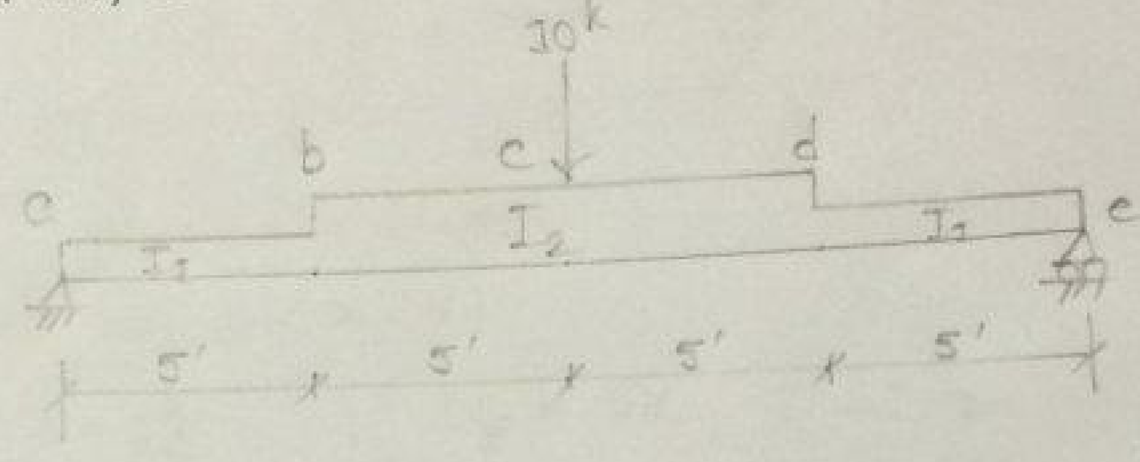
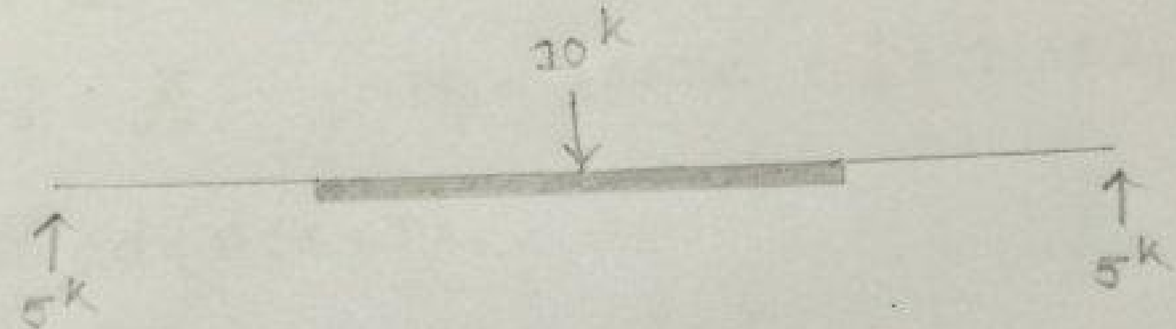
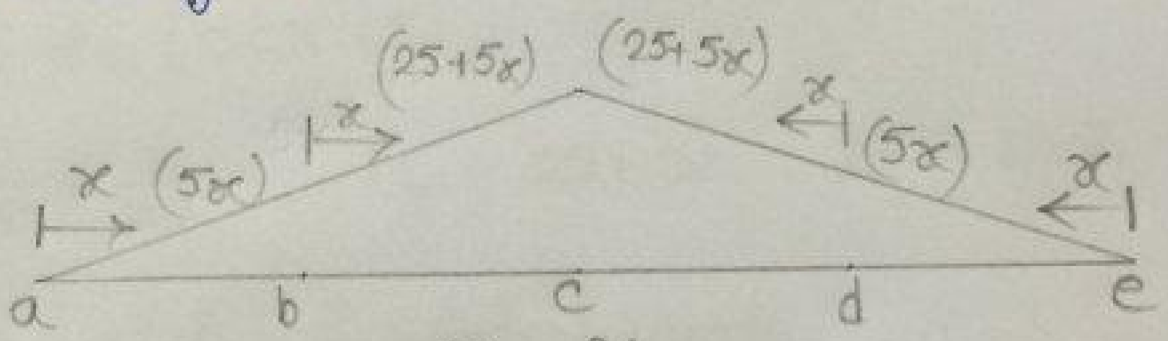
Assignment-10: Calculate change in slope at point 'a'. Given, $E = 30 \times 10^3$ ksi, $I_1 = 150$ in$^4$, $I_2 = 200$ in$^4$.




Sol$^n$:

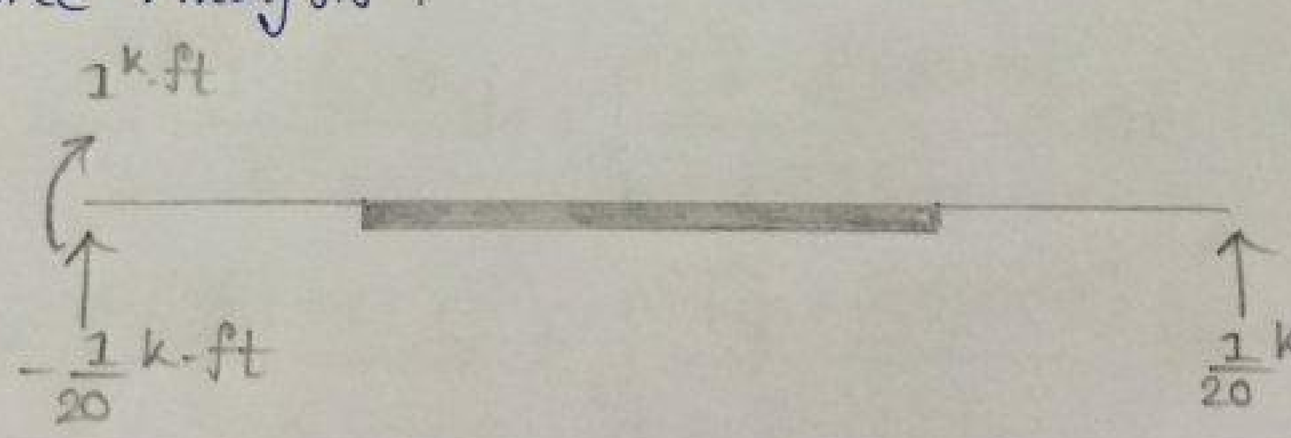


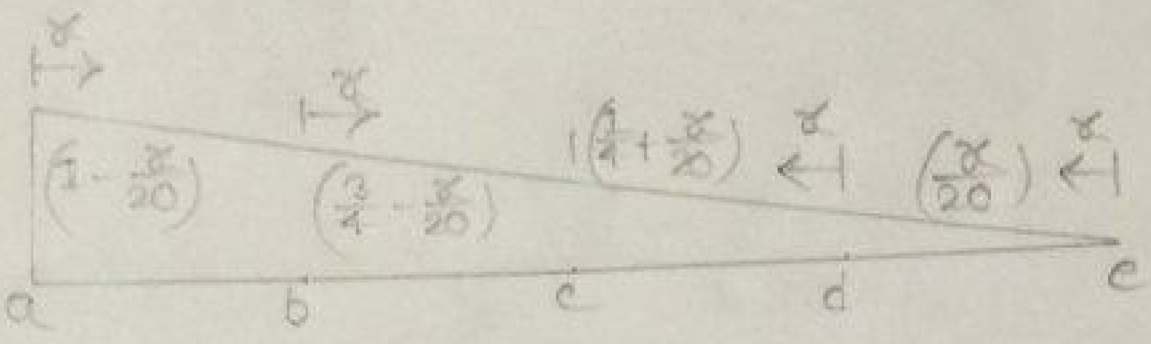

P-Force Analysis:




$M_P$-Diagram

Q-Force Analysis:






$M_Q$ - Diagram

Segment ab $(0 < x < 5)$

$M_Q = 1 - \frac{x}{20}$ ; $M_P = 5x$ ; $I = I_1$

Segment bc $(0 < x < 5)$

$M_Q = \frac{3}{4} - \frac{x}{20}$ ; $M_P = 25 + 5x$ ; $I = 1.33\, I_1$

Segment ed $(0 < x < 5)$

$M_Q = \frac{x}{20}$ ; $M_P = 5x$ ; $I = I_1$

Segment dc $(0 < x < 5)$

$M_Q = \frac{1}{4} + \frac{x}{20}$ ; $M_P = 25 + 5x$ ; $I = 1.33\, I_1$

Using principle of virtual-work,

$$\Sigma\, Q \cdot \Delta_a = \int \frac{M_Q M_P}{EI}\, dx$$

$$\Rightarrow \Delta_a = \int_a^b \frac{\left(1 - \frac{x}{20}\right) 5x}{EI_1}\, dx + \int_b^c \frac{\left(\frac{3}{4} - \frac{x}{20}\right)(25 + 5x)}{1.33\, EI_1}\, dx$$

$$+ \int_e^d \frac{\left(\frac{x}{20}\right) 5x}{EI_1}\, dx + \int_d^c \frac{\left(\frac{1}{4} + \frac{x}{20}\right)(25 + 5x)}{1.33\, EI_1}\, dx$$

$$\Rightarrow \Delta_a = \int_0^5 \frac{\left(5x - \frac{x^2}{4}\right)}{EI_1}dx + \int_0^5 \frac{\left(\frac{75}{4} - \frac{5x}{4} + \frac{15x}{4} - \frac{x^2}{4}\right)}{1.33EI_1}dx$$

$$\int_0^5 \frac{\frac{x^2}{4}}{EI_1}dx + \int_0^5 \frac{\left(\frac{25}{4} + \frac{5x}{4} + \frac{5x}{4} + \frac{x^2}{4}\right)}{1.33\,EI_1}dx$$

$$\Rightarrow \Delta_a = \frac{1}{EI_1}\int_0^5 \left(5x - \cancel{\frac{x^2}{4}} + \frac{225}{16} + \frac{15x}{8} - \cancel{\frac{3x^2}{16}} + \cancel{\frac{x^2}{4}} + \frac{75}{16} + \frac{15x}{8} + \cancel{\frac{3x^2}{16}}\right)dx$$

$$\Rightarrow \Delta_a = \frac{1}{EI_1}\int_0^5 \left(\frac{35x}{4} + \frac{75}{4}\right)dx$$

$$\Rightarrow \Delta_a = \frac{1}{EI_1}\left[\frac{35x^2}{8} + \frac{75x}{4}\right]_0^5$$

