{-}8)$$

14.8.4 — The maximum deflection D_s due to service loads, including $P\Delta$ effects, shall not exceed ℓ_c/**150**. The midheight deflection D_s shall be determined by:

$$D_s = \frac{(5M)\ell_c^2}{48E_cI_e} \quad (14\text{-}9)$$

$$M = \frac{M_{sa}}{1 - \frac{5P_s\ell_c^2}{48E_cI_e}} \quad (14\text{-}10)$$

I_e shall be calculated using the procedure of 9.5.2.3, substituting M for M_a. I_{cr} shall be evaluated using Eq. (14-7).

CODE

COMMENTARY

Notes

CHAPTER 15 — FOOTINGS

CODE

15.0 — Notation

A_g = gross area of section, in.^{2}
d_p = diameter of pile at footing base, in.
b = ratio of long side to short side of footing

15.1 — Scope

15.1.1 — Provisions of Chapter 15 shall apply for design of isolated footings and, where applicable, to combined footings and mats.

15.1.2 — Additional requirements for design of combined footings and mats are given in 15.10.

15.2 — Loads and reactions

15.2.1 — Footings shall be proportioned to resist the factored loads and induced reactions, in accordance with the appropriate design requirements of this code and as provided in Chapter 15.

15.2.2 — Base area of footing or number and arrangement of piles shall be determined from unfactored forces and moments transmitted by footing to soil or piles and permissible soil pressure or permissible pile capacity determined through principles of soil mechanics.

15.2.3 — For footings on piles, computations for moments and shears shall be permitted to be based on the assumption that the reaction from any pile is concentrated at pile center.

COMMENTARY

R15.0 — Notation

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

R15.1 — Scope

While the provisions of Chapter 15 apply to isolated footings supporting a single column or wall, most of the provisions are generally applicable to combined footings and mats supporting several columns or walls or a combination thereof.[15.1,15.2]

R15.2 — Loads and reactions

Footings are required to be proportioned to sustain the applied factored loads and induced reactions which include axial loads, moments, and shears that have to be resisted at the base of the footing or pile cap.

After the permissible soil pressure or the permissible pile capacity has been determined by principles of soil mechanics and in accord with the general building code, the size of the base area of a footing on soil or the number and arrangement of the piles should be established on the basis of unfactored (service) loads such as D, L, W, and E in whatever combination that governs the design.

Only the computed end moments that exist at the base of a column (or pedestal) need to be transferred to the footing; the minimum moment requirement for slenderness considerations given in 10.12.3.2 need not be considered for transfer of forces and moments to footings.

In cases in which eccentric loads or moments are to be considered, the extreme soil pressure or pile reaction obtained from this loading should be within the permissible values. Similarly, the resultant reactions due to service loads combined with moments, shears, or both, caused by wind or earthquake loads should not exceed the increased values that may be permitted by the general building code.

To proportion a footing or pile cap for strength, the contact soil pressure or pile reaction due to the applied factored loading (see 8.1.1) should be determined. For a single concentrically loaded spread footing, the soil reaction q_s due to the factored loading is $q_s = U/A_f$, where U is the factored concentric load to be resisted by the footing, and A_f is the

CODE

COMMENTARY

base area of the footing as determined by the principles stated in 15.2.2 using the unfactored loads and the permissible soil pressure.

q_s is a calculated reaction to the factored loading used to produce the same required strength conditions regarding flexure, shear, and development of reinforcement in the footing or pile cap, as in any other member.

In the case of eccentric loading, load factors may cause eccentricities and reactions that are different from those obtained by unfactored loads.

15.3 — Footings supporting circular or regular polygon shaped columns or pedestals

For location of critical sections for moment, shear, and development of reinforcement in footings, it shall be permitted to treat circular or regular polygon shaped concrete columns or pedestals as square members with the same area.

15.4 — Moment in footings

R15.4 — Moment in footings

15.4.1 — External moment on any section of a footing shall be determined by passing a vertical plane through the footing, and computing the moment of the forces acting over entire area of footing on one side of that vertical plane.

15.4.2 — Maximum factored moment for an isolated footing shall be computed as prescribed in 15.4.1 at critical sections located as follows:

(a) At face of column, pedestal, or wall, for footings supporting a concrete column, pedestal, or wall;

(b) Halfway between middle and edge of wall, for footings supporting a masonry wall;

(c) Halfway between face of column and edge of steel base plate, for footings supporting a column with steel base plate.

15.4.3 — In one-way footings and two-way square footings, reinforcement shall be distributed uniformly across entire width of footing.

15.4.4 — In two-way rectangular footings, reinforcement shall be distributed in accordance with 15.4.4.1 and 15.4.4.2.

R15.4.4 — In previous codes, the reinforcement in the short direction of rectangular footings should be distributed so that an area of steel given by Eq. (15-1) is provided in a band width equal to the length of the short side of the footing. The band width is centered about the column centerline.

CODE

15.4.4.1 — Reinforcement in long direction shall be distributed uniformly across entire width of footing.

15.4.4.2 — For reinforcement in short direction, a portion of the total reinforcement given by Eq. (15-1) shall be distributed uniformly over a band width (centered on centerline of column or pedestal) equal to the length of short side of footing. Remainder of reinforcement required in short direction shall be distributed uniformly outside center band width of footing.

$$\frac{\text{Reinforcement in band width}}{\text{Total reinforcement in short direction}} = \frac{2}{(\beta + 1)} \qquad (15\text{-}1)$$

15.5 — Shear in footings

15.5.1 — Shear strength of footings supported on soil or rock shall be in accordance with 11.12.

15.5.2 — Location of critical section for shear in accordance with Chapter 11 shall be measured from face of column, pedestal, or wall, for footings supporting a column, pedestal, or wall. For footings supporting a column or pedestal with steel base plates, the critical section shall be measured from location defined in 15.4.2(c).

15.5.3 — Where the distance between the axis of any pile to the axis of the column is more than two times the distance between the top of the pile cap and the top of the pile, the pile cap shall satisfy 11.12 and 15.5.4. Other pile caps shall satisfy one of 11.12, 15.5.4, or Appendix A. If Appendix A is used, the effective concrete compression strength of the struts, f_{cu}, shall be determined using A.3.2.2(b).

COMMENTARY

The remaining reinforcement required in the short direction is to be distributed equally over the two segments outside the band width, one-half to each segment.

R15.5 — Shear in footings

R15.5.1 and R15.5.2 — The shear strength of footings are determined for the more severe condition of 11.12.1.1 or 11.12.1.2. The critical section for shear is measured from the face of supported member (column, pedestal, or wall), except for supported members on steel base plates.

Computation of shear requires that the soil reaction q_s be obtained from the factored loads and the design be in accordance with the appropriate equations of Chapter 11.

Where necessary, shear around individual piles may be investigated in accordance with 11.12.1.2. If shear perimeters overlap, the modified critical perimeter b_o should be taken as that portion of the smallest envelope of individual shear perimeter that will actually resist the critical shear for the group under consideration. One such situation is illustrated in Fig. R15.5.

R15.5.3 — Pile caps supported on piles in more than one plane can be designed using three-dimensional strut-and-tie models satisfying Appendix A.[15.3] The effective concrete compressive strength is from A.3.2.2(b) because it is generally not feasible to provide confining reinforcement satisfying A.3.3.1 and A.3.3.2 in a pile cap.

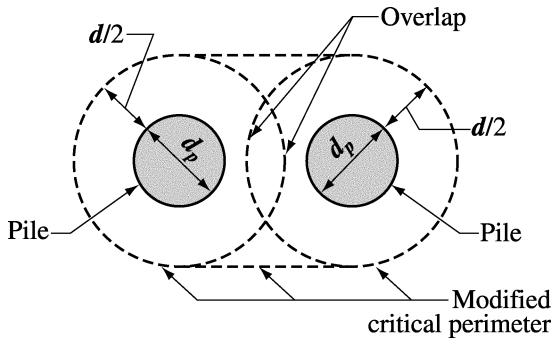


Fig. R15.5—Modified critical perimeter for shear with overlapping critical perimeters

CODE

15.5.4 —Computation of shear on any section through a footing supported on piles shall be in accordance with 15.5.4.1, 15.5.4.2, and 15.5.4.3.

15.5.4.1 — Entire reaction from any pile whose center is located $d_p/2$ or more outside the section shall be considered as producing shear on that section.

15.5.4.2 — Reaction from any pile whose center is located $d_p/2$ or more inside the section shall be considered as producing no shear on that section.

15.5.4.3 — For intermediate positions of pile center, the portion of the pile reaction to be considered as producing shear on the section shall be based on straight-line interpolation between full value at $d_p/2$ outside the section and zero value at $d_p/2$ inside the section.

15.6 — Development of reinforcement in footings

15.6.1 — Development of reinforcement in footings shall be in accordance with Chapter 12.

15.6.2 — Calculated tension or compression in reinforcement at each section shall be developed on each side of that section by embedment length, hook (tension only) or mechanical device, or a combination thereof.

15.6.3 — Critical sections for development of reinforcement shall be assumed at the same locations as defined in 15.4.2 for maximum factored moment, and at all other vertical planes where changes of section or reinforcement occur. See also 12.10.6.

15.7 — Minimum footing depth

Depth of footing above bottom reinforcement shall not be less than 6 in. for footings on soil, nor less than 12 in. for footings on piles.

15.8 — Transfer of force at base of column, wall, or reinforced pedestal

15.8.1 — Forces and moments at base of column, wall, or pedestal shall be transferred to supporting pedestal or footing by bearing on concrete and by reinforcement, dowels, and mechanical connectors.

COMMENTARY

R15.5.4 — When piles are located inside the critical sections d or $d/2$ from face of column, for one-way or two-way shear, respectively, an upper limit on the shear strength at a section adjacent to the face of the column should be considered. The *CRSI Handbook*[15.4] offers guidance for this situation.

R15.8 — Transfer of force at base of column, wall, or reinforced pedestal

Section 15.8 provides the specific requirements for force transfer from a column, wall, or pedestal (supported member) to a pedestal or footing (supporting member). Force transfer should be by bearing on concrete (compressive force only) and by reinforcement (tensile or compressive force). Reinforcement may consist of extended longitudinal bars, dowels, anchor bolts, or suitable mechanical connectors.

The requirements of 15.8.1 apply to both cast-in-place construction and precast construction. Additional require-

CODE

15.8.1.1 — Bearing stress on concrete at contact surface between supported and supporting member shall not exceed concrete bearing strength for either surface as given by 10.17.

15.8.1.2 — Reinforcement, dowels, or mechanical connectors between supported and supporting members shall be adequate to transfer:

(a) All compressive force that exceeds concrete bearing strength of either member;

(b) Any computed tensile force across interface.

In addition, reinforcement, dowels, or mechanical connectors shall satisfy 15.8.2 or 15.8.3.

15.8.1.3 — If calculated moments are transferred to supporting pedestal or footing, then reinforcement, dowels, or mechanical connectors shall be adequate to satisfy 12.17.

15.8.1.4 — Lateral forces shall be transferred to supporting pedestal or footing in accordance with shear-friction provisions of 11.7, or by other appropriate means.

COMMENTARY

ments for cast-in-place construction are given in 15.8.2. Section 15.8.3 gives additional requirements for precast construction.

R15.8.1.1 — Compressive force may be transmitted to a supporting pedestal or footing by bearing on concrete. For strength design, allowable bearing stress on the loaded area is equal to $\mathbf{0.85}\phi f_c'$, if the loaded area is equal to the area on which it is supported.

In the common case of a column bearing on a footing larger than the column, bearing strength should be checked at the base of the column and the top of the footing. Strength in the lower part of the column should be checked since the column reinforcement cannot be considered effective near the column base because the force in the reinforcement is not developed for some distance above the base, unless dowels are provided, or the column reinforcement is extended into the footing. The unit bearing stress on the column will normally be $\mathbf{0.85}\phi f_c'$. The permissible bearing strength on the footing may be increased in accordance with 10.17 and will usually be two times $\mathbf{0.85}\phi f_c'$. The compressive force that exceeds that developed by the permissible bearing strength at the base of the column or at the top of the footing should be carried by dowels or extended longitudinal bars.

R15.8.1.2 — All tensile forces, whether created by uplift, moment, or other means, should be transferred to supporting pedestal or footing entirely by reinforcement or suitable mechanical connectors. Generally, mechanical connectors would be used only in precast construction.

R15.8.1.3 — If computed moments are transferred from the column to the footing, the concrete in the compression zone of the column will be stressed to $\mathbf{0.85}f_c'$ under factored load conditions and, as a result, all the reinforcement will generally have to be doweled into the footing.

R15.8.1.4 — The shear-friction method given in 11.7 may be used to check for transfer of lateral forces to supporting pedestal or footing. Shear keys may be used, provided that the reinforcement crossing the joint satisfies 15.8.2.1, 15.8.3.1, and the shear-friction requirements of 11.7. In precast construction, resistance to lateral forces may be provided by shear-friction, shear keys, or mechanical devices.

CODE

15.8.2 — In cast-in-place construction, reinforcement required to satisfy 15.8.1 shall be provided either by extending longitudinal bars into supporting pedestal or footing, or by dowels.

15.8.2.1 — For cast-in-place columns and pedestals, area of reinforcement across interface shall be not less than 0.005 times gross area of supported member.

15.8.2.2 — For cast-in-place walls, area of reinforcement across interface shall be not less than minimum vertical reinforcement given in 14.3.2.

15.8.2.3 — At footings, it shall be permitted to lap splice No. 14 and No. 18 longitudinal bars, in compression only, with dowels to provide reinforcement required to satisfy 15.8.1. Dowels shall not be larger than No. 11 bar and shall extend into supported member a distance not less than the development length of No. 14 or No. 18 bars or the splice length of the dowels, whichever is greater, and into the footing a distance not less than the development length of the dowels.

15.8.2.4 — If a pinned or rocker connection is provided in cast-in-place construction, connection shall conform to 15.8.1 and 15.8.3.

15.8.3 — In precast construction, anchor bolts or suitable mechanical connectors shall be permitted for satisfying 15.8.1. Anchor bolts shall be designed in accordance with Appendix D.

15.8.3.1 — Connection between precast columns or pedestals and supporting members shall meet the requirements of 16.5.1.3(a).

15.8.3.2 — Connection between precast walls and supporting members shall meet the requirements of 16.5.1.3(b) and (c).

COMMENTARY

R15.8.2.1 and R15.8.2.2 — A minimum amount of reinforcement is required between all supported and supporting members to ensure ductile behavior. The code does not require that all bars in a column be extended through and be anchored into a footing. However, reinforcement with an area of 0.005 times the column area or an equal area of properly spliced dowels is required to extend into the footing with proper anchorage. This reinforcement is required to provide a degree of structural integrity during the construction stage and during the life of the structure.

R15.8.2.3 — Lap splices of No. 14 and No. 18 longitudinal bars in compression only to dowels from a footing are specifically permitted in 15.8.2.3. The dowel bars should be No. 11 or smaller in size. The dowel lap splice length should meet the larger of the two criteria: (a) be able to transfer the stress in the No. 14 and No. 18 bars, and (b) fully develop the stress in the dowels as a splice.

This provision is an exception to 12.14.2.1, which prohibits lap splicing of No. 14 and No. 18 bars. This exception results from many years of successful experience with the lap splicing of these large column bars with footing dowels of the smaller size. The reason for the restriction on dowel bar size is recognition of the anchorage length problem of the large bars, and to allow use of the smaller size dowels. A similar exception is allowed for compression splices between different size bars in 12.16.2.

R15.8.3.1 and R15.8.3.2 — For cast-in-place columns, 15.8.2.1 requires a minimum area of reinforcement equal to $\mathbf{0.005}A_g$ across the column-footing interface to provide some degree of structural integrity. For precast columns this requirement is expressed in terms of an equivalent tensile force that should be transferred. Thus, across the joint, $A_s f_y = \mathbf{200}A_g$ [see 16.5.1.3(a)]. The minimum tensile strength required for precast wall-to-footing connection [see 16.5.1.3(b)] is somewhat less than that required for columns, since an overload would be distributed laterally and a sudden failure would be less likely. Since the tensile strength values of 16.5.1.3 have been arbitrarily chosen, it is not necessary to include a strength reduction factor f for these calculations.

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15.8.3.3 — Anchor bolts and mechanical connections shall be designed to reach their design strength before anchorage failure or failure of surrounding concrete. Anchor bolts shall be designed in accordance with Appendix D.

15.9 — Sloped or stepped footings

15.9.1 — In sloped or stepped footings, angle of slope or depth and location of steps shall be such that design requirements are satisfied at every section. (See also 12.10.6.)

15.9.2 — Sloped or stepped footings designed as a unit shall be constructed to ensure action as a unit.

15.10 — Combined footings and mats

15.10.1 — Footings supporting more than one column, pedestal, or wall (combined footings or mats) shall be proportioned to resist the factored loads and induced reactions, in accordance with appropriate design requirements of the code.

15.10.2 — The Direct Design Method of Chapter 13 shall not be used for design of combined footings and mats.

15.10.3 — Distribution of soil pressure under combined footings and mats shall be consistent with properties of the soil and the structure and with established principles of soil mechanics.

COMMENTARY

R15.10 — Combined footings and mats

R15.10.1 — Any reasonable assumption with respect to the distribution of soil pressure or pile reactions can be used as long as it is consistent with the type of structure and the properties of the soil, and conforms with established principles of soil mechanics (see 15.1). Similarly, as prescribed in 15.2.2 for isolated footings, the base area or pile arrangement of combined footings and mats should be determined using the unfactored forces, moments, or both, transmitted by the footing to the soil, considering permissible soil pressures and pile reactions.

Design methods using factored loads and strength reduction factors ϕ can be applied to combined footings or mats, regardless of the soil pressure distribution.

Detailed recommendations for design of combined footings and mats are reported by ACI Committee 336.[15.1] See also Reference 15.2.

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COMMENTARY

Notes

CHAPTER 16 — PRECAST CONCRETE

CODE

16.0 — Notation

A_g = gross area of column, in.^{2}
ℓ = clear span, in.

16.1 — Scope

16.1.1 — All provisions of this code, not specifically excluded and not in conflict with the provisions of Chapter 16, shall apply to structures incorporating precast concrete structural members.

16.2 — General

16.2.1 — Design of precast members and connections shall include loading and restraint conditions from initial fabrication to end use in the structure, including form removal, storage, transportation, and erection.

16.2.2 — When precast members are incorporated into a structural system, the forces and deformations occurring in and adjacent to connections shall be included in the design.

16.2.3 — Tolerances for both precast members and interfacing members shall be specified. Design of precast members and connections shall include the effects of these tolerances.

COMMENTARY

R16.0 — Notation

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

R16.1 — Scope

R16.1.1 — See 2.1 for definition of precast concrete.

Design and construction requirements for precast concrete structural members differ in some respects from those for cast-in-place concrete structural members and these differences are addressed in this chapter. Where provisions for cast-in-place concrete applied to precast concrete, they have not been repeated. Similarly, items related to composite concrete in Chapter 17 and to prestressed concrete in Chapter 18 that apply to precast concrete are not restated.

More detailed recommendations concerning precast concrete are given in References 16.1 through 16.7. Tilt-up concrete construction is a form of precast concrete. It is recommended that Reference 16.8 be reviewed for tilt-up structures.

R16.2 — General

R16.2.1 — Stresses developed in precast members during the period from casting to final connection may be greater than the service load stresses. Handling procedures may cause undesirable deformations. Care should be given to the methods of storing, transporting, and erecting precast members so that performance at service loads and strength under factored loads meet code requirements.

R16.2.2 — The structural behavior of precast members may differ substantially from that of similar members that are cast-in-place. Design of connections to minimize or transmit forces due to shrinkage, creep, temperature change, elastic deformation, differential settlement, wind, and earthquake require special consideration in precast construction.

R16.2.3 — Design of precast members and connections is particularly sensitive to tolerances on the dimensions of individual members and on their location in the structure. To prevent misunderstanding, the tolerances used in design should be specified in the contract documents. The designer may specify the tolerance standard assumed in design. It is important to specify any deviations from accepted standards.

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The tolerances required by 7.5 are considered to be a minimum acceptable standard for reinforcement in precast concrete. The designer should refer to publications of the Precast/Prestressed Concrete Institute (PCI) (References 16.9, 16.10, 16.11) for guidance on industry established standard product and erection tolerances. Added guidance is given in Reference 16.12.

16.2.4 — In addition to the requirements for drawings and specifications in 1.2, (a) and (b) shall be included in either the contract documents or shop drawings:

(a) Details of reinforcement, inserts and lifting devices required to resist temporary loads from handling, storage, transportation, and erection;

(b) Required concrete strength at stated ages or stages of construction.

R16.2.4 — The additional requirements may be included in either contract documents or shop drawings, depending on the assignment of responsibility for design.

16.3 — Distribution of forces among members

R16.3 — Distribution of forces among members

16.3.1 — Distribution of forces that are perpendicular to the plane of members shall be established by analysis or by test.

R16.3.1 — Concentrated point and line loads can be distributed among members provided they have sufficient torsional stiffness and that shear can be transferred across joints. Torsionally stiff members such as hollow-core or solid slabs have more favorable load distribution properties than do torsionally flexible members such as double tees with thin flanges. The actual distribution of the load depends on many factors discussed in detail in References 16.13 through 16.19. Large openings can cause significant changes in distribution of forces.

16.3.2 — Where the system behavior requires in-plane forces to be transferred between the members of a precast floor or wall system, 16.3.2.1 and 16.3.2.2 shall apply.

16.3.2.1 — In-plane force paths shall be continuous through both connections and members.

16.3.2.2 — Where tension forces occur, a continuous path of steel or steel reinforcement shall be provided.

R16.3.2 — In-plane forces result primarily from diaphragm action in floors and roofs, causing tension or compression in the chords and shear in the body of the diaphragm. A continuous path of steel, steel reinforcement, or both, using lap splices, mechanical or welded splices, or mechanical connectors, should be provided to carry the tension, whereas the shear and compression may be carried by the net concrete section. A continuous path of steel through a connection includes bolts, weld plates, headed studs, or other steel devices. Tension forces in the connections are to be transferred to the primary reinforcement in the members.

In-plane forces in precast wall systems result primarily from diaphragm reactions and external lateral loads.

Connection details should provide for the forces and deformations due to shrinkage, creep, and thermal effects. Connection details may be selected to accommodate volume changes and rotations caused by temperature gradients and long-term deflections. When these effects are restrained, connections and members should be designed to provide adequate strength and ductility.

CODE

16.4 — Member design

16.4.1 — In one-way precast floor and roof slabs and in one-way precast, prestressed wall panels, all not wider than 12 ft, and where members are not mechanically connected to cause restraint in the transverse direction, the shrinkage and temperature reinforcement requirements of 7.12 in the direction normal to the flexural reinforcement shall be permitted to be waived. This waiver shall not apply to members that require reinforcement to resist transverse flexural stresses.

16.4.2 — For precast, nonprestressed walls the reinforcement shall be designed in accordance with the provisions of Chapters 10 or 14, except that the area of horizontal and vertical reinforcement each shall be not less than 0.001 times the gross cross-sectional area of the wall panel. Spacing of reinforcement shall not exceed 5 times the wall thickness nor 30 in. for interior walls nor 18 in. for exterior walls.

16.5 — Structural integrity

16.5.1 — Except where the provisions of 16.5.2 govern, the minimum provisions of 16.5.1.1 through 16.5.1.4 for structural integrity shall apply to all precast concrete structures.

16.5.1.1 — Longitudinal and transverse ties required by 7.13.3 shall connect members to a lateral load resisting system.

COMMENTARY

R16.4 — Member design

R16.4.1 — For prestressed concrete members not wider than 12 ft, such as hollow-core slabs, solid slabs, or slabs with closely spaced ribs, there is usually no need to provide transverse reinforcement to withstand shrinkage and temperature stresses in the short direction. This is generally true also for nonprestressed floor and roof slabs. The 12 ft width is less than that in which shrinkage and temperature stresses can build up to a magnitude requiring transverse reinforcement. In addition, much of the shrinkage occurs before the members are tied into the structure. Once in the final structure, the members are usually not as rigidly connected transversely as monolithic concrete, thus the transverse restraint stresses due to both shrinkage and temperature change are significantly reduced.

The waiver does not apply to members such as single and double tees with thin, wide flanges.

R16.4.2 — This minimum area of wall reinforcement, instead of the minimum values in 14.3, has been used for many years and is recommended by the PCI [16.4] and the Canadian Building Code.[16.20] The provisions for reduced minimum reinforcement and greater spacing recognize that precast wall panels have very little restraint at their edges during early stages of curing and develop less shrinkage stress than comparable cast-in-place walls.

R16.5 — Structural integrity

R16.5.1 — The provisions of 7.13.3 apply to all precast concrete structures. Sections 16.5.1 and 16.5.2 give minimum requirements to satisfy 7.13.3. It is not intended that these minimum requirements override other applicable provisions of the code for design of precast concrete structures.

The overall integrity of a structure can be substantially enhanced by minor changes in the amount, location, and detailing of member reinforcement and in the detailing of connection hardware.

R16.5.1.1 — Individual members may be connected into a lateral load resisting system by alternative methods. For example, a load-bearing spandrel could be connected to a diaphragm (part of the lateral load resisting system). Structural integrity could be achieved by connecting the spandrel into all or a portion of the deck members forming the diaphragm. Alternatively, the spandrel could be connected only to its supporting columns, which in turn is connected to the diaphragm.

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16.5.1.2 — Where precast elements form floor or roof diaphragms, the connections between diaphragm and those members being laterally supported shall have a nominal tensile strength capable of resisting not less than 300 lb per linear ft.

16.5.1.3 — Vertical tension tie requirements of 7.13.3 shall apply to all vertical structural members, except cladding, and shall be achieved by providing connections at horizontal joints in accordance with (a) through (c):

(a) Precast columns shall have a nominal strength in tension not less than $200 A_g$, in lb. For columns with a larger cross section than required by consideration of loading, a reduced effective area A_g, based on cross section required but not less than one-half the total area, shall be permitted;

(b) Precast wall panels shall have a minimum of two ties per panel, with a nominal tensile strength not less than 10,000 lb per tie;

(c) When design forces result in no tension at the base, the ties required by 16.5.1.3(b) shall be permitted to be anchored into an appropriately reinforced concrete floor slab on grade.

16.5.1.4 — Connection details that rely solely on friction caused by gravity loads shall not be used.

16.5.2 — For precast concrete bearing wall structures three or more stories in height, the minimum provisions of 16.5.2.1 through 16.5.2.5 shall apply.

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R16.5.1.2 — Diaphragms are typically provided as part of the lateral load resisting system. The ties prescribed in 16.5.1.2 are the minimum required to attach members to the floor or roof diaphragms. The tie force is equivalent to the service load value of 200 lb/ft given in the Uniform Building Code.

R16.5.1.3 — Base connections and connections at horizontal joints in precast columns and wall panels, including shear walls, are designed to transfer all design forces and moments. The minimum tie requirements of 16.5.1.3 are not additive to these design requirements. Common practice is to place the wall ties symmetrically about the vertical centerline of the wall panel and within the outer quarters of the panel width, wherever possible.

R16.5.1.4 — In the event of damage to a beam, it is important that displacement of its supporting members be minimized, so that other members will not lose their load-carrying capacity. This situation shows why connection details that rely solely on friction caused by gravity loads are not used. An exception could be heavy modular unit structures (one or more cells in cell-type structures) where resistance to overturning or sliding in any direction has a large factor of safety. Acceptance of such systems should be based on the provisions of 1.4.

R16.5.2 — The structural integrity minimum tie provisions for bearing wall structures, often called large panel structures, are intended to provide catenary hanger supports in case of loss of a bearing wall support, as shown by test.[16.21] Forces induced by loading, temperature change, creep, and wind or seismic action may require a larger amount of tie force. It is intended that the general precast concrete provisions of 16.5.1 apply to bearing wall structures less than three stories in height.

Minimum ties in structures three or more stories in height, in accordance with 16.5.2.1, 16.5.2.2, 16.5.2.3, 16.5.2.4, and 16.5.2.5, are required for structural integrity (Fig. R16.5.2). These provisions are based on PCI's recommendations for design of precast concrete bearing wall buildings.[16.22] Tie capacity is based on yield strength.

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COMMENTARY

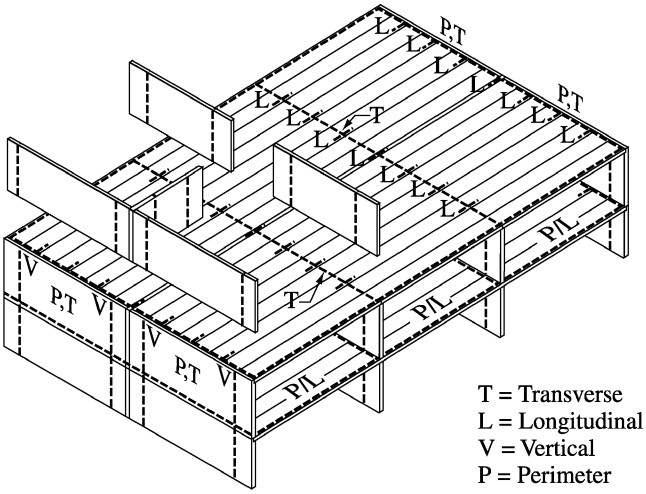


Fig. R16.5.2—Typical arrangement of tensile ties in large panel structures

16.5.2.1 — Longitudinal and transverse ties shall be provided in floor and roof systems to provide a nominal strength of 1500 lb per foot of width or length. Ties shall be provided over interior wall supports and between members and exterior walls. Ties shall be positioned in or within 2 ft of the plane of the floor or roof system.

R16.5.2.1 — Longitudinal ties may project from slabs and be lap spliced, welded, or mechanically connected, or they may be embedded in grout joints, with sufficient length and cover to develop the required force. Bond length for unstressed prestressing steel should be sufficient to develop the yield strength.[16.23] It is uncommon to have ties positioned in the walls reasonably close to the plane of the floor or roof system.

16.5.2.2 — Longitudinal ties parallel to floor or roof slab spans shall be spaced not more than 10 ft on centers. Provisions shall be made to transfer forces around openings.

16.5.2.3 — Transverse ties perpendicular to floor or roof slab spans shall be spaced not greater than the bearing wall spacing.

R16.5.2.3 — Transverse ties may be uniformly spaced either encased in the panels or in a topping, or they may be concentrated at the transverse bearing walls.

16.5.2.4 — Ties around the perimeter of each floor and roof, within 4 ft of the edge, shall provide a nominal strength in tension not less than 16,000 lb.

R16.5.2.4 —The perimeter tie requirements need not be additive with the longitudinal and transverse tie requirements.

16.5.2.5 — Vertical tension ties shall be provided in all walls and shall be continuous over the height of the building. They shall provide a nominal tensile strength not less than 3000 lb per horizontal foot of wall. Not less than two ties shall be provided for each precast panel.

16.6 — Connection and bearing design

R16.6 — Connection and bearing design

16.6.1 — Forces shall be permitted to be transferred between members by grouted joints, shear keys, mechanical connectors, reinforcing steel connections, reinforced topping, or a combination of these means.

R16.6.1 — The code permits a variety of methods for connecting members. These are intended for transfer of forces both in-plane and perpendicular to the plane of the members.

CODE

16.6.1.1 — The adequacy of connections to transfer forces between members shall be determined by analysis or by test. Where shear is the primary result of imposed loading, it shall be permitted to use the provisions of 11.7 as applicable.

16.6.1.2 — When designing a connection using materials with different structural properties, their relative stiffnesses, strengths, and ductilities shall be considered.

16.6.2 — Bearing for precast floor and roof members on simple supports shall satisfy 16.6.2.1 and 16.6.2.2.

16.6.2.1—The allowable bearing stress at the contact surface between supported and supporting members and between any intermediate bearing elements shall not exceed the bearing strength for either surface or the bearing element, or both. Concrete bearing strength shall be as given in 10.17.

16.6.2.2 — Unless shown by test or analysis that performance will not be impaired, (a) and (b) shall be met:

(a) Each member and its supporting system shall have design dimensions selected so that, after consideration of tolerances, the distance from the edge of the support to the end of the precast member in the direction of the span is at least 1/180 of the clear span ℓ, but not less than:

For solid or hollow-core slabs..... 2 in.
For beams or stemmed members ..... 3 in.

(b) Bearing pads at unarmored edges shall be set back a minimum of 1/2 in. from the face of the support, or at least the chamfer dimension at chamfered edges.

COMMENTARY

R16.6.1.2 — Various components in a connection (such as bolts, welds, plates, and inserts) have different properties that can affect the overall behavior of the connection.

R16.6.2.1 — When tensile forces occur in the plane of the bearing, it may be desirable to reduce the allowable bearing stress, provide confinement reinforcement, or both. Guidelines are provided in Reference 16.4.

R16.6.2.2 — This section differentiates between bearing length and length of the end of a precast member over the support (Fig. R16.6.2). Bearing pads distribute concentrated loads and reactions over the bearing area, and allow limited horizontal and rotational movements for stress relief. To prevent spalling under heavily loaded bearing areas, bearing pads should not extend to the edge of the support unless the edge is armored. Edges can be armored with anchored steel plates or angles. Section 11.9.7 gives requirements for bearing on brackets or corbels.

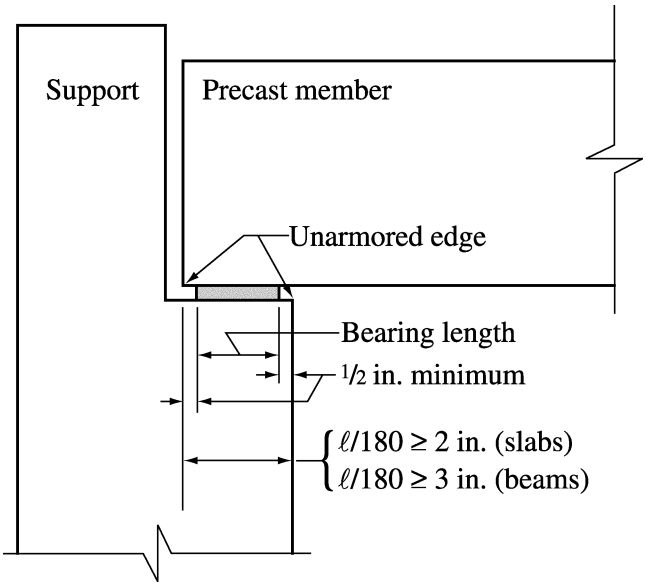


Fig. R16.6.2—Bearing length on support

CODE

16.6.2.3 — The requirements of 12.11.1 shall not apply to the positive bending moment reinforcement for statically determinate precast members, but at least one-third of such reinforcement shall extend to the center of the bearing length, taking into account permitted tolerances in 7.5.2.2 and 16.2.3.

16.7 — Items embedded after concrete placement

16.7.1 — When approved by the registered design professional, embedded items (such as dowels or inserts) that either protrude from the concrete or remain exposed for inspection shall be permitted to be embedded while the concrete is in a plastic state provided that 16.7.1.1, 16.7.1.2, and 16.7.1.3 are met.

16.7.1.1 — Embedded items are not required to be hooked or tied to reinforcement within the concrete.

16.7.1.2 — Embedded items are maintained in the correct position while the concrete remains plastic.

16.7.1.3 — The concrete is properly consolidated around the embedded item.

16.8 — Marking and identification

16.8.1 — Each precast member shall be marked to indicate its location and orientation in the structure and date of manufacture.

16.8.2 — Identification marks shall correspond to placing drawings.

16.9 — Handling

16.9.1 — Member design shall consider forces and distortions during curing, stripping, storage, transportation, and erection so that precast members are not overstressed or otherwise damaged.

16.9.2 — During erection, precast members and structures shall be adequately supported and braced to ensure proper alignment and structural integrity until permanent connections are completed.

COMMENTARY

R16.6.2.3 — It is unnecessary to develop positive bending moment reinforcement beyond the ends of the precast element if the system is statically determinate. Tolerances need to be considered to avoid bearing on plain concrete where reinforcement has been discontinued.

R16.7 — Items embedded after concrete placement

R16.7.1 — Section 16.7.1 is an exception to the provisions of 7.5.1. Many precast products are manufactured in such a way that it is difficult, if not impossible, to position reinforcement that protrudes from the concrete before the concrete is placed. Such items as ties for horizontal shear and inserts can be placed while the concrete is plastic, if proper precautions are taken. This exception is not applicable to reinforcement that is completely embedded, or to embedded items that will be hooked or tied to embedded reinforcement.

R16.9 — Handling

R16.9.1 — The code requires acceptable performance at service loads and adequate strength under factored loads. However, handling loads should not produce permanent stresses, strains, cracking, or deflections inconsistent with the provisions of the code. A precast member should not be rejected for minor cracking or spalling where strength and durability are not affected. Guidance on assessing cracks is given in PCI reports on fabrication and shipment cracks.[16.24, 16.25]

R16.9.2 — All temporary erection connections, bracing, shoring as well as the sequencing of removal of these items are shown on contract or erection drawings.

CODE

16.10 — Strength evaluation of precast construction

16.10.1 — A precast element to be made composite with cast-in-place concrete shall be permitted to be tested in flexure as a precast element alone in accordance with 16.10.1.1 and 16.10.1.2.

16.10.1.1 — Test loads shall be applied only when calculations indicate the isolated precast element will not be critical in compression or buckling.

16.10.1.2 — The test load shall be that load which, when applied to the precast member alone, induces the same total force in the tension reinforcement as would be induced by loading the composite member with the test load required by 20.3.2.

16.10.2 — The provisions of 20.5 shall be the basis for acceptance or rejection of the precast element.

COMMENTARY

R16.10 — Strength evaluation of precast construction

The strength evaluation procedures of Chapter 20 are applicable to precast members.

CHAPTER 17 — COMPOSITE CONCRETE FLEXURAL MEMBERS

CODE

17.0 — Notation

A_c = area of contact surface being investigated for horizontal shear, in.^{2}
A_v = area of ties within a distance s, in.^{2}
b_v = width of cross section at contact surface being investigated for horizontal shear, in.
d = distance from extreme compression fiber to centroid of tension reinforcement for entire composite section, in.
h = overall thickness of composite member, in.
s = spacing of ties measured along the longitudinal axis of the member, in.
V_{nh} = nominal horizontal shear strength, lb
V_u = factored shear force at section, lb
l = correction factor related to unit weight of concrete
r_v = ratio of tie reinforcement area to area of contact surface
= $A_v / b_v s$
f = strength reduction factor. See 9.3

17.1 — Scope

17.1.1 — Provisions of Chapter 17 shall apply for design of composite concrete flexural members defined as precast concrete, cast-in-place concrete elements, or both, constructed in separate placements but so interconnected that all elements respond to loads as a unit.

17.1.2 — All provisions of the code shall apply to composite concrete flexural members, except as specifically modified in Chapter 17.

17.2 — General

17.2.1 — The use of an entire composite member or portions thereof for resisting shear and moment shall be permitted.

17.2.2 — Individual elements shall be investigated for all critical stages of loading.

17.2.3 — If the specified strength, unit weight, or other properties of the various elements are different, properties of the individual elements or the most critical values shall be used in design.

COMMENTARY

R17.0 — Notation

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

R17.1 — Scope

R17.1.1 — The scope of Chapter 17 is intended to include all types of composite concrete flexural members. In some cases with fully cast-in-place concrete, it may be necessary to design the interface of consecutive placements of concrete as required for composite members. Composite structural steel-concrete members are not covered in this chapter. Design provisions for such composite members are covered in Reference 17.1.

R17.2 — General

CODE

17.2.4 — In strength computations of composite members, no distinction shall be made between shored and unshored members.

17.2.5 — All elements shall be designed to support all loads introduced prior to full development of design strength of composite members.

17.2.6 — Reinforcement shall be provided as required to minimize cracking and to prevent separation of individual elements of composite members.

17.2.7 — Composite members shall meet requirements for control of deflections in accordance with 9.5.5.

COMMENTARY

R17.2.4 — Tests have indicated that the strength of a composite member is the same whether or not the first element cast is shored during casting and curing of the second element.

R17.2.6 — The extent of cracking is dependent on such factors as environment, aesthetics, and occupancy. In addition, composite action should not be impaired.

R17.2.7 — The premature loading of precast elements can cause excessive creep and shrinkage deflections. This is especially so at early ages when the moisture content is high and the strength low.

The transfer of shear by direct bond is important if excessive deflection from slippage is to be prevented. A shear key is an added mechanical factor of safety but it does not operate until slippage occurs.

17.3 — Shoring

When used, shoring shall not be removed until supported elements have developed design properties required to support all loads and limit deflections and cracking at time of shoring removal.

R17.3 — Shoring

The provisions of 9.5.5 cover the requirements pertaining to deflections of shored and unshored members.

17.4 — Vertical shear strength

17.4.1 — When an entire composite member is assumed to resist vertical shear, design shall be in accordance with requirements of Chapter 11 as for a monolithically cast member of the same cross-sectional shape.

17.4.2 — Shear reinforcement shall be fully anchored into interconnected elements in accordance with 12.13.

17.4.3 — Extended and anchored shear reinforcement shall be permitted to be included as ties for horizontal shear.

17.5 — Horizontal shear strength

17.5.1 — In a composite member, full transfer of horizontal shear forces shall be ensured at contact surfaces of interconnected elements.

R17.5 — Horizontal shear strength

R17.5.1 — Full transfer of horizontal shear between segments of composite members should be ensured by horizontal shear strength at contact surfaces or properly anchored ties, or both.

CODE

17.5.2 — Unless calculated in accordance with 17.5.3, design of cross sections subject to horizontal shear shall be based on

$$V_u \leq \phi V_{nh} \quad (17\text{-}1)$$

where V_u is factored shear force at the section considered and V_{nh} is nominal horizontal shear strength in accordance with 17.5.2.1 through 17.5.2.5.

17.5.2.1 — When contact surfaces are clean, free of laitance, and intentionally roughened, shear strength V_{nh} shall not be taken greater than $80 b_v d$, in lb.

17.5.2.2 — When minimum ties are provided in accordance with 17.6, and contact surfaces are clean and free of laitance, but not intentionally roughened, shear strength V_{nh} shall not be taken greater than $80 b_v d$, in lb.

17.5.2.3 — When ties are provided in accordance with 17.6, and contact surfaces are clean, free of laitance, and intentionally roughened to a full amplitude of approximately 1/4 in., shear strength V_{nh} shall be taken equal to $(260 + 0.6 \rho_v f_y) \lambda b_v d$, in lb, but not greater than $500 b_v d$, in lb. Values for λ in 11.7.4.3 shall apply.

17.5.2.4 — When factored shear force V_u at section considered exceeds $\phi (500 b_v d)$, design for horizontal shear shall be in accordance with 11.7.4.

17.5.2.5 — When determining nominal horizontal shear strength over prestressed concrete elements, d shall be as defined or $0.8h$, whichever is greater.

17.5.3 — As an alternative to 17.5.2, horizontal shear shall be permitted to be determined by computing the actual change in compressive or tensile force in any segment, and provisions shall be made to transfer that force as horizontal shear to the supporting element. The factored horizontal shear force shall not exceed horizontal shear strength ϕV_{nh} as given in 17.5.2.1 through 17.5.2.4, where area of contact surface A_c shall be substituted for $b_v d$.

17.5.3.1 — When ties provided to resist horizontal shear are designed to satisfy 17.5.3, the tie area to tie spacing ratio along the member shall approximately reflect the distribution of shear forces in the member.

COMMENTARY

R17.5.2 — The nominal horizontal shear strengths V_{nh} apply when the design is based on the load factors and ϕ-factors of Chapter 9.

Prestressed members used in composite construction may have variations in depth of tension reinforcement along member length due to draped or depressed tendons. Because of this variation, the definition of d used in Chapter 11 for determination of vertical shear strength is also appropriate when determining horizontal shear strength.

R17.5.2.3 — The permitted horizontal shear strengths and the requirement of 1/4 in. amplitude for intentional roughness are based on tests discussed in References 17.2 through 17.4.

R17.5.3.1 — The distribution of horizontal shear stresses along the contact surface in a composite member will reflect the distribution of shear along the member. Horizontal shear failure will initiate where the horizontal shear stress is a maximum and will spread to regions of lower stress. Because the slip at peak horizontal shear resistance is small for a concrete-to-concrete contact surface, longitudinal redistribution of horizontal shear resistance is very limited. The spacing of the ties along the contact surface should, therefore, be such as to provide horizontal shear resistance distributed approximately as the shear acting on the member is distributed.

CODE

17.5.4 — When tension exists across any contact surface between interconnected elements, shear transfer by contact shall be permitted only when minimum ties are provided in accordance with 17.6.

17.6 — Ties for horizontal shear

17.6.1 — When ties are provided to transfer horizontal shear, tie area shall not be less than that required by 11.5.5.3, and tie spacing shall not exceed four times the least dimension of supported element, nor exceed 24 in.

17.6.2 — Ties for horizontal shear shall consist of single bars or wire, multiple leg stirrups, or vertical legs of welded wire fabric (plain or deformed).

17.6.3 — All ties shall be fully anchored into interconnected elements in accordance with 12.13.

COMMENTARY

R17.5.4 — Proper anchorage of ties extending across interfaces is required to maintain contact of the interfaces.

R17.6 — Ties for horizontal shear

The minimum areas and maximum spacings are based on test data given in References 17.2 through 17.6.

CHAPTER 18 — PRESTRESSED CONCRETE

CODE

18.0 — Notation

A = area of that part of cross section between flexural tension face and center of gravity of gross section, in.^{2}

A_{cf} = larger gross cross-sectional area of the slab-beam strips of the two orthogonal equivalent frames intersecting at a column of a two-way slab, in.^{2}

A_{ps} = area of prestressed reinforcement in tension zone, in.^{2}

A_s = area of nonprestressed tension reinforcement, in.^{2}

A_s' = area of compression reinforcement, in.^{2}

b = width of compression face of member, in.

c_c = clear cover from the nearest surface in tension to the surface of the flexural tension steel, in.

d = distance from extreme compression fiber to centroid of nonprestressed tension reinforcement, in.

d' = distance from extreme compression fiber to centroid of compression reinforcement, in.

d_p = distance from extreme compression fiber to centroid of prestressed reinforcement, in.

D = dead loads, or related internal moments and forces

e = base of Napierian logarithms

f_c' = specified compressive strength of concrete, psi

$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi

f_{ci}' = compressive strength of concrete at time of initial prestress, psi

$\sqrt{f_{ci}'}$ = square root of compressive strength of concrete at time of initial prestress, psi

f_{dc} = decompression stress. Stress in the prestressing steel when stress is zero in the concrete at the same level as the centroid of the tendons, ksi

f_{pc} = average compressive stress in concrete due to effective prestress force only (after allowance for all prestress losses), psi

f_{ps} = stress in prestressed reinforcement at nominal strength, psi

f_{pu} = specified tensile strength of prestressing steel, psi

f_{py} = specified yield strength of prestressing steel, psi

f_r = modulus of rupture of concrete, psi

f_{se} = effective stress in prestressed reinforcement (after allowance for all prestress losses), psi

COMMENTARY

R18.0 — Notation

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

CODE

f_t = extreme fiber stress in tension in the precompressed tensile zone, computed using gross section properties, psi

f_y = specified yield strength of nonprestressed reinforcement, psi

h = overall thickness of member, in.

K = wobble friction coefficient per foot of tendon

ℓ_x = length of prestressing steel element from jacking end to any point x, ft. See Eq. (18-1) and (18-2)

L = live loads, or related internal moments and forces

n = number of monostrand anchorage devices in a group

N_c = tensile force in concrete due to unfactored dead load plus live load $(D + L)$, lb

P_s = prestressing force at jacking end, lb

P_{su} = factored prestressing force at the anchorage device, lb

P_x = prestressing force at any point x, lb

s = center-to-center spacing of flexural tension steel near the extreme tension face, in. Where there is only one bar or tendon near the extreme tension face, s is the width of extreme tension face

a = total angular change of tendon profile in radians from tendon jacking end to any point x

b_1 = factor defined in 10.2.7.3

Df_{ps} = stress in prestressing steel at service loads less decompression stress, ksi

g_p = factor for type of prestressing steel
= 0.55 for f_{py}/f_{pu} not less than 0.80
= 0.40 for f_{py}/f_{pu} not less than 0.85
= 0.28 for f_{py}/f_{pu} not less than 0.90

l = correction factor related to unit weight of concrete (See 11.7.4.3)

m = curvature friction coefficient

r = ratio of nonprestressed tension reinforcement
= A_s/bd

$r¢$ = ratio of compression reinforcement
= A_s'/bd

r_p = ratio of prestressed reinforcement
= A_{ps}/bd_p

f = strength reduction factor. See 9.3

w = rf_y/f_c'

w' = $r'f_y/f_c'$

w_p = r_pf_{ps}/f_c'

w_w, w_{pw}, w_w' = reinforcement indices for flanged sections computed as for w, w_p, and $w¢$ except that b shall be the web width, and reinforcement area shall be that required to develop compressive strength of web only

COMMENTARY

The factored prestressing force P_{su} is the product of the load factor (1.2 from Section 9.2.5) and the maximum prestressing force allowed. Under 18.5.1 this is usually overstressing to $0.94 f_{py}$ but not greater than $0.80 f_{pu}$, which is permitted for short periods of time.

$$P_{su} = (1.2)(0.80) f_{pu} A_{ps}$$

$$= 0.96 f_{pu} A_{ps}$$

CODE

18.1 — Scope

18.1.1 — Provisions of Chapter 18 shall apply to members prestressed with wire, strands, or bars conforming to provisions for prestressing steel in 3.5.5.

18.1.2 — All provisions of this code not specifically excluded, and not in conflict with provisions of Chapter 18, shall apply to prestressed concrete.

18.1.3 — The following provisions of this code shall not apply to prestressed concrete, except as specifically noted: Sections 7.6.5, 8.10.2, 8.10.3, 8.10.4, 8.11, 10.5, 10.6, 10.9.1, and 10.9.2; Chapter 13; and Sections 14.3, 14.5, and 14.6, except that certain sections of 10.6 apply as noted in 18.4.4.

COMMENTARY

R18.1 — Scope

R18.1.1 — The provisions of Chapter 18 were developed primarily for structural members such as slabs, beams, and columns that are commonly used in buildings. Many of the provisions may be applied to other types of construction, such as, pressure vessels, pavements, pipes, and crossties. Application of the provisions is left to the judgment of the engineer in cases not specifically cited in the code.

R18.1.3 — Some sections of the code are excluded from use in the design of prestressed concrete for specific reasons. The following discussion provides explanation for such exclusions:

Section 7.6.5 — Section 7.6.5 of the code is excluded from application to prestressed concrete because the requirements for bonded reinforcement and unbonded tendons for cast-in-place members are provided in 18.9 and 18.12, respectively.

Sections 8.10.2, 8.10.3, and 8.10.4 — The empirical provisions of 8.10.2, 8.10.3, and 8.10.4 for T-beams were developed for nonprestressed reinforced concrete, and if applied to prestressed concrete would exclude many standard prestressed products in satisfactory use today. Hence, proof by experience permits variations.

By excluding 8.10.2, 8.10.3, and 8.10.4, no special requirements for prestressed concrete T-beams appear in the code. Instead, the determination of an effective width of flange is left to the experience and judgment of the engineer. Where possible, the flange widths in 8.10.2, 8.10.3, and 8.10.4 should be used unless experience has proven that variations are safe and satisfactory. It is not necessarily conservative in elastic analysis and design considerations to use the maximum flange width as permitted in 8.10.2.

Sections 8.10.1 and 8.10.5 provide general requirements for T-beams that are also applicable to prestressed concrete members. The spacing limitations for slab reinforcement are based on flange thickness, which for tapered flanges can be taken as the average thickness.

Section 8.11 — The empirical limits established for nonprestressed reinforced concrete joist floors are based on successful past performance of joist construction using standard joist forming systems. See R8.11. For prestressed joist construction, experience and judgment should be used. The provisions of 8.11 may be used as a guide.

Sections 10.5, 10.9.1, and 10.9.2 — For prestressed concrete, the limitations on reinforcement given in 10.5, 10.9.1, and 10.9.2 are replaced by those in 18.8.3, 18.9, and 18.11.2.

CODE

COMMENTARY

Section 10.6 —This section does not apply to prestressed members in its entirety. However, 10.6.4 and 10.6.7 are referenced in 18.4.4 pertaining to Class C prestressed flexural members.

Chapter 13 — The design of continuous prestressed concrete slabs requires recognition of secondary moments. Also, volume changes due to the prestressing force can create additional loads on the structure that are not adequately covered in Chapter 13. Because of these unique properties associated with prestressing, many of the design procedures of Chapter 13 are not appropriate for prestressed concrete structures and are replaced by the provisions of 18.12.

Sections 14.5 and 14.6 — The requirements for wall design in 14.5 and 14.6 are largely empirical, utilizing considerations not intended to apply to prestressed concrete.

18.2 — General

R18.2 — General

18.2.1 — Prestressed members shall meet the strength requirements of this code.

18.2.2 — Design of prestressed members shall be based on strength and on behavior at service conditions at all stages that will be critical during the life of the structure from the time prestress is first applied.

R18.2.1 and R18.2.2 — The design investigation should include all stages that may be significant. The three major stages are: (1) jacking stage, or prestress transfer stage—when the tensile force in the prestressing steel is transferred to the concrete and stress levels may be high relative to concrete strength; (2) service load stage—after long-term volume changes have occurred; and (3) the factored load stage—when the strength of the member is checked. There may be other load stages that require investigation. For example, if the cracking load is significant, this load stage may require study, or the handling and transporting stage may be critical.

From the standpoint of satisfactory behavior, the two stages of most importance are those for service load and factored load.

Service load stage refers to the loads defined in the general building code (without load factors), such as live load and dead load, while the factored load stage refers to loads multiplied by the appropriate load factors.

Section 18.3.2 provides assumptions that may be used for investigation at service loads and after transfer of the prestressing force.

18.2.3 — Stress concentrations due to prestressing shall be considered in design.

18.2.4 — Provisions shall be made for effects on adjoining construction of elastic and plastic deformations, deflections, changes in length, and rotations due to prestressing. Effects of temperature and shrinkage shall also be included.

CODE

18.2.5 — The possibility of buckling in a member between points where there is intermittent contact between the prestressing steel and an oversize duct, and buckling in thin webs and flanges shall be considered.

18.2.6 — In computing section properties before bonding of prestressing steel, effect of loss of area due to open ducts shall be considered.

18.3 — Design assumptions

18.3.1 — Strength design of prestressed members for flexure and axial loads shall be based on assumptions given in 10.2, except that 10.2.4 shall apply only to reinforcement conforming to 3.5.3.

18.3.2 — For investigation of stresses at transfer of prestress, at service loads, and at cracking loads, elastic theory shall be used with the assumptions of 18.3.2.1 and 18.3.2.2.

18.3.2.1 — Strains vary linearly with depth through the entire load range.

18.3.2.2 — At cracked sections, concrete resists no tension.

18.3.3 — Prestressed flexural members shall be classified as Class U, Class T, or Class C based on the computed extreme fiber stress f_t at service loads in the precompressed tensile zone, as follows:

(a) Class U: $f_t \le 7.5\sqrt{f'_c}$;
(b) Class T: $7.5\sqrt{f'_c} < f_t \le 12\sqrt{f'_c}$;
(c) Class C: $f_t > 12\sqrt{f'_c}$;

Prestressed two-way slab systems shall be designed as Class U.

COMMENTARY

R18.2.5 — Section 18.2.5 refers to the type of post-tensioning where the prestressing steel makes intermittent contact with an oversize duct. Precautions should be taken to prevent buckling of such members.

If the prestressing steel is in complete contact with the member being prestressed, or is unbonded with the sheathing not excessively larger than the prestressing steel, it is not possible to buckle the member under the prestressing force being introduced.

R18.2.6 — In considering the area of the open ducts, the critical sections should include those that have coupler sheaths that may be of a larger size than the duct containing the prestressing steel. Also, in some instances, the trumpet or transition piece from the conduit to the anchorage may be of such a size as to create a critical section. If the effect of the open duct area on design is deemed negligible, section properties may be based on total area.

In post-tensioned members after grouting and in pretensioned members, section properties may be based on effective sections using transformed areas of bonded prestressing steel and nonprestressed reinforcement gross sections, or net sections.

R18.3 — Design assumptions

R18.3.3 — This section defines three classes of behavior of prestressed flexural members. Class U members are assumed to behave as uncracked members. Class C members are assumed to behave as cracked members. The behavior of Class T members is assumed to be in transition between uncracked and cracked. The serviceability requirements for each class are summarized in Table R18.3.3. For comparison, Table R18.3.3 also shows corresponding requirements for nonprestressed members.

These classes apply to both bonded and unbonded prestressed flexural members, but prestressed two-way slab systems must be designed as Class U.

CODE

COMMENTARY

TABLE R18.3.3 — SERVICEABILITY DESIGN REQUIREMENTS

	Prestressed			Nonprestressed
	Class U	Class T	Class C	
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 18.3.4	Gross section 18.3.4	Cracked section 18.3.4	No requirement
Allowable stress at transfer	18.4.1	18.4.1	18.4.1	No requirement
Allowable compressive stress based on uncracked section properties	18.4.2	18.4.2	No requirement	No requirement
Tensile stress at service loads 18.3.3	$\le 7.5\sqrt{f_c'}$	$7.5\sqrt{f_c'} < f_t \le 12\sqrt{f_c'}$	No requirement	No requirement
Deflection calculation basis	9.5.4.1 Gross section	9.5.4.2 Cracked section, bilinear	9.5.4.2 Cracked section, bilinear	9.5.2, 9.5.3 Effective moment of inertia
Crack control	No requirement	No requirement	10.6.4 Modified by 18.4.4.1	10.6.4
Computation of Δf_{ps} or f_s for crack control	—	—	Crack section analysis	$M/(A_s \times$ lever arm), or $0.6f_y$
Side skin reinforcement	No requirement	No requirement	10.6.7	10.6.7

The precompressed tensile zone is that portion of the member cross section in which flexural tension occurs under dead and live loads. Prestressed concrete is usually designed so that the prestress force introduces compression into this zone, thus effectively reducing the magnitude of the tensile stress.

18.3.4 — For Class U and Class T flexural members, stresses at service loads shall be permitted to be calculated using the uncracked section. For Class C flexural members, stresses at service loads shall be calculated using the cracked transformed section.

R18.3.4 — A method for computing stresses in a cracked section is given in Reference 18.1.

18.3.5 — Deflections of prestressed flexural members shall be calculated in accordance with 9.5.4

R18.3.5 — Reference 18.2 provides information on computing deflections of cracked members.

18.4 — Serviceability requirements — Flexural members

R18.4 — Serviceability requirements — Flexural members

Permissible stresses in concrete address serviceability. Permissible stresses do not ensure adequate structural strength, which should be checked in conformance with other code requirements.

18.4.1 — Stresses in concrete immediately after prestress transfer (before time-dependent prestress losses) shall not exceed the following:

(a) Extreme fiber stress in compression ..... $0.60f'_{ci}$

(b) Extreme fiber stress in tension except as permitted in (c) ..... $3\sqrt{f'_{ci}}$

(c) Extreme fiber stress in tension at ends of simply supported members ..... $6\sqrt{f'_{ci}}$

Where computed tensile stresses exceed these values, bonded additional reinforcement (nonprestressed or prestressed) shall be provided in the tensile zone to

R18.4.1 — The concrete stresses at this stage are caused by the force in the prestressing steel at transfer reduced by the losses due to elastic shortening of the concrete, relaxation of the prestressing steel, seating at transfer, and the stresses due to the weight of the member. Generally, shrinkage and creep effects are not included at this stage. These stresses apply to both pretensioned and post-tensioned concrete with proper modifications of the losses at transfer.

R18.4.1(b) and (c) — The tension stress limits of $3\sqrt{f'_{ci}}$ and $6\sqrt{f'_{ci}}$ refer to tensile stress at locations other than the precompressed tensile zone. Where tensile stresses exceed the permissible values, the total force in the tensile stress

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resist the total tensile force in concrete computed with the assumption of an uncracked section.

18.4.2 — For Class U and Class T prestressed flexural members, stresses in concrete at service loads (based on uncracked section properties, and after allowance for all prestress losses) shall not exceed the following:

(a) Extreme fiber stress in compression due to prestress plus sustained load ..... $0.45f'_c$

(b) Extreme fiber stress in compression due to prestress plus total load ..... $0.60f'_c$

18.4.3 — Permissible stresses in 18.4.1 and 18.4.2 shall be permitted to be exceeded if shown by test or analysis that performance will not be impaired.

18.4.4 — For Class C prestressed flexural members not subject to fatigue or to aggressive exposure, the spacing of bonded reinforcement nearest the extreme tension face shall not exceed that given by 10.6.4.

For structures subject to fatigue or exposed to corrosive environments, special investigations and precautions are required.

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zone may be calculated and reinforcement proportioned on the basis of this force at a stress of $0.6f_y$, but not more than 30,000 psi. The effects of creep and shrinkage begin to reduce the tensile stress almost immediately; however, some tension remains in these areas after allowance is made for all prestress losses.

R18.4.2(a) and (b) — The compression stress limit of $0.45f'_c$ was conservatively established to decrease the probability of failure of prestressed concrete members due to repeated loads. This limit seemed reasonable to preclude excessive creep deformation. At higher values of stress, creep strains tend to increase more rapidly as applied stress increases.

The change in allowable stress in the 1995 code recognized that fatigue tests of prestressed concrete beams have shown that concrete failures are not the controlling criterion. Designs with transient live loads that are large compared to sustained live and dead loads have been penalized by the previous single compression stress limit. Therefore, the stress limit of $0.60f'_c$ permits a one-third increase in allowable compression stress for members subject to transient loads.

Sustained live load is any portion of the service live load that will be sustained for a sufficient period to cause significant time-dependent deflections. Thus, when the sustained live and dead loads are a large percentage of total service load, the $0.45f'_c$ limit of 18.4.2(a) may control. On the other hand, when a large portion of the total service load consists of a transient or temporary service live load, the increased stress limit of 18.4.2(b) may apply.

The compression limit of $0.45f'_c$ for prestress plus sustained loads will continue to control the long-term behavior of prestressed members.

R18.4.3 — This section provides a mechanism whereby development of new products, materials, and techniques in prestressed concrete construction need not be inhibited by code limits on stress. Approvals for the design should be in accordance with 1.4 of the code.

R18.4.4 — Spacing requirements for prestressed members with calculated tensile stress exceeding $12\sqrt{f'_c}$ were introduced in the 2002 edition of the code.

For conditions of corrosive environments, defined as an environment in which chemical attack (such as seawater, corrosive industrial atmosphere, or sewer gas) is encountered, cover greater than that required by 7.7.2 should be used, and tension stresses in the concrete reduced to eliminate possible cracking at service loads. The engineer should use judgment to determine the amount of increased cover and whether reduced tension stresses are required.

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18.4.4.1 — The spacing requirements shall be met by nonprestressed reinforcement and bonded tendons. The spacing of bonded tendons shall not exceed 2/3 of the maximum spacing permitted for nonprestressed reinforcement.

Where both reinforcement and bonded tendons are used to meet the spacing requirement, the spacing between a bar and a tendon shall not exceed 5/6 of that permitted by 10.6.4. See also 18.4.4.3.

18.4.4.2 — In applying Eq. (10-4) to prestressing tendons, Df_{ps} shall be substituted for f_s, where Df_{ps} shall be taken as the difference between the stress computed in the prestressing tendons at service loads based on a cracked section analysis and the decompression stress f_{dc} in the prestressing tendons. It shall be permitted to take f_{dc} equal to the effective prestress f_{se}. See also 18.4.4.3.

18.4.4.3 — The magnitude of Df_{ps} shall not exceed 36 ksi. When Df_{ps} is less than or equal to 20 ksi, the spacing requirements of 18.4.4.1 and 18.4.4.2 shall not apply.

18.4.4.4 — If the effective depth of a beam exceeds 36 in., the area of skin reinforcement consisting of reinforcement or bonded tendons shall be provided as required by 10.6.7.

18.5 — Permissible stresses in prestressing steel

18.5.1 — Tensile stress in prestressing steel shall not exceed the following:

(a) Due to prestressing steel jacking force.... $\mathbf{0.94}f_{py}$

but not greater than the lesser of $\mathbf{0.80}f_{pu}$ and the maximum value recommended by the manufacturer of prestressing steel or anchorage devices.

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R18.4.4.1 — Only tension steel nearest the tension face need be considered in selecting the value of c_c used in computing spacing requirements. To account for prestressing steel, such as strand, having bond characteristics less effective than deformed reinforcement, a 2/3 effectiveness factor is used.

For post-tensioned members designed as cracked members, it will usually be advantageous to provide crack control by the use of deformed reinforcement, for which the provisions of 10.6 may be used directly. Bonded reinforcement required by other provisions of this code may also be used as crack control reinforcement.

R18.4.4.2 — It is conservative to take the decompression stress f_{dc} equal to the effective prestress f_{se}.

R18.4.4.3 — The maximum limitation of 36 ksi for Df_{ps} and the exemption for members with Df_{ps} less than 20 ksi are intended to be similar to the code requirements before the 2002 edition.

R18.4.4.4 — The steel area of reinforcement, bonded tendons, or a combination of both may be used to satisfy this requirement.

R18.5 — Permissible stresses in prestressing steel

The code does not distinguish between temporary and effective prestressing steel stresses. Only one limit on prestressing steel stress is provided because the initial prestressing steel stress (immediately after transfer) can prevail for a considerable time, even after the structure has been put into service. This stress, therefore, should have an adequate safety factor under service conditions and cannot be considered as a temporary stress. Any subsequent decrease in prestressing steel stress due to losses can only improve conditions and no limit on such stress decrease is provided in the code.

R18.5.1 — With the 1983 code, permissible stresses in prestressing steel were revised to recognize the higher yield strength of low-relaxation wire and strand meeting the requirements of ASTM A 421 and A 416. For such prestressing steel, it is more appropriate to specify permissible stresses in terms of specified minimum ASTM yield strength rather than specified minimum ASTM tensile strength. For the low-relaxation wire and strands, with f_{py}

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(b) Immediately after prestress transfer..... $0.82f_{py}$

but not greater than $0.74f_{pu}$.

(c) Post-tensioning tendons, at anchorage devices and couplers, immediately after force transfer..... $0.70f_{pu}$

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equal to $0.90f_{pu}$, the $0.94f_{py}$ and $0.82f_{py}$ limits are equivalent to $0.85f_{pu}$ and $0.74f_{pu}$, respectively. In the 1986 supplement and in the 1989 code, the maximum jacking stress for low-relaxation prestressing steel was reduced to $0.80f_{pu}$ to ensure closer compatibility with the maximum prestressing steel stress value of $0.74f_{pu}$ immediately after prestress transfer. The higher yield strength of the low-relaxation prestressing steel does not change the effectiveness of tendon anchorage devices; thus, the permissible stress at post-tensioning anchorage devices and couplers is not increased above the previously permitted value of $0.70f_{pu}$. For ordinary prestressing steel (wire, strands, and bars) with f_{py} equal to $0.85f_{pu}$, the $0.94f_{py}$ and $0.82f_{py}$ limits are equivalent to $0.80f_{pu}$ and $0.70f_{pu}$, respectively, the same as permitted in the 1977 code. For bar prestressing steel with f_{py} equal to $0.80f_{pu}$, the same limits are equivalent to $0.75f_{pu}$ and $0.66f_{pu}$, respectively.

Because of the higher allowable initial prestressing steel stresses permitted since the 1983 code, final stresses can be greater. Designers should be concerned with setting a limit on final stress when the structure is subject to corrosive conditions or repeated loadings.

18.6 — Loss of prestress

R18.6 — Loss of prestress

18.6.1 — To determine effective prestress f_{se}, allowance for the following sources of loss of prestress shall be considered:

(a) Prestressing steel seating at transfer;

(b) Elastic shortening of concrete;

(c) Creep of concrete;

(d) Shrinkage of concrete;

(e) Relaxation of prestressing steel stress;

(f) Friction loss due to intended or unintended curvature in post-tensioning tendons.

R18.6.1 — For an explanation of how to compute prestress losses, see References 18.3 through 18.6. Lump sum values of prestress losses for both pretensioned and post-tensioned members that were indicated before the 1983 commentary are considered obsolete. Reasonably accurate estimates of prestress losses can be calculated in accordance with the recommendations in Reference 18.6, which include consideration of initial stress level ($0.7f_{pu}$ or higher), type of steel (stress-relieved or low-relaxation wire, strand, or bar), exposure conditions, and type of construction (pretensioned, bonded post-tensioned, or unbonded post-tensioned).

Actual losses, greater or smaller than the computed values, have little effect on the design strength of the member, but affect service load behavior (deflections, camber, cracking load) and connections. At service loads, overestimation of prestress losses can be almost as detrimental as underestimation, since the former can result in excessive camber and horizontal movement.

18.6.2 — Friction loss in post-tensioning tendons

R18.6.2 — Friction loss in post-tensioning tendons

18.6.2.1 — Effect of friction loss in post-tensioning tendons shall be computed by

$$P_s = P_x e^{(K\ell_x + ma)} \quad (18\text{-}1)$$

When $(K\ell_x + ma)$ is not greater than 0.3, effect of friction loss shall be permitted to be computed by

$$P_s = P_x(1 + K\ell_x + ma) \quad (18\text{-}2)$$

The coefficients tabulated in Table R18.6.2 give a range that generally can be expected. Due to the many types of prestressing steel ducts and sheathing available, these values can only serve as a guide. Where rigid conduit is used, the wobble coefficient K can be considered as zero. For large-diameter prestressing steel in semirigid type conduit, the wobble factor can also be considered zero. Values of the coefficients to be used for the particular types of prestressing steel and particular types of ducts should be obtained

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TABLE R18.6.2—FRICTION COEFFICIENTS FOR POST-TENSIONED TENDONS FOR USE IN EQ. (18-1) OR (18-2)

			Wobble coefficient, K	Curvature coefficient, m
Grouted tendons in metal sheathing		Wire tendons	0.0010-0.0015	0.15-0.25
		High-strength bars	0.0001-0.0006	0.08-0.30
		7-wire strand	0.0005-0.0020	0.15-0.25
Unbonded tendons	Mastic coated	Wire tendons	0.0010-0.0020	0.05-0.15
		7-wire strand	0.0010-0.0020	0.05-0.15
	Pre-greased	Wire tendons	0.0003-0.0020	0.05-0.15
		7-wire strand	0.0003-0.0020	0.05-0.15

18.6.2.2—Friction loss shall be based on experimentally determined wobble K and curvature m friction coefficients, and shall be verified during tendon stressing operations.

from the manufacturers of the tendons. An unrealistically low evaluation of the friction loss can lead to improper camber of the member and inadequate prestress. Overestimation of the friction may result in extra prestressing force. This could lead to excessive camber and excessive shortening of a member. If the friction factors are determined to be less than those assumed in the design, the tendon stressing should be adjusted to give only that prestressing force in the critical portions of the structure required by the design.

18.6.2.3 — Values of wobble and curvature friction coefficients used in design shall be shown on design drawings.

R18.6.2.3 — When the safety or serviceability of the structure may be involved, the acceptable range of prestressing steel jacking forces or other limiting requirements should either be given or approved by the structural engineer in conformance with the permissible stresses of 18.4 and 18.5.

18.6.3—Where loss of prestress in a member occurs due to connection of the member to adjoining construction, such loss of prestress shall be allowed for in design.

18.7 — Flexural strength

R18.7 — Flexural strength

18.7.1 — Design moment strength of flexural members shall be computed by the strength design methods of the code. For prestressing steel, f_{ps} shall be substituted for f_y in strength computations.

R18.7.1 — Design moment strength of prestressed flexural members may be computed using strength equations similar to those for nonprestressed concrete members. The 1983 code provided strength equations for rectangular and flanged sections, with tension reinforcement only and with tension and compression reinforcement. When part of the prestressing steel is in the compression zone, a method based on applicable conditions of equilibrium and compatibility of strains at a factored load condition should be used.

For other cross sections, the design moment strength ϕM_n is computed by an analysis based on stress and strain compatibility, using the stress-strain properties of the prestressing steel and the assumptions given in 10.2.

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18.7.2 — As an alternative to a more accurate determination of f_{ps} based on strain compatibility, the following approximate values of f_{ps} shall be permitted to be used if f_{se} is not less than $0.5f_{pu}$.

(a) For members with bonded tendons:

$$f_{ps} = f_{pu}\left\{1 - \frac{\gamma_p}{\beta_1}\left[\rho_p\frac{f_{pu}}{f'_c} + \frac{d}{d_p}(\omega - \omega')\right]\right\} \quad (18\text{-}3)$$

If any compression reinforcement is taken into account when calculating f_{ps} by Eq. (18-3), the term

$$\left[\rho_p\frac{f_{pu}}{f'_c} + \frac{d}{d_p}(\omega - \omega')\right]$$

shall be taken not less than 0.17 and d' shall be no greater than $0.15d_p$.

(b) For members with unbonded tendons and with a span-to-depth ratio of 35 or less:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{100\rho_p} \quad (18\text{-}4)$$

but f_{ps} in Eq. (18-4) shall not be taken greater than f_{py}, nor greater than $(f_{se} + 60{,}000)$.

(c) For members with unbonded tendons and with a span-to-depth ratio greater than 35:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{300\rho_p} \quad (18\text{-}5)$$

but f_{ps} in Eq. (18-5) shall not be taken greater than f_{py}, nor greater than $(f_{se} + 30{,}000)$.

18.7.3 — Nonprestressed reinforcement conforming to 3.5.3, if used with prestressing steel, shall be permitted to be considered to contribute to the tensile force and to be included in moment strength computations at a stress equal to the specified yield strength f_y. Other nonprestressed reinforcement shall be permitted to be included in strength computations only if a strain compatibility analysis is performed to determine stresses in such reinforcement.

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R18.7.2 — Eq. (18-3) may underestimate the strength of beams with high percentages of reinforcement and, for more accurate evaluations of their strength, the strain compatibility and equilibrium method should be used. Use of Eq. (18-3) is appropriate when all of the prestressed reinforcement is in the tension zone. When part of the prestressed reinforcement is in the compression zone, a strain compatibility and equilibrium method should be used.

By inclusion of the w' term, Eq. (18-3) reflects the increased value of f_{ps} obtained when compression reinforcement is provided in a beam with a large reinforcement index. When the term $[\rho_p f_{pu}/f'_c + (d/d_p)(w - w')]$ in Eq. (18-3) is small, the neutral axis depth is small, the compressive reinforcement does not develop its yield strength, and Eq. (18-3) becomes unconservative. This is the reason why the term $[\rho_p f_{pu}/f'_c + (d/d_p)(w - w')]$ in Eq. (18-3) may not be taken less than 0.17 if compression reinforcement is taken into account when computing f_{ps}. If the compression reinforcement is neglected when using Eq. (18-3), ω' is taken as zero, then the term $[\rho_p f_{pu}/f'_c + (d/d_p)w]$ may be less than 0.17 and an increased and correct value of f_{ps} is obtained.

When d' is large, the strain in compression reinforcement can be considerably less than its yield strain. In such a case, the compression reinforcement does not influence f_{ps} as favorably as implied by Eq. (18-3). For this reason, the applicability of Eq. (18-3) is limited to beams in which d' is less than or equal to $0.15d_p$.

The term $[\rho_p f_{pu}/f'_c + (d/d_p)(w - w')]$ in Eq. (18-3) may also be written $[\rho_p f_{pu}/f'_c + A_s f_y/(bd_p f'_c) - A_s' f_y/(bd_p f'_c)]$. This form may be more convenient, such as when there is no unprestressed tension reinforcement.

Eq. (18-5) reflects results of tests on members with unbonded tendons and span-to-depth ratios greater than 35 (one-way slabs, flat plates, and flat slabs).[18.7] These tests also indicate that Eq. (18-4), formerly used for all span-depth ratios, overestimates the amount of stress increase in such members. Although these same tests indicate that the moment strength of those shallow members designed using Eq. (18-4) meets the factored load strength requirements, this reflects the effect of the code requirements for minimum bonded reinforcement, as well as the limitation on concrete tensile stress that often controls the amount of prestressing force provided.

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18.8 — Limits for reinforcement of flexural members

18.8.1 — Prestressed concrete sections shall be classified as either tension-controlled, transition, or compression-controlled sections, in accordance with 10.3.3 and 10.3.4. The appropriate ϕ-factors from 9.3.2 shall apply.

R18.8 — Limits for reinforcement of flexural members

R18.8.1 — The net tensile strain limits for compression- and tension-controlled sections given in 10.3.3 and 10.3.4 apply to prestressed sections. These provisions take the place of maximum reinforcement limits used in the 1999 code.

The net tensile strain limits for tension-controlled sections given in 10.3.4 may also be stated in terms of w_p as defined in the 1999 and earlier editions of the code. The net tensile strain limit of 0.005 corresponds to $w_p = 0.32b_1$ for prestressed rectangular sections.

18.8.2 — Total amount of prestressed and nonprestressed reinforcement shall be adequate to develop a factored load at least 1.2 times the cracking load computed on the basis of the modulus of rupture f_r specified in 9.5.2.3. This provision shall be permitted to be waived for:

(a)Two-way, unbonded post-tensioned slabs; and

(b)Flexural members with shear and flexural strength at least twice that required by 9.2.

R18.8.2 — This provision is a precaution against abrupt flexural failure developing immediately after cracking. A flexural member designed according to code provisions requires considerable additional load beyond cracking to reach its flexural strength. Thus, considerable deflection would warn that the member strength is approaching. If the flexural strength were reached shortly after cracking, the warning deflection would not occur.

18.8.3 — Part or all of the bonded reinforcement consisting of bars or tendons shall be provided as close as practicable to the extreme tension fiber in all prestressed flexural members, except that in members prestressed with unbonded tendons, the minimum bonded reinforcement consisting of bars or tendons shall be as required by 18.9.

R18.8.3 — Some bonded steel is required to be placed near the tension face of prestressed flexural members. The purpose of this bonded steel is to control cracking under full service loads or overloads.

18.9 — Minimum bonded reinforcement

18.9.1 — A minimum area of bonded reinforcement shall be provided in all flexural members with unbonded tendons as required by 18.9.2 and 18.9.3.

R18.9 — Minimum bonded reinforcement

R18.9.1 — Some bonded reinforcement is required by the code in members prestressed with unbonded tendons to ensure flexural performance at ultimate member strength, rather than as a tied arch, and to limit crack width and spacing at service load when concrete tensile stresses exceed the modulus of rupture. Providing the minimum bonded reinforcement as stipulated in 18.9 helps to ensure adequate performance.

Research has shown that unbonded post-tensioned members do not inherently provide large capacity for energy dissipation under severe earthquake loadings because the member response is primarily elastic. For this reason, unbonded post-tensioned structural elements reinforced in accordance with the provisions of this section should be assumed to carry only vertical loads and to act as horizontal diaphragms between energy dissipating elements under earthquake loadings of the magnitude defined in 21.2.1.1. The minimum bonded reinforcement areas required by Eq. (18-6) and (18-8) are absolute minimum areas independent of grade of steel or design yield strength.

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18.9.2 — Except as provided in 18.9.3, minimum area of bonded reinforcement shall be computed by

$$A_s = 0.004A \quad (18\text{-}6)$$

18.9.2.1 — Bonded reinforcement required by Eq. (18-6) shall be uniformly distributed over precompressed tensile zone as close as practicable to extreme tension fiber.

18.9.2.2 — Bonded reinforcement shall be required regardless of service load stress conditions.

18.9.3 — For two-way flat slab systems, minimum area and distribution of bonded reinforcement shall be as required in 18.9.3.1, 18.9.3.2, and 18.9.3.3.

18.9.3.1 — Bonded reinforcement shall not be required in positive moment areas where computed concrete tensile stress at service load (after allowance for all prestress losses) does not exceed $2\sqrt{f'_c}$.

18.9.3.2 — In positive moment areas where computed tensile stress in concrete at service load exceeds $2\sqrt{f'_c}$, minimum area of bonded reinforcement shall be computed by

$$A_s = \frac{N_c}{0.5f_y} \quad (18\text{-}7)$$

where design yield strength f_y shall not exceed 60,000 psi. Bonded reinforcement shall be uniformly distributed over precompressed tensile zone as close as practicable to the extreme tension fiber.

18.9.3.3 — In negative moment areas at column supports, the minimum area of bonded reinforcement A_s in the top of the slab in each direction shall be computed by

$$A_s = 0.00075A_{cf} \quad (18\text{-}8)$$

Bonded reinforcement required by Eq. (18-8) shall be distributed between lines that are $1.5h$ outside opposite faces of the column support. At least four bars or wires shall be provided in each direction. Spacing of bonded reinforcement shall not exceed 12 in.

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R18.9.2 — The minimum amount of bonded reinforcement for members other than two-way flat slab systems is based on research comparing the behavior of bonded and unbonded post-tensioned beams.[18.8] Based on this research, it is advisable to apply the provisions of 18.9.2 also to one-way slab systems.

R18.9.3 — The minimum amount of bonded reinforcement in two-way flat slab systems is based on reports by ACI-ASCE Committee 423.[18.3,18.9] Limited research available for two-way flat slabs with drop panels[18.16] indicates that behavior of these particular systems is similar to the behavior of flat plates. Reference 18.15 was revised by Committee 423 in 1983 to clarify that Section 18.9.3 applies to two-way flat slab systems.

R18.9.3.1 — For usual loads and span lengths, flat plate tests summarized in the Committee 423 report[18.3] and experience since the 1963 code was adopted indicate satisfactory performance without bonded reinforcement in the areas described in 18.9.3.1.

R18.9.3.2 — In positive moment areas, where the concrete tensile stresses are between $2\sqrt{f'_c}$ and $6\sqrt{f'_c}$, a minimum bonded reinforcement area proportioned according to Eq. (18-7) is required.The tensile force N_c is computed at service load on the basis of an uncracked, homogeneous section.

R18.9.3.3 — Research on unbonded post-tensioned two-way flat slab systems evaluated by ACI-ASCE Committee 423[18.1,18.3,18.9,18.10] shows that bonded reinforcement in negative moment regions, proportioned on the basis of 0.075 percent of the cross-sectional area of the slab-beam strip, provides sufficient ductility and reduces crack width and spacing. To account for different adjacent tributary spans, Eq. (18-8) is given on the basis of the equivalent frame as defined in 13.7.2 and pictured in Fig. R13.7.2. For rectangular slab panels, Eq. (18-8) is conservatively based upon the larger of the cross-sectional areas of the two intersecting equivalent frame slab-beam strips at the column. This ensures that the minimum percentage of steel recommended by research is provided in both directions. Concentration of this reinforcement in the top of the slab directly over and immediately adjacent to the column is important. Research also shows that where low tensile stresses occur at service loads, satisfactory behavior has been achieved at factored

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18.9.4 — Minimum length of bonded reinforcement required by 18.9.2 and 18.9.3 shall be as required in 18.9.4.1, 18.9.4.2, and 18.9.4.3.

18.9.4.1 — In positive moment areas, minimum length of bonded reinforcement shall be one-third the clear span length and centered in positive moment area.

18.9.4.2 — In negative moment areas, bonded reinforcement shall extend one-sixth the clear span on each side of support.

18.9.4.3 — Where bonded reinforcement is provided for design moment strength in accordance with 18.7.3, or for tensile stress conditions in accordance with 18.9.3.2, minimum length also shall conform to provisions of Chapter 12.

18.10 — Statically indeterminate structures

18.10.1 — Frames and continuous construction of prestressed concrete shall be designed for satisfactory performance at service load conditions and for adequate strength.

18.10.2 — Performance at service load conditions shall be determined by elastic analysis, considering reactions, moments, shears, and axial forces induced by prestressing, creep, shrinkage, temperature change, axial deformation, restraint of attached structural elements, and foundation settlement.

18.10.3 — Moments used to compute required strength shall be the sum of the moments due to reactions induced by prestressing (with a load factor of 1.0) and the moments due to factored loads. Adjustment of the sum of these moments shall be permitted as allowed in 18.10.4.

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loads without bonded reinforcement. However, the code requires minimum bonded reinforcement regardless of service load stress levels to help ensure flexural continuity and ductility, and to limit crack widths and spacing due to overload, temperature, or shrinkage. Research on post-tensioned flat plate-to-column connections is reported in References 18.11, 18.12, 18.13, 18.14, and 18.15.

R18.9.4 — Bonded reinforcement should be adequately anchored to develop factored load forces. The requirements of Chapter 12 will ensure that bonded reinforcement required for flexural strength under factored loads in accordance with 18.7.3, or for tensile stress conditions at service load in accordance with 18.9.3.2, will be adequately anchored to develop tension or compression forces. The minimum lengths apply for bonded reinforcement required by 18.9.2 or 18.9.3.3, but not required for flexural strength in accordance with 18.7.3. Research[18.1] on continuous spans shows that these minimum lengths provide adequate behavior under service load and factored load conditions.

R18.10 — Statically indeterminate structures

R18.10.3 — For statically indeterminate structures, the moments due to reactions induced by prestressing forces, referred to as secondary moments, are significant in both the elastic and inelastic states. When hinges and full redistribution of moments occur to create a statically determinate structure, secondary moments disappear. However, the elastic deformations caused by a nonconcordant tendon change the amount of inelastic rotation required to obtain a given amount of moment redistribution. Conversely, for a beam with a given inelastic rotational capacity, the amount by which the moment at the support may be varied is changed by an amount equal to the secondary moment at the support due to prestressing. Thus, the code requires that secondary moments be included in determining design moments.

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To determine the moments used in design, the order of calculation should be: (a) determine moments due to dead and live load; (b) modify by algebraic addition of secondary moments; (c) redistribute as permitted. A positive secondary moment at the support caused by a tendon transformed downward from a concordant profile will reduce the negative moments near the supports and increase the positive moments in the midspan regions. A tendon that is transformed upward will have the reverse effect.

18.10.4 — Redistribution of negative moments in continuous prestressed flexural members

18.10.4.1 — Where bonded reinforcement is provided at supports in accordance with 18.9, it shall be permitted to increase or decrease negative moments calculated by elastic theory for any assumed loading, in accordance with 8.4.

18.10.4.2 — The modified negative moments shall be used for calculating moments at sections within spans for the same loading arrangement.

R18.10.4 — Redistribution of negative moments in continuous prestressed flexural members

The provisions for redistribution of negative moments given in 8.4 apply equally to prestressed members. See Reference 9.16 for a comparison of research results and to Section 18.10.4 of the 1999 ACI Code.

For the moment redistribution principles of 18.10.4 to be applicable to beams with unbonded tendons, it is necessary that such beams contain sufficient bonded reinforcement to ensure they will act as beams after cracking and not as a series of tied arches. The minimum bonded reinforcement requirements of 18.9 will serve this purpose.

18.11 — Compression members — Combined flexure and axial loads

18.11.1 — Prestressed concrete members subject to combined flexure and axial load, with or without nonprestressed reinforcement, shall be proportioned by the strength design methods of this code. Effects of prestress, creep, shrinkage, and temperature change shall be included.

R18.11 — Compression members — Combined flexure and axial loads

18.11.2 — Limits for reinforcement of prestressed compression members

18.11.2.1 — Members with average prestress f_{pc} less than 225 psi shall have minimum reinforcement in accordance with 7.10, 10.9.1 and 10.9.2 for columns, or 14.3 for walls.

18.11.2.2—Except for walls, members with average prestress f_{pc} equal to or greater than 225 psi shall have all tendons enclosed by spirals or lateral ties in accordance with (a) through (d):

(a) Spirals shall conform to 7.10.4;

(b) Lateral ties shall be at least No. 3 in size or welded wire fabric of equivalent area, and shall be spaced vertically not to exceed 48 tie bar or wire diameters, or the least dimension of the compression member;

R18.11.2 — Limits for reinforcement of prestressed compression members

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(c) Ties shall be located vertically not more than half a tie spacing above top of footing or slab in any story, and not more than half a tie spacing below the lowest horizontal reinforcement in members supported above;

(d) Where beams or brackets frame into all sides of a column, ties shall be terminated not more than 3 in. below lowest reinforcement in such beams or brackets.

18.11.2.3 — For walls with average prestress f_{pc} equal to or greater than 225 psi, minimum reinforcement required by 14.3 shall not apply where structural analysis shows adequate strength and stability.

18.12 — Slab systems

18.12.1 — Factored moments and shears in prestressed slab systems reinforced for flexure in more than one direction shall be determined in accordance with provisions of 13.7 (excluding 13.7.7.4 and 13.7.7.5), or by more detailed design procedures.

18.12.2 — Design moment strength of prestressed slabs required by 9.3 at every section shall be equal to or exceed the required strength considering 9.2, 18.10.3, and 18.10.4. Design shear strength of prestressed slabs at columns required by 9.3 shall be equal to or exceed the required strength considering 9.2, 11.1, 11.12.2, and 11.12.6.2.

18.12.3 — At service load conditions, all serviceability limitations, including limits on deflections, shall be met, with appropriate consideration of the factors listed in 18.10.2.

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R18.11.2.3 — The minimum amounts of reinforcement in 14.3 need not apply to prestressed concrete walls, provided the average prestress is 225 psi or greater and a structural analysis is performed to show adequate strength and stability with lower amounts of reinforcement.

R18.12 — Slab systems

R18.12.1 — Use of the equivalent frame method of analysis (see 13.7) or more precise analysis procedures is required for determination of both service and factored moments and shears for prestressed slab systems. The equivalent frame method of analysis has been shown by tests of large structural models to satisfactorily predict factored moments and shears in prestressed slab systems. (See References 18.11, 18.12, 18.13, 18.17, 18.18, and 18.19.) The referenced research also shows that analysis using prismatic sections or other approximations of stiffness may provide erroneous results on the unsafe side. Section 13.7.7.4 is excluded from application to prestressed slab systems because it relates to reinforced slabs designed by the direct design method, and because moment redistribution for prestressed slabs is covered in 18.10.4. Section 13.7.7.5 does not apply to prestressed slab systems because the distribution of moments between column strips and middle strips required by 13.7.7.5 is based on tests for nonprestressed concrete slabs. Simplified methods of analysis using average coefficients do not apply to prestressed concrete slab systems.

R18.12.2 — Tests indicate that the moment and shear strength of prestressed slabs is controlled by total prestessing steel strength and by the amount and location of nonprestressed reinforcement, rather than by tendon distribution. (See References 18.11, 18.12, 18.13, 18.17, 18.18, and 18.19.)

R18.12.3 — For prestressed flat slabs continuous over two or more spans in each direction, the span-thickness ratio generally should not exceed 42 for floors and 48 for roofs; these limits may be increased to 48 and 52, respectively, if calculations verify that both short- and long-term deflection, camber, and vibration frequency and amplitude are not objectionable.

Short- and long-term deflection and camber should be computed and checked against the requirements of serviceability of the structure.

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18.12.4 — For normal live loads and loads uniformly distributed, spacing of tendons or groups of tendons in one direction shall not exceed eight times the slab thickness, nor 5 ft. Spacing of tendons also shall provide a minimum average prestress (after allowance for all prestress losses) of 125 psi on the slab section tributary to the tendon or tendon group. A minimum of two tendons shall be provided in each direction through the critical shear section over columns. Special consideration of tendon spacing shall be provided for slabs with concentrated loads.

18.12.5 — In slabs with unbonded tendons, bonded reinforcement shall be provided in accordance with 18.9.3 and 18.9.4.

18.12.6 — In lift slabs, bonded bottom reinforcement shall be detailed in accordance with 13.3.8.6.

18.13 — Post-tensioned tendon anchorage zones

18.13.1 — Anchorage zone

The anchorage zone shall be considered as composed of two zones:

(a) The local zone is the rectangular prism (or equivalent rectangular prism for circular or oval anchorages) of concrete immediately surrounding the anchorage device and any confining reinforcement;

(b) The general zone is the anchorage zone as defined in 2.1 and includes the local zone.

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The maximum length of a slab between construction joints is generally limited to 100 to 150 ft to minimize the effects of slab shortening, and to avoid excessive loss of prestress due to friction.

R18.12.4 — This section provides specific guidance concerning tendon distribution that will permit the use of banded tendon distributions in one direction. This method of tendon distribution has been shown to provide satisfactory performance by structural research.

R18.13 — Post-tensioned tendon anchorage zones

Section 18.13 was extensively revised in the 1999 code and was made compatible with the 1996 AASHTO "Standard Specifications for Highway Bridges"[18.20] and the recommendations of NCHRP Report 356.[18.21]

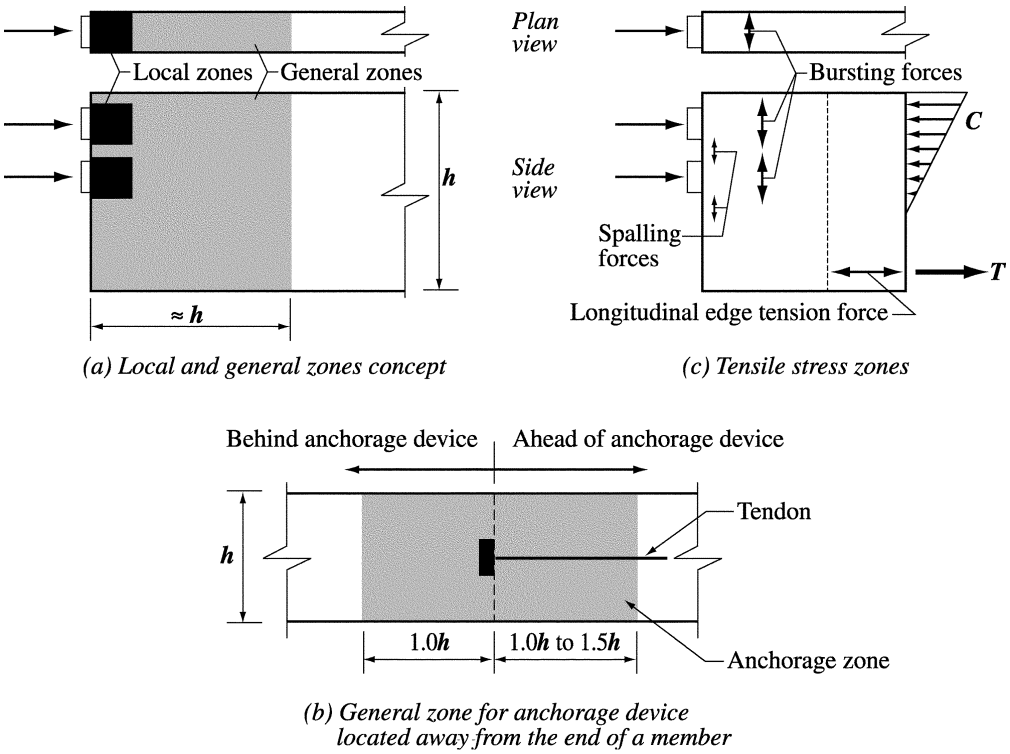
Following the adoption by AASHTO 1994 of comprehensive provisions for post-tensioned anchorage zones, ACI Committee 318 revised the code to be generally consistent with the AASHTO requirements. Thus, the highly detailed AASHTO provisions for analysis and reinforcement detailing are deemed to satisfy the more general ACI 318 requirements. In the specific areas of anchorage device evaluation and acceptance testing, ACI 318 incorporates the detailed AASHTO provisions by reference.

R18.13.1 — Anchorage zone

Based on the Principle of Saint-Venant, the extent of the anchorage zone may be estimated as approximately equal to the largest dimension of the cross section. Local zone and general zone are shown in Fig. R18.13.1(a). When anchorage devices located away from the end of the member are tensioned, large tensile stresses exist locally behind and ahead of the device. These tensile stresses are induced by incompatibility of deformations ahead of [as shown in Fig. R.18.13.1(b)] and behind the anchorage device. The entire shaded region should be considered, as shown in Fig. R18.13.1(b).

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(a) Local and general zones concept

(c) Tensile stress zones

(b) General zone for anchorage device located away from the end of a member

Fig. R18.13.1—Anchorage zones

18.13.2 — Local zone

18.13.2.1 — Design of local zones shall be based upon the factored prestressing force, P_{su}, and the requirements of 9.2.5 and 9.3.2.5.

18.13.2.2 — Local-zone reinforcement shall be provided where required for proper functioning of the anchorage device.

18.13.2.3 — Local-zone requirements of 18.13.2.2 are satisfied by 18.14.1 or 18.15.1 and 18.15.2.

R18.13.2 — Local zone

The local zone resists the very high local stresses introduced by the anchorage device and transfers them to the remainder of the anchorage zone. The behavior of the local zone is strongly influenced by the specific characteristics of the anchorage device and its confining reinforcement, and less influenced by the geometry and loading of the overall structure. Local-zone design sometimes cannot be completed until specific anchorage devices are determined at the shop drawing stage. When special anchorage devices are used, the anchorage device supplier should furnish the test information to show the device is satisfactory under AASHTO "Standard Specifications for Highway Bridges," Division II, Article 10.3.2.3 and provide information regarding necessary conditions for use of the device. The main considerations in local-zone design are the effects of the high bearing pressure and the adequacy of any confining reinforcement provided to increase the capacity of the concrete resisting bearing stresses.

18.13.3 — General zone

18.13.3.1 — Design of general zones shall be based upon the factored prestressing force, P_{su}, and the requirements of 9.2.5 and 9.3.2.5.

18.13.3.2 — General-zone reinforcement shall be provided where required to resist bursting, spalling, and longitudinal edge tension forces induced by

R18.13.3 — General zone

Within the general zone the usual assumption of beam theory that plane sections remain plane is not valid.

Design should consider all regions of tensile stresses that can be caused by the tendon anchorage device, including bursting, spalling, and edge tension as shown in Fig. R18.13.1(c). Also, the compressive stresses immediately ahead [as shown

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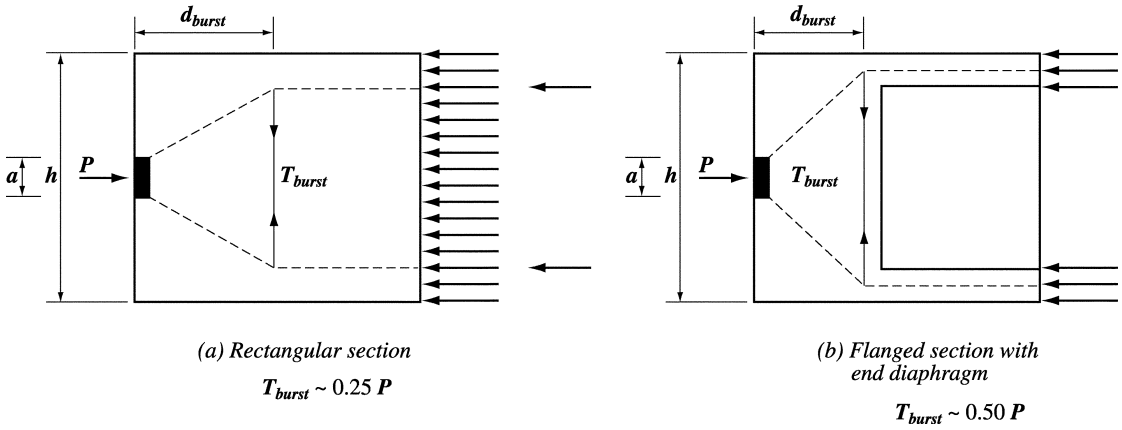


Fig. R18.13.3—Effect of cross section change

anchorage devices. Effects of abrupt change in section shall be considered.

18.13.3.3 — The general-zone requirements of 18.13.3.2 are satisfied by 18.13.4, 18.13.5, 18.13.6 and whichever one of 18.14.2 or 18.14.3 or 18.15.3 is applicable.

18.13.4 — Nominal material strengths

18.13.4.1 — Nominal tensile strength of bonded reinforcement is limited to f_y for nonprestressed reinforcement and to f_{py} for prestressed reinforcement. Nominal tensile stress of unbonded prestressed reinforcement for resisting tensile forces in the anchorage zone shall be limited to $f_{ps} = f_{se} + 10{,}000$.

18.13.4.2 — Except for concrete confined within spirals or hoops providing confinement equivalent to that corresponding to Eq. (10-5), nominal compressive strength of concrete in the general zone shall be limited to $0.7\,l\,f'_{ci}$.

18.13.4.3 — Compressive strength of concrete at time of post-tensioning shall be specified on the design drawings. Unless oversize anchorage devices sized to compensate for the lower compressive strength are used or the prestressing steel is stressed to no more than 50 percent of the final prestressing force, prestressing steel shall not be stressed until f_{ci}, as indicated by tests consistent with the curing of the member, is at least 4000 psi for multistrand tendons or at least 2500 psi for single-strand or bar tendons.

in Fig. R18.13.1(b)] of the local zone should be checked. Sometimes reinforcement requirements cannot be determined until specific tendon and anchorage device layouts are determined at the shop-drawing stage. Design and approval responsibilities should be clearly assigned in the project drawings and specifications.

Abrupt changes in section can cause substantial deviation in force paths. These deviations can greatly increase tension forces as shown in Fig. R18.13.3.

R18.13.4 — Nominal material strengths

Some inelastic deformation of concrete is expected because anchorage zone design is based on a strength approach. The low value for the nominal compressive strength for unconfined concrete reflects this possibility. For well-confined concrete, the effective compressive strength could be increased (See Reference 18.23). The value for nominal tensile strength of bonded prestressing steel is limited to the yield strength of the prestressing steel because Eq. (18-3) may not apply to these nonflexural applications. The value for unbonded prestressing steel is based on the values of 18.7.2 (b) and (c), but is somewhat limited for these short-length, nonflexural applications. Test results given in Reference 18.23 indicate that the compressive stress introduced by auxiliary prestressing applied perpendicular to the axis of the main tendons is effective in increasing the anchorage zone capacity. The inclusion of the l factor for lightweight concrete reflects its lower tensile strength, which is an indirect factor in limiting compressive stresses, as well as the wide scatter and brittleness exhibited in some lightweight concrete anchorage zone tests.

The designer is required to specify concrete strength at the time of stressing in the project drawings and specifications. To limit early shrinkage cracking, monostrand tendons are sometimes stressed at concrete strengths less than 2500 psi. In such cases, either oversized monostrand anchorages are used, or the strands are stressed in stages, often to levels 1/3 to 1/2 the final prestressing force.

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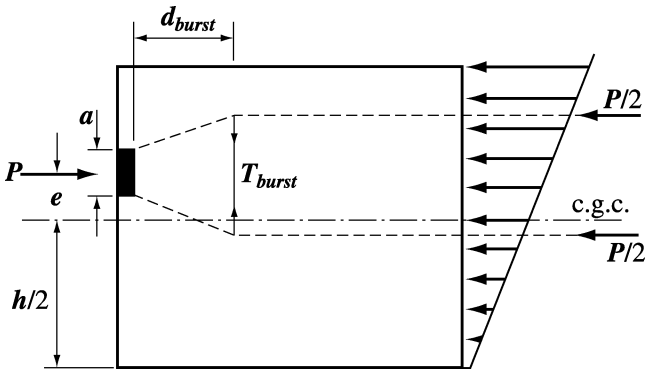


Fig. R18.13.5—Strut-and-tie model example

18.13.5 — Design methods

18.13.5.1 — The following methods shall be permitted for the design of general zones provided that the specific procedures used result in prediction of strength in substantial agreement with results of comprehensive tests:

(a) Equilibrium based plasticity models (strut-and-tie models);

(b) Linear stress analysis (including finite element analysis or equivalent); or

(c) Simplified equations where applicable.

18.13.5.2 — Simplified equations shall not be used where member cross sections are nonrectangular, where discontinuities in or near the general zone cause deviations in the force flow path, where minimum edge distance is less than 1-1/2 times the anchorage device lateral dimension in that direction, or where multiple anchorage devices are used in other than one closely spaced group.

R18.13.5 — Design methods

The list of design methods in 18.13.5.1 includes those procedures for which fairly specific guidelines have been given in References 18.20 and 18.21. These procedures have been shown to be conservative predictors of strength when compared to test results.[18.23] The use of strut-and-tie models is especially helpful for general zone design.[18.23] In many anchorage applications, where substantial or massive concrete regions surround the anchorages, simplified equations can be used except in the cases noted in 18.13.5.2.

For many cases, simplified equations based on References 18.20 and 18.21 can be used. Values for the magnitude of the bursting force, T_{burst}, and for its centroidal distance from the major bearing surface of the anchorage, d_{burst}, may be estimated from Eq. (R18-1) and (R18-2), respectively. The terms of Eq. (R18-1) and (R18-2) are shown in Fig. R18.13.5 for a prestressing force with small eccentricity. In the applications of Eq. (R18-1) and (R18-2), the specified stressing sequence should be considered if more than one tendon is present.

$$T_{burst} = 0.25\Sigma P_{su}\left(1 - \frac{a}{h}\right) \quad \text{(R18-1)}$$

$$d_{burst} = 0.5(h - 2e) \quad \text{(R18-2)}$$

where:

ΣP_{su} = the sum of the total factored prestressing force for the stressing arrangement considered, lb;
a = the depth of anchorage device or single group of closely spaced devices in the direction considered, in.;
e = the eccentricity (always taken as positive) of the anchorage device or group of closely spaced devices with respect to the centroid of the cross section, in.;
h = the depth of the cross section in the direction considered, in.

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Anchorage devices should be treated as closely spaced if their center-to-center spacing does not exceed 1.5 times the width of the anchorage device in the direction considered.

The spalling force for tendons for which the centroid lies within the kern of the section may be estimated as 2 percent of the total factored prestressing force, except for multiple anchorage devices with center-to-center spacing greater than 0.4 times the depth of the section. For large spacings and for cases where the centroid of the tendons is located outside the kern, a detailed analysis is required. In addition, in the post-tensioning of thin sections, or flanged sections, or irregular sections, or when the tendons have appreciable curvature within the general zone, more general procedures such as those of AASHTO Articles 9.21.4 and 9.21.5 will be required. Detailed recommendations for design principles that apply to all design methods are given in Article 9.21.3.4 of Reference 18.20.

18.13.5.3 — The stressing sequence shall be specified on the design drawings and considered in the design.

R18.13.5.3 — The sequence of anchorage device stressing can have a significant effect on the general zone stresses. Therefore, it is important to consider not only the final stage of a stressing sequence with all tendons stressed, but also intermediate stages during construction. The most critical bursting forces caused by each of the sequentially post-tensioned tendon combinations, as well as that of the entire group of tendons, should be taken into account.

18.13.5.4 — Three-dimensional effects shall be considered in design and analyzed using three-dimensional procedures or approximated by considering the summation of effects for two orthogonal planes.

R18.13.5.4 — The provision for three-dimensional effects was included to alert the designer to effects perpendicular to the main plane of the member, such as bursting forces in the thin direction of webs or slabs. In many cases these effects can be determined independently for each direction, but some applications require a fully three-dimensional analysis (for example diaphragms for the anchorage of external tendons).

18.13.5.5 — For anchorage devices located away from the end of the member, bonded reinforcement shall be provided to transfer at least $\mathbf{0.35}P_{su}$ into the concrete section behind the anchor. Such reinforcement shall be placed symmetrically around the anchorage devices and shall be fully developed both behind and ahead of the anchorage devices.

R18.13.5.5 — Where anchorages are located away from the end of a member, local tensile stresses are generated behind these anchorages [see Fig. R18.13.1(b)] due to compatibility requirements for deformations ahead of and behind the anchorages. Bonded tie-back reinforcement is required in the immediate vicinity of the anchorage to limit the extent of cracking behind the anchorage. The requirement $\mathbf{0.35}P_{su}$ was developed using 25 percent of the unfactored prestressing force being resisted by reinforcement at $\mathbf{0.6}\,f_y$.

18.13.5.6 — Where tendons are curved in the general zone, except for monostrand tendons in slabs or where analysis shows reinforcement is not required, bonded reinforcement shall be provided to resist radial and splitting forces.

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18.13.5.7 — Except for monostrand tendons in slabs or where analysis shows reinforcement is not required, minimum reinforcement with a nominal tensile strength equal to 2 percent of each factored prestressing force shall be provided in orthogonal directions parallel to the back face of all anchorage zones to limit spalling.

18.13.5.8 — Tensile strength of concrete shall be neglected in calculations of reinforcement requirements.

18.13.6 — Detailing requirements

Selection of reinforcement sizes, spacings, cover, and other details for anchorage zones shall make allowances for tolerances on the bending, fabrication, and placement of reinforcement, for the size of aggregate, and for adequate placement and consolidation of the concrete.

18.14 — Design of anchorage zones for monostrand or single 5/8 in. diameter bar tendons

18.14.1 — Local zone design

Monostrand or single 5/8 in. or smaller diameter bar anchorage devices and local zone reinforcement shall meet the requirements of ACI 423.6 or the special anchorage device requirements of 18.15.2.

18.14.2 — General-zone design for slab tendons

18.14.2.1 — For anchorage devices for 0.5 in. or smaller diameter strands in normalweight concrete slabs, minimum reinforcement meeting the requirements of 18.14.2.2 and 18.14.2.3 shall be provided unless a detailed analysis satisfying 18.13.5 shows such reinforcement is not required.

18.14.2.2 — Two horizontal bars at least No. 4 in size shall be provided parallel to the slab edge. They shall be permitted to be in contact with the front face of the anchorage device and shall be within a distance of **1/2*h*** ahead of each device. Those bars shall extend at least 6 in. either side of the outer edges of each device.

18.14.2.3 — If the center-to-center spacing of anchorage devices is 12 in. or less, the anchorage devices shall be considered as a group. For each group of six or more anchorage devices, ***n* + 1** hairpin bars or closed stirrups at least No. 3 in size shall be provided, where ***n*** is the number of anchorage devices. One hairpin bar or stirrup shall be placed

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R18.14 — Design of anchorage zones for monostrand or single 5/8 in. diameter bar tendons

R18.14.2 — General-zone design for slab tendons

For monostrand slab tendons, the general-zone minimum reinforcement requirements are based on the recommendations of ACI-ASCE Committee 423,[18.22] which shows typical details. The horizontal bars parallel to the edge required by 18.14.2.2 should be continuous where possible.

The tests on which the recommendations of Reference 18.24 were based were limited to anchorage devices for 1/2 in. diameter, 270 ksi strand, unbonded tendons in normalweight concrete. Thus, for larger strand anchorage devices and for all use in lightweight concrete slabs, ACI-ASCE Committee 423 recommended that the amount and spacing of reinforcement should be conservatively adjusted to provide for the larger anchorage force and smaller splitting tensile strength of lightweight concrete.[18.22]

Both References 18.21 and 18.22 recommend that hairpin bars also be furnished for anchorages located within 12 in. of slab corners to resist edge tension forces. The words "ahead of" in 18.14.2.3 have the meaning shown in Fig. R18.13.1.

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between each anchorage device and one on each side of the group. The hairpin bars or stirrups shall be placed with the legs extending into the slab perpendicular to the edge. The center portion of the hairpin bars or stirrups shall be placed perpendicular to the plane of the slab from **3*h*/8** to ***h*/2** ahead of the anchorage devices.

18.14.2.4 — For anchorage devices not conforming to 18.14.2.1, minimum reinforcement shall be based upon a detailed analysis satisfying 18.13.5.

18.14.3 — General-zone design for groups of monostrand tendons in beams and girders

Design of general zones for groups of monostrand tendons in beams and girders shall meet the requirements of 18.13.3 through 18.13.5.

18.15 — Design of anchorage zones for multistrand tendons

18.15.1 — Local zone design

Basic multistrand anchorage devices and local zone reinforcement shall meet the requirements of AASHTO "Standard Specification for Highway Bridges," Division I, Articles 9.21.7.2.2 through 9.21.7.2.4.

Special anchorage devices shall satisfy the tests required in AASHTO "Standard Specification for Highway Bridges," Division I, Article 9.21.7.3 and described in AASHTO "Standard Specification for Highway Bridges," Division II, Article 10.3.2.3.

18.15.2 — Use of special anchorage devices

Where special anchorage devices are to be used, supplemental skin reinforcement shall be furnished in the corresponding regions of the anchorage zone, in addition to the confining reinforcement specified for the anchorage device. This supplemental reinforcement shall be similar in configuration and at least equivalent in volumetric ratio to any supplementary skin reinforcement used in the qualifying acceptance tests of the anchorage device.

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In those cases where multistrand anchorage devices are used for slab tendons, 18.15 is applicable.

The bursting reinforcement perpendicular to the plane of the slab required by 18.14.2.3 for groups of relatively closely spaced tendons should also be provided in the case of widely spaced tendons if an anchorage device failure could cause more than local damage.

R18.14.3 — General-zone design for groups of monostrand tendons in beams and girders

Groups of monostrand tendons with individual monostrand anchorage devices are often used in beams and girders. Anchorage devices can be treated as closely spaced if their center-to-center spacing does not exceed 1.5 times the width of the anchorage device in the direction considered. If a beam or girder has a single anchorage device or a single group of closely spaced anchorage devices, the use of simplified equations such as those given in R18.13.5 is allowed, unless 18.13.5.2 governs. More complex conditions can be designed using strut-and-tie models. Detailed recommendations for use of such models are given in References 18.22 and 18.23 as well as in R18.13.5.

R18.15 — Design of anchorage zones for multistrand tendons

R18.15.1 — Local zone design

See R18.13.2.

R18.15.2 — Use of special anchorage devices

Skin reinforcement is reinforcement placed near the outer faces in the anchorage zone to limit local crack width and spacing. Reinforcement in the general zone for other actions (flexure, shear, shrinkage, temperature and similar) may be used in satisfying the supplementary skin reinforcement requirement. Determination of the supplementary skin reinforcement depends on the anchorage device hardware used and frequently cannot be determined until the shop-drawing stage.

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18.15.3 — General-zone design

Design for general zones for multistrand tendons shall meet the requirements of 18.13.3 through 18.13.5.

18.16 — Corrosion protection for unbonded tendons

18.16.1 — Unbonded prestressing steel shall be encased with sheathing. The prestressing steel shall be completely coated and the sheathing around the prestressing steel filled with suitable material to inhibit corrosion.

18.16.2 — Sheathing shall be watertight and continuous over entire length to be unbonded.

18.16.3 — For applications in corrosive environments, the sheathing shall be connected to all stressing, intermediate and fixed anchorages in a watertight fashion.

18.16.4 — Unbonded single strand tendons shall be protected against corrosion in accordance with ACI's "Specification for Unbonded Single-Strand Tendons (ACI 423.6)."

18.17 — Post-tensioning ducts

18.17.1 — Ducts for grouted tendons shall be mortar-tight and nonreactive with concrete, prestressing steel, grout, and corrosion inhibitor.

18.17.2 — Ducts for grouted single wire, single strand, or single bar tendons shall have an inside diameter at least 1/4 in. larger than the prestressing steel diameter.

18.17.3 — Ducts for grouted multiple wire, multiple strand, or multiple bar tendons shall have an inside cross-sectional area at least two times the cross-sectional area of the prestressing steel.

18.17.4 — Ducts shall be maintained free of ponded water if members to be grouted are exposed to temperatures below freezing prior to grouting.

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R18.16 — Corrosion protection for unbonded tendons

R18.16.1 — Suitable material for corrosion protection of unbonded prestressting steel should have the properties identified in Section 5.1 of Reference 18.23.

R18.16.2 — Typically, sheathing is a continuous, seamless, high-density polythylene material that is extruded directly onto the coated prestressing steel.

R18.16.4 — In the 1989 code, corrosion protection requirements for unbonded single strand tendons were added in accordance with the Post-Tensioning Institute's "Specification for Unbonded Single Strand Tendons." In the 2002 code, the reference changed to ACI 423.6.

R18.17 — Post-tensioning ducts

R18.17.4 — Water in ducts may cause distress to the surrounding concrete upon freezing. When strands are present, ponded water in ducts should also be avoided. A corrosion inhibitor should be used to provide temporary corrosion protection if prestressing steel is exposed to prolonged periods of moisture in the ducts before grouting.[18.24]

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18.18 — Grout for bonded tendons

18.18.1 — Grout shall consist of portland cement and water; or portland cement, sand, and water.

18.18.2 — Materials for grout shall conform to 18.18.2.1 through 18.18.2.4.

18.18.2.1 — Portland cement shall conform to 3.2.

18.18.2.2 — Water shall conform to 3.4.

18.18.2.3 — Sand, if used, shall conform to "Standard Specification for Aggregate for Masonry Mortar" (ASTM C 144) except that gradation shall be permitted to be modified as necessary to obtain satisfactory workability.

18.18.2.4 — Admixtures conforming to 3.6 and known to have no injurious effects on grout, steel, or concrete shall be permitted. Calcium chloride shall not be used.

18.18.3 — Selection of grout proportions

18.18.3.1 — Proportions of materials for grout shall be based on either (a) or (b):

(a) Results of tests on fresh and hardened grout prior to beginning grouting operations; or

(b) Prior documented experience with similar materials and equipment and under comparable field conditions.

18.18.3.2 — Cement used in the work shall correspond to that on which selection of grout proportions was based.

18.18.3.3 — Water content shall be minimum necessary for proper pumping of grout; however, water-cement ratio shall not exceed 0.45 by weight.

18.18.3.4 — Water shall not be added to increase grout flowability that has been decreased by delayed use of the grout.

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R18.18 — Grout for bonded tendons

Proper grout and grouting procedures are critical to post-tensioned construction.[18.25,18.26] Grout provides bond between the prestressing steel and the duct, and provides corrosion protection to the prestressing steel.

Past success with grout for bonded tendons has been with portland cement. A blanket endorsement of all cementitious materials (defined in 2.1) for use with this grout is deemed inappropriate because of a lack of experience or tests with cementitious materials other than portland cement and a concern that some cementitious materials might introduce chemicals listed as harmful to tendons in R18.18.2. Thus, portland cement in 18.18.1 and water-cement ratio in 18.18.3.3 are retained in the code.

R18.18.2 — The limitations on admixtures in 3.6 apply to grout. Substances known to be harmful to tendons, grout, or concrete are chlorides, fluorides, sulfites, and nitrates. Aluminum powder or other expansive admixtures, when approved, should produce an unconfined expansion of 5 to 10 percent. Neat cement grout is used in almost all building construction. Use of finely graded sand in the grout should only be considered with large ducts having large void areas.

R18.18.3 — Selection of grout proportions

Grout proportioned in accordance with these provisions will generally lead to 7-day compressive strength on standard 2 in. cubes in excess of 2500 psi and 28-day strengths of about 4000 psi. The handling and placing properties of grout are usually given more consideration than strength when designing grout mixtures.

CODE

18.18.4 — Mixing and pumping grout

18.18.4.1 — Grout shall be mixed in equipment capable of continuous mechanical mixing and agitation that will produce uniform distribution of materials, passed through screens, and pumped in a manner that will completely fill the ducts.

18.18.4.2 — Temperature of members at time of grouting shall be above 35 F and shall be maintained above 35 F until field-cured 2 in. cubes of grout reach a minimum compressive strength of 800 psi.

18.18.4.3 — Grout temperatures shall not be above 90 F during mixing and pumping.

18.19 — Protection for prestressing steel

Burning or welding operations in the vicinity of prestressing steel shall be performed so that prestressing steel is not subject to excessive temperatures, welding sparks, or ground currents.

18.20 — Application and measurement of prestressing force

18.20.1 — Prestressing force shall be determined by both of (a) and (b):

(a) Measurement of steel elongation. Required elongation shall be determined from average load-elongation curves for the prestressing steel used;

(b) Observation of jacking force on a calibrated gage or load cell or by use of a calibrated dynamometer.

Cause of any difference in force determination between (a) and (b) that exceeds 5 percent for pretensioned elements or 7 percent for post-tensioned construction shall be ascertained and corrected.

COMMENTARY

R18.18.4 — Mixing and pumping grout

In an ambient temperature of 35 F, grout with an initial minimum temperature of 60 F may require as much as 5 days to reach 800 psi. A minimum grout temperature of 60 F is suggested because it is consistent with the recommended minimum temperature for concrete placed at an ambient temperature of 35 F. Quickset grouts, when approved, may require shorter periods of protection and the recommendations of the suppliers should be followed. Test cubes should be cured under temperature and moisture conditions as close as possible to those of the grout in the member. Grout temperatures in excess of 90 F will lead to difficulties in pumping.

R18.20 — Application and measurement of prestressing force

R18.20.1 — Elongation measurements for prestressed elements should be in accordance with the procedures outlined in the "Manual for Quality Control for Plants and Production of Precast and Prestressed Concrete Products," published by the Precast/Prestressed Concrete Institute.[18.27]

Section 18.18.1 of the 1989 code was revised to permit 7 percent tolerance in prestressing steel force determined by gage pressure and elongation measurements for post-tensioned construction. Elongation measurements for post-tensioned construction are affected by several factors that are less significant, or that do not exist, for pretensioned elements. The friction along prestressing steel in post-tensioning applications may be affected to varying degrees by placing tolerances and small irregularities in tendon profile due to concrete placement. The friction coefficients between the prestressing steel and the duct are also subject to variation. The 5 percent tolerance that has appeared since the 1963 code was proposed by ACI-ASCE Committee 423 in 1958,[18.3] and primarily reflected experience with production of pretensioned concrete elements. Because the tendons for pretensioned elements are usually stressed in the air with minimal friction effects, the 5 percent tolerance for such elements was retained.

CODE

18.20.2 — Where the transfer of force from the bulkheads of pretensioning bed to the concrete is accomplished by flame cutting prestressing steel, cutting points and cutting sequence shall be predetermined to avoid undesired temporary stresses.

18.20.3 — Long lengths of exposed pretensioned strand shall be cut near the member to minimize shock to concrete.

18.20.4 — Total loss of prestress due to unreplaced broken prestressing steel shall not exceed 2 percent of total prestress.

COMMENTARY

R18.20.4 — This provision applies to all prestressed concrete members. For cast-in-place post-tensioned slab systems, a member should be that portion considered as an element in the design, such as the joist and effective slab width in one-way joist systems, or the column strip or middle strip in two-way flat plate systems.

18.21 — Post-tensioning anchorages and couplers

R18.21 — Post-tensioning anchorages and couplers

18.21.1 — Anchorages and couplers for bonded and unbonded tendons shall develop at least 95 percent of the specified breaking strength of the prestressing steel, when tested in an unbonded condition, without exceeding anticipated set. For bonded tendons, anchorages and couplers shall be located so that 100 percent of the specified breaking strength of the prestressing steel shall be developed at critical sections after the prestressing steel is bonded in the member.

R18.21.1 — In the 1986 interim code, the separate provisions for strength of unbonded and bonded tendon anchorages and couplers presented in 18.19.1 and 18.19.2 of the 1983 code were combined into a single revised 18.19.1 covering anchorages and couplers for both unbonded and bonded tendons. Since the 1989 code, the required strength of the tendon-anchorage or tendon-coupler assemblies for both unbonded and bonded tendons, when tested in an unbonded state, is based on 95 percent of the specified breaking strength of the prestressing steel in the test. The prestressing steel material should comply with the minimum provisions of the applicable ASTM specifications as outlined in 3.5.5. The specified strength of anchorages and couplers exceeds the maximum design strength of the prestressing steel by a substantial margin, and, at the same time, recognizes the stress-riser effects associated with most available post-tensioning anchorages and couplers. Anchorage and coupler strength should be attained with a minimum amount of permanent deformation and successive set, recognizing that some deformation and set will occur when testing to failure. Tendon assemblies should conform to the 2 percent elongation requirements in ACI 301[18.28] and industry recommendations.[18.14] Anchorages and couplers for bonded tendons that develop less than 100 percent of the specified breaking strength of the prestressing steel should be used only where the bond transfer length between the anchorage or coupler and critical sections equals or exceeds that required to develop the prestressing steel strength. This bond length may be calculated by the results of tests of bond characteristics of untensioned prestressing strand,[18.29] or by bond tests on other prestressing steel materials, as appropriate.

18.21.2 — Couplers shall be placed in areas approved by the engineer and enclosed in housing long enough to permit necessary movements.

CODE

18.21.3 — In unbonded construction subject to repetitive loads, special attention shall be given to the possibility of fatigue in anchorages and couplers.

18.21.4 — Anchorages, couplers, and end fittings shall be permanently protected against corrosion.

18.22 — External post-tensioning

18.22.1 — Post-tensioning tendons shall be permitted to be external to any concrete section of a member. The strength and serviceability design methods of this code shall be used in evaluating the effects of external tendon forces on the concrete structure.

18.22.2 — External tendons shall be considered as unbonded tendons when computing flexural strength unless provisions are made to effectively bond the external tendons to the concrete section along its entire length.

18.22.3 — External tendons shall be attached to the concrete member in a manner that maintains the desired eccentricity between the tendons and the concrete centroid throughout the full range of anticipated member deflection.

18.22.4 — External tendons and tendon anchorage regions shall be protected against corrosion, and the details of the protection method shall be indicated on the drawings or in the project specifications.

COMMENTARY

R18.21.3 — For discussion on fatigue loading, see Reference 18.30.

For detailed recommendations on tests for static and cyclic loading conditions for tendons and anchorage fittings of unbonded tendons, see Section 4.1.3 of Reference 18.9, and Section 15.2.2 of Reference 18.28.

R18.21.4 — For recommendations regarding protection see Sections 4.2 and 4.3 of Reference 18.9, and Sections 3.4, 3.6, 5, 6, and 8.3 of Reference 18.23.

R18.22 — External post-tensioning

External attachment of tendons is a versatile method of providing additional strength, or improving serviceability, or both, in existing structures. It is well suited to repair or upgrade existing structures and permits a wide variety of tendon arrangements.

Additional information on external post-tensioning is given in Reference 18.31.

R18.22.3 — External tendons are often attached to the concrete member at various locations between anchorages (such as midspan, quarter points, or third points) for desired load balancing effects, for tendon alignment, or to address tendon vibration concerns. Consideration should be given to the effects caused by the tendon profile shifting in relationship to the concrete centroid as the member deforms under effects of post-tensioning and applied load.

R18.22.4 — Permanent corrosion protection can be achieved by a variety of methods. The corrosion protection provided should be suitable to the environment in which the tendons are located. Some conditions will require that the prestressing steel be protected by concrete cover or by cement grout in polyethylene or metal tubing; other conditions will permit the protection provided by coatings such as paint or grease. Corrosion protection methods should meet the fire protection requirements of the general building code, unless the installation of external post-tensioning is to only improve serviceability.

CHAPTER 19 — SHELLS AND FOLDED PLATE MEMBERS

CODE

19.0 — Notation

E_c = modulus of elasticity of concrete, psi. See 8.5.1
f_c' = specified compressive strength of concrete, psi
$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi
f_y = specified yield strength of nonprestressed reinforcement, psi
h = thickness of shell or folded plate, in.
ℓ_d = development length, in.
f = strength reduction factor. See 9.3

19.1 — Scope and definitions

19.1.1 — Provisions of Chapter 19 shall apply to thin shell and folded plate concrete structures, including ribs and edge members.

19.1.2 — All provisions of this code not specifically excluded, and not in conflict with provisions of Chapter 19, shall apply to thin-shell structures.

19.1.3 — ***Thin shells*** — Three-dimensional spatial structures made up of one or more curved slabs or folded plates whose thicknesses are small compared to their other dimensions. Thin shells are characterized by their three-dimensional load-carrying behavior, which is determined by the geometry of their forms, by the manner in which they are supported, and by the nature of the applied load.

COMMENTARY

R19.0 — Notation

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

R19.1 — Scope and definitions

The code and commentary provide information on the design, analysis, and construction of concrete thin shells and folded plates. The process began in 1964 with the publication of a practice and commentary by ACI Committee 334,[19.1] and continued with the inclusion of Chapter 19 in the 1971 code. The 1982 revision of ACI 334R.1 reflected additional experience in design, analysis, and construction and was influenced by the publication of the **"Recommendations for Reinforced Concrete Shells and Folded Plates"** of the International Association for Shell and Spatial Structures (IASS) in 1979.[19.2]

Since Chapter 19 applies to concrete thin shells and folded plates of all shapes, extensive discussion of their design, analysis, and construction in the commentary is not possible. Additional information can be obtained from the references. Performance of shells and folded plates requires special attention to detail.[19.3]

R19.1.1 — Discussion of the application of thin shells in special structures such as cooling towers and circular prestressed concrete tanks may be found in the reports of ACI Committee 334[19.4] and ACI Committee 373.[19.5]

R19.1.3 — Common types of thin shells are domes (surfaces of revolution),[19.6,19.7] cylindrical shells,[19.7] barrel vaults,[19.8] conoids,[19.8] elliptical paraboloids,[19.8] hyperbolic paraboloids,[19.9] and groined vaults.[19.9]

CODE

19.1.4 — ***Folded plates*** — A special class of shell structure formed by joining flat, thin slabs along their edges to create a three-dimensional spatial structure.

19.1.5 — ***Ribbed shells*** — Spatial structures with material placed primarily along certain preferred rib lines, with the area between the ribs filled with thin slabs or left open.

19.1.6 — ***Auxiliary members*** — Ribs or edge beams that serve to strengthen, stiffen, or support the shell; usually, auxiliary members act jointly with the shell.

19.1.7 — ***Elastic analysis*** — An analysis of deformations and internal forces based on equilibrium, compatibility of strains, and assumed elastic behavior, and representing to a suitable approximation the three-dimensional action of the shell together with its auxiliary members.

19.1.8 — ***Inelastic analysis*** — an analysis of deformations and internal forces based on equilibrium, nonlinear stress-strain relations for concrete and reinforcement, consideration of cracking and time-dependent effects, and compatibility of strains. The analysis shall represent to a suitable approximation three-dimensional action of the shell together with its auxiliary members.

COMMENTARY

R19.1.4 — Folded plates may be prismatic,[19.6,19.7] nonprismatic,[19.7] or faceted. The first two types consist generally of planar thin slabs joined along their longitudinal edges to form a beam-like structure spanning between supports. Faceted folded plates are made up of triangular or polygonal planar thin slabs joined along their edges to form three-dimensional spatial structures.

R19.1.5 — Ribbed shells[19.8,19.9] generally have been used for larger spans where the increased thickness of the curved slab alone becomes excessive or uneconomical. Ribbed shells are also used because of the construction techniques employed and to enhance the aesthetic impact of the completed structure.

R19.1.6 — Most thin shell structures require ribs or edge beams at their boundaries to carry the shell boundary forces, to assist in transmitting them to the supporting structure, and to accommodate the increased amount of reinforcement in these areas.

R19.1.7 — Elastic analysis of thin shells and folded plates can be performed using any method of structural analysis based on assumptions that provide suitable approximations to the three-dimensional behavior of the structure. The method should determine the internal forces and displacements needed in the design of the shell proper, the rib or edge members, and the supporting structure. Equilibrium of internal forces and external loads and compatibility of deformations should be satisfied.

Methods of elastic analysis based on classical shell theory, simplified mathematical or analytical models, or numerical solutions using finite element,[19.10] finite differences,[19.8] or numerical integration techniques,[19.8,19.11] are described in the cited references.

The choice of the method of analysis and the degree of accuracy required depends on certain critical factors. These include: the size of the structure, the geometry of the thin shell or folded plate, the manner in which the structure is supported, the nature of the applied load, and the extent of personal or documented experience regarding the reliability of the given method of analysis in predicting the behavior of the specific type of shell[19.8] or folded plate.[19.7]

R19.1.8—Inelastic analysis of thin shells and folded plates can be performed using a refined method of analysis based on the specific nonlinear material properties, nonlinear behavior due to the cracking of concrete, and time-dependent effects such as creep, shrinkage, temperature, and load history. These effects are incorporated in order to trace the response and crack propagation of a reinforced concrete shell through the elastic, inelastic, and ultimate ranges. Such analyses usually require incremental loading and iterative procedures to converge on solutions that satisfy both equilibrium and strain compatibility. [19.12,19.13]

CODE

19.1.9 — ***Experimental analysis*** — An analysis procedure based on the measurement of deformations or strains, or both, of the structure or its model; experimental analysis is based on either elastic or inelastic behavior.

19.2 — Analysis and design

19.2.1 — Elastic behavior shall be an accepted basis for determining internal forces and displacements of thin shells. This behavior shall be permitted to be established by computations based on an analysis of the uncracked concrete structure in which the material is assumed linearly elastic, homogeneous, and isotropic. Poisson's ratio of concrete shall be permitted to be taken equal to zero.

19.2.2 — Inelastic analyses shall be permitted to be used where it can be shown that such methods provide a safe basis for design.

19.2.3 — Equilibrium checks of internal resistances and external loads shall be made to ensure consistency of results.

19.2.4 — Experimental or numerical analysis procedures shall be permitted where it can be shown that such procedures provide a safe basis for design.

19.2.5 — Approximate methods of analysis shall be permitted where it can be shown that such methods provide a safe basis for design.

COMMENTARY

R19.2 — Analysis and design

R19.2.1 — For types of shell structures where experience, tests, and analyses have shown that the structure can sustain reasonable overloads without undergoing brittle failure, elastic analysis is an acceptable procedure. The designer may assume that reinforced concrete is ideally elastic, homogeneous, and isotropic, having identical properties in all directions. An analysis should be performed for the shell considering service load conditions. The analysis of shells of unusual size, shape, or complexity should consider behavior through the elastic, cracking, and inelastic stages.

R19.2.2 — Several inelastic analysis procedures contain possible solution methods. [19.12,19.13]

R19.2.4 — Experimental analysis of elastic models[19.14] has been used as a substitute for an analytical solution of a complex shell structure. Experimental analysis of reinforced microconcrete models through the elastic, cracking, inelastic, and ultimate stages should be considered for important shells of unusual size, shape, or complexity.

For model analysis, only those portions of the structure that significantly affect the items under study need be simulated. Every attempt should be made to ensure that the experiments reveal the quantitative behavior of the prototype structure.

Wind tunnel tests of a scaled-down model do not necessarily provide usable results and should be conducted by a recognized expert in wind tunnel testing of structural models.

R19.2.5 — Solutions that include both membrane and bending effects and satisfy conditions of compatibility and equilibrium are encouraged. Approximate solutions that satisfy statics but not the compatibility of strains may be used only when extensive experience has proved that safe designs have resulted from their use. Such methods include beam-type analysis for barrel shells and folded plates having large ratios of span to either width or radius of curvature, simple membrane analysis for shells of revolution, and others in which the equations of equilibrium are satisfied, while the strain compatibility equations are not.

CODE

19.2.6 — In prestressed shells, the analysis shall also consider behavior under loads induced during prestressing, at cracking load, and at factored load. Where tendons are draped within a shell, design shall take into account force components on the shell resulting from the tendon profile not lying in one plane.

19.2.7 — The thickness of a shell and its reinforcement shall be proportioned for the required strength and serviceability, using either the strength design method of 8.1.1 or the design method of 8.1.2.

19.2.8 — Shell instability shall be investigated and shown by design to be precluded.

19.2.9 — Auxiliary members shall be designed according to the applicable provisions of the code. It shall be permitted to assume that a portion of the shell equal to the flange width, as specified in 8.10, acts with the auxiliary member. In such portions of the shell, the reinforcement perpendicular to the auxiliary member

COMMENTARY

R19.2.6 — If the shell is prestressed, the analysis should include its strength at factored loads as well as its adequacy under service loads, under the load that causes cracking, and under loads induced during prestressing. Axial forces due to draped tendons may not lie in one plane and due consideration should be given to the resulting force components. The effects of post-tensioning of shell supporting members should be taken into account.

R19.2.7 — The thin shell's thickness and reinforcement are required to be proportioned to satisfy the strength provisions of this code, and to resist internal forces obtained from an analysis, an experimental model study, or a combination thereof. Reinforcement sufficient to minimize cracking under service load conditions should be provided. The thickness of the shell is often dictated by the required reinforcement and the construction constraints, by 19.2.8, or by the code minimum thickness requirements.

R19.2.8 — Thin shells, like other structures that experience in-plane membrane compressive forces, are subject to buckling when the applied load reaches a critical value. Because of the surface-like geometry of shells, the problem of calculating buckling load is complex. If one of the principal membrane forces is tensile, the shell is less likely to buckle than if both principal membrane forces are compressive. The kinds of membrane forces that develop in a shell depend on its initial shape and the manner in which the shell is supported and loaded. In some types of shells, post-buckling behavior should be considered in determining safety against instability.[19.2]

Investigation of thin shells for stability should consider the effect of (1) anticipated deviation of the geometry of the shell surface as-built from the idealized, geometry, (2) large deflections, (3) creep and shrinkage of concrete, (4) inelastic properties of materials, (5) cracking of concrete, (6) location, amount, and orientation of reinforcement, and (7) possible deformation of supporting elements.

Measures successfully used to improve resistance to buckling include the provision of two mats of reinforcement—one near each outer surface of the shell, a local increase of shell curvatures, the use of ribbed shells, and the use of concrete with high tensile strength and low creep.

A procedure for determining critical buckling loads of shells is given in the IASS recommendations.[19.2] Some recommendations for buckling design of domes used in industrial applications are given in References 19.5 and 19.15.

CODE

shall be at least equal to that required for the flange of a T-beam by 8.10.5.

19.2.10 — Strength design of shell slabs for membrane and bending forces shall be based on the distribution of stresses and strains as determined from either an elastic or an inelastic analysis.

19.2.11 — In a region where membrane cracking is predicted, the nominal compressive strength parallel to the cracks shall be taken as $0.4f_c'$.

19.3 — Design strength of materials

19.3.1 — Specified compressive strength of concrete f_c' at 28 days shall not be less than 3000 psi.

19.3.2 — Specified yield strength of nonprestressed reinforcement f_y shall not exceed 60,000 psi.

19.4 — Shell reinforcement

19.4.1 — Shell reinforcement shall be provided to resist tensile stresses from internal membrane forces, to resist tension from bending and twisting moments, to limit shrinkage and temperature crack width and spacing, and as special reinforcement at shell boundaries, load attachments, and shell openings.

19.4.2 — Tensile reinforcement shall be provided in two or more directions and shall be proportioned such that its resistance in any direction equals or exceeds the component of internal forces in that direction.

Alternatively, reinforcement for the membrane forces in the slab shall be calculated as the reinforcement required to resist axial tensile forces plus the tensile force due to shear-friction required to transfer shear across any cross section of the membrane. The assumed coefficient of friction shall not exceed 1.0λ where $\lambda = 1.0$ for normalweight concrete, **0.85** for sand-lightweight concrete, and **0.75** for all-lightweight concrete. Linear interpolation shall be permitted when partial sand replacement is used.

COMMENTARY

R19.2.10 — The stresses and strains in the shell slab used for design are those determined by analysis (elastic or inelastic) multiplied by appropriate load factors. Because of detrimental effects of membrane cracking, the computed tensile strain in the reinforcement under factored loads should be limited.

R19.2.11 — When principal tensile stress produces membrane cracking in the shell, experiments indicate the attainable compressive strength in the direction parallel to the cracks is reduced.[19.16,19.17]

R19.4 — Shell reinforcement

R19.4.1 — At any point in a shell, two different kinds of internal forces may occur simultaneously: those associated with membrane action, and those associated with bending of the shell. The membrane forces are assumed to act in the tangential plane midway between the surfaces of the shell, and are the two axial forces and the membrane shears. Flexural effects include bending moments, twisting moments, and the associated transverse shears. Limiting membrane crack width and spacing due to shrinkage, temperature, and service load conditions is a major design consideration.

R19.4.2 — The requirement of ensuring strength in all directions is based on safety considerations. Any method that ensures sufficient strength consistent with equilibrium is acceptable. The direction of the principal membrane tensile force at any point may vary depending on the direction, magnitudes, and combinations of the various applied loads.

The magnitude of the internal membrane forces, acting at any point due to a specific load, is generally calculated on the basis of an elastic theory in which the shell is assumed as uncracked. The computation of the required amount of reinforcement to resist the internal membrane forces has been traditionally based on the assumption that concrete does not resist tension. The associated deflections, and the possibility of cracking, should be investigated in the serviceability phase of the design. Achieving this may require a working stress design for steel selection.

CODE

COMMENTARY

Where reinforcement is not placed in the direction of the principal tensile forces and where cracks at the service load level are objectionable, the computation of reinforcement may have to be based on a more refined approach[19.16,19.18,19.19] that considers the existence of cracks. In the cracked state, the concrete is assumed to be unable to resist either tension or shear. Thus, equilibrium is attained by equating tensile resisting forces in reinforcement and compressive resisting forces in concrete.

The alternative method to calculate orthogonal reinforcement is the shear-friction method. It is based on the assumption that shear integrity of a shell should be maintained at factored loads. It is not necessary to calculate principal stresses if the alternative approach is used.

19.4.3 — The area of shell reinforcement at any section as measured in two orthogonal directions shall not be less than the slab shrinkage or temperature reinforcement required by 7.12.

R19.4.3 — Minimum membrane reinforcement corresponding to slab shrinkage and temperature reinforcement are to be provided in at least two approximately orthogonal directions even if the calculated membrane forces are compressive in one or more directions.

19.4.4 — Reinforcement for shear and bending moments about axes in the plane of the shell slab shall be calculated in accordance with Chapters 10, 11, and 13.

19.4.5 — The area of shell tension reinforcement shall be limited so that the reinforcement will yield before either crushing of concrete in compression or shell buckling can take place.

R19.4.5 — The requirement that the tensile reinforcement yields before the concrete crushes anywhere is consistent with 10.3.3. Such crushing can also occur in regions near supports and for some shells where the principal membrane forces are approximately equal and opposite in sign.

19.4.6 — In regions of high tension, membrane reinforcement shall, if practical, be placed in the general directions of the principal tensile membrane forces. Where this is not practical, it shall be permitted to place membrane reinforcement in two or more component directions.

R19.4.6 — Generally, for all shells, and particularly in regions of substantial tension, the orientation of reinforcement should approximate the directions of the principal tensile membrane forces. However, in some structures it is not possible to detail the reinforcement to follow the stress trajectories. For such cases, orthogonal component reinforcement is allowed.

19.4.7 — If the direction of reinforcement varies more than 10 deg from the direction of principal tensile membrane force, the amount of reinforcement shall be reviewed in relation to cracking at service loads.

R19.4.7 — When the directions of reinforcement deviate significantly (more than 10 deg) from the directions of the principal membrane forces, higher strains in the shell occur to develop the capacity of reinforcement. This might lead to the development of unacceptable wide cracks. The crack width should be estimated and limited if necessary.

Permissible crack widths for service loads under different environmental conditions are given in the report of ACI Committee 224.[19.20] Crack width can be limited by an increase in the amount of reinforcement used, by reducing the stress at the service load level, by providing reinforcement in three or more directions in the plane of the shell, or by using closer spacing of smaller diameter bars.

CODE

19.4.8 — Where the magnitude of the principal tensile membrane stress within the shell varies greatly over the area of the shell surface, reinforcement resisting the total tension shall be permitted to be concentrated in the regions of largest tensile stress where it can be shown that this provides a safe basis for design. However, the ratio of shell reinforcement in any portion of the tensile zone shall be not less than 0.0035 based on the overall thickness of the shell.

19.4.9 — Reinforcement required to resist shell bending moments shall be proportioned with due regard to the simultaneous action of membrane axial forces at the same location. Where shell reinforcement is required in only one face to resist bending moments, equal amounts shall be placed near both surfaces of the shell even though a reversal of bending moments is not indicated by the analysis.

19.4.10 — Shell reinforcement in any direction shall not be spaced farther apart than 18 in. nor farther apart than five times the shell thickness. Where the principal membrane tensile stress on the gross concrete area due to factored loads exceeds $4\phi\sqrt{f'_c}$, reinforcement shall not be spaced farther apart than three times the shell thickness.

19.4.11 — Shell reinforcement at the junction of the shell and supporting members or edge members shall be anchored in or extended through such members in accordance with the requirements of Chapter 12, except that the minimum development length shall be $1.2\ell_d$ but not less than 18 in.

19.4.12 — Splice lengths of shell reinforcement shall be governed by the provisions of Chapter 12, except that the minimum splice length of tension bars shall be 1.2 times the value required by Chapter 12 but not less than 18 in. The number of splices in principal tensile reinforcement shall be kept to a practical minimum. Where splices are necessary they shall be staggered at least ℓ_d with not more than one-third of the reinforcement spliced at any section.

19.5 — Construction

19.5.1 — When removal of formwork is based on a specific modulus of elasticity of concrete because of stability or deflection considerations, the value of the modulus of elasticity E_c shall be determined from flexural tests of field-cured beam specimens. The number of test specimens, the dimensions of test beam specimens, and test procedures shall be specified by the engineer.

COMMENTARY

R19.4.8 — The practice of concentrating tensile reinforcement in the regions of maximum tensile stress has led to a number of successful and economical designs, primarily for long folded plates, long barrel vault shells, and for domes. The requirement of providing the minimum reinforcement in the remaining tensile zone is intended to limit crack width and spacing.

R19.4.9 — The design method should ensure that the concrete sections, including consideration of the reinforcement, are capable of developing the internal forces required by the equations of equilibrium.[19.21]The sign of bending moments may change rapidly from point to point of a shell. For this reason, reinforcement to resist bending, where required, is to be placed near both outer surfaces of the shell. In many cases, the thickness required to provide proper cover and spacing for the multiple layers of reinforcement may govern the design of the shell thickness.

R19.4.10 — The value of ϕ to be used is that prescribed in 9.3.2.1 for axial tension.

R19.4.11 and R19.4.12 — On curved shell surfaces it is difficult to control the alignment of precut reinforcement. This should be considered to avoid insufficient splice and development lengths. Sections 19.4.11 and 19.4.12 require extra reinforcement length to maintain the minimum lengths on curved surfaces.

R19.5 — Construction

R19.5.1 — When early removal of forms is necessary, the magnitude of the modulus of elasticity at the time of proposed form removal should be investigated to ensure safety of the shell with respect to buckling, and to restrict deflections.[19.3,19.22] The value of the modulus of elasticity E_c should be obtained from a flexural test of field-cured specimens. It is not sufficient to determine the modulus from the formula in 8.5.1, even if f'_c is determined for the field-cured specimen.

CODE

19.5.2 — The engineer shall specify the tolerances for the shape of the shell. If construction results in deviations from the shape greater than the specified tolerances, an analysis of the effect of the deviations shall be made and any required remedial actions shall be taken to ensure safe behavior.

COMMENTARY

R19.5.2 — In some types of shells, small local deviations from the theoretical geometry of the shell can cause relatively large changes in local stresses and in overall safety against instability. These changes can result in local cracking and yielding that may make the structure unsafe or can greatly affect the critical load producing instability. The effect of such deviations should be evaluated and any necessary remedial actions should be taken. Special attention is needed when using air supported form systems.[19.23]

PART 6 — SPECIAL CONSIDERATIONS

CHAPTER 20 — STRENGTH EVALUATION OF EXISTING STRUCTURES

CODE

20.0 — Notation

D = dead loads or related internal moments and forces

f_c' = specified compressive strength of concrete, psi

h = overall thickness of member, in.

L = live loads or related internal moments and forces

ℓ_t = span of member under load test, in. (The shorter span for two-way slab systems.) Span is the smaller of (a) distance between centers of supports, and (b) clear distance between supports plus thickness h of member. In Eq. (20-1), span for a cantilever shall be taken as twice the distance from support to cantilever end

D_{max} = measured maximum deflection, in. See Eq. (20-1)

D_{rmax} = measured residual deflection, in. See Eq. (20-2) and (20-3)

D_{fmax} = maximum deflection measured during the second test relative to the position of the structure at the beginning of the second test, in. See Eq. (20-3)

20.1 — Strength evaluation — General

20.1.1 — If there is doubt that a part or all of a structure meets the safety requirements of this code, a strength evaluation shall be carried out as required by the engineer or building official.

COMMENTARY

R20.0 — Notation

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

R20.1 — Strength evaluation — General

Chapter 20 does not cover load testing for the approval of new design or construction methods. (See 16.10 for recommendations on strength evaluation of precast concrete members.) Provisions of Chapter 20 may be used to evaluate whether a structure or a portion of a structure satisfies the safety requirements of this code. A strength evaluation may be required if the materials are considered to be deficient in quality, if there is evidence indicating faulty construction, if a structure has deteriorated, if a building will be used for a new function, or if, for any reason, a structure or a portion of it does not appear to satisfy the requirements of the code. In such cases, Chapter 20 provides guidance for investigating the safety of the structure.

If the safety concerns are related to an assembly of elements or an entire structure, it is not feasible to load test every element and section to the maximum. In such cases, it is appropriate that an investigation plan be developed to address the specific safety concerns. If a load test is described as part of the strength evaluation process, it is desirable for all parties

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involved to come to an agreement about the region to be loaded, the magnitude of the load, the load test procedure, and acceptance criteria before any load tests are conducted.

20.1.2 — If the effect of the strength deficiency is well understood and if it is feasible to measure the dimensions and material properties required for analysis, analytical evaluations of strength based on those measurements shall suffice. Required data shall be determined in accordance with 20.2.

R20.1.2 — Strength considerations related to axial load, flexure, and combined axial load and flexure are well understood. There are reliable theories relating strength and short-term displacement to load in terms of dimensional and material data for the structure.

To determine the strength of the structure by analysis, calculations should be based on data gathered on the actual dimensions of the structure, properties of the materials in place, and all pertinent details. Requirements for data collection are in 20.2.

20.1.3 — If the effect of the strength deficiency is not well understood or if it is not feasible to establish the required dimensions and material properties by measurement, a load test shall be required if the structure is to remain in service.

R20.1.3 — If the shear or bond strength of an element is critical in relation to the doubt expressed about safety, a test may be the most efficient solution to eliminate or confirm the doubt. A test may also be appropriate if it is not feasible to determine the material and dimensional properties required for analysis, even if the cause of the concern relates to flexure or axial load.

Wherever possible and appropriate, support the results of the load test by analysis.

20.1.4 — If the doubt about safety of a part or all of a structure involves deterioration, and if the observed response during the load test satisfies the acceptance criteria, the structure or part of the structure shall be permitted to remain in service for a specified time period. If deemed necessary by the engineer, periodic reevaluations shall be conducted.

R20.1.4 — For a deteriorating structure, the acceptance provided by the load test may not be assumed to be without limits in terms of time. In such cases, a periodic inspection program is useful. A program that involves physical tests and periodic inspection can justify a longer period in service. Another option for maintaining the structure in service, while the periodic inspection program continues, is to limit the live load to a level determined to be appropriate.

The length of the specified time period should be based on consideration of (a) the nature of the problem, (b) environmental and load effects, (c) service history of the structure, and (d) scope of the periodic inspection program. At the end of a specified time period, further strength evaluation is required if the structure is to remain in service.

With the agreement of all concerned parties, special procedures may be devised for periodic testing that do not necessarily conform to the loading and acceptance criteria specified in Chapter 20.

20.2 — Determination of required dimensions and material properties

R20.2 — Determination of required dimensions and material properties

This section applies if it is decided to make an analytical evaluation (see 20.1.2).

20.2.1 — Dimensions of the structural elements shall be established at critical sections.

R20.2.1 — Critical sections are where each type of stress calculated for the load in question reaches its maximum value.

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20.2.2 — Locations and sizes of the reinforcing bars, welded wire fabric, or tendons shall be determined by measurement. It shall be permitted to base reinforcement locations on available drawings if spot checks are made confirming the information on the drawings.

20.2.3 — If required, concrete strength shall be based on results of cylinder tests or tests of cores removed from the part of the structure where the strength is in doubt. Concrete strengths shall be determined as specified in 5.6.5.

20.2.4 — If required, reinforcement or prestressing steel strength shall be based on tensile tests of representative samples of the material in the structure in question.

20.2.5 — If the required dimensions and material properties are determined through measurements and testing, and if calculations can be made in accordance with 20.1.2, it shall be permitted to increase the strength reduction factor in 9.3, but the strength reduction factor shall not be more than:

Tension-controlled sections, as defined in 10.3.4	1.0
Compression-controlled sections, as defined in 10.3.3:	
Members with spiral reinforcement conforming to 10.9.3	0.85
Other reinforced members	0.8
Shear and/or torsion	0.8
Bearing on concrete	0.8

20.3 — Load test procedure

20.3.1 — Load arrangement

The number and arrangement of spans or panels loaded shall be selected to maximize the deflection and stresses in the critical regions of the structural elements of which strength is in doubt. More than one test load arrangement shall be used if a single arrangement will not simultaneously result in maximum values of the effects (such as deflection, rotation, or stress) necessary to demonstrate the adequacy of the structure.

COMMENTARY

R20.2.2 — For individual elements, amount, size, arrangement, and location should be determined at the critical sections for reinforcement or tendons, or both, designed to resist applied load. Nondestructive investigation methods are acceptable. In large structures, determination of these data for approximately 5 percent of the reinforcement or tendons in critical regions may suffice if these measurements confirm the data provided in the construction drawings.

R20.2.3 — The number of tests may depend on the size of the structure and the sensitivity of structural safety to concrete strength. In cases where the potential problem involves flexure only, investigation of concrete strength can be minimal for a lightly reinforced section ($\rho f_y / f'_c \leq 0.15$ for rectangular section).

R20.2.4 — The number of tests required depends on the uniformity of the material and is best determined by the engineer for the specific application.

R20.2.5 — Strength reduction factors given in 20.2.5 are larger than those specified in Chapter 9. These increased values are justified by the use of accurate field-obtained material properties, actual in-place dimensions, and well-understood methods of analysis.

The strength reduction factors in 20.2.5 were changed for the 2002 edition to be compatible with the load combinations and strength reduction factors of Chapter 9, which were revised at that time.

R20.3 — Load test procedure

R20.3.1—Load arrangement

It is important to apply the load at locations so that its effects on the suspected defect are a maximum and the probability of unloaded members sharing the applied load is a minimum. In cases where it is shown by analysis that adjoining unloaded elements will help carry some of the load, the load should be placed to develop effects consistent with the intent of the load factor.

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20.3.2 — Load intensity

The total test load (including dead load already in place) shall not be less than **0.85 (1.4*D* + 1.7*L*)**. It shall be permitted to reduce ***L*** in accordance with the requirements of the applicable general building code.

20.3.3 — A load test shall not be made until that portion of the structure to be subjected to load is at least 56 days old. If the owner of the structure, the contractor, and all involved parties agree, it shall be permitted to make the test at an earlier age.

20.4 — Loading criteria

20.4.1 — The initial value for all applicable response measurements (such as deflection, rotation, strain, slip, crack widths) shall be obtained not more than 1 h before application of the first load increment. Measurements shall be made at locations where maximum response is expected. Additional measurements shall be made if required.

20.4.2 — Test load shall be applied in not less than four approximately equal increments.

20.4.3 — Uniform test load shall be applied in a manner to ensure uniform distribution of the load transmitted to the structure or portion of the structure being tested. Arching of the applied load shall be avoided.

20.4.4 — A set of response measurements shall be made after each load increment is applied and after the total load has been applied on the structure for at least 24 h.

20.4.5 — Total test load shall be removed immediately after all response measurements defined in 20.4.4 are made.

20.4.6 — A set of final response measurements shall be made 24 h after the test load is removed.

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R20.3.2 — Load intensity

The required load intensity follows previous load test practice. The live load ***L*** may be reduced as permitted by the general building code governing safety considerations for the structure. The live load should be increased to compensate for resistance provided by unloaded portions of the structure in questions. The increase in live load is determined from analysis of the loading conditions in relation to the selected pass/fail criterion for the test.

Although the load combinations and strength reduction factors of Chapter 9 were revised for the 2002 edition, the test load intensity remained the same. It is considered appropriate for designs using the load combinations and strength reduction factors of Chapter 9 or Appendix C.

R20.4 — Loading criteria

R20.4.2 — Inspecting the structure after each load increment is advisable.

R20.4.3 — Arching refers to the tendency for the load to be transmitted nonuniformly to the flexural element being tested. For example, if a slab is loaded by a uniform arrangement of bricks with the bricks in contact, arching would results in reduction of the load on the slab near the midspan of the slab.

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20.5 — Acceptance criteria

20.5.1 — The portion of the structure tested shall show no evidence of failure. Spalling and crushing of compressed concrete shall be considered an indication of failure.

20.5.2 — Measured maximum deflections shall satisfy one of the following conditions:

$$D_{max} \le \frac{\ell_t^2}{20{,}000h} \quad (20\text{-}1)$$

$$D_{rmax} \le \frac{D_{max}}{4} \quad (20\text{-}2)$$

If the measured maximum and residual deflections do not satisfy Eq. (20-1) or (20-2), it shall be permitted to repeat the load test.

The repeat test shall be conducted not earlier than 72 h after removal of the first test load. The portion of the structure tested in the repeat test shall be considered acceptable if deflection recovery satisfies the condition:

$$D_{rmax} \le \frac{D_{fmax}}{5} \quad (20\text{-}3)$$

where D_{fmax} is the maximum deflection measured during the second test relative to the position of the structure at the beginning of the second test.

COMMENTARY

R20.5 — Acceptance criteria

R20.5.1 — A general acceptance criterion for the behavior of a structure under the test load is that it does not show evidence of failure. Evidence of failure includes cracking, spalling, or deflection of such magnitude and extent that the observed result is obviously excessive and incompatible with the safety requirements of the structure. No simple rules have been developed for application to all types of structures and conditions. If sufficient damage has occurred so that the structure is considered to have failed that test, retesting is not permitted since it is considered that damaged members should not be put into service even at a lower load rating.

Local spalling or flaking of the compressed concrete in flexural elements related to casting imperfections need not indicate overall structural distress. Crack widths are good indicators of the state of the structure and should be observed to help determine whether the structure is satisfactory. However, exact prediction or measurement of crack widths in reinforced concrete elements is not likely to be achieved under field conditions. Establish criteria before the test, relative to the types of cracks anticipated; where the cracks will be measured; how they will be measured; and approximate limits or criteria to evaluate new cracks or limits for the changes in crack width.

R20.5.2 — The deflection limits and the retest option follow past practice. If the structure shows no evidence of failure, recovery of deflection after removal of the test load is used to determine whether the strength of the structure is satisfactory. In the case of a very stiff structure, however, the errors in measurements under field conditions may be of the same order as the actual deflections and recovery. To avoid penalizing a satisfactory structure in such a case, recovery measurements are waived if the maximum deflection is less than $\ell_t^2/(20{,}000h)$. The residual deflection D_{rmax} is the difference between the initial and final (after load removal) deflections for the load test or the repeat load test.

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20.5.3 — Structural members tested shall not have cracks indicating the imminence of shear failure.

20.5.4 — In regions of structural members without transverse reinforcement, appearance of structural cracks inclined to the longitudinal axis and having a horizontal projection longer than the depth of the member at midpoint of the crack shall be evaluated.

20.5.5 — In regions of anchorage and lap splices, the appearance along the line of reinforcement of a series of short inclined cracks or horizontal cracks shall be evaluated.

20.6 — Provision for lower load rating

If the structure under investigation does not satisfy conditions or criteria of 20.1.2, 20.5.2, or 20.5.3, the structure shall be permitted for use at a lower load rating based on the results of the load test or analysis, if approved by the building official.

20.7 — Safety

20.7.1 — Load tests shall be conducted in such a manner as to provide for safety of life and structure during the test.

20.7.2 — No safety measures shall interfere with load test procedures or affect results.

COMMENTARY

R20.5.3 — Forces are transmitted across a shear crack plane by a combination of aggregate interlock at the interface of the crack that is enhanced by clamping action of transverse stirrup reinforcing and by dowel action of stirrups crossing the crack. As crack lengths increase to approach a horizontal projected length equal to the depth of the member and concurrently widen to the extent that aggregate interlock cannot occur, and as transverse stirrups if present begin to yield or display loss of anchorage so as to threaten their integrity, the member is assumed to be approaching imminent shear failure.

R20.5.4 — The intent of 20.5.4 is to make the professionals in charge of the test pay attention to the structural implication of observed inclined cracks that may lead to brittle collapse in members without transverse reinforcement.

R20.5.5 — Cracking along the axis of the reinforcement in anchorage zones may be related to high stresses associated with the transfer of forces between the reinforcement and the concrete. These cracks may be indicators of pending brittle failure of the element if they are associated with the main reinforcement. It is important that their causes and consequences be evaluated.

R20.6 — Provision for lower load rating

Except for load tested members that have failed under a test (see 20.5), the building official may permit the use of a structure or member at a lower load rating that is judged to be safe and appropriate on the basis of the test results.

CHAPTER 21 — SPECIAL PROVISIONS FOR SEISMIC DESIGN

CODE

21.0 — Notation

A_{ch} = cross-sectional area of a structural member measured out-to-out of transverse reinforcement, in.^{2}

A_{cp} = area of concrete section, resisting shear, of an individual pier or horizontal wall segment, in.^{2}

A_{cv} = gross area of concrete section bounded by web thickness and length of section in the direction of shear force considered, in.^{2}

A_g = gross area of section, in.^{2}

A_j = effective cross-sectional area within a joint, see 21.5.3.1, in a plane parallel to plane of reinforcement generating shear in the joint, in.^{2} The joint depth shall be the overall depth of the column. Where a beam frames into a support of larger width, the effective width of the joint shall not exceed the smaller of:
(a) beam width plus the joint depth
(b) twice the smaller perpendicular distance from the longitudinal axis of the beam to the column side. See 21.5.3.1

A_{sh} = total cross-sectional area of transverse reinforcement (including crossties) within spacing s and perpendicular to dimension h_c, in.^{2}

A_{vd} = total area of reinforcement in each group of diagonal bars in a diagonally reinforced coupling beam, in.^{2}

b = effective compressive flange width of a structural member, in.

b_w = web width, or diameter of circular section, in.

c = distance from the extreme compression fiber to neutral axis, see 10.2.7, calculated for the factored axial force and nominal moment strength, consistent with the design displacement d_u, resulting in the largest neutral axis depth, in.

c_1 = size of rectangular or equivalent rectangular column, capital, or bracket measured in the direction of the span for which moments are being determined, in.

c_t = dimension equal to the distance from the interior face of the column to the slab edge measured parallel to c_1, but not exceeding c_1, in.

d = effective depth of section, in.

d_b = bar diameter, in.

E = load effects of earthquake, or related internal moments and forces

f_c' = specified compressive strength of concrete, psi

$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi

COMMENTARY

R21.0 — Notation

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

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COMMENTARY

f_y = specified yield strength of reinforcement, psi

f_{yh} = specified yield strength of transverse reinforcement, psi

h = overall thickness of member, in.

h_c = cross-sectional dimension of column core measured center-to-center of confining reinforcement, in.

h_w = height of entire wall or of the segment of wall considered, in.

h_x = maximum horizontal spacing of hoop or crosstie legs on all faces of the column, in.

ℓ_d = development length for a straight bar, in.

ℓ_{dh} = development length for a bar with a standard hook as defined in Eq. (21-6), in.

ℓ_n = clear span measured face-to-face of supports, in.

ℓ_o = minimum length, measured from joint face along axis of structural member, over which transverse reinforcement must be provided, in.

ℓ_w = length of entire wall or of segment of wall considered in direction of shear force, in.

M_c = moment at the face of the joint, corresponding to the nominal flexural strength of the column framing into that joint, calculated for the factored axial force, consistent with the direction of the lateral forces considered, resulting in the lowest flexural strength, in.-lb. See 21.4.2.2

M_g = moment at the face of the joint, corresponding to the nominal flexural strength of the girder including slab where in tension, framing into that joint, in.-lb. See 21.4.2.2

M_{pr} = probable flexural strength of members, with or without axial load, determined using the properties of the member at the joint faces assuming a tensile strength in the longitudinal bars of at least $\mathbf{1.25}f_y$ and a strength reduction factor ϕ of 1.0, in.-lb

M_s = portion of slab moment balanced by support moment, in.-lb

M_u = factored moment at section, in.-lb

S_e = moment, shear, or axial force at connection corresponding with development of probable strength at intended yield locations, based on the governing mechanism of inelastic lateral deformation, considering both gravity and earthquake load effects

S_n = nominal flexural, shear, or axial strength of the connection

S_y = yield strength of connection, based on f_y, for moment, shear, or axial force

s = spacing of transverse reinforcement measured along the longitudinal axis of the structural member, in.

s_o = maximum spacing of transverse reinforcement, in.

s_x = longitudinal spacing of transverse reinforcement within the length ℓ_o, in.

V_c = nominal shear strength provided by concrete, lb

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V_e = design shear force determined from 21.3.4.1 or 21.4.5.1, lb
V_n = nominal shear strength, lb
V_u = factored shear force at section, lb
a = angle between the diagonal reinforcement and the longitudinal axis of a diagonally reinforced coupling beam
a_c = coefficient defining the relative contribution of concrete strength to wall strength. See Eq. (21-7)
d_u = design displacement, in.
r = ratio of nonprestressed tension reinforcement = A_s/bd
r_g = ratio of total reinforcement area to cross-sectional area of column
r_n = ratio of area of distributed reinforcement parallel to the plane of A_{cv} to gross concrete area perpendicular to that reinforcement
r_s = ratio of volume of spiral reinforcement to the core volume confined by the spiral reinforcement (measured out-to-out)
r_v = ratio of area of distributed reinforcement perpendicular to the plane of A_{cv} to gross concrete area A_{cv}.
f = strength reduction factor

COMMENTARY

21.1 — Definitions

Base of structure — Level at which earthquake motions are assumed to be imparted to a building. This level does not necessarily coincide with the ground level.

Boundary elements — Portions along structural wall and structural diaphragm edges strengthened by longitudinal and transverse reinforcement. Boundary elements do not necessarily require an increase in the thickness of the wall or diaphragm. Edges of openings within walls and diaphragms shall be provided with boundary elements as required by 21.7.6 or 21.9.5.3.

Collector elements — Elements that serve to transmit the inertial forces within structural diaphragms to members of the lateral-force-resisting systems.

Connection — A region that joins two or more members, of which one or more is precast.

Ductile connection — Connection that experiences yielding as a result of the design displacements.

Strong connection — Connection that remains elastic while adjoining members experience yielding as a result of the design displacements.

R21.1 — Definitions

CODE

Crosstie — A continuous reinforcing bar having a seismic hook at one end and a hook not less than 90 deg with at least a six-diameter extension at the other end. The hooks shall engage peripheral longitudinal bars. The 90 deg hooks of two successive crossties engaging the same longitudinal bars shall be alternated end for end.

Design displacement — Total lateral displacement expected for the design-basis earthquake, as required by the governing code for earthquake-resistant design.

Design load combinations — Combinations of factored loads and forces in 9.2.

Development length for a bar with a standard hook — The shortest distance between the critical section (where the strength of the bar is to be developed) and a tangent to the outer edge of the 90 deg hook.

Factored loads and forces — Loads and forces multiplied by appropriate load factors in 9.2.

Hoop — A closed tie or continuously wound tie. A closed tie can be made up of several reinforcement elements each having seismic hooks at both ends. A continuously wound tie shall have a seismic hook at both ends.

Joint — Portion of structure common to intersecting members. The effective area of the joint for shear strength computations is defined in 21.0 (See A_j).

Lateral-force resisting system — That portion of the structure composed of members proportioned to resist forces related to earthquake effects.

Lightweight aggregate concrete — All-lightweight or sand-lightweight aggregate concrete made with lightweight aggregates conforming to 3.3.

Moment frame — Frame in which members and joints resist forces through flexure, shear, and axial force. Moment frames shall be categorized as follows:

Intermediate moment frame — A cast-in-place frame complying with the requirements of 21.2.2.3 and 21.12 in addition to the requirements for ordinary moment frames.

Ordinary moment frame — A cast-in-place or precast concrete frame complying with the requirements of Chapters 1 through 18.

Special moment frame — A cast-in-place frame complying with the requirements of 21.2 through 21.5 or a precast frame complying with the requirements of 21.2 through 21.6. In addition, the requirements for ordinary moment frames shall be satisfied.

COMMENTARY

The design displacement is an index of the maximum lateral displacement expected in design for the design-basis earthquake. In documents such as the National Earthquake Hazards Reduction Provisions (NEHRP),[21.1] ASCE 7-95, the Uniform Building Code (UBC),[21.2] the BOCA/National Building Code (BOCA)[21.3] published by Building Officials and Code Administrators International, or the Standard Building Code (SBC)[21.4] published by Southern Building Code Congress International, the design-basis earthquake has approximately a 90 percent probability of nonexceedance in 50 years. In those documents, the design displacement is calculated using static or dynamic linear elastic analysis under code-specified actions considering effects of cracked sections, effects of torsion, effects of vertical forces acting through lateral displacements, and modification factors to account for expected inelastic response. The design displacement generally is larger than the displacement calculated from design-level forces applied to a linear-elastic model of the building.

The provisions of 21.6 are intended to result in a special moment frame constructed using precast concrete having minimum strength and toughness equivalent to that for a special moment frame of cast-in-place concrete.

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Plastic hinge region — Length of frame element over which flexural yielding is intended to occur due to design displacements, extending not less than a distance ***h*** from the critical section where flexural yielding initiates.

Seismic hook — A hook on a stirrup, hoop, or crosstie having a bend not less than 135 deg, except that circular hoops shall have a bend not less than 90 deg. Hooks shall have a six-diameter (but not less than 3 in.) extension that engages the longitudinal reinforcement and projects into the interior of the stirrup or hoop.

Special boundary elements — Boundary elements required by 21.7.6.2 or 21.7.6.3.

Specified lateral forces — Lateral forces corresponding to the appropriate distribution of the design base shear force prescribed by the governing code for earthquake-resistant design.

Structural diaphragms — Structural members, such as floor and roof slabs, that transmit inertial forces to lateral-force resisting members.

Structural trusses — Assemblages of reinforced concrete members subjected primarily to axial forces.

Structural walls — Walls proportioned to resist combinations of shears, moments, and axial forces induced by earthquake motions. A shearwall is a structural wall. Structural walls shall be categorized as follows:

Intermediate precast structural wall — A wall complying with all applicable requirements of Chapters 1 through 18 in addition to 21.13.

Ordinary reinforced concrete structural wall — A wall complying with the requirements of Chapters 1 through 18.

Ordinary structural plain concrete wall — A wall complying with the requirements of Chapter 22.

Special precast structural wall — A precast wall complying with the requirements of 21.8. In addition, the requirements for ordinary reinforced concrete structural walls and the requirements of 21.2 shall be satisfied.

Special reinforced concrete structural wall — A cast-in-place wall complying with the requirements of 21.2 and 21.7 in addition to the requirements for ordinary reinforced concrete structural walls.

Strut — An element of a structural diaphragm used to provide continuity around an opening in the diaphragm.

COMMENTARY

The provisions of 21.13 are intended to result in an intermediate precast structural wall having minimum strength and toughness equivalent to that for an ordinary reinforced concrete structural wall of cast-in-place concrete. A precast concrete wall satisfying only the requirements of Chapters 1 through 18 and not the additional requirements of 21.13 or 21.8 is considered to have ductility and structural integrity less than that for an intermediate precast structural wall.

The provisions of 21.8 are intended to result in a special precast structural wall having minimum strength and toughness equivalent to that for a special reinforced concrete structural wall of cast-in-place concrete.

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Tie elements — Elements that serve to transmit inertia forces and prevent separation of building components such as footings and walls.

21.2 — General requirements

21.2.1 — Scope

21.2.1.1 — Chapter 21 contains special requirements for design and construction of reinforced concrete members of a structure for which the design forces, related to earthquake motions, have been determined on the basis of energy dissipation in the nonlinear range of response. For applicable specified concrete strengths, see 1.1.1 and 21.2.4.1

21.2.1.2 — In regions of low seismic risk or for structures assigned to low seismic performance or design categories, the provisions of Chapters 1 through 18 and 22 shall apply except as modified by the provisions of this chapter. Where the design seismic loads are computed using provisions for intermediate or special concrete systems, the requirements of Chapter 21 for intermediate or special systems, as applicable, shall be satisfied.

21.2.1.3 — In regions of moderate seismic risk or for structures assigned to intermediate seismic performance or design categories, intermediate or special moment frames, or ordinary, intermediate, or special structural walls shall be used to resist forces induced by earthquake motions. Where the design seismic loads are computed using provisions for special concrete systems, the requirements of Chapter 21 for special systems, as applicable, shall be satisfied.

21.2.1.4 — In regions of high seismic risk or for structures assigned to high seismic performance or design categories, special moment frames, special structural walls, and diaphragms and trusses complying with 21.2 through 21.10 shall be used to resist forces induced by earthquake motions. Frame members not proportioned to resist earthquake forces shall comply with 21.11.

21.2.1.5 — A reinforced concrete structural system not satisfying the requirements of this chapter shall be permitted if it is demonstrated by experimental evidence and analysis that the proposed system will have strength and toughness equal to or exceeding those provided by a comparable monolithic reinforced concrete structure satisfying this chapter.

COMMENTARY

R21.2 — General requirements

R21.2.1 — Scope

Chapter 21 contains provisions considered to be the minimum requirements for a cast-in-place or precast concrete structure capable of sustaining a series of oscillations into the inelastic range of response without critical deterioration in strength. The integrity of the structure in the inelastic range of response should be maintained because the design forces defined in documents such as the IBC,[21.5] the UBC,[21.2] and the NEHRP[21.1] provisions are considered less than those corresponding to linear response at the anticipated earthquake intensity.[21.1,21.6-21.8]

As a properly detailed cast-in-place or precast concrete structure responds to strong ground motion, its effective stiffness decreases and its energy dissipation increases. These changes tend to reduce the response accelerations and lateral inertia forces relative to values that would occur were the structure to remain linearly elastic and lightly damped.[21.8] Thus, the use of design forces representing earthquake effects such as those in Reference 21.2 requires that the lateral-force resisting system retain a substantial portion of its strength into the inelastic range under displacement reversals.

The provisions of Chapter 21 relate detailing requirements to type of structural framing, earthquake risk level at the site, level of inelastic deformation intended in structural design, and use and occupancy of the structure. Earthquake risk levels traditionally have been classified as low, moderate, and high. The seismic risk level of a region or the seismic performance or design category of a structure is regulated by the legally adopted general building code or determined by local authority (see 1.1.8.3, R1.1.8.3, and Table R1.1.8.3). The 2000 IBC[21.5] and the 2000 NEHRP[21.9] provisions use the same terminology as the 1997 NEHRP[21.1] provisions.

The design and detailing requirements should be compatible with the level of energy dissipation (or toughness) assumed in the computation of the design seismic loads. The terms ordinary, intermediate, and special are specifically used to facilitate this compatibility. The degree of required toughness, and therefore the level of required detailing, increases for structures progressing from ordinary through intermediate to special categories. It is essential that structures in higher seismic zones or assigned to higher seismic performance or design categories possess a higher degree of toughness. It is permitted, however, to design for higher toughness in the lower seismic zones or design categories and take advantage of the lower design force levels.

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COMMENTARY

The provisions of Chapters 1 through 18 and 22 are intended to provide adequate toughness for structures in regions of low seismic risk, or assigned to ordinary categories. Therefore, it is not required to apply the provisions of Chapter 21 to lateral-force resisting systems consisting of ordinary structural walls.

Chapter 21 requires special details for reinforced concrete structures in regions of moderate seismic risk, or assigned to intermediate seismic performance or design categories. These requirements are contained in 21.2.1.3, 21.12, and 21.13. Although new provisions are provided in 21.13 for design of intermediate precast structural walls, general building codes that address seismic performance or design categories currently do not include intermediate structural walls.

Structures in regions of high seismic risk, or assigned to high seismic performance or design categories, may be subjected to strong ground shaking. Structures designed using seismic forces based upon response modification factors for special moment frames or special reinforced concrete structural walls are likely to experience multiple cycles of lateral displacements well beyond the point where reinforcement yields should the design earthquake ground shaking occur. The provisions of 21.2 through 21.11 have been developed to provide the structure with adequate toughness for this special response.

The requirements of Chapter 21 and 22 as they apply to various components of structures in regions of intermediate or high seismic risk, or assigned to intermediate or high seismic performance or design categories, are summarized in Table R21.2.1.

The special proportioning and detailing requirements in Chapter 21 are based predominantly on field and laboratory experience with monolithic reinforced concrete building structures and precast concrete building structures designed and detailed to behave like monolithic building structures. Extrapolation of these requirements to other types of cast-in-place or precast concrete structures should be based on evidence provided by field experience, tests, or analysis. ACI T1.1-01, “Acceptance Criteria for Moment Frames Based on Structural Testing,” can be used in conjunction with Chapter 21 to demonstrate that the strength and toughness of a proposed frame system equals or exceeds that provided by a comparable monolithic concrete system.

The toughness requirements in 21.2.1.5 refer to the concern for the structural integrity of the entire lateral-force resisting system at lateral displacements anticipated for ground motions corresponding to the design earthquake. Depending on the energy-dissipation characteristics of the structural system used, such displacements may be larger than for a monolithic reinforced concrete structure.

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TABLE R21.2.1— SECTIONS OF CHAPTERS 21 AND 22 TO BE SATISFIED*

Component resisting earthquake effect, unless otherwise noted	Level of seismic risk or assigned seismic performance or design categories (as defined in code section)	
	Intermediate (21.2.1.3)	High (21.2.1.4)
Frame members	21.12	21.2, 21.3, 21.4, 21.5
Structural walls and coupling beams	None	21.2, 21.7
Precast structural walls	21.13	21.2, 21.8
Structural diaphragms and trusses	None	21.2, 21.9
Foundations	None	21.2, 21.10
Frame members not proportioned to resist forces induced by earthquake motions	None	2.11
Plain concrete	22.4	22.4, 22.10.1

*In addition to requirements of Chapters 1 through 18 for structures at intermediate seismic risk (21.2.1.3), and for Chapters 1 through 17 for structures at high seismic risk (21.2.1.4).

21.2.2 — Analysis and proportioning of structural members

21.2.2.1 — The interaction of all structural and nonstructural members that materially affect the linear and nonlinear response of the structure to earthquake motions shall be considered in the analysis.

21.2.2.2 — Rigid members assumed not to be a part of the lateral-force resisting system shall be permitted provided their effect on the response of the system is considered and accommodated in the structural design. Consequences of failure of structural and nonstructural members, which are not a part of the lateral-force resisting system, shall also be considered.

21.2.2.3 — Structural members below base of structure that are required to transmit to the foundation forces resulting from earthquake effects shall also comply with the requirements of Chapter 21.

21.2.2.4 — All structural members assumed not to be part of the lateral-force resisting system shall conform to 21.11.

R21.2.2 — Analysis and proportioning of structural members

It is assumed that the distribution of required strength to the various components of a lateral-force resisting system will be guided by the analysis of a linearly elastic model of the system acted upon by the factored forces required by the governing code. If nonlinear response history analyses are to be used, base motions should be selected after a detailed study of the site conditions and local seismic history.

Because the design basis admits nonlinear response, it is necessary to investigate the stability of the lateral-force resisting system as well as its interaction with other structural and nonstructural members at displacements larger than those indicated by linear analysis. To handle this without having to resort to nonlinear response analysis, one option is to multiply by a factor of at least two the displacements from linear analysis by using the factored lateral forces, unless the governing code specifies the factors to be used as in References 21.1 and 21.2. For lateral displacement calculations, assuming all the horizontal structural members to be fully cracked is likely to lead to better estimates of the possible drift than using uncracked stiffness for all members.

The main concern of Chapter 21 is the safety of the structure. The intent of 21.2.2.1 and 21.2.2.2 is to draw attention to the influence of nonstructural members on structural response and to hazards from falling objects.

Section 21.2.2.3 alerts the designer that the base of the structure as defined in analysis may not necessarily correspond to the foundation or ground level.

In selecting member sizes for earthquake-resistant structures, it is important to consider problems related to congestion of reinforcement. The designer should ensure that all reinforcement can be assembled and placed and that concrete can be

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cast and consolidated properly. Use of upper limits of reinforcement ratios permitted is likely to lead to insurmountable construction problems especially at frame joints.

21.2.3 — Strength reduction factors

Strength reduction factors shall be as given in 9.3.4.

21.2.4 — Concrete in members resisting earthquake-induced forces

21.2.4.1 — Compressive strength f'_c of the concrete shall be not less than 3000 psi.

21.2.4.2 — Compressive strength of lightweight aggregate concrete used in design shall not exceed 5000 psi. Lightweight aggregate concrete with higher design compressive strength shall be permitted if demonstrated by experimental evidence that structural members made with that lightweight aggregate concrete provide strength and toughness equal to or exceeding those of comparable members made with normalweight aggregate concrete of the same strength.

R21.2.4 — Concrete in members resisting earthquake-induced forces

Requirements of this section refer to concrete quality in frames, trusses, or walls proportioned to resist earthquake-induced forces. The maximum design compressive strength of lightweight aggregate concrete to be used in structural design calculations is limited to 5000 psi, primarily because of paucity of experimental and field data on the behavior of members made with lightweight aggregate concrete subjected to displacement reversals in the nonlinear range. If convincing evidence is developed for a specific application, the limit on maximum compressive strength of lightweight aggregate concrete may be increased to a level justified by the evidence.

21.2.5 — Reinforcement in members resisting earthquake-induced forces

Reinforcement resisting earthquake-induced flexural and axial forces in frame members and in structural wall boundary elements shall comply with ASTM A 706. ASTM A 615 Grades 40 and 60 reinforcement shall be permitted in these members if:

(a) The actual yield strength based on mill tests does not exceed the specified yield strength by more than 18,000 psi (retests shall not exceed this value by more than an additional 3000 psi); and

(b) The ratio of the actual ultimate tensile strength to the actual tensile yield strength is not less than 1.25.

R21.2.5 — Reinforcement in members resisting earthquake-induced forces

Use of longitudinal reinforcement with strength substantially higher than that assumed in design will lead to higher shear and bond stresses at the time of development of yield moments. These conditions may lead to brittle failures in shear or bond and should be avoided even if such failures may occur at higher loads than those anticipated in design. Therefore, a ceiling is placed on the actual yield strength of the steel [see 21.2.5(a)].

The requirement for an ultimate tensile strength larger than the yield strength of the reinforcement [21.2.5(b)] is based on the assumption that the capability of a structural member to develop inelastic rotation capacity is a function of the length of the yield region along the axis of the member. In interpreting experimental results, the length of the yield region has been related to the relative magnitudes of ultimate and yield moments.[21.10] According to this interpretation, the larger the ratio of ultimate to yield moment, the longer the yield region. Chapter 21 requires that the ratio of actual tensile strength to actual yield strength is not less than 1.25. Members with reinforcement not satisfying this condition can also develop inelastic rotation, but their behavior is sufficiently different to exclude them from direct consideration on the basis of rules derived from experience with members reinforced with strain-hardening steel.

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21.2.6 — Mechanical splices

21.2.6.1 — Mechanical splices shall be classified as either Type 1 or Type 2 mechanical splices, as follows:

(a) Type 1 mechanical splices shall conform to 12.14.3.2;

(b) Type 2 mechanical splices shall conform to 12.14.3.2 and shall develop the specified tensile strength of the spliced bar.

21.2.6.2 — Type 1 mechanical splices shall not be used within a distance equal to twice the member depth from the column or beam face or from sections where yielding of the reinforcement is likely to occur as a result of inelastic lateral displacements. Type 2 mechanical splices shall be permitted to be used at any location.

21.2.7 — Welded splices

21.2.7.1 — Welded splices in reinforcement resisting earthquake-induced forces shall conform to 12.14.3.4 and shall not be used within a distance equal to twice the member depth from the column or beam face or from sections where yielding of the reinforcement is likely to occur as a result of inelastic lateral displacements.

21.2.7.2 — Welding of stirrups, ties, inserts, or other similar elements to longitudinal reinforcement that is required by design shall not be permitted.

21.2.8 — Anchoring to concrete

21.2.8.1 — Anchors resisting earthquake-induced forces in structures in regions of moderate or high seismic risk, or assigned to intermediate or high seismic performance or design categories shall conform to the additional requirements of D.3.3 of Appendix D.

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R21.2.6 — Mechanical splices

In a structure undergoing inelastic deformations during an earthquake, the tensile stresses in reinforcement may approach the tensile strength of the reinforcement. The requirements for Type 2 mechanical splices are intended to avoid a splice failure when the reinforcement is subjected to expected stress levels in yielding regions. Type 1 splices are not required to satisfy the more stringent requirements for Type 2 splices, and may not be capable of resisting the stress levels expected in yielding regions. The locations of Type 1 splices are restricted because tensile stresses in reinforcement in yielding regions can exceed the strength requirements of 12.14.3.2.

Recommended detailing practice would preclude the use of splices in regions of potential yield in members resisting earthquake effects. If use of mechanical splices in regions of potential yielding cannot be avoided, the designer should have documentation on the actual strength characteristics of the bars to be spliced, on the force-deformation characteristics of the spliced bar, and on the ability of the Type 2 splice to be used to meet the specified performance requirements.

R21.2.7 — Welded splices

R21.2.7.1 — Welding of reinforcement should be according to ANSI/AWS D1.4 as required in Chapter 3. The locations of welded splices are restricted because reinforcement tension stresses in yielding regions can exceed the strength requirements of 12.14.3.4.

R21.2.7.2 — Welding of crossing reinforcing bars can lead to local embrittlement of the steel. If welding of crossing bars is used to facilitate fabrication or placement of reinforcement, it should be done only on bars added for such purposes. The prohibition of welding crossing reinforcing bars does not apply to bars that are welded with welding operations under continuous, competent control as in the manufacture of welded wire fabric.

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21.3 — Flexural members of special moment frames

21.3.1 — Scope

Requirements of 21.3 apply to special moment frame members (a) resisting earthquake-induced forces and (b) proportioned primarily to resist flexure. These frame members shall also satisfy the conditions of 21.3.1.1 through 21.3.1.4.

21.3.1.1 — Factored axial compressive force on the member shall not exceed $(A_g f_c'/10)$.

21.3.1.2 — Clear span for the member shall not be less than four times its effective depth.

21.3.1.3 — The width-to-depth ratio shall not be less than 0.3.

21.3.1.4 — The width shall not be

(a) less than 10 in.; and

(b) more than the width of the supporting member (measured on a plane perpendicular to the longitudinal axis of the flexural member) plus distances on each side of the supporting member not exceeding three-fourths of the depth of the flexural member.

21.3.2 — Longitudinal reinforcement

21.3.2.1 — At any section of a flexural member, except as provided in 10.5.3, for top as well as for bottom reinforcement, the amount of reinforcement shall not be less than that given by Eq. (10-3) but not less than $200 b_w d/f_y$, and the reinforcement ratio r shall not exceed 0.025. At least two bars shall be provided continuously both top and bottom.

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R21.3 — Flexural members of special moment frames

R21.3.1 — Scope

This section refers to beams of special moment frames resisting lateral loads induced by earthquake motions. Any frame member subjected to a factored axial compressive force exceeding $(A_g f_c'/10)$ is to be proportioned and detailed as described in 21.4.

Experimental evidence[21.11] indicates that, under reversals of displacement into the nonlinear range, behavior of continuous members having length-to-depth ratios of less than four is significantly different from the behavior of relatively slender members. Design rules derived from experience with relatively slender members do not apply directly to members with length-to-depth ratios less than four, especially with respect to shear strength.

Geometric constraints indicated in 21.3.1.3 and 21.3.1.4 were derived from practice with reinforced concrete frames resisting earthquake-induced forces.[21.12]

R21.3.2 — Longitudinal reinforcement

Section 10.3.5 limits the net tensile strain, ε_t, thereby indirectly limiting the tensile reinforcement ratio in a flexural member to a fraction of the amount that would produce balanced conditions. For a section subjected to bending only and loaded monotonically to yielding, this approach is feasible because the likelihood of compressive failure can be estimated reliably with the behavioral model assumed for determining the reinforcement ratio corresponding to balanced failure. The same behavioral model (because of incorrect assumptions such as linear strain distribution, well-defined yield point for the steel, limiting compressive strain in the concrete of 0.003, and compressive stresses in the shell concrete) fails to describe the conditions in a flexural member subjected to reversals of displacements well into the inelastic range. Thus, there is little rationale for continuing to refer to balanced conditions in earthquake-resistant design of reinforced concrete structures.

R21.3.2.1 — The limiting reinforcement ratio of 0.025 is based primarily on considerations of steel congestion and, indirectly, on limiting shear stresses in girders of typical proportions. The requirement of at least two bars, top and bottom, refers again to construction rather than behavioral requirements.

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21.3.2.2 — Positive moment strength at joint face shall be not less than one-half of the negative moment strength provided at that face of the joint. Neither the negative nor the positive moment strength at any section along member length shall be less than one-fourth the maximum moment strength provided at face of either joint.

21.3.2.3 — Lap splices of flexural reinforcement shall be permitted only if hoop or spiral reinforcement is provided over the lap length. Maximum spacing of the transverse reinforcement enclosing the lapped bars shall not exceed $d/4$ or 4 in. Lap splices shall not be used

(a) within the joints;

(b) within a distance of twice the member depth from the face of the joint; and

(c) at locations where analysis indicates flexural yielding caused by inelastic lateral displacements of the frame.

21.3.2.4 — Mechanical splices shall conform to 21.2.6 and welded splices shall conform to 21.2.7.

21.3.3 — Transverse reinforcement

21.3.3.1 — Hoops shall be provided in the following regions of frame members:

(a) Over a length equal to twice the member depth measured from the face of the supporting member toward midspan, at both ends of the flexural member;

(b) Over lengths equal to twice the member depth on both sides of a section where flexural yielding is likely to occur in connection with inelastic lateral displacements of the frame.

21.3.3.2 — The first hoop shall be located not more than 2 in. from the face of a supporting member. Maximum spacing of the hoops shall not exceed (a), (b), (c) and (d):

(a) $d/4$;

(b) eight times the diameter of the smallest longitudinal bars;

(c) 24 times the diameter of the hoop bars; and

(d) 12 in.

21.3.3.3 — Where hoops are required, longitudinal bars on the perimeter shall have lateral support conforming to 7.10.5.3.

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R21.3.2.3 — Lap splices of reinforcement are prohibited at regions where flexural yielding is anticipated because such splices are not reliable under conditions of cyclic loading into the inelastic range. Transverse reinforcement for lap splices at any location is mandatory because of the likelihood of loss of shell concrete.

R21.3.3 —Transverse reinforcement

Transverse reinforcement is required primarily to confine the concrete and maintain lateral support for the reinforcing bars in regions where yielding is expected. Examples of hoops suitable for flexural members of frames are shown in Fig. R21.3.3.

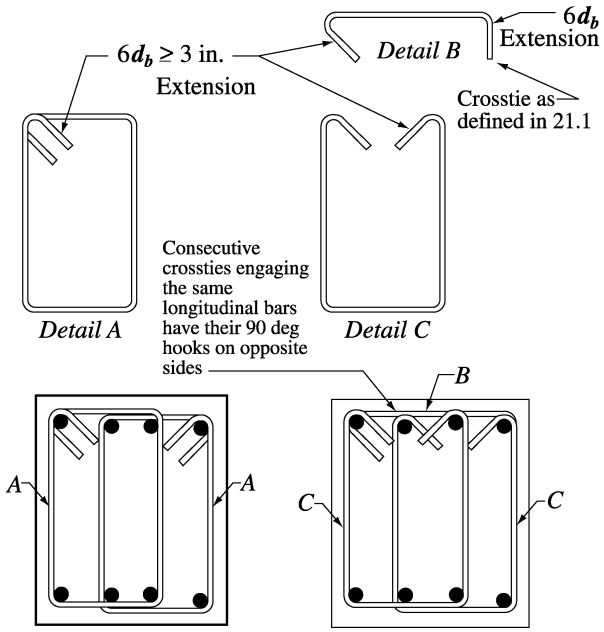


Fig. R21.3.3—Examples of overlapping hoops

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21.3.3.4 — Where hoops are not required, stirrups with seismic hooks at both ends shall be spaced at a distance not more than $d/2$ throughout the length of the member.

21.3.3.5 — Stirrups or ties required to resist shear shall be hoops over lengths of members in 21.3.3, 21.4.4, and 21.5.2.

21.3.3.6 — Hoops in flexural members shall be permitted to be made up of two pieces of reinforcement: a stirrup having seismic hooks at both ends and closed by a crosstie. Consecutive crossties engaging the same longitudinal bar shall have their 90 deg hooks at opposite sides of the flexural member. If the longitudinal reinforcing bars secured by the crossties are confined by a slab on only one side of the flexural frame member, the 90 deg hooks of the crossties shall be placed on that side.

21.3.4 — Shear strength requirements

21.3.4.1 — Design forces

The design shear force V_e shall be determined from consideration of the statical forces on the portion of the member between faces of the joints. It shall be assumed that moments of opposite sign corresponding to probable flexural moment strength M_{pr} act at the joint faces and that the member is loaded with the factored tributary gravity load along its span.

21.3.4.2 — Transverse reinforcement

Transverse reinforcement over the lengths identified in 21.3.3.1 shall be proportioned to resist shear assuming $V_c = 0$ when both of the following conditions occur:

(a) The earthquake-induced shear force calculated in accordance with 21.3.4.1 represents one-half or more of the maximum required shear strength within those lengths;

(b) The factored axial compressive force including earthquake effects is less than $A_g f_c'/20$.

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In the case of members with varying strength along the span or members for which the permanent load represents a large proportion of the total design load, concentrations of inelastic rotation may occur within the span. If such a condition is anticipated, transverse reinforcement also should be provided in regions where yielding is expected.

Because spalling of the concrete shell is anticipated during strong motion, especially at and near regions of flexural yielding, all web reinforcement should be provided in the form of closed hoops as defined in 21.3.3.5.

R21.3.4 — Shear strength requirements

R21.3.4.1 — Design forces

In determining the equivalent lateral forces representing earthquake effects for the type of frames considered, it is assumed that frame members will dissipate energy in the nonlinear range of response. Unless a frame member possesses a strength that is a multiple on the order of 3 or 4 of the design forces, it should be assumed that it will yield in the event of a major earthquake. The design shear force should be a good approximation of the maximum shear that may develop in a member. Therefore, required shear strength for frame members is related to flexural strengths of the designed member rather than to factored shear forces indicated by lateral load analysis. The conditions described by 21.3.4.1 are illustrated in Fig. R21.3.4.

Because the actual yield strength of the longitudinal reinforcement may exceed the specified yield strength and because strain hardening of the reinforcement is likely to take place at a joint subjected to large rotations, required shear strengths are determined using a stress of at least $1.25f_y$ in the longitudinal reinforcement.

R21.3.4.2 — Transverse reinforcement

Experimental studies[21.13, 21.14] of reinforced concrete members subjected to cyclic loading have demonstrated that more shear reinforcement is required to ensure a flexural failure if the member is subjected to alternating nonlinear displacements than if the member is loaded in only one direction: the necessary increase of shear reinforcement being higher in the case of no axial load. This observation is reflected in the code (see 21.3.4.2) by eliminating the term representing the contribution of concrete to shear strength. The added conservatism on shear is deemed necessary in locations where potential flexural hinging may occur. However, this stratagem, chosen for its relative simplicity, should not be inter-

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preted to mean that no concrete is required to resist shear. On the contrary, it may be argued that the concrete core resists all of the shear with the shear (transverse) reinforcement confining and strengthening the concrete. The confined concrete core plays an important role in the behavior of the beam and should not be reduced to a minimum just because the design expression does not explicitly recognize it.

Notes on Fig. R21.3.4:

1. Direction of shear force V_e depends on relative magnitudes of gravity loads and shear generated by end moments.

2. End moments M_{pr} based on steel tensile stress of $1.25 f_y$, where f_y is specified yield strength. (Both end moments should be considered in both directions, clockwise and counter-clockwise).

3. End moment M_{pr} for columns need not be greater than moments generated by the M_{pr} of the beams framing into the beam-column joints. V_e should not be less than that required by analysis of the structure.

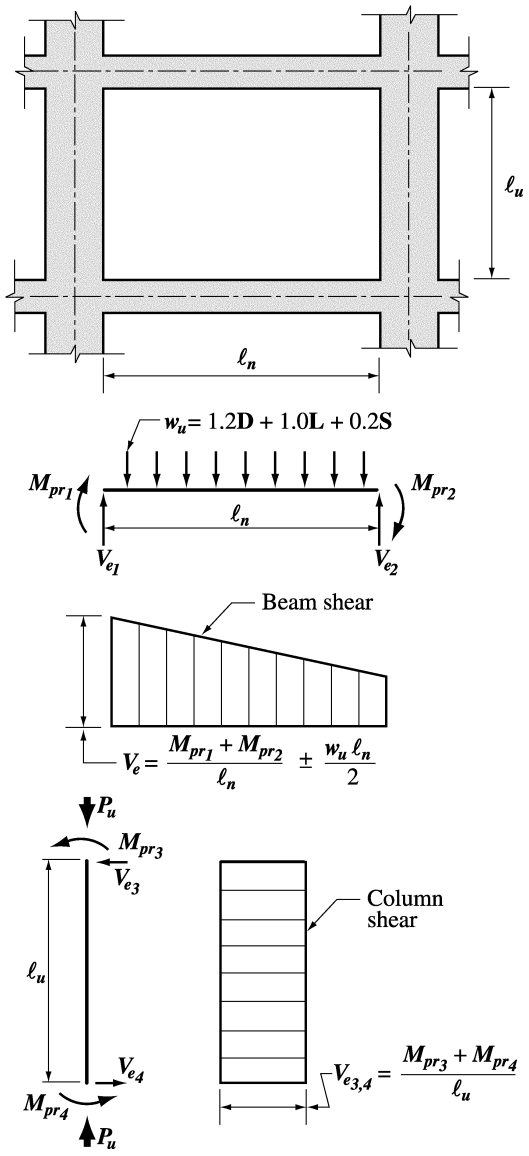


Fig. R21.3.4—Design shears for girders and columns

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21.4 — Special moment frame members subjected to bending and axial load

21.4.1 — Scope

The requirements of this section apply to special moment frame members (a) resisting earthquake-induced forces and (b) having a factored axial force exceeding $(A_g f_c'/10)$. These frame members shall also satisfy the conditions of 21.4.1.1 and 21.4.1.2.

21.4.1.1 — The shortest cross-sectional dimension, measured on a straight line passing through the geometric centroid, shall not be less than 12 in.

21.4.1.2 — The ratio of the shortest cross-sectional dimension to the perpendicular dimension shall not be less than 0.4.

21.4.2 — Minimum flexural strength of columns

21.4.2.1 — Flexural strength of any column proportioned to resist a factored axial compressive force exceeding $(A_g f_c'/10)$ shall satisfy 21.4.2.2 or 21.4.2.3.

Lateral strength and stiffness of columns not satisfying 21.4.2.2 shall be ignored in determining the calculated strength and stiffness of the structure, but such columns shall conform to 21.11.

21.4.2.2 — The flexural strengths of the columns shall satisfy Eq. (21-1)

$$\Sigma M_c \geq (6/5) \Sigma M_g \qquad (21\text{-}1)$$

ΣM_c = sum of moments at the faces of the joint corresponding to the nominal flexural strength of the columns framing into that joint. Column flexural strength shall be calculated for the factored axial force, consistent with the direction of the lateral forces considered, resulting in the lowest flexural strength.

ΣM_g = sum of moments at the faces of the joint corresponding to the nominal flexural strengths of the girders framing into that joint. In T-beam construction, where the slab is in tension under moments at the face of the joint, slab reinforcement within an effective slab width defined in 8.10 shall be assumed to contribute to flexural strength if the slab reinforcement is developed at the critical section for flexure.

Flexural strengths shall be summed such that the column moments oppose the beam moments. Eq. (21-1) shall be satisfied for beam moments acting in both directions in the vertical plane of the frame considered.

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R21.4 — Special moment frame members subjected to bending and axial load

R21.4.1 — Scope

Section 21.4.1 is intended primarily for columns of special moment frames. Frame members, other than columns, that do not satisfy 21.3.1 are to be proportioned and detailed according to this section.

The geometric constraints in 21.4.1.1 and 21.4.1.2 follow from previous practice.[21.12]

R21.4.2 — Minimum flexural strength of columns

The intent of 21.4.2.2 is to reduce the likelihood of yielding in columns that are considered as part of the lateral force resisting system. If columns are not stronger than beams framing into a joint, there is likelihood of inelastic action. In the worst case of weak columns, flexural yielding can occur at both ends of all columns in a given story, resulting in a column failure mechanism that can lead to collapse.

In 21.4.2.2, the nominal strengths of the girders and columns are calculated at the joint faces, and those strengths are compared directly using Eq. (21-1). The 1995 code required design strengths to be compared at the center of the joint, which typically produced similar results but with added computational effort.

When determining the nominal flexural strength of a girder section in negative bending (top in tension), longitudinal reinforcement contained within an effective flange width of a top slab that acts monolithically with the girder increases the girder strength. Research[21.15] on beam-column subassemblies under lateral loading indicates that using the effective flange widths defined in 8.10 gives reasonable estimates of girder negative bending strengths of interior connections at interstory displacement levels approaching 2 percent of story height. This effective width is conservative where the slab terminates in a weak spandrel.

If 21.4.2.2 cannot be satisfied at a joint, any positive contribution of the column or columns involved to the lateral strength and stiffness of the structure is to be ignored. Negative contributions of the column or columns should not be ignored. For example, ignoring the stiffness of the columns ought not be used as a justification for reducing the design base shear. If inclusion of those columns in the analytical model of the building results in an increase in torsional effects, the increase should be considered as required by the governing code.

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21.4.2.3 — If 21.4.2.2 is not satisfied at a joint, columns supporting reactions from that joint shall be provided with transverse reinforcement as specified in 21.4.4.1 through 21.4.4.3 over their full height.

21.4.3 — Longitudinal reinforcement

21.4.3.1 — The reinforcement ratio ρ_g shall not be less than 0.01 and shall not exceed 0.06.

21.4.3.2 — Mechanical splices shall conform to 21.2.6 and welded splices shall conform to 21.2.7. Lap splices shall be permitted only within the center half of the member length, shall be designed as tension lap splices, and shall be enclosed within transverse reinforcement conforming to 21.4.4.2 and 21.4.4.3.

21.4.4 — Transverse reinforcement

21.4.4.1 — Transverse reinforcement as required below in (a) through (e) shall be provided unless a larger amount is required by 21.4.3.2 or 21.4.5.

(a) The volumetric ratio of spiral or circular hoop reinforcement ρ_s shall not be less than that required by Eq. (21-2).

$$\rho_s = 0.12 f_c'/f_{yh} \quad (21\text{-}2)$$

and shall not be less than that required by Eq. (10-5).

(b) The total cross-sectional area of rectangular hoop reinforcement shall not be less than that required by Eq. (21-3) and (21-4).

$$A_{sh} = 0.3(sh_c f_c'/f_{yh})[(A_g/A_{ch}) - 1] \quad (21\text{-}3)$$

$$A_{sh} = 0.09 sh_c f_c'/f_{yh} \quad (21\text{-}4)$$

(c) Transverse reinforcement shall be provided by either single or overlapping hoops. Crossties of the same bar size and spacing as the hoops shall be permitted. Each end of the crosstie shall engage a peripheral longitudinal reinforcing bar. Consecutive crossties shall be alternated end for end along the longitudinal reinforcement.

(d) If the design strength of member core satisfies the requirement of the design loading combinations

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R21.4.3 — Longitudinal reinforcement

The lower limit of the reinforcement ratio is to control time-dependent deformations and to have the yield moment exceed the cracking moment. The upper limit of the section reflects concern for steel congestion, load transfer from floor elements to column especially in low-rise construction, and the development of high shear stresses.

Spalling of the shell concrete, which is likely to occur near the ends of the column in frames of typical configuration, makes lap splices in these locations vulnerable. If lap splices are to be used at all, they should be located near the midheight where stress reversal is likely to be limited to a smaller stress range than at locations near the joints. Special transverse reinforcement is required along the lap-splice length because of the uncertainty in moment distributions along the height and the need for confinement of lap splices subjected to stress reversals.[21.16]

R21.4.4 — Transverse reinforcement

Requirements of this section are concerned with confining the concrete and providing lateral support to the longitudinal reinforcement.

The effect of helical (spiral) reinforcement and adequately configured rectangular hoop reinforcement on strength and ductility of columns is well established.[21.17] While analytical procedures exist for calculation of strength and ductility capacity of columns under axial and moment reversals,[21.18] the axial load and deformation demands required during earthquake loading are not known with sufficient accuracy to justify calculation of required transverse reinforcement as a function of design earthquake demands. Instead, Eq. (10-5) and (21-3) are required, with the intent that spalling of shell concrete will not result in a loss of axial load strength of the column. Eq. (21-2) and (21-4) govern for large-diameter columns, and are intended to ensure adequate flexural curvature capacity in yielding regions.

Fig. R21.4.4 shows an example of transverse reinforcement provided by one hoop and three crossties. Crossties with a 90 deg hook are not as effective as either crossties with 135 deg hooks or hoops in providing confinement. Tests show that if crosstie ends with 90 deg hooks are alternated, confinement will be sufficient.

Sections 21.4.4.2 and 21.4.4.3 are interrelated requirements for configuration of rectangular hoop reinforcement. The requirement that spacing not exceed one-quarter of the minimum member dimension is to obtain adequate concrete confinement. The requirement that spacing not exceed six

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including earthquake effect, Eq. (21-3) and (10-5) need not be satisfied.

(e) If the thickness of the concrete outside the confining transverse reinforcement exceeds 4 in., additional transverse reinforcement shall be provided at a spacing not exceeding 12 in. Concrete cover on the additional reinforcement shall not exceed 4 in.

21.4.4.2 — Transverse reinforcement shall be spaced at a distance not exceeding (a) one-quarter of the minimum member dimension, (b) six times the diameter of the longitudinal reinforcement, and (c) s_x, as defined by Eq. (21-5).

$$s_x = 4 + \left(\frac{14 - h_x}{3}\right) \qquad (21\text{-}5)$$

The value of s_x shall not exceed 6 in. and need not be taken less than 4 in.

21.4.4.3 — Crossties or legs of overlapping hoops shall not be spaced more than 14 in. on center in the direction perpendicular to the longitudinal axis of the structural member.

21.4.4.4 — Transverse reinforcement in amount specified in 21.4.4.1 through 21.4.4.3 shall be provided over a length ℓ_o from each joint face and on both sides of any section where flexural yielding is likely to occur as a result of inelastic lateral displacements of the frame. The length ℓ_o shall not be less than (a), (b), and (c):

(a) the depth of the member at the joint face or at the section where flexural yielding is likely to occur;

(b) one-sixth of the clear span of the member; and

(c) 18 in.

21.4.4.5 — Columns supporting reactions from discontinued stiff members, such as walls, shall be provided with transverse reinforcement as required in 21.4.4.1 through 21.4.4.3 over their full height beneath the level at which the discontinuity occurs if the factored axial compressive force in these members, related to earthquake effect, exceeds $(A_g f'_c/10)$. Transverse reinforcement as required in 21.4.4.1 through 21.4.4.3 shall extend into the discontinued member for at least the development length of the largest longitudinal reinforcement in the column in accordance with 21.5.4. If the lower end of the column terminates on a wall, transverse reinforcement as required in 21.4.4.1 through 21.4.4.3 shall extend into the wall for at least the development length of the largest longitudinal bar in the column at the point of termination. If the column terminates on a footing or mat, transverse reinforcement as required in 21.4.4.1 through 21.4.4.3 shall extend at least 12 in. into the footing or mat.

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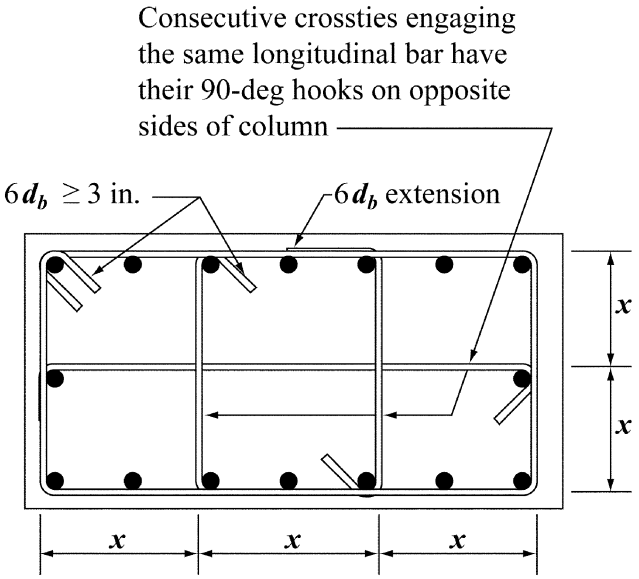
bar diameters is intended to restrain longitudinal reinforcement buckling after spalling. The 4 in. spacing is for concrete confinement; 21.4.4.2 permits this limit to be relaxed to a maximum of 6 in. if the spacing of crossties or legs of overlapping hoops is limited to 8 in.

The unreinforced shell may spall as the column deforms to resist earthquake effects. Separation of portions of the shell from the core caused by local spalling creates a falling hazard. The additional reinforcement is required to reduce the risk of portions of the shell falling away from the column.

Section 21.4.4.4 stipulates a minimum length over which to provide closely-spaced transverse reinforcement at the member ends, where flexural yielding normally occurs. Research results indicate that the length should be increased by 50 percent or more in locations, such as the base of the building, where axial loads and flexural demands may be especially high.[21.19]

Columns supporting discontinued stiff members, such as walls or trusses, may develop considerable inelastic response. Therefore, it is required that these columns have special transverse reinforcement throughout their length. This covers all columns beneath the level at which the stiff member has been discontinued, unless the factored forces corresponding to earthquake effect are low (see 21.4.4.5).

Field observations have shown significant damage to columns in the unconfined region near the midheight. The requirements of 21.4.4.6 are to ensure a relatively uniform toughness of the column along its length.



Note: $x \leq 14$"
h_x = maximum value of x on all column faces

Fig. R21.4.4—Example of transverse reinforcement in columns

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21.4.4.6 — Where transverse reinforcement, as specified in 21.4.4.1 through 21.4.4.3, is not provided throughout the full length of the column, the remainder of the column length shall contain spiral or hoop reinforcement with center-to-center spacing not exceeding the smaller of six times the diameter of the longitudinal column bars or 6 in.

21.4.5 — Shear strength requirements

21.4.5.1 — Design forces

The design shear force V_e shall be determined from consideration of the maximum forces that can be generated at the faces of the joints at each end of the member. These joint forces shall be determined using the maximum probable moment strengths M_{pr} of the member associated with the range of factored axial loads on the member. The member shears need not exceed those determined from joint strengths based on the probable moment strength M_{pr} of the transverse members framing into the joint. In no case shall V_e be less than the factored shear determined by analysis of the structure.

21.4.5.2 — Transverse reinforcement over the lengths ℓ_o, identified in 21.4.4.4, shall be proportioned to resist shear assuming $V_c = 0$ when both the following conditions occur:

(a) The earthquake-induced shear force, calculated in accordance with 21.4.5.1, represents one-half or more of the maximum required shear strength within those lengths;

(b) The factored axial compressive force including earthquake effects is less than $A_g f_c'/20$.

21.5 — Joints of special moment frames

21.5.1 — General requirements

21.5.1.1 — Forces in longitudinal beam reinforcement at the joint face shall be determined by assuming that the stress in the flexural tensile reinforcement is $1.25f_y$.

21.5.1.2 — Strength of joint shall be governed by the appropriate strength reduction factors in 9.3.

21.5.1.3 — Beam longitudinal reinforcement terminated in a column shall be extended to the far face of the confined column core and anchored in tension according to 21.5.4 and in compression according to Chapter 12.

21.5.1.4 — Where longitudinal beam reinforcement extends through a beam-column joint, the column dimension parallel to the beam reinforcement shall not

COMMENTARY

R21.4.4.6 — The provisions of 21.4.4.6 were added to the 1989 code to provide reasonable protection and ductility to the midheight of columns between transverse reinforcement. Observations after earthquakes have shown significant damage to columns in the nonconfined region, and the minimum ties or spirals required should provide a more uniform toughness of the column along its length.

R21.4.5 — Shear strength requirements

R21.4.5.1 — Design forces

The provisions of 21.3.4.1 also apply to members subjected to axial loads (for example, columns). Above the ground floor the moment at a joint may be limited by the flexural strength of the beams framing into the joint. Where beams frame into opposite sides of a joint, the combined strength may be the sum of the negative moment strength of the beam on one side of the joint and the positive moment strength of the beam on the other side of the joint. Moment strengths are to be determined using a strength reduction factor of 1.0 and reinforcing steel stress equal to at least $1.25f_y$. Distribution of the combined moment strength of the beams to the columns above and below the joint should be based on analysis. The value of M_{pr} in Fig. R21.3.4 may be computed from the flexural member strengths at the beam-column joints.

R21.5 — Joints of special moment frames

R21.5.1 — General requirements

Development of inelastic rotations at the faces of joints of reinforced concrete frames is associated with strains in the flexural reinforcement well in excess of the yield strain. Consequently, joint shear force generated by the flexural reinforcement is calculated for a stress of $1.25f_y$ in the reinforcement (see 21.5.1.1). A detailed explanation of the reasons for the possible development of stresses in excess of the yield strength in girder tensile reinforcement is provided in Reference 21.10.

R21.5.1.4 — Research[21.20-21.24] has shown that straight beam bars may slip within the beam-column joint during a series of large moment reversals. The bond stresses on

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be less than 20 times the diameter of the largest longitudinal bar for normalweight concrete. For lightweight concrete, the dimension shall be not less than 26 times the bar diameter.

21.5.2 — Transverse reinforcement

21.5.2.1 — Transverse hoop reinforcement in 21.4.4 shall be provided within the joint, unless the joint is confined by structural members in 21.5.2.2.

21.5.2.2 — Within the depth of the shallowest framing member, transverse reinforcement equal to at least one-half the amount required by 21.4.4.1 shall be provided where members frame into all four sides of the joint and where each member width is at least three-fourths the column width. At these locations, the spacing required in 21.4.4.2 shall be permitted to be increased to 6 in.

21.5.2.3 — Transverse reinforcement as required by 21.4.4 shall be provided through the joint to provide confinement for longitudinal beam reinforcement outside the column core if such confinement is not provided by a beam framing into the joint.

21.5.3 — Shear strength

21.5.3.1 — The nominal shear strength of the joint shall not be taken as greater than the values specified below for normalweight aggregate concrete.

For joints confined on all four faces ..... $20\sqrt{f'_c}A_j$

For joints confined on three faces or on two opposite faces ..... $15\sqrt{f'_c}A_j$

For others ..... $12\sqrt{f'_c}A_j$

A member that frames into a face is considered to provide confinement to the joint if at least three-quarters of the face of the joint is covered by the framing member. A joint is considered to be confined if such confining members frame into all faces of the joint.

COMMENTARY

these straight bars may be very large. To substantially reduce slip during the formation of adjacent beam hinging, it would be necessary to have a ratio of column dimension to bar diameter of approximately 1/32, which would result in very large joints. On reviewing the available tests, the limit of 1/20 of the column depth in the direction of loading for the maximum size of beam bars for normalweight concrete and a limit of 1/26 for lightweight concrete were chosen. Due to the lack of specific data, the modification for lightweight concrete used a factor of 1.3 from Chapter 12. These limits provide reasonable control on the amount of potential slip of the beam bars in a beam-column joint, considering the number of anticipated inelastic excursions of the building frames during a major earthquake. A thorough treatment of this topic is given in Reference 21.25.

R21.5.2 — Transverse reinforcement

However low the calculated shear force in a joint of a frame resisting earthquake-induced forces, confining reinforcement (see 21.4.4) should be provided through the joint around the column reinforcement (see 21.5.2.1). In 21.5.2.2, confining reinforcement may be reduced if horizontal members frame into all four sides of the joint. The 1989 code provided a maximum limit on spacing to these areas based on available data.[21.26-21.29]

Section 21.5.2.3 refers to a joint where the width of the girder exceeds the corresponding column dimension. In that case, girder reinforcement not confined by the column reinforcement should be provided lateral support either by a girder framing into the same joint or by transverse reinforcement.

R21.5.3 — Shear strength

The requirements in Chapter 21 for proportioning joints are based on Reference 21.10 in that behavioral phenomena within the joint are interpreted in terms of a nominal shear strength of the joint. Because tests of joints[21.20] and deep beams[21.11] indicated that shear strength was not as sensitive to joint (shear) reinforcement as implied by the expression developed by ACI Committee 326[21.30] for beams and adopted to apply to joints by ACI Committee 352,[21.10] Committee 318 set the strength of the joint as a function of only the compressive strength of the concrete (see 21.5.3) and to require a minimum amount of transverse reinforcement in the joint (see 21.5.2). The effective area of joint A_j is illustrated in Fig. R21.5.3. In no case is A_j greater than the column cross-sectional area.

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21.5.3.2 — For lightweight aggregate concrete, the nominal shear strength of the joint shall not exceed three-quarters of the limits given in 21.5.3.1.

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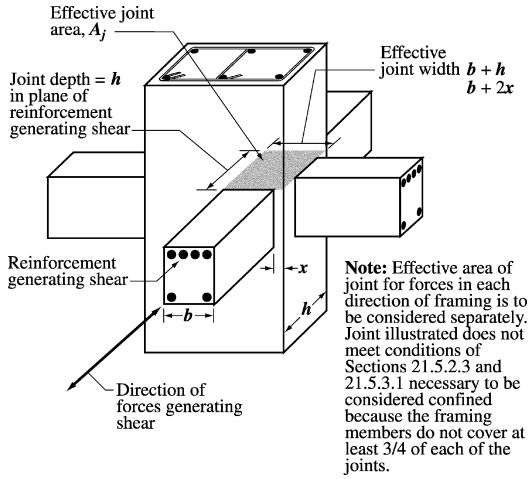


Fig. R21.5.3—Effective joint area

The three levels of shear strength required by 21.5.3.1 are based on the recommendation of ACI Committee 352.[21.10] Test data reviewed by the committee[21.28] indicate that the lower value given in 21.5.3.1 of the 1983 code was unconservative when applied to corner joints.

21.5.4 — Development length of bars in tension

21.5.4.1 — The development length ℓ_{dh} for a bar with a standard 90 deg hook in normalweight aggregate concrete shall not be less than the largest of $8d_b$, 6 in., and the length required by Eq. (21-6).

$$\ell_{dh} = f_y d_b / (65\sqrt{f'_c}) \qquad (21\text{-}6)$$

for bar sizes No. 3 through No. 11.

For lightweight aggregate concrete, the development length for a bar with a standard 90 deg hook shall not be less than the largest of $10d_b$, 7-1/2 in., and 1.25 times that required by Eq. (21-6).

The 90 deg hook shall be located within the confined core of a column or of a boundary element.

21.5.4.2 — For bar sizes No. 3 through No. 11, the development length ℓ_d for a straight bar shall not be less than (a) and (b):

(a) 2.5 times the length required by 21.5.4.1 if the depth of the concrete cast in one lift beneath the bar does not exceed 12 in.; and

(b) 3.5 times the length required by 21.5.4.1 if the depth of the concrete cast in one lift beneath the bar exceeds 12 in.

R21.5.4 — Development length of bars in tension

Minimum development length for deformed bars with standard hooks embedded in normalweight concrete is determined using Eq. (21-6). Eq. (21-6) is based on the requirements of 12.5. Because Chapter 21 stipulates that the hook is to be embedded in confined concrete, the coefficients 0.7 (for concrete cover) and 0.8 (for ties) have been incorporated in the constant used in Eq. (21-6). The development length that would be derived directly from 12.5 is increased to reflect the effect of load reversals.

The development length in tension for a reinforcing bar with a standard hook is defined as the distance, parallel to the bar, from the critical section (where the bar is to be developed) to a tangent drawn to the outside edge of the hook. The tangent is to be drawn perpendicular to the axis of the bar (see Fig. R12.5).

Factors such as the actual stress in the reinforcement being more than the yield force and the effective development length not necessarily starting at the face of the joint were implicitly considered in the development of the expression for basic development length that has been used as the basis for Eq. (21-6).

For lightweight aggregate concrete, the length required by Eq. (21-6) is to be increased by 25 percent to compensate for variability of bond characteristics of reinforcing bars in various types of lightweight aggregate concrete.

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21.5.4.3 — Straight bars terminated at a joint shall pass through the confined core of a column or of a boundary element. Any portion of the straight embedment length not within the confined core shall be increased by a factor of 1.6.

21.5.4.4 — If epoxy-coated reinforcement is used, the development lengths in 21.5.4.1 through 21.5.4.3 shall be multiplied by the applicable factor in 12.2.4 or 12.5.2.

COMMENTARY

Section 21.5.4.2 specifies the minimum development length for straight bars as a multiple of the length indicated by 21.5.4.1. Section 21.5.4.2(b) refers to top bars.

If the required straight embedment length of a reinforcing bar extends beyond the confined volume of concrete (as defined in 21.3.3, 21.4.4, or 21.5.2), the required development length is increased on the premise that the limiting bond stress outside the confined region is less than that inside.

$$\ell_{dm} = 1.6(\ell_d - \ell_{dc}) + \ell_{dc}$$

or

$$\ell_{dm} = 1.6\ell_d - 0.6\ell_{dc}$$

where:

ℓ_{dm} = required development length if bar is not entirely embedded in confined concrete;
ℓ_d = required development length for straight bar embedded in confined concrete (see 21.5.4.3);
ℓ_{dc} = length of bar embedded in confined concrete.

Lack of reference to No. 14 and No. 18 bars in 21.5.4 is due to the paucity of information on anchorage of such bars subjected to load reversals simulating earthquake effects.

21.6 — Special moment frames constructed using precast concrete

21.6.1 — Special moment frames with ductile connections constructed using precast concrete shall satisfy the requirements of (a) and (b) and all requirements for special moment frames constructed with cast-in-place concrete:

(a) The nominal shear strength for connections, V_n, computed according to 11.7.4, shall be greater than or equal to $2V_e$, where V_e is calculated according to 21.3.4.1 or 21.4.5.1; and

(b) Mechanical splices of beam reinforcement shall be located not closer than $h/2$ from the joint face and shall meet the requirements of 21.2.6.

21.6.2 — Special moment frames with strong connections constructed using precast concrete shall satisfy all requirements for special moment frames constructed with cast-in-place concrete, as well as the requirements of (a), (b), (c), and (d).

(a) Provisions of 21.3.1.2 shall apply to segments between locations where flexural yielding is intended to occur due to design displacements;

(b) Design strength of the strong connection ϕS_n shall be not less than S_e;

R21.6 — Special moment frames constructed using precast concrete

The detailing provisions in 21.6.1 and 21.6.2 are intended to produce frames that respond to design displacements essentially like monolithic special moment frames.

Precast frame systems composed of concrete elements with ductile connections are expected to experience flexural yielding in connection regions. Reinforcement in ductile connections can be made continuous by using Type 2 mechanical splices or any other technique that provides development in tension or compression of at least 125 percent of the specified yield strength f_y of bars and the specified tensile strength of bars.[21.31,21.32,21.33,21.34] Requirements for mechanical splices are in addition to those in 21.2.6 and are intended to avoid strain concentrations over a short length of reinforcement adjacent to a splice device. Additional requirements for shear strength are provided in 21.6.1 to prevent sliding on connection faces. Precast frames composed of elements with ductile connections may be designed to promote yielding at locations not adjacent to the joints. Therefore, design shear, V_e, as computed according to 21.3.4.1 or 21.4.5.1, may be conservative.

Precast concrete frame systems composed of elements joined using strong connections are intended to experience flexural yielding outside the connections. Strong connections include the length of the coupler hardware as shown in Fig. R21.6.2. Capacity-design techniques are used in

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(c) Primary longitudinal reinforcement shall be made continuous across connections and shall be developed outside both the strong connection and the plastic hinge region; and

(d) Column-to-column connections shall have design strength ϕS_n not less than $1.4S_e$. At column-to-column connections, the design flexural strength ϕM_n shall be not less than 0.4 times the maximum probable flexural strength M_{pr} for the column within the story height, and the design shear strength ϕV_n of the connection shall be not less than that determined by 21.4.5.1.

21.6.3 — Special moment frames constructed using precast concrete and not satisfying the requirements of 21.6.1 or 21.6.2 shall satisfy the requirements of ACI T1.1, "Acceptance Criteria for Moment Frames Based on Structural Testing," and the requirements of (a) and (b):

(a) Details and materials used in the test specimens shall be representative of those used in the structure; and

(b) The design procedure used to proportion the test specimens shall define the mechanism by which the frame resists gravity and earthquake effects, and shall establish acceptance values for sustaining that mechanism. Portions of the mechanism that deviate from code requirements shall be contained in the test specimens and shall be tested to determine upper bounds for acceptance values.

21.7 — Special reinforced concrete structural walls and coupling beams

21.7.1 — Scope

The requirements of this section apply to special reinforced concrete structural walls and coupling beams serving as part of the earthquake force-resisting system.

21.7.2 — Reinforcement

21.7.2.1 — The distributed web reinforcement ratios, ρ_v and ρ_n, for structural walls shall not be less than 0.0025, except if the design shear force does not exceed $A_{cv}\sqrt{f'_c}$, the minimum reinforcement for structural walls shall be permitted to be reduced to that required in 14.3. Reinforcement spacing each way in structural walls shall not exceed 18 in. Reinforcement provided for shear strength shall be continuous and shall be distributed across the shear plane.

COMMENTARY

21.6.2(b) to ensure the strong connection remains elastic following formation of plastic hinges. Additional column requirements are provided to avoid hinging and strength deterioration of column-to-column connections.

Strain concentrations have been observed to cause brittle fracture of reinforcing bars at the face of mechanical splices in laboratory tests of precast beam-column connections.[21.35] Designers should carefully select locations of strong connections or take other measures, such as debonding of reinforcing bars in highly stressed regions, to avoid strain concentrations that can result in premature fracture of reinforcement.

R21.6.3 — Precast frame systems not satisfying the prescriptive requirements of Chapter 21 have been demonstrated in experimental studies to provide satisfactory seismic performance characteristics.[21.36,21.37] ACI T1.1 defines a protocol for establishing a design procedure, validated by analysis and laboratory tests, for such frames. The design procedure should identify the load path or mechanism by which the frame resists gravity and earthquake effects. The tests should be configured to test critical behaviors, and the measured quantities should establish upper-bound acceptance values for components of the load path, which may be in terms of limiting stresses, forces, strains, or other quantities. The design procedure used for the structure should not deviate from that used to design the test specimens, and acceptance values should not exceed values that were demonstrated by the tests to be acceptable. Materials and components used in the structure should be similar to those used in the tests. Deviations may be acceptable if the engineer can demonstrate that those deviations do not adversely affect the behavior of the framing system.

R21.7 — Special reinforced concrete structural walls and coupling beams

R21.7.1 — Scope

This section contains requirements for the dimensions and details of special reinforced concrete structural walls and coupling beams. In the 1995 code, 21.6 also contained provisions for diaphragms. Provisions for diaphragms are in 21.9.

R21.7.2 — Reinforcement

Minimum reinforcement requirements (see 21.7.2.1) follow from preceding codes. The uniform distribution requirement of the shear reinforcement is related to the intent to control the width of inclined cracks. The requirement for two layers of reinforcement in walls carrying substantial design shears (see 21.7.2.2) is based on the observation that, under ordinary construction conditions, the probability of maintaining a single layer of reinforcement near the middle of the wall section is quite low. Furthermore, presence of reinforcement close to

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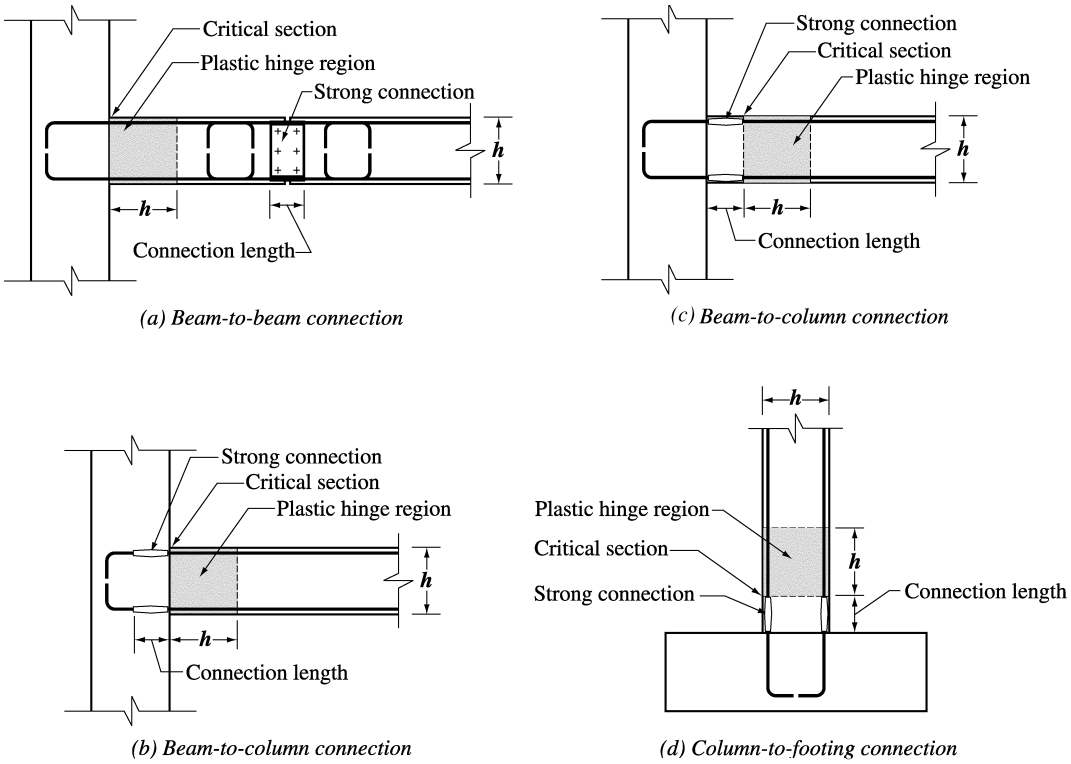


Fig. R21.6.2 — Strong connection examples

21.7.2.2 — At least two curtains of reinforcement shall be used in a wall if the in-plane factored shear force assigned to the wall exceeds $2A_{cv}\sqrt{f'_c}$.

21.7.2.3 — All continuous reinforcement in structural walls shall be anchored or spliced in accordance with the provisions for reinforcement in tension in 21.5.4.

21.7.3 — Design forces

The design shear force V_u shall be obtained from the lateral load analysis in accordance with the factored load combinations.

21.7.4 — Shear strength

21.7.4.1 — Nominal shear strength V_n of structural walls shall not exceed

$$V_n = A_{cv}(\alpha_c\sqrt{f'_c} + \rho_n f_y) \qquad (21\text{-}7)$$

where the coefficient α_c is 3.0 for $h_w/\ell_w \le 1.5$, is 2.0 for $h_w/\ell_w \ge 2.0$, and varies linearly between 3.0 and 2.0 for h_w/ℓ_w between 1.5 and 2.0.

the surface tends to inhibit fragmentation of the concrete in the event of severe cracking during an earthquake.

Because the actual forces in longitudinal reinforcing bars of stiff members may exceed the calculated forces, it is required (see 21.7.2.3) that all continuous reinforcement be developed fully.

R21.7.3 — Design forces

Design shears for structural walls are obtained from lateral load analysis with the appropriate load factors. However, the designer should consider the possibility of yielding in components of such structures, as in the portion of a wall between two window openings, in which case the actual shear may be in excess of the shear indicated by lateral load analysis based on factored design forces.

R21.7.4 — Shear strength

Eq. (21-7) recognizes the higher shear strength of walls with high shear-to-moment ratios.[21.10,21.30,21.38] The nominal shear strength is given in terms of the net area of the section resisting shear. For a rectangular section without openings, the term A_{cv} refers to the gross area of the cross section rather than to the product of the width and the effective depth. The definition of A_{cv} in Eq. (21-7) facilitates design calculations for walls with uniformly distributed reinforcement and walls with openings.

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21.7.4.2 — In 21.7.4.1, the value of ratio h_w/ℓ_w used for determining V_n for segments of a wall shall be the larger of the ratios for the entire wall and the segment of wall considered.

21.7.4.3 — Walls shall have distributed shear reinforcement providing resistance in two orthogonal directions in the plane of the wall. If the ratio h_w/ℓ_w does not exceed 2.0, reinforcement ratio r_v shall not be less than reinforcement ratio r_n.

21.7.4.4 — Nominal shear strength of all wall piers sharing a common lateral force shall not be assumed to exceed $8A_{cv}\sqrt{f'_c}$, where A_{cv} is the total cross-sectional area, and the nominal shear strength of any one of the individual wall piers shall not be assumed to exceed $10A_{cp}\sqrt{f'_c}$, where A_{cp} is the cross-sectional area of the pier considered.

21.7.4.5 — Nominal shear strength of horizontal wall segments and coupling beams shall be assumed not to exceed $10A_{cp}\sqrt{f'_c}$, where A_{cp} is the cross-sectional area of a horizontal wall segment or coupling beam.

21.7.5 — Design for flexure and axial loads

21.7.5.1 — Structural walls and portions of such walls subject to combined flexural and axial loads shall be designed in accordance with 10.2 and 10.3 except that 10.3.6 and the nonlinear strain requirements of 10.2.2 shall not apply. Concrete and developed longitudinal reinforcement within effective flange widths, boundary elements, and the wall web shall be considered effective. The effects of openings shall be considered.

21.7.5.2 — Unless a more detailed analysis is performed, effective flange widths of flanged sections shall extend from the face of the web a distance equal to the smaller of one-half the distance to an adjacent wall web and 25 percent of the total wall height.

COMMENTARY

A wall segment refers to a part of a wall bounded by openings or by an opening and an edge. Traditionally, a vertical wall segment bounded by two window openings has been referred to as a pier.

The ratio h_w/ℓ_w may refer to overall dimensions of a wall, or of a segment of the wall bounded by two openings, or an opening and an edge. The intent of 21.7.4.2 is to make certain that any segment of a wall is not assigned a unit strength larger than that for the whole wall. However, a wall segment with a ratio of h_w/ℓ_w higher than that of the entire wall should be proportioned for the unit strength associated with the ratio h_w/ℓ_w based on the dimensions for that segment.

To restrain the inclined cracks effectively, reinforcement included in r_n and r_v should be appropriately distributed along the length and height of the wall (see 21.7.4.3). Chord reinforcement provided near wall edges in concentrated amounts for resisting bending moment is not to be included in determining r_n and r_v. Within practical limits, shear reinforcement distribution should be uniform and at a small spacing.

If the factored shear force at a given level in a structure is resisted by several walls or several piers of a perforated wall, the average unit shear strength assumed for the total available cross-sectional area is limited to $8\sqrt{f'_c}$ with the additional requirement that the unit shear strength assigned to any single pier does not exceed $10\sqrt{f'_c}$. The upper limit of strength to be assigned to any one member is imposed to limit the degree of redistribution of shear force.

"Horizontal wall segments" in 21.7.4.5 refers to wall sections between two vertically aligned openings (see Fig. R21.7.4.5). It is, in effect, a pier rotated through 90 deg. A horizontal wall segment is also referred to as a coupling beam when the openings are aligned vertically over the building height.

R21.7.5 — Design for flexure and axial loads

R21.7.5.1 — Flexural strength of a wall or wall segment is determined according to procedures commonly used for columns. Strength should be determined considering the applied axial and lateral forces. Reinforcement concentrated in boundary elements and distributed in flanges and webs should be included in the strength computations based on a strain compatibility analysis. The foundation supporting the wall should be designed to develop the wall boundary and web forces. For walls with openings, the influence of the opening or openings on flexural and shear strengths is to be considered and a load path around the opening or openings should be verified. Capacity design concepts and strut-and-tie models may be useful for this purpose.[21.39]

R21.7.5.2 — Where wall sections intersect to form L-, T-, C-, or other cross-sectional shapes, the influence of the flange on the behavior of the wall should be considered by selecting appropriate flange widths. Tests[21.40] show that effective flange width increases with increasing drift level

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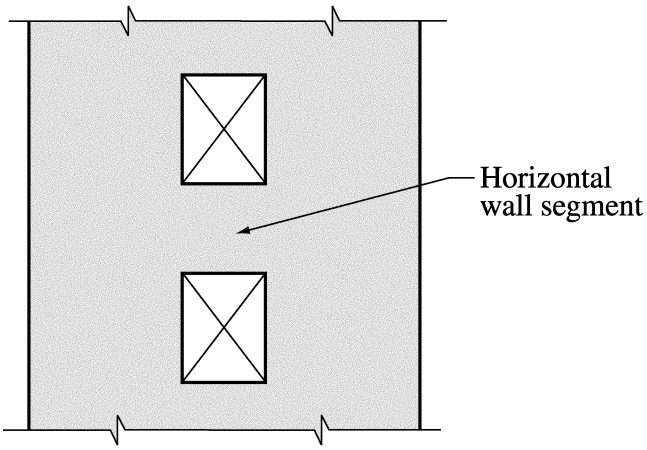


Fig. R21.7.4.5—Wall with openings

21.7.6 — Boundary elements of special reinforced concrete structural walls

21.7.6.1 — The need for special boundary elements at the edges of structural walls shall be evaluated in accordance with 21.7.6.2 or 21.7.6.3. The requirements of 21.7.6.4 and 21.7.6.5 also shall be satisfied.

21.7.6.2 — This section applies to walls or wall piers that are effectively continuous from the base of structure to top of wall and designed to have a single critical section for flexure and axial loads. Walls not satisfying these requirements shall be designed by 21.7.6.3.

(a) Compression zones shall be reinforced with special boundary elements where:

$$c \geq \frac{\ell_w}{600(\delta_u / h_w)} \qquad (21\text{-}8)$$

The quantity δ_u / h_w in Eq. (21.8) shall not be taken less than 0.007.

and the effectiveness of a flange in compression differs from that for a flange in tension. The value used for the effective compression flange width has little impact on the strength and deformation capacity of the wall; therefore, to simplify design, a single value of effective flange width based on an estimate of the effective tension flange width is used in both tension and compression.[21.40]

R21.7.6 — Boundary elements of special reinforced concrete structural walls

R21.7.6.1 — Two design approaches for evaluating detailing requirements at wall boundaries are included in 21.7.6.1. Section 21.7.6.2 allows the use of displacement-based design of walls, in which the structural details are determined directly on the basis of the expected lateral displacements of the wall. The provisions of 21.7.6.3 are similar to those of the 1995 code, and have been retained because they are conservative for assessing required transverse reinforcement at wall boundaries for many walls. Requirements of 21.7.6.4 and 21.7.6.5 apply to structural walls designed by either 21.7.6.2 or 21.7.6.3.

R21.7.6.2 — Section 21.7.6.2 is based on the assumption that inelastic response of the wall is dominated by flexural action at a critical, yielding section. The wall should be proportioned so that the critical section occurs where intended.

Eq. (21-8) follows from a displacement-based approach.[21.41,21.42] The approach assumes that special boundary elements are required to confine the concrete where the strain at the extreme compression fiber of the wall exceeds a critical value when the wall is displaced to the design displacement. The horizontal dimension of the special boundary element is intended to extend at least over the length where the compression strain exceeds the critical value. The height of the special boundary element is based on upper bound estimates of plastic hinge length and extends beyond the zone over which concrete spalling is likely to occur. The lower

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(b) Where special boundary elements are required by 21.7.6.2(a), the special boundary element reinforcement shall extend vertically from the critical section a distance not less than the larger of ℓ_w or $M_u/4V_u$.

21.7.6.3 — Structural walls not designed to the provisions of 21.7.6.2 shall have special boundary elements at boundaries and edges around openings of structural walls where the maximum extreme fiber compressive stress, corresponding to factored forces including earthquake effect, exceeds $0.2f'_c$. The special boundary element shall be permitted to be discontinued where the calculated compressive stress is less than $0.15f'_c$. Stresses shall be calculated for the factored forces using a linearly elastic model and gross section properties. For walls with flanges, an effective flange width as defined in 21.7.5.2 shall be used.

21.7.6.4 — Where special boundary elements are required by 21.7.6.2 or 21.7.6.3, (a) through (f) shall be satisfied:

(a) The boundary element shall extend horizontally from the extreme compression fiber a distance not less than the larger of $c - 0.1\ell_w$ and $c/2$;

(b) In flanged sections, the boundary element shall include the effective flange width in compression and shall extend at least 12 in. into the web;

(c) Special boundary element transverse reinforcement shall satisfy the requirements of 21.4.4.1 through 21.4.4.3, except Eq. (21-3) need not be satisfied;

(d) Special boundary element transverse reinforcement at the wall base shall extend into the support at least the development length of the largest longitudinal reinforcement in the special boundary element unless the special boundary element terminates on a footing or mat, where special boundary element transverse reinforcement shall extend at least 12 in. into the footing or mat;

(e) Horizontal reinforcement in the wall web shall be anchored to develop the specified yield strength f_y within the confined core of the boundary element;

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limit of 0.007 on the quantity d_u/h_w requires moderate wall deformation capacity for stiff buildings.

The neutral axis depth c in Eq. (21-8) is the depth calculated according to 10.2, except the nonlinear strain requirements of 10.2.2 need not apply, corresponding to development of nominal flexural strength of the wall when displaced in the same direction as d_u. The axial load is the factored axial load that is consistent with the design load combination that produces the displacement d_u.

R21.7.6.3 — By this procedure, the wall is considered to be acted on by gravity loads W and the maximum shear and moment induced by earthquake in a given direction. Under this loading, the compressed boundary at the critical section resists the tributary gravity load plus the compressive resultant associated with the bending moment.

Recognizing that this loading condition may be repeated many times during the strong motion, the concrete is to be confined where the calculated compressive stresses exceed a nominal critical value equal to $0.2f'_c$. The stress is to be calculated for the factored forces on the section assuming linear response of the gross concrete section. The compressive stress of $0.2f'_c$ is used as an index value and does not necessarily describe the actual state of stress that may develop at the critical section under the influence of the actual inertia forces for the anticipated earthquake intensity.

R21.7.6.4 — The value of $c/2$ in 21.7.6.4(a) is to provide a minimum length of the special boundary element. Where flanges are heavily stressed in compression, the web-to-flange interface is likely to be heavily stressed and may sustain local crushing failure unless special boundary element reinforcement extends into the web. Eq. (21-3) does not apply to walls.

Because horizontal reinforcement is likely to act as web reinforcement in walls requiring boundary elements, it should be fully anchored in boundary elements that act as flanges (21.7.6.4). Achievement of this anchorage is difficult when large transverse cracks occur in the boundary elements. Therefore, standard 90 deg hooks or mechanical anchorage schemes are recommended instead of straight bar development.

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COMMENTARY

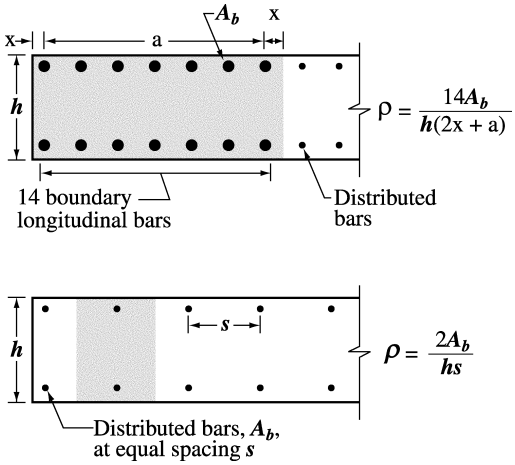


Fig. R21.7.6.5 — Longitudinal reinforcement ratios for typical wall boundary conditions

(f) Mechanical splices of longitudinal reinforcement of boundary elements shall conform to 21.2.6. Welded splices of longitudinal reinforcement of boundary elements shall conform to 21.2.7.

21.7.6.5 — Where special boundary elements are not required by 21.7.6.2 or 21.7.6.3, (a) and (b) shall be satisfied:

(a) If the longitudinal reinforcement ratio at the wall boundary is greater than $400/f_y$, boundary transverse reinforcement shall satisfy 21.4.4.1(c), 21.4.4.3, and 21.7.6.4(a). The maximum longitudinal spacing of transverse reinforcement in the boundary shall not exceed 8 in.;

(b) Except when V_u in the plane of the wall is less than $A_{cv}\sqrt{f'_c}$, horizontal reinforcement terminating at the edges of structural walls without boundary elements shall have a standard hook engaging the edge reinforcement or the edge reinforcement shall be enclosed in U-stirrups having the same size and spacing as, and spliced to, the horizontal reinforcement.

21.7.6.6 — Mechanical and welded splices of longitudinal reinforcement of boundary elements shall conform to 21.2.6 and 21.2.7.

R21.7.6.5 — Cyclic load reversals may lead to buckling of boundary longitudinal reinforcement even in cases where the demands on the boundary of the wall do not require special boundary elements. For walls with moderate amounts of boundary longitudinal reinforcement, ties are required to inhibit buckling. The longitudinal reinforcement ratio is intended to include only the reinforcement at the wall boundary as indicated in Fig. R21.7.6.5. A larger spacing of ties relative to 21.7.6.4(c) is allowed due to the lower deformation demands on the walls.

The addition of hooks or U-stirrups at the ends of horizontal wall reinforcement provides anchorage so that the reinforcement will be effective in resisting shear forces. It will also tend to inhibit the buckling of the vertical edge reinforcement. In walls with low in-plane shear, the development of horizontal reinforcement is not necessary.

21.7.7 — Coupling beams

21.7.7.1 — Coupling beams with aspect ratio $\ell_n/h \geq 4$, shall satisfy the requirements of 21.3. The provisions of 21.3.1.3 and 21.3.1.4(a) shall not be required if it can be shown by analysis that the beam has adequate lateral stability.

21.7.7.2 — Coupling beams with aspect ratio, $\ell_n/h < 4$, shall be permitted to be reinforced with two intersecting groups of diagonally placed bars symmetrical about the midspan.

R21.7.7 — Coupling beams

Coupling beams connecting structural walls can provide stiffness and energy dissipation. In many cases, geometric limits result in coupling beams that are deep in relation to their clear span. Deep coupling beams may be controlled by shear and may be susceptible to strength and stiffness deterioration under earthquake loading. Test results[21.43, 21.44] have shown that confined diagonal reinforcement provides adequate resistance in deep coupling beams.

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21.7.7.3 — Coupling beams with aspect ratio, $\ell_n/h < 2$, and with factored shear force V_u exceeding $4\sqrt{f_c'}\,A_{cp}$ shall be reinforced with two intersecting groups of diagonally placed bars symmetrical about the midspan, unless it can be shown that loss of stiffness and strength of the coupling beams will not impair the vertical load carrying capacity of the structure, or the egress from the structure, or the integrity of nonstructural components and their connections to the structure.

21.7.7.4 — Coupling beams reinforced with two intersecting groups of diagonally placed bars symmetrical about the midspan shall satisfy the following:

(a) Each group of diagonally placed bars shall consist of a minimum of four bars assembled in a core having sides measured to the outside of transverse reinforcement no smaller than $b_w/2$ perpendicular to the plane of the beam and $b_w/5$ in the plane of the beam and perpendicular to the diagonal bars;

(b) The nominal shear strength, V_n, shall be determined by

$$V_n = 2A_{vd}f_y \sin\alpha \le 10\sqrt{f_c'}\,A_{cp} \qquad (21\text{-}9)$$

(c) Each group of diagonally placed bars shall be enclosed in transverse reinforcement satisfying 21.4.4.1 through 21.4.4.3. For the purpose of computing A_g for use in Eq. (10-5) and (21-3), the minimum concrete cover as required in 7.7 shall be assumed on all four sides of each group of diagonally placed reinforcing bars;

(d) The diagonally placed bars shall be developed for tension in the wall;

(e) The diagonally placed bars shall be considered to contribute to nominal flexural strength of the coupling beam;

(f) Reinforcement parallel and transverse to the longitudinal axis shall be provided and, as a minimum, shall conform to 11.8.4 and 11.8.5.

21.7.8 — Construction joints

All construction joints in structural walls shall conform to 6.4 and contact surfaces shall be roughened as in 11.7.9.

21.7.9 — Discontinuous walls

Columns supporting discontinuous structural walls shall be reinforced in accordance with 21.4.4.5.

COMMENTARY

Experiments show that diagonally oriented reinforcement is effective only if the bars are placed with a large inclination. Therefore, diagonally reinforced coupling beams are restricted to beams having aspect ratio $\ell_n/h < 4$.

Each diagonal element consists of a cage of longitudinal and transverse reinforcement as shown in Fig. R21.7.7. The cage contains at least four longitudinal bars and confines a concrete core. The requirement on side dimensions of the cage and its core is to provide adequate toughness and stability to the cross section when the bars are loaded beyond yielding. The minimum dimensions and required reinforcement clearances may control the wall width.

When coupling beams are not used as part of the lateral force resisting system, the requirements for diagonal reinforcement may be waived. Nonprestressed coupling beams are permitted at locations where damage to these beams does not impair vertical load carrying capacity or egress of the structure, or integrity of the nonstructural components and their connections to the structure.

Tests in Reference 21.44 demonstrated that beams reinforced as described in Section 21.7.7 have adequate ductility at shear forces exceeding $10\sqrt{f_c'}\,b_w d$. Consequently, the use of a limit of $10\sqrt{f_c'}\,A_{cp}$ provides an acceptable upper limit.

When the diagonally oriented reinforcement is used, additional reinforcement in 21.7.7.4(f) is to contain the concrete outside the diagonal cores if the concrete is damaged by earthquake loading (Fig. R21.7.7).

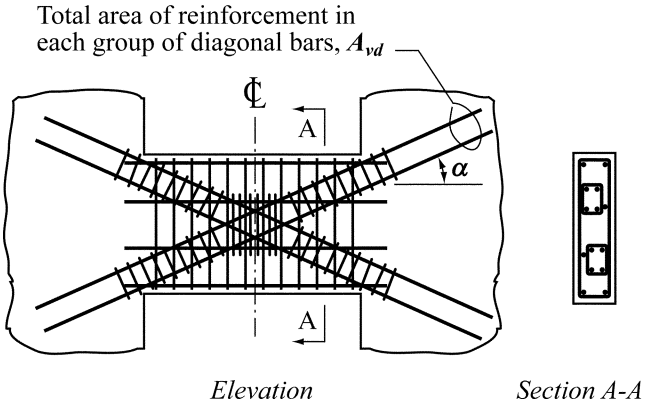


Fig. R21.7.7 — Coupling beam with diagonally oriented reinforcement

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21.8 — Special structural walls constructed using precast concrete

21.8.1 — Special structural walls constructed using precast concrete shall satisfy all requirements of 21.7 for cast-in-place special structural walls in addition to 21.13.2 and 21.13.3.

21.9 — Structural diaphragms and trusses

21.9.1 — Scope

Floor and roof slabs acting as structural diaphragms to transmit design actions induced by earthquake ground motions shall be designed in accordance with this section. This section also applies to struts, ties, chords, and collector elements that transmit forces induced by earthquakes, as well as trusses serving as parts of the earthquake force-resisting systems.

R21.9 — Structural diaphragms and trusses

R21.9.1 — Scope

Diaphragms as used in building construction are structural elements (such as a floor or roof) that provide some or all of the following functions:

(a) Support for building elements (such as walls, partitions, and cladding) resisting horizontal forces but not acting as part of the building vertical lateral-force-resisting system;

(b) Transfer of lateral forces from the point of application to the building vertical lateral-force-resisting system;

(c) Connection of various components of the building vertical lateral-force-resisting system with appropriate strength, stiffness, and toughness so the building responds as intended in the design.[21.45]

21.9.2 — Cast-in-place composite-topping slab diaphragms

A composite-topping slab cast in place on a precast floor or roof shall be permitted to be used as a structural diaphragm provided the topping slab is reinforced and its connections are proportioned and detailed to provide for a complete transfer of forces to chords, collector elements, and the lateral-force-resisting system. The surface of the previously hardened concrete on which the topping slab is placed shall be clean, free of laitance, and intentionally roughened.

R21.9.2 — Cast-in-place composite-topping slab diaphragms

A bonded topping slab is required so that the floor or roof system can provide restraint against slab buckling. Reinforcement is required to ensure the continuity of the shear transfer across precast joints. The connection requirements are introduced to promote a complete system with necessary shear transfers.

21.9.3 — Cast-in-place topping slab diaphragms

A cast-in-place noncomposite topping on a precast floor or roof shall be permitted to serve as a structural diaphragm, provided the cast-in-place topping acting alone is proportioned and detailed to resist the design forces.

R21.9.3 — Cast-in-place topping slab diaphragms

Composite action between the topping slab and the precast floor elements is not required, provided that the topping slab is designed to resist the design seismic forces.

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21.9.4 — Minimum thickness of diaphragms

Concrete slabs and composite topping slabs serving as structural diaphragms used to transmit earthquake forces shall not be less than 2 in. thick. Topping slabs placed over precast floor or roof elements, acting as structural diaphragms and not relying on composite action with the precast elements to resist the design seismic forces, shall have thickness not less than 2-1/2 in.

21.9.5 — Reinforcement

21.9.5.1 — The minimum reinforcement ratio for structural diaphragms shall be in conformance with 7.12. Reinforcement spacing each way in nonpost-tensioned floor or roof systems shall not exceed 18 in. Where welded wire fabric is used as the distributed reinforcement to resist shear in topping slabs placed over precast floor and roof elements, the wires parallel to the span of the precast elements shall be spaced not less than 10 in. on center. Reinforcement provided for shear strength shall be continuous and shall be distributed uniformly across the shear plane.

21.9.5.2 — Bonded tendons used as primary reinforcement in diaphragm chords or collectors shall be proportioned such that the stress due to design seismic forces does not exceed 60,000 psi. Precompression from unbonded tendons shall be permitted to resist diaphragm design forces if a complete load path is provided.

21.9.5.3 — Structural truss elements, struts, ties, diaphragm chords, and collector elements with compressive stresses exceeding $\mathbf{0.2}\,f'_c$ at any section shall have transverse reinforcement, as given in 21.4.4.1 through 21.4.4.3, over the length of the element. The special transverse reinforcement is allowed to be discontinued at a section where the calculated compressive strength is less than $\mathbf{0.15}f'_c$. Stresses shall be calculated for the factored forces using a linearly elastic model and gross-section properties of the elements considered.

21.9.5.4 — All continuous reinforcement in diaphragms, trusses, struts, ties, chords, and collector elements shall be anchored or spliced in accordance with the provisions for reinforcement tension as specified in 21.5.4.

21.9.5.5 — Type 2 splices are required where mechanical splices are used to transfer forces between the diaphragm and the vertical components of the lateral-force-resisting system.

COMMENTARY

R21.9.4 — Minimum thickness of diaphragms

The minimum thickness of concrete diaphragms reflects current practice in joist and waffle systems and composite topping slabs on precast floor and roof systems. Thicker slabs are required when the topping slab does not act compositely with the precast system to resist the design seismic forces.

R21.9.5 — Reinforcement

Minimum reinforcement ratios for diaphragms correspond to the required amount of temperature and shrinkage reinforcement (7.12). The maximum spacing for web reinforcement is intended to control the width of inclined cracks. Minimum average prestress requirements (7.12.3) are considered to be adequate to limit the crack widths in post-tensioned floor systems; therefore, the maximum spacing requirements do not apply to these systems.

The minimum spacing requirement for welded wire fabric in topping slabs on precast floor systems (see 21.9.5.1) is to avoid fracture of the distributed reinforcement during an earthquake. Cracks in the topping slab open immediately above the boundary between the flanges of adjacent precast members, and the wires crossing those cracks are restrained by the transverse wires.[21.46] Therefore, all the deformation associated with cracking should be accommodated in a distance not greater than the spacing of the transverse wires. A minimum spacing of 10 in. for the transverse wires is required in 21.9.5.1 to reduce the likelihood of fracture of the wires crossing the critical cracks during a design earthquake. The minimum spacing requirements do not apply to diaphragms reinforced with individual bars, because strains are distributed over a longer length.

Compressive stress calculated for the factored forces on a linearly elastic model based on gross section of the structural diaphragm is used as an index value to determine whether confining reinforcement is required. A calculated compressive stress of $\mathbf{0.2}f'_c$ in a member is assumed to indicate that integrity of the entire structure is dependent on the ability of that member to resist substantial compressive force under severe cyclic loading. Therefore, transverse reinforcement in 21.4.4 is required in such members to provide confinement for the concrete and the reinforcement (21.9.5.3).

The dimensions of typical structural diaphragms often preclude the use of transverse reinforcement along the chords. Reducing the calculated compressive stress by reducing the span of the diaphragm is considered to be a solution.

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21.9.6 — Design forces

The seismic design forces for structural diaphragms shall be obtained from the lateral load analysis in accordance with the design load combinations.

21.9.7 — Shear strength

21.9.7.1 — Nominal shear strength V_n of structural diaphragms shall not exceed

$$V_n = A_{cv}(2\sqrt{f'_c} + \rho_n f_y) \quad (21\text{-}10)$$

21.9.7.2 — Nominal shear strength V_n of cast-in-place composite-topping slab diaphragms and cast-in-place noncomposite topping slab diaphragms on a precast floor or roof shall not exceed the shear force

$$V_n = A_{cv}\rho_n f_y \quad (21\text{-}11)$$

where A_{cv} is based on the thickness of the topping slab. The required web reinforcement shall be distributed uniformly in both directions.

21.9.7.3 — Nominal shear strength shall not exceed $8A_{cv}\sqrt{f'_c}$ where A_{cv} is the gross cross-sectional area of the diaphragm.

21.9.8 — Boundary elements of structural diaphragms

21.9.8.1 — Boundary elements of structural diaphragms shall be proportioned to resist the sum of the factored axial forces acting in the plane of the diaphragm and the force obtained from dividing the factored moment at the section by the distance between the boundary elements of the diaphragm at that section.

21.9.8.2 — Splices of tensile reinforcement in the chords and collector elements of diaphragms shall develop the yield strength of the reinforcement. Mechanical and welded splices shall conform to 21.2.6 and 21.2.7, respectively.

21.9.8.3 — Reinforcement for chords and collectors at splices and anchorage zones shall have either:

(a) A minimum center-to-center spacing of three longitudinal bar diameters, but not less than 1-1/2 in., and a minimum concrete clear cover of two and one-half longitudinal bar diameters, but not less than 2 in.; or

(b) Transverse reinforcement as required by 11.5.5.3, except as required in 21.9.5.3.

COMMENTARY

R21.9.7 — Shear strength

The shear strength requirements for monolithic diaphragms, Eq. (21-10) in 21.9.7.1, are the same as those for slender structural walls. The term A_{cv} refers to the thickness times the width of the diaphragm. This corresponds to the gross area of the effective deep beam that forms the diaphragm. The shear reinforcement should be placed perpendicular to the span of the diaphragm.

The shear strength requirements for topping slab diaphragms are based on a shear friction model, and the contribution of the concrete to the nominal shear strength is not included in Eq. (21-9) for topping slabs placed over precast floor elements. Following typical construction practice, the topping slabs are roughened immediately above the boundary between the flanges of adjacent precast floor members to direct the paths of shrinkage cracks. As a result, critical sections of the diaphragm are cracked under service loads, and the contribution of the concrete to the shear capacity of the diaphragm may have already been reduced before the design earthquake occurs.

R21.9.8 — Boundary elements of structural diaphragms

For structural diaphragms, the design moments are assumed to be resisted entirely by chord forces acting at opposite edges of the diaphragm. Reinforcement located at the edges of collectors should be fully developed for its yield strength. Adequate confinement of lap splices is also required. If chord reinforcement is located within a wall, the joint between the diaphragm and the wall should be provided with adequate shear strength to transfer the shear forces.

Section 21.9.8.3 is intended to reduce the possibility of chord buckling in the vicinity of splices and anchorage zones.

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21.9.9 — Construction joints

All construction joints in diaphragms shall conform to 6.4 and contact surfaces shall be roughened as in 11.7.9.

21.10 — Foundations

21.10.1 — Scope

21.10.1.1 — Foundations resisting earthquake-induced forces or transferring earthquake-induced forces between structure and ground shall comply with 21.10 and other applicable code provisions.

21.10.1.2 — The provisions in this section for piles, drilled piers, caissons, and slabs on grade shall supplement other applicable code design and construction criteria. See 1.1.5 and 1.1.6.

21.10.2 — Footings, foundation mats, and pile caps

21.10.2.1 — Longitudinal reinforcement of columns and structural walls resisting forces induced by earthquake effects shall extend into the footing, mat, or pile cap, and shall be fully developed for tension at the interface.

21.10.2.2 — Columns designed assuming fixed-end conditions at the foundation shall comply with 21.10.2.1 and, if hooks are required, longitudinal reinforcement resisting flexure shall have 90 deg hooks near the bottom of the foundation with the free end of the bars oriented towards the center of the column.

21.10.2.3 — Columns or boundary elements of special reinforced concrete structural walls that have an edge within one-half the footing depth from an edge of the footing shall have transverse reinforcement in accordance with 21.4.4 provided below the top of the footing. This reinforcement shall extend into the footing a distance no less than the smaller of the depth of the footing, mat, or pile cap, or the development length in tension of the longitudinal reinforcement.

21.10.2.4 — Where earthquake effects create uplift forces in boundary elements of special reinforced concrete structural walls or columns, flexural reinforcement shall be provided in the top of the footing, mat or pile cap to resist the design load combinations, and shall not be less than required by 10.5.

21.10.2.5 — See 22.10 for use of plain concrete in footings and basement walls.

COMMENTARY

R21.10 — Foundations

R21.10.1 — Scope

Requirements for foundations supporting buildings assigned to high seismic performance or design categories were added to the 1999 code. They represent a consensus of a minimum level of good practice in designing and detailing concrete foundations including piles, drilled piers, and caissons. It is desirable that inelastic response in strong ground shaking occurs above the foundations, as repairs to foundations can be extremely difficult and expensive.

R21.10.2 — Footings, foundation mats, and pile caps

R21.10.2.2 — Tests[21.47] have demonstrated that flexural members terminating in a footing, slab or beam (a T-joint) should have their hooks turned inwards toward the axis of the member for the joint to be able to resist the flexure in the member forming the stem of the T.

R21.10.2.3 — Columns or boundary members supported close to the edge of the foundation, as often occurs near property lines, should be detailed to prevent an edge failure of the footing, pile cap, or mat.

R21.10.2.4 — The purpose of 21.10.2.4 is to alert the designer to provide top reinforcement as well as other required reinforcement.

R21.10.2.5 — In regions of high seismicity, it is desirable to reinforce foundations. Committee 318 recommends that foundation or basement walls be reinforced in regions of high seismicity.

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21.10.3 — Grade beams and slabs on grade

21.10.3.1 — Grade beams designed to act as horizontal ties between pile caps or footings shall have continuous longitudinal reinforcement that shall be developed within or beyond the supported column or anchored within the pile cap or footing at all discontinuities.

21.10.3.2 — Grade beams designed to act as horizontal ties between pile caps or footings shall be proportioned such that the smallest cross-sectional dimension shall be equal to or greater than the clear spacing between connected columns divided by 20, but need not be greater than 18 in. Closed ties shall be provided at a spacing not to exceed the lesser of one-half the smallest orthogonal cross-sectional dimension or 12 in.

21.10.3.3 — Grade beams and beams that are part of a mat foundation subjected to flexure from columns that are part of the lateral-force-resisting system shall conform to 21.3.

21.10.3.4 — Slabs on grade that resist seismic forces from walls or columns that are part of the lateral-force-resisting system shall be designed as structural diaphragms in accordance with 21.9. The design drawings shall clearly state that the slab on grade is a structural diaphragm and part of the lateral-force-resisting system.

21.10.4 — Piles, piers, and caissons

21.10.4.1 — Provisions of 21.10.4 shall apply to concrete piles, piers, and caissons supporting structures designed for earthquake resistance.

21.10.4.2 — Piles, piers, or caissons resisting tension loads shall have continuous longitudinal reinforcement over the length resisting design tension forces. The longitudinal reinforcement shall be detailed to transfer tension forces within the pile cap to supported structural members.

21.10.4.3 — Where tension forces induced by earthquake effects are transferred between pile cap or mat foundation and precast pile by reinforcing bars grouted or post-installed in the top of the pile, the grouting system shall have been demonstrated by test to develop at least 125 percent of the specified yield strength of the bar.

21.10.4.4 — Piles, piers, or caissons shall have transverse reinforcement in accordance with 21.4.4 at locations (a) and (b):

(a) At the top of the member for at least 5 times the

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R21.10.3 — Grade beams and slabs on grade

For seismic conditions, slabs on grade (soil-supported slabs) are often part of the lateral-force-resisting system and should be designed in accordance with this code as well as other appropriate standards or guidelines. See 1.1.6.

R21.10.3.2 — Grade beams between pile caps or footings can be separate beams beneath the slab on grade or can be a thickened portion of the slab on grade. The cross-sectional limitation and minimum tie requirements provide reasonable proportions.

R21.10.3.3 — Grade beams resisting seismic flexural stresses from column moments should have reinforcing details similar to the beams of the frame above the foundation.

R21.10.3.4 — Slabs on grade often act as a diaphragm to hold the building together at the ground level and minimize the effects of out-of-phase ground motion that may occur over the footprint of the building. In these cases, the slab on grade should be adequately reinforced and detailed. The design drawings should clearly state that these slabs on grade are structural members so as to prohibit sawcutting of the slab.

R21.10.4 — Piles, piers, and caissons

Adequate performance of piles and caissons for seismic loadings requires that these provisions be met in addition to other applicable standards or guidelines. See R1.1.5.

R21.10.4.2 — A load path is necessary at pile caps to transfer tension forces from the reinforcing bars in the column or boundary member through the pile cap to the reinforcement of the pile or caisson.

R21.10.4.3 — Grouted dowels in a blockout in the top of a precast concrete pile need to be developed, and testing is a practical means of demonstrating capacity. Alternatively, reinforcing bars can be cast in the upper portion of the pile, exposed by chipping of concrete and mechanically connected or welded to an extension.

R21.10.4.4 — During earthquakes, piles can be subjected to extremely high flexural demands at points of discontinuity, especially just below the pile cap and near the base of a soft or loose soil deposit. The 1999 code requirement for confinement reinforcement at the top of the pile is

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member cross-sectional dimension, but not less than 6 ft below the bottom of the pile cap;

(b) For the portion of piles in soil that is not capable of providing lateral support, or in air and water, along the entire unsupported length plus the length required in 21.10.4.4(a).

21.10.4.5 — For precast concrete driven piles, the length of transverse reinforcement provided shall be sufficient to account for potential variations in the elevation in pile tips.

21.10.4.6 — Concrete piles, piers, or caissons in foundations supporting one- and two-story stud bearing wall construction are exempt from the transverse reinforcement requirements of 21.10.4.4 and 21.10.4.5.

21.10.4.7 — Pile caps incorporating batter piles shall be designed to resist the full compressive strength of the batter piles acting as short columns. The slenderness effects of batter piles shall be considered for the portion of the piles in soil that is not capable of providing lateral support, or in air or water.

21.11 — Frame members not proportioned to resist forces induced by earthquake motions

21.11.1 — Frame members assumed not to contribute to lateral resistance shall be detailed according to 21.11.2 or 21.11.3 depending on the magnitude of moments induced in those members if subjected to the design displacement. If effects of design displacements are not explicitly checked, it shall be permitted to apply the requirements of 21.11.3.

21.11.2 — When the induced moments and shears under design displacements of 21.11.1 combined with the factored gravity moments and shears do not exceed the design moment and shear strength of the frame member, the conditions of 21.11.2.1, 21.11.2.2, and 21.11.2.3 shall be satisfied. The gravity load combinations of $(1.2D + 1.0L + 0.2S)$ or $0.9D$, whichever is critical, shall be used. The load factor on L shall be permitted to be reduced to 0.5 except for garages, areas occupied as places of public assembly, and all areas where the live load, L, is greater than 100 lb/ft^2.

21.11.2.1 — Members with factored gravity axial forces not exceeding $A_g f_c'/10$ shall satisfy 21.3.2.1.

COMMENTARY

based on numerous failures observed at this location in recent earthquakes. Transverse reinforcement is required in this region to provide ductile performance. The designer should also consider possible inelastic action in the pile at abrupt changes in soil deposits, such as changes from soft to firm or loose to dense soil layers. Where precast piles are to be used, the potential for the pile tip to be driven to an elevation different than that specified in the drawings needs to be considered when detailing the pile. If the pile reaches refusal at a shallower depth, a longer length of pile will need to be cut off. If this possibility is not foreseen, the length of transverse reinforcement required by 21.10.4.4 may not be available after the excess pile length is cut off.

R21.10.4.7 — Extensive structural damage has often been observed at the junction of batter piles and the buildings. The pile cap and surrounding structure should be designed for the potentially large forces that can be developed in batter piles.

R21.11 — Frame members not proportioned to resist forces induced by earthquake motions

The detailing requirements for members that are part of the lateral-force resisting system assume that the members may undergo deformations that exceed the yield limit of the member without significant loss of strength. Members that are not part of the designated lateral-force-resisting system are not required to meet all the detailing requirements of members that are relied on to resist lateral forces. They should, however, be able to resist the gravity loads at lateral displacements corresponding to the design level prescribed by the governing code for earthquake-resistant design. The design displacement is defined in 21.1.

Section 21.11 recognizes that actual displacements resulting from earthquake forces may be larger than the displacements calculated using the design forces and commonly used analysis models. Section 21.11.1 defines a nominal displacement for the purpose of prescribing detailing requirements. This section has been revised from the 1995 code to reflect changes from a working stress design approach to a strength design approach in governing codes for earthquake-resistant design.[21.2] Actual displacements may exceed the value of

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Stirrups shall be spaced not more than $d/2$ throughout the length of the member.

21.11.2.2 — Members with factored gravity axial forces exceeding $A_g f_c'/10$ shall satisfy 21.4.3, 21.4.4.1(c), 21.4.4.3, and 21.4.5. The maximum longitudinal spacing of ties shall be s_o for the full column height. The spacing s_o shall not be more than six diameters of the smallest longitudinal bar enclosed or 6 in., whichever is smaller.

21.11.2.3 — Members with factored gravity axial forces exceeding $0.35P_o$ shall satisfy 21.11.2.2 and the amount of transverse reinforcement provided shall be one-half of that required by 21.4.4.1 but shall not exceed a spacing s_o for the full height of the column.

21.11.3 — If the induced moment or shear under design displacements of 21.11.1 exceeds the design moment or shear strength of the frame member, or if induced moments are not calculated, the conditions of 21.11.3.1, 21.11.3.2, and 21.11.3.3 shall be satisfied.

21.11.3.1 — Materials shall satisfy 21.2.4 and 21.2.5. Mechanical splices shall satisfy 21.2.6 and welded splices shall satisfy 21.2.7.1.

21.11.3.2 — Members with factored gravity axial forces not exceeding $A_g f_c'/10$ shall satisfy 21.3.2.1 and 21.3.4. Stirrups shall be spaced at not more than $d/2$ throughout the length of the member.

21.11.3.3 — Members with factored gravity axial forces exceeding $A_g f_c'/10$ shall satisfy 21.4.3.1, 21.4.4, 21.4.5, and 21.5.2.1.

21.11.4 — Precast concrete frame members assumed not to contribute to lateral resistance, including their connections, shall satisfy (a), (b), and (c), in addition to 21.11.1 through 21.11.3:

(a) Ties specified in 21.11.2.2 shall be provided over the entire column height, including the depth of the beams;

(b) Structural integrity reinforcement, as specified in 16.5, shall be provided; and

(c) Bearing length at support of a beam shall be at least 2 in. longer than determined from calculations using bearing strength values from 10.17.

21.12 — Requirements for intermediate moment frames

21.12.1 — The requirements of this section apply to intermediate moment frames.

COMMENTARY

21.11.1. Section 21.11.2 prescribes detailing requirements intended to provide a system capable of sustaining gravity loads under moderate excursions into the inelastic range. Section 21.11.3 prescribes detailing requirements intended to provide a system capable of sustaining gravity loads under larger displacements.

Models used to determine design deflections of buildings should be chosen to produce results that conservatively bound the values expected during the design earthquake considering vertical, horizontal, and diaphragm systems as appropriate.

For gravity load factors, see R9.2.

The poor performance of some buildings with precast concrete gravity systems during the Northridge Earthquake was attributed to several factors addressed in 21.11.4. Columns should contain ties over their entire height, frame members not proportioned to resist earthquake forces should be tied together, and longer bearing lengths should be used to maintain integrity of the gravity system during shaking. The 2 in. increase in bearing length is based on an assumed 4 percent story drift ratio and 50 in. beam depth, and is considered to be conservative for the ground motions expected in high seismic zones. In addition to the provisions of 21.11.4, precast frame members assumed not to contribute to lateral resistance should also satisfy 21.11.1 through 21.11.3.

R21.12 — Requirements for intermediate moment frames

The objective of the requirements in 21.12.3 is to reduce the risk of failure in shear during an earthquake. The

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21.12.2 — Reinforcement details in a frame member shall satisfy 21.12.4 if the factored compressive axial load for the member does not exceed $A_g f_c'/10$. If the factored compressive axial load is larger, frame reinforcement details shall satisfy 21.12.5 unless the member has spiral reinforcement according to Eq. (10-5). If a two-way slab system without beams is treated as part of a frame resisting earthquake effect, reinforcement details in any span resisting moments caused by lateral force shall satisfy 21.12.6.

21.12.3 — Design shear strength of beams, columns, and two-way slabs resisting earthquake effect shall not be less than either (a) or (b):

(a) The sum of the shear associated with development of nominal moment strengths of the member at each restrained end of the clear span and the shear calculated for factored gravity loads;

(b) The maximum shear obtained from design load combinations that include earthquake effect ***E***, with ***E*** assumed to be twice that prescribed by the governing code for earthquake-resistant design.

21.12.4 — Beams

21.12.4.1 — The positive moment strength at the face of the joint shall be not less than one-third the negative moment strength provided at that face of the joint. Neither the negative nor the positive moment strength at any section along the length of the member shall be less than one-fifth the maximum moment strength provided at the face of either joint.

21.12.4.2 — At both ends of the member, hoops shall be provided over lengths equal to twice the member depth measured from the face of the supporting member toward midspan. The first hoop shall be located at not more than 2 in. from the face of the supporting member. Maximum hoop spacing shall not exceed the smallest of (a), (b), (c), or (d):

(a) $d/4$;

(b) Eight times the diameter of the smallest longitudinal bar enclosed;

(c) 24 times the diameter of the hoop bar;

(d) 12 in.

21.12.4.3 — Stirrups shall be placed at not more than $d/2$ throughout the length of the member.

21.12.5 — Columns

21.12.5.1 — Columns shall be spirally reinforced in accordance with 7.10.4 or shall conform with 21.12.5.2

COMMENTARY

designer is given two options by which to determine the factored shear force.

According to option (a) of 21.12.3, the factored shear force is determined from the nominal moment strength of the member and the gravity load on it. Examples for a beam and a column are illustrated in Fig. R21.12.3.

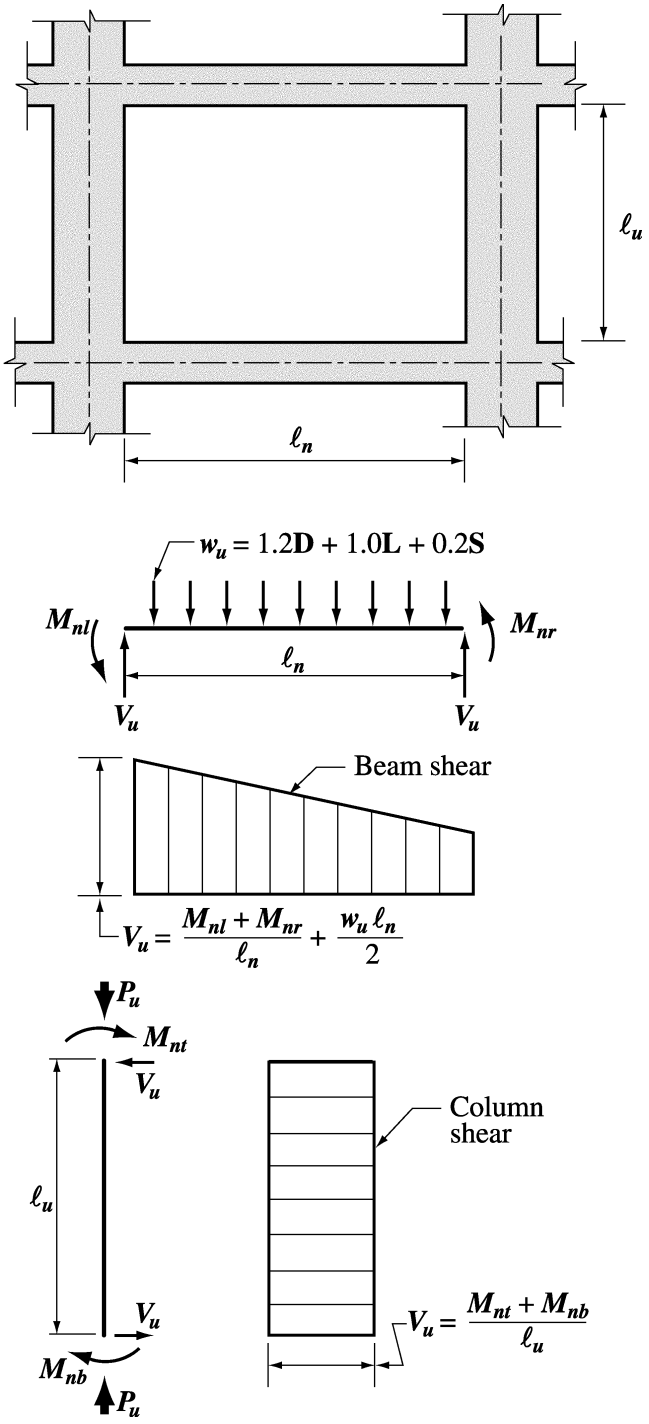


Fig. R21.12.3—Design shears for frames in regions of moderate seismic risk (see 21.12)

CODE

through 21.12.5.4. Section 21.12.5.5 shall apply to all columns.

21.12.5.2 — At both ends of the member, hoops shall be provided at spacing s_o over a length ℓ_o measured from the joint face. Spacing s_o shall not exceed the smallest of (a), (b), (c), and (d):

(a) Eight times the diameter of the smallest longitudinal bar enclosed;

(b) 24 times the diameter of the hoop bar;

(c) One-half of the smallest cross-sectional dimension of the frame member;

(d) 12 in.

Length ℓ_o shall not be less than the largest of (e), (f), and (g):

(e) One-sixth of the clear span of the member;

(f) Maximum cross-sectional dimension of the member;

(g) 18 in.

21.12.5.3 — The first hoop shall be located at not more than $s_o/2$ from the joint face.

21.12.5.4 — Outside the length ℓ_o, spacing of transverse reinforcement shall conform to 7.10 and 11.5.4.1.

21.12.5.5 — Joint transverse reinforcement shall conform to 11.11.2.

21.12.6 — Two-way slabs without beams

21.12.6.1 — Factored slab moment at support related to earthquake effect shall be determined for load combinations given in Eq. (9-5) and (9-7). All reinforcement provided to resist M_s, the portion of slab moment balanced by support moment, shall be placed within the column strip defined in 13.2.1.

21.12.6.2 — The fraction, defined by Eq. (13-1), of moment M_s, shall be resisted by reinforcement placed within the effective width specified in 13.5.3.2. Effective slab width for exterior and corner connections shall not extend beyond the column face a distance greater than c_t measured perpendicular to the slab span.

21.12.6.3 — Not less than one-half of the reinforcement in the column strip at support shall be placed within the effective slab width given in 13.5.3.2.

21.12.6.4 — Not less than one-quarter of the top reinforcement at the support in the column strip shall be continuous throughout the span.

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To determine the maximum beam shear, it is assumed that its nominal moment strengths ($\phi = 1.0$) are developed simultaneously at both ends of its clear span. As indicated in Fig. R21.12.3, the shear associated with this condition $[(M_{nl} + M_{nr})/\ell_n]$ added algebraically to the effect of the factored gravity loads indicates the design shear for of the beam. For this example, both the dead load w_D and the live load w_L have been assumed to be uniformly distributed.

Determination of the design shear for a column is also illustrated for a particular example in Fig. R21.12.3. The factored design axial load, P_u, should be chosen to develop the largest moment strength of the column.

In all applications of option (a) of 21.12.3, shears are required to be calculated for moment, acting clockwise and counterclockwise. Fig. R21.12.3 demonstrates only one of the two conditions that are to be considered for every member. Option (b) bases V_u on the load combination including the earthquake effect, E, which should be doubled. For example, the load combination defined by Eq. (9-5) would be:

$$U = 1.2D + 2.0E + 1.0L + 0.2S$$

where E is the value specified by the governing code.

Section 21.12.4 contains requirements for providing beams with a threshold level of toughness. Transverse reinforcement at the ends of the beam shall be hoops. In most cases, stirrups required by 21.12.3 for design shear force will be more than those required by 21.12.4. Requirements of 21.12.5 serve the same purpose for columns.

Section 21.12.6 applies to two-way slabs without beams, such as flat plates.

Using load combinations of Eq. (9-5) and (9-7) may result in moments requiring top and bottom reinforcement at the supports.

The moment M_s refers, for a given design load combination with E acting in one horizontal direction, to that portion of the factored slab moment that is balanced by the supporting members at a joint. It is not necessarily equal to the total design moment at support for a load combination including earthquake effect. In accordance with 13.5.3.2, only a fraction ($\gamma_f M_s$) of the moment M_s is assigned to the slab effective width. For edge and corner connections, flexural reinforcement perpendicular to the edge is not considered fully effective unless it is placed within the effective slab width.[21.48,49] See Fig. 21.12.6.1.

Application of the provisions of 21.12.6 is illustrated in Fig. R21.12.6.2 and Fig. R21.12.6.3.

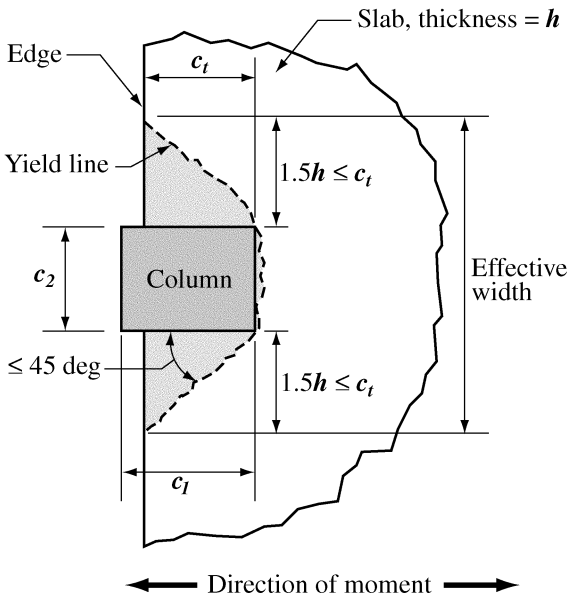
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21.12.6.5 — Continuous bottom reinforcement in the column strip shall be not less than one-third of the top reinforcement at the support in the column strip.

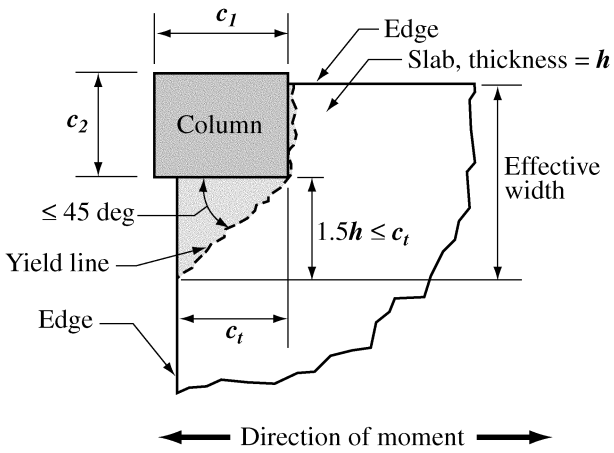
21.12.6.6 — Not less than one-half of all bottom middle strip reinforcement and all bottom column strip reinforcement at midspan shall be continuous and shall develop its yield strength at face of support as defined in 13.6.2.5.

21.12.6.7—At discontinuous edges of the slab all top and bottom reinforcement at support shall be developed at the face of support as defined in 13.6.2.5.

COMMENTARY

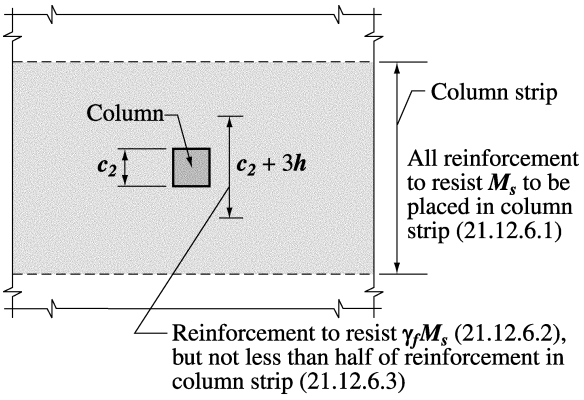


(a) Edge connection



(b) Corner connection

Fig. R21.12.6.1 — Effective width for reinforcement placement in edge and corner connections



Notes: (a) Applies to both top and bottom reinforcement
(b) See 13.0—Notation

Fig. R21.12.6.2 — Location of reinforcement in slabs

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COMMENTARY

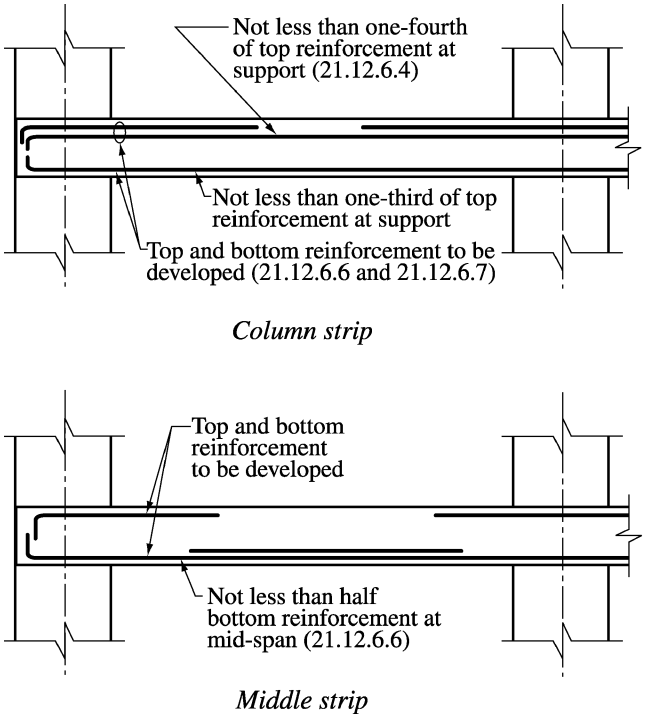


Fig. R21.12.6.3—Arrangement of reinforcement in slabs

21.12.6.8 — At the critical sections for columns defined in 11.12.1.2, two-way shear caused by factored gravity loads shall not exceed $0.4\phi V_c$, where V_c shall be calculated as defined in 11.12.2.1 for nonprestressed slabs and in 11.12.2.2 for prestressed slabs. It shall be permitted to waive this requirement if the contribution of the earthquake-induced factored two-way shear stress transferred by eccentricity of shear in accordance with 11.12.6.1 and 11.12.6.2 at the point of maximum stress does not exceed one-half of the stress ϕv_n permitted by 11.12.6.2.

R21.12.6.8 — The requirements apply to two-way slabs that are part of the primary lateral force-resisting system. Slab-column connections in laboratory tests[21.49] exhibited reduced lateral displacement ductility when the shear at the column connection exceeded the recommended limit.

21.13 — Intermediate precast structural walls

21.13.1 — The requirements of this section apply to intermediate precast structural walls used to resist forces induced by earthquake motions.

21.13.2 — In connections between wall panels, or between wall panels and the foundation, yielding shall be restricted to steel elements or reinforcement.

21.13.3 — Elements of the connection that are not designed to yield shall develop at least $1.5S_y$.

21.13 — Intermediate precast structural walls

Connections between precast wall panels or between wall panels and the foundation are required to resist forces induced by earthquake motions and to provide for yielding in the vicinity of connections. When Type 2 mechanical splices are used to directly connect primary reinforcement, the probable strength of the splice should be at least 1-1/2 times the specified yield strength of the reinforcement.

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COMMENTARY

Notes

PART 7 — STRUCTURAL PLAIN CONCRETE

CHAPTER 22 — STRUCTURAL PLAIN CONCRETE

CODE

22.0 — Notation

A_g = gross area of section, in.^{2}
A_1 = loaded area, in.^{2}
A_2 = the area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal, in.^{2}
b = width of member, in.
b_o = perimeter of critical section for shear in footings, in.
B_n = nominal bearing load, lb
f_c' = specified compressive strength of concrete, psi. See Chapter 5
$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi
f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi. See 5.1.4 and 5.1.5
h = overall thickness of member, in.
ℓ_c = vertical distance between supports, in.
M_n = nominal moment strength at section, in.-lb
M_u = factored moment at section, in.-lb
P_n = nominal strength of cross section subject to compression, lb
P_{nw} = nominal axial load strength of wall designed by 22.6.5, lb
P_u = factored axial load at given eccentricity, lb
S = elastic section modulus of section, in.^{3}
V_n = nominal shear strength at section, lb
V_u = factored shear force at section, lb
b_c = ratio of long side to short side of concentrated load or reaction area
f = strength reduction factor. See 9.3.5

22.1 — Scope

22.1.1 — This chapter provides minimum requirements for design and construction of structural plain concrete members (cast-in-place or precast) except in 22.1.1.1 and 22.1.1.2.

COMMENTARY

22.0 — Notation

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

R22.1 — Scope

Prior to the 1995 code, requirements for plain concrete were set forth in **"Building Code Requirements for Structural Plain Concrete (ACI 318.1-89) (Revised 1992)."** Requirements for plain concrete are now in the code.

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22.1.1.1 — Structural plain concrete basement walls shall be exempt from the requirements for special exposure conditions of 4.2.2.

22.1.1.2 — Design and construction of soil-supported slabs, such as sidewalks and slabs on grade, shall not be governed by the code unless they transmit vertical loads or lateral forces from other parts of the structure to the soil.

22.1.2 — For special structures, such as arches, underground utility structures, gravity walls, and shielding walls, provisions of this chapter shall govern where applicable.

22.2 — Limitations

22.2.1 — Provisions of this chapter shall apply for design of structural plain concrete members. See 2.1.

22.2.2 — Use of structural plain concrete shall be limited to (a), (b), or (c):

(a) Members that are continuously supported by soil or supported by other structural members capable of providing continuous vertical support;

(b) Members for which arch action provides compression under all conditions of loading;

(c) Walls and pedestals. See 22.6 and 22.8.

The use of structural plain concrete columns shall not be permitted.

22.2.3 — This chapter shall not govern design and installation of cast-in-place concrete piles and piers embedded in ground.

COMMENTARY

R22.1.1.1 — Section 22.1.1.1 exempts structural plain concrete walls from the requirements for special exposure conditions because of the numerous successful uses of concrete with 28-day compressive strengths of 2500 and 3000 psi in the basement walls of residences and other minor structures that did not meet the strength requirements of Table 4.2.2.

R22.1.1.2 — It is not within the scope of this code to provide design and construction requirements for nonstructural members of plain concrete such as soil-supported slabs (slabs on grade).

R22.2 — Limitations

R22.2.2 and R22.2.3 — Since the structural integrity of plain concrete members depends solely on the properties of the concrete, use of structural plain concrete members should be limited to members that are primarily in a state of compression, members that can tolerate random cracks without detriment to their structural integrity, and members where ductility is not an essential feature of design. The tensile strength of concrete can be recognized in design of members. Tensile stresses due to restraint from creep, shrinkage, or temperature effects are to be considered and sufficiently reduced by construction techniques to avoid uncontrolled cracks, or when uncontrolled cracks due to such restraint effects are anticipated to occur, they will not induce structural failure.

Plain concrete walls are permitted (see 22.6) without a height limitation. However, for multistory construction and other major structures, ACI Committee 318 encourages the use of walls designed in accordance with Chapter 14 (see R22.6).

Since plain concrete lacks the necessary ductility that columns should possess and because a random crack in an unreinforced column will most likely endanger its structural integrity, the code does not permit use of plain concrete for columns. It does allow its use for pedestals limited to a ratio of unsupported height to least lateral dimension of 3 or less (see 22.8.2).

Structural elements such as cast-in-place concrete piles and piers in ground or other material sufficiently stiff to provide adequate lateral support to prevent buckling are not covered by this code. Such elements are covered by the general building code.

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22.2.4 — Minimum strength

Specified compressive strength of plain concrete to be used for structural purposes shall be not less than given in 1.1.1.

22.3 — Joints

22.3.1 — Contraction or isolation joints shall be provided to divide structural plain concrete members into flexurally discontinuous elements. The size of each element shall limit control excessive buildup of internal stresses caused by restraint to movements from creep, shrinkage, and temperature effects.

22.3.2 — In determining the number and location of contraction or isolation joints, consideration shall be given to: influence of climatic conditions; selection and proportioning of materials; mixing, placing, and curing of concrete; degree of restraint to movement; stresses due to loads to which an element is subject; and construction techniques.

22.4 — Design method

22.4.1 — Structural plain concrete members shall be designed for adequate strength in accordance with the code, using load factors and design strength.

22.4.2 — Factored loads and forces shall be in combinations as in 9.2.

22.4.3 — Where required strength exceeds design strength, reinforcement shall be provided and the member designed as a reinforced concrete member in accordance with appropriate design requirements of the code.

22.4.4 — Strength design of structural plain concrete members for flexure and axial loads shall be based on a linear stress-strain relationship in both tension and compression.

22.4.5 — Tensile strength of concrete shall be permitted to be considered in design of plain concrete members when provisions of 22.3 have been followed.

22.4.6 — No strength shall be assigned to steel reinforcement that may be present.

COMMENTARY

R22.2.4 — Minimum strength

A minimum strength requirement for plain concrete construction is considered necessary because safety is based solely on strength and quality of concrete treated as a homogeneous material. Lean concrete mixtures may not produce adequately homogeneous material or well-formed surfaces.

R22.3 — Joints

Joints in plain concrete construction are an important design consideration. In reinforced concrete, reinforcement is provided to resist the stresses due to restraint of creep, shrinkage, and temperature effects. In plain concrete, joints are the only means of controlling and thereby relieving the buildup of such tensile stresses. A plain concrete member, therefore, should be small enough, or divided into smaller elements by joints, to control the buildup of internal stresses. The joint may be a contraction joint or an isolation joint. A minimum 25 percent reduction of member thickness is considered sufficient for contraction joints to be effective. The jointing should be such that no axial tension or flexural tension can be developed across a joint after cracking, if applicable, a condition referred to as flexural discontinuity. Where random cracking due to creep, shrinkage, and temperature effects will not affect the structural integrity, and is otherwise acceptable, such as transverse cracks in a continuous wall footing, transverse contraction or isolation joints are not necessary.

R22.4 — Design method

Plain concrete members are proportioned for adequate strength using factored loads and forces. When the design strength is exceeded, the section should be increased or the specified strength of concrete increased, or both, or the member designed as a reinforced concrete member in accordance with the code. The designer should note, however, that an increase in concrete section may have a detrimental effect; stress due to load will decrease but stresses due to creep, shrinkage, and temperature effects may increase.

R22.4.4 — Flexural tension may be considered in design of plain concrete members to sustain loads, provided the computed stress does not exceed the permissible stress, and construction, contraction, or isolation joints are provided to relieve the resulting tensile stresses due to restraint of creep, temperature, and shrinkage effects.

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22.4.7 — Tension shall not be transmitted through outside edges, construction joints, contraction joints, or isolation joints of an individual plain concrete element. No flexural continuity due to tension shall be assumed between adjacent structural plain concrete elements.

22.4.8 — When computing strength in flexure, combined flexure and axial load, and shear, the entire cross section of a member shall be considered in design, except for concrete cast against soil where overall thickness h shall be taken as 2 in. less than actual thickness.

22.5 — Strength design

22.5.1 — Design of cross sections subject to flexure shall be based on

$$\phi M_n \geq M_u \quad (22\text{-}1)$$

where

$$M_n = 5\sqrt{f'_c}\,S \quad (22\text{-}2)$$

if tension controls, and by

$$M_n = 0.85 f'_c\,S \quad (22\text{-}3)$$

if compression controls, where S is the corresponding elastic section modulus.

22.5.2 — Design of cross sections subject to compression shall be based on

$$\phi P_n \geq P_u \quad (22\text{-}4)$$

where P_u is factored load and P_n is nominal compression strength computed by

$$P_n = 0.60 f'_c\left[1-\left(\frac{\ell_c}{32h}\right)^2\right]A_1 \quad (22\text{-}5)$$

where A_1 is the loaded area.

22.5.3 — Members subject to combined flexure and axial load in compression shall be proportioned such that on the compression face:

$$P_u/\phi P_n + M_u/\phi M_n \leq 1 \quad (22\text{-}6)$$

and on the tension face:

$$M_u/S - P_u/A_g \leq 5\phi\sqrt{f'_c} \quad (22\text{-}7)$$

COMMENTARY

R22.4.8 — The reduced overall thickness h for concrete cast against earth is to allow for unevenness of excavation and for some contamination of the concrete adjacent to the soil.

R22.5 — Strength design

R22.5.2 — Eq. (22-5) is presented to reflect the general range of braced and restrained end conditions encountered in structural plain concrete elements. The effective length factor was omitted as a modifier of ℓ_c, the vertical distance between supports, since this is conservative for walls with assumed pin supports that are required to be braced against lateral translation as in 22.6.6.4.

R22.5.3 — Plain concrete members subject to combined flexure and axial compressive load are proportioned such that on the compression face:

$$\frac{P_u}{0.60\phi f'_c\left[1-\left(\frac{\ell_c}{32h}\right)^2\right]A_1} + \frac{M_u}{0.85\phi f'_c S} \leq 1$$

CODE

22.5.4 — Design of rectangular cross sections subject to shear shall be based on

$$\phi V_n \geq V_u \quad (22\text{-}8)$$

where V_u is factored shear and V_n is nominal shear strength computed by

$$V_n = \frac{4}{3}\sqrt{f'_c}\,bh \quad (22\text{-}9)$$

for beam action and by

$$V_n = \left[\frac{4}{3} + \frac{8}{3b_c}\right]\sqrt{f'_c}\,b_o h \quad (22\text{-}10)$$

for two-way action, but not greater than $2.66\sqrt{f'_c}\,b_o h$.

22.5.5 — Design of bearing areas subject to compression shall be based on

$$\phi B_n \geq P_u \quad (22\text{-}11)$$

where P_u is factored bearing load and B_n is nominal bearing strength of loaded area A_1 computed by

$$B_n = 0.85 f'_c A_1 \quad (22\text{-}12)$$

except when the supporting surface is wider on all sides than the loaded area, design bearing strength on the loaded area shall be multiplied by $\sqrt{A_2/A_1}$ but not more than 2.

22.5.6 — Lightweight concrete

22.5.6.1 — Provisions of 22.5 apply to normalweight concrete. When lightweight aggregate concrete is used, either (a) or (b) shall apply:

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and that on the tension face:

$$\left(\begin{matrix}\text{Calculated}\\ \text{bending stress}\end{matrix}\right) - \left(\begin{matrix}\text{Calculated}\\ \text{axial stress}\end{matrix}\right) \leq 5\phi\sqrt{f'_c}$$

R22.5.4 — Proportions of plain concrete members usually are controlled by tensile strength rather than shear strength. Shear stress (as a substitute for principal tensile stress) rarely will control. However, since it is difficult to foresee all possible conditions where shear may have to be investigated (such as shear keys), Committee 318 maintains the investigation of this basic stress condition. An experienced designer will soon recognize where shear is not critical for plain concrete members and will adjust design procedures accordingly.

The shear requirements for plain concrete assume an uncracked section. Shear failure in plain concrete will be a diagonal tension failure, occurring when the principal tensile stress near the centroidal axis becomes equal to the tensile strength of the concrete. Since the major portion of the principal tensile stress comes from the shear, the code safeguards against tensile failure by limiting the permissible shear at the centroidal axis as calculated from the equation for a section of homogeneous material:

$$v = VQ/Ib$$

where v and V are the shear stress and shear force, respectively, at the section considered, Q is the statical moment of the area outside the section being considered about centroidal axis of the gross section, I is the moment of inertia of the gross section, and b is the width where shear stress is being computed.

R22.5.6 — Lightweight concrete

See R11.2

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(a) When f_{ct} is specified and concrete is proportioned in accordance with 5.2, equations that include $\sqrt{f'_c}$ shall be modified by substituting $f_{ct}/6.7$ for $\sqrt{f'_c}$ wherever it appears in 22.5, but the value of $f_{ct}/6.7$ shall not exceed $\sqrt{f'_c}$;

(b) When f_{ct} is not specified, all values of $\sqrt{f'_c}$ in 22.5 shall be multiplied by 0.75 for all-lightweight concrete, and 0.85 for sand-lightweight concrete. Linear interpolation shall be permitted when partial sand replacement is used.

22.6 — Walls

22.6.1 — Structural plain concrete walls shall be continuously supported by soil, footings, foundation walls, grade beams, or other structural members capable of providing continuous vertical support.

22.6.2 — Structural plain concrete walls shall be designed for vertical, lateral, and other loads to which they are subjected.

22.6.3 — Structural plain concrete walls shall be designed for an eccentricity corresponding to the maximum moment that can accompany the axial load but not less than $0.10h$. If the resultant of all factored loads is located within the middle-third of the overall wall thickness, the design shall be in accordance with 22.5.3 or 22.6.5. Otherwise, walls shall be designed in accordance with 22.5.3.

22.6.4 — Design for shear shall be in accordance with 22.5.4.

22.6.5 — Empirical design method

22.6.5.1 — Structural plain concrete walls of solid rectangular cross section shall be permitted to be designed by Eq. (22-13) if the resultant of all factored loads is located within the middle-third of the overall thickness of wall.

22.6.5.2 — Design of walls subject to axial loads in compression shall be based on

$$\phi P_{nw} \geq P_u \qquad (22\text{-}13)$$

where P_u is factored axial load and P_{nw} is nominal axial load strength computed by

$$P_{nw} = 0.45 f'_c A_g \left[1 - \left(\frac{\ell_c}{32h}\right)^2\right] \qquad (22\text{-}14)$$

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R22.6 — Walls

Plain concrete walls are commonly used for basement wall construction for residential and light commercial buildings in low or nonseismic areas. Although the code imposes no absolute maximum height limitation on the use of plain concrete walls, designers are cautioned against extrapolating the experience with relatively minor structures and using plain concrete walls in multistory construction and other major structures where differential settlement, wind, earthquake, or other unforeseen loading conditions require the walls to possess some ductility and ability to maintain their integrity when cracked. For such conditions, ACI Committee 318 strongly encourages the use of walls designed in accordance with Chapter 14.

The provisions for plain concrete walls are applicable only for walls laterally supported in such a manner as to prohibit relative lateral displacement at top and bottom of individual wall elements (see 22.6.6.4). The code does not cover walls without horizontal support to prohibit relative displacement at top and bottom of wall elements. Such laterally unsupported walls are to be designed as reinforced concrete members in accordance with the code.

R22.6.5 — Empirical design method

When the resultant load falls within the middle-third of the wall thickness (kern of wall section), plain concrete walls may be designed using the simplified Eq. (22-14). Eccentric loads and lateral forces are used to determine the total eccentricity of the factored load P_u. If the eccentricity does not exceed $h/6$, Eq. (22-14) may be applied, and design performed considering P_u as a concentric load. The factored axial load P_u should not exceed the design axial load strength ϕP_{nw}. Eq. (22-14) reflects the range of braced and restrained end conditions encountered in wall design. The limitations of 22.6.6 apply whether the wall is proportioned by 22.5.3 or by the empirical method of 22.6.5.

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22.6.6 — Limitations

22.6.6.1 — Unless demonstrated by a detailed analysis, horizontal length of wall to be considered effective for each vertical concentrated load shall not exceed center-to-center distance between loads, nor width of bearing plus four times the wall thickness.

22.6.6.2 — Except as provided in 22.6.6.3, thickness of bearing walls shall be not less than 1/24 the unsupported height or length, whichever is shorter, nor less than 5-1/2 in.

22.6.6.3 — Thickness of exterior basement walls and foundation walls shall be not less than 7-1/2 in.

22.6.6.4 — Walls shall be braced against lateral translation. See 22.3 and 22.4.7.

22.6.6.5 — Not less than two No. 5 bars shall be provided around all window and door openings. Such bars shall extend at least 24 in. beyond the corners of openings.

22.7 — Footings

22.7.1 — Structural plain concrete footings shall be designed for factored loads and induced reactions in accordance with appropriate design requirements of this code and as provided in 22.7.2 through 22.7.8.

22.7.2 — Base area of footing shall be determined from unfactored forces and moments transmitted by footing to soil and permissible soil pressure selected through principles of soil mechanics.

22.7.3 — Plain concrete shall not be used for footings on piles.

22.7.4 — Thickness of structural plain concrete footings shall be not less than 8 in. See 22.4.8.

22.7.5 — Maximum factored moment shall be computed at (a), (b), and (c):

(a) At the face of the column, pedestal, or wall, for footing supporting a concrete column, pedestal, or wall;

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R22.7 — Footings

R22.7.4 — Thickness of plain concrete footings will be controlled by flexural strength (extreme fiber stress in tension not greater than $5f\sqrt{f'_c}$) rather than shear strength for the usual proportions of plain concrete footings. Shear rarely will control (see R22.5.4). For footings cast against soil, overall thickness h used for strength computations should be taken as 2 in. less than actual thickness to allow for unevenness of excavation and contamination of the concrete adjacent to soil as required by 22.4.8. Thus, for a minimum footing thickness of 8 in., calculations for flexural and shear stresses must be based on an overall thickness h of 6 in.

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(b) Halfway between center and face of the wall, for footing supporting a masonry wall;

(c) Halfway between face of column and edge of steel base plate, for footing supporting a column with steel base plate.

22.7.6 — Shear in plain concrete footings

22.7.6.1 — Maximum factored shear shall be computed in accordance with 22.7.6.2, with location of critical section measured at face of column, pedestal, or wall for footing supporting a column, pedestal, or wall. For footing supporting a column with steel base plates, the critical section shall be measured at location defined in 22.7.5(c).

22.7.6.2 — Shear strength of structural plain concrete footings in the vicinity of concentrated loads or reactions shall be governed by the more severe of two conditions:

(a) Beam action for footing, with a critical section extending in a plane across the entire footing width and located at a distance h from face of concentrated load or reaction area. For this condition, the footing shall be designed in accordance with Eq. (22-9);

(b) Two-way action for footing, with a critical section perpendicular to plane of footing and located so that its perimeter b_o is a minimum, but need not approach closer than $h/2$ to perimeter of concentrated load or reaction area. For this condition, the footing shall be designed in accordance with Eq. (22-10).

22.7.7 — Circular or regular polygon shaped concrete columns or pedestals shall be permitted to be treated as square members with the same area for location of critical sections for moment and shear.

22.7.8 — Factored bearing load on concrete at contact surface between supporting and supported member shall not exceed design bearing strength for either surface as given in 22.5.5.

22.8 — Pedestals

22.8.1 — Plain concrete pedestals shall be designed for vertical, lateral, and other loads to which they are subjected.

22.8.2 — Ratio of unsupported height to average least lateral dimension of plain concrete pedestals shall not exceed 3.

22.8.3 — Maximum factored axial load applied to plain concrete pedestals shall not exceed design bearing strength given in 22.5.5.

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R22.8 — Pedestals

The height-thickness limitation for plain concrete pedestals does not apply for portions of pedestals embedded in soil capable of providing lateral restraint.

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22.9 — Precast members

22.9.1 — Design of precast plain concrete members shall consider all loading conditions from initial fabrication to completion of the structure, including form removal, storage, transportation, and erection.

22.9.2 — Limitations of 22.2 apply to precast members of plain concrete not only to the final condition but also during fabrication, transportation, and erection.

22.9.3 — Precast members shall be connected securely to transfer all lateral forces into a structural system capable of resisting such forces.

22.9.4 — Precast members shall be adequately braced and supported during erection to ensure proper alignment and structural integrity until permanent connections are completed.

22.10 — Plain concrete in earthquake-resisting structures

22.10.1 — Structures designed for earthquake-induced forces in regions of high seismic risk or assigned to high seismic performance or design categories shall not have foundation elements of structural plain concrete, except as follows:

(a) For detached one- and two-family dwellings three stories or less in height and constructed with stud bearing walls, plain concrete footings without longitudinal reinforcement supporting walls and isolated plain concrete footings supporting columns or pedestals are permitted;

(b) For all other structures, plain concrete footings supporting cast-in-place reinforced concrete or reinforced masonry walls are permitted provided the footings are reinforced longitudinally with not less than two continuous reinforcing bars. Bars shall not be smaller than No. 4 and shall have a total area of not less than 0.002 times the gross cross-sectional area of the footing. Continuity of reinforcement shall be provided at corners and intersections;

(c) For detached one- and two-family dwellings three stories or less in height and constructed with stud bearing walls, plain concrete foundations or basement walls are permitted provided the wall is not less than 7-1/2 in. thick and retains no more than 4 ft of unbalanced fill.

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R22.9 — Precast members

Precast structural plain concrete members are subject to all limitations and provisions for cast-in-place concrete contained in this chapter.

The approach to contraction or isolation joints is expected to be somewhat different than for cast-in-place concrete since the major portion of shrinkage stresses takes place prior to erection. To ensure stability, precast members should be connected to other members. The connection should transfer no tension.

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Notes

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Notes

APPENDIX A — STRUT-AND-TIE MODELS

CODE

A.0 — Notation

a = shear span, equal to the distance between a load and a support in a structure, in.
A_c = the effective cross-sectional area at one end of a strut in a strut-and-tie model, taken perpendicular to the axis of the strut, in.^{2}
A_n = area of a face of a nodal zone or a section through a nodal zone, in.^{2}
A_{ps} = area of prestressed reinforcement in a tie, in.^{2}
A_{si} = area of surface reinforcement in the ith layer crossing a strut, in.^{2}
A_{st} = area of nonprestressed reinforcement in a tie, in.^{2}
A_s' = area of compression reinforcement in a strut, in.^{2}

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in.
f_c' = specified compressive strength of concrete, psi
f_{cu} = effective compressive strength of the concrete in a strut or a nodal zone, psi
f_s' = stress in compression reinforcement, psi
f_{se} = effective stress after losses in prestressed reinforcement, psi

f_y = specified yield strength of nonprestressed reinforcement, psi
F_n = nominal strength of a strut, tie, or nodal zone, lb
F_{nn} = nominal strength of a face of a nodal zone, lb
F_{ns} = nominal strength of a strut, lb
F_{nt} = nominal strength of a tie, lb
F_u = factored force acting in a strut, tie, bearing area, or nodal zone in a strut-and-tie model, lb

COMMENTARY

RA.0 — Notation

[Notation used only in the Commentary is listed here.]

Units of measurement are given in the notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

C = compression force acting on a nodal zone, lb

f_{si} = the stress in the ith layer of surface reinforcement, psi

ℓ_a = length in which anchorage of a tie should occur, in.
ℓ_b = width of bearing, in.

CODE

ℓ_n = clear span, in.

s_i = spacing of reinforcement in the ith layer adjacent to the surface of the member, in.

w_s = effective width of strut, in.

w_t = effective width of tie, in.

b_s = factor to account for the effect of cracking and confining reinforcement on the effective compressive strength of the concrete in a strut

b_n = factor to account for the effect of the anchorage of ties on the effective compressive strength of a nodal zone.

g_i = angle between the axis of a strut and the bars in the ith layer of reinforcement crossing that strut

Df_p = increase in stress in prestressing tendons due to factored loads, psi

e_s = the strain in the longitudinal reinforcement in a compression zone or a longitudinally reinforced strut

q = angle between the axis of a strut or compression field and the tension chord of the member

l = correction factor related to the unit weight of concrete. See 11.7.4.3

f = strength reduction factor

A.1 — Definitions

B-region — A portion of a member in which the plane sections assumption of flexure theory from 10.2.2 can be applied.

Discontinuity — An abrupt change in geometry or loading.

COMMENTARY

R = reaction, lb

T = tension force acting on a nodal zone, lb

w_s = effective width of a strut perpendicular to the axis of the strut, in.

w_t = effective width of concrete concentric with a tie, used to dimension nodal zone, in.

$w_{t,max}$ = maximum effective width of concrete concentric with a tie, in.

b_1 = factor defined in 10.2.7.3

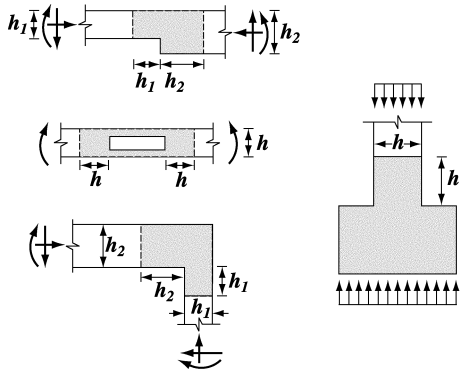
RA.1 — Definitions

B-region — In general, any portion of a member outside of a D-region is a B-region.

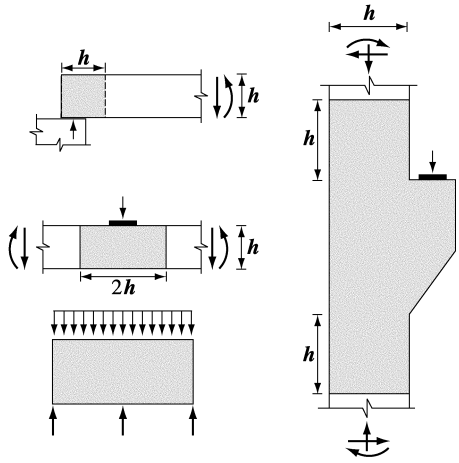
Discontinuity — A discontinuity in the stress distribution occurs at a change in the geometry of a structural element or at a concentrated load or reaction. St. Venant's principle indicates that the stresses due to axial load and bending approach a linear distribution at a distance approximately equal to the overall height of the member, h, away from the discontinuity. For this reason, discontinuities are assumed to extend a distance h from the section where the load or change in geometry occurs. Figure RA.1.1(a) shows typical geometric discontinuities, and Fig. RA.1.1(b) shows combined geometrical and loading discontinuities.

CODE

COMMENTARY



(a) Geometric discontinuities



(b) Loading and geometric discontinuities

Fig. RA.1.1—D-regions and discontinuities

D-region — The portion of a member within a distance equal to the member height ***h*** or depth ***d*** from a force discontinuity or a geometric discontinuity.

D-region — The shaded regions in Fig. RA.1.1(a) and (b) show typical D–regions.[A-1] The plane sections assumption of 10.2.2 is not applicable in such regions.

Each shear span of the beam in Fig. RA.1.2(a) is a D-region. If two D-regions overlap or meet as shown in Fig. RA.1.2 (b), they can be considered as a single D-region for design purposes. The maximum length-to-depth ratio of such a D-region would be approximately two. Thus, the smallest angle between the strut and the tie in a D-region is arctan 2 = 26.5 degrees, rounded to 25 degrees.

If there is a B-region between the D-regions in a shear span, as shown in Fig. RA.1.2(c), the strength of the shear span is governed by the strength of the B-region if the B- and D-regions have similar geometry and reinforcement.[A–2] This is because the shear strength of a B-region is less than the shear strength of a comparable D-region. Shear spans containing B-regions — the usual case in beam design — are designed for shear using the traditional shear design procedures from 11.1 through 11.5 ignoring D-regions.

CODE

COMMENTARY

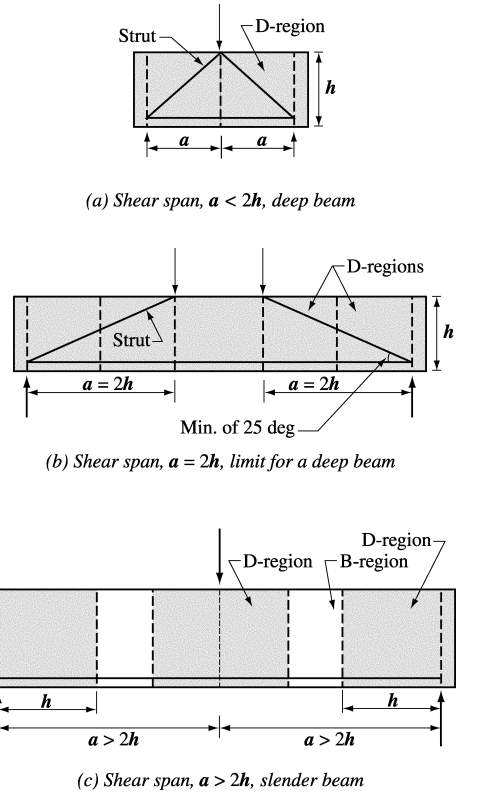


Fig. RA.1.2—Description of deep and slender beams.

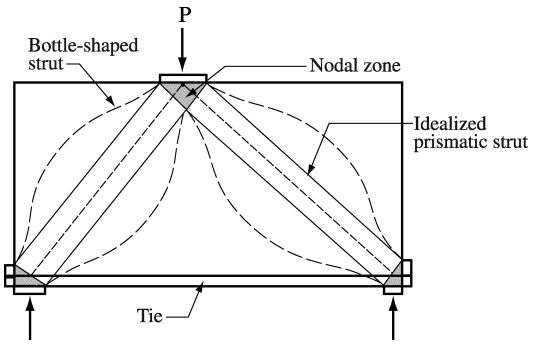


Fig. RA.1.3—Description of strut-and-tie model.

Deep beam — See 10.7.1 and 11.8.1.

Node — The point in a joint in a strut-and-tie model where the axes of the struts, ties, and concentrated forces acting on the joint intersect.

Nodal zone — The volume of concrete around a node that is assumed to transfer strut-and-tie forces through the node.

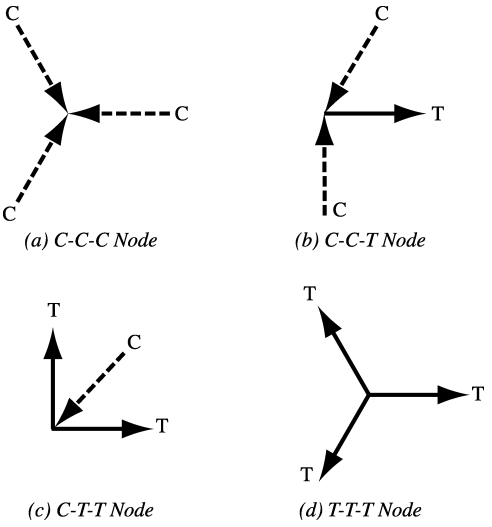
Deep beam — See Fig. RA.1.2(a), RA.1.2(b), and RA.1.3, and Sections 10.7 and 11.8.

Node — For equilibrium, at least three forces should act on a node in a strut-and-tie model, as shown in Fig. RA.1.4. Nodes are classified according to the signs of these forces. A ***C-C-C*** node resists three compressive forces, a ***C-C-T*** node resists two compressive forces and one tensile force, and so on.

Nodal zone — Historically, hydrostatic nodal zones as shown in Fig. RA.1.5 were used. These were largely superseded by what are called extended nodal zones, shown in Fig. RA.1.6.

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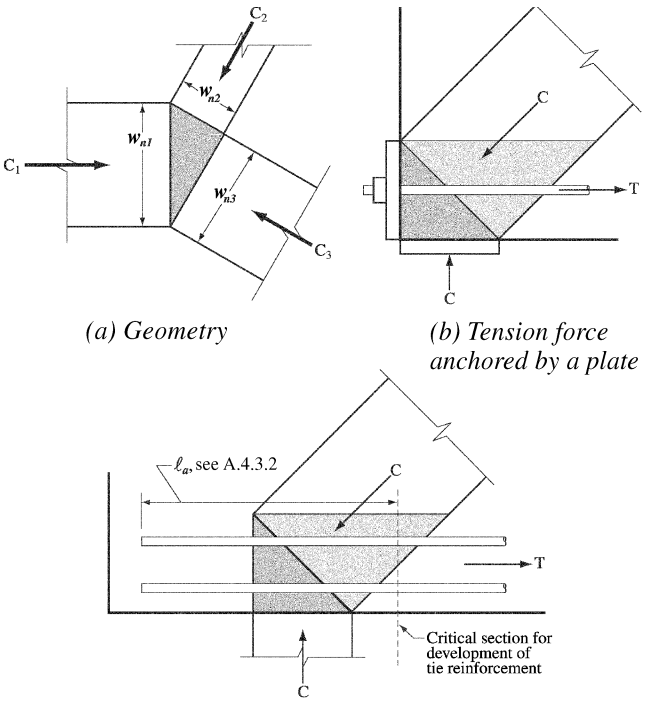
COMMENTARY



(a) C-C-C Node (b) C-C-T Node

(c) C-T-T Node (d) T-T-T Node

Fig. RA.1.4—Classification of nodes.



(a) Geometry (b) Tension force anchored by a plate

(c) Tension force anchored by bond

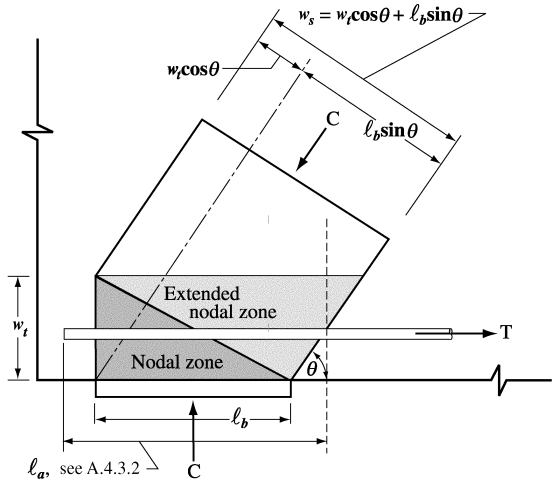
Fig. RA.1.5—Hydrostatic nodes.

A ***hydrostatic nodal zone*** has loaded faces perpendicular to the axes of the struts and ties acting on the node and has equal stresses on the loaded faces. Figure RA.1.5(a) shows a ***C-C-C*** nodal zone. If the stresses on the face of the nodal zone are the same in all three struts, the ratios of the lengths of the sides of the nodal zone, w_{n1}: w_{n2}: w_{n3} are in the same proportions as the three forces C_1: C_2: C_3. The faces of a hydrostatic nodal zone are perpendicular to the axes of the struts and ties acting on the nodal zone.

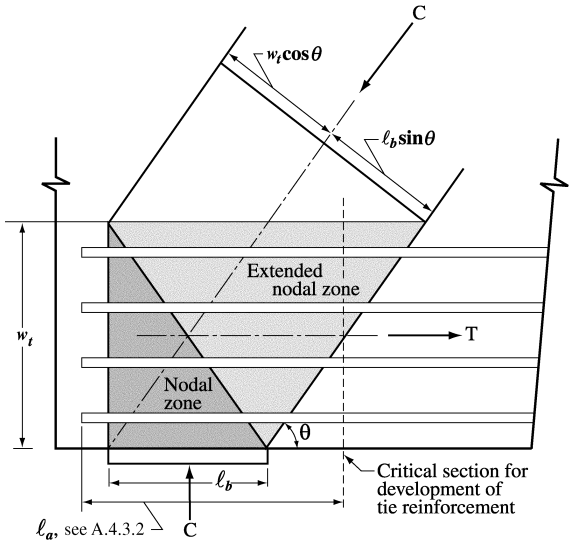
These nodal zones are called hydrostatic nodal zones because the in-plane stresses are the same in all directions.

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COMMENTARY



(a) One layer of steel



(b) Distributed steel

Fig. RA.1.6—Extended nodal zone showing the effect of the distribution of the force.

Strictly speaking, this terminology is incorrect because the in-plane stresses are not equal to the out-of-plane stresses.

A ***C-C-T*** nodal zone can be represented as a hydrostatic nodal zone if the tie is assumed to extend through the node to be anchored by a plate on the far side of the node, as shown in Fig. RA.1.5(b), provided that the size of the plate results in bearing stresses that are equal to the stresses in the struts. The bearing plate on the left side of Fig. RA.1.5(b) is used to represent an actual tie anchorage. The tie force can be anchored by a plate, or through development of straight or hooked bars, as shown in Fig. RA.1.5(c).

The shaded areas in Fig. RA.1.6(a) and (b) are extended nodal zones. An ***extended nodal zone*** is that portion of a member bounded by the intersection of the effective strut width, w_s, and the effective tie width, w_t (see RA.4.2).

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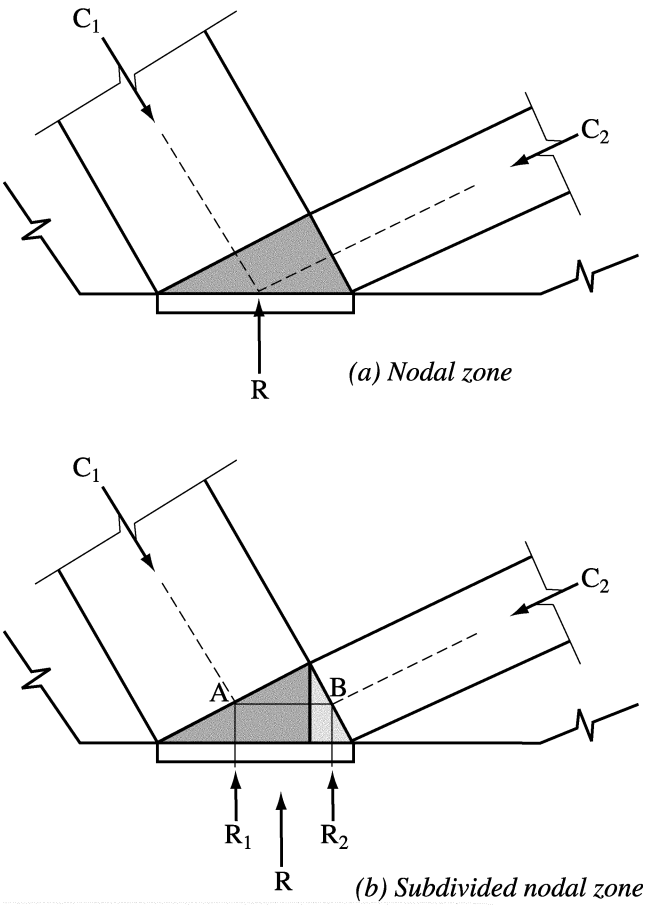


Fig. RA.1.7—Subdivision of nodal zone.

In the nodal zone shown in Fig. RA.1.7(a), the reaction $\boldsymbol{R}$ equilibrates the vertical components of the forces $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$. Frequently, calculations are easier if the reaction $\boldsymbol{R}$ is divided into $\boldsymbol{R_1}$, which equilibrates the vertical component of $\boldsymbol{C_1}$ and $\boldsymbol{R_2}$, which equilibrates the vertical component of the force $\boldsymbol{C_2}$, as shown in Fig. RA1.7(b).

Strut — A compression member in a strut-and-tie model. A strut represents the resultant of a parallel or a fan-shaped compression field.

Strut — In design, struts are usually idealized as prismatic compression members, as shown by the straight line outlines of the struts in Fig. RA.1.3. If the effective compression strength $\boldsymbol{f_{cu}}$ differs at the two ends of a strut, due either to different nodal zone strengths at the two ends, or to different bearing lengths, the strut is idealized as a uniformly tapered compression member.

Bottle-shaped strut — A strut that is wider at midlength than at its ends.

Bottle-shaped struts — A bottle-shaped strut is a strut located in a part of a member where the width of the compressed concrete at midlength of the strut can spread laterally.[A-1,A-3]

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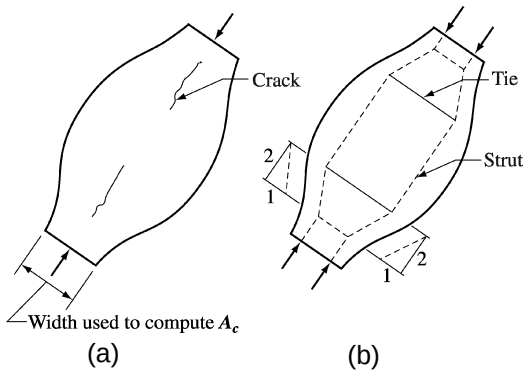


Fig. RA.1.8—Bottle-shaped strut: (a) cracking of a bottle-shaped strut; and (b) strut-and-tie model of a bottle-shaped strut.

The curved dashed outlines of the struts in Fig. RA.1.3 and the curved solid outlines in Fig. RA.1.8 approximate the boundaries of bottle-shaped struts. A split cylinder test is an example of a bottle-shaped strut. The internal lateral spread of the applied compression force in such a test leads to a transverse tension that splits the specimen.

To simplify design, bottle-shaped struts are idealized either as prismatic or as uniformly tapered, and crack-control reinforcement from A.3.3 is provided to resist the transverse tension. The amount of confining transverse reinforcement can be computed using the strut-and-tie model shown in Fig. RA.1.8(b) with the struts that represent the spread of the compression force acting at a slope of 1:2 to the axis of the applied compressive force. Alternatively for f_c' not exceeding 6000 psi, Eq. (A-4) can be used. The cross-sectional area A_c of a bottle-shaped strut, is taken as the smaller of the cross-sectional areas at the two ends of the strut. See Fig. RA.1.8(a).

Strut-and-tie model — A truss model of a structural member, or of a D-region in such a member, made up of struts and ties connected at nodes, capable of transferring the factored loads to the supports or to adjacent B-regions.

Strut-and-tie model — The components of a strut-and-tie model of a single-span deep beam loaded with a concentrated load are identified in Fig. RA.1.3. The cross-sectional dimensions of a strut or tie are designated as thickness and width, both perpendicular to the axis of the strut or tie. Thickness is perpendicular to the plane of the truss model, and width is in the plane of the truss model.

Tie — A tension member in a strut-and-tie model.

Tie — A tie consists of reinforcement or prestressing steel plus a portion of the surrounding concrete that is concentric with the axis of the tie. The surrounding concrete is included to define the zone in which the forces in the struts and ties are to be anchored. The concrete in a tie is not used to resist the axial force in the tie. Although not considered in design, the surrounding concrete will reduce the elongations of the tie, especially at service loads.

A.2 — Strut-and-tie model design procedure

RA.2 — Strut-and-tie model design procedure

A.2.1 — It shall be permitted to design structural concrete members, or D-regions in such members, by mod-

RA.2.1 — The truss model described in A.2.1 is referred to as a strut-and-tie model. Details of the use of strut-and-tie

CODE

eling the member or region as an idealized truss. The truss model shall contain struts, ties, and nodes as defined in A.1. The truss model shall be capable of transferring all factored loads to the supports or adjacent B-regions.

A.2.2 — The strut-and-tie model shall be in equilibrium with the applied loads and the reactions.

COMMENTARY

models are given in References A.1 through A.7. The design of a D-region includes the following four steps:

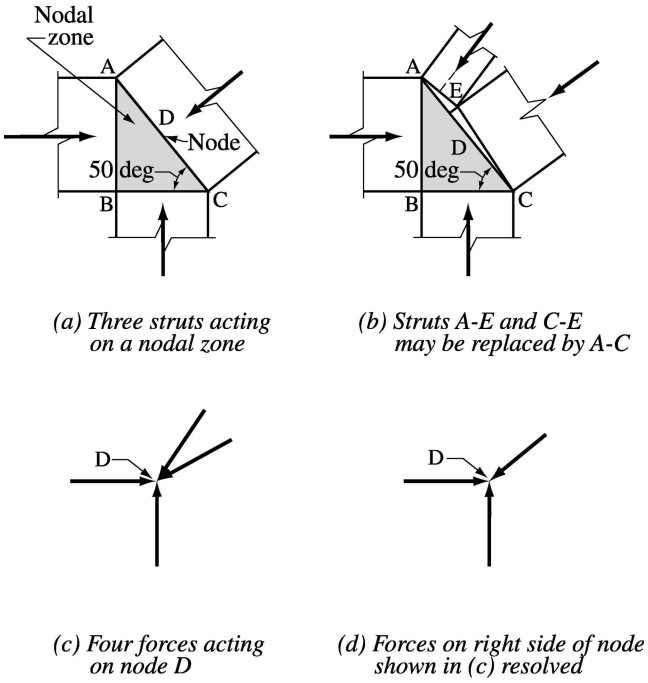
1. Define and isolate each D-region;

2. Compute resultant forces on each D-region boundary;

3. Select a truss model to transfer the resultant forces across the D-region. The axes of the struts and ties, respectively, are chosen to approximately coincide with the axes of the compression and tension fields. The forces in the struts and ties are computed.

4. The effective widths of the struts and nodal zones are determined considering the forces from Step 3 and the effective concrete strengths defined in A.3.2 and A.5.2, and reinforcement is provided for the ties considering the steel strengths defined in A.4.1. The reinforcement should be anchored in the nodal zones.

Strut-and-tie models represent strength limit states and designers should also comply with the requirements for serviceability in the code. Deflections of deep beams or similar members can be estimated using an elastic analysis to analyze the strut-and-tie model. In addition, the crack widths in a tie can be checked using 10.6.4, assuming the tie is encased in a prism of concrete corresponding to the area of tie from RA.4.2.



(a) Three struts acting on a nodal zone

(b) Struts A-E and C-E may be replaced by A-C

(c) Four forces acting on node D

(d) Forces on right side of node shown in (c) resolved

Fig. RA.2.3—Resolution of forces on a nodal zone.

CODE

A.2.3 — In determining the geometry of the truss, the dimensions of the struts, ties, and nodal zones shall be taken into account.

A.2.4 — Ties shall be permitted to cross struts. Struts shall cross or overlap only at nodes.

A.2.5 — The angle between the axes of any strut and any tie entering a single node shall not be taken as less than 25 degrees.

A.2.6 — Design of struts, ties, and nodal zones shall be based on

$$f F_n \geq F_u \quad \text{(A-1)}$$

where F_u is the force in a strut or tie, or the force acting on one face of a nodal zone, due to the factored loads; F_n is the nominal strength of the strut, tie, or nodal zone; and f is the strength reduction factor specified in 9.3.2.6.

A.3 — Strength of struts

A.3.1 — The nominal compressive strength of a strut without longitudinal reinforcement shall be taken as the smaller value of

$$F_{ns} = f_{cu} A_c \quad \text{(A-2)}$$

COMMENTARY

RA.2.3 — The struts, ties, and nodal zones making up the strut-and-tie model all have finite widths that should be taken into account in selecting the dimensions of the truss. Figure RA.2.3(a) shows a node and the corresponding nodal zone. The vertical and horizontal forces equilibrate the force in the inclined strut. If the stresses are equal in all three struts, a hydrostatic nodal zone can be used and the widths of the struts will be in proportion to the forces in the struts.

If more than three forces act on a nodal zone in a two-dimensional structure, as shown in Fig. RA.2.3(b), it is generally necessary to resolve some of the forces to end up with three intersecting forces. The strut forces acting on Faces A–E and C–E in Fig. RA.2.3(b) can be replaced with one force acting on face A–C. This force passes through the node at D.

Alternatively, the strut-and-tie model could be analyzed assuming all the strut forces acted through the node at D, as shown in Fig. RA.2.3(c). In this case, the forces in the two struts on the right side of Node D can be resolved into a single force acting through Point D, as shown in Fig. RA.2.3(d).

If the width of the support in the direction perpendicular to the member is less than the width of the member, transverse reinforcement may be required to restrain vertical splitting in the plane of the node. This can be modeled using a transverse strut-and-tie model.

RA.2.5 — The angle between the axes of struts and ties acting on a node should be large enough to mitigate cracking and to avoid incompatibilities due to shortening of the struts and lengthening of the ties occurring in almost the same directions. This limitation on the angle prevents modeling the shear spans in slender beams using struts inclined at less than 25 degrees from the longitudinal steel. See Reference A.6.

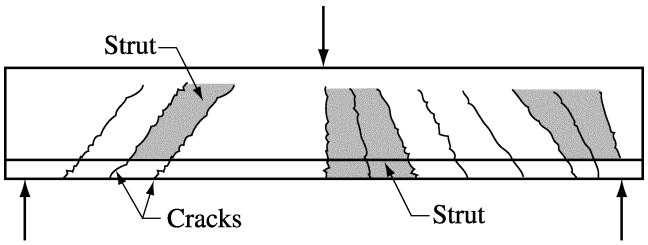
RA.2.6 — Factored loads are applied to the strut-and-tie model, and the forces in all the struts, ties, and nodal zones are computed. If several loading cases exist, each should be investigated. The strut-and-tie model, or models, are analyzed for the loading cases and, for a given strut, tie, or nodal zone, F_u is the largest force in that element for all loading cases.

RA.3 — Strength of struts

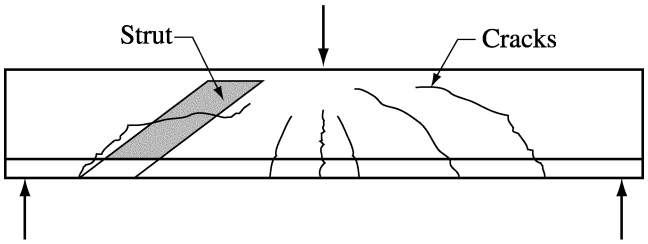
RA.3.1 — The width of strut w_s used to compute A_c is the smaller dimension perpendicular to the axis of the strut at the ends of the strut. This strut width is illustrated in Fig. RA.1.5(a) and Fig. RA.1.6(a) and (b). In two-dimensional

CODE

COMMENTARY



(a) Struts in a beam web with inclined cracks parallel to struts - Section A.3.2.4



(b) Struts crossed by skew cracks - Section A.3.2.4

Fig. RA.3.2—Type of struts.

at the two ends of the strut, where A_c is the cross-sectional area at one end of the strut, and f_{cu} is the smaller of (a) and (b):

(a) the effective compressive strength of the concrete in the strut given in A.3.2;

(b) the effective compressive strength of the concrete in the nodal zone given in A.5.2.

A.3.2 — The effective compressive strength of the concrete in a strut shall be taken as

$$f_{cu} = 0.85 b_s f_c' \qquad \text{(A-3)}$$

A.3.2.1 — For a strut of uniform cross-sectional area over its length ..... $b_s = 1.0$

A.3.2.2 — For struts located such that the width of the midsection of the strut is larger than the width at the nodes (bottle-shaped struts):

(a) with reinforcement satisfying A.3.3 ... $b_s = 0.75$

(b) without reinforcement satisfying A.3.3 ..... $b_s = 0.60\lambda$

where λ is given in 11.7.4.3.

structures, such as deep beams, the thickness of the struts may be taken as the width of the member.

RA.3.2 — The strength coefficient, $0.85f_c'$, in Eq. (A-3), represents the effective concrete strength under sustained compression, similar to that used in Eq. (10-1) and (10-2).

RA.3.2.1 — The value of b_s in A.3.2.1 applies to a strut equivalent to the rectangular stress block in a compression zone in a beam or column.

RA.3.2.2 — The value of b_s in A.3.2.2 applies to bottle-shaped struts as shown in Fig. RA.1.3. The internal lateral spread of the compression forces can lead to splitting parallel to the axis of the strut near the ends of the strut, as shown in Fig. RA.1.8. Reinforcement placed to resist the splitting force restrains crack width, allows the strut to resist more axial load, and permits some redistribution of force.

The value of b_s in A.3.2.2(b) includes the correction factor, λ, for lightweight concrete because the strength of a strut without transverse reinforcement is assumed to be limited to less than the load at which longitudinal cracking develops.

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A.3.2.3 — For struts in tension members, or the tension flanges of members ..... b_s = **0.40**

A.3.2.4 — For all other cases ..... b_s = **0.60**

A.3.3 — If the value of b_s specified in A.3.2.2(a) is used, the axis of the strut shall be crossed by reinforcement proportioned to resist the transverse tensile force resulting from the compression force spreading in the strut. It shall be permitted to assume the compressive force in the strut spreads at a slope of 2 longitudinal to 1 transverse to the axis of the strut.

A.3.3.1 — For f_c' not greater than 6000 psi, the requirement of A.3.3 shall be permitted to be satisfied by the axis of the strut being crossed by layers of reinforcement that satisfy

$$\sum \frac{A_{si}}{b s_i} \sin\gamma_i \geq 0.003 \qquad \text{(A-4)}$$

where A_{si} is the total area of reinforcement at spacing s_i in a layer of reinforcement with bars at an angle γ_i to the axis of the strut.

COMMENTARY

RA.3.2.3 — The value of b_s in A.3.2.3 applies, for example, to compression struts in a strut-and-tie model used to design the longitudinal and transverse reinforcement of the tension flanges of beams, box girders, and walls. The low value of b_s reflects that these struts need to transfer compression across cracks in a tension zone.

RA.3.2.4 — The value of b_s in A.3.2.4 applies to strut applications not included in A.3.2.1, A.3.2.2, and A.3.2.3. Examples are struts in a beam web compression field in the web of a beam where parallel diagonal cracks are likely to divide the web into inclined struts, and struts are likely to be crossed by cracks at an angle to the struts (see Fig. RA.3.2(a) and (b)). Section A.3.2.4 gives a reasonable lower limit on b_s except for struts described in A.3.2.2(b) and A.3.2.3.

RA.3.3 — The reinforcement required by A.3.3 is related to the tension force in the concrete due to the spreading of the strut, as shown in the strut-and-tie model in Fig. RA.1.8(b). Section RA.3.3 allows designers to use local strut-and-tie models to compute the amount of transverse reinforcement needed in a given strut. The compressive forces in the strut may be assumed to spread at a 2:1 slope, as shown in Fig. RA.1.8(b). For concrete strengths not exceeding 6000 psi, the amount of steel required by Eq. (A-4) is deemed to satisfy A.3.3.

Figure RA.3.3 shows two layers of reinforcement crossing a cracked strut. If the crack opens without shear slip along the crack, the vertical bars in the figure will cause a stress of

$$\frac{A_{s1} f_{s1}}{b s_1} \sin\gamma_1$$

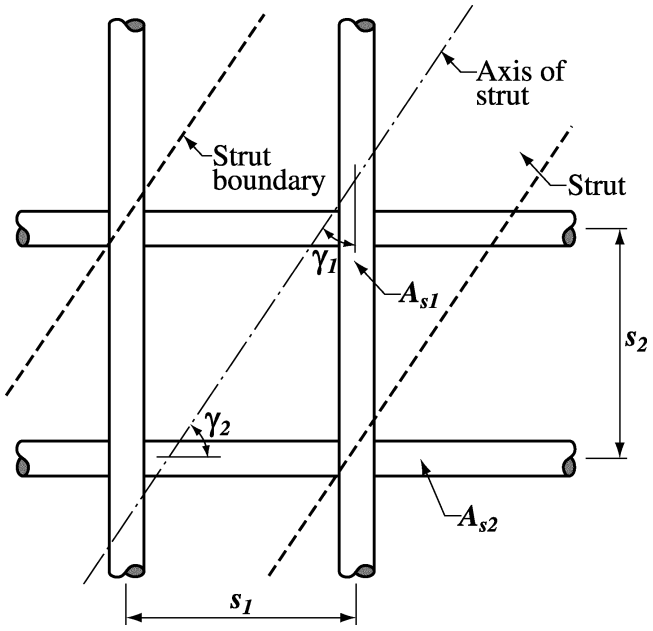


Fig. RA.3.3—Reinforcement crossing a strut.

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A.3.3.2 — The reinforcement required in A.3.3 shall be placed in either: two orthogonal directions at angles γ_1 and γ_2 to the axis of the strut, or in one direction at an angle γ to the axis of the strut. If the reinforcement is in only one direction, γ shall not be less than 40 deg.

A.3.4 — If documented by tests and analyses, it shall be permitted to use an increased effective compressive strength of a strut due to confining reinforcement.

A.3.5 — The use of compression reinforcement shall be permitted to increase the strength of a strut. Compression reinforcement shall be properly anchored, parallel to the axis of the strut, located within the strut, and enclosed in ties or spirals satisfying 7.10. In such cases, the strength of a longitudinally reinforced strut is:

$$F_{ns} = f_{cu}A_c + A'_s f'_s \quad \text{(A-5)}$$

A.4 — Strength of ties

A.4.1 —The nominal strength of a tie shall be taken as

$$F_{nt} = A_{st}f_y + A_{ps}\,(f_{se} + Df_p) \quad \text{(A-6)}$$

where $(f_{se} + Df_p)$ shall not exceed f_{py}, and A_{ps} is zero for nonprestressed members.

In Eq. (A–6), it shall be permitted to take Df_p equal to 60,000 psi for bonded prestressed reinforcement, or 10,000 psi for unbonded prestressed reinforcement. Other values of Df_p shall be permitted when justified by analysis.

A.4.2 — The axis of the reinforcement in a tie shall coincide with the axis of the tie in the strut-and-tie model.

COMMENTARY

perpendicular to the strut, where the subscript **1** refers to the vertical bars in Fig. RA.3.3. Equation (A-4) is written in terms of a reinforcement ratio rather than a stress to simplify the calculation. The summation adds the ratio resulting from the perpendicular bars, denoted by the subscript **2** in Fig. RA.3.3.

Often, the confinement reinforcement given in A.3.3 is difficult to place in three-dimensional structures such as pile caps. If this reinforcement is not provided, the value of f_{cu} given in A.3.2.2(b) is used.

RA.3.3.2 — In a corbel with a shear span-to-depth ratio less than 1.0, the confinement reinforcement required to satisfy A.3.3 is usually provided in the form of horizontal stirrups crossing the inclined compression strut, as shown in Fig. R.11.9.2.

RA.3.4 — The design of tendon anchorage zones for prestressed concrete sometimes uses confinement to enhance the compressive strength of the struts in the local zone. Confinement of struts is discussed in References A.4 and A.8.

RA.3.5 — The strength added by the reinforcement is given by the last term in Eq. (A-5). The stress f'_s in the reinforcement in a strut at nominal strength can be obtained from the strains in the strut when the strut crushes. For Grade 40 or 60 reinforcement, f'_s can be taken as f_y.

RA.4 — Strength of ties

RA.4.2 — The effective tie width assumed in design w_t can vary between the following limits, depending on the distribution of the tie reinforcement:

1. If the bars in the tie are in one layer, the effective tie width can be taken as the diameter of the bars in the tie plus twice the cover to the surface of the bars, as shown in Fig. RA.1.6(a); and

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A.4.3 — Tie reinforcement shall be anchored by mechanical devices, post-tensioning anchorage devices, standard hooks, or straight bar development as required by A.4.3.1 through A.4.3.4.

A.4.3.1 — Nodal zones shall develop the difference between the tie force on one side of the node and the tie force on the other side.

A.4.3.2 — At nodal zones anchoring one tie, the tie force shall be developed at the point where the centroid of the reinforcement in a tie leaves the extended nodal zone and enters the span.

A.4.3.3 — At nodal zones anchoring two or more ties, the tie force in each direction shall be developed at the point where the centroid of the reinforcement in the tie leaves the extended nodal zone.

A.4.3.4 — The transverse reinforcement required by A.3.3 shall be anchored in accordance with 12.13.

A.5 — Strength of nodal zones

A.5.1 — The nominal compression strength of a nodal zone shall be

$$F_{nn} = f_{cu}A_n \quad \text{(A-7)}$$

where f_{cu} is the effective compressive strength of the concrete in the nodal zone as given in A.5.2 and A_n is (a) or (b):

(a) the area of the face of the nodal zone that F_u acts on, taken perpendicular to the line of action of F_u, or

COMMENTARY

2. A practical upper limit of the tie width can be taken as the width corresponding to the width in a hydrostatic nodal zone, calculated as

$$w_{t,max} = F_{nt}/f_{cu}$$

where f_{cu} is the applicable effective compression strength of the nodal zone given in A.5.2. If the tie width exceeds the value from Item 1, the tie reinforcement should be distributed approximately uniform over the width and thickness of the tie, as shown in Fig. RA.1.6(b).

RA.4.3 — Anchorage of ties often requires special attention in nodal zones of corbels or in nodal zones adjacent to exterior supports of deep beams. The reinforcement in a tie should be anchored before it leaves the extended nodal zone at the point defined by the intersection of the centroid of the bars in the tie and the extensions of the outlines of either the strut or the bearing area. In Fig. RA.1.6(a) and (b), this occurs where the outline of the extended nodal zone is crossed by the centroid of the reinforcement in the tie. Some of the anchorage may be achieved by extending the reinforcement through the nodal zone as shown in Fig. RA.1.5(c), and developing it beyond the nodal zone. If the tie is anchored using 90 deg hooks, the hooks should be confined within the reinforcement extending into the beam from the supporting member to avoid cracking along the outside of the hooks in the support region.

In deep beams, hairpin bars spliced with the tie reinforcement can be used to anchor the tension tie forces at exterior supports, provided the beam width is large enough to accommodate such bars.

Figure RA.4.3 shows two ties anchored at a nodal zone. Development is required where the centroid of the tie crosses the outline of the extended nodal zone.

The development length of the tie reinforcement can be reduced through hooks, mechanical devices, additional confinement, or by splicing it with several layers of smaller bars.

RA.5 — Strength of nodal zones

RA.5.1 — If the stresses in all the struts meeting at a node are equal, a hydrostatic nodal zone can be used. The faces of such a nodal zone are perpendicular to the axes of the struts, and the widths of the faces of the nodal zone are proportional to the forces in the struts.

Assuming the principal stresses in the struts and ties act parallel to the axes of the struts and ties, the stresses on faces perpendicular to these axes are principal stresses, and A5.1(a) is used. If, as shown in Fig. RA.1.6(b), the face of a nodal zone is not perpendicular to the axis of the strut, there will be both shear stresses and normal stresses on the face of

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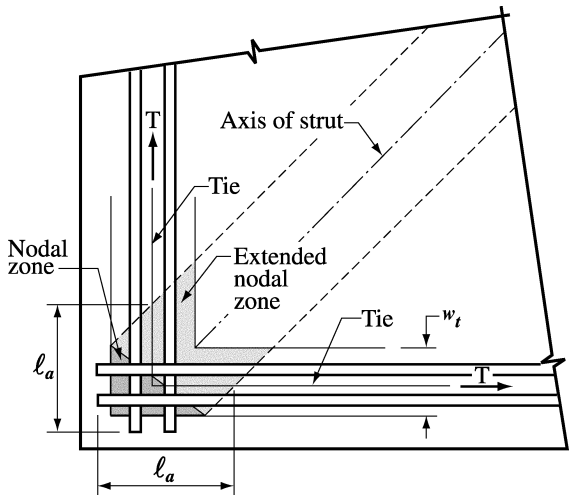


Fig. RA.4.3—Extended nodal zone anchoring two ties.

(b) the area of a section through the nodal zone, taken perpendicular to the line of action of the resultant force on the section.

the nodal zone. Typically, these stresses are replaced by the normal (principal compression) stress acting on the cross-sectional area A_c of the strut, taken perpendicular to the axis of the strut as given in A.5.1(a).

In some cases, A.5.1(b) requires that the stresses be checked on a section through a subdivided nodal zone. The stresses are checked on the least area section which is perpendicular to a resultant force in the nodal zone. In Fig. RA.1.7(b), the vertical face which divide the nodal zone into two parts is stressed by the resultant force acting along A-B. The design of the nodal zone is governed by the critical section from A.5.1(a) or A.5.1(b), whichever gives the highest stress.

A.5.2 — Unless confining reinforcement is provided within the nodal zone and its effect is supported by tests and analysis, the calculated effective compressive stress on a face of a nodal zone due to the strut-and-tie forces shall not exceed the value given by:

$$f_{cu} = 0.85\beta_n f_c' \quad \text{(A-8)}$$

where the value of β_n is given in A.5.2.1 through A.5.2.3.

A.5.2.1 — In nodal zones bounded by struts or bearing areas, or both..... $\beta_n = 1.0$;

A.5.2.2 — In nodal zones anchoring one tie ..... $\beta_n = 0.80$;

or

A.5.2.3 — In nodal zones anchoring two or more ties ..... $\beta_n = 0.60$.

RA.5.2 — The nodes in two-dimensional members, such as deep beams, can be classified as ***C-C-C*** if all the members intersecting at the node are in compression; as ***C-C-T*** nodes if one of the members acting on the node is in tension; and so on, as shown in Fig. RA.1.4. The effective compressive strength of the nodal zone is given by Eq. (A-8), as modified by A.5.2.1 through A.5.2.3 apply to ***C-C-C*** nodes, ***C-C-T*** nodes, and ***C-T-T*** or ***T-T-T*** nodes, respectively.

The β_n values reflect the increasing degree of disruption of the nodal zones due to the incompatibility of tension strains in the ties and compression strains in the struts. The stress on any face of the nodal zone or on any section through the nodal zone should not exceed the value given by Eq. (A-8), as modified by A.5.2.1 through A.5.2.3.

CODE

A.5.3 — In a three-dimensional strut-and-tie model, the area of each face of a nodal zone shall not be less than that given in A.5.1, and the shape of each face of the nodal zones shall be similar to the shape of the projection of the end of the struts onto the corresponding faces of the nodal zones.

COMMENTARY

RA.5.3 — This description of the shape and orientation of the faces of the nodal zones is introduced to simplify the calculations of the geometry of a three-dimensional strut-and-tie model.

APPENDIX B — ALTERNATIVE PROVISIONS FOR REINFORCED AND PRESTRESSED CONCRETE FLEXURAL AND COMPRESSION MEMBERS

CODE

COMMENTARY

B.0 — Notation

A_g = gross area of section, in.^{2}
A_s = area of tension reinforcement, in.^{2}
A_s' = area of compression reinforcement, in.^{2}
f_c' = specified compressive strength of concrete, psi
f_y = specified yield strength of nonprestressed reinforcement, ksi
d = distance from extreme compression fiber to centroid of tension reinforcement, in.
d' = distance from extreme compression fiber to centroid of compression reinforcement, in.
d_s = distance from extreme tension fiber to centroid of tension reinforcement, in.
h = overall thickness of member, in.
P_b = nominal axial load strength at balanced strain conditions, lb. See 10.3.2
P_n = nominal axial load strength at given eccentricity, lb
β_1 = factor defined in 10.2.7.3
ρ = reinforcement ratio for nonprestressed tension reinforcement
= A_s/bd
ρ' = reinforcement ratio for nonprestressed compression reinforcement
= A_s'/bd
ρ_b = reinforcement ratio producing balanced strain conditions. See 10.3.2.
ϕ = strength reduction factor
ω = $\rho f_y/f_c'$
ω' = $\rho' f_y/f_c'$
ω_p = $\rho_p f_{ps}/f_c'$
$\omega_w, \omega_{pw}, \omega_w'$ = reinforcement indexes for flanged sections computed as for ω, ω_p, and ω', except that b shall be the web width, and reinforcement area shall be that required to develop compressive strength of web only

B.1 — Scope

Design for flexure and axial load by provisions of Appendix B shall be permitted. When Appendix B is used in design, B.8.4, B.8.4.1, B.8.4.2, and B.8.4.3 shall replace the corresponding numbered sections in Chapter 8; B.10.3.3 shall replace 10.3.3, 10.3.4, and 10.3.5, except 10.3.5.1 shall remain; B.18.8.1, B.18.8.2, and B.18.8.3 shall replace the corresponding numbered

RB.1 — Scope

Reinforcement limits, strength reduction factors, ϕ, and moment redistribution in Appendix B differ from those in the main body of the code. Appendix B contains the reinforcement limits, strength reduction factors, ϕ, and moment redistribution used in the code for many years. Designs using the provisions of Appendix B satisfy the code, and are equally acceptable.

CODE

sections in Chapter 18; B.18.10.4, B.18.10.4.1, B.18.10.14.2, and B.18.10.4.3 shall replace 18.10.4, 18.10.4.1, and 18.10.4.2. If any section in this appendix is used, all sections in this appendix shall be substituted in the body of the code, and all other sections in the body of the code shall be applicable.

B.8.4 — Redistribution of negative moments in continuous nonprestressed flexural members

For criteria on moment redistribution for prestressed concrete members, see B.18.10.4.

B.8.4.1 — Except where approximate values for moments are used, it shall be permitted to increase or decrease negative moments calculated by elastic theory at supports of continuous flexural members for any assumed loading arrangement by not more than

$$20\left(1 - \frac{\rho - \rho'}{\rho_b}\right) \text{ percent}$$

B.8.4.2 — The modified negative moments shall be used for calculating moments at sections within the spans

B.8.4.3 — Redistribution of negative moments shall be made only when the section at which moment is reduced is so designed that ρ or $\rho - \rho'$ is not greater than $0.50\rho_b$, where

$$\rho_b = \frac{0.85\beta_1 f'_c}{f_y}\left(\frac{87{,}000}{87{,}000 + f_y}\right) \qquad \text{(B-1)}$$

B.10.3 — General principles and requirements

B.10.3.3 — For flexural members and members subject to combined flexure and compressive axial load when the design axial load strength ϕP_n is less than the smaller of $0.10 f'_c A_g$ or ϕP_b, the ratio of reinforcement, ρ, provided shall not exceed 0.75 of the ratio ρ_b that would produce balanced strain conditions for the section under flexure without axial load. For members with compression reinforcement, the portion of ρ_b equalized by compression reinforcement need not be reduced by the 0.75 factor.

COMMENTARY

When this appendix is used, the corresponding commentary sections apply. The load factors and strength reduction factors of either Chapter 9 or Appendix C are applicable.

RB.8.4 — Redistribution of negative moments in continuous nonprestressed flexural members

Moment redistribution is dependent on adequate ductility in plastic hinge regions. These plastic hinge regions develop at points of maximum moment and cause a shift in the elastic moment diagram. The usual results are reduction in the values of negative moments in the plastic hinge region and an increase in the values of positive moments from those computed by elastic analysis. Since negative moments are determined for one loading arrangement and positive moments for another, each section has a reserve capacity that is not fully utilized for any one loading condition. The plastic hinges permit the utilization of the full capacity of more cross sections of a flexural member at ultimate loads. Using conservative values of ultimate concrete strains and lengths of plastic hinges derived from extensive tests, flexural members with small rotation capacity were analyzed for moment redistribution up to 20 percent, depending on the reinforcement ratio. The results were found to be conservative (see Fig. RB.8.4). Studies by Cohn[B.1] and Mattock[B.2] support this conclusion and indicate that cracking and deflection of beams designed for moment redistribution are not significantly greater at service loads than for beams designed by the elastic theory distribution of moments. Also, these studies indicated that adequate rotation capacity for the moment redistribution allowed by the code is available if the members satisfy the code requirements. This appendix maintains the same limit on redistribution as used in previous code editions.

Moment redistribution may not be used for slab systems designed by the Direct Design Method (see 13.6.1.7).

RB.10.3 — General principles and requirements

RB.10.3.3 — The maximum amount of tension reinforcement in flexural members is limited to ensure a level of ductile behavior.

The nominal flexural strength of a section is reached when the strain in the extreme compression fiber reaches the limiting strain in the concrete. At ultimate strain of the concrete, the strain in the tension reinforcement could just reach the strain at first yield, be less than the yield strain (elastic), or exceed the yield strain (inelastic). The steel strain that exists at limiting concrete strain depends on the relative proportion of steel to concrete and material strengths f'_c and f_y.

CODE

COMMENTARY

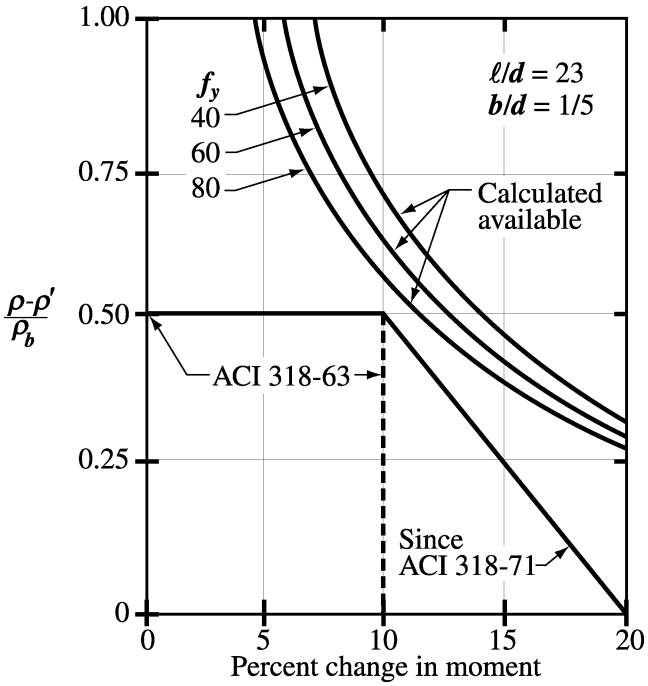


Fig. RB.8.4 — Permissible moment redistribution for minimum rotation capacity.

If $r(f_y/f'_c)$ is sufficiently low, the strain in the tension steel will greatly exceed the yield strain when the concrete strain reaches its limiting value, with large deflection and ample warning of impending failure (ductile failure condition). With a larger $r(f_y/f'_c)$, the strain in the tension steel may not reach the yield strain when the concrete strain reaches its limiting value, with consequent small deflection and little warning of impending failure (brittle failure condition). For design it is considered more conservative to restrict the nominal strength condition so that a ductile failure mode can be expected.

Unless unusual amounts of ductility are required, the $0.75r_b$ limitation will provide ductile behavior for most designs. One condition where greater ductile behavior is required is in design for redistribution of moments in continuous members and frames. Section B.8.4 permits negative moment redistribution. Since moment redistribution is dependent on adequate ductility in hinge regions, the amount of tension reinforcement in hinging regions is limited to $0.5r_b$.

For ductile behavior of beams with compression reinforcement, only that portion of the total tension steel balanced by compression in the concrete need be limited; that portion of the total tension steel where force is balanced by compression reinforcement need not be limited by the 0.75 factor.

CODE

B.18.1 — Scope

B.18.1.3 — The following provisions of this code shall not apply to prestressed concrete, except as specifically noted: Sections 7.6.5, B.8.4, 8.10.2, 8.10.3, 8.10.4, 8.11, B.10.3.3, 10.5, 10.6, 10.9.1, and 10.9.2; Chapter 13; and Sections 14.3, 14.5, and 14.6.

COMMENTARY

RB.18.1 — Scope

RB.18.1.3 — Some sections of the code are excluded from use in the design of prestressed concrete for specific reasons. The following discussion provides an explanation for such exclusions:

Section 7.6.5 — Section 7.6.5 is excluded from application to prestressed concrete since the requirements for bonded reinforcement and unbonded tendons for cast-in-place members are provided in 18.9 and 18.12, respectively.

Section B.8.4 — Moment redistribution for prestressed concrete is provided in B.18.10.4.

Sections 8.10.2, 8.10.3, and 8.10.4 — The empirical provisions of 8.10.2, 8.10.3, and 8.10.4 for T-beams were developed for conventionally reinforced concrete and, if applied to prestressed concrete, would exclude many standard prestressed products in satisfactory use today. Hence, proof by experience permits variations.

By excluding 8.10.2, 8.10.3, and 8.10.4, no special requirements for prestressed concrete T-beams appear in the code. Instead, the determination of an effective width of flange is left to the experience and judgment of the engineer. Where possible, the flange widths in 8.10.2, 8.10.3, and 8.10.4 should be used unless experience has proven that variations are safe and satisfactory. It is not necessarily conservative in elastic analysis and design considerations to use the maximum flange width as permitted in 8.10.2.

Sections 8.10.1 and 8.10.5 provide general requirements for T-beams that are also applicable to prestressed concrete units. The spacing limitations for slab reinforcement are based on flange thickness, which for tapered flanges can be taken as the average thickness.

Section 8.11 — The empirical limits established for conventionally reinforced concrete joist floors are based on successful past performance of joist construction using “standard” joist forming systems. See R8.11. For prestressed joist construction, experience and judgment should be used. The provisions of 8.11 may be used as a guide.

Sections B.10.3.2, 10.5, 10.9.1, and 10.9.2 — For prestressed concrete, the limitations on reinforcement given in B.10.3.2, 10.5, 10.9.1, and 10.9.2 are replaced by those in B.18.8, 18.9, and 18.11.2.

Section 10.6 — When originally prepared, the provisions of 10.6 for distribution of flexural reinforcement were not intended for prestressed concrete members. The behavior of a prestressed member is considerably different from that a nonprestressed member. Experience and judgment should be used for proper distribution of reinforcement in a prestressed member.

CODE

B.18.8 — Limits for reinforcement of flexural members

B.18.8.1 — Ratio of prestressed and nonprestressed reinforcement used for computation of moment strength of a member, except as provided in B.18.8.2, shall be such that w_p, $[w_p + (d/d_p)(w - w')]$, or $[w_{pw} + (d/d_p)(w_w - w_w')]$ is not greater than $0.36 b_1$, except as permitted in B.18.8.2.

B.18.8.2 — When a reinforcement ratio exceeds the limit specified in B.18.8.1 is provided, design moment strength shall not exceed the moment strength based on the compression portion of the moment couple.

B.18.8.3 — Total amount of prestressed and nonprestressed reinforcement shall be adequate to develop a factored load at least 1.2 times the cracking load computed on the basis of the modulus of rupture f_r in 9.5.2.3. This provision shall be permitted to be waived for:

(a) two-way, unbonded post-tensioned slabs; and

(b) flexural members with shear and flexural strength at least twice that required by 9.2.

COMMENTARY

Chapter 13 — The design of prestressed concrete slabs requires recognition of secondary moments induced by the undulating profile of the prestressing tendons. Also, volume changes due to the prestressing force can create additional loads on the structure that are not adequately covered in Chapter 13. Because of these unique properties associated with prestressing, many of the design procedures of Chapter 13 are not appropriate for prestressed concrete structures and are replaced by the provisions of 18.12.

Sections 14.5 and 14.6 — The requirements for wall design in 14.5 and 14.6 are largely empirical, utilizing considerations not intended to apply to prestressed concrete.

RB.18.8 — Limits for reinforcement of flexural members

RB.18.8.1 — The terms w_p, $[w_p + (d/d_p)(w - w')]$ and $[w_{pw} + (d/d_p)(w_w - w_w')]$ are each equal to $0.85a/d_p$, where a is the depth of the equivalent rectangular stress distribution for the section under consideration, as defined in 10.2.7.1. Use of this relationship can simplify the calculations necessary to check compliance with RB.18.8.1.

RB.18.8.2 — Design moment strength of over-reinforced sections may be computed using strength equations similar to those for nonprestressed concrete members. The 1983 code provided strength equations for rectangular and flanged sections.

RB.18.8.3 — This provision is a precaution against abrupt flexural failure developing immediately after cracking. A flexural member designed according to code provisions requires considerable additional load beyond cracking to reach its flexural strength. This additional load should result in considerable deflection that would warn when the member nominal strength is being approached. If the flexural strength is reached shortly after cracking, the warning deflection would not occur.

Due to the very limited extent of initial cracking in the negative moment region near columns of two-way flat plates, deflection under load does not reflect any abrupt change in stiffness as the modulus of rupture of concrete is reached.

Only at load levels beyond the factored loads is the additional cracking extensive enough to cause an abrupt change in the deflection under load. Tests have shown that it is not possible to rupture (or even yield) unbonded post-tensioning tendons in two-way slabs before a punching shear failure.[B.3-B.8] The use of unbonded tendons in combination with the minimum bonded reinforcement requirements of 18.9.3 and 18.9.4 has been shown to ensure post-cracking ductility and that a brittle failure mode will not develop at first cracking.

CODE

B.18.10 — Statically indeterminate structures

B.18.10.1 — Frames and continuous construction of prestressed concrete shall be designed for satisfactory performance at service load conditions and for adequate strength.

B.18.10.2 — Performance at service load conditions shall be determined by elastic analysis, considering reactions, moments, shears, and axial forces produced by prestressing, creep, shrinkage, temperature change, axial deformation, restraint of attached structural elements, and foundation settlement.

B.18.10.3 — Moments to be used to compute required strength shall be the sum of the moments due to reactions induced by prestressing (with a load factor of 1.0) and the moments due to factored loads. Adjustment of the sum of these moments shall be permitted as allowed in B.18.10.4.

B.18.10.4 — Redistribution of negative moments in continuous prestressed flexural members

B.18.10.4.1 — Where bonded reinforcement is provided at supports in accordance with 18.9, negative moments calculated by elastic theory for any assumed loading, arrangement shall be permitted to be increased or decreased by not more than

$$20\left[1-\frac{w_p+\frac{d}{d_p}(w-w\ell)}{0.36b_1}\right] \text{ percent}$$

COMMENTARY

RB.18.10 — Statically indeterminate structures

RB.18.10.3 — For statically indeterminate structures, the moments due to reactions induced by prestressing forces, referred to as secondary moments, are significant in both the elastic and inelastic states. When hinges and full redistribution of moments occur to create a statically determinate structure, secondary moments disappear. However, the elastic deformations caused by a nonconcordant tendon change the amount of inelastic rotation required to obtain a given amount of moment redistribution. Conversely, for a beam with a given inelastic rotation capacity, the amount by which the moment at the support may be varied is changed by an amount equal to the secondary moment at the support due to prestressing. Thus, the code requires that secondary moments be included in determining design moments.

To determine the moments used in design, the order of calculation should be: (a) determine moments due to dead load and live load; (b) modify by algebraic addition of secondary moments; and (c) redistribute as permitted. A positive secondary moment at the support caused by a tendon transformed downward from a concordant profile will reduce the negative moments near the supports and increase the positive moments in the midspan regions. A tendon that is transformed upward will have the reverse effect.

RB.18.10.4 — Redistribution of negative moments in continuous prestressed flexural members

As member strength is approached, inelastic behavior at some sections can result in a redistribution of moments in prestressed concrete beams and slabs. Recognition of this behavior can be advantageous in design under certain circumstances. A rigorous design method for moment redistribution is complex. However, recognition of moment redistribution can be accomplished by permitting a reasonable adjustment of the sum of the elastically calculated factored gravity load moments and the unfactored secondary moments due to prestress. The amount of adjustment should be kept within predetermined safety limits.

CODE

B.18.10.4.2 — The modified negative moments shall be used for calculating moments at sections within spans for the same loading arrangement.

B.18.10.4.3 — Redistribution of negative moments shall be made only when the section at which moment is reduced is so designed that w_p, $[w_p + (d/d_p)(w - w')]$ or $[w_{pw} + (d/d_p)(w_w - w_w')]$, whichever is applicable, is not greater than $0.24b_1$.

COMMENTARY

The amount of redistribution allowed depends on the ability of the critical sections to deform inelastically by a sufficient amount. Serviceability is addressed in 18.4. The choice of $0.24b_1$ as the largest tension reinforcement index, w_p, $[w_p + (d/d_p)(w - w')]$ or $[w_{pw} + (d/d_p)(w_w - w_w')]$, for which redistribution of moments is allowed, is in agreement with the requirements for nonprestressed concrete of $0.5r_b$ stated in B.8.4.

The terms w_p, $[w_p + (d/d_p)(w - w')]$ and $[w_{pw} + (d/d_p)(w_w - w_w')]$ appear in B.18.10.4.1 and B.18.10.4.3 and are each equal to $0.85a/d_p$, where a is the depth of the equivalent rectangular stress distribution for the section under consideration, as defined in 10.2.7.1. Use of this relationship can simplify the calculations necessary to determine the amount of moment redistribution permitted by B.18.10.4.1 and to check compliance with the limitation on flexural reinforcement contained in B.18.10.4.3.

For the moment redistribution principles of B.18.10.4 to be applicable to beams and slabs with unbonded tendons, it is necessary that such beams and slabs contain sufficient bonded reinforcement to ensure that they act as flexural members after cracking and not as a series of tied arches. The minimum bonded reinforcement requirements of 18.9 serve this purpose.

CODE COMMENTARY

Notes

APPENDIX C — ALTERNATIVE LOAD AND STRENGTH REDUCTION FACTORS

CODE

C.1 — General

C.1.1 — Structural concrete shall be permitted to be designed using the load combinations and strength reduction factors of Appendix C.

C.2 — Required strength

C.2.1 — Required strength U to resist dead load D and live load L shall be at least equal to

$$U = 1.4D + 1.7L \quad \text{(C-1)}$$

C.2.2 — If resistance to structural effects of a specified wind load W or earthquake load E is included in design, the following combinations shall be investigated to determine the greatest required strength U:

$$U = 0.75(1.4D + 1.7L) + (1.6W \text{ or } 1.0E) \quad \text{(C-2)}$$

and

$$U = 0.9D + (1.6W \text{ or } 1.0E) \quad \text{(C-3)}$$

but required strength U shall not be less than Eq. (C-1). Where wind load W has not been reduced by a directionality factor, it shall be permitted to use $1.3W$ in place of $1.6W$ in Eq. (C-2) and (C-3). Where earthquake load E is based on service-level seismic forces, $1.4E$ shall be used in place of $1.0E$ in Eq. (C-2) and (C-3).

C.2.3 — If resistance to earth pressure H is included in design, required strength U shall be at least equal to

$$U = 1.4D + 1.7L + 1.7H \quad \text{(C-4)}$$

except that where D or L reduce the effect of H, $0.9D$ shall be substituted for $1.4D$, and zero value of L shall be used to determine the greatest required strength U. For any combination of D, L, and H, required strength U shall not be less than Eq. (C-1).

COMMENTARY

RC.1 — General

RC.1.1 — In the 2002 code, the load and strength reduction factors formerly in Chapter 9 were revised and moved to this appendix. They have evolved since the early 1960s and are considered to be reliable for concrete construction.

RC.2 — Required strength

The wind load equation in ASCE 7-98 and IBC 2000[C.1] includes a factor for wind directionality that is equal to 0.85 for buildings. The corresponding load factor for wind in the load combination equations was increased accordingly (1.3/0.85 = 1.53, rounded up to 1.6). The code allows use of the previous wind load factor of 1.3 when the design wind load is obtained from other sources that do not include the wind directionality factor.

Model building codes and design load references have converted earthquake forces to strength level, and reduced the earthquake load factor to 1.0 (ASCE 7-93[C.2]; BOCA/NBC 93[C.3]; SBC 94[C.4]; UBC 97[C.5]; and IBC 2000[C.1]). The code requires use of the previous load factor for earthquake loads, approximately 1.4, when service-level earthquake forces from earlier editions of these references are used.

RC.2.3 — If effects H caused by earth pressure, groundwater pressure, or pressure caused by granular materials are included in design, the required strength equations become:

$$U = 1.4D + 1.7L + 1.7H$$

and where D or L reduce the effect of H

$$U = 0.9D + 1.7H$$

but for any combination of D, L, or H

$$U = 1.4D + 1.7L$$

CODE

C.2.4 — If resistance to loadings due to weight and pressure of fluids with well-defined densities and controllable maximum heights ***F*** is included in design, such loading shall have a load factor of 1.4, and be added to all loading combinations that include live load.

C.2.5 — If resistance to impact effects is taken into account in design, such effects shall be included with live load ***L***.

C.2.6 — Where structural effects ***T*** of differential settlement, creep, shrinkage, expansion of shrinkage-compensating concrete, or temperature change are significant in design, required strength ***U*** shall be at least equal to

$$U = 0.75(1.4D + 1.4T + 1.7L) \quad \text{(C-5)}$$

but required strength ***U*** shall not be less than

$$U = 1.4(D + T) \quad \text{(C-6)}$$

Estimations of differential settlement, creep, shrinkage, expansion of shrinkage-compensating concrete, or temperature change shall be based on realistic assessment of such effects occurring in service.

C.2.7 — For post-tensioned anchorage zone design, a load factor of 1.2 shall be applied to the maximum prestressing steel jacking force.

COMMENTARY

RC.2.4 — This section addresses the need to consider loading due to weight of liquid or liquid pressure. It specifies a load factor for such loadings with well-defined densities and controllable maximum heights equivalent to that used for dead load. Such reduced factors would not be appropriate where there is considerable uncertainty of pressures, as with groundwater pressures, or uncertainty as to the possible maximum liquid depth, as in ponding of water. See R8.2.

For well-defined fluid pressures, the required strength equations become:

$$U = 1.4D + 1.7L + 1.4F$$

and where ***D*** or ***L*** reduce the effect of ***F***

$$U = 0.9D + 1.4F$$

but for any combination of ***D***, ***L***, or ***F***

$$U = 1.4D + 1.7L$$

RC.2.5 — If the live load is applied rapidly, as may be the case for parking structures, loading docks, warehouse floors, elevator shafts, etc., impact effects should be considered. In all equations, substitute (***L*** + impact) for ***L*** when impact must be considered.

RC.2.6 — The designer should consider the effects of differential settlement, creep, shrinkage, temperature, and shrinkage-compensating concrete. The term “realistic assessment” is used to indicate that the most probable values, rather than the upper bound values, of the variables should be used.

Equation (C-6) is to prevent a design for load

$$U = 0.75\ (1.4D + 1.4T + 1.7L)$$

to approach

$$U = 1.05\ (D + T)$$

when live load is negligible.

RC.2.7 — The load factor of 1.2 applied to the maximum prestressing steel jacking force results in a design load of 113 percent of the specified prestressing steel yield strength but not more than 96 percent of the nominal ultimate strength of the tendon. This compares well with a maximum attainable jacking force, which is limited by the anchor efficiency factor.

CODE

COMMENTARY

C.3 — Design strength

RC.3 — Design strength

C.3.1 — Design strength provided by a member, its connections to other members, and its cross sections, in terms of flexure, axial load, shear, and torsion, shall be taken as the nominal strength calculated in accordance with requirements and assumptions of this code, multiplied by the strength reduction factors ϕ in C.3.2, C.3.4, and C.3.5.

RC.3.1 — The term "design strength" of a member, refers to the nominal strength calculated in accordance with the requirements stipulated in this code multiplied by a strength reduction factor ϕ that is always less than one.

The purposes of the strength reduction factor ϕ are: (1) to allow for the probability of understrength members due to variations in material strengths and dimensions; (2) to allow for inaccuracies in the design equations; (3) to reflect the degree of ductility and required reliability of the member under the load effects being considered; and (4) to reflect the importance of the member in the structure. For example, a lower ϕ is used for columns than for beams because columns generally have less ductility, are more sensitive to variations in concrete strength, and generally support larger loaded areas than beams. Furthermore, spiral columns are assigned a higher ϕ than tied columns since they have greater ductility or toughness.

C.3.2 — Strength reduction factor ϕ shall be as follows:

C.3.2.1 — Tension-controlled sections, as defined in 10.3.4 (See also C.3.2.7) 0.90

RC.3.2.1 — In applying C.3.2.1 and C.3.2.2, the axial tensions and compressions to be considered are those caused by external forces. Effects of prestressing forces are not included.

C.3.2.2 — Compression-controlled sections, as defined in 10.3.3:

(a) Members with spiral reinforcement conforming to 10.9.3..... 0.75

(b) Other reinforced members 0.70

RC.3.2.2 — Before the 2002 edition, the code gave the magnitude of the ϕ-factor for cases of axial load or flexure, or both, in terms of the type of loading. For these cases, the ϕ-factor is now determined by the strain conditions at a cross section, at nominal strength.

A lower ϕ-factor is used for compression-controlled sections than is used for tension-controlled sections because compression-controlled sections have less ductility, are more sensitive to variations in concrete strength, and generally occur in members that support larger loaded areas than members with tension-controlled sections. Members with spiral reinforcement are assigned a higher ϕ than tied columns since they have greater ductility or toughness.

For sections in which the net tensile strain in the extreme tension steel at nominal strength is between the limits for compression-controlled and tension-controlled sections, ϕ shall be permitted to be linearly increased from that for compression-controlled sections to 0.90 as the net tensile strain in the extreme tension steel at nominal strength increases from the compression-controlled strain limit to 0.005.

Alternatively, when Appendix B is used, for members in which f_y does not exceed 60,000 psi, with symmetric reinforcement, and with $(h - d' - d_s)/h$ not less than 0.70, ϕ shall be permitted to be increased linearly to 0.90 as ϕP_n decreases from $0.10 f'_c A_g$ to zero. For other reinforced members, ϕ shall be permitted to be increased linearly to 0.90 as ϕP_n decreases from $0.10 f'_c A_g$ or ϕP_b, whichever is smaller, to zero.

For sections subjected to axial load with flexure, design strengths are determined by multiplying both P_n and M_n by the appropriate single value of ϕ. Compression-controlled and tension-controlled sections are defined in 10.3.3 and 10.3.4 as those that have net tensile strain in the extreme tension steel at nominal strength less than or equal to the compression-controlled strain limit, and equal to or greater than 0.005, respectively. For sections with net tensile strain ε_t in the extreme tension steel at nominal strength between the above limits, the value of ϕ may be determined by linear interpolation, as shown in Fig. RC.3.2. The concept of net tensile strain ε_t is discussed in R10.3.3.

CODE

C.3.2.3 — Shear and torsion 0.85

C.3.2.4 — Bearing on concrete (except for post-tensioned anchorage zones and strut-and-tie models).. 0.70

C.3.2.5 — Post-tensioned anchorage zones 0.85

C.3.2.6 — Strut-and-tie models (Appendix A), and struts, ties, nodal zones, and bearing areas in such models..... 0.85

C.3.2.7 — Flexure sections without axial load in pretensioned members where strand embedment is less than the development length as provided in 12.9.1.1 0.85

COMMENTARY

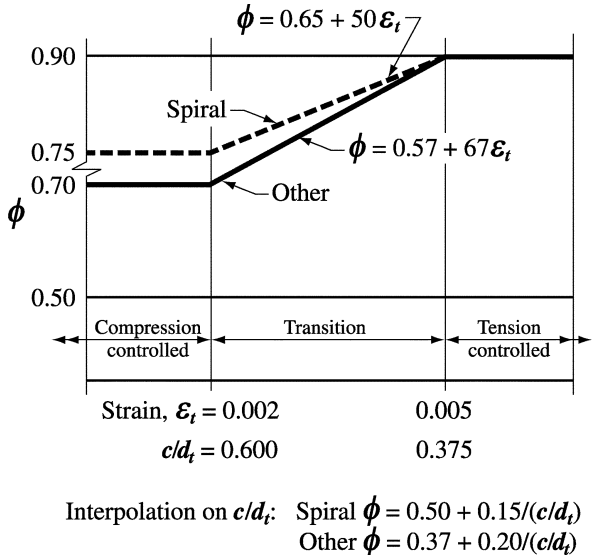


Fig. RC.3.2—Variation of ϕ with net tensile strain ε_t and c/d_t for Grade 60 reinforcement and for prestressing steel.

Since the compressive strain in the concrete at nominal strength is assumed in 10.2.3 to be 0.003, the net tensile strain limits for compression-controlled members may also be stated in terms of the ratio c/d_t, where c is the depth of the neutral axis at nominal strength, and d_t is the distance from the extreme compression fiber to the extreme tension steel. The c/d_t limits for compression-controlled and tension-controlled sections are 0.6 and 0.375, respectively. The 0.6 limit applies to sections reinforced with Grade 60 steel and to prestressed sections. Figure RC.3.2 also gives equations for ϕ as a function of c/d_t.

The net tensile strain limit for tension-controlled sections may also be stated in terms of the r/r_b as defined in the 1999 and earlier editions of the code. The net tensile strain limit of 0.005 corresponds to a r/r_b ratio of 0.63 for rectangular sections with Grade 60 reinforcement. For a comparison of these provisions with those of the body of the code, the 1999 ACI Code Section 9.3, see Reference C.6.

RC.3.2.5 — The ϕ-factor of 0.85 reflects the wide scatter of results of experimental anchorage zone studies. Since 18.13.4.2 limits the nominal compressive strength of unconfined concrete in the general zone to $0.7 f'_{ci}$, the effective design strength for unconfined concrete is $0.85 \times 0.7 f'_{ci} \approx 0.6 f'_{ci}$.

RC.3.2.7 — If a critical section occurs in a region where strand is not fully developed, failure may be by bond slip. Such a failure resembles a brittle shear failure; hence the requirement for a reduced ϕ.

CODE

C.3.3 — Development lengths specified in Chapter 12 do not require a ϕ-factor.

C.3.4 — In structures that rely on special moment resisting frames or special reinforced concrete structural walls to resist earthquake effects, the strength reduction factors ϕ shall be modified as given in (a) through (c):

(a) The strength reduction factor for shear shall be 0.60 for any structural member that is designed to resist earthquake effects if its nominal shear strength is less than the shear corresponding to the development of the nominal flexural strength of the member. The nominal flexural strength shall be determined considering the most critical factored axial loads and including earthquake effects;

(b) The strength reduction factor for shear in diaphragms shall not exceed the minimum strength reduction factor for shear used for the vertical components of the primary lateral-force-resisting system;

(c) The strength reduction factor for shear in joints and diagonally reinforced coupling beams shall be 0.85.

C.3.5 — Strength reduction factor ϕ for flexure, compression, shear, and bearing of structural plain concrete in Chapter 22 shall be 0.65.

COMMENTARY

RC.3.4 — Strength-reduction factors in C.3.4 are intended to compensate for uncertainties in estimation of strength of structural members in buildings. They are based primarily on experience with constant or steadily increasing applied load. For construction in regions of high seismic risk, some of the strength reduction factors have been modified in C.3.4 to account for the effects of displacements on strength into the nonlinear range of response on strength.

Section C.3.4(a) refers to brittle members, such as low-rise walls or portions of walls between openings, or diaphragms that are impractical to reinforce to raise their nominal shear strength above nominal flexural strength for the pertinent loading conditions.

Short structural walls were the primary vertical elements of the lateral-force-resisting system in many of the parking structures that sustained damage during the 1994 Northridge earthquake. Section C.3.4(b) requires the shear strength reduction factor for diaphragms to be 0.60 if the shear strength reduction factor for the walls is 0.60.

RC.3.5 — The strength-reduction factor ϕ for structural plain concrete design is the same for all strength conditions. Since both flexural tension strength and shear strength for plain concrete depend on the tensile strength characteristics of the concrete, with no reserve strength or ductility possible due to the absence of reinforcement, equal strength reduction factors for both bending and shear are considered appropriate.

CODE COMMENTARY

Notes

APPENDIX D — ANCHORING TO CONCRETE

CODE

D.0 — Notation

A_{brg} = bearing area of the head of stud or anchor bolt, in.^{2}

A_{No} = projected concrete failure area of one anchor, for calculation of strength in tension when not limited by edge distance or spacing, in.^{2} (see 5.2.1)

A_N = projected concrete failure area of an anchor or group of anchors, for calculation of strength in tension, as defined in, in.^{2} (see D.5.2.1) A_N shall not be taken greater than nA_{No}.

A_{se} = effective cross-sectional area of anchor, in.^{2}

A_{sl} = effective cross-sectional area of expansion or undercut anchor sleeve, if sleeve is within shear plane, in.^{2}

A_{Vo} = projected concrete failure area of one anchor, for calculation of strength in shear, when not limited by corner influences, spacing, or member thickness, in.^{2} (see D.6.2.1)

A_V = projected concrete failure area of an anchor or group of anchors, for calculation of strength in shear, in.^{2} (see D.6.2.1) A_V shall not be taken greater than nA_{Vo}.

c = distance from center of an anchor shaft to the edge of concrete, in.

c_1 = distance from the center of an anchor shaft to the edge of concrete in one direction, in.; where shear force is applied to anchor, c_1 is in the direction of the shear force.

c_2 = distance from center of an anchor shaft to the edge of concrete in the direction orthogonal to c_1, in.

COMMENTARY

RD.0 — Notation

See Fig. RD.5.2.1(a)

See Fig. RD.5.2.1(b)

A_{se} The effective cross-sectional area of an anchor should be provided by the manufacturer of expansion anchors with reduced cross-sectional area for the expansion mechanism. For threaded bolts, ANSI/ASME B1.1[D.1] defines A_{se} as:

$$A_{se} = \frac{\pi}{4}\left(d_o - \frac{0.9743}{n_t}\right)^2$$

where n_t is the number of threads per inch

See Fig. RD.6.2.1(a).

See Fig. RD.6.2.1(b).

See Fig. RD.6.2.1(a).