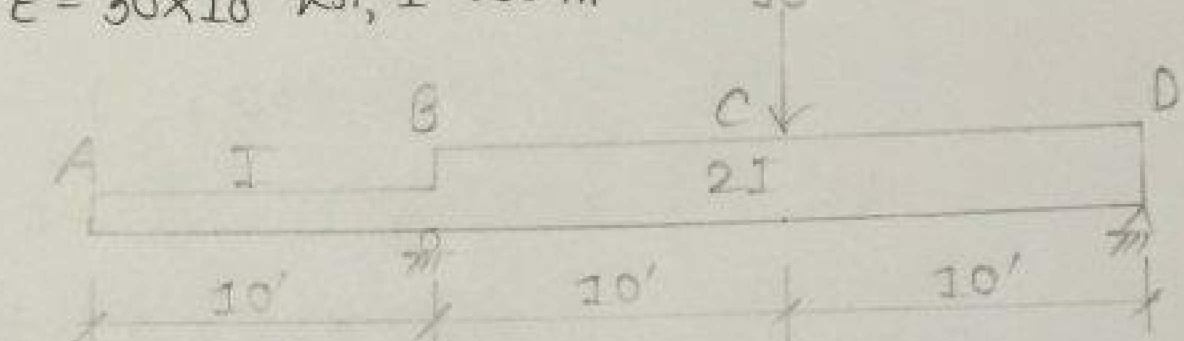
$$\Rightarrow \Delta_a = \frac{1}{30\times10^3\times150}\left[\frac{35\times(5)^2}{8} + \frac{75\times5}{4} + 0 - 0\right]\times\frac{(144)^2}{144}$$

$\therefore \Delta_a = +0.0065$ radian (clockwise)

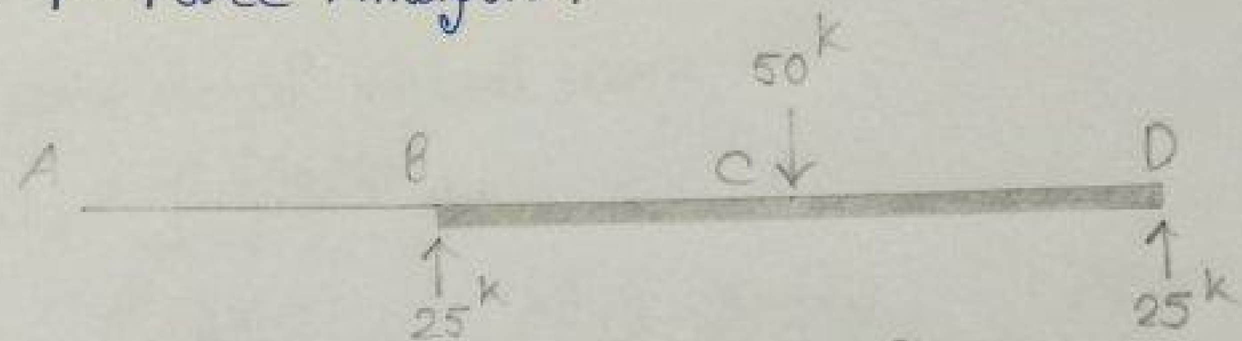
(Ans:)

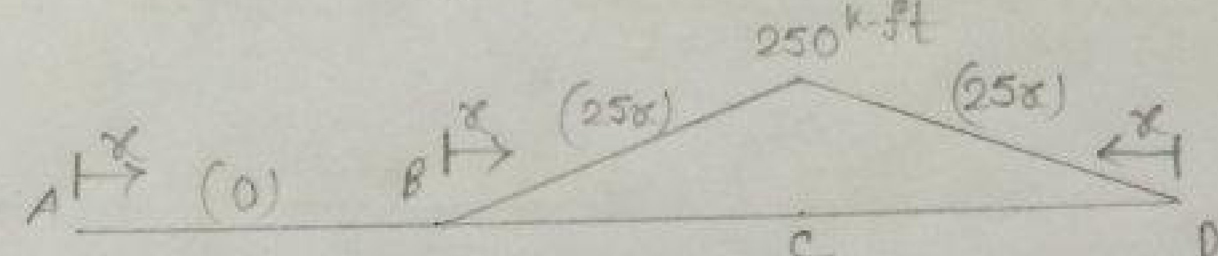
Assignment-11: Calculate change in slope at 'A'.

Given, $E = 30\times10^3$ ksi, $I = 300$ in$^4$



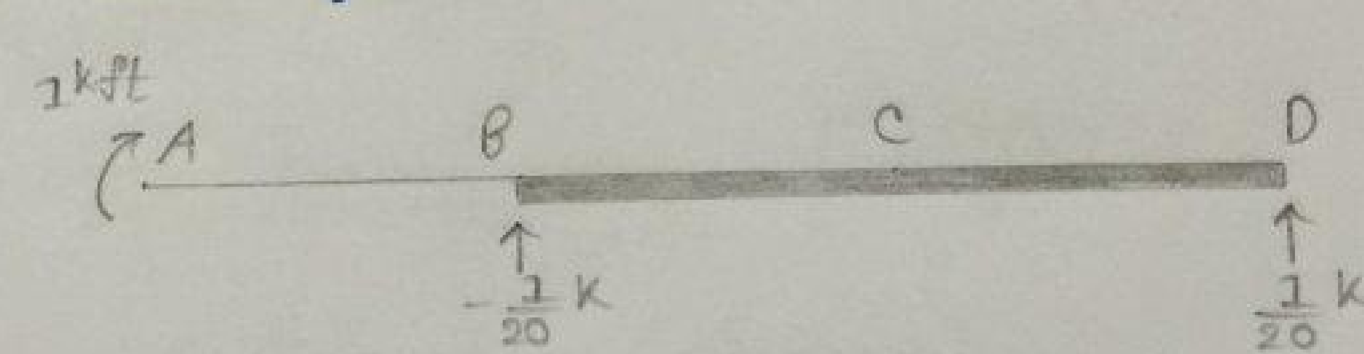

Sol$^n$: P-Force Analysis:

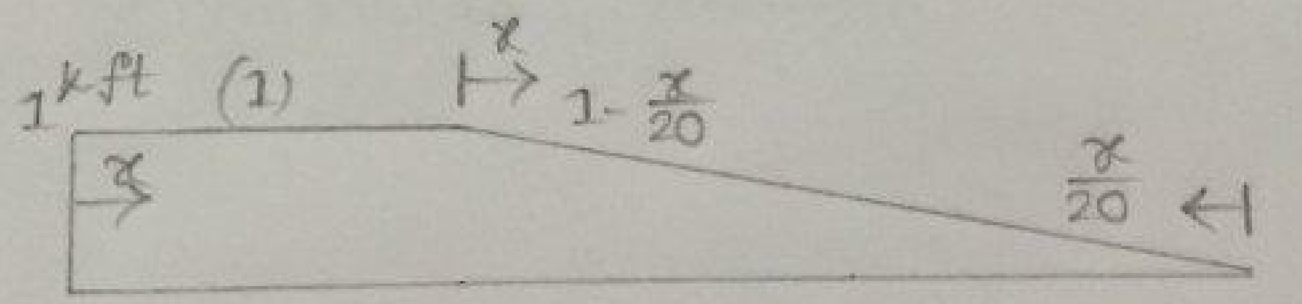



$M_P$- Diagram

Q- Force Analysis:




$M_Q$- Diagram

Segment AB $(0 < x < 10)$

$M_Q = 1 \quad ; M_P = 0 \quad ; I = I$

Segment BC $(0 < x < 10)$

$M_Q = 1 - \frac{x}{20} \quad ; M_P = 25x \quad ; I = 2I$

Segment DC $(0 < x < 10)$

$M_Q = \frac{x}{20} \quad ; M_P = 25x \quad ; I = 2I$

Using principle of virtual work,

$$\sum Q . \Delta_A = \int \frac{M_Q M_P}{EI} dx$$

$$\Rightarrow \Delta_A = \int_A^B \frac{1 \times 0}{EI} dx + \int_B^C \frac{\left(1 - \frac{x}{20}\right) 25x}{2EI} dx + \int_D^C \frac{\left(\frac{x}{20}\right) 25x}{2EI} dx$$

$$\Rightarrow \Delta_A = 0 + \int_0^{10} \frac{\left(25x - \frac{5x^2}{4}\right)}{2EI} dx + \int_0^{10} \frac{\frac{5x^2}{4}}{2EI} dx$$

$$\Rightarrow \Delta_A = \frac{1}{2EI} \int_0^{10} \left(25x - \cancel{\frac{5x^2}{4}} + \cancel{\frac{5x^2}{4}}\right) dx$$

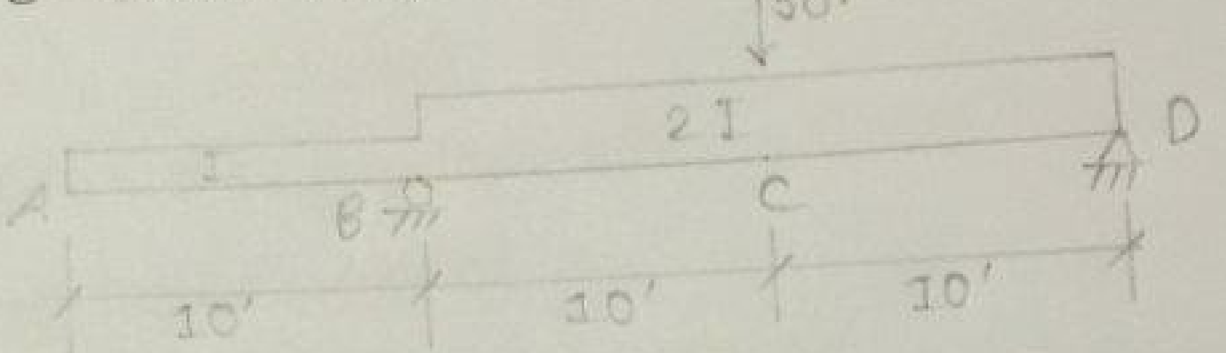
$$\Rightarrow \Delta_A = \frac{1}{2EI} \left[\frac{25x^2}{2}\right]_0^{10}$$

$$\Rightarrow \Delta_A = \frac{1}{2 \times (30 \times 10^3 \times 144) \times \frac{300}{(144)^2}} \left[\frac{25 \times (10)^2}{2}\right]$$

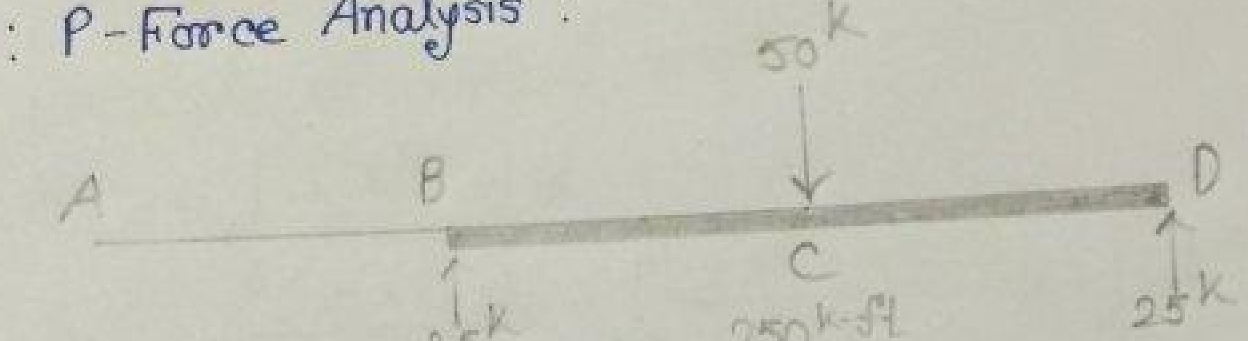
$\therefore \Delta_A = +0.01$ radian (clockwise)

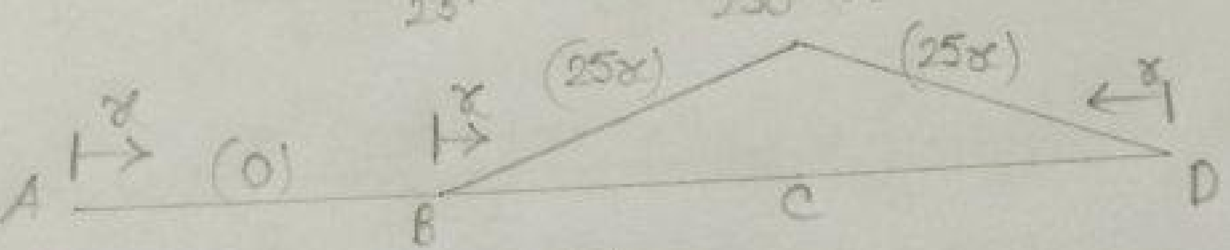
(Ans:)

Assignment - 12 : Compute vertical deflection at 'A'.
Given, $E = 30 \times 10^3$ ksi, $I = 300$ in$^4$.



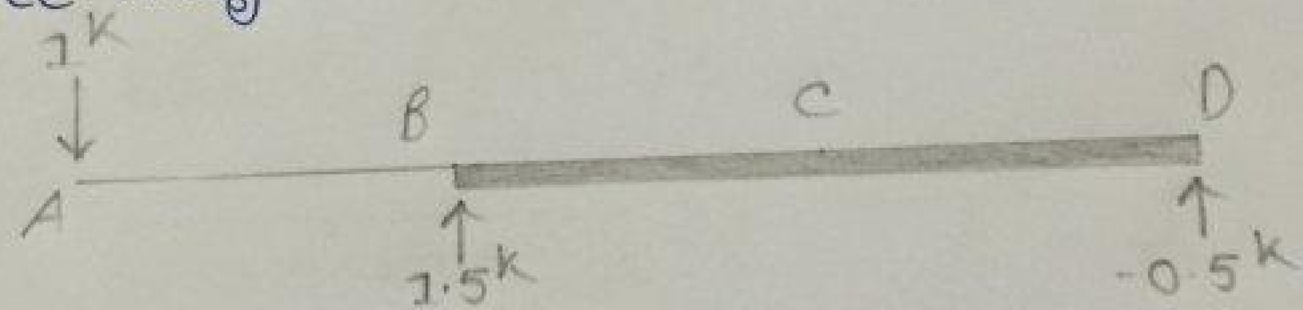

Sol$^n$ : P-Force Analysis :

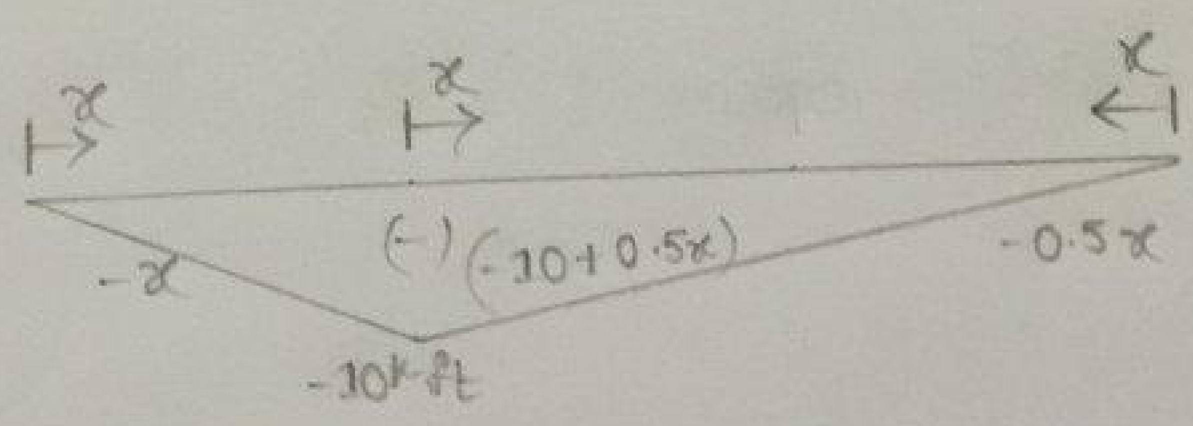



$M_P$ - Diagram

Q-Force Analysis :




$M_Q$ - Diagram

Segment AB $(0<x<10)$

$M_Q = -x$ ; $M_P = 0$ ; $I = I$

Segment BC $(0<x<10)$

$M_Q = (10+0.5x)$ ; $M_P = 25x$ ; $I = 2I$

Segment DC $(0<x<10)$

$M_Q = -0.5x$ ; $M_P = 25x$ ; $I = 2I$

Using principle of virtual work,

$$\Sigma Q \cdot \delta_{Av} = \int \frac{M_Q M_P}{EI} dx$$

$$\Rightarrow 1 \times \delta_{Av} = \int_A^B \frac{(-x)\times 0}{EI} dx + \int_B^C \frac{(10+0.5x)(25x)}{2EI} dx + \int_D^C \frac{(-0.5x)(25x)}{2EI} dx$$

$$\Rightarrow \delta_{Av} = 0 + \int_0^{10} \frac{(-250x + 12.5x^2)}{2EI} dx + \int_0^{10} \frac{-12.5x^2}{2EI} dx$$

$$\Rightarrow \delta_{Av} = \frac{1}{2EI} \int_0^{10} (-250x + \cancel{12.5x^2} - \cancel{12.5x^2}) dx$$

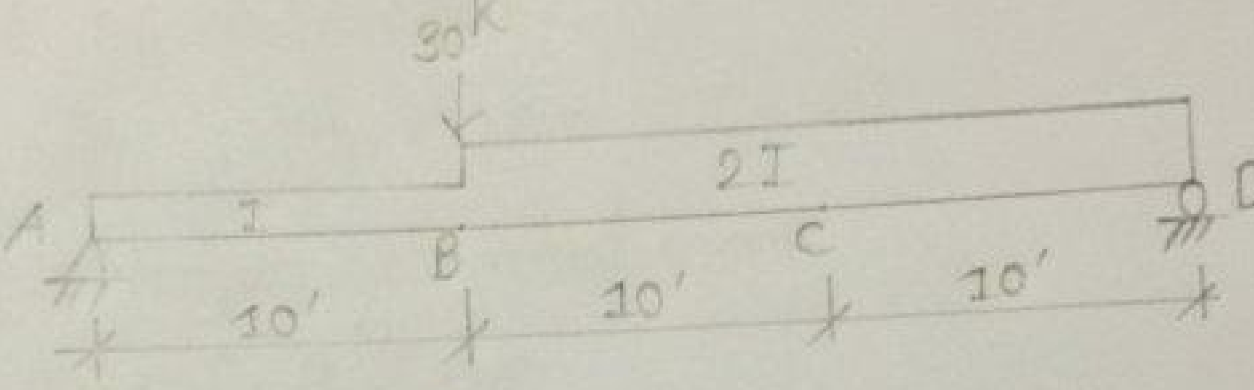
$$\Rightarrow \delta_{Av} = \frac{1}{2EI} \left[ -\frac{250x^2}{2} \right]_0^{10}$$

$$\Rightarrow \delta_{Av} = \frac{1}{2\times(30\times10^3\times144)\times\frac{300}{(144)^2}} \times \left[ -\frac{250\times(10)^2}{2} \right]$$