CODE

c_{max} = the largest edge distance, in.

c_{min} = the smallest edge distance, in.

d_o = outside diameter of anchor or shaft diameter of headed stud, headed bolt, or hooked bolt, in. (see also D.8.4)

d'_o = value substituted for d_o when an oversized anchor is used, in. (see D.8.4)

e_h = distance from the inner surface of the shaft of a J- or L-bolt to the outer tip of the J- or L-bolt, in.

e'_N = eccentricity of normal force on a group of anchors; the distance between the resultant tension load on a group of anchors in tension and the centroid of the group of anchors loaded in tension, in.; e'_N is always positive

e'_V = eccentricity of shear force on a group of anchors; the distance between the point of shear force application and the centroid of the group of anchors resisting shear in the direction of the applied shear, in.

f'_c = specified compressive strength of concrete, psi

f_{ct} = specified tensile strength of concrete, psi

f_r = modulus of rupture of concrete, psi (see 9.5.2.3)

f_t = calculated concrete tensile stress in a region of a member, psi

f_y = specified yield strength of anchor steel, psi

f_{ut} = specified tensile strength of anchor steel, psi

f_{utsl} = specified tensile strength of anchor sleeve, psi

h = thickness of member in which an anchor is anchored, measured parallel to anchor axis, in.

h_{ef} = effective anchor embedment depth, in. (see D.8.5)

k = coefficient for basic concrete breakout strength in tension

k_{cp} = coefficient for pryout strength

ℓ = load bearing length of anchor for shear, not to exceed $8d_o$, in.

= h_{ef} for anchors with a constant stiffness over the full length of the embedded section, such as headed studs or post-installed anchors with one tubular shell over the full length of the embedment depth

= $2d_o$ for torque-controlled expansion anchors with a distance sleeve separated from the expansion sleeve

n = number of anchors in a group

N_b = basic concrete breakout strength in tension of a single anchor in cracked concrete, lb (see 5.2.2)

COMMENTARY

e_N Actual eccentricity of a normal force on an attachment (See Fig. RD.5.2.4.)

h_{ef} Effective embedment depths for a variety of anchor types are shown in Fig. RD.0

CODE

N_{cb} = nominal concrete breakout strength in tension of a single anchor, lb (see D.5.2.1)

N_{cbg} = nominal concrete breakout strength in tension of a group of anchors, lb (see D.5.2.1)

N_n = nominal strength in tension, lb

N_p = pullout strength in tension of a single anchor in cracked concrete, lb (see D.5.3.4 or D.5.3.5)

N_{pn} = nominal pullout strength in tension of a single anchor, lb (see D.5.3.1)

N_{sb} = side-face blowout strength of a single anchor, lb

N_{sbg} = side-face blowout strength of a group of anchors, lb

N_s = nominal strength of a single anchor or group of anchors in tension as governed by the steel strength, lb (see D.5.1.1 or D.5.1.2)

N_u = factored tensile load, lb

s = anchor center-to-center spacing, in.

s_o = spacing of the outer anchors along the edge in a group, in.

t = thickness of washer or plate, in.

V_b = basic concrete breakout strength in shear of a single anchor in cracked concrete, lb (see D.6.2.2 or D.6.2.3)

V_{cb} = nominal concrete breakout strength in shear of a single anchor, lb (see D.6.2.1)

V_{cbg} = nominal concrete breakout strength in shear of a group of anchors, lb (see D.6.2.1)

V_{cp} = nominal concrete pryout strength, lb (see D.6.3)

V_n = nominal shear strength, lb

V_s = nominal strength in shear of a single anchor or group of anchors as governed by the steel strength, lb (see D.6.1.1 or D.6.1.2)

V_u = factored shear load, lb

f = strength reduction factor (see D.4.4 and D.4.5)

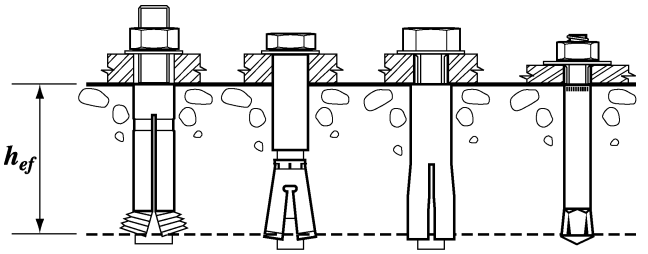
y_1 = modification factor, for strength in tension, to account for anchor groups loaded eccentrically (see D.5.2.4)

y_2 = modification factor, for strength in tension, to account for edge distances smaller than $1.5h_{ef}$ (see D.5.2.5)

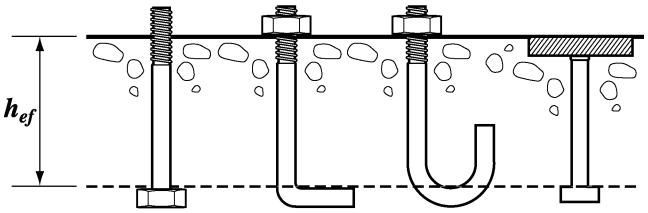
y_3 = modification factor, for strength in tension, to account for cracking (see D.5.2.6 and D.5.2.7)

y_4 = modification factor, for pullout strength, to

COMMENTARY



(a) Post-installed anchors



(b) Cast-in-place anchors

Fig. RD.0—Types of anchors.

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account for cracking (see D.5.3.1 and D.5.3.6)

y_5 = modification factor, for strength in shear, to account for anchor groups loaded eccentrically (see D.6.2.5)

y_6 = modification factor, for strength in shear, to account for edge distances smaller than $1.5c_1$ (see D.6.2.6)

y_7 = modification factor, for strength in shear, to account for cracking (see D.6.2.7)

D.1 — Definitions

Anchor — A steel element either cast into concrete or post-installed into a hardened concrete member and used to transmit applied loads, including headed bolts, hooked bolts (J- or L-bolt), headed studs, expansion anchors, or undercut anchors.

Anchor group — A number of anchors of approximately equal effective embedment depth with each anchor spaced at less than three times its embedment depth from one or more adjacent anchors.

Anchor pullout strength — The strength corresponding to the anchoring device or a major component of the device sliding out from the concrete without breaking out a substantial portion of the surrounding concrete.

Attachment — The structural assembly, external to the surface of the concrete, that transmits loads to or receives loads from the anchor.

Brittle steel element — An element with a tensile test elongation of less than 14 percent, or reduction in area of less than 30 percent, or both.

Cast-in anchor — A headed bolt, headed stud, or hooked bolt installed before placing concrete.

Concrete breakout strength — The strength corresponding to a volume of concrete surrounding the anchor or group of anchors separating from the member.

Concrete pryout strength — The strength corresponding to formation of a concrete spall behind a short, stiff anchor with an embedded base that is displaced in the direction opposite to the applied shear force.

Distance sleeve — A sleeve that encases the center part of an undercut anchor, a torque-controlled expansion anchor, or a displacement-controlled expansion anchor, but does not expand.

COMMENTARY

RD.1 — Definitions

Brittle steel element and ductile steel element The 14 percent elongation should be measured over the gage length specified in the appropriate ASTM standard for the steel.

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Ductile steel element — An element with a tensile test elongation of at least 14 percent and reduction in area of at least 30 percent. A steel element meeting the requirements of ASTM A 307 shall be considered ductile.

Edge distance — The distance from the edge of the concrete surface to the center of the nearest anchor.

Effective embedment depth — The overall depth through which the anchor transfers force to or from the surrounding concrete. The effective embedment depth will normally be the depth of the concrete failure surface in tension applications. For cast-in headed anchor bolts and headed studs, the effective embedment depth is measured from the bearing contact surface of the head. (See Fig. RD.0)

Expansion anchor — A post-installed anchor, inserted into hardened concrete that transfers loads to or from the concrete by direct bearing or friction or both. Expansion anchors may be torque-controlled, where the expansion is achieved by a torque acting on the screw or bolt; or displacement-controlled, where the expansion is achieved by impact forces acting on a sleeve or plug and the expansion is controlled by the length of travel of the sleeve or plug.

Expansion sleeve — The outer part of an expansion anchor that is forced outward by the center part, either by applied torque or impact, to bear against the sides of the predrilled hole.

Five percent fractile — A statistical term meaning 90 percent confidence that there is 95 percent probability of the actual strength exceeding the nominal strength.

Hooked bolt — A cast-in anchor anchored mainly by mechanical interlock from the 90-deg bend (L-bolt) or 180-deg bend (J-bolt) at its lower end, having a minimum e_h of $3d_o$.

Headed stud — A steel anchor conforming to the requirements of AWS D1.1 and affixed to a plate or similar steel attachment by the stud arc welding process before casting.

Post-installed anchor — An anchor installed in hardened concrete. Expansion anchors and undercut anchors are examples of post-installed anchors.

Projected area — The area on the free surface of the concrete member that is used to represent the larger base of the assumed rectilinear failure surface.

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Five percent fractile — The determination of the coefficient k associated with the 5 percent fractile, $\bar{x} - k\sigma$, depends on the number of tests, n, used to compute $\bar{x}$ and σ. Values of k range, for example, from 1.645 for $n = \infty$, to 2.010 for $n = 40$, and 2.568 for $n = 10$. With this definition of the 5 percent fractile, the nominal strength in D.4.2 is the same as the characteristic strength in ACI 355.2.

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Side-face blowout strength — The strength of anchors with deeper embedment but thinner side cover corresponding to concrete spalling on the side face around the embedded head while no major breakout occurs at the top concrete surface.

Specialty insert — Predesigned and prefabricated cast-in anchors specifically designed for attachment of bolted or slotted connections. Specialty inserts are often used for handling, transportation, and erection, but are also used for anchoring structural elements. Specialty inserts are not within the scope of this appendix.

Supplementary reinforcement — Reinforcement proportioned to tie a potential concrete failure prism to the structural member.

Undercut anchor — A post-installed anchor that develops its tensile strength from the mechanical interlock provided by undercutting of the concrete at the embedded end of the anchor. The undercutting is achieved with a special drill before installing the anchor or alternatively by the anchor itself during its installation.

COMMENTARY

D.2 — Scope

D.2.1 — This appendix provides design requirements for anchors in concrete used to transmit structural loads by means of tension, shear, or a combination of tension and shear between (a) connected structural elements; or (b) safety-related attachments and structural elements. Safety levels specified are intended for in-service conditions, rather than for short-term handling and construction conditions.

D.2.2 — This appendix applies to both cast-in anchors and post-installed anchors. Specialty inserts, through bolts, multiple anchors connected to a single steel plate at the embedded end of the anchors, adhesive or grouted anchors, and direct anchors such as powder or pneumatic actuated nails or bolts, are not included. Reinforcement used as part of the embedment shall be designed in accordance with other parts of the code.

D.2.3 — Headed studs and headed bolts having a geometry that has been demonstrated to result in a pullout strength in uncracked concrete equal or exceeding $\mathbf{1.4}N_p$ (where N_p is given by Eq. (D-13)) are included. Hooked bolts that have a geometry that has been demonstrated to result in a pullout strength without the benefit of friction in uncracked concrete equal

RD.2 — Scope

RD.2.1 — Appendix D is restricted in scope to structural anchors that transmit structural loads related to strength, stability, or life safety. Two types of applications are envisioned. The first is connections between structural elements where the failure of an anchor or an anchor group could result in loss of equilibrium or stability of any portion of the structure. The second is where safety-related attachments that are not part of the structure (such as sprinkler systems, heavy suspended pipes, or barrier rails) are attached to structural elements. The levels of safety defined by the combinations of load factors and f factors are appropriate for structural applications. Other standards may require more stringent safety levels during temporary handling.

RD.2.2 — The wide variety of shapes and configurations of specialty inserts makes it difficult to prescribe generalized tests and design equations for many insert types. Hence, they have been excluded from the scope of Appendix D. Adhesive anchors are widely used and can perform adequately. At this time, however, such anchors are outside the scope of this appendix.

RD.2.3 — Typical cast-in headed studs and headed bolts with geometries consistent with ANSI/ASME B1.1,[D.1] B18.2.1,[D.2] and B18.2.6[D.3] have been tested and proven to behave predictably, so calculated pullout values are acceptable. Post-installed anchors do not have predictable pullout capacities, and therefore are required to be tested. For a post-installed anchor to be used in conjunction with the

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or exceeding $1.4N_p$ (where N_p is given by Eq. (D-14)) are included. Post-installed anchors that meet the assessment requirements of ACI 355.2 are included. The suitability of the post-installed anchor for use in concrete shall have been demonstrated by the ACI 355.2 prequalification tests.

D.2.4 — Load applications that are predominantly high cycle fatigue or impact loads are not covered by this appendix.

D.3 — General requirements

D.3.1 — Anchors and anchor groups shall be designed for critical effects of factored loads as determined by elastic analysis. Plastic analysis approaches are permitted where nominal strength is controlled by ductile steel elements, provided that deformational compatibility is taken into account.

D.3.2 — The design strength of anchors shall equal or exceed the largest required strength calculated from the applicable load combinations in 9.2.

D.3.3 — When anchor design includes seismic loads, the additional requirements of D.3.3.1 through D.3.3.5 shall apply.

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requirements of this appendix, the results of the ACI 355.2 tests have to indicate that pullout failures exhibit an acceptable load-displacement characteristic or that pullout failures are precluded by another failure mode.

RD.2.4 — The exclusion from the scope of load applications producing high cycle fatigue or extremely short duration impact (such as blast or shock wave) are not meant to exclude seismic load effects. D.3.3 presents additional requirements for design when seismic loads are included.

RD.3 — General requirements

RD.3.1 — When the strength of an anchor group is governed by breakage of the concrete, the behavior is brittle and there is limited redistribution of the forces between the highly stressed and less stressed anchors. In this case, the theory of elasticity is required to be used assuming the attachment that distributes loads to the anchors is sufficiently stiff. The forces in the anchors are considered to be proportional to the external load and its distance from the neutral axis of the anchor group.

If anchor strength is governed by ductile yielding of the anchor steel, significant redistribution of anchor forces can occur. In this case, an analysis based on the theory of elasticity will be conservative. References D.4 to D.6 discuss nonlinear analysis, using theory of plasticity, for the determination of the capacities of ductile anchor groups.

RD.3.3 — Post-installed structural anchors are required to be qualified for moderate or high seismic risk zone usage by demonstrating the capacity to undergo large displacements through several cycles as specified in the seismic simulation tests of ACI 355.2. Because ACI 355.2 excludes plastic hinge zones, Appendix D is not applicable to the design of anchors in plastic hinge zones under seismic loads. In addition, the design of anchors in zones of moderate or high seismic risk is based on a more conservative approach by the introduction of 0.75 factor on the design strength ϕN_n and ϕV_n, and by requiring the system to have adequate ductility. Anchorage capacity should be governed by ductile yielding of a steel element. If the anchor cannot meet these ductility requirements, then the attachment is required to be designed so as to yield at a load well below the anchor capacity. In designing attachments for adequate ductility, the ratio of yield to ultimate load capacity should be considered. A connection element could yield only to result in a secondary failure as one or more elements strain harden and fail if the ultimate load capacity is excessive when compared to the yield capacity.

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D.3.3.1 —The provisions of Appendix D do not apply to the design of anchors in plastic hinge zones of concrete structures under seismic loads.

D.3.3.2 — In regions of moderate or high seismic risk, or for structures assigned to intermediate or high seismic performance or design categories, post-installed structural anchors for use under D.2.3 shall have passed the Simulated Seismic Tests of ACI 355.2.

D.3.3.3 — In regions of moderate or high seismic risk, or for structures assigned to intermediate or high seismic performance or design categories, the design strength of anchors shall be taken as $0.75\phi N_n$ and $0.75\phi V_n$, where ϕ is given in D.4.4 or D.4.5 and N_n and V_n are determined in accordance with D.4.1.

D.3.3.4 — In regions of moderate or high seismic risk, or for structures assigned to intermediate or high seismic performance or design categories, anchors shall be designed to be governed by tensile or shear strength of a ductile steel element, unless D.3.3.5 is satisfied.

D.3.3.5 — Instead of D.3.3.4, the attachment that the anchor is connecting to the structure shall be designed so that the attachment will undergo ductile yielding at a load level corresponding to anchor forces no greater than the design strength of anchors specified in D.3.3.3.

D.3.4 — All provisions for anchor axial tension and shear strength apply to normalweight concrete. When lightweight aggregate concrete is used, provisions for N_n and V_n shall be modified by multiplying all values of $\sqrt{f'_c}$ affecting N_n and V_n, by 0.75 for all-lightweight concrete and 0.85 for sand-lightweight concrete. Linear interpolation shall be permitted when partial sand replacement is used.

D.3.5 — The values of f'_c used for calculation purposes in this appendix shall not exceed 10,000 psi for cast-in anchors, and 8,000 psi for post-installed anchors. Testing is required for post-installed anchors when used in concrete with f'_c greater than 8,000 psi.

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Under seismic conditions, the direction of shear loading may not be predictable. The full shear load should be assumed in any direction for a safe design.

RD.3.3.1 —Section 3.1 of ACI 355.2 specifically states that the seismic test procedures do no simulate the behavior of anchors in plastic hinge zones. The possible higher level of cracking and spalling in plastic hinge zones are beyond the damage states for which Appendix D is applicable.

RD.3.5 — A limited number of tests of cast-in-place and post-installed anchors in high-strength concrete[D.7] indicate that the design procedures contained in this appendix become unconservative, particularly for cast-in anchors, at f'_c = 11,000 to 12,000 psi. Until further tests are available, an upper limit of f'_c = 10,000 psi has been imposed in the design of cast-in-place anchors. This is consistent with Chapters 11 and 12. The companion ACI 355.2 does not require testing of post-installed anchors in concrete with f'_c

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D.4 — General requirements for strength of anchors

D.4.1 — Strength design of anchors shall be based either on computation using design models that satisfy the requirements of D.4.2, or on test evaluation using the 5 percent fractile of test results for the following:

(a) steel strength of anchor in tension (D.5.1);
(b) steel strength of anchor in shear (D.6.1);
(c) concrete breakout strength of anchor in tension (D.5.2);
(d) concrete breakout strength of anchor in shear (D.6.2);
(e) pullout strength of anchor in tension (D.5.3);
(f) concrete side-face blowout strength of anchor in tension (D.5.4); and
(g) concrete pryout strength of anchor in shear (D.6.3).

In addition, anchors shall satisfy the required edge distances, spacings, and thicknesses to preclude splitting failure, as required in D.8.

D.4.1.1 — For the design of anchors, except as required in D.3.3,

$$\phi N_n \geq N_u \quad \text{(D-1)}$$

$$\phi V_n \geq V_u \quad \text{(D-2)}$$

D.4.1.2 — In Eq. (D-1) and (D-2), ϕN_n and ϕV_n are the lowest design strengths determined from all appropriate failure modes. ϕN_n is the lowest design strength in tension of an anchor or group of anchors as determined from consideration of ϕN_s, $\phi n N_{pn}$, either ϕN_{sb} or ϕN_{sbg}, and either ϕN_{cb} or ϕN_{cbg}. ϕV_n is the lowest design strength in shear of an anchor or a group of anchors as determined from consideration of ϕV_s, either ϕV_{cb} or ϕV_{cbg}, and ϕV_{cp}.

D.4.1.3 — When both N_u and V_u are present, interaction effects shall be considered in accordance with D.4.3.

D.4.2 — The nominal strength for any anchor or group of anchors shall be based on design models that result in predictions of strength in substantial agreement with results of comprehensive tests. The materials used in the tests shall be compatible with the materials used in the structure. The nominal strength shall be based on

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greater than 8000 psi because some post-installed anchors may have difficulty expanding in very high-strength concretes. Because of this, f'_c is limited to 8000 psi in the design of post-installed anchors unless testing is performed.

RD.4 — General requirements for strength of anchors

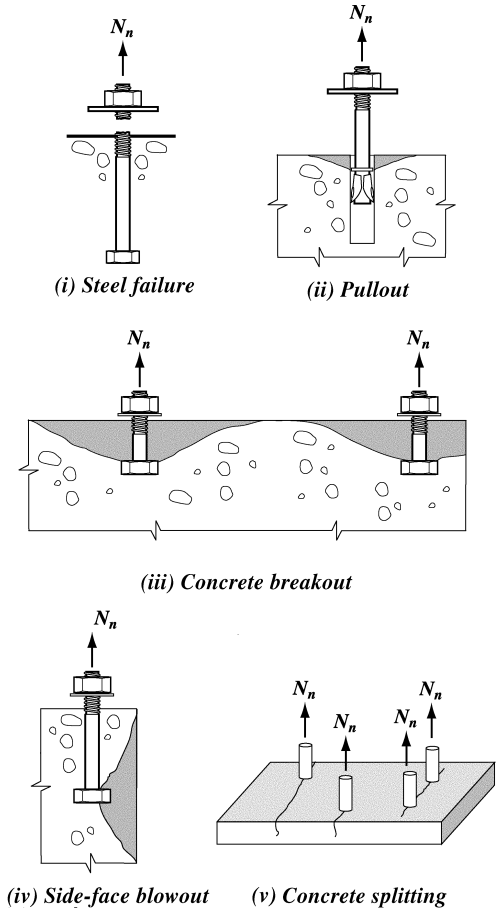
RD.4.1 — This section provides requirements for establishing the strength of anchors to concrete. The various types of steel and concrete failure modes for anchors are shown in Fig. RD.4.1(a) and RD.4.1(b) Comprehensive discussions of anchor failure modes are included in References D.8 to D.10. Any model that complies with the requirements of D.4.2 and D.4.3 can be used to establish the concrete-related strengths. For anchors such as headed bolts, headed studs, and post-installed anchors, the concrete breakout design methods of D.5.2 and D.6.2 are acceptable. The anchor strength is also dependent on the pullout strength of D.5.3, the side-face blowout strength of D.5.4, and the minimum spacings and edge distances of D.8. The design of anchors for tension recognizes that the strength of anchors is sensitive to appropriate installation; installation requirements are included in D.9. Some post-installed anchors are less sensitive to installation errors and tolerances. This is reflected in varied ϕ factors based on the assessment criteria of ACI 355.2.

Test procedures can also be used to determine the single-anchor breakout strength in tension and in shear. The test results, however, are required to be evaluated on a basis statistically equivalent to that used to select the values for the concrete breakout method "considered to satisfy" provisions of D.4.2. The basic strength cannot be taken greater than the 5 percent fractile. The number of tests has to be sufficient for statistical validity and should be considered in the determination of the 5 percent fractile.

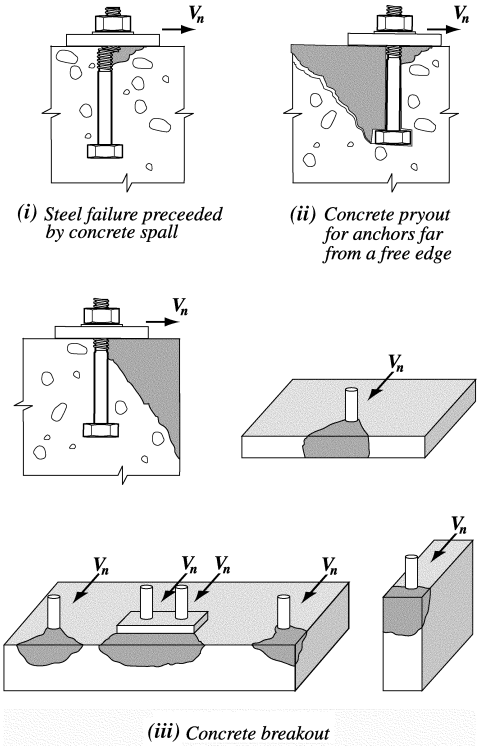
RD.4.2 and RD.4.3 — D.4.2 and D.4.3 establish the performance factors for which anchor design models are required to be verified. Many possible design approaches exist and the user is always permitted to "design by test" using D.4.2 as long as sufficient data are available to verify the model.

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COMMENTARY



(a) tensile loading



(b) shear loading

Fig. RD.4.1 — Failure modes for anchors.

CODE

the 5 percent fractile of the basic individual anchor strength. For nominal strengths related to concrete strength, modifications for size effects, the number of anchors, the effects of close spacing of anchors, proximity to edges, depth of the concrete member, eccentric loadings of anchor groups, and presence or absence of cracking shall be taken into account. Limits on edge distances and anchor spacing in the design models shall be consistent with the tests that verified the model.

D.4.2.1 — The effect of supplementary reinforcement provided to confine or restrain the concrete breakout, or both, shall be permitted to be included in the design models used to satisfy D.4.2.

D.4.2.2 — For anchors with diameters not exceeding 2 in., and tensile embedments not exceeding 25 in. in depth, the concrete breakout strength requirements shall be considered satisfied by the design procedure of D.5.2 and D.6.2.

COMMENTARY

RD.4.2.1 — The addition of supplementary reinforcement in the direction of the load, confining reinforcement, or both, can greatly enhance the strength and ductility of the anchor connection. Such enhancement is practical with cast-in anchors such as those used in precast sections.

The shear strength of headed anchors located near the edge of a member can be significantly increased with appropriate supplementary reinforcement. References D.8, D.11, and D.12 provide substantial information on design of such reinforcement. The effect of such supplementary reinforcement is not included in the ACI 355.2 anchor acceptance tests or in the concrete breakout calculation method of D.5.2 and D.6.2. The designer has to rely on other test data and design theories in order to include the effects of supplementary reinforcement.

For anchors exceeding the limitations of D.4.2.2, or for situations where geometric restrictions limit breakout capacity, or both, reinforcement oriented in the direction of load and proportioned to resist the total load within the breakout prism, and fully anchored on both sides of the breakout planes, may be provided instead of calculating breakout capacity.

The breakout strength of an unreinforced connection can be taken as an indication of the load at which significant cracking will occur. Such cracking can represent a serviceability problem if not controlled. (See RD.6.2.1)

RD.4.2.2 — The method for concrete breakout design included as "considered to satisfy" D.4.2 was developed from the Concrete Capacity Design (CCD) Method,[D.9,D.10] which was an adaptation of the k Method[D.13,D.14] and is considered to be accurate, relatively easy to apply, and capable of extension to irregular layouts. The CCD Method predicts the load capacity of an anchor or group of anchors by using a basic equation for tension, or for shear for a single anchor in cracked concrete, and multiplied by factors that account for the number of anchors, edge distance, spacing, eccentricity and absence of cracking. The limitations on anchor size and embedment length are based on the current range of test data.

The breakout strength calculations are based on a model suggested in the k Method. It is consistent with a breakout prism angle of approximately 35 degrees [Fig. RD.4.2.2(a) and (b)].

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COMMENTARY

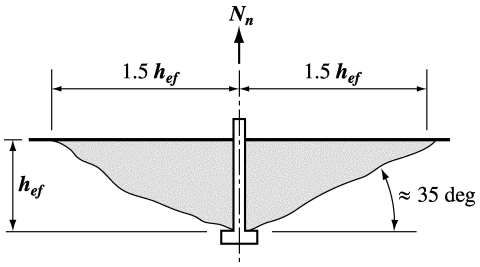


Fig. RD.4.2.2(a)—Breakout cone for tension

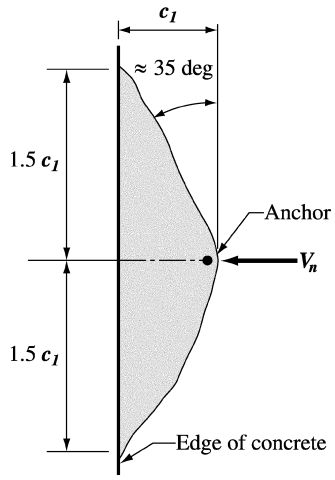


Fig. RD.4.2.2(b)—Breakout cone for shear

D.4.3 — Resistance to combined tensile and shear loads shall be considered in design using an interaction expression that results in computation of strength in substantial agreement with results of comprehensive tests. This requirement shall be considered satisfied by D.7.

D.4.4 — Strength reduction factor ϕ for anchors in concrete shall be as follows when the load combinations of 9.2 are used:

a) Anchor governed by strength of a ductile steel element

i) Tension loads	0.75
ii) Shear loads	0.65

b) Anchor governed by strength of a brittle steel element

i) Tension loads	0.65
ii) Shear loads	0.60

c) Anchor governed by concrete breakout, side-face blowout, pullout, or pryout strength

	Condition A	Condition B
i) Shear loads	0.75	0.70
ii) Tension loads		
Cast-in headed studs, headed bolts, or hooked bolts	0.75	0.70
Post-installed anchors with category as determined from ACI 355.2		

RD.4.4 — The ϕ factors for steel strength are based on using f_{ut} to determine the nominal strength of the anchor (see D.5.1 and D.6.1) rather than f_y as used in the design of reinforced concrete members. Although the ϕ factors for use with f_{ut} appear low, they result in a level of safety consistent with the use of higher ϕ factors applied to f_y. The smaller ϕ factors for shear than for tension do not reflect basic material differences but rather account for the possibility of a non-uniform distribution of shear in connections with multiple anchors. It is acceptable to have a ductile failure of a steel element in the attachment if the attachment is designed so that it will undergo ductile yielding at a load level no greater than 75 percent of the minimum design strength of an anchor (See D.3.3.4). For anchors governed by the more brittle concrete breakout or blowout failure, two conditions are recognized. If supplementary reinforcement is provided to tie the failure prism into the structural member (Condition A), more ductility is present than in the case where such supplementary reinforcement is not present (Condition B). Design of supplementary reinforcement is discussed in RD.4.2.1 and References D.8, D.11, D.12, and D.15. Further discussion of strength reduction factors is presented in RD.4.5.

The ACI 355.2 tests for sensitivity to installation procedures determine the category appropriate for a particular anchoring device. In the ACI 355.2 tests, the effects of variability in anchor torque during installation, tolerance on drilled hole size, energy level used in setting anchors, and for anchors approved for use in cracked concrete, increased

CODE

Category 1 (Low sensitivity to installation and high reliability)	0.75	0.65
Category 2 (Medium sensitivity to installation and medium reliability)	0.65	0.55
Category 3 (High sensitivity to installation and lower reliability)	0.55	0.45

Condition A applies where the potential concrete failure surfaces are crossed by supplementary reinforcement proportioned to tie the potential concrete failure prism into the structural member.

Condition B applies where such supplementary reinforcement is not provided, or where pullout or pryout strength governs.

D.4.5 — Strength reduction factor ϕ for anchors in concrete shall be as follows when the load combinations referenced in Appendix C are used:

a) Anchor governed by strength of a ductile steel element

i) Tension loads	0.80
ii) Shear loads	0.75

b) Anchor governed by strength of a brittle steel element

i) Tension loads	0.70
ii) Shear loads	0.65

c) Anchor governed by concrete breakout, side-face blowout, pullout, or pryout strength

	Condition A	Condition B
i) Shear loads	0.85	0.75
ii) Tension loads		
Cast-in headed studs, headed bolts, or hooked bolts	0.85	0.75
Post-installed anchors with category as determined from ACI 355.2		

COMMENTARY

crack widths are considered. The three categories of acceptable post-installed anchors are:

Category 1 — low sensitivity to installation and high reliability;

Category 2 — medium sensitivity to installation and medium reliability; and

Category 3 — high sensitivity to installation and lower reliability.

The capacities of anchors under shear loads are not as sensitive to installation errors and tolerances. Therefore, for shear calculations of all anchors, ϕ = 0.75 for Condition A and ϕ = 0.70 for Condition B.

RD.4.5 — As noted in R9.1, the 2002 code incorporated the load factors of ASCE 7-98 and the corresponding strength reduction factors provided in the 1999 Appendix C into 9.2 and 9.3, except that the factor for flexure has been increased. Developmental studies for the ϕ factors to be used for Appendix D were based on the 1999 9.2 and 9.3 load and strength reduction factors. The resulting ϕ factors are presented in D.4.5 for use with the load factors of the 2002 Appendix C. The ϕ factors for use with the load factors of the 1999 Appendix C were determined in a manner consistent with the other ϕ factors of the 1999 Appendix C. These ϕ factors are presented in D.4.4 for use with the load factors of 2002 9.2. Since developmental studies for ϕ factors to be used with Appendix D, for brittle concrete failure modes, were performed for the load and strength reduction factors now given in Appendix C, the discussion of the selection of these ϕ factors appears in this section.

Even though the ϕ factor for plain concrete in Appendix C uses a value of 0.65, the basic factor for brittle concrete failures (ϕ = 0.75) was chosen based on results of probabilistic studies[D.16] that indicated the use of ϕ = 0.65 with mean values of concrete-controlled failures produced adequate safety levels. Because the nominal resistance expressions used in this appendix and in the test requirements are based on the 5 percent fractiles, the ϕ = 0.65 value would be overly conservative. Comparison with other design procedures and probabilistic studies[D.16] indicated that the choice of ϕ = 0.75 was justified. For applications with supplementary reinforcement and more ductile failures (Condition A), the ϕ factors are increased. The value of ϕ = 0.85 is compatible with the level of safety for

CODE

Category 1 (Low sensitivity to installation and high reliability)	0.85	0.75
Category 2 (Medium sensitivity to installation and medium reliability)	0.75	0.65
Category 3 (High sensitivity to installation and lower reliability)	0.65	0.55

Condition A applies where the potential concrete failure surfaces are crossed by supplementary reinforcement proportioned to tie the potential concrete failure prism into the structural member.

Condition B applies where such supplementary reinforcement is not provided, or where pullout or pryout strength governs.

D.5 — Design requirements for tensile loading

D.5.1 — Steel strength of anchor in tension

D.5.1.1 — The nominal strength, N_s, of an anchor in tension as governed by the steel shall be evaluated by calculations based on the properties of the anchor material and the physical dimensions of the anchor.

D.5.1.2 — The nominal strength N_s of an anchor or group of anchors in tension shall not exceed:

$$N_s = nA_{se} f_{ut} \quad \text{(D-3)}$$

where f_{ut} shall not be taken greater than $1.9f_y$ or 125,000 psi.

COMMENTARY

shear failures in concrete beams, and has been recommended in the PCI Design Handbook[D.17] and by ACI 349.[D.15]

RD.5 — Design requirements for tensile loading

RD.5.1 — Steel strength of anchor in tension

RD.5.1.2 — The nominal tension strength of anchors is best represented by $A_{se} f_{ut}$ rather than $A_{se} f_y$ because the large majority of anchor materials do not exhibit a well-defined yield point. The American Institute of Steel Construction (AISC) has based tension strength of anchors on $A_{se} f_{ut}$ since the 1986 edition of their specifications. The use of Eq. (D-3) with 9.2 load factors and the f factors of D.4.4 give design strengths consistent with the AISC Load and Resistance Factor Design Specifications.[D.18]

The limitation of $1.9f_y$ on f_{ut} is to ensure that under service load conditions the anchor does not exceed f_y. The limit on f_{ut} of $1.9f_y$ was determined by converting the LRFD provisions to corresponding service level conditions. For Section 9.2, the average load factor of 1.4 (from $1.2D + 1.6L$) divided by the highest f factor (0.75 for tension) results in a limit of f_{ut}/f_y of 1.4/0.75 = 1.87. For Appendix C, the average load factor of 1.55 (from $1.4D + 1.7L$), divided by the highest f factor (0.80 for tension), results in a limit of f_{ut}/f_y of 1.55/0.8 = 1.94. For consistent results, the serviceability limitation of f_{ut} was taken as $1.9f_y$. If the ratio of f_{ut} to f_y

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D.5.2 — Concrete breakout strength of anchor in tension

D.5.2.1 — The nominal concrete breakout strength, N_{cb} or N_{cbg}, of an anchor or group of anchors in tension shall not exceed

for a single anchor:

$$N_{cb} = \frac{A_N}{A_{No}} \psi_2 \psi_3 N_b \qquad \text{(D-4)}$$

COMMENTARY

exceeds this value, the anchoring may be subjected to service loads above f_y under service loads. Although not a concern for standard structural steel anchors (maximum value of f_{ut}/f_y is 1.6 for ASTM A 307), the limitation is applicable to some stainless steels.

RD.5.2 — Concrete breakout strength of anchor in tension

RD.5.2.1 — The effects of multiple anchors, spacing of anchors, and edge distance on the nominal concrete breakout strength in tension are included by applying the modification factors A_N / A_{No} and ψ_2 in Eq. (D-4) and (D-5).

Figure RD.5.2.1(a) shows A_{No} and the development of Eq. (D-6). A_{No} is the maximum projected area for a single anchor. Figure RD.5.2.1(b) shows examples of the projected areas for various single-anchor and multiple-anchor arrangements. Because A_N is the total projected area for a group of anchors, and A_{No} is the area for a single anchor, there is no need to include n, the number of anchors, in Eq. (D-4) or

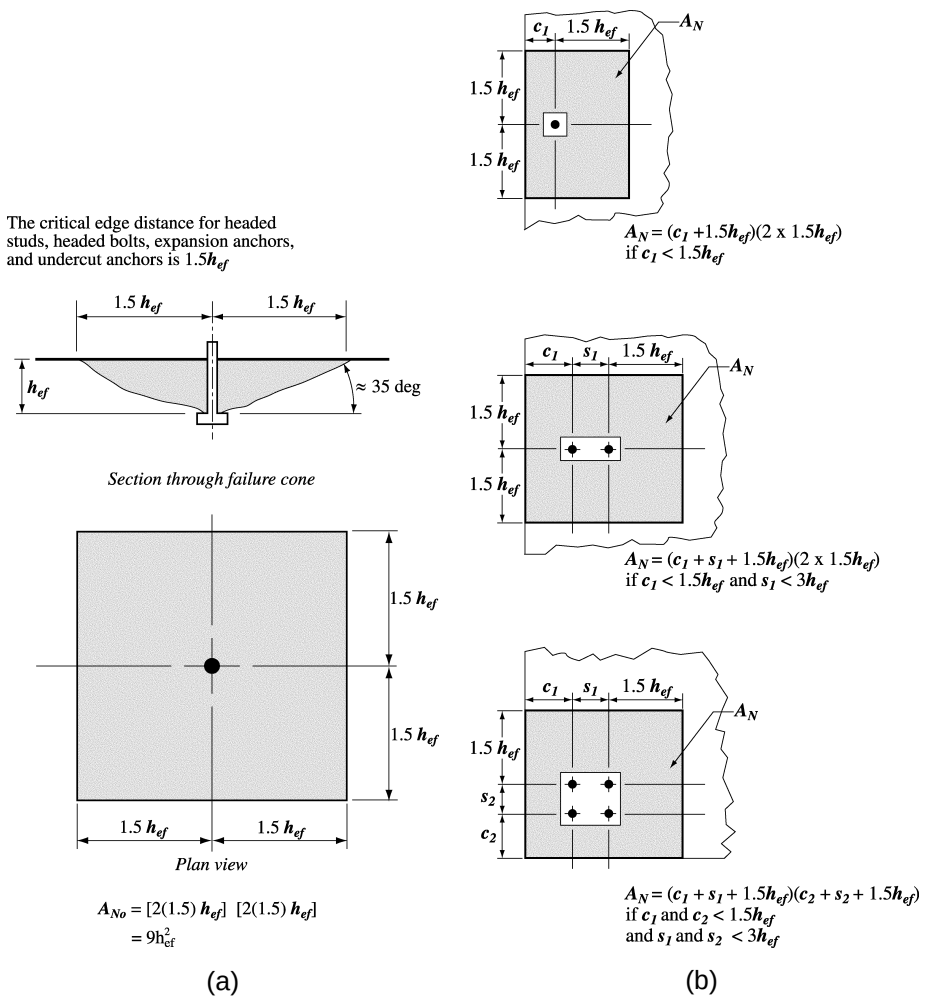


Fig. RD.5.2.1(a) — Calculation of $\mathbf{A_{No}}$*; and (b) projected areas for single anchors and groups of anchors and calculation of* $\mathbf{A_N}$

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for a group of anchors:

$$N_{cbg} = \frac{A_N}{A_{No}} \psi_1 \psi_2 \psi_3 N_b \quad \text{(D-5)}$$

A_N is the projected area of the failure surface for the anchor or group of anchors that shall be approximated as the base of the rectilinear geometrical figure that results from projecting the failure surface outward $1.5h_{ef}$ from the centerlines of the anchor, or in the case of a group of anchors, from a line through a row of adjacent anchors. A_N shall not exceed nA_{No}, where n is the number of tensioned anchors in the group. A_{No} is the projected area of the failure surface of a single anchor remote from edges:

$$A_{No} = 9h_{ef}^2 \quad \text{(D-6)}$$

D.5.2.2 — The basic concrete breakout strength N_b of a single anchor in tension in cracked concrete shall not exceed

$$N_b = k\sqrt{f'_c}\, h_{ef}^{1.5} \quad \text{(D-7)}$$

where
k = 24 for cast-in anchors; and
k = 17 for post-installed anchors.

Alternatively, for cast-in headed studs and headed bolts with 11 in. ≤ h_{ef} ≤ 25 in., the basic concrete breakout strength of a single anchor in tension in cracked concrete shall not exceed

$$N_b = 16\sqrt{f'_c}\, h_{ef}^{5/3} \quad \text{(D-8)}$$

D.5.2.3 — For the special case of anchors in an application with three or four edges along with the largest edge distance c_{max} £ $1.5h_{ef}$, the embedment depth h_{ef} used in Eq. (D-6) through (D-11) shall be limited to $c_{max}/1.5$.

COMMENTARY

(D-5). If anchor groups are positioned in such a way that their projected areas overlap, the value of A_N is required to be reduced accordingly.

RD.5.2.2 — The basic equation for anchor capacity was derived[D.9-D.11,D.14] assuming a concrete failure prism with an angle of about 35 degrees, considering fracture mechanics concepts.

The values of k in Eq. (D-7) were determined from a large database of test results in uncracked concrete[D.9] at the 5 percent fractile. The values were adjusted to corresponding k values for cracked concrete.[D.10,D.19] Higher k values for post-installed anchors may be permitted, provided they have been determined from product approval testing in accordance with ACI 355.2. When using k values from ACI 355.2 product approval reports, ψ_3 shall be taken as 1.0 because the published test results of the ACI 355.2 product approval tests provide specific k values for cracked or uncracked concrete. For anchors with a deep embedment (h_{ef} > 11 in.), some test evidence indicates the use of $h_{ef}^{1.5}$ can be overly conservative for some cases. Often such tests have been with selected aggregates for special applications. An alternative expression (Eq. (D-8)) is provided using $h_{ef}^{5/3}$ for evaluation of cast-in anchors with 11 in. ≤ h_{ef} ≤ 25 in. The limit of 25 in. corresponds to the upper range of test data. This expression can also be appropriate for some undercut post-installed anchors. D.4.2, however, should be used with test results to justify such applications.

RD.5.2.3 — For anchors influenced by three or more edges where any edge distance is less than $1.5h_{ef}$, the tensile breakout strength computed by the ordinary CCD Method, which is the basis for Eq. (D-7) and (D-8), gives misleading results. This occurs because the ordinary definitions of A_N/A_{No} do not correctly reflect the edge effects. If the value of h_{ef} is limited to $c_{max}/1.5$, however, where c_{max} is the largest of the influencing edge distances that are less than or equal to the actual $1.5h_{ef}$, this problem is corrected. As shown by Lutz,[D.20] this limiting value of h_{ef} is to be used in Eq. (D-6)

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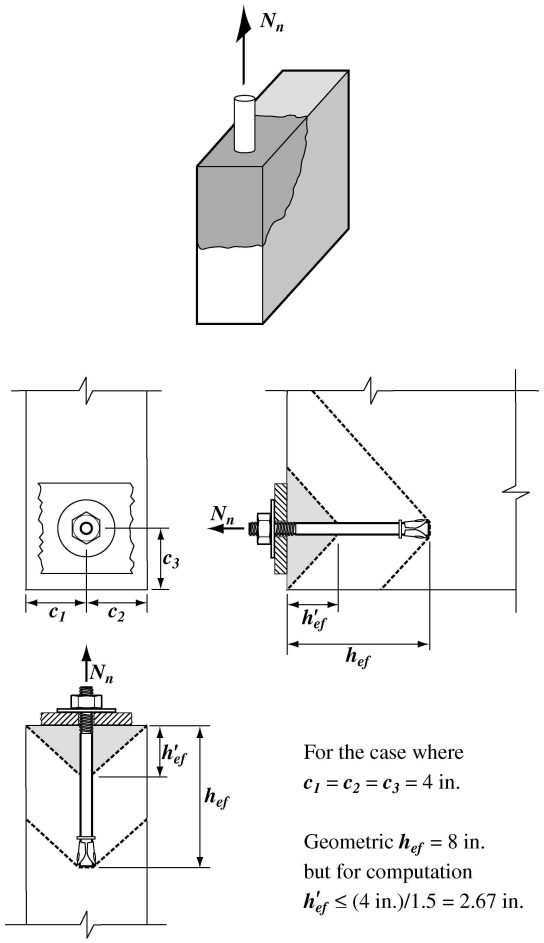


Fig. RD.5.2.3 — Failure surfaces in narrow members for different embedment depths.

to (D-11). This approach is best understood when applied to an actual case. Figure RD.5.2.3 shows how the failure surface has the same area for any embedment beyond the proposed limit on h_{ef} (taken as h'_{ef} in the figure). In this example, the proposed limit on the value of h_{ef} to be used in the computations where $h_{ef} = c_{max}/1.5$, results in $h_{ef} = h'_{ef}$ = 4 in./1.5 = 2.67 in. For this example, this would be the proper value to be used for h_{ef} in computing the resistance even if the actual embedment depth is larger.

D.5.2.4 — The modification factor for eccentrically loaded anchor groups is:

$$\psi_1 = \frac{1}{\left(1 + \frac{2e'_N}{3h_{ef}}\right)} \le 1 \qquad \text{(D-9)}$$

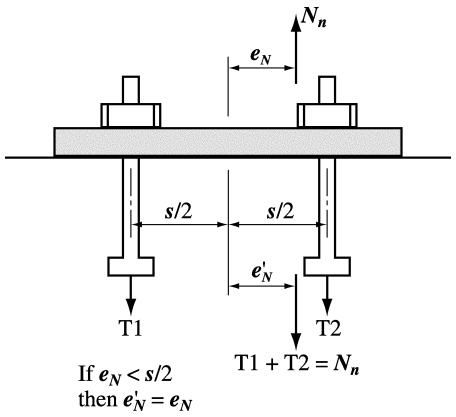
Eq. (D-9) is valid for $e'_N \le s/2$.

If the loading on an anchor group is such that only some anchors are in tension, only those anchors that

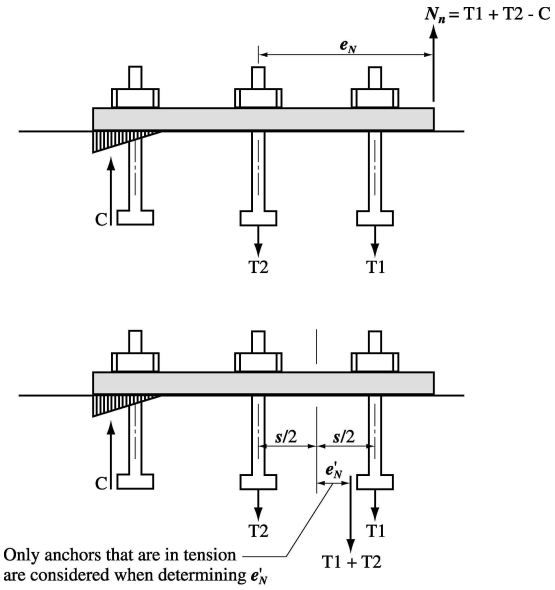
RD.5.2.4 — Figure RD.5.2.4(a) shows dimension $e'_N = e_N$ for a group of anchors that are all in tension but that have a resultant force eccentric with respect to the centroid of the anchor group. Groups of anchors can be loaded in such a way that only some of the anchors are in tension [Fig. RD.5.2.4(b)]. In this case, only the anchors in tension are to be considered in the determination of e'_N. The anchor loading has to be determined as the resultant anchor tension at an eccentricity with respect to the center of gravity of the anchors in tension. Equation (D-9) is limited to cases where $e'_N \le s/2$ to alert the designer that all anchors may not be in tension.

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COMMENTARY



(a) When all anchors in a group are in tension



(b) When only some anchors in a group are in tension

Fig. RD.5.2.4 — Definition of dimension $\mathbf{e'_N}$ *for group anchors.*

are in tension shall be considered when determining the eccentricity e'_N for use in Eq. (D-9).

In the case where eccentric loading exists about two axes, the modification factor, y_1, shall be computed for each axis individually and the product of these factors used as y_1 in Eq. (D-5).

D.5.2.5 — The modification factor for edge effects is:

$$y_2 = 1 \text{ if } c_{min} \geq 1.5h_{ef} \quad \text{(D-10)}$$

$$y_2 = 0.7 + 0.3\frac{c_{min}}{1.5h_{ef}} \text{ if } c_{min} < 1.5h_{ef} \quad \text{(D-11)}$$

RD.5.2.5 — If anchors are located close to an edge so that there is not enough space for a complete breakout prism to develop, the load-bearing capacity of the anchor is further reduced beyond that reflected in A_N/A_{No}. If the smallest side cover distance is greater than $1.5h_{ef}$, a complete prism can form and there is no reduction ($y_2 = 1$). If the side cover is less than $1.5h_{ef}$, the factor y_2 is required to adjust for the edge effect.[D.9]

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D.5.2.6 — When an anchor is located in a region of a concrete member where analysis indicates no cracking ($f_t < f_r$) at service load levels, the following modification factor shall be permitted:

ψ_3 = 1.25 for cast-in anchors

ψ_3 = 1.4 for post-installed anchors.

When analysis indicates cracking at service load levels, ψ_3 shall be taken as 1.0 for both cast-in anchors and post-installed anchors. Post-installed anchors shall be qualified for use in cracked concrete in accordance with ACI 355.2. The cracking in the concrete shall be controlled by flexural reinforcement distributed in accordance with 10.6.4, or equivalent crack control shall be provided by confining reinforcement.

D.5.2.7 — When an additional plate or washer is added at the head of the anchor, it shall be permitted to calculate the projected area of the failure surface by projecting the failure surface outward $1.5h_{ef}$ from the effective perimeter of the plate or washer. The effective perimeter shall not exceed the value at a section projected outward more than t from the outer edge of the head of the anchor, where t is the thickness of the washer or plate.

D.5.3 — Pullout strength of anchor in tension

D.5.3.1 — The nominal pullout strength N_{pn} of an anchor in tension shall not exceed:

$$N_{pn} = \psi_4 N_p \quad \text{(D-12)}$$

D.5.3.2 — For post-installed expansion and undercut anchors, the values of N_p shall be based on the 5 percent fractile of results of tests performed and evaluated according to ACI 355.2. It is not permissible to calculate the pullout strength in tension for such anchors.

D.5.3.3 — For single cast-in headed studs and headed bolts, it shall be permitted to evaluate the pullout strength in tension using D.5.3.4. For single J- or L-bolts, it shall be permitted to evaluate the pullout strength in tension using D.5.3.5. Alternatively, it shall be permitted to use values of N_p based on the 5 percent fractile of tests performed and evaluated in the same manner as the ACI 355.2 procedures but without the benefit of friction.

COMMENTARY

RD.5.2.6 — Post-installed and cast-in anchors that have not met the requirements for use in cracked concrete according to ACI 355.2 should be used in uncracked regions only. The analysis for the determination of crack formation should include the effects of restrained shrinkage (see 7.12.1.2). The anchor qualification tests of ACI 355.2 require that anchors in cracked concrete zones perform well in a crack that is 0.012 in. wide. If wider cracks are expected, confining reinforcement to control the crack width to about 0.012 in. should be provided.

The concrete breakout strengths given by Eq. (D-7) and (D-8) assume cracked concrete (that is, ψ_3 = 1.0) with $\psi_3 k$ = 24 for cast-in-place, and 17 for post-installed (cast-in 40 percent higher). When the uncracked concrete ψ_3 factors are applied (1.25 for cast-in, and 1.4 for post-installed), the results are $\psi_3 k$ factors of 30 for cast-in and 24 for post-installed (25 percent higher for cast-in). This agrees with field observations and tests that show cast-in anchor strength exceeds that of post-installed for both cracked and uncracked concrete.

RD.5.3 — Pullout strength of anchor in tension

RD.5.3.2 — The pullout strength equations given in D.5.3.4 and D.5.3.5 are only applicable to cast-in headed and hooked anchors;[D.8,D.21] they are not applicable to expansion and undercut anchors that use various mechanisms for end anchorage unless the validity of the pullout strength equations are verified by tests.

RD.5.3.3 — The pullout strength in tension of headed studs or headed bolts can be increased by providing confining reinforcement, such as closely spaced spirals, throughout the head region. This increase can be demonstrated by tests.

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D.5.3.4 — The pullout strength in tension of a single headed stud or headed bolt N_p for use in Eq. (D-12), shall not exceed:

$$N_p = A_{brg}8f_c' \quad \text{(D-13)}$$

D.5.3.5 — The pullout strength in tension of a single hooked bolt N_p for use in Eq. (D-12) shall not exceed:

$$N_p = 0.9f_c' e_h d_o \quad \text{(D-14)}$$

where: $3d_o \le e_h \le 4.5d_o$.

D.5.3.6 — For an anchor located in a region of a concrete member where analysis indicates no cracking ($f_t < f_r$) at service load levels, the following modification factor shall be permitted

$$y_4 = 1.4$$

Otherwise, y_4 shall be taken as 1.0.

D.5.4 — Concrete side-face blowout strength of a headed anchor in tension

D.5.4.1 — For a single headed anchor with deep embedment close to an edge ($c < 0.4h_{ef}$), the nominal side-face blowout strength N_{sb} shall not exceed:

$$N_{sb} = 160c\sqrt{A_{brg}}\sqrt{f_c'} \quad \text{(D-15)}$$

If the single headed anchor is located at a perpendicular distance, c_2, less than $3c$ from an edge, the value of N_{sb} shall be multiplied by the factor $(1 + c_2/c)/4$ where $1 \le c_2/c \le 3$.

D.5.4.2 — For multiple headed anchors with deep embedment close to an edge ($c < 0.4h_{ef}$) and spacing between anchors less than $6c$, the nominal strength of the group of anchors for a side-face blowout failure N_{sbg} shall not exceed:

$$N_{sbg} = \left(1 + \frac{s_o}{6c}\right)N_{sb} \quad \text{(D-16)}$$

where s_o = spacing of the outer anchors along the edge in the group; and N_{sb} is obtained from Eq. (D-15) without modification for a perpendicular edge distance.

COMMENTARY

RD.5.3.4 — Equation (D-13) corresponds to the load at which the concrete under the anchor head begins to crush.[D.8.D.15] It is not the load required to pull the anchor completely out of the concrete, so the equation contains no term relating to embedment depth. The designer should be aware that local crushing under the head will greatly reduce the stiffness of the connection, and generally will be the beginning of a pullout failure.

RD.5.3.5 — Equation (D-14) for hooked bolts was developed by Lutz based on the results of Reference D.21. Reliance is placed on the bearing component only, neglecting any frictional component because crushing inside the hook will greatly reduce the stiffness of the connection, and generally will be the beginning of pullout failure. The limits on e_h are based on the range of variables used in the three tests programs reported in Reference D.21.

RD.5.4 — Concrete side-face blowout strength of a headed anchor in tension

The design requirements for side-face blowout are based on the recommendations of Reference D.22. These requirements are applicable to headed anchors that usually are cast-in anchors. Splitting during installation rather than side-face blowout generally governs post-installed anchors, and is evaluated by the ACI 355.2 requirements.

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COMMENTARY

D.6 — Design requirements for shear loading

RD.6 — Design requirements for shear loading

D.6.1 — Steel strength of anchor in shear

RD.6.1 — Steel strength of anchor in shear

D.6.1.1 — The nominal strength of an anchor in shear as governed by steel V_s shall be evaluated by calculations based on the properties of the anchor material and the physical dimensions of the anchor.

D.6.1.2 — The nominal strength V_s of an anchor or group of anchors in shear shall not exceed (a) through (c):

(a) for cast-in headed stud anchors

$$V_s = nA_{se} f_{ut} \quad \text{(D-17)}$$

where f_{ut} shall not be taken greater than $1.9f_y$ or 125,000 psi.

(b) for cast-in headed bolt and hooked bolt anchors

$$V_s = n0.6A_{se} f_{ut} \quad \text{(D-18)}$$

where f_{ut} shall not be taken greater than $1.9f_y$ or 125,000 psi.

(c) for post-installed anchors

$$V_s = n(0.6A_{se} f_{ut} + 0.4A_{sl} f_{utsl}) \quad \text{(D-19)}$$

where f_{ut} shall not be taken greater than $1.9f_y$ or 125,000 psi.

D.6.1.3 — Where anchors are used with built-up grout pads, the nominal strengths of D.6.1.2 shall be multiplied by a 0.80 factor.

RD.6.1.2 — The nominal shear strength of anchors is best represented by $A_{se} f_{ut}$ for headed stud anchors and $0.6A_{se} f_{ut}$ for other anchors rather than a function of $A_{se} f_y$ because typical anchor materials do not exhibit a well-defined yield point. The use of Eq. (D-17) and (D-18) with 9.2 load factors and the f factors of D.4.4 give design strengths consistent with the AISC Load and Resistance Factor Design Specifications.[D.18]

The limitation of $1.9f_y$ on f_{ut} is to ensure that under service load conditions the anchor stress does not exceed f_y. The limit on f_{ut} of $1.9f_y$ was determined by converting the LRFD provisions to corresponding service level conditions as discussed in RD.5.1.2.

D.6.2 — Concrete breakout strength of anchor in shear

RD.6.2 — Concrete breakout strength of anchor in shear

D.6.2.1 — The nominal concrete breakout strength, V_{cb} or V_{cbg}, in shear of an anchor or group of anchors shall not exceed:

(a) for shear force perpendicular to the edge on a single anchor:

$$V_{cb} = \frac{A_V}{A_{Vo}} y_6 y_7 V_b \quad \text{(D-20)}$$

(b) for shear force perpendicular to the edge on a group of anchors:

RD.6.2.1 — The shear strength equations were developed from the CCD method. They assume a breakout cone angle of approximately 35 degrees (See Fig. RD.4.2.2(b)), and consider fracture mechanics theory. The effects of multiple anchors, spacing of anchors, edge distance, and thickness of the concrete member on nominal concrete breakout strength in shear are included by applying the reduction factor A_V/A_{Vo} in Eq. (D-20) and (D-21), and y_5 in Eq. (D-21). For anchors far from the edge, D.6.2 usually will not govern. For these cases, D.6.1 and D.6.3 often govern.

Fig. RD.6.2.1(a) shows A_{Vo} and the development of Eq. (D-22). A_{Vo} is the maximum projected area for a single

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$$V_{cbg} = \frac{A_V}{A_{Vo}} y_5 y_6 y_7 V_b \quad \text{(D-21)}$$

(c) for shear force parallel to an edge, V_{cb} or V_{cbg} shall be permitted to be twice the value for shear force determined from Eq. (D-20) or (D-21), respectively, with y_6 taken equal to 1.

(d) for anchors located at a corner, the limiting nominal concrete breakout strength shall be determined for each edge, and the minimum value shall be used.

V_b is the basic concrete breakout strength value for a single anchor. A_V is the projected area of the failure surface on the side of the concrete member at its edge for a single anchor or a group of anchors. It shall be permitted to evaluate this area as the base of a truncated half pyramid projected on the side face of the member where the top of the half pyramid is given by the axis of the anchor row selected as critical. The value of c_1 shall be taken as the distance from the edge to this axis. A_V shall not exceed nA_{Vo}, where n is the number of anchors in the group.

A_{Vo} is the projected area for a single anchor in a deep member and remote from edges in the direction perpendicular to the shear force. It shall be permitted to evaluate this area as the base of a half pyramid with a side length parallel to the edge of $3c_1$ and a depth of $1.5c_1$:

$$A_{Vo} = 4.5(c_1)^2 \quad \text{(D-22)}$$

Where anchors are located at varying distances from the edge and the anchors are welded to the attachment so as to distribute the force to all anchors, it shall be permitted to evaluate the strength based on the distance to the farthest row of anchors from the edge. In this case, it shall be permitted to base the value of c_1 on the distance from the edge to the axis of the farthest anchor row that is selected as critical, and all of the shear shall be assumed to be carried by this critical anchor row alone.

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anchor that approximates the surface area of the full breakout prism or cone for an anchor unaffected by edge distance, spacing or depth of member. Fig. RD.6.2.1(b) shows examples of the projected areas for various single anchor and multiple anchor arrangements. A_V approximates the full surface area of the breakout cone for the particular arrangement of anchors. Because A_V is the total projected area for a group of anchors, and A_{Vo} is the area for a single anchor, there is no need to include the number of anchors in the equation.

The assumption shown in the upper right example of Fig. RD.6.2.1(b), with the case for two anchors perpendicular to the edge, is a conservative interpretation of the distribution of the shear force on an elastic basis. If the anchors are welded to a common plate, when the anchor nearest the front edge begins to form a failure cone, shear load would be transferred to the stiffer and stronger rear anchor. For cases where nominal strength is not controlled by ductile steel elements, D.3.1 requires that load effects be determined by elastic analysis. The PCI Design Handbook approach[D.17] suggests in Section 6.5.2.2 that the increased capacity of the anchors away from the edge be considered. Because this is a reasonable approach, assuming that the anchors are spaced far enough apart so that the shear failure surfaces do not intersect,[D.11] D.6.2 allows such a procedure. If the failure surfaces do not intersect, as would generally occur if the anchor spacing s is equal to or greater than $1.5c_1$, then after formation of the near-edge failure surface, the higher capacity of the farther anchor would resist most of the load. As shown in the bottom right example in Fig. RD.6.2.1(b), it would be appropriate to consider the full shear capacity to be provided by this anchor with its much larger resisting failure surface. No contribution of the anchor near the edge is then considered. Checking the near-edge anchor condition is advisable to preclude undesirable cracking at service load conditions. Further discussion of design for multiple anchors is given in Reference D.8.

For the case of anchors near a corner subjected to a shear force with components normal to each edge, a satisfactory solution is to check independently the connection for each component of the shear force. Other specialized cases, such as the shear resistance of anchor groups where all anchors do not have the same edge distance, are treated in Reference D.11.

The detailed provisions of D.6.2.1(a) apply to the case of shear force directed towards an edge. When the shear force is directed away from the edge, the strength will usually be governed by D.6.1 or D.6.3.

The case of shear force parallel to an edge is shown in Fig. RD.6.2.1(c). A special case can arise with shear force parallel to the edge near a corner. In the example of a single anchor near a corner (See Fig. RD.6.2.1(d)), where the edge distance to the side c_2 is 40 percent or more of the distance

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D.6.2.2 — The basic concrete breakout strength V_b in shear of a single anchor in cracked concrete shall not exceed

$$V_b = 7\left(\frac{\ell}{d_o}\right)^{0.2}\sqrt{d_o}\sqrt{f'_c}(c_1)^{1.5} \quad \text{(D-23)}$$

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c_1 in the direction of the load, the shear strength parallel to that edge can be computed directly from Eq. (D-20) and (D-21) using c_1 in the direction of the load.

RD.6.2.2 — Like the concrete breakout tensile capacity, the concrete breakout shear capacity does not increase with the failure surface, which is proportional to $(c_1)^2$. Instead the capacity increases proportionally to $(c_1)^{1.5}$ due to size effect. The capacity is also influenced by the anchor stiffness and the anchor diameter.[D.9-D.11,D.14]

The constant, 7, in the shear strength equation was determined from test data reported in Reference D.9 at the 5 percent fractile adjusted for cracking.

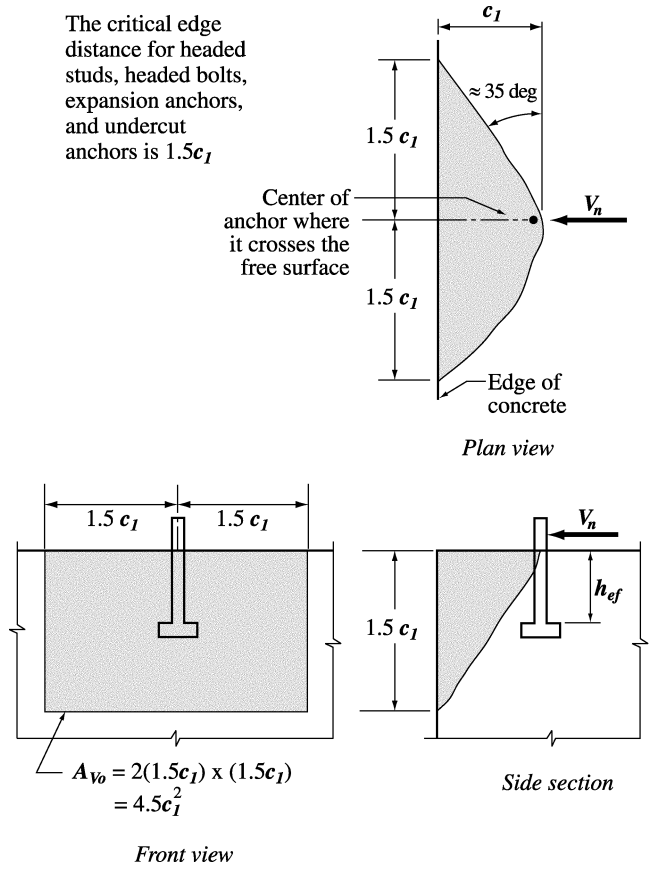


Fig. RD.6.2.1(a)—Calculation of A_{Vo}

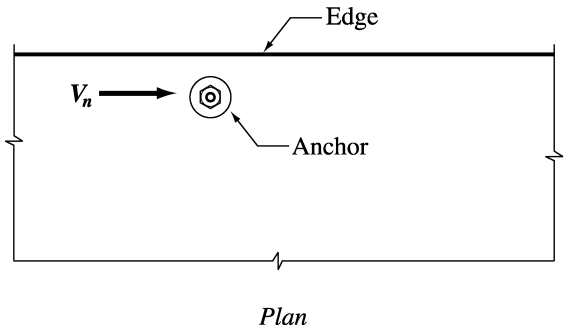


Fig. RD.6.2.1(c) — Shear force parallel to an edge

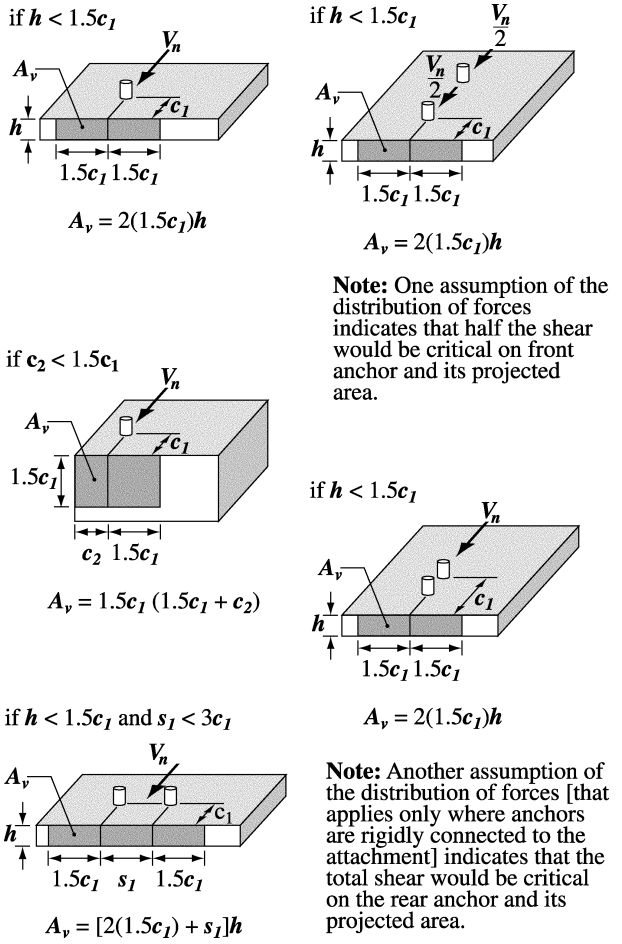


Fig. RD.6.2.1(b) — Projected area for single anchors and groups of anchors and calculation of A_v

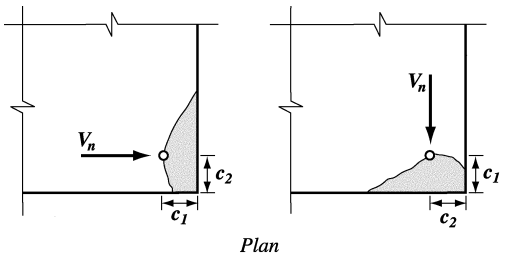


Fig. RD.6.2.1(d) — Shear force near a corner.

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D.6.2.3 — For cast-in headed studs, headed bolts, or hooked bolts that are continuously welded to steel attachments having a minimum thickness equal to the greater of 3/8 in. or half of the anchor diameter, the basic concrete breakout strength V_b in shear of a single anchor in cracked concrete shall not exceed

$$V_b = 8\left(\frac{\ell}{d_o}\right)^{0.2}\sqrt{d_o}\sqrt{f'_c}(c_1)^{1.5} \quad \text{(D-24)}$$

provided that:

(a) for groups of anchors, the strength is determined based on the strength of the row of anchors farthest from the edge;

(b) the center-to-center spacing of the anchors is not less than 2.5 in.; and

(c) supplementary reinforcement is provided at the corners if $c_2 \le 1.5h_{ef}$.

D.6.2.4 — For the special case of anchors influenced by three or more edges, the edge distance c_1 used in Eq. (D-22), (D-23), (D-24), (D-25), (D-26) and (D-27) shall be limited to $h/1.5$.

D.6.2.5 — The modification factor for eccentrically loaded anchor groups is

$$\psi_5 = \frac{1}{1 + \frac{2e'_v}{3c_1}} \le 1 \quad \text{(D-25)}$$

Equation (D-25) is valid for $e'_v \le s/2$.

D.6.2.6 — The modification factor for edge effect is

$$\psi_6 = 1.0 \quad \text{(D-26)}$$

if $c_2 \ge 1.5c_1$

$$\psi_6 = 0.7 + 0.3\frac{c_2}{1.5c_1} \quad \text{(D-27)}$$

if $c_2 < 1.5c_1$

COMMENTARY

RD.6.2.3 — For the special case of cast-in headed bolts continuously welded to an attachment, test data[D.23,D.24] show that somewhat higher shear capacity exists, possibly due to the stiff welding connection clamping the bolt more effectively than an attachment with an anchor gap. Because of this, the basic shear value for such anchors is increased. Limits are imposed to ensure sufficient rigidity. The design of supplementary reinforcement is discussed in References D.8, D.11, and D.12.

RD.6.2.4 — For anchors influenced by three or more edges where any edge distance is less than $1.5c_1$, the shear breakout strength computed by the basic CCD Method, which is the basis for Eq. (D-23) and (D-24), gives safe but misleading results. These special cases were studied for the κ Method[D.14] and the problem was pointed out by Lutz.[D.20] Similar to the approach used for tensile breakouts in D.5.2.3, a correct evaluation of the capacity is determined if the value of c_1 to be used in Eq. (D-22) to (D-27) is limited to $h/1.5$.

RD.6.2.5 — This section provides a modification factor for an eccentric shear force towards an edge on a group of anchors. If the shear load originates above the plane of the concrete surface, the shear should first be resolved as a shear in the plane of the concrete surface, with a moment that may or may not also cause tension in the anchors, depending on the normal force. Figure RD.6.2.5 defines the term e'_v for calculating the ψ_5 modification factor that accounts for the fact that more shear is applied on one anchor than the other, tending to split the concrete near an edge. If $e'_v \le s/2$, the CCD procedure is not applicable.

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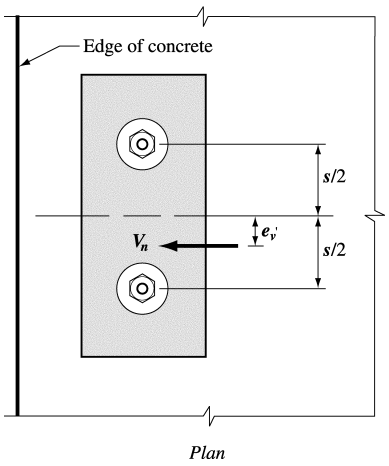


Fig. RD.6.2.5 — Definition of dimensions e'_V.

D.6.2.7 — For anchors located in a region of a concrete member where analysis indicates no cracking ($f_t < f_r$) at service loads, the following modification factor shall be permitted

$$y_7 = 1.4$$

For anchors located in a region of a concrete member where analysis indicates cracking at service load levels, the following modification factors shall be permitted:

y_7 **= 1.0** for anchors in cracked concrete with no supplementary reinforcement or edge reinforcement smaller than a No. 4 bar;

y_7 **= 1.2** for anchors in cracked concrete with supplementary reinforcement of a No. 4 bar or greater between the anchor and the edge; and

y_7 **= 1.4** for anchors in cracked concrete with supplementary reinforcement of a No. 4 bar or greater between the anchor and the edge, and with the supplementary reinforcement enclosed within stirrups spaced at not more than 4 in.

RD.6.2.7 — Torque-controlled and displacement-controlled expansion anchors are permitted in cracked concrete under pure shear loadings.

D.6.3 — Concrete pryout strength of anchor in shear

D.6.3.1 — The nominal pryout strength V_{cp} shall not exceed

$$V_{cp} = k_{cp}N_{cb} \quad \text{(D-28)}$$

where

k_{cp} = 1.0 for $h_{ef} < 2.5$ in.
k_{cp} = 2.0 for $h_{ef} \geq 2.5$ in.

and N_{cb} shall be determined from Eq. (D-4), lb.

RD.6.3 — Concrete pryout strength of anchor in shear

Reference D.9 indicates that the pryout shear resistance can be approximated as one to two times the anchor tensile resistance with the lower value appropriate for h_{ef} less than 2.5 in.

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COMMENTARY

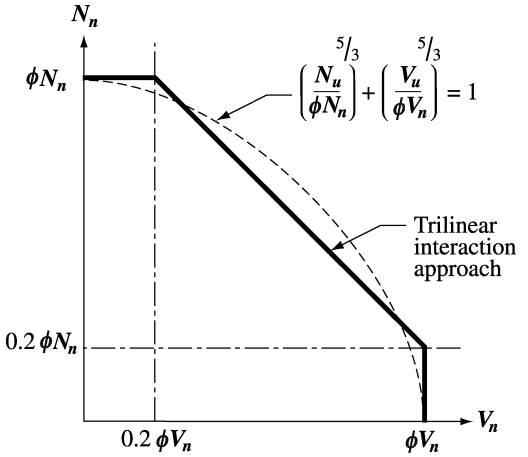


Fig. RD.7 — Shear and tensile load interaction equation.

D.7 — Interaction of tensile and shear forces

Unless determined in accordance with D.4.3, anchors or groups of anchors that are subjected to both shear and axial loads shall be designed to satisfy the requirements of D.7.1 through D.7.3. The value of ϕN_n shall be as required in D.4.1.2. The value of ϕV_n shall be as defined in D.4.1.2.

D.7.1 — If $V_u \le 0.2\phi V_n$, then full strength in tension shall be permitted: $\phi N_n \ge N_u$.

D.7.2 — If $N_u \le 0.2\phi N_n$, then full strength in shear shall be permitted: $\phi V_n \ge V_u$.

D.7.3 — If $V_u > 0.2\phi V_n$ and $N_u > 0.2\phi N_n$, then:

$$\frac{N_u}{\phi N_n} + \frac{V_u}{\phi V_n} \le 1.2 \qquad \text{(D-29)}$$

RD.7 — Interaction of tensile and shear forces

The shear-tension interaction expression has traditionally been expressed as:

$$\left(\frac{N_u}{N_n}\right)^{\alpha} + \left(\frac{V_u}{V_n}\right)^{\alpha} \le 1.0$$

where α varies from 1 to 2. The current trilinear recommendation is a simplification of the expression where $\alpha = 5/3$ (Fig. RD.7). The limits were chosen to eliminate the requirement for computation of interaction effects where very small values of the second force are present. Any other interaction expression that is verified by test data, however, can be used to satisfy D.4.3.

D.8 — Required edge distances, spacings, and thicknesses to preclude splitting failure

Minimum spacings and edge distances for anchors and minimum thicknesses of members shall conform to D.8.1 through D.8.5, unless supplementary reinforcement is provided to control splitting. Lesser values from product-specific tests performed in accordance with ACI 355.2 shall be permitted.

D.8.1 — Unless determined in accordance with D.8.4, minimum center-to-center spacing of anchors shall be $4d_o$ for untorqued cast-in anchors, and $6d_o$ for torqued cast-in anchors and post-installed anchors.

RD.8 — Required edge distances, spacings, and thicknesses to preclude splitting failure

The minimum spacings, edge distances, and thicknesses are very dependent on the anchor characteristics. Installation forces and torques in post-installed anchors can cause splitting of the surrounding concrete. Such splitting also can be produced in subsequent torquing during connection of attachments to anchors including cast-in anchors. The primary source of values for minimum spacings, edge distances, and thicknesses of post-installed anchors should be the product-specific tests of ACI 355.2. In some cases, however, specific products are not known in the design stage. Approximate values are provided for use in design.

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D.8.2 — Unless determined in accordance with D.8.4, minimum edge distances for cast-in headed anchors that will not be torqued shall be based on minimum cover requirements for reinforcement in 7.7. For cast-in headed anchors that will be torqued, the minimum edge distances shall be $6d_o$.

D.8.3 — Unless determined in accordance with D.8.4, minimum edge distances for post-installed anchors shall be based on the greater of the minimum cover requirements for reinforcement in 7.7, or the minimum edge distance requirements for the products as determined by tests in accordance with ACI 355.2, and shall not be less than 2.0 times the maximum aggregate size. In the absence of product-specific ACI 355.2 test information, the minimum edge distance shall be taken as not less than:

Undercut anchors	$6d_o$
Torque-controlled anchors	$8d_o$
Displacement-controlled anchors	$10d_o$

D.8.4 — For anchors where installation does not produce a splitting force and that will remain untorqued, if the edge distance or spacing is less than those specified in D.8.1 to D.8.3, calculations shall be performed by substituting for d_o a smaller value d_o' that meets the requirements of D.8.1 to D.8.3. Calculated forces applied to the anchor shall be limited to the values corresponding to an anchor having a diameter of d_o'.

D.8.5 — The value of h_{ef} for an expansion or undercut post-installed anchor shall not exceed the greater of either 2/3 of the member thickness or the member thickness less 4 in.

D.8.6 — Project drawings and project specifications shall specify use of anchors with a minimum edge distance as assumed in design.

COMMENTARY

RD.8.2 — Because the edge cover over a deep embedment close to the edge can have a significant effect on the side-face blowout strength of D.5.4, in addition to the normal concrete cover requirements, the designer may wish to use larger cover to increase the side-face blowout strength.

RD.8.3 — Drilling holes for post-installed anchors can cause microcracking. The requirement for a minimum edge distance twice the maximum aggregate size is to minimize the effects of such microcracking.

RD.8.4 — In some cases, it may be desirable to use a larger-diameter anchor than the requirements on D.8.1 to D.8.3 permit. In these cases, it is permissible to use a larger-diameter anchor provided the design strength of the anchor is based on a smaller assumed anchor diameter, d_o'.

RD.8.5 — This minimum thickness requirement is not applicable to through-bolts because they are outside the scope of Appendix D. In addition, splitting failures are caused by the load transfer between the bolt and the concrete. Because through-bolts transfer their load differently than cast-in or expansion and undercut anchors, they would not be subject to the same member thickness requirements. Post-installed anchors should not be embedded deeper than 2/3 of the member thickness.

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D.9 — Installation of anchors

D.9.1 — Anchors shall be installed in accordance with the project drawings and project specifications.

COMMENTARY

RD.9 — Installation of anchors

Many anchor performance characteristics depend on proper installation of the anchor. Anchor capacity and deformations can be assessed by acceptance testing under ACI 355.2. These tests are carried out assuming that the manufacturer's installation directions will be followed. Certain types of anchors can be sensitive to variations in hole diameter, cleaning conditions, orientation of the axis, magnitude of the installation torque, crack width, and other variables. Some of this sensitivity is indirectly reflected in the assigned f values for the different anchor categories, which depend in part on the results of the installation safety tests. Gross deviations from the ACI 355.2 acceptance testing results could occur if anchor components are incorrectly exchanged, or if anchor installation criteria and procedures vary from those recommended. Project specifications should require that anchors be installed according to the manufacturer's recommendations.

APPENDIX E — NOTATION

Code notation

a = depth of equivalent rectangular stress block as defined in 10.2.7.1, in., Chapters 10, 12

a = shear span, distance between concentrated load and face of support, in., Chapter 11

a = shear span, equal to the distance between a load and a support in a structure, in., Appendix A

A = area of that part of cross section between flexural tension face and center of gravity of gross section, in.^{2}, Chapter 18

A_b = area of an individual horizontal bar or wire, in.^{2}, Chapter 10

A_b = area of an individual bar, in.^{2}, Chapter 12

A_{brg} = bearing area of the head of stud or anchor bolt, in.^{2}, Appendix D

A_c = area of core of spirally reinforced compression member measured to outside diameter of spiral, in.^{2}, Chapter 10

A_c = area of concrete section resisting shear transfer, in.^{2}, Chapter 11

A_c = area of contact surface being investigated for horizontal shear, in.^{2}, Chapter 17

A_c = the effective cross-sectional area at one end of a strut in a strut-and-tie model, taken perpendicular to the axis of the strut, in.^{2}, Appendix A

A_{cf} = larger gross cross-sectional area of the slab-beam strips of the two orthogonal equivalent frames intersecting at a column of a two-way slab, in.^{2}, Chapter 18

A_{ch} = cross-sectional area of a structural member measured out-to-out of transverse reinforcement, in.^{2}, Chapter 21

A_{cp} = area enclosed by outside perimeter of concrete cross section, in.^{2} See 11.6.1, Chapter 11

A_{cp} = area of concrete section, resisting shear, of an individual pier or horizontal wall segment, in.^{2}, Chapter 21

A_{cv} = gross area of concrete section bounded by web thickness and length of section in the direction of shear force considered, in.^{2}, Chapter 21

A_f = area of reinforcement in bracket or corbel resisting factored moment, $[V_u a + N_{uc} (h - d)]$, in.^{2}, Chapter 11

A_g = gross area of column, in.^{2}, Chapter 16

A_g = gross area of section, in.^{2} For a hollow section, A_g is the area of the concrete only and does not include the area of the void(s). See 11.6.1., Chapter 11

A_g = gross area of section, in.^{2}, Chapters 9, 10, 14, 21, 22, 15, Appendix B

A_j = effective cross-sectional area within a joint, see 21.5.3.1, in a plane parallel to plane of reinforcement generating shear in the joint, in.^{2} The joint depth shall be the overall depth of the column. Where a beam frames into a support of larger width, the effective width of the joint shall not exceed the smaller of:
(a) beam width plus the joint depth
(b) twice the smaller perpendicular distance from the longitudinal axis of the beam to the column side. See 21.5.3.1, Chapter 21

A_h = area of shear reinforcement parallel to flexural tension reinforcement, in.^{2}, Chapter 11

A_ℓ = total area of longitudinal reinforcement to resist torsion, in.^{2}, Chapter 11

A_n = area of reinforcement in bracket or corbel resisting tensile force N_{uc}, in.^{2}, Chapter 11

A_n = area of a face of a nodal zone or a section through a nodal zone, in.^{2}, Appendix A

A_N = projected concrete failure area of an anchor or group of anchors, for calculation of strength in tension, as defined in D.5.2.1, in.^{2} A_N shall not be taken greater than nA_{No}. [See Fig. RD.5.1(b)], Appendix D

A_{No} = projected concrete failure area of one anchor, for calculation of strength in tension when not limited by edge distance or spacing, as defined in D.5.2.1, in.^{2} [See Fig. RD.5.1(a)], Appendix D

A_o = gross area enclosed by shear flow path, in.^{2}, Chapter 11

A_{oh} = area enclosed by centerline of the outermost closed transverse torsional reinforcement, in.^{2}, Chapter 11

A_{ps} = area of prestressed reinforcement in a tie, in.^{2}, Appendix A

A_{ps} = area of prestressed reinforcement in tension zone, in.^{2}, Chapters 11, 18

A_s = area of longitudinal tension reinforcement in wall segment, in.^{2}, Chapter 14

A_s = area of nonprestressed tension reinforcement, in.^{2}, Chapters 8, 10, 11, 12, 18

A_s = area of tension reinforcement, in.^{2}, Appendix B

A_s' = area of compression reinforcement, in.^{2}, Chapters 8, 9, 18, Appendix B

A_s' = area of compression reinforcement in a strut, in.^{2}, Appendix A

A_{se} = area of effective longitudinal tension reinforcement in wall segment, in.^{2}, as calculated by Eq. (14-8), Chapter 14

A_{se} = effective cross-sectional area of anchor, in.^{2}, Appendix D

A_{sh} = total cross-sectional area of transverse reinforcement (including crossties) within spacing s and perpendicular to dimension h_c, in.^{2}, Chapter 21

A_{si} = area of surface reinforcement in the ith layer crossing a strut, in.^{2}, Appendix A

A_{sl} = effective cross-sectional area of expansion or undercut anchor sleeve, if sleeve is within shear plane, in.^{2}, Appendix D

$A_{s,min}$ = minimum amount of flexural reinforcement, in.^{2} See 10.5, Chapter 10

A_{st} = total area of longitudinal reinforcement, (bars or steel shapes), in.^{2}, Chapter 10

A_{st} = area of nonprestressed reinforcement in a tie, in.^{2}, Appendix A

A_t = area of structural steel shape, pipe, or tubing in a composite section, in.^{2}, Chapter 10

A_t = area of one leg of a closed stirrup resisting torsion within a distance s, in.^{2}, Chapter 11

A_{tr} = total cross-sectional area of all transverse reinforcement which is within the spacing s and which crosses the potential plane of splitting through the reinforcement being developed, in.^{2}, Chapter 12

A_v = area of shear reinforcement within a distance s, or area of shear reinforcement perpendicular to flexural tension reinforcement within a distance s for deep flexural members, in.^{2}, Chapter 11

A_v = area of shear reinforcement within a distance s, in.^{2}, Chapter 12

A_v = area of ties within a distance s, in.^{2}, Chapter 17

A_V = projected concrete failure area of an anchor or group of anchors, for calculation of strength in shear, as defined in D.6.2.1, in.^{2} A_V shall not be taken greater than nA_{Vo}. [See Fig. RD.6.2(b)], Appendix D

A_{vd} = total area of reinforcement in each group of diagonal bars in a diagonally reinforced coupling beam, in.^{2}, Chapter 21

A_{vf} = area of shear-friction reinforcement, in.^{2}, Chapter 11

A_{vh} = area of shear reinforcement parallel to flexural tension reinforcement within a distance s_2, in.^{2}, Chapter 11

A_{Vo} = projected concrete failure area of one anchor, for calculation of strength in shear, when not limited by corner influences, spacing, or member thickness, as defined in D.6.2.1, in.^{2} [See Fig. RD.6.2(a)], Appendix D

A_w = area of an individual wire to be developed or spliced, in.^{2}, Chapter 12

A_1 = loaded area, in.^{2}, Chapters 10, 22

A_2 = the area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal, in.^{2}, Chapters 10, 22

b = width of compression face of member, in., Chapters 8-11, 18

b = width of member, in., Chapter 22, Appendix A

b = effective compressive flange width of a structural member, in., Chapter 21

B_n = nominal bearing load, lb, Chapter 22

b_o = perimeter of critical section for slabs and footings, in., Chapters 11, 22

b_t = width of that part of cross section containing the closed stirrups resisting torsion, in., Chapter 11

b_v = width of cross section at contact surface being investigated for horizontal shear, in., Chapter 17

b_w = web width, in., Chapter 10

b_w = web width, or diameter of circular section, in., Chapters 11, 12, 21

b_1 = width of the critical section defined in 11.12.1.2 measured in the direction of the span for which moments are determined, in., Chapters 11, 13

b_2 = width of the critical section defined in 11.12.1.2 measured in the direction perpendicular to b_1, in., Chapters 11, 13

c = distance from extreme compression fiber to neutral axis, in., Chapter 9

c = distance from extreme compression fiber to neutral axis, in., Chapters 10, 14, Appendix A

c = spacing or cover dimension, in. See 12.2.4, Chapter 12

c = distance from the extreme compression fiber to neutral axis, see 10.2.7, calculated for the factored axial force and nominal moment strength, consistent with the design displacement d_u, resulting in the largest neutral axis depth, in., Chapter 21

c = distance from center of an anchor shaft to the edge of concrete, in., Appendix D

C = cross-sectional constant to define torsional properties

$$= \sum\left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3}$$

The constant C for T- or L-sections shall be permitted to be evaluated by dividing the section into separate rectangular parts and summing the values of C for each part, Chapter 13

c_c = clear cover from the nearest surface in tension to the surface of the flexural tension reinforcement, in., Chapter 10

c_c = clear cover from the nearest surface in tension to the surface of the flexural tension steel, in., Chapter 18

C_m = a factor relating actual moment diagram to an equivalent uniform moment diagram, Chapter 10

c_{max}= the largest edge distance, in., Appendix D

c_{min}= the smallest edge distance, in., Appendix D

c_t = dimension equal to the distance from the interior face of the column to the slab edge measured parallel to c_1, but not exceeding c_1, in., Chapter 21

c_1 = size of rectangular or equivalent rectangular column, capital, or bracket measured in the direction of the span for which moments are being determined, in., Chapters 11, 13, 21

c_1 = distance from the center of an anchor shaft to the edge of concrete in one direction, in.; where shear force is applied to anchor, c_1 is in the direction of the shear force. [See Fig. RD.6.2(a)], Appendix D

c_2 = size of rectangular or equivalent rectangular column, capital, or bracket measured transverse to the direction of the span for which moments are being determined, in., Chapters 11, 13

c_2 = distance from center of an anchor shaft to the edge of concrete in the direction orthogonal to c_1, in., Appendix D

d = distance from extreme compression fiber to centroid of tension reinforcement, in., Chapters 7-10, 12, Appendix B

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, but need not be less than $0.80h$ for circular sections and prestressed members, in., Chapter 11

d = distance from extreme compression fiber to centroid of tension reinforcement for entire composite section, in., Chapter 17

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in., Chapter 14

d = distance from extreme compression fiber to centroid of nonprestressed tension reinforcement, in., Chapter 18

d = effective depth of section, in., Chapter 21

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in., Appendix A

d' = distance from extreme compression fiber to centroid of compression reinforcement, in., Chapter 18, Appendix B

D = dead loads, or related internal moments and forces, Chapters 9, 18, 20

d_b = nominal diameter of bar, wire, or prestressing strand, in., Chapters 7, 12

d_b = bar diameter, in., Chapter 21

d_o = outside diameter of anchor or shaft diameter of headed stud, headed bolt, or hooked bolt, in. (See also D.8.4), Appendix D

d_o' = value substituted for d_o when an oversized anchor is used, in. (See D.8.4), Appendix D

d_p = diameter of pile at footing base, in., Chapter 15

d_p = distance from extreme compression fiber to centroid of prestressed reinforcement, in., Chapter 18

d_s = distance from extreme tension fiber to centroid of tension reinforcement, in., Chapter 9, Appendix B

d_t = distance from extreme compression fiber to extreme tension steel, in., Chapters 9, 10

e = base of Napierian logarithms, Chapter 18

E = load effects of earthquake, or related internal moments and forces, Chapters 9, 21

E_c = modulus of elasticity of concrete, psi. See 8.5.1, Chapters 8-10, 14, 19

E_{cb} = modulus of elasticity of beam concrete, psi, Chapter 13

E_{cs} = modulus of elasticity of slab concrete, psi, Chapter 13

e_h = distance from the inner surface of the shaft of a J- or L-bolt to the outer tip of the J- or L-bolt, in., Appendix D

EI = flexural stiffness of compression member. See Eq. (10-12) and Eq. (10-13), in.^{2}-lb, Chapter 10

e_N' = eccentricity of normal force on a group of anchors; the distance between the resultant tension load on a group of anchors in tension and the centroid of the group of anchors loaded in tension, in.; e_N' is always positive [See Fig. RD.5.2(b) and (c)], Appendix D

E_s = modulus of elasticity of reinforcement, psi. See 8.5.2 and 8.5.3, Chapters 8, 10

e_V' = eccentricity of shear force on a group of anchors; the distance between the point of shear force application and the centroid of the group of anchors resisting shear in the direction of the applied shear, in., Appendix D

F = loads due to weight and pressures of fluids with well-defined densities and controllable maximum heights, or related internal moments and forces, Chapter 9

$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi, Chapters 9, 11, 12, 18, 19, 21, 22

f_c' = specified compressive strength of concrete, psi, Chapters 4, 5, 8-12, 14, 18-22, Appendixes A, B, D

$\sqrt{f'_{ci}}$ = square root of compressive strength of concrete at time of initial prestress, psi, Chapter 18

f'_{ci} = compressive strength of concrete at time of initial prestress, psi, Chapters 7, 18

f'_{cr} = required average compressive strength of concrete used as the basis for selection of concrete proportions, psi, Chapter 5

f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi, Chapters 5, 8, 9, 11, 12, 22

f_{ct} = specified tensile strength of concrete, psi, Appendix D

f_{cu} = effective compressive strength of the concrete in a strut or a nodal zone, psi, Appendix A

f_d = stress due to unfactored dead load, at extreme fiber of section where tensile stress is caused by externally applied loads, psi, Chapter 11

f_{dc} = decompression stress. Stress in the prestressing steel when stress is zero in the concrete at the same level as the centroid of the tendons, psi, Chapter 18

F_n = nominal strength of a strut, tie, or nodal zone, lb, Appendix A

F_{nn} = nominal strength of a face of a nodal zone, lb, Appendix A

F_{ns} = nominal strength of a strut, lb, Appendix A

F_{nt} = nominal strength of a tie, lb, Appendix A

f_{pc} = compressive stress in concrete (after allowance for all prestress losses) at centroid of cross section resisting externally applied loads or at junction of web and flange when the centroid lies within the flange, psi. (In a composite member, f_{pc} is resultant compressive stress at centroid of composite section, or at junction of web and flange when the centroid lies within the flange, due to both prestress and moments resisted by precast member acting alone), Chapter 11

f_{pc} = average compressive stress in concrete due to effective prestress force only (after allowance for all prestress losses), psi, Chapter 18

f_{pe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads, psi, Chapter 11

f_{ps} = stress in prestressed reinforcement at nominal strength, ksi, Chapters 12, 18

f_{pu} = specified tensile strength of prestressing steel, psi, Chapters 11, 18

f_{py} = specified yield strength of prestressing steel, psi, Chapter 18

f_r = modulus of rupture of concrete, psi, see 9.5.2.3, Chapter 18, Appendix D

f_s = calculated stress in reinforcement at service loads, ksi, Chapter 10

f'_s = stress in compression reinforcement, psi, Appendix A

f_{se} = effective stress in prestressed reinforcement (after allowance for all prestress losses), psi, Chapter 18

f_{se} = effective stress in prestressed reinforcement (after allowance for all prestress losses), ksi, Chapter 12

f_{se} = effective stress after losses in prestressed reinforcement, psi, Appendix A

f_t = extreme fiber stress in tension in the precompressed tensile zone, computed using gross section properties, psi, Chapter 18

f_t = calculated concrete tensile stress in a region of a member, psi, Appendix D

F_u = factored force acting in a strut, tie, bearing area, or nodal zone in a strut-and-tie model, lb, Appendix A

f_{ut} = specified tensile strength of anchor steel, psi, Appendix D

f_{utsl} = specified tensile strength of anchor sleeve, psi, Appendix D

f_y = specified yield strength of reinforcement, psi, Chapter 21

f_y = specified yield strength of anchor steel, psi, Appendix D

f_y = specified yield strength of nonprestressed reinforcement, psi, Chapters 3, 7-12, 14, 18, 19, Appendixes A, B

f_{yh} = specified yield strength of circular tie, hoop, or spiral reinforcement, psi, Chapter 11

f_{yh} = specified yield strength of transverse reinforcement, psi, Chapter 21

$f_{y\ell}$ = yield strength of longitudinal torsional reinforcement, psi, Chapter 11

f_{yt} = specified yield strength of transverse reinforcement, psi, Chapter 12

f_{yv} = yield strength of closed transverse torsional reinforcement, psi, Chapter 11

h = overall thickness of member, in., Chapters 9-14, 17, 18, 20-22, Appendix B

h = thickness of shell or folded plate, in., Chapter 19

h = height of member, in., Appendix A

h_t = effective height of tie, in., Appendix A

h = thickness of member in which an anchor is anchored, measured parallel to anchor axis, in., Appendix D

H = loads due to weight and pressure of soil, water in soil, or other materials, or related internal moments and forces, Chapter 9

h_c = cross-sectional dimension of column core measured center-to-center of confining reinforcement, in., Chapter 21

h_{ef} = effective anchor embedment depth, in. (See D.8.5 and Fig. RD.1), Appendix D

h_v = total depth of shearhead cross section, in., Chapter 11

h_w = total height of wall from base to top, in., Chapter 11

h_w = height of entire wall or of the segment of wall considered, in., Chapter 21

h_x = maximum horizontal spacing of hoop or crosstie legs on all faces of the column, in., Chapter 21

I = moment of inertia of section resisting externally applied factored loads, in.^{4}, Chapter 11

I_b = moment of inertia about centroidal axis of gross section of beam as defined in 13.2.4, in.^{4}, Chapter 13

I_{cr} = moment of inertia of cracked section transformed to concrete, in.^{4}, Chapters 9, 14

I_e = effective moment of inertia for computation of deflection, in.^{4}, Chapters 9, 14

I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in.^{4},Chapters 9, 10

I_s = moment of inertia about centroidal axis of gross section of slab, in.^{4}

= $h^3/12$ times width of slab defined in notations a and b_t, Chapter 13

I_{se} = moment of inertia of reinforcement about centroidal axis of member cross section, in.^{4}, Chapter 10

I_t = moment of inertia of structural steel shape, pipe, or tubing about centroidal axis of composite member cross section, in.^{4}, Chapter 10

k = effective length factor for compression members, Chapter 10

k = coefficient for basic concrete breakout strength in tension, Appendix D

k = effective length factor, Chapter 14

K = wobble friction coefficient per foot of tendon, Chapter 18

k_{cp} = coefficient for pryout strength, Appendix D

K_t = torsional stiffness of torsional member; moment per unit rotation. See R13.7.5., Chapter 13

K_{tr} = transverse reinforcement index

= $\frac{A_{tr}f_{yt}}{1500sn}$ (constant 1500 carries the unit lb/in.^{2}), Chapter 12

ℓ = span length of beam or one-way slab, as defined in 8.7; clear projection of cantilever, in., Chapter 9

ℓ = clear span, in., Chapter 16

ℓ = load bearing length of anchor for shear, not to exceed $8d_o$, in., Appendix D

= h_{ef} for anchors with a constant stiffness over the full length of the embedded section, such as headed studs or post-installed anchors with one tubular shell over the full length of the embedment depth, Appendix D

= $2d_o$ for torque-controlled expansion anchors with a distance sleeve separated from the expansion sleeve, Appendix D

L = live loads, or related internal moments and forces, Chapters 9, 18, 20

ℓ_a = additional embedment length at support or at point of inflection, in., Chapter 12

ℓ_c = length of compression member in a frame, measured from center to center of the joints in the frame, in., Chapter 10

ℓ_c = vertical distance between supports, in., Chapters 14, 22

ℓ_d = development length, in, Chapters 7, 12

ℓ_d = development length for a straight bar, in., Chapter 21

ℓ_d = development length of deformed bars and deformed wire in tension in., Chapter 12

ℓ_d = development length, in., Chapter 18

ℓ_{dc} = development length of deformed bars and deformed wire in compression, in., Chapter 12

ℓ_{dh} = development length of standard hook in tension, measured from critical section to outside end of hook (straight embedment length between critical section and start of hook [point of tangency] plus radius of bend and one bar diameter), in., Chapter 12

ℓ_{dh} = development length for a bar with a standard hook as defined in Eq. (21-6), in., Chapter 21

ℓ_{hb} = basic development length of standard hook in tension, in., Chapter 12

ℓ_n = length of clear span in long direction of two-way construction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases, in., Chapter 9

ℓ_n = clear span, in., Appendix A

ℓ_n = length of clear span in direction that moments are being determined, measured face-to-face of supports, in., Chapter 13

ℓ_n = clear span measured face-to-face of supports, in., Chapters 11, 21

ℓ_n = clear span for positive moment or shear and average of adjacent clear spans for negative moment, Chapter 8

ℓ_o = minimum length, measured from joint face along axis of structural member, over which transverse reinforcement must be provided, in., Chapter 21

L_r = roof live load, or related internal moments and forces, Chapter 9

ℓ_t = span of member under load test, in. (the shorter span for two-way slab systems). Span

is the smaller of (a) distance between centers of supports, and (b) clear distance between supports plus thickness h of member. In Eq. (20-1), span for a cantilever shall be taken as twice the distance from support to cantilever end, Chapter 20

ℓ_u = unsupported length of compression member, in., Chapter 10

ℓ_v = length of shearhead arm from centroid of concentrated load or reaction, in., Chapter 11

ℓ_w = horizontal length of wall, in., Chapters 11, 14

ℓ_w = length of entire wall or of segment of wall considered in direction of shear force, in., Chapter 21

ℓ_x = length of prestressing steel element from jacking end to any point x, ft. See Eq. (18-1) and (18-2), Chapter 18

ℓ_1 = length of span in direction that moments are being determined, measured center-to-center of supports, in., Chapter 13

ℓ_2 = length of span transverse to ℓ_1, measured center-to-center of supports. See also 13.6.2.3 and 13.6.2.4, in., Chapter 13

M = maximum unfactored moment due to service loads, including PD effects, in.-lb, Chapter 14

M_a = maximum moment in member at stage deflection is computed, in.-lb, Chapters 9, 14

M_c = factored moment to be used for design of compression member, in.-lb, Chapter 10

M_c = moment at the face of the joint, corresponding to the nominal flexural strength of the column framing into that joint, calculated for the factored axial force, consistent with the direction of the lateral forces considered, resulting in the lowest flexural strength, in.-lb. See 21.4.2.2, Chapter 21

M_{cr} = moment causing flexural cracking at section due to externally applied loads. See 11.4.2.1, Chapters, 11, 14

M_{cr} = cracking moment, in. See 9.5.2.3, Chapter 9

M_g = moment at the face of the joint, corresponding to the nominal flexural strength of the girder including slab where in tension, framing into that joint, in.-lb. See 21.4.2.2, Chapter 21

M_m = modified moment, in.-lb, Chapter 11

M_{max} = maximum factored moment at section due to externally applied loads, in.-lb, Chapter 11

M_n = nominal moment strength at section, in.-lb
= $A_s f_y (d - a/2)$, Chapter 12

M_n = nominal moment strength at section, in.-lb, Chapters 14, 22

M_o = total factored static moment, in.-lb, Chapter 13

M_p = required plastic moment strength of shearhead cross section, in.-lb, Chapter 11

M_{pr} = probable flexural strength of members, with or without axial load, determined using the properties of the member at the joint faces assuming a tensile strength in the longitudinal bars of at least $1.25 f_y$ and a strength reduction factor f of 1.0, in.-lb, Chapter 21

M_s = moment due to loads causing appreciable sway, in.-lb, Chapter 10

M_s = portion of slab moment balanced by support moment, in.-lb, Chapter 21

M_{sa} = maximum unfactored applied moment due to service loads, not including PD effects, in.-lb, Chapter 14

M_u = factored moment at section, in.-lb, Chapters 10, 11, 13, 21, 22

M_u = factored moment at section including PD effects, in.-lb, Chapter 14

M_{ua} = moment at the midheight section of the wall due to factored lateral and eccentric vertical loads, in.-lb, Chapter 14

M_v = moment resistance contributed by shearhead reinforcement, in.-lb, Chapter 11

M_1 = smaller factored end moment on a compression member, positive if member is bent in single curvature, negative if bent in double curvature, in.-lb, Chapter 10

M_{1ns} = factored end moment on a compression member at the end at which M_1 acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis, in.-lb, Chapter 10

M_{1s} = factored end moment on compression member at the end at which M_1 acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis, in.-lb, Chapter 10

M_2 = larger factored end moment on compression member, always positive, in.-lb, Chapter 10

$M_{2,min}$ = minimum value of M_2, in.-lb, Chapter 10

M_{2ns} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis, in.-lb, Chapter 10

M_{2s} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame

n = number of bars or wires being spliced or developed along the plane of splitting, Chapter 12

n = modular ratio of elasticity, but not less than 6
= E_s/E_c, Chapter 14

n = number of monostrand anchorage devices in a group, Chapter 18

n = number of anchors in a group, Appendix D

N_b = basic concrete breakout strength in tension of a single anchor in cracked concrete, as defined in D.5.2.2, lb, Appendix D

N_c = tensile force in concrete due to unfactored

dead load plus live load **(*D* + *L*)**, lb, Chapter 18

N_{cb} = nominal concrete breakout strength in tension of a single anchor, as defined in D.5.2.1, lb, Appendix D

N_{cbg} = nominal concrete breakout strength in tension of a group of anchors, as defined in D.5.2.1, lb, Appendix D

N_n = nominal strength in tension, lb, Appendix D

N_p = pullout strength in tension of a single anchor in cracked concrete, as defined in D.5.3.4 or D.5.3.5, lb, Appendix D

N_{pn} = nominal pullout strength in tension of a single anchor, as defined in D.5.3.1, lb, Appendix D

N_s = nominal strength of a single anchor or group of anchors in tension as governed by the steel strength, as defined in D.5.1.1 or D.5.1.2, lb, Appendix D

N_{sb} = side-face blowout strength of a single anchor, lb, Appendix D

N_{sbg} = side-face blowout strength of a group of anchors, lb, Appendix D

N_u = factored tensile load, lb, Appendix D

N_u = factored axial load normal to cross section occurring simultaneously with V_u *or* T_u; to be taken as positive for compression, lb, Chapter 11

N_{uc} = factored tensile force applied at top of bracket or corbel acting simultaneously with V_u, to be taken as positive for tension, lb, Chapter 11

P_b = nominal axial load strength at balanced strain conditions, lb. See 10.3.2, Chapters 9, 10, Appendix B

P_c = critical load, lb. See Eq. (10-10), Chapter 10

p_{cp} = outside perimeter of the concrete cross section, in. See 11.6.1, Chapter 11

p_h = perimeter of centerline of outermost closed transverse torsional reinforcement, in., Chapter 11

P_n = nominal strength of cross section subject to compression, lb, Chapter 22

P_n = nominal axial load strength at given eccentricity, lb, Chapters 9, 10, Appendix B

P_o = nominal axial load strength at zero eccentricity, lb, Chapter 10

P_{nw} = nominal axial load strength of wall designed by 22.6.5, lb, Chapter 22

P_{nw} = nominal axial load strength of wall designed by 14.4, lb, Chapter 14

P_s = unfactored axial load at the design (midheight) section including effects of self-weight, lb, Chapter 14

P_s = prestressing force at jacking end, lb, Chapter 18

P_{su} = factored prestressing force at the anchorage device, lb, Chapter 18

P_u = factored axial load at given eccentricity, lb $\le \phi P_n$, Chapter 10

P_u = factored axial load, lb, Chapter 14

P_x = prestressing force at any point x, lb, Chapter 18

Q = stability index for a story. See 10.11.4, Chapter 10

r = radius of gyration of cross section of a compression member, in., Chapter 10

R = rain load, or related internal moments and forces, Chapter 9

s = standard deviation, psi, Chapter 5

s = spacing of shear or torsion reinforcement in direction parallel to longitudinal reinforcement, in., Chapter 11

s = maximum center-to-center spacing of transverse reinforcement within ℓ_d, in., Chapter 12

s = center-to-center spacing of flexural tension reinforcement nearest to the extreme tension face, in. (where there is only one bar or wire nearest to the extreme tension face, s is the width of the extreme tension face.), Chapter 10

s = spacing of transverse reinforcement measured along the longitudinal axis of the structural member, in., Chapter 21

s = spacing of ties measured along the longitudinal axis of the member, in., Chapter 17

s = center-to-center spacing of flexural tension steel near the extreme tension face, in. Where there is only one bar or tendon near the extreme tension face, s is the width of extreme tension face, Chapter 18

s = anchor center-to-center spacing, in., Appendix D

S = snow load, or related internal moments and forces, Chapter 9

S = elastic section modulus of section, in.^{3}, Chapter 22

S_e = moment, shear, or axial force at connection corresponding with development of probable strength at intended yield locations, based on the governing mechanism of inelastic lateral deformation, considering both gravity and earthquake load effects, Chapter 21

s_i = spacing of reinforcement in the ith layer adjacent to the surface of the member, in., Appendix A

S_n nominal flexural, shear, or axial strength of the connection, Chapter 21

s_o = maximum spacing of transverse reinforcement, in., Chapter 21

s_o = spacing of the outer anchors along the edge in a group, in., Appendix D

s_{sk} = spacing of skin reinforcement, in., Chapter 10

s_w = spacing of wire to be developed or spliced, in., Chapter 12

s_x = longitudinal spacing of transverse reinforcement within the length ℓ_o, in., Chapter 21

S_y = yield strength of connection, based on f_y, for moment, shear, or axial force, Chapter 21

s_1 = spacing of vertical reinforcement in wall, in., Chapter 11

s_2 = spacing of shear or torsion reinforcement in direction perpendicular to longitudinal reinforcement—or spacing of horizontal reinforcement in wall, in., Chapter 11

t = thickness of a wall of a hollow section, in., Chapter 11

t = thickness of washer or plate, in., Appendix D

T = cumulative effect of temperature, creep, shrinkage, differential settlement, and shrinkage-compensating concrete, Chapter 9

T_n = nominal torsional moment strength, in.-lb, Chapter 11

T_u = factored torsional moment at section, in.-lb, Chapter 11

U = required strength to resist factored loads or related internal moments and forces, Chapter 9

V_b = basic concrete breakout strength in shear of a single anchor in cracked concrete, as defined in D.6.2.2 or D.6.2.3, lb, Appendix D

V_c = nominal shear strength provided by concrete, lb, See 11.12.2.1, Chapters 8, 11, 13, 21

V_{cb} = nominal concrete breakout strength in shear of a single anchor, as defined in D.6.2.1, lb, Appendix D

V_{cbg} = nominal concrete breakout strength in shear of a group of anchors, as defined in D.6.2.1, lb, Appendix D

V_{ci} = nominal shear strength provided by concrete when diagonal cracking results from combined shear and moment, lb, Chapter 11

V_{cp} = nominal concrete pryout strength, as defined in D.6.3, lb, Appendix D

V_{cw} = nominal shear strength provided by concrete when diagonal cracking results from excessive principal tensile stress in web, lb, Chapter 11

V_d = shear force at section due to unfactored dead load, lb, Chapter 11

V_e = design shear force determined from 21.3.4.1 or 21.4.5.1, lb, Chapter 21

V_i = factored shear force at section due to externally applied loads occurring simultaneously with M_{max}, lb, Chapter 11

V_n = nominal shear strength, lb, Chapters 11, 21, Appendix C

v_n = nominal shear stress, psi. See 11.12.6.2, Chapter 11

V_n = nominal shear strength at section, lb, Chapter 22

V_{nh} = nominal horizontal shear strength, lb, Chapter 17

V_p = vertical component of effective prestress force at section, lb, Chapter 11

V_s = nominal shear strength provided by shear reinforcement, lb, Chapter 11

V_s = nominal strength in shear of a single anchor or group of anchors as governed by the steel strength, as defined in D.6.1.1 or D.6.1.2, lb, Appendix D

V_u = factored shear force at section, lb, Chapters 11-13, 17, 21

V_u = factored shear load, lb, Appendix D

V_u = factored horizontal shear in a story, lb, Chapter 10

W = wind load, or related internal moments and forces, Chapter 9

w_c = unit weight of concrete, lb/ft^3, Chapters 8, 9

w_d = factored dead load per unit area, Chapter 13

w_ℓ = factored live load per unit area, Chapter 13

w_u = factored load per unit length of beam or per unit area of slab, Chapter 8

w_u = factored load per unit area, Chapter 13

x = shorter overall dimension of rectangular part of cross section, in., Chapter 13

y = longer overall dimension of rectangular part of cross section, in., Chapter 13

y_t = distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension, in., Chapters 9, 11

α = angle between inclined stirrups and longitudinal axis of member, Chapter 11

α = reinforcement location factor. See 12.2.4, Chapter 12

α = ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels (if any) on each side of the beam, Chapter 13

$$= \frac{E_{cb} I_b}{E_{cs} I_s}$$

α = total angular change of tendon profile in radians from tendon jacking end to any point x, Chapter 18

α = angle between the diagonal reinforcement and the longitudinal axis of a diagonally reinforced coupling beam, Chapter 21

α_c = coefficient defining the relative contribution of concrete strength to wall strength. See Eq. (21-7), Chapter 21

α_f = angle between shear-friction reinforcement and shear plane, Chapter 11

α_m = average value of α for all beams on edges of a panel, Chapter 9

α_s = constant used to compute V_c in slabs and footings, Chapter 11

α_v = ratio of flexural stiffness of shearhead arm to that of the surrounding composite slab section. See 11.12.4.5, Chapter 11

α_1 = α in direction of ℓ_1, Chapter 13

α_2 = α in direction of ℓ_2, Chapter 13

β = ratio of clear spans in long to short direction of two-way slabs, Chapter 9

β = coating factor. See 12.2.4, Chapter 12

b = ratio of long side to short side of footing, Chapter 15

b_b = ratio of area of reinforcement cut off to total area of tension reinforcement at section, Chapter 12

b_1 = factor defined in 10.2.7.3, Chapters 8, 10, 18, Appendixes A, B

b_c = ratio of long side to short side of concentrated load or reaction area, Chapters 11, 22

b_d = (a) for nonsway frames, b_d is the ratio of the maximum factored axial sustained load to the maximum factored axial load associated with the same load combination;
(b) for sway frames, except as required in (c) of this definition, b_d is the ratio of the maximum factored sustained shear within a story to the maximum factored shear in that story; and
(c) for stability checks of sway frames carried out in accordance with 10.13.6, b_d is the ratio of the maximum factored sustained axial load to the maximum factored axial load, Chapter 10

b_n = factor to account for the effect of the anchorage of ties on the effective compressive strength of a nodal zone, Appendix A

b_p = constant used to compute V_c in prestressed slabs, Chapter 11

b_s = factor to account for the effect of cracking and confining reinforcement on the effective compressive strength of the concrete in a strut, Appendix A

b_t = ratio of torsional stiffness of edge beam section to flexural stiffness of a width of slab equal to span length of beam, center-to-center of supports, Chapters 9, 13

$$= \frac{E_{cb}C}{2E_{cs}I_s}$$

g_i = angle between the axis of a strut and the bars in the ith layer of reinforcement crossing that strut, Appendix A

g = reinforcement size factor. See 12.2.4, Chapter 12

g_f = fraction of unbalanced moment transferred by flexure at slab-column connections. See 13.5.3.2, Chapters 11, 13

g_p = factor for type of prestressing steel, Chapter 18
= 0.55 for f_{py}/f_{pu} not less than 0.80
= 0.40 for f_{py}/f_{pu} not less than 0.85
= 0.28 for f_{py}/f_{pu} not less than 0.90

g_v = fraction of unbalanced moment transferred by eccentricity of shear at slab-column connections. See 11.12.6.1, Chapters 11, 13
= $1 - g_f$

d_{ns} = moment magnification factor for frames braced against sidesway, to reflect effects of member curvature between ends of compression member, Chapter 10

d_s = moment magnification factor for frames not braced against sidesway, to reflect lateral drift resulting from lateral and gravity loads, Chapter 10

d_u = design displacement, in., Chapter 21

e_t = net tensile strain in extreme tension steel at nominal strength, Chapters 8-10

D_{fmax} = maximum deflection measured during the second test relative to the position of the structure at the beginning of the second test, in. See Eq. (20-3), Chapter 20

Df_p = increase in stress in prestressing steel due to factored loads, psi, Appendix A

Df_{ps} = stressing in prestressing steel at service loads less decompression stress, ksi, Chapter 18

D_{max} = measured maximum deflection, in. See Eq. (20-1), Chapter 20

D_o = relative lateral deflection between the top and bottom of a story due to V_u, computed using a first-order elastic frame analysis and stiffness values satisfying 10.11.1, in., Chapter 10

D_{rmax} = measured residual deflection, in. See Eq. (20-2) and (20-3), Chapter 20

D_s = maximum deflection at or near midheight due to service loads, in., Chapter 14

D_u = deflection at midheight of wall due to factored loads, in., Chapter 14

e_s = the strain in the longitudinal reinforcement in a compression zone or a longitudinally reinforced strut, Appendix A

h = number of identical arms of shearhead, Chapter 11

q = angle of compression diagonals in truss analogy for torsion, Chapter 11

q = angle between the axis of a strut or compression field and the tension chord of the member, Appendix A

l = lightweight aggregate concrete factor. See 12.2.4, Chapter 12

l = multiplier for additional long-term deflection as defined in 9.5.2.5, Chapter 9

l = correction factor related to unit weight of concrete. See 11.7.4.3, Chapters 11, 17, 18, Appendix A

m = curvature friction coefficient, Chapter 18

m = coefficient of friction. See 11.7.4.3, Chapter 11

x = time-dependent factor for sustained load. See 9.5.2.5, Chapter 9

r = ratio of nonprestressed tension reinforcement
= A_s/bd Chapters 8-11, 13, 18, 21, Appendix B

r = ratio of tension reinforcement
= $A_s/(\ell_w d)$, Chapter 14

r' = ratio of nonprestressed compression reinforcement

= A_s'/bd, Chapters 8, 9, Appendix B

$r¢$ = ratio of compression reinforcement

= A_s'/bd, Chapter 18

r_b = reinforcement ratio producing balanced strain conditions. See 10.3.2, Chapters 8-10, 13, 14, Appendix B

r_g = ratio of total reinforcement area to cross-sectional area of column, Chapter 21

r_h = ratio of horizontal shear reinforcement area to gross concrete area of vertical section, Chapter 11

r_n = ratio of vertical shear reinforcement area to gross concrete area of horizontal section, Chapter 11

r_n = ratio of area of distributed reinforcement parallel to the plane of A_{cv} to gross concrete area perpendicular to that reinforcement, Chapter 21

r_p = ratio of prestressed reinforcement

= A_{ps}/bd_p, Chapter 18

r_s = ratio of volume of spiral reinforcement to total volume of core (out-to-out of spirals) of a spirally reinforced compression member, Chapter 10

r_s = ratio of volume of spiral reinforcement to the core volume confined by the spiral reinforcement (measured out-to-out), Chapter 21

r_v = ratio of tie reinforcement area to area of contact surface, Chapter 17

= $A_v/b_v s$

r_v = ratio of area of distributed reinforcement perpendicular to the plane of A_{cv} to gross concrete area A_{cv}, Chapter 21

r_w = $A_s/b_w d$, Chapter 11

f = strength reduction factor, Chapters 8-11, 13, 14, 17-19, 21, 22, Appendixes A, B, D

f_K = stiffness reduction factor. See R10.12.3, Chapter 10

y_1 = modification factor, for strength in tension, to account for anchor groups loaded eccentrically, as defined in D.5.2.4, Appendix D

y_2 = modification factor, for strength in tension, to account for edge distances smaller than $1.5h_{ef}$, as defined in D.5.2.5, Appendix D

y_3 = modification factor, for strength in tension, to account for cracking, as defined in D.5.2.6 and D.5.2.7, Appendix D

y_4 = modification factor, for pullout strength, to account for cracking, as defined in D.5.3.1 and D.5.3.6, Appendix D

y_5 = modification factor, for strength in shear, to account for anchor groups loaded eccentrically, as defined in D.6.2.5, Appendix D

y_6 = modification factor, for strength in shear, to account for edge distances smaller than $1.5c_1$, as defined in D.6.2.6, Appendix D

y_7 = modification factor, for strength in shear, to account for cracking, as defined in D.6.2.7, Appendix D

w = rf_y/f_c', Chapter 18. Appendix B

w' = $r'f_y/f_c'$, Chapter 18

w_p = $r_p f_{ps}/f_c'$, Chapter 18

w_w, w_{pw}, w_w' = reinforcement indexes for flanged sections computed as for w, w_p, and $w¢$ except that b shall be the web width, and reinforcement area shall be that required to develop compressive strength of web only, Chapter 18

Commentary notation

a = depth of rectangular stress block, in., Appendix A

A_{se} = The effective cross-sectional area of an anchor should be provided by the manufacturer of expansion anchors with reduced cross-sectional area for the expansion mechanism., Appendix D For threaded bolts, ANSI/ASME B1.1[D.1] defines A_{se} as:

$$A_{se} = \frac{p}{4}\left(d_o - \frac{0.9743}{n_t}\right)^2$$

where n_t is the number of threads per in.

C = compression force acting on a nodal zone, lb, Appendix A

e_N = Actual eccentricity of a normal force on an attachment, Appendix D

f_{si} = the stress in the ith layer of surface reinforcement, psi, Appendix A

h = overall height of member, in., Appendix A

h_{ef} = effective embedment depths for a variety of anchor types are shown in Fig. RD.1, Appendix D

h_t = effective height of concrete concentric with a tie, Appendix A used to dimension nodal zone, in., Appendix A

$h_{t,max}$ = maximum effective height of concrete concentric with a tie, in., Appendix A

ℓ_a = length in which anchorage of a tie must occur, in., Appendix A

ℓ_b = width of bearing, in., Appendix A

R = reaction, lb, Appendix A

T = tension force acting on a nodal zone, lb, Appendix A

w_s = width of a strut perpendicular to the axis of the strut, in., Appendix A

APPENDIX F — STEEL REINFORCEMENT INFORMATION

As an aid to users of the ACI Building Code, information on sizes, areas, and weights of various steel reinforcement is presented.

ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, $in.^2$	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

ASTM STANDARD PRESTRESSING TENDONS

Type*	Nominal diameter, in.	Nominal area, $in.^2$	Nominal weight, lb/ft
Seven-wire strand (Grade 250)	1/4 (0.250)	0.036	0.122
	5/16 (0.313)	0.058	0.197
	3/8 (0.375)	0.080	0.272
	7/16 (0.438)	0.108	0.367
	1/2 (0.500)	0.144	0.490
	(0.600)	0.216	0.737
Seven-wire strand (Grade 270)	3/8 (0.375)	0.085	0.290
	7/16 (0.438)	0.115	0.390
	1/2 (0.500)	0.153	0.520
	(0.600)	0.217	0.740
Prestressing wire	0.192	0.029	0.098
	0.196	0.030	0.100
	0.250	0.049	0.170
	0.276	0.060	0.200
Prestressing bars (plain)	3/4	0.44	1.50
	7/8	0.60	2.04
	1	0.78	2.67
	1-1/8	0.99	3.38
	1-1/4	1.23	4.17
	1-3/8	1.48	5.05
Prestressing bars (deformed)	5/8	0.28	0.98
	3/4	0.42	1.49
	1	0.85	3.01
	1-1/4	1.25	4.39
	1-3/8	1.58	5.56

* Availability of some tendon sizes should be investigated in advance.

ASTM STANDARD WIRE REINFORCEMENT

W & D size		Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft	Area, in.^{2}/ft of width for various spacings						
					Center-to-center spacing, in.						
Plain	Deformed				2	3	4	6	8	10	12
W31	D31	0.628	0.310	1.054	1.86	1.24	0.93	0.62	0.465	0.372	0.31
W30	D30	0.618	0.300	1.020	1.80	1.20	0.90	0.60	0.45	0.366	0.30
W28	D28	0.597	0.280	0.952	1.68	1.12	0.84	0.56	0.42	0.336	0.28
W26	D26	0.575	0.260	0.934	1.56	1.04	0.78	0.52	0.39	0.312	0.26
W24	D24	0.553	0.240	0.816	1.44	0.96	0.72	0.48	0.36	0.288	0.24
W22	D22	0.529	0.220	0.748	1.32	0.88	0.66	0.44	0.33	0.264	0.22
W20	D20	0.504	0.200	0.680	1.20	0.80	0.60	0.40	0.30	0.24	0.20
W18	D18	0.478	0.180	0.612	1.08	0.72	0.54	0.36	0.27	0.216	0.18
W16	D16	0.451	0.160	0.544	0.96	0.64	0.48	0.32	0.24	0.192	0.16
W14	D14	0.422	0.140	0.476	0.84	0.56	0.42	0.28	0.21	0.168	0.14
W12	D12	0.390	0.120	0.408	0.72	0.48	0.36	0.24	0.18	0.144	0.12
W11	D11	0.374	0.110	0.374	0.66	0.44	0.33	0.22	0.165	0.132	0.11
W10.5		0.366	0.105	0.357	0.63	0.42	0.315	0.21	0.157	0.126	0.105
W10	D10	0.356	0.100	0.340	0.60	0.40	0.30	0.20	0.15	0.12	0.10
W9.5		0.348	0.095	0.323	0.57	0.38	0.285	0.19	0.142	0.114	0.095
W9	D9	0.338	0.090	0.306	0.54	0.36	0.27	0.18	0.135	0.108	0.09
W8.5		0.329	0.085	0.289	0.51	0.34	0.255	0.17	0.127	0.102	0.085
W8	D8	0.319	0.080	0.272	0.48	0.32	0.24	0.16	0.12	0.096	0.08
W7.5		0.309	0.075	0.255	0.45	0.30	0.225	0.15	0.112	0.09	0.075
W7	D7	0.298	0.070	0.238	0.42	0.28	0.21	0.14	0.105	0.084	0.07
W6.5		0.288	0.065	0.221	0.39	0.26	0.195	0.13	0.097	0.078	0.065
W6	D6	0.276	0.060	0.204	0.36	0.24	0.18	0.12	0.09	0.072	0.06
W5.5		0.264	0.055	0.187	0.33	0.22	0.165	0.11	0.082	0.066	0.055
W5	D5	0.252	0.050	0.170	0.30	0.20	0.15	0.10	0.075	0.06	0.05
W4.5		0.240	0.045	0.153	0.27	0.18	0.135	0.09	0.067	0.054	0.045
W4	D4	0.225	0.040	0.136	0.24	0.16	0.12	0.08	0.06	0.048	0.04
W3.5		0.211	0.035	0.119	0.21	0.14	0.105	0.07	0.052	0.042	0.035
W3		0.195	0.030	0.102	0.18	0.12	0.09	0.06	0.045	0.036	0.03
W2.9		0.192	0.029	0.098	0.174	0.116	0.087	0.058	0.043	0.035	0.029
W2.5		0.178	0.025	0.085	0.15	0.10	0.075	0.05	0.037	0.03	0.025
W2		0.159	0.020	0.068	0.12	0.08	0.06	0.04	0.03	0.024	0.02
W1.4		0.135	0.014	0.049	0.084	0.056	0.042	0.028	0.021	0.017	0.014

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Portland Cement Association



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with Design Applications



EB702

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NOTES ON

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BUILDING CODE REQUIREMENTS FOR STRUCTURAL CONCRETE

with Design Applications

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About the building on the cover

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Preface

The first edition of this reference manual was developed to aid users in applying the provisions of the 1971 edition of "Building Code Requirements for Reinforced Concrete (ACI 318-71)." The second through fifth editions updated the material in conformity with provisions of the 1977 code edition, the 1980 code supplement, and the 1983 and 1989 code editions, respectively. The sixth and seventh editions addressed the 1995 and 1999 editions of "Building Code Requirements for Structural Concrete (ACI 318-95) and (ACI 318-99)." Through seven editions, much of the initial material has been revised to better emphasize the subject matter, and new chapters added to assist the designer in proper application of the ACI 318 design provisions.

This eighth edition reflects the contents of "Building Code Requirements for Structural Concrete (ACI 318-02)." The text and design examples have been revised to reflect, where possible, comments received from users of the "Notes" who suggested improvements in wording, identified errors, and recommended items for inclusion or deletion.

The primary purpose for publishing this manual is to assist the engineer and architect in the proper application of the ACI 318-02 design standard. The emphasis is placed on "how-to-use" the code. For complete background information on the development of the code provisions, the reader is referred to the "Commentary on Building Code Requirements for Structural Concrete (ACI 318R-02)" which, starting with the 1989 edition, has been published together with the code itself under the same cover.

This manual is also a valuable aid to educators, contractors, materials and products manufacturers, building code authorities, inspectors, and others involved in the design, construction, and regulation of concrete structures.

Although every attempt has been made to impart editorial consistency to the thirty-four chapters, some inconsistencies probably still remain. A few typographical and other errors are probably also to be found. PCA would be grateful to any reader who would bring such errors and inconsistencies to our attention. Other suggestions for improvement are also most sincerely welcome.

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The following Portland Cement Association staff members have contributed to the update of this document:

Attila Beres	Salah M. Elgammal	David A. Fanella
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The magnitude of changes adopted in 2002 code necessitated the assistance of several consultants:

Kenneth B. Bondy, Consulting Structural Engineer, West Hills, CA, updated Part 26, Prestressed Slab Systems
Ronald A. Cook, University of Florida, Gainesville, FL, developed Part 34, Anchoring to Concrete
James Doyle, Consultant, Chaplin, KY, updated Part 10, Deflections
S.K. Ghosh, S.K. Ghosh Associates Inc., Northbrook, IL, helped update Part 29, Special Provisions for Seismic Design, and reviewed Part 1, General Requirements
Robert F. Mast, BERGER/ABAM, Federal Way, WA, updated Parts 5, 6, 7, 8, 24, and 25, all pertaining to application of the Unified Design Provisions

Dale McFarlane of PCA staff and Cheryl Taylor, Consultant, played a crucial role in the production of this publication. They were responsible for the word processing, layout and formatting of this large and complex manuscript. Their assistance is very much appreciated.

Finally, sincere gratitude must be expressed to the authors and contributors of various parts of the first seven editions of the “Notes.” Their initial work is carried over into this edition, although their names are no longer separately identified with the various parts.

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1

General Requirements

A significant renaming of the ACI 318 standard took place with the 1995 edition; in the document title, "Reinforced Concrete," was changed to "Structural Concrete" in recognition of the new Chapter 22 - Structural Plain Concrete. Prior to the '95 code, design and construction requirements for structural members of plain concrete were contained in a separate companion document to ACI 318, designated ACI 318.1. The ACI 318.1 code requirements for structural plain concrete are now incorporated in Chapter 22.

1.1* SCOPE

As the name implies, *Building Code Requirements for Structural Concrete (ACI 318-02)* is meant to be adopted by reference in a general building code, to regulate the design and construction of buildings and structures of concrete. Section 1.1.1 emphasizes the intent and format of the ACI 318 document and its status as part of a legally adopted general building code. The ACI 318 code has no legal status unless adopted by a state or local jurisdiction having power to regulate building design and construction through a legally appointed building official. It is also recognized that when the ACI code is made part of a legally adopted general building code, that general building code may modify some provisions of ACI 318 to reflect local conditions and requirements. For areas where there is no general building code, there is no law to make ACI 318 the "code." In such cases, the ACI code defines minimum acceptable standards of design and construction practice, even though it has no legal status.

A provision new to ACI 318-02 has been added to 1.1.1 that specifies that the minimum specified compressive strength of concrete must be not less than 2500 psi. This provision has also been included in 5.1.1. While the commentary does not explain why this provision was added, it was most likely included because an identical requirement has been in *The BOCA National Building Code* (NBC), and *Standard Building Code* (SBC) for several editions, and it is also in the 2000 *International Building Code* (IBC).

Also new to 1.1.1 is a statement that "No maximum specified compressive strength (of concrete) shall apply unless restricted by a specific code provision." The impetus for adding this was the fact that some local jurisdictions, most notably in southern California, were in effect, if not formally, imposing maximum limits on strength of concrete used in structures in regions of high seismic risk (UBC Seismic Zone 3 or 4). Committee 318 felt that it was advisable to add the statement to make it known to regulators that limitations on concrete strength are considered when new code provisions are introduced, and unless concrete strength is specifically limited by other provisions of ACI 318, no maximum upper limit on strength is deemed necessary. The Committee has been making adjustments in the standard on an ongoing basis to account for sometimes differing properties of high-strength concrete.

In the past, most jurisdictions in the United States have adopted one of the three following model building codes to regulate building design and construction. *The BOCA National Building Code* (NBC), published by the Building Officials and Code Administrators International[1.1], is used primarily in the northeastern states; the

*Section numbers correspond to those of ACI 318-02.

Standard Building Code (SBC), published by the Southern Building Code Congress International[1.2], is used primarily in the southeastern states; and the *Uniform Building Code* (UBC), published by the International Conference of Building Officials[1.3], is used mainly in the central and western United States. All three of these model codes use the ACI 318 standard to regulate design and construction of structural elements of concrete in buildings or other structures. *The BOCA National Building Code* and the *Standard Building Code* adopt ACI 318 primarily by reference, incorporating only the construction requirements (Chapter 4 through 7) of ACI 318 directly within Chapter 19 of their documents. The *Uniform Building Code* reprints ACI 318 in its entirety in Chapter 19. It is essential that designers of concrete buildings in jurisdictions regulated by the UBC refer to Chapter 19, as some ACI 318 provisions are modified and some provisions are added to reflect, in most cases, more stringent seismic design requirements. To clearly distinguish where the UBC differs from ACI 318, the differing portions of UBC Chapter 19 are printed in italics.

Many states and local jurisdictions that formally adopted one of the three older model building codes, have either adopted or have announced their intent to adopt the *International Building Code* (IBC), developed by the International Code Council[1.4]. The 2000 edition (first edition) of the IBC adopts ACI 318-99 by reference, although portions of Chapters 3 – 7 of ACI 318 have been included in IBC Sections 1903 - 1907. A few modifications have been made to the ACI 318 provisions and these are indicated by the text printed in italics. Additional modifications to provisions in other Chapters of ACI 318 are contained in IBC Section 1908. Most of these were necessary to coordinate ACI 318 provisions for seismic design (Chapter 21) with the IBC's seismic design provisions, or to add structural systems of precast concrete for use in structures in regions of high seismic risk, or assigned to high seismic performance or design categories (IBC seismic design category D, E or F) that were not provided for in ACI 318.

As this book goes to press, a fifth model building code is nearing completion. The code is being developed by the National Fire Protection Association (NFPA) and its first edition is slated for printing in the summer of 2002. While the process is not complete, it appears that the code will adopt ACI 318-02 by reference to regulate the design and construction of concrete structures. As with the IBC, a few modifications to ACI 318 will be included in the code's concrete chapter in order to coordinate ACI 318 provisions for seismic design (Chapter 21) with the seismic design provisions of ASCE 7, which the NFPA code will adopt by reference for its structural loads provisions.

Whichever model code governs the design, the prudent designer should always refer to the governing model code for any modifications and additions to the ACI 318 code requirements for structural concrete.

Seismic Design Practice — Earthquake design requirements in two of the four U.S. model codes are based on the 1991 edition of the *NEHRP (National Earthquake Hazards Reduction Program) Recommended Provisions for the Development of Seismic Regulations for New Buildings*.[1.5] *The BOCA National Building Code* (NBC) and the *Standard Building Code* (SBC) have incorporated the NEHRP recommended provisions into the codes, with relatively few modifications. The *Uniform Building Code* (UBC), published by the International Conference of Building Officials which traditionally follows the lead of the Structural Engineers Association of California (SEAOC), has its seismic provisions based on the *Recommended Lateral Force Requirements and Commentary*[1.6] (the SEAOC "Blue Book") published by the Seismology Committee of SEAOC. The latest SEAOC recommendations have adopted many of the features of the NEHRP provisions.

The designer should be aware that there are important differences in design methodologies between the UBC and the NBC and SBC for earthquake design. Even with the different design methodologies, it is important to note that a building designed under the NBC or SBC earthquake design criteria and the UBC criteria will provide a similar level of safety and that the two sets of provisions (NBC and SBC versus UBC) are substantially equivalent.[1.7]

The seismic design provisions of the 2000 edition of the *International Building Code*, are based on the 1997 edition of the *NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Struc-*

tures.[1.8] Major differences between the IBC and the NBC and SBC seismic provisions, that are based on the '91 NEHRP Provisions[1.5], include:

1. Seismic ground motion maps of the 1991 edition have been replaced with spectral response acceleration maps at periods of 0.2 second and 1.0 second.

2. The 1991 maps gave ground motion parameters that had a 10% probability of exceedance in 50 years (i.e., approximately a 475-year return period). The 1997 maps are based on a maximum considered earthquake (MCE), and for most regions the MCE ground motion is defined with a uniform likelihood of exceedance of 2% in 50 years (return period of about 2500 years).

3. Seismic detailing requirements, that were triggered by building use and estimated ground motion on rock in the '91 edition, have been revised to include the amplifying effects of soft soils overlying rock. This may require buildings on soft soils in areas that have traditionally been considered to be at low or moderate seismic risk to be detailed for moderate and high seismic risk, respectively.

4. In the '91 edition, the amplifying effects of soft soils were ignored in calculating the design base shear for short period buildings. These effects are now taken into consideration, and will result in significant increases in base shear for short period buildings on soft soils in areas at low seismic risk.

5. A reliability/redundancy factor has been introduced for building in regions of high seismic risk. This was done to force designers to either add redundancies to the seismic force resisting system or pay a penalty in the form of designing for a higher base shear.

6. All buildings will be required to be designed for a minimum base shear of 1% of total gravity dead load, and other loads required to be considered a part of *W* for purposes of computing the base shear. Seismic design of buildings in areas with negligible or very low seismic risk (e.g., Seismic Zone 0, SPC A) has traditionally not been required by building codes. This new requirement will mean that in areas where seismic design has traditionally been ignored (e.g., south Florida, and south Texas), designers will need to check to make sure that this does not control the design of the lateral force resisting system. Seismic forces instead of wind may control design of the lateral force resisting system of larger concrete buildings, such as parking structures, or long narrow buildings, such as hotels/motels.

For a comprehensive comparison of the major differences between the seismic design requirements of the 2000 IBC, and the latest editions of the NBC, SBC and UBC, see *Impact of the Seismic design Provisions of the International Building Code*[1.9].

Differences in Design Methodology — The UBC earthquake design force level is based on the seismic zone, the structural system, and the building use (occupancy). These design considerations are used to determine a design base shear. As the anticipated level of ground shaking increases, the design base shear increases. Similarly, as the need for post-disaster functionality increases, the design base shear is increased.

As with the UBC, the IBC, NBC and SBC provisions increase the design base shear as the level of ground shaking increases. In the NBC and SBC this is done not through a seismic zone factor Z, but through a coefficient A_v representing effective peak velocity-related acceleration or a coefficient A_a representing effective peak acceleration (for definitions of these terms, see the Commentary to the NEHRP Provisions[1.5]). These two quantities are given on separate contour maps that take the place of the seismic zoning map of the UBC. NBC and SBC utilize a "seismic performance category" (SPC) that takes into account the level of seismicity and the building occupancy. In the IBC, the A_a and A_v maps are replaced with spectral response acceleration maps at periods of 0.2 second and 1.0 second. The IBC replaces the "seismic performance category" of the NBC and SBC with "seismic design category" (SDC). This is more than a change of terminology because in addition to considering the occupancy of the structure and the estimated ground motion on rock, also considered is the modification of ground motion due to the amplifying effects of soft soils overlying rock. Based on the SPC or

SDC of the building, different design criteria such as drift limits and detailing requirements are specified. Like the UBC, the IBC, NBC and SBC earthquake provisions factor into design the effects of site geology and soil characteristics and the type and configuration of the structural framing system.

Another major difference between the provisions of the 1994 and earlier editions of the UBC and those of the IBC, NBC and SBC is in the magnitude of the design base shear. The designer should note that the earthquake design forces of the IBC, NBC and SBC, and the 1994 and earlier editions of the UBC cannot be compared by simply looking at the numbers, since one set of numbers is based on strength design and the other set is based on working or allowable stress design. IBC, NBC and SBC design earthquake forces are strength level while pre-1997 UBC forces are service load level. The difference shows up in the magnitude of the response modification coefficient, commonly called the "R" factor. In the IBC, NBC and SBC provisions, the term is R; in the pre-1997 UBC it is R_w, with the "w" subscript signifying "working" load level design forces. The difference also becomes apparent in the load factors to be applied to the earthquake force effects (E). In the IBC, NBC and SBC, the load factor for earthquake force effects is 1.0. In the pre-1997 UBC, for reinforced concrete design, a load factor of 1.4 is applied to the earthquake force effects. Thus, for reinforced concrete, when comparing the base shear calculated by the pre-1997 UBC with that calculated by the 2000 IBC, or 1993, 1996 or 1999 NBC, or the 1994, 1997 or 1999 SBC, the designer must multiply the UBC base shear by 1.4.

The seismic design force of the 1997 UBC is at strength level, rather than service level. The change was accomplished by changing the former response modification factors, R_w, to strength-based R-factors, similar to those found in the NBC and SBC. Since the load combinations of Section 9.2 of ACI 318-95, reproduced in Section 1909.2 of the 1997 UBC, are intended to be used with service level loads, the UBC had to adopt strength-based load combinations that are intended to be used with strength level seismic forces. Therefore, the 1997 UBC requires that when concrete elements are to be designed for seismic forces or the effects thereof, the strength-based load combinations of UBC Section 1612.2.1 must be used. These load combinations are based on the load combinations of ASCE 7-95[1.10]. The 1997 UBC also requires that when concrete elements are being $_a$designed for seismic forces or the effects thereof using the UBC load combination, a multiplier of 1.1 must be applied to amplify the required strengths. This was felt to be necessary at the time because of a presumed incompatibility between the strength reduction factors of Section 9.3 of ACI 318 and the strength design load combination of ASCE 7-95 that are incorporated into the 1997 UBC. After actual seismic designs were performed using the 1997 UBC provisions, it was discovered that use of the 1.1 multiplier resulted in overly conservative designs when compared to the 1994 UBC. Based on a study of the appropriateness of using the multiplier, the SEAOC Seismology Committee has gone on record recommending that it not be used. For additional information on this subject, see Ref. 1.11.

The vertical distribution of base shear along the height of a building also differs between the UBC and the IBC, NBC and SBC, and remains unchanged from the 1994 to the 1997 UBC. For shorter buildings (with a fundamental period less than or equal to 0.7 second), the UBC requires that the design base shear be distributed throughout the height at each floor level in proportion to the story weights and the heights of the stories above the building base (in accordance with the first mode of vibration of the building). For taller buildings (with fundamental period greater than 0.7 second), the design base shear is divided into two parts. The first part is applied as a concentrated force at the top of the building (to account for higher modes of vibration), with the magnitude being in proportion to the fundamental period of the building, this concentrated force need not exceed 25% of the design base shear. The remainder of the design base shear is distributed as specified for shorter buildings. In the IBC, NBC and SBC, a fraction of the base shear is applied at a story in proportion to the product of story weight and story height (above the base) raised to the power k, where k is a coefficient based on building period. The IBC specifies a k of 1 (linear distribution of V) for $T < 0.5$ sec. It specifies a k of 2 (parabolic distribution of V) for $T > 2.5$ sec. For 0.5 sec. $< T <$ 2.5 sec., two choices are available. One may interpolate between a linear and a parabolic distribution by finding a k-value between 1 and 2, depending upon the period; or one may use a parabolic distribution ($k = 2$), which is always more conservative.

Lastly, the detailing requirements, also termed ductility or toughness requirements, which are applicable to structures in regions of high seismic risk, or assigned to high seismic performance or design categories, are similar in the three model codes. These requirements are essential to impart to buildings the ability to deform beyond the elastic limit and to undergo many cycles of extreme stress reversals. Fortunately, for reinforced concrete structures, all four model codes adopt the ACI 318 standard including Chapter 21 — Special Provisions for Seismic Design. However, the designer will need to refer to the governing model code for any modifications to the ACI 318 seismic requirements. Portions of UBC Chapter 19 that differ substantially from the ACI are printed in italics. The NBC and SBC also include some modifications to the ACI document, most notably for prestressed concrete structures assigned to SPC D or E. Likewise, the IBC includes modifications to ACI 318 in Section 1908, most of which recognize precast concrete systems not in Chapter 21 of ACI 318 for use in structures assigned to SDC D, E or F.

Metric in Concrete Construction — Metric is back. In 1988, federal law mandated the metric system as the preferred system of measurement in the United States. In July 1990, by executive order, all federal agencies were required to develop specific timetables for transition to metric. Some federal agencies involved in construction generally agreed to institute the use of metric units in the design of federal construction by January 1994.

The current editions of the four model codes feature both inch-pound (U.S. Customary) and SI-metric (Systeme International) units. The "soft" metric equivalents are given in the four model codes, generally in parentheses after the English units.

It is noteworthy that when metric conversion was first proposed in the 1970s, some of the standards-writing organizations began preparing metric editions of some of their key documents. The American Concrete Institute first published a "hard" metric companion edition to the ACI 318 standard, ACI 318M-83, in 1983. The current ACI 318 standard is available as ACI 318-02 (U.S. Customary units) and ACI 318M-02 (SI-metric units). Within the same time period, the American Society for Testing and Materials (ASTM) published metric companions to many of its ASTM standards. For example, Standard Specifications A 615M and A 706M for steel bars for concrete reinforcement were developed as metric companions to A 615 and A 706. The older editions of these metric standards are in rounded metric (hard metric) numbers and included ASTM standard metric reinforcing bars. Due to the expense of maintaining two inventories, one for bars in inch-pound units and another for bars in hard metric units, reinforcing bar manufacturers convinced the standards writers to do away with the hard metric standards and develop metric standards based upon soft conversion of ASTM standard inch-pound bars. The latest editions of the ASTM metric reinforcing bar standards reflect this philosophy. Since all federally financed projects have to be designed and constructed in metric, bar manufacturers decided in 1997 that rather than produce the same bars with two different systems of designating size and strength (i.e., inch-pound and metric), they would produce bars with only one system of marking and that would be the system prescribed for the soft metric converted bars. Thus, it is now commonplace to see reinforcing bars with metric size and strength designations on a job that was designed in inch-pound units. It is important to remember that if this occurs on your job, the bars are identical to the inch-pound bars that were specified, except for the markings designating size and strength.

This Eighth edition of the "Notes" is presented in the traditional U.S. Customary units. Largely because of the large volume of this text, unlike in most other PCA publications, no soft metric conversion has been included.

1.1.6 Soil-Supported Slabs

Prior to the 1995 edition of the code, it did not explicitly state whether soil-supported slabs, commonly referred to as slabs-on-grade or slabs-on-ground, were regulated by the code or not. They were explicitly excluded from the 1995 edition of ACI 318 "…unless the slab transmits vertical loads from other portions of the structure to the soil." The 1999 edition expanded the scope by regulating slabs-on-grade that "… transmit vertical loads or *lateral forces* from other portions of the structure to the soil." Mat foundation slabs and other slabs on ground which help support the structure vertically and/or transfer lateral forces from the supported structure to the soil should be designed according to the applicable provisions of the code, especially Chapter 15 - Footings. The

design methodology for typical slabs-on-grade differs from that for building elements, and is addressed in References 1.12 and 1.13. Reference 1.12 describes the design and construction of concrete floors on ground for light industrial, commercial, warehouse, and heavy industrial buildings. Reference 1.13 gives guidelines for slab thickness design for concrete floors on grade subject to loadings suitable for factories and warehouses.

In addition to the modification to 1.1.6, a new Section 21.8, Foundations, was added in Chapter 21 — Special Provisions for Seismic Design in the 1999 edition. Due to sections being added to Chapter 21 in the 2002 edition, these provisions are now in 21.10. Section 21.10.3.4 indicates that "slabs on grade that resist seismic forces from walls or columns that are part of the lateral-force-resisting system shall be designed as structural diaphragms in accordance with 21.9." In this location of Chapter 21, the provisions only apply in regions of high seismic risk, or to structures assigned to high seismic performance or design categories. In regions of low or moderate seismic risk, or for structures assigned to low or intermediate seismic performance or design categories, the provisions of Chapters 1 through 18, or Chapter 22 apply to such slabs, by virtue of the new provision in 1.1.6 (see Table 1-3).

1.1.8 Special Provisions for Earthquake Resistance

Since publication of the 1989 code, the special provisions for seismic design have been located in the main body of the code to ensure adoption of the special seismic design provisions when a jurisdiction adopts the ACI code as part of its general building code. With the continuing high interest nationally in the proper design of buildings for earthquake performance, the code's emphasis on seismic design of concrete buildings continues with this edition. Chapter 21 represents the latest in special seismic detailing of reinforced concrete buildings for earthquake performance.

For an in-depth treatise on the design of concrete buildings for earthquake forces, the designer is referred to *Design of Concrete Buildings for Earthquake and Wind Forces*.[1.14] The text focuses on the proper design and detailing of reinforced concrete buildings for earthquake performance under the 1994 edition of the *Uniform Building Code* (UBC). The book includes a comparison of shearwall design using the design procedures of the 1991 UBC and the revised provisions of the 1994 UBC. Since major changes occurred between the 1994 and 1997 editions of the UBC as discussed above, a new book titled *Design of Concrete Buildings for Earthquake and Wind Forces According to the 1997 Uniform Building Code*[1.15] was developed. It discusses the major differences in the design requirements between the 1994 and 1997 editions of the UBC. Three different types of concrete structural framing systems are designed and detailed for earthquake forces representing regions of high seismicity (Seismic Zones 3 and 4). Although the design examples focus on regions of high seismicity, one chapter discusses the detailing requirements for structures located in regions of low, moderate, and high seismicity. Design of the basic structural systems for wind is also illustrated. As in this "Notes" publication, the emphasis is placed on "how-to-use" the various seismic design and detailing provisions of the latest and possibly the last UBC.

PCA publication, *Design of Low-Rise Concrete Buildings for Earthquake Forces*[1.16], is a companion document to that described above; however, its focus is on designing concrete buildings under the 1996 and 1997 editions of *The BOCA National Building Code* (NBC) and *Standard Building Code* (SBC), respectively. As indicated previously, the seismic provisions of the latest editions of the NBC and SBC are almost identical, and are based on the 1991 edition of the *NEHRP Recommended Provisions for the Development of Seismic Regulations for New Buildings*.[1.5] With the two exceptions noted below, the book is also applicable to the 1993 and 1999 editions of the NBC, and the 1994 and 1999 editions of the SBC. The only difference between the loading requirements of the 1993, and the 1996 and 1999 NBC, is that the load combinations to be used for seismic design under the 1993 edition of the NBC are identical to those that must be used under all three editions of the SBC. Whereas, the 1996 and 1999 NBC adopt by reference the strength design load combinations of ASCE 7-95[1.9]. The second exception is that different editions of ACI 318 are adopted by the various editions of the codes as illustrated in the table below.

Model Code	Edition	Edition of ACI 318 adopted by Model Code
NBC	1993	1989, Revised 1992
	1996	1995
	1999	1995
SBC	1994	1989
	1997	1995
	1999	1995
IBC	2000	1999
IBC	2003*	2002
NFPA 5000	2002*	2002

*These documents had not been completed at press time; however, based on the latest information it is anticipated that the 2002 edition of ACI 318 will be referenced in these codes.

Since designing for seismic forces in areas that have traditionally adopted the NBC or SBC is relatively new, the book will provide excellent background information for the structural engineer. Since the overwhelming majority of all buildings constructed in this country are low-rise, that is the focus of this book. For its purpose, low-rise is defined as less than 65 feet in height or having a fundamental period of vibration of less than 0.7 second. Although not specifically written for it, the book will also serve as a valuable source of information for the engineer designing under the IBC.

To assist the designer in understanding and using the special detailing requirements of Chapter 21 of the Code, PCA has developed a publication titled *Seismic Detailing of Concrete Buildings*[1.17]. Numerous tables and figures illustrate the provisions for buildings located in regions of moderate and high seismic risk – IBC seismic design categories C, D, E and F. While the book is based on the '99 edition of the Code, which is referenced by the 2000 IBC, most of the provisions are applicable to ACI 318-02.

1.1.8.1 Structures at Low Seismic Risk — For concrete structures located in regions of low seismic risk or assigned to low seismic performance or design categories (no or minor risk of damage), no special design or detailing is required; thus, the general requirements of the code, excluding Chapter 21, apply. Concrete structures proportioned by the general requirements of the code are considered to have a level of toughness adequate for low earthquake intensities.

The designer should be aware that the general requirements of the code include several provisions specifically intended to improve toughness, in order to increase resistance of concrete structures to earthquake and other catastrophic or abnormal loads. For example, when a beam is part of the lateral force resisting system of a structure, a portion of the positive moment reinforcement must be anchored at supports to develop its yield strength (see 12.11.2). Similarly, hoop reinforcement must be provided in certain types of beam-column connections (see 11.11.2). Other design provisions introduced since publication of the 1971 code, such as those requiring minimum shear reinforcement (see 11.5.5) and improvements in bar anchorage and splicing details (Chapter 12), also increase toughness and the ability of concrete structures to withstand reversing loads due to earthquakes. With publication of the 1989 code, provisions addressing special reinforcement for structural integrity (see 7.13) were added, to enhance the overall integrity of concrete structures in the event of damage to a major supporting element or abnormal loading.

1.1.8.2 Structures at Moderate/Intermediate or High Seismic Risk — For concrete structures located in regions of moderate seismic risk, or assigned to intermediate seismic performance or design categories (moderate risk of damage), 21.12 includes certain reinforcing details, in addition to those contained in Chapters 1 through 18, that are applicable to reinforced concrete moment frames (beam-column or slab-column framing systems) required to resist earthquake effects. Reflecting terminology that has been in use in the model codes over at least the past decade, frames detailed in accordance with 21.12 are now referred to as *Intermediate*

Moment Frames. These so-called "intermediate" reinforcement details will serve to accommodate an appropriate level of inelastic behavior if the frame is subjected to an earthquake of such magnitude as to require it to perform inelastically. There are no design or detailing requirements in addition to those of Chapters 1 through 18 for other structural components of structures at moderate seismic risk (including structural walls (shearwalls)) regardless of whether they are assumed in design to be a part of the seismic-force-resisting system or not. Structural walls proportioned by the general requirements of the code are considered to have sufficient toughness at drift levels anticipated in regions of moderate seismicity.

The type of framing system provided for earthquake resistance in a structure at moderate seismic risk will dictate whether any special reinforcement details need to be incorporated in the structure.

If the lateral load resisting system consists of moment frames, the details of 21.12 for *Intermediate Moment Frames* must be provided, and 21.2.2.3 shall also apply. Note that even if a load combination including wind load effects (see 9.2.1) governs design versus a load combination including earthquake force effects, the intermediate reinforcement details must still be provided to ensure a limited level of toughness in the moment resisting frames. Whether or not the specified earthquake forces govern design, the frames are the only defense against the effects of an earthquake.

For a combination frame-shearwall structural system, inclusion of the intermediate details will depend on how the earthquake loads are "assigned" to the shearwalls and the frames. If the total earthquake forces are assigned to the shearwalls, the intermediate detailing of 21.12 is not required for the frames. If frame-shearwall interaction is considered in the analysis, with some of the earthquake forces to be resisted by the frames, then the intermediate details of 21.12 are required to toughen up the frame portion of the dual framing system. Model codes have traditionally considered a dual system to be one in which at least 25% of the design lateral forces are capable of being resisted by the moment frames. If structural walls resist total gravity and lateral load effects, no intermediate details are required; the general requirements of the code apply.

For concrete structures located in regions of high seismic risk, or assigned to high seismic performance or design categories (major risk of damage), all structural components must satisfy the applicable special proportioning and detailing requirements of Chapter 21 (excluding 21.12). If, for purposes of design, some of the frame members are not considered as part of the lateral force resisting system, special consideration is still required in the proportioning and detailing of these frame members (see 21.11). The special provisions for seismic design of Chapter 21 are intended to provide a monolithic reinforced concrete structure with adequate toughness to respond inelastically under severe earthquake motions.

Unlike previous editions of the Code, the 2002 edition specifically addresses precast concrete systems for use in structures in regions of moderate or high seismic risk, or in structures assigned to intermediate or high seismic performance or design categories. A special moment frame can either be cast-in-place or erected with precast elements. A precast concrete special moment frame must comply with all the requirements for cast-in-place frames (21.2 through 21.5), plus 21.6. In addition, the requirements for an ordinary moment frame must be satisfied (Chapters 1 through 18). Since there is no provision for an intermediate moment frame of precast elements, by implication such frames erected in structures in regions of moderate seismic risk, or assigned to intermediate seismic performance or design categories must either be special moment frames, or be qualified under the performance criteria of 21.2.1.5. In the 2002 code, the definition of "ordinary moment frame" has been revised to clarify that such a frame can either be cast-in-place or constructed with precast elements, both of which must comply with Chapters 1 through 18.

Two new precast structural walls have been added; an intermediate precast structural wall, and a special precast structural wall. The intermediate precast structural wall must comply with Chapters 1 through 18, plus 21.13. Section 21.13 does not address the wall itself, but covers the connection between individual wall panels, and the connection of wall panels to the foundation. Wherever precast wall panels are used to resist seismic lateral forces in structures in regions of moderate seismic risk or assigned to intermediate seismic performance or design

categories, they must comply with the requirements for an intermediate precast structural wall, or special precast structural wall. By implication, a wall composed of precast elements designed in accordance with Chapters 1 through 18, but not complying with either of these requirements can only be used in structures in regions of low seismic risk, or in structures assigned to low seismic performance or design categories.

The special precast structural wall must comply with Chapters 1 through 18, plus 21.2, 21.7, 21.13.2 and 21.13.3. Wherever precast wall panels are used to resist seismic lateral forces in structures in regions of high seismic risk, or assigned to high seismic performance or design categories, they must comply with the requirements for a special precast structural wall.

The ACI 318 proportioning and detailing requirements for lateral force resisting structural systems of reinforced concrete are summarized in Table 1-1.

Table 1-1 Sections of Code to be Satisfied

Component resisting earthquake effect unless otherwise noted		Level of Seismic Risk or Assigned Seismic Performance of Design Categories as Defined in Code Section Indicated		
		Low 21.2.1.2	Intermediate 21.2.1.3	High 21.2.1.4
Frame members	Cast-in-Place	Ch. 1–18	Ch. 1–18, 21.2.2.3, 21.12	Ch. 1–18, 21.2–21.5
	Precast	Ch. 1–18	Note 1	Ch. 1–18, 21.2–21.6
Structural walls and coupling beams	Cast-in-Place	Ch. 1-18, or Ch. 22	Ch. 1–18	Ch. 1–18. 21.2, 21.7
	Precast	Ch. 1–18, or Ch. 22	Ch. 1–18, 21.13	Ch. 1–18, 21.2, 21.7, 21.8
Structural diaphragms and trusses		Ch. 1–18	Ch. 1–18	Ch. 1–18, 21.2, 21.9
Foundations		Ch. 1–18, or Ch. 22	Ch. 1–18	Ch. 1–18, 21.2, 21.10
Frame members assumed not to resist earthquake forces		Ch. 1–18	Ch. 1–18	Ch. 1–18, 21.11

Note 1: There are no provisions for constructing an intermediate moment frame with precast elements. See 21.2.1.5.

1.1.8.3 Seismic Risk Level Specified in General Building Code — This code has traditionally addressed levels of seismic risk as “low,” “moderate,” or “high.” Precise definitions of seismic risk levels are under the jurisdiction of the general building code, and have traditionally been designated by zones (related to intensity of ground shaking). The model codes specify which sections of Chapter 21 must be satisfied, based on the seismic risk level. As a guide, in the absence of specific requirements in the general building code, seismic risk levels and seismic zones generally correlate as follows:

Seismic Risk Level	Seismic Zone
Low	0 and 1
Moderate	2
High	3 and 4

The above correlation of seismic risk levels and seismic zones refers to the *Uniform Building Code*[1.3].

However, with the adoption of the 1991 NEHRP Provisions into *The BOCA National Building Code* and the *Standard Building Code*, the designer will need to refer to the governing model code to determine appropriate seismic risk level and corresponding special provisions for earthquake resistance. The NBC, SBC, and ’91 NEHRP Provisions, on which the seismic design requirements of the two model codes are based, assign a building to a Seismic Performance Category (SPC). The SPC expresses risk in terms of the nature and use of the building and the expected ground shaking on rock at the building site. To determine the SPC of a structure, one must first determine its Seismic Hazard Exposure Group. Essential facilities are assigned to Seismic Hazard

Exposure Group III, assembly buildings and other structures with a large number of occupants are assigned to Group II. Buildings and other structures not assigned to Group II or III, are considered Group I (see the governing code for more precise definitions of these Seismic Hazard Exposure Groups). The next step is to determine the effective peak velocity-related acceleration coefficient, A_v, given on a contour map that forms part of the NBC and the SBC. With these two items, the structure's SPC can be determined from a table in the governing code that is similar to Table 1-2, which is reproduced from the 1991 NEHRP Provisions.

Table 1-2 Seismic Performance Category[1.5]

Value of A_v	Seismic Hazard Exposure Group		
	I	II	III
$A_v < 0.05$	A	A	A
$0.05 \leq A_v < 0.10$	B	B	C
$0.10 \leq A_v < 0.15$	C	C	C
$0.15 \leq A_v < 0.20$	C	D	D
$0.20 \leq A_v$	D	D	E

The 2000 edition of the *International Building Code*[1.4], the seismic design requirements of which are based on the 1997 NEHRP Provisions[1.8], expresses seismic risk in a fashion similar to that of the NBC and SBC, but with one important difference. The IBC also considers the amplifying effects of softer soils on ground shaking in assigning seismic risk. The terminology used in the IBC for assigning risk, detailing and other purposes is the Seismic Design Category (SDC). The SDC of a building is determined in a manner similar to the SPC in the NBC and SBC. First the building is assigned to a Seismic Use Group (SUG), which is the same as the Seismic Hazard Exposure Group of the NBC and the SBC. At this point the IBC process becomes more involved. Instead of determining one mapped value of expected ground shaking, two spectral response acceleration values are determined from two different maps; one for a short (0.2 second) period and the other for a period of 1 second. These values are then adjusted for site soil effects and multiplied by two-thirds to arrive at design spectral acceleration values. Knowing the SUG and the design spectral acceleration values, one enters two different tables to determine the SDC based on each design value. The governing SDC is the higher of the two, if they differ.

As a guide, for purposes of applicability of special seismic proportioning and detailing requirements, Table 1-3 shows the correlation between UBC seismic zones; the NBC, SBC and 1991 NEHRP Seismic Performance Categories; and the 2000 IBC and 1997 NEHRP Seismic Design Categories.

1.2 DRAWINGS AND SPECIFICATIONS

If the design envisioned by the engineer is to be properly implemented in the field, adequate information needs to be included on the drawings or in the specifications, collectively known as the construction documents. The code has for many editions included a list of items that need to be shown on the construction documents.

1.2.1 Items Required to be Shown

The information required to be included as a part of the construction documents remains essentially unchanged from the 1999 code; however, item "e" has been expanded to require that anchors be shown on the drawings. Enough information needs be shown so anchors can be installed with the embedment depth and edge distances the engineer assumed in the design. In addition, where "supplemental reinforcement" (see definition in D.1) was assumed in the design, the location of the reinforcement with respect to the anchors needs to be indicated.

Table 1-3 — Correlation Between Seismic Risk Levels of ACI 318 and Other Codes and Standards

Code, Standard or Resource Document And Edition	Level of Seismic Risk or Assigned Seismic Performance or Design Categories as Defined in Code Section		
	Low (21.2.1.2)	Moderate/Intermediate (21.2.1.3)	High (21.2.1.4)
BOCA National Building Code 1993, 1996, 1999	SPC[1] A, B	SPC C	SPC D, E
Standard Building Code 1994, 1997, 1999	SPC A, B	SPC C	SPC D, E
Uniform Building Code 1991, 1994, 1997	Seismic Zone 0, 1	Seismic Zone 2	Seismic Zone 3, 4
International Building Code 2000, 2003	SDC[2] A, B	SDC C	SDC D, E, F
ASCE[3] 7-93, 7-95	SPC[1] A, B	SPC C	SPC D, E
NEHRP[4] 1991, 1994	SPC[1] A, B	SPC C	SPC D, E
ASCE[3] 7-98, 7-02	SDC[2] A, B	SDC C	SDC D, E, F
NEHRP[5] 1997	SDC[2] A, B	SDC C	SDC D, E, F

1. *SPC = Seismic Performance Category as defined in building code, standard or resource document*
2. *SDC = Seismic Design Category as defined in building code, standard or resource document*
3. *Minimum Design Loads for Buildings and Other Structures*
4. *NEHRP (National Earthquake Hazards Reduction Program) Recommended Provisions for Seismic Regulations for New Buildings*
5. *NEHRP (National Earthquake Hazards Reduction Program) Recommended Provisions for Seismic Regulations for New Buildings and Other Structures*

1.3 INSPECTION

The ACI code requires that concrete construction be inspected as required by the legally adopted general building code. In the absence of inspection requirements in the general building code or in an area where a building code has not been adopted, the provisions of 1.3 may serve as a guide to providing an acceptable level of inspection for concrete construction. In cases where the building code is silent on this issue or a code has not been adopted, concrete construction, at a minimum; should be inspected by a licensed design professional, someone under the supervision of a licensed design professional, or a qualified inspector. Individuals professing to be qualified to perform these inspections should be required to demonstrate their competence by becoming certified. A voluntary certification program for inspectors of concrete construction has been established by the American Concrete Institute (ACI), Building Officials and Code Administrators International (BOCA), International Conference of Building Officials (ICBO), and Southern Building Code Congress International (SBCCI). Other similar certification programs may also exist.

The four model codes, adopted extensively in the U.S. to regulate building design and construction, require varying degrees of inspection of concrete construction. However, administrative provisions such as these are frequently amended when the model code is adopted locally. The engineer should refer to the specific inspection requirements contained in the general building code having jurisdiction over the construction.

In addition to periodic inspections performed by the building official or his representative, inspections of concrete structures by special inspectors may be required; see discussion below on 1.3.5. The engineer should check the local building code or with the local building official to ascertain if special inspection requirements exist within the jurisdiction where the construction will be occurring. Degree of inspection and inspection responsibility should be set forth in the contract documents. However, it should be pointed out that most codes

*Commentary section numbers are preceded by an "R" (e.g., R1.3.5 refers to Comentary Section R1.3.5).

with provisions for special inspections do not permit the contractor to retain the special inspector. Normally they require that the owner enter into a contract with the special inspector. Therefore, if the frequency and type of inspections are shown in the project's construction documents, it should be made clear that the costs for providing these services are not to be included in the bid of the general contractor.

1.3.4 Records of Inspection

Inspectors and inspection agencies will need to be aware of the wording of 1.3.4. Records of inspection must be preserved for two years after completion of a project, or longer if required by the legally adopted general building code. Preservation of inspection records for a minimum two-year period after completion of a project is to ensure that records are available, should disputes or discrepancies arise subsequent to owner acceptance or issuance of a certificate of occupancy, concerning workmanship or any violations of the approved construction documents, or the general building code requirements.

1.3.5 Special Inspections

Continuous inspection is required for placement of all reinforcement and concrete for special moment frames (beam and column framing systems) resisting earthquake-induced forces in structures located in regions of high seismic risk, or in structures assigned to high seismic performance or design categories. Special moment frames of cast-in-place concrete must comply with the 21.2 - 21.5. Special moment frames constructed with precast concrete elements must comply with the additional requirements of 21.6. For information on how the model building codes in use in the U.S. assign seismic risk, see Table 1-3. The code stipulates that the inspections must be made by a qualified inspector under the supervision of the engineer responsible for the structural design or under the supervision of an engineer with demonstrated capability for supervising inspection of special moment frames resisting seismic forces in regions of high seismic risk, or in structures assigned to high seismic performance or design categories. R1.3.5* indicates that qualification of inspectors should be acceptable to the jurisdiction enforcing the general building code.

This inspection requirement is patterned after similar provisions contained in the *The BOCA National Building Code* (NBC), *International Building Code* (IBC), *Standard Building Code* (SBC), and the *Uniform Building Code* (UBC), referred to in those codes as "special inspections." The specially qualified inspector must "demonstrate competence for inspection of the particular type of construction requiring special inspection." See Section 1.3 above for information on a voluntary certification program for concrete special inspectors. Duties and responsibilities of the special inspector are further outlined as follows:

1. Observe the work for conformance with the approved construction documents.
2. Furnish inspection reports to the building official, the engineer or architect of record, and other designated persons.
3. Submit a final inspection report indicating whether the work was in conformance with the approved construction documents and acceptable workmanship.

The requirement for special inspections by a specially qualified special inspector has long been part of the *Uniform Building Code*; however, it is relatively new in the NBC and the SBC. With the adoption of the NEHRP recommended earthquake provisions by the IBC, NBC and the SBC, the need for special inspections comes to the forefront. An integral part of the NEHRP provisions is the requirement for special inspections of the seismic-force resisting systems of buildings in intermediate and high seismic performance or design categories.

By definition, special inspection by a special inspector implies continuous inspection of construction. For concrete construction, special inspection is required during placement of all reinforcing steel, during the taking of samples of concrete used for fabricating strength test cylinders, and during concrete placing operations. The special inspector need not be present during the entire time reinforcing steel is being placed, provided final inspection of the in-place reinforcement is performed prior to concrete placement. Generally, special inspections are not required for certain concrete work when the building official determines that the construction is of

a minor nature or that no special hazard to public safety exists. Special inspections are also not required for precast concrete elements manufactured under plant control where the plant has been prequalified by the building official to perform such work without special inspections.

Another "inspection" requirement in the IBC and UBC that is not part of the NBC or the SBC is the concept of "structural observation". Under the UBC, structural observation is required for buildings located in high seismic risk areas (Seismic Zone 3 or 4). Under the IBC, it is required for more important structures assigned to seismic design category D, E or F, or sited in an area where the basic wind speed exceeds 110 miles per hour (3-second gust speed). Under the UBC, the owner is required to retain the engineer or architect in responsible charge of the structural design work or another engineer or architect designated by the engineer or architect responsible for the structural design, to perform visual observation of the structural framing system at significant stages of construction and upon completion, for general conformance to the approved plans and specifications. Under the IBC, any registered design professional qualified to perform the work can be retained for the purpose of making structural observations. At the completion of the project, and prior to issuance of the certificate of occupancy, the engineer or architect is required to submit a statement in writing to the building official indicating that the site visits have been made and noting any deficiencies that have not been corrected.

With ever increasing interest in inspection of new building construction in the U.S., especially in high seismic risk areas, the designer will need to review the inspection requirements of the governing general building code, and ascertain the role of the engineer in the inspection of the construction phase.

REFERENCES

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1.2 *Standard Building Code*, Southern Building Code Congress International, Birmingham, AL, 1999.

1.3 *Uniform Building Code*, International Conference of Building Officials, Whittier, CA, 1997.

1.4 *International Building Code*, 2000 Edition, International Code Council, Inc., Falls Church, VA, 2000.

1.5 *NEHRP (National Earthquake Hazards Reduction Program) Recommended Provisions for the Development of Seismic Regulations for New Buildings, Part 1-Provisions, Part 2-Commentary*, Building Seismic Safety Council, Washington, D.C., 1991.

1.6 *Recommended Lateral Force Requirements and Commentary*, Structural Engineers Association of California, San Francisco, CA, 1999.

1.7 *Report and Findings — Uniform Building Code Provisions Compared With the NEHRP Provisions*, prepared for National Institute of Standards and Technology, prepared by International Conference of Building Officials, Whittier, CA, July 1992.

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1.9 Ghosh, S.K., Impact of the Seismic Design Provisions of the International Building Code, Structures and Code Institute, Northbrook, IL, 2001 (PCA Publication LT254).

1.10 American Society of Civil Engineers (1995), *Minimum Design Loads for Buildings and Other Structures*, ASCE 7-95 Standard, ASCE, New York, NY.

1.11 Ghosh, S. K. (1998), "Design of Reinforced Concrete Buildings Under the 1997 UBC," *Building Standards, May/June 1998*, pp. 20-24.

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1.13 *Slab Thickness Design for Industrial Concrete Floors on Grade*, Publication IS195.01D, Portland Cement Association, Skokie, IL, Revised 1996.

1.14 Ghosh, S. K., Domel, A. W. Jr., and Fanella, D. A., *Design of Concrete Buildings for Earthquake and Wind Forces*, Publication EB113.02D, Portland Cement Association, Skokie, IL, 1995.

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1.17 Fanella, D.A., Seismic Detailing of Concrete Buildings, Publication SP382, Portland Cement Association, Skokie, IL, 2000.

2

Materials, Concrete Quality

CHAPTER 3—MATERIALS

UPDATE FOR THE ’02 CODE

1. New to the ’02 Edition of the Code is recognition of ASTM C 1157 (Performance Specification for Blended hydraulic Cement) which focuses on performance rather than the chemical composition of the different types of cements.

2. ASTM A 996 (Specification for Rail-Steel and Axle-Steel Deformed Bars for Concrete Reinforcement) has replaced ASTM A 616 and ASTM A 617, for rail and axle steels, respectively.

3.1 TESTS OF MATERIALS

Provisions in 3.1.3 (and in 1.3.4) require the inspecting engineer and architect responsible for maintaining availability of complete test records during construction. The provisions of 3.1.3 also require that records of tests of materials and of concrete must be retained by the inspector for two years after completion of a project, or longer if required by the legally adopted building code. Retention of test records for a minimum two-year period after completion of a project is to ensure that records are available, should questions arise (subsequent to owner acceptance or issuance of the certificate of occupancy) concerning quality of materials and of concrete, or concerning any violations of the approved plans and specifications or of the building code.

This is required because engineers and architect do not normally inspect concrete, whereas inspectors are typically hired for this purpose. The term “inspector” is defined in 1.3.1. For many portions of the United States, the term “inspector” may be assumed to be the “special inspector”, as defined in the legally adopted building codes. When a special inspector is not employed, other arrangements with the code official will be necessary to insure the availability and retention of the test records.

3.2 CEMENTS

Cement used in the work must correspond to that on which the selection of concrete proportions for strength and other properties was based. This may simply mean the same type of cement or it may mean cement from the same source. In the case of a plant that has determined the standard deviation from tests involving cements from several sources, the former would apply. The latter would be the case if the standard deviation of strength tests used in establishing the required target strength was based on one particular type of cement from one particular source.

New in ACI 318-02 is recognition of ASTM C 1157 (Performance Specification for Blended hydraulic Cement). The ASTM C 1157 standard differs from ASTM C 150 and ASTM C 595 in that it does not establish the chemical composition of the different types of cements. However, individual constituents used to manufacture ASTM C 1157

cements must comply with the requirements specified in the standard. The standard also provides for several optional requirements, including one for cement with low reactivity to alkali-reactive aggregates.

Shrinkage-compensating concrete, made using expansive cement, minimizes the potential for drying shrinkage cracks. Expansive cement expands slightly during the early hardening period after initial setting. When expansion is restrained by reinforcement, expansive cement concrete can also be used to (1) compensate for volume decrease due to drying shrinkage, (2) induce tensile stress in the reinforcement (post-tensioning), and (3) stabilize the long term dimensions of post-tensioned structures with respect to original design. The major advantage of using expansive cement in concrete is in the control and reduction of drying shrinkage cracks.

The proportions of the concrete mix assume additional importance when expansive cement is used in conjunction with some admixtures. The beneficial effects of using expansive cement may be less or may have the opposite effect when some admixtures are used in concrete containing expansive cement. Section 3.6.8 flags this concern. Trial mixtures should be made with the selected admixtures and other ingredients of expansive cement concrete to observe the effects of the admixtures on the properties of the fresh and the hardened concrete.

Also, when expansive cement concrete is specified, the design professional must consider certain aspects of the design that may be affected. Code sections related to such design considerations include:

- Section 8.2.4 - Effects of forces due to expansion of shrinkage-compensating concrete must be given consideration in addition to all the other effects listed.
- Section 9.2.7 - Structural effects due to expansion of shrinkage-compensating concrete must be included in T which is included in the load combinations of Eq. (9-2).

3.3 AGGREGATES

The nominal maximum aggregate size is limited to (i) one-fifth the narrowest dimension between sides of forms, (ii) one-third the depth of the slab, and (iii) three-quarters the minimum clear spacing between reinforcing bars or prestressing tendons or ducts. The limitations on nominal maximum aggregate size may be waived if the workability and methods of consolidation of the concrete are such that the concrete can be placed without honeycomb or voids. The engineer must decide whether the limitations on maximum size of aggregate may be waived.

3.4 WATER

Water used in reinforced concrete (prestressed or nonprestressed) or in concrete that is to have aluminum embedments or is to be cast against stay-in-place galvanized metal forms should not contain deleterious amounts of chloride ion. The chief concern over a high chloride content in mixing water is the possible effect of chloride ions on the corrosion of embedded reinforcing steel or prestressing tendons. Limits on chloride ion content contributed from the ingredients including water, aggregates, cement, and admixtures are given in Chapter 4, Table 4.4.1. An in-depth discussion of this and other topics covered in Chapters 3 through 5 of the code may be found in Ref. 2.1.

Water containing less than 2000 parts per million (ppm) of total dissolved solids can generally be used satisfactorily for making concrete. Water containing more than 2000 ppm of dissolved solids should be tested for its effect on strength and time of set. Seawater is not suitable for use in making concrete that will contain steel

reinforcement, due to risk of corrosion of the reinforcement. Such corrosion is accelerated in warm and humid environments.

3.5 STEEL REINFORCEMENT

3.5.2 Welding of Reinforcement

ACI 318-02 references the latest edition of the Structural Welding Code for Reinforcing Steel - ANSI/AWS D1.4-2000. All welding of reinforcing bars must be performed in strict compliance with the D1.4 requirements. Recent revisions to D1.4 deserve notice. Most notably, the preheat requirements for billet steel bars (A 615) require consideration if the chemical composition of the bars is not known. See discussion on 12.14.3 in Part 4.

The engineer should especially note the welding restrictions of 21.2.7 for reinforcement in earthquake force resisting structural members in buildings in regions of high seismic risk. Because these structural elements may perform beyond the elastic range of response, under potentially extreme effects of major earthquakes, welding of reinforcing steel, especially welded splices, must be performed in strict adherence with the welding procedures outlined in ANSI/AWS D1.4. These procedures include adequate inspection.

Section R3.5.2 provides guidance on welding to existing reinforcing bars (which lack mill test reports) and on field welding of cold drawn wire and welded wire. Cold drawn wire is used as spiral reinforcement, and wires or welded wire may occasionally be field welded. Special attention is necessary when welding cold drawn wire to address possible loss of its yield strength and ductility. Electric resistance welding, as covered by ASTM A 185 and A 497, is an acceptable welding procedure used in the manufacture of welded wire fabric. When welded splices are used in lieu of required laps, pull tests of representative samples or other methods should be specified to determine that an acceptable level of specified strength of steel is provided. “Tack” welding (welding of cross bars) of deformed bars or wire reinforcement is not permitted unless authorized by the engineer (see 7.5.4).

The last paragraph of R3.5.2 states that welding of wire is not covered in ANSI/AWS D1.4. Actually, ANSI/AWS D1.4 addresses the welding of all forms of steel reinforcement, but lacks certain critical information for wire or welded wire fabric (e.g., preheats and electrode selection are not discussed). However, it is recommended that field welding of wire and welded wire fabric follow the applicable provisions of ANSI/AWS D1.4, such as certification of welders, inspection procedures, and other applicable welding procedures.

3.5.3 Deformed Reinforcement

Only deformed reinforcement as defined in Chapter 2 may be used for nonprestressed reinforcement, except that plain bars and plain wire may be used for spiral reinforcement. Welded plain wire fabric is included under the code definition of deformed reinforcement. Reinforcing bars rolled to ASTM A 615 specifications (billet steel) are the most commonly specified for construction. For ACI 318-02, rail and axle steels (ASTM A 616 and ASTM A 617, respectively) have been deleted and replaced by ASTM A 996 (Specification for Rail-Steel and Axle-Steel Deformed Bars for Concrete Reinforcement). Reinforcing bars conforming to ASTM A 996 are designated Type R steel, and must meet more restrictive provisions for bend tests than was required in ASTM A 616 and ASTM A 617. Deformed reinforcement meeting ASTM A 996 will be marked with the letter *R*. Rail steel (ASTM A 996) is not generally available, except in a few areas of the country.

ASTM A 706 covers low-alloy steel deformed bars (Grade 60 only) intended for special applications where welding or bending or both are of importance. Reinforcing bars conforming to A 706 should be specified wherever critical or extensive welding of reinforcement is required, and for use in reinforced concrete structures located in regions of high seismic risk where more bendability and controlled ductility are required. The special provisions of Chapter 21 for seismic design require that reinforcement resisting earthquake-induced flexural and axial forces in frame members and in wall boundary elements forming parts of structures located in regions of high seismic risk comply with ASTM A 706 (see 21.2.5). Grades 40 and 60 ASTM A 615 bars are also permitted in these members

if (a) the ratio of actual ultimate tensile strength to the actual tensile yield strength is not less than 1.25, and (b) the actual yield strength based on mill tests does not exceed the specified yield strength by more than 18,000 psi (retests are not permitted to exceed the specified yield strength by more than an additional 3000 psi).

Before specifying A 706 reinforcement, local availability should be investigated. Most rebar producers can make A 706 bars, but generally not in quantities less than one heat of steel for each bar size ordered. A heat of steel varies from 50 to 200 tons, depending on the mill. A 706 in lesser quantities of single bar sizes may not be immediately available from any single producer. Notably, A 706 is being specified more and more for reinforced concrete structures in high seismic risk areas (see Table 1-1 in Part 1). Not only are structural engineers specifying it for use in earthquake-resisting elements of buildings, but also for reinforced concrete bridge structures. Also, A 706 has long been the choice of precast concrete producers because it is easier and more cost effective for welding, especially in the various intricate bearing details for precast elements. This increased usage should impact favorably on the availability of this low-alloy bar.

Section 9.4 permits designs based on a yield strength of reinforcement up to a maximum of 80,000 psi. Currently there is no ASTM specification for a Grade 80 reinforcement. However, deformed reinforcing bars No. 11, No. 14, and No. 18 with a yield strength of 75,000 psi (Grade 75) are included in the ASTM A 615 specification. The provisions in the A 615 specification for Grade 75 bars are compatible with the ACI code requirements, including the requirement that yield strength correspond to a strain of 0.35 percent (see 3.5.3.2). The designer may simply specify Grade 75 to meet A 615 with no special requirements, or he may delineate exceptions. Certified mill test reports should be obtained from the supplier when Grade 75 bars are used. Before specifying Grade 75, local availability should be investigated. The higher yield strength No. 11, No. 14, and No. 18 bars are intended primarily as column reinforcement. They are used in conjunction with higher strength concrete to reduce the size of columns in high-rise buildings and other applications where high capacity columns are required. Wire used to manufacture both plain and deformed welded wire fabric can have a specified yield strength in excess of 60,000 psi. It is permissible to take advantage of the higher yield strength provided the specified yield strength, f_y, used in the design corresponds to the stress at a strain of 0.35 percent.

A number of manufacturers of reinforcing bars announced during the first quarter of 1997 that they were beginning to phase in the production of soft metric bars. Generally, they intended to shift their production entirely to soft metric bars, and several producers expected to be in full metric production by mid-1997. When the shift to producing soft metric bars is completed, it is expected that they will discontinue production of inch-pound bars.

The physical dimensions (i.e., diameter, and height and spacing of deformations) of the soft-metric bars are no different than the inch-pound bars that have been manufactured for many years. The only difference will be that the bar size mark that is rolled onto the bar will be based on SI metric units. Metric bar sizes and bar marks are based on converting the bar's inch-pound diameter to millimeters and rounding to the nearest millimeter. For example, a No. 4, or 1⁄2-in. diameter bar, becomes a No. 13 bar since its diameter is 12.7 mm. See Table 2-1 for a complete listing of all 11 ASTM standard reinforcing bar sizes.

ASTM standard specifications A 615, A 706 and A 996 have requirements for bars in both inch-pound and SI metric units; therefore, they have dual designations (e.g., ASTM A 706/A 706M). Each specification provides criteria for one or more grades of steel, which are summarized in Table 2-2.

The minimum required yield strength of the steel used to produce the bars has been changed slightly within the ASTM metric bar specifications. For example, the latest editions of the applicable ASTM bar specifications have a Grade 420, or 420 megapascals (MPa) minimum yield strength, which corresponds to approximately 60,900 psi (1,000 psi = 6.895 MPa). Soft converting the typically used Grade 60 or 60,000 psi yield strength steel would have resulted in a metric yield strength of 413.7 MPa. Presently there is a movement within the bar manufacturing industry to "tweak" the minimum yield strength to 415 MPa, or approximately 60,200 psi, so that it is more closely aligned with the existing inch-pound yield strength of 60,000 psi.

Table 2-1 Inch-Pound and Soft Metric Bar Sizes

Inch-Pound		Metric	
Size No.	Dia. (in.)	Size No.	Dia. (mm)
3	0.375	10	9.5
4	0.500	13	12.7
5	0.625	16	15.9
6	0.750	19	19.1
7	0.875	22	22.2
8	1.000	25	25.4
9	1.128	29	28.7
10	1.270	32	32.3
11	1.410	36	35.8
14	1.693	43	43.0
18	2.257	57	57.3

Table 2-2 ASTM Specifications - Grade and Min. Yield Strength

ASTM Specification	Grade/Minimum Yield Strength	
	Inch-Pound (psi)	Metric (MPa)
A 615 and A 615M	40/40,000 60/60,000 75/75,000	300/300 420/420 520/520
A 616 and A 616M	50/50,000 60/60,000	350/350 420/420
A 617 and A 617M	40/40,000 60/60,000	300/300 420/420
A 706/A 706M	60/60,000	520/520

When design and construction proceed in accordance with the ACI 318 Code, using customary inch-pound units, the use of soft metric bars will have only a very small effect on the design strength or allowable load-carrying capability of members. For example, where the design strength of a member is a function of the steel's specified yield strength, f_y, the use of soft metric bars increases the strength approximately 1.5% [(420 - 413.7)/413.7].

3.5.3.5 - 3.5.3.6 Welded Wire Fabric—On occasion, building department plan reviewers have questioned the use of welded wire fabric as structural reinforcement for reinforced concrete, especially when specified as an alternative to conventional reinforcing bars. This usually occurs during the construction phase when reinforcing bars that were initially indicated on the structural drawings are replaced with welded wire fabric through a change order. The code officials' concern probably stems from the commonly accepted industry terminology for welded wire fabric used as "nonstructural" reinforcement for the control of crack widths for slabs-on-ground. That welded wire fabric is commonly referred to as "building fabric."

Wire sizes for welded wire fabric range from W1.4 (10 gauge) to W4 (4 gauge). Plain wire is denoted by the letter "W" followed by a number indicating cross-sectional area in hundredths of a square inch. Styles for fabric used to control crack widths in residential and light industrial slabs-on-grade are 6 x 6 W1.4 x W1.4, 6 x 6 W2 x W2, 6 x 6 W2.9 x W2.9 and 6 x 6 W4 x W4. These welded wire fabrics weigh 0.21 lb, 0.30, 0.42 lb and 0.55 lb per square foot respectively, and are manufactured in rolls, although they are also available in sheets. These "building fabrics" that use smaller wire sizes are not typically used as an alternative to conventional reinforcing bars. Welded wire fabric used for structural reinforcement is typically made with a wire size larger than W4 and is referred to as "structural fabric".

Substitution of welded wire fabric for reinforcing bars may be requested for construction or economic considerations. Whatever the reason, both types of reinforcement, either made with welded wire or reinforcing bars are equally recognized and permitted by the code for structural reinforcement. Both welded deformed wire fabric and welded plain wire fabric are included under the code definition for deformed reinforcement. Welded deformed wire fabric utilizes wire deformations plus welded intersections for bond and anchorage. (Deformed wire is denoted by the letter "D" followed by a number indicating cross-sectional area in hundredths of a square inch.) Welded plain wire fabric bonds to concrete by positive mechanical anchorage at each wire intersection. This difference in bond and anchorage for plain versus deformed fabric is reflected in the development of lap splices provisions of Chapter 12.

3.5.3.7 Coated Reinforcement—Appropriate references to the ASTM specifications for coated reinforcement, A 767 (galvanized) and A 775 (epoxy-coated), are included in the code to reflect increased usage of coated bars. Coated welded wire fabric is available with an epoxy coating (ASTM A 884), with wire galvanized before welding (ASTM A 641), and with welded wire galvanized after welding (ASTM A 123). The most common coated bars and welded wire are epoxy-coated reinforcement for corrosion protection. Epoxy-coated reinforcement provides a viable corrosion protection system for reinforced concrete structures. In recent years, usage of epoxy-coated reinforcement has spread to many types of reinforced concrete construction such as parking garages (exposed to deicing salts), wastewater treatment plants, marine structures, and other facilities located near coastal areas where the risk of corrosion of reinforcement is higher because of exposure to seawater—particularly if the climate is warm and humid.

Designers specifying epoxy-coated reinforcing bars should clearly outline in the project specifications special hardware and handling methods to minimize damage to the epoxy coating during handling, transporting, and placing coated bars, and placing of concrete.[2.3, 2.4] Special hardware and handling methods include:

1. Using nylon lifting slings, or padded wire rope slings.
2. Using spreader bars for lifting bar bundles, or lifting bundles at the third points with nylon or padded slings. Bundling bands should be made of nylon, or be padded.
3. Storing coated bars on padded or wooden cribbing.
4. Not dragging coated bars over the ground, or over other bars.
5. Minimizing walking on coated bars and dropping tools or other construction materials during or after placing the bars.
6. Using bar supports of an organic material or wire bar supports coated with an organic material such as epoxy or vinyl compatible with concrete.
7. Using epoxy- or plastic-coated tie wire, or nylon-coated tie wire to minimize damage or cutting into the bar coating.
8. Setting up, supporting and moving concrete conveying and placing equipment carefully to minimize damage to the bar coating.

Project specifications should also address field touch-up of the epoxy coating after bar placement. Permissible coating damage and repair are included in the ASTM A 775 and in Ref. 2.3. Reference 2.4 contains suggested project specification provisions for epoxy-coated reinforcing bars.

The designer should be aware that epoxy-coated reinforcement requires increased development and splice lengths for bars in tension (see 12.2.4.3).

3.6 ADMIXTURES

3.6.9 Silica Fume

Silica fume (ASTM C 1240) gets its name because it extracted from the fumes of electric furnaces that produce ferrosilicon or silicon metal. By the time it is collected and prepared as an admixture for concrete it has become a very finely divided solid-microsilica. Silica fume is generally used in concrete for one or more of the following reasons. When used in conjunction with high-range water reducing admixtures, it makes it possible to produce concrete with compressive strengths of 20,000 psi (138 MPa) or higher. It is also used to achieve a very dense cement paste matrix to reduce the permeability of concrete. This provides better corrosion protection to reinforcing steel, particularly when the concrete will be subject to direct or indirect applications of deicing chemicals, such as in bridge decks or in parking garages.

Mix proportioning, production methods (mixing and handling), and the placing and curing procedures for silica fume concrete require a more concentrated quality control effort than for conventional concretes. It is imperative that the engineer, concrete supplier, and the contractor work as a team to ensure consistently high quality when silica fume concrete is specified.

Note, concrete containing silica fume can be almost black, dark gray, or practically unchanged from the color of cement, depending on the dosage of silica fume. The greatest differences in color will occur in concretes made with cements that are light in color. Mix proportions may also affect variations in color. If color difference is a problem (architectural concrete), the darkest brand of cement available should be used, and different trial mixtures should be tried during the mix design process.

CHAPTER 4—DURABILITY REQUIREMENTS

UPDATE FOR THE ’02 CODE

A footnote has been added to Tables 4.2.2 and 4.3.1 regarding the durability of structures subject to sulfate and other special exposure conditions. In those situations, the lowest applicable maximum water/cementitious materials ratio, and highest applicable minimum f'_c should be used. Table 4.2.2 is concerned with Requirements for Special Exposure Conditions, while Table 4.3.1 covers Requirements for Concrete Exposed to Sulfate-Containing Solutions.

GENERAL CONSIDERATIONS

The special exposure conditions addressed by the code are located exclusively in Chapter 4 - Durability Requirements, to emphasize the importance of special exposures on concrete durability. The code provisions for concrete proportioning and strength evaluation are located in Chapter 5 - Concrete Quality, Mixing, and Placing. As stated in 5.1.1, selection of concrete proportions must be established to provide for both (a) resistance to special exposures as required by Chapter 4, and (b) conformance with the strength requirements of Chapter 5.

Resistance to special exposures is addressed in 4.2 - Freeze-thaw exposure, 4.3 - Sulfate exposure, and 4.4 - Corrosion protection of reinforcement. Conformance with strength test requirements is addressed in 5.6 - Evaluation and acceptance of concrete. Depending on design and exposure requirements, the lower of the water-cementitious materials ratios required for the structural design requirements and for the concrete exposure conditions must be specified (see Example 2.1)

Unacceptable deterioration of concrete structures in many areas due to severe exposure to freezing and thawing, to deicing salts used for snow and ice removal, to sulfate in soil and water, and to chloride exposure have warranted a stronger code emphasis on the special exposure requirements. Chapter 4 directs special attention to the need for considering concrete durability, in addition to concrete strength.

In the context of the code, durability refers to the ability of concrete to resist deterioration from the environment or the service in which it is placed. Properly designed and constructed concrete should serve its intended function without significant distress throughout its service life. The code, however, does not include provisions for especially severe exposures such as to acids or high temperatures, nor is it concerned with aesthetic considerations such as surface finishes. Items like these, which are beyond the scope of the code, must be covered specifically in the project specifications. Concrete ingredients and proportions must be selected to meet the minimum requirements stated in the code and the additional requirements of the construction documents.

In addition to the proper selection of cement, adequate air entrainment, maximum water-cementitious materials ratio, and limiting chloride ion content of the materials, other requirements essential for durable concrete exposed to adverse environments include: low slump, adequate consolidation, uniformity, adequate cover of reinforcement, and sufficient moist curing to develop the potential properties of the concrete.

4.1 WATER-CEMENTITIOUS MATERIALS RATIO

The traditional "water-cement" ratio was renamed "water-cementitious materials ratio" starting with the 1989 edition of code in recognition of the other cementitious materials permitted by the code to satisfy the code limitations on w/c for concrete durability. The notation, "w/c", is commonly used for "water-cementitious materials" in the term w/c ratio. The definition for "cementitious materials" (see Chapter 2) permits the use of cementitious materials other than portland cement and blended hydraulic cements to satisfy the code's w/c limitation. For the calculation of w/c ratios, as limited by the code, the "cementitious materials" may include:

- portland cement (ASTM C 150)
- blended hydraulic cement (ASTM C 595 and ASTM C 1157)
- expansive hydraulic cement (ASTM C 845)

either by themselves, or in combination with:

- fly ash (ASTM C 618)
- raw or calcinated natural pozzolans (ASTM C 618)
- ground granulated blast-furnace slag (ASTM C 989)
- silica fume (ASTM C 1240)

4.2 FREEZING AND THAWING EXPOSURES

For concrete that will be exposed in a moist condition to freezing and thawing or deicer salts, air-entrained concrete must be specified with minimum air contents for severe and moderate exposure, as set forth in Table 4.2.1. Project specifications should allow the air content of the delivered concrete to be within (-1.5) and (+1.5) percentage points of Table 4.2.1 target values. Severe exposure is a cold climate where the concrete may be exposed almost continuously to wet freeze-thaw conditions or where deicing salts are used. Examples include pavements, bridge decks, sidewalks, and parking garages. For severe exposure conditions, the code also imposes a maximum limit on the w/c ratio and minimum f'_c (See 4.4.2). A moderate exposure is a cold climate where concrete may be exposed to freezing, but will only occasionally be exposed to moisture prior to freezing, and where no deicing salts are used. Examples are certain exterior walls, beams, girders and slabs not in direct contact with soil.

Intentionally entraining air in concrete significantly improves the resistance of hardened concrete to freezing when exposed to water and deicing salts. Concrete that is dry or contains only a small amount of moisture is essentially not affected by even a large number of cycles of freezing and thawing. Sulfate resistance is also improved by air entrainment.

The entrainment of air in concrete can be accomplished by adding an air-entraining admixture at the mixer, by

using an air-entraining cement, or by a combination of both. Air-entraining admixtures, added at the mixer, must conform to ASTM C 260 (3.6.4); air-entraining cements must comply with the specifications in ASTM C 150 and C 595 (3.2.1). Air-entraining cements are sometimes difficult to obtain; and their use has been decreasing as the popularity of air-entraining admixtures has increased. ASTM C 94, Standard Specifications for Ready Mixed Concrete, which is adopted by reference in the ACI code (5.8.2), requires that air content tests be conducted. The frequency of these tests is the same as required for strength evaluation. Samples of concrete must be obtained in accordance with ASTM C 172 and tested in accordance with ASTM C 173 or C 231.

Freeze-thaw resistance is also significantly improved when concrete has a low water-cementitious materials ratio. Normal weight concrete that will be exposed to freeze-thaw conditions while wet and exposed to deicing salts must be proportioned so that both the maximum w/c ratio and the minimum strength as set forth in Table 4.2.2. This helps to ensure that the w/c ratio required for durability will actually be obtained in the field. The introductory commentary statement to code Chapter 4 sets the theme for this code change... "Selection of an f'_c which is consistent with the water-cementitious materials ratio selected for durability will help ensure that the required water-cementitious materials ratio is actually obtained in the field." As noted, it is difficult to accurately determine the w/c ratio of the concrete during production and especially as a jobsite control. The minimum strengths required for durability effectively transfers quality control from a w/c ratio limitation to a more easily controlled specified compressive strength.

A minimum strength only is specified for lightweight aggregate concrete, due to the variable absorption characteristics of lightweight aggregates, which makes calculation of w/c ratios impractical.

Concrete used in water-retaining structures or exposed to severe exposure conditions as described above must be virtually impermeable or watertight. Low permeability not only improves freezing and thawing resistance, especially in the presence of deicing salts, but also improves the resistance of concrete to chloride ion penetration. Concrete that is intended to have low permeability to water must be proportioned so that the specified w/c ratio does not exceed 0.50. If concrete is to be exposed to freezing and thawing in a moist condition, the specified w/c ratio must be no more than 0.45. Also, for corrosion protection of reinforcement in concrete exposed to deicing salts (wet freeze-thaw conditions), and in concrete exposed to seawater (including seawater spray), the concrete must be proportioned so that the specified w/c ratio does not exceed 0.40.

For the above exposure conditions, the corresponding minimum concrete strengths indicated in Table 4.2.2 must also be satisfied. Design Example 2.1 illustrates mix proportioning to satisfy both a w/c ratio and a strength requirement for concrete durability.

4.2.3 Concrete Exposed to Deicing Chemicals

Table 4.2.3 limits the type and amount of portland cement replacement permitted in concrete exposed to deicing salts. The amount of fly ash or other pozzolan, or both, is limited to 25 percent of the total weight of cementitious materials. Slag and silica fume are similarly limited to 50 percent and 10 percent, respectively, of the total weight. If fly ash (or other pozzolan) plus slag and silica fume are used as partial cement replacement, the total weight of the combined replacement materials cannot exceed 50 percent of the total weight of cementitious materials, with the maximum percentage of each type of replacement not to exceed the individual percentage limitations. If slag is excluded from the cement replacement combination, the total weight of the combined replacement cannot exceed 35 percent, with the individual percentages of each also not to be exceeded.

As an example: If a reinforced concrete element is to be exposed to deicing salts, Table 4.2.2 limits the w/c ratio to 0.40. If the mix design requires 280 lb of water to produce an air-entrained concrete mix of a given slump, the total weight of cementitious materials cannot be less than 280/0.40 = 700 lbs. The 700 lbs of "cementitious materials" may be all portland cement or a combination of portland cement and fly ash, pozzolan, slag, or silica fume.

If fly ash is used as portland cement replacement, the maximum amount of fly ash is limited to 0.25 (700) = 175

lbs, maintaining the same w/c = 280/(525+175) = 0.40.

If slag is the total replacement, the maximum is limited to 0.50 (700) = 350 lbs, with w/c = 280/(350+350) = 0.40.

If the cement replacement is a combination of fly ash and slag, the maximum amount of the combination is limited to 0.50 (700) = 350 lbs, with the fly ash portion limited to 0.25 (700) = 175 lbs of the total combination, with w/c = 280/(350+175+175) = 0.40.

If the cement replacement is a combination of fly ash and silica fume (a common practice in high performance concrete), the maximum amount of the combination is limited to 0.35 (700) = 245 lbs, and the silica fume portion limited to 0.10 (700) = 70 lbs, with w/c = 280/(385+245+70) = 0.40.
Obviously, other percentages of cement replacement can be used so long as the combined and individual percentages of Table 4.2.3 are not exceeded.

It should be noted that the portland cement replacement limitations apply only to concrete exposed to the potential damaging effects of deicing chemicals. Research has indicated that fly ash, slag, and silica fume can reduce concrete permeability and chloride ingress by providing a more dense and impermeable cement paste. As to the use of fly ash and other pozzolans, and especially silica fume, it is noteworthy that these cement replacement admixtures are commonly used in high performance concrete (HPC) to decrease permeability and increase strength.

4.3 SULFATE EXPOSURES

Sulfate attack of concrete can occur when it is exposed to soil, seawater, or groundwater having a high sulfate content. Measures to reduce sulfate attack include the use of sulfate-resistant cement. The susceptibility to sulfate attack is greater for concrete exposed to moisture, such as in foundations and slabs on ground, and in structures directly exposed to seawater. For concrete that will be exposed to sulfate attack from soil or water, sulfate-resisting cement must be specified. Table 4.3.1 lists the appropriate types of sulfate-resisting cements and maximum water-cementitious materials ratios and corresponding minimum concrete strengths for various exposure conditions. Degree of exposure is based on the amount of water-soluble sulfate concentration in soil or on the amount of sulfate concentration in water. Note that Table 4.3.1 lists seawater under “moderate exposure,” even though it generally contains more than 1500 ppm of sulfate concentration. The reason is that the presence of chlorides in seawater inhibits the expansive reaction that is characteristic of sulfate attack.[2.1]

In selecting a cement type for sulfate resistance, the principal consideration is the tricalcium aluminate (C_3A) content. Cements with low percentages of C_3A are especially resistant to soils and waters containing sulfates. Where precaution against moderate sulfate attack is important, as in drainage structures where sulfate concentrations in groundwater are higher than normal, but not necessarily severe (0.10 - 0.20 percent), Type II portland cement (maximum C_3A content of eight percent per ASTM C 150) should be specified.

Type V portland cement should be specified for concrete exposed to severe sulfate attack—principally where soils or groundwaters have a high sulfate content. The high sulfate resistance of Type V cement is attributed to its low tricalcium aluminate content (maximum C_3A content of five percent).

Certain blended cements (C 595) also provide sulfate resistance. Other types of cement produced with low C_3A contents are usable in cases of moderate to severe sulfate exposure. Sulfate resistance also increases with air-entrainment and increasing cement contents (low water-cementitious materials ratios).

Before specifying a sulfate resisting cement, its availability should be checked. Type II cement is usually available, especially in areas where resistance to moderate sulfate attack is needed. Type V cement is available only in particular areas where it is needed to resist severe and very severe sulfate environments. Blended cements may not be available in many areas.

4.4 CORROSION PROTECTION OF REINFORCEMENT

Chlorides can be introduced into concrete through its ingredients: mixing water, aggregates, cement, and admixtures, or through exposure to deicing salts, seawater, or salt-laden air in coastal environments. The chloride ion content limitations of Table 4.4.1 are to be applied to the chlorides contributed by the concrete ingredients, not to chlorides from the environment surrounding the concrete (chloride ion ingress). Chloride ion limits are the responsibility of the concrete production facility which must ensure that the ingredients used in the production of concrete (cement, water, aggregate, and admixtures) result in concrete with chloride ion contents within the limits given for different exposure conditions. When testing is performed to determine chloride ion content of the individual ingredients, or samples of the hardened concrete, test procedures must conform to ASTM C 1218, as indicated in 4.4.1. In addition to a high chloride content, oxygen and moisture must be present to induce the corrosion process. The availability of oxygen and moisture adjacent to embedded steel will vary with the in-service exposure condition, which varies between structures, and between different parts of the same structure.

If significant amounts of chlorides may be introduced into the hardened concrete from the concrete materials to be used, the individual concrete ingredients, including water, aggregates, cement, and any admixtures, must be tested to ensure that the total chloride ion concentration contributed from the ingredients does not exceed the limits of Table 4.4.1. These limits have been established to provide a threshold level to avoid corrosion of the embedded reinforcement prior to service exposure. Chloride limits for corrosion protection also depend upon the type of construction and the environment to which the concrete is exposed during its service life, as indicated in Table 4.4.1.

Chlorides are present in variable amounts in all of the ingredients of concrete. Both water soluble and insoluble chlorides exist; however, only water soluble chlorides induce corrosion. Tests are available for determining either the water soluble chloride content or the total (soluble plus insoluble) chloride content. The test for soluble chloride is more time-consuming and difficult to control, and is therefore more expensive than the test for total chloride. An initial evaluation of chloride content may be obtained by testing the individual concrete ingredients for total (soluble plus insoluble) chloride content. If the total chloride ion content is less than that permitted by Table 4.4.1, water-soluble chloride need not be determined. If the total chloride content exceeds the permitted value, testing of samples of the hardened concrete for water-soluble chloride content will need to be performed for direct comparison with Table 4.4.1 values. Some of the soluble chlorides in the ingredients will react with the cement during hydration and become insoluble, further reducing the soluble chloride ion content, the corrosion-inducing culprit. Of the total chloride ion content in hardened concrete, only about 50 to 85 percent is water soluble; the rest is insoluble. Note that hardened concrete should be at least 28 days of age before sampling.

Chlorides are among the more abundant materials on earth, and are present in variable amounts in all of the ingredients of concrete. Potentially high chloride-inducing materials and conditions include: use of seawater as mixing water or as washwater for aggregates, since seawater contains significant amounts of sulfates and chlorides; use of marine-dredged aggregates, since such aggregates often contain salt from the seawater; use of aggregates that have been contaminated by salt-laden air in coastal areas; use of admixtures containing chloride, such as calcium chloride; and use of deicing salts where salts may be tracked onto parking structures by vehicles. The engineer needs to be cognizant of the potential hazard of chlorides to concrete in marine environments or other exposures to soluble salts. Research has shown that the threshold value for a water soluble chloride content of concrete necessary for corrosion of embedded steel can be as low as 0.15 percent by weight of cement. When chloride content is above this threshold value, corrosion is likely if moisture and oxygen are readily available. If chloride content is below the threshold value, the risk of corrosion is low.

Depending on the type of construction and the environment to which it is exposed during its service life, and the amount and extent of protection provided to limit chloride ion ingress, the chloride level in concrete may increase with age and exposure. Protection against chloride ion ingress from the environment is addressed in 4.4.2, with reference to Table 4.2.2. A maximum water-cementitious materials ratio of 0.40 and a minimum strength of 5000 psi must be provided for corrosion protection of “reinforcement in concrete exposed to chlorides from deicing chemicals, brackish water, seawater or spray from these sources.” Resistance to corrosion of

embedded steel is also improved with an increase in the thickness of concrete cover. Section R7.7.5 recommends a minimum concrete cover of 2 in. for cast-in-place walls and slabs, and 2-1/2 in. for other members, when concrete will be exposed to external sources of chlorides in service. For plant-produced precast members, the corresponding recommended minimum concrete covers are 1-1/2 in. and 2 in., respectively.

Other methods of reducing environmentally caused corrosion include the use of epoxy-coated reinforcing steel, corrosion-inhibiting admixtures, surface treatments, and cathodic protection. Epoxy coating of reinforcement prevents chloride ions from reaching the steel. Corrosion-inhibiting admixtures attempt to chemically arrest the corrosive reaction. Surface treatments attempt to stop or reduce chloride ion penetration at the exposed concrete surface. Cathodic protection methods reverse the corrosion current flow through the concrete and reinforcing steel. It should be noted that, depending on the potential severity of the chloride exposure, and the type and importance of the construction, more than one of the above methods may be combined to provide "added" protection. For example, in prestressed parking deck slabs in cold climates where deicing salts are used for snow and ice removal, all conventional reinforcement and the post-tensioning tendons may be epoxy-coated, with the entire tendon system including the anchorages encapsulated in a watertight protective system especially manufactured for aggressive environments. In addition, special high performance (impermeable) concrete may be used, with the entire deck surface covered with a multi-layer membrane surface treatment. Such extreme protective measures may be cost-effective, considering the alternative. Performance tests for chloride permeability of concrete mixtures may also be used to assure corrosion resistance. ASTM C 1202, which has been introduced starting with the 2002 edition of the code, provides a test method for an electrical indication of concrete's ability to resist chloride ion penetration. It is based on AASHTO T 277-83, which was previously referenced in the code.

CHAPTER 5—CONCRETE QUALITY, MIXING, AND PLACING

UPDATE FOR THE '02 CODE

1. A sentence has been added to the end of 5.1.1 that will require that the specified compressive strength, f'_c, of concrete shall be a minimum of 2500 psi.

2. Several significant revisions are made to the design overstrengths and acceptance criterion for concrete that has a specified compressive strength, f'_c, greater than 5000 psi. They include a revision in the calculation for the required average compressive strength, f'_{cr}, in new Table 5.3.2.1 (where test data is available to determine the standard deviation) and in existing Table 5.3.2.2 (where test data is not available). Corresponding changes to the acceptance criterion are made to 5.6.3.3(b), 5.6.5.1 and 5.6.5.2. The current acceptance criterion for all concrete strengths requires an investigation of low-strength when strength tests falling below f'_c by more than 500. The new acceptance criteria for concrete in excess of 5000 psi requires an investigation when strength tests falling below f'_c by more than 10 percent of f'_c. This new criterion will allow larger test strength variations than 500 psi (e.g., 600 psi for f'_c = 6000 psi; 700 psi for f'_c = 7000; 800 psi for f'_c = 8000 psi) that are appropriate based analysis of extensive test data on high strength concrete over the past two decades. The revisions were recommended by the ACI 363 Committee (High Strength Concrete), the ACI 214 Committee (Evaluation of Strength Data) and the ACI 211 Committee (Concrete Proportioning) to provide a more reasonable and realistic criterion for high strength concrete.

3. Preparation and storage of test cores after drilling and the prescribed time limits for their testing have also been revised (see changes to 5.6.5.3).

5.1.1 Concrete Proportions for Strength

Concrete mix designs are proportioned for strength based on probabilistic concepts that are intended to ensure that adequate strength will be developed in the concrete. It is emphasized in 5.1.1 that the average strength of concrete produced must exceed the larger of the value of f'_c specified for the structural design requirements and the minimum strength required for the special exposure conditions set forth in Chapter 4. Concrete proportioned

by the code's probabilistic approach may produce strength tests which fall below the specified compressive strength, f'_c. Section 5.1.1 introduces this concept by noting that it is the code's intent to "minimize frequency of strength below f'_c." If a concrete strength test falls below f'_c, the acceptability of this lower strength concrete is provided for in Section 5.6.2.3.

A minimum 2500 psi specified compressive strength, f'_c, was added to the '02 Edition of the code. This will make the code consistent with minimum provisions that are contained in several national model building codes, including the *International Building Code* (IBC).

5.1.3 Test Age for Strength of Concrete

Section 5.1.3 permits f'_c to be based on tests at ages other than the customary 28 days. If other than 28 days, the test age for f'_c must be indicated on the design drawings or in the specifications. Higher strength concretes, exceeding 6000 psi compression strength, are often used in tall buildings; and concretes having strengths of 20,000 psi have been used. Test ages for the higher strength concretes can justifiably be longer than the customary 28 days. In high-rise structures requiring high-strength concrete, the process of construction is such that the columns of the lower floors are not fully loaded until a year or more elapses after commencement of construction. For this reason, specified compressive strengths, f'_c, based on 56- or 90-day test results are commonly specified.

5.2 SELECTION OF CONCRETE PROPORTIONS

Recommendations for proportioning concrete mixtures are given in detail in *Design and Control of Concrete Mixtures*.[2.1] Recommendations for selecting proportions for concrete are also given in detail in "Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete" (ACI 211.1)[2.5] and "Standard Practice for Selecting Proportions for Structural Lightweight Concrete" (ACI 211.2).[2.6]

The use of field experience or laboratory trial batches (see 5.3) is the preferred method for selecting concrete mixture proportions. When no prior experience or trial batch data are available, permission may be granted by the registered design professional to base concrete proportions on "other experience or information" as prescribed in 5.4.

5.3 PROPORTIONING ON THE BASIS OF FIELD EXPERIENCE AND/OR TRIAL MIXTURES

5.3.1 Standard Deviation

For establishing concrete mixture proportions, emphasis is placed on the use of laboratory trial batches or field experience as the basis for selecting the required water-cementitious materials ratio. The code emphasizes a statistical approach to establishing the target strength f'_{cr} required to ensure attainment of the specified compressive strength, f'_c. If an applicable standard deviation, s, from strength tests of the concrete is known, the target strength level for which the concrete must be proportioned is established. Otherwise, the proportions must be selected to produce a conservative target strength sufficient to allow for a high degree of variability in strength test results. For background information on statistics as it relates to concrete, see "Recommended Practice for Evaluation of Compression Test Results of Concrete"[2.7] and "Statistical Product Control."[2.8]

Concrete used in background tests to determine standard deviation is considered to be "similar" to that specified, if it was made with the same general types of ingredients, under no more restrictive conditions of control over material quality and production methods than are specified to exist on the proposed work, and if its specified strength does not deviate by more than 1000 psi from that f'_c specified. A change in the type of concrete or a significant increase in the strength level may increase the standard deviation. Such a situation might occur with

a change in the type of aggregate; i.e., from natural aggregate to lightweight aggregate or vice versa, or with a change from non-air-entrained concrete to air-entrained concrete. Also, there may be an increase in standard deviation when the average strength level is raised by a significant amount, although the increment in standard deviation should be somewhat less than directly proportional to the strength increase. When there is reasonable doubt as to its reliability, any estimated standard deviation used to calculate the required average strength should always be on the conservative (high) side.

Standard deviations are normally established by at least 30 consecutive tests on representative materials. If less than 30, but at least 15 tests are available, Section 5.3.1.2 provides for a proportional increase in the calculated standard deviation as the number of consecutive tests decrease from 29 to 15.

Statistical methods provide valuable tools for assessing the results of strength tests. It is important that concrete technicians understand the basic language of statistics and be capable of effectively utilizing the tool to evaluate strength test results.

Figure 2-1 illustrates several fundamental statistical concepts. Data points represent six (6) strength test results* from consecutive tests on a given class of concrete. The horizontal line represents the average of tests that is designated $\overline{X}$. The average is computed by adding all test values and dividing by the number of values summed; i.e., in Fig. 2-1:

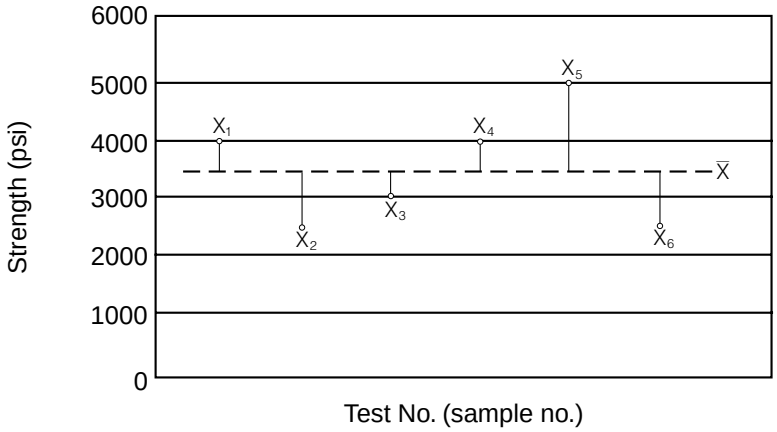


Figure 2-1 Illustration of Statistical Terms

$$\overline{X} = (4000 + 2500 + 3000 + 4000 + 5000 + 2500)/6 = 3500 \text{ psi}$$

The average $\overline{X}$ gives an indication of the overall strength level of the concrete tested.

It would also be informative to have a single number which would represent the variability of the data about the average. The up and down deviations from the average (3500 psi) are given as vertical lines in Fig. 2-1. If one were to accumulate the total length of the vertical lines without regard to whether they are up or down, and divide that total length by the number of tests, the result would be the average length, or the average distance from the average strength:

$$(500 + 1000 + 500 + 500 + 1500 + 1000)/6 = 833 \text{ psi}$$

This is one measure of variability. If concrete test results were quite variable, the vertical lines would be long. On the other hand, if the test results were close, the lines would be short.

In order to emphasize the impact of a few very high or very low test values, statisticians recommend the use of the square of the vertical line lengths. The square root of the sum of the squared lengths divided by one less than

* A strength test result is the average of the strengths of two cylinders made from the same batch of concrete and tested at the same time.

the number of tests (some texts use the number of tests) is known as the standard deviation. This measure of variability is commonly designated by the letter s. Mathematically, s is expressed as:

$$s = \sqrt{\frac{\Sigma \left(X - \overline{X}\right)^2}{n - 1}}$$

where

s = standard deviation, psi
Σ indicates summation
X = an individual strength test result, psi
$\overline{X}$ = average strength, psi
n = number of tests

For example, for the data in Fig. 2-1, the standard deviation would be:

$$s = \sqrt{\frac{\left(X_1 - \overline{X}\right)^2 + \left(X_2 - \overline{X}\right)^2 + \left(X_3 - \overline{X}\right)^2 + \left(X_4 - \overline{X}\right)^2 + \left(X_5 - \overline{X}\right)^2 + \left(X_6 - \overline{X}\right)^2}{6 - 1}}$$

which is calculated below.

Deviation $\left(X - \overline{X}\right)$ (length of vertical lines)					$\left(X - \overline{X}\right)^2$ (length squared)
4000	-	3500	=	+ 500	+ 250,000
2500	-	3500	=	- 1000	+ 1,000,000
3000	-	3500	=	- 500	+ 250,000
4000	-	3500	=	+ 500	+ 250,000
5000	-	3500	=	+ 1500	+ 2,250,000
2500	-	3500	=	- 1000	+ 1,000,000
				Total	+ 5,000,000

$$s = \sqrt{\frac{5,000,000}{5}} = 1{,}000 \text{ psi (a very large value)}$$

For concrete strengths in the range of 3000 to 4000 psi, the expected standard deviation, representing different levels of quality control, will range as follows:

Standard Deviation	Representing
300 to 400 psi	Excellent Quality Control
400 to 500 psi	Good
500 to 600 psi	Fair
> 600 psi	Poor Quality Control

For the very-high-strength, so called high-performance concrete (HPC), with strengths in excess of 10,000 psi, the expected standard deviation will range as follows:

300 to 500 psi	Excellent Quality Control
500 to 700 psi	Good
> 700 psi	Poor Quality Control

Obviously, it would be time consuming to actually calculate s in the manner described above. Most hand-held scientific calculators are programmed to calculate standard deviation directly. The appropriate mathematical equations are programmed into the calculator with the user simply entering the statistical data (test values), then pressing the appropriate function key to obtain standard deviation directly. Example 2.2 illustrates a typical statistical evaluation of strength test results.

The coefficient of variation is simply the standard deviation expressed as a percentage of the average value. The mathematical formula is:

$$V = \frac{s}{\bar{X}} \times 100\%$$

For the test results of Fig. 2-1:

$$V = \frac{1000}{3500} \times 100 = 29\%$$

Standard deviation may be computed either from a single group of successive tests of a given class of concrete or from two groups of such tests. In the latter case, a statistical average value of standard deviation is to be used, calculated by usual statistical methods as follows:

$$s_3 = \sqrt{\frac{(n_1 - 1)(s_1)^2 + (n_2 - 1)(s_2)^2}{n_{total} - 2}}$$

where

n_1 = number of samples in group 1

n_2 = number of samples in group 2

$n_{total} = n_1 + n_2$

s_1 or s_2 is calculated as follows:

$$s = \sqrt{\frac{(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + \ldots + (X_n - \bar{X})^2}{n - 1}}$$

For ease of computation,

$$s = \sqrt{\frac{X_1^2 + X_2^2 + X_3^2 + \ldots + X_n^2 - n\bar{X}^2}{n - 1}}$$

or

$$s = \sqrt{\frac{\left(X_1^2 + X_2^2 + X_3^2 + \ldots + X_n^2\right) - \dfrac{(X_1 + X_2 + X_3 + \ldots + X_n)^2}{n}}{n - 1}}$$

where X_1, X_2, X_3,…X_n are the individual strength test results and n is the total number of strength tests.

5.3.2 Required Average Strength

Where the concrete production facility has a record based on at least 30 consecutive strength tests representing materials and conditions similar to those expected (or a record based on 15 to 29 consecutive tests with the calculated standard deviation modified by the applicable factor from Table 5.3.1.2), the strength used as the basis for selecting concrete proportions for specified compressive strengths, f'_c, equal to or greater than 5000 psi must be the larger of:

$$f'_{cr} = f'_c + 1.34s \qquad \textit{Table 5.3.2.1, (5-1)}$$

and

$$f'_{cr} = f'_c + 2.33s - 500 \qquad \textit{Table 5.3.2.1, (5-2)}$$

For specified compressive strengths, f'_c over 5000, the strength used as the basis for selecting concrete proportions must be the larger of:

$$f'_{cr} = f'_c + 1.34s \qquad \textit{Table 5.3.2.1, (5-1)}$$

and

$$f'_{cr} = 0.90f'_c + 2.33s \qquad \textit{Table 5.3.2.1, (5-3)}$$

If the standard deviation is unknown, the required average strength f'_{cr} used as the basis for selecting concrete proportions must be determined from Table 5.3.2.2:

For f'_c	less than 3000 psi	$f'_{cr} = f'_c + 1000$ psi
	between 3000 and 5000 psi	$f'_{cr} = f'_c + 1200$ psi
	greater than 5000 psi	$f'_{cr} = f'_c + 1.10f'_c + 700$ psi

Formulas for calculating the required target strengths are based on the following criteria:

1. A probability of 1 in 100 that the average of 3 consecutive strength tests will be below the specified strength, f'_c: $f'_{cr} = f'_c + 1.34s$, and

2. A probability of 1 in 100 that an individual strength test will be more than 500 psi below the specified strength f'_c, : $f'_{cr} = f'_c + 2.33s - 500$ (for concrete strengths not over 5000 psi), and

3. A probability of 1 in 100 that an individual strength test will be more than $0.90f'_c$ below the specified strength f'_c (for concrete strengths in excess of 5000 psi): $f'_{cr} = 0.90f'_c + 2.33s$.

Criterion (1) will produce a higher required target strength than Criterion (2) for low to moderate standard deviations, up to 500 psi. For higher standard deviations, however, Criterion (2) will govern.

The average strength provisions of Section 5.3.2 are intended to provide an acceptable level of assurance that concrete strengths are satisfactory when viewed on the following basis: (1) the average of strength tests over an appreciable time period (three consecutive tests) is equal to or greater than the specified compressive strength, f'_c; or (2) an individual strength test is not more than 500 psi below (for concrete strengths not over 5000 psi); or (3) an individual strength test is not more than $0.90f'_c$ below (for concrete strengths in excess of 5000 psi).

5.3.3 Documentation of Average Strength

Mix approval procedures are necessary to ensure that the concrete furnished will actually meet the strength requirements. The steps in a mix approval procedure can be outlined as follows:

1. Determine the expected standard deviation from past experience.

   a. This is done by examining a record of 30 consecutive tests made on a similar mix.

   b. If it is difficult to find a similar mix on which 30 tests have been conducted, the standard deviation can be computed from two mixes, if the total number of tests equals or exceeds 30. The standard deviations are computed separately and then averaged by the statistical averaging method already described.

2. Use the standard deviation to select the appropriate target strength from the larger of Table 5.3.2.1 (5-1), (5-2) and (5-3).

   a. For example, if the standard deviation is 450 psi, then overdesign must be by the larger of:

      1.34 (450) = 603 psi
      2.33 (450) - 500 = 549 psi

      Thus, for a 3000 psi specified strength, the average strength used as a basis for selecting concrete mixture proportions must be 3600 psi.

   b. Note that if no acceptable test record is available, the average strength must be 1200 psi greater than f'_c (i.e., 4200 psi average for a specified 3000 psi concrete), see Table 5.3.2.2.

3. Furnish data to document that the mix proposed for use will give the average strength needed. This may consist of:

   a. A record of 30 tests of field concrete. This would generally be the same test record that was used to document the standard deviation, but it could be a different set of 30 results; or

   b. Laboratory strength data obtained from a series of trial batches.

Where the average strength documentation for strengths over 5000 psi are based on laboratory trail mixtures, it is permitted to increase f'_{cr} calculated in Table 5.3.2.2 to allow for a reduction in strength from laboratory trials to actual concrete production.

Section 5.3.3.2(c) permits tolerances on slump and air content when proportioning by laboratory trial batches. The tolerance limits are stated at maximum permitted values, because most specifications, regardless of form, will permit establishing a maximum value for slump or air content. The wording also makes it clear that these tolerances on slump and air content are to be applied only to laboratory trial batches and not a record of tests on field concrete. Selection of concrete proportions by trial mixtures is illustrated in Example 2.3.

5.4 PROPORTIONING WITHOUT FIELD EXPERIENCE OR TRIAL MIXTURES

When no field or trail mixture data are available, “other experience or information” may be used to select a water-cementitious materials ratio. This mixture proportioning option, however, is permitted only when approved by the project engineer/architect. Note that this option must, of necessity, be conservative, requiring a rather high target overstrength (overdesign) of 1200 psi. If, for example, the specified strength is 3000 psi, the strength used as the basis for selecting concrete mixture proportions (water-cementitious materials ratio) must be based on 4200 psi. In the interest of economy of materials, the use of this option for mix proportioning should be limited to relatively small projects where the added cost of obtaining trial mixture data is not warranted. Note also that this alternative applies only for specified compressive strengths of concrete up to 5000 psi; for higher concrete strengths, proportioning by field experience or trial mixture data is required. The ‘99 Edition of the code limited the maximum strength proportioned without field experience or trial mixtures to 4000 psi.

5.6 EVALUATION AND ACCEPTANCE OF CONCRETE

5.6.1 Laboratory and Field Technicians

The concrete test procedures prescribed in the code require personnel with specific knowledge and skills. Experience has shown that only properly trained field technicians and laboratory personnel who have been certified under nationally recognized programs can consistently meet the standard of control that is necessary to provide meaningful test results. In the '02 code, provisions in 5.6.1 were added to require that tests performed on fresh concrete at the job site and procedures required to prepare concrete specimens for strength tests must be performed by a "qualified field testing technician". Commonly performed field test which will require qualified field testing technicians include sampling of fresh concrete; testing for slump, air content and temperature; and making and curing test specimens. Field technicians in charge of these duties may be qualified through certification in the ACI Concrete Field Testing Technician – Grade I Certification Program.

Section 5.6.1 also requires that "qualified laboratory technicians" must perform all required laboratory tests. Laboratory technicians performing concrete testing may be qualified by receiving certification in accordance with requirements of ACI Concrete Laboratory Testing Technician, Concrete Strength Testing Technician, or the requirements of ASTM C 1007.

The following discussion on Chapter 5 of code addresses the selection of concrete mixture proportions for strength, based on probabilistic concepts.

5.6.2 Frequency of Testing

Proportioning concrete by the probabilistic basis of the code requires that a statistically acceptable number of concrete strength tests be provided. Requiring that strength tests be performed according to a prescribed minimum frequency provides a statistical basis.

The code minimum frequency criterion for taking samples for strength tests** , based on a per day and a per project criterion (the more stringent governs***) for each class of concrete, is summarized below.

5.6.2.1 Minimum Number of Strength Tests Per Day—This number shall be no less than:

- Once per day, nor less than,
- Once for each 150 cu yds of concrete placed, nor less than,
- Once for each 5000 sq ft of surface area of slabs or walls placed.

5.6.2.2 Minimum Number of Strength Tests Per Project —This number shall not be less than:

- Five strength tests from five (5) randomly selected batches or from each batch if fewer than five batches.

If the total quantity of concrete placed on a project is less than 50 cu yds, 5.6.2.3 permits strength tests to be waived by the building official.

According to the ASTM Standard for making concrete test specimens in the field (ASTM C 31), test cylinders should be 6 ×12 in., unless required otherwise by the project specifications.

With the increased use of very-high-strength concretes (in excess of 10,000 psi), the standard 6 × 12 in. cylinder requires very high capacity testing equipment which is not readily available in many testing laboratories. Con-

** Strength test = average of two cylinder strengths (see 5.6.1.4).

*** On a given project, if total volume of concrete is such that frequency of testing required by 5.6.1.1 would provide less than five tests for a given class of concrete, the per project criterion will govern.

sequently, most projects that specify very-high-strength concrete specifically permit the use of the smaller 4 × 8 in. cylinders for strength specimens. The 4 × 8 in. cylinder requires about one-half the testing capacity of the 6 × 12 in. specimen. Also, most precast concrete producers use the 4 × 8 in. cylinders for in-house concrete quality control.

It should be noted that the total number of cylinders cast for a project will normally exceed the code minimum number needed to determine acceptance of concrete strength (two cylinders per strength test). A prudent total number for a project may include additional cylinders for information (7-day tests) or to be field cured to check early strength development for form stripping, plus one or two in reserve, should a low cylinder break occur at the 28-day acceptance test age.

Example 2.4 illustrates the above frequency criteria for a large project (5.6.2.1 controls). Example 2.5 illustrates a smaller project (5.6.2.2 controls).

5.6.3.3 Acceptance of Concrete—The strength level of an individual class of concrete is considered satisfactory if both of the following criteria are met:

1. No single test strength (the average of the strengths of at least two cylinders from a batch) shall be more than 500 psi below the specified compressive strength when f'_c is 5000 psi or less (i.e., less than 2500 psi for a specified 3000 psi concrete strength); or is more than 10 percent below f'_c if over 5000 psi.

2. The average of any three consecutive test strengths must equal or exceed the specified compressive strength f'_c.

Examples 2.6 and 2.7 illustrate "acceptable" and "low strength" strength test results, respectively, based on the above code acceptance criteria.

5.6.5 Investigation of Low-Strength Test Results

If the average of three strength test results in a row is below the specified strength, steps must be taken to increase the strength level of the concrete (see 5.6.3.4). If a single strength test result falls more than 500 psi below the specified strength when f'_c is 5000 or less, or is more than 10 percent below f'_c if over 5000 psi, there may be more serious problems, and an investigation is required according to the procedures outlined in 5.6.5 to ensure structural adequacy.

Note that for acceptance of concrete, a single strength test result (one "test") is always the average strength of 2 cylinders broken at the designated test age, usually 28 days. Due to the many potential variables in the production and handling of concrete, concrete acceptance is never based on a single cylinder break. Two major reasons for low strength test results[2.9] are: (1) improper handling and testing – found to contribute to the majority of low strength investigations; and (2) reduced concrete strength due to an error in production, or the addition of too much water to the concrete on the job. The latter usually occurs because of delays in placement or requests for wet or high slump concrete. High air content due to an over-dosage of air entraining admixture at the batch plant has also contributed to low strength.

If low strength is reported, it is imperative that the investigation follows a logical sequence of possible cause and effect. All test reports should be reviewed and results analyzed before any action is taken. The pattern of strength test results should be studied for any clue to the cause. Is there any indication of actual violation of the specifications? Look at the slump, air content, concrete and ambient temperatures, number of days cylinders were left in the field and under what curing conditions, and any reported cylinder defects.

If the deficiency justifies investigation, testing accuracy should be verified first, and then the structural requirements compared with the measured strength. Of special interest in the early investigation should be the handling and testing of the test cylinders. Minor discrepancies in curing cylinders in mild weather will probably not affect

strength much, but if major violations occur, large reductions in strength may result. Almost all deficiencies involving handling and testing of cylinders will lower strength. A number of simultaneous violations may contribute to significant reductions. Examples include: extra days in the field; curing over 80°F; frozen cylinders; impact during transportation; delay in moist curing at the lab; improper caps; and insufficient care in breaking cylinders.

For in-place concrete investigation, it is essential to know where in the structure the "tested concrete" is located and which batch (truck) the concrete is from. This information should be part of the data recorded at the time the test cylinders were molded. If test results are found deficient, in-place strength testing may be necessary to ascertain compliance with the code and construction documents. If strength is greater than that actually needed, there is little point in investigating the in-place strength. However, if testing procedures conform to the standards and the test results indicate that concrete strength is lower than required for the member in question, further investigation of the in-place concrete may be required (see 5.6.5).

The laboratory should be held responsible for deficiencies in its procedures. Use of qualified lab personnel is essential. Personnel sampling concrete, making test cylinders and operating lab equipment must be qualified by the ACI certification program or equivalent (see 5.6.1).

If core testing should be required, core drilling from the area in question should be performed according to the procedures outlined in ASTM C 42. The testing of cores requires great care in the operation itself and in the interpretation of the results. Detailed procedures are given in ASTM C 42. The following highlights proper core drilling and testing procedures:

1. Wait 14 days (minimum) before core drilling.
2. Drill 3 cores from the questionable area.
3. Drill cores with a diamond bit.
4. Drill core with a diameter of 2-1/2 in. (minimum) or 2 $\times$ maximum aggregate size.
5. Avoid any reinforcing steel in the drilled cores.
6. Drill a minimum core length of 1 $\times$ core diameter, but preferably 2 $\times$ core diameter.
7. If possible, drill completely through member.
8. Allow 2 in. extra length at the core end to be broken out.
9. Use wooden wedges to remove end portions to be broken out.
10. Saw broken ends to plane surfaces.
11. If concrete is dry under service conditions, air dry the cores for 7 days (60 to 70°F, 60% relative humidity). Test the cores dry.
12. If concrete is wet under service conditions, soak the cores in water (73.4 $\pm$ 3°F) for 40 hours. Test the cores wet.
13. Cap the core ends with 1/8 in. thick (or less) capping material.
14. Accurately center the core in the testing machine.
15. Correct the strength for length-to-diameter ratio less than 2, as shown below (interpolate between listed values):

Length-to-Diameter Ratio	Strength Correction Factor
1.94 - 2.10	1.00
1.75	0.98
1.50	0.96
1.25	0.93
1.00	0.87

In addition to the procedures contained in ASTM C 42, a new paragraph was added to the Commentary to 5.6.5 in the '02 Edition of the code to caution that when a water-cooled bit is used to obtain cores, the coring process

causes a moisture gradient between the exterior and interior of the core during, which will adversely affect the core's compressive strength. Thus, a restriction on the commencement of core testing is imposed to provide a minimum time for the moisture gradient to dissipate.

There are several significant changes to the '02 Edition of the code that affect the storage and testing of drilled cores. The provisions in 5.6.5.3 have been completely revised to require that immediately after drilling, cores must have any surface water removed by wiping and be placed in watertight bags or containers prior for transportation and storage. The cores must be tested no earlier than 48 hours, nor more than 7 days after coring unless approved by the registered design professional. In prior editions, storage conditions and restrictions on when testing could be performed were different for concrete in structures that would be "dry" or "superficially wet" under service conditions.

In evaluating core test results, the fact that core strengths may not equal the strength specified for molded cylinders should not be a cause for concern. Specified cylinder strengths allow a large margin for the unknowns of placement and curing conditions in the field as well as for normal variability. For cores actually taken from the structure, the unknowns have already exerted their effect, and the margin of measured strength above required strength can logically be reduced.

Section 5.6.5.4 states that the concrete will be considered structurally adequate if the average strength of three cores is at least 85 percent of f'_c, with no single core strength less than 75 percent of the specified compressive strength. The concrete can be considered acceptable from the standpoint of strength if the core test results for a given location meet these requirements. The structural engineer should examine cases where core strength values fail to meet the above criteria, to determine if there is cause for concern over structural adequacy. If the results of properly made core tests are so low as to leave structural integrity in doubt, further action may be required.

As a last resort, load tests may be required to check the adequacy of structural members, which are seriously in doubt. Generally such tests are suited only for flexural members—floors, beams, and the like—but they may sometimes be applied to other members. In any event, load testing is a highly specialized endeavor that should be performed and interpreted only by an engineer fully qualified in the proper techniques. Load testing procedures and criteria for their interpretation are given in code Chapter 20.

In those rare cases where a structural element fails the load test or where structural analysis of unstable members indicates an inadequacy, appropriate corrective measures must be taken. The alternatives, depending on individual circumstances, are:

- Reducing the load rating to a level consistent with the concrete strength actually obtained.
- Augmenting the construction to bring its load-carrying capacity up to original expectations. This might involve adding new structural members or increasing the size of existing members.
- Replacing the unacceptable concrete.

REFERENCES

2.1 *Design and Control of Concrete Mixtures*, 13th Edition, Publication EB001T, Portland Cement Association, Skokie, IL, 1988.

2.2 ASTM D 3963-99, "Standard Specification for Fabrication and Jobsite Handling of Epoxy-Coated Reinforcing Steel Bars", American Society for Testing and Materials, West Conshohocken, Pennsylvania, 1999.

2.3 ASTM A 775-01, "Standard Specification for Epoxy-Coated Steel Reinforcing Bars", American Society for Testing and Materials, West Conshohocken, Pennsylvania, , 2001.

2.4 ASTM A 934-01, “Standard Specification for Epoxy-Coated Prefabricated Steel Reinforcing Bars,” American Society for Testing and Materials, West Conshohocken, Pennsylvania, , 2001.

2.5 ACI Committee 211.1, “Standard Practice for Selecting Proportions for Normal, Heavyweight and Mass Concrete (ACI 211.1-91 (Reapproved 97), Revised 1985),” American Concrete Institute, Farmington Hills, 1997.

2.6 ACI Committee 211.2, “Standard Practice for Selecting Proportions for Structural Lightweight Concrete (ACI 211.2-98),” American Concrete Institute, Farmington Hills, 1998.

2.7 ACI Committee 214, “Recommended Practice for Evaluation of Compression Test Results of Concrete (ACI 214-77, Reapproved 1997),” American Concrete Institute, Farmington Hills, 1997.

2.8 “Guideline Manual to Quality Assurance / Quality Control” NRMCA Publication 190, National Ready Mixed Concrete Association, Silver Springs, MD 1999.

2.9 “What, Why & How? Low Concrete Cylinder Strength,” *Concrete in Practice*, CIP-9, National Ready Mixed Concrete Association, Silver Spring, MD, 2000.

Example 2.1—Selection of Water-Cementitious Materials Ratio for Strength and Durability

Concrete is required for a loading dock slab that will be exposed to moisture in a severe freeze-thaw climate, but not subject to deicers. A compressive strength f'_c of 3000 psi is specified for structural design. Type I cement with 3/4-in. maximum size normal weight aggregate is specified.

Calculations and Discussion	Code Reference

1. Determine the required minimum strength and maximum w/c ratio for the proposed concrete work to satisfy both design strength and exposure requirements. — *5.2.1*

   For concrete exposed to freezing and thawing in a moist condition, Table 4.2.2 requires a maximum water-cementitious materials ratio of 0.45, and a minimum strength f'_c of 4500 psi. — *4.2.2*

   Since the required strength for the exposure conditions is greater than the required strength for structural design (f'_c = 3000 psi), the strength for the exposure requirements (f'_c = 4500 psi) governs.

2. Select a w/c ratio to satisfy the governing required strength, f'_c = 4500 psi.

   Concrete exposed to freezing and thawing must be air-entrained, with air content indicated in Table 4.2.1. For concrete in a cold climate and exposed to wet freeze-thaw conditions, a target air content of 6% is required for a 3/4-in. maximum size aggregate. — *4.2.1*

   Selection of water-cementitious materials ratio for required strength should be based on trial mixtures or field data made with actual job materials, to determine the relationship between w/c ratio and strength. — *5.3*

   Assume that the strength test data of Example 2.2, with an established standard deviation of 353 psi, represent materials and conditions similar to those expected for the proposed concrete work: — *5.3.1.1*

   a. normal weight, air-entrained concrete
   b. specified strength (4000 psi) within 1000 psi of that required for the proposed work (4500 psi)
   c. 30 strength test results.

   For a standard deviation of 353 psi, the required average compressive strength f'_c to be used as the basis for selection of concrete proportions must be the larger of — *5.3.2*

   $f'_{cr} = f'_c + 1.34s = 4500 + 1.34\,(353) = 5000$ psi, or — *Eq. 5-1*

   $f'_{cr} = f'_c + 2.33s - 500 = 4500 + 2.33\,(353) - 500 = 4800$ psi — *Eq. 5-2*

   Therefore, $f'_{cr} = 5000$ psi.

Note: The average strength required for the mix design should equal the specified strength plus an allowance to account for variations in materials; variations in methods of mixing, transporting, and placing the concrete; and variations in making, curing, and testing concrete cylinder specimens. For this example, with a standard deviation of 353 psi, an allowance of 500 psi for all those variations is made.

Typical trial mixture or field data strength curves are given in Ref. 2.1. Using the field data strength curve, Fig. 7-1 of Ref. 2.1, reproduced in Fig. 2-2, the required water-cementitious materials ratio (w/c) approximately equals 0.38 for an f'_{cr} of 5000 psi. (Use of the typical data curve of Fig. 2-2 is for illustration purposes only; a w/c versus required strength curve that is reflective of local materials and conditions should be used in an actual design situation.)

Since the required w/c ratio of 0.38 for the 4500 psi specified strength is less than the 0.45 required by Table 4.2.2, the 0.38 value must be used to establish the mixture proportions. Note that the specified strength, f'_c = 4500 psi, is the strength that is expected to be equaled or exceeded by the average of any set of three consecutive strength tests, with no individual test more than 500 psi below the specified 4500 psi strength. *5.6.3.3*

As a follow up to this example, the test records of Example 2.2 could probably be used (by the concrete producer) to demonstrate that the concrete mix for which the records were generated will produce the required average strength f'_{cr} of the concrete work for this project. For the purpose of documenting the average strength potential of the concrete mix, the concrete producer need only select 10 consecutive tests from the total of 30 tests that represent a higher average than the required average of 5000 psi. Realistically, the average of the total 30 test results (4835 psi) is close enough to qualify the same concrete mix for the proposed work. *5.3.3*

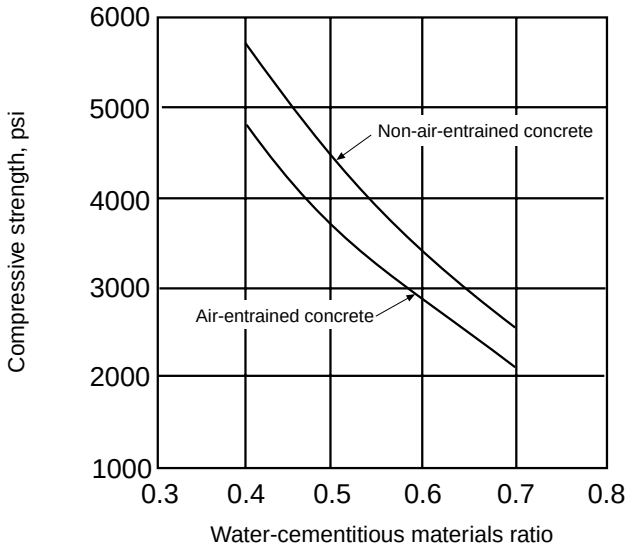


Figure 2-2 Typical Trial Mixture or Field Data Strength Curves

Example 2.2—Strength Test Data Report

Calculate the mean and standard deviation for the 30 strength tests results given below, using the formula for standard deviation given in R5.3.1. The project specifications call for column concrete to be normal weight, air-entrained, with a specified strength of 4000 psi.

Test No.	Date of Test	28-Day #1	28-Day #2	28-Day Average	28-Day Average (3-Consecutive)
1	05-March-99	4640	4770	4705	
2	06-March-99	4910	5100	5005	
3	10-March-99	4570	4760	4665	4792
4	12-March-99	4800	5000	4900	4857
5	13-March-99	5000	4900	4950	4838
6	17-March-99	4380	4570	4475	4775
7	19-March-99	4630	4820	4725	4717
8	21-March-99	4800	4670	4735	4645
9	25-March-99	5020	4940	4980	4813
10	28-March-99	4740	4900	4820	4845
11	30-March-99	4300	4110	4205	4668
12	02-April-99	4280	3620	3950	4325
13	05-April-99	4740	4880	4810	4322
14	08-April-99	4870	5040	4955	4592
15	09-April-99	4590	4670	4630	4798
16	15-April-99	4420	4690	4555	4713
17	16-April-99	4980	5070	5025	4737
18	19-April-99	4900	4860	4880	4820
19	20-April-99	5690	5570	5630	5178
20	22-April-99	5310	5310	5310	5273
21	24-April-99	5080	4970	5025	5322
22	28-April-99	4640	4440	4540	4958
23	01-May-99	5090	5080	5085	4883
24	03-May-99	5430	5510	5470	5032
25	07-May-99	5290	5360	5325	5293
26	10-May-99	4700	4770	4735	5177
27	11-May-99	4880	5040	4960	5007
28	15-May-99	5000	4890	4945	4880
29	16-May-99	4810	4670	4740	4882
30	18-May-99	4250	4400	4325	4670

Calculations and Discussion

Code Reference

Computation of the mean strength and standard deviation is shown in the following table. The standard deviation of 353 psi represents excellent quality control for the specified 4000 psi concrete.

Note that the concrete supplied for this concrete work satisfies the acceptance criteria of 5.6.3.3; no single strength test (28-day average) of two cylinders falls below the specified strength (4000 psi) by more than 500 psi (3500 psi), and the average of each set of 3 consecutive strength tests exceeds the specified strength (4000 psi).

Test No.	28-day Strength, X, psi	$X - \bar{X}$, psi	$(X - \bar{X})^2$
1	4705	-130	16,900
2	5005	170	28,900
3	4665	-170	28,900
4	4900	65	4,225
5	4950	115	13,225
6	4475	-360	129,600
7	4725	-110	12,100
8	4735	-100	10,000
9	4980	145	21,025
10	4820	-15	225
11	4205	-630	396,900
12	3950	-885	783,225
13	4810	-25	625
14	4955	100	10,000
15	4630	-205	42,025
16	4555	-280	78,400
17	5025	190	36,100
18	4880	45	2,025
19	5630	795	632,025
20	5310	475	225,625
21	5025	190	36,100
22	4540	-295	87,025
23	5085	250	62,500
24	5470	635	403,225
25	5325	490	240,100
26	4735	-100	10,000
27	4960	125	15,625
28	4945	110	12,100
29	4740	-95	9,025
30	4325	-510	260,100
Σ	145,060		3,607,850

Number of Tests = *30*
Maximum Strength = *5630 psi*
Minimum Strength = *3950 psi*

$$\textit{Mean Strength} = \frac{145{,}060}{30} = 4835 \text{ psi}$$

$$\textit{Standard Deviation} = \sqrt{\frac{3{,}607{,}850}{29}} = 353 \text{ psi}$$

The single low strength test (3950 psi) results from the very low break for cylinder #2 (3620 psi). The large disparity between cylinder #2 and cylinder #1 (4280 psi), both from the same batch, would seem to indicate a possible problem with the handling and testing procedures for cylinder #2.

Interestingly, the statistical data from the 30 strength test results can be filed for use on subsequent projects to establish a mix design, where the concrete work calls for normal weight, air-entrained concrete with a specified strength within 1000 psi of the specified 4000 psi value (3000 to 5000 psi). The target strength for mix proportioning would be calculated using the 353 psi standard deviation in code Eqs. (5-1) and (5-2). The low standard deviation should enable the "ready-mix company" to produce an economical mix for similar concrete work. The strength test data of this example are used to demonstrate that the concrete mix used for this project qualifies for the proposed concrete work of Example 2.1.

Example 2.3—Selection of Concrete Proportions by Trial Mixtures

Establish a water-cementitious materials ratio for a concrete mixture on the basis of the specified compressive strength of the concrete to satisfy the structural design requirements.

Project Specifications:

f'_c = 3000 psi (normal weight) at 28 days

3/4-in. max. size aggregate

5% total air content

4 in. max slump

Kona sand and gravel

Type I Portland Cement

Assume no strength test records are available to establish a target strength for selection of concrete mixture proportions. The water-cementitious materials ratio is to be determined by trial mixtures. See 5.3.3.2.

Calculations and Discussion	Code Reference
1. Without strength test results, use Table 5.3.2.2 to establish a target strength, f'_{cr}.	*5.3.2.2*
For f'_c = 3000 psi, $f'_{cr} = f'_c + 1200 = 3000 + 1200 = 4200$ psi	
2. Trial Mixture Procedure	*5.3.3.2*
Trial mixtures should be based on the same materials as proposed for the concrete work. Three (3) concrete mixtures with three (3) different water-cementitious materials ratios (w/c) should be made to produce a range of strengths that encompass the target strength f'_{cr}. The trial mixtures should have a slump within ± 0.75 in. of the maximum specified (3.25 to 4.75 in.), and a total air content within ± 0.5% of the maximum permitted by the project specifications (4.5 to 5.5%). Three (3) test cylinders per trial mixture should be made and tested at 28 days. The test results are then plotted to produce a strength versus w/c ratio curve to be used to establish an appropriate w/c ratio for the target strength f'_{cr}.	
To illustrate the trial mixture procedure, assume trial mixtures and test data as shown in Table 2-3. Based on the strength test results plotted in Fig. 2-3 for the three trial mixtures, the maximum w/c ratio to be used as the basis for proportioning the concrete mixture, by interpolation, is 0.49.	
Using a water-cementitious materials ratio of 0.49 to produce a concrete with a specified strength of 3000 psi results in a significant overdesign. Referring to Fig. 2-2, Example 2.1, for a w/c ratio of 0.49, a strength level approximating 3800 psi can be expected for air-entrained concrete. The required extent of mix overdesign, when sufficient strength data are not available to establish a standard deviation, should be apparent.	

Table 2-3 Trial Mixture Data

Trial Mixtures	Batch No. 1	Batch No. 2	Batch No. 3
Selected w/c ratio	0.45	0.55	0.65
Measured slump, in.	3.75	4.25	4.50
Measured air content, %	4.4	5.3	4.8
Test results, psi:			
Cylinder #1	4650	3900	2750
Cylinder #2	4350	3750	2900
Cylinder #3	4520	3650	2850
Average	4510	3770	2830

As strength test data become available during construction, the amount by which the value of f'_{cr} must exceed the specified value of f'_c (1200 psi) may be reduced using a standard deviation calculated from the actual job test data, producing a more economical concrete mix. *5.5*

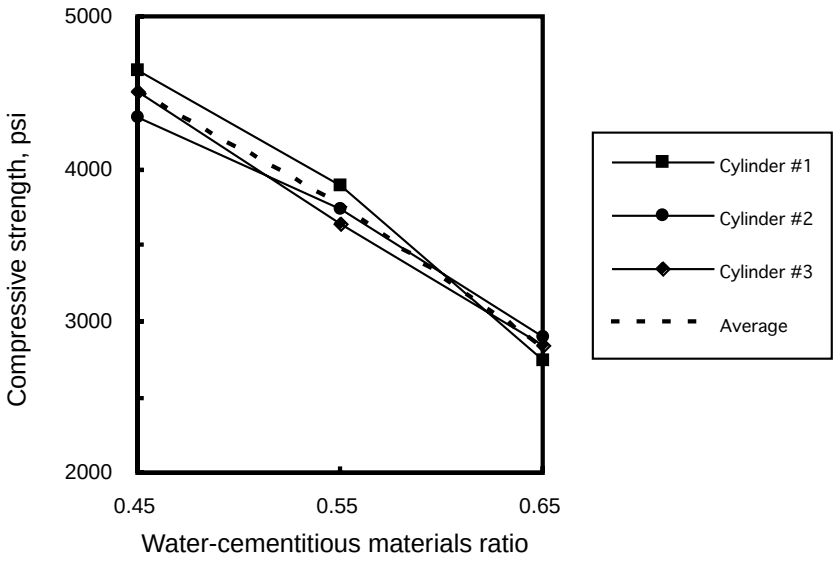


Figure 2-3 Trial Mixture Strength Curve

Example 2.4—Frequency of Testing

Determine the minimum number of test cylinders that must be cast to satisfy the code minimum sampling frequency for strength tests. Concrete placement = 200 cu yd per day for 7 days, transported by 10 cu yd truck mixers. This is a larger project where the minimum number of test cylinders per day of concrete placement (see 5.6.2.1) is greater than the minimum number per project (see 5.6.2.2).

Calculations and Discussion	Code Reference
1. Total concrete placed on project = 200 (7) = 1400 cu yd	
2. Total truck loads (batches) required ≈ 1400/10 ≈ 140	
3. Truck loads required to be sampled per day = 200/150 = 1.3	*5.6.2.1*
4. 2 truck loads must be sampled per day	
5. Total truck loads required to be sampled for project = 2 (7) = 14	
6. Total number of cylinders required to be cast for project = 14 (2 cylinders per test) = 28 (minimum)	*5.6.2.4*

It should be noted that the total number of cylinders required to be cast for this project represents a code required minimum number only that is needed for determination of acceptable concrete strength. Addition cylinders should be cast to provide for 7-day breaks, to provide field cured specimens to check early strength development for form removal or for determining when to post-tension prestressing tendons, and to keep one or two in reserve, should a low cylinder break occur at 28-day.

Example 2.5—Frequency of Testing

Determine the minimum number of test cylinders that must be cast to satisfy the code minimum sampling frequency for strength tests. Concrete is to be placed in a 100 ft × 75 ft × 7-1/2 in. slab, and transported by 10 cu yd truck mixers. This is a smaller project where the minimum required number of test cylinders is based on the frequency criteria of 5.6.2.2.

Calculations and Discussion	Code Reference
1. Total surface area placed = 100 × 75 = 7500 sq ft	
2. Total concrete placed on project = $7500 \times 7.5 \times \frac{1\text{ ft}}{12\text{ in.}}/27$ = 174 cu yd	
3. Total truck loads (batches) required ≈ 174/10 ≈ 18	
4. Required truck loads sampled per day = 174/150 = 1.2 = 7500/5000 = 1.5	*5.6.2.1*
5. But not less than 5 truck loads (batches) per project	*5.6.2.2*
6. Total number of cylinders cast for project = 5 (2 cylinders per test) = 10 (minimum)	*5.6.2.4*

It should again be noted that the total number of cylinders cast represents a code required minimum number only for acceptance of concrete strength. A more prudent total number for a project may include additional cylinders.

Example 2.6—Acceptance of Concrete

The following table lists strength test data from 5 truck loads (batches) of concrete delivered to the job site. For each batch, two cylinders were cast and tested at 28 days. The specified strength of the concrete f'_c is 4000 psi. Determine the acceptability of the concrete based on the strength criteria of 5.6.3.3.

Test No.	Cylinder #1	Cylinder #2	Test Average	Average of 3 Consecutive Tests
1	4110	4260	4185	—
2	3840	4080	3960	—
3	4420	4450	4435	4193
4	3670	3820	3745	4047
5	4620	4570	4595	4258

Calculations and Discussion	Code Reference

The average of the two cylinder breaks for each batch represents a single strength test result. Even though the lowest of the five strength test results (3745 psi) is below the specified strength of 4000 psi, the concrete is considered acceptable because it is not below the specified value by more than 500 psi for concrete with an f'_c not over 5000 psi; i.e., not below 3500 psi. The second acceptance criterion, based on the average of three (3) consecutive tests, is also satisfied by the three consecutive strength test averages shown. The procedure to evaluate acceptance based on 3 strength test results in a row is shown in the right column. The 4193 psi value is the average of the first 3 consecutive test results: (4185 + 3960 + 4435)/3 = 4193 psi. The average of the next 3 consecutive tests is calculated as (3960 + 4435 + 3745)/3 = 4047 psi, after the 4185 psi value is dropped from consideration. The average of the next 3 consecutive values is calculated by dropping the 3960 psi value. For any number of strength test results, the consecutive averaging is simply a continuation of the above procedure. Thus, based on the code acceptance criteria for concrete strength, the five strength tests results are acceptable, both on the basis of individual test results and the average of three consecutive test results.

Example 2.7—Acceptance of Concrete

The following table lists strength test data from 5 truck loads (batches) of concrete delivered to the job site. For each batch, two cylinders were cast and tested at 28 days. The specified strength of the concrete f'_c is 4000 psi. Determine the acceptability of the concrete based on the strength criteria of 5.6.3.3.

Test No.	Cylinder #1	Cylinder #2	Test Average	Average of 3 Consecutive Tests
1	3620	3550	3585	—
2	3970	4060	4015	—
3	4080	4000	4040	3880*
4	4860	4700	4780	4278
5	3390	3110	3250**	4023

**Average of 3 consecutive tests low.*
***One test more than 500 psi below specified value.*

Calculations and Discussion	Code Reference

Investigation of low-strength test results is addressed in 5.6.5. If average of three "tests" in a row dips below the specified strength, steps must be taken to increase the strength of the concrete. If a "test" falls more than 500 psi below the specified strength for concrete with an f'_c not over 5000 psi, there may be more serious problems, requiring an investigation to ensure structural adequacy; and again, steps taken to increase the strength level. For investigations of low strength, it is imperative that the location of the questionable concrete in the structure be known, so that the engineer can make an evaluation of the low strength on the structural adequacy of the member or element. *5.6.5*

Based on experience,[2.9] the major reasons for low strength test results are (1) improper sampling and testing, and (2) reduced concrete quality due to an error in production, or the addition of too much water to the concrete at the job site, caused by delays in placement or requests for wet or high slump concrete. High air content can also be a cause of low strength.

The test results for the concrete from Truck 5 are below the specified value, especially the value for cylinder #2, with the average strength being only 3250 psi. (Note that no acceptance decisions are based on the single low cylinder break of 3110 psi. Due to the many variables in the production, sampling and testing of concrete, acceptance or rejection is always based on the average of at least 2 cylinder breaks.)

3

Details of Reinforcement

UPDATE FOR THE '02 CODE

1. Section 7.5.2.2 was modified to clarify the required reinforcement placement tolerances at discontinuous ends of members and resolve conflicts between the code and ACI 117, *Standard Tolerances for Concrete Construction and Materials*.

2. Section 7.7.3.1 was relocated to Section 7.7.2 and renamed Cast-in-place concrete (prestressed). In addition, Section 7.7.2 was renumbered to Section 7.7.3, renamed Precast concrete (manufactured under plant control conditions) and modified to add prestressing tendons and wire reinforcement to the concrete cover requirements. These changes clarify the cover requirements between prestressed cast-in-place concrete and precast concrete members manufactured under plant conditions.

3. Section 7.7.5.1 has been added to require that the concrete cover be increased by 50% for Class C prestressed concrete members exposed to corrosive environments, if the precompressed fibers are in tension under sustained loads.

4. Section 7.10.5.6 has been added to give requirements for confinement of anchor bolts placed in the top of columns and pedestals, for attachment of other structural members.

5. Section 7.13.2 has been modified to permit mechanical and welded splices in continuous structural integrity reinforcement in beams and to enhance the effectiveness of transverse reinforcement details in perimeter beams.

GENERAL CONSIDERATIONS

Good reinforcement details are vital to satisfactory performance of reinforced concrete structures. Standard practice for reinforcing steel details has evolved gradually. The Building Code Committee (ACI 318) continually collects reports of research and practice related to structural concrete, suggests new research needed, and translates the results into specific code provisions for details of reinforcement.

The ACI Detailing Manual[3.1] provides recommended methods and standards for preparing design drawings, typical details, and drawings for fabrication and placing of reinforcing steel in reinforced concrete structures. Separate sections of the manual define responsibilities of both the engineer and the reinforcing bar detailer. The CRSI Manual of Standard Practice[3.2] provides recommended industry practices for reinforcing steel. As an aid to designers, Recommended Industry Practices for Estimating, Detailing, Fabrication, and Field Erection of Reinforcing Materials are included in Ref. 3.2, for direct reference in project drawings and specifications. The WRI Structural Welded Wire Fabric Detailing Manual[3.3] provides information on detailing welded wire fabric reinforcement systems.

7.1 STANDARD HOOKS

Requirements for standard hooks and minimum finished inside bend diameters for reinforcing bars are illustrated in Tables 3-1 and 3-2. The standard hook details for stirrups and ties apply to No. 8 and smaller bar sizes only.

Table 3-1 Standard Hooks for Primary Reinforcement

Bar size, No.	Min. finished bend dia.[(a)]
3 through 8	$6d_b$
9, 10, 11	$8d_b$
14 and 18	$10d_b$

[(a)] Measured on inside of bar.

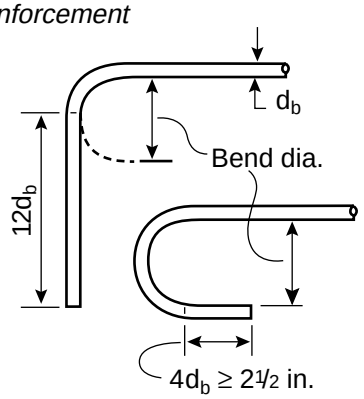
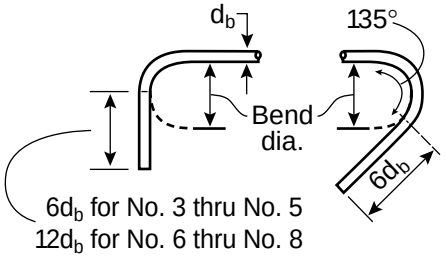


Table 3-2 Standard Hooks for Stirrups and Tie Reinforcement

Bar size, No.	Min. finished bend dia.[(b)]
3 through 5	$4d_b$
6 through 8	$6d_b$

[(b)] Measured on inside of bar.



Moment resisting frames used to resist seismic lateral forces in regions of high seismic risk or in structures assigned to high seismic performance or design categories (see Table 1-1), must be designed as special moment frames as defined in 21.1. In special moment frames, detailing of transverse reinforcement in beams and columns must comply with 21.3.3 and 21.4.4, respectively. Except for circular hoops which are required to have hooks with a 90-deg bend on the free ends, the ends of hoops and crossed ties must terminate in hooks with 135-deg bends. These hooks are necessary to effectively anchor the free ends within the confined core so satisfactory performance is achieved in areas of members where inelastic behavior may occur. See Part 29 of this publication for discussion and illustrations of this special detailing requirement.

7.2 MINIMUM BEND DIAMETERS

Minimum bend diameter for a reinforcing bar is defined as "the diameter of bend measured on the *inside* of the bar." Minimum bend diameters, expressed as multiples of bar diameters, are dependent on bar size; for No. 3 to No. 8 bars, the minimum bend diameter is 6 bar diameters; for No. 9 to No. 11 bars, the minimum bend diameter is 8 bar diameters; and for No. 14 and No. 18 bars, the minimum bend diameter is 10 bar diameters. Exceptions to these provisions are:

1. For stirrups and ties in sizes No. 5 and smaller, the minimum bend diameter is 4 bar diameters. For No. 6 through No. 8 stirrups and ties, the minimum bend diameter is 6 bar diameters.

2. For welded wire fabric used for stirrups and ties, the inside diameter of the bend must not be less than four wire diameters for deformed wire larger than D6 and two wire diameters for all other wire. Welded intersections must be at least four wire diameters away from bends with inside diameters of less than eight wire diameters.

7.3 BENDING

All reinforcement must be bent cold unless otherwise permitted by the engineer. For unusual bends, special fabrication including heating may be required and the engineer must give approval to the techniques used.

7.3.2 Field Bending of Reinforcing Bars

Reinforcing bars partially embedded in concrete are frequently subjected to bending and straightening in the field. Protruding bars often must be bent to provide clearance for construction operations. Field bending and straightening may also be required because of incorrect fabrication or accidental bending. According to 7.3.2, bars partially embedded in concrete must not be field bent without authorization of the engineer unless shown on the plans. Recent tests[3.4] provide guidelines for field bending and straightening, and heating if necessary, of bars partially embedded in concrete. As an aid to the engineer on proper procedure, the recommendations of Ref. 3.4 are stated below. ASTM A615 Grade 60 deformed bars were used in the experimental work on which the recommendations are based.

1. Field bending/straightening should be limited to bar sizes No. 11 and smaller. Heat should be applied for bending/straightening bar sizes No. 6 through No. 11, or for bending/straightening bar sizes No. 5 and smaller when those bars have been previously bent. Previously unbent bars of sizes No. 5 and smaller may be bent/straightened without heating.

2. A bending tool with bending diameter as shown in Table 3-3(a) should be used. Any bend should be limited to 90 degrees.

3. In applying heat for field bending/straightening, the steel temperature should be at or above the minimum temperature shown in Table 3-3(b) at the end of the heating operation, and should not exceed the maximum temperature shown during the heating operation.

4. In applying heat for field bending/straightening, the entire length of the portion of the bar to be bent (or the entire length of the bend to be straightened) should be heated plus an additional 2 in. at each end. For bars larger than No. 9, two heat tips should be used simultaneously at opposite sides of the bar to assure a uniform temperature throughout the thickness of the bar.

5. Before field bending/straightening, the significance of possible reductions in the mechanical properties of bent/straightened bars, as indicated in Table 3-3(c), should be evaluated.

7.5 PLACING REINFORCEMENT

7.5.1 Support for Reinforcement

Support for reinforcement, including tendons and post-tensioning ducts, is required to adequately secure the reinforcement against displacement during concrete placement. The CRSI Manual of Standard Practice[3.2] gives an in-depth treatise on types and typical sizes of supports for reinforcement. Types and typical sizes of wire bar supports are illustrated in Table 3-4. In addition to wire bar supports, bar supports are also available in precast concrete, cementitious fiber-reinforced and all plastic materials. If the concrete surface will be exposed during service, consideration must be given to the importance of the appearance of the concrete surface and the environment to which it will be exposed. For example, if the concrete surface will be exposed directly to the weather or to a humid environment, it is likely that rust spots or stains will eventually show if unprotected bright steel bar

Table 3-3 Field Bending and Straightening of Reinforcing Bars[3.4]

(a) Ratio of Bend Diameter to Bar Diameter

Bar Size, No.	Bend inside diameter/bar diameter	
	Not Heated	Heated
3, 4, 5	8	8
6, 7, 8, 9	Not permitted	8
10, 11	Not permitted	10

(b) Temperature Limits for Heating Bars

Bar Size, No.	Minimum Temperature (°F)	Maximum Temperature (°F)
3, 4	1200	1250
5, 6	1350	1400
7, 8, 9	1400	1450
10, 11	1450	1500

(c) Percent Reduction in Mechanical Properties of Bent and Straightened Bars

Bending Condition	Bar Size, No.	% Yield Strength Reduction	% Ultimate Tensile Strength Reduction	% Elongation Reduction
Cold	3, 4	—	—	20
Cold	5	5	—	30
Hot	All sizes	10	10	20

supports are used. As outlined in the CRSI manual, bar supports are available in three classes of protection, depending on their expected exposure and the amount of corrosion protection required. Based on current industry practice, the available classes of protection are:

Class 1 Maximum Protection
Plastic protected bar supports intended for use in situations of moderate to severe exposure and/or situations requiring light grinding or sandblasting of the concrete surface.

Class 1A Maximum Protection (For Use With Epoxy-Coated Reinforcement Bars) Epoxy-, vinyl-, or plastic coated bright basic wire bar supports intended for use in situations of moderate to maximum exposure where no grinding or sandblasting of the concrete surface is required. Generally, they are used when epoxy-coated reinforcing bars are required.

Class 2 Moderate Protection
Stainless steel protected steel wire bar supports intended for use in situations of moderate exposure and/or situations requiring light grinding or sandblasting of the concrete surface. The bottom of each leg is protected with a stainless steel tip.

Class 3 No Protection
Bright basic wire bar supports with no protection against rusting. Unprotected wire bar supports are intended for use in situations where surface blemishes can be tolerated, or where supports do not come into contact with a concrete surface which is exposed.

The engineer will need to specify the proper class of protection in the project specifications. It should be noted that the support system for reinforcement is usually detailed on the reinforcement placing drawings prepared by the "rebar" fabricator. The support system, including the proper class of protection, should be reviewed by the engineer, noting that the bar support size also dictates the proper cover for the reinforcement.

Use of epoxy-coated reinforcing bars will require bar supports made of a dielectrical material, or wire supports coated with a dielectrical material such as epoxy or vinyl, which is compatible with concrete. See discussion on 3.5.3.7, in Part 2 of this document, concerning special hardware and handling to minimize damage to the epoxy coating during handling, transporting, and placing epoxy-coated bars.

Commentary R7.5.1 emphasizes the importance of rigidly supporting the beam stirrups, in addition to the main flexural reinforcement, directly on the formwork. If not supported directly, foot traffic during concrete placement can push the web reinforcement down onto the forms, resulting in loss of cover and potential corrosion problems. It should be noted that the CRSI Manual of Standard Practice[3.2], often referenced in the design documents for placing reinforcing bars, does not specifically address this need for direct web reinforcement support. The placing drawings, usually prepared by the bar fabricator, should show a typical section or detail, so that this support requirement is clear and not overlooked by the ironworkers.

A word of caution on reinforcement displacement during concrete placing operations. If concrete placement is by pumping, it is imperative that the pipelines and the pipeline support system be supported above and independently of the chaired reinforcement by “chain-chairs” or other means. There must be no contact, direct or indirect, with the chaired reinforcement; otherwise, the surging action of the pipeline during pumping operations can, and most assuredly will, completely dislodge the reinforcement. This potential problem is especially acute in relatively thin slab members, especially those containing tendons, where the vertical placement of the reinforcement is most critical. The project specifications should specifically address this potential concrete placement problem.

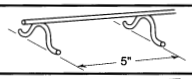
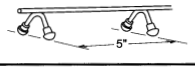
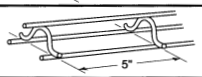
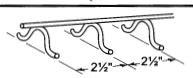
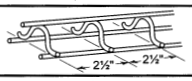
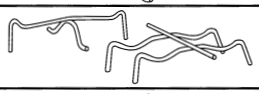
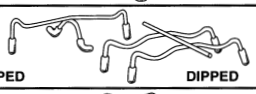
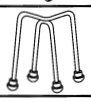
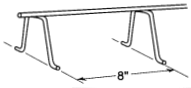
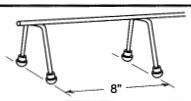
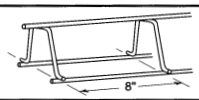
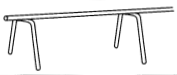
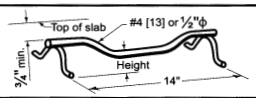
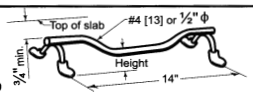
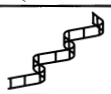
7.5.2 Tolerances in Placing Reinforcement

The code provides tolerances applied simultaneously to concrete cover and member effective depth, d. With dimension “d” being the most structurally important dimension, any deviation in this dimension, especially for members of lesser depth, can have an adverse effect on the strength provided in the completed construction. The permitted variation from the effective depth d takes this strength reduction into account, with a smaller permitted variation for the shallower members. The permitted tolerances are also established to reflect common construction techniques and practices. The critical dimensional tolerances for locating the longitudinal reinforcement are illustrated in Table 3-5, with two exceptions:

1. Tolerance for clear distance to formed soffits must not exceed minus 1/4 in.

2. Tolerance for cover must not exceed minus one-third the minimum concrete cover required in the design drawings and specifications. See Example 3.1

Table 3-4 Types and Sizes of Wire Bar Supports[3.2]

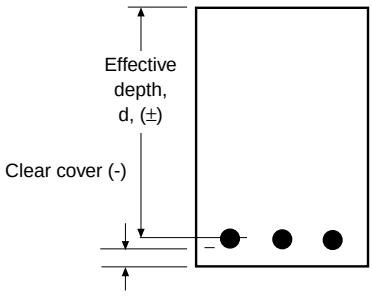
SYMBOL	BAR SUPPORT ILLUSTRATION	BAR SUPPORT ILLUSTRATION PLASTIC CAPPED OR DIPPED	TYPE OF SUPPORT	TYPICAL SIZES
SB		 CAPPED	Slab Bolster	¾, 1, 1½, and 2 in. heights in 5 ft and 10 ft lengths
SBU*			Slab Bolster Upper	Same as SB
BB		 CAPPED	Beam Bolster	1, 1½, 2 to 5 in. heights in increments of ¼ in. in lengths of 5 ft
BBU*			Beam Bolster Upper	Same as BB
BC		 DIPPED	Individual Bar Chair	¾, 1, 1½, and 1¾ in. heights
JC		 DIPPED DIPPED	Joist Chair	4, 5, and 6 in. widths and ¾, 1 and 1½ in. heights
HC		 CAPPED	Individual High Chair	2 to 15 in. heights in increments of ¼ in.
HCM*			High Chair for Metal Deck	2 to 15 in. heights in increments of ¼ in.
CHC		 CAPPED	Continuous High Chair	Same as HC in 5 ft and 10 ft lengths
CHCU*			Continuous High Chair Upper	Same as CHC
CHCM*			Continuous High Chair for Metal Deck	Up to 5 in. heights in increments of ¼ in.
JCU**		 DIPPED	Joist Chair Upper	14 in. span; heights –1 in. thru +3½ in. vary in ¼ in. increments
CS			Continuous Support	1½ to 12 in. in increments of ¼ in. in lengths of 6'-8"
SBC			Single Bar Centralizer (Friction)	6 in. to 24 in. diameter

*Usually available in Class 3 only, except on special order.
**Usually available in Class 3 only, with upturned or end-bearing legs.

1 in. = 25.4 mm
1 ft = 304.8 mm

Table 3-5 Critical Dimensional Tolerances for Placing Reinforcement

Effective Depth d	Tolerance on d	Tolerance on Min. Cover
d ≤ 8 in.	± 3/8 in.	- 3/8 in.
d > 8 in.	± 1/2 in.	- 1/2 in.



For ends of bars and longitudinal location of bends, the tolerance is ± 2 in., except at discontinuous ends of corbels and brackets where the tolerance is ± 1/2 in. At the discontinuous ends of other members the tolerance is permitted to be + 1 in. The tolerance for minimum cover in 7.5.2.1 shall also apply. These tolerances are illustrated in Fig. 3-1.

Note that a plus (+) tolerance increases the dimension and a minus (-) tolerance decreases the dimension. Where only a minus tolerance is indicated, there is no limit in the other direction. Quality control during construction should be based on the more restrictive of related tolerances.

In addition to the code prescribed rebar placing tolerances, the engineer should be familiar with ACI Standard 117, *Standard Tolerances for Concrete Construction and Materials*.[3.5] ACI 117 includes tolerances for all measured dimensions, quantities and concrete properties used in concrete construction. The ACI 117 document is intended to be used by direct reference in the project specifications; therefore it is written in a specification format.

The designer must specify and clearly identify cover tolerances as the needs of the project dictate. For example, if concrete is to be exposed to a very aggressive environment, such as deicing salts, where the amount of concrete cover to the reinforcement may be a critical durability consideration, the engineer may want to indicate closer tolerances on concrete cover than those permitted by the code, or alternatively, specify a larger cover in recognition of expected variation in the placing of the reinforcement.

7.5.4 "Tack" Welding

Note that welding of crossing bars (tack welding) for assembly of reinforcement is prohibited except as specifically authorized by the engineer. By definition, a tack weld is a small spotweld to facilitate fabrication or field installation of reinforcement, and is not intended as a structural weld. Tack welding can lead to local embrittlement of the steel, and should never be done on reinforcement required by design. As noted in 3.5.2, all welding of reinforcement must conform to controlled welding procedures specified in AWS D1.4, including proper preheat (if required), and welding with electrodes meeting requirements of final welds.

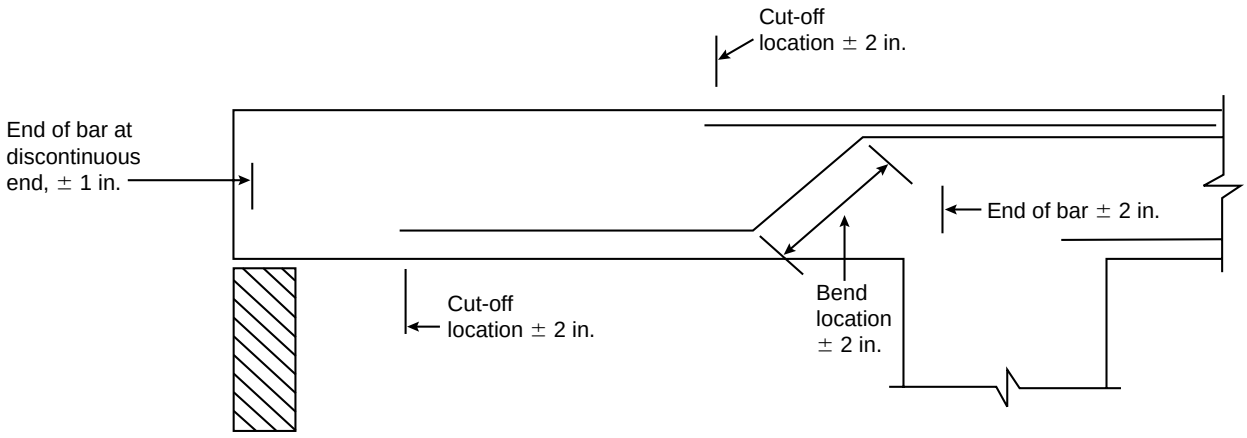


Figure 3-1 Tolerances for Bar Bend and Cutoff Locations

7.6 SPACING LIMITS FOR REINFORCEMENT

Spacing (clear distance) between bars must be as follows:

Minimum Spacing

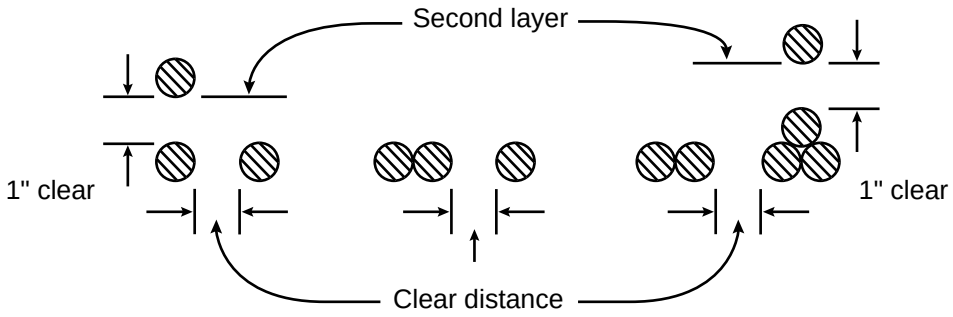
For members with parallel bars in a layer, the clear spacing between bars must be at least one bar diameter but not less than 1 in.; and for reinforcement in two or more layers, bars in the upper layers must be directly above bars in the bottom layer, with at least 1 in. clear vertically between layers. For spirally reinforced and tied reinforced compression members, the clear distance between longitudinal bars must be at least 1-1/2 bar diam-

eters, but not less than 1-1/2 in. These spacing requirements also apply to clear distance between contact-lap-spliced single or bundled bars and adjacent splices or bars. Section 3.3.2, which contains spacing requirements based on maximum nominal aggregate size, may be applicable. Clear distances between bars are illustrated in Table 3-6.

Maximum Spacing

In walls and slabs other than concrete joists, primary flexural reinforcement must not be spaced greater than 3 times the wall or slab thickness nor 18 in.

Table 3-6 Clear Distances Between Bars, Bundles, or Tendons



Reinforcement Type	Type Member	Clear Distance
Deformed bars	Flexural members	$d_b \geq 1$ in.
	Compression members, tied or spirally reinforced	$d_b \geq 1$ in.
Pretensioning tendons	Wires	$4d_b$
	Strands	$3d_b$

7.6.6 Bundled Bars

For isolated situations requiring heavy concentration of reinforcement, bundles of standard bar sizes can save space and reduce congestion for placement and consolidation of concrete. In those situations, bundling of bars in columns is a means to better locating and orienting the reinforcement for increased column capacity; also, fewer ties are required if column bars are bundled.

Bundling of bars (parallel reinforcing bars in contact, assumed to act as a unit) is permitted, but only if such bundles are enclosed by ties or stirrups. Some limitations are placed on the use of bundled bars as follows:

1. No. 14 and No. 18 bars cannot be bundled in beams.

2. If individual bars in a bundle are cut off within the span of beams, such cutoff points must be staggered at least 40 bar diameters.

3. A maximum of two bundled bars in any one plane is implied (three or four adjacent bars in one plane are not considered as bundled bars).

4. For spacing and concrete cover, a unit of bundled bars must be treated as a single bar with an area equivalent to the total area of all bars in the bundle. Equivalent diameters of bundled bars are given in Table 3-7.

5. A maximum of four bars may be bundled (See Fig. 3-2).

6. Bundled bars must be enclosed within stirrups or ties.

Table 3-7 Equivalent Diameters of Bundled Bars, in.

Bar Size, No.	Bar Diameter	2-Bar Bundle	3-Bar Bundle	4-Bar Bundle
6	0.750	1.06	1.30	1.50
7	0.875	1.24	1.51	1.75
8	1.000	1.42	1.74	2.01
9	1.128	1.60	1.95	2.26
10	1.270	1.80	2.20	2.54
11	1.410	1.99	2.44	2.82
14	1.693	2.39	2.93	3.39

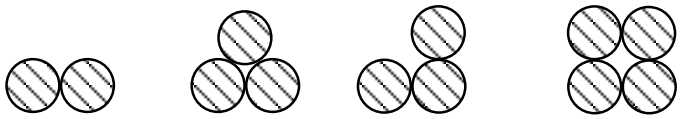


Figure 3-2 Possible Reinforcing Bar Bundling Schemes

7.6.7 Prestressing Steel and Ducts

Prior to the ’99 code, distances between prestressed steel were specified in terms of minimum clear distances. The ’99 code specified distances between prestressed steel in terms of minimum center-to-center spacing and requires $4d_b$ for strands and $5d_b$ for wire. When the compressive strength of the concrete at the time of prestress transfer, f'_{ci} is 4000 psi or greater, the minimum center-to-center spacing can be reduced to 1-3/4 in. for strands 1/2-in. nominal diameter or smaller and 2 in. for strands 0.6-in. nominal diameter. These changes were made as a result of research sponsored by the Federal Highway Administration. Center-to-center spacing is now specified because that is the way it was measured in the research. In addition, converting to clear spacing is awkward and unnecessary, and templates used by precast manufacturers have always been fabricated based on center-to-center dimensions. Closer vertical spacing or bundling of prestressed steel is permitted in the middle portion of the span if special care in design and fabrication is employed. Post-tensioning ducts may be bundled if concrete can be satisfactorily placed and provision is made to prevent the tendons from breaking through the duct when tensioned.

7.7 CONCRETE PROTECTION FOR REINFORCEMENT

Concrete cover or protection requirements are specified for members cast against earth, in contact with earth or weather, and for interior members not exposed to weather. Starting with the ’02 code, the location of the cover requirements for cast-in-place concrete (prestressed) was reorganized. Cast-in-place concrete (prestressed) immediately follows cast-in-place (nonprestressed). They are then followed by the cover requirements for precast concrete manufactured under plant conditions. In some cases slightly reduced cover or protection is permitted under the conditions for cast-in-place (prestressed) and precast concrete manufactured under plant control control conditions than permitted for cast-in-place concrete (non-prestressed). The term “manufactured under plant controlled conditions” does not necessarily mean that precast members must be manufactured in a plant. Structural elements precast at the job site (e.g., tilt-up concrete walls) will also qualify for the lesser cover if the control of form dimensions, placing of reinforcement, quality of concrete, and curing procedure are equivalent to those normally expected in a plant operation. Larger diameter bars, bundled bars, and prestressed tendons require slightly greater cover. Corrosive environments or fire protection may also warrant increased cover. Section 18.3.3, which was introduced in the ’02 code, requires that prestressed flexural members be classified as Class U (uncracked), Class C (cracked), or Class T (transition between uncracked and cracked). Section 7.7.5.1,

also new to the ’02 code, requires the cover of 7.7.2 be increased 50% for prestressed members classified as Class C or T where the members are exposed to corrosive environments or other severe exposure conditions. The requirement to increase the cover by 50% may be waived if the precompressed zone is not in tension under sustained load. The designer should take special note of the commentary recommendations (R7.7.5) for increased cover where concrete will be exposed to external sources of chlorides in service, such as deicing salts and seawater. As noted in R7.7, alternative methods of protecting the reinforcement from weather may be used if they provide protection equivalent to the additional concrete cover required in 7.7.1(b), 7.7.2(b), and 7.7.3.1(a), as compared to 7.7.1(c), 7.7.2(c), and 7.7.3.1(b), respectively.

7.8 SPECIAL REINFORCEMENT DETAILS FOR COLUMNS

Section 7.8 covers the special detailing requirements for offset bent longitudinal bars and steel cores of composite columns.

When column offsets are necessary, longitudinal bars may be bent, subject to the following limitations:

1. Slope of the inclined portion of an offset bar with respect to the axis of column must not exceed 1 in 6 (see Fig. 3-3).

2. Portions of bar above and below an offset must be parallel to axis of column.

3. Horizontal support at offset bends must be provided by lateral ties, spirals, or parts of the floor construction. Ties or spirals, if used, shall be placed not more than 6 in. from points of bend (see Fig. 3-3). Horizontal support provided must be designed to resist 1-1/2 times the horizontal component of the computed force in the inclined portion of an offset bar.

4. Offset bars must be bent before placement in the forms.

5. When a column face is offset 3 in. or more, longitudinal column bars parallel to and near that face must not be offset bent. Separate dowels, lap spliced with the longitudinal bars adjacent to the offset column faces, must be provided (see Fig. 3-3). In some cases, a column might be offset 3 in. or more on some faces, and less than 3 in. on the remaining faces, which could possibly result in some offset bent longitudinal column bars and some separate dowels being used in the same column.

Steel cores in composite columns may be detailed to allow transfer of up to 50 percent of the compressive load in the core by direct bearing. The remainder of the load must be transferred by welds, dowels, splice plates, etc. This should ensure a minimum tensile capacity similar to that of a more common reinforced concrete column.

7.9 CONNECTIONS

Enclosures must be provided for splices of continuing reinforcement, and for end anchorage of reinforcement terminating at beam and column connections. This confinement may be provided by the surrounding concrete or internal closed ties, spirals, or stirrups.

7.10 LATERAL REINFORCEMENT FOR COMPRESSION MEMBERS

7.10.4 Spirals

Minimum diameter of spiral reinforcement in cast-in-place construction is 3/8 in. and the clear spacing must be between the limits of 1 in. and 3 in. This requirement does not preclude the use of a smaller minimum diameter

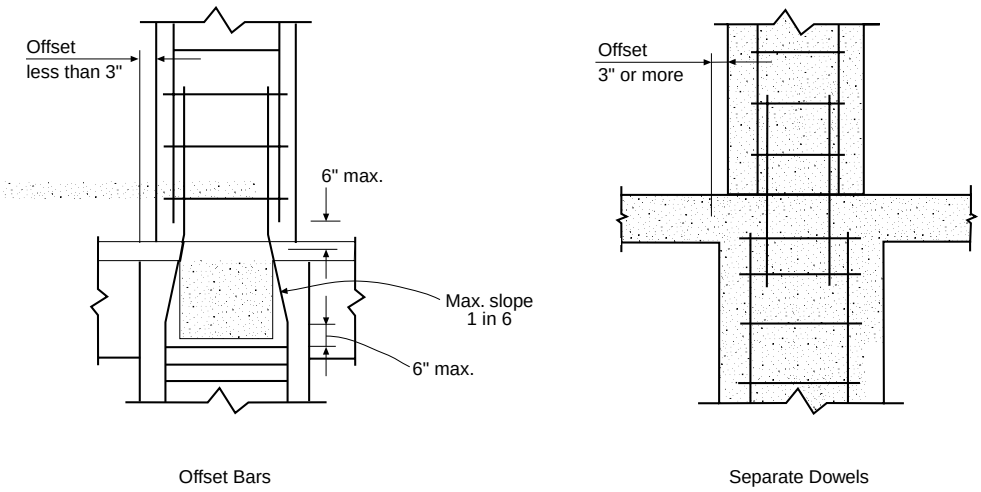


Figure 3-3 Special Column Details

for precast units. Beginning with the ’99 code, full mechanical splices complying with 12.14.3 were allowed. Previously, only lap splices and full welded splices were permitted. Editions of the code prior to the ’99 required lap splices to be 48 bar or wire diameters, regardless of whether the bar or wire was plain or deformed, or uncoated or epoxy-coated. The ’99 code was revised to require that lap splices of plain uncoated and epoxy-coated deformed bar or wire be 72 bar or wire diameters. The required lap splice length for plain uncoated and epoxy-coated deformed bar or wire is permitted to be reduced to 48 bar or wire diameters provided the ends of the lapped bars or wires terminate in a standard 90 degree hook as required for stirrups and ties (7.1.3). The lap splice length for deformed uncoated bar or wire remains unchanged at 48 bar or wire diameters, as did the requirement that the minimum lap splice length be not less than 12 in. Anchorage of spiral reinforcement must be provided by 1-1/2 extra turns at each end of a spiral unit.

Spiral reinforcement must extend from the top of footing or slab in any story to the level of the lowest horizontal reinforcement in slabs, drop panels, or beams above. If beams or brackets do not frame into all sides of the column, ties must extend above the top of the spiral to the bottom of the slab or drop panel (see Fig. 3-4). In columns with capitals, spirals must extend to a level where the diameter or width of capital is twice that of the column.

Spirals must be held firmly in place, at proper pitch and alignment, to prevent displacement during concrete placement. Prior to ACI 318-89, the code specifically required spacers for installation of column spirals. Section 7.10.4.9 now simply states that “spirals shall be held firmly in place and true to line.” This performance provision permits alternative methods, such as tying, to hold the fabricated cage in place during construction, which is current practice in most areas where spirals are used. The original spacer requirements were moved to the commentary to provide guidance where spacers are used for spiral installation. Note that the project specifications should cover the spacer requirements (if used) or the tying of the spiral reinforcement.

7.10.5 Ties

In tied reinforced concrete columns, ties must be located at no more than half a tie spacing above the floor slab or footing and at no more than half a tie spacing below the lowest horizontal reinforcement in the slab or drop panel above. If beams or brackets frame from four directions into a column, ties may be terminated not more than 3 in. below the lowest horizontal reinforcement in the shallowest of such beams or brackets (see Fig. 3-5). Minimum size of lateral ties in tied reinforced columns is related to the size of the longitudinal bars. Minimum tie sizes are No. 3 for non-prestressed longitudinal bars No. 10 and smaller, and No. 4 for No. 11 longitudinal bars and larger and for bundled bars. The following restrictions also apply: spacing must not exceed 16 longitudinal bar diameters, 48 tie bar diameters, or the least dimension of the column; every corner bar and alternate bar must have lateral support provided by the corner of a tie or crosstie with an included angle of not more than 135 deg. No unsupported bar shall be farther than 6 in. from a supported bar (see Fig. 3-6). Note that the 6-in. clear distance is measured along the tie.

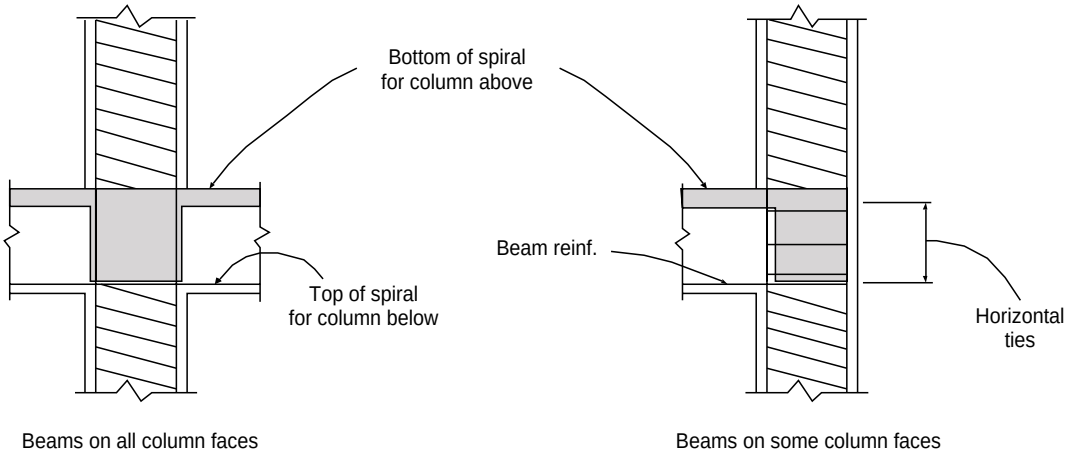


Figure 3-4 Termination of Spirals

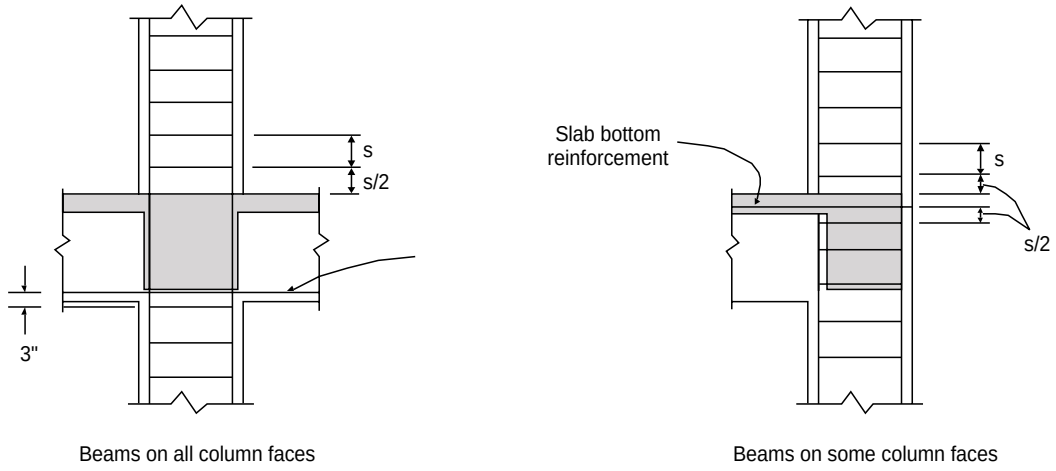


Figure 3-5 Termination of Column Ties

Welded wire and continuously wound bars or wire reinforcement of equivalent area may be used for ties. Where main reinforcement is arranged in a circular pattern, it is permissible to use complete circular ties at the specified spacing. This provision allows the use of circular ties at a spacing greater than that specified for spirals in spirally reinforced columns. Anchorage at the end of a continuously wound bar or wire reinforcement should be by a standard hook or by one additional turn of the tie pattern.

Where anchor bolts are provided in the tops of columns or pedestals to attach other structural members, the code requires that these bolts be confined by lateral reinforcement that is also surrounding at least four of the vertical bars in the column or pedestal for continuity of the load transfer at the connection. The lateral ties are required to be a minimum of two No. 4 or three No. 3 bars and must be distributed within the top 5 in. of the column or pedestal.

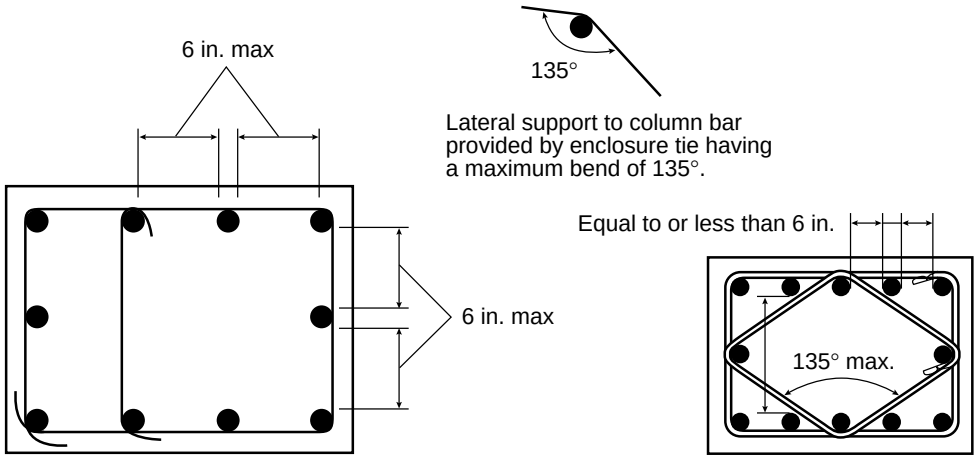


Figure 3-6 Lateral Support of Column Bars by Ties

7.11 LATERAL REINFORCEMENT FOR FLEXURAL MEMBERS

Where compression reinforcement is used to increase the flexural strength of a member (10.3.5.1), or to control long-term deflection [Eq. (9.11)], 7.11.1 requires that such reinforcement be enclosed by ties or stirrups. Requirements for size and spacing of the ties or stirrups are the same as for ties in tied columns. Welded wire fabric of equivalent area may be used. The ties or stirrups must extend throughout the distance where the compression reinforcement is required for flexural strength or deflection control. Section 7.11.1 is interpreted not to apply to reinforcement located in a compression zone to help assemble the reinforcing cage or hold the web reinforcement in place during concrete placement.

Enclosing reinforcement required by 7.11.1 is illustrated by the U-shaped stirrup in Fig. 3-7; the continuous bottom portion of the stirrup satisfies the enclosure intent of 7.11.1 for the two bottom bars shown. A completely closed stirrup is ordinarily not necessary, except in cases of high moment reversal, where reversal conditions require that both top and bottom longitudinal reinforcement be designed as compression reinforcement.

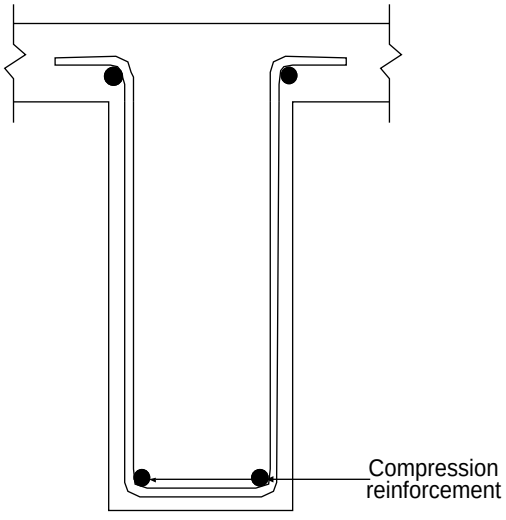


Figure 3-7 Enclosed Compression Reinforcement

Torsion reinforcement, where required, must consist of completely closed stirrups, closed ties, spirals, or closed cages of welded wire fabric as required by 11.6.4.

7.11.3 Closed Ties or Stirrups

According to 7.11.3, a closed tie or stirrup is formed either in one piece with overlapping 90-deg or 135-deg end hooks around a longitudinal bar, or in one or two pieces with a Class B lap splice, as illustrated in Fig. 3-8. The one-piece closed stirrup with overlapping end hooks is not practical for placement. Neither of the closed stirrups shown in Fig. 3-8 is considered effective for members subject to high torsion. Tests have shown that, with high torsion, loss of concrete cover and subsequent loss of anchorage result if the 90-deg hook and lap splice details are used where confinement by external concrete is limited. See Fig. 3-9. The ACI Detailing Manual[3.1] recommends the details illustrated in Fig. 3-10 for closed stirrups used as torsional reinforcement.

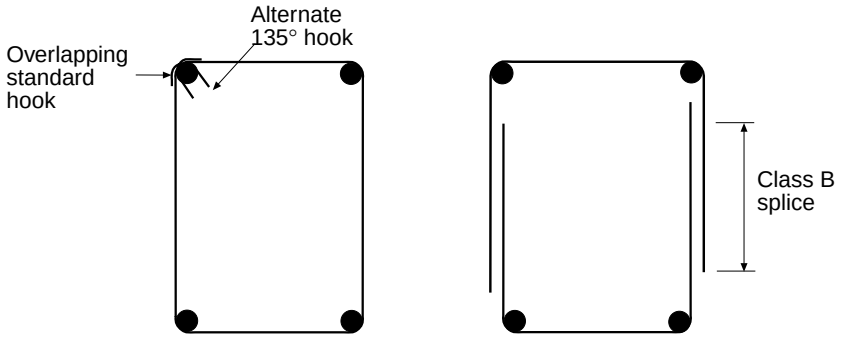


Figure 3-8 Code Definition of Closed Tie or Stirrup

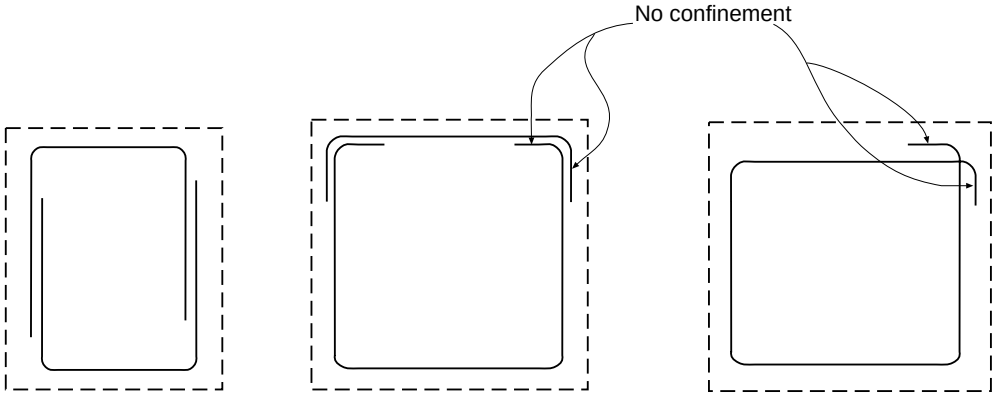


Figure 3-9 Closed Stirrup Details Not Recommended for Members Subject to High Torsion

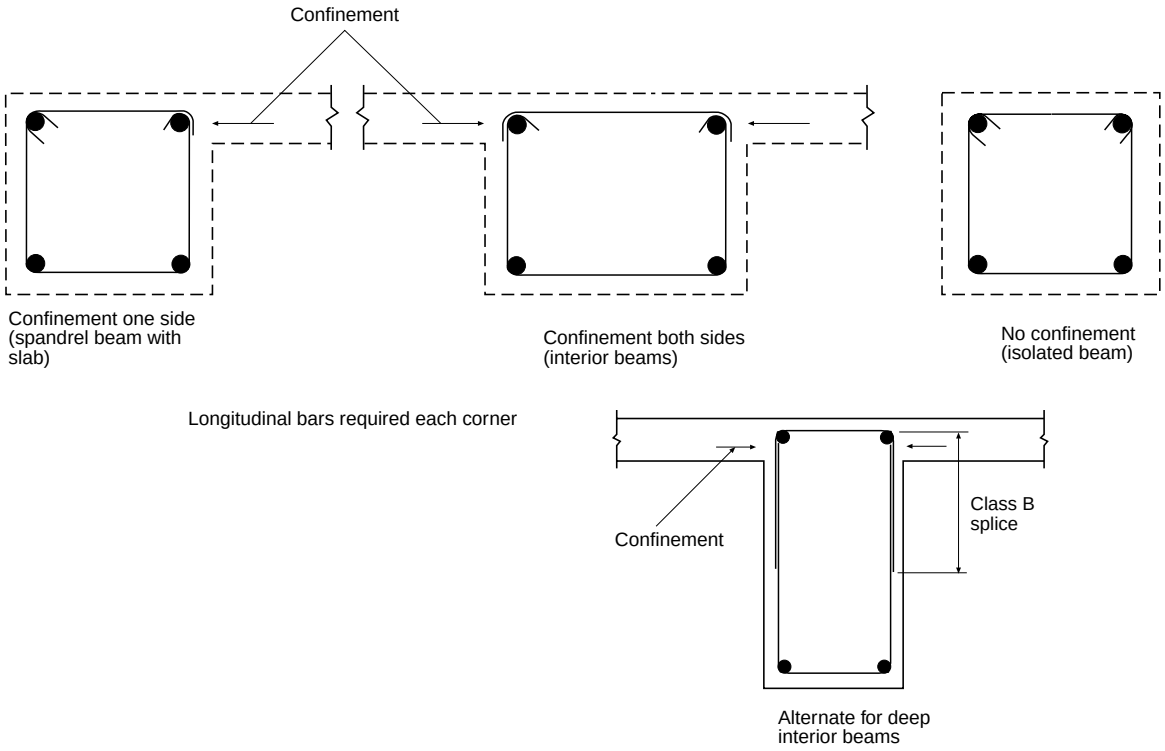


Figure 3-10 Two-Piece Closed Stirrup Details [3.1]
Recommended for Members Subject to High Torsion

7.12 SHRINKAGE AND TEMPERATURE REINFORCEMENT

Minimum shrinkage and temperature reinforcement normal to primary flexural reinforcement is required for structural floor and roof slabs (not slabs on ground) where the flexural reinforcement extends in one direction only. Minimum steel ratios, based on the gross concrete area, are:

1. 0.0020 for Grades 40 and 50 deformed bars;

2. 0.0018 for Grade 60 deformed bars or welded wire fabric;

3. $0.0018 \times 60{,}000/f_y$ for reinforcement with a yield strength greater than 60,000 psi; but not less than 0.0014.

Spacing of shrinkage and temperature reinforcement must not exceed 5 times the slab thickness nor 18 in. Splices and end anchorages of such reinforcement must be designed for the full specified yield strength. The minimum steel ratios cited above do not apply where prestressed steel is used.

Bonded or unbonded prestressing tendons may be used for shrinkage and temperature reinforcement in structural slabs (7.12.3). The tendons must provide a minimum average compressive stress of 100 psi on the gross concrete area, based on effective prestress after losses. Spacing of tendons must not exceed 6 ft. When the spacing is greater than 54 in., additional bonded reinforcement must be provided at slab edges.

7.13 REQUIREMENTS FOR STRUCTURAL INTEGRITY

Structures capable of safely supporting all conventional design loads may suffer local damage from severe local abnormal loads, such as explosions due to gas or industrial liquids; vehicle impact; impact of falling objects; and local effects of very high winds such as tornadoes. Generally, such abnormal loads or events are not design

considerations. The overall integrity of a reinforced concrete structure to withstand such abnormal loads can be substantially enhanced by providing relatively minor changes in the detailing of the reinforcement. The intent of 7.13 is to improve the redundancy and ductility of structures. This is achieved by providing, as a minimum, some continuity reinforcement or tie between horizontal framing members. In the event of damage to a major supporting element or an abnormal loading event, the integrity reinforcement is intended to confine any resulting damage to a relatively small area, thus improving overall stability.

It is not the intent of 7.13 that a structure be designed to resist general collapse caused by gross misuse or to resist severe abnormal loads acting directly on a large portion of the structure. General collapse of a structure as the result of abnormal events such as wartime or terrorist bombing, and landslides, are beyond the scope of any practical design.

7.13.1 General Structural Integrity

Since accidents and misuse are normally unforseeable events, they cannot be defined precisely; likewise, providing general structural integrity to a structure is a requirement that cannot be stated in simple terms. The performance provision..."members of a structure shall be effectively tied together to improve integrity of the overall structure," will require a level of judgment on the part of the design engineer, and will generate certain differing opinions among engineers as to how to effectively provide a general structural integrity solution for a particular framing system. It is obvious that all conditions that might be encountered in design cannot be specified in the code. The code, however, does set forth specific examples of certain reinforcing details for cast-in-place joists, beams, and two-way slab construction.

With damage to a support, top reinforcement which is continuous over the support, but not confined by stirrups, will tend to tear out of the concrete and will not provide the catenary action needed to bridge the damaged support. By making a portion of the bottom reinforcement in beams continuous over supports, some catenary action can be provided. By providing some continuous top and bottom reinforcement in edge or perimeter beams, an entire structure can be tied together; also, the continuous tie provided to perimeter beams of a structure will toughen the exterior portion of a structure, should an exterior column be severely damaged. Other examples of ways to detail for required integrity of a framing system to carry loads around a severely damaged member can be cited. The design engineer will need to evaluate his particular design for specific ways of handling the problem. The concept of providing general structural integrity is discussed in the Commentary of ASCE 7, *Minimum Design Loads for Buildings and Other Structures*.[3.6] The reader is referred to that document for further discussion of design concepts and details for providing general structural integrity.

7.13.2 Cast-in-Place Joists and Beams

Since 1989, the code requires continuous reinforcement in beams around the perimeter of the structure for structural integrity. The required amount is a minimum of one-sixth the tension reinforcement for negative moment at the support and one-fourth of the tension reinforcement for positive moment at the midspan. In either case the '02 code requires a minimum of two bars. Also starting with the '02 code, mechanical and welded splices are explicitly permitted for splicing of continuous reinforcement in cast-in-place joists and beams. Figures 3-11 through 3-13 illustrate the required reinforcing details for the general case of cast-in-place joists and beams.

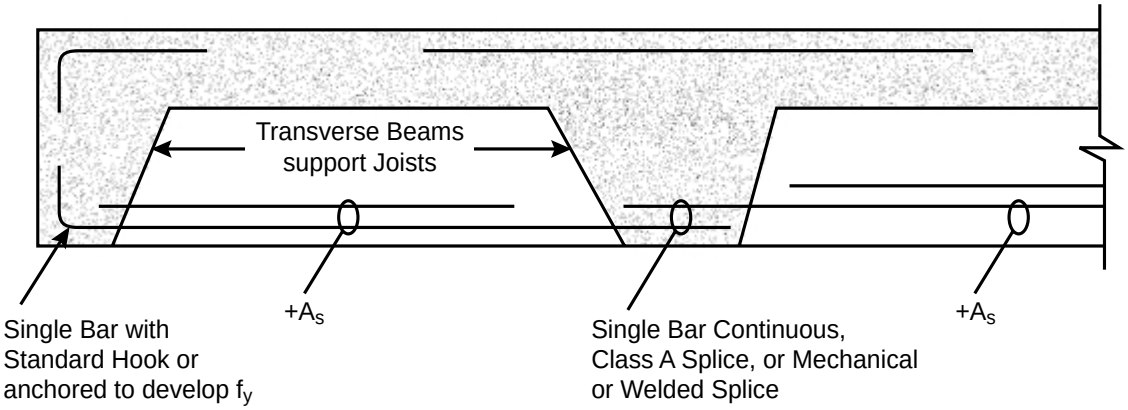
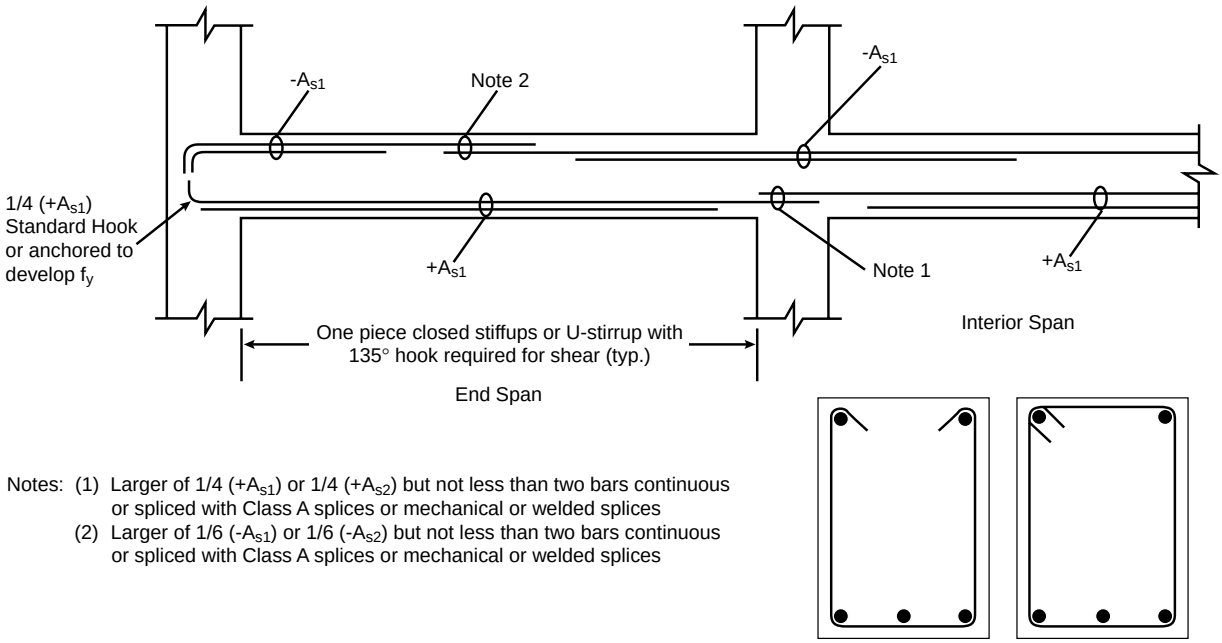
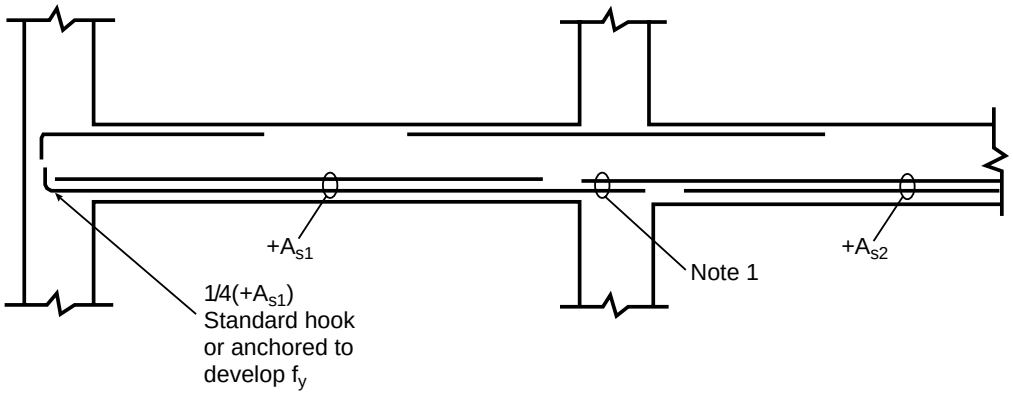


Figure 3-11 Continuity Reinforcement for Joist Construction



Notes: (1) Larger of 1/4 (+A_{s1}) or 1/4 (+A_{s2}) but not less than two bars continuous or spliced with Class A splices or mechanical or welded splices
(2) Larger of 1/6 (-A_{s1}) or 1/6 (-A_{s2}) but not less than two bars continuous or spliced with Class A splices or mechanical or welded splices

Figure 3-12 Continuity Reinforcement for Perimeter Beams



Note: (1) Larger of 1/4 (+As1) or 1/4 (+As2) continuous or spliced with Class A splices or mechanical or welded splices

Figure 3-13 Continuity Reinforcement for Other Beams without Closed Stirrups

7.13.3 Precast Concrete Construction

While the requirements for structural integrity introduced in ACI 318-89 were prescriptive for cast-in-place construction, the '89 code provided only performance requirements for precast construction. This approach was made necessary because precast structures can be built in a lot of different ways. The code requires tension ties for precast concrete buildings of all heights. Connections that rely solely on friction due to gravity forces are not permitted.

The general requirement for structural integrity (7.13.1) states that "...members of a structure shall be effectively tied together...". The '89 commentary cautioned that for precast concrete construction, connection details should be arranged so as to minimize the potential for cracking due to restrained creep, shrinkage, and temperature movements. Ref. 3.7 contains information on industry practice for connections and detailing requirements. Prescriptive requirements recommended by the PCI for precast concrete bearing wall buildings are given in Ref. 3.8. Prescriptive structural integrity requirements for precast concrete structures were introduced for the first time in Chapter 16 of ACI 318-95 (see discussion in Part 23 of this publication).

7.13.4 Lift-Slab Construction

Section 7.13.4 refers the code user to 13.3.8.6 and 18.12.6 for lift-slab construction.

REFERENCES

3.1 *ACI Detailing Manual* — 1994, Publication SP-66(94), American Concrete Institute, Detroit, MI, 1994.

3.2 *Manual of Standard Practice*, 27th edition, Concrete Reinforcing Steel Institute, Schaumburg, IL, 2001.

3.3 S*tructural Welded Wire Fabric Detailing Manual*, WWR-600, Wire Reinforcement Institute, McLean, VA, 1994.

3.4 Babaei, K., and Hawkins, N.M., "Field Bending and Straightening of Reinforcing Steel," *Concrete International: Design and Construction*, V. 14, No. 1, January 1992.

3.5 *Standard Specification for Tolerances for Concrete Construction and Materials and Commentary*, ACI 117/117-90, American Concrete Institute, Detroit, MI, 1990.

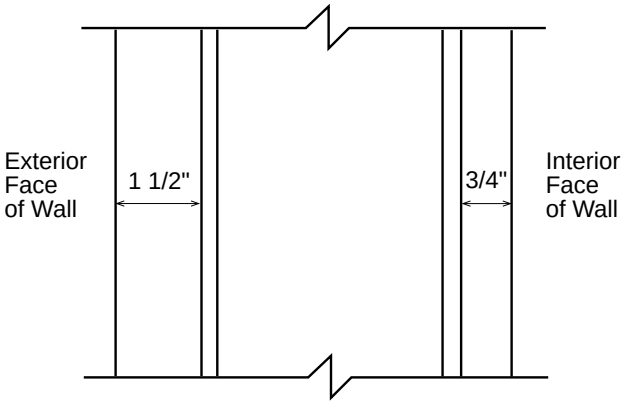
3.6 *Minimum Design Loads for Buildings and Other Structures*, (ASCE 7-98), American Society of Civil Engineers, Reston, VA, 1998.

3.7 *Design and Typical Details of Connections for Precast and Prestressed Concrete*, Publication MNL-123-88, Precast/Prestressed Concrete Institute, Chicago, IL, 1988.

3.8 PCI Building Code Committee, "Proposed Design Requirements for Precast Concrete," *PCI Journal*, V. 31, No. 6, Nov.-Dec. 1986, pp. 32-47.

Example 3.1—Placing Tolerance for Rebars

For the wall section shown below, with specified clear concrete cover indicated, determine the minimum cover permitted in construction, including the code tolerances on concrete cover.



Calculations and Discussion	Code Reference

Tolerance on concrete cover is minus 1/2 in., but in no case may the tolerance be more than 1/3 the specified concrete cover. *7.5.2.1*

1. For the exterior face, a measured 1 in. cover (1-1/2 - 1/2) is permitted. Actual bar placement may be within 1 in. of the side forms.

2. For the interior face, a measured 1/2 in. cover (3/4 - 1/4) is permitted. For the 3/4 in. specified cover, the tolerance limit is (1/3) (3/4) = 1/4 in. < 1/2 in.

As noted in the ACI 117 Standard[3.5], tolerances are a means to establish permissible variation in dimension and location, giving both the designer and the contractor parameters within which the work is to be performed. They are the means by which the designer conveys to the contractor the performance expectations.

4

Development and Splices of Reinforcement

UPDATE FOR THE '02 CODE

Most of the changes made to Chapter 12 clarify existing provisions. The equations in 12.2 for the development length ℓ_d for deformed bars and deformed wires in tension have been slightly modified: the nominal diameter d_b now appears on the right-hand-side of the equation. The section title for 12.3 now includes deformed wire. In this section, the definition of the basic development length ℓ_{db} has been removed; for clarity, the development length ℓ_{dc} for deformed bars and deformed wires in compression has been introduced. Similarly, 12.5 has been modified to further explain the requirements for development of standard hooks in tension.

The expression for development length ℓ_d of prestressing strands has moved from the commentary to new Eq. (12-2) in 12.9. This replaces the unnumbered version of this equation that was formerly in this section. New section 12.9.1.1 provides provisions when it is permissible to have embedment less than the development length. Section 12.9.2 has expanded to emphasize that it may be unconservative to only investigate sections required to develop full design strength. The commentary to this section gives guidance for determining capacity at sections where strands are not fully developed.

Section 12.15.4 for mechanical and welded splices that do not meet the requirements of 12.14.3.2 or 12.14.3.4 has been modified for clarity.

GENERAL CONSIDERATIONS

The development length concept for anchorage of deformed bars and deformed wire in tension, is based on the attainable average bond stress over the length of embedment of the reinforcement. This concept requires the specified minimum lengths or extensions of reinforcement beyond all locations of peak stress in the reinforcement. Such peak stresses generally occur in flexural members at the locations of maximum stress and where adjacent reinforcement terminates or is bent.

The strength reduction factor φ is not used in Chapter 12 of the code since the specified development lengths already include an allowance for understrength.

12.1 DEVELOPMENT OF REINFORCEMENT—GENERAL

Development length or anchorage of reinforcement is required on both sides of a location of peak stress at each section of a reinforced concrete member. In continuous members, for example, reinforcement typically continues for a considerable distance on one side of a critical stress location so that detailed calculations are usually required only for the side where the reinforcement is terminated.

Until further research is completed and to ensure ductility and safety of structures built with high strength concrete, starting with the 1989 code, the term $\sqrt{f'_c}$ has been limited to 100 psi. Existing design equations for development of straight bars in tension and compression, and standard hooks in tension, are all a function of $\sqrt{f'_c}$. These equations were developed from results of tests on reinforcing steel embedded in concrete with compressive strengths of 3000 to 6000 psi. ACI Committee 318 was prudent in limiting $\sqrt{f'_c}$ at 100 psi pending completion of tests to verify applicability of current design equations to bars in high strength concrete.

12.2 DEVELOPMENT OF DEFORMED BARS AND DEFORMED WIRE IN TENSION

The provisions of 12.2 are based on the work of Orangun, Jirsa, and Breen[4.1] and Sozen and Moehle.[4.2] Development length of straight deformed bars and wires in tension, expressed in terms of bar or wire diameter, is given in 12.2.3 by the general equation:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b} \right)} \right) d_b \qquad \textit{Eq. (12-1)}$$

where

ℓ_d = development length, in.

d_b = nominal diameter of bar or wire, in.

f_y = specified yield strength of nonprestressed bar or wire, psi

f'_c = specified compressive strength of concrete, psi

α = **reinforcement location factor**
= 1.3 for horizontal reinforcement so placed that more than 12 in. of fresh concrete is cast below the bar being developed or spliced
= 1.0 for other reinforcement

β = **coating factor**
= 1.5 for epoxy-coated bars or wires with cover less than $3d_b$ or clear spacing less than $6d_b$
= 1.2 for all other epoxy-coated bars or wires
= 1.0 for uncoated reinforcement

The product of α and β need not be taken greater than 1.7.

γ = **reinforcement size factor**
= 0.8 for No. 6 and smaller bars and deformed wires
= 1.0 for No. 7 and larger bars

λ = **lightweight aggregate concrete factor**
= 1.3 when lightweight aggregate concrete is used, or
= $6.7\sqrt{f'_c}/f_{ct}$, but not less than 1.0, when f_{ct} is specified
= 1.0 for normal weight concrete

c = **spacing or cover dimension**, in.
= the smaller of (1) distance from center of bar or wire being developed to the nearest concrete surface, and (2) one-half the center-to-center spacing of bars or wires being developed

K_{tr} = **transverse reinforcement index**

$$= \frac{A_{tr} f_{yt}}{1500sn}$$

where

A_{tr} = total cross-sectional area of all transverse reinforcement which is within the spacing s and which crosses the potential plane of splitting through the reinforcement being developed, $in.^2$

f_{yt} = specified yield strength of transverse reinforcement, psi

s = maximum spacing of transverse reinforcement within ℓ_d, center-to-center, in.

n = number of bars or wires being developed along the plane of splitting

Note that the term $\left(\frac{c + K_{tr}}{d_b}\right)$ cannot be taken greater than 2.5 (12.2.3) to safeguard against pullout type failures.

In the 1989 and earlier editions of the code, the expression $0.03 d_b f_y / \sqrt{f'_c}$ was specified to prevent pullout type failures.

As a design simplification, it is conservative to assume $K_{tr} = 0$, even if transverse reinforcement is present. If a clear cover of $2d_b$ and a clear spacing between bars being developed of $4d_b$ is provided, variable "c" would equal $2.5d_b$. For the preceding conditions, even if $K_{tr} = 0$, the term $\left(\frac{c + K_{tr}}{d_b}\right)$ would equal 2.5.

The term $\left(\frac{c + K_{tr}}{d_b}\right)$ in the denominator of Eq. (12-1) accounts for the effects of small cover, close bar spacing, and confinement provided by transverse reinforcement. To further simplify computation of ℓ_d, preselected values for term $\left(\frac{c + K_{tr}}{d_b}\right)$ were chosen starting with the 1995 code. As a result, Equation (12-1) can take the simplified forms specified in 12.2.2, and shown below in Table 4-1. For discussion purposes only, the four equations are identified in this table as Equations A through D. Note that these identifiers do not appear in the code.

In Eqs. A and B, the term $\left(\frac{c + K_{tr}}{d_b}\right) = 1.5$, while in Eqs. C and D, $\left(\frac{c + K_{tr}}{d_b}\right) = 1.0$. Equations A and C include a reinforcement size factor $\gamma = 0.8$. The 20 percent reduction is based on comparisons with past provisions and numerous test results.

Equations A and B can only be applied if one of the following two different sets of conditions is satisfied:

Table 4-1 Development Lengths ℓ_d Specified in 12.2.2

	No. 6 and smaller bars and deformed wires	No. 7 and larger bars
Clear spacing of reinforcement being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the code minimum or Clear spacing of reinforcement being developed or spliced not less than $2d_b$ and clear cover not less than d_b	(Eq. A) $\left(\frac{f_y \alpha\beta\lambda}{25\sqrt{f'_c}}\right) d_b$	(Eq. B) $\left(\frac{f_y \alpha\beta\lambda}{20\sqrt{f'_c}}\right) d_b$
Other cases	(Eq. C) $\left(\frac{3f_y \alpha\beta\lambda}{50\sqrt{f'_c}}\right) d_b$	(Eq. D) $\left(\frac{3f_y \alpha\beta\lambda}{40\sqrt{f'_c}}\right) d_b$

Set # 1

The following three conditions must simultaneously be satisfied:

1. The clear spacing of reinforcement being developed or spliced should not be less than the diameter of reinforcement being developed, d_b,
2. The clear cover for reinforcement being developed should not be less than d_b, and
3. Minimum amount of stirrups or ties throughout ℓ_d should not be less than the minimum values specified in 11.5.5.3 for beams or 7.10.5 for columns.

Set #2

The following two conditions must simultaneously be satisfied:

1. The clear spacing of reinforcement being developed or spliced should not be less than $2d_b$, and
2. The clear cover should not be less than d_b.

If all the conditions of Set #1 or of Set #2 cannot be satisfied, then Eqs. C or D must be used. Note that Eq. D is identical to Eq. (12-1) with $\left(\frac{c + K_{tr}}{d_b}\right) = 1.0$ and reinforcement size factor $\gamma = 1.0$.

Although Eqs. A through D are easier to use than Eq. (12-1), the term $\left(\frac{c + K_{tr}}{d_b}\right)$ can only assume the value of 1.0 (Eqs. C and D) or 1.5 (Eqs. A and B). On the other hand, Eq. (12-1) may require a little extra effort, but the value of expression $\left(\frac{c + K_{tr}}{d_b}\right)$ can be as high as 2.5. Therefore, the development lengths ℓ_d computed by Eq. (12-1) could be substantially shorter than development lengths computed from the simplified equations of 12.2.2.

The development lengths of Table 4-1 can be further simplified for specific conditions. For example, for Grade 60 reinforcement (f_y = 60,000 psi) and different concrete compressive strengths, assuming normal weight concrete ($\lambda = 1.0$) and uncoated ($\beta = 1.0$)bottom bars or wires ($\alpha = 1.0$), values of ℓ_d as a function of d_b can be determined as shown in Table 4-2.

Table 4-2 Development Length ℓ_d for Grade 60, Uncoated, Bottom Reinforcement in Normal Weight Concrete

	f'_c psi	No. 6 and smaller bars and deformed wires	No. 7 and larger bars
Clear spacing of bars being developed or spliced not less than d_b, clear cover not less than d_b, and beam stirrups or column ties throughout ℓ_d not less than the code minimum or Clear spacing of bars being developed or spliced not less than $2d_b$ and clear cover not less than d_b	3000	$44d_b$	$55d_b$
	4000	$38d_b$	$47d_b$
	5000	$34d_b$	$42d_b$
	6000	$31d_b$	$39d_b$
	8000	$27d_b$	$34d_b$
	10,000	$24d_b$	$30d_b$
Other cases	3000	$66d_b$	$82d_b$
	4000	$57d_b$	$71d_b$
	5000	$51d_b$	$64d_b$
	6000	$46d_b$	$58d_b$
	8000	$40d_b$	$50d_b$
	10,000	$36d_b$	$45d_b$

As in previous editions of the code, development length of straight deformed bars or wires, including all modification factors must not be less than 12 in.

12.2.5 Excess Reinforcement

Development length may be reduced by the ratio [(A_s required)/(A_s provided)] when excess reinforcement is provided in a flexural member. Note that this reduction does not apply when the full f_y development is required, as for tension lap splices in 7.13, 12.15.1, and 13.3.8.5, development of positive moment reinforcement at supports in 12.11.2, and for development of shrinkage and temperature reinforcement according to 7.12.2.3. Note also that this reduction in development length is not permitted for reinforcement in structures located in regions of high seismic risk or for structures assigned to high seismic performance or design categories (see 21.2.1.4).

Development length ℓ_d computed after applying the excess reinforcement reduction of 12.2.5 must not be less than 12 in.

12.3 DEVELOPMENT OF DEFORMED BARS AND DEFORMED WIRE IN COMPRESSION

Shorter development lengths are required for bars in compression than in tension since the weakening effect of flexural tension cracks in the concrete is not present. The development length for deformed bars or deformed wire in compression is $\ell_{dc} = 0.02 d_b f_y / \sqrt{f'_c}$, but not less than $0.0003 d_b f_y$ or 8 in. In the 1999 code, this was defined as the basic compression development length ℓ_{db}. Note that ℓ_{dc} may be reduced where excess reinforcement is provided (12.3.3(a)) and where "confining" ties or spirals are provided around the reinforcement (12.3.3(b)). Note that the tie and spiral requirements to permit the 25 percent reduction in development length are somewhat more restrictive than those required for "regular" column ties in 7.10.5 and less restrictive than those required for spirals in 7.10.4. For reference, compression development lengths for Grade 60 bars are given in Table 4-3.

Table 4-3 Compression Development Length ℓ_{dc} (inches) for Grade 60 Bars

Bar Size No.	f'_c (Normal Weight Concrete), psi		
	3000	4000	≥ 4444 *
3	8.2	7.1**	6.8**
4	11.0	9.5	9.0
5	13.7	11.9	11.3
6	16.4	14.2	13.5
7	19.2	16.6	15.8
8	21.9	19.0	18.0
9	24.7	21.4	20.3
10	27.8	24.1	22.9
11	30.9	26.8	25.4
14	37.1	32.1	30.5
18	49.4	42.8	40.6

* *For $f'_c \geq 4444$ psi, minimum basic development length $0.0003 d_b f_y$ governs; for Grade 60 bars, $\ell_{dc} = 18 d_b$.*

** *Development length ℓ_{dc} (including applicable modification factors) must not be less than 8 in.*

12.4 DEVELOPMENT OF BUNDLED BARS

Increased development length for individual bars within a bundle, whether in tension or compression, is required when 3 or 4 bars are bundled together. The additional length is needed because the grouping makes it more difficult to mobilize resistance to slippage from the "core" between the bars. The modification factor is 1.2 for a 3-bar bundle, and 1.33 for a 4-bar bundle. Other pertinent requirements include 7.6.6.4 concerning cut-off points of individual bars within a bundle, and 12.14.2.2 relating to lap splices of bundled bars.

Where the factors of 12.2 are based on bar diameter d_b, a unit of bundled bars must be treated as a single bar of a diameter derived from the total equivalent area. See Table 3-7 in Part 3 of this document.

12.5 DEVELOPMENT OF STANDARD HOOKS IN TENSION

The current provisions for hooked bar development were first introduced in the 1983 code. They represented a major departure from the hooked-bar anchorage provisions of earlier codes in that they uncoupled hooked-bar anchorages from straight bar development and gave total hooked-bar embedment length directly. The current provisions not only simplify calculations for hook anchorage lengths but also result in a required embedment length considerably less, especially for the larger bar sizes, than that required by earlier codes. Provisions are given in 12.5 for determining the development length of deformed bars with standard end hooks. End hooks can only be considered effective in developing bars in tension, and not in compression (see 12.1.1 and 12.5.5). Only "standard" end hooks (see 7.1) are considered; anchorage capacity of end hooks with larger end diameters cannot be determined by the provisions of 12.5.

In applying the hook development provisions, the first step is to calculate the development length of the hooked bar, ℓ_{dh} from 12.5.2. This length is then multiplied by the applicable modification factor or factors of 12.5.3. Note that in the 1999 code, the basic development length ℓ_{hb} for a hooked bar was determined, which could be modified by the factors in 12.5.3. Development length ℓ_{dh} is measured from the critical section to the outside end of the standard hook, i.e., the straight embedment length between the critical section and the start of the hook, plus the radius of bend of the hook, plus one-bar diameter. For reference, Fig. 4-1 shows ℓ_{dh} and the standard hook details (see 7.1) for all standard bar sizes. For 180 deg hooks normal to exposed surfaces, the embedment length should provide for a minimum distance of 2 in. beyond the tail of the hook.

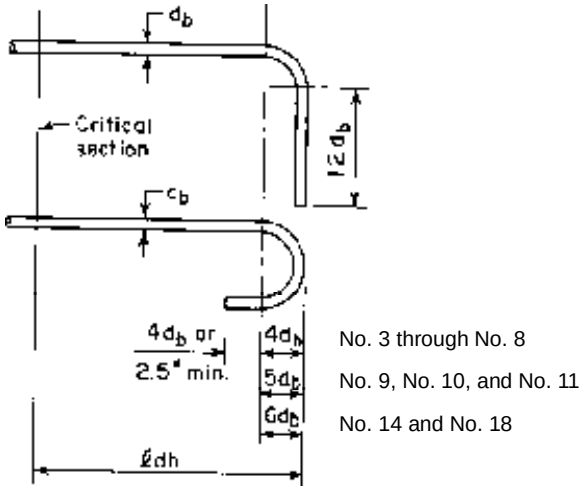


Figure 4-1 Development Length ℓ_{dh} of Standard Hooks

12.5.2 Development Length ℓ_{dh}

The development length, ℓ_{dh}, for standard hooks in tension is given in 12.5.2 as:

$$\ell_{dh} = \left(\frac{0.02 \beta \lambda f_y}{\sqrt{f'_c}} \right) d_b$$

where $\beta = 1.2$ for epoxy-coated reinforcement[4.3] and $\lambda = 1.3$ for lightweight aggregate concrete. For other cases, β and λ are equal to 1.0.

Table 4-4 lists the development length of hooked bars embedded in normal weight concrete with different specified compressive strengths and uncoated Grade 60 reinforcing bars.

*Table 4-4 Development Length ℓ_{dh} (inches) of Standard Hooks for Uncoated Grade 60 Bars**

Bar Size No.	f'_c (Normal Weight Concrete), psi					
	3000	4000	5000	6000	8000	10,000
3	8.2	7.1	6.4	5.8	5.0	4.5
4	11.0	9.5	8.5	7.7	6.7	6.0
5	13.7	11.9	10.6	9.7	8.4	7.5
6	16.4	14.2	12.7	11.6	10.1	9.0
7	19.2	16.6	14.8	13.6	11.7	10.5
8	21.9	19.0	17.0	15.5	13.4	12.0
9	24.7	21.4	19.1	17.5	15.1	13.5
10	27.8	24.1	21.6	19.7	17.0	15.2
11	30.9	26.8	23.9	21.8	18.9	16.9
14	37.1	32.1	28.7	26.2	22.7	20.3
18	49.5	42.8	38.3	35.0	30.3	27.1

* Development length ℓ_{dh} (including modification factors) must not be less than the larger of $8d_b$ or 6 in.

12.5.3 Modification Factors

The ℓ_{dh} modification factors listed in 12.5.3 account for:

- Favorable confinement conditions provided by increased cover (12.5.3(a))
- Favorable confinement provided by transverse ties or stirrups to resist splitting of the concrete (12.5.3(b) and (c))

- More reinforcement provided than required by analysis (12.5.3(d))

The side cover (normal to plane of hook), and the cover on bar extension beyond 90 deg hook referred to in 12.5.3(a) are illustrated in Fig. 4-2.

Note that requirements for 90-deg and 180-deg hooks are clarified in 12.5.3 of the 2002 code. Figures R12.5.3 (a) and R12.5.3 (b) illustrate the cases where the modification factor of 12.5.3 (b) may be used.

After multiplying the development length ℓ_{dh} by the applicable modification factor or factors, the resulting development length ℓ_{dh} must not be less than the larger of $8d_b$ or 6 in.

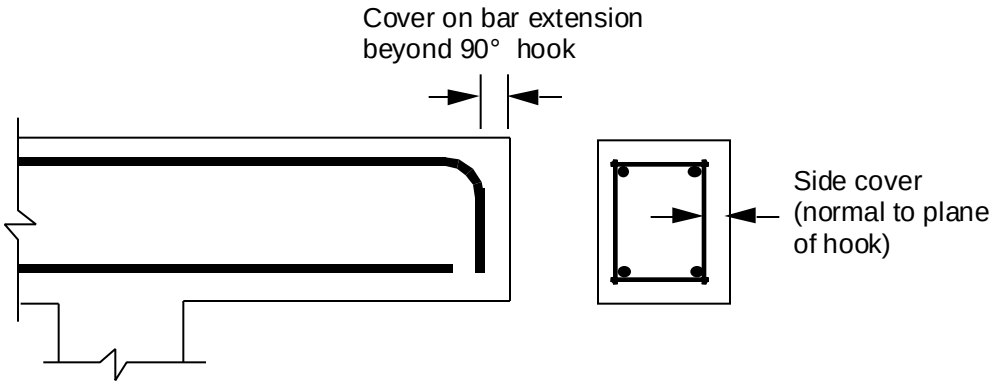


Figure 4-2 Concrete Covers Referenced in 12.5.3(a)

12.5.4 Standard Hook at Discontinuous Ends

Section 12.5.4 is a special provision for hooked bars terminating at discontinuous ends of members, such as at the ends of simply-supported beams, at free ends of cantilevers, and at ends of members framing into a joint where the member does not extend beyond the joint. If the full strength of a hooked bar must be developed, and both side cover and top (or bottom) cover over the hook are less than 2.5 in., 12.5.4 requires the hook to be enclosed within ties or stirrup for the full development length, ℓ_{dh}. Spacing of the ties or stirrup must not exceed $3d_b$, where d_b is the diameter of the hooked bar. In addition, the modification factor of 0.8 for confinement provided by ties or stirrups (12.5.3(b) and (c)) does not apply to the special condition covered by 12.5.4. At discontinuous ends of slabs with concrete confinement provided by the slab continuous on both sides normal to the plane of the hook, the provisions of 12.5.4 do not apply.

12.6 MECHANICAL ANCHORAGE

Section 12.6 permits the use of mechanical devices for development of reinforcement, provided their adequacy without damaging the concrete has been confirmed by tests. Section 12.6.3 reflects the concept that development of reinforcement may consist of a combination of mechanical anchorage plus additional embedment length of the reinforcement. For example, when a mechanical device cannot develop the design strength of a bar, additional embedment length must be provided between the mechanical device and the critical section.

12.7 DEVELOPMENT OF WELDED DEFORMED WIRE FABRIC IN TENSION

For welded deformed wire fabric, development length is measured from the critical section to the end of the wire. As specified in 12.7.1, development of welded deformed wire is computed as the product of ℓ_d from 12.2.2 or 12.2.3 times a wire fabric factor from 12.7.2 or 12.7.3. Where provided reinforcement is more than required, development length can be reduced by 12.2.5. In applying 12.2.2 or 12.2.3 to epoxy-coated deformed wire fabric, a coating factor $\beta = 1.0$ can be used. The resulting development length ℓ_d cannot be less than 8 in.,

except in computation of lap splice lengths (see 12.18) and development of web reinforcement (see 12.13). Figure 4-3 shows the development length requirements for welded deformed wire fabric.

To apply the wire fabric factor of 12.7.2 to the development length of deformed wire fabric requires at least one cross wire located within the development length at a distance no less than 2 in. from the critical section. The wire fabric factor given in 12.7.2 is the greater of $(f_y - 35{,}000)/f_y$ or $5d_b/s_w$, but need not be taken greater than 1.0.

If there is no cross wire within the development length, or the cross wire is less than 2 in. from the critical section, the development length of welded deformed wire fabric must be computed from 12.2.2 or 12.2.3. For this condition, the wire fabric factor must be taken equal to 1.0 (see 12.7.3).

According to ASTM A497, welded deformed steel wire fabric may consist solely of deformed steel wire (ASTM A496), or welded deformed steel wire fabric (ASTM A496) in one direction in combination with plain steel wire (ASTM A82) in the orthogonal direction. In the latter case, the fabric must be developed according to 12.8 for plain wire fabric.

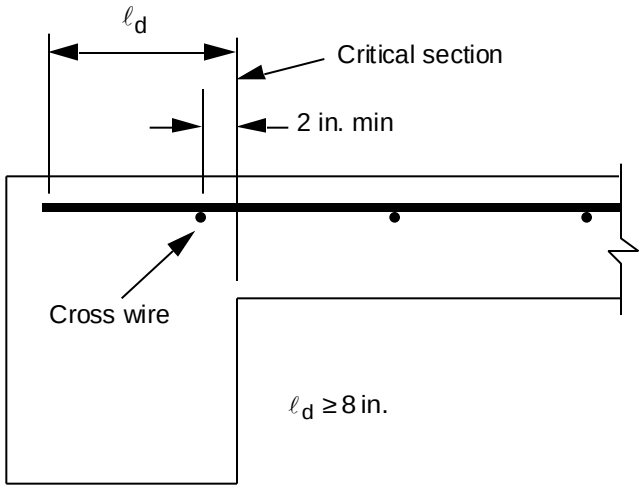


Figure 4-3 Development of Welded Deformed Wire Fabric

12.8 DEVELOPMENT OF WELDED PLAIN WIRE FABRIC IN TENSION

For welded plain wire fabric, the development length is measured from the point of critical section to the outermost cross wire. Full development of plain fabric ($A_w f_y$) is achieved by embedment of at least two cross wires beyond the critical section, with the closer cross wire located not less than 2 in. from the critical section. Section 12.8 further requires that the length of embedment from critical section to outermost cross wire not be less than $\ell_d = 0.27 A_w f_y \lambda / \left(s_w \sqrt{f'_c}\right)$, nor less than 6 in. If more reinforcement is provided than that required by analysis, the development length ℓ_d may be reduced by the ratio of (A_s required)/(A_s provided). The 6 in. minimum development length does not apply to computation of lap splice lengths (see 12.19). Figure 4-4 shows the development length requirements for welded plain wire fabric.

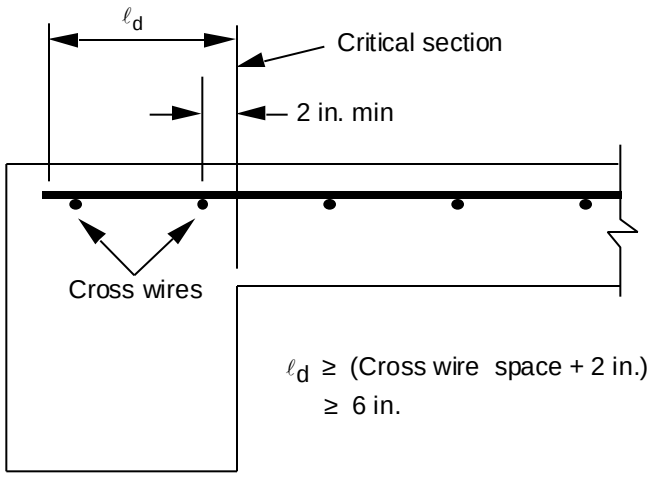


Figure 4-4 Development of Welded Plain Wire Fabric

For fabrics made with smaller wires, embedment of two cross wires, with the closer cross wire not less than 2 in. from the critical section, is usually adequate to develop the full yield strength of the anchored wires. Fabrics made with larger (closely spaced) wires will require a longer embedment ℓ_d .

For example, check fabric 6 × 6-W4 × W4 with f'_c = 3000 psi, f_y = 60,000 psi, and normal weight concrete (λ = 1.0).

$$\ell_d = 0.27 \times (A_w/s_w) \times \left(f_y/\sqrt{f'_c}\right) \times \lambda$$

$$= 0.27 \times (0.04/6) \times (60{,}000/\sqrt{3000}) \times 1.0 = 1.97 \text{ in.}$$

$$< 6 \text{ in.}$$

$$< (1 \text{ space} + 2 \text{ in.}) \quad \text{governs}$$

Two cross wire embedment plus 2 in. is satisfactory (see Fig. 4-5).

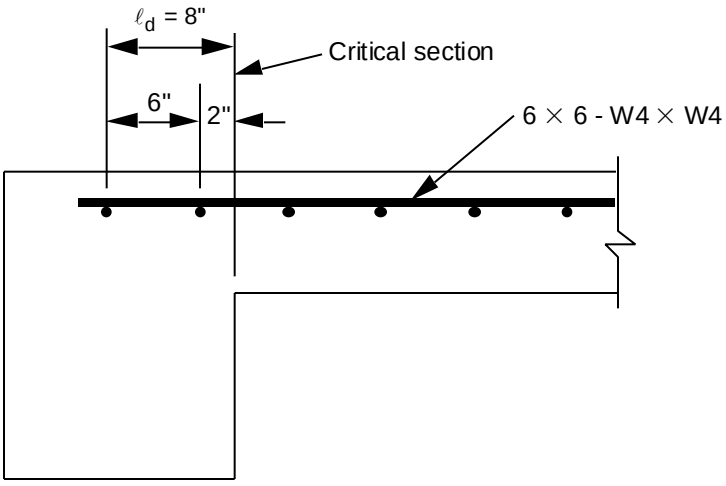


Figure 4-5 Development of 6 × 6-W4 × W4 Fabric

Check fabric 6 × 6-W20 × W20:

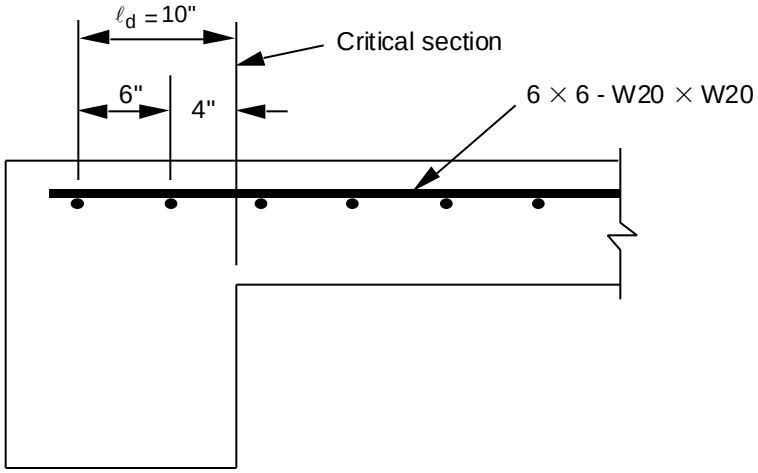
$$\ell_d = 0.27 \times (0.20/6) \times \left(60{,}000/\sqrt{3000}\right) \times 1.0 = 9.9 \text{ in.}$$

> 6 in.

$>$ (1 space + 2 in.)

As shown in Fig. 4-6, an additional 2 in. beyond the two cross wires plus 2 in. embedment is required to fully develop the W20 fabric. If the longitudinal spacing is reduced to 4 in. (4 × 6-W20 × W20), a minimum ℓ_d of 15 in. is required for full development, i.e. 3 cross wires plus 3 in. embedment.

References 4.4 and 4.5 provide design aids for welded wire fabric, including development length tables for both deformed and plain welded wire fabric.



Note: If end support is not wide enough for straight embedment, the development length ℓ_d may be bent down (hooked) into support.

Figure 4-6 Development of 6 × 6-W20 × W20 Fabric

12.9 DEVELOPMENT OF PRESTRESSING STRAND

Prestressed concrete members may be either pretensioned or post-tensioned. In post-tensioned applications, development of tendons is accomplished through mechanical anchorage. Tendons may include strands, wires or high-strength bars.

In pretensioned members, tendons typically consist of seven-wire strands. Development length ℓ_d (in inches) of strands is specified in 12.9.1 and is computed from Eq. (12-2), which was formerly in R12.9:

$$\ell_d = \left(\frac{f_{se}}{3}\right) d_b + \left(f_{ps} - f_{se}\right) d_b \qquad \textit{Eq. (12-2)}$$

where f_{ps} = stress in prestressed reinforcement at nominal strength, ksi

f_{se} = effective stress in prestressed reinforcement after all prestress losses, ksi

d_b = nominal diameter of strand, in.

The expressions in parentheses are dimensionless.

The term $\left(\frac{f_{se}}{3}\right) d_b$ represents the transfer length of the strand (ℓ_t), i.e., the distance over which the strand

should be bonded to the concrete to develop f_{se} in the strand. The second term, $\left(f_{ps} - f_{se}\right) d_b$, represents the flexural bond length, i.e., the additional length over which the strand should be bonded so that a stress f_{ps} may develop in the strand at nominal strength of the member.

Where bonding of one or more strands does not extend to the end of the member, critical sections may be at locations other than those where full design strength is required (see 12.9.2). In such cases, a more detailed analysis may be required. Similarly, where heavy concentrated loads occur within the strand development length, critical sections may occur away from the section that is required to develop full design strength.

Note that two times the development length specified in 12.9.1 is required for "debonded" strands (12.9.3) when the member is designed allowing tension in the precompressed tensile zone under service load conditions.

In some pretensioned applications, total member length may be shorter than two times the development length. This condition may be encountered in very short precast, prestressed concrete members. In such cases, the strands will not be able to develop f_{ps}. Maximum usable stress in underdeveloped strands can be derived as illustrated in Fig. 4-7. The maximum strand stress, f_{max}, at distance ℓ_x from girder end can be determined for the condition of $\ell_t < \ell_x < \ell_d$ as follows:

$$f_{max} = f_{se} + \Delta f$$

$$= f_{se} + \frac{(f_{ps} - f_{se})}{(f_{ps} - f_{se})\, d_b} \left(\ell_x - \frac{f_{se}}{3} d_b \right)$$

$$= f_{se} + \frac{\ell_x}{d_b} - \frac{f_{se}}{3}$$

Therefore,

$$f_{max} = \frac{\ell_x}{d_b} + \frac{2}{3} f_{se}$$

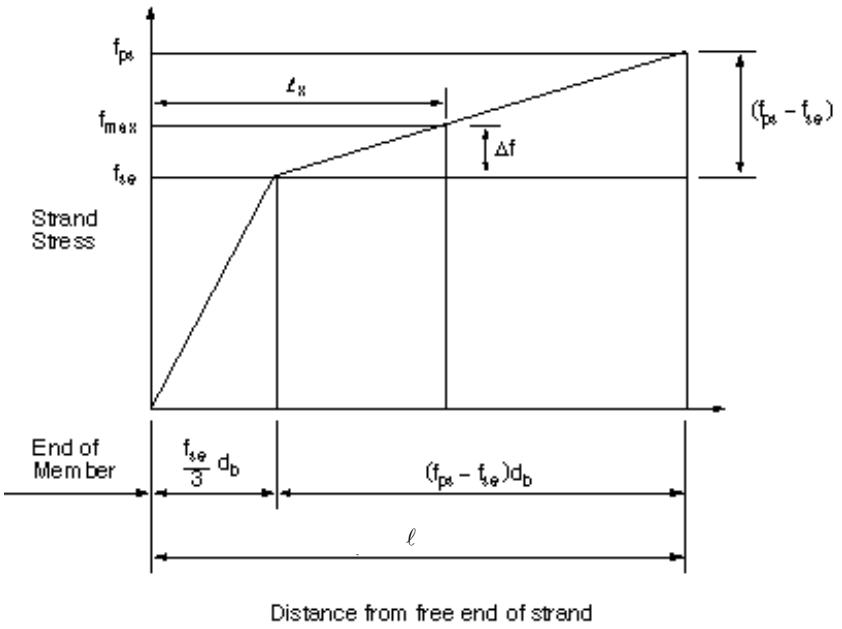
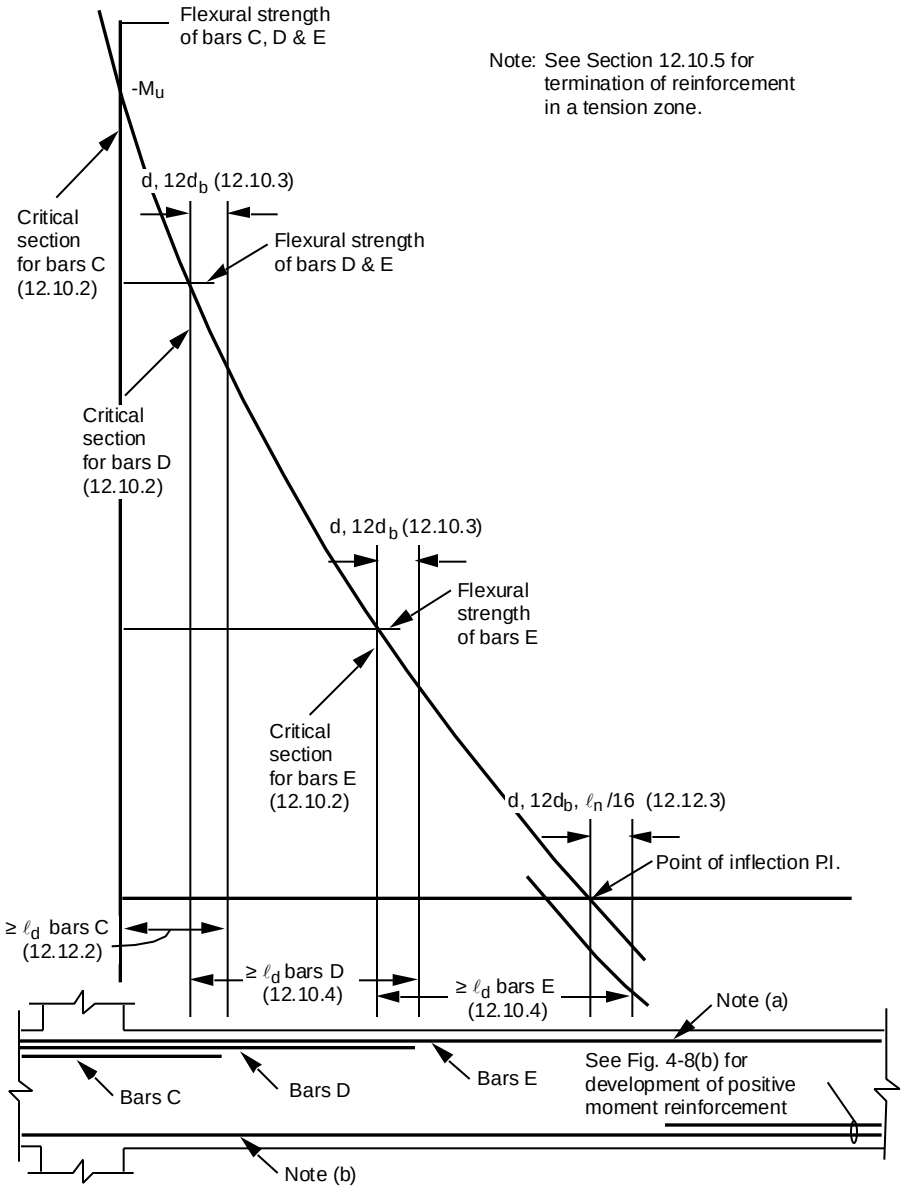


Figure 4-7 Strand Transfer and Development Lengths

12.10 DEVELOPMENT OF FLEXURAL REINFORCEMENT—GENERAL

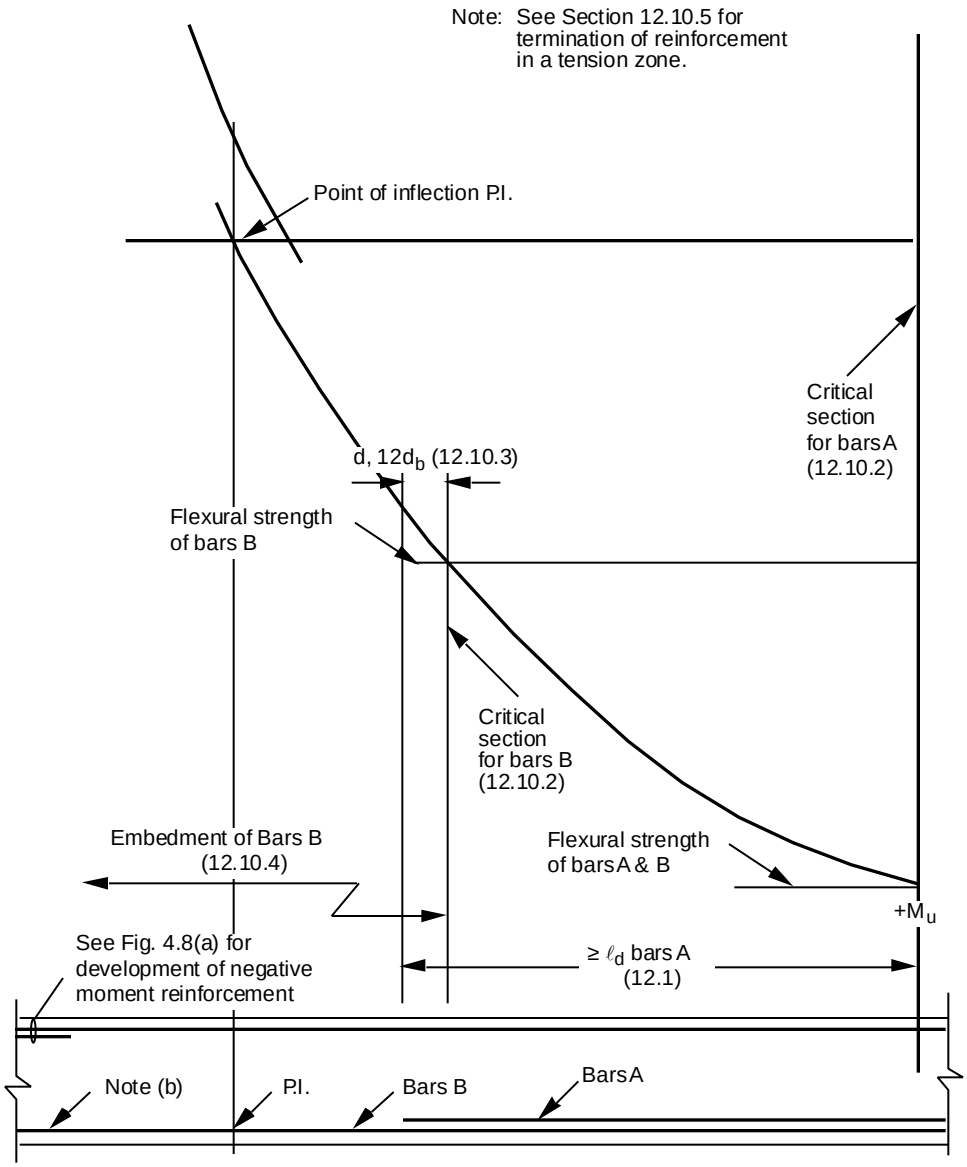
Section 12.10 gives the basic requirements for providing development length of reinforcement from the points of maximum or critical stress. Figures 4-8(a) and (b) illustrate typical critical sections and code requirements for development and termination of flexural reinforcement in a continuous beam. Points of maximum positive and negative moments $\left(M_u^+ \text{ and } M_u^-\right)$ are critical sections, from which adequate anchorage ℓ_d must be provided. Critical sections are also at points within the span where adjacent reinforcement is terminated; continuing bars must have adequate anchorage ℓ_d from the theoretical cut-off points of terminated bars (see 12.10.4). Note also that terminated bars must be extended beyond the theoretical cut-off points in accordance with 12.10.3. This extension requirement is to guard against possible shifting of the moment diagram due to load variation, settlement of supports, and other unforeseen changes in the moment conditions. Development lengths ℓ_d are determined from 12.2.



Note (a): Portion of total negative reinforcement $\left(A_s^-\right)$ must be continuous (or spliced with a Class A splice or a mechanical or welded splice satisfying 12.14.3) along full length of perimeter beams (7.13.2.2).

(a) Negative Moment Reinforcement

Figure 4-8 Development of Positive and Negative Moment Reinforcement



Note (b): Portion of total positive reinforcement $\left(A_s^+\right)$ must be continuous (or spliced with a Class A splice or a mechanical or welded splice satisfying 12.14.3) along full length of perimeter beams and of beams without closed stirrups (7.13.2.2). See also 7.13.2.4.

(b) Positive Moment Reinforcement

Figure 4-8 Development of Positive and Negative Moment Reinforcement — continued —

Sections 12.10.1 and 12.10.5 address the option of anchoring tension reinforcement in a compression zone. Research has confirmed the need for restrictions on terminating bars in a tension zone. When flexural bars are cut off in a tension zone, flexural cracks tend to open early. If the shear stress in the area of bar cut-off and tensile stress in the remaining bars at the cut-off location are near the permissible limits, diagonal tension cracking tends to develop from the flexural cracks. One of the three alternatives of 12.10.5 must be satisfied to reduce the possible occurrence of diagonal tension cracking near bar cut-offs in a tension zone. Section 12.10.5.2 requires excess stirrup area over that required for shear and torsion. Requirements of 12.10.5 are not intended to apply to tension splices.

Section 12.10.6 is for end anchorage of tension bars in special flexural members such as brackets, members of variable depth, and others where bar stress, f_s, does not decrease linearly in proportion to a decreasing moment.

In Fig. 4-9, the development length ℓ_d into the support is probably less critical than the required development length. In such a case, safety depends primarily on the outer end anchorage provided. A welded cross bar of equal diameter should provide an effective end anchorage. A standard end hook in the vertical plane may not be effective because an essentially plain concrete corner might exist near the load and could cause localized failure. Where brackets are wide and loads are not applied too close to the corners, U-shaped bars in a horizontal plane provide effective end hooks.

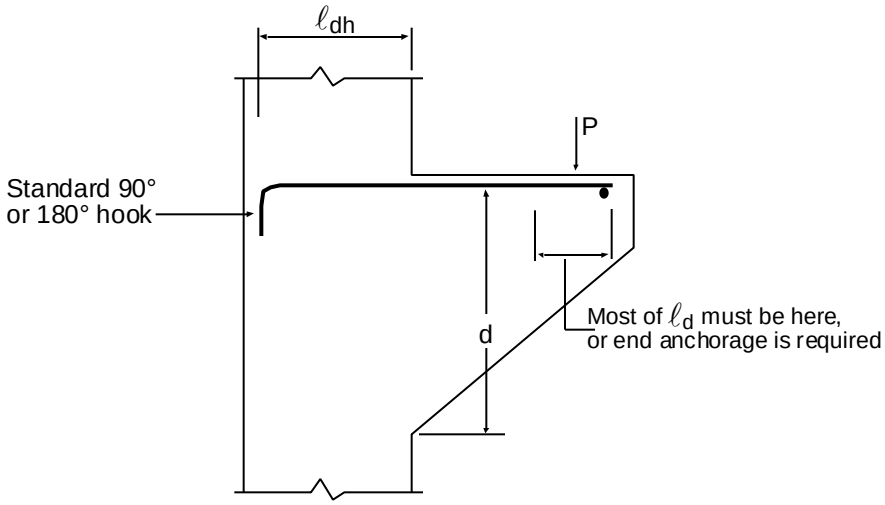


Figure 4-9 Special Member Largely Dependent on End Anchorage

12.11 DEVELOPMENT OF POSITIVE MOMENT REINFORCEMENT

To further guard against possible shifting of moments due to various causes, 12.11.1 requires specific amounts of positive moment reinforcement to be extended along the same face of the member into the support, and for beams, to be embedded into the support at least 6 in. The specified amounts are one-third for simple members and one-fourth for continuous members. In Fig. 4-8(b), for example, the area of Bars "B" would have to be at least one-fourth of the area of reinforcement required at the point of maximum positive moment M_u^+.

Section 12.11.2 is intended to assure ductility in the structure under severe overload, as might be experienced in a strong wind or earthquake. In a lateral load resisting system, full anchorage of the reinforcement extended into the support provides for possible stress reversal under such overload. Anchorage must be provided to develop the full yield strength in tension at the face of the support. The provision will require such members to have bottom bars lapped at interior supports or hooked at exterior supports. The full anchorage requirement does not apply to any excess reinforcement provided at the support.

Section 12.11.3 limits bar sizes for the positive moment reinforcement at simple supports and at points of inflection. In effect, this places a design restraint on flexural bond stress in areas of small moment and large shear. Such a condition could exist in a heavily loaded beam of short span, thus requiring large size bars to be developed within a short distance. Bars should be limited to a diameter such that the development length ℓ_d computed for f_y according to 12.2 does not exceed $(M_n/V_u) + \ell_a$ (12.11.3). The limit on bar size at simple supports is waived if the bars have standard end hooks or mechanical anchorages terminating beyond the centerline of the support. Mechanical anchorages must be equivalent to standard hooks.

The length (M_n/V_u) corresponds to the development length of the maximum size bar permitted by the previously used flexural bond equation. The length (M_n/V_u) may be increased 30% when the ends of the bars are confined by a compressive reaction, such as provided by a column below, but not when a beam frames into a girder.

For the simply-supported beam shown in Fig. 4-10, the maximum permissible ℓ_d for Bars "a" is 1.3 $M_n/V_u + \ell_a$. This has the effect of limiting the size of bar to satisfy flexural bond. Even though the total embedment length from the critical section for Bars "a" is greater than 1.3 $M_n/V_u + \ell_a$, the size of Bars "a" must be limited so that $\ell_d \leq 1.3\ M_n/V_u + \ell_a$. Note that M_n is the nominal flexural strength of the cross-section (without the φ factor). As noted previously, larger bar sizes can be accommodated by providing a standard hook or mechanical anchorage at the end of the bar within the support. At a point of inflection (see Fig. 4-11), the positive moment reinforcement must have a development length ℓ_d, as computed by 12.2, not to exceed the value of $(M_n/V_u) + \ell_a$, with ℓ_a not greater than d or $12d_b$, whichever is greater.

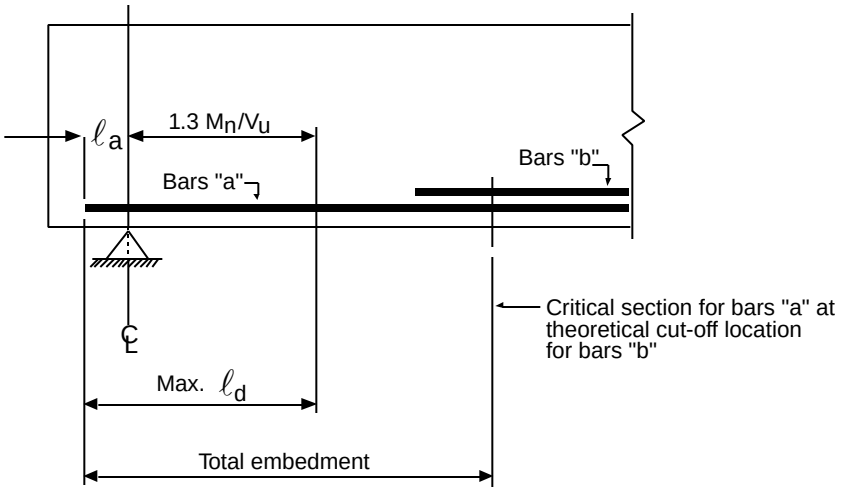


Figure 4-10 Development Length Requirements at Simple Support (straight bars)

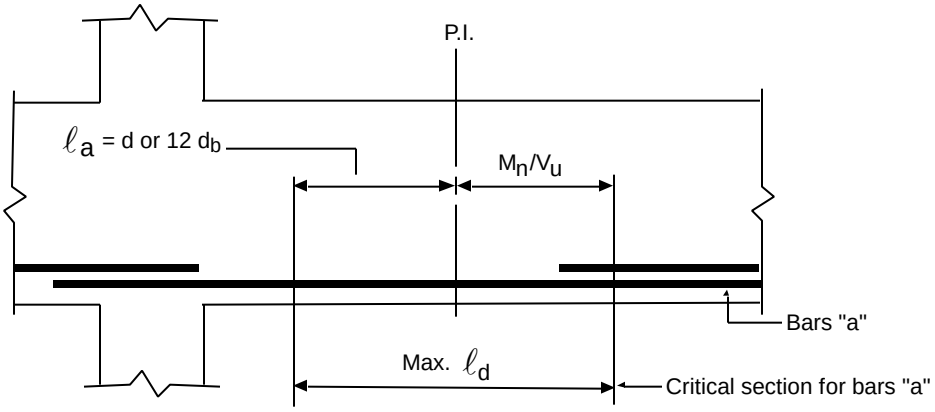
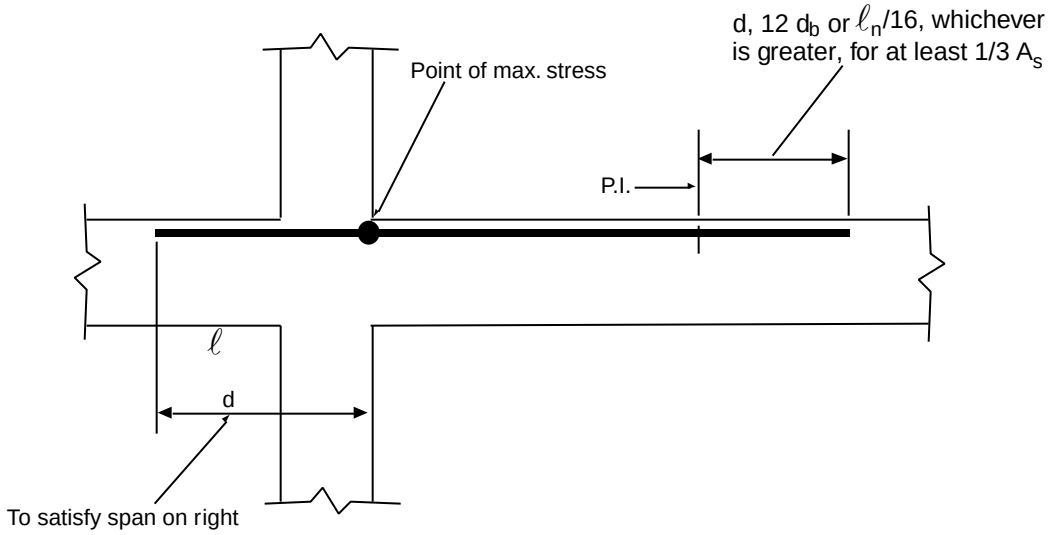


Figure 4-11 Concept for Determining Maximum Size of Bars "a" at Point of Inflection (12.11.3)

Sections 12.11.4 and 12.12.4 address development of positive and negative moment reinforcement in deep flexural members. The provisions specify that at simple supports of deep beams, positive moment tension reinforcement should be anchored to develop its specified yield strength f_y in tension at the face of the support. However, if the design is carried out using the strut-and-tie method of Appendix A, this reinforcement shall be anchored in accordance with A.4.3. At interior supports of deep beams, both positive and negative moment tension reinforcement shall be continuous or be spliced with that of the adjacent spans.

12.12 DEVELOPMENT OF NEGATIVE MOMENT REINFORCEMENT

The requirements in 12.12.3 guard against possible shifting of the moment diagram at points of inflection. At least one-third of the negative moment reinforcement provided at a support must be extended a specified embedment length beyond a point of inflection. The embedment length must be the effective depth of the member d, $12d_b$, or 1/16 the clear span, whichever is greater, as shown in Figs. 4-8 and 4-12. The area of Bars "E" in Fig. 4-8(a) must be at least one-third the area of reinforcement provided for $-M_u$ at the face of the support. Anchorage of top reinforcement in tension beyond interior support of continuous members usually becomes part of the adjacent span top reinforcement, as shown in Fig. 4-12.



(Usually such anchorage becomes part of adjacent beam reinforcement)

Figure 4-12 Anchorage into Adjacent Beam

Standard end hooks are an effective means of developing top bars in tension at exterior supports as shown in Fig. 4-13. Code requirements for development of standard hooks are discussed above in 12.5.

12.13 DEVELOPMENT OF WEB REINFORCEMENT

Stirrups must be properly anchored so that the full tensile force in the stirrup can be developed at or near mid-depth of the member. To function properly, stirrups must be extended as close to the compression and tension surfaces of the member as cover requirements and proximity of other reinforcement permit (12.13.1). It is equally important for stirrups to be anchored as close to the compression face of the member as possible because flexural tension cracks initiate at the tension face and extend towards the compression zone as member strength is approached.

The ACI code anchorage details for stirrups have evolved over many editions of the code and are based primarily on past experience and performance in laboratory tests. For No. 5 bar and smaller, stirrup anchorage is provided by a standard stirrup hook (90 deg bend plus $6d_b$ extension at free end of bar)* around a longitudinal bar (12.13.2.1). The same anchorage detail is permitted for the larger stirrup bar sizes, No. 6, No. 7, and No. 8, in Grade 40. Note that for the larger bar sizes, the 90 deg hook detail requires a $12d_b$ extension at the free end of the bar (7.1.3(b)). Fig. 4-14 illustrates the anchorage requirement for U-stirrups fabricated from deformed bars and deformed wire.

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* For structures located in regions of high seismic risk, stirrups required to be hoops must be anchored with a 135-deg bend plus $6d_b$ (but not less than 3 in.) extension. See definition of seismic hook in 21.1.

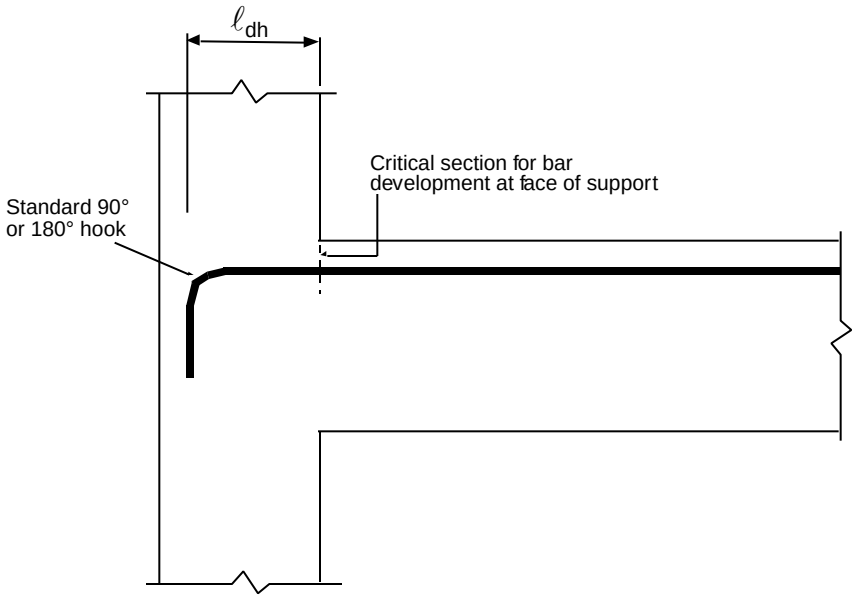


Figure 4-13 Anchorage into Exterior Support with Standard Hook

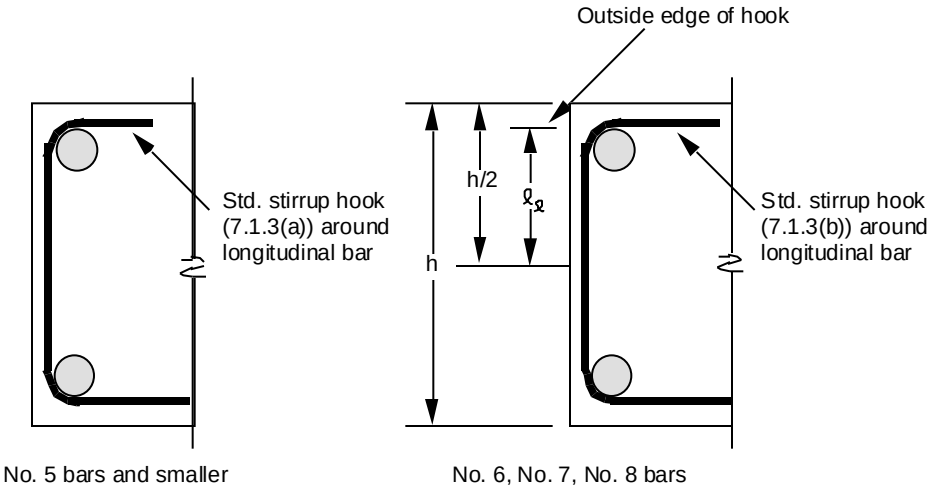


Figure 4-14 Anchorage Details for U-Stirrups (Deformed Bars and Deformed Wires)

For the larger stirrup bar sizes (No. 6, No. 7, or No. 8) in Grade 60, in addition to a standard stirrup hook, an embedment of $0.014 d_b f_y / \sqrt{f'_c}$ between midheight of member and outside end of hook is required. The available embedment length, denoted ℓ_ℓ, must be checked to ensure adequate anchorage at the higher bar force (see 12.13.2.2). The embedment length required is illustrated in Fig. 4-14 and listed in Table 4-5. Minimum depth of member required to accommodate No. 6, No. 7, or No. 8 stirrups fabricated in Grade 60 is also shown in Table 4-6. For practical size of beams where the loads are of such magnitude to require No. 6, No. 7, or No. 8 bar sizes for shear reinforcement, the embedment length required should be easily satisfied, and the designer need only be concerned with providing a standard stirrup hook around a longitudinal bar for proper stirrup end anchorage.

Provisions of 12.13.2.3 covering the use of welded plain wire fabric as simple U-stirrups are shown in Fig. 4-15. Requirements for stirrup anchorage (12.13.2.4) detail for straight single leg stirrups formed with welded plain or deformed wire fabric is shown in Fig. 4-16. Anchorage of the single leg is provided primarily by the longitudinal wires. Use of welded wire fabric for shear reinforcement has become commonplace in the precast, prestressed concrete industry.

Table 4-5 Embedment Length ℓ_ℓ (in.) for Grade 60 Stirrups

Bar Size No.	Concrete Compressive Strength f'_c, psi					
	3000	4000	5000	6000	8000	10,000
6	11.5	10.0	8.9	8.1	7.0	6.3
7	13.4	11.6	10.4	9.5	8.2	7.4
8	15.3	13.3	11.9	10.8	9.4	8.4

Table 4-6 Minimum Depth of Member (in.) to Accommodate Grade 60 No. 6, No. 7, and No. 8 Stirrups

Clear cover to stirrup (in.)	Bar Size No.	Concrete Compressive Strength f'_c, psi					
		3000	4000	5000	6000	8000	10,000
1-1/2	6	26	23	21	20	17	16
	7	30	27	24	22	20	18
	8	34	30	27	25	22	20
2	6	27	24	22	21	18	17
	7	31	28	25	23	21	19
	8	35	31	28	26	23	21

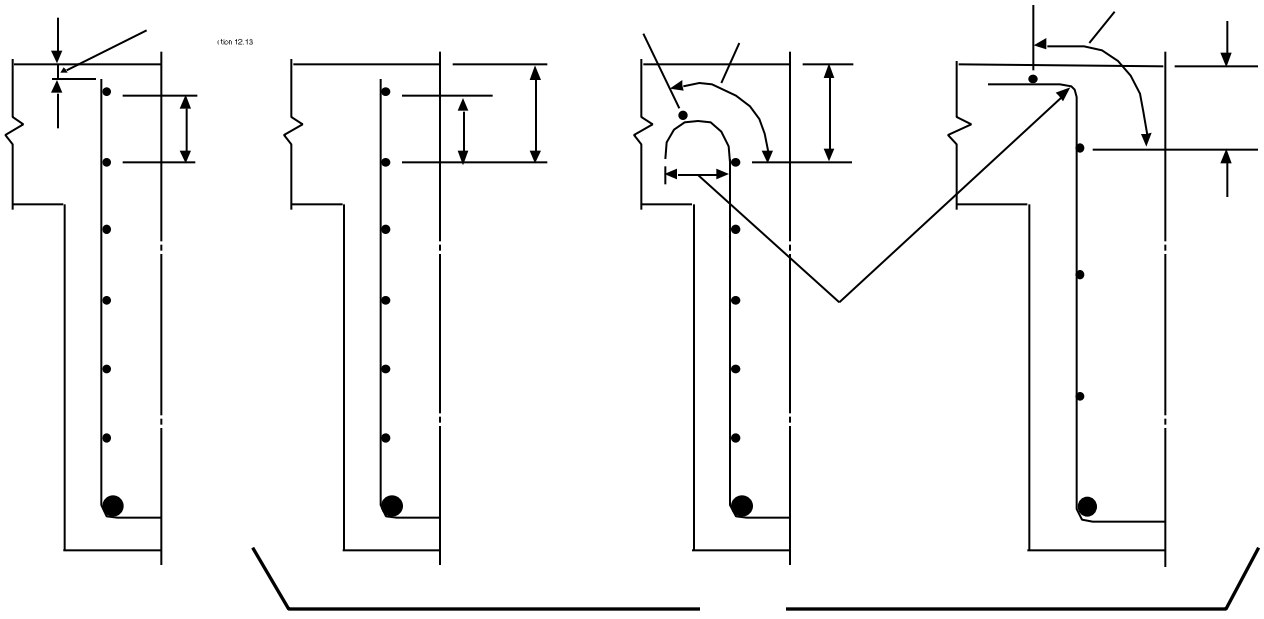


Figure 4-15 Anchorage Details for Welded Plain Wire Fabric U-Stirrups (12.13.2.3)

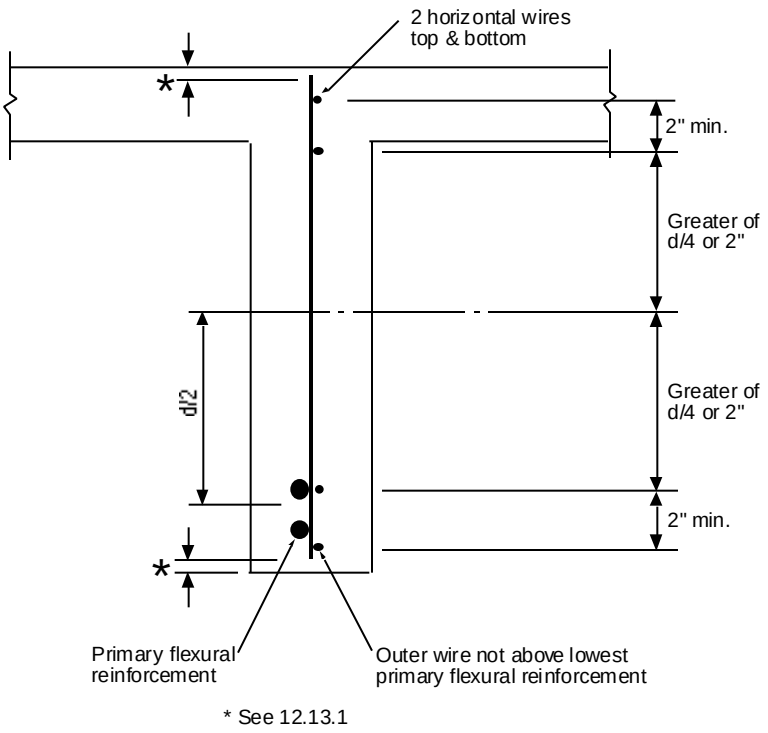


Figure 4-16 Anchorage Details for Welded Wire Fabric Single Leg Stirrups (12.13.2.4)

Note that 12.13.3 requires that each bend in the continuous portion of U-stirrups must enclose a longitudinal bar. This requirement is usually satisfied for simple U-stirrups, but requires special attention in bar detailing when multiple U-stirrups are used.

Clarifications of anchorage of web reinforcement made in the 1989 code eliminated the possibility of anchoring web reinforcement without hooking the stirrup around a longitudinal bar. Inquiries have shown that some designers routinely use small bars in joists without hooking them around a longitudinal bar, particularly a continuously bent single leg stirrup called a W-stirrup, accordion stirrup, or snake. To recognize this practice, 12.13.2.5 was introduced starting with the 1995 code.

12.13.4 Anchorage for Bent-Up Bars

Section 12.13.4 gives anchorage requirements for longitudinal (flexural) bars bent up to resist shear. If the bent-up bars are extended into a tension region, the bent-up bars must be continuous with the longitudinal reinforcement. If the bent-up bars are extended into a compression region, the required anchorage length beyond mid-depth of the member (d/2) must be based on that part of f_y required to satisfy Eq. (11-17). For example, if f_y = 60,000 psi and calculations indicate that 30,000 psi is required to satisfy Eq. (11-17), the required anchorage length $\ell'_d = (30{,}000/60{,}000)\ell_d$, where ℓ_d is the tension development length for full f_y per 12.2. Fig. 4-17 shows the required anchorage length ℓ'_d.

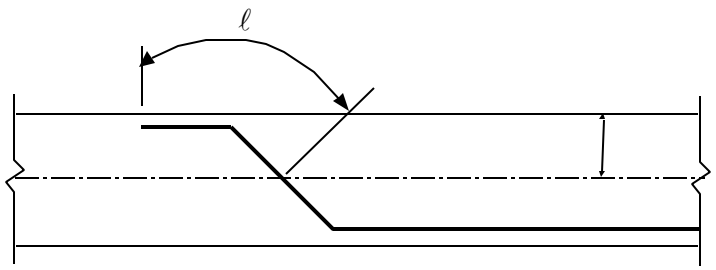


Figure 4-17 Anchorage for Bent-Up Bars

12.13.5 Closed Stirrups or Ties

Section 12.13.5 gives requirements for lap splicing double U-stirrups or ties (without hooks) to form a closed stirrup. Legs are considered properly spliced when the laps are $1.3\ell_d$ as shown in Fig. 4-18, where ℓ_d is determined from 12.2.

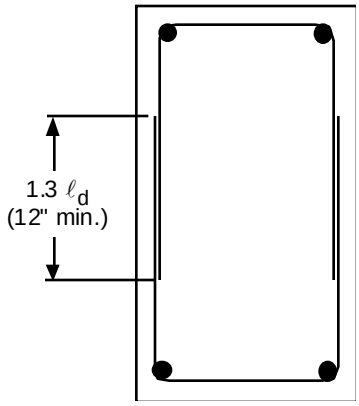
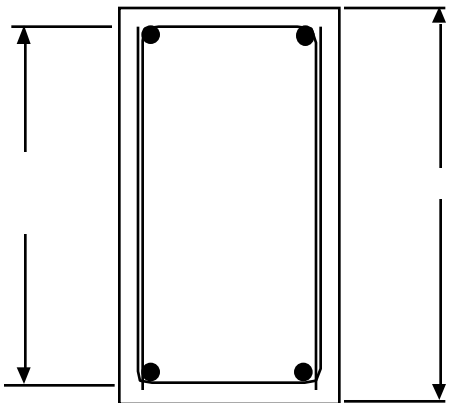


Figure 4-18 Overlapping U-Stirrups to Form Closed Unit

Alternatively, if a lap splice of $1.3\ell_d$ cannot fit within the depth of shallow members, provided that depth of members is at least 18 in., double U-stirrups may be used if each leg extends the full available depth of the member and the force in each leg does not exceed 9000 lb ($A_b f_y \leq 9000$ lb.; see Fig. 4-19).



If stirrups are designed for the full yield strength f_y, No. 3 and 4 stirrups of Grade 40 and only No. 3 of Grade 60 satisfy the 9000 lb limitation.

Figure 4-19 Lap Splice Alternative for U-Stirrups

12.14 SPLICES OF REINFORCEMENT—GENERAL

The splice provisions require the engineer to show clear and complete splice details in the contract documents. The structural drawings, notes and specifications should clearly show or describe all splice locations, types permitted or required, and for lap splices, length of lap required. The engineer cannot simply state that all splices shall be in accordance with the ACI 318 code. This is because many factors affect splices of reinforcement, such as the following for tension lap splices of deformed bars:

- bar size
- bar yield strength
- concrete compressive strength
- bar location (top bars or other bars)
- normal weight or lightweight aggregate concrete
- spacing and cover of bars being developed
- enclosing transverse reinforcement
- epoxy coating
- number of bars spliced at one location

It is virtually impossible for a reinforcing bar detailer to know what splices are required at a given location in a structure, unless the engineer explicitly illustrates or defines the splice requirements. Section 12.14.1 states: "Splices of reinforcement shall be made only as required or permitted on the design drawings, or in specifications, or as authorized by the engineer."

Two industry publications are suggested as design reference material for proper splicing of reinforcement. Reference 4.4 provides design aid data in the use of welded wire fabric, including development length and splice length tables for both deformed and plain wire fabric. Reference 4.5 provides accepted practices in splicing reinforcement; use of lap, mechanical, and welded splices are described, including simplified design data for lap splice lengths.

12.14.2 Lap Splices

Lap splices are not permitted for bars larger than No. 11, either in tension or compression, except:

- No. 14 and No. 18 bars in compression only may be lap spliced to No. 11 and smaller bars (12.16.2), and
- No. 14 and No. 18 bars in compression only may be lap spliced to smaller size footing dowels (15.8.2.3).

Section 12.14.2.2 gives the provisions for lap splicing of bars in a bundle (tension or compression). The lap lengths required for individual bars within a bundle must be increased by 20 percent and 33 percent for 3- and 4-bar bundles, respectively. Overlapping of individual bar splices within a bundle is not permitted. Two bundles must not be lap-spliced as individual bars.

Bars in flexural members may be spliced by noncontact lap splices. To prevent a possible unreinforced section in a spaced (noncontact) lap splice, 12.14.2.3 limits the maximum distance between bars in a splice to one-fifth the lap length, or 6 in. whichever is less. Contact lap splices are preferred for the practical reason that when the bars are wired together, they are more easily secured against displacement during concrete placement.

12.14.3 Mechanical and Welded Splices

Section 12.14.3 permits the use of mechanical or welded splices. A full mechanical splice must develop, in tension or compression, at least 125 percent of the specified yield strength of the bar (12.14.3.2). In a full welded splice, the bars must develop in tension at least 125 percent of the specified yield strength of the bar (12.4.3.4). ANSI/AWS D1.4 allows indirect welds where the bars are not butted. Note that ANSI/AWS D1.4 indicates that wherever practical, direct butt splices are preferable for No. 7 and larger bars. Use of mechanical or welded splices having less than 125 percent of the specified yield strength of the bar is limited to No. 5 and smaller bars (12.14.3.5) in regions of low computed stress. Mechanical and welded splices not meeting 12.14.3.2 and 12.14.3.4 are limited to No. 5 and smaller bars due to the potentially brittle nature of failure at these welds.

Section 12.14.3.3 requires all welding of reinforcement to conform to *Structural Welding Code-Reinforcing Steel* (ANSI/AWS D1.4). Section 3.5.2 requires that the reinforcement to be welded must be indicated on the drawings, and the welding procedure to be used must be specified. To carry out these code requirements properly, the engineer should be familiar with provisions in ANSI/AWS D1.4 and the ASTM specifications for reinforcing bars.

The standard rebar specifications ASTM A615, A616 and A617 do not address weldability of the steel. No limits are given in these specifications on the chemical elements that affect weldability of the steels. A key item in ANSI/AWS D1.4 is carbon equivalent (C.E.). The minimum preheat and interpass temperatures specified in ANSI/AWS D1.4 are based on C.E. and bar size. Thus, as indicated in 3.5.2 and R3.5.2, when welding is required, the ASTM A615, A616 and A617 rebar specifications must be supplemented to require a report of the chemical composition to assure that the welding procedure specified is compatible with the chemistry of the bars.

ASTM A706 reinforcing bars are intended for welding. The A706 specification contains restrictions on chemical composition, including carbon, and C.E. is limited to 0.55 percent. The chemical composition and C.E. must be reported. By limiting C.E. to 0.55 percent, little or no preheat is required by ANSI/AWS D1.4. Thus, the engineer does not need to supplement the A706 specification when the bars are to be welded. However, before specifying ASTM A706 reinforcing bars, local availability should be investigated.

Reference 4.5 contains a detailed discussion of welded splices. Included in the discussion are requirements for other important items such as field inspection, supervision, and quality control.

The ANSI/AWS D1.4 document covers the welding of reinforcing bars only. For welding of wire to wire, and of wire or welded wire fabric to reinforcing bars or structural steels, such welding should conform to applicable provisions of ANSI/AWS D1.4 and to supplementary requirements specified by the engineer. Also, the engineer should be aware that there is a potential loss of yield strength and ductility of low carbon cold-drawn wire if wire is welded by a process other than controlled resistance welding used in the manufacture of welded wire fabric.

In the discussion of 7.5 in Part 3 of this document, it was noted that welding of crossing bars (tack welding) is not permitted for assembly of reinforcement unless authorized by the engineer. An example of tack welding would be a column cage where the ties are secured to the longitudinal bars by small arc welds. Such welding can cause a metallurgical notch in the longitudinal bars, which may affect the strength of the bars. Tack welding seems to be particularly detrimental to ductility (impact resistance) and fatigue resistance. Reference 4.5 recommends: "Never permit field welding of crossing bars ('tack' welding, 'spot' welding, etc.). Tie wire will do the job without harm to the bars."

12.15 SPLICES OF DEFORMED BARS AND DEFORMED WIRE IN TENSION

Tension lap splices of deformed bars and deformed wire are designated as Class A and B with the length of lap being a multiple of the tensile development length ℓ_d. The two-level splice classification (Class A & B) is intended to encourage designers to splice bars at points of minimum stress and to stagger lap splices along the length of the bars to improve behavior of critical details.

The development length ℓ_d (12.2) used in the calculation of lap length must be that for the full f_y because the splice classifications already reflect any excess reinforcement at the splice location (factor of 12.2.5 for excess A_s must not be used). The minimum length of lap is 12 in.

For lap splices of slab and wall reinforcement, effective clear spacing of bars being spliced at the same location is taken as the clear spacing between the spliced bars (R12.15.1). This clear spacing criterion is illustrated in Fig. 4-20(a). Spacing for noncontact lap splices (spacing between lapped bars not greater than (1/5) lap length nor 6 in.) should be considered the same as for contact lap splices. For lap splices of column and beam bars, effective clear spacing between bars being spliced will depend on the orientation of the lapped bars; see Fig. 4-20(b) and (c), respectively.

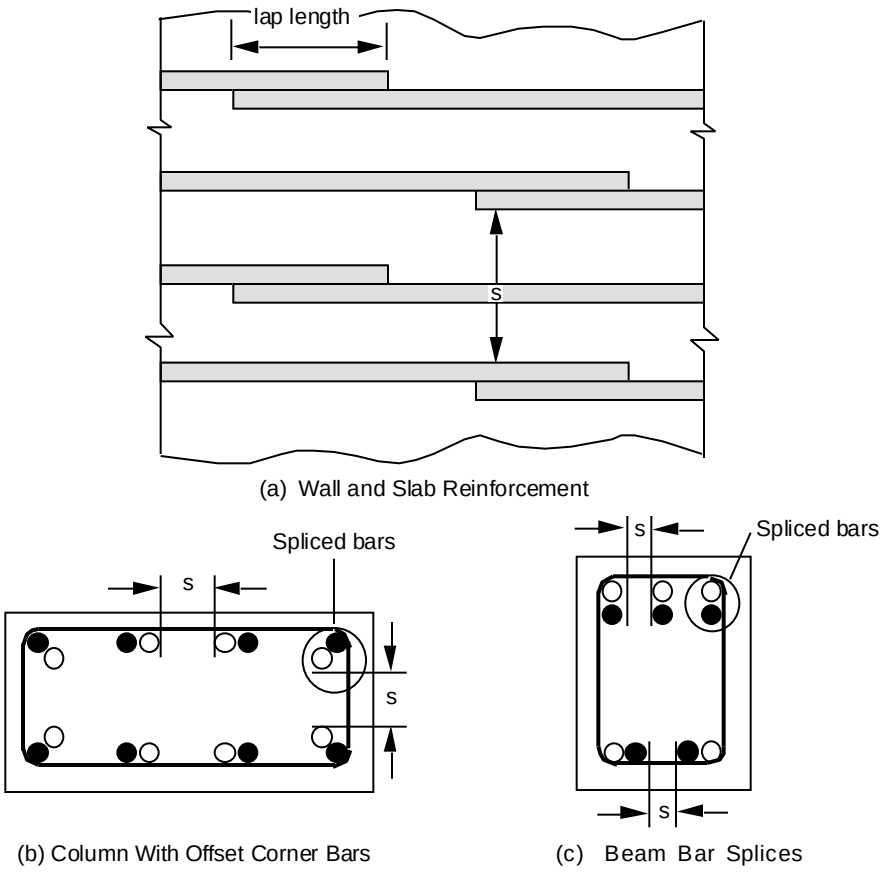


Figure 4-20 Effective Clear Spacing of Spliced Bars

The designer must specify the class of tension lap splice to be used. The class of splice depends on the magnitude of tensile stress in the reinforcement and the percentage of total reinforcement to be lap spliced within any given splice length as shown in Table 4-7. If the area of tensile reinforcement provided at the splice location is more than twice that required for strength (low tensile stress) and 1/2 or less of the total steel area is lap spliced within the required splice length, a Class A splice may be used. Both splice conditions must be satisfied, otherwise, a Class B splice must be used. In other words, if the area of reinforcement provided at the splice location is less than twice that required for strength (high tensile stress) and/or more than 1/2 of the total area is to be spliced within the lap length, a Class B splice must be used.

Table 4-7 Tension Lap Splice Conditions (at splice location)

CLASS A...1.0ℓ_d	CLASS B...1.3ℓ_d
(A_s provided) ≥ 2 (A_s required) and percent A_s spliced ≤ 50	All other conditions

Mechanical or welded splices conforming to 12.14.3 may be used in lieu of tension lap splices. Section R12.15.3 clarifies that such splices need not be staggered although such staggering is encouraged where the area of reinforcement provided is less than twice that required by analysis.

Section 12.15.4 emphasizes that mechanical and welded splices not meeting the requirements of 12.14.3.2 and 12.14.3.4, respectively, are only allowed for No. 5 bars and smaller, and only if certain conditions are met (see 12.15.4.1 and 12.15.4.2).

Splices in tension tie members are required to be made with a full mechanical or welded splice with a 30 in. stagger between adjacent bar splices. See definition of "tension tie member" in R12.15.5.

12.16 SPLICES OF DEFORMED BARS IN COMPRESSION

Since bond behavior of reinforcing bars in compression is not complicated by the potential problem of transverse tension cracking in the concrete, compression lap splices do not require such strict provisions as those specified for tension lap splices. Tests have shown that the strength of compression lap splices depends primarily on end bearing of the bars on the concrete, without a proportional increase in strength even when the lap length is doubled. Thus, the code requires significant longer lap length for bars with a yield strength greater than 60,000 psi.

12.16.1 Compression Lap Splices

Calculation of compression lap splices was simplified starting with the '89 code by removing the redundant calculation for development length in compression. For compression lap splices, 12.16.1 requires the minimum lap length to be simply $0.0005 d_b f_y$ for f_y = 60,000 psi or less, but not less than 12 in. For reinforcing bars with a yield strength greater than 60,000 psi, a minimum lap length of $(0.0009 f_y - 24)\, d_b$ but not less than 12 in. is specified. Lap splice lengths must be increased by one-third for concrete with a specified compressive strength less than 3000 psi.

As noted in the discussion of 12.14.2, No. 14 and No. 18 bars may be lap spliced, in compression only, to No. 11 and smaller bars or to smaller size footing dowels. Section 12.16.2 requires that when bars of a different size are lap spliced in compression, the length of lap must be the compression development length of the larger bar, or the compression lap splice length of the smaller bar, whichever is the longer length.

12.16.4 End-Bearing Splices

Section 12.16.4 specifies the requirements for end-bearing compression splices. End-bearing splices are only permitted in members containing closed ties, closed stirrups or spirals (12.16.4.3). Section R12.16.4.1 cautions the engineer in the use of end-bearing splices for bars inclined from the vertical. End-bearing splices for compression bars have been used almost exclusively in columns and the intent is to limit use to essentially vertical bars because of the field difficulty of getting adequate end bearing on horizontal bars or bars significantly inclined from the vertical. Mechanical or welded splices are also permitted for compression splices and must meet the requirements of 12.14.3.2 or 12.14.3.4, respectively.

12.17 SPECIAL SPLICE REQUIREMENTS FOR COLUMNS

The special splice requirements for columns were significantly simplified in the '89 code. The column splice requirements simplify the amount of calculations that are required compared to previous provisions by assuming that a compression lap splice (12.17.2.1) has a tensile capacity of at least one-fourth f_y (R12.17).

The column splice provisions are based on the concept of providing some tensile resistance at all column splice locations even if analysis indicates compression only at a splice location. In essence, 12.17 establishes the required tensile strength of spliced longitudinal bars in columns. Lap splices, butt-welded splices, mechanical or end-bearing splices may be used.

12.17.2 Lap Splices in Columns

Lap splices are permitted in column bars subject to compression or tension. Type of lap splice to be used will depend on the bar stress at the splice location, compression or tension, and magnitude if tension, due to all factored load combinations considered in the design of the column. Type of lap splice to be used will be gov-

erned by the load combination producing the greatest amount of tension in the bars being spliced. The design requirements for lap splices in column bars can be illustrated by a typical column load-moment strength interaction as shown in Fig. 4-21.

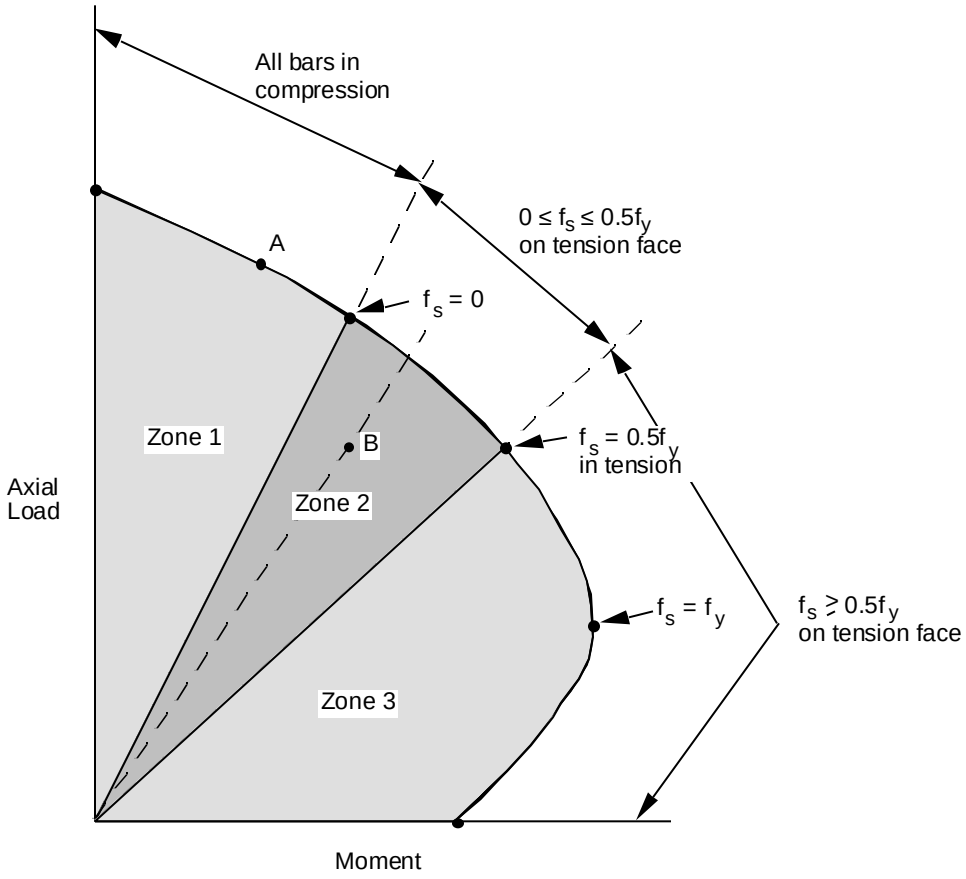


Figure 4-21 Special Splice Requirements for Columns

Bar stress at various locations along the strength interaction curve define segments of the strength curve where the different types of lap splices may be used. For factored load combinations along the strength curve, bar stress can be readily calculated to determine type of lap splice required. However, a design dilemma exists for load combinations that do not fall exactly on the strength curve (below the strength curve) as there is no simple exact method to calculate bar stress for this condition.

A seemingly rational approach is to consider factored load combinations below the strength curve as producing bar stress of the same type, compression or tension, and of the same approximate magnitude as that produced along the segment of the strength curve intersected by radial lines (lines of equal eccentricity) through the load combination point. This assumption becomes more exact as the factored load combinations being investigated fall nearer to the actual strength interaction curve of the column. Using this approach, zones of "bar stress" can be established as shown in Fig. 4-21.

For factored load combinations in Zone 1 of Fig. 4-21, all column bars are considered to be in compression. For load combinations in Zone 2 of the figure, bar stress on the tension face of the column is considered to vary from zero to $0.5f_y$ in tension. For load combinations in Zone 3, bar stress on the tension face is considered to be greater than $0.5f_y$ in tension. Type of lap splice to be used will then depend on which zone, or zones, all factored load combinations considered in the design of the column are located. The designer need only locate the factored load combinations on the load-moment strength diagram for the column and bars selected in the design to determine type of lap splice required. Use of load-moment design charts in this manner will greatly facilitate the

design of column bar splices. For example, if factored gravity load combination governed design of the column, say Point A in Fig. 4-21, where all bars are in compression, but a load combination including wind, say Point B in Fig. 4-21, produces some tension in the bars, the lap splice must be designed for a Zone 2 condition (bar stress is tensile but does not exceed $0.5f_y$ in tension).

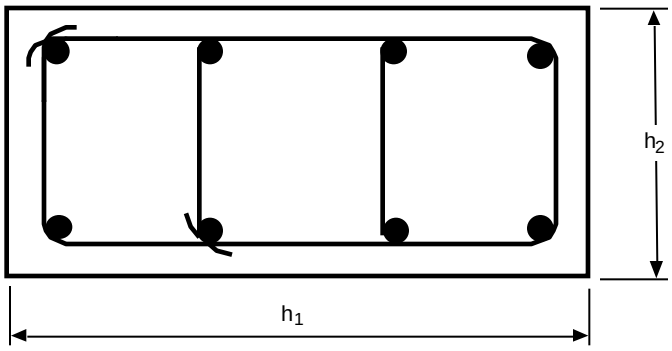
The design requirements for lap splices in columns are summarized in Table 4-8. Note that the compression lap splice permitted when all bars are in compression (see 12.17.2.1) considers a compression lap length adequate as a minimum tensile strength requirement. See Example 4.8 for design application of the lap splice requirements for columns.

Table 4-8 Lap Splices in Columns

12.17.2.1—Bar stress in compression (Zone 1)*	Use compression lap splice (12.16) modified by factor of 0.83 for ties (12.17.2.4) or 0.75 for spirals (12.17.2.5).
12.17.2.2—Bar stress ≤ $0.5f_y$ in tension (Zone 2)*	Use Class B tension lap splice (12.15) if more than 1/2 of total column bars spliced at same location. or Use Class A tension lap splice (12.15) if not more than 1/2 of total column bars spliced at same location. Stagger alternate splices by ℓ_d.
12.17.2.3—Bar stress > $0.5f_y$ in tension (Zone 3)*	Use Class B tension lap splice (12.15).

** For Zones 1, 2, and 3, see Fig. 4-21.*

Sections 12.17.2.4 and 12.17.2.5 provide reduction factors for the compression lap splice when the splice is enclosed throughout its length by ties (0.83 reduction factor) or by a spiral (0.75 reduction factor). Spirals must meet the requirements of 7.10.4 and 10.9.3. When ties are used to reduce the lap splice length, the ties must have a minimum effective area of 0.0015hs. The tie legs in both directions must provide the minimum effective area to permit the 0.83 modification factor. See Fig. 4-22. The 12 in. minimum lap length also applies to these permitted reductions.



(perpendicular to h_1 dimension) 4 tie bar areas ≥ $0.0015h_1s$
(perpendicular to h_2 dimension) 2 tie bar areas ≥ $0.0015h_2s$

Figure 4-22 Application of 12.17.2.4

With the "basic" lap length for compression lap splices a function of bar diameter d_b and bar yield strength f_y, and three modification factors for ties and spirals and for lower concrete strength, it is convenient to establish compression lap splices simply as a multiple of bar diameter.

For Grade 60 bars ..... $30d_b$
enclosed within ties ..... $25d_b$
enclosed within spirals ..... $22.5d_b$
For Grade 75 bars ..... $43.5d_b$
enclosed within ties ..... $36d_b$
enclosed within spirals ..... $33d_b$

but not less than 12 in. For f'_c less than 3000 psi, multiply by a factor of 1.33. Compression lap splice tables for the standard bar sizes can be readily developed using the above values.

12.17.3 Mechanical or Welded Splices in Columns

Mechanical or welded splices are permitted in column bars where bar stress is either compressive or tensile for all factored load combinations (Zones 1, 2, and 3 in Fig. 4-21). "Full" mechanical or "full" welded splices must be used; that is, the mechanical or welded splice must develop at least 125 percent of the bar yield strength, $1.25A_bf_y$. Use of mechanical or welded splices of lesser strength is permitted for splicing bars No. 5 and smaller in tension, in accordance with 12.15.4.

12.17.4 End Bearing Splices in Columns

End bearing splices are permitted for column bars stressed in compression for all factored load combinations (Zone 1 in Fig. 4-21). Even though there is no calculated tension, a minimum tensile strength of the continuing (unspliced) bars must be maintained when end bearing splices are used. Continuing bars on each face of the column must provide a tensile strength of $A_sf_y/4$, where A_s is the total area of bars on that face of the column. Thus, not more than 3/4 of the bars can be spliced on each face of the column at any one location. End bearing splices must be staggered or additional bars must be added at the splice location if more than 3/4 of the bars are to be spliced.

12.18 SPLICES OF WELDED DEFORMED WIRE FABRIC IN TENSION

For tension lap splices of deformed wire fabric, the code requires a minimum lap length of $1.3\ell_d$, but not less than 8 in. Lap length is measured between the ends of each fabric sheet. The development length ℓ_d is the value calculated by the provisions in 12.7. The code also requires that the overlap measured between the outermost cross wires be at least 2 in. Figure 4-23 shows the lap length requirements.

If there are no cross wires within the splice length, the provisions in 12.15 for deformed wire must be used to determine the length of the lap.

Section 12.18.3 provides requirements for splicing welded wire fabric, including deformed wires in one direction and plain wires in the orthogonal direction.

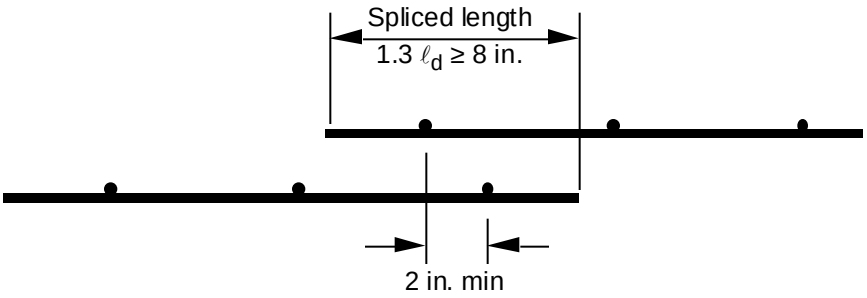


Figure 4-23 Lap Splice Length for Deformed Wire Fabric

12.19 SPLICES OF WELDED PLAIN WIRE FABRIC IN TENSION

The minimum length of lap for tension lap splices of plain wire fabric is dependent upon the ratio of the area of reinforcement provided to that required by analysis. Lap length is measured between the outermost cross wires of each fabric sheet. The required lap lengths are shown in Fig. 4-24.

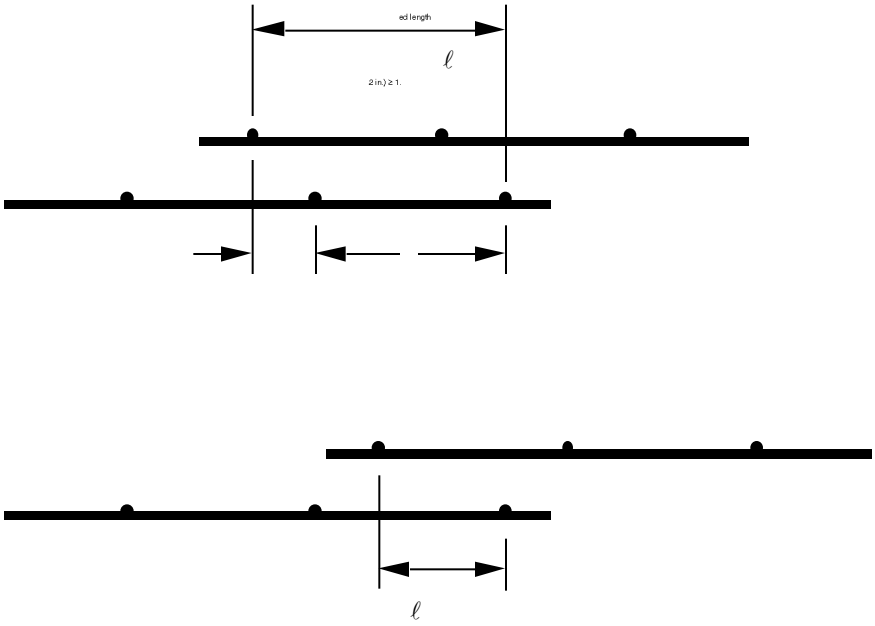


Figure 4-24 Lap Splice Length for Plain Wire Fabric

CLOSING REMARKS

One additional comment concerning splicing of temperature and shrinkage reinforcement at the exposed surfaces of walls or slabs: one must assume all temperature and shrinkage reinforcement to be stressed to the full specified yield strength f_y. The purpose of this reinforcement is to prevent excess cracking. At some point in the member, it is likely that cracking will occur, thus fully stressing the temperature and shrinkage reinforcement. Therefore, all splices in temperature and shrinkage reinforcement must be assumed to be those required for development of yield tensile strength. A Class B tension lap splice must be provided for this steel.

REFERENCES

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4.2 Sozen, M.A., and Moehle, J.P., "Selection of Development and Lap-Splice Lengths of Deformed Reinforcing Bars in Concrete Subjected to Static Loads," Report to PCA and CRSI, PCA R&D Serial No. 1868, March 1990.

4.3 Hamad, B. S., Jirsa, J. O., and D'Abreu, N. I., "Effect of Epoxy Coating on Bond and Anchorage of Reinforcement in Concrete Structures," Research Report 1181-1F, Center for Transportation Research, University of Texas at Austin, Dec. 1990, 242 pp.

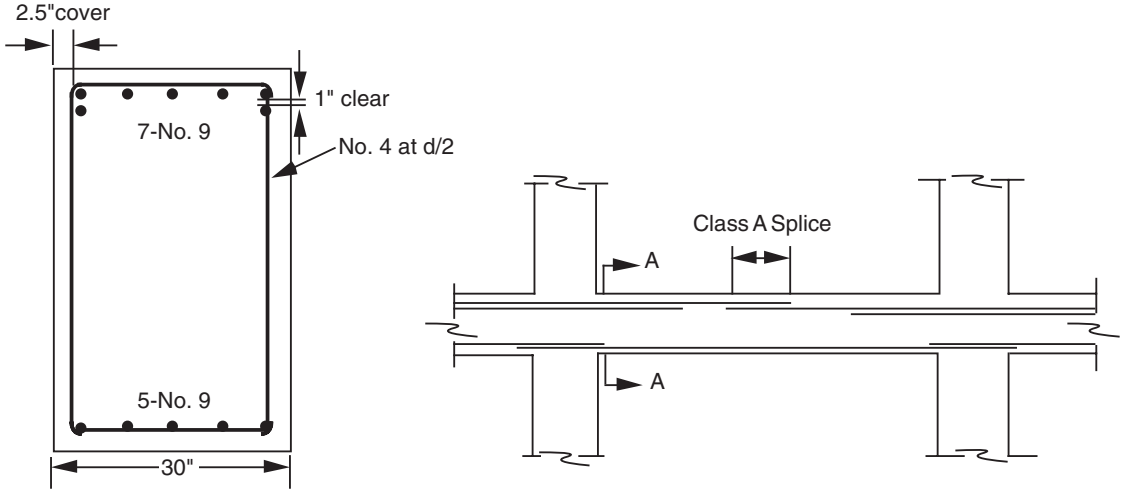
4.4 *Manual of Standard Practice, Stuctural Welded Wire Reinforcement,* WWR-500, 5th Edition, Wire Reinforcement Institute, Findlay, OH, 1999, 27 pp.

4.5 *Reinforcement Anchorages and Splices*, 4th Edition, Concrete Reinforcing Steel Institute, Schaumburg, IL, 1997.

Example 4.1—Development of Bars in Tension

A beam at the perimeter of the structure has 7-No. 9 top bars over the support. Structural integrity provisions require that at least one-sixth of the tension reinforcement be made continuous, but not less than 2 bars (7.13.2.2). Bars are to be spliced with a Class A splice at midspan. Determine required length of Class A lap splice for the following two cases:

Case A - Development computed from 12.2.2
Case B - Development computed from 12.2.3

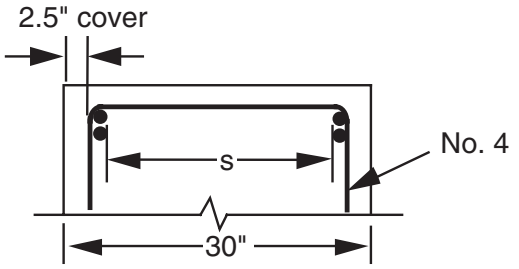


Assume:

Lightweight concrete
2.5 in. clear cover to stirrups
Epoxy-coated bars
f'_c = 4000 psi
f_y = 60,000 psi
b = 30 in. (with bar arrangement as shown)

Calculations and Discussion	Code Reference
It is assumed that development of negative moment reinforcement has been satisfied and, therefore, top bars are stopped away from midspan.	12.12.3
Minimum number of top bars to be made continuous for structural integrity is 1/6 of 7 bars provided, i.e., 7/6 bars or, a minimum of 2 bars. Two corner bars will be spliced at midspan.	7.13.2.2
Class A lap splice requires a $1.0\ell_d$ length of bar lap	12.15.1
Nominal diameter of No. 9 bar = 1.128 in.	
CASE A - Section 12.2.2	
Refer to Table 4-1. For bars No. 7 and larger, either Eq. B or Eq. D apply. To determine if Eq. B or Eq. D governs, determine clear cover and clear spacing for bars being developed.	

Clear spacing between spliced bars (corner bars)
= [30 - 2 (cover) - 2 (No. 4 stirrup) - 2 (No. 9 bar)]
= [30 - 2 (2.5) - 2 (0.5) - 2(1.128)]
= 21.7 in.
= $19.3d_b$



Clear cover to spliced bar = 2.5 + 0.5 = 3.0 in. = $2.7d_b$

As clear spacing > $2d_b$ and clear cover > d_b, Eq. B applies.

$$\ell_d = \left(\frac{f_y \alpha\beta\lambda}{20\sqrt{f'_c}}\right) d_b$$ *12.2.2*

α = 1.3 for top bar *12.2.4*

β = 1.5 for epoxy-coated bar with cover less than $3d_b$ *12.2.4*

$\alpha\beta = 1.3 \times 1.5 = 1.95$; however, product of α and β need not be taken greater than 1.7. *12.2.4*

λ = 1.3 for lightweight aggregate concrete *12.2.4*

$$\ell_d = \frac{60{,}000\ (1.7)\ (1.3)}{20\sqrt{4000}}\ (1.128)$$

= 118.3 in.

Class A splice = $1.0\ell_d$ = 118.3 in.

CASE B - Section 12.2.3

Application of Eq. (12-1) requires a little more computations, but can result in smaller development lengths.

$$\ell_d = \left(\frac{3}{40}\ \frac{f_y}{\sqrt{f'_c}}\ \frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b}\right)}\right) d_b$$ *Eq. (12-1)*

Parameter "c" is the smaller of (1) distance from center of bar being developed to the nearest concrete surface, and (2) one-half the center-to-center spacing of bars being developed. Also, note that the term $\left(\frac{c + K_{tr}}{d_b}\right)$ cannot exceed 2.5.

Distance from center of bar or wire being developed to the nearest concrete surface
= clear cover to spliced bar + 1/2 bar diameter

$= 2.7d_b + 0.5d_b = 3.2d_b$

Center-to-center spacing = clear spacing + $1.0d_b = 19.3d_b + 1.0d_b = 20.3d_b$

Therefore, c is the smaller of $3.2d_b$ and 0.5 $(20.3d_b)$, i.e. $3.2d_b$

No need to compute K_{tr} as c/d_b is greater than 2.5

$\gamma = 1.0$ for No. 7 bar and larger

$$\ell_d = \frac{3\,(60{,}000)\,(1.7)\,(1.0)\,(1.3)}{40\sqrt{4000}\,(2.5)}\,(1.128)$$

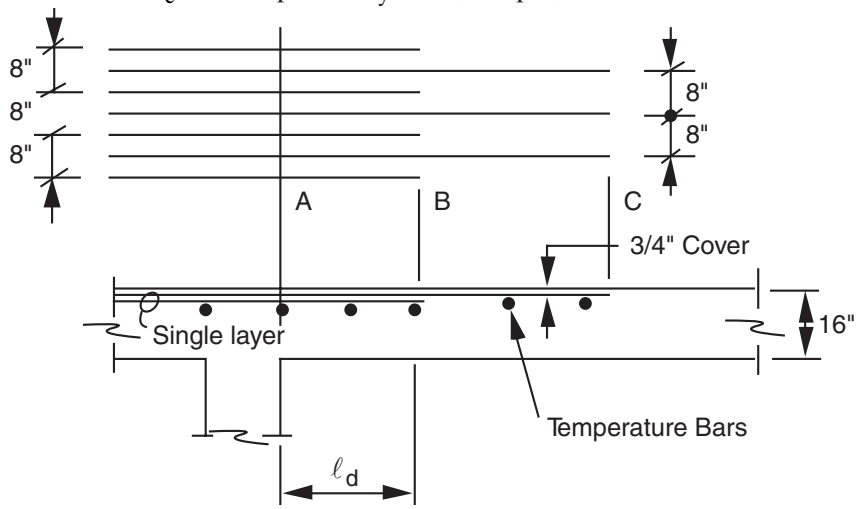
$= 71.0$ in.

Class A splice $= 1.0\ell_d = 71.0$ in.

The extra computations required to satisfy the general Eq. (12-1) of 12.2.3 can lead to substantial reductions in tension development or splice lengths compared to values computed from the simplified procedure of 12.2.2.

Example 4.2—Development of Bars in Tension

Calculate required tension development length for the No. 8 bars (alternate short bars) in the "sand-lightweight" one-way slab shown below. Use f'_c = 4000 psi and f_y = 60,000 psi, and uncoated bars.



Calculations and Discussion	Code Reference

Calculations for this example will be performed using 12.2.2 and 12.2.3.

Assume short bars are developed within distance AB while long bars are developed within BC.

Nominal diameter of No. 8 bar is 1.00 in.

A. Development length by 12.2.2

Center-to-center spacing of bars being developed = 8 in. = $8d_b$

Clear cover = 0.75 in. = $0.75d_b$

As clear cover is less than d_b, and bar size is larger than No. 7, Eq. D of Table 4-1 applies.

$$\ell_d = \left(\frac{3 f_y \alpha \beta \lambda}{40\sqrt{f'_c}} \right) d_b \qquad 12.2.2$$

α = 1.3 for top bar

β = 1.0 for uncoated bars

λ = 1.3 for lightweight concrete

$$\ell_d = \frac{3\,(60{,}000)\,(1.3)\,(1.0)\,(1.3)}{40\sqrt{4000}}\,(1.0) = 120.3 \text{ in.}$$

B. Development length by 12.2.3

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b} \right)} \right) d_b$$ *Eq. (12-1)*

α = 1.3 for top bar

β = 1.0 for uncoated bars

γ = 1.0 for No. 7 and larger bars

λ = 1.3 for lightweight concrete

Center-to-center spacing of bars being developed = 8 in. = $8d_b$
Clear spacing between bars being developed = 8 - 1 = 7 in. = $7d_b$

Clear cover = 0.75 in. = $0.75d_b$
Distance "c" from center of bar to concrete surface = 0.75 + 0.5 = 1.25 in. = $1.25d_b$ (governs)

= $8d_b/2 = 4d_b$ (center -to-center spacing/2)

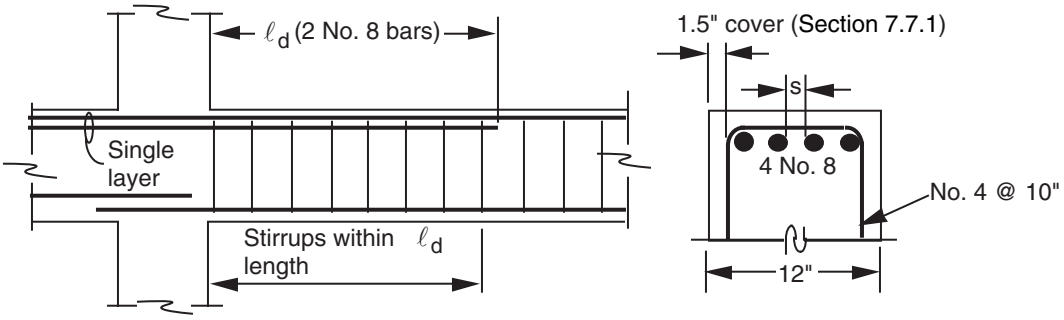
c = $1.25d_b$ (computed above)

K_{tr} = 0 (no transverse reinforcement)

$$\ell_d = \frac{3\,(60{,}000)\,(1.3)\,(1.0)\,(1.0)\,(1.3)}{40\sqrt{4000}\,(1.25)}\,(1.0) = 96.2 \text{ in.}$$

Example 4.3—Development of Bars in Tension

Calculate required development length for the inner 2 No. 8 bars in the beam shown below. The 2 No. 8 outer bars are to be made continuous along full length of beam. Use f'_c = 4000 psi (normal weight concrete) and f_y = 60,000 psi, and uncoated bars. Stirrups provided satisfy the minimum code requirements for beam shear reinforcement.



Calculations and Discussion	Code Reference

Calculations for this example will be performed using 12.2.2 and 12.2.3.

Nominal diameter of No. 8 bar = 1.00 in.

A. Development length by 12.2.2

Clear spacing = [12 - 2 (cover) - 2 (No. 4 stirrups) - 4 (No. 8 bars)]/3 spaces
= [12 - 2 (1.5) - 2 (0.50) - 4 (1.00)]/3
= 1.33 in.
= $1.33d_b$

Clear cover = 1.5 + 0.5 = 2.0 in. = $2d_b$

Refer to Table 4-1. Clear spacing between bars being developed more than d_b, clear cover more than d_b, and minimum stirrups provided. Eq. B of Table 4-1 applies.

$$\ell_d = \left(\frac{f_y \alpha\beta\lambda}{20\sqrt{f'_c}}\right) d_b \qquad \textit{12.2.2}$$

α = 1.3 for top bar

β = 1.0 for uncoated bars

λ = 1.0 for normal weight concrete

$$\ell_d = \frac{(60{,}000)\ (1.3)\ (1.0)\ (1.0)}{20\sqrt{4000}}\ (1.0) = 61.7 \text{ in.}$$

B. Development length by 12.2.3

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b} \right)} \right) d_b$$

Eq. (12-1)

α = 1.3 for top bar

β = 1.0 for uncoated bars

γ = 1.0 for No. 7 and larger bars

λ = 1.0 for normal weight concrete

Clear spacing = $1.33d_b$
Center-to-center spacing of bars being developed = 1.33 + 1.0 in. = 2.33 in. = $2.33d_b$

Clear cover = 1.50 + 0.5 = 2.0 in. = $2d_b$
Distance from center of bar to concrete surface = 1.5 + 0.5 + 0.5 = 2.5 in. = $2.5d_b$

c = the smaller of (1) distance from center of bar being developed to the nearest concrete surface ($2.5d_b$), and of (2) one-half the center-to-center spacing of bars being developed ($2.33d_b/2$ = $1.17d_b$)

$c = 1.17d_b$

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500sn}$$

A_{tr} (2-No. 4) = $2 \times 0.2 = 0.4$ in.^{2}

s = 10 in. spacing of stirrups

n = 2 bars being developed

$$K_{tr} = \frac{0.4\,(60{,}000)}{1500\,(10)\,(2)} = 0.80 \text{ in.} = 0.80d_b$$

$$\left(\frac{c + K_{tr}}{d_b} \right) = \frac{1.17 + 0.80}{1.0} = 1.97 < 2.5 \quad \text{O.K.}$$

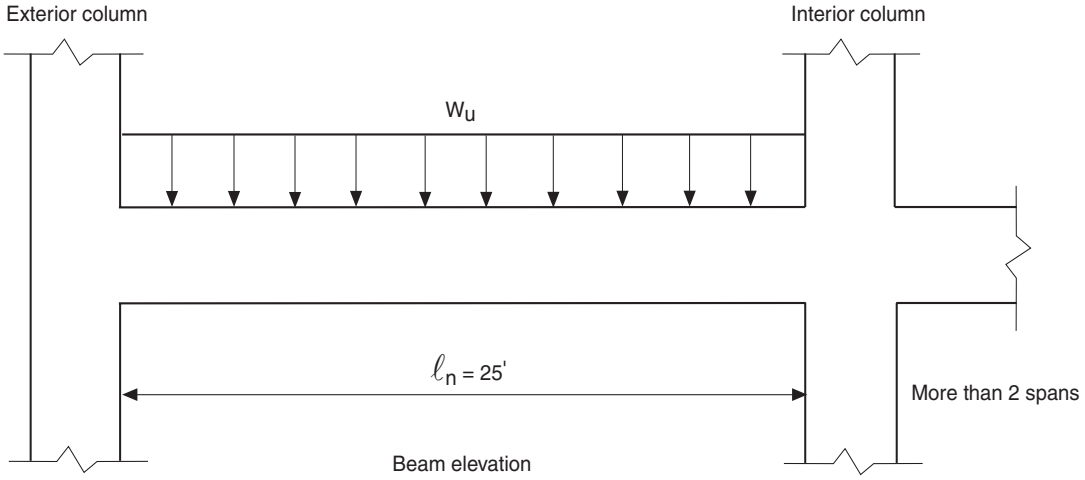
12.2.3

$$\ell_d = \frac{3\,(60{,}000)\,(1.3)\,(1.0)\,(1.0)\,(1.0)}{40\,\sqrt{4000}\,(1.97)}\,(1.0) = 47.0 \text{ in.}$$

Example 4.4—Development of Flexural Reinforcement

Determine lengths of top and bottom bars for the exterior span of the continuous beam shown below. Concrete is normal weight and bars are Grade 60. Total uniformly distributed factored gravity load on beam is w_u = 6.0 kips/ft (including weight of beam).

f'_c = 4000 psi
f_y = 60,000 psi
b = 16 in.
h = 22 in.
Concrete cover = 1 1/2 in.



Calculations and Discussion	Code Reference

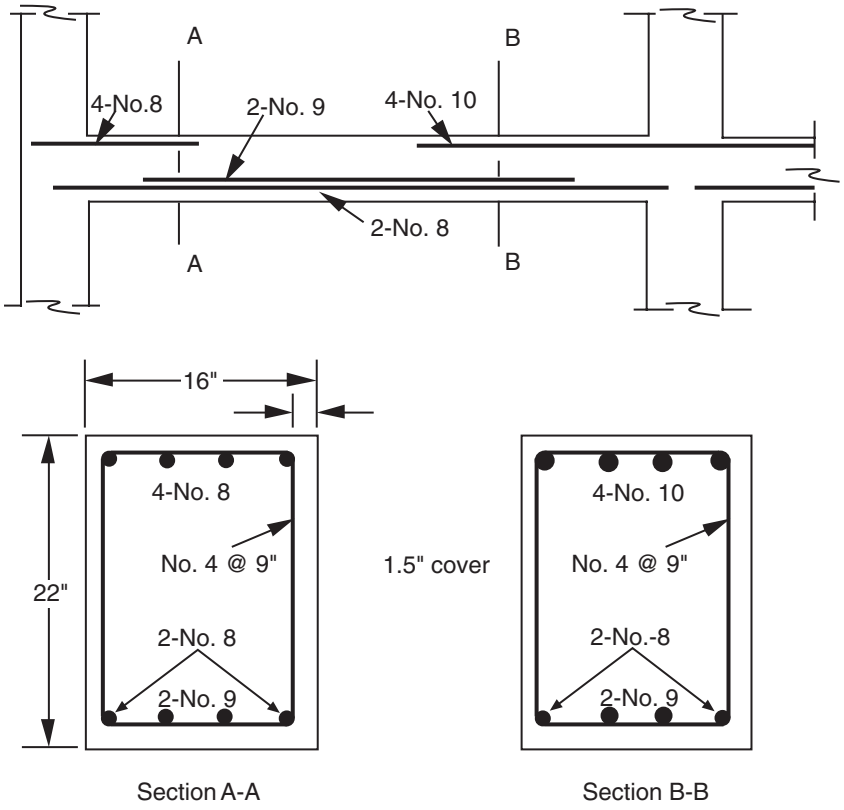
1. Preliminary design for moment and shear reinforcement

   a. Use approximate analysis for moment and shear — *8.3.3*

Location	Factored moments & shears
Interior face of exterior support	$-M_u = w_u\ell_n^2/16 = 6\ (25^2)/16 = -234.4$ ft - kips
End span positive	$+M_u = w_u\ell_n^2/14 = 6\ (25^2)/14 = 267.9$ ft - kips
Exterior face of first interior support	$-M_u = w_u\ell_n^2/10 = 6\ (25^2)/10 = -375.0$ ft - kips
Exterior face of first interior support	$V_u = 1.15w_u\ell_n/2 = 1.15\ (6)\ (25)/2 = 86.3$ kips

   b. Determine required flexural reinforcement using procedures of Part 7 of this publication. With 1.5 in. cover, No. 4 bar stirrups, and No. 9 or No. 10 flexural bars, d ≈ 19.4 in.

M_u	A_s required	Bars	A_s provided
-234.4 ft-kips	2.93 in.^{2}	4 No. 8	3.16 in.^{2}
+267.9 ft-kips	3.40 in.^{2}	2 No. 8 2 No. 9	3.58 in.^{2}
-375.0 ft-kips	5.01 in.^{2}	4 No. 10	5.08 in.^{2}



c. Determine required shear reinforcement

V_u at "d" distance from face of support: *11.1.3.1*

$V_u = 86.3 - 6\,(19.4/12) = 76.6$ kips

$\phi V_c = \phi\left(2\sqrt{f'_c}\,b_w d\right) = 0.75 \times 2\sqrt{4{,}000} \times 16 \times 19.4\,/1{,}000 = 29.5$ kips *1.1.3.1*

Try No. 4 U-stirrups @ 7 in. spacing $< s_{max} = \dfrac{d}{2} = 9.7$ in. *11.5.4.1*

$\phi V_s = \dfrac{\phi A_v f_y d}{s} = 0.75\,(0.40)\,(60)\,(19.4)/7 = 49.9$ kips *11.5.6.2*

$\phi V_n = \phi V_c + \phi V_s = 29.5 + 49.9 = 79.4$ kips > 76.6 kips O.K.

Distance from support where stirrups not required:

$$V_u < \frac{\phi V_c}{2} = \frac{29.5}{2} = 14.8 \text{ kips}$$ *11.5.5.1*

$V_u = 86.3 - 6x = 14.8$ kips

$x = 11.9$ ft $\approx$ 1/2 span

Use No. 4 U-stirrups @ 7 in. (entire span)

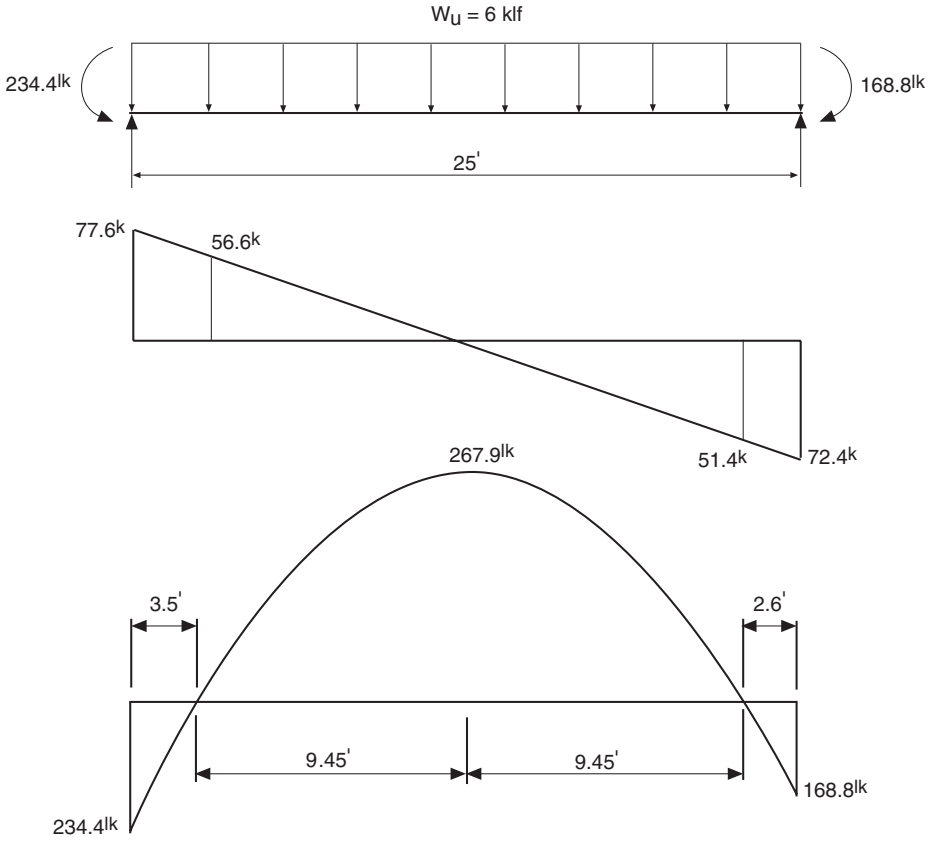
2. Bar lengths for bottom reinforcement

   a. Required number of bars to be extended into supports. *12.11.1*

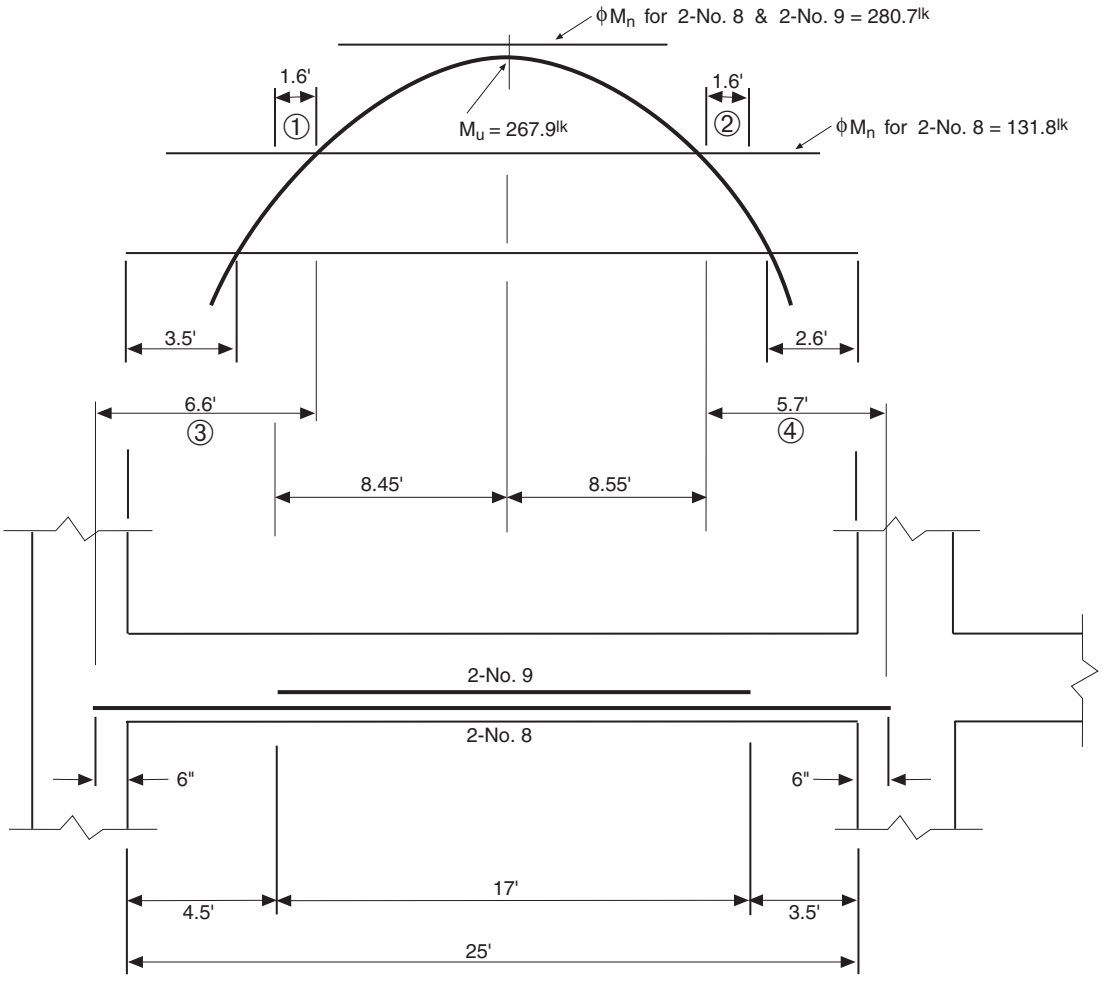
   One-fourth of ($+A_s$) must be extended at least 6 in. into the supports. With a longitudinal bar required at each corner of the stirrups (12.13.3), at least 2 bars should be extended full length. Extend the 2-No. 8 bars full span length (plus 6 in. into the supports) and cut off the 2-No. 9 bars within the span.

   b. Determine cut-off locations for the 2 No. 9 bars and check other development requirements.

   Shear and moment diagrams for loading condition causing maximum factored positive moment are shown below.



The positive moment portion of the M_u diagram is shown below at a larger scale, including the design moment strengths ϕM_n for the total positive A_s (2-No. 8 and 2-No. 9) and for 2-No. 8 bars separately. For 2-No. 8 and 2-No. 9, ϕM_n = 280.7 ft-kips. For 2-No. 8, ϕM_n = 131.8 ft-kips.



As shown, the 2-No. 8 bars extend full span length plus 6 in. into the supports. The 2-No. 9 bars are cut off tentatively at 4.5 ft and 3.5 ft from the exterior and interior supports, respectively. These tentative cutoff locations are determined as follows:

Dimensions (1) and (2) must be the larger of d or $12d_b$: *12.10.3*

d = 19.4 in. = 1.6 ft (governs)

$12d_b$ = 12 (1.128) = 13.5 in.

Dimensions (3) and (4) must be equal to or larger than ℓ_d. *12.10.4*

Within the development length ℓ_d, only 2-No. 8 bars are being developed (2-No. 9 bars are already developed in length 8.45 ft)

Development for No. 8 corner bars, see Table 4-2.

$\ell_d = 47d_b = 47\ (1.0) = 47$ in. $= 3.9$ ft

Dimension (3): 6.6 ft > 3.9 ft O.K.
Dimension (4): 5.7 ft > 3.9 ft O.K.

Check required development length ℓ_d for 2-No. 9 bars. Note that 2-No. 8 bars are already developed in length 4 ft from bar end.

Clear spacing between 2-No. 9 bars

$[16 - 2(1.5) - 2(0.5) - 2(1.0) - 2(1.128)]/3 = 2.58$ in. $= 2.29d_b > 2d_b$

For No. 9 bar, $\ell_d = 47d_b$ *Table 4-2*

$= 47\ (1.128) = 53$ in. $= 4.4$ ft < 8.45 ft O.K.

For No. 8 bars, check development requirements at points of inflection (PI): *12.11.3*

$$\ell_d \le \frac{M_n}{V_u} + \ell_a$$ *Eq. (12-3)*

For 2-No. 8 bars, $M_n = 131.8/0.9 = 146.4$ ft-kips

At left PI, $V_u = 77.6 - 6(3.5) = 56.6$ kips

ℓ_a = larger of $12d_b = 12\ (1.0) = 12$ in. or $d = 19.4$ in. (governs)

$$\ell_d \le \frac{146.4 \times 12}{56.6} + 19.4 = 50.5 \text{ in.}$$

For No. 8 bars, $\ell_d = 47$ in. < 50.5 in. O.K.

At right PI, $V_u = 56.8$ kips; by inspection, the development requirements for the No. 8 bars are O.K.

With both tentative cutoff points located in a zone of flexural tension, one of the three conditions of 12.10.5 must be satisfied.

At left cutoff point (4.5 ft from support):

$V_u = 77.6 - (4.5 \times 6) = 50.6$ kips

$\phi V_n = 79.4$ kips (No. 4 U-stirrups @ 7 in.)

$2/3\ (79.4) = 52.9$ kips $>$ 50.6 kips O.K. *12.10.5.1*

For illustrative purposes, determine if the condition of 12.10.5.3 is also satisfied:

M_u = 54.1 ft-kips at 4.5 ft from support

A_s required = 0.63 in.^{2}

For 2-No. 8 bars, A_s provided = 1.58 in.^{2}

1.58 in.^{2} > 2 (0.63) = 1.26 in.^{2} O.K. *12.10.5.3*

3/4 (79.4) = 59.6 kips > 50.6 kips O.K. *12.10.5.3*

Therefore, 12.10.5.3 is also satisfied at cutoff location.

At right cutoff point (3.5 ft from support):

V_u = 72.4 - (3.5 × 6) = 51.4 kips

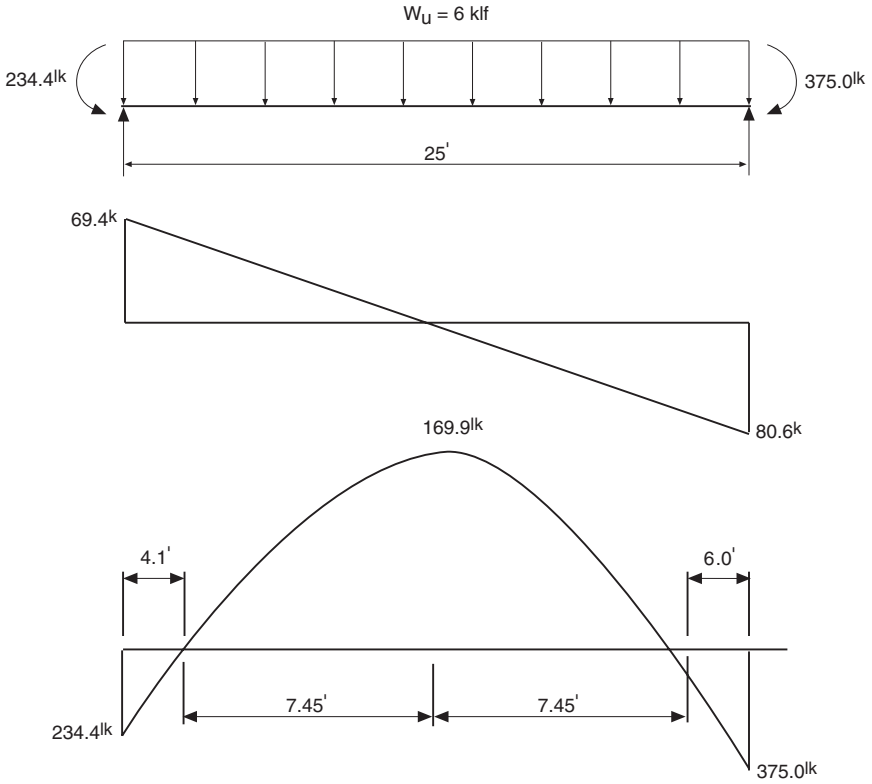
2/3 (ϕV_n) = 52.9 kips > 51.4 kips O.K. *12.10.5.1*

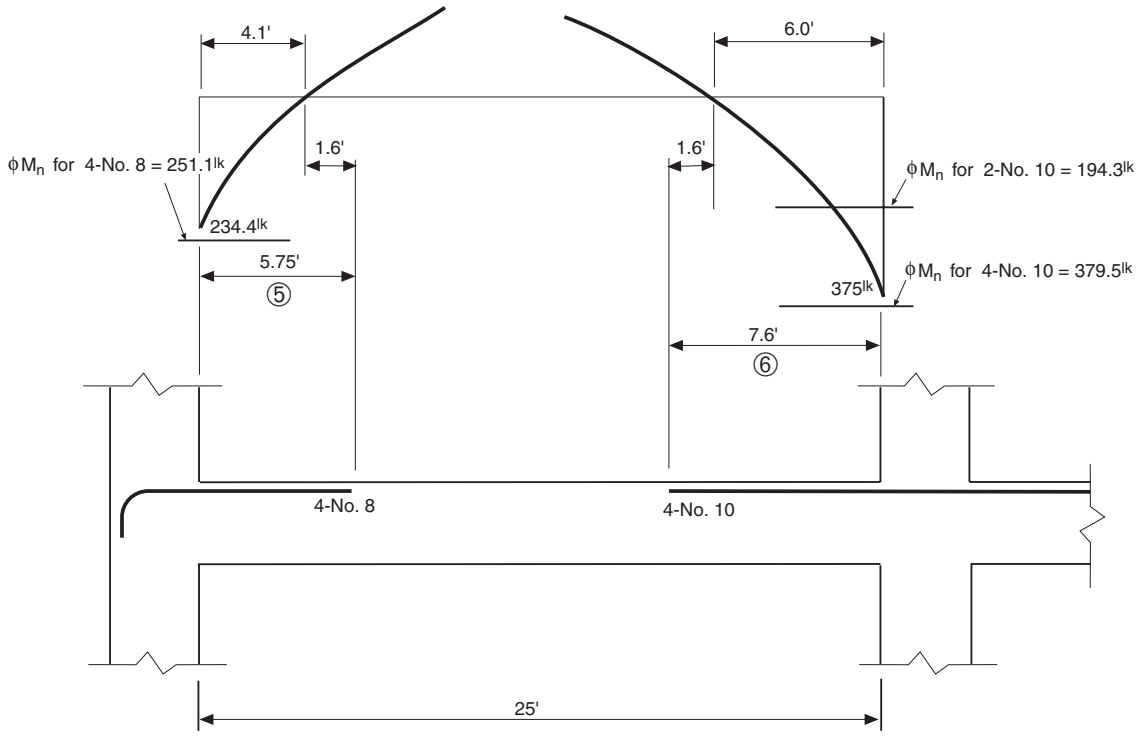
Summary: The tentative cutoff locations for the bottom reinforcement meet all code development requirements. The 2-No. 9 bars × 17 ft would have to be placed unsymmetrically within the span. To assure proper placing of the No. 9 bars, it would be prudent to specify a 18 ft length for symmetrical bar placement within the span, i.e., 3.5 ft from each support. The ends of the cut off bars would then be at or close to the points of inflection, thus, eliminating the need to satisfy the conditions of 12.10.5 when bars are terminated in a tension zone. The recommended bar arrangement is shown at the end of the example.

3. Bar lengths for top reinforcement

Shear and moment diagrams for loading condition causing maximum factored negative moments are shown below.

The negative moment portions of the M_u diagram are also shown below at a larger scale, including the design moment strengths ϕM_n for the total negative A_s at each support (4-No. 8 at exterior support and 4-No. 10 at interior support) and for 2-No. 10 bars at the interior support. For 4-No. 8, ϕM_n = 251.1 ft-kips. For 4-No. 10, ϕM_n = 379.5 ft-kips. For 2-No. 10, ϕM_n = 194.3 ft-kips.


W_u = 6 klf
234.4lk
375.0lk
25'
69.4k
80.6k
169.9lk
4.1'
6.0'
7.45'
7.45'
234.4lk
375.0lk


4.1'
6.0'
ϕM_n for 4-No. 8 = 251.1lk
1.6'
1.6'
ϕM_n for 2-No. 10 = 194.3lk
234.4lk
5.75'
⑤
375lk
ϕM_n for 4-No. 10 = 379.5lk
7.6'
⑥
4-No. 8
4-No. 10
25'

4. Development requirements for 4-No. 8 bars at exterior support

   a. Required number of bars to be extended.

   One-third of ($-A_s$) provided at supports must be extended beyond the point of inflection a distance equal to the greater of d, $12d_b$, or $\ell_n/16$. *12.12.3*

   d = 19.4 in. = 1.6 ft (governs)

   $12d_b = 12\ (1.0) = 12.0$ in.

   $\ell_n/16 = 25 \times 12/16 = 18.75$ in.

   Since the inflection point is located only 4.1 ft from the support, total length of the No. 8 bars will be relatively short even with the required 1.6 ft extension beyond the point of inflection. Check required development length ℓ_d for a cutoff location at 5.75 ft from face of support.

   Dimension (5) must be at least equal to ℓ_d *12.12.2*

   For No. 8 bars, $\ell_d = 47d_b = 47\ (1.0) = 47$ in. *Table 4-2*

   With 4-No. 8 bars being developed at same location (face of support):

   Including top bar effect, $\ell_d = 1.3\ (47) = 61.1$ in.

   For No. 8 top bars, $\ell_d = 61.1$ in. = 5.1 ft < 5.75 ft O.K.

   b. Anchorage into exterior column.

   The No. 8 bars can be anchored into the column with a standard end hook. From Table 4-4, ℓ_{dh} = 19.0 in. The required ℓ_{dh} for the hook could be reduced if excess reinforcement is considered:

   $$\frac{(A_s \text{ required})}{(A_s \text{ provided})} = \frac{2.93}{3.16} = 0.93$$ *12.5.3(c)*

   $\ell_{dh} = 19 \times 0.93 = 17.7$ in.

   Overall depth of column required would be 17.7 + 2 = 19.7 in.

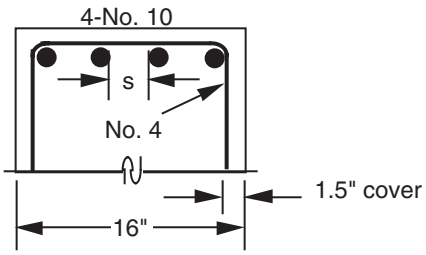
5. Development requirements for 4-No. 10 bars at interior column

   a. Required extension for one-third of ($-A_s$) *12.12.3*

d = 19.4 in. = 1.6 ft (governs)

$12d_b$ = 12 (1.27) = 15.24 in.

$\ell_n/16$ = 18.75 in.



For No. 10 bars, clear spacing = [16 - 2 (1.5) - 2 (0.5) - 4 (1.27)]/3
= 2.31 in. = $1.82d_b > d_b$
Center-to-center spacing = $2.82d_b$

Cover = 1.5 + 0.5 = 2.0 in. = $1.57d_b > d_b$

Distance from center of bar to concrete surface = $1.57d_b + 0.5d_b = 2.07d_b$

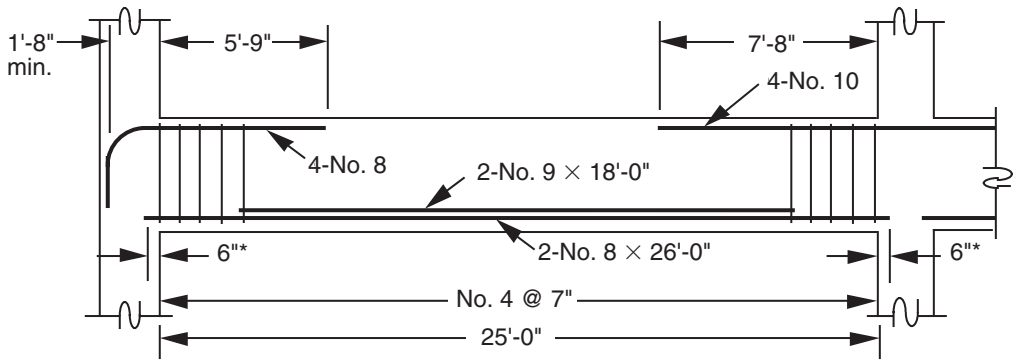
With minimum shear reinforcement provided and including top bar effect

ℓ_d = 1.3 ($47d_b$) *Table 4-2*

= 1.3 (47) (1.27) = 77.6 in.

Dimension (6) = 6.0 ft + 1.6 ft = 7.6 ft > ℓ_d = 77.6 in = 6.5 ft O.K.

6. Summary: Selected bar lengths for the top and bottom reinforcement shown below.



*see 12.11.1

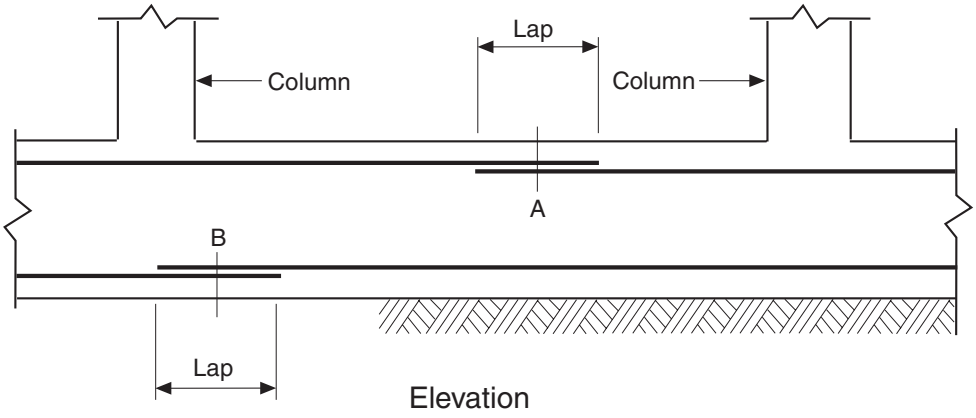
7. Supplementary Requirements

If the beam were part of a primary lateral load resisting system, the 2-No. 8 bottom bars extending into the supports would have to be anchored to develop the bar yield strength at the face of supports. At the exterior column, anchorage can be provided by a standard end hook. Minimum width of support (overall column depth) required for anchorage of the No. 8 bar with a standard hook is a function of the development length ℓ_{dh} from Table 4-4, and the appropriate modification factors (12.5.3). *12.11.2*

At the interior column, the 2-No. 8 bars could be extended ℓ_d distance beyond the face of support into the adjacent span or lap spliced with extended bars from the adjacent span. Consider a Class A lap splice adequate to satisfy the intent of 12.11.2.

Example 4.5—Lap Splices in Tension

Design the tension lap splices for the grade beam shown below.



Elevation

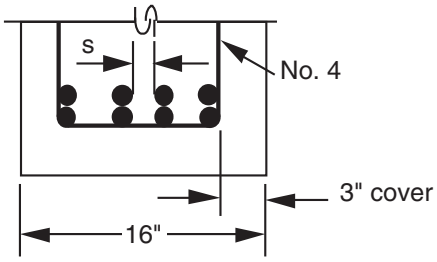
f'_c = 4000 psi
f_y = 60,000 psi, uncoated bars
b = 16 in.
h = 30 in.
Bar cover = 3.0 in.
4-No. 9 bars top and bottom (continuous)
No. 4 stirrups @ 14 in. (entire span)
$+M_u$ @B = 340 ft-kips
$-M_u$ @A = 120 ft-kips

Preferably, splices should be located away from zones of high tension. For a typical grade beam, top bars should be spliced under the columns, and bottom bars about midway between columns. Even though in this example the splice at A is not a preferred location, the moment at A is relatively small. Assume for illustration that the splices must be located as shown.

Calculations and Discussion	Code Reference

Calculations for this example will be performed using 12.2.3.

Nominal diameter of No. 9 bar = 1.128 in.



Assuming all bars are spliced at the same location

Clear spacing = [16 - 2 (cover) - 2 (No. 4 stirrups) - 4 (No. 9 bars)]/3 spaces
= [16 - 2 (3.0) - 2 (0.50) - 4 (1.128)]/3
= 1.50 in.
= $1.33d_b$

Center-to-center spacing of bars being developed = 1.50 + 1.128 = 2.63 in. = $2.33d_b$

Clear cover = 3.0 + 0.5 = 3.5 in. = $3.1d_b$

Distance from center of bar to concrete surface = 3.0 + 0.5 + (1.128/2) = 4.1 in. = $3.6d_b$

c = the smaller of (1) distance from center of bar being developed to the nearest concrete surface and (2) one-half the center-to-center spacing of bars being developed

$c = 3.6d_b$

$= 2.33d_b/2 = 1.17d_b$ (governs)

Lap Splice of Bottom Reinforcement at Section B

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b}\right)} \right) d_b$$ *Eq. (12-1)*

α = 1.0 for bottom bar *12.2.4*

β = 1.0 for uncoated bars

γ = 1.0 for No. 7 and larger bars

λ = 1.0 for normal weight concrete

$c = 1.17d_b$ (computed above)

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500sn}$$

A_{tr} = area of 2-No. 4 stirrups = 2 (0.2) = 0.4 $in.^2$
s = 14 in. spacing
n = 4 bars being developed

$$K_{tr} = \frac{0.4\ (60{,}000)}{1500\ (14)\ (4)} = 0.29 \text{ in.} = 0.26d_b$$

$$\left(\frac{c + K_{tr}}{d_b}\right) = 1.17 + 0.26 = 1.43 < 2.5 \quad \text{O.K.}$$ *12.2.3*

$$\ell_d = \frac{3\ (60{,}000)\ (1.0)\ (1.0)\ (1.0)\ (1.0)}{40\sqrt{4{,}000}\ (1.43)} (1.128) = 56.1 \text{ in.}$$

A_s required ($+M_u$ @ B = 340 ft-kips) = 3.11 in.^{2}

A_s provided (4 No. 9 bars) = 4.00 in.^{2}

$$\frac{A_s \text{ provided}}{A_s \text{ required}} = \frac{4.00}{3.11} = 1.29 < 2$$

Class B splice required = $1.3\ell_d$ 12.15.1

12.15.2

Note: Even if lap splices were staggered (A_s spliced = 50%), a Class B splice must be used with (A_s provided/A_s required) < 2

Class B Splice = $1.3\ell_d$ = 1.3 (56.1) = 72.9 in. = 6.1ft

It is better practice to stagger alternate lap splices. As a result, the clear spacing between spliced bars will be increased with a potential reduction of development length.

Clear spacing = 2 (1.50) + 1.128 = 4.13 in. = $3.66d_b$

Center-to-center spacing of bars being developed = $3.66d_b + d_b = 4.66d_b$

Distance from center of bar to concrete surface = $3.6d_b$

Thus, $c = \frac{4.66d_b}{2} = 2.33d_b$

$$K_{tr} = \frac{2\ (0.2)\ (60{,}000)}{(1500)\ (14)\ (2)} = 0.57 \text{ in.} = 0.51d_b$$

Therefore, $\left(\frac{c + K_{tr}}{d_b}\right) = 2.33 + 0.51 = 2.84 > 2.5$ Use 2.5. 12.2.3

$$\ell_d = \frac{3\ (60{,}000)\ (1.0)\ (1.0)\ (1.0)\ (1.0)}{40\sqrt{4000}\ (2.5)}\ (1.128) = 32.1 \text{ in.}$$

Class B splice = 1.3 (32.1) = 41.7 in. = 3.5 ft

Use 3 ft-6 in. lap splice @ B and stagger alternate lap splices.

Lap Splice of Top Reinforcement at Section A

As size of top and bottom reinforcement is the same, computed development and splice lengths for top bars will be equal to that of the bottom bars increased by the 1.3 multiplier for top bars. In addition, because positive and negative factored moments are different, the ratio of provided to required reinforcement may affect the type of splice as demonstrated below.

A_s required ($+M_u$ @ A = 120 ft-kips) = 1.05 in.^{2}

A_s provided/A_s required = 4.00/1.05 = 3.81 > 2

If alternate lap splices are staggered at least a lap length (A_s spliced = 50%):

Class A splice may be used = $1.0\ell_d$ *12.15.2*

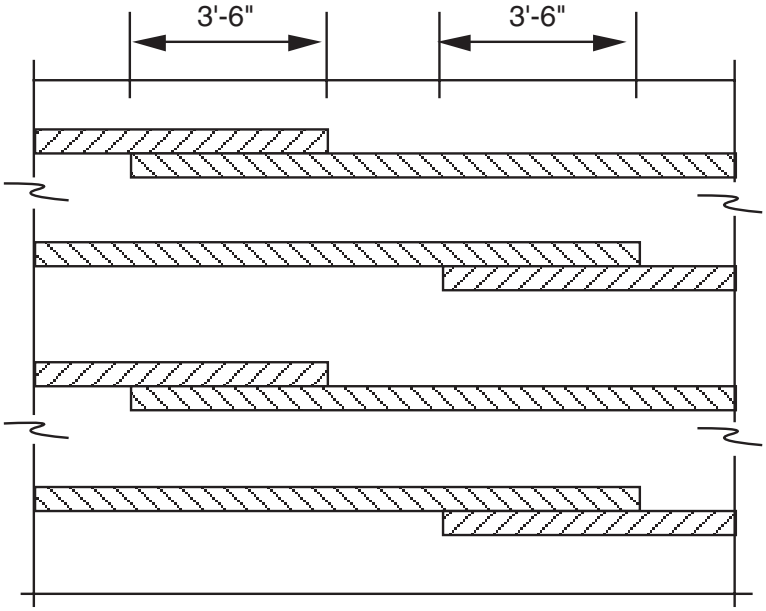
If all bars are lap spliced at the same location (within req'd lap length):

Class B splice must be used = $1.3\ell_d$

Assuming splices are staggered, the top bar multiplier will be 1.3.

Class A splice = 1.3 (1.0) (32.1) = 41.7 in. = 3.5 ft

Use 3 ft-6 in. lap splice @ A also, and stagger alternate lap splices.



Alternate lap splice stagger arrangement
(Note: bar laps are positioned vertically)

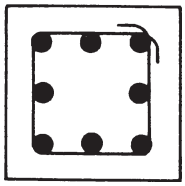
Example 4.6—Lap Splices in Compression

The following two examples illustrate typical calculations for compression lap splices in tied and spirally reinforced columns.

Calculations and Discussion	Code Reference

1. Design a compression lap splice for the tied column shown below. Assume all bars in compression for factored load combinations considered in design (Zone 1 in Fig. 4-21). See also Table 4-8.

   b = 16 in.
   h = 16 in.
   f'_c = 4000 psi
   f_y = 60,000 psi
   8-No. 9 bars



   a. Determine lap splice length: *12.16.1*

      For f_y = 60,000 psi:

      Length of lap = $0.0005 f_y d_b$, but not less than 12 in.

      = 0.0005 (60,000) 1.128 = 34 in.

   b. Determine column tie requirements to allow an 0.83 reduced lap length: *12.17.2.4*

      Required column ties: No. 3 @ 16 in. o.c. *7.10.5.2*

      Required spacing of No. 3 ties for reduced lap length:

      effective area of ties $\geq$ 0.0015hs

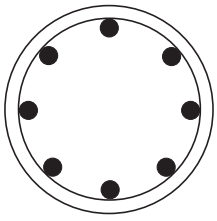
      $(2 \times 0.11) \geq 0.0015 \times 16s$

      s = 9.2 in.

      Spacing of the No. 3 ties must be reduced to 9 in. o.c. throughout the lap splice length to allow a lap length of 0.83 (34 in.) = 28 in.

2. Determine compression lap splice for spiral column shown.

   f'_c = 4000 psi
   f_y = 60,000 psi
   8-No. 9 bars
   No. 3 spirals



Example 4.6 (cont'd)	Calculations and Discussion	Code Reference

a. Determine lap splice length — *12.16.1*

For bars enclosed within spirals, "basic" lap splice length may be multiplied by a factor of 0.75. — *12.17.2.5*

For f_y = 60,000 psi:

lap = 0.75(34) = 26 in.

Note: End bearing, welded, or mechanical connections may also be used. — *12.16.3* *12.16.4*

Example 4.7—Lap Splices in Columns

Design the lap splice for the tied column detail shown.

- Continuing bars from column above (4-No. 8 bars)
- Offset bars from column below (4-No. 8 bars)

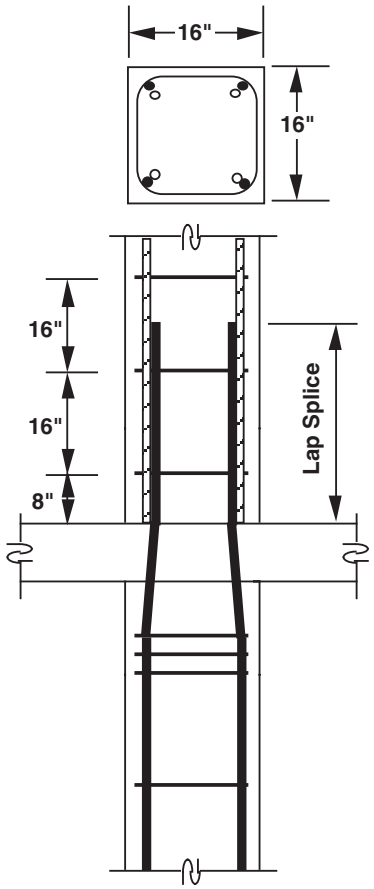
f'_c = 4000 psi (normal weight)
f_y = 60,000 psi
b = h = 16 in.
4-No. 8 bars (above and below floor level)
No. 3 ties @ 16 in.
Cover = 1.5 in.

Lap splice to be designed for the following factored load combinations:

1. P_u = 465 kips
   M_u = 20 ft-kips

2. P_u = 360 kips
   M_u = 120 ft-kips

3. P_u = 220 kips
   M_u = 100 ft-kips



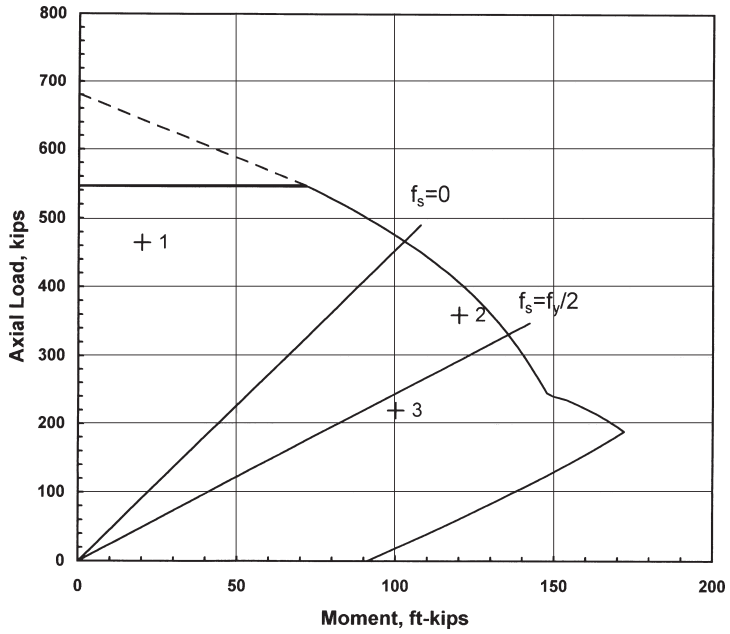
Calculations and Discussion	Code Reference

1. Determine type of lap splice required. — *12.17.2*

   Type of lap splice to be used depends on the bar stress at the splice location due to all factored load combinations considered in the design of the column. For design purposes, type of lap splice will be based on which zone, or zones, of bar stress all factored load combinations are located on the column load-moment strength diagram. See discussion for 12.17.2, and Fig. 4-21. The load-moment strength diagram (column design chart) for the 16 × 16 column with 4-No. 8 bars is shown below, with the three factored load combinations considered in the design of the column located on the interaction strength diagram.

   Note that load combination (2) governed the design of the column (selection of 4-No. 8 bars). — *Table 4-8*

   For load combination (1), all bars are in compression (Zone 1), and a compression lap splice could be used. For load combination (2), bar stress is not greater than $0.5f_y$ (Zone 2), so a Class B tension lap splice is required; or, a Class A splice may be used if alternate lap splices are staggered. For load combination (3), bar stress is greater than $0.5f_y$ (Zone 3), and a Class B splice must be used.



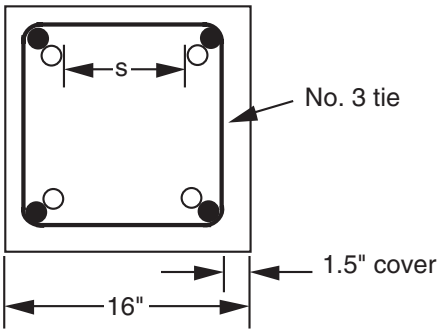
Interaction Diagram for 16 in. × 16 in. Column

Lap splice required for the 4-No. 8 bars must be based on the load combination producing the greatest amount of tension in the bars; for this example, load combination (3) governs the type of lap splice to be used.

Class B splice required = $\ell_d = 1.3\ell_{db}$ 12.15.1

2. Determine lap splice length

Determine tension development length by 12.2.3.



Nominal diameter of No. 8 bar = 1.00 in.

Clear spacing between bars being developed is large and will not govern.

Clear cover = 1.5 + 0.375 = 1.875 in. = $1.875d_b$

Distance from center of bar to concrete surface = 1.875 + 0.5 = 2.375 in. = $2.375d_b$

c = the smaller of (1) distance from center of bar being developed to the nearest concrete surface, and of (2) one-half the center-to-center spacing of bars being developed

$c = 2.375 d_b$

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b} \right)} \right) d_b$$ *Eq. (12-1)*

α = 1.0 for vertical bar *12.2.4*

β = 1.0 for uncoated bars

γ = 1.0 for No. 7 and larger bars

λ = 1.0 for normal weight concrete

$c = 2.375 d_b$

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500sn}$$

A_{tr} = area of 2-No. 3 ties

s = 16 in. spacing

n = 2 bars being developed on one column face

$$K_{tr} = \frac{2\ (0.11)\ (60{,}000)}{1500\ (16)\ (2)} = 0.275 \text{ in.} = 0.275\ d_b$$

$$\left(\frac{c + K_{tr}}{d_b} \right) = 2.375 + 0.275 = 2.65 > 2.5 \quad \text{Use } 2.5$$ *12.2.3*

$$\ell_d = \frac{3\ (60{,}000)\ (1.0)\ (1.0)\ (1.0)\ (1.0)}{40\sqrt{4000}\ (2.5)}\ (1.00) = 28.5 \text{ in.}$$

Class B splice = 1.3(28.5) = 37 in.

Use 37 in. lap splice for the 4 No. 8 bars at the floor level indicated.

5

Design Methods and Strength Requirements

UPDATE FOR THE '02 CODE

The 2002 code contains major revisions to Chapters 8, 9, and 10. The provisions in Appendices B and C of the 1999 code have been moved to the body of the 2002 code, and the provisions displaced by this move are now placed in the new Appendixes B and C of the 2002 code.

The changes affect the moment redistribution provisions in Chapter 8. The load factors and strength reduction factors in Chapter 9 are changed. The design methods for flexural and compression members in Chapter 10 are modified. Some provisions in Chapter 18, Prestressed Concrete, are also modified.

The load factors in Chapter 9 have been revised to be those of ASCE 7. These are the same load factors that are used in IBC 2000 for the strength design of other materials. In general, those load factors are lower than those in previous editions of the ACI code. The strength reduction factors ϕ to be used with these load factors were formerly given in Appendix C of the 1995 and 1991 codes. These strength reduction factors are now moved into Chapter 9.

In general, the combination of the lower load factors and lower capacity reduction factors in the new Chapter 9 produce approximately the same result as designs by previous codes. However, there is one important difference with design of flexural members by the 2002 code: for tension-controlled flexural members (defined below), the ϕ factor remains unchanged at 0.9. This, combined with lower load factors, produces a reduced strength requirement of approximately 10 percent for many flexural members.

The new Appendix C in the 2002 code contains the load factors and ϕ factors formerly given in Chapter 9. Use of these former load and ϕ factors is permitted by use of Appendix C.

Chapters 8, 9, 10, and 18 have been revised to include the provisions of Appendix B, Unified Design Provision, of the 1995 and 1999 codes. These provisions were developed in order to unify the design of flexural and compression members, and to unify the strength design of nonprestressed and prestressed members.

Sections are classified based on the net tensile strain in the extreme tension steel at nominal strength. Tension controlled sections are defined as those with a tensile steel strain of 0.005 or more. These sections are designed with a strength reduction factor ϕ of 0.9. Compression-controlled sections are defined as those with a net tensile steel strain of 0.002 or less, and are designed using a ϕ factor of 0.65 or 0.7 for compression members. For sections with net tensile strain between these limits, an equation for a transition in the ϕ factor is provided. For prestressed sections, the net tensile strain is exclusive of the prestress strain.

The moment redistribution provisions of Chapter 8 have been modified, to apply equally to nonprestressed and prestressed concrete. The new moment redistribution provisions are described in Part 9 of this book.

8.1 DESIGN METHODS

Two philosophies of design for reinforced concrete have long been prevalent. Working Stress Design was the principal method used from the early 1900s until the early 1960s. Since publication of the 1963 edition of the ACI code, there has been a rapid transition to Ultimate Strength Design, largely because of its more rational approach. Ultimate strength design, referred to in the code as the Strength Design Method (SDM) is conceptually more realistic in its approach to structural safety.

The 1956 ACI code (ACI 318-56) was the first code edition which officially recognized and permitted the ultimate strength method of design. Recommendations for the design of reinforced concrete structures by ultimate strength theories were included in an appendix.

The 1963 ACI code (ACI 318-63) treated the working stress and the ultimate strength methods on an equal basis. However, a major portion of the working stress method was modified to reflect ultimate strength behavior. The working stress provisions of the 1963 code, relating to bond, shear and diagonal tension, and combined axial compression and bending, had their basis in ultimate strength.

The 1971 ACI code (ACI 318-71) was based entirely on the strength approach for proportioning reinforced concrete members, except for a small section (8.10) devoted to what was called an Alternate Design Method (ADM). The ADM was not applicable to the design of prestressed concrete members. Even in that section, the service load capacities (except for flexure) were given as various percentages of the ultimate strength capacities of other parts of the code. The transition to ultimate strength theories for reinforced concrete design was essentially complete in the 1971 ACI code, with ultimate strength design definitely established as being preferred.

In the 1977 ACI code (ACI 318-77) the ADM was relegated to Appendix B. The appendix location served to separate and clarify the two methods of design, with the main body of the code devoted exclusively to the SDM. The ADM was retained in all editions of the code from 1977 to the 1999 edition, where it was found in Appendix A. The ADM method was deleted from the 2002 code. It is still referenced in Commentary Section R1.1 of the 2002 code. The general serviceability requirements of the main body of the code, such as the provisions for deflection and crack control, must always be satisfied. Because earthquake forces computed in accordance with the latest editions of model building codes in use in the U.S. are strength-level loads, those codes prohibit the use of the ADM of ACI 318 for load combinations which include seismic forces or their effects. See 8.1.2 below.

A modification to the SDM, referred to as the Unified Design Provisions, was added to the ‘95 edition of the code. In keeping with tradition, the method was added as Appendix B. The provisions apply to the design of nonprestressed and prestressed members subject to flexure and axial loads. The Unified Design Provisions were incorporated into the body of the 2002 code. See 8.1.2 below.

8.1.1 Strength Design Method

The Strength Design Method requires that the design strength of a member at any section should equal or exceed the required strength calculated by the code-specified factored load combinations. In general,

$$\text{Design Strength} \geq \text{Required Strength (U)}$$

where

$$\text{Design Strength} = \text{Strength Reduction Factor } (\phi) \times \text{Nominal Strength}$$

ϕ = Strength reduction factor that accounts for (1) the probability of understrength of a member due to variations in material strengths and dimensions, (2) inaccuracies in the design equations, (3) the degree of ductility and required reliability of the loaded member, and (4) the importance of the member in the structure (see 9.3.2).

Nominal Strength = Strength of a member or cross-section calculated using assumptions and strength equations of the Strength Design Method before application of any strength reduction factors.

Required Strength (U) = Load factors x Service load effects. The required strength is computed in accordance with the load combinations in 9.2.

Load Factor = Overload factor due to probable variation of service loads.

Service Load = Load specified by general building code (unfactored).

Notation

Required strength:

M_u = factored flexural moment (required flexural strength)
P_u = factored axial load (required axial load strength) at given eccentricity
V_u = factored shear force (required shear strength)
T_u = factored torsional moment (required torsional strength)

Nominal strength:

M_n = nominal flexural moment strength
M_b = nominal flexural moment strength at balanced strain conditions
P_n = nominal axial load strength at given eccentricity
P_o = nominal axial load strength at zero eccentricity
P_b = nominal axial load strength at balanced strain conditions
V_n = nominal shear strength
V_c = nominal shear strength provided by concrete
V_s = nominal shear strength provided by shear reinforcement
T_n = nominal torsional moment strength

Design Strength:

ϕM_n = design flexural moment strength
ϕP_n = design axial load strength at given eccentricity
ϕV_n = design shear strength = $\phi (V_c + V_s)$
ϕT_n = design torsional moment strength

Section R2.1 gives an in-depth discussion on many of the concepts in the Strength Design Method.

8.1.2 Unified Design Provisions

A modification to the Strength Design Method for reinforced and prestressed concrete flexural and compression members was introduced in 1995 in Appendix B. This appendix introduced substantial changes in the design for flexure and axial loads. Reinforcement limits, strength reduction factors ϕ, and moment redistribution were affected.

The Unified Design method is similar to the Strength Design Method in that it uses factored loads and strength reduction factors to proportion the members. The main difference is that in the Unified Design Provisions, a concrete section is defined as either compression-controlled or tension-controlled, depending on the magnitude of the net tensile strain in the reinforcement closest to the tension face of a member. The ϕ factor is then determined by the strain conditions at a section at nominal strength. Prior to these provisions, the ϕ factors were specified for cases of axial load or flexure or both in terms of the type of loading.

It is important to note that the Unified Design Provisions do not alter nominal strength calculations. The major differences occur in checking reinforcement limits for flexural members, determining the ϕ factor for columns with small axial load, and computing redistributed moments. Most other applicable provisions in the body of the 1999 code apply to design using the 2002 code.

The code sections displaced by the Unified Design Provisions are now located in Appendix B. These former provisions are still permitted to be used.

In general, theUnified Design Provisions provide a rational means for designing reinforced and prestressed flexural and compression members, and produce results similar to those obtained from the Strength Design Method. The examples in Part 10 and Ref. 5.1 illustrate the use of this new design method.

9.1 STRENGTH AND SERVICEABILITY—GENERAL

9.1.1 Strength Requirements

The basic criterion for strength design as indicated in 9.1.1 is as follows:

Design Strength $\geq$ Required Strength

Strength Reduction Factor (ϕ) x Nominal Strength $\geq$ Load Factor x Service Load Effects

All structural members and sections must be proportioned to meet the above criterion under the most critical load combination for all possible actions (flexure, axial load, shear, etc.):

$$\phi P_n \geq P_u$$

$$\phi M_n \geq M_u$$

$$\phi V_n \geq V_u$$

$$\phi T_n \geq T_u$$

The above criterion provides for the margin of structural safety in two ways:

1. It decreases the strength by multiplying the nominal strength with the appropriate strength reduction factor

ϕ, which is always less than 1. The nominal strength is computed by the code procedures assuming that the member or the section will have the exact dimensions and material properties assumed in the computations. For example, the nominal flexural moment strength for the singly reinforced section shown in Fig. 5-1 is:

$$M_n = A_s f_y (d - a/2)$$

and the design flexural moment strength is

$$\phi M_n = \phi[A_s f_y (d - a/2)]$$

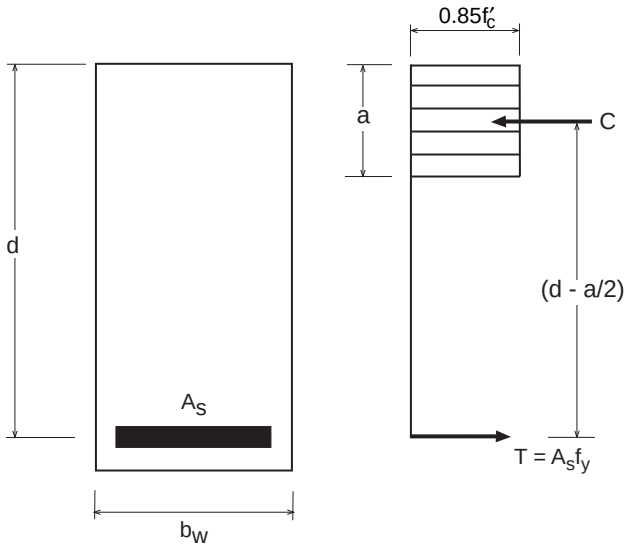


Figure 5-1 Singly Reinforced Section

2. It increases the required strength by using factored loads or the factored internal moments and forces. Factored loads are defined in 2.1 as service loads multiplied by the appropriate load factors. The loads to be used are described in 8.2. Thus, the required flexural strength of the section shown in Fig. 5-1 for dead and live loads is:

$$M_u = 1.2M_d + 1.6\,M_\ell \geq 1.4M_d \qquad \textit{Eqs. (9-1) \& (9-2)}$$

where M_d and M_ℓ are the moments due to service dead and live loads, respectively.

Thus, the strength design requirement for this section becomes:

$$[A_s f_y (d - a/2)] \geq 1.2M_d + 1.6\,M_\ell \geq 1.4M_d$$

Similarly, for shear acting on the section, the criterion for strength design can be stated as:

$$\phi V_n = \phi (V_c + V_s) \geq V_u$$

$$\phi \left[2\sqrt{f'_c}\, b_w d + \frac{A_v f_y d}{s} \right] \geq 1.2V_d + 1.6\,V_\ell \geq 1.4V_d$$

The following are the reasons for requiring strength reduction factors and load factors in strength design: [5.2]

1. The strength reduction of materials or elements is required because:

   a. Material strengths may differ from those assumed in design because of:

      - Variability in material strengths—the compression strength of concrete as well as the yield strength and ultimate tensile strength of reinforcement are variable.
      - Effect of speed of testing—the strengths of both concrete and steel are affected by the rate of loading.
      - In situ strength vs. specimen strength—the strength of concrete in a structure is somewhat different from the strength of the same concrete in a control specimen.
      - Effect of variability of shrinkage stresses or residual stresses—the variability of the residual stresses due to shrinkage may affect the cracking load of a member, and is significant where cracking is the critical limit state. Similarly, the transfer of compression loading from concrete to steel due to creep and shrinkage in columns may lead to premature yielding of the compression steel, possibly resulting in instability failures of slender columns with small amounts of reinforcement.

   b. Member dimensions may vary from those assumed, due to construction/fabrication errors. The following are significant:

      - Rolling and fabrication tolerances in reinforcing bars.
      - Geometric errors in cross-section and errors in placement of reinforcement.

   c. Assumptions and simplifications in design equations, such as use of the rectangular stress block and the maximum usable strain of concrete equal to 0.003, introduce both systematic and random errors.

   d. The use of discrete bar sizes leads to variations in the actual capacity of members.

2. The load factors are required for possible overloading because:

   a. Magnitudes of loads may vary from those assumed. Dead loads may vary because of:

      - Variations in member sizes.
      - Variations in material density.
      - Structural and nonstructural alterations.

      Live loads vary considerably from time to time and from building to building.

   b. Uncertainties exist in the calculation of load effects—the assumptions of stiffnesses, span lengths, etc., and the inaccuracies involved in modeling three-dimensional structures for structural analysis lead to differences between the stresses which actually occur in a building and those estimated in the designer's analysis.

3. Strength reduction and load increase are also required because the consequences of failure may be severe. A number of factors should be considered:

   a. The type of failure, warning of failure, and existence of alternative load paths.
   b. Potential loss of life.
   c. Costs to society in lost time, lost revenue, or indirect loss of life or property due to failure.
   d. The importance of the structural element in the structure.
   e. Cost of replacing the structure.

By way of background to the numerical values of load factors and strength reduction factors specified in the code, it may be worthwhile reproducing the following paragraph from Ref. 5.2:

> "The ACI ... design requirements ... are based on an underlying assumption that if the probability of understrength members is roughly 1 in 100 and the probability of overload is roughly 1 in 1000, the probability of overload on an understrength structure is about 1 in 100,000. Load factors were derived to achieve this probability of overload. Based on values of concrete and steel strength corresponding to probability of 1 in 100 of understrength, the strengths of a number of typical sections were computed. The ratio of the strength based on these values to the strength based on nominal strengths of a number of typical sections were arbitrarily adjusted to allow for the consequences of failure and the mode of failure and the mode of failure of a particular type of member, and for a number of other sources of variation in strength."

An Appendix to Ref. 5.2 traces the history of development of the current ACI load and strength reduction factors.

9.1.2 Serviceability Requirements

The provisions for adequate strength do not necessarily ensure acceptable behavior of the member at service load levels. Therefore, the code includes additional requirements to provide satisfactory service load performance.

There is not always a clear separation between the provisions for strength and those for serviceability. For actions other than flexure, the detailing provisions in conjunction with the strength requirements are meant to ensure adequate performance at service loads. For flexural action, there are special serviceability requirements concerning deflection, distribution of reinforcement, and permissible stresses in prestressed concrete. A consideration of service load deflections is particularly important in view of the extended use of high-strength materials and less conservative methods of design which result in increasingly slender reinforced concrete members.

9.1.3 Appendix C

The load factors and strength reduction factors used in the 1999 and earlier codes are now placed in Appendix C of the 2002 code. Use of Appendix C is permitted by 9.1.3. However, it is not permissible to combine the load factors in Chapter 9 with the strength reduction factors in Appendix C.

9.2 REQUIRED STRENGTH

As previously stated, the required strength U is expressed in terms of factored loads, or their related internal moments and forces. Factored loads are the service-level loads specified in the general building code, multiplied by appropriate load factors in 9.2. It is important to recognize that earthquake forces computed in accordance with the latest editions of the model buildings codes in use in the U. S. are strength-level forces. Specifically, seismic forces calculated under the 1993 and later editions of *The BOCA National Building Code*, the 1994 and later editions of the *Standard Building Code*, and the 1997 *Uniform Building Code* are strength-level forces. In addition, the 2000 *International Building Code* (IBC) developed by the International Code Council has seismic provisions that are strength-level forces.

This development has created confusion within the structural engineering profession since when designing in concrete one must use some load combinations from ACI 318 and others from the governing building code. To assist the structural engineer in understanding the various load combinations and their proper application to design of concrete structural elements governed by one of these codes, a new publication was recently released

by PCA. *Strength Design Load Combinations for Concrete Elements*[5.3] provides background on the use of the ACI 318 factored load combinations. In addition, it cites the load combinations in the most recent editions of the model codes, including the IBC, that must be used for seismic design.

Section 9.2 prescribes load factors for specific combinations of loads. A list of these combinations is below. The numerical value of the load factor assigned to each type of load is influenced by the degree of accuracy with which the load can usually be assessed, the variation which may be expected in the load during the lifetime of a structure and the probability of simultaneous occurrence of different load types. Hence, dead loads, because they can usually be more accurately determined and are less variable, are assigned a lower load factor (1.2) as compared to live loads (1.6). Also, weight and pressure of liquids with well-defined densities and controllable maximum heights are assigned a reduced load factor of 1.2 due the lesser probability of overloading. A higher load factor of 1.6 is required for earth and groundwater pressures due to considerable uncertainty of their magnitude and recurrence. Note that while most usual combinations of loads are included, it should not be assumed that all cases are covered. Section 9-2 contains load combination as follows:

$$U = 1.4(D + F) \qquad \textit{Eq. (9-1)}$$

$$U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R) \qquad \textit{Eq. (9-2)}$$

$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.8W) \qquad \textit{Eq. (9-3)}$$

$$U = 1.2D + 1.6W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R) \qquad \textit{Eq. (9-4)}$$

$$U = 1.2D + 1.0E + 1.0L + 0.2S \qquad \textit{Eq. (9-5)}$$

$$U = 0.9D + 1.6W + 1.6H \qquad \textit{Eq. (9-6)}$$

$$U = 0.9D + 1.0E + 1.6H \qquad \textit{Eq. (9-7)}$$

where:

- D = dead loads, or related internal moments and forces
- E = load effects of seismic forces, or related internal moments and forces
- F = loads due to weight and pressures of fluids with well-defined densities and controllable maximum heights, or related internal moments and forces
- H = loads due to weight and pressure of soil, water in soil, or other materials, or related internal moments and forces
- L = live loads, or related internal moments and forces
- L_r = roof live load, or related internal moments and forces
- R = rain load, or related internal moments and forces
- S = snow load, or related internal moments and forces
- T = cumulative effect of temperature, creep, shrinkage, differential settlement, and shrinkage-compensating concrete
- U = required strength to resist factored loads or related internal moments and forces
- W = wind load, or related internal moments and forces

Note that in Eqs. (9-1) through (9-7), the effect of one or more loads not acting simultaneously must also be investigated.

Exceptions to the load combination are as follows:

1. The load factor on L in Eq. (9-3), (9-4), and (9-5) shall be permitted to be reduced to 0.5 except for garages, areas occupied as places of public assembly, and all areas where the live load L is greater than 100 lb/ft^2.
2. Where wind load W has not been reduced by a directionality factor, it shall be permitted to use 1.3W in place of 1.6W in Eq. (9-4) and (9-6). Note that the wind load equation in ASCE 7-98 and IBC 2000 includes a factor for wind directionality that is equal to 0.85 for buildings. The corresponding load factor for wind in the load combination equations was increased accordingly (1.3/0.85 = 1.53 rounded up to 1.6). The code allows use of the previous wind load factor of 1.3 when the design wind load is obtained from other sources that do not include the wind directionality factor.
3. Where earthquake load E is based on service-level seismic forces, 1.4E shall be used in place of 1.0E in Eq. (9-5) and (9-7).
4. The load factor on H shall be set equal to zero in Eq. (9-6) and (9-7) if the structural action due to H counteracts that due to W or E. Where lateral earth pressure provides resistance to structural actions from other forces, it shall not be included in H but shall be included in the design resistance.

Other consideration related to load combination are as follows:

1. Resistance to impact effects, where applicable, shall be included with live load (9.2.2).
2. Differential settlement, creep, shrinkage, expansion of shrinkage-compensating concrete, or temperature change shall be based on a realistic assessment of such effects occurring in service (9.2.3).
3. For a structure in a flood zone, the flood load and load combinations of ASCE 7 shall be used (9.2.4).
4. For post-tensioned anchorage zone design, a load factor of 1.2 shall be applied to the maximum prestressing steel jacking force (9.2.5).

As can be seen, the load combinations are more complex than in previous editions of the ACI code. For many members, the loads considered are dead, live, wind, and earthquake. A conservative simplification can be made by including the roof live load L_r with the live load L, and using the higher load factor for L or L_r in each of the equations. Where the F, H, R, S, and T loads are not considered, the seven equations simplify to those given in Table 5-1 below.

Table 5-1 Required Strength for Simplified Load Combinations

Loads	Required Strength	Code Eq. No.
Dead (D) and Live (L)	1.4D 1.2D + 1.6L	9-1 9-2
Dead, Live, and Wind (W)	1.2D + 1.6L + 0.8W 1.2D + 1.0L + 1.6W 0.9D + 1.6W	9-3 9-4 9-6
Dead, Live, and Earthquake (E)	1.2D + 1.0L + 1.0E 0.9D + 1.0E	9-5 9-7

While considering gravity loads (dead and live), a designer using the code moment coefficients (same coefficients for dead and live loads—8.3.3) has three choices: (1) multiplying the loads by the appropriate load factors, adding them into the total factored load, and then computing the forces and moments due to the total load, (2) computing the effects of factored dead and live loads separately, and then superimposing the effects, or (3) computing the effects of unfactored dead and live loads separately, multiplying the effects by the appropriate load

factors, and then superimposing them. Under the principle of superposition, all three procedures yield the same answer. To a designer doing a more exact analysis using different coefficients for dead and live loads (pattern loading for live loads), choice (1) does not exist. While considering gravity as well as lateral loads, load effects (due to factored or unfactored loads), of course, have to be computed separately before any superposition can be made.

In determining the required strength for combinations of loads, due regard must be given to the proper sign (positive or negative), since one type of loading may produce effects that either add to or counteract the effect of another load. Even though Eqs. (9-6) and (9-7) have a positive sign preceding the wind (W) or earthquake (E) load, the combinations are to be used when wind or earthquake forces or effects counteract dead loads. When the effects of gravity loads and wind (W) or earthquake (E) loads are additive, Eqs. (9-4), (9-5), and (9-6) must be used.

Consideration must be given to various combinations of loads in determining the most critical design combination. This is of particular importance when strength is dependent on more than one load effect, such as strength under combined moment and axial load, or the shear strength of members carrying axial load.

9.3 DESIGN STRENGTH

9.3.1 Nominal Strength vs. Design Strength

The design strength provided by a member, its connections to other members, and its cross-section, in terms of flexure, axial load, shear, and torsion, is equal to the nominal strength calculated in accordance with the provisions and assumptions stipulated in the code, multiplied by a strength reduction factor ϕ, which is less than unity. The rules for computing the nominal strength are based generally on conservatively chosen limiting states of stress, strain, cracking or crushing, and conform to research data for each type of structural action. An understanding of all aspects of the strengths computed for various actions can only be obtained by reviewing the background to the code provisions.

9.3.2 Strength Reduction Factors

The ϕ factors prescribed for structural concrete in 9.3.2 are listed in Table 5-2. The reasons for use of strength reduction factors have been given in earlier sections.

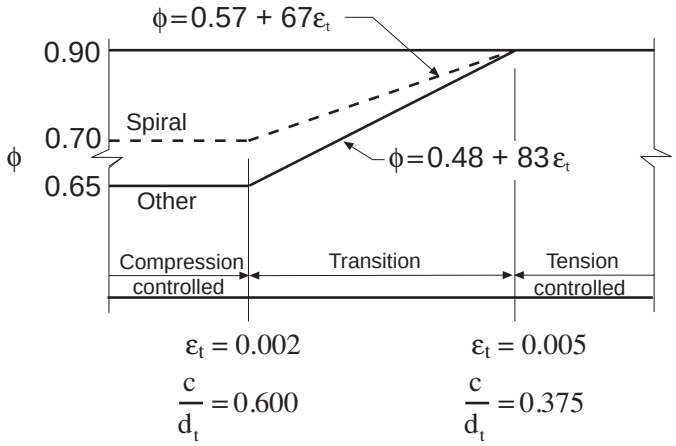
Note that a lower ϕ factor is used for compression-controlled sections than for tension-controlled sections because columns generally have less ductility and are more sensitive to variations in concrete strength. Additionally, the consequences of failure of a column would generally be more severe than those for failure of a beam. Furthermore, spiral columns are assigned a higher f factor than tied columns because the former have greater toughness and ductility.

Table 5-2 Strength Reduction Factors ϕ in the Strength Design Method

Tension-controlled sections	0.90
Compression-controlled sections Members with spiral reinforcement conforming to 10.9.3 Other reinforced members	 0.70 0.65
Shear and torsion	0.75
Bearing on concrete (except for post-tensioned anchorage zones)	0.65
Post-tensioned anchorage zones	0.85
Strut-and-tie models (Appendix A)	0.75

Tension-controlled sections and compression-controlled sections are defined in 10.3.3. See Part 7 for detailed discussion.

The code permits a linear transition in ϕ between the limits for tension-controlled and compression-controlled sections. For sections in which the net tensile strain in the extreme tension steel at nominal strength is between the limits for compression-controlled and tension-controlled section, ϕ shall be permitted to be linearly increased from that for compression-controlled sections to 0.90 as the net tensile strain in the extreme tension steel at nominal strength increases from the compression-controlled strain limit to 0.005. This is best illustrated by Figure 5-2.



Interpolation on c/d_t: Spiral $\phi = 0.37 + 0.20/(c/d_t)$
Other $\phi = 0.23 + 0.25/(c/d_t)$

Figure 5-2 Variation of ϕ with Net Tensile ε_t and c/d_t for Grade 60 Reinforcement and for Prestressing Steel

For members subject to flexure and axial load, the design strengths are determined by multiplying both P_n and M_n by the appropriate single value of ϕ.

9.3.3 Development Lengths for Reinforcement

Development lengths for reinforcement, as specified in Chapter 12, do not require a strength reduction modification. Likewise, ϕ factors are not required for splice lengths, since these are expressed in multiples of development lengths.

9.3.5 Structural Plain Concrete

This section was introduced in the '95 code and specifies that the strength reduction factor $\phi = 0.65$ be used for flexure, compression, shear, and bearing of plain concrete. This is because both the flexural tension strength and the shear strength of plain concrete depend on the tensile strength characteristics of concrete having no reserve strength or ductility in the absence of steel reinforcement.

9.4 DESIGN STRENGTH FOR REINFORCEMENT

An upper limit of 80,000 psi is placed on the yield strength of reinforcing steels other than prestressing tendons. A steel strength above 80,000 psi is not recommended because the yield strain of 80,000 psi steel is about equal to the maximum usable strain of concrete in compression. Currently there is no ASTM specification for Grade 80

reinforcement. However, No. 11, No. 14, and No. 18 deformed reinforcing bars with a yield strength of 75,000 psi (Grade 75) are included in ASTM A615.

In accordance with 3.5.3.2, use of reinforcing bars with a specified yield strength f_y exceeding 60,000 psi requires that f_y be the stress corresponding to a strain of 0.35 percent. ASTM A615 for Grade 75 bars includes the same requirement. The 0.35 percent strain requirement also applies to welded wire fabric with wire having a specified yield strength greater than 60,000 psi. Higher-yield-strength wire is available and a value of f_y greater than 60,000 psi can be used in design, provided compliance with the 0.35 percent strain requirement is certified.

There are limitations on the yield strength of reinforcement in other sections of the code:

1. Sections 11.5.2, 11.6.3.4, and 11.7.6: The maximum f_y that may be used in design for shear, combined shear and torsion, and shear friction is 60,000 psi, except that f_y up to 80,000 psi may be used only for shear reinforcement consisting of welded deformed wire fabric meeting the requirements of ASTM A497.

2. Sections 19.3.2 and 21.2.5: The maximum specified f_y is 60,000 psi in shells, folded plates and structures governed by the special seismic provisions of Chapter 21.

In addition, the deflection provisions of 9.5 and the limitations on distribution of flexural reinforcement of 10.6 will become increasingly critical as f_y increases.

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5.1 Mast, R. F., "Unified Design Provisions for Reinforced and Prestressed Concrete Flexural and Compression Members," *ACI Structural Journal*, Vol. 89, No. 2, March-April 1992, pp. 185-199.

5.2. MacGregor, J. G., "Safety and Limit States Design for Reinforced Concrete," *Canadian Journal of Civil Engineering*, Vol. 3, No. 4, December 1976, pages 484-513.

5.3 *Strength Design Load Combinations for Concrete Elements*, Publication IS521, Portland Cement Association, Skokie, Illinois, 1998.

6

General Principles of Strength Design

UPDATE TO THE '02 CODE

The Unified Design Provisions, formerly in Appendix B, are now incorporated in the body of the code. These provisions use the maximum tensile strain as a measure of behavior of a member or cross-section. Formerly, the percentage of the balanced reinforcement ratio ρ_b was used as a measure of behavior.

GENERAL CONSIDERATIONS

Historically, ultimate strength was the earliest method used in design, since the ultimate load could be measured by test without a knowledge of the magnitude or distribution of internal stresses. Since the early 1900s, experimental and analytical investigations have been conducted to develop ultimate strength design theories that would predict the ultimate load measured by test. Some of the early theories that resulted from the experimental and analytical investigations are reviewed in Fig. 6-1.

Structural concrete and reinforcing steel both behave inelastically as ultimate strength is approached. In theories dealing with the ultimate strength of reinforced concrete, the inelastic behavior of both materials must be considered and must be expressed in mathematical terms. For reinforcing steel with a distinct yield point, the inelastic behavior may be expressed by a bilinear stress-strain relationship (Fig. 6-2). For concrete, the inelastic stress distribution is more difficult to measure experimentally and to express in mathematical terms.

Studies of inelastic concrete stress distribution have resulted in numerous proposed stress distributions as outlined in Fig. 6-1. The development of our present ultimate strength design procedures has its basis in these early experimental and analytical studies. Ultimate strength of reinforced concrete in American design specifications is based primarily on the 1912 and 1932 theories (Fig. 6-1).

INTRODUCTION TO UNIFIED DESIGN PROVISIONS

The Unified Design Provisions that are now in the main body of the code do not alter nominal strengths. The nominal strength of a section is the same as it was using previous calculations. However, the Unified Design Provisions do alter the calculations of design strengths, which are reduced from nominal strengths by the capacity reduction factor ϕ.

The following definitions are related to the Unified Design Provisions, and are given in Chapter 2 of the code. These definitions are briefly explained here, with further detailed discussion under the relevant code section.

1. Net tensile strain: The tensile strain at nominal strength exclusive of strains due to effective prestress, creep, shrinkage, and temperature. The phrase "at nominal strength" in the definition means at the time the concrete in compression reaches its assumed strain limit of 0.003 (10.2.3). The "net tensile strain" is the strain caused by bending moments and axial loads, exclusive of strain caused by prestressing and by volume changes. The net tensile strain is that normally calculated in nominal strength calculations.

2. Extreme tension steel: The reinforcement (prestressed or nonprestressed) that is the farthest from the extreme compression fiber. The symbol d_t is used to stand for the depth from the extreme compression fiber to the extreme tensile steel. The net tensile strain in the extreme tension steel is simply the maximum tensile steel strain due to external loads.

3. Compression-controlled strain limit: The net tensile strain at balanced strain conditions; see 10.3.2. The definition of balanced strain conditions in 10.3.2 is unchanged from previous editions of the code. However, 10.3.3 permits the compression-controlled strain limit for Grade 60 reinforcement and for prestressed reinforcement to be set equal to 0.002.

4. Compression-controlled section: A cross-section in which the net tensile strain in the extreme tension steel at nominal strength is less than or equal to the compression-controlled strain limit. The strength reduction factor ϕ for compression-controlled sections is set at 0.65 or 0.7 in 9.3.2.2.

5. Tension-controlled section: A cross-section in which the net tensile strain in the extreme tension steel at nominal strength is greater than or equal to 0.005. The strength reduction factor ϕ for tension-controlled sections is set at 0.9 in 9.3.2.1. However, former editions of the code permitted a ϕ of 0.9 to be used for flexural members with reinforcement ratios of 0.75 of the balanced reinforcement ration ρ_b. For rectangutions, the corresponding net tensile strain ε_t is 0.00376. Using the new reduced load factors, the use of ϕ of 0.9 is permitted only for less heavily reinforced sections with $\varepsilon_t \geq 0.005$.

The use of these definitions is described under 8.4, 9.2, 10.3, and 18.8.

10.2 DESIGN ASSUMPTIONS

10.2.1 Equilibrium of Forces and Compatibility of Strains

Computation of the strength of a member or cross-section by the Strength Design Method requires that two basic conditions be satisfied: (1) static equilibrium and (2) compatibility of strains.

The first condition requires that the compressive and tensile forces acting on the cross-section at "ultimate" strength be in equilibrium, and the second condition requires that compatibility between the strains in the concrete and the reinforcement at "ultimate" conditions must also be satisfied within the design assumptions permitted by the code (see 10.2).

The term "ultimate" is used frequently in reference to the Strength Design Method; however, it should be realized that the "nominal" strength computed under the provisions of the code may not necessarily be the actual ultimate value. Within the design assumptions permitted, certain properties of the materials are neglected and other conservative limits are established for practical design. These contribute to a possible lower "ultimate strength" than that obtained by test. The computed nominal strength should be considered a code-defined strength only. Accordingly, the term "ultimate" is not used when defining the computed strength of a member. The term "nominal" strength is used instead.

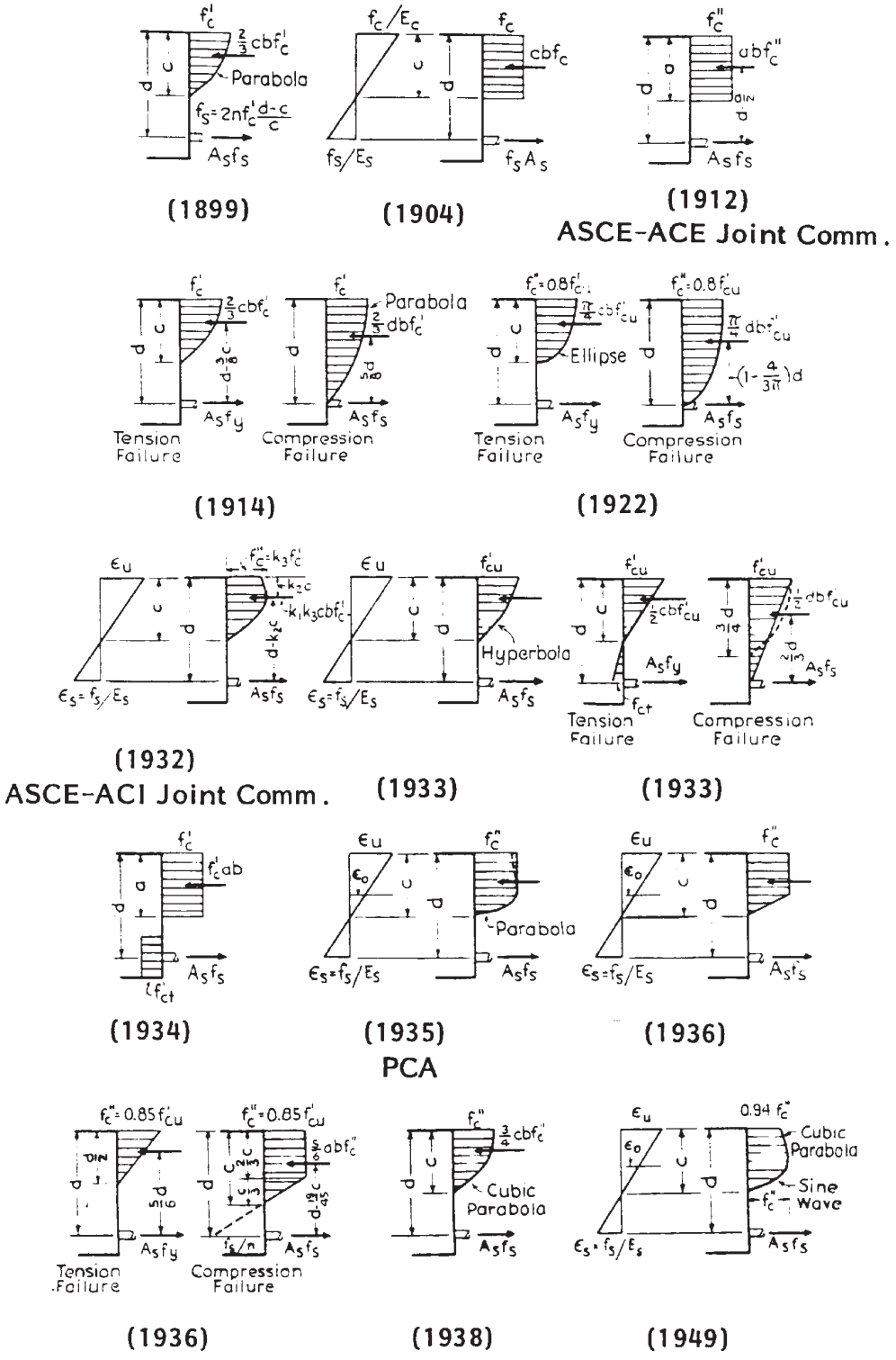


Figure 6-1 Development of Ultimate Strength Theories of Flexure

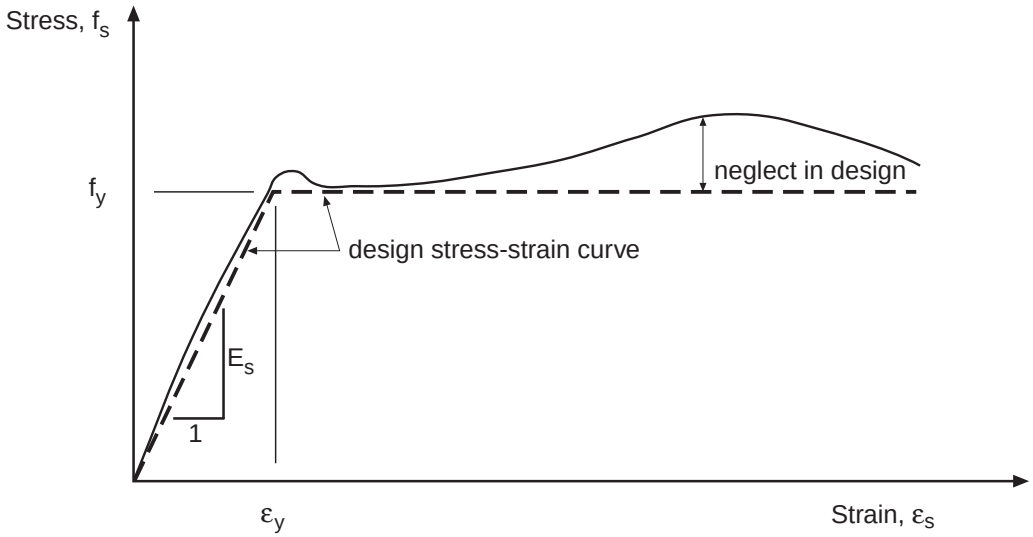


Figure 6-2 Stress-Strain Relationship for Reinforcement

Furthermore, in discussing the strength method of design for reinforced concrete structures, attention must be called to the difference between loads on the structure as a whole and load effects on the cross-sections of individual members. Elastic methods of structural analysis are used first to compute service load effects on the individual members due to the action of service loads on the entire structure. Only then are the load factors applied to the service load effects acting on the individual cross-sections. Inelastic (or limit) methods of structural analysis, in which design load effects on the individual members are determined directly from the ultimate test loads acting on the whole structure, are not considered. Section 8.4, however, does permit a limited redistribution of negative moments in continuous members. The provisions of 8.4 recognize the inelastic behavior of concrete structures and constitute a move toward "limit design." This subject is presented in Part 8.

The computed "nominal strength" of a member must satisfy the design assumptions given in 10.2.

10.2.2 Design Assumption #1

Strain in reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis.

In other words, plane sections normal to the axis of bending are assumed to remain plane after bending. Many tests have confirmed that the distribution of strain is essentially linear across a reinforced concrete cross-section, even near ultimate strength. For reinforcement, this assumption has been verified by numerous tests to failure of eccentrically loaded compression members and members subjected to bending only.

The assumed strain conditions at ultimate strength of a rectangular and circular section are illustrated in Fig. 6-3. Both the strain in the reinforcement and in the concrete are directly proportional to the distance from the neutral axis. This assumption is valid over the full range of loading—zero to ultimate. As shown in Fig. 6-3, this assumption is of primary importance in design for determining the strain (and the corresponding stress) in the reinforcement.

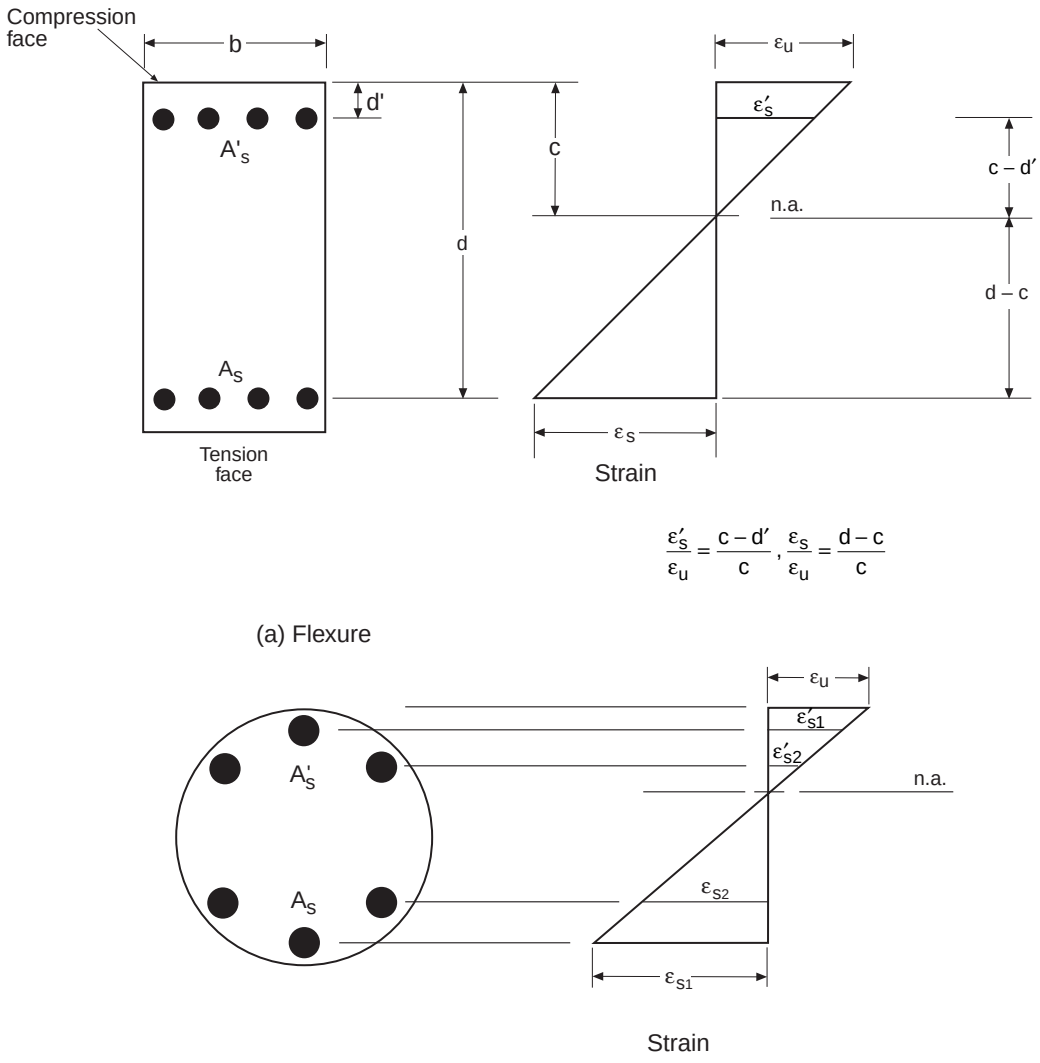


Figure 6-3 Assumed Strain Variation

10.2.3 Design Assumption #2

Maximum usable strain at extreme concrete compression fiber shall be assumed equal to $\varepsilon_u = 0.003$.

The maximum concrete compressive strain at crushing of the concrete has been measured in many tests of both plain and reinforced concrete members. The test results from a series of reinforced concrete beam and column specimens, shown in Fig. 6-4, indicate that the maximum concrete compressive strain varies from 0.003 to as high as 0.008. However, the maximum strain for practical cases is 0.003 to 0.004; see stress-strain curves in Fig. 6-5. Though the maximum strain decreases with increasing compressive strength of concrete, the 0.003 value allowed for design is reasonably conservative. The codes of some countries specify a value of 0.0035 for design, which makes little difference in the computed strength of a member.

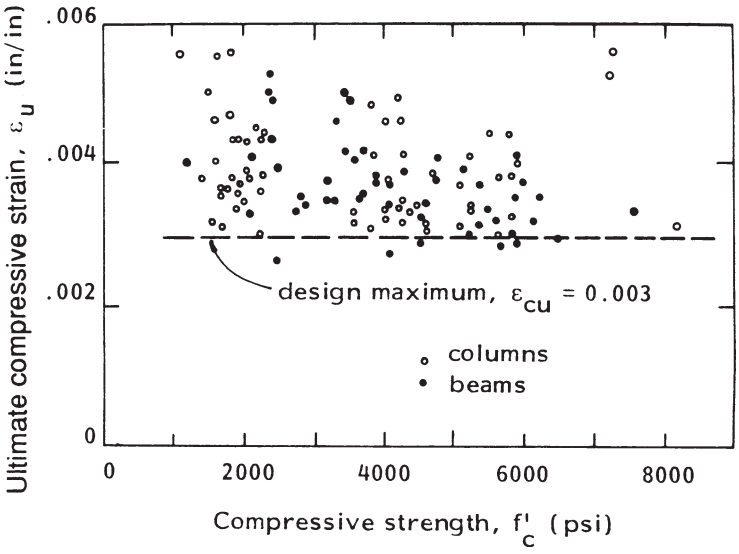


Figure 6-4 Maximum Concrete Compressive Strain, ε_u from Tests of Reinforced Concrete Members

10.2.4 Design Assumption #3

Stress in reinforcement f_s below the yield strength f_y shall be taken as E_s times the steel strain ε_s. For strains greater than f_y/E_s, stress in reinforcement shall be considered independent of strain and equal to f_y.

For deformed reinforcement, it is reasonably accurate to assume that below the yield stress, the stress in the reinforcement is proportional to strain. For practical design, the increase in strength due to the effect of strain hardening of the reinforcement is neglected for strength computations; see actual vs. design stress-strain relationship of steel in Fig. 6-2.

The force developed in the tensile or compressive reinforcement is a function of the strain in the reinforcement ε_s, such that:

when $\varepsilon_s \leq \varepsilon_y$ (yield strain):

$$f_s = E_s \varepsilon_s$$

$$A_s f_s = A_s E_s \varepsilon_s$$

when $\varepsilon_s \geq \varepsilon_y$:

$$f_s = E_s \varepsilon_y = f_y$$

$$A_s f_s = A_s f_y$$

where ε_s is the value from the strain diagram at the location of the reinforcement; see Fig. 6-3. For design, the modulus of elasticity of steel reinforcement, E_s, is taken as 29,000,000 psi (see 8.5.2).

10.2.5 Design Assumption #4

Tensile strength of concrete shall be neglected in flexural calculations of reinforced concrete.

The tensile strength of concrete in flexure, known as the modulus of rupture, is a more variable property than the compressive strength, and is about 8% to 12% of the compressive strength. The generally accepted value for design is $7.5\sqrt{f'_c}$ for normal-weight concrete. This tensile strength in flexure is neglected in strength design. For practical percentages of reinforcement, the resulting computed strengths are in good agreement with test results. For very small percentages of reinforcement, neglecting the tensile strength of concrete is conservative. It should be realized, however, that the strength of concrete in tension is important in cracking and deflection (serviceability) considerations.

10.2.6 Design Assumption #5

Relationship between concrete compressive stress distribution and concrete strain shall be assumed to be rectangular, trapezoidal, parabolic, or any other shape that results in prediction of strength in substantial agreement with results of comprehensive tests.

This assumption recognizes the inelastic stress distribution in concrete at high stresses. As maximum stress is approached, the stress-strain relationship of concrete is not a straight line (stress is not proportional to strain). The general stress-strain behavior of concrete is shown in Fig. 6-5. The shape of the curves is primarily a function of concrete strength and consists of a rising curve from zero stress to a maximum at a compressive strain between 0.0015 and 0.002, followed by a descending curve to an ultimate strain (corresponding to crushing of the concrete) varying from 0.003 to as high as 0.008. As discussed under Design Assumption #2, the code sets the maximum usable strain at 0.003 for design. The curves show that the stress-strain behavior for concrete becomes notably nonlinear at stress levels exceeding $0.5\,f'_c$.

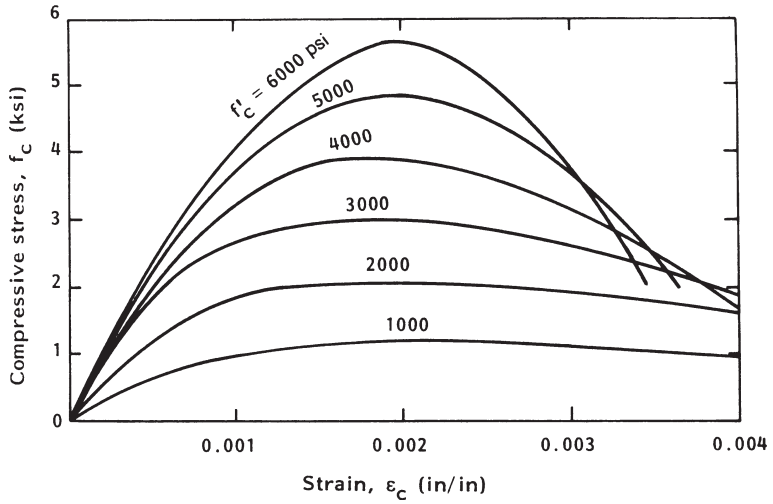


Figure 6-5 Typical Stress-Strain Curves for Concrete

The actual distribution of concrete compressive stress in a practical case is complex and usually not known. However, research has shown that the important properties of the concrete stress distribution can be approximated closely using any one of several different forms of stress distributions (see Fig. 6-1). The three most common stress distributions are the parabolic, the trapezoidal, and the rectangular, each giving reasonable results. At the theoretical ultimate strength of a member in flexure, the compressive stress distribution should conform closely to the actual variation of stress, as shown in Fig. 6-6. In this figure, the maximum stress is indicated by $k_3\,f'_c$, the average stress is indicated by $k_1k_3\,f'_c$, and the depth of the centroid of the approximate parabolic distribution from the extreme compression fiber by k_2c, where c is the neutral axis depth.

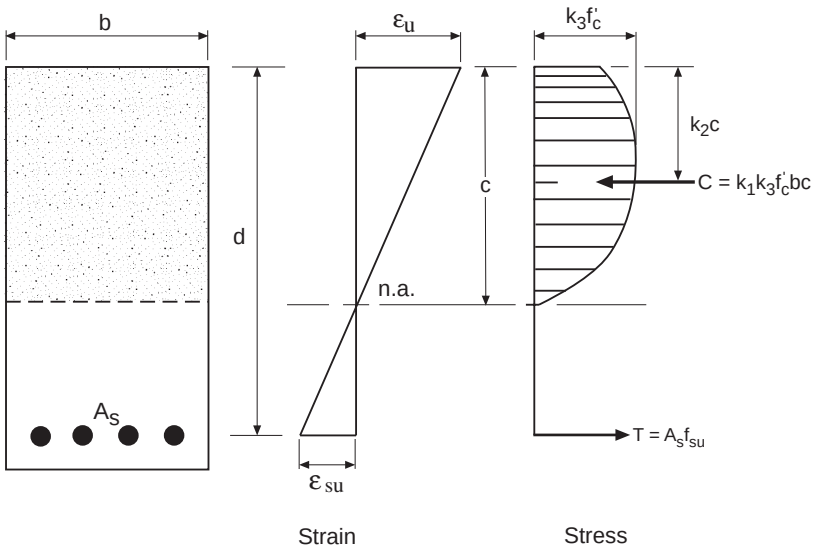


Figure 6-6 Actual Stress-Strain Conditions at Nominal Strength in Flexure

For the stress conditions at ultimate, the nominal moment strength, M_n, may be computed by equilibrium of forces and moments in the following manner:

From force equilibrium (Fig. 6-6):

$$C = T$$

or, $$k_1k_3 f'_c bc = A_sf_{su}$$

so that $$c = \frac{A_sf_{su}}{k_1k_3f'_cb}$$

From moment equilibrium:

$$M_n = (C \text{ or } T)(d - k_2c) = A_sf_{su}\left(d - \frac{k_2}{k_1k_3}\frac{A_sf_{su}}{f'_cb}\right) \quad (1)$$

The maximum strength is assumed to be reached when the strain in the extreme compression fiber is equal to the crushing strain of the concrete, ε_u. When crushing occurs, the strain in the tension reinforcement, ε_{su}, may be either larger or smaller than the yield strain, $\varepsilon_y = f_y/E_s$, depending on the relative proportion of reinforcement to concrete. If the reinforcement amount is low enough, yielding of the steel will occur prior to crushing of the concrete (ductile failure condition). With a very large quantity of reinforcement, crushing of the concrete will occur first, allowing the steel to remain elastic (brittle failure condition). The code has provisions which are intended to ensure a ductile mode of failure by limiting the amount of tension reinforcement. For the ductile failure condition, f_{su} equals f_y, and Eq. (1) becomes:

$$M_n = A_sf_y\left(d - \frac{k_2}{k_1k_3}\frac{A_sf_y}{f'_cb}\right) \quad (2)$$

If the quantity $k_2/(k_1k_3)$ is known, the moment strength can be computed directly from Eq. (2). It is not neces-

sary to know the values of k_1, k_2, and k_3 individually. Values for the combined term, as well as the individual k_1 and k_2 values, have been established from tests and are shown in Fig. 6-7. As shown in the figure, $k_2/(k_1k_3)$ varies from about 0.55 to 0.63. Computation of the flexural strength based on the approximate parabolic stress distribution of Fig. 6-6 may be done using Eq. (2) with given values of $k_2/(k_1k_3)$. However, for practical design purposes, a method based on simple static equilibrium is desirable.

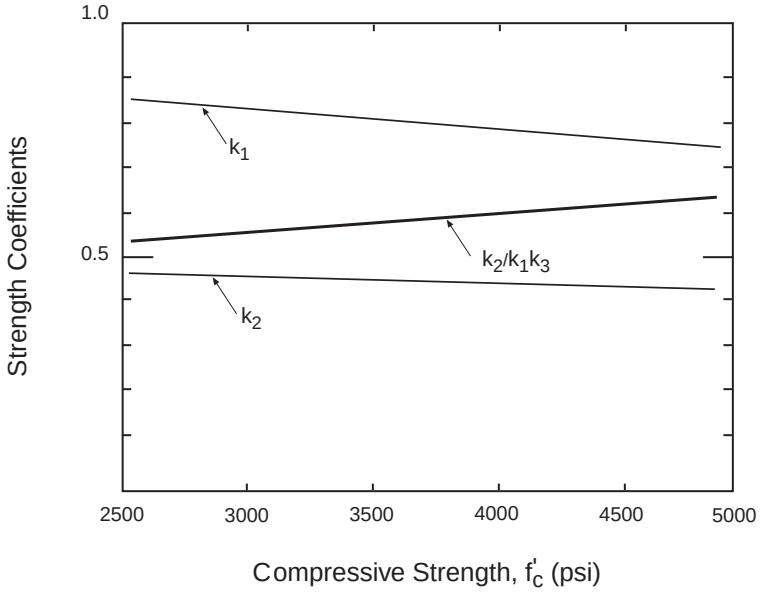


Figure 6-7 Stress-Block Parameters

The Portland Cement Association adopted the parabolic stress-strain relationship shown in Fig. 6-8 for much of its experimental and analytical research work. "More exact" stress distributions such as this one have their greatest application with computers and are not recommended for longhand calculations. Most of the recent PCA publications and computer software related to structural concrete design are based either entirely on the rectangular stress block or give the user the choice to use either the rectangular or the parabolic stress block.

10.2.7 Design Assumption #6

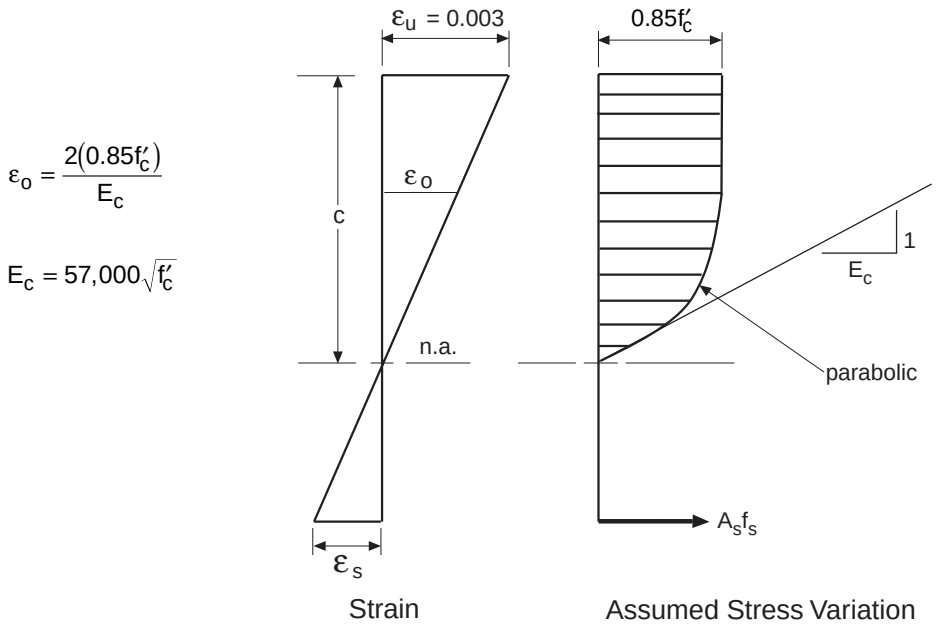
Requirements of 10.2.6 may be considered satisfied by an equivalent rectangular concrete stress distribution defined as follows: A concrete stress of 0.85 f'_c shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross-section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain. Distance c from the fiber of maximum compressive strain to the neutral axis shall be measured in a direction perpendicular to that axis. Fraction β_1 shall be taken as 0.85 for strengths f'_c up to 4000 psi and shall be reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi, but β_1 shall not be taken less than 0.65.

The code allows the use of a rectangular compressive stress block to replace the more exact stress distributions of Fig. 6-6 (or Fig. 6-8). The equivalent rectangular stress block, shown in Fig. 6-9, assumes a uniform stress of 0.85 f'_c over a depth $a = \beta_1 c$, determined so that $a/2 = k_2 c$. The constant β_1 is equal to 0.85 for concrete with $f'_c \le 4000$ psi and reduces by 0.05 for each additional 1000 psi of f'_c in excess of 4000 psi. For high-strength concretes, above 8000 psi, a lower limit of 0.65 is placed on the β_1 factor. Variation in β_1 vs. concrete strength f'_c is shown in Fig. 6-10.

The need for a β_1 factor is caused by the variation in shape of the stress-strain curve for different concrete strengths, as shown in Fig. 6-5. For concrete strengths up to 4,000 psi, the shape and centroid of the actual concrete stress block can reasonably be approximated by a rectangular stress block with a uniform stress of 0.85 f'_c and a depth of 0.85 times the depth to the neutral axis. That is to say, with a β_1 of 0.85.

Higher strength concretes have a more linear shape, with less inelastic behavior. For a good approximation of the stress block for concretes with strengths above 4,000 psi, the ratio β_1 of rectangular stress block depth to neutral axis depth needs to be reduced. Thus, the 1963 code required that β_1"shall be reduced continuously at a rate of 0.05 for each 1,000 psi of strength in excess of 4,000 psi.

As time went by and much higher concrete strengths came into use, it was realized that this reduction in β_1 should not go on indefinitely. Indeed, at 17,000 psi, β_1 would be zero. Very high strengths have a stress block that approaches a triangular shape. This almost-triangular stress block is best approximated by a rectangular stress block with β_1 = 0.65. Thus, in the 1977 and later codes, β_1 was set at 0.65 for concrete strengths of 8,000 psi and above.



Concrete

$$0 < \varepsilon_c < \varepsilon_o \quad : \quad f_c = 0.85f'_c\left[2\left(\frac{\varepsilon_c}{\varepsilon_o}\right) - \left(\frac{\varepsilon_c}{\varepsilon_o}\right)^2\right]$$

$$\varepsilon_c \geq \varepsilon_o \quad : \quad f_c = 0.85f'_c$$

Steel

$$\varepsilon_s \leq \varepsilon_y \quad : \quad f_s = \varepsilon_s E_s$$

$$\varepsilon_s > \varepsilon_y \quad : \quad f_s = f_y$$

$$E_s = 29{,}000{,}000\,\text{psi}$$

Figure 6-8 Stress-Strain Relationship Adopted by PCA

Using the equivalent rectangular stress distribution (Fig. 6-9), and assuming that the reinforcement yields prior to crushing of the concrete ($\varepsilon_s > \varepsilon_y$), the nominal moment strength M_n may be computed by equilibrium of forces and moments.

From force equilibrium:

$$C = T$$

or, $$0.85 f'_c ba = A_s f_y$$

so that $$a = \frac{A_s f_y}{0.85 f'_c b}$$

From moment equilibrium:

$$M_n = (C \text{ or } T)\left(d - \frac{a}{2}\right) = A_s f_y \left(d - \frac{a}{2}\right)$$

Substituting a from force equilibrium,

$$M_n = A_s f_y \left(d - 0.59 \frac{A_s f_y}{f'_c b}\right) \qquad (3)$$

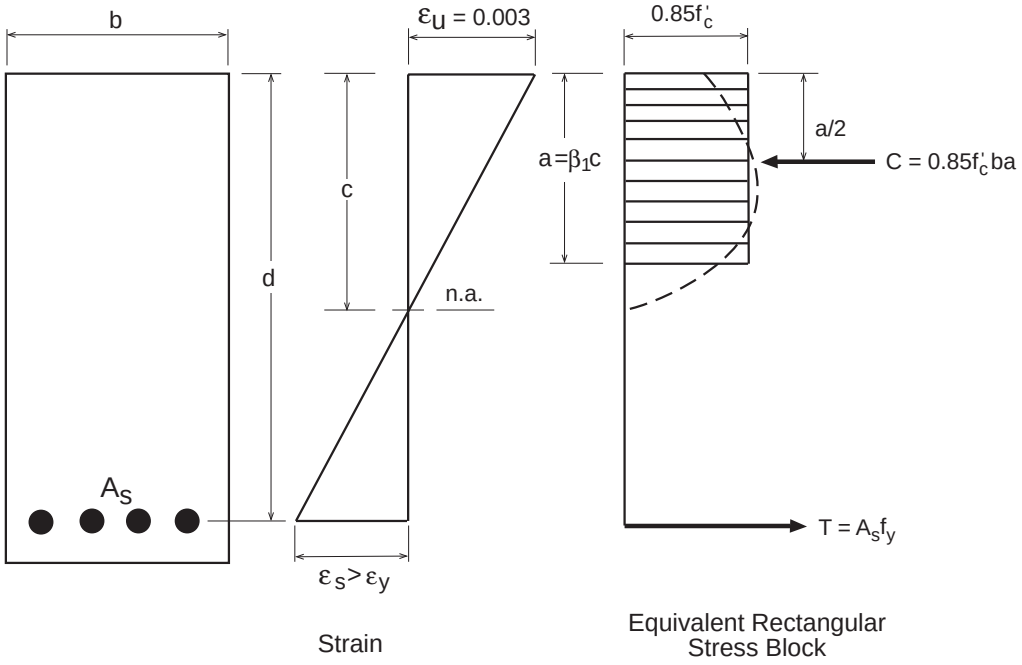


Figure 6-9 Equivalent Rectangular Concrete Stress Distribution (ACI)

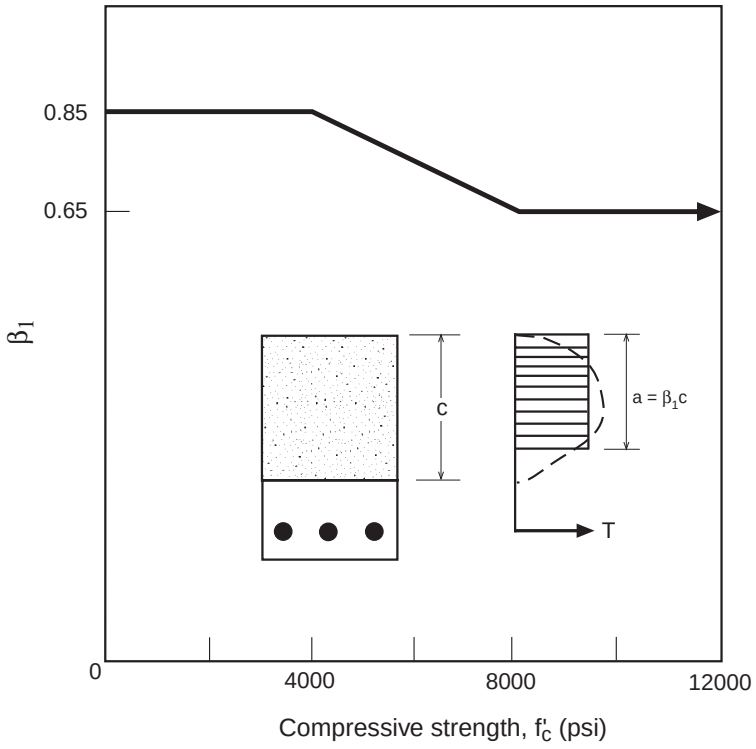


Figure 6-10 Strength Factor β_1

Note that the 0.59 value corresponds to $k_2/(k_1k_3)$ of Eq. (2). Substituting $A_s = \rho bd$, Eq. (3) may be written in the following nondimensional form:

$$\text{let } \omega = \rho \frac{f_y}{f'_c}$$

$$\frac{M_n}{bd^2 f'_c} = \rho \frac{f_y}{f'_c}\left(1 - 0.59\,\rho \frac{f_y}{f'_c}\right) \qquad (4)$$

$$= \omega(1 - 0.59\omega)$$

As shown in Fig. 6-11, Eq. (4) is "in substantial agreement with the results of comprehensive tests." However, it must be realized that the rectangular stress block does not represent the actual stress distribution in the compression zone at ultimate, but does provide essentially the same strength results as those obtained in tests. Computation of moment strength using the equivalent rectangular stress distribution and static equilibrium is illustrated in Example 6.1.

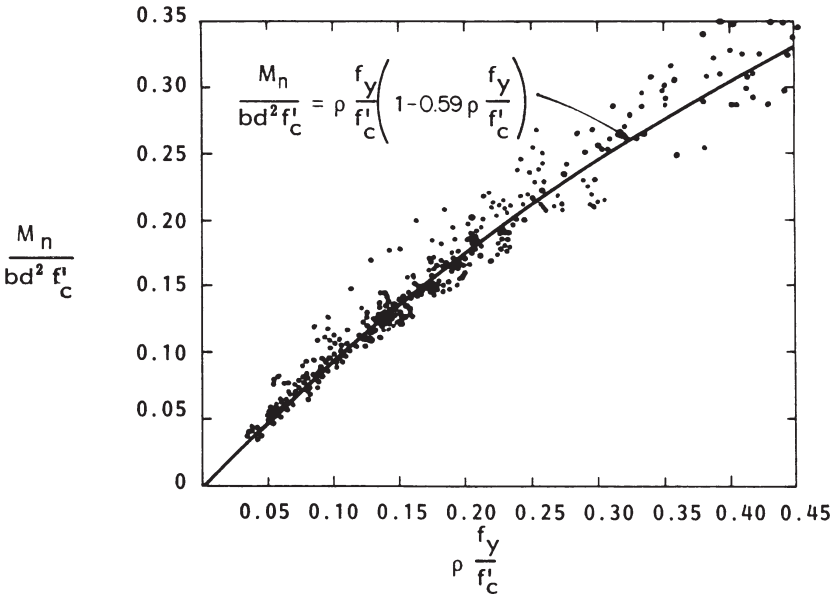


Figure 6-11 Tests of 364 Beams Controlled by Tension ($\varepsilon_s > \varepsilon_y$)

10.3 GENERAL PRINCIPLES AND REQUIREMENTS

10.3.1 Nominal Flexural Strength

Nominal strength of a member or cross-section subject to flexure (or to combined flexure and axial load) must be based on equilibrium and strain compatibility using the design assumptions of 10.2. Nominal strength of a cross-section of any shape, containing any amount and arrangement of reinforcement, is computed by applying the force and moment equilibrium and strain compatibility conditions in a manner similar to that used to develop the nominal moment strength of the rectangular section with tension reinforcement only, as illustrated in Fig. 6-9. Using the equivalent rectangular concrete stress distribution, expressions for nominal moment strength of rectangular and flanged sections (typical sections used in concrete construction) are summarized as follows:

a. Rectangular section with tension reinforcement only (see Fig. 6-9):

   Expressions are given above under Design Assumption #6 (10.2.7).

b. Flanged section with tension reinforcement only:

   When the compression flange thickness is equal to or greater than the depth of the equivalent rectangular stress block a, moment strength M_n is calculated by Eq. (3), as for a rectangular section with width equal to the flange width. When the compression flange thickness h_f is less than a, the nominal moment strength M_n is (see Fig. 6-12):

$$M_n = (A_s - A_{sf}) f_y \left(d - \frac{a}{2}\right) + A_{sf} f_y \left(d - \frac{h_f}{2}\right) \qquad (5)$$

where

A_{sf} = area of reinforcement required to equilibrate compressive strength of overhanging flanges

$= 0.85\, f'_c\, (b - b_w)\, h_f/f_y$

$a = (A_s - A_{sf})\, f_y/0.85\, f'_c\, b_w$

b = width of effective flange (see 8.10)

b_w = width of web

h_f = thickness of flange

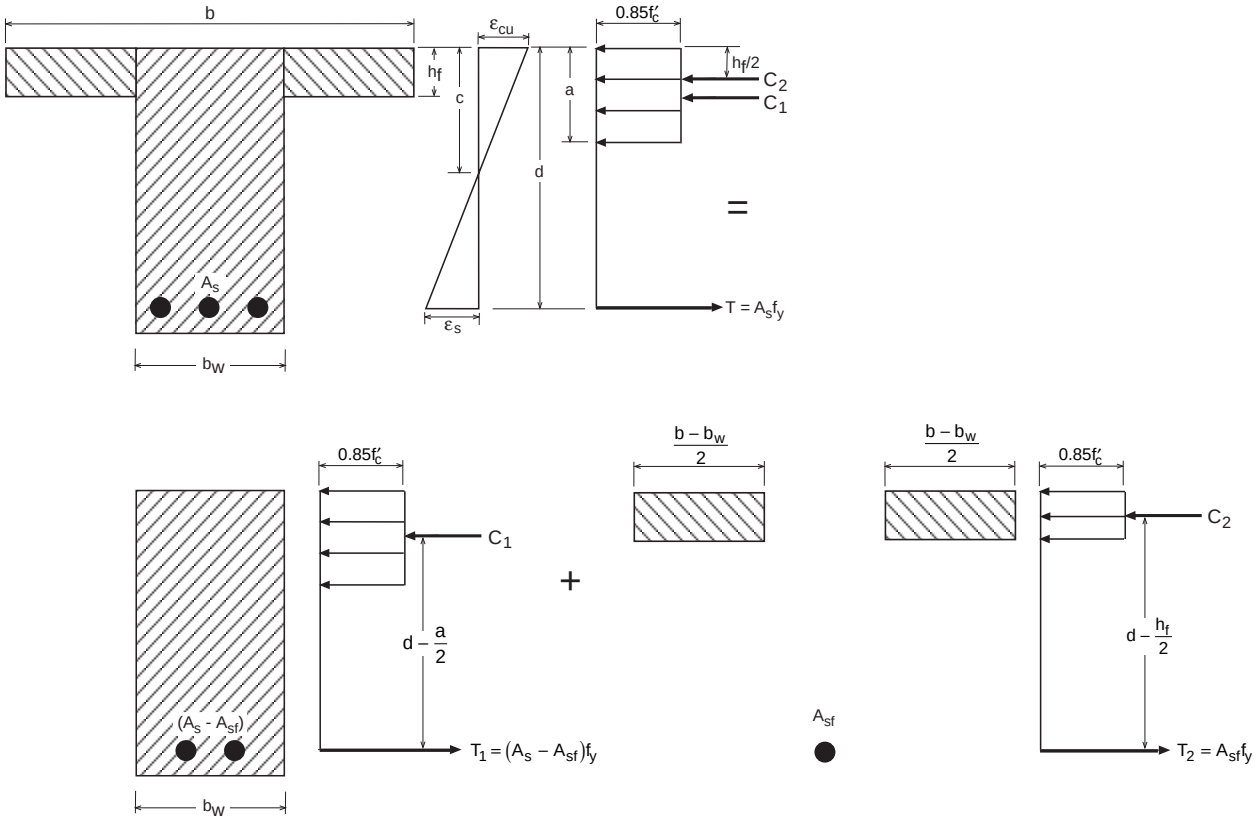


Figure 6-12 Strain and Equivalent Stress Distribution for Flanged Section

c. Rectangular section with compression reinforcement:

For a doubly reinforced section with compression reinforcement A'_s, two possible situations can occur (see Fig. 6-13):

i. Compression reinforcement A'_s yields:

$$f'_s = f_y$$

$$a = \frac{(A_s - A'_s)\, f_y}{0.85 f'_c b} \qquad (6)$$

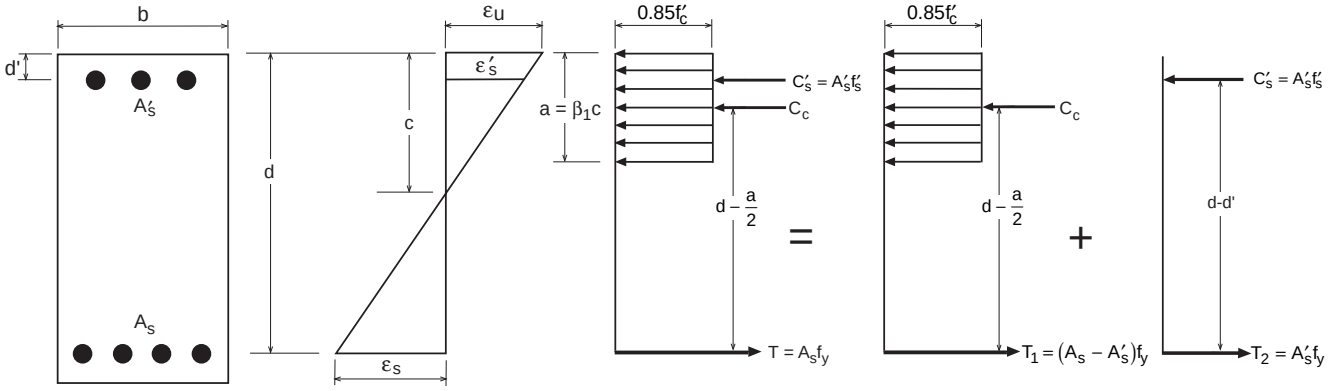


Figure 6-13 Strain and Equivalent Stress Distribution of Doubly Reinforced Rectangular Section

The nominal moment strength is:

$$M_n = (A_s - A'_s) f_y (d - \frac{a}{2}) + A'_s f_y (d - d') \quad (7)$$

Note that A'_s yields when the following (for Grade 60 reinforcement, with $\varepsilon_y = 0.00207$) is satisfied:

$$d'/c \le 0.31$$

where $c = \dfrac{a}{\beta_1}$

ii. Compression reinforcement does not yield:

$$f'_s = \varepsilon'_s E_s = \varepsilon_u E_s \left(\frac{c - d'}{c} \right) < f_y \quad (8)$$

The neutral axis depth c can be determined from the following quadratic equation:

$$c^2 - \frac{(A_s f_y - 87A'_s)\, c}{0.85\beta_1 f'_c b} - \frac{87A'_s d'}{0.85\beta_1 f'_c b} = 0$$

where f'_c and f_y have the units of ksi. The nominal moment strength is:

$$M_n = 0.85 f'_c ab (d - \frac{a}{2}) + A'_s f'_s (d - d') \quad (9)$$

where

$$a = \beta_1 c$$

Alternatively, the contribution of compression reinforcement may be neglected and the moment strength calculated by Eq. (3), as for a rectangular section with tension reinforcement only.

d. For other cross-sections, the nominal moment strength M_n is calculated by a general analysis based on equilibrium and strain compatibility using the design assumptions of 10.2.

e. Nominal flexural strength M_n of a cross-section of a composite flexural member consisting of cast-in-place and precast concrete is computed in a manner similar as that for a regular reinforced concrete section. Since the "ultimate" strength is unrelated to the sequence of loading, no distinction is made between shored and unshored members in strength computations (see 17.2.4).

10.3.2 Balanced Strain Condition

A balanced strain condition exists at a cross-section when the maximum strain at the extreme compression fiber just reaches $\varepsilon_u = 0.003$ simultaneously with the first yield strain of $\varepsilon_s = \varepsilon_y = f_y / E_s$ in the tension reinforcement. This balanced strain condition is shown in Fig. 6-14.

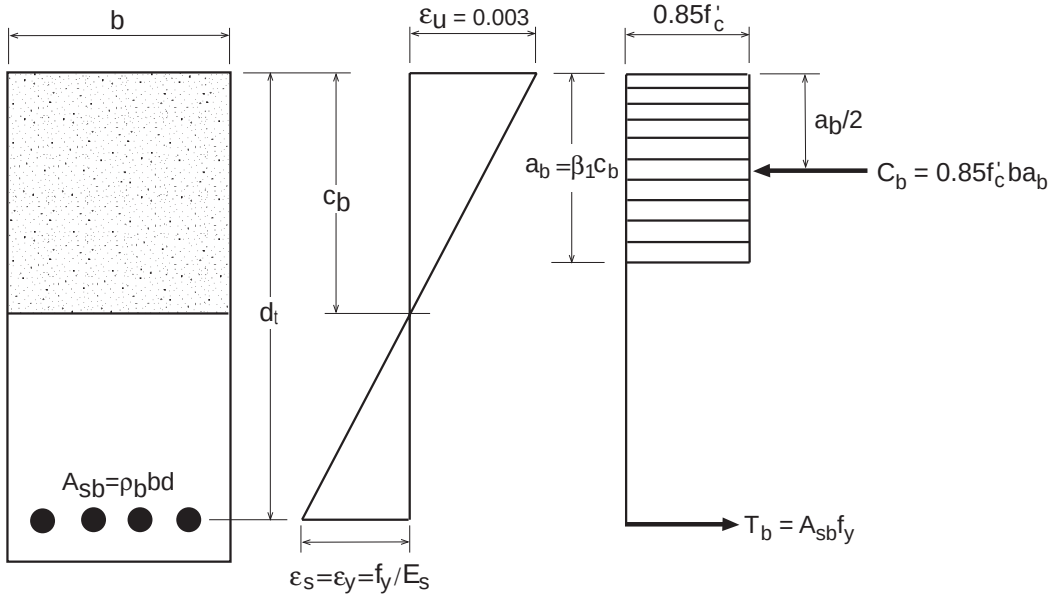


Figure 6-14 Balanced Strain Condition in Flexure

The ratio of neutral axis depth c_b to extreme depth d_t to produce a balanced strain condition in a section with tension reinforcement only may be obtained by applying strain compatibility conditions. Referring to Fig. 6-14, for the linear strain condition:

$$\frac{c_b}{d} = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$$

$$= \frac{0.003}{0.003 + f_y / 29{,}000{,}000} = \frac{0.003}{0.003 + \varepsilon_y}$$

Note that for Grade 60 steel, 10.3.3 permits the steel strain ε_y to be rounded to 0.002. Substituting into the above equation, the ratio $c_b/d_t = 0.6$. This value applies to all sections with Grade 60 steel, not just to rectangular sections.

10.3.3 Compression-Controlled Sections

Sections are compression-controlled when the net tensile strain in the extreme tension steel is equal to or less than the compression-controlled strain limit at the time the concrete in compression reaches its assumed strain limit of 0.003. The compression-controlled strain limit is the net tensile strain in the reinforcement at balanced strain conditions. For Grade 60 reinforcement, and for all prestressed reinforcement, it shall be permitted to set the compression-controlled strain limit equal to 0.002.

Note that when other grades of reinforcement are used, the compression-controlled strain limit is not 0.002. This changes the compression-controlled strain limit, and that changes the "transition" equations for the strength reduction factor given in Fig. 9.2 in Part 5.

10.3.4 Tension-Controlled Sections and Transition

Sections are tension-controlled when the net tensile strain in the extreme tension steel is equal to or greater than 0.005 just as the concrete in compression reaches its assumed strain limit of 0.003. Sections with net tensile strain in the extreme tension steel between the compression-controlled strain limit and 0.005 constitute a transition region between compression-controlled and tension-controlled sections.

Figure 6-15 shows the stress and strain conditions at the limit for tension-controlled sections. This limit is important because it is the limit for the use of $\phi = 0.9$ (9.3.2.1). Critical parameters at this limit are given a subscript t. Referring to Fig. 6-15, by similar triangles:

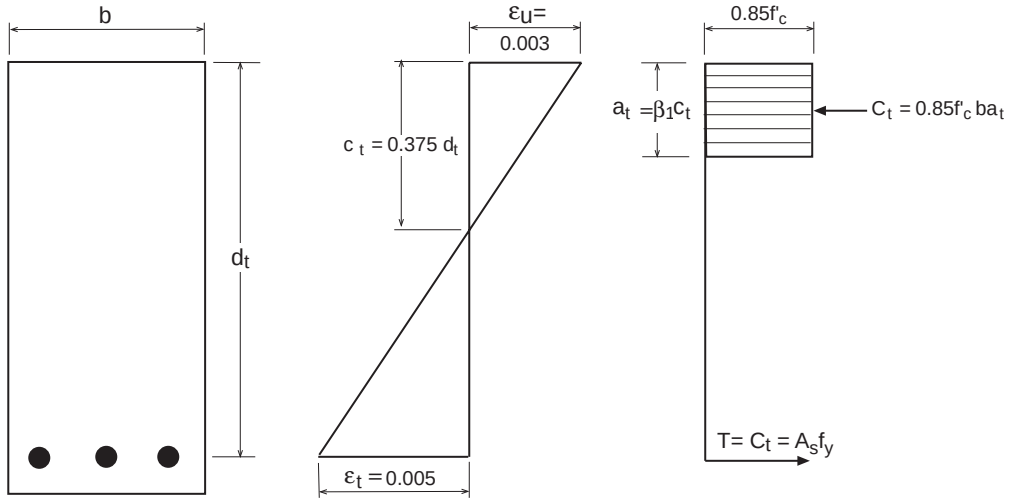


Figure 6-15 Strains at Tension-Controlled Limit

$$c_t = 0.375 d_t$$

$$a_t = \beta_1 c_t = 0.375 \beta_1 d_t$$

$$C_t = 0.85 f'_c b a_t = 0.319 \beta_1 f'_c b d_t$$

$$T = A_s f_y = C_t$$

$$A_s = 0.319 \beta_1 f'_c b d_t / f_y$$

$$\rho_t = A_s / (b d_t) = 0.319 \beta_1 f'_c / f_y \qquad (10)$$

$$\omega_t = \frac{\rho_t f_y}{f'_c} = 0.319 \beta_1 \qquad (11)$$

$$M_{nt} = \omega_t (1 - 0.59 \omega_t) f'_c b d_t^2 \qquad \text{from Eq. (4)}$$

$$R_{nt} = \frac{M_{nt}}{b d_t^2} = \omega_t (1 - 0.59 \omega_t) f'_c \qquad (12)$$

Values for ρ_t, ω_t, and R_{nt} are given in Table 6-1.

Table 6-1 Design Parameters at Strain Limit of 0.005 for Tension-Controlled Sections

			$f'_c = 3000$ $\beta_1 = 0.85$	$f'_c = 4000$ $\beta_1 = 0.85$	$f'_c = 5000$ $\beta_1 = 0.80$	$f'_c = 6000$ $\beta_1 = 0.75$	$f'_c = 8000$ $\beta_1 = 0.65$	$f'_c = 10{,}000$ $\beta_1 = 0.65$
R_{nt}			683	911	1084	1233	1455	1819
ϕR_{nt}			615	820	975	1109	1310	1637
ω_t			0.2709	0.2709	0.2550	0.2391	0.2072	0.2072
ρ_t	Grade	40	0.02032	0.02709	0.03187	0.03586	0.04144	0.05180
		60	0.01355	0.01806	0.02125	0.02391	0.02762	0.03453
		75	0.01084	0.01445	0.01700	0.01912	0.02210	0.02762

10.3.5 Maximum Reinforcement for Flexural Members

The 2002 Code defines reinforcement limits in terms of net tensile strain, ε_t, instead of the balanced ratio ρ/ρ_b that was used formerly. For rectangular sections with one layer of Grade 60 steel, a simple relationship between ε_t and ρ/ρ_b exists (see Fig. 6-16):

$$c = \frac{0.003d}{\varepsilon_t + 0.003}$$

$$a = \beta_1 c = \frac{0.003\beta_1 d}{\varepsilon_t + 0.003}$$

At balanced:

$$a_b = \frac{0.003\beta_1 d}{(60/29{,}000) + 0.003} = 0.592\,\beta_1 d$$

$$\frac{\rho}{\rho_b} = \frac{a}{a_b} = \frac{0.00507}{\varepsilon_t + 0.003}$$

or,

$$\varepsilon_t = \frac{0.00507}{\rho/\rho_b} - 0.003$$

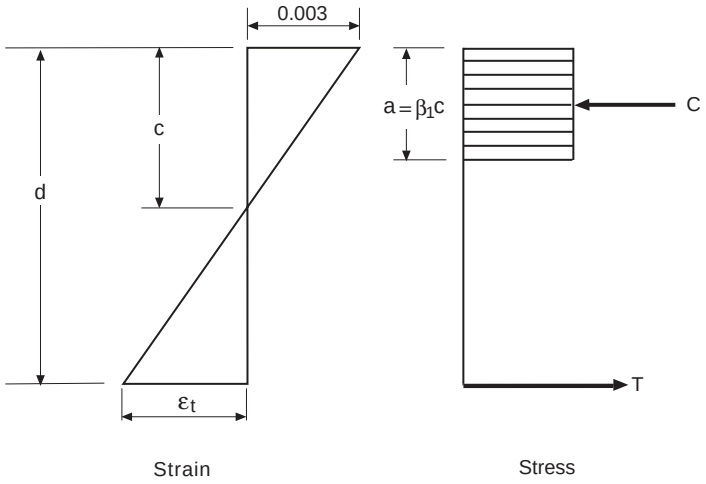


Figure 6-16 Strain and Stress Relationship

This relationship is shown graphically in Fig. 6-17.

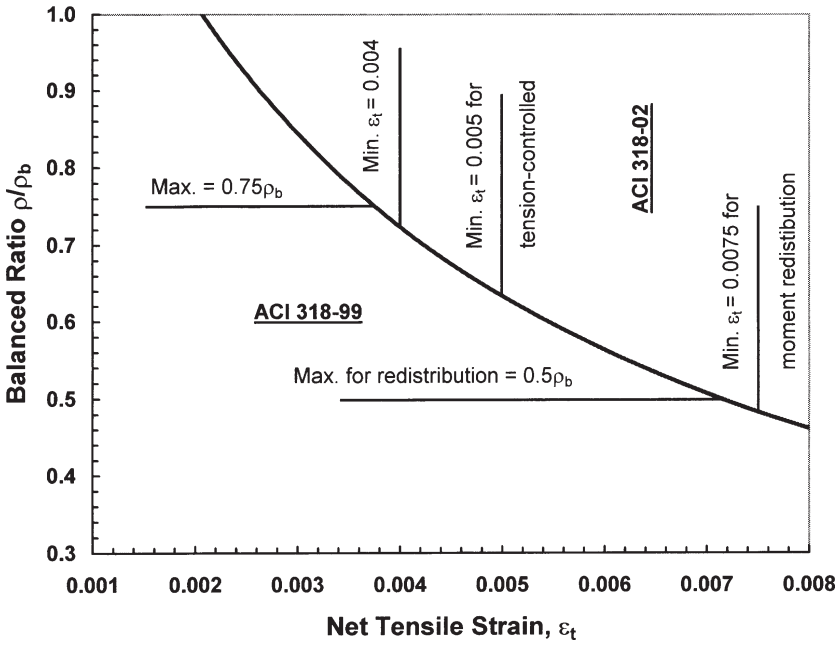


Figure 6-17 Relationship Between Balanced Ratio and Net Tensile Strain

The 2002 code limits the maximum reinforcement in a flexural member (with axial load less than $0.1\,f'_cA_g$) to that which would result in a net tensile strain ε_t at nominal strength not less than 0.004. This compares to the former code limit of $0.75\,\rho_b$, which results in an ε_t of 0.00376. Furthermore, at the net tensile strain limit of 0.004, the ϕ factor is reduced to 0.812. For heavily reinforced members, the overall safety factor (load factor/ϕ) is about the same as by 318-99, despite the reduced load factors. See Fig. 6-18.

The strength of tension-controlled sections is clearly controlled by steel strength, which is less variable than concrete strength. For tension-controlled flexural members, the 2002 code permits a ϕ of 0.9 to be used, despite the reduced load factors. As Fig. 6-18 shows, the new code reduces the strength requirement by about 10 percent for tension-controlled sections.

As discussed in Part 7 of this book, it is almost always advantageous to limit the net tensile strain in flexural members to a minimum of 0.005, even though the code permits higher amounts of reinforcement producing lower net tensile strains. Where member size is limited and extra strength is needed, it is best to use compression reinforcement to limit the net tensile strain so that the section is tension-controlled.

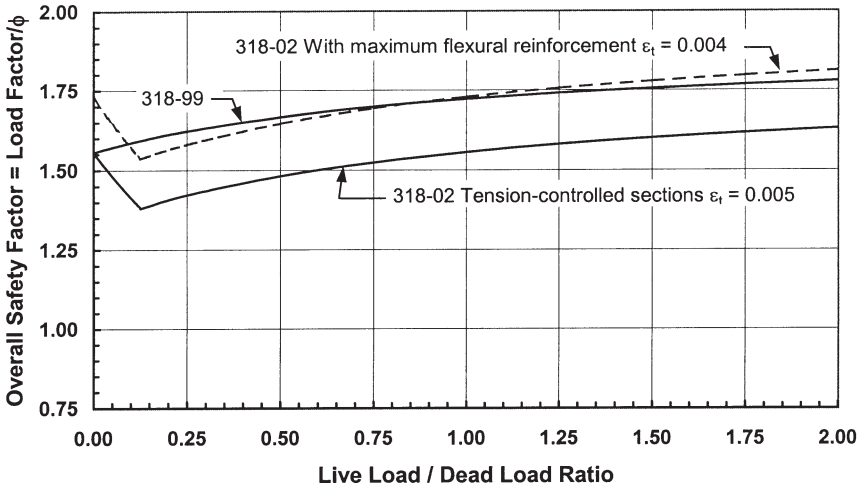


Figure 6-18 Overall Safety Factor for Flexural Members

10.3.6 Maximum Axial Load Strength

The strength of a member in pure compression (zero eccentricity) is computed by:

$$P_o = 0.85 f'_c A_g + f_y A_{st}$$

where A_{st} is the total area of reinforcement and A_g is the gross area of the concrete section. Refinement in concrete area can be considered by subtracting the area of concrete displaced by the steel:

$$P_o = 0.85 f'_c (A_g - A_{st}) + f_y A_{st} \quad (13)$$

Pure compression strength P_o represents a hypothetical loading condition. Prior to the 1977 ACI code, all compression members were required to be designed for a minimum eccentricity of 0.05h for spirally reinforced members or 0.10h for tied reinforced members (h = overall thickness of member). The specified minimum eccentricities were originally intended to serve as a means of reducing the axial design load strength of a section in pure compression and were included to, (1) account for accidental eccentricities, not considered in the analysis, that may exist in a compression member, and (2) recognize that concrete strength is less than f'_c at sustained high loads.

Since the primary purpose of the minimum eccentricity requirement was to limit the axial load strength for design of compression members with small or zero computed end moments, the 1977 code was revised to accomplish this directly by limiting the axial load strength to 85% and 80% of the axial load strength at zero eccentricity (P_o) for spiral and tied reinforcement columns, respectively.

For spirally reinforced members,

$$P_{n(max)} = 0.85 P_o = 0.85 [0.85 f'_c (A_g - A_{st}) + f_y A_{st}] \quad (14)$$

For tied reinforced members,

$$P_{n(max)} = 0.80 P_o = 0.80 [0.85 f'_c (A_g - A_{st}) + f_y A_{st}] \quad (15)$$

The maximum axial load strength, $P_{n(max)}$, is illustrated in Fig. 6-19. In essence, design within the cross-hatched portion of the load-moment interaction diagram is not permitted. The 85% and 80% values approximate the axial load strengths at e/h ratios of 0.05 and 0.10 specified in the 1971 code for spirally reinforced and tied reinforced members, respectively (see Example 6.3). The designer should note that R10.3.5 states that "Design aids and computer programs based on the minimum eccentricity requirement of the 1963 and 1971 ACI Building Codes may be considered equally applicable for usage."

The current provisions for maximum axial load strength also eliminate the concerns expressed by engineers about the excessively high minimum design moments required for large column sections, and the often asked question as to whether the minimum moments were required to be transferred to other interconnecting members (beams, footings, etc.).

Note that a minimum moment (minimum eccentricity requirement) for slender compression members in a braced frame is given in 10.12.3.2. If factored column moments are very small or zero, the design of these columns must be based on the minimum moment P_u (0.6 + 0.03h).

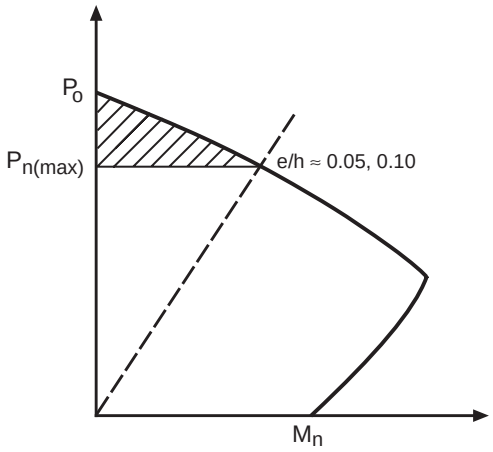
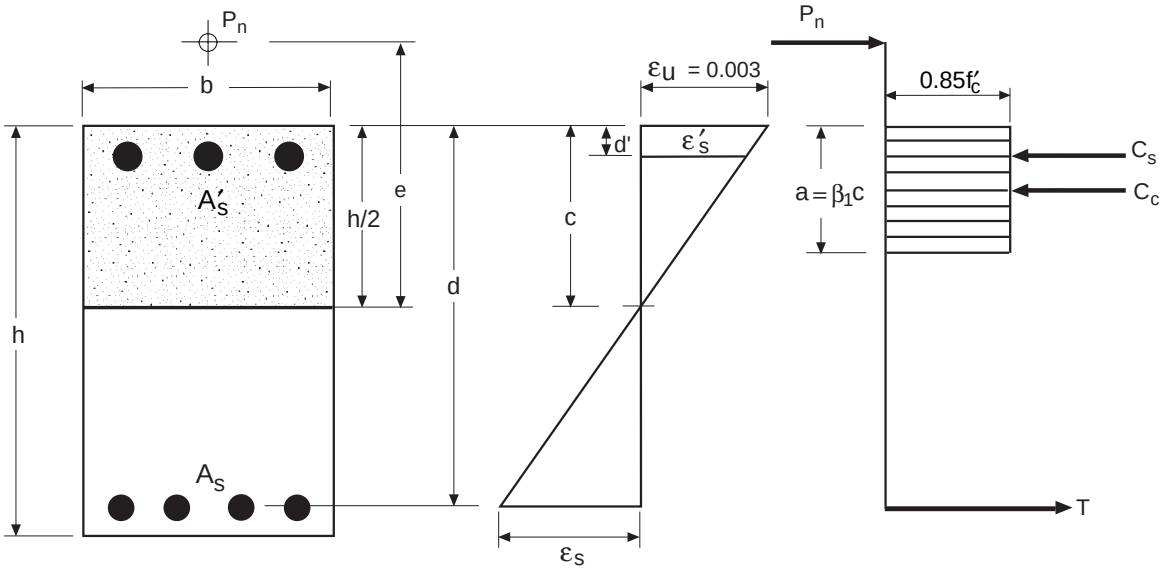


Figure 6-19 Maximum Axial Load Strength (10.3.6)

10.3.7 Nominal Strength for Combined Flexure and Axial Load

The strength of a member or cross-section subject to combined flexure and axial load, M_n and P_n, must satisfy the same two conditions as required for a member subject to flexure only, (1) static equilibrium and (2) compatibility of strains. Equilibrium between the compressive and tensile forces includes the axial load P_n acting on the cross-section. The general condition of the stress and the strain in concrete and steel at nominal strength of a member under combined flexure and axial compression is shown in Fig. 6-20. The tensile or compressive force developed in the reinforcement is determined from the strain condition at the location of the reinforcement.



$\varepsilon_s < \varepsilon_y$ (Compression Controls)

$\varepsilon_s = \varepsilon_y$ (Balanced Condition)

$\varepsilon_s > \varepsilon_y$ (Tension Controls)

Figure 6-20 Strain and Equivalent Stress Distribution for Section Subject to Combined Flexure and Axial Load

Referring to Fig. 6-20,

$$T = A_s f_s = A_s (E_s \varepsilon_s) \qquad \text{when } \varepsilon_s < \varepsilon_y$$

or $$T = A_s f_y \qquad \text{when } \varepsilon_s \geq \varepsilon_y$$

$$C_s = A'_s f'_s = A'_s (E_s \varepsilon'_s) \qquad \text{when } \varepsilon'_s < \varepsilon_y$$

or $$C_s = A'_s f_y \qquad \text{when } \varepsilon'_s \geq \varepsilon_y$$

$$C_c = 0.85 f'_c ba$$

The combined load-moment strength (P_n and M_n) may be computed by equilibrium of forces and moments.

From force equilibrium:

$$P_n = C_c + C_s - T \tag{16}$$

From moment equilibrium about the mid-depth of the section:

$$M_n = P_n e = C_c \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \left(\frac{h}{2} - d'\right) + T \left(d - \frac{h}{2}\right) \tag{17}$$

For a known strain condition, the corresponding load-moment strength, P_n and M_n, can be computed directly. Assume the strain in the extreme tension steel, A_s, is at first yield ($\varepsilon_s = \varepsilon_y$). This strain condition with simultaneous strain of 0.003 in the extreme compression fiber defines the "balanced" load-moment strength, P_b and M_b, for the cross-section.

For the linear strain condition:

$$\frac{c_b}{d} = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} = \frac{0.003}{0.003 + f_y / 29{,}000{,}000} = \frac{87{,}000}{87{,}000 + f_y}$$

so that $$a_b = \beta_1 c_b = \left(\frac{87{,}000}{87{,}000 + f_y}\right) \beta_1 d$$

Also $$\frac{c_b}{c_b - d'} = \frac{\varepsilon_u}{\varepsilon'_s}$$

so that $$\varepsilon'_s = 0.003 \left(1 - \frac{d'}{c_b}\right) = 0.003 \left[1 - \frac{d'}{d} \left(\frac{87{,}000 + f_y}{87{,}000}\right)\right]$$

and $$f'_{sb} = E_s \varepsilon'_s = 87{,}000 \left[1 - \frac{d'}{d} \left(\frac{87{,}000 + f_y}{87{,}000}\right)\right]$$ but not greater than f_y

From force equilibrium:

$$P_b = 0.85 f'_c b a_b + A'_s f'_{sb} - A_s f_y \tag{18}$$

From moment equilibrium:

$$M_b = P_b e_b = 0.85 f'_c b a_b \left(\frac{h}{2} - \frac{a}{2}\right) + A'_s f'_{sb} \left(\frac{h}{2} - d'\right) + A_s f_y \left(\frac{d}{2} - h\right) \quad (19)$$

The "balanced" load-moment strength defines only one of many load-moment combinations possible over the full range of the load-moment interaction relationship of a cross-section subject to combined flexure and axial load. The general form of a strength interaction diagram is shown in Fig. 6-21. The load-moment combination may be such that compression exists over most or all of the section, so that the compressive strain in the concrete reaches 0.003 before the tension steel yields ($\varepsilon_s < \varepsilon_y$) (compression-controlled segment); or the load combination may be such that tension exists over a large portion of the section, so that the strain in the tension steel is greater than the yield strain ($\varepsilon_s > \varepsilon_y$) when the compressive strain in the concrete reaches 0.003 (tension-controlled segment). The "balanced" strain condition ($\varepsilon_s = \varepsilon_y$) divides these two segments of the strength curve. The linear strain variation for the full range of the load-moment interaction relationship is illustrated in Fig. 6-21.

Under pure compression, the strain is uniform over the entire cross-section and equal to 0.003. With increasing load eccentricity (moment), the compressive strain at the "tension face" gradually decreases to zero, then becomes tensile, reaching the yield strain ($\varepsilon_s = \varepsilon_y$) at the balanced strain condition. For this range of strain variations, the strength of the section is controlled by compression (ε_s = -0.003 to ε_y). Beyond the balanced strain condition, the steel strain gradually increases up to the state of pure flexure corresponding to an infinite load eccentricity ($e = \infty$). For this range of strain variations, strength is controlled by tension ($\varepsilon_s > \varepsilon_y$). With increasing eccentricity, more and more tension exists over the cross-section. Each of the many possible strain conditions illustrated in Fig. 6-22 describes a point, P_n and M_n, on the load-moment curve (Fig. 6-21). Calculation of P_n and M_n for four different strain conditions along the load-moment strength curve is illustrated in Example 6.4.

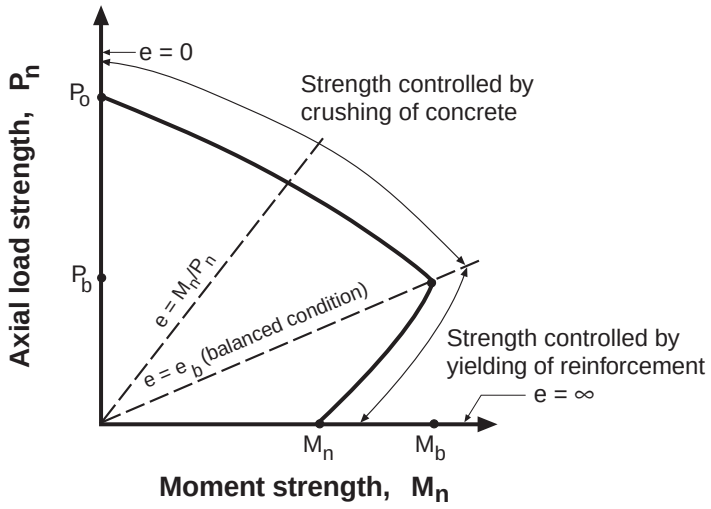


Figure 6-21 Axial Load-Moment Interaction Diagram

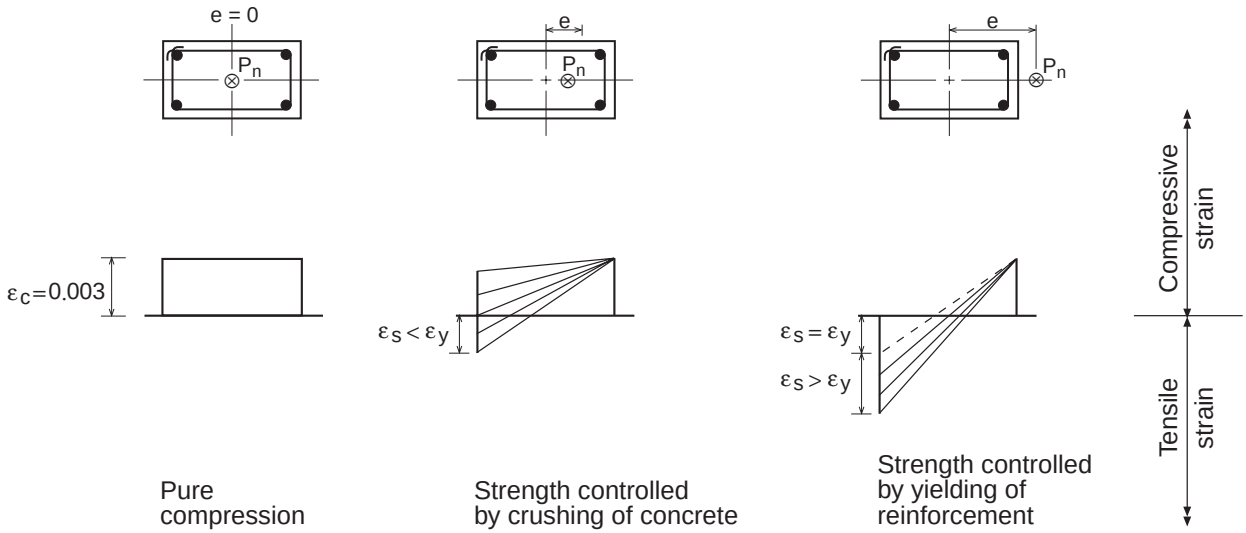


Figure 6-22 Strain Variation for Full Range of Load-Moment Interaction

10.5 MINIMUM REINFORCEMENT OF FLEXURAL MEMBERS

Members with cross-sections much larger than required for strength could fail suddenly because of small amounts of tensile reinforcement. The computed moment strength of such sections, assuming reinforced concrete behavior and using cracked section analyses, becomes less than that of a corresponding unreinforced concrete section computed from its modulus of rupture. To prevent failure in such situations, a minimum amount of tensile reinforcement is specified in 10.5.

The minimum reinforcement ratio $\rho_{min} = 200/f_y$ was originally derived to provide the same 0.5% minimum (for mild steel grade) as required in earlier versions of the ACI code. This minimum reinforcement is adequate for concrete strengths of about 4000 psi and less. The '95 version of the code recognizes that $\rho_{min} = 200/f_y$ may not be sufficient for f'_c greater than about 5000 psi. The code has accordingly revised 10.5.1 and 10.5.2 to specify the following minimum amounts of steel:

At every section of flexural members where tensile reinforcement is required,

$$A_{s,min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \geq \frac{200}{f_y} b_w d \qquad \textit{Eq. (10-3)}$$

Note that $3\sqrt{f'_c}$ and 200 are equal when $f'_c = 4444$ psi. Thus, $3\sqrt{f'_c}\, b_w d / f_y$ controls when $f'_c > 4444$ psi; otherwise, $200\, b_w d / f_y$ controls.

Equation (10-4) of the '99 code was removed and replaced with the following statement, which says the same thing as the former Eq. (10-4):

> 10.5.2 For statically determinate members with a flange in tension, the area $A_{s,min}$ shall be equal to or greater than the value given by Eq. (10-3) with b_w replaced by either $2b_w$ or the width of the flange, whichever is smaller.

Subsequent equations in Chapter 10 were renumbered.

Note that the requirements of 10.5.1 and 10.5.2 need not be applied if at every section the area of tensile reinforcement provided is at least one-third greater than that required by analysis (see 10.5.3).

10.15 TRANSMISSION OF COLUMN LOADS THROUGH FLOOR SYSTEM

Section 10.15.3 was modified in the '02 code to read:

> For columns laterally supported on four sides by beams of approximately equal depth or by slabs, it shall be permitted to base strength of the column on an assumed concrete strength in the column joint equal to 75 percent of column concrete strength plus 35 percent of floor concrete strength. In the application of 10.15.3, the ratio of column concrete strength to slab concrete strength shall not be taken greater than 2.5 for design.

The last sentence was added. This effectively limits the assumed column strength to a maximum of 2.225 times the floor concrete strength.

10.17 BEARING STRENGTH ON CONCRETE

Code-defined bearing strength (P_{nb}) of concrete is expressed in terms of an average bearing stress of $0.85\,f'_c$ over a bearing area (loaded area) A_1. When the supporting concrete area is wider than the loaded area on all sides, the surrounding concrete acts to confine the loaded area, resulting in an increase in the bearing strength of the supporting concrete. With confining concrete, the bearing strength may be increased by the factor $\sqrt{A_2 / A_1}$, but not greater than 2, where $\sqrt{A_2 / A_1}$ is a measure of the confining effect of the surrounding concrete. Evaluation of the strength increase factor $\sqrt{A_2 / A_1}$ is illustrated in Fig. 6-23.

For the usual case of a supporting concrete area considerably greater than the loaded area $\left(\sqrt{A_2 / A_1} > 2\right)$, the permissible bearing stress is $2\,(0.85\,f'_c)$.

Referring to Fig. 6-24,

a. For the supported surface (column):

$$P_{nb} = 0.85\,f'_c A_1$$

where f'_c is the specified strength of the column concrete.

b. For supporting surface (footing):

$$P_{nb} = \sqrt{\frac{A_2}{A_1}}\,(0.85 f'_c A_1), \quad \sqrt{\frac{A_2}{A_1}} \le 2.0$$

where f'_c is the specified strength of the footing concrete.

The design bearing strength is ϕP_{nb}, where, for bearing on concrete, $\phi = 0.65$. When the bearing strength is exceeded, reinforcement must be provided to transfer the excess load.

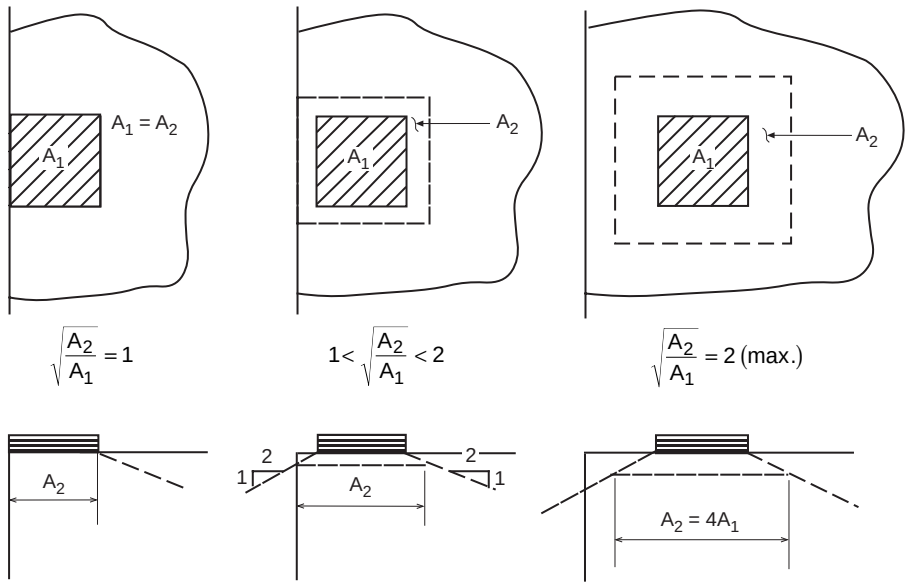


Figure 6-23 Measure of Confinement $\sqrt{A_2/A_1} \leq 2$ Provided by Surrounding Concrete

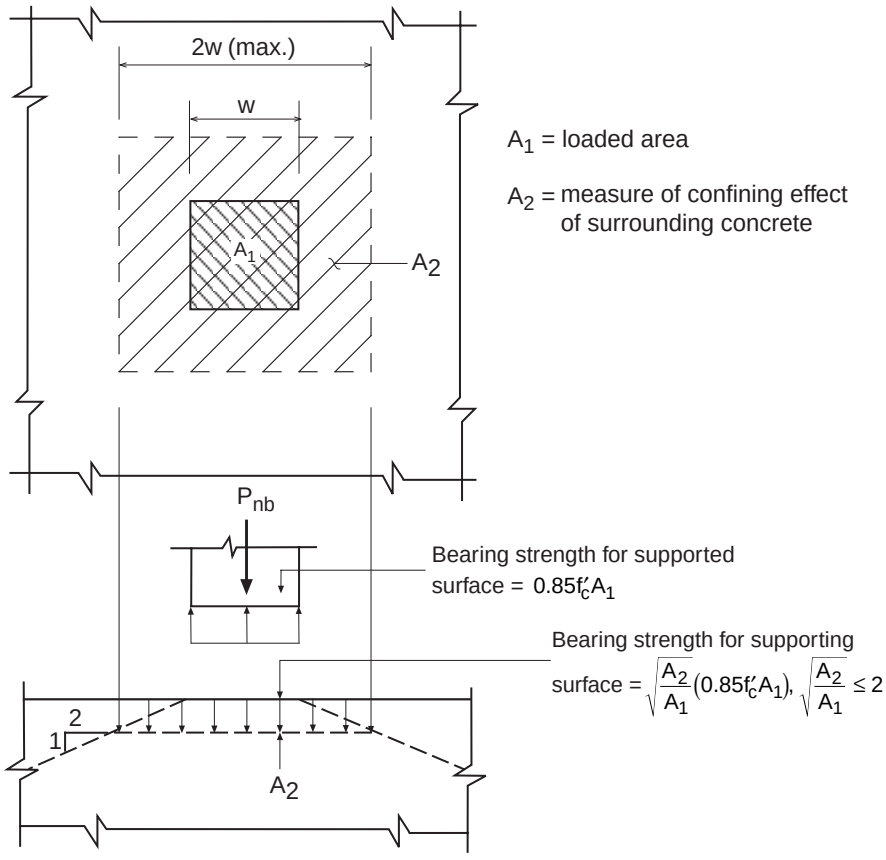


Figure 6-24 Nominal Bearing Strength of Concrete (10.15)

REFERENCES

6.1 Hognestad, E., Hanson, N.W., and McHenry, D., "Concrete Stress Distribution in Ultimate Strength Design," *ACI Journal, Proceedings* Vol. 52, December 1955, pp. 455-479; also *PCA Development Department Bulletin D6*.

6.2 Hognestad, E., "Ultimate Strength of Reinforced Concrete in American Design Practice," Proceedings of a Symposium on the Strength of Concrete Structures, London, England, May 1955; also *PCA Development Department Bulletin D12*.

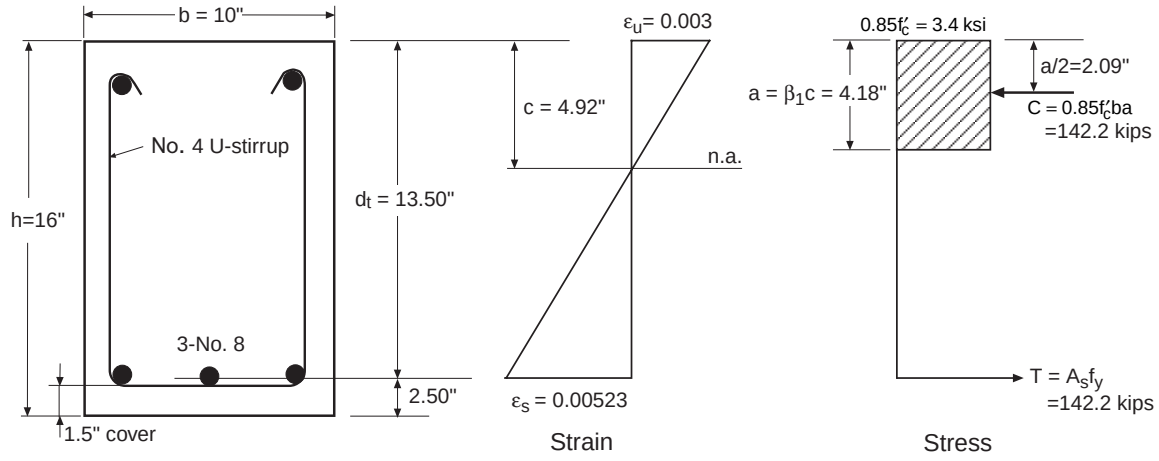
6.3 Hognestad, E., "Confirmation of Inelastic Stress Distribution in Concrete," *Journal of the Structural Division, Proceedings ASCE*, Vol. 83, No. ST2, March 1957; pp. 1189-1—1189-17 also *PCA Development Department Bulletin D15*.

6.4 Mattock, A.H., Kriz, L.B., and Hognestad, E., "Rectangular Concrete Stress Distribution in Ultimate Strength Design," *ACI Journal, Proceedings*, Vol. 57, February 1961, pp. 875-928; also *PCA Development Department Bulletin D49*.

6.5 Wang, C.K., and Salmon, C.G., *Reinforced Concrete Design*, Fourth Edition, Harper & Row Publishers, New York, N.Y. 1985.

Example 6.1—Moment Strength Using Equivalent Rectangular Stress Distribution

For the beam section shown, calculate moment strength based on static equilibrium using the equivalent rectangular stress distribution shown in Fig. 6-9. Assume $f'_c = 4000$ psi and $f_y = 60{,}000$ psi. For simplicity, neglect hanger bars.



Calculations and Discussion	Code Reference

1. Define rectangular concrete stress distribution. *10.2.7*

   $d = d_t = 16 - 2.5 = 13.50$ in. *10.0*

   $A_s = 3 \times 0.79 = 2.37$ in.^{2}

   Assuming $\varepsilon_s > \varepsilon_y$,

   $T = A_s f_y = 2.37 \times 60 = 142.2$ kips *10.2.4*

   $$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{142.2}{0.85 \times 4 \times 10} = 4.18 \text{ in.}$$

2. Determine net tensile strain ε_s and ϕ

   $$c = \frac{a}{\beta_1} = \frac{4.18}{0.85} = 4.92 \text{ in.}$$

   $$\varepsilon_s = \left(\frac{d_t - c}{c}\right)0.003 = \left(\frac{13.50 - 4.92}{4.92}\right)0.003 = 0.00523 > 0.005$$

   Therefore, section is tension-controlled *10.3.4*

   $\phi = 0.9$ *9.3.2.1*

   $\varepsilon_s = 0.00523 > 0.004$ which is minimum for flexural members *10.3.5*

   This also confirms that $\varepsilon_s > \varepsilon_y$ at nominal strength.

3. Determine nominal moment srength, M_n, and design moment strength, ϕM_n.

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) = 142.2\ (13.50 - 2.09) = 1{,}622.3 \text{ in.-kips} = 135.2 \text{ ft-kips}$$

$$\phi M_n = 0.9(135.2) = 121.7 \text{ ft-kips}$$ 9.3.2.1

4. Minimum reinforcement.

$$A_{s,min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \geq \frac{200 b_w d}{f_y}$$ Eq. (10-3)

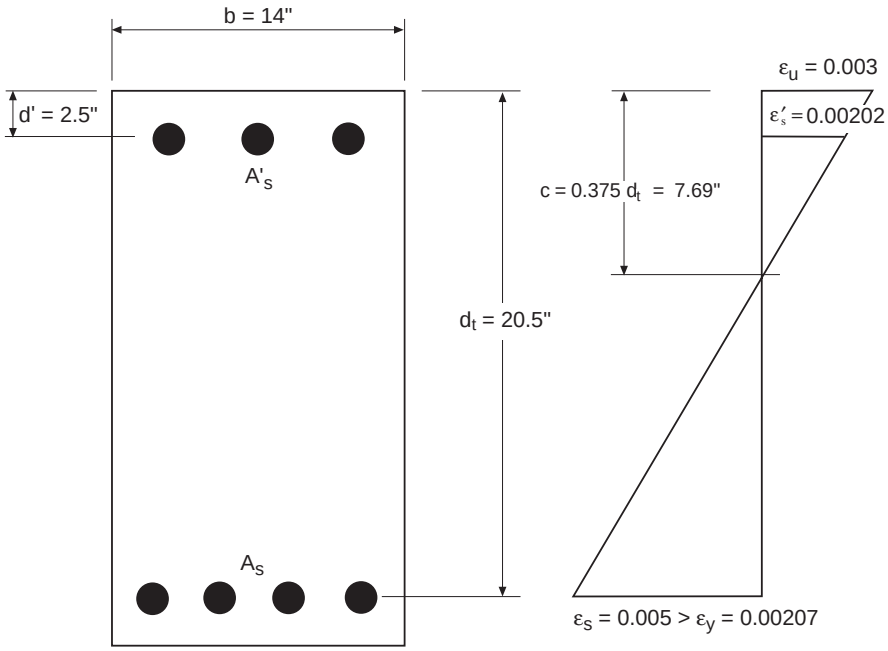
Since $f'_c < 4444$ psi, $200 b_w d / f_y$ governs:

$$\frac{200 b_w d}{f_y} = \frac{200 \times 10 \times 13.50}{60{,}000} = 0.45 \text{ in.}^2$$

A_s (provided) = 2.37 in.^{2} > $A_{s,min}$ = 0.45 in.^{2} O.K.

Example 6.2—Design of Beam with Compression Reinforcement

A beam cross-section is limited to the size shown. Determine the required area of reinforcement for a factored moment M_u = 516 ft-kips. f'_c = 4000 psi, f_y = 60,000 psi.



Calculations and Discussion	Code Reference

1. Check if compression reinforcement is required, using $\phi = 0.9$

$$M_n = M_u/\phi = 516/0.9 = 573 \text{ ft-kips}$$

$$R_n = \frac{M_n}{bd^2} = \frac{573\times12\times1000}{14\times20.5^2} = 1169$$

This exceeds the maximum R_{nt} of 911 for tension-controlled sections of 4000 psi concrete. (see Table 6-1.) Also, it appears likely that two layers of tension reinforcement will be necessary. But, for simplicity, assume that $d_t = d$.

2. Find the nominal strength moment M_{nt} resisted by the concrete section, without compression reinforcement, and M'_n to be resisted by the compression reinforcement.

$$M_{nt} = R_n bd^2 = 911\times14\times20.5^2/(1000\times12) = 447 \text{ ft-kips}$$

$$M'_n = M_n - M_{nt} = 573 - 447 = 126 \text{ ft-kips}$$

3. Determine the required compression steel

The strain in the compression steel at nominal strength is just below yield strain, as shown in the strain diagram above.

$$f'_s = E_s\varepsilon_s = 29{,}000{,}000\times0.00202 = 58{,}700 \text{ psi} = 58.7 \text{ ksi}$$

$$A'_s = \frac{M'_n}{f'_s(d-d')} = \frac{126 \times 12}{58.7\ (20.5-2.5)} = 1.43 \text{ in.}^2$$

4. Determine the required tension steel

$$A_s = \rho_t(bd) + A'_s\left(f'_s / f_y\right)$$

From Table 6-1, $\rho_t = 0.01806$, so that

$$A_s = 0.01806(14)(20.5) + 1.43(58.7/60)$$

$$= 5.18 + 1.40 = 6.58 \text{ in.}^2$$

5. Alternative solution

Required nominal strength

$$M_n = \frac{M_u}{\phi} = \frac{516}{0.9} = 573 \text{ ft-kips}$$

a. Determine maximum moment without compression reinforcement M_{nt}, using $\phi = 0.9$:

$c = 0.375 d_t = 0.375 \times 20.5 = 7.69$ in.

$a = \beta_1 c = 0.85 \times 7.69 = 6.54$ in.

$C = T = 3.4 \times 6.54 \times 14 = 311.3$ kips

$$M_{nt} = T\left(d_t - \frac{a}{2}\right) = 311.3\left(20.5 - \frac{6.54}{2}\right) = 5363.7 \text{ kip-in.} = 447.0 \text{ kip-ft}$$

b. Required area of tension steel to develop M_{nt}:

$$A_{s,nt} = \frac{311.3}{60} = 5.19 \text{ in.}^2$$

c. Additional moment (573-447 = 126 ft-kips) must be developed in T-C couple between tension steel and compression steel.

Additional tension steel required: $\Delta A_s = \dfrac{126 \times 12}{(20.5-2.5) \times 60} = 1.40 \text{ in.}^2$

Total tension steel required: $A_s = 5.19 + 1.40 = 6.59 \text{ in.}^2$

Compression steel required: $A'_s = \dfrac{126 \times 12}{(20.5-2.5) \times 58.7} = 1.43 \text{ in.}^2$

6. Comparison to Example 6.2 of *Notes on ACI 318-99* designed by ACI 318-99:

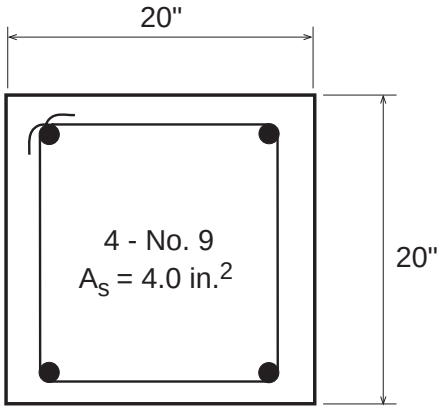
   Example 6.2 of *Notes on ACI 318-99* was designed by the 1999 code for an M_u of 580 ft-kips. By the 2002 code, assuming a live-to-dead load ratio of 0.5 for this beam, the beam could be designed as a tension-controlled section for an M_u of 516 ft-kips. The results for the required reinforcement are

	by 318-99	by 318-02
Compression reinforcement A'_s	1.49 in.^{2}	1.43 in.^{2}
Tension reinforcement A_s	7.63 in.^{2}	6.58 in.^{2}

   The reduction in tension reinforcement is a result of the lower load factors in the 2002 code. However, the compression reinforcement requirement is about the same. This is caused by the need for ductility in order to use the ϕ of 0.9 for flexure.

Example 6.3—Maximum Axial Load Strength vs. Minimum Eccentricity

For the tied reinforced concrete column section shown below, compare the nominal axial load strength P_n equal to $0.80P_o$ with P_n at 0.1h eccentricity. f'_c = 5000 psi, f_y = 60,000 psi.



Calculations and Discussion	Code Reference

Prior to ACI 318-77, columns were required to be designed for a minimum eccentricity of 0.1h (tied) or 0.05h (spiral). This required tedious computations to find the axial load strength at these minimum eccentricities. With the 1977 ACI code, the minimum eccentricity provision was replaced with a maximum axial load strength: $0.80P_o$ (tied) or $0.85P_o$ (spiral). The 80% and 85% values were chosen to approximate the axial load strengths at e/h ratios of 0.1 and 0.05, respectively.

1. In accordance with the minimum eccentricity criterion:

   At e/h = 0.10: P_n = 1543 kips (computer solution)

2. In accordance with maximum axial load strength criterion: *10.3.5.2*

   $P_{n(max)} = 0.80P_o = 0.80\,[0.85\,f'_c\,(A_g - A_{st}) + f_yA_{st}]$ *Eq. (10-2)*

   $= 0.80\,[0.85 \times 5\,(400 - 4.0) + (60 \times 4.0)] = 1538$ kips

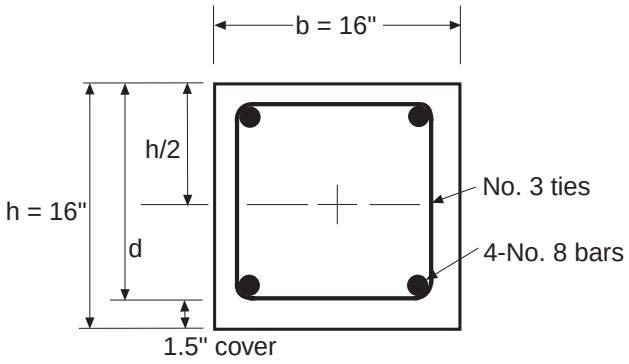
Depending on material strengths, size, and amount of reinforcement, the comparison will vary slightly. Both solutions are considered equally acceptable.

Example 6.4—Load-Moment Strength, P_n and M_n, for Given Strain Conditions

For the column section shown, calculate the load-moment strength, P_n and M_n, for four strain conditions:

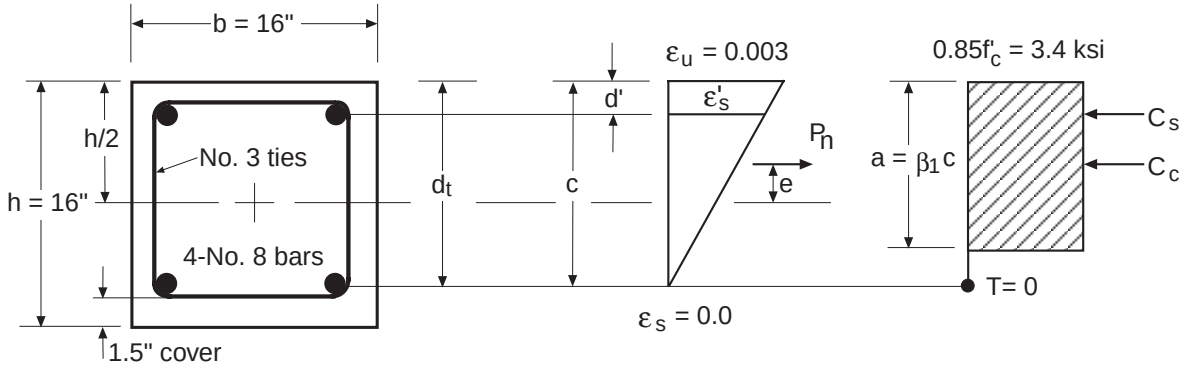
1. Bar stress near tension face of member equal to zero, $f_s = 0$
2. Bar stress near tension face of member equal to $0.5f_y$ ($f_s = 0.5f_y$)
3. At limit for compression-controlled section ($\varepsilon_t = 0.002$)
4. At limit for tension-controlled sections ($\varepsilon_t = 0.005$).

Use $f'_c = 4000$ psi, and $f_y = 60{,}000$ psi.



Calculations and Discussion	Code Reference

1. Load-moment strength, P_n and M_n, for strain condition 1:



$A_s = A'_s = 2(0.79) = 1.58$ in.^{2} Strain Condition - 1 Stress

a. Define stress distribution and determine force values. *10.2.7*

$d' = \text{Cover} + \text{No. 3 tie dia.} + \frac{d_b}{2} = 1.5 + 0.375 + 0.5 = 2.38$ in.

$d_t = 16 - 2.38 = 13.62$ in.

Since $\varepsilon_s = 0$, $c = d_t = 13.62$ in. *10.2.7.2*

$a = \beta_1 c = 0.85\,(13.62) = 11.58$ in. *10.2.7.1*

where $\beta_1 = 0.85$ for $f'_c = 4000$ psi *10.2.7.3*

$C_c = 0.85 f'_c ba = 0.85 \times 4 \times 16 \times 11.58 = 630.0$ kips *10.2.7*

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29{,}000} = 0.00207$$ *10.2.4*

From strain compatibility:

$$\varepsilon'_s = \varepsilon_u \left(\frac{c - d'}{c}\right) = 0.003 \left(\frac{13.62 - 2.38}{13.62}\right) = 0.00248 > \varepsilon_y = 0.00207$$ *10.2.2*

Compression steel has yielded.

$C_s = A'_s f_y = 1.58\,(60) = 94.8$ kips

b. Determine P_n and M_n from static equilibrium.

$P_n = C_c + C_s = 630.0 + 94.8 = 724.8$ kips *Eq. (16)*

$$M_n = P_n e = C_c \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \left(\frac{h}{2} - d'\right)$$ *Eq. (17)*

$= 630\,(8.0 - 5.79) + 94.8\,(8.0 - 2.38) = 1925.1$ in.-kips $= 160.4$ ft-kips

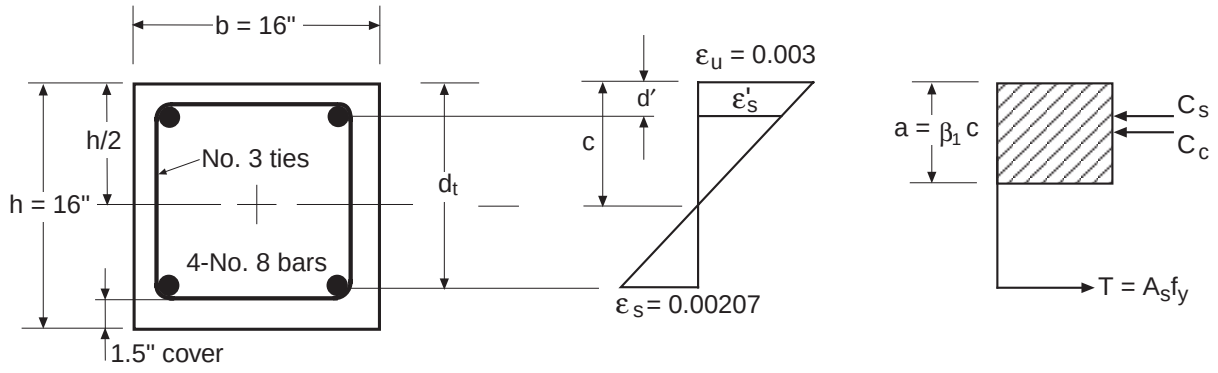
$$e = \frac{M_n}{P_n} = \frac{1925.1}{724.8} = 2.66 \text{ in.}$$

Therefore, for strain condition $\varepsilon_s = 0$:

Design axial load strength, $\phi P_n = 0.65\,(724.8) = 471.1$ kips *9.3.2.2*

Design moment strength, $\phi M_n = 0.65\,(160.4) = 104.3$ ft-kip

2. Load-moment strength, P_n and M_n, for strain condition 2:



Strain Condition - 3

a. Define stress distribution and determine force values. *10.2.7*

$d' = 2.38$ in., $d_t = 13.62$ in.

From strain compatibility:

$$\frac{c}{0.003} = \frac{d_t - c}{0.5\varepsilon_y}$$

$$c = \frac{0.003 d_t}{0.5\varepsilon_y + 0.003} = \frac{0.003 \times 13.62}{0.00104 + 0.003} = 10.13 \text{ in.}$$

Strain in compression reinforcement:

$$\varepsilon'_s = \varepsilon_u \left(\frac{c - d'}{c}\right) = 0.003 \left(\frac{10.13 - 2.38}{10.13}\right) = 0.00230 > \varepsilon_y = 0.00207$$

Compression steel has yielded.

$a = \beta_1 c = 0.85\,(10.13) = 8.61$ in. *10.2.7.1*

$C_c = 0.85 f'_c ba = 0.85 \times 4 \times 16 \times 8.61 = 468.4$ kips *10.2.7*

$C_s = A'_s f_y = 1.58\,(60) = 94.8$ kips

$T = A_s f_s = A_s\,(0.5 f_y) = 1.58\,(30) = 47.4$ kips

b. Determine P_n and M_n from static equilibrium. *Eq. (16)*

$$P_n = C_c + C_s - T = 468.4 + 94.8 - 47.4 = 515.8 \text{ kips}$$ *Eq. (17)*

$$M_n = P_n e = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_s\left(\frac{h}{2} + d'\right) + T\left(d - \frac{h}{2}\right)$$

$$= 468.4\,(8.0 - 4.31) + 94.8\,(8.0 - 2.38) + 47.4\,(13.62 - 8.0)$$

$$= 2527.6 \text{ in.-kips} = 210.6 \text{ ft-kips}$$

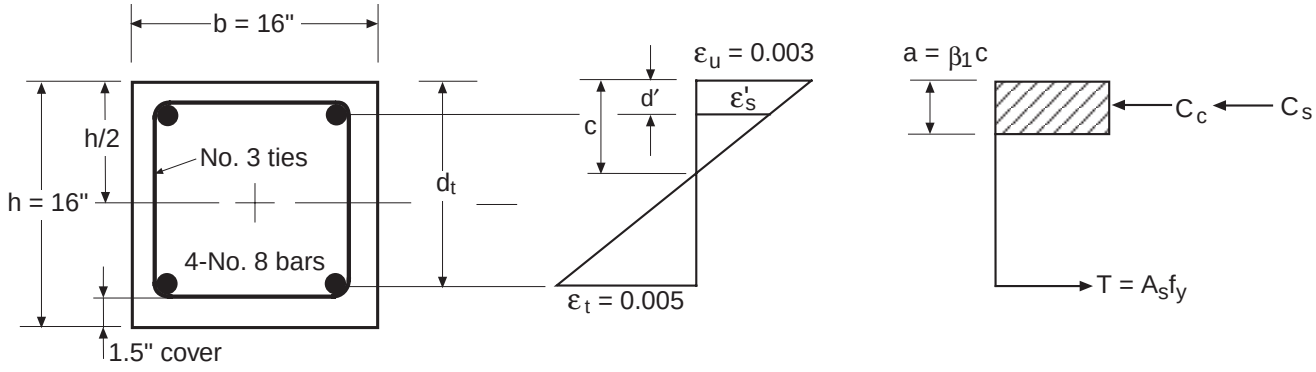
$$e = \frac{M_n}{P_n} = \frac{2527.6}{515.8} = 4.90 \text{ in.}$$

Therefore, for strain condition $\varepsilon_s = 0.5\,\varepsilon_y$:

Design axial load strength, $\phi P_n = 0.65\,(515.8) = 335.3$ kips *9.3.2.2*

Design moment strength, $\phi M_n = 0.65\,(210.6) = 136.9$ ft-kips

3. Load-moment strength, P_n and M_n, for strain condition 3:



Strain Condition - 4

a. Define stress distribution and determine force values. *10.2.7*

$d' = 2.38$ in., $d = 13.62$ in.

From strain compatibility:

$$\frac{c}{0.003} = \frac{d_t - c}{\varepsilon_y}$$

$$c = \frac{0.003 d_t}{\varepsilon_y + 0.003} = \frac{0.003 \times 13.62}{0.00207 + 0.003} = 8.06 \text{ in.}$$

Note: The code permits the use of 0.002 as the strain limit for compression-controlled sections with Grade 60 steel. It is slightly conservative, and more consistent, to use the yeild strain of 0.00207.

Strain in compression reinforcement:

$$\varepsilon'_s = \varepsilon_u \left(\frac{c - d'}{c} \right) = 0.003 \left(\frac{8.06 - 2.38}{8.06} \right) = 0.00211 > \varepsilon_y = 0.00207$$

Compression steel has yielded.

$a = \beta_1 c = 0.85 (8.06) = 6.85$ in. *10.2.7.1*

$C_c = 0.85 f'_c ba = 0.85 \times 4 \times 16 \times 6.85 = 372.7$ kips *10.2.7*

$C_s = A'_s f_y = 1.58 (60) = 94.8$ kips

$T = A_s f_s = A_s f_y = 1.58 (60) = 94.8$ kips

b. Determine P_n and M_n from static equilibrium. *Eq. (16)*

$P_n = C_c + C_s - T = 372.7 + 94.8 - 94.8 = 372.7$ kips *Eq. (17)*

$$M_n = P_n e = C_c \left(\frac{h}{2} - \frac{a}{2} \right) + C_s \left(\frac{h}{2} + d' \right) + T \left(d - \frac{h}{2} \right)$$

$= 372.7 (8.0 - 3.43) + 94.8 (8.0 - 2.38) + 94.8 (13.62 - 8.0)$

$= 2770.5$ in.-kips $= 230.9$ ft-kips

$$e = \frac{M_n}{P_n} = \frac{2770.5}{372.7} = 7.43 \text{ in.}$$

Therefore, for strain condition $\varepsilon_s = \varepsilon_y$:

Design axial load strength, $\phi P_n = 0.65 (372.7) = 242.3$ kips *9.3.2.2*

Design moment strength, $\phi M_n = 0.65 (230.9) = 150.1$ ft-kips

b. Determine P_n and M_n from static equilibrium. *Eq. (16)*

$$P_n = C_c + C_s - T = 236.2 + 73.5 - 94.8 = 214.9 \text{ kips} \quad \textit{Eq. (17)}$$

$$M_n = P_n e = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_s\left(\frac{h}{2} + d'\right) + T\left(d - \frac{h}{2}\right)$$

$$= 236.2\,(8.0 - 2.32) + 73.5\,(8.0 - 2.38) + 94.8\,(13.62 - 8.0)$$

$$= 2287.4 \text{ in.-kips} = 190.6 \text{ ft-kips}$$

$$e = \frac{M_n}{P_n} = \frac{2287.4}{214.9} = 10.64 \text{ in.}$$

Therefore, for strain condition $\varepsilon_s = 0.005$:

Design axial load strength, $\phi P_n = 0.9\,(214.9) = 193.4$ kips *9.3.2.2*

Design moment strength, $\phi M_n = 0.9\,(190.6) = 171.5$ ft-kips

A complete interaction diagram for this column is shown in Fig. 6-25.

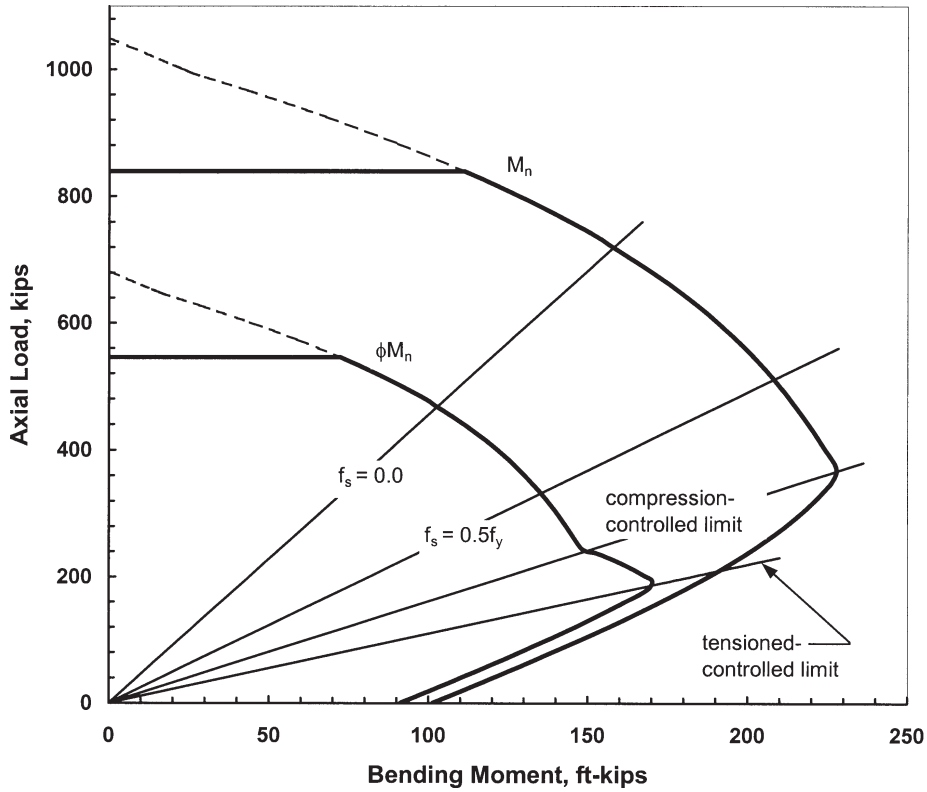


Figure 6-25 Interaction Diagram

7

Design for Flexure and Axial Load

GENERAL CONSIDERATIONS—FLEXURE

For design or investigation of members subjected to flexure (beams and slabs), the nominal strength of the member cross-section (M_n) must be reduced by the strength reduction factor ϕ to obtain the design strength (ϕM_n) of the section. The design strength (ϕM_n) must be equal to or greater than the required strength (M_u). In addition, the serviceability requirements for deflection control (9.5) and distribution of reinforcement for crack control (10.6) must also be satisfied.

Examples 7.1 through 7.7 illustrate proper application of the various code provisions that govern design of members subject to flexure. The design examples are prefaced by step-by-step procedures for design of rectangular sections with tension reinforcement only, rectangular sections with multiple layers of steel, rectangular sections with compression reinforcement, and flanged sections with tension reinforcement only.

DESIGN OF RECTANGULAR SECTIONS WITH TENSION REINFORCEMENT ONLY[7.1]

In the design of rectangular sections with tension reinforcement only (Fig. 7-1), the conditions of equilibrium are:

1. Force equilibrium:

$$C = T \tag{1}$$

$$0.85f'_c ba = A_s f_y = \rho b d f_y$$

$$a = \frac{A_s f_y}{0.85f'_c b} = \frac{\rho d f_y}{0.85f'_c}$$

2. Moment equilibrium:

$$M_n = (C \text{ or } T)\left(d - \frac{a}{2}\right)$$

$$M_n = \rho b d f_y \left[d - \frac{0.5\rho d}{0.85}\frac{f_y}{f'_c}\right] \tag{2}$$

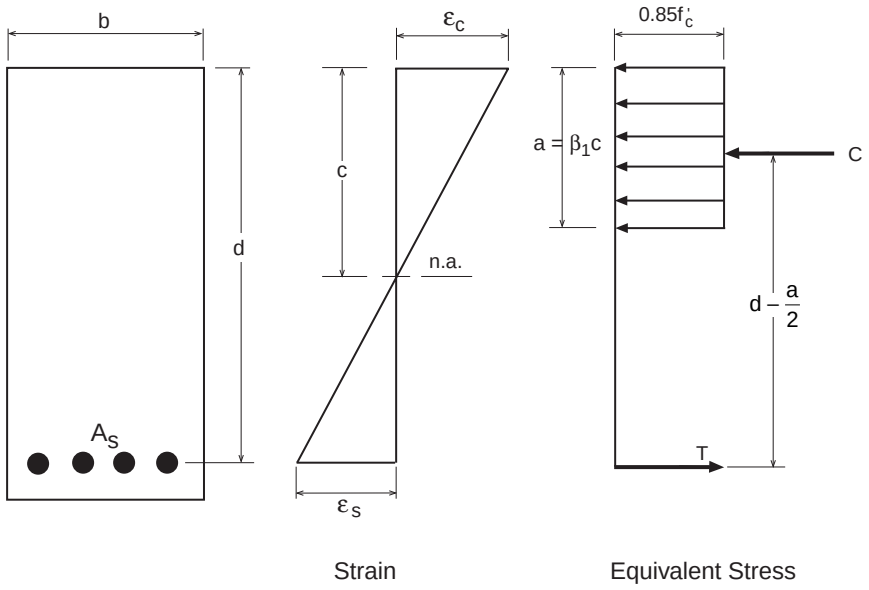


Figure 7-1 Strain and Equivalent Stress Distribution in Rectangular Section

A nominal strength coefficient of resistance R_n is obtained when both sides of Eq. (2) are divided by bd^2:

$$R_n = \frac{M_n}{bd^2} = \rho f_y \left(1 - \frac{0.5\rho f_y}{0.85f'_c}\right) \tag{3}$$

When b and d are preset, ρ is obtained by solving the quadratic equation for R_n:

$$\rho = \frac{0.85f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85f'_c}}\right) \tag{4}$$

The relationship between ρ and R_n for Grade 60 reinforcement and various values of f'_c is shown in Fig. 7-2.

Equation (3) can be used to determine the steel ratio ρ given M_u or vice-versa if the section properties b and d are known. Substituting $M_n = M_u/\phi$ into Eq. (3) and dividing each side by f'_c:

$$\frac{M_u}{\phi f'_c bd^2} = \frac{\rho f_y}{f'_c}\left(1 - \frac{0.5\rho f_y}{0.85f'_c}\right)$$

Define $\omega = \dfrac{\rho f_y}{f'_c}$

Substituting ω into the above equation:

$$\frac{M_u}{\phi f'_c bd^2} = \omega(1 - 0.59\omega) \tag{5}$$

Table 7-1, based on Eq. (5), has been developed in order to serve as a design aid for either design or investigation of sections having tension reinforcement only where b and d are known.

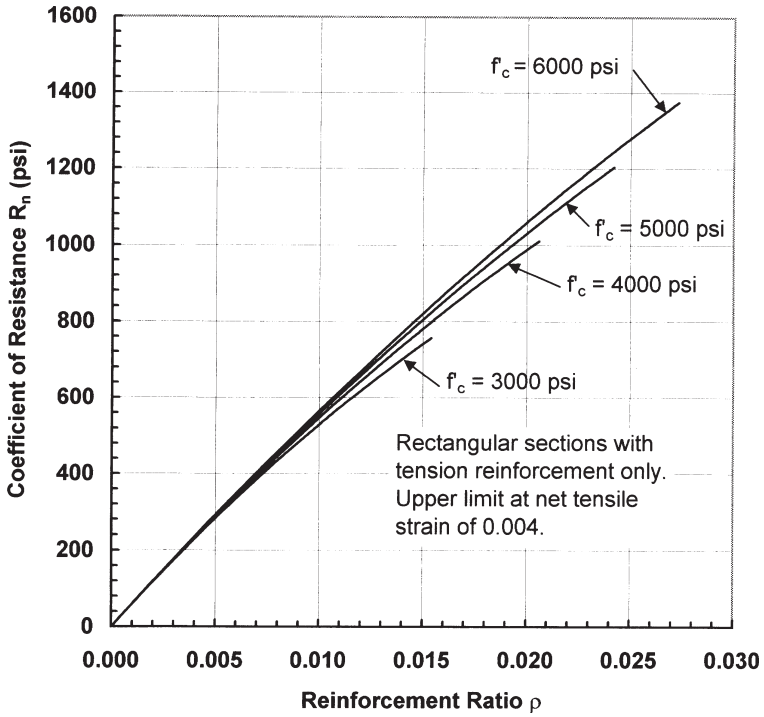


Figure 7-2 Strength Curves (R_n vs. ρ) for Grade 60 Reinforcement

Table 7-1 Flexural Strength $M_u / \phi f'_c bd^2$ or $M_n / f'_c bd^2$ of Rectangular Sections with Tension Reinforcement Only

ω	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
0.00	0	.0010	.0020	.0030	.0040	.0050	.0060	.0070	.0080	0090
0.01	.0099	.0109	.0119	.0129	.0139	.0149	.0159	.0168	.0178	.0188
0.02	.0197	.0207	.0217	.0226	.0236	.0246	.0256	.0266	.0275	.0285
0.03	.0295	.0304	.0314	.0324	.0333	.0343	.0352	.0362	.0372	.0381
0.04	.0391	.0400	.0410	.0420	.0429	.0438	.0448	.0457	.0467	.0476
0.05	.0485	.0495	.0504	.0513	.0523	.0532	.0541	.0551	.0560	.0569
0.06	.0579	.0588	.0597	.0607	.0616	.0626	.0634	.0643	.0653	.0662
0.07	.0671	.0680	.0689	.0699	.0708	.0717	.0726	.0735	.0744	.0753
0.08	.0762	.0771	.0780	.0789	.0798	.0807	.0816	.0825	.0834	.0843
0.09	.0852	.0861	.0870	.0879	.0888	.0897	.0906	.0915	.0923	.0932
0.10	.0941	.0950	.0959	.0967	.0976	.0985	.0994	.1002	.1001	.1020
0.11	.1029	.1037	.1046	.1055	.1063	.1072	.1081	.1089	.1098	.1106
0.12	.1115	.1124	.1133	.1141	.1149	.1158	.1166	.1175	.1183	.1192
0.13	.1200	.1209	.1217	.1226	.1234	.1243	.1251	.1259	.1268	.1276
0.14	.1284	.1293	.1301	.1309	.1318	.1326	.1334	.1342	.1351	.1359
0.15	.1367	.1375	.1384	.1392	.1400	.1408	.1416	.1425	.1433	.1441
0.16	.1449	.1457	.1465	.1473	.1481	.1489	.1497	.1506	.1514	.1522
0.17	.1529	.1537	.1545	.1553	.1561	.1569	.1577	.1585	.1593	.1601
0.18	.1609	.1617	.1624	.1632	.1640	.1648	.1656	.1664	.1671	.1679
0.19	.1687	.1695	.1703	.1710	.1718	.1726	.1733	.1741	.1749	.1756
0.20	.1764	.1772	.1779	.1787	.1794	.1802	.1810	.1817	.1825	.1832
0.21	.1840	.1847	.1855	.1862	.1870	.1877	.1885	.1892	.1900	.1907
0.22	.1914	.1922	.1929	.1937	.1944	.1951	.1959	.1966	.1973	.1981
0.23	.1988	.1995	.2002	.2010	.2017	.2024	.2031	.2039	.2046	.2053
0.24	.2060	.2067	.2075	.2082	.2089	.2096	.2103	.2110	.2117	.2124
0.25	.2131	.2138	.2145	.2152	.2159	.2166	.2173	.2180	.2187	.2194
0.26	.2201	.2208	.2215	.2222	.2229	.2236	.2243	.2249	.2256	.2263
0.27	.2270	.2277	.2284	.2290	.2297	.2304	.2311	.2317	.2324	.2331
0.28	.2337	.2344	.2351	.2357	.2364	.2371	.2377	.2384	.2391	.2397
0.29	.2404	.2410	.2417	.2423	.2430	.2437	.2443	.2450	.2456	.2463
0.30	.2469	.2475	.2482	.2488	.2495	.2501	.2508	.2514	.2520	.2527

$M_n / f'_c bd^2 = \omega(1 - 0.59\omega)$, *where* $\omega = \rho f_y / f'_c$

For design: Using factored moment M_u, enter table with $M_u / \phi f'_c bd^2$; find ω and compute steel percentage $\rho = \omega f'_c / f_y$

For investigation: Enter table with $\omega = \rho f_y / f'_c$; find value of $M_n / f'_c bd^2$ and solve for nominal strength, M_n.

Figure 7-3 shows the effect of the strength reduction factor ϕ. In particular, it shows what happens when the limit for tension-controlled sections with a ϕ of 0.9 is passed. As can be seen from Fig. 7-3, there is no benefit in designing a flexural member that is below the tension-controlled strain limit of 0.005. Any gain in strength with higher reinforcement ratios is canceled by the reduction in the strength reduction factor ϕ at higher reinforcement ratios. Therefore, flexural members should be designed as tension-controlled sections.

One might wonder "why even permit higher amounts of reinforcement and lower net tensile strains if there is no advantage?" In many cases, the provided steel is above the optimum at the limit for tension-controlled sections. The "flat" portion of the curve in Fig. 7-3 allows the designer to provide excess reinforcement above that required (considering discrete bar sizes) without being penalized for "being above a code limit."

Although flexural members should almost always be designed as tension-controlled sections with $\varepsilon_t \geq 0.005$, it often happens that columns with small axial load and large bending moments are in the "transition region" with ε_t between 0.002 and 0.005, and ϕ is somewhere between that for compression-controlled sections and that for tension-controlled sections.

Columns are normally designed using interaction charts or tables. The "breakpoint" for ε_t of 0.005 and $\phi = 0.9$ may fall above or below the zero axial load line on the interaction diagrams.

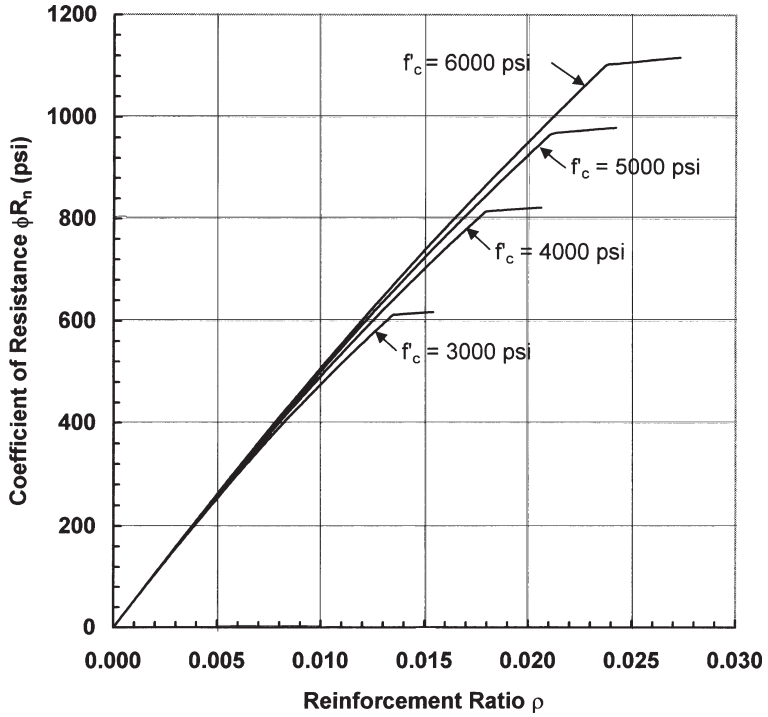


Figure 7-3 Design Strength Curves (ϕR_n vs. ρ) for Grade 60 Reinforcement

DESIGN PROCEDURE FOR SECTIONS WITH TENSION REINFORCEMENT ONLY

Step 1: Select an approximate value of tension reinforcement ratio ρ equal to or less than ρ_t, but greater than the minimum (10.5.1), where the reinforcement ratio ρ_t is given by:

$$\rho_t = \frac{0.319\beta_1 f'_c}{f_y}$$

where $\beta_1 = 0.85$ for $f'_c \leq 4000$ psi

$$= 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \text{ for } 4000 \text{ psi} < f'_c < 8000 \text{ psi}$$

$$= 0.65 \text{ for } f'_c \geq 8000 \text{ psi}$$

Values of ρ_t are given in Table 6-1.

Step 2: With ρ preset ($\rho_{min} \leq \rho \leq \rho_t$) compute bd^2 required:

$$bd^2 \text{ (required)} = \frac{M_u}{\phi R_n}$$

where $R_n = \rho f_y\left(1 - \frac{0.5\rho f_y}{0.85 f'_c}\right)$, $\phi = 0.90$ for flexure with $\rho \leq \rho_t$, and M_u = applied factored moment (required flexural strength)

Step 3: Size the member so that the value of bd^2 provided is greater than or equal to the value of bd^2 required.

Step 4: Based on the provided bd^2, compute a revised value of ρ by one of the following methods:

1. By Eq. (4) where $R_n = M_u/\phi bd^2$ (exact method)
2. By strength curves such as those shown in Fig. 7-2 and Fig. 7-3. Values of ρ are given in terms of $R_n = M_u/\phi bd^2$ for Grade 60 reinforcement.
3. By moment strength tables such as Table 7-1. Values of $\omega = \rho f_y/f'_c$ are given in terms of moment strength $M_u/\phi f'_c bd^2$.
4. By approximate proportion

$$\rho \approx (\text{original } \rho)\frac{(\text{revised } R_n)}{(\text{original } R_n)}$$

Note from Fig. 7-2 that the relationship between R_n and ρ is approximately linear.

Step 5: Compute required A_s:

A_s = (revised ρ) (bd provided)

When b and d are preset, the required A_s is computed directly from:

$A_s = \rho$ (bd provided)

where ρ is computed using one of the methods outlined in Step 4.

DESIGN PROCEDURE FOR SECTIONS WITH MULTIPLE LAYERS OF STEEL

The simple and conservative way to design a beam with two layers of tension steel is to take d_t equal to d, the depth to the centroid of all the tension steel. However, the code does permit the designer to take advantage of the fact that d_t, measured to the center of the layer farthest from the compression face, is greater than d. The only time this would be necessary is when designing at or very close to the strain limit of 0.005 for tension-controlled sections.

Figure 7-4 shows strain and stress diagrams for a section with multiple layers of steel with the extreme steel layer at the tension-controlled strain limit of 0.005. Let ρ_2 stand for the maximum ρ (based on d) for this section.

$$\rho_2 = \frac{C}{f_y b d}$$

However,

$$\rho_t = \frac{C}{f_y b d_t}$$

Therefore,

$$\frac{\rho_2}{\rho_t} = \frac{d_t}{d}$$

$$\rho_2 = \rho_t\left(\frac{d_t}{d}\right) \qquad (6)$$

Additional information can be found in the strain diagram of Fig. 7-4. The yield strain of Grade 60 reinforcement is 0.00207. By similar triangles, any Grade 60 steel that is within 0.366 d_t of the bottom layer will be at yield. This is almost always the case, unless steel is distributed on the side faces. Also, compression steel will be at yield if it is within $0.116d_t$ (or, 0.31c) of the compression face.

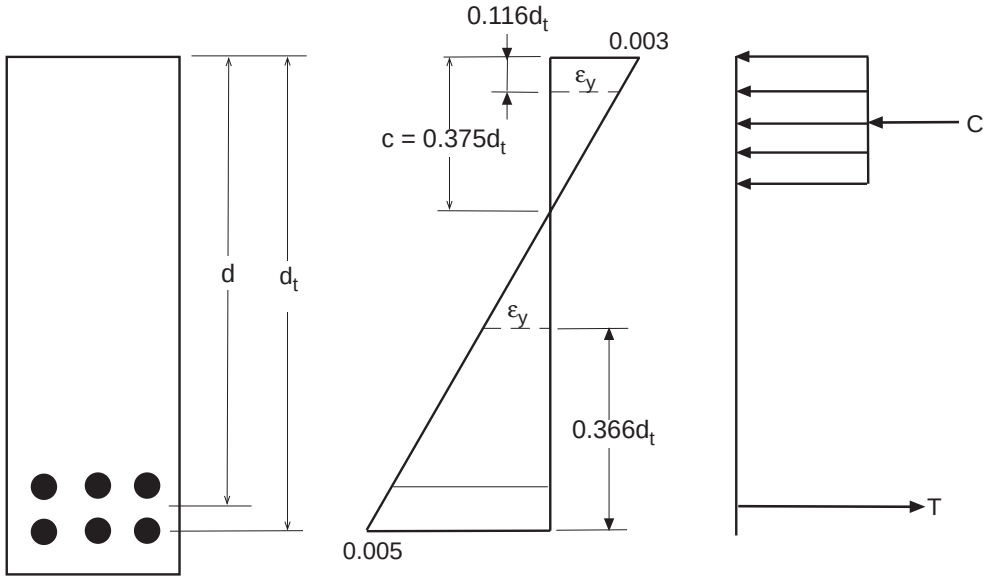


Figure 7-4 Multiple Layers of Reinforcement

DESIGN PROCEDURE FOR RECTANGULAR SECTIONS WITH COMPRESSION REINFORCEMENT (see Part 6)

Steps are summarized for the design of rectangular beams (with b and d preset) requiring compression reinforcement (see Example 7.3).

Step 1. Check to see if compression reinforcement is needed. Compute

$$R_n = \frac{M_n}{bd^2}$$

Compare this to the maximum R_n for tension-controlled sections given in Table 6-1. If R_n exceeds this, use compression reinforcement.

If compression reinforcement is needed, it is likely that two layers of tension reinforcement will be needed. Estimate d_t/d ratio.

Step 2. Find the nominal moment strength resisted by a section without compression reinforcement, and the additional moment strength M'_n to be resisted by the compression reinforcement and by added tension reinforcement.

From Table 6-1, find ρ_t. Then, using Eq. (6);

$$\rho = \rho_t\left(\frac{d_t}{d}\right)$$

$$\omega = \rho\frac{f_y}{f'_c}$$

Determine M_{nt} from Table 7-1.

Compute moment strength to be resisted by compression reinforcement:

$$M'_n = M_n - M_{nt}$$

Step 3. Check yielding of compression reinforcement

If $d'/c < 0.31$, compressive reinforcement has yielded and $f'_s = f_y$

See Part 6 to determine f'_s when the compression reinforcement does not yield.

Step 4. Determine the total required reinforcement, A'_s and A_s

$$A'_s = \frac{M'_n}{(d-d')f'_s}$$

$$A_s = \frac{M'_n}{(d-d')f_y} + \rho bd$$

Step 5: Check moment capacity

$$\phi M_n = \phi\left[\left(A - A_s'\right) f_y \left(d - \frac{a}{2}\right) + A_s' f_y \left(d - d'\right)\right] \geq M_u$$

where

$$a = \frac{\left(A_s - A_s'\right) f_y}{0.85 f_c' b}$$

DESIGN PROCEDURE FOR FLANGED SECTIONS WITH TENSION REINFORCEMENT (see Part 6)

Steps are summarized for the design of flanged sections with tension reinforcement only (see Examples 7.4 and 7.5).

Step 1: Determine effective flange width b according to 8.10.

Using Table 7-1, determine the depth of the equivalent stress block a, assuming rectangular section behavior with b equal to the flange width (i.e., $a \leq h_f$):

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{\rho d f_y}{0.85 f_c'} = 1.18 \omega d$$

where ω is obtained from Table 7-1 for $M_u / \phi f_c' b d^2$. Assume tension-controlled section with $\phi = 0.9$.

Step 2: If $a \leq h_f$, determine the reinforcement as for a rectangular section with tension reinforcement only. If $a > h_f$, go to step 3.

Step 3: If $a > h_f$, compute the required reinforcement A_{sf} and the moment strength ϕM_{nf} corresponding to the overhanging beam flange in compression:

$$A_{sf} = \frac{C_f}{f_y} = \frac{0.85 f_c' \left(b - b_w\right) h_f}{f_y}$$

$$\phi M_{nf} = \phi\left[A_{sf} f_y \left(d - \frac{h_f}{2}\right)\right]$$

Step 4: Compute the required moment strength to be carried by the beam web:

$$M_{uw} = M_u - \phi M_{nf}$$

Step 5: Using Table 7-1, compute the reinforcement A_{sw} required to develop the moment strength to be carried by the web:

$$A_{sw} = \frac{0.85 f_c' b_w a_w}{f_y}$$

where $a_w = 1.18\omega_w d$ with ω_w obtained from Table 7-1 for $M_{uw}/\phi f'_c b_w d^2$.

Alternatively, obtain A_{sw} from the following:

$$A_{sw} = \frac{\omega_w f'_c b_w d}{f_y}$$

Step 6: Determine the total required reinforcement:

$$A_s = A_{sf} + A_{sw}$$

Step 7: Check to see if section is tension-controlled, with $\phi = 0.9$.

$$c = a_w / \beta_1$$

If $c/d_t \leq 0.375$, section is tension-controlled
If $c/d_t > 0.375$, add compression reinforcement

Step 8: Check moment capacity:

$$\phi M_n = \phi\left[(A_s - A_{sf}) f_y \left(d - \frac{a_w}{2}\right) + A_{sf} f_y \left(d - \frac{h_f}{2}\right)\right] \geq M_u$$

where $A_{sf} = \dfrac{0.85 f'_c (b - b_w) h_f}{f_y}$

$$a_w = \frac{(A_s - A_{sf}) f_y}{0.85 f'_c b_w}$$

GENERAL CONSIDERATIONS—FLEXURE AND AXIAL LOAD

Design or investigation of a short compression member is based primarily on the strength of its cross-section. Strength of a cross-section under combined flexure and axial load must satisfy both force equilibrium and strain compatibility (see Part 6). The combined nominal axial load and moment strength (P_n, M_n) is then multiplied by the appropriate strength reduction factor ϕ to obtain the design strength (ϕP_n, ϕM_n) of the section. The design strength must be equal to or greater than the required strength:

$$(\phi P_n, \phi M_n) \geq (P_u, M_u)$$

All members subjected to combined flexure and axial load must be designed to satisfy this basic criterion. Note that the required strength (P_u, M_u) represents the structural effects of the various combinations of loads and forces to which a structure may be subjected; see Part 5 for discussion on 9.2.

A "strength interaction diagram" can be generated by plotting the design axial load strength ϕP_n against the corresponding design moment strength ϕM_n; this diagram defines the "usable" strength of a section at different eccentricities of the load. A typical design load-moment strength interaction diagram is shown in Fig. 7-5, illustrating the various segments of the strength curve permitted for design. The "flat-top" segment of the design strength curve defines the limiting axial load strength $P_{n(max)}$; see Part 5 for a discussion on 10.3.6. As

the design axial load strength ϕP_n decreases, a transition occurs between the compression-controlled limit and the tension-controlled limit, as shown in the figure. Example 6.4 illustrates the construction of an interaction diagram.

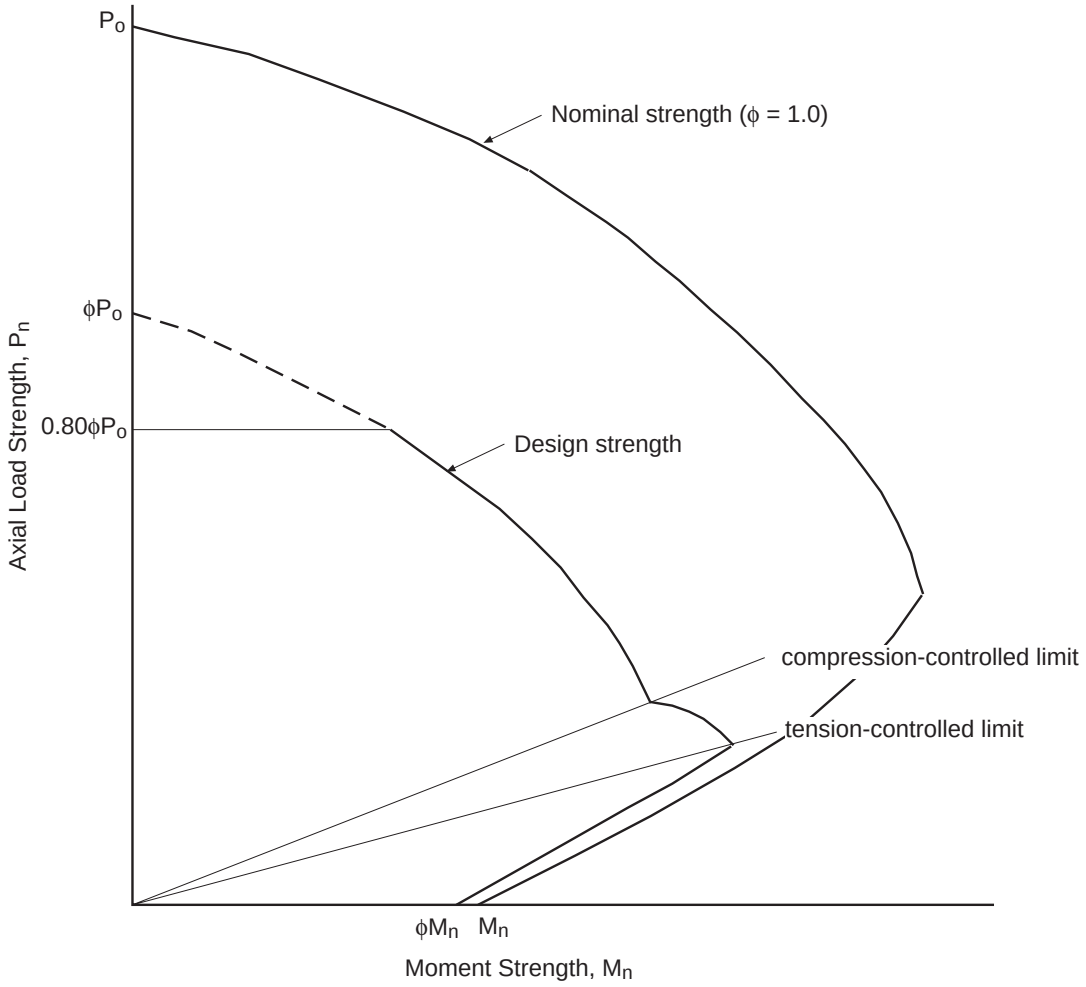


Figure 7-5 Design Load-Moment Strength Diagram (tied column)

GENERAL CONSIDERATIONS—BIAXIAL LOADING

Biaxial bending of columns occurs when the loading causes bending simultaneously about both principal axes. The commonly encountered case of such loading occurs in corner columns. Design for biaxial bending and axial load is mentioned in R10.3.6 and R10.3.7. Section 10.11.6 addresses moment magnifiers for slenderness consideration of compression members under biaxial loading. Section R10.3.6 states that "corner and other columns exposed to known moments about each axis simultaneously should be designed for biaxial bending and axial load." Two methods are recommended for combined biaxial bending and axial load design: the Reciprocal Load Method and the Load Contour Method. Both methods, and an extension of the Load Contour Method (PCA Load Contour Method), are presented below.

BIAXIAL INTERACTION STRENGTH

A uniaxial interaction diagram defines the load-moment strength along a single plane of a section under an axial load P and a uniaxial moment M. The biaxial bending resistance of an axially loaded column can be represented schematically as a surface formed by a series of uniaxial interaction curves drawn radially from the P axis (see

Fig. 7-6). Data for these intermediate curves are obtained by varying the angle of the neutral axis (for assumed strain configurations) with respect to the major axes (see Fig. 7-7).

The difficulty associated with the determination of the strength of reinforced columns subject to combined axial load and biaxial bending is primarily an arithmetic one. The bending resistance of an axially loaded column about a particular skewed axis is determined through iterations involving simple but lengthy calculations. These extensive calculations are compounded when optimization of the reinforcement or cross-section is sought.

For uniaxial bending, it is customary to utilize design aids in the form of interaction curves or tables. However, for biaxial bending, because of the voluminous nature of the data and the difficulty in multiple interpolations, the development of interaction curves or tables for the various ratios of bending moments about each axis is impractical. Instead, several approaches (based on acceptable approximations) have been developed that relate the response of a column in biaxial bending to its uniaxial resistance about each major axis.

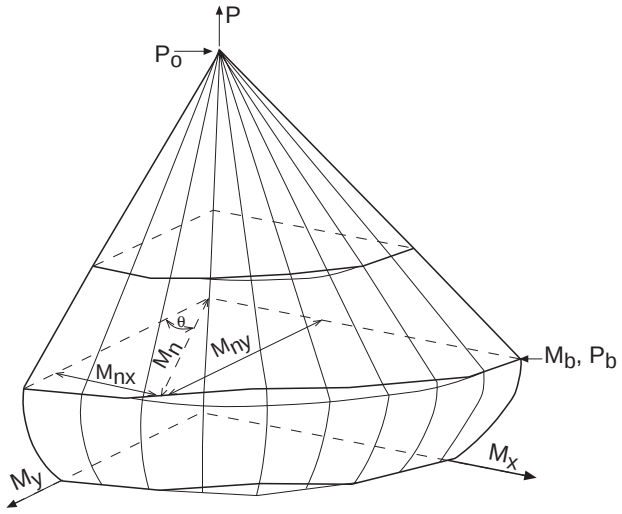


Figure 7-6 Biaxial Interaction Surface

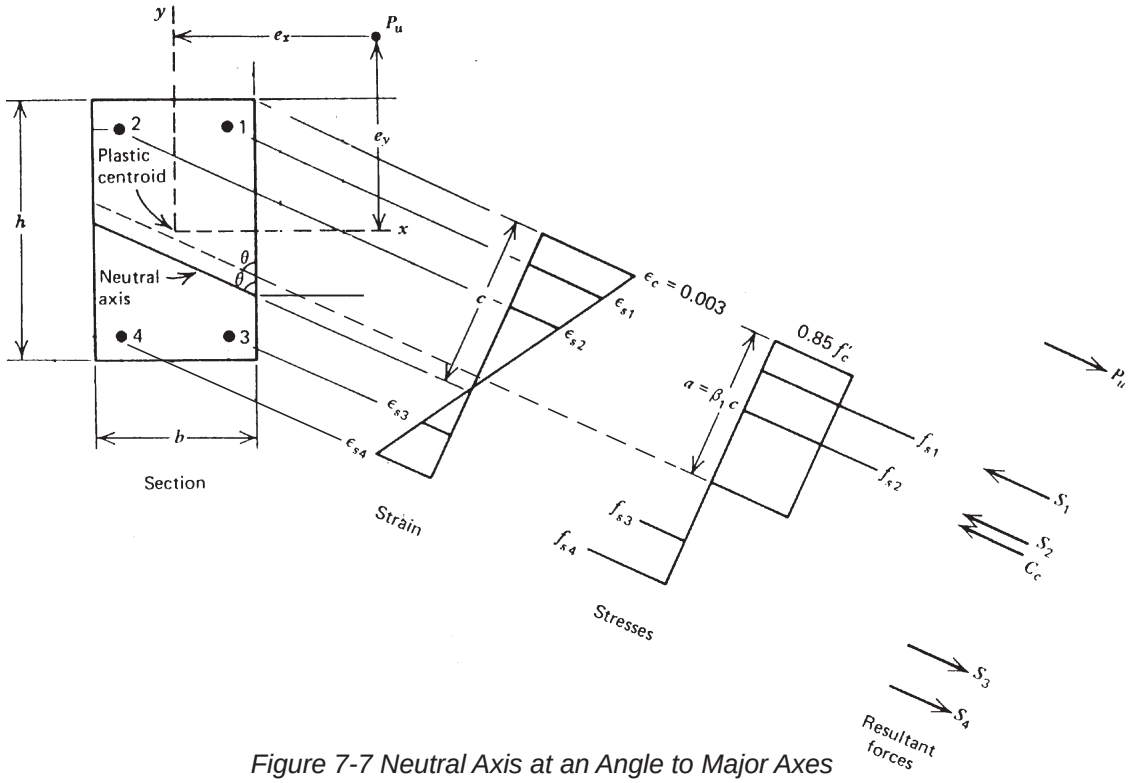
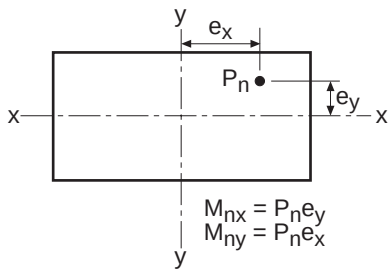


Figure 7-7 Neutral Axis at an Angle to Major Axes

FAILURE SURFACES

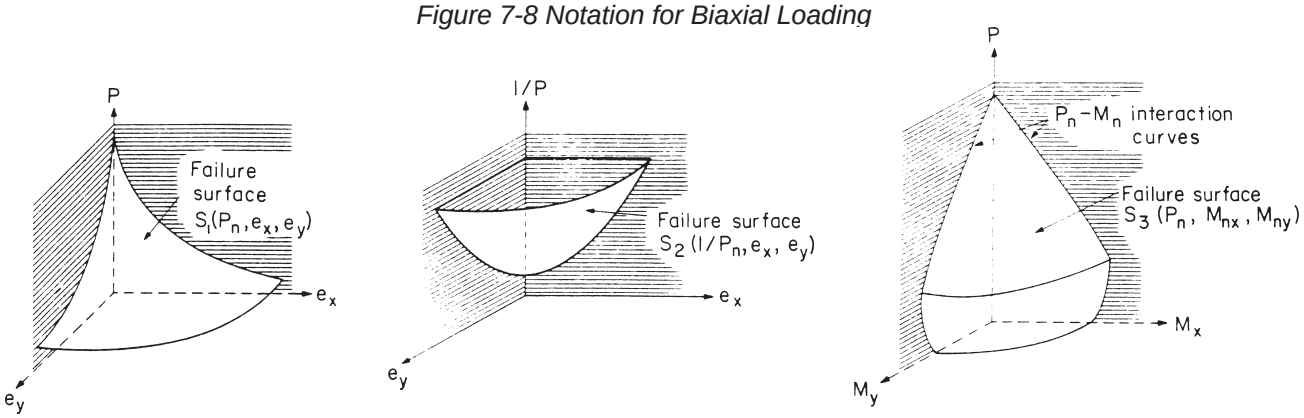
The nominal strength of a section under biaxial bending and compression is a function of three variables P_n , M_{nx} and M_{ny} which may be expressed in terms of an axial load acting at eccentricities $e_x = M_{ny}/P_n$ and $e_y = M_{nx}/P_n$ as shown in Fig. 7-8. A failure surface may be described as a surface produced by plotting the failure load P_n as a function of its eccentricities e_x and e_y, or of its associated bending moments M_{ny} and M_{nx}. Three types of failure surfaces have been defined.[7.4, 7.5, 7.6] The basic surface S_1 is defined by a function which is dependent upon the variables P_n, e_x and e_y, as shown in Fig. 7-9(a). A reciprocal surface can be derived from S_1 in which the reciprocal of the nominal axial load P_n is employed to produce the surface S_2 $(1/P_n, e_x, e_y)$ as illustrated in Fig. 7-9(b). The third type of failure surface, shown in Fig. 7-9(c), is obtained by relating the nominal axial load P_n to the moments M_{nx} and M_{ny} to produce surface S_3 (P_n, M_{nx}, M_{ny}). Failure surface S_3 is the three-dimensional extension of the uniaxial interaction diagram previously described.

A number of investigators have made approximations for both the S_2 and S_3 failure surfaces for use in design and analysis.[7.6 - 7.10] An explanation of these methods used in current practice, along with design examples, is given below.



Reinforcing bars not shown

Figure 7-8 Notation for Biaxial Loading



(a) Failure Surface S_1 (b) Reciprocal Failure Surface S_2 (c) Failure Surface S_3

Figure 7-9 Failure Surfaces

A. Bresler Reciprocal Load Method

This method approximates the ordinate $1/P_n$ on the surface S_2 $(1/P_n, e_x, e_y)$ by a corresponding ordinate $1/P'_n$ on the plane $S'_2(1/P'_n, e_x, e_y)$, which is defined by the characteristic points A, B and C, as indicated in Fig. 7-10. For any particular cross-section, the value P_o (corresponding to point C) is the load strength under pure axial compression; P_{ox} (corresponding to point B) and P_{oy} (corresponding to point A) are the load strengths under uniaxial eccentricities e_y and e_x, respectively. Each point on the true surface is approximated by a different plane; therefore, the entire surface is approximated using an infinite number of planes.

The general expression for axial load strength for any values of e_x and e_y is as follows:[7.6]

$$\frac{1}{P_n} \approx \frac{1}{P'_n} = \frac{1}{P_{ox}} + \frac{1}{P_{oy}} - \frac{1}{P_o}$$

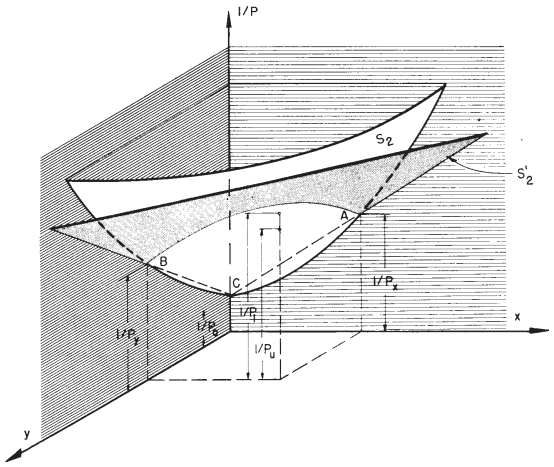


Figure 7-10 Reciprocal Load Method

Rearranging variables yields:

$$P_n \approx \frac{1}{\frac{1}{P_{ox}} + \frac{1}{P_{oy}} - \frac{1}{P_o}} \qquad (7)$$

where

P_{ox} = Maximum uniaxial load strength of the column with a moment of $M_{nx} = P_n e_y$

P_{oy} = Maximum uniaxial load strength of the column with a moment of $M_{ny} = P_n e_x$

P_o = Maximum axial load strength with no applied moments

This equation is simple in form and the variables are easily determined. Axial load strengths P_o , P_{ox} , and P_{oy} are determined using any of the methods presented above for uniaxial bending with axial load. Experimental results have shown the above equation to be reasonably accurate when flexure does not govern design. The equation should only be used when:

$$P_n \geq 0.1 f'_c A_g \qquad (8)$$

B. Bresler Load Contour Method

In this method, the surface S_3 (P_n , M_{nx}, M_{ny}) is approximated by a family of curves corresponding to constant values of P_n. These curves, as illustrated in Fig. 7-11, may be regarded as “load contours.”

The general expression for these curves can be approximated[7.6] by a nondimensional interaction equation of the form

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\alpha} + \left(\frac{M_{ny}}{M_{noy}}\right)^{\beta} = 1.0 \qquad (9)$$

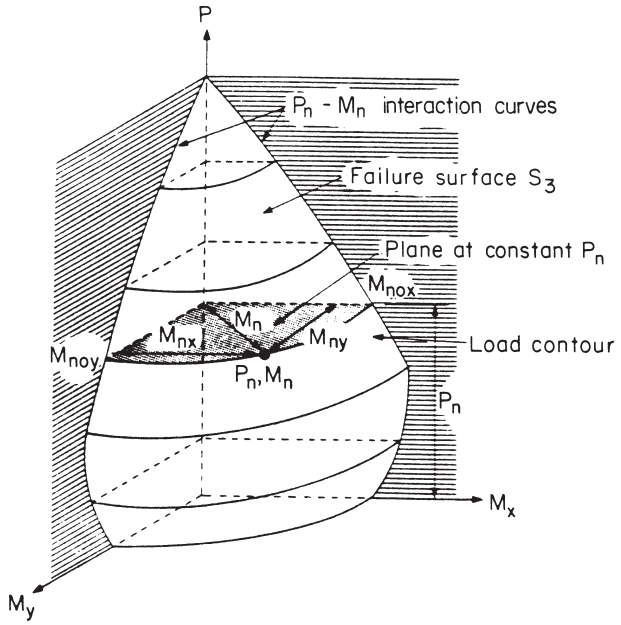


Figure 7-11 Bresler Load Contours for Constant P_n on Failure Surface S_3

where M_{nx} and M_{ny} are the nominal biaxial moment strengths in the direction of the x and y axes, respectively. Note that these moments are the vectorial equivalent of the nominal uniaxial moment M_n. The moment M_{nox} is the nominal uniaxial moment strength about the x-axis, and M_{noy} is the nominal uniaxial moment strength about the y-axis. The values of the exponents α and β are a function of the amount, distribution and location of reinforcement, the dimensions of the column, and the strength and elastic properties of the steel and concrete. Bresler[7.6] indicates that it is reasonably accurate to assume that $\alpha = \beta$; therefore, Eq. (9) becomes

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\alpha} + \left(\frac{M_{ny}}{M_{noy}}\right)^{\alpha} = 1.0 \qquad (10)$$

which is shown graphically in Fig. 7-12.

When using Eq. (10) or Fig. 7-12, it is still necessary to determine the α value for the cross-section being designed. Bresler indicated that, typically, α varied from 1.15 to 1.55, with a value of 1.5 being reasonably accurate for most square and rectangular sections having uniformly distributed reinforcement.

With α set at unity, the interaction equation becomes linear:

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}} = 1.0 \qquad (11)$$

Equation (11), as shown in Fig. 7-12, would always yield conservative results since it underestimates the column capacity, especially for high axial loads or low percentages of reinforcement. It should only be used when

$$P_n < 0.1 f'_c A_g \qquad (12)$$

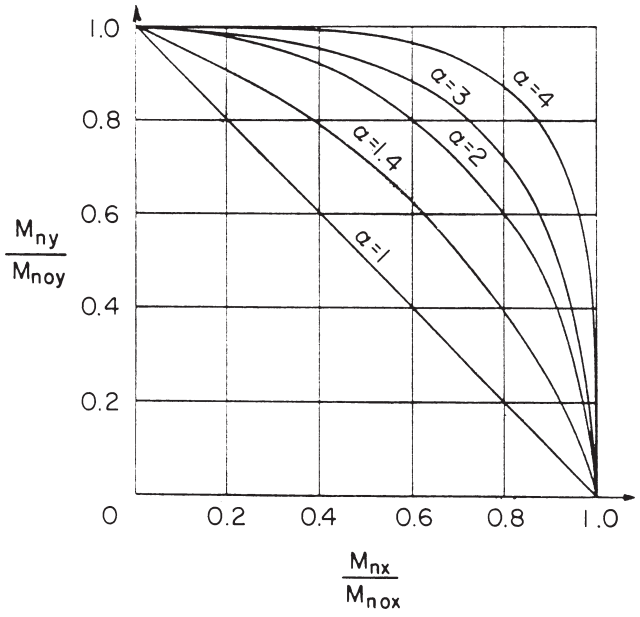


Figure 7-12 Interaction Curves for Bresler Load Contour Method (Eq.(9))

C. PCA Load Contour Method

The PCA approach described below was developed as an extension of the Bresler Load Contour Method. The Bresler interaction equation [Eq. (10)] was chosen as the most viable method in terms of accuracy, practicality, and simplification potential.

A typical Bresler load contour for a certain P_n is shown in Fig. 7-13(a). In the PCA method,[7.11] point B is defined such that the nominal biaxial moment strengths M_{nx} and M_{ny} at this point are in the same ratio as the uniaxial moment strengths M_{nox} and M_{noy}. Therefore, at point B

$$\frac{M_{nx}}{M_{ny}} = \frac{M_{nox}}{M_{noy}} \qquad (13)$$

When the load contour of Fig. 7-13(a) is nondimensionalized, it takes the form shown in Fig. 7-13(b), and the point B will have x and y coordinates of β. When the bending resistance is plotted in terms of the dimensionless parameters P_n/P_o, M_{nx}/M_{nox}, M_{ny}/M_{noy} (the latter two designated as the relative moments), the generated failure surface S_4 (P_n/P_o, M_{nx}/M_{nox}, M_{ny}/M_{noy}) assumes the typical shape shown in Fig. 7-13(c). The advantage of expressing the behavior in relative terms is that the contours of the surface (Fig. 7-13(b))—i.e., the intersection formed by planes of constant P_n/P_o and the surface—can be considered for design purposes to be symmetrical about the vertical plane bisecting the two coordinate planes. Even for sections that are rectangular or have unequal reinforcement on the two adjacent faces, this approximation yields values sufficiently accurate for design.

The relationship between α from Eq. (10) and β is obtained by substituting the coordinates of point B from Fig. 7-13(a) into Eq. (10), and solving for α in terms of β. This yields:

$$\alpha = \frac{\log 0.5}{\log \beta}$$

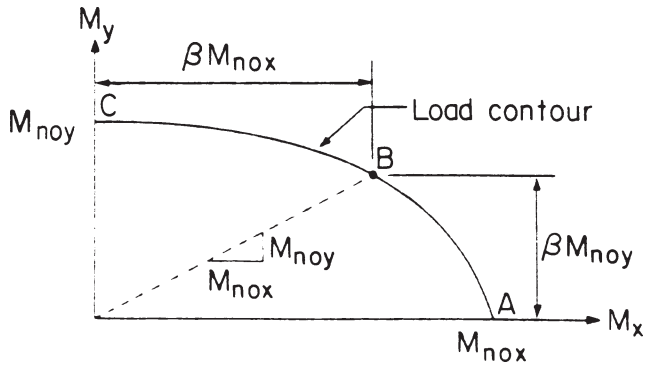


Figure 7-13(a) Load Contour of Failure Surface s_3 along Plane of Constant P_n

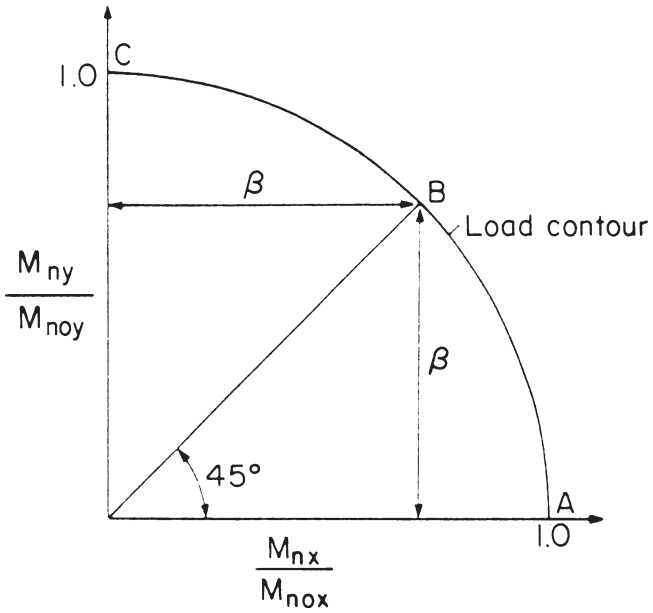


Figure 7-13(b) Nondimensional Load Contour at Constant P_n

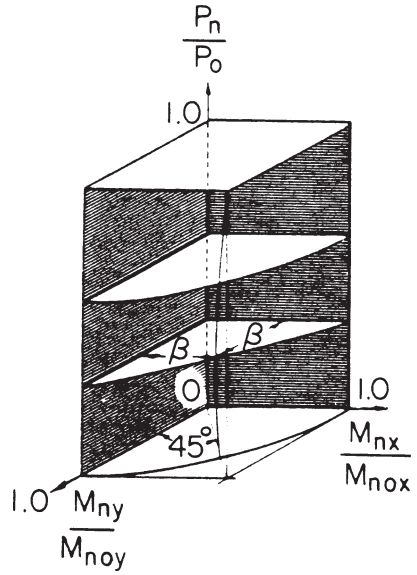


Figure 7-13(c) Failure Surface $S_4\left(\frac{P_n}{P_o}, \frac{M_{nx}}{M_{nox}}, \frac{M_{ny}}{M_{noy}}\right)$

Thus, Eq. (10) may be written as:

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\left(\frac{\log 0.5}{\log \beta}\right)} + \left(\frac{M_{ny}}{M_{noy}}\right)^{\left(\frac{\log 0.5}{\log \beta}\right)} = 1.0 \qquad (14)$$

For design convenience, a plot of the curves generated by Eq. (14) for nine values of β are given in Fig. 7-14. Note that when β = 0.5, its lower limit, Eq. (14) is a straight line joining the points at which the relative moments equal 1.0 along the coordinate planes. When β = 1.0, its upper limit, Eq. (14) is two lines, each of which is parallel to one of the coordinate planes.

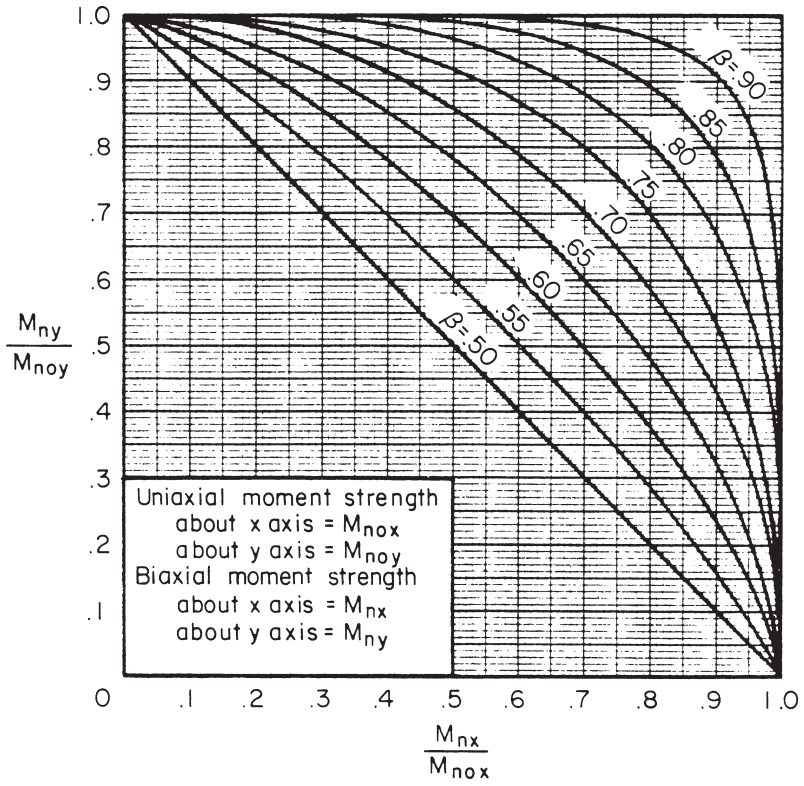
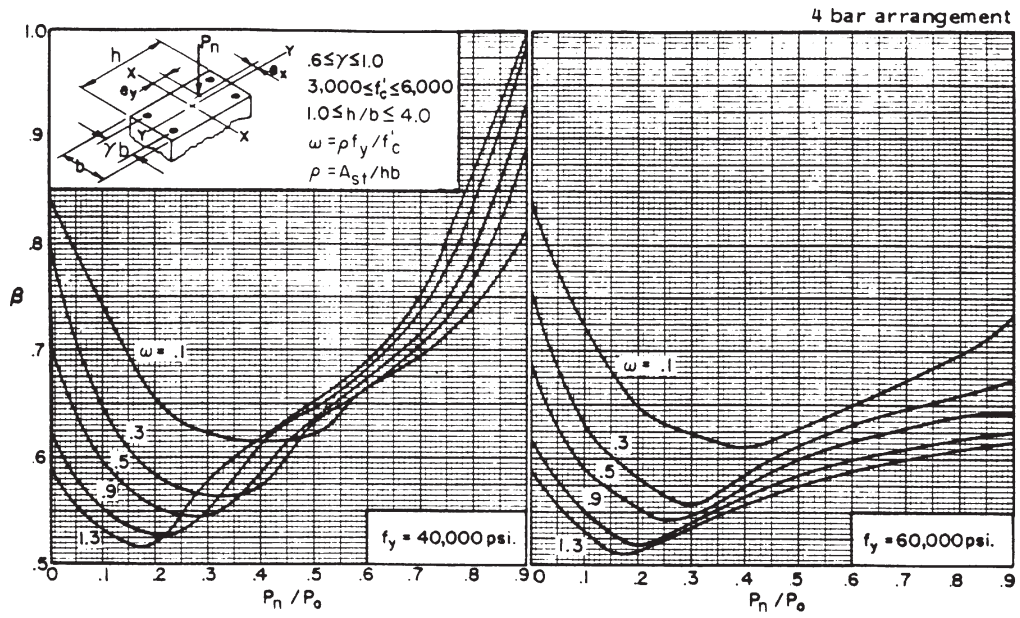


Figure 7-14 Biaxial Moment Strength Relationship

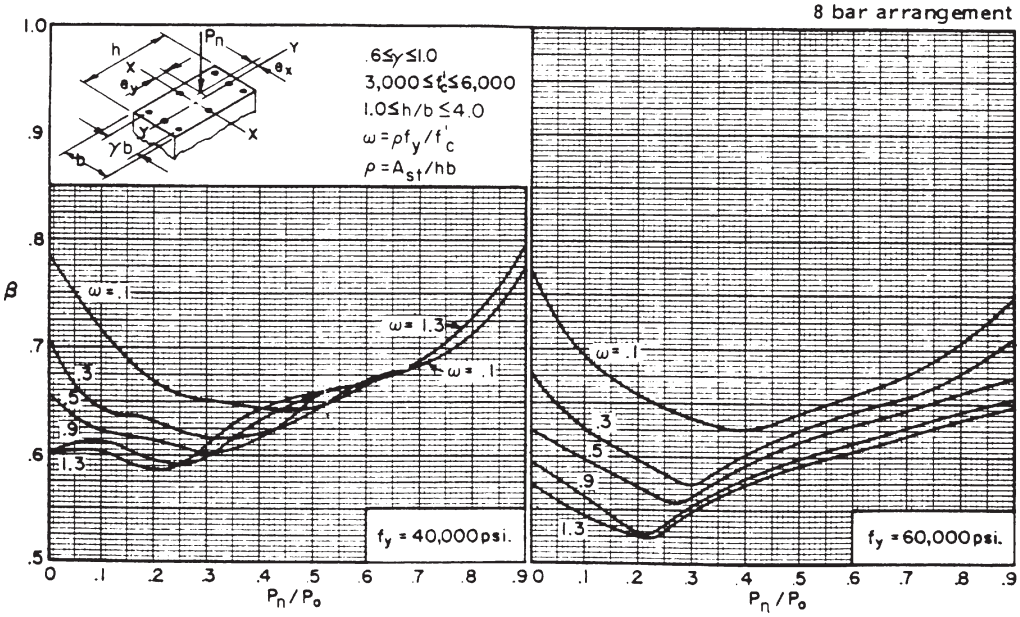
Values of β were computed on the basis of 10.2, utilizing a rectangular stress block and the basic principles of equilibrium. It was found that the parameters γ, b/h, and f'_c had minor effect on the β values. The maximum difference in β was about 5% for values of P_n/P_o ranging from 0.1 to 0.9. The majority of the β values, especially in the most frequently used range of P_n/P_o, did not differ by more than 3%. In view of these small differences, only envelopes of the lowest β values were developed for two values of f_y and different bar arrangements, as shown in Figs. 7-15 and 7-16.

As can be seen from Figs. 7-15 and 7-16, β is dependent primarily on the ratio P_n/P_o and to a lesser, though still significant extent, on the bar arrangement, the reinforcement index ω and the strength of the reinforcement.

Figure 7-14, in combination with Figs. 7-15 and 7-16, furnish a convenient and direct means of determining the biaxial moment strength of a given cross-section subject to an axial load, since the values P_o, M_{nox}, and M_{noy} can be readily obtained by methods described above.



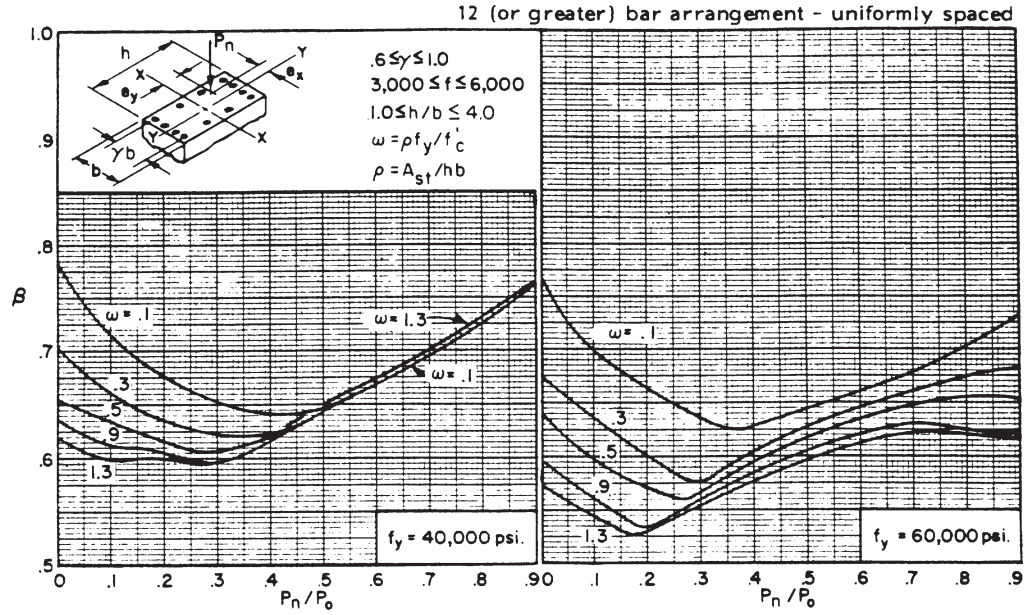
(a) 4 Bar Arrangement



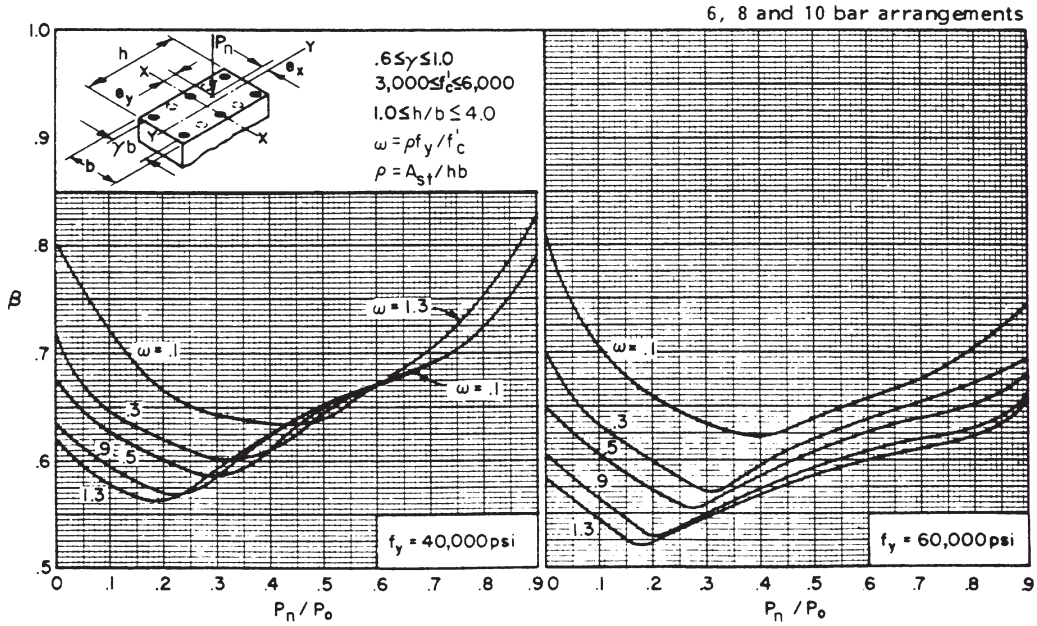
(b) 8 Bar Arrangement

Figure 7-15 Biaxial Design Constants

While investigation of a given section has been simplified, the determination of a section which will satisfy the strength requirements imposed by a load eccentric about both axes can only be achieved by successive analyses of assumed sections. Rapid and easy convergence to a satisfactory section can be achieved by approximating the curves in Fig. 7-14 by two straight lines intersecting at the 45 degree line, as shown in Fig. 7-17.



(a) 6, 8, and 10 Bar Arrangement



(b) 12 (or greater) Bar Arrangement

Figure 7-16 Biaxial Design Constants

By simple geometry, it can be shown that the equation of the upper lines is:

$$\frac{M_{nx}}{M_{nox}}\left(\frac{1-\beta}{\beta}\right) + \frac{M_{ny}}{M_{noy}} = 1 \text{ for } \frac{M_{ny}}{M_{nx}} > \frac{M_{noy}}{M_{nox}} \qquad (15)$$

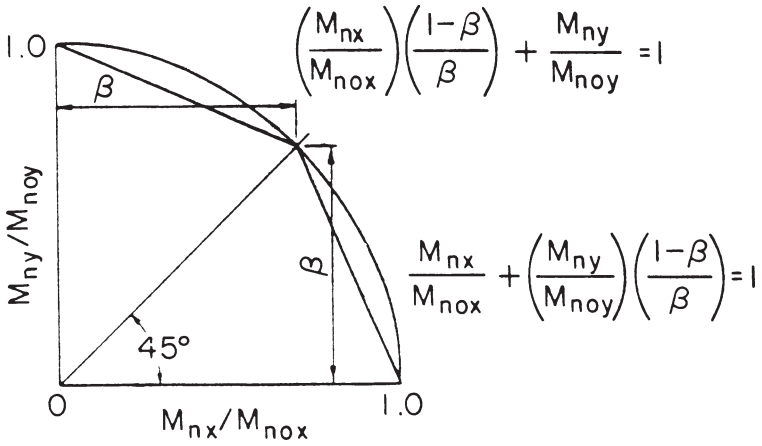


Figure 7-17 Bilinear Approximation of Nondimensionalized Load Contour (Fig. 7-13(b))

which can be restated for design convenience as follows:

$$M_{nx}\left(\frac{M_{noy}}{M_{nox}}\right)\left(\frac{1\ -\ \beta}{\beta}\right)\ +\ M_{ny}\ =\ M_{noy} \qquad (16)$$

For rectangular sections with reinforcement equally distributed on all faces, Eq. (16) can be approximated by:

$$M_{nx}\ \frac{b}{h}\left(\frac{1\ -\ \beta}{\beta}\right)\ +\ M_{ny}\ \approx\ M_{noy} \qquad (17)$$

The equation of the lower line of Fig. 7-17 is:

$$\frac{M_{nx}}{M_{nox}}\ +\ \frac{M_{ny}}{M_{noy}}\left(\frac{1\ -\ \beta}{\beta}\right)\ =\ 1\ \text{for}\ \frac{M_{ny}}{M_{nx}}\ <\ \frac{M_{noy}}{M_{nox}} \qquad (18)$$

or $$M_{nx}\ +\ M_{ny}\left(\frac{M_{nox}}{M_{noy}}\right)\left(\frac{1\ -\ \beta}{\beta}\right)\ =\ M_{nox} \qquad (19)$$

For rectangular sections with reinforcement equally distributed on all faces,

$$M_{nx}\ +\ M_{ny}\ \frac{h}{b}\left(\frac{1\ -\ \beta}{\beta}\right)\ \approx\ M_{nox} \qquad (20)$$

In design Eqs. (17) and (20), the ratio b/h or h/b must be chosen and the value of β must be assumed. For lightly loaded columns, β will generally vary from 0.55 to about 0.70. Hence, a value of 0.65 for β is generally a good initial choice in a biaxial bending analysis.

MANUAL DESIGN PROCEDURE

To aid the engineer in designing columns for biaxial bending, a procedure for manual design is outlined below:

1. Choose the value of β at 0.65 or use Figs. 7-15 and 7-16 to make an estimate.

2. If M_{ny}/M_{nx} is greater than b/h, use Eq. (17) to calculate an approximate equivalent uniaxial moment strength M_{noy}. If M_{ny}/M_{nx} is less than b/h, use Eq. (20) to calculate an approximate equivalent uniaxial moment strength M_{nox}.

3. Design the section using any of the methods presented above for uniaxial bending with axial load to provide an axial load strength P_n and an equivalent uniaxial moment strength M_{noy} or M_{nox}.

4. Verify the section chosen by any one of the following three methods:

   a. Bresler Reciprocal Load Method:

$$P_n \leq \frac{1}{\frac{1}{P_{ox}} + \frac{1}{P_{oy}} - \frac{1}{P_o}} \qquad (7)$$

   b. Bresler Load Contour Method:

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}} \leq 1.0 \qquad (11)$$

   c. PCA Load Contour Method: Use Eq. (14) or,

$$\frac{M_{nx}}{M_{nox}}\left(\frac{1-\beta}{\beta}\right) + \frac{M_{ny}}{M_{noy}} \leq 1.0 \quad \text{for} \quad \frac{M_{ny}}{M_{nx}} > \frac{M_{noy}}{M_{nox}} \qquad (15)$$

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}}\left(\frac{1-\beta}{\beta}\right) \leq 1.0 \quad \text{for} \quad \frac{M_{ny}}{M_{nx}} < \frac{M_{noy}}{M_{nox}} \qquad (18)$$

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Example 7.1—Design of Rectangular Beam with Tension Reinforcement Only

Select a rectangular beam size and required reinforcement A_s to carry service load moments $M_D = 56$ ft-kips and $M_L = 35$ ft-kips. Select reinforcement to control flexural cracking.

$f'_c = 4000$ psi
$f_y = 60{,}000$ psi

Calculations and Discussion	Code Reference

1. To illustrate a complete design procedure for rectangular sections with tension reinforcement only, a minimum beam depth will be computed using the maximum reinforcement permitted for tension-controlled flexural members, ρ_t. The design procedure will follow the method outlined on the preceding pages. *10.3.4*

   Step 1. Determine maximum tension-controlled reinforcement ratio for material strengths $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

   $\rho_t = 0.01806$ from Table 6-1

   Step 2. Compute bd^2 required.

   Required moment strength:

   $$M_u = (1.2 \times 56) + (1.6 \times 35) = 123.2 \text{ ft-kips}$$ *Eq. (9-2)*

   $$R_n = \rho f_y \left(1 - \frac{0.5\rho f_y}{0.85 f'_c}\right)$$

   $$= (0.01806 \times 60{,}000)\left(1 - \frac{0.5 \times 0.01806 \times 60{,}000}{0.85 \times 4000}\right) = 911 \text{ psi}$$

   $$bd^2 \text{ (required)} = \frac{M_u}{\phi R_n} = \frac{123.2 \times 12 \times 1000}{0.90 \times 911} = 1803 \text{ in.}^3$$

   Step 3. Size member so that bd^2 provided $\geq bd^2$ required.

   Set $b = 10$ in. (column width)

   $$d = \sqrt{\frac{1803}{10}} = 13.4 \text{ in.}$$

   Minimum beam depth $\approx 13.4 + 2.5 = 15.9$ in.

For moment strength, a 10 × 16 in. beam size is adequate. However, deflection is an essential consideration in designing beams by the Strength Design Method. Control of deflection is discussed in Part 10.

Step 4. Using the 16 in. beam depth, compute a revised value of ρ. For illustration, ρ will be computed by all four methods outlined earlier.

$d = 16 - 2.5 = 13.5$ in.

1. By Eq. (4) (exact method):

$$R_n = \frac{M_u}{\phi(bd^2 \text{ provided})} = \frac{123.2 \times 12 \times 1000}{0.90\,(10 \times 13.5^2)} = 901 \text{ psi}$$

$$\rho = \frac{0.85f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85f'_c}}\right)$$

$$= \frac{0.85 \times 4}{60}\left(1 - \sqrt{1 - \frac{2 \times 901}{0.85 \times 4000}}\right) = 0.0178$$

2. By strength curves such as shown in Fig. 7-2:

for $R_n = 901$ psi, $\rho \approx 0.0178$

3. By moment strength tables such as Table 7-1:

$$\frac{M_u}{\phi f'_c bd^2} = \frac{123.2 \times 12 \times 1000}{0.90 \times 4000 \times 10 \times 13.5^2} = 0.2253$$

$\omega \approx 0.2676$

$$\rho = \frac{\omega f'_c}{f_y} = 0.2676 \times \frac{4}{60} = 0.0178$$

4. By approximate proportion:

$$\rho \approx (\text{original } \rho)\frac{(\text{revised } R_n)}{(\text{original } R_n)}$$

$$\rho = 0.01806 \times \frac{901}{911} = 0.0179$$

Step 5. Compute A_s required.

$A_s = (\text{revised } \rho)(\text{bd provided})$

$= 0.0178 \times 10 \times 13.5 = 2.40 \text{ in.}^2$

Compare results to those obtained for Example 10.1 in *Notes on ACI 318-99,* designed by the '99 code.

	by 318-99	by 318-02
M_u	138 ft-k	123 ft-k
ϕ	0.9	0.9
Beam size (bxd)	10 x 16 in.	10 x 16 in.
A_s required	2.78 in.^{2}	2.40 in.^{2}

The reduction in A_s is a direct result of the fact that, with lower load factors, ϕ is not required to be reduced from 0.9 for flexure in tension-controlled members.

2. A review of the correctness of the computations can be made by considering statics.

$T = A_s f_y = 2.40 \times 60 = 144.0$ kips

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{144.0}{0.85 \times 4 \times 10} = 4.24 \text{ in.}$$

Design moment strength:

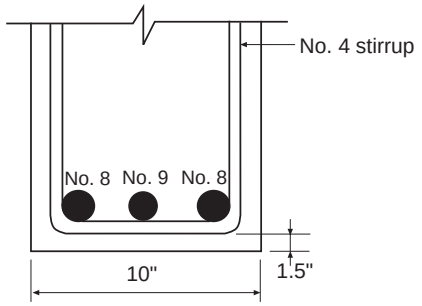
$$\phi M_n = \phi\left[A_s f_y\left(d - \frac{a}{2}\right)\right] = 0.9\left[144.0\left(13.5 - \frac{4.24}{2}\right)\right]$$

$= 1{,}475$ in.-kips $= 122.9$ ft-kips $\approx$ required $M_u = 123.2$ ft-kips O.K.

3. Select reinforcement to satisfy distribution of flexural reinforcement requirements of 10.6. *10.6*

A_s required $= 2.40$ in.^{2}

For illustrative purposes, select 1-No. 9 and 2-No. 8 bars ($A_s = 2.40$ in.^{2}). For practical design and detailing, one bar size for total A_s is preferable.



$c_c = 1.5 + 0.5 = 2.0$ in.

Maximum spacing allowed,

$$s = \frac{540}{f_s} - 2.5\, c_c$$ *Eq. (10-4)*

Use $f_s = 36$ ksi

$$s = \frac{540}{36} - (2.5 \times 2.0) = 10 \text{ in.}$$

or, refer to Table 9-1: for $f_s = 36$ and $c_c = 2$, s = 10 in.

$$\text{Spacing provided} = \frac{1}{2}\left\{10 - 2\left(1.5 + 0.5 + \frac{1.0}{2}\right)\right\}$$

$$= 2.50 \text{ in.} < 10 \text{ in.} \quad \text{O.K.}$$

Example 7.2—Design of One-Way Solid Slab

Determine required thickness and reinforcement for a one-way slab continuous over two or more equal spans. Clear span ℓ_n = 18 ft.

f'_c = 4000 psi
f_y = 60,000 psi
Service loads: w_d = 75 psf (assume 6-in. slab), w_ℓ = 50 psf

Calculations and Discussion	Code Reference

1. Compute required moment strengths using approximate moment analysis permitted by 8.3.3. Design will be based on end span.

   Factored load $w_u = (1.2 \times 75) + (1.6 \times 50) = 170$ psf — *Eq. (9-2)*

   Positive moment at discontinuous end integral with support:

   $+M_u = w_u \ell_n^2/14 = 0.170 \times 18^2/14 = 3.93$ ft-kips/ft — *8.3.3*

   Negative moment at exterior face of first interior support:

   $-M_u = w_u \ell_n^2/10 = 0.170 \times 18^2/10 = 5.51$ ft-kips/ft — *8.3.3*

2. Determine required slab thickness. — *10.3.3*

   Choose a reinforcement percentage ρ equal to about $0.5\rho_t$, or one-half the maximum permitted for tension-controlled sections, to have reasonable deflection control.

   From Table 6-1, for f'_c = 4000 psi and f_y = 60,000 psi: $\rho_t = 0.01806$

   Set $\rho = 0.5\,(0.01806) = 0.00903$

   Design procedure will follow method outlined earlier:

$$R_n = \rho f_y \left(1 - \frac{0.5\rho f_y}{0.85 f'_c}\right)$$

$$= (0.00903 \times 60{,}000)\left(1 - \frac{0.5 \times 0.00903 \times 60{,}000}{0.85 \times 4000}\right) = 499 \text{ psi}$$

$$\text{Required } d = \sqrt{\frac{M_u}{\phi R_n b}} = \sqrt{\frac{5.51 \times 12{,}000}{0.90 \times 499 \times 12}} = 3.50 \text{ in.}$$

   Assuming No. 5 bars, required h = 3.50 + 0.31/2 + 0.75 = 4.41 in.

   The above calculations indicate a slab thickness of 4.5 in. is adequate. However, Table 9-5(a) indicates a minimum thickness of $\ell/24 \geq 9$ in., unless deflections are computed. Also note that Table 9-5(a) is applicable only to "members not supporting or attached to partitions or other construction likely to be damaged by large deflections." Otherwise deflections must be computed.

For purposes of illustration, the required reinforcement will be computed for h = 4.5 in., d = 3.59 in.

3. Compute required negative moment reinforcement.

$$R_n = \frac{M_u}{\phi bd^2} = \frac{5.51 \times 12 \times 1000}{0.9 \times 12 \times 3.59^2} = 475$$

$$\rho \approx 0.00903\left(\frac{475}{499}\right) = 0.00860$$

$-A_s$ (required) $= \rho bd = 0.00860 \times 12 \times 3.59 = 0.37$ in.^{2}/ft

Use No. 5 @ 10 in. ($A_s = 0.37$ in.^{2}/ft)

4. For positive moment, use Table 7-1:

$$\frac{M_u}{\phi f'_c bd^2} = \frac{3.93 \times 12{,}000}{0.9 \times 4000 \times 12 \times 3.59^2} = 0.0847$$

From Table 7-1, $\omega \approx 0.090$

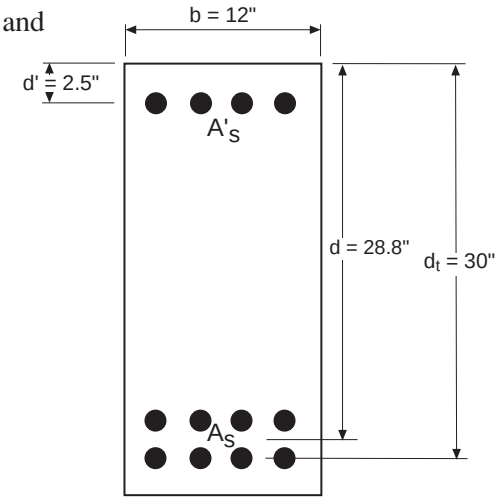
$$\rho = \frac{\omega f'_c}{f_y} = 0.090 \times \frac{4}{60} = 0.006$$

$+A_s$ (required) $= \rho bd = 0.006 \times 12 \times 3.59 = 0.258$ in.^{2}/ft

Use No. 4 @ 9 in. ($A_s = 0.27$ in.^{2}/ft) or No. 5 @ 12 in. ($A_s = 0.31$ in.^{2}/ft)

Example 7.3—Design of Rectangular Beam with Compression Reinforcement

A beam cross-section is limited to the size shown. Determine the required area of reinforcement for service load moments M_D = 430 ft-kips and M_L = 175 ft-kips. Check crack control requirements of 10.6.



f'_c = 4000 psi
f_y = 60,000 psi

Calculations and Discussion	Code Reference

1. Determine required reinforcement.

Step 1. Determine if compression reinforcement is needed.

$$M_u = 1.2M_D + 1.6M_L = 796 \text{ ft-kips}$$ *Eq. (9-2)*
$$M_n = M_u/\phi = 796/0.9 = 884 \text{ ft-kips}$$

$$R_n = \frac{M_n}{bd^2} = \frac{884 \times 12 \times 1000}{12 \times 30^2} = 982$$

This exceeds the maximum R_n of 911 for tension-controlled sections of 4000 psi concrete, without compression reinforcement. (see Table 6-1.) Also, it appears likely that two layers of tension reinforcement will be necessary. Estimate d = d_t - 1.2 in. = 28.8 in.

Step 2. Find the nominal strength moment resisted by the concrete section, without compression reinforcement.

ρ_t = 0.01806 from Table 6-1

$$\rho = \rho_t\left(\frac{d_t}{d}\right) = 0.01806\left(\frac{30}{28.8}\right) = 0.01881$$ *(6)*

$$\omega = \rho\frac{f_y}{f'_c} = 0.01881 \times \frac{60}{4} = 0.282$$

$\frac{M_{nt}}{f'_c bd^2}$ = 0.2351 from Table 7-1

$M_{nt} = 0.2351 \times 4 \times 12 \times 28.8^2 = 9{,}360$ in.-kips = 780 ft-kips resisted by the concrete

Required moment strength to be resisted by the compression reinforcement:

$M'_n = 884 - 780 = 104$ ft-kips

Step 3. Determine the compression steel stress f'_s.

Check yielding of compression reinforcement. Since the section was designed at the tension-controlled net tensile strain limit $\varepsilon_t = 0.005$, $c/d_t = 0.375$

$c = 0.375 d_t = 0.375 \times 30 = 11.25$ in.

$d'/c = 2.5/11.25 = 0.22 < 0.31$

Compression reinforcement yields at the nominal strength ($f'_s = f_y$)

Step 4. Determine the total required reinforcement:

$$A'_s = \frac{M'_n}{f_y(d-d')}$$

$$= \frac{104 \times 12 \times 1000}{60{,}000(28.8-2.5)} = 0.79 \text{ in.}^2$$

$A_s = 0.79 + \rho bd$

$= 0.79 + (0.01881 \times 12 \times 28.8) = 7.29$ in.^{2}

Step 5. Check moment capacity.

When the compression reinforcement yields:

$$a = \frac{(A_s - A'_s)f_y}{0.85 f'_c b} = \frac{6.50 \times 60}{0.85 \times 4 \times 12} = 9.56 \text{ in.}$$

$$\phi M_n = \phi\left[(A_s - A'_s) f_y \left(d - \frac{a}{2}\right) + A'_s f_y (d - d')\right]$$

$$= 0.9\left[6.50 \times 60\left(28.8 - \frac{9.56}{2}\right) + (0.79 \times 60)(28.8 - 2.5)\right]/12$$

$= 796$ ft-kips $= M_u = 796$ ft-kips O.K.

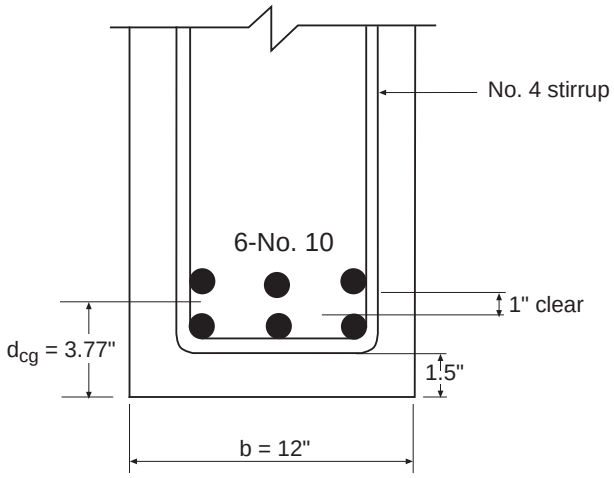
2. Select reinforcement to satisfy control of flexural cracking criteria of 10.6.

Compression reinforcement:

Select 2-No. 6 bars ($A'_s = 0.88$ in.$^2 > 0.79$ in.^{2})

Tension reinforcement:

Select 6-No. 10 bars in two layers ($A_s = 7.62$ in.$^2 > 7.29$ in.^{2})



Maximum spacing allowed,

$$s = \frac{540}{f_s} - 2.5\, c_c$$ *Eq. (10-4)*

$c_c = 1.5 + 0.5 = 2.0$ in.

Use $f_s = 36$ ksi

$$s = \frac{540}{36} - (2.5 \times 2) = 10 \text{ in.}$$

$$\text{Spacing provided} = \frac{1}{2}\left\{12 - 2\left(1.5 + 0.5 + \frac{1.27}{2}\right)\right\}$$

$= 4.68$ in. < 10 in. O.K.

4. Stirrups or ties are required throughout distance where compression reinforcement is required for strength. 7.11.1

Max. spacing = 16 × long. bar dia. = 16 × 0.75 = 12 in. (governs) 7.10.5.2

= 48 × tie bar dia. = 48 × 0.5 = 24 in.

= least dimension of member = 12 in.

Use s_{max} = 12 in. for No. 4 stirrups

Compare required steel to that in Example 10.3 of *Notes on ACI 318-99*.

	by 318-99	**by 318-02**	**by 318-02 assuming $d_t = d$**
M_u	900 ft-k	796 ft-k	796 ft-k
ϕ	0.9	0.9	0.9
A'_s required	0.47 in.^{2}	0.79 in.^{2}	0.97
A_s required	8.17 in.^{2}	7.29 in.^{2}	7.21
Total reinforcement	8.67 in.^{2}	8.08 in.^{2}	8.18

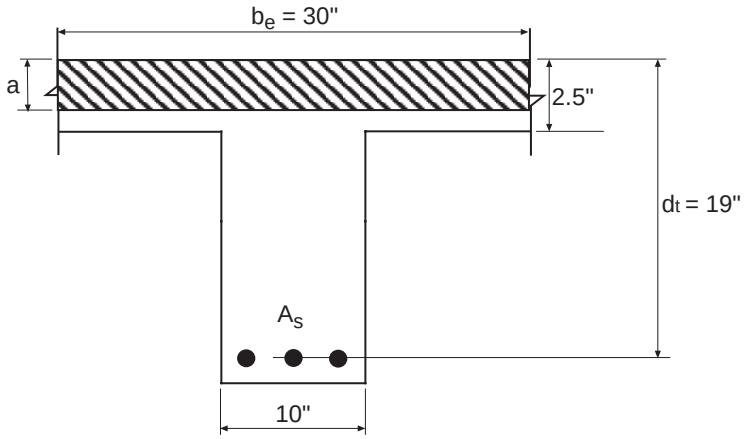
Somewhat more sompression steel is needed, but there is a significant reduction in the tension steel using 318-02.

Also shown are the results of a calculation (not shown) using the simplfying assumption of $d = d_t$. The extra steel is only 1.2 percent.

Example 7.4—Design of Flanged Section with Tension Reinforcement Only

Select reinforcement for the T-section shown, to carry service dead and live load moments of M_D = 72 ft-kips and M_L = 88 ft-kips.

f'_c = 4000 psi
f_y = 60,000 psi



Calculations and Discussion	Code Reference

1. Determine required flexural strength.

   $M_u = (1.2 \times 72) + (1.6 \times 88) = 227$ ft-kips — *Eq. (9-2)*

2. Using Table 7-1, determine depth of equivalent stress block a, as for a rectangular section. Assume $\phi = 0.9$.

$$\frac{M_u}{\phi f'_c b d^2} = \frac{227 \times 12}{0.9 \times 4 \times 30 \times 19^2} = 0.0699$$

From Table 7-1, $\omega \approx 0.073$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{\rho d f_y}{0.85 f'_c} = 1.18 \omega d = 1.18 \times 0.073 \times 19 = 1.64 \text{ in.} < 2.5 \text{ in.}$$

With $a < h_f$, determine A_s as for a rectangular section (see Ex. 7.5 for the case when $a > h_f$).

Check ϕ:

$$c = a/\beta_1 = 1.64/0.85 = 1.93 \text{ in.}$$

$c/d_t = 1.93/19 = 0.102 < 0.375$

Section is tension-controlled, and $\phi = 0.9$.

3. Compute A_s required.

$A_s f_y = 0.85 f'_c b a$

$$A_s = \frac{0.85 \times 4 \times 30 \times 1.64}{60} = 2.78 \text{ in.}^2$$

Alternatively,

$$A_s = \rho b d = \omega \frac{f'_c}{f_y} b d$$

$$= 0.073 \times \frac{4}{60} \times 30 \times 19 = 2.77 \text{ in.}^2$$

Try 3-No. 9 bars ($A_s = 3.0$ in.^{2}).

4. Check minimum required reinforcement. *10.5*

For $f'_c < 4444$ psi, *Eq. (10-3)*

$$\rho_{min} = \frac{200}{f_y} = \frac{200}{60{,}000} = 0.0033$$

$$\frac{A_s}{b_w d} = \frac{3.0}{30 \times 19} = 0.0053 > 0.0033 \quad \text{O.K.}$$

5. Check distribution of reinforcement. *10.6*

Maximum spacing allowed,

$$s = \frac{540}{f_s} - 2.5\, c_c \qquad \textit{Eq. (10-4)}$$

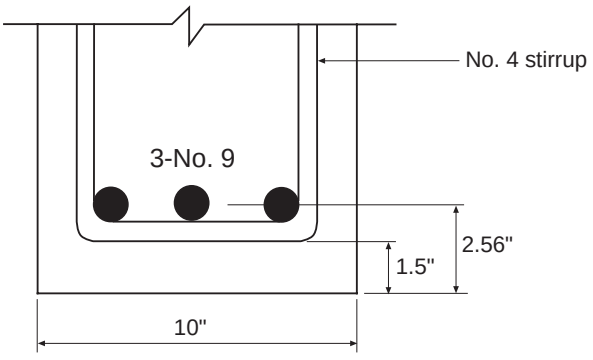
$c_c = 1.5 + 0.5 = 2.0$ in.

Use $f_s = 36$ ksi

$$s = \frac{540}{36} - (2.5 \times 2.0) = 10 \text{ in.}$$

$$\text{Spacing provided} = \frac{1}{2}\left\{10 - 2\left(1.5 + 0.5 + \frac{1.128}{2}\right)\right\}$$

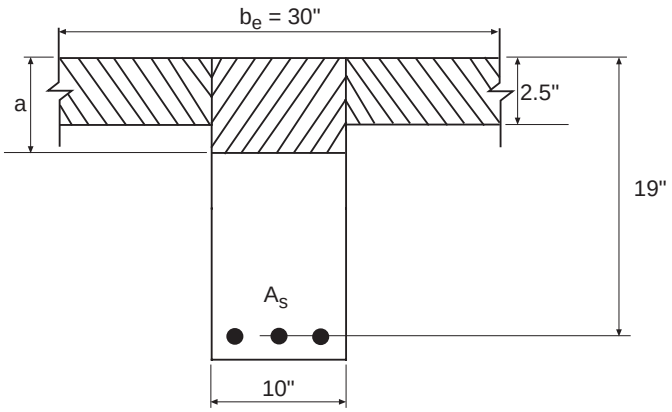
$= 2.44$ in. < 10 in. O.K.



Example 7.5—Design of Flanged Section with Tension Reinforcement Only

Select reinforcement for the T-section shown, to carry a factored moment of M_u = 400 ft-kips.

f'_c = 4000 psi
f_y = 60,000 psi



Calculations and Discussion	Code Reference

1. Determine required reinforcement.

   Step 1. Using Table 7-1, determine depth of equivalent stress block a, as for a rectangular section.

   Assume tension-controlled section, $\phi = 0.9$.

   $$M_n = M_u/\phi = 400/0.9 = 444 \text{ ft-kips}$$

   Assume a < 2.5 in.

   $$\frac{M_n}{f'_c bd^2} = \frac{444 \times 12}{4 \times 30 \times 19^2} = 0.123$$

   From Table 7-1, $\omega \approx 0.134$

   $$a = \frac{A_s f_y}{0.85 f'_c b} = 1.18\omega d$$

   $$= 1.18 \times 0.134 \times 19 = 3.0 \text{ in.} > 2.5 \text{ in.}$$

   Step 2. Since the value of a as a rectangular section exceeds the flange thickness, the equivalent stress block extends in the web, and the design must be based on T-section behavior. See Example 7.4 when a is less than the flange depth.

   Step 3. Compute required reinforcement A_{sf} and nominal moment strength M_{nf} corresponding to the overhanging beam flange in compression (see Part 6).

   Compressive strength of flange

$C_f = 0.85f'_c\,(b - b_w)\,h_f$

$= 0.85 \times 4\,(30 - 10)\,2.5 = 170$ kips

Required A_{sf} to equilibrate C_f :

$$A_{sf} = \frac{C_f}{f_y} = \frac{170}{60} = 2.83 \text{ in.}^2$$

Nominal moment strength of flange:

$$M_{nf} = \left[A_{sf} f_y \left(d - \frac{h_f}{2}\right)\right]$$

$= [2.83 \times 60\,(19 - 1.25)]/12 = 251$ ft-kips

Step 4. Required nominal moment strength to be carried by beam web:

$M_{nw} = M_n - M_{nf} = 444 - 251 = 193$ ft-kips

Step 5. Using Table 7-1, compute reinforcement A_{sw} required to develop moment strength to be carried by the web.

$$\frac{M_{nw}}{f'_c b d^2} = \frac{193 \times 12}{4 \times 10 \times 19^2} = 0.1604$$

From Table 7-1, $\omega_w \approx 0.179$

$$\rho_w = 0.179 \times \frac{4}{60} = 0.01193$$

Step 6. Check to see if section is tension-controlled, with $\phi = 0.9$:

$\rho_t = 0.01806$ from Table 6-1

Therefore, $\rho_w < \rho_t$ and section is tension-controlled ($\phi = 0.9$)

$A_{sw} = \rho_w b d = 0.01193 \times 10 \times 19 = 2.27$ in.^{2}

Step 7. Total reinforcement required to carry factored moment $M_u = 400$ ft-kips:

$A_s = A_{sf} + A_{sw} = 2.83 + 2.27 = 5.10$ in.^{2}

:

Step 8. Check moment capacity.

$$\phi M_n = \phi\left[(A_s - A_{sf})\, f_y \left(d - \frac{a_w}{2}\right) + A_{sf} f_y \left(d - \frac{h_f}{2}\right)\right]$$

$$a_w = \frac{(A_s - A_{sf})\, f_y}{0.85 f'_c b_w}$$

$$= \frac{(5.10 - 2.83) \times 60}{0.85 \times 4 \times 10} = 4.01 \text{ in.}$$

$$\phi M_n = 0.9 \left[(5.10 - 2.83)\, 60 \left(19 - \frac{4.01}{2}\right) + (2.83 \times 60) \left(19 - \frac{2.5}{2}\right)\right]/12$$

$$= 400 \text{ ft-kips} = M_u = 400 \text{ ft-kips} \quad \text{O.K.}$$

2. Select reinforcement to satisfy crack control criteria. *10.6*

Try 5-No. 9 bars in two layers ($A_s = 5.00$ in.^{2})
(2% less than required, assumed sufficient)

Maximum spacing allowed,

$$s = \frac{540}{f_s} - 2.5\, c_c$$ *Eq. (10-4)*

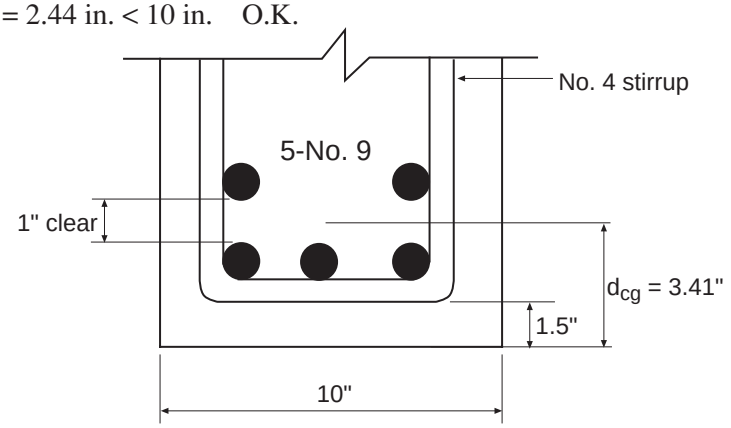
$c_c = 1.5 + 0.5 = 2.0$ in.

Use $f_s = 36$ ksi

$$s = \frac{540}{36} - (2.5 \times 2.0) = 10 \text{ in.}$$

$$\text{Spacing provided} = \frac{1}{2}\left\{10 - 2\left(1.5 + 0.5 + \frac{1.128}{2}\right)\right\}$$

$= 2.44$ in. < 10 in. O.K.



Note: Two layers of reinforcement are required, which may not have been recognized when d was assumed to be 19 in. Also, the provided steel is slightly less than required. Therefore, the overall height should be a little more than $d + d_{cg} = 22.41$ in., or the steel should be increased.

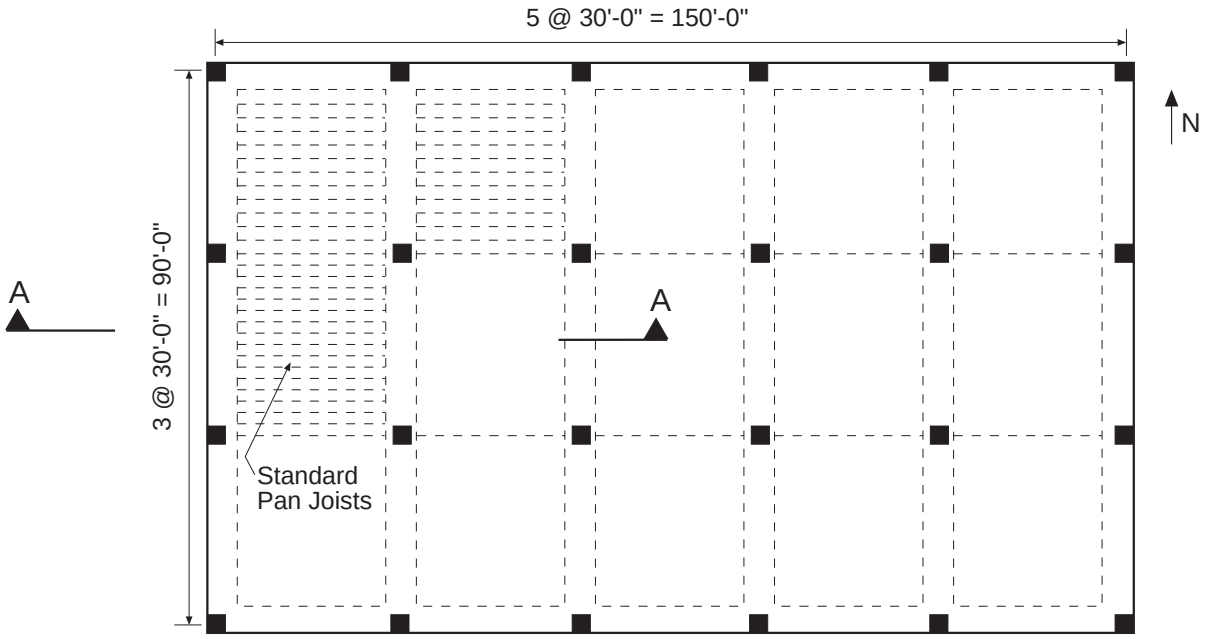
Example 7.6—Design of One-Way Joist

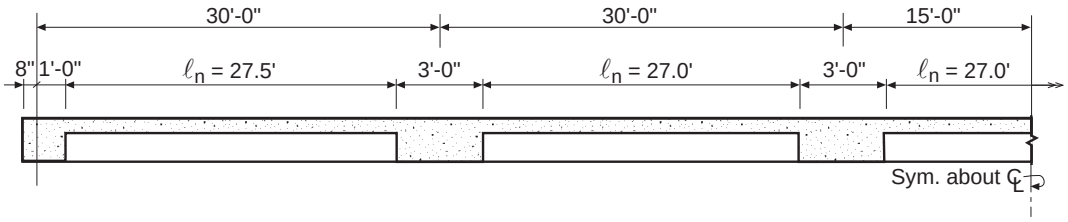
Determine the required depth and reinforcement for the one-way joist system shown below. The joists are 6 in. wide and are spaced 36 in. o.c. The slab is 3.5 in. thick.

f'_c = 4000 psi
f_y = 60,000 psi
Service DL = 130 psf (assumed total for joists and beams plus superimposed dead loads)
Service LL = 60 psf

Width of spandrel beams = 20 in.
Width of interior beams = 36 in.

Columns: interior = 18 × 18 in.
exterior = 16 × 16 in.
Story height (typ.) = 13 ft





Section A-A

1. Compute the factored moments at the faces of the supports and determine the depth of the joists.

$w_u = [(1.2 \times 0.13) + (1.6 \times 0.06)] \times 3 = 0.756$ kips/ft *Eq. (9-2)*

Using the approximate coefficients, the factored moments along the span are summarized in the table below. *8.3.3*

Location	M_u (ft-kips)
End span	
Ext. neg.	$w_u\ell_n^2/24 = 0.756 \times 27.5^2/24 = 23.8$
Pos.	$w_u\ell_n^2/14 = 0.756 \times 27.5^2/14 = 40.8$
Int. neg.	$w_u\ell_n^2/10 = 0.756 \times 27.25^2/10 = 56.1$
Interior span	
Pos.	$w_u\ell_n^2/16 = 0.756 \times 27^2/16 = 34.4$
Neg.	$w_u\ell_n^2/11 = 0.756 \times 27^2/11 = 50.1$

For reasonable deflection control, choose a reinforcement ratio ρ equal to about one-half ρ_t. From Table 6-1, $\rho_t = 0.01806$.

Set $\rho = 0.5 \times 0.01806 = 0.00903$

Determine the required depth of the joist based on $M_u = 56.1$ ft-kips:

$$\omega = \frac{\rho f_y}{f'_c} = \frac{0.00903 \times 60}{4} = 0.1355$$

From Table 7-1, $M_u/\phi f'_c bd^2 = 0.1247$

$$d = \sqrt{\frac{M_u}{\phi f'_c b_w (0.1247)}} = \sqrt{\frac{56.1 \times 12}{0.9 \times 4 \times 6 \times 0.1247}} = 15.8 \text{ in.}$$

$h \approx 15.8 + 1.25 = 17.1$ in.

From Table 9-5(a), the minimum required thickness of the joist is

$$h_{min} = \frac{\ell}{18.5} = \frac{30 \times 12}{18.5} = 19.5 \text{ in.}$$

Use a 19.5-in. deep joist (16 + 3.5).

2. Compute required reinforcement.

a. End span, exterior negative

$$\frac{M_u}{\phi f'_c b d^2} = \frac{23.8 \times 12}{0.9 \times 4 \times 6 \times 18.25^2} = 0.0397$$

From Table 7-1, $\omega \approx 0.041$

$$A_s = \frac{\omega b d f'_c}{f_y} = \frac{0.041 \times 6 \times 18.25 \times 4}{60} = 0.30 \text{ in.}^2$$

For $f'_c < 4444$ psi, use

$$A_{s,\,min} = \frac{200 b_w d}{f_y} = \frac{200 \times 6 \times 18.25}{60{,}000} = 0.37 \text{ in.}^2 > A_s$$ *Eq. (10-3)*

Distribute bars uniformly in top slab:

$$A_s = \frac{0.37}{3} = 0.123 \text{ in.}^2/\text{ft}$$

Use No. 3 @ 10 in. ($A_s = 0.13$ in.^{2}/ft)

b. End span, positive

$$\frac{M_u}{\phi f'_c b d^2} = \frac{40.8 \times 12}{0.9 \times 4 \times 36 \times 18.25^2} = 0.0113$$

From Table 7-1, $\omega \approx 0.012$

$$A_s = \frac{\omega b d f'_c}{f_y} = \frac{0.012 \times 36 \times 18.25 \times 4}{60} = 0.53 \text{ in.}^2$$

Check rectangular section behavior:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{0.53 \times 60}{0.85 \times 4 \times 36} = 0.26 \text{ in.} < 3.5 \text{ in. O.K.}$$

Use 2-No. 5 bars ($A_s = 0.62$ in.^{2})

c. End span, interior negative

$$\frac{M_u}{\phi f'_c b d^2} = \frac{56.1 \times 12}{0.9 \times 4 \times 6 \times 18.25^2} = 0.0936$$

From Table 7-1, $\omega \approx 0.100$

$$A_s = \frac{\omega b d f'_c}{f_y} = \frac{0.100 \times 6 \times 18.25 \times 4}{60} = 0.73 \text{ in.}^2$$

Distribute reinforcement uniformly in slab:

$$A_s = \frac{0.73}{3} = 0.24 \text{ in.}^2/\text{ft}$$

Use No. 5 @ 12 in. for crack control considerations in slabs (see Table 9-1).

d. The reinforcement for the other sections is obtained in a similar fashion. The following table summarizes the results. Note that at all sections, the requirements in 10.6 for crack control are satisfied.

Location	M_u	A_s	Reinforcement
End span	(ft-kips)	(in.^{2})	
Ext. neg.	23.8	0.37	No. 3@10 in.
Pos.	40.8	0.53	2-No. 5
Int. neg.	56.1	0.73	No. 5@12 in.*
Interior span			
Pos.	34.4	0.42	2-No. 5
Neg.	50.1	0.65	No. 5@12 in.*

**Maximum 12 in. spacing required for crack control in slab.*

e. The slab reinforcement normal to the ribs is often located at mid-depth of the slab to resist both positive and negative moments.

$$\text{Use } M_u = \frac{w_u \ell_n^2}{12} = \frac{0.185 \times 2.5^2}{12} = 0.096 \text{ ft-kips}$$

where $w_u = 1.2\,(44 + 30) + 1.6\,(60)$

$= 185 \text{ psf} = 0.185 \text{ kips/ft}^2$

$$\frac{M_u}{\phi f'_c b d^2} = \frac{0.096 \times 12}{0.9 \times 4 \times 12 \times 1.75^2} = 0.0087$$

From Table 7-1, $\omega \approx 0.0087$

$$A_s = \frac{\omega b d f'_c}{f_y} = \frac{0.0087 \times 12 \times 1.75 \times 4}{60} = 0.01 \text{ in.}^2/\text{ft}$$

For slabs, minimum reinforcement is governed by the provisions in 7.12.2.1:

$A_{s,min} = 0.0018 \times 12 \times 3.5 = 0.08$ in.^{2}/ft

$s_{max} = 5h = 5 \times 3.5 = 17.5$ in. (governs) *7.12.2.2*

$= 18$ in.

Use No. 3 @ 16 in. ($A_s = 0.08$ in.^{2}/ft)

3. Shear at supports must be checked. Since the joists meet the requirements in 8.11, the contribution of the concrete to shear strength V_c is permitted to be 10% more than that specified in Chapter 11. *8.11.8*

Example 7.7—Design of Continuous Beams

Determine the required depth and reinforcement for the support beams along the interior column line in Example 7.6. The width of the beams is 36 in.

f'_c = 4000 psi
f_y = 60,000 psi
Service DL = 130 psf (assumed total for joists and beams plus superimposed dead loads)
Service LL = 60 psf

Columns: interior = 18 × 18 in.
exterior = 16 × 16 in.
Story height (typ.) = 13 ft

Calculations and Discussion	Code Reference

1. Compute the factored moments at the faces of the supports and determine the depth of the beam.

$w_u = [(1.2 \times 0.13) + (1.6 \times 0.06)] \times 30 = 7.56$ kips/ft — *Eq. (9-2)*

Using the approximate coefficients, the factored moments along the span are summarized in the table below. — *8.3.3*

Location	M_u (ft-kips)
End span	
Ext. neg.	$w_u\ell_n^2/16 = 7.56 \times 28.58^2/16 = 385.9$
Pos.	$w_u\ell_n^2/14 = 7.56 \times 28.58^2/14 = 441.1$
Int. neg.	$w_u\ell_n^2/10 = 7.56 \times 28.54^2/10 = 615.8$
Interior span	
Pos.	$w_u\ell_n^2/16 = 7.56 \times 28.50^2/16 = 383.8$

For overall economy, choose a beam depth equal to the joist depth used in Example 7.6.

Check the 19.5-in. depth for M_u = 615.8 ft-kips:

From Table 6-2,

$$\phi R_{nt} = 820 = \frac{M_{ut}}{bd^2}$$

$$M_{ut} = 820 \times 36 \times 17^2/1000 = 8531 \text{ in.-kips} = 711 \text{ ft-kips}$$

$M_u < M_{ut}$

Section will be tension-controlled without compresion reinforcement.

Check beam depth based on deflection criteria in Table 9.5(a):

$$h_{min} = \frac{\ell}{18.5} = \frac{30 \times 12}{18.5} = 19.5 \text{ in.} \quad \text{O.K.}$$

Use a 36 × 19.5 in. beam.

2. Compute required reinforcement:

a. End span, exterior negative

$$\frac{M_u}{\phi f'_c b d^2} = \frac{385.9 \times 12}{0.9 \times 4 \times 36 \times 17^2} = 0.1236$$

From Table 7-1, $\omega \approx 0.134$

$$A_s = \frac{\omega b d f'_c}{f_y} = \frac{0.134 \times 36 \times 17 \times 4}{60} = 5.47 \text{ in.}^2$$

For $f'_c < 4444$ psi, use

$$A_{s,min} = \frac{200 b_w d}{f_y} = \frac{200 \times 36 \times 17}{60{,}000} = 2.04 \text{ in.}^2$$ *Eq. (10-3)*

Use 7-No. 8 bars ($A_s = 5.53$ in.^{2})

Check distribution of flexural reinforcement requirements of 10.6.

Maximum spacing allowed,

$$s = \frac{540}{f_s} - 2.5\, c_c$$ *Eq. (10-4)*

$c_c = 1.5 + 0.5 = 2.0$ in.

Use $f_s = 0.6 f_y = 36$ ksi

$$s = \frac{540}{36} - (2.5 \times 2.0) = 10 \text{ in.}$$

$$\text{Spacing provided} = \frac{1}{6}\left\{36 - 2\left(1.5 + 0.5 + \frac{1.0}{2}\right)\right\}$$

$$= 5.17 \text{ in.} < 10 \text{ in.} \quad \text{O.K.}$$

b. End span, positive

$$\frac{M_u}{\phi f'_c b d^2} = \frac{441.1 \times 12}{0.9 \times 4 \times 36 \times 17^2} = 0.1413$$

From Table 7-1, $\omega \approx 0.156$

$$A_s = \frac{\omega b d f'_c}{f_y} = \frac{0.156 \times 36 \times 17 \times 4}{60} = 6.37 \text{ in.}^2$$

Use 11-No. 7 bars ($A_s = 6.60$ in.^{2})

Note that this reinforcement satisfies the cracking requirements in 10.6.4, and fits adequately within the beam width. It can also conservatively be used at the midspan section of the interior span.

c. End span, interior negative

$$\frac{M_u}{\phi f'_c b d^2} = \frac{615.8 \times 12}{0.9 \times 4 \times 36 \times 17^2} = 0.1973$$

From Table 7-1, $\omega \approx 0.228$

$$A_s = \frac{\omega b d f'_c}{f_y} = \frac{0.228 \times 36 \times 17 \times 4}{60} = 9.30 \text{ in.}^2$$

Use 10-No. 9 bars ($A_s = 10.0$ in.^{2})

This reinforcement is adequate for cracking and spacing requirements as well.

Example 7.8—Design of a Square Column for Biaxial Loading

Determine the required square tied column size and reinforcement for the factored load and moments given. Assume the reinforcement is equally distributed on all faces.

$P_u = 1200$ kips, $M_{ux} = 300$ ft-kips, $M_{uy} = 125$ ft-kips

$f'_c = 5000$ psi, $f_y = 60{,}000$ psi

Calculations and Discussion	Code Reference

1. Determine required nominal strengths, assuming compression-controlled behavior: 9.3.2.2(b)

$$P_n = \frac{P_u}{\phi} = \frac{1200}{0.65} = 1846 \text{ kips}$$

$$M_{nx} = \frac{M_{ux}}{\phi} = \frac{300}{0.65} = 461.5 \text{ ft-kips}$$

$$M_{ny} = \frac{M_{uy}}{\phi} = \frac{125}{0.65} = 192.3 \text{ ft-kips}$$

2. Assume $\beta = 0.65$

3. Determine an equivalent uniaxial moment strength M_{nox} or M_{noy}.

$$\frac{M_{ny}}{M_{nx}} = \frac{192.3}{465.1} = 0.42 \text{ is less than } \frac{b}{h} = 1.0 \text{ (square column)}$$

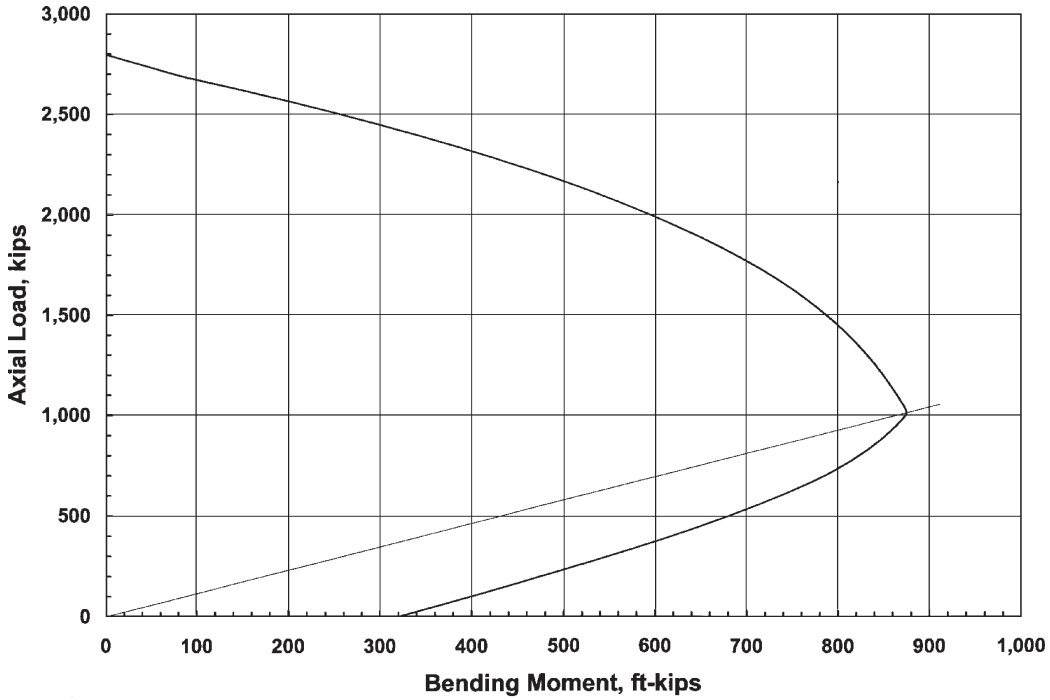
Therefore, using Eq. (19)

$$M_{nox} \approx M_{nx} + M_{ny}\frac{h}{b}\left(\frac{1-\beta}{\beta}\right)$$

$$= 461.5 + \left[192.3 \times (1.0)\left(\frac{1-0.65}{0.65}\right)\right] = 565.1 \text{ ft-kips}$$

4. Assuming a 24 in. square column, determine the reinforcement required to provide an axial load strength $P_n = 1846$ kips and an equivalent uniaxial moment strength $M_{nox} = 565.1$ ft-kips

The figure below is an interaction diagram for this column with 4-No. 11 bars. The section is adequate with this reinforcement for (P_n, M_{nox}).



5. Selected section will now be checked for biaxial strength by each of the three methods presented in the discussion.

   a. Bresler Reciprocal Load Method

   Check $P_n \geq 0.1 f'_c A_g$ *(8)*

   1714 kips $>$ 0.1 (5) (576) = 288 kips O.K.

   To employ this method, P_o, P_{ox}, and P_{oy} must be determined.

   $P_o = 0.85 f'_c (A_g - A_{st}) + A_{st} f_y$

   $= 0.85 (5) (576 - 6.24) + 6.24 (60) = 2796$ kips

   P_{ox} is the uniaxial load strength when only M_{nx} acts on the column. From the interaction diagram, P_{ox} = 2225 kips when M_{nx} = 461.5 ft-kips.

   Similarly, P_{oy} = 2575 kips when M_{ny} = 192.3 ft-kips. Note that both P_{ox} and P_{oy} are greater than the balanced axial force, so that the section is compression-controlled.

   Using the above values, Eq. (7) can now be evaluated:

   $$P_n = 1846 \text{ kips} \leq \frac{1}{\frac{1}{P_{ox}} + \frac{1}{P_{oy}} - \frac{1}{P_o}}$$

$$< \frac{1}{\frac{1}{2225} + \frac{1}{2575} - \frac{1}{2796}} = 2083 \text{ kips} \quad \text{O.K}$$

b. Bresler Load Contour Method

Due to a lack of available data, a conservative α value of 1.0 is chosen. Although $P_u > 0.1 f'_c A_g$, the necessary calculations will be carried out for example purposes. Since the section is symmetrical, M_{nox} is equal to M_{noy}.

From the interaction diagram, $M_{nox} = 680$ ft-kips for $P_n = 1846$ kips.

Using the above value, Eq. (11) can now be evaluated:

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}} = \frac{461.5}{680} + \frac{192.3}{680} = 0.68 + 0.28 = 0.96 < 1.0 \quad \text{O.K.}$$

c. PCA Load Contour Method

To employ this method, P_o, M_{nox}, M_{noy} and the true value of β must first be found.

$$P_o = 0.85 f'_c (A_g - A_{st}) + A_{st} f_y$$

$$= 0.85 (5) (576 - 6.24) + 6.24 (60) = 2796 \text{ kips}$$

Since the section is symmetrical, M_{nox} and M_{noy} are equal.

From the interaction diagram, $M_{nox} = 680$ ft-kips for $P_n = 1846$ kips.

Having found P_o and using ρ_g (actual), the true β value is determined as follows:

$$\frac{P_n}{P_o} = \frac{1846}{2796} = 0.66, \quad \omega = \frac{\rho_g f_y}{f'_c} = \frac{(6.24/24^2)}{5} = 0.13$$

From Fig. 7-15(a), read $\beta = 0.66$

Using the above values, Eq. (13) can now be evaluated:

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\left(\frac{\log 0.5}{\log \beta}\right)} + \left(\frac{M_{ny}}{M_{noy}}\right)^{\left(\frac{\log 0.5}{\log \beta}\right)} \leq 1.0$$

$\log 0.5 = -0.3$

$\log \beta = \log 0.66 = -0.181$

$$\frac{\log 0.5}{\log \beta} = 1.66$$

$$\left(\frac{461.5}{680}\right)^{1.66} + \left(\frac{192.3}{680}\right)^{1.66} = 0.53 + 0.12 = 0.65 < 1.0 \quad \text{O.K.}$$

This section can also be checked using the bilinear approximation.

Since $\frac{M_{ny}}{M_{nx}} < \frac{M_{noy}}{M_{nox}}$, Eq. (17) should be used.

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}}\left(\frac{1 - \beta}{\beta}\right) = \frac{461.5}{680} + \frac{192.3}{680}\left(\frac{1 - 0.66}{0.66}\right)$$

$$= 0.68 + 0.15 = 0.83 < 1.0 \quad \text{O.K.}$$

8

Moment Redistribution

UPDATE FOR THE '02 CODE

In the 2002 code, the unified design provisions for moment redistribution are now placed in 8.4, and the former provisions moved to B.8.4, where they may still be used. The new provisions apply equally to nonprestressed and prestressed concrete. Figure R8.4 in the new commentary shows how the new provisions relate to the former provisions, and to calculated available capacity for nonprestressed sections. A similar figure for prestressed sections may be found in Reference 8.1.

8.4 REDISTRIBUTION OF NEGATIVE MOMENTS IN CONTINUOUS FLEXURAL MEMBERS

Section 8.4 permits a redistribution of negative moments in continuous flexural members if the net tensile strain exceeds a specified amount. This provision recognizes the inelastic behavior of concrete structures and constitutes a move toward "limit design."

A maximum 10 percent adjustment of negative moments was first permitted in the 1963 ACI Code. Experience with the use of that provision, though satisfactory, was still conservative. The 1971 code increased the maximum adjustment percentage. The increase was justified by additional knowledge of ultimate and service load behavior obtained from tests and analytical studies.

Application of 8.4 will permit, in many cases, substantial reduction in total reinforcement required without reducing safety, and reduce reinforcement congestion in negative moment regions.

According to 8.9, continuous members must be designed to resist more than one configuration of live loads. An elastic analysis is performed for each loading configuration, and an envelope moment value is obtained for the design of each section. Thus, for any of the loading conditions considered, certain sections in a given span will reach the ultimate moment while others will have reserve capacity. Tests have shown that a structure can continue to carry additional loads if the sections that reached their moment capacities continue to rotate as plastic hinges and redistribute the moments to other sections until a collapse mechanism forms.

Recognition of this additional load capacity beyond the intended original design suggests the possibility of redesign with resulting savings in material. Section 8.4 allows a redesign by decreasing or increasing the elastic negative moments for each loading condition (with the corresponding changes in positive moment required by statics). These moment changes may be such as to reduce both the maximum positive and negative moments in the final moment envelope. In order to ensure proper rotation capacity, the net tensile strain in the sections at the support must conform to 8.4. Example 8.1 illustrates this requirement.

In certain cases, the primary benefit to be derived from 8.4 will be simply a reduction of negative moment at the supports, to avoid reinforcement congestion or reduce concrete dimensions.

Limits of applicability of 8.4 may be summarized as follows:

1. Provisions apply to continuous nonprestressed and prestressed flexural members.
2. Provisions do not apply to members designed by the approximate moments of 8.3.3, or to slab systems designed by the Direct Design Method (13.6.1.7).
3. Bending moments must be determined by analytical methods, such as moment distribution, slope deflection, etc. Redistribution is not allowed for moments determined through approximate methods.
4. Redistribution is only permitted when the net tensile strain is not less than 0.0075 (8.4.3).
5. Maximum allowable percentage increase or decrease of negative moment is equal to 1000 ε_t, but not more than 20 percent (8.4.1).
6. Adjustment of negative moments is made for each loading configuration considered. Members are then proportioned for the maximum adjusted moments resulting from all loading conditions.
7. Adjustment of negative support moments for any span requires adjustment of positive moments in the same span (8.4.2). A decrease of a negative support moment requires a corresponding increase in the positive span moment for equilibrium.
8. Static equilibrium must be maintained at all joints before and after moment redistribution.
9. In the case of unequal negative moments on the two sides of a fixed support (i.e., where adjacent spans are unequal), the difference between these two moments is taken into the support. Should either or both of these negative moments be adjusted, the resulting difference between the adjusted moments is taken into the support.
10. Moment redistribution may be carried out for additional cycles. After each cycle of redistribution, a new allowable percentage increase or decrease in negative moment is calculated. Usually, one cycle is enough.

In the new 8.4, the permissible percentage redistribution is defined in terms of the net tensile strain ε_t. A relationship exists between the steel strain ε_t at depth d and the nondimensional coefficient of resistance R_n/f'_c or $M_n/(f'_c bd^2)$. Thus, when the required nominal moment strength M_n is known, the steel strain ε_t may be found directly. The derivation follows (see Fig. 8-1):

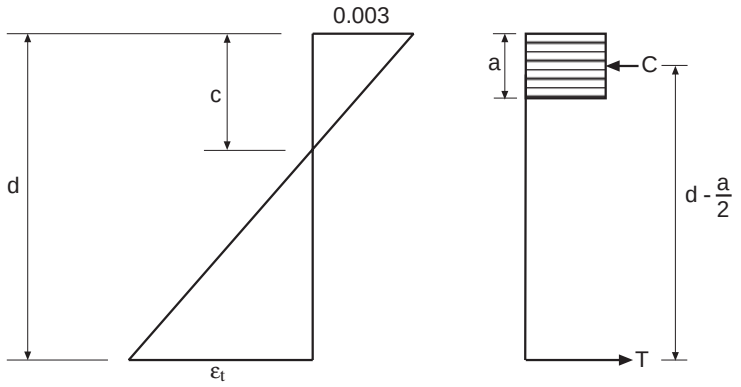


Figure 8-1 Strains and Stresses

$$\text{Let } r = \frac{0.003}{\varepsilon_t + 0.003}$$

By similar triangles: c = rd, and $a = \beta_1 c = \beta_1 rd$

$C = 0.85 f'_c ba$

$$M_n = C\left(d - \frac{a}{2}\right) = 0.85 f'_c b \beta_1 rd\left(d - \frac{\beta_1 rd}{2}\right) = f'_c bd^2 \times 0.85\beta_1 r\left(1 - \frac{\beta_1 r}{2}\right)$$

$$\frac{M_n}{f'_c bd^2} = 0.85\beta_1 r\left(1 - \frac{\beta_1 r}{2}\right) \qquad \textit{Eq. (1)}$$

Note that Eq. (1) does not involve steel strength. Equation (1) is valid for use with all types of steel, including prestressing steel. Figure 8-2 shows the relationship between permissible redistribution, net tensile strain, and coefficient of resistance.

The following procedure may be utilized to determine the permissable moment redistribution.

1. Determine factored bending moments at supports by analytical methods. Compute coefficient of resistance $M_n / f'_c bd^2$, or $M_u / \left(\phi f'_c bd^2\right)$ at supports.
2. Enter Fig. 8-2 with value of $M_u / \left(\phi f'_c bd^2\right)$. Read up to appropriate curve, and read left to find permissible percent redistribution.
3. Adjust support moments, and corresponding positive moments to satisfy equilibrium.

It usually happens that the steel provided using discrete bar sizes is somewhat more than that required. This reduces ε_t and the permissible percent redistribution slightly. However, the excess steel increases the strength far more than the change in percent redistribution. For example, referring to Fig. 8-2, the curve for 4,000 psi concrete shows a coefficient of resistance of 0.112 when $\varepsilon_t = 0.015$ and a 15 percent redistribution. If so much extra steel were provided that ε_t was reduced to 0.010, with a permissible redistribution of 10 percent, the coefficient of resistance increases from 0.112 to 0.150. Thus, a 5 percent reduction in permissible redistribution is accompanied by a 34 percent increase in strength. Consequently, it is not necessary to calculate the slight reduction in permissible redistribution, because it is offset by a far greater increase in strength.

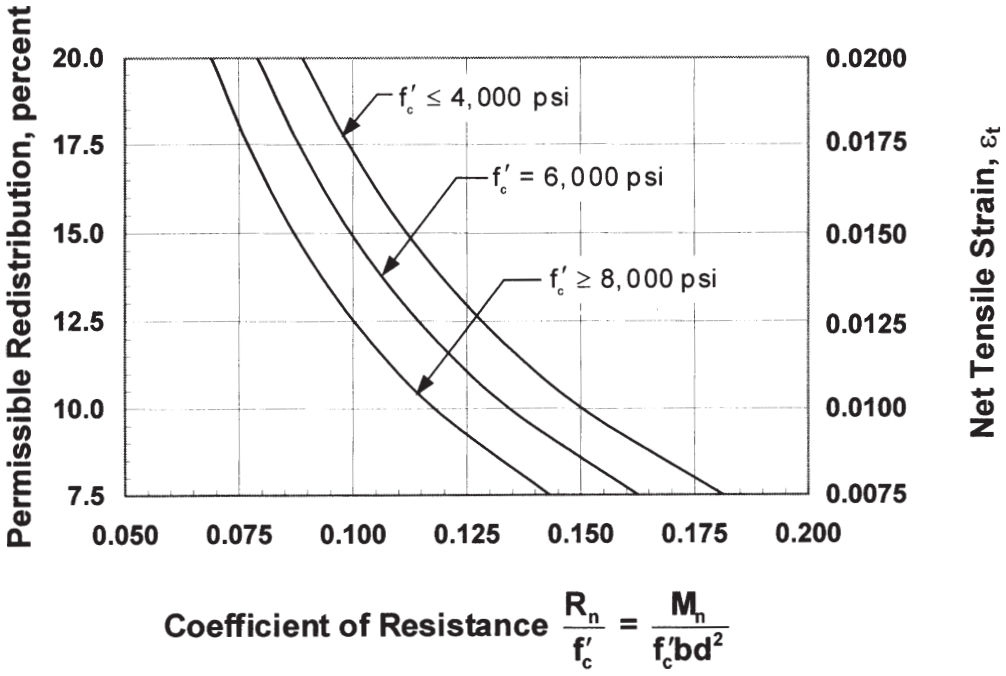


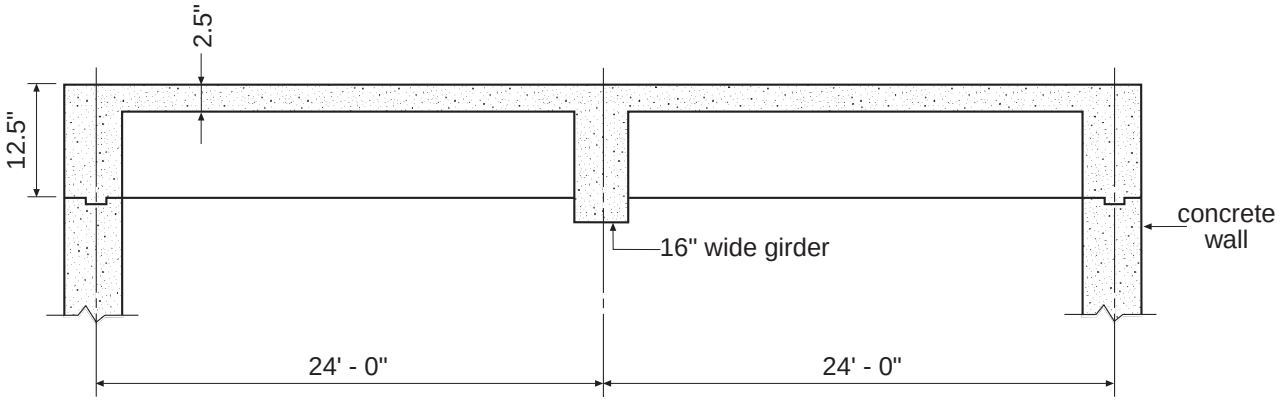
Figure 8-2 Permissible Moment Redistribution

REFERENCE

8.1 Mast, R.F., (1992), "Unified Design Provisions for Reinforced and Prestressed Concrete Flexural and Compression Members,""*ACI Structural Journal*, V. 89, pp. 185-199.

Example 8.1—Moment Redistribution

Determine required reinforcement for the one-way joist floor shown, using moment redistribution to reduce total reinforcement.



Joist-slab: 10 + 2.5 × 5 + 25 (10-in. deep form + 2.5-in. slab, 5-in. wide form spaced @ 25 in. o.c.)

f'_c = 4000 psi
f_y = 60,000 psi
DL = 80 psf
LL = 100 psf

For simplicity, continuity at concrete walls is not considered.

Calculations and Discussion	Code Reference

1. Determine factored loads.

   $U = 1.2D + 1.6L$ — *Eq. (9-2)*

   $w_d = 1.2 \times 0.08 \times 25/12 = 0.200$ kips/ft

   $w_\ell = 1.6 \times 0.10 \times 25/12 = 0.333$ kips/ft

   $w_u = 0.533$ kips/ft per joist

2. Obtain moment diagrams by elastic analysis.

   Consider three possible load patterns:

   Load pattern I: Factored DL and LL on both spans.
   Load pattern II: Factored DL and LL on one span and factored DL only on the other span.
   Load pattern III: Reverse of load pattern II.

   The elastic moment diagrams for these load cases are shown in the figure (moments shown in ft-kips).

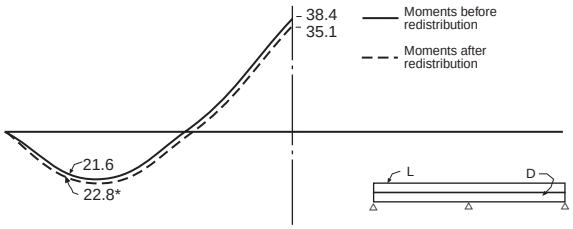


Fig. (a) Load Pattern I

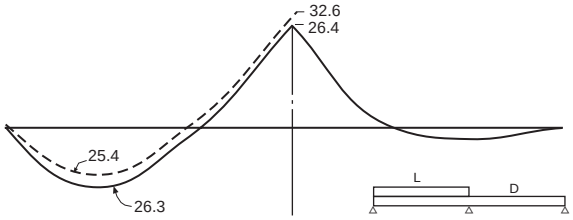


Fig. (b) Load Patterns II & III (Reverse of II)

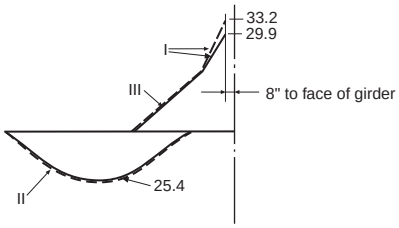
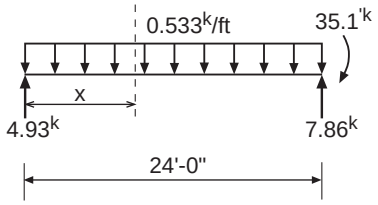


Fig. (c) Factored Moment Envelope

*Calculation of 22.8'k adjusted positive moment



V = 0 @ x = 9.25 ft

$$M \text{ @ } x = 9.25 \text{ ft} = (4.93 \times 9.25) - 0.533 \times \frac{9.25^2}{2}$$

$$= 22.8'^{k}$$

3. Redistribution of negative moments.

   a. Load pattern I:

   The intent is to decrease the negative moment at the support to obtain a new moment envelope.

   From load pattern I: M_u = -33.2 ft-kips at face of girder.

   For b = 5 in., and d = 11.5 in.:

   $$\frac{M_u}{\phi f'_c b d^2} = 0.167;$$ enter Fig. 8-2 to obtain permissible reduction = 8.5 percent

   Decreasing the negative moment M_u = -38.4 ft-kips in Fig. (a) by 8.5%, redistributed moment diagrams are obtained as shown by the dashed lines in Fig. (a).

   The maximum span moment correspondingly increases to 22.8 ft-kips by equilibrium (see calculation in figure).

b. Load pattern II:

The elastic moment diagram of Load Pattern II is compared with the redistributed moment diagram of Load Pattern I. For savings in span positive moment reinforcement, it is desirable to reduce the span positive moment of 26.3 ft-kips. This can be achieved through redistribution of the negative moment at the support by increasing it by 8.5%, to 26.4 × 1.085 = 28.6 ft-kips. As a result the positive moment is reduced from 26.3 to 25.4 ft-kips.

4. Design factored moments.

From the redistributed moment envelope, factored moments and required reinforcement are determined as shown in the following table.

Table 8-1 Summary of Final Design

Section	Load Pattern		Required Steel		Provided Steel		Redistribution, percent
	I	II	A_s (in.^{2})	ρ	A_s (in.^{2})	ρ **	
Support Moment* (ft-kips)	-29.9	—	0.64	0.0116	1-No. 6 1-No. 5	0.0130 (b = 5 in.)	-8.5
Midspan Moment (ft-kips)	—	25.4	0.50	0.0017	1-No. 4 1-No. 5	0.0018*** (b = 25 in.)	+8.5 at support

**Calculated at face of support.*

***Check* $\rho_{min} = \dfrac{3\sqrt{f'_c}}{f_y} = 0.0032$

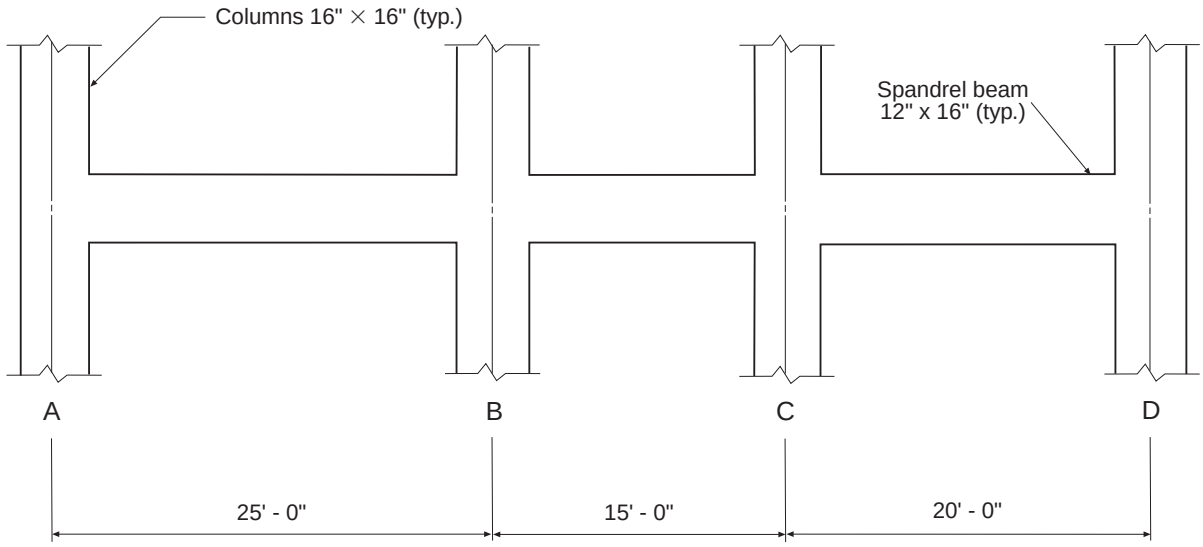
$\rho_{min} = \dfrac{200}{60{,}000} = 0.0033$ *(governs)*

***** $\rho = \dfrac{A_s}{b_w d} = \dfrac{0.51}{5 \times 11.5} = 0.0089 > \rho_{min}$

Final note: Moment redistribution has permitted a reduction of 8.5% in the negative moment. Similarly, the positive span moment has been reduced through redistribution of the negative support moment. Therefore, a total steel saving was obtained without reduction in safety.

Example 8.2—Moment Redistribution

Determine the required reinforcement for the spandrel beam at an intermediate floor level as shown, using moment redistribution to reduce total reinforcement required.



Columns = 16 × 16 in.
Story height = 10 ft
Spandrel beam = 12 × 16 in.
f'_c = 4000 psi
f_y = 60,000 psi
DL = 1167 lb/ft
LL = 450 lb/ft

Calculations and Discussion	Code Reference

1. Determine factored loads.

   $U = 1.2D + 1.6L$ — *Eq. (9-2)*

   $w_d = 1.2 \times 1.167 = 1.4$ kips/ft

   $w_\ell = 1.6 \times 0.45 = 0.72$ kips/ft

   $w_u = 2.12$ kips/ft

2. Determine the elastic bending moment diagrams for the five load patterns shown in Figs. (a) to (e) and the maximum moment envelope values for all load patterns. — *8.9.2*

   (Maximum negative moments at supports and positive mid-span moments were determined by computer analysis for each of the five loading configurations.)

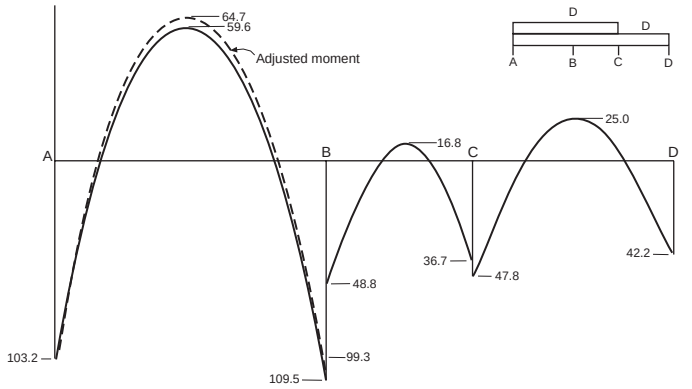


Fig. (a) Load Pattern I (moments in ft-kips)

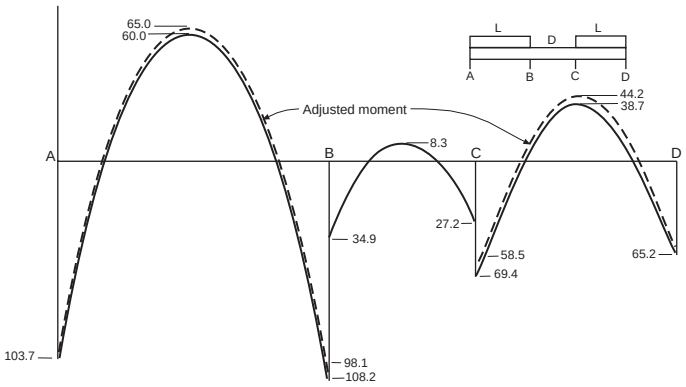


Fig. (b) Load Pattern II (moments in ft-kips)

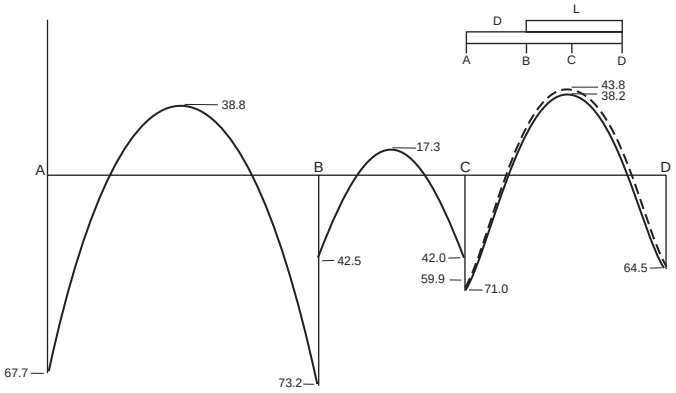


Fig. (c) Load Pattern III (moments in ft-kips)

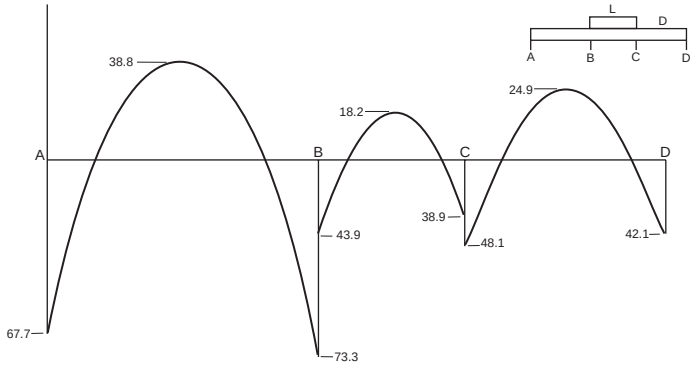


Fig.(d) Load Pattern IV (moments in ft-kips)

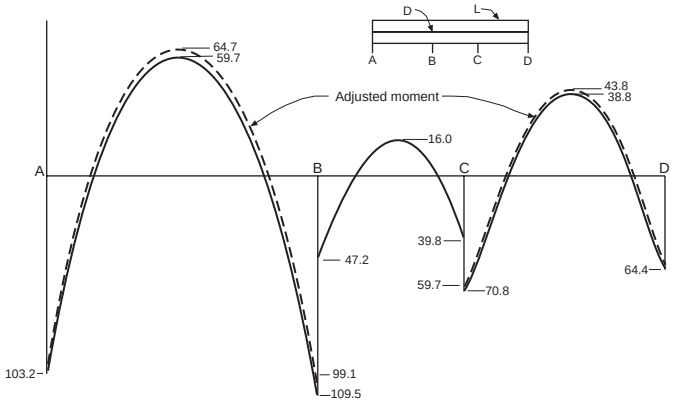


Fig. (e) Load Pattern V (moments in ft-kips)

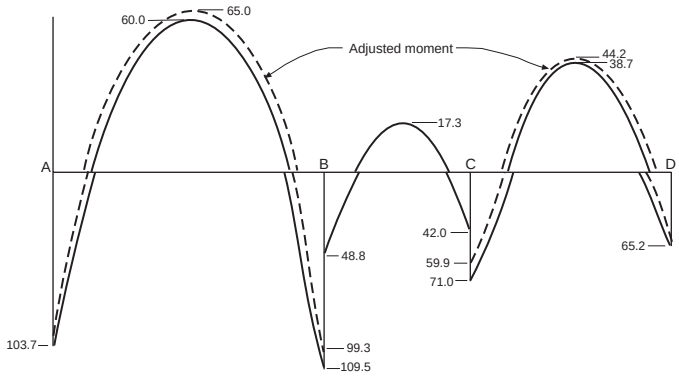


Fig. (f) Maximum Moment Envelopes for Pattern Loading
(moments in ft-kips)

3. Determine maximum allowable percentage increase or decrease in negative moments:

use d = 13.5 in.; cover = 2 in. *7.7.1*

Calculate $\frac{M_u}{\phi f'_c bd^2}$ and find percent redistribution from Fig. 8-2.

Table 8-2(a) Moment Adjustments at Supports

Support	Factored Moment M_u (ft-kips)	$\frac{M_u}{\phi f'_c bd^2}$	% Allowable Adjustment
A	-103.7	0.158	9.3
B	-109.5	0.167	8.5
C	-71.0	0.108	15.7
D	-65.2	0.099	17.4

4. Adjustment of moments.

Note: Adjustment of negative moments, either increase or decrease, is a decision to be made by the engineer. In this example, it was decided to reduce the negative moments at supports B and C and accept the increase in the corresponding positive moments, and not to adjust the negative moments at the exterior supports A and D.

Referring to Figs. (a) through (e), the following adjustments in moments are made.

Load Pattern I — Fig. (a)
M_B = -109.5 ft-kips (adjustment = 9.3%)
Reduction to M_B = -109.5 × 0.093 = -10.2 ft-kips
Adjusted M_B = -109.5 - (-10.2) = -99.3 ft-kips

Increase in positive moment in span A-B:
M_A = -103.2 ft-kips
Adjusted M_B = -99.3 ft-kips

Mid-span ordinate on line M_A to M_B = $\frac{-103.2+(-99.3)}{2}$ = -101.3 ft-kips

Moment due to uniform load = $w_u \ell^2/8$ = 2.12 × $25^2/8$ = 166.0 ft-kips
Adjusted positive moment at mid-span = 166.0 + (-101.3) = +64.7 ft-kips

The adjusted moment is approximately equal to the maximum positive moment. Similar calculations are made to determine the adjusted support and mid-span moments for span CD. Moments are also adjusted for other load patterns for spans AB and CD and the results of the additional calculations indicating the adjusted moments are shown in the following table.

Table 8-2(b) Maximum Moments for One Cycle of Redistribution (moments in ft-kips)

	Load Pattern I		Load Pattern II		Load Pattern III		Load Pattern V	
Location	M_u	M_{adj}	M_u	M_{adj}	M_u	M_{adj}	M_u	M_{adj}
A	-103.2	—	-103.7*	—	-67.7	—	-103.2	—
Mid-Span A-B	+59.6	+64.7	+60.0	+65.0*	+38.8	—	+59.7	+64.7
B	-109.5	-99.3*	-108.2	-98.1	-73.2	—	-109.3	-99.1
Mid-Span B-C	+16.8	—	+8.3	—	+17.3*	—	+16.0	—
C	-47.8	—	-69.4	-58.5	-71.0	-59.9*	-70.8	-59.7
Mid-Span C-D	+25.0	—	+38.7	+44.2*	+38.2	+43.8	+38.8	+43.8
D	-42.2	—	-65.2*	—	-64.5	—	-64.4	—

**Final design moment after redistribution.*

No adjustment is made to the elastic support moments obtained from load pattern IV. The negative support moments are less than the maximum adjusted moments from the other loading patterns and, therefore, would not control the maximum adjusted moment envelope.

5. After the adjusted moments have been determined analytically, the adjusted bending moment diagrams for each loading pattern can be determined. The adjusted moment curves were determined graphically and are indicated by the dashed lines in Figs. (a) to (e).

6. An adjusted maximum moment envelope can now be obtained from the adjusted moment curves as shown in Fig. (f) by dashed lines.

7. Final steel ratios ρ can now be obtained on the basis of the adjusted moments.

   From the redistributed moment envelopes of Fig. (f), the design factored moments and the required reinforcement are obtained as shown in Table 8-2(c).

8. Second cycle of moment redistribution.

   Should the engineer decide additional adjustment of the moments is warranted, further cycles of redistribution may be made. The procedure to be followed is similar to that used for the first cycle except that the coefficient of resistance in the second cycle is now computed based on the moment from the first cycle.

Table 8-2(c) Summary of Final Design

Location	Moment (ft-kips)	Load Case	Required		Provided	
			A_s (in.^{2})	ρ	A_s (in.^{2})	ρ**
Mid-Span A-B	+65.0	II	1.14	0.0070	1-No.5 2-No.6	0.0073
Support B*	-99.3	I	1.81	0.0112	3-No.7	0.0111
Support C*	-59.9	III	1.05	0.0065	2-No. 5 1-No. 6	0.0065
Mid-Span C-D	+44.2	II	0.76	0.0047	1-No. 4 2-No. 5	0.0051

**Centerline moments. May use moments at face of support.*

$$**\text{Check } \rho_{min} = \frac{3\sqrt{f'_c}}{f_y} = \frac{3\sqrt{4000}}{60{,}000} = 0.0032$$

$$\rho_{min} = \frac{200}{f_y} = \frac{200}{60{,}000} = 0.0033 \text{ (governs)}$$

For this design example, the design moments were not reduced to the face of the column because the intent was to illustrate moment redistribution alone. In Example 8.1, both moment redistribution and moments at the face of the support were used in the design.

9

Distribution of Flexural Reinforcement

UPDATE FOR THE '02 CODE

A significant change to 10.6.4 affecting the distribution of flexural reinforcement for control of cracking was made in the 1999 code edition. The new method directly aims at limiting the spacing of reinforcement closest to the tension face. Previously, it was assumed that crack control would be achieved by limiting the z-factors, which indirectly limited the reinforcement spacing (see Appendix 9A). The new method replaces Eq. (10-5) for computation of z-factor with an equation for the maximum reinforcement spacing s. This maximum reinforcement spacing depends only on the steel stress f_s at service load, and clear cover c_c of the flexural reinforcement from the nearest surface in tension.

GENERAL CONSIDERATIONS

Provisions of 10.6 require proper distribution of tension reinforcement in beams and one-way slabs to control flexural cracking. Structures built in the past using Working Stress Design methods and reinforcement with a yield strength of 40,000 psi or less had low tensile stresses in the reinforcement at service loads. Laboratory investigations have shown that cracking is generally in proportion to the steel tensile stress. Thus, with low tensile stresses in the reinforcement at service loads, these structures exhibited few flexural cracking problems.

With the advent of high-strength steels having yield stresses of 60,000 psi and higher, and with the use of Strength Design methods which allow higher stresses in the reinforcement, control of flexural cracking has assumed more importance. For example, if a beam were designed using Working Stress Design and a steel yield strength of 40,000 psi, the stress in the reinforcement at service loads would be about 20,000 psi. Using Strength Design and a steel yield strength of 60,000 psi, the stress at service loads could be as high as 36,000 psi. If flexural cracking is indeed proportional to steel tensile stress, then it is quite evident that the criteria for crack control must be included in the design process.

Early investigations of crack width in beams and members subject to axial tension indicated that crack width was proportional to steel stress and bar diameter, but was inversely proportional to reinforcement percentage. More recent research using deformed bars has confirmed that crack width is proportional to steel stress. However, other variables such as the quality of concrete and concrete cover were also found to be important. It should be kept in mind that there are large variations in crack widths, even in careful laboratory-controlled work. For this reason, only a simple crack control expression, designed to give reasonable reinforcing details that are in accord with laboratory work and practical experience, is presented in the code.

EFFECT OF NEW LOAD FACTORS

Steel stress at working load is the most important factor in determining crack width. As a result of the new load factors in 9.2.1, the strength requirement for tension-controlled flexural members is reduced by about 10 percent, as shown in Fig. 6-16. This has the effect of increasing steel stress at service load.

For a member with live load equal to half the dead load, the overall safety factor using ACI 318-99 is 1.67, implying a service load steel stress of 1/1.67 = 0.6 f_y at service load. This is the basis of the permission to assume service load f_s to be 60 percent of f_y (10.6.4).

Using the new code, the overall safety for the same live-to-dead ratio is 1.48, implying a steel stress at service load of 1/1.48 = 0.67 f_y. However, 10.6.4 still permits the use of f_s = 0.6 f_y for crack control. The result is that crack widths may be 12 to 13 percent wider for tension-controlled sections using the new code.

Crack widths are not predictable within 12 to 13 percent, and the 2002 code permits one to ignore this increase. It is important to note that cracks will (on average) be 12 to 13 percent wider using the new code.

A designer may wish to limit the crack widths to that experienced using the former code. This may be done by using a "default" value for steel stress of 0.67 f_y or 40 ksi, instead of 0.6 f_y. This is a designer's option; it is not a code requirement.

10.6 BEAMS AND ONE-WAY SLABS

10.6.4 Distribution of Tension Reinforcement

There are three perceived reasons that were identified early on for limiting the crack widths in concrete. These are appearance, corrosion, and water tightness. The three seldom apply simultaneously in a particular structure. Appearance is important for concrete exposed to view such as wall panels. Corrosion is important for concrete exposed to aggressive environments. Water tightness may be required for marine/sanitary structures. Appearance requires limiting of crack widths on the surface. This can be ensured by locating the reinforcement as close as possible to the surface (by using small cover) to prevent cracks from widening. Corrosion control, on the other hand, is obtained by using better quality concrete and by increasing the thickness of concrete cover. Water tightness requires severe limits on crack widths, applicable only to specialty structures. Thus, it should be recognized that a single provision, such as Eq. (10-4) of this code, may not be sufficient to address the control of cracking for all the three different reasons of appearance, corrosion, and water tightness.

There is a strong correlation between surface crack width and cover d_c, as shown in Fig. 9-1. For a particular magnitude of strain in the steel, the larger the cover, the larger will be the surface crack width affecting the appearance. From 1971 through 1995, the code specified limiting of z-factors based on the concept that the width of surface cracks needs to be limited. The specified values of z = 175 and 145 kips/in. for interior and exterior exposures, respectively, corresponded to the limiting crack widths of 0.016 and 0.013 in. (see Appendix 9A). It was assumed that by limiting the crack width to these values, one would achieve corrosion protection. But in order to comply with the specified z-value limits, the method essentially encouraged reduction of the reinforcement cover, which could be detrimental to corrosion protection. Furthermore, the method severely penalized structures with covers more than 2 in. by either reducing the spacing or the service load stress of the reinforcement.

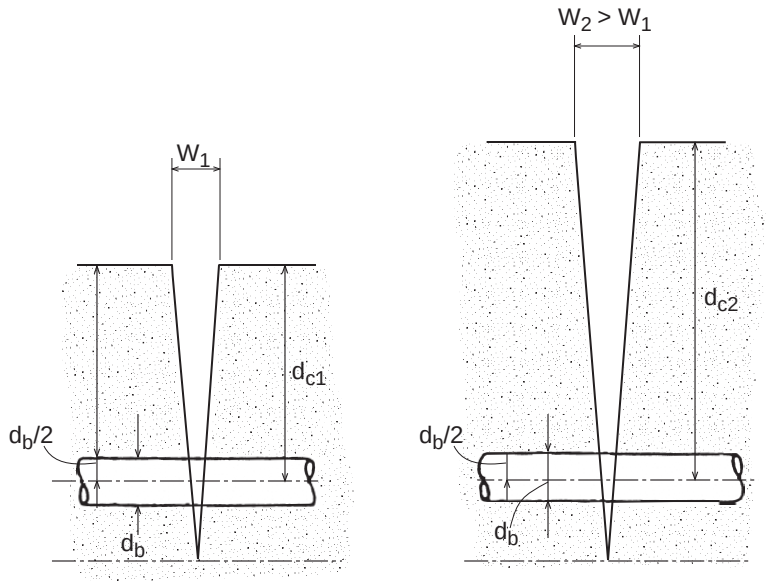


Figure 9-1 Crack Width for Different Cover Thicknesses.

The role of cracks in the corrosion of reinforcement has been found to be controversial. Research [9.1 & 9.2] shows that corrosion is not clearly correlated with surface crack widths in the range normally found with reinforcement stresses at service load level. In fact, it is weakly related to the earlier codes' surface crack width limits of 0.013 to 0.016 in. Further, it has been found that actual crack widths in structures are highly variable. A scatter of the order of ± 50% is observed. This prompted investigation of altermatives to the z factor limits for exterior and interior exposure, as given in the previous edition of the code (see Appendix 9A).

Addressing some of the limitations of the previous approach, a simple and more practical equation has been adopted starting with the 1999 code, which directly limits the maximum reinforcement spacing. The new method is intended to control surface cracks to a width that is generally acceptable in practice but may vary widely in a given structure. The new method, for this reason, does not purport to predict crack widths in the field. According to the new method, the spacing of reinforcement closest to a tension surface shall not exceed that given by

$$s = (540 / f_s) - 2.5c_c \qquad \textit{Eq. (10-4)}$$

but not greater than $12(36 / f_s)$

where s = center-to-center spacing of flexural tension reinforcement nearest to the extreme tension face, in. (where there is only one bar or wire nearest to the extreme tension face, s is the width of the extreme tension face).

f_s = calculated stress (ksi) in reinforcement at service load computed as the unfactored moment divided by the product of steel area and internal moment arm. It is permitted to take f_s as 60 percent of specified yield strength of nonprestressed reinforcement.

c_c = clear cover from the nearest surface in tension to the surface of flexural tension reinforcement, in.

Figure 9-2 compares maximum reinforcement spacing according to the 1999 edition of the code with that of the 1995 edition for slabs with a single layer of reinforcement. Note that the provisions of the 1999 edition significantly relax the spacing requirement for large cover, between 2 in. and 4 in. Note also that contrary to the 1995 provision, this spacing is independent of the exposure condition.

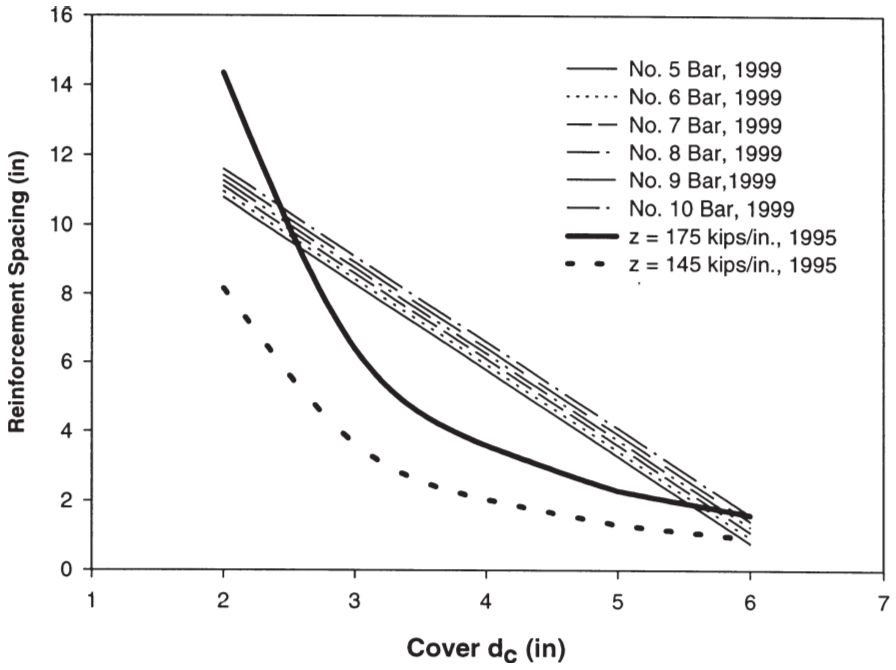


Figure 9-2 Comparison of Maximum Reinforcement Spacing for Slabs per ACI 10.6.4 1995 and 1999

For the usual case of beams with Grade 60 reinforcement with 2 in. clear cover to the tension face and assuming $f_s = 0.6 \times 60 = 36$ ksi, the maximum bar spacing is 10 in. Using the upper limit of Eq. (10-4), the maximum spacing allowed, irrespective of the cover, is 12 in. for $f_s = 36$ ksi. The spacing limitation is independent of the bar size used. Thus for a required amount of flexural reinforcement, this approach would encourage use of smaller bar sizes to satisfy the spacing criteria of Eq. (10-4).

Although Eq. (10-4) is easy to solve, it is convenient to have a table showing maximum spacing of reinforcement for various amounts of clear cover (see Table 9-1 below).

Table 9-1 Maximum Spacing of Reinforcement

	Clear Cover (in.)							
Steel Stress	3/4	1	1-1/4	1-1/2	1-3/4	2	2-1/2	3
f_S = 36 ksi	12	12	11.88	11.25	10.68	10	8.75	7.5
f_S = 40 ksi	10.8	10.8	10.38	9.75	9.18	8.5	7.25	6

10.6.5 Corrosive Environments

As described under 10.6.4, data are not available regarding crack width beyond which a danger of corrosion exists. Exposure tests indicate that concrete quality, adequate compaction, and ample cover may be of greater importance for corrosion protection than crack width at the concrete surface. The requirements of 10.6.4 do not apply to structures subject to very aggressive exposure or designed to be watertight. Special precautions are required and must be investigated for such cases.

10.6.6 Distribution of Tension Reinforcement in Flanges of T-Beams

For control of flexural cracking in the flanges of T-beams, the flexural tension reinforcement must be distributed over a flange width not exceeding the effective flange width (8.10) or 1/10 of the span, whichever is smaller. If the effective flange width is greater than 1/10 the span, some additional longitudinal reinforcement, as illustrated in Fig. 9-3, must be provided in the outer portions of the flange (see Example 9.2).