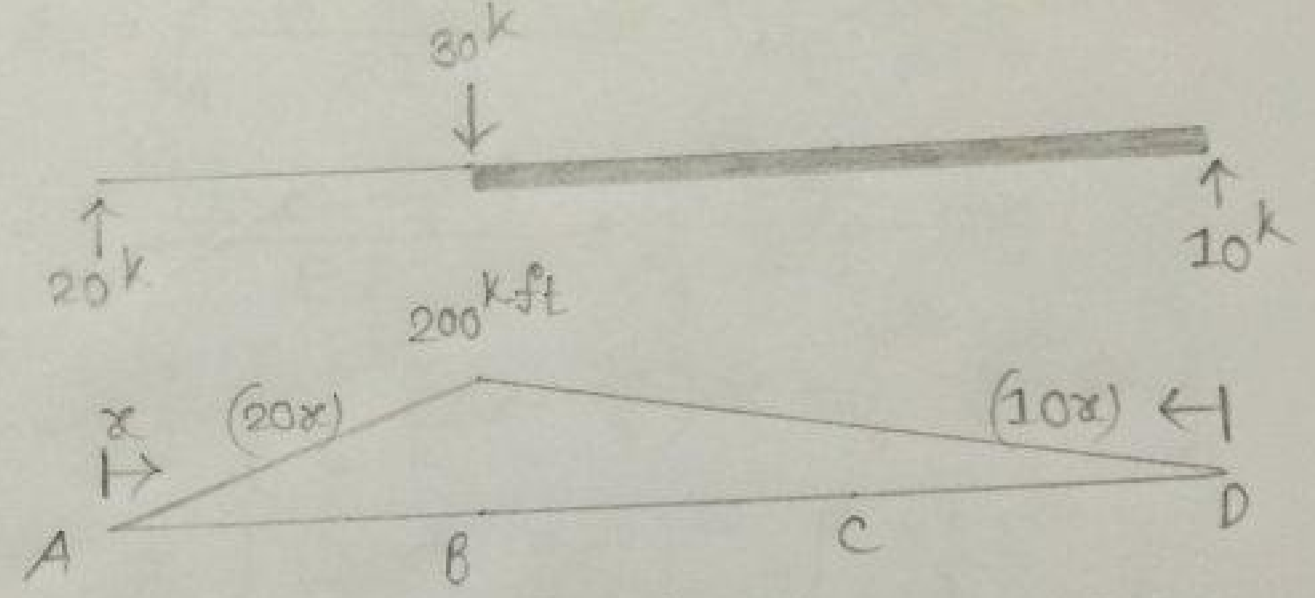
$$\therefore \delta_{Av} = -0.1\,ft \;(\uparrow \text{upward deflection})$$

(Ans:)

Assignment-13: Compute change in slope at 'A'. Given, $E = 30,000$ ksi, $I = 300$ in$^4$.

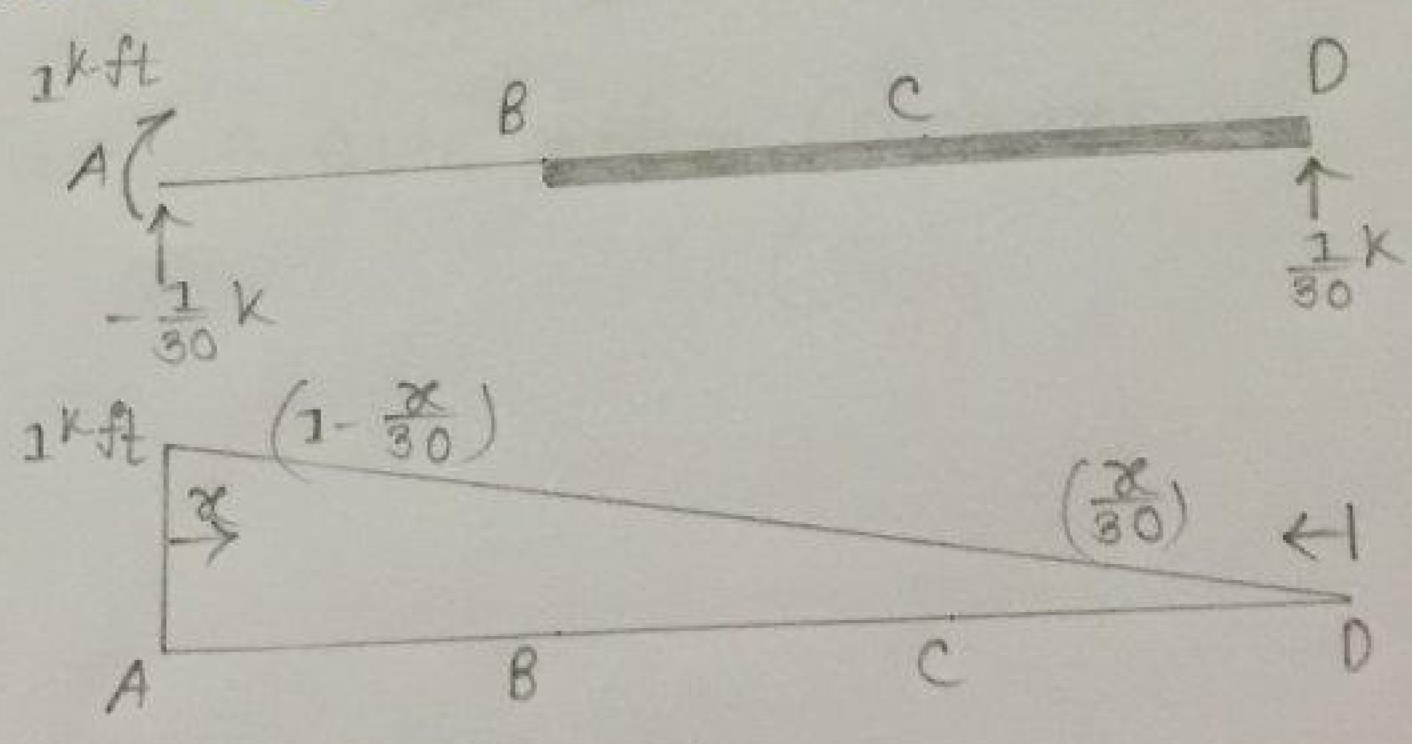



Sol$^n$: P-Force Analysis:




$M_P$ - Diagram

Q-Force Analysis:




$M_Q$ - Diagram

Segment AB $(0<x<10)$

$M_Q = 1 - \frac{x}{30}$ ; $M_P = 20x$ ; $I = I$

Segment DB $(0<x<20)$

$M_Q = \frac{x}{30}$ ; $M_P = 10x$ ; $I = 2I$

Using principle of virtual work,

$$\sum Q \cdot \Delta_A = \int \frac{M_Q M_P}{EI} dx$$

$$\Rightarrow 1 \times \Delta_A = \int_A^B \frac{\left(1 - \frac{x}{30}\right)(20x)}{EI} dx + \int_D^B \frac{\left(\frac{x}{30}\right)(10x)}{2EI} dx$$

$$\Rightarrow \Delta_A = \int_0^{10} \frac{\left(20x - \frac{2x^2}{3}\right)}{EI} + \int_0^{20} \frac{\frac{x^2}{3}}{2EI} dx$$

$$\Rightarrow \Delta_A = \frac{1}{EI} \left\{ \int_0^{10} \left(20x - \frac{2x^2}{3}\right) dx + \int_0^{20} \frac{x^2}{6} dx \right\}$$

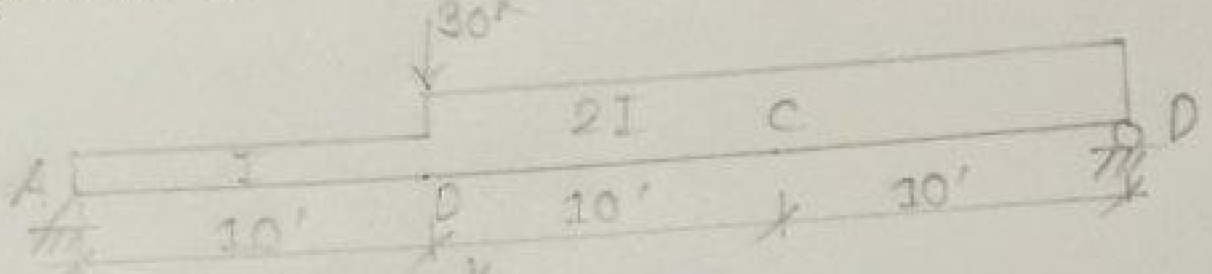
$$\Rightarrow \Delta_A = \frac{1}{EI} \left\{ \left[\frac{20x^2}{2} - \frac{2x^3}{3\times 3}\right]_0^{10} + \left[\frac{x^3}{6\times 3}\right]_0^{20} \right\}$$

$$\Rightarrow \Delta_A = \frac{1}{(30\times 10^3 \times 144) \times \frac{300}{(144)^2}} \times \left[10\times(10)^2 - \frac{2}{3\times 3}(10)^3 + \frac{(20)^3}{18}\right]$$

$\therefore \Delta_A = +0.0196$ radian (clockwise)

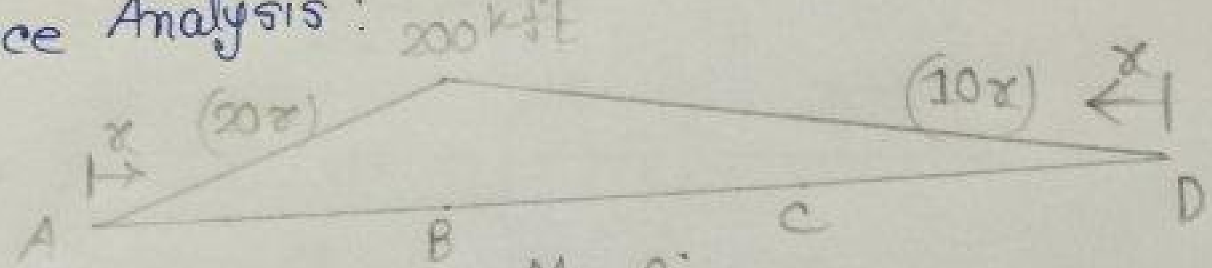
(Ans:)

Assignment-14 : Compute change in slope at D. Given, $E = 30{,}000$ ksi, $I = 300 \text{ in}^4$.