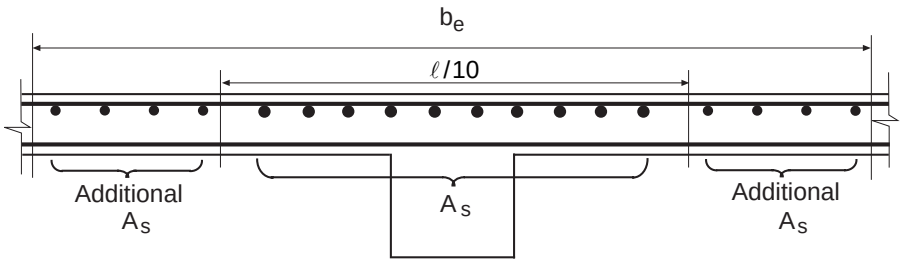


Figure 9-3 Negative Moment Reinforcement for Flanged Floor Beams

10.6.7 Crack Control Reinforcement in Deep Flexural Members

In the past, several cases of wide cracks developing on side faces of deep beams between the main reinforcement and neutral axis (Fig. 9-4(a)) have been observed. These cracks are attributed to the absence of any skin reinforcement, as a result of which cracks in the web widen more as compared to the cracks at the level of flexural tension reinforcement (Fig. 9-4(a)). For deep flexural members with effective depth d exceeding 36 in., additional longitudinal reinforcement for crack control must be distributed along the side faces over the full depth of the flexural tension zone (see Fig. 9-4(b)).

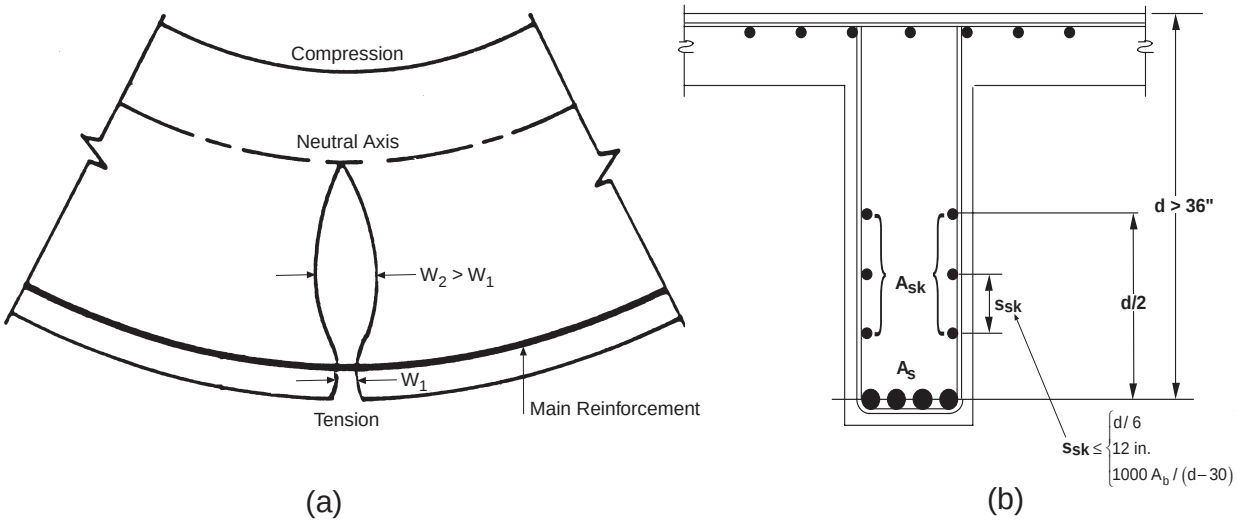


Figure 9-4 (a) Side Face Cracking and (b) Crack Control "Skin" Reinforcement for Deep Beams

The required skin reinforcement (see Fig. 9-4(b)) must be uniformly distributed along both sides of the member within the flexural tension zone, considered to extend over a distance d/2 nearest the main tension reinforcement. The spacing is not to exceed the least of d/6, 12 in., and 1000 A_b/(d-30). The [1000 A_b/(d-30)] requirement, where A_b is the area of an individual bar of wire, was added in the 2002 code, replacing a previous formula for area of skin reinforcement. Table 9-2 gives maximum spacing and minimum bar area of that spacing.

Table 9-2 Spacing and Bar Size for Skin Reinforcement

Depth, d in.	Maximum Spacing s_{sk} in.	Minimum Bar Area A_b at Maximum Spacing, $in.^2$
36	6	0.036
42	7	0.084
48	8	0.144
60	10	0.300
72	12	0.504
84	12	0.648

The total area of skin reinforcement provided on both faces need not be greater than one-half the total area of the main tensile reinforcement.

Note that the provisions of 10.6 do not directly apply to prestressed concrete members, as the behavior of a prestressed member is considerably different from that of a nonprestressed member. Requirements for proper distribution of reinforcement in prestressed members are given in Chapter 18 of the code and Part 24 of this book.

13.4 TWO-WAY SLABS

Control of flexural cracking in two-way slabs, including flat plates and flat slabs, is usually not a problem, and is not specifically covered in the code. However, 13.3.2 restricts spacing of slab reinforcement at critical moment sections to 2 times the slab thickness, and the area of reinforcement in each direction for two-way slab systems must not be less than that required for shrinkage and temperature (7.12). These limitations are intended in part to control cracking. Also, the minimum thickness requirements for two-way construction for deflection control (9.5.3) indirectly serve as a control on excessive cracking.

REFERENCES

9.1 Darwin, David et al, "Debate: Crack Width, Cover and Corrosion," *Concrete International,* Vol. 7, No. 5, May 1985, American Concrete Institute, Detroit, MI, pp. 20-35.

9.2 Oesterle, R.G., "The Role of Concrete Cover in Crack Control Crieria and Corrosion Protection," RD Serial No. 2054, Portland Cement Association, Skokie, IL, 1997.

Appendix 9A
Distribution of Tension Reinforcement According to the 1995 Code

The 1995 edition of the code required that when the yield strength of the reinforcement exceeds 40,000 psi, detailing of the flexural tension reinforcement must be such that the following equation is satisfied:

$$z = f_s\sqrt[3]{d_c A} \le 175 \text{ kip/in.} \quad \text{for interior exposures} \qquad \textit{Eq. (10-5)}$$
$$\le 145 \text{ kip/in.} \quad \text{for exterior exposures}$$

where f_s = calculated stress in the reinforcement at service loads (ksi). According to 10.6.4, it may be taken as $0.6f_y$.

d_c = thickness of the concrete cover measured from the extreme tension fiber to the center of the bar or wire located closest to it (in.)

A = effective tension area of concrete surrounding the flexural tension reinforcement and having the same centroid as that reinforcement, divided by the number of bars or wires ($in.^2$)

A plot of z versus d_cA is shown in Fig. 9A-1. When the flexural reinforcement consists of different bar or wire sizes, the number of bars or wires is computed as the total area of reinforcement divided by the area of the largest bar or wire used (Fig. 9A-2). This definition is satisfactory for all reinforcement details except bundled bars. For guidance in determining the number of equivalent bars to use in calculating A for bundled bars, consult Ref. 9A.1.

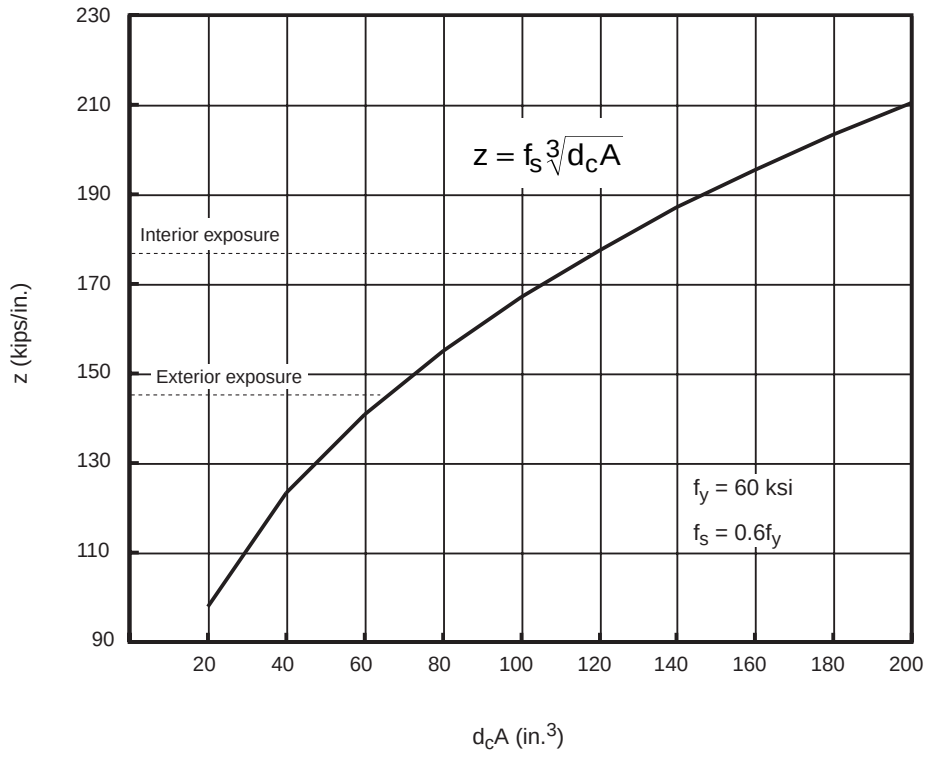
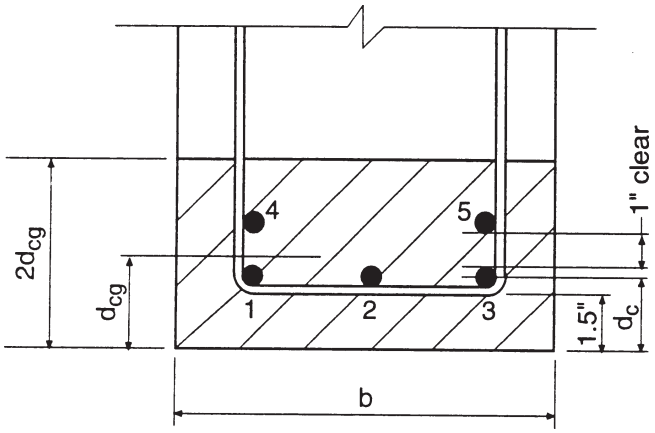


Figure 9A-1 Design Curve for z, for Distribution of Flexural Reinforcement



Examples for determining A

1. Bars 1, 2, and 3 are No. 10 (area = 1.27 in.^{2})
   Bars 4 and 5 are No. 9 (area = 1.00 in.^{2})
   Equivalent no. of No. 10 bars
   = (3 × 1.27 + 2 × 1.00)/1.27 = 4.57
   A = $2d_{cg} \times b/4.57$
   d_{cg} = center of gravity of steel

2. Bars 1, 3, 4, and 5 are No. 8 (area = 0.79 in.^{2})
   Bar 2 is No. 9 (area = 1.00 in.^{2})
   Equivalent no. of No. 9 bars
   = (4 × 0.79 + 1.00)/1.00 = 4.16
   A = $2d_{cg} \times b/4.16$
   d_{cg} = center of gravity of steel

Figure 9A-2 Effective Tension Area A, When Different Bar Sizes Are Used

Equation (10-5) is written in a form emphasizing reinforcing details rather than crack width itself. The equation will provide a distribution of the flexural reinforcement that should ensure reasonable control of flexural cracking—a large number of small diameter bars at close spacing. Equation (10-5) was derived from the Gergely-Lutz expression $w/0.076\beta = f_s\sqrt[3]{d_c A}$, where w is the crack width in units of 0.001 in., and β is the ratio of the distances to the neutral axis from the extreme tension fiber and from the centroid of the reinforcement. To simplify practical design of beams, an approximate value of 1.2 for β was used in Eq. (10-5). The numerical limitations of z = 175 and 145 kips/in. for interior and exterior exposure, respectively, correspond to limiting crack widths of 0.016 and 0.013 in. Since derivation of the original expression, additional tests have indicated that the crack width expression is also applicable to one-way slabs, with a value of β about 1.35. Accordingly, R10.6.4 suggests that the maximum value of z for one-way slabs be reduced by the ratio of 1.2/1.35, which yields z = 156 kips/in. for interior exposures and 129 kips/in. for exterior exposures. Similar adjustments for other cases where the value of β exceeds 1.2 may be required.

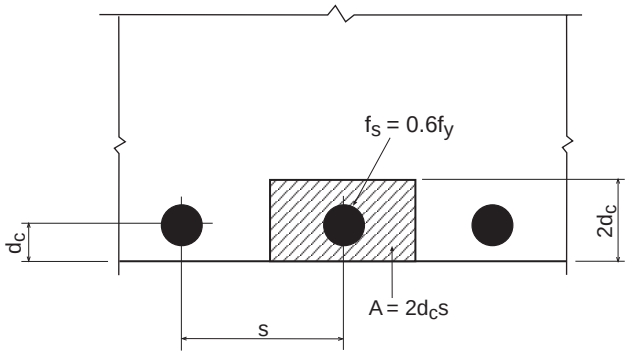


Figure 9A-3 Single Layer Reinforcement Criteria Used to Develop Tables 9A-1 and 9A-2

The required maximum spacing for crack control is determined for a single layer of reinforcement as follows (see Fig. 9A-3):

$$z = f_s\sqrt[3]{d_c A} = f_s\sqrt[3]{d_c\,(2d_c s)}$$

or

$$s = \frac{z^3}{2f_s^3 d_c^2}$$

where d_c = clear cover + 1/2 (bar diameter). The above equation was used to produce the values for maximum bar spacing in Tables 9A-1 and 9A-2 for Grade 60 reinforcement. The tables are based on a service load stress $f_s = 0.6f_y$, as permitted by 10.6.4. Computations for f_s would yield about $0.56f_y$ for a dead-to-live load ratio of 0.5, and $0.60f_y$ for a dead-to-live load ratio of 2.

Table 9A-1 Maximum Bar Spacing in Beams for Crack Control (Grade 60 Reinforcement)*

Bar Size	Exterior Exposure z = 145 kips/in.			Interior Exposure z = 175 kips/in.		
	Clear cover (in.)			Clear cover (in.)		
	1.5	2	3	1.5	2	3
No. 4	10.7	6.5	3.1	18.8	11.3	5.4
No. 5	9.9	6.1	3.0	17.5	10.7	5.2
No. 6	9.3	5.8	2.9	16.3	10.2	5.0
No. 7	8.7	5.5	2.8	15.3	9.7	4.9
No. 8	8.2	5.2	2.7	14.4	9.2	4.7
No. 9	7.7	5.0	2.6	13.5	8.7	4.5
No. 10	7.2	4.7	2.5**	12.6	8.3	4.3
No. 11	6.7	4.5	2.4**	11.8	7.8	4.2

**Values in inches, $f_S = 0.6f_Y = 36$ ksi, single layer of reinforcement.*

***Spacing less than permitted by 7.6.1*

Table 9A-2 Maximum Bar Spacing in One-Way Slabs for Crack Control (Grade 60 Reinforcement)*

Bar Size	Exterior Exposure z = 129 kips/in.				Interior Exposure z = 156 kips/in.			
	Clear cover (in.)				Clear cover (in.)			
	0.75	1	1.5	2	0.75	1	1.5	2
No. 4	—	14.7	7.5	4.5	—	—	13.3	8.0
No. 5	—	13.4	7.0	4.3	—	—	12.4	7.6
No. 6	—	12.2	6.5	4.1	—	—	11.6	7.2
No. 7	16.3	11.1	6.1	3.9	—	—	10.8	6.8
No. 8	14.7	10.2	5.8	3.7	—	—	10.2	6.5
No. 9	13.3	9.4	5.4	3.5	—	16.6	9.6	6.2
No. 10	12.0	8.6	5.0	3.3	—	15.2	8.9	5.9
No. 11	10.9	7.9	4.7	3.1	—	14.0	8.4	5.6

**Values in inches, $f_S = 0.6f_Y = 36$ ksi, single layer of reinforcement. Spacing should not exceed 3 times slab thickness nor 18 in. (7.6.5). No value indicates spacing greater than 18 in.*

REFERENCE

9A.1 Lutz, L.A., "Crack Control Factor for Bundled Bars and for Bars of Different Sizes," *ACI Journal, Proceedings* Vol. 71, No. 1, January 1974, pp. 9-10.

Example 9.1—Distribution of Reinforcement for Effective Crack Control

Assume a 16 in. wide beam with A_s (required) = 3.00 in.^{2}, and f_y = 60,000 psi. Select various bar arrangements to satisfy Eq. (10-5) for control of flexural cracking.

Calculations and Discussion	Code Reference

1. For 2-No. 11 bars (A_s = 3.12 in.^{2})

   c_c = 1.5 + 0.5 = 2.0 in. (No. 4 stirrup)

   use f_s = 0.6 f_y = 36 ksi

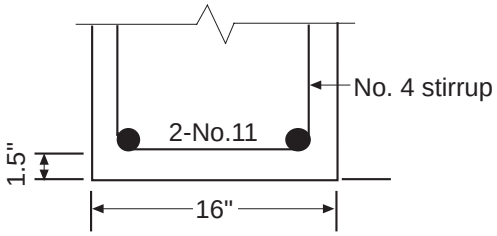
   Maximum spacing allowed,

   $$s = \frac{540}{36} - (2.5 \times 2.0) = 10 \text{ in. Eq. (10-4)}$$

   $$12\left(\frac{36}{f_s}\right) = 12 \text{ in.} > 10 \text{ in.}$$

   $$\text{spacing provided} = 16 - 2\left(1.5 + 0.5 + \frac{1.41}{2}\right)$$

   = 10.6 in. > 10 in. N.G.



2. For 4-No. 8 bars (A_s = 3.16 in.^{2})

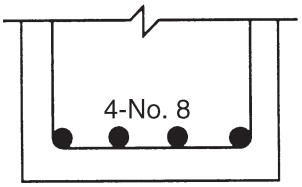
   c_c = 2.0 in. (No. 4 stirrup)

   Maximum spacing allowed,

   s = 10 in. [Eq. (10-4)]

   $$\text{spacing provided} = \frac{1}{3}\left[16 - 2\left(1.5 + 0.5 + \frac{1.0}{2}\right)\right]$$

   = 3.7 in. < 10 in. O.K.



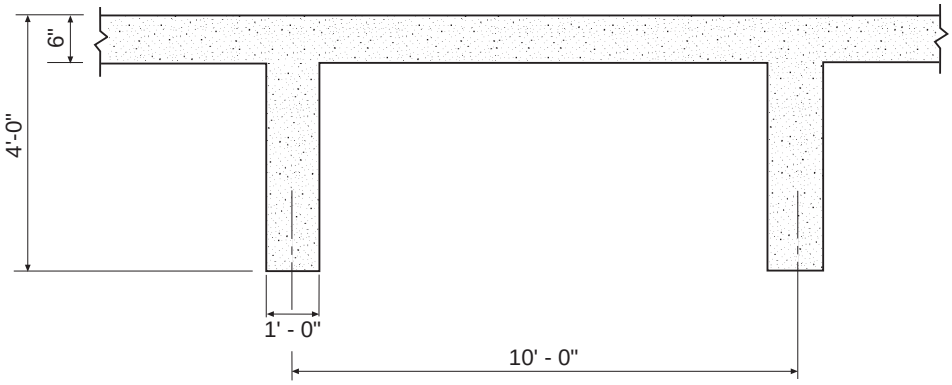
Example 9.2—Distribution of Reinforcement in Deep Flexural Member with Flanges

Select reinforcement for the T-section shown below.

Span: 50 ft continuous f'_c = 4000 psi
f_y = 60,000 psi

Service load moments:

Positive Moment	Negative Moment
M_d = +265 ft-kips	M_d = -280 ft-kips
M_ℓ = +680 ft-kips	M_ℓ = -750 ft-kips



Calculations and Discussion	Code Reference

1. Distribution of positive moment reinforcement

   a. M_u = 1.2 (265) + 1.6(680) = 1406 ft-kips — *Eq. (9-2)*

   Assuming 2 layers of No. 11 bars with 1.5 in. clear cover and No. 4 stirrups,

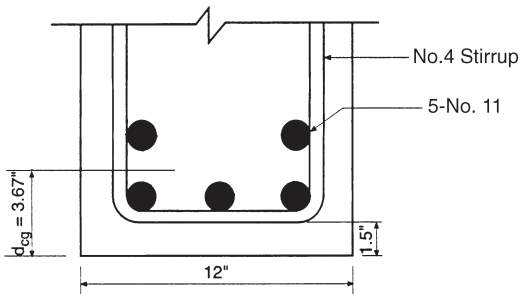
   $$d_{cg} = \frac{(3 \times 1.56)(2.71) + (2 \times 1.56)(5.12)}{(5 \times 1.56)} = 3.67 \text{ in.}$$

   d = 48 - 3.67 = 44.3 in.

   Effective width = 108 in. — *8.10.2*

   A_s required = 7.18 in.^{2}

   Try 5-No. 11 (A_s = 7.80 in.^{2})



b. Clear cover to the tension reinforcement

$c_c = 1.5 + 0.5 = 2.0$ in. *10.0*

Stress in reinforcement at service load: *10.6.4*

$$f_s = \frac{+M}{jdA_s} = \frac{(265+680)12}{0.87 \times 44.3 \times 7.80} = 37.7 \text{ ksi}$$

$$s = \left(\frac{540}{f_s}\right) - 2.5\, c_c \qquad \textit{Eq. (10-4)}$$

$$= \frac{540}{37.7} - (2.5 \times 2) = 9.3 \text{ in.}$$

$$12\left(\frac{36}{f_s}\right) = 12\left(\frac{36}{37.7}\right)$$

$= 11.5$ in. > 9.3 in. O.K.

$$\text{Spacing provided} = \frac{1}{2}\left[12 - 2\left(1.5 + 0.5 + \frac{1.41}{2}\right)\right]$$

$= 3.3$ in. < 9.3 in. O.K.

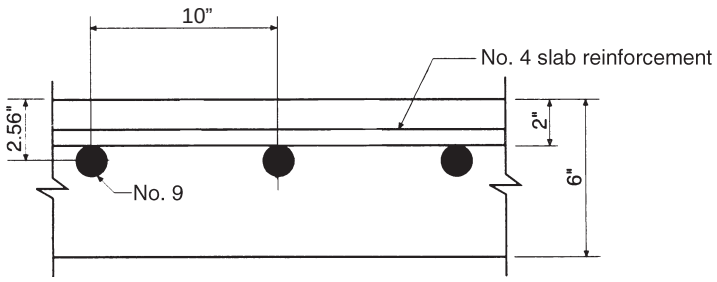
2. Distribution of negative moment reinforcement

a. $M_u = 1.2\,(280) + 1.6\,(750) = 1536$ ft-kips

A_s required $= 8.76$ in.^{2}

Effective width for tension reinforcement $= 1/10 \times 50 \times 12 = 60$ in. < 108 in. *10.6.6*

Try 9-No. 9 bars @ ≈ 10 in. ($A_s = 9.0$ in.^{2})



b. $c_c = 2.0$ in.

In lieu of computations for f_s at service load, use $f_s = 0.6f_y$ as permitted in 10.6.4

Maximum spacing allowed,

$$s = \frac{540}{36} - (2.5 \times 2.0) = 10 \text{ in.} = 10 \text{ in.} \quad \text{O.K.}$$ *Eq. (10-4)*

c. Longitudinal reinforcement in slab outside 60-in. width. *10.6.6*

For crack control outside the 60-in. width, use shrinkage and temperature reinforcement according to 7.12. *7.12*

For Grade 60 reinforcement, $A_s = 0.0018 \times 12 \times 6 = 0.130$ in.^{2}/ft

Use No. 4 bars @ 18 in. ($A_s = 0.133$ in.^{2}/ft)

3. Side face crack control reinforcement ($d > 36$ in.) *10.6.7*

$$s_{max} = \frac{d}{6} = \frac{44.3}{6} = 7.4 \text{ in.} \quad \text{(governs)}$$

$= 12$ in.

$= 1000A_b/(d - 30) = 1000 \times 0.11/(44.3 - 30) = 7.7$ in.

Use No. 3 bars @ 6 in.

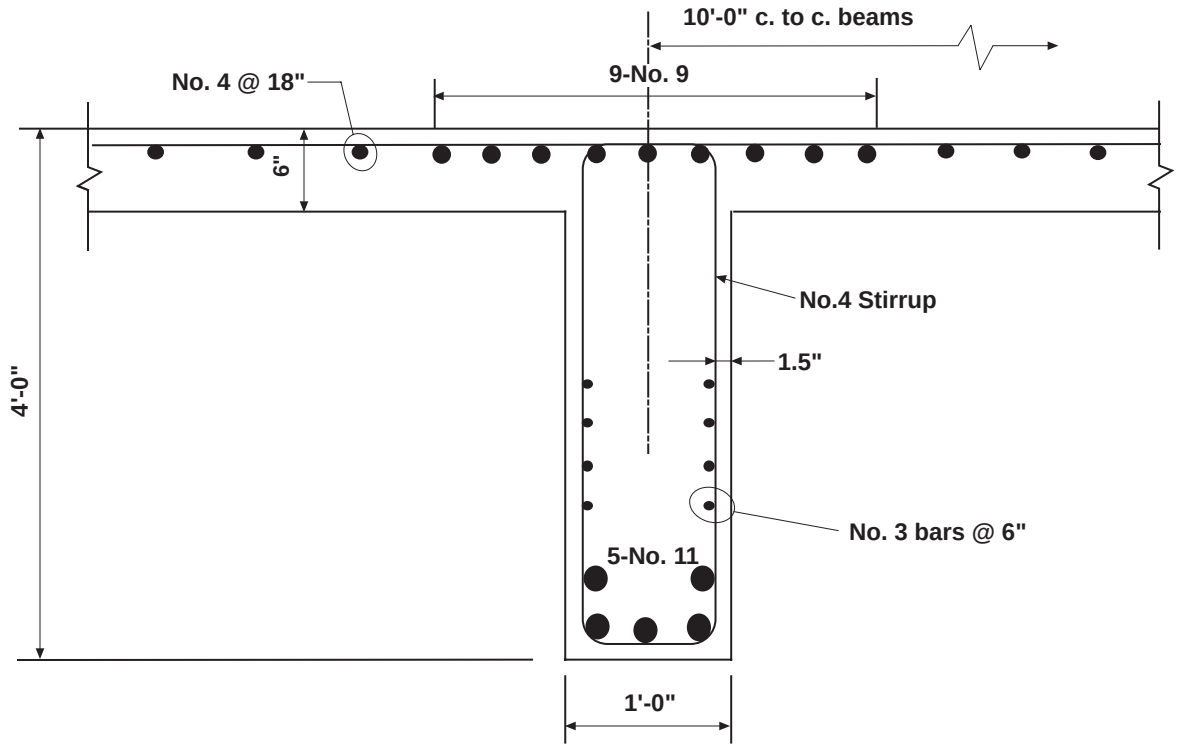
Use 4-No. 3 bars uniformly spaced along each side face of beam, extending > d/2 beyond the centroid of tension reinforcement.

Total area of skin reinforcement is 8-No. 3 bars

$8 \times 0.11 = 0.88$ in.$^2 < A_s/2 = 3.9$ in.^{2} O.K.

Provide similar No. 3 @ 6 in. in the upper half of the depth in the negative moment region.

4. Detail section as shown below.



10

Deflections

UPDATE FOR THE '02 CODE

The 2002 edition of ACI 318 expands 9.5.4 pertaining to deflections of prestressed concrete members. Specifically, existing 9.5.4.2 and 9.5.4.3 are renumbered as 9.5.4.3 and 9.5.4.4 to make room for a new 9.5.4.2. The new section addresses deflection calculations of cracked and transition (Class C and Class T) prestressed flexural members, as defined in 18.3.3. Calculations may be based on either a bilinear moment-deflection relationship or on the traditional effective moment of inertia.

GENERAL CONSIDERATIONS

The ACI code provisions for control of deflections are concerned only with deflections that occur at service load levels under static conditions and may not apply to loads with strong dynamic characteristics such as those due to earthquakes, transient winds, and vibration of machinery. Because of the variability of concrete structural deformations, designers must not place undue reliance on computed estimates of deflections. In most cases, the use of relatively simple procedures for estimating deflections is justified. In-depth treatments of the subject of deflection control, including more refined methods for computing deformations, may be found in Refs. 10.1 and 10.2.

9.5 CONTROL OF DEFLECTIONS

Two methods are given in the code for controlling deflections of one-way and two-way flexural members. Deflections may be controlled directly by limiting computed deflections [see Table 9.5(b)] or indirectly by means of minimum thickness [Table 9.5(a) for one-way systems, and Table 9.5(c) and Eqs. (9-12) and (9-13) for two-way systems.]

9.5.2.1 Minimum Thickness for Beams and One-Way Slabs (Nonprestressed)—Deflections of beams and one-way slabs supporting loads commonly experienced in buildings will normally be satisfactory when the minimum thickness from Table 9.5(a) (reproduced in Table 10-1) are met or exceeded.

The designer should especially note that this minimum thickness requirement is intended only for members **not** supporting or attached to partitions or other construction likely to be damaged by large deflections. For all other members, deflections need to be computed.

9.5.2.2 Immediate Deflection of Beams and One-Way Slabs (Nonprestressed)—Initial or short-term deflections of beams and one-way slabs occur immediately on the application of load to a structural member. The principal factors that affect the immediate deflection (see Ref. 10.3) of a member are:

a. magnitude and distribution of load,
b. span and restraint condition,
c. section properties and steel percentage,

d. material properties, and
e. amount and extent of flexural cracking.

Table 10-1 Minimum Thickness for Nonprestressed Beams and One-Way Slabs (Grade 60 Reinforcement and Normal Weight Concrete)

Member	Minimum Thickness, h			
	Simply Supported	One End Continuous	Both Ends Continuous	Cantilever
One-Way Slabs	ℓ/20	ℓ/24	ℓ/28	ℓ/10
Beams	ℓ/16	ℓ/18.5	ℓ/21	ℓ/8

(1) For f_y other than 60,000 psi, multiply tabulated values by (0.4 + f_y/100,000) e.g., for grade 40 reinforcement, multiply values by 0.80.

(2) For structural lightweight concrete, multiply values by (1.65 - 0.005w_c) but not less than 1.09, where w_c is the unit weight in lb per cu ft.

The following concrete properties strongly influence the behavior of reinforced flexural members under short-time loads: compressive strength (f'_c), modulus of elasticity (E_c) and modulus of rupture (f_r). The modulus of elasticity particularly shows more variation with concrete quality, concrete age, stress level, and rate or duration of load.

The idealized short-term deflection of a typical reinforced concrete beam is shown in Fig. 10-1. There are two distinct phases of behavior: (i) uncracked behavior, when the applied moment (M_a) is less than the cracking moment (M_{cr}); and (ii) cracked behavior, when the applied moment (M_a) is greater than the cracking moment (M_{cr}). Two different values for the moment of inertia would therefore be used for calculating the deflections: the gross moment of inertia (I_g) for the uncracked section, and the reduced moment of inertia for the cracked section (I_{cr}).

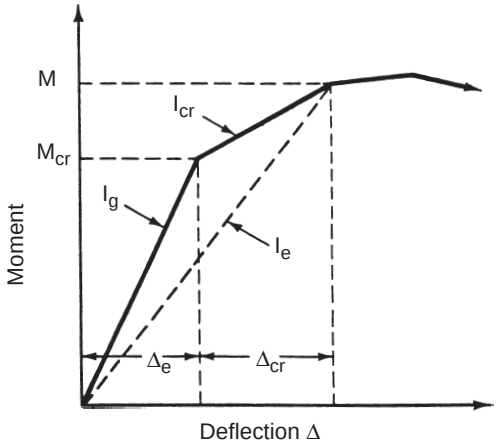


Figure 10-1 Bilinear Moment-Deflection Relationship[10.4]

For the uncracked rectangular beam shown in Fig. 10-2, the gross moment of inertia is used ($I_g = bh^3/12$). The moment of inertia of a cracked beam with tension reinforcement (I_{cr}) is computed in the following manner:

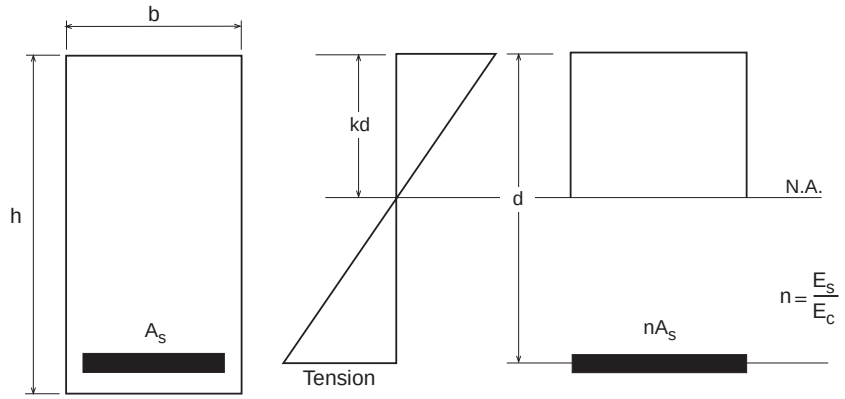


Figure 10-2 Cracked Transformed Section of Singly Reinforced Beam

Taking moment of areas about the neutral axis

$$b \times kd \times \frac{kd}{2} = nA_s (d - kd)$$

use $$B = \frac{b}{nA_s}$$

$$kd = \frac{\sqrt{2Bd + 1} - 1}{B}$$

Moment of inertia of cracked section about neutral axis,

$$I_{cr} = \frac{b (kd)^3}{3} + nA_s (d - kd)^2$$

Expressions for computing the cracked moment of inertia for sections with compression reinforcement and flanged sections, which are determined in a similar manner, are given in Table 10-2.

9.5.2.3, 9.5.2.4 Effective Moment of Inertia for Beams and One-Way Slabs (Nonprestressed)—The flexural rigidity EI of a beam may not be constant along its length because of varying amounts of steel and cracking at different sections along the beam. This, and other material related sources of variability, makes the exact prediction of deflection difficult in practice.

The effective moment of inertia of cantilevers, simple beams, and continuous beams between inflection points is given by

$$I_e = (M_{cr}/M_a)^3 I_g + [1 - (M_{cr}/M_a)^3] I_{cr} \leq I_g \qquad \textit{Eq. (9-8)}$$

where $$M_{cr} = f_r I_g/y_t \qquad \textit{Eq. (9-9)}$$

M_a = maximum service load moment (unfactored) at the stage for which deflections are being considered

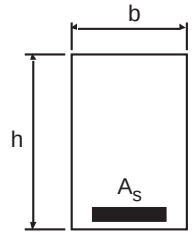
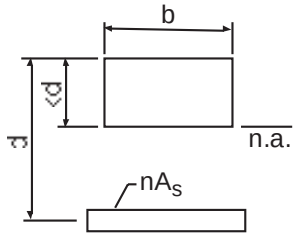
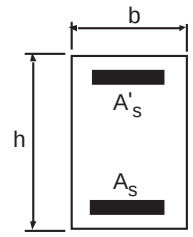
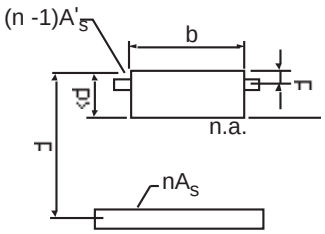
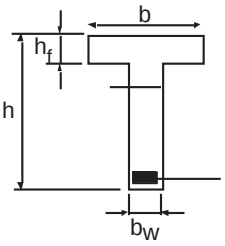
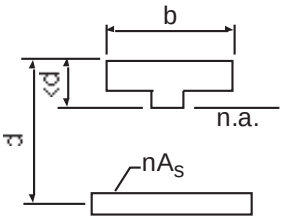
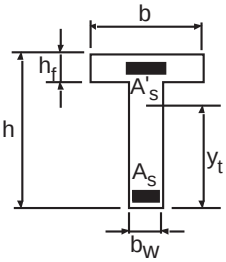
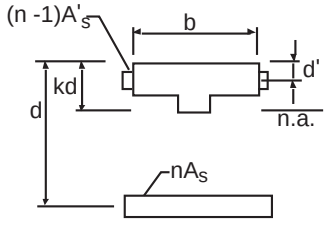
$$f_r = 7.5\sqrt{f'_c} \text{ for normal weight concrete} \qquad \textit{Eq. (9-10)}$$

For lightweight concrete, f_r is modified according to 9.5.2.3.

The effective moment of inertia I_e provides a transition between the well-defined upper and lower bounds of I_g and I_{cr} as a function of the level of cracking represented by M_a/M_{cr}. The equation empirically accounts for the effect of tension stiffening—the contribution of uncracked concrete between cracks in regions of low tensile stress.

For each load combination being considered, such as dead load or dead plus live load, deflections should be calculated using an effective moment of inertia [Eq. (9-8)] computed with the appropriate service load moment, M_a. The incremental deflection caused by the addition of load, such as live load, is then computed as the difference between deflections computed for any two load combinations.

Table 10-2 Gross and Cracked Moment of Inertia of Rectangular and Flanged Section

Gross Section	Cracked Transformed Section	Gross and Cracked Moment of Inertia
	 Without compression steel	$n = \frac{E_s}{E_c}$ $B = \frac{b}{(nA_s)}$ $I_g = \frac{bh^3}{12}$ Without compression steel $kd = \left(\sqrt{2dB+1}-1\right)/B$ $I_{cr} = b(kd)^3/3 + nA_s(d-kd)^2$
	 With compression steel	With compression steel $r = (n-1)A'_s/(nA_s)$ $kd = \left[\sqrt{2dB(1+rd'/d)+(1+r)^2}-(1+r)\right]/B$ $I_{cr} = b(kd)^3/3 + nA_s(d-kd)^2 + (n-1)A'_s(kd-d')^2$
	 Without compression steel	$n = \frac{E_s}{E_c}$ $C = b_w/(nA_s)$, $f = h_f(b-b_w)/(nA_s)$, $y_t = h - 1/2\left[(b-b_w)h_f^2 + b_w h^2\right]/\left[(b-b_w)h_f + b_w h\right]$ $I_g = (b-b_w)h_f^3/12 + b_w h^3/12 + (b-b_w)h_f(h-h_f/2-y_t)^2 + b_w h(y_t-h/2)^2$ Without compression steel $kd = \left[\sqrt{C(2d+h_f f)+(1+f)^2}-(1+f)\right]/C$ $I_{cr} = (b-b_w)h_f^3/12 + b_w(kd)^3/3 + (b-b_w)h_f(kd-h_f/2)^2 + nA_s(d-kd)^2$
	 With compression steel	With compression steel $kd = \left[\sqrt{C(2d+h_f f+2rd')+(f+r+1)^2}-(f+r+1)\right]/C$ $I_{cr} = (b-b_w)h_f^3/12 + b_w(kd)^3/3 + (b-b_w)h_f(kd-h_f/2)^2 + nA_s(d-kd)^2 + (n-1)A'_s(kd-d')2$

For prismatic members (including T-beams with different cracked sections in positive and negative moment regions), I_e may be determined at the support section for cantilevers and at the midspan section for simple and continuous spans. The use of the midspan section properties for continuous prismatic members is considered satisfactory in approximate calculations primarily because the midspan rigidity has the dominant effect on deflections. Alternatively, for continuous prismatic and nonprismatic members, 9.5.2.4 suggests using the average I_e at the critical positive and negative moment sections. The '83 commentary on 9.5.2.4 suggested the following approach to obtain improved results:

Beams with one end continuous:

$$\text{Avg. } I_e = 0.85I_m + 0.15\,(I_{cont.end}) \qquad (1)$$

Beams with both ends continuous:

$$\text{Avg. } I_e = 0.70I_m + 0.15\,(I_{e1} + I_{e2}) \qquad (2)$$

where I_m refers to I_e at the midspan section

I_{e1} and I_{e2} refer to I_e at the respective beam ends.

Moment envelopes based on the approximate moment coefficients of 8.3.3 are accurate enough to be used in computing both positive and negative values of I_e (see Example 10.2). For a single heavy concentrated load, only the midspan I_e should be used.

The initial or short-term deflection (Δ_i) for cantilevers and simple and continuous beams may be computed using the following elastic equation given in the '83 commentary on 9.5.2.4. For continuous beams, the midspan deflection may usually be used as an approximation of the maximum deflection.

$$\Delta_i = K\,(5/48)\,M_a\ell^2/E_cI_e \qquad (3)$$

where M_a is the support moment for cantilevers and the midspan moment (when K is so defined) for simple and continuous beams

ℓ is the span length as defined in 8.7.

For uniformly distributed loading w, the theoretical values of the deflection coefficient K are shown in Table 10-3.

Since deflections are logically computed for a given continuous span based on the same loading pattern as for maximum positive moment, Eq. (3) is thought to be the most convenient form for a deflection equation.

9.5.2.5 Long-Term Deflection of Beams and One-Way Slabs (Nonprestressed)—Beams and one-way slabs subjected to sustained loads experience long-term deflections. These deflections may be two to three times as large as the immediate elastic deflection that occurs when the sustained load is applied. The long-term deflection is caused by the effects of shrinkage and creep, the formation of new cracks and the widening of earlier cracks. The principal factors that affect long-term deflections (see Ref. 10.3) are:

a. stresses in concrete
b. amount of tensile and compressive reinforcement
c. member size
d. curing conditions
e. temperature
f. relative humidity
g. age of concrete at the time of loading
h. duration of loading

Table 10-3 Deflection Coefficient K

		K
1.	Cantilevers (deflection due to rotation at supports not included)	2.40
2.	Simple beams	1.0
3.	Continuous beams	$1.2-0.2\,M_o/M_a$
4.	Fixed-hinged beams (midspan deflection)	0.80
5.	Fixed-hinged beams (maximum deflection using maximum moment)	0.74
6.	Fixed-fixed beams	0.60

For other types of loading, K values are given in Ref. 8.2.

M_o = Simple span moment at midspan $\left(\frac{w\ell^2}{8}\right)$

M_a = Net midspan moment.

The effects of shrinkage and creep must be approximated because the strain and stress distribution varies across the depth and along the span of the beam. The concrete properties (strength, modulus of elasticity, shrinkage and creep) also vary with mix composition, curing conditions and time. Two approximate methods for estimating long-term deflection appear below.

ACI 318 Method

According to 9.5.2.5, additional long-term deflections due to the combined effects of shrinkage and creep from sustained loads $\Delta_{(cp+sh)}$ may be estimated by multiplying the immediate deflection caused by the sustained load $(\Delta_i)_{sus}$ by the factor λ; i.e.

$$\Delta_{(cp\,+\,sh)} = \lambda(\Delta_i)_{sus} \qquad (4)$$

where $$\lambda = \frac{\xi}{1 + 50\rho'} \qquad \text{Eq. (9-11)}$$

Values for ξ are given in Table 10-4 for different durations of sustained load. Figure R9.5.2.5 in the commentary to the code shows the variation of ξ for periods up to 5 years. The compression steel $\rho' = A'_s/bd$ is computed at the support section for cantilevers and the midspan section for simple and continuous spans. Note that sustained loads include dead load and that portion of live load that is sustained. See R9.5.1.

Table 10-4 Time-Dependent Factor ξ (9.5.2.5)

Sustained Load Duration	ξ
5 years and more	2.0
12 months	1.4
6 months	1.2
3 months	1.0

Alternate Method

Alternatively, creep and shrinkage deflections may be computed separately using the following expressions from Refs. 10.2, 10.5, and 10.6. The procedure is summarized in Section 2.6.2 of Ref.10.4.

$$\Delta_{cp} = \lambda_{cp} (\Delta_i)_{sus} \quad (5)$$

$$\Delta_{sh} = K_{sh} \phi_{sh} \ell^2 \quad (6)$$

where

$\lambda_{cp} = k_r C_t$;

$k_r = 0.85/(1 + 50\rho')$

C_t = time dependent creep coefficient (Table 2.1 or Eq. 2.7 of Ref. 10.4)

K_{sh} = shrinkage deflection constant (Table 10-5)

$\phi_{sh} = A_{sh}(\varepsilon_{sh})_t / h$

A_{sh} = shrinkage deflection multiplier (Figure 10-3 or Eq. 6.1 below)

$(\varepsilon_{sh})_t$ time dependent shrinkage ratio (Table 2.1 or Eq. 2.8 & 2.9 of Ref. 10.4)

ℓ = beam span length

h = beam depth

The ultimate value of the creep coefficient C_t,denoted as C_u, is dependent on the factors a through h listed above. Likewise, the ultimate value of the time dependent shrinkage strain depends on the varying conditions and is designated $(\varepsilon_{sh})_u$. Typical values for the two properties are discussed in Section 2.3.4 of ACI 435 (Ref. 10.4).

In Ref. 10.4, the ultimate creep coefficient is dependent on six factors:

a. relative humidity
b. age of concrete at load application
c. minimum member dimension
d. concrete consistency
e. fine aggregate content
f. air content

Standard conditions for these six variables are 40% R.H., 3 days (steam cured) or 7 days (moist cured), 6 in. least dimension, 3 in. slump, 50% fine aggregate and 6% air content. For the case of standard conditions, C_u is equal to 2.35. Correction factors are presented in Fig. 2.1 of Ref. 10.4, to adjust the value of C_u for non-standard conditions.

Two variations from standard conditions that might be encountered in normal construction are for relative humidity of 70% and load application taking place at an age of 20 days. The correction factor for the relative humidity is given by the following:

$$K_h^c = 1.27 - 0.0067H$$

where H is the relative humidity in percent. For the case of 70% relative humidity,

$$K_h^c = 1.27 - 0.0067(70) = 0.80$$

Correction for the time of load application is given in the following two expressions for steam or moist curing conditions:

$$K_{to}^c = 1.13(t^{-0.095}) \quad \text{(Steam Cured)}$$

$$K_{to}^c = 1.25(t^{-0.118}) \quad \text{(Moist Cured)}$$

where t is the age of load application in days. For t = 20 days the two equations give 0.85 and 0.88 respectively. The average is 0.865.

If it is assumed that all other conditions remain constant the ultimate creep coefficient for the condition of 70% relative humidity and load application at 20 days becomes, according to the methodology indicated:

$$C_u = (0.80)(0.865)(2.35) = 1.63$$

By comparison, the value for C_u suggested in the 1978 edition of ACI 435, based on relative humidity of 70%, age at load application of 20 days and minimum dimension of 6 in. (the standard case) was $C_u = 1.60$.

An evaluation of ultimate creep strain can also be made. In Ref. 10.4 it is stated that $(\varepsilon_{sh})_u$ is dependent on a set of factors similar to those that affect the ultimate creep coefficient. In particular, the five conditions, and their standard values, are as follows:

a. relative humidity – 40%
b. minimum member dimension – 6 in.
c. fine aggregate content – 50%
d. cement content – 1200 kg/m^3
e. air content – 6%

For standard conditions, the ultimate shrinkage strain is 780 x 10^{-6}. Keeping all applicable conditions the same as used in evaluation of the ultimate creep and use of a cement factor of 6 bags per cubic yard (335 kg/m^3), calculation of the appropriate correction factors yields:

$$K_h^s = 1.4 - 0.01H = 1.4 - (0.01)(70) = 0.70 \quad \text{(relative humidity)}$$

$$K_b^s = 0.75 + 0.000214B = 0.75 + (0.000214)(335) = 0.82 \quad \text{(cement content)}$$

Application of the product of the two corrections to the standard value gives:

$$(\varepsilon_{sh})_u = (0.70)(0.82)(780 \times 10^{-6}) = 448 \times 10^{-6}$$

This value compares with 400 x 10^{-6} suggested in the 1978 edition of ACI 435.

In summary, an estimate of the values of C_u and $(\varepsilon_{sh})_u$ can be obtained for non-prestressed flexural members using the methodology presented in Section 2.3.4 of Ref. 10.4.

Once the ultimate values for creep and shrinkage are determined, the relationships between these ultimate values and the values at earlier times can be estimated by Eqs. 2.7, 2.8 and 2.9 of ACI 435R[10.4]. The expressions are reproduced below:

$$C_t = \left(\frac{t^{0.6}}{10 + t^{0.6}}\right) C_u \qquad \textit{Eq. (2.7) of ACI 435R}$$

Where t represents time, in days, after application of load.

For moist cured concrete, the shrinkage relationship is:

$$(\varepsilon_{sh})_t = \left(\frac{t}{35 + t}\right)(\varepsilon_{sh})_u \qquad \textit{Eq. (2.8) of ACI 435R}$$

(t is in days minus 7 after placement)

and for steam cured concrete:

$$(\varepsilon_{sh})_t = \left(\frac{t}{55 + t}\right)(\varepsilon_{sh})_u \qquad \textit{Eq. (2.9) of ACI 435R}$$

(t is in days minus 3 after placement)

Comparison of the values for the time dependent creep coefficients and shrinkage strains given in Table 2.1 of ACI 435R and those that result from Eqs. 2.7, 2.8 and 2.9 shows that the values obtained by the two methods vary slightly, particularly for the lower values of time, t. Since the calculation of deflections in concrete structures involves considerable approximation, the use of the time dependent quantities obtained either from the table or from the equations is considered acceptable.

A_{sh} may be taken directly from Fig. 10-3 or computed by the following set of equations which are given in Section 2.6.2 of ACI 435:

$$\begin{aligned} A_{sh} &= 0.7 \cdot (\rho - \rho')^{\frac{1}{3}} \cdot \left(\frac{\rho - \rho'}{\rho}\right)^{\frac{1}{2}} && \text{for } \rho - \rho' \le 3.0 \\ &= 0.7 \cdot \rho^{\frac{1}{3}} && \text{for } \rho' = 0 \\ &= 1.0 && \text{for } \rho - \rho' \ge 3.0 \end{aligned} \qquad (6.1)$$

In the above equations, both ρ and ρ' are expressed in <u>percent</u>, not in decimal fraction as is usual. The ratios are also expressed in <u>percent</u> for determination of A_{sh} from Figure 10-3.

Values for the shrinkage deflection coefficient K_{sh} are given in Table 10-5, assuming equal positive and negative shrinkage curvatures with an inflection point at the quarter-point of continuous spans, which is generally satisfactory for deflection computation.

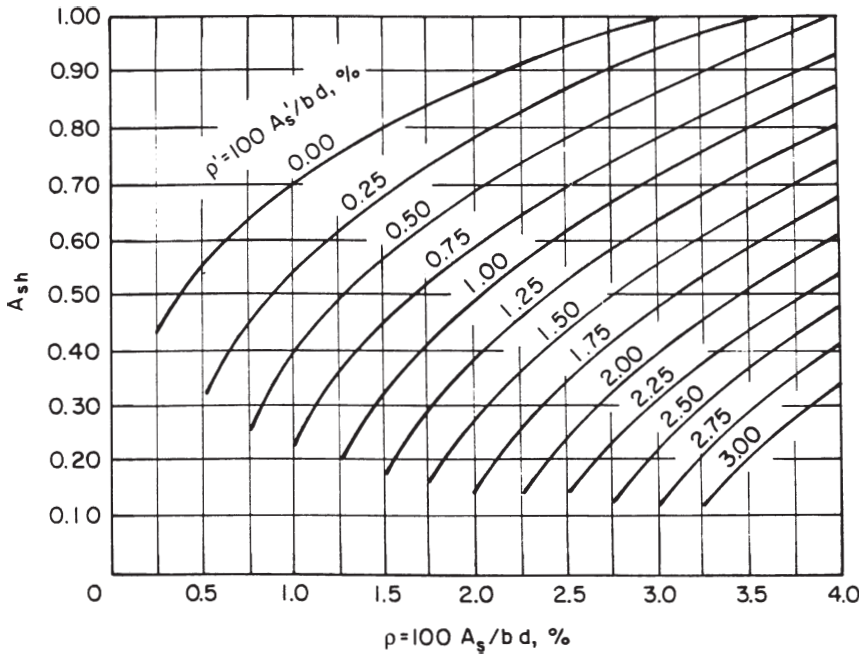


Figure 10-3 Values of A_{sh} for Calculating Shrinkage Deflection

Table 10-5 Shrinkage Deflection Coefficient K_{sh}

	K_{sh}
Cantilevers	0.50
Simple Spans	0.13
Spans with One End Continuous—Multi-Span Beams	0.09
Spans with One End Continuous—Two-Span Beams	0.08
Spans With Both Ends Continuous	0.07

The reinforcement ratios ρ and ρ' used in determining A_{sh} from Fig. 10-3, refer to the support section of cantilevers and the midspan section of simple and continuous beams. For T-beams, use $\rho = 100\ (\rho + \rho_w)/2$ and a similar calculation for any compression steel ρ' in determining A_{sh}, where $\rho_w = A_s/b_w d$. See Example 10.2.

As to the choice of computing creep and shrinkage deflections by Eq. (9-11) or separately by Eqs. (5) and (6), the combined ACI calculation is simpler but provides only a rough approximation, since shrinkage deflections are only indirectly related to the loading (primarily by means of the steel content). One case in which the separate calculation of creep and shrinkage deflections may be preferable is when part of the live load is considered as a sustained load.

All procedures and properties for computing creep and shrinkage deflections apply equally to normal weight and lightweight concrete.

9.5.2.6 Deflection Limits—Deflections computed using the preceding methods are compared to the limits given in Table 9.5(b). The commentary gives information for the correct application of these limits, including consideration of deflections occurring prior to installation of partitions.

9.5.3 Two-Way Construction (Nonprestressed)

Deflections of two-way slab systems with and without beams, drop panels, and column capitals need not be computed when the minimum thickness requirements of 9.5.3 are met. The minimum thickness requirements include the effects of panel location (interior or exterior), panel shape, span ratios, beams on panel edges, supporting columns and capitals, drop panels, and the yield strength of the reinforcing steel.

Table 10-6 Minimum Thickness of Slabs without Interior Beams (Table 9.5(c))

Yield strength, f_y psi*	Without drop panels†			With drop panels†		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams††		Without edge beams	With edge beams††	
40,000	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{40}$	$\frac{\ell_n}{40}$
60,000	$\frac{\ell_n}{30}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$
75,000	$\frac{\ell_n}{28}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{34}$	$\frac{\ell_n}{34}$

* *For values of reinforcement yield strength between the values given in the table, minimum thickness shall be determined by linear interpolation.*

† *Drop panel is defined in 13.3.7.1 and 13.3.7.2.*

†† *Slabs with beams between columns along exterior edges. The value of α for the edge beam shall not be less than 0.8.*

Section 9.5.3.2 provides minimum thickness requirements for two-way slab systems without beams between interior columns (flat plates and flat slabs). The minimum thickness is determined directly as a function of span length using Table 9.5(c). The section also provides minimum values for slabs with and without drop panels. The values given in Table 9.5(c) represent the upper limit of slab thicknesses given by Eqs. (9-12) and (9-13). The minimum thickness requirements of 9.5.3.2 are illustrated in Fig. 10-4.

Section 9.5.3.3 provides minimum thickness requirements for two-way slab systems with beams supporting all sides of a panel. It should be noted that these provisions are intended to apply only to two-way systems, that is, systems in which the ratio of long to short span is not greater than 2. For slabs that do not satisfy this limitation, Eqs. (9-12) and (9-13) may give unreasonable results. For such cases, 9.5.2 should be used.

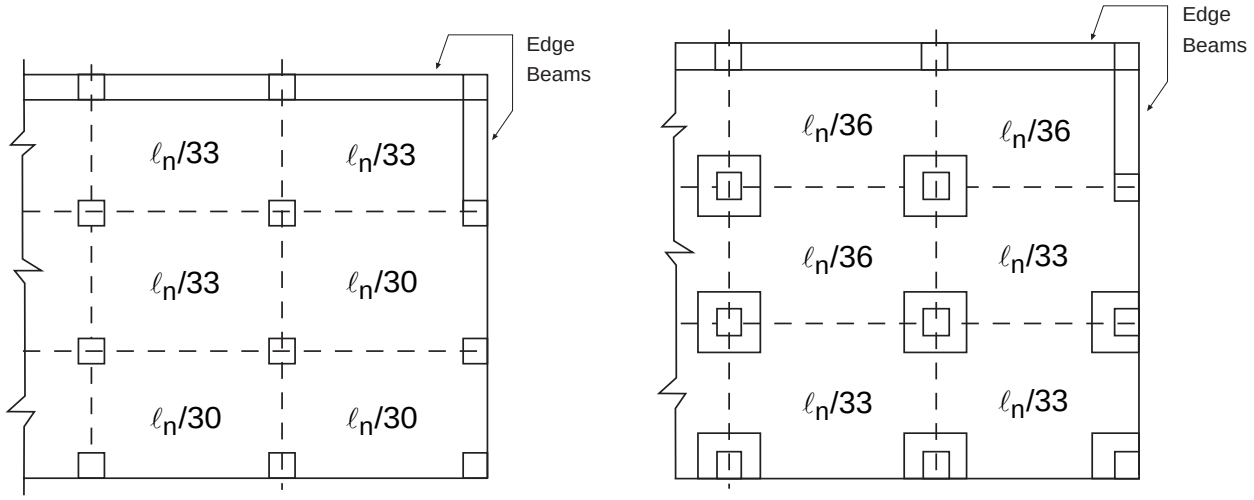


Figure 10-4 Minimum Thickness of Slabs without Interior Beams (Grade 60 Reinforcement)

Figure 10-5 may be used to simplify minimum thickness calculations for two-way slabs. It should be noted in Fig. 10-5 that the difference between the controlling minimum thickness for square panels and rectangular panels having a 2-to-1 panel side ratio is not large.

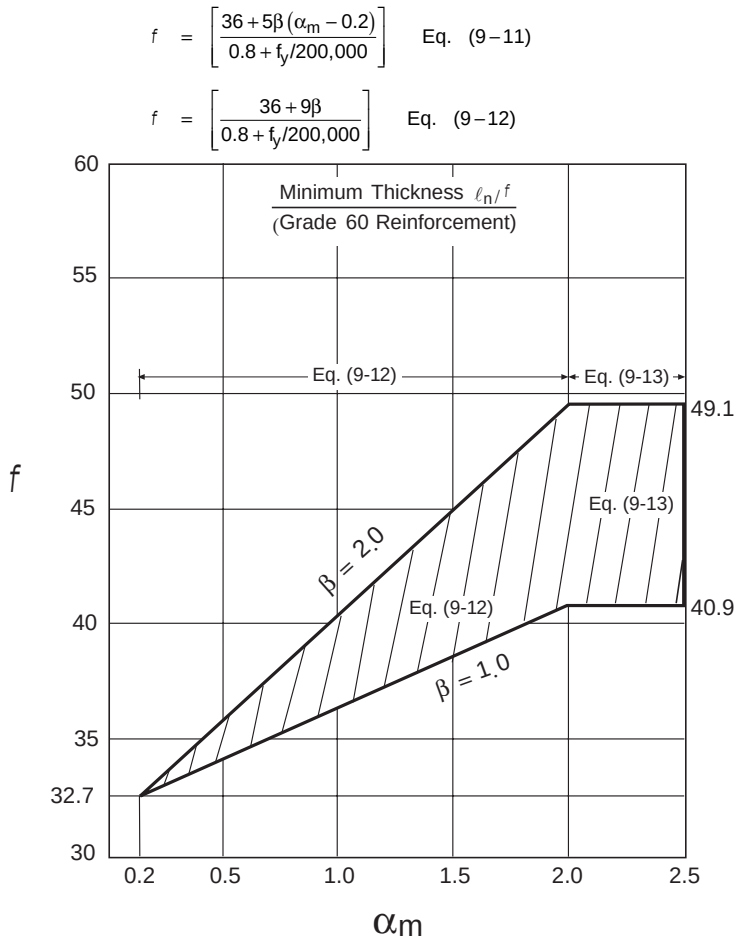


Figure 10-5 Minimum Thickness for Two-Way Beam Supported Slabs

9.5.3.4 Deflection of Nonprestressed Two-Way Slab Systems—

Initial or Short-Term Deflection: An approximate procedure[10.2, 10.7] that is compatible with the Direct Design and Equivalent Frame Methods of code Chapter 13 may be used to compute the initial or short-term deflection of two-way slab systems. The procedure is essentially the same for flat plates, flat slabs, and two-way beam-supported slabs, after the appropriate stiffnesses are computed. The midpanel deflection is computed as the sum of the deflection at midspan of the column strip or column line in one direction, Δ_{cx} or Δ_{cy}, and deflection at midspan of the middle strip in the orthogonal direction, Δ_{mx} or Δ_{my} (see Fig. 10-6). The column strip is the width on each side of column center line equal to 1/4 of the smaller panel dimension. The middle strip is the central portion of the panel which is bounded by two column strips.

For square panels,

$$\Delta = \Delta_{cx} + \Delta_{my} = \Delta_{cy} + \Delta_{mx} \qquad (7)$$

For rectangular panels, or for panels that have different properties in the two directions, the average Δ of the two directions is used:

$$\Delta = \left[\left(\Delta_{cx} + \Delta_{my}\right) + \left(\Delta_{cy} + \Delta_{mx}\right)\right]/2 \qquad (8)$$

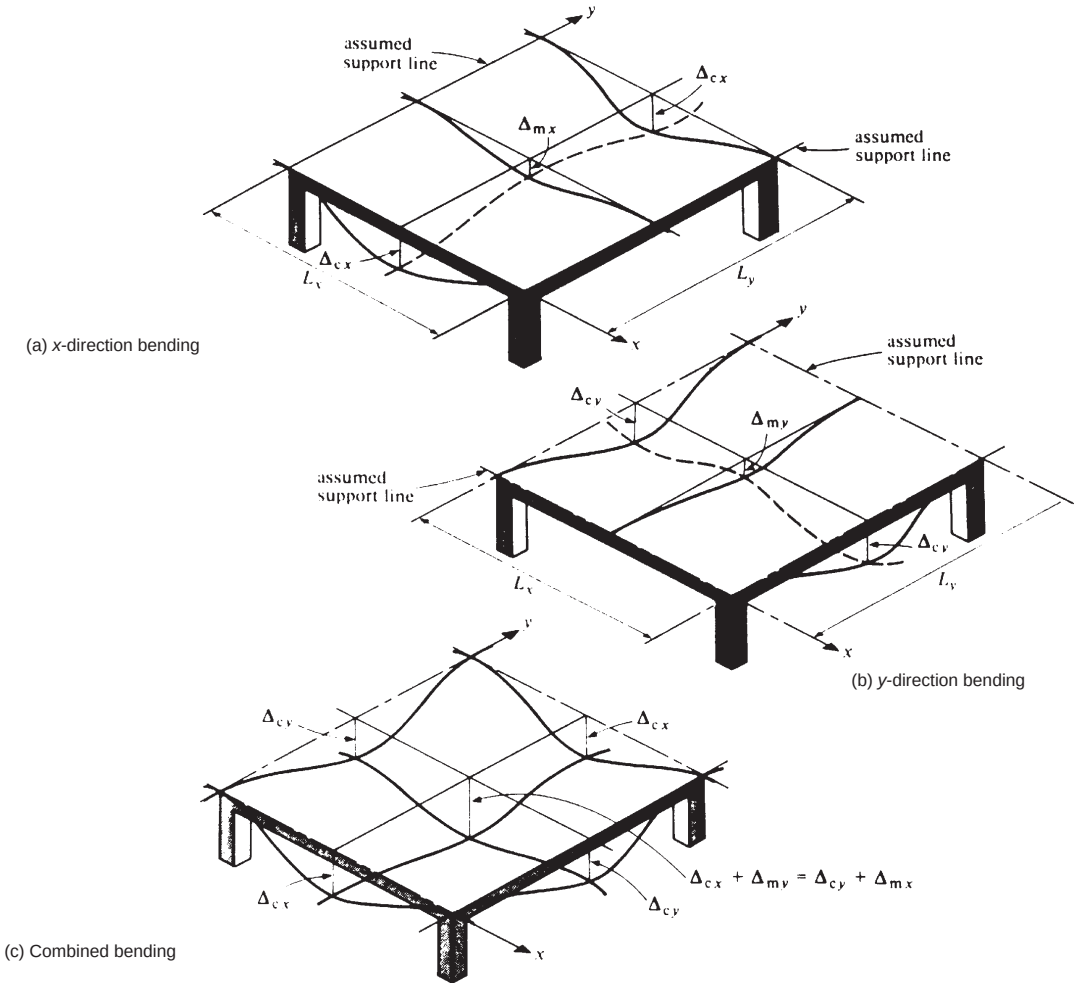


Figure 10-6 Basis for Equivalent Frame Method of Deflection Analysis of Two-Way Slab Systems, with or without Beams

The midspan deflection of the column strip or middle strip in an equivalent frame is computed as the sum of three parts: deflection of panel assumed fixed at both ends, plus deflection of panel due to the rotation at the two support lines. In the x direction, the deflections would be computed using the following expressions:

$$\Delta_{cx} = \text{Fixed } \Delta_{cx} + (\Delta\theta_1)_{cx} + (\Delta\theta_2)_{cx} \qquad \text{for column strip} \qquad (9)$$

$$\Delta_{mx} = \text{Fixed } \Delta_{mx} + (\Delta\theta_1)_{mx} + (\Delta\theta_2)_{mx} \qquad \text{for middle strip}$$

While these equations and the following discussion address only the computation of deflections in the x direction, similar computations to determine Δ_{cy} and Δ_{my} would be necessary to compute deflections in the y direction.

The first step in the process of computing Fixed Δ_{cx} and Fixed Δ_{mx} is to compute the midspan fixed-end deflection of the full-width equivalent frame under uniform loading, given by

$$\text{Fixed } \Delta_{frame} = \frac{w\ell^4}{384\, E_c I_{frame}} \qquad (10)$$

where w = uniformly distributed load × full width

The effect of different stiffnesses in positive and negative moment regions [primarily when using drop panels and/or I_e in Eq. (9-8)] can be included by using an average moment of inertia as given by Eqs. (1) and (2).

The midspan fixed-end deflection of the column and middle strips is then computed by multiplying Fixed Δ_{frame} (Eq. (10)) by the M/EI ratio of the strips (column or middle) to the full-width frame.

$$\text{Fixed } \Delta_{c,m} = (LDF)_{c,m} \text{ Fixed } \Delta_{frame} \frac{(EI)_{frame}}{(EI)_{c,m}} \quad \text{for column or middle strip} \qquad (11)$$

where $(LDF)_{c,m} = \dfrac{M_{c,m}}{M_{frame}}$ = lateral distribution factor

The distribution of the total factored static moment, M_o,, to the column and middle strips is prescribed in 13.6.4 . In particular, 13.6.4.1, 13.6.4.2 and 13.6.4.4 provide tables which allocate fractions of M_o to the interior and exterior negative moment regions and the positive moment region, respectively, for column strips. The percent of the total not designated for the column strips is allocated to the middle strips. That is, for example, if 75 percent of M_o is designated for the interior negative moment of a column strip, the corresponding moment in the middle strip will be required to sustain 25 percent of M_o. The following expressions provide linear interpolation between the tabulated values given in 13.6.4.1, 13.6.4.2 and 13.6.4.4. Note that all expressions are given as percentages of M_o:

$$M^-_{ext} = 100 - 10\beta_t + 12\beta_t (\alpha_1 \ell_2/\ell_1) (1- \ell_2/\ell_1) \qquad \text{(Exterior negative moment, \% } M_o)$$

$$M^-_{int} = 75 + 30 (\alpha_1 \ell_2/\ell_1) (1- \ell_2/\ell_1) \qquad \text{(Interior negative moment, \% } M_o)$$

$$M^+ = 60 + 30 (\alpha_1 \ell_2/\ell_1) (1.5- \ell_2/\ell_1) \qquad \text{(Positive moment, \% } M_o)$$

In application of the above expressions, if the actual value of $\alpha_1 \ell_2/\ell_1$ exceeds 1.0, the value 1.0 is used. Similarly, if β_t exceeds 2.5, the value 2.5 is used.

In order to calculate the lateral distribution factors (LDF), three cases should be considered:

a. strips for interior panels
b. strips in edge panels parallel to the edge
c. strips in edge panels perpendicular to the edge

Note that in corner panels, *Case c* is used for strips in either direction as there is an exterior negative moment at each outer panel edge. In all cases, the strip moment, used in determination of the LDFs, is taken as the average of the positive and negative moment. Thus, the following formulas are obtained for the three cases:

Case a: $LDF = {}^1/_2 (M^-_{int} + M^+)$
Case b: $LDF = {}^1/_2 (M^-_{int} + M^+)$
Case c: $LDF = {}^1/_2 [\, {}^1/_2 (M^-_{int} + M^-_{ext}) + M^+]$

These lateral distribution factors apply to column strips and are expressed in percentages of the total panel moment M_o. The corresponding factors for the middle strips are determined, in general, as follows:

$$LDF_{mid} = 100 - LDF_{col}$$

The remaining terms in Eq. (9), the midspan deflection of column strip or middle strip caused by rotations at the ends $\left((\Delta\theta_1)_{cx}, (\Delta\theta_1)_{mx}, \text{etc.}\right)$, must now be computed. If the ends of the column at the floor above and below are assumed fixed (usual case for an equivalent frame analysis) or ideally pinned, the rotation of the column at the floor in question is equal to the net applied moment divided by the stiffness of the equivalent column.

$$\theta_{frame} = \theta_c = \theta_m = \frac{(M_{net})_{frame}}{K_{ec}} \quad (12)$$

where K_{ec} = equivalent column stiffness (see 13.7.4)

The midspan deflection of the column strip subjected to a rotation of θ_1 radians at one end with the opposite end fixed is

$$(\Delta\theta_1)_c = \frac{\theta_1 \ell}{8} \quad (13)$$

The additional deflection terms for the column and middle strips would be computed similarly.

Because θ in Eq. (12) is based on gross section properties, while the deflection calculations are based on I_e, Eq. (14) may be used instead of Eq. (13) for consistency:

$$(\Delta\theta_1)_c = \theta_1 \left(\frac{\ell}{8}\right)\left(\frac{I_g}{I_e}\right)_{frame} \quad (14)$$

Direct Design Method: The deflection computation procedure described above has been expressed in terms of the equivalent frame method for moment analysis. However, it is equally suited for use with the direct design method in which coefficients are used to calculate moments at critical sections instead of using elastic frame analysis as in case of the equivalent frame method. In the direct design method, design moments are computed using clear spans. When determining deflections due to rotations at the ends of a member, these moments should theoretically be corrected to obtain moments at the center of the columns. However, this difference is generally small and may be neglected. In the case of flat plates and flat slabs, the span measured between the column centerlines is thought to be more appropriate than the clear span for deflection computations.

If all spans are equal and are identically loaded, the direct design method will give no unbalanced moments and rotations except at an exterior column. Therefore, in these cases, rotations need be considered only at the exterior columns. When live load is large compared to the dead load (not usually the case), end rotations may be computed by a simple moment-area procedure in which the effect of pattern loading may be included.

Effective Moment of Inertia: The effective moment of inertia given by Eq. (9-8) is recommended for computing deflections of partially cracked two-way construction. An average I_e of the positive and negative regions in accordance with Eqs. (1) and (2) may also be used.

For the typical cracking locations found empirically, the following moment of inertia values have been shown to be applicable in most cases.

	Case	Inertia
a.	Slabs without beams (flat plates, flat slabs)	
	(i) All dead load deflections—	I_g
	(ii) Dead-plus-live load deflections:	
	For the column strips in both directions—	I_e
	For the middle strips in both directions—	I_g
b.	Slab with beams (two-way beam-supported slabs)	
	(i) All dead load deflections—	I_g
	(ii) Dead-plus-live load deflections:	
	For the column strips in both directions—	I_g
	For the middle strips in both directions—	I_e

The I_e of the equivalent frame in each direction is taken as the sum of the column and middle strip I_e values.

Long-Term Deflection: Since the available data on long-term deflections of two-way construction is too limited to justify more elaborate procedures, the same procedures as those used for one-way members are recommended. Equation (9-11) may be used with $\xi = 2.5$ for sustained loading of five years or longer duration.

9.5.4 Prestressed Concrete Construction

Typical span-depth ratios for general use in design of prestressed members are given in the PCI Design Handbook[10.8] and summarized in Ref. 10.2 from several sources. Starting with the 2002 edition of ACI 318, the Building Code classifies prestressed concrete flexural members, in 18.3.3, as Class U (uncracked), and Classes T (transition) and C (cracked.) For Class U flexural members, deflections must be calculated based on the moment of inertia of the gross section I_g. For Classes T and C, deflections must be computed based on a cracked transformed section analysis or on a bilinear moment-deflection relationship. Reference 10.9 provides a procedure to compute deflection of cracked prestressed concrete members.

Deflection of Noncomposite Prestressed Members—The ultimate (in time) camber and deflection of prestressed members may be computed based on a procedure described in Ref. 10.2. The procedure includes the use of I_e for partially prestressed members (Ref. 10.8) as a suggested method of satisfying 18.4.2(d) for deflection analysis when the computed tensile stress exceeds the modulus of rupture, but does not exceed $12\sqrt{f'_c}$. For detailed information on the deflection of cracked prestressed beams and on the deflection of composite prestressed beams, see Refs. 10.2 and 10.9.

The ultimate deflection of noncomposite prestressed members is obtained as (Refs. 10.2 and 10.10):

$$\Delta_u = \underbrace{-\Delta_{po}}_{(1)} + \underbrace{\Delta_o}_{(2)} \underbrace{- \left[- \frac{\Delta P_u}{P_o} + (k_r C_u) \left(1 - \frac{\Delta P_u}{2P_o} \right) \right] \Delta_{po}}_{(3)} + \underbrace{(k_r C_u)\, \Delta_o}_{(4)} + \underbrace{\Delta_s}_{(5)} + \underbrace{(\beta_s k_r C_u)\, \Delta_s}_{(6)} + \underbrace{\Delta_\ell}_{(7)} + \underbrace{(\Delta_{cp})_\ell}_{(8)} \qquad (15)$$

Term (1) is the initial camber due to the initial prestressing moment after elastic loss, $P_o e$. For example, $\Delta_{po} = P_o e \ell^2 / 8 E_{ci} I_g$ for a straight tendon.

Term (2) is the initial deflection due to self-weight of the beam. $\Delta_o = 5 M_o \ell^2 / 48 E_{ci} I_g$ for a simple beam, where M_o = midspan self-weight moment.

Term (3) is the creep (time-dependent) camber of the beam due to the prestressing moment. This term includes the effects of creep and loss of prestress; that is, the creep effect under variable stress. Average values of the prestress loss ratio after transfer (excluding elastic loss), $(P_o - P_e)/P_e$, are about 0.18, 0.21, and 0.23 for normal, sand, and all-lightweight concretes, respectively. An average value of $C_u = 2.0$ might be reasonable for the creep factor due to ultimate prestress force and self-weight. The k_r factor takes into account the effect of any nonprestressed tension steel in reducing time-dependent camber, using Eq. (16). It is also used in the PCI Design Handbook[8.8] in a slightly different form.

$$k_r = 1/\left[1 + (A_s/A_{ps})\right] \quad \text{for } A_s/A_{ps} < 2 \qquad (16)$$

When $k_r = 1$, Terms (1) + (3) can be combined as:

$$-\Delta_{po} - \left[-\Delta_{po} + \Delta_{pe} + C_u\left(\frac{\Delta_{po} + \Delta_{pe}}{2}\right)\right] = -\Delta_{pe} - C_u\left(\frac{\Delta_{po} + \Delta_{pe}}{2}\right)$$

Term (4) is the creep deflection due to self-weight of the beam. Use the same value of C_u as in Term (3). Since creep due to prestress and self-weight takes place under the combined stresses caused by them, the effect of any nonprestressed tension steel in reducing the creep deformation is included in both the camber Term (3) and the deflection Term (4).

Term (5) is the initial deflection of the beam under a superimposed dead load. $\Delta_s = 5M_s\ell^2/48E_cI_g$ for a simple beam, where M_s = midspan moment due to superimposed dead load (uniformly distributed).

Term (6) is the creep deflection of the beam caused by a superimposed dead load. k_r is the same as in Terms (3) and (4), and is included in this deflection term for the same reason as in Term (4). An average value of $C_u = 1.6$ is recommended, as in Eq. (7) for nonprestressed members, assuming load application at 20 days after placement. β_s is the creep correction factor for the age of the beam concrete when the superimposed dead load is applied at ages other than 20 days (same values apply for normal as well as lightweight concrete): $\beta_s = 1.0$ for age 3 weeks, 0.96 for age 1 month, 0.89 for age 2 months, 0.85 for age 3 months, and 0.83 for age 4 months.

Term (7) is the initial live load deflection of the beam. $\Delta_\ell = 5M_\ell\ell^2/48E_cI_g$ for a simple beam under uniformly distributed live load, where M_ℓ = midspan live load moment. For uncracked members, $I_e = I_g$. For partially cracked noncomposite and composite members, see Refs. 10.2 and 10.3. See also Example 8.5 for a partially cracked case.

Term (8) is the live load creep deflection of the beam. This deflection increment may be computed as $\left(\Delta_{cp}\right)_\ell = \left(M_s/M_\ell\right) C_u\Delta_\ell$, where M_s is the sustained portion of the live load moment and $C_u = 1.6$, for load application at 20 days or multiplied by the appropriate β_s, as in Term (6).

An alternate method of calculation of long-term camber and deflection is the so-called *PCI Multiplier Method* which is presented in both Ref. 10.4 and Ref.10.8. In that procedure the various instantaneous components of camber or deflection are simply multiplied by the appropriate tabulated coefficients to obtain the additional contributions due to long term effects. The coefficients are given in Table 3.4 of Ref. 10.4 or Table 4.8.2 of Ref. 10.8.

9.5.5 Composite Construction

The ultimate (in time) deflection of unshored and shored composite flexural members may be computed by methods discussed in Refs. 10.2 and 10.10. The methods are reproduced in the following text for both unshored and shored construction. Subscripts 1 and 2 are used to refer to the slab (or effect of the slab, such as under slab dead load) and the precast beam, respectively. Examples 10.6 and 10.7 demonstrate the beneficial effect of shoring in reducing deflections.

9.5.5.1 Shored Construction—For shored composite members, where the dead and live load is resisted by the full composite section, the minimum thicknesses of Table 9.5(a) apply as for monolithic structural members.

The calculation of deflections for shored composite beams is essentially the same as for monolithic beams, except for the deflection due to shrinkage warping of the precast beam, which is resisted by the composite section after the slab has hardened, and the deflection due to differential shrinkage and creep of the composite beam. These effects are represented by Terms (3) and (4) in Eq. (17).

$$\underset{(1)}{\Delta_u} = \underset{(2)}{(\Delta_i)_{1+2} + 1.80k_r\,(\Delta_i)_{1+2}} + \underset{(3)}{\Delta_{sh}\frac{I_2}{I_c}} + \underset{(4)}{\Delta_{ds}} + \underset{(5)}{(\Delta_i)_\ell} + \underset{(6)}{(\Delta_{cp})_\ell} \qquad (17)$$

When $k_r = 0.85$ (neglecting any effect of slab compression steel) and Δ_{ds} is assumed to be equal to $(\Delta_i)_{1+2}$, Eq. (17) reduces to Eq. (18).

$$\Delta_u = \underbrace{3.53\,(\Delta_i)_{1+2}}_{(1+2+4)} + \underbrace{\Delta_{sh}\frac{I_2}{I_c}}_{(3)} + \underbrace{(\Delta_i)_\ell}_{(5)} + \underbrace{(\Delta_{cp})_\ell}_{(6)} \qquad (18)$$

Term (1) is the initial or short-term deflection of the composite beam due to slab plus precast beam dead load (plus partitions, roofing, etc.), using Eq. (3), with $M_a = M_1 + M_2$ = midspan moment due to slab plus precast beam dead load. For computing $(I_e)_{1+2}$ in Eq. (1), M_a refers to the moment $M_1 + M_2$, and M_{cr}, I_g, and I_{cr} to the composite beam section at midspan.

Term (2) is the creep deflection of the composite beam due to the dead load in Term (1), using Eq. (5). The value of C_u to be used must be a combination of that for the slab and that for the beam. In the case of the slab, an adjusted value of $C_u = 1.74$, based on the shores being removed at 10 days of age for a moist-cured slab, may be used. The beam may be older than 20 days (the standard condition) when the loads are applied, however $C_u = 1.60$ may be used conservatively. An average of the two values may be used as an approximation. For other times of load application, the adjustments can be made in similar fashion using the correction factors, β_s, listed previously in the description of Term (6) of Eq. (15). Index ρ' refers to any compression steel in the slab at midspan when computing k_r.

Term (3) is the shrinkage deflection of the composite beam after the shores are removed, due to the shrinkage of the precast beam concrete, but not including the effect of differential shrinkage and creep which is given by Term (4). Equation (6) may be used to compute Δ_{sh}. Assuming the slab is cast at a precast (steam-cured) beam concrete age of 2 months and that shores are removed about 10 days later. At that time, the shrinkage in the beam is approximately 36% of the ultimate, according to Table 2.1 of ACI 435. The shrinkage strain subsequent to that time will be $(\varepsilon_{sh})_{rem} = (1 - 0.36)\,(\varepsilon_{sh})_u$. That value should be used in Eq. (6) to calculate the deflection component in this Term.

Term (4) is the deflection due to differential shrinkage and creep. As an approximation, $\Delta_{ds} = (\Delta_i)_{1+2}$ may be used.

Term (5) is the initial or short-term live load deflection of the composite beam, using Eq. (3). The calculation of the incremental live load deflection follows the same procedure as that for a monolithic beam. This is the same as in the method described in connection with Term (9) of Eq. (19) discussed below.

Term (6) is the creep deflection due to any sustained live load, using Eq. (5). In computing this component of deflection, use of an ultimate creep coefficient, $C_u = 1.6$ is conservative. The creep coefficient may be reduced by the factor β_s defined in Term (6) of Eq. (15).

These procedures suggest using midspan values only, which may normally be satisfactory for both simple composite beams and those with a continuous slab as well. See Ref. 10.10 for an example of a continuous slab in composite construction.

9.5.5.2 Unshored Construction—For unshored composite construction, if the thickness of a nonprestressed precast member meets the minimum thickness requirements, deflections need not be computed. Section 9.5.5.2 also states that, if the thickness of an unshored nonprestressed composite member meets the minimum thickness requirements, deflections occurring after the member becomes composite need not be computed, but the long-term deflection of the precast member should be investigated for the magnitude and duration of load prior to beginning of effective composite action.

$$\Delta_u = \underbrace{(\Delta_i)_2}_{(1)} + \underbrace{0.77k_r\,(\Delta_i)_2}_{(2)} + \underbrace{0.83k_r\,(\Delta_i)_2\frac{I_2}{I_c}}_{(3)} + \underbrace{0.36\Delta_{sh}}_{(4)} + \underbrace{0.64\Delta_{sh}\,\frac{I_2}{I_c}}_{(5)}$$

$$+ \underbrace{(\Delta_i)_1}_{(6)} + \underbrace{1.22k_r\,(\Delta_i)_1\,\frac{I_2}{I_c}}_{(7)} + \underbrace{\Delta_{ds}}_{(8)} + \underbrace{(\Delta_i)_\ell}_{(9)} + \underbrace{(\Delta_{cp})_\ell}_{(10)} \qquad (19)$$

With $k_r = 0.85$ (no compression steel in the precast beam) and Δ_{ds} assumed to be equal to $0.50\,(\Delta_i)_1$, Eq. (19) reduces to Eq. (20).

$$\Delta_u = \underbrace{\left(1.65 + 0.71\,\frac{I_2}{I_c}\right)(\Delta_i)_2}_{(1+2+3)} + \underbrace{\left(0.36 + 0.64\,\frac{I_2}{I_c}\right)\Delta_{sh}}_{(4+5)}$$

$$+ \underbrace{\left(1.50 + 1.04\,\frac{I_2}{I_c}\right)(\Delta_i)_1}_{(6+7+8)} + \underbrace{(\Delta_i)_\ell}_{(9)} + \underbrace{(\Delta_{cp})_\ell}_{(10)} \qquad (20)$$

In Eqs. (19) and (20), the parts of the total creep and shrinkage occurring before and after slab casting are based on the assumption of a precast beam age of 20 days when its dead load is applied and of 2 months when the composite slab is cast.

Term (1) is the initial or short-term dead load deflection of the precast beam, using Eq. (3), with $M_a = M_2$ = midspan moment due to the precast beam dead load. For computing $(I_e)_2$ in Eq. (9-9), M_a refers to the precast beam dead load, and M_{cr}, I_g, and I_{cr} to the precast beam section at midspan.

Term (2) is the dead load creep deflection of the precast beam up to the time of slab casting, using Eq. (5), with $C_t = 0.48 \times 1.60 = 0.77$ (for 20 days to 2 months; Table 2.1 of ACI 435; for slabs cast at other than 60 days, the appropriate values from Table 2.1 should be used), and the ρ' refers to the compression steel in the precast beam at midspan when computing k_r.

Term (3) is the creep deflection of the composite beam following slab casting, due to the precast beam dead load, using Eq. (5), with the long term creep being the balance after the slab is cast, $C_t = 1.60 - 0.77 = 0.83$. As indicated in Term (3), if the slab is cast at time other than 2 months, C_t will be as determined from Table 2.1 of ACI 435 and the value of C_t to be used for this term will be found as the difference between 1.60 and the value used for Term (2). ρ' is the same as in Term (2). The ratio I_2/I_c modifies the initial stress (strain) and accounts for the effect of the composite section in restraining additional creep curvature (strain) after the composite section becomes effective. As a simple approximation, $I_2/I_c = [(I_2/I_c)_g + (I_2/I_c)_{cr}]/2$ may be used.

Term (4) is the deflection due to shrinkage warping of the precast beam up to the time of slab casting, using Eq. (6), with $(\varepsilon_{sh})_t = 0.36(\varepsilon_{sh})_u$ at age 2 months for steam cured concrete (assumed to be the usual case for precast beams) The multiplier 0.36 is obtained from Table 2.1 of Ref. 10.4. As in the previous two terms, if the slab is cast at time different from 2 months after beam manufacture, the percentage of the ultimate shrinkage strain should be adjusted to reflect the appropriate value from Table 2.1 of ACI 435. $(\varepsilon_{sh})_u = 400 \times 10^{-6}$ in./in.

Term (5) is the shrinkage deflection of the composite beam following slab casting, due to the shrinkage of the precast beam concrete, using Eq.(6), with $\varepsilon_{sh} = 0.64(\varepsilon_{sh})_u$. This term does not include the effect of differential shrinkage and creep, which is given by Term (8). I_2/I_c is the same as in Term (3).

Term (6) is the initial or short-term deflection of the precast beam under slab dead load, using Eq. (3), with the incremental deflection computed as follows: $(\Delta_i)_1 = (\Delta_i)_{1+2} - (\Delta_i)_2$, where $(\Delta_i)_2$ is the same as in Term (1). For computing $(I_e)_{1+2}$ and $(\Delta_i)_{1+2}$ in Eqs. (9-8) and (3), $M_a = M_1 + M_2$ due to the precast beam plus slab dead load at midspan, and M_{cr}, I_g, and I_{cr} refer to the precast beam section at midspan. When partitions, roofing, etc., are placed at the same time as the slab, or soon thereafter, their dead load should be included in M_1 and M_a.

Term (7) is the creep deflection of the composite beam due to slab dead load using Eq. (5), with $C_u = \beta_s$ x 1.60. For loading age of 2 months, $\beta_s = 0.89$ is the appropriate correction factor as noted in Term(6) of Eq. (15). For loading at other times, the appropriate value of β_s should be used. In this term, the initial strains, curvatures and deflections under slab dead load were based on the precast section only. Hence the creep curvatures and deflections refer to the precast beam concrete, although the composite section is restraining the creep curvatures and deflections, as mentioned in connection with Term (3). k_r is the same as in Term (2), and I_2/I_c is the same as in Term (3).

Term (8) is the deflection due to differential shrinkage and creep. As an approximation, $\Delta_{ds} = 0.50\ (\Delta_i)_1$, may be used.

Term (9) is the initial or short-term deflection due to live load (and other loads applied to the composite beam and not included in Term (6)) of the composite beam, using Eq. (4), with the incremental deflection estimated as follows: $(\Delta_i)_\ell = (\Delta_i)_{d+\ell} - (\Delta_i)_d$, based on the composite section. This is thought to be a conservative approximation, since the computed $(\Delta_i)_d$ is on the low side and thus the computed $(\Delta_i)_\ell$ is on the high side, even though the incremental loads are actually resisted by different sections (members). This method is the same as for Term (5) of Eq. (17), and the same as for a monolithic beam. Alternatively, Eq. (3) may be used with $M_a = M_1$ and $I_e = (I_c)_{cr}$ as a simple rough approximation. The first method is illustrated in Example 8.7 and the alternative method in Example 8.6.

Term (10) is the creep deflection due to any sustained live load applied to the composite beam, using Eq. (5), with $C_u = \beta_s$ x 1.60. As in the other cases, β_s is given for various load application times in the explanation of Term (6) of Eq. (15). ρ' refers to any compression steel in the slab at midspan when computing k_r. This Term corresponds to Term (6) in Eqs. (17) and (18).

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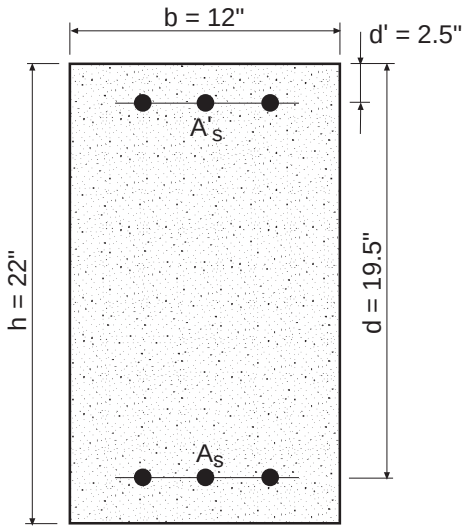
Example 10.1—Simple-Span Nonprestressed Rectangular Beam

Required: Analysis of short-term deflections, and long-term deflections at ages 3 months and 5 years (ultimate value)

Data:

f'_c = 3000 psi (normal weight concrete)
f_y = 40,000 psi
A_s = 3-No. 7 = 1.80 in.^{2}
E_s = 29,000,000 psi
$\rho = A_s/bd = 0.0077$
A'_s = 3-No. 4 = 0.60 in.^{2}
$\rho' = A'_s/bd = 0.0026$

(A'_s not required for strength)
Superimposed dead load (not including beam weight) = 120 lb/ft
Live load = 300 lb/ft (50% sustained)
Span = 25 ft



Calculations and Discussion	Code Reference

1. Minimum beam thickness, for members not supporting or attached to partitions or other construction likely to be damaged by large deflections:

$$h_{min} = \left(\frac{\ell}{16}\right)$$ *Table 9.5(a)*

multiply by 0.8 for f_y = 40,000 psi steel

$$h_{min} = \frac{25 \times 12}{16} \times 0.8 = 15 \text{ in.} < 22 \text{ in.} \quad \text{O.K.}$$

2. Moments:

$w_d = 0.120 + (12)(22)(0.150)/144 = 0.395$ kips/ft

$$M_d = \frac{w_d \ell^2}{8} = \frac{(0.395)(25)^2}{8} = 30.9 \text{ ft-kips}$$

$$M_\ell = \frac{w_\ell \ell^2}{8} = \frac{(0.300)(25)^2}{8} = 23.4 \text{ ft-kips}$$

$M_{d+\ell} = 54.3$ ft-kips

$M_{sus} = M_d + 0.50M_\ell = 30.9 + (0.50)(23.4) = 42.6$ ft-kips

3. Modulus of rupture, modulus of elasticity, modular ratio:

$$f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{3000} = 411 \text{ psi}$$ *Eq. (9-10)*

$$E_c = w_c^{1.5}\, 33\sqrt{f'_c} = (150)^{1.5}\, 33\sqrt{3000} = 3.32 \times 10^6 \text{ psi}$$ *8.5.1*

$$n_s = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3.32 \times 10^6} = 8.7$$

4. Gross and cracked section moments of inertia, using Table 8-2:

$$I_g = \frac{bh^3}{12} = \frac{(12)\,(22)^3}{12} = 10{,}650 \text{ in.}^4$$

$$B = \frac{b}{(nA_s)} = \frac{12}{(8.7)\,(1.80)} = 0.766 \text{ in.}$$

$$r = \frac{(n - 1)\,A'_s}{(nA_s)} = \frac{(7.7)\,(0.60)}{(8.7)\,(1.80)} = 0.295$$

$$kd = \left[\sqrt{2dB\,(1 + rd'/d) + (1 + r)^2} - (1 + r)\right]/B$$

$$= \left[\sqrt{(2)\,(19.5)\,(0.766)\left\{1 + \frac{0.295 \times 2.5}{19.5}\right\} + (1.295)^2} - 1.295\right]/0.766 = 5.77 \text{ in.}$$

$$I_{cr} = \frac{bk^3d^3}{3} + nA_s\,(d - kd)^2 + (n - 1)\,A'_s\,(kd - d')^2$$

$$= \frac{(12)\,(5.77)^3}{3} + (8.7)\,(1.80)\,(19.5 - 5.77)^2 + (7.7)\,(0.60)\,(5.77 - 2.5)^2$$

$$= 3770 \text{ in.}^4$$

$$\frac{I_g}{I_{cr}} = 2.8$$

5. Effective moments of inertia, using Eq. (9-7):

$$M_{cr} = \frac{f_r I_g}{y_t} = [(411)\,(10{,}650)/(11)]/(12{,}000) = 33.2 \text{ ft-kips}$$ *Eq. (9-9)*

a. Under dead load only

$\frac{M_{cr}}{M_d} = \frac{33.2}{30.9} > 1$. Hence $(I_e)_d = I_g = 10{,}650$ in.^{4}

b. Under sustained load

$$\left(\frac{M_{cr}}{M_{sus}}\right)^3 = \left(\frac{33.2}{42.6}\right)^3 = 0.473$$

$(I_e)_{sus} = (M_{cr}/M_a)^3 I_g + [1 - (M_{cr}/M_a)^3] I_{cr} \le I_g$ *Eq. (9-8)*

$= (0.473)(10{,}650) + (1 - 0.473)(3770)$

$= 7025$ in.^{4}

c. Under dead + live load

$$\left(\frac{M_{cr}}{M_{d+\ell}}\right)^3 = \left(\frac{33.2}{54.3}\right)^3 = 0.229$$

$(I_e)_{d+\ell} = (0.229)(10{,}650) + (1 - 0.229)(3770)$

$= 5345$ in.^{4}

6. Initial or short-time deflections, using Eq. (3): *9.5.2.2* *9.5.2.3*

$$(\Delta_i)_d = \frac{K\,(5/48)\,M_d \ell^2}{E_c\,(I_e)_d} = \frac{(1)\,(5/48)\,(30.9)\,(25)^2\,(12)^3}{(3320)\,(10{,}650)} = 0.098 \text{ in.}$$

K = 1 for simple spans (see Table 8-3)

$$(\Delta_i)_{sus} = \frac{K\,(5/48)\,M_{sus} \ell^2}{E_c\,(I_e)_{sus}} = \frac{(1)\,(5/48)\,(42.6)\,(25)^2\,(12)^3}{(3320)\,(7025)} = 0.205 \text{ in.}$$

$$(\Delta_i)_{d+\ell} = \frac{K\,(5/48)\,M_{d+\ell} \ell^2}{E_c\,(I_e)_{d+\ell}} = \frac{(1)\,(5/48)\,(54.3)\,(25)^2\,(12)^3}{(3320)\,(5345)} = 0.344 \text{ in.}$$

$(\Delta_i)_\ell = (\Delta_i)_{d+\ell} - (\Delta_i)_d = 0.344 - 0.098 = 0.246$ in.

Allowable Deflections (Table 9.5(b)):

Flat roofs not supporting and not attached to nonstructural elements likely to be damaged by large deflections—

$$(\Delta_i)_\ell \le \frac{\ell}{180} = \frac{300}{180} = 1.67 \text{ in.} > 0.246 \text{ in.} \quad \text{O.K.}$$

Floors not supporting and not attached to nonstructural elements likely to be damaged by large deflections—

$$(\Delta_i)_\ell \le \frac{\ell}{360} = \frac{300}{360} = 0.83 \text{ in.} > 0.246 \text{ in.} \quad \text{O.K.}$$

7. Additional long-term deflections at ages 3 mos. and 5 yrs. (ultimate value):

Combined creep and shrinkage deflections, using Eqs. (9-10) and (4):

Duration	ξ	$\lambda = \frac{\xi}{1 + 50\rho'}$	$(\Delta_i)_{sus}$ in.	$(\Delta_i)_\ell$ in.	$\Delta_{cp} + \Delta_{sh} = \lambda(\Delta_i)_{sus}$ in.	$\Delta_{cp} + \Delta_{sh} + (\Delta_i)_\ell$ in.
5-years	2.0	1.77	0.205	0.246	0.363	0.61
3-months	1.0	0.89	0.205	0.246	0.182	0.43

Separate creep and shrinkage deflections, using Eqs. (5) and (6):

For $\rho = 0.0077$; $\rho' = 0.0026$

For $\rho = 100\rho = 0.77$ and $\rho' = 100\rho' = 0.26$, read $A_{sh} = 0.455$ (Fig. 8-3) and $K_{sh} = 0.125$ for simple spans (Table 8-5).

Duration	C_t	$\lambda_{cp} = \frac{0.85C_t}{1+50\rho'}$	$\Delta_{cp} = \lambda_{cp}(\Delta_i)_{sus}$ in.	ε_{sh} in./in.	$\phi_{sh} = \frac{A_{sh}\varepsilon_{sh}}{h}$ 1/in.	$\Delta_{sh} = K_{sh}\phi_{sh}\ell^2$ in.	$\Delta_{cp} + \Delta_{sh} + (\Delta_i)_\ell$ in.
5-years	1.6 (ultimate)	1.20	0.246	400×10^{-6}	$\frac{0.455 \times 400 \times 10^{-6}}{22} = 8.27 \times 10^{-6}$	$\frac{1}{8} \times 8.27 \times 10^{-6} \times (25 \times 12)^2 = 0.093$	$0.246 + 0.093 + 0.246 = 0.59$
3-months	$0.56 \times 1.6 = 0.9$	0.68	0.14	$0.6 \times 400 \times 10^{-6} = 240 \times 10^{-6}$	4.96×10^{-6}	= 0.0558	$0.14 + 0.056 + 0.246 = 0.44$

Allowable Deflection Table 9.5(b):

Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections (very stringent limitation).

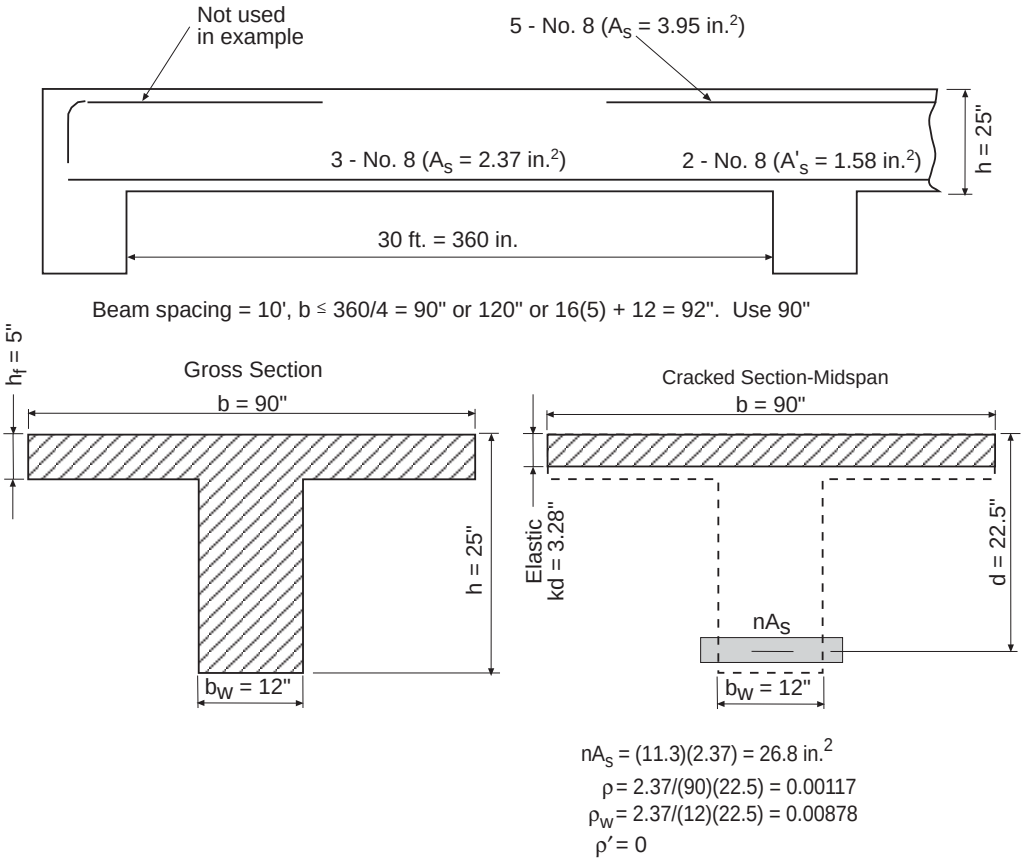
$$\Delta_{cp} + \Delta_{sh} + (\Delta_i)_\ell \le \frac{\ell}{480} = \frac{300}{480} = 0.63 \text{ in.} \quad \text{O.K. by both methods}$$

Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections.

$$\Delta_{cp} + \Delta_{sh} + (\Delta_i)_\ell \le \frac{\ell}{240} = \frac{300}{240} = 1.25 \text{ in.} \quad \text{O.K. by both methods}$$

Example 10.2—Continuous Nonprestressed T-Beam

Required: Analysis of short-term and ultimate long-term deflections of end-span of multi-span beam shown below.



Data:

f'_c = 4000 psi (sand-lightweight concrete)
f_y = 50,000 psi
w_c = 120 pcf

Beam spacing = 10 ft
Superimposed Dead Load (not including beam weight) = 20 psf
Live Load = 100 psf (30% sustained)

(A'_s is not required for strength)

Beam will be assumed to be continuous at one end only for h_{min} in Table 9.5(a), for Avg. I_e in Eq. (1), and for K_{sh} in Eq. (6), since the exterior end is supported by a spandrel beam. The end span might be assumed to be

Calculations and Discussion	Code Reference

1. Minimum thickness, for members not supporting or attached to partitions or other construction likely to be damaged by large deflections:

$h_{min} = \frac{\ell}{18.5}$ Table 9.5(a)

Modifying factors = 1.09 for w_c = 120 pcf [footnote (a) Table 9.5(a)]

= 0.9 for f_y = 50,000 psi [footnote (b) Table 9.5(a)]

$h_{min} = \left(\frac{360}{18.5}\right)(0.90)(1.09) = 19.1 \text{ in.} < h = 25 \text{ in.}$ O.K.

2. Loads and moments:

$w_d = (20 \times 10) + (120)(12 \times 20 + 120 \times 5)/144 = 900$ lb/ft

$w_\ell = (100 \times 10) = 1000$ lb/ft

In lieu of a moment analysis, the ACI approximate moment coefficients may be used as follows: Pos. $M = w\ell_n^2/14$ for positive I_e and maximum deflection, Neg. $M = w\ell_n^2/10$ for negative I_e. 8.3.3

a. Positive Moments

$$\text{Pos. } M_d = \frac{w_d \ell_n^2}{14} = \frac{(0.900)(30)^2}{14} = 57.9 \text{ ft-kips}$$

$$\text{Pos. } M_\ell = \frac{(1.000)(30)^2}{14} = 64.3 \text{ ft-kips}$$

$$\text{Pos. } M_{d+\ell} = 57.9 + 64.3 = 122.2 \text{ ft-kips}$$

$$\text{Pos. } M_{sus} = M_d + 0.30M_\ell = 57.9 + (0.30)(64.3) = 77.2 \text{ ft-kips}$$

b. Negative Moments

$$\text{Neg. } M_d = \frac{w_d \ell_n^2}{10} = \frac{(0.900)(30)^2}{10} = 81.0 \text{ ft-kips}$$

$$\text{Neg. } M_\ell = \frac{(1.000)(30)^2}{10} = 90.0 \text{ ft-kips}$$

$$\text{Neg. } M_{d+\ell} = 81.0 + 90.0 = 171.0 \text{ ft-kips}$$

$$\text{Neg. } M_{sus} = M_d + 0.30M_\ell = 81.0 + (0.30)(90.0) = 108.0 \text{ ft-kips}$$

3. Modulus of rupture, modulus of elasticity, modular ratio:

$f_r = (0.85)(7.5)\sqrt{f'_c} = 6.38\sqrt{4000} = 404$ psi (0.85 for 9.5.2.3(b)) Eq. (9-10)

$E_c = w_c^{1.5}\, 33\sqrt{f'_c} = (120)^{1.5}\, 33\sqrt{4000} = 2.74 \times 10^6$ psi — *8.5.1*

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{2.74 \times 10^6} = 10.6$$

4. Gross and cracked section moments of inertia:

   a. Positive moment section

$$y_t = h - (1/2)\,[(b - b_w)\, h_f^2 + b_w h^2]/[(b - b_w)\, h_f + b_w h]$$

$$= 25 - (1/2)\,[(78)\,(5)^2 + (12)(25)^2]/[(78)\,(5) + (12)\,(25)]$$

$$= 18.15 \text{ in.}$$

$$I_g = (b - b_w)\, h_f^3/12 + b_w h^3/12 + (b - b_w)\, h_f\, (h - h_f/2 - y_t)^2 + b_w h\, (y_t - h/2)^2$$

$$= (78)\,(5)^3/12 + (12)\,(25)^3/12 + (78)\,(5)\,(25 - 2.5 - 18.15)^2$$

$$+ (12)\,(25)\,(18.15 - 12.5)^2 = 33{,}390 \text{ in.}^4$$

$$B = \frac{b}{nA_s} = \frac{90}{(10.6)\,(2.37)} = 3.58/\text{in.} \quad \text{(Table 10-2)}$$

$$kd = \frac{\sqrt{2dB + 1} - 1}{B} = \frac{\sqrt{(2)\,(22.5)\,(3.58) + 1} - 1}{3.58}$$

$$= 3.28 \text{ in.} < h_f = 5 \text{ in.}$$

Hence, treat as a rectangular compression area.

$$I_{cr} = bk^3d^3/3 + nA_s\,(d - kd)^2 = (90)\,(3.28)^3/3 + (10.6)\,(2.37)\,(22.5 - 3.28)^2$$

$$= 10{,}340 \text{ in.}^4$$

   b. Negative moment section

$$I_g = \frac{12 \times 25^3}{12} = 15{,}625 \text{ in.}^4$$

$I_{cr} = 11{,}161$ in.^{4} (similar to Example 8.1, for b = 12 in., d = 22.5 in., d′ = 2.5 in., $A_s = 3.95$ in.^{2}, $A'_s = 1.58$ in.^{2})

5. Effective moments of inertia, using Eqs. (9-8) and (1):

   a. Positive moment section:

$M_{cr} = f_r I_g / y_t = [(404)\,(33{,}390)/(18.15)]/12{,}000 = 61.9$ ft-kips — *Eq. (9-9)*

$M_{cr}/M_d = 61.9/57.9 > 1$. Hence $(I_e)_d = I_g = 33{,}390$ in.^{4}

$(M_{cr}/M_{sus})^3 = (61.9/77.2)^3 = 0.515$

$(I_e)_{sus} = (M_{cr}/M_a)^3 I_g + [1 - (M_{cr}/M_a)^3] I_{cr} \le I_g$ *Eq. (9-8)*

$= (0.515)(33{,}390) + (1 - 0.515)(10{,}340) = 22{,}211$ in.^{4}

$\left(M_{cr}/M_{d+\ell}\right)^3 = (61.9/122.2)^3 = 0.130$

$(I_e)_{d+\ell} = (0.130)(33{,}390) + (1 - 0.130)(10{,}340) = 13{,}336$ in.^{4}

b. Negative moment section:

$M_{cr} = [(404)(15{,}625)/(12.5)]/12{,}000 = 42.1$ ft-kips *Eq. (9-9)*

$(M_{cr}/M_d)^3 = (42.1/81.0)^3 = 0.14$

$(I_e)_d = (0.14)(15{,}625) + (1 - 0.14)(11{,}161) = 11{,}786$ in.^{4} *Eq. (9-8)*

$(M_{cr}/M_{sus})^3 = (42.1/108.0)^3 = 0.06$

$(I_e)_{sus} = (0.06)(15{,}625) + (1 - 0.06)(11{,}161) = 11{,}429$ in.^{4} *Eq. (9-8)*

$\left(M_{cr}/M_{d+\ell}\right)^3 = (42.1/171.0)^3 = 0.015$

$(I_e)_{d+\ell} = (0.015)(15{,}625) + (1 - 0.015)(11{,}161) = 11{,}228$ in.^{4} *Eq. (9-8)*

c. Average inertia values:

Avg. $(I_e) = 0.85 I_m + 0.15 (I_{cont.\ end})$ *Eq. (1)*

Avg. $(I_e)_d = (0.85)(33{,}390) + (0.15)(11{,}786) = 30{,}149$ in.^{4}

Avg. $(I_e)_{sus} = (0.85)(22{,}211) + (0.15)(11{,}429) = 20{,}594$ in.^{4}

Avg. $(I_e)_{d+\ell} = (0.85)(13{,}336) + (0.15)(11{,}228) = 13{,}020$ in.^{4}

6. Initial or short-time deflections, with midspan I_e and with avg. I_e: *9.5.2.4*

$$\left(\Delta_i\right) = K\left(\frac{5}{48}\right)\frac{M_a \ell^2}{E_c I_e}$$ *Eq. (3)*

$K = 1.20 - 0.20 M_o/M_a = 1.20 - (0.20)\left(w\ell_n^2/8\right)/\left(w\ell_n^2/14\right) = 0.850$ *(Table 10-3)*

$$\left(\Delta_i\right)_d = \frac{K\,(5/48)\,M_d \ell^2}{E_c\,(I_e)_d} = \frac{(0.85)\,(5/48)\,(57.9)\,(30)^2\,(12)^3}{(2740)\,(33{,}390)} = 0.087 \text{ in.}$$

$= 0.096$ in., using avg. $I_e = 30{,}149$ in.^{4}

$$(\Delta_i)_{sus} = \frac{K(5/48)M_{sus}\ell^2}{E_c(I_e)_{sus}} = \frac{(0.85)(5/48)(77.2)(30)^2(12)^3}{(2740)(22{,}211)} = 0.175 \text{ in.}$$

$= 0.188$ in., using avg. $I_e = 20{,}594$ in.^{4}

$$(\Delta_i)_{d+\ell} = \frac{K(5/48)M_{d+\ell}\ell^2}{E_c(I_e)_{d+\ell}} = \frac{(0.85)(5/48)(122.2)(30)^2(12)^3}{(2740)(13{,}336)} = 0.460 \text{ in.}$$

$= 0.472$ in., using avg. $I_e = 13{,}020$ in.^{4}

$$(\Delta_i)_\ell = (\Delta_i)_{d+\ell} - (\Delta_i)_d = 0.460 - 0.087 = 0.373 \text{ in.}$$

$= 0.472 - 0.096 = 0.376$ in., using avg. I_e from Eq. (1)

Allowable deflections Table 9.5(b):

For flat roofs not supporting and not attached to nonstructural elements likely to be damaged by large deflections — $(\Delta_i)_\ell \le \ell/180 = 2.00$ in. > 0.376 in. O.K

For floors not supporting and not attached to nonstructural elements likely to be damaged by large deflections — $(\Delta_i)_\ell \le \ell/360 = 360/360 = 1.00$ in. O.K.

7. Ultimate long-term deflections:

Using ACI Method with combined creep and shrinkage effects:

$$\lambda = \frac{\xi}{1 + 50\rho'} = \frac{2.0 \text{ (ultimate value)}}{1 + 0} = 2.0 \qquad \textit{Eq. (9-11)}$$

$$\Delta_{(cp+sh)} = \lambda(\Delta_i)_{sus} = (2.0)(0.175) = 0.350 \text{ in.} \qquad \textit{Eq. (4)}$$

$$\Delta_{(cp+sh)} + (\Delta_i)\ell = 0.350 + 0.373 = 0.723 \text{ in.}$$

$= [2(0.188) + 0.376] = 0.752$ using avg. I_e from Eq. (1).

Using Alternate Method with separate creep and shrinkage deflections:

$$\lambda_{cp} = \frac{0.85C_u}{1 + 50\rho'} = \frac{(0.85)\ (1.60)}{1 + 0} = 1.36$$

$$\Delta_{cp} = \lambda_{cp}\,(\Delta_i)_{sus} = (1.36)0.175 = 0.238 \text{ in.} \qquad \textit{Eq. (5)}$$

$= 1.36(0.188) = 0.256$ in., using avg. I_e Eq. (1).

$$\rho = 100\left(\frac{\rho + \rho_w}{2}\right) = 100\left(\frac{2.37}{90 \times 22.5} + \frac{2.37}{12 \times 22.5}\right)/2$$

$$= 100\,(0.00117 + 0.00878)/2 = 0.498\%$$

A_{sh} (from Fig. 10-3) $= 0.555$

$$\phi_{sh} = A_{sh}\frac{(\varepsilon_{sh})_u}{h} = \frac{(0.555)\,(400 \times 10^{-6})}{25} = 8.88 \times 10^{-6}/\text{in.}$$

$$\Delta_{sh} = K_{sh}\phi_{sh}\ell^2 = (0.090)\left(8.88 \times 10^{-6}\right)(30)^2\,(12)^2 = 0.104 \text{ in.} \qquad \textit{Eq. (6)}$$

$$\Delta_{cp} + \Delta_{sh+}\left(\Delta_i\right)_\ell = 0.238 + 0.104 + 0.373 = 0.715 \text{ in.}$$

$= (0.256 + 0.104 + 0.376) = 0.736.$, using avg. I_e from Eq. (1).

Allowable deflections Table 9.5(b):

For roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections (very stringent limitation) —

$$\Delta_{cp} + \Delta_{sh} + \left(\Delta_i\right)_\ell \le \frac{\ell}{480} = \frac{360}{480} = 0.75 \text{ in.}$$

All results O.K.

For roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections —

$$\Delta_{cp} + \Delta_{sh} + \left(\Delta_i\right)_\ell \le \frac{\ell}{240} = \frac{360}{240} = 1.50 \text{ in.}$$ All results O.K.

Example 10.3—Slab System Without Beams (Flat Plate)

Required: Analysis of short-term and ultimate long-term deflections of a corner panel.

Data:

Flat plate with no edge beams, designed by Direct Design Method
Slab f'_c = 3000 psi, Column f'_c = 5000 psi, (normal weight concrete)
f_y = 40,000 psi
Square panels—15 × 15 ft center-to-center of columns
Square columns—14 × 14 in., Clear span, ℓ_n = 15 - 1.17 = 13.83 ft
Story height = 10 ft., Slab thickness, h = 6 in.
The reinforcement in the column strip negative moment regions consists of No. 5 bars at 7.5 in. spacing. Therefore, the total area of steel in a 90-in. strip (half the panel length) is given by:

$A_s = (90/7.5)(0.31) = 3.72$ sq. in.

The distance from the compressive side of the slab to the center of the steel is:

d = 4.62 in

Middle Strip reinforcement and d values are not required for deflection computations, since the slab remains uncracked in the middle strips.
Superimposed Dead Load = 10 psf
Live Load = 50 psf
Check for 0% and 40% Sustained Live Load

Calculations and Discussion	Code Reference

1. Minimum thickness: *9.5.3.2*

   From Table 8-6, with Grade 40 steel:

   Interior panel $h_{min} = \frac{\ell_n}{36} = (13.83 \times 12)/36 = 4.61$ in.

   Exterior panel $h_{min} = \frac{\ell_n}{33} = (13.83 \times 12)/33 = 5.03$ in.

   Since the actual slab thickness is 6 in., deflection calculations are not required; however, as an illustration, deflections will be checked for a corner panel, to make sure that all allowable deflections per Table 9.5(b) are satisfied.

2. Comment on trial design with regard to deflections:

   Based on the minimum thickness limitations versus the actual slab thickness, it appears likely that computed deflections will meet most or all of the code deflection limitations. It turns out that all are met.

3. Modulus of rupture, modulus of elasticity, modular ratio:

$f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{3000} = 411$ psi *Eq. (9-10)*

$E_{cs} = w_c^{1.5}\, 33\sqrt{f'_c} = (150)^{1.5}\, 33\sqrt{3000} = 3.32 \times 10^6$ psi *8.5.1*

$E_{cc} = (150)^{1.5}\, 33\sqrt{5000} = 4.29 \times 10^6$ psi

$$n = \frac{E_s}{E_{cs}} = \frac{29}{3.32} = 8.73$$

4. Service load moments and cracking moment:

$w_d = 10 + (150)\,(6.0)/12 = 85.0$ psf

$(M_o)_d = w_d \ell_2 \ell_n^2/8 = (85.0)\,(15)\,(13.83)^2/8000 = 30.48$ ft-kips

$(M_o)_{d+\ell} = w_d \ell_2 \ell_n^2/8 = (85.0 + 50.0)\,(15)\,(13.83)^2/8000 = 48.41$ ft-kips

$(M_o)_{sus} = (85 + 0.4 \times 50)\,(15)(13.83)^2/8000 = 37.65$ ft-kips

The moments are distributed to the ends and centers of the column and middle strips according to the coefficients in the tables of Sections 13.6.3.3, 13.6.4.1, 13.6.4.2 and 13.6.4.4. In this case, the span ratio, ℓ_2/ℓ_1, is equal to 1.0. The multipliers of the panel moment, M_o, that are used to make the distribution in an end span are given in the following table:

	Ext. Negative	Positive	Int. Negative
Total Panel	0.26	0.52	0.70
Col. Strip	(1.0)(0.26)	(0.60)(0.52)	(0.75)(0.70)
Mid. Strip	(1.0-1.0)(0.26)	(1.0-0.60)(0.52)	(1.0-0.75)(0.70)

The resulting moments applied to the external and internal ends and to the center span of the column and middle strips are given in the following tables:

Dead Load Moments, ft-kips

	Ext. Negative	Positive	Int. Negative
Total Panel	7.93	15.85	21.34
Col. Strip	7.93	9.51	16.00
Mid. Strip	0	6.34	5.34

Dead Load + Live Load Moments, ft-kips

	Ext. Negative	Positive	Int. Negative
Total Panel	12.59	25.18	33.89
Col. Strip	12.59	15.10	25.41
Mid. Strip	0	10.07	8.47

Sustained Load Moments, ft-kips (Dead Load + 40% Live Load)

	Ext. Negative	Positive	Int. Negative
Total Panel	9.79	19.58	26.36
Col. Strip	9.79	11.75	19.77
Mid. Strip	0	7.83	6.59

The gross moment of inertia of a panel, referred to as the total equivalent frame moment of inertia is:

$$I_{frame} = \ell_s h^3/12 = (15 \times 12)(6)^3/12 = 3{,}240 \text{ in.}^4$$

For this case, the moment of inertia of a column strip or a middle strip is equal to half of the moment of inertia of the total equivalent frame:

$$I_g = {}^1/_2(3240) = 1{,}620 \text{ in.}^4$$

The cracking moment for either a column strip or a middle strip is obtained from the standard flexure formula based on the uncracked section as follows:

$$(M_{cr})_{c/2} = (M_{cr})_{m/2} = f_r I_g/y_t = (411)\,(15 \times 12)\,(6.0)^3/(4)\,(12)\,(3.0)\,(12{,}000)$$

$$= 9.25 \text{ ft-kips}$$

5. Effective moments of inertia:

A comparison of the tabulated applied moments with the cracking moment shows that the apportioned moment at all locations, except at the interior support of the column strips for the live load and sustained load cases, is less than the cracking moment under the imposed loads. The cracked section moment of inertia is, therefore, only required for the column strips in the negative moment zones. Formulas for computation of the cracked section moment of inertia are obtained from Table 10-2:

$$B = \frac{b}{nA_s} = \frac{\frac{1}{2}(15 \times 12)}{8.73 \times 3.72} = 2.77\left(\frac{1}{\text{in.}}\right)$$

$$kd = \frac{\sqrt{2\,dB + 1} - 1}{B} = \frac{\sqrt{2 \times 4.62 \times 2.77 + 1} - 1}{2.77} = 1.50 \text{ in.}$$

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d - kd)^2 = \frac{90 \times 1.50^3}{3} + 8.73 \times 3.72\,(4.62 - 1.50)^2 = 417 \text{ in.}^4$$

To obtain an equivalent moment of inertia for the cracked location, apply the Branson modification to the moments of inertia for cracked and uncracked sections. The approximate moment of inertia in the cracked sections is given by the general formula in Equation (9-8) of ACI 318. From the tables developed in Section 4 above, the ratios of the dead load plus live load and sustained load moments to the cracking moment are found as follows:

For live dead load plus live load:

$$\frac{M_{cr}}{M_a} = \frac{18.50}{25.41} = 0.728$$

$$\left(\frac{M_{cr}}{M_a}\right)^3 = 0.386$$

and for the sustained load case (dead load plus 40% live load):

$$\frac{M_{cr}}{M_a} = \frac{18.50}{19.77} = 0.936$$

$$\left(\frac{M_{cr}}{M_a}\right)^3 = 0.819$$

The equivalent moment of inertia for the two cases are now computed by Eq. (9-8) of ACI 318:

For dead load plus live load:

$$I_e = (0.386)1620 + (1\text{-}0.386)(417) = 881 \text{ in.}^4$$

For sustained load (dead load + 40% live load):

$$I_e = (0.819)1620 + (1\text{-}0.819)(417) = 1402 \text{ in.}^4$$

Finally, the equivalent moment of inertia for the uncracked sections is just the moment of inertia of the gross section, I_g.

To obtain an average moment of inertia for calculation of deflection, the "end" and "midspan" values are then combined according to Equation (1):

For dead load plus live load:

$$\text{Avg. } I_e = 0.85(1620) + 0.15(881)] = 1509 \text{ in.}^4$$

For sustained load (dead load + 40% live load):

$$\text{Avg. } I_e = 0.85(1620) + 0.15(1402)] = 1587 \text{ in.}^4$$

To obtain the equivalent moment of inertia for the "equivalent frame", which consists of a column and a middle strip, add the average moments of inertia for the respective strips. For the middle strips, the moment of inertia is that of the gross section, I_g, and for the column strips, the average values computed above are used:

For dead load only:

$$(I_e)_{frame} = 1620 + 1620 = 3240 \text{ in.}^4$$

For dead load plus live load:

$$(I_e)_{frame} = 1620 + 1509 = 3129 \text{ in.}^4$$

For dead load plus 40% live load:

$$(I_e)_{frame} = 1620 + 1587 = 3207 \text{ in.}^4$$

Note: In this case, where a corner panel is considered, there is only half of a column strip along the two outer edges. However, the section properties for half a strip are equal to half of those for a full strip; also, the applied moments to the edge strip are half those applied to an interior strip. Consequently, deflections calculated for either a half or for a full column strip are the same. Strictly, these relationships only apply because all panels are of equal dimensions in both directions. If the panels are not square or if adjacent panels are of differing dimensions, additional calculations would be necessary.

6. Flexural stiffness (K_{ec}) of an exterior equivalent column:

$K_b = 0$ (no beams) *R13.7.4*

The stiffness of the equivalent exterior column is determined by combining the stiffness of the upper and lower columns at the outer boundary of the floor with the torsional stiffness offered by a strip of the floor slab, parallel to the edge normal to the direction of the equivalent frame and extending the full panel length between columns. In the case of a corner column, the length is, of course, only half the panel length. The width of the strip is equal to the column dimension normal to the direction of the equivalent frame (ACI 318, R13.7.5).

The column stiffness is computed on the basis of the rotation resulting from application of a moment to the simply supported end of a propped cantilever, M = 4EI/L. In this case the result is:

$$K_c = 4E_{cc}I_c/\ell_c = 4E_{cc}\,[(14)^4/(12)]/[(10)\,(12)] = 106.7E_{cc}$$

Since the columns above and below the slab are equal in dimension, the total stiffness of the columns is twice that of a single column:

$$\Sigma K_c = 2K_c = (2)\,(106.7E_{cc}) = 213.4E_{cc}$$

The torsional stiffness of the slab strip is calculated according to the methodology set out in R13.7.5 of ACI 318, $K_t = \Sigma 9E_{cs}/L_2(1\text{-}c_2/L_2)3$. The cross-sectional torsional constant, C, is defined in Section 13.0 of ACI 318.

$$C = (1 - 0.63\ x/y)\ (x^3y/3) = (1 - 0.636 \times 6.0/14)\ (6.0^3 \times 14/3) = 735.8\ in.^4$$

$$K_t = \frac{\Sigma 9E_{cs}C}{\ell_2\ (1 - c_2/\ell_2)^3} = \frac{(2)\ (9)\ E_{cs}\ (735.8)}{(15)\ (12)\left(1 - \frac{14}{15 \times 12}\right)^3} = 93.9E_{cs}$$

For Ext. Frame, $K_t = 93.9E_{cs}/2 = 47E_{cs}$, $E_{cc} = (4.29/3.32)\ E_{cs} = 1.292E_{cs}$

The equivalent column stiffness is obtained by treating the column stiffness and the torsional member stiffness as springs in series:

$$K_{ec} = \frac{1}{\frac{1}{\Sigma K_c} + \frac{1}{K_t}} = \frac{E_{cs}}{\left(\frac{1}{213.4 \times 1.292}\right) + \left(\frac{1}{93.9}\right)} = 70E_{cs} = 19{,}370\ \text{ft-kips/rad}$$

$$\text{For Ext. Frame, } K_{ec} = \frac{E_{cs}}{\left(\frac{1}{213.4 \times 1.292}\right) + \left(\frac{1}{47.0}\right)} = 40.1E_{cs} = 11{,}090\ \text{ft-kips/rad}$$

7. Deflections, using Eqs. (7) to (14):

$$\text{Fixed } \Delta_{frame} = w\ell_2\ell^4/384E_{cs}I_{frame} \qquad \textit{Eq. (10)}$$

$$(\text{Fixed } \Delta_{frame})_{d,d+\ell} = \frac{(85.0 \text{ or } 135.0 \text{ or } 105.0)\ (15)^5(12)^3}{(384)\left(3.32 \times 10^6\right)(3240 \text{ or } 3129 \text{ or } 3207)}$$

$$= 0.027 \text{ in., } 0.044 \text{ in.; } 0.034$$

$$\text{Fixed } \Delta_{c,m} = (LDF)_{c,m}\ (\text{Fixed } \Delta_{frame})\ (I_{frame}/I_{c,m}) \qquad \textit{Eq. (11)}$$

These deflections are distributed to the column and middle strips in the ratio of the total applied moment to the beam stiffness (M/EI) of the respective strips to that of the complete frame. As shown in Step 4 above, the fraction of bending moment apportioned to the column or middle strips varies between the ends and the midspan. Therefore, in approximating the deflections by this method, the average moment allocation fraction (Lateral Distribution Factor - LDF) is used. In addition, since the equivalent moment of inertia changes whenever the cracking moment is exceeded, an average moment of inertia is utilized. This average moment of inertia is computed on the basis of Equation (9-8) from ACI 318 and Eq. (1) of this chapter. Finally, since the modulus of elasticity is constant throughout the slab, the term E occurs in both the numerator and the denominator and is, therefore, omitted. The LDFs are calculated as follows:

For the column strip:

$$LDF_c = {}^1/_2\,[\ {}^1/_2(\ M^-_{int} + M^-_{ext}\ ) + M^+] = {}^1/_2[\ {}^1/_2(0.75+1.00)+0.60] = 0.738$$

For the middle strip:

$$LDF_m = 1 - LDF_c = 0.262$$

(Fixed $\Delta_c)_d$ = (0.738) (0.027) (2) = 0.040 in.

(Fixed $\Delta_c)_{d+\ell}$ = (0.738) (0.044) (3129/1509) = 0.067 in.

(Fixed $\Delta_c)_{\ell}$ = 0.067 - 0.040 = 0.027 in.

(Fixed $\Delta_c)_{sus}$ = (0.738)(0.034)(3207/1587) = 0.051 in

(Fixed $\Delta_m)_d$ = (0.262) (0.027) (2) = 0.014 in.

(Fixed $\Delta_m)_{d+\ell}$ = (0.262) (0.044) (3129/1620) = 0.022 in.

(Fixed $\Delta_m)_{\ell}$ = 0.022 - 0.014 = 0.008 in.

(Fixed $\Delta_m)_{sus}$ = (0.0.262)(0.034)(3207/1587) = 0.018 in

In addition to the fixed end displacement found above, an increment of deflection must be added to each due to the actual rotation that occurs at the supports. The magnitude of the increment is equal to qL/8. The rotations, q, are determined as the net moments at the column locations divided by the effective column stiffnesses. In this case, the column strip moment at the corner column of the floor is equal to half of 100% of 0.26 x M_o (ACI 318, Sec. 13.6.3.313 and Sec.13.6.4.2). Because the column strip at the edge of the floor is only half as wide as an interior column strip, only half of the apportioned moment acts. The net moments at other columns are either quite small or zero. Therefore they are neglected. The net moments on a corner column for the three loading cases are:

$$(M_{net})_d = {}^1/_2 \times 0.26 \times 1.00 \times (M_o)_d = {}^1/_2\ [(0.26)(1.00)](30.48) = 3.96 \text{ ft-kips}$$

$$(M_{net})_{d+\ell} = {}^1/_2 \times 0.26 \times 1.00 \times (M_o)_{d+\ell} = {}^1/_2\ [(0.26)(1.00)](48.41) = 6.29 \text{ ft-kips}$$

$$(M_{net})_{sus} = {}^1/_2 \times 0.26 \times 1.00 \times (M_o)_{sus} = {}^1/_2\ [(0.26)(1.00)](37.65) = 4.89 \text{ ft-kips}$$

For both column and middle strips,

End θ_d = $(M_{net})_d$/avg. K_{ec} = 3.96/11,090 = 0.000357 rad *Eq. (12)*

End $\theta_{d+\ell}$ = 6.29/11,090 = 0.000567 rad

End θ_{sus} = 4.89/11090 = 0.000441 rad

$\Delta\theta = (\text{End } \theta)\ (\ell/8)\ (I_g/I_e)_{frame}$ *Eq. (14)*

$(\Delta\theta)_d = (0.000357)\ (15)\ (12)\ (1)/8 = 0.008$ in.

$(\Delta\theta)_{d+\ell} = (0.000567)\ (15)\ (12)\ (1620/1509)/8 = 0.014$ in.

$(\Delta\theta)_\ell = 0.014 - 0.008 = 0.006$ in.

$(\Delta\theta)_{sus} = (0.000441)(15)(12)/8 = 0.010$ in.

The deflections due to rotation calculated above are for column strips. The deflections due to end rotations for the middle strips will be assumed to be equal to that in the column strips. Therefore, the strip deflections are calculated by the general relationship:

$\Delta_{c,m} = \text{Fixed } \Delta_{c,m} + (\Delta\theta)$ *Eq.(9)*

$(\Delta_c)_d = 0.040 + 0.008 = 0.048$ in.

$(\Delta_m)_d = 0.014 + 0.008 = 0.022$ in.

$(\Delta_c)_\ell = 0.027 + 0.006 = 0.033$ in.

$(\Delta_m)_\ell = 0.008 + 0.006 = 0.014$ in.

$(\Delta_c)_{sus} = 0.051 + (0.010) = 0.061$ in.

$(\Delta_m)_{sus} = 0.018 + (0.010) = 0.028$ in.

$\Delta = \Delta_{cx} + \Delta_{my} =$ midpanel deflection of corner panel *Eq. (7)*

$(\Delta_i)_d = 0.048 + 0.022 = 0.070$ in.

$(\Delta_i)_\ell = 0.033 + 0.014 = 0.047$ in.

$(\Delta_i)_{sus} = 0.061 + 0.028 = 0.089$ in.

The long term deflections may be calculated using Eq. (9-11) of ACI 318 (Note: $\rho' = 0$):

For dead load only:

$(\Delta_{cp+sh})_d = 2.0 \times (\Delta_i)_d = (2)(0.070) = 0.140$ in.

For sustained load (dead load + 40% live load)

$(\Delta_{cp+sh})_{sus} = 2.0 \times (\Delta_i)_{sus} = (2)(0.109) = 0.218$ in.

The long term deflection due to sustained load plus live load is calculated as:

$(\Delta_{cp+sh})_{sus} + (\Delta_i)_{\ell} = 0.218 + 0.047 = 0.265$ in.

These computed deflections are compared with the code allowable deflections in Table 9.5(b) as follows:

Flat roofs not supporting and not attached to nonstructural elements likely to be damaged by large deflections—

$(\Delta_i)_{\ell} \leq (\ell_n \text{ or } \ell)/180 = (13.83 \text{ or } 15)\ (12)/180 = 0.92$ in. or 1.00 in., versus 0.047 in. O.K.

Floors not supporting and not attached to nonstructural elements likely to be damaged by large deflections—

$(\Delta_i)_{\ell} \leq (\ell_n \text{ or } \ell)/360 = 0.46$ in. or 0.50 in., versus 0.047 in. O.K.

Roof or floor construction supporting or attached to non-structural elements likely to be damaged by large deflections—

$\Delta_{(cp+sh)} + (\Delta_i)_{\ell} \leq (\ell_n \text{ or } \ell)/480 = 0.35$ in. or 0.38 in., versus 0.265 in. O.K.

Roof or floor construction supporting or attached to non-structural elements not likely to be damaged by large deflections—

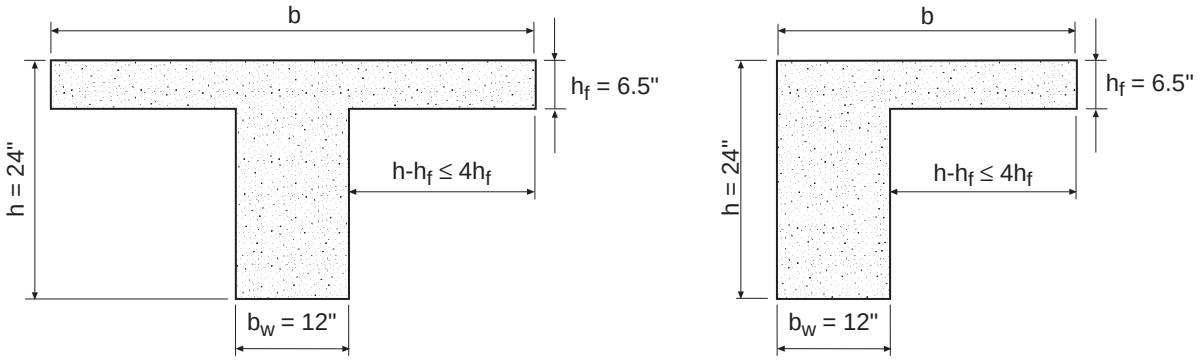
$\Delta_{(cp+sh)} + (\Delta_i)_{\ell} \leq (\ell_n \text{ or } \ell)/240 = 0.69$ in. or 0.75 in., versus 0.265 in. O.K.

All computed deflections are found to be satisfactory in all four categories.

Example 10.4—Two-Way Beam Supported Slab System

Required: Minimum thickness for deflection control

Data:



f_y = 60,000 psi, slab thickness h_f = 6.5 in.
Square panels—22 × 22 ft center-to-center of columns
All beams—b_w = 12 in. and h = 24 in. ℓ_n = 22 - 1 = 21 ft
It is noted that f'_c and the loading are not required in this analysis.

Calculations and Discussion	Code Reference

1. Effective width b and section properties, using Table 8-2:

   a. Interior Beam

   $I_s = (22)(12)(6.5)^3/12 = 6040 \text{ in.}^4$

   $h - h_f = 24 - 6.5 = 17.5 \text{ in.} \le 4h_f = (4)(6.5) = 26 \text{ in.}$ O.K.

   Hence, $b = 12 + (2)(17.5) = 47 \text{ in.}$

   $y_t = h - (1/2)[(b - b_w) h_f^2 + b_w h^2]/[(b - b_w) h_f + b_w h]$

   $= 24 - (1/2)[(35)(6.5)^2 + (12)(24)^2]/[(35)(6.5) + (12)(24)]$

   $= 15.86 \text{ in.}$

   $I_b = (b - b_w) h_f^3/12 + b_w h^3/12 + (b - b_w) h_f (h - h_f/2 - y_t)^2 + b_w h (y_t - h/2)^2$

   $= (35)(6.5)^3/12 + (12)(24)^3/12 + (35)(6.5)(24 - 3.25 - 15.86)^2 +$

   $(12)(24)(15.86 - 12)^2 = 24{,}360 \text{ in.}^4$

   $\alpha = E_{cb} I_b / E_{cs} I_s = I_b / I_s = 24{,}360/6040 = 4.03$

b. Edge Beam

$I_s = (11)(12)(6.5)^3/12 = 3020 \text{ in.}^4$

$b = 12 + (24 - 6.5) = 29.5 \text{ in.}$

$y_t = 24 - (1/2)[(17.5)(6.5)^2 + (12)(24)^2]/[(17.5)(6.5) + (12)(24)] = 14.48 \text{ in.}$

$I_b = (17.5)(6.5)^3/12 + (12)(24)^3/12 + (17.5)(6.5)(24 - 3.25 - 14.48)^2 +$

$(12)(24)(14.48 - 12)^2 = 20,470 \text{ in.}^4$

$\alpha = I_b/I_s = 20,470/3020 = 6.78$

α_m and β values:

α_m (average value of α for all beams on the edges of a panel):

Interior panel — $\alpha_m = 4.03$

Side panel — $\alpha_m = [(3)(4.03) + 6.78]/4 = 4.72$

Corner panel — $\alpha_m = [(2)(4.03) + (2)(6.78)]/4 = 5.41$

For square panels, β = ratio of clear spans in the two directions = 1

2. Minimum thickness: *9.5.3.3*

Since $\alpha_m > 2.0$ for all panels, Eq. (9-12) applies.

$$h_{min} = \frac{\ell_n \left(0.8 + \frac{f_y}{200,000}\right)}{36 + 9\beta} \qquad \textit{Eq. (9-13)}$$

$$= \frac{(21 \times 12)\left(0.8 + \frac{60,000}{200,000}\right)}{36 + 9(1)} = 6.16 \text{ in.} \quad \text{(all panels)}$$

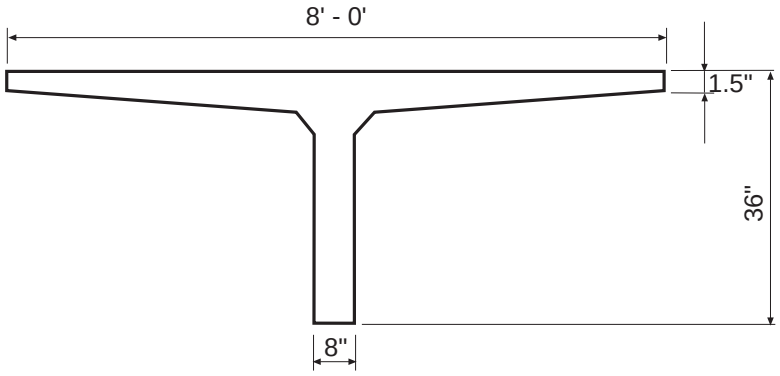
Hence, the slab thickness of 6.5 in. > 6.16 in. is satisfactory for all panels, and deflections need not be checked.

Example 10.5—Simple-Span Prestressed Single T-Beam

Required: Analysis of short-term and ultimate long-term camber and deflection.

Data:

8ST36 (Design details from PCI Handbook 3rd Edition, 1985)
Span = 80 ft, beam is partially cracked
f'_{ci} = 3500 psi, f'_c = 5000 psi (normal weight concrete)
f_{pu} = 270,000 psi
E_p = 27,000,000 psi
14 - 1/2 in. dia. depressed (1 Pt.) strands
4 - 1/2 in. dia. nonprestressed strands
(Assume same centroid when computing I_{cr})
P_i = (0.7) (14) (0.153) (270) = 404.8 kips
P_o = (0.90) (404.8) = 364 kips
P_e = (0.78) (404.8) = 316 kips
e_e = 11.15 in., e_c = 22.51 in.
y_t = 26.01 in., A_g = 570 in.^{2}, I_g = 68,920 in.^{4}
Self weight, w_o = 594 lb/ft
Superimposed DL, w_s = (8)(10 psf) = 80 lb/ft is applied at age 2 mos (β_s = 0.76 in Term (6) of Eq. (15))
Live load, w_ℓ = (8)(51 psf) = 408 lb/ft
Capacity is governed by flexural strength



Calculations and Discussion	Code Reference

1. Span-depth ratios (using PCI Handbook):

   Typical span-depth ratios for single T beams are 25 to 35 for floors and 35 to 40 for roofs, versus (80)(12)/36 = 27, which indicates a relatively deep beam. It turns out that all allowable deflections in Table 9.5(b) are satisfied.

2. Moments for computing deflections:

$$M_o = \frac{w_o \ell^2}{8} = \frac{(0.594)\,(80)^2}{8} = 475 \text{ ft-kips}$$

($\times$ 0.96 = 456 ft-kips at 0.4ℓ for computing stresses and I_e— tendons depressed at one point)

$$M_s = \frac{w_s \ell^2}{8} = \frac{(0.080)\,(80)^2}{8} = 64 \text{ ft-kips (61 ft-kips at } 0.4\ell)$$

$$M_\ell = \frac{w_\ell \ell^2}{8} = \frac{(0.408)\,(80)^2}{8} = 326 \text{ ft-kips (313 ft-kips at } 0.4\ell)$$

3. Modulus of rupture, modulus of elasticity, moment of inertia:

$$f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{5000} = 530 \text{ psi}$$ *Eq. (9-10)*

$$E_{ci} = w_c^{1.5}\, 33\sqrt{f'_{ci}} = (150)^{1.5}\, 33\sqrt{3500} = 3.59 \times 10^6 \text{ psi}$$ *8.5.1*

$$E_c = w_c^{1.5}\, 33\sqrt{f'_c} = (150)^{1.5}\, 33\sqrt{5000} = 4.29 \times 10^6 \text{ psi}$$

$$n = \frac{E_p}{E_c} = \frac{27 \times 10^6}{4.29 \times 10^6} = 6.3$$

The moment of inertia of the cracked section, at 0.4ℓ, can be obtained by the approximate formula given in Eq. 4.8.2 of the PCI Handbook:

$$I_{cr} = nA_{st}d^2\left(1 - 1.6\sqrt{n\rho}\right) = (6.3)\,(18 \times 0.153)\,(30.23)^2\left(1 - 1.6\sqrt{6.3 \times 0.000949}\right)$$

$$= 13{,}890 \text{ in}^4 \text{ (at } 0.4\ell)$$

It may be shown that the cracked section moment of inertia calculated by the formulas given in Table 10.2 is very close to the value obtained by the approximate method shown above. The results differ by approximately 1%; therefore either method is suitable for this case.

4. Determination of classification of beams

In order to classify the beam according to the requirements of ACI Section 18.3.3, the maximum flexural stress is calculated and compared to the modulus rupture to determine its classification. The classifications are defined as follows:

Class U: $f_t \le 7.5\sqrt{f'_c}$

Class T: $7.5\sqrt{f'_c} < f_t \le 12\sqrt{f'_c}$

Class C: $f_t > 12\sqrt{f'_c}$

The three classes refer to uncracked (U), transition (T) and cracked (C) behavior.

The maximum tensile stress due to service loads and prestressing forces are calculated by the standard formula for beams subject to bending moments and axial loads. It may be shown that the maximum bending stresses in a prestressed beam occur at approximately 0.4ℓ. In the following, the bending moments are those that occur at 0.4ℓ. The eccentricity of the prestressing force at 0.4ℓ, e = 20.24 in, is obtained by linear interpolation between the end eccentricity, e_e = 11.15 in and that at the center, e_e = 22.51. The calculation proceeds as follow:

$$M_{tot} = M_d + M_\ell$$

$$f_t = \frac{M_{tot} y_t}{I_g} - \frac{P_e e \cdot y_t}{I_g} - \frac{P_e}{A_g}$$

$$= [(456 + 61 + 313)(12) - (316)(20.24)]\ [(26.01)/68{,}920] - 316/570$$

$$f_t = 791 \text{ psi}$$

Check the ratio of calculated tensile stress to the square root of f'_c:

$$\frac{f_t}{\sqrt{f'_c}} = \frac{791}{\sqrt{5000}} = 11.2$$

The ratio is between 7.5 and 12, therefore according to the definitions of Section 18.3.3 of ACI 318, the beam classification is T, transition region. Table R18.3.3 requires that deflections for this classification be based on the cracked section properties assuming bi-linear moment deflection behavior; Section 9.5.4.2 of the code allows either a bi-linear moment-deflection approach or calculation of deflections on the basis of an equivalent moment of inertia determined according to Eq. (9-8).

5. Determine live load moment that causes first cracking:

Check the tensile stress due to dead load and prestressing forces only. As noted previously, the maximum tensile stresses occur at approximately 0.4ℓ:

$$f_t = [(456 + 61)(12) - (316)(20.24)]\ [(26.01)/68920] - 316/570 = -627 \text{ psi}$$

Since the sign is negative, compressive stress is indicated. Therefore, the section is uncracked under the dead load plus prestressing forces and dead load deflections can be based on the moment of inertia of the gross concrete cross section. It was shown above that the maximum tensile stress due to combined dead load plus live load equals 791 psi, which exceeds the modulus of rupture, f_r = 530 psi

Therefore, the live load deflections must be computed on the basis of a cracked section analysis because the behavior is inelastic after the addition of full live load. In particular, Table R18.3.3 of ACI 318 requires that bilinear behavior be utilized to determine deflections in such cases, however. Section 9.5.4.2 permits deflections to be computed either on the basis of bilinear behavior or on the basis of an effective moment of inertia.

In order to calculate the deflection assuming bilinear behavior, it is first necessary to determine the fraction of the total live load that causes first cracking. That is, to find the portion of live load that will just produce a maximum tensile stress equal to f_r. The desired value of live load moment can be obtained by re-arranging the equation used above to determine the total tensile stress (for classification), and setting the tensile stress equal to f_r. The moment value is obtained as follows (Note: Quantities calculated at 0.4ℓ):

$$\text{Live Load Cracking Moment} = \frac{f_r I_g}{y_t} + P_e e + \frac{P_e I_g}{A_g y_t} - M_d$$

= (530)68920)/(12000)(26.01) + 316(20.24/12) +[(316/570)(68920/26.01)]/12 - 517

= 117 + 533 +122 - 517

= 255 ft-kips

The fraction of the live load cracking moment to the total live load moment is:

255/313 = 0.815

6. Camber and Deflection, using Eq. (15):

Term (1) — $\Delta_{po} = \frac{P_o (e_c - e_e)\ell^2}{12E_{ci}I_g} + \frac{P_o e_e \ell^2}{8E_{ci}I_g}$ (from PCI Handbook for single point depressed strands)

$$= \frac{(364)\,(22.51 - 11.15)\,(80)^2\,(12)^2}{(12)\,(3590)\,(68{,}920)} + \frac{(364)\,(11.15)\,(80)^2\,(12)^2}{(8)\,(3590)\,(68{,}920)}$$

= 3.17 in.

Term (2) — $\Delta_o = \frac{5M_o\ell^2}{48E_{ci}I_g} = \frac{(5)\,(475)\,(80)^2\,(12)^3}{(48)\,(3590)\,(68{,}590)} = 2.21$ in.

Term (3) — $k_r = 1/[1 + (A_s/A_{ps})] = 1/[1 + (4/14)] = 0.78$

$$\left[-\frac{\Delta P_u}{P_o} + (k_r C_u)\left(1 - \frac{\Delta P_u}{2P_o}\right)\right]\Delta_{po}$$

The increment in prestressing force is:

$\Delta P_u = P_o - P_e = 364 - 316 = 48$ kips

It follows that:

$\Delta P_u / P_o = 48/364 = 0.13$

Therefore, the deflection is:

$= [-0.13 + (0.78 \times 2.0)(1 - 0.065)](3.17) = 4.21$ in.

Term (4) — $(k_r C_u)\ \Delta_o = (0.78)(2.0)(2.21) = 3.45$ in.

Term (5) — $\Delta_s = \dfrac{5M_s\ell^2}{48E_cI_g} = \dfrac{(5)(64)(80)^2(12)^3}{(48)(4290)(68{,}920)} = 0.25$ in.

Term (6) — $(\beta_s k_r C_u)\Delta_s = (0.76)(0.78)(1.6)(0.25) = 0.24$ in.

Term (7) – Initial deflection due to live load.

The ratio of live load cracking moment to total live load moment was found previously. To calculate the deflection according to bi-linear behavior, the deflection due to the portion of the live load below the cracking value is based on the gross moment of inertia; the deflection due to the remainder of the live load is based on the cracked section moment of inertia. Also, the deflections are based on moments at the center of the span even though the moment that caused initial cracking was evaluated at 0.4ℓ.

The deflection formula used is the standard expression:

$$\Delta = \frac{5\ ML^2}{48\ EI}$$

For the portion of the live load applied below the cracking moment load, the value of M is the value calculated above, 255 ft-kips and the moment of inertia is that of the gross section:

$$\Delta_{\ell 1} = 5(255)(80)^2(12)^3/48(3590)(68590) = 1.19 \text{ in}$$

Deflection due to the remainder of the live load is calculated similarly, with a moment of 313-255 = 58 ft-kips and the cracked section moment of inertia, 13,890 in^4:

$$\Delta_{\ell 2} = 5(58)(80)^2(12)^3/48(3590)(13{,}890) = 1.34 \text{ in}$$

The total live load deflection is the sum of the previous two components:

$$\Delta_\ell = 1.19 + 1.34 = 2.53 \text{ in.}$$

It can be verified by a separate calculation that the deflection based on the full live load moment and the effective moment of inertia, calculated by Eq. 9-8 of ACI 318, is slightly less than that calculated here on the basis of a bi-linear moment-deflection relationship.

Combined results and comparisons with code limitations

$$\Delta_u = \underset{(1)}{-3.17} + \underset{(2)}{2.21} \underset{(3)}{-4.21} + \underset{(4)}{3.45} + \underset{(5)}{0.25} + \underset{(6)}{0.24} + \underset{(7)}{2.53} = 1.30 \text{ in.} \downarrow$$

Initial Camber = $\Delta_{po} - \Delta_o$ = 3.17 - 2.21 = 0.96 in. ↑ versus 1.6 in. at erection in PCI Handbook

Residual Camber = $\Delta_\ell - \Delta_u$ = 2.53 - 0.87 = 1.66 in. ↑ versus 1.1 in.

Time-Dependent plus Superimposed Dead Load and Live Load Deflection

= - 4.21 + 3.45 + 0.25 + 0.24 + 2.53 = 2.26 in. or

= $\Delta_u - \left(\Delta_o - \Delta_{po}\right)$ = 0.87 - (- 0.96) = 2.26 in. ↓

These computed deflections are compared with the allowable deflections in Table 9.5(b) as follows:

ℓ/180 = (80) (12)/180 = 5.33 in. versus Δ_ℓ = 2.53 in. O.K.

ℓ/360 = (80) (12)/360 = 2.67 in. versus Δ_ℓ = 2.53 in. O.K.

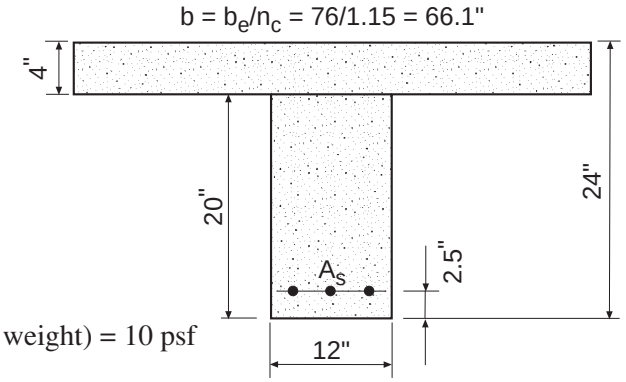
ℓ/480 = (80) (12)/480 = 2.00 in. versus Time-Dep. etc. = 2.26 in. O.K.

Note that the long term deflection occurring after attachment of non-structural elements (2.26 in) exceeds the L/480 limit. It actually meets L/425. Since the L/480 limit only applies in case of *nonstructural elements likely to be damaged by large deflections*, the particular use of the beam would have to be considered in order to make a judgment on the acceptability of the computed deflections. Refer to the footnotes following Table 9.5(b) of ACI 318.

Example 10.6—Unshored Nonprestressed Composite Beam

Required: Analysis of short-term and ultimate long-term deflections.

Data:



Normal weight concrete
Slab f'_c = 3000 psi
Precast beam f'_c = 4000 psi
f_y = 40,000 psi
A_s = 3-No. 9 = 3.00 in.^{2}
E_s = 29,000,000 psi
Superimposed Dead Load (not including beam and slab weight) = 10 psf
Live Load = 75 psf (20% sustained)
Simple span = 26 ft = 312 in., spacing = 8 ft = 96 in.
b_e = 312/4 = 78.0 in., or spacing = 96.0 in., or 16(4) + 12 = 76.0 in.

Calculations and Discussion	Code Reference

1. Minimum thickness for members not supporting or attached to partitions or other construction likely to be damaged by large deflections:

$$h_{min} = \left(\frac{\ell}{16}\right)(0.80 \text{ for } f_y) = \left(\frac{312}{16}\right)(0.80) = 15.6 \text{ in.} < h = 20 \text{ in. or } 24 \text{ in.}$$ *Table 9.5(a)*

2. Loads and moments:

w_1 = (10 psf) (8) + (150 pcf) (96) (4)/144 = 480 lb/ft

w_2 = (150 pcf) (12) (20)/144 = 250 lb/ft

w_ℓ = (75 psf) (8) = 600 lb/ft

$M_1 = w_1\ell^2/8 = (0.480)(26)^2/8 = 40.6$ ft-kips

$M_2 = w_2\ell^2/8 = (0.250)(26)^2/8 = 21.1$ ft-kips

$M_\ell = w_\ell\ell^2/8 = (0.600)(26)^2/8 = 50.7$ ft-kips

3. Modulus of rupture, modulus of elasticity, modular ratio:

$(E_c)_1 = w_c^{1.5}\, 33\sqrt{f'_c} = (150)^{1.5}\, 33\sqrt{3000} = 3.32 \times 10^6$ psi *8.5.1*

$(f_r)_2 = 7.5\sqrt{f'_c} = 7.5\sqrt{4000} = 474$ psi *Eq. (9-10)*

$(E_c)_2 = (150)^{1.5}\, 33\sqrt{4000} = 3.83 \times 10^6$ psi *8.5.1*

$$n_c = \frac{(E_c)_2}{(E_c)_1} = \frac{3.83}{3.32} = 1.15$$

$$n = \frac{E_s}{(E_c)_2} = \frac{29}{3.83} = 7.56$$

4. Gross and cracked section moments of inertia, using Table 10-2:

Precast Section

$I_g = (12)(20)^3/12 = 8000 \text{ in.}^4$

$B = b/(nA_s) = 12/(7.56)(3.00) = 0.529/\text{in.}$

$kd = \left(\sqrt{2dB + 1} - 1\right)/B = \left[\sqrt{(2)(17.5)(0.529) + 1} - 1\right]/0.529 = 6.46 \text{ in.}$

$I_{cr} = bk^3d^3/3 + nA_s(d - kd)^2 = (12)(6.46)^3/3 + (7.56)(3.00)(17.5 - 6.46)^2 = 3840 \text{ in.}^4$

Composite Section

$y_t = h - (1/2)[(b - b_w)h_f^2 + b_wh^2]/[(b - b_w)h_f + b_wh]$

$= 24 - (1/2)[(54.1)(4)^2 + (12)(24)^2]/[(54.1)(4) + (12)(24)] = 16.29 \text{ in.}$

$I_g = (b - b_w)h_f^3/12 + b_wh^3/12 + (b - b_w)h_f(h - h_f/2 - y_t)^2 + b_wh(y_t - h/2)^2$

$= (54.1)(4)^3/12 + (12)(24)^3/12 + (54.1)(4)(24 - 2 - 16.29)^2$

$+ (12)(24)(16.29 - 12)^2 = 26{,}470 \text{ in.}^4$

$B = b/(nA_s) = 66.1/(7.56)(3.00) = 2.914$

$d = \left(\sqrt{2dB + 1} - 1\right)/B = \left[\sqrt{(2)(21.5)(2.914) + 1} - 1\right]/2.914$

$= 3.51 \text{ in.} < h_f = 4 \text{ in.}$ Hence, treat as a rectangular compression area.

$I_{cr} = bk^3d^3/3 + nA_s(d - kd)^2 = (66.1)(3.51)^3/3 + (7.56)(3.00)(21.5 - 3.51)^2$

$= 8295 \text{ in.}^4$

$I_2/I_c = [(I_2/I_c)_g + (I_2/I_c)_{cr}]/2 = [(8000/26{,}470) + (3840/8295)]/2 = 0.383$

5. Effective moments of inertia, using Eq. (9-8):

For Term (1), Eq. (19)—Precast Section,

$M_{cr} = f_rI_g/y_t = (474)(8000)/(10)(12{,}000) = 31.6 \text{ ft-kips}$ *Eq. (9-9)*

$M_{cr}/M_2 = 31.6/21.1 > 1$. Hence $(I_e)_2 = I_g = 8000 \text{ in.}^4$

For Term (6), Eq. (19)—Precast Section,

$[M_{cr}/(M_1 + M_2)]^3 = [31.6/(40.6 + 21.1)]^3 = 0.134$

$(I_e)_{1+2} = (M_{cr}/M_a)^3 I_g + [1 - (M_{cr}/M_a)^3] I_{cr} \le I_g$ *Eq. (9-8)*

$= (0.134)(8000) + (1 - 0.134)(3840) = 4400 \text{ in.}^4$

6. Deflection, using Eq. (19):

Term (1) — $(\Delta_i)_2 = \dfrac{K\,(5/48)\,M_2 \ell^2}{(E_c)_2\,(I_e)_2} = \dfrac{(1)\,(5/48)\,(21.1)\,(26)^2\,(12)^3}{(3830)\,(8000)} = 0.084$ in.

Term (2) — $k_r = 0.85$ (no compression steel in precast beam).

$0.77 k_r (\Delta_i)_2 = (0.77)(0.85)(0.084) = 0.055$ in.

Term (3) — $0.83 k_r (\Delta_i)_2 \dfrac{I_2}{I_c} = (0.83)(0.85)(0.084)(0.383) = 0.023$ in.

Term (4) — $K_{sh} = 1/8$. Precast Section: $\rho = (100)(3.00)/(12)(17.5) = 1.43\%$

From Fig. 8-3, $A_{sh} = 0.789$

$\phi_{sh} = A_{sh} (\varepsilon_{sh})_u / h = (0.789)\left(400 \times 10^{-6}\right)/20 = 15.78 \times 10^{-6}$/in.

$\Delta_{sh} = K_{sh} \phi_{sh} \ell^2 = (1/8)\left(15.78 \times 10^{-6}\right)(26)^2\,(12)^2 = 0.192$ in.

The ratio of shrinkage strain at 2 months to the ultimate is 0.36 per Table 2.1 of Ref. 10.4 Therefore the shrinkage deflection of the precast beam at 2 months is:

$0.36 \Delta_{sh} = (0.36)(0.192) = 0.069$ in.

Term (5) — $0.64 \Delta_{sh} \dfrac{I_2}{I_c} = (0.64)(0.192)(0.383) = 0.047$ in.

Term (6) — $(\Delta_i)_1 = \dfrac{K\,(5/48)\,(M_1 + M_2)\,\ell^2}{(E_c)_2\,(I_e)_{1+2}} - (\Delta_i)_2$

$= \dfrac{(1)\,(5/48)\,(40.6 + 21.1)\,(26)^2\,(12)^3}{(3830)\,(4400)} - 0.088 = 0.358$ in.

Term (7) —Creep deflection of the composite beam due to slab dead load. The slab is cast at 2 months. Therefore, the fraction of the creep coefficient, C_u, is obtained my multiplying the value under standard conditions of 1.60 by a b_s value of 0.89 (See explanation of Term (6) in Eq. (15)). The total creep of the beam is reduced by the ratio of the moment of inertia of the beam to the moment of inertia of the composite section. k_r is, as before, taken as 0.85:

$$(0.89)(1.60)k_r(\Delta_i)_1 \frac{I_2}{I_c} = (0.89)\,(1.60)\,(0.85)\,(0.358)\,(0.383) = 0.166 \text{ in.}$$

Term (8) —Due to the fact that the beam and the slab were cast at different times, there will be some contribution to the total deflection due to the tendency of the two parts to creep and shrink at different rates. It is noted in Table 2.1 of ACI 435R-95 (Ref. 10.4) that the creep and shrinkage at a time of 2 months is almost half of the total. Consequently, behavior of the composite section will be affected by this different age. The proper calculation of the resulting deflection is very complex. In this example, the deflection due to differing age concrete is approximated as one-half of the dead load deflection of the beam due to the slab dead load. Readers are cautioned that this procedure results in only a rough estimate. Half of the dead load deflection is

$$\Delta_{ds} = 0.50\,(\Delta_i)_1 = (0.50)\,(0.358) = 0.179 \text{ in. (rough estimate)}$$

Term (9) — Using the alternative method

$$(\Delta_i)_\ell = \frac{K\,(5/48)\,M_\ell \ell^2}{(E_c)_2\,(I_c)_{cr}} = \frac{(1)\,(5/48)\,(50.7)\,(26)^2\,(12)^3}{(3830)\,(8295)} = 0.194 \text{ in.}$$

Term (10) — $k_r = 0.85$ (neglecting the effect of any compression steel in slab)

$$(\Delta_{cp})_\ell = k_r C_u \left[0.20\,(\Delta_i)_\ell\right]$$

$$= (0.85)\,(1.60)\,(0.20 \times 0.194) = 0.053 \text{ in.}$$

In Eq. (19), $\Delta_u = 0.084 + 0.055 + 0.023 + 0.069 + 0.047 + 0.358 + 0.166 + 0.179 + 0.194 + 0.053$

$= 1.23$ in.

Checking Eq. (20) (same solution),

$$\Delta_u = \left(1.65 + 0.71\,\frac{I_2}{I_c}\right)(\Delta_i)_2 + \left(0.36 + 0.64\,\frac{I_2}{I_c}\right)\Delta_{sh} +$$

$$\left(1.05 + 1.21\frac{I_2}{I_c}\right)(\Delta_i)_1 + (\Delta_i)_\ell + (\Delta_{cp})_\ell$$

$= (1.65 + 0.71 \times 0.383)\,(0.084) + (0.36 + 0.64 \times 0.383)\,(0.192)$
$+ (1.50 + 1.21 \times 0.383)\,(0.358) + 0.194 + 0.053$
$= 1.23$ in. (same as above)Assuming nonstructural elements are installed after the composite slab has hardened,

$\Delta_{cp} + \Delta_{sh} + (\Delta_i)_\ell$ = Terms (3) + (5) + (7) + (8) + (9) + (10)

$= 0.023 + 0.047 + 0.166 + 0.179 + 0.194 + 0.053 = 0.66$ in.

Comparisons with the allowable deflections in Table 9.5(b) are shown at the end of Design Example 8.7.

Example 10.7—Shored Nonprestressed Composite Beam

Required: Analysis of short-term and ultimate long-term deflections, to show the beneficial effect of shoring in reducing deflections.

Data: Same as in Example 8.6, except that shored construction is used.

Calculations and Discussion	Code Reference

1. Effective moments of inertia for composite section, using Eq. (9-8):

   $M_{cr} = f_r I_g / y_t = (474)\,(26{,}470)/(16.29)\,(12{,}000) = 64.2$ ft-kips — *Eq. (9-9)*

   $M_{cr}/(M_1 + M_2) = [64.2/(40.6 + 21.1)] = 1.04 > 1$

   Hence $(I_e)_{1+2} = I_g = 26{,}470$ in.^{4}

   In Term (5), Eq. (17)—Composite Section,

   $[M_{cr}/(M_1 + M_2 + M_\ell)]^3 = [64.2/(40.6 + 21.1 + 50.7)]^3 = 0.186$

   $(I_e)_{d+\ell} = (M_{cr}/M_a)^3 I_g + [1 - (M_{cr}/M_a)^3] I_{cr} \le I_g$ — *Eq. (9-8)*

   $= (0.186)\,(26{,}470) + (1 - 0.186)\,(8295) = 11{,}675$ in.^{4}

   versus the alternative method of Example 8.6 where $I_e = (I_c)_{cr} = 8295$ in.^{4} was used with the live load moment directly.

2. Deflections, using Eqs. (17) and (18):

   $$\text{Term (1)} — (\Delta_i)_{1+2} = \frac{K\,(5/48)\,(M_1 + M_2)\ell^2}{(E_c)_2\,(I_e)_{1+2}}$$

   $$= \frac{(1)\,(5/48)\,(40.6 + 21.1)\,(26)^2\,(12)^3}{(3830)\,(26{,}470)} = 0.074 \text{ in.}$$

   Term (2) —Creep deflection due to total dead load of beam and slab. The value of C_u for the beam is taken to be 1.60. Consider the value of C_u for the slab to be slightly higher. For shores removed at 10 days, it may be shown by comparison of the correction factors, K_{to}^c for 10 and 20 day load applications (Section 2.3.4, ACI 435, Ref. 10.4) that the ultimate creep coefficient for the slab is approximately 1.74. k_r is conservatively assumed to have a value of 0.85.

   The average creep coefficient for the composite section is:

   Avg.$C_u = {}^1/_2(1.60 + 1.74) = 1.67$

   $1.67 k_r (\Delta_i)_{1+2} = (1.67)\,(0.85)\,(0.074) = 0.105$ in.

Term (3) — Shrinkage deflection of the precast beam after shores are removed. As indicated in Term 4 of Example 8.6, the fraction of shrinkage of the precast beam at 2 months is 0.36. The shores are assumed to be removed about 10 days after the 2-month point. Therefore, consider the remaining fraction of shrinkage is 1-0.36 = 0.64. Recall that the ultimate shrinkage, $(\varepsilon_{sh})_u = 400 \times 10^{-6}$. Utilize the result found for Δ_{sh} in Term (4) of Example 10.6:

Remaining $(e_{sh}) = (0.64)(\ 400 \times 10^{-6}\ ) = 256 \times 10^{-6}$

$$\Delta_{sh} \frac{I_2}{I_c} = (256/400)\ (0.192)\ (0.383) = 0.047 \text{ in.}$$

Term (4) — Deflection due to differences in shrinkage and creep in the beam and slab. This is a complex issue. For this example, assume that the magnitude of this component is approximated by the initial dead load deflection of the composite section.

$$\Delta_{ds} = (\Delta_i)_{l+2} = 0.074 \text{ in. (rough estimate)}$$

$$\text{Term (5)} — (\Delta_i)_\ell = \frac{K\ (5/48)\ (M_1 + M_2 + M_\ell)\ell^2}{(E_c)_2\ (I_e)_{d+\ell}} - (\Delta_i)_{l+2}$$

$$= \frac{(1)\ (5/48)\ (40.6 + 21.1 + 50.7)\ (26)^2\ (12)^3}{(3830)\ (11{,}675)} - 0.074 \text{ in.} = 0.232 \text{ in.}$$

Term (6) — $k_r = 0.85$ (neglecting the effect of any compression steel in slab),

$$(\Delta_{cp})_\ell = k_r C_u\ [0.20\ (\Delta_i)_\ell] = (0.85)\ (1.60)\ (0.20 \times 0.232) = 0.063 \text{ in.}$$

In Eq. (17), $\Delta_u = 0.074 + 0.105 + 0.047 + 0.074 + 0.232 + 0.063 = 0.60$ in.

versus 1.23 in. with unshored construction.

This shows the beneficial effect of shoring in reducing the total deflection.

Checking by Eq. (18) (same solution),

$$\Delta_u = 3.42\ (\Delta_i)_{l+2} + \Delta_{sh} \frac{I_2}{I_c} + (\Delta_i)_\ell + (\Delta_{cp})_\ell$$

$$= (3.42)\ (0.074) + 0.046 + 0.232 + 0.063 = 0.60 \text{ in. (same as above)}$$

Assuming that nonstructural elements are installed after shores are removed,

$$\Delta_{cp} + \Delta_{sh} + (\Delta_i)_\ell = \Delta_u - (\Delta_i)_{l+2} = 0.60 - 0.07 = 0.53 \text{ in.}$$

Comparison of Results of Examples 8.6 and 8.7

The computed deflections of $(\Delta_i)_\ell = 0.19$ in. in Example 10.6 and 0.23 in. in Example 10.7; and $\Delta_{cp} + \Delta_{sh} + (\Delta_i)_\ell = 0.66$ in. in Example 10.6 and 0.53 in. in Example 10.7 are compared with the allowable deflections in Table 9.5(b) as follows:

Flat roofs not supporting and not attached to nonstructural elements likely to be damaged by large deflections—

$(\Delta_i)_\ell \leq \ell/180 = 312/180 = 1.73$ in. O.K.

Floors not supporting and not attached to nonstructural elements likely to be damaged by large deflections—

$(\Delta_i)_\ell \leq \ell/360 = 312/360 = 0.87$ in. O.K.

Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections (very stringent limitation)—

$\Delta_{cp} + \Delta_{sh} + (\Delta_i)_\ell \leq \ell/480 = 312/480 = 0.65$ in.

Note that the long term deflection occurring after attachment of non-structural elements (0.66 in) exceeds the L/480 limit. It actually meets L/473. Since the L/480 limit only applies in case of *nonstructural elements likely to be damaged by large deflections*, the particular use of the beam would have to be considered in order to make a judgment on the acceptability of the computed deflections. Refer to the footnotes following Table 9.5(b) of ACI 318

Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections—

$\Delta_{cp} + \Delta_{sh} + (\Delta_i)_\ell \leq \ell/240 = 312/240 = 1.30$ in. O.K.

11

Design for Slenderness Effects

GENERAL CONSIDERATIONS

Design of columns consists essentially of selecting an adequate column cross-section with reinforcement to support required combinations of factored axial loads P_u and factored (primary) moments M_u, including consideration of column slenderness (secondary moments).

Column slenderness is expressed in terms of its slenderness ratio $k\ell_u/r$, where k is an effective length factor (dependent on rotational and lateral restraints at the ends of the column), ℓ_u is the unsupported column length, and r is the radius of gyration of the column cross-section. In general, a column is slender if its applicable cross-sectional dimension is small in comparison to its length.

For design purposes, the term "short column" is used to denote a column that has a strength equal to that computed for its cross-section, using the forces and moments obtained from an analysis for combined bending and axial load. A "slender column" is defined as a column whose strength is reduced by second-order deformations (secondary moments). By these definitions, a column with a given slenderness ratio may be considered a short column for design under one set of restraints, and a long column under another set. With the use of higher strength concrete and reinforcement, and with more accurate analysis and design methods, it is possible to design smaller cross-sections, resulting in members that are more slender. The need for reliable and rational design procedures for slender columns thus becomes a more important consideration in column design.

A short column may fail due to a combination of moment and axial load that exceeds the strength of the cross-section. This type of a failure is known as "material failure." As an illustration, consider the column shown in Fig. 11-1. Due to loading, the column has a deflection Δ which will cause an additional (secondary) moment in the column. From the free body diagram, it can be seen that the maximum moment in the column occurs at section A-A, and is equal to the applied moment plus the moment due to member deflection, which is $M = P(e + \Delta)$.

Failure of a short column can occur at any point along the strength interaction curve, depending on the combination of applied moment and axial load. As discussed above, some deflection will occur and a "material failure" will result when a particular combination of load P and moment $M = P(e + \Delta)$ intersects the strength interaction curve.

If a column is very slender, it may reach a deflection due to axial load P and a moment Pe such that deflections will increase indefinitely with an increase in the load P. This type of failure is known as a "stability failure," as shown on the strength interaction curve.

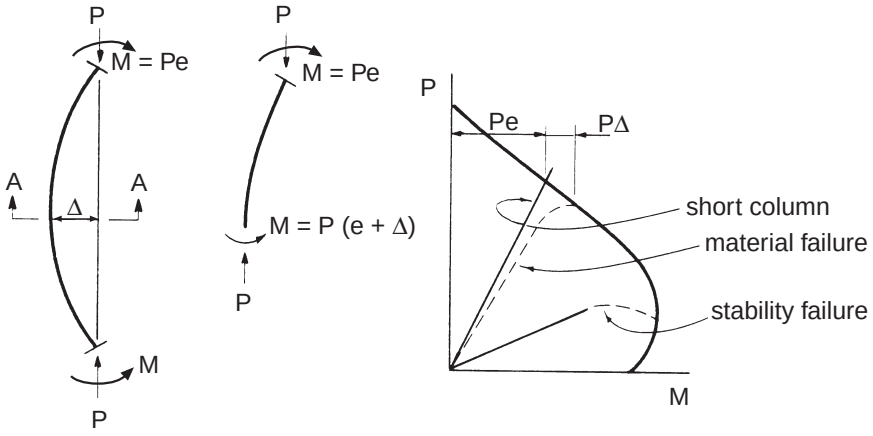


Figure 11-1 Strength Interaction for Slender Columns

The basic concept on the behavior of straight, concentrically loaded, slender columns was originally developed by Euler more than 200 years ago. It states that a member will fail by buckling at the critical load $P_c = \pi^2 EI/(\ell_e)^2$, where EI is the flexural stiffness of the member cross-section, and ℓ_e is the effective length, which is equal to $k\ell_u$. For a "stocky" short column, the value of the buckling load will exceed the direct crushing strength (corresponding to material failure). In members that are more slender (i.e., members with larger $k\ell_u/r$ values), failure may occur by buckling (stability failure), with the buckling load decreasing with increasing slenderness (see Fig. 11-2).

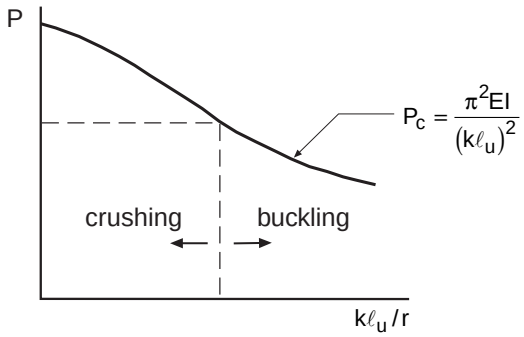


Figure 11-2 Failure Load as a Function of Column Slenderness

As shown above, it is possible to depict slenderness effects and amplified moments on a typical strength interaction curve. Hence, a "family" of strength interaction diagrams for slender columns with varying slenderness ratios can be developed, as shown in Fig. 11-3. The strength interaction diagram for $k\ell_u/r = 0$ corresponds to the combinations of moment and axial load where strength is not affected by member slenderness (short column strength).

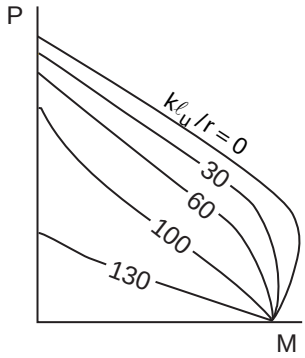
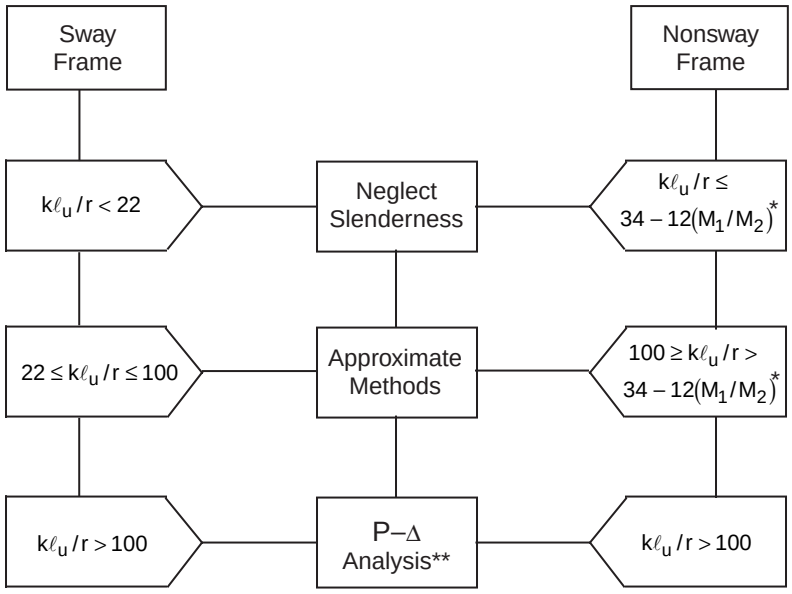


Figure 11-3 Strength Interaction Diagrams for Slender Columns

CONSIDERATION OF SLENDERNESS EFFECTS

Slenderness limits are prescribed for both nonsway and sway frames, including design methods permitted for each slenderness range. Lower-bound slenderness limits are given, below which secondary moments may be disregarded and only axial load and primary moment need be considered to select a column cross-section and reinforcement (short column design). It should be noted that for ordinary beam and column sizes and typical story heights of concrete framing systems, effects of slenderness may be neglected for more than 90 percent of columns in nonsway frames and around 40 percent of columns in sway frames. For moderate slenderness ratios, an approximate analysis of slenderness effects based on a moment magnifier (see 10.12 and 10.13) is permitted. For columns with high slenderness ratios, a more exact second-order analysis is required (see 10.11.5), taking into account material nonlinearity and cracking, as well as the effects of member curvature and lateral drift, duration of the loads, shrinkage and creep, and interaction with the supporting foundation. No upper limits for column slenderness are prescribed. The slenderness ratio limits in 10.12.2 for nonsway frames and 10.13.2 for sway frames, and design methods permitted for consideration of column slenderness, are summarized in Fig. 11-4.



* $34 - 12(M_1/M_2) \le 40$

** Permitted for any slenderness ratio

Figure 11-4 Consideration of Column Slenderness

10.10 SLENDERNESS EFFECTS IN COMPRESSION MEMBERS

10.10.1 Second-Order Frame Analysis

The code encourages the use of a second-order frame analysis or P-Δ analysis for consideration of slenderness effects in compression members. Generally, the results of a second-order analysis give more realistic values of the moments than those obtained from an approximate analysis by 10.12 or 10.13. For sway frames, the use of second-order analyses will generally result in a more economical design. Procedures for carrying out a second-order analysis are given in Commentary Refs. 10.24-10.29. The reader is referred to R10.10.1, which discusses minimum requirements for an adequate second-order analysis under 10.10.1.

If more exact analyses are not feasible or practical, 10.10.2 permits an approximate moment magnifier method to account for column slenderness. Note, however, that for all compression members with a column slenderness ratio $(k\ell_u/r)$ greater than 100 (see Fig. 11-4), a more exact analysis as defined in 10.10.1 must be used for consideration of slenderness effects.

10.11 APPROXIMATE EVALUATION OF SLENDERNESS EFFECTS

The moment magnification factor δ is used to magnify the primary moments to account for increased moments due to member curvature and lateral drift. The moment magnifier δ is a function of the ratio of the applied axial load to the critical or buckling load of the column, the ratio of the applied moments at the ends of the column, and the deflected shape of the column.

10.11.1 Section Properties for Frame Analysis

According to 10.11.1, the factored axial loads P_u, the factored moments at the column ends M_1 and M_2, and the lateral story deflections Δ_o shall be computed using an elastic first-order frame analysis taking into account cracked regions along the length of the members. It is obviously not economically feasible to perform such calculations even for small structures. Thus, the section properties given in 10.11.1 and summarized in Table 11-1 may be used in the analysis to account for cracking. The values of E, I, and A have been chosen from the results of frame tests and analyses as outlined in code Reference 10.28. It is important to note that for service load analysis of the structure, it is satisfactory to multiply the moments of inertia given in Table 11-1 by 1/0.70 = 1.43 (R10.11.1). Also, the moments of inertia must be divided by $(1 + \beta_d)$ in the case when sustained lateral loads act on the structure (for example, lateral loads resulting from earth pressure) or when the gravity load stability check made in accordance with 10.13.6 is performed.

Table 11-1 Section Properties for Frame Analysis

	Modulus of Elasticity	Moment of Inertia†	Area
Beams	E_c from 8.5.1	0.35 I_g	1.0A_g
Columns		0.70 I_g	
Walls - uncracked		0.70 I_g	
Walls - cracked		0.35 I_g	
Flat plates and flat slabs		0.25 I_g	

†Divide by $(1+\beta_d)$ when sustained lateral loads act or for stability checks made in accordance with 10.13.6. For service load analyses, multiply by 1/0.70 = 1.43.

10.11.2 Radius of Gyration

In general, the radius of gyration, r, is $\sqrt{I_g/A_g}$. In particular, r may be taken as 0.30 times the dimension in the direction of analysis for a rectangular section and 0.25 times the diameter of a circular section, as shown in Fig. 11-5.

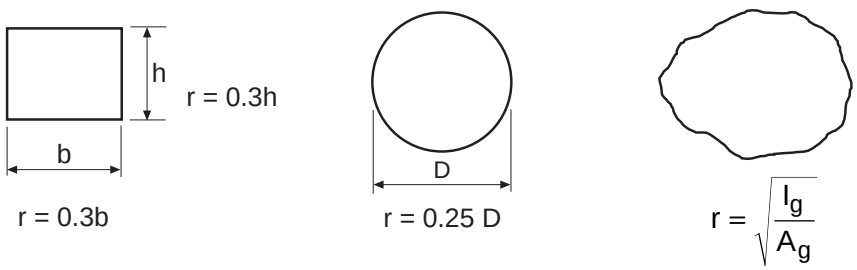


Figure 11-5 Radius of Gyration, r

10.11.3, 10.12.1 Unsupported and Effective Lengths of Compression Members

The unsupported length ℓ_u of a column, defined in 10.11.3, is the clear distance between lateral supports as shown in Fig. 11-6. Note that the length ℓ_u may be different for buckling about each of the principal axes of the column cross-section. The basic Euler equation for critical buckling load can be expressed as $P_c = \pi^2 EI/(\ell_e)^2$, where ℓ_e is the effective length $k\ell_u$. The basic equations for the design of slender columns were derived for hinged ends, and thus, must be modified to account for the effects of end restraint. Effective column length $k\ell_u$, as contrasted to actual unbraced length ℓ_u, is the term used in estimating slender column strength, and considers end restraints as well as nonsway and sway conditions.

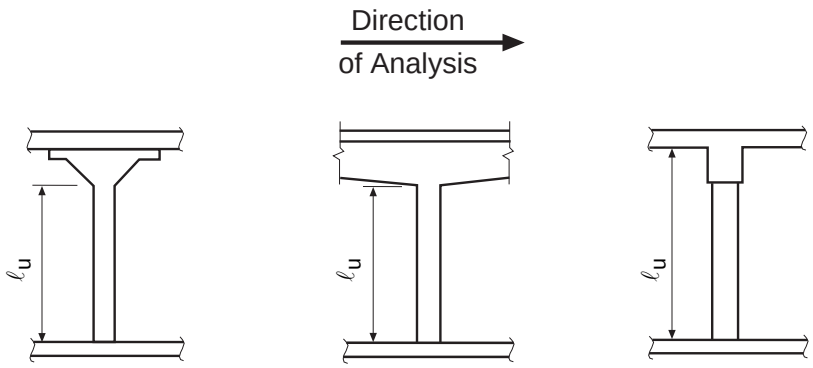


Figure 11-6 Unsupported Length, ℓ_u

At the critical load defined by the Euler equation, an originally straight member buckles into a half-sine wave as shown in Fig. 11-7(a). In this configuration, an additional moment P-Δ acts at every section, where Δ is the lateral deflection at the specific location under consideration along the length of the member. This deflection continues to increase until the bending stress caused by the increasing moment (P-Δ), plus the original compression stress caused by the applied loading, exceeds the compressive strength of concrete and the member fails. The effective length ℓ_e (= $k\ell_u$) is the length between pinned ends, between zero moments or between inflection points. For the pinned condition illustrated in Fig. 11-7(a), the effective length is equal to the unsupported length ℓ_u. If the member is fixed against rotation at both ends, it will buckle in the shape depicted in Fig. 11-7(b); inflection points will occur at the locations shown, and the effective length ℓ_e will be one-half of the unsupported length. The critical buckling load P_c for the fixed-end condition is four times that for a pin-end condition. Rarely are columns in actual structures either hinged or fixed; they are partially restrained against rotation by members framing into the column, and thus the effective length is between $\ell_u/2$ and ℓ_u, as shown in Fig. 11-7(c) as long as the lateral displacement of one end of the column with respect to the other end is prevented. The actual value of the effective length depends on the rigidity of the members framing into the top and bottom ends of the column.

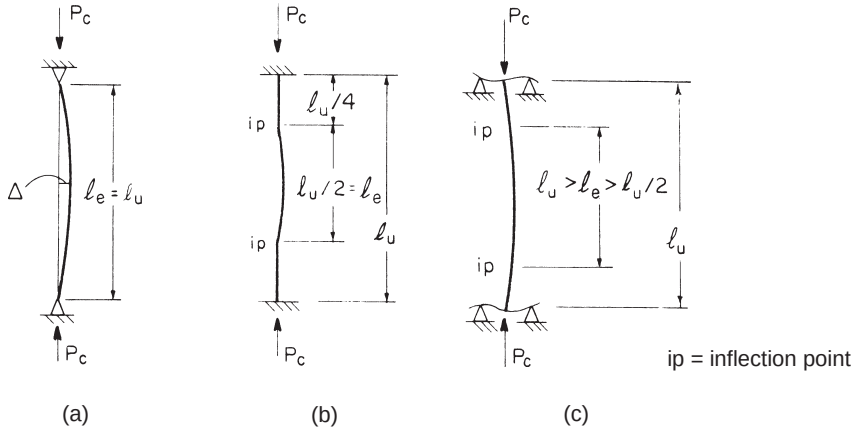


Figure 11-7 Effective Length, ℓ_e (Nonsway Condition)

A column that is fixed at one end and entirely free at the other end (cantilever) will buckle as shown in Fig. 11-8(a). The upper end will deflect laterally relative to the lower end; this is known as sidesway. The deflected shape of such a member is similar to one-half of the sinusoidal deflected shape of the pin-ended member illustrated in Fig. 11-7(a). Thus, the effective length is equal to twice the actual length. If the column is fixed against rotation at both ends but one end can move laterally with respect to the other, it will buckle as shown in Fig. 11-8(b). The effective length ℓ_e will be equal to the actual length ℓ_u, with an inflection point (ip) occurring as shown. The buckling load of the column in Fig. 11-8(b), where sidesway is not prevented, is one-quarter that of the column in Fig. 11-7(b), where sidesway is prevented. As noted above, the ends of columns are rarely either completely hinged or completely fixed, but rather are partially restrained against rotation by members framing into the ends of the columns. Thus, the effective length will vary between ℓ_u and infinity, as shown in Fig. 11-8(c). If restraining members (beams or slab) are very rigid as compared to the column, the buckling in Fig. 11-8(b) is approached. If, however, the restraining members are quite flexible, a hinged condition is approached at both ends and the column(s), and possibly the structure as a whole, approaches instability.

In typical reinforced concrete structures, the designer is rarely concerned with single members, but rather with rigid framing systems consisting of beam-column and slab-column assemblies. The buckling behavior of a frame that is not braced against sidesway can be illustrated by the simple portal frame shown in Fig. 11-9. Without lateral restraint at the upper end, the entire (unbraced) frame is free to move sideways. The bottom end may be pinned or partially restrained against rotation. In general, the effective length ℓ_e depends on the degree of rotational restraint at the ends of the column and $\ell_u < \ell_e < \infty$.

In summary, the following comments can be made:

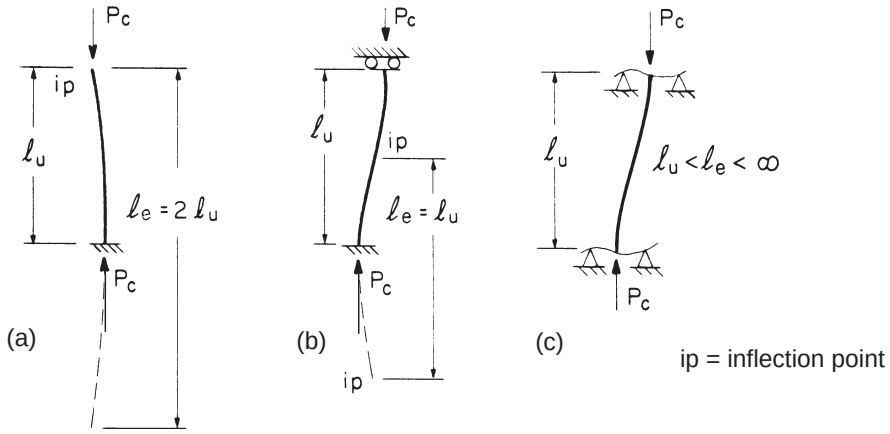


Figure 11-8 Effective Length, ℓ_e (Sway Condition)

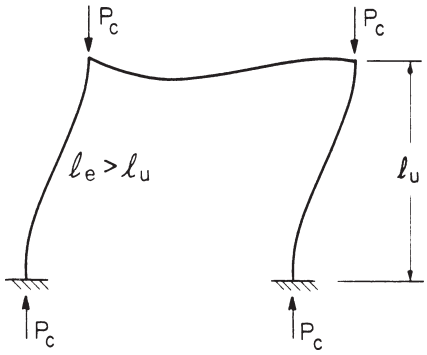


Figure 11-9 Rigid Frame (Sway Condition)

1. For compression members in a nonsway frame, the effective length ℓ_e falls between $\ell_u/2$ and ℓ_u, where ℓ_u is the actual unsupported length of the column.

2. For compression members in a sway frame, the effective length ℓ_e is always greater than the actual length of the column ℓ_u, and may be $2\ell_u$ and higher. In this case, a value of k less than 1.2 normally would not be realistic.

3. Use of the alignment charts shown in Figs. 11-10 and 11-11 (also given in Fig. R10.12.1) allows graphical determination of the effective length factors for compression members in nonsway and sway frames, respectively. If both ends of a column in a nonsway frame have minimal rotational stiffness, or approach $\psi = \infty$, then k = 1.0. If both ends have or approach full fixity, $\psi = 0$, and k = 0.5. If both ends of a column in a sway frame have minimal rotational stiffness, or approach $\psi = \infty$, then $k = \infty$. If both ends have or approach full fixity, $\psi = 0$, then k = 1.0.

An alternative method for computing the effective length factors for compression members in nonsway and sway frames is contained in R10.12.1. For compression members in a nonsway frame, an upper bound to the effective length factor may be taken as the smaller of the values given by the following two expressions, which are given in the 1992 British Standard Code of Practice (ACI Refs. 10.33 and 10.34):

$$k = 0.7 + 0.05(\psi_A + \psi_B) \leq 1.0$$

$$k = 0.85 + 0.05\,\psi_{min} \leq 1.0$$

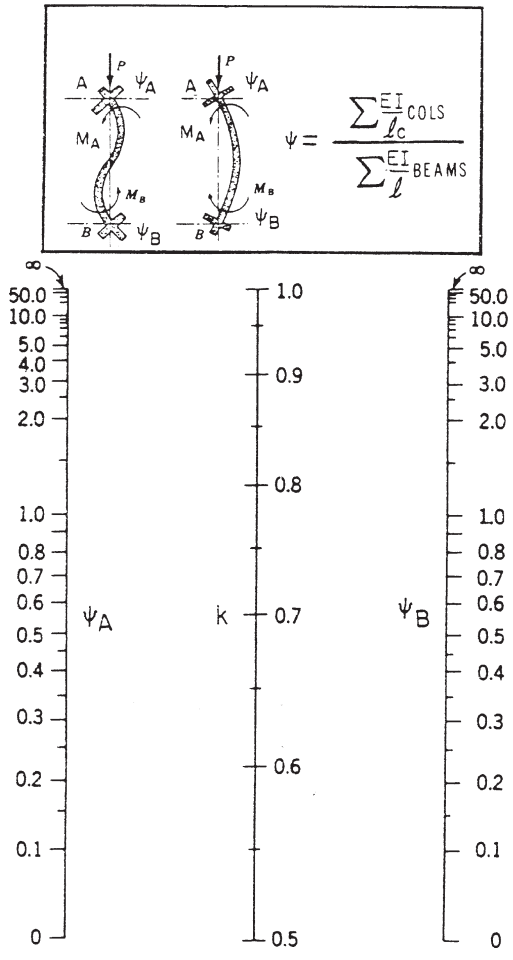


Figure 11-10 Effective Length Factors for Compression Members in a Nonsway Frame

where ψ_A and ψ_B are the values of ψ at the ends of the column and ψ_{min} is the smaller of the two values.

For compression members in a sway frame restrained at both ends, the effective length factor may be taken as (ACI Ref. 10.32):

$$\text{For } \psi_m < 2, \quad k = \frac{20 - \psi_m}{20}\sqrt{1 + \psi_m}$$

$$\text{For } \psi_m \geq 2, \quad k = 0.9\sqrt{1 + \psi_m}$$

where ψ_m is the average of the ψ values at the two ends of the column.

For compression members in a sway frame hinged at one end, the effective length factor may be taken as (ACI Refs. 10.33 and 10.34):

$$k = 2.0 + 0.3\psi$$

where ψ is the column-to-beam stiffness ratio at the restrained end.

In determining the effective length factor k from Figs. 11-10 and 11-11, or from the Commentary equations, the rigidity (EI) of the beams (or slabs) and columns shall be calculated based on the values given in 10.11.1.

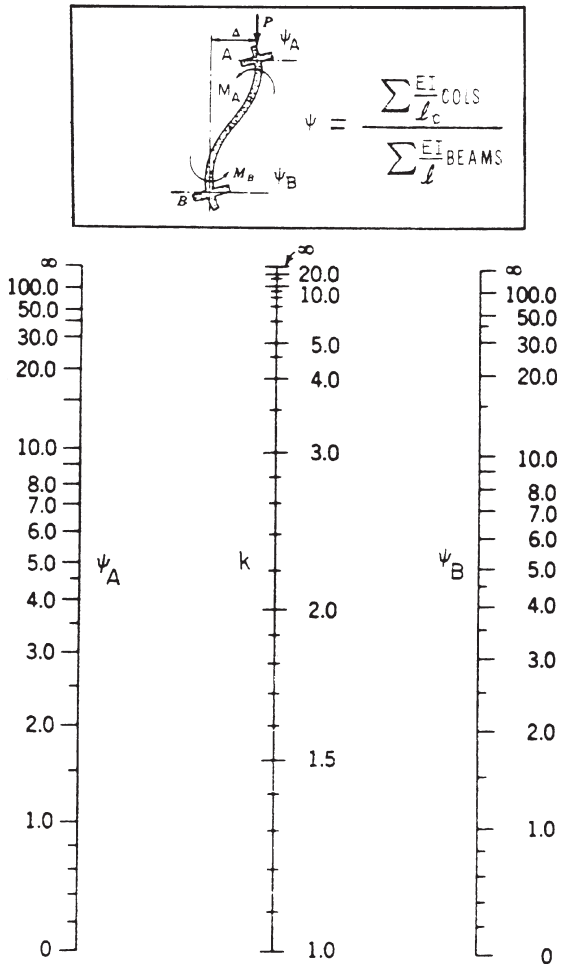


Figure 11-11 Effective Length Factors for Compression Members in a Sway Frame

10.11.4 Nonsway Versus Sway Frames

In actual structures, there is rarely a completely nonsway or sway condition. If it is not readily apparent by inspection, 10.11.4.1 and 10.11.4.2 give two possible ways of determining if a frame is nonsway or not. According to 10.11.4.1, a column in a structure can be considered nonsway if the column end moments due to second-order effects do not exceed 5 percent of the first-order end moments. According to 10.11.4.2, it is also permitted to assume a story within a structure is nonsway if:

$$Q = \frac{\Sigma P_u \Delta_o}{V_u \ell_c} \leq 0.05 \qquad \textit{Eq. (10-6)}$$

where

Q = stability index for a story
ΣP_u = total factored vertical load in the story corresponding to the lateral loading case for which ΣP_u is greatest (R10.11.4)
V_u = total story shear
Δ_o = first-order relative deflection between the top and bottom of the story due to V_u
ℓ_c = column length, measured from center-to-center of the joints in the frame

Note that Eq. (10-6) is not applicable when $V_u = 0$.

10.11.6 Moment Magnifier δ for Biaxial Bending

When biaxial bending occurs in a column, the computed moments about each of the principal axes must be magnified. The magnification factors δ are computed considering the buckling load P_c about each axis separately, based on the appropriate effective lengths and the related stiffness ratios of columns to beams in each direction. Thus, different buckling capacities about the two axes are reflected in different magnification factors. The moments about each of the two axes are magnified separately, and the cross-section is then proportioned for an axial load P_u and magnified biaxial moments.

10.12.2, 10.13.2 Consideration of Slenderness Effects

For compression members in a nonsway frame, effects of slenderness may be neglected when $k\ell_u/r$ is less than or equal to 34 - 12 (M_1/M_2), where M_2 is the larger end moment and M_1 is the smaller end moment. The ratio M_1/M_2 is positive if the column is bent in single curvature, negative if bent in double curvature. Note that M_1 and M_2 are factored end moments obtained by an elastic frame analysis and that the term [34-12M_1/M_2] shall not be taken greater than 40. For compression members in a sway frame, effects of slenderness may be neglected when $k\ell_u/r$ is less than 22 (10.13.2). The moment magnifier method may be used for columns with slenderness ratios exceeding these lower limits.

The upper slenderness limit for columns that may be designed by the approximate moment magnifier method is $k\ell_u/r$ equal to 100 (10.11.5). When $k\ell_u/r$ is greater than 100, an analysis as defined in 10.10.1 must be used, taking into account the influence of axial loads and variable moment of inertia on member stiffness and fixed-end moments, the effect of deflections on the moments and forces, and the effects of duration of loading (sustained load effects). Criteria for consideration of column slenderness are summarized in Fig. 11-4.

The lower slenderness ratio limits will allow a large number of columns to be exempt from slenderness consideration. Considering the slenderness ratio $k\ell_u/r$ in terms of ℓ_u/h for rectangular columns, the effects of slenderness may be neglected in design when ℓ_u/h is less than 10 for compression members in a nonsway frame and with zero restraint at both ends. This lower limit increases to 18 for a column in double curvature with equal end moments and a column-to-beam stiffness ratio equal to one at each end. For columns with minimal or zero

restraint at both ends, a value of k equal to 1.0 should be used. For stocky columns restrained by flat slab floors, k ranges from about 0.95 to 1.0 and can be conservatively estimated as 1.0. For columns in beam-column frames, k ranges from about 0.75 to 0.90, and can be conservatively estimated as 0.90. If the initial computation of the slenderness ratio based on estimated values of k indicates that effects of slenderness must be considered in the design, a more accurate value of k should be calculated and slenderness re-evaluated. For a compression member in a sway frame with a column-to-beam stiffness ratio equal to 1.0 at both ends, effects of slenderness may be neglected when ℓ_u/h is less than 5. This value reduces to 3 if the beam stiffness is reduced to one-fifth of the column stiffness at each end of the column. Thus, beam stiffnesses at the top and bottom of a column of a high-rise structure where sidesway is not prevented by structural walls or other means will have a significant effect on the degree of slenderness of the column.

The upper limit on the slenderness ratio of $k\ell_u/r$ equal to 100 corresponds to an ℓ_u/h equal to 30 for a compression member in a nonsway frame with zero restraint at both ends. This ℓ_u/h limit increases to 39 for a column-to-beam stiffness ratio of 1.0 at each end.

10.12.3 Moment Magnification—Nonsway Frames

The approximate slender column design equations contained in 10.12.3 for nonsway frames are based on the concept of a moment magnifier δ_{ns} which amplifies the larger factored end moment M_2 on a compression member. The column is then designed for the factored axial load P_u and the amplified moment M_c where M_c is given by:

$$M_c = \delta_{ns} M_2 \qquad \textit{Eq. (10-8)}$$

where

$$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \geq 1.0 \qquad \textit{Eq. (10-9)}$$

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \qquad \textit{Eq. (10-10)}$$

The critical load P_c is computed for a nonsway condition using an effective length factor k of 1.0 or less. When k is determined from the alignment charts or the equations in R10.12, the values of E and I from 10.11.1 must be used in the computations. Note that the 0.75 factor in Eq. (10-9) is a stiffness reduction factor (see R10.12.3).

In defining the critical column load P_c, the difficult problem is the choice of a stiffness parameter EI which reasonably approximates the stiffness variations due to cracking, creep, and the nonlinearity of the concrete stress-strain curve. In lieu of a more exact analysis, EI shall be taken as:

$$EI = \frac{(0.2E_cI_g + E_sI_{se})}{1 + \beta_d} \qquad \textit{Eq. (10-11)}$$

or

$$EI = \frac{0.4E_cI_g}{1 + \beta_d} \qquad \textit{Eq. (10-12)}$$

The second of these two equations is a simplified approximation to the first. Both equations approximate the lower limits of EI for practical cross-sections and, thus, are conservative. The approximate nature of the EI equations is shown in Fig. 11-12 where they are compared with values derived from moment-curvature diagrams for the case when there is no sustained load ($\beta_d = 0$).

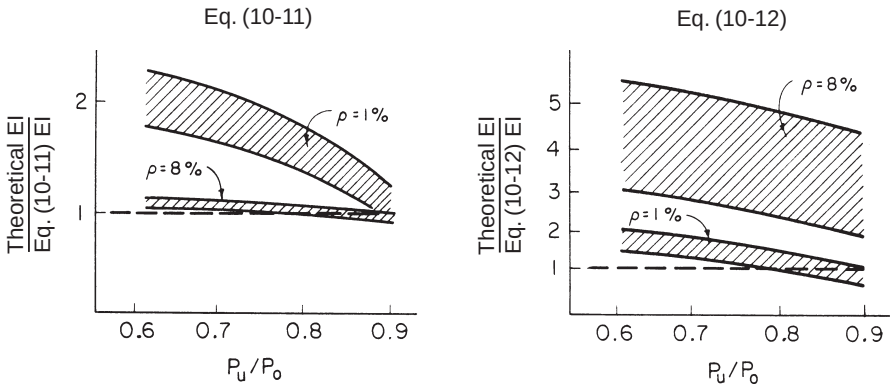


Figure 11-12 Comparison of Equations for EI with EI Values from Moment-Curvature Diagrams

Equation (10-11) represents the lower limit of the practical range of stiffness values. This is especially true for heavily reinforced columns. As noted above, Eq. (10-12) is simpler to use but greatly underestimates the effect of reinforcement in heavily reinforced columns (see Fig. 11-12).

Both EI equations were derived for small e/h values and high P_u/P_o values, where the effect of axial load is most pronounced. The term P_o is the nominal axial load strength at zero eccentricity.

For reinforced concrete columns subjected to sustained loads, creep of concrete transfers some of the load from the concrete to the steel, thus increasing steel stresses. For lightly reinforced columns, this load transfer may cause compression steel to yield prematurely, resulting in a loss in the effective value of EI. This is taken into account by dividing EI by $(1 + \beta_d)$. For nonsway frames, β_d is defined as follows (see 10.0):

$$\beta_d = \frac{\text{Maximum factored axial sustained load}}{\text{Maximum factored axial load associated with the same load combination}}$$

For composite columns in which a structural steel shape makes up a large percentage of the total column cross-section, load transfer due to creep is not significant. Accordingly, only the EI of the concrete portion should be reduced by $(1 + \beta_d)$ to account for sustained load effects.

The term C_m is an equivalent moment correction factor. For members without transverse loads between supports, C_m is (10.12.3.1):

$$C_m = 0.6 + 0.4\left(\frac{M_1}{M_2}\right) \geq 0.4 \qquad \textit{Eq. (10-13)}$$

For members with transverse loads between supports, it is possible that the maximum moment will occur at a section away from the ends of a member. In this case, the largest calculated moment occurring anywhere along the length of the member should be magnified by δ_{ns}, and C_m must be taken as 1.0. Figure 11-13 shows some values of C_m, which are a function of the deformed shape of the column and the end restraints.

If the computed column moment M_2 in Eq. (10-8) is small or zero, design of a nonsway column must be based on the minimum moment $M_{2,min}$ (10.12.3.2):

$$M_{2,min} = P_u\,(0.6 + 0.03h) \qquad \textit{Eq. (10-14)}$$

For members where $M_{2,min} > M_2$, the value of C_m shall either be taken equal to 1.0, or shall be computed by Eq. (10-13) using the ratio of the actual computed end moments M_1 and M_2.

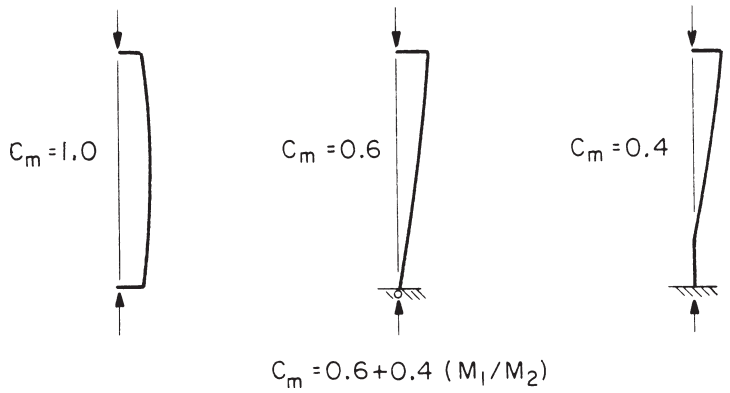


Figure 11-13 Moment Factor C_m

10.13.3 Moment Magnification—Sway Frames

The design of sway frames for slenderness consists essentially of three steps:

1. The magnified sway moments $\delta_s M_s$ are computed in one of three ways:
   a. A second-order elastic frame analysis (10.13.4.1)
   b. An approximate second-order analysis (10.13.4.2)
   c. An approximate magnifier method from previous ACI codes (10.13.4.3)

2. The magnified sway moments $\delta_s M_s$ are added to the unmagnified nonsway moments M_{ns} at each end of the column (10.13.3):

$$M_1 = M_{1ns} + \delta_s M_{1s} \qquad \textit{Eq. (10-15)}$$

$$M_2 = M_{2ns} + \delta_s M_{2s} \qquad \textit{Eq. (10-16)}$$

   The nonsway moments M_{1ns} and M_{2ns} are computed using a first-order elastic analysis.

3. If the column is slender and subjected to high axial loads, it must be checked to see whether moments at points between the column ends are larger than those at the ends. According to 10.13.5, this check is performed using the nonsway magnifier δ_{ns} with P_c computed assuming k = 1.0 or less.

10.13.4 Calculation of $\delta_s M_s$

As noted above, there are three different ways to compute the magnified sway moments $\delta_s M_s$. If a second-order elastic analysis is used to compute $\delta_s M_s$, the deflections must be representative of the stage immediately prior to the ultimate load. Thus, the values of EI given in 10.11.1 must be used in the second-order analysis. Note that I must be divided by $(1 + \beta_d)$ where for sway frames, β_d is defined as follows (see 10.0):

$$\beta_d = \frac{\text{Maximum factored sustained shear within a story}}{\text{Maximum factored shear in that story}}$$

For wind or earthquake loads, $\beta_d = 0$. An example of a non-zero β_d may occur when members are subjected to earth pressure.

Section 10.13.4.2 allows an approximate second-order analysis to determine $\delta_s M_s$. In this case, the solution of the infinite series that represents the iterative P-Δ analysis for second-order moments is given as follows:

$$\delta_s M_s = \frac{M_s}{1 - Q} \geq M_s \qquad \textit{Eq. (10-17)}$$

where

Q = stability index for a story

$$= \frac{\Sigma P_u \Delta_o}{V_u \ell_c} \qquad \textit{Eq. (10-7)}$$

Note that Eq. (10-17) closely predicts the second-order moments in a sway frame until δ_s exceeds 1.5. For the case when $\delta_s > 1.5$, $\delta_s M_s$ must be computed using 10.13.4.1 or 10.13.4.3.

The code also allows $\delta_s M_s$ to be determined using the magnified moment procedure that was given in previous ACI codes (10.13.4.3):

$$\delta_s M_s = \frac{M_s}{1 - \dfrac{\Sigma P_u}{0.75 \Sigma P_c}} \geq M_s \qquad \textit{Eq. (10-18)}$$

where

ΣP_u = summation of all the vertical loads in a story

ΣP_c = summation of the critical loads for all sway-resisting columns in a story

It is important to note that the moment magnification in the columns farthest from the center of twist in a building subjected to significant torsional displacement may be underestimated by the moment magnifier procedure. A three-dimensional second-order analysis should be considered in such cases.

10.13.5 Location of Maximum Moment

When the unmagnified nonsway moments at the ends of the column are added to the magnified sway moments at the same points, one of the resulting total end moments is usually the maximum moment in the column. However, for slender columns with high axial loads, the maximum moment may occur between the ends of the column. A simple way of determining if this situation occurs or not is given in 10.13.5: if an individual compression member has

$$\frac{\ell_u}{r} > \frac{35}{\sqrt{\dfrac{P_u}{f'_c A_g}}} \qquad \textit{Eq. (10-19)}$$

the maximum moment will occur at a point between the ends of the column. In this case, M_2, which is defined in Eq. (10-16), must be magnified by the nonsway moment magnifier given in Eq. (10-9). The column is then designed for the factored axial load P_u and the moment M_c, where M_c is computed from the following:

$$M_c = \delta_{ns} M_2 \qquad \textit{Eq. (10-8)}$$

$$= \left(\frac{C_m}{1 - \frac{P_u}{0.75P_c}} \right) (M_{2ns} + \delta_s M_{2s})$$

Note that k is determined according to 10.12.1 and $\delta_{ns} \geq 1.0$.

10.13.6 Structural Stability Under Gravity Loads

For sway frames, the possibility of sidesway instability of the structure as a whole under factored gravity loads must be investigated. This is checked in three different ways, depending on the method that is used in determining $\delta_s M_s$:

1. When $\delta_s M_s$ is computed by a second-order analysis (10.13.4.1), the following expression must be satisfied:

$$\frac{\text{Second-order lateral deflections}}{\text{First-order lateral deflections}} \leq 2.5$$

   Note that these deflections are based on the applied loading of $1.4P_D$ and $1.7P_L$ plus lateral load. The frame should be analyzed twice for this set of applied loads: the first analysis should be a first-order analysis and the second should be a second-order analysis. The lateral load may be the actual lateral loads used in design or it may be a single lateral load applied to the top of the frame. In any case, the lateral load(s) should be large enough to give deflections that can be compared accurately.

2. When $\delta_s M_s$ is computed by the approximate second-order analysis (10.13.4.2), then

$$Q = \frac{\Sigma P_u \Delta_o}{V_u \ell_c} \leq 0.60$$

   where the value of Q is evaluated using $1.4P_D$ and $1.7P_L$. Note that the above expression is equivalent to $\delta_s = 2.5$. The values of V_u and Δ_o may be determined using the actual or any arbitrary set of lateral loads. The above stability check is satisfied if the value of Q computed in 10.11.4.2 is less than or equal to 0.2.

3. When $\delta_s M_s$ is computed using the expressions from previous ACI codes (10.13.4.3), the stability check is satisfied when

$$0 < \delta_s \leq 2.5$$

   In this case, ΣP_u and ΣP_c correspond to the factored dead and live loads.

It is important to note that in each of the three cases above, β_d shall be taken as the following:

$$\beta_d = \frac{\text{Maximum factored sustained axial load}}{\text{Maximum factored axial load}}$$

10.13.7 Moment Magnification for Flexural Members

The strength of a laterally unbraced frame is governed by the stability of the columns and by the degree of end restraint provided by the beams in the frame. If plastic hinges form in the restraining beams, the structure approaches a mechanism and its axial load capacity is drastically reduced. Section 10.13.7 requires that the restraining flexural members (beams or slabs) have the capacity to resist the magnified column moments. The ability of the moment magnifier method to provide a good approximation of the actual magnified moments at the member ends in an unbraced frame is a significant improvement over the reduction factor method for long columns prescribed in earlier ACI codes to account for member slenderness in design.

SUMMARY OF DESIGN EQUATIONS

The following summary of equations for the design of slender columns subjected to dead, live and wind loads, in both nonsway and sway frames may be helpful. Examples 11.1 and 11.2 illustrate the application of these equations for the design of columns in nonsway and sway frames, respectively.

- **Nonsway Frames**

1. Determine the factored load combinations per 9.2.

   It is assumed in the examples that follow that the load factor for live load is 0.5 (i.e. condition 9.2.1(a) applies) and that the wind load has been reduced by a directionality factor (9.2.1(b)).

   Note that the factored moments $M_{u,top}$ and $M_{u,bot}$ at the top and bottom of the column, respectively, are to be determined using a first-order frame analysis, based on the cracked section properties of the members.

2. For each load combination, determine M_c, where M_c is the largest factored column end moment, including slenderness effects (if required). Note that M_c may be determined by one of the following methods:

   a. Second-order (P-Δ) analysis (10.10.1)

   b. Magnified moment method (only if $k\ell_u/r \le 100$; see 10.12 and step (3) below)

   Determine the required column reinforcement for the critical load combination determined in step (1) above. Each load combination consists of P_u and M_c.

3. Magnified moment method (10.12):

   Slenderness effects can be neglected when

$$\frac{k\ell_u}{r} \le 34 - 12\left(\frac{M_1}{M_2}\right) \qquad \textit{Eq. (10-7)}$$

   where $[34-12\,M_1/M_2\,] \le 40$. The term M_1/M_2 is positive if the column is bent in single curvature, negative if bent in double curvature.

   When slenderness effects need to be considered, determine M_c for each load combination:

$$M_c = \delta_{ns} M_2 \qquad \textit{Eq. (10-8)}$$

   where

$$M_2 = \text{larger of } M_{u,bot} \text{ and } M_{u,top}$$

$$\ge P_u\,(0.6 + 0.03h) \qquad \textit{10.12.3.2}$$

$$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \ge 1.0 \qquad \textit{Eq. (10-9)}$$

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \qquad \textit{Eq. (10-10)}$$

$$EI = \frac{(0.2E_cI_g + E_sI_{se})}{1 + \beta_d} \qquad \textit{Eq. (10-11)}$$

or

$$EI = \frac{0.4E_cI_g}{1 + \beta_d} \qquad \textit{Eq. (10-12)}$$

$$\beta_d = \frac{\text{Maximum factored axial sustained load}}{\text{Maximum factored axial load associated with the same load combination}} \qquad \textit{10.0}$$

$$C_m = 0.6 + 0.4\left(\frac{M_1}{M_2}\right) \geq 0.4 \quad \text{(without transverse loads)} \qquad \textit{Eq. (10-13)}$$

$$= 1.0 \quad \text{(with transverse loads)}$$

The effective length factor k shall be taken as 1.0, or may be determined from analysis (i.e., alignment chart or equations given in R10.12). In the latter case, k shall be based on the E and I values used in 10.11.1 (see 10.12.1).

- **Sway Frames**

1. Determine the factored load combinations.

   a. Gravity (dead and live) loads

      The moments $(M_{u,bot})_{ns}$ and $(M_{u,top})_{ns}$ at the bottom and top of column, respectively, are to be determined using an elastic first-order frame analysis, based on the cracked section properties of the members.

      The moments M_1 and M_2 are the smaller and the larger of the moments $(M_{u,bot})_{ns}$ and $(M_{u,top})_{ns}$, respectively. The moments M_{1ns} and M_{2ns} are the factored end moments at the ends at which M_1 and M_2 act, respectively.

   b. Gravity (dead and live) plus wind loads

      The total moments at the top and bottom of the column are $M_{u,top} = (M_{u,top})_{ns} + (M_{u,top})_s$ and $M_{u,bot} = (M_{u,bot})_{ns} + (M_{u,bot})_s$, respectively. The moments M_1 and M_2 are the smaller and the larger of the moments $M_{u,top}$ and $M_{u,bot}$, respectively. Note that at this stage, M_1 and M_2 do not include slenderness effects. The moments M_{1ns} and M_{1s} are the factored nonsway and sway moments, respectively, at the end of the column at which M_1 acts, while M_{2ns} and M_{2s} are the factored nonsway and sway moments, respectively, at the end of the column at which M_2 acts.

   c. Gravity (dead) plus wind loads

      The definitions for the moments in this load combination are the same as given above for part 1(b).

   d. The effects due to wind forces acting equal and opposite to the ones in the initial direction of analysis must also be considered in the load combinations given in parts 1(b) and 1(c) above.

2. Determine the required column reinforcement for the critical load combination determined in step (1) above. Each load combination consists of P_u, M_1, and M_2, where now M_1 and M_2 are the total factored end moments, including slenderness effects. Note that if the critical load P_c is computed using EI from Eq. (10-11), it is necessary to estimate first the column reinforcement. Moments M_1 and M_2 are determined by one of the following methods:

   a. Second-order (P-Δ) analysis (10.10.1)

   b. Magnified moment method (only if $k\ell_u/r \le 100$; see 10.13 and step 3 below)

3. Magnified moment method (see 10.13):

   Slenderness effects can be neglected when

$$\frac{k\ell_u}{r} < 22 \qquad \textit{10.13.2}$$

   When slenderness effects need to be considered:

$$M_1 = M_{1ns} + \delta_s M_{1s} \qquad \textit{Eq. (10-15)}$$

$$M_2 = M_{2ns} + \delta_s M_{2s} \qquad \textit{Eq. (10-16)}$$

   The moments $\delta_s M_{1s}$ and $\delta_s M_{2s}$ are to be computed by one of the following methods (10.13.4):

   a. Second-order (P-Δ) analysis (see 10.13.4.1)

   b. Approximate second-order analysis (10.13.4.2)

$$\delta_s M_s = \frac{M_s}{1 - Q} \ge M_s, \quad 1.0 \le \delta_s \le 1.5 \qquad \textit{Eq. (10-17)}$$

      where

$$Q = \frac{\Sigma P_u \Delta_o}{V_u \ell_c} \qquad \textit{Eq. (10-6)}$$

   c. Approximate magnifier method given in previous ACI codes (see 10.13.4.3):

$$\delta_s M_s = \frac{M_s}{1 - \dfrac{\Sigma P_u}{0.75 \Sigma P_c}} \ge M_s \qquad \textit{Eq. (10-18)}$$

      where

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \qquad \textit{Eq. (10-10)}$$

$$EI = \frac{(0.2E_c I_g + E_s I_{se})}{1 + \beta_d} \qquad \textit{Eq. (10-11)}$$

      or

$$EI = \frac{0.4E_cI_g}{1 + \beta_d} \qquad \textit{Eq. (10-12)}$$

The effective length factor k must be greater than 1.0 and shall be based on the E and I values given in 10.11.1 (see 10.13.1).

4. Check if the maximum moment occurs at the ends of the column or between the ends of the column (10.13.5). If

$$\frac{\ell_u}{r} > \frac{35}{\sqrt{\frac{P_u}{f'_cA_g}}} \qquad \textit{Eq. (10-19)}$$

the column must be designed for the factored axial load P_u and the moment M_c, where

$$M_c = \delta_{ns}M_2$$

$$= \left(\frac{C_m}{1 - \frac{P_u}{0.75P_c}} \right) (M_{2ns} + \delta_s M_{2s})$$

In this case, k is determined according to the provisions in 10.12.1 and $\delta_{ns} \geq 1.0$.

5. Check the possibility of sidesway instability under gravity loads (10.13.6):

   a. When $\delta_s M_s$ is computed from 10.13.4.1:

$$\frac{\text{Second-order lateral deflections}}{\text{First-order lateral deflections}} \leq 2.5$$

   based on a loading of $1.4P_D$ and $1.7P_L$ plus lateral load.

   b. When $\delta_s M_s$ is computed from 10.13.4.2:

$$Q = \frac{\Sigma P_u \Delta_o}{V_u \ell_c} \leq 0.60$$

   based on a loading of $1.4P_D$ and $1.7P_L$ plus lateral load.

   c. When $\delta_s M_s$ is computed from 10.13.4.3:

$$0 < \delta_s \leq 2.5$$

   where δ_s is computed using ΣP_u and ΣP_c corresponding to a loading of $1.4P_D$ and $1.7P_L$.

In all three cases, β_d shall be taken as:

$$\beta_d = \frac{\text{Maximum factored sustained axial load}}{\text{Maximum factored axial load}}$$

Reference 11.1 gives the derivation of the design equations for the slenderness provisions outlined above.

REFERENCE

11.1 MacGregor, J. G., "Design of Slender Concrete Columns—Revisited," *ACI Structural Journal,* V. 90, No. 3, May-June 1993, pp. 302-309.

Example 11.1—Slenderness Effects for Columns in a Nonsway Frame

Design columns A3 and C3 in the first story of the 10-story office building shown below. The clear height of the first story is 21 ft-4 in., and is 11 ft-4 in. for all of the other stories. Assume that the lateral load effects on the building are caused by wind, and that the dead loads are the only sustained loads. Other pertinent design data for the building are as follows:

Material properties:

Concrete:

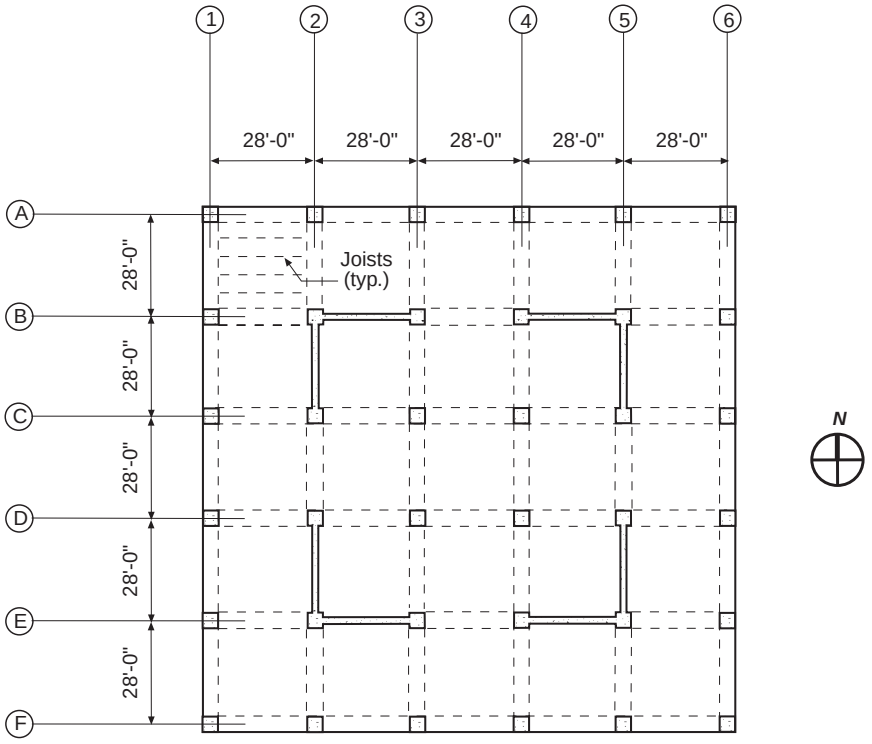
Floors: f'_c = 4,000 psi, w_c = 150 pcf

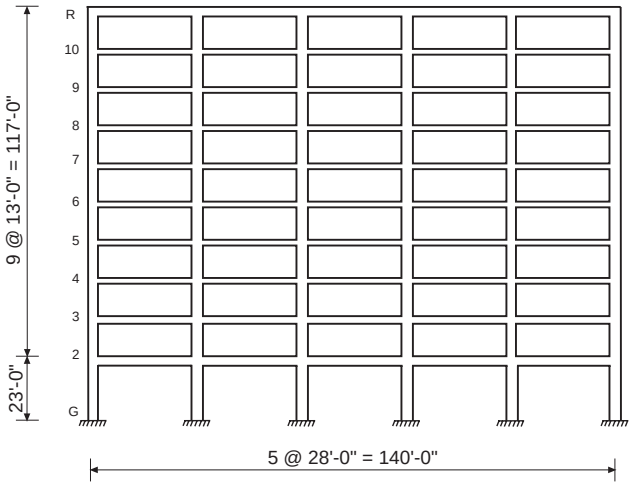
Columns and walls: f'_c = 6,000 psi, w_c = 150 pcf

Reinforcement: f_y = 60 ksi

Beams: 24 × 20 in.
Exterior columns: 20 × 20 in.
Interior columns: 24 × 24 in.
Shearwalls: 12 in.

Weight of floor joists = 86 psf
Superimposed dead load = 32 psf
Roof live load = 30 psf
Floor live load = 50 psf
Wind loads computed according to ASCE 7.





1. Factored axial loads and bending moments for columns A3 and C3 in the first story

Column A3

Load Case			Axial Load (kips)	Bending Moment (ft-kips)	
				Top	Bottom
		Dead (D)	718.0	79.0	40.0
		Live (L)*	80.0	30.3	15.3
		Roof live load (L_r)	12.0	0.0	0.0
		Wind (W)	±8.0	±1.1	±4.3
Eq.	No.	Load Combination			
9-1	1	1.4D	1,005.2	110.6	56.0
9-2	2	1.2D + 1.6L + 0.5L_r	995.6	143.3	72.5
9-3	3	1.2D + 0.5L + 1.6L_r	920.8	110.0	55.7
	4	1.2D + 1.6L_r + 0.8W	887.2	95.7	51.4
	5	1.2D + 1.6L_r - 0.8W	874.4	93.9	44.6
9-4	6	1.2D + 0.5L + 0.5L_r + 1.6W	920.4	111.7	62.5
	7	1.2D + 0.5L + 0.5L_r - 1.6W	894.8	108.2	48.8
9-6	8	0.9D + 1.6W	659.0	72.9	42.9
	9	0.9D - 1.6W	633.4	69.3	29.1

*includes live load reduction per ASCE 7

Column C3

Load Case			Axial Load (kips)	Bending Moment (ft-kips)	
				Top	Bottom
		Dead (D)	1,269.0	1.0	0.7
		Live (L)*	147.0	32.4	16.3
		Roof live load (L_r)	24.0	0.0	0.0
		Wind (W)	±3.0	±2.5	±7.7
Eq.	No.	Load Combination			
9-1	1	1.4D	1,776.6	1.4	1.0
9-2	2	1.2D + 1.6L + 0.5L_r	1,770.0	53.0	26.9
9-3	3	1.2D + 0.5L + 1.6L_r	1,634.7	17.4	9.0
	4	1.2D + 1.6L_r + 0.8W	1,563.6	3.2	7.0
	5	1.2D + 1.6L_r - 0.8W	1,558.8	-0.8	-5.3
9-4	6	1.2D + 0.5L + 0.5L_r + 1.6W	1,613.1	21.4	21.3
	7	1.2D + 0.5L + 0.5L_r - 1.6W	1,603.5	13.4	-3.3
9-6	8	0.9D + 1.6W	1,146.9	4.9	13.0
	9	0.9D - 1.6W	1,137.3	-3.1	-11.7

*includes live load reduction per ASCE 7

Note that Columns A3 and C3 are bent in double curvature.

2. Determine if the frame at the first story is nonsway or sway

The results from an elastic first-order analysis using the section properties prescribed in 10.11.1 are as follows:

ΣP_u = total vertical load in the first story corresponding to the lateral loading case for which ΣP_u is greatest

The total building loads are: D = 37,371 kips, L = 3,609 kips, and L_r = 602 kips.
The maximum ΣP_u is determined from Eq. (9-4):

$$\Sigma P_u = (1.2 \times 37{,}371) + (0.5 \times 3{,}609) + (0.5 \times 602) + 0 = 46{,}951 \text{ kips}$$

V_u = factored story shear in the first story corresponding to the wind loads
$= 1.6 \times 324.3 = 518.9$ kips — *Eq. (9-4), (9-6)*

Δ_o = first-order relative deflection between the top and bottom of the first story due to V_u
$= 1.6 \times (0.03 - 0) = 0.05$ in.

Stability index $Q = \dfrac{\Sigma P_u \Delta_o}{V_u \ell_c} = \dfrac{46{,}951 \times 0.05}{518.9 \times [(23 \times 12) - (20/2)]} = 0.02 < 0.05$ — *Eq. (10-6)*

Since Q < 0.05, the frame at the first story level is considered nonsway. — *10.11.4.2*

3. Design of column C3

a. Determine if slenderness effects must be considered.

Using an effective length factor k = 1.0, — *10.12.1*

$$\frac{k\ell_u}{r} = \frac{1.0 \times 21.33 \times 12}{0.3 \times 24} = 35.6$$

Since the column is bent in double curvature, the least value of $34 - 12\left(\dfrac{M_1}{M_2}\right)$ is obtained from load combination no. 5:

$$34 - 12\left(\frac{M_1}{M_2}\right) = 34 - 12\left(\frac{-(-0.8)}{-5.3}\right) = 35.8 < 40$$

Slenderness effects need not be considered for column C3 since $k\ell_u/r < 34 - 12\,(M_1/M_2)$. — *10.12.2*
Note that there is no need to compute a more exact value for k in this case since the column is nonslender for k = 1.0.

b. Determine required reinforcement.

For the 24 × 24 in. column, try 16-No. 7 bars.

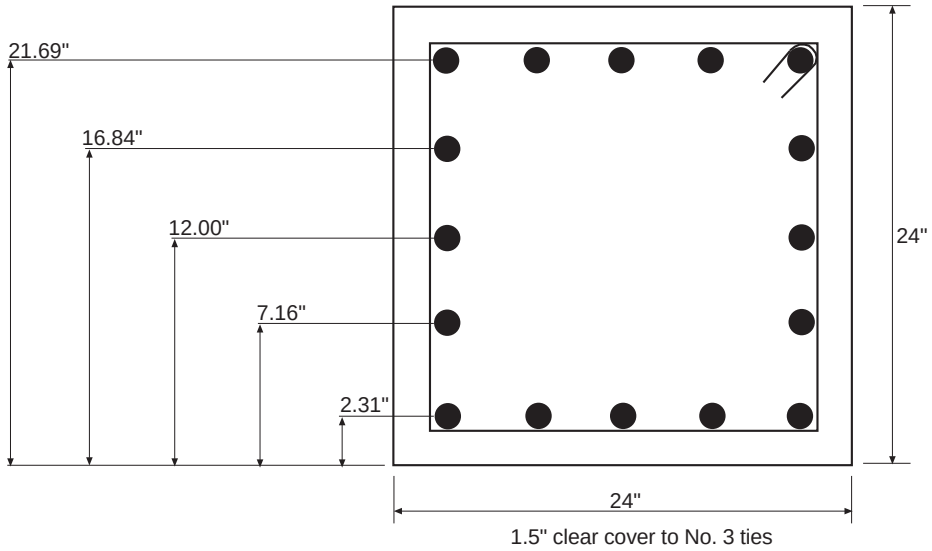
Determine maximum allowable axial compressive force, $\phi P_{n,\,max}$:

$$\phi P_{n,max} = 0.80\phi\left[0.85f'_c\left(A_g - A_{st}\right) + f_y A_{st}\right]$$ *Eq. (10-2)*

$$= (0.80 \times 0.65)\left[(0.85 \times 6)\left(24^2 - 9.6\right) + (60 \times 9.6)\right]$$

$$= 1{,}801.6 \text{ kips} > \text{maximum } P_u = 1{,}776.6 \text{ kips} \quad \text{O.K.}$$

The following table contains results from a strain compatibility analysis, where compressive strains are taken as positive (see Part 6 and 7).



No.	P_u (kips)	M_u (ft-kips)	c (in.)	ε_t	ϕ	ϕP_n (kips)	ϕM_n (ft-kips)
1	1,776.6	1.4	25.92	0.00049	0.65	1,776.6	367.2
2	1,770.0	53.0	25.83	0.00048	0.65	1,770.0	371.0
3	1,634.7	17.4	23.86	0.00027	0.65	1,634.7	447.0
4	1,563.6	7.0	22.85	0.00015	0.65	1,563.6	480.9
5	1,558.8	5.3	22.78	0.00014	0.65	1,558.8	483.2
6	1,613.1	21.4	23.55	0.00024	0.65	1,613.1	457.8
7	1,603.5	13.4	23.41	0.00022	0.65	1,603.5	462.5
8	1,146.9	13.0	17.25	-0.00077	0.65	1,146.9	609.9
9	1,137.3	11.7	17.13	-0.00080	0.65	1,137.3	611.7

Therefore, since $\phi M_n > M_u$ for all $\phi P_n = P_u$, use a 24 × 24 in. column with 16-No. 7 bars ($\rho_g = 1.7\%$).

4. Design of column A3

a. Determine if slenderness effects must be considered.

Determine k from the alignment chart of Fig. 11-10 or from Fig. R10.12.1:

$$I_{col} = 0.7\left(\frac{20^4}{12}\right) = 9{,}333 \text{ in.}^4$$ *10.11.1(b)*

$$E_c = 57{,}000\ \frac{\sqrt{6{,}000}}{1{,}000} = 4{,}415 \text{ ksi}$$ 8.5.1

For the column below level 2:

$$\left(\frac{E_c I}{\ell_c}\right) = \frac{4{,}415 \times 9{,}333}{[(23 \times 12) - (20/2)]} = 155 \times 10^3 \text{ in.-kips}$$

For the column above level 2:

$$\left(\frac{E_c I}{\ell_c}\right) = \frac{4{,}415 \times 9{,}333}{13 \times 12} = 264 \times 10^3 \text{ in.-kips}$$

$$I_{beam} = 0.35\left(\frac{24 \times 20^3}{12}\right) = 5{,}600 \text{ in.}^4$$ 10.11.1(b)

$$\frac{EI}{\ell} = \frac{57\sqrt{4{,}000} \times 5{,}600}{28 \times 12} = 60 \times 10^3 \text{ in.-kips}$$

$$\psi_A = \frac{\Sigma E_c I/\ell_c}{\Sigma E_c I/\ell} = \frac{155 + 264}{60} = 7.0$$

Assume $\psi_B = 1.0$ (column essentially fixed at base)

From Fig. R10.12.1(a), k = 0.86.

Therefore, for column A3 bent in double curvature, the least $34 - 12\left(\frac{M_1}{M_2}\right)$ is obtained from load combination no. 9:

$$34 - 12\left(\frac{-29.1}{69.3}\right) = 39.0$$

$$\frac{k\ell_u}{r} = \frac{0.86 \times 21.33 \times 12}{0.3 \times 20} = 36.7 < 39.0$$

For column A3 bent in single curvature, the least $34 - 12\left(\frac{M_1}{M_2}\right)$ is obtained from load combination no. 8:

$$\frac{k\ell_u}{r} = 36.7 > 34 - 12\left(\frac{42.9}{72.9}\right) = 26.9$$

Therefore, column slenderness need not be considered for column A3 if bent in double curvature. However, to illustrate the design procedure including slenderness effects for nonsway columns, assume single curvature bending.

b. Determine total moment M_c (including slenderness effects) for each load combination.

$$M_c = \delta_{ns} M_2$$ *Eq. (10-8)*

where

$$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \geq 1.0$$ *Eq. (10-9)*

The following table summarizes magnified moment computations for column A3 for all load combinations, followed by detailed calculations for combination no. 6 to illustrate the procedure.

No.	P_u (kips)	M_2 (ft-kips)	β_d	EI x 10^6 (kip-in.^{2})	P_c (kips)	C_m	δ_{ns}	$M_{2,min}$ (ft-kips)	M_c (ft-kips)
1	1,005.2	110.6	1.00	9.88	2,013	0.80	2.40	100.5	265.6
2	995.6	143.3	0.87	10.60	2,158	0.80	2.08	99.6	298.6
3	920.8	110.0	0.94	10.21	2,080	0.80	1.96	92.1	215.3
4	887.2	95.7	0.97	10.03	2,042	0.82	1.94	88.7	185.3
5	874.4	93.9	0.99	9.96	2,028	0.79	1.86	87.4	174.5
6	920.4	111.7	0.94	10.21	2,079	0.82	2.01	92.0	224.6
7	894.8	108.2	0.96	10.07	2,051	0.78	1.87	89.5	201.8
8	659.0	72.9	0.98	9.98	2,033	0.84	1.47	65.9	107.2
9	633.4	69.3	1.02	9.78	1,993	0.77	1.33	63.3	92.4

Load combination no. 6:

$$U = 1.2D + 0.5L + 0.5\,L_r + 1.6W$$

$$C_m = 0.6 + 0.4\left(\frac{M_1}{M_2}\right) \geq 0.4$$ *Eq. (10-13)*

$$= 0.6 + 0.4\left(\frac{62.5}{111.7}\right) = 0.82$$

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2}$$ *Eq. (10-10)*

$$EI = \frac{(0.2E_cI_g + E_sI_{se})}{1 + \beta_d}$$ *Eq. (10-11)*

$$E_c = 57{,}000\,\frac{\sqrt{6{,}000}}{1{,}000} = 4{,}415 \text{ ksi}$$ *8.5.1*

$$I_g = \frac{20^4}{12} = 13{,}333 \text{ in.}^4$$

$E_s = 29{,}000$ ksi 8.5.2

Assuming 8-No. 8 bars with 1.5 in. cover to No. 3 ties:

$$I_{se} = 2\left[(3\times 0.79)\left(\frac{20}{2} - 1.5 - 0.375 - \frac{1.00}{2}\right)^2\right] = 276 \text{ in.}^4$$

Since the dead load is the only sustained load,

$$\beta_d = \frac{1.2P_D}{1.2P_D + 0.5P_L + 0.5P_{L_r} + 1.6P_w}$$

$$= \frac{1.2\times 718}{(1.2\times 718) + (0.5\times 80) + (0.5\times 12) + (1.6\times 8)} = 0.94$$ 10.0

$$EI = \frac{(0.2\times 4{,}415\times 13{,}333) + (29{,}000\times 276)}{1 + 0.94} = 10.21 \times 10^6 \text{ kip-in.}^2$$

From Eq. (10-12):

$$EI = \frac{0.4E_cI_g}{1 + \beta_d}$$

$$= \frac{0.4\times 4{,}415\times 13{,}333}{1 + 0.94} = 12.14 \times 10^6 \text{ kip-in.}^2$$

Using EI from Eq. (10-11), the critical load P_c is:

$$P_c = \frac{\pi^2 \times 10.21\times 10^6}{(0.86\times 21.33\times 12)^2} = 2{,}079 \text{ kips}$$

Therefore, the moment magnification factor δ_{ns} is:

$$\delta_{ns} = \frac{0.82}{1 - \frac{920.4}{0.75\times 2{,}079}} = 2.01$$

Check minimum moment requirement:

$M_{2,min} = P_u (0.6 + 0.03h)$ *Eq. (10-14)*

$= 920.4 [0.6 + (0.03 \times 20)]/12$

$= 92.0$ ft-kips $< M_2 = 111.7$ ft-kips

Therefore,

$M_c = 2.01 \times 111.7 = 224.6$ ft-kips

c. Determine required reinforcement.

For the 20 x 20 in. column, try 8-No. 8 bars.

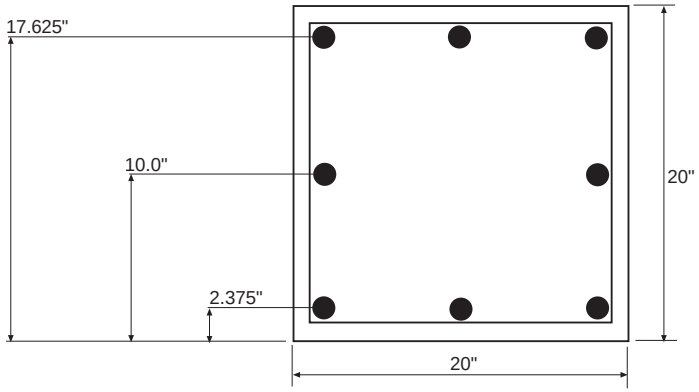
Determine maximum allowable axial compressive force, $\phi P_{n,max}$:

$\phi P_{n,max} = 0.80\phi \left[0.85f'_c\left(A_g - A_{st}\right) + f_y A_{st}\right]$ *Eq. (10-2)*

$= (0.80 \times 0.65)\left[(0.85 \times 6)\left(20^2 - 6.32\right) + (60 \times 6.32)\right]$

$= 1{,}241.2$ kips $>$ maximum $P_u = 1{,}005.2$ kips O.K.

The following table contains results from a strain compatibility analysis, where compressive strains are taken as positive (see Parts 6 and 7).



No.	P_u (kips)	M_u (ft-kips)	c (in.)	ε_t	ϕ	ϕP_n (kips)	ϕM_n (ft-kips)
1	1,005.2	265.6	17.81	0.00003	0.65	1,005.2	298.0
2	995.6	298.6	17.64	0.00000	0.65	995.6	301.1
3	920.8	215.3	16.42	-0.00022	0.65	920.8	321.4
4	887.2	185.3	15.88	-0.00033	0.65	887.2	329.3
5	874.4	174.5	15.67	-0.00037	0.65	874.4	332.1
6	920.4	224.6	16.41	-0.00022	0.65	920.4	321.6
7	894.8	201.8	16.00	-0.00030	0.65	894.8	327.6
8	659.0	107.2	12.36	-0.00128	0.65	659.0	364.8
9	633.4	92.4	12.00	-0.00141	0.65	633.4	367.2

Therefore, since $\phi M_n > M_u$ for all $\phi P_n = P_u$, use a 20 $\times$ 20 in. column with 8-No. 8 bars ($\rho_g = 1.6\%$).

Example 11.2—Slenderness Effects for Columns in a Sway Frame

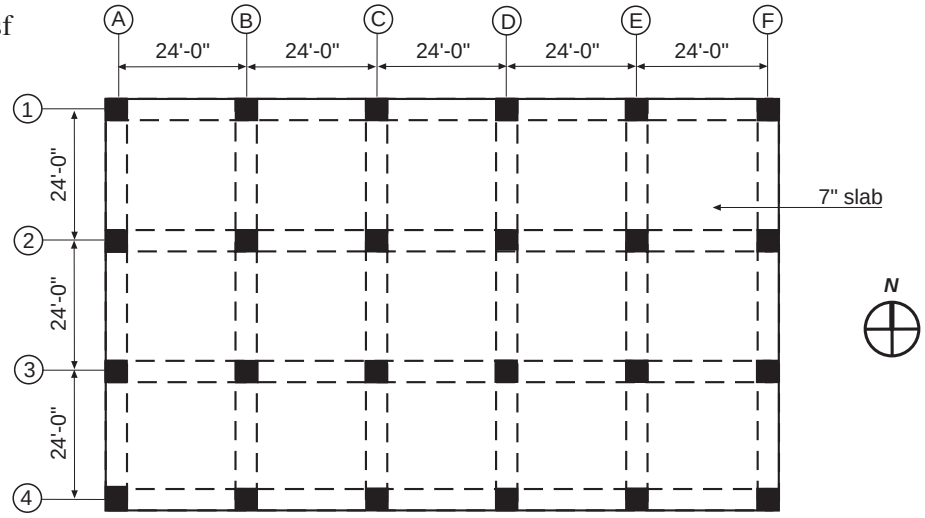
Design columns C1 and C2 in the first story of the 12-story office building shown below. The clear height of the first story is 13 ft-4 in., and is 10 ft-4 in. for all of the other stories. Assume that the lateral load effects on the building are caused by wind, and that the dead loads are the only sustained loads. Other pertinent design data for the building are as follows:

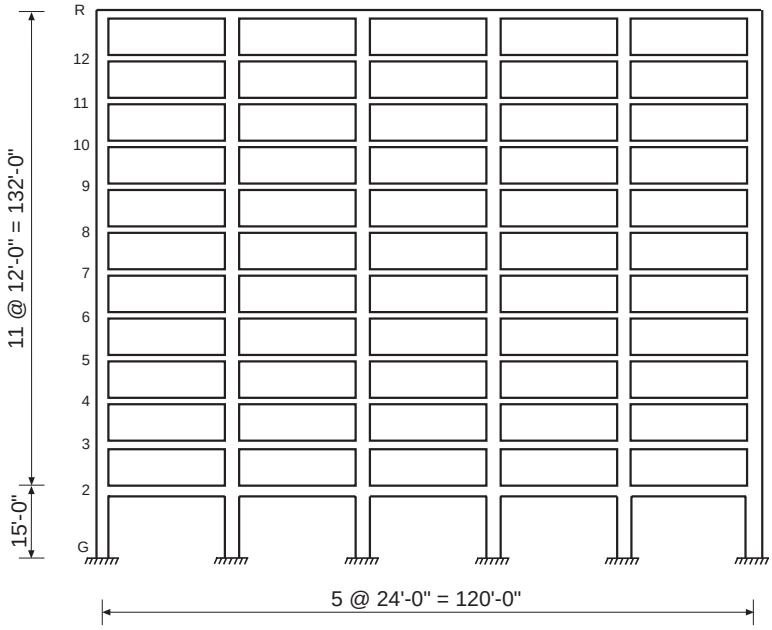
Material properties:

Concrete: = 6,000 psi for columns in the bottom two stories (w_c = 150 pcf)
= 4,000 psi elsewhere (w_c = 150 pcf)
Reinforcement: f_y = 60 ksi

Beams: 24 × 20 in.
Exterior columns: 22 × 22 in.
Interior columns: 24 × 24 in.

Superimposed dead load = 30 psf
Roof live load = 30 psf
Floor live load = 50 psf
Wind loads computed according to ASCE 7





1. Factored axial loads and bending moments for columns C1 and C2 in the first story

Since this is a symmetrical frame, the gravity loads will not cause appreciable sidesway.

Column C1

Load Case	Axial Load (kips)	Bending moment (ft-kips) Top	Bottom
Dead (D)	622.4	34.8	17.6
Live (L)*	73.9	15.4	7.7
Roof live load (L_r)	8.6	0.0	0.0
Wind (W) (N-S)	-48.3	17.1	138.0
Wind (W) (S-N)	48.3	-17.1	-138.0

	No.	Load Combination				M_1	M_2	M_{1ns}	M_{2ns}	M_{1s}	M_{2s}
9-1	1	1.4D	871.4	48.7	24.6	24.6	48.7	24.6	48.7	– – –	– – –
9-2	2	1.2D + 1.6L + 0.5L_r	869.4	66.4	33.4	33.4	66.4	33.4	66.4	– – –	– – –
9-3	3	1.2D + 0.5L + 1.6L_r	797.6	49.5	25.0	25.0	49.5	25.0	49.5	– – –	– – –
	4	1.2D + 1.6L_r + 0.8W	722.0	55.4	131.5	55.4	131.5	41.8	21.1	13.7	110.4
	5	1.2D + 1.6L_r - 0.8W	799.3	28.1	-89.3	28.1	-89.3	41.8	21.1	-13.7	-110.4
9-4	6	1.2D + 0.5L + 0.5L_r + 1.6W	710.9	76.8	245.8	76.8	245.8	49.5	25.0	27.4	220.8
	7	1.2D + 0.5L + 0.5L_r - 1.6W	865.4	22.1	-195.8	22.1	-195.8	49.5	25.0	-27.4	-220.8
9-6	8	0.9D + 1.6W	482.9	58.7	236.6	58.7	236.6	31.3	15.8	27.4	220.8
	9	0.9D - 1.6W	637.4	4.0	-205.0	4.0	-205.0	31.3	15.8	-27.4	-220.8

*includes live load reduction per ASCE 7

Column C2

Load Case	Axial Load (kips)	Bending moment (ft-kips) Top	Bottom
Dead (D)	1,087.6	-2.0	-1.0
Live (L)*	134.5	-15.6	-7.8
Roof live load (L_r)	17.3	0.0	0.0
Wind (W) (N-S)	-0.3	43.5	205.0
Wind (W) (S-N)	0.3	-43.5	-205.0

	No.	Load Combination				M_1	M_2	M_{1ns}	M_{2ns}	M_{1s}	M_{2s}
9-1	1	1.4D	1,522.6	-2.8	-1.4	-1.4	-2.8	-1.4	-2.8	– – –	– – –
9-2	2	1.2D + 1.6L + 0.5L_r	1,529.0	-27.4	-13.7	-13.7	-27.4	-13.7	-27.4	– – –	– – –
9-3	3	1.2D + 0.5L + 1.6L_r	1,400.1	-10.2	-5.1	-5.1	-10.2	-5.1	-10.2	– – –	0.0
	4	1.2D + 1.6L_r + 0.8W	1,332.6	32.4	162.8	32.4	162.8	-2.4	-1.2	34.8	164.0
	5	1.2D + 1.6L_r - 0.8W	1,333.0	-37.2	-165.2	-37.2	-165.2	-2.4	-1.2	-34.8	-164.0
9-4	6	1.2D + 0.5L + 0.5L_r + 1.6W	1,380.5	59.4	322.9	59.4	322.9	-10.2	-5.1	69.6	328.0
	7	1.2D + 0.5L + 0.5L_r - 1.6W	1,381.5	-79.8	-333.1	-79.8	-333.1	-10.2	-5.1	-69.6	-328.0
9-6	8	0.9D + 1.6W	978.4	67.8	327.1	67.8	327.1	-1.8	-0.9	69.6	328.0
	9	0.9D - 1.6W	979.3	-71.4	-328.9	-71.4	-328.9	-1.8	-0.9	-69.6	-328.0

*includes live load reduction per ASCE 7

2. Determine if the frame at the first story is nonsway or sway

The results from an elastic first-order analysis using the section properties prescribed in 10.11.1 are as follows:

ΣP_u = total vertical load in the first story corresponding to the lateral loading case for which ΣP_u is greatest

The total building loads are: D = 17,895 kips, L = 1,991 kips, L_r = 270 kips. The maximum ΣP_u is from Eq. (9-4):

$\Sigma P_u = (12 \times 17{,}895) + (0.5 \times 1{,}991) + (0.5 \times 270) + 0 = 22{,}605$ kips

V_u = factored story shear in the first story corresponding to the wind loads

$= 1.6 \times 302.6 = 484.2$ kips *Eq. (9-4), (9-6)*

Δ_o = first-order relative deflection between the top and bottom of the first story due to V_u
$= 1.6 \times (0.28 - 0) = 0.45$ in.

$$\text{Stability index } Q = \frac{\Sigma P_u \Delta_o}{V_u \ell_c} = \frac{22{,}605 \times 0.45}{484.2 \times [(15 \times 12) - (20/2)]} = 0.12 > 0.05$$ *Eq. (10-6)*

Since $Q > 0.05$, the frame at the first story level is considered sway. *10.11.4.2*

3. Design of column C1

a. Determine if slenderness effects must be considered.

Determine k from alignment chart in R10.12.1.

$$I_{col} = 0.7\left(\frac{22^4}{12}\right) = 13{,}665 \text{ in.}^4$$ *10.11.1*

$$E_c = 57{,}000\,\frac{\sqrt{6{,}000}}{1{,}000} = 4{,}415 \text{ ksi}$$ *8.5.1*

For the column below level 2:

$$\frac{E_c I}{\ell_c} = \frac{4{,}415 \times 13{,}665}{(15 \times 12) - 10} = 355 \times 10^3 \text{ in.-kips}$$

For the column above level 2:

$$\frac{E_c I}{\ell_c} = \frac{4{,}415 \times 13{,}665}{12 \times 12} = 419 \times 10^3 \text{ in.-kips}$$

$$I_{beam} = 0.35\left(\frac{24 \times 20^3}{12}\right) = 5{,}600 \text{ in.}^4$$ *10.11.1*

$$\text{For the beam: } \frac{E_c I}{\ell_c} = \frac{57\sqrt{4{,}000} \times 5{,}600}{24 \times 12} = 70 \times 10^3 \text{ in.-kips}$$

$$\psi_A = \frac{\Sigma E_c I/\ell_c}{\Sigma E_c I/\ell} = \frac{355 + 419}{70} = 11.1$$

Assume $\psi_B = 1.0$ (column essentially fixed at base)

From the alignment chart (Fig. R10.12.1(b)), k = 1.9.

$$\frac{k\ell_u}{r} = \frac{1.9 \times 13.33 \times 12}{0.3 \times 22} = 46 > 22$$ *10.13.2*

Thus, slenderness effects must be considered.

b. Determine total moment M_2 (including slenderness effects) and the design load combinations, using the approximate analysis of 10.13.4.2.

The following table summarizes magnified moment computations for column C1 for all load combinations, followed by detailed calculations for combinations no. 4 and 5 to illustrate the procedure.

No.	Load Combination	ΣP_u (kips)	Δ_o (in.)	V_u (kips)	Q	δ_s	M_{2ns} (ft-kips)	M_{2s} (ft-kips)	M_2 (ft-kips)
1	1.4D	25,053	– – –	– – –	– – –	– – –	48.7	– – –	48.7
2	1.2D + 1.6L + 0.5L_r	24,795	– – –	– – –	– – –	– – –	66.4	– – –	66.4
3	1.2D + 0.5L + 1.6L_r	22,902	– – –	– – –	– – –	– – –	49.5	– – –	49.5
4	1.2D + 1.6L_r + 0.8W	21,906	0.22	242.1	0.12	1.14	21.1	110.4	147.0
5	1.2D + 1.6L_r - 0.8W	21,906	0,22	242.1	0.12	1.14	21.1	-110.4	-104.8
6	1.2D + 0.5L + 0.5L_r + 1.6W	22,605	0.45	484.2	0.12	1.14	25.0	220.8	276.7
7	1.2D + 0.5L + 0.5L_r - 1.6W	22,605	0.45	484.2	0.12	1.14	25.0	-220.8	-226.8
8	0.9D + 1.6W	16,106	0.45	484.2	0.09	1.10	15.8	220.8	257.9
9	0.9D - 1.6W	16,106	0.45	484.2	0.09	1.10	15.8	-220.8	-226.2

$$M_2 = M_{2ns} + M_{2s}$$ *Eq. (10-16)*

$$\delta_s M_{2s} = \frac{M_{2s}}{I - Q} \geq M_{2s}$$ *Eq. (10-17)*

For load combinations no. 4 and 5:

$$U = 1.2D + 1.6L_r \pm 0.8W$$

$$\Sigma P_u = (1.2 \times 17{,}895) + (1.6 \times 270) \pm 0 = 21{,}906 \text{ kips}$$

$$\Delta_o = 0.8 \times (0.28 - 0) = 0.22 \text{ in.}$$

$$V_u = 0.8 \times 302.6 = 240.1 \text{ kips}$$

$$\ell_c = (15 \times 12) - (20/2) = 170 \text{ in.}$$

$$Q = \frac{\Sigma P_u \Delta_o}{V_u \ell_c} = \frac{21{,}906 \times 0.22}{240.1 \times 170} = 0.12$$

$$\delta = \frac{1}{1-Q} = \frac{1}{1-0.12} = 1.14$$

- For sidesway from north to south (load combination no. 4):

  $\delta_s M_{2s} = 1.14 \times 110.4 = 125.9$ ft-kips

  $M_2 = M_{2ns} + \delta_s M_{2s} = 21.1 + 125.9 = 147.0$ ft-kips

  $P_u = 722.0$ kips

- For sidesway from south to north (load combination no. 5):

  $\delta_s M_{2s} = 1.14 \times (-110.4) = -125.9$ ft-kips

  $M_2 = 21.1 - 125.9 = -104.8$ ft-kips

  $P_u = 799.3$ kips

c. For comparison purposes, recompute $\delta_s M_{2s}$ using the magnified moment method outlined in 10.13.4.3

$$\delta_s M_{2s} = \frac{M_{2s}}{1 - \dfrac{\Sigma P_u}{0.75 \Sigma P_c}} \geq M_{2s} \qquad \textit{Eq. (10-18)}$$

The critical load P_c is calculated from Eq. (10-10) using k from 10.13.1 and EI from Eq. (10-11) or (10-12). Since the reinforcement is not known as of yet, use Eq. (10-12) to determine EI.

For each of the 12 exterior columns along column lines 1 and 4 (i.e., the columns with one beam framing into them in the direction of analysis), k was determined in part 3(a) above to be 1.9.

$$EI = \frac{0.4 E_c I}{1 + \beta_d} = \frac{0.4 \times 4{,}415 \times 22^4}{12(1 + 0)} = 34.5 \times 10^6 \text{ in.}^2\text{-kips} \qquad \textit{Eq. (10-12)}$$

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} = \frac{\pi^2 \times 34.5 \times 10^6}{(1.9 \times 13.33 \times 12)^2} = 3{,}686 \text{ kips} \qquad \textit{Eq. (10-10)}$$

For each of the exterior columns A2, A3, F2, and F3, (i.e., the columns with two beams framing into them in the direction of analysis):

$$\psi_A = \frac{355 + 419}{2 \times 70} = 5.5$$

$$\psi_B = 1.0$$

From the alignment chart, k = 1.75.

$$P_c = \frac{\pi^2 \times 34.5 \times 10^6}{(1.75 \times 13.33 \times 12)^2} = 4{,}345 \text{ kips}$$ *Eq. (10-10)*

For each of the 8 interior columns:

$$I_{col} = 0.7\left(\frac{24^4}{12}\right) = 19{,}354 \text{ in.}^4$$ *10.11.1*

For the column below level 2:

$$\frac{E_c I}{\ell_c} = \frac{4{,}415 \times 19{,}354}{(15 \times 12) - 10} = 503 \times 10^3 \text{ in.-kips}$$

For the column above level 2:

$$\frac{E_c I}{\ell_c} = \frac{4{,}415 \times 19{,}354}{12 \times 12} = 593 \times 10^3 \text{ in.-kips}$$

$$\psi_A = \frac{503 + 593}{2 \times 70} = 7.8$$

$$\psi_B = 1.0$$

From the alignment chart, k = 1.82.

$$EI = 0.4 \times 4{,}415 \times \frac{24^2}{12} = 48.8 \times 10^6 \text{ in.-kips}$$ *Eq. (10-12)*

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} = \frac{\pi^2 \times 48.8 \times 10^6}{(1.82 \times 13.33 \times 12)^2} = 5{,}683 \text{ kips}$$

Therefore,

$$\Sigma P_c = 12(3{,}686) + 4(4{,}345) + 8(5{,}683) = 107{,}076 \text{ kips}$$

The following table summarizes magnified moment computations for column C1 using 10.13.4.3 for all load conditions. The table is followed by detailed calculations for combinations no. 4 and 5 to illustrate the procedure.

No.	Load Combination	ΣP_u (kips)	δ_s (in.)	M_{2ns} (ft-kips)	M_{2s} (ft-kips)	M_2 (ft-kips)
1	1.4D	25,053	– – –	48.7	– – –	48.7
2	1.2D + 1.6L + 1.6L_r	24,795	– – –	66.4	– – –	66.4
3	1.2D + 0.5L + 1.6L_r	22,903	– – –	49.5	– – –	49.5
4	1.2D + 1.6L_r + 0.8W	21,908	1.38	21.1	110.4	173.5
5	1.2D + 1.6L_r - 0.8W	21,908	1.38	21.1	-110.4	-131.3
6	1.2D + 0.5L + 0.5L_r + 1.6W	22,605	1.39	25.0	220.8	331.9
7	1.2D + 0.5L + 0.5L_r - 1.6W	22,605	1.39	25.0	-220.8	-281.9
8	0.9D + 1.6W	16,106	1.25	15.8	220.8	292.0
9	0.9D - 1.6W	16,106	1.25	15.8	-220.8	-260.3

For load combinations No. 4 and 5:

$$U = 1.2D + 1.6L_r \pm 0.8W$$

$$\delta_s = \frac{1}{1 - \frac{\Sigma P_u}{0.75 \Sigma P_c}} = \frac{1}{1 - \frac{21{,}908}{0.75 \times 107{,}076}} = 1.38$$

- For sidesway from north to south (load combination no. 4):

  $\delta_s M_{2s} = 1.38 \times 110.4 = 152.4$ ft-kips

  $M_2 = 21.1 + 152.4 = 173.5$ ft-kips

  $P_u = 722.0$ kips

- For sidesway from south to north (load combination no. 5):

  $\delta_s M_{2s} = 1.38 \times (-110.4) = -152.4$ ft-kips

  $M_2 = 21.1 - 152.4 = -131.3$ ft-kips

  $P_u = 799.3$ kips

A summary of the magnified moments for column C1 for all load combinations is provided in the following table.

No.	Load Combination	P_u (kips)	10.13.4.2		10.13.4.3	
			δ_s	M_2 (ft-kips)	δ_s	M_2 (ft-kips)
1	1.4D	871.4	– – –	48.7	– – –	48.7
2	1.2D + 1.6L + 0.5L_r	869.4	– – –	66.4	– – –	66.4
3	1.2D + 0.5L + 1.6L_r	797.6	– – –	49.5	– – –	49.5
4	1.2D + 1.6L_r + 0.8W	722.0	1.14	147.0	1.38	173.5
5	1.2D + 1.6L_r - 0.8W	799.3	1.14	-104.8	1.38	-131.3
6	1.2D + 0.5L + 0.5L_r + 1.6W	710.9	1.14	276.7	1.39	331.9
7	1.2D + 0.5L + 0.5L_r - 1.6W	865.4	1.14	-226.8	1.39	-281.9
8	0.9D + 1.6W	482.9	1.10	257.9	1.25	292.0
9	0.9D - 1.6W	637.4	1.10	-226.2	1.25	-260.3

d. Determine required reinforcement.

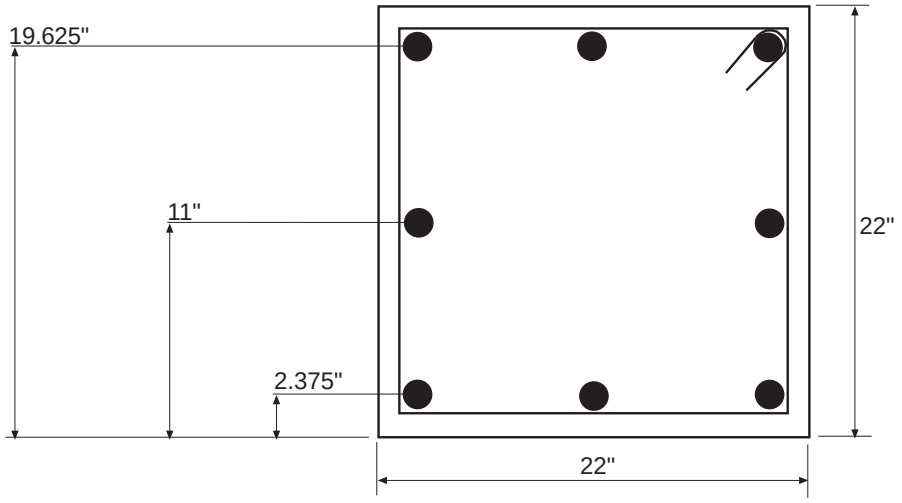
For the 22 × 22 in. column, try 8-No. 8 bars. Determine maximum allowable axial compressive force, $\phi P_{n,max}$:

$$\phi P_{n,max} = 0.80\phi\left[0.85f'_c\left(A_g - A_{st}\right) + f_y A_{st}\right]$$

$$= (0.80 \times 0.65)[(0.85 \times 6)\,(22^2 - 6.32) + (60 \times 6.32)]$$

$$= 1{,}464.0 \text{ kips} > \text{maximum } P_u = 871.4 \text{ kips} \quad \text{O.K.}$$

The following table contains results from a strain compatibility analysis, where compressive strains are taken as positive (see Parts 6 and 7). Use $M_u = M_2$ from the approximate $P-\Delta$ method in 10.13.4.2.



No.	P_u (kips)	M_u (ft-kips)	c (in.)	ε_t	ϕ	ϕP_n (kips)	ϕM_n (ft-kips)
1	871.4	48.7	14.85	-0.00096	0.65	871.4	459.4
2	869.4	66.4	14.82	-0.00097	0.65	869.4	459.7
3	797.6	49.5	13.75	-0.00128	0.65	797.6	468.2
4	722.0	147.0	12.75	-0.00162	0.65	722.0	474.1
5	799.3	-104.8	13.78	-0.00127	0.65	799.3	468.0
6	710.9	276.7	12.61	-0.00167	0.65	710.9	474.8
7	865.4	-226.8	14.76	-0.00099	0.65	865.4	460.2
8	482.9	257.9	7.36	-0.00500	0.90	482.9	557.2
9	637.4	-226.2	11.68	-0.00204	0.65	637.4	478.8

Therefore, since $\phi M_n > M_u$ for all $\phi P_n = P_u$, use a 22 × 22 in. column with 8-No. 8 bars ($\rho_g = 1.3\%$). The same reinforcement is also adequate for the load combinations from the magnified moment method of 10.13.4.3.

4. Design of column C2

a. Determine if slenderness effects must be considered.

In part 3(c), k was determined to be 1.82 for the interior columns. Therefore,

$$\frac{k\ell_u}{r} = \frac{1.82 \times 13.33 \times 12}{0.3 \times 24} = 40.4 > 22$$ *10.13.2*

Slenderness effects must be considered.

b. Determine total moment M_2 (including slenderness effects) and the design load combinations, using the approximate analysis of 10.13.4.2.

The following table summarizes magnified moment computation for column C2 for all load combinations, followed by detailed calculations for combinations no. 4 and 5 to illustrate the procedure.

No.	Load Combination	ΣP_u (kips)	Δ_o (in.)	V_u (kips)	Q	δ_s	M_{2ns} (ft-kips)	M_{2s} (ft-kips)	M_2 (ft-kips)
1	1.4D	25,053	– – –	– – –	– – –	– – –	-2.8	– – –	-2.8
2	1.2D + 1.6L + 0.5L_r	24,795	– – –	– – –	– – –	– – –	-27.4	– – –	-27.4
3	1.2D + 0.5L + 1.6L_r	22,902	– – –	– – –	– – –	– – –	-10.2	– – –	-10.2
4	1.2D + 1.6L_r + 0.8W	21,906	0.22	242.1	0.12	1.14	-1.2	164.0	185.8
5	1.2D + 1.6L_r - 0.8W	21,906	0.22	242.1	0.12	1.14	-1.2	-164.0	-188.2
6	1.2D + 0.5L + 0.5L_r + 1.6W	22,605	0.45	484.2	0.12	1.14	-5.1	328.0	368.8
7	1.2D + 0.5L + 0.5L_r - 1.6W	22,605	0.45	484.2	0.12	1.14	-5.1	-328.0	-379.0
8	0.9D + 1.6W	16,106	0.45	484.2	0.09	1.10	-0.9	328.0	358.6
9	0.9D - 1.6W	16,106	0.45	484.2	0.09	1.10	-0.9	-328.0	-360.4

$$M_2 = M_{2ns} + M_{2s}$$ *Eq. (10-16)*

$$\delta_s M_{2s} = \frac{M_{2s}}{1-Q} \geq M_{2s}$$ *Eq. (10-17)*

For load combinations no. 4 and 5:

$$U = 1.2D + 1.6L_r \pm 0.8W$$

From part 3(b), δ_s was determined to be 1.14.

- For sidesway from north to south (load combination no. 4):

  $\delta_s M_{2s} = 1.14 \times 164 = 187.0$ ft-kips

  $M_2 = M_{2ns} + \delta_s M_{2s} = -1.2 + 187.0 = 185.8$ ft-kips

  $P_u = 1{,}332.6$ kips

- For sidesway from south to north (load combination no. 5):

  $\delta_s M_{2s} = 1.14 \times (-164) = -187.0$ ft-kips

  $M_2 = -1.2 - 187.0 = -188.2$ ft-kips

  $P_u = 1{,}333.0$ kips

c. For comparison purposes, recompute $\delta_s M_{2s}$ using the magnified moment method outlined in 10.13.4.3. Use the values of ΣP_u, ΣP_c, and δ_s computed in part 3(c).

No.	Load Combination	ΣP_u (kips)	δ_s (in.)	M_{2ns} (ft-kips)	M_{2s} (ft-kips)	M_2 (ft-kips)
1	1.4D	25,053	– – –	-2.8	– – –	-2.8
2	1.2D + 1.6L + 0.5L_r	24,795	– – –	-27.4	– – –	-27.4
3	1.2D + 0.5L + 1.6L_r	22,903	– – –	-10.2	– – –	-10.2
4	1.2D + 1.6L_r + 0.8W	21,908	1.38	-1.2	164.0	225.1
5	1.2D + 1.6L_r - 0.8W	21,908	1.38	-1.2	-164.0	-227.5
6	1.2D + 0.5L + 0.5L_r + 1.6W	22,605	1.39	-5.1	328.0	451.4
7	1.2D + 0.5L + 0.5L_r - 1.6W	22,605	1.39	-5.1	-328.0	-461.6
8	0.9D + 1.6W	16,106	1.25	-0.9	328.0	409.4
9	0.9D - 1.6W	16,106	1.25	-0.9	-328.0	-411.2

$$U = 1.2D + 1.6L_r \pm 0.8W$$

$\delta_s = 1.38$ from part 3(c)

- For sidesway from north to south (load combination no. 4):

$\delta_s M_{2s} = 1.38 \times 164.0 = 226.3$ ft-kips

$M_2 = -1.2 + 226.3 = 225.1$ ft-kips

$P_u = 1{,}332.6$ kips

- For sidesway from south to north (load combination no. 5):

  $\delta_s M_{2s} = 1.38 \times (-164.0) = -226.3$ ft-kips

  $M_2 = -1.2 - 226.3 = -227.5$ ft-kips

  $P_u = 1{,}333.0$ kips

A summary of the magnified moments for column C2 under all load combinations is provided in the following table.

No.	Load Combination	P_u (kips)	10.13.4.2		10.13.4.3	
			δ_s	M_2 (ft-kips)	δ_s	M_2 (ft-kips)
1	1.4D	1,522.6	– – –	-2.8	– – –	-2.8
2	1.2D + 1.6L + 0.5L_r	1,529.0	– – –	-27.4	– – –	-27.4
3	1.2D + 0.5L + 1.6L_r	1,400.1	– – –	-10.2	– – –	-10.2
4	1.2D + 1.6L_r + 0.8W	1,332.6	1.14	185.8	1.38	225.1
5	1.2D + 1.6L_r - 0.8W	1,333.0	1.14	-188.2	1.38	-227.5
6	1.2D + 0.5L + 0.5L_r + 1.6W	1,380.5	1.14	368.8	1.39	451.4
7	1.2D + 0.5L + 0.5L_r - 1.6W	1,381.5	1.14	-379.0	1.39	-461.6
8	0.9D + 1.6W	978.4	1.10	358.6	1.25	409.4
9	0.9D - 1.6W	979.3	1.10	-360.4	1.25	-411.2

d. Determine required reinforcement.

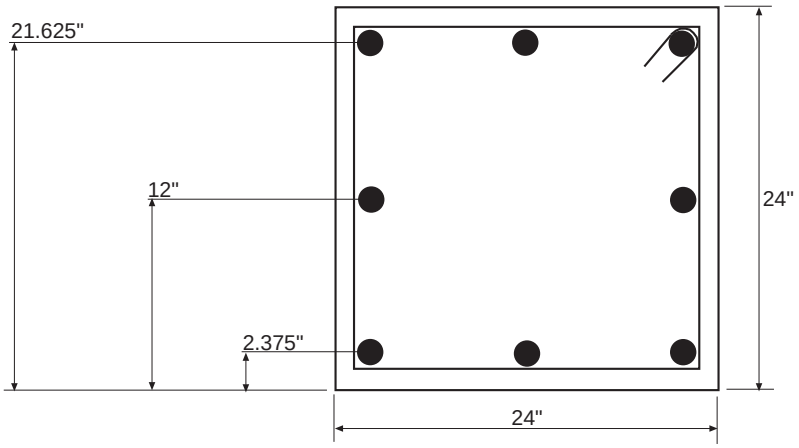
For the 24 × 24 in. column, try 8-No. 8 bars. Determine maximum allowable axial compressive force, $\phi P_{n,max}$:

$$\phi P_{n,max} = 0.80\phi\left[0.85f'_c\left(A_g - A_{st}\right) + f_y A_{st}\right]$$

$$= (0.80 \times 0.65)[(0.85 \times 6)\,(24^2 - 6.32) + (60 \times 6.32)]$$

$$= 1{,}708 \text{ kips} > \text{maximum } P_u = 1{,}529.0 \text{ kips} \quad \text{O.K.}$$

The following table contains results from a strain compatibility analysis, where compressive strains are taken as positive (see Parts 6 and 7). Use $M_u = M_2$ from the approximate $P-\Delta$ method in 10.13.4.2.



No.	P_u (kips)	M_u (ft-kips)	c (in.)	ε_t	ϕ	ϕP_n (kips)	ϕM_n (ft-kips)
1	1,522.6	-2.8	23.30	0.00022	0.65	1,522.6	438.1
2	1,529.0	-27.4	23.39	0.00023	0.65	1,529.0	435.3
3	1,400.1	-10.2	21.49	-0.00002	0.65	1,400.1	489.7
4	1,332.6	185.8	20.50	-0.00016	0.65	1,332.6	513.3
5	1,333.0	-188.2	20.51	-0.00016	0.65	1,333.0	513.1
6	1,380.5	368.8	21.20	-0.00006	0.65	1,380.5	496.9
7	1,381.5	-379.0	21.22	-0.00005	0.65	1,381.5	496.4
8	978.4	358.6	15.52	-0.00118	0.65	978.4	587.1
9	979.3	-360.4	15.46	-0.00120	0.65	979.3	587.5

Therefore, since $\phi M_n > M_u$ for all $\phi P_n = P_u$, use a 24 × 24 in. column with 8-No. 8 bars ($\rho_g = 1.1\%$). The same reinforcement is also adequate for the load combinations from the magnified moment method of 10.13.4.3.

5. Determine if the maximum moment occurs at the end of the member. *10.13.5*

For column C1:

$$\frac{\ell_u}{r} = \frac{13.33 \times 12}{0.3 \times 22} = 24.2 < \frac{35}{\sqrt{\frac{871.4}{6 \times 22^2}}} = 63.9$$ *Eq. (10-19)*

For column C2:

$$\frac{\ell_u}{r} = \frac{13.33 \times 12}{0.3 \times 24} = 22.2 < \frac{35}{\sqrt{\frac{1{,}529.0}{6 \times 24^2}}} = 52.6$$

Therefore, for columns C1 and C2, the maximum moment occurs at one of the ends, and the total moment M_2 does not have to be further magnified by δ_{ns}.

6. Check for sidesway instability of the structure. *10.13.6*

   a. When using 10.13.4.2 to compute $\delta_s M_s$, the value of Q evaluated using factored gravity loads shall not exceed 0.60. Note that for stability checks, all moments of inertia must be divided by $(1 + \beta_d)$ 10.11.1 where, for this story

$$\beta_d = \frac{\text{Maximum factored sustained axial load}}{\text{Maximum factored axial load}}$$

$$= \frac{\Sigma 1.4 P_D}{\Sigma P_u} = \frac{1.4 \times 17{,}895}{(1.4 \times 17{,}895) + (1.7 \times 2{,}261)} = 0.87$$

$$1 + \beta_d = 1.87$$

   Dividing all of the moments of inertia by $(1 + \beta_d)$ is equivalent to increasing the deflections, and consequently Q, by $(1 + \beta_d)$. Thus, at the second floor level,

$$Q = 1.87 \times 0.12 = 0.22 < 0.60$$

   Therefore, the structure is stable at this level. In fact, computing the modified Q at each floor level shows that the entire structure is stable.

   b. When using 10.13.4.3 to compute $\delta_s M_s$, the value of δ_s computed using ΣP_u and ΣP_c corresponding to the factored dead and live loads shall be positive and shall not exceed 2.5. For the stability check, the values of EI must be divided by $(1 + \beta_d)$. Thus, the values of P_c must be recomputed considering the effects of β_d.

$$\Sigma P_c = \frac{107{,}076}{1 + 0.87} = 57{,}260 \text{ kips}$$

$$\text{and } \delta_s = \frac{1}{1 - \dfrac{(1.4 \times 17{,}895) + (1.7 \times 2{,}261)}{0.75 \times 57{,}260}} = 3.1 > 2.5$$

   The structure is unstable when the magnified moment method of 10.13.4.3 is used.

12

Shear

UPDATE FOR THE ‘02 CODE

Changes in the 2002 edition of the code include the following:

1. Reference to Appendix A, Strut-and-Tie Models, is given in 11.11.1. This allows the use of the new provisions of Appendix A in the shear design of disturbed regions.
2. The minimum shear reinforcement requirements for members with a concrete compressive strength greater than 10,000 psi have been revised. Section 11.1.2.1 refers to revised sections 11.5.5.3 and 11.5.5.4 for minimum areas of shear reinforcement.
3. For both prestressed and nonprestressed members, the minimum area of shear reinforcement has been revised, and is now a function of the concrete compressive strength.

GENERAL CONSIDERATIONS

The relatively abrupt nature of a “shear” failure, as compared to a ductile flexural failure, makes it desirable to design members so that strength in shear is equal to, or greater than, strength in flexure. To ensure a ductile flexural failure, the code (1) limits the minimum and maximum amount of longitudinal reinforcement and (2) requires a minimum amount of shear reinforcement in all flexural members if the factored shear force V_u exceeds one-half of the shear strength provided by the concrete, ($V_u > 0.5\phi V_c$), except for certain types of construction (11.5.5.1).

The determination of the amount of shear reinforcement is based on a modified form of the truss analogy. The truss analogy assumes that shear reinforcement resists the total transverse shear. Considerable research has indicated that shear strength provided by concrete V_c can be assumed equal to the shear causing inclined cracking; therefore, shear reinforcement need be designed to carry only the excess shear.

Only shear design for nonprestressed members with clear-span-to-overall-depth ratios greater than 4 is considered in Part 12. Also included is horizontal shear design in composite concrete flexural members, which is covered separately in the second half of Part 12. Shear design for deep flexural members, which have clear-span-to-overall-depth ratios less than 4, is presented in Part 17. Shear design of prestressed members is discussed in Part 25. The alternate shear design method of Appendix A, Strut-and-Tie Models, is discussed in Part 32.

11.1 SHEAR STRENGTH

Design provisions for shear are presented in terms of shear forces (rather than stresses) to be compatible with the other design conditions for the strength design method, which are expressed in terms of loads, moments, and forces.

Accordingly, shear is expressed in terms of the factored shear force V_u, using the basic shear strength requirement:

Design shear strength $\geq$ Required shear strength

$$\phi V_n \geq V_u \qquad \textit{Eq. (11-1)}$$

The nominal shear strength V_n is computed by:

$$V_n = V_c + V_s \qquad \textit{Eq. (11-2)}$$

where V_c is the nominal shear strength provided by concrete and V_s is the nominal shear strength provided by shear reinforcement.

Equation (11-2) can be substituted into Eq. (11-1) to obtain:

$$\phi V_c + \phi V_s \geq V_u$$

The required shear strength at any section is computed using Eqs. (11-1) and (11-2), where the factored shear force V_u is obtained by applying the load factors specified in 9.2. The strength reduction factor, $\phi = 0.75$, is specified in 9.3.2.3.

11.1.1.1 Web Openings

Often it is necessary to modify structural components of buildings to accommodate necessary mechanical and electrical service systems. Passing these services through openings in the webs of floor beams within the floor-ceiling sandwich eliminates a significant amount of dead space and results in a more economical design. However, the effect of the openings on the shear strength of the floor beams must be considered, especially when such openings are located in regions of high shear near supports. In 11.1.1.1, the code requires the designer to consider the effect of openings on the shear strength of members. Because of the many variables such as opening shape, size, and location along the span, specific design rules are not stated. However, references are given for design guidance in R11.1.1.1. Generally, it is desirable to provide additional vertical stirrups adjacent to both sides of a web opening, except for small isolated openings. The additional shear reinforcement can be proportioned to carry the total shear force at the section where an opening is located. Example 12.5 illustrates application of a design method recommended in Ref. 12.1.

11.1.2 Limit on $\sqrt{f'_c}$

Concrete shear strength equations presented in Chapter 11 of the Code are a function of $\sqrt{f'_c}$, and had been verified experimentally for members with concrete compressive strength up to 10,000 psi. Due to a lack of test data for members with $f'_c > 10{,}000$ psi, 11.1.2 limits the value of $\sqrt{f'_c}$ to 100 psi, except as allowed in 11.1.2.1.

Section 11.1.2 does not prohibit the use of concrete with $f'_c > 10{,}000$ psi; it merely directs the engineer not to count on any strength in excess of 10,000 psi when computing V_c, unless minimum shear reinforcement is provided in accordance with 11.1.2.1.

It should be noted that prior to the 2002 Code, minimum area of transverse reinforcement is independent of the concrete strength. Recent tests, however, indicate that an increase in the minimum amount of transverse reinforcement is required for members with high-strength concrete to prevent sudden shear failures when inclined cracking occurs. Thus, to account for this, minimum transverse reinforcement requirements are now a function of $\sqrt{f'_c}$.

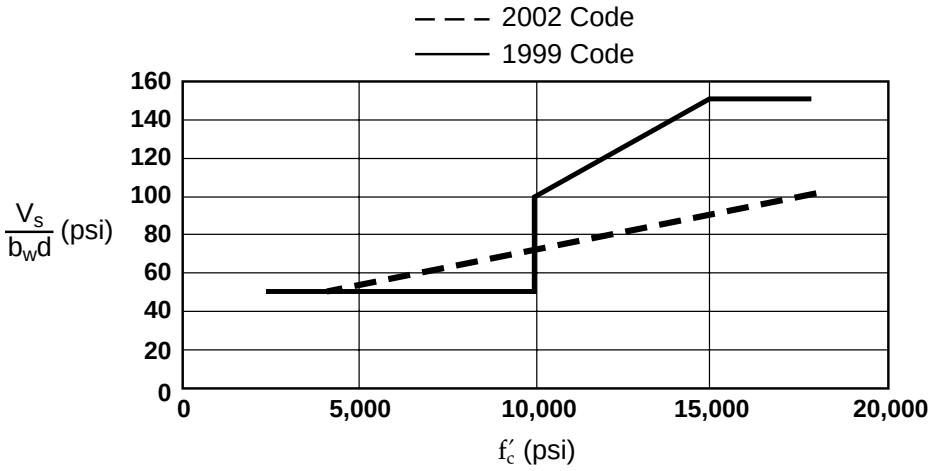


Figure 12-1 Minimum Shear Reinforcement Requirements per Eq. (11-13)

Figure 12-1 illustrates the required minimum shear reinforcement in accordance with 11.5.5.3 and Eq. (11-13) per the 1999 and 2002 Codes. As can be seen from the figure, Eq. (11-13) of the 2002 Code provides for a gradual increase in the minimum area of transverse reinforcement, while maintaining the minimum value of the 1999 Code.

11.1.3 Computation of Maximum Factored Shear Force

Section 11.1.3 describes three conditions that shall be satisfied in order to compute the maximum factored shear force V_u in accordance with 11.1.3.1 for nonprestressed members:

1. Support reaction, in direction of applied shear force, introduces compression into the end regions of the member.
2. Loads are applied at or near the top of the member.
3. No concentrated load occurs between the face of the support and the location of the critical section, which is a distance d from the face of the support (11.1.3.1).

When the conditions of 11.1.3 are satisfied, sections along the length of the member located less than a distance d from the face of the support are permitted to be designed for the shear force V_u computed at a distance d from the face of the support. See Fig. 12-2 (a), (b), and (c) for examples of support conditions where 11.1.3 would be applicable.

Conditions where 11.1.3 cannot be applied include: (1) members framing into a supporting member in tension, see Fig. 12-2 (d); (2) members loaded near the bottom, see Fig. 12-2 (e); and (3) members subjected to an abrupt change in shear force between the face of the support and a distance d from the face of the support, see Fig. 12-2 (f). In all of these cases, the critical section for shear must be taken at the face of the support. Additionally, in the case of Fig. 12-2 (d), the shear within the connection must be investigated and special corner reinforcement should be provided.

One other support condition is noteworthy. For brackets and corbels, the shear at the face of the support V_u must be considered, as shown in Fig. 12-3. However, these elements are more appropriately designed for shear using the shear-friction provisions of 11.7. See Part 15 for design of brackets and corbels.

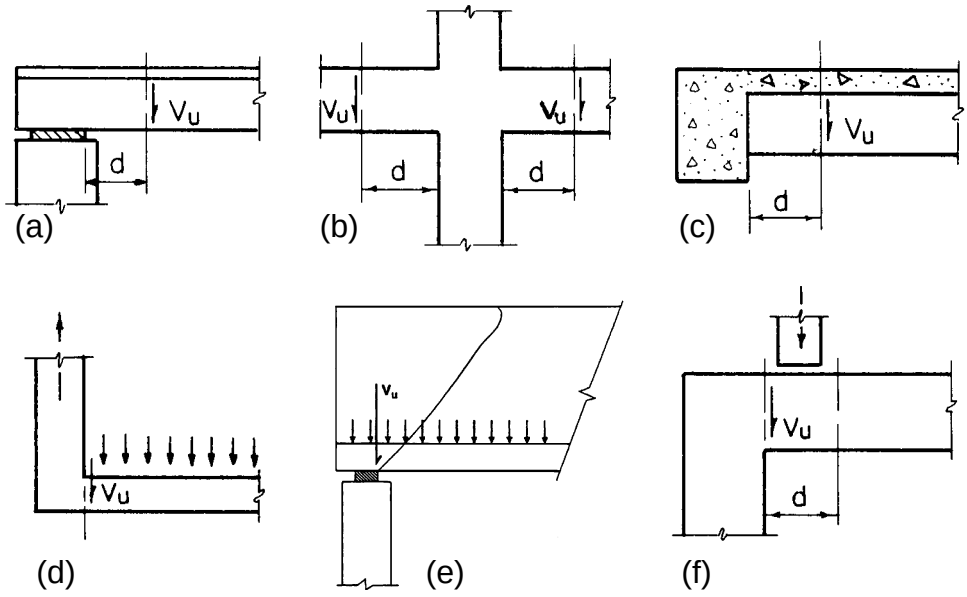


Figure 12-2 Typical Support Conditions for Locating Factored Shear Force V_u

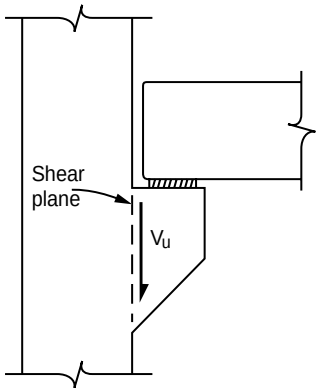


Figure 12-3 Critical Shear Plane for Brackets and Corbels

11.2 LIGHTWEIGHT CONCRETE

Since the shear strength of lightweight aggregate concrete may be less than that of normal weight concrete with equal compressive strength, adjustments in the value of V_c, as computed for normal weight concrete, are necessary.

Except for 11.5.4.3, 11.5.6.9, 11.6.3.1, 11.12.3.2, and 11.12.4.8, when average splitting tensile strength f_{ct} is specified, $f_{ct} / 6.7$ is substituted for $\sqrt{f'_c}$ in all equations of Chapter 11. However, the value of $f_{ct} / 6.7$ cannot be taken greater than $\sqrt{f'_c}$. When f_{ct} is not specified, $\sqrt{f'_c}$ is reduced using a multiplier of 0.75 for all-lightweight concrete or 0.85 for sand-lightweight concrete. Linear interpolation between these multipliers is allowed when partial sand replacement is used.

11.3 SHEAR STRENGTH PROVIDED BY CONCRETE FOR NONPRESTRESSED MEMBERS

When computing the shear strength provided by concrete for members subject to shear and flexure only, designers have the option of using either the simplified equation, $V_c = 2\sqrt{f'_c}\, b_w d$ (Eq. (11-3)), or the more elaborate expression given by Eq. (11-5). In computing V_c from Eq. (11-5), it should be noted that V_u and M_u are the values which occur simultaneously at the section considered. A maximum value of 1.0 is prescribed for the ratio $V_u d/M_u$ to limit V_c near points of inflection where M_u is zero or very small.

For members subject to shear and flexure with axial compression, a simplified V_c expression is given in 11.3.1.2, with an optional more elaborate expression for V_c available in 11.3.2.2. For members subject to shear, flexure and significant axial tension, 11.3.1.3 requires that shear reinforcement must be provided to resist the total shear unless the more detailed analysis of 11.3.2.3 is performed. Note that N_u represents a tension force in Eq. (11-8) and is therefore taken to be negative.

No precise definition is given for "significant axial tension." If there is uncertainty about the magnitude of axial tension, it may be desirable to carry all applied shear by shear reinforcement.

Figure 12-4 shows the variation of shear strength provided by concrete, V_c as function of $\sqrt{f'_c}$, $V_u d/M_u$, and reinforcement ratio ρ_w.

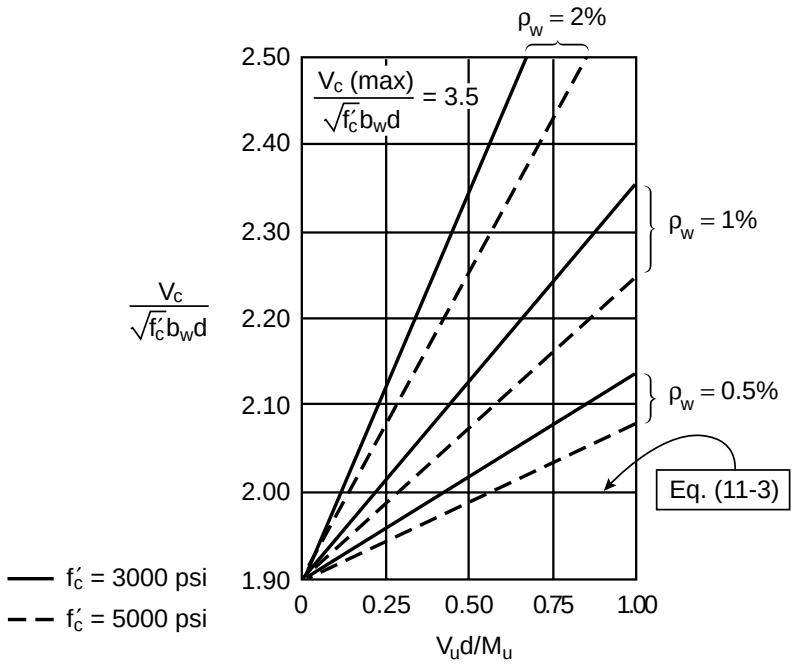


Figure 12-4 Variation of $V_c / \sqrt{f'_c}\, b_w d$ with f'_c, ρ_w, and $V_u d/M_u$ using Eq. (11-5)

Figure 12-5 shows the approximate range of values of V_c for sections under axial compression, as obtained from Eqs. (11-5) and (11-6). Values correspond to a 6 ×12 in. beam section with an effective depth of 10.8 in. The curves corresponding to the alternate expressions for V_c given by Eqs. (11-4) and (11-7), as well as that corresponding to Eq. (11-8) for members subject to axial tension, are also indicated.

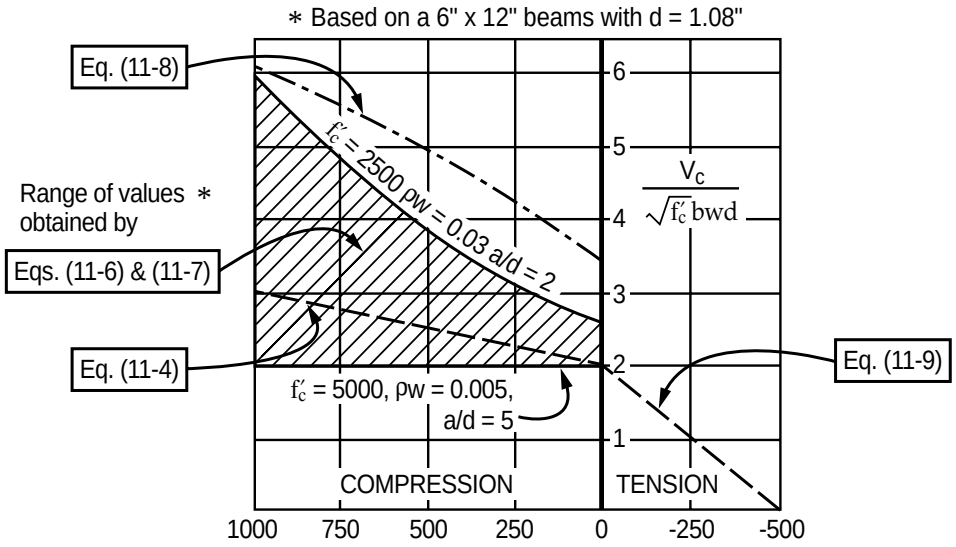


Figure 12-5 Comparison of Design Equations for Shear and Axial Load

Figure 12-6 shows the variation of V_c with N_u/A_g and f'_c for sections subject to axial compression, based on Eq. (11-4). For the range of N_u/A_g values shown, V_c varies from about 49% to 57% of the value of V_c as defined by Eq. (11-7).

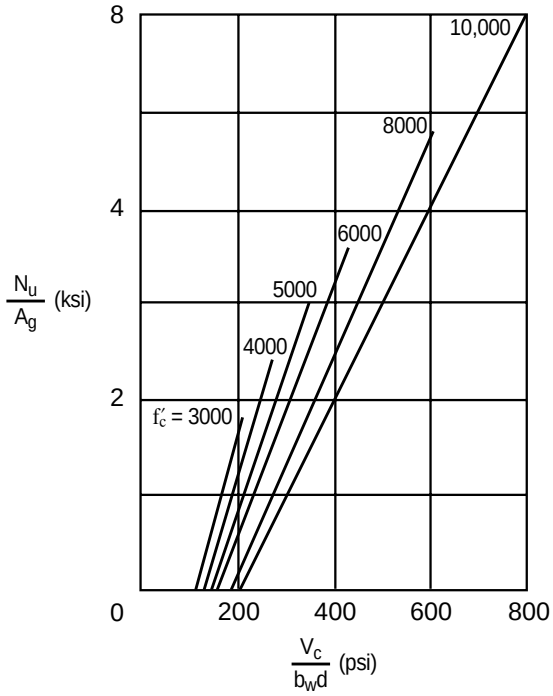


Figure 12-6 Variation of V_c/b_wd with f'_c and N_u/A_g using Eq. (11-4)

11.5 SHEAR STRENGTH PROVIDED BY SHEAR REINFORCEMENT

11.5.1 Types of Shear Reinforcement

Several types and arrangements of shear reinforcement permitted by 11.5.1.1 and 11.5.1.2 are illustrated in Fig. 12-7. Spirals, circular ties, or hoops are explicitly recognized as types of shear reinforcement starting with the 1999 code. Vertical stirrups are the most common type of shear reinforcement. Inclined stirrups and longitudinal bent bars are rarely used as they require special care during placement in the field.

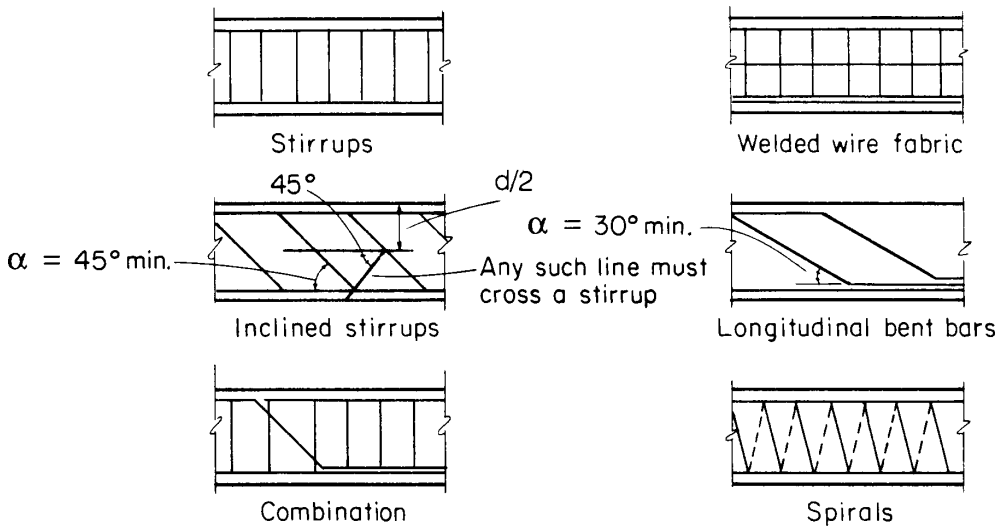


Figure 12-7 Types and Arrangements of Shear Reinforcement

11.5.3 Anchorage Details for Shear Reinforcement

To be fully effective, shear reinforcement must extend as close to full member depth as cover requirements and proximity of other reinforcement permit (12.13.1), and be anchored at both ends to develop the design yield strength of the shear reinforcement. The anchorage details prescribed in 12.13 are presumed to satisfy this development requirement.

11.5.4 Spacing Limits for Shear Reinforcement

Spacing of stirrups and wires (individual or in wire fabric) used as shear reinforcement, placed perpendicular to axis of member, must not exceed one-half the effective depth of the member (d/2), nor 24 in. When the quantity $\phi V_s = (V_u - \phi V_c)$ exceeds $\phi 4\sqrt{f'_c}\, b_w d$, maximum spacing must be reduced by one-half to (d/4) or 12 in. Note also that the value of (ϕV_s) shall not exceed $\phi 8\sqrt{f'_c}\, b_w d$ (11.5.6.9). For situations where the required shear strength exceeds this limit, the member size or the strength of the concrete may be increased to provide additional shear strength provided by concrete.

11.5.5 Minimum Shear Reinforcement

When the factored shear force V_u exceeds one-half the shear strength provided by concrete ($V_u > \phi V_c/2$), a minimum amount of shear reinforcement must be provided in concrete flexural members, except for slabs and footings, joists defined by 8.11, and wide, shallow beams (11.5.5.1). When required, the minimum shear reinforcement for nonprestressed members is

$$A_v = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_y} \qquad \textit{Eq. (11-13)}$$

but not less than $A_v = \frac{50 b_w s}{f_y}$

Minimum shear reinforcement is now a function of the concrete compressive strength in the 2002 Code. Equation (11-13) provides a gradual increase in the minimum required A_v, while maintaining the previous minimum value of 50 $b_w s / f_y$.

Note that spacing of minimum shear reinforcement must not exceed d/2 or 24 in.

11.5.6 Design of Shear Reinforcement

When the factored shear force V_u exceeds the shear strength provided by concrete, ϕV_c, shear reinforcement must be provided to carry the excess shear. The code provides an equation that defines the required shear strength provided by reinforcement V_s in terms of the area A_v, yield strength f_y, and spacing s of the shear reinforcement (Eq. (11-15)). The equation is based on a truss model with an angle of inclination of compression diagonals of 45 deg.

To assure correct application of the strength reduction factor, ϕ, equations for directly computing required shear reinforcement A_v are developed below. For shear reinforcement placed perpendicular to the member axis, the following method may be used to determine the required area of shear reinforcement A_v, spaced at a distance s:

$$\phi V_n \geq V_u \qquad \textit{Eq. (11-1)}$$

where $V_n = V_c + V_s$ *Eq. (11-2)*

and $V_s = \frac{A_v f_y d}{s}$ *Eq. (11-15)*

Substituting V_s into Eq. (11-2) and V_n into Eq. (11-1), the following equation is obtained:

$$\phi V_c + \frac{\phi A_v f_y d}{s} \geq V_u$$

Solving for A_v,

$$A_v = \frac{(V_u - \phi V_c)\, s}{\phi f_y d}$$

Similarly, when inclined stirrups are used as shear reinforcement,

$$A_v = \frac{(V_u - \phi V_c)\, s}{\phi f_y (\sin\alpha + \cos\alpha)\, d}$$

where α is the angle between the inclined stirrup and longitudinal axis of member (see Fig. 12-7).

When shear reinforcement consists of a single bar or group of parallel bars, all bent-up at the same distance from the support,

$$A_v = \frac{(V_u - \phi V_c)}{f_y \sin\alpha}$$

where α is the angle between the bent-up portion and longitudinal axis of member, but not less than 30 deg (see Fig. 12-7). For this case, the quantity (V_u - ϕV_c) must not exceed $\phi 3\sqrt{f'_c}\,b_w d$.

Design Procedure for Shear Reinforcement

Design of a nonprestressed concrete beam for shear involves the following steps:

1. Determine maximum factored shear force V_u at critical sections of the member per 11.1.3 (see Fig. 12-2).
2. Determine shear strength provided by the concrete ϕV_c per Eq. (11-3): $\phi V_c = \phi 2\sqrt{f'_c}\,b_w d$ where $\phi = 0.75$ (9.3.2.3).
3. Compute V_u - ϕV_c at the critical section. If V_u - $\phi V_c > \phi 8\sqrt{f'_c}\,b_w d$, increase the size of the section or the concrete compressive strength.
4. Compute the distance from the support beyond which minimum shear reinforcement is required (i.e., where ($V_u = \phi V_c$), and the distance from the support beyond which the concrete can carry the total shear force (i.e., where $V_u = \phi V_c/2$).
5. Use Table 12-1 to determine the required area of vertical stirrups A_v or stirrup space s at a few controlling sections along the length of the member, which includes the critical sections.

   Where stirrups are required, it is usually more expedient to select a bar size and type (e.g., No. 3 U-stirrups (2 legs)) and determine the required spacing. Larger stirrup sizes at wider spacings are usually more cost effective than smaller stirrup sizes at closer spacings because the latter requires disproportionately high costs for fabrication and placement. Changing the stirrup spacing as few times as possible over the required length also results in cost savings. If possible, no more than three different stirrup spacings should be specified, with the first stirrup located 2 in. from the face of the support.

Table 12-1 Provisions for Shear Design

		$V_u \le \phi V_c/2$	$\phi V_c \ge V_u > \phi V_c/2$	$V_u > \phi V_c$
Required area of stirrups, A_v		none	$0.75\sqrt{f'_c}\,\frac{b_w s}{f_y} \ge \frac{50 b_w s}{f_y}$	$\frac{(V_u - \phi V_c)s}{\phi f_y d}$
Stirrup spacing, s	Required	—	$\frac{A_v f_y}{0.75\sqrt{f'_c}\,b_w} \le \frac{A_v f_y}{50 b_w}$	$\frac{\phi A_v f_y d}{V_u - \phi V_c}$
	Maximum	—	$d/2 \le 24$ in.	$d/2 \le 24$ in. for $(V_u - \phi V_c) \le \phi 4\sqrt{f'_c}\,b_w d$
				$d/4 \le 12$ in. for $(V_u - \phi V_c) > \phi 4\sqrt{f'_c}\,b_w d$

The shear strength requirements are illustrated in Fig. 12-8.

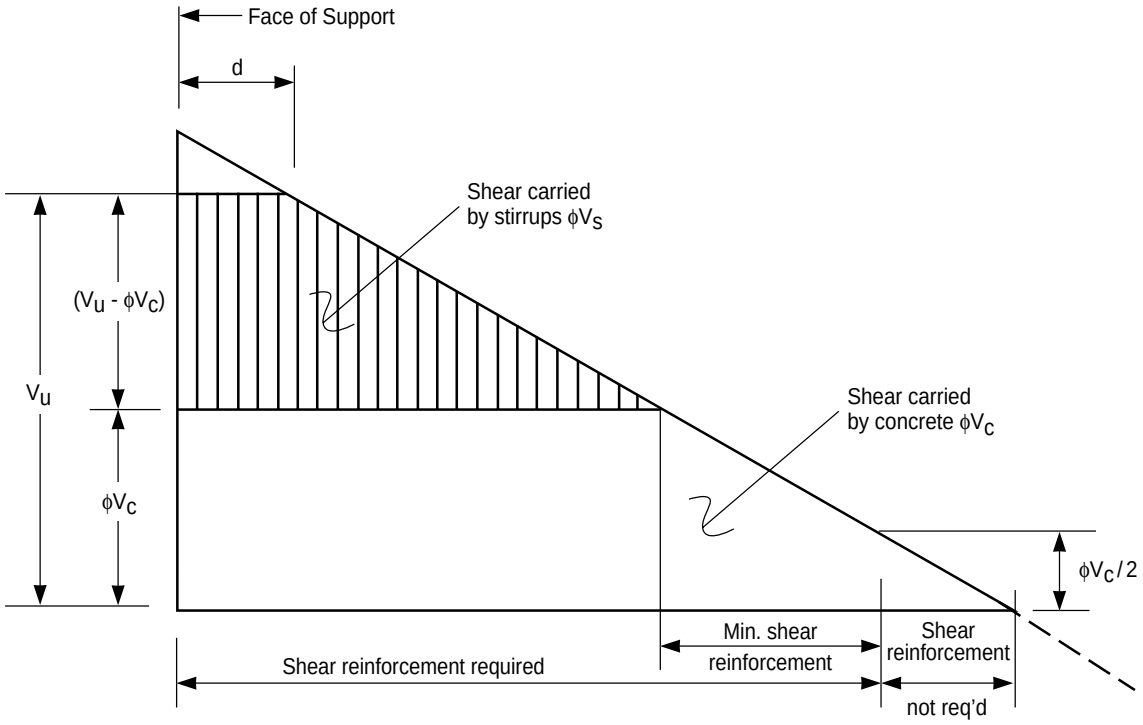


Figure 12-8 Shear Strength Requirements

The expression for shear strength provided by shear reinforcement ϕV_s can be assigned specific force values for a given stirrup size and strength of reinforcement. The selection and spacing of stirrups can be simplified if the spacing is expressed as a function of the effective depth d instead of numerical values. Practical limits of stirrup spacing generally vary from s = d/2 to s = d/4, since spacing closer than d/4 is not economical. With one intermediate spacing at d/3, a specific value of ϕV_s can be derived for each stirrup size and spacing as follows:

For vertical stirrups:

$$\phi V_s = \frac{\phi A_v f_y d}{s} \qquad \textit{Eq. (11-7)}$$

Substituting d/n for s, where n = 2, 3, and 4

$$\phi V_s = \phi A_v f_y n$$

Thus, for No. 3 U-stirrups @ s = d/2, f_y = 60 ksi and ϕ = 0.75

$$\phi V_s = 0.75\,(2 \times 0.11)\,60 \times 2 = 19.8 \text{ kips, say } 19 \text{ kips}$$

Values of ϕV_s given in Table 12-2 may be used to select shear reinforcement. Note that the ϕV_s values are independent of member size and concrete strength. Selection and spacing of stirrups using the design values for $\phi V_s = (V_u - \phi V_c)$ can be easily solved by numerical calculation or graphically. See Example 12.1.

Table 12-2 Shear Strength ϕV_s for Given Bar Sizes and Spacings

Spacing	Shear Strength ϕV_s (kips)					
	No. 3 U-Stirrups*		No. 4 U-Stirrups*		No. 5 U-Stirrups*	
	Grade 40	Grade 60	Grade 40	Grade 60	Grade 40	Grade 60
d/2	13	19	24	36	37	55
d/3	19	29	36	54	55	83
d/4	26	39	48	72	74	111

** Stirrups with 2 legs (double values for 4 legs, etc.)*

CHAPTER 17 — COMPOSITE CONCRETE FLEXURAL MEMBERS

17.4 VERTICAL SHEAR STRENGTH

Section 17.4.1 of the Code permits the use of the entire composite flexural member to resist the design vertical shear as if the member were monolithically cast. Therefore, the requirements of Code Chapter 11 apply.

Section 17.4.3 permits the use of vertical shear reinforcement to serve as ties for horizontal shear reinforcement, provided that the vertical shear reinforcement is extended and anchored in accordance with applicable provisions.

17.5 HORIZONTAL SHEAR STRENGTH

In composite flexural members, horizontal shear forces are caused by the moment gradient resulting from vertical shear force. These horizontal shear forces act over the interface of interconnected elements that form the composite member.

Section 17.5.1 requires full transfer of the horizontal shear forces by friction at the contact surface, properly anchored ties, or both. Unless calculated in accordance with 17.5.3, the factored applied horizontal shear force $V_u \le \phi V_{nh}$, where ϕV_{nh} is the horizontal shear strength (17.5.2).

The horizontal shear strength is $\phi V_{nh} = 80 b_v d$ for intentionally roughened contact surfaces without the use of ties (friction only), and for surfaces that are not intentionally roughened with the use of minimum ties provided in accordance with 17.6 (17.5.2.1 and 17.5.2.2). When ties per 17.6 are provided, and the contact surface is intentionally roughened to a full amplitude of approximately 1/4 in., the horizontal shear strength is:

$$\phi V_{nh} = (260 + 0.6\rho_v f_y)\lambda b_v d \le 500 b_v d \text{ (17.5.2.3)}.$$

The expression for V_{nh} in 17.5.2.3 accounts for the effect of the quantity of reinforcement crossing the interface by including ρ_v, which is the ratio of tie reinforcement area to area of contact surface, or $\rho_v = A_v/b_v s$. It also incorporates the correction factor λ to account for lightweight aggregate concrete per 11.7.4.3. It should also be noted that for concrete compressive strength $f'_c \le 4444$ psi, the minimum tie reinforcement per Eq. (11-13) is $\rho_v f_y = 50$ psi; substituting this into the above expression, $V_{nh} = 290\lambda b_v d$. The upper limit of $500 b_v d$ corresponds to $\rho_v f_y = 400$ psi in the case of normal weight concrete (i.e., $\lambda = 1$).

When in computing the horizontal shear strength of a composite flexural member, the following apply:

1. When $V_u > \phi(500 b_v d)$, the shear friction method of 11.7.4 must be used (17.5.2.4). Refer to Part 14 for further details on the application of 11.7.4.
2. No distinction shall be made between shored or unshored members (17.2.4). Tests have indicated that the strength of a composite member is the same whether or not the first element cast is shored or not.
3. Composite members must meet the appropriate requirements for deflection control per 9.5.6.
4. The contact surface shall be clean and free of laitance. Intentionally roughened surface may be achieved by scoring the surface with a stiff bristled broom. Heavy raking or grooving of the surface may be sufficient to achieve "full 1/4 in. amplitude."
5. The effective depth d is defined as the distance from the extreme compression fiber to the centroid of the tension reinforcement for the entire composite section (17.5.2.5). For prestressed member, the effective depth shall be the greater of d or 0.8 times the depth of the full composite section. This makes the definition of d for composite sections consistent with shear design in other parts of the Code. This section is significant for the design of prestressed member where the effective depth often varies along the span.

The code also presents an alternative method for horizontal shear design in 17.5.3. The horizontal shear force that must be transferred across the interface between parts of a composite member is taken to be the change in internal compressive or tensile force, parallel to the interface, in any segment of a member. When this method is used, the limits of 17.5.2.1 through 17.5.2.4 apply, with the contact area A_c substituted for the quantity $b_v d$ in the expressions. Section 17.5.3.1 also requires that the reinforcement be distributed to approximately reflect the variation in shear force along the member. This requirement emphasizes the difference between the design of composite members on concrete and on steel. Slip between the steel beam and composite concrete slab at maximum strength is large, which permits redistribution of the shear force along the member. In concrete members with a composite slab, the slip at maximum strength is small and redistribution of shear resistance along the member is limited. Therefore, distribution of horizontal shear reinforcement must be based on the computed distribution of factored horizontal shear in concrete composite flexural members.

17.6 TIES FOR HORIZONTAL SHEAR

According to 17.6.3, ties are required to be "fully anchored" into interconnected elements "in accordance with 12.13." Figure 12-9 shows some tie details that have been used successfully in testing and design practice. Figure 12-9(a) shows an extended stirrup detail used in tests of Ref. 12.3. Use of an embedded "hairpin" tie, as illustrated in Fig. 12-9(b), is common practice in the precast, prestressed concrete industry. Many precast products are manufactured in such a way that it is difficult to position tie reinforcement for horizontal shear before concrete is placed. Accordingly, the ties are embedded in the plastic concrete as permitted by 16.7.1.

Shear reinforcement that extends from previously-cast concrete and is adequately anchored into the composite portion of a member (Fig. 12-9(c)) may be used as reinforcement (ties) to resist horizontal shear (17.4.3). Therefore, this reinforcement may be used to satisfy requirements for both vertical and horizontal shear.

Example 12.6 illustrates design for horizontal shear.

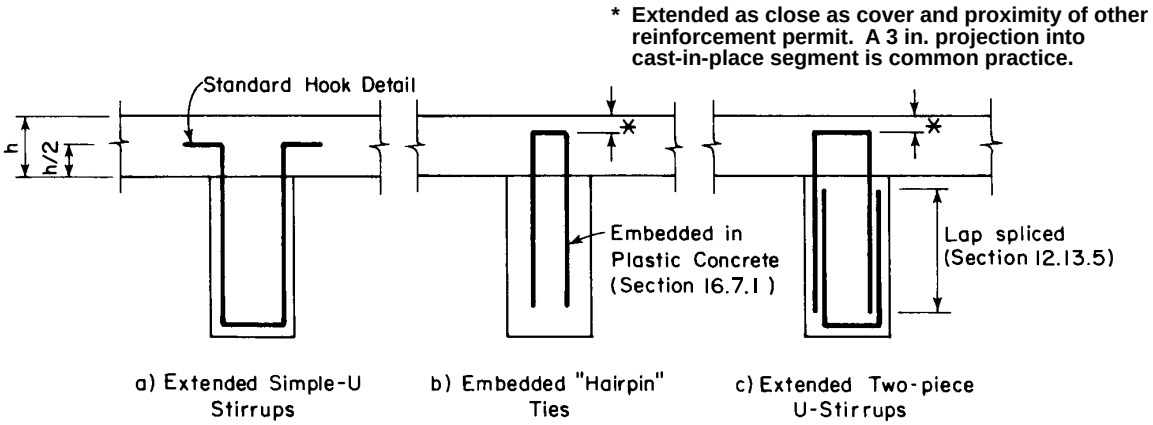


Figure 12-9 Ties for Horizontal Shear

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12.2 Hanson, N.W., *Precast-Prestressed Concrete Bridges 2. Horizontal Shear Connections*, Development Department Bulletin D35, Portland Cement Association, Skokie, IL, 1960.

12.3 Roller, J.J., and Russell, H.G., "Shear Strength of High Strength Concrete Beams with Web Reinforcement," *ACI Structural Journal*, Vol. 87, No. 2, March-April 1990, pp. 191-198.

Example 12.1—Design for Shear - Members Subject to Shear and Flexure Only

Determine required size and spacing of vertical U-stirrups for a 30-foot span, simply supported beam.

b_w = 13 in.
d = 20 in.
f'_c = 3000 psi
f_y = 40,000 psi
w_u = 4.5 kips/ft

Calculations and Discussion	Code Reference

For the purpose of this example, the live load will be assumed to be present on the full span, so that design shear at centerline of span is zero. (A design shear greater than zero at midspan is obtained by considering partial live loading of the span.) Using design procedure for shear reinforcement outlined in this part:

1. Determine factored shear forces

   @ support: $V_u = 4.5\ (15) = 67.5$ kips

   @ distance d from support:

   $V_u = 67.5 - 4.5\ (20/12) = 60$ kips — *11.1.3.1*

2. Determine shear strength provided by concrete

   $\phi V_c = \phi 2\sqrt{f'_c}\ b_w d$ — *Eq. (11-3)*

   $\phi = 0.75$ — *9.3.2.3*

   $\phi V_c = 0.75\ (2)\ \sqrt{3000} \times 13 \times 20 / 1000 = 21.4$ kips

   $V_u = 60$ kips $> \phi V_c = 21.4$ kips

   Therefore, shear reinforcement is required. — *11.1.1*

3. Compute $V_u - \phi V_c$ at critical section.

   $V_u - \phi V_c = 60 - 21.4 = 38.6$ kips $< \phi 8\sqrt{f'_c}\, b_w d = 85.4$ kips O.K. — *11.5.6.9*

4. Determine distance x_c from support beyond which minimum shear reinforcement is required ($V_u = \phi V_c$):

$$x_c = \frac{V_u \text{ @ support} - \phi V_c}{w_u} = \frac{67.5 - 21.4}{4.5} = 10.2 \text{ ft}$$

Determine distance x_m from support beyond which concrete can carry total shear force ($V_u = \phi V_c / 2$):

$$x_m = \frac{V_u \text{ @ support} - (\phi V_c / 2)}{w_u} = \frac{67.5 - (21.4/2)}{4.5} = 12.6 \text{ ft}$$

5. Use Table 12-1 to determine required spacing of vertical U-stirrups.

At critical section, $V_u = 60$ kips $> \phi V_c = 21.4$ kips

$$s \text{ (req'd)} = \frac{\phi A_v f_y d}{V_u - \phi V_c}$$ *Eq. (11-15)*

Assuming No. 4 U-stirrups ($A_v = 0.40$ in.^{2}),

$$s \text{ (req'd)} = \frac{0.75 \times 0.40 \times 40 \times 20}{38.6} = 6.2 \text{ in.}$$

Check maximum permissible spacing of stirrups:

s (max) $\le d/2 = 20/2 = 10$ in. (governs) *11.5.4.1*

≤ 24 in. since $V_u - \phi V_c = 38.6$ kips $< \phi 4\sqrt{f'_c}\, b_w d = 42.7$ kips

Maximum stirrup spacing based on minimum shear reinforcement:

$$s \text{ (max)} \le \frac{A_v f_y}{0.75\sqrt{f'_c} b_w} = \frac{0.4 \times 40{,}000}{0.75\sqrt{3{,}000}(13)} = 30 \text{ in.}$$ *11.5.5.3*

$$\le \frac{A_v f_y}{50 b_w} = \frac{0.4 \times 40{,}000}{50 \times 13} = 24.6 \text{ in.}$$

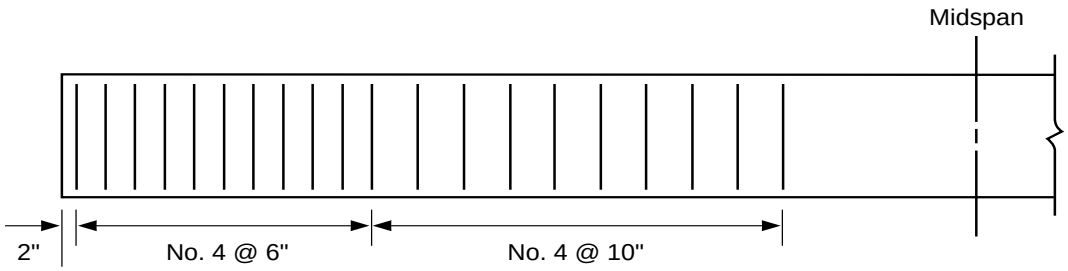
Determine distance x from support beyond which 10 in. stirrup spacing may be used:

$$10 = \frac{0.75 \times 0.4 \times 40{,}000 \times 20}{V_u - 21.4}$$

$V_u - 21.4 = 24$ kips or $V_u = 24 + 21.4 = 45.4$ kips

$$x = \frac{67.5 - 45.4}{4.5} = 4.9 \text{ ft}$$

Stirrup spacing using No. 4 U-stirrups:



6. As an alternate procedure, use simplified method presented in Table 12-2 to determine stirrup size and spacing.

   At critical section,

   $\phi V_s = V_u - \phi V_c = 60 - 21.4 = 38.6$ kips

   From Table 12-2 for Grade 40 stirrups:

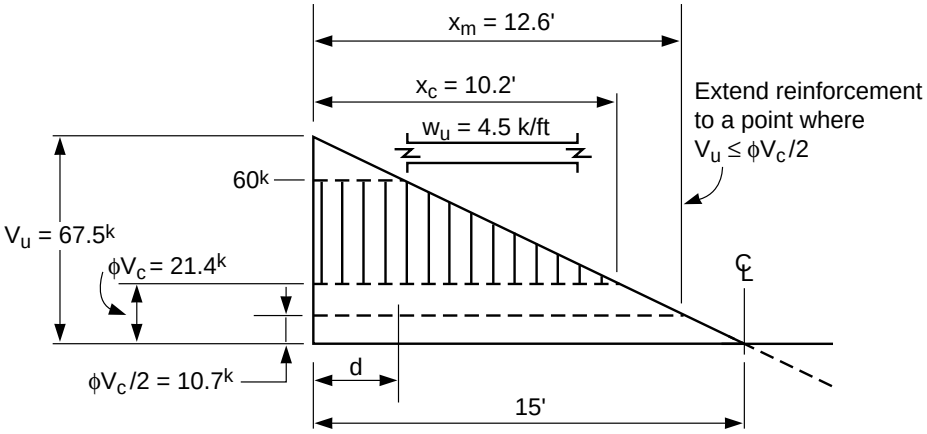
   No. 4 U-stirrups @ d/4 provides $\phi V_s = 48$ kips

   No. 4 U-stirrups @ d/3 provides $\phi V_s = 36$ kips

   By interpolation, No. 4 U-stirrups @ d/3.22 = 38.6 kips

   Stirrup spacing = d/3.22 = 20/3.22 = 6.2 in.

   Stirrup spacing along length of beam is determined as shown previously.



Example 12.2—Design for Shear - with Axial Tension

Determine required spacing of vertical U-stirrups for a beam subject to axial tension.

f'_c = 3600 psi (sand-lightweight concrete, f_{ct} not specified)
f_y = 40,000 psi
M_d = 43.5 ft-kips
M_ℓ = 32.0 ft-kips
V_d = 12.8 kips
V_ℓ = 9.0 kips
N_d = -2.0 kips (tension)
N_ℓ = -15.2 kips (tension)

Calculations and Discussion	Code Reference

1. Determine factored loads — *9.2.1*

   $M_u = 1.2\ (43.5) + 1.6\ (32.0) = 103.4$ ft-kips — *Eq. (9-2)*

   $V_u = 1.2\ (12.8) + 1.6\ (9.0) = 29.8$ kips

   $N_u = 1.2\ (-2.0) + 1.6\ (-15.2) = -26.7$ kips (tension)

2. Determine shear strength provided by concrete

   Since average splitting tensile strength f_{ct} is not specified, $\sqrt{f'_c}$ is reduced by a factor of 0.85 (sand-lightweight concrete) — *11.2.1.2*

   $$\phi V_c = \phi 2\left[1 + \frac{N_u}{500A_g}\right] 0.85\sqrt{f'_c}\, b_w d$$ — *Eq. (11-8)*

   $\phi = 0.75$ — *9.3.2.3*

   $$\phi V_c = (0.75)\, 2\left[1 + \frac{(-26{,}700)}{500\ (18\times 10.5)}\right] 0.85\sqrt{3600}\ (10.5)\ 16/1000 = 9.2 \text{ kips}$$

3. Check adequacy of cross-section.

   $(V_u - \phi V_c) \le \phi 8\ (0.85)\sqrt{f'_c}\, b_w d$ — *11.5.6.9*

   Note: 0.85 is a factor for lightweight concrete per 11.2.1.2

   $(V_u - \phi V_c) = 29.8 - 9.2 = 20.6$ kips

   $\phi 8(0.85)\sqrt{f'_c}\, b_w d = 0.75\times 8\times 0.85\sqrt{3600}\times 10.5\times 16/1000 = 51.4$ kips $>$ 20.6 kips O.K.

4. Determine required spacing of U-stirrups

   Assuming No. 3 U-stirrups (A_v = 0.22 in.^{2}),

   $$s \text{ (req'd)} = \frac{\phi A_v f_y d}{(V_u - \phi V_c)}$$

   $$= \frac{0.75 \times 0.22 \times 40 \times 16}{20.6} = 5.1 \text{ in.}$$

5. Determine maximum permissible spacing of stirrups

   $V_u - \phi V_c$ = 20.6 kips

   $\phi 4(0.85)\sqrt{f'_c} b_w d$ = 25.4 kips > 20.6 kips *11.2.1.2 & 11.5.4.3*

   Therefore, provisions of 11.5.4.1 apply.

   s (max) of vertical stirrups ≤ d/2 = 8 in. (governs) *11.5.4.1*
   or ≤ 24 in.

   s (max) of No. 3 U-stirrups corresponding to minimum reinforcement area requirements:

   $$s \text{ (max)} = \frac{A_v f_y}{0.75(0.85)\sqrt{f'_c} b_w} = \frac{0.22 \times 40{,}000}{0.75 \times 0.85 \times \sqrt{3600} \times 10.5} = 21.9 \text{ in.}$$ *11.5.5.3*

   $$s \text{ (max)} = \frac{A_v f_y}{50 b_w} = \frac{0.22\ (40{,}000)}{50\ (10.5)} = 16.8 \text{ in.}$$

   s (max) = 8 in. (governs)

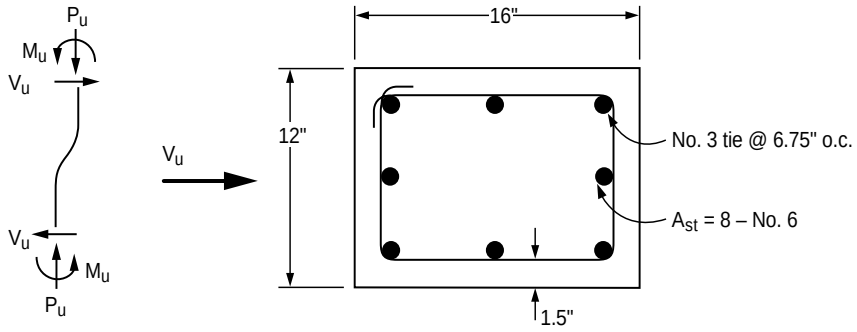
   Summary:

   Use No. 3 vertical stirrups @ 5.0 in. spacing.

Example 12.3—Design for Shear - with Axial Compression

A tied compression member has been designed for the given load conditions. However, the original design did not take into account the fact that under a reversal in the direction of lateral load (wind), the axial load, due to the combined effects of gravity and lateral loads, becomes P_u = 10 kips, with essentially no change in the values of M_u and V_u. Check shear reinforcement requirements for the column under (1) original design loads and (2) reduced axial load.

M_u = 86 ft-kips
P_u = 160 kips
V_u = 20 kips
f'_c = 4000 psi
f_y = 40,000 psi



Calculations and Discussion	Code Reference

Condition 1: $P_u = N_u = 160$ kips

1. Determine shear strength provided by concrete

   $d = 16 - [1.5 + 0.375 + (0.750/2)] = 13.75$ in.

   $$\phi V_c = \phi 2\left[1 + \frac{N_u}{2000A_g}\right]\sqrt{f'_c}\, b_w d$$ *Eq. (11-4)*

   $\phi = 0.75$ *9.3.2.3*

   $$\phi V_c = 0.75\,(2)\left[1 + \frac{160{,}000}{2000\,(16\times 12)}\right]\sqrt{4000}\,(12)(13.75)/1000 = 22.2 \text{ kips}$$

   $\phi V_c = 22.2$ kips $> V_u = 20$ kips

2. Since $V_u = 20$ kips $> \phi V_c/2 = 11.1$ kips, minimum shear reinforcement requirements must be satisfied. *11.5.5.1*

   No. 3 stirrups ($A_v = 0.22$ in.^{2})

   $$s\,(\max) = \frac{A_v f_y}{0.75\sqrt{f'_c}\, b_w} = \frac{0.22\,(40{,}000)}{0.75\sqrt{4000}\,(12)} = 15.5 \text{ in.}$$ *Eq. (11-13)*

   $$s\,(\max) = \frac{A_v f_y}{50 b_w} = \frac{0.22\,(40{,}000)}{50\,(12)} = 14.7 \text{ in.}$$

   s (max) = d/2 = 13.75/2 = 6.9 in. (governs) *11.5.4.1*

   Therefore, use of s = 6.75 in. is satisfactory.

Condition 2: $P_u = N_u = 10$ kips

1. Determine shear strength provided by concrete.

$$\phi V_c = 0.75\,(2)\left[1 + \frac{(10{,}000)}{2000\,(16\times12)}\right] \times \sqrt{4000}\,(12)(13.75)/1000 = 16.1 \text{ kips}$$ *Eq. (11-4)*

$\phi V_c = 16.1$ kips $< V_u = 20$ kips

Shear reinforcement must be provided to carry excess shear.

2. Determine maximum permissible spacing of No. 3 ties

$$s\,(\max) = \frac{d}{2} = \frac{13.75}{2} = 6.9 \text{ in.}$$ *11.5.4.1*

Maximum spacing, d/2, governs for Conditions 1 and 2.

3. Check total shear strength with No. 3 @ 6.75 in.

$$\phi V_s = \phi A_v f_y \frac{d}{s} = \frac{0.75\,(0.22)\,(40)\,(13.75)}{6.75} = 13.4 \text{ kips}$$ *Eq. (11-15)*

$\phi V_c + \phi V_s = 16.1 + 13.4 = 29.5$ kips $> V_u = 20$ kips O.K.

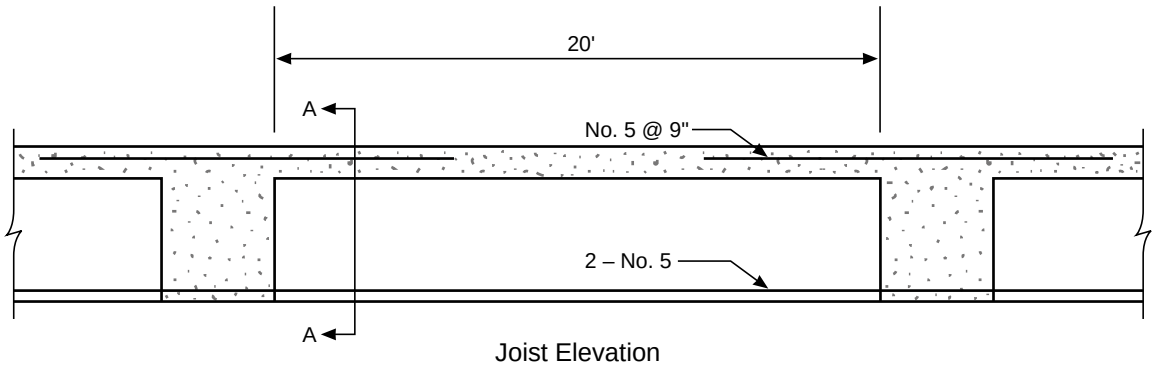
Example 12.4—Design for Shear - Concrete Floor Joist

Check shear requirements in the uniformly loaded floor joist shown below.

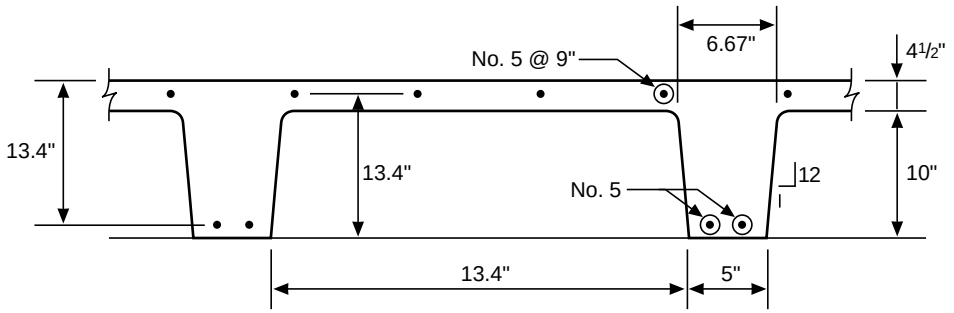
f'_c = 4000 psi
f_y = 40,000 psi
w_d = 77 psf
w_ℓ = 120 psf

Assumed longitudinal reinforcement:

Bottom bars: 2 – No. 5
Top bars: No. 5 @ 9 in.



Joist Elevation



Section

Calculations and Discussion	Code Reference

1. Determine factored load.

w_u = [1.2 (77) + 1.6 (120)] 35/12 = 830 lb/ft — *Eq. (9-2)*

2. Determine factored shear force.

   @ distance d from support: *11.1.3.1*

   $V_u = 0.83\,(10) - 0.83\,(13.4/12) = 7.4$ kips *8.3.3*

3. Determine shear strength provided by concrete.

   According to 8.11.8, V_c may be increased by 10 percent.

   Average width of joist web $b_w = (6.67 + 5) / 2 = 5.83$ in.

   $\phi V_c = 1.1\phi 2\sqrt{f'_c}\, b_w d$ *Eq. (11-3)*

   $\phi = 0.75$ *9.3.2.3*

   $\phi V_c = 1.1\,(0.75)\,2\sqrt{4000}\,(5.83)\,(13.4)/1000 = 8.2$ kips

   $\phi V_c = 8.2$ kips $> V_u = 7.4$ kips O.K.

   Note that minimum shear reinforcement is not required for joist construction defined by 8.11. *11.5.5.1(b)*

   Alternatively, calculate V_c using Eq. (11-5)

   Compute ρ_w and $V_u d/M_u$ at distance d from support:

   $$\rho_w = \frac{A_s}{b_w d} = \frac{(2 \times 0.31)}{5\,(13.4)} = 0.0093$$

   $$M_u \text{ @ face of support} = \frac{w_u \ell_n^2}{11} = \frac{0.83\,(20)^2}{11} = 30.2 \text{ ft - kips}$$

   $$M_u \text{ @ d} = \frac{w_u \ell_n^2}{11} + \frac{w_u d^2}{2} - \frac{w_u \ell_n d}{2}$$

   $$= 30.2 + \frac{(0.83)(13.4/12)^2}{2} - \frac{(0.83)(20)(13.4/12)}{2} = 21.5 \text{ ft - kips}$$

   $$\frac{V_u d}{M_u} = \frac{7.4(13.4/12)}{21.5} = 0.38 < 1.0 \quad \text{O.K.}$$ *11.3.2.1*

   $$\phi V_c = \phi\, 1.1 \left(1.9\sqrt{f'_c} + 2500\,\rho_w \frac{V_u d}{M_u}\right) b_w d \le \phi(1.1)\,3.5\sqrt{f'_c}\, b_w d$$

   $$= 0.75(1.1)\left[1.9\sqrt{4{,}000} + 2500(0.0093)(0.38)\right](5.83)(13.4)/1{,}000$$

   $$= 8.3 \text{ kips} < 0.75(1.1)(3.5)\sqrt{4{,}000}\,(5.83)(13.4)/1{,}000 = 14.3 \text{ kips} \quad \text{O.K.}$$

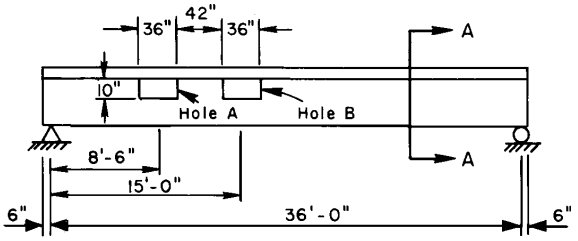
   $\phi V_c = 8.3$ kips $> V_u = 7.4$ kips O.K.

Example 12.5—Design for Shear - Shear Strength at Web Openings

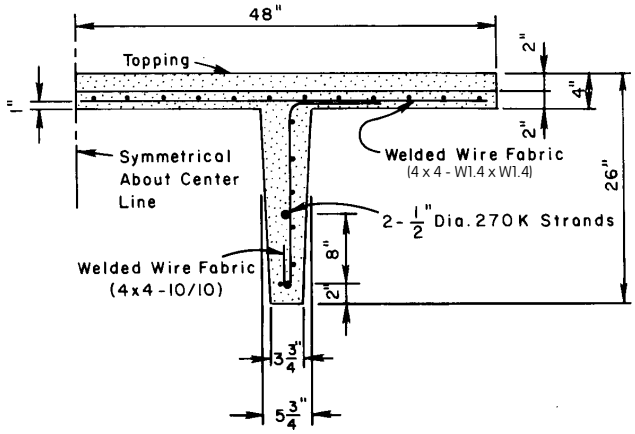
The simply supported prestressed double tee beam shown below has been designed without web openings to carry a live load of 50 lb/ft^2 (w_u = 1520 lb/ft). Two 10-in.-deep by 36-in.-long web openings are required for passage of mechanical and electrical services. Investigate the shear strength of the beam at web opening A.

This design example is based on an experimental and analytical investigation reported in Ref. 12.1.

Beam f'_c = 6000 psi
Topping f'_c = 3000 psi
f_{pu} = 270,000 psi
f_y = 60,000 psi



Elevation



Section A-A
(one-half of the double tee section)

Calculations and Discussion	Code Reference

This example treats only the shear strength considerations for the web opening. Other strength considerations need to be investigated, such as: to avoid slip of the prestressing strand, openings must be located outside the required strand development length, and strength of the struts to resist flexure and axial loads must be checked. The reader is referred to the complete design example in Ref. 12.1 for such calculations. The design example in Ref. 12.1 also illustrates procedures for checking service load stresses and deflections around the openings.

1. Determine factored moment and shear at center of opening A. Since double tee is symmetric about centerline, consider one-half of double tee section.

   $$w_u = \frac{1520}{2} = 760 \text{ lb/ft per tee}$$

   $$M_u = 0.760\,(36/2)\,(8.5) - 0.760\,(8.5)^2/2 = 88.8 \text{ ft-kips}$$

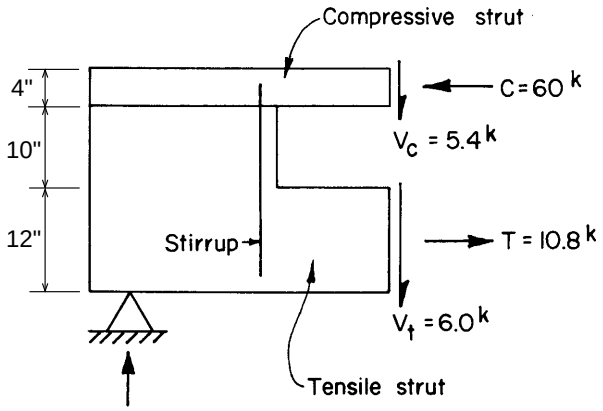
   $$V_u = 0.760\,(36/2) - 0.760\,(8.5) = 7.2 \text{ kips}$$

2. Determine required shear reinforcement adjacent to opening. Vertical stirrups must be provided adjacent to both sides of web opening. The stirrups should be proportioned to carry the total shear force at the opening.

$$A_v = \frac{V_u}{\phi f_y} = \frac{7200}{0.75 \times 60{,}000} = 0.16 \text{ in.}^2$$

Use No. 3 U-stirrup, one on each side of opening (A_v = 0.22 in.^{2})

3. Using a simplified analytical procedure developed in Ref. 12.1, the axial and shear forces acting on the "struts" above and below opening A are calculated. Results are shown in the figure below. The reader is referred to the complete design example in Ref. 12.3 for the actual force calculations. Axial forces should be accounted for in the shear design of the struts.



4. Investigate shear strength for tensile strut.

V_u = 6.0 kips

N_u = -10.8 kips *11.0*

d = 0.8h = 0.8 (12) = 9.6 in. *11.0*

b_w = average width of tensile strut = [3.75 + (3.75 + 2 × 12/22)]/2 = 4.3 in.

$$V_c = 2\left(1 + \frac{N_u}{500A_g}\right)\sqrt{f'_c}\, b_w d$$ *Eq. (11-8)*

$$= 2\left(1 - \frac{10{,}800}{500 \times 4.3 \times 12}\right)\sqrt{6000}\,(4.3)\,(9.6)/1000 = 3.72 \text{ kips}$$

$\phi V_c = 0.75\,(3.72) = 2.8$ kips

$V_u = 6.0$ kips $> \phi V_c = 2.8$ kips

Therefore, shear reinforcement is required in tensile strut.

$$A_v = \frac{(V_u - \phi V_c)\, s}{\phi f_y d}$$

$$= \frac{(6.0 - 2.8)\, 9}{0.75 \times 60 \times 9.6} = 0.07 \text{ in.}^2$$

where $s = 0.75h = 0.75 \times 12 = 9$ in. *11.5.4.1*

Use No. 3 single leg stirrups at 9-in. centers in tensile strut, ($A_v = 0.11$ in.^{2}). Anchor stirrups around prestressing strands with 180 deg. bend at each end.

5. Investigate shear strength for compressive strut.

$V_u = 5.4$ kips

$N_u = 60$ kips

$d = 0.8h = 0.8\,(4) = 3.2$ in.

$b_w = 48$ in.

$$V_c = 2\left(1 + \frac{N_u}{2000A_g}\right)\sqrt{f'_c}\, b_w d \qquad \textit{Eq. (11-4)}$$

$$= 2\left(1 + \frac{60{,}000}{2000 \times 48 \times 4}\right)\sqrt{3000}\,(48)\,(3.2)/1000 = 19.5 \text{ kips}$$

$\phi V_c = 0.75\,(19.5) = 14.6$ kips

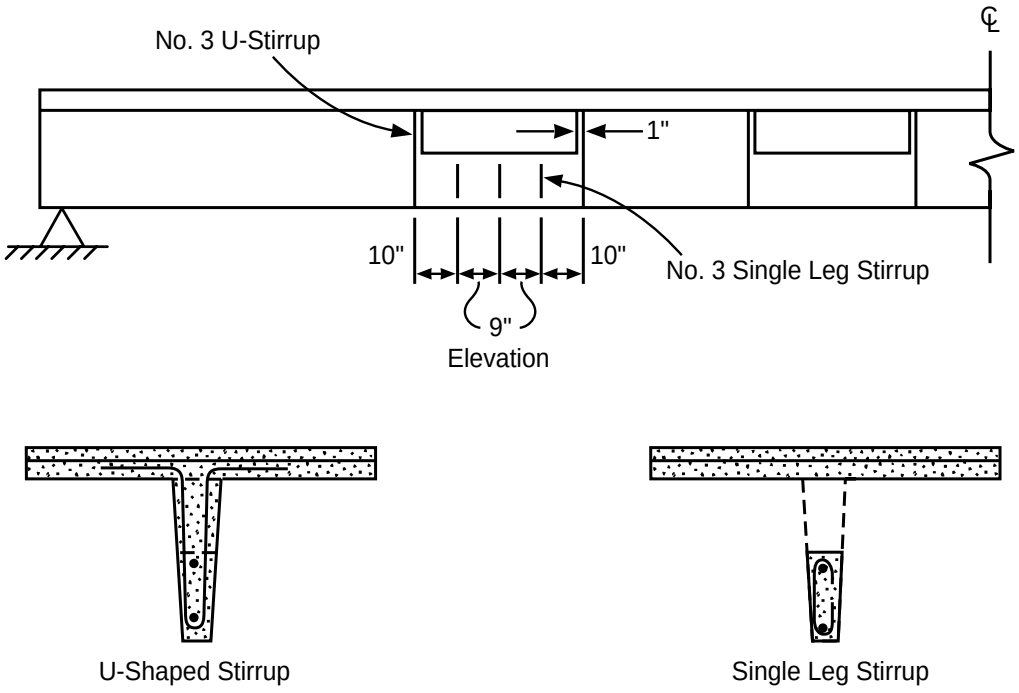
$V_u = 5.4$ kips $< \phi V_c = 14.6$ kips

Therefore, shear reinforcement is not required in compressive strut.

6. Design Summary - See reinforcement details below.

   a. Use U-shaped No. 3 stirrup adjacent to both edges of opening to contain cracking within the struts.

b. Use single-leg No. 3 stirrups at 9-in. centers as additional reinforcement in the tensile strut.

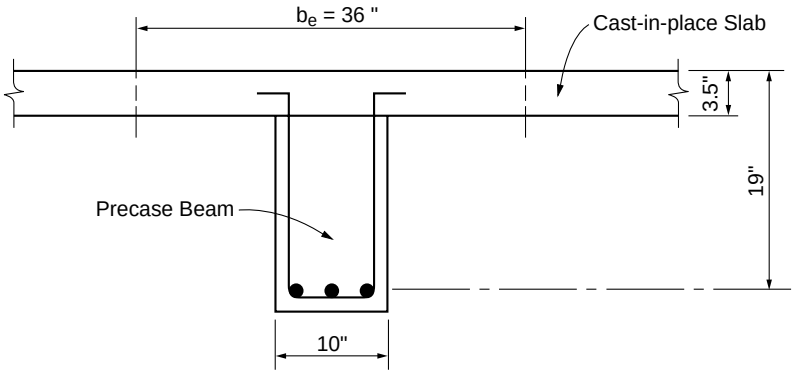


Details of Additional Reinforcement

A similar design procedure is required for opening B.

Example 12.6—Design for Horizontal Shear

For the composite slab and precast beam construction shown, design for transfer of horizontal shear at contact surface of beam and slab for the three cases given below. Assume the beam is simply supported with a span of 30 feet.



f'_c = 3000 psi (normal weight concrete)
f_y = 60,000 psi

Calculations and Discussion	Code Reference

Case I: Service dead load = 315 lb/ft
Service live load = 235 lb/ft

1. Determine factored shear force V_u at span end.

$V_u = 1.2D + 1.6L$ *Eq. (9-2)*

$= 1.2\ (0.315)\ (30/2) + 1.6\ (0.235)\ (30/2) = 11.3$ kips

At a distance d from face of support: *11.1.3.1*

$$V_u = 11.3 - \frac{19}{12}\ [1.2\ (0.315) + 1.6\ (0.235)] = 10.1 \text{ kips}$$

2. Determine horizontal shear strength. *17.5.2*

$V_u \leq \phi V_{nh}$ *Eq. (17-1)*

$\phi V_{nh} = \phi\ (80 b_v d)$ *17.5.2.1 & 17.5.2.2*

$= 0.75\ (80 \times 10 \times 19)/1000 = 11.4$ kips

$V_u = 10.1$ kips $\leq \phi V_{nh} = 11.4$ kips

Therefore, design in accordance with either 17.5.2.1 or 17.5.2.2:

Note: For either condition, top surface of precast beam must be cleaned and free of laitance prior to placing slab concrete.

If top surface of precast beam is intentionally roughened, no ties are required. *17.5.2.1*

If top surface of precast beam is not intentionally roughened, minimum ties are required in accordance with 17.6. *17.5.2.2*

3. Determine required minimum area of ties. *17.6*

$$A_v = \frac{0.75\sqrt{f'_c}\,b_w d}{f_y} \geq \frac{50 b_w s}{f_y}$$ *17.6.1, 11.5.5.3*

where s (max) = 4 (3.5) = 14 in. < 24 in. *17.6.1*

$$A_v = \frac{0.75\sqrt{3{,}000}(10)(14)}{60{,}000} = 0.096 \text{ in.}^2 \text{ at 14 in. o.c.}$$

$$\text{Min. } A_v = \frac{50 \times 10 \times 14}{60{,}000} = 0.117 \text{ in.}^2 \text{ at 14 in. o.c.}$$

or 0.10 in.^{2}/ft

Case II: Service dead load = 315 lb/ft
Service live load = 1000 lb/ft

1. Determine factored shear force V_u at span end.

$V_u = 1.2\ (0.315)\ (15) + 1.6\ (1.0)\ (15) = 29.7$ kips

At a distance d from face of support *11.1.3.1*

$$V_u = 29.7 - \frac{19}{12}\,[1.2\ (0.315) + 1.6\ (1.0)] = 26.6 \text{ kips}$$

2. Determine horizontal shear strength. *17.5.2*

$V_u = 26.6$ kips $> \phi V_{nh} = \phi\ (80 b_v d) = 11.4$ kips

Therefore, 17.5.2.3 must be satisfied. Minimum ties are required as computed above ($A_v = 0.10$ in.^{2}/ft).

$$V_{nh} = \phi\ (260 + 0.6\rho_v f_y)\ \lambda b_v d$$ *17.5.2.3*

where $\rho_v = \frac{A_v}{b_v s} = \frac{0.10 \text{ in.}^2}{10 \text{ in. } (12 \text{ in.})}$

$= 0.00083$

$\lambda = 1.0$ (normal weight concrete) *11.7.4.3*

$\phi V_{nh} = 0.75 (260 + 0.6 (0.00083) 60{,}000) (1.0 \times 10 \times 19) = 0.75 (290) 190 = 41.3$ kips

$\phi V_{nh} = 41.3 \text{ kips} < \phi (500 b_v d)/1000 = 71.3 \text{ kips}$ O.K. *17.5.2.3*

$V_u = 26.6 \text{ kips} < \phi V_{nh} = 41.3 \text{ kips}$

Therefore, design in accordance with 17.5.2.3:

Contact surface must be intentionally roughened to "a full amplitude of approximately 1/4-in.," and minimum ties provided in accordance with 17.6.

3. Compare tie requirements with required vertical shear reinforcement at span end.

$V_u = 26.6$ kips

$V_c = 2\sqrt{f'_c}\, b_w d = 2\sqrt{3000} \times 10 \times 19/1000 = 20.8$ kips *Eq. (11-3)*

$V_u \le \phi (V_c + V_s) = \phi V_c + \phi A_v f_y \frac{d}{s}$ *Eq. (11-15)*

Solving for A_v/s:

$$\frac{A_v}{s} = \frac{V_u - \phi V_c}{\phi f_y d} = \frac{26.6 - (0.75) \times (20.8)}{0.75 \times 60 \times 19} = 0.013 \text{ in.}^2/\text{in.}$$

$s_{max} = \frac{19}{2} = 9.5 \text{ in.} < 24 \text{ in.}$ *11.5.4.1*

$A_v = 0.013 \times 9.5 = 0.12 \text{ in.}^2$

Provide No. 3 U-stirrups @ 9.5 in. o.c. ($A_v = 0.28$ in.^{2}/ft). This exceeds the minimum ties required for horizontal shear ($A_v = 0.10$ in.^{2}/ft) so the No. 3 U-stirrups @ 9.5 in. o.c. are adequate to satisfy both vertical and horizontal shear reinforcement requirements. Ties must be adequately anchored into the slab by embedment or hooks. See Fig. 12-9.

Case III: Service dead load = 315 lb/ft
Service live load = 3370 lb/ft

1. Determine factored shear force V_u at span end.

$V_u = 1.2\ (0.315)\ (15) + 1.6\ (3.37)\ (15) = 86.6$ kips

At distance d from support

$$V_u = 86.6 - \frac{19}{12}\ [1.2\ (0.315) + 1.6\ (3.37)] = 77.5 \text{ kips}$$

$V_u = 77.5$ kips $> \phi\ (500 b_v d) = 0.75\ (500 \times 10 \times 19)/1000 = 71.3$ kips *17.5.2.4*

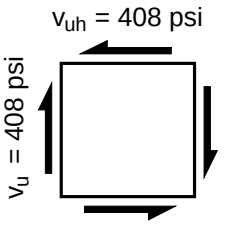
Since V_u exceeds $\phi\ (500 b_v d)$, design for horizontal shear must be in accordance with 11.7.4 - Shear-Friction. Shear along the contact surface between beam and slab is resisted by shear-friction reinforcement across and perpendicular to the contact surface.

As required by 17.5.3.1, a varied tie spacing must be used, based on the actual shape of the horizontal shear distribution. The following method seems reasonable and has been used in the past:

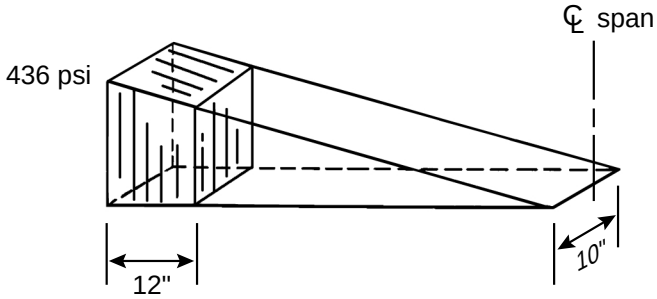
Converting the factored shear force to a unit stress, the factored horizontal shear stress at a distance d from span end is:

$$v_{uh} = \frac{V_u}{b_v d} = \frac{77.5}{10 \times 19} = 0.408 \text{ ksi}$$

The shear "stress block" diagram may be shown as follows:



State of stress at span end (point A)



Assume that the horizontal shear is uniform per foot of length, then the shear transfer force for the first foot is:

$V_{uh} = 0.408 \times 10 \times 12 = 49.0$ kips

Required area of shear-friction reinforcement is computed by Eqs. (11-1) and (11-25):

$$V_{uh} \le \phi V_n = \phi A_{vf} f_y \mu$$ *Eq. (11-25)*

$$A_{vf} = \frac{V_{uh}}{\phi f_y \mu}$$

If top surface of precast beam is intentionally roughened to approximately 1/4 in., $\mu = 1.0$. *11.7.4.3*

$$A_{vf} = \frac{49.0}{0.75 \times 60 \times 1.0} = 1.09 \text{ in.}^2 / \text{ft}$$

With No. 5 double leg stirrups, $A_{vf} = 0.62 \text{ in.}^2$

$$s = \frac{0.62 \times 12}{1.09} = 6.8 \text{ in.}$$

Use No. 5 U-stirrups @ 6.5 in. o.c. for a minimum distance of d + 12 in. from span end.

If top surface of precast beam is not intentionally roughened, $\mu = 0.6$. *11.7.4.3*

$$A_{vf} = \frac{49.0}{0.75 \times 60 \times 0.6} = 1.81 \text{ in.}^2 / \text{ft}$$

$$s = \frac{0.62 \times 12}{1.81} = 4.1 \text{ in.}$$

Use No. 5 U-Stirrups @ 4 in. o.c. for a minimum distance of d + 12 in. from span end.

This method can be used to determine the tie spacing for each successive one-foot length. The shear force will vary at each one-foot increment and the tie spacing can vary accordingly to a maximum of 14 in. toward the center of the span.

Note: Final tie details are governed by vertical shear requirements.

13

Torsion

UPDATE FOR THE '02 CODE

In the 2002 edition of the code, new equations are provided in 11.6.1 and 11.6.2 to determine the threshold torsion and the factored torsional moment, respectively, for nonprestressed members subjected to an axial tensile or compressive force. Also in 11.6.1, the minimum torque limits of the 1999 code were modified to apply to hollow sections.

The mimimum area of transverse closed stirrups prescribed in 11.6.5.2 has also been modified to account for high-strength concrete. This modification is consistent with modified provisions of 11.5.5.3 for shear.

BACKGROUND

Design provisions for torsion were first introduced in the 1971 code. With the exception of a change in format in the 1977 document, the requirements have remained essentially unchanged through the 1989 code. These first generation provisions applied only to reinforced, nonprestressed concrete members. The design procedure for torsion was analogous to that for shear. Torsional strength consisted of a contribution from concrete (T_c) and a contribution from stirrups and longitudinal reinforcement, based on a 45 deg truss analogy.

The design provisions for torsion were completely revised in the 1995 code. The new procedure, for solid and hollow members, is based on a thin-walled tube, space truss analogy. This unified approach applies equally to reinforced and prestressed concrete members. Background of the torsion provisions has been summarized by MacGregor and Ghoneim.[13.1] Design aids and design examples for structural concrete members subject to torsion are presented in Ref. 13.2.

For design purposes, the center portion of a solid beam can conservatively be neglected. This assumption is supported by test results reported in Ref. 13.1. Therefore, the beam is idealized as a tube. Torsion is resisted through a constant shear flow q (force per unit length of wall centerline) acting around the centerline of the tube as shown in Fig. 13-1(a). From equilibrium of external torque T and internal stresses:

$$T = 2A_o q = 2A_o \tau t \qquad (1)$$

Rearranging Eq. (1)

$$q = \tau t = \frac{T}{2A_o} \qquad (2)$$

where τ = shear stress, assumed uniform, across wall thickness

t = wall thickness

T = applied torque

A_o = area enclosed within the tube centerline [see Fig. 13-1(b)]

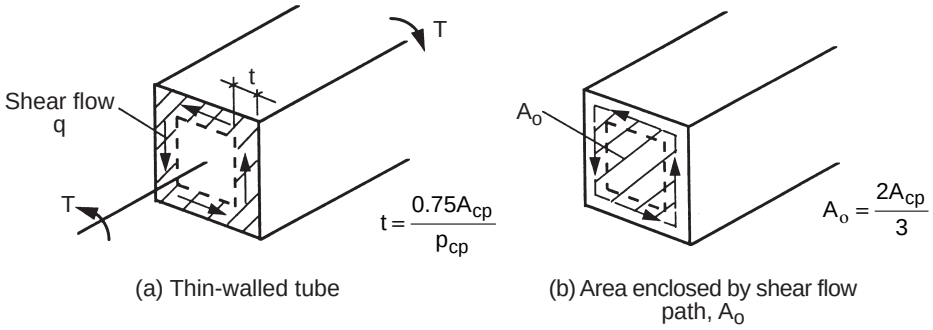


Figure 13-1 Thin-Wall Tube Analogy

When a concrete beam is subjected to a torsional moment causing principal tension larger than $4\sqrt{f'_c}$, diagonal cracks spiral around the beam. After cracking, the tube is idealized as a space truss as shown in Fig. 13-2. In this truss, diagonal members are inclined at an angle θ. Inclination of the diagonals in all tube walls is the same. Note that this angle is not necessarily 45 deg. The resultant of the shear flow in each tube wall induces forces in the truss members. A basic concept for structural concrete design is that concrete is strong in compression, while steel is strong in tension. Therefore, in the truss analogy, truss members that are in tension consist of steel reinforcement or "tension ties." Truss diagonals and other members that are in compression consist of concrete "compression struts." Forces in the truss members can be determined from equilibrium conditions. These forces are used to proportion and detail the reinforcement.

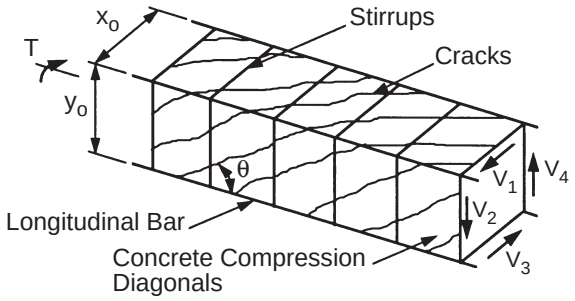


Figure 13-2 Space Truss Analogy

Figure 13-3 depicts a free body extracted from the front vertical wall of the truss of Fig. 13-2. Shear force V_2 is equal to the shear flow q (force per unit length) times the height of the wall y_o. Stirrups are designed to yield when the maximum torque is reached. The number of stirrups intersected is a function of the stirrup spacing s and the horizontal projection $y_o \cot\theta$ of the inclined surface. From vertical equilibrium:

$$V_2 = \frac{A_t f_{yv}}{s} \cot\theta \qquad (3)$$

As the shear flow (force per unit length) is constant over the height of the wall,

$$V_2 = q y_o = \frac{T}{2A_o} y_o \qquad (4)$$

Substituting for V_2 in Eqs. (3) and (4),

$$T = \frac{2A_oA_tf_{yv}}{s}\cot\theta \qquad (5)$$

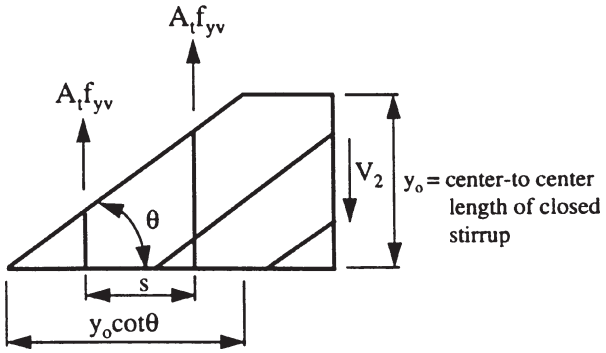


Figure 13-3 Free Body Diagram for Vertical Equilibrium

A free body diagram for horizontal equilibrium is shown in Fig. 13-4. The vertical shear force V_i in wall "i" is equal to the product of the shear flow q times the length of the wall y_i. Vector V_i can be resolved into two components: a diagonal component with an inclination θ equal to the angle of the truss diagonals, and a horizontal component equal to:

$$N_i = V_i\cot\theta$$

Force N_i is centered at the midheight of the wall since q is constant along the side of the element. Top and bottom chords of the free body of Fig. 13-4 are subject to a force $N_i/2$ each. Internally, it is assumed that the longitudinal steel yields when the maximum torque is reached. Summing the internal and external forces in the chords of all the space truss walls results in:

$$\Sigma A_{\ell i}f_{yl} = A_\ell f_{y\ell} = \Sigma N_i = \Sigma V_i\cot\theta = \Sigma q y_i\cot\theta = \Sigma\frac{T}{2A_o}y_i\cot\theta = \frac{T}{2A_o}\cot\theta\Sigma y_i$$

where $A_\ell f_{y\ell}$ is the yield force in all longitudinal reinforcement required for torsion.

Rearranging the above equation,

$$T = \frac{2A_oA_\ell f_{y\ell}}{2\,(x_o + y_o)\cot\theta} \qquad (6)$$

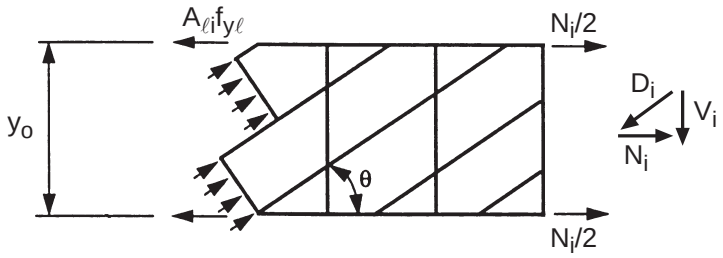


Figure 13-4 Free Body Diagram for Horizontal Equilibrium

11.6.1 Threshold Torsion

Torsion can be neglected if the factored torque T_u is less than $\phi T_{cr}/4$, where T_{cr} is the cracking torque. The cracking torque corresponds to a principal tensile stress of $4\sqrt{f'_c}$. Prior to cracking, thickness of the tube wall "t" and the area enclosed by the wall centerline "A_o" are related to the uncracked section geometry based on the following assumptions:

$$t = \frac{3A_{cp}}{4p_{cp}} \tag{7}$$

$$A_o = \frac{2A_{cp}}{3} \quad \text{(before cracking)} \tag{8}$$

where A_{cp} = area enclosed by outside perimeter of concrete cross-section, in.^{2}

p_{cp} = outside perimeter of concrete cross-section, in.

A_o = area within centerline of the thin-wall tube, in.^{2}

Equations (7) and (8) apply to the uncracked section. For spandrel beams and other members cast monolithically with a slab, parts of the slab overhangs contribute to torsional resistance. Size of effective portion of slab to be considered with the beam is illustrated in Fig. R13.2.4.

Substituting for t from Eq. (7), A_o from Eq. (8), and taking $\tau = 4\sqrt{f'_c}$ in Eq. (1), the cracking torque for nonprestressed members can be derived:

$$T_{cr} = 4\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right) \tag{9}$$

For prestressed concrete members, based on a Mohr's Circle analysis, the principal tensile stress of $4\sqrt{f'_c}$ is reached at $\sqrt{1 + \frac{f_{pc}}{4\sqrt{f'_c}}}$ times the corresponding torque for nonprestressed members. Therefore, the cracking torque for prestressed concrete members is computed as:

$$T_{cr} = 4\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1 + \frac{f_{pc}}{4\sqrt{f'_c}}} \tag{10}$$

where f_{pc} = compressive stress in concrete, due to prestress, at centroid of section (also see 11.0)

Similarly, for nonprestressed members subjected to an applied axial force, the principal tensile stress of $4\sqrt{f'_c}$ is reached at $\sqrt{1+\frac{N_u}{4A_g\sqrt{f'_c}}}$ times the corresponding torque, so that the cracking torque is:

$$T_{cr} = 4\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\sqrt{f'_c}}} \tag{11}$$

where N_u = factored axial force normal to the cross-section (positive for compression)

A_g = gross area of section. For a hollow section, A_g is the area of the concrete only and does not include the area of the void(s) (see 11.6.1).

According to 11.6.1, design for torsion can be neglected if $T_u < \frac{\phi T_{cr}}{4}$, i.e.:

For nonprestressed members:

$$T_u < \phi\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right) \tag{12}$$

For prestressed members:

$$T_u < \phi\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\sqrt{f'_c}}} \tag{13}$$

For nonprestressed members subjected to an axial tensile or compressive force:

$$T_u < \phi\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\sqrt{f'_c}}} \tag{14}$$

It is important to note that A_g is to be used in place of A_{cp} in Eqs. (12) through (14) for hollow sections, where for torsion, a hollow section is defined as having one or more longitudinal voids such that $A_g/A_{cp} < 0.95$ (see R11.6.1). The quantity A_g in this case is the area of the concrete only (i.e., the area of the void(s) are not included), based on the outer boundaries prescribed in 13.2.4. The provisions of 11.6.1 were modified in the 2002 code to apply to hollow sections, since the tests of code Ref. 11.29 indicate that the cracking torque of a hollow section is approximately (A_g/A_{cp}) times the cracking torque of a solid section with the same outside dimensions.

11.6.2 Equilibrium and Compatibility - Factored Torsional Moment T_u

Whether a reinforced concrete member is subject to torsion only, or to flexure combined with shear, the stiffness of that member will decrease after cracking. The reduction in torsional stiffness after cracking is much larger than the reduction in flexural stiffness after cracking. If the torsional moment T_u in a member cannot be reduced by redistribution of internal forces in the structure, that member must be designed for the full torsional moment T_u (11.6.2.1). This is referred to as "equilibrium torsion." See Fig. R11.6.2.1. If redistribution of internal forces can occur, as in indeterminate structures, the design torque can be reduced. This type of torque is referred to as "compatibility torsion." See Fig. R11.6.2.2. Members subject to compatibility torsion need not be designed for a torque larger than the product of the cracking torque times the strength reduction factor ϕ (0.75 for torsion, see 9.3.2.3). For cases of compatibility torsion where $T_u > \phi T_{cr}$ the member can be designed for ϕT_{cr} only, provided redistribution of internal forces is accounted for in the design of the other members of the structure (11.6.2.2). Cracking torque T_{cr} is computed by Eq. (9) for nonprestressed members, by Eq. (10) for prestressed members, and by Eq. (11) for nonprestressed members subjected to an axial tensile or compressive force. For hollow sections, A_{cp} shall not be replaced with A_g in these equations (11.6.2.2). Multiplying the cracking torque by (A_g/A_{cp}) a second time reflects the transition from the circular interaction between the inclined cracking loads in shear and torsion for solid members, to the approximately linear interaction for thin-walled hollow sections.

11.6.2.4-11.6.2.5 Critical Section—In nonprestressed members, the critical section for torsion design is at distance "d" (effective depth) from the face of support. Sections located at a distance less than d from the face of support must be designed for the torque at distance d from the support. Where a cross beam frames into a girder at a distance less than d from the support, a concentrated torque occurs in the girder within distance d. In such cases, the design torque

must be taken at the face of support. The same rule applies to prestressed members, except that h/2 replaces distance d, where h is the overall height of member. In composite members, h is the overall height of the composite section.

11.6.3 Torsional Moment Strength

The design torsional strength should be equal to or greater than the required torsional strength:

$$\phi T_n \geq T_u \qquad \textit{Eq. (11-20)}$$

The nominal torsional moment strength in terms of stirrup yield strength was derived above (see Eq.(5)):

$$T_n = \frac{2A_o A_t f_{yv}}{s} \cot\theta \qquad \textit{Eq. (11-21)}$$

where $A_o = 0.85A_{oh}$ (this is an assumption for simplicity)

A_{oh} = area enclosed by centerline of the outermost closed transverse torsional reinforcement as illustrated in Fig. 13-5

θ = angle of compression diagonal, ranges between 30 and 60 deg. It is suggested in 11.6.3.6 to use 45 deg for nonprestressed members and 37.5 deg for prestressed members with prestress force greater than 40 percent of tensile strength of the longitudinal reinforcement.

Note that the definition of A_o used in Eq. (8) was for the uncracked section. Also note that nominal torsional strength T_n is reached after cracking and after the concrete member has undergone considerable twisting rotation. Under these large deformations, part of the concrete cover may have spalled. For this reason, when computing area A_o corresponding to T_n, the concrete cover is ignored. Thus, parameter A_o is related to A_{oh}, the area enclosed by centerline of the outermost closed transverse torsional reinforcement. Area A_o can be determined through rigorous analysis (Ref. 13.3) or simply assumed equal to $0.85A_{oh}$. See 11.6.3.6.

Substituting for T from Eq. (5) into Eq. (6) and replacing $2(x_o + y_o)$ with p_h (perimeter of centerline of outermost closed transverse torsional reinforcement), the longitudinal reinforcement required to resist torsion is computed as a function of the transverse reinforcement:

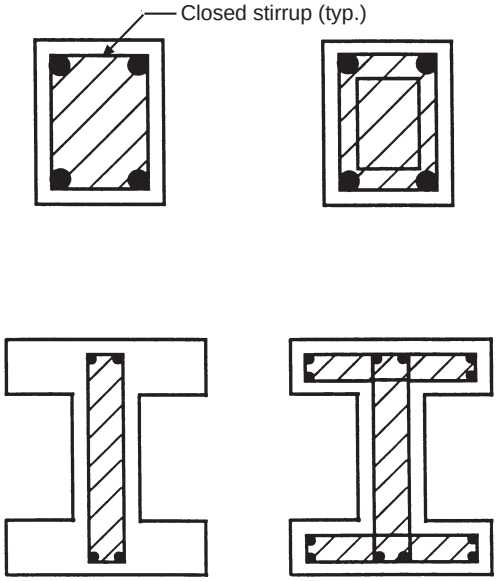


Figure 13-5 Definition of A_{oh}

$$A_\ell = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yv}}{f_{y\ell}}\right) \cot^2\theta \qquad \textit{Eq. (11-22)}$$

Note that term (A_t/s) used in Eq. (11-22) is that due to torsion only, and is computed from Eq. (11-21). In members subject to torsion combined with shear, flexure or axial force, the amount of longitudinal and transverse reinforcement required to resist all actions must be determined using the principle of superposition. See 11.6.3.8 and R11.6.3.8. In members subject to flexure, area of longitudinal torsion reinforcement in the flexural compression zone may be reduced to account for the compression due to flexure (11.6.3.9). In prestressed members, the longitudinal reinforcement required for torsion may consist of tendons with a tensile strength $A_{ps}f_{ps}$ equivalent to the yield force of mild reinforcement, $A_\ell f_{y\ell}$,computed by Eq. (11-22).

To reduce unsightly cracking and prevent crushing of the concrete compression struts, 11.6.3.1 prescribes an upper limit for the maximum stress due to shear and torsion, analogous to that due to shear only. In solid sections, stresses due to shear act over the full width of the section, while stresses due to torsion are assumed resisted by a thin-walled tube. See Fig. R11.6.3.1(b). Thus, 11.6.3.1 specifies an elliptical interaction between stresses due to shear and those due to torsion for solid sections as follows:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \qquad \textit{Eq. (11-18)}$$

For hollow sections, the stresses due to shear and torsion are directly additive on one side wall [see Fig. R11.6.3.1(a)]. Thus, the following linear interaction is specified:

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right) \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \qquad \textit{Eq. (11-19)}$$

In Eqs. (11-18) and (11-19), V_c is the contribution of concrete to shear strength of nonprestressed (see 11.3) or prestressed (see 11.4) concrete members.

When applying Eq. (11-19) to a hollow section, if the actual wall thickness t is less than A_{oh}/p_h, the actual wall thickness should be used instead of A_{oh}/p_h (11.6.3.3).

11.6.4 Details of Torsional Reinforcement

Longitudinal and transverse reinforcement are required to resist torsion. Longitudinal reinforcement may consist of mild reinforcement or prestressing tendons. Transverse reinforcement may consist of stirrups, welded wire fabric, or spiral reinforcement. To control widths of diagonal cracks, the design yield strength of nonprestressed reinforcement must not exceed 60,000 psi (11.6.3.4).

In the truss analogy illustrated in Fig. 13-2, the diagonal compression strut forces bear against the longitudinal corner reinforcement. In each wall, the component of the diagonal struts, perpendicular to the longitudinal reinforcement is transferred from the longitudinal reinforcement to the transverse reinforcement. It has been observed in torsional tests of beams loaded to destruction that as the maximum torque is reached, the concrete cover spalls.[13.3] The forces in the compression struts outside the stirrups, i.e. within the concrete cover, push out the concrete shell. Based on this observation, 11.6.4.1 specifies that the stirrups should be closed, with 135 deg hooks. Stirrups with 90 deg hooks become ineffective when the concrete cover spalls. Similarly, lapped U-shaped stirrups have been found to be inadequate for resisting torsion again due to lack of support when the concrete cover spalls. For hollow sections, the distance from the centerline of the transverse torsional reinforcement to the inside face of the wall of the hollow section must not be less than $0.5A_{oh}/p_h$ (11.6.4.4).

11.6.5 Minimum Torsion Reinforcement

In general, to ensure ductility of reinforced and prestressed concrete members, minimum reinforcement is specified for flexure (10.5) and for shear (11.5.5). Similarly, minimum transverse and longitudinal reinforcement is specified in 11.6.5 whenever $T_u > \phi T_{cr}/4$. Usually, a member subject to torsion will also be simultaneously subjected to shear. The minimum area of stirrups for shear and torsion is computed from:

$$\left(A_v + 2A_t\right) = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_y} \geq \frac{50 b_w s}{f_{yv}} \qquad \textit{Eq. (11-23)}$$

which now accounts for higher strength concretes (see 11.5.5.3 and R11.6.5.3).

The minimum area of longitudinal reinforcement is computed from:

$$A_{\ell,\min} = \frac{5\sqrt{f'_c}A_{cp}}{f_{y\ell}} - \left(\frac{A_t}{s}\right) p_h \frac{f_{yv}}{f_{y\ell}} \qquad \textit{Eq. (11-24)}$$

but A_t/s (due to torsion only) must not be taken less than $25b_w/f_{yv}$.

11.6.6 Spacing of Torsion Reinforcement

Spacing of stirrups must not exceed the smaller of $p_h/8$ and 12 in. For a square beam subject to torsion, this maximum spacing is analogous to a spacing of about d/2 in a beam subject to shear.

The longitudinal reinforcement required for torsion must be distributed around the perimeter of the closed stirrups, at a maximum spacing of 12 in. In the truss analogy, the compression struts push against the longitudinal reinforcement which transfers the transverse forces to the stirrups. Thus, the longitudinal bars should be inside the stirrups. There should be at least one longitudinal bar or tendon in each corner of the stirrups to help transmit the forces from the compression struts to the transverse reinforcement. To avoid buckling of the longitudinal reinforcement due to the transverse component of the compression struts, the longitudinal reinforcement must have a diameter not less than 1/24 of the stirrup spacing, but not less than a No. 3 bar (11.6.6.2).

REFERENCES

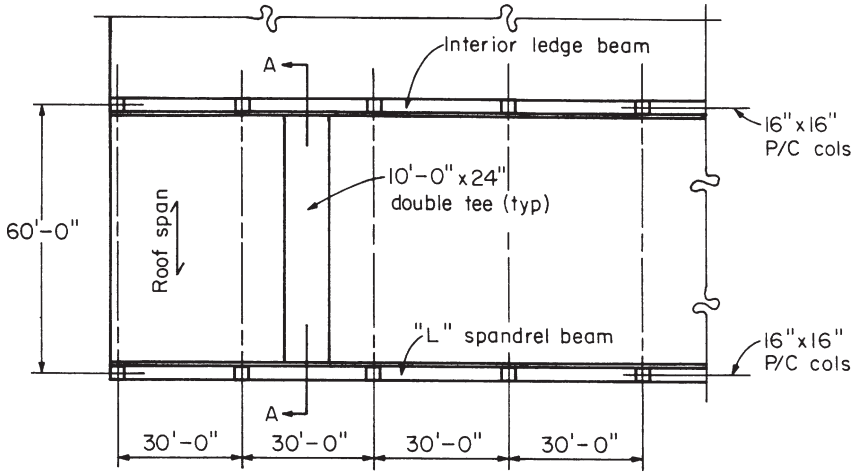
13.1 MacGregor, J.G., and Ghoneim, M.G., "Design for Torsion," *ACI Structural Journal*, March-April 1995, pp. 211-218.

13.2 Fanella, D. A. and Rabbat, B.G., *Design of Concrete Beams for Torsion*, Engineering Bulletin EB106.02D, Portland Cement Association, Skokie, IL 1997.

13.3 Collins, M.P. and Mitchell, D., *Prestressed Concrete Structures*, Prentice Hall, Englewood Cliffs, NJ, 1991, 766 pp.

Example 13.1—Precast Spandrel Beam Design for Combined Shear and Torsion

Design a precast, nonprestressed concrete spandrel beam for combined shear and torsion. Roof members are simply supported on spandrel ledge. Spandrel beams are connected to columns to transfer torsion. Continuity between spandrel beams is not provided.



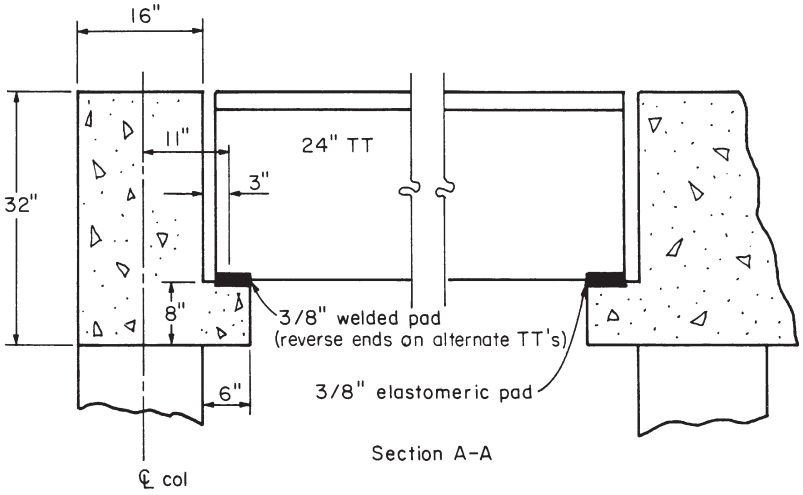
Partial plan of precast roof system

Design Criteria:
Live load = 30 lb/ft^2
Dead load = 64 lb/ft^2 (double tee + insulation + roofing)
f'_c = 5000 psi (w_c = 150 pcf)
f_y = 60,000 psi

Roof members are 10 ft wide double tee units, 24 in. deep. Design of these units is not included in this design example. For lateral support, alternate ends of roof members are fixed to supporting beams.

Calculations and Discussion	Code Reference

1. Assume double tee loading on spandrel beam as uniform. Calculate factored loading M_u, V_u, T_u for spandrel beam.



Dead load:

Superimposed	= (0.064) (60)/2	= 1.92
Spandrel	= [(1.33) (2.67) + (0.5) (0.67)] 0.150	= 0.58
	Total	= 2.50 kips/ft

Live load = (0.030) (60)/2 = 0.90 kips/ft

Factored load = (1.2) (2.50) + (1.6) (0.90) = 4.4 kips/ft *9.2.1*

At center of span, $M_u = \frac{4.4 \times 30^2}{8} = 495$ ft-kips

End shear V_u = (4.4) (30)/2 = 66.0 kips

Torsional factored load = 1.2 (1.92) + 1.6 (0.90) = 3.7 kips/ft

Eccentricity of double tee reactions relative to centerline of spandrel beam = 8 + 3 = 11 in.

End torsional moment $T_u = 3.7 \left(\frac{30}{2}\right)\left(\frac{11}{12}\right) = 50.9$ ft-kips

Critical section for torsion is at distance "d" from face of support. *11.6.2.4*

Assume d = 29.5 in.

Critical section for shear is also at distance "d" from face of support. *11.1.3.1*

Therefore, critical section is at 29.5 + 8 = 37.5 in. from column centerline.

At critical section: [15.0 - (37.5/12) = 11.88 ft from midspan]

V_u = 66.0 (11.88/15) = 52.3 kips

T_u = 50.9 (11.88/15) = 40.3 ft-kips

The spandrel beam must be designed for the full factored torsional moment since it is required to maintain equilibrium. *11.6.2.1*

2. Check if torsion may be neglected *11.6.1*

Torsion may be neglected if $T_u < \frac{\phi T_{cr}}{4}$

ϕ = 0.75 *9.3.2.3*

$T_{cr} = 4\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)$ *Eq. (9)*

A_{cp} = area enclosed by outside perimeter of spandrel beam, including the ledge
= (16) (32) + (6) (8) = 512 + 48 = 560 in.^{2}

p_{cp} = outside perimeter of spandrel beam
= 2 (16 + 32) + 2 (6) = 108 in.

The limiting value to ignore torsion is:

$$\phi\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right) = 0.75\sqrt{5000}\left(\frac{560^2}{108}\right)\frac{1}{12{,}000} = 12.8 \text{ ft-kips} < 40.3 \text{ ft-kips} \qquad \textit{Eq.(12)}$$

Torsion must be considered.

3. Determine required area of stirrups for torsion

Design torsional strength must be equal to or greater than the required torsional strength:

$$\phi T_n \geq T_u \qquad \textit{Eq. (11-20)}$$

where

$$T_n = \frac{2A_oA_tf_{yv}}{s}\cot\theta \qquad \textit{Eq. (11-21)}$$

$A_o = 0.85A_{oh}$

A_{oh} = area enclosed by centerline of the outermost closed transverse torsional reinforcement

Assuming 1.25 in. cover (precast concrete exposed to weather) and No. 4 stirrup *7.7.3(a)*

A_{oh} = (13) (29) + (6) (5) = 407 in.^{2}

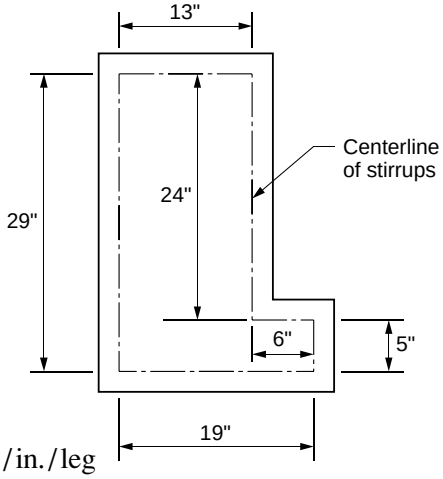
A_o = 0.85 (407) = 346 in.^{2}

For nonprestressed member, use θ = 45 deg *11.6.3.6(a)*

Substituting in Eqs. (11-20) and (11-21)

$$\frac{A_t}{s} = \frac{T_u}{2\phi A_o f_{yv}\cot\theta}$$

$$\frac{A_t}{s} = \frac{(40.3)\ (12{,}000)}{2\ (0.75)\ (346)\ (60{,}000)\ (1.0)} = 0.016 \text{ in.}^2/\text{in./leg}$$



4. Calculate required area of stirrups for shear

$$V_c = 2\sqrt{f'_c}\, b_w d \qquad \text{Eq. (11-3)}$$

$$= 2\sqrt{5000}\,(16)\,(29.5)/1000$$

$$= 66.8 \text{ kips}$$

From Eqs. (11-1) and (11-2)

$$V_s = \frac{V_u}{\phi} - V_c = \frac{52.3}{0.75} - 66.8 = 2.9 \text{ kips}$$

$$\frac{A_v}{s} = \frac{V_s}{f_{yv} d} = \frac{2.9}{60\,(29.5)} = 0.002 \text{ in.}^2/\text{in./leg}$$

5. Determine combined shear and torsion stirrup requirements 11.6.3.8

$$\frac{A_t}{s} + \frac{A_v}{2s} = 0.016 + \frac{0.002}{2} = 0.017 \text{ in.}^2/\text{in./leg}$$

Try No. 3 bar, $A_b = 0.11$ in.^{2}

$$s = \frac{0.11}{0.017} = 6.5 \text{ in.} \quad \text{Use 6 in. minimum spacing.}$$

6. Check maximum stirrup spacing

For torsion spacing must not exceed $p_h/8$ or 12 in.: 11.6.6

$$p_h = 2\,(13 + 29) + 2\,(6) = 96 \text{ in.}$$

$$\frac{p_h}{8} = \frac{96}{8} = 12 \text{ in.}$$

For shear, spacing must not exceed d/2 or 24 in. $\left(V_s = 2.9 \text{ kips} < 4\sqrt{f'_c} b_w d = 133.6 \text{ kips}\right)$: 11.5.4.1, 11.5.4.3

$$\frac{d}{2} = \frac{29.5}{2} = 14.75 \text{ in.}$$

Use 6 in. minimum and 12 in. maximum spacing.

7. Check minimum stirrup area

$$\left(A_v + 2A_t\right) = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_{yv}} = 0.75\sqrt{5{,}000}\,\frac{(16)(12)}{60{,}000} = 0.17 \text{ in.}^2$$

$$> \frac{50 b_w s}{f_{yv}} = \frac{50\,(16)\,(12)}{60{,}000} = 0.16 \text{ in.}^2 \qquad \textit{Eq. (11-23)}$$

Area provided = 2 (0.11) = 0.22 in.^{2} > 0.17 in.^{2} O.K.

8. Determine stirrup layout

Since both shear and torsion are zero at the center of span, and are assumed to vary linearly to the maximum value at the critical section, the start of maximum stirrup spacing can be determined by simple proportion.

$$\frac{s \text{ (critical)}}{s \text{ (maximum)}}\,(11.88) = \frac{6}{12}\,(11.88) = 5.94 \text{ ft, say 6 ft from midspan.}$$

9. Check for crushing of the concrete compression struts *11.6.3.1*

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \qquad \textit{Eq. (11-18)}$$

$$\sqrt{\left(\frac{52{,}300}{(16)\,(29.5)}\right)^2 + \left(\frac{(40.3 \times 12{,}000)\,(96)}{1.7\,(407)^2}\right)^2} = 199 \text{ psi} < 10\phi\sqrt{f'_c} = 530 \text{ psi O.K.}$$

10. Calculate longitudinal torsion reinforcement *11.6.3.7*

$$A_\ell = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yv}}{f_{y\ell}}\right) \cot^2\theta \qquad \textit{Eq. (11-22)}$$

$$A_\ell = (0.016)\,(96)\left(\frac{60}{60}\right)(1.0) = 1.54 \text{ in.}^2$$

Check minimum area of longitudinal reinforcement

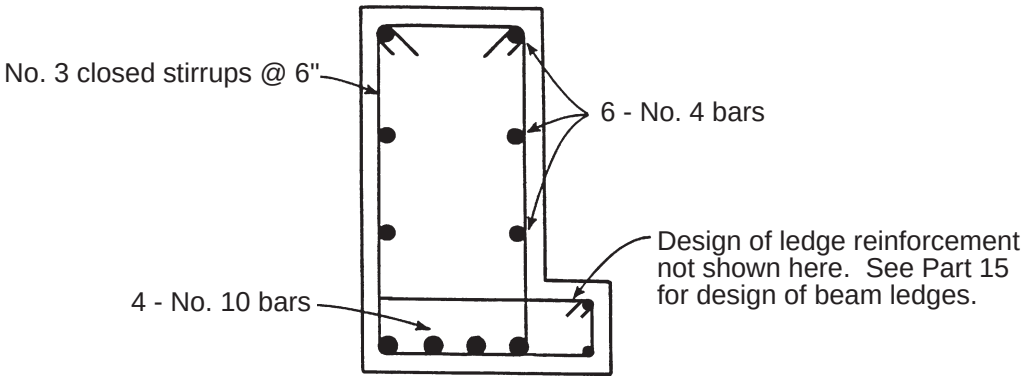
$$A_{\ell,\min} = \frac{5\sqrt{f'_c}\,A_{cp}}{f_{y\ell}} - \left(\frac{A_t}{s}\right) p_h \frac{f_{yv}}{f_{y\ell}} \qquad \textit{Eq. (11-24)}$$

$\left(\frac{A_t}{s}\right)$ must not be less than $\frac{25 b_w}{f_{yv}} = \frac{25\,(16)}{60{,}000} = 0.007$ in.^{2}/in. *11.6.5.3*

$$A_{\ell,min} = \frac{5\sqrt{5000}\,(560)}{60{,}000} - (0.016)\,(96) = 1.76 \text{ in.}^2 \quad \text{(governs)}$$

The longitudinal reinforcement required for torsion must be distributed around the perimeter of the closed stirrups, at a maximum spacing of 12 in. The longitudinal bars should be inside the stirrups. There should be at least one longitudinal bar in each corner of the stirrups. Select 10 bars. 11.6.6.2

Area of each longitudinal bar $= \frac{1.76}{10} = 0.176 \text{ in.}^2$ Use No. 4 bars



Reinforcement Details

11. Size combined longitudinal reinforcement

Use No. 4 bars in sides and top corners of spandrel beam. Note that two of the ten longitudinal bars (bars next to vertical face of ledge) required for torsion are to be combined with the ledge reinforcement. Design of the ledge reinforcement is not shown here. See Part 15 of this document for design of beam ledges.

Determine required flexural reinforcement, assuming tension-controlled behavior.

$\phi = 0.90$ 9.3.2
From Part 7,

$$R_n = \frac{M_u}{\phi b d^2} = \frac{495 \times 12{,}000}{0.9 \times 16 \times 29.5} = 474 \text{ psi}$$

$$\rho = \frac{0.85 f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right)$$

$$= \frac{0.85 \times 5}{60}\left(1 - \sqrt{1 - \frac{2 \times 474}{0.85 \times 5{,}000}}\right) = 0.0084$$

$A_s = \rho bd = 0.0084 \times 16 \times 29.5 = 3.96 \text{ in.}^2$

At midspan, provide (2/10) of the longitudinal torsion reinforcement in addition to the flexural reinforcement.

$\left(\frac{2}{10}\right)(1.76) + 3.96 = 4.31 \text{ in.}^2$

At end of span, provide (2/10) of the longitudinal torsion reinforcement plus at least (1/3) the positive reinforcement for flexure: 12.11.1

$\left(\frac{2}{10}\right)(1.76) + \left(\frac{3.96}{3}\right) = 1.67 \text{ in.}^2$

Use 4-No. 10 bars ($A_s = 5.08 \text{ in.}^2 > 4.31 \text{ in.}^2$)

Check if section is tension-controlled, based on provided reinforcement.

From a strain compatibility analysis, conservatively assuming that the section is subjected to flexure only,

$\varepsilon_t = 0.0117 > 0.0050$

Therefore, section is tension-controlled, and $\phi = 0.90$. 10.3.4

Note that for strain compatability analysis including the effects of torsion, see Ref. 13.3.

Extend 2-No. 10 bars to end of girder ($A_s = 2.54 \text{ in.}^2 > 1.67 \text{ in.}^2$)

Note that the longitudinal torsion reinforcement must be adequately anchored.

12. Check required area of beam stirrups used as "hanger" reinforcement for beam ledge.

Sufficient stirrups in beam section must be available to act also as hanger reinforcement for the beam ledge. See Part 15 for design of beam ledges.

Reaction from one double tee stem (for 10 ft wide double tee, stems are 5 ft on center.)

$R_u = [1.2\,(1.92) + 1.6\,(0.9)]\,5 = 18.7 \text{ kips/stem}$

$A_v \text{ (per effective width of ledge)} = \frac{R_u}{\phi f_{yv}} = \frac{18.7}{0.75\,(60)} = 0.416 \text{ in.}^2/\text{stem}$

(Note that $\phi = 0.75$ for beam ledge reinforcement per Part 15)

The effective width of ledge over which hanger forces can be distributed may be evaluated from Ref. 15.5. For the ledge loading dimensions of this example, $b_e = 26$ in.

$$\frac{A_v}{s} = \frac{0.416}{26} = 0.016 \text{ in.}^2/\text{in.}$$

For No. 3 stirrups, $s_{max} = \frac{0.11}{0.016} = 6.8$ in.

The No. 3 stirrups @ 6 in., computed earlier in Step 6, must be used for the full span length to act also as hanger reinforcement for the beam ledge.

14

Shear Friction

GENERAL CONSIDERATIONS

With the publication of ACI 318-83, 11.7 was completely rewritten to expand the shear-friction concept to include applications (1) where the shear-friction reinforcement is placed at an angle other than 90 degrees to the shear plane, (2) where concrete is cast against concrete not intentionally roughened, and (3) with lightweight concrete. In addition, a performance statement was added to allow "any other shear-transfer design methods" substantiated by tests. It is noteworthy that 11.9 refers to 11.7 for the direct shear-transfer in brackets and corbels; see Part 15.

11.7 SHEAR-FRICTION

The shear-friction concept provides a convenient tool for the design of members for direct shear where it is inappropriate to design for diagonal tension, as in precast connections, and in brackets and corbels. The concept is simple to apply and allows the designer to visualize the structural action within the member or joint. The approach is to assume that a crack has formed at an expected location, as illustrated in Fig. 14-1. As slip begins to occur along the crack, the roughness of the crack surface forces the opposing faces of the crack to separate. This separation is resisted by reinforcement (A_{vf}) across the assumed crack. The tensile force ($A_{vf}f_y$) developed in the reinforcement by this strain induces an equal and opposite normal clamping force, which in turn generates a frictional force ($A_{vf}f_y\mu$) parallel to the crack to resist further slip.

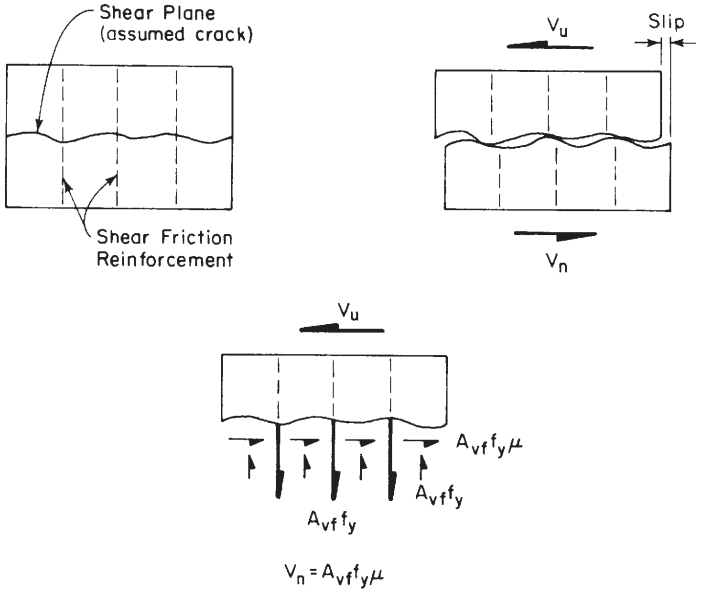


Figure 14-1 Idealization of the Shear-Friction Concept

11.7.1 Applications

Shear-friction design is to be used where direct shear is being transferred across a given plane. Situations where shear-friction design is appropriate include the interface between concretes cast at different times, an interface between concrete and steel, and connections of precast constructions, etc. Example locations of direct shear transfer and potential cracks for application of the shear-friction concept are shown in Fig. 14-2 for several types of members. Successful application of the concept depends on proper selection of location of the assumed slip or crack. In typical end or edge bearing applications, the crack tends to occur at an angle of about 20 degrees to the vertical (see Example 14.2).

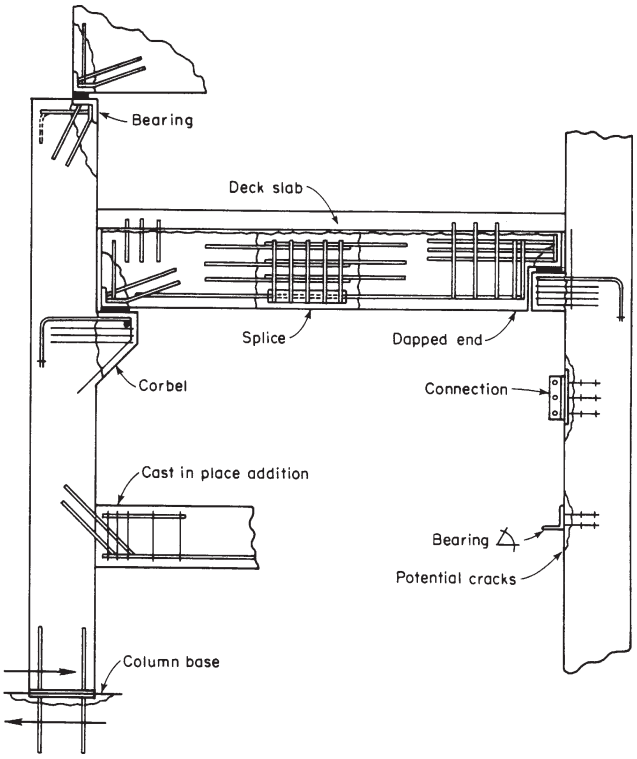


Figure 14-2 Applications of the Shear-Friction Concept and Potential Crack Locations

11.7.3 Shear-Transfer Design Methods

The shear-friction design method presented in 11.7.4 is based on the simplest model of shear-transfer behavior, resulting in a conservative prediction of shear-transfer strength. Other more comprehensive shear-transfer relationships provide closer predictions of shear-transfer strength. The performance statement of 11.7.3 "…any other shear-transfer design methods…" includes the other methods within the scope and intent of 11.7. However, it should be noted that the provisions of 11.7.5 through 11.7.10 apply to whatever shear-transfer method is used. One of the more comprehensive methods is outlined in R11.7.3. Application of the "Modified Shear-Friction Method" is illustrated in Part 15, Example 15.2. The 1992 edition of the code introduced in 17.5.2.3 a modified shear-friction equation. It applies to the interface shear between precast concrete and cast-in-place concrete.

11.7.4 Shear-Friction Design Method

As with the other shear design applications, the code provisions for shear-friction are presented in terms of shear-transfer strength V_n for direct application in the basic shear strength relation:

Required shear-transfer strength $\leq$ Design shear-transfer strength

$$V_u \leq \phi V_n \qquad \textit{Eq. (11-1)}$$

Note that ϕ is 0.75 for shear and torsion (9.3.2.3). Furthermore, it is recommended that $\phi = 0.75$ be used for all design calculations involving shear-friction, where shear effects predominate. For example, 11.9.3.1 specifies the use of $\phi = 0.75$ for all design calculations in accordance with 11.9 (brackets and corbels). The nominal shear strength V_n is computed as:

$$V_n = A_{vf} f_y \mu \qquad \textit{Eq. (11-25)}$$

Combining Eqs. (11-1) and (11-25), the required shear-transfer strength for shear-friction reinforcement perpendicular to the shear plane is:

$$V_u \leq \phi A_{vf} f_y \mu$$

The required area of shear-friction reinforcement, A_{vf}, can be computed directly from:

$$A_{vf} = \frac{V_u}{\phi f_y \mu}$$

The condition where shear-friction reinforcement crosses the shear-plane at an angle α_f other than 90 degrees is illustrated in Fig. 14-3. The tensile force $A_{vf}f_y$ is inclined to the crack and must be resolved into two components: (1) a clamping component $A_{vf}f_y \sin\alpha_f$ with an associated frictional force $A_{vf}f_y \sin\alpha_f \mu$, and (2) a component parallel to the crack that directly resists slip equal to $A_{vf}f_y \cos\alpha_f$. Adding the two components resisting slip, the nominal shear-transfer strength becomes:

$$V_n = A_{vf}f_y \sin\alpha_f \mu + A_{vf}f_y \cos\alpha_f$$

$$= A_{vf}f_y (\mu \sin\alpha_f + \cos\alpha_f)$$

Substituting this into Eq. (11-1):

$$V_u \leq \phi [A_{vf}f_y \sin\alpha_f \mu + A_{vf}f_y \cos\alpha_f] \qquad \textit{Eq. (11-26)}$$

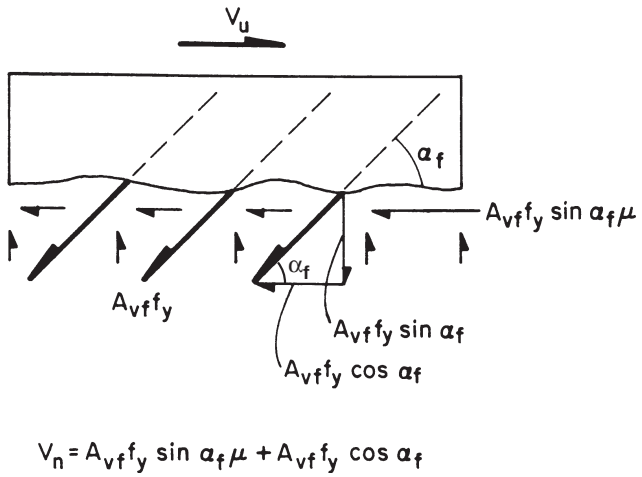


Figure 14-3 Idealization of Inclined Shear-Friction Reinforcement

For shear reinforcement inclined to the crack, the required area of shear-friction reinforcement, A_{vf}, can be computed directly from:

$$A_{vf} = \frac{V_u}{\phi f_y (\mu \sin\alpha_f + \cos\alpha_f)}$$

Note that Eq. (11-26) applies only when the shear force V_u produces tension in the shear-friction reinforcement.

The shear-friction method assumes that all shear resistance is provided by friction between crack faces. The actual mechanics of resistance to direct shear are more complex, since dowel action and the apparent cohesive strength of the concrete both contribute to direct shear strength. It is, therefore, necessary to use artificially high values of the coefficient of friction μ in the direct shear-friction equations so that the calculated shear strength will be in reasonable agreement with test results. Use of these high coefficients gives predicted strengths that are a conservative lower bound to test data, as shown in Fig. 14-4. The modified shear-friction design method given in R11.7.3 is one of several more comprehensive methods which provide closer estimates of the shear-transfer strength.

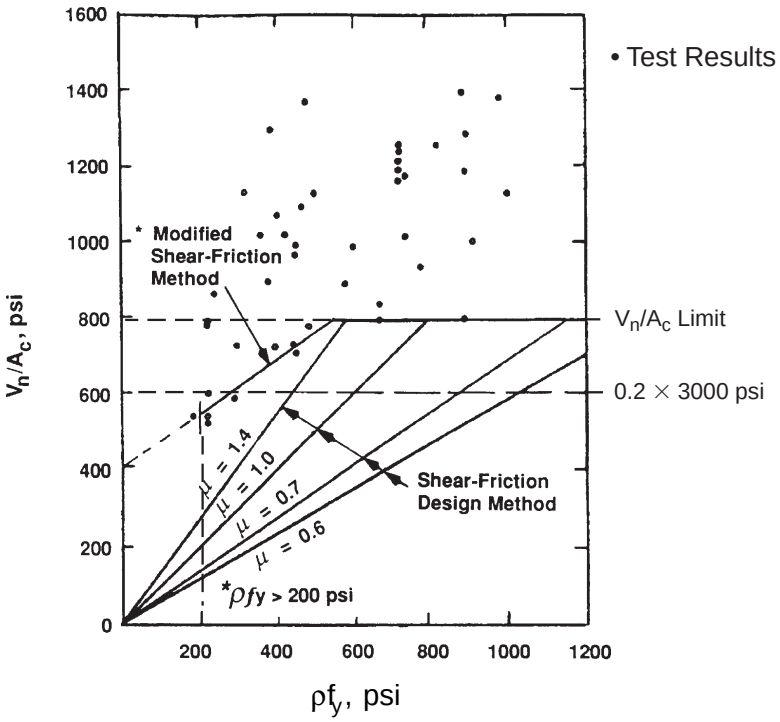


Figure 14-4 Effect of Shear-Friction Reinforcement on Shear Transfer Strength

11.7.4.3 Coefficient of Friction—The "effective" coefficients of friction, μ, for the various interface conditions include a parameter λ which accounts for the somewhat lower shear strength of all-lightweight and sand-lightweight concretes. For example, the μ value for all lightweight concrete (λ = 0.75) placed against hardened concrete not intentionally roughened is 0.6 (0.75) = 0.45. The coefficient of friction for different interface conditions is as follows:

Concrete placed monolithically	1.4λ
Concrete placed against hardened concrete with surface intentionally roughened as specified in 11.7.9	1.0λ
Concrete placed against hardened concrete not intentionally roughened	0.6λ
Concrete anchored to as-rolled structural steel by headed studs or by reinforcing bars (see 11.7.10)	0.7λ

where λ = 1.0 for normal weight concrete, 0.85 for sand-lightweight concrete, and 0.75 for "all light weight" concrete.

11.7.5 Maximum Shear-Transfer Strength

The shear-transfer strength V_n cannot be taken greater than $0.2 f'_c$, nor 800 psi times the area of concrete section resisting shear transfer. This upper limit on V_n effectively limits the maximum reinforcement, as shown in Fig. 14-4. Also, for lightweight concretes, 11.9.3.2.2 limits the shear-transfer strength V_n along the shear plane for design applications with low shear span-to-depth ratios a/d, such as brackets and corbels. This further restriction on lightweight concrete is illustrated in Example 14.1.

11.7.7 Normal Forces

Equations (11-25) and (11-26) assume that there are no forces other than shear acting on the shear plane. A certain amount of moment is almost always present in brackets, corbels, and other connections due to eccentricity of loads or applied moments at connections. In case of moments acting on a shear plane, the flexural tension stresses and flexural compression stresses are in equilibrium. There is no change in the resultant compression $A_{vf}f_y$ acting across the shear plane and the shear-transfer strength is not changed. It is therefore not necessary to provide additional reinforcement to resist the flexural tension stresses, unless the required flexural tension reinforcement exceeds the amount of shear-transfer reinforcement provided in the flexural tension zone.

Joints may also carry a significant amount of tension due to restrained shrinkage or thermal shortening of the connected members. Friction of bearing pads, for example, can cause appreciable tensile forces on a corbel supporting a member subject to shortening. Therefore, it is recommended, although not generally required, that the member be designed for a minimum direct tensile force of at least $0.2V_u$ in addition to the shear. This minimum force is required for design of connections such as brackets or corbels (see 11.9.3.4), unless the actual force is accurately known. Reinforcement must be provided for direct tension according to 11.7.7, using $A_s = N_{uc}/\phi f_y$, where N_{uc} is the factored tensile force.

Since direct tension perpendicular to the assumed crack (shear plane) detracts from the shear-transfer strength, it follows that compression will add to the strength. Section 11.7.7 acknowledges this condition by allowing a "permanent net compression" to be added to the shear-friction clamping force, $A_{vf}f_y$. It is recommended, although not required, to use a reduction factor of 0.9 for strength contribution from such compressive loads.

11.7.8 — 11.7.10 Additional Requirements

Section 11.7.8 requires that the shear-friction reinforcement be "appropriately placed" along the shear plane. Where no moment acts on the shear plane, uniform distribution of the bars is proper. Where a moment exists, the reinforcement should be distributed in the flexural tension zone.

Reinforcement should be adequately embedded on both sides of the shear plane to develop the full yield strength of the bars. Since space is limited in thin walls, corbels, and brackets, it is often necessary to use special anchorage details such as welded plates, angles, or cross bars. Reinforcement should be anchored in confined concrete. Confinement may be provided by beam or column ties, "external" concrete, or special added reinforcement.

In 11.7.9, if coefficient of friction μ is taken equal to 1.0λ, concrete at the interface must be roughened to a full amplitude of approximately 1/4 in. This can be accomplished by raking the plastic concrete or by bushhammering or chiseling hardened concrete surfaces.

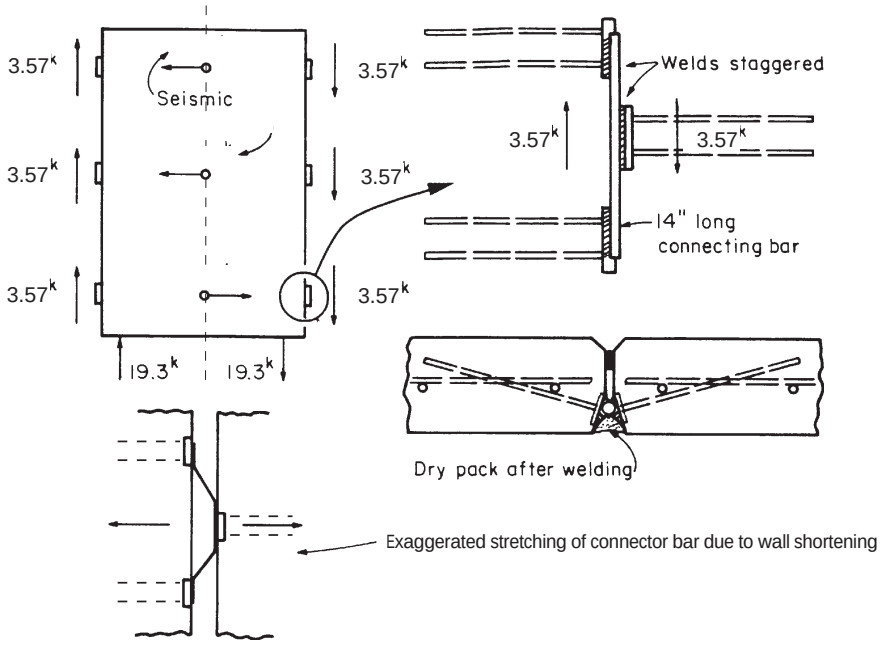
A final requirement of 11.7.10, often overlooked, is that structural steel interfaces must be clean and free of paint. This requirement is based on tests to evaluate the friction coefficient for concrete anchored to unpainted structural steel by studs or reinforcing steel ($\mu = 0.7$). Data are not available for painted surfaces. If painted surfaces are to be used, a lower value of μ would be appropriate.

DESIGN EXAMPLES

In addition to Examples 14.1 and 14.2 of this part, shear-friction design is also illustrated for direct shear-transfer in brackets and corbels (see Part 15), horizontal shear transfer between composite members (see Part 12) and at column/footing connections (see Part 22).

Example 14.1—Shear-Friction Design

A tilt-up wall panel is subject to the factored seismic shear forces shown below. Design the shear anchors assuming lightweight concrete, w_c = 95 pcf. f'_c = 4000 psi and f_y = 60,000 psi.



Calculations and Discussion	Code Reference

1. Design anchor steel using shear-friction method.

   Center plate is most heavily loaded. Try 2 in. × 4 in. × 1/4 in. plate.

   V_u = 3570 lb

   $V_u \le \phi V_n$ *Eq. (11-1)*

   $V_u \le \phi(A_{vf} f_y \mu)$ *Eq. (11-25)*

   For unpainted steel in contact with all lightweight concrete (95 pcf):

   $\mu = 0.7\lambda = 0.7 \times 0.75 = 0.525$ *11.7.4.3*

   $\phi = 0.75$ *9.3.2.3*

   Solving for $A_{vf} = \dfrac{V_u}{\phi f_y \mu} = \dfrac{3570}{0.75\ (60{,}000)(0.525)} = 0.15\ \text{in.}^2$

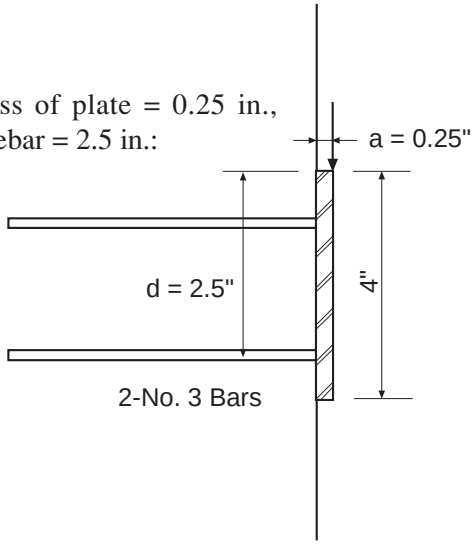
   Use 2-No. 3 bars per plate (A_{vf} = 0.22 in.^{2})

   Note: Weld bars to plates to develop full f_y. Length of bar must be adequate to fully develop bar.

Check maximum shear-transfer strength permitted for connection. For lightweight aggregate concrete: *11.9.3.2.2*

$$V_{n(max)} = \left[0.2 - 0.07\left(\frac{a}{d}\right)\right] f'_c b_w d \text{ or } \left[800 - 280\left(\frac{a}{d}\right)\right] b_w d$$

For the purposes of the above equations, assume a = thickness of plate = 0.25 in., and d = distance from edge of plate to center of farthest attached rebar = 2.5 in.:



$$\frac{a}{d} \approx \frac{0.25}{2.5} = 0.1$$

Assume, for the purposes of the above equations, that $b_w d = A_c$ = contact area of plate:

$b_w d = A_c = 2 \times 4 = 8$ in.^{2}

$V_{n(max)} = [0.2 - 0.07\ (0.1)]\ (4000)\ (8) = 6176$ lb

or $V_n = [800 - (280 \times 0.1)]\ (8) = 6176$ lb

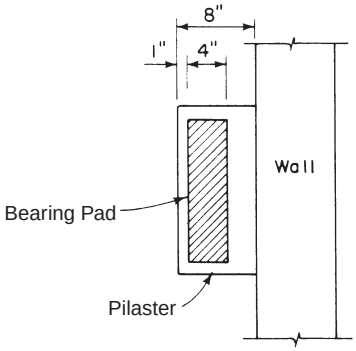
$\phi V_{n(max)} = 0.75\ (6176) = 4632$ lb

$V_u = 3570$ lb $\leq \phi V_{n(max)} = 4632$ lb O.K. *Eq. (11-1)*

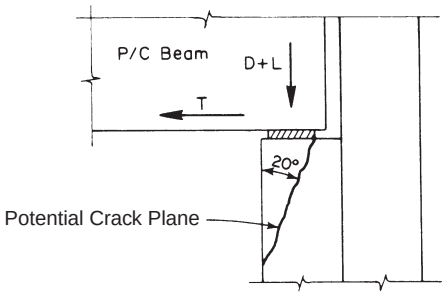
Use 2 in. $\times$ 4 in. $\times \frac{1}{4}$ in. plates, with 2-No. 3 bars.

Example 14.2—Shear-Friction Design (Inclined Shear Plane)

For the pilaster beam support shown, design for shear transfer across the potential crack plane. Assume a crack at an angle of about 20 degrees to the vertical, as shown below. Beam reactions are D = 25 kips, L = 30 kips. Use T = 20 kips as an estimate of shrinkage and temperature change effects. f'_c = 3500 psi and f_y = 60,000 psi.



Plan



Elevation

Calculations and Discussion	Code Reference

1. Factored loads to be considered:

   Beam reaction $R_u = 1.2D + 1.6L = 1.2\,(25) + 1.6\,(30) = 30 + 48 = 78$ kips — *Eq. (9-2)*

   Shrinkage and temperature effects $T_u = 1.6\,(20) = 32$ kips (governs) — *11.9.3.4*
   but not less than $0.2\,(R_u) = 0.2\,(78) = 15.6$ kips

   Note that the live load factor of 1.6 is used with T, due to the low confidence level in determining shrinkage and temperature effects occurring in service. Also, a minimum value of 20 percent of the beam reaction is considered (see 11.9.3.4 for corbel design).

2. Evaluate force conditions along potential crack plane.

   Direct shear transfer force along shear plane:

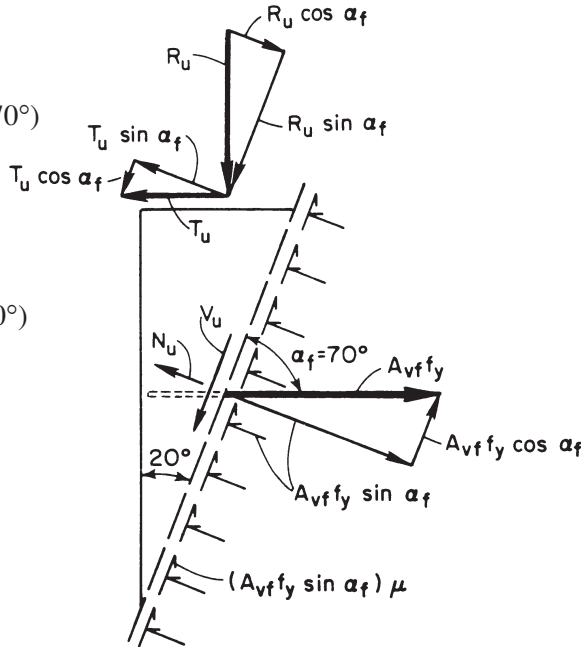
   $V_u = R_u \sin\alpha_f + T_u \cos\alpha_f = 78\,(\sin 70°) + 32\,(\cos 70°)$

   $= 73.3 + 11.0 = 84.3$ kips

   Net tension (or compression) across shear plane:

   $N_u = T_u \sin\alpha_f - R_u \cos\alpha_f = 32\,(\sin 70°) - 78\,(\cos 70°)$

   $= 30.1 - 26.7 = 3.4$ kips (net tension)



If the load conditions were such as to result in net compression across the shear plane, it still should not have been used to reduce the required A_{vf}, because of the uncertainty in evaluating the shrinkage and temperature effects. Also, 11.7.7 permits a reduction in A_{vf} only for "permanent" net compression.

3. Shear-friction reinforcement to resist direct shear transfer. Use μ for concrete placed monolithically.

$$A_{vf} = \frac{V_u}{\phi f_y (\mu \sin\alpha_f + \cos\alpha_f)}$$ *Eq. (11-26)*

$$\mu = 1.4\lambda = 1.4 \times 1.0 = 1.4$$ *11.7.4.3*

$$A_{vf} = \frac{84.3}{0.75 \times 60 (1.4 \sin 70° + \cos 70°)} = 1.13 \text{ in.}^2$$ *[μ from 11.7.4.3]*

4. Reinforcement to resist net tension.

$$A_n = \frac{N_u}{\phi f_y (\sin\alpha_f)} = \frac{3.4}{0.75 \times 60 (\sin 70°)} = 0.08 \text{ in.}^2$$

Since failure is primarily controlled by shear, use $\phi = 0.75$ (see 11.9.3.1 for corbel design).

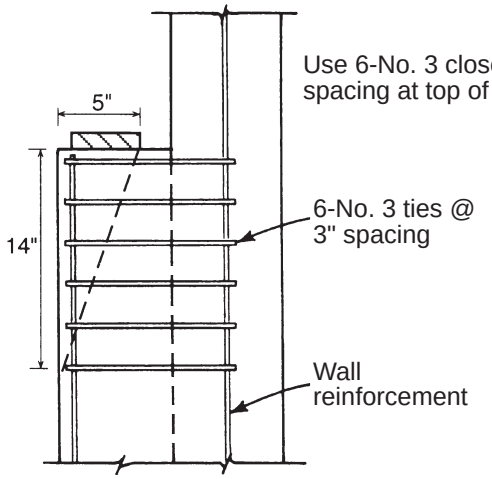
5. Add A_{vf} and A_n for total area of required reinforcement. Distribute reinforcement uniformly along the potential crack plane.

$A_s = 1.13 + 0.08 = 1.21 \text{ in.}^2$

Use No. 3 closed ties (2 legs per tie)

Number required = 1.21 / [2 (0.11)] = 5.5, say 6.0 ties

Ties should be distributed along length of potential crack plane; approximate length = 5/(tan20°) ≈ 14 in.



6. Check reinforcement requirements for dead load only plus shrinkage and temperature effects. Use 0.9 load factor for dead load to maximize net tension across shear plane.

$R_u = 0.9D = 0.9\ (25) = 22.5$ kips, $T_u = 32$ kips

$V_u = 22.5\ (\sin 70°) + 32\ (\cos 70°) = 21.1 + 11.0 = 32.1$ kips

$N_u = 32\ (\sin 70°) - 22.5\ (\cos 70°) = 30.1 - 7.7 = 22.4$ kips (net tension)

$$A_{vf} = \frac{32.1}{0.75 \times 60\ (1.4 \sin 70° + \cos 70°)} = 0.43 \text{ in.}^2$$

$$A_n = \frac{22.4}{0.75 \times 60 \times \sin 70°} = 0.53 \text{ in.}^2$$

$A_s = 0.43 + 0.53 = 0.96 \text{ in.}^2 < 1.21 \text{ in.}^2$

Therefore, original design for full dead load + live load governs.

7. Check maximum shear-transfer strength permitted

$V_{n(max)} = \left[0.2 f'_c A_c\right]$ or $\left[800 A_c\right]$ *11.7.5*

Taking the width of the pilaster to be 16 in.:

$$A_c = \left(\frac{5}{\sin 20°}\right) \times 16 = 234 \text{ in.}^2$$

$V_{n(max)} = 0.2\ (3500)\ (234)/1000 = 164$ kips (governs)

or $V_{n(max)} = 800\ (234)/1000 = 187$ kips

$\phi V_{n(max)} = 0.75\ (164) = 123$ kips

$V_u = 84.3 \text{ kips} \le \phi V_{n(max)} = 123$ kips O.K. *Eq. (11-1)*

15

Brackets, Corbels and Beam Ledges

UPDATE FOR THE ’02 CODE

Prior to the ’02 code, the provisions of 11.9 were applicable only to brackets and corbels with a shear span-to-depth ratio a/d less than or equal to 1.0. The ’02 code allows the use of the provisions of Appendix A, strut-and-tie models, to design brackets and corbels, with a/d ratios less than 2.0 (11.9.1.) See Part 17.

11.9 LIMITATIONS OF BRACKET AND CORBEL PROVISIONS

The design procedure for brackets and corbels recognizes the deep beam or simple truss action of these short-shear-span members, as illustrated in Fig. 15-1. Four potential failure modes shown in Fig. 15-1 shall be prevented: (1) Direct shear failure at the interface between bracket or corbel and supporting member; (2) Yielding of the tension tie due to moment and direct tension; (3) Crushing of the internal compression “strut;” and (4) Localized bearing or shear failure under the loaded area.

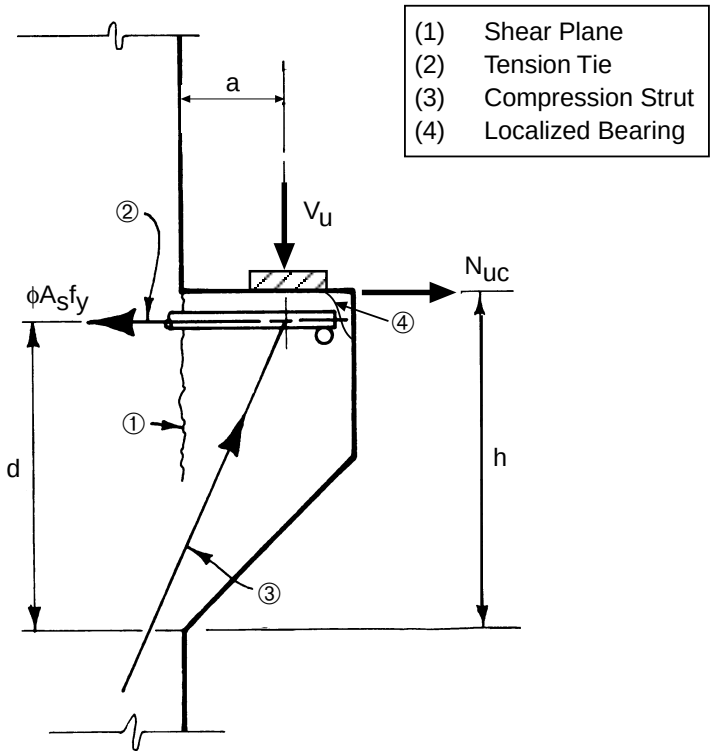


Figure 15-1 Structural Action of Corbel

For brackets and corbels with a shear span-to-depth ratio a/d less than 2, the provision of Appendix A may be used for design. The provisions of 11.9.3 and 11.9.4 are permitted with a/d 1 and the horizontal force N_{uc} V_n.

Regardless which design method is used, the provisions of 11.9.2, 11.9.3.2.1, 11.9.3.2.2, 11.9.5, 11.9.6, and 11.9.7 must be satisfied.

When a/d is greater than 2.0, brackets and corbels shall be designed as cantilevers subjected to the applicable provisions of flexure and shear.

11.9.1 - 11.9.5 Design Provisions

The critical section for design of brackets and corbels is taken at the face of the support. This section should be designed to resist simultaneously a shear V_u, a moment $M_u = V_u a + N_{uc} (h - d)$, and a horizontal tensile force N_{uc} (11.9.3). The value of N_{uc} must be not less than $0.2V_u$, unless special provisions are made to avoid tensile forces (11.9.3.4). This minimum value of N_{uc} is established to account for the uncertain behavior of a slip joint and/or flexible bearings. Also, the tension force N_{uc} typically is due to indeterminate causes such as restrained shrinkage or temperature stresses. In any case it shall be treated as a live load with load factor of 1.6 (11.9.3.4). Since corbel and bracket design is predominantly controlled by shear, 11.9.3.1 specifies that the strength reduction factor ϕ shall be taken equal to 0.75 for all design conditions.

For normal weight concrete, shear strength V_n is limited to the smaller of $0.2 f'_c b_w d$ and $800 b_w d$ (11.9.3.2). For lightweight concrete, V_n is limited by the provisions of 11.9.3.2.2, which are somewhat more restrictive than those for normal weight concrete. Tests show that for lightweight concrete, V_n is a function of f'_c and a/d.

For brackets and corbels, the required reinforcement is:

A_{vf} = area of shear-friction reinforcement to resist direct shear V_u, computed in accordance with 11.7 (11.9.3.2).

A_f = area of flexural reinforcement to resist moment $M_u = V_u a + N_{uc} (h - d)$, computed in accordance with 10.2 and 10.3 (11.9.3.3).

A_n = area of tensile reinforcement to resist direct tensile force N_{uc}, computed in accordance with 11.9.3.4.

Actual reinforcement is to be provided as shown in Fig. 15-2 and includes:

A_s = primary tension reinforcement

A_h = shear reinforcement (closed stirrups or ties)

This reinforcement is provided such that total amount of reinforcement $A_s + A_h$ crossing the face of support is the greater of (a) $A_{vf} + A_n$, and (b) $3A_f/2 + A_n$ to satisfy criteria based on test results.[15.1]

If case (a) controls (i.e., $A_{vf} > 3A_f/2$):

$$A_s = A_{vf} + A_n - A_h$$

$$= A_{vf} + A_n - 0.5 (A_s - A_n) \qquad \textit{11.9.4}$$

or $A_s = 2A_{vf}/3 + A_n$ (primary tension reinforcement)

and $A_h = (0.5) (A_s - A_n) = A_{vf}/3$ (closed stirrups or ties) *11.9.4*

If case (b) controls (i.e., $3A_f/2 > A_{vf}$):

$$A_s = 3A_f/2 + A_n - A_h$$

$$= 3A_f/2 + A_n - 0.5\,(A_s - A_n)$$

or $A_s = A_f + A_n$ (primary tension reinforcement)

and $A_h = (0.5)\,(A_s - A_n) = A_f/2$ (closed stirrups or ties)

In both cases (a) and (b), $A_h = (0.5)\,(A_s - A_n)$ determines the amount of shear reinforcement to be provided as closed stirrups parallel to A_s and uniformly distributed within (2/3)d adjacent to A_s per 11.9.4.

A minimum ratio of primary tension reinforcement $\rho_{min} = 0.04\,f'_c/f_y$ is required to ensure ductile behavior after cracking under moment and direct tensile force (11.9.5).

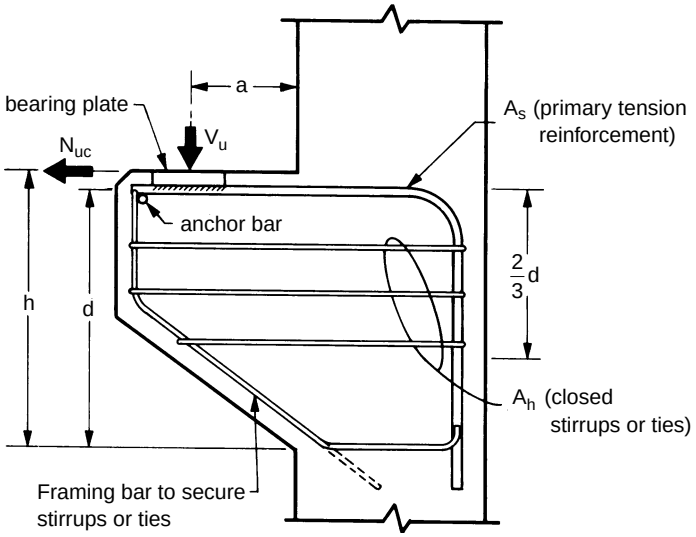


Figure 15-2 Corbel Reinforcement

BEAM LEDGES

Beam with ledges shall be designed for the overall member effects of flexure, shear, axial forces, and torsion, as well as for local effects in the vicinity of the ledge (Refs. 15.2-15.6). The design of beam ledges is not specifically addressed by the code. This section addresses only local failure modes and reinforcement requirements to prevent such failure.

Design of beam ledges is somewhat similar to that of a bracket or corbel with respect to loading conditions. Additional design considerations and reinforcement details need to be considered in beam ledges. Accordingly, even though not specifically addressed by the code, special design of beam ledges is included in this Part. Some failure modes discussed above for brackets and corbels are also shown for beam ledges in Fig. 15-3. However, with beam ledges, two additional failure modes shall be considered (see Fig. 15-3): (5) separation between ledge and beam web near the top of the ledge in the vicinity of the ledge load and (6) punching shear. The vertical load applied to the ledge is resisted by a compression strut. In turn, the vertical component of the inclined compression strut must be picked up by the web stirrups (stirrup legs A_v adjacent to the side face of the web) acting as "hanger" reinforcement to carry the ledge load to the top of beam. At the reentrant corner of the ledge to web intersection, a diagonal crack would extend to the stirrup and run downward next to the stirrup. Accordingly, a slightly larger shear span, a_f, is used to compute the moment due to V_u. Therefore, the critical section for moment is taken at center of beam stirrups, not at face of beam. Also, for beam ledges, the internal moment arm should not be taken greater than 0.8h for flexural strength.

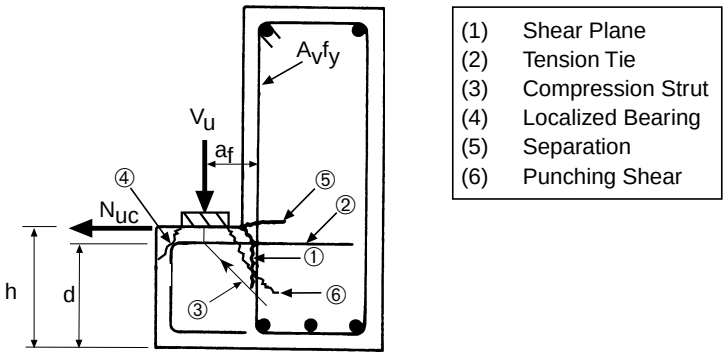


Figure 15-3 Structural Action of Beam Ledge

The design procedure described in this section is based on investigations performed by Mirza and Furlong (Refs. 15.3 to 15.5). The key information needed by the designer is establishing the effective width of ledge for each of the potential failure modes. These effective widths were determined by Mirza and Furlong through analytical investigations, with results verified by large scale testing. Design of beam ledges can also be performed by the strut-and-tie procedure (refer to Part 32 for discussion).

Design to prevent local failure modes requires consideration of the following actions:

1. Shear V_u

2. Horizontal tensile force N_{uc} greater or equal to $0.2V_u$, but not greater than V_u.

3. Moment $M_u = V_u a_f + N_{uc}(h-d)$

Reinforcement for the different failure modes is determined based on the effective widths or critical sections discussed below. In all cases, the required strengths (V_u, M_u, or N_u) should be less than or equal to the design strengths (ϕV_n, ϕM_n, or ϕN_n). The strength reduction factor ϕ is taken equal to 0.75 for all actions, as for brackets and corbels. The strength requirements for different failure modes are shown below for normal weight concrete. When lightweight aggregate concrete is used, modifications should be made per 11.2.

a. Shear Friction

Parameters affecting determination of the shear friction reinforcement are illustrated in Figure 15-4.

$$V_u \quad 0.2\phi f'_c (W + 4a)\, d \qquad (1)$$

$$\phi \mu A_{vf} f_y$$

where

d = effective depth of ledge from centroid of top layer of ledge transverse reinforcement to the bottom of the ledge (see Fig. 15-4)

μ = coefficient of friction per 11.7.4.3.

Note that per 11.7.5, $0.2 f'_c$ 800 psi.

If $(W + 4a) > S$, then V_u $0.2\ \phi f'_c Sd$, where S is the distance between center of adjacent bearings on the same ledge.

At ledge ends, V_u $0.2\ \phi f'_c (2c)d$, where c is the distance from center of end bearing to the end of the ledge. However, 2c should be less than or equal to the smaller of W + 4a and S.

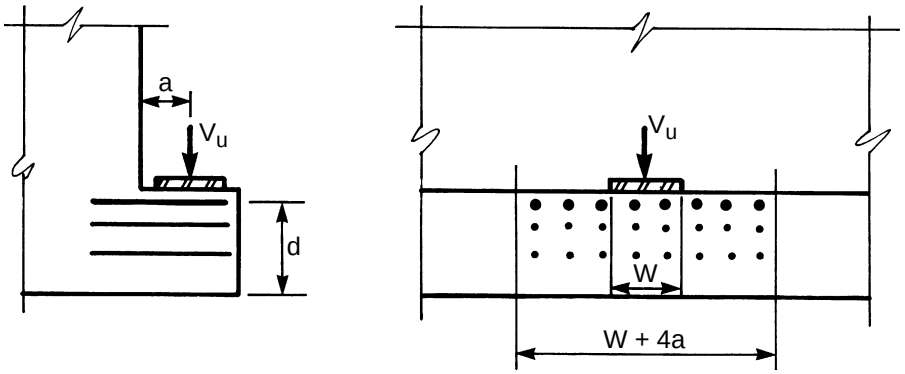


Figure 15-4 Shear Friction

b. Flexure

Conditions for flexure and direct tension are shown in Figure 15-5.

$$V_u a_f + N_{uc}(h-d) \quad \phi A_f f_y (jd) \qquad (2)$$

$$N_{uc} \quad \phi A_n f_y$$

The primary tension reinforcement A_s should equal the greater of $(A_f + A_n)$ or $(2A_{vf}/3 + A_n)$. If $(W + 5a_f) >$ S, reinforcement should be placed over distance S. At ledge ends, reinforcement should be placed over distance (2c), where c is the distance from the center of the end bearing to the end of the ledge, but not more than 1/2 $(W + 5a_f)$. Reference 15.5 recommends taking jd = 0.8d.

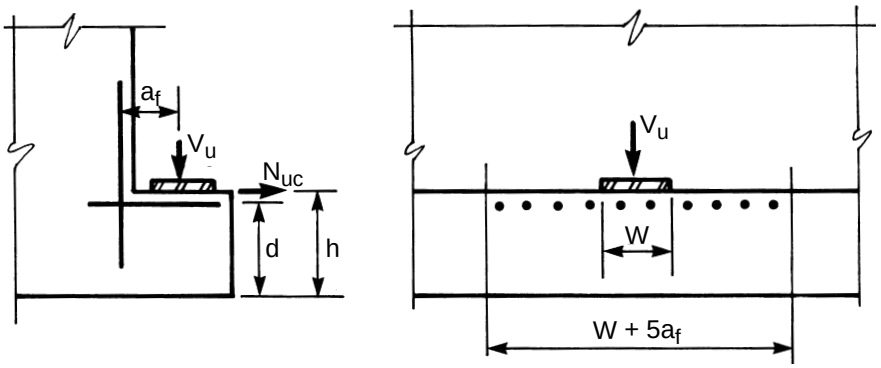


Figure 15-5 Flexure and Direct Tension

c. Punching Shear

Critical perimeter for punching shear design is illustrated in Fig. 15-6.

$$V_u \leq 4\phi\sqrt{f'_c}\,(W + 2L + 2d_f)\,d_f \qquad (3)$$

where d_f = effective depth of ledge from top of ledge to center of bottom transverse reinforcement (see Fig. 15-6)

Truncated pyramids from adjacent bearings should not overlap. At ledge ends,

$$V_u \leq 4\phi\sqrt{f'_c}\,(W + L + d_f)\,d_f$$

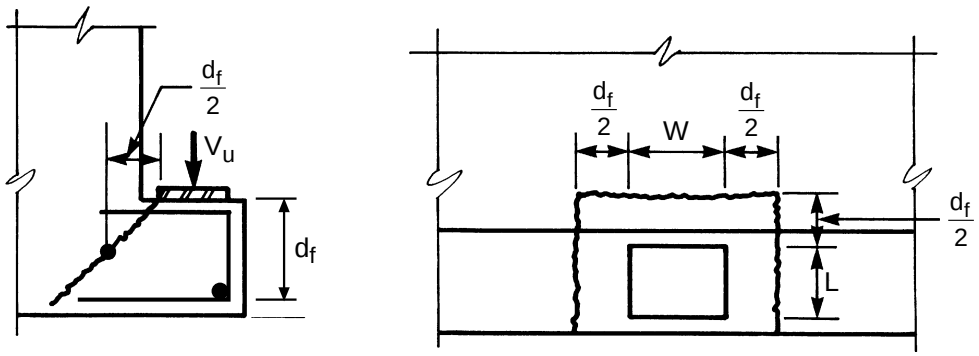


Figure 15-6 Punching Shear

d. Hanger Reinforcement

Hanger reinforcement should be proportioned to satisfy strength. Furthermore, serviceability criteria should be considered when the ledge is subjected to a large number of live load repetitions, as in parking garages and bridges. As shown in Figure 15-7, strength is governed by

$$V_u \quad \phi \frac{A_v f_y}{s} S \qquad (4)$$

where A_v = area of one leg of hanger reinforcement

S = distance between ledge loads

s = spacing of hanger reinforcement

Serviceability is governed by

$$V \quad \frac{A_v (0.5 f_y)}{s} (W + 3a) \qquad (5)$$

where V is the reaction due to service dead load and live load.

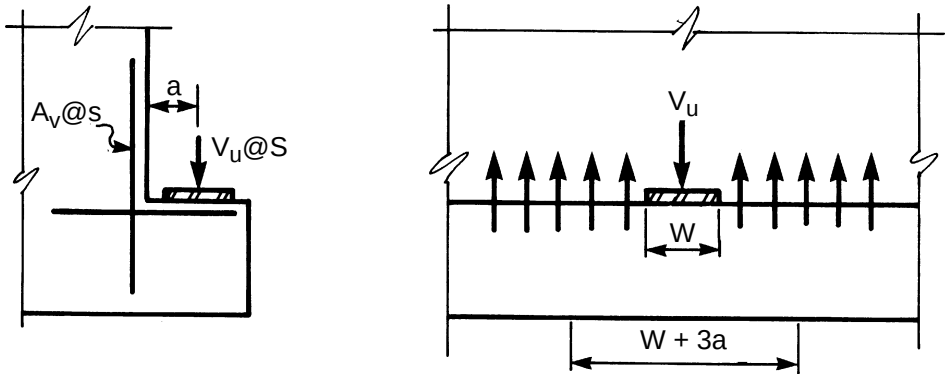


Figure 15-7 Hanger Reinforcement to Prevent Separation of Ledge from Stem

In addition, hanger reinforcement in inverted tees is governed by consideration of the shear failure mode depicted in Figure 15-8:

$$2V_u \le 2\left[2\phi\sqrt{f'_c}\,b_f d'_f\right] + \phi\frac{A_v f_y}{s}(W + 2d'_f) \qquad (6)$$

where d'_f = flange depth from top of ledge to center of bottom longitudinal reinforcement (see Fig. 15-8)

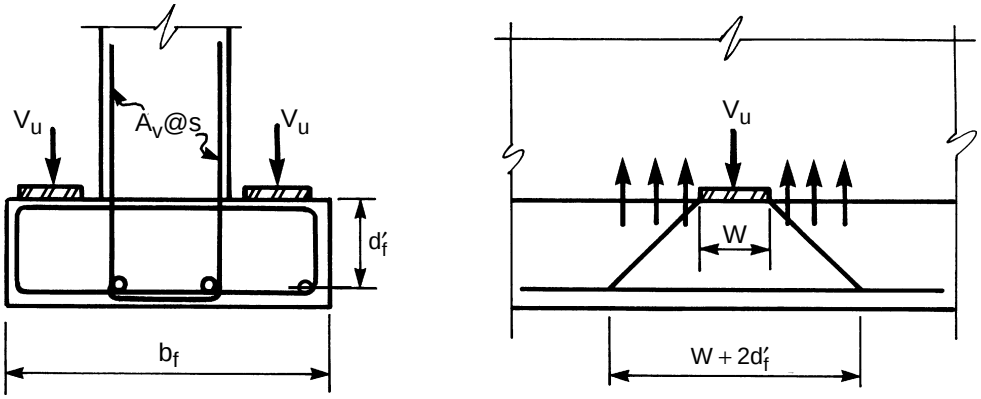


Figure 15-8 Hanger Reinforcement to Prevent Partial Separation of Ledge from Stem and Shear of the Ledge

11.9.6 Development and Anchorage of Reinforcement

All reinforcement must be fully developed on both sides of the critical section. Anchorage within the support is usually accomplished by embedment or hooks. Within the bracket or corbel, the distance between load and support face is usually too short, so that special anchorage must be provided at the outer ends of both primary reinforcement A_s and shear reinforcement A_h. Anchorage of A_s is normally provided by welding an anchor bar of equal size across the ends of A_s (Fig. 15-9(a)) or welding to an armor angle. In the former case, the anchor bar must be located beyond the edge of the loaded area. Where anchorage is provided by a hook or a loop in A_s, the load must not project beyond the straight portion of the hook or loop (Fig. 15-9(b)). In beam ledges, anchorage may be provided by a hook or loop, with the same limitation on the load location (Fig. 15-10). Where a corbel or beam ledge is designed to resist specific horizontal forces, the bearing plate should be welded to A_s.

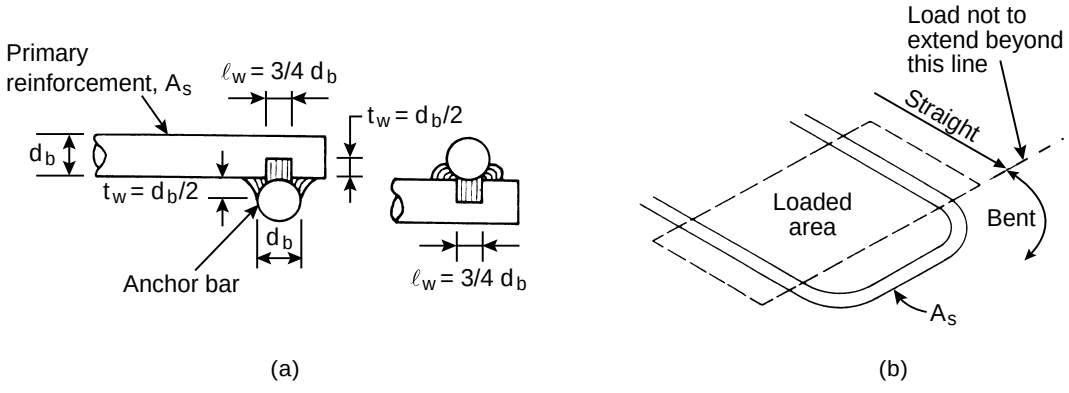


Figure 15-9 Anchorage Details Using (a) Cross-Bar Weld and (b) Loop Bar Detail

The closed stirrups or ties used for A_h must be similarly anchored, usually by engaging a "framing bar" of the same diameter as the closed stirrups or ties (see Fig. 15-2).

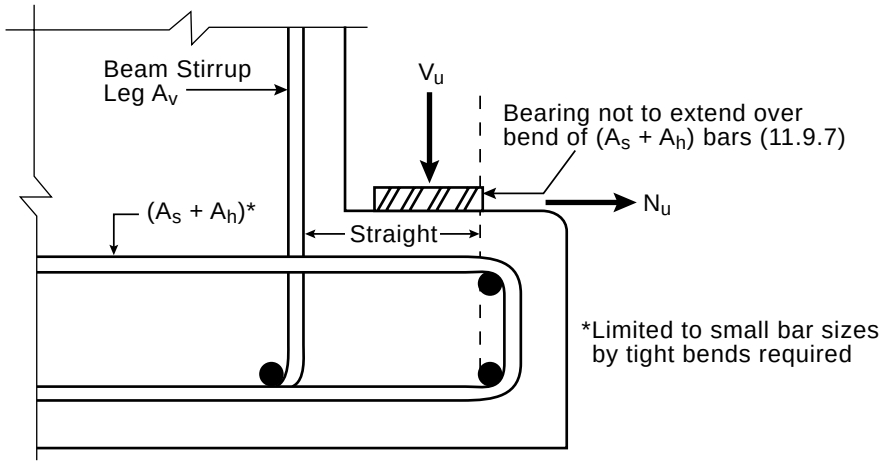


Figure 15-10 Bar Details for Beam Ledge

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15.1 Mattock, Alan H., Chen, K.C., and Soongswang, K., "The Behavior of Reinforced Concrete Corbels," *PCI Journal*, Prestressed Concrete Institute, V. 21, No. 2, Mar.-Apr. 1976, pp. 52-77.

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15.4 Mirza, Sher Ali, and Furlong, Richard W., "Serviceability Behavior and Failure Mechanisms of Concrete Inverted T-Beam Bridge Bent Caps," *ACI Journal, Proceedings*, V. 80, No. 4, July-Aug. 1983, pp. 294-304.

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15.6 "Design of Concrete Beams for Torsion," Portland Cement Association, Skokie, Illinois, 1999.

Example 15.1—Corbel Design

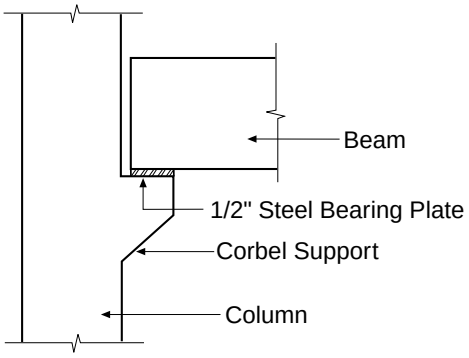
Design a corbel with minimum dimensions to support a beam as shown below. The corbel is to project from a 14-in. square column. Restrained creep and shrinkage create a horizontal force of 20 kips at the welded bearing.

f'_c = 5000 psi (normal weight)

f_y = 60,000 psi

Beam reactions:

DL = 24 kips
LL = 37.5 kips
T = 20 kips



Calculations and Discussion	Code Reference

1. Size bearing plate based on bearing strength on concrete according to 10.17. Width of bearing plate = 14 in.

$V_u = 1.2\,(24) + 1.6\,(37.5) = 88.8$ kips — *Eq. (9-2)*

$V_u \quad \phi P_{nb} = \phi(0.85 f'_c A_1)$ — *10.17.1*

$\phi = 0.65$ — *9.3.2.4*

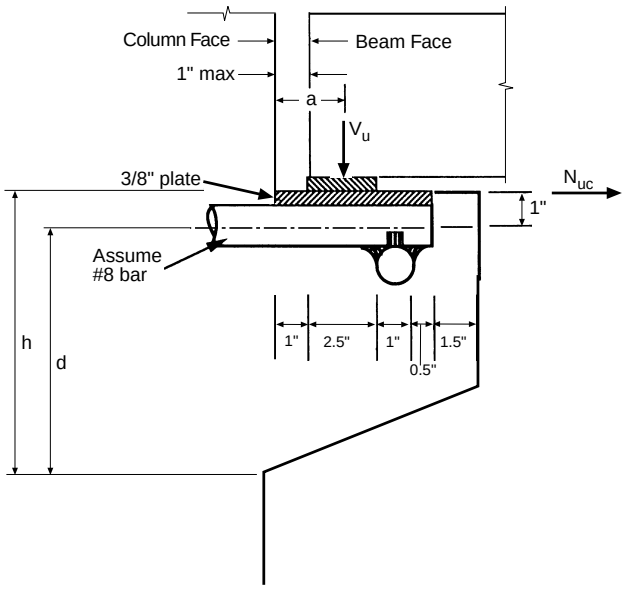
$88.8 = 0.65\,(0.85 \times 5 \times A_1) = 2.763A_1$

$$A_1 = \frac{88.8}{2.763} = 32.14 \text{ in.}^2$$

$$\text{Bearing length} = \frac{32.14}{14} = 2.30 \text{ in.}$$

Use 2.5 in. × 14 in. bearing plate.

2. Determine shear span 'a' with 1 in. max. clearance at beam end. Beam reaction is assumed at third point of bearing plate to simulate rotation of supported girder and triangular distribution of stress under bearing pad.



$$a = \frac{2}{3}(2.5) + 1.0 = 2.67 \text{ in.}$$

Use a = 3 in. maximum.

Detail cross bar just outside outer bearing edge.

3. Determine total depth of corbel based on limiting shear-transfer strength V_n.

   V_n is the least of $V_n = 800 b_w d$ (governs) *11.9.3.2.1*

   or $V_n = 0.2 f'_c b_w d = (0.2 \times 5000) b_w d = 1000 b_w d$

   Thus, $V_u \quad \phi V_n = \phi (800 b_w d)$

   $$\text{Required } d = \frac{88{,}800}{0.75\ (800 \times 14)} = 10.57 \text{ in.}$$

   Assuming No. 8 bar, 3/8 in. steel plate, plus tolerance,

   $h = 10.57 + 1.0 = 11.57$ in. Use $h = 12$ in.

   For design, $d = 12.0 - 1.0 = 11.0$ in.

   $\frac{a}{d} = 0.27 < 1$ O.K. *11.9.1*

   Also, $N_{uc} = 1.6 \times 20 = 32.0$ kips (treat as live load)

   $N_{uc} < V_u = 88.8$ kips O.K.

4. Determine shear-friction reinforcement A_{vf}. *11.9.3.2*

   $$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{88.8}{0.75\ (60)\ (1.4 \times 1)} = 1.41 \text{ in.}^2$$ *11.7.4.1*

5. Determine direct tension reinforcement A_n.

   $$A_n = \frac{N_{uc}}{\phi f_y} = \frac{32.0}{0.75 \times 60} = 0.71 \text{ in.}^2$$ *11.9.3.1*

6. Determine flexural reinforcement A_f. *11.7.4.3*

   $M_u = V_u a + N_{uc} (h - d) = 88.8\ (3) + 32\ (12 - 11) = 298.4$ in.-kips *11.9.3.3*

   Find A_f using conventional flexural design methods or conservatively use $j_u d = 0.9d$.

   $$A_f = \frac{298.4}{0.75\ (60)\ (0.9 \times 11)} = 0.67 \text{ in.}^2$$

   Note that for all design calculations, $\phi = 0.75$ *11.9.3.1*

7. Determine primary tension reinforcement A_s. *11.9.3.5*

$\frac{2}{3}A_{vf} = \frac{2}{3}(1.41) = 0.94$ in.^{2} > $A_f = 0.67$ in.^{2}; Therefore, $\frac{2}{3}A_{vf}$ controls design

$A_s = \frac{2}{3}A_{vf} + A_n = 0.94 + 0.71 = 1.65$ in.^{2}

Use 2-No. 9 bars, $A_s = 2.0$ in.^{2}

Check minimum reinforcement: *11.9.5*

$$\rho_{min} = 0.04\left(\frac{f'_c}{f_y}\right) = 0.04\left(\frac{5}{60}\right) = 0.0033$$

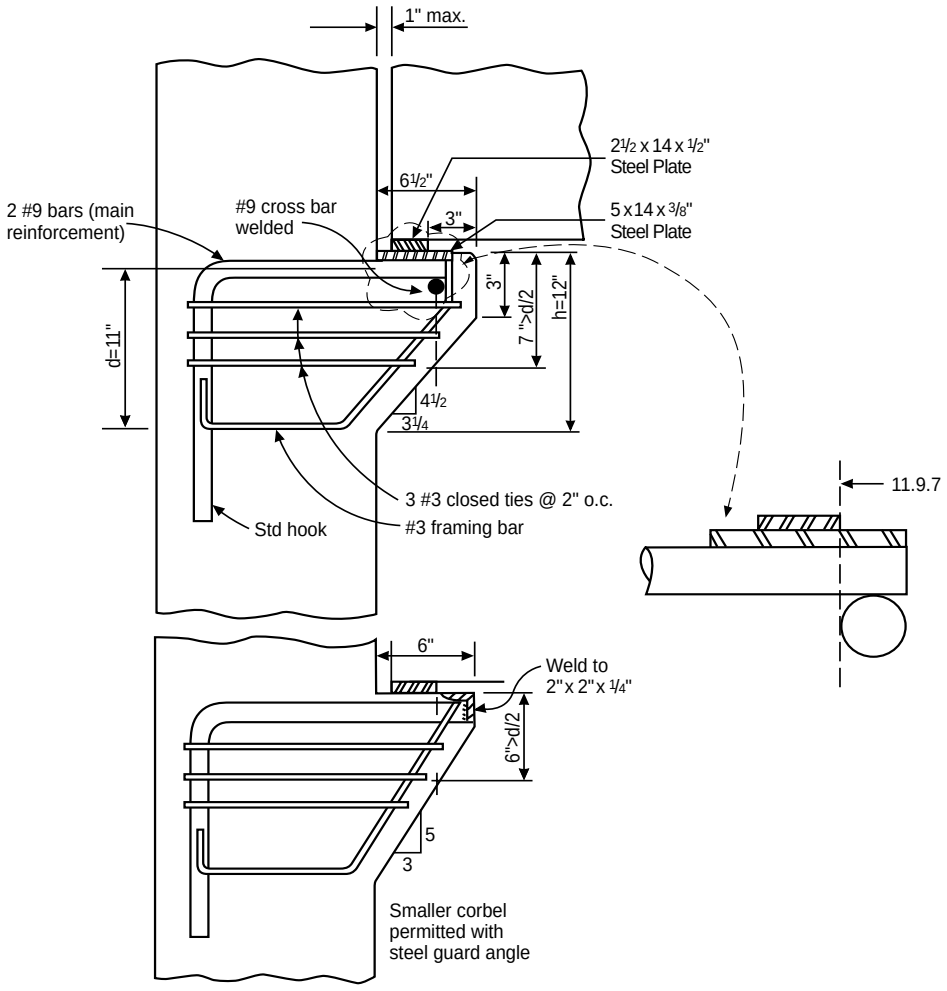
$A_{s(min)} = 0.0033\,(14)\,(11) = 0.51$ in.^{2} < $A_s = 2.0$ in.^{2} O. K.

8. Determine shear reinforcement A_h *11.9.4*

$A_h = 0.5\,(A_s - A_n) = 0.5\,(2.0 - 0.71) = 0.65$ in.^{2}

Use 3-No. 3 stirrups, $A_h = 0.66$ in.^{2}

Distribute stirrups in two-thirds of effective corbel depth adjacent to A_s.



Example 15.2—Corbel Design . . . Using Lightweight Concrete and Modified Shear-Friction Method

Design a corbel to project from a 14-in.-square column to support the following beam reactions:

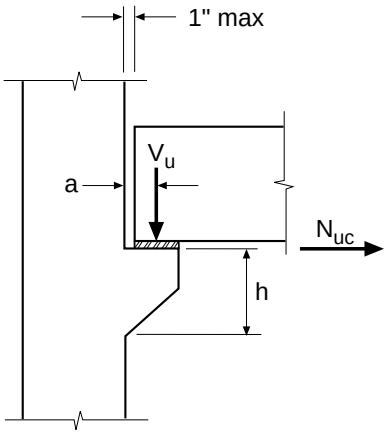
dead load = 32 kips

live load = 30 kips

horizontal force = 24 kips

f'_c = 4000 psi (all lightweight)

f_y = 60,000 psi



Calculations and Discussion	Code Reference

1. Size bearing plate

   $V_u = 1.2\,(32) + 1.6\,(30) = 86.4$ kips — *Eq. (9-2)*

   $V_u \quad \phi P_{nb} = \phi\,(0.85\,f'_c A_1)$ — *10.17.1*

   $\phi = 0.65$ — *9.3.2.4*

   $86.4 = 0.65\,(0.85 \times 4 \times A_1)$

   Solving, $A_1 = 39.1$ in.^{2}

   Length of bearing required $= \dfrac{39.1}{14} = 2.8$ in.

   Use 14 in. × 3 in. bearing plate.

2. Determine a.

   Assume beam reaction to act at outer third point of bearing plate, and 1 in. gap between back edge of bearing plate and column face. Therefore:

   $a = 1 + \dfrac{2}{3}\,(3) = 3$ in.

3. Determine total depth of corbel based on limiting shear-transfer strength V_n. For easier placement of reinforcement and concrete, try h = 15 in. Assuming No. 8 bar:

   $d = 15 - 0.5 - 0.375 = 14.13$ in., say 14 in.

   $\dfrac{a}{d} = \dfrac{3}{14} = 0.21 < 1.0$ — *11.9.1*

   $N_{uc} = 1.6 \times 24 = 38.4$ kips $< V_u = 86.4$ kips O.K.

For lightweight concrete and $f'_c = 4000$ psi, V_n is the least of: *11.9.3.2.2*

$$V_n = \left(800 - 280\,\frac{a}{d}\right) b_w d = [800 - (280 \times 0.21)]\,14 \times \frac{14}{1000} = 145.3 \text{ kips}$$

$$V_n = \left(0.2 - 0.07\,\frac{a}{d}\right) f'_c b_w d = \left[0.2 - 0.07\,(0.21)\right](4{,}000)\,(14)\,\frac{14}{1000} = 145.3 \text{ kips}$$

$\phi V_n = 0.75\,(145.3) = 109.0 \text{ kips} > V_u = 86.4 \text{ kips}$ O.K.

4. Determine shear-friction reinforcement A_{vf}. *11.9.3.2*

Using a Modified Shear-Friction Method as permitted by 11.7.3 (see R11.7.3):

$$V_n = 0.8A_{vf}f_y + K_1 b_w d, \text{ with } \frac{A_{vf} f_y}{b_w d} \text{ not less than 200 psi}$$

For all lightweight concrete, $K_1 = 200$ psi *R11.7.3*

$V_u \quad \phi V_n = \phi\,(0.8A_{vf}f_y + 0.2b_w d)$

Solving for A_{vf}:

$$A_{vf} = \frac{V_u - \phi(0.2b_w d)}{\phi(0.8f_y)}, \text{ but not less than } 0.2 \times \frac{b_w d}{f_y}$$

$$= \frac{86.4 - (0.75 \times 0.2 \times 14 \times 14)}{0.75\,(0.8 \times 60)} = 1.58 \text{ in.}^2 \text{ (governs)}$$

$$\text{but not less than } 0.2 \times \frac{b_w d}{f_y} = 0.2 \times \frac{14 \times 14}{60} = 0.65 \text{ in.}^2$$

For comparison, compute A_{vf} by Eq. (11-25): *11.7.4.3*

For lightweight concrete,

$\mu = 1.4\lambda = 1.4\,(0.75) = 1.05$

$$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{86.4}{0.75 \times 60 \times 1.05} = 1.83 \text{ in.}^2 > 1.58 \text{ in.}^2$$

Note: Modified shear-friction method presented in R11.7.3 would give a closer estimate of shear-transfer strength than the conservative shear-friction method in 11.7.4.1.

5. Determine flexural reinforcement A_f. *11.9.3.3*

$M_u = V_u a + N_{uc}\,(h - d) = 86.4\,(3) + 38.4\,(15 - 14.0) = 297.6$ in.-kips

Find A_f using conventional flexural design methods, or conservatively use $j_u d = 0.9d$

$$A_f = \frac{M_u}{\phi f_y j_u d} = \frac{297.6}{0.75 \times 60 \times 0.9 \times 14} = 0.53 \text{ in.}^2$$

Note that for all design calculations, $\phi = 0.75$ *11.9.3.1*

6. Determine direct tension reinforcement A_n. *11.9.3.4*

$$A_n = \frac{N_{uc}}{\phi f_y} = \frac{38.4}{0.75 \times 60} = 0.85 \text{ in.}^2$$

7. Determine primary tension reinforcement A_s. *11.9.3.5*

$\left(\frac{2}{3}\right)A_{vf} = \left(\frac{2}{3}\right) 1.83 = 1.22 \text{ in.}^2 > A_f = 0.53 \text{ in.}^2$; Therefore, $\left(\frac{2}{3}\right)A_{vf}$ controls design.

$$A_s = \left(\frac{2}{3}\right)A_{vf} + A_n = 1.22 + 0.85 = 2.07 \text{ in.}^2$$

Use 3-No. 8 bars, $A_s = 2.37 \text{ in.}^2$

Check $A_{s(min)} = 0.04 \left(\frac{4}{60}\right) 14 \times 14 = 0.52 \text{ in.}^2 < A_s = 2.37 \text{ in.}^2$ O. K. *11.9.5*

8. Determine shear reinforcement A_h. *11.9.4*

$$A_h = 0.5 (A_s - A_n) = 0.5 (2.37 - 0.85) = 0.76 \text{ in.}^2$$

Use 4-No. 3 stirrups, $A_h = 0.88 \text{ in.}^2$

The shear reinforcement is to be placed within two-thirds of the effective corbel depth adjacent to A_s.

$s_{max} = \left(\frac{2}{3}\right)\frac{14}{4} = 2.33$ in. Use 2¼ in. o.c. stirrup spacing.

9. Corbel details

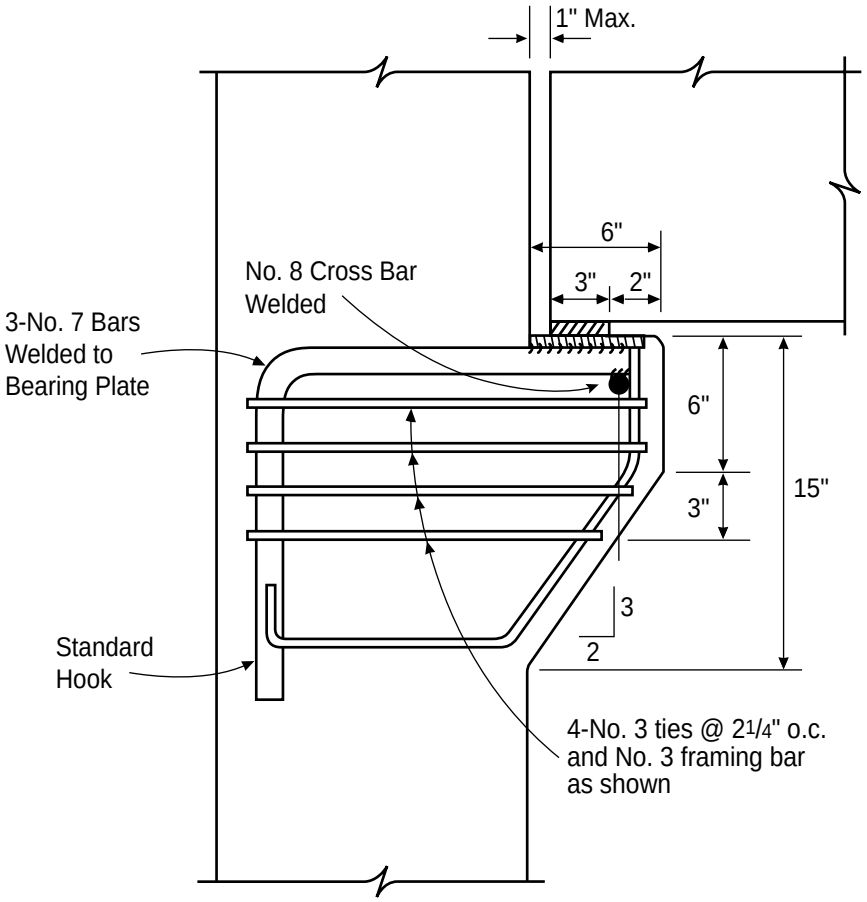
Corbel will project $(1 + 3 + 2) = 6$ in. from column face.

Use 6-in. depth at outer face of corbel, then depth at outer edge of bearing plate will be

$6 + 3 = 9 \text{ in.} > \frac{14}{2} = 7.0 \text{ in.}$ O.K. *11.9.2*

A_s to be anchored at front face of corbel by welding a No. 8 bar transversely across ends of A_s bars. *11.9.6*

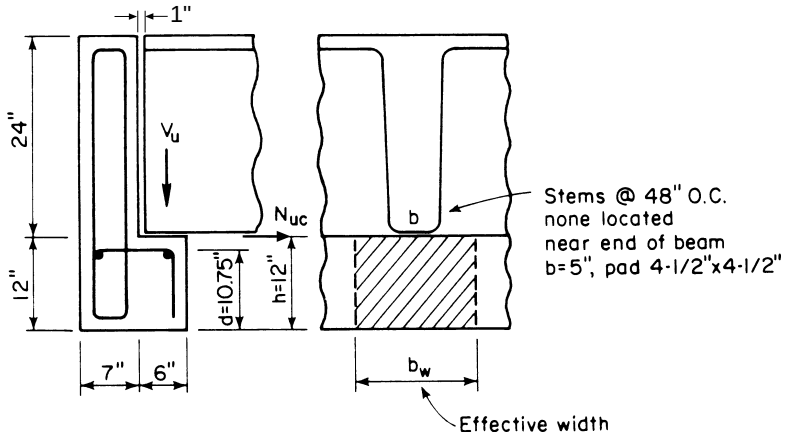
A_s must be anchored within column by standard hook.

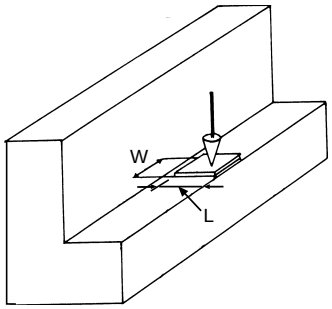

1" Max.
6"
3"
2"
No. 8 Cross Bar
Welded
3-No. 7 Bars
Welded to
Bearing Plate
6"
3"
15"
3
2
Standard
Hook
4-No. 3 ties @ 2¼" o.c.
and No. 3 framing bar
as shown

Example 15.3—Beam Ledge Design

f'_c = 5000 psi (normal weight)

f_y = 60,000 psi





The L-beam shown is to support a double-tee parking deck spanning 64 ft. Maximum service loads per stem are: DL = 11.1 kips; LL = 6.4 kips; total load = 17.5 kips. The loads may occur at any location on the L-beam ledge except near beam ends. The stems of the double-tees rest on 4.5 in. × 4.5 in. × 1/4 in. neoprene bearing pads (1000 psi maximum service load).

Design in accordance with the code provisions for brackets and corbels may require a wider ledge than the 6 in. shown. To maintain the 6-in. width, one of the following may be necessary: (1) Use of a higher strength bearing pad (up to 2000 psi); or (2) Anchoring primary ledge reinforcement A_s to an armor angle.

This example will be based on the 6-in. ledge with 4.5-in.-square bearing pad. At the end of the example an alternative design will be shown.

Note: This example illustrates design to prevent potential local failure modes. In addition, ledge beams should be designed for global effects, not considered in this example. For more details see References 15.2 to 15.6.

Calculations and Discussion	Code Reference

1. Check 4.5 × 4.5 in. bearing pad size (1000 psi maximum service load).

   Capacity = 4.5 × 4.5 × 1.0 = 20.3 kips > 17.5 kips O.K.

2. Determine shear spans and effective widths for both shear and flexure [Ref. 15.3 to 15.5].
   The reaction is considered at outer third point of the bearing pad.

   a. For shear friction

   $$a = 4.5\left[\frac{2}{3}\right] + 1.0 = 4 \text{ in.}$$

   Effective width = W + 4a = 4.5 + 4 (4) = 20.5 in.

b. For flexure, critical section is at center of the hanger reinforcement (A_v)

Assume 1 in. cover and No. 4 bar stirrups

$a_f = 4 + 1 + 0.25 = 5.25$ in.

Effective width $= W + 5a_f = 4.5 + 5\,(5.25) = 30.75$ in.

3. Check concrete bearing strength.

$V_u = 1.2\,(11.1) + 1.6\,(6.4) = 23.6$ kips — *Eq. (9-2)*

$\phi P_{nb} = \phi(0.85\, f'_c A_1)$ — *10.17.1*

$\phi = 0.65$ — *9.3.2.4*

$\phi P_{nb} = 0.65\,(0.85 \times 5 \times 4.5 \times 4.5) = 55.9$ kips $>$ 23.6 kips O.K.

4. Check effective ledge section for maximum nominal shear-transfer strength V_n. — *11.9.3.2.1*

For $f'_c = 5000$ psi: $V_n(\text{max}) = 800 b_w d$, where $b_w = (W + 4a) = 20.5$ in.

$$V_n = \frac{800\ (20.5)\ (10.75)}{1000} = 176.3 \text{ kips}$$

$\phi = 0.75$ — *11.9.3.1*

$\phi V_n = 0.75\,(176.3) = 132.2$ kips $>$ 23.6 kips O.K.

5. Determine shear-friction reinforcement A_{vf}. — *11.9.3.2*

$$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{23.6}{0.75\ (60)\ 1.4} = 0.37 \text{ in.}^2\text{/per effective width of 20.5 in.}$$ — *11.7.4.1*

where $\mu = 1.4$ — *11.7.4.3*

6. Check for punching shear (Eq. (3))

$V_u \quad 4\phi\sqrt{f'_c}\ (W + 2L + 2d_f)\, d_f$

$W = L = 4.5$ in.

$d_f \approx 10$ in. (assumed)

$4\phi\sqrt{f'_c}\ (3W + 2d_f)\, d_f = 4 \times 0.75 \times \sqrt{5000}\ [(3 \times 4.5) + (2 \times 10)] \times 10/1000$

$= 71.1$ kips $>$ 23.6 kips

7. Determine reinforcement to resist direct tension A_n. Unless special provisions are made to reduce direct tension, N_u should be taken not less than $0.2V_u$ to account for unexpected forces due to restrained long-time deformation of the supported member, or other causes. When the beam ledge is designed to resist specific horizontal forces, the bearing plate should be welded to the tension reinforcement A_s. *11.9.3.4*

$N_u = 0.2V_u = 0.2\ (23.6) = 4.7$ kips

$$A_n = \frac{N_u}{\phi f_y} = \frac{4.7}{0.75\ (60)} = 0.10 \text{ in.}^2\text{/per effective width of 30.75 in. (0.003 in.}^2\text{/in.)}$$

8. Determine flexural reinforcement A_f.

$M_u = V_u a_f + N_u\ (h - d) = 23.6\ (5.25) + 4.7\ (12 - 10.75) = 129.8$ in.-kips

Find A_f using conventional flexural design methods. For beam ledges, Ref. 15.5 recommends to use $j_u d = 0.8d$. *11.9.3.3*

$\phi = 0.75$ *11.9.3.1*

$$A_f = \frac{129.8}{0.75\ (60)\ (0.8 \times 10.75)} = 0.34 \text{ in.}^2\text{/per 30.75 in. width} = 0.011 \text{ in.}^2\text{/in.}$$

9. Determine primary tension reinforcement A_s. *11.9.3.5*

$$\left(\frac{2}{3}\right)A_{vf} = \left(\frac{2}{3}\right)0.37 = 0.25 \text{ in.}^2\text{/per 20.5 in. width} = 0.012 \text{ in.}^2\text{/in.}$$

$$A_s = \left(\frac{2}{3}\right)A_{vf} + A_n = 0.012 + 0.003 = 0.015 \text{ in.}^2\text{/in. (governs)}$$

$A_s = A_f + A_n = 0.011 + 0.003 = 0.014$ in.^{2}/in.

Check $A_{s(min)} = 0.04\left(\frac{f'_c}{f_y}\right)$ d per in. width *11.9.5*

$$= 0.04\left(\frac{5}{60}\right)10.75 = 0.036 \text{ in.}^2\text{/in.} > 0.015 \text{ in.}^2\text{/in.}$$

For typical shallow ledge members, minimum A_s by 11.9.5 will almost always govern.

10. Determine shear reinforcement A_h.

$A_h = 0.5\ (A_s - A_n) = 0.5\ (0.036 - 0.003) = 0.017$ in.^{2}/in. *11.9.4*

11. Determine final size and spacing of ledge reinforcement.

For $A_s = 0.036$ in.^{2}/in.:

Try No. 5 bars ($A = 0.31$ in.^{2})

$$s_{max} = \frac{0.31}{0.036} = 8.6 \text{ in.}$$

Use No. 5 @ 8 in.

$A_h = 0.017$ in.^{2}/in. For ease of constructability, provide reinforcement A_h at same spacing of 8 in.

Provide No. 4 ($A = 0.2$ in.^{2}) @ 8 in. within 2/3d adjacent to A_s.

12. Check required area of hanger reinforcement.

For strength (Eq. (4)):

$$A_v = \frac{V_u s}{\phi f_y S}$$

For s = 8 in. and S = 48 in.

$$A_v = \frac{23.6 \times 8}{0.75 \times 60 \times 48} = 0.09 \text{ in.}^2$$

For serviceability (Eq. (5)):

$$A_v = \frac{V}{0.5 f_y} \times \frac{s}{(W + 3a)}$$

$V = 11.1 + 6.4 = 17.5$ kips

$W + 3a = 4.5 + (3 \times 4) = 16.5$ in.

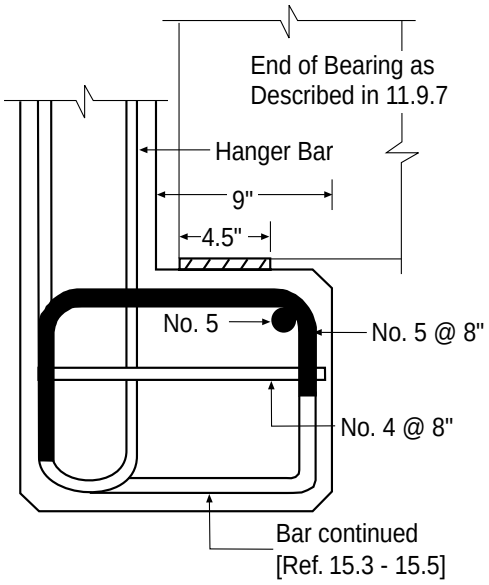
$$A_v = \frac{17.5}{0.5 \times 60} \times \frac{8}{16.5} = 0.28 \text{ in.}^2 \text{ (governs)}$$

No. 5 hanger bars @ 8 in. are required

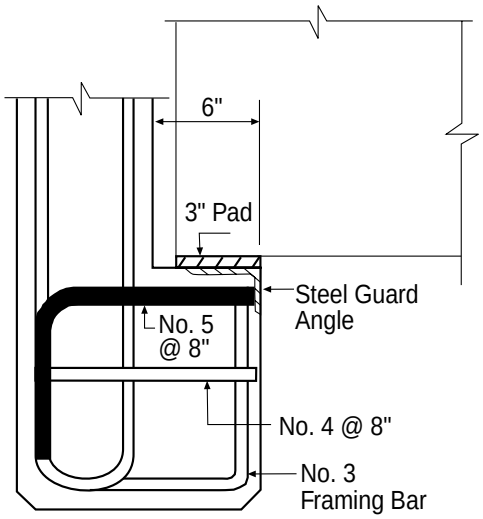
Sufficient stirrups for combined shear and torsion must be provided for global effects in the ledge beam. (See Refs. 15.5 and 15.6)

13. Reinforcement Details

In accordance with 11.9.7, bearing area (4.5 in. pad) must not extend beyond straight portion of beam ledge reinforcement, nor beyond inside edge of transverse anchor bar. With a 4.5 in. bearing pad, this requires that the width of ledge be increased to 9 in. as shown below. Alternately, a 6 in. ledge with a 3 in. medium strength pad (1500 psi) and the ledge reinforcement welded to an armor angle would satisfy the intent of 11.9.7.



9 in. Ledge Detail



6 in. Ledge Detail
(Alternate)

16

Shear in Slabs

UPDATE FOR THE ’02 CODE

The update mainly covers revised provisions for shear reinforcement and shearheads in slabs. Specific provisions that were revised or added are:

- Revised 11.12.3 to include minimum slab thickness required with shear reinforcement.
- Revised 11.12.3.1 to define the computed area of shear reinforcement.
- New 11.12.3.3 to provide details on placement of shear reinforcement.
- New 11.12.3.4 to specify anchorage of shear reinforcement.
- Revised 11.12.6.2 to specify the termination of shear reinforcement at slab-column connection designed for direct shear and moment transfer.

11.12 SPECIAL PROVISIONS FOR SLABS AND FOOTINGS

The provisions of 11.12 must be satisfied for shear design in slabs and footings. Included are requirements for critical shear sections, nominal shear strength of concrete, and shear reinforcement.

11.12.1 Critical Shear Section

In slabs and footing, shear strength in the vicinity of columns, concentrated loads, or reactions is governed by the more severe of two conditions:

- Wide-beam action, or one-way shear, as evaluated by provisions 11.1 through 11.5.
- Two-way action, as evaluated by 11.12.2 through 11.12.6.

Analysis for wide-beam action considers the slab to act as a wide beam spanning between columns. The critical section extends in a plane across the entire width of the slab and is taken at a distance d from the face of the support (11.12.1.1); see Fig. 16-1. In this case, the provisions of 11.1 through 11.5 must be satisfied. Except for long, narrow slabs, this type of shear is seldom a critical factor in design, as the shear force is usually well below the shear capacity of the concrete. However, it must be checked to ensure that shear strength is not exceeded.

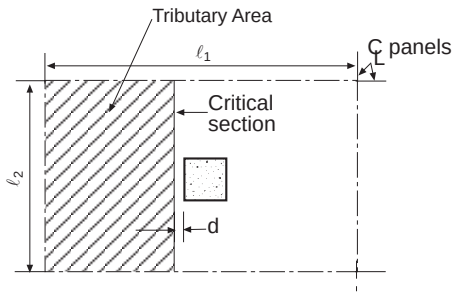


Fig. 16-1 Trbutary Area and Critical Section for Wide-Beam Shear

Two-way or "punching" shear is generally the more critical of the two types of shear in slab systems supported directly on columns. Depending on the location of the column, concentrated load, or reaction, failure can occur along two, three, or four sides of a truncated cone or pyramid. The perimeter of the critical section b_o is located in such a manner that it is a minimum, but need not approach closer than a distance d / 2 from edges or corners of columns, concentrated loads, or reactions, or from changes in slab thickness such as edges of capitals or drop panels (11.12.1.2); see Fig. 16-2. In this case the provisions of 11.12.2 through 11.12.6 must be satisfied. It is important to note that it is permissable to use a rectangular perimeter b_o to define the critical section for square or rectangular columns, concentrated loads, or reaction areas (11.12.1.3).

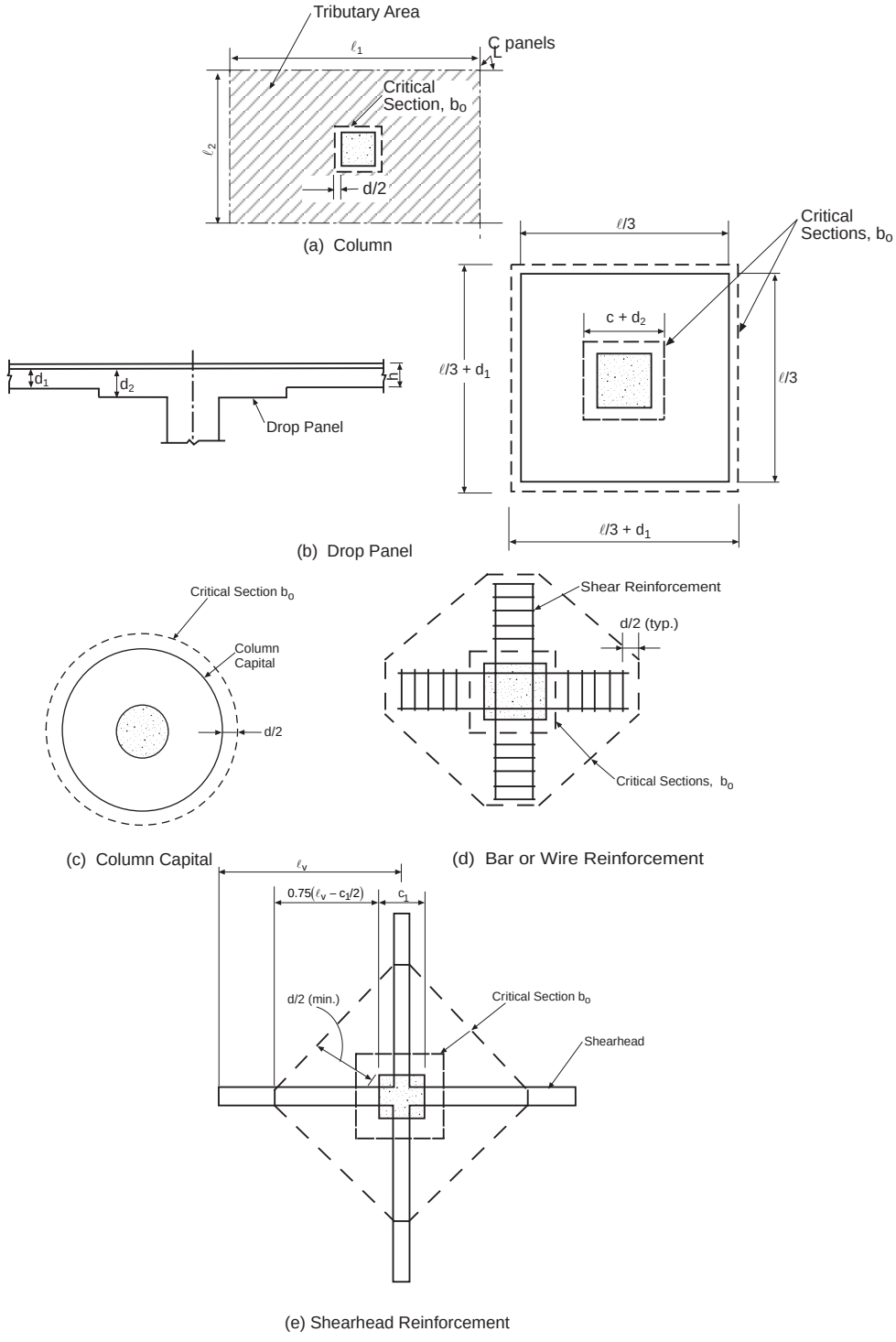


Fig. 16-2 Tributary Areas and Critical Sections for Two-Way Shear

11.12.2 Shear Strength Requirement for Two-Way Action

In general, the factored shear force V_u at the critical shear section shall be less than or equal to the shear strength ϕV_n:

$$\phi V_n \geq V_u \qquad \textit{Eq. (11-1)}$$

where the nominal shear strength V_n is:

$$V_n = V_c + V_s \qquad \textit{Eq. (11-2)}$$

and

V_c = nominal shear strength provided by concrete, computed in accordance with 11.12.2.1 if shear reinforcement is not used or 11.12.3.1 if shear reinforcement is used.

V_s = nominal shear strength provided by reinforcement, if required, computed in accordance with 11.12.3 if bars, wires, or stirrups are used, or 11.2.4 if shearheads are used. Where moment is transferred between the slab and the column in addition to direct shear, 11.2.6 shall apply.

11.12.2.1 Nominal shear strength provided by concrete V_c for slabs without shear reinforcement

The shear stress provided by concrete at a section v_c is a function of the concrete compressive stress f'_c, and is limited to $4\sqrt{f'_c}$ for square columns. The nominal shear strength provided by concrete V_c is obtained by multiplying v_c by the area of concrete section resisting shear transfer, which is equal to the perimeter of the critical shear section b_o multiplied by the effective depth of the slab d:

$$V_c = 4\sqrt{f'_c}\, b_o d \qquad \textit{Eq. (11-35)}$$

Tests have indicated that the value of $4\sqrt{f'_c}$ is unconservative when the ratio of the long and short sides of a rectangular column or loaded area β_c is larger than 2.0. In such cases, the shear stress on the critical section varies as shown in Fig. 16-3. Equation (11-33) accounts for the effect of β_c on the concrete shear strength:

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f'_c}\, b_o d \qquad \textit{Eq. (11-33)}$$

From Fig. 16-3, it can be seen that for $\beta_c \leq 2.0$ (i.e., square or nearly square column or loaded area), two-way shear action governs, and the maximum concrete shear stress v_c is $4\sqrt{f'_c}$. For values of β_c value larger than 2.0, the concrete stress decreases linearly to a minimum $2\sqrt{f'_c}$, which is equivalent to shear stress for one-way shear.

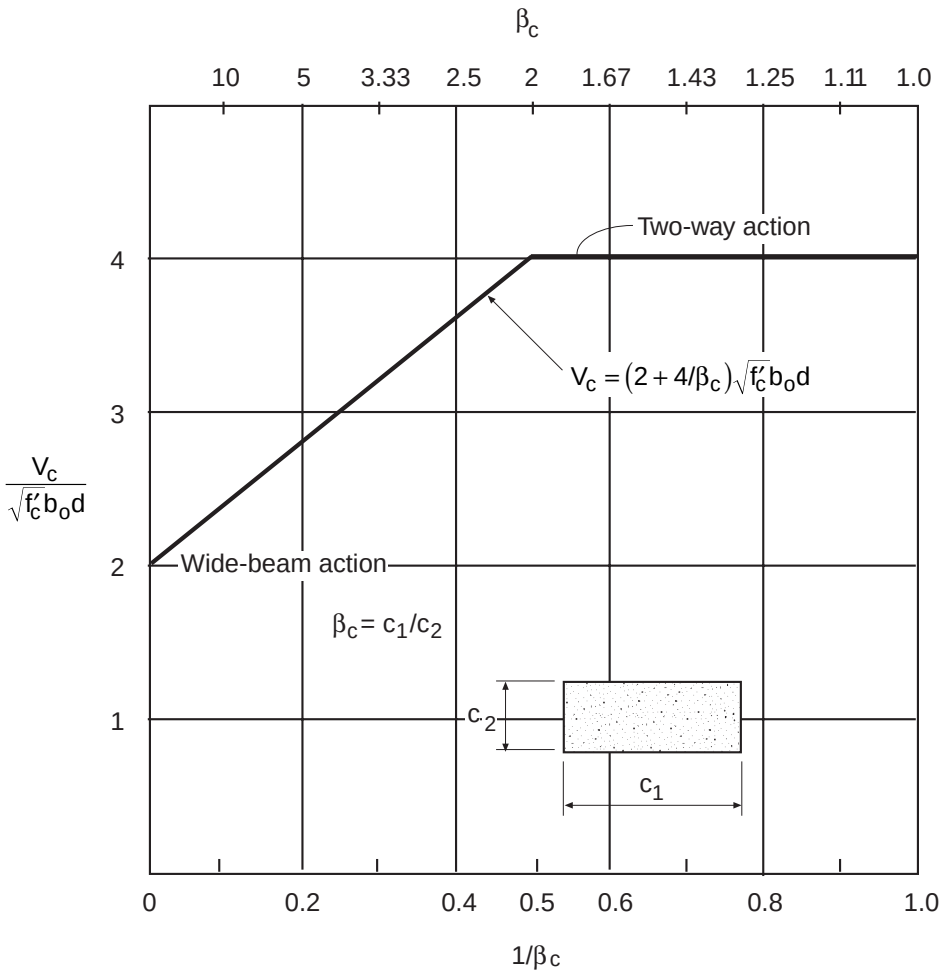


Fig. 16-3 Effect of β_c on Concrete Shear Strength

Other tests have indicated that v_c decreases as the ratio b_o/d increases. Equation (11-34) accounts for the effect of b_o/d on the concrete shear strength:

$$V_c = \left(\frac{\alpha_s d}{b_o} + 2 \right) \sqrt{f'_c} b_o d \qquad \textit{Eq. (11-34)}$$

Figure 16-4 illustrates the effect of b_o/d for interior, edge, and corner columns, where α_s equals 40, 30, and 20, respectively. For an interior column with $b_o/d \leq 2.0$, the maximum permissible shear stress is $4\sqrt{f'_c}$; see Fig. 16-4. Once $b_o/d > 2.0$, the shear stress decreases linearly to $2\sqrt{f'_c}$ at b_o/d equal to infinity.

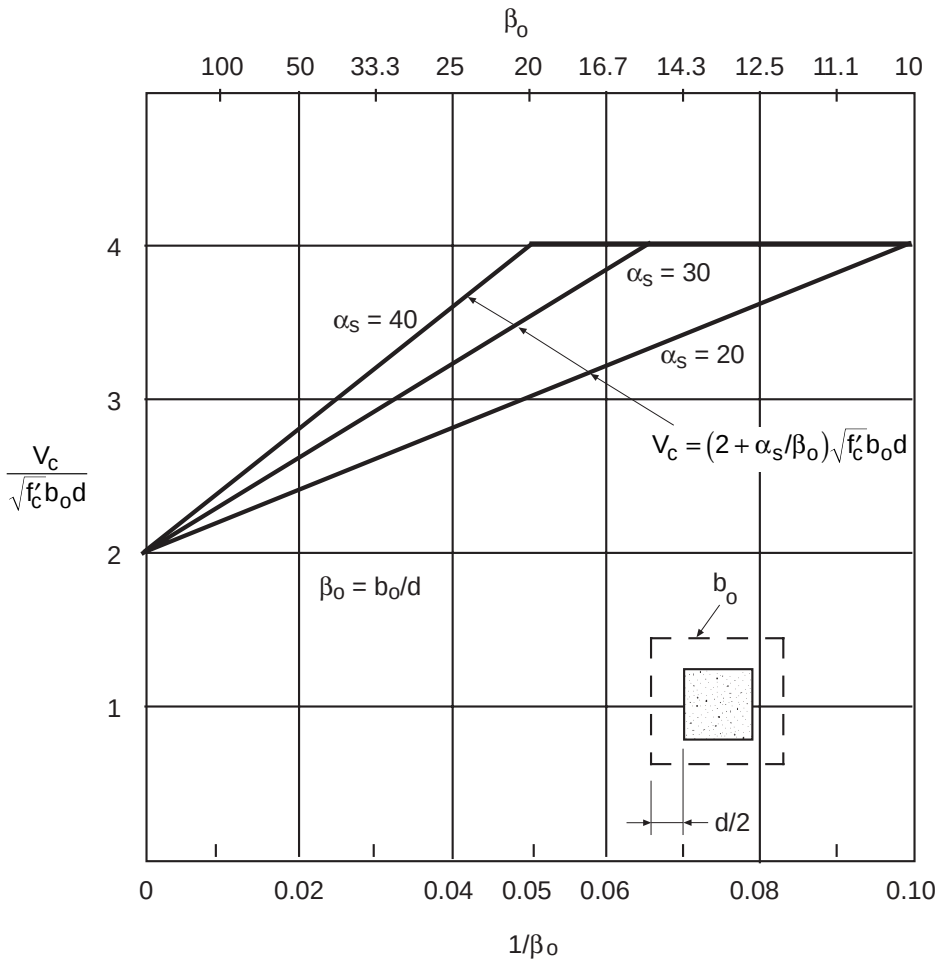


Fig. 16-4 Effect of b_o / d on Concrete Shear Strength

Note that reference to interior, edge, and corner column does not suggest column location in a building, but rather refers to the number of sides of the critical section available to resist the shear stress. For example, a column that is located in the interior of a building, with one side at the edge of an opening, shall be evaluated as an edge column.

The concrete nominal shear strength for two-way shear action of slabs without shear reinforcement is the least of Eqs. (11-33), (11-34), and (11-35) (11.12.2.1). Note that if lightweight concrete is used, 11.2 shall apply.

11.12.3 Shear Strength Provided by Bars, Wires, and Single or Multiple-Leg Stirrups

The use of bars, wires, or single or multiple-leg stirrups as shear reinforcement in slabs is permitted provided that the effective depth of the slab is greater than or equal to 6 inches, but not less than 16 times the shear reinforcement bar diameter (11.12.3). Suggested rebar shear reinforcement consist of properly anchored single-leg, multiple-leg, or closed stirrups that are engaging longitudinal reinforcement at both the top and bottom of the slab (11.12.3.4); see Fig. R11.12.3 (a), (b), (c).

With the use of shear reinforcement, the nominal shear strength provided by concrete V_c shall not be taken greater than $2\sqrt{f'_c}b_o d$ (11.12.3.1), and nominal shear strength V_n is limited to $6\sqrt{f'_c}b_o d$ (11.12.3.2). Thus, V_s must not be greater than $4\sqrt{f'_c}b_o d$.

The area of shear reinforcement A_v is computed from Eq. (11-15), and is equal to the cross-sectional area of all legs of reinforcement on one peripheral line that is geometrically similar to the perimeter of the column section (11.12.3.1):

$$A_v = \frac{V_s s}{f_y d} \qquad \textit{Eq. (11-15)}$$

The spacing limits of 11.12.3.3 correspond to slab shear reinforcement details that have been shown to be effective. These limits are as follows (see Fig. 16-5):

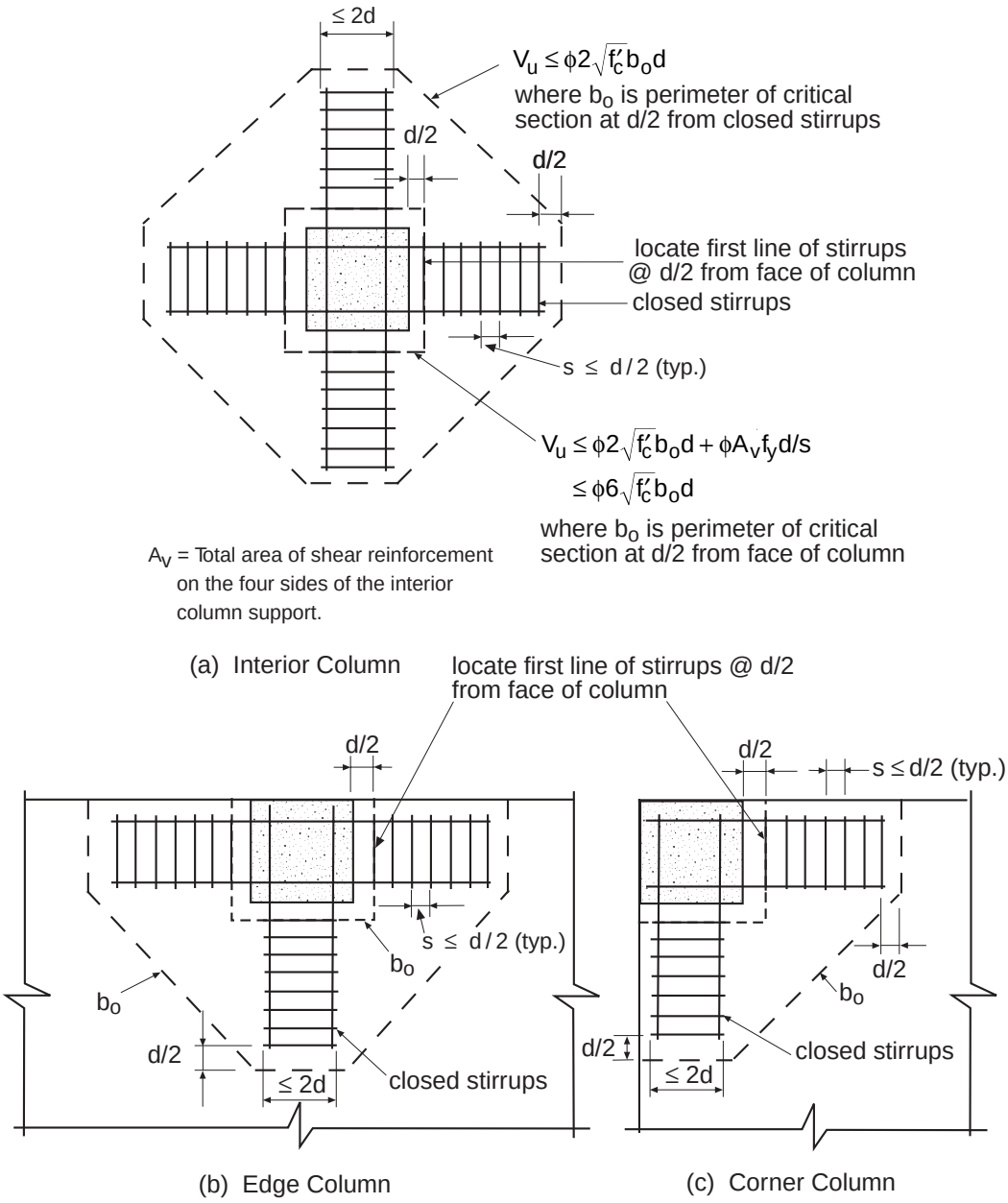


Fig. 16-5 Design and Detailing Criteria for Slabs with Stirrups

1. The first line of stirrups surrounding the column shall be placed at distance not exceeding d/2 from the column face.
2. The spacing between adjacent legs in the first line of shear reinforcement shall not exceed 2d.
3. The spacing between successive lines of shear reinforcement that surround the column shall not exceed d / 2.
4. The shear reinforcement can be terminated when $V_u \le \phi 2\sqrt{f'_c} b_o d$ (11.12.3.1).

Proper anchorage of the shear reinforcement is achieved by satisfying the provisions of 12.13 (11.12.3.4). Refer to Fig. R11.12.3 and Part 4 for additional details on stirrup anchorage. It should be noted that anchorage requirements of 12.13 may be difficult for slabs thinner than 10 inches. Application of shear reinforcement design using bars or stirrups is illustrated in Example 16.3.

Where moment transfer is significant between the column and the slab, it is recommended to use closed stirrups in a pattern as symmetrical as possible around the column (R11.12.3).

11.12.4 Shear Strength Provided by Shearheads

The provisions of 11.12.4 permit the use of structural steel sections such as I- or channel-shaped sections (shearheads) as shear reinforcement in slabs, provided the following criteria are satisfied:

1. Each arm of the shearhead shall be welded to an identical perpendicular arm with full penetration welds and each arm must be continuous within the column section (11.12.4.1); see Fig. 16-6 (a).
2. Shearhead depth shall not exceed 70 times the web thickness of the steel shape (11.12.4.2); see Fig. 16-6 (b).
3. Ends of each shearhead arm is permitted to be cut at angles not less than 30 deg with the horizontal, provided the tapered section is adequate to resist the shear force at that location (11.12.4.3); see Fig. 16-6 (b).
4. All compression flanges of steel shapes shall be located within 0.3d of compression surface of slab, which in the case of direct shear, is the distance measured from the bottom of the slab (11.12.4.4); see Fig. 16-6 (b).
5. The ratio α_v of the flexural stiffness of the steel shape to surrounding composite cracked slab section of width $c_2 + d$ shall not be less than 0.15 (11.12.4.5); see Fig. 16-6 (c).
6. The required plastic moment strength M_p is computed from the following equation (11.12.4.6):

$$\phi M_p = \frac{V_u}{2\eta}\left[h_v + \alpha_v(\ell_v - 0.5c_1)\right] \qquad \textit{Eq. (11-37)}$$

where:

M_p = plastic moment strength for each shearhead arm required to ensure that the ultimate shear is attained as the moment strength of the shearhead is reached.

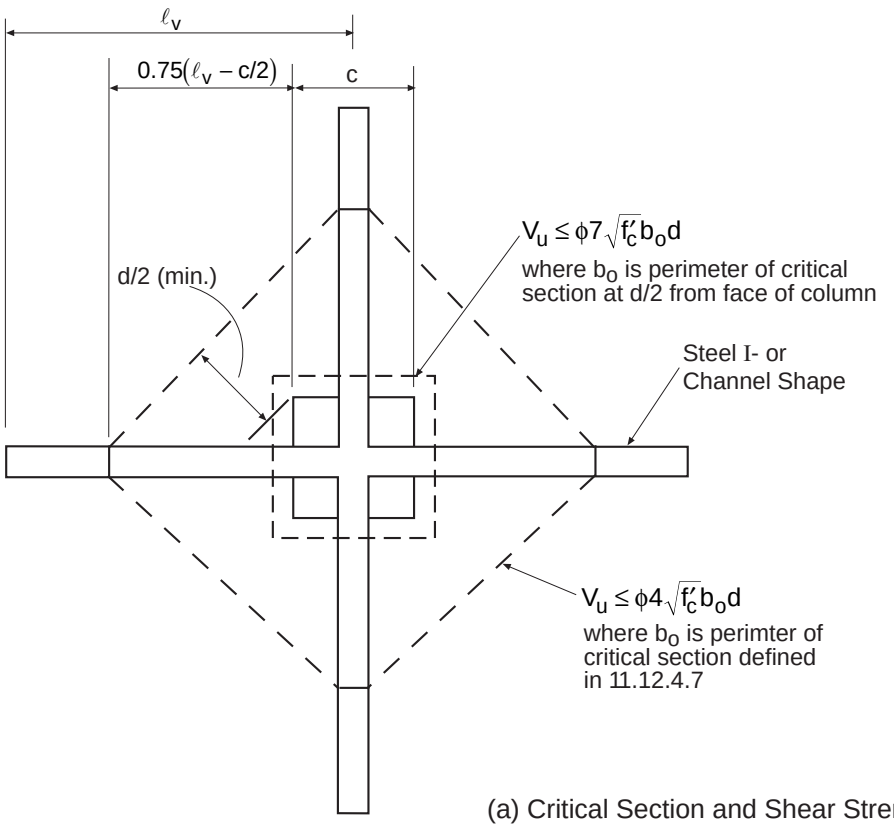
ϕ = strength reduction factor for tension-controlled members, equal to 0.9 per 9.3.2.3.

η = number of shearhead arms; see Fig. R11.12.4.7.

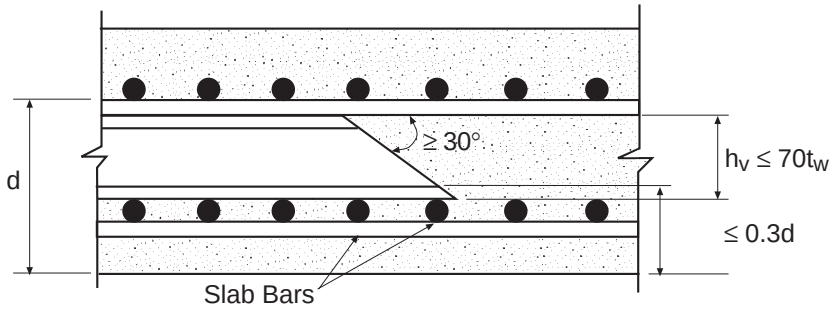
ℓ_v = minimum required length of shearhead arm per 11.12.4.7 and 11.12.4.8; see Fig. R11.12.4.7.

h_v = depth of shearhead cross-section; see Fig. 16-6 (b).

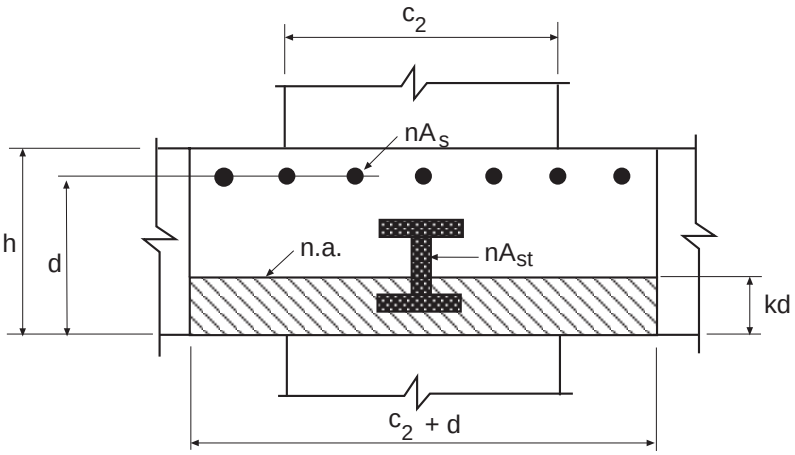
7. The critical section for shear shall be perpendicular to the plane of the slab and shall cross each shearhead arm at three-quarters of the distance $(\ell_v - 0.5c_1)$ from the column face to the end of the shearhead arm. The critical section shall be located per 11.12.1.2(a) (11.12.4.7); see Fig. R11.12.4.7.



(a) Critical Section and Shear Strength



(b) Shearhead Details



(c) Properties of Cracked Composite Slab Section

Fig. 16-6 Design and Detailing Criteria for Slabs with Shearhead Reinforcement

8. The nominal shear strength V_n shall be less than or equal to $4\sqrt{f'_c}b_o d$ on the critical section defined by 11.12.4.7, and $7\sqrt{f'_c}b_o d$ at d / 2 distance from the column face (11.12.4.8); see Fig. 16-6 (a).

9. Section 11.12.4.9 permits the shearheads to contribute in resisting the slab design moment in the column strip. The moment resistance M_v contributed to each column strip shall be the minimum of:

   a. $\dfrac{\phi\alpha_v V_u}{2\eta}\left(\ell_v - 0.5c_1\right)$ *Eq.(11-38)*

   b. $0.30M_u$ of the total factored moment in each column strip
   c. the change in column strip moment over the length ℓ_v
   d. the value of M_p computed by Eq. (11-37).

When direct shear and moment are transferred between slab and column, the provisions of 11.12.6 must be satisfied in addition to the above criteria. In slabs with shearheads, integrity steel shall be provided in accordance with 13.3.8.6. Application on the design of shearheads as shear reinforcement is illustrated in Example 16.3.

Other Type of Shear Reinforcement

Slab shear reinforcement consisting of vertical bars mechanically anchored at each end by a plate or a head capable of developing the yield strength of the bars have been used successfully (R11.12.3); see Fig. 16-7. This type of shear reinforcement for slabs can be advantageous when considering their ease of installation and the cost of placement, compared to other types of slab shear reinforcement. Extensive tests, methods of design, and worked-out design examples are presented in Refs. 16.1 through 16.4.

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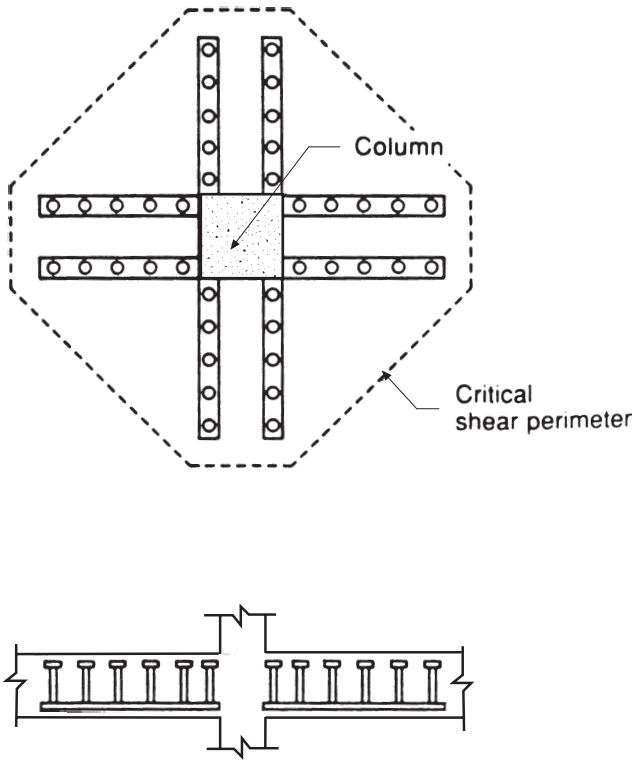


Fig. 16-7 Shear Reinforcement by Headed Studs

11.12.5 Effect of Openings in Slabs on Shear Strength

The effect of openings in slabs on concrete shear strength shall be considered when the opening is located: (1) anywhere within a column strip of a flat slab system and (2) within 10 times the slab thickness from a concentrated load or reaction area. Slab opening effect is evaluated by reducing the perimeter of the critical section b_o by a length equal to the projection of the opening enclosed by two-lines extending from the centroid of the column and tangent to the opening; see Fig 16-8 (a). For slabs with shear reinforcement, the ineffective portion of the perimeter b_o is one-half of that without shear reinforcement; see Fig. 16-8 (b). The one-half factor is interpreted to apply equally to shearhead reinforcement and bar or wire reinforcement as well. Effect of opening in slabs on flexural strength is discussed in Part 18.

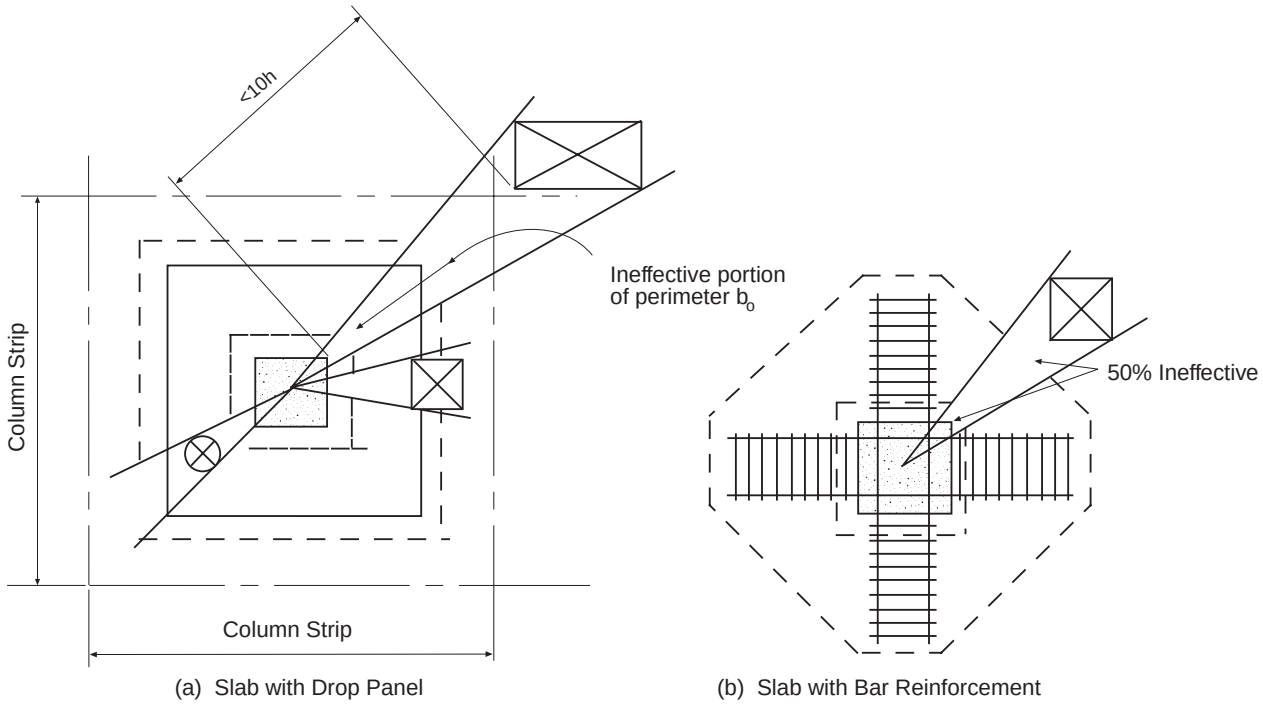


Fig. 16-8 Effect of Openings in Slabs on Shear Strength

11.12.6 Moment Transfer at Slab-Column Connections

For various loading conditions, unbalanced moment M_u can occur at the slab-column connections. For slabs without beams between supports, the transfer of unbalanced moment is one of the most critical design conditions for two-way slab systems. Shear strength at an exterior slab-column connection (without spandrel beam) is especially critical, because the total exterior negative moment must be transferred to the column, which is in addition to the direct shear due to gravity loads; see Fig. 16-9. The designer should not take this aspect of two-way slab design lightly. Two-way slab systems usually are fairly "forgiving" in the event of an error in the amount and or distribution of flexural reinforcement; however, little or no forgiveness is to be expected if shear strength provisions are not fully satisfied.

Note that the provisions of 11.12.6 (or 13.5.3) do not apply to slab systems with beams framing into the column support. When beams are present, load transfer from the slab through the beams to the columns is considerably less critical. Shear strength in slab systems with beams is covered in 13.6.8.

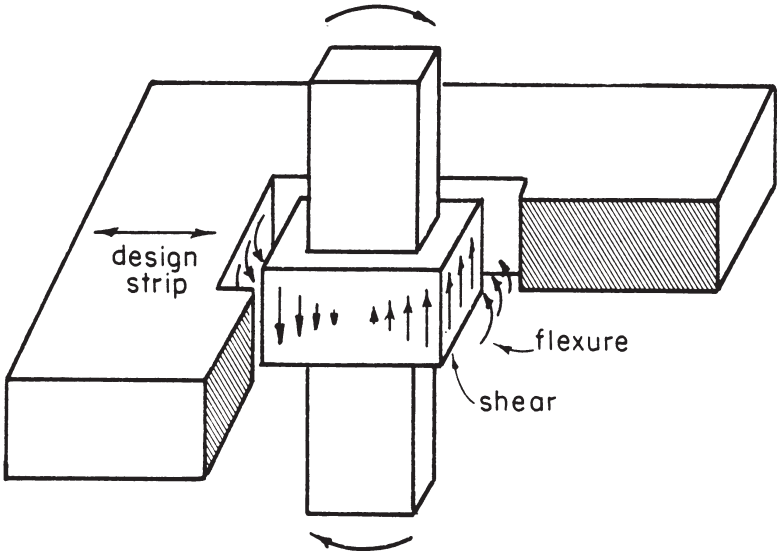


Fig. 16-9 Direct Shear and Moment Transfer

11.12.6.1 Distribution of Unbalanced Moment

The code specifies that the unbalanced moment at a slab-column connection must be transferred from the slab (without beams) to the column by eccentricity of shear in accordance with 11.12.6 and by flexure in accordance with 13.5.3 (11.12.6.1). Studies (Ref. 16.7) of moment transfer between slabs and square columns found that $0.6M_u$ is transferred by flexure across the perimeter of the critical section b_o defined by 11.12.1.2, and $0.4M_u$ by eccentricity of shear about the centroid of the critical section. For a rectangular column, the portion of moment transferred by flexure $\gamma_f M_u$ increases as the dimension of the column that is parallel to the applied moment increases. The fraction of unbalanced moment transferred by flexure γ_f is:

$$\gamma_f = \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} \qquad \textit{Eq. (13-1)}$$

and the fraction of unbalanced moment transferred by eccentricity of shear is:

$$\gamma_v = 1 - \gamma_f \qquad \textit{Eq. (11-39)}$$

where b_1 and b_2 are the dimensions of the perimeter of the critical section, with b_1 parallel to the direction of analysis; see Fig. 16-10. The relationship of the parameters presented into Eqs. (13-1) and (11-39) is graphically illustrated in Fig. 16-11. Modification or adjustment of γ_f and thus γ_v, is permitted in accordance with 13.5.3.3 for any two-way slab system, except for prestressed slabs. The following modifications are applicable, provided that the reinforcement ratio in the slab within the effective width defined in 13.5.3.2 does not exceed $0.375\rho_b$:

- For unbalanced moments about an axis parallel to the slab edge at exterior supports (i.e., bending perpendicular to the edge), it is permitted to take $\gamma_f = 1.0$ provided that $V_u \le 0.75\phi V_c$ at an edge column or $V_u \le 0.5\phi V_c$ at a corner column.
- For unbalanced moments at interior supports and for unbalanced moments about an axis transverse to the edge of exterior supports (i.e., bending parallel to the edge), it is permitted to increase γ_f by up to 25%, provided that $V_u \le 0.4\phi V_c$.

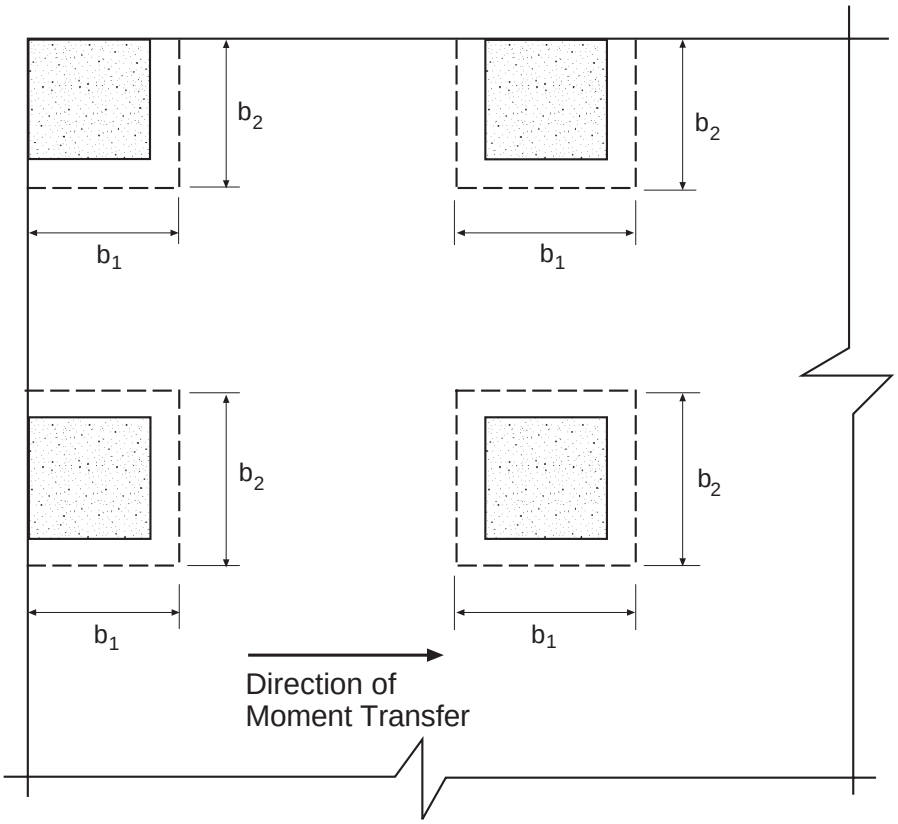


Fig. 16-10 Parameters b_1 and b_2 for Eqs. (11-39) and (13-1)

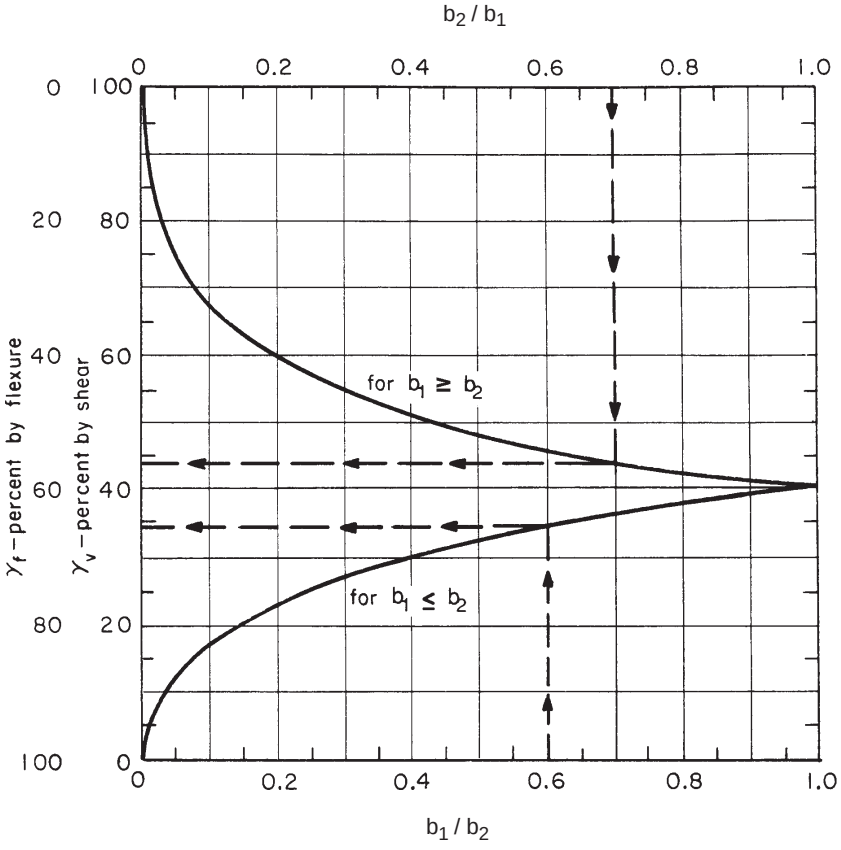


Fig. 16-11 Graphical Solution of Eqs. (13-1) and (11-39)

The unbalanced moment transferred by eccentricity of shear is $\gamma_v M_u$, where M_u is the unbalanced moment at the centroid of the critical section. The unbalanced moment M_u at an exterior support of an end span will generally not be computed at the centroid of the critical transfer section in the frame analysis. When the Direct Design Method of Chapter 13 is utilized, moments are computed at the face of the support. Considering the approximate nature of the procedure to evaluate the stress distribution due to moment-shear transfer, it seems unwarranted to consider a change in moment to the transfer centroid; use of the moment values from frame analysis (centerline of support) or from 13.6.3.3 (face of support) is accurate enough.

Unbalanced moment transfer between an edge column and a slab without edge beams requires special consideration when slabs are analyzed for gravity loads using the moment coefficients of the Direct Design Method. In this case, unbalanced moment M_u must be set equal to $0.3M_o$ (13.6.3.6), where M_o is the total factored static moment in the span. Therefore, the fraction of unbalanced moment transferred by shear is $\gamma_v M_u = \gamma_v(0.3M_o)$. See Part 19 for further discussion of that special shear strength requirement and its application in Example 19.1. If the Equivalent Frame Method is used, the unbalanced moment is equal to the computed frame moment.

11.12.6.2 Shear Stresses and Strength Computation

Assuming that shear stress resulting from moment transfer by eccentricity of shear varies linearly about the centroid of the critical section defined in 11.12.1.2, the factored shear stresses at the faces of the critical section due to the direct shear V_u and the unbalanced moment transferred by eccentricity of shear $\gamma_v M_u$ are (see Fig. 16-12, and R11.12.6.2):

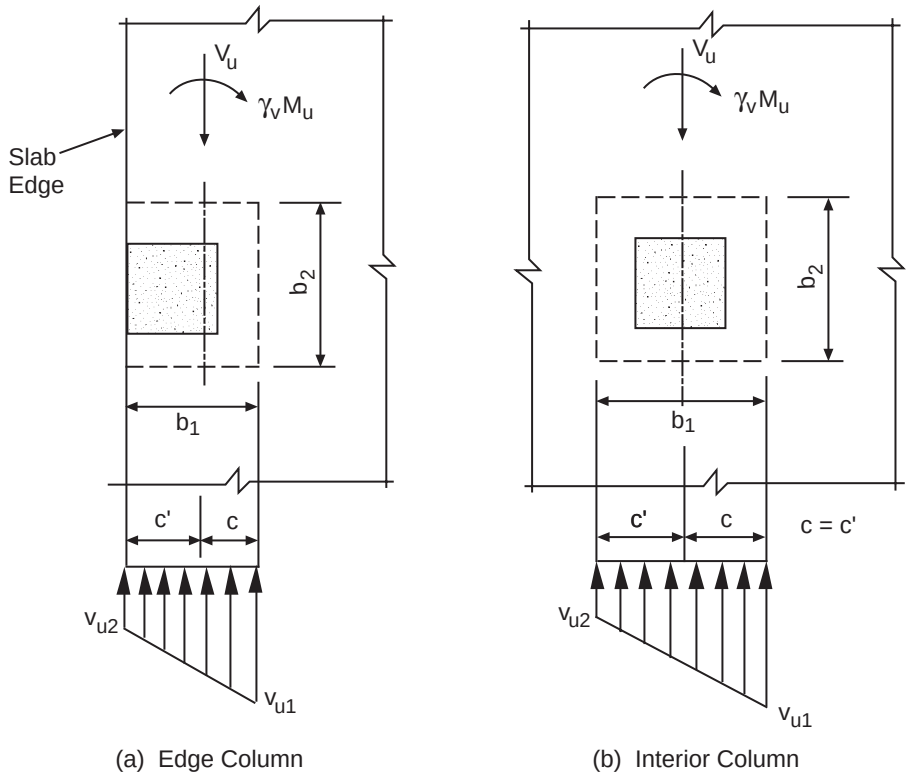


Fig. 16-12 Shear Stress Distribution due to Moment-Shear Transfer at Slab-Column Connection

$$v_{u1} = \frac{V_u}{A_c} + \frac{\alpha_v M_u c}{J} \qquad \textit{Eq. (1)}$$

$$v_{u2} = \frac{V_u}{A_c} - \frac{\gamma_v M_u c'}{J} \qquad \textit{Eq. (2)}$$

where: A_c = area of concrete section resisting shear transfer, equal to the perimeter b_o multiplied by the effective depth d

J = property of critical section analogous to polar moment of inertia of segments forming area A_c.

c and c' = distances from centroidal axis of critical section to the perimeter of the critical section in the direction of analysis

Expressions for $A_c, c, c', J/c$, and J/c', are contained in Fig. 16-13 for rectangular columns and Fig. 16-14 for circular interior columns.

The maximum shear stress v_{u1} computed from Eq. (1) shall not exceed ϕv_n, where ϕv_n is determined from the following (11.12.6.2):

a. For slabs without shear reinforcement: $\phi v_n = \phi v_c$, where ϕv_n is the minimum of:

$$\phi v_c = \phi\left(2 + \frac{4}{\beta_c}\right)\sqrt{f'_c} \qquad \textit{Eq. (11-33)}$$

$$\phi v_c = \phi\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f'_c} \qquad \textit{Eq. (11-34)}$$

$$\phi v_c = \phi 4\sqrt{f'_c} \qquad \textit{Eq. (11-35)}$$

b. For slabs with shear reinforcement other than shearheads, ϕv_n is computed from (11.12.3):

$$\phi v_n = \phi\left(2\sqrt{f'_c} + \frac{A_v f_y}{b_o s}\right) \le \phi 6\sqrt{f'_c} \qquad \textit{Eqs. (11-15), (11.12.3.1), and (11.12.3.2)}$$

where A_v is the total area of shear reinforcement provided on the column sides and b_o is the perimeter of the critical section located at d / 2 distance away from the column perimeter, as defined by 11.12.1.2 (a). Due to the variation in shear stresses, as illustrated in Fig. 16-12, the computed area of shear reinforcement, if required, may be different from one column side to the other. The required area of shear reinforcement due to shear stress v_{u1} at its respective column side is:

$$A_v = \left(v_{u1} - \phi v_c\right)\frac{(c + d)s}{\phi f_y} \qquad \textit{Eq. (3)}$$

where $(c+d)$ is an effective "beam" width and $v_c = 2\sqrt{f'_c}$. However, R11.12.3 recommends symmetrical placement if shear reinforcement on all column sides. Thus, with symmetrical shear reinforcement assumed on all sides of the column, the required area A_v may be computed from:

$$A_v = (v_{u1} - \phi v_c)\frac{b_o s}{\phi f_y} \qquad \textit{Eq. (4)}$$

where A_v is the total area of required shear reinforcement to be extended from the sides of the column, and b_o is the perimeter of the critical section located at d/2 from the column perimeter. With symmetrical reinforcement on all column sides, the reinforcement extending from the column sides with lesscomputed shear stress provides torsional resistance in the strip of slab perpendicular to the direction of analysis.

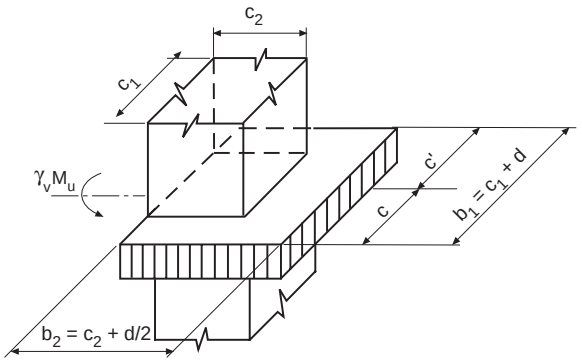
c. For slabs with shearheads as shear reinforcement, ϕv_n is computed from:

$$\phi v_n = \phi 4\sqrt{f'_c} \geq v_{u1} \qquad \textit{11.12.6.3}$$

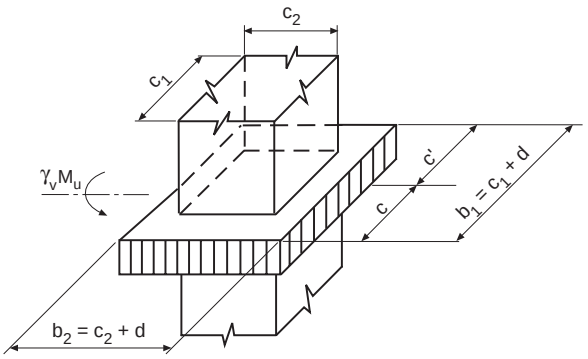
$$v_{u1} = \frac{V_u}{b_o d} + \frac{\gamma_v M_u c}{J} \leq \phi 4\sqrt{f'_c} \qquad \textit{Eq. (1)}$$

where b_o is the perimeter of the critical section defined in 11.12.4.7, c and J are section properties of the critical section located at d / 2 from the column perimeter (11.12.6.3), V_u is the direct shear force acting on the critical section defined in 11.12.4.7, and $\gamma_v M_u$ is the unbalanced moment transferred by eccentricity of shear acting about the centroid of the critical section defined in 11.12.1.2(a). Note that this seemingly inconsistent summation of shear stresses occurring at two different critical shear sections is conservative and justified by tests (see R11.12.6.3). At the critical section located d / 2 from the column perimeter, v_u shall not exceed $\phi 7\sqrt{f'_c}$ (11.12.4.8); see Fig. 16-5.

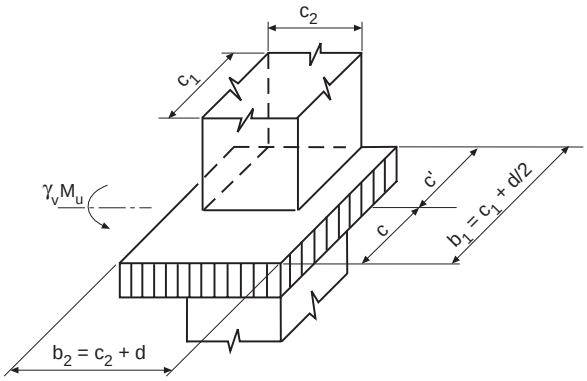
Case A: Edge Column (Bending parallel to edge)



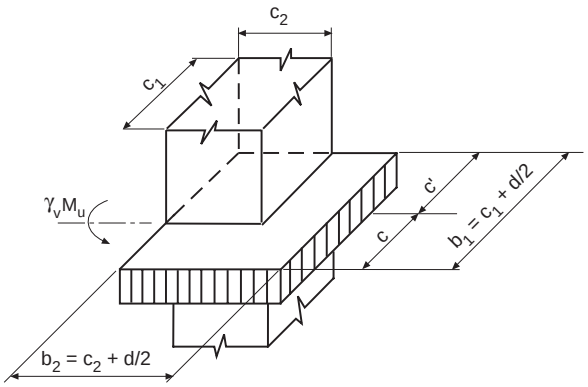
Case B: Interior Column



Case C: Edge Column (Bending perpendicular to edge)



Case D: Corner Column



Case	Area of critical section, A_c	Modulus of critical section		c	c'
		J/c	J/c'		
A	$(b_1+2b_2)d$	$\frac{b_1d(b_1+6b_2)+d^3}{6}$	$\frac{b_1d(b_1+6b_2)+d^3}{6}$	$\frac{b_1}{2}$	$\frac{b_1}{2}$
B	$2(b_1+b_2)d$	$\frac{b_1d(b_1+3b_2)+d^3}{3}$	$\frac{b_1d(b_1+3b_2)+d^3}{3}$	$\frac{b_1}{2}$	$\frac{b_1}{2}$
C	$(2b_1+b_2)d$	$\frac{2\,b_1^2d(b_1+2b_2)+d^3(2b_1+b_2)}{6b_1}$	$\frac{2\,b_1^2d(b_1+2b_2)+d^3(2b_1+b_2)}{6(b_1+b_2)}$	$\frac{b_1^2}{2b_1+b_2}$	$\frac{b_1(b_1+b_2)}{2b_1+b_2}$
D	$(b_1+b_2)d$	$\frac{b_1^2d(b_1+4b_2)+d^3(b_1+b_2)}{6b_1}$	$\frac{b_1^2d(b_1+4b_2)+d^3(b_1+b_2)}{6(b_1+2b_2)}$	$\frac{b_1^2}{2(b_1+b_2)}$	$\frac{b_1(b_1+2b_2)}{2(b_1+b_2)}$

Fig. 16-13 Section Properties for Shear Stress Computations – Rectangular Columns

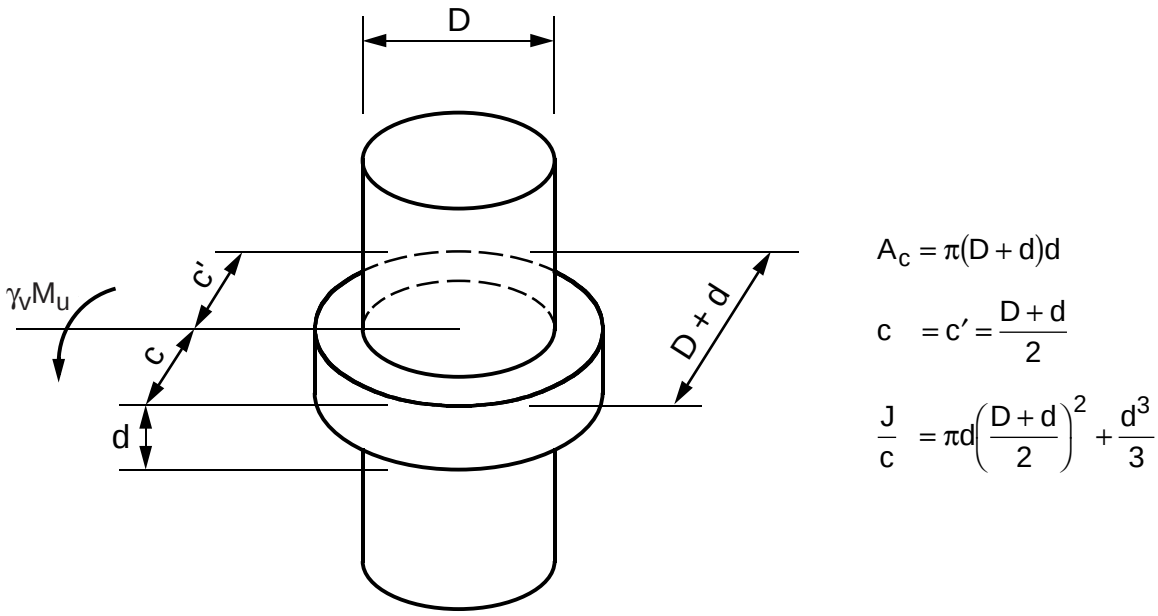


Fig. 16-14 Section Properties for Shear Stress Computations – Circular Interior Column

REFERENCES

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16.4 Hammill, N. and Ghali, A., "Punching Shear Resistance of Corner Slab-Column Connections," *ACI Structural Journal*, Vol. 91, No. 6, November-December 1994, pp. 697-707.

16.5 Ghosh, S.K., "Aspects of Design of Reinforced Concrete Flat Plate Slab Systems," *Analysis and Design of High-Rise Concrete Buildings*, SP-97, American Concrete Institute, Detroit, MI, 1985, pp. 139-157.

16.6 *Simplified Design - Reinforced Concrete Buildings of Moderate Size and Height*, 2nd edition, Portland Cement Association, Skokie, IL, 1993.

16.7 Hanson, N.W., and Hanson, J.M., "Shear and Moment Transfer Between Concrete Slabs and Columns," Journal, PCA Research and Development Labs, V-10, No. 1, Jan. 1968, pp. 2-16.

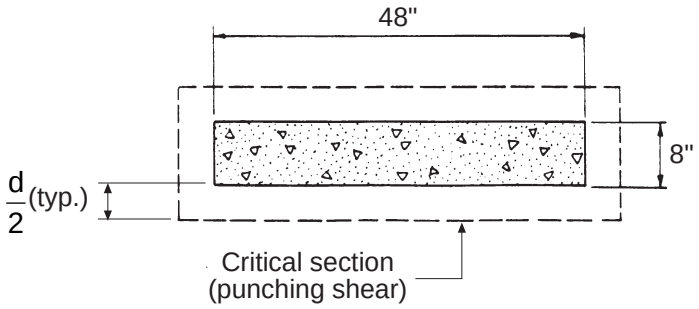
Example 16.1—Shear Strength of Slab at Column Support

Determine two-way action shear strength at an interior column support of a flat plate slab system for the following design conditions.

Column dimensions = 48 in. × 8 in.

Slab effective depth d = 6.5 in.

Specified concrete strength f'_c = 4,000 psi



Calculations and Discussion	Code Reference

1. Two-way action shear (punching shear) without shear reinforcement:

$V_u \le \phi V_n$ — *Eq. (11-1)*

$\le \phi V_c$ — *11.12.2*

2. Effect of loaded area aspect ratio β_c:

$$\phi V_c = \phi\left(2 + \frac{4}{\beta_c}\right)\sqrt{f'_c}\, b_o d$$ — *Eq. (11-35)*

where $\beta_c = \frac{48}{8} = 6$ — *11.12.2.1*

$b_o = 2\,(48 + 6.5 + 8 + 6.5) = 138$ in. — *11.12.1.2*

$\phi = 0.75$ — *9.3.2.3*

$\phi V_c = 0.75 \times 138 \times 6.5/1{,}000 = 113.5$ kips

3. Effect of perimeter area aspect ratio β_o:

$$\phi V_c = \phi\left(2 + \frac{\alpha_s}{\beta_o}\right)\sqrt{f'_c}\, b_o d$$ — *Eq.(11-36)*

where $\alpha_s = 40$ for interior column support — *11.12.2.1*

$$\beta_o = \frac{b_o}{d} = \frac{138}{6.5} = 21.2$$

$\phi V_c = 0.75 \times 138 \times 6.5/1{,}000 = 165.4$ kips

4. Excluding effect of β_c and β_o:

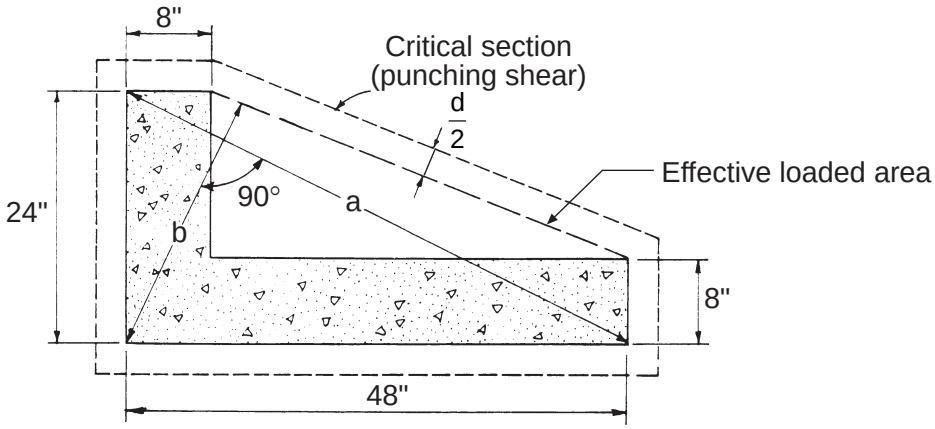
$$\phi V_c = \phi 4\sqrt{f'_c}\, b_o d$$ *Eq. (11-37)*

$$= 0.75 \times 4 \times \sqrt{4{,}000} \times 138 \times 6.5/1{,}000 = 170.2 \text{ kips}$$

5. The shear strength ϕV_n is the smallest of the values computed above, i.e., $\phi V_n = 113.5$ kips.

Example 16.2—Shear Strength for Non-Rectangular Support

For the L-shaped interior column support shown, check punching shear strength for a factored shear force of V_u = 125 kips. Use f'_c = 4,000 psi. Effective slab depth = 5.5 in.



Calculations and Discussion	Code Reference

1. For shapes other than rectangular, R11.12.2.1 recommends that β_c be taken as the ratio of the longest overall dimension of the effective loaded area a to the largest overall dimension of the effective loaded area b, measured perpendicular to a: *R11.12.2.1*

$$\beta_c = \frac{a}{b} = \frac{54}{25} = 2.16$$

For the critical section shown, b_o = 141 in. *11.12.1.2*

Scaled dimensions of the drawings are used, and should be accurate enough

2. Two-way action shear (punching shear) without shear reinforcement:

$$V_u \le \phi V_n \qquad \textit{Eq. (11-1)}$$

$$\le \phi V_c \qquad \textit{11.12.2}$$

where the nominal shear strength V_c without shear reinforcement is the lesser of values given by Eqs. (11-33) and (11-34), but not greater than $4\sqrt{f'_c}\, b_o d$:

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f'_c}\, b_o d \qquad \textit{Eq. (11-33)}$$

$$= \left(2 + \frac{4}{2.16}\right)\sqrt{4{,}000} \times 141 \times 5.5 / 1{,}000 = 188.9 \text{ kips}$$

$$V_c = \left(2 + \frac{\alpha_s}{\beta_o}\right)\sqrt{f'_c}\, b_o d \qquad \textit{Eq. (11-34)}$$

where α_s = 40 for interior column support *11.12.2.1*

$$\beta_o = \frac{b_o}{d} = \frac{141}{5.5} = 25.6$$

$$V_c = \left(2 + \frac{40}{25.6}\right)\sqrt{4{,}000} \times 141 \times 5.5 / 1{,}000 = 174.7 \text{ kips}$$

$V_c = 4\sqrt{f'_c}\, b_o d$ *Eq. (11-35)*

$= 4\sqrt{4{,}000} \times 141 \times 5.5 / 1{,}000 = 196.2$ kips

$\phi V_c = 0.75\ (174.7) = 131$ kips

$V_u = 125$ kips $< \phi V_c = 131$ kips O.K.

Example 16.3—Shear Strength of Slab with Shear Reinforcement

Consider an interior panel of a flat plate slab system supported by a 12-in. square column. Panel size $\ell_1 = \ell_2 =$ 21 ft. Determine shear strength of slab at column support, and if not adequate, increase the shear strength by shear reinforcement. Overall slab thickness h = 7.5 in. (d = 6 in.).

f'_c = 4,000 psi

f_y = 60,000 psi (bar reinforcement)

f_y = 36,000 psi (structural steel)

Superimposed factored load = 160 psf

Column strip negative moment M_u = 175 ft-kips

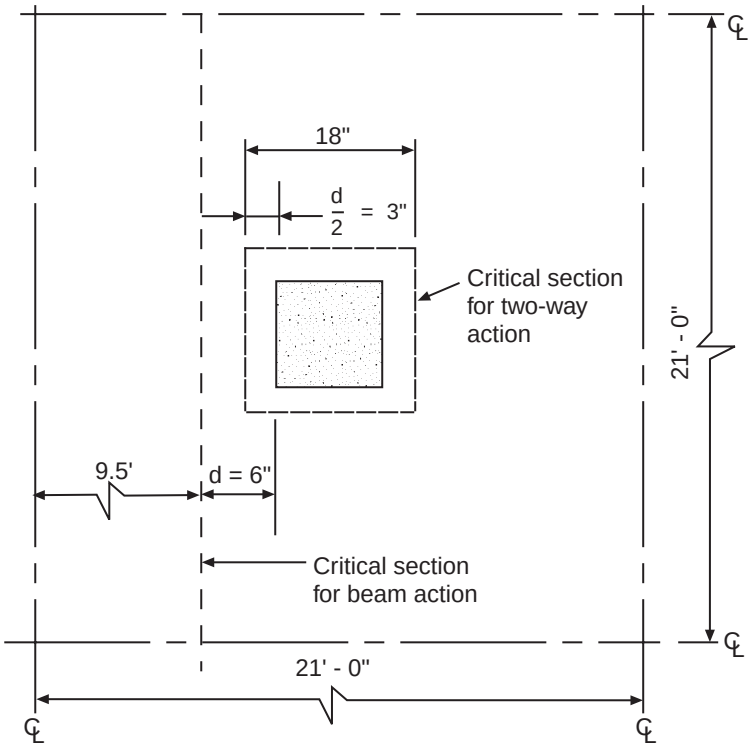
Calculations and Discussion	Code Reference

1. Wide-beam action shear and two-way action shear (punching shear) without shear reinforcement: *11.12.2*

   $V_u \le \phi V_n$ *Eq. (11-1)*

   $\le \phi V_c$ *11.12.2*

   a. Since there are no shear forces at the center lines of adjacent panels, tributary areas and critical sections for slab shear are as shown below.



For 7.5-in. slab, factored dead load $w_d = 1.2 \times \frac{7.5}{12} \times 150 = 113$ psf — 9.2.1

$w_u = 113 + 160 = 273$ psf

a. Wide-Beam Action Shear.

Investigation of wide-beam action shear strength is made at the critical section at a distance d from face of column support. — 11.1.3.1

$V_u = 0.273\ (9.5 \times 21) = 54.5$ kips

$V_c = 2\sqrt{f'_c}\, b_w d = 2\sqrt{4{,}000}\ (21 \times 21) \times 6/1{,}000 = 191.3$ kips — Eq. (11-3)

$\phi = 0.75$ — 9.3.2.3

$\phi V_c = 0.75\ (191.3) = 143.5$ kips $> V_u = 54.5$ kips O.K.

Wide-beam action will rarely control the shear strength of two-way slab systems.

b. Two-Way Action Shear.

Investigation of two-way action shear strength is made at the critical section b_o located at d/2 from the column perimeter. Total factored shear force to be transferred from slab to column: — 11.12.1.2(a)

$V_u = 0.273\ (21^2 - 1.5^2) = 119.8$ kips

Shear strength V_c without shear reinforcement: — 11.12.2.1

$b_o = 4\ (18) = 72$ in. — 11.12.1.2(a)

$\beta_c = \frac{12}{12} = 1.0 < 2$ — Eq. (11-35)

$\beta_o = \frac{b_o}{d} = \frac{72}{6} = 12 < 20$ — Eq. (11-36)

$V_c = 4\sqrt{f'_c}\, b_o d = 4\sqrt{4{,}000} \times 72 \times 6/1{,}000 = 109.3$ kips

$\phi = 0.75$ — 9.3.2.3

$\phi V_c = 0.75\ (109.3) = 82$ kips $< V_u = 119.8$ kips N.G.

Shear strength of slab is not adequate to transfer the factored shear force $V_u = 119.8$ kips from slab to column support. Shear strength may be increased by:

i. increasing concrete strength f'_c
ii. increasing slab thickness at column support, i.e., using a drop panel
iii. providing shear reinforcement (bars, wires, or steel I- or channel-shapes)

The following parts of the example will address all methods to increase shear strength.

2. Increase shear strength by increasing strength of slab concrete:

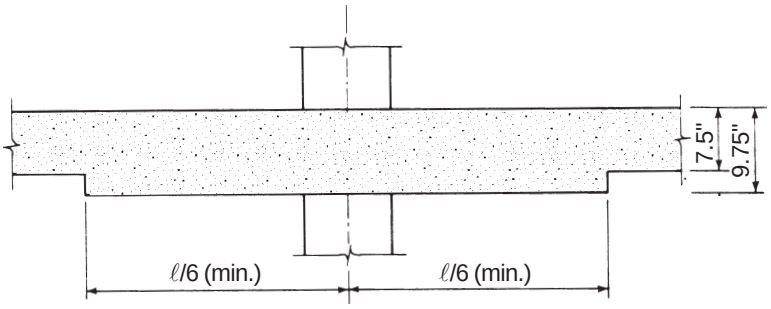
   $V_u \leq \phi V_n$ *Eq. (11-1)*

   $119{,}800 \leq 0.75\left(4\sqrt{f'_c} \times 72 \times 6\right)$

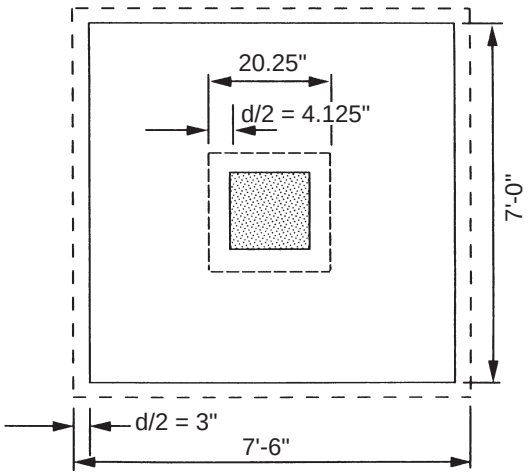
   Solving, $f'_c = 8{,}545$ psi

3. Increase shear strength by increasing slab thickness at column support with drop panel:

   Provide drop panel in accordance with 9.5.3.4 and 13.4.7 (see Fig. 18-18). Minimum overall slab thickness at drop panel = 1.25 (7.5) = 9.375-in. Try a 9.75 in. slab thickness (2.25-in. projection below slab*; d ≈ 8.25 in.). Minimum distance from centerline of column to edge of drop panel = 21/6 = 3.5 ft. Try 7 × 7 ft drop panel.



Drop Panel Section



   a. Investigate shear strength at critical section b_o located at d/2 from column perimeter.

      Total factored shear force to be transferred —

* See Chapter 9 (Design Considerations for Economical Formwork) in Ref. 16.6.

For 2.25-in. drop panel projection, $w_d = 1.2 \times \frac{2.25}{12} \times 150 = 34$ psf

$V_u = 0.273\,(21^2 - 1.69^2) + 0.034\,(7^2 - 1.69^2) = 119.6 + 1.6 = 121.2$ kips

$b_o = 4\,(12 + 8.25) = 81$ in. *11.12.1.2(a)*

$\beta_c = 1.0 < 2$

$\beta_o = \frac{b_o}{d} = \frac{81}{8.25} = 9.8 < 20$

$\phi V_c = \phi 4\sqrt{f'_c}\, b_o d$ *Eq. (11-35)*

$= 0.75 \times 4\sqrt{4{,}000} \times 81 \times 8.25 > V_u = 121.2$ kips O.K.

b. Investigate shear strength at critical section b_o located at d/2 from edge of drop panel.

Total factored shear force to be transferred —

$V_u = 0.273\,(21^2 - 7.5^2) = 105.0$ kips

$b_o = 4\,(84 + 6) = 360$ in. *11.12.1.2(b)*

$\beta_c = \frac{84}{84} = 1.0 < 2$ *Eq. (11-35)*

$\beta_o = \frac{b_o}{d} = \frac{360}{6} = 60 > 20$

$\phi V_c = \phi\left(2 + \frac{\alpha_s}{\beta_o}\right)\sqrt{f'_c} b_o d = \phi\left(2 + \frac{40}{60}\right)\sqrt{f'_c} b_o d = \phi 2.67\sqrt{f'_c} b_o d$ *Eq. (11-36)*

$= 0.75 \times 2.67\sqrt{4{,}000} \times 360 \times 6 / 1{,}000 = 273.2$ kips $> V_u = 105.0$ kips O.K.

Note the significant decrease in potential shear strength at edge of drop panel due to large β_o.

A 7 × 7 ft drop panel with a 2.25-in. projection below the slab will provide adequate shear strength for the superimposed factored loads of 160 psf.

4. Increase shear strength by bar reinforcement (see Figs. R11.12.3(a) and 16-5):

   a. Check effective depth d — *11.12.3*

   Assuming No. 3 stirrups ($d_b = 0.375$ in.),

   $$d = 6 \text{ in.} \geq \begin{cases} 6 \text{ in. O.K.} \\ 16 \times 0.375 = 6 \text{ in. O.K.} \end{cases}$$

   b. Check maximum shear strength permitted with bars. — *11.12.3.2*

   $V_u \leq \phi V_n$ — *Eq. (11-1)*

   $$\phi V_n = \phi\left(6\sqrt{f'_c}\, b_o d\right) = 0.75\left(6\sqrt{4{,}000} \times 72 \times 6\right)/1{,}000 = 123.0 \text{ kips}$$

   $V_u = 0.273\,(21^2 - 1.5^2) = 119.8 \text{ kips} < \phi V_n = 123.0 \text{ kips}$ O.K.

   c. Determine shear strength provided by concrete with bar shear reinforcement. — *11.12.3.1*

   $$V_c = 2\sqrt{f'_c}\, b_o d = 2\sqrt{4{,}000} \times 72 \times 6 / 1{,}000 = 54.6 \text{ kips}$$

   $\phi V_c = 0.75\,(54.6) = 41.0$ kips

   d. Design shear reinforcement in accordance with 11.5.

   Required area of shear reinforcement A_v is computed by

   $$A_v = \frac{(V_u - \phi V_c)\, s}{\phi f_y d}$$

   Assumes = 3 in. (maximum spacing permitted = d/2) — *11.5.4.1*

   $$A_v = \frac{(119.8 - 41.0) \times 3}{0.75 \times 6.0 \times 6} = 0.88 \text{ in.}^2$$

   where A_v is total area of shear reinforcement required on the four sides of the column (see Fig. 16-5).

   $$A_v \text{ (per side)} = \frac{0.88}{4} = 0.22 \text{ in.}^2$$

e. Determine distance from sides of column where stirrups may be terminated (see Fig. 16-5).

$V_u \le \phi V_c$ *Eq. (11-1)*

$\le \phi 2\sqrt{f'_c}\, b_o d$

For square column (see sketch below),

$b_o = 4\left(12 + a\sqrt{2}\right)$

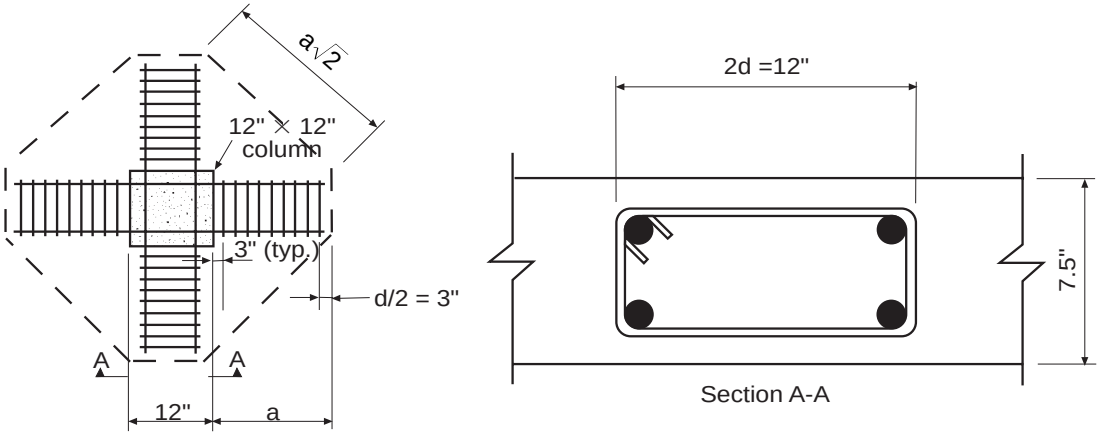
$119{,}800 \le 0.75 \times 2\sqrt{4000} \times 4\left(12 + a\sqrt{2}\right) \times 6$

Solving, $a = 28.7$ in.

Note that the above is a conservative estimate, since V_u at the perimeter of the critical section shown below is considerably lower than 119.8 kips.

Stirrups may be terminated at d/2 = 3 in. inside the critical perimeter b_o.

Use 9-No. 3 closed stirrups @ 3 in. spacing ($A_v = 0.22$ in.^{2}) along each column line as shown below.



5. Increase shear strength by steel I shapes (shearheads): *11.12.4*

a. Check maximum shear strength permitted with steel shapes (see Fig. 18-8). *11.12.4.8*

$V_u = 0.273\,(21^2 - 1.5^2) = 119.8$ kips

$V_u \le \phi V_n$ *Eq. (11-1)*

$\phi V_n = \phi\left(7\sqrt{f'_c} b_o d\right)$ *11.12.1.2(a)*

$\le 0.75\left(7\sqrt{4{,}000} \times 72 \times 6\right)/1{,}000 = 143.4 \text{ kips} > V_u = 119.8 \text{ kips}$ O.K.

b. Determine minimum required perimeter b_o of a critical section at shearhead ends with shear strength limited to $V_n = 4\sqrt{f'_c}\, b_o d$ (see Fig. 16-6 (b)).

$V_u \le \phi V_n$ *Eq. (11-1)*

$119{,}800 \le 0.75\left(4\sqrt{4{,}000} \times b_o \times 6\right)$

Solving, $b_o = 105.2$ in. *11.12.4.7*

c. Determine required length of shearhead arm ℓ_v to satisfy $b_o = 105.2$ in. at $0.75\ (\ell_v - c_1/2)$. *11.12.4.7*

$$b_o \approx 4\sqrt{2}\left[\frac{c_1}{2} + \frac{3}{4}\left(\ell_v - \frac{c_1}{2}\right)\right] \text{ (see Fig. 16-6 (b))}$$

With $b_o = 105.2$ in. and $c_1 = 12$ in., solving, $\ell_v = 22.8$ in.

Note that the above is a conservative estimate, since V_u at the perimeter of the critical section considered is considerably lower than 119.8 kips.

d. To ensure that premature flexural failure of shearhead does not occur before shear strength of slab is reached, determine required plastic moment strength M_p of each shearhead arm.

$$\phi M_p = \frac{V_u}{2\eta}\left[h_v + \alpha_v\left(\ell_v - \frac{c_1}{2}\right)\right]$$ *Eq. (11-39)*

For a four (identical) arm shearhead, $\eta = 4$; assuming $h_v = 4$ in. and $\alpha_v = 0.25$: *11.12.4.5*

$$\phi M_p = \frac{119.8}{2(4)}\left[4 + 0.25\left(23.6 - \frac{12}{2}\right)\right] = 125.8 \text{ in.-kips}$$

$\phi = 0.9$ (tension-controlled member) *9.3.2.1*

Required $M_p = \dfrac{125.8}{0.9} = 139.8$ in.-kips

Try W4 × 13 (plastic modulus $Z_x = 6.28$ in.^{3}) A36 steel shearhead

$M_p = Z_x f_y = 6.28\ (36) = 226.1$ in.-kips $>$ 139.8 in.-kips O.K.

e. Check depth limitation of W4 × 13 shearhead. *11.12.4.2*

$70 t_w = 70\ (0.280) = 19.6$ in. $> h_v = 4.16$ in. O.K.

f. Determine location of compression flange of steel shape with respect to compression surface of slab, assuming 3/4-in. cover and 2 layers of No. 5 bars. *11.12.4.4*

$0.3d = 0.3\ (6) = 1.8$ in. $< 0.75 + 2\ (0.625) = 2$ in. N.G.

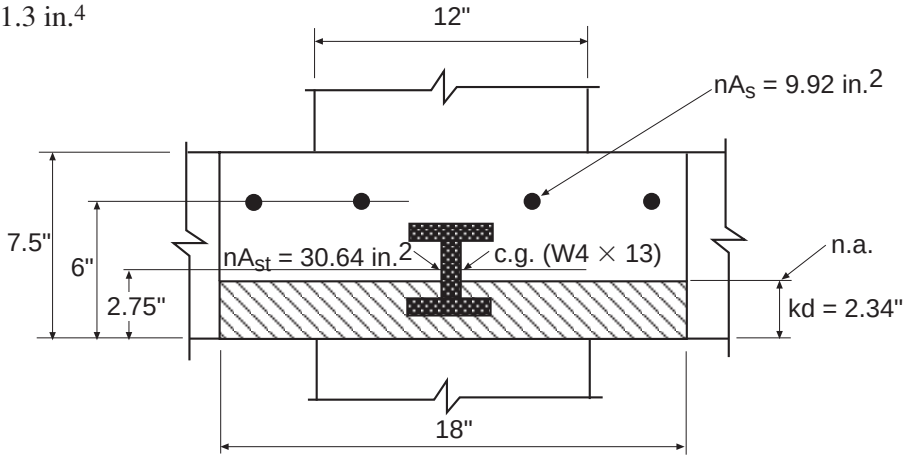
Therefore, both layers of the No. 5 bars in the bottom of the slab must be cut.

g. Determine relative stiffness ratio α_v.

For the W4 × 13 shape:

A_{st} = 3.83 in.^{2}

I_s = 11.3 in.^{4}



A_s provided for M_u = 175 ft-kips is No. 5 @ 5 in.

c.g. of W4 × 13 from compression face = 0.75 + 2 = 2.75 in.

Effective slab width = c_2 + d = 12 + 6 = 18 in.

Transformed section properties:

For f'_c = 4,000 psi, use $n = \frac{E_s}{E_c} = \frac{29{,}000}{3605} = 8$

Steel transformed to equivalent concrete:

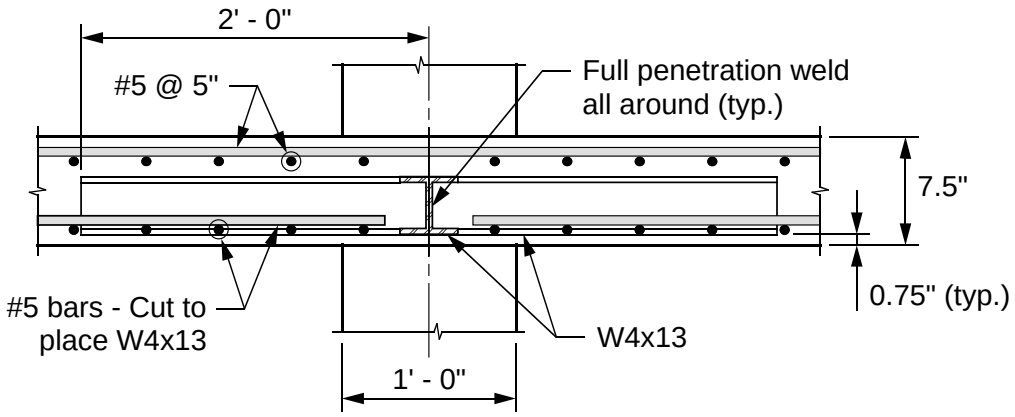
$nA_s = 8\ (4 \times 0.31) = 9.92$ in.^{2}

$nA_{st} = 8\ (3.83) = 30.64$ in.^{2}

Neutral axis of composite cracked slab section may be obtained by equating the static moments of the transformed areas.

$$\frac{18\ (kd)^2}{2} = 30.64\ (2.75 - kd) + 9.92\ (6 - kd)$$

Solving, kd = 2.34 in.



Final Details of Shearhead Reinforcement

$$\text{Composite I} = \frac{18\,(2.34)^3}{3} + n\,(I_s \text{ steel shape}) + 9.92\,(3.66)^2 + 30.64\,(0.41)^2$$

$$= 76.9 + 8\,(11.3) + 132.9 + 5.2 = 305.4 \text{ in.}^4$$

$$\alpha_v = \frac{nI_s}{I_{composite}} = \frac{8 \times 11.3}{305.4} = 0.30 > 0.15 \qquad \text{O.K.}$$

Therefore, W4 × 13 section satisfies all code requirements for shearhead reinforcement.

h. Determine contribution of shearhead to negative moment strength of column strip. *11.12.4.9*

$$M_v = \frac{\phi\alpha_v V_u}{2\eta}\left(\ell_v - \frac{c_1}{2}\right)$$ *Eq. (11-40)*

$$= \frac{0.9 \times 0.30 \times 119.8}{2 \times 4}(25 - 6) = 76.8 \text{ in.-kips} = 6.4 \text{ ft-kips}$$

However, M_v must not exceed either M_p = 139.8 in.-kips or 0.3 × 175 × 12 = 630 in.-kips, or the change in column strip moment over the length ℓ_v. For this design, approximately 4% of the column strip negative moment may be considered resisted by the shearhead reinforcement.

Example 16.4—Shear Strength of Slab with Transfer of Moment

Consider an exterior (edge) panel of a flat plate slab system supported by a 16-in. square column. Determine shear strength for transfer of direct shear and moment between slab and column support. Overall slab thickness h = 7.25 in. (d ≈ 6.0 in.). Assume that the Direct Design Method is used for analysis of the slab. Consider two loading conditions:

1. Total factored shear force V_u = 30 kips

   Total factored static moment M_o in the end span = 96 ft-kips

2. V_u = 60 kips

   M_o = 170 ft-kips

   f'_c = 4,000 psi

   f_y = 60,000 psi

Calculations and Discussion	Code Reference

1. Section properties for shear stress computations:

   Referring to Fig. 16-13, edge column bending perpendicular to edge (Case C),

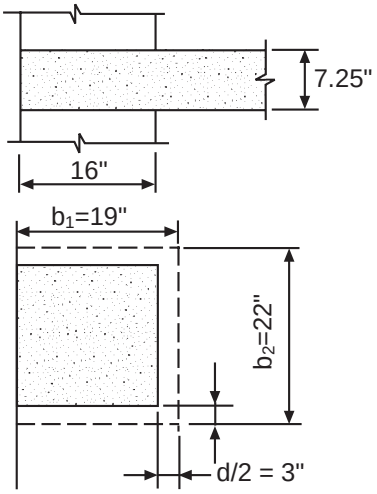
$$b_1 = c_1 + \frac{d}{2} = 16 + \frac{6}{2} = 19.0 \text{ in.}$$

$$b_2 = c_2 + d = 16 + 6 = 22.0 \text{ in.}$$

$$b_o = 2\,(19.0) + 22 = 60.0 \text{ in.}$$

$$c = \frac{b_1^2}{2b_1 + b_2}$$

$$= \frac{19.0^2}{(2\times 19.0) + 22.0} = 6.02 \text{ in.}$$



$$A_c = (2b_1 + b_2)\,d = 360 \text{ in.}^2$$

$$\frac{J}{c} = \frac{\left[2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)\right]}{6b_1} = 2{,}508 \text{ in.}^3$$

$$c' = b_1 - c = 19 - 6.02 = 12.98 \text{ in.}$$

$$\frac{J}{c'} = \left(\frac{J}{c}\right)\left(\frac{c}{c'}\right) = 2{,}508\left(\frac{6.02}{12.98}\right) = 1{,}163 \text{ in.}^3$$

2. Loading condition (1), $V_u = 30$ kips, $M_o = 96$ ft-kips:

   a. Portion of unbalanced moment to be transferred by eccentricity of shear. *11.12.6.1*

   $\gamma_v = 1 - \gamma_f$ *Eq. (11-39)*

   For unbalanced moments about an axis parallel to the edge at exterior supports, the value of γ_f can be taken equal to 1.0 provided that $V_u \le 0.75\phi V_c$. *13.5.3.3*

   $V_c = 4\sqrt{f'_c}\, b_o d$ *Eq. (11-35)*

   $= 4\sqrt{4{,}000} \times 60 \times 6.0/1{,}000 = 91.1$ kips

   $\phi = 0.75$ *9.3.2.3*

   $0.75\phi V_c = 0.75 \times 0.75 \times 91.1 = 51.2 \text{ kips} > V_u = 30 \text{ kips}$

   Therefore, all of the unbalanced moment at the support may be considered transferred by flexure (i.e., $\gamma_f = 1.0$ and $\gamma_v = 0$). Note that γ_f can be taken as 1.0 provided that ρ within the effective slab width $3h + c_2 = 21.75 + 16 = 37.75$ in. is not greater than $0.375\rho_b$.

   b. Check shear strength of slab without shear reinforcement.

   Combined shear stress along inside face of critical transfer section.

   $$v_{u1} = \frac{V_u}{A_c} + \frac{\gamma_v M_u c}{J} = \frac{30{,}000}{360} + 0 = 83.3 \text{ psi}$$

   Permissible shear stress:

   $\phi v_n = \phi 4\sqrt{f'_c} = 0.75\left(4\sqrt{4{,}000}\right) = 189.7 \text{ psi} > v_{u1} = 83.3 \text{ psi}$ O.K.

   Slab shear strength is adequate for the required shear and moment transfer between slab and column.

   Design for the portion of unbalanced moment transferred by flexure $\gamma_f M_u$ must also be considered. See Example 19.1 when using the Direct Design Method. See Example 20.1 for the Equivalent Frame Method. *13.5.3.2*

   For the Direct Design Method, $\gamma_f M_u = 1.0 \times (0.26\, M_o) = 25$ ft-kips to be transferred over the effective width of 37.75 in., provided that ρ within the 37.75-in. width $\le 0.375\, \rho_b$. *13.6.3.3* *13.5.3.3*

3. Loading condition (2), V_u = 60 kips, M_o = 170 ft-kips:

a. Check shear strength of slab without shear reinforcement.

Portion of unbalanced moment to be transferred by eccentricity of shear. *11.12.6.1*

$0.75\phi V_c = 51.2 \text{ kips} < V_u = 60 \text{ kips}$ *13.5.3.3*

Therefore, $\gamma_v = 1 - \gamma_f$ *Eq. (11-39)*

$$\gamma_f = \frac{1}{1+\frac{2}{3}\sqrt{\frac{b_1}{b_2}}} = \frac{1}{1+\frac{2}{3}\sqrt{\frac{19.0}{22.0}}} = 0.62$$ *Eq. (13-1)*

$\gamma_v = 1 - 0.62 = 0.38$

For the Direct Design Method, the unbalanced moment M_u to be used in the shear stress computation for the edge column = $0.3M_o = 0.3 \times 170 = 51.0$ ft-kips. *13.6.3.6*

Combined shear stress along inside face of critical transfer section,

$$v_{u1} = \frac{V_u}{A_c} + \frac{\gamma_v M_u c}{J}$$

$$= \frac{60{,}000}{360} + \frac{0.38 \times 51.0 \times 12{,}000}{2{,}508}$$

$$= 166.7 + 92.7 = 259.4 \text{ psi}$$

$\phi v_n = 189.7 \text{ psi} < v_{u1} = 259.4 \text{ psi}$ N.G.

Shear reinforcement must be provided to carry excess shear stress; provide either bar reinforcement or steel I- or channel-shapes (shearheads).

Increase slab shear strength by bar reinforcement.

b. Check maximum shear stress permitted with bar reinforcement. *11.12.3.2*

Check effective depth, d *11.12.3*

Assuming No. 3 stirrups (d_b = 0.375 in.),

$$d = 6 \text{ in.} \geq \begin{cases} 6 \text{ in. O.K.} \\ 16 \times 0.375 = 6 \text{ in. O.K.} \end{cases}$$

$$v_{u1} \leq \phi 6\sqrt{f'_c}$$

$$\phi v_n = 0.75\left(6\sqrt{4{,}000}\right) = 284.6 \text{ psi}$$

$$v_{u1} = 259.4 \text{ psi} < \phi v_n = 284.6 \text{ psi} \quad \text{O.K.}$$

c. Determine shear stress carried by concrete with bar reinforcement. *11.12.3.1*

$$\phi V_c = \phi 2\sqrt{f'_c} = 0.75\left(2\sqrt{4{,}000}\right) = 94.9 \text{ psi}$$

d. With symmetrical shear reinforcement on all sides of column, required A_v is

$$A_v = \frac{(v_{u1} - \phi v_c)\, b_o s}{\phi f_y} \qquad \textit{Eq. (4)}$$

where b_o is perimeter of critical section located at d/2 from column perimeter

$b_o = 2(19) + 22 = 60$ in. and

$$s = \frac{d}{2} = 3.0 \text{ in.}$$

$$A_v = \frac{(259.4 - 94.9) \times 60 \times 3.0}{0.75 \times 60{,}000} = 0.66 \text{ in.}^2$$

A_v is total area of shear reinforcement required on the three sides of the column.

$$A_v \text{ (per side)} = \frac{0.66}{3} = 0.22 \text{ in.}^2$$

Use No. 3 closed stirrups @ 3.0 in. spacing (A_v = 0.22 in.^{2})

A check on the calculations can easily be made; for No. 3 closed stirrups @ 3.0 in.:

$$\phi (v_c + v_s) = \phi\left(2\sqrt{f'_c} + \frac{A_v f_y}{b_o s}\right) \qquad \textit{Eq. (11-41)}$$

$$= 0.75\left[2\sqrt{4{,}000} + \frac{(3\times 0.22)\times 60{,}000}{60\times 3.0}\right]$$

$= 0.75\ (126.5 + 220.0) = 259.9$ psi $> v_{u1} = 259.4$ psi O.K.

e. Determine distance from sides of column where stirrups may be terminated.

$V_u \le \phi V_c$ *11.12.3.1*

$\phi V_c = \phi 2\sqrt{f'_c} b_o d$

where $b_o = 2a\sqrt{2} + (3 \times 16)$

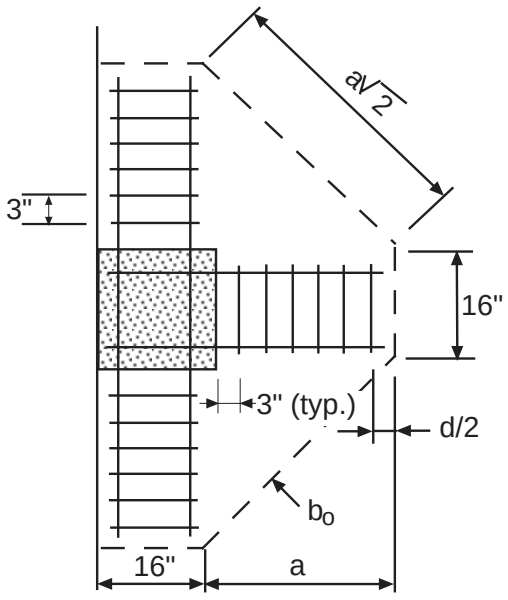
$$60{,}000 \le 0.75\times 2\sqrt{4{,}000}\left(2a\sqrt{2} + 48\right)6.0$$

Solving, a = 20.3 in.

Note that the above is a conservative estimate, since V_u at the perimeter of the critical section considered is considerably lower than 60 kips.

No. of stirrups required = (20.3 - d/2)/3.0 = 5.8
(Stirrups may be terminated at d/2 = 3.0 in. inside perimeter b_o)

Use 6-No. 3 closed stirrups @ 3.0 in. spacing along the three sides of the column.
Use similar stirrup detail as for Example 16.3.



17

Strut-and-Tie Models

UPDATE FOR '02 CODE

The 2002 edition of ACI 318 Appendix A introduces the codified form of the strut-and-tie method in the US building code. While truss analogies were implicit in the design provisions for shear and torsion of earlier editions of the code, and allowance was given to "methods that satisfy equilibrium and strength requirements," this is the first consistent treatment of the subject.

GENERAL

The strut-and-tie model provides a novel design approach, applicable to an array of design problems that do not have an explicit design solution in the body of the code. This method requires the designer to consciously select a realistic load path within the structural member in the form of an idealized truss. Rational detailing of the truss elements and compliance with equilibrium assures the safe transfer of loads to the supports or to other regions designed by conventional procedures. While solutions provided with this powerful design and analysis tool are not unique, they represent a conservative lower bound approach. As opposed to some of the prescriptive formulations in the body of ACI 318, the very visual, rational strut-and-tie model of Appendix A gives insight into detailing needs of irregular regions of concrete structures and promotes ductility.

The design methodology presented in Appendix A is largely based on the seminal articles on the subject by Schlaich et al.[17.1] and Marti[17.2]. Since publication of these papers, the strut-and-tie method has received increased attention by researchers and textbook writers (Collins and Mitchell[17.3], MacGregor[17.4]). The present form of Appendix A does not include explicit serviceability provisions (such as deflection or crack control).

A.1 DEFINITIONS

The strut-and tie design procedure calls for the distinction of two types of zones in a concrete component depending on the characteristics of stress fields at each location. Thus, structural members are divided into B-regions and D-regions.

B-regions represent portions of a member in which the "plane section" assumptions of the classical beam theory can be applied.
D-regions are all the zones outside the B-regions where cross-sectional planes are not expected to remain plane upon loading. D-regions are typically assumed at portions of a member where **discontinuities** (or disturbances) of stress distribution occur due to concentrated forces (loads or reactions) or abrupt changes of geometry. Based on St. Venant's Principle, the normal stresses (due to axial load and bending) approach quasi-linear distribution at a distance approximately equal to the larger of the overall height (h) and width of the member, away from the

location of the concentrated force or geometric irregularity. Figure 17-1 shows typical discontinuities, D-Regions (cross-hatched areas), and B-Regions.

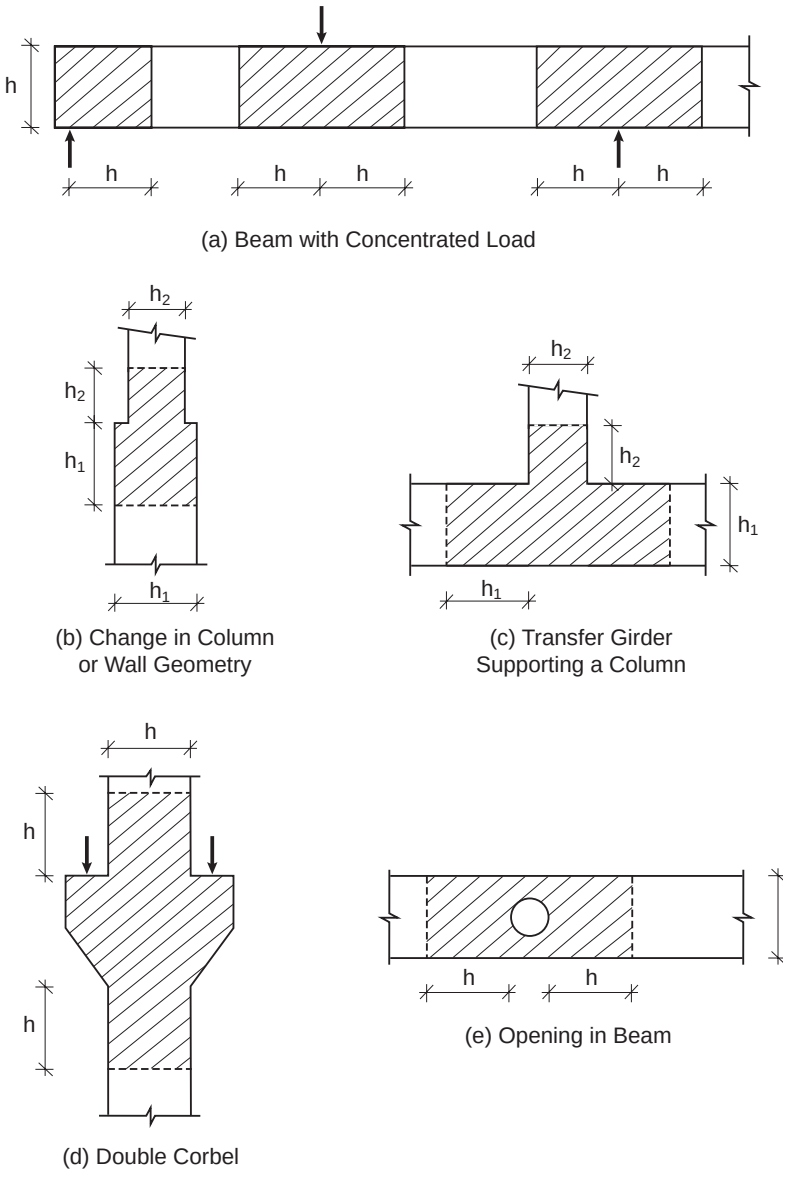


Figure 17-1 Load and Geometric Discontinuities

While B-regions can be designed with the traditional methods (ACI 318 Chapters 10 and 11), the **strut-and-tie model** was primarily introduced to facilitate the design of D-regions, and can be extended to the B-regions as well. The strut-and-tie model depicts the D-region of the structural member with a truss system consisting of compression struts and tension ties connected at nodes as shown in Fig. 17-2. This truss system is designed to transfer the factored loads to the supports or to adjacent B-regions. At the same time, forces in the truss members should maintain equilibrium with the applied loads and reactions.

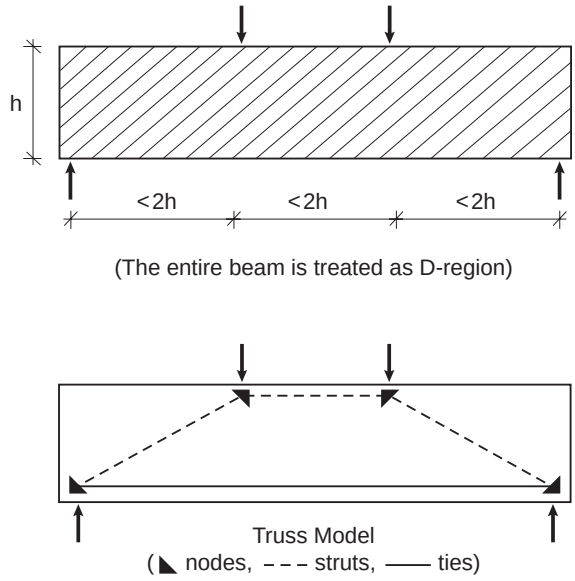


Figure 17-2 Strut-and-Tie Model

Struts are the compression elements of the strut-and-tie model representing the resultants of a compression field. Both parallel and fan shaped compression fields can be modeled by their resultant compression struts as shown in Fig. 17-3.

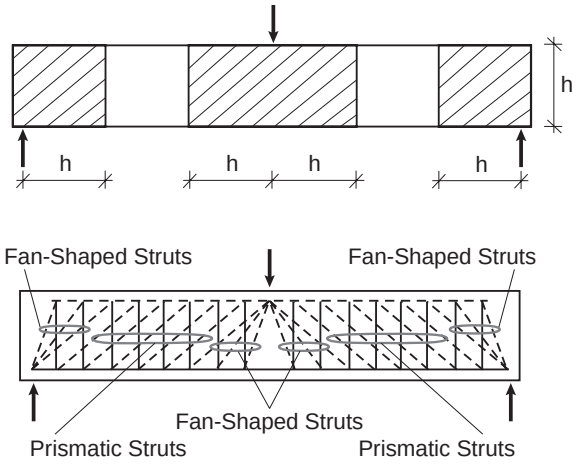


Figure 17-3 Prismatic and Fan-Shaped Struts

Typically compression struts would take a bottle-shape wherever the strut can spread laterally at mid-length. As a design simplification, prismatic compression members commonly idealize struts, however, other shapes are also possible. If the effective concrete compression strength (f_{cu}) is different at the opposite ends of a strut, a linearly tapering compression member is suggested. This condition may occur, if at the two ends of the strut the nodal zones have different strengths or different bearing lengths. Should the compression stress be high in the strut, reinforcement may be necessary to prevent splitting due to transverse tension. (The splitting cracks developing in cylinders supported on edge, and loaded in compression are good examples of the internal lateral spread of the compressive stress trajectories).

Ties consist of conventional deformed steel, or prestressing steel, or both, plus a portion of the surrounding concrete that is concentric with the axis of the tie. The surrounding concrete is not considered to resist axial

force in the model. However, it reduces the elongation of the tie (tension stiffening), in particular, under service loads. It also defines the zone in which the forces in the struts and ties are to be anchored.

Nodes are the intersection points of the axes of the struts, ties and concentrated forces, representing the joints of a strut-and-tie model. To maintain equilibrium, at least three forces should act on a given node of the model. Nodes are classified depending on the sign of the forces acting upon them (e.g., a C-C-C node resists three compression forces, a C-T-T node resists one compression forces and two tensile forces, etc.) as shown in Fig. 17-4



Figure 17-4 Classification of Nodes

A nodal zone is the volume of concrete that is assumed to transfer strut-and tie forces through the node. The early strut-and-tie models used hydrostatic nodal zones, which were lately superseded by extended nodal zones.

The faces of a **hydrostatic nodal zone** are perpendicular to the axes of the struts and ties acting on the node, as depicted in Fig. 17-5. The term hydrostatic refers to the fact that the in-plane stresses are the same in all directions. (Note that in a true hydrostatic stress state the out-of plane stresses should be also equal). Assuming identical stresses on all faces of a C-C-C nodal zone with three struts implies that the ratios of the lengths of the sides of the nodal zones ($w_{n1} : w_{n2} : w_{n3}$) are proportional to the magnitude of the strut forces ($C_1 : C_2 : C_3$).

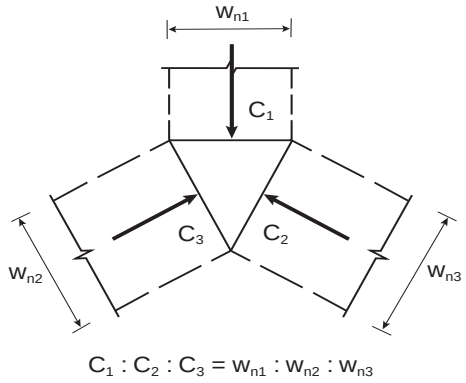


Figure 17-5 Hydrostatic Nodal Zone

The extended nodal zone is a portion of a member bounded by the intersection of the effective strut width, w_s, and the effective tie width, w_t. This is illustrated in Fig. 17-6.

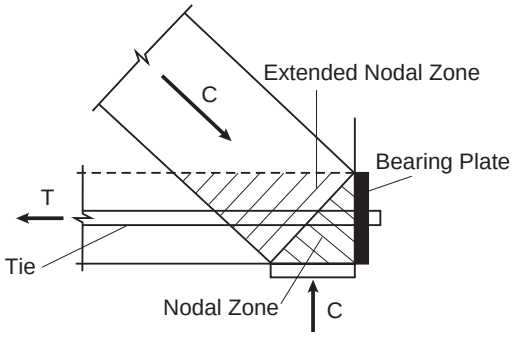


Figure 17-6 Extended Nodal Zone

A.2 STRUT-AND-TIE MODEL DESIGN PROCEDURE

A design with the strut-and-tie model typically involves the following steps:

1. Define and isolate D-regions.
2. Compute resultant forces on each D-region boundary.
3. Devise a truss model to transfer the resultant forces across the D-region. The axes of the struts and ties, respectively, are oriented to approximately coincide with the axes of the compression and tension stress fields.
4. The forces in the truss members are calculated.
5. The effective widths of the struts and nodal zones are determined considering the forces from the previous steps and the effective concrete strengths (defined in A.3.2 and A.5.2). Strength checks are based on

$$\phi F_n \geq F_u \qquad \textit{Eq. (A-1)}$$

   where F_u is the largest factored force obtained from the applicable load combinations, F_n is the nominal strength of the strut or node, and the ϕ factor is listed in 9.3.2.6 as 0.75 for ties, strut, nodal zones and bearing areas of strut-and-tie models.
6. Reinforcement is provided for the ties considering the steel strengths defined in A.4.1. The reinforcement is detailed to provide proper anchorage in the nodal zones

In addition to the strength limit states, represented by the strut-and-tie model, the structural members should be checked for the serviceability requirements. Traditional elastic analysis can be used for deflection checks. Crack control can be verified using provisions of 10.6.4, assuming that the tie is encased in a prism of concrete corresponding to the area of tie (per RA.4.2).

There are usually several strut-and-tie models that can be devised for a given structural member and loading condition. Models that satisfy the serviceability requirements the best, have struts and ties that follow the compressive and tensile stress trajectories, respectively. Certain construction rules of strut-and-tie models, e.g., "the angle between the axes of any strut and any tie entering a single node shall not be taken as less than 25 deg" (A.2.5) are imposed to mitigate potential cracking problems and to avoid incompatibilities due to shortening of the struts and lengthening of the ties in almost the same direction.

A.3 STRENGTH OF STRUTS

The nominal compressive strength of a strut without longitudinal reinforcement shall be taken as

$$F_{ns} = f_{cu} A_c \qquad \textit{Eq. (A-2)}$$

to be calculated at the weaker end of the compression member. A_c is the cross-sectional area at the end of the strut. In typical two-dimensional members the width of the strut (w_s) can be taken as the width of the member. The effective compressive strength of the concrete (f_{cu}) for this purpose shall be taken as the lesser of the concrete strengths at the two sides of the nodal zone/strut interface. Section A.3.2 specifies the calculation of f_{cu} for the strut (detailed below), while A.5.2 provides for the same in the nodal zone (detailed later).

The effective compressive strength of the concrete in a strut is calculated, similarly to basic strength equations, as:

$$f_{cu} = 0.85\beta_s f_c' \qquad \textit{Eq. (A-3)}$$

The β_s factor accounts for the effect of cracking and possible presence of transverse reinforcement. The strength of the concrete in a strut can be computed with $\beta_s = 1.0$ for struts that have uniform cross sectional area over their

length. This is quasi-equivalent to the rectangular stress block in the compression zone of a beam or column. For bottle-shaped struts (Fig. 17-7) with reinforcement placed to resist the splitting forces (satisfying A3.3) $\beta_s = 0.75$ or without adequate confinement to resist splitting forces $\beta_s = 0.6\lambda$ (where λ is a correction factor (per 11.7.4.3) for lightweight concrete.)

For compression struts passing through cracks of a tensile zone β_s is reduced to 0.4. Examples of these are strut-and-tie models used to design the longitudinal and transverse reinforcement of the tension flanges of beams, box-girders and walls. For all other cases (e.g., in beam webs where struts are likely to be crossed by inclined cracks), the β_s factor can be conservatively taken as 0.6.

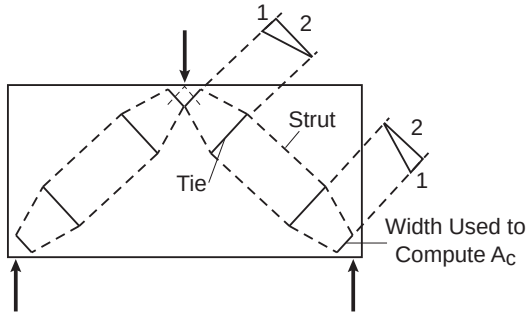


Figure 17-7 Bottle Shaped Compression Strut

Section A.3.3 addresses cases when transverse reinforcement is provided to cross the bottle-shaped struts. The compression forces in the strut may be assumed to spread at a slope 2:1. The rebars are intended to resist the transverse tensile forces resulting from the compression force spreading in the strut. They may be placed in one layer (when the γ angle between the rebar and the axis of the strut is at least 40 deg) or in two orthogonal layers.

To allow for $\beta_s = 0.75$, for concrete strength not exceeding 6000 psi, the reinforcement ratio needed to cross the strut is:

$$\Sigma \frac{A_{si}}{bs_i} \sin\gamma_i \geq 0.003 \qquad \textit{Eq. (A-4)}$$

where A_{si} is the total area of reinforcement at spacing s_i in a layer of reinforcement with bars at an angle γ_i to the axis of the strut (shown in Fig. 17-8). Often, this reinforcement ratio cannot be provided due to space limitations. In those cases $\beta_s = 0.6\lambda$ shall be used.

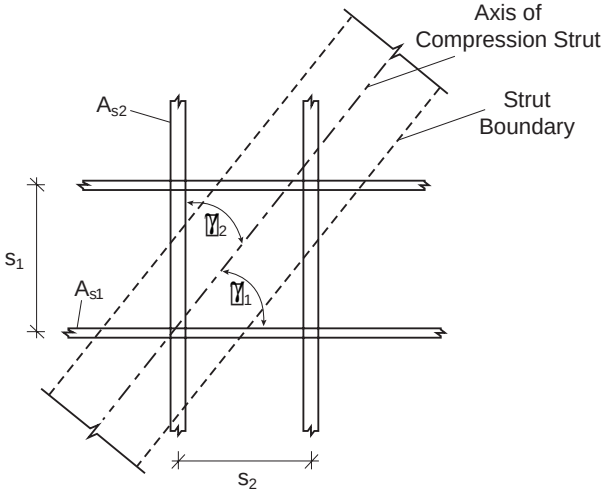


Figure 17-8 Layers of Reinforcement to Restrain Splitting Cracks of Struts

If substantiated by test and analyses, increased effective compressive strength of a strut due to confining reinforcement may be used (e.g., at anchorage zones of prestressing tendons). This topic is discussed in detail in Refs. 17.5 and 17.6.

Additional strength can be provided to the struts by including compression reinforcement parallel to the axis of the strut. These bars shall be properly anchored and enclosed by ties or spirals per 7.10. The compressive strength of these longitudinally reinforced struts can be calculated as:

Eq(A-5)

$$F_{ns} = f_{cu}A_c + A_s'f_s' \qquad \text{Eq. (A-5)}$$

where f_s' is the stress in the longitudinal strut reinforcement at nominal strength. It can be either obtained from strain analyses at the time the strut crushes or estimated as $f_s' = f_y$ for Grade 40 and 60 rebars.

A.4 STRENGTH OF TIES

The nominal strength of a tie is calculated as the sum of yield strength of the conventional reinforcement plus the force in the prestressing steel:

$$F_{nt} = A_{st}f_y + A_{ps}\left(f_{se} + \Delta f_p\right) \qquad \text{Eq. (A-6)}$$

Note, that A_{ps} is zero if there is no prestressing present in the tie, and the actual prestressing stress $(f_{se} + \Delta f_p)$ should not exceed the yield stress f_{py} of the prestressing steel. Also, if not calculated, the code allows to estimate the increase in prestressing steel stress due to factored loads Δf_p, as 60,000 psi for bonded prestressed reinforcement, or 10,000 psi for unbonded prestressed reinforcement.

Since the intent of having a tie is to provide for a tension element in a truss, the axis of the reinforcement centroid shall coincide with the axis of the tie assumed in the model. Depending on the distribution of the tie reinforcement, the effective tie width (w_t) may vary between the following limits:

- The minimum width for configurations where only one layer of reinforcement provided in a tie, w_t can be taken as the diameter of the bars in the tie plus twice the concrete cover to the surface of the ties. Should the tie be wider than this, the reinforcement shall be distributed evenly over the width.
- The upper limit is established as the width corresponding to the width in a hydrostatic nodal zone, calculated as

  $w_{t,max} = F_{nt}/f_{cu}$

  where f_{cu} is the applicable effective compression strength of a nodal zone discussed below.

Nodes shall be able to develop the difference between the forces of truss members connecting to them. Thus, besides providing adequate amount of tie reinforcement, special attention shall be paid to proper anchorage. Anchorage can be achieved using mechanical devices, post-tensioning anchorage devices, standard hooks, or straight bar embedment. The reinforcement in a tie should be anchored before it leaves the extended nodal zone, i.e., at the point defined by the intersection of the centroid of the bars in the tie and the extensions of the outlines of either the strut or the bearing area as shown on Fig.17-9. For truss layouts where more than one tie intersect at a node, each tie force shall be developed at the point where the centroid of the reinforcement in the tie leaves the extended nodal zone. (Note, that transverse reinforcement required by A3.3 shall be anchored according to the provisions of 12.13).
In many cases the structural configuration does not allow to provide for the straight development length for a tie.

For such cases, anchorage is provided through mechanical devices, hooks, or splicing with several layers of smaller bars. These options often require a wider structural member and/or additional confinement reinforcement (e.g., to avoid cracking along the outside of the hooks).

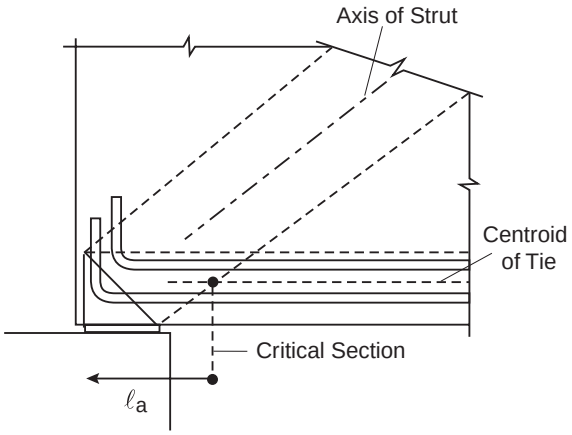


Figure 17-9 Anchorage of Tie Reinforcement

A.5 STRENGTH OF NODAL ZONES

The nominal compression strength at the face of a nodal zone or at any section through the nodal zone shall be:

$$F_{nn} = f_{cu}A_n \qquad \textit{Eq. (A-7)}$$

where A_n is taken as the area of the face of the nodal zone that the strut force F_u acts on, if the face is perpendicular to the line of action of F_u. If the nodal zone is limited by some other criteria, the node-to-strut interface may not be perpendicular to the axis of the strut, therefore, the axial stresses in the compression-only strut will generate both shear and normal stresses acting on the interface. In those cases, the A_n parameter shall be the area of a section, taken through the nodal zone perpendicular to the strut axis.

The strut-and-tie model is applicable to three-dimensional situations as well. In order to keep calculations simple, A5.3 allows the area of the nodal faces to be less than that described above and the shape of each face of the nodal zones shall be similar to the shape of the projection of the end of the struts onto the corresponding faces of the nodal zones.

The effective compressive strength of the concrete in the nodal zone (f_{cu}) is calculated as:

$$f_{cu} = 0.85\beta_n f_c' \qquad \textit{Eq. (A-8)}$$

and shall not exceed the effective concrete compressive strength on the face of a nodal zone due to the strut-and-tie model forces, unless confining reinforcement is provided within the nodal zone and its effect is evidenced by tests and analysis. The sign of forces acting on the node influences the capacity at the nodal zones as reflected by the β_n value. The presence of tension stresses due to ties decreases the nodal zone concrete strength.

β_n = 1.0 in nodal zones bounded by struts or bearing areas (e.g., C-C-C nodes)
β_n = 0.8 in nodal zones anchoring one tie (e.g., C-C-T nodes)
β_n = 0.6 in nodal zones anchoring two or more ties (e.g., C-T-T or T-T-T nodes).

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Example 17.1—Design of Deep Flexural Member by the Strut-and-Tie Model

Determine the required reinforcement for the simply supported transfer girder shown in Fig. 17-10. The single column at midspan subjects the girder to 180 kips dead load and 250 kips live load.

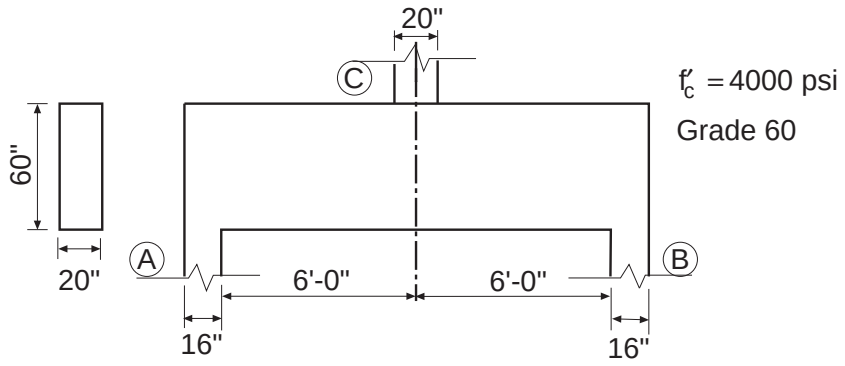


Figure 17-10 Spandrel Beam

Calculations and Discussion	Code Reference

1. Calculate factored load and reactions

   The transfer girder dead load is conservatively lumped to the column load at midspan. Transfer girder dead load is:

   5(20/12) [6 + 6 + (32/12)] 0.15 = 18.5 kips

   $P_u = 1.2D + 1.6L = 1.2 \times (18.5 + 180) + 1.6 \times 250 = 640$ kips — *Eq. (9-2)*

   $R_A = R_B = 640/2 = 320$ kips

2. Determine if this beam satisfies the definition of a "deep beam" — *10.7.1*, *11.8.1*

   Overall girder height h = 5 ft

   Clear span ℓ_n = 12 ft

   $$\frac{\ell_n}{h} = \frac{12}{5} = 2.4 < 4$$

   Member is a "deep beam" and will be designed using Appendix A.

3. Check the maximum shear capacity of the cross section

   $V_u = 320$ kips

   $\phi V_n = \phi\left(10\sqrt{f'_c}\, b_w d\right)$ — *1.8.3*

$= 0.75(10\sqrt{4000} \times 20 \times 54)/1000 = 512 \text{ kips} > V_u$ O.K.

4. Establish truss model

Assume that the nodes coincide with the centerline of the columns (supports), and are located 5 in. from the upper or lower edge of the beam as shown in Fig. 17-11. The strut-and-tie model consists of two struts (A-C and B-C), one tie (A-B), and three nodes (A, B, and C). In addition, columns at A and B act as struts representing reactions. The vertical strut at the top of Node C represents the applied load.

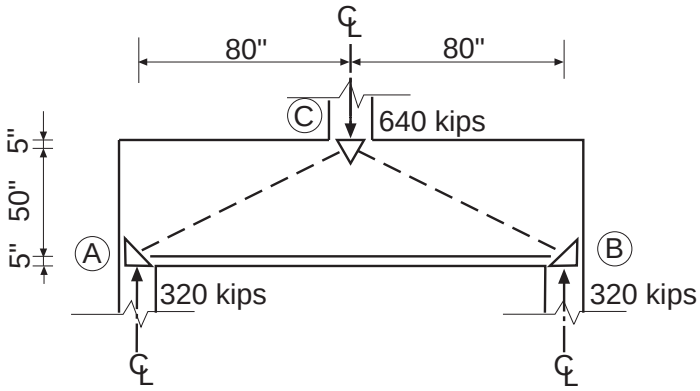


Figure 17-11 Preliminary Truss Layout

The length of the diagonal struts $= \sqrt{50^2 + 80^2} = 94.3$ in.

The force in the diagonal struts $= 320\dfrac{94.3}{50} = 603$ kips

The force in the horizontal tie $= 320\dfrac{80}{50} = 512$ kips

Verify the angle between axis of strut and tie entering Node A.

The angle between the diagonal struts and the horizontal tie = $\tan^{-1}$ (50/80) = 32° > 25° O.K. *A.2.5*

5. Calculate the effective concrete strength (f_{cu}) for the struts assuming that reinforcement is provided per A.3.3 to resist splitting forces. (See Step 9)

For the "bottle-shaped" Struts A-C & B-C

$f_{cu} = 0.85\ \beta_s f'_c = 0.85 \times 0.75 \times 4000 = 2550$ psi *Eq. (A-3)*

where $\beta_s = 0.75$ per A.3.2.2(a)

Note, this effective compressive strength cannot exceed the strength of the nodes at both ends of the strut. See A.3.1.

The vertical struts at A, B, and C have uniform cross-sectional area throughout their length.

$\beta_s = 1.0$ *A.3.2.1*

$f_{cu} = 0.85 \times 1.0 \times 4000 = 3400$ psi

6. Calculate the effective concrete strength (f_{cu}) for Nodal Zones A, B, and C

Nodal Zone C is bounded by three struts. So this is a C-C-C nodal zone with $\beta_n = 1.0$ *A.5.2.1*

$f_{cu} = 0.85 \times \beta_n \times f'_c = 0.85 \times 1.0 \times 4000 = 3400$ psi *Eq. (A-8)*

Nodal Zones A and C are bounded by two struts and a tie. For a C-C-T node:

$\beta_n = 0.80$

$f_{cu} = 0.85\beta_n f'_c = 0.85 \times 0.80 \times 4000 = 2720$ psi

7. Check strength at Node C

Assume that a hydrostatic nodal zone is formed at Node C. This means that the faces of the nodal zone are perpendicular to the axis of the respective struts, and that the stresses are identical on all faces.

To satisfy the strength criteria for all three struts and the node, the minimum nodal face dimension is determined based on the least strength value of $f_{cu} = 2550$ psi. The same strength value will be used for Nodes A and B as well.

The strength checks for all components of the strut and tie model are based on $\phi F_n \geq F_u$ *Eq. (A-1)*

where $\phi = 0.75$ for struts, ties, and nodes. *9.3.2.6*

The length of the horizontal face of Nodal Zone C is calculated as

$$\frac{640{,}000}{0.75 \times 2550 \times 20} = 16.7 \text{ in. (less than column width of 20 in.)}$$

The length of the other faces, perpendicular to the diagonal struts, can be obtained from proportionality:

$$16.7 \frac{603}{640} = 15.7 \text{ in.}$$

8. Check the truss geometry.

At Node C

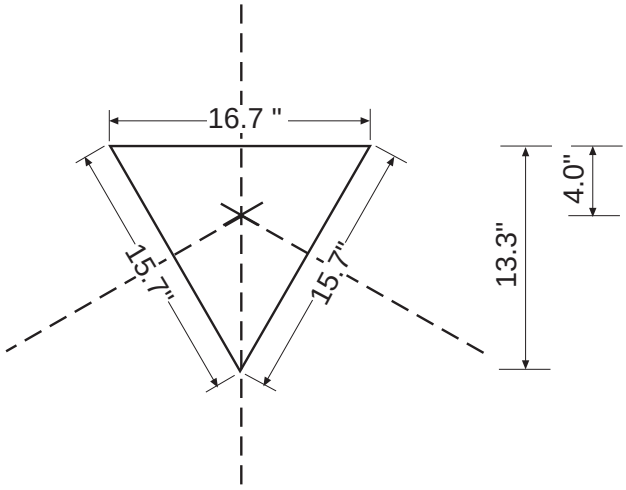


Figure 17-12 Geometry of Node C

The center of the nodal zone is at 4.0 in. from the top of the beam, which is very close to the assumed 5 in.

At Node A

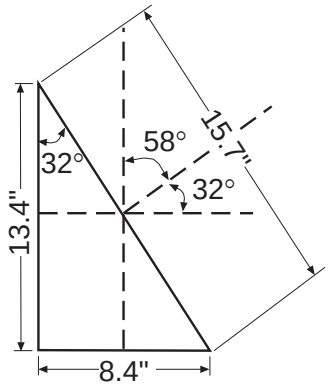


Figure 17-13 Geometry of Node A

The horizontal tie should exert a force on this node to create a stress of 2550 psi. Thus size of the vertical face of the nodal zone is

$$\frac{512{,}000}{0.75 \times 2550 \times 20} = 13.4 \text{ in.}$$

The center of the tie is located 13.4/2 = 6.7 in. from the bottom of the beam. This is reasonably close to the 5 in. originally assumed, so no further iteration is warranted.

Width of node at Support A

$$\frac{320{,}000}{0.75 \times 2550 \times 20} = 8.4 \text{ in.}$$

9. Provide vertical and horizontal reinforcement to resist splitting of diagonal struts.

The angle between the vertical ties and the struts is 90°- 32° = 58° (sin 58° = 0.85)

Try two overlapping No. 4 ties @ 12 in. O.C. (to accomodate the longitudinal tie reinforcement designed in Step 10, below.

$$\frac{A_{si}}{bs}\sin\gamma_i = \frac{4 \times 0.20}{20 \times 12} \times 0.85 = 0.00283$$ *Eq. (A-4)*

and No. 5 horizontal bars @ 12 in. O.C. (sin 32° = 0.53)

$$\frac{2 \times 0.31}{20 \times 12} \times 0.53 = 0.00137$$

$$\Sigma\frac{A_{si}}{bs}\sin\gamma_i = 0.00283 + 0.00137 = 0.0042 > 0.003 \text{ O.K.}$$ *Eq. (A-4)*

10. Provide horizontal reinforcing steel for the tie

$$A_{s,req} = \frac{F_u}{\phi f_y} = \frac{512}{0.75 \times 60} = 11.4 \text{ in.}^2$$

Select 16 - No. 8 $A_s = 12.64$ in.^{2}

These bars must be properly anchored. The anchorage is to be measured from the point where the tie exits the extended nodal zone as shown in Fig. 17-14.

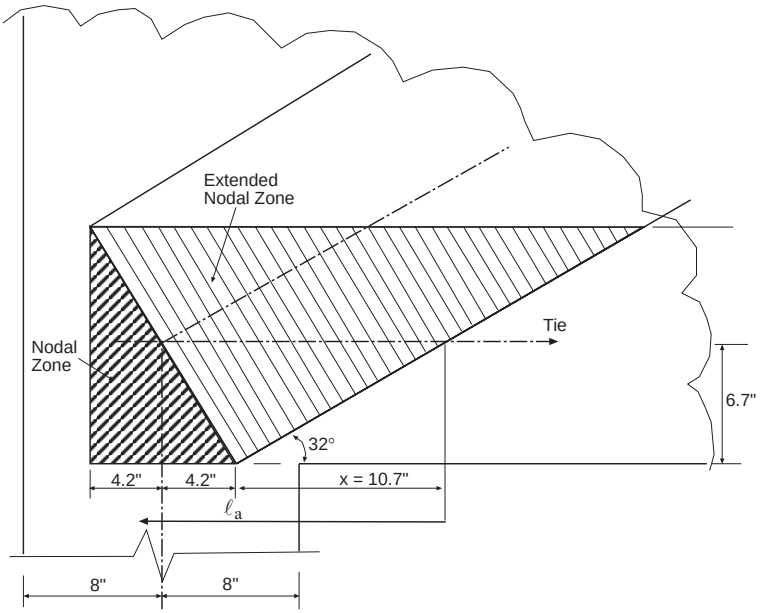


Figure 17-14 Development of Tie Reinforcement Within the Extended Nodal Zone

Distance x = 6.7/tan 32 = 10.7 in.

Available space for a straight bar embedment

10.7 + 4.2 + 8 - 2.0 (cover) = 20.9 in.

This length is inadequate to develop a straight No. 8 bar.

Development length for a No. 8 bar with a standard 90 deg. hook

$\ell_{dh} = (0.02\beta\lambda f_y / \sqrt{f'_c})\, d_b$ 12.5.2

$= \left(0.02(1.0)(1.0)60{,}000 / \sqrt{4000}\right) 1.0$

= 19.0 < 20.9 in. O.K.

Note: the 90 deg. hooks will be enclosed within the column reinforcement that extends in the transfer girder. (Fig. 17-15) By providing adequate cover and transverse confinement, the development length of the standard hook could be reduced by the modifiers of 12.5.3.

Less congested reinforcement schemes can be devised with reinforcing steel welded to bearing plates, or with the use of prestressing steel.

Comments:

The discrepancy in the vertical location of the nodes results in a negligible (about 1.5 percent) difference in the truss forces. Thus, another iteration is not warranted.

There are several alternative strut-and-tie models that could have been selected for this problem. An alternative truss layout is illustrated in Fig. 17-16

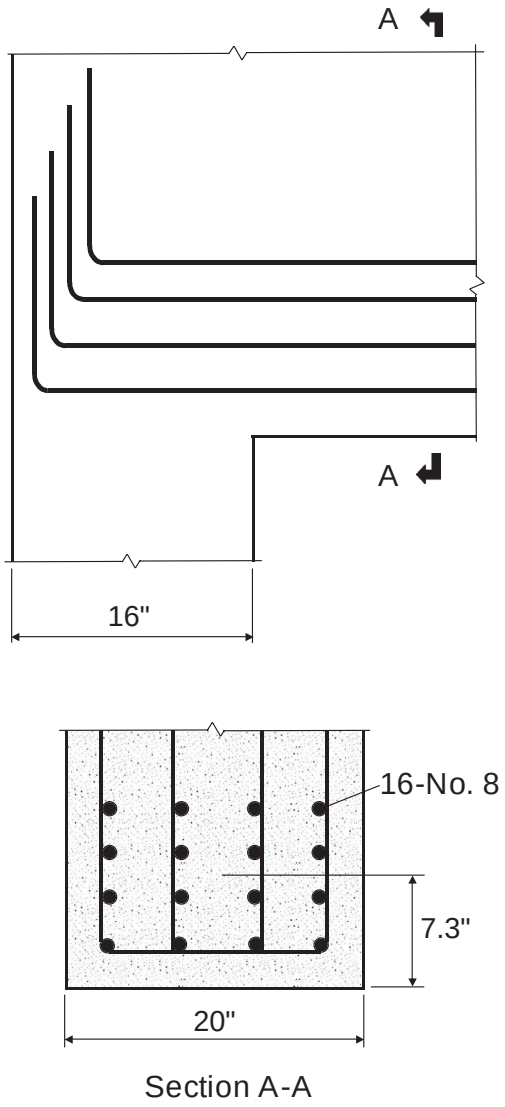


Figure 17-15 Detail of Tie Reinforcement

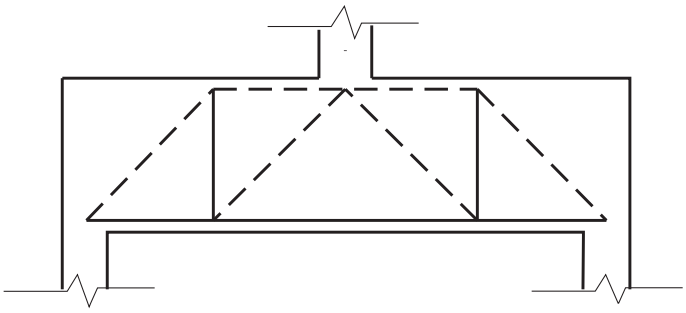


Figure 17-16 Alternative Strut-and-Tie Model

Example 17.2—Design of Column Corbel

Design the single corbel of the 16 in. × 16 in. reinforced concrete column for a vertical force V_u = 60 kips and horizontal force N_u = 12 kips. Assume f'_c = 5000 psi, and Grade 60 reinforcing steel.

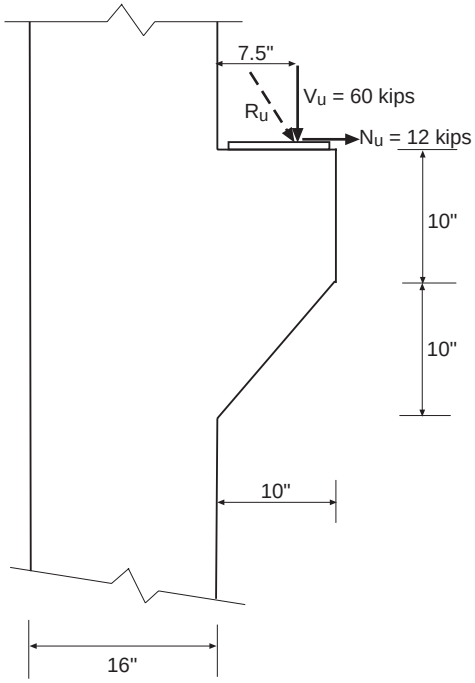


Figure 17-17 Design of Corbel

Calculations and Discussion	Code Reference

1. Establish the geometry of a trial truss and calculate force demand in members.

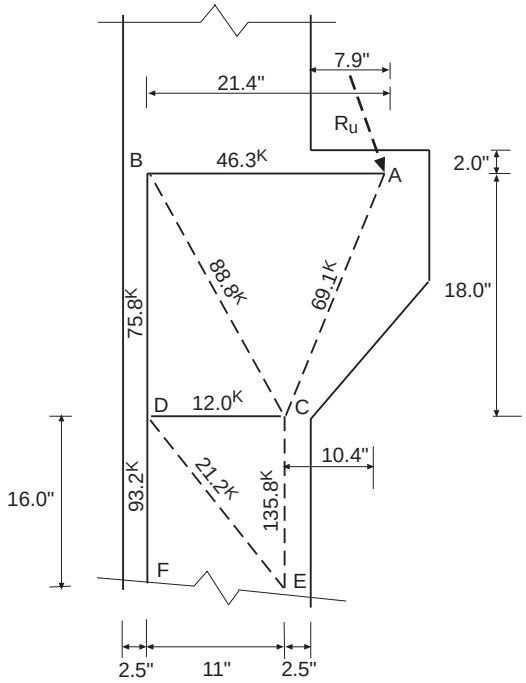


Figure 17-18 Truss Layout

2. Provide reinforcement for ties

Use $\phi = 0.75$ 9.3.2.6

The nominal strength of ties is to be taken as:

$$F_{nt} = A_{st}f_y + A_{ps}\left(f_{se} + \Delta f_p\right)$$

where the last term can be ignored for nonprestressed reinforcement

Tie AB $F_u = 46.3$ kips

$$A_{st} = \frac{F_u}{\phi f_y} = \frac{46.3}{0.75 \times 60} = 1.03 \text{ in.}^2$$ Provide 4 - No. 5 $A_{st} = 1.24 \text{ in.}^2$

Tie CD $F_u = 12.0$ kips

$$A_{st} = \frac{12.0}{0.75 \times 60} = 0.27 \text{ in.}^2$$ Provide No. 4 tie $A_{st} = 0.40 \text{ in.}^2$

Tie BD & DF $P_u = 93.2$ kips

$$A_{st} = \frac{93.2}{0.75 \times 60} = 2.07 \text{ in.}^2$$ Provide steel in addition of the vertical column reinforcement

This reinforcement may be added longitudinal bar or a rebar bent at Node A, that is used as Tie AB as well.

3. Calculate strut widths

It is assumed that reinforcement will be provided in compliance with A.3.3, so a $\beta_s = 0.75$ can be used

$$f_{cu} = 0.85\,\beta_s f'_c = 0.85 \times 0.75 \times 5000 = 3187 \text{ kips}$$

$$\phi f_{cu} = 0.75 \times 3187 = 2390 \text{ psi}$$

Calculate the width of struts required

Strut AC

$$w = \frac{69{,}100}{16 \times 2390} = 1.81 \text{ in.}$$

Strut BC

$$w = \frac{88{,}800}{16 \times 2390} = 2.32 \text{ in.}$$

Strut CE

$$w = \frac{135{,}800}{16 \times 2390} = 3.55 \text{ in.}$$

Strut DE

$$w = \frac{21.2}{16 \times 2390} = 0.55 \text{ in.}$$

The width of the struts will fit within the concrete column with the corbel.

Provide confinement reinforcement for the struts per A.3.3 in the form of horizontal ties The angle of the diagonal struts to the horizontal hoops is 58°. Provide No. 4 hoops at 4.5 in. on center.

$$\frac{A_s}{bs} \sin\gamma = \frac{2 \times 0.20}{24 \times 4.5} \sin 58° = 0.0031$$

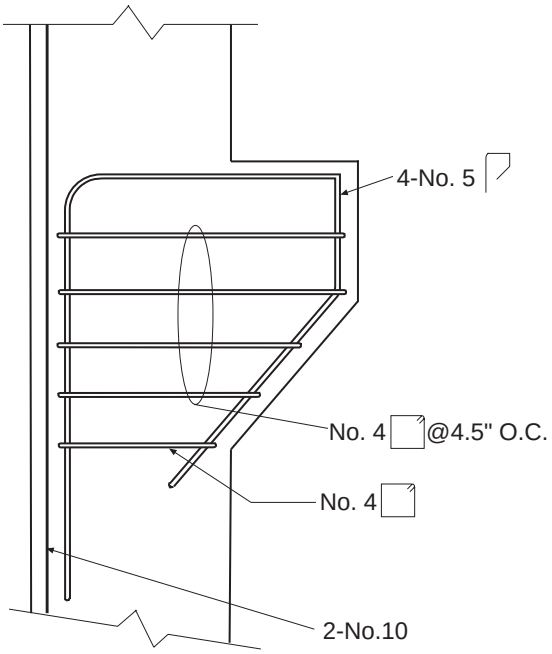


Figure 17-19 Reinforcement Details

18

Two-Way Slab Systems

UPDATE FOR THE '02 CODE

One minor change in Chapter 13 occurs in 13.3.8.5: mechanical or welded tension splices satisfying 12.14.3 are now allowed for bottom bars in the column strip.

13.1 SCOPE

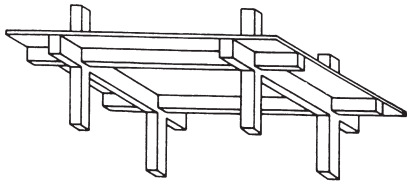
Figure 18-1 shows the various types of two-way reinforced concrete slab systems in use at the present time that may be designed according to Chapter 13.

A solid slab supported on beams on all four sides (Fig. 18-1(a)) was the original slab system in reinforced concrete. With this system, if the ratio of the long to the short side of a slab panel is two or more, load transfer is predominantly by bending in the short direction and the panel essentially acts as a one-way slab. As the ratio of the sides of a slab panel approaches unity (or as panel approaches a square shape), significant load is transferred by bending in both orthogonal directions, and the panel should be treated as a two-way rather than a one-way slab.

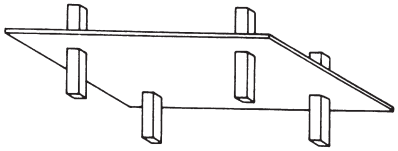
As time progressed and technology evolved, the column-line beams gradually began to disappear. The resulting slab system consisting of solid slabs supported directly on columns is called the flat plate (Fig. 18-1(b)). The two-way flat plate is very efficient and economical and is currently the most widely used slab system for multi-story construction, such as motels, hotels, dormitories, apartment buildings, and hospitals. In comparison to other concrete floor/roof systems, flat plates can be constructed in less time and with minimum labor costs because the system utilizes the simplest possible formwork and reinforcing steel layout. The use of flat plate construction also has other significant economic advantages. For instance, because of the shallow thickness of the floor system, story heights are automatically reduced, resulting in smaller overall height of exterior walls and utility shafts; shorter floor-to-ceiling partitions; reductions in plumbing, sprinkler, and duct risers; and a multitude of other items of construction. In cities like Washington, D.C., where the maximum height of buildings is restricted, the thin flat plate permits the construction of the maximum number of stories in a given height. Flat plates also provide for the most flexibility in the layout of columns, partitions, small openings, etc. An additional advantage of flat plate slabs that should not be overlooked is their inherent fire resistance. Slab thickness required for structural purposes will, in most cases, provide the fire resistance required by the general building code, without having to apply spray-on fire proofing, or install a suspended ceiling. This is of particular importance where job conditions allow direct application of the ceiling finish to the flat plate soffit, eliminating the need for suspended ceilings. Additional cost and construction time savings are then possible as compared to other structural systems.

The principal limitation on the use of flat plate construction is imposed by shear around the columns (13.5.4). For heavy loads or long spans, the flat plate is often thickened locally around the columns creating what are known as drop panels. When a flat plate incorporates drop panels, it is called a flat slab (Fig. 18-1(c)). Also for reasons of

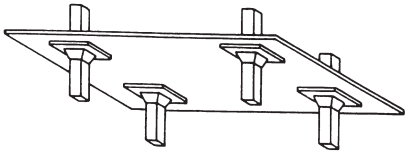
shear around the columns, the column tops are sometimes flared, creating column capitals. For purposes of design, a column capital is part of the column, whereas a drop panel is part of the slab (13.7.3 and 13.7.4).



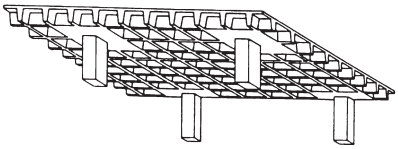
(a) Two-Way Beam-Supported Slab



(b) Flat Plate



(c) Flat Slab



(d) Waffle Slab (Two-Way Joist Slab)

Figure 18-1 Types of Two-Way Slab Systems

Waffle slab construction (Fig. 18-1(d)) consists of rows of concrete joists at right angles to each other with solid heads at the column (needed for shear strength). The joists are commonly formed by using standard square "dome" forms. The domes are omitted around the columns to form the solid heads. For design purposes, waffle slabs are considered as flat slabs with the solid heads acting as drop panels (13.1.3). Waffle slab construction allows a considerable reduction in dead load as compared to conventional flat slab construction since the slab thickness can be minimized due to the short span between the joists. Thus, it is particularly advantageous where the use of long span and/or heavy loads is desired without the use of deepened drop panels or support beams. The geometric shape formed by the joist ribs is often architecturally desirable.

13.1.4 Deflection Control—Minimum Slab Thickness

Minimum thickness/span ratios enable the designer to avoid extremely complex deflection calculations in routine designs. Deflections of two-way slab systems need not be computed if the overall slab thickness meets the minimum requirements specified in 9.5.3. Minimum slab thicknesses for flat plates, flat slabs, and waffle slabs based on Table 9.5(c), and two-way beam-supported slabs based on Eqs. (9-12) and (9-13) are summarized in Table 18-1, where ℓ_n is the clear span length in the long direction of a two-way slab panel. The tabulated values are the controlling minimum thicknesses governed by interior, side, or corner panels assuming a constant slab thickness for all panels making up a slab system. Practical edge beam sizes will usually provide beam-to-slab stiffness ratios α greater than the minimum specified value of 0.8. A "standard" size drop panel that would

allow a 10% reduction in the minimum required thickness of a flat slab floor system is illustrated in Fig. 18-2. Note that a drop of larger size and depth may be used if required for shear strength; however, a corresponding lesser slab thickness is not permitted unless deflections are computed.

For design convenience, minimum thicknesses for the six types of two-way slab systems listed in Table 18-1 are plotted in Fig. 18-3.

Refer to Part 10 for a general discussion on control of deflections for two-way slab systems, including design examples of deflection calculations for two-way slabs.

Table 18-1 Minimum Thickness for Two-Way Slab Systems (Grade 60 Reinforcement)

Two-Way Slab System	α_m	β	Minimum h
Flat Plate	—	≤ 2	$\ell_n/30$
Flat Plate with Spandrel Beams[1] [Min. h = 5 in.]	—	≤ 2	$\ell_n/33$
Flat Slab[2]	—	≤ 2	$\ell_n/33$
Flat Slab[2] with Spandrel beams[1] [Min. h = 4 in.]	—	≤ 2	$\ell_n/36$
Two-Way Beam-Supported Slab[3]	≤ 0.2	≤ 2	$\ell_n/30$
	1.0	1	$\ell_n/33$
		2	$\ell_n/36$
	≥ 2.0	1	$\ell_n/37$
		2	$\ell_n/44$
Two-Way Beam-Supported Slab[1,3]	≤ 0.2	≤ 2	$\ell_n/33$
	1.0	1	$\ell_n/36$
		2	$\ell_n/40$
	≥ 2.0	1	$\ell_n/41$
		2	$\ell_n/49$

[1] *Spandrel beam-to-slab stiffness ratio $\alpha \geq 0.8$ (9.5.3.3)*

[2] *Drop panel length $\geq \ell/3$, depth $\geq 1.25h$ (13.3.7)*

[3] *Min. h = 5 in. for $\alpha_m \leq 2.0$; min. h = 3.5 in. for $\alpha_m > 2.0$ (9.5.3.3)*

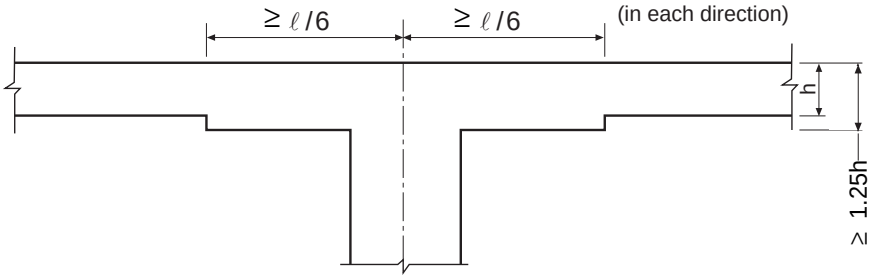


Figure 18-2 Drop Panel Details (13.3.7)

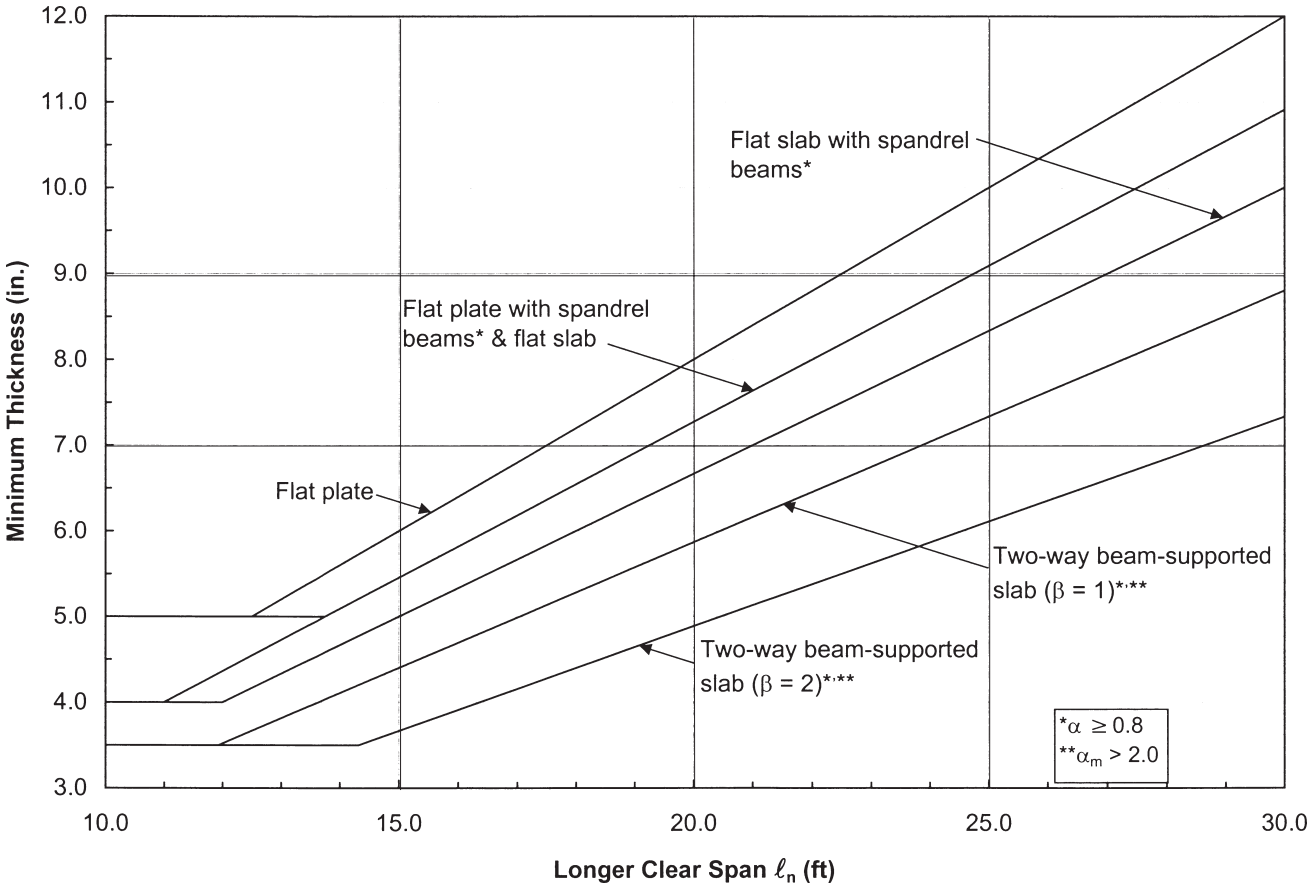


Figure 18-3 Minimum Slab Thickness for Two-Way Slab Systems (see Table 18-1)

13.2 DEFINITIONS

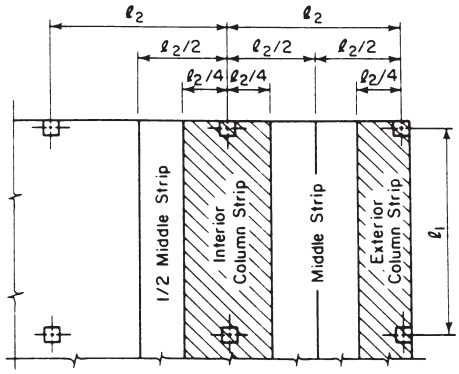
13.2.1 Design Strip

For analysis of a two-way slab system by either the Direct Design Method (13.6) or the Equivalent Frame Method (13.7), the slab system is divided into design strips consisting of a column strip and half middle strip(s) as defined in 13.2.1 and 13.2.2, and as illustrated in Fig. 18-4. The column strip is defined as having a width equal to one-half the transverse or longitudinal span, whichever is smaller. The middle strip is bounded by two column strips. Some judgment is required in applying the definitions given in 13.2.1 for column strips with varying span lengths along the design strip.

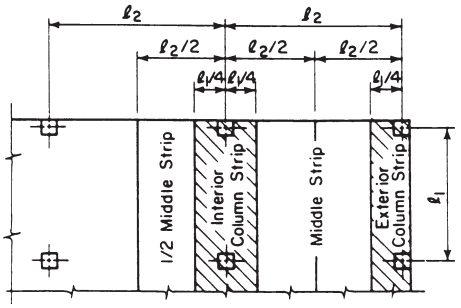
The reason for specifying that the column strip width be based on the shorter of ℓ_1 or ℓ_2 is to account for the tendency for moment to concentrate about the column line when the span length of the design strip is less than its width.

13.2.4 Effective Beam Section

For slab systems with beams between supports, the beams include portions of the slab as flanges, as shown in Fig. 18-5. Design constants and stiffness parameters used with the Direct Design and Equivalent Frame analysis methods are based on the effective beam sections shown.



(a) Column Strip for $\ell_2 \leq \ell_1$



(b) Column Strip for $\ell_2 > \ell_1$

Figure 18-4 Definition of Design Strips

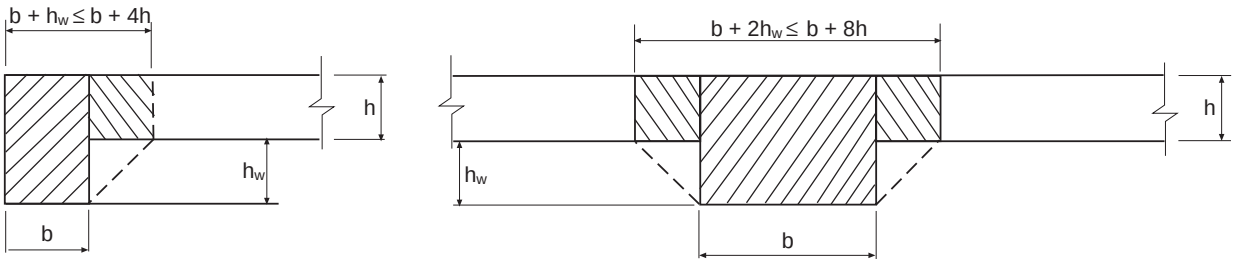


Figure 18-5 Effective Beam Section (13.2.4)

13.3 SLAB REINFORCEMENT

- Minimum area of reinforcement in each direction for two-way slab systems = 0.0018bh (b = slab width, h = total thickness) for Grade 60 bars for either top or bottom steel (13.3.1).
- Maximum bar spacing is 2h, but not more than 18 in. (13.3.2).
- Minimum extensions for reinforcement in slabs without beams (flat plates and flat slabs) are prescribed in Fig. 13.3.8 (13.3.8.1).

Note that the reinforcement details of Fig. 13.3.8 do not apply to two-way slabs with beams between supports or to slabs in non-sway or sway frames resisting lateral loads. For those slabs, a general analysis must be made according to Chapter 12 of the Code to determine bar lengths based on the moment variation but shall not be less than those prescribed in Fig. 13.3.8 (13.3.8.4). Reinforcement details for bent bars were deleted from Fig. 13.3.8 in the '89 code in view of their rare usage in today's construction. Designers who wish to use bent bars in two-way slabs (without beams) should refer to Fig. 13.4.8 of the '83 code.

According to 13.3.6, special top and bottom reinforcement must be provided at the exterior corners of a slab with spandrel beams that have a value of α greater than 1.0. The reinforcement must be designed for a moment equal to the largest positive moment per unit width in the panel, and must be placed in a band parallel to the diagonal in the top of the slab and a band perpendicular to the diagonal in the bottom of the slab (Fig. 18-6 (a)); alternatively, it may be placed in two layers parallel to the edges of the slab in both the top and bottom of the slab (Fig. 18-6 (b)). Additionally, the reinforcement must extend at least one-fifth of the longer span in each direction from the corner.

In slabs without beams, all bottom bars in the column strip shall be continuous or spliced with class A splices or with mechanical or welded splices satisying 12.14.3 (13.3.8.5) to provide some capacity for the slab to span to an adjacent support in the event a single support is damaged. Additionally, at least two of these continuous bottom bars shall pass through the column and be anchored at exterior supports. In lift-slab construction and slabs with shearhead reinforcement, clearance may be inadequate and it may not be practical to pass the column strip bottom reinforcing bars through the column. In these cases, two continuous bonded bottom bars in each direction shall pass as close to the column as possible through holes in the shearhead arms or, in the case of lift-slab construction, within the lifting collar (13.3.8.6). This condition was initially addressed in the 1992 Code and was further clarified in 1999.

13.4 OPENINGS IN SLAB SYSTEMS

The code permits openings of any size in any slab system, provided that an analysis is performed that demonstrates that both strength and serviceability requirements are satisfied (13.4.1). For slabs without beams; the analysis of 13.4.1 is waived when the provisions of 13.4.2.1 through 13.4.2.4 are met:

- In the area common to intersecting middle strips, openings of any size are permitted (13.4.2.1).
- In the area common to intersecting column strips, maximum permitted opening size is one-eighth the width of the column strip in either span (13.4.2.2).
- In the area common to one column strip and one middle strip, maximum permitted opening size is limited such that only a maximum of one-quarter of slab reinforcement in either strip may be interrupted (13.4.2.3).

The total amount of reinforcement required for the panel without openings, in both directions, shall be maintained; thus, reinforcement interrupted by the opening must be replaced on each side of the opening. Figure 18-7 illustrates the provisions of 13.4.2 for slabs with $\ell_2 > \ell_1$. Refer to Part 16 for a discussion on the effect of openings in slabs without beams on concrete shear strength (13.4.2.4).

13.5 DESIGN PROCEDURES

Section 13.5.1 permits design (analysis) of two-way slab systems by any method that satisfies code-defined strength requirements (9.2 and 9.3), and all applicable code serviceability requirements, including specified limits on deflections (9.5.3).

13.5.1.1 Gravity Load Analysis—Two methods of analysis of two-way slab systems under gravity loads are addressed in Chapter 13: the simpler Direct Design Method (DDM) of 13.6, and the more complex Equivalent Frame Method (EFM) of 13.7. The Direct Design Method is an approximate method using moment coeffi cients, while the Equivalent Frame (elastic analysis) Method is more exact. The approximate analysis procedure

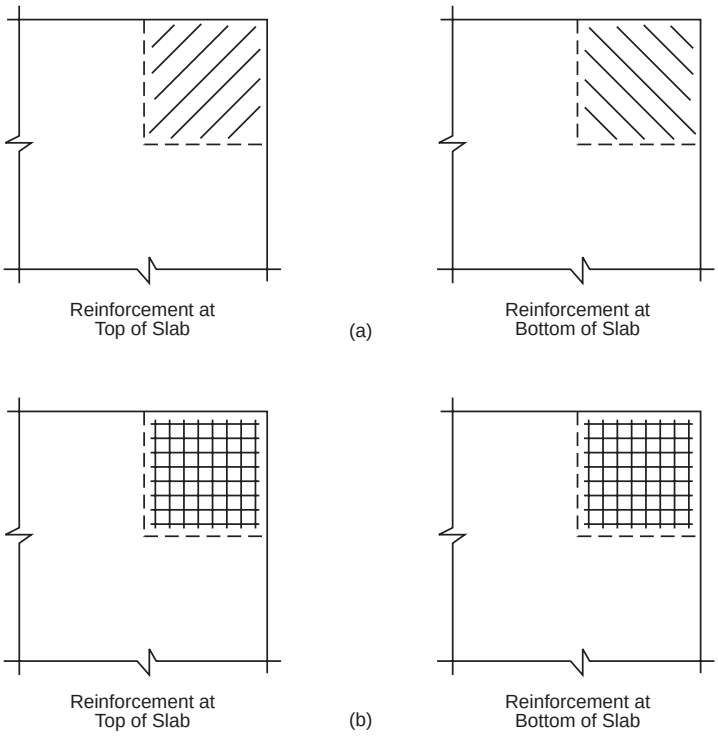


Figure 18-6 Special Reinforcement Required at Corners of Beam-Supported Slabs

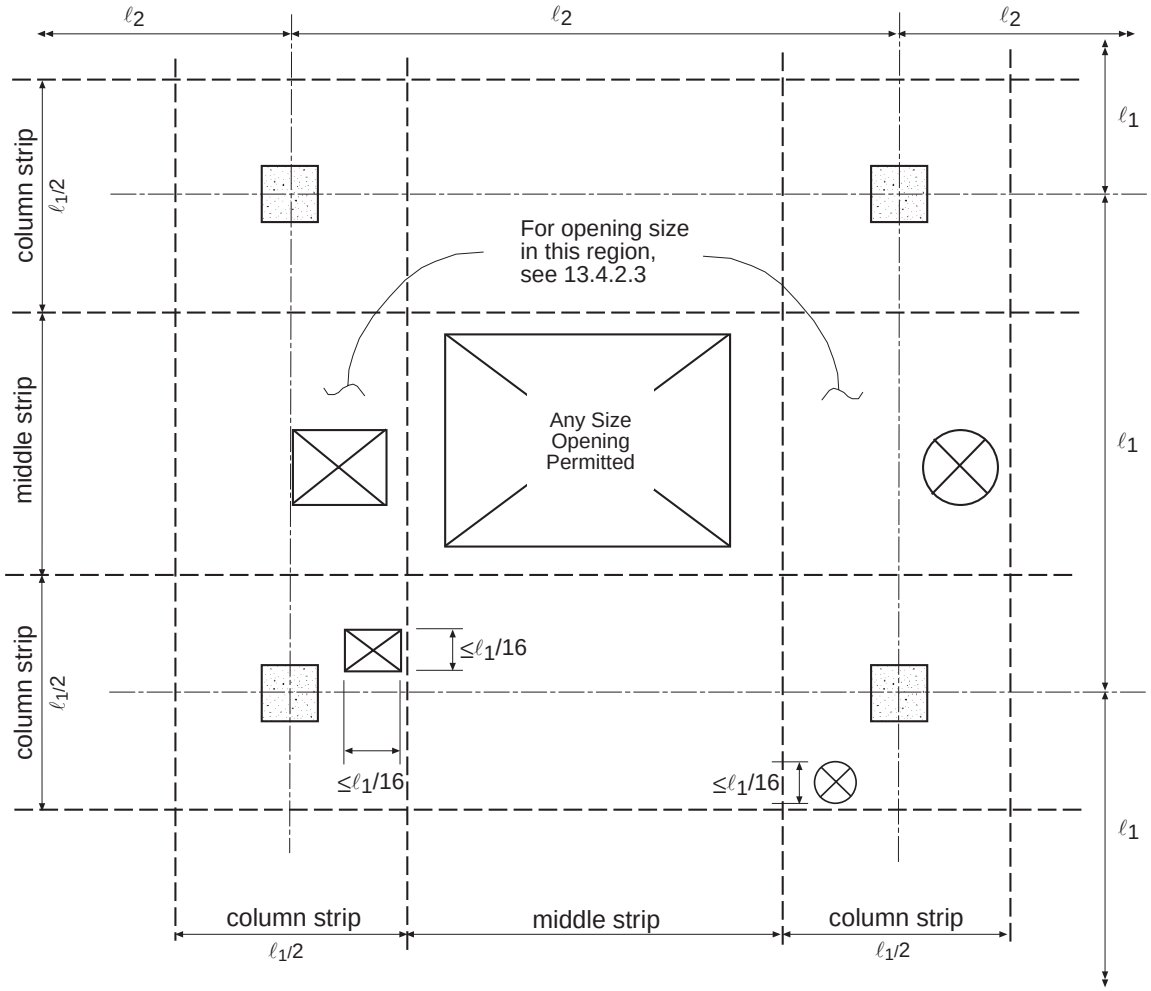


Figure 18-7 Permitted Openings in Slab Systems without Beams for $\ell_2 > \ell_1$

of the Direct Design Method will give reasonably conservative moment values for the stated design conditions for slab systems within the limitations of 13.6.1.

Both methods are for analysis under gravity loads only, and are limited in application to buildings with columns and/or walls laid out on a basically orthogonal grid, i.e., where column lines taken longitudinally and transversely through the building are mutually perpendicular. Both methods are applicable to slabs with or without beams between supports. Note that neither method applies to slab systems with beams spanning between other beams; the beams must be located along column lines and be supported by columns or other essentially nondeflecting supports at the corners of the slab panels.

13.5.1.2 Lateral Load Analysis—For lateral load analysis of frames, the model of the structure may be based upon any approach that is shown to satisfy equilibrium and geometric compatibility and to be in reasonable agreement with test data. Acceptable approaches include plate-bending finite-element models, effective beam width models, and equivalent frame models. The stiffness values for frame members used in the analysis must reflect effects of slab cracking, geometric parameters, and concentration of reinforcement.

During the life of the structure, ordinary occupancy loads and volume changes due to shrinkage and temperature effects will cause cracking of slabs. To ensure that lateral drift caused by wind or earthquakes is not underestimated, cracking of slabs must be considered in stiffness assumptions for lateral drift calculations.

The stiffness of slab members is affected not only by cracking, but also by other parameters such as ℓ_2/ℓ_1, c_1/ℓ_1, c_2/c_1, and on concentration of reinforcement in the slab width defined in 13.5.3.2 for unbalanced moment transfer by flexure. This added concentration of reinforcement increases stiffness by preventing premature yielding and softening in the slab near the column supports. Consideration of the actual stiffness due to these factors is important for lateral load analysis because lateral displacement can significantly affect the moments in the columns, especially in tall moment frame buildings. Also, actual lateral displacement for a single story, or for the total height of a building is an important consideration for building stability and performance.

Cracking reduces stiffness of the slab-beams as compared with that of an uncracked floor. The magnitude of the loss of stiffness due to cracking will depend on the type of slab system and reinforcement details. For example, prestressed slab systems with reduced slab cracking due to prestressing, and slab systems with large beams between columns will lose less stiffness than a conventional reinforced flat plate system.

Prior to the 1999 code, the commentary indicated stiffness values based on Eq. (9-8) were reasonable. However, this was deleted from the commentary in 1999, since factors such as volume change effects and early age loading are not adequately represented in Eq. (9-8) . Since it is difficult to evaluate the effect of cracking on stiffness, it is usually sufficient to use a lower bound value. On the assumption of a fully cracked slab with minimum reinforcement at all locations, a stiffness for the slab-beam equal to one-fourth that based on the gross area of concrete ($K_{sb}/4$) should be reasonable. A detailed evaluation of the effect of cracking may also be made. Since slabs normally have more than minimum reinforcement and are not fully cracked, except under very unusual conditions, the one-fourth value should be expected to provide a safe lower bound for stiffness under lateral loads. See R13.5.1.2 for guidance on stiffness assumption for lateral load analysis.

Moments from an Equivalent Frame (or Direct Design) analysis for gravity loading may be combined with moments from a lateral load analysis (13.5.1.3). Alternatively, the Equivalent Frame Analysis can be used for lateral load analysis, if modified to account for reduced stiffness of the slab-beams.

For both vertical and lateral load analyses, moments at critical sections of the slab-beams are transversely distributed in accordance with 13.6.4 (column strips) and 13.6.6 (middle strips).

13.5.4 Shear in Two-Way Slab Systems

If two-way slab systems are supported by beams or walls, the slab shear is seldom a critical factor in design, as the shear force at factored loads is generally well below the shear strength of the concrete.

In contrast, when two-way slabs are supported directly by columns as in flat plates or flat slabs, shear around the columns is of critical importance. Shear strength at an exterior slab-column connection (without edge beams) is especially critical because the total exterior negative slab moment must be transferred directly to the column. This aspect of two-way slab design should not be taken lightly by the designer. Two-way slab systems will normally be found to be quite "forgiving" if an error in the distribution or even in the amount of flexural reinforcement is made, but there will be no forgiveness if the required shear strength is not provided.

For slab systems supported directly by columns, it is advisable at an early stage in design to check the shear strength of the slab in the vicinity of columns as illustrated in Fig. 18-8.

Two types of shear need to be considered in the design of flat plates or flat slabs supported directly on columns. The first is the familiar one-way or beam-type shear, which may be critical in long narrow slabs. Analysis for beam shear considers the slab to act as a wide beam spanning between the columns. The critical section is taken at a distance d from the face of the column. Design against beam shear consists of checking for satisfaction of the requirement indicated in Fig. 18-9(a). Beam shear in slabs is seldom a critical factor in design, as the shear force is usually well below the shear strength of the concrete.

Two-way or "punching" shear is generally the more critical of the two types of shear in slab systems supported directly on columns. Punching shear considers failure along the surface of a truncated cone or pyramid around a column. The critical section is taken perpendicular to the slab at a distance d/2 from the perimeter of a column. The shear force V_u to be resisted can be easily calculated as the total factored load on the area bounded by panel centerlines around the column, less the load applied within the area defined by the critical shear perimeter (see Fig. 18-8).

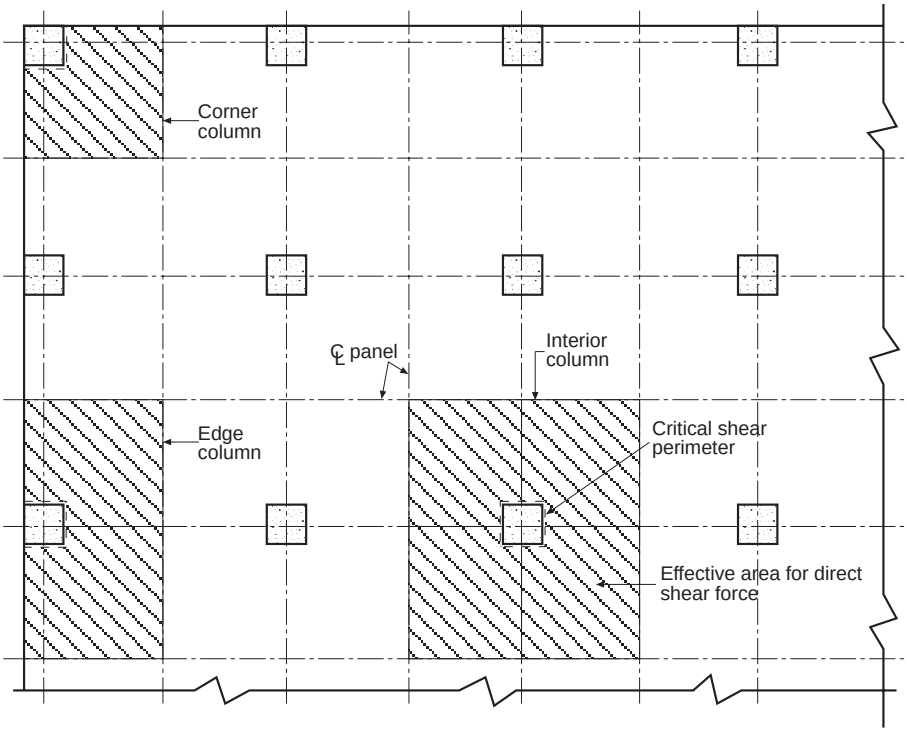


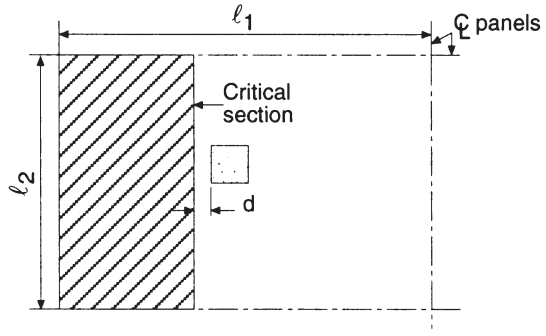
Figure 18-8 Critical Locations for Slab Shear Strength

In the absence of significant moment transfer from the slab to the column, design against punching shear consists of making sure that the requirement of Fig. 18-9(b) is satisfied. For practical design, only direct shear (uniformly distributed around the perimeter b_o) occurs around interior slab-column supports where no (or insignificant) moment is to be transferred from the slab to the column. Significant moments may have to be carried when unbalanced gravity loads on either side of an interior column or horizontal loading due to wind must be transferred from the slab to the column. At exterior slab-column supports, the total exterior slab moment from gravity loads (plus any lateral load moments due to wind or earthquake) must be transferred directly to the column.

13.5.3 Transfer of Moment in Slab-Column Connections

Transfer of moment between a slab and a column takes place by a combination of flexure (13.5.3) and eccentricity of shear (11.12.6.1). Shear due to moment transfer is assumed to act on a critical section at a distance d/2 from the face of the column (the same critical section around the column as that used for direct shear transfer; see Fig. 18-9(b). The portion of the moment transferred by flexure is assumed to be transferred over a width of slab equal to the transverse column width c_2, plus 1.5 times the slab thickness (1.5h) on either side of the column (13.5.3.2). Concentration of negative reinforcement is to be used to resist moment on this effective slab width. The combined shear stress due to direct shear and moment transfer often governs the design, especially at the exterior slab-column supports.

The portions of the total unbalanced moment M_u to be transferred by eccentricity of shear and by flexure are given by Eqs. (11-39) and (13-1), respectively, where $\gamma_v M_u$ is considered transferred by eccentricity of shear, and $\gamma_f M_u$ is considered transferred by flexure. At an interior square column with $b_1 = b_2$, 40% of the moment is transferred by eccentricity of shear ($\gamma_v M_u = 0.40 M_u$), and 60% by flexure ($\gamma_f M_u = 0.60 M_u$), where M_u is the transfer moment at the centroid of the critical section. The moment M_u at the exterior slab-column support will generally not be computed at the centroid of the critical transfer section. In the Equivalent Frame analysis, moments are computed at the column centerline. In the Direct Design Method, moments are computed at the face of support. Considering the approximate nature of the procedure used to evaluate the stress distribution due to moment transfer, it seems unwarranted to consider a change in moment to the critical section centroid; use of the moment values at column centerline (EFM) or at face of support (DDM) directly would usually be accurate enough.

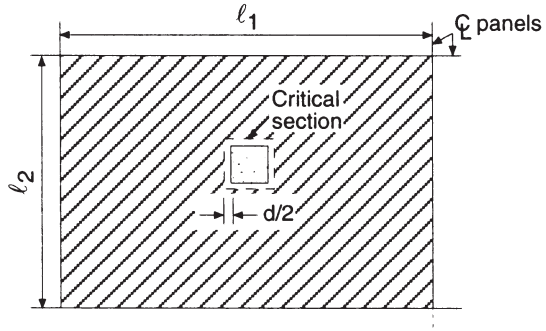


(a) Beam shear

$V_u \le \phi V_c$

$\le \phi 2\sqrt{f'_c}\ \ell_2 d$

where V_u is factored shear force (total factored load on shaded area).



(b) Two-way shear

$V_u \le \phi V_c$

where:

$$\phi V_c = \text{least of} \begin{cases} \phi\left(2 + \dfrac{4}{\beta_c}\right)\sqrt{f'_c}\ b_o d \\ \phi\left(\dfrac{\alpha_s d}{b_o} + 2\right)\sqrt{f'_c}\ b_o d \\ \phi 4\sqrt{f'_c}\ b_o d \end{cases}$$

V_u = factored shear force (total factored load on shaded area)
b_o = perimeter of critical section
β_c = long side/short side of reaction area
α_s = constant (11.12.2.1 (b))

Figure 18-9 Direct Shear at an Interior Slab-Column Support (see Fig. 18-8)

The factored shear stress on the critical transfer section is the sum of the direct shear and the shear caused by moment transfer,

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v M_u c}{J}$$

For slabs supported on square columns, shear stress v_u must not exceed $\phi 4\sqrt{f'_c}$.

Computation of the combined shear stress involves the following properties of the critical transfer section:

A_c = area of critical section

c = distance from centroid of critical section to face of section where stress is being computed

J = property of critical section analogous to polar moment of inertia

The above properties are given in Part 16. Note that in the case of flat slabs, two different critical sections need to be considered in punching shear calculations, as shown in Fig. 18-10.

Unbalanced moment transfer between slab and an edge column (without spandrel beams) requires special consideration when slabs are analyzed by the Direct Design Method for gravity loads. See discussion on 13.6.3.6 in Part 19.

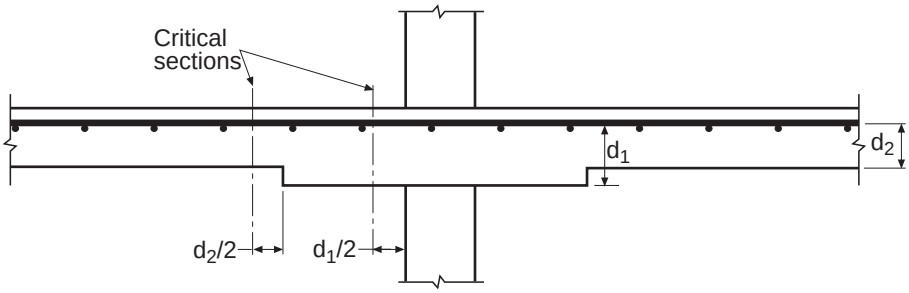


Figure 18-10 Critical Shear-Transfer Sections for Flat Slabs

The provisions of 13.5.3.3 were introduced in the '95 Code. At exterior supports, for unbalanced moments about an axis parallel to the edge, the portion of moment transferred by eccentricity of shear, $\gamma_v M_u$, may be reduced to zero provided that the factored shear at the support (excluding the shear produced by moment transfer) does not exceed 75 percent of the shear strength ϕV_c defined in 11.12.2.1 for edge columns or 50 percent for corner columns. Tests indicate that there is no significant interaction between shear and unbalanced moment at the exterior support in such cases. It should be noted that as $\gamma_v M_u$ is decreased, $\gamma_f M_u$ is increased.

Tests of interior supports have indicated that some flexibility in distributing unbalanced moment by shear and flexure is also possible, but with more severe limitations than for exterior supports. For interior supports, the unbalanced moment transferred by flexure is permitted to be increased up to 25 percent provided that the factored shear (excluding the shear caused by moment transfer) at an interior support does not exceed 40 percent of the shear strength ϕV_c defined in 11.12.2.1.

Note that the above modifications are permitted only when the reinforcement ratio ρ within the effective slab width defined in 13.5.3.2 is less than or equal to $0.375\rho_b$. This provision is intended to improve ductile behavior of the column-slab joint.

SEQUEL

The Direct Design Method and the Equivalent Frame Method for gravity load analysis of two-way slab systems are treated in detail in the following Parts 19 and 20, respectively.

19

Two-Way Slabs — Direct Design Method

GENERAL CONSIDERATIONS

The Direct Design Method is an approximate procedure for analyzing two-way slab systems subjected to gravity loads only. Since it is approximate, the method is limited to slab systems meeting the limitations specified in 13.6.1. Two-way slab systems not meeting these limitations must be analyzed by more accurate procedures such as the Equivalent Frame Method, as specified in 13.7. See Part 20 for discussion and design examples using the Equivalent Frame Method.

With the publication of ACI 318-83, the Direct Design Method for moment analysis of two-way slab systems was greatly simplified by eliminating all stiffness calculations for determining design moments in an end span. A table of moment coefficients for distribution of the total span moment in an end span (13.6.3.3) replaced the expressions for distribution as a function of the stiffness ratio α_{ec}. As a companion change, the then approximate Eq. (13-4) for unbalanced moment transfer between the slab and an interior column was also simplified through elimination of the α_{ec} term. With these changes, the Direct Design Method became a truly direct design procedure, with all design moments determined directly from moment coefficients. Also, a new 13.6.3.6 was added, addressing a special provision for shear due to moment transfer between a slab without beams and an edge column when the approximate moment coefficients of 13.6.3.3 are used. See discussion on 13.6.3.6 below. Through the 1989 (Revised 1992) edition of the code and commentary, R13.6.3.3 included a "Modified Stiffness Method" reflecting the original distribution, and confirming that design aids and computer programs based on the original distribution as a function of the stiffness ratio α_{ec} were still applicable for usage. The "Modified Stiffness Method" was dropped from R13.6.3.3 in the 1995 edition of the code and commentary.

PRELIMINARY DESIGN

Before proceeding with the Direct Design Method, a preliminary slab thickness h needs to be determined for control of deflections according to the minimum thickness requirements of 9.5.3. Table 18-1 and Fig. 18-3 can be used to simplify minimum thickness computations.

For slab systems without beams, it is advisable at this stage in the design process to check the shear strength of the slab in the vicinity of columns or other support locations in accordance with the special shear provision for slabs (11.12). See discussion on 13.5.4 in Part 18.

Once a slab thickness has been selected, the Direct Design Method, which is essentially a three-step analysis procedure, involves: (1) determining the total factored static moment for each span, (2) dividing the total factored static moment between negative and positive moments within each span, and (3) distributing the negative and the positive moment to the column and the middle strips in the transverse direction.

For analysis, the slab system is divided into design strips consisting of a column strip and two half-middle strip(s) as defined in 13.2.1 and 13.2.2, and as illustrated in Fig. 19-1. Some judgment is required in applying the definitions given in 13.2.1 for slab systems with varying span lengths along the design strip.

13.6.1 Limitations

The Direct Design Method applies within the limitations illustrated in Fig. 19-2:

1. There must be three or more continuous spans in each direction;

2. Slab panels must be rectangular with a ratio of longer to shorter span (centerline-to-centerline of supports) not greater than 2;

3. Successive span lengths (centerline-to-centerline of supports) in each direction must not differ by more than 1/3 of the longer span;

4. Columns must not be offset more than 10% of the span (in direction of offset) from either axis between centerlines of successive columns;

5. Loads must be uniformly distributed, with the unfactored or service live load not more than 2 times the unfactored or service dead load ($L/D \leq 2$);

6. For two-way beam-supported slabs, relative stiffness of beams in two perpendicular directions must satisfy the minimum and maximum requirements given in 13.6.1.6; and

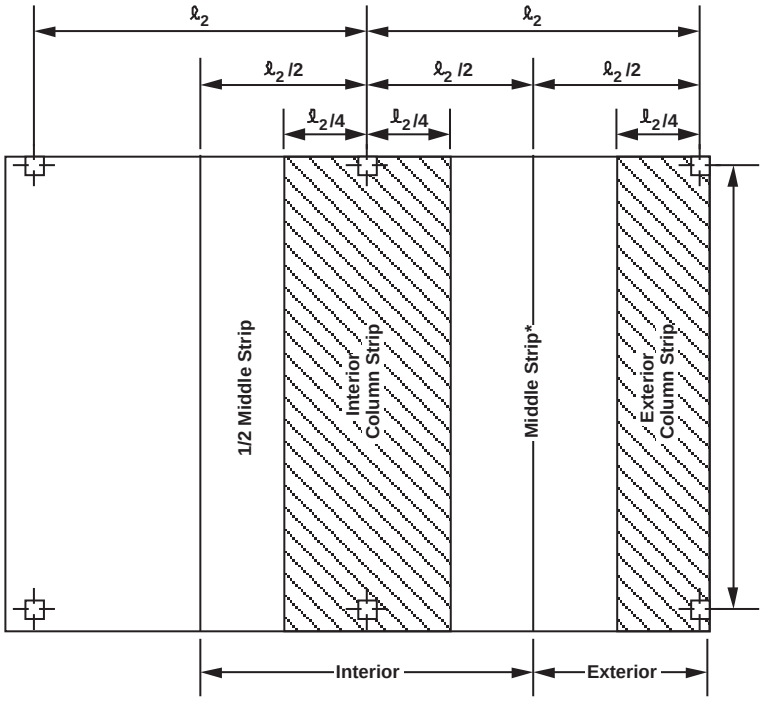
7. Redistribution of negative moments by 8.4 is not permitted.

13.6.2 Total Factored Static Moment for a Span

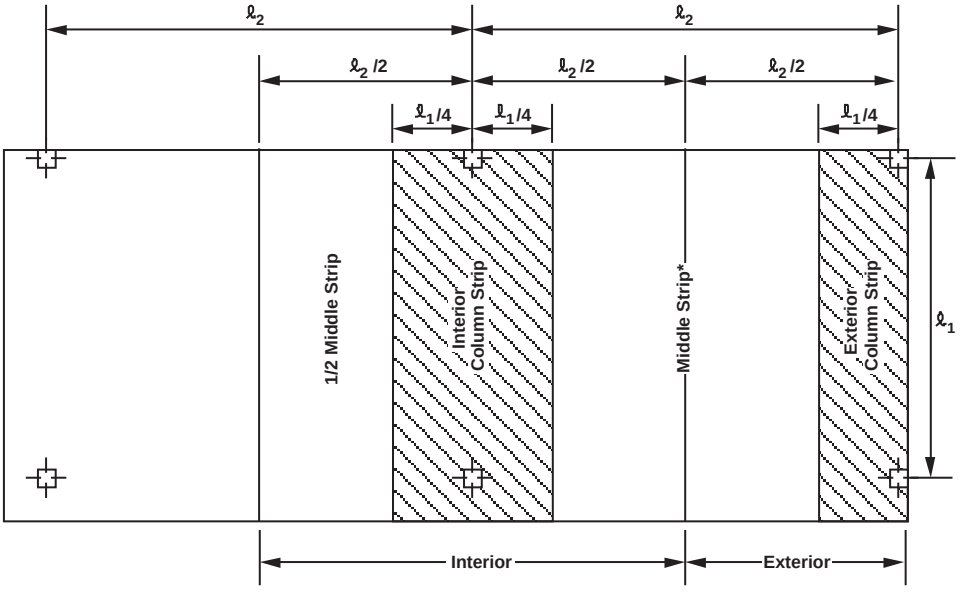
For uniform loading, the total design moment M_o for a span of the design strip is calculated by the simple static moment expression:

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} \qquad \textit{Eq. (13-3)}$$

where w_u is the factored combination of dead and live loads (psf), $w_u = 1.2w_d + 1.6w_\ell$. The clear span ℓ_n (in the direction of analysis) is defined in a straightforward manner for columns or other supporting elements of rectangular cross-section. The clear span starts at the face of support. Face of support is defined as shown in Fig. 19-3. One limitation requires that the clear span not be taken as less than 65% of the span center-to-center of supports (13.6.2.5). The length ℓ_2 is simply the span (centerline-to-centerline) transverse to ℓ_n; however, when the span adjacent and parallel to an edge is being considered, the distance from edge of slab to panel centerline is used for ℓ_2 in calculation of M_o (13.6.2.4).



(a) Column strip for $\ell_2 \leq \ell_1$



(b) Column strip for $\ell_2 > \ell_1$

* When edge of exterior design strip is supported by a wall, the factored moment resisted by this middle strip is defined in 13.6.6.3.

Figure 19-1 Definition of Design Strips

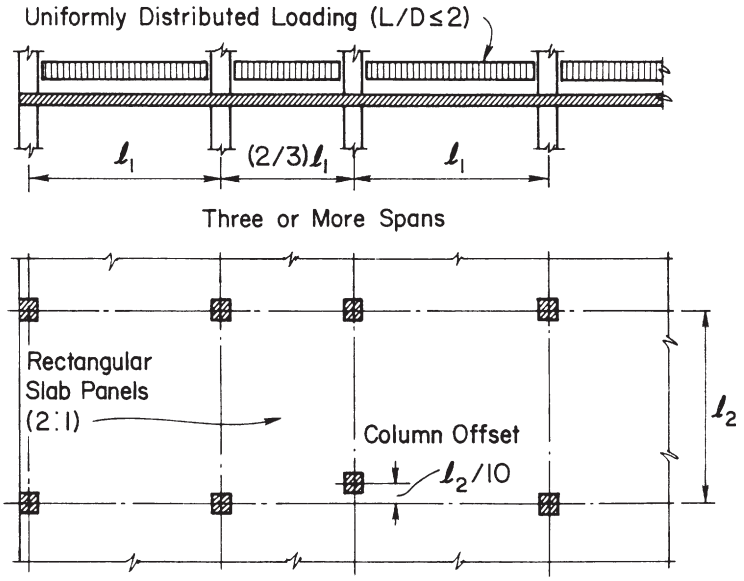
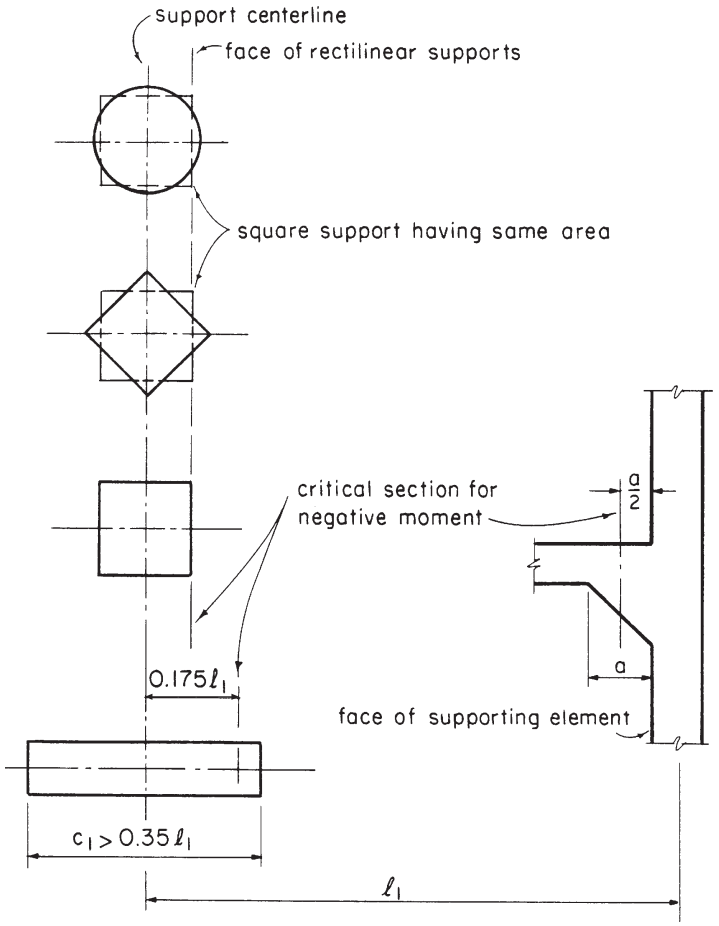


Figure 19-2 Conditions for Analysis by Coefficients



(a) Interior & Exterior Column or Wall Supports

(b) Exterior Supports with Brackets or Corbels

Figure 19-3 Critical Sections for Negative Design Moment

13.6.3 Negative and Positive Factored Moments

The total static moment for a span is divided into negative and positive design moments as shown in Fig. 19-4. End span moments in Fig. 19-4 are shown for a flat plate or flat slab without spandrels (slab system without beams between interior supports and without edge beam). For other end span conditions, the total static moment M_o is distributed as shown in Table 19-1.

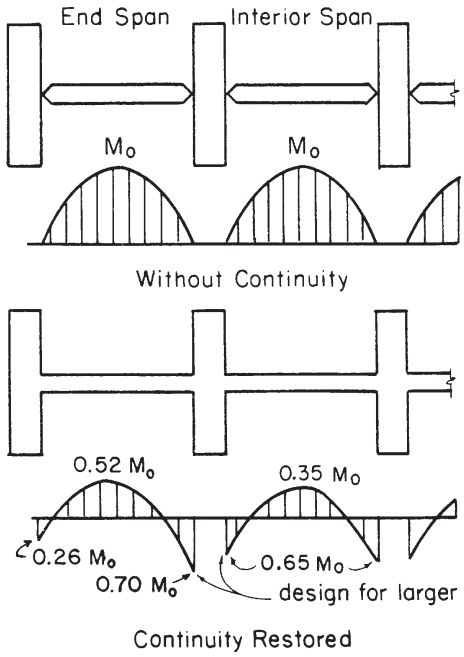


Figure 19-4 Design Strip Moments

Table 19-1 Distribution of Total Static Moment for an End Span

Factored Moment	(1)	(2)	(3)	(4)	(5)
			Flat Plates and Flat Slabs		
	Slab Simply Supported on Concrete or Masonry Wall	Two-Way Beam-Supported Slabs	Without Edge Beam	With Edge Beam	Slab Monolithic with Concrete Wall
Interior Negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior Negative	0	0.16	0.26	0.30	0.65

13.6.3.6 Special Provision for Load Transfer Between Slab and an Edge Column—For columns supporting a slab without beams, load transfer directly between the slab and the supporting columns (without intermediate load transfer through beams) is one of the more critical design conditions for the flat plate or flat slab system. Shear strength of the slab-column connection is critical. This aspect of two-way slab design should not be taken lightly by the designer. Two-way slab systems are fairly "forgiving" of an error in the distribution or even in the amount of flexural reinforcement; however, there is little or no forgiveness if a critical error in the provision of shear strength is made. See Part 16 for special provisions for direct shear and moment transfer at slab-column connections.

Section 13.6.3.6 addresses the potentially critical moment transfer between a beamless slab and an edge column. To ensure adequate shear strength when using the approximate end-span moment coefficients of 13.6.3.3, the 1989 edition of the code required that the full nominal strength M_n provided by the column strip be used in determining the fraction of unbalanced moment transferred by the eccentricity of shear (γ_v) in accordance with 11.12.6 (for end spans without edge beams, the column strip is proportioned to resist the total exterior negative factored moment). This requirement was changed in ACI 318-95. The moment $0.3M_o$ instead of M_n of the column strip must be used in determining the fraction of unbalanced moment transferred by the eccentricity of shear. The total reinforcement provided in the column strip includes the additional reinforcement concentrated over the column to resist the fraction of unbalanced moment transferred by flexure, $\gamma_f M_u = \gamma_f (0.26M_o)$, where the moment coefficient (0.26) is from 13.6.3.3, and γ_f is given by Eq. (13-1).

13.6.4 Factored Moments in Column Strips

The amounts of negative and positive factored moments to be resisted by a column strip, as defined in Fig. 19-1, depends on the relative beam-to-slab stiffness ratio and the panel width-to-length ratio in the direction of analysis. An exception to this is when a support has a large transverse width.

The column strip at the exterior of an end span is required to resist the total factored negative moment in the design strip unless edge beams are provided.

When the transverse width of a support is equal to or greater than three quarters (3/4) of the design strip width, 13.6.4.3 requires that the negative factored moment be uniformly distributed across the design strip.

The percentage of total negative and positive factored moments to be resisted by a column strip may be determined from the tables in 13.6.4.1 (interior negative), 13.6.4.2 (exterior negative) and 13.6.4.4 (positive), or from the following expressions:

Percentage of negative factored moment at interior support to be resisted by column strip

$$= 75 + 30\left(\frac{\alpha_1 \ell_2}{\ell_1}\right)\left(1 - \frac{\ell_2}{\ell_1}\right) \qquad (1)$$

Percentage of negative factored moment at exterior support to be resisted by column strip

$$= 100 - 10\beta_t + 12\beta_t\left(\frac{\alpha_1 \ell_2}{\ell_1}\right)\left(1 - \frac{\ell_2}{\ell_1}\right) \qquad (2)$$

Percentage of positive factored moment to be resisted by column strip

$$= 60 + 30\left(\frac{\alpha_1 \ell_2}{\ell_1}\right)\left(1.5 - \frac{\ell_2}{\ell_1}\right) \qquad (3)$$

Note: When $\alpha_1 \ell_2/\ell_1 > 1.0$, use 1.0 in above equations. When $\beta_t > 2.5$, use 2.5 in Eq. (2) above.

For slabs without beams between supports ($\alpha_1 = 0$) and without edge beams ($\beta_t = 0$), the distribution of total negative moments to column strips is simply 75 and 100 percent for interior and exterior supports, respectively, and the distribution of total positive moment is 60 percent. For slabs with beams between supports, distribution depends on the beam-to-slab stiffness ratio; when edge beams are present, the ratio of torsional stiffness of edge beam to flexural stiffness of slab also influences distribution. Figs. 19-6, 19-7, and 19-8 simplify evaluation of the beam-to-slab stiffness ratio α. To evaluate β_t, stiffness ratio for edge beams, Table 19-2 simplifies calculation of the torsional constant C.

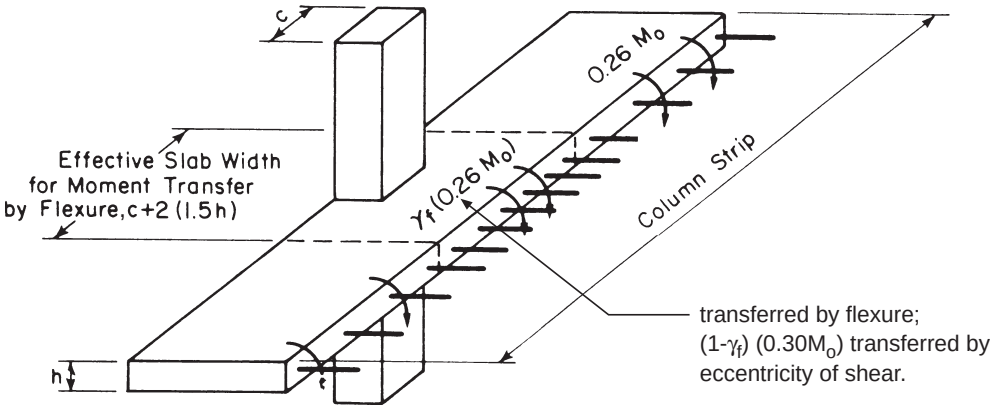


Figure 19-5 Transfer of Negative Moment at Exterior Support Section of Slab without Beams

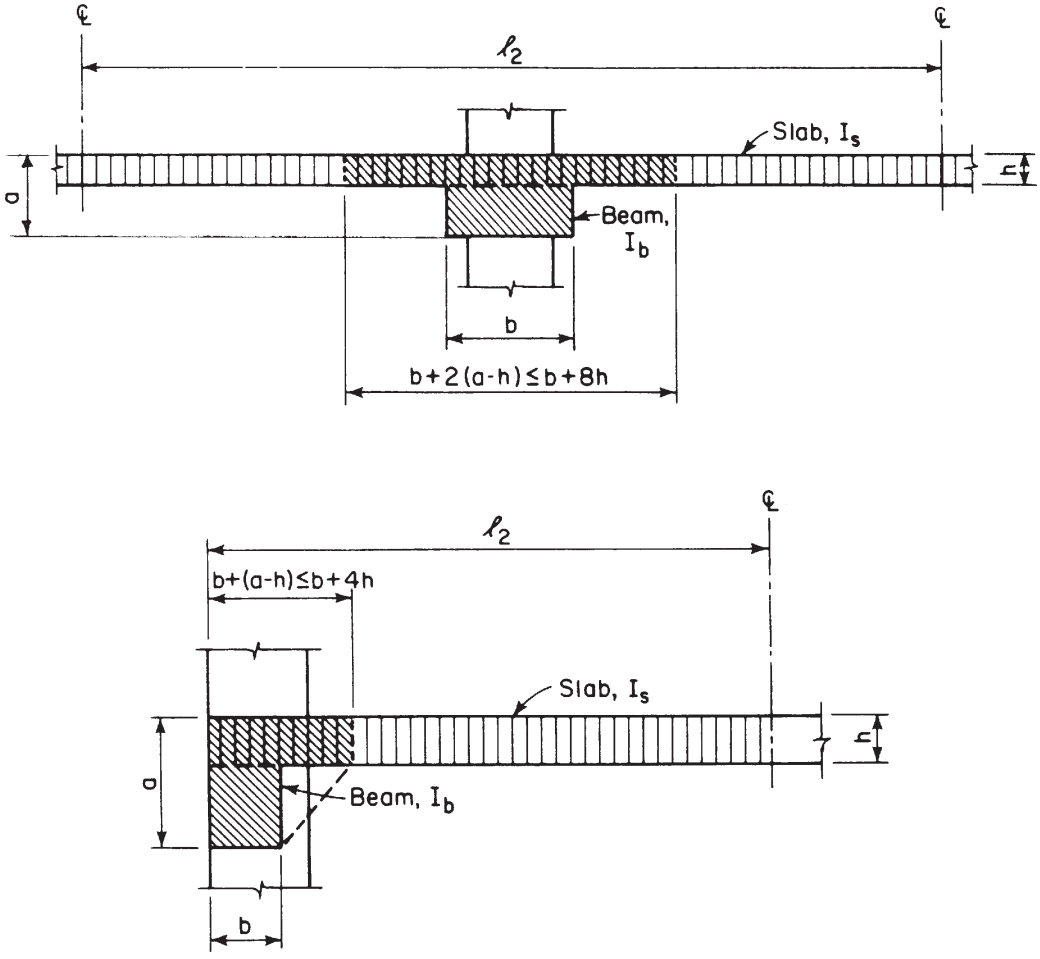


Figure 19-6 Effective Beam and Slab Sections for Computation of Stiffness Ratio α

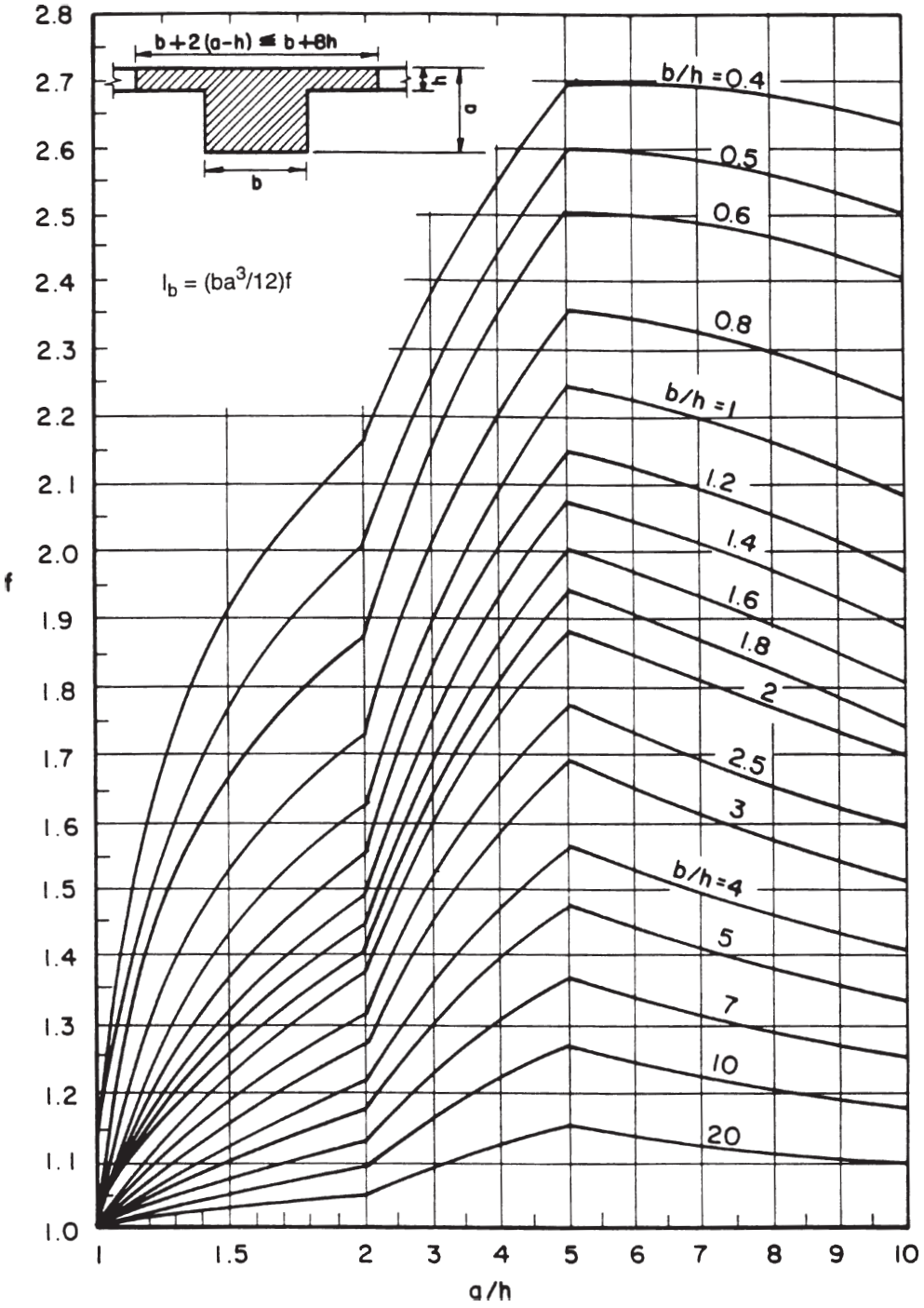


Figure 19-7 Beam Stiffness (Interior Beams)

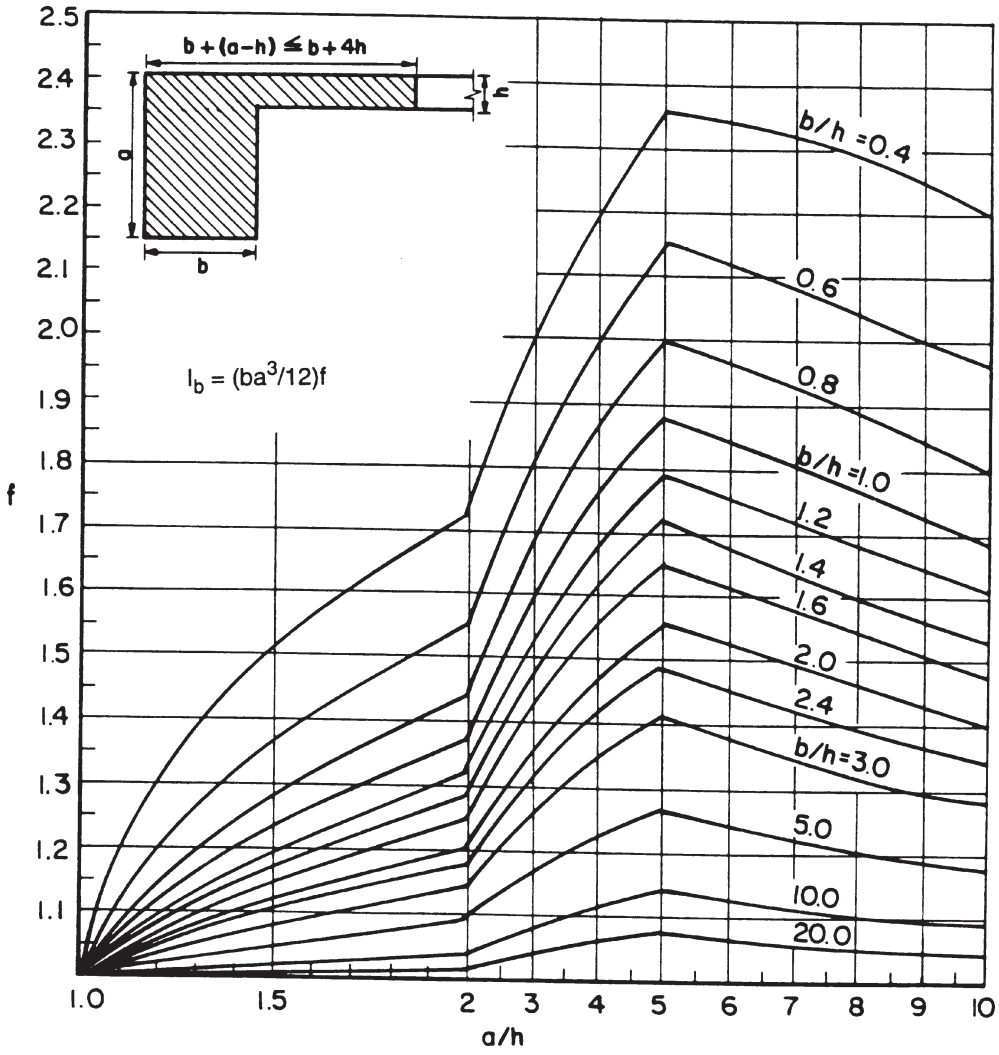
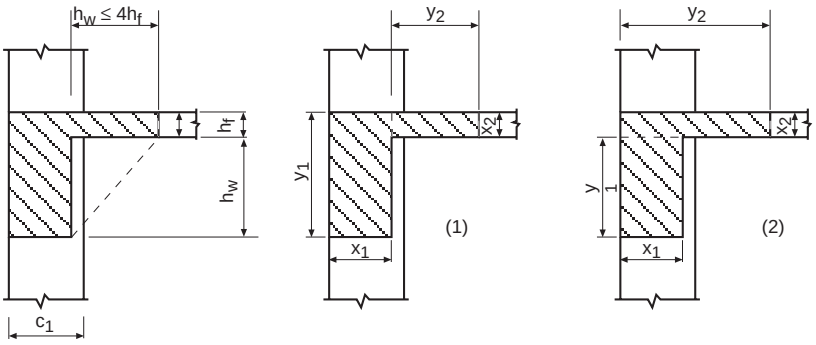


Figure 19-8 Beam Stiffness (Edge Beams)

Table 19-2 Design Aid for Computing C, Cross-Sectional Constant Defining Torsional Properties



Edge beam (ACI 13.2.4)

Use larger value of C computed from (1) or (2)

y	x*									
	4	5	6	7	8	9	10	12	14	16
12	202	369	592	868	1,118	1,538	1,900	2,557		
14	245	452	736	1,096	1,529	2,024	2,566	3,709	4,738	
16	288	534	880	1,325	1,871	2,510	3,233	4,861	6,567	8,083
18	330	619	1,024	1,554	2,212	2,996	3,900	6,013	8,397	10,813
20	373	702	1,167	1,782	2,553	3,482	4,567	7,165	10,226	13,544
22	416	785	1,312	2,011	2,895	3,968	5,233	8,317	12,055	16,275
24	458	869	1,456	2,240	3,236	4,454	5,900	9,459	13,885	19,005
27	522	994	1,672	2,583	3,748	5,183	6,900	11,197	16,628	23,101
30	586	1,119	1,888	2,926	4,260	5,912	7,900	12,925	19,373	27,197
33	650	1,243	2,104	3,269	4,772	6,641	8,900	14,653	22,117	31,293
36	714	1,369	2,320	3,612	5,284	7,370	9,900	16,381	24,860	35,389
42	842	1,619	2,752	4,298	6,308	8,828	11,900	19,837	30,349	43,581
48	970	1,869	3,183	4,984	7,332	10,286	13,900	23,293	35,836	51,773
54	1,098	2,119	3,616	5,670	8,356	11,744	15,900	26,749	41,325	59,965
60	1,226	2,369	4,048	6,356	9,380	13,202	17,900	30,205	46,813	68,157

** Small side of a rectangular cross-section with dimensions x and y.*

13.6.5 Factored Moments in Beams

When a design strip contains beams between columns, the factored moment assigned to the column strip must be distributed between the slab and the beam portions of the column strip. The amount of the column strip factored moment to be resisted by the beam varies linearly between zero and 85 percent as $\alpha_1 \ell_2 / \ell_1$ varies between zero and 1.0. When $\alpha_1 \ell_2 / \ell_1$ is equal to or greater than 1.0, 85 percent of the total column strip moment must be resisted by the beam. In addition, the beam section must resist the effects of loads applied directly to the beam, including weight of beam stem projecting above or below the slab.

13.6.6 Factored Moments in Middle Strips

Factored moments not assigned to the column strips must be resisted by the two half-middle strips comprising the design strip. An exception to this is a middle strip adjacent to and parallel with an edge supported by a wall, where the moment to be resisted is twice the factored moment assigned to the half middle strip corresponding to the first row of interior supports (see Fig. 19-1).

13.6.9 Factored Moments in Columns and Walls

Supporting columns and walls must resist any negative moments transferred from the slab system.

For interior columns (or walls), the approximate Eq. (13-4) may be used to determine the unbalanced moment transferred by gravity loading, unless an analysis is made considering the effects of pattern loading and unequal adjacent spans. The transfer moment is computed directly as a function of span length and gravity loading. For the more usual case with equal transverse and adjacent spans, Eq. (13-4) reduces to

$$M_u = 0.07\left(0.5w_\ell \ell_2 \ell_n^2\right) \qquad (4)$$

where, w_ℓ = factored live load, psf

ℓ_2 = span length transverse to ℓ_n

ℓ_n = clear span length in the direction of analysis

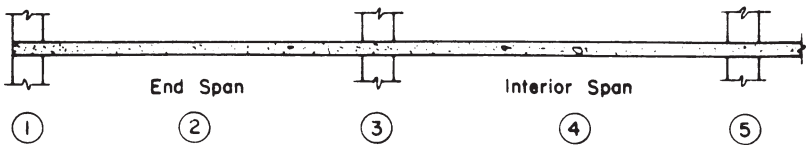
At exterior column or wall supports, the total exterior negative factored moment from the slab system (13.6.3.3) is transferred directly to the supporting members. Due to the approximate nature of the moment coefficients, it seems unwarranted to consider the change in moment from face of support to centerline of support; use the moment values from 13.6.3.3 directly.

Columns above and below the slab must resist the unbalanced support moment based on the relative column stiffnesses—generally, in proportion to column lengths above and below the slab. Again, due to the approximate nature of the moment coefficients of the Direct Design Method, the refinement of considering the change in moment from centerline of slab-beam to top or bottom of column seems unwarranted.

DESIGN AID — DIRECT DESIGN MOMENT COEFFICIENTS

Distribution of the total free-span moment M_o into negative and positive moments, and then into column and middle strip moments, involves direct application of moment coefficients to the total moment M_o. The moment coefficients are a function of location of span (interior or end), slab support conditions, and type of two-way slab system. For design convenience, moment coefficients for typical two-way slab systems are given in Tables 19-3 through 19-7. Tables 19-3 through 19-6 apply to flat plates or flat slabs with differing end support conditions. Table 19-7 applies to two-way slabs supported on beams on all four sides. Final moments for the column strip and the middle strip are directly tabulated.

Table 19-3 Design Moment Coefficients for Flat Plate or Flat Slab Supported Directly on Columns

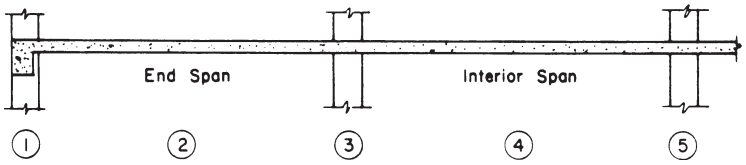


Slab Moments	End Span			Interior Span	
	(1)	(2)	(3)	(4)	(5)
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Total Moment	$0.26M_o$	$0.52M_o$	$0.70M_o$	$0.35M_o$	$0.65M_o$
Column Strip	$0.26M_o$	$0.31M_o$	$0.53M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	0	$0.21M_o$	$0.17M_o$	$0.14M_o$	$0.16M_o$

Note: All negative moments are at face of support.

The moment coefficients of Table 19-4 (flat plate with edge beams) are valid for $\beta_t \geq 2.5$. The coefficients of Table 19-7 (two-way beam-supported slabs) apply for $\alpha_1 \ell_2/\ell_1 \geq 1.0$ and $\beta_t \geq 2.5$. Many practical beam sizes will provide beam-to-slab stiffness ratios such that $\alpha_1 \ell_2/\ell_1$ and β_t will be greater than these limits, allowing moment coefficients to be taken directly from the tables, without further consideration of stiffnesses and interpolation for moment coefficients. However, if beams are present, the two stiffness parameters α_1 and β_t will need to be evaluated. For two-way slabs, and for $E_{cb} = E_{cs}$, the stiffness parameter α_1 is simply the ratio of the moments of inertia of the effective beam and slab sections in the direction of analysis, $\alpha_1 = I_b/I_s$, as illustrated in Fig. 19-6. Figures 19-7 and 19-8 simplify evaluation of the α_1 term.

Table 19-4 Design Moment Coefficients for Flat Plate or Flat Slab with Edge Beams



Slab Moments	End Span			Interior Span	
	(1)	(2)	(3)	(4)	(5)
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Total Moment	$0.30M_o$	$0.50M_o$	$0.70M_o$	$0.35M_o$	$0.65M_o$
Column Strip	$0.23M_o$	$0.30M_o$	$0.53M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	$0.07M_o$	$0.20M_o$	$0.17M_o$	$0.14M_o$	$0.16M_o$

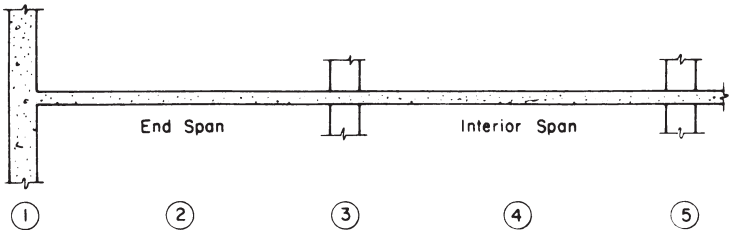
Notes: (1) All negative moments are at face of support.
(2) Torsional stiffness of edge beam is such that $\beta_t \geq 2.5$. For values of β_t less than 2.5, exterior negative column strip moment increases to $(0.30 - 0.03\ \beta_t)M_o$.

For $E_{cb} = E_{cs}$, relative stiffness provided by an edge beam is reflected by the parameter $\beta_t = C/2I_s$, where I_s is the moment of inertia of the effective slab section spanning in the direction of ℓ_1 and having a width equal to ℓ_2, i.e., $I_s = \ell_2 h^3/12$. The constant C pertains to the torsional stiffness of the effective edge beam cross-section. It is found by dividing the beam section into its component rectangles, each having a smaller dimension x and a larger dimension y, and by summing the contributions of all the parts by means of the equation:

$$C = \Sigma \left(1 - \frac{0.63x}{y}\right)\left(\frac{x^3 y}{3}\right) \qquad (5)$$

The subdivision can be done in such a way as to maximize C. Table 19-2 simplifies calculation of the torsional constant C.

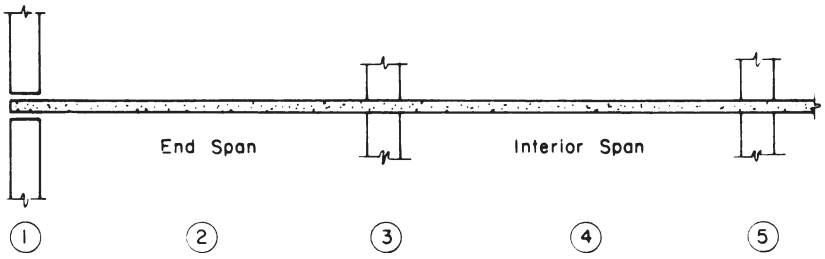
Table 19-5 Design Moment Coefficients for Flat Plate or Flat Slab with End Span Integral with Wall



Slab Moments	End Span			Interior Span	
	(1)	(2)	(3)	(4)	(5)
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Total Moment	$0.65M_o$	$0.35M_o$	$0.65M_o$	$0.35M_o$	$0.65M_o$
Column Strip	$0.49M_o$	$0.21M_o$	$0.49M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	$0.16M_o$	$0.14M_o$	$0.16M_o$	$0.14M_o$	$0.16M_o$

Note: All negative moments are at face of support.

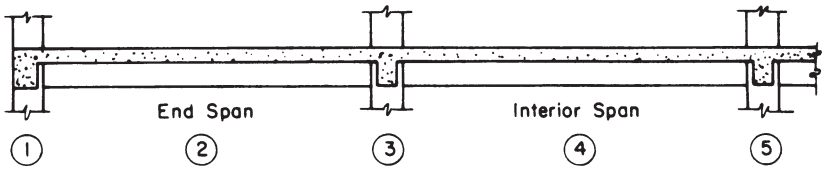
Table 19-6 Design Moment Coefficients for Flat Plate or Flat Slab with End Span Simply Supported on Wall



Slab Moments	End Span			Interior Span	
	(1)	(2)	(3)	(4)	(5)
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Total Moment	0	$0.63M_o$	$0.75M_o$	$0.35M_o$	$0.65M_o$
Column Strip	0	$0.38M_o$	$0.56M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	0	$0.25M_o$	$0.19M_o$	$0.14M_o$	$0.16M_o$

Note: All negative moments are at face of support.

Table 19-7 Design Moment Coefficients for Two-Way Beam-Supported Slab



Span Ratio ℓ_2/ℓ_1	Slab and Beam Moments	End Span			Interior Span	
		(1)	(2)	(3)	(4)	(5)
		Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
	Total Moment	$0.16M_o$	$0.57M_o$	$0.70M_o$	$0.35M_o$	$0.65M_o$
0.5	Column Strip Beam	$0.12M_o$	$0.43M_o$	$0.54M_o$	$0.27M_o$	$0.50M_o$
	Slab	$0.02M_o$	$0.08M_o$	$0.09M_o$	$0.05M_o$	$0.09M_o$
	Middle Strip	$0.02M_o$	$0.06M_o$	$0.07M_o$	$0.03M_o$	$0.06M_o$
1.0	Column Strip Beam	$0.10M_o$	$0.37M_o$	$0.45M_o$	$0.22M_o$	$0.42M_o$
	Slab	$0.02M_o$	$0.06M_o$	$0.08M_o$	$0.04M_o$	$0.07M_o$
	Middle Strip	$0.04M_o$	$0.14M_o$	$0.17M_o$	$0.09M_o$	$0.16M_o$
2.0	Column Strip Beam	$0.06M_o$	$0.22M_o$	$0.27M_o$	$0.14M_o$	$0.25M_o$
	Slab	$0.01M_o$	$0.04M_o$	$0.05M_o$	$0.02M_o$	$0.04M_o$
	Middle Strip	$0.09M_o$	$0.31M_o$	$0.38M_o$	$0.19M_o$	$0.36M_o$

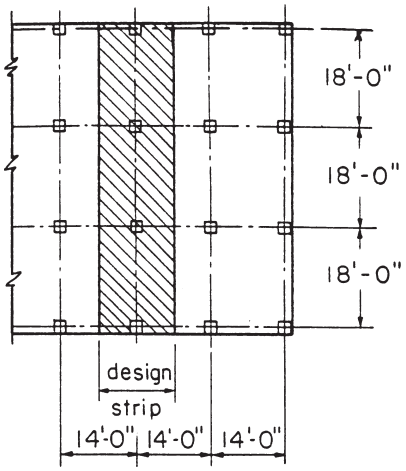
Notes: (1) All negative moments are at face of support.

(2) Torsional stiffness of edge beam is such that $\beta_t \geq 2.5$

(3) $\alpha_1 \ell_2/\ell_1 \geq 1.0$

Example 19.1—Two-Way Slab without Beams Analyzed by the Direct Design Method

Use the Direct Design Method to determine design moments for the flat plate slab system in the direction shown, for an intermediate floor.



Story height = 9 ft
Column dimensions = 16 × 16 in.
Lateral loads to be resisted by shear walls
No edge beams
Partition weight = 20 psf
Service live load = 40 psf
f'_c = 4,000 psi, normal weight concrete
f_y = 60,000 psi

Also determine the reinforcement and shear requirements at an exterior column.

Calculations and Discussion	Code Reference

1. Preliminary design for slab thickness h:

   a. Control of deflections.

   For slab systems without beams (flat plate), the minimum overall thickness h with Grade 60 reinforcement is (see Table 18-1): — *9.5.3.2, Table 9.5(c)*

   $$h = \frac{\ell_n}{30} = \frac{200}{30} = 6.67 \text{ in.} \quad \text{Use } h = 7 \text{ in.}$$

   where ℓ_n is the length of clear span in the long direction = 216 - 16 = 200 in.

   This is larger than the 5 in. minimum specified for slabs without drop panels. — *9.5.3.2(a)*

   b. Shear strength of slab.

   Use an average effective depth, d ≈ 5.75 in. (3/4-in. cover and No. 4 bar)

   Factored dead load, $w_d = 1.2\ (87.5 + 20) = 129$ psf

   Factored live load, $w_\ell = 1.6 \times 40 = 64$ psf

   Total factored load, $w_u = 193$ psf

   Investigation for wide-beam action is made on a 12-in. wide strip at a distance d from the face of support in the long direction (see Fig.19-9). — *11.12.1.1*

   $V_u = 0.193 \times 7.854 = 1.5$ kips

   $V_c = 2\sqrt{f'_c}\, b_w d$ — *Eq. (11-3)*

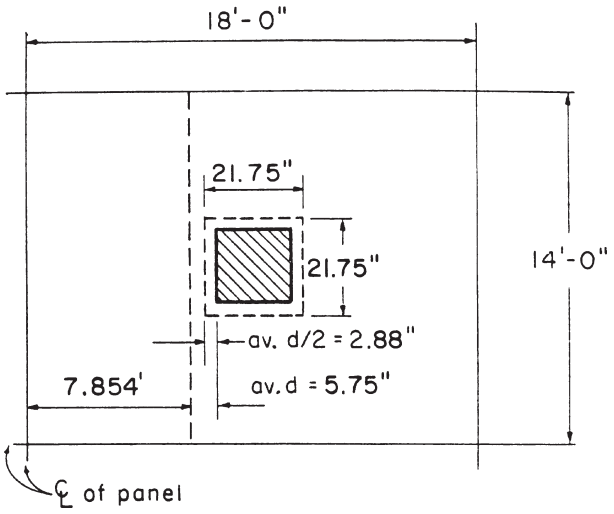


Figure 19-9 Critical Sections for One-Way and Two-Way Shear

$$= \frac{2\sqrt{4,000} \times 12 \times 5.75}{1,000} = 8.73 \text{ kips}$$

$\phi V_c = 0.75 \times 8.73 = 6.6 \text{ kips} > V_u = 1.5 \text{ kips}$ O.K.

Since there are no shear forces at the centerline of adjacent panels (see Fig. 19-9), the shear strength in two-way action at d/2 distance around a support is computed as follows:

$V_u = 0.193 [(18 \times 14) - 1.81^2)] = 48.0 \text{ kips}$

$V_c = 4\sqrt{f'_c}\, b_o d$ (for square columns) *Eq. (11-35)*

$$= \frac{4\sqrt{4,000} \times (4 \times 21.75) \times 5.75}{1,000} = 126.6 \text{ kips}$$

$V_u = 48.0 \text{ kips} < \phi V_c = 0.75 \times 126.6 \text{ kips} = 95.0 \text{ kips}$ O.K.

Therefore, preliminary design indicates that a 7 in. slab is adequate for control of deflection and shear strength.

2. Check applicability of Direct Design Method: *13.6.1*

   There is a minimum of three continuous spans in each direction *13.6.1.1*

   Long-to-short span ratio is 1.29 < 2.0 *13.6.1.2*

Successive span lengths are equal — *13.6.1.3*

Columns are not offset — *13.6.1.4*

Loads are uniformly distributed with service live-to-dead load ratio of 0.37 < 2.0 — *13.6.1.5*

Slab system is without beams — *13.6.1.6*

3. Factored moments in slab:

   a. Total factored moment per span. — *13.6.2*

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} \qquad \textit{Eq. (13-3)}$$

$$= \frac{0.193 \times 14 \times 16.67^2}{8} = 93.6 \text{ ft-kips}$$

   b. Distribution of the total factored moment M_o per span into negative and positive moments, and then into column and middle strip moments. This distribution involves direct application of the moment coefficients to the total moment M_o. Referring to Table 19-3 (flat plate without edge beams), — *13.6.3, 13.6.4, 13.6.6*

	Total Moment (ft-kips)	Column Strip Moment (ft-kips)	Moment (ft-kips) in Two Half-Middle Strips*
End Span:			
Exterior Negative	$0.26M_o = 24.3$	$0.26M_o = 24.3$	0
Positive	$0.52M_o = 48.7$	$0.31M_o = 29.0$	$0.21M_o = 19.7$
Interior Negative	$0.70M_o = 65.5$	$0.53M_o = 49.6$	$0.17M_o = 15.9$
Interior Span:			
Positive	$0.35M_o = 32.8$	$0.21M_o = 19.7$	$0.14M_o = 13.1$
Negative	$0.65M_o = 60.8$	$0.49M_o = 45.9$	$0.16M_o = 15.0$

**That portion of the total moment M_o not resisted by the column strip is assigned to the two half-middle strips.*

Note: The factored moments may be modified by 10 percent, provided the total factored static moment in any panel is not less than that computed from Eq. (13-3). This modification is omitted here. — *13.6.7*

4. Factored moments in columns: — *13.6.9*

   a. Interior columns, with equal spans in the direction of analysis and (different) equal spans in the transverse direction.

$$M_u = 0.07\left(0.5 w_\ell \ell_2 \ell_n^2\right) \qquad \textit{Eq. (13-4)}$$

$$= 0.07\,(0.5 \times 1.6 \times 0.04 \times 14 \times 16.67^2) = 8.7 \text{ ft-kips}$$

With the same column size and length above and below the slab,

$$M_c = \frac{8.7}{2} = 4.35 \text{ ft-kips}$$

This moment is combined with the factored axial load (for each story) for design of the interior columns.

b. Exterior columns.

Total exterior negative moment from slab must be transferred directly to the columns: $M_u = 24.3$ ft-kips. With the same column size and length above and below the slab,

$$M_c = \frac{24.3}{2} = 12.15 \text{ ft-kips}$$

This moment is combined with the factored axial load (for each story) for design of the exterior column.

5. Check slab flexural and shear strength at exterior column

a. Total flexural reinforcement required for design strip:

i. Determine reinforcement required for strip moment $M_u = 24.3$ ft-kips

Assume tension-controlled section (ϕ=0.9) 9.3.2

Column strip width $b = \frac{14 \times 12}{2} = 84$ in. 13.2.1

$$R_n = \frac{M_u}{\phi b d^2} = \frac{24.3 \times 12{,}000}{0.9 \times 84 \times 5.75^2} = 117 \text{ psi}$$

$$\rho = \frac{0.85 f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right)$$

$$= \frac{0.85 \times 4}{60}\left(1 - \sqrt{1 - \frac{2 \times 117}{0.85 \times 4{,}000}}\right) = 0.0020$$

$$A_s = \rho b d = 0.0020 \times 84 \times 5.75 = 0.96 \text{ in.}^2$$

$\rho_{min} = 0.0018$ 13.3.1

Min. $A_s = 0.0018 \times 84 \times 7 = 1.06 \text{ in.}^2 > 0.96 \text{ in.}^2$

Number of No. 4 bars $= \dfrac{1.06}{0.2} = 5.3$, say 6 bars

Maximum spacing $s_{max} = 2h = 14 \text{ in.} < 18 \text{ in.}$ *13.3.2*

Number of No.4 bars based on $s_{max} = \dfrac{84}{14} = 6$

Verify tension-controlled section:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(6 \times 0.2) \times 60}{0.85 \times 4 \times 84} = 0.25 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.25}{0.85} = 0.29 \text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right) d_t - 0.003$$

$$= \left(\frac{0.003}{0.29}\right) 5.75 - 0.003 = 0.057 > 0.005$$ *10.3.4*

Therefore, section is tension-controlled.

Use 6-No. 4 bars in column strip.

ii. Check slab reinforcement at exterior column for moment transfer between slab and column

Portion of unbalanced moment transferred by flexure = $\gamma_f M_u$

From Fig. 16-13, Case C:

$$b_1 = c_1 + \frac{d}{2} = 16 + \frac{5.75}{2} = 18.88 \text{ in.}$$

$$b_2 = c_2 + d = 16 + 5.75 = 21.75 \text{ in.}$$

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} = \frac{1}{1 + (2/3)\sqrt{18.88/21.75}} = 0.62$$

$\gamma_f M_u = 0.62 \times 24.3 = 15.1$ ft-kips

Note that the provions of 13.5.3.3 may be utilized; however, they are not in this example.

Assuming tension-controlled behavior, determine required area of reinforcement for $\gamma_f M_u = 15.1$ ft-kips:

Effective slab width $b = c_2 + 3h = 16 + 3\,(7) = 37$ in. *13.5.3.2*

$$R_n = \frac{M_u}{\phi b d^2} = \frac{15.1 \times 12{,}000}{0.9 \times 37 \times 5.75^2} = 165 \text{ psi}$$

$$\rho = \frac{0.85 f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right)$$

$$= \frac{0.85 \times 4}{60}\left(1 - \sqrt{1 - \frac{2 \times 165}{0.85 \times 4000}}\right) = 0.0028$$

$$A_s = 0.0028 \times 37 \times 5.75 = 0.60 \text{ in.}^2$$

$$\text{Min. } A_s = 0.0018 \times 37 \times 7 = 0.47 \text{ in.}^2 < 0.60 \text{ in.}^2$$

$$\text{Number of No. 4 bars} = \frac{0.60}{0.2} = 3$$

Verify tension-controlled section:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3 \times 0.2) \times 60}{0.85 \times 4 \times 37} = 0.29 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.29}{0.85} = 0.34 \text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{0.34}\right) 5.75 - 0.003 = 0.048 > 0.005$$

Therefore, section is tension-controlled.

Provide the required 3-No. 4 bars by concentrating 3 of the column strip bars (6-No. 4) within the 37 in. slab width over the column. For symmetry, add one additional No. 4 bar outside of 37-in. width.

Note that the column strip section remains tension-controlled with the addition of 1-No. 4 bar.

iii. Determine reinforcement required for middle strip.

Since all of the moment at exterior columns is transferred to the column strip, provide minimum reinforcement in middle strip:

$$\text{Min. } A_s = 0.0018 \times 84 \times 7 = 1.06 \text{ in.}^2$$

$$\text{Number of No. 4 bars} = \frac{1.06}{0.2} = 5.3, \text{ say } 6$$

Maximum spacing $s_{max} = 2h = 14 \text{ in.} < 18 \text{ in.}$ *13.3..2*

$$\text{Number of No. 4 bars based on } s_{max} = \frac{84}{14} = 6$$

Provide No. 4 @ 14 in. in middle strip.

b. Check combined shear stress at inside face of critical transfer section: *11.12.6.1*

For shear strength equations, see Part 16.

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v M_u}{J/c}$$

Factored shear force at exterior column:

$$V_u = 0.193\left[(14 \times 9.667) - \left(\frac{18.88 \times 21.75}{144}\right)\right] = 25.6 \text{ kips}$$

When the end span moments are determined from the Direct Design Method, the fraction of unbalanced moment transferred by eccentricity of shear must be $0.3M_o = 0.3 \times 93.6 = 28.1 \text{ ft-kips}$. *13.6.3.6*

$$\gamma_v = 1 - \gamma_f = 1 - 0.62 = 0.38$$ *Eq(11-3)*

From Fig. 16-13, critical section propeties for edge column bending perpendicular to edge (Case C):

$$A_c = (2b_1 + b_2)\,d = [(2 \times 18.88) + 21.75] \times 5.75 = 342.2 \text{ in.}^2$$

$$\frac{J}{c} = \frac{2b_1^2 d\,(b_1 + 2b_2) + d^3\,(2b_1 + b_2)}{6b_1}$$

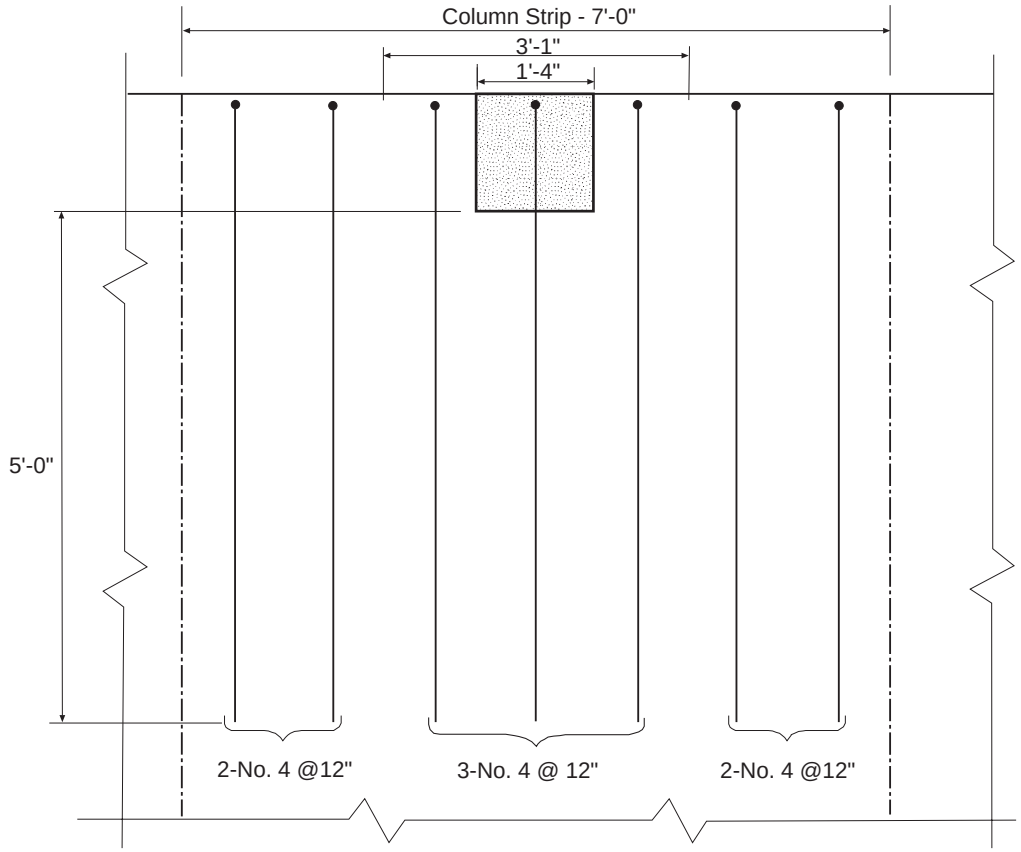
$$= \frac{2(18.88)^2(5.75)\,[18.88 + (2 \times 21.75)] + 5.75^3\,[(2 \times 18.88) + 21.75]}{6 \times 18.88}$$

$$= 2{,}357 \text{ in.}^3$$

$$v_u = \frac{25{,}600}{342.2} + \frac{0.38 \times 28.1 \times 12{,}000}{2{,}357}$$

$$= 74.8 + 54.4 = 129.2 \text{ psi}$$

Allowable shear stress $\phi v_n = \phi 4\sqrt{f'_c} = 0.75 \times 4\sqrt{4{,}000} = 189.7 \text{ psi} > v_u$ O.K. *1.12..6.2*

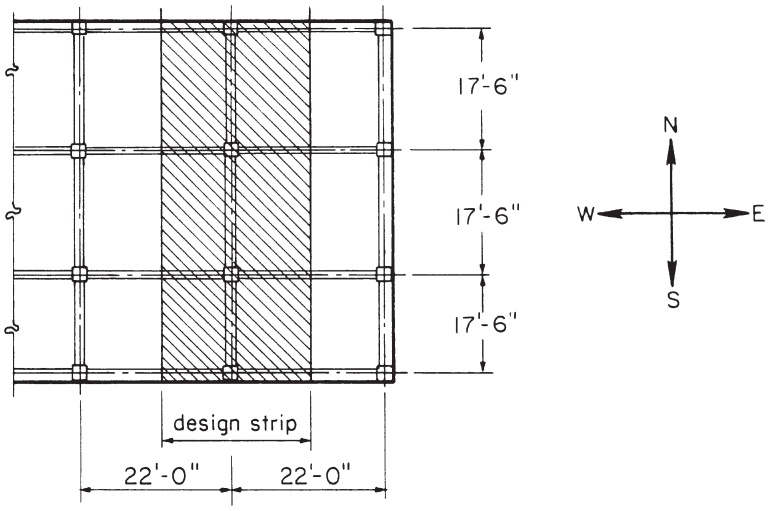


Example 19.2—Two-Way Slab with Beams Analyzed by the Direct Design Method

Use the Direct Design Method to determine design moments for the slab system in the direction shown, for an intermediate floor.

Story height = 12 ft
Edge beam dimensions = 14 × 27 in.
Interior beam dimensions = 14 × 20 in.
Column dimensions = 18 × 18 in.
Slab thickness = 6 in.
Service live load = 100 psf

f'_c = 4,000 psi (for all members), normal weight concrete
f_y = 60,000 psi



Calculations and Discussion	Code Reference

1. Preliminary design for slab thickness h: *9.5.3*

   Control of deflections.

   With the aid of Figs. 19-6, 19-7, and 19-8, beam-to-slab flexural stiffness ratio α is computed as follows:

   NS edge beams:

   $\ell_2 = \quad = 141$ in.

   $$\frac{a}{h} = \frac{27}{6} = 4.5$$

   $$\frac{b}{h} = \frac{14}{6} = 2.33$$

   From Fig. 19-8, f = 1.47

   $$I_b = \left(\frac{ba^3}{12}\right) f$$

   $$I_s = \frac{\ell_2 h^3}{12}$$

$$\alpha = \frac{E_{cb}I_b}{E_{cs}I_s} = \frac{I_b}{I_s}$$ 13.0

$$= \left(\frac{b}{\ell_2}\right)\left(\frac{a}{h}\right)^3 f$$

$$= \left(\frac{14}{141}\right)\left(\frac{27}{6}\right)^3 (1.47) = 13.30$$

EW edge beams:

$$\ell_2 = \frac{17.5 \times 12}{2} + \frac{18}{2} = 114 \text{ in.}$$

$$\alpha = \left(\frac{14}{114}\right)\left(\frac{27}{6}\right)^3 (1.47) = 16.45$$

NS interior beams:

$$\ell_2 = 22 \text{ ft} = 264 \text{ in.}$$

$$\frac{a}{h} = \frac{20}{6} = 3.33$$

$$\frac{b}{h} = \frac{14}{6} = 2.33$$

From Fig. 19-7, f = 1.61

$$\alpha = \left(\frac{14}{264}\right)\left(\frac{20}{6}\right)^3 1.61 = 3.16$$

EW interior beams:

$\ell_2 = 17.5$ ft $= 210$ in.

$$\alpha = \left(\frac{14}{210}\right)\left(\frac{20}{6}\right)^3 (1.61) = 3.98$$

Since $\alpha > 2.0$ for all beams, Eq. (9-13) will control minimum thickness. 9.5.3.3

Therefore,

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{200{,}000}\right)}{36 + 9\beta}$$ Eq. (9-12)

$$= \frac{246\left(0.8 + \frac{60{,}000}{200{,}000}\right)}{36 + 9\,(1.28)} = 5.7 \text{ in.}$$

where

$$\beta = \frac{\text{clear span in the long direction}}{\text{clear span in the short direction}} = \frac{20.5}{16} = 1.28$$

ℓ_n = clear span in long direction measured face to face of columns = 20.5 ft = 246 in.

Use 6 in. slab thickness

2. Check applicability of Direct Design Method: 13.6.1

There is a minimum of three continuous spans in each direction 13.6.1.1
Long-to-short span ratio is 1.26 < 2.0 13.6.1.2
Successive span lengths are equal 13.6.1.3
Columns are not offset 13.6.1.4
Loads are uniformly distributed with service live-to-dead ratio of 1.33 < 2.0 13.6.1.5

Check relative stiffness for slab panel: 13.6.1.6

Interior Panel:

$\alpha_1 = 3.16$ $\ell_2 = 264$ in.

$\alpha_2 = 3.98$ $\ell_1 = 210$ in.

$$\frac{\alpha_1 \ell_2^2}{\alpha_2 \ell_1^2} = \frac{3.16 \times 264^2}{3.98 \times 210^2} = 1.25$$ $0.2 < 1.25 < 5.0$ O.K. Eq. (13-2)

Exterior Panel:

$\alpha_1 = 3.16$ $\ell_2 = 264$ in.

$\alpha_2 = 16.45$ $\ell_1 = 210$ in.

$$\frac{\alpha_1 \ell_2^2}{\alpha_2 \ell_1^2} = \frac{3.16 \times 264^2}{16.45 \times 210^2} = 0.3$$ $0.2 < 0.3 < 5.0$ O.K.

Therefore, use of Direct Design Method is permitted.

3. Factored moments in slab:

Total factored moment per span — *13.6.2*

$$\text{Average weight of beams stem} = \frac{14 \times 14}{144} \times \frac{150}{22} = 9.3 \text{ psf}$$

$$\text{Weight of slab} = \frac{6}{12} \times 150 = 75 \text{ psf}$$

$$w_u = 1.2(75 + 9.3) + 1.6(100) = 261 \text{ psf}$$ — *Eq. (9-2)*

$$\ell_n = 17.5 - \frac{18}{12} = 16 \text{ ft}$$

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8}$$

$$= \frac{0.261 \times 22 \times 16^2}{8} = 183.7 \text{ ft-kips}$$

Distribution of moment into negative and positive moments:

Interior span: — *13.6.3.2*

$$\text{Negative moment} = 0.65\, M_o = 0.65 \times 183.7 = 119.4 \text{ ft-kips}$$
$$\text{Positive moment} = 0.35\, M_o = 0.35 \times 183.7 = 64.3 \text{ ft-kips}$$

End span: — *13.6.3.3*

$$\text{Exterior negative} = 0.16\, M_o = 0.16 \times 183.7 = 29.4 \text{ ft-kips}$$
$$\text{Positive} = 0.57\, M_o = 0.57 \times 183.7 = 104.7 \text{ ft-kips}$$
$$\text{Interior negative} = 0.70\, M_o = 0.7 \times 183.7 = 128.6 \text{ ft-kips}$$

Note: The factored moments may be modified by 10 percent, provided the total factored static moment in any panel is not less than that computed from Eq. (13-3). This modification is omitted here. — *13.6.7*

4. Distribution of factored moments to column and middle strips: — *13.6.4*

Percentage of total negative and positive moments to column strip.

At interior support:

$$75 + 30\left(\frac{\alpha_1 \ell_2}{\ell_1}\right)\left(1 - \frac{\ell_2}{\ell_1}\right) = 75 + 30\,(1 - 1.26) = 67\% \qquad \textit{Eq. (1)}$$

where α_1 was computed earlier to be 3.16 (see NS interior beam above)

At exterior support:

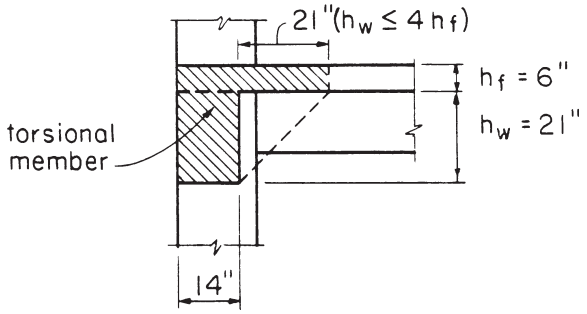
$$100 - 10\beta_t + 12\beta_t\left(\frac{\alpha_1 \ell_2}{\ell_1}\right)\left(1 - \frac{\ell_2}{\ell_1}\right) = 100 - 10\,(1.88) + 12\,(1.88)\,(1 - 1.26) = 75\% \qquad \textit{Eq. (2)}$$

where

$$\beta_t = \frac{C}{2I_s} = \frac{17{,}868}{2 \times 4752} = 1.88$$

$$I_s = \frac{\ell_2 h^3}{12} = 4{,}752 \text{ in.}^4$$

C is taken as the larger value computed (with the aid of Table 19-2) for the torsional member shown below.



x_1 = 14 in.	x_2 = 6 in.	x_1 = 14 in.	x_2 = 6 in.
y_1 = 21 in.	y_2 = 35 in.	y_1 = 27 in.	y_2 = 21 in.
C_1 = 11,141 in.^{4}	C_2 = 2248 in.^{4}	C_1 = 16,628 in.^{4}	C_2 = 1240 in.^{4}
ΣC = 11,141 + 2248 = 13,389 in.^{4}		ΣC = 16,628 + 1240 = 17,868 in.^{4}	

Positive moment:

$$60 + 30\left(\frac{\alpha_1 \ell_2}{\ell_1}\right)\left(1.5 - \frac{\ell_2}{\ell_1}\right) = 60 + 30\,(1.5 - 1.26) = 67\% \qquad \textit{Eq. (3)}$$

Factored moments in column strips and middle strips are summarized as follows:

	Factored Moment (ft-kips)	Column Strip		Moment (ftkips) in Two Half-Middle Strips[2]
		Percent	Moment[1] (ft-kips)	
End Span:				
Exterior Negative	29.4	75	22.1	7.3
Positive	104.7	67	70.1	34.6
Interior Negative	128.6	67	86.2	42.4
Interior Span:				
Negative	119.4	67	80.0	39.4
Positive	64.3	67	43.1	21.2

[1] *Since $\alpha_1 \ell_2 / \ell_1 > 1.0$, beams must be proportioned to resist 85 percent of column strip moment per 13.6.5.1.*

[2] *That portion of the factored moment not resisted by the column strip is assigned to the half-middle strips.*

5. Factored moments in columns: *13.6.9*

   a. Interior columns, with equal spans in the direction of analysis and (different) equal spans in the transverse direction. *13.6.9*

$$M_u = 0.07\left(0.5 w_\ell \ell_2 \ell_n^2\right) \qquad \textit{Eq. (13-4)}$$

$$= 0.07\,(0.5 \times 1.6 \times 0.1 \times 22 \times 16^2) = 31.5 \text{ ft-kips}$$

With the same column size and length above and below the slab,

$$M_c = \frac{31.5}{2} = 15.8 \text{ ft-kips}$$

This moment is combined with the factored axial load (for each story) for design of the interior columns.

   b. Exterior columns.

The total exterior negative moment from the slab beam is transferred to the exterior columns; with the same column size and length above and below the slab system:

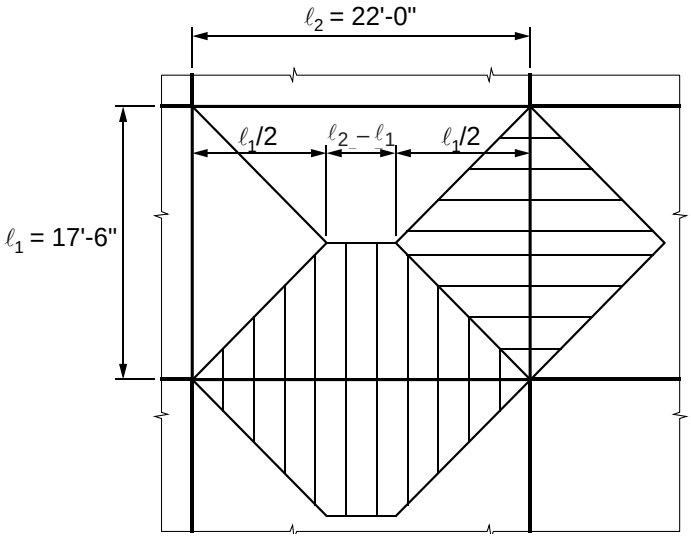
$$M_c = \frac{29.4}{2} = 14.7 \text{ ft-kips}$$

6. Shear strength:

a. Beams.

Since $\alpha_1 \ell_2/\ell_1 > 1$ for all beams, they must resist total shear (b_w = 14 in., d = 17 in.). *13.6.8.1*

Only interior beams will be checked here, because they carry much higher shear forces than the edge beams.





NS beams

$$V_u = \frac{1}{2} w_u \ell_1 \frac{\ell_1}{2} = \frac{w_u \ell_1^2}{4}$$

EW beams

$$V_u = \frac{1}{2} w_u \frac{\ell_1}{2} \frac{\ell_1}{2} 2 + w_u (\ell_2 - \ell_1) \frac{\ell_1}{2}$$

$$= \frac{w_u \ell_1}{4} (\ell_1 + 2\ell_2 - 2\ell_1) = \frac{w_u \ell_1}{4} (2\ell_2 - \ell_1)$$

NS Beams:

$$V_u = \frac{w_u \ell_1^{\,2}}{4} = \frac{0.261\,(17.5)^2}{4} = 20.0 \text{ kips}$$

$$\phi V_c = \phi 2\sqrt{f'_c}\, b_w d \qquad \textit{Eq. (11-3)}$$

$$= 0.75 \times 2\sqrt{4{,}000} \times 14 \times 17/1{,}000 = 22.6 \text{ kips} > V_u$$

Provide minimum shear reinforcement per 11.5.5.3. *11.5.5.1*

EW Beams:

$$V_u = \frac{w_u \ell_1 (2\ell_2 - \ell_1)}{4}$$

$$= \frac{0.261 \times 17.5 \left[(2 \times 22) - 17.5\right]}{4} = 30.3 \text{ kips} > \phi V_c = 22.6 \text{ kips} \quad \text{N.G.}$$

Required shear strength to be provided by shear reinforcement:

$$V_s = (V_u - \phi V_c)/\phi = (30.3 - 22.6)/0.75 = 10.3 \text{ kips}$$

b. Slabs (b_w = 12 in., d = 5 in.). *13.6.8.4*

$$w_u = (1.2 \times 75) + (1.6 \times 100) = 250 \text{ psf}$$

$$V_u = \frac{w_u \ell_1}{2} = \frac{0.25 \times 17.5}{2} = 2.2 \text{ kips}$$

$$\phi V_c = \phi 2\sqrt{f'_c}\, b_w d$$

$$= 0.75 \times 2\sqrt{4{,}000} \times 12 \times 5/1{,}000 = 5.7 \text{ kips} > V_u = 2.2 \text{ kips} \quad \text{O.K.}$$

Shear strength of slab is adequate without shear reinforcement.

7. Edge beams must be designed to resist moment not transferred to exterior columns by interior beams, in accordance with 11.6.

20

Two-Way Slabs—Equivalent Frame Method

GENERAL CONSIDERATIONS

The Equivalent Frame Method of analysis converts a three-dimensional frame system with two-way slabs into a series of two-dimensional frames (slab-beams and columns), with each frame extending the full height of the building, as illustrated in Fig. 20-1. The width of each equivalent frame extends to mid-span between column centerlines. The complete analysis of the two-way slab system for a building consists of analyzing a series of equivalent interior and exterior frames spanning longitudinally and transversely through the building. For gravity loading, the slab-beams at each floor or roof (level) may be analyzed separately, with the far ends of attached columns considered fixed (13.7.2.5).

The Equivalent Frame Method of elastic analysis applies to buildings with columns laid out on a basically orthogonal grid, with column lines extending longitudinally and transversely through the building. The analysis method is applicable to slabs with or without beams between supports.

The Equivalent Frame Method may be used for lateral load analysis if the stiffnesses of frame members are modified to account for cracking and other relevant factors. See discussion on 13.3.1.2 in Part 18.

PRELIMINARY DESIGN

Before proceeding with Equivalent Frame analysis, a preliminary slab thickness h needs to be determined for control of deflections, according to the minimum thickness requirements of 9.5.3. Table 18-1 and Fig. 18-3 may be used to simplify minimum thickness computations. For slab systems without beams, it is advisable at this stage of design to check the shear strength of the slab in the vicinity of columns or other support locations, according to the special provisions for slabs of 11.12. See discussion on 13.5.4 in Part 18.

13.7.2 Equivalent Frame

Application of the frame definitions given in 13.7.2, 13.2.1, and 13.2.2 is illustrated in Figs. 20-1 and 20-2. Some judgment is required in applying the definitions given in 13.2.1 for slab systems with varying span lengths along the design strip. Members of the equivalent frame are slab-beams and torsional members (transverse horizontal members) supported by columns (vertical members). The torsional members provide moment transfer between the slab-beams and the columns. The equivalent frame members are illustrated in Fig. 20-3. The initial step in the frame analysis requires that the flexural stiffness of the equivalent frame members be determined.

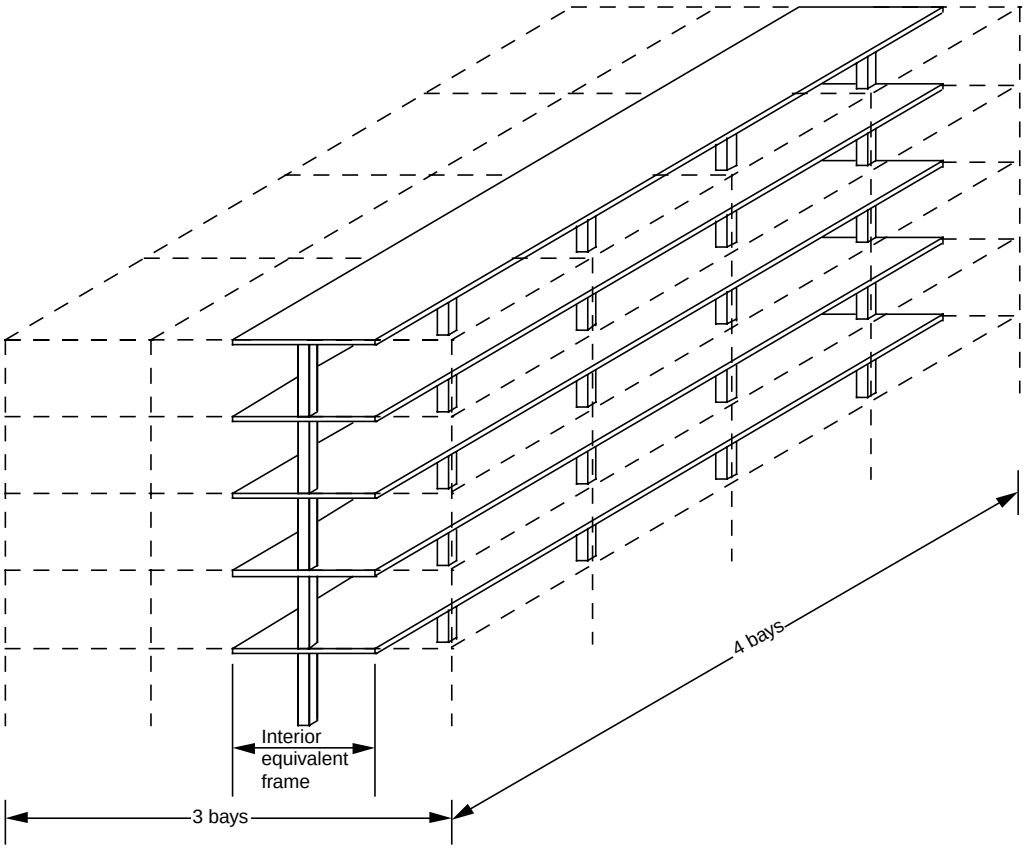


Figure 20-1 Equivalent Frames for 5-Story Building

13.7.3 Slab-Beams

Common types of slab systems with and without beams between supports are illustrated in Figs. 20-4 and 20-5. Cross-sections for determining the stiffness of the slab-beam members K_{sb} between support centerlines are shown for each type. The equivalent slab-beam stiffness diagrams may be used to determine moment distribution constants and fixed-end moments for Equivalent Frame analysis.

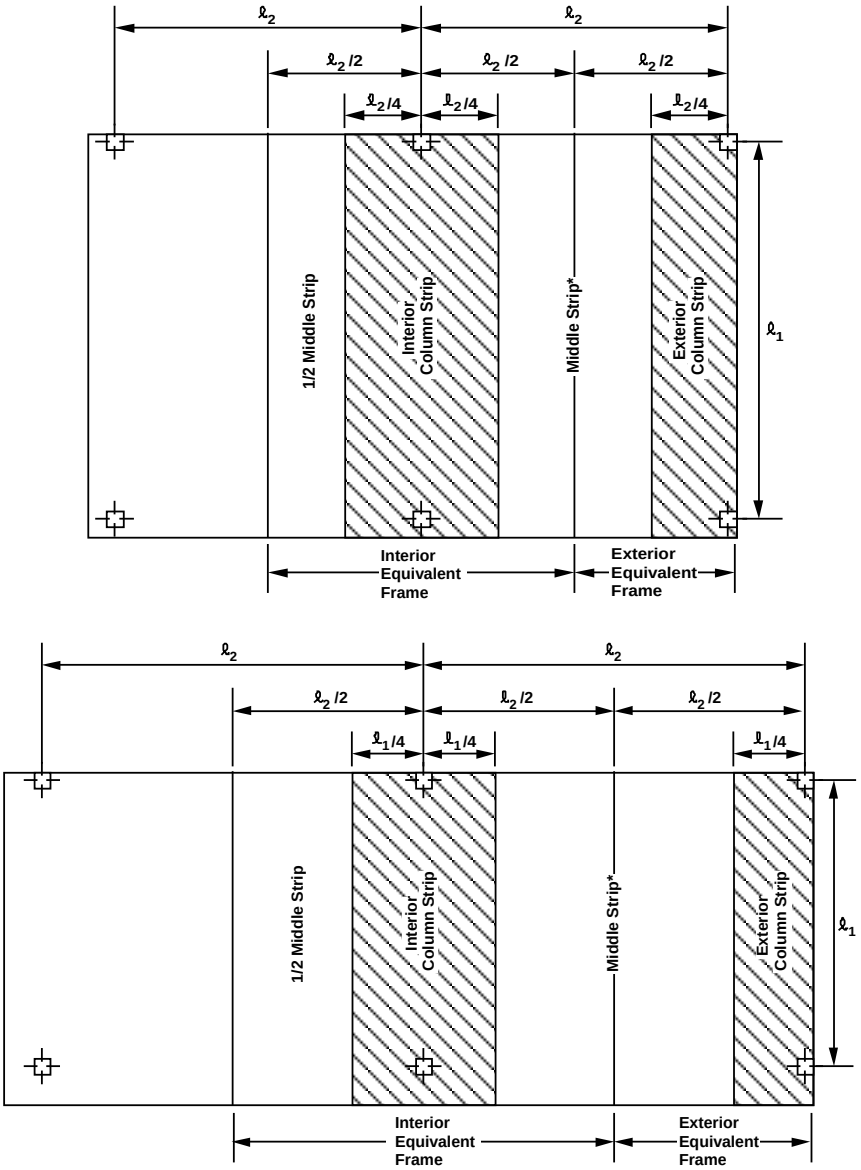
Stiffness calculations are based on the following considerations:

a. The moment of inertia of the slab-beam between faces of supports is based on the gross cross-sectional area of the concrete. Variation in the moment of inertia along the axis of the slab-beam between supports must be taken into account (13.7.3.2).

b. A support is defined as a column, capital, bracket or wall. Note that a beam is not considered a supporting member for the equivalent frame (13.7.3.3).

c. The moment of inertia of the slab-beam from the face of support to the centerline of support is assumed equal to the moment of inertia of the slab-beam at the face of support, divided by the quantity $(1 - c_2/\ell_2)^2$ (13.7.3.3).

The magnification factor $1/(1 - c_2/\ell_2)^2$ applied to the moment of inertia between support face and support centerline, in effect, makes each slab-beam at least a haunched member within its length. Consequently, stiffness and carryover factors and fixed-end moments based on the usual assumptions of uniform prismatic members cannot be applied to the slab-beam members.



*When edge of exterior design strip is supported by a wall, the factored moment resisted by this middle strip is defined in 13.6.6.3.

Figure 20-2 Design Strips of Equivalent Frame

Tables A1 through A6 in Appendix 20A at the end of this chapter give stiffness coefficients, carry-over factors, and fixed-end moment coefficients for different geometric and loading configurations. A wide range of column size-to-span ratios in both longitudinal and transverse directions is covered in the tables. Table A1 can be used for flat plates and two-way slabs with beams. Tables A2 through A5 are intended to be used for flat slabs and waffle slabs with various drop (solid head) depths. Table A6 covers the unusual case of a flat plate combined with a flat slab. Fixed-end moment coefficients are provided for both uniform and partially uniform loads. Partial load coefficients were developed for loads distributed over a length of span equal to $0.2\ell_1$. However, loads acting over longer portions of span may be considered by summing the effects of loads acting over each $0.2\ell_1$ interval. For example, if the partial loading extends over $0.6\ell_1$, then the coefficients corresponding to three consecutive $0.2\ell_1$ intervals are to be added. This provides flexibility in the arrangement of loading. For concentrated loads, a high intensity of partial loading may be considered at the appropriate location, and assumed to be distributed over $0.2\ell_1$. For parameter values in between those listed, interpolation may be made. Stiffness diagrams are shown on each table. With appropriate engineering judgment, different span conditions may be considered with the help of information given in these tables.

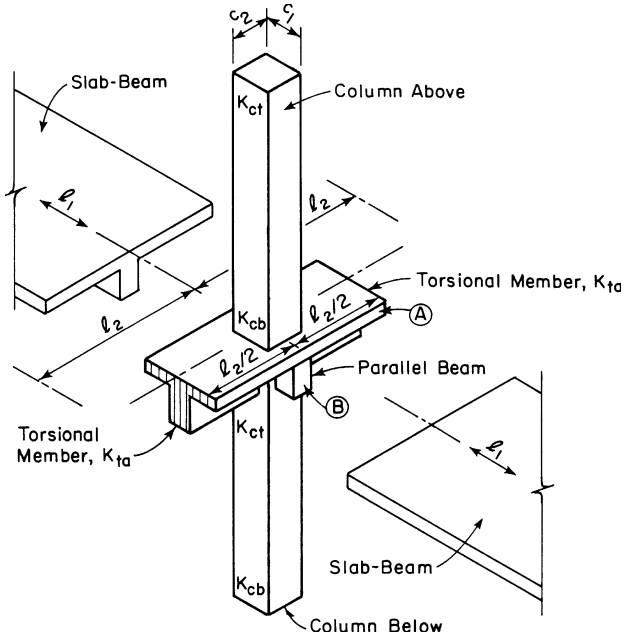


Figure 20-3 Equivalent Frame Members

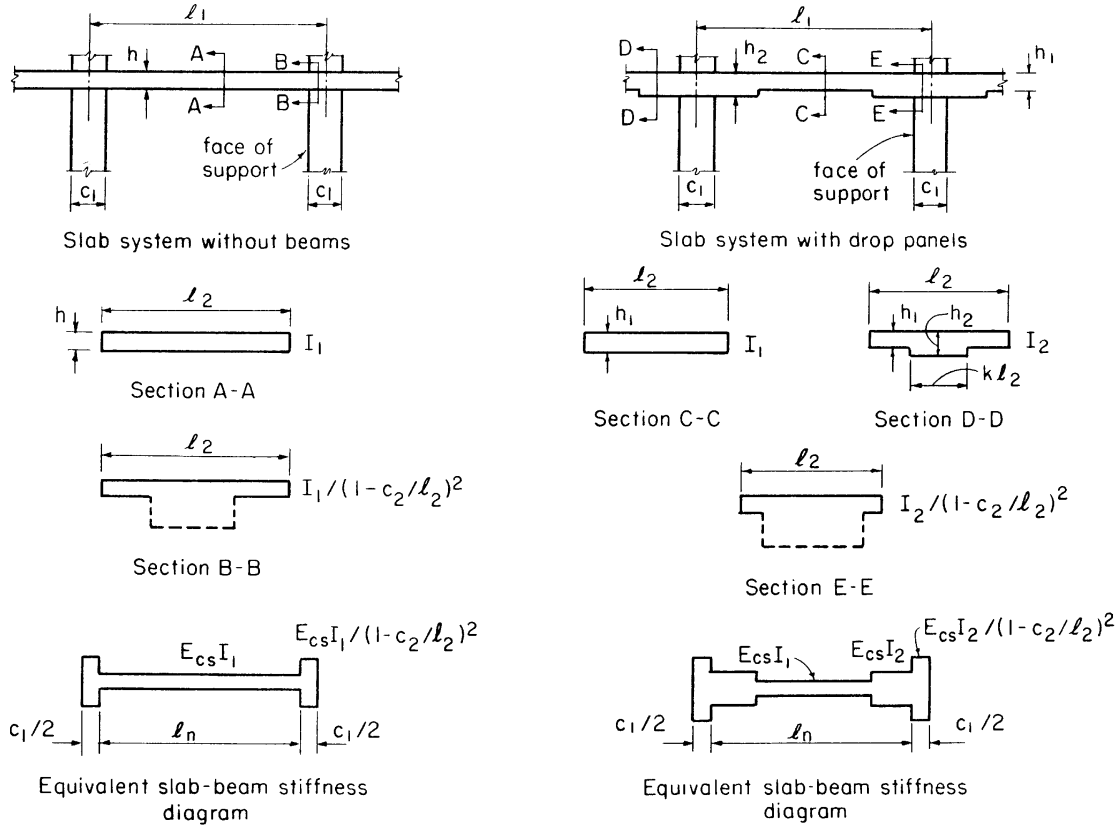


Figure 20-4 Sections for Calculating Slab-Beam Stiffness K_{sb}

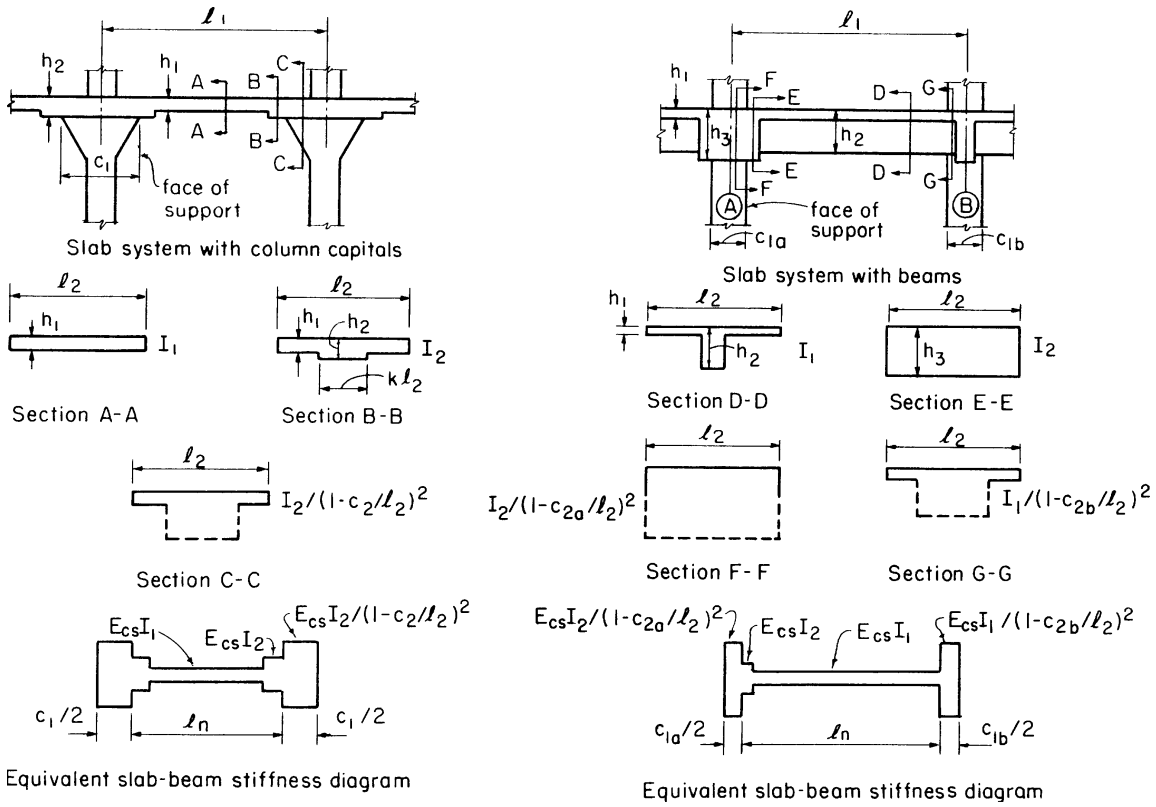


Figure 20-5 Sections for Calculating Slab-Beam Stiffness K_{sb}

13.7.4 Columns

Common types of column end support conditions for slab systems are illustrated in Fig. 20-6. The column stiffness is based on a height of column ℓ_c measured from the mid-depth of the slab above to the mid-depth of the slab below. The column stiffness diagrams may be used to determine column flexural stiffness, K_c. The stiffness diagrams are based on the following considerations:

a. The moment of inertia of the column outside the slab-beam joint is based on the gross cross-sectional area of the concrete. Variation in the moment of inertia along the axis of the column between slab-beam joints is taken into account. For columns with capitals, the moment of inertia is assumed to vary linearly from the base of the capital to the bottom of the slab-beam (13.7.4.1 and 13.7.4.2).

b. The moment of inertia is assumed infinite ($I = \infty$) from the top to the bottom of the slab-beam at the joint. As with the slab-beam members, the stiffness factor K_c for the columns cannot be based on the assumption of uniform prismatic members (13.7.4.3).

Table A7 in Appendix 20A can be used to determine the actual column stiffnesses and carry-over factors.

13.7.5 Torsional Members

Torsional members for common slab-beam joints are illustrated in Fig. 20-7. The cross-section of a torsional member is the largest of those defined by the three conditions given in 13.7.5.1. The governing condition (a), (b), or (c) is indicated below each illustration in Fig. 20-7.

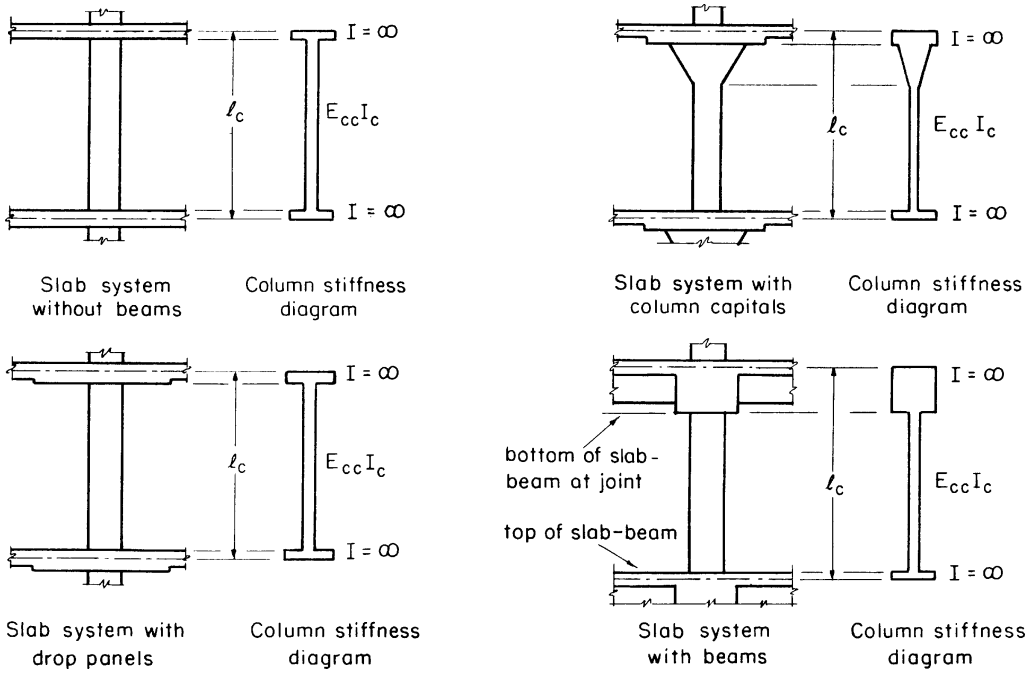


Figure 20-6 Sections for Calculating Column Stiffness K_c

The stiffness K_t of the torsional member is calculated by the following expression:

$$K_t = \Sigma \left[\frac{9E_{cs}C}{\ell_2 \left[1 - (c_2/\ell_2)\right]^3} \right] \qquad (1)$$

where the summation extends over torsional members framing into a joint: two for interior frames, and one for exterior frames.

The term C is a cross-sectional constant that defines the torsional properties of each torsional member framing into a joint:

$$C = \Sigma \left[1 - 0.63 \left(\frac{x}{y} \right) \right] \frac{x^3 y}{3} \qquad (2)$$

where x is the shorter dimension of a rectangular part and y is the longer dimension of a rectangular part.

The value of C is computed by dividing the cross section of a torsional member into separate rectangular parts and summing the C values for the component rectangles. It is appropriate to subdivide the cross section in a manner that results in the largest possible value of C. Application of the C expression is illustrated in Fig. 20-8.

If beams frame into the support in the direction moments are being determined, the torsional stiffness K_t given by Eq. (1) needs to be increased as follows:

$$K_{ta} = \frac{K_t I_{sb}}{I_s}$$

where K_{ta} = increased torsional stiffness due to the parallel beam (note parallel beam shown in Fig. 20-3)

I_s = moment of inertia of a width of slab equal to the full width between panel centerlines, ℓ_2, excluding that portion of the beam stem extending above and below the slab (note part A in Fig. 20-3).

$$= \frac{\ell_2 h^3}{12}$$

I_{sb} = moment of inertia of the slab section specified for I_s including that portion of the beam stem extending above and below the slab (for the parallel beam illustrated in Fig. 20-3, I_{sb} is for the full tee section shown).

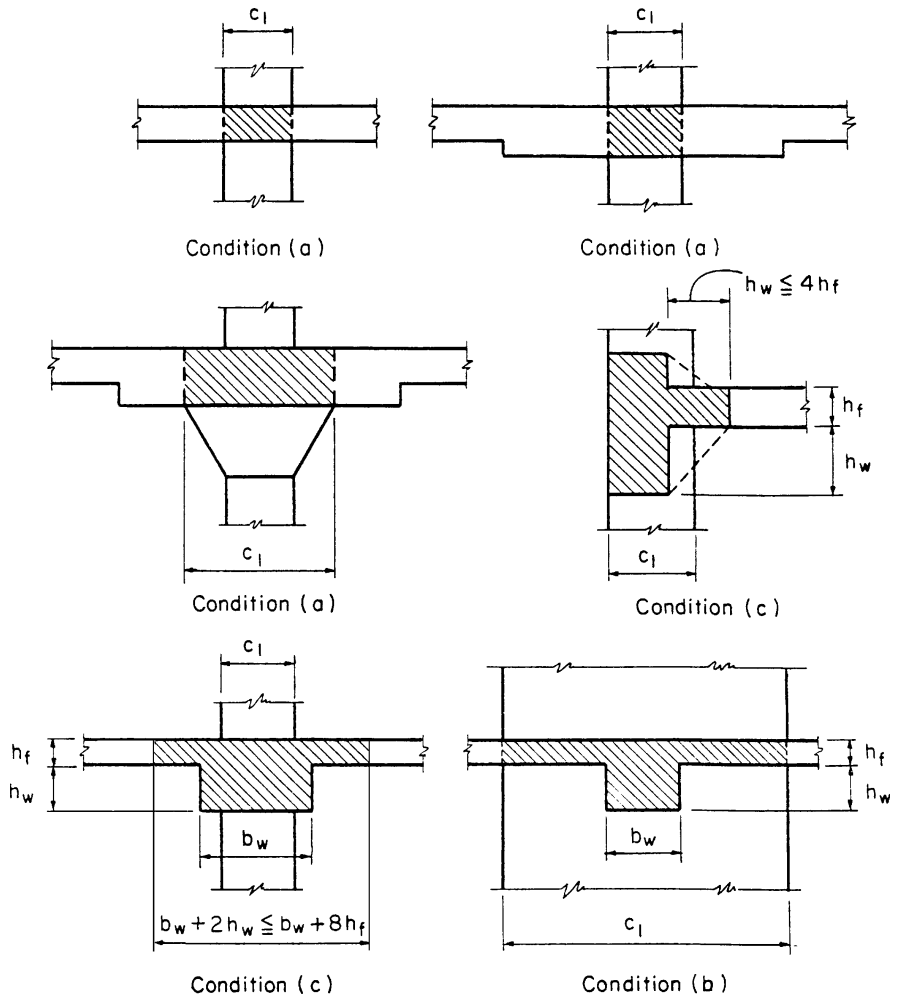


Figure 20-7 Torsional Members

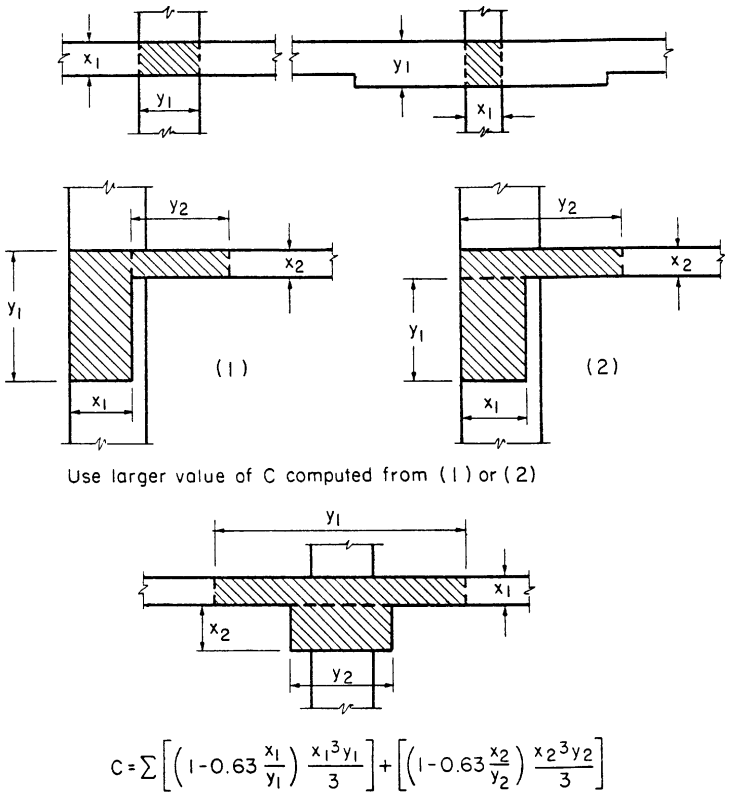


Figure 20-8 Cross-Sectional Constant C, Defining Torsional Properties of a Torsional Member

Equivalent Columns (R13.7.4)

With the publication of ACI 318-83, the equivalent column concept of defining a single-stiffness element consisting of the actual columns above and below the slab-beams plus an attached transverse torsional member was eliminated from the code. With the increasing use of computers for two-way slab analysis by the Equivalent Frame Method, the concept of combining stiffnesses of actual columns and torsional members into a single stiffness has lost much of it attractiveness. The equivalent column was, however, retained in the commentary until the 1989 edition of the code, as an aid to analysis where slab-beams at different floor levels are analyzed separately for gravity loads, especially when using moment distribution or other hand calculation procedures for the analysis. While the equivalent column concept is still recognized by R13.7.5, the detailed procedure contained in the commentary since the '83 edition for calculating the equivalent column stiffness, K_{ec}, was deleted from R13.7.5 of the '95 code.

Both Examples 20.1 and 20.2 utilize the equivalent column concept with moment distribution for gravity load analysis.

The equivalent column concept modifies the column stiffness to account for the torsional flexibility of the slab-to-column connection which reduces its efficiency for transmission of moments. An equivalent column is illustrated in Fig. 20-3. The equivalent column consists of the actual columns above and below the slab-beams, plus "attached" torsional members on both sides of the columns, extending to the centerlines of the adjacent panels. Note that for an edge frame, the attached torsional member is on one side only. The presence of parallel beams will also influence the stiffness of the equivalent column.

The flexural stiffness of the equivalent column K_{ec} is given in terms of its inverse, or flexibility, as follows:

$$\frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{\Sigma K_t}$$

For computational purposes, the designer may prefer that the above expression be given directly in terms of stiffness as follows:

$$K_{ec} = \frac{\Sigma K_c \times \Sigma K_t}{\Sigma K_c + \Sigma K_t}$$

Stiffnesses of the actual columns, K_c, and torsional members, K_t must comply with 13.7.4 and 13.7.5.

After the values of K_c and K_t are determined, the equivalent column stiffness K_{ec} is computed. Using Fig. 20-3 for illustration,

$$K_{ec} = \frac{(K_{ct} + K_{cb})(K_{ta} + K_{ta})}{K_{ct} + K_{cb} + K_{ta} + K_{ta}}$$

where K_{ct} = flexural stiffness at top of lower column framing into joint,

K_{cb} = flexural stiffness at bottom of upper column framing into joint,

K_{ta} = torsional stiffness of each torsional member, one on each side of the column, increased due to the parallel beam (if any).

13.7.6 Arrangement of Live Load

In the usual case where the exact loading pattern is not known, the maximum factored moments are developed with loading conditions illustrated by the three-span partial frame in Fig. 20-9, and described as follows:

a. When the service live load does not exceed three-quarters of the service dead load, only loading pattern (1) with full factored live load on all spans need be analyzed for negative and positive factored moments.

b. When the service live-to-dead load ratio exceeds three-quarters, the five loading patterns shown need to be analyzed to determine all factored moments in the slab-beam members. Loading patterns (2) through (5) consider partial factored live loads for determining factored moments. However, with partial live loading, the factored moments cannot be taken less than those occurring with full factored live load on all spans; hence load pattern (1) needs to be included in the analysis.

For slab systems with beams, loads supported directly by the beams (such as the weight of the beam stem or a wall supported directly by the beams) may be inconvenient to include in the frame analysis for the slab loads, $w_d + w_\ell$. An additional frame analysis may be required with the beam section designed to carry these loads in addition to the portion of the slab moments assigned to the beams.

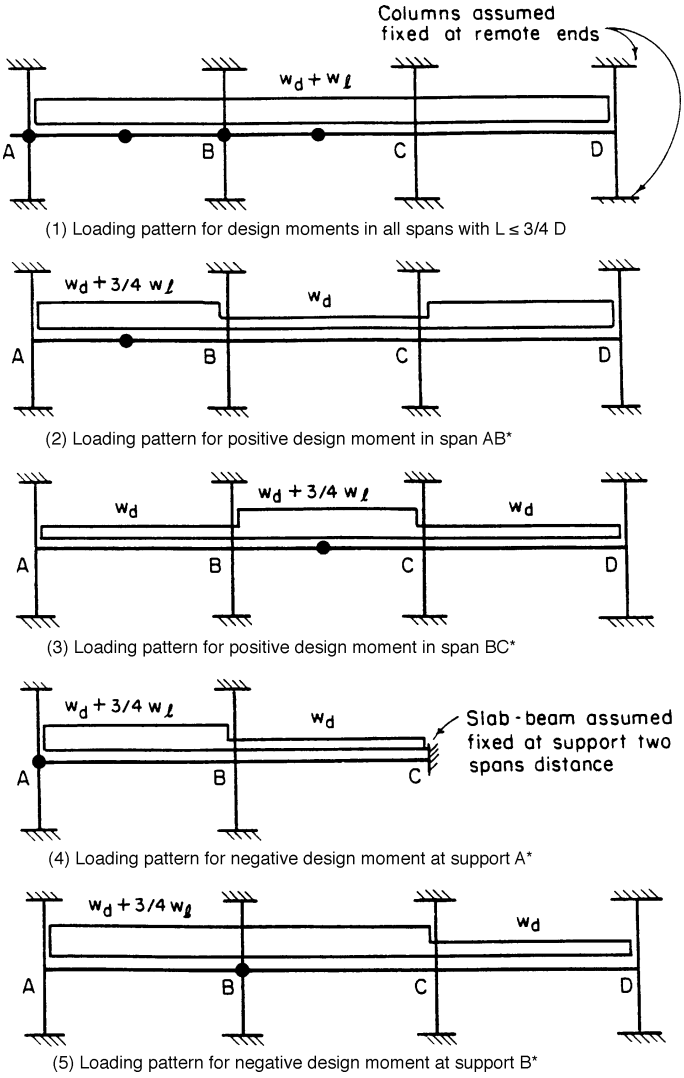


Figure 20-9 Partial Frame Analysis for Vertical Loading

13.7.7 Factored Moments

Moment distribution is probably the most convenient hand calculation method for analyzing partial frames involving several continuous spans with the far ends of upper and lower columns fixed. The mechanics of the method will not be described here, except for a brief discussion of the following two points: (1) the use of the equivalent column concept to determine joint distribution factors and (2) the proper procedure to distribute the equivalent column moment obtained in the frame analysis to the actual columns above and below the slab-beam joint. See Examples 20.1 and 20.2.

A frame joint with stiffness factors K shown for each member framing into the joint is illustrated in Fig. 20-10. Expressions are given below for the moment distribution factors DF at the joint, using the equivalent column stiffness, K_{ec}. These distribution factors are used directly in the moment distribution procedure.

Equivalent column stiffness,

$$K_{ec} = \frac{\Sigma K_c \times \Sigma K_t}{\Sigma K_c + \Sigma K_t}$$

$$= \frac{(K_{ct} + K_{cb})(K_t + K_t)}{K_{ct} + K_{cb} + K_t + K_t}$$

Slab-beam distribution factor,

$$\text{DF (span 2-1)} = \frac{K_{b1}}{K_{b1} + K_{b2} + K_{ec}}$$

$$\text{DF (span 2-3)} = \frac{K_{b2}}{K_{b1} + K_{b2} + K_{ec}}$$

Equivalent column distribution factor (unbalanced moment from slab-beam),

$$\text{DF} = \frac{K_{ec}}{K_{b1} + K_{b2} + K_{ec}}$$

The unbalanced moment determined for the equivalent column in the moment distribution cycles is distributed to the actual columns above and below the slab-beam in proportion to the actual column stiffnesses at the joint. Referring to Fig. 20-10:

$$\text{Portion of unbalanced moment to upper column} = \frac{K_{cb}}{(K_{cb} + K_{ct})}$$

$$\text{Portion of unbalanced moment to lower column} = \frac{K_{ct}}{(K_{cb} + K_{ct})}$$

The "actual" columns are then designed for these moments.

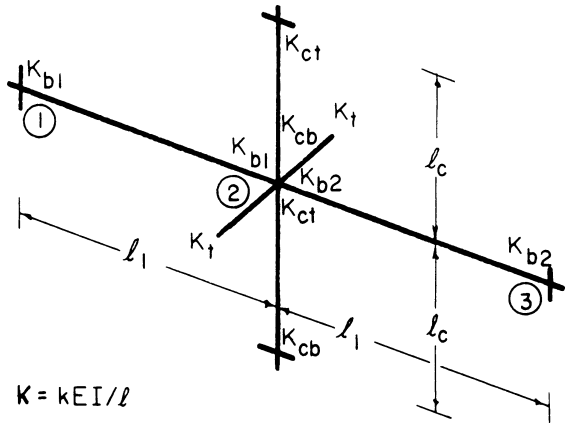


Figure 20-10 Moment Distribution Factors DF

13.7.7.1 - 13.7.7.3 Negative Factored Moments—Negative factored moments for design must be taken at faces of rectilinear supports, but not at a distance greater than $0.175\ell_1$ from the center of a support. This absolute value is a limit on long narrow supports in order to prevent undue reduction in design moment. The support member is defined as a column, capital, bracket or wall. Non-rectangular supports should be treated as square supports having the same cross-sectional area. Note that for slab systems with beams, the faces of beams are not considered face-of-support locations. Locations of the critical section for negative factored moment for various support conditions are illustrated in Fig. 20-11. Note the special requirements illustrated for exterior supports.

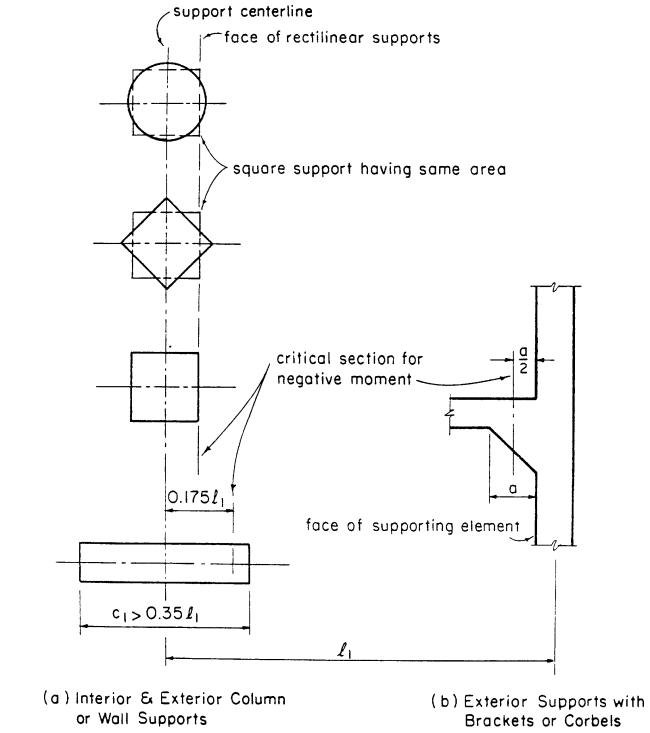


Figure 20-11 Critical Sections for Negative Factored Moment

13.7.7.4 Moment Redistribution—Should a designer choose to use the Equivalent Frame Method to analyze a slab system that meets the limitations of the Direct Design Method, the factored moments may be reduced so that the total static factored moment (sum of the average negative and positive moments) need not exceed M_o computed by Eq. (13-3). This permissible reduction is illustrated in Fig. 20-12.

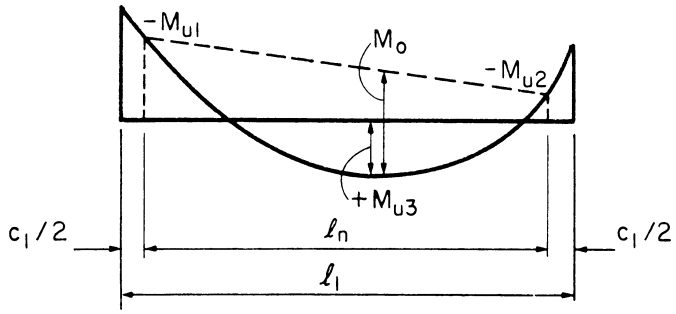


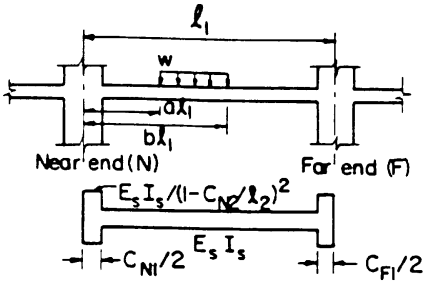
Figure 20-12 Total Static Design Moment for a Span

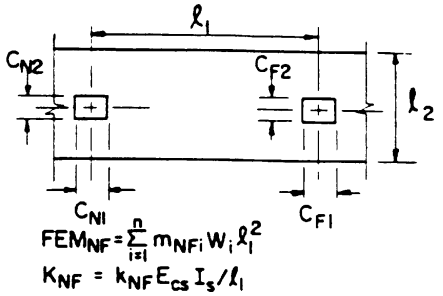
Since the Equivalent Frame Method of analysis is not an approximate method, the moment redistribution allowed in 8.4 may be used. Excessive cracking may result if these provisions are imprudently applied. The burden of judgment is left to the designer as to what, if any, redistribution is warranted.

13.7.7.5 Factored Moments in Column Strips and Middle Strips—Negative and positive factored moments may be distributed to the column strip and the two half-middle strips of the slab-beam in accordance with 13.6.4, 13.6.5 and 13.6.6, provided that the requirement of 13.6.1.6 is satisfied. See discussion on 13.6.4, 13.6.5, 13.6.6 in Part 19.