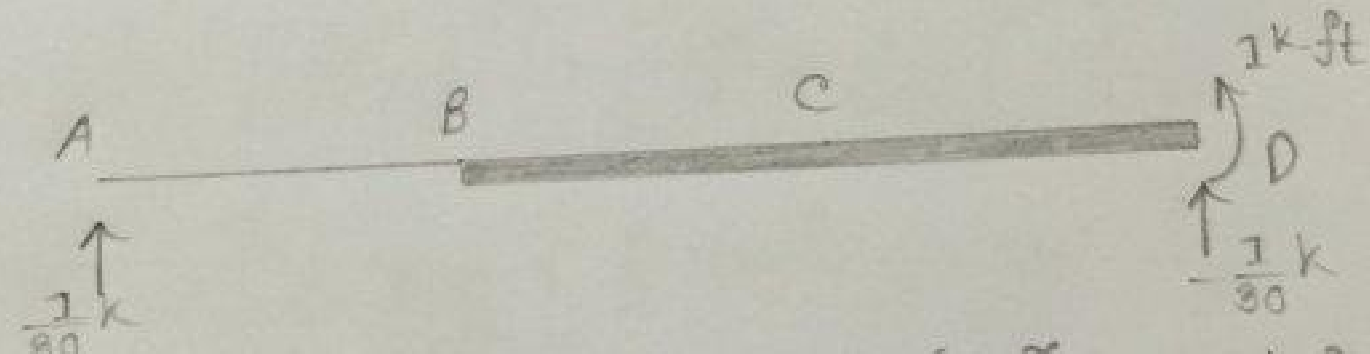

Sol$^n$ :

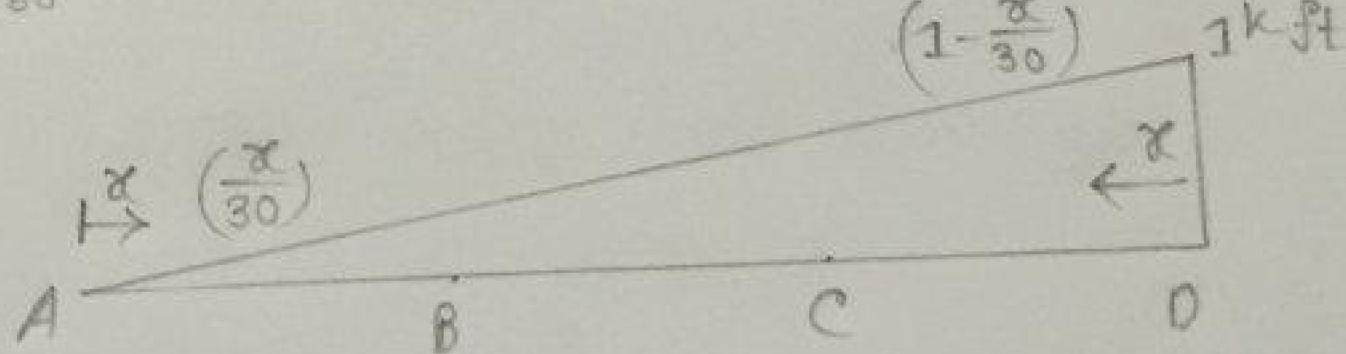

P- Force Analysis :




Mp- Diagram

Q- Force Analysis :




MQ - Diagram

Segment AB $(0 < x < 10)$

$M_Q = \frac{x}{30}$ ; $M_P = 20x$ ; $I = I$

Segment DB $(0 < x < 20)$

$M_Q = 1 - \frac{x}{30}$ ; $M_P = 10x$ ; $I = 2I$

Using principle of virtual work,

$$\Sigma Q . \Delta_D = \int \frac{M_Q M_P}{EI} dx$$

$$\Rightarrow 1 \times \Delta_D = \int_A^B \frac{\left(\frac{x}{30}\right)(20x)}{EI} dx + \int_D^B \frac{\left(1 - \frac{x}{30}\right)(10x)}{2EI} dx$$

$$\Rightarrow \Delta_D = \int_0^{10} \frac{\frac{2x^2}{3}}{EI} dx + \int_0^{20} \frac{\left(10x - \frac{x^2}{3}\right)}{2EI} dx$$

$$\Rightarrow \Delta_D = \frac{1}{EI} \left\{ \int_0^{10} \frac{2x^2}{3} dx + \int_0^{20} \left(5x - \frac{x^2}{6}\right) dx \right\}$$

$$\Rightarrow \Delta_D = \frac{1}{EI} \left\{ \left[\frac{2x^3}{3 \times 3}\right]_0^{10} + \left[\frac{5x^2}{2} - \frac{x^3}{6 \times 3}\right]_0^{20} \right\}$$

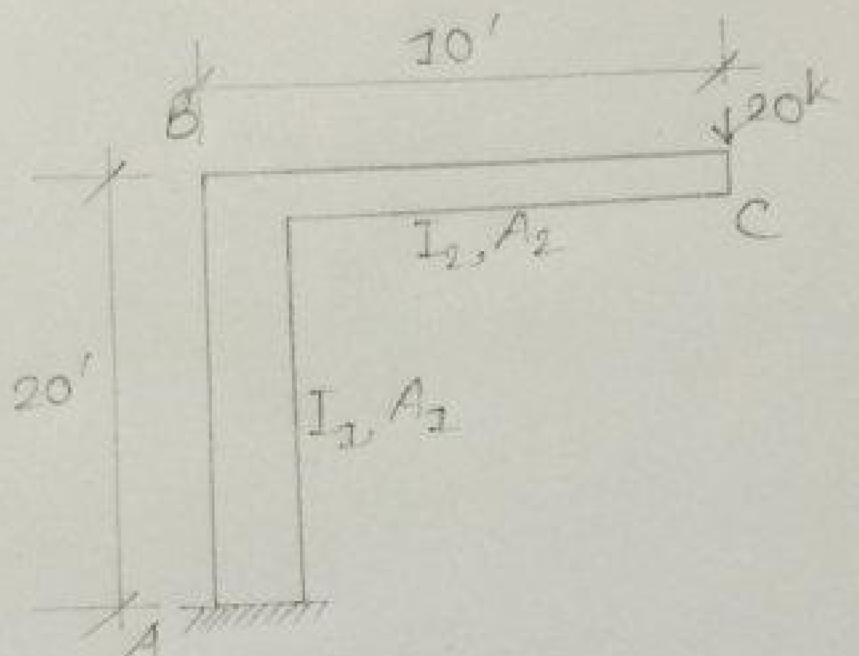
$$\Rightarrow \Delta_D = \frac{1}{(30 \times 10^3 \times 144) \times \frac{300}{(144)^2}} \left\{ \frac{2}{9} \times (10)^3 + \frac{5}{2} \times (20)^2 - \frac{(20)^3}{18} \right\}$$

$\therefore \Delta_D = +0.0124$ radian (anti-clockwise)

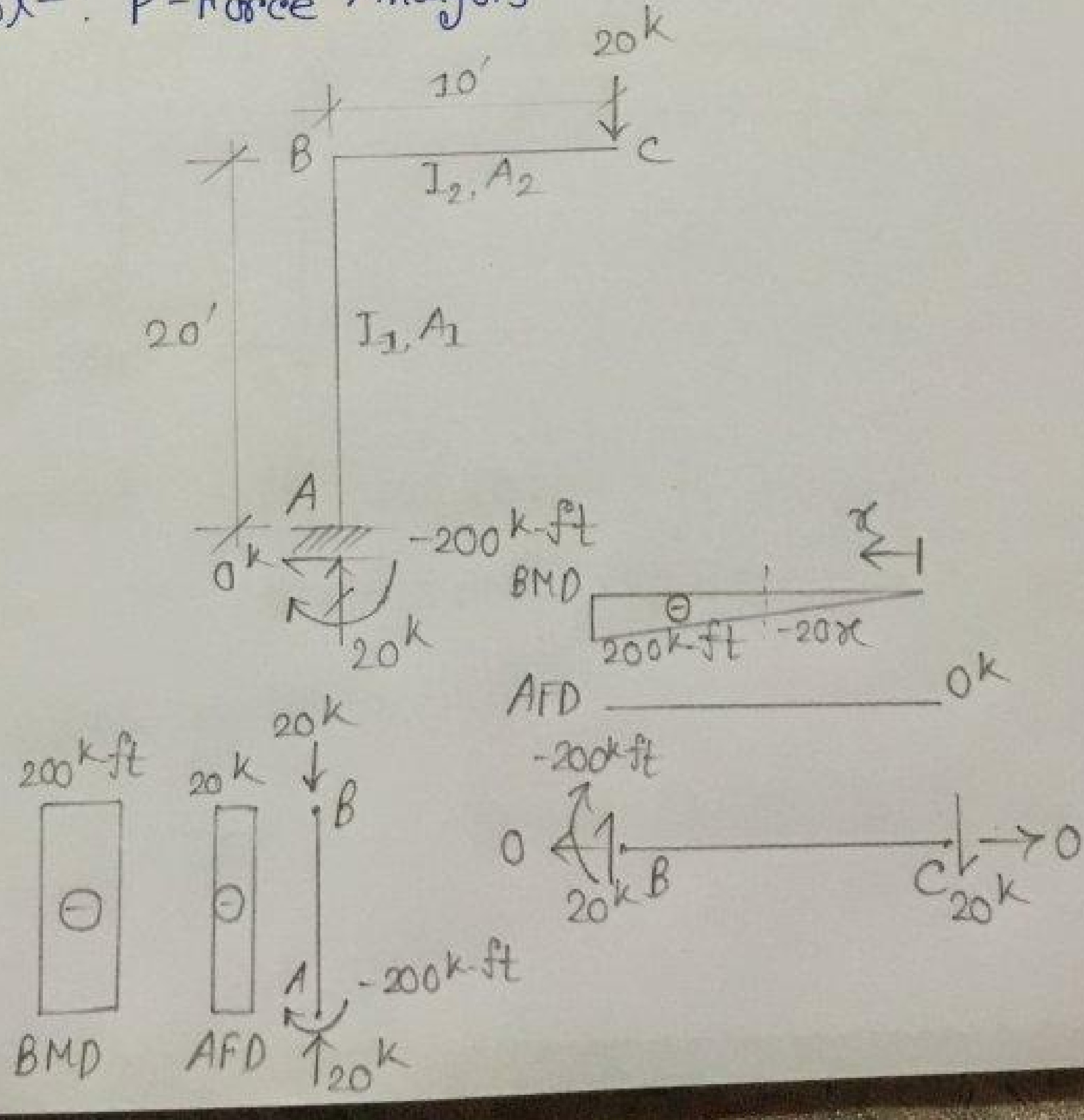
(Ans:)

Assignment-15: Compute horizontal deflection at 'B'.