APPENDIX 20A DESIGN AIDS FOR MOMENT DISTRIBUTION CONSTANTS

Table A1 Moment Distribution Constants for Slab-Beam Members

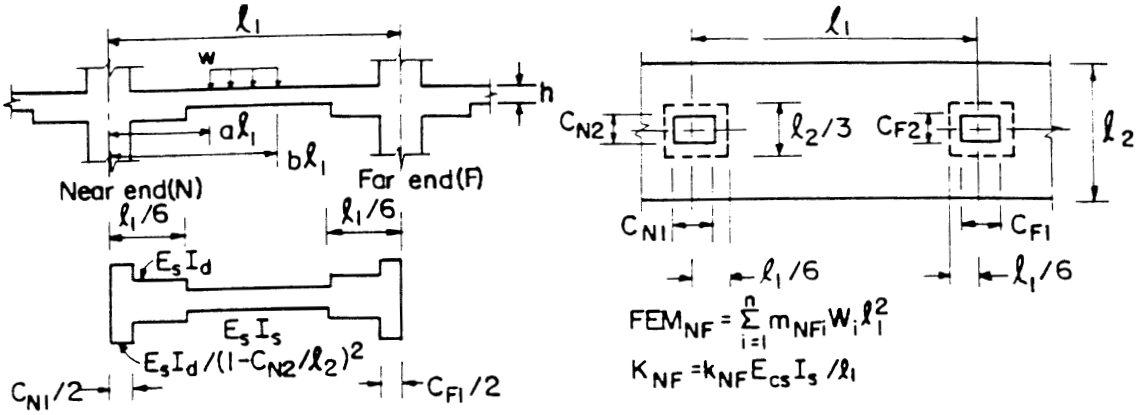




C_{N1}/ℓ_1	C_{N2}/ℓ_2	Stiffness Factors k_{NF}	Carry Over Factors C_{NF}	Unif. Load Fixed end M. Coeff. (m_{NF})	Fixed end moment Coeff. (m_{NF}) for (b—a) = 0.2				
					a = 0.0	a = 0.2	a = 0.4	a = 0.6	a = 0.8
$C_{F1} = C_{N1}$; $C_{F2} = C_{N2}$									
0.00	—	4.00	0.50	0.0833	0.0151	0.0287	0.0247	0.0127	0.00226
	0.00	4.00	0.50	0.0833	0.0151	0.0287	0.0247	0.0127	0.00226
	0.10	4.18	0.51	0.0847	0.0154	0.0293	0.0251	0.0126	0.00214
0.10	0.20	4.36	0.52	0.0860	0.0158	0.0300	0.0255	0.0126	0.00201
	0.30	4.53	0.54	0.0872	0.0161	0.0301	0.0259	0.0125	0.00188
	0.40	4.70	0.55	0.0882	0.0165	0.0314	0.0262	0.0124	0.00174
	0.00	4.00	0.50	0.0833	0.0151	0.0287	0.0247	0.0127	0.00226
	0.10	4.35	0.52	0.0857	0.0155	0.0299	0.0254	0.0127	0.00213
0.20	0.20	4.72	0.54	0.0880	0.0161	0.0311	0.0262	0.0126	0.00197
	0.30	5.11	0.56	0.0901	0.0166	0.0324	0.0269	0.0125	0.00178
	0.40	5.51	0.58	0.0921	0.0171	0.0336	0.0276	0.0123	0.00156
	0.00	4.00	0.50	0.0833	0.0151	0.0287	0.0247	0.0127	0.00226
	0.10	4.49	0.53	0.0863	0.0155	0.0301	0.0257	0.0128	0.00219
0.30	0.20	5.05	0.56	0.0893	0.0160	0.0317	0.0267	0.0128	0.00207
	0.30	5.69	0.59	0.0923	0.0165	0.0334	0.0278	0.0127	0.00190
	0.40	6.41	0.61	0.0951	0.0171	0.0352	0.0287	0.0124	0.00167
	0.00	4.00	0.50	0.0833	0.0151	0.0287	0.0247	0.0127	0.00226
	0.10	4.61	0.53	0.0866	0.0154	0.0302	0.0259	0.0129	0.00225
0.40	0.20	5.35	0.56	0.0901	0.0158	0.0318	0.0271	0.0131	0.00221
	0.30	6.25	0.60	0.0936	0.0162	0.0337	0.0284	0.0131	0.00211
	0.40	7.37	0.64	0.0971	0.0168	0.0359	0.0297	0.0128	0.00195
$C_{F1} = 0.5C_{N1}$; $C_{F2} = 0.5C_{N2}$									
0.00	—	4.00	0.50	0.0833	0.0151	0.0287	0.0247	0.0127	0.0023
	0.00	4.00	0.50	0.0833	0.0151	0.0287	0.0247	0.0127	0.0023
	0.10	4.16	0.51	0.0857	0.0155	0.0296	0.0254	0.0130	0.0023
0.10	0.20	4.31	0.52	0.0879	0.0158	0.0304	0.0261	0.0133	0.0023
	0.30	4.45	0.54	0.0900	0.0162	0.0312	0.0267	0.0135	0.0023
	0.40	4.58	0.54	0.0918	0.0165	0.0319	0.0273	0.0138	0.0023
	0.00	4.00	0.50	0.0833	0.0151	0.0287	0.0247	0.0127	0.0023
	0.10	4.30	0.52	0.0872	0.0156	0.0301	0.0259	0.0132	0.0023
0.20	0.20	4.61	0.55	0.0912	0.0161	0.0317	0.0272	0.0138	0.0023
	0.30	4.92	0.57	0.0951	0.0167	0.0332	0.0285	0.0143	0.0024
	0.40	5.23	0.58	0.0989	0.0172	0.0347	0.0298	0.0148	0.0024
	0.00	4.00	0.50	0.0833	0.0151	0.0287	0.0247	0.0127	0.0023
	0.10	4.43	0.53	0.0881	0.0156	0.0305	0.0263	0.0134	0.0023
0.30	0.20	4.89	0.56	0.0932	0.0161	0.0324	0.0281	0.0142	0.0024
	0.30	5.40	0.59	0.0986	0.0167	0.0345	0.0300	0.0150	0.0024
	0.40	5.93	0.62	0.1042	0.0173	0.0367	0.0320	0.0158	0.0025
	0.00	4.00	0.50	0.0833	0.0151	0.0287	0.0247	0.0127	0.0023
	0.10	4.54	0.54	0.0884	0.0155	0.0305	0.0265	0.0135	0.0024
0.40	0.20	5.16	0.57	0.0941	0.0159	0.0326	0.0286	0.0145	0.0025
	0.30	5.87	0.61	0.1005	0.0165	0.0350	0.0310	0.0155	0.0025
	0.40	6.67	0.64	0.1076	0.0170	0.0377	0.0336	0.0166	0.0026
$C_{F1} = 2C_{N1}$; $C_{F2} = 2C_{N2}$									
0.00	—	4.00	0.50	0.0833	0.0151	0.0287	0.0247	0.0127	0.0023
	0.00	4.00	0.50	0.0833	0.0150	0.0287	0.0247	0.0127	0.0023
0.10	0.10	4.27	0.51	0.0817	0.0153	0.0289	0.0241	0.0116	0.0018
	0.20	4.56	0.52	0.0798	0.0156	0.0290	0.0234	0.0103	0.0013
	0.00	4.00	0.50	0.0833	0.0151	0.0287	0.0247	0.0127	0.0023
0.20	0.10	4.49	0.51	0.0819	0.0154	0.0291	0.0240	0.0114	0.0019
	0.20	5.11	0.53	0.0789	0.0158	0.0293	0.0228	0.0096	0.0014

APPENDIX 20A DESIGN AIDS FOR MOMENT DISTRIBUTION CONSTANTS (cont'd)

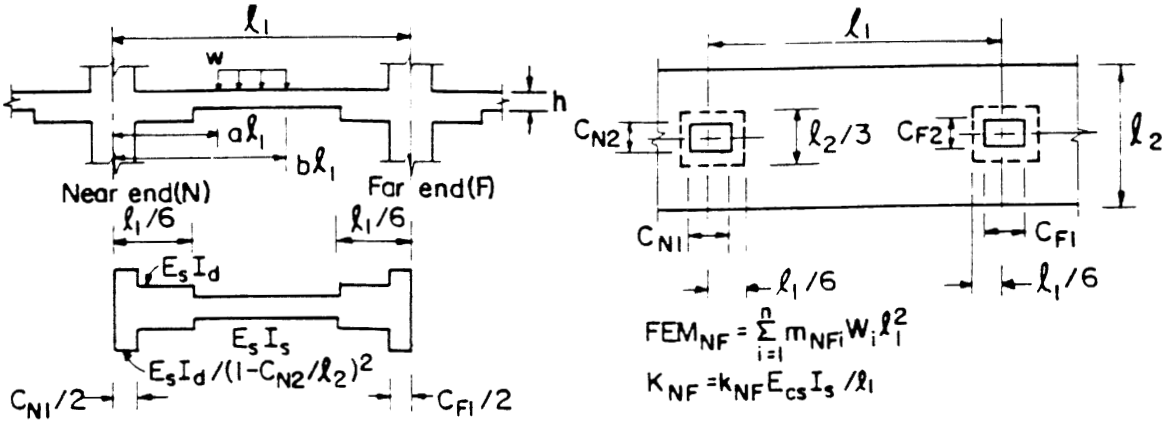
Table A2 Moment Distribution Constants for Slab-Beam Members (Drop thickness = 0.25h)



C_{N1}/ℓ_1	C_{N2}/ℓ_2	Stiffness Factors k_{NF}	Carry Over Factors C_{NF}	Unif. Load Fixed end M. Coeff. (m_{NF})	Fixed end moment Coeff. (m_{NF}) for (b—a) = 0.2				
					a = 0.0	a = 0.2	a = 0.4	a = 0.6	a = 0.8
$C_{F1} = C_{N1}$; $C_{F2} = C_{N2}$									
0.00	—	4.79	0.54	0.0879	0.0157	0.0309	0.0263	0.0129	0.0022
0.10	0.00	4.79	0.54	0.0879	0.0157	0.0309	0.0263	0.0129	0.0022
	0.10	4.99	0.55	0.0890	0.0160	0.0316	0.0266	0.0128	0.0020
	0.20	5.18	0.56	0.0901	0.0163	0.0322	0.0270	0.0127	0.0019
	0.30	5.37	0.57	0.0911	0.0167	0.0328	0.0273	0.0126	0.0018
0.20	0.00	4.79	0.54	0.0879	0.0157	0.0309	0.0263	0.0129	0.0022
	0.10	5.17	0.56	0.0900	0.0161	0.0320	0.0269	0.0128	0.0020
	0.20	5.56	0.58	0.0918	0.0166	0.0332	0.0276	0.0126	0.0018
	0.30	5.96	0.60	0.0936	0.0171	0.0344	0.0282	0.0124	0.0016
0.30	0.00	4.79	0.54	0.0879	0.0157	0.0309	0.0263	0.0129	0.0022
	0.10	5.32	0.57	0.0905	0.0161	0.0323	0.0272	0.0128	0.0021
	0.20	5.90	0.59	0.0930	0.0166	0.0338	0.0281	0.0127	0.0019
	0.30	6.55	0.62	0.0955	0.0171	0.0354	0.0290	0.0124	0.0017
$C_{F1} = 0.5C_{N1}$; $C_{F2} = 0.5C_{N2}$									
0.00	—	4.79	0.54	0.0879	0.0157	0.0309	0.0263	0.0129	0.0022
0.10	0.00	4.79	0.54	0.0879	0.0157	0.0309	0.0263	0.0129	0.0022
	0.10	4.96	0.55	0.0900	0.0160	0.0317	0.0269	0.0131	0.0022
	0.20	5.12	0.56	0.0920	0.0164	0.0325	0.0276	0.0134	0.0022
0.20	0.00	4.79	0.54	0.0879	0.0157	0.0309	0.0263	0.0129	0.0022
	0.10	5.11	0.56	0.0914	0.0162	0.0323	0.0275	0.0133	0.0022
	0.20	5.43	0.58	0.0950	0.0167	0.0337	0.0286	0.0138	0.0022
$C_{F1} = 2C_{N1}$; $C_{F2} = 2C_{N2}$									
0.00	—	4.79	0.54	0.0879	0.0157	0.0309	0.0263	0.0129	0.0022
0.10	0.00	4.79	0.54	0.0879	0.0157	0.0309	0.0263	0.0129	0.0022
	0.10	5.10	0.55	0.0860	0.0159	0.0311	0.0256	0.0117	0.0017

APPENDIX 20A DESIGN AIDS FOR MOMENT DISTRIBUTION CONSTANTS (cont'd)

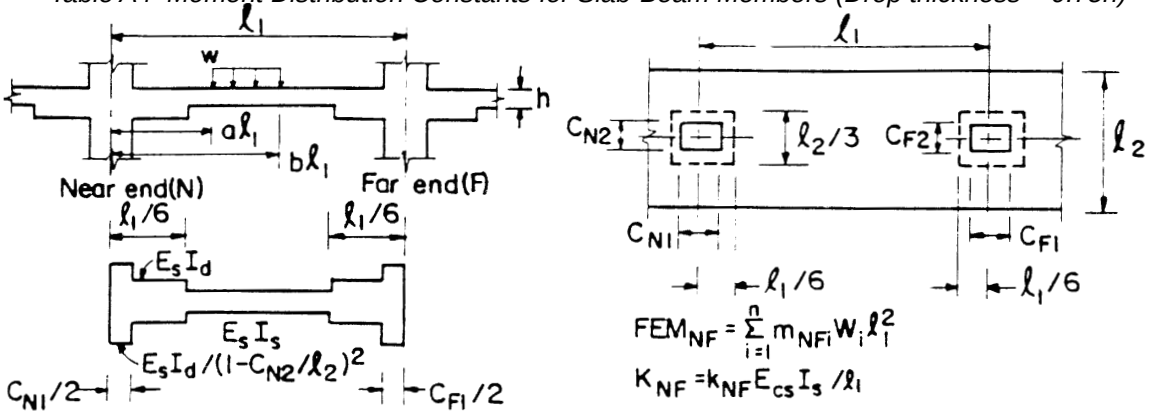
Table A3 Moment Distribution Constants for Slab-Beam Members (Drop thickness = 0.50h)



C_{N1}/ℓ_1	C_{N2}/ℓ_2	Stiffness Factors k_{NF}	Carry Over Factors C_{NF}	Unif. Load Fixed end M. Coeff. (m_{NF})	Fixed end moment Coeff. (m_{NF}) for (b—a) = 0.2				
					a = 0.0	a = 0.2	a = 0.4	a = 0.6	a = 0.8
$C_{F1} = C_{N1}$; $C_{F2} = C_{N2}$									
0.00	—	5.84	0.59	0.0926	0.0164	0.0335	0.0279	0.0128	0.0020
0.10	0.00	5.84	0.59	0.0926	0.0164	0.0335	0.0279	0.0128	0.0020
	0.10	6.04	0.60	0.0936	0.0167	0.0341	0.0282	0.0126	0.0018
	0.20	6.24	0.61	0.0940	0.0170	0.0347	0.0285	0.0125	0.0017
	0.30	6.43	0.61	0.0952	0.0173	0.0353	0.0287	0.0123	0.0016
0.20	0.00	5.84	0.59	0.0926	0.0164	0.0335	0.0279	0.0128	0.0020
	0.10	6.22	0.61	0.0942	0.0168	0.0346	0.0285	0.0126	0.0018
	0.20	6.62	0.62	0.0957	0.0172	0.0356	0.0290	0.0123	0.0016
	0.30	7.01	0.64	0.0971	0.0177	0.0366	0.0294	0.0120	0.0014
0.30	0.00	5.84	0.59	0.0926	0.0164	0.0335	0.0279	0.0128	0.0020
	0.10	6.37	0.61	0.0947	0.0168	0.0348	0.0287	0.0126	0.0018
	0.20	6.95	0.63	0.0967	0.0172	0.0362	0.0294	0.0123	0.0016
	0.30	7.57	0.65	0.0986	0.0177	0.0375	0.0300	0.0119	0.0014
$C_{F1} = 0.5C_{N1}$; $C_{F2} = 0.5C_{N2}$									
0.00	—	5.84	0.59	0.0926	0.0164	0.0335	0.0279	0.0128	0.0020
0.10	0.00	5.84	0.59	0.0926	0.0164	0.0335	0.0279	0.0128	0.0020
	0.10	6.00	0.60	0.0945	0.0167	0.0343	0.0285	0.0130	0.0020
	0.20	6.16	0.60	0.0962	0.0170	0.0350	0.0291	0.0132	0.0020
0.20	0.00	5.84	0.59	0.0926	0.0164	0.0335	0.0279	0.0128	0.0020
	0.10	6.15	0.60	0.0957	0.0169	0.0348	0.0290	0.0131	0.0020
	0.20	6.47	0.62	0.0987	0.0173	0.0360	0.0300	0.0134	0.0020
$C_{F1} = 2C_{N1}$; $C_{F2} = 2C_{N2}$									
0.00	—	5.84	0.59	0.0926	0.0164	0.0335	0.0279	0.0128	0.0020
0.10	0.00	5.84	0.59	0.0926	0.0164	0.0335	0.0279	0.0128	0.0020
	0.10	6.17	0.60	0.0907	0.0166	0.0337	0.0273	0.0116	0.0015

APPENDIX 20A DESIGN AIDS FOR MOMENT DISTRIBUTION CONSTANTS (cont'd)

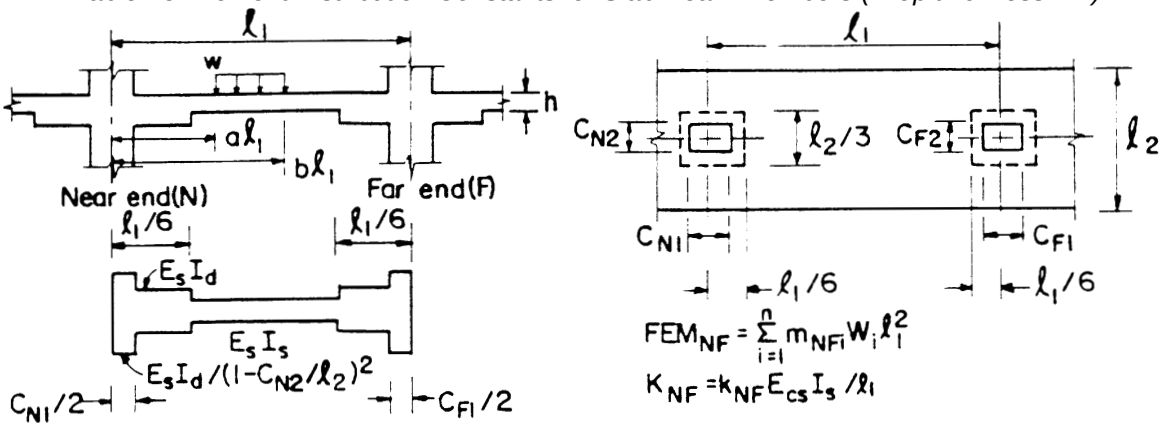
Table A4 Moment Distribution Constants for Slab-Beam Members (Drop thickness = 0.75h)



C_{N1}/ℓ_1	C_{N2}/ℓ_2	Stiffness Factors k_{NF}	Carry Over Factors C_{NF}	Unif. Load Fixed end M. Coeff. (m_{NF})	Fixed end moment Coeff. (m_{NF}) for (b—a) = 0.2				
					a = 0.0	a = 0.2	a = 0.4	a = 0.6	a = 0.8
$C_{F1} = C_{N1}$; $C_{F2} = C_{N2}$									
0.00	—	6.92	0.63	0.0965	0.0171	0.0360	0.0293	0.0124	0.0017
0.10	0.00	6.92	0.63	0.0965	0.0171	0.0360	0.0293	0.0124	0.0017
	0.10	7.12	0.64	0.0972	0.0174	0.0365	0.0295	0.0122	0.0016
	0.20	7.31	0.64	0.0978	0.0176	0.0370	0.0297	0.0120	0.0014
	0.30	7.48	0.65	0.0984	0.0179	0.0375	0.0299	0.0118	0.0013
0.20	0.00	6.92	0.63	0.0965	0.0171	0.0360	0.0293	0.0124	0.0017
	0.10	7.12	0.64	0.0977	0.0175	0.0369	0.0297	0.0121	0.0015
	0.20	7.31	0.65	0.0988	0.0178	0.0378	0.0301	0.0118	0.0013
	0.30	7.48	0.67	0.0999	0.0182	0.0386	0.0304	0.0115	0.0011
0.30	0.00	6.92	0.63	0.0965	0.0171	0.0360	0.0293	0.0124	0.0017
	0.10	7.29	0.65	0.0981	0.0175	0.0371	0.0299	0.0121	0.0015
	0.20	7.66	0.66	0.0996	0.0179	0.0383	0.0304	0.0117	0.0013
	0.30	8.02	0.68	0.1009	0.0182	0.0394	0.0309	0.0113	0.0011
$C_{F1} = 0.5C_{N1}$; $C_{F2} = 0.5C_{N2}$									
0.00	—	6.92	0.63	0.0965	0.0171	0.0360	0.0293	0.0124	0.0017
0.10	0.00	6.92	0.63	0.0965	0.0171	0.0360	0.0293	0.0124	0.0017
	0.10	7.08	0.64	0.0980	0.0174	0.0366	0.0298	0.0125	0.0017
	0.20	7.23	0.64	0.0993	0.0177	0.0372	0.0302	0.0126	0.0016
0.20	0.00	6.92	0.63	0.0965	0.0171	0.0360	0.0293	0.0124	0.0017
	0.10	7.21	0.64	0.0991	0.0175	0.0371	0.0302	0.0126	0.0017
	0.20	7.51	0.65	0.1014	0.0179	0.0381	0.0310	0.0128	0.0016
$C_{F1} = 2C_{N1}$; $C_{F2} = 2C_{N2}$									
0.00	—	6.92	0.63	0.0965	0.0171	0.0360	0.0293	0.0124	0.0017
0.10	0.00	6.92	0.63	0.0965	0.0171	0.0360	0.0293	0.0124	0.0017
	0.10	7.26	0.64	0.0946	0.0173	0.0361	0.0287	0.0112	0.0013

APPENDIX 20A DESIGN AIDS FOR MOMENT DISTRIBUTION CONSTANTS (cont'd)

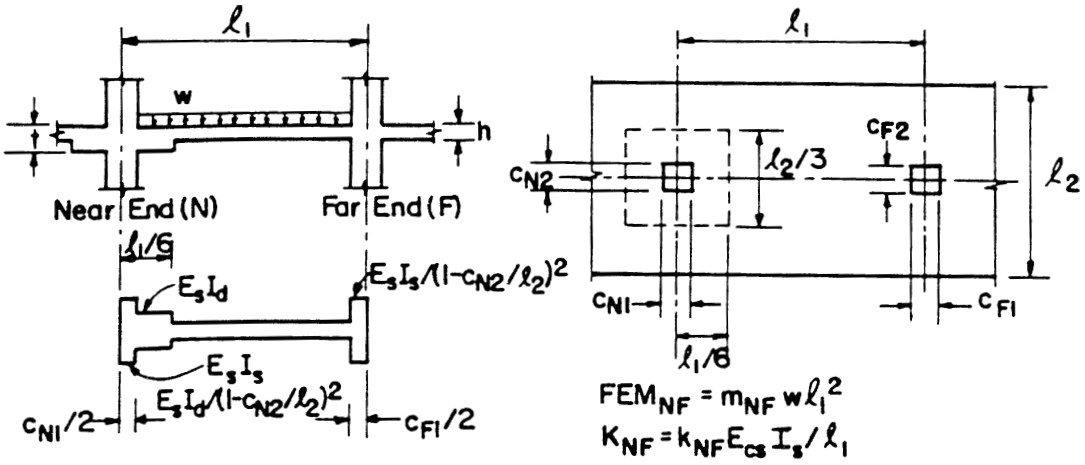
Table A5 Moment Distribution Constants for Slab-Beam Members (Drop thickness = h)



C_{N1}/ℓ_1	C_{N2}/ℓ_2	Stiffness Factors k_{NF}	Carry Over Factors C_{NF}	Unif. Load Fixed end M. Coeff. (m_{NF})	Fixed end moment Coeff. (m_{NF}) for (b—a) = 0.2				
					a = 0.0	a = 0.2	a = 0.4	a = 0.6	a = 0.8
$C_{F1} = C_{N1}$; $C_{F2} = C_{N2}$									
0.00	—	7.89	0.66	0.0993	0.0177	0.0380	0.0303	0.0118	0.0014
0.10	0.00	7.89	0.66	0.0993	0.0177	0.0380	0.0303	0.0118	0.0014
	0.10	8.07	0.66	0.0998	0.0180	0.0385	0.0305	0.0116	0.0013
	0.20	8.24	0.67	0.1003	0.0182	0.0389	0.0306	0.0115	0.0012
	0.30	8.40	0.67	0.1007	0.0183	0.0393	0.0307	0.0113	0.0011
0.20	0.00	7.89	0.66	0.0993	0.0177	0.0380	0.0303	0.0118	0.0014
	0.10	8.22	0.67	0.1002	0.0180	0.0388	0.0306	0.0115	0.0012
	0.20	8.55	0.68	0.1010	0.0183	0.0395	0.0309	0.0112	0.0011
	0.30	9.87	0.69	0.1018	0.0186	0.0402	0.0311	0.0109	0.0009
0.30	0.00	7.89	0.66	0.0993	0.0177	0.0380	0.0303	0.0118	0.0014
	0.10	8.35	0.67	0.1005	0.0181	0.0390	0.0307	0.0115	0.0012
	0.20	8.82	0.68	0.1016	0.0184	0.0399	0.0311	0.0111	0.0011
	0.30	9.28	0.70	0.1026	0.0187	0.0409	0.0314	0.0107	0.0009
$C_{F1} = 0.5C_{N1}$; $C_{F2} = 0.5C_{N2}$									
0.00	—	7.89	0.66	0.0993	0.0177	0.0380	0.0303	0.0118	0.0014
0.10	0.00	7.89	0.66	0.0993	0.0177	0.0380	0.0303	0.0118	0.0014
	0.10	8.03	0.66	0.1006	0.0180	0.0386	0.0307	0.0119	0.0014
	0.20	8.16	0.67	0.1016	0.0182	0.0390	0.0310	0.0120	0.0014
0.20	0.00	7.89	0.66	0.0993	0.0177	0.0380	0.0303	0.0118	0.0014
	0.10	8.15	0.67	0.1014	0.0181	0.0389	0.0310	0.0120	0.0014
	0.20	8.41	0.68	0.1032	0.0184	0.0398	0.0316	0.0121	0.0013
$C_{F1} = 2C_{N1}$; $C_{F2} = 0.5C_{N2}$									
0.00	—	7.89	0.66	0.0993	0.0177	0.0380	0.0303	0.0118	0.0014
0.10	0.00	7.79	0.66	0.0993	0.0177	0.0380	0.0303	0.0118	0.0014
	0.10	8.20	0.67	0.0981	0.0179	0.0382	0.0297	0.0113	0.0010

APPENDIX 20A DESIGN AIDS FOR MOMENT DISTRIBUTION CONSTANTS (cont'd)

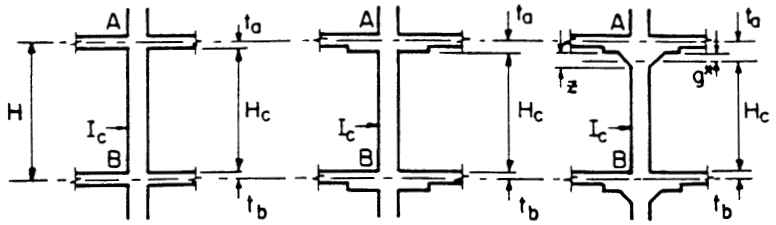
Table A6 Moment Distribution Constants for Slab-Beam Members
(Column dimensions assumed equal at near end and far end —$c_{F1} = c_{N1}$, $c_{F2} = c_{N2}$)



C_1/ℓ_1	C_2/ℓ_2	t = 1.5h						t = 2h					
		k_{NF}	C_{NF}	m_{NF}	k_{FN}	C_{FN}	m_{FN}	k_{NF}	C_{NF}	m_{NF}	k_{FN}	C_{FN}	m_{FN}
0.00	—	5.39	0.49	0.1023	4.26	0.60	0.0749	6.63	0.49	0.1190	4.49	0.65	0.0676
0.10	0.00	5.39	0.49	0.1023	4.26	0.60	0.0749	6.63	0.49	0.1190	4.49	0.65	0.0676
	0.10	5.65	0.52	0.1012	4.65	0.60	0.0794	7.03	0.54	0.1145	5.19	0.66	0.0757
	0.20	5.86	0.54	0.1012	4.91	0.61	0.0818	7.22	0.56	0.1140	5.43	0.67	0.0778
	0.30	6.05	0.55	0.1025	5.10	0.62	0.0838	7.36	0.56	0.1142	5.57	0.67	0.0786
0.20	0.00	5.39	0.49	0.1023	4.26	0.60	0.0749	6.63	0.49	0.1190	4.49	0.65	0.0676
	0.10	5.88	0.54	0.1006	5.04	0.61	0.0826	7.41	0.58	0.1111	5.96	0.66	0.0823
	0.20	6.33	0.58	0.1003	5.63	0.62	0.0874	7.85	0.61	0.1094	6.57	0.67	0.0872
	0.30	6.75	0.60	0.1008	6.10	0.64	0.0903	8.18	0.63	0.1093	6.94	0.68	0.0892
0.30	0.00	5.39	0.49	0.1023	4.26	0.60	0.075	6.63	0.49	0.1190	4.49	0.65	0.0676
	0.10	6.08	0.56	0.1003	5.40	0.61	0.085	7.76	0.62	0.1087	6.77	0.67	0.0873
	0.20	6.78	0.61	0.0996	6.38	0.63	0.092	8.49	0.66	0.1055	7.91	0.68	0.0952
	0.30	7.48	0.64	0.0997	7.25	0.65	0.096	9.06	0.68	0.1047	8.66	0.69	0.0991

APPENDIX 20A DESIGN AIDS FOR MOMENT DISTRIBUTION CONSTANTS (cont'd)

Table A7 Stiffness and Carry-Over Factors for Columns



$K_c = k\frac{EI_c}{H}$

For values of k_{BA} and c_{BA} read (t_a/t_b) as (t_b/t_a)

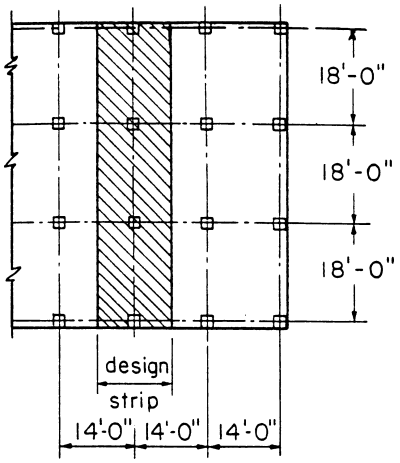
*g can be approximated as z/2.

t_a/t_b	H/H_c	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
0.00	k_{AB}	4.20	4.40	4.60	4.80	5.00	5.20	5.40	5.60	5.80	6.00
	C_{AB}	0.57	0.65	0.73	0.80	0.87	0.95	1.03	1.10	1.17	1.25
0.2	k_{AB}	4.31	4.62	4.95	5.30	5.65	6.02	6.40	6.79	7.20	7.62
	C_{AB}	0.56	0.62	0.68	0.74	0.80	0.85	0.91	0.96	1.01	1.07
0.4	k_{AB}	4.38	4.79	5.22	5.67	6.15	6.65	7.18	7.74	8.32	8.94
	C_{AB}	0.55	0.60	0.65	0.70	0.74	0.79	0.83	0.87	0.91	0.94
0.6	k_{AB}	4.44	4.91	5.42	5.96	6.54	7.15	7.81	8.50	9.23	10.01
	C_{AB}	0.55	0.59	0.63	0.67	0.70	0.74	0.77	0.80	0.83	0.85
0.8	k_{AB}	4.49	5.01	5.58	6.19	6.85	7.56	8.31	9.12	9.98	10.89
	C_{AB}	0.54	0.58	0.61	0.64	0.67	0.70	0.72	0.75	0.77	0.79
1.0	k_{AB}	4.52	5.09	5.71	6.38	7.11	7.89	8.73	9.63	10.60	11.62
	C_{AB}	0.54	0.57	0.60	0.62	0.65	0.67	0.69	0.71	0.73	0.74
1.2	k_{AB}	4.55	5.16	5.82	6.54	7.32	8.17	9.08	10.07	11.12	12.25
	C_{AB}	0.53	0.56	0.59	0.61	0.63	0.65	0.66	0.68	0.69	0.70
1.4	k_{AB}	4.58	5.21	5.91	6.68	7.51	8.41	9.38	10.43	11.57	12.78
	C_{AB}	0.53	0.55	0.58	0.60	0.61	0.63	0.64	0.65	0.66	0.67
1.6	k_{AB}	4.60	5.26	5.99	6.79	7.66	8.61	9.64	10.75	11.95	13.24
	C_{AB}	0.53	0.55	0.57	0.59	0.60	0.61	0.62	0.63	0.64	0.65
1.8	k_{AB}	4.62	5.30	6.06	6.89	7.80	8.79	9.87	11.03	12.29	13.65
	C_{AB}	0.52	0.55	0.56	0.58	0.59	0.60	0.61	0.61	0.62	0.63
2.0	k_{AB}	4.63	5.34	6.12	6.98	7.92	8.94	10.06	11.27	12.59	14.00
	C_{AB}	0.52	0.54	0.56	0.57	0.58	0.59	0.59	0.60	0.60	0.61
2.2	k_{AB}	4.65	5.37	6.17	7.05	8.02	9.08	10.24	11.49	12.85	14.31
	C_{AB}	0.52	0.54	0.55	0.56	0.57	0.58	0.58	0.59	0.59	0.59
2.4	k_{AB}	4.66	5.40	6.22	7.12	8.11	9.20	10.39	11.68	13.08	14.60
	C_{AB}	0.52	0.53	0.55	0.56	0.56	0.57	0.57	0.58	0.58	0.58
2.6	k_{AB}	4.67	5.42	6.26	7.18	8.20	9.31	10.53	11.86	13.29	14.85
	C_{AB}	0.52	0.53	0.54	0.55	0.56	0.56	0.56	0.57	0.57	0.57
2.8	k_{AB}	4.68	5.44	6.29	7.23	8.27	9.41	10.66	12.01	13.48	15.07
	C_{AB}	0.52	0.53	0.54	0.55	0.55	0.55	0.56	0.56	0.56	0.56
3.0	k_{AB}	4.69	5.46	6.33	7.28	8.34	9.50	10.77	12.15	13.65	15.28
	C_{AB}	0.52	0.53	0.54	0.54	0.55	0.55	0.55	0.55	0.55	0.55
3.2	k_{AB}	4.70	5.48	6.36	7.33	8.40	9.58	10.87	12.28	13.81	15.47
	C_{AB}	0.52	0.53	0.53	0.54	0.54	0.54	0.54	0.54	0.54	0.54
3.4	k_{AB}	4.71	5.50	6.38	7.37	8.46	9.65	10.97	12.40	13.95	15.64
	C_{AB}	0.51	0.52	0.53	0.53	0.54	0.54	0.54	0.53	0.53	0.53
3.6	k_{AB}	4.71	5.51	6.41	7.41	8.51	9.72	11.05	12.51	14.09	15.80
	C_{AB}	0.51	0.52	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.52
3.8	k_{AB}	4.72	5.53	6.43	7.44	8.56	9.78	11.13	12.60	14.21	15.95
	C_{AB}	0.51	0.52	0.53	0.53	0.53	0.53	0.53	0.52	0.52	0.52
4.0	k_{AB}	4.72	5.54	6.45	7.47	8.60	9.84	11.21	12.70	14.32	16.08
	C_{AB}	0.51	0.52	0.52	0.53	0.53	0.52	0.52	0.52	0.52	0.51
4.2	k_{AB}	4.73	5.55	6.47	7.50	8.64	9.90	11.27	12.78	14.42	16.20
	C_{AB}	0.51	0.52	0.52	0.52	0.52	0.52	0.52	0.51	0.51	0.51
4.4	k_{AB}	4.73	5.56	6.49	7.53	8.68	9.95	11.34	12.86	14.52	16.32
	C_{AB}	0.51	0.52	0.52	0.52	0.52	0.52	0.51	0.51	0.51	0.50
4.6	k_{AB}	4.74	5.57	6.51	7.55	8.71	9.99	11.40	12.93	14.61	16.43
	C_{AB}	0.51	0.52	0.52	0.52	0.52	0.52	0.51	0.51	0.50	0.50
4.8	k_{AB}	4.74	5.58	6.53	7.58	8.75	10.03	11.45	13.00	14.69	16.53
	C_{AB}	0.51	0.52	0.52	0.52	0.52	0.51	0.51	0.50	0.50	0.49
5.0	k_{AB}	4.75	5.59	6.54	7.60	8.78	10.07	11.50	13.07	14.77	16.62
	C_{AB}	0.51	0.51	0.52	0.52	0.51	0.51	0.51	0.50	0.49	0.49
6.0	k_{AB}	4.76	5.63	6.60	7.69	8.90	10.24	11.72	13.33	15.10	17.02
	C_{AB}	0.51	0.51	0.51	0.51	0.50	0.50	0.49	0.49	0.48	0.47
7.0	k_{AB}	4.78	5.66	6.65	7.76	9.00	10.37	11.88	13.54	15.35	17.32
	C_{AB}	0.51	0.51	0.51	0.50	0.50	0.49	0.48	0.48	0.47	0.46
8.0	k_{AB}	4.78	5.68	6.69	7.82	9.07	10.47	12.01	13.70	15.54	17.56
	C_{AB}	0.51	0.51	0.50	0.50	0.49	0.49	0.48	0.47	0.46	0.45
9.0	k_{AB}	4.79	5.69	6.71	7.86	9.13	10.55	12.11	13.83	15.70	17.74
	C_{AB}	0.50	0.50	0.50	0.50	0.49	0.48	0.47	0.46	0.45	0.45
10.0	k_{AB}	4.80	5.71	6.74	7.89	9.18	10.61	12.19	13.93	15.83	17.90
	C_{AB}	0.50	0.50	0.50	0.49	0.48	0.48	0.47	0.46	0.45	0.44

Example 20.1—Two-Way Slab Without Beams Analyzed by Equivalent Frame Method

Using the Equivalent Frame Method, determine design moments for the slab system in the direction shown, for an intermediate floor.

Story height = 9 ft
Column dimensions = 16 × 16 in.
Lateral loads to be resisted by shear walls
No edge beams
Partition weight = 20 psf
Service live load = 40 psf
f'_c = 4,000 psi (for slabs), normal weight concrete
f'_c = 6,000 psi (for columns), normal weight concrete
f_y = 60,000 psi



Calculations and Discussion	Code Reference

1. Preliminary design for slab thickness h:

   a. Control of deflections.

   For flat plate slab systems, the minimum overall thickness h with Grade 60 reinforcement is (see Table 18-1): *9.5.3.2*

   $$h = \frac{\ell_n}{30} = \frac{200}{30} = 6.67 \text{ in.}$$ *Table 9.5 (a)*

   but not less than 5 in. *9.5.3.2(a)*

   where ℓ_n = length of clear span in the long direction = 216 - 16 = 200 in.

   Try 7 in. slab for all panels (weight = 87.5 psf)

   b. Shear strength of slab.

   Use average effective depth d = 5.75 in. (3/4 in. cover and No. 4 bar)

   Factored dead load, w_d = 1.2 (87.5 + 20) = 129 psf *9.2.1*
   Factored live load, w_ℓ = 1.6 × 40 = 64 psf
   Total factored load = 193 psf

   For wide beam action consider a 12-in. wide strip taken at d distance from the face of support in the long direction (see Fig. 20-13). *11.12.1.1*

   $V_u = 0.193 \times 7.854 = 1.5$ kips

   $V_c = 2\sqrt{f'_c}\, b_w d$

$\phi V_c = 0.75 \times 2\sqrt{4,000} \times 12 \times 5.75/1,000 = 6.6 \text{ kips} > V_u$ O.K. *9.3.2.3*

For two-way action, since there are no shear forces at the centerlines of adjacent panels, the shear strength at d/2 distance around the support is computed as follows:

$V_u = 0.193\,[(18 \times 14) - 1.81^2] = 48.0 \text{ kips}$

$V_c = 4\sqrt{f'_c}\, b_o d$ (for square interior column) *Eq. (11-35)*

$= 4\sqrt{4,000}\,(4 \times 21.75) \times 5.75/1,000 = 126.6 \text{ kips}$

$\phi V_c = 0.75 \times 126.6 = 95.0 \text{ kips} > V_u$ O.K. *9.3.2.3*

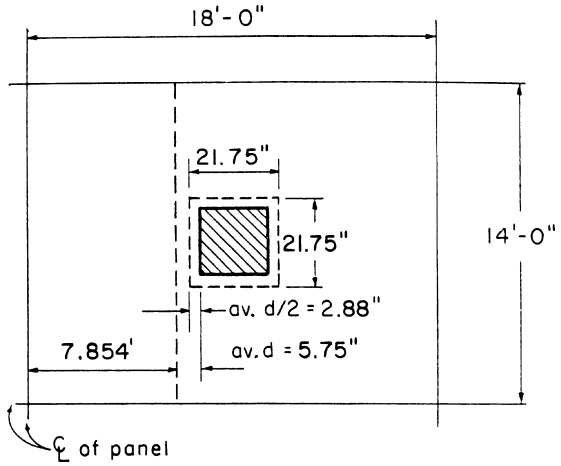


Figure 20-13 Critical Sections for Shear for Example Problem

Preliminary design indicates that a 7 in. overall slab thickness is adequate for control of deflections and shear strength.

2. Frame members of equivalent frame:

Determine moment distribution factors and fixed-end moments for the equivalent frame members. The moment distribution procedure will be used to analyze the partial frame. Stiffness factors k, carry over factors COF, and fixed-end moment factors FEM for the slab-beams and column members are determined using the tables of Appendix 20-A. These calculations are shown here.

a. Flexural stiffness of slab-beams at both ends, K_{sb}.

$$\frac{c_{N1}}{\ell_1} = \frac{16}{(18 \times 12)} = 0.07, \quad \frac{c_{N2}}{\ell_2} = \frac{16}{(14 \times 12)} = 0.1$$

For $c_{F1} = c_{N1}$ and $c_{F2} = c_{N2}$, $k_{NF} = k_{FN} = 4.13$ by interpolation from Table A1 in Appendix 20A.

Thus, $K_{sb} = k_{NF} \frac{E_{cs}I_s}{\ell_1} = 4.13 \frac{E_{cs}I_s}{\ell_1}$ *Table A1*

$= 4.13 \times 3.60 \times 10^6 \times 4{,}802/216 = 331 \times 10^6$ in.-lb

where $I_s = \frac{\ell_2 h^3}{12} = \frac{168(7)^3}{12} = 4{,}802$ in.^{4}

$E_{cs} = 57{,}000\sqrt{f'_c} = 57{,}000\sqrt{4{,}000} = 3.60 \times 10^6$ psi *8.5.1*

Carry-over factor COF = 0.509, by interpolation from Table A1.

Fixed-end moment FEM = $0.0843 w_u \ell_2 \ell_1^2$, by interpolation from Table A1.

b. Flexural stiffness of column members at both ends, K_c.

Referring to Table A7, Appendix 20A, $t_a = 3.5$ in., $t_b = 3.5$ in.,

H = 9 ft = 108 in., $H_c = 101$ in., $t_a/t_b = 1$, $H/H_c = 1.07$

Thus, $k_{AB} = k_{BA} = 4.74$ by interpolation.

$K_c = 4.74 E_{cc} I_c/\ell_c$ *Table A7*

$= 4.74 \times 4.42 \times 10^6 \times 5461/108 = 1059 \times 10^6$ in.-lb

where $I_c = \frac{c^4}{12} = \frac{(16)^4}{12} = 5{,}461$ in.^{4}

$E_{cs} = 57{,}000\sqrt{f'_c} = 57{,}000\sqrt{6{,}000} = 4.42 \times 10^6$ psi *8.5.1*

$\ell_c = 9$ ft $= 108$ in.

c. Torsional stiffness of torsional members, K_t.

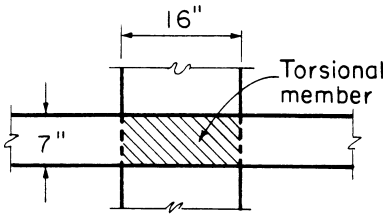
$K_t = \frac{9E_{cs}C}{\left[\ell_2 (1 - c_2/\ell_2)^3\right]}$ *R.13.7.5*

$= \frac{9 \times 3.60 \times 10^6 \times 13.25}{168(0.905)^3} = 3.45 \times 10^6$ in.-lb

where C = Σ (1 - 0.63 x/y) ($x^3y/3$) 13.0

= (1 - 0.63 × 7/16) (7^3 × 16/3) = 1,325 $in.^4$

c_2 = 16 in. and ℓ_2 = 14 ft = 168 in.



Condition (a) of Fig. 20-7

d. Equivalent column stiffness K_{ec}.

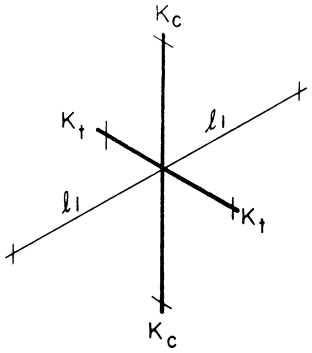
$$K_{ec} = \frac{\Sigma K_c \times \Sigma K_t}{\Sigma K_c + \Sigma K_t}$$

$$= \frac{(2 \times 1{,}059)(2 \times 345)}{[(2 \times 1{,}059) + (2 \times 345)]}$$

= 520 × 10^6 in.-lb

where ΣK_t is for two torsional members, one on each side of column, and ΣK_c is for the upper and lower columns at the slab-beam joint of an intermediate floor.

e. Slab-beam joint distribution factors DF.



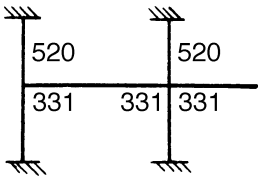
At exterior joint,

$$DF = \frac{331}{(331 + 520)} = 0.389$$

At interior joint,

$$DF = \frac{331}{(331 + 331 + 520)} = 0.280$$

COF for slab-beam = 0.509



3. Partial frame analysis of equivalent frame:

Determine maximum negative and positive moments for the slab-beams using the moment distribution method. Since the service live load does not exceed three-quarters of the service dead load, design moments are assumed to occur at all critical sections with full factored live load on all spans. 13.7.6.2

$$\frac{L}{D} = \frac{40}{(87.5 + 20)} = 0.37 < \frac{3}{4}$$

a. Factored load and fixed-end moments.

Factored dead load w_d = 1.2 (87.5 + 20) = 129 psf Eq. (9-2)

Factored live load w_ℓ = 1.6 (40) = 64 psf Eq. (9-2)

Factored load $w_u = w_d + w_\ell = 193$ psf

FEM's for slab-beams $= m_{NF} w_u \ell_2 \ell_1^2$ (Table A1, Appendix 20A)
$= 0.0843\ (0.193 \times 14)\ 18^2 = 73.8$ ft-kips

b. Moment distribution. Computations are shown in Table 20-1. Counterclockwise rotational moments acting on the member ends are taken as positive. Positive span moments are determined from the following equation:

M_u (midspan) $= M_o - (M_{uL} + M_{uR})/2$

where M_o is the moment at midspan for a simple beam.

When the end moments are not equal, the maximum moment in the span does not occur at midspan, but its value is close to that at midspan.

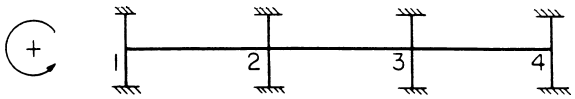
Positive moment in span 1-2:

$+M_u = (0.193 \times 14)\ 18^2/8 - (46.5 + 84.0)/2 = 44.2$ ft-kips

Positive moment in span 2-3:

$+M_u = (0.193 \times 14)\ 18^2/8 - (76.3 + 76.3)/2 = 33.1$ ft-kips

Table 20-1 Moment Distribution for Partial Frame



Joint	1	2		3		4
Member	1-2	2-1	2-3	3-2	3-4	4-3
DF	0.389	0.280	0.280	0.280	0.280	0.389
COF	0.509	0.509	0.509	0.509	0.509	0.509
FEM COM*	+73.8 0 +0.21 +0.2	-73.8 -14.6 0 -0.4	+73.8 0 -2.1 -0.2	-73.8 0 +2.1 +0.2	+73.8 +14.6 0 +0.4	-73.8 0 -2.1 -0.2
Σ DM**	+76.1 -29.6	-88.8 +4.8	+71.5 +4.8	-71.5 -4.8	+88.8 -4.8	-76.1 +29.6
Neg. M	+46.5	-84.0	+76.3	-76.3	+84.0	-46.5
M @ midspan	44.2		33.1		44.2	

Footnotes:

* *Carry-over moment, COM, is the negative product of the distribution factor, carry-over factor and unbalanced joint moment carried to the opposite end of span.*

** *Distributed moment, DM, is the negative product of the distribution factor and the unbalanced joint moment.*

4. Design moments:

Positive and negative factored moments for the slab system in the direction of analysis are plotted in Fig. 20-14. The negative design moments are taken at the faces of rectilinear supports but not at distances greater than $0.175\ell_1$ from the centers of supports. *13.7.7.1*

$$\frac{16 \text{ in.}}{2} = 0.67 \text{ ft} < 0.175 \times 18 = 3.2 \text{ ft} \quad \text{(Use face of support location)}$$

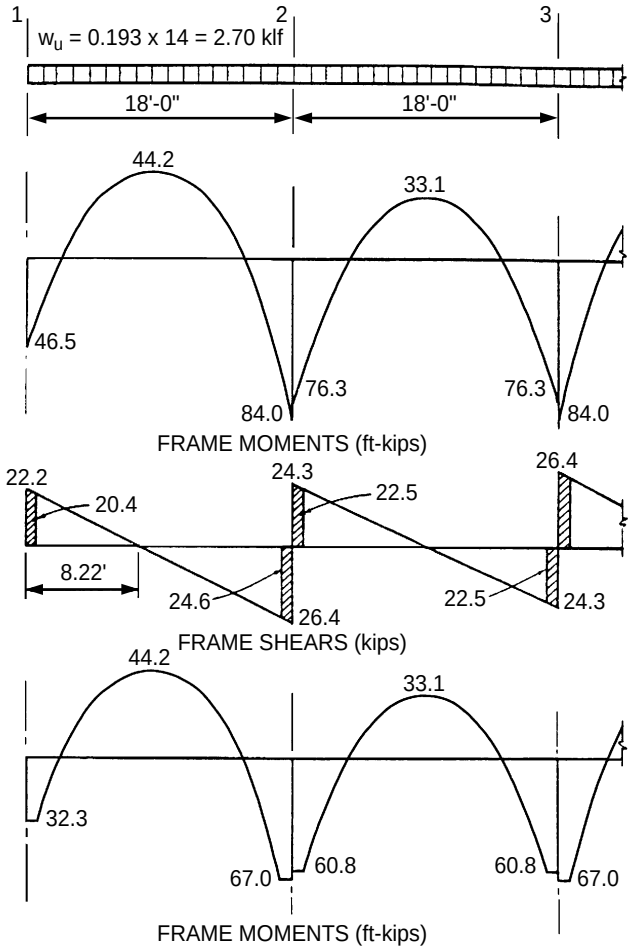


Figure 20-14 Positive and Negative Design Moments for Slab-Beam (All Spans Loaded with Full Factored Live Load)

5. Total factored moment per span:

Slab systems within the limitations of 13.6.1 may have the resulting moments reduced in such proportion that the numerical sum of the positive and average negative moments need not be greater than: *13.7.7.4*

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} = 0.193 \times 14 \times (16.67)^2 / 8 = 93.9 \text{ ft-kips}$$

End spans: 44.2 + (32.3 + 67.0) / 2 = 93.9 ft-kips

Interior span: 33.1 + (60.8 + 60.8) / 2 = 93.9 ft-kips

It may be seen that the total design moments from the Equivalent Frame Method yield a static moment equal to that given by the static moment expression used with the Direct Design Method.

6. Distribution of design moments across slab-beam strip: *13.7.7.5*

The negative and positive factored moments at critical sections may be distributed to the column strip and the two half-middle strips of the slab-beam according to the proportions specified in 13.6.4 and 13.6.6. The requirement of 13.6.1.6 does not apply for slab systems without beams, $\alpha = 0$. Distribution of factored moments at critical sections is summarized in Table 20-2.

Table 20-2 Distribution of Factored Moments

	Factored Moment (ft-kips)	Column Strip		Moment (ft-kips) in Two Half-Middle Strips**
		Percent*	Moment (ft-kips)	
End Span:				
Exterior Negative	32.3	100	32.3	0.0
Positive	44.2	60	26.5	17.7
Interior Negative	67.0	75	50.3	16.7
Interior Span:				
Negative	60.8	75	45.6	15.2
Positive	33.1	60	19.9	13.2

** For slab systems without beams*

*** That portion of the factored moment not resisted by the column strip is assigned to the two half-middle strips.*

7. Column moments:

The unbalanced moment from the slab-beams at the supports of the equivalent frame are distributed to the actual columns above and below the slab-beam in proportion to the relative stiffnesses of the actual columns. Referring to Fig. 20-14, the unbalanced moment at joints 1 and 2 are:

Joint 1 = +46.5 ft-kips

Joint 2 = -84.0 + 76.3 = -7.7 ft-kips

The stiffness and carry-over factors of the actual columns and the distribution of the unbalanced moments to the exterior and interior columns are shown in Fig. 20-15. The design moments for the columns may be taken at the juncture of column and slab.

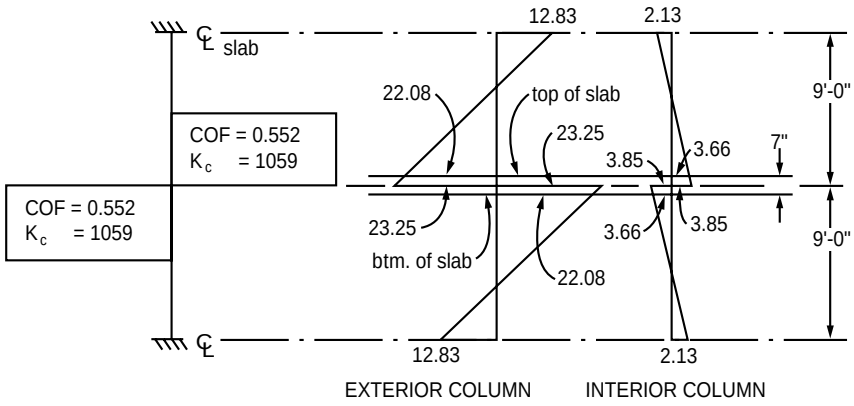


Figure 20-15 Column Moments (Unbalanced Moments from Slab-Beam)

In summary:

Design moment in exterior column = 22.08 ft-kips

Design moment in interior column = 3.66 ft-kips

8. Check slab flexural and shear strength at exterior column

   a. Total flexural reinforcement required for design strip:

      i. Determine reinforcement required for column strip moment M_u = 32.3 ft-kips

      Assume tension-controlled section (ϕ = 0.9) *9.3.2.1*

      Column strip width $b = \frac{14 \times 12}{2} = 84$ in. *13.2.1*

      $$R_u = \frac{M_u}{\phi b d^2} = \frac{32.3 \times 12{,}000}{0.9 \times 84 \times 5.75^2} = 155 \text{ psi}$$

      $$\rho = \frac{0.85 f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2R_u}{0.85 f'_c}}\right)$$

      $$= \frac{0.85 \times 4}{60}\left(1 - \sqrt{1 - \frac{2 \times 155}{0.85 \times 4{,}000}}\right) = 0.0026$$

      $A_s = \rho b d = 0.0026 \times 84 \times 5.75 = 1.28$ in.^{2}

      $\rho_{min} = 0.0018$ *13.3.1*

      Min $A_s = 0.0018 \times 84 \times 7 = 1.06$ in.^{2} < 1.28 in.^{2}

      Number of No. 4 bars = $\frac{1.28}{0.2} = 6.4$, say 7 bars

      Maximum spacing $s_{max} = 2h = 14$ in. < 18 in. *13.3.2*

$$c = \frac{a}{\beta_1} = \frac{0.29}{0.85} = 0.34 \text{ in.}$$

$$\epsilon_t = \left(\frac{0.003}{c}\right) d_t - 0.003$$

$$= \left(\frac{0.003}{0.34}\right) 5.75 - 0.003 = 0.048 > 0.005$$

Therefore, section is tension-controlled. *10.3.4*

Use 7-No. 4 bars in column strip.

ii. Check slab reinforcement at exterior column for moment transfer between slab and column

Portion of unbalanced moment transferred by flexure = $\gamma_f M_u$ *13.5.3.2*

From Fig. 16-13, Case C:

$$b_1 = c_1 + \frac{d}{2} = 16 + \frac{5.75}{2} = 18.88 \text{ in.}$$

$$b_2 = c_2 + d = 16 + 5.75 = 21.75 \text{ in.}$$

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} \quad \textit{Eq. (13-1)}$$

$$= \frac{1}{1 + (2/3)\sqrt{18.88/21.75}} = 0.62$$

$\gamma_f M_u = 0.62 \times 32.3 = 20.0$ ft-kips

Note that the provisions of 13.5.3.3 may be utilized; however, they are not in this example.

Assuming tension-controlled behavior, determine required area of reinforcement for $\gamma_f M_u = 20.0$ ft-kips

Effective slab width $b = c_2 + 3h = 16 + 3(7) = 37$ in. *13.5.3.2*

$$R_u = \frac{M_u}{\phi b d^2} = \frac{20 \times 12{,}000}{0.9 \times 37 \times 5.75^2} = 218 \text{ psi}$$

$$\rho = \frac{0.85 f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2R_u}{0.85 f'_c}}\right)$$

$$= \frac{0.85 \times 4}{60}\left(1-\sqrt{1-\frac{2 \times 218}{0.85 \times 4{,}000}}\right) = 0.0038$$

$A_s = 0.0038 \times 37 \times 5.75 = 0.80$ in.^{2}

Min. $A_s = 0.0018 \times 37 \times 7 = 0.47$ in.^{2} $<$ 0.80 in.^{2} *13.3.1*

Number of No. 4 bars $= \frac{0.80}{0.20} = 4$

Verify tension-controlled section:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(4 \times 0.2) \times 60}{0.85 \times 4 \times 37} = 0.38 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.38}{0.85} = 0.45 \text{ in.}$$

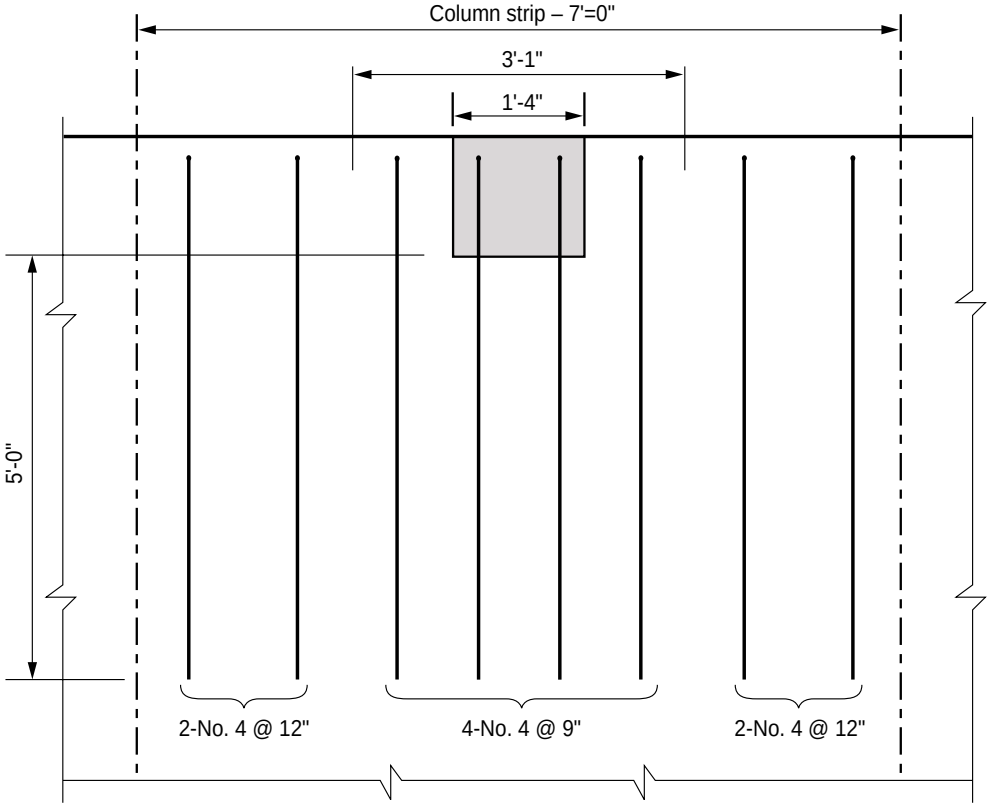
$$\epsilon_t = \left(\frac{0.003}{0.45}\right)5.75 - 0.003 = 0.035 > 0.005$$

Therefore, section is tension-controlled. *10.3.4*

Provide the required 4-No. 4 bars by concentrating 4 of the column strip bars (7-No. 4) within the 37 in. slab width over the column. For symmetry, add one additional No. 4 bar outside of 37-in. width.

Note that the column strip section remains tension-controlled with the addition of 1-No. 4 bar.

The reinforcement details at the edge column are shown below.



iii. Determine reinforcement required for middle strip.

Provide minimum reinforcement, since $M_u = 0$ (see Table 20-2).

Min. $A_s = 0.0018 \times 84 \times 7 = 1.06$ in.^{2}

Maximum spacing $s_{max} = 2h = 14$ in. < 18 in. *13.3.2*

Provide No. 4 @ 14 in. in middle strip.

b. Check combined shear stress at inside face of critical transfer section *11.12.6.1*

For shear strength equations, see Part 16.

$$V_u = \frac{V_u}{A_c} + \frac{\gamma_v M_u}{J / C}$$

From Example 19.1, $V_u = 25.6$ kips

When factored moments are determined by an accurate method of frame analysis, such as the Equivalent Frame Method, unbalanced moment is taken directly from the results of the frame analysis. Also, considering the approximate nature of the moment transfer analysis procedure, assume the unbalanced moment M_u is at the centroid of the critical transfer section.

Thus, M_u = 32.3 ft-kips (see Table 20-2)

$\gamma_v = 1-\gamma_f = 1-062 = 0.38$ *Eq. (11-39)*

From Example 19.1, critical section properties:

A_c = 342.2 in.^{2}

J/c = 2,357 in.^{3}

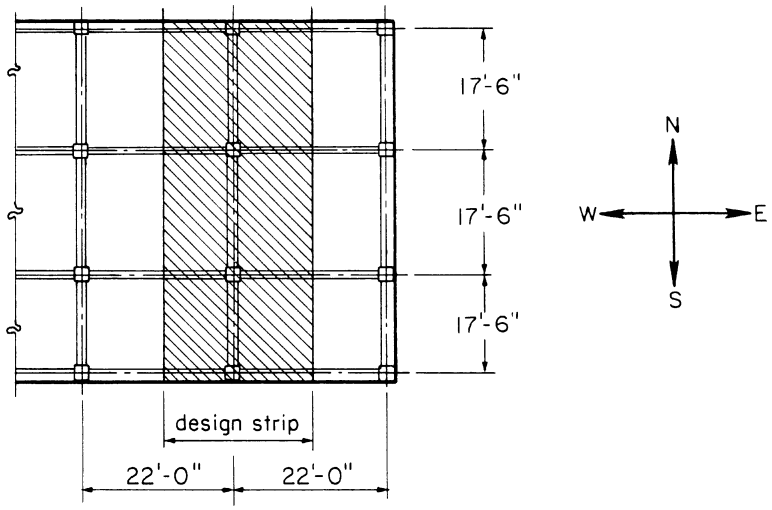
$$v_u = \frac{25{,}600}{342.2} + \frac{0.38 \times 32.3 \times 12{,}000}{2{,}357}$$

$$= 74.8 + 62.5 = 137.3 \text{ psi}$$

Allowable shear stress $\phi v_n = \phi 4\sqrt{f'_c}$ = 189.7 psi > v_u O.K. *11.12.6.2*

Example 20.2—Two-Way Slab with Beams Analyzed by Equivalent Frame Method

Using the Equivalent Frame Method, determine design moments for the slab system in the direction shown, for an intermediate floor.



Story height = 12 ft
Edge beam dimensions = 14 × 27 in.
Interior beam dimensions = 14 × 20 in.
Column dimensions = 18 × 18 in.
Service live load = 100 psf

f'_c = 4,000 psi (for all members), normal weight concrete
f_y = 60,000 psi

Calculations and Discussion	Code Reference

1. Preliminary design for slab thickness h.

   Control of deflections: *9.5.3.3*

   From Example 19.2, the beam-to-slab flexural stiffness ratios α are:

   α = 13.30 (NS edge beam)

   = 16.45 (EW edge beam)

   = 3.16 (NS interior beam)

   = 3.98 (EW interior beam)

   Since all $\alpha > 2.0$ (see Fig. 8-2), Eq. (9-13) will control. Therefore,

   $$h = \frac{\ell_n (0.8 + f_y / 200{,}000)}{36 + 9\beta}$$ *Eq. (9-12)*

$$= \frac{246\,(0.8 + 60{,}000/200{,}000)}{36 + 9\,(1.28)} = 5.7 \text{ in.}$$

where ℓ_n = clear span in long direction = 20.5 ft = 246 in.

$$\beta = \frac{\text{clear span in long direction}}{\text{clear span in short direction}} = \frac{20.5}{16.0} = 1.28$$

Use 6 in. slab thickness.

2. Frame members of equivalent frame.

Determine moment distribution constants and fixed-end moment coefficients for the equivalent frame members. The moment distribution procedure will be used to analyze the partial frame for vertical loading. Stiffness factors k, carry-over factors COF, and fixed-end moment factors FEM for the slab-beams and column members are determined using the tables of Appendix 20-A. These calculations are shown here.

a. Slab-beams, flexural stiffness at both ends K_{sb}:

$$\frac{c_{N1}}{\ell_1} = \frac{18}{17.5 \times 12} = 0.0857 \approx 0.1$$

$$\frac{c_{N2}}{\ell_2} = \frac{18}{22 \times 12} = 0.0682$$

Referring to Table A1, Appendix 20A,

$$K_{sb} = \frac{4.11 E_c I_{sb}}{\ell_1} = 4.11 \times 25{,}387 E_c/(17.5 \times 12) = 497 E_c$$

where I_{sb} is the moment of inertia of slab-beam section shown in Fig. 20-16 and computed with the aid of Fig. 20-21 at the end of this Example.

$I_{sb} = 2.72\,(14 \times 20^3)/12 = 25{,}387 \text{ in.}^4$

Carry-over factor COF = 0.507

Fixed-end moment, FEM = $0.0842 w_u \ell_2 \ell_1^2$

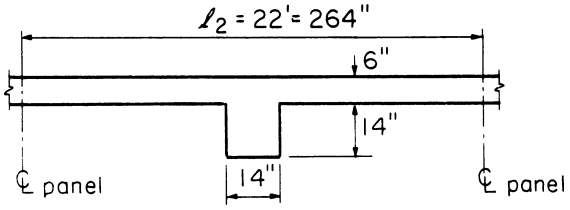


Figure 20-16 Cross-Section of Slab-Beam

b. Column members, flexural stiffness K_c:

t_a = 17 in, t_b = 3 in., t_a/t_b = 5.67

H = 12 ft = 144 in., H_c = 144 - 17 - 3 = 124 in.

H/H_c = 1.16 for interior columns

t_a = 24 in., t_b = 3 in., t_a/t_b = 8.0

H = 12 ft = 144 in., H_c = 144 - 24 - 3 = 117 in.

H/H_c = 1.23 for exterior columns

Referring to Table A7, Appendix 20A,

For interior columns:

$$K_{ct} = \frac{6.82E_cI_c}{\ell_c} = \frac{6.82 \times 8748E_c}{144} = 414E_c$$

$$K_{cb} = \frac{4.99E_cI_c}{\ell_c} = \frac{4.99 \times 8748E_c}{144} = 303E_c$$

For exterior columns:

$$K_{ct} = \frac{8.57E_cI_c}{\ell_c} = \frac{8.57 \times 8748E_c}{144} = 512E_c$$

$$K_{cb} = \frac{5.31E_cI_c}{\ell_c} = \frac{5.31 \times 8748E_c}{144} = 323E_c$$

where $I_c = \frac{(c)^4}{12} = \frac{(18)^4}{12}$ = 8,748 in.^{4}

ℓ_c = 12 ft = 144 in.

c. Torsional members, torsional stiffness K_t:

$$K_t = \frac{9E_cC}{\ell_2 (1 - c_2/\ell_2)^3}$$ *R13.7.5*

where $C = \Sigma(1 - 0.63\ x/y)\ (x^3y/3)$ *13.0*

For interior columns:

$K_t = 9E_c \times 11{,}698/[264\ (0.932)^3] = 493E_c$

where $1 - \frac{c_2}{\ell_2} = 1 - \frac{18}{(22 \times 12)} = 0.932$

C is taken as the larger value computed with the aid of Table 19-2 for the torsional member shown in Fig. 20-17.

$x_1 = 14$ in. $x_2 = 6$ in. $y_1 = 14$ in. $y_2 = 42$ in. $C_1 = 4{,}738$ $C_2 = 2{,}752$	$x_1 = 14$ in. $x_2 = 6$ in. $y_1 = 20$ in. $y_2 = 14$ in. $C_1 = 10{,}226$ $C_2 = 736$
$\Sigma C = 4{,}738 + 2{,}752 = 7{,}490$ in.^{4}	$\Sigma C = 10{,}226 + 736 \times 2 = 11{,}698$ in.^{4}

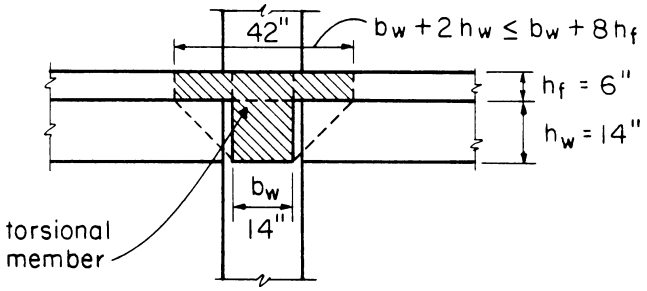


Figure 20-17 Attached Torsional Member at Interior Column

For exterior columns:

$K_t = 9E_c \times 17{,}868/[264\ (0.932)^3] = 752E_c$

where C is taken as the larger value computed with the aid of Table 19-2 for the torsional member shown in Fig. 20-18.

$x_1 = 14$ in. $x_2 = 6$ in. $y_1 = 21$ in. $y_2 = 35$ in. $C_1 = 11{,}141$ $C_2 = 2{,}248$	$x_1 = 14$ in. $x_2 = 6$ in. $y_1 = 27$ in. $y_2 = 21$ in. $C_1 = 16{,}628$ $C_2 = 1{,}240$
$\Sigma C = 11{,}141 + 2{,}248 = 13{,}389$ in.^{4}	$\Sigma C = 16{,}628 + 1{,}240 = 17{,}868$ in.^{4}

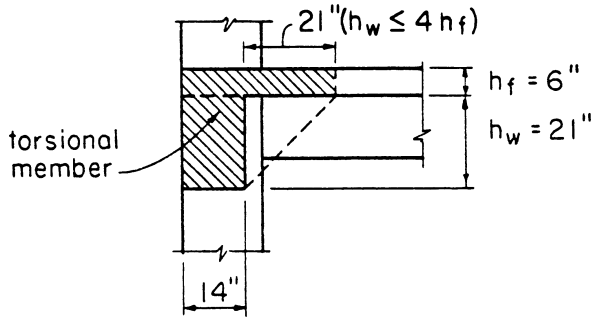


Figure 20-18 Attached Torsional Member at Exterior Column

d. Increased torsional stiffness K_{ta} due to parallel beams:

For interior columns:

$$K_{ta} = \frac{K_t I_{sb}}{I_s} = \frac{493E_c \times 25,387}{4,752} = 2,634E_c$$

For exterior columns:

$$K_{ta} = \frac{752E_c \times 25,387}{4,752} = 4,017E_c$$

where I_s = moment of inertia of slab-section shown in Fig. 20-19.

$= 264\ (6)^3/12 = 4,752\ \text{in.}^4$

I_{sb} = moment of inertia of full T-section shown in Fig. 20-19 and computed with the aid of Fig. 20-21

$= 2.72\ (14 \times 20^3/12) = 25,387\ \text{in.}^4$

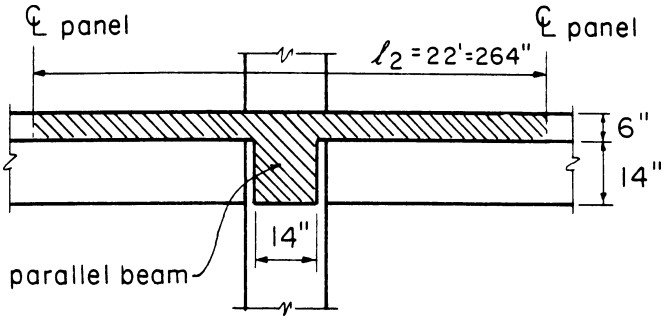


Figure 20-19 Slab-Beam in the Direction of Analysis

e. Equivalent column stiffness, K_{ec}:

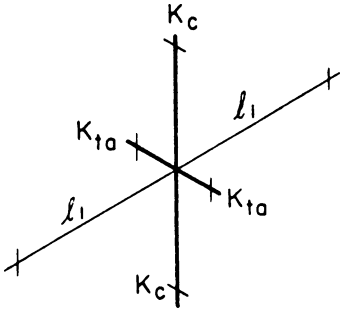
$$K_{ec} = \frac{\Sigma K_c \times \Sigma K_{ta}}{\Sigma K_c + \Sigma K_{ta}}$$

where ΣK_{ta} is for two torsional members, one on each side of column, and ΣK_c is for the upper and lower columns at the slab-beam joint of an intermediate floor.

For interior columns:

$$K_{ec} = \frac{(303E_c + 414E_c)(2 \times 2,634E_c)}{(303E_c + 414E_c) + (2 \times 2,634E_c)} = 631E_c$$

For exterior columns:



$$K_{ec} = \frac{(323E_c + 521E_c)(2 \times 4,017E_c)}{(323E_c + 521E_c) + (2 \times 4,017E_c)} = 764E_c$$

f. Slab-beam joint distribution factors DF:

At exterior joint:

$$DF = \frac{497E_c}{(497E_c + 764E_c)} = 0.394$$

At interior joint:

$$DF = \frac{497E_c}{(497E_c + 497E_c + 631E_c)} = 0.306$$

COF for slab-beam = 0.507

3. Partial frame analysis of equivalent frame.

Determine maximum negative and positive moments for the slab-beams using the moment distribution method.

With a service live-to-dead load ratio:

$$\frac{L}{D} = \frac{100}{75} = 1.33 > \frac{3}{4}$$

the frame will be analyzed for five loading conditions with pattern loading and partial live load as allowed by 13.7.6.3 (see Fig. 20-9 for an illustration of the five load patterns considered). *13.7.6.3*

a. Factored loads and fixed-end moments:

Factored dead load, $w_d = 1.2\ (75 + 9.3) = 101$ psf

$\left(\frac{14 \times 14}{144} \times \frac{150}{22} = 9.3\right.$ psf is weight of beam stem per foot divided by $\left.\ell_2\right)$

Factored live load , $w_\ell = 1.6\ (100) = 160$ psf

Factored load, $w_u = w_d + w_\ell = 261$ psf

FEM for slab-beams $= m_{NF} w_u \ell_2 \ell_1^2$ (Table A1, Appendix 20A)

FEM due to $w_d + w_\ell$ $= 0.0842\ (0.261 \times 22)\ 17.5^2 = 148.1$ ft-kips

FEM due to $w_d + 3/4w_\ell$ = 0.0842 (0.221 × 22) 17.5^2 = 125.4 ft-kips

FEM due to w_d only = 0.0842 (0.101 × 22) 17.5^2 = 57.3 ft-kips

b. Moment distribution for the five loading conditions is shown in Table 20-3. Counter-clockwise rotational moments acting on member ends are taken as positive. Positive span moments are determined from the equation:

$M_{u(midspan)} = M_o - (M_{uL} + M_{uR})/2$

where M_o is the moment at midspan for a simple beam.

When the end moments are not equal, the maximum moment in the span does not occur at midspan, but its value is close to that at midspan.

Positive moment in span 1-2 for loading (1):

$+M_u = (0.261 \times 22)\ 17.5^2/8 - (93.1 + 167.7)/2 = 89.4$ ft-kips

The following moment values for the slab-beams are obtained from Table 20-3. Note that according to 13.7.6.3, the design moments shall be taken not less than those occurring with full factored live load on all spans.

Maximum positive moment in end span

= the larger of 89.4 or 83.3 = 89.4 ft-kips

Maximum positive moment in interior span*

= the larger of 66.2 or 71.3 = 71.3 ft-kips

Maximum negative moment at end support

= the larger of 93.1 or 86.7 = 93.1 ft-kips

Maximum negative moment at interior support of end span

= the larger of 167.7 or 145.6 = 167.7 ft-kips

Maximum negative moment at interior support of interior span

= the larger of 153.6 or 139.2 = 153.6 ft-kips

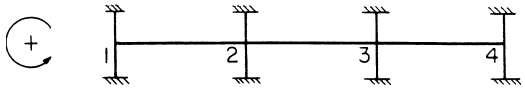
4. Design moments.

Positive and negative factored moments for the slab system in the transverse direction are plotted in Fig. 20-20. The negative factored moments are taken at the face of rectilinear supports at distances not greater than $0.175\ell_1$ from the center of supports. *13.7.7.1*

$\frac{18 \text{ in.}}{2}$ = 0.75 ft < 0.175 × 17.5 = 3.1 ft (Use face of support location).

* This is the only moment governed by the pattern loading with partial live load. All other maximum moments occur with full factored live load on all spans.

Table 20-3 Moment Distribution for Partial Frame (Transverse Direction)



Joint	1	2		3		4
Member	1-2	2-1	2-3	3-2	3-4	4-3
DF	0.394	0.306	0.306	0.306	0.306	0.394
COF	0.507	0.507	0.507	0.507	0.507	0.507

(1) All spans loaded with full factored live load

FEC	+148.1	-148.1	+148.1	-148.1	+148.1	-148.1
COM*		-29.6			+29.6	
COM	+4.6		-4.6	+4.6		-4.6
COM	+0.7	-0.9	-0.7	+0.7	+0.9	-0.7
COM	+0.3	-0.2	-0.3	+0.3	+0.2	-0.3
Σ	+153.7	-178.8	+142.5	-142.5	+178.8	-153.7
DM**	+60.6	+11.1	+11.1	-11.1	-11.1	-60.6
Total	+93.1	-167.7	+153.6	-153.6	+167.7	-93.1

(2) First and third spans loaded with 3/4 factored live load

FEC	+125.4	-125.4	+57.3	-57.3	+125.4	-125.4
COM*	+10.6	-25.1	-10.6	+10.6	+25.1	-10.6
COM	+5.5	-2.1	-5.5	+5.5	+2.1	-5.5
COM	+1.2	-1.1	-1.2	+1.2	+1.1	-1.2
COM	+0.4	-0.3	-0.4	+0.4	+0.3	-0.4
Σ	+143.1	-154.0	+39.6	-39.6	+154.0	-143.1
DM**	-56.4	+35.0	+35.0	-35.0	-35.0	+56.4
Total	+86.7	-119.0	+74.6	-74.6	+119.0	-86.7
Midspan M	83.3				83.3	

(3) Center span loaded with 3/4 factored live load

FEC	+57.3	-57.3	+125.4	-125.4	+57.3	-57.3
COM*	-10.6	-11.5	+10.6	-10.6	+11.5	+10.6
COM	+0.3	+2.1	+0.3	+0.3	-2.1	-0.3
COM	-0.3	-0.1	-0.3	-0.3	+0.1	+0.3
Σ	+46.7	-66.8	+136.0	-130.6	+66.8	-46.7
DM**	-18.4	-21.2	-21.2	+21.2	+21.2	+18.4
Total	+28.3	-88.0	+114.8	-114.8	+88.0	-28.3
Midspan M			71.3			

Table cont'd on next page

Table 20-3 Moment Distribution for Partial Frame (Transverse Direction) — continued —

(4) First span loaded with 3/4 factored live load and beam-slab assumed fixed at support two spans away

FEC COM* COM COM	+125.4 +10.6 +3.9 +0.3	-125.4 -25.1 -2.1 -0.8	+57.3	-57.3 +10.6 +3.9 +0.3		
Σ DM**	+140.2 -55.2	-153.4 +29.4	+57.3 +29.4			
Total	+85.0	-124.0	+86.7	-42.5		

(5) First and second span loaded with 3/4 factored live load

FEC COM* COM COM	+125.4 +2.3 +0.3	-125.4 -25.1 -0.2	+125.4 +10.6 -1.8 -0.1	-125.4 +2.3 +0.3	+57.3 +11.5 -2.1 +0.4	-57.3 +10.6 -1.8 -0.1
Σ DM**	+128.0 -50.4	-150.7 +5.1	+134.1 +5.1	-122.8 +17.0	+67.1 +17.0	-48.6 +19.2
Total	+77.6	-145.6	+139.2	-105.8	+84.1	-29.4

**Carry-over moment, COM, is the negative product of the distribution factor, carry-over and unbalanced joint moment carried to the opposite end of span.*

***Distributed moment, DM, is the negative product of the distribution factor and the unbalanced joint moment.*

5. Total factored moment per span. *13.7.7.4*

Slab systems within the limitations of 13.6.1 may have the resulting moments reduced in such proportion that the numerical sum of the positive and average negative moments are not greater than the total static moment M_o given by Eq. (13-3). Check limitations of 13.6.1.6 for relative stiffness of beams in two perpendicular directions.

For interior panel (see Example 19.2):

$$\frac{\alpha_1 \ell_2^2}{\alpha_2 \ell_1 2} = \frac{316\ (22)^2}{3.98\ (17.5)^2} = 1.25$$ *13.6.1.6*

$0.2 < 1.25 < 5.0$ O.K.

For exterior panel (see Example 19.2):

$$\frac{3.16\ (22)^2}{16.45\ (17.5)^2} = 0.30$$

$0.2 < 0.30 < 5.0$ O.K.

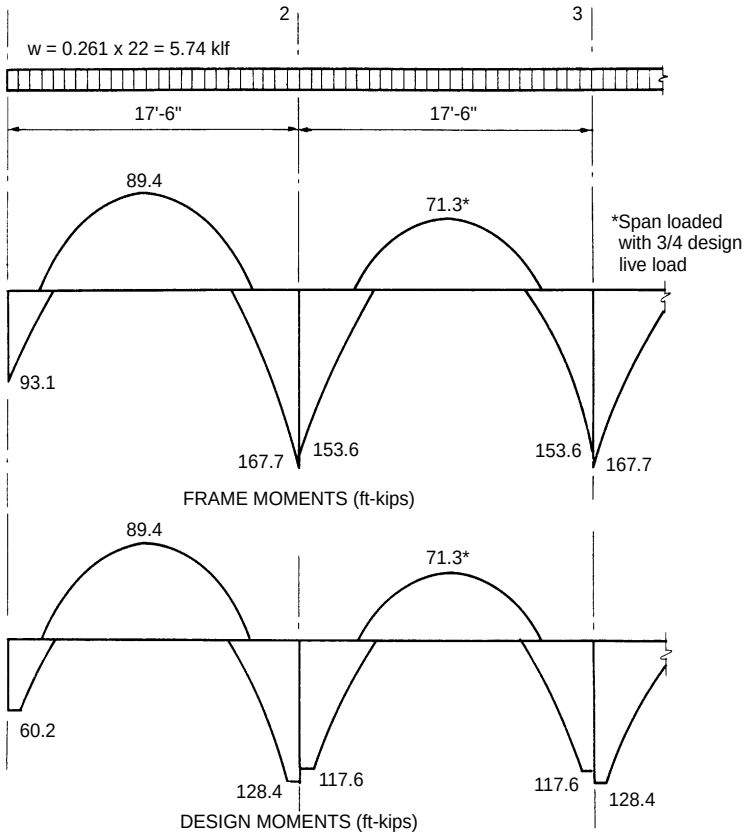


Figure 20-20 Positive and Negative Design Moments for Slab-Beam (All Spans Loaded with Full Factored Live Load Except as Noted)

All limitations of 13.6.1 are satisfied and the provisions of 13.7.7.4 may be applied.

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} = \frac{0.261 \times 22 \times 16^2}{8} = 183.7 \text{ ft-kips}$$ *Eq. (13-3)*

End span: $89.4 + (60.2 + 128.4)/2 = 183.7$ ft-kips

Interior span: $71.3 + (117.6 + 117.6)/2 = 188.9$ ft-kips

To illustrate proper procedure, the interior span factored moments may be reduced as follows:

Permissible reduction $= 183.7/188.9 = 0.9725$

Adjusted negative design moment $= 117.6 \times 0.9725 = 114.4$ ft-kips
Adjusted positive design moment $= 71.3 \times 0.9725 = 69.3$ ft-kips
$M_o = 183.7$ ft-kips

6. Distribution of design moments across slab-beam strip. *13.7.7.5*

Negative and positive factored moments at critical sections may be distributed to the column strip, beam and two-half middle strips of the slab-beam according to the proportions specified in 13.6.4, 13.6.5 and 13.6.6, if requirement of 13.6.1.6 is satisfied.

a. Since the relative stiffnesses of beams are between 0.2 and 5.0 (see step No. 5), the moments can be distributed across slab-beams as specified in 13.6.4, 13.6.5 and 13.6.6.

b. Distribution of factored moments at critical section:

$$\frac{\ell_2}{\ell_1} = \frac{22}{17.5} = 1.257$$

$$\frac{\alpha_1 \ell_2}{\ell_1} = 3.16 \times 1.257 = 3.97$$

$$\beta_t = \frac{C}{2I_s} = \frac{17{,}868}{(2 \times 4752)} = 1.88$$

where $I_s = \frac{22 \times 12 \times 6^3}{12} = 4{,}752$ in.^{4}

C = 17,868 in.^{4} (see Fig. 22-18)

Factored moments at critical sections are summarized in Table 20-4.

Table 20-4 Distribution of Design Moments

	Factored Moment (ft-kips)	Column Strip		Moment (ft-kips) in Two Half-Middle Strips**
		Percent*	Moment (ft-kips)	
End Span:				
Exterior Negative	60.2	75	45.2	15.0
Positive	89.4	67	59.9	29.5
Interior Negative	128.4	67	86.0	42.4
Interior Span:				
Negative	117.6	67	78.8	38.8
Positive	71.3	67	47.8	23.5

** Since $\alpha_1 \ell_2 / \ell 1 > 1.0$ beams must be proportioned to resist 85 percent of column strip moment per 13.6.5.1.*
*** That portion of the factored moment not resisted by the column strip is assigned to the two half-middle strips.*

7. Calculations for shear in beams and slab are performed in Example 19.2, Part 19.

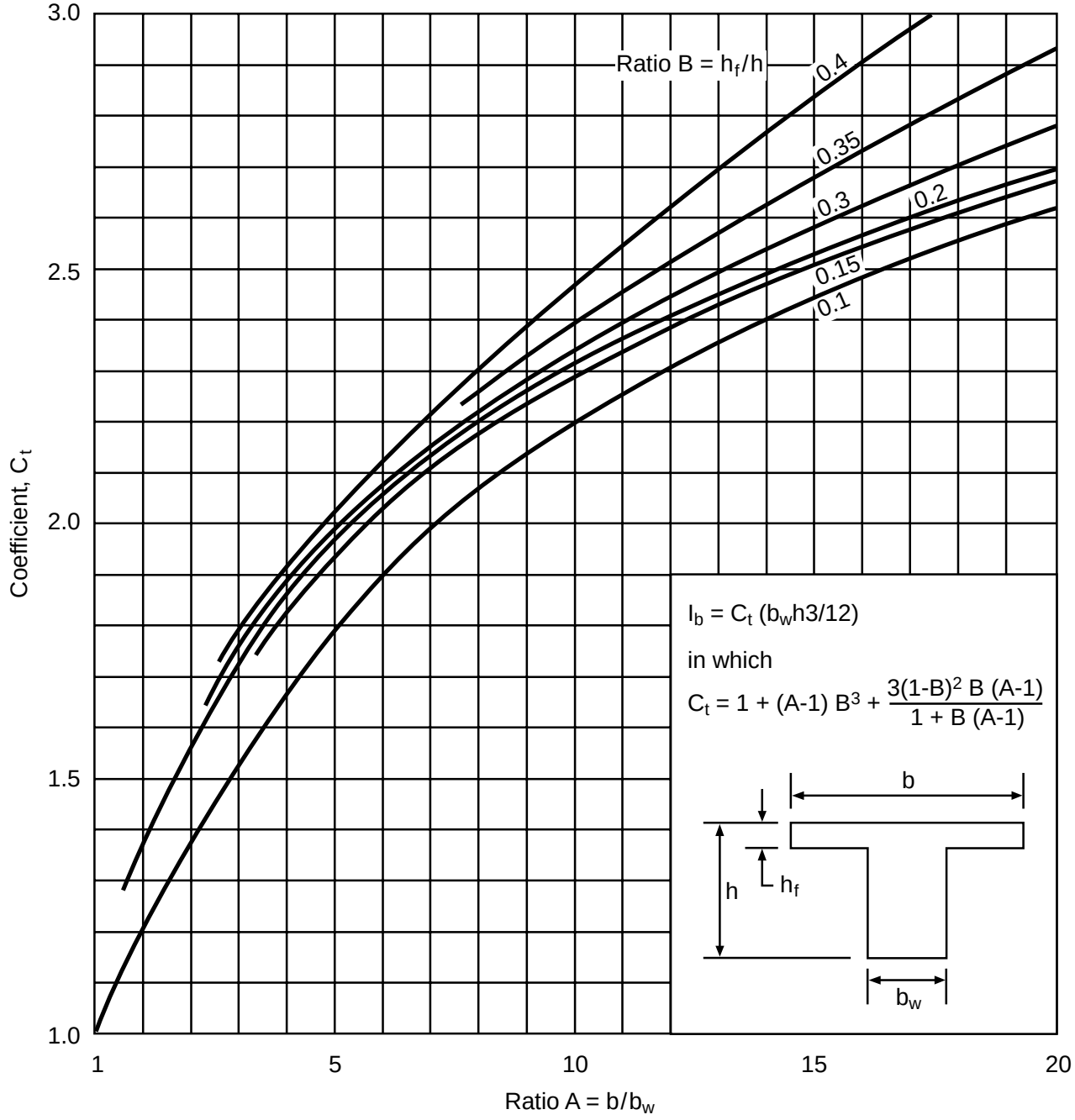


Figure 20-21 Coefficient C_t for Gross Moment of Inertia of Flanged Sections (Flange on One or Two Sides)

21

Walls

UPDATE FOR THE '02 CODE

The stiffness factor (ϕ) in Eqs. (14-5) and (14-6) is replaced with its numerical value of 0.75 in order to eliminate confusion between strength factor ϕ and stiffness factor.

14.1 SCOPE

Chapter 14 contains the provisions for the design of walls subjected to axial loads, with or without flexure (14.1.1). Cantilever retaining walls with minimum horizontal reinforcement according to 14.3.3 are designed according to the flexural design provisions of Chapter 10 (14.1.2).

14.2 GENERAL

According to 14.2.2, walls shall be designed in accordance with the provisions of 14.2, 14.3, and either 14.4, 14.5, or 14.8. Section 14.4 contains the requirements for walls designed as compression members using the strength design provisions for flexure and axial loads of Chapter 10. Any wall may be designed by this method, and no minimum wall thicknesses are prescribed.

Section 14.5 contains the Empirical Design Method which applies to walls of solid rectangular cross-section with resultant loads for all applicable load combinations falling within the middle third of the wall thickness at all sections along the height of the wall. Minimum thicknesses of walls designed by this method are contained in 14.5.3. Walls of nonrectangular cross-section, such as ribbed wall panels, must be designed by the provisions of 14.4, or if applicable, 14.8.

Section 14.8 contains the provisions of the Alternate Design Method, which are applicable to simply supported, axially loaded members subjected to out-of-plane uniform lateral loads, with maximum moments and deflections occurring at mid-height. Also, the wall cross-section must be constant over the height of the panel. No minimum wall thicknesses are prescribed for walls designed by this method.

All walls must be designed for the effects of shear forces. Section 14.2.3 requires that the design for shear must be in accordance with 11.10, the special shear provisions for walls. The required shear reinforcement may exceed the minimum wall reinforcement prescribed in 14.3.

For a wall subjected to a series of point loads, the horizontal length of the wall that is considered effective for each concentrated load is the least of the center-to-center distance between loads and width of bearing plus four times the wall thickness (14.2.4). Columns built integrally with walls shall conform to 10.8.2 (14.2.5). Walls shall be properly anchored into all intersecting elements, such as floors, columns, other walls, and footings (14.2.6).

Section 15.8 provides the requirements for force transfer between a wall and a footing. Note that for cast-in-place walls, the required area of reinforcement across the interface shall not be less than the minimum vertical reinforcement given in 14.3.2 (15.8.2.2).

14.3 MINIMUM WALL REINFORCEMENT

The minimum wall reinforcement provisions apply to walls designed according to 14.4, 14.5, or 14.8, unless a greater amount is required to resist horizontal shear forces in the plane of the wall according to 11.10.9.

Walls must contain both vertical and horizontal reinforcement. The minimum ratio of vertical reinforcement area to gross concrete area is (1) 0.0012 for deformed bars not larger than No. 5 with $f_y \geq 60{,}000$ psi, or for welded wire fabric (plain or deformed) not larger than W31 or D31, or (2) 0.0015 for all other deformed bars (14.3.2). The minimum ratio of horizontal reinforcement is (1) 0.0020 for deformed bars not larger than No. 5 with $f_y \geq 60{,}000$ psi, or for welded wire fabric (plain or deformed) not larger than W31 or D31, or (2) 0.0025 for all other deformed bars (14.3.3).

The minimum wall reinforcement required by 14.3 is provided primarily for control of cracking due to shrinkage and temperature stresses. Also, the minimum vertical wall reinforcement required by 14.3.2 does not substantially increase the strength of a wall above that of a plain concrete wall. It should be noted that the reinforcement and minimum thickness requirements of 14.3 and 14.5.3 may be waived where structural analysis shows adequate strength and wall stability (14.2.7). This required condition may be satisfied by a design using the structural plain concrete provisions in Chapter 22 of the code.

For walls thicker than 10 in., except for basement walls, reinforcement in each direction shall be placed in two layers (14.3.4).

Spacing of vertical and horizontal reinforcement shall not exceed 18 in. nor three times the wall thickness (14.3.5).

According to 14.3.6, lateral ties for vertical reinforcement are not required as long as the vertical reinforcement is not required as compression reinforcement or the area of vertical reinforcement does not exceed 0.01 times the gross concrete area.

A minimum of two No. 5 bars shall be provided around all window and door openings, with minimum bar extension beyond the corner of opening equal to the greater of bar development length or 24 in. (14.3.7).

14.4 WALLS DESIGNED AS COMPRESSION MEMBERS

When the limitations of 14.5 or 14.8 are not satisfied, walls must be designed as compression members by the strength design provisions in Chapter 10 for flexure and axial loads. The minimum reinforcement requirements of 14.3 apply to walls designed by this method. Vertical wall reinforcement need not be enclosed by lateral ties (as for columns) when the conditions of 14.3.6 are satisfied. All other code provisions for compression members apply to walls designed by Chapter 10.

As with columns, the design of walls is usually difficult without the use of design aids. Wall design is further complicated by the fact that slenderness is a consideration in practically all cases. A second-order analysis, which takes into account variable wall stiffness, as well as the effects of member curvature and lateral drift, duration of the loads, shrinkage and creep, and interaction with the supporting foundation, is specified in 10.10.1. In lieu of that procedure, the approximate evaluation of slenderness effects prescribed in 10.11 may be used (10.10.2).

It is important to note that Eqs. (10-11) and (10-12) for EI in the approximate slenderness method were not originally derived for members with a single layer of reinforcement. For members with a single layer of reinforcement, the following expression for EI has been suggested in Ref. 21.2:

$$EI = \frac{E_c I_g}{\beta}\left(0.5 - \frac{e}{h}\right) \geq 0.1\,\frac{E_c I_g}{\beta} \qquad \textit{Eq. (1)}$$

$$\leq 0.4\,\frac{E_c I_g}{\beta}$$

where

E_c = modulus of elasticity of concrete
I_g = moment of inertia of gross concrete section about the centroidal axis, neglecting reinforcement
e = eccentricity of the axial loads and lateral forces for all applicable load combinations
h = overall thickness of wall
$\beta = 0.9 + 0.5\beta_d^2 - 12\rho \geq 1.0$
β_d = ratio of dead load to total load
ρ = ratio of area of vertical reinforcement to gross concrete area

The definition of β_d, included in Eqs. (10-11) and (10-12) for EI, depends on the frame being non-sway or sway. According to 10.0, β_d for non-sway frames is the ratio of the maximum factored axial sustained load to the maximum factored axial load associated with the same load combination. For consistency, the same definition of β_d seems appropriate for the EI expressions for walls in Eq. (1). Note that if it is determined by the provisions of 10.11.4 that a sway condition exists, $\beta_d = 0$ for the case of lateral loads that are not sustained (10.0).

Figure 21-1 shows the comparison of flexural stiffness (EI) by Code Eq. (10-12) and Eq. (1) in terms of $E_c I_g$. The ratio of $EI/E_c I_g$ is plotted as a function of e/h for several values of β_d, for a constant reinforcement ratio ρ of 0.0015. Note that Code Eq. (10-12) assumes EI to be independent of e/h and appears to overestimate the wall stiffness for larger eccentricities. For walls designed by Chapter 10 with slenderness evaluation by 10.11, Eq. (1) is recommended in lieu of Code Eq. (10-12) for determining wall stiffness. Example 21.1 illustrates this method for a tilt-up wall panel.

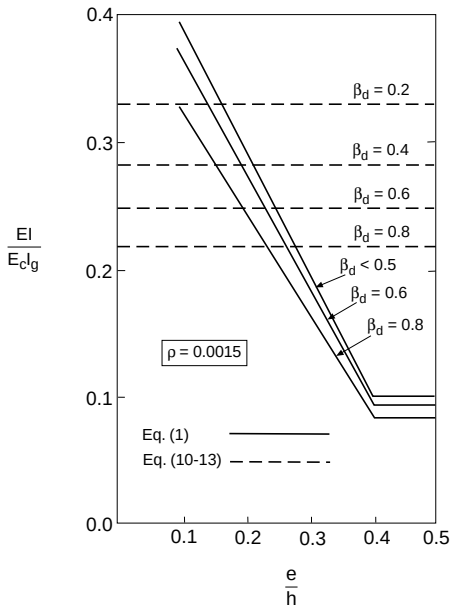


Figure 21-1 Stiffness EI of Walls

When wall slenderness exceeds the limit for application of the approximate slenderness evaluation method of 10.11 ($k\ell_u/h > 30$), 10.10.1 must be used to determine the slenderness effects (10.11.5). The wall panels currently used in some building systems, especially in tilt-up wall construction, usually fall in this high slenderness category. The slenderness analysis must account for the influence of variable wall stiffness, the effects of deflections on the moments and forces, and the effects of load duration.

14.5 EMPIRICAL DESIGN METHOD

The Empirical Design Method may be used for the design of walls if the resultant of all applicable loads falls within the middle one-third of the wall thickness (eccentricity e ≤ h/6), and the thickness is at least the minimum prescribed in 14.5.3 (see Fig. 21-2). Note that in addition to any eccentric axial loads, the effect of any lateral loads on the wall must be included to determine the total eccentricity of the resultant load. The method applies only to walls of solid rectangular cross-section.

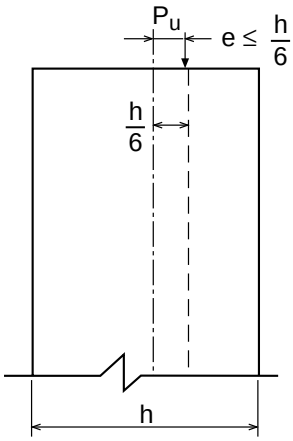


Figure 21-2 Design of Walls by Empirical Design Method (14.5)

Primary application of this method is for relatively short walls subjected to vertical loads only. Application becomes extremely limited when lateral loads need to be considered, because the total load eccentricity must not exceed h/6. Walls not meeting these criteria must be designed as compression members for axial load and flexure by the provisions of Chapter 10 (14.4) or, if applicable, by the Alternate Design Method of 14.8.

When the total eccentricity e does not exceed h/6, the design is performed considering P_u as a concentric axial load. The factored axial load P_u must be less than or equal to the design axial load strength ϕP_{nw} computed by Eq. (14-1):

$$P_u \leq \phi P_{nw}$$

$$\leq 0.55\phi f'_c A_g \left[1 - \left(\frac{k\ell_c}{32h}\right)^2\right] \qquad \textit{Eq. (14-1)}$$

where

ϕ = strength reduction factor = 0.7
A_g = gross area of wall section
k = effective length factor defined in 14.5.2
ℓ_c = vertical distance between supports

Equation (14-1) takes into consideration both load eccentricity and slenderness effects. The eccentricity factor 0.55 was originally selected to give strengths comparable to those given by Chapter 10 for members with axial load applied at an eccentricity of h/6.

In order to use Eq. (14-1), the wall thickness h must not be less than 1/25 times the supported length or height, whichever is shorter, nor less than 4 in. (14.5.3.1). Exterior basement walls and foundation walls must be at least 7-1/2 in. thick (14.5.3.2).

With the publication of the 1980 supplement of ACI 318, Eq. (14-1) was modified to reflect the general range of end conditions encountered in wall design, and to allow for a wider range of design applications. The wall

strength equation in previous codes was based on the assumption that the top and bottom ends of the wall are restrained against lateral movement, and that rotation restraint exists at one end, so as to have an effective length factor between 0.8 and 0.9. Axial load strength values could be unconservative for pinned-pinned end conditions, which can exist in certain walls, particularly of precast and tilt-up applications. Axial strength could also be overestimated where the top end of the wall is free and not braced against translation. In these cases, it is necessary to reflect the proper effective length in the design equation. Equation (14-1) allows the use of different effective length factors k to address this situation. The values of k have been specified in 14.5.2 for commonly occurring wall end conditions. Equation (14-1) will give the same results as the 1977 Code Eq. (14-1) for walls braced against translation at both ends and with reasonable base restraint against rotation. Reasonable base restraint against rotation implies attachment to a member having a flexural stiffness EI/ ℓ at least equal to that of the wall. Selection of the proper k for a particular set of support end conditions is left to the judgment of the engineer.

Figure 21-3 shows typical axial load-moment strength curves for 8-, 10-, and 12-in. walls with f'_c = 4,000 psi and f_y = 60,000 psi.[21.3] The curves yield eccentricity factors (ratios of strength under eccentric loading to that under concentric loading) of 0.562, 0.568, and 0.563 for the 8-, 10-, and 12-in. walls with e = h/6 and ρ = 0.0015.

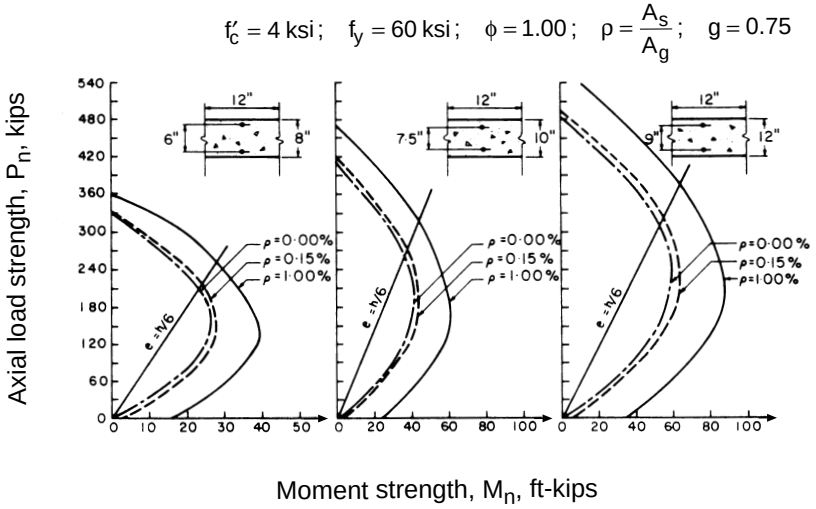


Figure 21-3 Typical Load-Moment Strength Curves for 8-, 10-, and 12-in. Walls

Figure R14.5 in the Commentary shows a comparison of the strengths obtained from the Empirical Design Method and Sect. 14.4 for members loaded at the middle third of the thickness with different end conditions.

Example 21.2 illustrates application of the Empirical Design Method to a bearing wall supporting precast floor beams.

14.8 ALTERNATE DESIGN OF SLENDER WALLS

The alternate design method for walls is based on the experimental research reported in Ref. 21.4. This method has appeared in the Uniform Building Code (UBC) since 1988, and is contained in the 2000 International Building Code (IBC)[21.5]. It is important to note that the provisions of 14.8 differ from those in the UBC and IBC in the following ways: (1) nomenclature and wording has been changed to comply with ACI 318 style, (2) the procedure has been limited to out-of-plane flexural effects on simply supported wall panels with maximum moments and deflections occurring at midspan, and (3) the procedure has been made as compatible as possible with the provisions of 9.5.2.3 for obtaining the cracking moment and the effective moment of inertia.

According to 14.8.1, the provisions of 14.8 are considered to satisfy 10.10 when flexural tension controls the design of a wall. The following limitations apply to the alternate design method (14.8.2):

1. The wall panel shall be simply supported, axially loaded, and subjected to an out-of-plane uniform lateral load. The maximum moments and deflections shall occur at the mid-height of the wall (14.8.2.1).
2. The cross-section is constant over the height of the panel (14.8.2.2).
3. The reinforcement ratio ρ shall be less than or equal to $0.6\rho_b$ (14.8.2.2), where ρ_b is the reinforcement ratio producing balanced strain conditions; see Eq. (B-1) in Appendix B of the code (14.8.2.3).
4. Reinforcement shall provide a design moment strength ϕM_n greater than or equal to M_{cr}, where M_{cr} is the moment causing flexural cracking due to the applied lateral and vertical loads. Note that M_{cr} shall be obtained using the modulus of rupture f_r given by Eq. (9-10) (14.8.2.4).
5. Concentrated gravity loads applied to the wall above the design flexural section shall be distributed over a width equal to the lesser of (a) the bearing width plus a width on each side that increases at a slope of 2 vertical to 1 horizontal down to the design flexural section or (b) the spacing of the concentrated loads. Also, the distribution width shall not extend beyond the edges of the wall panel (14.8.2.5) (see Fig. 21-4).
6. The vertical stress P_u/A_g at the mid-height section shall not exceed $0.06 f'_c$ (14.8.2.6).

When one or more of these conditions are not satisfied, the wall must be designed by the provisions of 14.4.

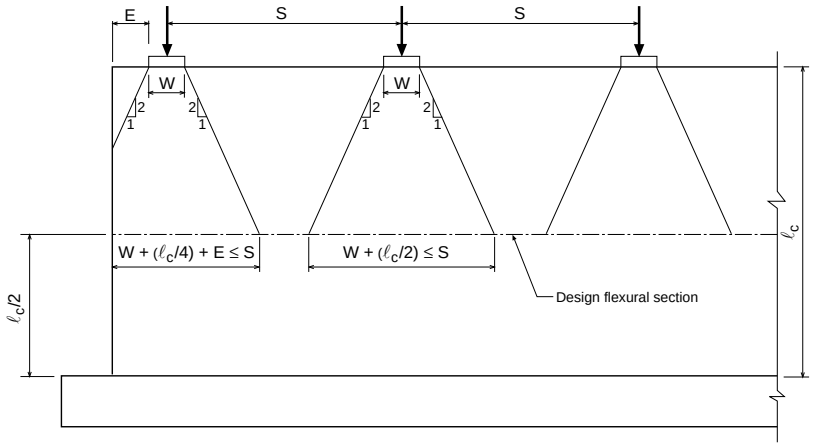


Figure 21-4 Distribution Width of Concentrated Gravity Loads (14.8.2.5)

According to 14.8.3, the design moment strength ϕM_n for combined flexure and axial loads at the mid-height cross-section must be greater than or equal to the total factored moment M_u at this section. The factored moment M_u includes P-Δ effects and is defined as follows:

$$M_u = M_{ua} + P_u \Delta_u \qquad \textit{Eq. (14-4)}$$

where M_{ua} = factored moment at the mid-height section of the wall due to factored lateral and eccentric vertical loads

P_u = factored axial load

Δ_u = deflection at the mid-height of the wall due to the factored loads

$$= 5M_u \ell_c^2/(0.75)48E_cI_{cr} \qquad \textit{Eq. (14-5)}$$

ℓ_c = vertical distance between supports

E_c = modulus of elasticity of concrete (8.5)

I_{cr} = moment of inertia of cracked section transformed to concrete

$$= nA_{se}(d - c)^2 + (\ell_w c^3/3) \qquad \textit{Eq. (14-7)}$$

n = modular ratio of elasticity = $E_s / E_c \geq 6$

E_s = modulus of elasticity of nonprestressed reinforcement

A_{se} = area of effective longitudinal tension reinforcement in the wall segment

$= (P_u + A_s f_y)/f_y$ *Eq. (14-8)*

A_s = area of longitudinal tension reinforcement in the wall segment

f_y = specified yield stress of nonprestressed reinforcement

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement

c = distance from extreme compression fiber to neutral axis

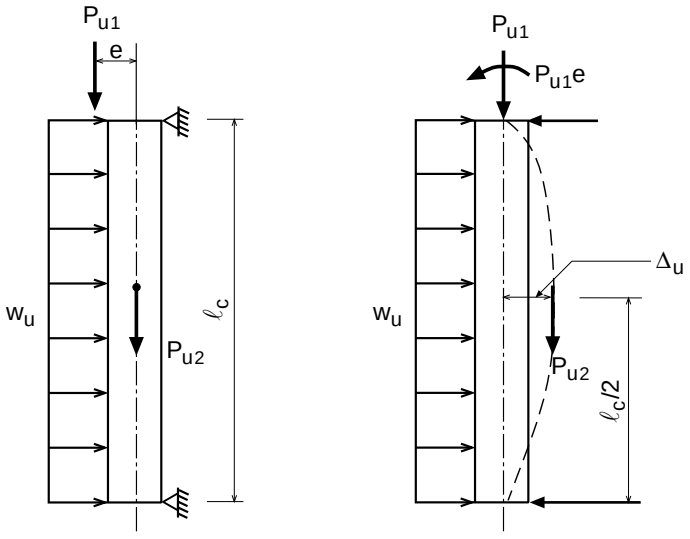
ℓ_w = horizontal length of the wall

Note that Eq. (14-4) includes the effects of the factored axial loads and lateral load (M_{ua}), as well as the P-Δ effects ($P_u \Delta_u$).

Substituting Eq. (14-5) for Δ_u into Eq. (14-4) results in the following equation for M_u:

$$M_u = \frac{M_{ua}}{1 - \dfrac{5 P_u \ell_c^2}{(0.75) 48 E_c I_{cr}}} \qquad \textit{Eq. (14-6)}$$

Figure 21-5 shows the analysis of the wall according to the provisions of 14.8 for the case of additive lateral and gravity load effects.



P_{u1} = factored applied gravity load

P_{u2} = factored self-weight of the wall (total)

e = eccentricity of applied gravity load

w_u = factored uniform lateral load

$$P_u = P_{u1} + \frac{P_{u2}}{2}$$

$$M_u = M_{ua} + P_u \Delta_u = \frac{M_{ua}}{1 - \dfrac{5 P_u \ell_c^2}{(0.75) 48 E_c I_{cr}}}$$

$$M_{ua} = \frac{w_u \ell_c^2}{8} + \frac{P_{u1} e}{2}$$

$$M_u = \frac{w_u \ell_c^2}{8} + \frac{P_{u1} e}{2} + \left(P_{u1} + \frac{P_{u2}}{2} \right) \Delta_u$$

$$\Delta_u = \frac{5 M_u \ell_c^2}{(0.75) 48 E_c I_{cr}}$$

Figure 21-5 Analysis of Wall According to 14.8

The design moment strength ϕM_n of the wall can be determined from the following equation:

$$\phi M_n = \phi A_{se} f_y \left(d - \frac{a}{2} \right) \qquad \textit{Eq. (2)}$$

where

$$a = \frac{A_{se} f_y}{0.85 f'_c \ell_w}$$

and ϕ is determined in accordance with 9.3.2.

In addition to satisfying the strength requirement of Eq. (14-3), the deflection requirement of 14.8.4 must also be satisfied. In particular, the maximum deflection Δ_s due to service loads, including P-Δ effects, shall not exceed $\ell_c/150$, where Δ_s is:

$$\Delta_s = \frac{5M\ell_c^2}{48E_c I_e} \qquad \textit{Eq. (14-9)}$$

where M = maximum unfactored moment due to service loads, including P-Δ effects

$$= \frac{M_{sa}}{1 - \dfrac{5P_s\ell_c^2}{48E_c I_e}} \qquad \textit{Eq. (14-10)}$$

and M_{sa} = maximum unfactored applied moment due to service loads, not including P-Δ effects

P_s = unfactored axial load at the design (mid-height) section including effects of self-weight

I_e = effective moment of inertia evaluated using the procedure of 9.5.2.3, substituting M for M_a.

It is important to note that Eq. (14-10) does not provide a closed form solution for M, since I_e is a function of M. Thus, an iterative process is required to determine Δ_s.

Example 21.3 illustrates the design of a nonprestressed precast wall panel by the alternated design method.

11.10 SPECIAL SHEAR PROVISIONS FOR WALLS

For most low-rise buildings, horizontal shear forces acting in the plane of walls are small, and can usually be neglected in design. Such in-plane forces, however, become an important design consideration in high-rise buildings. Design for shear shall be in accordance with the special provisions for walls in 11.10 (14.2.3). Example 21.4 illustrates in-plane shear design of walls, including design for flexure.

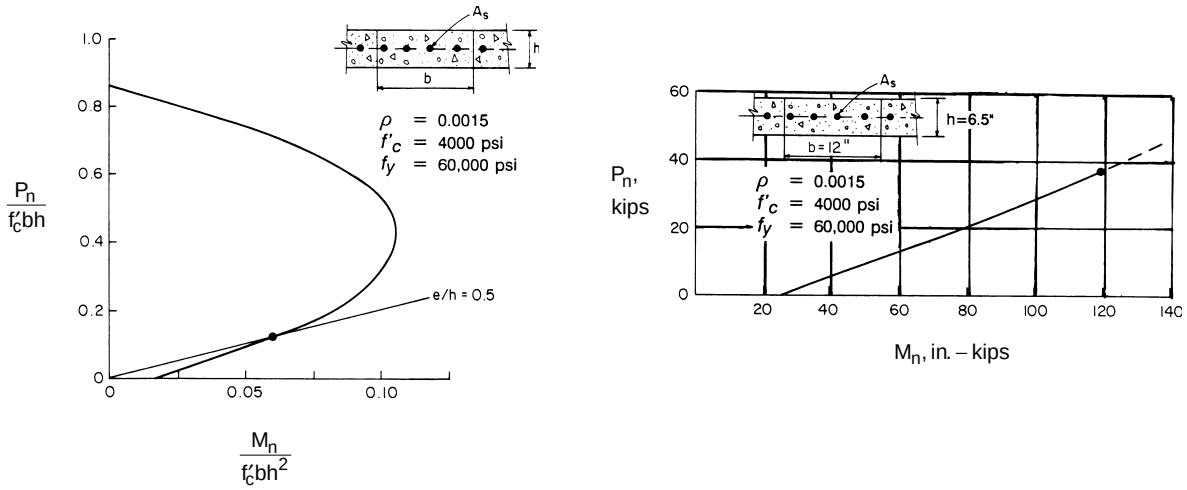
DESIGN SUMMARY

A trial procedure for wall design is suggested: first assume a wall thickness h and a reinforcement ratio ρ. Based on these assumptions, check the trial wall for the applied loading conditions.

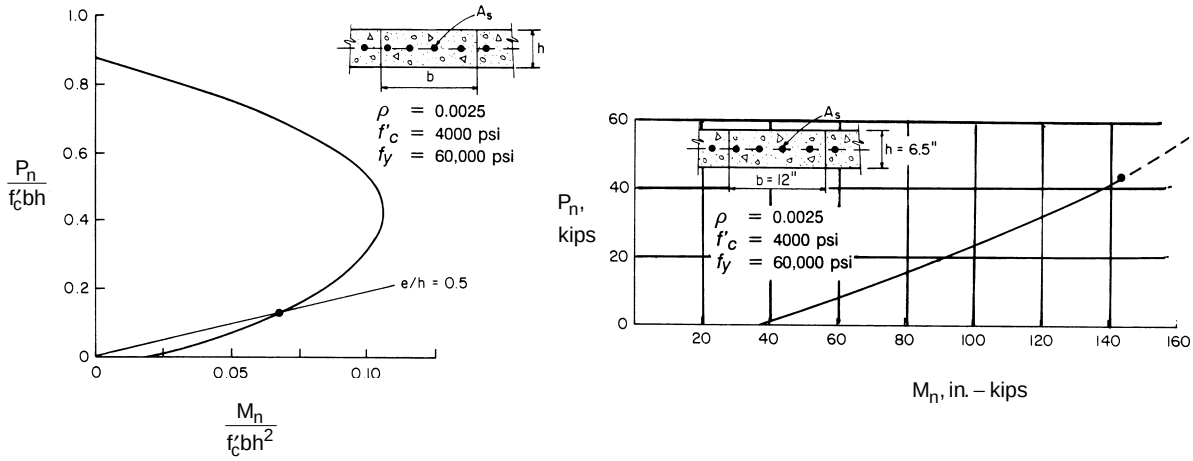
It is not within the scope of Part 21 to include design aids for a broad range of wall and loading conditions. The intent is to present examples of various design options and aids. The designer can, with reasonable effort, produce design aids to fit the range of conditions usually encountered in practice. For example, strength interaction diagrams such as those plotted in Fig. 21-6(a) (ρ = 0.0015) and Fig. 21-6(b) (ρ = 0.0025) can be helpful

design aids for evaluation of wall strength. The lower portions of the strength interaction diagrams are also shown for 6.5-in. thick walls. Design charts, such as the one shown in Fig. 21-7 can also be developed for specific walls. Figure 21-8 may be used to select wall reinforcement.

Prestressed walls are not covered specifically in Part 21. Prestressing of walls is advantageous for handling (precast panels) and for increased buckling resistance. For design of prestressed walls, the designer should consult Ref. 21.6.



(a) Reinforcement Ratio ρ = 0.0015



(b) Reinforcement Ratio ρ = 0.0025

Figure 21-6 Axial Load-Moment Interaction Diagram for Walls (f'_c = 4000 psi, f_y = 60 ksi)

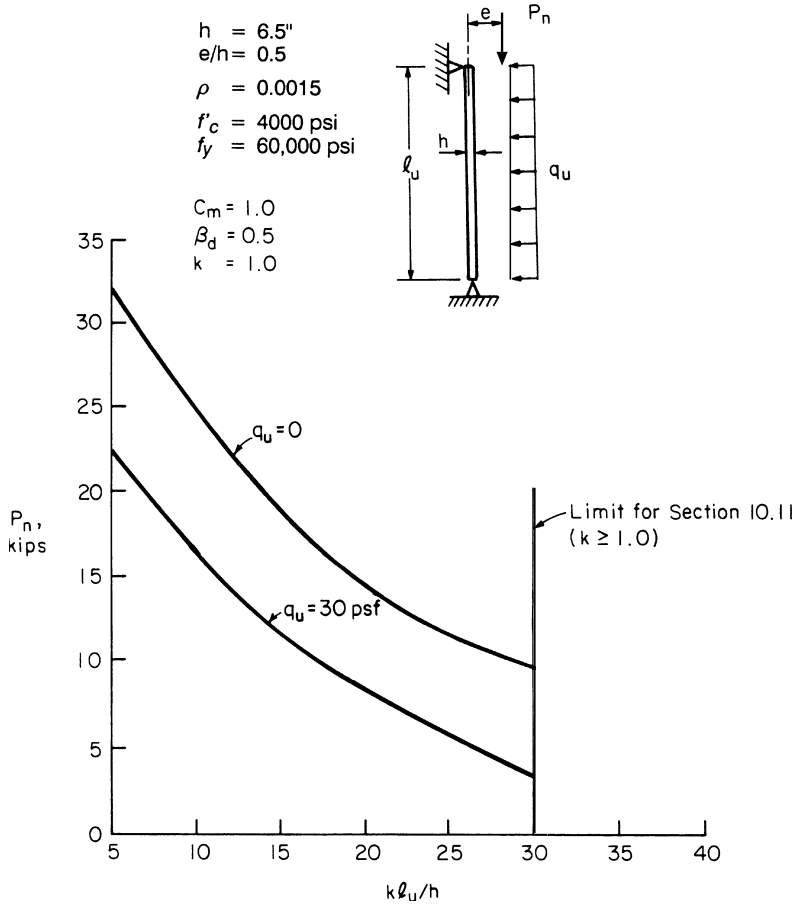


Figure 21-7 Design Chart for 6.5-in. Wall

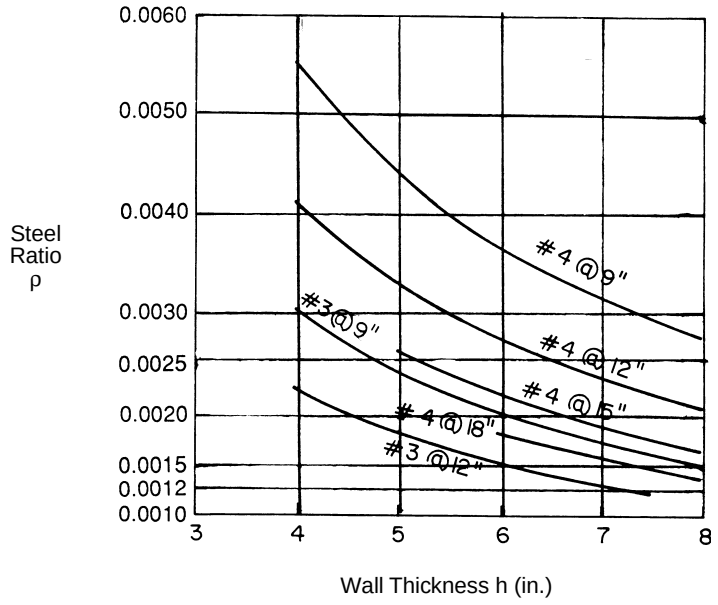


Figure 21-8 Design Aid for Wall Reinforcement

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21.2 MacGregor, J.G., "Design and Safety of Reinforced Concrete Compression Members," paper presented at International Association for Bridge and Structural Engineering Symposium, Quebec, 1974.

21.3 Kripanaryanan, K.M., "Interesting Aspects of the Empirical Wall Design Equation," *ACI Journal*, Proceedings Vol. 74, No. 5, May 1977, pp. 204-207.

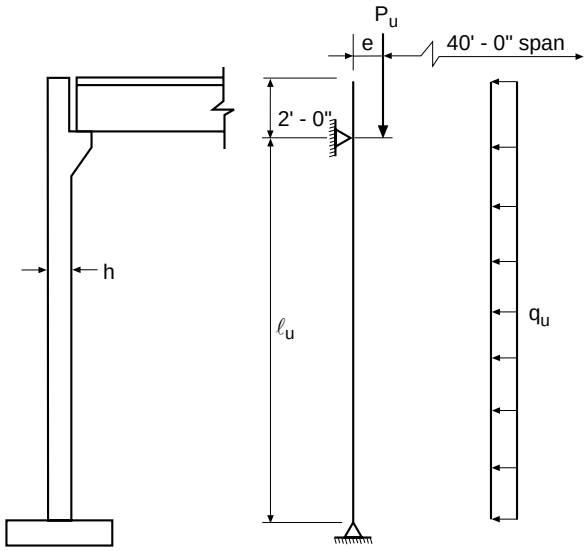
21.4 Athey, J.W., Ed., "Test Report on Slender Walls," Southern California Chapter of the American Concrete Institute and Structural Engineers Association of Southern California, Los Angeles, CA, 1982.

21.5 *2000 International Building Code*, International Code Council, Falls Church, VA, 2000.

21.6 *PCI Design Handbook - Precast and Prestressed Concrete*, 5th Edition, Prestressed Concrete Institute, Chicago, IL, 1999.

Example 21.1—Design of Tilt-up Wall Panel by Chapter 10 (14.4)

Design of the wall shown is required. The wall is restrained at the top edge, and the roof load is supported through 4 in. tee stems spaced at 4 ft on center.



Design data:

Roof dead load = 50 psf
Roof live load = 20 psf
Wind load = 20 psf
Unsupported length of wall ℓ_u = 16 ft
Effective length factor k = 1.0 (pinned-pinned end condition)
Concrete f'_c = 4,000 psi (w_c = 150 pcf)
Reinforcing steel f_y = 60,000 psi
Assume non-sway condition.

Calculations and Discussion	Code Reference

1. Trial wall selection

   Try h = 6.5 in. with assumed e = 6.75 in.

   Try a single layer of No. 4 @ 12 in. vertical reinforcement (A_s = 0.20 in.^{2}/ft) at centerline of wall

   For a 1-ft wide design strip:

   $$\rho = \frac{A_s}{bh} = \frac{0.20}{(12 \times 6.5)} = 0.0026 > 0.0012 \quad \text{O.K.}$$ *14.3.2 (a)*

2. Effective wall length for roof reaction *14.2.4*

   Bearing width + 4 (wall thickness) = 4 + 4 (6.5) = 30 in. = 2.5 ft (governs)

   Center-to-center distance between stems = 4 ft

3. Roof loading per foot width of wall

Dead load = $\left[50 \times \left(\frac{4}{2.5}\right)\right] \times \frac{40}{2}$ = 1,600 plf

Live load = $\left[20 \times \left(\frac{4}{2.5}\right)\right] \times \frac{40}{2}$ = 640 plf

Wall dead load at mid-height = $\frac{6.5}{12} \times \left(\frac{16}{2} + 2\right) \times 150$ = 813 plf

4. Factored load combinations

Load comb. 1: $U = 1.2D + 0.5L_r$ *Eq. (9-2)*
$P_u = 1.2\ (1.6 + 0.81) + 0.5\ (0.64) = 2.9 + 0.3 = 3.2$ kips
$M_u = 1.2\ (1.6 \times 6.75) + 0.5\ (0.64 \times 6.75) = 15.1$ in.-kips
$\beta_d = 2.9/3.2 = 0.91$

Load comb. 2: $U = 1.2D + 1.6L_r + 0.8W$ *Eq. (9-3)*
$P_u = 1.2\ (1.6 + 0.81) + 1.6\ (0.64) + 0 = 2.9 + 1.0 = 3.9$ kips
$M_u = 1.2\ (1.6 \times 6.75) + 1.6\ (0.64 \times 6.75) + 0.8\ (0.02 \times 16^2 \times 12/8)$
$= 26.0$ in.-kips
$\beta_d = 2.9/3.9 = 0.74$

Load comb. 3: $U = 1.2D + 1.6W + 0.5L_r$ *Eq. (9-4)*
$P_u = 1.2\ (1.6 + 0.81) + 0 + 0.5\ (0.64) = 3.2$ kips
$M_u = 1.2\ (1.6 \times 6.75) + 1.6\ (0.02 \times 16^2 \times 12/8) + 0.5\ (0.64 \times 6.75)$
$= 27.4$ in.-kips
$\beta_d = 2.9/3.2 = 0.91$

Load comb. 4: $U = 0.9D + 1.6W$ *Eq. (9-6)*
$P_u = 0.9\ (1.6 + 0.81) + 0 = 2.2$ kips
$M_u = 0.9\ (1.6 \times 6.75) + 1.6\ (0.02 \times 16^2 \times 12/8)$
$= 22.0$ in.-kips
$\beta_d = 2.2/2.2 = 1.0$

5. Check wall slenderness

$\frac{k\ell_u}{r} = \frac{1.0\ (16 \times 12)}{(0.3 \times 6.5)} = 98.5 < 100$ *10.11.5*

where $r = 0.3h$ *10.11.12*

Therefore, 10.11 may be used to account for slenderness effects.

6. Calculate magnified moments for non-sway case — *10.12*

$M_c = \delta_{ns} M_2$ — *Eq. (10-8)*

$$\delta_{ns} = \frac{C_m}{1 - \left(\frac{P_u}{0.75P_c}\right)} \geq 1$$ — *Eq. (10-9)*

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2}$$ — *Eq. (10-10)*

$$EI = \frac{E_c I_g}{\beta}\left(0.5 - \frac{e}{h}\right) \geq 0.1\frac{E_c I_g}{\beta}$$ — *Eq. (1)*

$$\leq 0.4\frac{E_c I_g}{\beta}$$

$$\frac{e}{h} = \frac{6.75}{6.5} = 1.04 > 0.5$$

Thus, $EI = 0.1\left(\frac{E_c I_g}{\beta}\right)$

$E_c = 57{,}000\sqrt{4{,}000} = 3.605 \times 10^6$ psi — *8.5.1*

$$I_g = \frac{12 \times 6.5^3}{12} = 274.6 \text{ in.}^4$$

$\beta = 0.9 + 0.5\beta_d^2 - 12\rho \geq 1.0$

$= 0.9 + 0.5\beta_d^2 - 12\,(0.0026)$

$= 0.869 + 0.5\beta_d^2 \geq 1.0$

$$EI = \frac{0.1 \times 3.605 \times 10^6 \times 274.6}{\beta} = \frac{99 \times 10^6}{\beta} \text{ lb-in.}^2$$

$$P_c = \frac{\pi^2 \times 99 \times 10^6}{\beta(16 \times 12)^2 \times 1{,}000} = \frac{26.5}{\beta} \text{ kips}$$

C_m = 1.0 for members with transverse loads between supports *10.12.3.1*

Determine magnified moment M_c for each load case.

Load Comb.	P_u (kips)	$M_2 = M_u$ (in.-kips)	β_d	β	EI (lb-in.^{2})	P_c (kips)	δ_{ns}	M_c (in.-kips)
1	3.2	15.1	0.91	1.28	77 x 10^6	20.7	1.26	19.0
2	3.9	26.0	0.74	1.14	87 x 10^6	23.2	1.29	33.5
3	3.2	27.4	0.91	1.28	77 x 10^6	20.7	1.26	34.5
4	2.2	22.0	1.00	1.37	72 x 10^6	19.3	1.18	26.0

7. Check design strength vs. required strength

Assume that the section is tension-controlled for each load combination, i.e., $\varepsilon_t \geq 0.005$ and $\phi = 0.90$. *10.3.4* *9.3.2*

The following table contains a summary of the strain compatibility analysis for each load combination, based on the assumption above:

Load Comb.	$P_n = P_u/\phi$ (kips)	a (in.)	c (in.)	ε_t (in./in.)
1	3.6	0.38	0.45	0.0187
2	4.3	0.40	0.47	0.0177
3	3.6	0.38	0.45	0.0187
4	2.4	0.35	0.42	0.0205

For example, the strain in the reinforcement ε_t is computed for load combination No. 2 as follows:

$P_n = 0.85 f'_c ba - A_s f_y$ *10.3.1* *10.2.1*

$4.3 = (0.85)(4)(12)a - (0.2)(60) = 40.8a - 12$

$a = 0.40$ in.

$c = a/\beta_1 = 0.4/0.85 = 0.47$ in. *10.2.7.1* *10.2.7.3*

$$\varepsilon_t = \frac{0.003}{c}\left(d - c\right)$$ *10.2.2*

$$= \frac{0.003}{0.47}\left(3.25 - 0.47\right)$$

$= 0.0177 > 0.0050 \rightarrow$ tension-controlled section *10.3.4*

Note that the strain in the reinforcement for each of the load combinations is greater than 0.0050, so that the assumption of tension-controlled sections ($\phi = 0.90$) is correct.

For each load combination, the required nominal strength will be compared to the computed design strength. The results are tabulated below.

Load Comb.	Required Nominal Strength		Design Strength M_n (in.-kips)
	$P_n = P_u/\phi$ (kips)	$M_n = M_c/\phi$ (in.-kips)	
1	3.6	21.1	47.7
2	4.3	37.2	49.7
3	3.6	38.3	47.7
4	2.4	28.9	44.2

For example, the design strength M_n is computed for load combination No. 2 as follows:

$$M_n = 0.85\, f'_c\, ba \left(\frac{h}{2} - \frac{a}{2}\right) - A_s f_y \left(\frac{h}{2} - d_t\right)$$

$$= 0.85(4)(12)(0.40)\left(\frac{6.5}{2} - \frac{0.40}{2}\right) - 0.2(60)\left(\frac{6.5}{2} - 3.25\right)$$

$$= 49.7 \text{ in.-kips}$$

The wall is adequate with the No. 4 @ 12 in. since the design strength is greater than the required nominal strength for all load combinations.

Example 21.2—Design of Bearing Wall by Empirical Design Method (14.5)

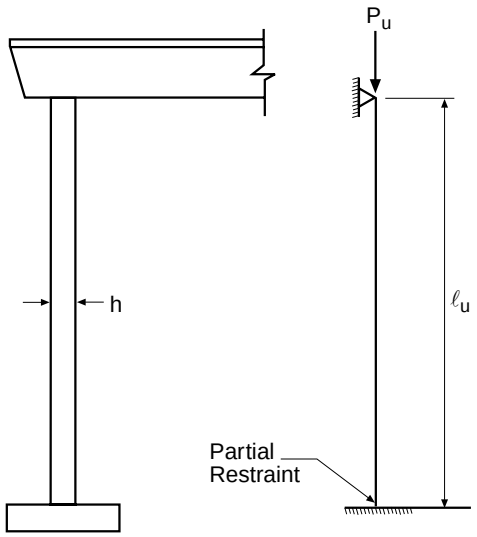
A concrete bearing wall supports a floor system of precast single tees spaced at 8 ft on centers. The stem of each tee section is 8 in. wide. The tees have full bearing on the wall. The height of the wall is 15 ft, and the wall is considered laterally restrained at the top.

Design Data:

Floor beam reactions: dead load = 28 kips
live load = 14 kips

f'_c = 4,000 psi
f_y = 60,000 psi

Neglect weight of wall



Calculations and Discussion	Code Reference

The general design procedure is to select a trial wall thickness h, then check the trial wall for the applied loading conditions.

1. Select trial wall thickness h

   $h \geq \frac{\ell_u}{25}$ but not less than 4 in. *14.5.3.1*

   $\geq \frac{15 \times 12}{25} = 7.2$ in.

   Try h = 7.5 in.

2. Calculate factored loading

   $P_u = 1.2D + 1.6L$ *Eq. (9-2)*

   $= 1.2\ (28) + 1.6\ (14) = 33.6 + 22.4 = 56.0$ kips

3. Check bearing strength of concrete

   Assume width of stem for bearing equal to 7 in., to allow for beveled bottom edges.

   Loaded area $A_1 = 7 \times 7.5 = 52.5$ in.^{2}

   Bearing capacity $= \phi(0.85f'_cA_1) = 0.65\ (0.85 \times 4 \times 52.5) = 116$ kips > 56.0 kips O.K. *10.17.1*

4. Calculate design strength of wall

Effective horizontal length of wall per tee reaction = $\begin{cases} 8 \times 12 = 96 \text{ in.} \\ 7 + 4(7.5) = 37 \text{ in.} \quad \text{(governs)} \end{cases}$ *14.2.4*

k = 0.8 *14.5.2*

$$\phi P_{nw} = 0.55\phi f'_c A_g \left[1 - \left(\frac{k\ell_c}{32h}\right)^2\right] \qquad \textit{Eq. (14-1)}$$

$$= 0.55 \times 0.70 \times 4\,(37 \times 7.5)\left[1 - \left(\frac{0.8 \times 15 \times 12}{32 \times 7.5}\right)^2\right]$$

= 273 kips > 56 kips O.K.

The 7.5-in. wall is adequate, with sufficient margin for possible effect of load eccentricity.

5. Determine single layer of reinforcement

Based on 1-ft width of wall and Grade 60 reinforcement (No. 5 and smaller):

Vertical $A_s = 0.0012 \times 12 \times 7.5 = 0.108$ in.^{2}/ft *14.3.2*

Horizontal $A_s = 0.0020 \times 12 \times 7.5 = 0.180$ in.^{2}/ft *14.3.3*

Spacing = $\begin{cases} 3h = 3 \times 7.5 = 22.5 \text{ in.} \\ 18 \text{ in.} \quad \text{(governs)} \end{cases}$ *14.3.5*

Vertical A_s: use No. 4 @ 18 in. on center (A_s = 0.13 in.^{2}/ft)

Horizontal A_s: use No. 4 @ 12 in. on center (A_s = 0.20 in.^{2}/ft)

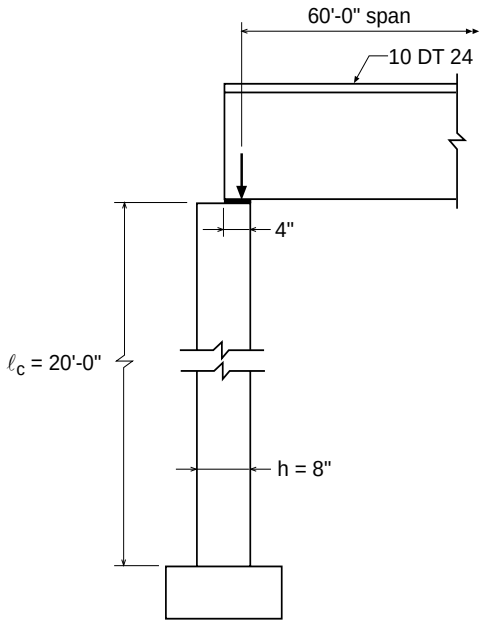
Design aids such as the one in Fig. 21-8 may be used to select reinforcement directly.

Example 21.3—Design of Precast Panel by the Alternate Design Method (14.8)

Determine the required vertical reinforcement for the precast wall panel shown below. The roof loads are supported through the 3.75 in. webs of the 10DT24 which are spaced 5 ft on center.

Design data:

Weight of 10DT24 = 468 plf
Roof dead load = 20 psf
Roof live load = 30 psf
Wind load = 30 psf
Concrete f'_c = 4,000 psi (w_c = 150 pcf)
Reinforcing steel f_y = 60,000 psi



Calculations and Discussion	Code Reference

1. Trial wall section

Try h = 8 in.
Try a single layer of No. 4 @ 9 in. vertical reinforcement (A_s = 0.27 in.^{2}/ft) at centerline of wall.

For a 1-ft wide design strip: ρ (gross) = $\frac{A_s}{\ell_w h} = \frac{0.27}{12 \times 8} = 0.0028 > 0.0012$ O.K. *14.3.2*

Check maximum reinforcement ratio: *14.8.2.3*

$$\rho_b = \frac{0.85\beta_1 f'_c}{f_y}\left(\frac{87{,}000}{87{,}000 + f_y}\right)$$ *Eq. (B-1)*

$$= \frac{0.85 \times 0.85 \times 4{,}000}{60{,}000} \times \frac{87{,}000}{87{,}000 + 60{,}000} = 0.0285$$

$0.6\rho_b = 0.6 \times 0.0285 = 0.0171 > \rho = \frac{A_s}{bd} = \frac{0.27}{12 \times 4} = 0.0056$ O.K. *14.8.2.3*

2. Distribution width of interior concentrated loads at mid-height of wall (see Fig. 21-4)

$$W + \frac{\ell_c}{2} = \frac{3.75}{12} + \frac{20}{2} = 10.3 \text{ ft}$$ *14.8.2.5*

S = 5.0 ft (governs)

3. Roof loading per foot width of wall

$$\text{Dead load} = \left[\frac{468}{2} + (20 \times 5)\right]\left(\frac{60}{2}\right) = 10{,}020 \text{ lbs/5 ft} = 2{,}004 \text{ plf}$$

$$\text{Live load} = (30 \times 5)\left(\frac{60}{2}\right) = 4{,}500 \text{ lbs/5 ft} = 900 \text{ plf}$$

$$\text{Wall dead load} = \frac{8}{12} \times 20 \times 150 = 2{,}000 \text{ plf}$$

$$\text{Eccentricity of the roof loads about the panel center line} = \frac{2}{3} \times 4 = 2.7 \text{ in.}$$

4. Factored load combinations at mid-height of wall (see Fig. 21-5)

a. Load comb. 1: $U = 1.2D + 0.5L_r$ *Eq. (9-2)*

$$P_u = P_{u1} + \frac{P_{u2}}{2}$$

$$P_{u1} = (1.2 \times 2.0) + (0.5 \times 0.9) = 2.4 + 0.5 = 2.9 \text{ kips}$$

$$P_{u2} = 1.2 \times 2.0 = 2.4 \text{ kips}$$

$$P_u = 2.9 + \frac{2.4}{2} = 4.1 \text{ kips}$$

$$M_u = \frac{M_{ua}}{1 - \dfrac{5P_u\ell_c^2}{(0.75)\ 48E_cI_{cr}}}$$ *Eq. (14-6)*

$$M_{ua} = \frac{w_u\ell_c^2}{8} + \frac{P_{u1}e}{2} = 0 + \frac{2.9 \times 2.7}{2} = 3.9 \text{ in.-kips}$$

$$E_c = 57{,}000\sqrt{4{,}000} = 3{,}605{,}000 \text{ psi}$$ *8.5.1*

$$I_{cr} = nA_{se}(d - c)^2 + \frac{\ell_w c^3}{3}$$ *Eq. (14-7)*

$$n = \frac{E_s}{E_c} = \frac{29,000}{3,605} = 8.0$$

$$A_{se} = \frac{P_u + A_s f_y}{f_y} = \frac{4.1 + \left(0.27 \times 60\right)}{60} = 0.34 \text{ in.}^2/\text{ft}$$ *Eq. (14-8)*

$$a = \frac{A_{se} f_y}{0.85 f'_c \ell_w} = \frac{0.34 \times 60}{0.85 \times 4 \times 12} = 0.50 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.50}{0.85} = 0.59 \text{ in.}$$

Therefore,

$$I_{cr} = 8.0 \times 0.34 \times (4 - 0.59)^2 + \frac{12 \times 0.59^3}{3} = 32.5 \text{ in.}^4$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right) d_t - 0.003$$

$$= \left(\frac{0.003}{0.59}\right)(4) - 0.003 = 0.0173 > 0.005$$

Therefore, section is tension-controlled *10.3.4*

$\phi = 0.9$ *9.3.2*

$$M_u = \frac{3.9}{1 - \frac{5 \times 4.1 \times \left(20 \times 12\right)^2}{0.75 \times 48 \times 3,605 \times 32.5}} = 5.4 \text{ in.-kips}$$ *Eq. (14-6)*

b. Load comb. 2: $U = 1.2D + 1.6L_r + 0.8W$ *Eq. (9-3)*

$P_{u1} = (1.2 \times 2) + (1.6 \times 0.9) = 3.8$ kips

$P_{u2} = 1.2 \times 2.0 = 2.4$ kips

$P_u = 3.8 + \frac{2.4}{2} = 5.0$ kips

$$M_{ua} = \frac{w_u \ell_c^2}{8} + \frac{P_{u1}e}{2} = \frac{0.8 \times 0.030 \times 20^2}{8} + \frac{3.8 \times (2.7/12)}{2}$$

$$= 1.2 + 0.4 = 1.6 \text{ ft-kips} = 19.2 \text{ in.-kips}$$

$$A_{se} = \frac{5.0 + (0.27 \times 60)}{60} = 0.35 \text{ in.}^2/\text{ft}$$ *Eq. (14-8)*

$$a = \frac{0.35 \times 60}{0.85 \times 4 \times 12} = 0.51 \text{ in.}$$

$$c = \frac{0.51}{0.85} = 0.60 \text{ in.}$$

Therefore,

$$I_{cr} = 8.0 \times 0.35 \times (4 - 0.60)^2 + \frac{12 \times 0.60^3}{3} = 33.2 \text{ in.}^4$$ *Eq. (14-7)*

$$\varepsilon_t = \left(\frac{0.003}{0.60}\right)(4) - 0.003 = 0.0170 > 0.005$$

$$\phi = 0.9$$ *9.3.2*

$$M_u = \frac{19.2}{1 - \frac{5 \times 5.0 \times (20 \times 12)^2}{0.75 \times 48 \times 3{,}605 \times 33.2}} = 28.8 \text{ in.-kips}$$ *Eq. (14-6)*

c. Load comb. 3: $U = 1.2D + 1.6W + 0.5L_r$ *Eq. (9-4)*

$P_{u1} = (1.2 \times 2.0) + (0.5 \times 0.9) = 2.9$ kips

$P_{u2} = 1.2 \times 2.0 = 2.4$ kips

$P_u = 2.9 + \frac{2.4}{2} = 4.1$ kips

$$M_{ua} = \frac{1.6 \times 0.03 \times 20^2}{8} + \frac{2.9 \times (2.7/12)}{2}$$

$$= 2.4 + 0.3 = 2.7 \text{ ft-kips} = 32.4 \text{ in.-kips}$$

$$A_{se} = \frac{4.1 + (0.27 \times 60)}{60} = 0.34 \text{ in.}^2/\text{ft}$$ *Eq. (14-8)*

$$a = \frac{0.34 \times 60}{0.85 \times 4 \times 12} = 0.5 \text{ in.}$$

$$c = \frac{0.5}{0.85} = 0.59 \text{ in.}$$

Therefore,

$$I_{cr} = 8 \times 0.34 \times (4 - 0.59)^2 + \frac{12 \times 0.59^3}{3} = 32.5 \text{ in.}^4$$

$\phi = 0.9$ as in load combination 1

$$M_u = \frac{32.4}{1 - \frac{5 \times 4.1 \times (20 \times 12)^2}{0.75 \times 48 \times 3{,}605 \times 32.5}} = 45.0 \text{ in.-kips}$$

d. Load comb. 4: U = 0.9D + 1.6W *Eq. (9-6)*

$P_{u1} = 0.9 \times 2.0 = 1.8$ kips

$P_{u2} = 0.9 \times 2.0 = 1.8$ kips

$$P_u = 1.8 + \frac{1.8}{2} = 2.7 \text{ kips}$$

$$M_{ua} = \frac{1.6 \times 0.030 \times 20^2}{8} + \frac{1.8 \times (2.7 / 12)}{2} = 2.6 \text{ ft-kips} = 31.2 \text{ in.-kips}$$

$$A_{se} = \frac{2.7 + (0.27 \times 60)}{60} = 0.32 \text{ in.}^2\text{/ft}$$ *Eq. (14-8)*

$$a = \frac{0.32 \times 60}{0.85 \times 4 \times 12} = 0.47 \text{ in.}$$

$$c = \frac{0.47}{0.85} = 0.55 \text{ in.}$$

Therefore,

$$I_{cr} = 8.0 \times 0.32 \times (4 - 0.55)^2 + \frac{12 \times 0.55^3}{3} = 31.1 \text{ in.}^4$$ *Eq. (14-7)*

$$\varepsilon_t = \left(\frac{0.003}{c}\right) d_t - 0.003 = \left(\frac{0.003}{0.55}\right)(4) - 0.003 = 0.0188 > 0.005$$

$\phi = 0.9$ *9.3.2*

$$M_u = \frac{31.2}{1 - \frac{5 \times 2.7 \times (20 \times 12)^2}{0.75 \times 48 \times 3{,}605 \times 31.1}} = 38.7 \text{ in.-kips}$$ *Eq. (14-6)*

5. Determine M_{cr}

$$I_g = \frac{1}{12} \ell_w b^3 = \frac{1}{12} \times 12 \times 8^3 = 512 \text{ in.}^4$$

$$y_t = \frac{8}{2} = 4 \text{ in.}$$

$$f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{4{,}000} = 474.3 \text{ psi}$$ *Eq. (9-9)*

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{474.3 \times 512}{4 \times 1{,}000} = 60.7 \text{ in.-kips}$$

6. Check design moment strength ϕM_n

a. Load comb. 1

$$M_n = A_{se} f_y (d - \frac{a}{2}) = 0.34 \times 60 \times \left(4 - \frac{0.5}{2}\right) = 76.5 \text{ in.-kips}$$

$\phi M_n = 0.9 \times 76.5 = 68.9$ in.-kips $>$ $M_u = 5.4$ in.-kips O.K. *14.8.3*
$>$ $M_{cr} = 60.7$ in.-kips O.K. *14.8.2.4*

b. Load comb. 2

$$M_n = 0.35 \times 60 \times \left(4 - \frac{0.51}{2}\right) = 78.7 \text{ in.-kips}$$

$\phi M_n = 0.9 \times 78.7 = 70.8$ in.-kips $>$ $M_u = 28.8$ in.-kips O.K. *14.8.3*
$>$ $M_{cr} = 60.7$ in.-kips O.K. *14.8.2.4*

c. Load comb. 3

$$M_n = 0.34 \times 60 \times \left(4 - \frac{0.5}{2}\right) = 76.5 \text{ in.-kips}$$

$\phi M_n = 0.9 \times 76.5 = 68.9$ in.-kips $>$ $M_u = 45.0$ in.-kips O.K.
$>$ $M_{cr} = 60.7$ in.-kips O.K.

d. Load comb. 4

$$M_n = 0.32 \times 60 \times \left(4 - \frac{0.47}{2}\right) = 72.3 \text{ in.-kips}$$

$\phi M_n = 0.9 \times 72.3 = 65.1$ in.-kips $> M_u = 38.7$ in.-kips O.K. *14.8.3*
$> M_{cr} = 60.7$ in.-kips O.K. *14.8.2.4*

7. Check vertical stress at mid-height section

Load comb. 2 governs:

$$\frac{P_u}{A_g} = \frac{5{,}000}{8 \times 12} = 52.1 \text{ psi} < 0.06 f'_c = 0.06 \times 4{,}000 = 240 \text{ psi}$$ O.K. *14.8.2.6*

8. Check mid-height deflection Δ_s

$$\Delta_s = \frac{5M\ell_c^2}{48E_cI_e}$$ *Eq. (14-9)*

$$M = \frac{M_{sa}}{1 - \frac{5P_s\ell_c^2}{48E_cI_e}}$$ *Eq. (14-10)*

Using Δ_s from Eq. (14-9), Eq. (14-10) can be rewritten as follows:

$M = M_{sa} + P_s\Delta_s$

$$M_{sa} = \frac{w\ell_c^2}{8} + \frac{P_{s1}e}{2} = \frac{0.030 \times 20^2}{8} + \frac{(2.0 + 0.9)(2.7/12)}{2} = 1.8 \text{ ft-kips} = 21.6 \text{ in.-kips}$$

$$P_s = P_{s1} + \frac{P_{s2}}{2} = (2.0 + 0.9) + \frac{2.0}{2} = 3.9 \text{ kips}$$

$$I_e = \left(\frac{M_{cr}}{M}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M}\right)^3\right] I_{cr}$$ *Eq. (9-8)*

Since I_e is a function of M, no closed form solution for Δ_s is possible. Determine Δ_s by iterative procedure.

$$\text{Assume } \Delta_s = \frac{\ell_c}{150} = \frac{20 \times 12}{150} = 1.6 \text{ in.}$$

$M = 21.6 + (3.9 \times 1.6) = 27.8$ in.-kips

Since $M_{cr} = 60.7$ in.-kips $> M = 27.8$ in.-kips, $I_e = I_g = 512$ in.^{4}

$$M = \frac{21.6}{1 - \frac{5 \times 3.9 \times (20 \times 12)^2}{48 \times 3{,}605 \times 512}} = 21.9 \text{ in.-kips}$$ *Eq. (14-10)*

$$\Delta_s = \frac{5 \times 21.9 \times (20 \times 12)^2}{48 \times 3{,}605 \times 512} = 0.07 \text{ in.}$$ *Eq. (14-9)*

No further iterations are required since $I_e = I_g$.

Therefore,

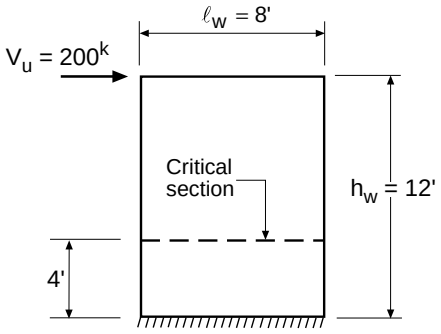
$$\Delta_s = 0.07 \text{ in.} < \frac{\ell_c}{150} = \frac{20 \times 12}{150} = 1.6 \text{ in.} \quad \text{O.K.}$$

The wall is adequate with No. 4 @ 9 in. vertical reinforcement.

Example 21.4—Shear Design of Wall

Determine the shear and flexural reinforcement for the wall shown.

h = 8 in.
f'_c = 3,000 psi
f_y = 60,000 psi



Calculations and Discussion	Code Reference

1. Check maximum shear strength permitted

   $\phi V_n = \phi 10\sqrt{f'_c}hd$ *11.10.3*

   where $d = 0.8\ell_w = 0.8 \times 8 \times 12 = 76.8$ in. *11.10.4*

   $\phi V_n = 0.75 \times 10\sqrt{3{,}000} \times 8 \times 76.8/1{,}000 = 252.4$ kips > V_u = 200 kips O.K.

2. Calculate shear strength provided by concrete V_c

   Critical section for shear: *11.10.7*

   $\frac{\ell_w}{2} = \frac{8}{2} = 4$ ft (governs)

   or

   $\frac{h_w}{2} = \frac{12}{2} = 6$ ft

   $V_c = 3.3\sqrt{f'_c}hd + \frac{N_u d}{4\ell_w}$ *Eq. (11-29)*

   $= 3.3\sqrt{3{,}000} \times 8 \times 76.8/1{,}000 + 0 = 111$ kips

   or

$$V_c = \left[0.6\sqrt{f'_c} + \frac{\ell_w \left(1.25\sqrt{f'_c} + \frac{0.2N_u}{\ell_w h} \right)}{\frac{M_u}{V_u} - \frac{\ell_w}{2}} \right] hd$$ *Eq. (11-30)*

$$= \left[0.6\sqrt{3,000} + \frac{96\left(1.25\sqrt{3,000} + 0\right)}{96 - 48} \right] \left(\frac{8 \times 76.8}{1,000} \right) = 104 \text{ kips (governs)}$$

where $M_u = (12 - 4)\, V_u = 8V_u$ ft-kips $= 96V_u$ in.-kips

3. Determine required horizontal shear reinforcement

$V_u = 200$ kips $> \phi V_c/2 = 0.75\,(104)/2 = 39.0$ kips *11.10.8*

Shear reinforcement must be provided in accordance with 11.10.9.

$V_u \le \phi V_n$ *Eq. (11-1)*

$\le \phi (V_c + V_s)$ *Eq. (11-2)*

$$\le \phi V_c + \frac{\phi A_v f_y d}{s_2}$$ *Eq. (11-31)*

$$\frac{A_v}{s_2} = \frac{(V_u - \phi V_c)}{\phi f_y d}$$

$$= \frac{[200 - (0.75 \times 104)]}{0.75 \times 60 \times 76.8} = 0.0353$$

For 2-No. 3: $s_2 = \dfrac{2 \times 0.11}{0.0353} = 6.2$ in.

2-No. 4: $s_2 = \dfrac{2 \times 0.20}{0.0353} = 11.3$ in.

2-No. 5: $s_2 = \dfrac{2 \times 0.31}{0.0353} = 17.6$ in.

Try 2-No. 4 @ 10 in.

$$\rho_h = \frac{A_v}{A_g} = \frac{2 \times 0.20}{8 \times 10} = 0.0050 > 0.0025 \quad \text{O.K.}$$ *11.10.9.2*

Maximum spacing = $\begin{cases} \frac{\ell_w}{5} = \frac{8 \times 12}{5} = 19.2 \text{ in.} \\ 3h = 3 \times 8 = 24.0 \text{ in.} \\ 18.0 \text{ in. (governs)} \end{cases}$ *11.10.9.3*

Use 2-No. 4 @ 10 in.

4. Determine vertical shear reinforcement

$$\rho_n = 0.0025 + 0.5\left(2.5 - \frac{h_w}{\ell_w}\right)(\rho_h - 0.0025) \geq 0.0025$$ *Eq. (11-32)*

$$= 0.0025 + 0.5\,(2.5 - 1.5)\,(0.0050 - 0.0025)$$

$$= 0.0038$$

Maximum spacing = $\begin{cases} \frac{\ell_w}{3} = \frac{8 \times 12}{3} = 32 \text{ in.} \\ 3h = 3 \times 8 = 24.0 \text{ in.} \\ 18.0 \text{ in. (governs)} \end{cases}$ *11.10.9.5*

Use 2-No. 4 @ 13 in. ($\rho_n = 0.0038$)

5. Design for flexure

$M_u = V_u h_w = 200 \times 12 = 2{,}400$ ft-kips

Assume tension-controlled section ($\phi = 0.90$) *9.3.2*
with $d = 0.8\ell_w = 0.8 \times 96 = 76.8$ in. *11.10.4*
(Note: an exact value of d will be determined by a strain compatibility analysis below)

$$R_n = \frac{M_u}{\phi b d^2} = \frac{2{,}400 \times 12{,}000}{0.9 \times 8 \times 76.8^2} = 678 \text{ psi}$$

$$\rho = \frac{0.85 f'_c}{fy}\left(1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right)$$

$$= \frac{0.85 \times 3}{60}\left(1 - \sqrt{1 - \frac{2 \times 678}{0.85 \times 3{,}000}}\right) = 0.0134$$

$A_s = \rho b d = 0.0134 \times 8 \times 76.8 = 8.24 \text{ in.}^2$

Try 9-No. 8 (A_s = 7.11 in.^{2}) at each end of wall, which provides less area of steel than that determined based on d = $0.8\ell_w$.

Check moment strength of wall with 9-No. 8 bars using a strain compatibility analysis (see figure below for reinforcement layout).

From strain compatibility analysis (including No. 4 vertical bars):

c = 13.1 in.

d = 81.0 in.

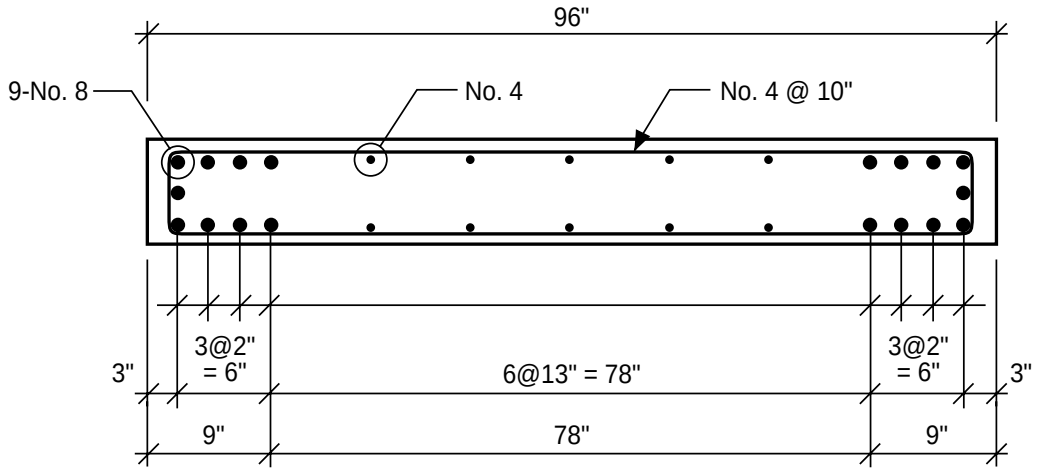
$\varepsilon_t = 0.0182 > 0.0050$

Therefore, section is tension-controlled as assumed and $\phi = 0.90$.

M_n = 3,451 ft-kips

$\phi M_n = 0.9 \times 3{,}451 = 3{,}106$ ft-kips > 2,400 ft-kips O.K.

Use 9-No. 8 bars each side (A_s = 7.11 in.^{2})



22

Footings

UPDATE FOR THE '02 CODE

Revised requirements for shear reinforcement in footings are contained in 11.12.3 to improve safety against punching shear failure.

GENERAL CONSIDERATIONS

Provisions of Chapter 15 apply primarily for design of footings supporting a single column (isolated footings) and do not provide specific design provisions for footings supporting more than one column (combined footings). The code states that combined footings shall be proportioned to resist the factored loads and induced reactions in accordance with the appropriate design requirements of the code. Detailed discussion of combined footing design is beyond the scope of Part 22. However, as a general design approach, combined footings may be designed as beams in the longitudinal direction and as an isolated footing in the transverse direction over a defined width on each side of the supported columns. Code references 15.1 and 15.2 are suggested for detailed design recommendations for combined footings.

15.2 LOADS AND REACTIONS

Footings must be designed to safely resist the effects of the applied factored axial loads, shears and moments. The size (base area) of a footing or the arrangement and number of piles is determined based on the permissible soil pressure or permissible pile capacity, respectively. The permissible soil or pile capacity is determined by principles of soil mechanics in accordance with general building codes. The following procedure is specified for footing design:

1. The footing size (plan dimensions) or the number and arrangement of piles is to be determined on the basis of unfactored (service) loads (dead, live, wind, earthquake, etc.) and the permissible soil pressure or pile capacity (15.2.2).

2. After having established the plan dimensions, the depth of the footing and the required amount of reinforcement are determined based on the appropriate design requirements of the code (15.2.1). The service pressures and the resulting shear and moments are multiplied by the appropriate load factors specified in 9.2 and are used to proportion the footing.

For purposes of analysis, an isolated footing may be assumed to be rigid, resulting in a uniform soil pressure for concentric loading, and a triangular or trapezoidal soil pressure distribution for eccentric loading (combined axial and bending effect). Only the computed bending moment that exists at the base of the column or pedestal is to be transferred to the footing. The minimum moment requirement for slenderness considerations in 10.12.3.2 need not be transferred to the footing (R15.2).

15.4 MOMENT IN FOOTINGS

At any section of a footing, the external moment due to the base pressure shall be determined by passing a vertical plane through the footing and computing the moment of the forces acting over the entire area of the footing on one side of the vertical plane. The maximum factored moment in an isolated footing is determined by passing a vertical plane through the footing at the critical sections shown in Fig. 22-1 (15.4.2). This moment is subsequently used to determine the required area of flexural reinforcement in that direction.

In one-way square or rectangular footings and two-way square footings, flexural reinforcement shall be distributed uniformly across the entire width of the footing (15.4.3). For two-way rectangular footings, the reinforcement must be distributed as shown in Table 22-1 (15.4.4).

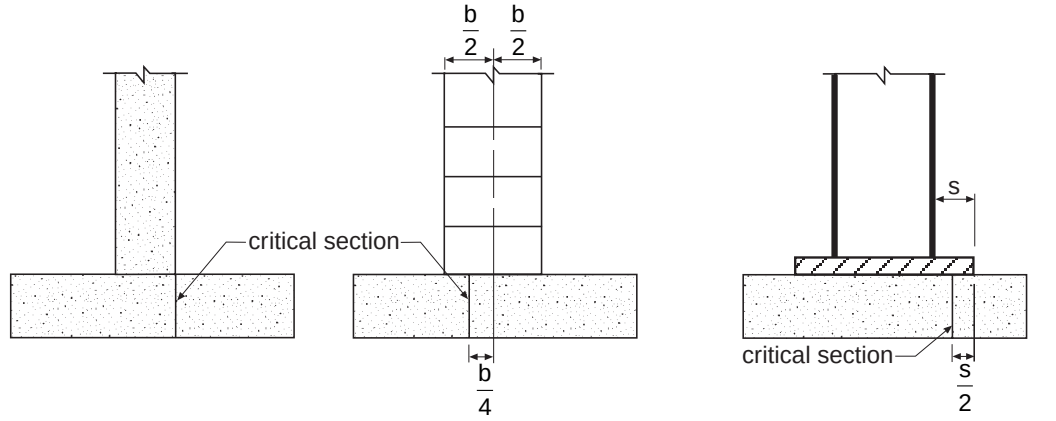


Figure 22-1 Critical Location for Maximum Factored Moment in an Isolated Footing (15.4.2)

Table 22-1 Distribution of Flexural Reinforcement

Footing Type	Square Footing	Rectangular Footing
One-way	s (typ.), L, L *(15.4.3)*	s (typ.), B, L *(15.4.3)*
Two-way	s (typ.), L, L *(15.4.3)*	A_{sL}, A_{s2}, A_{s1}, A_{s2}, $@s_2$, $@s_1$, $@s_2$, A_{sB} $@s_3$, B, B/2, B/2, L; $A_{s1} = \left(\frac{2}{\beta+1}\right) A_{sL}$; $A_{s2} = \frac{A_{sL} - A_{s1}}{2}$; $\beta = \frac{L}{B}$ *(15.4.4)*

15.5 SHEAR IN FOOTINGS

Shear strength of a footing supported on soil or rock in the vicinity of the supported member (column or wall) must be determined for the more severe of the two conditions stated in 11.12. Both wide-beam action (11.12.1.1) and two-way action (11.12.1.2) must be checked to determine the required footing depth. Beam action assumes that the footing acts as a wide beam with a critical section across its entire width. If this condition is the more severe, design for shear proceeds in accordance with 11.1 through 11.5. Even though wide-beam action rarely controls the shear strength of footings, the designer must ensure that shear strength for beam action is not exceeded. Two-way action for the footing checks "punching" shear strength. The critical section for punching shear is a perimeter b_o around the supported member with the shear strength computed in accordance with 11.12.2.1. Tributary areas and corresponding critical sections for wide-beam action and two-way action for an isolated footing are illustrated in Fig. 22-2. Note that it is permissible to use a critical section with four straight sides for square or rectangular columns (11.12.1.3).

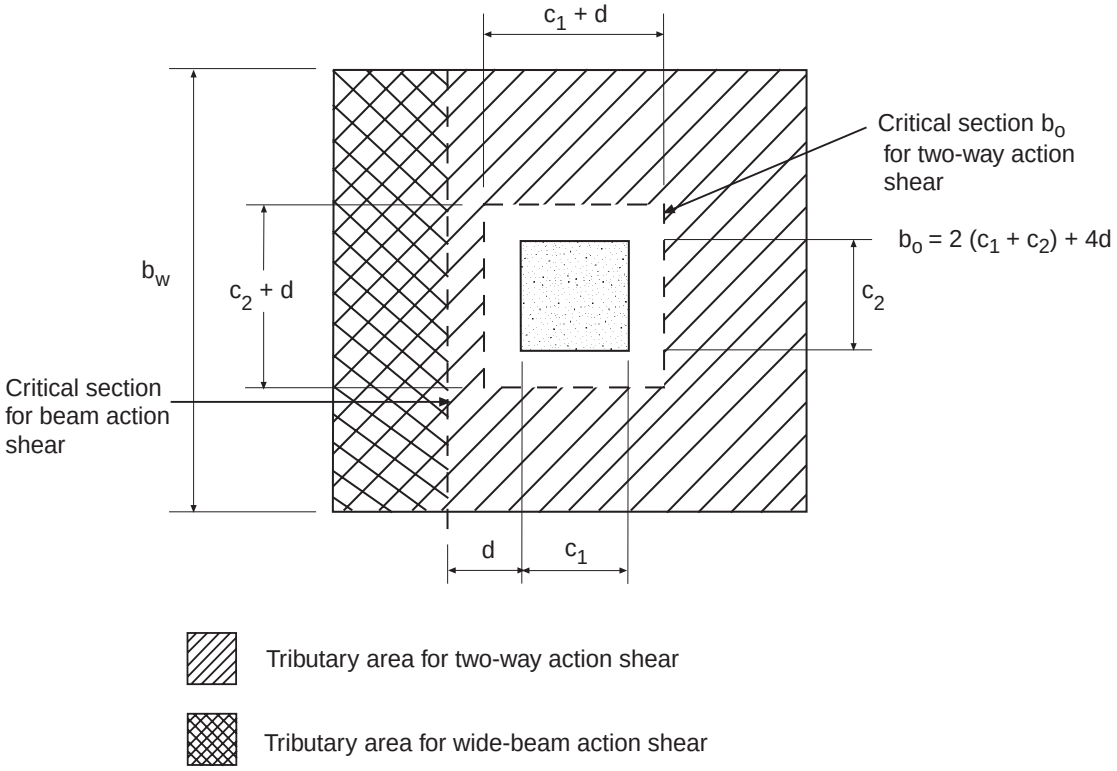


Figure 22-2 Tributary Areas and Critical Sections for Shear

In the design of a footing for two-way action, V_c is the smallest value obtained from Eqs. (11-33), (11-34), and (11-35). Eq. (11-35) established the upper limit of V_c at $4\sqrt{f'_c}\, b_o d$. Eq. (11-33) accounts for the effect of β_c, which is the ratio of the long side to the short side of the column, concentrated load, or reaction area. As β_c increases the concrete shear strength decreases (see Fig. 22-3). Eq. (11-34) was developed to account for the effect of b_o/d, and is based on tests that indicated shear strength decreases as b_o/d increases.

If the factored shear force V_u at the critical section exceeds the governing shear strength ϕV_c given by the minimum of Eqs. (11-33), (11-34), or (11-35), shear reinforcement must be provided. For shear reinforcement consisting of bars or wires and single- or multiple-leg stirrups, the shear strength may be increased to a maximum value of $6\sqrt{f'_c}\, b_o d$ (11.12.3.2), provided the footing has an effective depth d greater than or equal to 6 in., but not less than 16 times the shear reinforcement bar diameter (11.12.3). However, shear reinforcement must be designed to carry the shear in excess of $2\sqrt{f'_c}\, b_o d$ (11.12.3.1).

For footing design (without shear reinforcement), the shear strength equations may be summarized as follows:

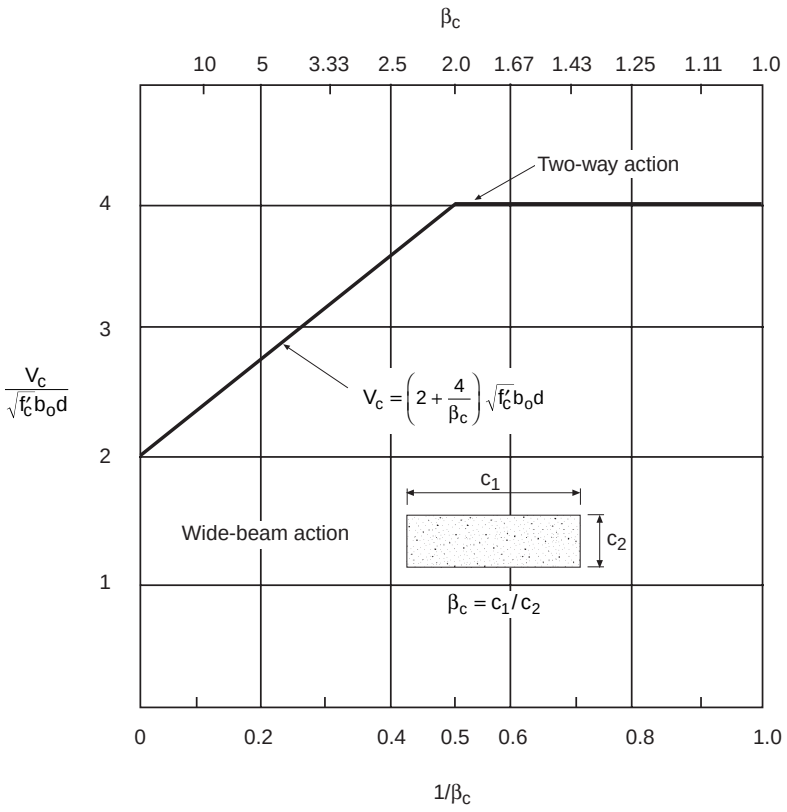


Figure 22-3 Shear Strength of Concrete in Footings

- Wide beam action

$$V_u \le \phi V_n \qquad \textit{Eq. (11-1)}$$

$$\le \phi\left(2\sqrt{f'_c}\, b_w d\right) \qquad \textit{Eq. (11-3)}$$

where b_w and V_u are computed for the critical section defined in 11.12.1.1 (see Fig. 22-2).

- Two-way action

$$V_u \le \text{minimum of} \begin{cases} \left(2 + \dfrac{4}{\beta_c}\right)\sqrt{f'_c}\, b_o d & \textit{Eq. (11-33)} \\ \left(\dfrac{\alpha_s d}{b_o} + 2\right)\sqrt{f'_c}\, b_o d & \textit{Eq. (11-34)} \\ 4\sqrt{f'_c}\, b_o d & \textit{Eq. (11-35)} \end{cases}$$

where

β_c = ratio of long side to short side of the column, concentrated load or reaction area

α_s = 40 for interior columns
= 30 for edge columns
= 20 for corner columns

b_o = perimeter of critical section shown in Fig. 22-2

15.8 TRANSFER OF FORCE AT BASE OF COLUMN, WALL, OR REINFORCED PEDESTAL

With the publication of ACI 318-83, 15.8 addressing transfer of force between a footing and supported member (column, wall, or pedestal) was revised to address both cast-in-place and precast construction. Section 15.8.1 gives general requirements applicable to both cast-in-place and precast construction. Sections 15.8.2 and 15.8.3 give additional rules for cast-in-place and precast construction, respectively. For force transfer between a footing and a precast column or wall, anchor bolts or mechanical connectors are specifically permitted by 15.8.3, with anchor bolts to be designed in accordance with Appendix D. (Prior to the '83 code, connection between a precast member and footing required either longitudinal bars or dowels crossing the interface, contrary to common practice.) Also note that walls are specifically addressed in 15.8 for force transfer to footings.

Section 15.8.3 contains requirements for the connection between precast columns and walls to supporting members. This section refers to 16.5.1.3 for minimum connection strength. Additionally, for precast columns with larger cross-sectional areas than required for loading, it is permitted to use a reduced effective area based on the cross-section required, but not less than one-half the total area when determining the nominal strength in tension.

The minimum tensile strength of a connection between a precast wall panel and its supporting member is required to have a minimum of two ties per panel with a minimum nominal tensile capacity of 10 kips per tie (16.5.1.3(b)).

All forces applied at the base of a column or wall (supported member) must be transferred to the footing (supporting member) by bearing on concrete and/or by reinforcement. Tensile forces must be resisted entirely by reinforcement. Bearing on concrete for both supported and supporting member must not exceed the concrete bearing strength permitted by 10.17 (see discussion on 10.17 in Part 6).

For a supported column, the bearing capacity ϕP_{nb} is

$$\phi P_{nb} = \phi\left(0.85 f'_c A_1\right) \qquad \textit{10.17.1}$$

where

f'_c = compressive strength of the column concrete

A_1 = loaded area (column area)

ϕ = 0.65 *9.3.2.4*

For a supporting footing,

$$\phi P_{nb} = \phi\left(0.85 f'_c A_1\right)\sqrt{\frac{A_2}{A_1}} \le 2\phi\left(0.85 f'_c A_1\right)$$

where

f'_c = compressive strength of the footing concrete

A_2 = area of the lower base of the largest frustrum of a pyramid, cone, or tapered wedge contained wholly within the footing and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal (see Fig. R10.17).

Example 22.4 illustrates the design for force transfer at the base of a column.

When bearing strength is exceeded, reinforcement must be provided to transfer the excess load. A minimum area of reinforcement must be provided across the interface of column or wall and footing, even where concrete

bearing strength is not exceeded. With the force transfer provisions addressing both cast-in-place and precast construction, including force transfer between a wall and footing, the minimum reinforcement requirements are based on the type of supported member, as shown in Table 22-2.

Table 22-2 Minimum Reinforcement for Force Transfer Between Footing and Supported Member

	Cast-in-Place	Precast
Columns	$0.005A_g$ (15.8.2.1)	$\frac{200A_g}{f_y}$ (16.5.1.3 (a))
Walls	see 14.3.2 (15.8.2.2)	see 16.5.1.3(b) and (c)

For cast-in-place construction, reinforcement may consist of extended reinforcing bars or dowels. For precast construction, reinforcement may consist of anchor bolts or mechanical connectors. Reference 22.1 devotes an entire chapter on connection design for precast construction.

The shear-friction design method of 11.7.4 should be used for horizontal force transfer between columns and footings (15.8.1.4; see Example 22.6). Consideration of some of the lateral force being transferred by shear through a formed shear key is questionable. Considerable slip is required to develop a shear key. Shear keys, if provided, should be considered as an added mechanical factor of safety only, with no design shear force assigned to the shear key.

PLAIN CONCRETE PEDESTALS AND FOOTINGS

Plain concrete pedestals and footings are designed in accordance with Chapter 22. See Part 30 for an in-depth discussion and examples.

REFERENCE

22.1 *PCI Design Handbook—Precast and Prestressed Concrete*, 5th Edition, Precast/Prestressed Concrete Institute, Chicago, IL, 1999.

Example 22.1—Design for Base Area of Footing

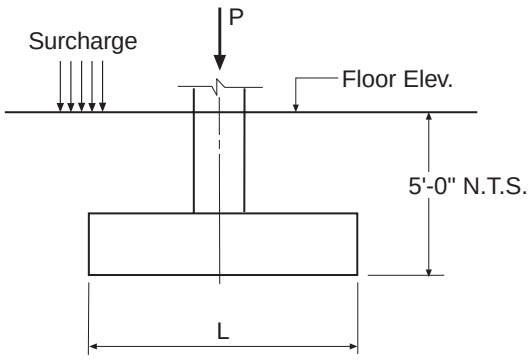
Determine the base area A_f required for a square spread footing with the following design conditions:

Service dead load = 350 kips
Service live load = 275 kips
Service surcharge = 100 psf

Assume average weight of soil and concrete above footing base = 130 pcf

Permissible soil pressure = 4.5 ksf

Column dimensions = 30 × 12 in.



Calculations and Discussion	Code Reference

1. Determination of base area:

   The base area of the footing is determined using service (unfactored) loads with the net permissible soil pressure.

   Total weight of surcharge = (0.130 × 5) + 0.100 = 0.750 ksf

   Net permissible soil pressure = 4.5 - 0.75 = 3.75 ksf

   Required base area of footing: *15.2.2*

   $$A_f = \frac{350 + 275}{3.75} = 167 \text{ ft}^2$$

   Use a 13 × 13 ft square footing (A_f = 169 ft^2)

2. Factored loads and soil reaction:

   To proportion the footing for strength (depth and required reinforcement) factored loads are used. *15.2.1*

   $P_u = 1.2\,(350) + 1.6\,(275) = 860$ kips *Eq. (9-2)*

   $$q_s = \frac{P_u}{A_f} = \frac{860}{169} = 5.10 \text{ ksf}$$

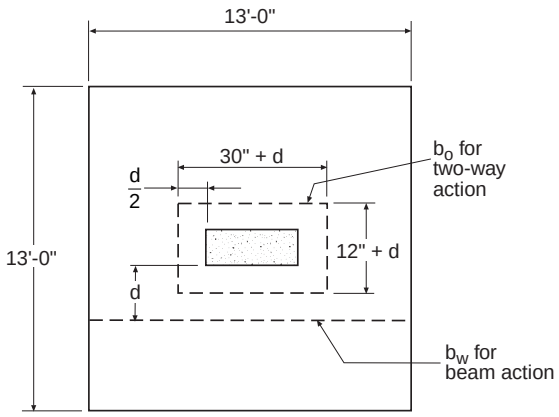
Example 22.2—Design for Depth of Footing

For the design conditions of Example 22.1, determine the overall thickness of footing required.

f'_c = 3000 psi

P_u = 860 kips

q_s = 5.10 ksf



Calculations and Discussion	Code Reference

Determine depth based on shear strength without shear reinforcement. Depth required for shear usually controls the footing thickness. Both wide-beam action and two-way action for strength computation need to be investigated to determine the controlling shear criteria for depth. *11.12*

Assume overall footing thickness = 33 in. and average effective thickness d = 28 in. = 2.33 ft

1. Wide-beam action:

   $V_u = q_s \times$ tributary area

   b_w = 13 ft = 156 in.

   Tributary area = 13 (6.0 - 2.33) = 47.7 ft^2

   $V_u = 5.10 \times 47.7 = 243$ kips

   $\phi V_n = \phi\left(2\sqrt{f'_c}\, b_w d\right)$ *Eq. (11-3)*

   $= 0.75\left(2\sqrt{3000} \times 156 \times 28\right)/1000$ *9.3.2.3*

   = 359 kips > V_u O.K.

2. Two-way action:

   $V_u = q_s \times$ tributary area

   $$\text{Tributary area} = \left[(13 \times 13) - \frac{(30 + 28)\,(12 + 28)}{144}\right] = 152.9\ ft^2$$

   $V_u = 5.10 \times 152.9 = 780$ kips

$$\frac{V_c}{\sqrt{f'_c}\,b_o d} = \text{minimum of} \begin{cases} 2 + \dfrac{4}{\beta_c} & \text{Eq. (11-33)} \\ \dfrac{\alpha_s d}{b_o} + 2 & \text{Eq. (11-34)} \\ 4 & \text{Eq. (11-35)} \end{cases}$$

$b_o = 2\,(30 + 28) + 2\,(12 + 28) = 196$ in.

$$\beta_c = \frac{30}{12} = 2.5$$

$$\frac{b_o}{d} = \frac{196}{28} = 7$$

$\alpha_s = 40$ for interior columns

$$\frac{V_c}{\sqrt{f'_c}\,b_o d} = \begin{cases} 2 + \dfrac{4}{2.5} = 3.6 \quad \text{(governs)} \\ \dfrac{40}{7} + 2 = 7.7 \\ 4 \end{cases}$$

$$\phi V_c = 0.75 \times 3.6\sqrt{3000} \times 196 \times 28/1000$$

$$= 812 \text{ kips} > V_u = 780 \text{ kips} \quad \text{O.K.}$$

Example 22.3—Design for Footing Reinforcement

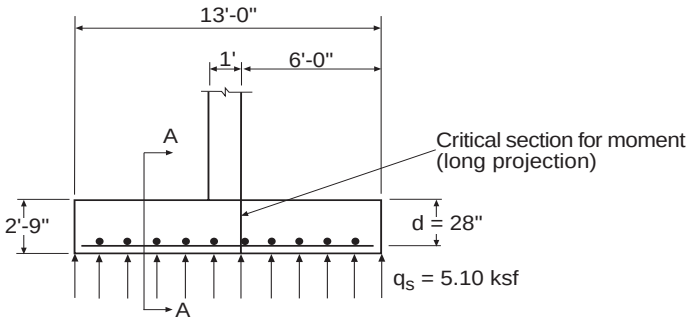
For the design conditions of Example 22.1, determine required footing reinforcement.

f'_c = 3000 psi

f_y = 60,000 psi

P_u = 860 kips

q_s = 5.10 ksf



Calculations and Discussion	Code Reference

1. Critical section for moment is at face of column — *15.4.2*

   $M_u = 5.10 \times 13 \times 6^2/2 = 1193$ ft-kips

2. Compute required A_s assuming tension-controlled section ($\phi = 0.9$) — *10.3.4, 9.3.2.1*

$$\text{Required } R_n = \frac{M_u}{\phi b d^2} = \frac{1193 \times 12 \times 1000}{0.9 \times 156 \times 28^2} = 130 \text{ psi}$$

$$\rho = \frac{0.85 f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right)$$

$$= \frac{0.85 \times 3}{60}\left(1 - \sqrt{1 - \frac{2 \times 130}{0.85 \times 3000}}\right) = 0.0022$$

$$p \text{ (gross area)} = \frac{d}{h} \times 0.0022 = \frac{28}{33} \times 0.0022 = 0.0019$$

Check minimum A_s required for footings of uniform thickness; for Grade 60 reinforcement: — *10.5.4*

$\rho_{min} = 0.0018 < 0.0019$ O.K. — *7.12.2*

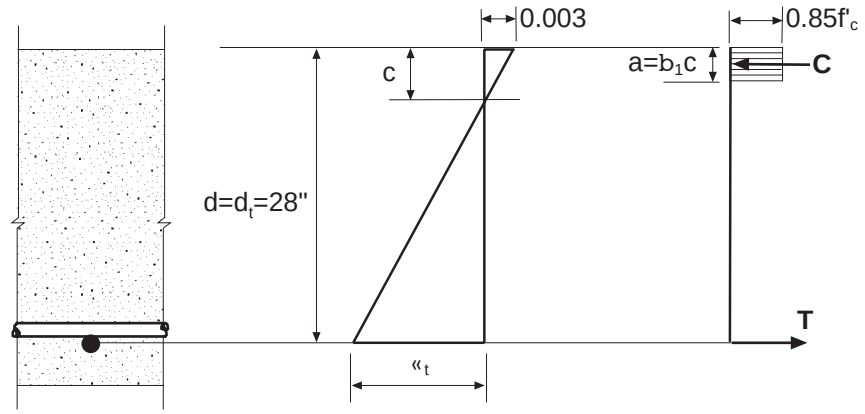
Required $A_s = \rho b d$

$A_s = 0.0022 \times 156 \times 28 = 9.60$ in.^{2}

Try 13-No. 8 bars (A_s = 10.27 in.^{2}) each way

Note that a lesser amount of reinforcement is required in the perpendicular direction due to lesser M_u , but for ease of placement, the same uniformly distributed reinforcement will be used each way (see Table 22-1). Also note that d_t = 27 in. for perpendicular direction.

3. Check net tensile strain (ε_t)



SECTION A-A Strain Diagram Force Diagram

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$= \frac{10.27 \times 60}{0.85 \times 3 \times 156} = 1.55$$

$$c = \frac{a}{\beta_1} = \frac{1.55}{0.85} = 1.82$$

$$\frac{\varepsilon_t + 0.003}{d_t} = \frac{0.003}{c}$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right) d_t - 0.003$$

$$= \frac{0.003}{1.82} \times 28 - 0.003 = 0.043 > 0.004$$ *10.3.4*

Therefore, section is tension-controlled and initial assumption is valid, O.K.

Thus, use 13-No. 8 bars each way.

4. Check development of reinforcement. *15.6*

Critical section for development is the same as that for moment (at face of column). *15.6.3*

$$\ell_d = \left[\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b}\right)}\right] d$$ *Eq. (12-1)*

Clear cover (bottom and side) = 3.0 in.

Center-to-center bar spacing = $\frac{156 - 2(3) - 2(0.5)}{12}$ = 12.4 in.

12.2.4

$$c = \text{minimum of} \begin{cases} 3.0 + 0.5 = 3.5 \text{ in.} \quad \text{(governs)} \\ \frac{12.4}{2} = 6.2 \text{ in.} \end{cases}$$

K_{tr} = 0 (no transverse reinforcement)

$\frac{c + K_{tr}}{d_b} = \frac{3.5 + 0}{1.0} = 3.5 > 2.5$, use 2.5 — 12.2.3

α = 1.0 (less than 12 in. of concrete below bars) — 12.2.4

β = 1.0 (uncoated reinforcement)

$\alpha\beta = 1.0 < 1.7$

γ = 1.0 (larger than No. 7 bars)

λ = 1.0 (normal weight concrete)

$$\ell_d = \left[\frac{3}{40} \frac{60{,}000}{\sqrt{3000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.5} \right] \times 1.0 = 32.9 \text{ in.} > 12.0 \text{ in. O.K.}$$

Since ℓ_d = 32.9 in. is less than the available embedment length in the short direction $\left(\frac{156}{2} - \frac{30}{2} - 3 = 60 \text{ in.} \right)$, the No. 8 bars can be fully developed.

Use 13-No. 8 each way.

Example 22.4—Design for Transfer of Force at Base of Column

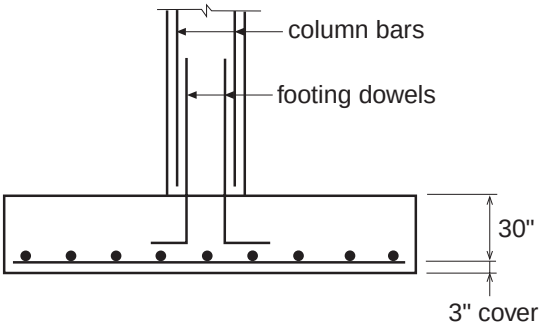
For the design conditions of Example 22.1, check force transfer at interface of column and footing.

f'_c (column) = 5000 psi

f'_c (footing) = 3000 psi

f_y = 60,000 psi

P_u = 860 kips



Calculations and Discussion	Code Reference

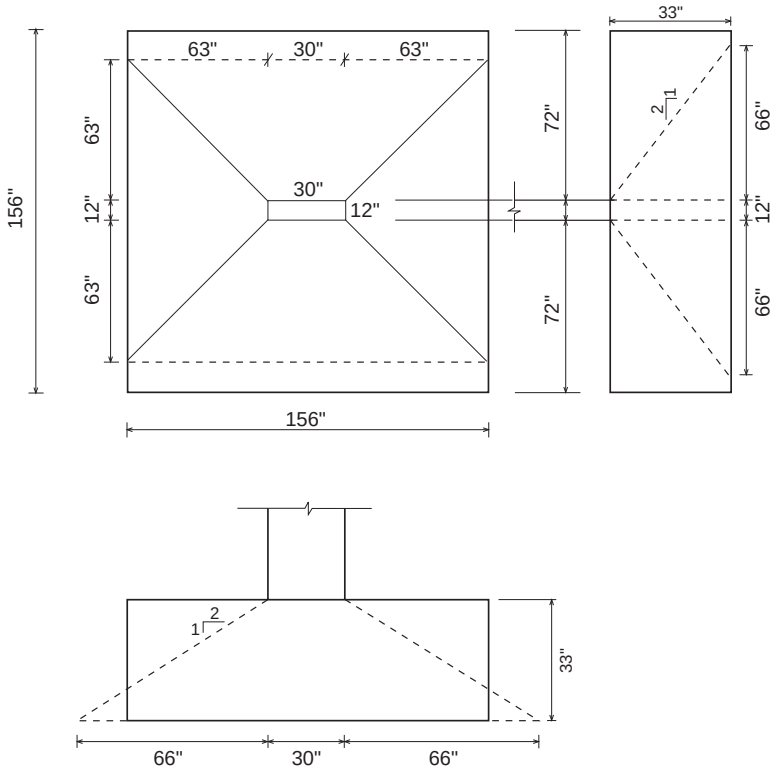
1. Bearing strength of column (f'_c = 5000 psi): *15.8.1.1*

   $\phi P_{nb} = \phi(0.85 f'_c A_1)$ *10.17.1*

   $= 0.65\,(0.85 \times 5 \times 12 \times 30) = 995 \text{ kips} > P_u = 860 \text{ kips}$ O.K. *9.3.2.4*

2. Bearing strength of footing (f'_c = 3000 psi): *15.8.1.1*

   The bearing strength of the footing is increased by a factor $\sqrt{A_2/A_1} \le 2$ due to the large footing area permitting a greater distribution of the column load. *10.17.1*



A_1 is the column (loaded) area and A_2 is the plan area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal. For the 30 × 12 in. column supported on the 13 × 13 ft square footing, $A_2 = (66 + 12 + 66) \times (63 + 30 + 63)$.

$$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{144 \times 156}{30 \times 12}} = 7.9 > 2, \text{ use } 2$$

Note that bearing on the column concrete will always govern until the strength of the column concrete exceeds twice that of the footing concrete.

$$\phi P_{nb} = 2\left[\phi(0.85 f'_c A_1)\right]$$

$$= 2\,[0.65\,(0.85 \times 3 \times 12 \times 30)] = 1193 \text{ kips} > P_u = 860 \text{ kips} \quad \text{O.K.}$$

3. Required dowel bars between column and footing:

Even though bearing strength on the column and footing concrete is adequate to transfer the factored loads, a minimum area of reinforcement is required across the interface. *15.8.2.1*

A_s (min) = 0.005 (30 × 12) = 1.80 in.^{2}

Provide 4-No. 7 bars as dowels (A_s = 2.40 in.^{2})

4. Development of dowel reinforcement in compression: *12.3.2*

In column:

$$\ell_{dc} = \left(\frac{0.02 f_y}{\sqrt{f'_c}}\right) d_b \geq \left(0.0003 f_y\right) d_b$$

For No. 7 bars:

$$\ell_{dc} = \left(\frac{0.02 \times 60{,}000}{\sqrt{5000}}\right) 0.875 = 14.9 \text{ in.}$$

$$\ell_{dc(min)} = 0.0003 \times 60{,}000 \times 0.875 = 15.8 \text{ in.} \quad \text{(governs)}$$

In footing:

$$\ell_{dc} = \left(\frac{0.02 \times 60{,}000}{\sqrt{3000}}\right) 0.875 = 19.2 \text{ in.} \quad \text{(governs)}$$

$$\ell_{dc(min)} = 0.0003 \times 60{,}000 \times 0.875 = 15.8 \text{ in.}$$

Available length for development in footing

= footing thickness - cover - 2 (footing bar diameter) - dowel bar diameter

= 33 - 3 - 2 (1.0) - 0.875 = 27.1 in. > 19.2 in.

Therefore, the dowels can be fully developed in the footing.

Example 22.5—Design for Transfer of Force by Reinforcement

For the design conditions given below, provide for transfer of force between the column and footing.

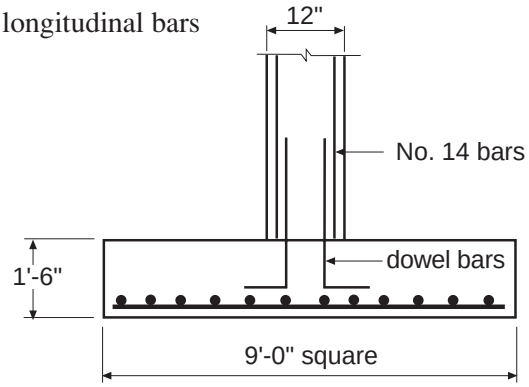
12×12 in. tied reinforced column with 4-No. 14 longitudinal bars

f'_c = 4000 psi (column and footing)

f_y = 60,000 psi

P_D = 200 kips

P_L = 100 kips



Calculations and Discussion	Code Reference

1. Factored load $P_u = (1.2 \times 200) + (1.6 \times 100) = 400$ kips — *Eq. (9-2)*

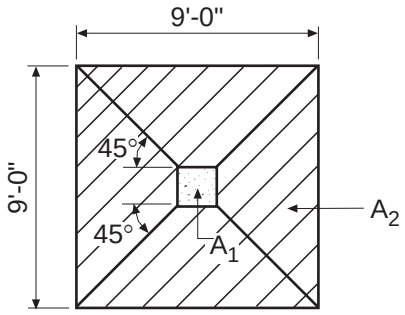
2. Bearing strength on column concrete: — *15.8.1.1*

$$\phi P_{nb} = \phi\left(0.85 f'_c A_1\right) = 0.65\left(0.85 \times 4 \times 12 \times 12\right)$$ — *10.17.1*

$= 318.2$ kips $< P_u = 400$ kips N.G.

The column load cannot be transferred by bearing on concrete alone. The excess load (400 - 318.2 = 81.8 kips) must be transferred by reinforcement. — *15.8.1.2*

3. Bearing strength on footing concrete: — *15.8.1.1*



$$\phi P_{nb} = \sqrt{\frac{A_2}{A_1}}\left[\phi(0.85 f'_c A_1)\right]$$

$$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{9 \times 9}{1 \times 1}} = 9 > 2, \text{ use } 2$$

$\phi P_{nb} = 2\,(318.2) = 636.4$ kips > 400 kips O.K.

4. Required area of dowel bars: — *15.8.1.2*

$$A_s \text{ (required)} = \frac{\left(P_u - \phi P_{nb}\right)}{\phi f_y}$$

$$= \frac{81.8}{0.65 \times 60} = 2.10 \text{ in.}^2$$

A_s (min) $= 0.005\,(12 \times 12) = 0.72$ in.^{2} — *15.8.2.1*

Try 4-No. 8 bars ($A_s = 3.16$ in.^{2})

5. Development of dowel reinforcement

a. For development into the column, the No. 14 column bars may be lap spliced with the No. 8 footing dowels. The dowels must extend into the column a distance not less than the development length of the No. 14 column bars or the lap splice length of the No. 8 footing dowels, whichever is greater. 15.8.2.3

For No. 14 bars:

$$\ell_{dc} = \left(\frac{0.02f_y}{\sqrt{f'_c}}\right) d_b = \left(\frac{0.02 \times 60{,}000}{\sqrt{4000}}\right) 1.693 = 32.1 \text{ in.} \quad \text{(governs)}$$

$$\ell_{dc(min)} = \left(0.0003f_y\right) d_b = 0.0003 \times 60{,}000 \times 1.693 = 30.5 \text{ in.}$$

For No. 8 bars:

lap length $= 0.0005 f_y d_b$ 12.16.1

$= 0.0005 \times 60{,}000 \times 1.0 = 30$ in.

Development length of No. 14 bars governs.

The No. 8 dowel bars must extend not less than 33 in. into the column.

b. For development into the footing, the No. 8 dowels must extend a full development length. 15.8.2.3

$$\ell_{dc} = \left(\frac{0.02f_y}{\sqrt{f'_c}}\right) d_b = \left(\frac{0.02 \times 60{,}000}{\sqrt{4000}}\right) \times 1.0 = 19.0 \text{ in.} \quad \text{(governs)}$$

$$\ell_{dc(min)} = \left(0.0003f_y\right) d_b = 0.0003 \times 60{,}000 \times 1.0 = 18.0 \text{ in.}$$

This length may be reduced to account for excess reinforcement. 12.3.3(a)

$$\frac{A_s \text{ (required)}}{A_s \text{ (provided)}} = \frac{2.10}{3.16} = 0.66$$

Required $\ell_d = 19 \times 0.66 = 12.5$ in.

Available length for dowels development $\approx 18 - 5 = 13$ in. > 12.5 in. required, O.K.

,

Note: In case the available development length is less than the required development length, either increase footing depth or use larger number of smaller size dowels. Also note that if the footing dowels are bent for placement on top of the footing reinforcement (as shown in the figure), the bent portion cannot be considered effective for developing the bars in compression (12.5.5).

Example 22.6—Design for Transfer of Horizontal Force at Base of Column

For the column and footing of Example 22.5, design for transfer of a horizontal factored force of 85 kips acting at the base of the column.

Design data:

Footing: size = 9 × 9 ft
thickness = 1ft-6 in.

Column: size = 12 × 12 in. (tied)
4-No. 14 longitudinal reinforcement

f'_c = 4000 psi (footing and column)

f_y = 60,000 psi

Calculations and Discussion	Code Reference

1. The shear-friction design method of 11.7 is applicable. *15.8.1.4*

   Check maximum shear transfer permitted: *11.7.5*

   $V_u \le \phi(0.2f'_cA_c)$ but not greater than $\phi(800A_c)$

   $\phi V_n = 0.75\,(0.2 \times 4 \times 12 \times 12) = 86.4$ kips

   $\phi(800A_c) = 0.75 \times 800 \times 12 \times 12/1000 = 86.4$ kips

   $V_u = 85 \text{ kips} < \phi(0.2f'_cA_c)$ and $\phi(800A_c)$ O.K.

   The shear transfer of 85 kips is permitted at the base of 12 × 12 in. column.

   Strength requirement for shear:

   $V_u \le \phi V_n$ *Eq. (11-1)*

   $V_n = V_u/\phi = A_{vf}f_y\mu$ *Eq. (11-25)*

   Use $\mu = 0.6$ (concrete not intentionally roughened) *11.7.4.3*

   and $\phi = 0.75$ (shear)

   $$\text{Required } A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{85}{0.75 \times 60 \times 0.6} = 3.15 \text{ in.}^2$$ *Eq. (11-25)*

   A_s (provided) = 3.16 in.^{2} O.K.

   Therefore, use 4-No. 8 dowels (A_s = 3.16 in.^{2})

If the 4-No. 8 dowels were not adequate for transfer of horizontal shear, the footing concrete in contact with the column concrete could be roughened to an amplitude of approximately 1/4 in. to take advantage of the higher coefficient of friction of 1.0:

$$\text{Required } A_{vf} = \frac{85}{0.75 \times 60 \times 1.0} = 1.89 \text{ in.}^2$$

2. Tensile development of No. 8 dowels, as required by 11.7.8

a. Within the column

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b} \right)} \right) d_b \qquad \text{Eq. (12-1)}$$

Clear cover to No. 8 bar $\approx$ 3.25 in.

Center-to-center bar spacing of No. 8 bars $\approx$ 4.5 in.

$$c = \text{minimum of} \begin{cases} 3.25 + 0.5 = 3.75 \text{ in.} \\ \frac{4.5}{2} = 2.25 \text{ in.} \quad \text{(governs)} \end{cases} \qquad 12.2.4$$

Assume $K_{tr} = 0$ (conservatively consider no transverse reinforcement)

$$\frac{c + K_{tr}}{d_b} = \frac{2.25 + 0}{1.0} = 2.25 < 2.5, \text{ use } 2.25 \qquad 12.2.3$$

$\alpha = 1.0$

$\beta = 1.0$

$\alpha\beta = 1.0 < 1.7$

$\gamma = 1.0$

$\lambda = 1.0$

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{\sqrt{4000}} \frac{1.0 \times 1.0 \times 1.0}{2.25} \right) \times 1.0 = 31.6 \text{ in.}$$

Provide at least 32 in. of embedment into the column.

b. Within the footing

Use standard hooks at the ends of the No. 8 bars

$\ell_{hb} = \left(0.02\beta\lambda f_y / \sqrt{f'_c}\right) d_b$ 12.5.2

$= \left(0.02 \times 1.0 \times 1.0 \times \frac{60{,}000}{\sqrt{4000}}\right) \times 1.0 = 19.0$ in.

Modifications: 12.5.3

cover normal to plane of 90° hook > 2.5 in.

cover on bar extension beyond hook ≥ 2 in.

$\ell_{dh} = 0.7 \times 19 = 13.3$ in. 12.5.3(a)

Min. $\ell_{dh} = 8 \times d_b = 8$ in. < 13.3 in. 12.5.1

Available development length =

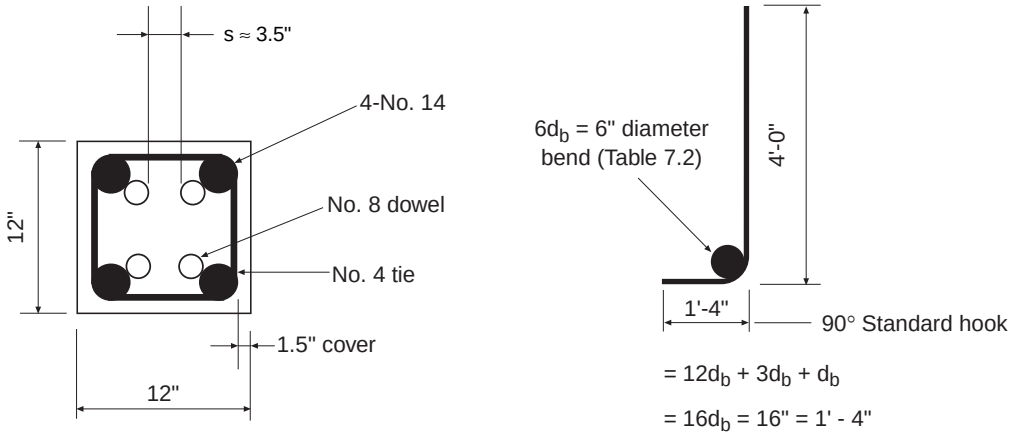
= 18 - 5 = 13 in. < 13.3 in. N.G.

Increase footimg depth by 2 in. Total depth = 20 in.

Use 15 in. hook embedment into footing to secure dowels at the footing reinforcement.

Total length of No. 8 dowel = 32 + 15 = 47 in. Use 4 ft-0 in. long dowels.

Note: The top of the footing at the interface between column and footing must be clean and free of laitance before placement of the column concrete. 11.7.9



No. 8 dowel detail

Example 22.7—Design for Depth of Footing on Piles

For the footing supported on the piles shown, determine the required thickness of the footing (pile cap).

Footing size = 8.5 × 8.5 ft

Column size = 16 × 16 in.

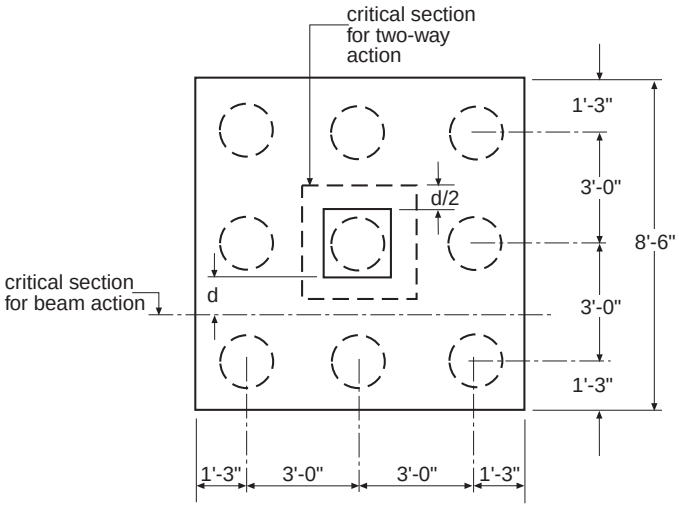
Pile diameter = 12 in.

f'_c = 4000 psi

Load per pile:

P_D = 20 kips

P_L = 10 kips



Calculations and Discussion	Code Reference

1. Depth required for shear usually controls footing thickness. Both wide-beam action and two-way action for the footing must be investigated. *11.12*

   Assume an overall footing thickness of 1 ft-9 in. with an average d ≈ 14 in. *15.7*

2. Factored pile loading:

   $P_u = 1.2\,(20) + 1.6\,(10) = 40$ kips *Eq. (9-2)*

3. Strength requirements for shear

   $V_u \le \phi V_n$ *Eq. (11-1)*

   a. Wide-beam action for footing: *11.12.1.1*

   3 piles fall within tributary area

   V_u (neglecting footing wt.) = 3 × 40 = 120 kips

   $\phi V_n = \phi\left(2\sqrt{f'_c}\, b_w d\right)$ *Eq. (11-3)*

   b_w = 8 ft-6 in. = 102 in.

   $\phi V_n = 0.75\left(2\sqrt{4000} \times 102 \times 14\right)/1000 = 135.4 \text{ kips} > V_u = 120 \text{ kips}$ O.K.

b. Two-way action: *11.12.1.2*

8 piles fall within the tributary area

$V_u = 8 \times 40 = 320$ kips

$$\frac{V_c}{\sqrt{f'_c}\, b_o d} = \text{minimum of} \begin{cases} 2 + \dfrac{4}{\beta_c} & \textit{Eq. (11-35)} \\ \dfrac{\alpha_s d}{b_o} + 2 & \textit{Eq. (11-36)} \\ 4 & \textit{Eq. (11-37)} \end{cases}$$

$$\beta_c = \frac{16}{16} = 1.0$$

$b_o = 4\,(16 + 14) = 120$ in.

$\alpha_s = 40$ for interior columns

$$\frac{b_o}{d} = \frac{120}{14} = 8.6 \qquad \textit{Eq. (11-33)}$$

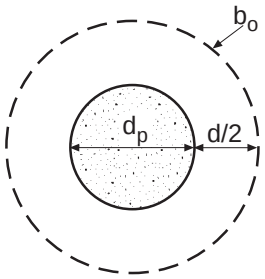
Eq. (11-34)

Eq. (11-35)

$$\frac{V_c}{\sqrt{f'_c}\, b_o d} = \begin{cases} 2 + \dfrac{4}{1} = 6 \\ \dfrac{40}{8.6} + 2 = 6.7 \\ 4 \quad \text{(governs)} \end{cases}$$

$$\phi V_c = 0.75 \times 4\sqrt{4000} \times 120 \times 14/1000$$

$= 319$ kips $\cong V_u = 320$ kips O.K.



4. Check "punching" shear strength at corner piles. With piles spaced at 3 ft-0 in. on center, critical perimeters do not overlap.

$V_u = 40$ kips per pile

$$\frac{V_c}{\sqrt{f'_c}\,b_o d} = \text{minimum of} \begin{cases} 2 + \dfrac{4}{\beta_c} & \textit{Eq. (11-33)} \\ \dfrac{\alpha_s d}{b_o} + 2 & \textit{Eq. (11-34)} \\ 4 & \textit{Eq. (11-35)} \end{cases}$$

$\beta_c = 1.0$ (square reaction area of equal area)

$b_o = \pi(12 + 14) = 81.7$ in.

$\alpha_s = 20$ *11.12.2.1*

$$\frac{b_o}{d} = \frac{81.7}{14} = 5.8$$

$$\frac{V_c}{\sqrt{f'_c}\,b_o d} = \begin{cases} 2 + \dfrac{4}{1} = 6 \\ \dfrac{20}{5.8} + 2 = 5.4 \\ 4 \quad \text{(governs)} \end{cases}$$

$\phi V_c = 0.75 \times 4\sqrt{4000} \times 81.7 \times 14/1000 = 217 \text{ kips} > V_u = 40 \text{ kips}$ O.K.

23

Precast Concrete

GENERAL CONSIDERATIONS

Chapter 16 was completely rewritten for the 1995 code. Previous editions of Chapter 16 were largely performance oriented. The current chapter is more prescriptive, although the word "instructive" may be more appropriate, as the chapter provides much more guidance to the designer of structures which incorporate precast concrete. Not only does the chapter itself provide more requirements and guidelines, but the commentary contains some 25 references, as opposed to 4 in the 1989 code, thus encouraging the designer to make maximum use of the available literature.

The increase in instructive material is most notable in 16.5, Structural Integrity. Requirements for structural integrity were introduced in 7.13 of the 1989 code. For precast construction, this section required only that tension ties be provided in all three orthogonal directions (two horizontal and one vertical) and around the perimeter of the structure, without much further guidance. Reference 23.1 was given for precast bearing wall buildings. The recommendations given in that reference are now codified in 16.5.2. Section 16.5.1 applies primarily to precast structures other than bearing wall buildings and, as is the case with most of the rewritten Chapter 16, is largely a reflection of time-tested industry practice. Note that tilt-up concrete construction is a form of precast concrete. Reference 23.3 addresses all phases of design and construction of tilt-up concrete structures.

16.2 GENERAL

The code requires that precast members and connections be designed for "... loading and restraint conditions from initial fabrication to end use in the structure ..." Often, especially in the case of wall panels, conditions during handling are far more severe than those experienced during service. For this reason, and also because practices and details vary among manufacturers, precast concrete components are most often designed by specialty engineers employed by the manufacturer. Calculations, as well as shop drawings (16.2.4) are then submitted to the engineer-of-record for approval. This procedure is also usually followed in the design of connections. For more information on the relationship between the engineer-of-record and the specialty engineer, see Refs. 23.4 and 23.5.

As stated above, since 1995, the code encourages the use of other publications for the design of precast concrete structures. References 23.2, 23.6, and 23.7 are particularly useful to the designer. Of these, the most widely used is Ref. 23.7, the *PCI Design Handbook*.

Section 16.2.3 states that tolerances must be specified. This is usually done by reference to Industry documents[23.8, 23.9, 23.10], as noted in the commentary. Design of precast concrete members and connections is particularly sensitive to tolerances. Therefore, they should be specified in the contract documents together with required concrete strengths at different stages of construction [16.2.4(b)].

16.3 DISTRIBUTION OF FORCES AMONG MEMBERS

Section 16.3.1 covers distribution of forces perpendicular to the plane of members. Most of the referenced research relates to hollow-core slabs, and is also applicable to solid slabs that are connected by continuous grout keys. Members that are connected together by other means, such as weld plates on double tees, have through extensive use been found to be capable of distributing concentrated loads to adjacent members. It is common to assume for design purposes that up to 25% of a concentrated load can be transferred to each adjacent member; the connections should be designed accordingly. Since load transfer is dependent on compatible deflections, less distribution occurs nearer the support, as shown in Fig. 23-1(a) for hollow core slabs. Flanges of double tees are also designed for transverse load distribution over an effective width as illustrated in Fig. 23-1(b). Other types of decks may not necessarily follow the same pattern, because of different torsional resistance properties, but the same principles are applicable. A typical design is shown in Example 23.1.

Compatibility of the deflections of adjacent units is an important design consideration. For example, in the driving lane of a precast concrete double-tee parking deck, even if each member has adequate strength to carry a full wheel load, it is undesirable for each unit to deflect independently. It is common practice to use more closely spaced connections in those cases, to assure sharing of the load and to eliminate differential deflections between members.

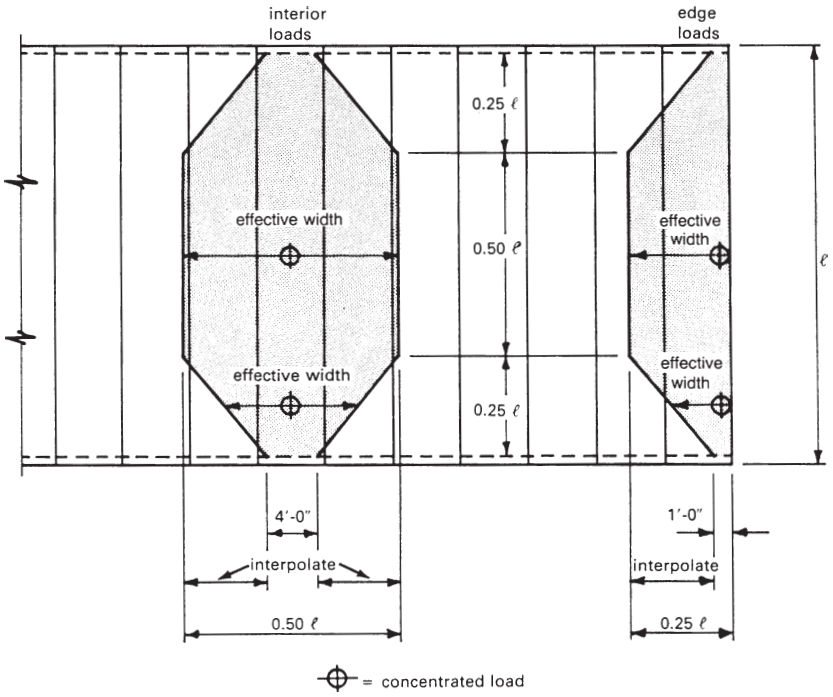
Section 16.3.2 covers distribution of in-plane forces. It requires a continuous load path for such forces. If these forces are tensile, they must be resisted by steel or steel reinforcement. Since these in-plane forces are usually caused by lateral loads such as those due to wind or earthquakes, and since such lateral loads can occur in any direction, nearly all of the continuous load path must be provided by steel or steel reinforcement. In that respect, the reinforcement and connections designed to meet the requirements of this section may also provide the continuous ties required by 16.5.

16.4 MEMBER DESIGN

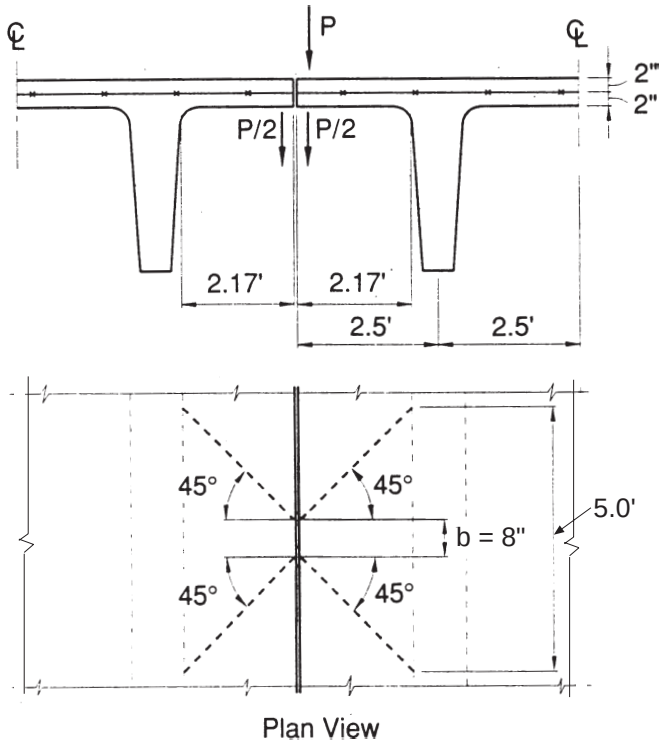
This section is primarily concerned with minimum reinforcement requirements for precast members. Section 16.4.1 waives transverse steel requirements in prestressed concrete members 12 ft or less in width, except where such steel is required for flexure. The commentary notes that an example of an exception to the waiver is the flanges of single and double tees. This section is intended primarily for hollow core and solid slabs where the transverse connection is typically grouted joints.

Section 16.4.2 reduces the minimum reinforcement in precast, nonprestressed walls, from that required for cast-in-place reinforced concrete walls in Chapter 14, to 0.001 times the gross cross-sectional area of the walls, in accordance with common industry practice. This is in recognition of the fact that much of the shrinkage of precast members occurs prior to attachment to the structure. Spacing of reinforcement in precast walls should not exceed 5 times the wall thickness or 30 in. for interior walls or 18 in. for exterior walls.

When wall panels are load-bearing, they are usually designed as compression members in accordance with Chapter 10. When they are not load-bearing (and often even when they are), the stresses during handling are usually critical. In those cases, it is common to place the lifting and dunnage points so that the stresses during handling do not exceed the modulus of rupture (with a safety factor), especially for architectural precast panels. If cracking is likely, crack control reinforcement in accordance with 10.6.4 is required.



(a) Hollow core slab



(b) Double tee

Figure 23-1 Assumed Load Distribution[23.7]

16.5 STRUCTURAL INTEGRITY

The provisions of 16.5.1.1 are intended to assure that there is a continuous load path from every precast member to the lateral load resisting system. The commentary gives several examples of how this may be accomplished. Section 16.5.1.2 is adopted from a similar requirement in the Uniform Building Code, as explained in the commentary.

Section 16.5.1.3 gives requirements for vertical tension ties. The requirement for columns in 16.5.1.3(a) applies not only to connections of columns to footings, but also to such connections as column splices. The 10,000 lb requirement for each of at least two ties per wall panel in 16.5.1.3(b) is from the *PCI Design Handbook*[23.7], and is the numerical equivalent of a common connection used in the precast concrete industry for many years. This is strictly an empirical value, and is intended to apply only to the dimensions of the hardware items in the connection, without including eccentricities in the design. Section 16.5.1.3(c) permits this connection to be into a reinforced concrete floor slab, as is common with tilt-up construction.

Section 16.5.2 in essence codifies some of the recommendations of Ref. 23.11, which gives numerical values for tension ties in bearing wall buildings. This report is based on a series of tests conducted at the Portland Cement Association's laboratories[23.12] in the late 1970s.

16.6 CONNECTION AND BEARING DESIGN

Section 16.6.1 lists the several ways that precast members can be connected, and then allows design by analysis or test. Special mention is made of 11.7, Shear Friction, as this is a commonly used analysis/design tool. See Part 16 of this document. Examples on application of shear friction design procedure are also given in *PCI Design Handbook*[23.7] and *Design and Typical Details of Connections for Precast and Prestressed Concrete*. [23.6]

Section 16.6.2 describes several important considerations when designing for bearing of precast elements. The minimum bearing lengths of 16.6.2.2(a) are particularly important. It should be emphasized that these are minimum values, and that the structure should be detailed with significantly longer bearing lengths, to allow for tolerance in placement. Section 16.6.2.3 makes it clear that positive moment reinforcement need not extend out of the end of the member, provided it goes at least to the center of the bearing.

16.7 ITEMS EMBEDDED AFTER CONCRETE PLACEMENT

In precasting plants, it has long been common practice to place certain embedded items in the concrete after it has been cast. This practice is recognized in this section and constitutes an exception to the provisions of 7.5.1. Conditions to embed items in the concrete while it is in a plastic state are: (1) embedded item is not required to be hooked or tied to reinforcement within the concrete, (2) embedded item is secured in its position until concrete hardens, and (3) concrete is properly consolidated around each embedment.

16.8 MARKING AND IDENTIFICATION

The purpose of identification marks on precast members is to facilitate construction and avoid placing errors. Each precast member should be marked to show date of manufacture and should be identified according to placing drawings.

16.9 HANDLING

This section re-emphasizes the general requirement of 16.2.1. Handling stresses and deformations must be considered during the design of precast concrete members. Erection steps and hardware required for each step must be shown on contract or erection drawings.

16.10 STRENGTH EVALUATION OF PRECAST CONSTRUCTION

It is always desirable, and certainly safer and more economical, to test a suspect precast concrete member before it is integrated into the structure. This new section describes how Chapter 20 provisions can be applied to this case. The test loads specified in 20.3.2 must be adjusted to simulate the load portion carried by the suspect member when it is in the final, composite mode. The acceptance criteria of 20.5 apply to the isolated precast member.

REFERENCES

23.1 PCI Committee on Building Code and PCI Technical Activities Committee, "Proposed Design Requirements for Precast Concrete," *PCI Journal*, V. 31, No. 6, November-December 1986, pp. 32-47.

23.2 ACI Committee 550, "Design Recommendations for Precast Concrete Structures," (ACI 550R-96, Reapproved 2000). American Concrete Institute, Farmington Hills, MI, 1996. Also in ACI Manual of Concrete Practice, Part 6.

23.3 ACI Committee 551, "Tilt-Up Concrete Structures," (ACI 551R-92, Reapproved 1997), American Concrete Institute, Farmington Hills, MI, 1992. Also in *ACI Manual of Concrete Practice*, Part 6.

23.4 The Case Task Group on Specialty Engineering, "National Practice Guidelines for Specialty Structural Engineers," Council of American Structural Engineers, Washington, DC, 1994, 11 pp.

23.5 ACI Committee on Responsibility in Concrete Construction, "Guidelines for Authorities and Responsibilities in Concrete Design and Construction," *Concrete International*, Vol. 17, No. 9, September 1995, pp. 66-69.

23.6 "Design and Typical Details of Connections for Precast and Prestressed Concrete," MNL-123-88, 2nd Edition, Precast/Prestressed Concrete Institute, Chicago, 1988, 270 pp.

23.7 "PCI Design Handbook—Precast and Prestressed Concrete," MNL-120-92, 5th Edition, Precast/Prestressed Concrete Institute, Chicago, 1999, 630 pp.

23.8 "Manual for Quality Control for Plants and Production of Precast and Prestressed Concrete Products," MNL-116-99, 4th Edition, Precast/Prestressed Concrete Institute, Chicago, 1999, 340 pp.

23.9 "Manual for Quality Control for Plants and Production of Architectural Precast Concrete," MNL-117-96, 3rd Edition, Precast/Prestressed Concrete Institute, Chicago, 1996, 226 pp.

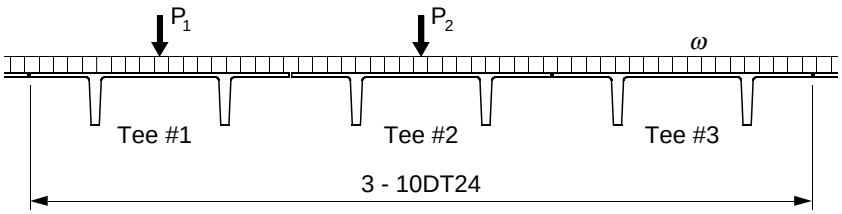
23.10 PCI Committee on Tolerances, "Tolerances for Precast and Prestressed Concrete," *PCI Journal*, V. 30, No. 1, January-February 1985, pp. 26-112.

23.11 PCI Committee on Precast Concrete Bearing Wall Buildings, "Considerations for the Design of Precast Concrete Bearing Wall Buildings to Withstand Abnormal Loads," *PCI Journal*, V. 21, No. 2, March-April 1976, pp. 18-51.

23.12 "Design and Construction of Large-Panel Concrete Structures," six reports, EB100D, 1976-1980, 762 pp.; three studies, EB102D, 1980, 300 pp.; Portland Cement Association, Skokie, IL.

Example 23.1—Load Distribution in Double Tees

Required: Compute the factored moments and shears for each of three double tees of the following roof:



Given:
Double tees = 10DT24 (self weight = 468 plf), h = 24 in.
Span = 60 ft
Superimposed DL = 15 psf, LL = 30 psf
Concentrated dead load on Tee #1, P_1 = 20 kips @ 3 ft from left support
Concentrated dead load on Tee #2, P_2 = 20 kips @ midspan

Calculations and Discussion	Code Reference

1. Assume:

   Concentrated dead load P_1 cannot be distributed to adjacent tees since it is near the support.

   Concentrated dead load P_2 is distributed, with 25 percent to adjacent tees and 50 percent to the tee supporting the load, i.e.

   0.25 (20 kips) = 5 kips to Tee #1
   0.50 (20 kips) = 10 kips to Tee #2
   0.25 (20 kips) = 5 kips to Tee #3

2. Factored uniform dead and live loads, for each tee

   DL = 468 + 15 (10 ft width) = 0.618 kip/ft

   LL = 30 (10 ft width) = 0.30 kip/ft

   w_u = 1.2D + 1.6L — *Eq. (9-2)*

   = 1.2 (0.618) + 1.6 (0.30) = 1.222 kip/ft

3. Factored moments and shears for Tee #1

   Factored concentrated dead load next to support = 1.2 (20) = 24 kips

   Factored concentrated dead load at midspan = 1.2 (5) = 6 kips

   w_u = 1.222 kip/ft

Reaction at left support $= \frac{57}{60}(24) + \frac{6}{2} + \frac{1.222(60)}{2} = 62.46$ kips

For prestressed concrete members, design for shear at distance h/2 *11.1.3.2*

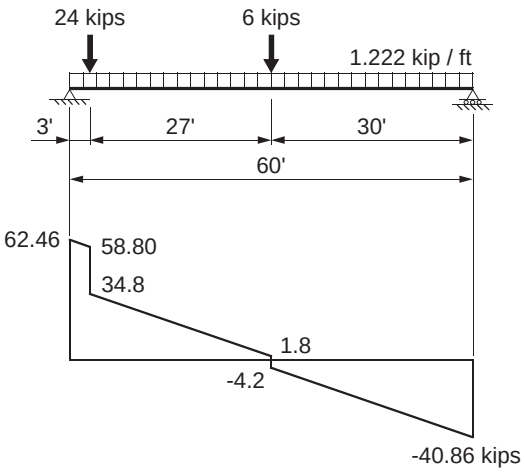
V_u (left) = 62.46 - 1.222 = 61.24 kips

Reaction at right support $= \frac{3}{60}(24) + \frac{6}{2} + \frac{1.222(60)}{2} = 40.86$ kips

At distance h/2, V_u (right) = 40.86 - 1.222 = 39.64 kips

Maximum moment is at midspan

M_u (max) = 40.86 (30) - 1.222 (30) (15) = 676 ft-kips



4. Factored moments and shears for Tee #2

$$M_u = \frac{w_u \ell^2}{8} + \frac{P\ell}{4}$$

$$= \frac{1.222(60)^2}{8} + \frac{1.2(10)(60)}{4} = 730 \text{ ft-kips}$$

Maximum reaction $= \frac{w_u \ell}{2} + \frac{P}{2}$

$$= \frac{1.222(60)}{2} + \frac{1.2(10)}{2} = 42.66 \text{ kips}$$

At distance h/2, V_u = 42.66 - 1.222 = 41.44 kips

5. Factored moments and shears for Tee #3

$$M_u = \frac{1.222(60)^2}{8} + \frac{1.2(5)(60)}{4} = 640 \text{ ft-kips}$$

Maximum reaction $= \frac{1.222(60)}{2} + \frac{1.2(5)}{2} = 39.66$ kips

At distance h/2, V_u = 39.66 - 1.222 = 38.44 kips

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Prestressed Concrete — Flexure

UPDATE FOR THE ’02 CODE

For ACI 318-02, the changes to prestressed concrete design are more extensive than in the last few editions of the code. The major changes are:

- The provisions of former Appendix B, relating to Unified Design Provisions, are now incorporated in the body of the code. Changes in Chapters 8, 9, 10, and 18 affect prestressed concrete design. Reinforcement limits, moment redistribution, and determination of the strength reduction factor ϕ are affected.
- Cracking at service load is now permitted in the new Class C flexural members, for which additional serviceability requirements apply. This new provision permits design using any combination of prestressing steel and reinforcement.
- Explicit definitions for “tendon” and “prestressing steel” are revised to reflect industry practice. They are used consistently throughout the code.
- Revisions to Chapter 7, giving separate (and different) concrete cover requirements for cast-in-place prestressed concrete and for prestressed concrete manufactured under plant control conditions.

This part (chapter) describes the changes in Chapter 18 of the 2002 code.

GENERAL CONSIDERATIONS

In prestressed members, compressive stresses are introduced into the concrete to reduce tensile stresses resulting from applied loads including the self weight of the member (dead load). Prestressing steel, such as strands, bars, or wires, is used to impart compressive stresses to the concrete. Pretensioning is a method of prestressing in which the tendons are tensioned before concrete is placed and the prestressing force is primarily transferred to the concrete through bond. Post-tensioning is a method of prestressing in which the tendons are tensioned after the concrete has hardened and the prestressing force is primarily transferred to the concrete through the end anchorages.

The act of prestressing a member introduces “prestressing loads” to the member. The induced prestressing loads, acting in conjunction with externally applied loads, must provide serviceability and strength to the member beginning immediately after prestress force transfer and continuing throughout the life of the member. Prestressed structures must be analyzed taking into account prestressing loads, service loads, temperature, creep, shrinkage and the structural properties of all materials involved.

The code states that all provisions of the code apply to prestressed concrete, unless they are in conflict with Chapter 18 or are specifically excluded. The exclusions, listed in 18.1.2 and 18.1.3, are necessary because some empirical or simplified analytical methods employed elsewhere in the code may not adequately account for the effects of prestressing forces.

Deflections of prestressed members calculated according to 9.5.4 should not exceed the values listed in Table 9.5(b). According to 9.5.4, prestressed concrete members, like any other concrete members, should be designed to have adequate stiffness to prevent deformations which may adversely affect the strength or serviceability of the structure.

PRESTRESSING MATERIALS

The most commonly used prestressing material in the United States is Grade 270 ksi low-relaxation, seven-wire strand, defined by ASTM A 416. The most common size is 1/2-in., although there is increasing use of 0.6-in. strand, especially for post-tensioning. The properties of these strands are as follows:

Nominal Diameter, in.	1/2	0.6
Area, sq. in.	0.153	0.217
Breaking stress f_{pu}, ksi	270	270
Breaking strength, kips	41.3	58.6
Jacking stress, ksi = 0.75 f_{pu}	202.5	202.5

Virtually identical metric strands are used in metric countries.

The Prestressed Concrete Institute's *PCI Design Handbook*, 5th edition, Ref. 24.1, gives a standard stress-strain curve for this material, as shown in Fig. 24-1.

NOTATION

The following symbols were added to 18.0, Notation, in ACI 318-02. They are used in 18.4.4, which deals with serviceability requirements for cracked prestressed flexural members.

Δf_{ps} = stress in prestressing steel at service loads less decompression stress, ksi

f_{dc} = decompression stress. Stress in the prestressing steel when stress is zero in the concrete at the same level as the centroid of the tendons, psi

s = center-to-center spacing of flexural tension steel near the extreme tension face, in. Where there is only one bar or tendon near the extreme tension face, s is the width of extreme tension face

The following new and revised definitions were placed in Chapter 2. They are consistently used in Chapter 18 and throughout the code, resulting in numerous editorial modifications.

Prestressing steel — High-strength steel element, such as wire, bar, or strand, or a bundle of such elements, used to impart prestress forces to concrete.

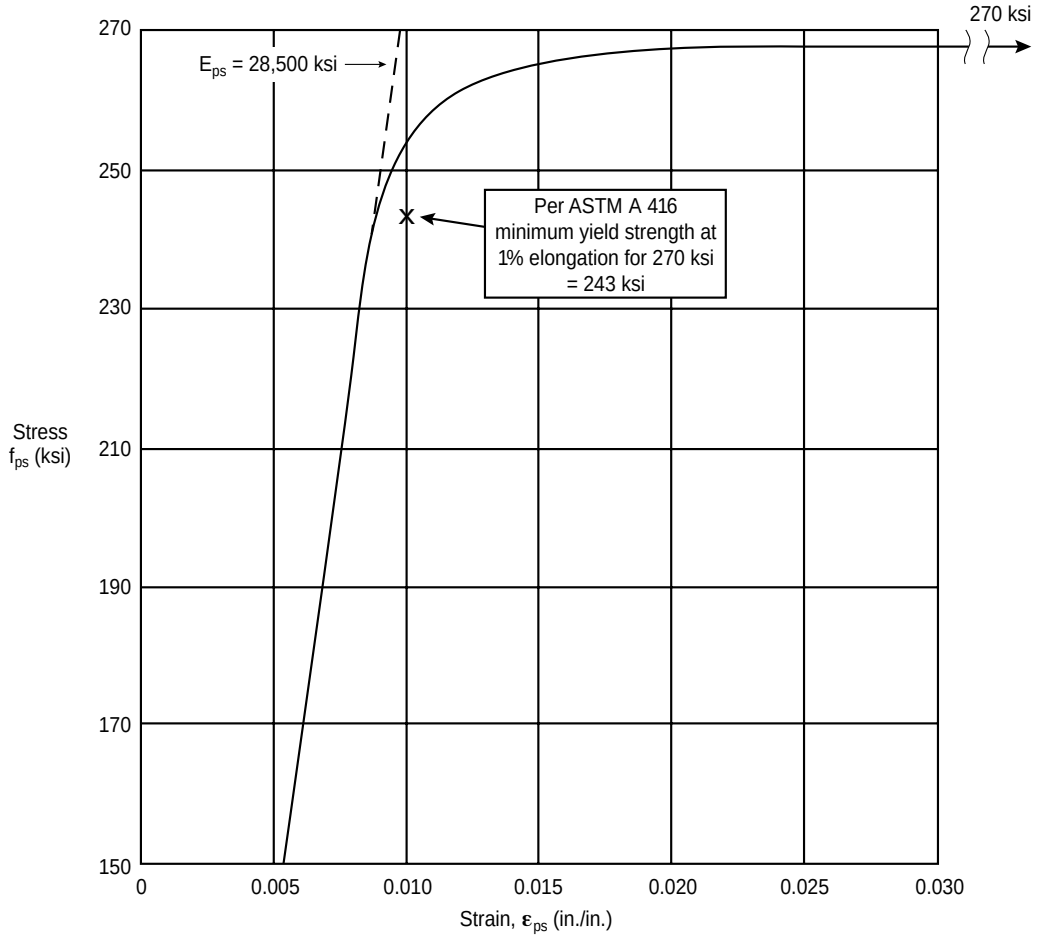
Tendon — In pretensioned applications, the tendon is the prestressing steel. In post-tensioned applications, the tendon is a complete assembly consisting of anchorages, prestressing steel, and sheathing with coating for unbonded applications or ducts with grout for bonded applications.

Bonded tendon — Tendon in which prestressing steel is bonded to concrete either directly or through grouting.

Unbonded tendon — Tendon in which the prestressing steel is prevented from bonding to the concrete and is free to move relative to the concrete. The prestressing force is permanently transferred to the concrete at the tendon ends by the anchorages only.

Duct — A conduit (plain or corrugated) to accommodate prestressing steel for post-tensioned installation. Requirements for post-tensioning ducts are given in 18.17.

Sheathing — A material encasing prestressing steel to prevent bonding of the prestressing steel with the surrounding concrete, to provide corrosion protection, and to contain the corrosion inhibiting coating.



The above curve can be approximated by the following equations: $\varepsilon_{ps} \le 0.0086$: $f_{ps} = 28{,}500\ \varepsilon_{ps}$ (ksi)

$$\varepsilon_{ps} > 0.0086: \quad f_{ps} = 270 - \frac{0.04}{\varepsilon_{ps} - 0.007} \text{ (ksi)}$$

Figure 24-1 Stress-Strain Curve for Grade 270, Low Relaxation Strand[24.1]

18.2 GENERAL

The code specifies strength and serviceability requirements for all concrete members, prestressed or nonprestressed. This section requires that, for prestressed members, both strength and behavior at service conditions must be checked. All load stages that may be critical during the life of the structure, beginning with the transfer of the prestressing force to the member and including handling and transportation, must be considered.

This section also calls attention to several structural issues specific to prestressed concrete structures that must be considered in design:

18.2.3...Stress concentrations. See 18.13 for requirements for post-tensioned anchorages.

18.2.4...Compatibility of deformation with adjoining construction. An example of the effect of prestressing on adjoining parts of a structure is the need to include moments caused by axial shortening of prestressed floors in the design of the columns which support the floors.

18.2.5...Buckling of prestressed members. This section addresses the possibility of buckling of any part of a member where prestressing tendons are not in contact with the concrete. This can occur when prestressing steel is in an oversize duct, and with external prestressing described in 18.22.

18.2.6...Section properties. The code requires that the area of open post-tensioning ducts be deducted from section properties prior to bonding of prestressing tendons. For pretensioned members and post-tensioned members after grouting, the commentary allows the use of gross section properties, or effective section properties that may include the transformed area of bonded tendons and nonprestressed reinforcement.

18.3 DESIGN ASSUMPTIONS

In applying fundamental structural principles (equilibrium, stress-strain relations, and geometric compatibility) to prestressed structures, certain simplifying assumptions can be made. For computation of strength (18.3.1), the basic assumptions given for nonprestressed members in 10.2 apply, except that 10.2.4 applies only to nonprestressed reinforcement. For investigation of service load conditions, the "elastic theory" (referring to the linear variation of stress with strain) may be used. Where concrete is cracked, the concrete resists no tension. For analysis at service load conditions, the moduli of elasticity for concrete and nonprestressed reinforcement are given in 8.5. The modulus of elasticity for prestressed reinforcement is not given but can generally be taken as described in Fig. 24-1.

A new Section 18.3.3 defines three classes of prestressed flexural members, as follows:

Uncracked	Class U:	$f_t \le 7.5\sqrt{f'_c}$
Transition	Class T:	$7.5\sqrt{f'_c} < f_t \le 12\sqrt{f'_c}$
Cracked	Class C:	$f_t > 12\sqrt{f'_c}$

Table 24-1 summarizes the applicable requirements for these three classes of prestressed flexural members and, for comparison, for nonprestressed flexural members as well.

Class U and Class T members correspond to those designed by 18.4.2(c) and 18.4.2(d), respectively, of ACI 318-99 and earlier editions of the code. In ACI 318-99, 18.4.2(d) required deflections to be checked by a cracked section analysis if tensile stresses exceeded $6\sqrt{f'_c}$, but the section was not assumed to be cracked unless the stress exceeded $7.5\sqrt{f'_c}$. This inconsistency was eliminated by setting the dividing tensile stress between Classes U and T at $7.5\sqrt{f'_c}$.

The new Class C permits design using any combination of prestressing steel and reinforcement. It "fills the gap" between prestressed and nonprestressed concrete. For Class C members, a cracked section analysis or stresses is required by 18.3.4; whereas, for Class T members, an approximate cracked section analysis is required for deflection only. Unfortunately, a cracked section stress analysis for combined flexure and axial load (from the prestress) is complex. Reference 24.2 gives one method of accomplishing this.

Section 18.3.3 requires that prestressed two-way slab systems be designed as Class U. ACI 318-99 had a similar requirement.

Table 24-1 Serviceability Design Requirements

	Prestressed			Nonprestressed
	Class U	Class T	Class C	
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 18.3.4	Gross section 18.3.4	Cracked section 18.3.4	No requirement
Allowable stress at transfer	18.4.1	18.4.1	18.4.1	No requirement
Allowable compressive stress based on uncracked section properties	18.4.2	18.4.2	No requirement	No requirement
Tensile stress at service loads 18.3.3	$\le 7.5\sqrt{f'_c}$	$7.5\sqrt{f'_c} < f_t \le 12\sqrt{f'_c}$	No requirement	No requirement
Deflection calculation basis	9.5.4.1 Gross section	9.5.4.2 Cracked section, bilinear	9.5.4.2 Cracked section, bilinear	9.5.2, 9.5.3 Effective moment of inertia
Crack control	No requirement	No requirement	10.6.4 Modified by 18.4.4.1	10.6.4
Computation of Δf_{ps} or f_s for crack control	—	—	Cracked section analysis	M/(A_s x lever arm), or $0.6f_y$
Side skin reinforcement	No requirement	No requirement	10.6.7	10.6.7

18.4 SERVICEABILITY REQUIREMENTS — FLEXURAL MEMBERS

Both concrete and prestressing tendon stresses are limited to ensure satisfactory behavior immediately after transfer of prestress and at service loads. The code provides different permissible stresses for conditions immediately after prestress transfer (before time-dependent losses) and for conditions at service loads (after all prestress losses have occurred).

For conditions immediately after prestress transfer, the code allows: extreme fiber compressive stress of $0.60f'_{ci}$; extreme fiber tensile stress of $3\sqrt{f'_{ci}}$, except $6\sqrt{f'_{ci}}$ is permitted at the ends of simply supported members. Where tensile stress exceeds the permissible values, bonded nonprestressed reinforcement shall be provided to resist the total tensile force assuming an uncracked section.

The permissible compressive stress due to prestress plus total service loads is limited to $0.60f'_c$. A permissible stress equal to $0.45f'_c$ has been added for the condition of prestress plus sustained loads. It should be noted that the "sustained loads" mentioned in 18.4.2(a) include any portion of the live load that will be sustained for a sufficient period to cause significant time-dependent deflections.

Concrete tensile stress limitations for Class U and T at service loads apply to the "precompressed" tensile zone which is that portion of the member cross-section in which flexural tension occurs under dead and live loads.

For Class C prestressed members, crack control is accomplished through a steel spacing requirement based on that of 10.6.4 and Eq. (10-4) for nonprestressed concrete. Eq. (10-4) is modified by 18.4.4. The maximum spacing between tendons is reduced to 2/3 of that permitted for bars, to account for lesser bond, compared to deformed bars. The quantity Δf_{ps}, the stress in the prestressing steel at service loads less the decompression stress f_{dc} is the stress in the prestressing steel when the stress is zero in the concrete at the same level as the centroid of the tendons. The code permits f_{dc} to be conservatively taken as the effective prestress f_{se}. The following shows Eq. (10-4), and as modified by 18.4.4.

Eq. (10-4) in 10.6.4:

$$s = \frac{540}{f_s} - 2.5c_c$$

As modified by 18.4.4:

$$s = \frac{2}{3}\left(\frac{540}{\Delta f_{ps}} - 2.5c_c\right)$$

The quantity of Δf_{ps} shall not exceed 36 ksi. If Δf_{ps} is not greater than 20 ksi, the above spacing limits need not apply.

When both reinforcement and bonded tendons are used to meet the spacing requirement, the spacing between a bar and a tendon shall not exceed 5/6 of that given by Eq. (10-4).

If the effective depth of a Class C beam exceeds 36 in., skin reinforcement consisting of reinforcement or bonded tendons shall be provided as required by 10.6.7.

18.5 PERMISSIBLE STRESSES IN PRESTRESSING STEEL

The permissible tensile stresses in all types of prestressing steel, in terms of the specified minimum tensile strength f_{pu}, are summarized as follows:

a. Due to tendon jacking force: ..... $0.94f_{py}$ but not greater than $0.80f_{pu}$
low-relaxation wire and strands ($f_{py} = 0.90f_{pu}$) ..... $0.80f_{pu}$
stress-relieved wire and strands, and plain bars (ASTM A722) ($f_{py} = 0.85f_{pu}$) ..... $0.80f_{pu}$
deformed bars (ASTM A722) ($f_{py} = 0.80f_{pu}$) ..... $0.75f_{pu}$

b. Immediately after prestress transfer: ..... $0.82f_{py}$ but not greater than $0.74f_{pu}$
low-relaxation wire and strands ($f_{py} = 0.90f_{pu}$) ..... $0.74f_{pu}$
stress-relieved wire and strands, and plain bars ($f_{py} = 0.85f_{pu}$) ..... $0.70f_{pu}$
deformed bars ($f_{py} = 0.80f_{pu}$) ..... $0.66f_{pu}$

c. Post-tensioning tendons, at anchorages and couplers, immediately after tendon anchorage .... $0.70f_{pu}$

Note that the permissible stresses given in 18.5.1(a) and (b) apply to both pretensioned and post-tensioned tendons. Pretensioned tendons are often jacked to 75 percent of f_{pu}. This will result in a stress below 0.74 f_{pu} after transfer.

18.6 LOSS OF PRESTRESS

A significant factor which must be considered in design of prestressed members is the loss of prestress due to various causes. These losses can dramatically affect the behavior of a member at service loads. Although calculation procedures and certain values of creep strain, friction factors, etc., may be recommended, they are at best only an estimate. For the design of members whose behavior (deflection in particular) is sensitive to prestress losses, the engineer should establish through tests the time-dependent properties of materials to be used in the analysis/design of the structure. Refined analyses should then be performed to estimate the prestress losses. Specific provisions for computing friction loss in post-tensioning tendons are provided in 18.6.2. Allowance for other types of prestress losses are discussed in Ref. 24.1. Note that the designer is required to show on the design drawings the magnitude and location of prestressing forces as required by 1.2.1(g).

ESTIMATING PRESTRESS LOSSES

Lump sum values of prestress losses that were widely used as a design estimate of prestress losses prior to the '83 code edition (35,000 psi for pretensioning and 25,000 psi for post-tensioning) are now considered obsolete. Also, the lump sum values may not be adequate for some design conditions.

Reference 24.3 offers guidance to compute prestress losses and it is adaptable to computer programs. It allows step-by-step computation of losses which is necessary for rational analysis of deformations. The method is too tedious for hand calculations.

Reference 24.4 presents a reasonably accurate and easy procedure for estimating prestress losses due to various causes for pretensioned and post-tensioned members with bonded and unbonded tendons. The procedure, which is intended for practical design applications under normal design conditions, is summarized below. The simple equations enable the designer to estimate the prestress loss from each source rather than using a lump sum value. The reader is referred to Ref. 24.4 for an in-depth discussion of the procedure, including sample computations for typical prestressed concrete beams. Quantities used in loss computations are defined in the summary of notation which follows this section.

COMPUTATION OF LOSSES

Elastic Shortening of Concrete (ES)

For members with bonded tendons:

$$ES = K_{es} E_s \frac{f_{cir}}{E_{ci}} \qquad (1)$$

where $K_{es} = 1.0$ for pretensioned members

$K_{es} = 0.5$ for post-tensioned members where tendons are tensioned in sequential order to the same tension. With other post-tensioning procedures, the value for K_{es} may vary from 0 to 0.5.

$$f_{cir} = K_{cir} f_{cpi} - f_g \qquad (2)$$

where $K_{cir} = 1.0$ for post-tensioned members

$K_{cir} = 0.9$ for pretensioned members.

For members with unbonded tendons:

$$ES = K_{es} E_s \frac{f_{cpa}}{E_{ci}} \qquad (1a)$$

Creep of Concrete (CR)

For members with bonded tendons:

$$CR = K_{cr} \frac{E_s}{E_c} (f_{cir} - f_{cds}) \qquad (3)$$

where $K_{cr} = 2.0$ for pretensioned members

$K_{cr} = 1.6$ for post-tensioned members

For members made of sand lightweight concrete the foregoing values of K_{cr} should be reduced by 20 percent.

For members with unbonded tendons:

$$CR = K_{cr} \frac{E_s}{E_c} f_{cpa} \qquad (3a)$$

Shrinkage of Concrete (SH)

$$SH = 8.2 \times 10^{-6} K_{sh} E_s \left(1 - 0.06 \frac{V}{S}\right) (100 - RH) \qquad (4)$$

where K_{sh} = 1.0 for pretensioned members

K_{sh} is taken from Table 24-2 for post-tensioned members.

Table 24-2 Values of K_{sh} for Post-Tensioned Members

Time, days*	1	3	5	7	10	20	30	60
K_{sh}	0.92	0.85	0.80	0.77	0.73	0.64	0.58	0.45

**Time after end of moist curing to application of prestress*

Relaxation of Tendons (RE)

$$RE = [K_{re} - J (SH + CR + ES)] C \qquad (5)$$

where the values of K_{re}, J, and C are taken from Tables 24-3 and 24-4.

Table 24-3 Values of K_{re} and J

Type of Tendon	Kre (psi)	J
270 Grade stress-relieved strand or wire	20,000	0.15
250 Grade stress-relieved strand or wire	18,500	0.14
240 or 235 Grade stress-relieved wire	17,600	0.13
270 Grade low-relaxation strand	5000	0.040
250 Grade low-relaxation wire	4630	0.037
240 or 235 Grade low-relaxation wire	4400	0.035
145 or 160 Grade stress-relieved bar	6000	0.05

Table 24-4 Values of C

f_{pi}/f_{pu}	Stressed relieved strand or wire	Stress-relieved bar or low relaxation strand or wire
0.80		1.28
0.79		1.22
0.78		1.16
0.77		1.11
0.76		1.05
0.75	1.45	1.00
0.74	1.36	0.95
0.73	1.27	0.90
0.72	1.18	0.85
0.71	1.09	0.80
0.70	1.00	0.75
0.69	0.94	0.70
0.68	0.89	0.66
0.67	0.83	0.61
0.66	0.78	0.57
0.65	0.73	0.53
0.64	0.68	0.49
0.63	0.63	0.45
0.62	0.58	0.41
0.61	0.53	0.37
0.60	0.49	0.33

Friction

Computation of friction losses is covered in 18.6.2. When the tendon is tensioned, the friction losses computed can be checked with reasonable accuracy by comparing the measured tendon elongation and the prestressing force applied by the tensioning jack.

SUMMARY OF NOTATION

A_c = area of gross concrete section at the cross-section considered
A_{ps} = total area of prestressing steel
C = a factor used in Eq. (5), see Table 24-3
CR = stress loss due to creep of concrete
e = eccentricity of center of gravity of prestressing steel with respect to center of gravity of concrete at the cross-section considered
E_c = modulus of elasticity of concrete at 28 days
E_{ci} = modulus of elasticity of concrete at time prestress is applied
E_s = modulus of elasticity of prestressing steel. Usually 28,000,000 psi
ES = stress loss due to elastic shortening of concrete
f_{cds} = stress in concrete at center of gravity of prestressing steel due to all superimposed permanent dead loads that are applied to the member after it has been prestressed
f_{cir} = net compressive stress in concrete at center of gravity of prestressing steel immediately after the prestress has been applied to the concrete. See Eq. (2).
f_{cpa} = average compressive stress in the concrete along the member length at the center of gravity of the prestressing steel immediately after the prestress has been applied to the concrete
f_{cpi} = stress in concrete at center of gravity of prestressing steel due to P_{pi}

f_g = stress in concrete at center of gravity of prestressing steel due to weight of structure at time prestress is applied

f_{pi} = stress in prestressing steel due to P_{pi}, = P_{pi}/A_{ps}

f_{pu} = specified tensile strength of prestressing steel, psi

I_c = moment of inertia of gross concrete section at the cross-section considered

J = a factor used in Eq. (5), see Table 24-2

K_{cir} = a factor used in Eq. (2)

K_{cr} = a factor used in Eq. (3)

K_{es} = a factor used in Eqs. (1) and (1a)

K_{re} = a factor used in Eq. (5), see Table 24-2

M_d = bending moment due to dead weight of member being prestressed and to any other permanent loads in place at time of prestressing

M_{ds} = bending moment due to all superimposed permanent dead loads that are applied to the member after it has been prestressed

P_{pi} = prestressing force in tendons at critical location on span after reduction for losses due to friction and seating loss at anchorages but before reduction for ES, CR, SH, and RE

RE = stress loss due to relaxation of prestressing steel

RH = average relative humidity surrounding the concrete member (see Fig. 24-1)

SH = stress loss due to shrinkage of concrete

V/S = volume to surface ratio, usually taken as gross cross-sectional area of concrete member divided by its perimeter

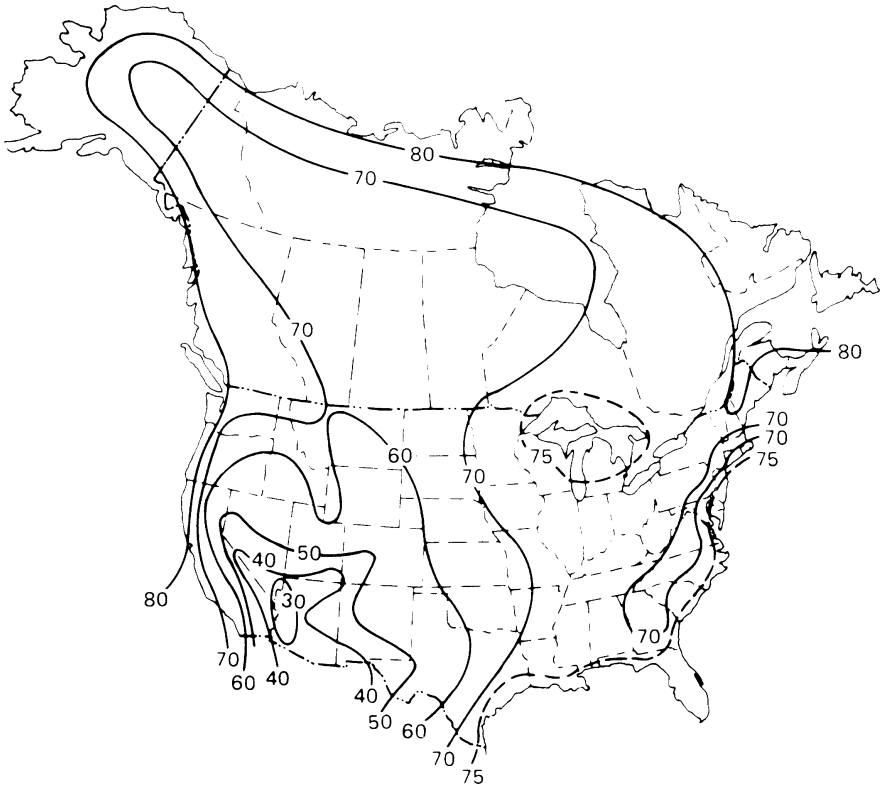


Figure 24-2 Annual Average Ambient Relative Humidity

18.7 FLEXURAL STRENGTH

The flexural strength of prestressed members can be calculated using the same assumptions as for nonprestressed members. Prestressing steel, however, does not have a well defined yield point as does mild reinforcement. As a prestressed cross-section reaches its flexural strength (defined by a maximum compressive concrete strain of

0.003), stress in the prestressed reinforcement at nominal strength, f_{ps}, will vary depending on the amount of prestressing. The value of f_{ps} can be obtained using the conditions of equilibrium, stress-strain relations, and strain compatibility (Design Example 24-4 illustrates the procedure). However, the analysis is quite cumbersome, especially in the case of unbonded tendon. For bonded prestressing, the compatibility of strains can be considered at an individual section, while for unbonded tendon, compatibility relations can be written only at the anchorage points and will depend on the entire cable profile and member loading. To avoid such lengthy calculations, the code allows f_{ps} to be obtained by the approximate Eqs. (18-3), (18-4), and (18-5).

For members with bonded prestressing steel, an approximate value of f_{ps} given by Eq. (18-3) may be used for flexural members reinforced with a combination of prestressed and nonprestressed reinforcement (partially prestressed members), taking into account effects of any nonprestressed tension reinforcement (ω), any compression reinforcement (ω'), the concrete compressive strength f'_c, rectangular stress block factor β_1, and an appropriate factor for type of prestressing material used (γ_p). For a fully prestressed member (with no nonprestressed tension or compression reinforcement), Eq. (18-3) reduces to:

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\rho_p\frac{f_{pu}}{f'_c}\right)$$

where γ_p = 0.55 for deformed bars $\left(f_{py}/f_{pu} \geq 0.80\right)$

= 0.40 for stress-relieved wire and strands, and plain bars $\left(f_{py}/f_{pu} \geq 0.85\right)$

= 0.28 for low-relaxation wire and strands $\left(f_{py}/f_{pu} \geq 0.90\right)$

and β_1, as defined in 10.2.7.3,

β_1 = 0.85 for $f'_c \leq$ 4000 psi
= 0.80 for f'_c = 5000 psi
= 0.75 for f'_c = 6000 psi
= 0.70 for f'_c = 7000 psi
= 0.65 for $f'_c \geq$ 8000 psi

Eq. (18-3) can be written in nondimensional form as follows:

$$\omega_p = \omega_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\omega_{pu}\right) \qquad (6)$$

where $$\omega_p = \frac{A_{ps}f_{ps}}{bd_pf'_c} \qquad (7)$$

$$\omega_{pu} = \frac{A_{ps}f_{pu}}{bd_pf'_c} \qquad (8)$$

The moment strength of a prestressed member with bonded tendons may be computed using Eq. (18-3) only when all of the prestressed reinforcement is located in the tension zone. When part of the prestressed reinforcement is located in the compression zone of a cross-section, Eq. (18-3), involving d_p, is not valid. Flexural strength for such a condition must be computed by a general analysis based on strain compatibility and equilibrium, using the stress-strain properties of the prestressing steel and the assumptions given in 10.2.

For members with unbonded prestressing steel, an approximate value of f_{ps} given by Eqs. (18-4) and (18-5) may be used. Eq. (18-5) applies to members with high span-to-depth ratios (> 35), such as post-tensioned one-way slabs, flat plates and flat slabs.

With the value of f_{ps} known, the nominal moment strength of a rectangular section, or a flanged section where the stress block is within the compression flange, can be calculated as follows:

$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) = A_{ps}f_{ps}\left(d_p - 0.59\,\frac{A_{ps}f_{ps}}{bf'_c}\right) \qquad (9)$$

where a = the depth of the equivalent rectangular stress block $= \dfrac{A_{ps}f_{ps}}{0.85bf'_c}$ (10)

or in nondimensional terms:

$$R_n = \omega_p\,(1 - 0.59\omega_p) \qquad (11)$$

where $R_n = \dfrac{M_n}{b(d_p)^2 f'_c}$ (12)

18.8 LIMITS FOR REINFORCEMENT OF FLEXURAL MEMBERS

The former reinforcement limit of $0.36\beta_1$ used in the 1999 Code and earlier edition has been replaced by the classification of tension-controlled, transition, or compression-controlled based on net tensile strain. These classifications are defined in 10.3.3 and 10.3.4, with appropriate ϕ-factors in 9.3.2. These requirements are the same as those for nonprestressed concrete.

Figure 24-3 shows the relationship between the coefficient of resistance $\phi M_n/(bd^2)$ and the reinforcement ratio ρ_p for prestressed flexural members. Grade 270 ksi prestressing steel has a useful strength 4.5 times that of Grade 60 reinforcement. Compare Fig. 24-3 to Fig. 7-3. Higher concrete strengths are normally used with prestressed concrete, so Fig. 24-3 shows curves for f'_c from 5000 to 8000 psi; whereas, Fig. 7-3 shows curves for f'_c from 3000 to 6000 psi. The curves for f'_c of 5000 and 6000 psi are almost identical in the two figures.

In both figures, the curves have a break point corresponding to a net tensile strain of 0.005. Beyond that point, the reduction in ϕ in the transition region almost cancels the benefit of increased reinforcement index. For both nonprestressed and prestressed concrete, the best design is to stay in the tension-controlled region, using compression reinforcement, if necessary, to maintain the net tensile strain, $\boldsymbol{\varepsilon}_t$ at 0.005 or more.

As in previous ACI 318 codes, there is no absolute limit on the reinforcement index for prestressed members. But it will always be advantageous to design the tension-controlled region at critical sections, as there is little or no gain in design strength in the transition region.

Critical parameters at the tension-controlled limit may be tabulated. The effective prestress f_{se} will normally be at least 0.6 f_{pu}, or 162 ksi, if a jacking stress of 0.75 f_{pu} is used. This amounts to a 20 percent loss. The total steel strain when $\boldsymbol{\varepsilon}_t$ = 0.005 is equal to 162/28,500 + 0.005 = 0.01068. Using this equation for steel stress shown in Fig. 24-1, f_{ps} = 270 - 0.04/(0.01068-0.007) = 259 ksi. This is valid when d_t is taken equal to d_p.

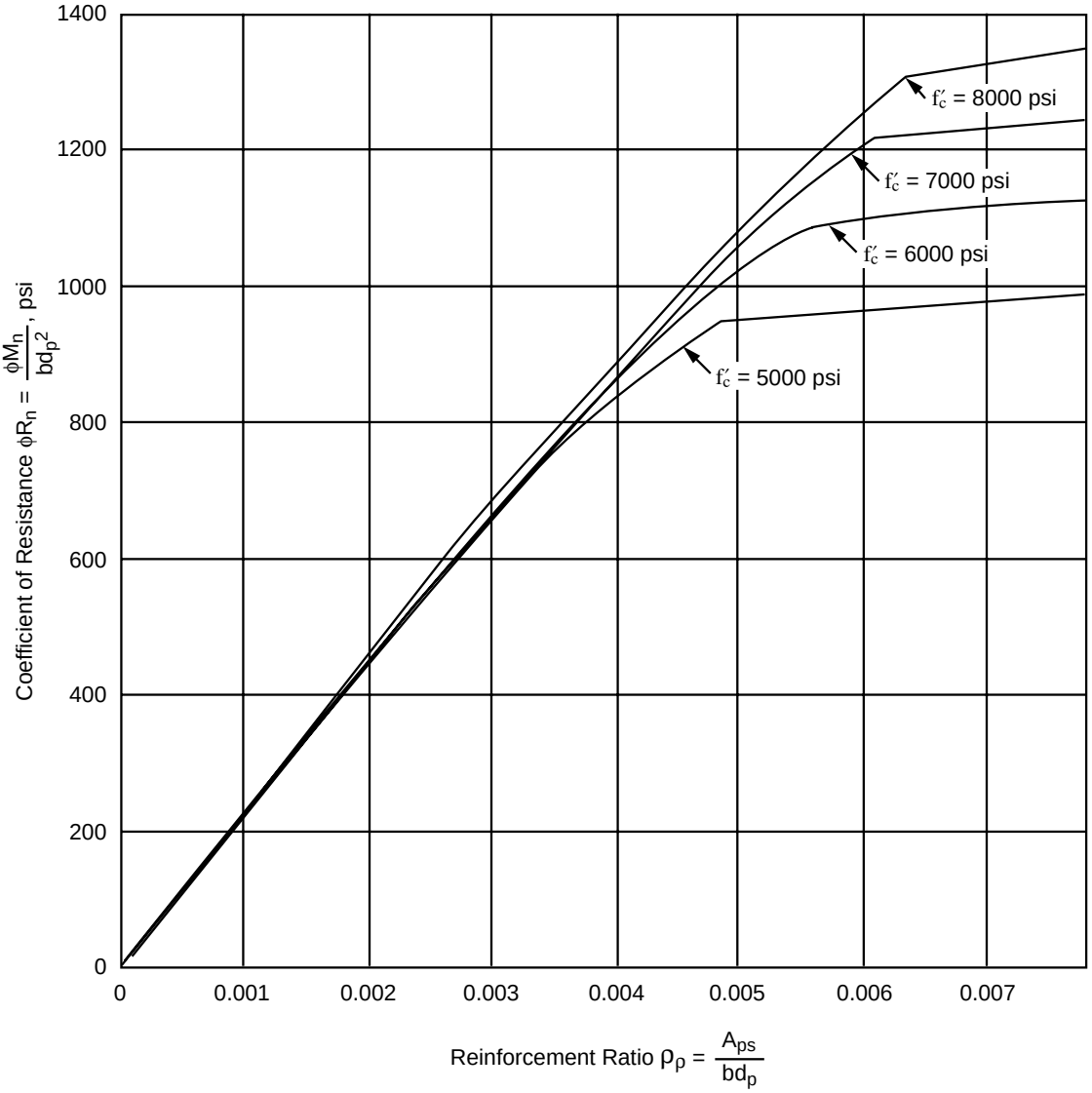


Figure 24-3 Design Strength Curves (ϕR_n vs. ρ_p) for Type 270k Low Relaxation Strand

Table 24-5 shows design parameters for prestressed sections at the tension-controlled strain limit, indicated by the added subscript t. The rows for R_{nt}, ϕ_{nt}, and ω_{pt} are identical to those in Table 6-1. The row for ω_{put} shows values slightly higher than ω_{pt}, because ω_{put} is based on f_{pu} of 270 ksi; whereas, ω_{put} is based on f_{ps} of 259 ksi. The final row for ρ_{pt} shows values much smaller than for ρ_t in Table 6-1, because of the much higher strength of the prestressing strand.

The following is a short-cut procedure for finding the flexural strength of sections in which the Grade 270 ksi low-relaxation prestressing steel can reasonably be assumed to be in one layer with $d_p = d_t$, and with $f_{se} \geq 162$ ksi.

1. Assume section is at tension-controlled limit, and $f_{ps} = 259$ ksi.
2. Compute steel tension T and equal compressive force C.
3. Find depth of stress block a and depth to neutral axis c.
4. Is $c/d_p \leq 0.375$? If so, proceed. If not, add compression steel to make $c/d_p \leq 0.375$.
5. Compute provided design strength $\phi M_n = 0.9(T)(d-a/2)$.

Table 24-5 Design Parameters at Strain Limit of 0.005 for Tension-Controlled Sections

	f'_c = 3000 β_1 = 0.85	f'_c = 4000 β_1 = 0.85	f'_c = 5000 β_1 = 0.80	f'_c = 6000 β_1 = 0.75	f'_c = 8000 β_1 = 0.65	f'_c = 10,000 β_1 = 0.65
R_{nt}	683	911	1084	1233	1455	1819
ϕR_{nt}	615	820	975	1109	1310	1637
ω_{pt}	0.2709	0.2709	0.2550	0.2391	0.2072	0.2072
ω_{put}	0.2823	0.2823	0.2657	0.2491	0.2159	0.2159
ρ_{pt}	0.00314	0.00418	0.00492	0.00554	0.00640	0.00800

For $f_{se} \geq 162$ ksi in low-relaxation Grade 270 ksi strand

6. If provided $\phi M_n \geq$ required, stop. Section is adequate. If not proceed.

7. If the deficiency in provided ϕM_n is more than 4 percent, steel must be added. If deficiency is less than 4 percent, strain compatibility may be used in an attempt to find a higher f_{ps} in order to justify adequacy of the section.

Section 18.8.2 requires the total amount of prestressed and nonprestressed reinforcement of flexural members to be adequate to develop a design moment strength at least equal to 1.2 times the cracking moment strength ($\phi M_n = 1.2 M_{cr}$), where M_{cr} is computed by elastic theory using a modulus of rupture equal to $7.5\sqrt{f'_c}$. The provisions of 18.8.2 are analogous to 10.5 for nonprestressed members. They are intended as a precaution against abrupt flexural failure resulting from rupture of the prestressing tendons immediately after cracking. The provision ensures that cracking will occur before flexural strength is reached, and by a large enough margin so that significant deflection will occur to warn that the ultimate capacity is being approached. The typical bonded prestressed member will have a fairly large margin between cracking strength and flexural strength, but the designer must be certain by checking it.

The cracking moment M_{cr} for a prestressed member is determined by summing all the moments that will cause a stress in the bottom fiber equal to the modulus of rupture f_r. Referring to Fig. 24-4 for an unshored prestressed composite member taking compression as negative and tension as positive:

$$-\left(\frac{P_{se}}{A_c}\right) - \left(\frac{P_{se}e}{S_b}\right) + \left(\frac{M_d}{S_b}\right) + \left(\frac{M_a}{S_c}\right) = +f_r$$

Solving for $M_a = \left(f_r + \frac{P_{se}}{A_c} + \frac{P_{se}e}{S_b}\right) S_c - M_d \left(\frac{S_c}{S_b}\right)$

Since $M_{cr} = M_d + M_a$

$$M_{cr} = \left(f_r + \frac{P_{se}}{A_c} + \frac{P_{se}e}{S_b}\right) S_c - M_d \left(\frac{S_c}{S_b} - 1\right) \qquad (13)$$

For a prestressed member alone (without composite slab), $S_c = S_b$. Therefore, M_{cr} reduces to

$$M_{cr} = \left(f_r + \frac{P_{se}}{A_c}\right) S_b + P_{se}e \qquad (14)$$

Examples 24.6 and 24.7 illustrate computation of the cracking moment strength of prestressed members.

Note that an exception in 18.8.2 waives the $1.2M_{cr}$ requirement for (a) two-way, unbonded post-tensioned slabs, and (b) flexural members with shear and flexural strength at least twice that required by 9-2.

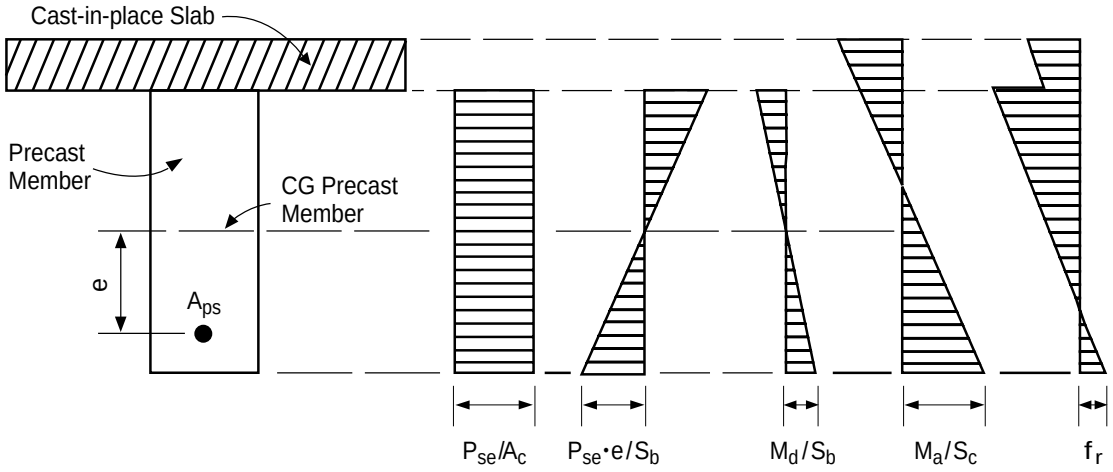
For flexural strength: $\phi M_n \geq 2M_u \geq 2\,(1.2M_d + 1.6M\ell)$

For shear strength: $\phi V_n \geq 2V_u \geq 2\,(1.2V_d + 1.6V\ell)$

The $1.2M_{cr}$ provision often requires excessive reinforcement for certain prestressed flexural members especially for short span hollow-core members. The exception is intended to limit the amount of additional reinforcement required to amounts that provide for ductility, and is comparable in concept to those for nonprestressed members in 10.5.3.

Introduced in the 1999 edition of the code, the waiver of the $1.2M_{cr}$ provision for two-way, unbonded post-tensioned slabs brings the code in line with current practices which have been shown to be technically sound and safe (Ref. 24.5).

Section 18.8.3 of ACI 318-02 introduces a new qualitative requirement stating that some bonded reinforcement or tendons must be placed as close to the tension face as is practicable.



A_{ps} = area of prestressed reinforcement in tensile zone
A_c = area of precast member
S_b = section modulus for bottom of precast member
S_c = section modulus for bottom of composite member
P_{se} = effective prestress force
e = eccentricity of prestress force
M_d = dead load moment of composite member
M_a = additional moment to cause a stress in bottom fiber equal to modulus of rupture f_r

Figure 24-4 Stress Conditions for Evaluating Cracking Moment Strength

18.9 MINIMUM BONDED REINFORCEMENT

A minimum amount of bonded reinforcement is desirable in members with unbonded tendons. Reference to R18.9 is suggested.

For all flexural members with unbonded prestressing tendons, except two-way solid slabs, a minimum area of bonded reinforcement computed by Eq. (18-6) must be uniformly distributed over the precompressed tensile zone as close as practicable to the extreme tension fiber. Figure 24-6 illustrates application of Eq. (18-6).

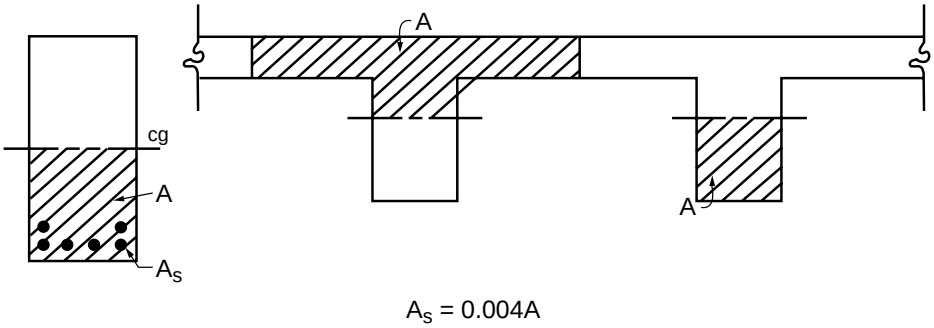


Figure 24-5 Bonded Reinforcement for Flexural Members

For solid slabs, the special provisions of 18.9.3 apply. Depending on the tensile stress in the concrete at service loads, the requirements for positive moment areas of solid slabs are illustrated in Fig. 24-5(a). Formerly, 18.9.3 applied only to flat plates. Starting with ACI 318-02, it also applies to two-way flat slab systems with drop panels.

The requirement for minimum area of bonded reinforcement in two-way flat plates at column supports was revised in the 1999 code edition to reflect the intent of the original research recommendations (Ref. 24.5). This revision increases the minimum reinforcement requirement over interior columns for rectangular panels in one direction, and, for square panels, doubles the minimum reinforcement requirement over exterior columns normal to the slab edge. Figure 24-6(b) illustrates the minimum bonded reinforcement requirements for the negative moment areas at column supports. The bonded reinforcement must be located within the width $c_2 + 2\,(1.5h)$ as shown, with a minimum of four bars spaced at not more than 12 in. Similarly, minimum bonded reinforcement should be provided parallel to slab edge.

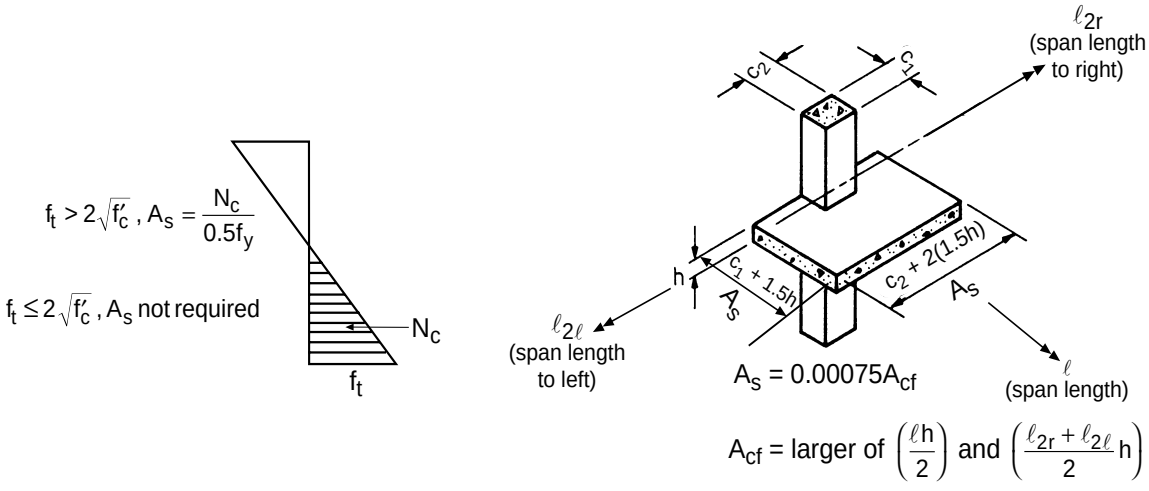


Figure 24-6 Bonded Reinforcement for Flat Plates

18.10.4 Moment Redistribution

The former special provisions for moment redistribution in continuous prestressed flexural members are now replaced by a reference to 8.4, which applies equally to prestressed and nonprestressed members.

18.11 COMPRESSION MEMBERS — COMBINED FLEXURE AND AXIAL LOADS

Provisions of the code for calculating the strength of prestressed members are the same as for members without prestressing. Additional considerations include (1) accounting for prestressing strains, and (2) using an appropriate stress-strain relation for the prestressing tendons. Example 24.7 illustrates the calculation procedure.

For compression members with an average concrete stress due to prestressing of less than 225 psi, minimum nonprestressed reinforcement must be provided (18.11.2.1). For compression members with an average concrete stress due to prestressing equal to or greater than 225 psi, 18.11.2.2 requires that all prestressing tendons be enclosed by spirals or lateral ties, except for walls.

REFERENCES

24.1 "PCI Design Handbook – Precast and Prestressed Concrete," MNL 120-92 5th Edition, Precast/Prestressed Concrete Institute, Chicago, 1999, 630 pp.

24.2 Mast, R. F., "Analysis of Cracked Prestressed Sections: A Practical Approach," *PCI Journal*, Vol. 43, No. 4, July-August 1975, pp. 43-75.

24.3 PCI Committee on Prestress Losses, "Recommendations for Estimating Prestress Losses," *PCI Journal*, Vol. 20, No. 4, July-August 1975, pp. 43-75.

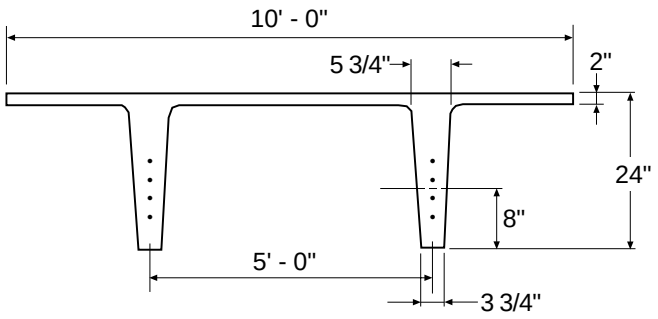
24.4 Zia, Paul, et al., "Estimating Prestress Losses," *Concrete International: Design and Construction*, Vol. 1, No. 6, June 1979, pp. 32-38.

24.5 ACI 423.3R-96 Report, "Recommendations for Concrete Members Prestressed with Unbonded Tendons," American Concrete Institute, Detroit, Michigan.

Example 24.1—Estimating Prestress Losses

For the simply supported double-tee shown below, estimate loss of prestress using the procedures of Ref. 24.4, as outlined earlier under "Computation of Losses." Assume the unit is manufactured in Green Bay, WI.

live load = 40 psf
roof load = 20 psf
dead load = 47 psf = 468 plf
span = 48 ft
f'_{ci} = 3500 psi
f'_c = 5000 psi
8 - 0.5 in. diameter low-relaxation strands
A_{ps} = 8 (0.153 in.^{2}) = 1.224 in.^{2}
e = 9.77 in. (all strands straight)
f_{pu} = 270,000 psi
f_{py} = 0.90f_{pu}
jacking stress = 0.74f_{pu} = 200 ksi



Assume the following for loss computations:
E_{ci} = 3590 ksi
E_c = 4290 ksi
E_s = 28,500 ksi

Section Properties
A_c = 449 in.^{2}
I_c = 22,469 in.^{4}
y_b = 17.77 in.
y_t = 6.23 in.
V/S = 1.35 in.

Calculations and Discussion	Code Reference

1. Elastic Shortening of Concrete (ES); using Eq. (1) of Part 26

$$ES = K_{es}E_s \frac{f_{cir}}{E_{ci}} = 1.0\,(28{,}500)\,\frac{0.725}{3590} = 5.8 \text{ ksi}$$

where

K_{es} = 1.0 for pretensioned members

$f_{cir} = K_{cir}f_{cpi} - f_g$

$$= K_{cir}\left(\frac{P_{pi}}{A_c} + \frac{P_{pi}e^2}{I_c}\right) - \frac{M_d e}{I_c}$$

$$= 0.9\left(\frac{245}{449} + \frac{245 \times 9.77^2}{22{,}469}\right) - \frac{1617 \times 9.77}{22{,}469} = 0.725 \text{ ksi}$$

K_{cir} = 0.9 for pretensioned members

$P_{pi} = 0.74 f_{pu} A_{ps} = 0.74\ (270)\ (1.224) = 245$ kips

$M_d = 0.468 \times 48^2 \times \frac{12}{8} = 1617$ in.-kips (dead load of unit)

2. Creep of Concrete (CR); using Eq. (3)

$CR = K_{cr} \frac{E_s}{E_c} (f_{cir} - f_{cds}) = 2.0 \times \frac{28{,}500}{4290} (0.725 - 0.30) = 5.6$ ksi

where $f_{cds} = M_{ds} \frac{e}{I_c} = 691 \times \frac{9.77}{22{,}469} = 0.30$ ksi

$M_{ds} = 0.02 \times 10 \times 48^2 \times \frac{12}{8} = 691$ in.-kips (roof load only)

and $K_{cr} = 2.0$ for pretensioned members.

3. Shrinkage of Concrete (SH); using Eq. (4)

$SH = 8.2 \times 10^{-6} K_{sh} E_s \left(1 - 0.06 \frac{V}{S}\right) (100 - RH)$

$= 8.2 \times 10^{-6} \times 1.0 \times 28{,}500\ (1 - 0.06 \times 1.35)\ (100 - 75) = 5.4$ ksi

RH = average relative humidity surrounding the concrete member from Fig. 24-1. For Green Bay, Wisconsin, RH = 75%

and $K_{sh} = 1.0$ for pretensioned members.

4. Relaxation of Tendon Stress (RE); using Eq. (5)

$RE = [K_{re} - J\ (SH + CR + ES)]\ C$

$= [5 - 0.04\ (5.4 + 5.6 + 5.8)]\ 0.95 = 4.1$ ksi

where, for 270 Grade low-relaxation strand:

$K_{re} = 5$ ksi (Table 24-3)

$J = 0.040$ (Table 24-3)

$C = 0.95$ (Table 24-4 for $\frac{f_{pi}}{f_{pu}} = 0.74$)

5. Total allowance for loss of prestress

$ES + CR + SH + RE = 5.8 + 5.6 + 5.4 + 4.1 = 20.9$ ksi *18.6.1*

6. Stress, f_p, and force, P_p, immediately after transfer.

   Assume that one-fourth of relaxation loss occurs prior to release.

   $f_p = 0.74f_{pu}$ - (ES + 1/4 RE)

   $= 0.74\ (270) - (5.8 + 1/4\ (4.1)) = 193.0$ ksi

   $P_p = f_pA_{ps} = 193.0 \times 1.224 = 236$ kips

7. Effective prestress stress f_{se} and effective prestress force P_e after all losses

   $f_{se} = 0.74f_{pu}$ - allowance for all prestress losses

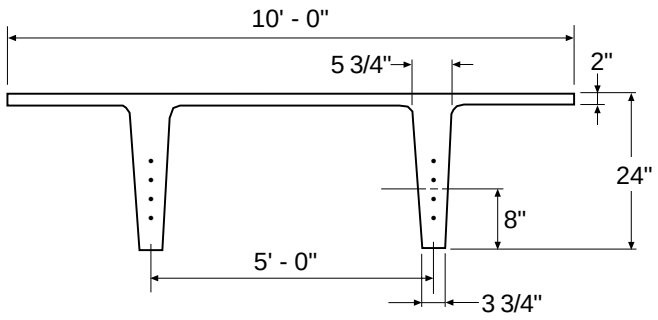
   $= 0.74\ (270) - 20.9 = 179$ ksi

   $P_e = f_{se}A_{ps} = 179 \times 1.224 = 219$ kips

Example 24.2—Investigation of Stresses at Prestress Transfer and at Service Load

For the simply supported double-tee considered in Example 24.1, check all permissible concrete stresses immediately after prestress transfer and at service load assuming the unit is used for roof framing. Use losses computed in Example 24.1.

live load = 40 psf
roof load = 20 psf
dead load = 47 psf = 468 plf
span = 48 ft
f'_{ci} = 3500 psi
f'_c = 5000 psi
8 - 0.5 in. diameter low-relaxation strands
A_{ps} = 8 (0.153 in.^{2}) = 1.224 in.^{2}
e = 9.77 in. (all strands straight)
f_{pu} = 270,000 psi
f_{py} = $0.90f_{pu}$
jacking stress = $0.74f_{pu}$ = 200 ksi
stress after transfer = 193 ksi
force after transfer = P_p = 1.224 × 193 = 236 kips



Section Properties
A_c = 449 in.^{2}
I_c = 22,469 in.^{4}
y_b = 17.77 in.
y_t = 6.23 in.
V/S = 1.35 in.

Calculations and Discussion	Code Reference
1. Calculate permissible stresses in concrete.	18.4
At prestress transfer (before time-dependent losses):	18.4.1
Compression: $0.60f'_{ci}$ = 0.60(3500) = 2100 psi	
Tension: $6\sqrt{f'_{ci}}$ = 355 psi (at ends of simply supported members; otherwise $3\sqrt{f'_{ci}}$)	
At service load (after allowance for all prestress losses):	18.4.2
Compression: $0.45f'_c$ = 2250 psi - Due to sustained loads	
Compression: $0.60f'_c$ = 3000 psi - Due to total loads	
Tension: $12\sqrt{f'_c}$ = 849 psi	18.3.3(b)
2. Calculate service load moments at midspan:	

$$M_d = \frac{w_d \ell^2}{8} = \frac{0.468 \times 48^2}{8} = 134.8 \text{ ft-kips (beam dead load)}$$

$$M_{ds} = \frac{w_{ds} \ell^2}{8} = \frac{0.02 \times 10 \times 48^2}{8} = 57.6 \text{ ft-kips (roof dead load)}$$

$$M_{sus} = M_d + M_{ds} = 134.8 + 57.6 = 192.4 \text{ ft-kips (sustained load)}$$

$$M_\ell = \frac{w_\ell \ell^2}{8} = \frac{0.04 \times 10 \times 48^2}{8} = 115.2 \text{ ft-kips (live load)}$$

$$M_{tot} = M_d + M_{ds} + M_\ell = 134.8 + 57.6 + 115.2 = 307.6 \text{ ft-kips (total load)}$$

3. Calculate service load moments at transfer point

Assume transfer point located at $50d_b = 25$ in. from end of beam. Assume distance from end of beam to center of support is 4 in. Therefore, $x = 25 - 4 = 21$ in. = 1.75 ft. *11.4.3*

$$M_d = \frac{w_d x}{2}(\ell - x) = \frac{0.468 \times 1.75}{2}(48 - 1.75) = 18.9 \text{ ft-kips (beam dead load)}$$

Additional moment calculations at this location are unnecessary because conditions immediately after release govern at this location.

4. Calculate extreme fiber stresses by "linear elastic theory" which leads to the following well known formulas:

$$f_t = \frac{P}{A} - \frac{Pey_t}{I} + \frac{My_t}{I}$$

$$f_b = \frac{P}{A} + \frac{Pey_b}{I} - \frac{My_b}{I}$$

where, from Example 24.1

$P = P_p = 236$ kips (immediately after transfer)

$P = P_e = 219$ kips (at service load)

Table 24-4 Stresses in Concrete Immediately after Prestress Transfer (psi)

	At Assumed Transfer Point		At Mid Span	
	Top	Bottom	Top	Bottom
P_p/A P_pey/I M_dy/I	+526 -639 +63	+526 +1824 -180	+526 -639 +448	+526 +1824 -1279
Total	-50 (O.K.)	+2170 (say O.K.)	+335 (O.K.)	+1071 (O.K.)
Permissible	-355	+2100	+2100	+2100

Compression (+)
Tension (-)

Table 24-5 Stresses in Concrete at Service Loads (psi)

	At Midspan – Sustained Loads		At Midspan – Total Loads	
	Top	Bottom	Top	Bottom
P_e/A P_eey/I My/I	+488 -594 +640	+488 +1695 -1826	+488 -594 +1023	+488 +1695 -2919
Total	+534 (O.K.)	+357 (O.K.)	+917 (O.K.)	-736 (O.K.)
Permissible	+2250	+2250	+3000	-849

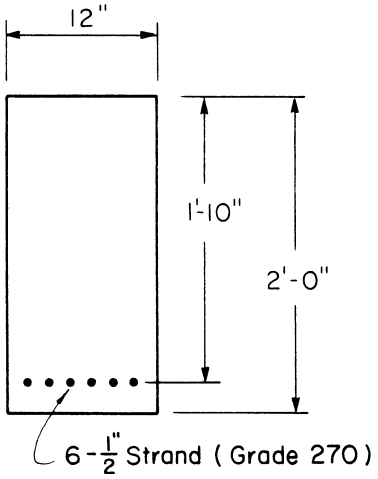
Compression (+)
Tension (-)

Example 24.3—Flexural Strength of Prestressed Member Using Approximate Value for f_{ps}

Calculate the nominal moment strength of the prestressed member shown.

f'_c = 5000 psi

f_{pu} = 270,000 psi (low-relaxation strands; $f_{py} = 0.90 f_{pu}$)



Calculations and Discussion	Code Reference

1. Calculate stress in prestressed reinforcement at nominal strength using approximate value for f_{ps}. For a fully prestressed member, Eq. (18-3) reduces to:

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\rho_p\frac{f_{pu}}{f'_c}\right)$$ *Eq. (18-3)*

$$= 270\left(1 - \frac{0.28}{0.80} \times 0.00348 \times \frac{270}{5}\right) = 252 \text{ ksi}$$

where

γ_p = 0.28 for $\frac{f_{py}}{f_{pu}}$ = 0.90 for low-relaxation strand

β_1 = 0.80 for f'_c = 5000 psi *10.2.7.3*

$\rho_p = \frac{A_{ps}}{bd_p} = \frac{6 \times 0.153}{12 \times 22} = 0.00348$

2. Calculate nominal moment strength from Eqs. (9) and (10) of Part 24

   Compute the depth of the compression block:

   $$a = \frac{A_{ps}f_{ps}}{0.85bf'_c} = \frac{0.918 \times 252}{0.85 \times 12 \times 5} = 4.54 \text{ in.} \qquad \text{Eq. (10)}$$

   $$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) \qquad \text{Eq. (9)}$$

   $$M_n = 0.918 \times 252\left(22 - \frac{4.54}{2}\right) = 4565 \text{ in-kips} = 380 \text{ ft-kips}$$

3. Check to see if tension controlled *10.3.4*

   $$c/d_p = (a/\beta_1)/d_p = \left(\frac{4.54}{0.80}\right)/22$$

   $c/d_p = 0.258 < 0.375$ *R9.3.2.2*

   Tension controlled $\phi = 0.9$

Example 24.4—Flexural Strength of Prestressed Member Based on Strain Compatibility

The rectangular beam section shown below is reinforced with a combination of prestressed and nonprestressed strands. Calculate the nominal moment strength using the strain compatibility (moment-curvature) method.

f'_c = 5000 psi

f_{pu} = 270,000 psi (low-relaxation strand; f_{py} = $0.9f_{pu}$)

E_{ps} = 28,500 ksi

jacking stress = $0.74f_{pu}$

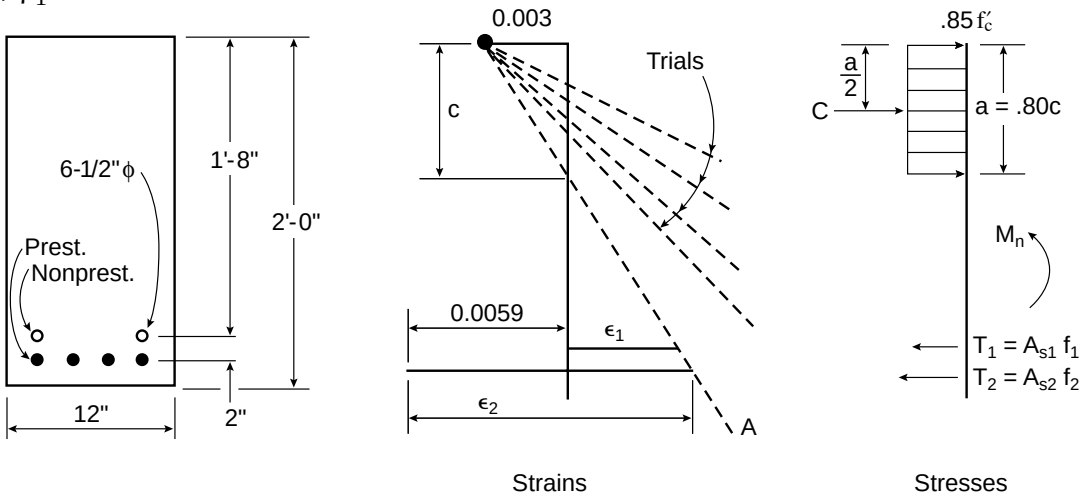
losses = 31.7 ksi (calculated by method of Ref. 24.4. See 18.6 — Loss of Prestress for procedure.)

Calculations and Discussion	Code Reference

1. Calculate effective strain in prestressing steel.

   $\varepsilon = (0.74f_{pu} - \text{losses})/E_{ps} = (0.74 \times 270 - 31.7)/28{,}500 = 0.0059$

2. Draw strain diagram at nominal moment strength, defined by the maximum concrete compressive strain of 0.003 and an assumed distance to the neutral axis, c. For f'_c = 5000, β_1 = 0.80. *18.3.1*



3. Obtain equilibrium of horizontal forces.

   The "strain line" drawn above from point 0 must be located to obtain equilibrium of horizontal forces:

   $C = T_1 + T_2$

   To compute T_1 and T_2, strains ε_1 and ε_2 are used with the stress-strain relation for the strand to determine the corresponding stresses f_1 and f_2. Equilibrium is obtained using the following iterative procedure:

a. assume c (location of neutral axis)

b. compute ε_1 and ε_2

c. obtain f_1 and f_2 from the equations in Fig. 24-1.

d. compute $a = \beta_1 c$

e. compute $C = 0.85 f'_c ab$

f. compute T_1 and T_2

g. check equilibrium using $C = T_1 + T_2$

h. if $C < T_1 + T_2$, increase c, or vice versa and return to step b of this procedure. Repeat until satisfactory convergence is achieved.

Estimate a neutral axis location for first trial. Estimate stressed strand at 260 ksi, unstressed strand at 200 ksi.

$T = \Sigma A_{ps} f_s = 0.306\ (200) + 0.612\ (260) = 220$ kips $= C$

$a = C/(0.85\ f'_c\ b) = 220/(0.85 \times 5 \times 12) = 4.32$ in.

$c = a/\beta_1 = 4.32/0.80 = 5.4$ in. Use c = 5.4 in. for first try

The following table summarizes the iterations required to solve this problem:

Trial No.	c in.	ε_1	ε_2	f_1 ksi	f_2 ksi	a in.	C kips	T_1 kips	T_2 kips	$T_1 + T_2$ kips
1	5.4	0.0081	0.0151	231	265	4.32	220	71	162	233
2 O.K.	5.6	0.0077	0.0147	220	265	4.48	228.5	67	162	229

4. Calculate nominal moment strength.

Using C = 228.5 kips, T_1 = 67 kips and T_2 = 162 kips, the nominal moment strength can be calculated as follows by taking moments about T_2:

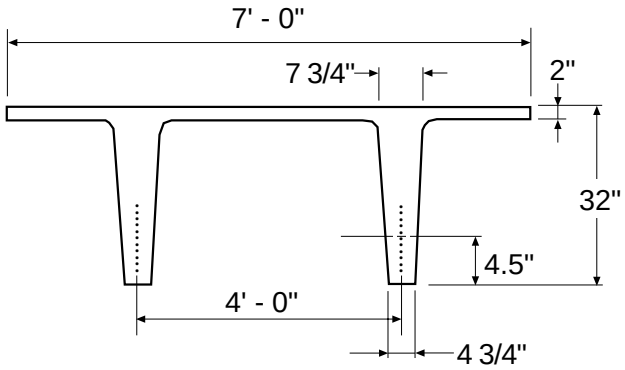
$M_n = [((d_2 - a/2) \times C) - ((d_2 - d_1) \times T_1)]/12$

$= [(22 - (4.48/2)) \times 228.5 - (22 - 20) \times 67)]/12 = 365$ ft-kips

Example 24.5—Tension-Controlled Limit for Prestressed Flexural Member

For the double tee section shown below, check limits for the prestressed reinforcement provided.

f'_c = 5000 psi
22 - 0.5 in. diameter low-relaxation strands
A_{ps} = 22 (0.153 in.^{2}) = 3.366 in.^{2}
f_{pu} = 270,000 psi
f_{py} = 0.90f_{pu}



Calculations and Discussion	Code Reference

Example No. 24.5.1

1. Calculate stress in prestressed reinforcement at nominal strength using Eqs. (6) and (8).

$$\omega_{pu} = \frac{A_{ps}f_{pu}}{bd_p f'_c} = \frac{3.366 \times 270}{84 \times 27.5 \times 5} = 0.079$$

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\omega_{pu}\right) = 270\left(1 - \frac{0.28}{0.8} \times 0.079\right) = 263 \text{ ksi}$$ *Eq. (18-3)*

where

γ_p = 0.28 for low-relaxation strands

β_1 = 0.80 for f'_c = 5000 psi *10.2.7.3*

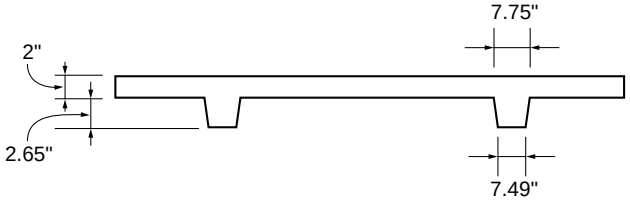
2. Calculate required depth of concrete stress block.

$$a = \frac{A_{ps}f_{ps}}{0.85bf'_c} = \frac{3.366 \times 263}{0.85 \times 84 \times 5} = 2.48 \text{ in.} > h_f = 2 \text{ in.}$$

3. Calculate area of compression zone.

$$A_c = \frac{A_{ps}f_{ps}}{0.85f'_c} = \frac{3.366 \times 263}{0.85 \times 5} = 208.3 \text{ in.}^2$$

4. Find depth a of stress block, and c.



$A = 2 \times 84 = 168$ in.^{2}
$+ 2 \times 2.65 \times \frac{7.75 + 7.49}{2} = 40.4$
208.4 in.^{2}

a = 4.65 in.
c = a/β_1 = 4.65/0.8 = 5.81 in.

5. Check to see if tension-controlled — *10.3.4*

   $c/d_p = 5.81/27.5 = 0.211 < 0.375$ — *R9.3.2.2*

   Section is tension-controlled

Note: In Step 1, Eq. 18-3 was used to find f_{ps}. But, with the stress block in the web, the value of ω_{pu} used in Eq. (18-5) was not correct, although the error is small in this case. A strain compatibility analysis gives c = 6.01 in. and f_{ps} = 266 ksi.

Example No. 24.5.2

Check the limits of reinforcement using a 3 in. thick flange on the member in Example 24.5.1. The overall depth remains 32 in.

1. f_{ps} = 263 ksi *No change from Example 24.5.1*

2. a = 2.48 *No change from Example 24.5.1, Step 2*

   < h_f = 3 in.

   Since the stress block is entirely within the flange, the section acts effectively as a rectangular section.

3. Check c/d_p ratio

   $c = a/\beta_1 = 2.48/0.8 = 3.10$ in.

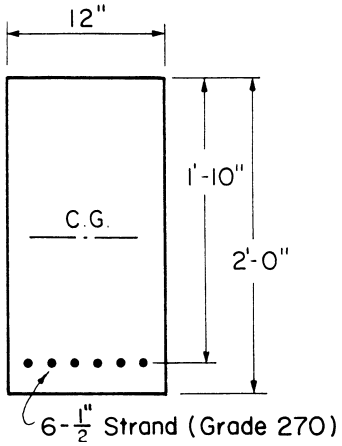
   $c/d_p = 3.10/27.5 = 0.113 < 0.375$ — *R9.3.2.2*

   Section is tension controlled

Example 24.6—Cracking Moment Strength and Minimum Reinforcement Limit for Non-composite Prestressed Member

For the non-composite prestressed member of Example 24.3, calculate the cracking moment strength and compare it with the design moment strength to check the minimum reinforcement limit.

f'_c = 5000 psi
f_{pu} = 270,000 psi
jacking stress = $0.70f_{pu}$
Assume 20% losses



Calculations and Discussion	Code Reference

1. Calculate cracking moment strength using Eq. (14) developed in Part 24.

$$M_{cr} = \left(f_r + \frac{P_{se}}{A_c}\right) S_b + (P_{se} \times e) \qquad \text{Eq. (14)}$$

$f_r = 7.5\sqrt{f'_c} = 530$ psi — *Eq. (9-9)*

Assuming 20% losses:

$P_{se} = 0.8 \times [6 \times 0.153 \times 0.7 \times 270] = 139$ kips

$$S_b = \frac{bh^2}{6} = \frac{12 \times 24^2}{6} = 1152 \text{ in.}^3$$

$A_c = bh = 12 \times 24 = 288 \text{ in.}^2$

$e = 12 - 2 = 10$ in.

$$M_{cr} = \left(0.530 + \frac{139}{288}\right) 1152 + (139 \times 10) = 2557 \text{ in.-kips} = 213 \text{ ft-kips}$$

Note that cracking moment strength needs to be determined for checking minimum reinforcement per 18.8.3.

2. Section 18.8.3 requires that the total reinforcement (prestressed and nonprestressed) must be adequate to develop a design moment strength at least equal to 1.2 times the cracking moment strength. From Example 24.3, M_n = 380 ft-kips.

Example 24.6 (cont'd)	Calculations and Discussion	Code Reference

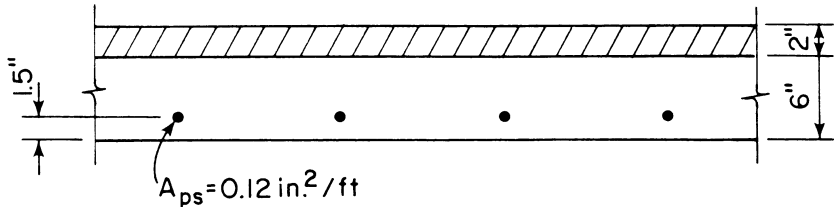
$\phi M_n \geq 1.2M_{cr}$ *18.8.3*

0.9 (380) > 1.2 (213)

342 > 256 O.K.

Example 24.7—Cracking Moment Strength and Minimum Reinforcement Limit for Composite Prestressed Member

For the 6 in. precast solid flat slab with 2 in. composite topping, calculate the cracking moment strength. The slab is supported on bearing walls with 15 ft span.



Section properties per foot of width:

A_c = 72 in.^{2} (precast slab)

S_b = 72 in.^{3} (precast slab)

S_c = 132.7 in.^{3} (composite section)

f'_c = 5000 psi (all-lightweight concrete, w_c = 125 pcf)

f_{pu} = 250,000 psi (stress-relieved strand)

jacking stress = $0.70f_{pu}$

Assume 25% losses

Calculations and Discussion	Code Reference

1. Calculate cracking moment strength using Eq. (13) developed for unshored composite members. All calculations are based on one foot width of slab. *18.8.3*

$$M_{cr} = \left(f_r + \frac{P_{se}}{A_c} + \frac{P_{se}e}{S_b}\right)S_c - M_d\left(\frac{S_c}{S_b} - 1\right) \qquad (13)$$

$f_r = 0.75\left(7.5\sqrt{5000}\right)$ = 398 psi reduced for all-lightweight concrete *9.5.2.3*

Assuming 25% losses:

P_{se} = 0.75 (0.12 × 0.7 × 250) = 15.75 kips

e = 3 - 1.5 = 1.5 in.

w_d = (6 + 2)/12 × 125 = 83 psf = 0.083 ksf (weight of precast slab + composite topping)

$$M_d = \frac{w_d\ell^2}{8} = \frac{0.083 \times 15^2}{8} = 2.33 \text{ ft-kips} = 28.0 \text{ in.-kips}$$

$$M_{cr} = \left(0.398 + \frac{15.75}{72} + \frac{15.75 \times 1.5}{72}\right)132.7 - 28.0\left(\frac{132.7}{72} - 1\right)$$

$$= 125.4 - 23.6 = 101.8 \text{ in.-kips}$$

2. Calculate design moment strength and compare with cracking moment strength. All calculations based on one foot width of slab.

$A_{ps} = 0.12$ in.^{2}, $d_p = 8.0 - 1.5 = 6.5$ in.

$$\rho_p = \frac{A_{ps}}{bd_p} = \frac{0.12}{12 \times 6.5} = 0.00154$$

With no additional tension or compression reinforcement, Eq. (18-3) reduces to:

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\rho_p\frac{f_{pu}}{f'_c}\right) = 250\left(1 - \frac{0.4}{0.8} \times 0.00154 \times \frac{250}{5}\right) = 240.4 \text{ ksi}$$

$$a = \frac{A_{ps}f_{ps}}{0.85f'_c b} = \frac{0.12 \times 240.4}{0.85 \times 5 \times 12} = 0.57 \text{ in.}$$

$M_n = A_{ps}f_{ps}(d_p - a/2) = 0.12 \times 240.4\,(6.5 - 0.57/2) = 179.3$ in.-kips

$\phi M_n = 0.9\,(179.3) = 161.4$ in.-kips

$\phi M_n \geq 1.2\,(M_{cr})$ *18.8.3*

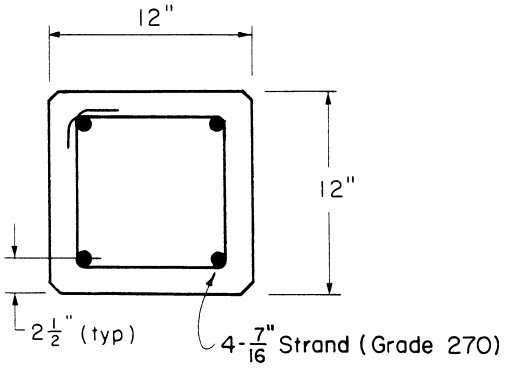
$161.4 > 1.2\,(101.8) = 122.2$ O.K.

Example 24.8—Prestressed Compression Member

For the short column shown, calculate the nominal strength M_n for a nominal axial load P_n = 30 kips.

Calculate design strength.

f'_c = 5000 psi
f_{pu} = 270,000 psi (low-relaxation strand)
jacking stress = $0.70f_{pu}$
Assume 10% losses



Calculations and Discussion	Code Reference

Eq. 18-3 should not be used when prestressing steel is in the compression zone. The same "strain compatibility" procedure used for flexure must be used here. The only difference is that for columns the load P_n must be included in the equilibrium of axial forces.

1. Calculate effective prestress.

   $f_{se} = 0.9 \times 0.7f_{pu} = 0.9 \times 0.7 \times 270 = 170$ ksi

   $P_e = A_{ps}f_{se} = 4 \times 0.115 \times 170 = 78.2$ kips

2. Calculate average prestress on column section.

   $$f_{pc} = \frac{P_e}{A_g} = \frac{78.2}{12^2} = 0.54 \text{ ksi}$$

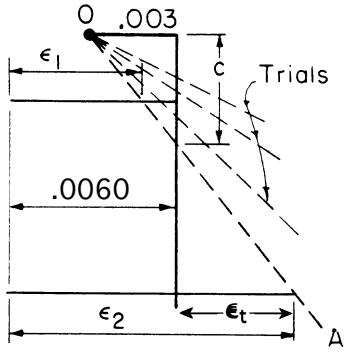
   Minimum reinforcement as per 10.9.1 not required because f_{pc} = 0.54 ksi > 0.225 ksi. *18.11.2.1*

   Since f_{pc} = 0.54 ksi > 0.225 ksi, lateral ties satisfying the requirements of 18.11.2.2 must enclose all prestressing tendons.

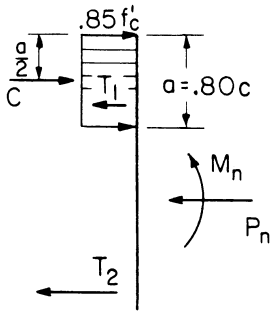
3. Calculate effective strain in prestressing steel.

   $$\varepsilon = \frac{f_{se}}{E_p} = \frac{170}{28{,}500} = 0.0060$$

4. Draw strain diagram at nominal moment strength, defined by the maximum concrete compressive strain of 0.003 and an assumed distance to the neutral axis, c. For f'_c = 5000 psi, β_1 = 0.80.



Strains



$C = ha\ .85\ f'_c$
$T_1 = A_{ps1}\ f_1$
$T_2 = A_{ps2}\ f_2$

Stresses

5. Obtain equilibrium of axial forces. The strain line OA drawn above, must be such that equilibrium of axial forces exists.

   $C = T_1 + T_2 + P_n$

   This can be done by trial-and-error as outlined in Example 24.4. Assuming different values of c, the following trial table is obtained:

Trial No.	c in.	ε_1	ε_2	f_1* ksi	f_2* ksi	a in.	C kips	T_1 kips	T_2 kips	$T_1 + T_2 + P_n$ kips
1	3.0	0.0055	0.0125	157	263	2.40	122.4	36.1	60.4	126.5
2	3.2	0.0053	0.0119	152	261	2.56	130.6	35.0	60.2	135.2
3 O.K.	3.1	0.0054	0.0122	154	262	2.48	126.5	35.5	60.3	125.8

**From equation in Fig. 24-1.*

6. Calculate nominal moment strength.

   Using C = 126.5 kips (from the sum of the other forces), P_n = 30 kips, T_1 = 35.5 kips, and T_2 = 60.3 kips, the moment strength can be calculated as follows by taking moments about P_n, located at the centroid of the section:

   $M_n = [((h/2 - a/2) \times C) - ((h/2 - 2.5) \times T_1) + ((h/2 - 2.5) \times T_2)]/12$

   $= [(4.76 \times 126.5) - (3.5 \times 35.5) + (3.5 \times 60.3)]/12 = 57.4$ ft-kips

7. Calculate design strength

   $\varepsilon_t = \varepsilon_2 - 0.0060 = 0.0122 - 0.0060 = 0.0062 > 0.005$

   Section is tension-controlled $\phi = 0.9$ *10.3.4*

   $\phi P_n = .9 \times 30 = 27$ kips *9.3.2.1*

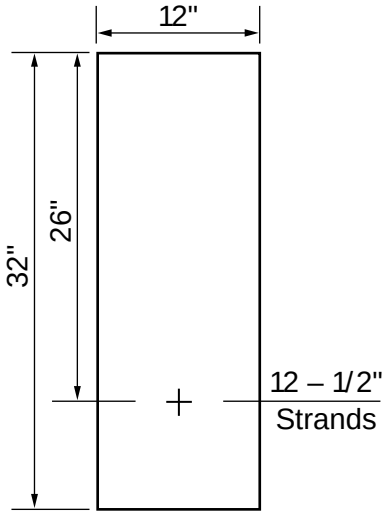
   $\phi M_n = .9 \times 57.4 = 51.7$ ft-kips

Example 24.9—Cracked Section Design When Tension Exceeds $12\sqrt{f'_c}$

Do the serviceability analysis for the beam shown.

f'_c = 6000 psi
depth d_p = 26 in.
effective prestress f_{se} = 150 ksi
decompression stress f_{dc} = 162 ksi
span = 40 ft

	w k/ft	Midspan moments in.-k
Self-weight	0.413	992
Additional dead load	1.000	2400
Live load	1.250	3000
Sum	2.663	6392



Calculations and Discussion	Code Reference

1. Check tension at service loads, based on gross section.

   $P = A_{ps}f_{se} = 1.836 \times 150 = 275.4$ kip

   $P/A = 275.4/384 = 0.717$ ksi

   $Pe/S = 275.4 \times 10/2048 = 1.345$

   $S = bh^2/6 = 12(32)^2/6 = 2.048$ in.^{3}

   $\Sigma M/S = 6392/2048 = -3.121$

   -1.059 ksi tension

   $12\sqrt{f'_c} = 12\sqrt{f'_c} = 930$ psi $= 0.930$ ksi

   Tension exceeds $12\sqrt{f'_c}$. Design as a Class C member — *18.3.3(c)*

2. A cracked section stress analysis is required — *18.3.4*

   The analysis will make use of cracked transformed section properties, like those used in the past days of working stress analysis of ordinary (nonprestressed) reinforced concrete. The area of steel elements is replaced by a "transformed" area of concrete equal to n times the actual steel area, where n is the ratio of the modulus of elasticity of steel to that of concrete.

   The modular ratio $n = E_{ps}/E_c = 28{,}500/4415 = 6.455$

   where $E_c = 57{,}000\sqrt{f'_c} = 57{,}000\sqrt{6000} = 4415$ ksi — *8.5.1*

   The transformed steel area A_t is:

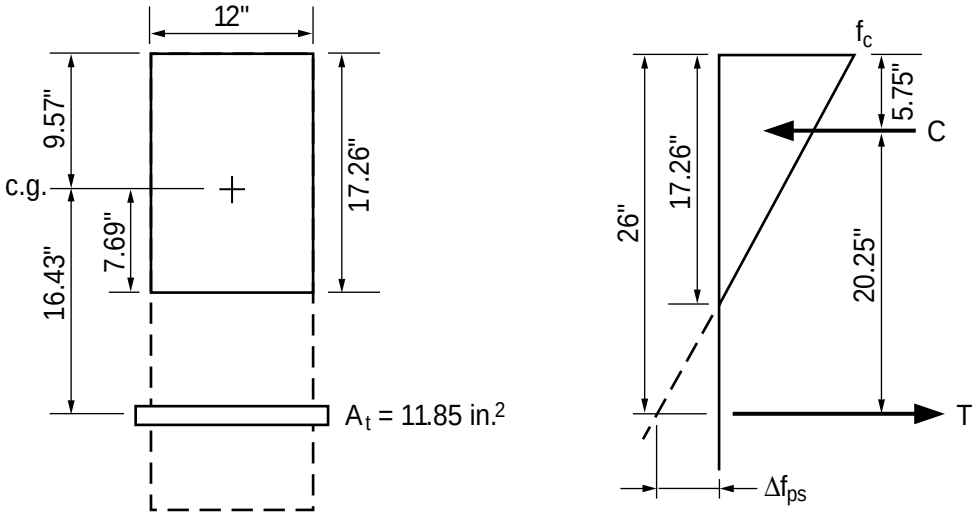
   $A_t = nA_{ps} = 6.455 \times 1.836 = 11.85$ in.^{2}

The force P_{dc} at decompression (when the stress in the concrete at the same level as the prestressing steel is zero) is:

$$P_{dc} = A_{ps}f_{dc} = 1.836 \times 162 = 297.4 \text{ kips}$$

3. The stress analysis of a cracked section with axial load (from the prestress) requires, at best, the solution of a cubic equation. A more general approach is to find a neutral axis location that satisfies horizontal force equilibrium and produces the given bending moment. Reference 24.2 gives one way to accomplish this. It is too lengthy to be presented in detail here.

   The results give a neutral axis depth c of 17.26 in., with a concrete stress f_c of 3.048 ksi and a transformed steel stress $\Delta f_{ps}/n$ of 1.545 ksi. The actual Δf_{ps} is $1.545 \times 6.455 = 9.97$ ksi.



4. The transformed section properties are

   $A = 219 \text{ in.}^2$

   $I = 8524 \text{ in.}^4$

   $y_t = 9.57$ in.

5. Equilibrium may be checked manually,

   $C = f_c bc/2 = 3.048\ (12)(17.26)/2$ = 315.7k

   C acts at top kern of compression zone

   = $d_c/3$ for rectangular area

   $17.26/3 = 5.75$ in.

   $T = P_{dc} + \Delta f_{ps}\ (A_{ps}) = 297.4 + 9.97\ (1.836)$ = 315.7k = C Check

   M = C or T × lever arm

   $= 315.7 \times 20.25$ = 6392 in.-kips Check

6. Check limits on Δf_{ps}

   Δf_{ps} is less than code limit of 36 ksi O.K. *18.4.3.3*

   Δf_{ps} is less than 20 ksi, so the spacing requirements of 18.4.4.1 and 18.4.4.2 need not be applied. *18.4.3.3*

7. Check deflection

   Live load deflection calculations based on a cracked section analysis are required for Class C members. *9.5.4.2*

   Use the "bilinear moment-deflection relationship," as described in Ref. 24.1 *9.5.4.2*

8. Find cracking moment M_{cr}, using P_{dc}

   $P/A + Pe/S + M_{cr}/S = f_r$

   modulus of rupture $f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{6000} = 581$ psi *9.5.2.3*

   $297/384 + 297 \times 10/2048 + 0.581 = M_{cr}/2048$

   $M_{cr} = 5750$ in.-kips

   $M_d = 3392$

   2358 = live load moment applied to gross section

   balance of M_ℓ 642 = live load moment applied to cracked section

9. Compute deflections before and after cracking

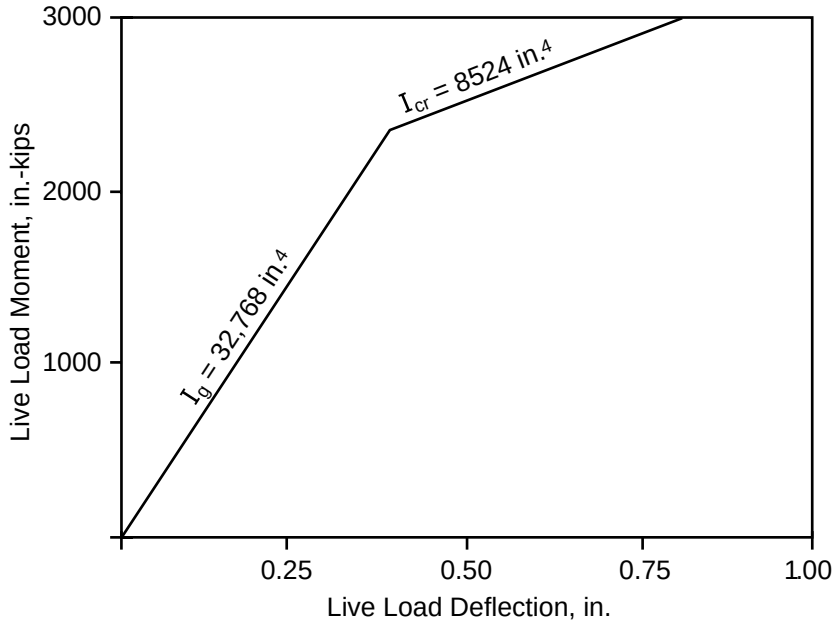
$$\Delta_L = \frac{5}{48}\frac{2358\,L^2}{EI_g} + \frac{5}{48}\frac{642\,L^2}{EI_{cr}}$$

$$= \frac{5}{48}\frac{2358 \times 480^2}{4415 \times 32768} + \frac{5}{48}\frac{642 \times 480^2}{4415 \times 8524}$$

$\Delta_L = 0.39 + 0.41 = 0.80$ in.

Δ_L is < (L/360 = 480/365 = 1.33 in.) O.K. *9.5.2.6* *Table 9.5(b)*

The live load deflection is shown graphically below.


3000
2000
1000
Live Load Moment, in.-kips
I_{cr} = 8524 in.^{4}
I_g = 32,768 in.^{4}
0.25
0.50
0.75
1.00
Live Load Deflection, in.

25

Prestressed Concrete Shear

UPDATE FOR THE '02 CODE

Code requirements for shear in prestressed concrete given in 11.4 were not changed in the 2002 code. A minor editorial change was made in 11.4.3, to be consistent in the use of the terms "prestressing steel" and "tendon."

Of course, shear design is affected by the revised load and ϕ factors in Chapter 9.

GENERAL CONSIDERATIONS

The basic equations for shear design of prestressed concrete, Eqs. (11-10), (11-11), and (11-12), were introduced in the 1963 code. Although well founded on test results, they have been found difficult to apply in practice. A simplified Eq. (11-9) was introduced in the 1971 code.

In order to understand Eqs. (11-10) and (11-12), it is best to review the principles on which ACI shear design is based. These principles are empirical, based on a large number of tests.

- The shear resisted by concrete and the shear resisted by stirrups are additive.
- The shear resisted by the concrete after shear cracks form is at least equal to the shear existing in the concrete at the location of the shear crack at the time the shear crack forms.

How does one compute the shear resisted by the concrete at the time a shear crack forms? There are two possibilities.

1. Web shear. A diagonal shear crack originates in the web, near the neutral axis, caused by principal tension in the web.
2. Flexure-shear. A crack starts as a flexural crack on the tension face of a flexural member. It then extends up into the web, and develops into a diagonal shear crack. This can happen at a much lower principal tensile stress than that causing a web shear crack, because of the tensile stress concentration at the tip of the crack.

Web Shear

The apparent tensile strength of concrete in direct tension is about $4\sqrt{f'_c}$. When the principal tension at the center of gravity of the cross section reaches $4\sqrt{f'_c}$, a web shear crack will occur. Section 11.4.2.2 states "…V_{cw} shall be computed as the shear…that results in a principal tensile stress of $4\sqrt{f'_c}$ …"

The compression from the prestress helps to reduce the principal tension. The computation of principal tension due to combined shear and compression can be somewhat tedious. The code gives a simplified procedure.

$$V_{cw} = (3.5\sqrt{f'_c} + 0.3f_{pc})\,b_w d + V_p \qquad \textit{Eq. (11-12)}$$

The term V_p in Eq. (11-12) is the vertical component of the tension in the prestressing tendons. This is additive for web shear strength (but not for flexure-shear strength).

A comparison to test results is shown below.

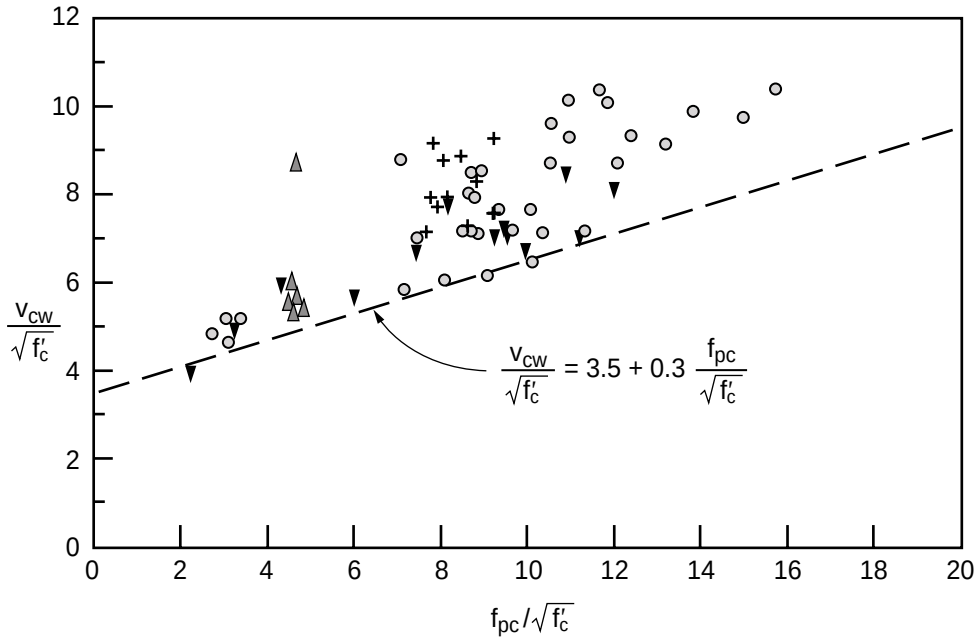


Figure 25-1 Diagonal Cracking in Regions not Previously Cracked

The compression from prestressing increases the shear strength by 30 percent of the P/A level, f_{pc}, of compression.

For nonprestressed beams, the principal tension at the center of gravity of the section is equal to the shear. Why does Eq. (11-3) for shear in nonprestressed members permit only $2\sqrt{f'_c}$ shear resisted by the concrete? Because shear strength is reduced by flexural cracking. In nonprestressed beams, shear is almost always influenced by flexural tension. But, prestressing reduces the flexural cracking.

Flexure-Shear in Prestressed Concrete

In prestressed beams, flexural cracking is delayed by the prestress – usually until loaded beyond service load. It is worthwhile to account for the beneficial effects of prestressing.

In the 1950s, it was thought that draping strands would increase shear strength, by the vertical component V_p of the prestressing force. Tests showed just the opposite. Why? Because draping the strands reduces the flexural cracking strength in the shear span.

The tests were done with concentrated loads; whereas, the dead load of the beam was a uniform load. For this reason, when the shear design method was developed from the test results, the dead load and test load shears were treated separately.

Flexure-Shear

Equation (11-10) is the equation for shear resistance provided by the concrete, as governed by flexural cracks that develop into shear cracks. The shear strength of the concrete at a given cross section is taken equal to the shear at the section at the time a flexural crack occurs, plus a small increment of shear which transforms the flexural crack into an inclined crack. Equation (10-11) may be expressed in words as follows.

V_{ci} = shear existing at the time of flexural cracking plus an added increment to convert it into a shear crack. The added increment is $0.6 b_w d \sqrt{f'_c}$.

The shear existing at the time of flexural cracking is the dead load shear V_d plus the added shear $V_i M_{cr}/M_{max}$.

What is the origin of the term $V_i M_{cr}/M_{max}$?

The term V_i is the factored ultimate shear at the section, less the dead load shear.

The term M_{cr} is the added moment (over and above stresses due to prestress and dead load) causing $6\sqrt{f'_c}$ tension in the extreme fiber.

The added moment M_{cr} is calculated by finding the bottom fiber stress f_{pe} due to prestress, subtracting the bottom fiber stress f_d due to dead loads, adding $6\sqrt{f'_c}$ tension, and multiplying the result by the section modulus for the section resisting live loads. This is Eq. (11-11) of the code.

$$M_{cr} = (I/y_t)(6\sqrt{f'_c} + f_{pe} - f_d) \qquad \textit{Eq. (11-11)}$$

Note: In the above discussion, "bottom" means "tension side" for continuous members.

The term M_{max} is the factored ultimate moment of the section, less the dead load moment.

To better understand the meaning of these terms and their use in Eq. (11-10), refer to Fig. 25-2.

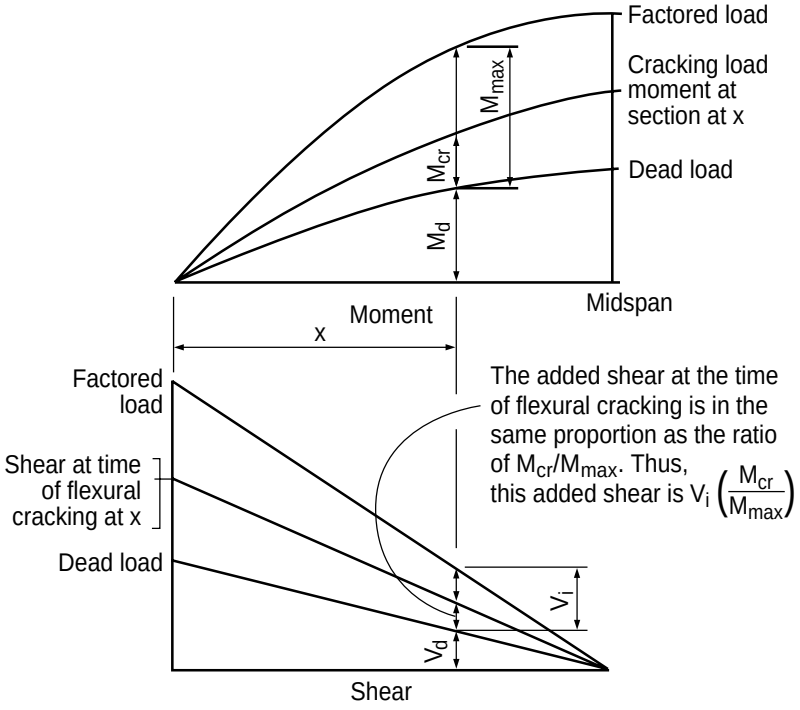


Figure 25-2 Origin of ($V_i M_{cr}/M_{max}$) Term in Eq. (11-10)

The quantity $V_i M_{cr}/M_{max}$ is the shear due to an added load (over and above the dead load) which causes the tensile stress in the extreme fiber to reach $6\sqrt{f'_c}$. The added load is applied to the composite section (if composite).

After a flexural crack forms, a small amount of additional shear is needed to transform the crack into a shear crack. This is determined empirically, as shown in Fig. 25-3.

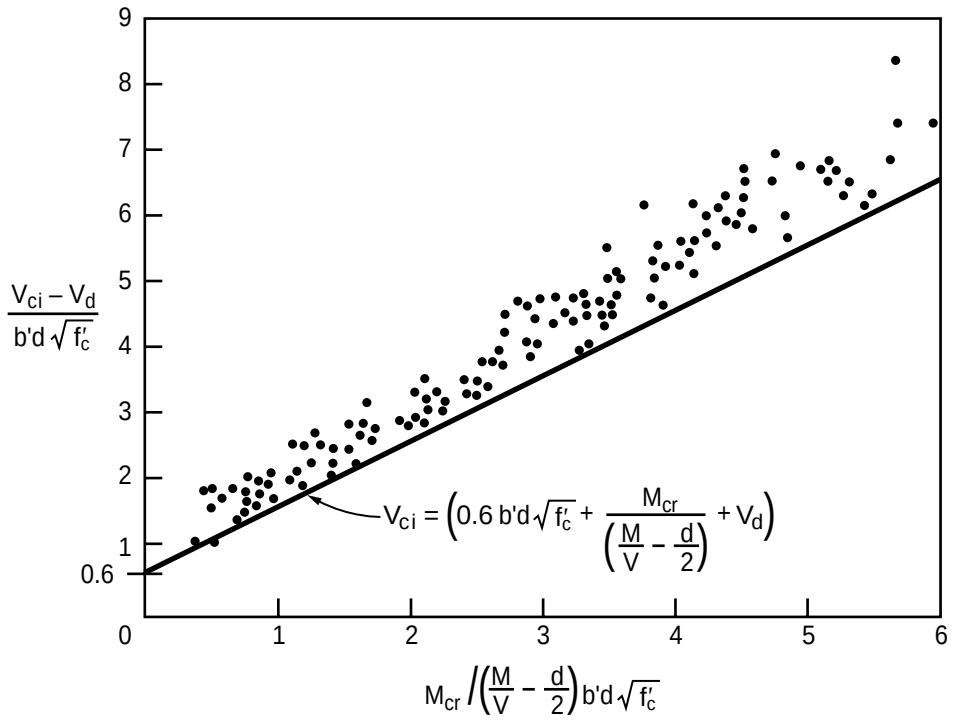


Figure 25-3 Diagonal Cracking in those Regions of Beams Previously Cracked in Flexure

The intercept at 0.6 produces the first term in Eq. (11-10), $0.6b_w d\sqrt{f'_c}$.

Note: The quantity "–d/2" shown in the expressions of Fig. 25-3 was later dropped, as a conservative simplication.

The notation used in Eqs. (11-10) and (11-11) is follows.

M_{cr} = moment causing flexural cracking at section due to externally applied loads

M_{max} = maximum factored moment at section due to externally applied loads

V_i = factored shear force at section due to externally applied loads occurring simultaneously with M_{max}.

Unfortunately, the subscripts are confusing.

M_{cr} is not the total cracking moment. It is not the same as M_{cr} that is used to check for minimum reinforcement in Example 24.6.

M_{max} is not the total factored moment. It is the total factored moment less the dead load moment.

It would seem that V_i and M_{max} should have the same subscript, because they both relate to the differences between the same two loadings.

To make matters worse, the term "externally applied loads" is ambiguous. Apparently, dead load is not regarded as "externally applied," perhaps because the weight comes from the "internal" mass of the member. But, R11.4.2 says that superimposed dead load on a composite section should be considered an externally applied load. The commentary explains a good reason for this, but the confusion still exists.

The shear strength must be checked at various locations along the shear span, a process that is tedious. For manual shear calculations, the simplified process described in 11.4.1 is adequate for most cases.

11.0 NOTATION

For prestressed members, the depth d used in shear calculations is defined as follows.

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, but need not be less than **0.80*h*** for circular sections and prestressed members, in.

The commentary points out that when using the minimum d of 0.8h, it is desirable to have some steel close to the tension face, enclosed by stirrups.

11.1 SHEAR STRENGTH FOR PRESTRESSED MEMBERS

The basic requirement for shear design of prestressed concrete members is the same as for reinforced concrete members: the design shear strength ϕV_n must be greater than the factored shear force V_u at all sections (11.1).

$$V_u \leq \phi V_n \qquad \textit{Eq. (11-1)}$$

For both reinforced and prestressed concrete members, the nominal shear strength V_n is the sum of two components: the nominal shear strength provided by concrete V_c and the nominal shear strength provided by shear reinforcement V_s.

$$V_n = V_c + V_s \qquad \textit{Eq. (11-2)}$$

Therefore,

$$V_u \leq \phi V_c + \phi V_s$$

The nominal shear strength provided by concrete V_c is assumed to be equal to the shear existing at the time an inclined crack forms in the concrete.

Beginning with the 1977 code, shear design provisions have been presented in terms of shear forces V_n, V_c, and V_s, to better clarify application of the material strength reduction factor ϕ for shear design. In force format, the ϕ factor is directly applied to the material strengths, i.e., ϕV_c and ϕV_s.

11.1.2 Concrete Strength

Section 11.1.2 restricts the concrete strength that can be used in computing the concrete contribution because of a lack of shear test data for high strength concrete. The limit does not allow $\sqrt{f'_c}$ to be greater than 100 psi, which corresponds to f'_c = 10,000 psi. Note, the limit is expressed in terms of $\sqrt{f'_c}$, as it denotes diagonal tension. The limit can be exceeded if minimum shear reinforcement is provided as specified in 11.1.2.1.

11.1.3 Location for Computing Maximum Factored Shear

Section 11.1.3 allows the maximum factored shear V_u to be computed at a distance from the face of the support when all of the following conditions are satisfied:

a. the support reaction, in the direction of the applied shear, introduces compression into the end regions of the member,

b. loads are applied at or near the top of the member, and

c. no concentrated load occurs between the face of the support and the critical section.

For prestressed concrete sections, 11.1.3.2 states that the critical section for computing the maximum factored shear V_u is located at a distance of h/2 from the face of the support. This differs from the provisions for rein-

forced concrete members, in which the critical section is located at d from the face of the support. For more details concerning maximum factored shear force at supports, see Part 12.

11.2 LIGHTWEIGHT CONCRETE

The adjustments to shear strength for lightweight concrete given in 11.2 apply equally to prestressed and reinforced concrete members.

11.4 SHEAR STRENGTH PROVIDED BY CONCRETE FOR PRESTRESSED MEMBERS

Section 11.4 provides two approaches to determining the nominal shear strength provided by concrete V_c. A simplified approach is presented in 11.4.1 with a more detailed approach presented in 11.4.2. In both cases, the shear strength provided by concrete is assumed to be equal to the shear existing at the time an inclined crack forms in the concrete.

11.4.1 Simplified Method

The use of this simplified method is limited to prestressed members with an effective prestress force not less than 40 percent of the tensile strength of the flexural reinforcement, which may consist of only prestressed reinforcement or a combination of prestressed and conventional reinforcement.

$$V_c = \left(0.6\sqrt{f'_c} + 700\,\frac{V_u d}{M_u}\right) b_w d \qquad \textit{Eq. (11-9)}$$

but need not be less than $2\sqrt{f'_c}\, b_w d$.

V_c must not exceed $5\sqrt{f'_c}\, b_w d$ or V_{cw} (11.4.2.2) computed considering the effects of transfer length (11.4.3) and debonding (11.4.4) which apply in regions near the ends of pretensioned members.

It should be noted that for the term $V_u d/M_u$ in Eq. (11-9), d must be taken as the actual distance from the extreme compression fiber to the centroid of the prestressed reinforcement rather than the 0.8h allowed elsewhere in the code.

The shear strength must be checked at various locations along the shear span. The commentary notes that for simply supported members subjected to uniform loads, the quantity of $V_u d/M_u$ may be expressed as:

$$\frac{V_u d}{M_u} = \frac{d(\ell - 2x)}{x(\ell - x)}$$

Figure 25-4, useful for a graphical solution, is also given in the commentary.

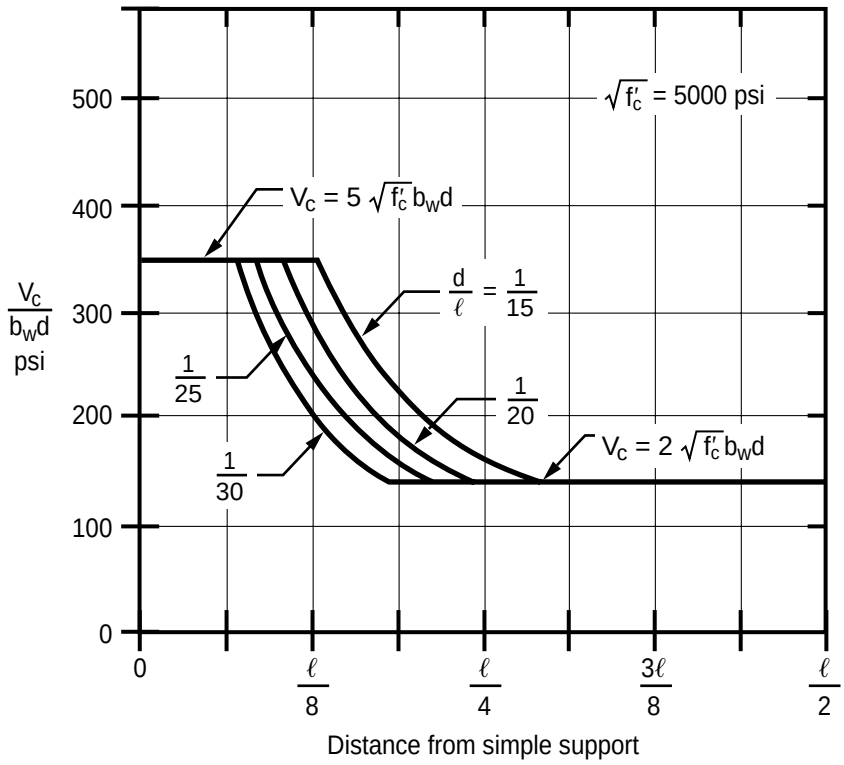


Figure 25-4 Application of Eq. (11-9) to Uniformly Loaded Prestressed Members (Fig. R11.4.1)

The use of this figure is illustrated in Example 25-2. Additional figures for graphical solutions of shear strength are given in Ref. 25.1.

11.4.2 Detailed Method

The origin of this method is discussed under General Considerations, at the beginning of Part 25.

Two types of inclined cracking have been observed in prestressed concrete members: flexure-shear cracking and web-shear cracking. Since the nominal shear strength from concrete is assumed to be equal to the shear causing inclined cracking of the concrete, the detailed method provides equations to determine the nominal shear strength for both types of cracking.

The two types of inclined cracking are illustrated in Fig. 25-5 which is found in R11.4.2. The nominal shear strength provided by concrete V_c is taken as the lesser shear causing the two types of cracking, which are discussed below. The detailed expressions for V_c in 11.4.2 may be difficult to apply without design aids, and should be used only when the simplified expression for V_c in 11.4.1 is not adequate.

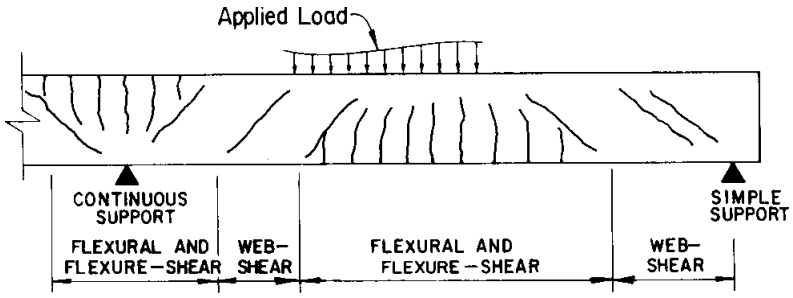


Figure 25-5 Types of Cracking in Concrete Beams (Fig. R11.4.2)

11.4.2.1 Flexure-Shear Cracking, V_{ci}—Flexure-shear cracking occurs when flexural cracks, which are initially vertical, become inclined under the influence of shear. The shear at which this occurs can be taken as

$$V_{ci} = 0.6\sqrt{f'_c}\, b_w d + V_d + \frac{V_i M_{cr}}{M_{max}} \qquad \textit{Eq. (11-10)}$$

Note that V_{ci} need not be taken less than $1.7\sqrt{f'_c}\, b_w d$.

The added moment M_{cr} to cause flexural cracking is computed using the equation

$$M_{cr} = \left(\frac{I}{y_t}\right)\left(6\sqrt{f'_c} + f_{pe} - f_d\right) \qquad \textit{Eq. (11-11)}$$

where f_{pe} is the compressive stress in concrete due to effective prestress force only (after allowance for all prestress losses) at the extreme fiber of the section where tensile stress is caused by externally applied loads.

V_{ci} usually governs for members subject to uniform loading. The total nominal shear strength V_{ci} is assumed to be the sum of three parts:

1. the shear force required to transform a flexural crack into an inclined crack — $0.6\sqrt{f'_c}\, b_w d$;
2. the unfactored dead load shear force — V_d; and
3. the portion of the remaining factored shear force that will cause a flexural crack to initially occur — $V_i M_{cr}/M_{max}$.

For non-composite members, V_d is the shear force caused by the unfactored dead load. For composite members, V_d is computed using the unfactored self weight plus unfactored superimposed dead load.

The load combination used to determine V_i and M_{max} is the one that causes maximum moment at the section under consideration. The value V_i is the factored shear force resulting from the externally applied loads occurring simultaneously with M_{max}. For composite members, V_i may be determined by subtracting V_d from the shear force resulting from the total factored loads, V_u. Similarly, $M_{max} = M_n - M_d$. When calculating the cracking moment M_{cr}, the load used to determine f_d is the same unfactored load used to compute V_d.

11.4.2.2 Web-Shear Cracking, V_{cw} — Web-shear cracking occurs when the principal diagonal tension in the web exceeds the tensile strength of the concrete. This shear is approximately equal to

$$V_{cw} = \left(3.5\sqrt{f'_c} + 0.3f_{pc}\right) b_w d + V_p \qquad \textit{Eq. (11-12)}$$

where f_{pc} is the compressive stress in concrete (after allowance for all prestress losses) at the centroid of the cross-section resisting externally applied loads or at the junction of the web and flange when the centroid lies within the flange.

V_p is the vertical component of the effective prestress force, which is present only when strands are draped or deflected.

The expression for web shear strength V_{cw} usually governs for heavily prestressed beams with thin webs, especially when the beam is subject to large concentrated loads near simple supports. Eq. (11-12) predicts the shear strength at first web-shear cracking.

An alternate method for determining the web shear strength V_{cw} is to compute the shear force corresponding to dead load plus live load that results in a principal tensile stress of $4\sqrt{f'_c}$ at the centroidal axis of the member, or at the interface of web and flange when the centroidal axis is located in the flange. This alternate method may be

advantageous when designing members where shear is critical. Note the limitation on V_{cw} in the end regions of pretensioned members as provided in 11.4.3 and 11.4.4.

11.4.3, 11.4.4 Special Considerations for Pretensioned Members

Section 11.4.3 applies to situations where the critical section located at h/2 from the face of the support is within the transfer length of the prestressing tendons. This means that the full effective prestress force is not available for contributing to the shear strength. A reduced value of effective prestress force must be used using linear interpolation between no stress in the tendons at the end of the member to full effective prestress at the transfer length from the end of the member, which is taken to be 50 diameters (d_b) for strand and $100d_b$ for a single wire.

Section 11.4.4 is provided to ensure that the effect on shear strength of reduced prestress is properly taken into account when bonding of some of the tendons is intentionally prevented (debonding) near the ends of a pretensioned member, as permitted by 12.9.3.

11.5 SHEAR STRENGTH PROVIDED BY SHEAR REINFORCEMENT FOR PRESTRESSED MEMBERS

The design of shear reinforcement for prestressed members is the same as for reinforced nonprestressed concrete members discussed in Part 12, except that V_c is computed differently (as discussed above) and another minimum shear reinforcement requirement applies (11.5.5.4). Therefore, see Part 12 for a complete discussion of design of shear reinforcement.

11.5.5.1 The code permits a slightly wider spacing of (3/4)h (instead of d/2) for prestressed members, because the shear crack inclination is lower in prestressed members.

As permitted by 11.5.5.2, shear reinforcement may be omitted in any member if shown by physical tests that the required strength can be developed without shear reinforcement. Section 11.5.5.2 clarifies conditions for appropriate tests. Also, commentary discussion gives further guidance on appropriate tests to meet the intent of 11.5.5.2. The commentary also calls attention to the need for sufficient stirrups in all thin-web, post-tensioned members to support the tendons in the design profile, and to provide reinforcement for tensile stresses in the webs resulting from local deviations of the tendons from the design tendon profile.

11.5.5.4 Minimum Reinforcement for Prestressed Members—For prestressed members, minimum shear reinforcement is computed as the smaller of Eqs. (11-13) and (11-14). However, Eq. (11-13) will generally give a higher minimum than Eq. (11-14). Note that Eq. (11-14) may not be used for members with an effective prestress force less than 40 percent of the tensile strength of the prestressing reinforcement.

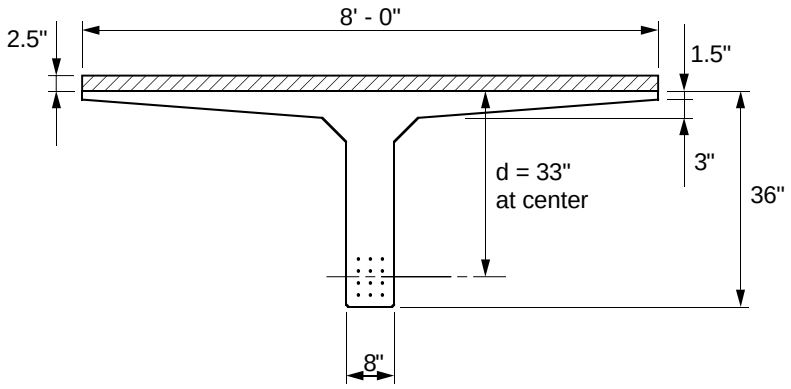
REFERENCE

25.1 "PCI Design Handbook – Precast and Prestressed Concrete," MNL 120-92 5th Edition, Precast/Prestressed Concrete Institute, Chicago, 1999, 630 pp.

Example 25.1—Design for Shear (11.4.1)

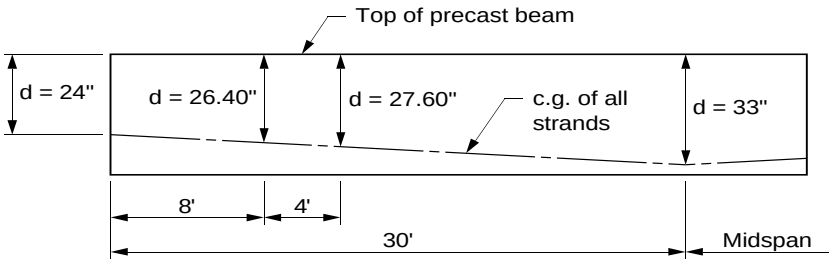
For the prestressed single tee shown, determine shear requirements using V_c by Eq. (11-9).

Precast concrete: f'_c = 5000 psi (sand lightweight, w_c = 120 pcf)
Topping concrete: f'_c = 4000 psi (normal weight, w_c = 150 pcf)
Prestressing steel: Twelve 1/2-in. dia. 270 ksi strands (single depression at midspan)
Span = 60 ft (simple)
Dead load = 725 lb/ft (includes topping)
Live load = 720 lb/ft
f_{se} (after all losses) = 150 ksi



Precast Section:
A = 570 in.^{2}
I = 68,917 in.^{4}
y_b = 26.01 in.
y_t = 9.99 in.

Composite Section:
y_{bc} = 29.27 in.



Strand Profile in Precast Girder

Strand Profile in Precast Girder

Calculations and Discussion	Code Reference

1. Determine factored shear force V_u at various locations along the span. The results are shown in Fig. 27-2.

2. Determine shear strength provided by concrete V_c using Eq. (11-9). The effective prestress f_{se} is greater than 40 percent of f_{pu} (150 ksi > 0.40 × 270 = 108 ksi). Note that the value of d need not be taken less than 0.8h for shear strength computations. Typical computations using Eq. (11-9) for a section 8 feet from support are as follows, assuming the shear is entirely resisted by the web of the precast section: *11.4.1* *11.0*

$$w_u = 1.2\,(0.725) + 1.6\,(0.720) = 2.022 \text{ kips/ft}$$

$$V_u = \left[\left(\frac{60}{2}\right) - 8\right] 2.022 = 44.5 \text{ kips}$$

$M_u = 30 \times 2.022 \times 8 - 2.022 \times 8 \times 4 = 421$ ft-kips

For the non-composite section, at 8 ft from support, determine distance d to centroid of tendons.

d = 26.40 in. (see strand profile)

For composite section, d = 26.4 + 2.5 = 28.9 in. < 0.8h = 30.8 in. use d = 30.8 in.

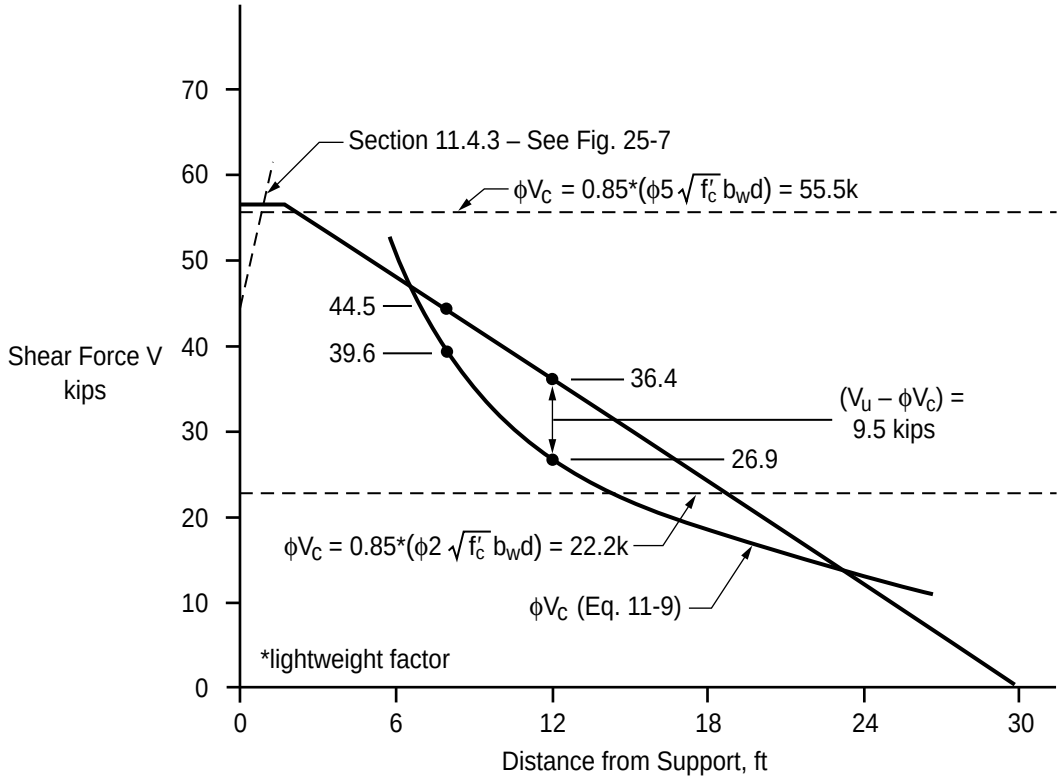


Figure 25-6 Shear Force Variation Along Member

$$V_c = \left(0.6\sqrt{f'_c} + 700\,\frac{V_u d}{M_u}\right) b_w d$$ *Eq. (11-9)*

but not less than $2\sqrt{f'_c}\, b_w d$ *11.4.1*

nor greater than $5\sqrt{f'_c}\, b_w d$ *11.4.1*

Since the precast section utilizes sand lightweight concrete, all $\sqrt{f'_c}$ terms must be reduced by the factor 0.85. *11.2.1.2*

Note: Total effective depth, d = 28.9 in., must be used in $V_u d/M_u$ term rather than 0.8h which is used elsewhere. *11.4.1*

$$V_c = \left(0.6 \times 0.85\sqrt{5000} + 700 \times 44.5 \times 28.90 / \left(421 \times 12\right)\right) 8 \times 30.8$$

$$= (36 + 178)\ 8 \times 30.8 = 52.8 \text{ kips} \quad \text{(governs)}$$

$$\geq 2 \times 0.85\sqrt{5000} \times 8 \times 30.8 = 29.6 \text{ kips}$$

$$\leq 5 \times 0.85\sqrt{5000} \times 8 \times 30.8 = 74.0 \text{ kips}$$

$\phi V_c = 0.75 \times 52.8 = 39.6$ (see Fig. 25-6) *11.2.1.2*

Note: For members simply supported and subject to uniform loading, $V_u d/M_u$ in. Eq. (11-9) becomes a simple function of d/ℓ, where ℓ is the span length,

$$V_c = \left[0.6\sqrt{f'_c} + 700d \frac{(\ell - 2x)}{x\,(\ell - x)}\right] b_w d \qquad \textit{Eq. (11-9)}$$

where x is the distance from the support to the section being investigated. At 8 feet from the support,

$$V_c = \left[0.6 \times 0.85\sqrt{5000} + 700 \times 28.90 \frac{(60 - 16)}{8\,(60 - 8)\,12}\right] 8 \times 30.8 = 52.8 \text{ kips}$$

3. In the end regions of pretensioned members, the shear strength provided by concrete V_c may be limited by the provisions of 11.4.3. For this design, 11.4.3 does not apply because the section at h/2 is farther out into the span than the bond transfer length (see Fig. 25-7). The following will, however, illustrate typical calculations to satisfy 11.4.3. Compute V_c at face of support, 10 in. from end of member.

Bond transfer length for 1/2-in. diameter strand = 50 (0.5) = 25 in. *11.4.3*

Prestress force at 10 in. location: $P_{se} = (10/25)\ 150 \times 0.153 \times 12 = 110.2$ kips

Vertical component of prestress force at 10 in. location:

$$\text{slope} = \frac{(d_{CL} - d_{end})}{\frac{\ell}{2}} = \frac{(33 - 24)}{30 \times 12} = 0.025$$

$$V_p \approx P \times \text{slope} = (110.2)\ (0.025) = 2.8 \text{ kips}$$

For composite section, d = 28.90 in., use 0.8h = 30.8 in. *11.4.2.3*

M_d (unfactored weight of precast unit + topping) = 214.4 in.-kips

Distance of composite section centroid above the centroid of precast unit,

$c = y_{bc} - y_b = 29.27 - 26.01 = 3.26$ in.

Tendon eccentricity, $e = d_{end} + 10 \text{ in.} \times \text{slope} - y_t = 24 + 10 \times 0.025 - 9.99$

$= 14.26$ in. below the centroid of the precast section

$$f_{pc} \text{ (see notation definition)} = \frac{P}{A_g} - (Pe)\frac{c}{I_g} + M_d \frac{c}{I_g}$$

$$= \frac{110.2}{570} - 110.2\,(14.26)\left(\frac{3.26}{68{,}917}\right) + 214.4\left(\frac{3.26}{68{,}917}\right) = 129 \text{ psi}$$

where A_g and I_g are for the precast section alone.

$$V_{cw} = \left(3.5\sqrt{f'_c} + 0.3f_{pc}\right) b_w d + V_p \qquad \textit{Eq. (11-12)}$$

$$= \left(3.5 \times 0.85\sqrt{5000} + 0.3 \times 129\right) 8 \times 30.8 + 2800 = 63.4 \text{ kips}$$

$\phi V_{cw} = 0.75 \times 63.4 = 47.6$ kips

The results of this analysis are shown graphically in Fig. 25-7.

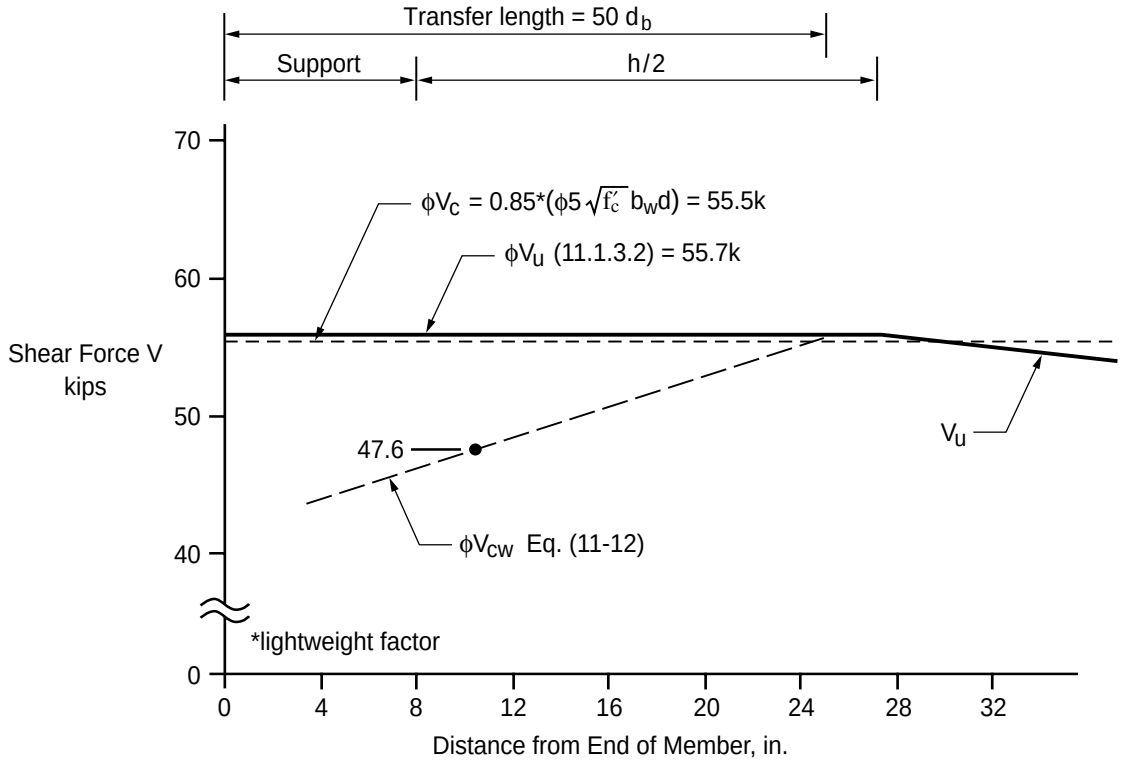


Figure 25-7 Shear Force Variation at End of Member

4. Compare factored shear V_u with shear strength provided by concrete ϕV_c. Where $V_u > \phi V_c$, shear reinforcement must be provided to carry the excess. Minimum shear reinforcement requirement should also be checked.

   Shear reinforcement required at 12 ft from support is calculated as follows:

   $d = 30.10$ in. (use in $V_u d/M_u$ term)

   $M_u = 30 \times 2.24 \times 12 - 2.24 \times 12 \times 6 = 645$ ft-kips

   $$V_u = \left[\left(\frac{60}{2}\right) - 12\right] 2.022 = 36.4 \text{ kips}$$

   $$V_c = (0.6 \times 0.85\sqrt{5000} + 700 \times 40.3 \times 30.10/(645 \times 12))\, 8 \times 30.8 = 35.9 \text{ kips}$$

   $$\phi V_c = 0.75 \times 35.9 = 26.9 \text{ kips}$$

   $$A_v = \frac{(V_u - \phi V_c)\, s}{\phi f_y d} = \frac{(36.4 - 26.9)\, 12}{0.75 \times 60 \times 30.8} = 0.082 \text{ in.}^2 / \text{ft}$$

   Check minimum required by 11.5.5.3 and 11.5.5.4.

   $$A_v(\text{min}) = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_y} = 0.75\sqrt{5000}\left(\frac{8 \times 12}{60{,}000}\right) = 0.085 \text{ in.}^2 / \text{ft} \qquad \textit{Eq. (11-13)}$$

   but not less than $50\dfrac{b_w s}{f_y}$ (not controlling for $f'_c > 4444$ psi)

   $$A_v(\text{min}) = \frac{A_{ps}}{80}\,\frac{f_{pu}}{f_y}\,\frac{s}{d}\,\sqrt{\frac{d}{b_w}} \qquad \textit{Eq. (11-14)}$$

   $$= \frac{1.84}{80} \times \frac{270}{60} \times \frac{12}{30.8}\sqrt{\frac{30.8}{8}} = 0.079 \text{ in.}^2/\text{ft}$$

   The lesser A_v (min) from Eqs. (11-13) and (11-14) may be used

   The required A_v is very slightly above minimum A_v

   Maximum stirrup spacing = 3/4d = (3/4) × 30.8 = 23.1 in.

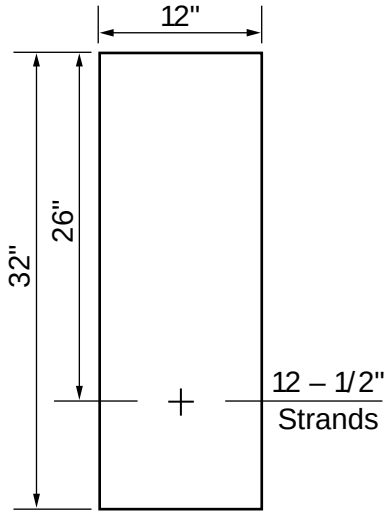
   Use No. 3 stirrups @ 18 in. for entire member length. ($A_v = 0.147$ in.^{2}/ft)

Example 25.2—Shear Design Using Fig. 25-4

Determine the shear reinforcement for the beam of Example 24.9

f'_c = 6000 psi
depth d_p = 26 in.
effective prestress f_{se} = 150 ksi
decompression stress f_{dc} = 162 ksi
span = 40 ft.

	w k/ft	Midspan moments in.-k
Self-weight	0.413	992
Additional dead load	1.000	2400
Live load	1.250	3000
Sum	2.663	6392



Calculations and Discussion	Code Reference

1. Calculate factored shear at support

$$V_u = 1.2D + 1.6L = [1.2(0.413 + 1.000) + 1.6(1.250)] \times \frac{40}{2}$$

= 73.9 kips

2. Prepare to use Fig. 25-4

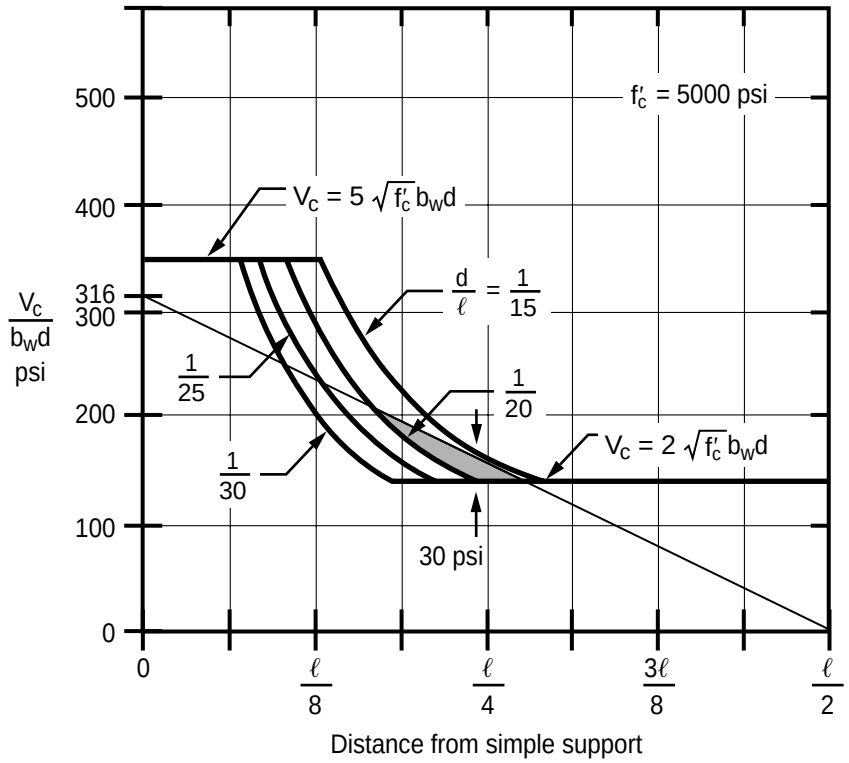
Note: Figure 25-4 is for f'_c = 5000 psi. Its use for f'_c = 6000 psi will be about 10 percent conservative.

$d/\ell = 26/480 = 1/18.5$

Use curve for $\ell/d = 1/20$

$$\frac{V_u}{\phi b_w d} = \frac{73.9}{0.75 \times 12 \times 26} = 0.316 \text{ ksi} = 316 \text{ psi}$$

3. Draw line for required nominal shear strength on Fig. 25-4, and find V_s required



The area where shear reinforcement is required is shaded. The maximum nominal shear stress to be resisted by shear reinforcement is 29 psi.

$$Vs = 0.03 \text{ ksi} \times b \times d = 0.030 \times 12 \times 26 = 9.4 \text{ kips}$$

$$A_v = \frac{V_s s}{f_y d} = \frac{9.4 \times 12}{60 \times 26} = 0.07 \text{ in.}^2/\text{ft}$$ *Eq. (11-15)*

4. Check minimum reinforcement.

$$A_v = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_y}, \text{ but not less than}$$ *Eq. (11-13)*

$$50\frac{b_w s}{f_y}$$

$0.75\sqrt{6000} = 58.1$ controls

$$A_v = 58.1 \times 12 \times 12/60{,}000 = 0.14 \text{ in.}^2/\text{ft}$$

$$A_v = \frac{A_{ps} f_{pu} s}{80 f_y d}\sqrt{\frac{d}{b_w}}$$ *Eq. (11-14)*

$$A_v = \frac{1.836 \times 270 \times 12}{80 \times 60 \times 26}\sqrt{\frac{26}{12}} = 0.07 \text{ in.}^2/\text{ft}$$

The lesser of A_v by Eqs. (11-13) and (11-14) may be used, but not less than A_v required.

5. Select stirrups

   A_v = 0.07 in.^{2}/ft

   Maximum s = (3/4)d ≤ 24 in. *11.5.4.1*

   s = (3/4)(26) = 19.5 in.

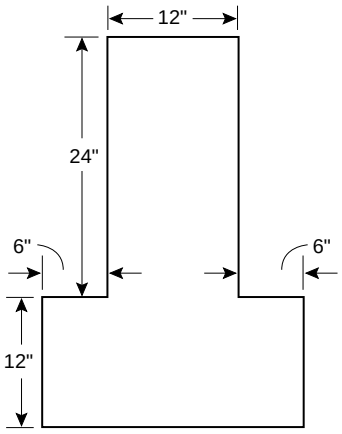
   Use twin No. 3 @ 18 in.

   A_v = 0.22/1.5 = 0.15 in.^{2}/ft O.K.

   This is required where V_u exceeds $\phi V_c/2$ *11.5.5.1*
   Most designers would provide it for the full length of the member.

Example 25.3—Shear Design Using 11.4.2

For the simple span pretensioned ledger beam shown, determine shear requirements using V_c by Eqs. (11-10) and (11-12).



$A = 576$ in.^{2} $w_d = 5.486$ kips/ft
$I = 63{,}936$ in.^{4} $w_\ell = 5.00$ kips/ft
$h = 36$ in.
$y_b = 15$ in.
$f'_c = 6$ ksi
$\ell = 24$ feet
16 1/2 in. Grade 270 ksi strands, $P = 396.6$ kips
$e_{end} = e_{msp}$ (midspan) $= 10$ in.

Calculations and Discussion	Code Reference

A systematized procedure is needed, to expedite the calculations.

1. Determine midspan moments and end shears

   $M_d = w_d \ell^2/8 = 5.486 \times 24^2/8 = 395$ ft-kips $= 4740$ in.-kips

   $M_\ell = w_\ell \ell^2/8 = 5.00 \times 24^2/8 = 360$ ft-kips $= 4320$ in.-kips

   $M_u = 1.2\, M_d + 1.6\, M_\ell = 1.2 \times 4740 + 1.6 \times 4320 = 12{,}600$ in.-kips — *Eq. (9-2)*

   $M_{max} = M_u - M_d = 12{,}600 - 4740 = 7860$ in.-kips — *11.0*

   $V_d = w_d \ell/2 = 5.486 \times 24/2 = 65.8$ kips

   V_ℓ - $w_\ell \ell/2 = 5 \times 24/2 = 60.0$ kips

   $V_u = 1.2\, V_d + 1.6\, V_\ell = 1.2 \times 65.8 + 1.6 \times 60 = 175.0$ kips — *Eq. (9-2)*

   $V_i = V_u - V_d = 175 - 65.8 = 109.2$ kips — *11.0*

2. Define factors for converting midspan moments and end shears to moments and shears at a distance x/ℓ from support, for $x/\ell = 0.3$.

   V factor $= 1–2(x/\ell) = 1–2(0.3) = 0.4$

   M factor $= 4(x/\ell – (x/\ell)^2) = 4 \times (0.3–0.3^2) = 0.84$

3. Compute V_3, the third term in Eq. (11-10)

$P/A = 396.6/576 = 0.689$

$Pe/S_b = 396.6 \times 10/4262 = 0.930$

$-M_d/S_b = 0.84\, M_d\,(msp)/S_b = -0.934$

$+6\sqrt{f'_c} = 6\sqrt{6000} = 465 = 0.465$

1.150 ksi

$M_{cr} = S_b\,(1.150\text{ ksi}) = 4900$ in.-kips — *Eq. (11-11)*

$V_i = 0.4V_i\,(end) = 0.4 \times 109.2 = 43.7$ kips

$M_{max} = 0.84\, M_{max}\,(msp) = 0.84 \times 7860 = 6602$ in.-kips

$$V_3 = \frac{V_i M_{cr}}{M_{max}} = \frac{43.7 \times 4900}{6602} = 32.4 \text{ kips}$$ — *Eq. (11-10)*

4. Compute the remaining terms V_1 and V_2 in Eq. (11-10), and solve for V_{ci} — *Eq. (11-10)*

d = 31, but not less than 0.8d = 28.8. Use d = 31 — *11-0*

$V_1 = 0.6 b_w d \sqrt{f'_c} = 0.6 \times 12 \times 31\sqrt{6000} = 17.3$k

$V_2 = V_d = 0.4(V_d end) = 0.4 \times 65.8 = 26.3$ kips

$V_{ci} = V_1 + V_2 + V_3 = 17.3 + 26.3 + 32.4 = 76.0$ kips — *Eq. (11-10)*

5. Compute V_u, and find V_s to be resisted by stirrups

$V_u = 0.4\, V_u\,(end) = 0.4 \times 175.0 = 70$ kips

ϕ for shear = 0.75 — *9.3.2.3*

$V_s = V_n - V_c = V_u/\phi - V_c = 70/0.75 - 76 = 17.3$ kips — *Eq. (11-2)*

6. Find required stirrups

$$A_v = \frac{V_s s}{f_y d} = \frac{17.3 \times 12}{60 \times 31} = 0.11 \text{ in.}^2/\text{ft}$$

Minimum requirements

$A_v = 0.75\sqrt{f'_c}\, b_w s/f_y$ when $f'_c > 4444$ psi — *Eq. (11-13)*

$= 0.75\sqrt{6000} \times 12 \times 12/60{,}000 = 0.14$ in.^{2}

$$A_v = \frac{A_{ps} f_{pu} s}{80\, f_y d}\sqrt{\frac{d}{b_w}} = \frac{2.448 \times 270 \times 12}{80 \times 60 \times 31}\sqrt{\frac{31}{12}} = 0.086 \text{ in.}^2$$ — *Eq. (11-13)*

The minimum need only be the lesser of that required by Eqs. (11-13) or (11-14). — *11.5.5.4*

So, the required A_v of 0.09 in.^{2}/ft controls

Maximum spacing = (3/4)d = (3/4)31 = 23.25 in.

Say, twin No. 3 at 18 in., $A_v = 2 \times 0.11/1.5 = 0.15$ in.^{2}/ft.

7. Compute required shear reinforcement at support

Because the ledger beam is loaded on the ledges, not "near the top," shear must be checked at the support, not at h/2 from the support for prestressed members. *11.1.3*

At the support, the prestress force P is assumed to be zero, for simplicity.

$V_{cw} = (3.5\sqrt{f'_c} + 0.3\, f_{pc})\, b_w d + V_p$ *Eq. (11-12)*

$= (3.5\sqrt{6000}) \times 12 \times 31 = 100.9$ kips

$V_s = V_n - V_c = V_u/\phi - V_c = 175/0.75 - 100.9$ *Eq. (11-12)*

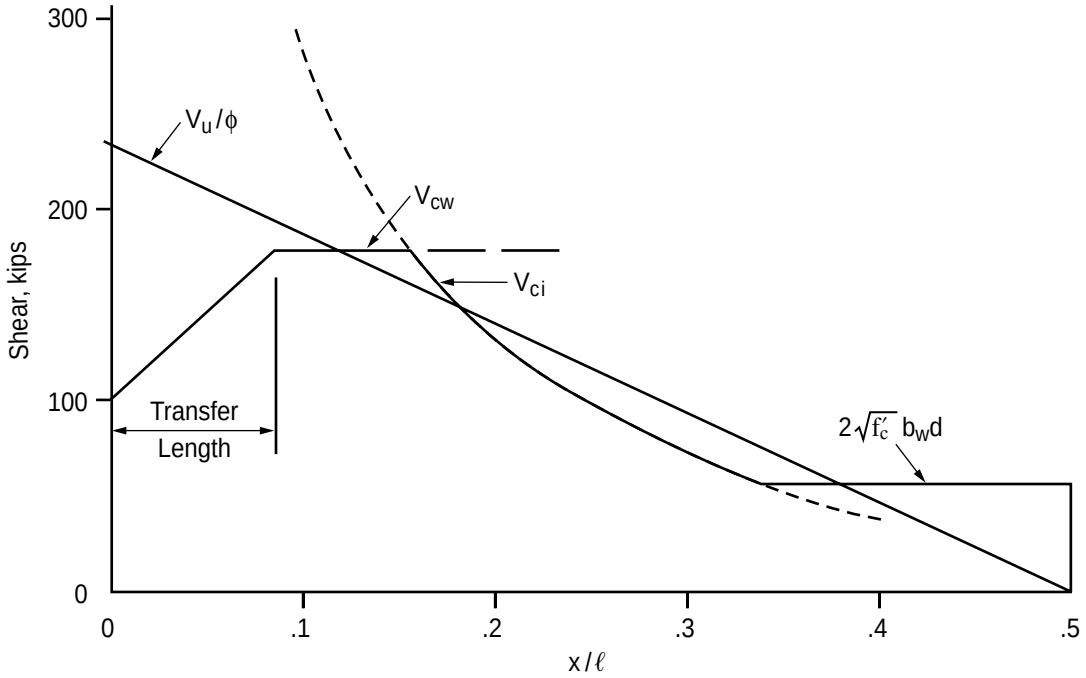
$V_s = 132.4$k

$$A_v = \frac{V_s s}{f_y d} = \frac{132.4 \times 12}{60 \times 31} = 0.85 \text{ in.}^2$$ *Eq. (11-14)*

Say twin No. 4 at 4 in., $A_v = .40/.33 = 1.20$ in.^{2}/ft near end.

Referring to Step 6, this is above minimum requirements.

8. Repeat the processes described above for various sections along the shear span (not shown). The results are shown below.



Note: Minimum V_c of $2\sqrt{f'_c}\, b_w d$ permitted by 11.4.1 was used.

9. Notes:

   1. A spreadsheet can be set up, with each column containing data for various values of x/ℓ and the shear and moment factors in Step 2.

   2. For members with draped tendons, additional factors for varying eccentricity, depth, and tendon slope (for computing V_p) need to be added in Step 2.

   3. For composite members, the portions of dead load applied before and after composite behavior is obtained need to be separated. The dead load applied after the beam becomes composite should not be included in V_d and M_d terms. See R11.4.2.

26

Prestressed Slab Systems

UPDATE FOR THE '02 CODE

Section 18.3.3 defines two-way prestressed slab systems as Class U. This permits the calculation of concrete service load flexural stresses using an uncracked section (18.3.4), and results in an increase in allowable concrete flexural tensile stress from $6\sqrt{f'_c}$ to $7.5\sqrt{f'_c}$.

The wording of 18.9.3 was revised to indicate that the minimum bonded reinforcing steel requirements in this Section apply to all two-way flat slab systems, not just flat plates.

Section 18.10.4.1 permits redistribution of negative moments in continuous prestressed flexural members in accordance with new 8.4, provided bonded reinforcement in accordance with 18.9 is provided at supports. This is a significant departure from previous codes.

Load factors and strength reduction factors, ϕ, have been extensively revised in Chapter 9 and are discussed in other parts of this document. The new factors have been incorporated in Design Example 26.1.

INTRODUCTION

Four code sections are particularly significant with respect to analysis and design of prestressed slab systems:

Section 11.12.2—Shear strength of prestressed slabs.

Section 11.12.6—Shear strength of prestressed slabs with moment transfer.

Section 18.4.2—Permissible compressive stresses.

Section 18.7.2—Determination of f_{ps} for calculation of flexural strength.

Section 18.12—Prestressed slab systems.

Discussion of each of these code sections is presented below, followed by Example 26.1 of a post-tensioned flat plate. The design example illustrates application of the above code sections as well as general applicability of the code to analysis and design of post-tensioned flat plates.

11.12.2 Shear Strength

Section 11.12.2 contains specific provisions for calculation of shear strength in two-way prestressed concrete systems. At columns of two-way prestressed slabs (and footings) utilizing unbonded tendons and meeting the bonded reinforcement requirements of 18.9.3, the shear strength V_n must not be taken greater than the shear strength V_c computed in accordance with 11.12.2.1 or 11.12.2.2, unless shear reinforcement is provided in accordance with 11.12.3 or 11.12.4. Section 11.12.2.2 gives the following value of the shear strength V_c at columns of two-way prestressed slabs:

$$V_c = \left(\beta_p \sqrt{f'_c} + 0.3f_{pc}\right) b_o d + V_p \qquad \textit{Eq. (11-36)}$$

Equation (11-36) includes the term β_p which is the smaller of 3.5 or ($\alpha_s d/b_o + 1.5$). The term $\alpha_s d / b_o$ is to account for a decrease in shear strength affected by the perimeter area aspect ratio of the column, where α_s is to be taken as 40 for interior columns, 30 for edge columns, and 20 for corner columns. f_{pc} is the average value of f_{pc} for the two directions, and V_p is the vertical component of all effective prestress forces crossing the critical section. If the shear strength is computed by Eq. (11-36), the following must be satisfied; otherwise, 11.12.2.1 for nonprestressed slabs applies:

a. no portion of the column cross-section shall be closer to a discontinuous edge than 4 times the slab thickness,

b. f'_c in Eq. (11-36) shall not be taken greater than 5000 psi, and

c. f_{pc} in each direction shall not be less than 125 psi, nor be taken greater than 500 psi.

In accordance with the above limitations, shear strength Eqs. (11-33), (11-34), and (11-35) for nonprestressed slabs are applicable to columns closer to the discontinuous edge than 4 times the slab thickness. The shear strength V_c is the lesser of the values given by these three equations. For usual design conditions (slab thicknesses and column sizes), the controlling shear strength at edge columns will be $4\sqrt{f'_c} b_o d$.

11.12.6 Shear Strength with Moment Transfer

For moment transfer calculations, the controlling shear stress at columns of two-way prestressed slabs with bonded reinforcement in accordance with 18.9.3 is governed by Eq. (11-36), which could be expressed as a shear stress for use in Eq. (11-40) as follows:

$$v_c = \beta_p \sqrt{f'_c} + 0.3f_{pc} + \frac{V_p}{b_o d} \qquad \textit{Eq. (11-36)}$$

If the permissible shear stress is computed by Eq. (11-36), the following must be satisfied:

a. no portion of the column cross-section shall be closer to a discontinuous edge than 4 times the slab thickness,

b. f'_c in Eq. (11-36) shall not be taken greater than 5000 psi, and

c. f_{pc} in each direction shall not be less than 125 psi, nor be taken greater than 500 psi.

For edge columns under moment transfer conditions, the controlling shear stress will be the same as that permitted for nonprestressed slabs. For usual design conditions, the governing shear stress at edge columns will be $4\sqrt{f'_c}$.

18.4.2 Permissible Compressive Stresses

In 1995, Section 18.4.2 increased the permissible concrete service load flexural compressive stress under total load from $0.45f'_c$ to $0.60f'_c$, but imposed a new limit of $0.45f'_c$ for sustained load. This involves some judgment on the part of designers in determining the appropriate sustained load

18.7.2 f_{ps} for Unbonded Tendons

In prestressed elements with unbonded tendons having a span/depth ratio greater than 35, the stress in the prestressed reinforcement at nominal strength is given by:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{300\rho_p} \qquad \textit{Eq. (18-5)}$$

but not greater than f_{py}, nor $(f_{se} + 30{,}000)$.

Nearly all prestressed one-way slabs and flat plates will have span/depth ratios greater than 35. Equation (18-5) provides values of f_{ps} which are generally 15,000 to 20,000 psi lower than the values of f_{ps} given by Eq. (18-4) which was derived primarily from results of beam tests. These lower values of f_{ps} are more compatible with values of f_{ps} obtained in more recent tests of prestressed one-way slabs and flat plates. Application of Eq. (18-5) is illustrated in Example 26.l.

18.12 SLAB SYSTEMS

Section 18.12 provides analysis and design procedures for two-way prestressed slab systems, including the following requirements:

1. Use of the Equivalent Frame Method of 13.7 (excluding 13.7.7.4 and 13.7.7.5), or more detailed analysis procedures, is required for determination of factored moments and shears in prestressed slab systems. According to References 26.1 and 26.4, for two-way prestressed slabs, the equivalent frame slab-beam strips would not be divided into column and middle strips as for a typical nonprestressed two-way slab, but would be designed as a total beam strip.

2. Spacing of tendons or groups of tendons in one direction shall not exceed 8 times the slab thickness nor 5 ft. Spacing of tendons shall also provide a minimum average prestress, after allowance for all prestress losses, of 125 psi on the slab section tributary to the tendon or tendon group. Special consideration must be given to tendon spacing in slabs with concentrated loads.

3. A minimum of two tendons shall be provided in each direction through the critical shear section over columns. This provision, in conjunction with the limits on tendon spacing outlined in item 2 above, provides specific guidance for distributing tendons in prestressed flat plates in accordance with the "banded" pattern illustrated in Fig. 26-1. This method of tendon installation is widely used and greatly simplifies detailing and installation procedures.

Calculation of equivalent frame properties is illustrated in Example 26.1. Tendon distribution is also discussed in this example.

References 26.1 and 26.4 illustrate application of ACI 318 requirements for design of one-way and two-way post-tensioned slabs, including detailed design examples.

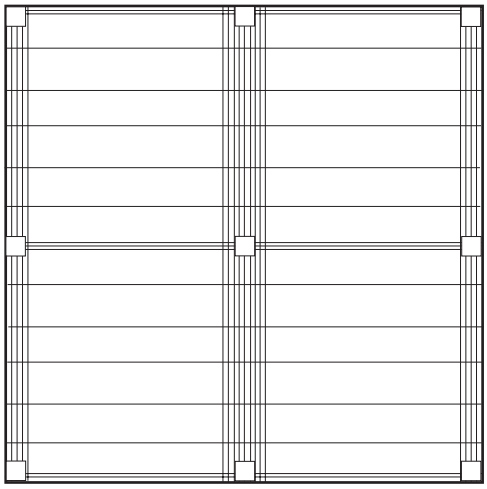


Figure 26-1 Banded Tendon Distribution

REFERENCES

26.1 *Design of Post-Tensioned Slabs*, Post-Tensioning Institute, 2nd. ed., Phoenix, AZ, 1995.

26.2 *Continuity in Concrete Building Frames*, Portland Cement Association, Skokie, IL, 1959.

26.3 *Estimating Prestress Losses,* Zia, P., Preston, H. K., Scott, N. L., and Workman, E. B., Concrete International, :Design and Construction, V. 1, No. 6, June 1979, pp. 32-38.

26.4 *Design Fundamentals of Post-Tensioned Concrete Floors,* Aalami, B. O., and Bommer, A., Post-Tensioning Institute, Phoenix, AZ, 1999

Example 26.1—Two-Way Prestressed Slab System

Design a typical transverse equivalent frame strip of the prestressed flat plate with partial plan and section shown in Figure 26-2.

f'_c = 4000 psi; w = 150 pcf (slab and columns)

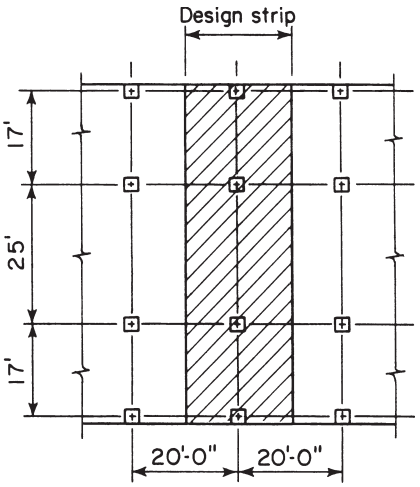
f_y = 60,000 psi

f_{pu} = 270,000 psi

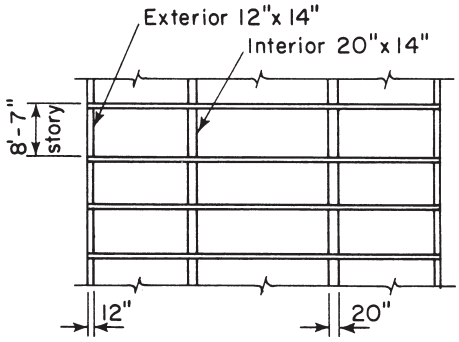
Live load = 40 psf
Partition load = 15 psf

Reduce live load in accordance with general building code. For this example live load is reduced in accordance with IBC 2000, Section 1607.9.2.

Required minimum concrete cover to tendons 1.5 in. from the bottom of the slab in end spans, 0.75 in. top and bottom elsewhere.



PARTIAL PLAN



SECTION

Figure 26-2

Calculations and Discussion	Code Reference

1. Slab Thickness

   For two-way prestressed slabs, a span/depth ratio of 45 typically results in overall economy and provides satisfactory structural performance.[26.1]

   Slab thickness:

   Longitudinal span: 20 × 12/45 = 5.3 in.
   Transverse span: 25 × 12/45 = 6.7 in.

   Use 6-1/2 in. slab.

   Slab weight = 81 psf
   Partition load = 15 psf
   Total dead load = 81 + 15 = 96 psf

   Span 2:
   Reduced live load (IBC 1607.9.2)
   Live load = 40(1-0.08(500-150)/100) = 29 psf
   Factored dead load = 1.2 × 96 = 115 psf
   Factored live load = 1.6 × 29 = 47 psf

Total load = 125 psf, unfactored
= 162 psf, factored

Spans 1 and 3:
Reduced live load (IBC 1607.9.2)
Live load = 40(1-0.08(340-150)/100) = 34 psf
Factored dead load = 1.2 × 96 = 115 psf
Factored live load = 1.6 × 34 = 55 psf
Total load = 130 psf, unfactored
= 170 psf, factored

2. Design Procedure

Assume a set of loads to be balanced by parabolic tendons. Analyze an equivalent frame subjected to the net downward loads according to 13.7. Check flexural stresses at critical sections, and revise load balancing tendon forces as required to obtain permissible flexural stresses according to 18.3.3 and 18.4.

When final forces are determined, obtain frame moments for factored dead and live loads. Calculate secondary moments induced in the frame by post-tensioning forces, and combine with factored load moments to obtain design factored moments. Provide minimum bonded reinforcement in accordance with 18.9.

Check design flexural strength and increase nonprestressed reinforcement if required by strength criteria. Investigate shear strength, including shear due to vertical load and due to moment transfer, and compare total to permissible values calculated in accordance with 11.12.2.

3. Load Balancing

Arbitrarily assume the tendons will balance 80% of the slab weight (0.8 × 0.081 = 0.065 ksf) in the controlling span (Span 2), with a parabolic tendon profile of maximum permissible sag, for the initial estimate of the required prestress force F_e:

Maximum tendon sag in Span 2 = 6.5-1-1 = 4.5 in.

$$F_e = \frac{w_{bal}L^2}{8a} = \frac{0.8(0.081)(25)^2(12)}{8(4.5)} = 13.5 \text{ kips/ft}$$

Assume 1/2 in. diameter (cross-sectional area = 0.153 in.^{2}), 270 ksi seven-wire low relaxation strand tendons with 14 ksi long-term losses (Reference 26.3). Effective force per tendon is 0.153 [(0.7 × 270) - 14] = 26.8 kips, where the tensile stress in the tendons immediately after tendon anchorage = $0.70f_{pu}$. *18.5.1(c)*

For a 20-ft bay, 20 × 13.5/26.8 = 10.1 tendons.

Use 10-1/2 in. diameter tendons/bay

$F_e = 10 \times 26.8/20 = 13.4$ kips/ft

$f_{pc} = F_e/A = 13.4/(6.5 \times 12) = 0.172$ ksi

Actual balanced load in Span 2:

$$w_{bal} = \frac{8F_e a}{L^2} = \frac{8(13.4)(4.5)}{12 \times 25^2} = 0.064 \text{ ksf}$$

Adjust tendon profile in Spans 1 and 3 to balance same load as in Span 2:

$$a = \frac{w_{bal}L^2}{8F_e} = \frac{0.064(17)^2(12)}{8(13.4)} = 2.1 \text{ in.}$$

Midspan cgs = (3.25+5.5)/2-2.1 = 2.275 in say 2.25 in.
Actual sag in Spans 1 and 3 = (3.25+5.5)/2-2.25 = 2.125 in.
Actual balanced load in Spans 1 and 3 =

$$w_{bal} = \frac{8(13.4)(2.125)}{17^2(12)} = 0.066 \text{ ksf}$$

4. Tendon Profile

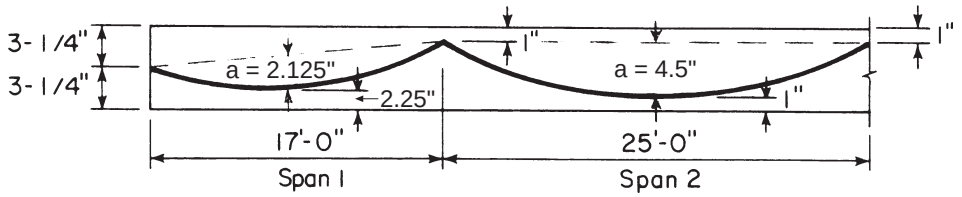


Figure 26-3

Net load causing bending:
Span 2:

$w_{net} = 0.125 - 0.064 = 0.061$ ksf

Spans 1 and 3:

$w_{net} = 0.130 - 0.066 = 0.064$ ksf

5. Equivalent Frame Properties — *13.7*

a. Column stiffness. — *13.7.4*

Column stiffness, including effects of "infinite" stiffness within the slab-column joint (rigid connection), may be calculated by classical methods or by simplified methods which are in close agreement. The following approximate stiffness K_c will give results within five percent of "exact" values.[26.1]

$K_c = 4EI/(\ell - 2h)$

where ℓ = center-to-center column height and h = slab thickness.

For exterior columns (14 × 12 in.):

$I = 14 \times 12^3/12 = 2016 \text{ in.}^4$

$E_{col}/E_{slab} = 1.0$

$K_c = (4 \times 1.0 \times 2016)/[103 - (2 \times 6.5)] = 90 \text{ in.}^3$

$\Sigma K_c = 2 \times 90 = 180 \text{ in.}^3$ (joint total)

Stiffness of torsional members is calculated as follows: — *13.7.5*

$C = (1 - 0.63\ x/y)\ x^3y/3$ — *13.0*

$= [1 - (0.63 \times 6.5/12)]\ (6.5^3 \times 12)/3 = 724 \text{ in.}^4$

$$K_t = \frac{9CE_{cs}}{\ell_2 \left(1 - c_2/\ell_2\right)^3}$$ — *R13.7.5*

$$= \frac{9 \times 724 \times 1.0}{(20 \times 12)\ (1 - 1.17/20)^3} = 32.5 \text{ in.}^3$$

$\Sigma K_t = 2 \times 32.5 = 65 \text{ in.}^3$ (joint total)

Exterior equivalent column stiffness (see ACI 318R-89, R13.7.4):

$1/K_{ec} = 1/\Sigma K_t + 1/\Sigma K_c$

$K_{ec} = (1/65 + 1/180)^{-1} = 48 \text{ in.}^3$

For interior columns (14 × 20 in.):

$I = 14 \times 20^3/12 = 9333 \text{ in.}^4$

$K_c = (4 \times 1.0 \times 9333)/[103 - (2 \times 6.5)] = 415 \text{ in.}^3$

$\Sigma K_c = 2 \times 415 = 830 \text{ in.}^3$ (joint total)

$C = [1 - (0.63 \times 6.5/20)] (6.5^3 \times 20)/3 = 1456 \text{ in.}^4$

$$K_t = \frac{9 \times 1456 \times 1.0}{240 (1 - 1.17/20)^3} = 65 \text{ in.}^3$$

$\Sigma K_t = 2 \times 65 = 130 \text{ in.}^3$ (joint total)

$K_{ec} = (1/130 + 1/830)^{-1} = 112 \text{ in.}^3$

b. Slab-beam stiffness. 13.7.3

Slab stiffness, including effects of infinite stiffness within slab-column joint, can be calculated by the following approximate expression.[26.1]

$K_s = 4EI/(\ell_1 - c_1/2)$

where ℓ_1 = length of span in direction of analysis measured center-to-center of supports and c_1 = column dimension in direction of ℓ_1.

At exterior column:

$K_s = (4 \times 1.0 \times 20 \times 6.5^3)/[(17 \times 12) - 12/2] = 111 \text{ in.}^3$

At interior column (spans 1 & 3):

$K_s = (4 \times 1.0 \times 20 \times 6.5^3)/[(17 \times 12) - 20/2] = 113 \text{ in.}^3$

At interior column (span 2):

$K_s = (4 \times 1.0 \times 20 \times 6.5^3)/[(25 \times 12) - 20/2] = 76 \text{ in.}^3$

c. Distribution factors for analysis by moment distribution.

Slab distribution factors:

At exterior joints = 111/(111 + 48) = 0.70

At interior joints for spans 1 and 3 = 113/(113 + 76 +112) = 0.37

At interior joints for span 2 = 76/301 = 0.25

6. Moment Distribution—Net Loads

Since the nonprismatic section causes only very small effects on fixed-end moments and carryover factors, fixed-end moments will be calculated from FEM = $wL^2/12$ and carryover factors will be taken as COF = 1/2.

For Spans 1 and 3, net load FEM = $0.064 \times 17^2/12 = 1.54$ ft-kips

For Span 2 net load FEM = $0.061 \times 25^2/12 = 3.18$ ft-kips

Note that since live load is less than three-quarters dead load, patterned or "skipped" live load is not required. Maximum factored moments are based upon full live load on all spans simultaneously. (13.7.6.2)

Table 26-1 Moment Distribution—Net Loads
(all moments are in ft-kips)

DF	0.70	0.37	0.25
FEM	-1.54	-1.54	-3.18
Distribution	+1.08	-0.61	+0.41
Carry-over	+0.31	-0.54	-0.21
Distribution	-0.22	+0.12	-0.08
Final	-0.37	-2.57	-3.06

7. Check Net Stresses (tension positive, compression negative)

a. At interior face of interior column:

Moment at column face = centerline moment + $Vc_1/3$ (see Ref. 26.2):

$$-M_{max} = -3.06 + \frac{1}{3}\left(\frac{0.061 \times 25}{2}\right)\left(\frac{20}{12}\right)$$

$$= -2.64 \text{ ft-kips}$$

$$S = bh^2/6 = 12 \times 6.5^2/6 = 84.5 \text{ in.}^3$$

$$f_{t,b} = -f_{pc} \pm \frac{M_{net}}{S_{t,b}} = -0.172 \pm \frac{12x2.64}{84.5} = 0.172 \pm 0.375 = +0.203, -0.547 \text{ ksi}$$

Allowable Tension = $7.5\sqrt{4000} = 0.474$ ksi (18.3.3)
At top 0.203 ksi applied < 0.474 allowable OK

Allowable compression under total load = $0.60f'_c = 0.6 \times 4000 = 2.4$ ksi (18.4.2(b))
At bottom 0.547 ksi applied < 2.4 ksi allowable OK

Allowable compression under sustained load = $0.45 \times 4000 = 1.8$ ksi (18.4.2(a))
0.547 ksi applied under total load < 1.8 ksi allowable under sustained load OK
(regardless of value of sustained load).

b. At midspan of Span 2:

$+M_{max} = (0.061 \times 25^2/8) - 3.18 = +1.59$ ft-kips

$$f_{t,b} = -f_{pc} \mp \frac{M_{net}}{S_{t,b}} = -0.172 \mp \frac{12x1.59}{84.5} = -0.172 \mp 0.226 = -0.398,\ +0.054 \text{ ksi}$$

Compression at top 0.398 < 1.8 ksi allowable sustained load < 2.4 ksi allowable total load O.K. Tension at bottom 0.054 ksi applied < 0.474 ksi allowable O.K.

When the tensile stress exceeds $2\sqrt{f'_c}$ in positive moment areas, the total tensile force N_c must be carried by bonded reinforcement. For this slab, $2\sqrt{4000} = 0.126$ ksi $>$ 0.054 ksi. Therefore, positive moment bonded reinforcement is not required. When it is, the calculation for the required amount of bonded reinforcement is done as follows (refer to Figure 26-4). 18.9.3.2

$$y = \frac{f_t}{f_t + f_c}(h) \text{ in.}$$

$$N_c = \frac{12(y)(f_t)}{2} \text{ kips/ft}$$

$$A_s = \frac{N_c}{0.5f_y} \text{ in.}^2/\text{ft}$$

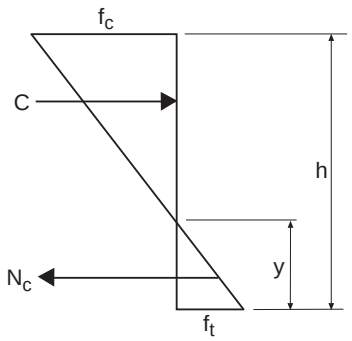


Figure 26-4

Determine minimum bar lengths for this reinforcement in accordance with *18.9.4* (Note that conformance to *Chapter 12* is also required.)

Calculate deflections under total loads using usual elastic methods and gross concrete section properties *(9.5.4)*. Limit ***computed*** deflections to those specified in Table *9.5(b)*.

This completes the service load portion of the design.

8. Flexural Strength

a. Calculation of design moments.

Design moments for statically indeterminate post-tensioned members are determined by combining frame moments due to factored dead and live loads with secondary moments induced into the frame by the tendons. The load balancing approach directly includes both primary and secondary effects, so that for service conditions only "net loads" need be considered.

At design flexural strength, the balanced load moments are used to determine secondary moments by subtracting the primary moment, which is simply $F_e \times e$, at each support. For multistory buildings where typical vertical load design is combined with varying moments due to lateral loading, an efficient design approach would be to analyze the equivalent frame under each case of dead, live, balanced, and lateral loads, and combine the cases for each design condition with appropriate load factors. For this example, the balanced load moments are determined by moment distribution as follows:

For spans 1 and 3, balanced load FEM $= 0.066 \times 17^2/12 = 1.59$ ft-kips

For span 2, balanced load FEM $= 0.064 \times 25^2/12 = 3.33$ ft-kips

Table 26-2 Moment Distribution—Balanced Loads (all moments are in ft-kips)

DF	0.70	0.37	0.25
FEM	+1.59	+1.59	+3.33
Distribution	-1.11	+0.64	-0.44
Carry-over	-0.32	+0.56	+0.22
Distribution	+0.22	-0.13	+0.09
Final	+0.38	+2.66	+3.20

Since the balanced load moment includes both primary (M_1) and secondary (M_2) moments, secondary moments can be found from the following relationship:

$M_{bal} = M_1 + M_2$, or $M_2 = M_{bal} - M_1$

The primary moment M_1 equals F_e x e at any point ("e" is the distance between the cgs and the cgc, the "eccentricity" of the prestress force).

Thus, the secondary moments are:

At an exterior column:

$M_2 = 0.38 - (13.4 \times 0/12) = 0.38$ ft-kips

At an interior column:

Spans 1 and 3,

$M_2 = 2.66 - 13.4\ (3.25 - 1.0)/12 = 0.15$ ft-kips

Span 2,

$M_2 = 3.20 - (13.4 \times 2.25)/12 = 0.69$ ft-kips

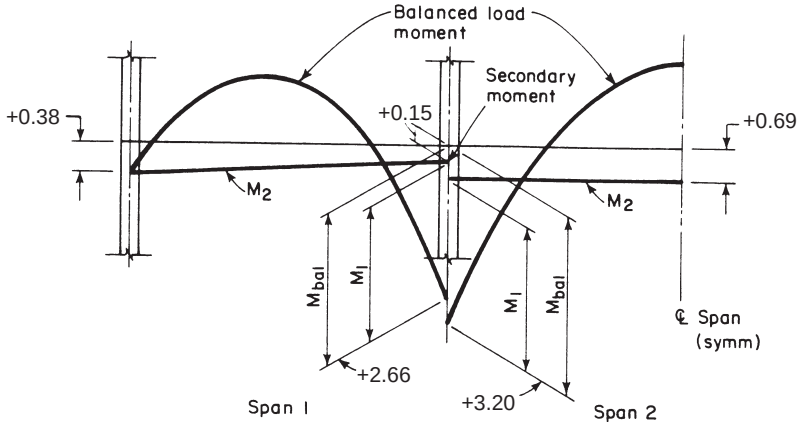


Figure 26-5

Factored load moments:

Spans 1 and 3: $w_u = 170$ psf

Span 2: $w_u = 162$ psf

For spans 1 and 3, factored load FEM $= 0.170 \times 17^2/12 = 4.09$ ft-kips

For span 2, factored load FEM $= 0.162 \times 25^2/12 = 8.44$ ft-kips

Table 26-3 Moment Distribution—Factored Loads (all moments are in ft-kips)

DF	0.70	0.37	0.25
FEM	-4.09	-4.09	-8.44
Distribution	+2.86	-1.61	+1.09
Carry-over	+0.81	-1.43	-0.55
Distribution	-0.57	+0.33	-0.22
Final	-0.99	-6.80	-8.12

Combine the factored load and secondary moments to obtain the total negative design moments. The results are given in Table 26-4.

Table 26-4 Design Moments at Face of Column (all moments are in ft-kips)

	Span 1		Span 2
Factored load moments	-0.99	-6.80	-8.12
Secondary moments	+0.38	+0.15	+0.69
Moments at column centerline	-0.61	-6.65	-7.43
Moment reduction to face of column, $Vc_1/3$	+0.48	+0.80	+1.13
Design moments at face of column	-0.13	-5.85	-6.30

Calculate total positive design moments at interior of span:

For span 1,

$V_{ext} = (0.170 \times 17/2) - (6.65 - 0.61)/17$

$= 1.45 - 0.36 = 1.09$ kips/ft

$V_{int} = 1.45 + 0.36 = 1.81$ kips/ft

Distance x to location of zero shear and maximum positive moment from centerline of exterior column:

$x = 1.09/0.170 = 6.42$ ft

End span positive moment $= (0.5 \times 1.09 \times 6.42) - 0.61 = 2.89$ ft-kips/ft (including M_2)

For span 2,

$V = 0.162 \times 25/2 = 2.03$ kips/ft

Interior span positive moment $= -7.43 + (0.5 \times 2.03 \times 12.5) = 5.26$ ft-kips/ft (including M_2)

b. Calculation of flexural strength.

Check slab at interior support. Section 18.9.3.3 requires a minimum amount of bonded reinforcement in negative moment areas at column supports regardless of service load stress levels. More than the minimum may be required for flexural strength. The minimum amount is to help ensure flexural continuity and ductility, and to control cracking due to overload, temperature, or shrinkage.

$A_s = 0.00075A_{cf}$ *Eq. (18-8)*

where

A_{cf} = larger cross-sectional area of the slab-beam strips of the two orthogonal equivalent frames intersecting at a column of a two-way slab.

$$A_s = 0.00075 \times 6.5 \times \left(\frac{17 + 25}{2}\right) \times 12 = 1.23 \text{ in.}^2$$

Try 6-No. 4 bars. Space bars at 6 in. on center, so that they are within the column width plus 1.5 times slab thickness on either side of column. *18.9.3.3*

Bar length $= [2 \times (25 - 20/12)/6] + 20/12 = 9$ ft-5 in. *18.9.4.2*

For average one-foot strip:

$A_s = 6 \times 0.20/20 = 0.06$ in.^{2}/ft

Initial check of flexural strength will be made considering this reinforcement.

Calculate stress in tendons at nominal strength:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{300\rho_p}$$ *Eq. (18-5)*

With 10 tendons in 20 ft bay:

$\rho_p = A_{ps}/bd_p = 10 \times 0.153/(20 \times 12 \times 5.5) = 0.00116$

$f_{se} = (0.7 \times 270) - 14 = 175$ ksi *18.5.1, 18.6, Reference 3*

$f_{ps} = 175 + 10 + 4/(300x0.00116) = 175 + 10 + 12 = 197$ ksi

f_{ps} shall not be taken greater than $f_{py} = 0.85f_{pu} = 230$ ksi >197
or $f_{se} + 30 = 205$ ksi > 197 OK *18.7.2(c)*

$A_{ps}f_{ps} = 10 \times 0.153 \times 197/20 = 15.1$ kips/ft

$A_sf_y = 0.06 \times 60 = 3.6$ kips/ft

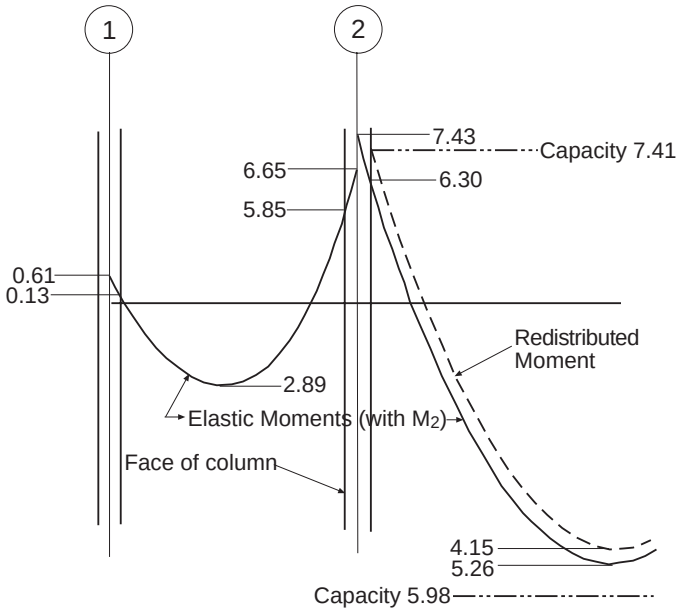


Figure 26-6 Moments in ft-kips

$$a = \frac{A_{ps}f_{ps} + A_s f_y}{0.85 f'_c b} = \frac{15.1 + 3.6}{0.85 \times 4 \times 12} = 0.46 \text{ in.}$$

$$c = a/\beta_1 = 0.46/0.85 = 0.54 \text{ in.}$$

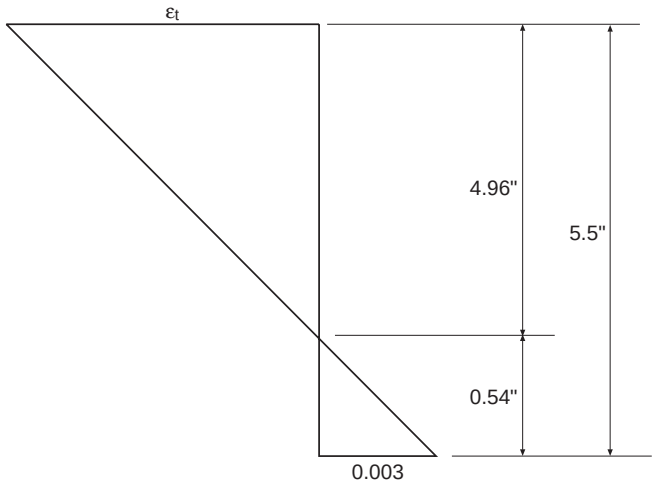


Figure 26-7 Strain Diagram at Interior Support

$\varepsilon_t = (5.5\text{-}0.54) \times 0.003/0.54 = 0.028$ therefore tension controlled $\phi = 0.9$ *9.3.2, 10.3.4*

Since the bars and tendons are in the same layer:

$$\left(d - \frac{a}{2}\right) = \left(5.5 - \frac{0.46}{2}\right)/12 = 0.44 \text{ ft}$$

$\phi M_n = 0.9 \times (15.1 + 3.6) \times 0.44 = 7.41$ ft-kips/ft > 6.30 ft-kips/ft OK. *9.3.2.1*

Since there is excess negative moment capacity available, use moment redistribution to increase the negative moment and minimize the positive moment demand in Span 2. Note that the actual inelastic moment redistribution occurs at the positive moment section of Span 2.

Permissible change in negative moment = $1000\varepsilon_t = 1000(0.028) = 28\% > 20\%$ max *18.10.4.1 8.4*

Available increase in negative moment = $0.2 \times 6.30 = 1.26$ ft-kips/ft

Actual increase in negative moment = Minimum capacity – Elastic Negative Moment
= 7.41 – 6.30 = 1.11 ft-kips/ft < 1.26 available O.K.

Minimum design positive moment in Span 2 = 5.26 – 1.11 = 4.15 ft-kips/ft

Capacity at midspan of Span 2 (no bonded reinforcement required):

$A_{ps}f_{ps}$ = 15.1 kips/ft

$$a = \frac{15.1}{0.85 \times 4 \times 12} = 0.37 \text{ in.}$$

$$\frac{c}{d_t} = \frac{\frac{0.37}{0.85}}{5.5} = 0.079 < 0.375, \text{ therefore tension controlled.}$$ 9.3.2.2 10.3.4

$$\left(d - \frac{a}{2}\right) = \frac{5.5 - \frac{0.37}{2}}{12} = 0.44 \text{ ft}$$

At center of span,

ϕM_n = 0.9 × (15.1) × 0.44 = 5.98 ft-kips/ft > 4.15 OK at midspan

Check positive moment capacity in Span 1:

$$\left(d - \frac{a}{2}\right) = \frac{(6.5 - 2.25) - \frac{0.37}{2}}{12} = 0.39 \text{ ft}$$

$$\frac{c}{d_t} = \frac{\frac{0.37}{0.85}}{4.25} = 0.102 < 0.375, \text{ therefore, tension controlled}$$ 9.3.2.2 10.3.4

ϕM_n = 0.9 × (15.1) × 0.39 = 5.30 ft-kips/ft > 2.89 OK at midspan

Exterior columns:

A_s minimum = 0.00075 × 20 × 12 × 6.5 = 1.17 in^2 use 6-#4 bars

A_s = 6 × 0.2/20 = 0.06 in^2/ft

$A_s f_y$ = 0.06 × 60 = 3.6 kips/ft

ρ_p = 10 × 0.153/(12 × 20 × 3.25) = 0.00196

f_{ps} = 175 + 10 + 4/(300 × 0.00196) = 192 ksi

$A_s f_{ps} = 10 \times 0.153 \times 192/20 = 14.7$ kips/ft

$$a = \frac{14.7 + 3.6}{0.85 \times 4 \times 12} = 0.45 \text{ in}$$

$\varepsilon_t = (5.5\text{-}0.53) \times 0.003/0.53 = 0.028$, therefore, tension controlled, $\phi = 0.9$ *9.3.2* *10.3.4*

Tendons:

$$\left(d - \frac{a}{2}\right) = \frac{(3.25) - \frac{0.45}{2}}{12} = 0.25 \text{ ft}$$

Rebar:

$$\left(d - \frac{a}{2}\right) = \frac{(5.5) - \frac{0.45}{2}}{12} = 0.44 \text{ ft}$$

$\phi M_n = 0.9 \times [(14.7 \times 0.25) + (3.6 \times 0.44)] = 4.73$ ft-kips/ft > 0.13 OK

This completes the design for flexural strength.

9. Shear and Moment Transfer Strength at Exterior Column *11.12.6* *13.5.3*

a. Shear and moment transferred at exterior column.

$V_u = (0.170 \times 17/2) - (6.65 - 0.61)/17 = 1.09$ kips/ft

Assume building enclosure is masonry and glass, weighing 0.40 kips/ft.

Total slab shear at exterior column:

$V_u = [(1.2 \times 0.40) + 1.09] \quad 20 = 31.4$ kips

Transfer moment = 20 (0.61) = 12.2 ft-kips
(factored moment at exterior column centerline = 0.61 ft-kips/ft)

b. Combined shear stress at inside face of critical transfer section.

For shear strength equations, see Part 16.

R11.12.6.2

where (referring to Table 16-2: edge column-bending perpendicular to edge)

$d \approx 0.8 \times 6.5 = 5.2$ in.

$c_1 = 12$ in.

$c_2 = 14$ in.

$b_1 = c_1 + d/2 = 14.6$ in.

$b_2 = c_2 + d = 19.2$ in.

$$c = \frac{b_1^2}{(2b_1 + b_2)} = 4.40 \text{ in.}$$

$A_c = (2b_1 + b_2)\, d = 252$ in.^{2}

$J/c = [2b_1 d\,(b_1 + 2b_2) + d^3\,(2b_1 + b_2)/b_1]/6 = 1419$ in^3

$\gamma_v = 1 - \gamma_f$ *Eq. (11-39)*

$$= 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} = 0.37$$ *13.5.3.2*

$$v_u = \frac{31400}{252} + \frac{0.37 \times 12.2 \times 12000}{1419} = 163 \text{ psi}$$

c. Permissible shear stress (for members without shear reinforcement). *11.12.6.2*

$\phi v_n = \phi V_c / (b_o d)$ *Eq. (11-20)*
where V_c is defined in 11.12.2.1 or 11.12.2.2

For edge columns:

$$\phi v_n = \phi 4\sqrt{f'_c} = 0.85 \times 4\sqrt{4000} = 215 \text{ psi} > 163 \quad \text{O.K.}$$ *11.12.2.1*

d. Check moment transfer strength. *13.5.3*

Although the transfer moment is small, for illustrative purposes, check the moment strength of the effective slab width (width of column plus 1.5 times the slab thickness on each side) for moment transfer. Assume that of the 10 tendons required for the 20 ft bay width, 3 tendons are anchored within the column and are bundled together across *13.5.3.2*

the building. This amount should be noted on the design drawings. Besides providing flexural strength, this prestress force will act directly on the critical section for shear and improve shear strength. As previously shown, a minimum amount of bonded reinforcement is required at all columns. For the exterior column, the required area is:

$A_s = 0.00075A_{cf} = 0.00075 \times 6.5 \times 20 \times 12 = 1.17$ in.^{2} *Eq. (18-8)*

Use 6-No. 4 bars, 5 ft in length (including standard end hook).

Calculate stress in tendons:

Effective slab width = 14 + 2 (1.5 × 6.5) = 33.5 in.

$$\rho_p = \frac{3 \times 0.153}{33.5 \times 3.25} = 0.0042$$

f_{ps} = 175 + 10 + 4/(300x0.0042) = 188.2 ksi

Corresponding prestress force = 3 × 0.153 × 188.2 = 86.4 kips

$A_s f_y = 6 \times 0.20 \times 60 = 72.0$ kips

$A_{ps}f_{ps} + A_s f_y = 158.4$ kips

a = 158.4/(0.85x4x33.5) = 1.39 in.

tendon (d_p- a/2) = (3.25 - 1.39/2)/12 = 0.21 ft

rebar (d - a/2) = (5.5 - 1.39/2)/12 = 0.40 ft

$\phi M_n = 0.9 [(86.4 \times 0.21) + (72 \times 0.40)] = 42.25$ ft-kips

$$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} = 0.63$$ *Eq. (13-1)*

$\gamma_f M_u = 0.63 (12.2) = 7.69$ ft-kips << 42.25 ft-kips O.K.

10. Shear and Moment Transfer Strength at Interior Column *11.12.6 13.5.3*

a. Shear and moment transferred at interior column.

Direct shear and moment to the left and right of interior columns is calculated in Step 8 above.

$V_u = (1.81 + 2.03)\ 20 = 76.8$ kips

Transfer moment $= 20\ (7.43 - 6.65) = 15.6$ ft-kips

b. Combined shear stress at face of critical transfer section. For shear strength equations, see Part 16.

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v M_u c}{J}$$ *R11.12.6.2*

where (referring to Table 16-1: interior column)

$d \approx 0.8 \times 6.5 = 5.2$ in.

$c_1 = 20$ in.

$c_2 = 14$ in.

$b_1 = c_1 + d = 25.2$ in.

$b_2 = c_2 + d = 19.2$ in.

$A_c = 2\ (b_1 + b_2)\ d = 462$ in.^{2}

$J/c = [b_1 d\ (b_1 + 3b_2) + d^3]\ /3 = 3664$ in.^{3}

$\gamma_v = 1 - \gamma_f$ *Eq. (11-39)*

$$= 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} = 0.43$$ *13.5.3.2*

$$v_u = \frac{76{,}800}{462} + \frac{0.43 \times 15.6 \times 12{,}000}{3664} = 188 \text{ psi}$$

c. Permissible shear stress.

For interior columns, Eq. (11-36) applies: *11.12.2.2*

$$\phi v_c = \phi\left(\beta_p \sqrt{f'_c} + 0.3 f_{pc} + \frac{V_p}{b_o d}\right)$$ *Eq. (11-36)*

where $\beta_p = \left(\frac{\alpha_s d}{b_o} + 1.5\right)$ but not greater than 3.5

$b_o = 2\,[(20 + 5.2) + (14 + 5.2)] = 88.8$ in.

$\alpha_s = 40$ for interior columns

$d = 5.2$ in.

$$\beta_p = \frac{40 \times 5.2}{88.8} + 1.5 = 3.8 > 3.5, \text{ use } 3.5$$

V_p is the shear carried through the critical transfer section by the tendons. For thin slabs, the V_p term must be carefully evaluated, as field placing practices can have a great effect on the profile of the tendons through the critical section. Conservatively, this term may be taken as zero. *R11.12.2.2*

$$\phi v_c = 0.85\left[3.5\sqrt{4000} + (0.3 \times 172)\right] = 232 \text{ psi} > 188 \text{ psi} \quad \text{O.K.}$$

d. Check moment transfer strength. *13.5.3*

$$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} = 0.57$$ *Eq. (13-1)*

Moment transferred by flexure within width of column plus 1.5 times slab thickness on each side = 0.57 (15.6) = 8.89 ft-kips. *13.5.3.2*

Effective slab width = 14 + 2 (1.5 × 6.5) = 33.5 in.

Say $A_{ps}f_{ps} = 86.4$ kips (same as exterior column)

$A_s = 0.00075A_{cf} = 0.00075 \times 6.5 \times (17 + 25)/2 \times 12 = 1.23$ in.^{2} *Eq. (18-8)*

Use 6-No. 4 bars ($A_s = 1.20$ in.^{2})

$A_s f_y = 1.20 \times 60 = 72.0$ kips

$A_{ps}f_{ps} + A_s f_y = 86.4 + 72.0 = 158.4$ kips

$$a = \frac{158.4}{0.85 \times 4 \times 33.5} = 1.39 \text{ in.}$$

$(d - a/2) = (5.5 - 1.39/2)/12 = 0.40$ ft

$\phi M_n = 0.9\ (158.4 \times 0.40) = 57.0$ ft-kips >> 8.89 ft-kips O.K.

This completes the shear design.

11. Distribution of tendons.

In accordance with 18.12.4, the 10 tendons per 20 ft bay will be distributed in a group of 3 tendons directly through the column with the remaining 7 tendons spaced at 2 ft-6 in. on center (4.6 times slab thickness). Tendons in the perpendicular direction will be placed in a narrow band through and immediately adjacent to the columns.

27

Shells and Folded Plate Members

INTRODUCTION

Chapter 19, concerning shells and folded plate members, was completely updated for ACI 318-83. Sections 19.2.10 and 19.2.11 were added to the '95 edition. In its present form, Chapter 19 reflects the current state-of-the-knowledge in analysis and design of folded plates and shells. It includes guidance on analysis methods appropriate for different types of shell structures, and provides specific direction as to design and proper placement of shell reinforcement. The Commentary on Chapter 19 should be helpful to designers; its contents reflect current information, including an extended reference listing.

GENERAL CONSIDERATIONS

Code requirements for shells and folded plates must, of necessity, be somewhat general in nature as compared to the provisions for other types of structures where the practice of design has been firmly established. Chapter 19 is specific in only a few critical areas inherent to shell design; otherwise, it refers to standard provisions of the code. It should be noted that strength design is permitted for shell structures, even though most of the shells in this country have been built using working stress design procedures.

The code, the commentary, and the list of references are an excellent source of information and guidance on shell design. The list of references, however, does not exhaust the possible sources of design assistance.

1. Chapter 19 covers the design of a large class of concrete structures that are quite different from the ordinary slab, beam and column construction. Structural action varies from shells with considerable bending in the shell portions (folded plates and barrel shells) to those with very little bending except at the junction of shell and support (hyperbolic paraboloids and domes of revolution). The problems of shell design, therefore, cannot be lumped together, as each type has its own peculiar attributes that must be thoroughly understood by the designer. Even shells classified under one type, such as the hyperbolic paraboloid, vary greatly in their structural action. Studies have shown that gabled hyperbolic paraboloids, for example, are much more complex than the simple membrane theory would indicate. This is one explanation for the lack of a rigid set of rules in the code for the design of shells and folded plate structures.

2. For the reasons given above, design of a shell requires considerable lead time to gain an understanding of the design problems for the particular type of shell. An attempt to design a shell without proper study may invite poor performance. Design of shell structures requires the ability to think in terms of three-dimensional space; this is only gained by study and experience. The conceptual stage is the most critical period in shell design, since this is when vital decisions on form and dimensions must be made.

3. Strength of shell structures is inherent in their shape and is not created by boosting the performance of materials to their limit as in the case of other types of concrete structures such as conventional and prestressed concrete beams. Therefore, the design stresses in the concrete should not be raised to their highest acceptable values, except where required for very large structures. Deflections are normally not a problem if the stresses are low.

4. Shell size is a very important determinant in the analytical precision required for its design. Short spans (up to 60 ft) can be designed using approximate methods such as the beam method for barrel shells, provided the exterior shell elements are properly supported by beams and columns. However, the limits and approximations of any method must be thoroughly understood. Large spans may require much more elaborate analyses. For example, a large hyperbolic paraboloid (150-ft span or more) may require a finite element analysis.

Application of the following code provisions warrants further explanation.

19.2 ANALYSIS AND DESIGN

19.2.6 Prestressed Shells

The components of force produced by prestressing tendons draped in a thin shell must be taken into account in the design. In the case of a barrel shell, it should be noted that the tendon does not lie in one plane, as shown in Fig. 27-1.

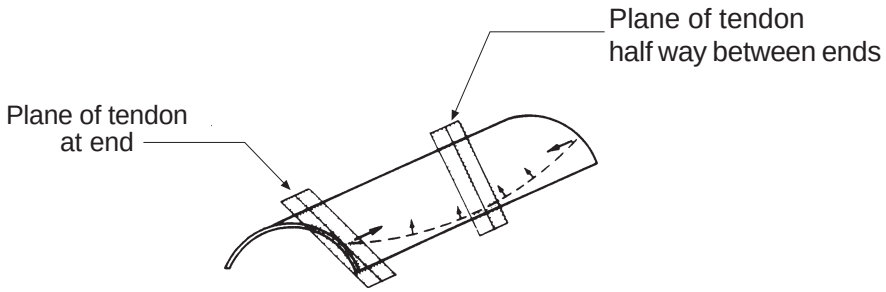


Figure 27-1 Draped Prestressing Tendon in Barrel Shell

19.2.7 Design Method

The Strength Design Method is permitted for the design of shells, but it should be noted that for slab elements intersecting at an angle, and having high tensile stresses at inside corners, the ultimate strength is greatly reduced from that at the center of a concrete slab. Therefore, special attention should be given to the reinforcement used in these areas, and thickness should be greater than the minimum allowed by the strength method.

19.4 SHELL REINFORCEMENT

19.4.6 Membrane Reinforcement

For shells with essentially membrane forces, such as hyperbolic paraboloids and domes of revolution, it is usually convenient to place the reinforcement in the direction of the principal forces. Even though folded plates and barrel shells act essentially as longitudinal beams (traditionally having vertical stirrups as shear reinforcement), an orthogonal pattern of reinforcement (diagonal bars) is much easier to place and also assures end anchorage in the barrel or folded plate. With diagonal bars, five layers of reinforcement may be required at some points.

The direction of principal stresses near the supports is usually about 45 degrees, so that equal areas of reinforcement are needed in each direction to satisfy the requirements of 19.4.4. For illustration, Fig. 27-2 shows a plot of the principal membrane forces in a barrel shell with a span of 60 ft, a rise of 6.3 ft, a thickness of 3.5 in., and a snow load of 25 psf and a roof load of 10 psf. Forces, due to service loads, are shown in kips per linear foot.

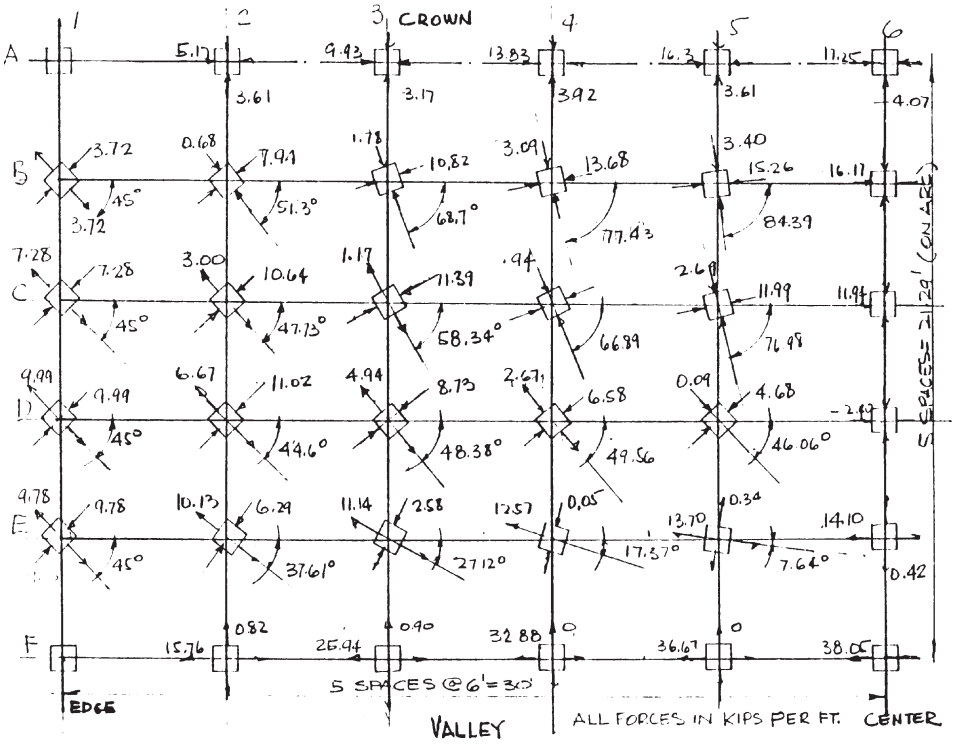


Figure 27-2 Principal Membrane Forces and Direction for 60-ft Span Barrel Shell

19.4.8 Concentration of Reinforcement

In the case of long barrel shells (or domes) it is often desirable to concentrate tensile reinforcement near the edges rather than distribute the reinforcement over the entire tensile zone. When this is done, a minimum amount of reinforcement equal to 0.0035bh must be distributed over the remaining portion of the tensile zone, as shown in Fig. 27-3. This amount in practical terms is twice the minimum steel requirement for shrinkage and temperature stresses.

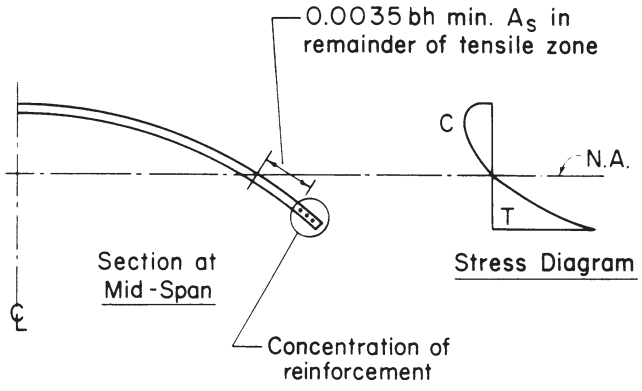


Figure 27-3 Concentration of Shell Reinforcement

19.4.10 Spacing of Reinforcement

Maximum permissible spacing of reinforcement is the smaller of 5 times the shell thickness and 18 in. Therefore, for shells less than 3.6 in. thick, 5 times the thickness controls. For thicker shells, the spacing of bars must not exceed 18 in.

28

Strength Evaluation of Existing Structures

UPDATE FOR THE '02 CODE

The strength reduction factors to be used for analytical strength evaluations in 20.2.5 are revised for compatibility with the new strength reduction factors and load factors introduced in Chapter 9. Note, the total test load prescribed in 20.3.2 has remained unchanged in spite of changes to the load factors prescribed for design in 9.2.

INTRODUCTION

Chapter 20 was revised in 1995 to flag the need to monitor during load tests not only deflections, but also cracks related to shear and/or bond, along with spalling and crushing of the concrete. In cases involving deterioration of the structure, acceptance of a building should be based on a load test. Further, the acceptance should include a time limit. Periodic inspections and strength reevaluations should be specified depending on the nature of the deterioration. When structure dimensions, size and location of reinforcement, and material properties are known, higher strength reduction factors were introduced in ACI 318-95 for analytical evaluations of the strength of existing structures.

Strength evaluation of an existing structure requires experience and sound engineering judgment. Chapter 20 provides guidance for investigating the safety of a structure when:

1. Materials of a building are considered to be deficient in quality.
2. There is evidence indicating faulty construction.
3. A building has deteriorated.
4. A building will be used for a new function.
5. A building or a portion of it does not appear to satisfy the requirements of the code.

The provisions of Chapter 20 should not be used for approval of special systems of design and construction. Approval of such systems is covered in 1.4.

References 28.1 and 28.2 published by the Concrete Reinforcing Steel Institute (CRSI) are suggested additional guides for strength evaluation of existing structures. Information about reinforcing steel found in old reinforced concrete structures is given in CRSI Engineering Data Report Number 11.[28.3]

20.1 STRENGTH EVALUATION - GENERAL

Strength evaluation of structures can be performed analytically or experimentally. Applicability of the analytical procedure depends on whether the source of deficiency is critical to the structure's strength under: (1) flexural and/or axial load, or (2) shear and/or bond. The behavior and strengths of structural concrete under flexural and/or axial load strengths can be accurately predicted based on Navier's hypothesis of "plane section before loading remains plane after loading." On the other hand, available theories and models are not as reliable to predict the shear and bond behavior and strengths of structural concrete. Code provisions for one- and two-way shear, and for bond are semi-empirical. Shear and bond failures can be brittle.

Analytical strength evaluations suffice for acceptance of buildings if two conditions are met (20.1.2). First, the source of deficiency should be critical to flexural, axial load, or combined flexural and axial load strengths. It cannot be critical to shear or bond strengths. Second, it should be possible to establish the actual building dimensions, size and location of reinforcement, and material properties. If both conditions are not met, strength evaluations should be determined by a load test as prescribed in 20.3. If causes of concern relate to flexure or axial load, but it is not possible or feasible to determine material properties, a physical test may be appropriate. Analytical evaluations of shear strength are not precluded if they are "well understood." If shear or bond strength is critical to the safety concerns, physical test may be the most efficient solution. Wherever possible and appropriate, it is desirable to support the results of the load tests by analysis (R20.1.3).

If the safety concerns are due to deterioration, strength evaluation may be through a load test. If the building satisfies the acceptance criteria of 20.5, the building should be allowed to remain in service for a specified period of time as a function of the nature of the deterioration. Periodic reevaluations of the building should be conducted.

20.2 DETERMINATION OF REQUIRED DIMENSIONS AND MATERIAL PROPERTIES

If strength evaluation of a building is performed through analysis, actual dimensions, location and size of reinforcement, and material properties should be established. Measurements should be taken at critical sections where calculated stress would reach a maximum value. When shop drawings are available, spot checks should be made to confirm location and size of reinforcing bars shown on the drawings. Nondestructive testing techniques are available to determine location and size of reinforcement, and estimate the strength of concrete. Unless they are already known, actual properties of reinforcing steel or prestressing tendons should be determined from samples extracted from the structure.

An analytical strength evaluation requires the use of the load factors of 9.2 and the strength reduction factors of 20.2.5. One of the purposes of the strength reduction factors ϕ given in R9.3.1 is "to allow for the probability of understrength members due to variations in material strengths and dimensions." When actual member dimensions, size and location of reinforcement, and concrete and reinforcing steel properties are measured, Chapter 20 specifies higher strength reduction factors. A comparison of the strength reduction factors of 20.2.5 to those of 9.3 is given in Table 28-1. The ratios of strength reduction factors of Chapter 20 to those of Chapter 9 are listed in the last column of the table. For analytical evaluation of columns and bearing on concrete, strength reduction factors ϕ of 20.2.5 are about 20 percent higher than those of 9.3. For flexure in beams and axial tension, the increase is 11 percent, while for shear and torsion it is 6 percent.

An increase in strength reduction factors, as specified in Chapter 20, results in an increase in computed member strengths. Nominal axial compressive strength of columns is in great part a function of the product of the column cross sectional area and the concrete compressive strength. As concrete compressive strength is subject to large variability, the strength reduction factors of Chapter 9 are lower for axial compression than for flexure. Because the actual concrete compressive strength is measured for strength evaluation of existing structures (20.1.2), a higher increase in strength reduction factor ϕ is specified for columns i20.2.5.

Table 28-1 Comparison of Strength Reduction Factors

	Strength reduction factor		
	Ch. 20	Ch. 9	Ch. 20/Ch. 9
Tension-controlled sections, as defined in 10.3.4	1.00	0.90	1.11
Compression-controlled sections, as defined in 10.3.3			
Members with spiral reinforcement conforming to 10.9.3	0.85	0.75	1.21
Other reinforced members	0.80	0.70	1.23
Shear and torsion	0.80	0.75	1.07
Bearing on concrete	0.80	0.65	1.23

20.3 LOAD TEST PROCEDURE

The number and arrangement of spans or panels loaded should be selected to maximize the deflection and stresses in the critical regions of the structural elements of which strength is in doubt (20.3.1). If adjoining elements are expected to contribute to the load carrying capacity, magnitude of the test load or placement should be adjusted to compensate for this contribution. As in earlier editions of the code, the total test load is specified at 0.85 (1.4D + 1.7L), where D is the sum of dead loads or related internal moments and forces, and L is defined as the live loads or related internal moments and forces. The total test load includes the dead load already in place (20.3.2). The portion of the structure being load tested should be at least 56 days old, unless all concerned parties agree to conduct the test at an earlier age (20.3.3).

Note, the load factors and strength reduction factors were revised in 9.2 and 9.3, respectively. In spite of these changes, the test load intensity prescribed in 20.3.2 has remained unchanged. Committee 318 felt that it was appropriate to maintain the same factored test load intensity, for designs complying with the new load factors and strength reduction factors of Chapter 9.

20.4 LOADING CRITERIA

Loading criteria are specified in 20.4. Initial values of all response measurements (deflection, strain, crack width, etc.) should be read and recorded not more than one hour before load application. When simulating uniformly distributed loads, arching of the applied loads must be avoided. Figure 28-1 illustrates arching action.[28.1] Sufficient gap should be provided between loading stacks so as to prevent contact, and hence arching, after member deflection, while assuring stability of the test loads.

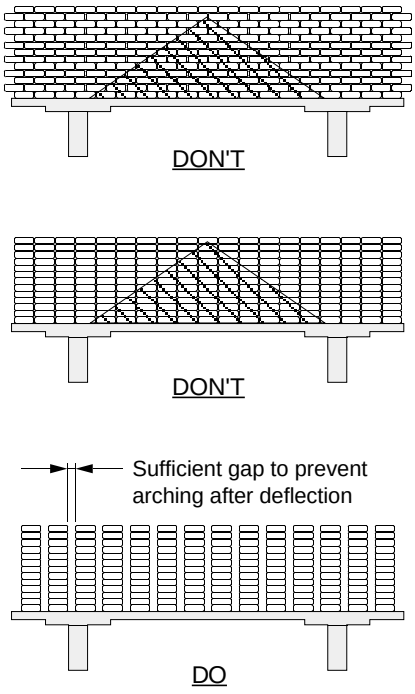


Figure 28-1 Arching Effect Shifts Applied Load to Ends of Span

Test load should be applied in not less than four approximately equal increments. A set of test load and response measurements is to be recorded after each load increment and after the total load has been applied for at least 24 hr. A set of final response measurements is to be recorded 24 hr after the test load is removed.

20.5 ACCEPTANCE CRITERIA

Evidence of failure includes spalling or crushing of concrete (20.5.1), excessive deflections (20.5.2), shear cracks (20.5.3 and 20.5.4), and bond cracks (20.5.5). No simple rules can be developed for application to all types of structures and conditions. However, in members without transverse reinforcement, projection of diagonal (inclined) cracks on an axis parallel to the longitudinal axis of the member should be monitored. If the projection of any diagonal crack is longer than the member depth at mid length of the crack, the member may be deficient in shear. If sufficient damage has occurred so that the structure is considered to have failed that test, retesting is not permitted since it is considered that damaged members should not be put into service even at a lower rating.

Deflection criteria must satisfy the following conditions (20.5.2):

1. When maximum deflection exceeds $\ell_t^2/(20{,}000h)$, the percentage recovery must be at least 75 percent after 24 hr, where

   h = overall thickness of member, in.

   ℓ_t = span of member under load test, in. (The shorter span for two-way slab systems.) Span is the smaller of (a) distance between centers of supports, and (b) clear distance between supports plus thickness h of member. Span for a cantilever must be taken as twice the distance from support to cantilever end, in.

2. When maximum deflection is less than $\ell_t^2/(20{,}000h)$, recovery requirement is waived. Figures 28-2 and 28-3 illustrate application of the limiting deflection criteria to the first load test. Figure 28-2 illustrates the limiting deflection versus member thickness for a sample span of 20 ft. Figure 28-3 depicts the limiting deflection versus span for a member 8 in. thick.

3. Members failing to meet the 75 percent recovery criterion may be retested.

4. Before retesting, 72 hr must have elapsed after load removal. On retest, the recovery must be 80 percent.

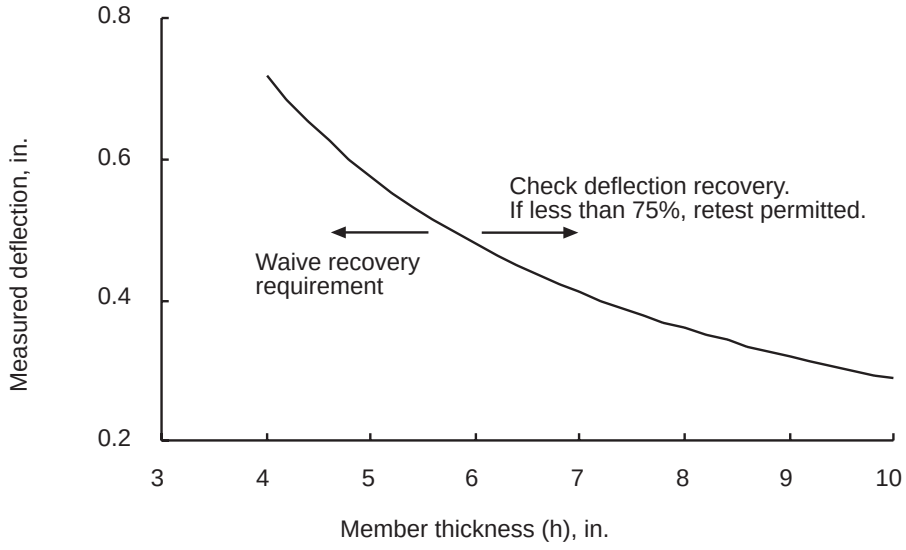


Figure 28-2 Load Testing Acceptance Criteria for Members with Span Length ℓ_t = 20 ft

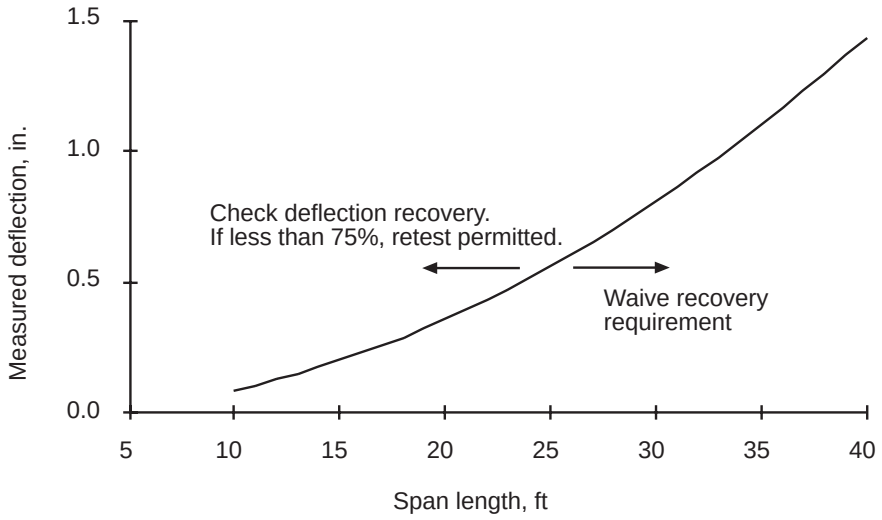


Figure 28-3 Load Testing Criteria for Members with Overall Thickness h = 8 in.

20.6 PROVISION FOR LOWER LOAD RATING

If analytical strength evaluations (20.1.2) indicate that a structure is inadequate, if the deflections of 20.5.2 are exceeded, or if cracks criteria of 20.5.3 are not met, the structure can be used for a lower load rating, if approved by the building official.

20.7 SAFETY

During load testing, shoring normally must be provided under the loaded members to assure safety. The shoring must not interfere with the test procedure or affect the test results. At no time during the load test should the deformed structure touch or bear against the shoring.

REFERENCES

28.1 "Applications of ACI 318 Load Test Requirements," Structural Bulletin No. 16, Concrete Reinforcing Steel Institute, Schaumburg, IL, November 1987.

28.2 "Proper Load Tests Protect the Public," Engineering Data Report Number 27, Concrete Reinforcing Steel Institute, Schaumburg, IL.

28.3 "Evaluation of Reinforcing Steel in Old Reinforced Concrete Structures," Engineering Data Report Number 11, Concrete Reinforcing Steel Institute, Schaumburg, IL.

29

Special Provisions for Seismic Design

UPDATE FOR THE '02 CODE

Many significant changes have been made in Chapter 21. Among these changes are new provisions for the use of precast concrete in regions of moderate and high seismic risk. New sections have been added and existing sections have been renumbered to accommodate these new provisions.

Section 21.1 contains many new definitions, most of them related to precast concrete. Included are new definitions of intermediate and special precast structural walls, ductile and strong connections, and plastic hinge regions. Additionally, the definitions of ordinary moment frame and special moment frame have been modified to include cast-in-place and precast concrete. Reference to cast-in-place concrete is now included in the definitions of intermediate moment frames and special reinforced concrete structural walls.

Editorial revisions in 21.2.1 clarify the use of structural framing systems in regions of moderate and high seismic risk. A new table in R21.2.1 provides a summary of the requirements of Chapters 21 and 22 as they apply to various structural components. Section 21.2.1.1 now includes a requirement on applicable specified concrete strengths, which essentially removes any barriers on the use of high-strength concrete unless restricted by specific code provisions. In 21.2.4.2, the maximum allowed compressive strength of lightweight aggregate concrete used in design was increased from 4,000 psi to 5,000 psi. New section 21.2.8 contains provisions for anchors resisting earthquake-induced forces in structures located in regions of moderate or high seismic risk.

Design requirements for special moment frames constructed using precast concrete are given in new section 21.6. Requirements are provided for special moment frames with ductile and strong connections. Also included is an alternative design procedure for special moment frames that do not satisfy the prescriptive requirements of 21.6.1 and 21.6.2.

The provisions for coupling beams, which are now in 21.7.7, have been modified. The aspect ratio, which was previously defined based on the effective depth of the member, is now defined based on the overall thickness. The equation for the nominal shear strength in 21.7.7.4 is now a function of the overall thickness of the member as well.

New section 21.8 contains design requirements for special structural walls constructed using precast concrete.

The provisions for structural diaphragms and trusses are now in 21.9. Editorial revisions to 21.9.8.3 were made to clarify the spacing and cover requirements for chord and collector reinforcement at splices and anchorage zones.

The design provisions for foundations supporting buildings in regions of high seismic risk are now in 21.10.

Section 21.11 now contains the provisions for frame members not proportioned to resist forces induced by earthquake motions. New requirements are provided in 21.11.4 for precast concrete frame members assumed not to contribute to lateral resistance. These provisions are also applicable to their connections.

Requirements for intermediate moment frames have moved to new section 21.12. According to 21.12.4.2, hoops instead of stirrups are now required at the ends of beams over a distance of at least twice the member depth. This change occurred based on recent experimental research and observed behavior in actual earthquakes. Also, revised requirements for transverse reinforcement in columns are contained in 21.12.5. New provisions in 21.12.6.2 address the effectiveness of flexural reinforcement perpendicular to the edge at edge and corner connections in two-way slabs. Section 21.12.6.8 contains new provisions to help prevent punching shear failures in two-way slabs during earthquakes.

Design requirements for intermediate precast structural walls are in new section 21.13. Included are provisions for connections between precast wall panels and wall panels and foundations.

BACKGROUND

Special provisions for earthquake resistance were first introduced into the 1971 edition of the ACI code in Appendix A, and were included with minor revisions in ACI 318-77. The original provisions of Appendix A were intended to apply only to reinforced concrete structures located in regions of high seismicity, and designed with a substantial reduction in total lateral seismic forces (as compared with the elastic response forces), in anticipation of inelastic structural behavior. Also, several changes were incorporated into the main body of the 1971 code specifically to improve toughness, in order to increase the resistance of concrete structures to earthquakes or other catastrophic loads. While Appendix A was meant for application to lateral load resisting frames and walls in regions of high seismicity, the main body of the code was supposed to be sufficient for regions where there is a probability of only moderate or light earthquake damage.

The special provisions of Appendix A were extensively revised for the 1983 code, to reflect current knowledge and practice of the design and detailing of monolithic reinforced concrete structures for earthquake resistance. Appendix A to ACI 318-83 for the first time included special detailing for frames in zones of moderate seismic risk.

The special provisions for earthquake resistance formed Chapter 21 of the 1989 code edition. This move from an appendix into the main body of the code was made for reasons discussed in Part 1 of this publication.

For buildings located in regions of low seismic risk, or for structures assigned to low seismic performance or design categories, no special design or detailing is required; the general requirements of Chapters 1 through 18 and 22 of the code apply. Concrete structures proportioned by Chapters 1 through 18 and 22 of the code are expected to provide a level of toughness adequate for low earthquake intensities.

For buildings located in regions of moderate seismic risk, or for structures assigned to intermediate seismic performance or design categories, moment frames proportioned to resist earthquake effects require some special reinforcement details. The special details apply only to frames (beams, columns, and slabs) to which earthquake-induced forces have been assigned in design. There are no special requirements for structural walls provided to resist the effects of wind and earthquakes, or for nonstructural components of buildings located in regions of moderate seismic risk, except for the connection requirements in 21.13 for intermediate precast structural walls. Structural walls proportioned by the non-seismic provisions of the code are considered to have sufficient toughness at drift levels anticipated in regions of moderate seismicity.

For buildings located in regions of high seismic risk, or for structures assigned to high seismic performance or design categories, where damage to construction has a high probability of occurrence, all building components, structural and nonstructural, must satisfy the requirements of 21.2 through 21.10. The special proportioning and detailing provisions of Chapter 21 are intended to provide a concrete structure with adequate toughness to respond inelastically under severe earthquake motions.

GENERAL CONSIDERATIONS

Economical earthquake-resistant design should aim at providing appropriate dynamic characteristics in structures so that acceptable response levels would result under the design earthquake. The structural properties which can be modified to achieve the desired results are the magnitude and distribution of stiffness and mass and the relative strengths of the structural members.

In some structures, such as slender free-standing towers or smoke stacks which depend for their stability on the stiffness of the single element making up the structure, or in nuclear containment buildings where a more-than-usual conservatism in design is required, yielding of the principal elements in the structure cannot be tolerated. In such cases, the design needs to be based on an essentially elastic response to moderate-to-strong earthquakes, with the critical stresses limited to the range below yield.

In most buildings, particularly those consisting of frames and other multiply-redundant systems, however, economy is achieved by allowing yielding to take place in some members under moderate-to-strong earthquake motion.

The performance criteria implicit in most earthquake code provisions require that a structure be able to:

1. Resist earthquakes of minor intensity without damage; a structure would be expected to resist such frequent but minor shocks within its elastic range of stresses.

2. Resist moderate earthquakes with negligible structural damage and some nonstructural damage; with proper design and construction, it is expected that structural damage due to the majority of earthquakes will be repairable.

3. Resist major catastrophic earthquakes without collapse; some structural and nonstructural damage is expected.

The above performance criteria allow only for the effects of a typical ground shaking. The effects of landslides, subsidence or active faulting in the immediate vicinity of the structure, which may accompany an earthquake, are not considered.

While no clear quantitative definition of the above earthquake intensity ranges has been given, their use implies the consideration not only of the actual intensity level but also of their associated probability of occurrence with reference to the expected life of a structure.

The principal concern in earthquake-resistant design is the provision of adequate strength and ductility to assure life safety, i.e., prevention of collapse under the most intense earthquake expected at a site during the life of a structure. Observations of building behavior in recent earthquakes, however, have made engineers increasingly aware of the need to ensure that buildings that house facilities essential to post-earthquake operations—such as hospitals, power plants, fire stations and communication centers—not only survive without collapse, but remain operational after an earthquake. This means that such buildings should suffer a minimum amount of damage. Thus, damage control is at times added to life safety as a second design consideration.

Often, damage control becomes desirable from a purely economic point of view. The extra cost of preventing severe damage to the nonstructural components of a building, such as partitions, glazing, ceiling, elevators and other mechanical systems, may be justified by the savings realized in replacement costs and from continued use of a building after a strong earthquake.

The principal steps involved in the earthquake-resistant design of a typical concrete structure according to building code provisions are as follows:

1. Determination of design earthquake forces

    a. calculation of base shear corresponding to computed or estimated fundamental period of vibration of the structure (a preliminary design of the structure is assumed here)

b. distribution of the base shear over the height of the building

2. Analysis of the structure under the (static) lateral earthquake forces calculated in step 1, as well as under gravity and wind loads, to obtain member design forces.

3. Designing members and joints for the critical combinations of gravity and lateral (wind or seismic) loads, and detailing them for ductile behavior.

It is important to note that some buildings are required to be designed by a dynamic, rather than a static, lateral force procedure when one or more criteria of the static procedure are not satisfied.

In the 2000 International Building Code (IBC)[29.1], as well as in the model codes that preceeded it, the design base shear represents the total horizontal seismic force that may be assumed acting parallel to the axis of the structure considered. The force in the other horizontal direction is usually assumed to act non-concurrently. Depending on the building and the seismic zone or seismic performance or design category, the seismic forces may need to be applied in the direction that produces the most critical load effect. The requirement that orthogonal effects be considered in the proportioning of a structural element may be satisfied by designing the element for 100 percent of the prescribed seismic forces in one direction plus 30 percent of the prescribed forces in the perpendicular direction. The combination requiring the greater component strength must be used for design. The vertical component of the earthquake ground motion is included in the load combinations involving earthquake forces that are prescribed in the IBC. Special provisions are also required for structural elements that are susceptible to vertical earthquake forces (cantilever beams and slabs; prestressed members).

The code-specified design lateral forces have a general distribution that is compatible with the typical envelope of maximum horizontal shears indicated by elastic dynamic analyses for regular structures. However, the code forces are substantially smaller than those that would be developed in a structure subjected to the anticipated earthquake intensity, if the structure were to respond elastically to such ground excitation. Thus, buildings designed under the present codes would be expected to undergo fairly large deformations when subjected to a major earthquake. These large deformations will be accompanied by yielding in many members of the structure, which is the intent of the codes. The reduced code-specified forces must be coupled with additional requirements for the design and detailing of members and their connections in order to ensure sufficient deformation capacity in the inelastic range.

The capacity of a structure to deform in a ductile manner (i.e., to deform beyond the yield limit without significant loss of strength), allows such a structure to dissipate a major portion of the energy from an earthquake without collapse. Laboratory tests have demonstrated that cast-in-place and precast concrete members and their connections, designed and detailed by the present codes, do possess the necessary ductility to allow a structure to respond inelastically to earthquakes of major intensity without significant loss of strength.

21.2 GENERAL REQUIREMENTS

21.2.1 Scope

Sections 21.2.1.2 through 21.2.1.4 contain the required detailing requirements based on the structural framing system, seismic risk level at the site, level of energy dissipation planned in the structural design, and the occupancy of the building.

Traditionally, seismic risk levels have been classified as low, moderate, and high. The seismic risk level of a region, or the seismic performance or design category of a building, is regulated by the legally adopted building code of the region or is determined by a local authority (1.1.8.3). Table R1.1.8.3 contains a summary of the seismic risk levels, seismic performance categories (SPC), and seismic design categories (SDC) specified in the IBC, three model building codes, as well as other resource documents (see R21.2.1).

The provisions of Chapters 1 through 18 and Chapter 22 of ACI 318 apply to structures in regions of low seismic risk or to structures assigned to low seismic performance or design categories (21.2.1.2). The design and detailing requirements of these chapters are intended to provide adequate toughness for structures in these regions or assigned to these categories. Ordinary moment frames (cast-in-place or precast) and ordinary structural walls are the structural systems that can be utilized. It is important to note that the requirements of Chapter 21 apply when the design seismic forces are computed using provisions for intermediate or special concrete systems.

In regions of moderate seismic risk or for structures assigned to satisfy intermediate seismic performance or design categories, intermediate or special moment frames, or ordinary, intermediate, or special structural walls shall be used (21.2.1.3). Provisions for intermediate moment frames and intermediate precast structural walls are contained in 21.12 and 21.13, respectively.

Special moment frames (cast-in-place or precast), special structural walls (cast-in-place or precast), and diaphragms and trusses complying with 21.2 through 21.10 shall be used in regions of high seismic risk or for structures assigned to satisfy high seismic performance or design categories (21.2.1.4). The provisions of 21.2 through 21.11 have been developed to provide adequate toughness should the design earthquake occur.

The requirements of Chapter 21 as they apply to various structural components are summarized in Table R21.2.1.

21.2.2 Analysis and Proportioning of Structural Members

The interaction of all structural and nonstructural components affecting linear and nonlinear structural response are to be considered in the analysis (21.2.2.1). Consequences of failure of structural and nonstructural components not forming part of the lateral force resisting system shall also be considered (21.2.2.2). The intent of 21.2.2.1 and 21.2.2.2 is to draw attention to the influence of nonstructural components on structural response and to hazards from falling objects.

Section 21.2.2.3 alerts the designer to the fact that the base of the structure as defined in analysis may not necessarily correspond to the foundation or ground level. It requires that structural members below base, which transmit forces resulting from earthquake effects to the foundation, shall also comply with the requirements of Chapter 21.

Even though some element(s) of a structure may not be considered part of the lateral force resisting system, the effect on all elements due to the design displacements must be considered (21.2.2.4).

21.2.3 Strength Reduction Factors

The strength reduction factors of 9.3.2 are not based on the observed behavior of cast-in-place or precast concrete members under load or displacement cycles simulating earthquake effects. Some of those factors have been modified in 9.3.4 in view of the effects on strength due to large displacement reversals into the inelastic range of response.

Section 9.3.4(a) refers to members such as low-rise walls or portions of walls between openings which are proportioned such as to make it impractical to raise their nominal shear strength above the shear corresponding to nominal flexural strength for the pertinent loading conditions.

21.2.4, 21.2.5 Limitations on Materials

A minimum specified concrete compressive strength f'_c of 3,000 psi and a maximum specified reinforcement yield strength f_y of 60,000 psi are mandated. These limits are imposed as reasonable bounds on the variation of material properties, particularly with respect to their unfavorable effects on the sectional ductilities of members in which they are used. A decrease in the concrete strength and an increase in the yield strength of the tensile

reinforcement tend to decrease the ultimate curvature and hence the sectional ductility of a member subjected to flexure.

The new statement added to 21.2.1.1, referencing 1.1.1, helps to clarify that no maximum specified compressive strength applies. Limitations on the compressive strength of lightweight aggregate concrete is discussed below.

There is evidence suggesting that lightweight concrete ranging in strength up to 12,500 psi can attain adequate ultimate strain capacities. Testing to examine the behavior of high strength, lightweight concrete under high intensity, cyclic shear loads, including a critical study of bond characteristics, has not been extensive in the past. However, there are test data showing that properly designed lightweight concrete columns, with concrete strength ranging up to 6,200 psi, maintained ductility and strength when subjected to large inelastic deformations from load reversals. It was felt by Committee 318 that a limit of 5,000 psi on the strength of lightweight concrete was advisable, pending further testing of high strength lightweight concrete members under reversed cyclic loading. Note that lightweight concrete with a higher design compressive strength is allowed if it can be demonstrated by experimental evidence that structural members made with that lightweight concrete possess strength and toughness equal to or exceeding those of comparable members made with normal weight concrete of the same strength.

Chapter 21 requires that reinforcement for resisting flexure and axial forces in frame members and wall boundary elements be ASTM A 706 Grade 60 low alloy steel, which is intended for application where welding or bending, or both, are important. However, ASTM A 615 billet steel bars of Grade 40 or 60 may be used in these members if the following two conditions are satisfied:

$$\text{actual } f_y \leq \text{specified } f_y + 18{,}000 \text{ psi}$$

$$\frac{\text{actual ultimate tensile stress}}{\text{actual } f_y} \geq 1.25$$

The first requirement helps to limit the magnitude of the actual shears that can develop in a flexural member above that computed on the basis of the specified yield value when plastic hinges form at the ends of a beam. Note that retests shall not exceed this value by more than an additional 3,000 psi. The second requirement is intended to ensure steel with a sufficiently long yield plateau.

In the "strong column-weak beam" frame intended by the code, the relationship between the moment strengths of columns and beams may be upset if the beams turn out to have much greater moment strengths than intended. Thus, the substitution of Grade 60 steel of the same area for specified Grade 40 steel in beams can be detrimental. The shear strength of beams and columns, which is generally based on the condition of plastic hinges forming at the ends of the members, may become inadequate if the moment strengths of member ends should be greater than intended as a result of the steel having a substantially greater yield strength than specified.

21.2.6 Mechanical Splices

Section 21.2.6 contains provisions for mechanical splices. According to 21.2.6.1, a Type 1 mechanical splice shall conform to 12.14.3.2, i.e., the splice shall develop in tension or compression at least 125 percent of the specified yield strength f_y of the reinforcing bar. A Type 2 mechanical splice shall also conform to 12.14.3.2 and shall develop the specified tensile strength of the spliced bar.

During an earthquake, the tensile stresses in the reinforcement may approach the tensile strength of the reinforcement as the structure undergoes inelastic deformations. Thus, Type 2 mechanical splices can be used at any location in a member (21.2.6.2). The locations of Type 1 mechanical splices are restricted since the tensile stresses in the reinforcement in yielding regions of the member can exceed the strength requirements of 12.14.3.2. Consequently, Type 1 mechanical splices are not permitted within a distance equal to twice the member depth from the face of the column or beam or from sections where yielding of the reinforcement is likely to occur due to inelastic lateral displacements (21.2.6.2).

21.2.7 Welded Splices

The requirements for welded splices are in 21.2.7. Welded splices shall conform to the provisions of 12.14.3.4, i.e., the splice shall develop at least 125 percent of the specified yield strength f_y of the reinforcing bar (21.2.7.1). Similar to Type 1 mechanical splices, welded splices are not permitted within a distance equal to twice the member depth from the face of the column or beam or from sections where yielding of the reinforcement is likely to occur due to inelastic lateral displacements; in yielding regions of the member, the tensile stresses in the reinforcement can exceed the strength requirements of 12.14.3.4 (21.2.7.1).

According to 21.2.7.2, welding of stirrups, ties, inserts or other similar elements to longitudinal reinforcement that is required by design is not permitted. Welding of crossing reinforcing bars can lead to local embrittlement of the steel. If such welding will facilitate fabrication or field installation, it must be done only on bars added expressly for construction. Note that this provision does not apply to bars that are welded with welding operations under continuous competent control, as is the case in the manufacture of welded wire fabric.

21.2.8 Anchoring to Concrete

The requirements in this new section pertain to anchors resisting earthquake-induced forces in structures located in regions of moderate or high seismic risk. The design of such anchors must conform to the additional requirements of D.3.3 of Appendix D. See Part 34 for additional information.

21.3 FLEXURAL MEMBERS OF SPECIAL MOMENT FRAMES

The lefthand column of Table 29-1 contains the requirements for flexural members of special moment frames (as noted above, special moment frames, which can be cast-in-place or precast, are required in regions of high seismic risk or for structures assigned to satisfy high seismic performance or design categories). These requirements typically apply to beams of frames and other flexural members with negligible axial loads (21.3.1). Special precast moment frames must also satisfy the provisions of 21.6, which are discussed below. For comparison purposes, Table 29-1 also contains the corresponding requirements for flexural members of intermediate and ordinary cast-in-place moment frames. See Chapter 16 and Part 23 for additional information on precast systems.

21.3.1 Scope

Flexural members of special moment frames must meet the general requirements of 21.3.1.1 through 21.3.1.4. These limitations have been guided by experimental evidence and observations of reinforced concrete frames that have performed well in past earthquakes. Members must have sufficient ductility and provide efficient moment transfer to the supporting columns. Note that columns subjected to bending and having a factored axial load $P_u \le A_g f'_c/10$ may be designed as flexural members, where A_g is the gross area of the section.

Table 29-1 Flexural Members of Frames

	Special Moment Frames	Intermediate and Ordinary CIP Moment Frames
General	Flexural frame members shall satisfy the following conditions: • Factored axial compressive force ≤ $A_g f_c'$ / 10 • Clear span ≥ 4 × effective depth • Width to depth ratio ≥ 0.3 • Width ≥ 10 in. • Width ≤ width of supporting member + distances on each side of the supporting member not exceeding three-fourths of the depth of the flexural member **21.3.1**	Intermediate — Factored axial compressive force ≤ $A_g f_c'$ / 10. **21.12.2** Ordinary — No similar requirements.
Flexural Requirements	Minimum reinforcement shall not be less than $\frac{3\sqrt{f_c'}b_w d}{f_y}$ and $\frac{200\, b_w d}{f_y}$ at any section, top and bottom, unless provisions in 10.5.3 are satisfied. **21.3.2.1**	Same requirement except minimum reinforcement need only be provided at sections where tensile reinforcment is required by analysis, except as provided in 10.5.2, 10.5.3, and 10.5.4. **10.5**
	The reinforcement ratio (ρ) shall not exceed 0.025. **21.3.2.1**	The net tensile strain ε_t at nominal strength shall not be less than 0.004. **10.3.5**
	At least two bars shall be provided continuously at both top and bottom of section. **21.3.2.1**	Provide minimum structural integrity reinforcement. **7.13**
	Positive moment strength at joint face ≥ 1/2 negative moment strength at that face of the joint. **21.3.2.2**	Intermediate — Positive moment strength at joint face ≥ 1/3 negative moment strength at that face of the joint. **21.12.4.1** Ordinary — No similar requirement.
	Neither the negative nor the positive moment strength at any section along the member shall be less than 1/4 the maximum moment strength provided at the face of either joint. **21.3.2.2**	Intermediate — Same requirement except only need to provide 1/5 of the maximum moment strength at the face of either joint at any section along the member. **21.12.4.1** Ordinary — No similar requirement.

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A_g = gross area of section

b_w = width of web

d = effective depth of section

f_c' = compressive strength of concrete

f_y = yield strength of reinforcement

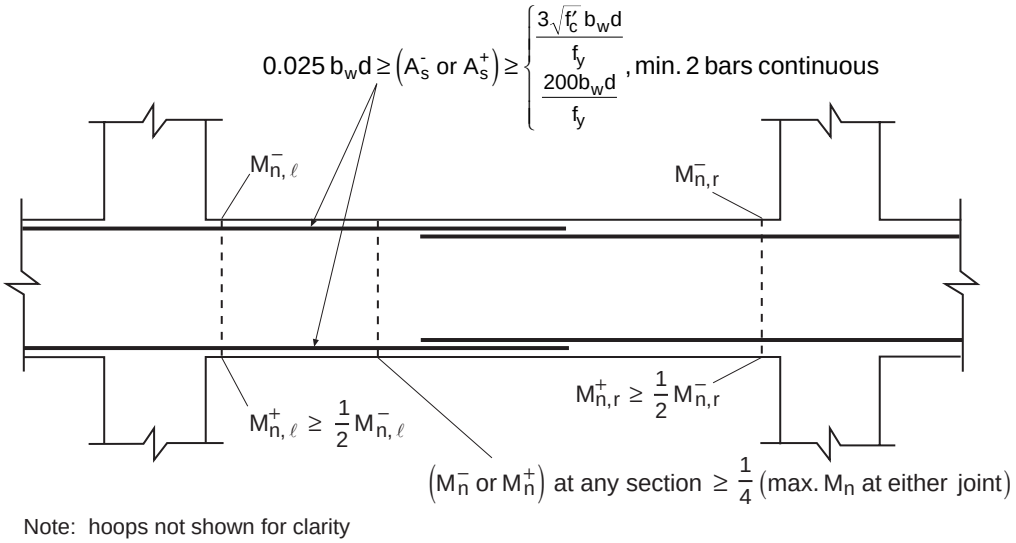
Table 29-1 Flexural Members of Frames (cont'd)

	Special Moment Frames	Intermediate and Ordinary CIP Moment Frames
Splices	Lap splices of flexural reinforcement are permitted only if hoop or spiral reinforcement is provided over the lap length. Hoop and spiral reinforcment spacing shall not exceed *d*/4 or 4 in. Mechanical splices shall conform to 21.2.6 and welded splices shall conform to 21.2.7. **21.3.2.3, 21.3.2.4**	There is no requirement that splices be enclosed in hoops.
	Lap splices are not to be used: • Within joints. • Within a distance of twice the member depth from the face of the joint. • At locations where analysis indicates flexural yielding caused by inelastic lateral displacements of the frame. **21.3.2.3**	No similar requirement.
Transverse Reinforcement	Hoops are required over a length equal to twice the member depth from the face of the supporting member toward midspan at both ends of the flexural member. **21.3.3.1**	Intermediate — Same requirement. . **21.12.4.2** Ordinary — No similar requirement.
	Hoops are required over lengths equal to twice the member depth on both sides of a section where flexural yielding may occur in connection with inelastic lateral displacements of the frame. **21.3.3.1**	Reinforcement for flexural members subject to stress reversals shall consist of closed stirrups extending around flexural reinforcement. Also, provide minimum structural integrity reinforcement. **7.11.2, 7.13**
	Where hoops are required, the spacing shall not exceed: *d*/4 8 × diameter of smallest longitudinal bar 24 × diameter of hoop bars 12 in. The first hoop shall be located not more than 2 in. from the face of the supporting member. **21.3.3.2**	Intermediate — Same requirement. **21.12.4.2** Ordinary — No similar requirement.
	Where hoops are required, longitudinal bars on the perimeter shall have lateral support conforming to 7.10.5.3. **21.3.3.3**	No similar requirement.
	Where hoops are not required, stirrups with seismic hooks at both ends shall be spaced at a distance not more than *d*/2 throughout the length of the member. **21.3.3.4**	Intermediate — Similar requirement except that seismic hooks are not required. **21.12.4.3**
	Transverse reinforcment must also be proportioned to resist the design shear force, neglecting the contribution of concrete to shear strength if certain conditions are met. **21.3.4**	Intermediate — Transverse reinforcement must also be proportioned to resist the design shear force. **21.12.3** Ordinary — Provide sufficient transverse reinforcement for shear and torsion. **11.5, 11.6**

21.3.2 Flexural Reinforcement

The reinforcement requirements for flexural members of special moment frames are shown in Fig. 29-1. To allow for the possibility of the positive moment at the end of a beam due to earthquake-induced lateral displacements exceeding the negative moment due to gravity loads, 21.3.2.2 requires a minimum positive moment strength at the ends of the beam equal to at least 50 percent of the corresponding negative moment strength. The minimum moment strength at any section of the beam is based on the moment strength at the faces of the supports. These requirements ensure strength and ductility under large lateral displacements. The limiting ratio of 0.025 is based primarily on considerations of steel congestion and also on limiting shear stresses in beams of typical proportions. The requirement that at least two bars be continuous at both the top and bottom of the section is for construction purposes.

The flexural requirements for flexural members of intermediate moment frames are similar to those shown in Fig. 29-1 (see Table 29-1).



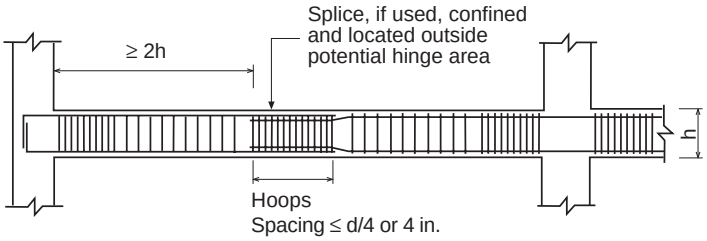
Note: hoops not shown for clarity

Figure 29-1 Reinforcement Requirements for Flexural Members of Special Moment Frames

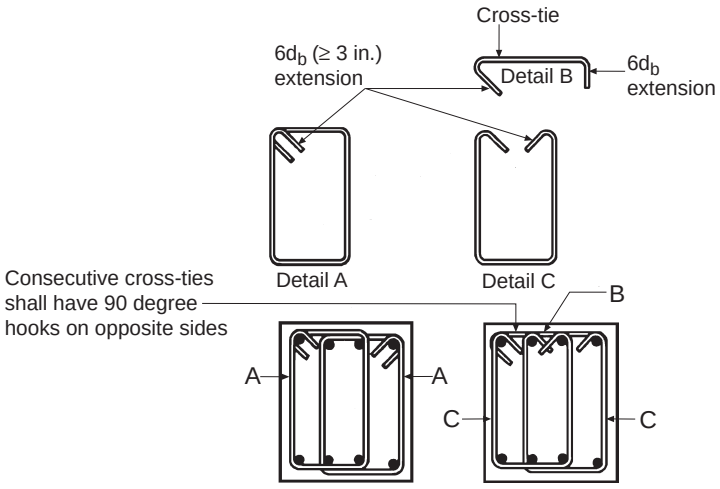
Lap splices of flexural reinforcement must be placed at locations away from potential hinge areas subjected to stress reversals under cyclic loading (see Fig. 29-2). Where lap splices are used, they should be designed as tension lap splices and must be properly confined. Mechanical splices and welded splices must conform to 21.2.6 and 21.2.7, respectively.

21.3.3 Transverse Reinforcement

Adequate confinement is required at ends of flexural members, where plastic hinges are likely to form, in order to ensure sufficient ductility of the members under reversible loads. Transverse reinforcement is also required at these locations to assist the concrete in resisting shear and to maintain lateral support for the reinforcing bars. For flexural members of special moment frames, the transverse reinforcement for confinement must consist of hoops as shown in Fig. 29-3. Hoops must be used for confinement in flexural members of intermediate moment frames as well (21.12.4.2). Shear strength requirements of flexural members are given in 21.3.4 for special moment frames and 21.12.3 for intermediate moment frames.

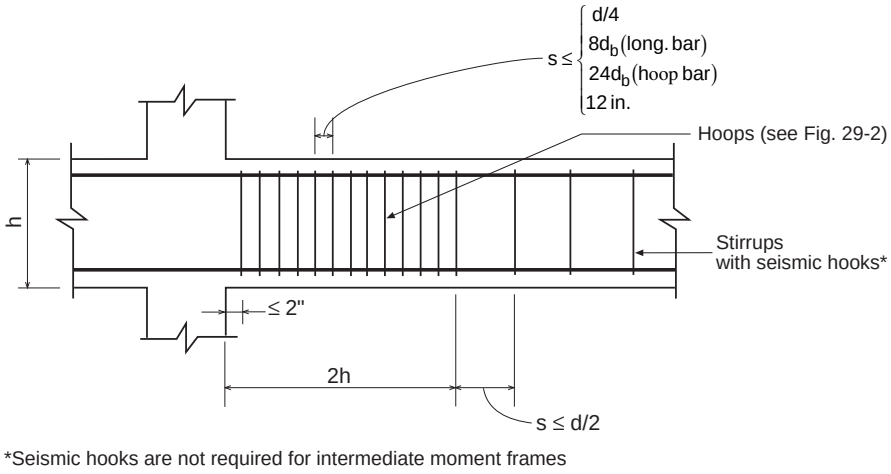


Typical Splice Details



Single- and Two-Piece Hoops

Figure 29-2 Splices and Hoop Reinforcement for Flexural Members of Special Moment Frames



*Seismic hooks are not required for intermediate moment frames

Figure 29-3 Transverse Reinforcement for Flexural Members of Special and Intermediate Moment Frames

21.3.4 Shear Strength Requirements

Typically, larger forces than those prescribed by the governing building code are induced in structural members during an earthquake. Designing for shear forces from a combined gravity and lateral load analysis using the code-prescribed load combinations is not conservative since in actuality, the reinforcement may be stressed beyond its yield strength resulting in larger than anticipated shear forces. Adequate shear reinforcement must be

provided so as to preclude shear failure prior to the development of plastic hinges at the ends of the beam. Thus, a flexural member of a special moment frame must be designed for the shear forces associated with probable moment strengths M_{pr} acting at the ends and the factored tributary gravity load along its span (21.3.4.1). The probable moment strength M_{pr} is associated with plastic hinging in the flexural member, and is defined as the strength of the beam with the stress in the reinforcing steel equal to $1.25f_y$ and a strength reduction factor of 1.0:

$$M_{pr} = A_s(1.25f_y)\left(d - \frac{a}{2}\right) \quad \text{(rectangular section with tension reinforcement only)}$$

where $a = \dfrac{A_s(1.25f_y)}{0.85f'_c b}$

Note that sidesway to the right and to the left must both be considered to obtain the maximum shear force (see Fig. 29-4). The use of $1.25f_y$ for the stress in the reinforcing steel reflects the strong likelihood that the deformation in the tensile reinforcement will be in the strain hardening range. By taking $1.25f_y$ as the stress in the reinforcement and 1.0 as the strength reduction factor, the chance of shear failure preceding flexural yielding is reduced.

In determining the required shear reinforcement over the lengths identified in 21.3.3.1, the contribution of the shear strength of the concrete V_c is taken as zero if the shear force from seismic loading is one-half or more of the maximum required shear strength and the factored axial compressive force including earthquake effects is less than $A_g f'_c/20$ (21.3.4.2). The purpose of this requirement is not to imply that the concrete does not contribute to the shear carrying capacity, but rather to provide adequate shear reinforcement to ensure flexural failure. Note that the strength reduction factor ϕ to be used is 0.75 (see 9.3.4 and 9.3.2.3).

Shear reinforcement shall be in the form of hoops over the lengths specified in 21.3.3.1 (21.3.3.5); at or near regions of flexural yielding, spalling of the concrete shell is very likely to occur. Details of hoop reinforcement are given in 21.3.3.6 (see Fig. 29-2). Where hoops are not required, stirrups with seismic hooks at both ends may be used (21.3.3.4, 21.3.3.5). A minimum amount of transverse reinforcement is required throughout the entire length of flexural members to safeguard against any loading cases that were unaccounted for in design.

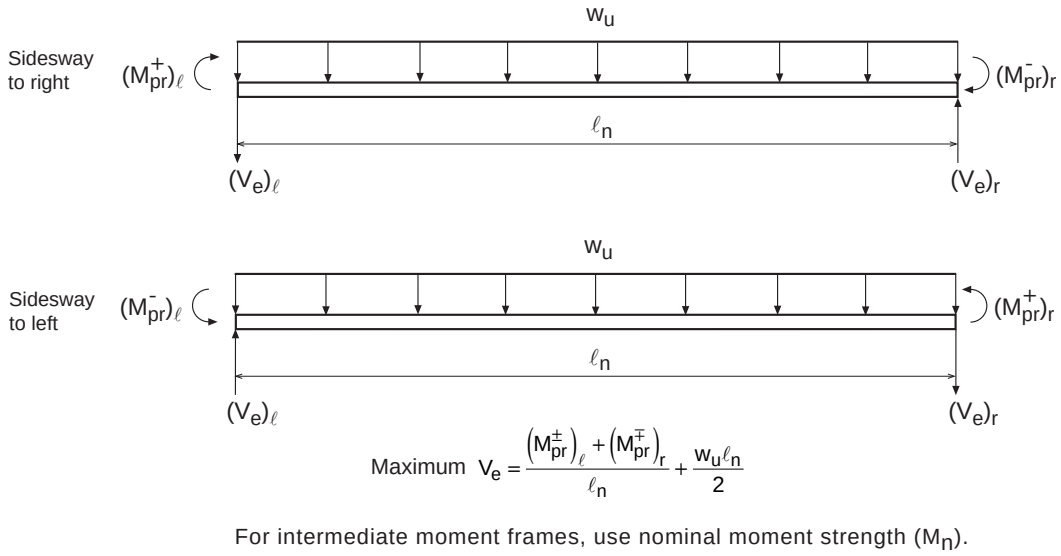


Figure 29-4 Design Shear Forces for Flexural Members of Special Moment Frames

The transverse reinforcement provided within the lengths specified in 21.3.3.1 shall satisfy the requirement for confinement or shear, whichever governs.

A similar analysis is required for frame members of intermediate moment frames except that the nominal moment strength M_n of the member is used instead of the probable moment strength (21.12.3). Also in 21.12.3 is an alternate procedure where the earthquake effects are doubled in lieu of using the nominal moment strength.

21.4 SPECIAL MOMENT FRAME MEMBERS SUBJECTED TO BENDING AND AXIAL LOAD

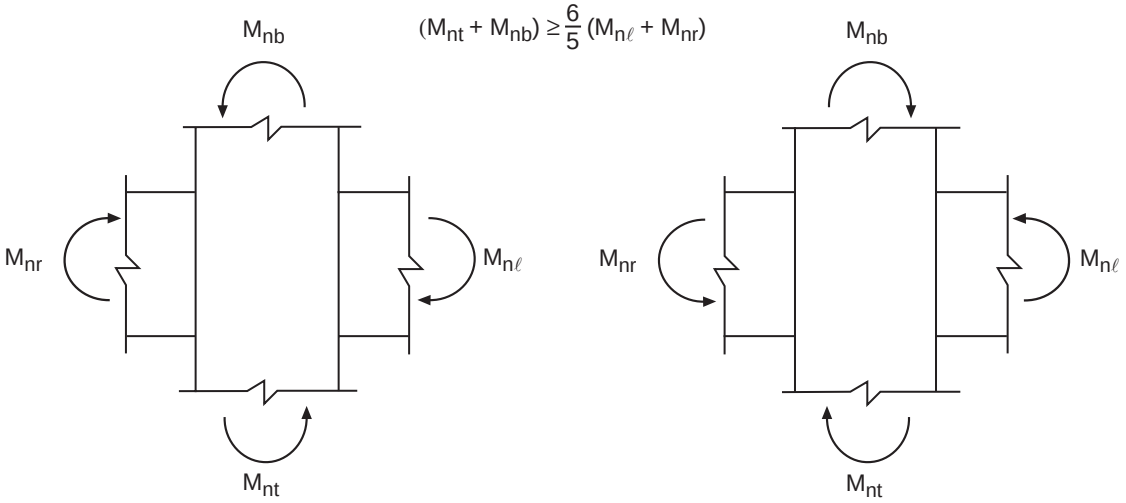
The left hand column of Table 29-2 contains the requirements for special moment frame members subjected to combined bending and axial loads. These requirements would typically apply to columns of frames and other flexural members which carry a factored axial load $P_u > A_g f'_c/10$. For comparison purposes, Table 29-2 also contains the corresponding requirements for intermediate and ordinary cast-in-place moment frames.

21.4.1 Scope

Section 21.4.1 is intended primarily for columns of special moment frames. Frame members other than columns that do not satisfy 21.3.1 are proportioned and detailed according to 21.4. The geometric constraint requirements are based on results from previous practice and tests.

21.4.2 Minimum Flexural Strength of Columns

Columns must be provided with sufficient strength so that they will not yield prior to the beams. Lateral sway caused by column hinging may result in excessive damage. Yielding of the columns prior to the beams could also result in total collapse of the structure. For these reasons, columns are designed with 20% higher flexural strength as compared to beams at the same joint, as shown in Fig. 29-5. In 21.4.2.2, nominal strengths of the columns and girder are calculated at the joint faces, and those strengths are used in Eq. (21-1). The column flexural strength is calculated for the factored axial load, consistent with the direction of the lateral forces considered, resulting in the lowest flexural strength.



Subscripts ℓ, r, t, and b stand for left support, right support, top of column, and bottom of column, respectively.

Figure 29-5 "Strong Column-Weak Beam" Frame Requirements for Special Moment Frames

When computing the nominal flexural strength of girders in T-beam construction, slab reinforcement within an effective slab width defined in 8.10 shall contribute to the flexural strength if the slab reinforcement is developed at the critical section for flexure. Research has shown that using the effective flange width in 8.10 gives reasonable estimates of the negative bending strength of girders at interior joints subjected to interstory displacements approaching 2% of the story height.

Table 29-2 Frame Members Subjected to Bending and Axial Loads

	Special Moment Frames	Intermediate and Ordinary CIP Moment Frames
General	Frame members under this classification must meet the following requirements: • Factored axial compressive force > $A_g f_c'$ / 10. • Shortest cross-sectional dimension ≥ 12 in. • Ratio of shortest cross-sectional dimension to perpendicular dimension ≥ 0.4. **21.4.1**	Intermediate — Factored axial compressive force > $A_g f_c'$ / 10. **21.12.2** Ordinary — No similar requirements.
Flexural Requirements	The flexural strengths of columns shall satisfy the following: $\Sigma M_c \geq (6/5)\ \Sigma M_g$ where ΣM_c = sum of moments at the faces of the joint, corresponding to the nominal flexural strength of the columns. ΣM_g = sum of moments at the faces of the joint, corresponding to the nominal flexural strength of the girders. In T-beam construction, slab reinforcement within an effective slab width defined in 8.10 shall contribute to flexural strength. If this requirement is not satisfied, the lateral strength and stiffness of the column shall not be considered when determining the strength and stiffness of the structure, and the column shall conform to 21.11; also, the column must have transverse reinforcement over its full height as specified in 21.4.4.1 through 21.4.4.3. **21.4.2**	No similar requirements.
	The reinforcement ratio (ρ_g) shall not be less than 0.01 and shall not exceed 0.06. **21.4.3.1**	The reinforcement ratio (ρ_g) shall not be less than 0.01 and shall not exceed 0.08. **10.9**
Splices	Mechanical splices shall conform to 21.2.6 and welded splices shall conform to 21.2.7. Lap splices are permitted only within the center half of the member length, must be tension lap splices, and shall be enclosed within transverse reinforcement conforming to 21.4.4.2 and 21.4.4.3. **21.4.3.2**	There is no restriction on the location of splices which are typically located just above the floor for ease of construction. However, for intermediate moment frames, splices may be located within the center half of the member as in special moment frames. Splices can be either Class A or B and shall have transverse reinforcement where reversal of flexural stress is anticipated.
Transverse Reinforcement	The transverse reinforcement requirements discussed in the following five items on the next page need only be provided over a length (ℓ_o) from each joint face and on both sides of any section where flexural yielding is likely to occur. The length (ℓ_o) shall not be less than: depth of member 1/6 clear span 18 in. **21.4.4.4**	Intermediate — The length (ℓ_o) is the same as for special moment frames. **21.12.5.2** Ordinary — No similar requirements.

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A_c = area of core of spirally reinforced compression member measured to outside diameter of spiral

A_{ch} = cross-sectional area of member measured out-to-out of transverse reinforcement

A_g = gross area of section

f_c' = compressive strength of concrete

f_{yh} = yield stress of transverse reinforcement

h = smallest cross-sectional dimension of the member

h_c = cross-sectional dimension of column core measured center-to-center of the confining reinforcement

h_x = maximum horizontal spacing of hoop or crosstie legs on all faces of the column

s = spacing of transverse reinforcement

s_x = longitudinal spacing of transverse reinforcement within the length ℓ_o.

Table 29-2 Frame Members Subjected to Bending and Axial Loads (cont'd)

	Special Moment Frame	Intermediate and Ordinary CIP Moment Frames
Transverse Reinforcement (continued)	Ratio of spiral reinforcement (ρ_s) shall not be less than the value given by: $\rho_s = 0.12\frac{f_c'}{f_{yh}} \geq 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_{yh}}$ **21.4.4.1**	Ratio of spiral reinforcement (ρ_s) shall not be less than the value given by: $\rho_s = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_{yh}}$ and shall conform to the provisions in 7.10.4. **10.9.3**
	Total cross-sectional area of rectangular hoop reinforcement for confinement (A_{sh}) shall not be less than that given by the following two equations: $A_{sh} = 0.3\left(sh_c f_c' / f_{yh}\right)\left[\left(A_g / A_{ch}\right) - 1\right]$ $A_{sh} = 0.09\left(sh_c f_c' / f_{yh}\right)$ **21.4.4.1**	Transverse reinforcement must be provided to satisfy both shear and lateral support requirements for longitudinal bars. **7.10.5, 11.1**
	If the thickness of the concrete outside the confining transverse reinforcement exceeds 4 in., additional transverse reinforcement shall be provided at a spacing ≤ 12 in. Concrete cover on the additional reinforcement shall not exceed 4 in. **21.4.4.1**	No similar requirements.
	Transverse reinforcement shall be spaced at distances not exceeding h/4, 6 × longitudinal bar diameter, 4 in. ≤ $s_x = 4 + [(14 - h_x)/3] \leq 6$ in. **21.4.4.2**	Intermediate—Maximum spacing s_o is 8 × smallest longitudinal bar diameter, 24 × hoop bar diameter, h/2, or 12 in. First hoop to be located no further than s_o/2 from the joint face. **21.12.5.2** Ordinary — No similar requirement.
	Cross ties or legs of overlapping hoops shall not be spaced more than 14 in. on center in the direction perpendicular to the longitudinal axis of the member. Vertical bars shall not be farther than 6 in. clear from a laterally supported bar. **21.4.4.3, 7.10.5.3**	Vertical bars shall not be farther than 6 in. clear from a laterally supported bar. **7.10.5.3**
	Where the transverse reinforcement as discussed above is no longer required, the remainder of the column shall contain spiral or hoop reinforcement spaced at distances not to exceed 6 × longitudinal bar diameter 6 in. **21.4.4.6**	Intermediate — Outside the length ℓ_o, spacing of transverse reinforcement shall conform to 7.10 and 11.5.4.1. **21.12.5.4** Ordinary — Transverse reinforcement to conform to 7.10 and 11.5.4.1.
	Transverse reinforcement must also be proportioned to resist the design shear force (V_e). **21.4.5**	Intermediate — Transverse reinforcement must also be proportioned to resist the design shear forces specified in 21.12.3. Ordinary — Provide sufficient transverse reinforcement for shear. **11.5.4, 11.5.6**
	Columns supporting reactions from discontinued stiff members, such as walls, shall have transverse reinforcement as specified in 21.4.4.1 through 21.4.4.3 over their full height, if the factored axial compressive force, including earthquake effects, exceeds ($A_g f_c' / 10$). This transverse reinforcement shall extend into the discontinued member for at least the development length of the largest longitudinal reinforcement in the column in accordance with 21.5.4. If the column terminates on a footing or mat, the transverse reinforcement shall extend at least 12 in. into the footing or mat. **21.4.4.5**	No similar requirement.

If Eq. (21-1) is not satisfied at a joint, columns supporting reactions from that joint are to be provided with transverse reinforcement as specified in 21.4.4 over their full height (21.4.2.3), and shall be ignored in determining the calculated strength and stiffness of the structure (21.4.2.1). These columns must also conform to the provisions for frame members not proportioned to resist earthquake motions as given in 21.11 (21.4.2.1).

No similar provisions are required for intermediate or ordinary moment frames.

21.4.3 Longitudinal Reinforcement

The maximum allowable reinforcement ratio is reduced from 8% to 6% for columns in special moment frames (21.4.3.1). This lower ratio prevents congestion of steel, which reduces the chance of improperly placed concrete. It also prevents the development of large shear stresses in the columns. Typically, providing a reinforcement ratio larger than about 3% is not practical or economical.

Mechanical splices shall conform to 21.2.6 and welded splices shall conform to 21.2.7 (21.4.3.2). When lap splices are used, they are permitted only within the center half of the member length and are to be designed as tension lap splices (see Fig. 29-6). Transverse reinforcement conforming to 21.4.4.2 and 21.4.4.3 is required along the length of the lap splice.

There are no restrictions on the location of lap splices in intermediate or ordinary moment frames.

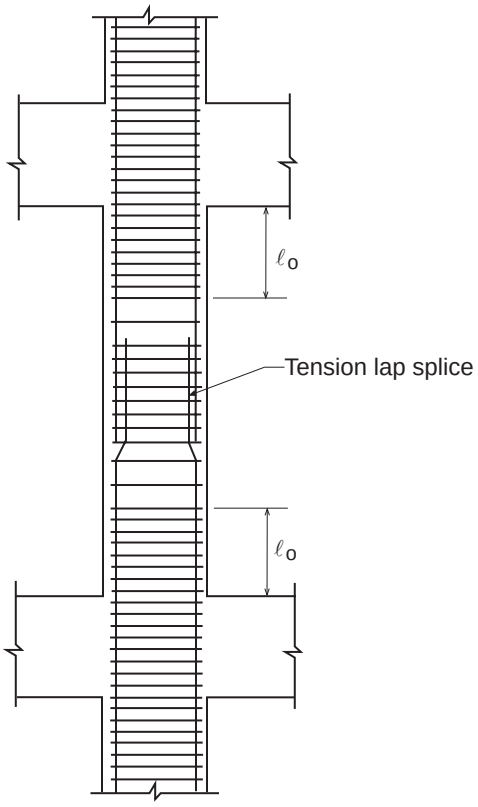
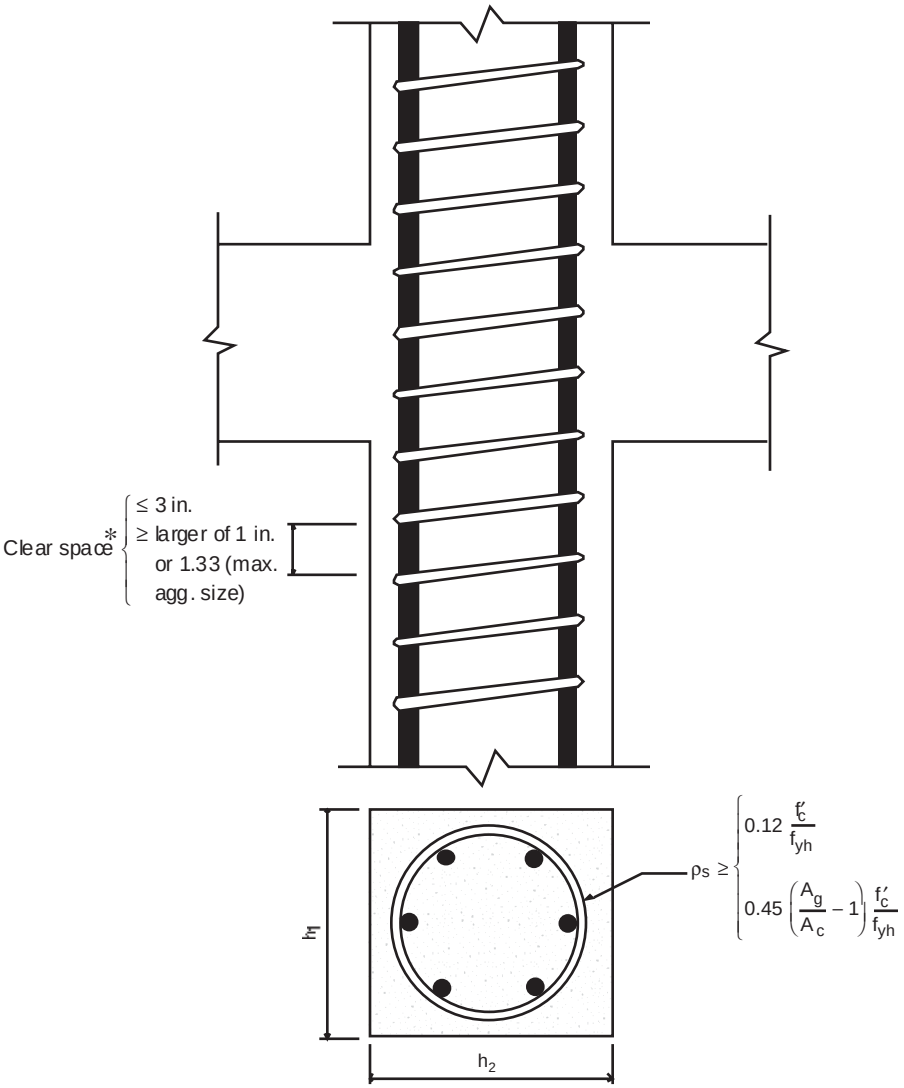


Figure 29-6 Typical Lap Splice Details of Columns in Special Moment Frames

21.4.4 Transverse Reinforcement

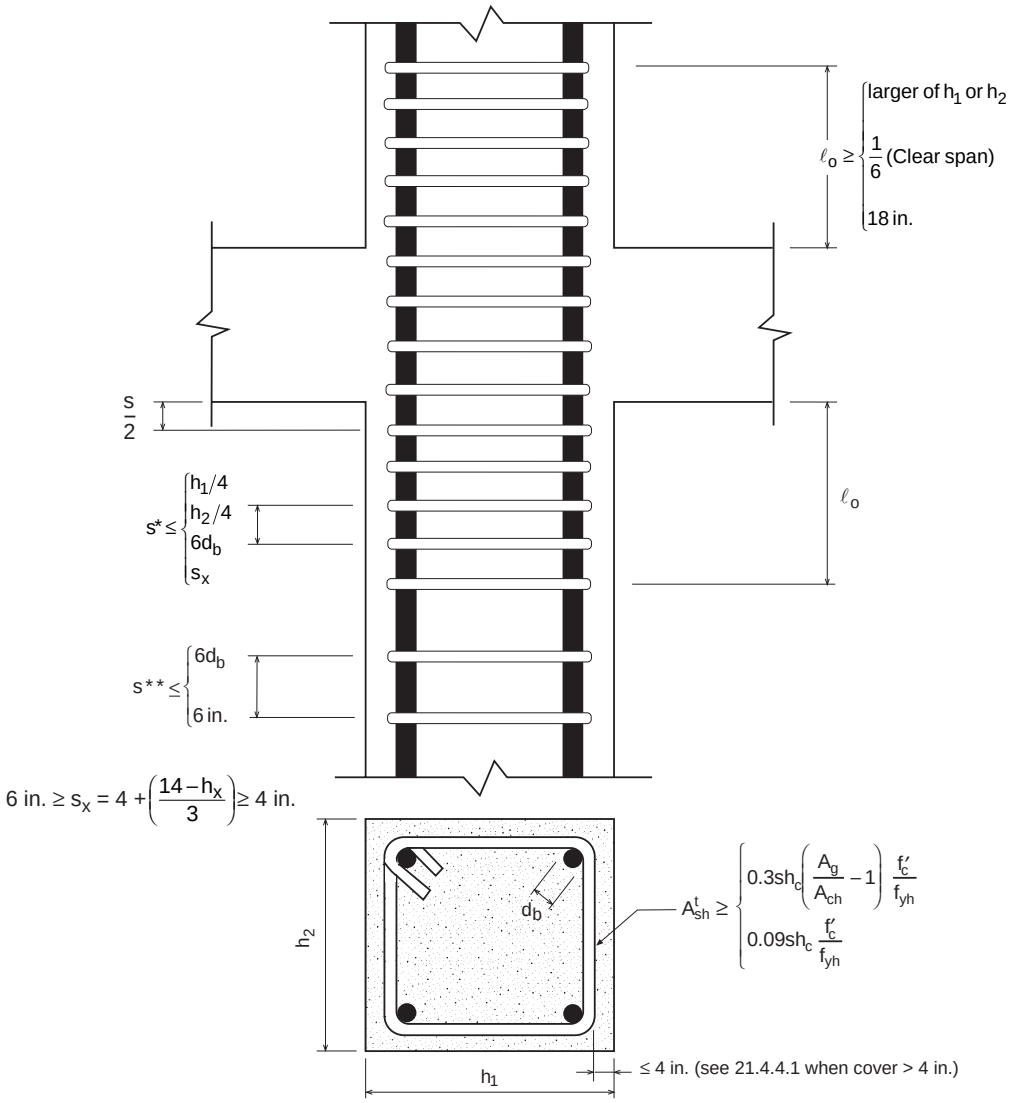
Column ends require adequate confinement to ensure column ductility in the event of hinge formation. They also require adequate shear reinforcement in order to prevent shear failure prior to the development of the flexural yielding of the section. The correct amount, spacing, and location of the transverse reinforcement must be provided so that both the confinement and shear strength requirements are satisfied. For special moment

frames, the transverse reinforcement must be spiral or circular hoop reinforcement or rectangular hoop reinforcement, as shown in Fig. 29-7. Spiral reinforcement is generally the most efficient form of confinement reinforcement; however, the extension of the spirals into the beam-column joint may cause some construction difficulties.



*Clear spacing for spiral reinforcement. Circular hoops to be spaced per 21.4.4.2.

Figure 29-7 Confinement Requirements at Column Ends
(a) spiral or circular hoop reinforcement



*For intermediate moment frames, $s \leq$ { 8 × smallest long. bar diameter; 24 × transverse bar diameter; $\frac{1}{2}$ × smaller of h_1 or h_2; 12 in. }

**For intermediate moment frames, s shall conform to 7.10 and 11.5.4.1

†Confinement requirements for special moment frames need not be satsified for intermediate moment frames

Figure 29-7 Confinement Requirements at Column Ends
(b) rectangular hoop reinforcement

Figure 29-8 shows an example of transverse reinforcement provided by one hoop and three crossties. Crossties with 90-deg hooks are not as effective as crossties with 135-deg hooks. Confinement will be sufficient if crosstie ends with 90-deg hooks are alternated.

The requirements of 21.4.4.2 and 21.4.4.3 must be satisfied for the configuration of rectangular hoop reinforcement. The requirement that spacing not exceed one-quarter of the minimum member dimension is to obtain adequate concrete confinement. Restraining longitudinal reinforcement buckling after spalling is the rationale behind the spacing being limited to 6 bar diameters. Section 21.4.4.2 permits the 4 in. spacing for confinement to be relaxed to a maximum of 6 in. if the horizontal spacing of crossties or legs of overlapping hoops is limited to 8 in.

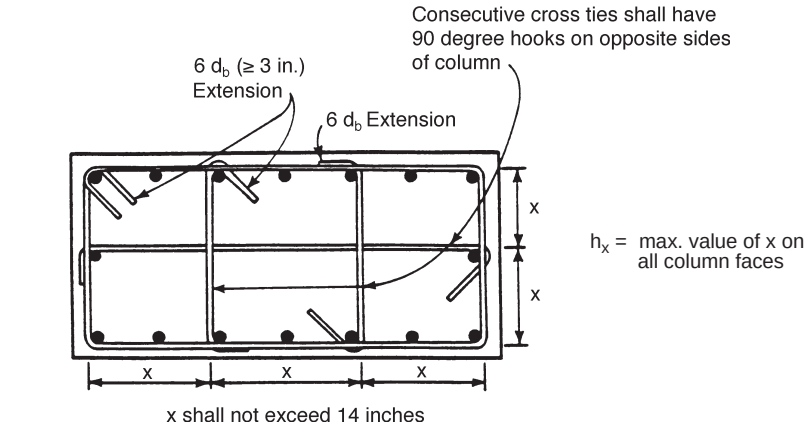


Figure 29-8 Transverse Reinforcement in Columns

Additional transverse reinforcement at a maximum spacing of 12 in. is required when concrete thickness outside the confining transverse reinforcement exceeds 4 in. This additional reinforcement will help reduce the risk of portions of the shell falling away from the column. The required amount of such reinforcement is not specifically indicated; the 1997 UBC [29.2] specifies a minimum amount equal to that which is required for columns that are not part of the lateral force resisting system.

For columns supporting discontinued stiff members (such as walls) as shown in Fig. 29-9, the transverse reinforcement needs to be provided over the full height of the column and must be extended at least the development length of the largest longitudinal bars in the column into the discontinued member (wall). The transverse reinforcement must also extend at least 12 in. into the footing or mat, if the column terminates on a footing or mat.

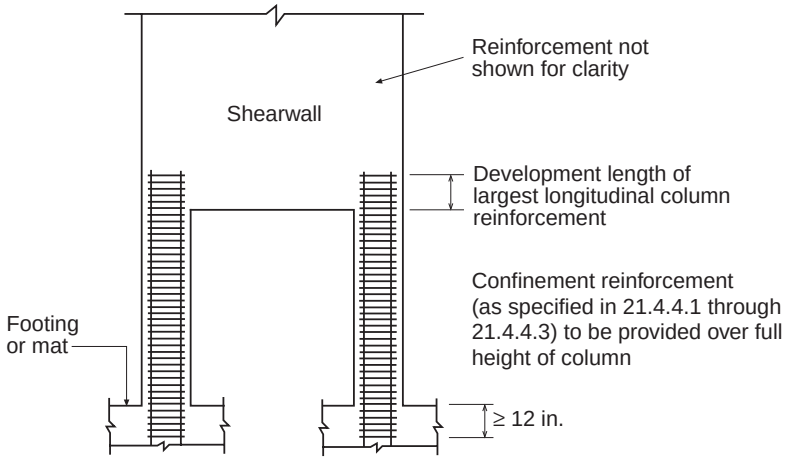


Figure 29-9 Columns Supporting Discontinued Stiff Members

Similar provisions are required for intermediate moment frames (see Fig. 29-7).

21.4.5 Shear Strength Requirements

In addition to satisfying confinement requirements, the transverse reinforcement in columns must resist the maximum shear forces associated with the formation of plastic hinges in the frame (21.4.5.1). Although the provisions of 21.4.2 are intended to have most of the inelastic deformation occur in the beams, the provisions of 21.4.5.1 recognize that hinging can occur in the column. Thus, as in the case of beams, the shear reinforcement in the columns is based on the probable moment strengths M_{pr} that can be developed at the ends of the column. The probable moment strength is to be the maximum consistent with the range of factored axial loads on the column; sidesway to the right and to the left must both be considered (see Fig. 29- 10).

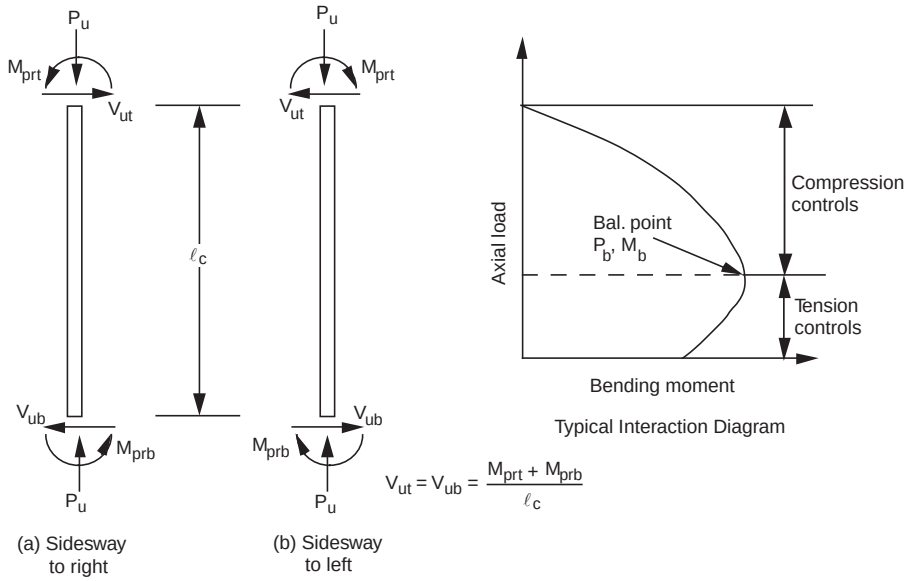


Figure 29-10 Loading Cases for Design of Shear Reinforcement in Columns of Special Moment Frames

Section 21.4.5.1 points out that the column shear forces need not exceed those determined from joint strengths based on the probable moment strengths of the beams framing into the joint. When beams frame on opposite sides of a joint, the combined probable moment strength may be taken as the sum of the negative probable moment strength of the beam on one side of the joint and the positive probable moment strength of the beam on the other side. The combined probable moment strength of the beams is then distributed appropriately to the columns above and below the joint, and the shear forces in the column are computed based on this distributed moment. It is important to note that in no case is the shear force in the column to be taken less than the factored shear force determined from analysis of the structure under the code-prescribed seismic forces (21.4.5.1).

Provisions for proportioning the transverse reinforcement are contained in 21.4.5.2. As in the case of beams, the strength reduction factor ϕ to be used is 0.75 (see 9.3.4 and 9.3.2.3).

The shear forces in intermediate frame members subjected to combined bending and axial force are determined in the same manner as for flexural members, i.e., nominal moment strengths are used to compute the shear forces (21.12.3).

21.5 JOINTS OF SPECIAL MOMENT FRAMES

The overall integrity of a structure is dependent on the behavior of the beam-column joint. Degradation of the joint can result in large lateral deformations which can cause excessive damage or even failure. The lefthand column of Table 29-3 contains the requirements for joints of special moment frames. For intermediate and ordinary cast-in-place frames, the beam-column joints do not require the special design and detailing requirements as for special moment frames. Although special detailing is not required for intermediate moment frame joints, it may be prudent to provide the same details as required for special moment frame joints.

Slippage of the longitudinal reinforcement in a beam-column joint can lead to an increase in the joint rotation. Longitudinal bars must be continued through the joint or must be properly developed for tension (21.5.4) and compression (Chapter 12) in the confined column core. The minimum column size requirement of 21.5.1.4 reduces the possibility of failure from loss of bond during load reversals beyond the yield point of the steel.

21.5.2 Transverse Reinforcement

The transverse reinforcement in a beam-column connection is intended to provide adequate confinement of the concrete to ensure its ductile behavior and to allow it to maintain its vertical load-carrying capacity even after spalling of the outer shell.

Table 29-3 Joints of Frames

	Special Moment Frames	Intermediate and Ordinary CIP Moment Frames
Longitudinal Beam Reinforcement	Beam longitudinal reinforcement terminated in a column shall be extended to the far face of the confined column core and anchored in tension according to 21.5.4 and in compression according to Chapter 12. **21.5.1.3**	No similar requirement.
	Where longitudinal beam reinforcement extends through a joint, the column dimension parallel to the beam reinforcement shall not be less than 20 times the diameter of the largest longitudinal bar for normal weight concrete. For lightweight aggregate concrete, this dimension shall be not less than 26 times the bar diameter. **21.5.1.4**	No similar requirements.
Transverse Reinforcement	The transverse hoop reinforcement required for column ends (21.4.4) shall be provided within the joint, unless the joint is confined by structural members as specified in 21.5.2.2. If members frame into all four sides of the joint and the member width at the column face is at least 3/4 the column width, the transverse reinforcement can be reduced to 50% of the requirements of 21.4.4.1 within the depth of the shallowest member. The spacing required in 21.4.4.2 shall not exceed 6 in. at these locations. **21.5.2.1, 21.5.2.2**	No similar requirement.

f'_c = *compressive strength of concrete*

f_y = *yield strength of reinforcement*

Table 29-3 Joints of Frames (cont'd)

	Special Moment Frames	Intermediate and Ordinary CIP Moment Frames
Shear Strength	The nominal shear strength of the joint shall not exceed the forces specified below for normal weight aggregate concrete. For joints confined on all four faces ..... $20\sqrt{f'_c}\,A_j$ For joints confined on three faces or on two opposite faces ..... $15\sqrt{f'_c}\,A_j$ For other joints: ..... $12\sqrt{f'_c}\,A_j$ where: A_j = effective cross-sectional area within a joint in a plane parallel to the plane of the reinforcement generating shear in the joint. The joint depth shall be the overall depth of the column. Where a beam frames into a support of larger width, the effective width of the joint shall not exceed the smaller of: 1. Beam width plus the joint depth. 2. Beam width plus twice the smaller perpendicular distance from the longitudinal axis of the beam to the column side. A joint is considered to be confined if confining members frame into all faces of the joint. A member that frames into a face is considered to provide confinement at the joint if at least 3/4 of the face of the joint is covered by the framing member. **21.5.3**	Although it is not required, it may be prudent to check the shear strength of the joint in intermediate moment frames. The force in the longitudinal beam reinforcement may be taken as $1.0f_y$ rather than the $1.25f_y$ required for special moment frames.
	In determining shear forces in the joints, forces in the longitudinal beam reinforcement at the joint face shall be calculated by assuming that the stress in the flexural tensile reinforcement is $1.25f_y$. **21.5.1.1**	No similar requirement.
	For lightweight aggregate concrete, the nominal shear strength of the joint shall not exceed 3/4 of the limits given in 21.5.3.1. **21.5.3.2**	No similar requirement.

Minimum confinement reinforcement of the same amount required for potential hinging regions in columns, as specified in 21.4.4, must be provided within a beam-column joint around the column reinforcement, unless the joint is confined by structural members as specified in 21.5.2.2.

For joints confined on all four faces, a 50% reduction in the amount of confinement reinforcement is allowed. A member that frames into a face is considered to provide confinement to the joint if at least three-quarters of the face of the joint is covered by the framing member. The code further allows that where a 50% reduction in the amount of confinement reinforcement is permissible, the spacing specified in 21.4.4.2(b) may be increased to 6 in. (21.5.2.2).

The minimum amount of confinement reinforcement, as noted above, must be provided through the joint regardless of the magnitude of the calculated shear force in the joint. The 50% reduction in the amount of confinement reinforcement allowed for joints having horizontal members framing into all four sides recognizes the beneficial effect provided by these members in resisting the bursting pressures that can be generated within the joint.

21.5.3 Shear Strength

The most significant factor in determining the shear strength of a beam-column joint is the effective area A_j of the joint, as shown in Fig. 29-11. For joints that are confined by beams on all four faces, the shear strength of the joint is equal to $20\sqrt{f'_c}\,A_j$. If the joint is confined only on three faces, or on two opposite faces, the strength must be reduced by 25% to $15\sqrt{f'_c}\,A_j$. For other cases, the shear strength is equal to $12\sqrt{f'_c}\,A_j$. It is important to note that the shear strength is a function of the concrete strength and the cross-sectional area only. Test results show that the shear strength of the joint is not altered significantly with changes in transverse reinforcement, provided a minimum amount of such reinforcement is present. Thus, only the concrete strength or the member size can be modified if the shear strength of the beam-column joint is inadequate. The strength reduction factor ϕ for shear in joints is 0.85 (9.3.4).

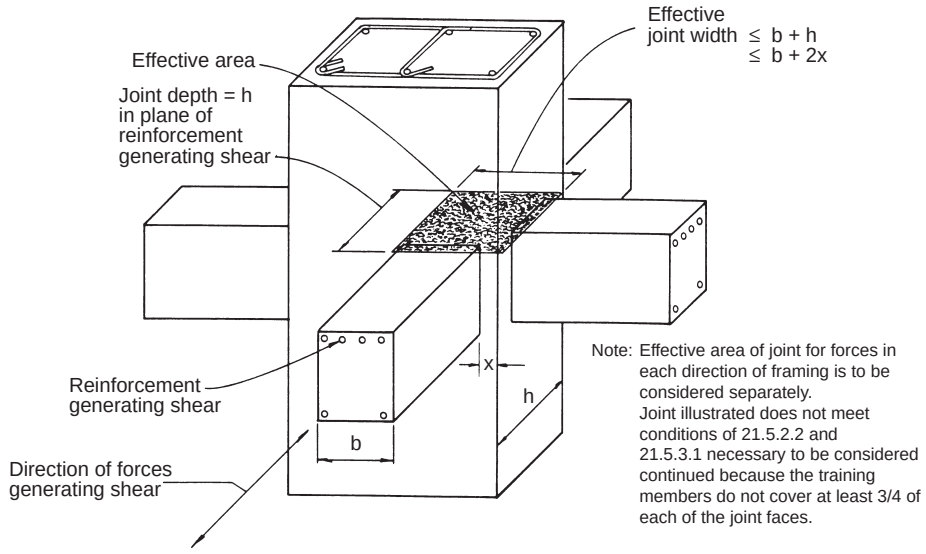


Figure 29-11 Effective Area of Joint (A_j)

The larger the tension force in the steel, the greater the shear in the joint (Fig. 29-12). Thus, the tensile force in the reinforcement is conservatively taken as $1.25f_yA_s$. The multiplier of 1.25 takes into account the likelihood that due to strain hardening and actual strengths higher than the specified yield strengths, a larger tensile force may develop in the bars, resulting in a larger shear force.

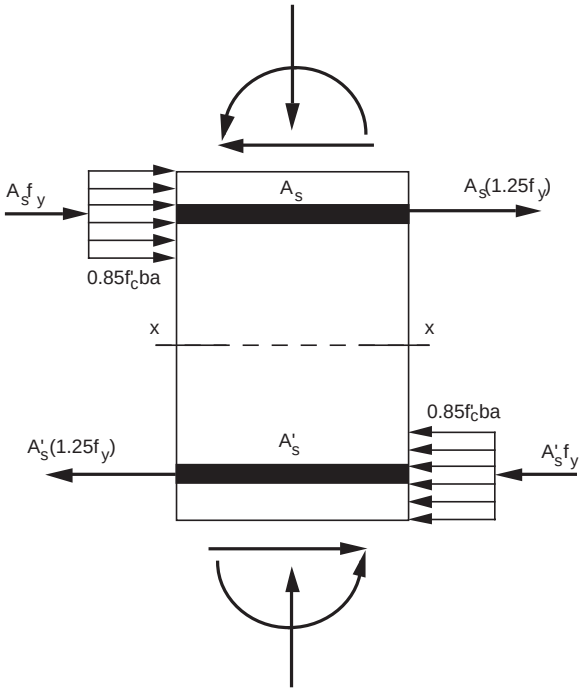


Figure 29-12 Horizontal Shear in Beam-Column Connection

21.5.4 Development Length of Bars in Tension

A standard 90-degree hook located within the confined core of a column or boundary element is depicted in Fig. 29-13. Equation (21-6), based on the requirements of 12.5, includes the factors for hooks enclosed in ties (0.8), satisfaction of minimum cover requirements (0.7), a cyclic load factor (1.1), and a factor of 1.25 for overstrength in the reinforcing steel. The equation for the development length in 12.5.2 $\left[\left(0.02\beta\lambda f_y / \sqrt{f'_c}\right) d_b\right]$ is multiplied by these factors to obtain the equation that is given in 21.5.4.1 for uncoated reinforcing bars with $f_y = 60{,}000$ psi embedded in normal weight concrete:

$$\ell_{dh} = \frac{0.8 \times 0.7 \times 1.25 \times 1.1 \times 0.02 \times 1.0 \times 1.0 \times 60{,}000 \times d_b}{\sqrt{f'_c}}$$

$$= \frac{924 d_b}{\sqrt{f'_c}} = \frac{f_y d_b}{65\sqrt{f'_c}}$$

For bar sizes No. 3 through No. 11, the development length ℓ_{dh} for a bar with a standard 90-deg hook in normal weight aggregate concrete shall not be less than the largest of $8d_b$, 6 in., and the length obtained from Eq. (21-6) shown above. For lightweight aggregate concrete, the development length shall be increased by 25%.

The development length for No. 11 straight bars and smaller is determined by multiplying the development length for hooked bars required by 21.5.4.1 by (a) two-and-a-half (2.5) if the depth of the concrete cast in one lift beneath the bar does not exceed 12 in., and (b) three-and-a-half (3.5) if the depth of the concrete cast in one lift beneath the bar exceeds 12 in. (21.5.4.2). If a portion of a straight bar is not located within the confined core of a column or boundary element, the length of that bar shall be increased by an additional 60%. Provisions for epoxy-coated bars are given in 21.5.4.4.

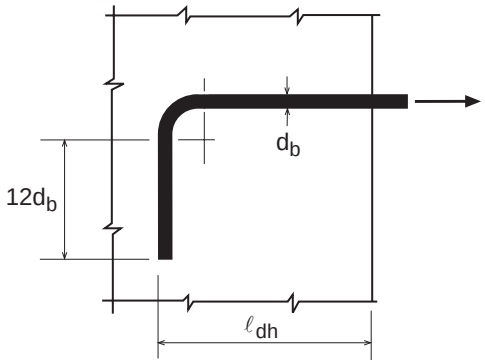


Figure 29-13 Standard 90-Degree Hook

21.6 SPECIAL MOMENT FRAMES CONSTRUCTED USING PRECAST CONCRETE

In addition to the requirements of 21.2 through 21.5, special moment frames constructed using precast concrete must satisfy the requirements of 21.6. The detailing provisions in 21.6.1 for frames with ductile connections and 21.6.2 for frames with strong connections are intended to produce frames that respond to design displacements essentially like cast-in-place special moment frames. Section 21.6.3 provides a design procedure for special moment frames that do not satisfy the appropriate prescriptive requirements of Chapter 21.

21.6.1 Special Moment Frames with Ductile Connections

Special moment frames with ductile connections are designed and detailed so that flexural yielding occurs within the connection regions. Type II mechanical splices or any other technique that provides development in tension or compression of at least $1.25f_y$ and the specified tensile strength of the bars can be used to make the reinforcement continuous in the connections.

According to 21.6.1(a), the nominal shear strength V_n at the connection must be computed in accordance with the shear-friction design method of 11.7.4. In order to help prevent sliding at the faces of the connection, V_n must be greater than or equal to $2V_e$, where V_e is the design shear force in the beams that is computed according to 21.3.4.1 or the design shear force in the columns that is computed according to 21.4.5.1. Since the ductile connections may be at locations that are not adjacent to the joints, using V_e may be conservative.

Mechanical splices of beam reinforcement must satisfy the requirements of 21.2.6 and must be located at least h/2 from the face of the joint, where h is the overall depth of the beam. This additional requirement is intended to avoid strain concentrations over a short length of reinforcement adjacent to a splice device.

21.6.2 Special Moment Frames with Strong Connections

Special moment frames with strong connections are designed and detailed so that flexural yielding occurs away from the connection regions. Examples of beam-to-beam, beam-to-column, and column-to-footing connections are shown in Fig. R21.6.2.

According to 21.6.2(a), the geometric constraint in 21.3.1.2 related to the clear span to effective depth ratio must be satisfied for any segment between locations where flexural yielding is intended to occur due to design displacements.

To ensure that strong connections remain elastic and do not slip following the formation of plastic hinges, the design strength of the connection, ϕS_n, in both flexure and shear must be greater than or equal to the bending

moment and shear force, S_e, respectively, corresponding to the development of probable flexural or shear strengths at intended locations of flexural or shear yielding (21.6.2(b)). These provisions are illustrated in Figs. 29-14 and 29-15 for a beam-to-beam and a beam-to-column strong connection, respectively, with sidesway to the right. Sidesway to the left must also be considered.

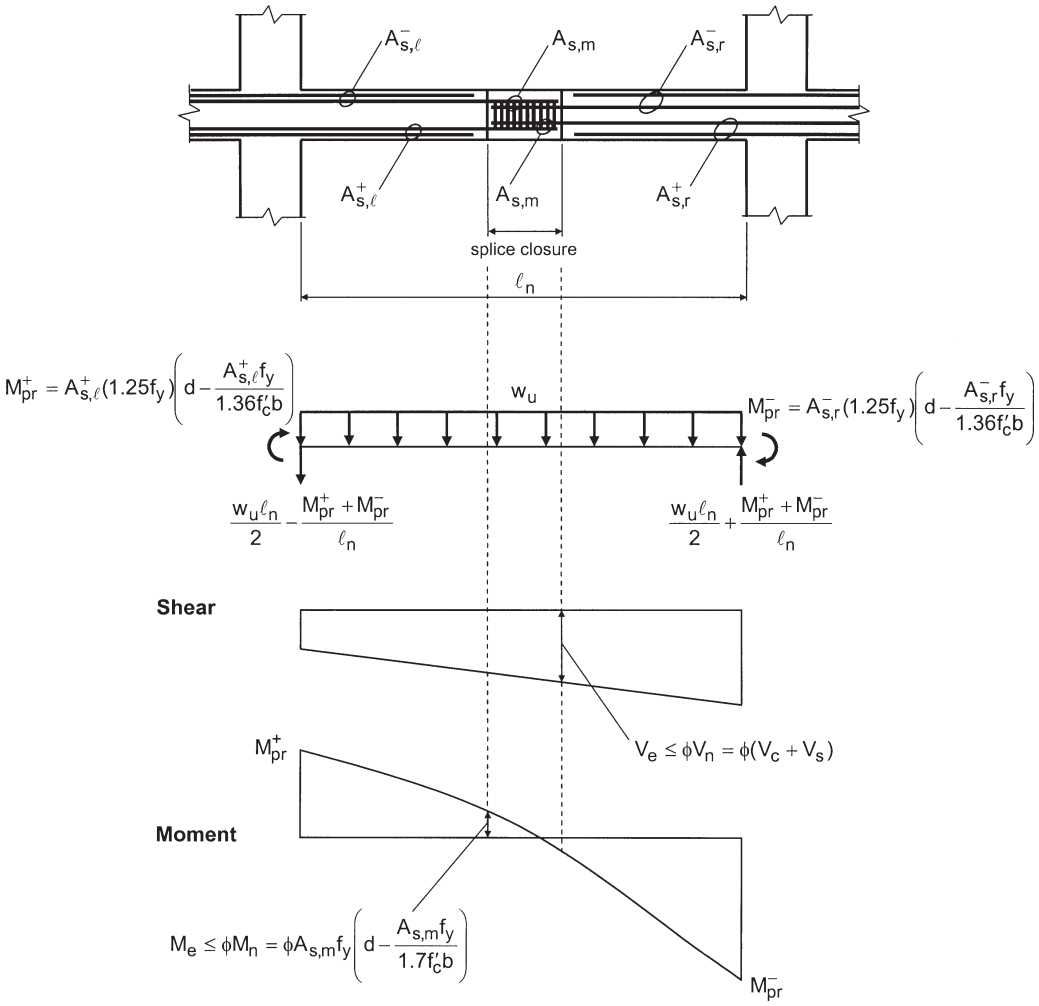
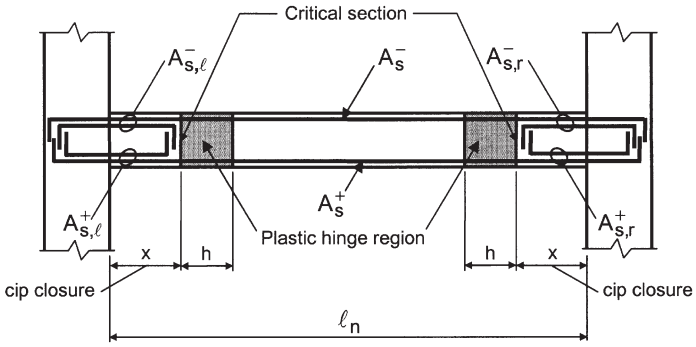
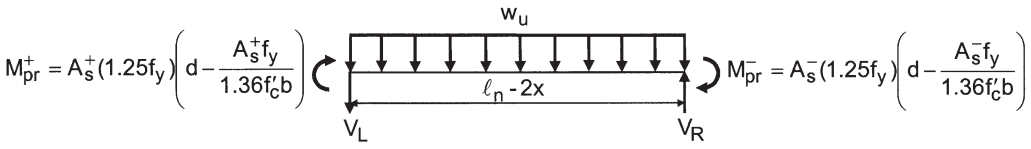


Fig. 29-14 Design Requirements for Beam-to-Beam Strong Connections near Midspan

Section 21.6.2(c) requires that primary longitudinal reinforcement be continuous across connections and be developed outside both the strong connection and the plastic hinge region. Laboratory tests of precast beam-column connections showed that strain concentrations caused brittle fracture of reinforcing bars at the face of mechanical splices. To avoid this premature fracture, designers should carefully select the locations of strong connections or take other measures, such as using debonded reinforcement in highly stressed regions.

The column-to-column connection requirements of 21.6.2(d) are provided to avoid hinging and strength deterioration of these connections. For columns above the ground floor level, the moments at a joint may be limited by the flexural strengths of the beams framing into that joint (21.4.2.2). Dynamic inelastic analysis and studies of strong ground motion have shown that for a strong column-weak beam deformation mechanism, the beam end moments are not equally divided between the top and bottom columns, even where columns have equal stiffness.[29.3] From an elastic analysis, the moments would be distributed as shown in Fig. 29-16, while the actual distribution is likely to be as shown in Fig. 29-17.





$$V_L = \frac{w_u(\ell_n - 2x)}{2} - \frac{M_{pr}^+ + M_{pr}^-}{\ell_n - 2x}$$

$$V_R = \frac{w_u(\ell_n - 2x)}{2} + \frac{M_{pr}^+ + M_{pr}^-}{\ell_n - 2x}$$

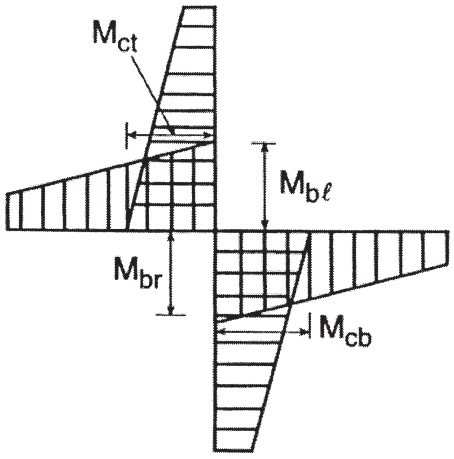
$$M_{e,\ell}^+ = M_{pr}^+ + V_L x$$

$$M_{e,r}^- = M_{pr}^- + V_R x$$

$$\phi M_{n,\ell}^+ = \phi A_{s,\ell}^+ f_y \left(d - \frac{A_{s,\ell}^+ f_y}{1.7f_c' b}\right) \geq M_{e,\ell}^+$$

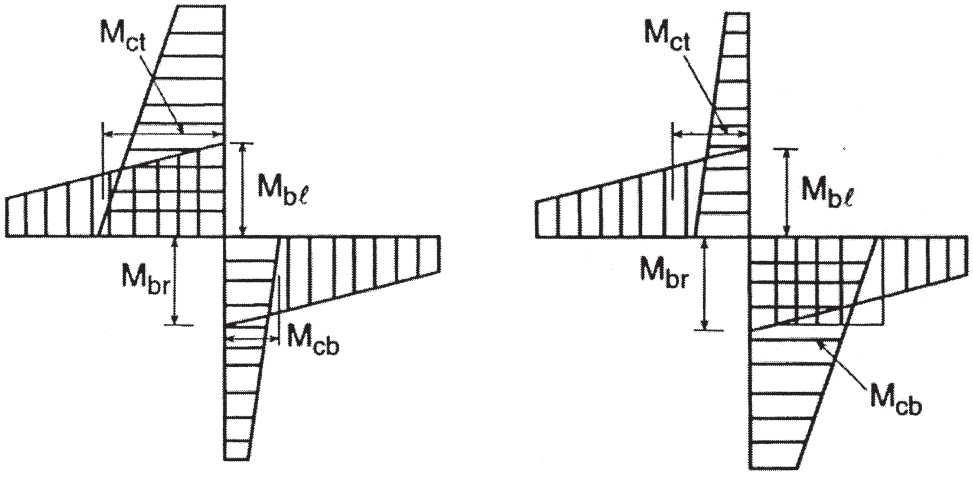
$$\phi M_{n,r}^- = \phi A_{s,r}^- f_y \left(d - \frac{A_{s,r}^- f_y}{1.7f_c' b}\right) \geq M_{e,r}^-$$

Fig. 29-15 Design Requirements for Beam-to-Column Strong Connections



$$M_{b\ell} + M_{br} = M_{cb} + M_{ct}$$

Fig. 29-16 Bending Moments at Beam-to-Column Connection – Elastic Analysis



$M_{b\ell} + M_{br} = M_{cb} + M_{ct}$

Fig. 29-17 Bending Moments at Beam-to-Column Connection – Inelastic Analysis

Figure 29-18 shows the distribution of the elastic moments M_E (dashed lines) due to the seismic forces and the corresponding envelopes of dynamic moments ωM_E (solid lines) over the full column height, where ω is a dynamic amplification factor. In regions outside of the middle third of the column height, ω is to be taken as 1.4 (21.6.2(d)); thus, connections within these regions must be designed such that $\phi M_n \geq 1.4 M_E$. For connections located within the middle third of the column height, 21.6.2(d) requires that $\phi M_n \geq 0.4 M_{pr}$, where M_{pr} is the maximum probable flexural strength of the column within the story height. Also, the design shear strength ϕV_n of the connection must be greater than or equal to the design shear force V_E computed according to 21.4.5.1.

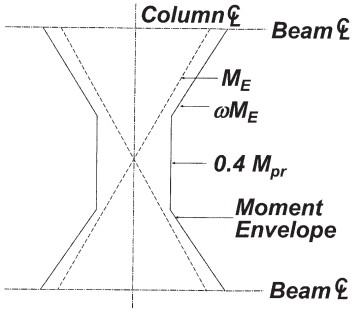


Fig. 29-18 Moment Envelope at Column-to-Column Connection

21.6.3 Non-emulative Design

It has been demonstrated in experimental studies that special moment frames constructed using precast concrete that do not satisfy the provisions of 21.6.1 for frames with ductile connections or 21.6.2 for frames with strong connections provide satisfactory seismic performance characteristics. For these frames, the requirements of ACI T1.1 Acceptance Criteria for Moment Frames Based on Structural Testing [29.4], as well as the provisions of 21.6.3(a) and 21.6.3(b), must be satisfied.

ACI T1.1 defines minimum acceptance criteria for weak beam/strong column moment frames designed for regions of high seismic risk that do not satisfy the prescriptive requirements of Chapter 21 of ACI 318-99. According to ACI T1.1, acceptance of such frames as special moment frames must be validated by analysis and laboratory tests.

Prior to testing, a design procedure must be developed for prototype moment frames that have the same generic form as those for which acceptance is sought (see 4.0 of ACI T1.1). The design procedure should account for the effects of material nonlinearity (including cracking), deformations of members and connections, and reversed cyclic loadings, and be used to proportion the test modules (see 5.0 for requirements for the test modules). It is also important to note that the overstrength factor (column-to-beam strength ratio) used for the columns of the prototype frame should not be less than 1.2, which is specified in 21.4.2.2 of ACI 318-99.

The test method is described in 7.0. In short, the test modules are to be subjected to a sequence of displacement-controlled cycles that are representative of the drifts expected during the design earthquake for that portion of the frame that is represented by the test module. Figure R5.1 of ACI T1.1 illustrates connection configurations for interior and exterior one-way joints and, if applicable, corner joints that must be tested as a minimum.

The first loading cycle shall be within the linear elastic response range of the module. Subsequent drift ratios are to be between 1.25 and 1.5 times the previous drift ratio, with 3 fully reversed cycles applied at each drift ratio. Testing continues until the drift ratio equals or exceeds 0.035. Cyclic deformation history that satisfies 7.0 is illustrated in Fig. R7.0. Drift ratio is defined in Fig. R2.1.

Section 9.0 provides the detailed acceptance criteria that apply to each module of the test program. The performance of the test module is deemed satisfactory when these criteria are met for both directions of response.

The first criterion is that the test module must attain a lateral resistance greater than or equal to nominal lateral resistance E_n (see 1.0 for definition of E_n) before the drift ratio exceeds the allowable story drift limitation of the governing building code (see Fig. R9.1). This criterion helps provide adequate initial stiffness.

In order to provide weak beam/strong column behavior, the second criterion requires that the maximum lateral resistance E_{max} recorded in the test must be less than or equal to λE_n where λ is the specified overstrength factor for the test column, which must be greater than or equal to 1.2. Commentary section R9.1.2 provides a detailed discussion on this requirement. Also see Fig. R9.1.

The third criterion requires that characteristics of the third complete cycle for each test module, at a drift ratio greater than or equal to 0.035, must satisfy 3 criteria regarding peak force value, relative energy dissipation ratio, and drift at zero stiffness. The first of these criteria limits the level of strength degradation, which is inevitable at high cyclic-drift ratios. A maximum strength degradation of $0.25E_{max}$ is specified (see Fig. R9.1). The second of these criteria sets a minimum level of damping for the frame as a whole by requiring that the relative energy dissipation ratio β be greater than or equal to 1/8. If β is less than 1/8, oscillations may continue for a long time after an earthquake, resulting in low-cycle fatigue effects and possible excessive displacements. The definition of β is illustrated in Fig. R2.4. The third of these criteria helps ensure adequate stiffness around zero drift ratio. The structure would be prone to large displacements following a major earthquake if this stiffness becomes too small. A hysteresis loop for the third cycle between peak drift ratios of 0.035 that has the form shown in Fig. R9.1 is acceptable. An unacceptable hysteresis loop form is shown in Fig. R9.1.3 where the stiffness around zero drift ratio is unacceptably small for positive, but not for negative, loading.

As noted above, 21.6.3 has additional requirements to those in ACI T1.1. According to 21.6.3(a), the details and materials used in the test specimen shall be representative of those used in the actual structure. Section 21.6.3(b) stipulates additional requirements for the design procedure. Specifically, the design procedure must identify the load path or mechanism by which the frame resists the effects due to gravity and earthquake forces and shall establish acceptable values for sustaining that mechanism. Any portions of the mechanism that deviate from code requirements shall be contained in the test specimens and shall be tested to determine upper bounds for

acceptance values. In other words, deviations are acceptable if it can be demonstrated that they do adversely affect the performance of the framing system.

21.7 SPECIAL REINFORCED CONCRETE STRUCTURAL WALLS AND COUPLING BEAMS

When properly proportioned so that they possess adequate lateral stiffness to reduce interstory distortions due to earthquake-induced motions, structural walls (also called shearwalls) reduce the likelihood of damage to the nonstructural elements of a building. When used with rigid frames, a system is formed that combines the gravity-load-carrying efficiency of the rigid frame with the lateral-load-resisting efficiency of the structural wall.

Observations of the comparative performance of rigid-frame buildings and buildings stiffened by structural walls during earthquakes have pointed to the consistently better performance of the latter. The performance of buildings stiffened by properly designed structural walls has been better with respect to both safety and damage control. The need to ensure that critical facilities remain operational after a major tremor and the need to reduce economic losses from structural and nonstructural damage, in addition to the primary requirement of life safety (i.e., no collapse), has focused attention on the desirability of introducing greater lateral stiffness into earthquake-resistant multistory structures. Structural walls, which have long been used in designing for wind resistance, offer a logical and efficient solution to the problem of lateral stiffening of multistory buildings.

Structural walls are normally much stiffer than regular frame elements and are therefore subjected to correspondingly greater lateral forces due to earthquake motions. Because of their relatively greater depth, the lateral deformation capacities of walls are limited, so that, for a given amount of lateral displacement, structural walls tend to exhibit greater apparent distress than frame members. However, over a broad period range, a structure with structural walls, which is substantially stiffer and hence has a shorter period than a structure with frames, will suffer less lateral displacement than the frame, when subjected to the same ground motion intensity. Structural walls with a height-to-horizontal length ratio, h_w/ℓ_w, in excess of 2 behave essentially as vertical cantilever beams and should therefore be designed as flexural members, with their strength governed by flexure rather than by shear.

Isolated structural walls or individual walls connected to frames will tend to yield first at the base where the bending moment is the greatest. Coupled walls, i.e., two or more walls linked by short, rigidly-connected beams at the floor levels, on the other hand, have the desirable feature that significant energy dissipation through inelastic action in the coupling beams can be made to precede hinging at the bases of the walls.

The lefthand column of Table 29-4 contains the requirements for special reinforced concrete structural walls (recall that special reinforced concrete structural wall are required in regions of high seismic risk or for structures assigned to high seismic performance or design categories). For comparison purposes, the requirements of ordinary reinforced concrete structural walls are also contained in Table 29-4.

21.7.2 Reinforcement

Special reinforced structural walls are to be provided with reinforcement in two orthogonal directions in the plane of the wall (see Fig. 29-19). The minimum reinforcement ratio for both the longitudinal and transverse reinforcement is 0.0025, unless the design shear force does not exceed $A_{cv}\sqrt{f'_c}$, where A_{cv} is the net area of concrete bounded by the web thickness and the length of the wall in the direction of analysis; in this case, the minimum reinforcement must not be less than that given in 14.3. The reinforcement provided for shear strength must be continuous and distributed uniformly across the shear plane with a maximum spacing of 18 in. At least two curtains of reinforcement are required if the in-plane factored shear force assigned to the wall exceeds $A_{cv}\sqrt{f'_c}$. This serves to reduce fragmentation and premature deterioration of the concrete under load reversals into the inelastic range. Uniform distribution of reinforcement across the height and horizontal length of the wall helps control the width of the inclined (diagonal) cracks.

Table 29-4 Structural Walls

	Special Reinforced Concrete Structural Wall	Ordinary Reinforced Concrete Structural Wall
Reinforcement	The distributed web reinforcement ratios ρ_v and ρ_n shall not be less than 0.0025. If the design shear force $V_u \leq A_{cv}\sqrt{f'_c}$, provide minimum reinforcement per 14.3. **21.7.2.1**	Minimum vertical reinforcement ratio = 0.0012 for No. 5 bars or smaller = 0.0015 for No. 6 bars or larger Minimum horizontal reinforcement ratio = 0.0020 for No. 5 bars or smaller = 0.0025 for No. 6 bars or larger **14.3**
	At least two curtains of reinforcement shall be used in a wall if the in-plane factored shear force (V_u) assigned to the wall exceeds $2A_{cv}\sqrt{f'_c}$. **21.7.2.2**	Walls more than 10 in. thick require two curtains of reinforcement (except basement walls). **14.3.4**
	Reinforcement spacing each way shall not exceed 18 in. **21.7.2.1**	Reinforcement spacing shall not exceed: 3 × wall thickness 18 in. **14.3.5**
	All continuous reinforcement in structural walls shall be anchored or spliced in accordance with the provisions for reinforcement in tension as specified in 21.5.4. **21.7.2.3**	The development lengths, spacing, and anchorage of reinforcement shall be as per Chapters 12, 14, and 15.

— continued on next page —

A_{cp} = area of concrete section, resisting shear, of an individual pier or horizontal wall segment

A_{cv} = gross area of concrete section bounded by web thickness and length of section in the direction of shear force considered

A_g = gross area of section

b_w = width of web

d = effective depth of section

f'_c = compressive strength of concrete

f_{yh} = yield strength of transverse reinforcement

h = overall thickness of member

h_w = height of entire wall or segment of wall considered

ℓ_w = length of entire wall or segment of wall in direction of shear force

s = spacing of transverse reinforcement

s_2 = spacing of horizontal reinforcement in walls

ρ_n = ratio of area of distributed reinforcement parallel to the plane of A_{cv} to gross concrete area perpendicular to that reinforcement

ρ_v = ratio of area of distributed reinforcment perpendicualr to the plane of A_{cv} to gross concrete area A_{cv}

Table 29-4 Structural Walls (cont'd)

	Special Reinforced Concrete Structural Wall	Ordinary Reinforced Concrete Structural Wall
Shear Strength	The nominal shear strength (V_n) for structural walls shall not exceed: $V_n = A_{cv}\left(\alpha_c\sqrt{f'_c} + \rho_n f_y\right)$ where α_c is 3.0 for $h_w / \ell_w \le 1.5$, is 2.0 for $h_w / \ell_w \ge 2.0$, and varies linearly between 3.0 and 2.0 for h_w / ℓ_w between 1.5 and 2.0. **21.7.4.1**	The nominal shear strength (V_n) for walls can be calculated using the following methods: $V_c = 3.3\sqrt{f'_c}\,hd + \frac{N_u d}{4\ell_w}$ or $V_c = \left[0.6\sqrt{f'_c} + \frac{\ell_w\left(1.25\sqrt{f'_c} + 0.2\frac{N_u}{\ell_w h}\right)}{\frac{M_u}{V_u} - \frac{\ell_w}{2}}\right] hd$ where $\frac{M_u}{V_u} - \frac{\ell_w}{2} \ge 0$ $V_s = \frac{A_v f_y d}{s_2}$ $V_n = V_c + V_s$ **11.10**
	Walls shall have distributed shear reinforcement providing resistance in two orthogonal directions in the plane of the wall. If the ratio (h_w / ℓ_w) does not exceed 2.0, reinforcement ratio (ρ_v) shall not be less than reinforcement ratio (ρ_n). **21.7.4.3**	Ratio (ρ_h) of horizontal shear reinforcement area to gross concrete area of vertical section shall not be less than 0.0025. **11.10.9.2** Spacing of horizontal shear reinforcement (s_2) shall not exceed $\ell_w/5$, $3h$, nor 18 in. **11.10.9.3** The minimum vertical reinforcement ratio (ρ_n) is a function of (h_w / ℓ_w) and of the horizontal reinforcement as shown below: $\rho_n = 0.0025 + 0.5\left(2.5 - \frac{h_w}{\ell_w}\right)(\rho_h - 0.0025) \ge 0.0025$ Also, the vertical shear reinforcement ratio need not exceed the required horizontal reinforcement ratio. **11.10.9.4** Spacing of vertical shear reinforcement (s_1) shall not exceed $\ell_w/3$, $3h$, nor 18 in. **11.10.9.5**
	Nominal shear strength of all wall piers sharing a common lateral force shall not be assumed to exceed $8A_{cv}\sqrt{f'_c}$, and the nominal shear strength of any one of the individual wall piers shall not be assumed to exceed $10A_{cp}\sqrt{f'_c}$. **21.7.4.4**	No similar requirement.
	Nominal shear strength of horizontal wall segments and coupling beams shall not be assumed to exceed $10A_{cp}\sqrt{f'_c}$. **21.7.4.5**	This limitation also exists for ordinary walls, except A_{cp} is replaced by hd where d may be taken equal to $0.8\ell_w$. **11.10.3**

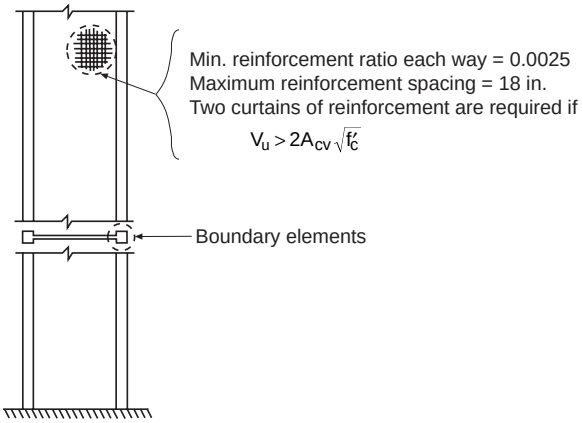


Figure 29-19 Structural Wall Design and Detailing Requirements

21.7.3 Design Forces

A condition similar to that used for the shear design of beams and columns is not as readily established for structural walls, primarily because the shear force at any section is significantly influenced by the forces and deformations at the other sections. Unlike the flexural behavior of beams and columns in a frame, with the forces and deformations determined primarily by the displacements in the end joints, the flexural deformation at any section of a structural wall is substantially influenced by the displacements at locations away from the section under consideration. Thus, for structural walls, the design shear force is determined from the lateral load analysis in accordance with the factored load combinations (21.7.3). The possibility of local yielding, as in the portion of a wall between two window openings, must also be considered; the actual shear forces may be much greater than that indicated by the lateral load analysis based on the factored design forces.

21.7.4 Shear Strength

The nominal shear strength V_n of structural walls is given in 21.7.4.1. The equation for V_n recognizes the higher shear strength of walls with high ratios of shear to moment. Additional requirements for wall segments and wall piers are contained in 21.7.4.2, 21.7.4.4, and 21.7.4.5.

The strength reduction factor ϕ is determined in accordance with 9.3.4. Note that ϕ for shear must be 0.60 for any structural member that is designed to resist earthquake effects if its nominal shear strength is less than the shear corresponding to the development of the nominal flexural strength of the member. This is applicable to brittle members, such as low-rise walls or portions of walls between openings, which are impractical to reinforce to raise their nominal shear strength above the nominal flexural strength for the pertinent loading conditions.

Walls are to be provided with distributed shear reinforcement in two orthogonal directions in the plane of the wall (21.7.4.3). If the ratio of the height of the wall to the length of the wall is less than or equal to 2.0, the reinforcement ratio ρ_v shall be greater than or equal to the reinforcement ratio ρ_n.

21.7.5 Design for Flexural and Axial Loads

Structural walls subjected to combined flexural and axial loads shall be designed in accordance with 10.2 and 10.3, excluding 10.3.6 and the nonlinear strain requirements of 10.2.2 (21.7.5.1). This procedure is essentially the same as that commonly used for columns. Reinforcement in boundary elements and distributed in flanges and webs must be included in the strain compatibility analysis. Openings in walls must also be considered.

Provisions for the influence of flanges for wall sections forming L-, T-, C-, or other cross-sectional shapes are in 21.7.5.2. Effective flange widths shall be assumed to extend from the face of the web a distance equal to the smaller of one-half the distance to an adjacent wall web and 25 percent of the total wall height.

21.7.6 Boundary Elements of Special Reinforced Concrete Structural Walls

Two approaches for evaluating the need for special boundary elements at the edges of structural walls are provided in 21.7.6. Section 21.7.6.2 allows the use of a displacement-based approach. In this method, the wall is displaced an amount equal to the expected design displacement, and special boundary elements are required to confine the concrete when the strain at the extreme compression fiber of the wall exceeds a critical value. Confinement is required over a horizontal length equal to at least the length where the compressive strain exceeds the critical value (21.7.6.4). This approach is applicable to walls or wall piers that are essentially continuous in cross-section over the entire height of the wall and designed to have one critical section for flexure and axial loads, i.e., the inelastic response of the wall is dominated by flexure at a critical, yielding section (21.7.6.2).

According to 21.7.6.2, compression zones shall include special boundary elements when

$$c \geq \frac{\ell_w}{600(\delta_u/h_w)}, \quad \delta_u/h_w \geq 0.007 \qquad \textit{Eq. (21-8)}$$

where c = distance from the extreme compression fiber to the neutral axis per 10.2.7 calculated for the factored axial force and nominal moment strength, consistent with the design displacement δ_u, resulting in the largest neutral axis depth

ℓ_w = length of the entire wall or segment of wall considered in the direction of the shear force

δ_u = design displacement

h_w = height of entire wall or of the segment of wall considered

The design displacement δ_u is the total lateral displacement expected for the design-basis earthquake, as specified by the governing code for earthquake-resistant design. In the three model building codes, ASCE 7-95, and the 1997 NEHRP Provisions, the design-basis earthquake has approximately a 10 percent chance of being exceeded in 50 years. In those documents, the design displacement is computed using a static or dynamic linear-elastic analysis under code-specified actions. Considered in the analysis are the effects of cracking, torsion, P-Δ effects, and modification factors to account for expected inelastic response. In particular, δ_u is determined by multiplying the deflections from an elastic analysis by a deflection amplification factor, which is given in the governing code. The deflection amplification factor, which depends on the type of seismic force-resisting system, is used to increase the elastic deflections to levels that would be expected for the design-basis earthquake. The lower limit of 0.007 on the quantity δ_u / h_w is specified to require moderate wall deformation capacity for stiff buildings.

Typically, the reinforcement for a structural wall section is determined first for the combined effects of bending and axial load, and shear forces in accordance with the provisions outlined above for all applicable load combinations. The distance c can then be obtained from a strain compatibility analysis for each load combination that includes seismic effects, considering sidesway to the left and to the right. The largest c is used in Eq. (21-8) to determine if special boundary elements are required or not.

When special boundary elements are required, they must extend horizontally from the extreme compression fiber a distance not less than the larger of $c - 0.1\ell_w$ and $c/2$ (21.7.6.4(a); see Fig. 29-20). In the vertical direction, the special boundary elements must extend from the critical section a distance greater than or equal to the larger of ℓ_w or $M_u/4V_u$ (21.7.6.2). This distance is based on upper bound estimates of plastic hinge lengths, and is beyond the zone over which concrete spalling is likely to occur.

The second approach for evaluating the need special boundary elements is contained in 21.7.6.3. These provisions have been retained from earlier editions of the code since they are conservative for assessing transverse reinforcement requirements at wall boundaries for many walls. Compression zones shall include special boundary elements where the maximum extreme fiber stress corresponding to the factored forces, including earthquake effects, exceeds $0.2 f'_c$ (see Fig. 29-21). Special boundary elements can be discontinued where the compressive stress is less than $0.15 f'_c$. Note that the stresses are calculated assuming a linear response of the gross concrete section.

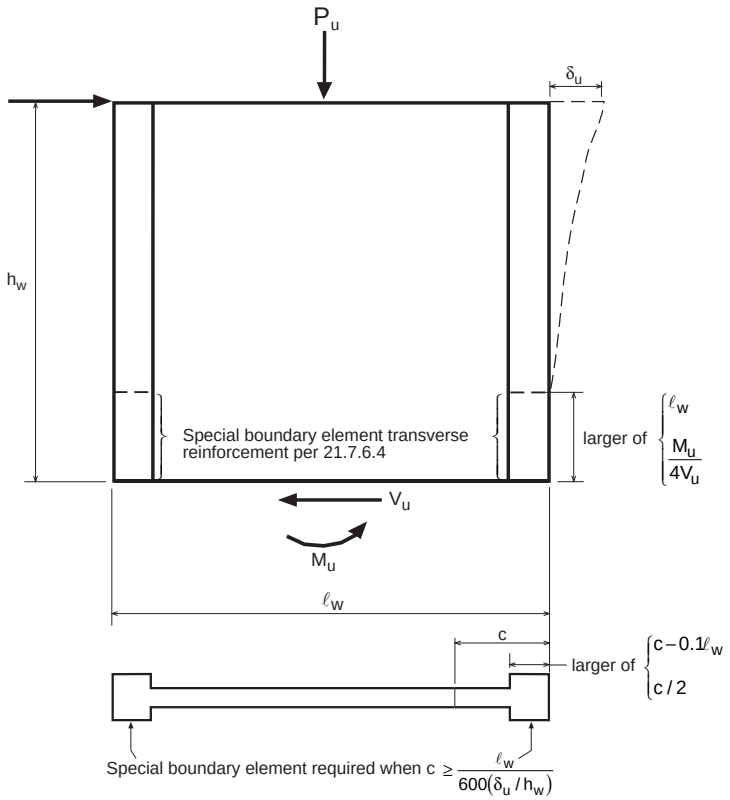


Figure 29-20 Special Boundary Element Requirements per 21.7.6.2

Section 21.7.6.4 contains the details of the reinforcement when special boundary elements are required by 21.7.6.2 or 21.7.6.3. The transverse reinforcement must satisfy the same requirements as for special moment frame members subjected to bending and axial load (21.4.4.1 through 21.4.4.3), excluding Eq. (21-3) (21.7.6.4(c); see Fig. 29-22). Also, the transverse reinforcement shall extend into the support a distance not less than the development length of the largest longitudinal bar in the special boundary element; for footings or mats, the transverse reinforcement shall extend at least 12 in. into the footing or mat (21.7.6.4(d)). Horizontal reinforcement in the wall web shall be anchored within the confined core of the boundary element to develop its specified yield strength (21.7.6.4(e)). To achieve this anchorage, 90-deg hooks or mechanical anchorages are recommended. Mechanical splices and welded splices of the longitudinal reinforcement in the boundary elements shall conform to 21.2.6 and 21.2.7, respectively (21.7.6.4(f)).

When special boundary elements are not required, the provisions of 21.7.6.5 must be satisfied. For the cases when the longitudinal reinforcement ratio at the wall boundary is greater than $400/f_y$, transverse reinforcement, spaced not more than 8 in. on center, shall be provided that satisfies 21.4.4.1(c), 21.4.4.3, and 21.7.6.4(c) (21.7.6.5(a)). This requirement helps in preventing buckling of the longitudinal reinforcement that can be caused by cyclic load reversals. The longitudinal reinforcement ratio to be used includes only the reinforcement at the end of the wall as indicated in Fig. R21.7.6.5. Horizontal reinforcement terminating at the edges of structural walls must be properly anchored per 21.7.6.5(b) in order for the reinforcement to be effective in resisting shear and to help in preventing buckling of the vertical edge reinforcement. The provisions of 21.7.6.5(b) are not required to be satisfied when the factored shear force V_u is less than $A_{cv}\sqrt{f'_c}$.

21.7.7 Coupling Beams

When adequately proportioned and detailed, coupling beams between structural walls can provide an efficient means of energy dissipation under seismic forces, and can provide a higher degree of overall stiffness to the structure. Due to their relatively large depth to clear span ratio, ends of coupling beams are usually subjected to large inelastic rotations. Adequate detailing and shear reinforcement are necessary to prevent shear failure and to ensure ductility and energy dissipation.

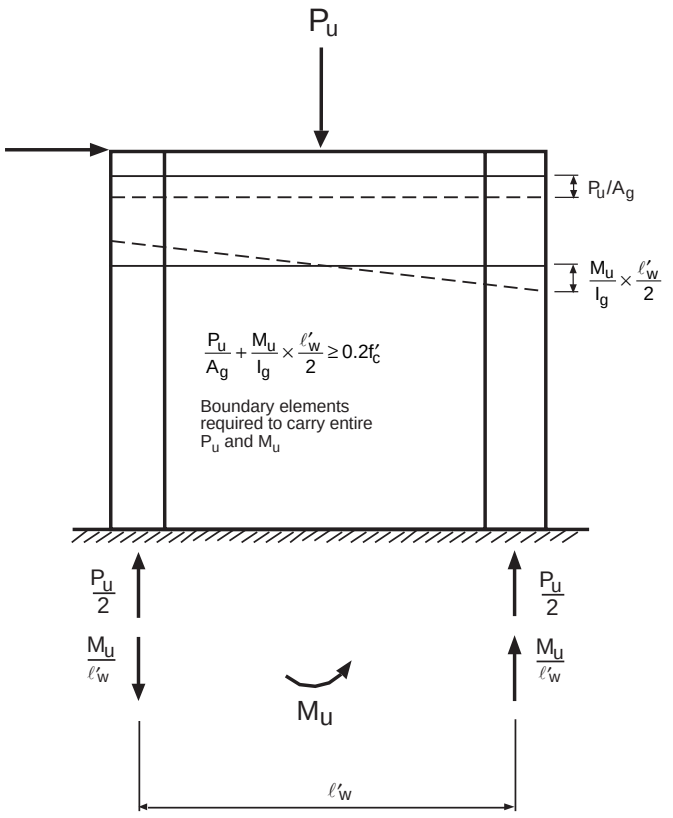


Figure 29-21 Special Boundary Element Requirements per 21.7.6.3

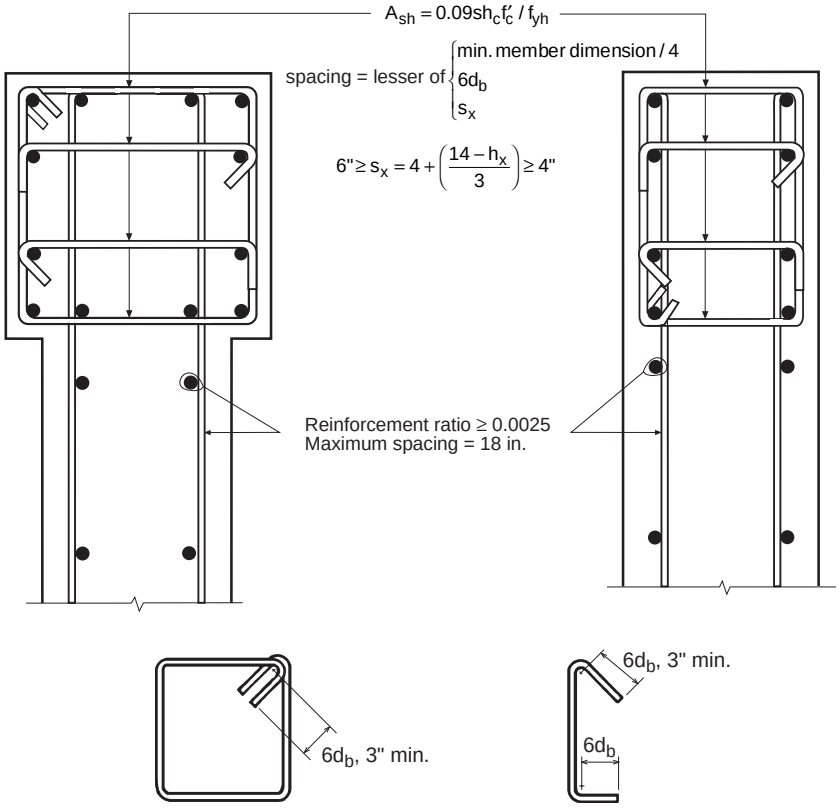


Fig. 29-22 Reinforcement Details for Special Boundary Elements

Coupling beams with $\ell_n/h \geq 4$ shall satisfy the requirement of 21.3 for flexural members of special moment frames, excluding 21.3.1.3 and 21.3.1.4(a) if it can be shown that the beam has adequate lateral stability (21.7.7.1). When $\ell_n/h < 4$, coupling beams with two intersecting groups of diagonally-placed bars symmetrical about the midspan is permitted (21.7.7.2). The diagonal bars are required for deep coupling beams ($\ell_n/h < 2$) with a factored shear force V_u greater than $4\sqrt{f'_c}A_{cp}$, unless it can be shown otherwise that safety and stability are not compromised (21.7.7.3). Experiments have shown that diagonally oriented reinforcement is effectve only if the bars can be placed at a large inclination.

Note that in the 2002 code, h replaces d in the definition of the aspect ratio (clear span/depth) and A_{cp} replaces $b_w d$ in the shear equations. The first change simplifies the code requirements, since d is not always readily known for beams with multiple layers of reinforcement. The second change removes an inconsistency between 21.6.4.5 and 21.6.7.4 of the 1999 code; A_{cp} is now consistently used in 21.7.4.5 and 21.7.7.4.

Section 21.7.7.4 contains the reinforcement details for the two intersecting groups of diagonally placed bars. Figure 29-23 provides a summary of these requirements. The requirement on side dimensions of the cage and its core is to provide adequate toughness and stability when the bars are stressed beyond yielding. The nominal shear strength of a coupling beam is computed from the following (21.7.7.4(b)):

$$V_n = 2A_{vd}f_y \sin\alpha \leq 10\sqrt{f'_c}A_{cp} \qquad \textit{Eq. (21-9)}$$

The additional reinforcement specified in 21.7.7.4(f) is used to confine the concrete outside of the diagonal cores.

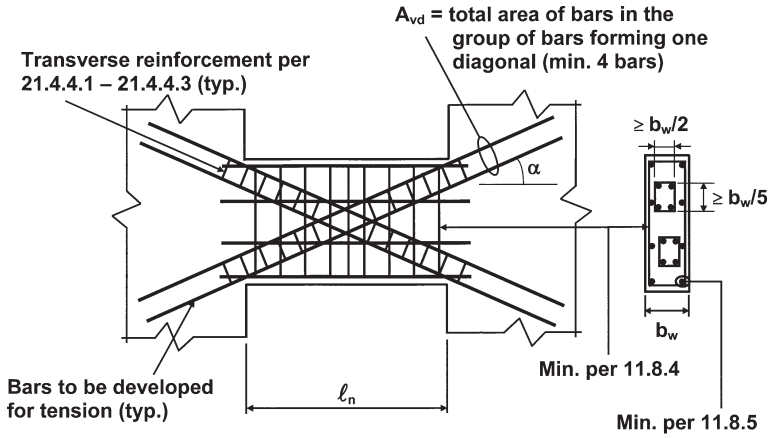


Figure 29-23 Coupling Beam with Diagonally Oriented Reinforcement

21.8 SPECIAL STRUCTURAL WALLS CONSTRUCTED USING PRECAST CONCRETE

According to 21.8.1, special structural walls constructed using precast concrete shall satisfy all requirements of 21.7 for cast-in-place special structural walls and the requirements in 21.13.2 and 21.13.3 for intermediate precast structural walls. Thus, the lefthand column of Table 29-4 may be utilized. The provisions for intermediate precast structural walls are discussed below. Note that the provisions of 21.8 do not apply to tilt-up walls.

21.9 STRUCTURAL DIAPHRAGMS AND TRUSSES

In building construction, diaphragms are structural elements, such as floor or roof slabs, that perform some or all of the following functions:

- Provide support for building elements such as walls, partitions, and cladding, and resist horizontal forces but not act as part of the vertical lateral force resisting system
- Transfer lateral forces to the vertical lateral force resisting system
- Interconnect various components of the lateral force resisting system with appropriate strength, stiffness, and toughness to permit deformation and rotation of the building as a unit

Section 21.9.4 prescribes a minimum thickness of 2 in. for concrete slabs and composite topping slabs serving as diaphragms used to transmit earthquake forces. The minimum thickness is based on what is currently used in joist and waffle slab systems and composite topping slabs on precast floor and roof systems. A minimum of 2.5 in. is required for topping slabs placed over precast floor or roof systems that do not act compositely with the precast system to resist the seismic forces.

Sections 21.9.2 and 21.9.3 provide design criteria for cast-in-place diaphragms. For the case of a cast-in-place composite topping slab on a precast floor or roof system, bonding is required so that the floor or roof system can provide restraint against slab buckling; also, reinforcement is required to ensure shear transfer across the precast joints. Composite action is not required for a cast-in-place topping slab on a precast floor or roof system, provided the topping slab acting alone is designed to resist the seismic forces.

21.9.5 Reinforcement

The minimum reinforcement ratio for structural diaphragms is the same as that required by 7.12 for temperature and shrinkage reinforcement. The maximum reinforcement spacing of 18 in. is intended to control the width of inclined cracks. Sections 21.9.5.1 and 21.9.5.2 contain provisions for welded wire fabric used in topping slabs placed over precast floor and roof elements and bonded prestressing tendons used as primary reinforcement in diaphragm chords or collectors, respectively.

According to 21.9.5.3, structural truss elements, struts, ties, diaphragm chords, and collector elements shall have transverse reinforcement as specified in 21.4.4.1 through 21.4.4.3 when the compressive stress at any section exceeds $0.2\,f'_c$. Note that compressive stress is calculated for the factored forces using a linearly elastic model and gross section properties. The special transverse reinforcement is no longer required where the compressive stress is less than $0.15\,f'_c$.

The provisions of 21.5.4 are to be used for anchoring and splicing all continuous reinforcement in diaphragms, trusses, struts, ties, chords, and collector elements (21.9.5.4). Also, where mechanical splices are used to transfer forces between a diaphragm and the vertical components of the lateral force resisting system, Type 2 splices are required (21.9.5.5).

21.9.7 Shear Strength

The shear strength requirements for monolithic structural diaphragms are similar as those for structural walls. In particular, the nominal shear strength V_n is computed from:

$$V_n = A_{cv}\left(2\sqrt{f'_c} + \rho_n f_y\right) \le 8A_{cv}\sqrt{f'_c} \qquad \textit{Eq. (21-10)}$$

where A_{cv} is the thickness times the width of the diaphragm. Shear reinforcement should be placed perpendicular to the span of the diaphragm.

The nominal shear strength V_n for topping slab diaphragms does not include the contribution from the concrete:

$$V_n = A_{cv}\rho_n f_y \le 8A_{cv}\sqrt{f'_c} \qquad \textit{Eq. (21-11)}$$

Typically, topping slabs are scored immediately above the boundary between the flanges of adjacent precast floor members to control the location of shrinkage cracks. Thus, these weakened sections of the diaphragm are

cracked under service load conditions, and the contribution of the concrete to the overall shear strength of the diaphragm may have already been reduced before the design earthquake occurs. For this reason, the contribution of the concrete is taken as zero. The required web reinforcement shall be uniformly distributed in both directions.

21.9.8 Boundary Elements of Structural Diaphragms

According to 21.9.8.1, boundary elements of structural diaphragms are to be proportioned to resist the sum of the factored axial forces acting in the plane of the diaphragm and the force obtained from dividing the factored moment at the section by the distance between the boundary elements of the diaphragm at that section. It is assumed that the factored flexural moments are resisted entirely by chord forces acting at opposite edges of the diaphragm (see Fig. 29-24). It is essential that splices of tensile reinforcement located in the chord and collector elements be fully developed and adequately confined (21.9.8.2). Mechanical and welded splices shall conform to 21.2.6 and 21.2.7, respectively. If chord reinforcement is located within a wall, the joint between the diaphragm and the wall should be provided with adequate shear strength to transfer the shear forces.

Reinforcement details for chords and collectors at splices and anchorage zones are given in 21.9.8.3.

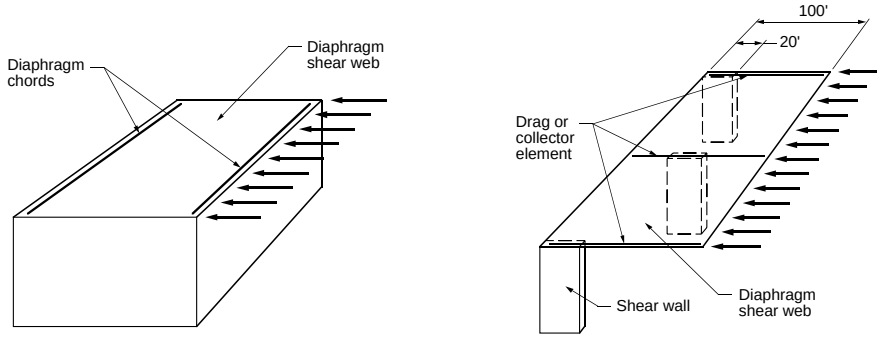


Figure 29-24 Diaphragm Chord and Collector Elements

21.10 FOUNDATIONS

Requirements for foundations supporting buildings in regions of high seismic risk or assigned to high seismic or performance categories are contained in 21.10. It is important to note that the foundations must also comply with all other applicable provisions of the code. For piles, drilled piers, caissons, and slabs on grade, the provisions of 21.10 shall supplement other applicable design and construction criteria (see also 1.1.5 and 1.1.6).

21.10.2 Footings, Foundation Mats, and Pile Caps

Detailing requirements are contained in 21.10.2.1 through 21.10.2.4 for footings, mats, and pile caps supporting columns or walls, and are illustrated in Fig. 29-25.

21.10.3 Grade Beams and Slabs on Grade

Grade beams that are designed as ties between pile caps or footings shall have continuous reinforcement that is developed within or beyond the supported column, or shall be anchored within the pile cap or footing at discontinuities (21.10.3.1).

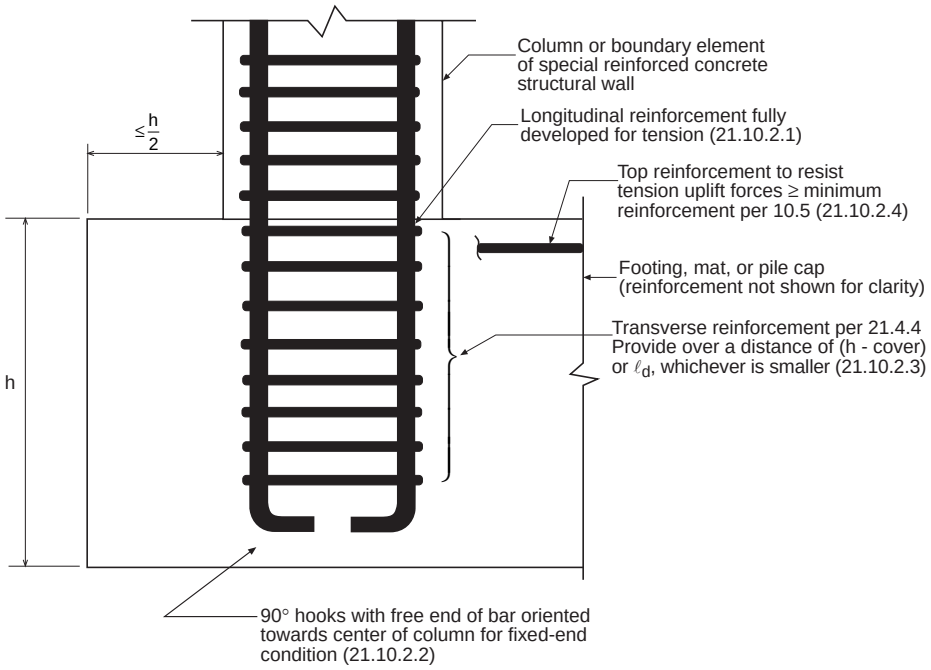


Figure 29-25 Reinforcement Details for Footings, Mats, and Pile Caps per 21.10.2

Section 21.10.3.2 contains geometrical and reinforcement requirements. The smallest cross-sectional dimension of the grade beam shall be greater than or equal to the clear spacing between the connected columns divided by 20; however, this dimension need not be greater than 18 in. Closed ties shall be provided over the length of the beam spaced at a maximum of one-half the smallest orthogonal cross-sectional dimension of the beam or 12 in., whichever is smaller. Both of these provisions are intended to provide reasonable beam proportions.

According to 21.10.3.3, grade beams and beams that are part of a mat foundation that is subjected to flexure from columns that are part of the lateral force resisting system shall have reinforcing details conforming to 21.3 for flexural members of special moment frames.

Slabs on grade shall be designed as diaphragms according to the provisions of 21.9 when they are subjected to seismic forces from walls or columns that are part of the lateral force resisting system (21.10.3.4). Such slabs shall be designated as structural members on the design drawings for obvious reasons.

21.10.4 Piles, Piers, and Caissons

When piles, piers, or caissons are subjected to tension forces from earthquake-induced effects, a proper load path is required to transfer these forces from the longitudinal reinforcement of the column or boundary element through the pile cap to the reinforcement of the pile or caisson. Thus, continuous longitudinal reinforcement is required over the length resisting the tensile forces, and it must be properly detailed to transfer the forces through the elements (21.10.4.2). When grouted or post-installed reinforcing bars are used to transfer tensile forces between the pile cap or mat foundation and a precast pile, a test must be performed to ensure that the grouting system can develop at least 125 percent of the specified yield strength of the reinforcing bar (21.10.4.3). In lieu of a test, reinforcing bars can be cast in the upper portion of a pile, exposed by later chipping away the concrete, and then mechanically connected or welded to achieve the proper extension.

Transverse reinforcement in accordance with 21.4.4 is required at the top of piles, piers, and caissons over a length equal to at least 5 times the cross-sectional dimension of the member, but not less than 6 ft below the bottom of the pile cap (21.10.4.4(a)). This requirement is based on numerous failures that were observed in earthquakes just below the pile cap, and provides ductility in this region of the pile. Also, for portions of piles in soil that is not capable of providing lateral support, or for piles in air or water, the entire unsupported length plus

the length specified in 21.10.4.4(a) must be confined with transverse reinforcement per 21.4.4 (21.10.4.4(b)). Additional requirements for precast concrete driven piles, foundations supporting one- and two-story stud bearing wall construction, and pile caps with batter piles are contained in 21.10.4.5 through 21.10.4.7.

21.11 FRAME MEMBERS NOT PROPORTIONED TO RESIST FORCES INDUCED BY EARTHQUAKE MOTIONS

In regions of high seismic risk or for structures assigned to high seismic performance or design categories, frame members that are assumed not to contribute to the lateral resistance shall comply with special requirements of 21.11. Specifically, these members are detailed depending on the magnitude of the moments that are induced when they are subjected to the design displacements. This requirement is intended to enable the gravity load system to maintain its vertical load carrying when subjected to the maximum lateral displacement expected for the design-basis earthquake. Although it is considered unnecessary that these elements of the gravity load system be designed for moments and shears related to the lateral forces, it has been recognized that they have to be designed to be adequate for the vertical loads and also the induced moments resulting from the actual displacements.

The following summarizes the requirements of 21.11:

(1) Compute moments and shears (E) in all elements that are not part of the lateral force resisting system due to the design displacement δ_u. The displacement δ_u is determined based on the provisions of the governing building code. In the IBC, three model building codes, ASCE 7-95, and the 1997 NEHRP Provisions, δ_u is determined from the design-basis earthquake (i.e., an earthquake with approximately a 90% probability of non-exceedance in 50 years) using a static or dynamic linear-elastic analysis, and considering the effects of cracked sections, torsion, P-Δ effects, and modification factors to account for expected inelastic response. In particular, δ_u is determined by multiplying the deflections from an elastic analysis by a deflection amplification factor, which is given in the governing code for various seismic-force-resisting systems.

(2) Determine the factored moment M_u and the factored shear V_u in each of the elements that are not part of the lateral force resisting system from the more critical of the following load combinations:

$$U = 1.2D + 1.0L + 0.2S + E$$

$$U = 0.9D + E$$

The local factor on L can be reduced to 0.5 except for garages, areas occupied as places of public assembly, and all areas where L > 100 psf.

Note that the E-values (moments and shears) in the above expressions are determined in step 1 above.

(3) If $M_u \le \phi M_n$ and $V_u \le \phi V_n$ for an element that is not part of the lateral force resisting system, and if such an element is subjected to factored gravity axial forces $P_u \le A_g f'_c/10$, it must satisfy the longitudinal reinforcement requirements in 21.3.2.1; in addition, stirrups spaced at no more than d/2 must be provided throughout the length of the member. If such an element is subjected to $P_u > A_g f'_c/10$ where $P_u \le 0.35P_o$ (where P_o is the nominal axial load strength at zero eccentricity), it must conform to 21.4.3, 21.4.4.1(c), 21.4.4.3, and 21.4.5. In addition, ties at a maximum spacing of s_o must be provided throughout the height of the column, where s_o must not exceed the smaller of six times the smallest longitudinal bar diameter or 6 in. If the factored gravity axial force $P_u > 0.35P_o$, the requirements of 21.11.2.3 must be satisfied and the amount of transverse reinforcement provided shall be one-half of that required by 21.4.4.1 not to exceed a spacing of s_o for the full column height.

(4) If M_u or V_u determined in step 2 for an element that is not part of the lateral force resisting system exceeds ϕM_n or ϕV_n, or if induced moments and shears due to the design displacements are not calculated, then the structural materials must satisfy 21.2.4 and 21.2.5, and splices of reinforcement must satisfy 21.2.6 and

21.2.7. If such an element is subjected to $P_u \le A_g f'_c/10$, it must conform to 21.3.2.1 and 21.3.4; in addition, stirrups spaced at no more than d/2 must be provided throughout the length of the member. If such an element is subjected to $P_u > A_g f'_c/10$, it must be provided with full ductile detailing in conformance with 21.4.3.1, 21.4.4, 21.4.5, and 21.5.2.1. Note that the requirements of 21.4.3.1 were added in 2002.

Precast concrete frame members assumed not to contribute to lateral resistance must also conform to 21.11.1 through 21.11.3. In addition, the following requirements of 21.11.4 must be satisfied: (a) ties specified in 21.11.2.2 must be provided over the entire column height, including the depth of the beams; (b) structural integrity reinforcement of 16.5 must be provided in all members; and (c) bearing length at the support of a beam must be at least 2 in. longer than the computed bearing length according to 10.17. The 2 in. increase in bearing length is based on an assumed 4% story drift ratio and a 50 in. beam depth, and is considered to be conservative for ground motions expected in high seismic zones.

21.12 REQUIREMENTS FOR INTERMEDIATE MOMENT FRAMES

For comparison purposes with the requirements for special moment frames, the provisions for beams (21.12.4) and columns (21.12.5) in intermediate moment frames have been presented in Table 29-1 and Table 29-2, respectively. The shear provisions of 21.12.3 are also included in the tables.

As was noted above, hoops instead of stirrups are now required at both ends of beams for a distance not less than 2h from the faces of the supports. The likelihood of spalling and loss of shell concrete in some regions of the frame are high. Both observed behavior under actual earthquakes and experimental research have shown that the transverse reinforcement will open at the ends and lose the ability to confine the concrete unless it is bent around the longitudinal reinforcement and its ends project into the core of the element. Similar provisions are now given in 21.12.5 for columns.

Two-way slabs without beams are acceptable lateral force resisting systems in regions of low or moderate seismic risk, or for structures assigned to low or moderate seismic performance or design categories. They are not permitted in regions of high seismic risk or for structures assigned to high seismic performance or design categories. Table 29-5, Fig. 29-26, and Fig. 29-27 summarize the detailing requirements for two-way slabs. Provisions for two-way slabs of ordinary moment frames are also presented in Table 29-5.

The provisions of 21.12.6.2 for the band width within which flexural moment transfer reinforcement must be placed at edge and corner slab-column connections are new in the 2002 code. For these connections, flexural moment-transfer reinforcement perpendicular to the edge is not considered fully effective unless it is placed within the specified narrow band width (see Fig. R21.12.6.1).

The shear strength requirements of 21.12.6.8 are also new in the 2002 code. Slab-column frames are susceptible to punching shear failures during earthquakes if the shear stresses due to gravity loads are high. Thus, a limit was inroduced on the allowable shear stress caused by gravity loads, which in turn permits the slab-column connection to have adequate toughness to withstand the anticipated inelastic moment transfer.

21.13 INTERMEDIATE PRECAST STRUCTURAL WALLS

This new section applies to intermediate precast structural walls used to resist forces induced by the design earthquake.

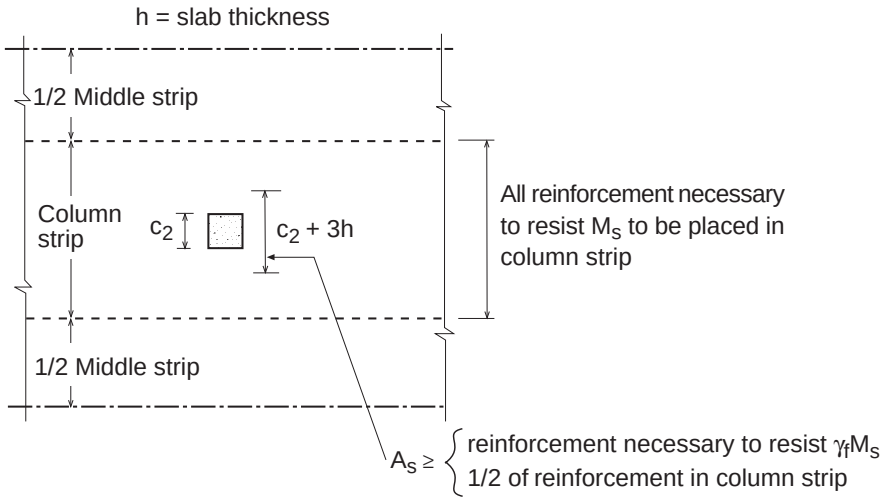
Connections between precast wall panels or between wall panels and the foundation are required to resist forces due to earthquake motions and must provide for yielding in the vicinity of the connection (21.13.2). When Type 2 mechanical splices are used for connecting the primary reinforcement, the strength of the splice should be greater than or equal to 1.5 times f_y of the reinforcement (21.13.3).

Note that the provisions of 21.13, like those of 21.8, do not apply to tilt-up walls.

*Table 29-5 Two-Way Slabs Without Beams**

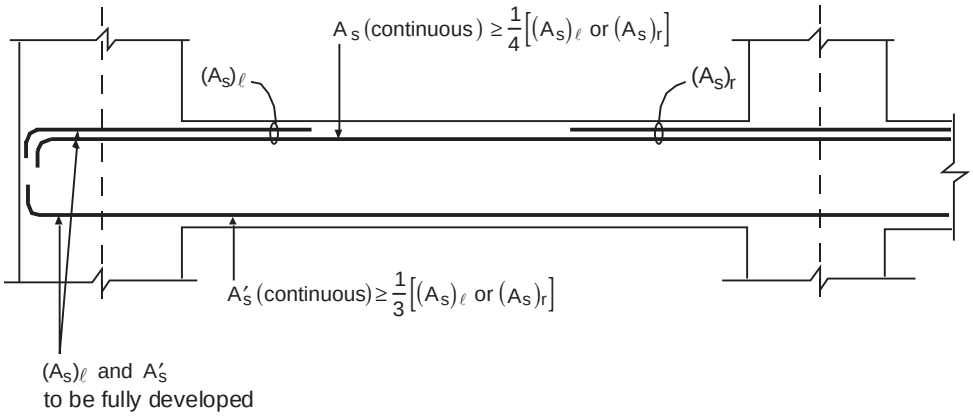
Intermediate — All reinforcement provided to resist M_s, the portion of slab moment balanced by the support moment, shall be placed within the column strip. **21.12.6.1** Ordinary — The middle strip is allowed to carry a portion of the unbalanced moment.
Intermediate — The fraction, defined by Eq. (13-1), of the moment M_s shall be resisted by reinforcement placed within the band width specified in 13.5.3.2. Band width for edge and corner connections shall not extend beyond the column face a distance greater than c_t measured perpendicular to the slab span. **21.12.6.2** Ordinary — Similar requirement, except band width for edge and corner connections does not apply.
Intermediate — Not less than one-half of the reinforcement in the column strip at the support shall be placed within the effective slab width specified in 13.5.3.2. **21.12.6.3** Ordinary — No similar requirement.
Intermediate — Not less than one-fourth of the top reinforcement at the support in the column strip shall be continuous throughout the span. **21.12.6.4** Ordinary — No similar requirement.
Intermediate — Continuous bottom reinforcement in the column strip shall not be less than one-third of the top reinforcement at the support in the column strip. **21.12.6.5** Intermediate — Not less than one-half of all bottom middle strip reinforcement and all bottom column strip reinforcement at midspan shall be continuous and shall develop its yield strength at the face of the support as defined in 13.6.2.5. **21.12.6.6** Ordinary — All bottom bars within the column strip shall be continuous or spliced with Class A tension splices or with mechanical or welded splices satisfying 12.14.3. **13.3.8.5**
Intermediate — At discontinuous edges of the slab, all top and bottom reinforcement at the support shall be developed at the face of support as defined in 13.6.2.5. **21.12.6.7** Ordinary — Positive moment reinforcement perpendicular to a discontinuous edge shall extend to the edge of the slab and have embedment at least 6 in. in spandrel beams, columns, or walls. Negative moment reinforcement perpendicular to a discontinuous edge must be anchored and developed at the face of the support according to provisions in Chapter 12. **13.3.3, 13.3.4**
Intermediate — At the critical sections for columns defined in 11.12.1.2, two-way shear caused by factored gravity loads shall not exceed $0.4\phi V_c$ where V_c is calculated by 11.12.2.1 for nonprestressed slabs and 11.12.2.2 for prestressed slabs. This requirement may be waived if the contribution of the earthquake-induced factored two-way shear stress transferred by eccentricity of shear in accordance with 11.12.6.1 and 11.12.6.2 at the point of maximum stress does not exceed (ϕv_n permitted by 11.12.6.2)/2. **21.12.6.8** Ordinary — No similar requirement.

* *Not permitted as part of the lateral force resisting system in regions of high seismic risk or for structures assigned to high seismic performance or design categories.*

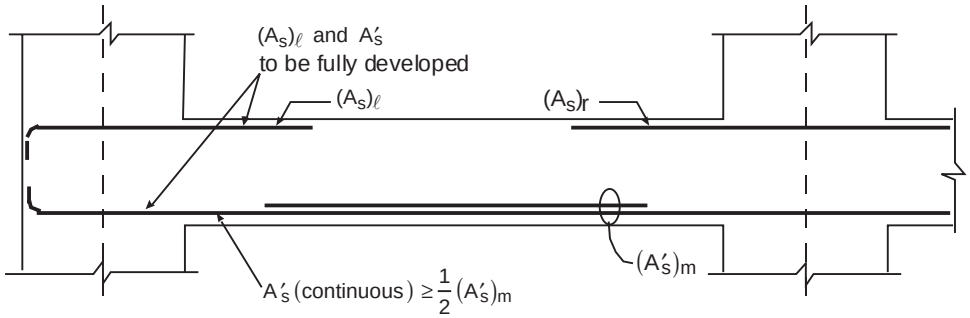


Note: applies to both top and bottom reinforcement

Figure 29-26 Location of Reinforcement in Two-way Slabs without Beams



(a) Column Strip



(b) Middle Strip

Figure 29-27 Details of Reinforcement in Two-way Slabs without Beams

REFERENCES

29.1 *International Building Code*, International Code Council, Falls Church, VA, 2000.

29.2 *Uniform Building Code*, Vol. 2, International Conference of Building Officials (ICBO), Whittier, CA, 1997.

29.3 Ghosh, S. K., Nakaki, S. D., and Krishnan, K., "Precast Structures in Regions of High Seismicity", PCI Journal, Nov.–Dec. 1997, pp. 76-93.

29.4 *Acceptance Criteria for Moment Frames Based on Structural Testing*, ACI T1.1-01, American Concrete Institute, Farmington Hills, MI, 2001.

Example 29.1—Design of a 12-Story Cast-in-Place Frame-Shearwall Building and its Components

This example, and the 5 examples that follow, illustrate the design and detailing requirements for typical members of a 12-story cast-in-place concrete building.

A typical plan and elevation of the structure are shown in Figs. 29-28(a) and (b). The columns and shearwalls have constant cross-sections throughout the height of the building*, and the bases of the lowest story segments are assumed fixed. The beams and the slabs also have the same dimensions at all floor levels. Although the member dimensions in this example are within the practical range, the structure itself is a hypothetical one, and has been chosen mainly for illustrative purposes. Other pertinent design data are as follows:

Material properties:

Concrete:	f'_c = 4,000 psi, w_c = 145 pcf
Reinforcement:	f_y = 60,000 psi

Service loads:

Live load:	Floors = 50 psf
	Additional average value to allow for heavier load on corridors = 25 psf
	Total average live load (floors) = 75 psf
	Roof = 20 psf
Superimposed dead load:	Average for partitions = 20 psf
	Ceiling and mechanical = 10 psf
	Total average superimposed dead load (floors) = 30 psf
	Roof = 10 psf

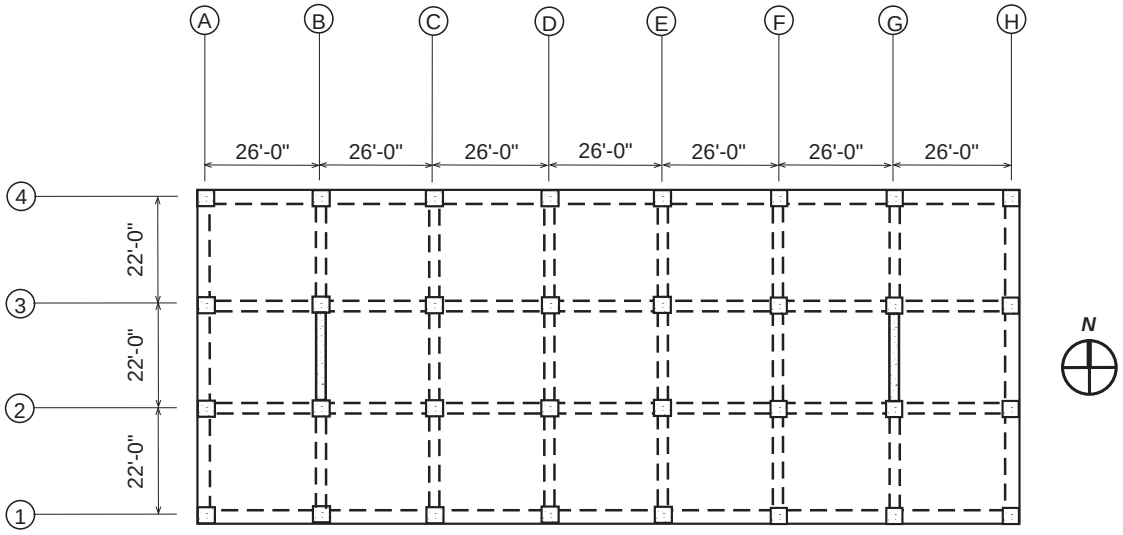
Seismic design data:

The building is located in a region of high seismic risk, and is assigned to a high seismic design or performance category.

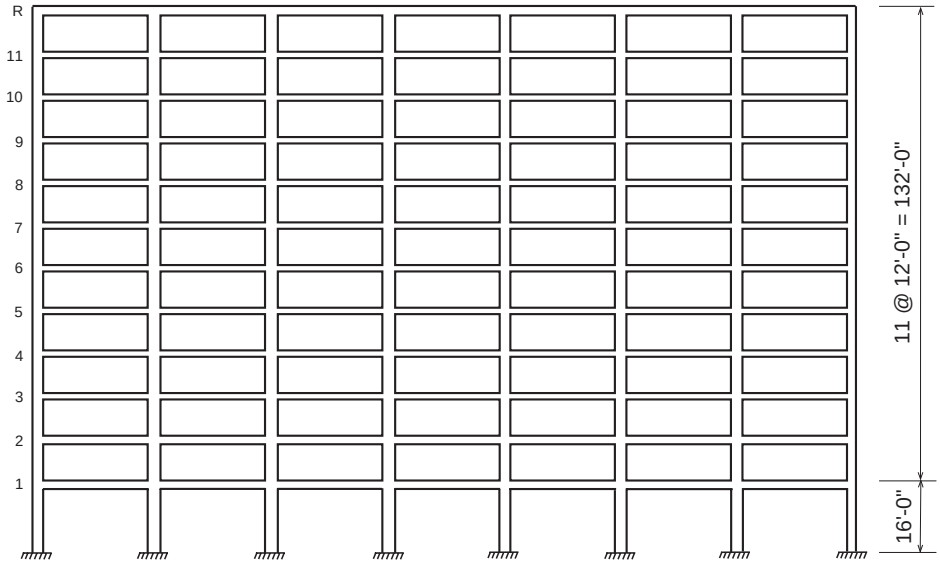
Dual system (special reinforced concrete structural walls with special moment frame) in the N-S direction

Special moment frame in the E-W direction

* The uniformity in member dimensions used in this example has been adopted mainly for simplicity.



(a) Typical floor plan



Edge columns: 24 x 24 in.
Interior columns: 30 x 30 in.
Beams: 20 x 24 in.
Slab: 8 in.
Walls: 18 in. web

(b) Longitudinal section

Figure 29-28 Example Building

1. Lateral analysis

   The computation of the seismic and wind design forces is beyond the scope of this example.

   A three-dimensional analysis of the building was performed in both the N-S and E-W directions for both seismic and wind load cases. The effects of the seismic loads governed; thus, load combinations containing the effects of wind loads are not considered in the following examples.

2. Gravity analysis

   The Equivalent Frame Method of 13.7 was used to determine the gravity load moments in the members.

   Cumulative service axial loads for the columns and walls were computed considering live load reduction according to ASCE 7.

Example 29.2—Proportioning and Detailing of Flexural Members of Building in Example 29.1

Design a beam on the first floor of a typical interior E-W frame of the example building (Fig. 29-28). The beam has dimensions of b = 20 in. and h = 24 in. (d = 21.5 in.). The slab is 8 in. thick. Use f'_c = 4,000 psi and f_y = 60,000 psi.

Calculations and Discussion	Code Reference

1. Check satisfaction of limitations on section dimensions.

   Factored axial compressive force on beams is negligible. O.K. — 21.3.1.1

   $\frac{\ell_n}{d} = \frac{(26\times12)-30}{21.5} = 13.1 > 4$ O.K. — 21.3.1.2

   $\frac{\text{width}}{\text{depth}} = \frac{20}{24} = 0.83 > 0.3$ O.K. — 21.3.1.3

   width = 20 in. > 10 in. O.K. — 21.3.1.4

   < width of supporting column + (1.5 × depth of beam)

   < 24 + (1.5 × 24) = 60 in. O.K.

2. Determine required flexural reinforcement.

   The required reinforcement for the beams on the first floor level is shown in Table 29-6. The provided areas of steel are within the limits specified in 21.3.2.1. Also given in Table 29-6 are the flexural moment strengths ϕM_n at each section. The positive moment strength at a joint face must be at least equal to 50% of the negative moment strength provided at that joint. At the exterior negative location, this provision is satisfied since the positive moment strength of 220.8 ft-kips is greater than 351.2/2 = 175.6 ft-kips. The provision is also satisfied at the interior negative location since 220.8 ft-kips is greater than 414.0/2 = 207.0 ft-kips. — 21.3.2.2

   Neither the negative nor the positive moment strength at any section along the length of the member shall be less than 25% of the maximum moment strength provided at the face of either joint. In this case, 25% of the maximum moment strength is equal to 414.0/4 = 103.5 ft-kips. Providing at least 2-No. 8 bars (ϕM_n = 147.9 ft-kips) or 2-No. 7 bars (ϕM_n = 113.2 ft-kips) at any section will satisfy this requirement. However, to satisfy the minimum reinforcement requirement of 21.3.2.1 (i.e., minimum A_s = 1.43 in.^{2}), a minimum of 2-No. 8 bars (A_s = 1.58 in.^{2}) or 3-No. 7 bars (A_s = 1.80 in.^{2}) must be provided at any section. This also automatically satisfies the requirement that 2 bars be continuous at both the top and bottom of any section. — 21.3.2.2, 21.3.2.1

Table 29-6 Required Reinforcement for Beams on Floor Level 1

Location		M_u (ft-kips)	A_s* (in.^{2})	Reinforcement*	ϕM_n** (ft-kips)
End Span	Ext. Neg.	-291.9	3.23	5-No. 8	351.2
		138.7	1.48	4-No. 7	220.8
	Positive	145.3	1.55	4-No. 7	220.8
	Int. Neg.	-366.2	4.14	6-No. 8	414.0
		120.1	1.43	4-No. 7	220.8
Interior Span	Positive	125.1	1.43	4-No. 7	220.8
	Negative	-354.3	3.99	5-No. 8	351.2
		135.7	1.45	4-No. 7	220.8

**Max A_s = 0.025 x 20 x 21.5 = 10.75 in.^{2} (21.3.2.1)*
Min. $A_s = 3\sqrt{4{,}000}$ x 20 x 21.5/60,000 = 1.36 in.^{2}
= 200 x 20 x 21.5/60,000 = 1.43 in.^{2} (governs)
***Does not include slab reinforcement*

3. Calculate required length of anchorage of flexural reinforcement in exterior column.

Beam longitudinal reinforcement terminated in a column shall be extended to the far face of the confined column core and shall be anchored in tension according to 21.5.4 and in compression according to Chapter 12. *21.5.1.3*

Minimum development length ℓ_{dh} for a bar with a standard 90-deg hook in normal weight concrete is

$$\ell_{dh} = \frac{f_y d_b}{65\sqrt{f'_c}} \qquad 21.5.4.1$$

$$\geq 8d_b$$

$$\geq 6 \text{ in.}$$

A standard hook is defined as a 90-deg bend plus a $12d_b$ extension at the free end of the bar. *7.1.2*

For the No. 8 top bars (bend diameter $\geq 6d_b$): *7.2.1*

$$\ell_{dh} = \begin{cases} (60{,}000 \times 1.00)/(65\sqrt{4{,}000}) = 14.6 \text{ in.} & (\text{governs}) \\ 8 \times 1.00 = 8 \text{ in.} \\ 6 \text{ in.} \end{cases}$$

For the No. 7 bottom bars (bend diamter $\geq 6d_b$):

$$\ell_{dh} = \begin{cases} (60{,}000 \times 0.875)/(65\sqrt{4{,}000}) = 12.8 \text{ in.} & (\text{governs}) \\ 8 \times 0.875 = 7 \text{ in.} \\ 6 \text{ in.} \end{cases}$$

Note that the development length ℓ_{dh} is measured from the near face of the column to the far edge of the vertical 12-bar-diameter extension (see Fig. 29-29).

When reinforcing bars extend through a joint, the column dimension must be at least 20 times the diameter of the largest longitudinal bar for normal weight concrete. In this case, the minimum required column dimension is 20 × 1.0 = 20 in., which is less than each of the two column widths that are provided. *21.5.1.4*

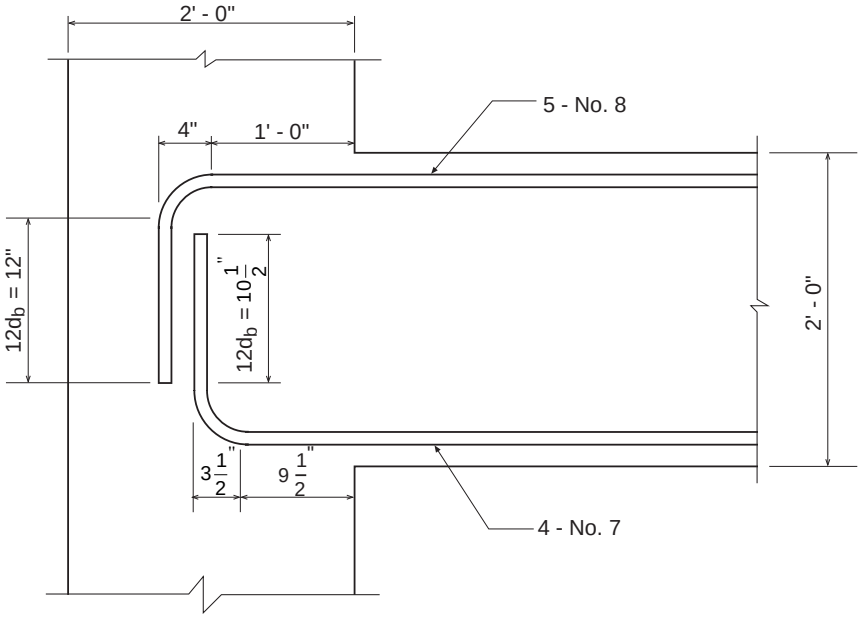


Figure 29-29 Detail of Flexural Reinforcement Anchorage at Exterior Column

4. Determine shear reinforcement requirements.

Design for shear forces corresponding to end moments are obtained by assuming the stress in the tensile flexural reinforcement equal to $1.25f_y$ and a strength reduction factor, $\phi = 1.0$ (probable flexural strength), plus factored tributary gravity loads. *21.3.4.1*

The following equation can be used to compute M_{pr}*:

$$M_{pr} = A_s\,(1.25f_y)\left(d - \frac{a}{2}\right)$$

$$\text{where } a = \frac{A_s\left(1.25f_y\right)}{0.85f'_c b}$$

* The slab reinforcement within the effective slab width defined in 8.10 is not included in the calculation of M_{pr} (note that this reinforcement must be included when computing the flexural strength of the beam when checking the requirements of 21.4.2). It is unlikely that all or even most of the reinforcement within the slab effective width away from the beam will yield when subjected to the forces generated from the design-basis earthquake. Furthermore, including the slab reinforcement in the calculation of M_{pr} would result in a major deviation from how members have been designed in the past. In particular, the magnitude of the negative probable moment strength of the beam would significantly increase if the slab reinforcement were included. This in turn would have a significant impact on the shear strength requirements of the beam (21.3.4) and most likely the columns framing into the joint as well (21.4.5). Such significant increases seem unwarranted when compared to the approximate provisions in previous editions of the ACI Code and other codes.

For example, for sidesway to the right, the interior joint must be subjected to the negative moment M_{pr} which is determined as follows:

For 6-No. 8 top bars, $A_s = 6 \times 0.79 = 4.74$ in.^{2}

$$a = \frac{A_s(1.25f_y)}{0.85f'_c b} = \frac{4.74 \times 1.25 \times 60}{0.85 \times 4 \times 20} = 5.23 \text{ in.}$$

$$M_{pr} = A_s(1.25f_y)\left(d - \frac{a}{2}\right) = 4.74 \times 1.25 \times 60 \times \left(21.5 - \frac{5.23}{2}\right) = 6{,}713.6 \text{ in.-kips} = 559.5 \text{ ft-kips}$$

Similarly, for the exterior joint, the positive moment M_{pr} based on the 4-No. 7 bottom bars is equal to 302.6 ft-kips. The probable flexural strengths for sidesway to the left can be obtained in a similar fashion.

The factored gravity load at midspan is:

$$w_D = \left[\frac{8}{12}(145) + 30\right] \times 22 + \frac{16 \times 20}{144}(145) = 3{,}109 \text{ lbs/ft}$$

$$w_L = 75 \times 22 = 1{,}650 \text{ lbs/ft}$$

$$w_u = 1.2w_D + 0.5w_L = 4.56 \text{ kips/ft}$$

Figure 29-30 shows the exterior beam and the shear forces due to the gravity loads. Also shown are the probable flexural strengths M_{pr} at the joint faces for sidesway to the right and to the left and the corresponding shear forces due to these moments. Note that the maximum combined design shear forces are larger than those obtained from the structural analysis. *21.3.4.2*

The shear strength of concrete V_c is to be taken as zero when the earthquake-induced shear force calculated in accordance with 21.3.4.1 is greater than or equal to 50% of the total shear force and the factored axial compressive forces is less than $A_g f'_c/20$ where A_g is the gross area of the beam. The beam carries negligible axial forces, and the maximum earthquake-induced shear force, which is equal to 36.3 kips (see Fig. 29-30), is greater than one-half the total design shear force which is equal to $0.5 \times 68.1 = 34.1$ kips. Thus, V_c must be taken equal to zero. The maximum shear force V_s is:

$$\phi V_s = V_u - \phi V_c$$

$$V_s = \frac{V_u}{\phi} - V_c$$

$$= \frac{68.1}{0.75} - 0 = 90.8 \text{ kips}$$

where the strength reduction factor ϕ was taken as 0.75. 9.3.4

Shear strength contributed by shear reinforcement shall not exceed $(V_s)_{max}$: 11.5.6.9

$$(V_s)_{max} = 8\sqrt{f'_c}\,b_w d = 8\sqrt{4{,}000} \times 20 \times 21.5/1{,}000 = 217.6 \text{ kips} > 90.8 \text{ kips} \quad \text{O.K.}$$

Also, V_s is less than $4\sqrt{f'_c}\,b_w d = 108.8$ kips. 11.5.4.3

Required spacing of No. 3 closed stirrups (hoops) for a factored shear force of 90.8 kips is: 11.5.6.2

$$s = \frac{A_v f_y d}{V_s} = \frac{(4 \times 0.11) \times 60 \times 21.5}{90.8} = 6.3 \text{ in.}$$

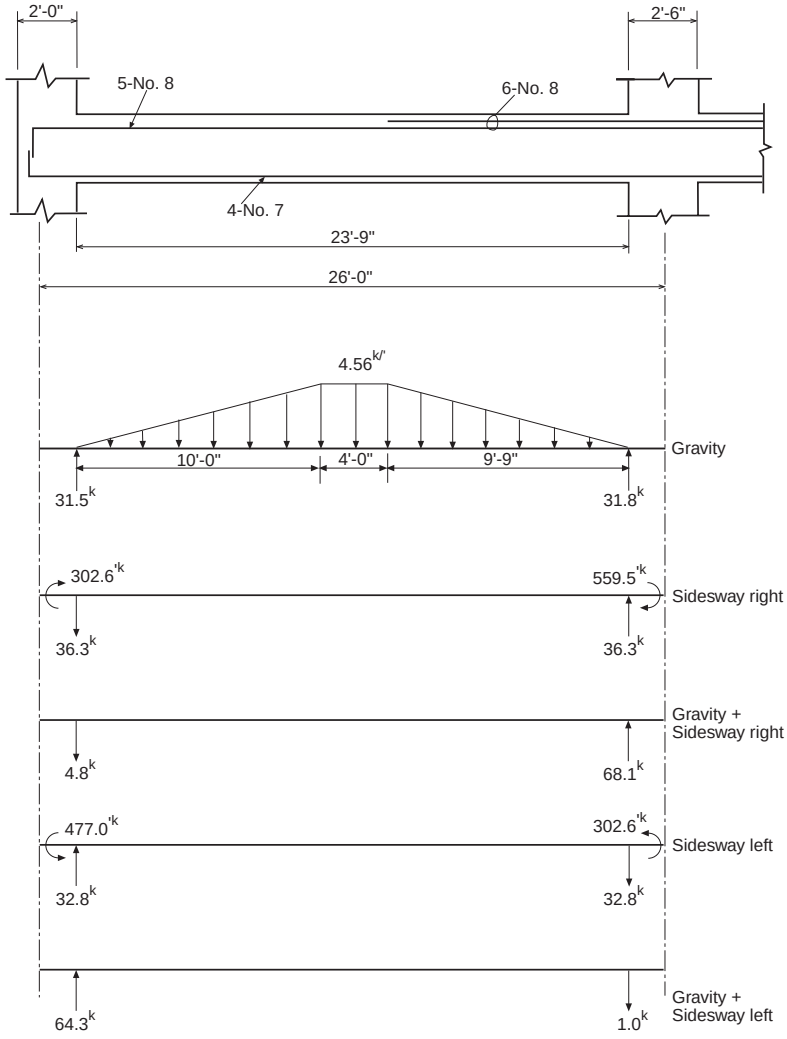


Figure 29-30 Design Shear Forces for Exterior Beam on Floor Level 1

Note that 4 legs are required for lateral support of the longitudinal bars. *21.3.3.3*

Maximum allowable hoop spacing (s_{max}) within a distance of 2h = 2 × 24 = 48 in. from the face of the support is the smallest of the following: *21.3.3.2*

$$s_{max} = \frac{d}{4} = \frac{21.5}{4} = 5.4 \text{ in. (governs)}$$

= 8 × (diameter of smallest longitudinal bar) = 8 × 0.875 = 7.0 in.

= 24 × (diameter of hoop bar) = 24 × 0.375 = 9.0 in.

= 12 in.

Therefore, hoops must be spaced at 5 in. on center with the first one located at 2 in. from the face of the support. Eleven hoops are to be placed at this spacing.

Where hoops are no longer required, stirrups with seismic hooks at both ends may be used. At a distance of 52 in. from the face of the interior support, V_u = 63.7 kips. *21.3.3.4*

With $V_c = 2\sqrt{4{,}000} \times 20 \times 21.5/1{,}000 = 54.4$ kips, the spacing required for No. 3 stirrups with two legs is 9.3 in. < d/2 = 10.8 in.

A 9 in. spacing, starting at 52 in. from the face of the support will be sufficient for the remaining portion of the beam.

5. Negative reinforcement cutoff points.

For the purpose of determining cutoff points for the negative reinforcement at the interior support, a moment diagram corresponding to the probable flexural strengths at the beam ends and 0.9 times the dead load on the span will be used. The cutoff point for four of the six No. 8 bars at the top will be determined. *Eq. (9-7)*

With the design flexural strength of a section with 2-No. 8 top bars = 147.9 ft-kips (calculated using $f_s = f_y$ = 60 ksi and ϕ = 0.9), the distance from the face of the support to where the moment under the loading considered equals 147.9 ft-kips is readily obtained by summing moments about section a-a in Fig. 29-31, and equating these to 147.9 ft-kips:

$$\frac{x}{2}\left(\frac{2.8x}{9.75}\right)\left(\frac{x}{3}\right) - 55.8x + 559.5 = 147.9$$

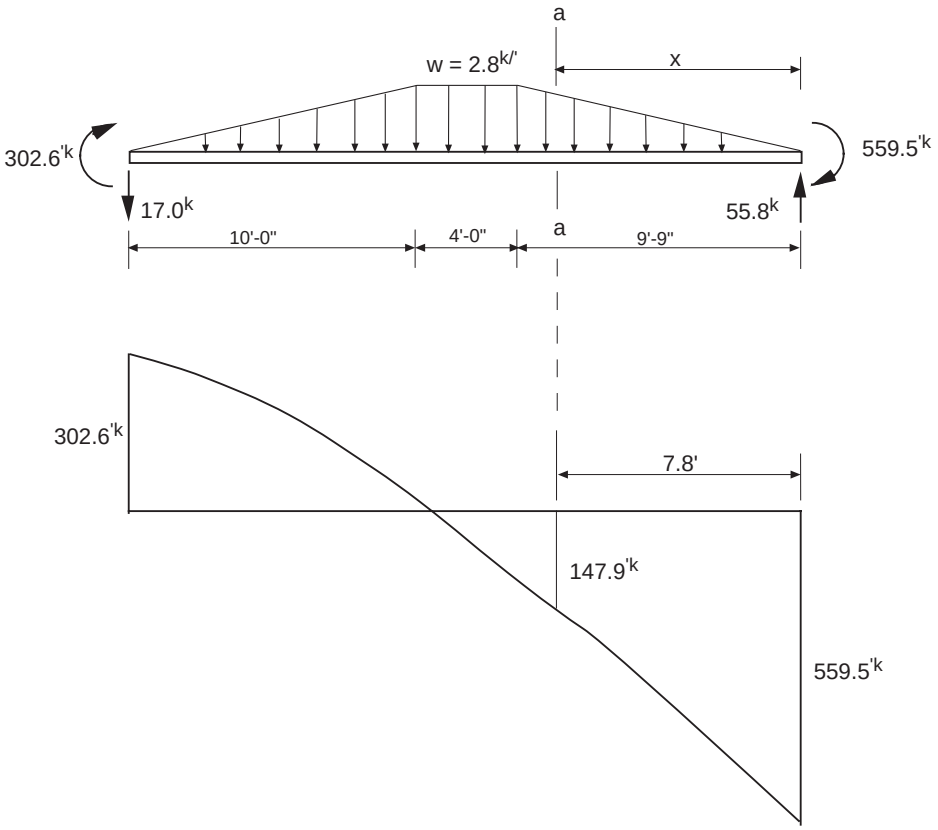


Figure 29-31 Moment Diagram for Cutoff Location of Negative Bars at Interior Support

Solving for x gives a distance of 7.8 ft. The 4-No. 8 bars must extend a distance d = 21.5 in. or $12d_b = 12 \times 1.0 = 12$ in. beyond the distance x. Thus, from the face of the support, the total bar length must be at least equal to 7.8 + (21.5/12) = 9.6 ft. Also, the bars must extend a full development length ℓ_d beyond the face of the support: *12.10.3* *12.10.4*

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b} \right)} \right) d_b \qquad \textit{Eq. (12-1)}$$

where α = reinforcement location factor = 1.3 (top bar)

β = coating factor = 1.0 (uncoated reinforcement)

γ = reinforcement size factor = 1.0 (No. 8 bar)

λ = lightweight aggregate concrete factor = 1.0 (normal weight concrete)

$$c = \text{spacing or cover dimension} = \begin{cases} 1.5 + 0.375 + \dfrac{1.0}{2} = 2.375 \text{ in.} \\ \dfrac{20 - 2(1.5 + 0.375) - 1.0}{2 \times 5} = 1.525 \text{ in. (governs)} \end{cases}$$

K_{tr} = transverse reinforcement index = 0 (conservative)

$$\frac{c + K_{tr}}{d_b} = \frac{1.525 + 0}{1.0} = 1.525$$

$$\ell_d = \frac{3}{40} \times \frac{60{,}000}{\sqrt{4{,}000}} \times \frac{1.3 \times 1.0 \times 1.0 \times 1.0}{1.525} \times 1.0 = 60.7 \text{ in.} = 5.1 \text{ ft} < 9.6 \text{ ft}$$

The total required length of the 4-No. 8 bars must be at least 9.6 ft beyond the face of the support.

Flexural reinforcement shall not be terminated in a tension zone unless one or more of the conditions of 12.10.5 are satisfied. In this case, the point of inflection is approximately 11.25 ft from the face of the right support which is greater than 9.6 ft. The 4-No. 8 bars can not be terminated here unless one of the conditions of 12.10.5 is satisfied.

Check if the factored shear force V_u at the cutoff point does not exceed two-thirds of ϕV_n. *12.10.5.1*
For No. 3 stirrups spaced at 9 in. on center that are provided in this region:

$$\phi V_n = \phi(V_s + V_c) = 0.75 \times \left(\frac{0.22 \times 60 \times 21.5}{9} + 54.4\right) = 64.5 \text{ kips}$$

$\frac{2}{3}\phi V_n = 43.0$ kips $> V_u = 42.7$ kips at 9.6 ft from face of support

Since $2\phi V_n/3 > V_u$, the cutoff point for the 4-No. 8 bars can be 9.6 ft beyond the face of the interior support.

The cutoff point for three of the 5-No. 8 bars at the exterior support can be determined in a similar fashion. These bars can be cut off at 8.4 ft from the face of the exterior support.

6. Flexural reinforcement splices.

Lap splices of flexural reinforcement must not be placed within a joint, within a distance 2h from faces of supports or within regions of potential plastic hinging. Note that all lap splices have to be confined by hoops or spirals with a maximum spacing or pitch of d/4 or 4 in. over the length of the lap. Lap splices will be determined for the No. 7 bottom bars. *21.3.2.3*

Since all of the bars will be spliced within the required length, use a Class B splice. *12.15.2*

Required length of splice = $1.3\ell_d \geq 12$ in. *12.15.1*

where

$$\ell_d = \left(\frac{3}{40}\,\frac{f_y}{\sqrt{f'_c}}\,\frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b}\right)}\right) d_b$$ *Eq. (12-1)*

reinforcement location factor α = 1.0 (other than top bars) *12.2.4*

coating factor β = 1.0 (uncoated bars)

reinforcement size factor γ = 1.0 (No. 7 bar)

lightweight aggregate concrete factor λ = 1.0 (normal weight concrete)

$$c = 1.5 + 0.375 + \frac{0.875}{2} = 2.31 \text{ in.} \quad \text{(governs)}$$

$$= \frac{1}{2}\left[\frac{20 - 2(1.5 + 0.375) - 0.875}{3}\right] = 2.56 \text{ in.}$$

$$K_{tr} = \frac{A_{tr} f_{yt}}{1{,}500sn} = \frac{(2 \times 0.11)(60{,}000)}{1{,}500 \times 4.0 \times 4} = 0.55$$

$$\frac{c + K_{tr}}{d_b} = \frac{2.31 + 0.55}{0.875} = 3.3 > 2.5, \text{ use } 2.5$$

Therefore,

$$\ell_d = \frac{3}{40} \times \frac{60{,}000}{\sqrt{4{,}000}} \times \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.5} \times 0.875 = 24.9 \text{ in.}$$

Class B splice length = 1.3 × 24.9 = 32.4 in.

7. Reinforcement details for the beam are shown in Fig. 29-32.

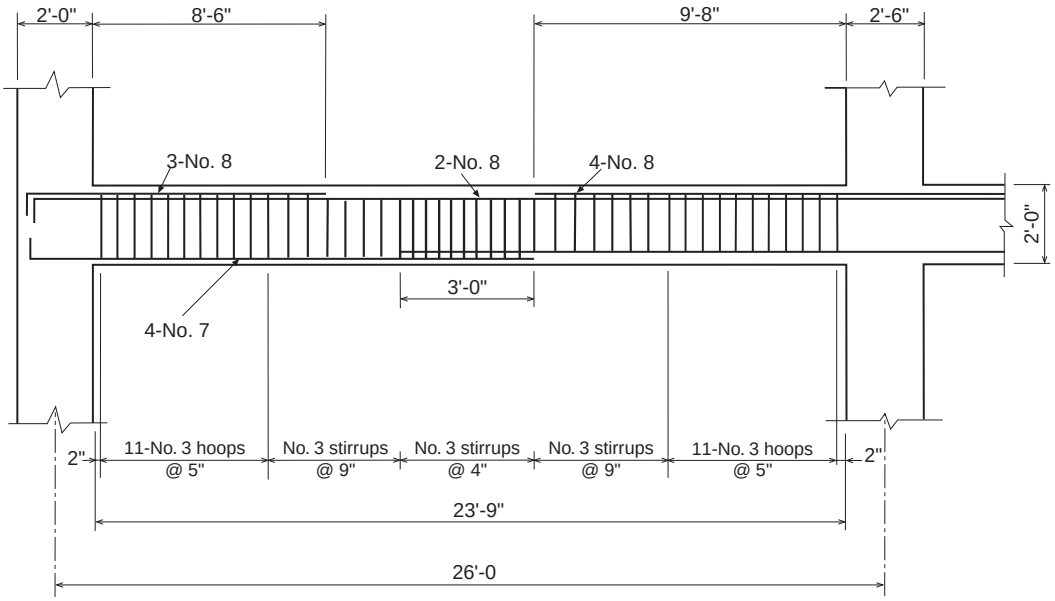


Figure 29-32 Reinforcement Details for Exterior Beam on Floor Level 1

Example 29.3—Proportioning and Detailing of Columns of Building in Example 29.1

Determine the required reinforcement for an edge column supporting the first floor of a typical E-W interior frame. The column dimensions have been established at 24-in. square. Use $f'_c = 4{,}000$ psi and $f_y = 60{,}000$ psi.

Calculations and Discussion	Code Reference

Table 29-7 contains a summary of the factored axial loads and bending moments for an edge column in the first floor level for seismic forces in the E-W direction.

From Table 29-7, maximum $P_u = 1{,}012$ kips

21.4.1

$P_u = 1{,}012 \text{ kips} > A_g f'_c/10 = (24 \times 24) \times 4/10 = 230 \text{ kips}$

Thus, the provisions of 21.4 governing special moment frame members subjected to bending and axial load apply.

Table 29-7 Summary of Factored Axial Loads and Bending Moments for an Edge Column in the First Floor for Seismic Forces in the E-W Direction

Load Combination	Axial Load, P_u (kips)	Bending Moment, M_u (ft-kips)
1.2D + 0.5L	1,002.9	-78.2
1.2D + 0.5L + E	722.8 1,012.0	166.4 -275.6
0.9D + E	459.8 749.0	188.1 -253.9

1. Check satisfaction of limitations on section dimensions.

   • Shortest cross-sectional dimension = 24 in. > 12 in. O.K. 21.4.1.1

   • Ratio of shortest cross-sectional dimension to perpendicular dimension = 1.0 > 0.4 O.K. 21.4.1.2

2. Determine required longitudinal reinforcement.

   Based on the load combinations in Table 29-7, a 24 × 24 in. column with 8-No. 8 bars ($\rho_g = 1.10\%$) is adequate for the column supporting the first floor level.

   Note that $0.01 < \rho_g \leq 0.06$ O.K. 21.4.3.1

3. Nominal flexural strength of columns relative to that of beams in E-W direction.

$$\Sigma M_c \text{ (columns)} \geq \frac{6}{5}\Sigma M_g \text{ (beams)} \qquad 21.4.2.2$$

The nominal flexural strength M_n of the beam framing into the column must include the slab reinforcement within an effective slab width equal to: 21.4.2.2

$(16 \times 8) + 20 = 148$ in.
$22 \times 12 = 264$ in. 8.10.2
$(26 \times 12)/4 = 78$ in. (governs)

The minimum required A_s in the 78-in. effective width is equal to $0.0018 \times 78 \times 8 = 1.12$ in.^{2}, which corresponds to 6-No. 4 bars @ 78/6 = 13 in. spacing. This spacing is less than the maximum bar spacing (= 2h = 16 in.). Provide No. 4 @ 13 in. at both the top and bottom of the slab (according to Fig. 13.3.8, 100 percent of both the top and bottom reinforcement in the column strip must be continuous or anchored at the support).

A strain compatibility analysis of the section yields M_n^- of the beam equal to 632 ft-kips.

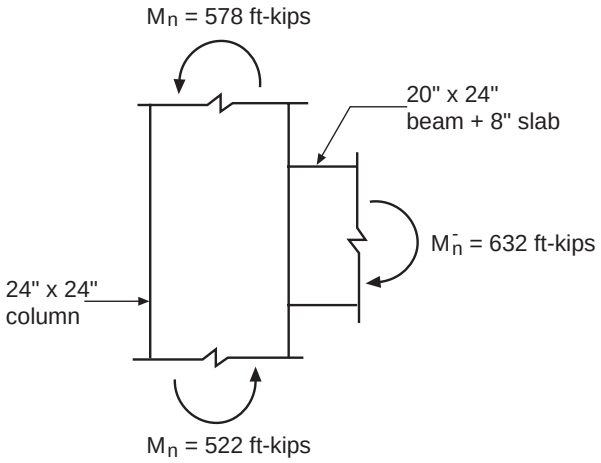
For the lower end of the upper column framing into the joint, the minimum nominal flexural strength is 578 ft-kips, which corresponds to P_u = 922 kips. Similarly, the minimum M_n is 522 ft-kips for the upper end of the lower column framing into the joint; this corresponds to P_u = 1,012 kips.

Therefore,

$\Sigma M_c = 578 + 522 = 1{,}100$ ft-kips

$\Sigma M_g = 632$ ft-kips

$1{,}100 \text{ ft-kips} > \frac{6}{5} \times 632 = 758$ ft-kips O.K.



4. Nominal flexural strength of columns relative to that of beams in the N-S direction.

The beams in the N-S direction framing into columns at the first floor level require 4-No. 7 bars at both the top and bottom of the section.

The nominal flexural strength M_n of the beams framing into the column must include the slab reinforcement within an effective slab width equal to:

$(22 \times 12)/12 + 20 = 42$ in. (governs)

$(6 \times 8) + 20 = 68$ in.

$(23.75 \times 12)/2 + 20 = 162.5$ in.

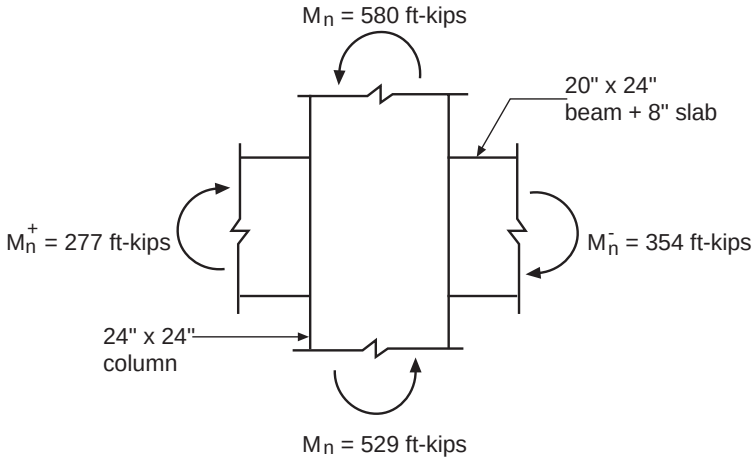
The minimum A_s in the 42-in. effective width is equal to $0.0018 \times 42 \times 8 = 0.6$ in.^{2}, which corresponds to 3-No. 4 bars @ 42/3 = 14 in. spacing. This spacing is less than the maximum bar spacing (= 2h = 16 in.). Provide No. 4 @ 14 in. at both the top and bottom of the slab (according to Fig. 13.3.8, 100 percent of both the top and bottom reinforcement in the column strip must be continuous or anchored at the support).

A strain compatibility analysis of the section yields $M_n^- = 354$ ft-kips and $M_n^+ = 277$ ft-kips.

For the lower end of the upper column framing into the joint, the minimum nominal flexural strength is 580.4 ft-kips, which corresponds to $P_u = 918$ kips. Similarly, the minimum M_n is 528.6 ft-kips for the upper end of the lower column framing into the joint; this corresponds to $P_u = 1{,}003$ kips.

$\Sigma M_g = 354 + 277 = 631$ ft-kips

$\Sigma M_c = 580 + 529 = 1{,}109$ ft-kips $> \frac{6}{5}\Sigma M_g = \frac{6}{5} \times 631 = 757$ ft-kips O.K. *21.4.4.4 Eq. (21-1)*



5. Determine transverse reinforcement requirements.

a. Confinement reinforcement (see Fig. 29-7(b)).

Transverse reinforcement for confinement is required over a distance ℓ_o from the column ends where

$$\ell_o \geq \begin{cases} \text{depth of member} = 24 \text{ in.} \\ 1/6 \text{ (clear height)} = (14 \times 12)/6 = 28 \text{ in. (governs)} \\ 18 \text{ in.} \end{cases}$$ *21.4.4.4*

Maximum allowable spacing of rectangular hoops assuming No. 4 hoops with a crosstie in both directions: *21.4.4.2*

s_{max} = 0.25 (smallest dimension of column) = 0.25 × 24 = 6 in.

= 6 (diameter of longitudinal bar) = 6 × 1.0 = 6 in.

$$= s_x = 4 + \left(\frac{14 - h_x}{3}\right) = 4 + \left(\frac{14 - 11}{3}\right) = 5 \text{ in.} \quad \begin{matrix} < 6 \text{ in.} \\ > 4 \text{ in.} \end{matrix} \quad \text{(governs)}$$

$$\text{where } h_x = \frac{24 - 2\left(1.5 + 0.5 + \frac{1.0}{2}\right)}{2} + 2\left(\frac{1.0}{2} + \frac{0.5}{2}\right) = 11 \text{ in.}$$

Required cross-sectional area of confinement reinforcement in the form of hoops:

$$A_{sh} \geq \begin{cases} 0.3 s h_c \left[\frac{A_g}{A_{ch}} - 1\right] \frac{f'_c}{f_{yh}} & \textit{Eq. (21-3)} \\ 0.09 s h_c \frac{f'_c}{f_{yh}} & \textit{Eq. (21-4)} \end{cases}$$

where

s = spacing of transverse reinforcement (in.)

h_c = cross-sectional dimension of column core, measured center-to-center of confining reinforcement (in.) = 24 - 2(1.5 + 0.25) = 20.5 in.

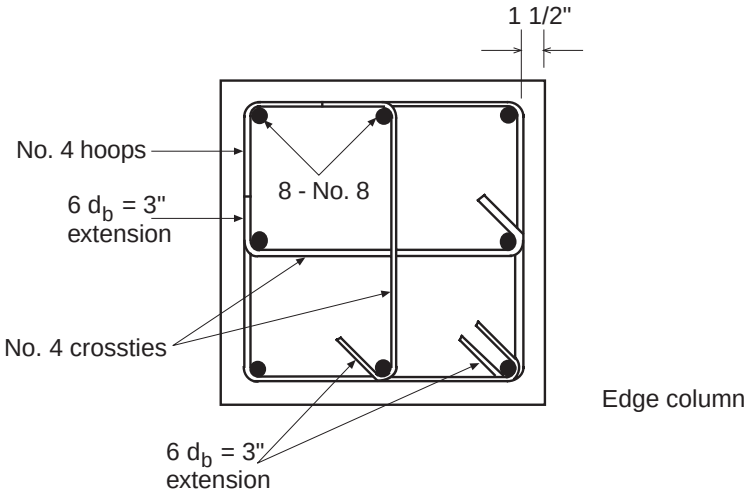
A_{ch} = core area of column section, measured outside to-outside of transverse reinforcement (in.^{2}) = [24 - (2 × 1.5)]^2 = 441 in.^{2}

f_{yh} = specified yield strength of transverse reinforcement (psi)

For a hoop spacing of 5 in. and f_{yh} = 60,000 psi, the required cross-sectional area is:

$$A_{sh} \geq \begin{cases} (0.3 \times 5 \times 20.5)\left(\frac{576}{441} - 1\right)\frac{4{,}000}{60{,}000} = 0.63 \text{ in.}^2 \quad (\text{governs}) \\ (0.09 \times 5 \times 20.5)\frac{4{,}000}{60{,}000} = 0.62 \text{ in.}^2 \end{cases}$$

No. 4 hoops with one crosstie, as shown in the sketch below, provides $A_{sh} = 3 \times 0.20 = 0.60$ in.$^2 < 0.63$ in.^{2} Reduce hoop spacing to 4.5 in. so that governing $A_{sh} = 0.57$ in.$^2 <$ provided $A_{sh} = 0.60$ in.^{2} *21.4.4.3*



b. Transverse reinforcement for shear.

As in the design of shear reinforcement for beams, the design shear for columns is based not on the factored shear forces obtained from a lateral load analysis but rather on the nominal flexural strengths provided in the columns. The column design shear forces shall be determined from the consideration of the maximum forces that can be developed at the faces of the joints, with the probable flexural strengths calculated for the factored axial compressive forces resulting in the largest moments acting at the joint faces. *21.4.5*

The largest probable flexural strength that may develop in the column can conservatively be assumed to correspond to the balanced point of the column interaction diagram.

With the strength reduction factors equal to 1.0 and $f_y = 1.25 \times 60 = 75$ ksi, the moment corresponding to balanced failure is 742 ft-kips. Thus, $V_u = (2 \times 742)/14 = 106$ kips.

The shear forces at the top of the column need not exceed those determined from joint strengths based on the probable flexural strength M_{pr} of the members framing into the joint. For seismic forces in the E-W direction, the negative probable flexural strength of the beam framing into the joint at the face of the edge column is 477.0 ft-kips (see Fig. 29-30). 21.4.5.1

Distribution of this moment to the columns is proportional to EI/ℓ of the columns above and below the joint. Since the columns above and below the joint have the same cross-section, reinforcement, and concrete strength, EI is a constant, and the moment is distributed according to 1/ℓ. Therefore, the moment at the top of the first story column is

$$477.0\left(\frac{12}{12 + 16}\right) = 204.4 \text{ ft-kips}$$

It is possible for the base of the first story column to develop the probable flexural strength of 742.0 ft-kips. Thus, the shear force is

$$V_u = \frac{204.4 + 742.0}{14} = 67.6 \text{ kips}$$

For seismic forces in the N-S direction, the negative probable flexural strength of the beam framing into one side of the column is 302.6 ft-kips (4-No. 7 top bars). The positive probableflexural strength of the beam framing into the other side of the column is also 302.6 ft-kips (4-No. 7 bottom bars). Therefore, at the top of the first story column, the moment is

$$(2 \times 302.6)\left(\frac{12}{12 + 16}\right) = 259.4 \text{ ft-kips}$$

The shear force is

$$V_u = \frac{259.4 + 742.0}{14} = 71.5 \text{ kips}$$

Both of these shear forces are greater than those obtained from analysis.

Since the factored axial forces are greater than $A_g f'_c / 20 = 115$ kips, the shear strength of the concrete may be used: 21.4.5.2

$$V_c = 2\sqrt{f'_c}bd\left(1 + \frac{N_u}{2{,}000A_g}\right)$$ Eq. (11-4)

Conservatively using the minimum axial load from Table 29-7,

$$V_c = \frac{2\sqrt{4{,}000}\,(24 \times 17.7)}{1{,}000}\left[1 + \frac{459{,}800}{2{,}000 \times (24)^2}\right] = 75.2 \text{ kips}$$

$$V_s = \frac{A_v f_y d}{s} = \frac{(3 \times 0.20) \times 60 \times 17.7}{4.5} = 141.6 \text{ kips}$$

$$\phi(V_c + V_s) = 0.75(75.2 + 141.6) = 162.6 \text{ kips} > V_u = 71.5 \text{ kips} \quad \text{O.K.}$$

Thus, the transverse reinforcement spacing over the distance $\ell_o = 28$ in. near the column ends required for confinement is also adequate for shear.

The remainder of the column length must contain hoop reinforcement with center-to-center spacing not to exceed either six times the diameter of the column longitudinal bars (= 6 × 1.0 = 6.0 in.) or 6 in. *21.4.4.6*

Use No. 4 hoops and crossties spaced at 4.5 in. within a distance of 28 in. from the column ends and No. 4 hoops spaced at 6 in. or less over the remainder of the column.

6. Minimum length of lap splices of column vertical bars.

The location of lap splices of column bars must be within the center half of the member length. Also, the splices are to be designed as Class B tension splices. Transverse reinforcement at 4.5 in. is to be provided over the full lap splice length. *21.4.3.2*

Required length of Class B splice $= 1.3\ell_d$ *12.15.1*

where

$$\ell_d = \left(\frac{3}{40}\,\frac{f_y}{\sqrt{f'_c}}\,\frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b}\right)}\right) d_b \qquad \textit{Eq. (12-1)}$$

reinforcement location factor $\alpha = 1.0$ (other than top bars) *12.2.4*

coating factor $\beta = 1.0$ (uncoated bars)

reinforcement size factor $\gamma = 1.0$ (No. 7 and larger bars)

lightweight aggregate concrete factor $\lambda = 1.0$ (normal weight concrete)

$$c = 1.5 + 0.5 + \frac{1.0}{2} = 2.5 \text{ in.} \quad \text{(governs)}$$

$$= \frac{1}{2}\left[\frac{24 - 2(1.5 + 0.5) - 1.0}{2}\right] = 4.75 \text{ in.}$$

$$K_{tr} = \frac{A_{tr} f_{yt}}{1{,}500sn} = \frac{(3 \times 0.20)(60{,}000)}{1{,}500 \times 4.5 \times 3} = 1.8$$

where A_{tr} is for 3-No. 4 bars, s (the maximum spacing of transverse reinforcement within ℓ_d) = 4.5 in., and n (number of bars being developed) = 3

$$\frac{c + K_{tr}}{d_b} = \frac{2.5 + 1.8}{1.0} = 4.3 > 2.5, \text{ use } 2.5$$

Therefore,

$$\ell_d = \frac{3}{40} \times \frac{60{,}000}{\sqrt{4{,}000}} \times \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.5} \times 1.0 = 28.5 \text{ in.}$$

Class B splice length = 1.3 × 28.5 = 37.1 in.

Use a 3 ft-2 in. splice length.

7. Reinforcement details for the column are shown in Fig. 29-33. Note that for practical purposes, a 4.5-in. hoop spacing is used over the entire length of the column.

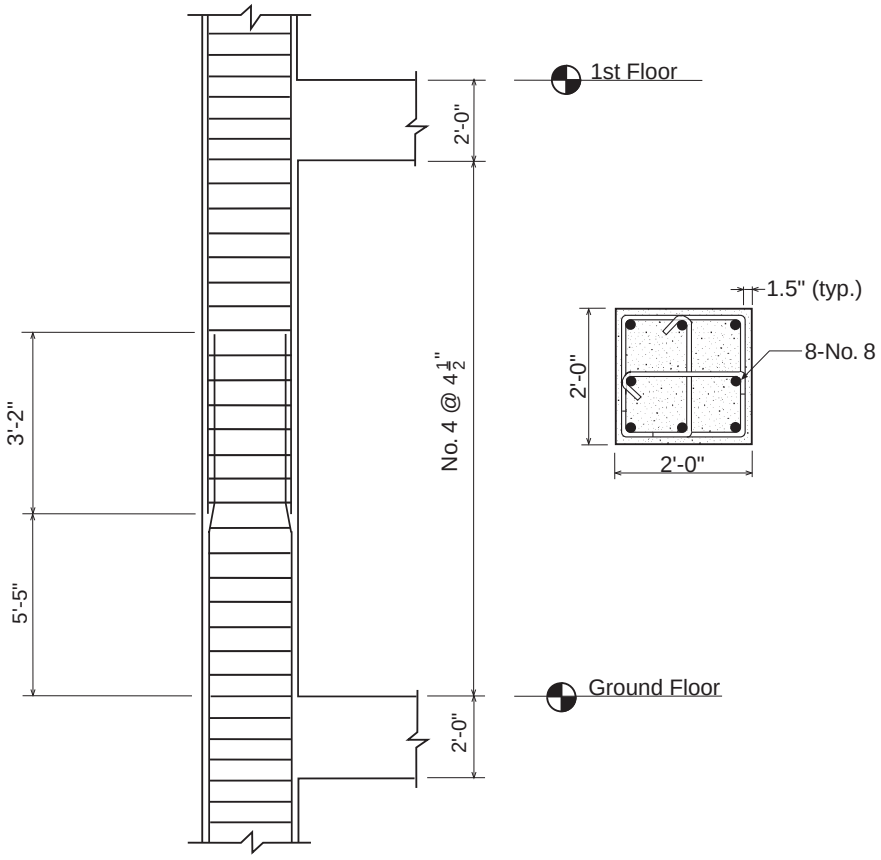


Figure 29-33 Reinforcement Details for Edge Column Supporting Level 1

Example 29.4—Proportioning and Detailing of Exterior Beam-Column Connection of Building in Example 29.1

Determine the transverse reinforcement and shear strength requirements for an exterior beam-column connection between the beam considered in Example 29.2 and the column of Example 29.3. Assume the joint to be located at the first floor level.

Calculations and Discussion	Code Reference

1. Transverse reinforcement for confinement.

   Section 21.5.2.1 requires the same amount of confinement reinforcement within the joint as for the length ℓ_o at column ends, unless the joint is confined by beams framing into all vertical faces of the column. A member that frames into a face is considered to provide confinement if at least three-quarters of the face of the joint is covered by the framing member.

   In the case of the beam-column joint considered here, beams frame into only three sides of the column. In Example 29.3, confinement requirements at column ends were satisfied by No. 4 hoops with crossties spaced at 4.5 in.

2. Check shear strength of joint in E-W direction.

   The shear force across section x-x (see Fig. 29-34) of the joint is obtained as the difference between the tensile force from the top flexural reinforcement of the framing beam (stressed to $1.25f_y$) and the horizontal shear from the column above. *21.5.1.1*

   $$T = A_s\,(1.25f_y) = (5 \times 0.79)\,(1.25 \times 60) = 296 \text{ kips}$$

   An estimate of the horizontal shear from the column, V_h, can be obtained by assuming that the beams in the adjoining floors are also deformed so that plastic hinges form at their junctions with the column, with M_{pr} (beam) = 477.0 ft-kips (see Fig. 29-30). By further assuming that the end moments in the beams are resisted by the columns above and below the joint inversely proportional to the column lengths, the average horizontal shear in the column is approximately:

   $$V_h = \frac{2 \times 477.0}{12 + 16} = 34.1 \text{ kips}$$

   Thus, the net shear at section x-x of the joint is $V_u = 296 - 34.1 = 261.9$ kips. Section 21.5.3.1 gives the nominal shear strength of a joint as a function of the area of the joint cross-section, A_j, and the degree of confinement by framing beams. For the joint confined on three faces considered here (note: beam width = 20 in. > 0.75 (column width) = 0.75 × 24 = 18 in.) :

   $$\phi V_c = \phi 15 \sqrt{f'_c}\, A_j$$ *21.5.3.1* *9.3.4*

   $$= 0.85 \times 15 \sqrt{4{,}000} \times 24^2 / 1{,}000 = 464.5 \text{ kips} > 261.9 \text{ kips} \quad \text{O.K.}$$

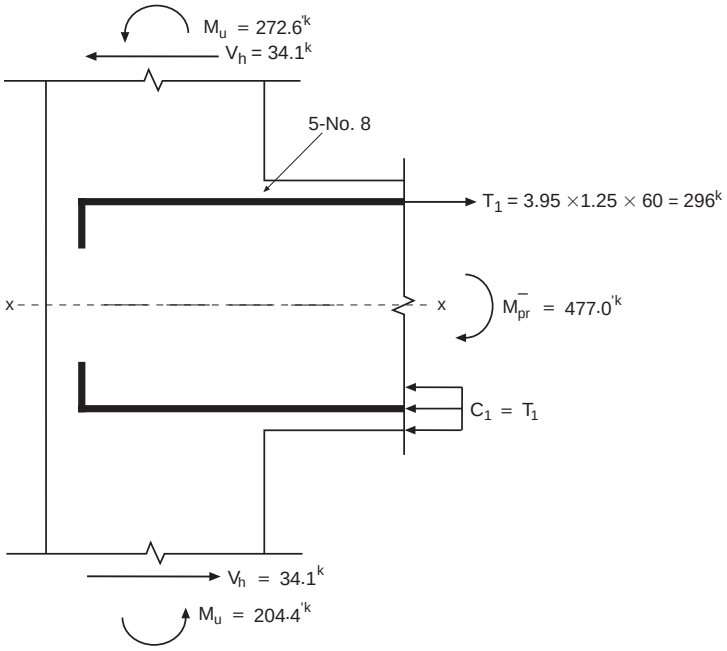


Figure 29-34 Shear Analysis of Exterior Beam-Column Joint in E-W Direction

3. Check shear strength of joint in N-S direction.

The shear force across section x-x (see Fig. 29-35) of the joint is determined as follows:

$T_1 = A_s(1.25f_y) = (4 \times 0.60)(1.25 \times 60) = 180$ kips

The negative and positive probable flexural strengths at the joint are 302.6 ft-kips (4-No. 7 bars top and bottom).

The average horizontal shear in the column is:

$$V_u = \frac{2(302.6 + 302.6)}{12 + 16} = 43.2 \text{ kips}$$

Thus, the net shear at section x-x of the joint is

$V_u = T_1 + C_2 - V_u = 180 + 180 - 43.2 = 316.8$ kips $< \phi V_c = 464.5$ kips O.K.

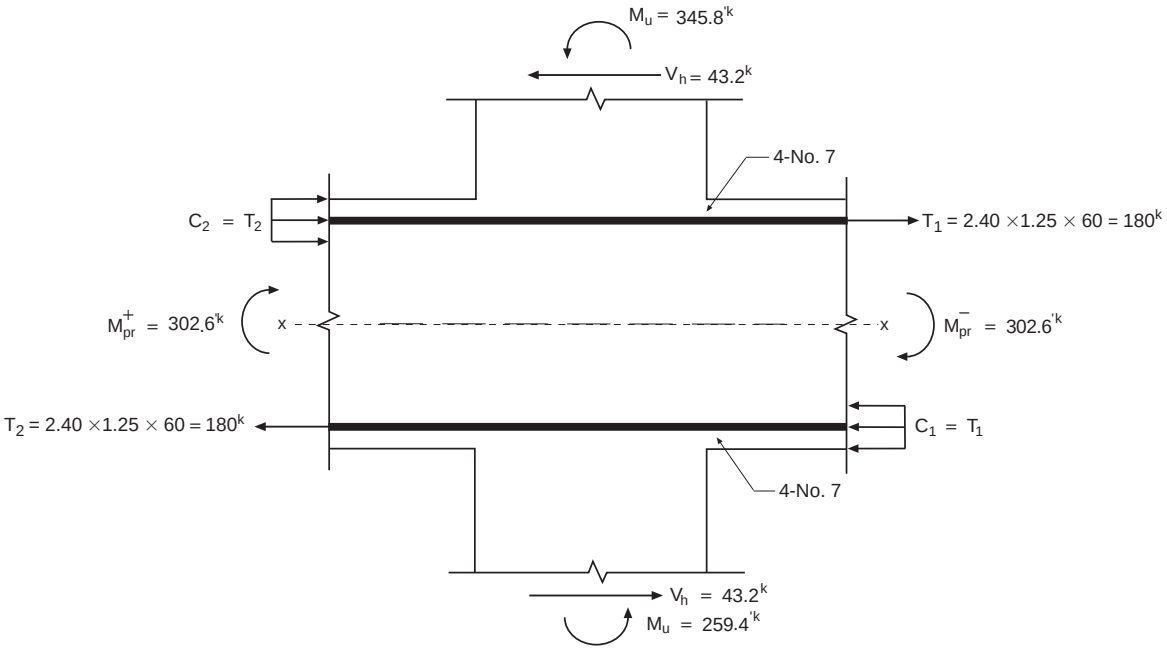


Figure 29-35 Shear Analysis of Exterior Beam-Column Joint in N-S Direction

Note that if the shear strength of the concrete in the joint as calculated above were inadequate, any adjustment would have to take the form of either an increase in the column cross-section (and hence A_j) or an increase in the beam depth (to reduce the amount of flexural reinforcement required and hence the tensile force T) since transverse reinforcement is considered not to have a significant effect on shear strength.

4. Reinforcement details for the exterior joint are shown in Fig. 29-36.

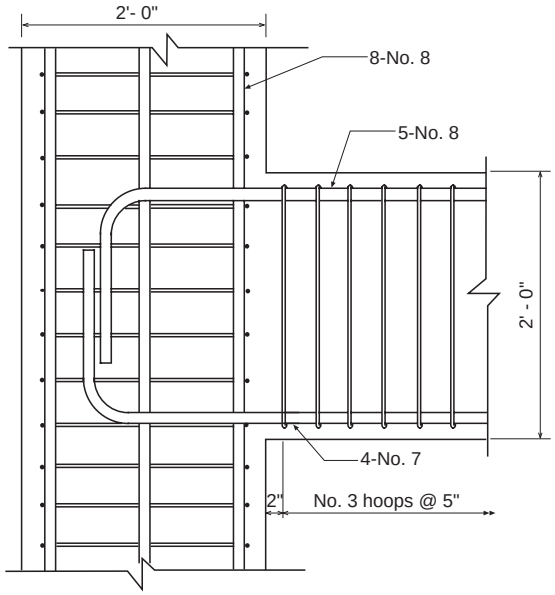


Figure 29-36 Reinforcement Details of Exterior Joint

Example 29.5—Proportioning and Detailing of Interior Beam-Column Connection of Building in Example 29.1

Determine the transverse reinforcement and shear strength requirements for the interior beam-column connection at the first floor of the interior E-W frame considered in the previous examples. The column is 30-in. square and is reinforced with 12-No. 8 bars. The beams have dimensions of b = 20 in. and d = 21.5 in. and are reinforced as noted in Example 29.2 (see Fig. 29-32).

Calculations and Discussion	Code Reference

1. Determine transverse reinforcement requirements.

   a. Confinement reinforcement

   Maximum allowable spacing of rectangular hoops assuming No. 4 hoops with two crossties in both directions:

   s_{max} = 0.25 (least dimension of column) = 0.25 × 30 = 7.5 in. — *21.4.4.2*

   = 6 (diameter of longitudinal bar) = 6 × 1.00 = 6 in.

   $$= s_x = 4 + \left(\frac{14 - h_x}{3}\right) = 4 + \left(\frac{14 - 9.83}{3}\right) = 5.4 \text{ in.} < 6 \text{ in.} \quad \text{(governs)}$$

   $$> 4 \text{ in.}$$

   $$\text{where } h_x = \frac{30 - 2\left(1.5 + 0.5 + \frac{1.00}{2}\right)}{3} + 2\left(\frac{1.00}{2} + \frac{1}{4}\right) = 9.83 \text{ in.}$$

   With a hoop spacing of 5 in., the required cross-sectional area of confinement reinforcement in the form of hoops is:

   $$A_{sh} \geq \begin{cases} 0.3sh_c\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f'_c}{f_{yh}} = (0.3 \times 5 \times 26.5)\left(\frac{900}{729} - 1\right)\frac{4{,}000}{60{,}000} = 0.62 \text{ in.}^2 & \textit{Eq. (21-3)} \\ 0.09sh_c\frac{f'_c}{f_{yh}} = (0.09 \times 5 \times 26.5)\frac{4{,}000}{60{,}000} = 0.80 \text{ in.}^2 \quad \text{(governs)} & \textit{Eq. (21-4)} \end{cases}$$

   Since the joint is framed by beams having widths = 20 in. < 3/4 width of column = 22.5 in. on all four sides, it is not considered confined and a 50% reduction in the amount of confinement reinforcement indicated above is not allowed. — *21.5.2.2*

   No. 4 hoops spaced at 5 in. on center provide A_{sh} = 0.20 × 4 = 0.80 in.^{2}

   b. Transverse reinforcement for shear

   Following the same procedure in Example 29.4, the shear forces in the column are obtained for seismic forces in the E-W and N-S directions.

The largest probable flexural strength that may develop in the column can conservatively be assumed to correspond to the balanced point of the column interaction diagram.

With the strength reduction factor equal to 1.0 and $f_y = 1.25 \times 60 = 75$ psi, the moment corresponding to balanced failure is 1,438 ft-kips. Thus, $V_u = (2 \times 1{,}438)/14 = 205.4$ kips.

The shear forces at the top of the column need not exceed those determined from joint strengths based on the M_{pr} of the beams framing into the joint. *21.4.5.1*

For seismic forces in the E-W direction, M_{pr}^{-} of the beam framing into the joint at the face of the interior column is 477.0 ft-kips (5-No. 8 top bars). The M_{pr}^{+} is 302.6 ft-kips (4-No. 7 bottom bars) based on the beam framing into the other face of the joint. Distributing the moment to the columns in proportion to $1/\ell$, the moment at the top of the first story column is:

$$(477.0 + 302.6)\left(\frac{12}{12 + 16}\right) = 334.1 \text{ ft-kips}$$

It is possible for the base of the first story column to develop M_{pr} of the column. Thus, the shear force is:

$$V_u = \frac{334.1 + 1{,}438}{14} = 126.6 \text{ kips}$$

For seismic forces in the N-S direction, M_{pr}^{-} of the beam is 302.6 ft-kips (4-No. 7 top bars) and M_{pr}^{+} is 302.6 ft-kips (4-No. 7 bottom bars). Therefore, at the top of the first story column, the moment is:

$$(2 \times 302.6)\left(\frac{12}{12 + 16}\right) = 259.4 \text{ ft-kips}$$

The shear force is:

$$V_u = \frac{259.4 + 1{,}438}{14} = 121.2 \text{ kips}$$

Both of these shear forces are greater than those obtained from analysis.

Since the factored axial forces are greater than $A_g f'_c/20 = 180$ kips, the shear strength of the concrete may be used: *21.4.5.2*

$$V_c = 2\sqrt{f'_c}bd\left(1 + \frac{N_u}{2{,}000\,A_g}\right) \qquad \textit{Eq.(11-4)1}$$

Conservatively using the minimum axial force on the column:

$$V_c = \frac{2\sqrt{4{,}000}\,(30 \times 27.5)}{1{,}000}\left[1 + \frac{1{,}192{,}700}{2{,}000 \times (30)^2}\right] = 173.5 \text{ kips}$$

$$\phi V_c = 0.75 \times 173.5 = 130.1 \text{ kips} > V_u = 126.6 \text{ kips} \quad \text{O.K.}$$

Thus, the transverse reinforcement spacing over the distance $\ell_o = 28$ in. near the column ends required for confinement is also adequate for shear.

Use No. 4 hoops and crossties spaced at 5 in. at the ends of the column.

2. Check shear strength of joint in E-W direction

Following the same procedure as used in Example 29.4, the forces affecting the horizontal shear across a section near mid-depth of the joint shown in Fig. 29-37 are obtained.

Net shear force across section x-x $= T_1 + C_2 - V_h = 296.3 + 180.0 - 55.7 = 420.6 \text{ kips} = V_u$

Shear strength of joint, noting that the joint is not confined on all faces is (i.e., beam width = 20 in. < 0.75 (column width) = 0.75 × 30 = 22.5 in.):

$$\phi V_c = \phi 12\sqrt{f'_c}\,A_j$$

$$= 0.85 \times 12\sqrt{4{,}000} \times 30^2 / 1{,}000 = 580.6 \text{ kips} > 420.6 \text{ kips} \quad \text{O.K.} \qquad \textit{21.5.3.1}$$

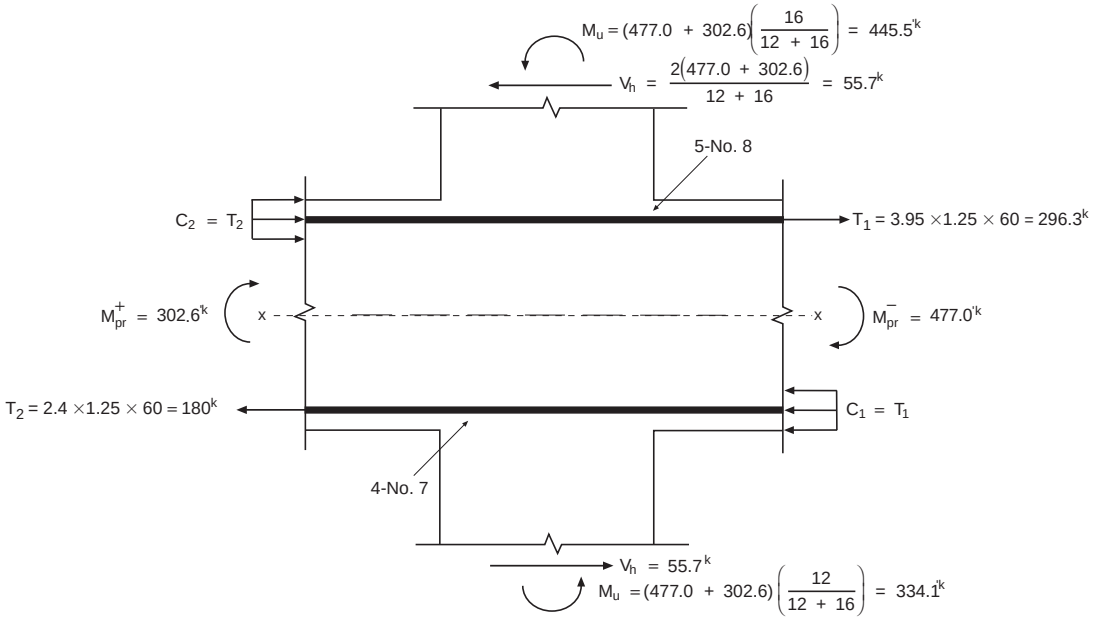


Figure 29-37 Shear Analysis of Interior Beam-Column Joint in E-W Direction

3. Check shear strength of joint in N-S direction

At both the top and bottom of the beam, 4-No. 7 bars are required (M_{pr} = 302.6 ft-kips).

Net shear force across section x-x = $T_1 + C_2 - V_u = 180 + 180 - 43.2 = 316.8$ kips = V_u

where $T_1 = C_2 = 2.4 \times 1.25 \times 60 = 180$ kips

$V_u = 2(302.6 + 302.6)/(12 + 16) = 43.2$ kips

$\phi V_c = \phi 12\sqrt{f'_c} A_j = 580.6 \text{ kips} > V_u = 316.8 \text{ kips}$ O.K.

Example 29.6—Proportioning and Detailing of Structural Wall of Building in Example 29.1

Design the wall section at the first floor level of the building in Example 29.1. At the base of the wall, M_u = 49,142 ft-kips and V_u = 812 kips.

Calculations and Discussion	Code Reference

1. Determine minimum longitudinal and transverse reinforcement requirements in the wall.

   a. Check if two curtains of reinforcement are required.

   Two curtains of reinforcement shall be provided in a wall if the in-plane factored shear force assigned to the wall exceeds $2A_{cv}\sqrt{f'_c}$, where A_{cv} is the cross-sectional area bounded by the web thickness and the length of section in the direction of the shear force considered. — *21.7.2.2*

   $2A_{cv}\sqrt{f'_c} = 2 \times 18 \times 24.5 \times 12 \times \sqrt{4{,}000}/1{,}000 = 669$ kips $< V_u = 812$ kips

   Therefore, two curtains of reinforcement are required.

   Note that $V_u = 812$ kips < upper limit on shear strength $= \phi 8A_{cv}\sqrt{f'_c} = 2{,}008$ kips — *21.7.4.4*

   b. Required longitudinal and transverse reinforcement in wall.

   Minimum distributed web reinforcement ratios = 0.0025 with max. spacing = 18 in. — *21.7.2.1*

   With A_{cv} (per foot of wall) = 18 × 12 = 216 in.^{2}, minimum required area of reinforcement in each direction per foot of wall = 0.0025 × 216 = 0.54 in.^{2}/ft

   Assuming No. 5 bars in two curtains ($A_s = 2 \times 0.31 = 0.62$ in.^{2}), required spacing is

   $$s = \frac{0.62}{0.54} \times 12 = 13.8 \text{ in.} < 18 \text{ in.}$$

3. Determine reinforcement requirements for shear. — *21.7.4*

   Assume two curtains of No. 5 bars spaced at 12 in. on center. Shear strength of wall:

   $$\phi V_n = \phi A_{cv}\left(\alpha_c \sqrt{f'_c} + \rho_n f_y\right)$$ — *Eq. (21-7)*

   where $\phi = 0.75$ and $\alpha_c = 2.0$ for $h_w/\ell_w = 148/24.5 = 6 > 2$

   $A_{cv} = 18 \times 24.5 \times 12 = 5{,}292$ in.^{2}

   $$\rho_n = \frac{0.62}{18 \times 12} = 0.0029$$

   $\phi V_n = (0.75 \times 5{,}292)\,[2\sqrt{4{,}000} + (0.0029 \times 60{,}000)] / 1{,}000 = 1{,}193$ kips > 812 kips O.K.

Therefore, use two curtains of No. 5 bars spaced at 12 in. on center in the horizontal direction.

The reinforcement ratio ρ_v shall not be less than the ratio ρ_n when the ratio h_w/ℓ_w is less than 2.0. Since h_w/ℓ_w is equal to 6.0, the minimum reinforcement ratio will be used. *21.7.4.3*

Use 2 curtains of No. 5 bars spaced at 12 in. on center in the vertical direction.

3. Determine reinforcement requirements for combined flexural and axial loads.

Structural walls subjected to combined flexural and axial loads shall be designed in accordance with 10.2 and 10.3 except that 10.3.6 and the nonlinear strain requirements of 10.2.2 do not apply. *21.7.5.1*

Assume that each 30 × 30 in. column at the end of the wall is reinforced with 24-No. 11 bars. It was determined above that 2-No. 5 bars at a spacing of 12 in. are required as vertical reinforcement in the web. With this reinforcement, the wall is adequate to carry the factored load combinations per 9.2.

4. Determine if special boundary elements are required.

The need for special boundary elements at the edges of structural walls shall be evaluated in accordance with 21.7.6.2 or 21.7.6.3. The provisions of 21.7.6.2 are used in this example. *21.7.6.1*

Compression zones shall be reinforced with special boundary elements where

$$c \geq \frac{\ell_w}{600(\delta_u / h_w)}, \quad \delta_u / h_w \geq 0.007$$ *Eq. (21-8)*

In this case, $\ell_w = 24.5$ ft = 294 in., $h_w = 148$ ft = 1,776 in., $\delta_u = 13.5$ in. and $\delta_u / h_w = 0.0076 > 0.007$. Therefore, special boundary elements are required if c is greater than or equal to 294/(600 × 0.0076) = 64.5 in.

The distance c to be used in Eq. (21-8) is the largest neutral axis depth calculated for the factored axial force and nominal moment strength consistent with the design displacement δ_u. From a strain compatibility analysis, the largest c is equal to 68.1 in. corresponding to an axial load of 3,649 kips and nominal moment strength of 97,302 ft-kips, which is greater than 64.5 in. Thus, special boundary elements are required.

The special boundary element shall extend horizontally from the extreme compression fiber a distance not less than $c - 0.1\ell_w = 68.1 - (0.1 \times 294) = 38.7$ in. (governs) or c/2 = 68.1/2 = 34.1 in. Considering the placement of the vertical bars in the web, confine 45 in. at both ends of the wall. *21.7.6.4(a)*

5. Determine special boundary element transverse reinforcement.

Transverse reinforcement shall satisfy the requirements of 21.4.4.1 through 21.4.4.3 except Eq. (21-3) need not be satisfied. *21.7.6.4(c)*

- Confinement of 30 × 30 in. boundary elements

  Maximum allowable spacing of rectangular hoops assuming No. 4 hoops and crossties around every longitudinal bar in both directions of the 30 × 30 in. boundary elements:

  s_{max} = 0.25 (minimum member dimension) = 0.25 × 30 = 7.5 in.
  = 6 (diameter of longitudinal bar) = 6 × 1.41 = 8.5 in.

  $$= s_x = 4 + \left(\frac{14 - h_x}{3}\right) = 4 + \left(\frac{14 - 6.0}{3}\right) = 6.7 \text{ in.} \geqslant 6.0 \text{ in.; use 6 in. (governs)}$$

  where h_x = maximum horizontal spacing of hoop or crosstie legs on all faces of the 30 × 30 in. boundary element.

  Required cross-sectional area of transverse reinforcement in the 30 × 30 in. boundary elements, assuming s = 6.0 in.:

  $$A_{sh} = \frac{0.09 s h_c f'_c}{f_y} = \frac{0.09 \times 6.0 \times \left[30 - (2 \times 1.5) - 0.5\right] \times 4}{60} = 0.95 \text{ in.}^2$$

  No. 4 hoops with crossties around every longitudinal bar in the 30 × 30 in. boundary elements provide $A_{sh} = 7 \times 0.2 = 1.40 \text{ in.}^2 > 0.95 \text{ in.}^2$

- Confinement of web

  Maximum allowable spacing of No. 5 transverse reinforcement:

  s_{max} = 0.25 (minimum member dimension) = 0.25 × (45 - 30) = 3.75 in. (governs)
  = 6 (diameter of longitudinal bar) = 6 × 0.625 = 3.75 in.

  $$= s_x = 4 + \left(\frac{14 - 13.25}{3}\right) = 4.25 \text{ in.}$$

  For confinement in the direction parallel to the wall, assuming s = 3.0 in.:

  h_c = 18 - (2 × 1.5) - 0.625 = 14.375 in.

  $$A_{sh} = \frac{0.09 \times 3.0 \times 14.375 \times 4}{60} = 0.26 \text{ in.}^2$$

  Using 2-No. 5 horizontal bars, $A_{sh} = 2 \times 0.31 = 0.62 \text{ in.}^2 > 0.26 \text{ in.}^2$

  For confinement in the direction perpendicular to the wall:

  h_c = 45 - 30 = 15 in.

  $$A_{sh} = \frac{0.09 \times 3.0 \times 15 \times 4}{60} = 0.27 \text{ in.}^2$$

  With a No. 5 hoop and crosstie, $A_{sh} = 2 \times 0.31 = 0.62 \text{ in.}^2 > 0.27 \text{ in.}^2$

The transverse reinforcement of the boundary element shall extend vertically a distance of ℓ_w = 24.5 ft (governs) or $M_u/4V_u$ = 49,142/(4 × 812) = 15.1 ft from the critical section. *21.7.6.2(b)*

6. Determine required development and splice lengths.

All continuous reinforcement in shearwalls shall be anchored or spliced in accordance with the provisions for reinforcement in tension as given in 21.5.4. *21.7.2.3*

a. Lap splice for No. 11 vertical bars in boundary elements.* In lieu of 21.5.4, the provisions of Chapter 12 are utilized since in this case, Chapter 12 yields longer splice lengths (conservative).

Class B splices are designed for the No. 11 vertical bars.

Required length of Class B splice = $1.3\ell_d$ *12.15.1*

where

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b} \right)} \right) d_b$$ *Eq. (12-1)*

reinforcement location factor α = 1.0 (other than top bars) *12.2.4*

coating factor β = 1.0 (uncoated bars)

reinforcement size factor γ = 1.0 (No. 7 and larger bars)

lightweight aggregate concrete factor λ = 1.0 (normal weight concrete)

Assume no more than 50% of the bars spliced at any one location.

$$c = 1.5 + 0.5 + \frac{1.41}{2} = 2.7 \text{ in.} \quad \text{(governs)}$$

$$= \frac{1}{2}\left[\frac{30 - 2(1.5 + 0.5) - 1.41}{3} \right] = 4.1 \text{ in.}$$

$$K_{tr} = \frac{A_{tr} f_{yt}}{1{,}500sn} = \frac{(4 \times 0.20)(60{,}000)}{1{,}500 \times 6.0 \times 4} = 1.3$$

where A_{tr} is for 4-No. 5 bars, s = 6.0 in., and n (number of bars being developed) = 4 in one layer at one location.

$$\frac{(c + K_{tr})}{d_b} = \frac{(2.7 + 1.3)}{1.41} = 2.8 > 2.5, \text{ use } 2.5$$

* The use of mechanical connectors may be considered as an alternative to lap splices for these large bars.

Therefore,

$$\ell_d = \frac{3}{40} \times \frac{60{,}000}{\sqrt{4{,}000}} \times \frac{1.0}{2.5} \times 1.41 = 40.1 \text{ in.}$$

Class B splice length = $1.3 \times 40.1 = 52.2$ in.

Use a 4 ft-6 in. splice length.

Note that the splices must be staggered at least 24 in. *12.15.4.1*

b. Lap splice for No. 5 vertical bars in wall web. The provisions of Chapter 12 are utilized instead of 21.5.4, since 21.5.4 assumes the bars are confined, which they are not in the wall web.

Again assuming no more than 50% of bars spliced at any one location, the length of the Class B splice is determined as follows:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b} \right)} \right) d_b \qquad \textit{Eq. (12-1)}$$

reinforcement location factor α = 1.0 (other than top bars) *12.2.4*

coating factor β = 1.0 (uncoated bars)

reinforcement size factor γ = 0.8 (No. 6 and smaller bars)

lightweight aggregate concrete factor λ = 1.0 (normal weight concrete)

$$c = 0.75 + 0.625 + \frac{0.625}{2} = 1.7 \text{ in.} \quad \text{(governs)}$$

$$= \frac{1}{2} \times 12 = 6 \text{ in.}$$

$$K_{tr} = 0$$

$$\frac{(c + K_{tr})}{d_b} = \frac{1.7}{0.625} = 2.7 > 2.5, \text{ use } 2.5$$

Therefore,

$$\ell_d = \frac{3}{40} \times \frac{60{,}000}{\sqrt{4{,}000}} \times \frac{0.8}{2.5} \times 0.625 = 14.2 \text{ in.}$$

Class B splice length = 1.3 × 14.2 = 18.5 in.

Use a 1 ft-8 in. splice length.

c. Development length for No. 5 horizontal bars in wall assuming no hooks are used within boundary element. *21.5.4*

Since it is reasonable to assume that the depth of concrete cast in one lift beneath a horizontal bar will be greater than 12 in., the required development length shall not be less than 3.5 times the length required by 21.5.4.1:

$$\ell_{dh} = \frac{f_y d_b}{65\sqrt{f'_c}} = \frac{60{,}000 \times 0.625}{65\sqrt{4{,}000}} = 9.1 \text{ in.} > 8d_b = 5 \text{ in.} > 6 \text{ in.}$$ *Eq. (21-6)*

Thus, required development length ℓ_d = 3.5 × 9.1 = 32 in.

This length cannot be accommodated within the confined core of the boundary element, thus hooks are needed.

Anchor horizontal bars to longitudinal reinforcement in boundary element. *21.7.6.4 (e)*

No lap splices would be required for the No. 5 horizontal bars (full length bars weigh approximately 25 lbs. and are easily installed).

7. Reinforcement details for structural wall are shown in Fig. 29-38.

Note that the No. 5 bars at 3 in. that are required for confinement in the direction parallel to the web are developed into the boundary element and into the web beyond the face of the 2'–6" boundary element (see Fig. 29-38(b)).

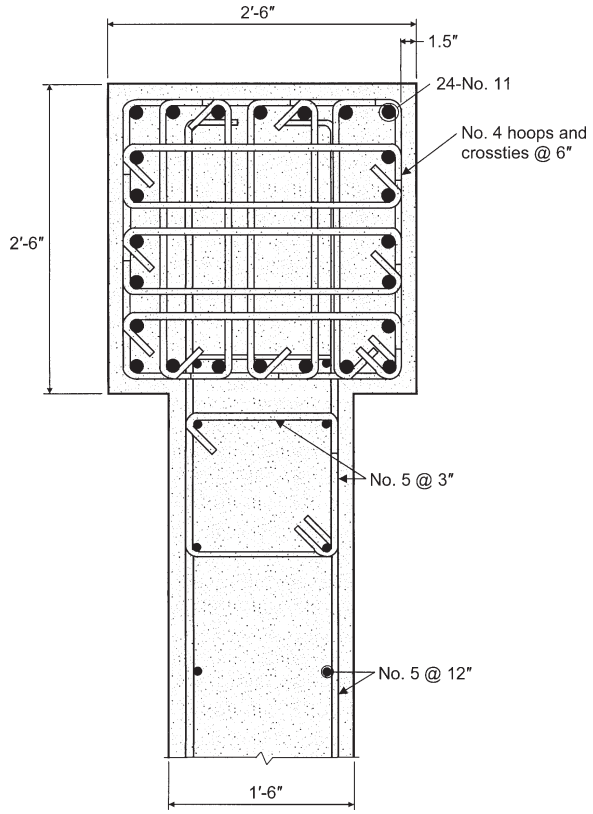


Figure 29-38(a) Reinforcement Details for the Structural Wall

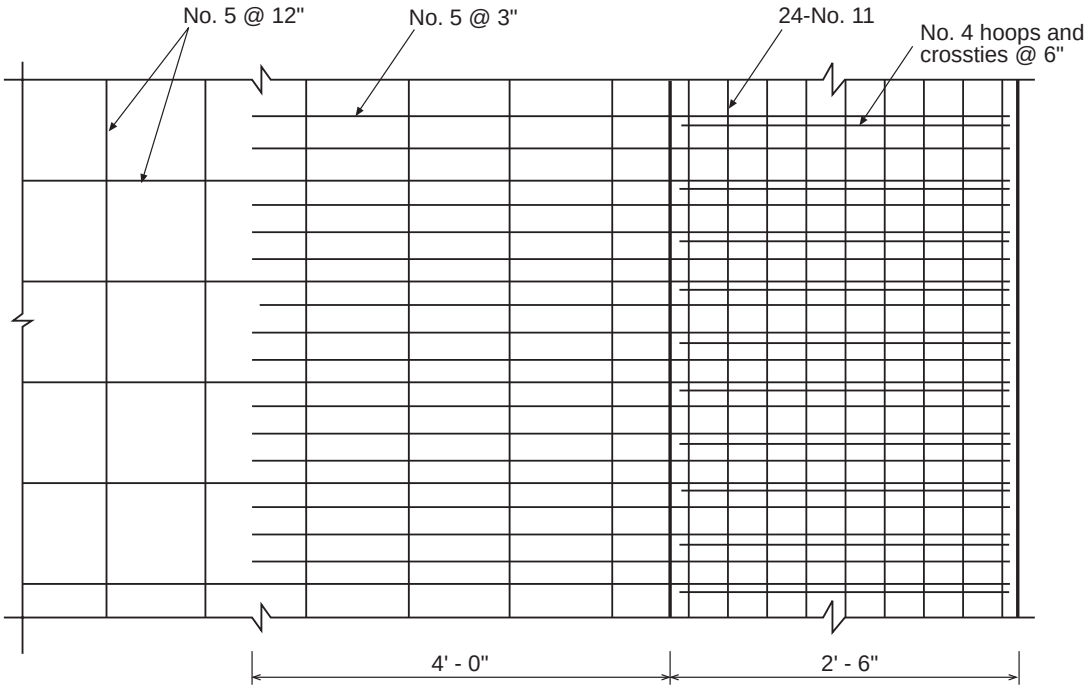


Figure 29-38(b) Reinforcement Details for the Structural Wall

Example 29.7—Design of 12-Story Precast Frame Building using Strong Connections*

This example illustrates the design and detailing requirements for typical beam-to-beam, column-to-column, and beam-to-column connections for the precast building shown in Fig. 29-39. In particular, details are developed for: (1) a strong connection near midspan of an interior beam that is part of an interior frame on the third floor level, (2) a column-to-column connection at mid-height between levels 2 and 3 of an interior column stack that is part of an interior frame, and (3) a strong connection at the interface between an exterior beam at the second floor level of an exterior frame and the continuous corner column to which it is connected. Pertinent design data are as follows:

Material Properties:

Concrete (w_c = 150 pcf): f'_c = 6,000 psi for columns in the bottom six stories
= 4,000 psi elsewhere

Reinforcement: f_y = 60,000 psi

Service Loads:

Live load = 50 psf
Superimposed dead load = 42.5 psf

Member Dimensions:

Beams in N-S direction: 24 × 26 in.
Beams in E-W direction: 24 × 20 in.
Columns: 24 × 24 in.
Slab: 7 in.

Calculations and Discussion	Code Reference

1. Seismic design forces

   The computation of the seismic design forces is beyond the scope of this example. Traditional analysis methods can be used for precast frames, although care should be taken to approximate the component stiffness in a way that is appropriate for the precast components being used. For emulation design (as illustrated in this example), it is reasonable to model the beams and columns as if they were monolithic concrete.

2. Strong connection near beam midspan

   a. Required flexural reinforcement

      Special moment frames with strong connections constructed using precast concrete shall satisfy all requirements for special moment frames constructed with cast-in-place concrete, in addition to the provisions of 21.6.2. — *21.6.2*

      The required reinforcement for the beams on the third floor level is shown in Table 29-8. The design moments account for all possible load combinations per 9.2.1, and the provided areas of steel are within the limits specified in 21.3.2.1. Also given in Table 29-8 are the flexural moment strengths ϕM_n at each section. Note that at each location, the section is tension-controlled, so that ϕ = 0.9. — *9.3.2.1*

*This example has been adopted from: Ghosh, S.K., Nakaki, S.D., and Krishnan, K., "Precast Structures in Regions of High Seismicity: 1997 UBC Design Provisions", PCI Journal, Vol. 42, No. 6, November-December 1997, pp. 76-93.

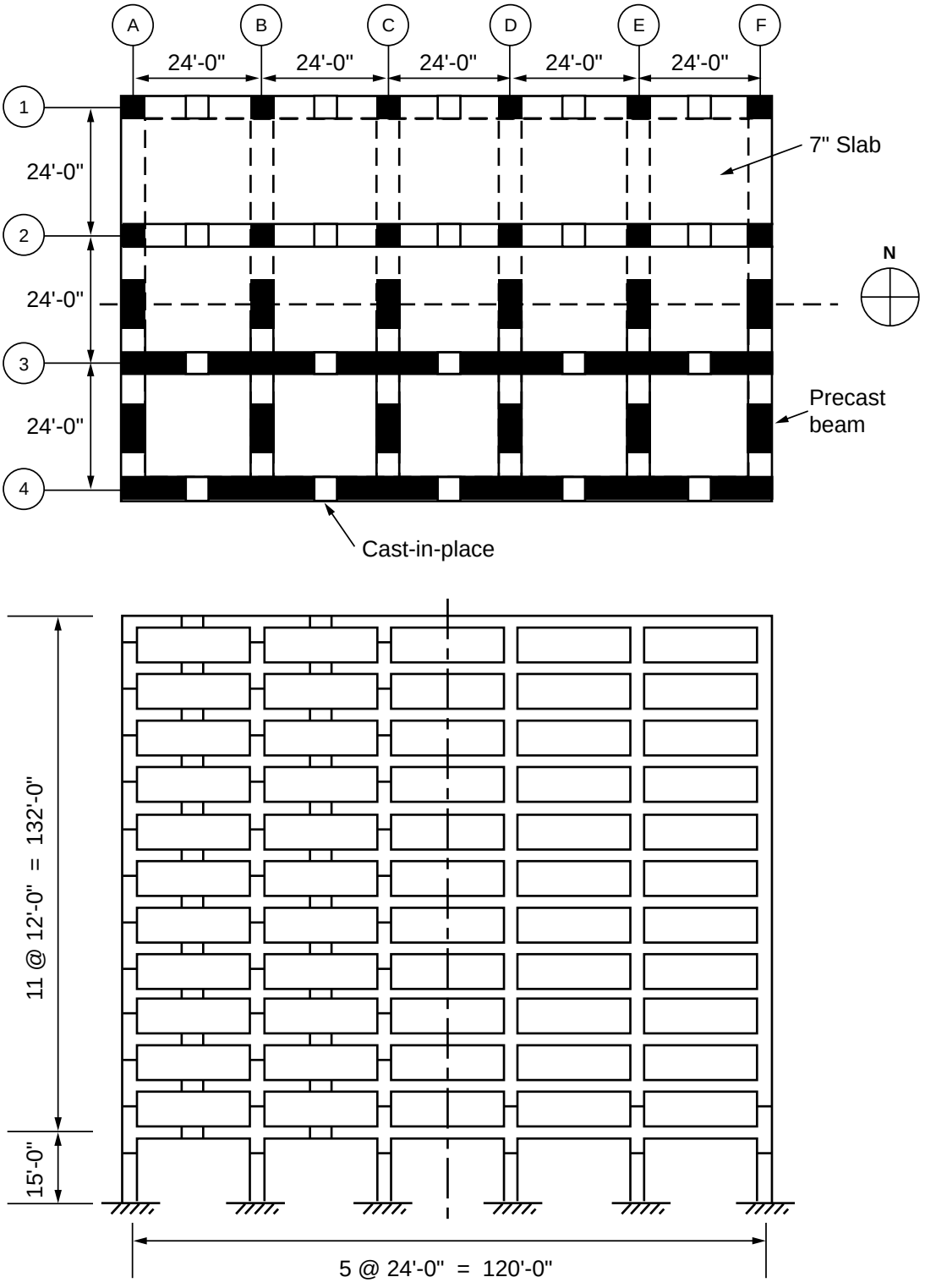


Fig 29-39 Example Building

Table 29-8 Required Reinforcement for E-W Third Floor Beam

Location	M_u (ft-kips)	A_s* (in.^{2})	Reinforcement	ϕM_n (ft-kips)
Supports	-510.8	7.79	8-No. 9	-522.0
	+311.7	4.38	5-No. 9	+351.0
Interior	+63.0	1.40	2-No. 9	+150.3

*Max. A_s $= 0.025 \times 24 \times 17.44 = 10.46$ in.^{2} (21.3.2.1)
Min. A_s $= 3\sqrt{4{,}000} \times 24 \times 17.44/60{,}000 = 1.32$ in.^{2} (10.5.1)
$= 200 \times 24 \times 17.44/60{,}000 = 1.40$ in.^{2} (governs)

Three No. 9 bars are made continuous at the top and bottom throughout the spans, providing negative and positive design moment strengths of 220.6 ft-kips.

Check provisions of 21.3.2.2 for moment strength along span of beam:

At the supports, ϕM_n^+ (5-No. 9) = 351.0 ft-kips > $\phi M_n^-/2$ = 261.0 ft-kips O.K. *21.3.2.2*

At other sections, ϕM_n (3-No. 9) = 220.6 ft-kips > $\phi M_n^-/4$ = 130.5 ft-kips O.K.

b. Lap splice length *12.2.1*

Lap splices of flexural reinforcement must not be placed within a joint, within a distance 2h from faces of supports or within regions of potential plastic hinging. Note that all lap splices must be confined by hoops or spirals with a maximum spacing or pitch of d/4 = 4.4 in. or 4 in. (governs) over the length of the lap. Lap splice lengths will be determined for the No. 9 top and bottom bars. *21.3.2.3*

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\alpha\beta\gamma\lambda}{\frac{c + K_{tr}}{d_b}} \right) d_b \qquad \textit{Eq. (12-1)}$$

where $\frac{c + K_{tr}}{d_b} \le 2.5$

α = 1.3 for top bars; α = 1.0 for other bars *12.2.4*

β = 1.0 for uncoated bars

γ = 1.0 for No. 7 and larger bars

λ = 1.0 for normal weight concrete

$c = 1.5 + 0.5 + \frac{1.128}{2} = 2.56$ in. (governs)

$= \frac{24 - 2(1.5 + 0.5) - 1.128}{2 \times 2} = 4.72$ in.

$\frac{c}{d_b} = \frac{2.56}{1.128} = 2.27$, which makes it reasonable to take $\frac{c + K_{tr}}{d_b} = 2.5$

Thus, for top bars:

$$\ell_d = \frac{3}{40}\frac{60{,}000}{\sqrt{4{,}000}}\frac{1.3}{2.5}d_b = 37d_b = 37 \times 1.128 = 41.7 \text{ in.}$$

For bottom bars:

$$\ell_d = \frac{3}{40}\frac{60{,}000}{\sqrt{4{,}000}}\frac{1.0}{2.5}d_b = 28.5d_b = 28.5 \times 1.128 = 32.2 \text{ in.}$$

Note that 2-No. 9 top bars are adequate in the interior of the span, i.e., ϕM_n(2-No. 9) = 150.3 ft-kips > $\phi M_n^-/4$ = 130.5 ft-kips. Thus, the top bar development length can be reduced by an excess reinforcement factor of (A_s required/A_s provided) = 2/3: *12.2.5*

$\ell_d = 2/3 \times 41.7 = 27.8$ in.

Since all of the reinforcement is spliced at the same location, Type B splices are to be used for both the top and bottom bars. *12.15.2*

Type B splice length = $1.3 \times 32.2 = 41.9$ in. > 12 in. for the bottom bar *12.15.1*

Provide 3 ft-6 in. splice length for both the top and bottom bars.

c. Reinforcing bar cutoff points

For the purpose of determining the cutoff points for the reinforcement, a moment diagram corresponding to the probable moment strengths at the beam ends and 0.9 times the dead load on the span will be used, since this will result in the longest bar lengths. The cutoff point for 5 of the 8-No. 9 bars at the top will be determined.

Determine probable moment strengths M_{pr}^+ and M_{pr}^- with $f_s = 1.25f_y = 75$ ksi and $\phi = 1.0$, ignoring compression steel. *21.0*

For 5-No.9 bottom bars:

$$a = \frac{A_s f_s}{0.85 f'_c b} = \frac{5 \times 75}{0.85 \times 4 \times 24} = 4.6 \text{ in.}$$

$$M_{pr}^+ = A_s f_s\left(d - \frac{a}{2}\right) = (5 \times 75)\left(17.44 - \frac{4.6}{2}\right)/12 = 473.1 \text{ ft-kips}$$

where d = 20 – 1.5 (clear cover) – 0.5 (diameter of No. 4 stirrup) – 0.564 (diameter of No. 9 bar/2) = 17.44 in.

Similarly, for 8-No. 9 top bars: M_{pr}^- = 688.2 ft-kips

Dead load on beam:

$$w_D = \left(\frac{7}{12} \times 0.150 \times 24\right) + (0.0425 \times 24) + \left(\frac{24 \times 13 \times 0.150}{144}\right) = 3.45 \text{ kips/ft at midspan}$$

$$0.9w_D = 0.9 \times 3.45 = 3.11 \text{ kips/ft}$$

The distance from the face of the interior support to where the moment under the loading considered equals ϕM_n(3-No. 9) = 220.6 ft-kips is readily obtained by summing moments about section A-A (see Fig. 29-40):

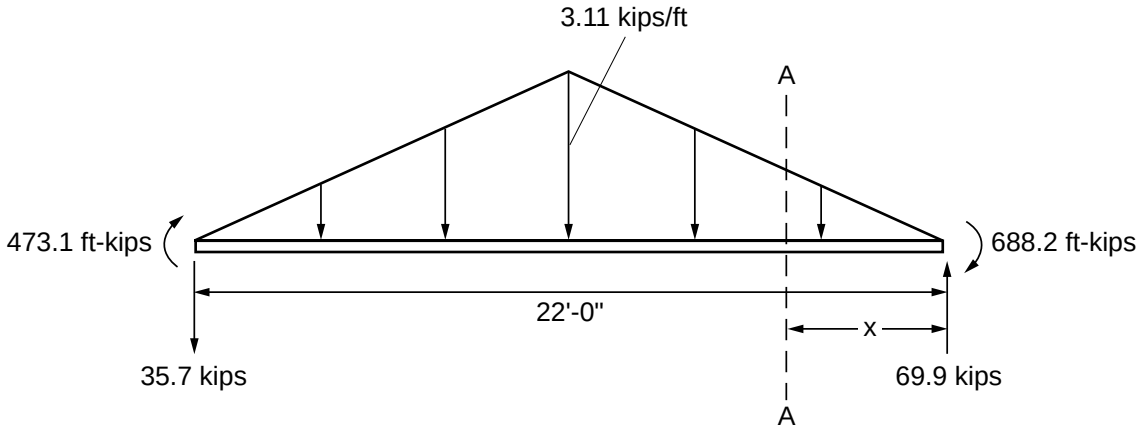


Fig. 29-40 Cutoff Location of Negative Bars

$$\frac{x}{2}\left(\frac{3.11x}{11}\right)\left(\frac{x}{3}\right) + 688.2 - 220.6 - 69.9x = 0$$

Solving for x gives a distance of 6.91 ft from the face of the support.

The 5-No. 9 bars must extend a distance d = 17.44 in. or $12d_b$ = 13.54 in. beyond the distance x. Thus, from the face of the support, the total bar length must be at least equal to 6.91 + (17.44/12) = 8.4 ft. Also, the bars must extend a full development length ℓ_d beyond the face of the support: *12.10.3* *12.10.4*

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\frac{c + K_{tr}}{d_b}}\right) d_b \qquad \textit{Eq. (12-1)}$$

where $\frac{c + K_{tr}}{d_b} \le 2.5$

α = 1.3 for top bars *12.2.4*

β = 1.0 for uncoated bars

γ = 1.0 for No. 7 and larger bars

λ = 1.0 for normal weight concrete

c = 2.56 in. or $\frac{24-2(1.5+0.5)-1.128}{2\times 7}$ = 1.35 in. (governs)

K_{tr} = 0 (conservative)

$$\frac{c+K_{tr}}{d_b}=\frac{1.35+0}{1.128}=1.2$$

$$\ell_d=\frac{3}{40}\frac{60{,}000}{\sqrt{4{,}000}}\frac{1.3}{1.2}d_b=77d_b=77\times 1.128=86.9 \text{ in.}=7.2 \text{ ft} < 8.4 \text{ ft}$$

The total required length of the 5-No. 9 bars must be at least 8.4 ft beyond the face of the supports.

Flexural reinforcement shall not be terminated in a tension zone unless one or more of the conditions of 12.10.5 are satisfied. In this case, the point of inflection is approximately 10.7 ft from the face of the right support, which is greater than 8.4 ft. The 5-No. 9 bars can not be terminated here unless one of the conditions of 12.10.5 is satisfied.

Check if the factored shear force V_u at the cutoff point does not exceed two-thirds of ϕV_n. In this region of the beam, it can be shown that No. 4 stirrups @ 8 in. are required. However, No. 4 stirrups @ 6 in. will be provided to satisfy 12.10.5(a).

$$\phi V_n+\phi(V_c+V_s)=0.75\times\left(2\sqrt{4{,}000}\times 24\times 17.44+\frac{0.4\times 60{,}000\times 17.44}{6}\right)/1{,}000=92.0 \text{ kips}$$

$$\frac{2}{3}\phi V_n=61.3 \text{ kips} > V_u = 60.0 \text{ kips at 8.4 ft from face of support}$$

Since $2\phi V_u/3 > V_u$, the cutoff point for the 5-No. 9 bars can be 8.4 ft beyond the face of the interior support.

The cutoff point for 2 of the 5-No. 9 bottom bars can be determined in a similar fashion. These bars can be cut off at 8.4 ft from the face of the exterior support as well, which is short of the splice closure.

d. Check connection strength

For strong connections: $\phi S_n \geq S_e$ *21.6.2(b)*

where S_n = nominal flexural or shear strength of the connection *21.0*

S_e = moment or shear at connection corresponding to development of probable strength at intended yield locations, based on the governing mechanism of inelastic lateral deformation, considering both gravity and earthquake load effects

At the connection,

$$\phi V_n = \phi(V_c + V_s) = 0.75\times\left(2\sqrt{4{,}000}\times 24\times 17.44+\frac{0.4\times 60{,}000\times 17.44}{4}\right)/1{,}000 = 118.2 \text{ kips}$$

Gravity load on beam:

$1.2w_D + 0.5w_L = (1.2 \times 3.45) + (0.5 \times 0.05 \times 24) = 4.74$ kips/ft *Eq. (9-5)*

Maximum shear force at connection due to gravity and earthquake load effects occurs at 9.125 ft from face of right support (see Fig. 29-41):

$$\text{Max. } V_e = 78.9 - \left(\frac{1}{2} \times 9.125 \times \frac{4.74 \times 9.125}{11}\right) = 61.0 \text{ kips} < \phi V_n = 118.2 \text{ kips} \quad \text{O.K.}$$

At the connection, ϕM_n (3-No. 9) = 220.6 ft-kips

Maximum moment at connection due to gravity and earthquake load effects occurs at 9.125 ft from face of left support (see Fig. 29-41):

$$\text{Max. } M_e = 473.1 - (26.8 \times 9.125) - \left(\frac{1}{2} \times 9.125\right)\left(\frac{4.74 \times 9.125}{11}\right)\left(\frac{1}{3} \times 9.125\right) = 174.0 \text{ ft-kips}$$

$< \phi M_n = 220.6$ ft-kips O.K.

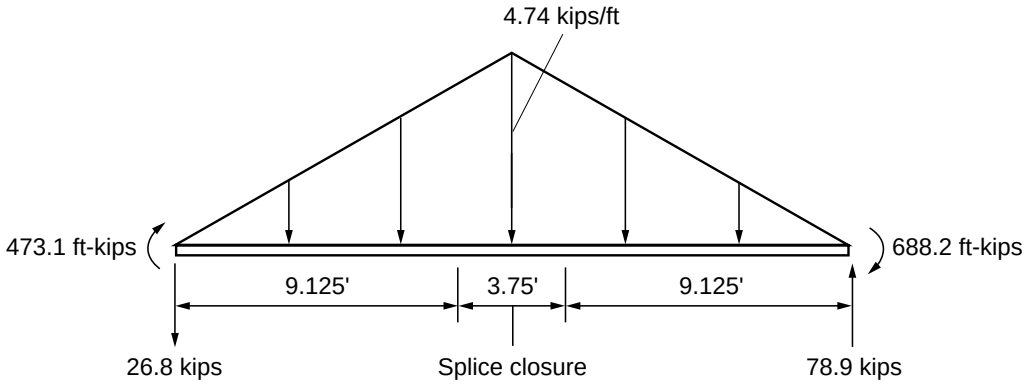
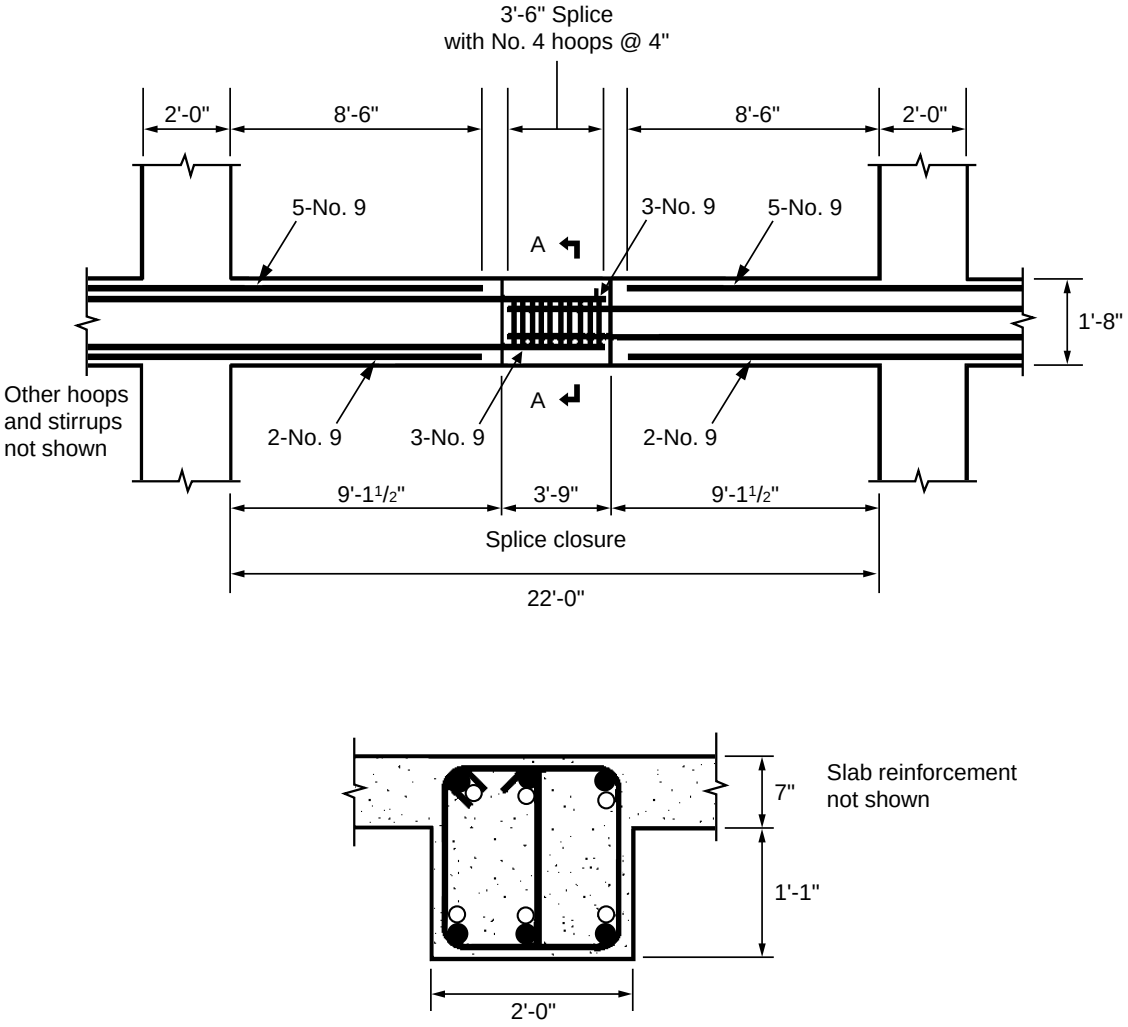


Fig. 29-41 Connection Strength

e. Reinforcement details

The reinforcement details for the beam are shown in Fig. 29-42.



Section A-A

Fig. 29-42 Reinforcement Details for Beam-to-Beam Connection

3. Column-to-column connection at mid-height

   a. Determine required longitudinal reinforcement

      A summary of the design forces for the interior column between levels 2 and 3, which is part of an interior longitudinal frame, is contained in Table 29-9. The design forces account for all possible load combinations per 9.2.1.

Table 29-9 Design Forces for Interior Column between the Second and Third Floors

Load Combination	Axial load, P_u (kips)	Moment, M_u (ft-kips)		Shear, V_u (kips)
		Top	Bottom	
1.2D + 1.6L	1,402.6	-8.0	5.5	1.2
1.2D + 0.5L + E	1,609.8	-408.3	467.5	70.7
	1,195.4	392.3	-456.5	73.1
0.9D + E	1,125.8	-405.5	466.3	71.1
	711.4	395.1	-457.8	72.7

It can be shown that 12-No. 10 bars are adequate for all load combinations.

Check longitudinal reinforcement ratio:

$$\rho_g = \frac{A_{st}}{bh} = \frac{12 \times 1.27}{24 \times 24} = 0.0265$$

$\rho_{min} = 0.01 < \rho_g = 0.0265 < \rho_{max} = 0.06$ O.K. *21.4.3.1*

b. Nominal flexural strength of columns relative to that of beams

$$\Sigma M_c \text{ (columns)} \geq \frac{6}{5} \Sigma M_g \text{ (beams)}$$ *21.4.2.2*

For the top end of the lower column framing into the joint between the second and third floor levels, M_n = 1,182.3 ft-kips, which corresponds to P_u = 711.4 kips. Similarly, for the bottom end of the upper column framing into the same joint, M_n = 1,168.4 ft-kips, which corresponds to P_u = 655.5 kips.

Thus,

ΣM_c = 1,182.3 + 1,168.4 = 2,350.7 ft-kips

The nominal flexural strength M_n of the beam framing into the column must include the slab reinforcement within an effective slab width equal to:

16(slab thickness) + beam width = (16 × 7) + 24 = 136 in. *8.10.2*
Center-to-center beam spacing = 24 × 12 = 288 in.
Span/4 = (24 × 12)/4 = 72 in. (governs)

The minimum required A_s in the 72-in. effective width = 0.0018 × 72 × 7 = 0.91 in.^{2} which corresponds to 5-No. 4 bars @ 72/5 = 14.4 in. Since maximum bar spacing = 2h = 14 in., provide No. 4 @ 14 in. at both the top and bottom of the slab (according to ACI 318 Fig. 13.3.8, 100 percent of both the top and bottom reinforcement in the column strip must be continuous or anchored at the support).

From a strain compatibility analysis, $M_n^- = 736.0$ ft-kips and $M_n^+ = 459.0$ ft-kips

Thus,

$\Sigma M_g = 736.0 + 459.0 + 1{,}195.0$ ft-kips

$2{,}350.7 \text{ ft-kips} > \frac{6}{5} \times 1{,}195.0 = 1{,}434.0 \text{ ft-kips}$ O.K. *Eq. (21-1)*

The intent of 21.4.2.2 is to prevent a story mechanism, rather than prevent local yielding in a column. The 6/5 factor is clearly insufficient to prevent column yielding if the adjacent beams both hinge. Therefore, confinement reinforcement is required in the potential hinge regions of a frame column.

c. Minimum connection strength

At column-to-column connections, $\phi M_n \geq 0.4 M_{pr}$ when bars are spliced within the middle third of the clear column height. *21.6.2(d)*

For the column between the second and third floor levels with $P_u = 711.4$ kips, it can be shown from a strain compatibility analysis that $M_{pr} = 1{,}244.1$ ft-kips.

Also, as indicated above, $M_n = 1{,}182.3$ ft-kips for $P_u = 711.4$ kips. From a strain compatibility analysis, $\varepsilon_t = 0.00223$ so that $\phi = 0.48 + (83 \times 0.00223) = 0.67$. *9.3.2.2*

Therefore,

$\phi M_n = 0.67 \times 1{,}182.3 = 792.1$ ft-kips $> 0.4 M_{pr} = 0.4 \times 1{,}244.1 = 497.6$ ft-kips O.K.

Splice all twelve bars at mid-height, as shown in Fig. 29-43.

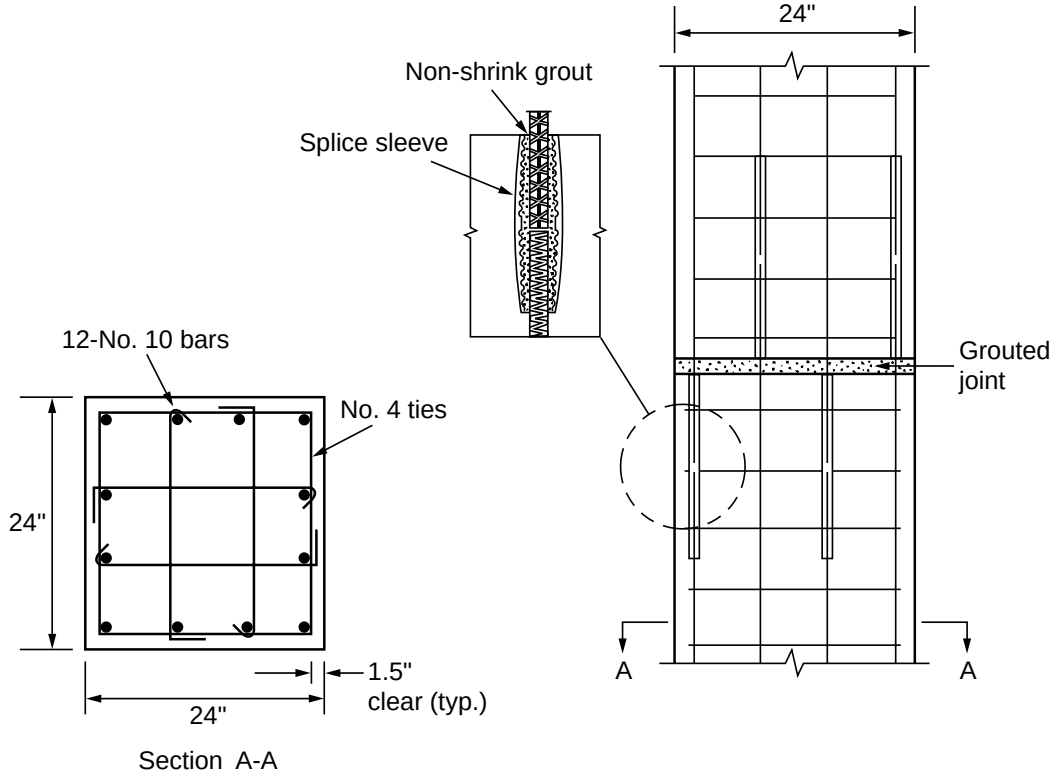


Fig. 29-43 Reinforcement Details for Column-to-Column Connection

4. Column-face strong connection in beam

A strong connection is to be designed at the interface between a precast beam at the second floor level of the building that forms the exterior span of an exterior transverse frame and the continuous corner column to which it is connected.

a. Required flexural reinforcement

From the combined effects of gravity and earthquake forces, the required flexural reinforcement at the top of the beam is 5-No. 9 bars and is 4-No. 9 bars at the bottom. All possible load combinations of 9.2.1 were considered.

b. Strength design of connection

The beam-to-column connection similar to the one depicted in ACI 318 Fig. R21.6.2(c) will be provided.

The strong connection must be designed for the probable moment strength of the beam plus the moment at the face of the column due the shear force at the critical section. *21.6.2*

Determine probable moment strengths M_{pr}^{+} and M_{pr}^{-} with $f_s = 1.25f_y = 75$ ksi and $\phi = 1.0$, ignoring compression steel. *21.0*

For 4-No. 9 bottom bars:

$$a = \frac{A_s f_s}{0.85 f'_c b} = \frac{4 \times 75}{0.85 \times 4 \times 24} = 3.7 \text{ in.}$$

$$M_{pr}^{+} = A_s f_s \left(d - \frac{a}{2}\right) = (4 \times 75)\left(23.44 - \frac{3.7}{2}\right)/12 = 539.8 \text{ ft-kips}$$

where d = 26 – 1.5 (clear cover) – 0.5 (diameter of No. 4 stirrup) – 0.564 (diameter of No. 9 bar/2) = 23.44 in.

Similarly, for 5-No. 9 top bars: $M_{pr}^{-} = 660.6$ ft-kips

Assuming a 2 ft-6 in. cast-in-place closure, the shear forces at the critical sections, and the moments at the connections can be determined for the two governing load combinations as follows (see Fig. 29-44).

Load combination 1: U = 1.2D + 0.5L + E *Eq. (9-5)*

$$w_D = \left(\frac{7}{12} \times 0.150 \times 13\right) + (0.0425 \times 13) + \left(\frac{24 \times 19 \times 0.150}{144}\right) = 2.17 \text{ kips/ft at midspan}$$

$$w_L = 0.05 \times 13 = 0.65 \text{ kips/ft at midspan}$$

$$w_{u,\,mid} = (1.2 \times 2.17) + (0.5 \times 0.65) = 2.93 \text{ kips/ft}$$

and

$$w_{u,\,end} = 2.93 \times \frac{2.5}{11} = 0.67 \text{ kips/ft}$$

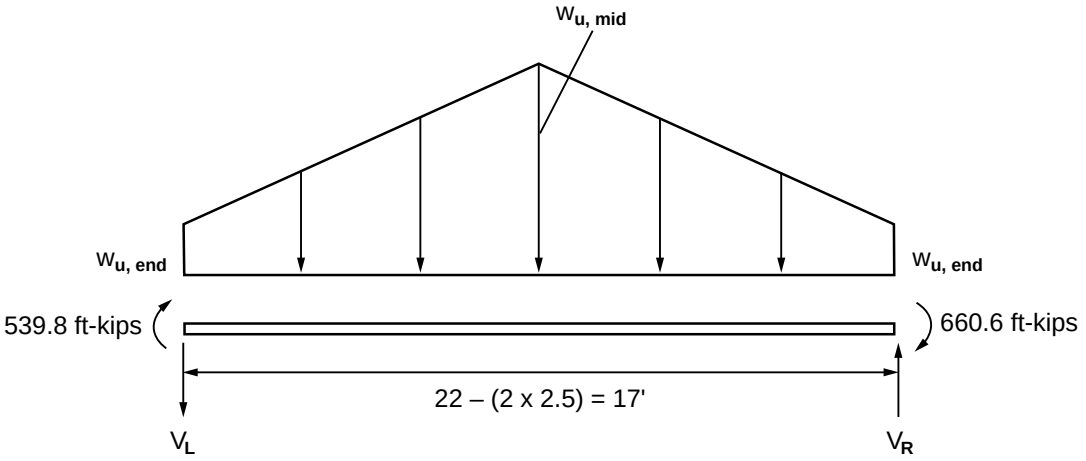


Fig. 29-44 Shear Forces at Critical Sections

From Fig. 29-44:

$$V_R(17) = \left(0.67 \times 17 \times \frac{17}{2}\right) + \left[\frac{1}{2} \times 17 \times (2.93 - 0.67) \times \frac{17}{2}\right] + 539.8 + 660.6$$

or, $V_R = 85.9$ kips

$$V_L = 85.9 - (0.67 \times 17) - \frac{1}{2}[(2.93 - 0.67) \times 17] = 55.3 \text{ kips}$$

$$M^{+}_{e,\ell} = 539.8 + (55.3 \times 2.5) = 678.1 \text{ ft-kips}$$

$$M^{-}_{e,r} = 660.6 + (85.9 \times 2.5) = 875.4 \text{ ft-kips}$$

Load combination 2: U = 0.9D + E *Eq. (9-7)*

$$w_{u,mid} = 0.9 w_D = 0.9 \times 2.17 = 1.95 \text{ kips/ft}$$

and

$$w_{u,end} = 1.95 \times \frac{2.5}{11} = 0.44 \text{ kips/ft}$$

From Fig. 29-44:

$$V_R(17) = \left(0.44 \times 17 \times \frac{17}{2}\right) + \left[\frac{1}{2} \times 17 \times (1.95 - 0.44) \times \frac{17}{2}\right] + 539.8 + 660.6$$

or, $V_R = 80.8$ kips

$$V_L = 80.8 - (0.44 \times 17) - \frac{1}{2}[(1.95 - 0.44) \times 17] = 60.5 \text{ kips}$$

$$M^+_{e,\ell} = 539.8 + (60.5 \times 2.5) = 691.1 \text{ ft-kips}$$

$$M^-_{e,r} = 660.6 + (80.8 \times 2.5) = 862.6 \text{ ft-kips}$$

Thus, the governing moments at the connections are

$$M^+_{e,\ell} = 691.1 \text{ ft - kips and } M^-_{e,r} = 875.4 \text{ ft-kips}$$

At the bottom of the connection, provide an additional 4-No. 9 bars to the 4-No. 9 bars (2 layers) and at the top of the section, provide an additional 5-No. 9 bars to the 5-No. 9 bars (2 layers). From a strain compatibility analysis considering all of the reinforcement in the section:

$\phi M^+_n = 728.8$ ft - kips $> M^+_{e,\ell} = 691.1$ ft-kips O.K.

$\phi M^-_n = 888.2$ ft - kips $> M^-_{e,r} = 875.4$ ft-kips O.K.

For both the positive and negative moment capacities, the strain in the extreme tension steel was determined from the strain compatibility analysis to be greater than 0.005 so that the section is tension-controlled.

Maximum reinforcement ratio = $\dfrac{10 \times 1.0}{24 \times 22.33} = 0.019 < 0.025$ O.K. *21.3.2.1*

where the effective depth d was determined from the strain compatibility analysis.

c. Anchorage and splices

Per 21.5.4.1, the minimum development length for a bar with a standard 90-degree hook in normal weight aggregate concrete is:

$$\ell_{dh} = \frac{f_y d_b}{65\sqrt{f'_c}} = \frac{(60{,}000)(1.128)}{65\sqrt{4{,}000}} = 16.5 \text{ in.}$$ *Eq. (21-6)*

Figure 29-45 shows the reinforcement details for the connection.

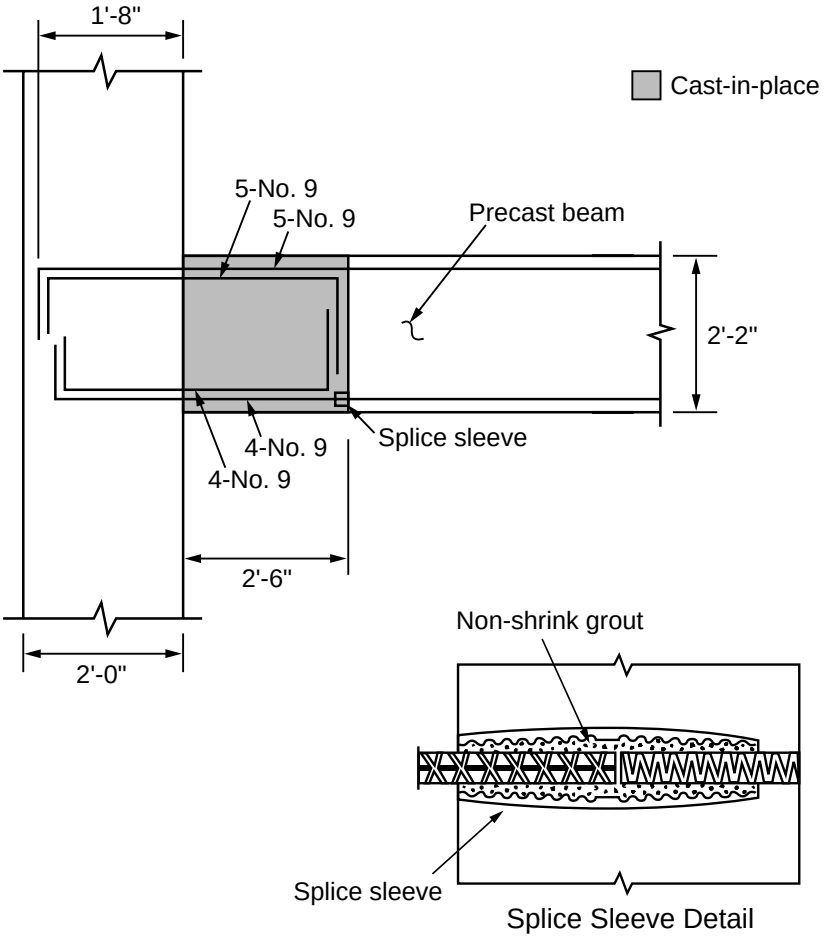


Fig. 29-45 Reinforcement Details for Connection

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Structural Plain Concrete

UPDATE FOR THE ‘02 CODE

The most significant change to Chapter 22 resulted from a change to another chapter. In 9.3.5 the strength reduction factor for plain concrete was reduced from 0.65 to 0.55. This will result in a 15% reduction in the design strengths of structural members of plain concrete. This reduction will be somewhat, but not completely offset by lower load factors in the load combinations of 9.2. Both of these changes will be mostly negated since the load combinations and strength reduction factors of 9.2 and 9.3, respectively, of the ’99 ACI code have been relocated to Appendix C and their continued use is permitted by 9.1.3. For additional discussion of this topic, see Part 5 and 22.4 in this part.

Section 22.2.4 of the 1999 ACI code indicated that the specified compressive strength of concrete used in structural plain concrete could not be less than 2500 psi. This provision was in Chapter 22 because there was no minimum strength requirement for concrete used in reinforced concrete members unless specific code provisions imposed it. The strength limit has been removed from 22.2.4 and a reference to 1.1.1 has been added. Section 1.1.1 and 5.1.1 of the 2002 ACI code include the 2500 psi minimum strength limitation, and by virtue of its placement in these two locations, it applies to all concrete, both reinforced and plain.

In 22.5.1, Eq. (22-3) has been added for calculating the nominal moment strength where governed by compression. In prior editions this equation was shown only in the Commentary. This has resulted in all equations after (22-3) of the code being renumbered.

In the ’99 edition, provisions were introduced to address the use of structural plain concrete members in structures in areas of high seismic risk, or in structures assigned to high seismic performance or design categories. In this edition, the provision allowing plain concrete footing with nominal longitudinal reinforcing steel was revised to indicate that the footing must support either a cast-in-place reinforced concrete wall or reinforced masonry wall.

BACKGROUND

With publication of the 1983 edition of ACI 318, provisions for structural plain concrete were incorporated into the code by reference. The document referenced was ACI 318.1, *Building Code Requirements for Structural Plain Concrete*. This method of regulating plain concrete continued with the 1989 edition of ACI 318. For the 1995 edition, the provisions formally contained in the ACI 318.1 standard were incorporated into Chapter 22 of the code and publication of ACI 318.1 was discontinued. While the presentation of some provisions is different, few technical changes have been made since the 1989 edition of ACI 318.1. Technical changes that were made are discussed at the appropriate location in this part.

22.1, 22.2 SCOPE AND LIMITATIONS

By definition, structural plain concrete is concrete in members that either contains no reinforcement or contains less reinforcement than the minimum amount specified for reinforced concrete in other chapters of ACI 318 and

Appendices A through C (22.2.1). The designer should take special note of 22.2.2. Since the structural integrity of structural plain concrete members depends solely on the properties of the concrete, it limits the use of plain concrete to: members that are continuously supported by soil or by other structural members capable of providing vertical support continuous throughout the length of the plain concrete member; members in which arch action assures compression under all conditions of loading; and walls and pedestals. Chapter 22 of ACI 318 contains specific design provisions for structural plain concrete walls, footings and pedestals.

Section 22.1.1.2 indicates that sidewalks and other slabs-on-grade are not regulated by the code unless they transmit vertical loads from other parts of the structure to the soil. Section 1.1.6 also stipulates that if a slab transmits lateral forces from the structure to the soil, it also must comply with the code. In addition, 22.2.3 points out that the design and construction of portions of structural plain concrete foundation piers and cast-in-place piles embedded in ground capable of providing adequate lateral support are not governed by Chapter 22. Provisions for these elements are typically found in the general building code.

22.3 JOINTS

Structural plain concrete members must be small enough or provided with contraction (control) joints so as to create elements that are flexurally discontinuous (22.3.1). This requires that the build-up of tensile stresses due to external loads and internal loads, such as from drying shrinkage, temperature and moisture changes, and creep, must be limited to permissible values. Section 22.3.2 emphasizes several items that will influence the size of elements and, consequently, the spacing of contraction joints. These include: climatic conditions; selection and proportioning of materials; mixing, placing and curing of concrete; degree of restraint to movement; stresses due to external and internal loads to which the element is subjected; and construction techniques. Where contraction joints are provided, the member thickness must be reduced a minimum of 25% if the joint is to be effective. For additional information on drying shrinkage of concrete, other causes of volume changes of concrete, and the use of contraction joints to relieve stress build-up, see Refs. 30.1 through 30.3.

While not a part of the provisions, R22.3 gives an exception to the above requirement for contraction joints. It indicates that where random cracking due to creep, shrinkage and temperature effects will not affect the structural integrity, and is otherwise acceptable, such as transverse cracks in a continuous wall footing, contraction joints are not necessary.

22.4 DESIGN METHOD

As for reinforced concrete designed in accordance with Chapters 1 through 21, the provisions of Chapter 22 are based on the strength design methodology. Load combinations and load factors are found in 9.2, and are the same as those used for the design of reinforced concrete. The load combinations and load factors in the 1999 and earlier ACI codes have been replaced with load combinations and load factors from ASCE 7-98. The strength reduction factor, ϕ, is found in 9.3.5. It has been reduced from 0.65 (found in the 1999 and earlier editions of the ACI code) to 0.55 and applies for all stress conditions (i.e., flexure, compression, shear and bearing). Everything else remaining the same, the reduction in ϕ results in a 15.4% decrease in the design strength. Although some load factors in 9.2 are less than those in C.2, they have not been reduced enough to completely compensate for the lower design strength. While each case needs to be investigated, generally speaking a more economical design will be obtained by using the load and strength reduction factors of Appendix C. If snow loads or roof live loads are included in the controlling gravity load, depending on the magnitude of these loads with respect to floor live load, use of the load and strength reduction factors of Chapter 9 may be more economical.

To quickly determine which one of the two sets of load and strength reduction factors should be used, compute the governing load/load effect (e.g., P_u or M_u) using the load factors in 9.2 and C.2. These values can then be divided by the corresponding strength reduction factors from 9.3.5 and C.3.5, respectively, to determine the nominal loads/load effects. Satisfying the lower nominal load/load effect may be more economical.

Numerous figures and tables are provided in the main body of this part to assist the user in designing structural members of plain concrete. They are based on the load factors and strength reduction factor (0.55) of Chapter 9. An appendix to this part contains similar figures and tables based on the load factors and strength reduction

factor (0.65) found in Appendix C. To facilitate comparing companion figures and tables, their assigned numbers are the same except those corresponding to the appendix, they are prefaced with the letter "C."

A linear stress-strain relationship in both tension and compression is assumed for members subject to flexure and axial loads. The allowable stress design procedures contained in Appendix A - Alternate Design Method of the 1999 and earlier editions of the code do not apply to structural members of plain concrete. That Appendix has been removed from the 2002 code.

Where the provisions for contraction joints and/or size of members have been observed in accordance with 22.3, tensile strength of plain concrete is permitted to be considered (22.4.5). Tension is not to be considered beyond the outside edges of the panel, contraction joints or construction joints, nor is flexural tension allowed to be assumed between adjacent structural plain concrete elements (22.4.7).

Section 22.4.8 permits the entire cross-section to be considered effective in resisting flexure, combined flexure and axial load, and shear; **except that for concrete cast on the ground, such as a footing, the overall thickness, h, shall be assumed to be 2 in. less than actual**. The commentary indicates that this provision is necessary to allow for unevenness of the excavation and for some contamination of the concrete adjacent to the soil. No strength shall be assigned to any steel reinforcement that may be present (22.4.6).

As in the past, 22.2.4, through its reference to 1.1.1, requires that the minimum specified compressive strength of concrete, f'_c, used in design of structural plain concrete elements shall not be less than 2500 psi. This provision is considered necessary due to the fact that safety and load-carrying capability is based solely on the strength and durability of the concrete.

22.5 STRENGTH DESIGN

Permissible stresses of ACI 318.1-89 were replaced with formulas for calculating nominal strengths for flexure, compression, shear and bearing. The nominal moment strength, M_n, is given by:

$$M_n = 5\sqrt{f'_c}\, S \qquad \textit{Eq. (22-2)}$$

for flexural tension controlled sections, and

$$M_n = 0.85 f'_c S \qquad \textit{Eq. (22-3)}$$

for flexural compression controlled sections.

The nominal axial compression strength, P_n, is given by:

$$P_n = 0.60 f'_c \left[1 - \left(\frac{\ell_c}{32h}\right)^2\right] A_1 \qquad \textit{Eq. (22-5)}$$

Note that the effective length factor, k, is missing from the numerator of the ratio $\ell_c/32h$. This change from ACI 318.1 was made because it was felt that it is always conservative to assume k = 1, which is based on both ends being fixed against translation. Also, it was recognized that it is difficult to obtain fixed connections in typical types of construction utilizing structural plain concrete walls. If a connection fixed against rotation is provided at one or both ends, the engineer can always assume k = 0.8 as in the past. However, before doing so the engineer should verify that the member providing rotational restraint has a flexural stiffness EI/ℓ at least equal to that of the wall.

For members subject to combined flexural and axial compression, two interaction equations are given and both must be satisfied. For the compression face:

$$\frac{P_u}{\phi P_n} + \frac{M_u}{\phi M_n} \leq 1 \qquad \textit{Eq. (22-6)}$$

where $M_n = 0.85f'_c S$

and for the tension face:

$$\frac{M_u}{S} - \frac{P_u}{A_g} \leq 5\phi\sqrt{f'_c} \qquad \textit{Eq. (22-7)}$$

The nominal moment strength, M_n, for use in Eq. (22-6) (i.e., $0.85\,f'_c S$) is more conservative than in the 1989 edition of ACI 318.1 in which it was $f'_c S$. Other nominal strengths of Chapter 22 are consistent with those calculated using permissible stresses of ACI 318.1-89.

The nominal shear strength, V_n, is given by:

$$V_n = \frac{4}{3}\sqrt{f'_c}\,bh \qquad \textit{Eq. (22-9)}$$

for beam action, and by

$$V_n = \left[\frac{4}{3} + \frac{8}{3\beta_c}\right]\sqrt{f'_c}\,b_o h \leq 2.66\sqrt{f'_c}\,b_o h \qquad \textit{Eq. (22-10)}$$

for two-way action, or punching shear.

In Eq. (22-10), the expression $[4/3 + 8/(3\beta_c)]$ reduces the nominal shear strength for concentrated loads with long-to-short-side ratios β_c greater than 2. Where the ratio is equal to or less than 2, the expression takes on the maximum permitted value of 2.66.

The equations for computing nominal flexural and shear strengths apply to normal weight aggregate concrete. If lightweight aggregate concrete is used, the strengths may need to be reduced in accordance with 22.5.6. Where the average splitting tensile strength of lightweight concrete, f_{ct}, is specified and concrete is proportioned in accordance with 5.2 of ACI 318, $\sqrt{f'_c}$ shall be replaced with $f_{ct}/6.7$, but the value of $f_{ct}/6.7$ shall not exceed $\sqrt{f'_c}$. Where f_{ct} is not specified, the value of $\sqrt{f'_c}$ shall be multiplied by 0.75 for all lightweight aggregate concrete and by 0.85 for sand-lightweight aggregate concrete, with linear interpolation permitted for mix designs using partial sand replacement.

Nominal bearing strength, B_n, is given by:

$$B_n = 0.85f'_c A_1 \qquad \textit{Eq. (22-11)}$$

where A_1 is the loaded area. If the supporting surface is wider on all sides than A_1, the bearing strength may be increased by $\sqrt{A_2/A_1}$, but by not more than 2. A_2 is the area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area, A_1, and having side slopes of 1 vertical to 2 horizontal. See Part 6 for determination of A_2.

22.6 WALLS

22.6.5 Empirical Design Method

The provisions offer two alternatives for designing plain concrete walls. The simpler of the two is referred to as the *empirical design method*. It is only permitted for walls of solid rectangular cross-section where the resultant of all factored loads falls within the middle one-third of the overall thickness of the wall. In determining the effective eccentricity, the moment induced by lateral loads must be considered in addition to any moment induced by the eccentricity of the axial load. Limiting the eccentricity to one-sixth the wall thickness assures that all portions of the wall remain under compression. Under the empirical design method, the nominal axial load strength, P_{nw}, is determined from:

$$P_{nw} = 0.45f'_c\left[1 - \left(\frac{\ell_c}{32h}\right)^2\right]A_g \qquad \textit{Eq. (22-14)}$$

This is a single-strength equation considering only the axial load. Moments due to eccentricity of the applied axial load and/or lateral loads can be ignored since an eccentricity not exceeding h/6 is assumed.

To assist the code user in the design of plain concrete walls using the empirical design method, Fig. 30-1 has been provided. By entering the figure with the required axial load strength, one can select the wall thickness that will yield a design axial load strength, ϕP_{nw}, that is equal to or greater than required. For intermediate values of f'_c, the required wall thickness can be determined by interpolation.

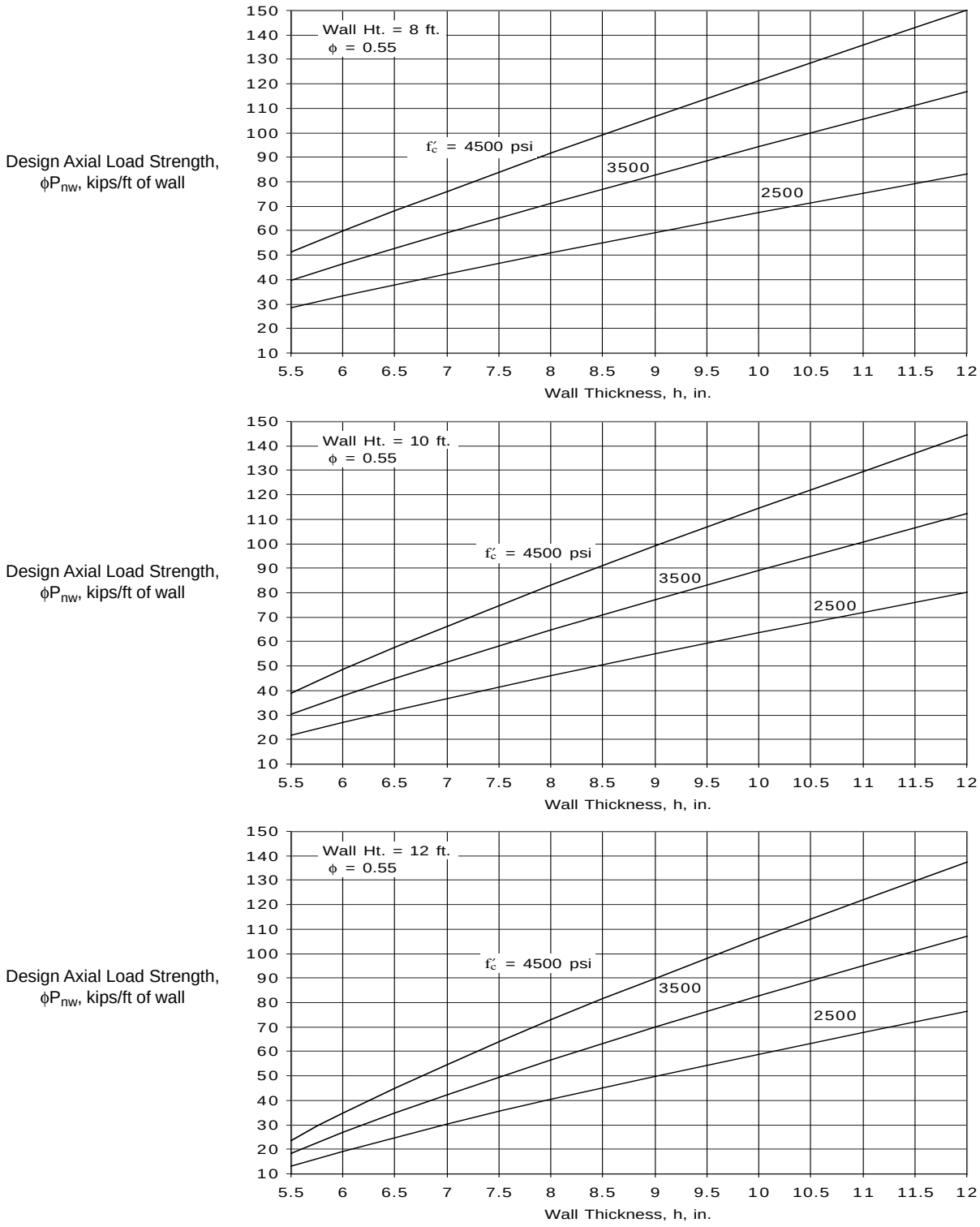


Figure 30-1 Design Axial Load Strength, P_{nw}, of Plain Concrete Walls using the Empirical Design Method

22.6.3 Combined Flexure and Axial Load

The second method, which may be used for all loading conditions, must be used where the resultant of all factored loads falls outside of the middle one-third of the wall thickness (i.e., e > h/6). In this procedure the wall must be proportioned to satisfy the provisions for combined flexure and axial loads of interaction Eqs. (22-6) and (22-7). Where the effective eccentricity is less than 10% of the wall thickness, h, an assumed eccentricity of not less than 0.10h is required.

To utilize this method, one must generally proceed on a trial and error basis by assuming a wall thickness and specified compressive strength of concrete, f'_c, and determine if the two interaction equations are satisfied. This process can proceed on a more structured basis if it is first determined whether Eq. (22-6) or (22-7) will control. Each equation can be rearranged to solve for M_u. Then, by setting the two equations equal to one another, rearranging to get both terms with P_u on the left side, and introducing constants to make units consistent, the resulting equation (1), which is shown below, can be solved for the axial load, P_u. The computed value of P_u is the axial load at which the moment strength is greatest and is the same regardless of whether Eq. (22-6) or (22-7) is used. If the required axial load strength is less than the computed value of P_u, the design is governed by Eq. (22-7). Conversely, if the required axial load strength is greater than the computed valued of P_u, the design is governed by Eq. (22-6). The equation for solving for P_u is:

$$\frac{SP_u}{12A_g} + \frac{M_n P_u}{P_n} = \phi M_n - \frac{5\phi\sqrt{f'_c}\,S}{12{,}000} \qquad (1)$$

where axial loads are in kips, moments are in ft-kips, section modulus is in in.^{3}, area is in in.^{2}, and $\sqrt{f'_c}$ is in psi.

If it is determined that Eq. (22-6) governs, equation (1) can be rearranged, with A_g and S expressed in terms of h. Quadratic equation (2) can be solved for the required wall thickness h:

$$0.06\phi\sqrt{f'_c}\,h^2 + P_u h - 72M_u = 0 \qquad (2)$$

where the axial load is in kips, the moment is in ft-kips, $\sqrt{f'_c}$ is in psi, and the thickness is in inches. If the required wall thickness is more than that assumed, another iteration is necessary. If it is significantly less than assumed, it may be advisable to repeat the process to determine if a more economical thickness and/or concrete strength can be justified.

If Eq. (22-6) governs, the required wall thickness is best determined by trial and error. Several iterations may be necessary before the most economical design solution is achieved.

The design process can be greatly simplified with the use of axial load-moment strength curves such as those shown in Figs. 30-2 and 30-3. To use the curves, enter with the known factored axial load, P_u, and determine if the design moment strength, ϕM_n, equals or exceeds the required factored moment strength, M_u. Of course, the curves can also be used by entering with the required factored moment strength, M_u, and determining if the design axial load strength, ϕP_n, equals or exceeds the required factored axial load strength, P_u.

If the effective eccentricity due to all factored loads is less than 0.10h, the design axial load strength, ϕP_n, is determined by projecting horizontally to the left from the intersection of the line labeled “e = h/10” and the curve representing the specified compressive strength of concrete, f'_c. For example, Fig. 30-2 shows that for an 8-in. wall, 8 ft in height constructed of concrete with a specified compressive strength, f'_c, of 2500 psi, the design axial load strength, ϕP_n, is approximately 68 kips/ft of wall. This assumes that the wall is loaded concentrically and there are no lateral loads to induce moments (i.e., $\phi M_n = 0$). However, when the axial load is applied at the required minimum eccentricity of 0.10h, the design axial load strength is reduced to approximately 37 kips/ft of wall. The moment corresponding to the 37-kip load being applied at the minimum eccentricity of 0.10h is approximately 3.3 ft-kips/ft of wall.

A line labeled “e = h/6” has also been included on Figs. 30-2 and 30-3 to assist the user in identifying when the effective eccentricity exceeds this value. If the intersection of the axial load, P_u, and moment, M_u, lies to the right of the line, a portion of the wall is under tension due to the induced moment.

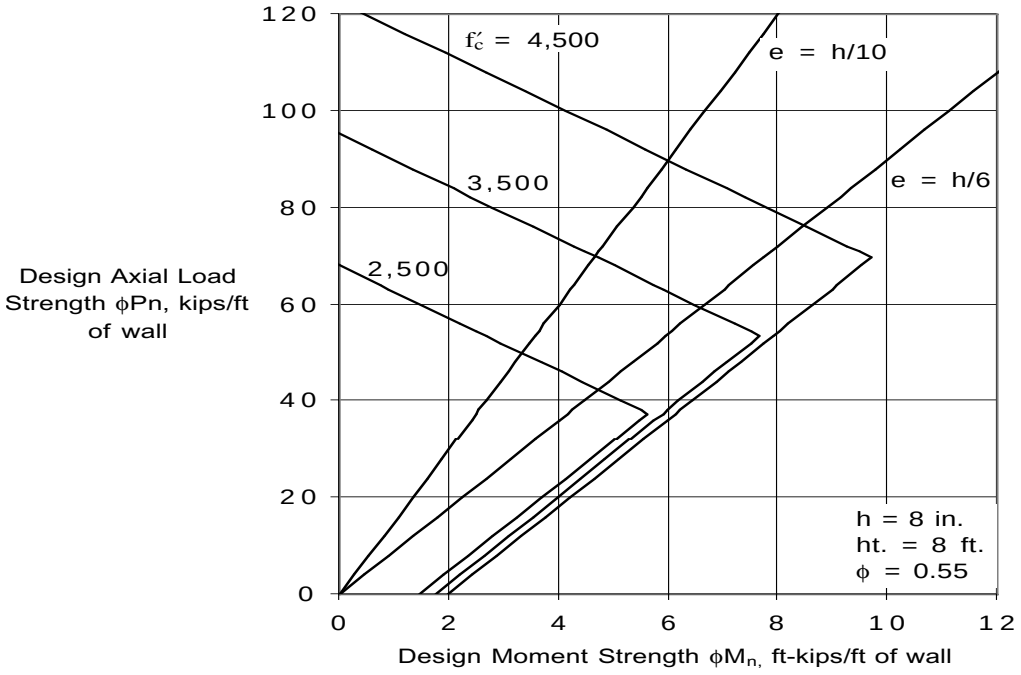


Figure 30-2 Design Strength Interaction Diagrams for 8.0-in. Wall, 8 ft in Height

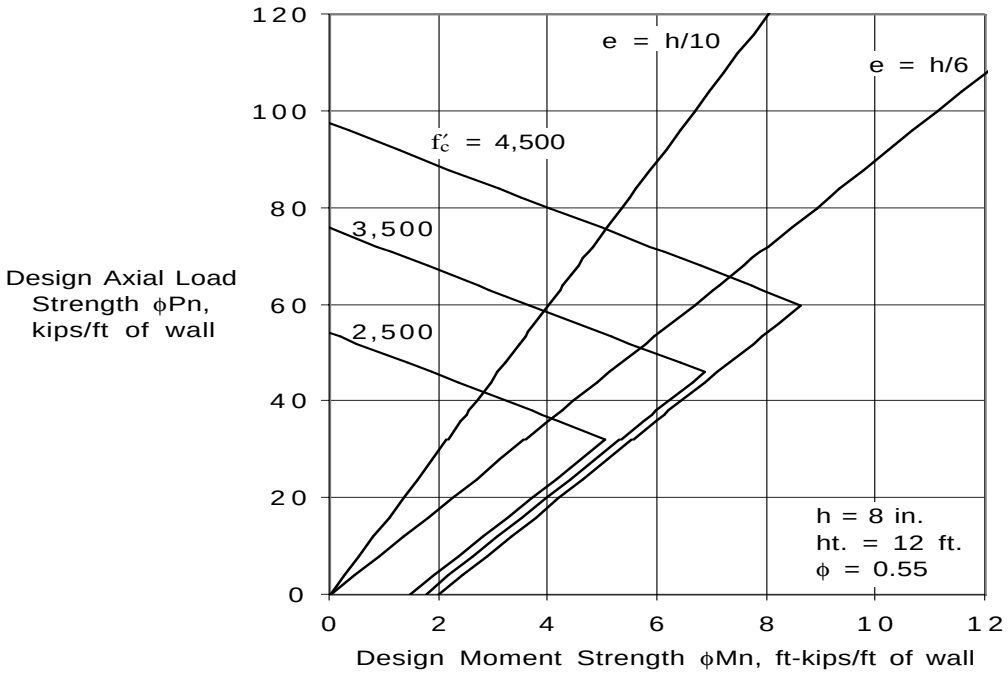


Figure 30-3 Design Strength Interaction Diagrams for 8.0-in. Wall, 12 ft in Height

Walls of plain concrete are typically used as basement walls and above grade walls in residential and small commercial buildings. In most cases the axial loads are small compared to the design axial load compressive strength, ϕP_n, of the wall. Therefore, Figs. 30-4 through 30-6 have been developed which include only the lower range of values of axial loads from Figs. 30-2 and 30-3. Where small axial loads are acting in conjunction with moments, the design is governed by flexural tension [Eq. (22-7)] rather than by combined axial and flexural compression [Eq. (22-6)]. An

examination of Eq. (22-7) will reveal that the design moment strength for lightly-loaded walls is not a function of the wall's height; therefore, the format of Figs. 30-4 through 30-6 is somewhat different than that of Figs. 30-2 and 30-3. To assist the user in verifying that the wall being designed is controlled by Eq. (22-7) instead of Eq. (22-6), Figs. 30-7 through 30-9 have been provided. These figures show the value of the design axial load strength, ϕP_n, that corresponds to the maximum value of the design moment strength, ϕM_n. For example, Fig. 30-7 shows an 8-in. wall 8 ft high has a design axial load strength, ϕP_n, of approximately 37.4 kips/ft of wall when a moment equal to the maximum design moment strength, ϕM_n, is applied. From Fig. 30-2 the maximum design moment strength, ϕM_n, is approximately 5.6 ft-kips/ft of wall when a factored load of approximately 37 kips/ft of wall is applied. When using Figs. 30-4 through 30-6, the user should always verify that the required axial load strength, P_u, is less than the value determined from Figs. 30-7 through 30-9. Also, the provisions of 22.6.6.2 should not be overlooked. They require that the thickness of the wall be not less than the larger of 1/24 the unsupported height or length of the wall and 5-1/2 in. A close examination of Figs. 30-7 through 30-9 will show that in almost every case covered, the design axial load strength exceeds 15 kips/ft of wall.

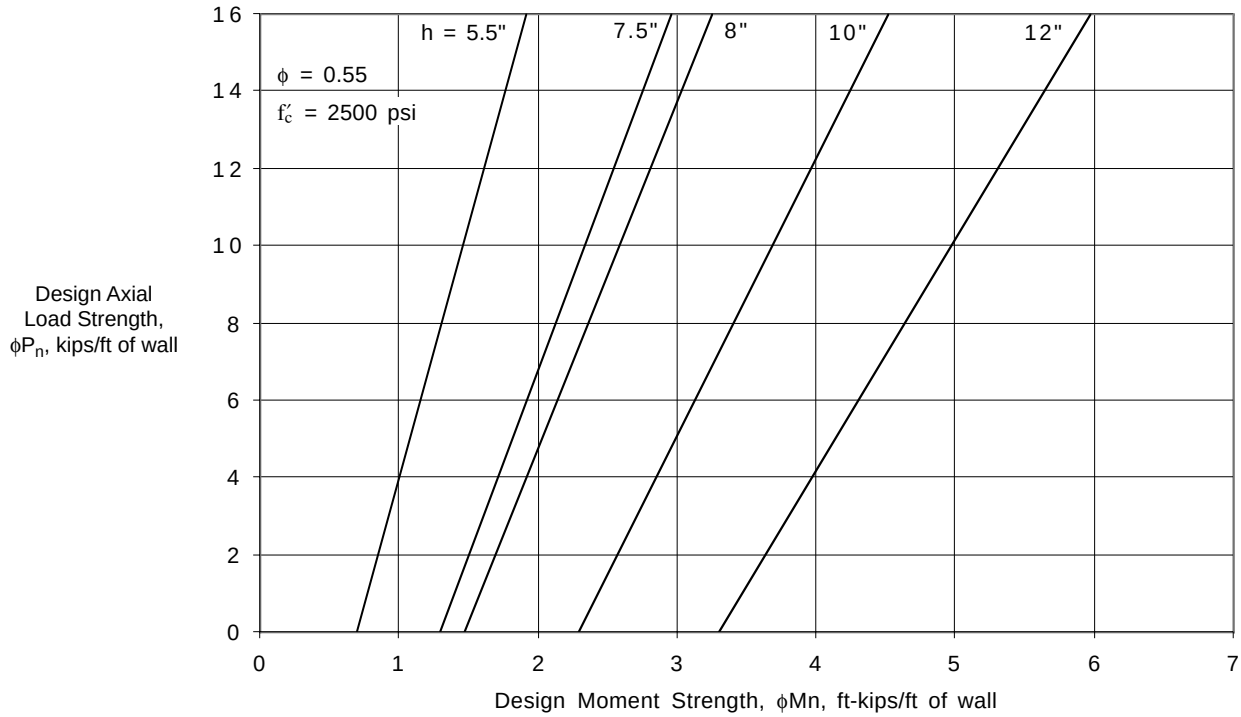


Figure 30-4 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls (f'_c = 2500 psi)

In typical construction of basement or foundation walls retaining unbalanced backfill, some walls of the building are generally nonload bearing walls. In this case, especially where the wall may be backfilled before all the dead load that will eventually be on the wall is in place, it is prudent to design the wall assuming no axial load is acting in conjunction with the lateral soil load. For this condition, equation (2) simplifies to:

$$0.6\phi\sqrt{f'_c}\,h^2 - 72M_u = 0 \qquad (3)$$

In this form, the equation can be rearranged to solve for required wall thickness:

$$h = (72M_u/0.06\phi\sqrt{f'_c})^{1/2} \qquad (4)$$

or to solve for required specified compressive strength of concrete:

$$f'_c = (72M_u/0.06\phi h^2)^2 \qquad (5)$$

In equations (3), (4), and (5), f'_c is in psi, h is in inches, and M_u is in ft-kips/ft of wall

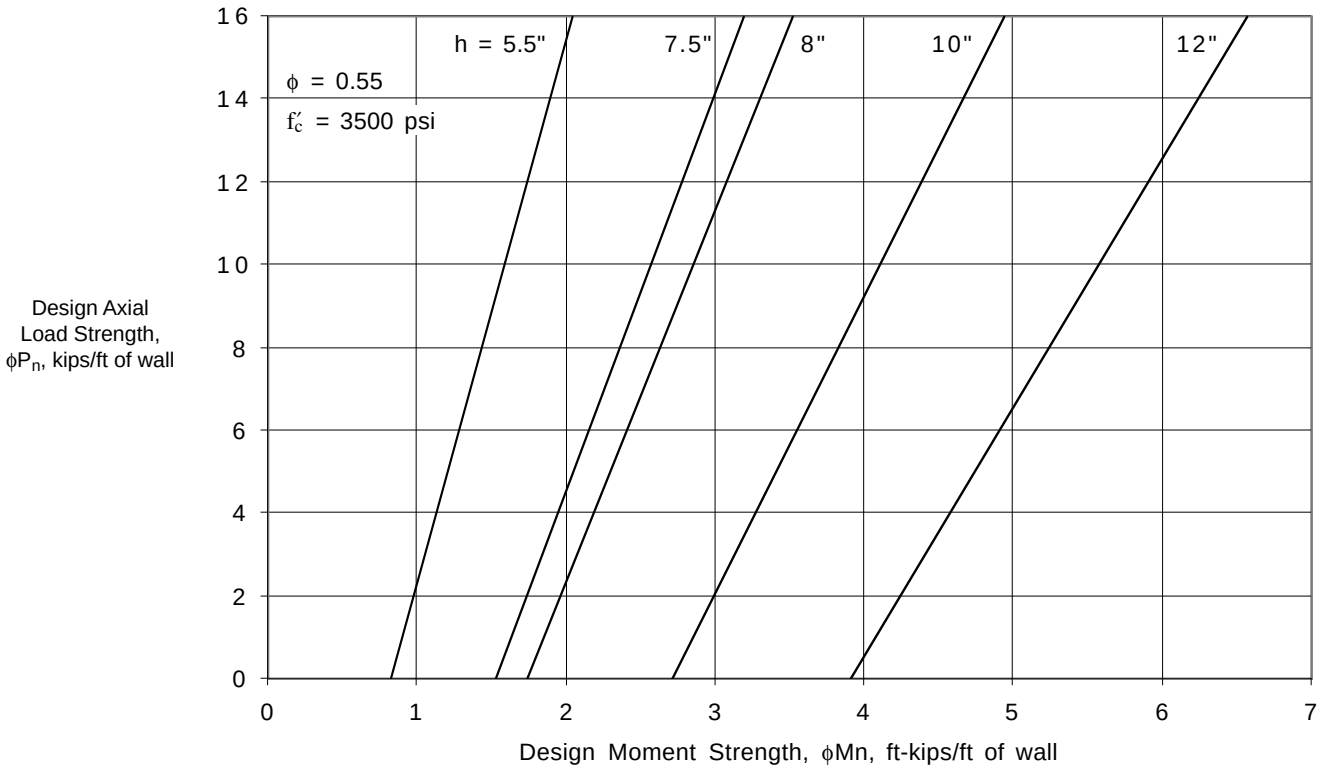


Figure 30-5 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls (f'_c = 3500 psi)

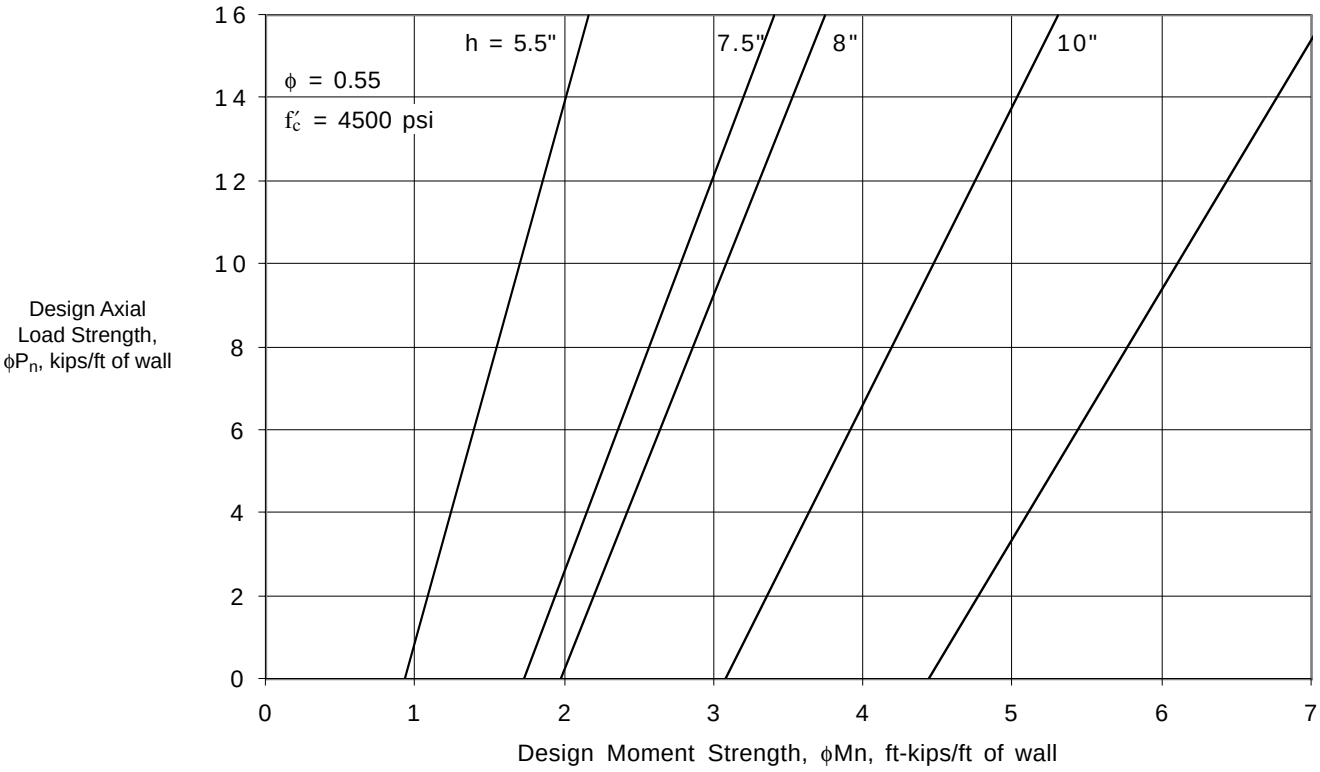


Figure 30-6 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls (f'_c = 4500 psi)

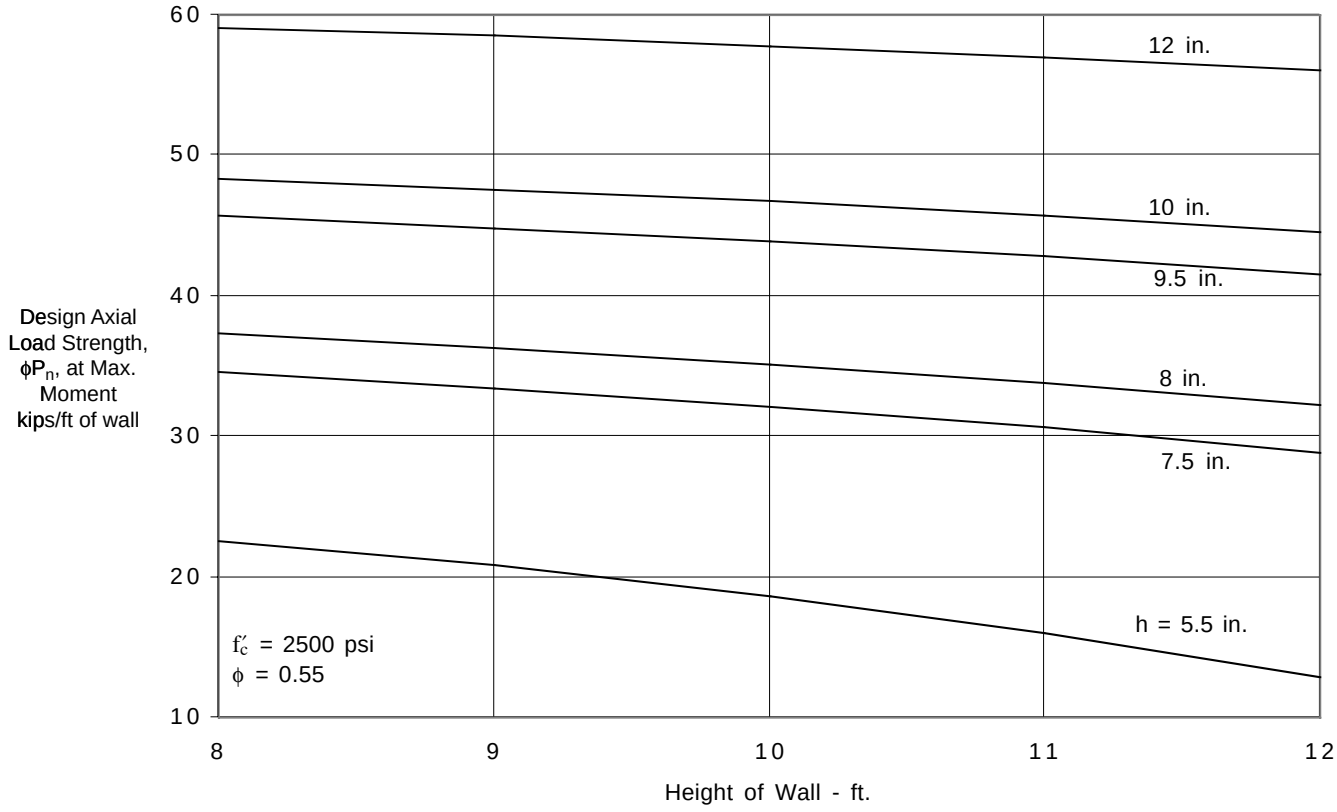


Figure 30-7 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength (f'_c = 2500 psi)

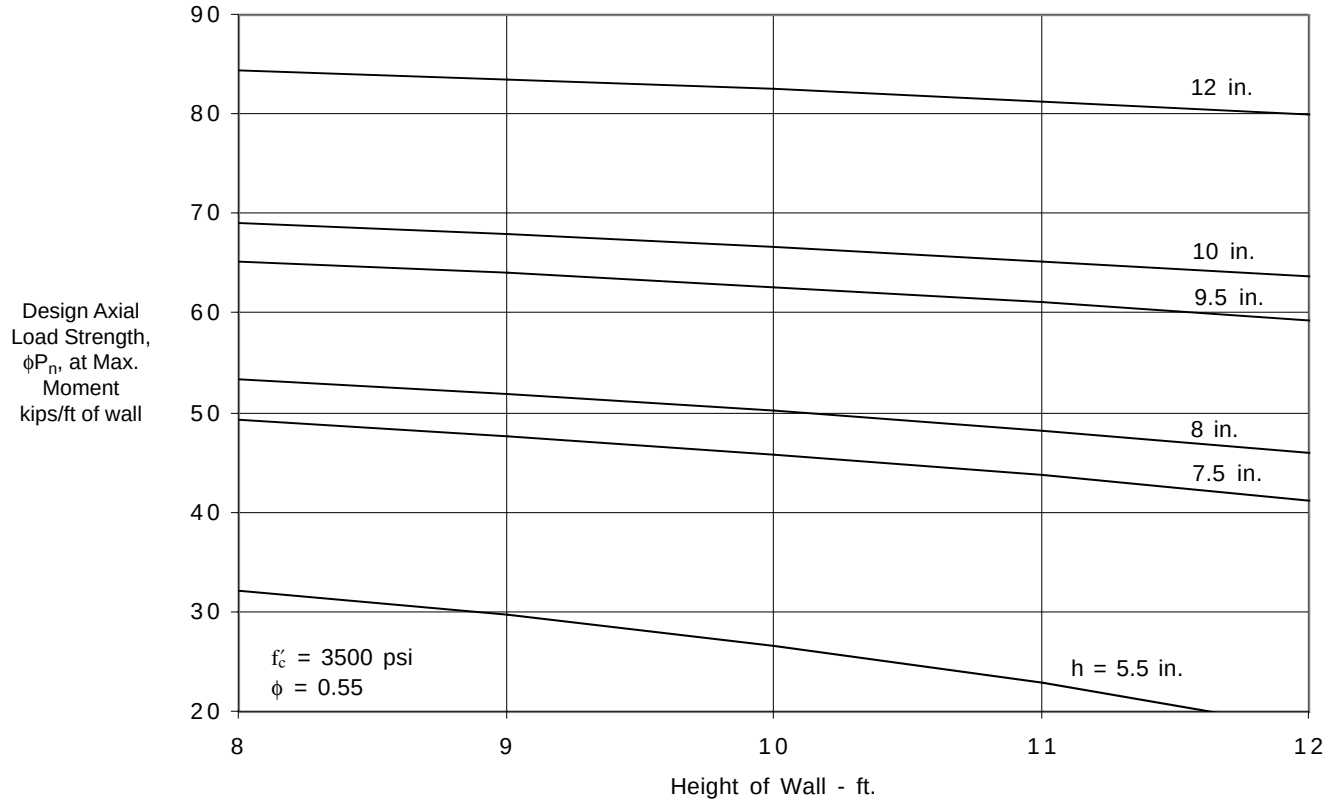


Figure 30-8 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength (f'_c = 3500 psi)

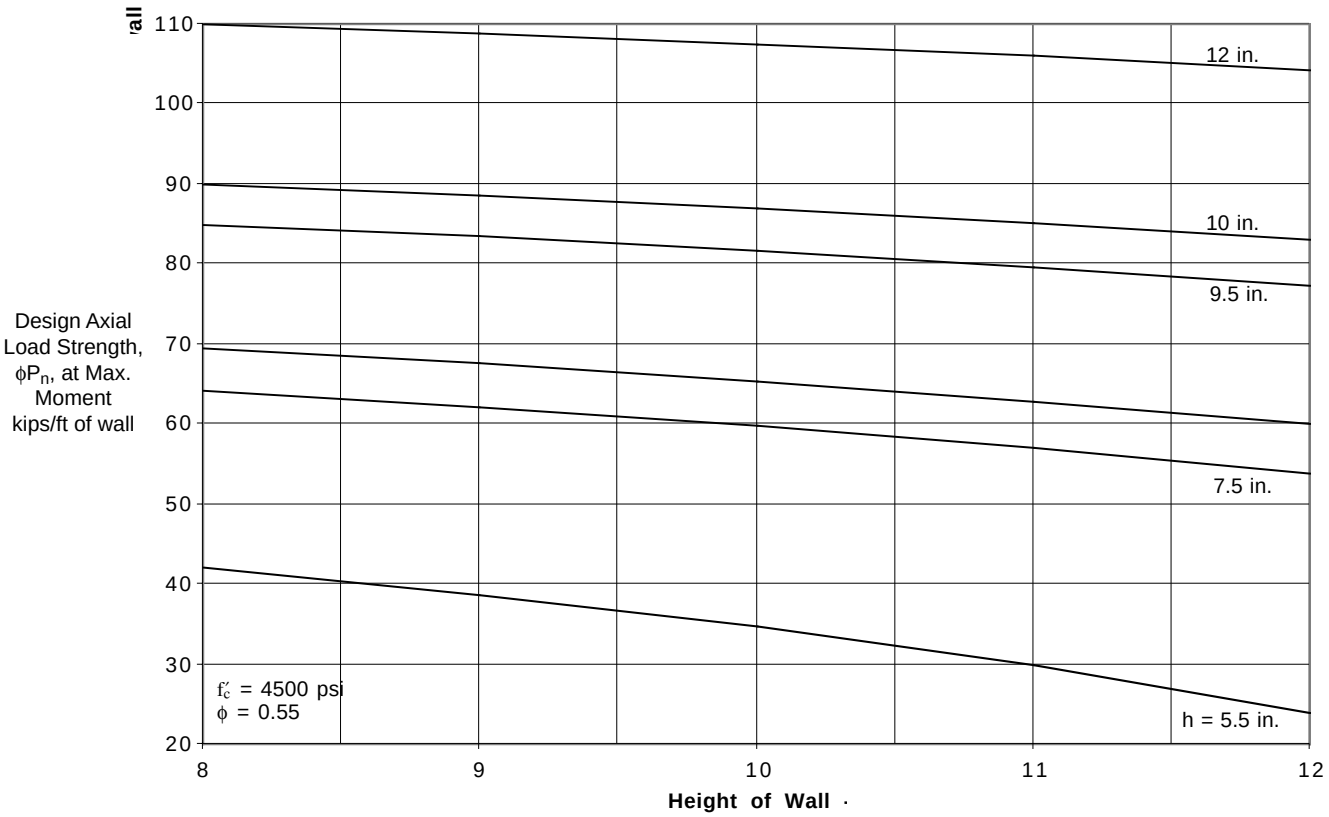


Figure 30-9 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength (f'_c = 4500 psi)

Comparison of the Two Methods

Since 22.6.5.1 limits use of the empirical method to cases where the resultant of all factored loads falls in the middle one-third of the wall, one would assume that this method is more conservative for effective eccentricities less that one-sixth the wall thickness. However, this is not the case. The empirical method has an implied eccentricity factor [ratio of strength given by Eq. (22-14) to that given by Eq. (22-5)] of 0.75 (i.e., 0.45/0.60). However, the curves of Figs. 30-2 and 30-3 have an eccentricity factor (ratio of strength where load is applied at eccentricity of 0.10h to that of load applied concentrically) of 0.725 for 2500 psi concrete. Therefore, for effective eccentricities of less than one-sixth the wall thickness, the empirical method will yield a greater nominal axial load strength (i.e., $\phi P_{nw} > \phi P_n$).

Section 22.6.6.3 requires that exterior basement and foundation walls must be not less than 7-1/2 in. thick. Some building codes permit these walls to be 5-1/2 in. in thickness. Section 22.6.6.2 requires the thickness of other walls to be not less than 5-1/2 in., but not less than 1/24 the unsupported height or length of the wall, whichever is shorter.

Other limitations of 22.6.6 must also be observed. They include: the wall must be braced against lateral translation (22.6.6.4); and not less than 2-No. 5 bars, extending at least 24 in. beyond the corners, shall be provided around all door and window openings (22.6.6.5).

To facilitate the design of simply-supported walls subject to lateral loads from wind and/or soil, Tables 30-1 and 30-2 have been provided. The tables give factored moments due to various combinations of wind and soil lateral loads, and varying backfill heights. The tables also accommodate exterior walls completely above grade (no lateral load due to backfill), as well as walls that are not subject to lateral loading from wind. Tables 30-1 and 30-2 are to be used with the load factors of 9.2, and Tables C30-1 and C30-2 are to be used with the load factors of C.2. Note that the only difference between the two tables in each set is that the first table was developed using

a load factor on wind of 1.6; whereas, the second table utilizes a load factor of 1.3. In each set of tables the load factor on the lateral load due to the soil is the same; either 1.6 or 1.7. Table 30-1 or C30-1 is to be used where load combinations in 9.2. or C.2, respectively, are being investigated where the wind load has been reduced by a directionality factor as in the IBC and ASCE 7-98 [9.2.1(b)]. Table 30-2 or C30-2 is to be used where load combinations in 9.2 or C.2, respectively, are being investigated where the wind load has not been reduced by a directionality factor, such as in the NBC, SBC, and UBC, and in editions of ASCE 7 prior to 1998.

For exterior walls partially above and below grade, the moments in the table assume that the wind load is acting in the same direction as the lateral load due to the soil (i.e., inward). Most contemporary wind design standards require that exterior walls be designed for both inward and outward acting pressures due to wind. Generally, the higher absolute value of wind loading occurs where the wall is under negative pressure (i.e., the force is acting outward on the wall). Section 9.2.1(d) stipulates that where earth pressure counteracts wind load, which is the case with wind acting outward, the load factor on H must be set equal to zero in load combination Eq. (9-6). Except for situations where the backfill height is small compared to the overall wall height, and depending upon the relative magnitude of the design wind pressure and lateral soil pressure, the moment due to lateral soil loads and inward-acting wind of Tables 30-1 and 30-2 will generally apply. For situations where outward-acting wind controls, it is simpler to design the wall as though the full height of the wall is exposed to the outward-acting wind pressure. Tables 30-1, 30-2, C30-1, and C30-2 can be used in this manner by assuming the backfill height is zero.

Before designing a structural plain concrete wall that will be resisting wind uplift and/or overturning forces, the appropriate load combinations of 9.2 or C.2 need to be investigated. If the entire wall cross-section will be in tension due to the factored axial and lateral forces, the wall must be designed as a reinforced concrete wall, or other means must be employed to transfer the uplift forces to the foundation. This condition can occur frequently in the design of walls supporting lightweight roof systems subject to net uplift forces from wind loads.

22.7 FOOTINGS

It is common practice throughout the United States, including high seismic risk areas, to use structural plain concrete footings for the support of walls for all types of structures. In addition, plain concrete is frequently used for footings supporting columns and pedestals, particularly in residential construction. These uses of structural plain concrete are permitted by Chapter 22; however, 22.7.3 prohibits the use of structural plain concrete footings which are supported on piles. In addition, contemporary building codes also have limitations on the use of structural plain concrete footings for structures in regions of moderate seismic risk, or for structures assigned to intermediate seismic performance or design categories. See section below on 22.10 for limitations on the use of structural plain concrete footings for structures in regions of high seismic risk, or for structures assigned to high seismic performance or design categories.

Many architects and engineers specify that two No. 4 or No. 5 longitudinal bars be included in footings supporting walls. However, typically these footings have no reinforcement in the transverse direction, or the amount provided is less than that required by the code to consider the footing reinforced. Such footings must be designed as structural plain concrete, since in the transverse direction the footing is subjected to flexural and possibly shear stresses due to the projection of the footing beyond the face of the supported member.

The base area of footings must be determined from unfactored loads and moments, using permissible soil bearing pressures. Once the base area of the footing is selected, factored loads and moments are used to proportion the thickness of the footing to satisfy moment and, where applicable, shear strength requirements. Sections 22.7.5 and 22.7.6.1 define the critical sections for computing factored moments and shears. The locations are summarized in Table 30-3. Figure 22-2 illustrates the location of the critical sections for beam action and two-way action shear for a footing supporting a column or pedestal.

Footings must be proportioned to satisfy the requirements for moment in accordance with Eq. (22-2). For footings supporting columns, pedestals or concrete walls, if the projection of the footing beyond the face of the supported member does not exceed the footing thickness, h, it is not necessary to check for beam action shear

since the location of the critical section for calculating shear falls outside the footing. Where beam action shear must be considered, the requirements of Eq. (22-9) must be satisfied. In addition, for footings supporting columns, pedestals or other concentrated loads, if the projection of the footing beyond the critical section exceeds h/2, it is necessary to determine if the requirements of Eq. (22-10) are satisfied for two-way action (punching) shear. Generally, flexural strength will govern the thickness design of plain concrete footings; however, the engineer should not overlook the possibility that beam action shear or two-way action shear may control. It must be remembered that the provisions of 22.4.8 require that for plain concrete members cast on soil, the thickness, h, used to compute flexural and shear strengths is the overall thickness minus 2 in. Thus, for a footing with an overall thickness of 8 in. (which is the minimum overall thickness permitted by 22.7.4), the thickness, h, used to compute strengths is 6 in. Some building codes permit 6-in. thick footings for residential and other small buildings. In this case the thickness, h, for strength computation purposes is 4 in.

Table 30-1 Factored Moments Induced in Walls by Lateral Soil and/or Wind Load (ft-kips/linear ft)
(For Use with Chapter 9 Load Factors: Soil 1.6, Wind 1.6)

Wall Ht. (ft.)	Back fill Ht. (ft.)	Unfactored Design Lateral Soil Load (psf per foot of depth)																			
		30	30	30	30	30	45	45	45	45	45	60	60	60	60	60	100	100	100	100	100
		Unfactored Design Wind Pressure (psf)																			
		0	10	20	40	80	0	10	20	40	80	0	10	20	40	80	0	10	20	40	80
8	0	0.00	0.13	0.26	0.51	1.02	0.00	0.13	0.26	0.51	1.02	0.00	0.13	0.26	0.51	1.02	0.00	0.13	0.26	0.51	1.02
8	1	0.01	0.13	0.25	0.50	1.00	0.01	0.13	0.25	0.50	1.00	0.01	0.13	0.26	0.50	1.00	0.02	0.14	0.26	0.51	1.01
8	2	0.05	0.14	0.26	0.48	0.93	0.08	0.16	0.27	0.50	0.95	0.10	0.18	0.29	0.51	0.96	0.17	0.23	0.34	0.56	1.00
8	3	0.15	0.21	0.29	0.48	0.85	0.23	0.28	0.35	0.53	0.90	0.31	0.36	0.42	0.59	0.95	0.51	0.56	0.62	0.75	1.10
8	4	0.33	0.37	0.41	0.51	0.78	0.49	0.53	0.57	0.66	0.90	0.65	0.69	0.73	0.82	1.02	1.09	1.12	1.16	1.25	1.42
8	5	0.57	0.59	0.62	0.67	0.79	0.85	0.87	0.90	0.95	1.06	1.13	1.16	1.18	1.23	1.34	1.88	1.91	1.93	1.98	2.09
8	6	0.86	0.88	0.89	0.91	0.96	1.30	1.31	1.32	1.34	1.39	1.73	1.74	1.75	1.78	1.83	2.88	2.89	2.90	2.93	2.98
8	7	1.21	1.21	1.21	1.22	1.23	1.81	1.81	1.82	1.82	1.84	2.41	2.42	2.42	2.43	2.44	4.02	4.03	4.03	4.04	4.05
8	8	1.58	1.58	1.58	1.58	1.58	2.36	2.36	2.36	2.36	2.36	3.15	3.15	3.15	3.15	3.15	5.26	5.26	5.26	5.26	5.26
9	0	0.00	0.16	0.32	0.65	1.30	0.00	0.16	0.32	0.65	1.30	0.00	0.16	0.32	0.65	1.30	0.00	0.16	0.32	0.65	1.30
9	1	0.01	0.16	0.32	0.64	1.27	0.01	0.16	0.32	0.64	1.27	0.01	0.17	0.32	0.64	1.27	0.02	0.17	0.33	0.65	1.28
9	2	0.05	0.18	0.32	0.62	1.20	0.08	0.20	0.34	0.63	1.22	0.10	0.21	0.36	0.65	1.23	0.17	0.27	0.40	0.69	1.27
9	3	0.16	0.24	0.36	0.61	1.12	0.24	0.31	0.42	0.67	1.17	0.32	0.39	0.48	0.72	1.23	0.53	0.60	0.68	0.88	1.37
9	4	0.34	0.40	0.47	0.65	1.05	0.51	0.57	0.63	0.78	1.17	0.69	0.74	0.80	0.94	1.30	1.14	1.20	1.26	1.38	1.66
9	5	0.60	0.65	0.69	0.78	1.01	0.91	0.95	0.99	1.08	1.28	1.21	1.25	1.29	1.38	1.57	2.01	2.05	2.09	2.18	2.36
9	6	0.94	0.96	0.99	1.04	1.16	1.41	1.43	1.46	1.51	1.62	1.88	1.90	1.93	1.98	2.09	3.13	3.15	3.18	3.23	3.33
9	7	1.33	1.35	1.36	1.38	1.43	2.00	2.01	2.03	2.05	2.10	2.67	2.68	2.69	2.72	2.77	4.45	4.46	4.47	4.50	4.55
9	8	1.78	1.78	1.78	1.79	1.80	2.66	2.67	2.67	2.68	2.69	3.55	3.56	3.56	3.57	3.58	5.92	5.92	5.93	5.93	5.95
9	9	2.24	2.24	2.24	2.24	2.24	3.37	3.37	3.37	3.37	3.37	4.49	4.49	4.49	4.49	4.49	7.48	7.48	7.48	7.48	7.48
10	0	0.00	0.20	0.40	0.80	1.60	0.00	0.20	0.40	0.80	1.60	0.00	0.20	0.40	0.80	1.60	0.00	0.20	0.40	0.80	1.60
10	1	0.01	0.20	0.40	0.79	1.57	0.01	0.20	0.40	0.79	1.57	0.01	0.20	0.40	0.79	1.58	0.02	0.21	0.41	0.80	1.58
10	2	0.05	0.22	0.40	0.77	1.51	0.08	0.23	0.42	0.78	1.52	0.11	0.25	0.43	0.80	1.54	0.18	0.30	0.48	0.84	1.58
10	3	0.16	0.28	0.44	0.76	1.43	0.25	0.35	0.50	0.82	1.48	0.33	0.42	0.56	0.87	1.53	0.55	0.64	0.74	1.03	1.67
10	4	0.36	0.44	0.54	0.80	1.35	0.54	0.61	0.70	0.93	1.47	0.71	0.79	0.87	1.08	1.60	1.19	1.27	1.34	1.52	1.96
10	5	0.64	0.70	0.76	0.91	1.31	0.95	1.01	1.08	1.22	1.55	1.27	1.33	1.39	1.53	1.82	2.12	2.18	2.24	2.37	2.64
10	6	1.00	1.04	1.09	1.18	1.39	1.50	1.54	1.59	1.68	1.87	2.00	2.04	2.09	2.18	2.36	3.33	3.38	3.42	3.51	3.69
10	7	1.44	1.47	1.49	1.55	1.66	2.16	2.19	2.22	2.27	2.38	2.88	2.91	2.94	2.99	3.10	4.81	4.83	4.86	4.91	5.02
10	8	1.95	1.96	1.97	2.00	2.05	2.92	2.93	2.95	2.97	3.02	3.89	3.91	3.92	3.94	3.99	6.49	6.50	6.52	6.54	6.59
10	9	2.50	2.50	2.51	2.51	2.53	3.75	3.75	3.76	3.76	3.78	5.00	5.00	5.01	5.01	5.03	8.33	8.34	8.34	8.35	8.36
10	10	3.08	3.08	3.08	3.08	3.08	4.62	4.62	4.62	4.62	4.62	6.16	6.16	6.16	6.16	6.16	10.26	10.26	10.26	10.26	10.26
12	0	0.00	0.29	0.58	1.15	2.30	0.00	0.29	0.58	1.15	2.30	0.00	0.29	0.58	1.15	2.30	0.00	0.29	0.58	1.15	2.30
12	1	0.01	0.29	0.57	1.14	2.28	0.01	0.29	0.57	1.14	2.28	0.01	0.29	0.58	1.14	2.28	0.02	0.30	0.58	1.15	2.29
12	2	0.06	0.30	0.58	1.12	2.21	0.08	0.32	0.59	1.14	2.22	0.11	0.34	0.61	1.15	2.24	0.18	0.39	0.65	1.20	2.28
12	3	0.17	0.36	0.61	1.12	2.13	0.26	0.43	0.67	1.17	2.18	0.34	0.50	0.73	1.23	2.23	0.57	0.70	0.90	1.38	2.38
12	4	0.38	0.51	0.71	1.15	2.06	0.57	0.69	0.86	1.28	2.18	0.76	0.88	1.02	1.42	2.30	1.26	1.38	1.51	1.83	2.66
12	5	0.69	0.80	0.92	1.25	2.01	1.03	1.14	1.25	1.53	2.25	1.37	1.48	1.59	1.84	2.51	2.29	2.39	2.50	2.73	3.26
12	6	1.10	1.19	1.28	1.49	2.03	1.65	1.74	1.83	2.02	2.46	2.20	2.28	2.37	2.56	2.97	3.66	3.75	3.84	4.02	4.40
12	7	1.61	1.68	1.75	1.89	2.20	2.42	2.49	2.55	2.69	2.99	3.23	3.29	3.36	3.50	3.78	5.38	5.45	5.51	5.65	5.92
12	8	2.22	2.27	2.31	2.41	2.61	3.34	3.38	3.43	3.52	3.71	4.45	4.49	4.54	4.63	4.82	7.41	7.46	7.50	7.59	7.78
12	9	2.92	2.94	2.97	3.03	3.14	4.37	4.40	4.43	4.48	4.59	5.83	5.86	5.89	5.94	6.05	9.72	9.75	9.77	9.83	9.94
12	10	3.68	3.69	3.70	3.73	3.78	5.51	5.53	5.54	5.56	5.62	7.35	7.36	7.38	7.40	7.45	12.25	12.27	12.28	12.30	12.35
12	11	4.48	4.49	4.49	4.50	4.51	6.73	6.73	6.73	6.74	6.75	8.97	8.97	8.98	8.98	8.99	14.95	14.95	14.95	14.96	14.97
12	12	5.32	5.32	5.32	5.32	5.32	7.98	7.98	7.98	7.98	7.98	10.64	10.64	10.64	10.64	10.64	17.74	17.74	17.74	17.74	17.74

Table 30-2 Factored Moments Induced in Walls by Lateral Soil and/or Wind Load (ft-kips/linear ft)
(For Use with Chapter 9 Load Factors: Soil 1.6, Wind 1.3x)

Wall Ht. (ft.)	Back fill Ht. (ft.)	Unfactored Design Lateral Soil Load (psf per foot of depth)																			
		30	30	30	30	30	45	45	45	45	45	60	60	60	60	60	100	100	100	100	100
		Unfactored Design Wind Pressure (psf)																			
		0	10	20	40	80	0	10	20	40	80	0	10	20	40	80	0	10	20	40	80
8	0	0.00	0.10	0.21	0.42	0.83	0.00	0.10	0.21	0.42	0.83	0.00	0.10	0.21	0.42	0.83	0.00	0.10	0.21	0.42	0.83
8	1	0.01	0.10	0.21	0.41	0.81	0.01	0.11	0.21	0.41	0.81	0.01	0.11	0.21	0.41	0.81	0.02	0.11	0.21	0.42	0.82
8	2	0.05	0.12	0.21	0.40	0.76	0.08	0.14	0.23	0.41	0.78	0.10	0.16	0.25	0.43	0.79	0.17	0.22	0.30	0.47	0.83
8	3	0.15	0.20	0.26	0.41	0.71	0.23	0.27	0.32	0.46	0.76	0.31	0.35	0.40	0.52	0.81	0.51	0.55	0.60	0.70	0.96
8	4	0.33	0.36	0.39	0.47	0.68	0.49	0.52	0.55	0.62	0.80	0.65	0.68	0.72	0.78	0.94	1.09	1.12	1.15	1.21	1.35
8	5	0.57	0.59	0.61	0.65	0.74	0.85	0.87	0.89	0.93	1.02	1.13	1.15	1.17	1.21	1.30	1.88	1.90	1.92	1.96	2.05
8	6	0.86	0.87	0.88	0.90	0.94	1.30	1.31	1.32	1.34	1.38	1.73	1.74	1.75	1.77	1.81	2.88	2.89	2.90	2.92	2.96
8	7	1.21	1.21	1.21	1.22	1.23	1.81	1.81	1.82	1.82	1.83	2.41	2.42	2.42	2.43	2.44	4.02	4.03	4.03	4.04	4.05
8	8	1.58	1.58	1.58	1.58	1.58	2.36	2.36	2.36	2.36	2.36	3.15	3.15	3.15	3.15	3.15	5.26	5.26	5.26	5.26	5.26
9	0	0.00	0.13	0.26	0.53	1.05	0.00	0.13	0.26	0.53	1.05	0.00	0.13	0.26	0.53	1.05	0.00	0.13	0.26	0.53	1.05
9	1	0.01	0.13	0.26	0.52	1.03	0.01	0.13	0.26	0.52	1.03	0.01	0.14	0.26	0.52	1.04	0.02	0.14	0.27	0.53	1.04
9	2	0.05	0.15	0.27	0.51	0.98	0.08	0.17	0.29	0.52	1.00	0.10	0.19	0.30	0.54	1.01	0.17	0.24	0.35	0.58	1.06
9	3	0.16	0.22	0.32	0.52	0.93	0.24	0.30	0.38	0.57	0.98	0.32	0.38	0.44	0.63	1.04	0.53	0.59	0.65	0.80	1.18
9	4	0.34	0.39	0.44	0.58	0.90	0.51	0.56	0.61	0.72	1.02	0.69	0.73	0.78	0.88	1.15	1.14	1.19	1.23	1.33	1.55
9	5	0.60	0.64	0.67	0.75	0.92	0.91	0.94	0.97	1.04	1.20	1.21	1.24	1.27	1.34	1.49	2.01	2.05	2.08	2.15	2.29
9	6	0.94	0.96	0.98	1.02	1.11	1.41	1.43	1.45	1.49	1.58	1.88	1.90	1.92	1.96	2.05	3.13	3.15	3.17	3.21	3.29
9	7	1.33	1.34	1.35	1.37	1.42	2.00	2.01	2.02	2.04	2.08	2.67	2.68	2.69	2.71	2.75	4.45	4.46	4.47	4.49	4.53
9	8	1.78	1.78	1.78	1.79	1.80	2.66	2.67	2.67	2.68	2.69	3.55	3.56	3.56	3.56	3.57	5.92	5.92	5.93	5.93	5.94
9	9	2.24	2.24	2.24	2.24	2.24	3.37	3.37	3.37	3.37	3.37	4.49	4.49	4.49	4.49	4.49	7.48	7.48	7.48	7.48	7.48
10	0	0.00	0.16	0.33	0.65	1.30	0.00	0.16	0.33	0.65	1.30	0.00	0.16	0.33	0.65	1.30	0.00	0.16	0.33	0.65	1.30
10	1	0.01	0.16	0.32	0.64	1.28	0.01	0.17	0.32	0.64	1.28	0.01	0.17	0.33	0.65	1.28	0.02	0.17	0.33	0.65	1.29
10	2	0.05	0.18	0.33	0.63	1.23	0.08	0.20	0.35	0.65	1.24	0.11	0.22	0.36	0.66	1.26	0.18	0.27	0.41	0.71	1.30
10	3	0.16	0.25	0.38	0.64	1.18	0.25	0.32	0.44	0.70	1.23	0.33	0.40	0.50	0.75	1.28	0.55	0.62	0.70	0.92	1.43
10	4	0.36	0.42	0.50	0.70	1.14	0.54	0.60	0.67	0.84	1.27	0.71	0.78	0.84	0.99	1.40	1.19	1.25	1.31	1.45	1.77
10	5	0.64	0.69	0.74	0.85	1.15	0.95	1.00	1.05	1.16	1.41	1.27	1.32	1.37	1.48	1.71	2.12	2.17	2.22	2.32	2.53
10	6	1.00	1.04	1.07	1.15	1.31	1.50	1.54	1.57	1.64	1.80	2.00	2.04	2.07	2.14	2.29	3.33	3.37	3.40	3.47	3.62
10	7	1.44	1.46	1.48	1.53	1.62	2.16	2.18	2.21	2.25	2.34	2.88	2.90	2.93	2.97	3.06	4.81	4.83	4.85	4.89	4.98
10	8	1.95	1.96	1.97	1.99	2.03	2.92	2.93	2.94	2.96	3.00	3.89	3.90	3.91	3.93	3.98	6.49	6.50	6.51	6.53	6.57
10	9	2.50	2.50	2.51	2.51	2.52	3.75	3.75	3.75	3.76	3.77	5.00	5.00	5.00	5.01	5.02	8.33	8.34	8.34	8.34	8.35
10	10	3.08	3.08	3.08	3.08	3.08	4.62	4.62	4.62	4.62	4.62	6.16	6.16	6.16	6.16	6.16	10.26	10.26	10.26	10.26	10.26
12	0	0.00	0.23	0.47	0.94	1.87	0.00	0.23	0.47	0.94	1.87	0.00	0.23	0.47	0.94	1.87	0.00	0.23	0.47	0.94	1.87
12	1	0.01	0.23	0.47	0.93	1.85	0.01	0.24	0.47	0.93	1.85	0.01	0.24	0.47	0.93	1.85	0.02	0.24	0.47	0.94	1.86
12	2	0.06	0.25	0.47	0.92	1.80	0.08	0.27	0.49	0.93	1.82	0.11	0.29	0.51	0.95	1.83	0.18	0.34	0.55	0.99	1.87
12	3	0.17	0.32	0.52	0.93	1.75	0.26	0.39	0.58	0.98	1.80	0.34	0.46	0.64	1.04	1.85	0.57	0.68	0.82	1.19	2.00
12	4	0.38	0.48	0.63	0.98	1.72	0.57	0.67	0.79	1.12	1.84	0.76	0.86	0.97	1.26	1.97	1.26	1.36	1.46	1.69	2.33
12	5	0.69	0.77	0.87	1.12	1.73	1.03	1.12	1.21	1.41	1.97	1.37	1.46	1.55	1.74	2.24	2.29	2.37	2.46	2.64	3.05
12	6	1.10	1.17	1.24	1.40	1.80	1.65	1.72	1.79	1.94	2.28	2.20	2.27	2.34	2.49	2.81	3.66	3.73	3.80	3.95	4.25
12	7	1.61	1.67	1.72	1.84	2.08	2.42	2.47	2.53	2.64	2.87	3.23	3.28	3.34	3.44	3.67	5.38	5.43	5.49	5.59	5.82
12	8	2.22	2.26	2.30	2.37	2.53	3.34	3.37	3.41	3.48	3.64	4.45	4.48	4.52	4.60	4.75	7.41	7.45	7.49	7.56	7.71
12	9	2.92	2.94	2.96	3.00	3.10	4.37	4.40	4.42	4.46	4.55	5.83	5.85	5.88	5.92	6.01	9.72	9.74	9.76	9.81	9.90
12	10	3.68	3.69	3.70	3.72	3.76	5.51	5.52	5.53	5.55	5.60	7.35	7.36	7.37	7.39	7.43	12.25	12.26	12.27	12.29	12.33
12	11	4.48	4.49	4.49	4.49	4.51	6.73	6.73	6.73	6.74	6.75	8.97	8.97	8.97	8.98	8.99	14.95	14.95	14.95	14.96	14.97
12	12	5.32	5.32	5.32	5.32	5.32	7.98	7.98	7.98	7.98	7.98	10.64	10.64	10.64	10.64	10.64	17.74	17.74	17.74	17.74	17.74

Figure 30-10 has been provided to aid in the selection of footing thickness to satisfy flexural strength requirements. The figure is entered with the *factored* soil bearing pressure. Project vertically upward to the curve that represents the length that the footing projects beyond the critical section at which the moment must be calculated (see Table 30-3). Read horizontally to the left to determine the minimum required footing thickness. Two (2) in. must be added to this value. The thicknesses in the figure are based on a specified compressive strength of concrete, f'_c of 2500 psi. For higher strength concrete, the thickness can be reduced by multiplying by the factor:

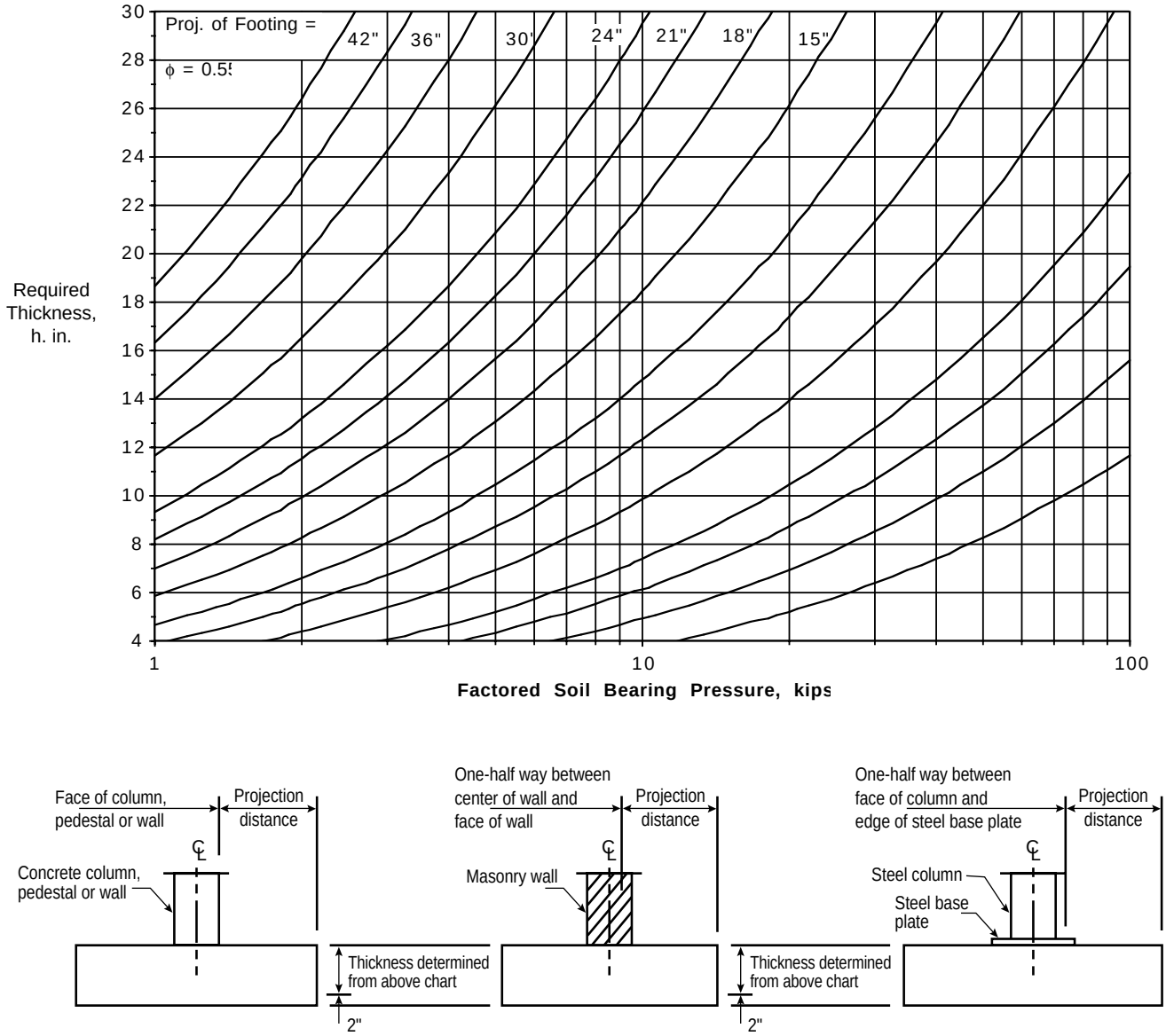
$$(2500/\text{specified compressive strength of concrete})^{0.25}$$

As the exponent in the equation suggests, a large increase in concrete strength results in only a small decrease in footing thickness. For example, doubling the concrete strength only reduces the thickness 16 percent.