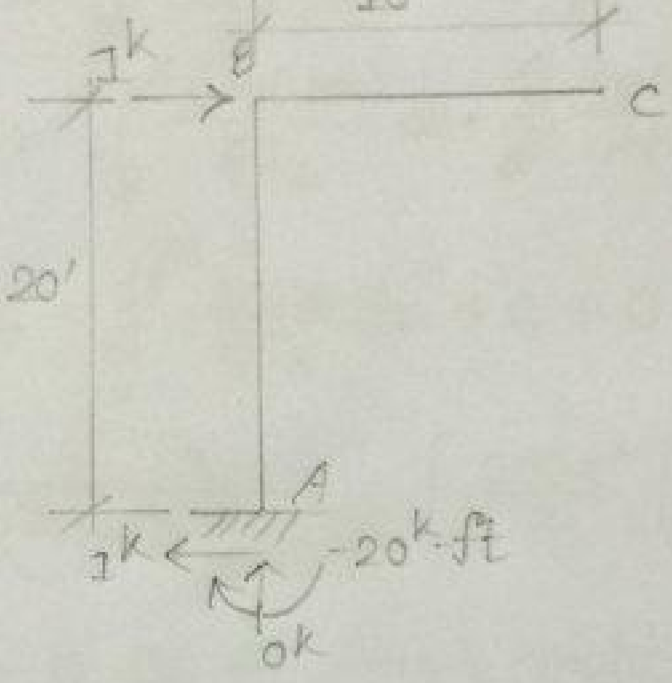
Given, $I_1 = 200\ in^4$, $I_2 = 150\ in^4$, $A_1 = 15\ in^2$, $A_2 = 10\ in^2$, $E = 30{,}000\ ksi$.

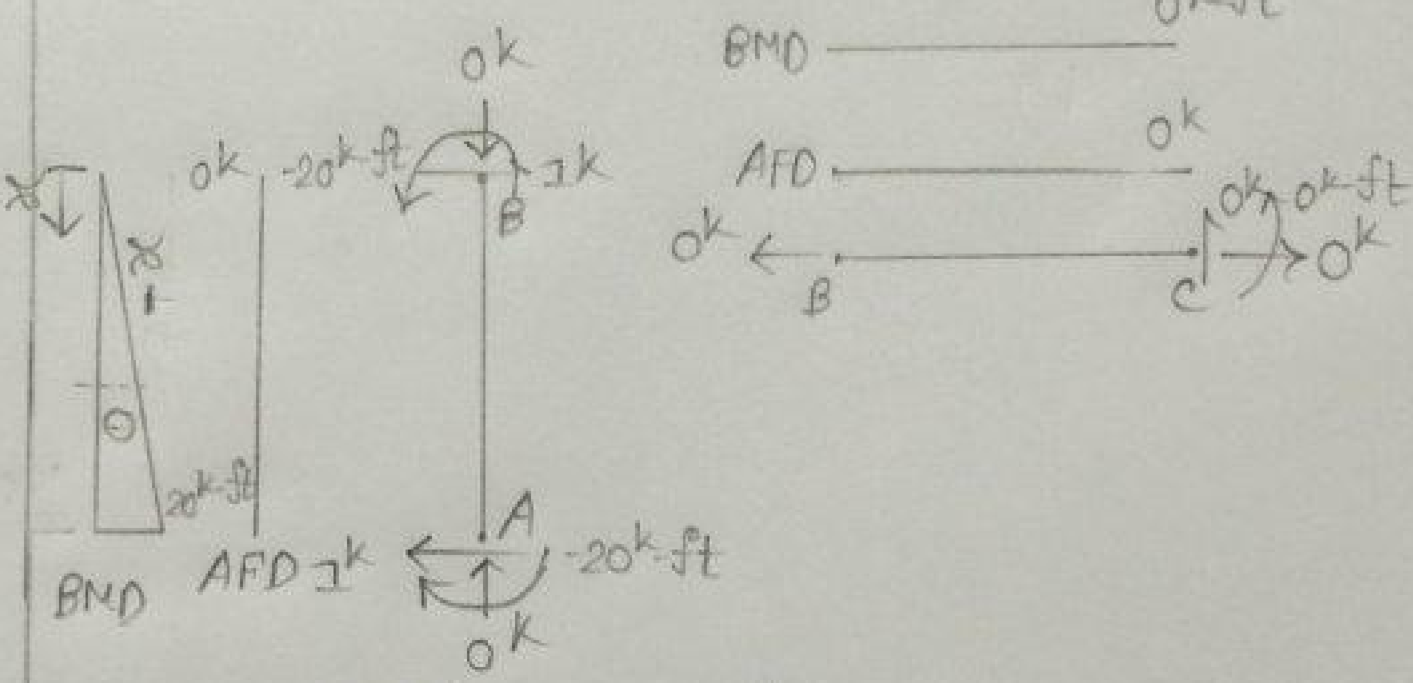



Sol$^n$: P-Force Analysis



Q- Force Analysis :




Segment (0 < $x$ < 20)

$M_Q = -x$ ; $M_P = -200^{k}\text{-}ft$ ; $F_Q = 0$ ; $F_P = 20^{k}$ ; $I = I_1$

Segment CB (0 < $x$ < 10)

$M_Q = 0$ ; $M_P = -20x$ ; $F_Q = 0$ ; $F_P = 0$ ; $I = \frac{3}{4} I_1$

Using principle of virtual work,

$$\Sigma Q \cdot \delta_{Bh} = \int \frac{M_Q M_P}{EI} dx + \Sigma \frac{F_Q F_P L}{AE}$$

$$\Rightarrow 1 \times \delta_{Bh} = \int_B^A \frac{(-x)(-200)}{EI_1} dx + \sum_B^A \frac{(0)(20)20}{AE} + \int_C^B \frac{(0)(-20x)}{\frac{3}{4} EI_1} dx + \sum_C^B \frac{(0)(0)(10)}{AE}$$

$$\Rightarrow 1 \times \delta_{Bh} = \int_0^{20} \frac{200x}{EI_1} dx + 0 + 0 + 0$$

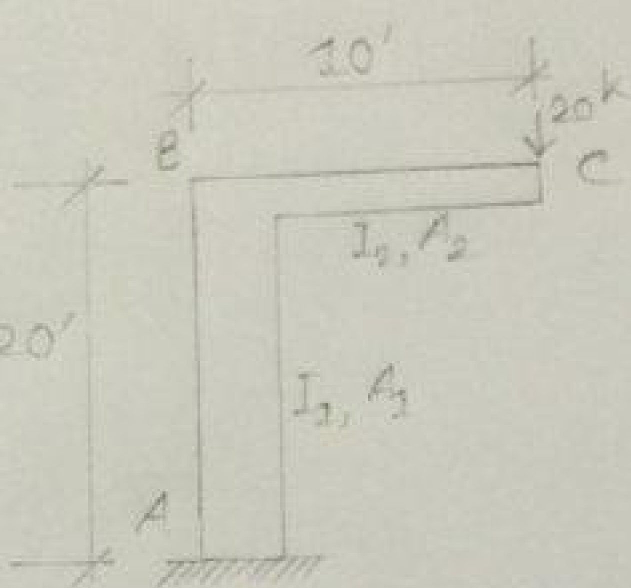
$$\Rightarrow \delta_{Bh} = \frac{1}{(30 \times 10^3 \times 144) \times \frac{200}{(144)^2}} \left[ \frac{200x^2}{2} \right]_0^{20}$$

$$\therefore \delta_{Bh} = 0.96 \text{ ft } (\rightarrow) \text{ rightward}$$

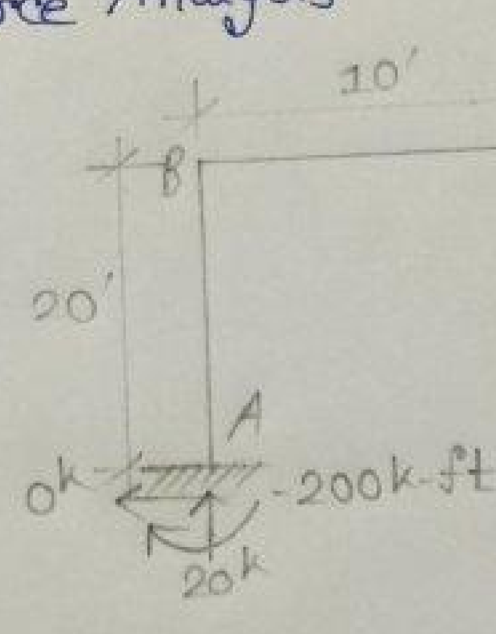
(Ans:)