*Table 30-3 Locations for Computing Moments and Shears in Footings**

Supported Member	Moment	Shear – Beam Action	Shear – Two-Way (punching)
Concrete Wall	at face of wall	h from face of wall	Not Applicable
Masonry Wall	1/2 way between center of wall and face of wall	h from face of wall	Not Applicable
Column or Pedestal	at face of column or pedestal	h from face of column or pedestal	h/2 from face of column or pedestal
Column with Steel Base Plate	1/2 way between face of column and edge of steel base plate	h from 1/2 way between face of column and edge of steel base plate	h/2 from 1/2 way between face of column edge of steel base plate

* h = thickness of footing for moment and shear computation purposes.



* For f'_c greater than 2500 psi, multiple thickness determined from above chart by $(2500/f'_c)^{0.25}$

Figure 30-10 Thickness of Footing Required to Satisfy Flexural Strength for Various Projection Distances, in. (f'_c = 2500 psi)*

22.8 PEDESTALS

Pedestals of plain concrete are permitted by 22.8.2 provided the unsupported height does not exceed three times the least plan dimension. The design must consider all vertical and lateral loads to which the pedestal will be subjected. The nominal bearing strength, B_n, must be determined from Eq. (22-12). Where moments are induced due to eccentricity of the axial load and/or lateral loads, the pedestal shall be designed for both flexural and axial loads and satisfy interaction Eqs. (22-6) and (22-7). In Eq. (22-6), P_n is replaced with B_n, nominal bearing strength.

Pedestal-like members with heights exceeding three times the least lateral dimension are defined as *columns* by the code and must be designed as reinforced concrete members. Columns of structural plain concrete are prohibited by Chapter 22.

Some contemporary building codes prohibit the use of structural plain concrete pedestals to resist seismic lateral forces in structures at moderate seismic risk or assigned to intermediate seismic performance or design categories. See Table 1-3 and section below on 22.10.

22.10 PLAIN CONCRETE IN EARTHQUAKE-RESISTING STRUCTURES

The ’99 edition of the code included a new section 22.10 to address a seismic design issue not covered previously. This concerns the use of plain concrete elements in structures subject to earthquake ground motions intense enough to cause significant structural damage to the elements or partial or total collapse of the structure. By default, the model building codes in use in the U.S. had assumed responsibility for this subject. The requirements, based on similar provisions in *The BOCA National Building Code*[30.4] and *Standard Building Code*[30.5], prohibit the use of structural plain concrete foundation elements for structures in regions at high seismic risk, or for structures assigned to high seismic performance or design categories, except for three specific cases cited in the provisions. See Table 1-3 for an explanation of how seismic risk assigned by the model building codes can be correlated to the requirements of ACI 318.

The provisions prohibit the use of structural plain concrete foundation elements forstructures assigned a “high” seismic risk in accordance with Table 1-3, except for the following specific cases. They are:

1. In detached one-and two-family dwellings not exceeding three stories in height and constructed with wood or steel stud bearing walls, the following are permitted:
    a. plain concrete footings supporting walls, columns or pedestals; and
    b. plain concrete foundation or basement walls provided
        i. the wall is not less than 7-1/2 in. thick, and
        ii. it retains no more than 4 ft of unbalanced fill.

2. In structures other than covered by 1 above, plain concrete footings supporting cast-in-place reinforced concrete walls or reinforced masonry walls are permitted provided the footing has at least two continuous No. 4 longitudinal reinforcing bars that provide an area of steel of not less than 0.002 times the gross transverse cross-sectional area of the footing. Continuity of reinforcement must be provided at corners and intersections. In the 2002 ACI code, the requirement was added to limit the use of this provision to situations where the supported wall is either of cast-in-place reinforced concrete or reinforced masonry.

Although Chapter 22 of the code has no limitations on the use of structural plain concrete elements for structures in areas of moderate seismic risk, or for structures assigned to intermediate seismic performance or

design categories in accordance with Table 1-3, model building codes in use in the U.S. either prohibit their use or generally require that some reinforcement be included to provide some ductility and tie the elements together. Where construction is contemplated in these areas, the legally adopted building code should be consulted to determine the specific limitations.

REFERENCES

30.1 *Joints in Walls Below Grade*, Portland Cement Association, Skokie, IL, 1982.

30.2 *Building Movements and Joints*, Portland Cement Association, Skokie, IL, 1982.

30.3 Kosmatka, Steven H., Kerkhoff, Beatrix, and Panarese, William C.; *Design and Control of Concrete Mixtures*, EB001, Fourteenth Edition, Portland Cement Association, Skokie, IL, 1994.

30.4 *The BOCA National Building Code*, Building Officials and Code Administrators International, Country Club Hills, IL, 1999.

30.5 *Standard Building Code*, Southern Building Code Congress International, Birmingham, AL 1999.

APPENDIX 30A

This Appendix includes figures and tables that parallel those of Part 30. The figures of this Appendix are compatible with the load factors and strength reduction factor (ϕ = 0.65) of ACI 318-02, Appendix C. (In the body of Part 30, figures and tables are compatible with load factors and strength reduction factor (ϕ = 0.55) of ACI 318-02, Chapter 9.)

Included are the following:

Table C30-1	*Table C30-1 Factored Moments Induced in Walls by Lateral Soil and/or Wind Loads (ft/kips/linear ft) (For Use with Appendix C Load Factors: Soil 1.7, Wind 1.6)*
Table C30-2	*Table C30-1 Factored Moments Induced in Walls by Lateral Soil and/or Wind Loads (ft/kips/linear ft) (For Use with Appendix C Load Factors: Soil 1.7, Wind 1.3)*
Figure C30-1(a-c)	*Figure C30-1 Design Axial Load Strength, P_{nw}, of Plain Concrete Walls using the Empirical Design Method*
Figure C30-2	*Figure C30-2 Design Strength Interaction Diagrams for 8.0-in Wall, 8 ft in Height*
Figure C30-3	*Figure C30-3 Design Strength Interaction Diagrams for 8.0-in Wall, 12 ft in Height*
Figure C30-4	*Figure C30-4 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls (f'_c = 2500 psi)*
Figure C30-5	*Figure C30-5 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls (f'_c = 3500 psi)*
Figure C30-6	*Figure C30-6 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls (f'_c = 4500 psi)*
Figure C30-7	*Figure C30-7 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength (f'_c = 2500 psi)*
Figure C30-8	*Figure C30-8 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength (f'_c = 3500 psi)*
Figure C30-9	*Figure C30-9 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength (f'_c = 4500 psi)*
Figure C30-10	*Figure C30-10 Thickness of Footing Required to Satisfy Flexural Strength for Various Projection Distances, in. (f'_c = 2500 psi*)*

Table C30-1 Factored Moments Induced in Walls by Lateral Soil and/or Wind Loads (ft/kips/linear ft)
(For Use with Appendix C Load Factors: Soil 1.7, Wind 1.6)

Wall Ht. (ft.)	Back fill Ht. (ft.)	Unfactored Design Lateral Soil Load (psf per foot of depth)																			
		30	30	30	30	30	45	45	45	45	45	60	60	60	60	60	100	100	100	100	100
		Unfactored Design Wind Pressure (psf)																			
		0	10	20	40	80	0	10	20	40	80	0	10	20	40	80	0	10	20	40	80
8	0	0.00	0.13	0.26	0.51	1.02	0.00	0.13	0.26	0.51	1.02	0.00	0.13	0.26	0.51	1.02	0.00	0.13	0.26	0.51	1.02
8	1	0.01	0.13	0.25	0.50	1.00	0.01	0.13	0.25	0.50	1.00	0.02	0.13	0.26	0.50	1.00	0.03	0.14	0.26	0.51	1.01
8	2	0.05	0.15	0.26	0.48	0.93	0.08	0.17	0.28	0.50	0.95	0.11	0.19	0.29	0.52	0.96	0.18	0.24	0.34	0.56	1.01
8	3	0.16	0.22	0.30	0.48	0.86	0.25	0.30	0.37	0.54	0.91	0.33	0.38	0.44	0.60	0.97	0.55	0.60	0.65	0.78	1.12
8	4	0.35	0.39	0.43	0.53	0.80	0.52	0.56	0.60	0.69	0.92	0.69	0.73	0.77	0.86	1.06	1.15	1.19	1.23	1.31	1.49
8	5	0.60	0.63	0.65	0.70	0.82	0.90	0.93	0.95	1.00	1.11	1.20	1.23	1.25	1.30	1.41	2.00	2.03	2.05	2.10	2.20
8	6	0.92	0.93	0.94	0.97	1.02	1.38	1.39	1.40	1.43	1.48	1.84	1.85	1.86	1.88	1.93	3.06	3.07	3.08	3.11	3.16
8	7	1.28	1.29	1.29	1.30	1.31	1.92	1.93	1.93	1.94	1.95	2.57	2.57	2.57	2.58	2.59	4.28	4.28	4.28	4.29	4.30
8	8	1.68	1.68	1.68	1.68	1.68	2.51	2.51	2.51	2.51	2.51	3.35	3.35	3.35	3.35	3.35	5.58	5.58	5.58	5.58	5.58
9	0	0.00	0.16	0.32	0.65	1.30	0.00	0.16	0.32	0.65	1.30	0.00	0.16	0.32	0.65	1.30	0.00	0.16	0.32	0.65	1.30
9	1	0.01	0.16	0.32	0.64	1.27	0.01	0.16	0.32	0.64	1.27	0.02	0.17	0.32	0.64	1.27	0.03	0.17	0.33	0.65	1.28
9	2	0.06	0.18	0.33	0.62	1.20	0.08	0.20	0.34	0.64	1.22	0.11	0.22	0.36	0.65	1.24	0.19	0.27	0.41	0.70	1.28
9	3	0.17	0.25	0.37	0.62	1.13	0.26	0.33	0.43	0.68	1.18	0.34	0.41	0.50	0.74	1.24	0.57	0.63	0.71	0.91	1.39
9	4	0.36	0.42	0.49	0.66	1.07	0.55	0.60	0.66	0.81	1.19	0.73	0.78	0.84	0.98	1.33	1.21	1.27	1.33	1.45	1.73
9	5	0.64	0.68	0.73	0.82	1.04	0.96	1.00	1.05	1.13	1.33	1.28	1.32	1.37	1.45	1.64	2.14	2.18	2.22	2.31	2.48
9	6	1.00	1.02	1.05	1.10	1.21	1.49	1.52	1.55	1.60	1.71	1.99	2.02	2.04	2.10	2.20	3.32	3.35	3.37	3.42	3.53
9	7	1.42	1.43	1.44	1.47	1.52	2.13	2.14	2.15	2.18	2.23	2.84	2.85	2.86	2.88	2.93	4.73	4.74	4.75	4.77	4.82
9	8	1.89	1.89	1.89	1.90	1.91	2.83	2.83	2.84	2.84	2.86	3.77	3.78	3.78	3.79	3.80	6.29	6.29	6.30	6.30	6.32
9	9	2.39	2.39	2.39	2.39	2.39	3.58	3.58	3.58	3.58	3.58	4.77	4.77	4.77	4.77	4.77	7.95	7.95	7.95	7.95	7.95
10	0	0.00	0.20	0.40	0.80	1.60	0.00	0.20	0.40	0.80	1.60	0.00	0.20	0.40	0.80	1.60	0.00	0.20	0.40	0.80	1.60
10	1	0.01	0.20	0.40	0.79	1.57	0.01	0.20	0.40	0.79	1.57	0.02	0.20	0.40	0.79	1.58	0.03	0.21	0.41	0.80	1.58
10	2	0.06	0.22	0.40	0.77	1.51	0.09	0.24	0.42	0.79	1.52	0.11	0.26	0.44	0.80	1.54	0.19	0.31	0.49	0.85	1.59
10	3	0.18	0.29	0.44	0.77	1.43	0.26	0.36	0.51	0.83	1.49	0.35	0.44	0.57	0.89	1.54	0.58	0.67	0.77	1.06	1.70
10	4	0.38	0.46	0.56	0.82	1.37	0.57	0.65	0.73	0.96	1.50	0.76	0.83	0.92	1.11	1.63	1.26	1.34	1.42	1.59	2.02
10	5	0.68	0.74	0.80	0.95	1.34	1.01	1.07	1.14	1.27	1.60	1.35	1.41	1.47	1.60	1.90	2.25	2.31	2.37	2.50	2.77
10	6	1.06	1.11	1.15	1.24	1.45	1.59	1.64	1.68	1.77	1.96	2.13	2.17	2.21	2.30	2.49	3.54	3.59	3.63	3.72	3.89
10	7	1.53	1.56	1.58	1.64	1.75	2.30	2.32	2.35	2.40	2.51	3.06	3.09	3.12	3.17	3.28	5.11	5.13	5.16	5.21	5.32
10	8	2.07	2.08	2.09	2.12	2.17	3.10	3.12	3.13	3.15	3.20	4.14	4.15	4.16	4.19	4.24	6.90	6.91	6.92	6.95	7.00
10	9	2.66	2.66	2.66	2.67	2.68	3.98	3.99	3.99	4.00	4.01	5.31	5.32	5.32	5.33	5.34	8.85	8.86	8.86	8.87	8.88
10	10	3.27	3.27	3.27	3.27	3.27	4.91	4.91	4.91	4.91	4.91	6.54	6.54	6.54	6.54	6.54	10.91	10.91	10.91	10.91	10.91
12	0	0.00	0.29	0.58	1.15	2.30	0.00	0.29	0.58	1.15	2.30	0.00	0.29	0.58	1.15	2.30	0.00	0.29	0.58	1.15	2.30
12	1	0.01	0.29	0.57	1.14	2.28	0.01	0.29	0.57	1.14	2.28	0.02	0.29	0.58	1.14	2.28	0.03	0.30	0.58	1.15	2.29
12	2	0.06	0.31	0.58	1.12	2.21	0.09	0.32	0.60	1.14	2.23	0.12	0.34	0.61	1.16	2.24	0.19	0.39	0.66	1.20	2.29
12	3	0.18	0.37	0.62	1.12	2.13	0.27	0.44	0.68	1.18	2.19	0.37	0.51	0.74	1.24	2.25	0.61	0.74	0.93	1.40	2.40
12	4	0.40	0.53	0.73	1.17	2.07	0.60	0.73	0.89	1.31	2.20	0.81	0.93	1.07	1.46	2.34	1.34	1.46	1.59	1.89	2.72
12	5	0.73	0.84	0.96	1.29	2.04	1.09	1.20	1.32	1.58	2.30	1.46	1.56	1.68	1.92	2.57	2.43	2.54	2.64	2.87	3.39
12	6	1.17	1.26	1.35	1.55	2.08	1.75	1.84	1.93	2.12	2.55	2.34	2.42	2.51	2.70	3.10	3.89	3.98	4.07	4.25	4.62
12	7	1.71	1.78	1.85	1.99	2.30	2.57	2.64	2.70	2.84	3.13	3.43	3.50	3.56	3.70	3.98	5.72	5.78	5.85	5.98	6.26
12	8	2.36	2.41	2.45	2.55	2.74	3.54	3.59	3.63	3.73	3.92	4.72	4.77	4.82	4.91	5.10	7.87	7.92	7.97	8.06	8.24
12	9	3.10	3.13	3.15	3.21	3.32	4.65	4.67	4.70	4.76	4.87	6.20	6.22	6.25	6.31	6.42	10.33	10.35	10.38	10.44	10.55
12	10	3.91	3.92	3.93	3.96	4.01	5.86	5.87	5.88	5.91	5.96	7.81	7.82	7.84	7.86	7.91	13.02	13.03	13.04	13.07	13.12
12	11	4.76	4.77	4.77	4.78	4.79	7.15	7.15	7.15	7.16	7.17	9.53	9.53	9.54	9.54	9.56	15.88	15.89	15.89	15.89	15.91
12	12	5.65	5.65	5.65	5.65	5.65	8.48	8.48	8.48	8.48	8.48	11.31	11.31	11.31	11.31	11.31	18.84	18.84	18.84	18.84	18.84

Table C30-2 Factored Moments Induced in Walls by Lateral Soil and/or Wind Loads (ft-kips/linear ft)
(For Use with Appendix C Load Factors: Soil 1.7, Wind 1.3)

Wall Ht. (ft.)	Back fill Ht. (ft.)	Unfactored Design Lateral Soil Load (psf per foot of depth)																			
		30	30	30	30	30	45	45	45	45	45	60	60	60	60	60	100	100	100	100	100
		Unfactored Design Wind Pressure (psf)																			
		0	10	20	40	80	0	10	20	40	80	0	10	20	40	80	0	10	20	40	80
8	0	0.00	0.10	0.21	0.42	0.83	0.00	0.10	0.21	0.42	0.83	0.00	0.10	0.21	0.42	0.83	0.00	0.10	0.21	0.42	0.83
8	1	0.01	0.11	0.21	0.41	0.81	0.01	0.11	0.21	0.41	0.81	0.02	0.11	0.21	0.41	0.81	0.03	0.12	0.22	0.42	0.82
8	2	0.05	0.13	0.22	0.40	0.76	0.08	0.15	0.23	0.42	0.78	0.11	0.17	0.25	0.43	0.80	0.18	0.23	0.30	0.48	0.84
8	3	0.16	0.21	0.27	0.41	0.72	0.25	0.29	0.34	0.47	0.77	0.33	0.37	0.42	0.54	0.83	0.55	0.59	0.63	0.73	0.99
8	4	0.35	0.38	0.41	0.49	0.69	0.52	0.55	0.58	0.65	0.82	0.69	0.72	0.76	0.82	0.98	1.15	1.18	1.22	1.28	1.42
8	5	0.60	0.62	0.64	0.68	0.78	0.90	0.92	0.94	0.98	1.07	1.20	1.22	1.24	1.28	1.37	2.00	2.02	2.04	2.08	2.17
8	6	0.92	0.93	0.94	0.96	1.00	1.38	1.39	1.40	1.42	1.46	1.84	1.85	1.86	1.88	1.92	3.06	3.07	3.08	3.10	3.14
8	7	1.28	1.29	1.29	1.29	1.30	1.92	1.93	1.93	1.93	1.95	2.57	2.57	2.57	2.58	2.59	4.28	4.28	4.28	4.29	4.30
8	8	1.68	1.68	1.68	1.68	1.68	2.51	2.51	2.51	2.51	2.51	3.35	3.35	3.35	3.35	3.35	5.58	5.58	5.58	5.58	5.58
9	0	0.00	0.13	0.26	0.53	1.05	0.00	0.13	0.26	0.53	1.05	0.00	0.13	0.26	0.53	1.05	0.00	0.13	0.26	0.53	1.05
9	1	0.01	0.13	0.26	0.52	1.03	0.01	0.13	0.26	0.52	1.03	0.02	0.14	0.27	0.52	1.04	0.03	0.14	0.27	0.53	1.04
9	2	0.06	0.15	0.27	0.51	0.98	0.08	0.17	0.29	0.53	1.00	0.11	0.19	0.31	0.54	1.02	0.19	0.25	0.36	0.59	1.06
9	3	0.17	0.23	0.32	0.52	0.94	0.26	0.31	0.39	0.58	0.99	0.34	0.40	0.46	0.65	1.05	0.57	0.62	0.68	0.83	1.21
9	4	0.36	0.41	0.46	0.59	0.91	0.55	0.59	0.64	0.75	1.05	0.73	0.77	0.82	0.93	1.18	1.21	1.26	1.31	1.40	1.62
9	5	0.64	0.68	0.71	0.78	0.95	0.96	1.00	1.03	1.10	1.25	1.28	1.32	1.35	1.42	1.57	2.14	2.17	2.21	2.27	2.41
9	6	1.00	1.02	1.04	1.08	1.17	1.49	1.52	1.54	1.58	1.67	1.99	2.01	2.03	2.08	2.16	3.32	3.34	3.36	3.41	3.49
9	7	1.42	1.43	1.44	1.46	1.50	2.13	2.14	2.15	2.17	2.21	2.84	2.85	2.86	2.88	2.92	4.73	4.74	4.75	4.77	4.81
9	8	1.89	1.89	1.89	1.90	1.91	2.83	2.83	2.84	2.84	2.85	3.77	3.78	3.78	3.79	3.80	6.29	6.29	6.30	6.30	6.31
9	9	2.39	2.39	2.39	2.39	2.39	3.58	3.58	3.58	3.58	3.58	4.77	4.77	4.77	4.77	4.77	7.95	7.95	7.95	7.95	7.95
10	0	0.00	0.16	0.33	0.65	1.30	0.00	0.16	0.33	0.65	1.30	0.00	0.16	0.33	0.65	1.30	0.00	0.16	0.33	0.65	1.30
10	1	0.01	0.16	0.32	0.64	1.28	0.01	0.17	0.32	0.64	1.28	0.02	0.17	0.33	0.65	1.28	0.03	0.17	0.33	0.65	1.29
10	2	0.06	0.18	0.33	0.63	1.23	0.09	0.20	0.35	0.65	1.25	0.11	0.22	0.37	0.67	1.26	0.19	0.28	0.42	0.71	1.31
10	3	0.18	0.26	0.38	0.65	1.18	0.26	0.34	0.45	0.71	1.24	0.35	0.42	0.52	0.77	1.30	0.58	0.65	0.73	0.94	1.45
10	4	0.38	0.44	0.52	0.72	1.16	0.57	0.63	0.70	0.87	1.29	0.76	0.82	0.89	1.03	1.43	1.26	1.33	1.39	1.52	1.84
10	5	0.68	0.73	0.78	0.89	1.18	1.01	1.06	1.11	1.22	1.46	1.35	1.40	1.45	1.55	1.78	2.25	2.30	2.35	2.45	2.66
10	6	1.06	1.10	1.13	1.21	1.37	1.59	1.63	1.66	1.74	1.89	2.13	2.16	2.20	2.27	2.41	3.54	3.58	3.61	3.68	3.83
10	7	1.53	1.55	1.57	1.62	1.71	2.30	2.32	2.34	2.38	2.47	3.06	3.08	3.11	3.15	3.24	5.11	5.13	5.15	5.19	5.28
10	8	2.07	2.08	2.09	2.11	2.15	3.10	3.11	3.12	3.14	3.18	4.14	4.15	4.16	4.18	4.22	6.90	6.91	6.92	6.94	6.98
10	9	2.66	2.66	2.66	2.67	2.68	3.98	3.99	3.99	3.99	4.01	5.31	5.31	5.32	5.32	5.33	8.85	8.86	8.86	8.86	8.87
10	10	3.27	3.27	3.27	3.27	3.27	4.91	4.91	4.91	4.91	4.91	6.54	6.54	6.54	6.54	6.54	10.91	10.91	10.91	10.91	10.91
12	0	0.00	0.23	0.47	0.94	1.87	0.00	0.23	0.47	0.94	1.87	0.00	0.23	0.47	0.94	1.87	0.00	0.23	0.47	0.94	1.87
12	1	0.01	0.24	0.47	0.93	1.85	0.01	0.24	0.47	0.93	1.85	0.02	0.24	0.47	0.93	1.85	0.03	0.25	0.48	0.94	1.86
12	2	0.06	0.26	0.48	0.92	1.80	0.09	0.27	0.49	0.94	1.82	0.12	0.29	0.51	0.95	1.84	0.19	0.35	0.56	1.00	1.88
12	3	0.18	0.33	0.53	0.93	1.75	0.27	0.40	0.59	0.99	1.81	0.37	0.48	0.65	1.05	1.87	0.61	0.71	0.85	1.22	2.02
12	4	0.40	0.51	0.65	1.00	1.73	0.60	0.70	0.82	1.15	1.86	0.81	0.90	1.01	1.30	2.00	1.34	1.44	1.54	1.77	2.39
12	5	0.73	0.82	0.91	1.15	1.76	1.09	1.18	1.27	1.48	2.02	1.46	1.54	1.63	1.83	2.31	2.43	2.52	2.60	2.78	3.18
12	6	1.17	1.24	1.31	1.47	1.85	1.75	1.82	1.89	2.05	2.38	2.34	2.41	2.48	2.63	2.94	3.89	3.96	4.03	4.18	4.48
12	7	1.71	1.77	1.82	1.94	2.18	2.57	2.63	2.68	2.79	3.02	3.43	3.48	3.54	3.65	3.87	5.72	5.77	5.82	5.93	6.15
12	8	2.36	2.40	2.44	2.51	2.67	3.54	3.58	3.62	3.69	3.85	4.72	4.76	4.80	4.87	5.02	7.87	7.91	7.95	8.02	8.17
12	9	3.10	3.12	3.14	3.19	3.28	4.65	4.67	4.69	4.74	4.83	6.20	6.22	6.24	6.28	6.37	10.33	10.35	10.37	10.42	10.50
12	10	3.91	3.92	3.93	3.95	3.99	5.86	5.87	5.88	5.90	5.94	7.81	7.82	7.83	7.85	7.89	13.02	13.03	13.04	13.06	13.10
12	11	4.76	4.77	4.77	4.78	4.79	7.15	7.15	7.15	7.16	7.17	9.53	9.53	9.53	9.54	9.55	15.88	15.88	15.89	15.89	15.90
12	12	5.65	5.65	5.65	5.65	5.65	8.48	8.48	8.48	8.48	8.48	11.31	11.31	11.31	11.31	11.31	18.84	18.84	18.84	18.84	18.84

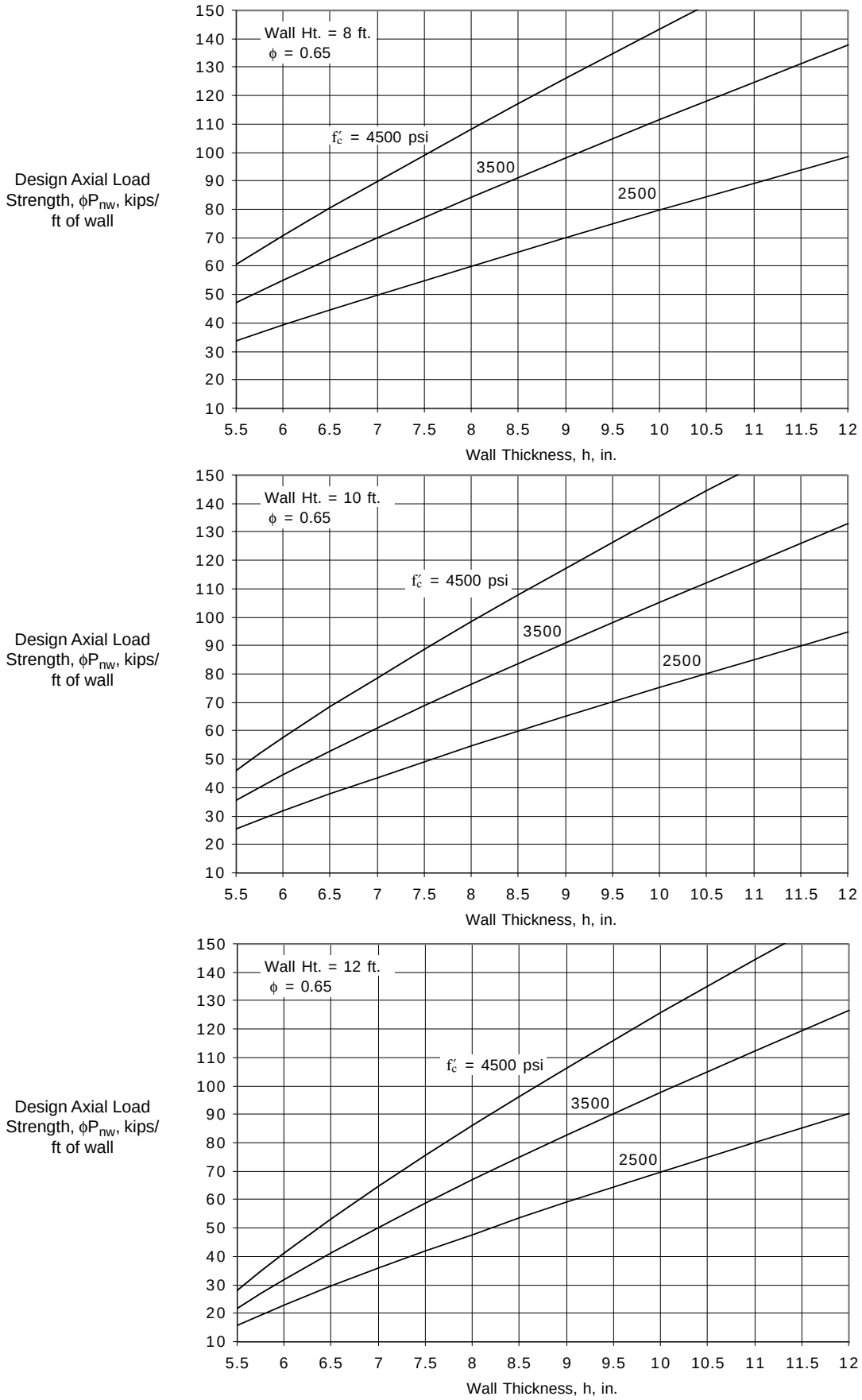


Figure C30-1 Design Axial Load Strength, P_{nw}, of Plain Concrete Walls using the Empirical Design Method

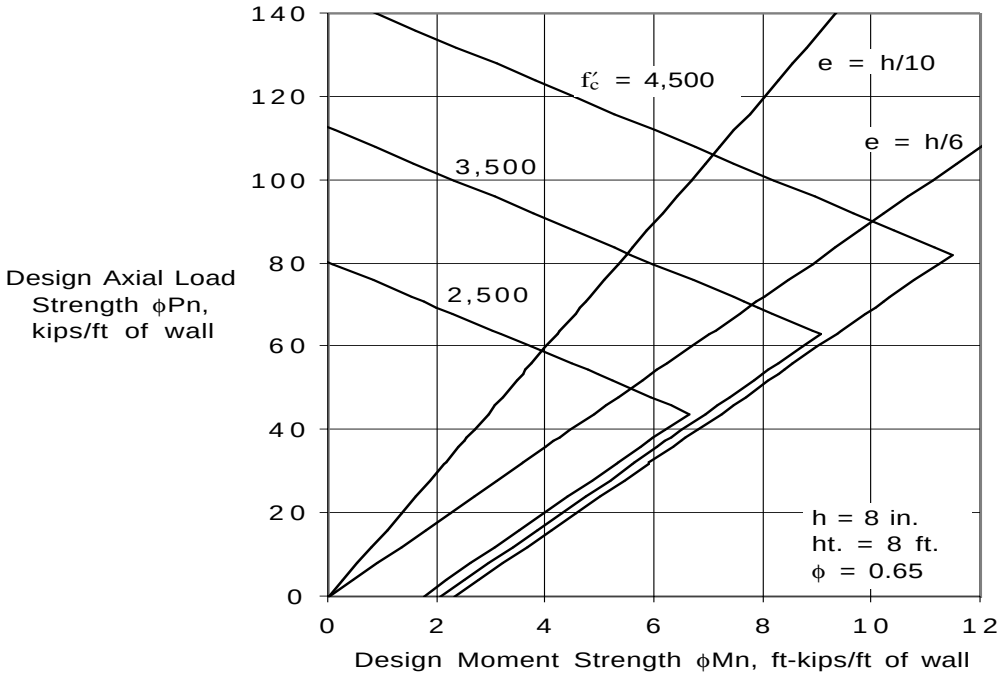


Figure C30-2 Design Strength Interaction Diagrams for 8.0-in. Wall, 8 ft in Height

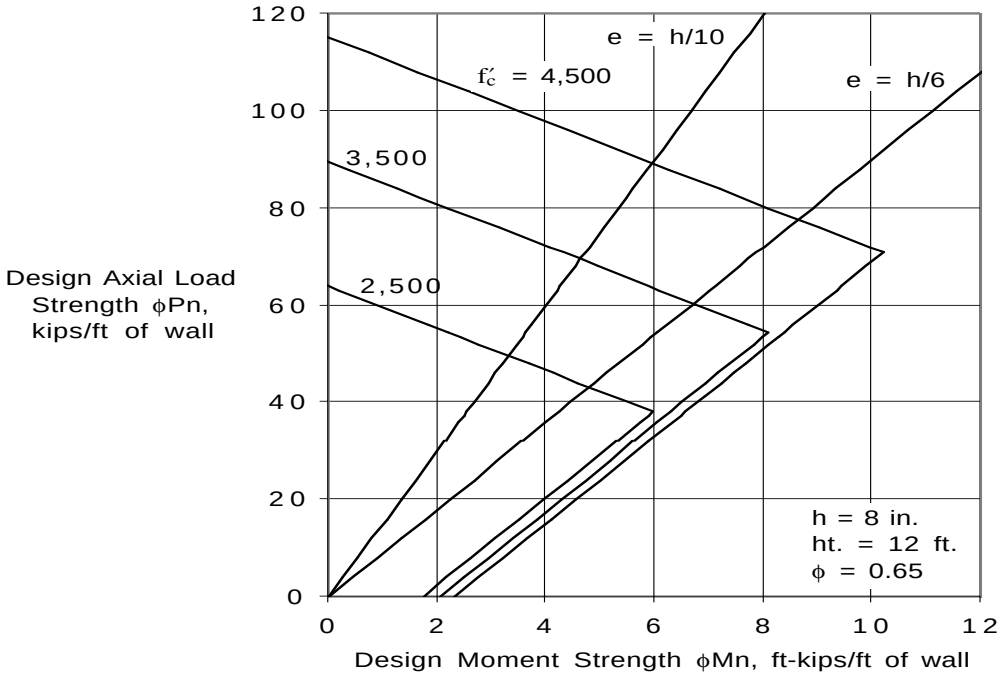


Figure C30-3 Design Strength Interaction Diagrams for 8.0-in. Wall, 12 ft in Height

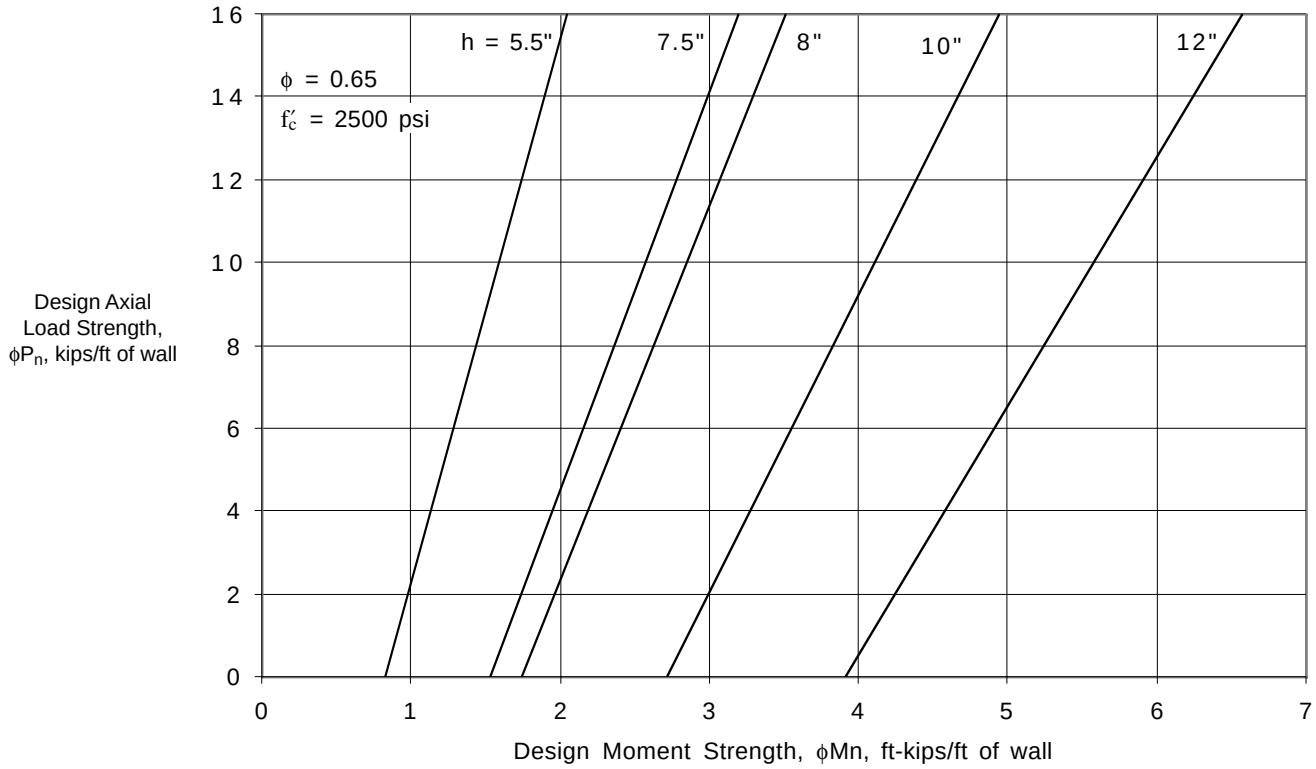


Figure C30-4 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls (f'_c = 2500 psi)

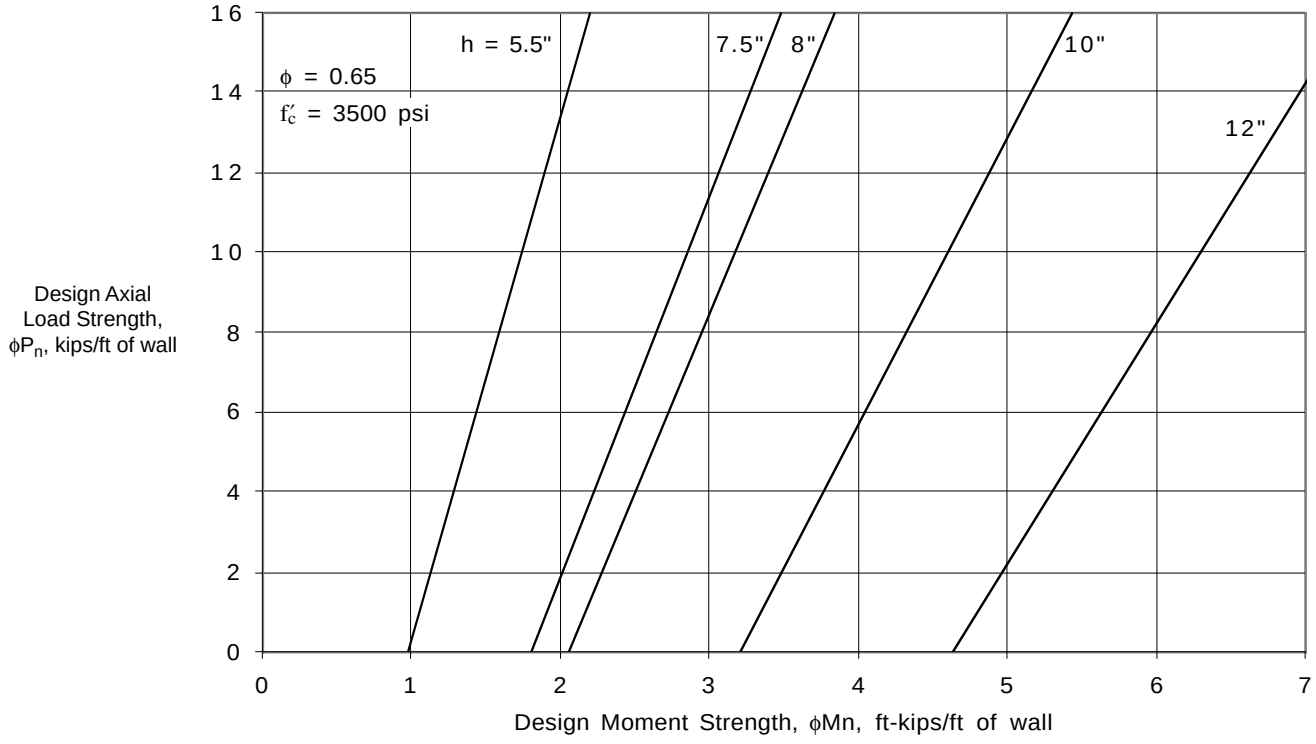


Figure C30-5 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls (f'_c = 3500 psi)

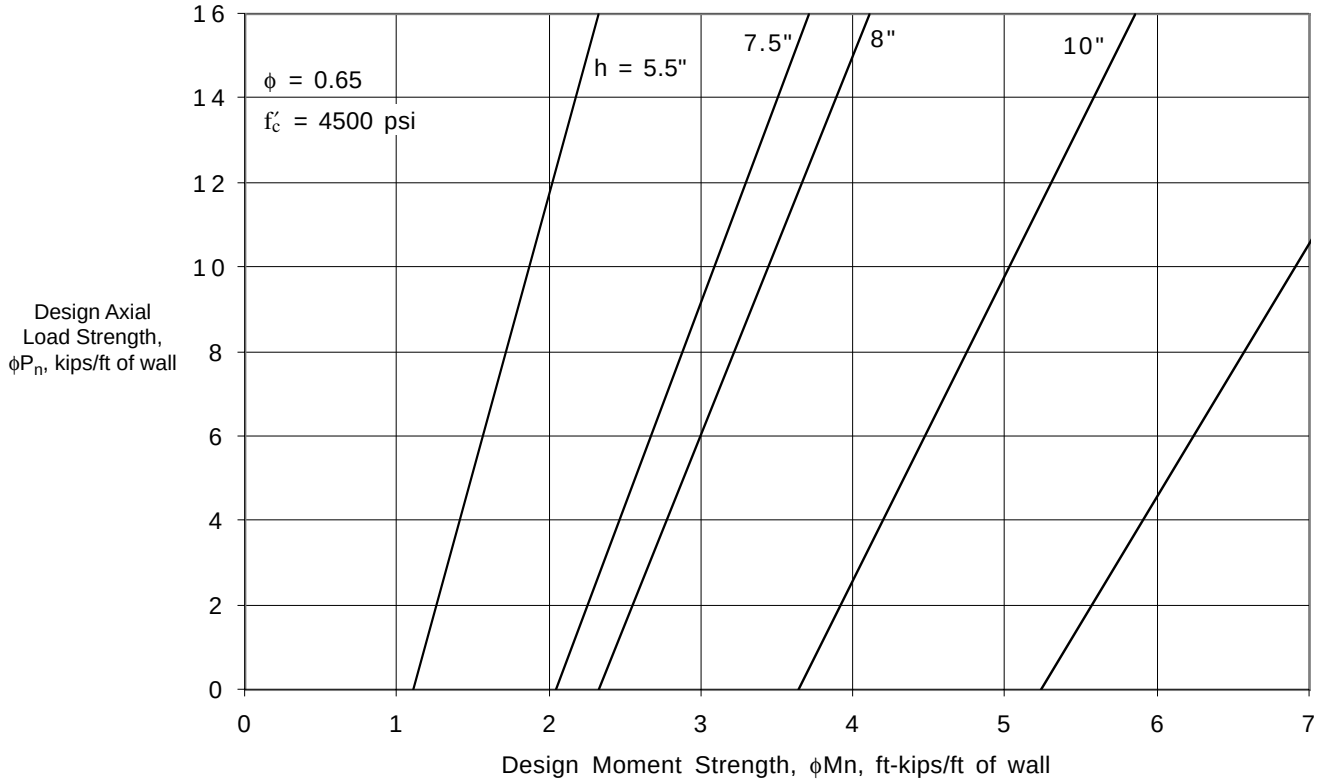


Figure C30-6 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls (f'_c = 4500 psi)

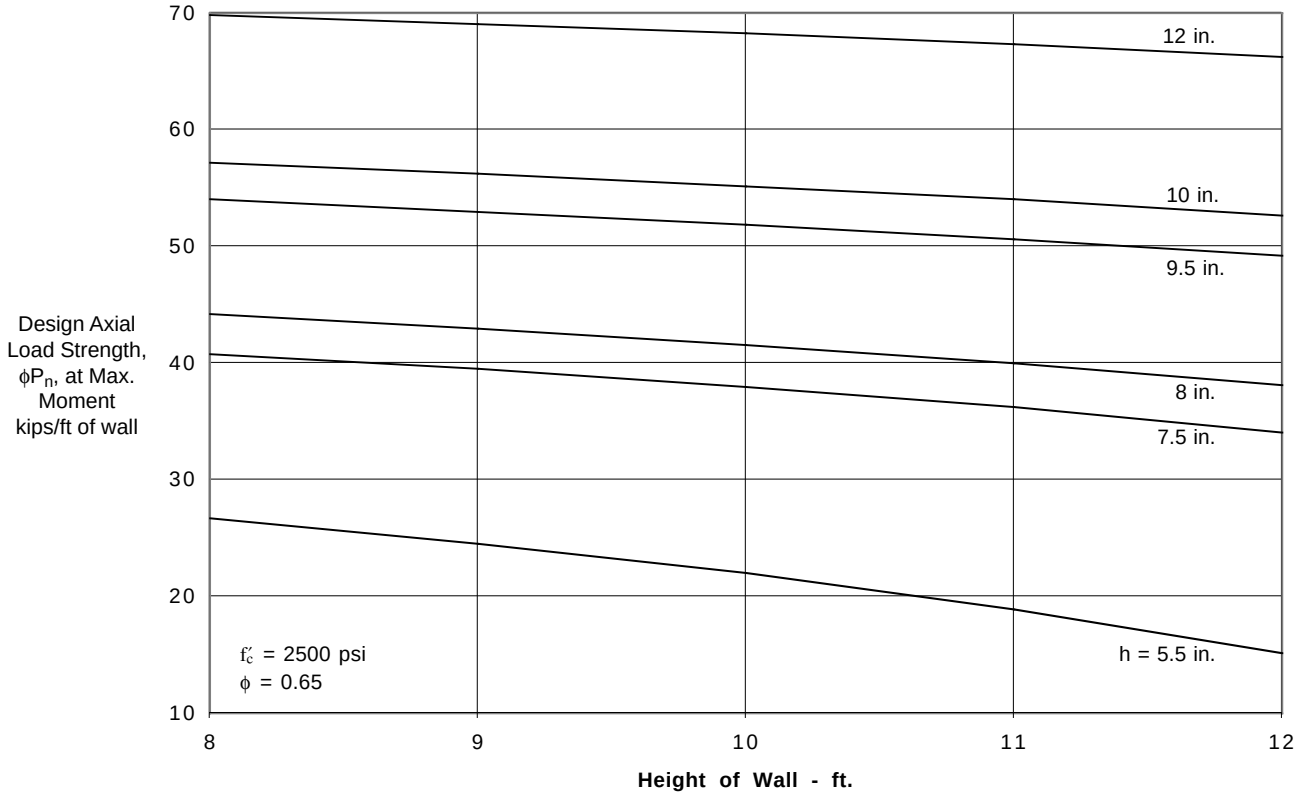


Figure C30-7 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength (f'_c = 2500 psi)

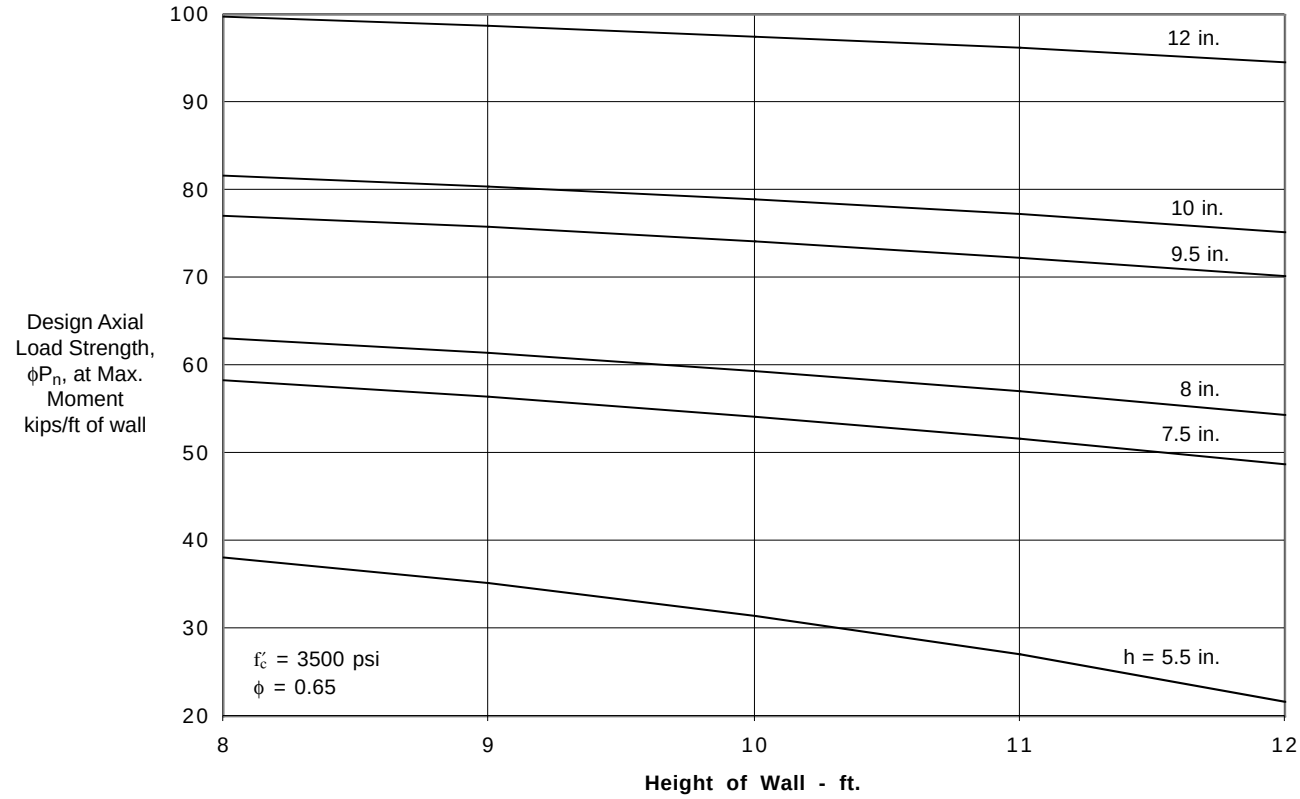


Figure C30-8 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength (f'_c = 3500 psi)

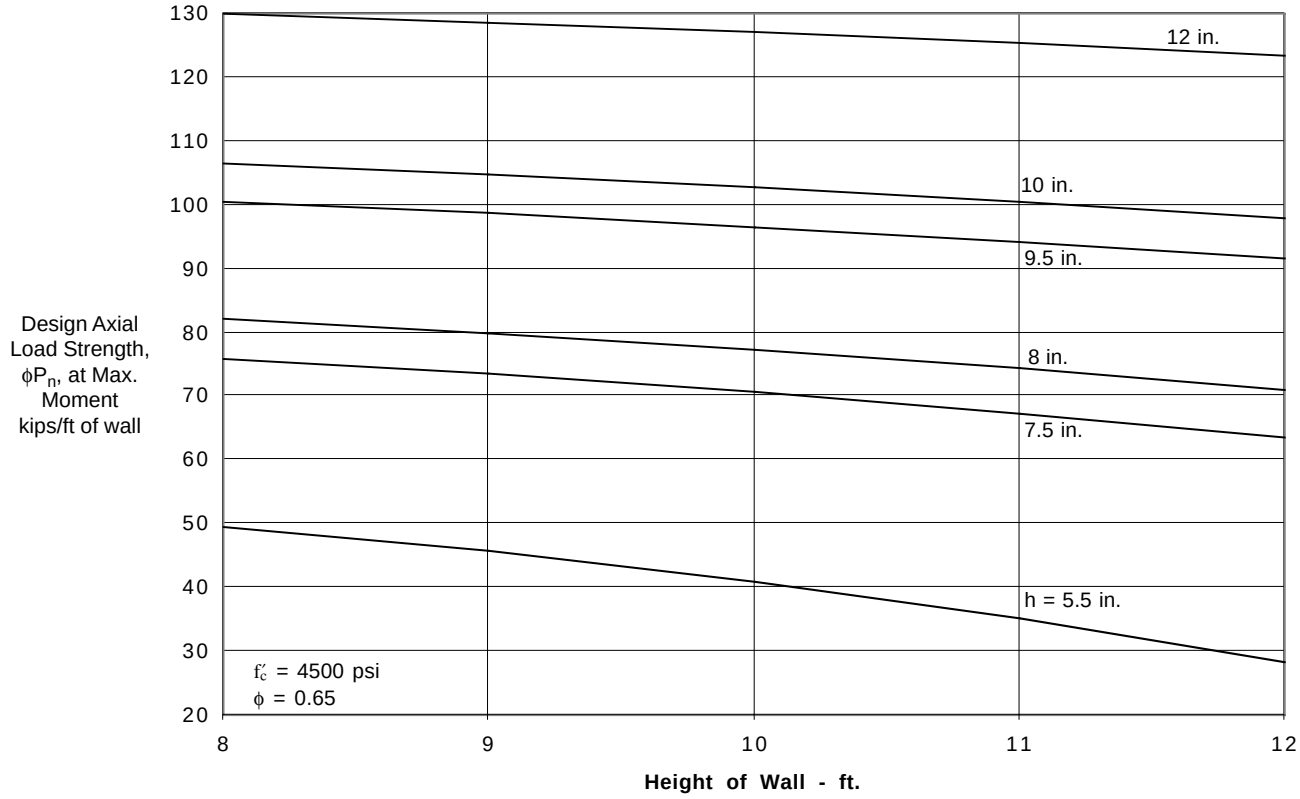
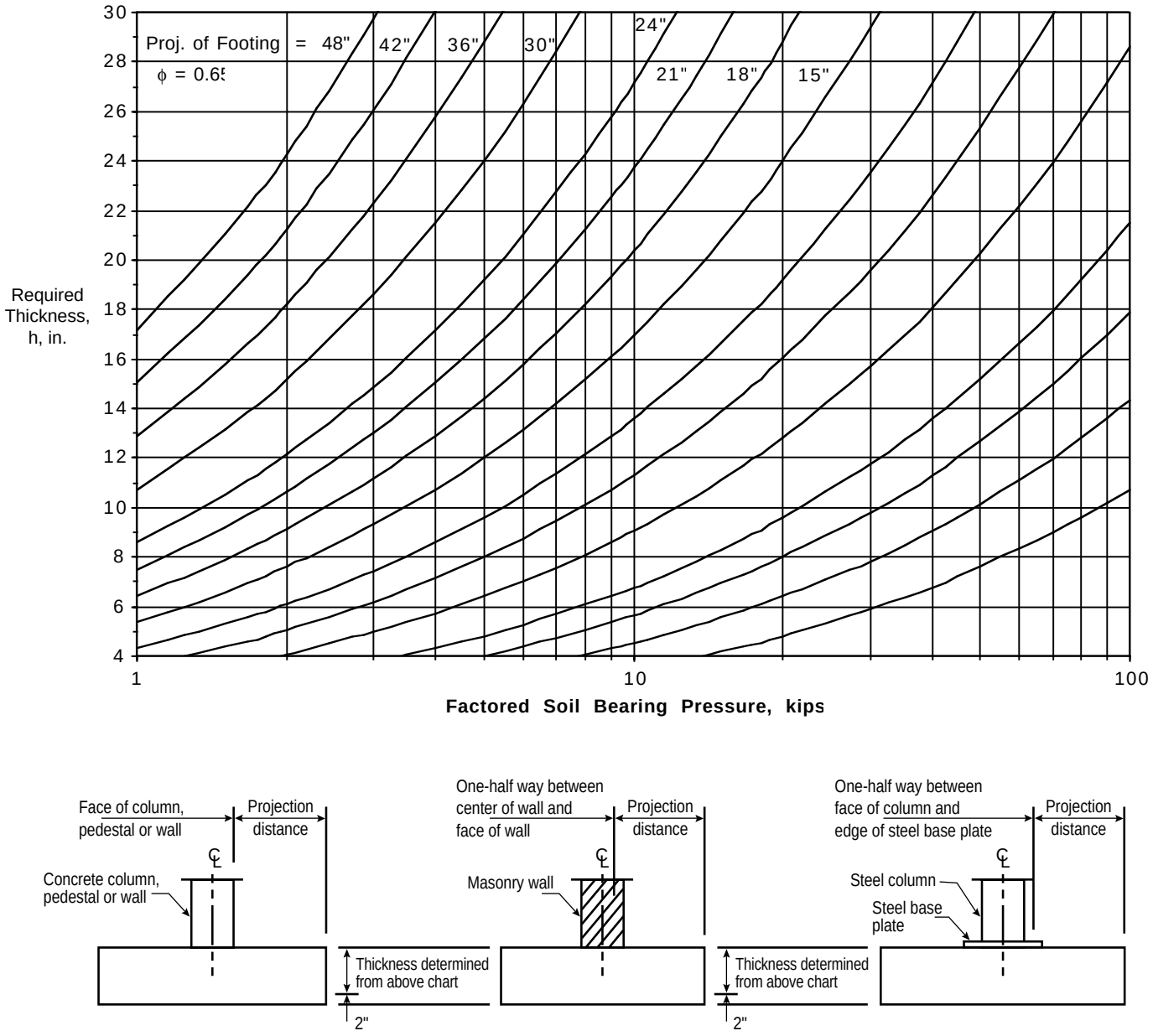


Figure C30-9 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength (f'_c = 4500 psi)



*For f'_c greater than 2500 psi, multiple thickness determined from above chart by $(2500/f'_c)^{0.25}$

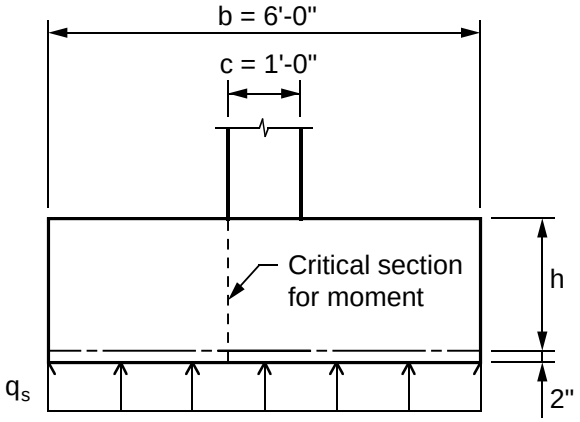
Figure C30-10 Thickness of Footing Required to Satisfy Flexural Strength for Various Projection Distances, in. (f'_c = 2500 psi)*

Example 30.1—Design of Plain Concrete Footing and Pedestal

Proportion a plain concrete square footing with pedestal for a residential occupancy building. Design in conformance with Chapter 22 using the load and strength reduction factors of Chapter 9. Perform a second design using the alternate load and strength reduction factors of Appendix C to determine which results in a more economical design.

Design data:

Service dead load = 40 kips
Service floor live load = 40 kips
Service roof live load = 7.5 kips
Service roof snow load = 10 kips
Service surcharge = 0
Pedestal dimensions = 12 ×12 in.
Permissible soil bearing pressure = 2.5 ksf
f'_c = 2500 psi



Calculations and Discussion	Code Reference

1. Determine base area of footing:

   The base area is determined by using unfactored service gravity loads and the permissible soil bearing pressure. *22.7.2*

   $$A_f = \frac{40 + 40 + 10}{2.5} = 36 \text{ ft}^2$$

   Use 6 × 6 ft square footing (A_f = 36 ft^2)

2. Determine the applicable load combinations that must be considered.

   To proportion the footing for strength, factored loads must be used. Two sets of load factors are provided; one in 9.2 and the other in C.2. Because of the significant difference between strength reduction factor, ϕ, to be used with each set of load factors (0.55 in 9.3.5 versus 0.65 in C.3.5), a design in accordance with each set of factors should be evaluated to determine which alternate with provide the more economical solution. *22.7.1, 9.2, 9.3.5, C.2, C.3.5*

   The required strength must at least equal the largest factored load determined from applicable load combinations. One of the following load combinations will govern:

   1. U = 1.2D + 1.6L + 0.5S *Eq. (9-2)*
   2. U = 1.2D + 0.5L + 1.6S *Eq. (9-3)*
   3. U = 1.2D + 0.5L + 0.5S *Eq. (9-4)*

   Note that in Combination 1, T is being neglected. In Combinations 2 and 3 the factor on L is 0.5 in accordance with 9.2.1(a). *9.2.1(a)*

3. Calculate the factored axial load, P_u, for each load combination.

   By observation it can be seen that either Combination 1 or 2 will yield the largest factored axial load.

   1. $P_u = 1.2D + 1.6L + 0.5S = 1.2(40) + 1.6(40) + 0.5(10) = 117$ kips — *Eq. (9-2)*

   2. $P_u = 1.2D + 0.5L + 1.6S = 1.2(40) + 0.5(40) + 1.6(10) = 64.8$ kips — *Eq. (9-3)*

   Use $P_u = 117$ kips

   Upon reviewing the applicable load combination of C.2, it is obvious that Eq. (C-1) will control:

   $P_u = 1.4D + 1.7L = 1.4\ (40) + 1.7\ (50) = 141$ kips — *Eq. (C-1)*

   To quickly determine which one of the two sets of load and strength reduction factors to use, compare the nominal axial load strength, P_n, required by the factored loads of 9.2 to that required by C.2. — *9.3.5, C.3.5*

   Chapter 9 $P_n = P_u/\phi = 117/0.55 = 212.7$ kips

   Appendix C $P_n = P_u/\phi = 141/0.65 = 216.9$ kips

   Since the nominal axial load strength required by the load and strength reduction factors of Chapter 9 is less than the corresponding load required by Appendix C, design according to Chapter 9 will be more economical. Regardless of the load and strength reduction factors used, the design procedures will be the same.

4. Calculate the factored soil bearing pressure.

   Since the footing must be proportioned for strength by using factored loads and induced reactions, the factored soil bearing pressure must be used. — *22.7.1*

   $$q_s = \frac{P_u}{A_f} = \frac{117}{36} = 3.25 \text{ ksf}$$

5. Determine the footing thickness required to satisfy moment strength.

   For plain concrete, flexural strength will usually control thickness. The critical section for calculating moment is at the face of the concrete pedestal (see figure above). — *22.7.5(a)*

   $$M_u = q_s\,(b)\left(\frac{b - c}{2}\right)\left(\frac{b - c}{4}\right)$$

   $$= 3.25\ (3)\ (2.5)^2 = 60.9 \text{ ft-kips}$$

   $\phi M_n \geq M_u$ — *Eq. (22-1)*

ϕ = 0.55 for all stress conditions — *9.3.5*

$$\phi M_n = 5\phi\sqrt{f'_c}S$$ — *Eq. (22-2)*

$$= \frac{5\,(0.55)\left(\sqrt{2500}\right)(6)\,(12)h^2}{(1000)\,(6)} \geq 60.9 \text{ ft-kips}$$

Solving for h:

$$h \geq \left[\frac{60.9\,(12)\,(1000)\,(6)}{5\,(0.55)\left(\sqrt{2500}\right)(6)\,(12)}\right]^{0.5} = 21.0 \text{ in.}$$

Alternate solution using Fig. 30-10:

Enter figure with factored soil bearing pressure (3.25 kips/sq. ft). Project upward to the distance that the footing projects beyond the face of the pedestal, which is 30 in. (36 - 6). The project horizontally and read approximately 21 in. as the required footing thickness.

For concrete cast on the soil, the bottom 2 in. of concrete cannot be considered for strength computations (the reduced overall thickness is to allow for unevenness of the excavation and for some contamination of the concrete adjacent to the soil). — *22.4.8*

Use overall footing thickness of 24 in.

6. Check for beam action shear. Use effective thickness of h = 22 in. = 1.83 ft.

The critical section for beam action shear is located a distance equal to the thickness, h, away from the face of the pedestal, or 0.67 ft (3 - 0.5 - 1.83) from the edge of the footing. — *22.7.6.1*, *22.7.6.2(a)*

$$V_u = q_s b\left[\left(\frac{b}{2}\right) - \left(\frac{c}{2}\right) - h\right] = 3.25\,(6)\,(0.67) = 13.07 \text{ kips}$$

$\phi V_n \geq V_u$ — *Eq. (22-8)*

$$V_n = \left(\frac{4}{3}\right)\sqrt{f'_c}\,bh$$ — *Eq. (22-9)*

$$\phi V_n = \frac{4\,(0.55)\left(\sqrt{2500}\right)(72)\,(22)}{(3)\,(1000)} = 58.08 \text{ kips} > 13.07 \text{ kips} \quad \text{O.K.}$$

7. Check for two-way action (punching) shear.

The critical section for two-way action shear is located a distance equal to one-half the footing thickness, h, away from the face of the pedestal. — *22.7.6.1*, *22.7.6.2(b)*

$$V_u = q_s [b^2 - (c + h)^2] = 3.25 \left[6^2 - \left(1 + \frac{22}{12}\right)^2\right] = 90.91 \text{ kips}$$

$\phi V_n \geq V_u$ *Eq. (22-8)*

$$V_n = \left[\frac{4}{3} + \frac{8}{3\beta_c}\right]\sqrt{f'_c}\, b_o h \leq 2.66\sqrt{f'_c}\, b_o h$$ *Eq. (22-10)*

where β_c is the ratio of the long-to-short side of the supported load. In this case, $\beta_c = 1$.

Since $\left[\left(\frac{4}{3}\right) + \left(\frac{8}{3}\right)\right] = 4.0 > 2.66$,

$V_n = 2.66\sqrt{f'_c}\, b_o h$

$$\phi V_n = \frac{2.66\,(0.55)\left(\sqrt{2500}\right)(34)(4)(22)}{1000} = 218.87 \text{ kips} > 90.91 \text{ kips} \quad \text{O.K.}$$

8. Check bearing strength of pedestal. *22.8.3*

$P_u = 141$ kips (from Step 3)

$\phi B_n \geq P_u$ *Eq. (22-11)*

$B_n = 0.85 f'_c A_1$ *Eq. (22-12)*

$$\phi B_n = \frac{0.85\,(0.55)\,(2500)\,(12 \times 12)}{1000} = 168.3 \text{ kips} > 117 \text{ kips} \quad \text{O.K.}$$

Example 30.2—Design of Plain Concrete Basement Wall

A plain concrete basement wall is to be used to support a 2-story residential occupancy building of wood frame construction with masonry veneer. The height of the wall is 10 ft (distance between the top of the concrete slab and the wood-framed floor, both of which provide lateral support of the wall). The backfill height is 7 ft and the wall is laterally restrained at the top. Design of the wall is required in accordance with Chapter 22 using the load and strength reduction factors of Chapter 9. Perform a second design using the alternate load and strength reduction factors of Appendix C to determine which results in a more economical design.

Design data:

Service dead load = 1.6 kips per linear foot
Service floor live load = 0.8 kips per linear foot
Service roof live load = 0.4 kips per linear foot
Service roof snow load = 0.3 kips per linear foot
Service lateral earth pressure = 60 psf/ft of depth
Service wind pressure = 20 psf inward, 25 psf outward
Assume factored roof dead load plus factored wind uplift load on roof (Eq. 9-6) = 0
Eccentricity of axial loads = 0

Calculations and Discussion	Code Reference

Design using load and strength reduction factors of Chapter 9.

1. The wall must be designed for vertical, lateral, and other loads to which it will be subjected. Therefore, determine the applicable load combinations that must be considered. — *22.6.2, 9.2*

   1. $U = 1.2D + 1.6L + 0.5L_r + 1.6H$ — *Eq. (9-2)*
   2. $U = 1.2D + 0.5L + 1.6L_r + 1.6H$ — *Eq. (9-3)*
   3. $U = 1.2D + 0.5L + 0.5L_r\ 1.6W + 1.6H$ — *Eq. (9-4)*
   4. $U = 0.9D + 1.6W + 1.6H$ — *Eq. (9-6)*
   5. $U = 1.6H$ — *Eq. (9-6)*

   Note that in Combination 1, T is being neglected. In Combinations 2 and 3 the factor on L is 0.5 in accordance with 9.2.1(a). In Combinations 2 and 4, 1.6H has been included since the lateral soil load will always be acting. In Combination 5, D has been omitted since this condition may occur during construction.

2. Calculate the axial load, P_u, for each load combination.

   1. $P_u = 1.2D + 1.6L + 0.5L_r = 1.2(1.6) + 1.6(0.8) + 0.5(0.4) = 3.40$ kips/ft — *Eq. (9-2)*
   2. $P_u = 1.2D + 0.5L + 1.6L_r = 1.2(1.6) + 0.5(0.8) + 1.6(0.4) = 2.96$ kips/ft — *Eq. (9-3)*
   3. $P_u = 1.2D + 0.5L + 0.5L_r = 1.2(1.6) + 0.5(0.8) + 0.5(0.4) = 2.52$ kips/ft — *Eq. (9-4)*

4. $P_u = 0.9D = 0.9(1.6) = 1.44$ kips/ft *Eq. (9-6)*

5. $P_u = 0$ *Eq. (9-6)*

3. Calculate the moment, M_u, for load combination.

1. M_u due to 1.6H *Eq. (9-2)*

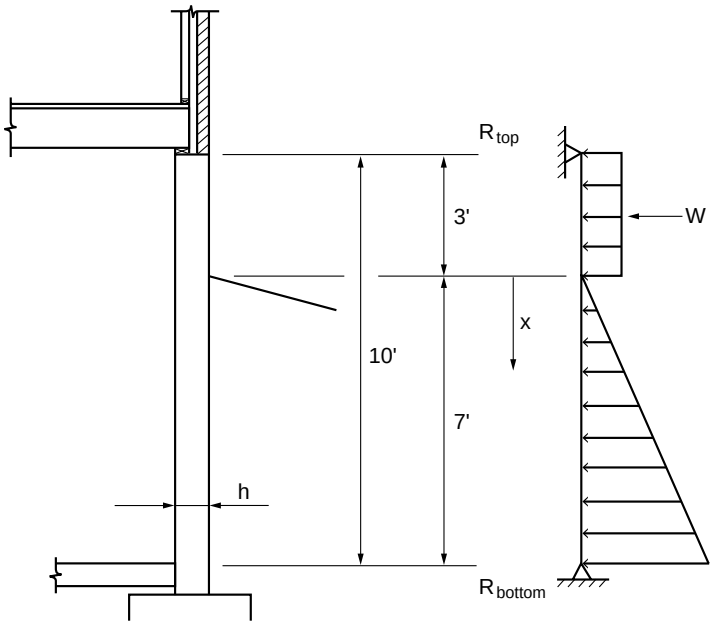
2. M_u due to 1.6H *Eq. (9-3)*

3. M_u due to 1.6W + 1.6H *Eq. (9-4)*

4. M_u due to 1.6W + 1.6H *Eq. (9-6)*

5. M_u due to 1.6H *Eq. (9-6)*

The maximum moment occurs at the location of zero shear. To determine this location with respect to the top of the wall, first calculate the reaction at the top of the wall. If wind is acting and in the same direction as the lateral soil load, and the resultant of the wind load, W, is greater than the reaction at the top of the wall, the location of zero shear is some distance "X" below the top of the wall (above the top of the backfill). Otherwise, the zero shear location will be some distance "X" below the top of the backfill. Next, sum the horizontal forces above "X" (the location of zero shear). Finally, solve for "X." See figure below.



For load Combinations 3 and 4, the reaction at the top of the wall is:

$$R_{top} = \frac{1.6\ (20)\ (3)\ (8.5)\ +\ [1.6\ (60)7^3]\ /\ 6}{10} = 630.4 \text{ plf}$$

W = 1.6(20)(3) = 96 plf

$W < R_{top}$

Therefore, location of zero shear is below top of backfill.

Sum horizontal forces at "X":

$630.4 - 1.6\,(20)\,(3) - [1.6\,(60)\,(X^2)]/2 = 0$

$630.4 - 96 - 48X^2 = 0$

Solving for distance "X":

$$X = \left(\frac{630.4 - 96}{48}\right)^{0.5} = 3.37 \text{ ft}$$

The point of zero shear is 3.0 + 3.37 = 6.37 ft from the top of the wall.

Note: It is generally simpler to compute the location of zero shear with respect to the top of the wall since doing so with respect to the bottom will involve solving a quadratic equation.

Compute the maximum moment, M_u, due to the wind and lateral soil loads:

$$M_u = \frac{630.4\,(6.37) - 1.6\,(20)\,(3)\,(6.37 - 1.5) - [1.6\,(60)\,(3.37)^3]/6}{1000} = 2.94 \text{ ft-kips/ft}$$

Alternately, the maximum moment can be determined from Table 30-1 since the load factor on wind is 1.6.

From Table 30-1, for a 10-ft high wall with 7 ft of backfill, for unfactored wind and soil loads of 20 psf and 60 psf/ft, respectively, the moment, M_u, is 2.94 ft-kips/ft, which is the same as calculated above.

Next determine from the moment Table 30-1 for load Combinatios 1, 2 and 5 (with no wind acting). From Table 30-1 for a 10-ft high wall with 7 ft of backfill, for unfactored wind and soil loads of 0 psf and 60 psf/ft, respectively, the moment, M_u, is 2.88 ft-kips/ft. Note that either Table 30-1 or 30-2 can be used since this load combination does not include wind. The moment is read from the zero column for "unfactored design wind pressure."

4. Calculate the effective eccentricities for load Combinations 1, 3, and 4 to determine if the wall can be designed by the empirical method (i.e., $e \le h/6$). Assume a conservative wall thickness of 12 in. *22.6.3* *22.6.5*

Allowable e = 12/6 = 2 in = 0.167 ft

For load Combination 1:

$$e = \frac{2.88}{3.40} = 0.85 \text{ ft} > 0.167 \text{ ft}$$

For load Combination 3:

$$e = \frac{2.94}{2.52} = 1.17 \text{ ft} > 0.167 \text{ ft}$$

For load Combination 4:

$$e = \frac{2.94}{1.44} = 2.04 \text{ ft} > 0.167 \text{ ft}$$

Since the effective eccentricity exceeds h/6, the wall cannot be designed by the empirical method. The wall must be designed taking into consideration both flexure and axial compression. *22.5.3* *22.6.3*

5. Determine the required wall thickness to satisfy the axial load and induced moments by using the appropriate interaction equation. The factored axial loads and moments for the various load combinations are summarized in the following table. *Eq. (22-6)* *Eq. (22-7)*

Load Combination	Axial Load, P_u, kips/ft	Moment, M_u, ft-kips/ft
1	3.40	2.88
2	2.96	2.88
3	2.52	2.94
4	1.44	2.94
5	0	2.88

By observation, the combination of very low axial load and relatively high moment will be governed by interaction Eq. (22-7). *22.5.3*

$$\frac{M_u}{S} - \frac{P_u}{A_g} \le 5\phi\sqrt{f'_c} \qquad \textit{Eq. (22-6)}$$

By rearranging the equation and substituting for S and A_g in terms of "h," the required thickness can be determined by solving the following quadratic equation (3):

$$0.06\,\phi\sqrt{f'_c}\,h^2 + P_u h - 72M_u = 0 \qquad (3)$$

Determine the required wall thickness, h, to satisfy load Combination 5 (ϕ 0.55, P_u = 0 and M_u = 2.88 ft-kips/ft), since this combination has the highest moment and lowest axial load. Since P_u = 0, equation (3) simplifies to equation (4). Use equation (4) to solve for required, "h," assuming f'_c = 4000 psi.

$$h = (72M_u/0.06\phi\sqrt{f'_c})^{1/2} = [((72)\,(2.88)) / ((0.6)\,(0.55)\,(\sqrt{4000}))]^{1/2} = 9.97 \text{ in.} \qquad (4)$$

Assume a 10-inch wall with f'_c = 4000 psi.

Check preliminary wall selection for all load combinations by interpolating between values from Figs. 30-5 and 30-6 for f'_c = 3500 and 4500 psi, respectively. The following table summarizes the required axial load, P_u, and moment, M_u, strengths for the various

load combinations, and indicates the approximate design moment strength, ϕM_n, determined from the figures based on a wall thickness of 10 in.

Load Combination	Axial Load, P_u kips/ft	Moment, M_u ft-kips/ft	Approximate Design Moment Strength,[1] ϕM_n ft-kips/ft	$\phi M_n / M_u$
1	3.40	2.88	Note 2	—
2	2.96	2.88	(3.1 + 3.5)/2 = 3.3	1.15
3	2.52	2.94	Note 2	—
4	1.44	2.94	(2.9 + 3.3)/2 = 3.1	1.05
5	0	2.88	(2.7 + 3.1)/2 = 2.9	1.01

[1] Values interpolated from Figs. 30-5 and 30-6.
[2] There is no need to evaluate this combination since another combination has the same moment and a lower axial load. Given equal moments, the combination with the lower axial load will govern.

Since the ratio of design moment strength, ϕM_n, to required moment strength, M_u, exceeds 1 in all cases, the 10-in. wall is adequate for axial loading and induced moments.

Tentatively, use 10-in. wall with concrete f'_c = 4000 psi and check for shear.

6. Check for shear strength.

Shear strength will rarely govern the design of a wall; nevertheless, it should not be overlooked. The shear will be greatest at the bottom of the wall. The critical section for calculating shear is located at wall thickness, h, above the top of the floor slab. *22.5.4* *22.6.4*

For shear, load Combinations 3 and 4, which are the same, will control. Calculate the reaction at the bottom of the wall.

$$R_{bottom} = \frac{[(1.6)(20)(3)^2/2] + [((1.6)(60)(7)^2/2)(3 + (2/3)(7))]}{10} = 1818 \text{ plf}$$

$$V_u = 1818 - 1.6(60)\{[(7 - 10/12)(10/12)] + [(10/12)^2/2]\}$$

$$= 1291 \text{ plf} = 1.29 \text{ klf}$$

$\phi V_n \geq V_u$ *Eq. (22-8)*

$$\phi V_n = \frac{4\phi\sqrt{f'_c}bh}{3}$$ *Eq. (22-9)*

$$= \frac{4(0.55)\left(\sqrt{4000}\right)(12)(10)}{3(1000)} = 5.57 \text{ klf} > 1.29 \text{ klf} \quad \text{O.K.}$$

7. Use 10-in. wall with specified compressive strength of concrete, f'_c = 4000 psi.

Design using load and strength reduction factors of Appendix C.

C1. The wall must be designed for vertical, lateral, and other loads to which it will be subjected. Therefore, determine the applicable load combinations that must be considered. *22.6.2 C.2*

1. $U = 0.75(1.4D + 1.7L) + 1.6W + 1.7H$ *Eq. (C-2)*
2. $U = 0.9D + 1.6W + 1.7H$ *Eq. (C-3)*
3. $U = 1.4D + 1.7L + 1.7H$ *Eq. (C-4)*
4. $U = 0.9D + 1.7H$ *Eq. (C-3)*
5. $U = 1.7H$ *Eq. (C-3)*

In Combinations 1 and 2, 1.7H has been added since lateral earth pressure will always be acting and for consistency with Eq. (9-2) and (9-3). For consistency with Eq. (9-6) and since dead load, D, may not be acting during early phases of construction, it has been omitted.

C2. Calculate the factored axial load, P_u, for each load combinations.

1. $P_u = 0.75\ (1.4D + 1.7L) = 0.75((1.4)(1.6) + (1.7)(1.2)) = 3.21$ kips/ft *Eq. (C-2)*
2. $P_u = 0.9D = 0.9(1.6) = 1.44$ kips/ft *Eq. (C-3)*
3. $P_u = 1.4D + 1.7L = 1.4(1.6) + 1.7(1.2) = 4.28$ kips/ft *Eq. (C-4)*
4. $P_u = 0.9D = 0.9(1.6) = 1.44$ kips/ft *Eq. (C-3)*
5. $P_u = 0$ *Eq. (C-3)*

C3. Calculate the factored moment, M_u, for each load combinations using Table 30-3.

1. $M_u = 1.6W + 1.7H = 3.12$ ft-kips/ft *Eq. (C-2)*
2. $M_u = 1.6W + 1.7H = 3.12$ ft-kips/ft *Eq. (C-3)*
3. $M_u = 1.7H = 3.06$ ft-kips/ft *Eq. (C-4)*
4. $M_u = 1.7H = 3.06$ ft-kips/ft *Eq. (C-3)*
5. $M_u = 1.7H = 3.06$ ft-kips/ft *Eq. (C-3)*

C4. By observation, the effective eccentricities exceed one-sixth the wall thickness; therefore, the empirical design procedure cannot be used. *22.6.5.1*

C5. Determine the required wall thickness by using the appropriate interaction equation. Since lightly-loaded walls are governed by Eq. (22-7), Figures 30-4 through C30-6 will be used to design the wall for axial loads and flexure. The following table shows for each load combination the factored axial load and moment, and corresponding approximate design moment strength, ϕM_n, and ratio of over- or under-strength based on the assumed wall thickness and concrete strength. Assume a wall thickness of 10 in. and $f'_c = 3000$ psi. *22.5.3 Eq. (22-6) Eq. (22-7)*

Load Combination	Axial Load, P_u kips/ft	Moment, M_u ft-kips/ft	Approximate Design Moment Strength,[1] ϕM_n ft-kips/ft	$\phi M_n / M_u$
1	3.21	3.12	Note 2	—
2	1.44	3.12	(2.9 + 3.4)/2 = 3.15	1.031
3	4.28	3.06	(3.3 + 3.8)/2 = 3.55[3]	1.16
4	1.44	3.06	Note 2	—
5	0	3.06	(2.7 + 3.2)/2 = 2.95	0.96

[1] Values interpolated from Figs. C30-4 and C30-5 based on 10-inch wall with 3000 psi concrete.
[2] There is no need to evaluate this combination since another combination has the same moment and a lower axial load. Given equal moments, the combination with the lower axial load will govern.
[3] While it was not necessary to check this load combination, it was done to illustrate that given equal induced moments, the combination with the higher axial, P_u, load yields a greater design moment strength, ϕM_n.

Since the 10-in. wall of 3000 psi concrete exceeds the requirements for all combinations except where no dead load is present (Combination 5), and in that case it is less than 4% under-strength, tentatively use these parameters.

C6. Check for shear strength.

As indicated for the first part of this design example in which the load and strength reduction factors of Chapter 9 were used, shear strength will rarely govern the design of a wall. Since in that example the wall was greatly over-designed for shear, there is no need to check for shear in this example. *22.5.4* *22.6.4*

C7. Use 10-in. wall with concrete specified compressive strength, $f'_c = 3000$ psi.

This example showed that use of the load and strength reduction factors of Appendix C resulted in a more economical design. To quickly determine which one of the two sets of load and strength reduction factors should be used, compare the nominal moment strength required by the factored loads of 9.2 to that required by C.2. The set requiring the lower nominal strength will generally be more economical.

Chapter 9 $M_n = M_u/\phi = 2.88/0.55 = 5.24$ ft-kips/ft

Appendix C $M_n = M_u/\phi = 3.06/0.65 = 4.71$ kips

Since the nominal moment strength required by the load and strength reduction factors of Appendix C is less than the corresponding strength required by Chapter 9, a design according to Appendix C will be more economical as the example shows. Regardless of the load and strength reduction factors used, the design procedures are the same.

31

Alternate (Working Stress) Design Method

UPDATE FOR ’02 CODE

ACI 318-99 Appendix A titled “Alternate Design Method” was deleted from the 2002 code. This design method was also known as Working Stress Design (WSD) or Allowable Stress Design (ASD). Although WSD was deleted from the code, ACI 318-02 Commentary Section R1.1 states, “The Alternate Design Method of the 1999 code may be used in place of applicable sections of the 2002 code.” Note, the Commentary is not mandatory language and, thus, does not bear legal status. Therefore, in jurisdictions that adopt ACI 318-02, designers that intend to design by the 1999 WSD are cautioned to first seek approval of the local building official of the jurisdiction where the structure will be built.

GENERAL CONSIDERATIONS

Prior to the 1956 edition of the code, the working stress design method, which was very similar to the alternate design method of Appendix A, was the only method available for design of reinforced concrete members. The (ultimate) strength design method was introduced as an appendix to the 1956 code. In the next edition of the code (1963), strength design was moved to the body of the code as an alternative to working stress design. Because of the widespread acceptance of the strength design method, the 1971 code covered the working stress method in less than one page. The working stress method was moved out of the body of the code and into an appendix with the 1983 edition of the code. The method then became referred to as the “alternate design method.” It remained an appendix through the 1999 code.

The alternate design method presented in Appendix A of the 1999 code is a method that seeks to provide adequate structural safety and serviceability by limiting stresses at service loads to certain prescribed limits. These “allowable stresses” are well within the range of elastic material behavior for concrete in compression and steel in tension (and compression). Concrete is assumed to be cracked and provide no resistance in tension. The stress in the concrete is represented by a linear elastic stress distribution. The steel is generally transformed into an equivalent area of concrete for design.

The alternate design method is identical to the “working stress design method” used prior to 1963 for members subject to flexure without axial loads. The procedures for the design of compression members with flexure, shear design, and bond stress and development of reinforcement follow the procedures of the strength design method of the body of the code with factors applied to reflect design at service loads. The design procedures of the alternate design method have not been updated as thoroughly as the remainder of the code.

The replacement of the working stress design method and alternate design method by the strength design method can be attributed to several factors including:

- the uniform treatment of all types of loads, i.e., all load factors are equal to unity. The different variability of different types of loads (dead and live load) is not acknowledged.

- the unknown factor of safety against failure (as discussed below)

- and the typically more conservative designs, which generally require more reinforcement or larger member sizes for the same design moments when compared to the strength design method.

It should be noted that in general, reinforced concrete members designed using working stresses, or the alternate design method, are less likely to have cracking and deflection problems than members designed using strength methods with Grade 60 reinforcement. This is due to the fact that with strength design using Grade 60 reinforcement, the stresses at service loads have increased significantly from what they were with working stress design.

Therefore, crack widths and deflection control are more critical in members designed using strength design methods because these factors are directly related to the stress in the reinforcement.

Today, the alternate design method is rarely used, except for a few special types of structures or by designers who are not familiar with strength design. Footings seem to be the members most often designed using the alternate design method. Note that ACI 350, Environmental Engineering Concrete Structures, governs the design of water retaining structures.

COMPARISON OF WORKING STRESS DESIGN WITH STRENGTH DESIGN

To illustrate the variability of the factor of safety against failure by the working stress design versus the strength design method, a rectangular and a T-section with dimensions shown in Figs. 31-1 and 31-2, respectively, were analyzed. In both cases, f'_c = 4000 psi, f_y = 60 ksi, and amount of reinforcement was varied between minimum flexural reinforcement per 10.5.1 and a maximum of $0.75\rho_b$ per 2002 Appendix B. Flexural strengths were computed using three procedures:

1. Nominal flexural strength, M_n, using the rectangular stress block of 10.2.7. Results are depicted by the solid lines.

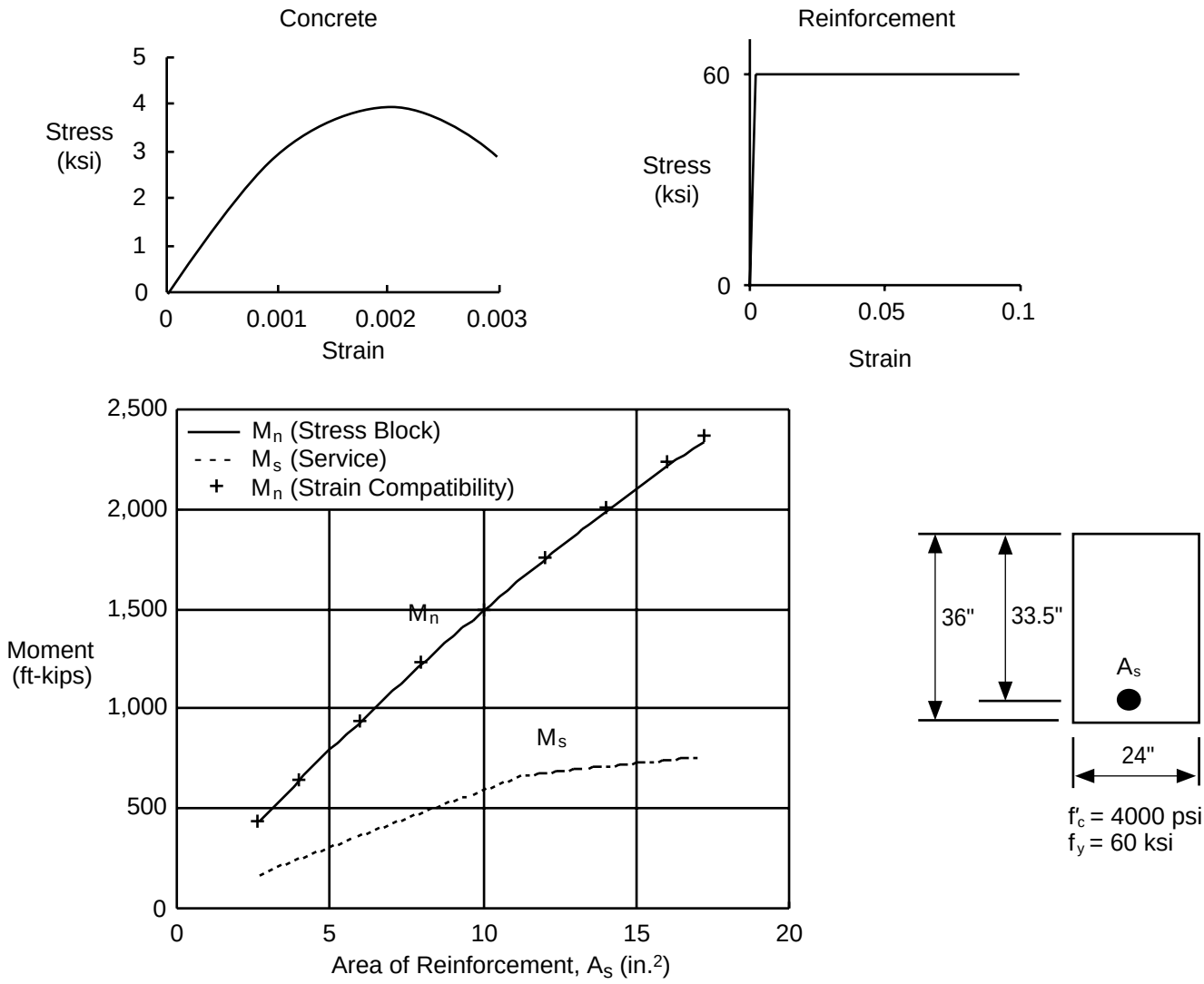
2. Nominal flexural strength based on equilibrium and compatibility. This detailed analysis was performed using program Response 2000[31.1] assuming representative stress-strain relationships for concrete and reinforcing steel are shown in Fig. 31.1. Results are depicted by symbol "+ ."

3. Working stress analysis using linear elastic stress-strain relationships for concrete and reinforcement, and permissible service load stresses of Appendix A of the 1999 code, as noted below. The results are depicted by the dashed lines for M_s.

Obervations:

(a) Flexural strength based on the rectangular stress block, M_n, is very similar to the prediction based on detailed analysis using strain compatibility and equilibrium.

(b) The factor of safety, represented by the ratio $\phi M_n/M_s$ is highly variable. For the rectangular sections, that ratio ranges between 2.3 and 2.8 while for the T-section it ranges between 2.3 and 2.4. In comparison, for flexural design using Chapter 9 load and strength reduction factors, the factor of safety (L.F./ϕ) ranges between 1.2/0.9 = 1.33 where dead load dominates, and 1.6/0.9 = 1.78 where live load dominates. For Appendix C load and strength reduction factors, those ratios are 1.4/0.9 = 1.56 and 1.7/0.9 = 1.89, respectively.



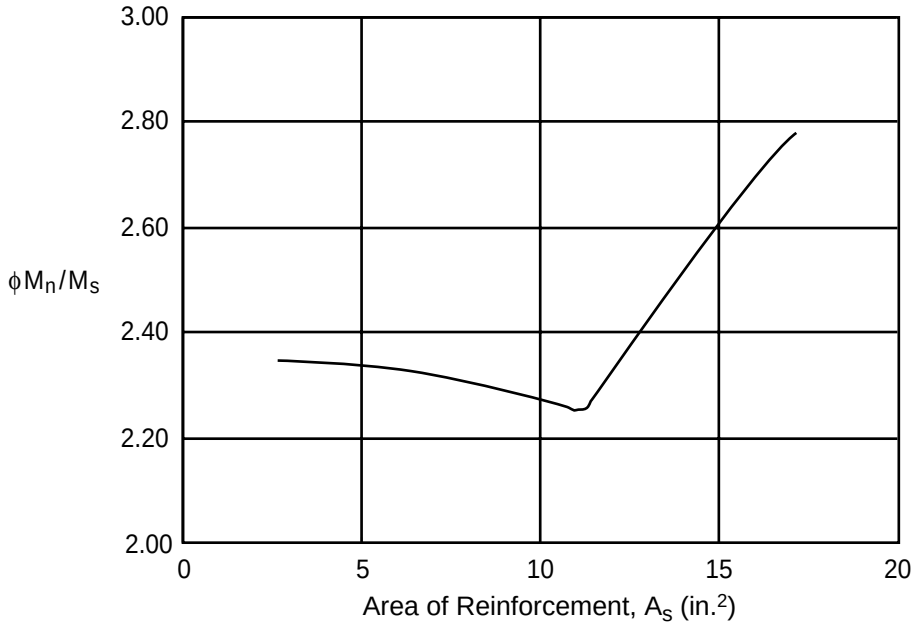
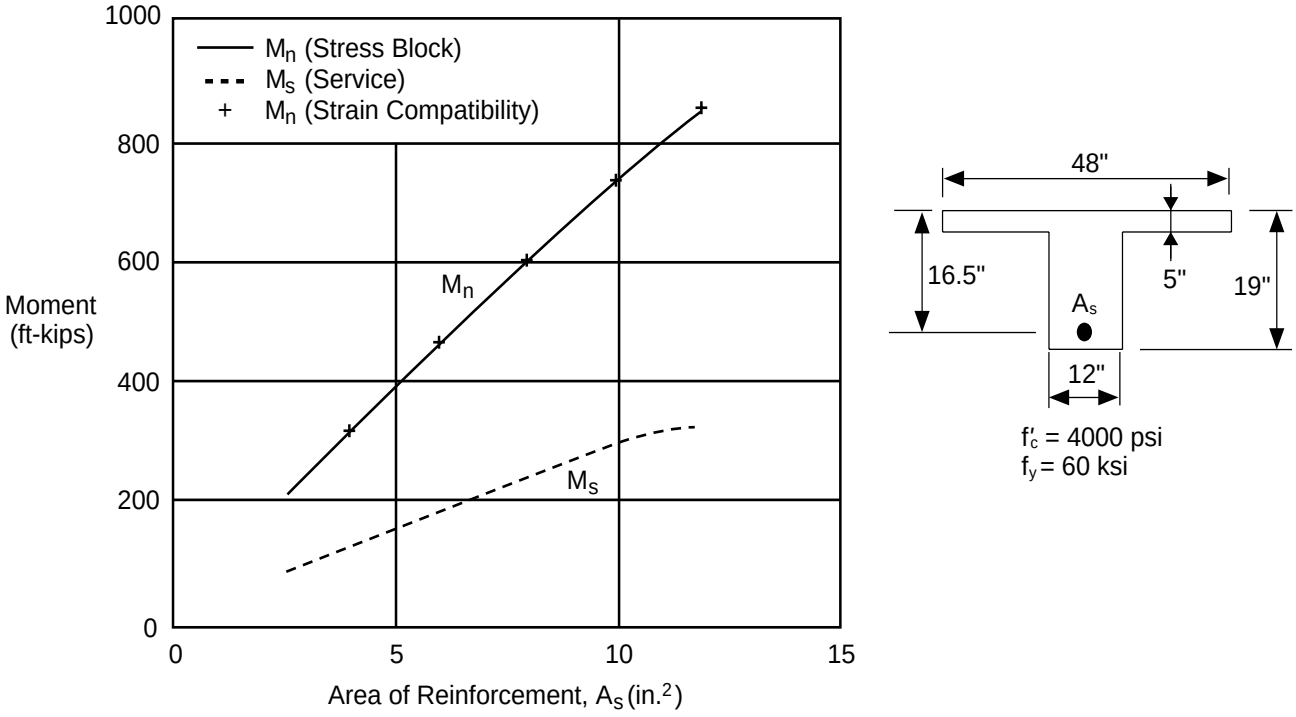
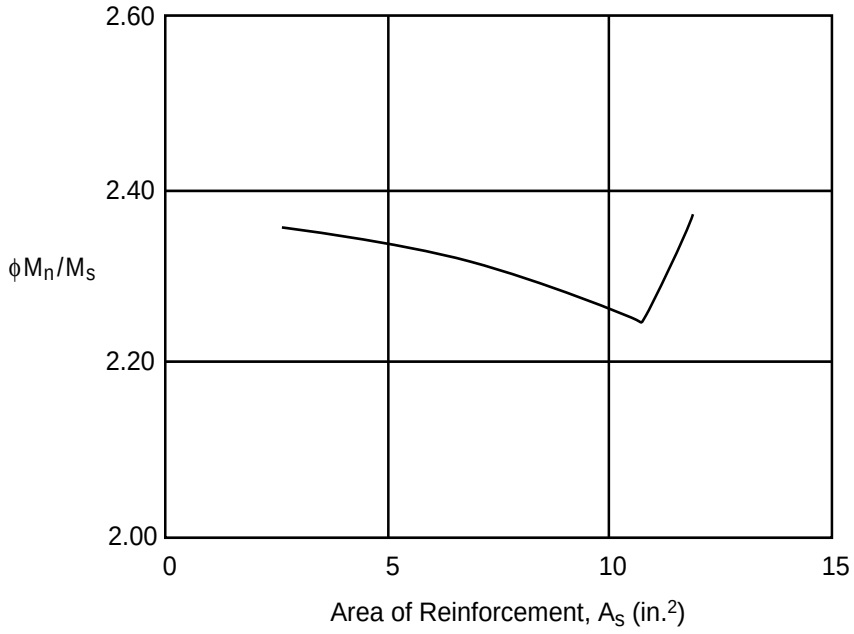


Figure 31-1 Rectangular section.





The following sections highlight provisions of Appendix A of the 1999 code.

SCOPE (A.1 OF '99 CODE)

The code specifies that any nonprestressed reinforced concrete member may be designed using the alternate design method of Appendix A. Prestressed concrete members are designed using a similar approach that is contained in code Chapter 18.

All other requirements of the code shall apply to members designed using the alternate design method, except the moment redistribution provisions of 8.4. This includes such items as distribution of flexural reinforcement and slenderness of compression members, as well as serviceability items such as control of deflections and crack control.

GENERAL (A.2 OF '99 CODE)

Load factors for all types of loads are taken to be unity for this design method. When wind and earthquake loads are combined with other loads, the member shall be designed to resist 75% of the total combined effect. This is similar to the provisions of the original working stress design method which allowed an overstress of one-third for load combinations including wind and earthquake.

When dead loads act to reduce the effects of other loads, 85% of the dead load may be used in computing load effects.

PERMISSIBLE SERVICE LOAD STRESSES (A.3 OF '99 CODE)

Concrete stresses at service loads must not exceed the following:

Flexure	Extreme fiber stress in compression	$0.45f'_c$
Bearing	On loaded area	$0.3f'_c$

Permissible concrete stresses for shear are also given in this section and in greater detail in A.7.

Tensile stresses in reinforcement at service loads must not exceed the following:

Grade 40 and 50 reinforcement	20,000 psi
Grade 60 reinforcement or greater and welded wire fabric (plain or deformed)	24,000 psi

Permissible tensile stresses for a special case are also given in A.3.2(c).

FLEXURE (A.5 OF '99 CODE)

Members are designed for flexure using the following assumptions:

- Strains vary linearly as the distance from the neutral axis. A non-linear distribution of strain must be used for deep members (see 10.7).
- Under service load conditions, the stress-strain relationship of concrete in compression is linear for stresses not exceeding the permissible stress.
- In reinforced concrete members, concrete resists no tension.
- The modular ratio, $n = E_s/E_c$, may be taken as the nearest whole number, but not less than 6. Additional provisions are given for lightweight concrete.
- In members with compression reinforcement, an effective modular ratio of $2E_s/E_c$ must be used to transform the compression reinforcement for stress computations. The stress in the compression reinforcement must not exceed the permissible tensile stress.

DESIGN PROCEDURE FOR FLEXURE

The following equations are used in the alternate design method for the flexural design of a member with a rectangular cross section, reinforced with only tension reinforcement. They are based on the assumptions stated

above and the notation defined in Fig. 31-3. See Refs. 33.2 and 33.3 or other texts on reinforced concrete design for derivation of these equations. Equations can also be developed for other cross sections, such as members with flanges or compression reinforcement.

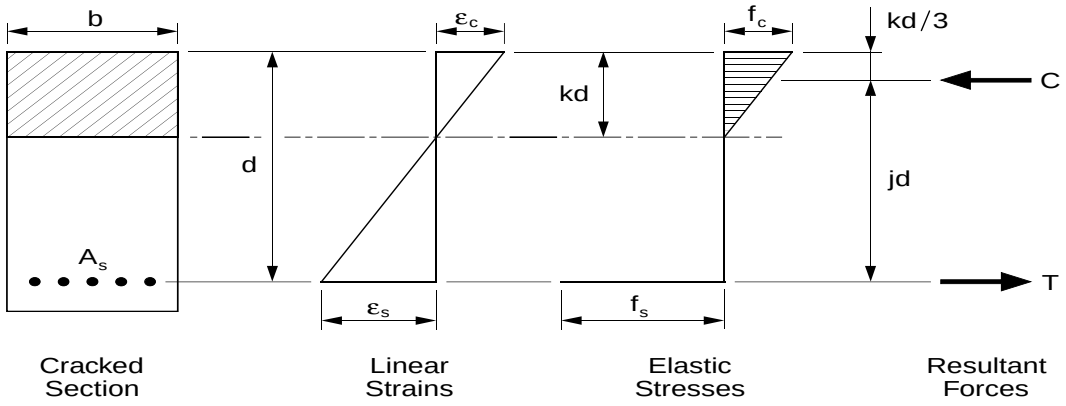


Figure 31-3 Assumptions for Alternate Design Method for Flexure

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

where

$$\rho = \frac{A_s}{bd}$$

$$n = \frac{E_s}{E_c}$$

$$j = 1 - \left(\frac{k}{3}\right)$$

$$f_s = \frac{M_s}{A_s jd}$$

$$f_c = \frac{2M_s}{kjbd^2}$$

A.7 SHEAR AND TORSION

Shear and torsion design in Appendix A of ACI 318-99 is based on the strength design methods of code Chapter 11 (’99 code) with reduced
coefficients that allow use of the equations for unfactored loads at service load conditions.

A complete set of equations is presented for shear design for the convenience of the user and in order to explicitly state the modified equations. Since the equations appear in the same form as in code Chapter 11, they will not be discussed here.

REFERENCES

33.1 Bentz, Evans C. and Collins, Michael P., "Response 2000 – Reinforced Concrete Sectional Analysis using the Modified Compression Field Theory." Downloadable at http://www.ecf.utoronto.ca/~bentz/r2k.htm

33.2 MacGregor, J.G., *Reinforced Concrete: Mechanics and Design*, 2nd Edition, Prentice Hall, Englewood Cliffs, NJ, 1997, 939 pp.

33.3 Leet, Kenneth, *Reinforced Concrete Design*, McGraw-Hill, New York, 1984, 544 pp.

Example 31.1—Design of Rectangular Beam with Tension Reinforcement Only

Given the rectangular beam of Example 10.1, modify the beam depth and/or required reinforcement to satisfy the permissible stresses of the alternate design method. The service load moments are: M_d = 56 ft-kips and $M\ell$ = 35 ft-kips.

f'_c = 4000 psi
f_y = 60,000 psi
A_s = 2.79 in.^{2}
b = 10 in.
h = 16 in.
d = 13.5 in.

Calculations and Discussion	Code Reference

1. To compare a design using the alternate design method to the load factor method of the code, check the service load stresses in concrete and steel in the design given in Example 10.1.

$M_s = M_d + M\ell = (56 + 35)\,(12) = 1092$ in.-kips

$$E_c = 57{,}000\sqrt{f'_c} = 57{,}000\sqrt{4000} = 3{,}605{,}000 \text{ psi}$$ 8.5.1

$$n = \frac{E_s}{E_c} = \frac{29{,}000{,}000}{3{,}605{,}000} = 8.04 \quad \text{Use } n = 8$$ A.5.4

$$\rho = \frac{A_s}{bd} = \frac{2.79}{(10 \times 13.5)} = 0.0207$$

$$\rho n = (0.0207)\,(8) = 0.1656$$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n = \sqrt{2(0.1656) + (0.1656)^2} - 0.1656 = 0.433$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.433}{3} = 0.856$$

$$f_s = \frac{M_s}{A_s jd} = \frac{1092}{[(2.79)\,(0.856)\,(13.5)]} = 33.88 \text{ ksi} > 24.0 \text{ ksi allowed} \quad \text{N.G.}$$ A.3.2

$$f_c = \frac{2M_s}{kjbd^2} = \frac{2\,(1092)}{[(0.433)\,(0.856)\,(10)\,(13.5)^2]} = 3.23 \text{ ksi} > 0.45\,(4.00) = 1.80 \text{ ksi allowed} \quad \text{N.G.}$$ A.3.1

Note: The above calculations are based on the assumption of linear-elastic material behavior. Since both f_c and f_s exceed the permissible stresses, increase the beam depth.

2. Check stresses in concrete and reinforcement with an increased member depth, but the same area of reinforcement.

h = 24 in. d = 21.5 in.

$$\rho = \frac{A_s}{bd} = \frac{2.79}{(10 \times 21.5)} = 0.0130$$

$\rho n = (0.0130)\ (8) = 0.1038$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n = \sqrt{2(0.1038) + (0.1038)^2} - 0.1038 = 0.364$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.364}{3} = 0.879$$

$$f_s = \frac{M_s}{A_s jd} = \frac{1092}{[(2.79)\ (0.879)\ (21.5)]} = 20.71 \text{ ksi} < 24.0 \text{ ksi allowed} \quad \text{O.K.}$$

$$f_c = \frac{2M_s}{kjbd^2} = \frac{2(1092)}{[(0.364)\ (0.879)\ (10)\ (21.5)^2]} = 1.48 \text{ ksi} < 0.45\ (4.0) = 1.80 \text{ ksi allowed} \quad \text{O.K.}$$

Note: It was necessary to increase the effective depth by nearly 60% in order to satisfy allowable stresses using the same quantity of reinforcement.

3. Compute the design moment strength, ϕM_n, of the modified member to determine the factor of safety (FS).

$$a = \frac{A_s f_y}{0.85 b f'_c} = \frac{(2.79)\ (60)}{[(0.85)\ (10)\ (4.00)]} = 4.92 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (2.79)\ (60)\left[21.5 - \left(\frac{4.92}{2}\right)\right] = 3190 \text{ in.-kips}$$

$\phi M_n = 0.9\ (3190) = 2870$ in.-kips

$$FS = \frac{\phi M_n}{M_s} = \frac{2870}{1092} = 2.63$$

32

Alternative Provisions for Reinforced and Prestressed Concrete Flexural and Compression Members

UPDATE FOR THE '02 CODE

New Appendix B contains the provisions that were displaced when the Unified Design Provisions, which were formerly in Appendix B, were incorporated in the main body of the 2002 code. In particular, provisions for moment redistribution in Chapter 8, design methods for flexural and compression members in Chapter 10, and corresponding provisions for prestressed concrete in Chapter 18 have been moved from the main body of the 1999 code to Appendix B of the 2002 code.

B.1 SCOPE

Section 8.1.2 allows the use of Appendix B to design reinforced and prestressed concrete flexural and compression members. Since it may be judged that an appendix is not an official part of a legal document unless it is specifically adopted, reference is made to Appendix B in the main body of the code in order to make it a legal part of the code.

Appendix B contains provisions for moment redistribution, design of flexural and compression members, and prestressed concrete that were in the main body of the code for many years prior to 2002. The use of these provisions is equally acceptable to those in the corresponding sections of the main body of the code.

Section B.1 contains the sections in Appendix B that replace those in the main body of the code when Appendix B is used in design. It must be emphasized that when any section of Appendix B is used, all sections of this appendix must be substituted in the main body of the code. All other sections in the body of the code are applicable.

According to RB.1, load factors and strength reduction factors of either Chapter 9 or new Appendix C (see Part 33) may be used. It is the intent that strength reduction factors given in Chapter 9 or Appendix C for tension-controlled sections be utilized for members subjected to bending only. Similarly, strength reduction factors for compression-controlled sections should be used for members subjected to flexure and axial load with ϕP_n greater than or equal to $0.10f'_cA_g$ or the balanced axial load ϕP_b, whichever is smaller (see 9.3.2.2 and C3.2.2). For other cases, ϕ can be increased linearly to 0.90 as ϕP_n decreases from $0.10f'_cA_g$ or ϕP_b, whichever is smaller, to zero (9.3.2.2 and C3.2.2).

B.8.4 REDISTRIBUTION OF NEGATIVE MOMENTS IN CONTINUOUS NONPRESTRESSED FLEXURAL MEMBERS

Section B.8.4 permits a redistribution of negative moments in continuous flexural members if reinforcement percentages do not exceed a specified amount.

A maximum 10 percent adjustment of negative moments was first permitted in the 1963 ACI Code (see Fig. RB.8.4). Experience with the use of that provision, though satisfactory, was still conservative. The 1971 code increased the maximum adjustment percentage to that shown in Fig. 32-1. The increase was justified by additional knowledge of ultimate and service load behavior obtained from tests and analytical studies. Appendix B of the 2002 code retains the same adjustment percentage criteria.

A comparison between the permitted amount of redistribution according to 8.4 and B.8.4 of the 2002 code as a function of the strain in the extreme tension steel ε_t is depicted in Figure 32-2.

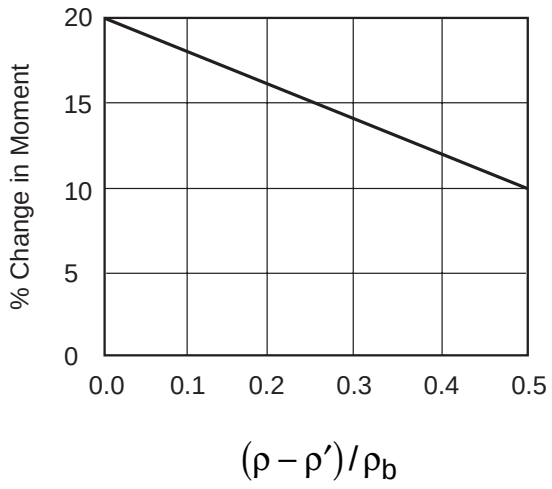


Figure 32-1 Permissible Moment Redistribution for Nonprestressed Members

Application of B.8.4 will permit, in many cases, substantial reduction in total reinforcement required without reducing safety, and reduce reinforcement congestion in negative moment regions.

According to 8.9, continuous members must be designed to resist more than one configuration of live loads. An elastic analysis is performed for each loading configuration, and an envelope moment value is obtained for the design of each section. Thus, for any of the loading conditions considered, certain sections in a given span will reach the ultimate moment, while others will have reserve capacity. Tests have shown that a structure can continue to carry additional loads if the sections that reached their moment capacities continue to rotate as plastic hinges and redistribute the moments to other sections until a collapse mechanism forms.

Recognition of this additional load capacity beyond the intended original design suggests the possibility of redesign with resulting savings in material. Section B.8.4 allows a redesign by decreasing or increasing the elastic negative moments for each loading condition (with the corresponding changes in positive moment required by statics). These moment changes may be such as to reduce both the maximum positive and negative moments in the final moment envelope. In order to ensure proper rotation capacity, the percentage of steel in the sections must conform to B.8.4, which is shown in Fig. 32-1.

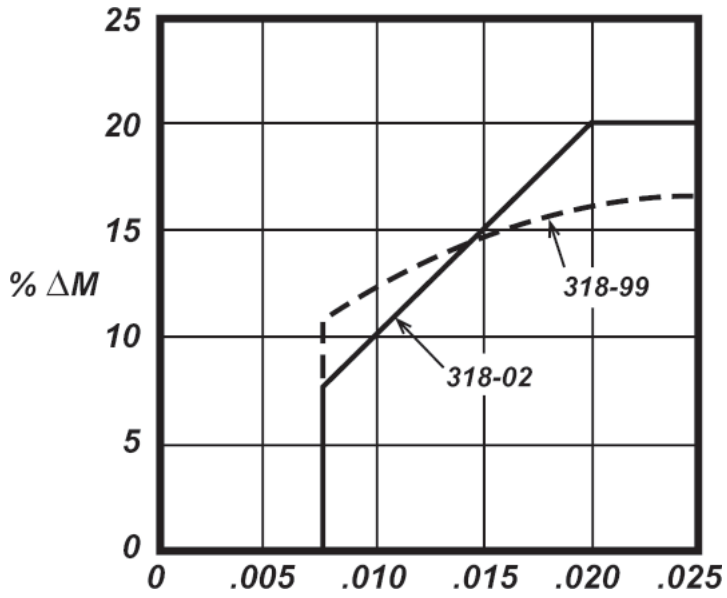


Figure 32-2 Comparison of Permissible Moment Redistribution for Nonprestressed Members

In certain cases, the primary benefit to be derived from B.8.4 will be simply a reduction of negative moment at the supports, to avoid reinforcement congestion or reduce concrete dimensions. In this case, the steel percentage must still conform to Fig. 32-1.

Limits of applicability of B.8.4 may be summarized as follows:

1. Provisions apply to continuous nonprestressed flexural members. Moment redistribution for prestressed members is addressed in B.18.10.4.

2. Provisions do not apply to members designed by the approximate moments of 8.3.3, or to slab systems designed by the Direct Design Method (see 13.6.1.7 and RB.8.4).

3. Bending moments must be determined by analytical methods, such as moment distribution, slope deflection, etc. Redistribution is not allowed for moments determined through approximate methods.

4. The reinforcement ratios ρ or (ρ -ρ′) at a cross-section where moment is to be adjusted must not exceed one-half of the balanced steel ratio, ρ_b, as defined by Eq. (B.8-1).

5. Maximum allowable percentage increase or decrease of negative moment is given by Eq. (B.8-1):

$$20\left(1-\frac{\rho-\rho'}{\rho_b}\right)$$

6. Adjustment of negative moments is made for each loading configuration considered. Members are then proportioned for the maximum adjusted moments resulting from all loading conditions.

7. Adjustment of negative support moments for any span requires adjustment of positive moments in the same span (B.8.4.2). A decrease of a negative support moment requires a corresponding increase in the positive span moment for equilibrium.

8. Static equilibrium must be maintained at all joints before and after moment redistribution.

9. In the case of unequal negative moments on the two sides of a fixed support (i.e., where adjacent spans are unequal), the difference between these two moments is taken into the support. Should either or both of these negative moments be adjusted, the resulting difference between the adjusted moments is taken into the support.

10. Moment redistribution may be carried out for as many cycles as deemed practical, provided that after each cycle of redistribution, a new allowable percentage increase or decrease in negative moment is calculated, based on the final steel ratios provided for the adjusted support moments from the previous cycle.

11. After the design is completed and the reinforcement is selected, the actual steel ratios provided must comply with Fig. 32-1 for the percent moment redistribution taken, to ensure that the requirements of B.8.4 are met.

Examples that illustrate these requirements can be found in Part 9 of Notes on ACI 318-99.

B.10.3 GENERAL PRINCIPLES AND REQUIREMENTS – NONPRESTRESSED MEMBERS

The flexural strength of a member is ultimately reached when the strain in the extreme compression fiber reaches the ultimate (crushing) strain of the concrete, ε_u. At that stage, the strain in the tension reinforcement could just reach the strain at first yield ($\varepsilon_s = \varepsilon_y = f_y/\varepsilon_u$), be less than the yield strain, or exceed the yield strain. Which steel strain condition exists at ultimate concrete strain depends on the relative proportion of reinforcement to concrete. If the steel amount is low enough, the strain in the tension steel will greatly exceed the yield strain ($\varepsilon_s >> \varepsilon_y$) when the concrete strain reaches ε_u, with large deflection and ample warning of impending failure (ductile failure condition). With a larger quantity of steel, the strain in the tension steel may not reach the yield strain ($\varepsilon_s < \varepsilon_y$) when the concrete strain reaches ε_u, which would mean small deflection and little warning of impending failure (brittle failure condition). For design it is desirable to restrict the ultimate strength condition so that a ductile failure mode would be expected.

The provisions of B.10.3.3 are intended to ensure a ductile mode of failure by limiting the amount of tension reinforcement to 75% of the balanced steel to ensure yielding of steel before crushing of concrete. The balanced steel will cause the strain in the tension steel to just reach yield strain when concrete reaches the crushing strain.

The maximum amount of reinforcement permitted in a rectangular section with tension reinforcement only is

$$\rho_{max} = 0.75\bar{\rho}_b = 0.75\left[0.85\beta_1 \frac{f'_c}{f_y} \times \frac{87{,}000}{87{,}000 + f_y}\right]$$

where $\bar{\rho}_b$ is the balanced reinforcement ratio for a rectangular section with tension reinforcement only.

The maximum amount of reinforcement permitted in a flanged section with tension reinforcement only is

$$\rho_{max} = 0.75\left[\frac{b_w}{b}\left(\bar{\rho}_b + \rho_f\right)\right]$$

where b_w = width of the web
b = width of the effective flange (see 8.10)

$\rho_f = A_{sf} / b_w d$

h_f = thickness of the flange

The maximum amount of reinforcement permitted in a rectangular section with compression reinforcement is (B10.3.3)

$$\rho_{max} = 0.75\bar{\rho}_b + \rho' \frac{f'_{sb}}{f_y}$$

where $\rho' = A'_s / bd$

A'_s = area of compression reinforcement

f'_{sb} = stress in compression reinforcement at balanced strain condition

$$= 87{,}000 - \frac{d'}{d}(87{,}000 + f_y) \le f_y$$

d' = distance from extreme compression fiber to centroid of compression reinforcement

Note that with compression reinforcement, the portion of ρ_b contributed by the compression reinforcement $(\rho' f'_{sb} / f_y)$ need not be reduced by the 0.75 factor. For ductile behavior of beams with compression reinforcement, only that portion of the total tension steel balanced by compression in the concrete $\left(\bar{\rho}_b\right)$ need be limited.

It should be realized that the limit on the amount of tension reinforcement for flexural members is a limitation for ductile behavior. Tests have shown that beams reinforced with the computed amount of balanced reinforcement actually behave in a ductile manner with gradually increasing deflections and cracking up to failure. Sudden compression failures do not occur unless the amount of reinforcement is considerably higher than the computed balanced amount.

One reason for the above is the limit on the ultimate concrete strain assumed at $\varepsilon_u = 0.003$ for design. The actual maximum strain based on physical testing may be much higher than this value. The 0.003 value serves as a lower bound on limiting strain. Unless unusual amounts of ductility are required, the $0.75\rho_b$ limitation will provide ample ductile behavior for most designs.

Example 7.1 provides a comparison of the design unified design method and the provisions of B.10.3 for a rectangular beam with tension reinforcement only. Similarly, a comparison is given for a rectangular beam with compression reinforcement in Example 7.3.

B18.1 SCOPE–PRESTRESSED CONCRETE

This section contains a list of the provisions in the code that do not apply to prestressed concrete. Section RB.18.1.3 provides detailed commentary and specific reasons on why some sections are excluded.

B.18.8 LIMITS FOR REINFORCEMENT OF PRESTRESSED FLEXURAL MEMBERS

The requirements of B.18.8 for percentage of reinforcement are illustrated in Fig. 32-3. Note that reinforcement can be added to provide a reinforcement index higher than $0.36\beta_1$; however, this added reinforcement cannot be assumed to contribute to the moment strength.

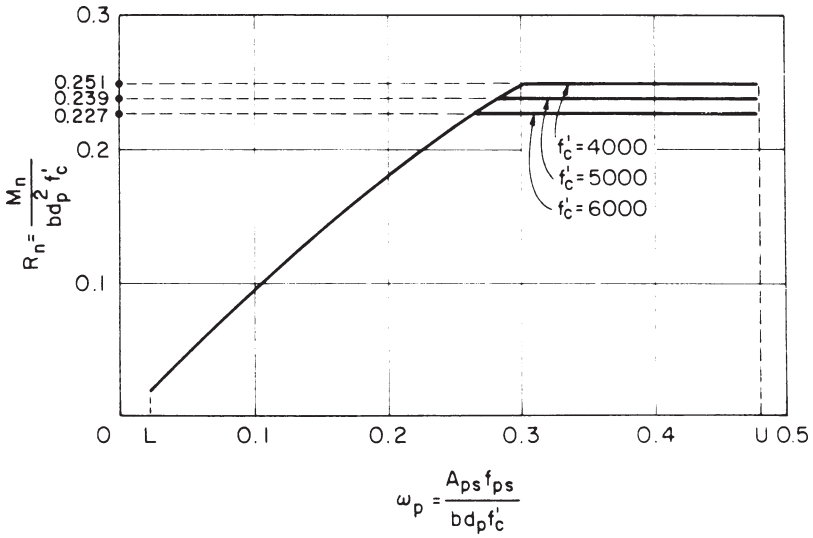


Figure 32-3 Permissible Limits of Prestressed Reinforcement and Influence on Moment Strength

Section B.18.8.3 requires the total amount of prestressed and nonprestressed reinforcement of flexural members to be adequate to develop a design moment strength at least equal to 1.2 times the cracking moment strength ($\phi M_n \geq 1.2M_{cr}$), where M_{cr} is computed by elastic theory using a modulus of rupture equal to $7.5\sqrt{f'_c}$ (see 9.5.2.3). The provisions of B.18.8.3 are analogous to 10.5 for nonprestressed members and are intended as a precaution against abrupt flexural failure resulting from rupture of the prestressing tendons immediately after cracking. The provision ensures that cracking will occur before flexural strength is reached, and by a large enough margin so that significant deflection will occur to warn that the ultimate capacity is being approached. The typical prestressed member will have a fairly large margin between cracking strength and flexural strength, but the designer must be certain by checking it.

The cracking moment M_{cr} for a prestressed member is determined by summing all the moments that will cause a stress in the bottom fiber equal to the modulus of rupture f_r. Refer to Part 24 for detailed equations to compute M_{cr} for prestressed members.

Note that an exception in B.18.8.3 waives the $1.2M_{cr}$ requirement for (a) two-way unbonded post-tensioned slabs, and (b) flexural members with shear and flexural strength at least twice that required by 9.2. See Part 24 for more information.

B.18.10.4 REDISTRIBUTION OF NEGATIVE MOMENTS IN CONTINUOUS PRESTRESSED FLEXURAL MEMBERS

Inelastic behavior at some sections of prestressed concrete beams and slabs can result in a redistribution of moments when member strength is approached. Recognition of this behavior can be advantageous in design

under certain circumstances. Although a rigorous design method for moment redistribution is complex, a rational method can be realized by permitting a reasonable adjustment of the sum of the elastically calculated factored gravity load moments and the unfactored secondary moments due to prestress. The amount of adjustment should be kept within predetermined safe limits.

According to B.18.10.4.1, the maximum allowable percentage increase or decrease of negative moment in a continuous prestressed flexural member is

$$20\left[1-\frac{\omega_p+\frac{d}{d_p}(\omega-\omega')}{0.36\beta_1}\right]$$

Note that redistribution of negative moments is allowed only when bonded reinforcement is provided at the supports in accordance with 18.9. The bonded reinforcement ensures that beams and slabs with unbonded tendons act as flexural members after cracking and not as a series of tied arches.

Similar to nonprestressed members, adjustment of negative support moments for any span requires adjustment of positive moments in the same span (B.18.10.4.2). A decrease of a negative support moment requires a corresponding increase in the positive span moment for equilibrium.

The amount of allowable redistribution depends on the ability of the critical sections to deform inelastically by a sufficient amount. Sections with larger amounts of reinforcement will not be able to undergo sufficient amounts of inelastic deformations. Thus, redistribution of negative moments is allowed only when the section is designed so that the appropriate reinforcement index is less than $0.24\beta_1$ (see B.18.10.4.3). This requirement is in agreement with the requirements of B.8.4 for nonprestressed members. Note that each of the expressions in B.18.10.4.3 is equal to $0.85a/d_p$ where a is the depth of the equivalent rectangular stress distribution for the section under consideration (see 10.2.7.1).

A comparison between the permitted amount of redistribution according to 18.10.4 and B.18.10.4 of the 2002 code as a function of the strain in the extreme tension steel ε_t is depicted in Figure 32-4.

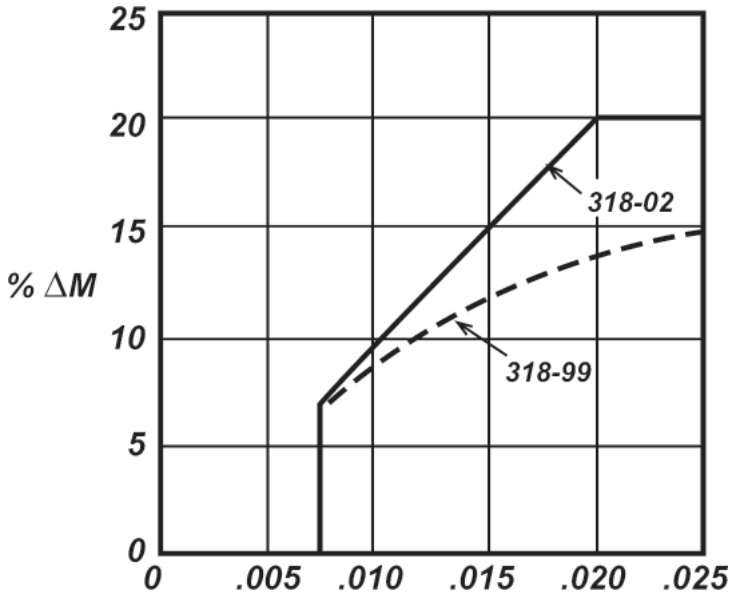


Figure 32-4 Comparison of Permissible Moment Redistribution for Prestressed Members

33

Alternative Load and Strength Reduction Factors

UPDATE FOR THE'02 CODE

New Appendix C contains revised versions of the load and strength reduction factors that were formerly in Chapter 9 for many years. Revisions were made to update some of the factors to recent code and design requirements.

C.1 GENERAL

Section 9.1.3 allows the use of load factor combinations and strength reduction factors of Appendix C to design structural members. Since it may be judged that an appendix is not an official part of a legal document unless it is specifically adopted, reference is made to Appendix C in the main body of the code in order to make it a legal part of the code.

The load and strength reduction factors in new Appendix C have evolved since the early 1960s when the strength design method was originally introduced in the code. Some of the factors have been changed from the values in the 1999 code for reasons stated below. In any case, these sets of factors are still considered to be reliable for the design of concrete structural members.

It is important to note that a consistent set of load and strength reduction factors must be utilized when designing members. It is not permissible to use the load factors of Chapter 9 in conjunction with the strength reduction factors of Appendix C.

C.2 REQUIRED STRENGTH

In general,

$$\text{Design Strength} \geq \text{Required Strength}$$

or

$$\text{Strength Reduction Factor} \times \text{Nominal Strength} \geq \text{Load Factor} \times \text{Service Load Effects}$$

Part 5 contains a comprehensive discussion on the philosophy of the strength design method, including the reasons why load factors and strength reduction factors are required.

Section C.2 prescribes load factors for specific combinations of loads. A list of these combinations is given in Table 33-1. The numerical value of the load factor assigned to each type of load is influenced by the degree of accuracy with which the load can usually be assessed, the variation which may be expected in the load during the lifetime of a structure, and the probability of simultaneous occurrence of different load types. Hence, dead loads, because they can usually be more accurately determined and are less variable, are assigned a lower load factor (1.4) as compared to live loads (1.7). Also, weight and pressure of liquids with well-defined densities and controllable maximum heights are assigned a reduced load factor of 1.4 due the lesser probability of overloading (see C.2.4). A higher load factor of 1.7 is required for earth and groundwater pressures due to considerable uncertainty of their magnitude and recurrence (see C.2.3). Note that while most usual combinations of loads are included, it should not be assumed that all cases are covered

Table 33-1 Required Strength for Different Load Combinations

Code Section	Loads†	Required Strength	Code Eq. No.
C.2.1	Dead (D) & Live (L)	U = 1.4D + 1.7L	C -1
C.2.2	Dead, Live & Wind (W)††	(i) U = 1.4D + 1.7L (ii) U = 0.75 (1.4D + 1.7L + 1.6W) (iii) U = 0.9D + 1.6W	C -1 C -2 C -3
C.2.2	Dead, Live & Earthquake (E) ††	(i) U = 1.4D + 1.7L (ii) U = 0.75 (1.4D + 1.7L + 1.0E) (iii) U = 0.9D + 1.0E	C -1 C -2 C -3
C.2.3	Dead, Live & Earth and Groundwater Pressure (H)*	(i) U = 1.4D + 1.7L (ii) U = 1.4D + 1.7L + 1.7H (iii) U = 0.9D + 1.7H where D or L reduces H	C -1 C -4
C.2.4	Dead, Live & Fluid Pressure (F)**	(i) U = 1.4D + 1.7L (ii) U = 1.4D + 1.7L + 1.4F (iii) U = 0.9D + 1.4F where D or L reduces F	C -1
C.2.5	Impact (I)***	In all of the above equations, substitute (L+I) for L when impact must be considered.	
C.2.6	Dead, Live and Effects from Differential Settlement, Creep, Shrinkage, Expansion of Shrinkage-Compensating Concrete, or Temperature (T)	(i) U = 1.4D + 1.7L (ii) U = 0.75 (1.4D + 1.4T + 1.7L) (iii) U = 1.4 (D + T)	C -1 C -5 C -6

† *D, L, W, E, H, F, and T represent the designated service loads or their corresponding effects such as moments, shears, axial forces, torsion, etc. Note: E is a service-level earthquake force.*

†† *Where wind load W has not been reduced by a directionality factor, it is permitted to use 1.3W in place of 1.6W in Eq. (C-2) and (C-3). Where earthquake load E is based on service-load seismic forces, 1.4E shall be used in place of 1.0E in Eq. (C-2) and (C-3).*

* *Weight and pressure of soil and water in soil. (Groundwater pressure is to be considered part of earth pressure with a 1.7 load factor.)*

** *Weight and pressure of fluids with well-defined densities and controllable maximum heights*

*** *Impact factor is required for design of parking structures, loading docks, warehouse floors, elevator shafts, etc.*

The load factors for wind and earthquake forces have changed from those in Chapter 9 of the 1999 code. Since the wind load equation in ASCE 7-98 and IBC 2000 includes a factor for wind directionality that is equal to 0.85 for buildings, the corresponding load factor for wind in the load combination equations was increased accordingly (1.3/0.85 = 1.53, rounded up to 1.6). The code allows use of the previous wind load factor of 1.3 when the design wind load is obtained from other sources that do not include the wind directionality factor.

The most recent model building codes and the 2000 IBC specify strength-level earthquake forces; thus, the earthquake load factor was reduced to 1.0. The code requires use of the previous load factor for earthquake loads, which is 1.4, when service-level earthquake forces from earlier editions of the model codes or other resource documents are used.

C.3 DESIGN STRENGTH

As noted above, the design strength of a member is the nominal strength of the member, which is determined in accordance with code requirements, multiplied by the appropriate strength reduction factor, ϕ. The purposes of the strength reduction factors are given in Part 5 and RC.3.

The ϕ-factors prescribed in C.3, which have changed from those given in Chapter 9 of the 1999 code, are contained in Table 33-2. Prior to the 2002 code, ϕ-factors were given in terms of the type of loading for members subjected to axial load, flexure, or combined flexure and axial load. Now, for these cases, the ϕ-factor is determined by the strain conditions at a cross-section at nominal strength. Figure RC3.2 shows the variation of ϕ with the net tensile strain ε_t for Grade 60 reinforcement and prestressing steel. The Unified Design Provisions are described in detail in Parts 5 and 6. As noted above, the ϕ-factors given in C.3 are consistent with the load factors given in C.2.

Table 33-2 Strength Reduction Factors ϕ in the Strength Design Method

Tension-controlled sections	0.90
Compression-controlled sections Members with spiral reinforcement conforming to 10.9.3 Other reinforced members	 0.75 0.70
Shear and torsion	0.85
Bearing on concrete (except for post-tensioned anchorage zones and strut-and-tie models)	0.70
Post-tensioned anchorage zones	0.85
Strut-and-tie models (Appendix A)	0.85

34

Anchoring to Concrete

UPDATE FOR ’02 CODE

Appendix D, Anchoring to Concrete, was introduced in ACI 318-02. It provides requirements for the design of anchorages to concrete using both cast-in-place and post-installed mechanical anchors.

INTRODUCTION

The following presents a brief overview regarding the development and publication of ACI 318-02 Appendix D. As of the late 1990’s, ACI 318 and the American Institute of Steel Construction LRFD and ASD Specifications were silent regarding the design of anchorage to concrete. ACI 349-85 Appendix B (Ref. 34.1) and the PCI Design Handbook (Ref. 34.2) provided the primary sources of design information for connections to concrete using cast-in-place anchors. The design of connections to concrete using post-installed anchors has typically been based on information provided by individual anchor manufacturers.

During the past several years, ACI Committee 318 has taken the lead in developing building code provisions for the design of anchorages to concrete using both cast-in-place and post-installed mechanical anchors. Committee 318 received support from ACI Committee 355 (ACI 355), Anchorage to Concrete, and ACI Committee 349, Concrete Nuclear Structures. Concurrent with the ACI 318 effort to develop design provisions, ACI 355 was involved with developing a test method for evaluating the performance of post-installed mechanical anchors in concrete. During the code cycle leading to ACI 318-99, a proposed Appendix D to ACI 318 dealing with the design of anchorages to concrete using both cast-in-place and post-installed mechanical anchors was approved by ACI 318. Final adoption of the proposed appendix awaited ACI 355 approval of a test method for evaluating the performance of post-installed mechanical anchors in concrete under the ACI consensus process.

Since ACI 355 was not able to complete the test method for evaluating post-installed mechanical anchors on time to meet the publication deadlines for the ACI 318-99 code, an attempt was made to process an ACI 318 Appendix D reduced in scope to only cast-in-place anchors (i.e., without post-installed mechanical anchors). However, there was not sufficient time to meet the deadlines established by the International Code Council for submittal of the published ACI 318-99 standard to be referenced in the International Building Code [IBC 2000 (Ref. 34.3)]. As a result, the anchorage to concrete provisions originally intended for ACI 318-99 Appendix D (excluding provisions for post-installed mechanical anchors) were submitted and approved for incorporation into Section 1913 of IBC 2000. Now that ACI 318-02 Appendix D, Anchoring to Concrete, which provides design requirements for both cast-in-place and post-installed mechanical anchors, and ACI 355.2, Evaluating the Performance of Post-Installed Mechanical Anchors (Ref. 34.4), have been published, it is anticipated that Section 1913 of the IBC will be replaced by a reference to ACI 318-02 Appendix D.

It should be noted that ACI 318-02 Appendix D does not address adhesive and grouted anchors. Based on the results of research conducted during the 1990’s, it is anticipated that design provisions for adhesive and grouted anchors will be incorporated into ACI 318-05.

HISTORICAL BACKGROUND OF DESIGN METHODS

The 45-deg cone method used in ACI 349 Appendix B (Ref. 34.1) and the PCI Design Handbook (Ref. 34.2) was developed in the mid 1970's. In the 1980's, comprehensive tests of different types of anchors with various embedment lengths, edge distances, and group effects were performed at the University of Stuttgart on both uncracked and cracked concrete. The Stuttgart test results led to the development of the Kappa (K) method that was introduced in ACI 349 and ACI 355 in the late 1980's. In the early 1990's, the K method was improved, and made user-friendlier at the University of Texas at Austin. This effort resulted in the Concrete Capacity Design (CCD) method. During this same period, an international database was assembled. During the mid 1990's, the majority of the work of ACI Committees 349 and 355 was to evaluate both the CCD method and the 45-deg cone method using the international database of test results. As a result of this evaluation, ACI Committees 318, 349, and 355 proceeded with implementation of the CCD method. The design provisions of ACI 318-02 Appendix D and ACI 349-01 Appendix B (Ref. 34.5) are based on the CCD method. Differences between the CCD method and the 45-deg cone method are discussed below.

GENERAL CONSIDERATIONS

The design of anchorages to concrete must address both strength of the anchor steel and that associated with the embedded portion of the anchors. The lesser of these two strengths will control the design.

The strength of the anchor steel depends on the steel properties and size of the anchor. The strength of the embedded portion of the anchorage depends on its embedment length, strength of the concrete, proximity to other anchors, distance to free edges, and the characteristics of the embedded end of the anchor (headed, hooked, expansion, undercut, etc.).

The primary difference between the ACI 318-02 Appendix D provisions and those of the 45-deg cone method lies in the calculation of the embedment capacity for concrete breakout (i.e., a concrete cone failure). In the 45-deg cone method, the calculation of breakout capacity is based on a 45-deg concrete cone failure model that results in an equation based on the embedment length squared (h_{ef}^2). The ACI 318-02 Appendix D provisions account for fracture mechanics and result in an equation for concrete breakout that is based on the embedment length to the 1.5 power ($h_{ef}^{1.5}$). Although the 45-deg concrete cone failure model gives conservative results for anchors with $h_{ef} \leq 6$ in., the ACI 318-02 Appendix D provisions have been shown to give a better prediction of embedment strength for both single anchors and for anchors influenced by edge and group effects.

In addition to better prediction of concrete breakout strength, the ACI 318-02 Appendix D provisions simplify the calculation of the effects of anchor groups and edges by using a rectangular area bounded by $1.5h_{ef}$ from each anchor and free edges rather than the overlapping circular cone areas typically used in the 45-deg cone method.

DISCUSSION OF DESIGN PROVISIONS

The following provides a section-by-section discussion of the design provisions of ACI 318-02 Appendix D. Section, equation, and figure numbers in the following discussion and examples refer to those used in ACI 318-02 Appendix D.

D.0 NOTATION AND D.1 DEFINITIONS

The notations and definitions presented are generally self-explanatory and are further explained in the text and figures of Appendix D. The following tables are provided as an aid to the designer in determining values for many of the variables:

Table 34-1: This table provides information on the types of materials typically specified for cast-in-place anchor applications. The table provides values for specified tensile strength f_{ut} and specified yield strength f_y as well as the elongation and reduction in area requirements necessary to determine if a material should be considered as a brittle or ductile steel element. As shown in Table 34-1, all typical anchor materials satisfy the ductile steel element requirements of D.1. When using cast-in-place anchor materials not given in Table 34-1, the designer should refer to the appropriate material specification to be sure the material falls within the ductile steel element definition. Some high strength materials may not meet these requirements and must be considered as brittle steel elements.

Table 34-1 Properties of Cast-in-Place Anchor Materials

Material specification[1]	Grade or type	Diameter (in.)	Tensile strength, for design f_{ut} (ksi)	Tensile strength, min. (ksi)	Yield strength, min.		Elongation, min.		Reduction of area, min., (%)
					ksi	method	%	length	
AWS D1.1[2]	B	1/2 – 1	60	60	50	0.2%	20	2"	50
ASTM A 307[3]	A C	≤ 4 ≤ 4	60 58	60 58-80	— 36	— —	18 23	2" 2"	— —
ASTM A 354[4]	BC BD	≤ 4 ≤ 4	125 125	125 150	109 130	0.2% 0.2%	16 14	2" 2"	50 40
ASTM A 449[5]	1	≤ 1 1 – 1-1/2 > 1-1/2	120 105 90	120 105 90	92 81 58	0.2% 0.2% 0.2%	14 14 14	4D 4D 4D	35 35 35
ASTM F 1554[6]	36 55 105	≤ 2 ≤ 2 ≤ 2	58 75 125	58-80 75-95 125-150	36 55 105	0.2% 0.2% 0.2%	23 21 15	2" 2" 2"	40 30 45

Notes:

1. The materials listed are commonly used for concrete anchors. Although other materials may be used (e.g., ASTM A 193 for high temperature applications, ASTM A 320 for low temperature applications), those listed are preferred for normal use. Structural steel bolting materials such as ASTM A 325 and ASTM A 490 are not typically available in the lengths needed for concrete anchorage applications.
2. *AWS D1.1-00 Structural Welding Code - Steel* - This specification covers welded headed studs or welded hooked studs (unthreaded). None of the other listed specifications cover welded studs.
3. *ASTM A 307-00 Standard Specification for Carbon Steel Bolts and Studs, 60,000 psi Tensile Strength* - This material is commonly used for concrete anchorage applications. Grade A is headed bolts and studs. Grade C is nonheaded bolts (studs), either straight or bent, and is equivalent to ASTM A 36 steel. Note that although a reduction in area requirement is not provided, A 307 may be considered a ductile steel element. Under the definition of "Ductile steel element" in D.1, the code states: "A steel element meeting the requirements of ASTM A 307 shall be considered ductile."
4. *ASTM A 354-01 Standard Specification for Quenched and Tempered Alloy Steel Bolts, Studs, and Other Externally Threaded Fasteners* - The strength of Grade BD is equivalent to ASTM A 490.
5. *ASTM A 449-00 Standard Specification for Quenched and Tempered Steel Bolts and Studs* - This specification is referenced by ASTM A 325 for "equivalent" anchor bolts.
6. *ASTM F 1554-99 Standard Specification for Anchor Bolts* - This specification covers straight and bent, headed and headless, anchor bolts in three strength grades. Anchors are available in diameters ≤ 4 in. but reduction in area requirements vary for anchors > 2 in.

Table 34-2: This table provides information on the effective cross-sectional area A_{se} and bearing area A_{brg} for threaded cast-in-place anchors up to 2 in. in diameter.

Table 34-3: This table provides a fictitious sample information table for post-installed mechanical anchors that have been tested in accordance with ACI 355.2. This type of table will be available from manufactures that have tested their products in accordance with ACI 355.2. The table provides all of the values necessary for design of a particular post-installed mechanical anchor. The design of post-installed mechanical anchors must be based on this type of table unless values assumed in the design are specified in the project specifications (e.g., the pullout strength N_p).

Table 34-2 Dimensional Properties of Threaded Cast-in-Place Anchors

Anchor Diameter (d_o) (in.)	Gross Area of Anchor (in.^{2})	Effective Area of Anchor (A_{se}) (in.^{2})	Bearing Area of Heads and Nuts (A_{brg}) (in.^{2})			
			Square	Heavy Square	Hex	Heavy Hex
0.250	0.049	0.032	0.142	0.201	0.117	0.167
0.375	0.110	0.078	0.280	0.362	0.164	0.299
0.500	0.196	0.142	0.464	0.569	0.291	0.467
0.625	0.307	0.226	0.693	0.822	0.454	0.671
0.750	0.442	0.334	0.824	1.121	0.654	0.911
0.875	0.601	0.462	1.121	1.465	0.891	1.188
1.000	0.785	0.606	1.465	1.855	1.163	1.501
1.125	0.994	0.763	1.854	2.291	1.472	1.851
1.250	1.227	0.969	2.228	2.773	1.817	2.237
1.375	1.485	1.160	2.769	3.300	2.199	2.659
1.500	1.767	1.410	3.295	3.873	2.617	3.118
1.750	2.405	1.900	—	—	—	4.144
2.000	3.142	2.500	—	—	—	5.316

Table 34-3 ACI 355.2 Sample Information Table for Post-Installed Mechanical Anchors

Undercut Anchor Data – Sample						
			Diameter			
Characteristic	Symbol	Units	3/8 in. Dia.	1/2 in. Dia.	5/8 in. Dia.	3/4 in. Dia.
Effective embedment depth	h_{ef}	in.	4	5	7-1/2	9-7/8
Outside diameter	d_o	in.	0.748	0.827	1.142	1.420
Effective bolt cross-sectional area – tension	A_{se}	in.^{2}	0.090	0.131	0.243	0.380
Steel shear capacity - mean value	V_s	lb	21,640	29,140	49,700	70,400
Bolt minimum specified yield strength	f_y	lb/in.^{2}	92,800	92,800	92,800	92,800
Bolt minimum specified ultimate strength	f_{ut}	lb/in.^{2}	116,000	116,000	116,000	116,000
Sleeve cross-sectional area	A_{sl}	in.^{2}	0.304	0.346	0.690	1.047
Sleeve ultimate tensile strength	f_{utsl}	lb/in.^{2}	123,300	123,300	101,500	79,800
Minimum spacing	s_{min}	in.	4	5	7.5	9-7/8
Installation torque	T_{min}	ft-lb	37	59	88	221
Critical edge distance - tension	c_{cr}	in.	6	7.5	11-1/4	14-3/4
Minimum edge distance - tension	c_{min}	in.	3.2	4	6	7-7/8
Minimum member thickness	h_{min}	in.	5-1/2	6-5/8	9-7/8	15
Category of anchor	—	—	1	1	1	1
Effectiveness factor	k	—	24	24	24	24
Pullout resistance-from tests-calculated characteristic value	N_p	lb	9100	12,730	23,390	35,530
Seismic resistance determined from tests – tension	N_{eq}	lb	5280	7977	14,310	22,872
Seismic resistance determined from tests – shear	V_{eq}	lb	10,820	14,570	24,850	35,200

As a further commentary on the five percent fractile in D.1 – Definitions, the five percent fractile is used to determine the nominal embedment strength of the anchor. It represents a value such that if 100 anchors are tested there is a 90% confidence that 95 of the anchors will exhibit strengths higher than the five percent fractile value. The five percent fractile is analogous to the use of f'_c for concrete strength and f_y for steel strength in the nominal strength calculations in other parts of the ACI 318 code. For example, ACI 318 Section 5.3 requires that the required average compressive strength of the concrete f'_{cr} be statistically greater than the specified value of f'_c used in design calculations. For steel, f_y represents the specified yield strength of the material. Since ASTM specifications give the minimum specified yield strength, the value of f_y used in design is in effect a zero percent fractile (i.e., the designer is ensured that the actual steel used will have a yield value higher than the minimum specified value). All embedment strength calculations in Appendix D are based on a nominal strength calculated using 5 percent fractile values (e.g., the k values used in calculating basic concrete breakout strength are based on the 5 percent fractile).

D.2 SCOPE

These provisions apply to cast-in-place and post-installed mechanical anchors (such as those illustrated in Fig. RD.0) that are used to transmit structural loads between structural elements and safety related attachments to structural elements. The type of anchors included are cast-in-place headed studs, headed bolts, hooked rods (J and L bolts), and post-installed mechanical anchors that have met the anchor assessment requirements of ACI 355.2[4]. Other types of cast-in-place anchors (e.g., specialty inserts) and post-installed anchors (e.g., adhesive, grouted, and pneumatically actuated nails or bolts) are currently excluded from the scope of Appendix D as well as post-installed mechanical anchors that have not met the anchor assessment requirements of ACI 355.2[4]. As noted in D.2.4, these design provisions do not apply to anchorages loaded with high cycle fatigue and impact loads.

D.3 GENERAL REQUIREMENTS

The analysis methods prescribed in D.3 to determine loads on individual anchors in multiple anchor applications depend on the type of loading, rigidity of the attachment base plate, and the embedment of the anchors.

For multiple-anchor connections loaded concentrically in pure tension, the applied tensile load may be assumed to be evenly distributed among the anchors if the base plate has been designed so as not to yield. Prevention of yielding in the base plate will ensure that prying action does not develop in the connection.

For multiple-anchor connections loaded with an eccentric tension load or moment, distribution of loads to individual anchors should be determined by elastic analysis unless calculations indicate that sufficient ductility exists in the embedment of the anchors to permit a redistribution of load among individual anchors. If sufficient ductility is provided, a plastic design approach may be used. The plastic design approach requires ductile steel anchors sufficiently embedded so that embedment failure will not occur prior to a ductile steel failure. The plastic design approach assumes that the tension load (either from eccentric tension or moment) is equally distributed among the tension anchors. For connections subjected to moment, the plastic design approach is analogous to multiple layers of flexural reinforcement in a reinforced concrete beam. If the multiple layers of steel are adequately embedded and are a sufficient distance from the neutral axis of the member, they may be considered to have reached yield.

For both the elastic and plastic analysis methods of multiple-anchor connections subjected to moment, the exact location of the compressive resultant cannot be accurately determined by traditional concrete beam analysis methods. This is true for both the elastic linear stress-strain method (i.e., the transformed area method) and the ACI 318 stress block method since plane sections do not remain plane. For design purposes, the compression resultant from applied moment may be assumed to be located one base plate thickness away from the compression element of the attached member unless base plate stiffeners are provided. If base plate stiffeners are provided, the compressive resultant may be assumed to be located at the leading edge of the base plate.

Sections D.3.3.1 to D.3.3.5 provide special requirements for anchor design that includes seismic loads. Appendix D should not be used for the design of anchors in plastic hinge zones where high levels of cracking and spalling may be expected due to a seismic event. The Appendix D design provisions and the anchor evaluation criteria of ACI 355.2 are based on cracks that might occur normally in concrete (the cracked concrete tests and simulated seismic tests in ACI 355.2 are based on anchor performance in cracks from 0.012 in. to 0.020 in.). In regions of moderate or high seismic risk, or for structures assigned to intermediate or high seismic performance or design categories (see Table 1-3 for equivalent terminology used in building codes) all values for ϕN_n and ϕV_n must be reduced by multiplying by an additional factor of 0.75. Further, the strength of the connection must be controlled by the strength of ductile steel elements and not the embedment strength or the strength of brittle steel elements unless the structural attachment has been designed to yield at a load no greater than the design strength of the anchors, reduced by the factor of 0.75. Section RD.3.3 provides a detailed discussion of these requirements.

D.4 GENERAL REQUIREMENTS FOR STRENGTH OF ANCHORS

This section provides a general discussion of the failure modes that must be considered in the design of anchorages to concrete. The section also provides strength reduction factors, ϕ, for each type of failure mode. The failure modes that must be considered include those related to the steel strength and those related to the strength of the embedment.

Failure modes related to steel strength are simply tensile failure [Fig. RD.4.1(a)(i)] and shear failure [Fig. RD.4.1(b)(i)] of the anchor steel. Anchor steel strength is relatively easy to compute but typically does not control the design of the connection unless there is a specific requirement that the steel strength of a ductile steel element control the design.

Embedment failure modes that must be considered are illustrated in Appendix D Fig. RD.4.1. They include:

- concrete breakout - a concrete cone failure emanating from the embedded end of tension anchors [Fig. RD.4.1(a)(iii)] or from the entry point of shear anchors located near an edge [Fig. RD.4.1(b)(iii)]
- pullout - a straight pullout of the anchor such as might occur for an anchor with a small head [Fig. RD.4.1(a)(ii)]
- side-face blowout - a spalling at the embedded head of anchors located near a free edge [Fig. RD.4.1(a)(iv)]
- concrete pryout - a shear failure mode that can occur with a short anchor popping out a wedge of concrete on the back side of the anchor [Fig. RD.4.1(b)(ii)]
- splitting - a tensile failure mode related to anchors placed in relatively thin concrete members [Fig. RD.4.1(a)(v)]

As noted in D.4.2, the use of any design model that results in predictions of strength that are in substantial agreement with test results is also permitted by the general requirements section. If the designer feels that the 45-deg cone method, or any other method satisfy this requirement he or she is permitted to use them. If not, the design provisions of the remaining sections of Appendix D should be used provided the anchor diameter does not exceed 2 in. and the embedment length does not exceed 25 in. These restrictions represent the upper limits of the database that the Appendix D design provisions are based on.

In the selection of the appropriate ϕ related to embedment failure modes, the presence of supplementary reinforcement designed to tie a potential failure prism to the structural member determines whether the ϕ for Condition A or Condition B applies. For the case of cast-in-place anchors loaded in shear directed toward a free edge, the supplementary reinforcement required for Condition A might be achieved by the use of hairpin reinforcement. It should be noted that for determining pullout and pryout strengths, N_{pn} or V_{cp}, respectively, D.4.4(c) indicates that Condition B applies in all cases regardless of whether supplementary reinforcement is provided or not. In the case of post-installed anchors it is doubtful that this type of reinforcement will have been provided and Condition B will normally apply. The selection of ϕ for post-installed anchors also depends on the anchor

category determined from the ACI 355.2 product evaluation tests. As part of the ACI 355.2 product evaluation tests, product reliability tests (i.e., sensitivity to installation variables) are performed and the results used to establish the appropriate category for the anchor. Since each post-installed mechanical anchor may be assigned a different category, product data tables resulting from ACI 355.2 testing should be referred to. Example data are shown in Table 34-3.

Table 34-4 summarizes the strength reduction factors, ϕ, to be used with the various governing conditions depending upon whether the load combinations of 9.2 or Appendix C are used.

D.5 DESIGN REQUIREMENTS FOR TENSILE LOADING

Methods to determine the nominal tensile strength as controlled by steel strength and embedment strength are presented in the section on tensile loading. The nominal tensile strength of the steel is based on the specified tensile strength of the steel Eq. (D-3). The nominal tensile strength of the embedment is based on (1) concrete breakout strength, Eq. (D-4) for single anchors or Eq. (D-5) for groups of anchors, (2) pullout strength, Eq. (D-12), or (3) side-face blowout strength, Eq. (D-15) for single anchors or Eq. (D-16) for groups. When combined with the appropriate strength reduction factors from D.4.4 or D.4.5, the smallest of these strengths will control the design tensile strength of the anchorage.

D.5.1 Steel Strength of Anchor in Tension

The tensile strength of the steel is determined from Eq. (D-3) using the effective cross-sectional area of the anchor A_{se} and the specified tensile strength of the anchor steel f_{ut}.

For cast-in-place anchors (i.e., threaded anchors, headed studs and hooked bars), the effective cross-sectional area of the anchor A_{se} is the net tensile stress area for threaded anchors and the gross area for headed studs that are welded to a base plate. These areas are provided in Table 34-2. For anchors of unusual geometry, the nominal steel strength may be taken as the lower 5% fractile of test results. For post-installed mechanical anchors the effective cross-sectional area of the anchor A_{se} must be determined from the results of the ACI 355.2 product evaluation tests. Example data are shown in Table 34-3.

The value of f_{ut} used in Eq. (D-3) is limited to $1.9f_y$ or 125,000 psi. The limit of $1.9f_y$ is intended to ensure that the anchor does not yield under service loads and typically is applicable only to stainless steel materials. The limit of 125,000 psi is based on the database used in developing the Appendix D provisions. Table 34-1 provides values for f_y and f_{ut} for typical anchor materials. Note that neither of the limits applies to the typical anchor materials given in Table 34-1. For anchors manufactured according to specifications having a range for specified tensile strength, f_{ut} (e.g., ASTM F 1554), the lower limit value should be used to calculate the design strength. For post-installed mechanical anchors, both f_y and f_{ut} must be determined from the results of the ACI 355.2 product evaluation tests. Example data are shown in Table 34-3.

D.5.2 Concrete Breakout Strength of Anchor in Tension

Figure RD.4.1(a)(iii) shows a typical concrete breakout failure (i.e., concrete cone failure) for a single headed cast-in-place anchor loaded in tension. Eq. (D-4) gives the concrete breakout strength for a single anchor while Eq. (D-5) gives the concrete breakout strength for a group of anchors in tension.

The individual terms in Eq. (D-4) and Eq. (D-5) are discussed below:

N_b: The basic concrete breakout strength for a single anchor located away from edges and other anchors (N_b) is given by Eq. (D-7) or Eq. (D-8). As previously noted, the primary difference between these equations and those of the 45-deg concrete cone method is the use of $h_{ef}^{1.5}$ in Eq. (D-7) [or alternatively $h_{ef}^{5/3}$ for anchors

Table 34-4 Strength Reduction Factors for Use with Appendix D

Strength Governed by	Strength Reduction Factor, ϕ, for use with Load Combinations in			
	Section 9.2		Appendix C	
Ductile steel element				
Tension, N_s	0.75		0.80	
Shear, V_s	0.65		0.75	
Brittle steel element				
Tension, N_s	0.65		0.70	
Shear, V_s	0.60		0.65	
Concrete	Condition		Condition	
	A	B	A	B
Shear				
Breakout, V_{cb} and V_{cbg}	0.75	0.70	0.85	0.75
Pryout, V_{cp}	0.70	0.70	0.75	0.75
Tension				
Cast-in headed studs, headed bolts, or hooked bolts				
Breakout and side face blowout, N_{cb}, N_{cbg}, N_{sb} and N_{sbg}	0.75	0.70	0.85	0.75
Pullout, N_{pn}	0.70	0.70	0.75	0.75
Post-installed anchors with category determined per ACI 355.2				
Category 1 (low sensitivity to installation and high reliability)				
Breakout and side face blowout, N_{cb}, N_{cbg}, N_{sb} and N_{sbg}	0.75	0.65	0.85	0.75
Pullout, N_{pn}	0.65	0.65	0.75	0.75
Category 2 (med. sensitivity to installation and med. reliability)				
Breakout and side face blowout, N_{cb}, N_{cbg}, N_{sb} and N_{sbg}	0.65	0.55	0.75	0.65
Pullout, N_{pn}	0.55	0.55	0.65	0.65
Category 3 (high sensitivity to installation and low reliability)				
Breakout and side face blowout, N_{cb}, N_{cbg}, N_{sb} and N_{sbg}	0.55	0.45	0.65	0.55
Pullout, N_{pn}	0.45	0.45	0.55	0.55

with $h_{ef} \geq 11$ in. in Eq. (D-8)] rather than h_{ef}^2. The use of $h_{ef}^{1.5}$ accounts for fracture mechanics principles and can be thought of as follows:

$$N_b = \frac{k\sqrt{f'_c}\, h_{ef}^2}{h_{ef}^{0.5}} \quad \left[\frac{\text{general } 45° \text{ concrete cone equation}}{\text{modification factor for fracture mechanics}}\right]$$

Resulting in:

$$N_b = k\sqrt{f'_c}\, h_{ef}^{1.5} \qquad \textit{Eq. (D-7)}$$

The fracture mechanics approach accounts for the high tensile stresses that exist at the embedded head of the anchor while other approaches (such as the 45-deg concrete cone method) assume a uniform distribution of stresses over the assumed failure surface.

The numeric constant k of 24 in Eq. (D-7) (or k of 16 in Eq. (D-8) if $h_{ef} \geq 11$ in.) is based on the 5% fractile of test results on headed cast-in-place anchors in cracked concrete. These k values must be used unless special testing has shown that higher values of k are applicable. Note that the crack width used in tests to establish these k values was 0.012 in. If larger crack widths are anticipated, confining reinforcement to control crack width to about 0.012 in. should be provided or special testing in larger cracks should be performed.

$\frac{A_N}{A_{No}}$: This factor accounts for adjacent anchors and/or free edges. For a single anchor located away from free edges, the A_{No} term is the projected area of a 35-deg failure plane, measured relative to the surface of the

concrete, and defined by a square with the sides $1.5h_{ef}$ from the centerline of the anchor [Fig. RD.5.2.1(a)]. The A_N term is a rectilinear projected area of the 35-deg failure plane at the surface of the concrete with sides $1.5h_{ef}$ from the centerline of the anchor(s) as limited by adjacent anchors and/or free edges. The definition of A_N is shown in Fig. RD.5.2.1(b). For a single anchor located at least $1.5h_{ef}$ from the closest free edge, A_N equals A_{No}.

Where a plate or washer is used to increase the bearing area of the head of an anchor, $1.5h_{ef}$ can be measured from the effective perimeter of the plate or washer where the effective perimeter is defined in D.5.2.7. Where a plate or washer is used, the projected area A_N can be based on $1.5h_{ef}$ measured from the effective perimeter of the plate or washer where the effective perimeter is defined in D.5.2.7 and shown in Fig. 34-1.

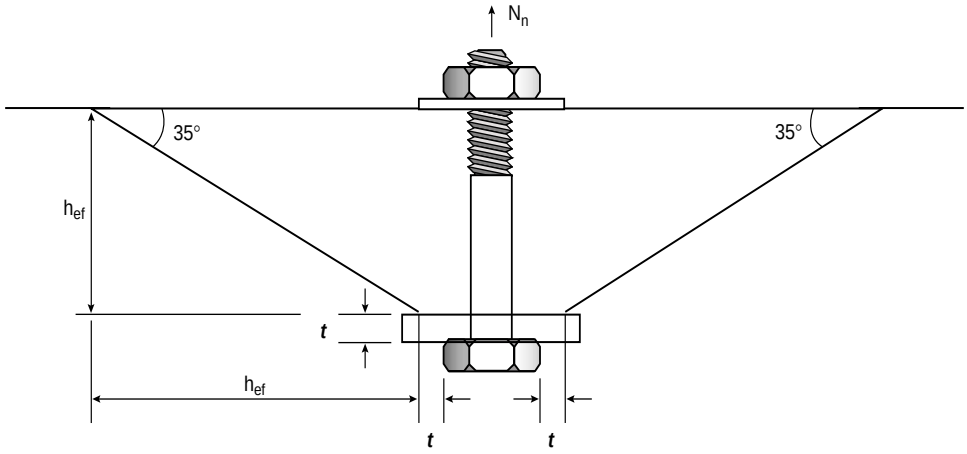


Figure 34-1 Effect of Washer Plate on Projected Area of Concrete Breakout

ψ_1 : This factor is applicable when multiple rows of tension anchors are present and the elastic design approach is used. In this case, the individual rows of tension anchors are assumed to carry different levels of load with the centerline of action of the applied tension load at an eccentricity (e_n') from the centroid of the tension anchors. If the plastic design approach is used, all tension anchors are assumed to carry the same load and the eccentricity factor, ψ_1, is taken as 1.0.

ψ_2 : This factor accounts for the non-uniform distribution of stresses when an anchor is located near a free edge of the concrete that are not accounted for by the $\frac{A_N}{A_{No}}$ term.

ψ_3 : This factor is taken as 1.0 if cracks in the concrete are likely to occur at the location of the anchor(s). If calculations indicate that concrete cracking is not likely to occur under service loads (e.g., $f_t < f_r$), then ψ_3 may be taken as 1.25 for cast-in-place anchors or 1.4 for post-installed anchors.

D.5.3 Pullout Strength of Anchor in Tension

A schematic of the pullout failure mode is shown in Fig. RD.4.1(a)(ii). The pullout strength of cast-in-place anchors is related to the bearing area at the embedded end of headed anchors, A_{brg}, and the properties of embedded hooks (e_h and d_o) for J-bolts and L-bolts. Obviously, if an anchor has no head or hook it will simply pull out of the concrete and not be able to achieve the concrete breakout strength associated with a full concrete cone failure (D.5.2). With an adequate head or hook size, pullout will not occur and the concrete breakout strength can be achieved. Eq. (D-12) provides the general requirement for pullout while Eq. (D-13) and Eq. (D-14) provide the specific requirements for headed and hooked anchors, respectively.

For headed anchors, the bearing area of the embedded head (A_{brg}) is the gross area of the head less the gross area of the anchor shaft (i.e., not the area of the embedded head). Washers or plates with an area larger than the head of an anchor can be used to increase the bearing area, A_{brg}, thus increasing the pullout strength (see D.5.2.7). In regions of moderate or high seismic risk, or for structures assigned to immediate or high seismic performance or design categories, where a headed bolt is being designed as a ductile steel element according to D.3.3.4, it may be necessary to use a bolt with a larger head or a washer in order to increase the design pullout strength, ϕN_{pn}, to assure that yielding of the steel takes place prior failure of the embedded portion of the anchor. Table 34-2 provides values for A_{brg} for standard bolt heads, nuts, and washers. Tables 34-5A, B and C can be used to quickly determine scenarios where the head of a bolt will not provide adequate pullout strength and will need to be increased in size.

For J-bolts and L-bolts, the minimum length of the hook measured from the inside surface of the shaft of the anchor is $3d_o$ while the maximum length for calculating pullout strength by Eq. (D-14) is $4.5d_o$. For other than high strength concrete, it is difficult to achieve design pullout strength of a hooked bolt that is equal to or greater than the design tensile strength of the steel. For example, a 1/2 in. diameter hooked bolt with the maximum hook length of $4.5d_o$ permitted in evaluating pullout strength in Eq. (D-14) requires that f'_c be at least 8700 psi to develop the design tensile strength of an ASTM A 307, Grade C, or ASTM F 1554, Grade 36 anchor (f_{ut} = 58,000 psi). This essentially prohibits the use of hooked bolts in many applications subject to seismic tensile loading due to the limitations of D.3.3.4 that the anchor strength must be governed by the ductile anchor steel.

For post-installed mechanical anchors, the value for the pullout strength, N_p, must be determined from the results of the ACI 355.2 product evaluation tests. Example data are shown in Table 34-3.

D.5.4 Concrete Side-Face Blowout Strength of Headed Anchor in Tension

The side-face blowout strength is associated with the lateral pressure that develops around the embedded end of headed anchors under load. Where the minimum edge distance for a single headed anchor is less than 0.4 h_{ef}, side-face blowout must be considered using Eq. (D-15). If an orthogonal free edge (i.e., an anchor in a corner) is located less than three times the distance from the anchor to the nearest edge) then an additional reduction factor of $[(1+ c_2/c)/4]$, where c is the distance to the nearest edge and c_2 is the distance to the orthogonal edge, must be applied to Eq. (D-15).

For multiple anchor groups, the side-face blowout strength is the given by Eq. (D-15) provided the spacing between individual anchors parallel to a free edge is greater than or equal to six times the distance to the free edge. If the spacing of the anchors in the group is less than six times the distance to the free edge, Eq. (D-16) must be used.

D.6 DESIGN REQUIREMENTS FOR SHEAR LOADING

Methods to determine the nominal shear strength as controlled by steel strength and embedment strength are specified in D.6. The nominal shear strength of the steel is based on the specified tensile strength of the steel using Eq. (D-17) for headed studs, Eq. (D-18) for headed and hooked bolts, and Eq. (D-19) for post-installed anchors. The nominal shear strength of the embedment is based on concrete breakout strength Eq. (D-20) for single anchors or Eq. (D-21) for groups of anchors) or pryout strength Eq. (D-28). When combined with the appropriate strength reduction factors from D.4.4, the smaller of these strengths will control the design shear strength of the anchorage.

D.6.1 Steel Strength of Anchor in Shear

For cast-in-place anchors, the shear strength of the steel is determined from Eq. (D-17) for headed studs and Eq. (D-18) for headed and hooked bolts using the effective cross-sectional area of the anchor, A_{se}, and the specified tensile strength of the anchor steel, f_{ut}. For post-installed mechanical anchors, the shear strength of the steel is determined from Eq (D-19) using the effective cross-sectional area of the anchor, A_{se}, and the specified tensile

strength of the anchor steel, f_{ut}. If the post-installed mechanical anchor is a sleeved anchor and if the sleeve extends across the shear plane, an additional term provided in Eq. (D-19) that uses the area of the sleeve, A_{sl}, and the specified tensile strength of the sleeve, f_{utsl}, may be used.

For cast-in-place anchors (i.e., headed anchors, headed studs and hooked bars), the effective cross-sectional area of the anchor (A_{se}) is the net tensile stress area for threaded anchors and the gross area for headed studs that are welded to a base plate. These areas are provided in Table 34-2. If the threads of headed anchors are located well above the shear plane (at least two diameters) the gross area of the anchor may be used for shear. For anchors of unusual geometry, the nominal steel strength may be taken as the lower 5% fractile of test results. For post-installed mechanical anchors the effective cross-sectional area of the anchor, A_{se}, and the effective cross-sectional of the sleeve, A_{sl}, must be determined from the results of the ACI 355.2 product evaluation tests. Example data are shown in Table 34-3.

The value of f_{ut} used in Eq. (D-17), Eq. (D-18), and Eq. (D-19) is limited to $1.9f_y$ or 125,000 psi. The limit of $1.9f_y$ is intended to ensure that the anchor does not yield under service loads and typically is applicable only to stainless steel materials. The limit of 125,000 psi is based on the database used in developing the Appendix D provisions. Table 34-1 provides values for f_y and f_{ut} for typical anchor materials. Note that neither of the limits applies to the typical anchor materials given in Table 34-1. For anchors manufactured according to specifications having a range for specified tensile strength, f_{ut} (e.g., ASTM F 1554), the lower limit value should be used to calculate the design strength. For post-installed mechanical anchors, f_y, f_{ut}, and f_{utsl} must be determined from the results of the ACI 355.2 product evaluation tests. Example data are shown in Table 34-3.

When built-up grout pads are present, the nominal shear strength values given by Eq. (D-17), Eq. (D-18), and Eq. (D-19) must be reduced by 20% to account for the flexural stresses developed in the anchor if the grout pad fractures upon application of the shear load.

D.6.2 Concrete Breakout Strength of Anchor in Shear

Fig. RD.4.1(b)(iii) shows typical concrete breakout failures for anchors loaded in shear directed toward a free edge. Eq. (D-20) gives the concrete breakout strength for a single anchor while Eq. (D-21) gives the concrete breakout strength for groups of anchors in shear. In cases where the shear is directed away from the free edge, the concrete breakout strength in shear need not be considered.

The individual terms in Eq. (D-20) and Eq. (D-21) are discussed below:

V_b : The basic concrete breakout strength for a single anchor in cracked concrete loaded in shear, directed toward a free edge (V_b) without any other adjacent free edges or limited concrete thickness is given by Eq. (D-23) for typical bolted connections and Eq. (D-24) for connections with welded studs or other anchors welded to the attached base plate. The primary difference between these equations and those using the 45-deg concrete cone method is the use of $c_1^{1.5}$ rather than c_1^2. The use of $c_1^{1.5}$ accounts for fracture mechanics principles in the same way that $h_{ef}^{1.5}$ does for tension anchors. The fracture mechanics approach accounts for the high tensile stresses that exist in the concrete at the point where the anchor first enters the concrete.

ℓ, d_o : The terms involving ℓ and d_o in Eq. (D-23) and Eq. (D-24) relate to the shear stiffness of the anchor. A stiff anchor is able to distribute the applied shear load further into the concrete than a flexible anchor.

$\frac{A_V}{A_{Vo}}$: This factor accounts for adjacent anchors, concrete thickness, and free edges. For a single anchor in a thick concrete member with shear directed toward a free edge, the A_{Vo} term is the projected area on the side of the free edge of a 35-deg failure plane radiating from the point where the anchor first enters the concrete and directed toward the free edge [see Fig. RD.6.2.1(a)]. The A_V term is a rectilinear projected

area of the 35-deg failure plane on the side of the free edge with sides 1.5 h_{ef} from the point where the anchor first enters the concrete as limited by adjacent anchors, concrete thickness and free edges. The definition of A_V is shown in Fig. RD.6.2.1(b).

ψ_5 : This factor applies when the applied shear load does not act through the centroid of the anchors loaded in shear [see Fig. RD.6.2.5]

ψ_6 : This factor accounts for the non-uniform distribution of stresses when an anchor is located in a corner that is not accounted for by the $\frac{A_V}{A_{Vo}}$ term [see Fig. RD.6.2.1(d)].

ψ_7 : This factor is taken as 1.0 if cracks in the concrete are likely to occur at the location of the anchor(s) and no supplemental reinforcement has been provided. If calculations indicate that concrete cracking is not likely to occur (e.g., $f_t < f_r$ at service loads), then ψ_7 may be taken as 1.4. Values of $\psi_7 > 1.0$ may be used if cracking at service loads is likely, provided No. 4 bar edge reinforcement is provided (see D.6.2.7).

D.6.3 Concrete Pryout Strength of Anchor in Shear

The concrete pryout strength of an anchor in shear may control when an anchor is both short and relatively stiff. Fig. RD.4.1(b)(ii) shows this failure mode. As a mental exercise, this failure mode may be envisioned by thinking of a No. 8 bar embedded 2 in. in concrete with 3 ft. of the bar sticking out. A small push at the top of the bar will cause the bar to "pryout" of the concrete.

D.7 INTERACTION OF TENSILE AND SHEAR FORCES

The interaction requirements for tension and shear are based on a trilinear approximation to the following interaction equation (see Fig. RD.7):

$$\left[\frac{N_u}{\phi N_n}\right]^{\frac{5}{3}} + \left[\frac{V_u}{\phi V_n}\right]^{\frac{5}{3}} = 1$$

In the trilinear simplification, D.7.1 permits the full value of ϕN_n if $V_u \le 0.2\ \phi V_n$ and Section D.7.2 permits the full value of ϕV_n if $N_u \le 0.2\ \phi N_n$. If both of these conditions are not satisfied, the linear interaction of Eq. (D-29) must be used.

The most important aspect of the interaction provisions is that both ϕN_n and ϕV_n are the smaller of the anchor strengths as controlled by the anchor steel or the embedment. Tests have shown that the interaction relationship is valid whether steel strength or embedment strength controls for ϕN_n or ϕV_n.

D.8 REQUIRED EDGE DISTANCES, SPACINGS, AND THICKNESSES TO PRECLUDE SPLITTING FAILURE

Section D.8 provides minimum edge distance, spacing, and member thickness requirements to preclude a possible splitting failure of the structural member. For untorqued cast-in-place anchors (e.g., headed studs or headed bolts that are not highly preloaded after the attachment is installed), the minimum edge distance and member thickness is controlled by the cover requirements of 7.7 and the minimum anchor spacing is $4d_o$. For torqued cast-in-place anchors (e.g., headed bolts that are highly pre-loaded after the attachment is installed), the minimum edge distance and spacing is $6d_o$ and the member thickness is controlled by the cover requirements of 7.7.

Post-installed mechanical anchors can exert large lateral pressures at the embedded expansion device during installation that can lead to a splitting failure. Minimum spacing, edge distance, and member thickness require-

ments for post-installed anchors should be determined from the product-specific test results developed in the ACI 355.2 product evaluation testing. Example data are shown in Table 34-3. In the absence of the product-specific test results, the following should be used: a minimum anchor spacing of $6d_o$; a minimum edge distance of $6d_o$ for undercut anchors, $8d_o$ for torque-controlled anchors, and $10d_o$ for displacement controlled anchors; and a minimum member thickness of $1.5h_{ef}$ but need not exceed h_{ef} plus 4 in. Examples of each of these types of anchors are shown in ACI 355.2 (Ref. 34.4). In all cases, the minimum edge distance and member thickness should meet the minimum cover requirements of 7.7.

For untorqued anchors, D.8.4 provides a method to use a large diameter anchor nearer to an edge or with closer spacing than that required by D.8.1 to D.8.3. In this case, a fictitious anchor diameter d'_o is used in evaluating the strength of the anchor and in determining the minimum edge and spacing requirements.

D.9 INSTALLATION OF ANCHORS

Cast-in-place anchors should be installed in accordance with construction documents. For threaded anchors, a metal or plywood template mounted above the surface of the concrete with nuts on each side of the template should be used to hold the anchors in a fixed position while the concrete is placed, consolidated, and hardens. Project specifications should require that post-installed anchors be installed in accordance with the manufacturer's installation instructions. As noted in RD.9, ACI 355.2 product evaluation testing is based on the manufacturer's installation instructions. As part of the ACI 355.2 product evaluation tests, product reliability tests (i.e., sensitivity to installation variables) are performed and the results are used to determine the category of the anchor to be used in the selection of the appropriate ϕ in D.4.4.

DESIGN TABLES FOR SINGLE CAST-IN ANCHORS

Tables have been provided to assist in the design of single anchors subject to tensile or shear loads. Tables 34-5A, B, and C provide design tensile strengths, ϕN_n, of single anchors in concrete with f'_c of 2500, 4000, and 6000 psi, respectively. Tables 34-6A, B, and C provide design shear strengths, ϕV_n, of single anchors in concrete with f'_c of 2500, 4000, and 6000 psi, respectively. A number of specified tensile strengths of steel, f_{ut}, are included to accommodate most anchor materials in use today. Notes accompany each group of tables that explain the assumptions used to develop the tables and how to adjust values for conditions that differ from those assumed.

According to D.8.2, minimum edge distances for cast-in headed anchors that will not be torqued must be based on minimum cover prescribed in 7.7. Thus, technically, concrete cover as low as 3/4 in. is permitted. If such a small cover is provided to the anchor shaft, the head of the anchor would end up having a cover smaller than 3/4 in. For corrosion protection, and in consideration of tolerances on placement (location and alignment) of anchors, it is recommended to provide a minimum concrete cover on cast-in anchors of 1-1/2 in. Tables 34-5 and 34-6 include design strengths for cast-in anchors with a minimum cover of 1-1/2 in.

NOTES FOR TENSION TABLES 34-5A, B AND C

NP – Not practical. Resulting edge distance, c, yields less than 3/4 in. cover.

All Notation are identical to those used in ACI 318-02 Appendix D, Section D.0.

1. Design strengths in table are for single cast-in anchors near one edge only. The values do not apply where the distance between adjacent anchors is less than $3h_{ef}$, or where the perpendicular distance, c_2, to the edge distance being considered, c_1, is less than $1.5h_{ef}$.

2. In regions of moderate or high seismic risk (UBC Zone 2, 3 or 4), or in structures assigned to intermediate or high seismic performance or design categories (IBC Seismic Design Category C, D, E or F), the design strengths in the table must be reduced by 25%. In addition, the anchor must be designed so strength is governed by a ductile steel element, unless D.3.3.5 is satisfied. Therefore, the design strengths based on the three concrete failure modes, ϕN_{cb}, ϕN_{pn}, and ϕN_{sb}, must exceed the design strength of the steel in tension, ϕN_s. This requirement effectively precludes the use of hooked anchor bolts in the seismic zones noted above.

3. For design purposes the tensile strength of the anchor steel, f_{ut}, must not exceed $1.9f_y$ or 125,000 psi.

4. Design strengths in table are based on strength reduction factor, ϕ, of Section D.4.4. Factored tensile load N_u must be computed from the load combinations of 9.2. Design strengths for concrete breakout, ϕN_{cb}, pullout, ϕN_{pn}, and sideface blowout, ϕN_{sb}, are based on Condition B. Where supplementary reinforcement is provided to satisfy Condition A, design strengths for ϕN_{cb} and ϕN_{sb} may be increased 7.1% to account for the increase in strength reduction factor from 0.70 to 0.75. This increase does not apply to pullout strength, ϕN_{pn}.

5. Design strengths for concrete breakout in tension, ϕN_{cb}, are based on N_b determined in accordance with Eq. (D-7) and apply to headed and hooked anchors. To determine the design strength of headed bolts with embedment depth, h_{ef}, greater than 11 in. in accordance with Eq. (D-8), multiply the table value by $[2(h_{ef}^{5/3})]/[3(h_{ef}^{1.5})]$.

6. Where analysis indicates that there will be no cracking at service load levels ($f_t < f_r$) in the region of the anchor, the design strengths for concrete breakout in tension, ϕN_{cb}, may be increased 25%.

7. The design strengths for pullout in tension, ϕN_{pn}, for headed bolts with diameter, d_o, less than 1-3/4 in. are based on bolts with regular hex heads. The design strengths for 1-3/4 and 2-in. bolts are based on heavy hex heads. For bolts with d_o less than 1-3/4 in. having heads with a larger bearing area, A_{brg}, than assumed, the design strengths may be increased by multiplying by the bearing area of the larger head and dividing by the bearing area of the regular hex head.

8. The design strengths for pullout in tension, ϕN_{pn}, for hooked bolts with hook-length, e_h, between 3 and 4.5 times diameter, d_o, may be determined by interpolation.

9. Where analysis indicates there will be no cracking at service load levels ($f_t < f_r$) in the region of the anchor, the design strengths for pullout in tension, ϕN_{pn}, may be increased 40%.

10. The design strengths for side-face blowout in tension, ϕN_{sb}, are applicable to headed bolts only and where edge distance, c, is less than $0.4h_{ef}$. The values for $0.4h_{ef}$ are shown for interpolation purposes only. The design strengths for bolts with diameter, d_o, less than 1-3/4 in. are based on bolts with regular hex heads. The design strengths for 1-3/4 and 2 in. bolts are based on bolts with heavy hex heads. For bolts with d_o less than 1-3/4 in. having heads with a larger bearing area, A_{brg}, than assumed, the design strengths may be increased by multiplying by the square root of the quotient resulting from dividing the bearing area of the larger head by the bearing area of the regular hex head ($\sqrt{A_{brg2} / A_{brg1}}$).

NOTES FOR SHEAR TABLES 34-6A, B AND C

NP – Not practical. Resulting edge distance, c, yields less than 3/4 in. cover.

All Notation are identical to those used in ACI 318-02 Appendix D, Section D.0.

1. Design strengths in table are for single cast-in anchors near one edge only. The values do not apply where the distance to an edge measured perpendicular to c_1 is less than $1.5c_1$. See Note 9.

   The values do not apply where the distance between adjacent anchors is less than $3c_1$, where c_1 is the distance from the center of the anchor to the edge in the direction of shear application.

2. In regions of moderate or high seismic risk (UBC Zone 2, 3 or 4), or in structures assigned to intermediate or high seismic performance or design categories (IBC Seismic Design Category C, D, E or F), the design strengths in the table must be reduced by 25%. In addition, the anchor must be designed so failure is initiated by a ductile steel element, unless D.3.3.5 is satisfied. This means that all the design strengths based on the two concrete failure modes, ϕV_{cb} and ϕV_{cp}, must equal or exceed the design strength of the steel in shear, ϕV_s.

3. Concrete pryout strength, ϕV_{cp}, is to be taken equal to tension breakout strength, ϕN_{cb}, where h_{ef} is less than 2.5 in., and to be taken as twice ϕN_{cb} where h_{ef} is equal to or greater than 2.5 in. Condition B (see D.4.4) must be assumed even where supplementary reinforcement qualifying for Condition A is present (i.e., strength reduction factor, ϕ, must be taken equal to 0.70).

4. For design purposes the tensile strength of the anchor steel, f_{ut}, must not exceed $1.9f_y$ or 125,000 psi.

5. Design strengths in table are based on strength reduction factor, ϕ, of Section D.4.4. Factored shear load V_u must be computed from the load combinations of 9.2. Design strengths for concrete breakout, ϕV_{cb}, are based on Condition B. Where supplementary reinforcement is provided to satisfy Condition A, design strengths may be increased 7.1% to account for the increase in strength reduction factor from 0.70 to 0.75.

6. Where analysis indicates that there will be no cracking at service load levels ($f_t < f_r$) in the region of the anchor, the design strengths for concrete breakout in shear, ϕV_{cb}, may be increased 40%.

7. In regions of members where analysis indicates cracking at service level loads, the strengths in the table for concrete breakout, ϕV_{cb}, may be increased in accordance with the factors in D.6.2.7 if edge reinforcement is provided in accordance with that section.

8. The design strengths for concrete breakout, ϕV_{cb}, are based on the shear load being applied perpendicular to the edge. If the load is applied parallel to the edge, the strengths may be increased 100%.

9. Where the anchor is located near a corner with an edge distance perpendicular to direction of shear, c_2, less than $1.5c_1$, design strengths for concrete breakout, ϕV_{cb}, shall be reduced by multiplying by modification factor, ψ_6, determined from Eq. (D-27). The calculated values in the table do not apply where two edge distances perpendicular to direction of shear, c_2, are less than $1.5c_1$. See D.6.2.4.

10. This value of thickness, h, is not practical since the head or hook would project below the bottom surface of the concrete. It was chosen to facilitate mental calculation of the actual edge distance, c_1, since the variable used in the calculation c_1 is a function of embedment depth, h_{ef}.

11. Linear interpolation for intermediate values of edge distance, c_1, is permissible. Linear interpolation for intermediate values of embedment depth, h_{ef}, is unconservative.

12. For 3/4 in. cover and for $c_1 = 0.25h_{ef}$ and $0.50h_{ef}$, see portion of table for $h = h_{ef}$.

13. For 3/4 in. cover and for $c_1 = 0.25h_{ef}$ and $0.50h_{ef}$, see portion of table for $h = h_{ef}$. For $c_1 = h_{ef}$, see portion of table for $h = 1.5h_{ef}$.

Table 34-5A. Design Strengths for Single Cast-In Anchors Subject to Tensile Loads (f'_c = 2500 psi)[1, 2, 4]
Notes pertaining to this table are given on Page 34-14

d_o in.	h_{ef} in.	ϕN_s - Tension Strength of Anchor							ϕN_{cb} - Tension Breakout[4, 5, 6]					ϕN_{pn} - Pullout[9]			ϕN_{sb} - Sideface Blowout[4, 10]		
		f_{ut} - for design purposes[3] - psi							c - edge distance in.					head[7]	"J" or "L" hook[8]		c - edge distance in.		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$\geq 1.5h_{ef}$		$e_h = 3d_o$	$e_h = 4.5d_o$	1-1/2-in. cover	$0.25h_{ef}$	$0.4h_{ef}$
1/4	2	1,392	1,440	1,800	2,160	2,520	2,880	3,000	1,580	NP	NP	1,782	2,376	1,638	295	443	3,113	NP	NP
	3	1,392	1,440	1,800	2,160	2,520	2,880	3,000	2,401	NP	NP	3,274	4,365	1,638	295	443	3,113	NP	NP
	4	1,392	1,440	1,800	2,160	2,520	2,880	3,000	3,336	NP	3,584	5,040	6,720	1,638	295	443	3,113	NP	NP
	5	1,392	1,440	1,800	2,160	2,520	2,880	3,000	4,371	NP	5,009	7,044	9,391	1,638	295	443	3,113	NP	3,831
	6	1,392	1,440	1,800	2,160	2,520	2,880	3,000	5,496	NP	6,584	9,259	12,345	1,638	295	443	3,113	NP	4,597
3/8	2	3,393	3,510	4,388	5,265	6,143	7,020	7,313	1,613	NP	NP	1,782	2,376	2,296	664	997	3,827	NP	NP
	3	3,393	3,510	4,388	5,265	6,143	7,020	7,313	2,438	NP	NP	3,274	4,365	2,296	664	997	3,827	NP	NP
	4	3,393	3,510	4,388	5,265	6,143	7,020	7,313	3,377	NP	3,584	5,040	6,720	2,296	664	997	3,827	NP	NP
	5	3,393	3,510	4,388	5,265	6,143	7,020	7,313	4,415	NP	5,009	7,044	9,391	2,296	664	997	3,827	NP	4,536
	6	3,393	3,510	4,388	5,265	6,143	7,020	7,313	5,543	NP	6,584	9,259	12,345	2,296	664	997	3,827	NP	5,443
1/2	2	6,177	6,390	7,988	9,585	11,183	12,780	13,313	1,646	NP	NP	1,782	2,376	4,074	1,181	1,772	5,287	NP	NP
	3	6,177	6,390	7,988	9,585	11,183	12,780	13,313	2,475	NP	NP	3,274	4,365	4,074	1,181	1,772	5,287	NP	NP
	4	6,177	6,390	7,988	9,585	11,183	12,780	13,313	3,418	NP	3,584	5,040	6,720	4,074	1,181	1,772	5,287	NP	NP
	5	6,177	6,390	7,988	9,585	11,183	12,780	13,313	4,459	NP	5,009	7,044	9,391	4,074	1,181	1,772	5,287	NP	6,042
	6	6,177	6,390	7,988	9,585	11,183	12,780	13,313	5,591	NP	6,584	9,259	12,345	4,074	1,181	1,772	5,287	NP	7,250
	7	6,177	6,390	7,988	9,585	11,183	12,780	13,313	6,806	6,806	8,297	11,668	15,557	4,074	1,181	1,772	5,287	5,287	8,458
	8	6,177	6,390	7,988	9,585	11,183	12,780	13,313	8,099	8,316	10,137	14,255	19,007	4,074	1,181	1,772	5,287	6,042	9,667
5/8	3	9,831	10,170	12,713	15,255	17,798	20,340	21,188	2,513	NP	NP	3,274	4,365	6,356	1,846	2,769	6,839	NP	NP
	4	9,831	10,170	12,713	15,255	17,798	20,340	21,188	3,459	NP	3,584	5,040	6,720	6,356	1,846	2,769	6,839	NP	NP
	5	9,831	10,170	12,713	15,255	17,798	20,340	21,188	4,504	NP	5,009	7,044	9,391	6,356	1,846	2,769	6,839	NP	7,547
	6	9,831	10,170	12,713	15,255	17,798	20,340	21,188	5,639	NP	6,584	9,259	12,345	6,356	1,846	2,769	6,839	NP	9,056
	7	9,831	10,170	12,713	15,255	17,798	20,340	21,188	6,857	NP	8,297	11,668	15,557	6,356	1,846	2,769	6,839	NP	10,565
	8	9,831	10,170	12,713	15,255	17,798	20,340	21,188	8,153	8,316	10,137	14,255	19,007	6,356	1,846	2,769	6,839	7,547	12,074
	9	9,831	10,170	12,713	15,255	17,798	20,340	21,188	9,522	9,923	12,096	17,010	22,680	6,356	1,846	2,769	6,839	8,490	13,584
	10	9,831	10,170	12,713	15,255	17,798	20,340	21,188	10,960	11,621	14,167	19,922	26,563	6,356	1,846	2,769	6,839	9,433	15,093
3/4	4	14,529	15,030	18,788	22,545	26,303	30,060	31,313	3,500	NP	3,584	5,040	6,720	9,156	2,658	3,987	8,491	NP	NP
	5	14,529	15,030	18,788	22,545	26,303	30,060	31,313	4,549	NP	5,009	7,044	9,391	9,156	2,658	3,987	8,491	NP	9,057
	6	14,529	15,030	18,788	22,545	26,303	30,060	31,313	5,687	NP	6,584	9,259	12,345	9,156	2,658	3,987	8,491	NP	10,869
	7	14,529	15,030	18,788	22,545	26,303	30,060	31,313	6,908	NP	8,297	11,668	15,557	9,156	2,658	3,987	8,491	NP	12,680
	8	14,529	15,030	18,788	22,545	26,303	30,060	31,313	8,207	8,316	10,137	14,255	19,007	9,156	2,658	3,987	8,491	9,057	14,492
	9	14,529	15,030	18,788	22,545	26,303	30,060	31,313	9,579	9,923	12,096	17,010	22,680	9,156	2,658	3,987	8,491	10,190	16,303
	10	14,529	15,030	18,788	22,545	26,303	30,060	31,313	11,020	11,621	14,167	19,922	26,563	9,156	2,658	3,987	8,491	11,322	18,115
	12	14,529	15,030	18,788	22,545	26,303	30,060	31,313	14,097	15,277	18,623	26,189	34,918	9,156	2,658	3,987	8,491	13,586	21,738
7/8	4	14,529	15,030	18,788	22,545	26,303	30,060	31,313	3,500	NP	3,584	5,040	6,720	9,156	2,658	3,987	8,491	NP	NP
	6	20,097	20,790	25,988	31,185	36,383	41,580	43,313	5,736	NP	6,584	9,259	12,345	12,474	3,618	5,426	10,242	NP	12,686
	8	20,097	20,790	25,988	31,185	36,383	41,580	43,313	8,261	8,316	10,137	14,255	19,007	12,474	3,618	5,426	10,242	10,572	16,915
	12	20,097	20,790	25,988	31,185	36,383	41,580	43,313	14,161	15,277	18,623	26,189	34,918	12,474	3,618	5,426	10,242	15,858	25,373
	15	20,097	20,790	25,988	31,185	36,383	41,580	43,313	19,235	21,350	26,026	36,600	48,800	12,474	3,618	5,426	10,242	19,822	31,716
	18	20,097	20,790	25,988	31,185	36,383	41,580	43,313	24,803	28,065	34,213	48,112	64,149	12,474	3,618	5,426	10,242	23,787	38,059
	25	20,097	20,790	25,988	31,185	36,383	41,580	43,313	39,505	45,938	56,000	78,750	105,000	12,474	3,618	5,426	10,242	33,037	52,860

Table 34-5A. Design Strengths for Single Cast-In Anchors Subject to Tensile Loads (f'_c = 2500 psi)[1, 2, 4] (cont'd.)
Notes pertaining to this table are given on Page 34-14

d_o in.	h_{ef} in.	ϕN_s - Tension Strength of Anchor							ϕN_{cb} - Tension Breakout[4, 5, 6]					ϕN_{pn} - Pullout[9]			ϕN_{sb} - Sideface Blowout[4, 10]		
		f_{ut} - for design purposes[3] - psi							c - edge distance in.					head[7]	"J" or "L" hook[8]		c - edge distance in.		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$\geq 1.5h_{ef}$		$e_h = 3d_o$	$e_h = 4.5d_o$	1-1/2-in. cover	$0.25h_{ef}$	$0.4h_{ef}$
1	6	26,361	27,270	34,088	40,905	47,723	54,540	56,813	5,784	NP	6,584	9,259	12,345	16,282	4,725	7,088	12,078	NP	14,494
	9	26,361	27,270	34,088	40,905	47,723	54,540	56,813	9,693	9,923	12,096	17,010	22,680	16,282	4,725	7,088	12,078	13,588	21,741
	12	26,361	27,270	34,088	40,905	47,723	54,540	56,813	14,226	15,277	18,623	26,189	34,918	16,282	4,725	7,088	12,078	18,118	28,988
	15	26,361	27,270	34,088	40,905	47,723	54,540	56,813	19,307	21,350	26,026	36,600	48,800	16,282	4,725	7,088	12,078	22,647	36,235
	18	26,361	27,270	34,088	40,905	47,723	54,540	56,813	24,881	28,065	34,213	48,112	64,149	16,282	4,725	7,088	12,078	27,176	43,482
	21	26,361	27,270	34,088	40,905	47,723	54,540	56,813	30,908	35,366	43,113	60,627	80,837	16,282	4,725	7,088	12,078	31,706	50,729
	25	26,361	27,270	34,088	40,905	47,723	54,540	56,813	39,595	45,938	56,000	78,750	105,000	16,282	4,725	7,088	12,078	37,745	60,392
1-1/8	6	33,191	34,335	42,919	51,503	60,086	68,670	71,531	5,833	NP	6,584	9,259	12,345	20,608	5,980	8,970	14,013	NP	16,306
	9	33,191	34,335	42,919	51,503	60,086	68,670	71,531	9,750	9,923	12,096	17,010	22,680	20,608	5,980	8,970	14,013	15,287	24,459
	12	33,191	34,335	42,919	51,503	60,086	68,670	71,531	14,291	15,277	18,623	26,189	34,918	20,608	5,980	8,970	14,013	20,383	32,612
	15	33,191	34,335	42,919	51,503	60,086	68,670	71,531	19,378	21,350	26,026	36,600	48,800	20,608	5,980	8,970	14,013	25,478	40,766
	18	33,191	34,335	42,919	51,503	60,086	68,670	71,531	24,958	28,065	34,213	48,112	64,149	20,608	5,980	8,970	14,013	30,574	48,919
	21	33,191	34,335	42,919	51,503	60,086	68,670	71,531	30,991	35,366	43,113	60,627	80,837	20,608	5,980	8,970	14,013	35,670	57,072
	25	33,191	34,335	42,919	51,503	60,086	68,670	71,531	39,685	45,938	56,000	78,750	105,000	20,608	5,980	8,970	14,013	42,464	67,943
1-1/4	6	42,152	43,605	54,506	65,408	76,309	87,210	90,844	5,882	NP	6,584	9,259	12,345	25,438	7,383	11,074	16,041	NP	18,117
	9	42,152	43,605	54,506	65,408	76,309	87,210	90,844	9,807	9,923	12,096	17,010	22,680	25,438	7,383	11,074	16,041	16,984	27,175
	12	42,152	43,605	54,506	65,408	76,309	87,210	90,844	14,355	15,277	18,623	26,189	34,918	25,438	7,383	11,074	16,041	22,646	36,233
	15	42,152	43,605	54,506	65,408	76,309	87,210	90,844	19,450	21,350	26,026	36,600	48,800	25,438	7,383	11,074	16,041	28,307	45,292
	18	42,152	43,605	54,506	65,408	76,309	87,210	90,844	25,036	28,065	34,213	48,112	64,149	25,438	7,383	11,074	16,041	33,969	54,350
	21	42,152	43,605	54,506	65,408	76,309	87,210	90,844	31,075	35,366	43,113	60,627	80,837	25,438	7,383	11,074	16,041	39,630	63,408
	25	42,152	43,605	54,506	65,408	76,309	87,210	90,844	39,776	45,938	56,000	78,750	105,000	25,438	7,383	11,074	16,041	47,179	75,486
1-3/8	6	50,460	52,200	65,250	78,300	91,350	104,400	108,750	5,931	NP	6,584	9,259	12,345	30,786	8,933	13,400	18,166	NP	19,930
	9	50,460	52,200	65,250	78,300	91,350	104,400	108,750	9,865	9,923	12,096	17,010	22,680	30,786	8,933	13,400	18,166	18,685	29,895
	12	50,460	52,200	65,250	78,300	91,350	104,400	108,750	14,420	15,277	18,623	26,189	34,918	30,786	8,933	13,400	18,166	24,913	39,860
	15	50,460	52,200	65,250	78,300	91,350	104,400	108,750	19,521	21,350	26,026	36,600	48,800	30,786	8,933	13,400	18,166	31,141	49,826
	18	50,460	52,200	65,250	78,300	91,350	104,400	108,750	25,114	28,065	34,213	48,112	64,149	30,786	8,933	13,400	18,166	37,369	59,791
	21	50,460	52,200	65,250	78,300	91,350	104,400	108,750	31,158	35,366	43,113	60,627	80,837	30,786	8,933	13,400	18,166	43,597	69,756
	25	50,460	52,200	65,250	78,300	91,350	104,400	108,750	39,866	45,938	56,000	78,750	105,000	30,786	8,933	13,400	18,166	51,902	83,043
1-1/2	12	61,335	63,450	79,313	95,175	111,038	126,900	132,188	14,486	15,277	18,623	26,189	34,918	36,638	10,631	15,947	20,383	27,178	43,484
	15	61,335	63,450	79,313	95,175	111,038	126,900	132,188	19,593	21,350	26,026	36,600	48,800	36,638	10,631	15,947	20,383	33,972	54,355
	18	61,335	63,450	79,313	95,175	111,038	126,900	132,188	25,192	28,065	34,213	48,112	64,149	36,638	10,631	15,947	20,383	40,766	65,226
	21	61,335	63,450	79,313	95,175	111,038	126,900	132,188	31,242	35,366	43,113	60,627	80,837	36,638	10,631	15,947	20,383	47,561	76,097
	25	61,335	63,450	79,313	95,175	111,038	126,900	132,188	39,957	45,938	56,000	78,750	105,000	36,638	10,631	15,947	20,383	56,620	90,592
1-3/4	12	82,650	85,500	106,875	128,250	149,625	171,000	178,125	14,616	15,277	18,623	26,189	34,918	58,016	14,470	21,705	27,075	34,199	54,719
	15	82,650	85,500	106,875	128,250	149,625	171,000	178,125	19,737	21,350	26,026	36,600	48,800	58,016	14,470	21,705	27,075	42,749	68,399
	18	82,650	85,500	106,875	128,250	149,625	171,000	178,125	25,348	28,065	34,213	48,112	64,149	58,016	14,470	21,705	27,075	51,299	82,079
	21	82,650	85,500	106,875	128,250	149,625	171,000	178,125	31,409	35,366	43,113	60,627	80,837	58,016	14,470	21,705	27,075	59,849	95,758
	25	82,650	85,500	106,875	128,250	149,625	171,000	178,125	40,138	45,938	56,000	78,750	105,000	58,016	14,470	21,705	27,075	71,249	113,998
2	12	108,750	112,500	140,625	168,750	196,875	225,000	234,375	14,747	15,277	18,623	26,189	34,918	74,424	18,900	28,350	32,279	38,735	61,976
	15	108,750	112,500	140,625	168,750	196,875	225,000	234,375	19,881	21,350	26,026	36,600	48,800	74,424	18,900	28,350	32,279	48,419	77,470
	18	108,750	112,500	140,625	168,750	196,875	225,000	234,375	25,504	28,065	34,213	48,112	64,149	74,424	18,900	28,350	32,279	58,102	92,964
	21	108,750	112,500	140,625	168,750	196,875	225,000	234,375	31,577	35,366	43,113	60,627	80,837	74,424	18,900	28,350	32,279	67,786	108,458
	25	108,750	112,500	140,625	168,750	196,875	225,000	234,375	40,320	45,938	56,000	78,750	105,000	74,424	18,900	28,350	32,279	80,698	129,116

Table 34-5B. Design Strengths for Single Cast-In Anchors Subject to Tensile Loads (f'_c = 4000 psi)[1, 2, 4]
Notes pertaining to this table are given on Page 34-14

d_o in.	h_{ef} in.	ϕN_s - Tension Strength of Anchor							ϕN_{cb} - Tension Breakout[4, 5, 6]					ϕN_{pn} - Pullout[9]			ϕN_{sb} - Sideface Blowout[4, 10]		
		f_{ut} - for design purposes[3] - psi							c - edge distance in.					head[7]	"J" or "L" hook[8]		c - edge distance in.		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$\geq 1.5h_{ef}$		$e_h = 3d_o$	$e_h = 4.5d_o$	1-1/2-in. cover	$0.25h_{ef}$	$0.4h_{ef}$
1/4	2	1,392	1,440	1,800	2,160	2,520	2,880	3,000	1,998	NP	NP	2,254	3,005	2,621	473	709	3,937	NP	NP
	3	1,392	1,440	1,800	2,160	2,520	2,880	3,000	3,037	NP	NP	4,141	5,521	2,621	473	709	3,937	NP	NP
	4	1,392	1,440	1,800	2,160	2,520	2,880	3,000	4,220	NP	4,533	6,375	8,500	2,621	473	709	3,937	NP	NP
	5	1,392	1,440	1,800	2,160	2,520	2,880	3,000	5,528	NP	6,336	8,910	11,879	2,621	473	709	3,937	NP	4,846
	6	1,392	1,440	1,800	2,160	2,520	2,880	3,000	6,952	NP	8,328	11,712	15,616	2,621	473	709	3,937	NP	5,815
3/8	2	3,393	3,510	4,388	5,265	6,143	7,020	7,313	2,040	NP	NP	2,254	3,005	3,674	1,063	1,595	4,841	NP	NP
	3	3,393	3,510	4,388	5,265	6,143	7,020	7,313	3,084	NP	NP	4,141	5,521	3,674	1,063	1,595	4,841	NP	NP
	4	3,393	3,510	4,388	5,265	6,143	7,020	7,313	4,271	NP	4,533	6,375	8,500	3,674	1,063	1,595	4,841	NP	NP
	5	3,393	3,510	4,388	5,265	6,143	7,020	7,313	5,584	NP	6,336	8,910	11,879	3,674	1,063	1,595	4,841	NP	5,737
	6	3,393	3,510	4,388	5,265	6,143	7,020	7,313	7,012	NP	8,328	11,712	15,616	3,674	1,063	1,595	4,841	NP	6,885
1/2	2	6,177	6,390	7,988	9,585	11,183	12,780	13,313	2,082	NP	NP	2,254	3,005	6,518	1,890	2,835	6,687	NP	NP
	3	6,177	6,390	7,988	9,585	11,183	12,780	13,313	3,131	NP	NP	4,141	5,521	6,518	1,890	2,835	6,687	NP	NP
	4	6,177	6,390	7,988	9,585	11,183	12,780	13,313	4,323	NP	4,533	6,375	8,500	6,518	1,890	2,835	6,687	NP	NP
	5	6,177	6,390	7,988	9,585	11,183	12,780	13,313	5,641	NP	6,336	8,910	11,879	6,518	1,890	2,835	6,687	NP	7,642
	6	6,177	6,390	7,988	9,585	11,183	12,780	13,313	7,072	NP	8,328	11,712	15,616	6,518	1,890	2,835	6,687	NP	9,171
	7	6,177	6,390	7,988	9,585	11,183	12,780	13,313	8,609	8,609	10,495	14,759	19,678	6,518	1,890	2,835	6,687	6,687	10,699
	8	6,177	6,390	7,988	9,585	11,183	12,780	13,313	10,245	10,518	12,823	18,032	24,042	6,518	1,890	2,835	6,687	7,642	12,228
5/8	3	9,831	10,170	12,713	15,255	17,798	20,340	21,188	3,179	NP	NP	4,141	5,521	10,170	2,953	4,430	8,651	NP	NP
	4	9,831	10,170	12,713	15,255	17,798	20,340	21,188	4,375	NP	4,533	6,375	8,500	10,170	2,953	4,430	8,651	NP	NP
	5	9,831	10,170	12,713	15,255	17,798	20,340	21,188	5,697	NP	6,336	8,910	11,879	10,170	2,953	4,430	8,651	NP	9,546
	6	9,831	10,170	12,713	15,255	17,798	20,340	21,188	7,133	NP	8,328	11,712	15,616	10,170	2,953	4,430	8,651	NP	11,455
	7	9,831	10,170	12,713	15,255	17,798	20,340	21,188	8,674	NP	10,495	14,759	19,678	10,170	2,953	4,430	8,651	NP	13,364
	8	9,831	10,170	12,713	15,255	17,798	20,340	21,188	10,313	10,518	12,823	18,032	24,042	10,170	2,953	4,430	8,651	9,546	15,273
	9	9,831	10,170	12,713	15,255	17,798	20,340	21,188	12,044	12,551	15,300	21,516	28,688	10,170	2,953	4,430	8,651	10,739	17,182
	10	9,831	10,170	12,713	15,255	17,798	20,340	21,188	13,864	14,700	17,920	25,200	33,600	10,170	2,953	4,430	8,651	11,932	19,091
3/4	4	14,529	15,030	18,788	22,545	26,303	30,060	31,313	4,428	NP	4,533	6,375	8,500	14,650	4,253	6,379	10,741	NP	NP
	5	14,529	15,030	18,788	22,545	26,303	30,060	31,313	5,754	NP	6,336	8,910	11,879	14,650	4,253	6,379	10,741	NP	11,457
	6	14,529	15,030	18,788	22,545	26,303	30,060	31,313	7,194	NP	8,328	11,712	15,616	14,650	4,253	6,379	10,741	NP	13,748
	7	14,529	15,030	18,788	22,545	26,303	30,060	31,313	8,738	NP	10,495	14,759	19,678	14,650	4,253	6,379	10,741	NP	16,040
	8	14,529	15,030	18,788	22,545	26,303	30,060	31,313	10,381	10,518	12,823	18,032	24,042	14,650	4,253	6,379	10,741	11,457	18,331
	9	14,529	15,030	18,788	22,545	26,303	30,060	31,313	12,116	12,551	15,300	21,516	28,688	14,650	4,253	6,379	10,741	12,889	20,622
	10	14,529	15,030	18,788	22,545	26,303	30,060	31,313	13,939	14,700	17,920	25,200	33,600	14,650	4,253	6,379	10,741	14,321	22,914
	12	14,529	15,030	18,788	22,545	26,303	30,060	31,313	17,831	19,324	23,556	33,126	44,168	14,650	4,253	6,379	10,741	17,185	27,497
7/8	4	14,529	15,030	18,788	22,545	26,303	30,060	31,313	4,428	NP	4,533	6,375	8,500	14,650	4,253	6,379	10,741	NP	NP
	6	20,097	20,790	25,988	31,185	36,383	41,580	43,313	7,255	NP	8,328	11,712	15,616	19,958	5,788	8,682	12,955	NP	16,047
	8	20,097	20,790	25,988	31,185	36,383	41,580	43,313	10,450	10,518	12,823	18,032	24,042	19,958	5,788	8,682	12,955	13,373	21,396
	12	20,097	20,790	25,988	31,185	36,383	41,580	43,313	17,913	19,324	23,556	33,126	44,168	19,958	5,788	8,682	12,955	20,059	32,094
	15	20,097	20,790	25,988	31,185	36,383	41,580	43,313	24,331	27,006	32,921	46,295	61,727	19,958	5,788	8,682	12,955	25,074	40,118
	18	20,097	20,790	25,988	31,185	36,383	41,580	43,313	31,374	35,500	43,276	60,857	81,142	19,958	5,788	8,682	12,955	30,088	48,141
	25	20,097	20,790	25,988	31,185	36,383	41,580	43,313	49,970	58,107	70,835	99,612	132,816	19,958	5,788	8,682	12,955	41,789	66,863

Table 34-5B. Design Strengths for Single Cast-In Anchors Subject to Tensile Loads (f'_c = 4000 psi)[1, 2, 4] (cont'd.)
Notes pertaining to this table are given on Page 34-14

d_o in.	h_{ef} in.	ϕN_s - Tension Strength of Anchor							ϕN_{cb} - Tension Breakout[4, 5, 6]					ϕN_{pn} - Pullout[9]			ϕN_{sb} - Sideface Blowout[4, 10]		
		f_{ut} - for design purposes[3] - psi							c - edge distance in.					head[7]	"J" or "L" hook[8]		c - edge distance in.		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$\geq 1.5h_{ef}$		$e_h = 3d_o$	$e_h = 4.5d_o$	1-1/2-in. cover	$0.25h_{ef}$	$0.4h_{ef}$
1	6	26,361	27,270	34,088	40,905	47,723	54,540	56,813	7,316	NP	8,328	11,712	15,616	26,051	7,560	11,340	15,278	NP	18,334
	9	26,361	27,270	34,088	40,905	47,723	54,540	56,813	12,260	12,551	15,300	21,516	28,688	26,051	7,560	11,340	15,278	17,188	27,500
	12	26,361	27,270	34,088	40,905	47,723	54,540	56,813	17,995	19,324	23,556	33,126	44,168	26,051	7,560	11,340	15,278	22,917	36,667
	15	26,361	27,270	34,088	40,905	47,723	54,540	56,813	24,421	27,006	32,921	46,295	61,727	26,051	7,560	11,340	15,278	28,646	45,834
	18	26,361	27,270	34,088	40,905	47,723	54,540	56,813	31,472	35,500	43,276	60,857	81,142	26,051	7,560	11,340	15,278	34,376	55,001
	21	26,361	27,270	34,088	40,905	47,723	54,540	56,813	39,096	44,735	54,534	76,688	102,251	26,051	7,560	11,340	15,278	40,105	64,168
	25	26,361	27,270	34,088	40,905	47,723	54,540	56,813	50,084	58,107	70,835	99,612	132,816	26,051	7,560	11,340	15,278	47,744	76,390
1-1/8	6	33,191	34,335	42,919	51,503	60,086	68,670	71,531	7,378	NP	8,328	11,712	15,616	32,973	9,568	14,352	17,725	NP	20,626
	9	33,191	34,335	42,919	51,503	60,086	68,670	71,531	12,333	12,551	15,300	21,516	28,688	32,973	9,568	14,352	17,725	19,337	30,939
	12	33,191	34,335	42,919	51,503	60,086	68,670	71,531	18,076	19,324	23,556	33,126	44,168	32,973	9,568	14,352	17,725	25,782	41,252
	15	33,191	34,335	42,919	51,503	60,086	68,670	71,531	24,511	27,006	32,921	46,295	61,727	32,973	9,568	14,352	17,725	32,228	51,565
	18	33,191	34,335	42,919	51,503	60,086	68,670	71,531	31,570	35,500	43,276	60,857	81,142	32,973	9,568	14,352	17,725	38,674	61,878
	21	33,191	34,335	42,919	51,503	60,086	68,670	71,531	39,201	44,735	54,534	76,688	102,251	32,973	9,568	14,352	17,725	45,119	72,191
	25	33,191	34,335	42,919	51,503	60,086	68,670	71,531	50,198	58,107	70,835	99,612	132,816	32,973	9,568	14,352	17,725	53,713	85,941
1-1/4	6	42,152	43,605	54,506	65,408	76,309	87,210	90,844	7,440	NP	8,328	11,712	15,616	40,701	11,813	17,719	20,290	NP	22,916
	9	42,152	43,605	54,506	65,408	76,309	87,210	90,844	12,405	12,551	15,300	21,516	28,688	40,701	11,813	17,719	20,290	21,484	34,374
	12	42,152	43,605	54,506	65,408	76,309	87,210	90,844	18,158	19,324	23,556	33,126	44,168	40,701	11,813	17,719	20,290	28,645	45,832
	15	42,152	43,605	54,506	65,408	76,309	87,210	90,844	24,602	27,006	32,921	46,295	61,727	40,701	11,813	17,719	20,290	35,806	57,290
	18	42,152	43,605	54,506	65,408	76,309	87,210	90,844	31,668	35,500	43,276	60,857	81,142	40,701	11,813	17,719	20,290	42,967	68,748
	21	42,152	43,605	54,506	65,408	76,309	87,210	90,844	39,307	44,735	54,534	76,688	102,251	40,701	11,813	17,719	20,290	50,129	80,206
	25	42,152	43,605	54,506	65,408	76,309	87,210	90,844	50,313	58,107	70,835	99,612	132,816	40,701	11,813	17,719	20,290	59,677	95,483
1-3/8	6	50,460	52,200	65,250	78,300	91,350	104,400	108,750	7,502	NP	8,328	11,712	15,616	49,258	14,293	21,440	22,978	NP	25,210
	9	50,460	52,200	65,250	78,300	91,350	104,400	108,750	12,478	12,551	15,300	21,516	28,688	49,258	14,293	21,440	22,978	23,634	37,815
	12	50,460	52,200	65,250	78,300	91,350	104,400	108,750	18,241	19,324	23,556	33,126	44,168	49,258	14,293	21,440	22,978	31,512	50,420
	15	50,460	52,200	65,250	78,300	91,350	104,400	108,750	24,693	27,006	32,921	46,295	61,727	49,258	14,293	21,440	22,978	39,391	63,025
	18	50,460	52,200	65,250	78,300	91,350	104,400	108,750	31,767	35,500	43,276	60,857	81,142	49,258	14,293	21,440	22,978	47,269	75,630
	21	50,460	52,200	65,250	78,300	91,350	104,400	108,750	39,412	44,735	54,534	76,688	102,251	49,258	14,293	21,440	22,978	55,147	88,235
	25	50,460	52,200	65,250	78,300	91,350	104,400	108,750	50,427	58,107	70,835	99,612	132,816	49,258	14,293	21,440	22,978	65,651	105,041
1-1/2	12	61,335	63,450	79,313	95,175	111,038	126,900	132,188	18,323	19,324	23,556	33,126	44,168	58,621	17,010	25,515	25,783	34,377	55,004
	15	61,335	63,450	79,313	95,175	111,038	126,900	132,188	24,783	27,006	32,921	46,295	61,727	58,621	17,010	25,515	25,783	42,972	68,755
	18	61,335	63,450	79,313	95,175	111,038	126,900	132,188	31,865	35,500	43,276	60,857	81,142	58,621	17,010	25,515	25,783	51,566	82,505
	21	61,335	63,450	79,313	95,175	111,038	126,900	132,188	39,518	44,735	54,534	76,688	102,251	58,621	17,010	25,515	25,783	60,160	96,256
	25	61,335	63,450	79,313	95,175	111,038	126,900	132,188	50,542	58,107	70,835	99,612	132,816	58,621	17,010	25,515	25,783	71,619	114,591
1-3/4	12	82,650	85,500	106,875	128,250	149,625	171,000	178,125	18,488	19,324	23,556	33,126	44,168	92,826	23,153	34,729	34,247	43,259	69,215
	15	82,650	85,500	106,875	128,250	149,625	171,000	178,125	24,965	27,006	32,921	46,295	61,727	92,826	23,153	34,729	34,247	54,074	86,519
	18	82,650	85,500	106,875	128,250	149,625	171,000	178,125	32,063	35,500	43,276	60,857	81,142	92,826	23,153	34,729	34,247	64,889	103,822
	21	82,650	85,500	106,875	128,250	149,625	171,000	178,125	39,730	44,735	54,534	76,688	102,251	92,826	23,153	34,729	34,247	75,704	121,126
	25	82,650	85,500	106,875	128,250	149,625	171,000	178,125	50,771	58,107	70,835	99,612	132,816	92,826	23,153	34,729	34,247	90,123	144,198
2	12	108,750	112,500	140,625	168,750	196,875	225,000	234,375	18,654	19,324	23,556	33,126	44,168	119,078	30,240	45,360	40,830	48,996	78,394
	15	108,750	112,500	140,625	168,750	196,875	225,000	234,375	25,148	27,006	32,921	46,295	61,727	119,078	30,240	45,360	40,830	61,245	97,992
	18	108,750	112,500	140,625	168,750	196,875	225,000	234,375	32,261	35,500	43,276	60,857	81,142	119,078	30,240	45,360	40,830	73,494	117,591
	21	108,750	112,500	140,625	168,750	196,875	225,000	234,375	39,942	44,735	54,534	76,688	102,251	119,078	30,240	45,360	40,830	85,743	137,189
	25	108,750	112,500	140,625	168,750	196,875	225,000	234,375	51,001	58,107	70,835	99,612	132,816	119,078	30,240	45,360	40,830	102,075	163,320

Table 34-5C. Design Strengths for Single Cast-In Anchors Subject to Tensile Loads (f'_c = 6000 psi)[1, 2, 4]
Notes pertaining to this table are given on Page 34-14

d_o in.	h_{ef} in.	ϕN_s - Tension Strength of Anchor							ϕN_{cb} - Tension Breakout[4, 5, 6]					ϕN_{pn} - Pullout[9]			ϕN_{sb} - Sideface Blowout[4, 10]		
		f_{ut} - for design purposes[3] - psi							c - edge distance in.					head[7]	"J" or "L" hook[8]		c - edge distance in.		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$\geq 1.5h_{ef}$		$e_h = 3d_o$	$e_h = 4.5d_o$	1-1/2-in. cover	$0.25h_{ef}$	$0.4h_{ef}$
1/4	2	1,392	1,440	1,800	2,160	2,520	2,880	3,000	2,447	NP	NP	2,761	3,681	3,931	709	1,063	4,822	NP	NP
	3	1,392	1,440	1,800	2,160	2,520	2,880	3,000	3,720	NP	NP	5,071	6,762	3,931	709	1,063	4,822	NP	NP
	4	1,392	1,440	1,800	2,160	2,520	2,880	3,000	5,168	NP	5,552	7,808	10,411	3,931	709	1,063	4,822	NP	NP
	5	1,392	1,440	1,800	2,160	2,520	2,880	3,000	6,771	NP	7,760	10,912	14,549	3,931	709	1,063	4,822	NP	5,935
	6	1,392	1,440	1,800	2,160	2,520	2,880	3,000	8,514	NP	10,200	14,344	19,125	3,931	709	1,063	4,822	NP	7,122
3/8	2	3,393	3,510	4,388	5,265	6,143	7,020	7,313	2,498	NP	NP	2,761	3,681	5,510	1,595	2,392	5,929	NP	NP
	3	3,393	3,510	4,388	5,265	6,143	7,020	7,313	3,777	NP	NP	5,071	6,762	5,510	1,595	2,392	5,929	NP	NP
	4	3,393	3,510	4,388	5,265	6,143	7,020	7,313	5,231	NP	5,552	7,808	10,411	5,510	1,595	2,392	5,929	NP	NP
	5	3,393	3,510	4,388	5,265	6,143	7,020	7,313	6,840	NP	7,760	10,912	14,549	5,510	1,595	2,392	5,929	NP	7,027
	6	3,393	3,510	4,388	5,265	6,143	7,020	7,313	8,588	NP	10,200	14,344	19,125	5,510	1,595	2,392	5,929	NP	8,432
1/2	2	6,177	6,390	7,988	9,585	11,183	12,780	13,313	2,550	NP	NP	2,761	3,681	9,778	2,835	4,253	8,190	NP	NP
	3	6,177	6,390	7,988	9,585	11,183	12,780	13,313	3,835	NP	NP	5,071	6,762	9,778	2,835	4,253	8,190	NP	NP
	4	6,177	6,390	7,988	9,585	11,183	12,780	13,313	5,295	NP	5,552	7,808	10,411	9,778	2,835	4,253	8,190	NP	NP
	5	6,177	6,390	7,988	9,585	11,183	12,780	13,313	6,908	NP	7,760	10,912	14,549	9,778	2,835	4,253	8,190	NP	9,360
	6	6,177	6,390	7,988	9,585	11,183	12,780	13,313	8,662	NP	10,200	14,344	19,125	9,778	2,835	4,253	8,190	NP	11,232
	7	6,177	6,390	7,988	9,585	11,183	12,780	13,313	10,544	10,544	12,854	18,076	24,101	9,778	2,835	4,253	8,190	8,190	13,104
	8	6,177	6,390	7,988	9,585	11,183	12,780	13,313	12,547	12,882	15,704	22,084	29,446	9,778	2,835	4,253	8,190	9,360	14,976
5/8	3	9,831	10,170	12,713	15,255	17,798	20,340	21,188	3,893	NP	NP	5,071	6,762	15,254	4,430	6,645	10,595	NP	NP
	4	9,831	10,170	12,713	15,255	17,798	20,340	21,188	5,359	NP	5,552	7,808	10,411	15,254	4,430	6,645	10,595	NP	NP
	5	9,831	10,170	12,713	15,255	17,798	20,340	21,188	6,978	NP	7,760	10,912	14,549	15,254	4,430	6,645	10,595	NP	11,691
	6	9,831	10,170	12,713	15,255	17,798	20,340	21,188	8,736	NP	10,200	14,344	19,125	15,254	4,430	6,645	10,595	NP	14,029
	7	9,831	10,170	12,713	15,255	17,798	20,340	21,188	10,623	NP	12,854	18,076	24,101	15,254	4,430	6,645	10,595	NP	16,367
	8	9,831	10,170	12,713	15,255	17,798	20,340	21,188	12,630	12,882	15,704	22,084	29,446	15,254	4,430	6,645	10,595	11,691	18,706
	9	9,831	10,170	12,713	15,255	17,798	20,340	21,188	14,751	15,372	18,739	26,352	35,136	15,254	4,430	6,645	10,595	13,152	21,044
	10	9,831	10,170	12,713	15,255	17,798	20,340	21,188	16,979	18,004	21,947	30,864	41,151	15,254	4,430	6,645	10,595	14,614	23,382
3/4	4	14,529	15,030	18,788	22,545	26,303	30,060	31,313	5,423	NP	5,552	7,808	10,411	21,974	6,379	9,568	13,155	NP	NP
	5	14,529	15,030	18,788	22,545	26,303	30,060	31,313	7,047	NP	7,760	10,912	14,549	21,974	6,379	9,568	13,155	NP	14,032
	6	14,529	15,030	18,788	22,545	26,303	30,060	31,313	8,811	NP	10,200	14,344	19,125	21,974	6,379	9,568	13,155	NP	16,838
	7	14,529	15,030	18,788	22,545	26,303	30,060	31,313	10,702	NP	12,854	18,076	24,101	21,974	6,379	9,568	13,155	NP	19,644
	8	14,529	15,030	18,788	22,545	26,303	30,060	31,313	12,714	12,882	15,704	22,084	29,446	21,974	6,379	9,568	13,155	14,032	22,451
	9	14,529	15,030	18,788	22,545	26,303	30,060	31,313	14,839	15,372	18,739	26,352	35,136	21,974	6,379	9,568	13,155	15,786	25,257
	10	14,529	15,030	18,788	22,545	26,303	30,060	31,313	17,071	18,004	21,947	30,864	41,151	21,974	6,379	9,568	13,155	17,540	28,064
	12	14,529	15,030	18,788	22,545	26,303	30,060	31,313	21,839	23,667	28,851	40,571	54,095	21,974	6,379	9,568	13,155	21,048	33,676
7/8	4	14,529	15,030	18,788	22,545	26,303	30,060	31,313	5,423	NP	5,552	7,808	10,411	21,974	6,379	9,568	13,155	NP	NP
	6	20,097	20,790	25,988	31,185	36,383	41,580	43,313	8,886	NP	10,200	14,344	19,125	29,938	8,682	13,023	15,866	NP	19,654
	8	20,097	20,790	25,988	31,185	36,383	41,580	43,313	12,798	12,882	15,704	22,084	29,446	29,938	8,682	13,023	15,866	16,378	26,205
	12	20,097	20,790	25,988	31,185	36,383	41,580	43,313	21,939	23,667	28,851	40,571	54,095	29,938	8,682	13,023	15,866	24,567	39,307
	15	20,097	20,790	25,988	31,185	36,383	41,580	43,313	29,799	33,075	40,320	56,700	75,600	29,938	8,682	13,023	15,866	30,709	49,134
	18	20,097	20,790	25,988	31,185	36,383	41,580	43,313	38,425	43,478	53,002	74,534	99,379	29,938	8,682	13,023	15,866	36,851	58,961
	25	20,097	20,790	25,988	31,185	36,383	41,580	43,313	61,200	71,166	86,755	121,999	162,665	29,938	8,682	13,023	15,866	51,181	81,890

Table 34-5C. Design Strengths for Single Cast-In Anchors Subject to Tensile Loads (f'_c = 6000 psi)[1, 2, 4] (cont'd.)
Notes pertaining to this table are given on Page 34-14

d_o in.	h_{ef} in.	ϕN_s - Tension Strength of Anchor							ϕN_{cb} - Tension Breakout[4, 5, 6]					ϕN_{pn} - Pullout[9]			ϕN_{sb} - Sideface Blowout[4, 10]		
		f_{ut} - for design purposes[3] - psi							c - edge distance in.					head[7]	"J" or "L" hook[8]		c - edge distance in.		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$\geq 1.5h_{ef}$		$e_h = 3d_o$	$e_h = 4.5d_o$	1-1/2-in. cover	$0.25h_{ef}$	$0.4h_{ef}$
1	6	26,361	27,270	34,088	40,905	47,723	54,540	56,813	8,961	NP	10,200	14,344	19,125	39,077	11,340	17,010	18,712	NP	22,454
	9	26,361	27,270	34,088	40,905	47,723	54,540	56,813	15,016	15,372	18,739	26,352	35,136	39,077	11,340	17,010	18,712	21,051	33,681
	12	26,361	27,270	34,088	40,905	47,723	54,540	56,813	22,039	23,667	28,851	40,571	54,095	39,077	11,340	17,010	18,712	28,068	44,908
	15	26,361	27,270	34,088	40,905	47,723	54,540	56,813	29,910	33,075	40,320	56,700	75,600	39,077	11,340	17,010	18,712	35,084	56,135
	18	26,361	27,270	34,088	40,905	47,723	54,540	56,813	38,545	43,478	53,002	74,534	99,379	39,077	11,340	17,010	18,712	42,101	67,362
	21	26,361	27,270	34,088	40,905	47,723	54,540	56,813	47,882	54,789	66,790	93,924	125,232	39,077	11,340	17,010	18,712	49,118	78,589
	25	26,361	27,270	34,088	40,905	47,723	54,540	56,813	61,340	71,166	86,755	121,999	162,665	39,077	11,340	17,010	18,712	58,474	93,559
1-1/8	6	33,191	34,335	42,919	51,503	60,086	68,670	71,531	9,036	NP	10,200	14,344	19,125	49,459	14,352	21,528	21,709	NP	25,261
	9	33,191	34,335	42,919	51,503	60,086	68,670	71,531	15,104	15,372	18,739	26,352	35,136	49,459	14,352	21,528	21,709	23,683	37,892
	12	33,191	34,335	42,919	51,503	60,086	68,670	71,531	22,139	23,667	28,851	40,571	54,095	49,459	14,352	21,528	21,709	31,577	50,523
	15	33,191	34,335	42,919	51,503	60,086	68,670	71,531	30,020	33,075	40,320	56,700	75,600	49,459	14,352	21,528	21,709	39,471	63,154
	18	33,191	34,335	42,919	51,503	60,086	68,670	71,531	38,665	43,478	53,002	74,534	99,379	49,459	14,352	21,528	21,709	47,365	75,784
	21	33,191	34,335	42,919	51,503	60,086	68,670	71,531	48,011	54,789	66,790	93,924	125,232	49,459	14,352	21,528	21,709	55,259	88,415
	25	33,191	34,335	42,919	51,503	60,086	68,670	71,531	61,480	71,166	86,755	121,999	162,665	49,459	14,352	21,528	21,709	65,785	105,256
1-1/4	6	42,152	43,605	54,506	65,408	76,309	87,210	90,844	9,112	NP	10,200	14,344	19,125	61,051	17,719	26,578	24,850	NP	28,066
	9	42,152	43,605	54,506	65,408	76,309	87,210	90,844	15,193	15,372	18,739	26,352	35,136	61,051	17,719	26,578	24,850	26,312	42,099
	12	42,152	43,605	54,506	65,408	76,309	87,210	90,844	22,239	23,667	28,851	40,571	54,095	61,051	17,719	26,578	24,850	35,083	56,132
	15	42,152	43,605	54,506	65,408	76,309	87,210	90,844	30,131	33,075	40,320	56,700	75,600	61,051	17,719	26,578	24,850	43,853	70,165
	18	42,152	43,605	54,506	65,408	76,309	87,210	90,844	38,786	43,478	53,002	74,534	99,379	61,051	17,719	26,578	24,850	52,624	84,198
	21	42,152	43,605	54,506	65,408	76,309	87,210	90,844	48,141	54,789	66,790	93,924	125,232	61,051	17,719	26,578	24,850	61,395	98,231
	25	42,152	43,605	54,506	65,408	76,309	87,210	90,844	61,620	71,166	86,755	121,999	162,665	61,051	17,719	26,578	24,850	73,089	116,942
1-3/8	6	50,460	52,200	65,250	78,300	91,350	104,400	108,750	9,188	NP	10,200	14,344	19,125	73,886	21,440	32,160	28,142	NP	30,876
	9	50,460	52,200	65,250	78,300	91,350	104,400	108,750	15,283	15,372	18,739	26,352	35,136	73,886	21,440	32,160	28,142	28,946	46,314
	12	50,460	52,200	65,250	78,300	91,350	104,400	108,750	22,340	23,667	28,851	40,571	54,095	73,886	21,440	32,160	28,142	38,595	61,751
	15	50,460	52,200	65,250	78,300	91,350	104,400	108,750	30,242	33,075	40,320	56,700	75,600	73,886	21,440	32,160	28,142	48,243	77,189
	18	50,460	52,200	65,250	78,300	91,350	104,400	108,750	38,906	43,478	53,002	74,534	99,379	73,886	21,440	32,160	28,142	57,892	92,627
	21	50,460	52,200	65,250	78,300	91,350	104,400	108,750	48,270	54,789	66,790	93,924	125,232	73,886	21,440	32,160	28,142	67,541	108,065
	25	50,460	52,200	65,250	78,300	91,350	104,400	108,750	61,760	71,166	86,755	121,999	162,665	73,886	21,440	32,160	28,142	80,406	128,649
1-1/2	12	61,335	63,450	79,313	95,175	111,038	126,900	132,188	22,441	23,667	28,851	40,571	54,095	87,931	25,515	38,273	31,578	42,103	67,365
	15	61,335	63,450	79,313	95,175	111,038	126,900	132,188	30,353	33,075	40,320	56,700	75,600	87,931	25,515	38,273	31,578	52,629	84,207
	18	61,335	63,450	79,313	95,175	111,038	126,900	132,188	39,027	43,478	53,002	74,534	99,379	87,931	25,515	38,273	31,578	63,155	101,048
	21	61,335	63,450	79,313	95,175	111,038	126,900	132,188	48,399	54,789	66,790	93,924	125,232	87,931	25,515	38,273	31,578	73,681	117,889
	25	61,335	63,450	79,313	95,175	111,038	126,900	132,188	61,901	71,166	86,755	121,999	162,665	87,931	25,515	38,273	31,578	87,715	140,345
1-3/4	12	82,650	85,500	106,875	128,250	149,625	171,000	178,125	22,643	23,667	28,851	40,571	54,095	139,238	34,729	52,093	41,944	52,982	84,771
	15	82,650	85,500	106,875	128,250	149,625	171,000	178,125	30,576	33,075	40,320	56,700	75,600	139,238	34,729	52,093	41,944	66,227	105,963
	18	82,650	85,500	106,875	128,250	149,625	171,000	178,125	39,269	43,478	53,002	74,534	99,379	139,238	34,729	52,093	41,944	79,472	127,156
	21	82,650	85,500	106,875	128,250	149,625	171,000	178,125	48,659	54,789	66,790	93,924	125,232	139,238	34,729	52,093	41,944	92,718	148,348
	25	82,650	85,500	106,875	128,250	149,625	171,000	178,125	62,182	71,166	86,755	121,999	162,665	139,238	34,729	52,093	41,944	110,378	176,605
2	12	108,750	112,500	140,625	168,750	196,875	225,000	234,375	22,846	23,667	28,851	40,571	54,095	178,618	45,360	68,040	50,006	60,008	96,012
	15	108,750	112,500	140,625	168,750	196,875	225,000	234,375	30,800	33,075	40,320	56,700	75,600	178,618	45,360	68,040	50,006	75,010	120,016
	18	108,750	112,500	140,625	168,750	196,875	225,000	234,375	39,511	43,478	53,002	74,534	99,379	178,618	45,360	68,040	50,006	90,012	144,019
	21	108,750	112,500	140,625	168,750	196,875	225,000	234,375	48,919	54,789	66,790	93,924	125,232	178,618	45,360	68,040	50,006	105,014	168,022
	25	108,750	112,500	140,625	168,750	196,875	225,000	234,375	62,463	71,166	86,755	121,999	162,665	178,618	45,360	68,040	50,006	125,016	200,026

Table 34-6A. Design Strengths for Single Cast-In Anchors Subject to Shear Loads (f'_c = 2500 psi)[1, 2, 3, 5]
Notes pertaining to this table are given on Page 34-15

d_o in.	h_{ef} in.	ϕV_s - Shear Strength of Anchor							ϕV_{cb} - Shear Breakout [5, 6, 7, 8, 9]												
		f_{ut} - for design purposes [4] - psi							$h = h_{ef}$ [10] and c_1 = [11]						$h = 1.5h_{ef}$ and c_1 = [11, 12]			$h = 2.25h_{ef}$ and c_1 = [11, 13]		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	$1.5h_{ef}$	$2h_{ef}$	$3h_{ef}$
1/4	2	724	749	936	1,123	1,310	1,498	1,560	316	NP	NP	350	429	606	525	643	910	965	1,114	1,364
	3	724	749	936	1,123	1,310	1,498	1,560	385	NP	NP	643	788	1,114	965	1,182	1,671	1,772	2,047	2,507
	4	724	749	936	1,123	1,310	1,498	1,560	385	NP	525	990	1,213	1,715	1,485	1,819	2,573	2,729	3,151	3,859
	5	724	749	936	1,123	1,310	1,498	1,560	385	NP	734	1,384	1,695	2,397	2,076	2,542	3,596	3,814	4,404	5,393
	6	724	749	936	1,123	1,310	1,498	1,560	385	NP	965	1,819	2,228	3,151	2,729	3,342	4,727	5,013	5,789	7,090
3/8	2	1,764	1,825	2,282	2,738	3,194	3,650	3,803	363	NP	NP	395	484	685	593	726	1,027	1,090	1,258	1,541
	3	1,764	1,825	2,282	2,738	3,194	3,650	3,803	499	NP	NP	788	965	1,364	1,182	1,447	2,047	2,171	2,507	3,070
	4	1,764	1,825	2,282	2,738	3,194	3,650	3,803	499	NP	643	1,213	1,485	2,101	1,819	2,228	3,151	3,342	3,859	4,727
	5	1,764	1,825	2,282	2,738	3,194	3,650	3,803	499	NP	899	1,695	2,076	2,936	2,542	3,114	4,404	4,671	5,393	6,606
	6	1,764	1,825	2,282	2,738	3,194	3,650	3,803	499	NP	1,182	2,228	2,729	3,859	3,342	4,093	5,789	6,140	7,090	8,683
1/2	2	3,212	3,323	4,154	4,984	5,815	6,646	6,923	403	NP	NP	431	528	747	647	792	1,120	1,188	1,372	1,680
	3	3,212	3,323	4,154	4,984	5,815	6,646	6,923	574	NP	NP	859	1,052	1,487	1,288	1,578	2,231	2,366	2,733	3,347
	4	3,212	3,323	4,154	4,984	5,815	6,646	6,923	608	NP	743	1,400	1,715	2,426	2,101	2,573	3,638	3,859	4,456	5,458
	5	3,212	3,323	4,154	4,984	5,815	6,646	6,923	608	NP	1,038	1,957	2,397	3,390	2,936	3,596	5,085	5,393	6,228	7,627
	6	3,212	3,323	4,154	4,984	5,815	6,646	6,923	608	NP	1,364	2,573	3,151	4,456	3,859	4,727	6,684	7,090	8,187	10,026
	7	3,212	3,323	4,154	4,984	5,815	6,646	6,923	608	608	1,719	3,242	3,971	5,615	4,863	5,956	8,423	8,934	10,316	12,635
	8	3,212	3,323	4,154	4,984	5,815	6,646	6,923	608	743	2,101	3,961	4,851	6,861	5,942	7,277	10,291	10,915	12,604	15,437
5/8	3	5,112	5,288	6,611	7,933	9,255	10,577	11,018	647	NP	NP	918	1,125	1,590	1,377	1,687	2,386	2,530	2,922	3,578
	4	5,112	5,288	6,611	7,933	9,255	10,577	11,018	685	NP	794	1,497	1,834	2,594	2,246	2,751	3,890	4,126	4,765	5,836
	5	5,112	5,288	6,611	7,933	9,255	10,577	11,018	716	NP	1,160	2,188	2,680	3,790	3,282	4,020	5,685	6,030	6,963	8,528
	6	5,112	5,288	6,611	7,933	9,255	10,577	11,018	716	NP	1,525	2,876	3,523	4,982	4,315	5,284	7,473	7,927	9,153	11,210
	7	5,112	5,288	6,611	7,933	9,255	10,577	11,018	716	NP	1,922	3,625	4,439	6,278	5,437	6,659	9,417	9,989	11,534	14,126
	8	5,112	5,288	6,611	7,933	9,255	10,577	11,018	716	830	2,349	4,429	5,424	7,671	6,643	8,136	11,506	12,204	14,092	17,259
	9	5,112	5,288	6,611	7,933	9,255	10,577	11,018	716	991	2,802	5,284	6,472	9,153	7,927	9,708	13,729	14,562	16,815	20,594
	10	5,112	5,288	6,611	7,933	9,255	10,577	11,018	716	1,160	3,282	6,189	7,580	10,720	9,284	11,370	16,080	17,055	19,694	24,120
3/4	4	7,555	7,816	9,770	11,723	13,677	15,631	16,283	761	NP	839	1,582	1,937	2,739	2,372	2,906	4,109	4,358	5,033	6,164
	5	7,555	7,816	9,770	11,723	13,677	15,631	16,283	796	NP	1,226	2,311	2,831	4,003	3,467	4,246	6,005	6,369	7,354	9,007
	6	7,555	7,816	9,770	11,723	13,677	15,631	16,283	826	NP	1,671	3,151	3,859	5,458	4,727	5,789	8,187	8,683	10,026	12,280
	7	7,555	7,816	9,770	11,723	13,677	15,631	16,283	826	NP	2,106	3,971	4,863	6,878	5,956	7,295	10,316	10,942	12,635	15,474
	8	7,555	7,816	9,770	11,723	13,677	15,631	16,283	826	910	2,573	4,851	5,942	8,403	7,277	8,912	12,604	13,369	15,437	18,906
	9	7,555	7,816	9,770	11,723	13,677	15,631	16,283	826	1,085	3,070	5,789	7,090	10,026	8,683	10,635	15,040	15,952	18,420	22,560
	10	7,555	7,816	9,770	11,723	13,677	15,631	16,283	826	1,271	3,596	6,780	8,304	11,743	10,170	12,455	17,615	18,683	21,574	26,422
	12	7,555	7,816	9,770	11,723	13,677	15,631	16,283	826	1,671	4,727	8,912	10,915	15,437	13,369	16,373	23,155	24,560	28,359	34,733
7/8	4	10,450	10,811	13,514	16,216	18,919	21,622	22,523	838	NP	878	1,656	2,029	2,869	2,485	3,043	4,304	4,565	5,271	6,455
	6	10,450	10,811	13,514	16,216	18,919	21,622	22,523	908	NP	1,750	3,300	4,042	5,716	4,950	6,063	8,574	9,094	10,501	12,861
	8	10,450	10,811	13,514	16,216	18,919	21,622	22,523	937	983	2,779	5,240	6,418	9,076	7,860	9,627	13,614	14,440	16,674	20,421
	12	10,450	10,811	13,514	16,216	18,919	21,622	22,523	937	1,805	5,105	9,627	11,790	16,674	14,440	17,685	25,010	26,528	30,631	37,516
	15	10,450	10,811	13,514	16,216	18,919	21,622	22,523	937	2,523	7,135	13,453	16,477	23,302	20,180	24,716	34,953	37,073	42,809	52,430
	18	10,450	10,811	13,514	16,216	18,919	21,622	22,523	937	3,316	9,379	17,685	21,660	30,631	26,528	32,489	45,947	48,734	56,273	68,921
	25	10,450	10,811	13,514	16,216	18,919	21,622	22,523	937	5,428	15,352	28,947	35,453	50,138	43,421	53,179	75,207	79,769	92,109	112,811

Table 34-6A. Design Strengths for Single Cast-In Anchors Subject to Shear Loads (f'_c = 2500 psi)[1, 2, 3, 5] (cont'd.)
Notes pertaining to this table are given on Page 34-15

d_o in.	h_{ef} in.	ϕV_s - Shear Strength of Anchor							ϕV_{cb} - Shear Breakout [5, 6, 7, 8, 9]												
		f_{ut} - for design purposes [4] - psi							$h = h_{ef}$ [10] and c_1 = [11]						$h = 1.5h_{ef}$ and c_1 = [11, 12]			$h = 2.25h_{ef}$ and c_1 = [11, 13]		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	$1.5h_{ef}$	$2h_{ef}$	$3h_{ef}$
1	6	13,708	14,180	17,726	21,271	24,816	28,361	29,543	992	NP	1,822	3,435	4,207	5,950	5,153	6,311	8,924	9,466	10,930	13,387
	9	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,050	1,253	3,545	6,684	8,187	11,578	10,026	12,280	17,366	18,420	21,269	26,050
	12	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,050	1,930	5,458	10,291	12,604	17,825	15,437	18,906	26,737	28,359	32,746	40,106
	15	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,050	2,697	7,627	14,382	17,615	24,911	21,574	26,422	37,366	39,633	45,764	56,050
	18	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,050	3,545	10,026	18,906	23,155	32,746	28,359	34,733	49,119	52,099	60,159	73,679
	21	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,050	4,467	12,635	23,824	29,179	41,265	35,737	43,768	61,898	65,652	75,809	92,846
	25	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,050	5,802	16,412	30,946	37,901	53,600	46,419	56,851	80,400	85,277	98,469	120,600
1-1/8	6	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,076	NP	1,887	3,559	4,358	6,164	5,338	6,538	9,245	9,806	11,323	13,868
	9	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,167	1,329	3,760	7,090	8,683	12,280	10,635	13,025	18,420	19,537	22,560	27,630
	12	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,167	2,047	5,789	10,915	13,369	18,906	16,373	20,053	28,359	30,079	34,733	42,539
	15	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,167	2,860	8,090	15,255	18,683	26,422	22,882	28,025	39,633	42,037	48,540	59,450
	18	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,167	3,760	10,635	20,053	24,560	34,733	30,079	36,840	52,099	55,259	63,808	78,149
	21	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,167	4,738	13,401	25,270	30,949	43,768	37,904	46,423	65,652	69,635	80,407	98,478
	25	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,167	6,154	17,407	32,823	40,200	56,851	49,235	60,300	85,277	90,450	104,442	127,915
1-1/4	6	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,161	NP	1,948	3,673	4,498	6,362	5,509	6,747	9,542	10,121	11,687	14,314
	9	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,259	1,372	3,881	7,317	8,962	12,674	10,976	13,443	19,011	20,165	23,284	28,517
	12	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,286	2,157	6,102	11,506	14,092	19,929	17,259	21,138	29,893	31,706	36,611	44,840
	15	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,286	3,015	8,528	16,080	19,694	27,851	24,120	29,541	41,777	44,311	51,166	62,665
	18	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,286	3,963	11,210	21,138	25,888	36,611	31,706	38,832	54,917	58,248	67,260	82,376
	21	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,286	4,994	14,126	26,636	32,623	46,136	39,955	48,934	69,204	73,401	84,757	103,805
	25	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,286	6,487	18,349	34,599	42,374	59,926	51,898	63,562	89,890	95,342	110,092	134,834
1-3/8	6	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,248	NP	2,004	3,779	4,629	6,546	5,669	6,943	9,819	10,415	12,026	14,729
	9	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,353	1,412	3,993	7,530	9,222	13,042	11,295	13,833	19,563	20,749	23,959	29,344
	12	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,409	2,263	6,400	12,067	14,780	20,901	18,101	22,169	31,352	33,254	38,398	47,028
	15	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,409	3,162	8,944	16,865	20,655	29,211	25,297	30,983	43,816	46,474	53,663	65,724
	18	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,409	4,157	11,757	22,169	27,152	38,398	33,254	40,728	57,598	61,092	70,542	86,396
	21	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,409	5,238	14,816	27,937	34,215	48,388	41,905	51,323	72,581	76,984	88,894	108,872
	25	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,409	6,804	19,244	36,287	44,443	62,851	54,431	66,664	94,277	99,996	115,465	141,416
1-1/2	12	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,535	2,363	6,684	12,604	15,437	21,831	18,906	23,155	32,746	34,733	40,106	49,119
	15	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,535	3,303	9,342	17,615	21,574	30,510	26,422	32,360	45,764	48,540	56,050	68,647
	18	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,535	4,342	12,280	23,155	28,359	40,106	34,733	42,539	60,159	63,808	73,679	90,238
	21	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,535	5,471	15,474	29,179	35,737	50,539	43,768	53,605	75,809	80,407	92,846	113,713
	25	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,535	7,106	20,100	37,901	46,419	65,646	56,851	69,628	98,469	104,442	120,600	147,704
1-3/4	12	42,978	44,460	55,575	66,690	77,805	88,920	92,625	1,743	2,475	7,001	13,201	16,167	22,864	19,801	24,251	34,296	36,377	42,004	51,444
	15	42,978	44,460	55,575	66,690	77,805	88,920	92,625	1,798	3,567	10,090	19,026	23,302	32,954	28,539	34,953	49,431	52,430	60,541	74,147
	18	42,978	44,460	55,575	66,690	77,805	88,920	92,625	1,798	4,689	13,264	25,010	30,631	43,319	37,516	45,947	64,979	68,921	79,583	97,468
	21	42,978	44,460	55,575	66,690	77,805	88,920	92,625	1,798	5,909	16,714	31,517	38,600	54,589	47,275	57,900	81,883	86,850	100,286	122,824
	25	42,978	44,460	55,575	66,690	77,805	88,920	92,625	1,798	7,676	21,710	40,938	50,138	70,906	61,406	75,207	106,359	112,811	130,262	159,538
2	12	56,550	58,500	73,125	87,750	102,375	117,000	121,875	1,960	2,576	7,287	13,740	16,828	23,799	20,610	25,242	35,698	37,863	43,721	53,547
	15	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,049	3,765	10,648	20,079	24,591	34,778	30,118	36,887	52,166	55,331	63,891	78,250
	18	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,076	5,013	14,180	26,737	32,746	46,310	40,106	49,119	69,465	73,679	85,077	104,198
	21	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,076	6,317	17,868	33,693	41,265	58,358	50,539	61,898	87,536	92,846	107,210	131,305
	25	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,076	8,206	23,209	43,764	53,600	75,802	65,646	80,400	113,702	120,600	139,256	170,554

Table 34-6B. Design Strengths for Single Cast-In Anchors Subject to Shear Loads (f'_c = 4000 psi)[1, 2, 3, 5]
Notes pertaining to this table are given on Page 34-15

d_o in.	h_{ef} in.	ϕV_s - Shear Strength of Anchor							ϕV_{cb} - Shear Breakout [5, 6, 7, 8, 9]												
		f_{ut} - for design purposes [4] - psi							$h = h_{ef}$ [10] and c_1 = [11]						$h = 1.5h_{ef}$ and c_1 = [11, 12]			$h = 2.25h_{ef}$ and c_1 = [11, 13]		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	$1.5h_{ef}$	$2h_{ef}$	$3h_{ef}$
1/4	2	724	749	936	1,123	1,310	1,498	1,560	399	NP	NP	443	542	767	664	814	1,151	1,220	1,409	1,726
	3	724	749	936	1,123	1,310	1,498	1,560	487	NP	NP	814	996	1,409	1,220	1,495	2,114	2,242	2,589	3,171
	4	724	749	936	1,123	1,310	1,498	1,560	487	NP	664	1,253	1,534	2,170	1,879	2,301	3,254	3,452	3,986	4,882
	5	724	749	936	1,123	1,310	1,498	1,560	487	NP	928	1,751	2,144	3,032	2,626	3,216	4,548	4,824	5,570	6,822
	6	724	749	936	1,123	1,310	1,498	1,560	487	NP	1,220	2,301	2,818	3,986	3,452	4,228	5,979	6,341	7,322	8,968
3/8	2	1,764	1,825	2,282	2,738	3,194	3,650	3,803	459	NP	NP	500	613	866	750	919	1,299	1,378	1,591	1,949
	3	1,764	1,825	2,282	2,738	3,194	3,650	3,803	631	NP	NP	996	1,220	1,726	1,495	1,831	2,589	2,746	3,171	3,883
	4	1,764	1,825	2,282	2,738	3,194	3,650	3,803	631	NP	814	1,534	1,879	2,657	2,301	2,818	3,986	4,228	4,882	5,979
	5	1,764	1,825	2,282	2,738	3,194	3,650	3,803	631	NP	1,137	2,144	2,626	3,714	3,216	3,939	5,570	5,908	6,822	8,355
	6	1,764	1,825	2,282	2,738	3,194	3,650	3,803	631	NP	1,495	2,818	3,452	4,882	4,228	5,178	7,322	7,766	8,968	10,983
1/2	2	3,212	3,323	4,154	4,984	5,815	6,646	6,923	510	NP	NP	545	668	944	818	1,002	1,417	1,502	1,735	2,125
	3	3,212	3,323	4,154	4,984	5,815	6,646	6,923	726	NP	NP	1,086	1,330	1,881	1,629	1,996	2,822	2,993	3,456	4,233
	4	3,212	3,323	4,154	4,984	5,815	6,646	6,923	769	NP	939	1,771	2,170	3,068	2,657	3,254	4,602	4,882	5,637	6,904
	5	3,212	3,323	4,154	4,984	5,815	6,646	6,923	769	NP	1,313	2,476	3,032	4,288	3,714	4,548	6,432	6,822	7,878	9,648
	6	3,212	3,323	4,154	4,984	5,815	6,646	6,923	769	NP	1,726	3,254	3,986	5,637	4,882	5,979	8,455	8,968	10,355	12,683
	7	3,212	3,323	4,154	4,984	5,815	6,646	6,923	769	769	2,175	4,101	5,023	7,103	6,151	7,534	10,655	11,301	13,049	15,982
	8	3,212	3,323	4,154	4,984	5,815	6,646	6,923	769	939	2,657	5,010	6,136	8,678	7,516	9,205	13,017	13,807	15,943	19,526
5/8	3	5,112	5,288	6,611	7,933	9,255	10,577	11,018	818	NP	NP	1,161	1,422	2,012	1,742	2,134	3,018	3,201	3,696	4,526
	4	5,112	5,288	6,611	7,933	9,255	10,577	11,018	867	NP	1,004	1,894	2,320	3,281	2,841	3,480	4,921	5,220	6,027	7,381
	5	5,112	5,288	6,611	7,933	9,255	10,577	11,018	906	NP	1,468	2,768	3,390	4,794	4,152	5,085	7,191	7,627	8,807	10,787
	6	5,112	5,288	6,611	7,933	9,255	10,577	11,018	906	NP	1,930	3,638	4,456	6,302	5,458	6,684	9,453	10,026	11,578	14,180
	7	5,112	5,288	6,611	7,933	9,255	10,577	11,018	906	NP	2,432	4,585	5,615	7,941	6,878	8,423	11,912	12,635	14,589	17,868
	8	5,112	5,288	6,611	7,933	9,255	10,577	11,018	906	1,050	2,971	5,602	6,861	9,703	8,403	10,291	14,554	15,437	17,825	21,831
	9	5,112	5,288	6,611	7,933	9,255	10,577	11,018	906	1,253	3,545	6,684	8,187	11,578	10,026	12,280	17,366	18,420	21,269	26,050
	10	5,112	5,288	6,611	7,933	9,255	10,577	11,018	906	1,468	4,152	7,829	9,588	13,560	11,743	14,382	20,340	21,574	24,911	30,510
3/4	4	7,555	7,816	9,770	11,723	13,677	15,631	16,283	963	NP	1,061	2,001	2,450	3,465	3,001	3,675	5,198	5,513	6,366	7,796
	5	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,007	NP	1,550	2,923	3,581	5,064	4,385	5,371	7,595	8,056	9,303	11,393
	6	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,044	NP	2,114	3,986	4,882	6,904	5,979	7,322	10,355	10,983	12,683	15,533
	7	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,044	NP	2,664	5,023	6,151	8,699	7,534	9,227	13,049	13,841	15,982	19,574
	8	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,044	1,151	3,254	6,136	7,516	10,629	9,205	11,273	15,943	16,910	19,526	23,915
	9	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,044	1,373	3,883	7,322	8,968	12,683	10,983	13,452	19,024	20,178	23,299	28,536
	10	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,044	1,608	4,548	8,576	10,503	14,854	12,864	15,755	22,281	23,633	27,289	33,422
	12	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,044	2,114	5,979	11,273	13,807	19,526	16,910	20,711	29,289	31,066	35,872	43,934
7/8	4	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,060	NP	1,111	2,095	2,566	3,629	3,143	3,849	5,444	5,774	6,667	8,165
	6	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,149	NP	2,214	4,174	5,113	7,230	6,262	7,669	10,845	11,503	13,283	16,268
	8	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,185	1,243	3,515	6,628	8,118	11,480	9,942	12,177	17,220	18,265	21,091	25,831
	12	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,185	2,283	6,458	12,177	14,913	21,091	18,265	22,370	31,636	33,555	38,746	47,454
	15	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,185	3,191	9,025	17,017	20,842	29,475	25,526	31,263	44,213	46,894	54,149	66,319
	18	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,185	4,194	11,863	22,370	27,398	38,746	33,555	41,096	58,119	61,644	71,181	87,178
	25	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,185	6,865	19,418	36,616	44,845	63,420	54,923	67,267	95,130	100,901	116,510	142,695

Table 34-6B. Design Strengths for Single Cast-In Anchors Subject to Shear Loads (f'_c = 4000 psi)[1, 2, 3, 5] (cont'd.)
Notes pertaining to this table are given on Page 34-15

d_o in.	h_{ef} in.	ϕV_s - Shear Strength of Anchor							ϕV_{cb} - Shear Breakout [5, 6, 7, 8, 9]											
		f_{ut} - for design purposes [4] - psi							$h = h_{ef}$ [10] and c_1 = [11]						$h = 1.5h_{ef}$ and c_1 = [11, 12]			$h = 2.25h_{ef}$ and c_1 = [11, 13]		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	$1.5h_{ef}$	$2h_{ef}$	$3h_{ef}$
1	6	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,254	NP	2,304	4,345	5,322	7,526	6,518	7,982	11,289	11,973	13,826	16,933
	9	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,329	1,585	4,484	8,455	10,355	14,645	12,683	15,533	21,967	23,299	26,904	32,950
	12	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,329	2,441	6,904	13,017	15,943	22,547	19,526	23,915	33,820	35,872	41,421	50,730
	15	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,329	3,411	9,648	18,192	22,281	31,510	27,289	33,422	47,265	50,132	57,888	70,898
	18	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,329	4,484	12,683	23,915	29,289	41,421	35,872	43,934	62,132	65,901	76,096	93,198
	21	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,329	5,650	15,982	30,136	36,909	52,197	45,204	55,363	78,295	83,044	95,891	117,442
	25	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,329	7,339	20,759	39,144	47,941	67,799	58,716	71,912	101,699	107,868	124,555	152,548
1-1/8	6	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,361	NP	2,387	4,501	5,513	7,796	6,752	8,269	11,695	12,404	14,323	17,542
	9	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,476	1,681	4,756	8,968	10,983	15,533	13,452	16,475	23,299	24,713	28,536	34,949
	12	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,476	2,589	7,322	13,807	16,910	23,915	20,711	25,365	35,872	38,048	43,934	53,808
	15	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,476	3,618	10,233	19,296	23,633	33,422	28,944	35,449	50,132	53,173	61,399	75,198
	18	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,476	4,756	13,452	25,365	31,066	43,934	38,048	46,599	65,901	69,898	80,711	98,851
	21	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,476	5,993	16,951	31,964	39,147	55,363	47,946	58,721	83,044	88,082	101,708	124,566
	25	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,476	7,785	22,018	41,518	50,849	71,912	62,277	76,274	107,868	114,411	132,110	161,801
1-1/4	6	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,469	NP	2,464	4,646	5,690	8,047	6,969	8,535	12,070	12,802	14,783	18,105
	9	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,593	1,735	4,909	9,256	11,336	16,032	13,884	17,004	24,048	25,506	29,452	36,071
	12	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,627	2,729	7,718	14,554	17,825	25,208	21,831	26,737	37,812	40,106	46,310	56,718
	15	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,627	3,814	10,787	20,340	24,911	35,229	30,510	37,366	52,844	56,050	64,721	79,266
	18	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,627	5,013	14,180	26,737	32,746	46,310	40,106	49,119	69,465	73,679	85,077	104,198
	21	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,627	6,317	17,868	33,693	41,265	58,358	50,539	61,898	87,536	92,846	107,210	131,305
	25	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,627	8,206	23,209	43,764	53,600	75,802	65,646	80,400	113,702	120,600	139,256	170,554
1-3/8	6	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,579	NP	2,535	4,781	5,855	8,280	7,171	8,783	12,420	13,174	15,212	18,631
	9	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,712	1,786	5,051	9,524	11,665	16,497	14,287	17,497	24,745	26,246	30,306	37,118
	12	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,782	2,862	8,095	15,264	18,695	26,438	22,896	28,042	39,658	42,063	48,571	59,487
	15	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,782	4,000	11,313	21,332	26,127	36,949	31,999	39,190	55,423	58,785	67,879	83,135
	18	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,782	5,258	14,872	28,042	34,345	48,571	42,063	51,517	72,856	77,275	89,230	109,284
	21	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,782	6,626	18,740	35,337	43,279	61,206	53,006	64,919	91,809	97,378	112,443	137,713
	25	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,782	8,606	24,342	45,900	56,216	79,501	68,850	84,324	119,252	126,486	146,053	178,878
1-1/2	12	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,942	2,989	8,455	15,943	19,526	27,614	23,915	29,289	41,421	43,934	50,730	62,132
	15	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,942	4,178	11,816	22,281	27,289	38,592	33,422	40,933	57,888	61,399	70,898	86,832
	18	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,942	5,492	15,533	29,289	35,872	50,730	43,934	53,808	76,096	80,711	93,198	114,143
	21	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,942	6,920	19,574	36,909	45,204	63,928	55,363	67,805	95,891	101,708	117,442	143,837
	25	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,942	8,989	25,425	47,941	58,716	83,037	71,912	88,074	124,555	132,110	152,548	186,832
1-3/4	12	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,205	3,131	8,855	16,698	20,450	28,921	25,046	30,675	43,382	46,013	53,131	65,073
	15	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,274	4,512	12,763	24,066	29,475	41,684	36,099	44,213	62,526	66,319	76,578	93,789
	18	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,274	5,932	16,777	31,636	38,746	54,795	47,454	58,119	82,193	87,178	100,665	123,289
	21	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,274	7,475	21,142	39,866	48,825	69,050	59,799	73,238	103,574	109,857	126,852	155,362
	25	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,274	9,709	27,462	51,782	63,420	89,690	77,674	95,130	134,535	142,695	164,770	201,802
2	12	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,479	3,259	9,217	17,380	21,286	30,103	26,070	31,929	45,155	47,894	55,303	67,732
	15	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,592	4,762	13,469	25,398	31,106	43,991	38,097	46,659	65,986	69,989	80,816	98,979
	18	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,626	6,341	17,936	33,820	41,421	58,578	50,730	62,132	87,868	93,198	107,615	131,801
	21	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,626	7,991	22,602	42,618	52,197	73,817	63,928	78,295	110,726	117,442	135,611	166,089
	25	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,626	10,380	29,358	55,358	67,799	95,882	83,037	101,699	143,823	152,548	176,147	215,735

Table 34-6C. Design Strengths for Single Cast-In Anchors Subject to Shear Loads (f'_c = 6000 psi)[1, 2, 3, 5]
Notes pertaining to this table are given on Page 34-15

d_o in.	h_{ef} in.	ϕV_s - Shear Strength of Anchor							ϕV_{cb} - Shear Breakout [5, 6, 7, 8, 9]												
		f_{ut} - for design purposes [4] - psi							$h = h_{ef}$ [10] and c_1 = [11]						$h = 1.5h_{ef}$ and c_1 = [11, 12]			$h = 2.25h_{ef}$ and c_1 = [11, 13]		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	$1.5h_{ef}$	$2h_{ef}$	$3h_{ef}$
1/4	2	724	749	936	1,123	1,310	1,498	1,560	489	NP	NP	542	664	939	814	996	1,409	1,495	1,726	2,114
	3	724	749	936	1,123	1,310	1,498	1,560	596	NP	NP	996	1,220	1,726	1,495	1,831	2,589	2,746	3,171	3,883
	4	724	749	936	1,123	1,310	1,498	1,560	596	NP	814	1,534	1,879	2,657	2,301	2,818	3,986	4,228	4,882	5,979
	5	724	749	936	1,123	1,310	1,498	1,560	596	NP	1,137	2,144	2,626	3,714	3,216	3,939	5,570	5,908	6,822	8,355
	6	724	749	936	1,123	1,310	1,498	1,560	596	NP	1,495	2,818	3,452	4,882	4,228	5,178	7,322	7,766	8,968	10,983
3/8	2	1,764	1,825	2,282	2,738	3,194	3,650	3,803	563	NP	NP	613	750	1,061	919	1,125	1,591	1,688	1,949	2,387
	3	1,764	1,825	2,282	2,738	3,194	3,650	3,803	772	NP	NP	1,220	1,495	2,114	1,831	2,242	3,171	3,363	3,883	4,756
	4	1,764	1,825	2,282	2,738	3,194	3,650	3,803	772	NP	996	1,879	2,301	3,254	2,818	3,452	4,882	5,178	5,979	7,322
	5	1,764	1,825	2,282	2,738	3,194	3,650	3,803	772	NP	1,393	2,626	3,216	4,548	3,939	4,824	6,822	7,236	8,355	10,233
	6	1,764	1,825	2,282	2,738	3,194	3,650	3,803	772	NP	1,831	3,452	4,228	5,979	5,178	6,341	8,968	9,512	10,983	13,452
1/2	2	3,212	3,323	4,154	4,984	5,815	6,646	6,923	625	NP	NP	668	818	1,157	1,002	1,227	1,735	1,840	2,125	2,602
	3	3,212	3,323	4,154	4,984	5,815	6,646	6,923	889	NP	NP	1,330	1,629	2,304	1,996	2,444	3,456	3,666	4,233	5,185
	4	3,212	3,323	4,154	4,984	5,815	6,646	6,923	942	NP	1,151	2,170	2,657	3,758	3,254	3,986	5,637	5,979	6,904	8,455
	5	3,212	3,323	4,154	4,984	5,815	6,646	6,923	942	NP	1,608	3,032	3,714	5,252	4,548	5,570	7,878	8,355	9,648	11,816
	6	3,212	3,323	4,154	4,984	5,815	6,646	6,923	942	NP	2,114	3,986	4,882	6,904	5,979	7,322	10,355	10,983	12,683	15,533
	7	3,212	3,323	4,154	4,984	5,815	6,646	6,923	942	942	2,664	5,023	6,151	8,699	7,534	9,227	13,049	13,841	15,982	19,574
	8	3,212	3,323	4,154	4,984	5,815	6,646	6,923	942	1,151	3,254	6,136	7,516	10,629	9,205	11,273	15,943	16,910	19,526	23,915
5/8	3	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,002	NP	NP	1,422	1,742	2,464	2,134	2,613	3,696	3,920	4,526	5,544
	4	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,061	NP	1,230	2,320	2,841	4,018	3,480	4,262	6,027	6,393	7,381	9,040
	5	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,110	NP	1,798	3,390	4,152	5,872	5,085	6,228	8,807	9,342	10,787	13,211
	6	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,110	NP	2,363	4,456	5,458	7,718	6,684	8,187	11,578	12,280	14,180	17,366
	7	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,110	NP	2,978	5,615	6,878	9,726	8,423	10,316	14,589	15,474	17,868	21,884
	8	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,110	1,286	3,638	6,861	8,403	11,883	10,291	12,604	17,825	18,906	21,831	26,737
	9	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,110	1,535	4,342	8,187	10,026	14,180	12,280	15,040	21,269	22,560	26,050	31,904
	10	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,110	1,798	5,085	9,588	11,743	16,607	14,382	17,615	24,911	26,422	30,510	37,366
3/4	4	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,180	NP	1,299	2,450	3,001	4,244	3,675	4,501	6,366	6,752	7,796	9,549
	5	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,233	NP	1,899	3,581	4,385	6,202	5,371	6,578	9,303	9,867	11,393	13,954
	6	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,279	NP	2,589	4,882	5,979	8,455	7,322	8,968	12,683	13,452	15,533	19,024
	7	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,279	NP	3,262	6,151	7,534	10,655	9,227	11,301	15,982	16,951	19,574	23,973
	8	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,279	1,409	3,986	7,516	9,205	13,017	11,273	13,807	19,526	20,711	23,915	29,289
	9	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,279	1,681	4,756	8,968	10,983	15,533	13,452	16,475	23,299	24,713	28,536	34,949
	10	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,279	1,969	5,570	10,503	12,864	18,192	15,755	19,296	27,289	28,944	33,422	40,933
	12	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,279	2,589	7,322	13,807	16,910	23,915	20,711	25,365	35,872	38,048	43,934	53,808
7/8	4	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,298	NP	1,361	2,566	3,143	4,445	3,849	4,714	6,667	7,072	8,165	10,001
	6	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,407	NP	2,711	5,113	6,262	8,855	7,669	9,392	13,283	14,089	16,268	19,924
	8	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,451	1,522	4,305	8,118	9,942	14,060	12,177	14,913	21,091	22,370	25,831	31,636
	12	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,451	2,796	7,909	14,913	18,265	25,831	22,370	27,398	38,746	41,096	47,454	58,119
	15	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,451	3,908	11,053	20,842	25,526	36,099	31,263	38,289	54,149	57,434	66,319	81,224
	18	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,451	5,137	14,530	27,398	33,555	47,454	41,096	50,332	71,181	75,499	87,178	106,771
	25	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,451	8,408	23,783	44,845	54,923	77,674	67,267	82,385	116,510	123,578	142,695	174,765

Table 34-6C. Design Strengths for Single Cast-In Anchors Subject to Shear Loads (f'_c = 6000 psi)[1, 2, 3, 5] (cont'd.)
Notes pertaining to this table are given on Page 34-15

d_o in.	h_{ef} in.	ϕV_s - Shear Strength of Anchor							ϕV_{cb} - Shear Breakout [5, 6, 7, 8, 9]												
		f_{ut} - for design purposes [4] - psi							$h = h_{ef}$ [10] and c_1 = [11]						$h = 1.5h_{ef}$ and c_1 = [11, 12]			$h = 2.25h_{ef}$ and c_1 = [11, 13]		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	$1.5h_{ef}$	$2h_{ef}$	$3h_{ef}$
1	6	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,536	NP	2,822	5,322	6,518	9,217	7,982	9,776	13,826	14,664	16,933	20,739
	9	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,627	1,942	5,492	10,355	12,683	17,936	15,533	19,024	26,904	28,536	32,950	40,356
	12	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,627	2,989	8,455	15,943	19,526	27,614	23,915	29,289	41,421	43,934	50,730	62,132
	15	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,627	4,178	11,816	22,281	27,289	38,592	33,422	40,933	57,888	61,399	70,898	86,832
	18	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,627	5,492	15,533	29,289	35,872	50,730	43,934	53,808	76,096	80,711	93,198	114,143
	21	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,627	6,920	19,574	36,909	45,204	63,928	55,363	67,805	95,891	101,708	117,442	143,837
	25	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,627	8,989	25,425	47,941	58,716	83,037	71,912	88,074	124,555	132,110	152,548	186,832
1-1/8	6	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,667	NP	2,924	5,513	6,752	9,549	8,269	10,128	14,323	15,192	17,542	21,485
	9	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,807	2,059	5,825	10,983	13,452	19,024	16,475	20,178	28,536	30,267	34,949	42,804
	12	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,807	3,171	8,968	16,910	20,711	29,289	25,365	31,066	43,934	46,599	53,808	65,901
	15	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,807	4,431	12,533	23,633	28,944	40,933	35,449	43,416	61,399	65,124	75,198	92,099
	18	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,807	5,825	16,475	31,066	38,048	53,808	46,599	57,072	80,711	85,607	98,851	121,067
	21	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,807	7,340	20,761	39,147	47,946	67,805	58,721	71,918	101,708	107,878	124,566	152,562
	25	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,807	9,534	26,967	50,849	62,277	88,074	76,274	93,416	132,110	140,124	161,801	198,165
1-1/4	6	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,799	NP	3,018	5,690	6,969	9,855	8,535	10,453	14,783	15,680	18,105	22,174
	9	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,951	2,126	6,012	11,336	13,884	19,635	17,004	20,826	29,452	31,239	36,071	44,178
	12	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,992	3,342	9,453	17,825	21,831	30,874	26,737	32,746	46,310	49,119	56,718	69,465
	15	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,992	4,671	13,211	24,911	30,510	43,147	37,366	45,764	64,721	68,647	79,266	97,081
	18	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,992	6,140	17,366	32,746	40,106	56,718	49,119	60,159	85,077	90,238	104,198	127,616
	21	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,992	7,737	21,884	41,265	50,539	71,473	61,898	75,809	107,210	113,713	131,305	160,815
	25	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,992	10,050	28,426	53,600	65,646	92,838	80,400	98,469	139,256	147,704	170,554	208,885
1-3/8	6	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,933	NP	3,105	5,855	7,171	10,141	8,783	10,756	15,212	16,135	18,631	22,818
	9	26,239	27,144	33,930	40,716	47,502	54,288	56,550	2,097	2,187	6,186	11,665	14,287	20,204	17,497	21,430	30,306	32,145	37,118	45,460
	12	26,239	27,144	33,930	40,716	47,502	54,288	56,550	2,183	3,505	9,914	18,695	22,896	32,380	28,042	34,345	48,571	51,517	59,487	72,856
	15	26,239	27,144	33,930	40,716	47,502	54,288	56,550	2,183	4,899	13,856	26,127	31,999	45,253	39,190	47,998	67,879	71,997	83,135	101,819
	18	26,239	27,144	33,930	40,716	47,502	54,288	56,550	2,183	6,440	18,214	34,345	42,063	59,487	51,517	63,095	89,230	94,643	109,284	133,845
	21	26,239	27,144	33,930	40,716	47,502	54,288	56,550	2,183	8,115	22,952	43,279	53,006	74,962	64,919	79,509	112,443	119,263	137,713	168,664
	25	26,239	27,144	33,930	40,716	47,502	54,288	56,550	2,183	10,540	29,813	56,216	68,850	97,369	84,324	103,275	146,053	154,913	178,878	219,080
1-1/2	12	31,894	32,994	41,243	49,491	57,740	65,988	68,738	2,378	3,661	10,355	19,526	23,915	33,820	29,289	35,872	50,730	53,808	62,132	76,096
	15	31,894	32,994	41,243	49,491	57,740	65,988	68,738	2,378	5,117	14,472	27,289	33,422	47,265	40,933	50,132	70,898	75,198	86,832	106,347
	18	31,894	32,994	41,243	49,491	57,740	65,988	68,738	2,378	6,726	19,024	35,872	43,934	62,132	53,808	65,901	93,198	98,851	114,143	139,796
	21	31,894	32,994	41,243	49,491	57,740	65,988	68,738	2,378	8,476	23,973	45,204	55,363	78,295	67,805	83,044	117,442	124,566	143,837	176,164
	25	31,894	32,994	41,243	49,491	57,740	65,988	68,738	2,378	11,009	31,139	58,716	71,912	101,699	88,074	107,868	152,548	161,801	186,832	228,822
1-3/4	12	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,701	3,834	10,845	20,450	25,046	35,421	30,675	37,570	53,131	56,354	65,073	79,697
	15	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,786	5,527	15,631	29,475	36,099	51,052	44,213	54,149	76,578	81,224	93,789	114,868
	18	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,786	7,265	20,548	38,746	47,454	67,110	58,119	71,181	100,665	106,771	123,289	150,997
	21	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,786	9,155	25,894	48,825	59,799	84,568	73,238	89,698	126,852	134,547	155,362	190,278
	25	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,786	11,891	33,634	63,420	77,674	109,847	95,130	116,510	164,770	174,765	201,802	247,156
2	12	56,550	58,500	73,125	87,750	102,375	117,000	121,875	3,036	3,991	11,289	21,286	26,070	36,869	31,929	39,105	55,303	58,658	67,732	82,955
	15	56,550	58,500	73,125	87,750	102,375	117,000	121,875	3,175	5,832	16,496	31,106	38,097	53,877	46,659	57,145	80,816	85,718	98,979	121,224
	18	56,550	58,500	73,125	87,750	102,375	117,000	121,875	3,216	7,766	21,967	41,421	50,730	71,744	62,132	76,096	107,615	114,143	131,801	161,423
	21	56,550	58,500	73,125	87,750	102,375	117,000	121,875	3,216	9,787	27,681	52,197	63,928	90,407	78,295	95,891	135,611	143,837	166,089	203,416
	25	56,550	58,500	73,125	87,750	102,375	117,000	121,875	3,216	12,712	35,956	67,799	83,037	117,431	101,699	124,555	176,147	186,832	215,735	264,221

REFERENCES

35.1 ACI Committee 349, “Code Requirements for Nuclear Safety Related Concrete Structures (ACI 349-85), Appendix B - Steel Embedment,” American Concrete Institute, Farmington Hills, Michigan.

35.2 PCI Design Handbook, 5th Edition, Precast/Prestressed Concrete Institute, Chicago, 1999.

35.3 International Building Code 2000 (IBC 2000), International Building Code Council, Falls Church, Virginia, May, 2000.

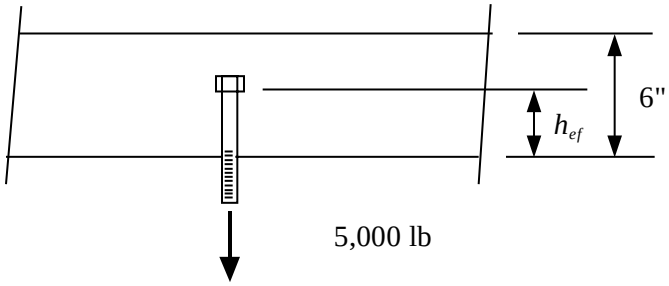
35.4 ACI Committee 355, “Evaluating the Performance of Post-Installed Mechanical Anchors in Concrete (ACI 355.2-01), American Concrete Institute, Farmington Hills, Michigan.

35.5 ACI Committee 349, “Code Requirements for Nuclear Safety Related Concrete Structures (ACI 349-01), Appendix B - Steel Embedment,” American Concrete Institute, Farmington Hills, Michigan.

Example 34.1—Single Headed Bolt in Tension Away from Edges

Design a single headed bolt installed in the bottom of a 6 in. slab to support a 5000 lb service dead load.

f'_c = 4000 psi



1. Determine factored design load (only dead load is present) *9.2*

   $N_u = 1.4\ (5000) = 7000$ lb

2. Determine anchor diameter and material *D.5.1*

   The strength of most anchors is likely to be controlled by the embedment strength rather than the steel strength. As a result, it is usually economical to design the anchor using a mild steel rather than a high strength steel. ASTM F 1554 "Standard Specification for Anchor Bolts, Steel, 36, 55, and 105-ksi Yield Strength," covers straight and bent, headed and headless, anchors in three strength grades.

   Assume an ASTM F1554 Grade 36 headed anchor for this example.

   The basic requirement for the anchor steel is:

   $\phi N_s \geq N_u$ *Eq. (D-1)* *D.4.1.2*

   where:

   $\phi = 0.75$ *D.4.4(a)*

   Per the Ductile Steel Element definition in D.1, ASTM F 1554 Grade 36 steel qualifies as a ductile steel element (23% minimum elongation in 2 in. which is greater than the 14% required and a minimum reduction in area of 40% that is greater than the 30% required, see Table 34.1). This results in $\phi = 0.75$ rather than $\phi = 0.65$ if the steel had not met the ductile steel element requirements.

   $N_s = nA_{se}f_{ut}$ *Eq. (D-3)*

   For design purposes, Eq. (D-1) with Eq. (D-3) may be rearranged as:

   $$A_{se} \geq \frac{N_u}{\phi\ n\ f_{ut}}$$

   where:

N_u = 7000 lbs
ϕ = 0.75
n = 1
f_{ut} = 58,000 psi

Per ASTM F 1554, Grade 36 has a specified minimum yield strength of 36 ksi and a specified tensile strength of 58-80 ksi (see Table 34-1). For design purposes, the minimum tensile strength of 58 ksi should be used.

Note: Per D.5.1.2, f_{ut} shall not be taken greater than $1.9f_y$ or 125,000 psi. For ASTM F 1554 Grade 36, $1.9f_y = 1.9(36{,}000) = 68{,}400$ psi, therefore use the specified minimum f_{ut} of 58,000 psi. *D.5.1.2*

Substituting:

$$A_{se} = \frac{7000}{0.75(1)(58{,}000)} = 0.161 \text{ in.}^2$$

Per Table 34-2, a 5/8 in. diameter threaded anchor will satisfy this requirement (A_{se} = 0.226 in.^{2}).

3. Determine the required embedment length (h_{ef}) based on concrete breakout *D.5.2*

The basic requirement for the single anchor embedment is:

$$\phi N_{cb} \geq N_u$$ *Eq. (D-1)* *D.4.1.2*

where:

ϕ = 0.70 *D.4.4(c)*

Condition B applies, no supplementary reinforcement has been provided to tie the failure prism associated with the concrete breakout failure mode of the anchor to the supporting structural member. This is likely to be the case for anchors loaded in tension attached to a slab. Condition A (with ϕ = 0.75) may apply when the anchoring is attached to a deeper member (such as a pedestal or beam) where there is space available to install supplementary reinforcement across the failure prism.

$$N_{cb} = \frac{A_N}{A_{No}} \psi_2 \psi_3 N_b$$ *Eq. (D-4)*

where:

$\frac{A_N}{A_{No}}$ and ψ_2 terms are 1.0 for single anchors away from edges

$$N_b = 24\sqrt{f'_c}\, h_{ef}^{1.5}$$ *Eq. (D-7)*

For design of a single anchor away from edges, Eq. (D-1) with Eq. (D-4) and Eq. (D-7) may be rearranged as:

$$h_{ef} = \left(\frac{N_u}{\phi \, \psi_3 \, 24\sqrt{f'_c}} \right)^{\frac{2}{3}}$$

where:

ψ_3 = 1.0 for locations where concrete cracking is likely to occur (i.e., the bottom of the slab) *D.5.2.6*

Substituting:

$$h_{ef} = \left(\frac{7000}{0.70(1.0)24\sqrt{4000}} \right)^{\frac{2}{3}} = 3.51 \text{ in.}$$

Select 4 in. embedment for this anchor.

Note: The case of a single anchor away from an edge is essentially the only case where h_{ef} can be solved for directly. Whenever edges or adjacent anchors are present, the solution for h_{ef} is iterative.

4. Determine the required head size for the anchor *D.5.3*

The basic requirement for pullout strength (i.e., the strength of the anchor related to the embedded anchor having insufficient bearing area so that the anchor pulls out without a concrete breakout failure) is:

$\phi N_{pn} \geq N_u$ *Eq. (D-1)* *D.4.1.2*

where:

ϕ = 0.70 *D.4.4(c)*

Condition B applies for pullout strength in all cases

$N_{pn} = \psi_4 \, N_p$ *Eq. (D-12)*

where:

$N_p = A_{brg} \, 8 \, f'_c$ *Eq. (D-13)*

ψ_4 = 1.0 for locations where concrete cracking is likely to occur (i.e., the bottom of the slab) *D.5.3.6*

For design purposes Eq. (D-1) with Eq. (D-12) and Eq. (D-13) may be rearranged as:

$$A_{brg} = \frac{N_u}{\phi \, \psi_4 \, 8 \, f'_c}$$

Substituting:

$$A_{brg} = \frac{7000}{0.70(1.0)(8)(4000)} = 0.313 \text{ in}^2$$

As shown in Table 34-2, any type of standard head (square, heavy square, hex, or heavy hex) is acceptable for this 5/8 in. diameter anchor. ASTM F 1554 specifies a hex head for Grade 36 bolts less that 1-1/2 in. in diameter.

5. Evaluate side-face blowout — *D.5.4*

Since this anchor is located far from a free edge of concrete ($c \geq 0.4\ h_{ef}$) this type of failure mode is not applicable.

6. Required edge distances, spacings, and thicknesses to preclude splitting failure — *D.8*

Since this is a cast-in-place anchor and is located far from a free edge of concrete, the only requirement is that the minimum cover requirements of Section 7.7 should be met. Assuming this is an interior slab, the requirements of Section 7.7 will be met with the 4 in. embedment length plus the head thickness. The head thickness for square, hex, and heavy hex heads and nuts are at most equal to the anchor diameter (refer to ANSI B.18.2.1 and ANSI B.18.2.2 for exact dimensions). This results in ~1-3/8 in. cover from the top of the anchor head to the top of the slab.

7. Summary:

Use an ASTM F 1554 Grade 36, 5/8 in. diameter headed anchor with a 4 in. embedment.

Alternate design using Table 34-5B

Note: Step numbers correspond to those in the main example above, but prefaced with "A".

Table 34-5B has been selected because it contains design tension strength values based on concrete with f'_c = 4000 psi. Table Note 4 indicates that the values in the table are based on Condition B (no supplementary reinforcement), and Notes 6 and 10 indicate that cracked concrete was assumed.

A2. Determine anchor diameter and material. — *D.5.1* *Eq. (D-3)*

Tentatively try a bolt complying with ASTM F 1554 Grade 36 with a f_{ut} of 58,000 psi. Using the factored tensile load from Step 1 above (7000 lb), and 58,000 psi as the value of f_{ut}, go down the column for 58,000 until an anchor size has a design tensile strength, ϕN_s, equal to or greater than 7000 lb. Table 34-6B shows that a 5/8 in. diameter bolt has a design tensile strength equal to

ϕN_s = 9831 lb > N_u = 7000 lb O.K.

Since this is greater than the required strength, tentatively use a 5/8 in. headed bolt.

A3. Determine the required embedment length (h_{ef}) based on concrete breakout strength (ϕN_{cb}) — *D.5.2, Eq. (D-4)*

Since the anchor will be far from an edge, use the column labeled "$>1.5h_{ef}$." In this case "far from an edge" means that the edge distance, c, must equal or exceed $1.5h_{ef}$. A 5/8 in. bolt with 3 in. embedment has a design tension breakout strength,

ϕN_{cb}, = 5521 lb < N_u = 7000 lb

A 5/8-in. bolt with 4 in. embedment has

ϕN_{cb} = 8500 lb > N_u = 7000 lb O.K.

Tentatively use 5/8 in. bolt with embedment depth of 4 in.

A4. Determine if the bearing area of the head of the 5/8-in. bolt, A_{brg}, is large enough to prevent anchor pullout (ϕN_{pn}). — *D.5.3, Eq. (D-12)*

Values for design tension pullout strength, ϕN_{pn}, in Table 34-6B for headed bolts with a diameter of less than 1-3/4 in. are based on a regular hex head (Table Note 7). Under the column labeled "head w/o washer," a 5/8 in. bolt has a design pullout strength,

ϕN_{pn} = 10,170 lb > N_u = 7000 lb O.K.

A5. Determine if the anchor has enough edge distance, c, to prevent side-face blow-out (ϕN_{sb}). — *D.5.4.1*

Since the anchor is farther from an edge than $0.4h_{ef}$ (0.4 x 4 in. = 1.6 in.), side-face blowout does not need to be considered.

A6. Required edge distances, spacings, and thicknesses to preclude splitting failure.

See Step 6 above.

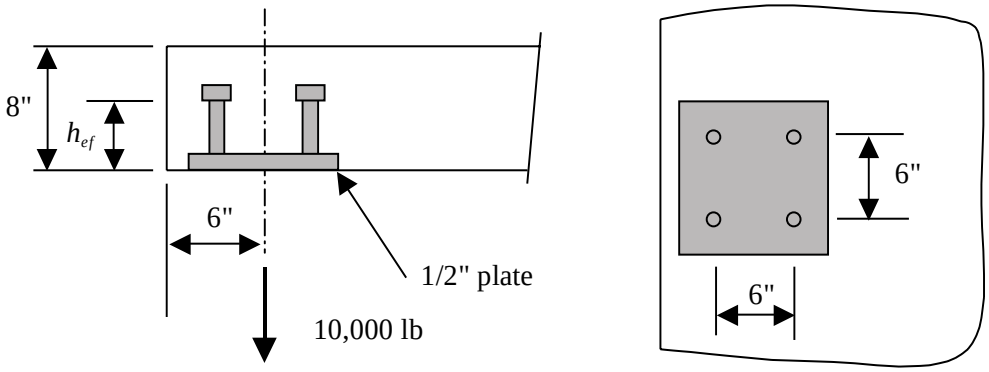
A7. Summary:

Use an ASTM F 1554 Grade 36, 5/8 in. diameter headed bolt with a 4 in. embedment.

Example 34.2—Group of Headed Studs in Tension Near an Edge

Design a group of four welded, headed studs spaced 6 in. on center each way and concentrically loaded with a 10,000 lb service dead load. The anchor group is to be installed in the bottom of an 8 in. thick slab with the centerline of the connection 6 in. from a free edge of the slab.

f'_c = 4000 psi



Calculations and Discussion	Code Reference

1. Determine factored design load — *9.2*

   $N_u = 1.4\ (10{,}000) = 14{,}000$ lb

2. Determine anchor diameter — *D.5.1*

   Assume AWS D1.1 Type B welded, headed studs.

   The basic requirement for the anchor steel is:

   $\phi N_s \geq N_u$ — *Eq. (D-1)*, *D.4.1.2*

   where:

   $\phi = 0.75$ — *D.4.4(a)i*

   Per the Ductile Steel Element definition in D.1, AWS D1.1 Type B studs qualify as a ductile steel element [20% minimum elongation in 2 in. which is greater than the 14% required and a minimum reduction in area of 50% that is greater than the 30% required, see Table 34-1)].

   $N_s = n\ A_{se}\ f_{ut}$ — *Eq. (D-3)*

   For design purposes, Eq. (D-1) with Eq. (D-3) may be rearranged as:

$$A_{se} = \frac{N_u}{\phi\ n\ f_{ut}}$$

where:

N_u = 14,000 lbs
ϕ = 0.75
n = 1
f_{ut} = 60,000 psi

Note: Per D5.1.2, f_{ut} shall not be taken greater than $1.9f_y$ or 125,000 psi. For AWS D1.1 headed studs, $1.9f_y = 1.9(50{,}000) = 95{,}000$ psi, therefore use the specified minimum f_{ut} of 60,000 psi. *D.5.1.2*

Substituting:

$$A_{se} = \frac{14{,}000}{0.75(4)(60{,}000)} = 0.078 \text{ in.}^2$$

Per Table 34-2, 1/2 in. diameter welded, headed studs will satisfy this requirement (A_{se} = 0.196 in.^{2}).

Note: Per AWS D1.1 Table 7.1, Type B welded studs are 1/2 in., 5/8 in., 3/4 in., 7/8 in., and 1 in. diameters. Although individual manufacturers may list smaller diameters they are not explicitly covered by AWS D1.1

3. Determine the required embedment length (h_{ef}) based on concrete breakout *D.5.2*

Two different equations are given for calculating concrete breakout strength; for single anchors Eq. (D-4) applies, and for anchor groups Eq. (D-5) applies. An "anchor group" is defined as:

"a number of anchors of approximately equal effective embedment depth with each anchor spaced at less than three times its embedment depth from one or more adjacent anchors." *D.1*

Since the spacing between anchors is 6 in., they must be treated as a group if the embedment depth exceeds 2 in. Although the embedment depth is unknown, at this point it will be assumed that the provisions for an anchor group will apply.

The basic requirement for embedment of a group of anchors is:

$\phi N_{cbg} \geq N_u$ *Eq. (D-1)* *D.4.1.2*

where:

ϕ = 0.70 *D.4.4(c)ii*

Condition B applies since no supplementary reinforcement has been provided (e.g., hairpin type reinforcement surrounding the anchors and anchored into the concrete).

$$N_{cbg} = \frac{A_N}{A_{No}} \psi_1 \psi_2 \psi_3 N_b$$ *Eq. (D-5)*

Since this connection is likely to be affected by both group effects and edge effects, the embedment length h_{ef} cannot be determined from a closed form solution. Therefore, an embedment length must be assumed. The strength of the connection is then determined and compared with the required strength.

Note: Welded studs are generally available in fixed lengths. Available lengths may be determined from manufacturers' catalogs. For example, the Nelson Stud web page (www.trw.com/nelsonstud) indicates an effective embedment of 4 in. for a standard 1/2 in. concrete anchor stud.

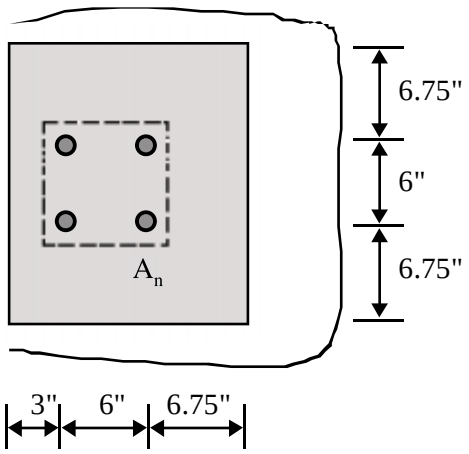
Assume an effective embedment length of h_{ef} = 4.5 in.

Note: The effective embedment length h_{ef} for the welded stud anchor is the effective embedment length of the stud plus the thickness of the embedded plate.

Evaluate the terms in Eq. (D-5) with h_{ef} = 4.5 in.

Determine A_N and A_{No} for the anchorage: *D.5.2.1*

A_N is the projected area of the failure surface as approximated by a rectangle with edges bounded by 1.5 h_{ef} (1.5 x 4.5 = 6.75 in. in this case) and free edges of the concrete from the centerlines of the anchors.



$$A_N = (3+6+6.75)(6.75+6+6.75) = 307 \text{ in.}^2$$

$A_{No} = 9\ h_{ef}^2 = 9\ (4.5)^2 = 182 \text{ in.}^2$ *Eq. (D-6)*

D.5.2.1

Check: $A_N \le nA_{No}$ $307 < 4(182)$ O.K.

Determine ψ_1: *D.5.2.4*

ψ_1 = 1.0 (no eccentricity in the connection)

Determine ψ_2 since $c < 1.5\ h_{ef}$ — *D.5.2.5*

$$\psi_2 = 0.7 + 0.3\frac{c_{min}}{1.5 h_{ef}}$$ — *Eq. (D-11)*

$$\psi_2 = 0.7 + 0.3\frac{3.0}{1.5(4.5)} = 0.83$$

Determine ψ_3: — *D.5.2.6*

$\psi_3 = 1.0$ for locations where concrete cracking is likely to occur (i.e., the bottom of the slab)

Determine N_b: — *D.5.2.2*

$$N_b = 24\ \sqrt{f'_c}\ h_{ef}^{1.5} = 24\ \sqrt{4000}\ (4.5)^{1.5} = 14,490\ \text{lb}$$ — *Eq. (D-7)*

Substituting into Eq. (D-5):

$$N_{cbg} = \left[\frac{307}{182}\right](1.0)(0.83)(1.0)(14,490) = 20,287\ \text{lb}$$

The final check on the assumption of $h_{ef} = 4.5$ in. is satisfied by meeting the requirements of Eq. (D-1):

(0.70) (20,287) ≥ 14,000

14,201 > 14,000 OK

Specify a 4 in. length for the welded, headed studs with the 1/2 in.-thick base plate.

4. Determine if welded stud head size is adequate for pullout — *D.5.3*

$$\phi N_{pn} \geq N_u$$ — *Eq. (D-1)*, *D.4.1.2*

where:

$\phi = 0.70$ — *D.4.4(c)ii*

Condition B applies for pullout strength in all cases.

$$N_{pn} = \psi_4 N_p$$ — *Eq. (D-12)*

where:

$$N_p = A_{brg}\ 8\ f'_c$$ — *Eq. (D-13)*

ψ_4 = 1.0 for locations where concrete cracking is likely to occur (i.e., the bottom of the slab)

For design purposes Eq. (D-1) with Eq. (D-12) and Eq. (D-13) may be rearranged as:

$$A_{brg} = \frac{N_u}{\phi\ \psi_4\ 8\ f'_c}$$

For the group of four studs the individual factored tension load N_u on each stud is:

$$N_u = \frac{14{,}000}{4} = 3500 \text{ lb}$$

Substituting:

$$A_{brg} = \frac{3500}{0.70\ (1.0)\ (8)\ (4000)} = 0.156 \text{ in.}^2$$

The bearing area of welded, headed studs should be determined from manufacturers' catalogs. As shown on the Nelson Stud web page the diameter of the head for a 1/2 in. diameter stud is 1 in.

$$A_{brg,\ provided} = \frac{\pi}{4}(1.0^2 - 0.5^2) = 0.589 \text{ in.}^2 > 0.156 \text{ in.}^2 \quad \text{O.K.}$$

5. Evaluate side-face blowout — *D.5.4*

Side-face blowout needs to be considered when the edge distance from the centerline of the anchor to the nearest free edge is less than $0.4h_{ef}$. For this example:

$0.4h_{ef}$ = 0.4 (4.5) = 1.8 in. < 3 in. actual edge distance O.K.

The side-face blowout failure mode is not applicable.

6. Required edge distances, spacings, and thickness to preclude splitting failure — *D.8*

Since a welded, headed anchor is not torqued the minimum cover requirements of 7.7 apply.

Per 7.7 the minimum clear cover for a 1/2 in. bar not exposed to earth or weather is 3/4 in. which is less than the 2-3/4 in. provided (3 - 1/4 = 2-3/4 in.) O.K.

7. Summary:

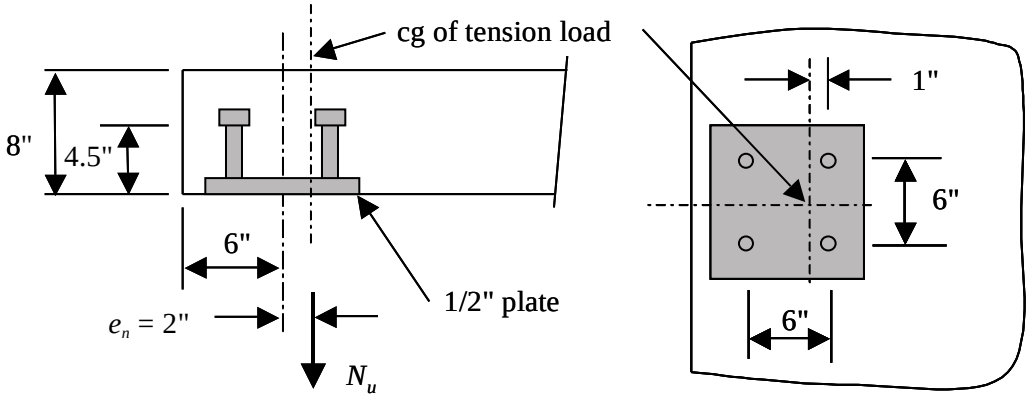
Use ASW D1.1 Type B 1/2 in. diameter welded studs with an effective embedment of 4.5 in. (4 in. from the stud plus 1/2 in. from the embedded plate).

Example 34.3—Group of Headed Studs in Tension Near an Edge with Eccentricity

Determine the factored tension load capacity (N_u) for a group of four 1/2 in. x 4 in. AWS D1.1 Type B headed studs spaced 6 in. on center each way and welded to a 1/2-in.-thick base plate. The centerline of the structural attachment to the base plate is located 2 in. off of the centerline of the base plate resulting in an eccentricity of the tension load of 2 in. The fastener group is installed in the bottom of an 8 in.-thick slab with the centerline of the connection 6 in. from a free edge of the slab.

Note: This is the configuration chosen as a solution for Example 34.2 to support a 14,000 lb factored tension load centered on the connection. The only difference is the eccentricity of the tension load. From Example 34.2, the spacing between anchors dictates that they be designed as an anchor group.

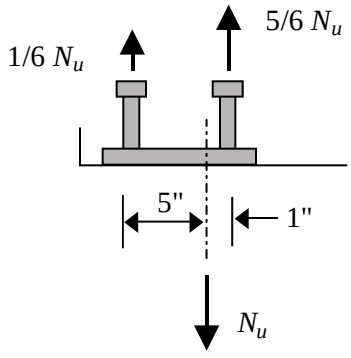
f'_c = 4000 psi



Calculations and Discussion	Code Reference

1. Determine distribution of loads to the anchors — *D.3.1*

   Assuming an elastic distribution of loads to the anchors, the eccentricity of the tension load will result in a higher force on the interior row of fasteners. Although the studs are welded to the base plate, their flexural stiffness at the joint with the base plate is minimal compared to that of the base plate. Therefore, assume a simple support condition for the base plate:



   The two interior studs will control the strength related to the steel ϕN_s and the pullout strength ϕN_{pn} (i.e., 5/6 N_u must be less than or equal to $\phi N_{s,\ 2\ studs}$ and $\phi N_{pn,\ 2\ studs}$). Rearranged:

   $N_u \leq 6/5\ \phi N_{s,\ 2\ studs}$ and $N_u \leq 6/5\ \phi N_{pn,\ 2\ studs}$)

2. Determine the design steel strength as controlled by the two anchors with the highest tensile load (ϕN_s) — *D.5.1*

$\phi N_s = \phi\ n\ A_{se}\ f_{ut}$ — *Eq. (D-3)*

where:

$\phi = 0.75$ — *D.4.4(a)i*

Per the Ductile Steel Element definition in D.1, AWS D1.1 Type B studs qualify as a ductile steel element [20% minimum elongation in 2 in. which is greater than the 14% required and a minimum reduction in area of 50% that is greater than the 30% required, see Table 34-1)].

n = 2 (for the two inner studs with the highest tension load)

$A_{se} = 0.196$ in.^{2} (see Table 34-2)

$f_{ut} = 60{,}000$ (see Table 34-1)

Substituting:

$\phi N_{s,\ 2studs} = 0.75\ (2)\ (0.196)\ (60{,}000) = 17{,}640$ lb

Therefore, the maximum N_u as controlled by the anchor steel is:

$\phi N_s = 6/5\ \phi N_{s,2\,studs} = 6/5\ (17{,}640) = 21{,}168$ lb

3. Determine design breakout strength (ϕN_{cbg}) — *D.5.2*

The only difference between concrete breakout strength in this example and Example 34.2 is the introduction of the eccentricity factor ψ_1.

From Example 34.2 with $\psi_1 = 1.0$:

$$\phi N_{cbg} = \phi \frac{A_N}{A_{No}} \psi_1\ \psi_2\ \psi_3\ N_b = 14{,}201\ \text{lb}$$ — *Eq. (D-5)*

Determine ψ_1 for this example ($e_N = e_N' = 2$ in $< s/2 = 3$ in): — *D.5.2.4*

$$\psi_1 = \frac{1}{\left(1 + \frac{2e'_N}{3h_{ef}}\right)}$$ — *Eq. (D-9)*

where:

$e'_N = 2$ in. (distance between centroid of anchor group and tension force)

$h_{ef} = 4.5$ in. (1/2 in. plate plus 4 in. embedment of headed stud)

Substituting:

$$\psi_1 = \frac{1}{\left(1 + \frac{2(2)}{3(4.5)}\right)} = 0.77$$

Therefore:

ϕN_{cbg} = (0.77) (14,201) = 10,935 lb

4. Determine the design pullout strength as controlled by the two anchors with the highest tensile load (ϕN_{pn}) — *D.5.3*

$\phi N_{pn,\ 1\ stud} = \phi\ \psi_4\ N_p = \phi\ \psi_4\ A_{brg}\ 8\ f'_c$ — *Eq. (D-12)* *Eq. (D-13)*

where:

ϕ = 0.70 – Condition B applies for pullout. — *D.4.4(c)ii*

ψ_4 = 1.0 for locations where concrete cracking is likely to occur (i.e., the bottom of the slab) — *D.5.3.6*

A_{brg} = 0.589 in.^{2} (see Step 4 of Example 35.2)

Substituting:

$\phi N_{pn,\ 1\ stud}$ = (0.70) (1.0) (0.589) (8.0) (4000) = 13,194 lb

For the two equally loaded inner studs:

$\phi N_{pn,2\ studs}$ = 2 (13,194) = 26,387 lb

Therefore, the maximum N_u as controlled by pullout is:

ϕN_{pn} = 6/5 $\phi N_{pn,\ 2\ studs}$ = 6/5 (26,387) = 31,664 lb

5. Evaluate side-face blowout — *D.5.4*

Side-face blowout needs to be considered when the edge distance from the centerline of the anchor to the nearest free edge is less than 0.4 h_{ef}. For this example:

0.4 h_{ef} = 0.4 (4.5) = 1.8 in. < 3 in. actual edge distance O.K.

The side-face blowout failure mode is not applicable.

6. Required edge distances, spacings, and thickness to preclude splitting failure — *D.8*

   Since a welded, headed anchor is not torqued the minimum cover requirements of 7.7 apply.

   Per 7.7 the minimum clear cover for a 1/2 in. bar not exposed to earth or weather is 3/4 in. which is less than the 2-3/4 in. provided (3 in. - 1/4 in. = 2-3/4 in.) O.K. — *7.7.1(c)*

7. Summary:

Steel strength, (ϕN_s):	21,168 lb
Embedment strength – concrete breakout, (ϕN_{cbg}):	10,935 lb ← controls
Embedment strength – pullout, (ϕN_{pn}):	31,664 lb
Embedment strength – side-face blowout, (ϕN_{sb}):	N/A

   The maximum factored tension load N_u for this anchorage is 10,935 lb

   Note: Example 34.2 with the same connection but without an eccentricity was also controlled by concrete breakout strength but had a factored load capacity of 14,201 lb (see Step 3 of Example 34.2).

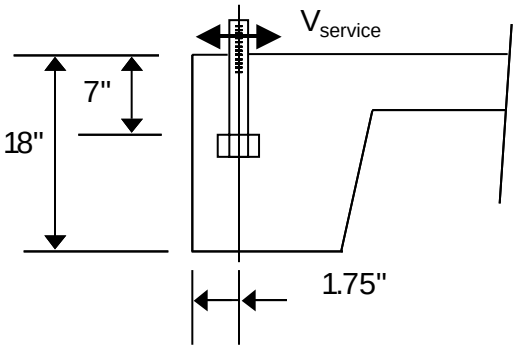
Example 34.4—Single Headed Bolt in Shear Near an Edge

Determine the reversible service wind load shear capacity for a single 1/2 in. diameter headed anchor with a 7 in. embedment installed with its centerline 1-3/4 in. from the edge of a concrete foundation.

Note: This is the minimum anchorage requirement at the foundation required by IBC 2000 Section 2308.6 for conventional light-frame wood construction. The 1-3/4 in. edge distance represents a typical connection at the base of wood framed walls using 2x4 members.

f'_c = 4000 psi

ASTM F 1554 Grade 36



Calculations and Discussion	Code Reference

1. This problem provides the anchor diameter, embedment length, and material properties, and requires computing the maximum service shear load capacity to resist wind load. In this case, it is best to first determine the controlling factored shear load, V_u, based on the smaller of the steel strength and embedment strength then as a last step determine the maximum service load. Step 6 of this example provides the conversion of the controlling factored shear load V_u to a service load due to wind.

2. Determine V_u as controlled by the anchor steel — *D.6.1*

   $\phi V_s \geq V_u$ — *Eq. (D-2)* *D.4.1.2*

   where:

   ϕ = 0.65 — *D.4.4(a)i*
   Per the Ductile Steel Element definition in D.1, ASTM F 1554 Grade 36 steel qualifies as a ductile steel element.

   $V_s = n\ 0.6\ A_{se}\ f_{ut}$ — *Eq. (D-18)*

   To determine V_u for the steel strength Eq. (D-2) can be combined with Eq. (D-18) to give:

   $V_u = \phi V_s = \phi\ n\ 0.6\ A_{se}\ F_{ut}$

where:

$\phi = 0.65$
$n = 1$
$A_{se} = 0.142$ in.^{2} for the 1/2 in. threaded bolt (Table 34-2)
$f_{ut} = 58{,}000$ psi

Per ASTM F 1554 Grade 36 has a specified minimum yield strength of 36 ksi and a specified tensile strength of 58-80 ksi (see Table 34-1). For design purposes, the minimum tensile strength of 58 ksi should be used.

Note: Per D5.1.2, f_{ut} shall not be taken greater than $1.9f_y$ or 125,000 psi. For ASTM F 1554 Grade 36, $1.9f_y = 1.9(36{,}000) = 68{,}400$ psi. Therefore, use the specified minimum f_{ut} of 58,000 psi.

Substituting, V_u as controlled by steel strength is:

$V_u = \phi V_s = 0.65\ (1)(0.6)(0.142)(58{,}000) = 3212$ lb

3. Determine V_u for embedment strength governed by concrete breakout strength with shear directed toward a free edge — *D.6.2*

$\phi V_{cb} \geq V_u$ — *Eq. (D-2)*, *D.4.1.2*

where:

$\phi = 0.70$ — *D.4.4(c)i*

No supplementary reinforcement has been provided (i.e., hairpin type reinforcement surrounding the anchor and anchored into the concrete).

$$V_{cb} = \frac{A_V}{A_{Vo}}\ \psi_6\ \psi_7\ V_b$$ — *Eq. (D-20)*

where:

$\frac{A_V}{A_{Vo}}$ and ψ_6 terms are 1.0 for single shear anchors not influenced by more than one free edge (i.e., the member thickness is greater than $1.5c_1$ and the distance to an orthogonal edge c_2 is greater than $1.5c_1$)

$\psi_7 = 1.0$ for locations where concrete cracking is likely to occur (i.e., the edge of the foundation is susceptible to cracks)

$$V_b = 7\left(\frac{\ell}{d_o}\right)^{0.2}\sqrt{d_o}\,\sqrt{f'_c}\,c_1^{1.5}$$ *Eq. (D-20)*

where:

ℓ = load bearing length of the anchor for shear, not to exceed $8d_o$ *D.0*

For this problem $8d_o$ will control since the embedment depth h_{ef} is 7 in.

$\ell = 8d_o = 8\ (0.5) = 4.0$ in.

To determine V_u for the embedment strength governed by concrete breakout strength Eq. (D-2) can be combined with Eq. (D-20) and Eq. (D-23) to give:

$$V_u = \phi V_{cb} = \phi\ \frac{A_V}{A_{Vo}}\psi_6\,\psi_7\,7\left(\frac{\ell}{d_o}\right)^{0.2}\sqrt{d_o}\,\sqrt{f'_c}\,c_1^{1.5}$$

Substituting, V_u for the embedment strength as controlled by concrete breakout strength is:

$$V_u = \phi V_{cb} = 0.70(1.0)(1.0)(1.0)(7)\left(\frac{8\ (0.5)}{(0.5)}\right)^{0.2}\sqrt{0.5}\ \sqrt{4000}\ (1.75)^{1.5} = 769\text{ lb}$$

4. Determine V_u for embedment strength governed by concrete pryout strength *D.6.3*

Note: The pryout failure mode is normally only a concern for shallow, stiff anchors. Since this example problem addresses both shear directed toward the free edge and shear directed inward from the free edge, the pryout strength will be evaluated.

$\phi V_{cp} \geq V_u$ *Eq. (D-2)* *D.4.1.2*

where:

$\phi = 0.70$ – Condition B applies for pryout strength in all cases *D.4.4(c)i*

$V_{cp} = k_{cp}\,N_{cb}$ *Eq. (D-28)*

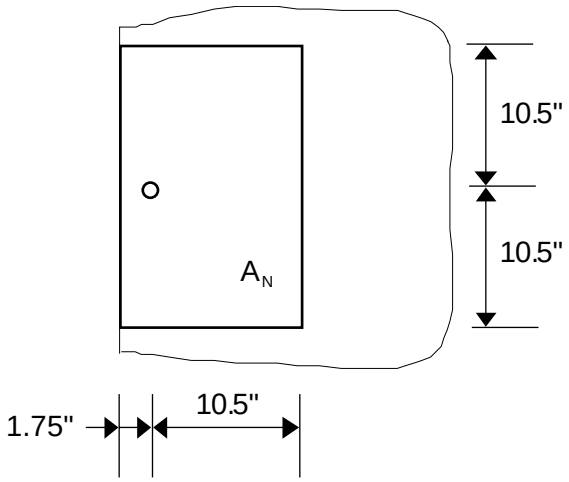
where:

$k_{cp} = 2.0$ for $h_{ef} \geq 2.5$ in.

$$N_{cb} = \frac{A_N}{A_{No}}\ \psi_2\ \psi_3\ N_b$$ *Eq. (D-4)*

Evaluate the terms of Eq. (D-4) for this problem:

A_N is the projected area of the tensile failure surface as approximated by a rectangle with edges bounded by 1.5 h_{ef} (1.5 x 7 = 10.5 in.) and free edges of the concrete from the centerline of the anchor.



A_N = (1.75 + 10.5)(10.5 + 10.5) = 257 in.^{2}

$A_{No} = 9\ h_{ef}^2 = 9\ (7.0)^2 = 441$ in.^{2} *Eq. (D-6)*

Determine ψ_2: *D.5.2.5*

$$\psi_2 = 0.7 + 0.3\frac{c_{min}}{1.5 h_{ef}}$$ *Eq. (D-11)*

$$\psi_2 = 0.7 + 0.3\frac{1.75}{1.5(7.0)} = 0.75$$

Determine ψ_3: *D.5.2.6*

ψ_3 = 1.0 for locations where concrete cracking is likely to occur (i.e., the edge of the foundation is susceptible to cracks)

Determine N_b for the fastening: *D.5.2.2*

$$N_b = 24\sqrt{f_c'}\ h_{ef}^{1.5} = 24\sqrt{4000}\left(7.0\right)^{1.5} = 28{,}112 \text{ lb}$$ *Eq. (D-7)*

Substituting into Eq. (D-4):

$$N_{cb} = \left[\frac{257}{441}\right](0.75)\ (1.0)\ (28{,}112) = 12{,}287 \text{ lb}$$

To determine V_u for the embedment strength governed by pryout strength Eq. (D-2) can be combined with Eq. (D-28) to give:

$V_u = \phi V_{cp} = \phi k_{cp} N_{cb}$

Substituting, V_u for the embedment strength governed by pryout is:

$V_u = \phi V_{cp} = 0.70\ (2.0)(12{,}287) = 17{,}202$ lb

5. Required edge distances, spacings, and thickness to preclude splitting failure — *D.8*

Since a headed anchor used to attach wood frame construction is not likely to be torqued significantly, the minimum cover requirements of 7.7 apply.

Per 7.7 the minimum clear cover for a 1/2 in. bar is 1-1/2 in. when exposed to earth or weather. The clear cover provided for the bolt is exactly 1-1/2 in. (1-3/4 in. to bolt centerline less one half bolt diameter). Note that the bolt head will have slightly less cover (1-3/16 in. for a hex head) say O.K. (note that this is within the minus 3/8 in. tolerance allowed for cover) — *7.7*, *7.5.2.1*

6. Summary:

The factored shear load ($V_u = \phi V_n$) based on steel strength and embedment strength (concrete breakout and pryout) can be summarized as:

Steel strength, (ϕV_s):	3212 lb
Embedment strength – concrete breakout, (ϕV_{cb}):	769 lb ← controls
Embedment strength – pryout, (ϕV_{cp}):	17,202 lb

In accordance with 9.2 the load factor for wind load is 1.6:

$$V_{service} = \frac{V_u}{1.6} = \frac{769}{1.6} = 481 \text{ lb}$$ — *9.2*

The reversible service load shear strength from wind load of the IBC 2000 Section 2308.6 minimum foundation connection for conventional wood-frame construction (1/2 in. diameter bolt embedded 7 in.) is 481 lb per bolt. The strength of the attached member (i.e., the 2x4 sill plate) also needs to be evaluated.

Note that this embedment strength is only related to the anchor being installed in concrete with a specified compressive strength of 4000 psi. In many cases, concrete used in foundations such as this is specified at 2500 psi, the minimum strength permitted by the code. Since the concrete breakout strength controlled the strength of the connection, a revised strength based on using 2500 psi concrete rather than the 4000 psi concrete used in the example can be determined as follows:

$$V_{service@2500} = 481\frac{\sqrt{2500}}{\sqrt{4000}} = 380 \text{ lb}$$

Alternate design using Table 34-6B.

Note: Step numbers correspond to those in the main example above, but prefaced with "A".

Table 34-6B has been selected because it contains design shear strength values based on concrete with f'_c = 4000 psi. Table Note 5 indicates that the values in the table are based on Condition B (no supplementary reinforcement), and Note 6 indicates that cracked concrete was assumed.

A2. Determine V_u as controlled by the anchor steel — *D.6.1, Eq. (D-18)*

From Step 2, use ASTM F 1554, Grade 36 headed bolt with f_{ut} = 58,000 psi. From Table 34-6B, for specified compressive strength of concrete, f'_c, = 4000 psi, determine the design shear strength, ϕV_s, for a 1/2-in. bolt.

ϕV_s = 3,212 lb

A3. Determine V_u for embedment strength governed by concrete breakout strength with shear directed toward a free edge — *D.6.2, Eq. (D-20)*

Determine the design concrete breakout strength in shear, ϕV_{cb}, based on 7-in. embedment, and an edge distance, c_1, of 1-3/4 in. In the table c_1, is a function of embedment depth, h_{ef}. Therefore, the edge distance is:

$c_1 = c_1/h_{ef} = 1.75/7 = 0.25h_{ef}$

From table, the design concrete breakout strength in shear is,

ϕV_{cb} = 769 lb

A4. Determine V_u for embedment strength governed by concrete pryout strength — *D.6.3*

Determine the design concrete pryout strength in shear, ϕV_{cp}, based on 7-in. embedment, and an edge distance of 1-3/4 in. This cannot be determined from Table 34-6B; however, since

$\phi V_{cp} = \phi k_{cp} N_{cb}$ — *Eq. (D-28)*

where k_{cp} = 2, since h_{ef} > 2.5 in., and N_{cb} can be determined from Table 34-5B.

Note that the values in Tables 34-5 and 34-6 are design strengths, thus they include the strength reduction factor, ϕ. Since Table 34-5B is based on Condition B (with no supplementary reinforcement), the ϕ-value used for the concrete tensile strength calculations was 0.70, which is the same as to be used to determine the concrete pryout strength in shear. Therefore, the design concrete breakout strength, ϕN_{cb}, value from Table 34-5B can be used above without adjustment. From Table 34-5B, for an edge distance, c, equal to $0.25h_{ef}$

ϕN_{cb} = 8609 lb

Substituting in Equation (D-28)

$\phi V_{cp} = k_{cp}\phi N_{cb} = (2)(8609) = 17{,}218$ lb

Note that the above value differs slightly from that obtained in Step #4 above. The table values are more precise due to rounding that occurred in the long-hand calculations.

A5. Required edge distances, spacings, and thickness to preclude splitting failure

See Step 5 above.

A6. Determine service wind shear load:

The factored shear load ($V_u = \phi V_n$) based on steel strength and embedment strength (concrete breakout and pryout) can be summarized as:

Steel strength, (ϕV_s):	3212 lb
Embedment strength - concrete breakout, (ϕV_{cb}):	769 lb← controls
Embedment strength - pryout, (ϕV_{cp}):	17,218 lb

From this point, the allowable service wind load shear capacity of the 1/2 in. anchor is determined as in Step 6 above.

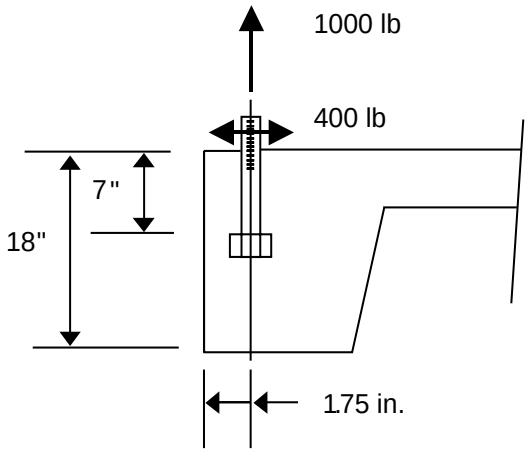
Example 34.5—Single Headed Bolt in Tension and Shear Near an Edge

Determine if a single 1/2 in. diameter hex headed anchor with a 7 in. embedment installed with its centerline 1-3/4 in. from the edge of a concrete foundation is adequate for a service tension load from wind of 1000 lb and reversible service shear load from wind of 400 lb.

Note: This is an extension of Example 34.4 that includes a tension load on the fastener as well as a shear load.

f'_c = 4000 psi

ASTM F 1554 Grade 36 hex head anchor



Calculations and Discussion	Code Reference

1. Determine the factored design loads — *9.2*

   N_u = 1.6 (1000) = 1600 lb

   V_u = 1.6 (400) = 640 lb

2. This is a tension/shear interaction problem where values for both the design tensile strength (ϕN_n) and design shear strength (ϕV_n) will need to be determined. ϕN_n is the smallest of the design tensile strengths as controlled by steel (ϕN_s), concrete breakout (ϕN_{cb}), pullout (ϕN_{pn}), and side-face blowout (ϕN_{sb}). ϕV_n is the smallest of the design shear strengths as controlled by steel (ϕV_s), concrete breakout (ϕV_{cb}), and pryout (ϕV_{cp}). — *D.7*

3. Determine the design tensile strength (ϕN_n) — *D.5*

   a. Steel strength, (ϕN_s): — *D.5.1*

      $\phi N_s = \phi \; n \; A_{se} \; f_{ut}$ — *Eq. (D-3)*

      where:

      ϕ = 0.75 — *D.4.4(a)i*

      Per the Ductile Steel Element definition in D.1, ASTM F 1554 Grade 36 steel qualifies as a ductile steel element.

A_{se} = 0.142 in.^{2} (see Table 34-2)

f_{ut} = 58,000 psi (see Table 34-1)

Substituting:

ϕN_s = 0.75 (1) (0.142) (58,000)) = 6177 lb

b. Concrete breakout strength (ϕN_{cb}): *D.5.2*

Since no supplementary reinforcement has been provided, ϕ = 0.70 *D.4.4(c)ii*

In the process of calculating the pryout strength for this fastener in Example 34.4 Step 4, N_{cb} for this fastener was found to be 12,287 lb

ϕN_{cb} = 0.70 (12,287) = 8601 lb

c. Pullout strength (ϕN_{pn}) *D.5.3*

$\phi N_{pn} = \phi \psi_4 N_p$ *Eq. (D-12)*

where:

ϕ = 0.70 – Condition B applies for pullout strength in all cases *D.4.4(c)ii*

ψ_4 = 1.0, cracking may occur at the edges of the foundation *D.5.3.6*

$N_p = A_{brg}\ 8\ f'_c$ *Eq. (D-13)*

A_{brg} = 0.291 in.^{2}, for 1/2 in. hex head bolt (see Table 34-2)

Pullout Strength (ϕN_{pn})

ϕN_{pn} = 0.70 (1.0) (0.291) (8) (4000) = 6518 lb

d. Concrete side-face blowout strength (ϕN_{sb}) *D.5.4*

The side-face blowout failure mode must be investigated when the edge distance (c) is less than 0.4 h_{ef} *D.5.4.1*

0.4 h_{ef} = 0.4 (7) = 2.80 in. > 1.75 in.

Therefore, the side-face blowout strength must be determined

$$\phi N_{sb} = \phi \left(160\ c \sqrt{A_{brg}}\ \sqrt{f'_c} \right)$$ *Eq. (D-15)*

where:

ϕ = 0.70, no supplementary reinforcement has been provided — *D.4.4(c)ii*

c = 1.75 in.

A_{brg} = 0.291 in.^{2}, for 1/2 in. hex head bolt (see Table 34-2)

Substituting:

$$\phi N_{sb} = 0.70\left(160\,(1.75)\sqrt{0.291}\sqrt{4000}\right) = 6687 \text{ lb}$$

Summary of steel strength, concrete breakout strength, pullout strength, and side-face blowout strength for tension:

Steel strength, (ϕN_s):	6177 lb ← controls	*D.5.1*
Embedment strength – concrete breakout, (ϕN_{cb}):	8601 lb	*D.5.2*
Embedment strength – pullout, (ϕN_{pn}):	6518 lb	*D.5.3*
Embedment strength – side-face blowout, (ϕN_{sb}):	6687 lb	*D.5.4*

Therefore:

ϕN_n = 6,177 lb

4. Determine the design shear strength (ϕV_n) — *D.6*

Summary of steel strength, concrete breakout strength, and pryout strength for shear from Example 35.4:

Steel strength, (ϕV_s):	3212 lb	*D.6.1*
Embedment strength – concrete breakout, (ϕV_{cb}):	769 lb ← controls	*D.6.2*
Embedment strength – pryout, (ϕV_{cp}):	17,202 lb	*D.6.3*

Therefore:

ϕV_n = 769 lb

5. Check tension and shear interaction — *D.7*

If $V_u \le 0.2\phi V_n$ then the full tension design strength is permitted — *D.7.1*

V_u = 640 lb

$0.2\phi V_n = 0.2\,(769) = 154 \text{ lb} < 640 \text{ lb}$

V_u exceeds $0.2\phi V_n$, the full tension design strength is not permitted

If $N_u \le 0.2\,\phi N_n$ then the full shear design strength is permitted — *D.7.2*

N_u = 1600 lb

$0.2\phi N_n = 0.2\,(6177) = 1235$ lb < 1600 lb

N_u exceeds $0.2\phi N_n$, the full shear design strength is not permitted

The interaction equation must be used — *D.7.3*

$$\frac{N_u}{\phi N_n} + \frac{V_u}{\phi V_n} \le 1.2$$ *Eq. (D-29)*

$$\frac{1600}{6177} + \frac{640}{769} = 0.26 + 0.83 = 1.09 < 1.2 \quad \text{O.K.}$$

6. Required edge distances, spacings, and thickness to preclude splitting failure — *D.8*

Since a headed anchor used to attach wood frame construction is not likely to be torqued significantly, the minimum cover requirements of 7.7 apply.

Per 7.7 the minimum clear cover for a 1/2 in. bar is 1-1/2 in. when exposed to earth or weather. The clear cover provided for the bolt is exactly 1-1/2 in. (1-3/4 in. to bolt centerline less one half bolt diameter). Note that the bolt head will have slightly less cover (1-3/16 in. for a hex head) say O.K. (note that this is within the minus 3/8 in. tolerance allowed for cover) — *7.7*, *7.5.2.1*, *D.5*

7. Summary

Use a 1/2 in. diameter ASTM F1554 Grade 36 hex headed anchor embedded 7 in.

Alternate design using Tables 34-5B and 34-6B

Note: Step numbers correspond to those in the main example above, but prefaced with "A".

Tables 34-5 and 34-6 have been selected because they contain design tension and shear values, respectively, based on concrete with f'_c = 4000 psi. Table Notes 4 and 5, respectively, indicate that the values in the tables are based on Condition B (no supplementary reinforcement). Cracked concrete is assumed in both tables (Table 34-5 Notes 6 and 10, and Table 34-6 Note 6).

A3. Determine the design tensile strength (ϕN_n): — *D.5.1*, *Eq. (D-3)*

A3a. Determine the design tensile strength of steel (ϕN_s):

Based on Step 3a, assume an ASTM F 1554, Grade 36 bolt, with a f_{ut} = 58,000 psi.

Using Table 34-6B, under the column for 58,000 a 1/2-in. diameter bolt has a design tensile strength,

ϕN_n, = 6177 lb.

A3b. Determine design concrete breakout strength (ϕN_{cb}): *D.5.2 Eq. (D-4)*

Since breakout strength varies with edge distance for anchors close to an edge ($c < 1.5h_{ef}$), determine the edge distance as a function of embedment depth. Since c = 1-3/4 in.

$c = c/h_{ef} = 1.75/7 = 0.25h_{ef}$

Under column labeled "$0.25h_{ef}$" for a 1/2 in. bolt with 7 in. embedment depth,

ϕN_{cb} = 8609 lb

Note that the above value differs slightly from that obtained in Step 3b above. The table values are more precise due to rounding that occurred in the long-hand calculations.

A3c. Determine design concrete pullout strength (ϕN_{pn}) *D.5.3 Eq. (D-12)*

From the table under the column labeled "head w/o washers" for a 1/2 in. bolt

ϕN_{pn} = 6518 lb

A3d. Determine design concrete side-face blowout strength (ϕN_{sb}) *D.5.4 D.5.4.1*

Side face blowout is not applicable where the edge distance is equal to or greater than $0.4h_{ef}$. In this case edge distance, c, as calculated above is $0.25h_{ef}$; therefore, it must be evaluated. From the table under the column labeled "$0.25h_{ef}$" for a 1/2 in. bolt with 7 in. embedment,

ϕN_{sb} = 6687 lb

Summary of steel strength, concrete breakout strength, pullout strength, and side-face blowout strength for tension:

Steel strength, (ϕN_s):	6177 lb ← controls
Embedment strength – concrete breakout, (ϕN_{cb}):	8609 lb
Embedment strength – pullout, (ϕN_{pn}):	6518 lb
Embedment strength – side-face blowout, (ϕN_{sb}):	6687 lb

Therefore:

ϕN_n = 6177 lb

A4. Determine the design shear strength (ϕV_n)

Summary of steel strength, concrete breakout strength, and pryout strength for shear from Step A6 of Example 34.4, alternate solution using Table 34-6B

Steel strength, (ϕV_s):	3212 lb
Embedment strength - concrete breakout, (ϕV_{cb}):	769 lb ← controls
Embedment strength - pryout, (ϕV_{cp}):	17,218 lb

Therefore:

ϕV_n = 769 lb

A5. Check tension and shear interaction.

See Step 5 above.

A6. Required edge distances, spacings, and thickness to preclude splitting failure

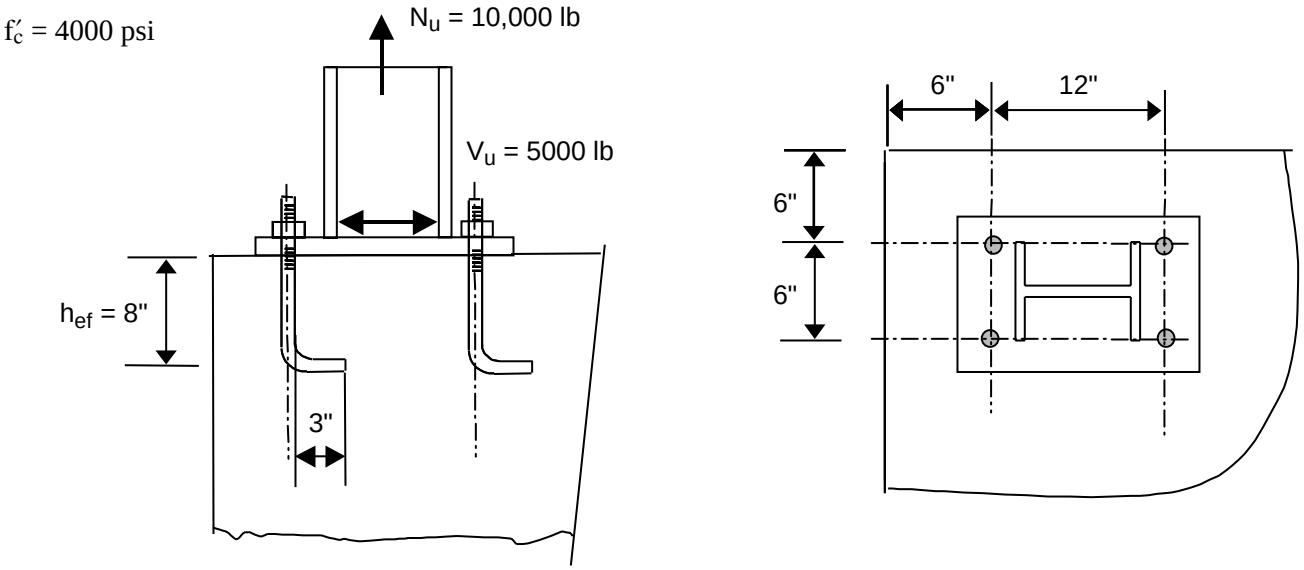
See Step 6 above.

A7. Summary

Use a 1/2 in. diameter ASTM A 307 Grade C hex headed bolt embedded 7 in.

Example 34.6—Group of L-Bolts in Tension and Shear Near Two Edges

Design a group of four L-bolts spaced as shown to support a 10,000 lb factored tension load and 5000 lb reversible factored shear load resulting from wind load. The connection is located at the base of a column in a corner of the building foundation.



Note: OSHA requirements implemented January 18, 2001 require that the column anchorage use a least four anchors and be able to sustain a minimum eccentric gravity load of 300 lb located 18 in. from the face of the extreme outer face of the column in each direction. The load is to be applied at the top of the column. The intent is that the column be able to sustain an iron worker hanging off the side of the top of the column. For more details, refer to http://www.osha-slc.gov/OshStd_data/1926_0755.html.

Calculations and Discussion	Code Reference
1. The solution to this example is found by assuming the size of the anchors, then checking compliance with the design provisions. Try four 5/8 in. ASTM F 1554 Grade 36 L-bolts with h_{ef} = 8 in. and a 3 in. extension, e_h, as shown in the figure.	
2. This is a tension/shear interaction problem where values for both the design tensile strength (ϕN_n) and design shear strength (ϕV_n) will need to be determined. ϕN_n is the smallest of the design tensile strengths as controlled by steel (ϕN_s), concrete breakout (ϕN_{cb}), pullout (ϕN_{pn}), and side-face blowout (ϕN_{sb}). ϕV_n is the smallest of the design shear strengths as controlled by steel (ϕV_s), concrete breakout (ϕV_{cb}), and pryout (ϕV_{cp}).	*D.7*
3. Determine the design tensile strength (ϕN_n)	*D.5*
a. Steel strength, (ϕN_s):	*D.5.1*

$\phi N_s = \phi \, n \, A_{se} \, f_{ut}$ *Eq. (D-3)*

where:

$\phi = 0.75$ *D.4.4(a)i*

Per Table 34-1, the ASTM F 1554 Grade 36 L-bolt meets the Ductile Steel Element definition of D.1.

$A_{se} = 0.226$ in.^{2} (see Table 34-2)

$f_{ut} = 58{,}000$ psi (see Table 34-1)

Substituting:

$\phi N_s = 0.75 \, (4) \, (0.226) \, (58{,}000) = 39{,}324$ lb

b. Concrete breakout strength (ϕN_{cbg}): *D.5.2*

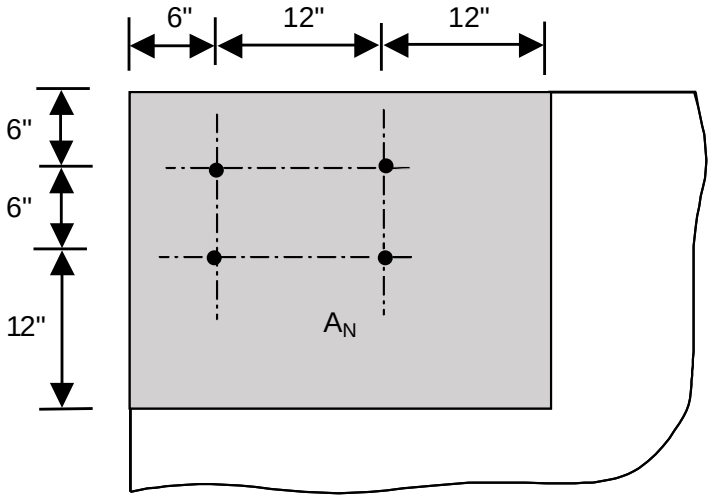
Since the spacing of the anchors is less than 3 times the effective embedment depth h_{ef} (3 x 8 = 24), the anchors must be treated as an anchor group. *D.1*

$$\phi N_{cbg} = \phi \frac{A_N}{A_{No}} \psi_1 \, \psi_2 \, \psi_3 \, N_b$$ *Eq. (D-5)*

Since no supplementary reinforcement has been provided, $\phi = 0.70$ *D.4.4(c)ii*

Determine A_N and A_{No}: *D.5.2.1*

A_N is the projected area of the failure surface as approximated by a rectangle with edges bounded by 1.5 h_{ef} (1.5 x 8.0 = 12.0 in. in this case) and free edges of the concrete from the centerlines of the anchors.



$A_N = (6 + 12 + 12)(6 + 6 + 12) = 720$ in.^{2}

$A_{No} = 9\ h_{ef}^2 = 9\ (8)^2 = 576$ in.^{2} — *Eq. (D-6)*

Check: $A_N \le nA_{No}$ $\quad 720 < 4(576)$ $\quad$ O.K.

Determine ψ_1: — *D.5.2.4*

$\psi_1 = 1.0$ (no eccentricity in the connection)

Determine ψ_2 ($c_{min} < 1.5\ h_{ef}$ $\quad 6 < (1.5\ (8))$): — *D.5.2.5*

$$\psi_2 = 0.7 + 0.3\frac{c_{min}}{1.5 h_{ef}}$$ — *Eq. (D-11)*

$$\psi_2 = 0.7 + 0.3\frac{6.0}{1.5\ (8.0)} = 0.85$$

Determine ψ_3: — *D.5.2.6*

$\psi_3 = 1.0$ for locations where concrete cracking is likely to occur (i.e., the edge of the foundation)

Determine N_b: — *D.5.2.2*

$$N_b = 24\ \sqrt{f'_c}\ h_{ef}^{1.5} = 24\ \sqrt{4000}\ (8.0)^{1.5} = 34{,}346 \text{ lb}$$ — *Eq. (D-7)*

Substituting into Eq. (D-5):

$$\phi N_{cbg} = 0.70\left[\frac{720}{576}\right](1.0)\ (0.85)\ (1.0)\ (34{,}346) = 25{,}545 \text{ lb}$$

c. Pullout strength (ϕN_{pn}) — *D.5.3*

$\phi N_{pn} = \phi\ \psi_4\ N_p$ — *Eq. (D-12)*

where:

$\phi = 0.70$, Condition B always applies for pullout strength — *D.4.4(c)ii*

$\psi_4 = 1.0$, cracking may occur at the edges of the foundation — *D.5.3.6*

N_p for the L-bolts:

$N_p = 0.9\ f'_c\ e_h\ d_o$ — *Eq. (D-14)*

e_h = maximum effective value of $4.5d_o$ = 4.5 (0.625) = 2.81 in.

$e_{h,provided}$ = 3 in. > 2.81 in., therefore use $e_h = 4.5d_o$ = 2.81 in. *D.5.3.5*

Substituting into Eq. (D-12) and Eq. (D-14) with 4 L-bolts (ϕN_{pn})

ϕN_{pn} = 4 (0.70) (1.0) (0.9) (4000) (2.81) (0.625) = 17,703 lb

Note: If 5/8 in. hex head bolts where used ϕN_{pn} would be significantly increased as shown below:

N_p for the hex head bolts:

$N_p = A_{brg}\ 8\ f'_c$ *Eq. (D-13)*

A_{brg} = 0.454 in.^{2}, for 5/8 in. hex head bolt (see Table 34-2)

Substituting into Eq. (D-12) and Eq. (D-13) with 4 bolts (ϕN_{pn})

ϕN_{pn} = 4 (0.70) (1.0) (0.454) (8) (4000) = 40,678 lb

The use of hex head bolts would increase the pullout capacity by a factor of 2.3 over that of the L-bolts.

d. Concrete side-face blowout strength (ϕN_{sb}) *D.5.4*

The side-face blowout failure mode must be investigated for headed anchors where the edge distance (c) is less than 0.4 h_{ef}. Since L-bolts are used here the side face blowout failure is not applicable. The calculation below is simply to show that if headed anchors were used the anchors are far enough from the edge that the side-face blowout strength is not applicable. *D.5.4.1*

0.4 h_{ef} = 0.4 (8) = 3.2 in. < 6.0 in.

Therefore, the side-face blowout strength is not applicable (N/A).

Summary of design strengths based on steel strength, concrete breakout strength, pullout strength, and side-face blowout strength for tension:

Steel strength, (ϕN_s):	39,324 lb	*D.5.1*
Embedment strength - concrete breakout, (ϕN_{cbg}):	25,545 lb	*D.5.2*
Embedment strength - pullout, (ϕN_{pn}):	17,703 lb ← controls	*D.5.3*
Embedment strength - side-face blowout, (ϕN_{sb}):	N/A	*D.5.4*

Therefore:

ϕN_n = 17,703 lb

Note: If hex head bolts were used the concrete breakout strength of 25,545 lb would control rather than the L-bolt pullout strength of 17,703 lb (i.e., 44% higher tensile capacity if hex head bolts were used).

4. Determine the design shear strength (ϕV_n) *D.6*

a. Steel strength, (ϕV_s): *D.6.1*

$\phi V_s = \phi \; n \; 0.6 \; A_{se} \; f_{ut}$ *Eq. (D-18)*

where:

$\phi = 0.65$ *D.4.4(a)ii*

Per Table 34-1, the ASTM F 1554 Grade 36 meets the Ductile Steel Element definition of Section D.1.

$A_{se} = 0.226$ in.^{2} (see Table 34-2)

$f_{ut} = 58{,}000$ psi (see Table 34-1)

Substituting:

$\phi V_s = 0.65 \; (4) \; (0.6) \; (0.226) \; (58{,}000) = 20{,}448$ lb

b. Concrete breakout strength (ϕV_{cbg}): *D.6.2*

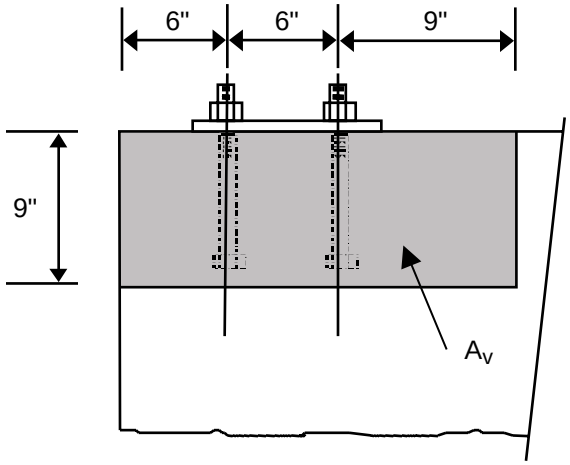
The two anchors located near the free edge toward which the shear is directed (when the shear acts from right to left) will control the concrete breakout strength. These anchors are assumed to carry one-half of the shear (see Fig. RD.6.2.1(b) upper right). The total breakout strength for shear will be taken as twice the value calculated for these two anchors. The reason for this is that although the two anchors away from the free edge may be able to develop a higher breakout strength, they will not have the opportunity to develop this strength since the two anchors nearest the edge will fail first. For the case of welded studs, the entire shear force may be assumed to be located at the two anchors farthest from the edge (see Fig. RD.6.2.1(b) lower right), however this assumption is not permitted for cast-in-place anchors that are installed through holes in the attached base plate. *RD.6.2.1*

$$\phi V_{cbg} = \phi \frac{A_V}{A_{Vo}} \psi_5 \; \psi_6 \; \psi_7 \; V_b$$ *Eq. (D-21)*

Since no supplementary reinforcement has been provided, $\phi = 0.70$ *D.4.4(c)i*

Determine A_V and A_{Vo}: *D.6.2.1*

A_V is the projected area of the shear failure surface on the free edge toward which shear is directed. The projected area is determined by a rectangle with edges bounded by 1.5 c_1 (1.5 x 6.0 = 9.0 in. in this case) and free edges of the concrete from the centerlines of the anchors and the surface of the concrete. Although the 1.5 c_1 distance is not specified in D.6.2.1, it is shown in Fig. RD.6.2.1(b).



$A_V = (6+6+9)(9) = 189$ in.^{2}

$A_{Vo} = 4.5\ c_1^2 = 4.5\ (6)^2 = 162$ in.^{2} *Eq. (D-22)*

Check: $A_V \le nA_{Vo}$ $189 < 2(162)$ O.K.

Determine ψ_5: *D.6.2.5*

$\psi_5 = 1.0$ (no eccentricity in the connection)

Determine ψ_6 ($c_2 < 1.5\ c_1$ $6 < [(1.5)\ (6)]$): *D.6.2.6*

$$\psi_6 = 0.7 + 0.3\frac{c_2}{1.5\ c_1}$$ *Eq. (D-27)*

$$\psi_6 = 0.7 + 0.3\frac{6.0}{1.5(6.0)} = 0.90$$

Determine ψ_7: *D.6.2.7*

$\psi_7 = 1.0$ for locations where concrete cracking is likely to occur (i.e., the edge of the foundation)

Determine V_b for one anchor:

$$V_b = 7\left(\frac{\ell}{d_o}\right)^{0.2}\sqrt{d_o}\ \sqrt{f_c'}\ c_1^{1.5}$$ *Eq. (D-23)*

where:

ℓ = load bearing length of the anchor for shear, not to exceed $8d_o$ — *D.0*

For this problem $8d_o$ will control:

$\ell = 8d_o = 8\ (0.625) = 5.0$ in. < 8 in. therefore, use $\ell = 8d_o = 5.0$ in.

Substituting into Eq. (D-23):

$$V_b = (7)\left(\frac{5.0}{0.625}\right)^{0.2} \sqrt{0.625}\ \sqrt{4000}\ \ 6.0^{1.5} = 7797 \text{ lb}$$

Substituting into Eq. (D-21) the design breakout strength of the two anchors nearest the edge toward which the shear is directed is:

$$\phi V_{cbg} = 0.70\left(\frac{189}{162}\right)(1.0)\ (0.90)\ (1.0)\ (7797) = 5731 \text{ lb}$$

The total breakout shear strength of the four anchor group is:

$\phi V_{cbg} = 2\ (5731) = 11{,}462$ lb

c. Pryout strength (ϕV_{cp}) — *D.6.3*

Note: The pryout failure mode is normally only a concern for shallow, stiff anchors. Since this example problem addresses both shear directed toward the free edge and shear directed away from the free edge, the pryout strength will be evaluated.

$\phi V_{cp} = \phi\ k_{cp}\ N_{cb}$ — *Eq. (D-28)*

where:

$\phi = 0.70$, Condition B always applies for pryout strength — *D.4.4(c)i*

$k_{cp} = 2.0$ for $h_{ef} \geq 2.5$ in.

$$N_{cb} = \frac{A_N}{A_{No}}\ \psi_2\ \psi_3\ N_b$$ — *Eq. (D-4)*

From Step 3(b) above and recognizing that $N_{cb} = N_{cbg}$ if no tension load eccentricity is present (i.e., $\psi_1 = 1.0$):

$$N_{cb} = \left[\frac{720}{576}\right](0.85)\,(1.0)\,(34{,}346) = 36{,}493 \text{ lb}$$

Substituting into Eq. (D-28):

ϕV_{cp} = 0.70 (2.0) (36,493) = 51,090 lb

Summary of design strengths based on steel strength, concrete breakout strength, and pryout strength for shear:

Steel strength, (ϕV_s):	20,448 lb	*D.6.1*
Embedment strength - concrete breakout, (ϕV_{cbg}):	11,462 lb ← controls	*D.6.2*
Embedment strength - pryout, (ϕV_{cp}):	51,090 lb	*D.6.3*

Therefore:

ϕV_n = 11,462 lb

5. Check tension and shear interaction *D.7*

If $V_u \le 0.2\phi V_n$ then the full tension design strength is permitted *D.7.1*

V_u = 5000 lb

$0.2\phi V_n$ = 0.2 (11,462) = 2,292 lb < 5000 lb

V_u exceeds $0.2\phi V_n$, the full tension design strength is not permitted

If $N_u \le 0.2\phi N_n$ then the full shear design strength is permitted *D.7.2*

N_u = 10,000 lb

$0.2\phi N_n$ = 0.2 (17,703) = 3544 lb < 10,000 lb

N_u exceeds $0.2\phi N_n$, the full shear design strength is not permitted

The interaction equation must be used. *D.7.3*

$$\frac{N_u}{\phi N_n} + \frac{V_u}{\phi V_n} \le 1.2 \qquad \textit{Eq. (D-29)}$$

$$\frac{10{,}000}{17{,}703} + \frac{5000}{11{,}462} = 1.00 < 1.2 \quad \text{O.K.}$$

6. Required edge distances, spacings, an thicknesses to preclude splitting failure — *D.8*

   Since cast-in-place L-bolts are not likely to be highly torqued, the minimum cover requirements of 7.7 apply.

   Per 7.7 the minimum clear cover for a 5/8 in. bar is 1-1/2 in. when exposed to earth or weather. The clear cover provided for the bolt exceeds this requirement with the 6 in. edge distance to the bolt centerline – O.K.

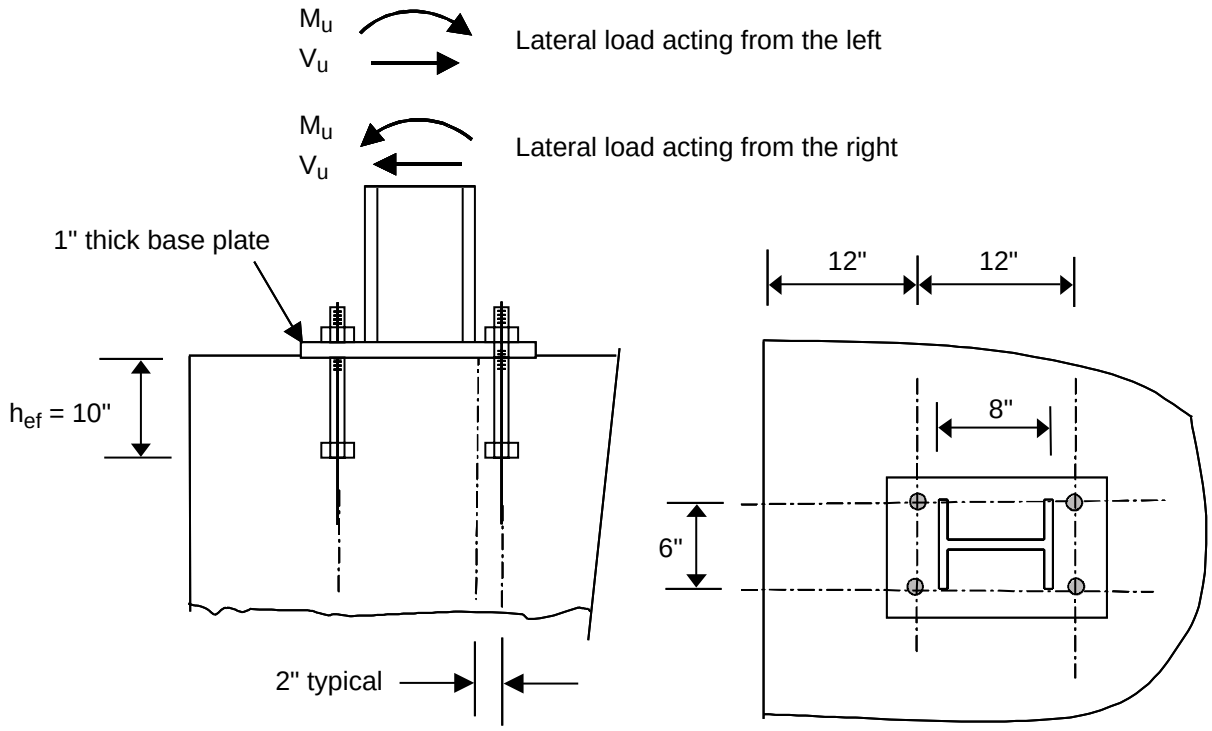
7. Summary

   Use 5/8 in. diameter ASTM F 1554 Grade 36 L-bolts with an embedment of 8 in. (measured to the upper surface of the L) and a 3 in. extension, e_h, as shown in the figure.

   Note: The use of hex head bolts rather than L-bolts would significantly increase the tensile strength of the connection. If hex head bolts were used, the design tensile strength would increase from 17,719 lb as controlled by the pullout strength of the L-bolts to 25,545 lb as controlled by concrete breakout for hex head bolts.

Example 34.7—Group of Headed Bolts in Moment and Shear Near an Edge in a Region of Moderate or High Seismic Risk

Design a group of four headed anchors spaced as shown for a reversible 19.5 k-ft factored moment and a 5.0 kip factored shear resulting from lateral seismic load in a region of moderate or high seismic risk. The connection is located at the base of an 8 in. steel column. f'_c = 4000 psi



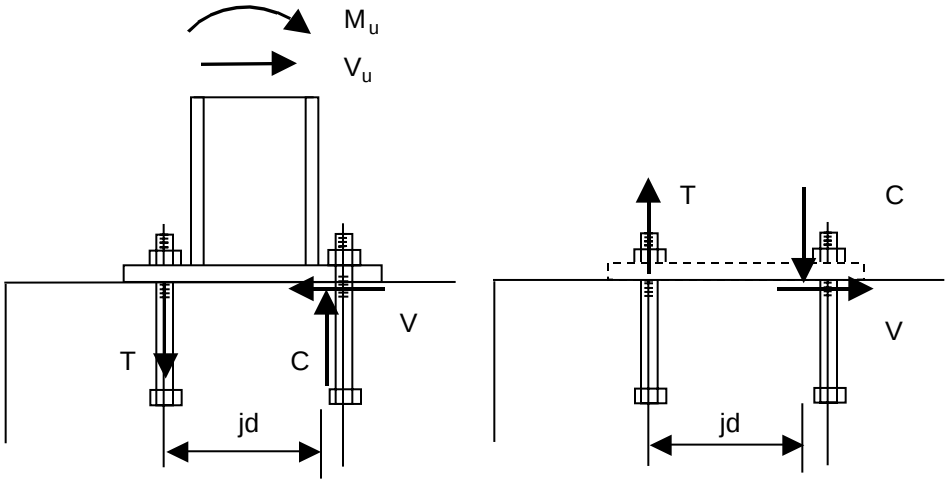
Calculations and Discussion	Code Reference

1. The solution to this example is found by assuming the size of the anchors, then checking for compliance with the design provisions for seismic loadings in regions of moderate or high seismic risk. For this example, assume four 3/4 in. ASTM F 1554 Grade 36 hex head anchors with h_{ef} = 10 in.

2. Since this connection is subjected to seismic load in a region of moderate or high seismic risk, the design tensile strength is $0.75\phi N_n$ and design shear strength is $0.75\phi V_n$. Unless the attachment has been designed to yield at a load lower than the design strength of the anchors (including the 0.75 factor), the strength of the anchors must be controlled by the tensile and shear strength of ductile steel elements. The design tensile strength ($0.75\phi N_n$) and design shear strength ($0.75\phi V_n$) will need to be D.4.1.2 determined. ϕN_n is the smallest of the design tensile strengths as controlled by steel (ϕN_s), concrete breakout (ϕN_{cb}), pullout (ϕN_{pn}), and side-face blowout (ϕN_{sb}). ϕV_n is the smallest of the design shear strengths as controlled by steel (ϕV_s), concrete breakout (ϕV_{cb}), and pryout (ϕV_{cp}). *D.3.3*

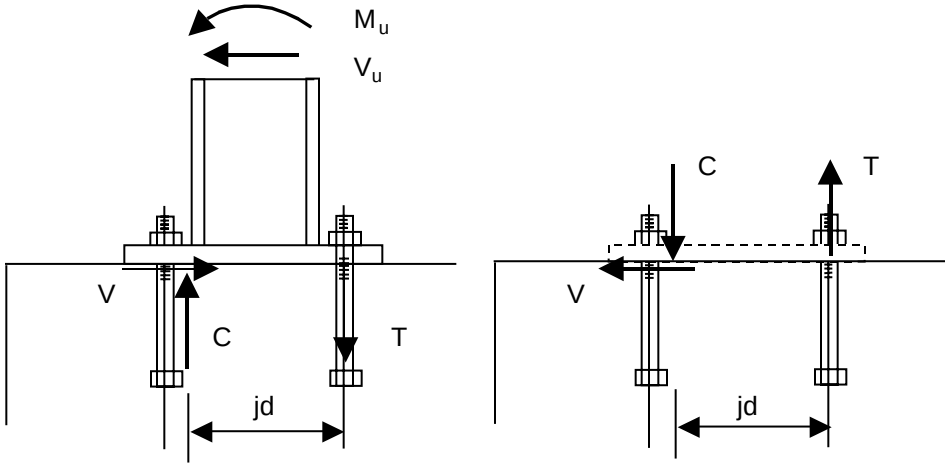
3. This problem involves the design of the connection of the steel column to the foundation for lateral loads coming from either the left or the right of the structure as shown below:

Lateral load acting from the left:



Resisting forces | Forces on embedment

Lateral load acting from the right:



Resisting forces | Forces on embedment

As shown in the figures above, due to the free edge on the left, the critical case for tension on the anchors occurs when the lateral load is acting from the left while the critical case for shear occurs when the lateral load is acting from the right.

4. Distribution of the applied moment and shear loads to the anchors

Tension in the anchors resulting from the applied moment – The exact location of the compressive resultant from the applied moment cannot be accurately determined *D.3*

by traditional concrete beam analysis methods. This is true for both the elastic linear stress-strain method (i.e., the transformed area method) and the ACI 318 stress block (strength design) method since plane sections do not remain plane and different cross-sections and materials are utilized on each side of the connection. Although these methods may be used they require additional analysis that is simply not warranted and in some cases can yield unconservative results for the location of the compressive resultant if relatively flexible base plates are used. References D.4, D.5, and D.6 provide further information. For design purposes, the compressive resultant from applied moment may be conservatively assumed to be located one base plate thickness away from the compression element of the attached member.

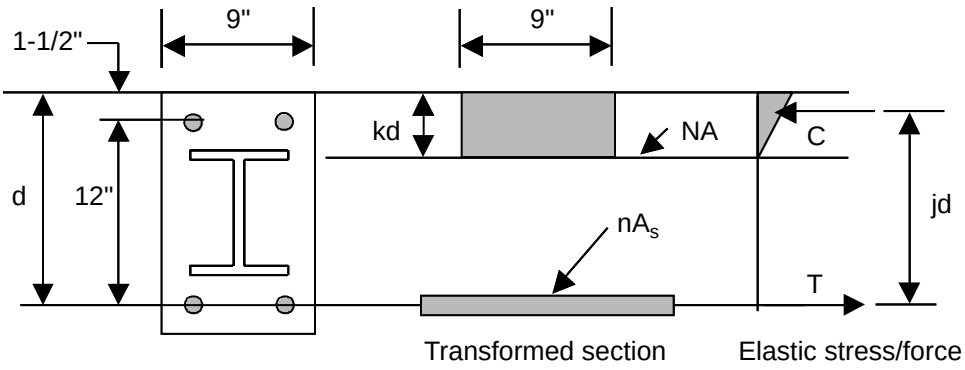
For this example, the estimate of the internal moment arm jd will be determined using both the simplified method based on the compressive resultant being located one base plate thickness from the compression element of the attached member and the transformed area method based on elastic analysis of a concrete beam. Conservatively, the smaller value of the internal moment arm jd will be used to verify the design.

Using the simplified, conservative method of assuming that the compressive resultant is located one base plate thickness (1 in.) away from the compression element of the 8 in. deep attached member:

jd = 2 + 8 + 1 = 11 in.

This value for the internal moment arm jd is simply the addition of the 2 in. between the tension anchors and the attached member, the 8 in. depth of the member and the 1 in. plate thickness.

Now for the more difficult approach. Assuming 1-1/2 in. edge distances from the centerline of the anchors to the edges of the base plate, the internal moment arm jd may be determined using the conventional transformed area method based on elastic analysis as follows:



Determine kd by summing moments about the neutral axis (NA) for the transformed section:

$$9\,kd\,\frac{kd}{2} = n\,A_s\,(d - kd)$$

where:

$$n = \frac{E_s}{E_c} = \frac{29{,}000{,}000}{57{,}000\sqrt{4000}} = 8.0$$

A_s = 2(0.334) = 0.668 in^2 (see Table 34-2)

d = 1.5 + 12 = 13.5"

Substituting and solving the quadratic equation for kd:

4.5 kd^2 = (8.0) (0.668) (13.5 – kd)

kd = 3.45 in.

Therefore, the internal moment arm jd using elastic beam theory is:

$$jd = d - \frac{kd}{3} = 13.5 - \frac{3.45}{3} = 12.4 \text{ in.}$$

Conservatively use jd = 11 in. as determined by simply assuming the compressive reaction to be located one base plate thickness away from the compression element of the attached member. Note that although the elastic beam analysis of the connection is included in this example, it is cumbersome and can lead to an overestimation of the internal moment arm jd for flexible base plates. See References D.4, D.5, and D.6 for further information.

By summing moments about the location of the compressive resultant (see figures in Step 3):

M_u = T (jd)

where:

M_u = 19.5 k-ft = 234,000 in.-lb

T = N_u (i.e., the factored tensile load acting on the anchors in tension)

jd = 11 in.

Rearranging and substituting:

$$N_u = \frac{M_u}{jd} = \frac{234{,}000}{11} = 21{,}273 \text{ lb}$$

Shear – Although the compressive resultant from the applied moment will allow for the development of a frictional shear resistance between the base plate and the concrete, the frictional resistance will be neglected for this example and the anchors on the compression side will be designed to transfer the entire shear. References D.4, D.5, and D.6 provide additional information regarding the contribution of friction to the shear strength.

V_u = 5000 lb on the two anchors on the compression side

5. Determine the design tensile strength for seismic load ($0.75\phi N_n$) — *D.5*

a. Steel strength, (ϕN_s): — *D.5.1*

$\phi N_s = \phi\ n\ A_{se}\ f_{ut}$ — *Eq. (D-3)*

where:

$\phi = 0.75$ — *D.4.4(a)i*

Per Table 34-1, the ASTM F 1554 Grade 36 bolt meets the Ductile Steel Element definition of Section D.1.

A_{se} = 0.334 in.^{2} (see Table 34-2)

f_{ut} = 58,000 psi (see Table 34-1)

Substituting:

$\phi N_s = 0.75\ (2)\ (0.334)\ (58{,}000) = 29{,}058$ lb

b. Concrete breakout strength (ϕN_{cbg}): — *D.5.2*

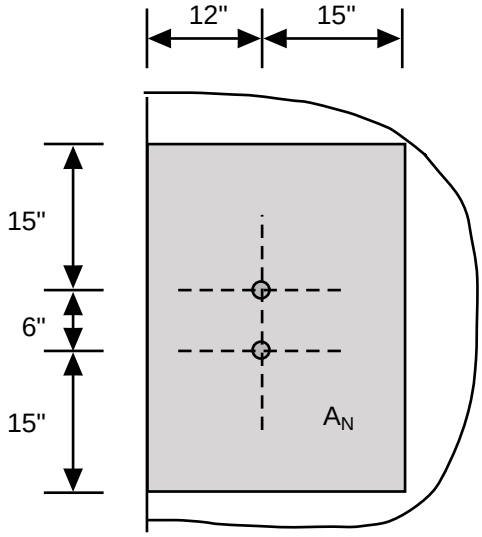
Since the spacing of the anchors is less than 3 times the effective embedment depth h_{ef} (3 x 10 in. = 30 in.), the anchors must be treated as an anchor group. — *D.1*

$$\phi N_{cbg} = \phi \frac{A_N}{A_{No}} \psi_1\ \psi_2\ \psi_3\ N_b$$ — *Eq. (D-5)*

Since no supplementary reinforcement has been provided, $\phi = 0.70$ — *D.4.4(c)ii*

Determine A_N and A_{No}: — *D.5.2.1*

A_N is the projected area of the failure surface as approximated by a rectangle with edges bounded by 1.5 h_{ef} (1.5 x 10.0 = 15.0 in.) and free edges of the concrete from the centerlines of the anchors.



$A_N = (12+15)(15 + 6 + 15) = 972$ in.^{2}

$A_{No} = 9\, h_{ef}^2 = 9\,(10)^2 = 900$ in.^{2} *Eq. (D-6)*

Check: $A_N \le nA_{No}$ $\quad 972 < 2(900)$ $\quad$ O.K.

Determine ψ_1: *D.5.2.4*

$\psi_1 = 1.0$ (no eccentricity in the connection)

Determine ψ_2: *D.5.2.5*

$$\psi_2 = 0.7 + 0.3\frac{c_{min}}{1.5\, h_{ef}}$$ *Eq. (D-11)*

$$\psi_2 = 0.7 + 0.3\frac{12.0}{1.5\,(10.0)} = 0.94$$

Determine ψ_3: *D.5.2.6*

$\psi_3 = 1.0$ for locations where concrete cracking is likely to occur (i.e., the edge of the foundation)

Determine N_b: *D.5.2.2*

$$N_b = 24\sqrt{f_c'}\; h_{ef}^{1.5} = 24\sqrt{4000}\,\left(10.0\right)^{1.5} = 48{,}000 \text{ lb}$$ *Eq. (D-7)*

Substituting into Eq. (D-5):

$$\phi N_{cbg} = 0.70\left[\frac{972}{900}\right](1.0)\,(0.94)\,(1.0)\,(48{,}000) = 34{,}111 \text{ lb}$$

c. Pullout strength (ϕN_{pn}) — *D.5.3*

$\phi N_{pn} = \phi\, \psi_4\, N_p$ — *Eq. (D-12)*

where:

$\phi = 0.70$, Condition B always applies for pullout strength — *D.4.4(c)ii*

$\psi_4 = 1.0$, cracking may occur at the edges of the foundation — *D.5.3.6*

N_p for the hex head bolts:

$N_p = A_{brg}\, 8\, f'_c$ — *Eq. (D-13)*

$A_{brg} = 0.654$ in.^{2}, for 3/4 in. hex head bolt (see Table 34-2)

Substituting into Eq. (D-12) and Eq. (D-13) with 2 bolts (ϕN_{pn})

$\phi N_{pn} = 2\,(0.70)\,(1.0)\,(0.654)\,(8)\,(4000) = 29{,}299$ lb

d. Concrete side-face blowout strength (ϕN_{sb}) — *D.5.4*

The side-face blowout failure mode must be investigated when the edge distance (c) is less than $0.4\, h_{ef}$ — *D.5.4.1*

$0.4\, h_{ef} = 0.4\,(10) = 4.0$ in. < 12.0 in.

Therefore, the side-face blowout strength is not applicable (N/A)

Summary of design strengths based on steel strength, concrete breakout strength, pullout strength, and side-face blowout strength for tension:

Steel strength, (ϕN_s):	29,058 lb ← controls	*D.5.1*
Embedment strength - concrete breakout, (ϕN_{cbg}):	34,111 lb	*D.5.2*
Embedment strength - pullout, (ϕN_{pn}):	29,299 lb	*D.5.3*
Embedment strength - side-face blowout, (ϕN_{sb}):	N/A	*D.5.4*

Therefore:

ϕN_n = 29,058 lb and is controlled by a ductile steel element as required in D.3.3.4

For seismic load in a region of moderate or high seismic risk, the design tensile strength is $0.75\phi N_n$: — *D.3.3.3*

$0.75\ \phi N_n = 0.75\ (29{,}058) = 21{,}794$ lb and is controlled by a ductile steel element

Check if $N_u \le 0.75\phi N_n$

21,273 lb < 21,794 lb O.K. for tension

6. Determine the design shear strength (ϕV_n) — *D.6*

   a. Steel strength, (ϕV_s): — *D.6.1*

   $\phi V_s = \phi\ n\ 0.6\ A_{se}\ f_{ut}$ — *Eq. (D-18)*

   where:

   $\phi = 0.65$ — *D.4.4(a)ii*

   Per Table 34-1, the ASTM F 1554 Grade 36 meets the Ductile Steel Element definition of Section D.1.

   $A_{se} = 0.334$ in.^{2} (see Table 34-2)

   $f_{ut} = 58{,}000$ psi (see Table 34-1)

   Substituting:

   $\phi\ V_s = 0.65\ (2)\ (0.6)\ (0.334)\ (58{,}000) = 15{,}110$ lb

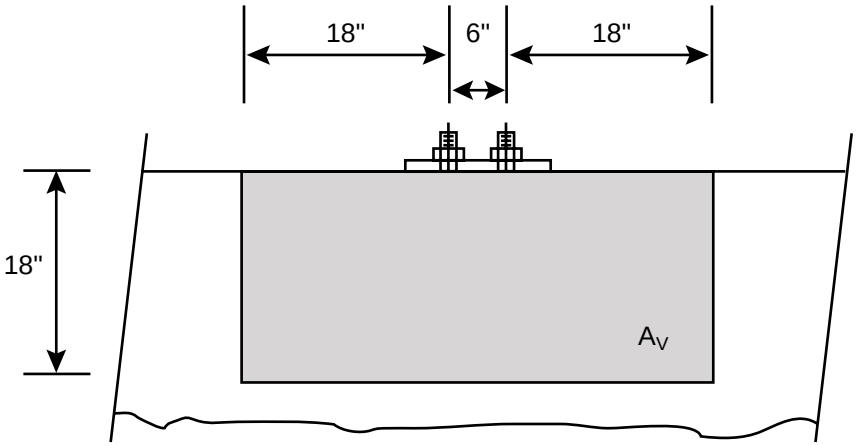
   b. Concrete breakout strength (ϕV_{cbg}): — *D.6.2*

   $$\phi V_{cbg} = \phi \frac{A_V}{A_{Vo}} \psi_5\ \psi_6\ \psi_7\ V_b$$ — *Eq. (D-21)*

   Since no supplementary reinforcement has been provided, $\phi = 0.70$ — *D.4.4(c)i*

   Determine A_V and A_{Vo}: — *D.6.2.1*

   A_V is the projected area of the shear failure surface on the free edge toward which shear is directed. The projected area is determined by a rectangle with edges bounded by 1.5 c_1 (1.5 x 12.0 = 18.0 in.) and free edges of the concrete from the centerlines of the anchors and surface of the concrete.



$A_V = (18+6+18)(18) = 756$ in.^{2}

$A_{Vo} = 4.5\ c_1^2 = 4.5\ (12)^2 = 648$ in.^{2} *Eq. (D-22)*

Check: $A_V \le nA_{Vo}$ $756 < 2(648)$ O.K.

Determine ψ_5: *D.6.2.5*

$\psi_5 = 1.0$ (no eccentricity in the connection)

Determine ψ_6: *D.6.2.6*

$\psi_6 = 1.0$ (no orthogonal free edge) *Eq. (D-26)*

Determine ψ_7:

$\psi_7 = 1.0$ for locations where concrete cracking is likely to occur (i.e., the edge of the foundation)

Determine V_b for an anchor:

$$V_b = 7\left(\frac{\ell}{d_o}\right)^{0.2}\sqrt{d_o}\ \sqrt{f'_c}\ c_1^{1.5}$$ *Eq. (D-23)*

where:

ℓ = load bearing length of the anchor for shear, not to exceed $8d_o$ *D.0*

For this problem $8d_o$ will control:

$\ell = 8d_o = 8\ (0.75) = 6.0$ in. < 10 in. therefore, use $8d_o$

Substituting into Eq. (D-23):

$$V_b = (7)\left(\frac{8\ (0.75)}{0.75}\right)^{0.2}\sqrt{0.75}\ \sqrt{4000}\ \ (12.0)^{1.5} = 24{,}157 \text{ lb}$$

Substituting into Eq. (D-21):

$$\phi V_{cbg} = 0.70\left(\frac{756}{648}\right)(1.0)\ (1.0)\ (1.0)\ (24{,}157) = 19{,}728 \text{ lb}$$

c. Pryout strength (ϕV_{cp}) *D.6.3*

Note: The pryout failure mode is normally only a concern for shallow, stiff anchors. Since this example problem addresses both shear directed toward the free edge and shear directed inward from the free edge, the pryout strength will be evaluated.

$\phi V_{cp} = \phi\ k_{cp}\ N_{cb}$ *Eq. (D-28)*

where:

$\phi = 0.70$, Condition B always applies for pryout strength *D.4.4(c)i*

$k_{cp} = 2.0$ for $h_{ef} > 2.5$ in.

$$N_{cb} = \frac{A_N}{A_{No}}\psi_2\ \psi_3\ N_b$$ *Eq. (D-4)*

$$N_{cb} = \left[\frac{972}{900}\right](0.94)\ (1.0)\ (48{,}000) = 48{,}730 \text{ lb}$$

Substituting into Eq. (D-28):

$\phi V_{cp} = 0.70\ (2.0)\ (48{,}730) = 68{,}222$ lb

Summary of design strengths based on steel strength, concrete breakout strength, and pryout strength for shear:

Steel strength, (ϕV_s):	15,110 lb ← controls	*D.6.1*
Embedment strength - concrete breakout, (ϕV_{cbg}):	19,728 lb	*D.6.2*
Embedment strength - pryout, (ϕV_{cp}):	68,222 lb	*D.6.3*

Therefore:

ϕV_n = 15,110 lb and is controlled by a ductile steel element as required in D.3.3.4

For seismic load in a region of moderate or high seismic risk, the design shear strength is $0.75\phi V_n$: *D.3.3.3*

$0.75\ \phi V_n$ = 0.75 (15,110) = 11,333 lb and is controlled by a ductile steel element

Check if $V_u \leq 0.75\phi V_n$

5000 lb < 11,333 lb O.K. for shear

7. Required edge distances, spacings, and thicknesses to preclude splitting failure *D.8*

   Since cast-in-place anchors are not likely to be highly torqued, the minimum cover requirements of 7.7 apply.

   Per 7.7 the minimum clear cover for a 3/4 in. bar is 1-1/2 in. when exposed to earth or weather. The clear cover provided for the bolt exceeds this requirement with the 12 in. edge distance to the bolt centerline O.K.

8. Summary

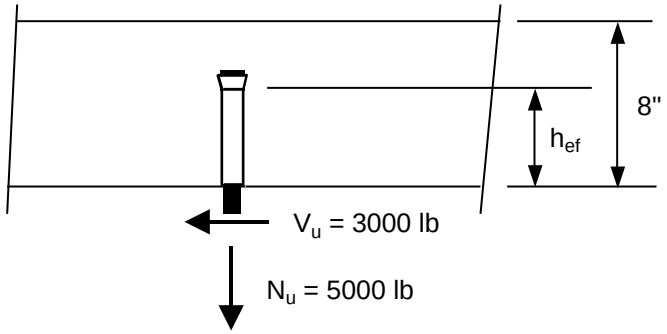
   Use 3/4 in. diameter ASTM F1554 Grade 36 hex head anchors with h_{ef} = 10 in.

Note: OSHA requirements implemented January 18, 2001 require that column anchorages use a least four anchors and be able to sustain a minimum eccentric gravity load of 300 pounds located 18 in. from the face of the extreme outer face of the column in each direction. The load is to be applied at the top of the column. The intent is that the column be able to sustain an iron worker hanging off the side of the top of the column. Refer to http://www.osha-slc.gov/OshStd_data/1926_0755.html for the requirement. This connection will satisfy the OSHA requirement but calculations are not included in the example.

Example 34.8—Single Post-Installed Anchor in Tension and Shear Away from Edges

Design a single post-installed mechanical anchor installed in the bottom of an 8 in. slab to support a factored 5000 lb tension load and a factored 3000 lb shear load (seismic loads from regions of moderate to high seismic risk are not included).

f'_c = 4000 psi



Note: This example for a single post-installed mechanical anchor is provided at the end of the design examples of Part 34 since the complicated calculations for groups, edge conditions, eccentricity, and tension/shear interaction covered in the previous examples for cast-in-place anchors are essentially the same as for post-installed mechanical anchors.

Similarities between post-installed mechanical anchors and cast-in-place anchors:

- For group and edge conditions, A_N, A_{No}, A_V, and A_{Vo} are determined in the same manner
- For eccentric loads, ψ_1 and ψ_5 are determined in the same manner
- For edge effects, ψ_2 and ψ_6 are determined in the same manner
- For anchors used in areas where concrete cracking may occur, ψ_3 and ψ_7 = 1.0

The unique properties of post-installed mechanical anchors are provided by the ACI 355.2 product evaluation report (refer to the sample in Table 34-3 for an example of an undercut post-installed mechanical anchor). The unique properties associated with each post-installed mechanical anchor product are:

- effective embedment length h_{ef}
- effective cross sectional area A_{se}
- specified yield strength f_y and specified ultimate strength f_{ut}
- effective cross-sectional area of the sleeve A_{sl} and specified ultimate strength of the sleeve f_{utsl} (to be used if the anchor sleeve extends through the shear plane)
- minimum edge distance c_{min} for the anchor
- minimum member thickness h_{min} for the anchor
- category of the anchor for determination of the appropriate ϕ factor for embedment strength
- coefficient for basic concrete breakout strength k for use in Eq. (D-7)
- pullout strength N_p of the anchor

Calculations and Discussion	Code Reference

1. The solution to this example is found by assuming the size of the anchor, then checking compliance with the design provisions. Try the fictitious 1/2 in. post-installed undercut mechanical anchor shown in Table 34-3.

2. This is a tension/shear interaction problem where values for both the design tensile strength (ϕN_n) and design shear strength (ϕV_n) will need to be determined. ϕN_n is the smallest of the design tensile strengths as controlled by steel (ϕN_s), concrete breakout (ϕN_{cb}), pullout (ϕN_{pn}), and side-face blowout (ϕN_{sb}). ϕV_n is the smallest of the design shear strengths as controlled by steel (ϕV_s), concrete breakout (ϕV_{cb}), and pryout (ϕV_{cp}). *D.7*

3. Determine the design tensile strength (ϕN_n) *D.5*

   a. Steel strength, (ϕN_s): *D.5.1*

   $\phi N_s = \phi \; n \; A_{se} \; f_{ut}$ *Eq. (D-3)*

   where:

   $\phi = 0.65$ *D.4.4*

   Currently, the ACI 355.2 product evaluation report does not include information on the elongation and minimum reduction in area of the anchor steel. These values are necessary for determination of whether the steel qualifies as a Ductile Steel Element or a Brittle Steel Element (see definitions in D.1). Conservatively, the steel will be assumed brittle for this example.

   $A_{se} = 0.131$ in.^{2} (see Table 34.3)

   $f_{ut} = 116{,}000$ psi (see Table 34.3)

   Note: Per D.5.1.2, f_{ut} shall not be taken greater than $1.9f_y$ or 125,000psi. From Table 34-3, $f_y = 92{,}800$ psi and $1.9f_y = 1.9(92{,}800) = 176{,}320$ psi, therefore use the specified minimum f_{ut} of 116,000 psi. *D.5.1.2*

   Substituting:

   $\phi N_s = 0.65 \; (1) \; (0.131) \; (116{,}000) = 9877$ lb

   b. Concrete breakout strength (ϕN_{cb}): *D.5.2*

   $$\phi N_{cb} = \phi \frac{A_N}{A_{No}} \psi_2 \; \psi_3 \; N_b$$ *Eq. (D-4)*

   where:

   $\phi = 0.65$ *D.4.4*

   From Table 34-3, this anchor is Category 1 and no supplementary reinforcement has been provided.

$\frac{A_N}{A_{No}}$ and ψ_2 terms are 1.0 for single anchors away from edges

ψ_3 = 1.0 for locations where concrete cracking is likely to occur (i.e., the bottom of the slab)

$N_b = k \sqrt{f_c'}\ h_{ef}^{1.5}$ *Eq. (D-7)*

where:

k = 24

Note: k = 17 for post-installed anchors unless the ACI 355.2 product evaluation report indicates a higher value may be used. For the case of this undercut anchor, k = 24 per Table 34-3. *RD.5.2.2*

h_{ef} = 5 in. (Table 34-3)

Therefore,

$N_b = 24 \sqrt{4000}\ 5^{1.5} = 16{,}970$ lb

Substituting:

ϕN_{cb} = 0.65 (1.0) (1.0) (1.0) (16,970) = 11,031 lb

c. Pullout strength (ϕN_{pn}) *D.5.3*

$\phi N_{pn} = \phi\ \psi_4\ N_p$ *Eq. (D-12)*

where:

ϕ = 0.65, Category 1 and no supplementary reinforcement has been provided *D.4.4*

ψ_4 = 1.0, cracking may occur at the edges of the foundation *D.5.3.6*

N_p = 12,730 lb (see Table 34.3)

Substituting:

ϕN_{pn} = 0.65 (1.0) (12,730) = 8275 lb

d. Concrete side-face blowout strength (ϕN_{sb}) *D.5.4*

This anchor is not located near any free edges therefore the side-face blowout strength is not applicable. *D.5.4.1*

Summary of steel strength, concrete breakout strength, pullout strength, and side-face blowout strength for tension:

Steel strength, (ϕN_s):	9877 lb	*D.5.1*
Embedment strength - concrete breakout, (ϕN_{cb}):	11,031 lb	*D.5.2*
Embedment strength - pullout, (ϕN_{pn}):	8265 lb ← controls	*D.5.3*
Embedment strength - side-face blowout, (ϕN_{sb}):	N/A	*D.5.4*

Therefore:

$\phi N_n = 8265$ lb

4. Determine the design shear strength (ϕV_n) *D.6*

a. Steel strength, (ϕV_s): *D.6.1*

$\phi V_s = \phi\ n\ (0.6 A_{se}\ f_{ut} + 0.4 A_{sl}\ f_{utsl})$ *Eq. (D-19)*

where:

$\phi = 0.60$

Currently, the ACI 355.2 product evaluation report does not include information on the elongation and minimum reduction in area of the anchor steel. These values are necessary for determination of whether the steel qualifies as a Ductile Steel Element or a Brittle Steel Element (see definitions in D.1). Conservatively, the steel will be assumed brittle for this example.

$A_{se} = 0.131$ in.^{2} (see Table 34-3)

$f_{ut} = 116{,}000$ psi (see Table 34-3)

Note: Per D.5.1.2, f_{ut} shall not be taken greater than $1.9 f_y$ or 125,000psi. From Table 3, $f_y = 92{,}800$ psi and $1.9 f_y = 1.9(92{,}800) = 176{,}320$ psi. Therefore, use the specified minimum f_{ut} of 116,000 psi.

For this example, it will be assumed that the anchor sleeve is installed flush with the surface of the concrete (i.e., the anchor sleeve does not extend through the shear plane).

Substituting:

$\phi V_s = 0.60\ (1)\ [(0.6)\ (0.131)\ (116{,}000) + 0)] = 5470$ lb

b. Concrete breakout strength (ϕV_{cbg}): *D.6.2*

This anchor is not located near any free edges therefore the concrete breakout for shear is not applicable.

c. Pryout strength (ϕV_{cp}) *D.6.3*

$\phi V_{cp} = \phi\, k_{cp}\, N_{cb}$ *Eq. (D-28)*

where:

$\phi = 0.65$, Category 1 and no supplementary reinforcement has been provided *D.4.4*

$k_{cp} = 2.0$ for $h_{ef} > 2.5$ in.

Eq. (D-4)

$$N_{cb} = \frac{A_N}{A_{No}} \psi_2\, \psi_3\, N_b$$

From Step 3 above:

$N_{cb} = (1.0)\,(1.0)\,(1.0)\,(16{,}970) = 16{,}970$ lb

Substituting into Eq. (D-28):

$\phi V_{cp} = 0.65\,(2.0)\,(16{,}970) = 22{,}061$ lb

Summary of steel strength, concrete breakout strength, and pryout strength for shear from Example 34.4:

Steel strength, (ϕV_s):	5470 lb ← controls	*D.6.1*
Embedment strength - concrete breakout, (ϕV_{cb}):	N/A	*D.6.2*
Embedment strength - pryout, (ϕV_{cp}):	22,061 lb	*D.6.3*

Therefore:

$\phi V_n = 5470$ lb

5. Check tension and shear interaction *D.7*

If $V_u \le 0.2\phi V_n$ then the full tension design strength is permitted *D.7.1*

$V_u = 3{,}000$ lb

$0.2\phi V_n = 0.2\,(5470) = 1094$ lb

V_u exceeds $0.2\phi V_n$, the full tension design strength is not permitted

If $N_u \le 0.2\phi N_n$ then the full shear design strength is permitted *D.7.2*

$N_u = 5000$ lb

$0.2\phi N_n = 0.2\,(8265) = 1653$ lb

N_u exceeds $0.2\phi N_n$, the full shear design strength is not permitted

The interaction equation must be used *D.7.3*

$$\frac{N_u}{\phi N_n}+\frac{V_u}{\phi V_n}\le 1.2 \qquad \textit{Eq. (D-29)}$$

$$\frac{5000}{8265}+\frac{3000}{5470}=0.60+0.55=1.15<1.2 \quad \text{O.K.}$$

6. Required edge distances, spacings, and thickness to preclude splitting failure *D.8*

Since this anchor is located away from edges, only the limits on embedment length h_{ef} related to member thickness are applicable. Per D.8.5, h_{ef} shall not exceed 2/3 of the member thickness or the member thickness less 4 in. D.8 does permit the use of larger values of h_{ef} provided product-specific tests have been performed in accordance with ACI 355.2.

As shown in Table 34-3, the ACI 355.2 product evaluation report for this anchor provides the minimum thickness as h_{min} = 6-5/8 in. which is less than the 8 in. provided O.K.

7. Summary

The fictitious 1/2 in. diameter post-installed undercut mechanical anchor shown in Table 34-3 is O.K. for the factored tension and shear loads.

Also from PCA

The following publications may be of interest to readers of this report:

Circular Concrete Tanks without Prestressing (IS072)
Column Shortening in Tall Buildings—Prediction and Compensation (EB108)
Concrete Floor Systems—Guide to Estimating and Economizing (SP041)
Concrete Structural Floor Systems and More (CD013)
Connections for Tilt-Up Wall Construction (EB110)
Design and Control of Concrete Mixtures (EB001)
Design of Concrete Buildings for Earthquake and Wind Forces (EB113)
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Design of Liquid-Containing Concrete Structures for Earthquake Forces (EB219)
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