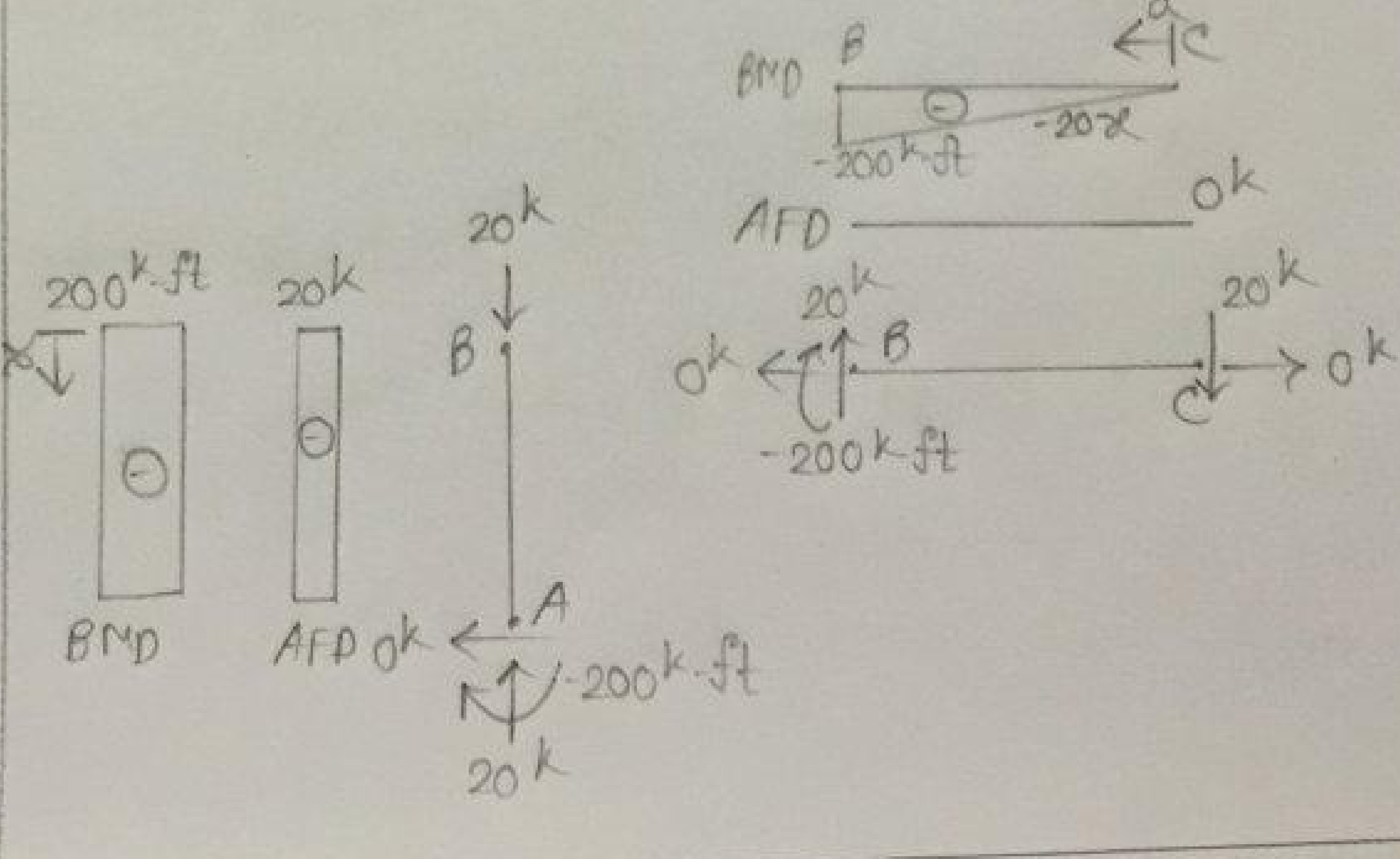
Assignment-16: Compute change in slope at 'C'.

Given, $I_1 = 200\ in^4$, $I_2 = 150\ in^4$, $A_1 = 15\ in^2$, $A_2 = 10\ in^2$, $E =$ 30,000 ksi.

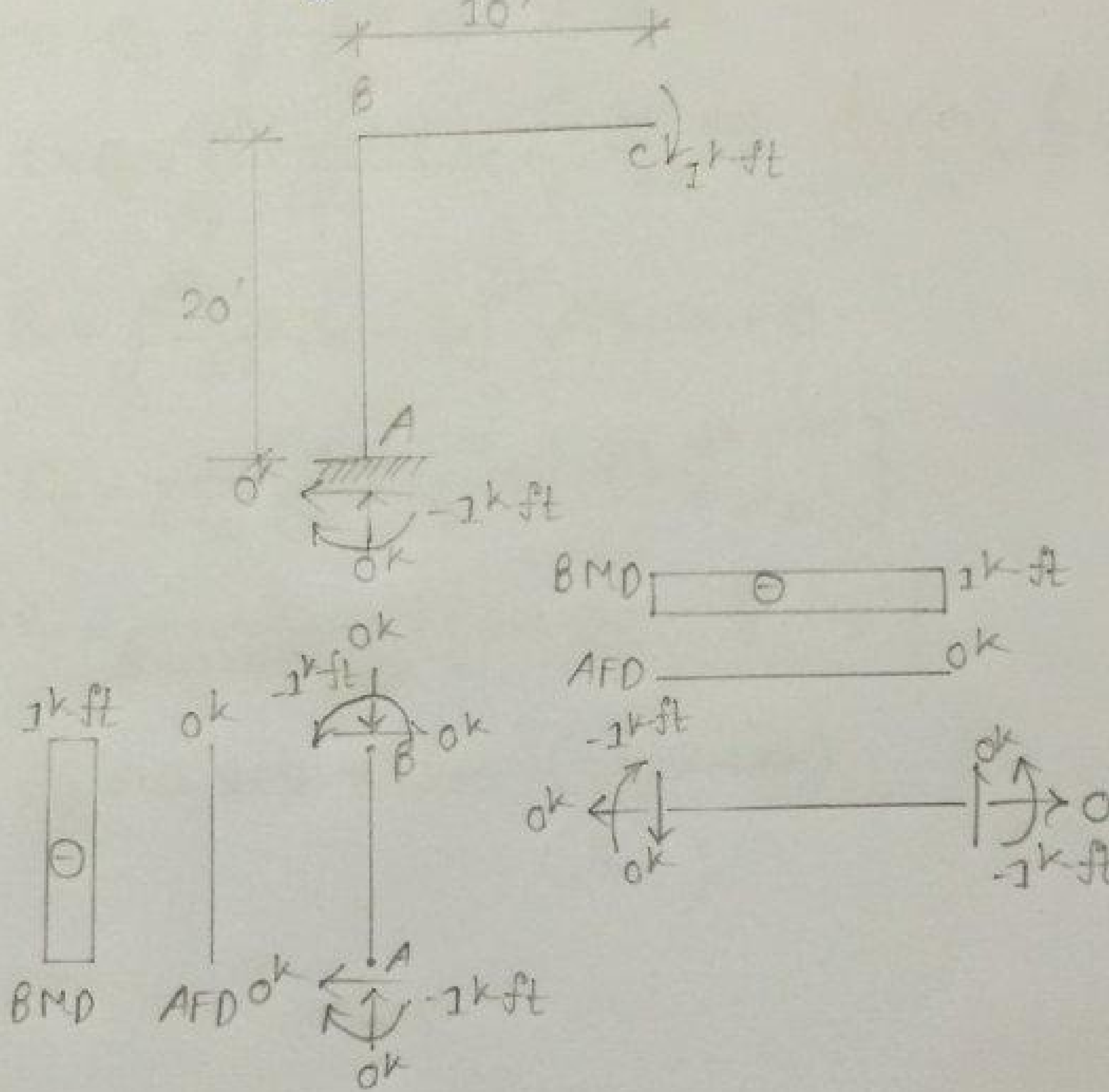



Sol$^n$: P- Force Analysis



## Q-Force Analysis:




Segment BA $(0 < x < 20)$

$M_Q = -1$ ; $M_P = -200$ ; $F_Q = 0$ ; $F_P = -20$ ; $I = I_1$

Segment CB $(0 < x < 10)$

$M_Q = -1$ ; $M_P = -20x$ ; $F_Q = 0$ ; $F_P = 0$ ; $I = \frac{3}{4} I_1$

Using principle of virtual work

$$\Sigma Q \cdot \theta_C = \int \frac{M_Q M_P}{EI} dx + \Sigma \frac{F_Q F_P L}{AE}$$

$$\Rightarrow 1 \times \theta_C = \int_B^A \frac{(-1)(-200)}{EI_1} dx + \sum_B^A \frac{(0)(-20)20}{AE} + \int_C^B \frac{(-1)(-20x)}{\frac{3}{4}EI_1} dx + \sum_C^B \frac{(0)(0)10}{AE}$$

$$\Rightarrow \theta_C = \int_0^{20} \frac{200}{EI_1} dx + 0 + \int_0^{10} \frac{20x}{\frac{3}{4}EI_1} dx + 0$$

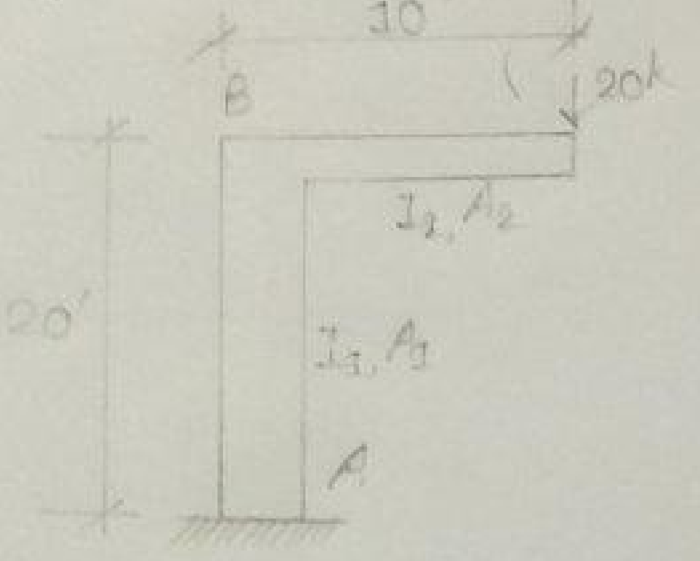
$$\Rightarrow \theta_C = \frac{1}{EI_1} \left\{ \left[200x\right]_0^{20} + \left[\frac{80}{3} \times \frac{x^2}{2}\right]_0^{10} \right\}$$

$$\Rightarrow \theta_C = \frac{1}{(30 \times 10^3 \times 144) \times \frac{200}{(144)^2}} \left\{ 200 \times (20) + \frac{40}{3} \times (10)^2 \right\}$$

$\therefore \theta_C = +0.128$ radian (clock-wise)

(Ans:)

Assignment - 17 : Compute vertical deflection at the mid-point of BC.




Given,

$I_1 = 200\ in^4$

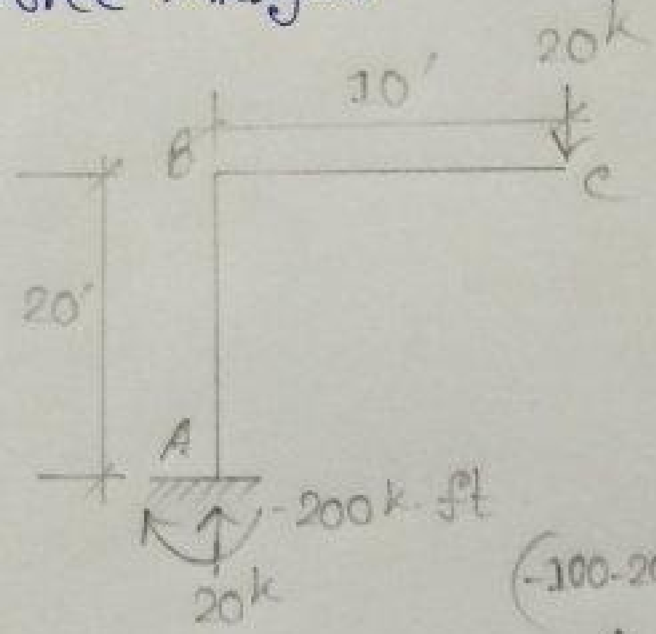
$I_2 = 150\ in^4$

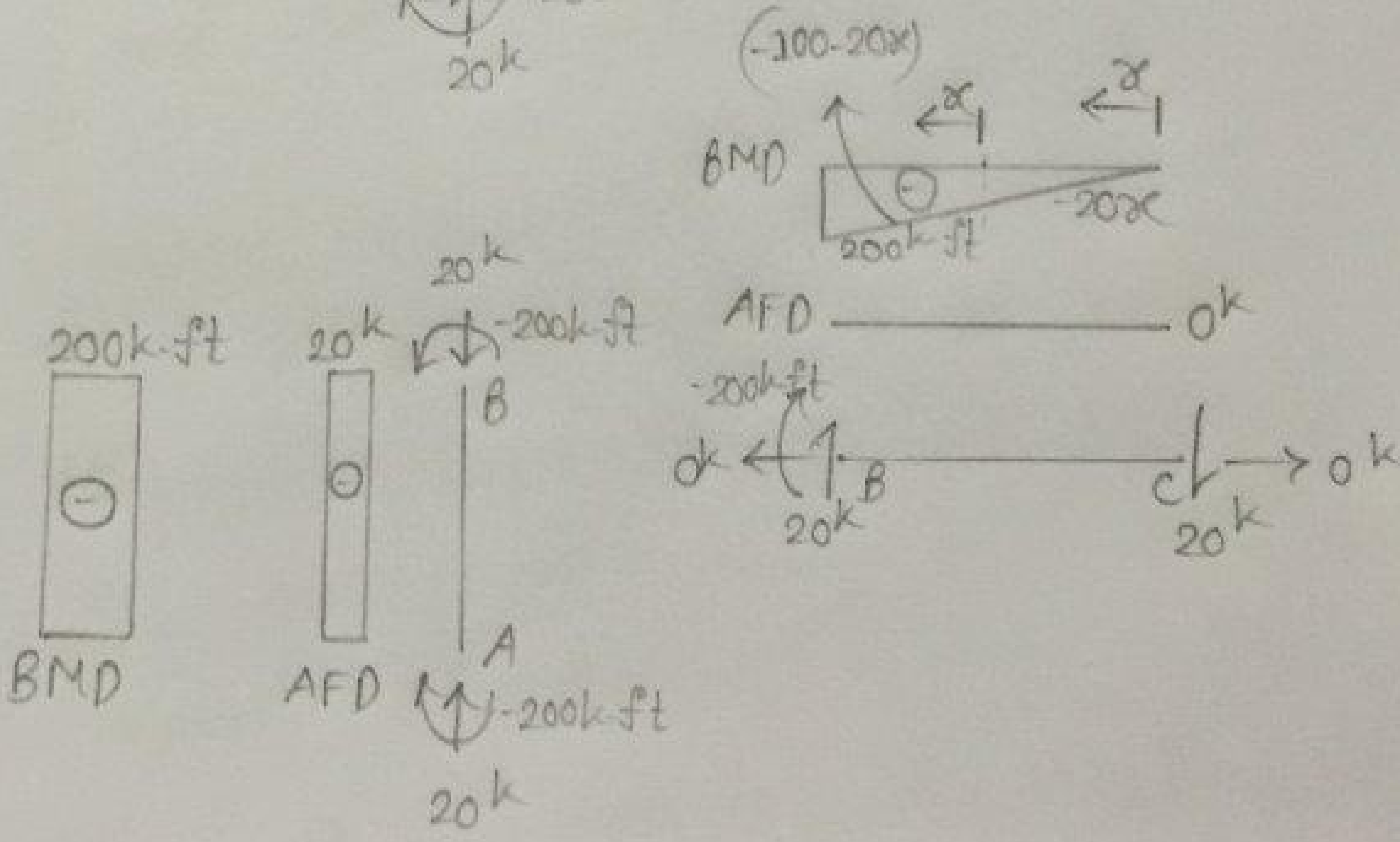
$A_1 = 15\ in^2$

$A_2 = 10\ in^2$

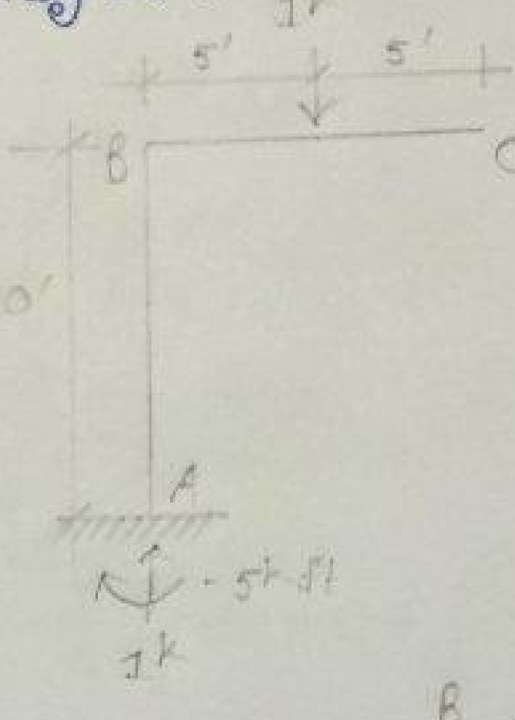
$E = 30{,}000\ ksi$

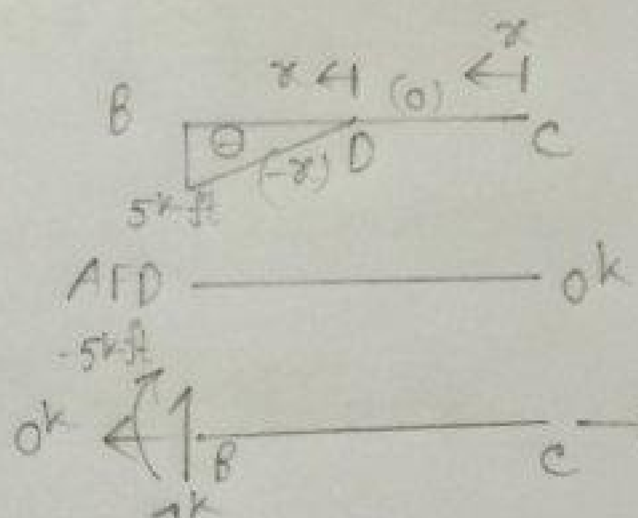
Soln : P- Force Analysis

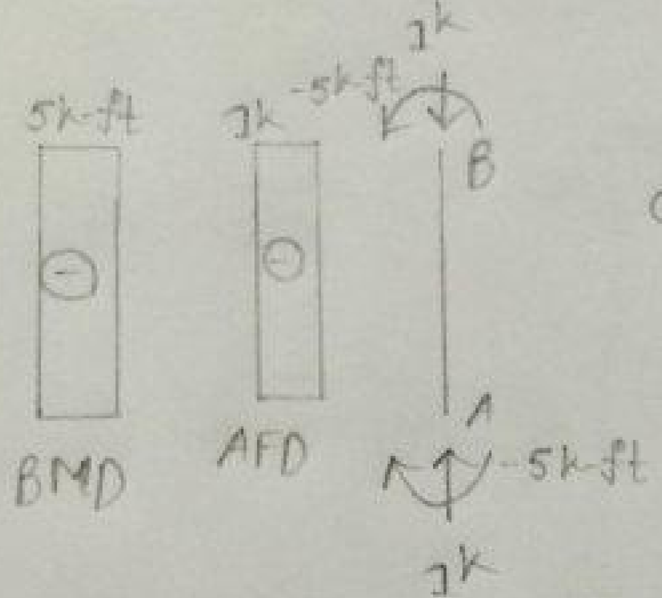


Q - Force Analysis :




Segment BA $(0<x<20)$

$M_Q = -5 \,;\; M_P = -200 \,;\; F_Q = -1 \,;\; F_P = -20 \,;\; I = I_1$

Segment CD $(0<x<5)$

$M_Q = 0 \,;\; M_P = -20x \,;\; F_Q = 0 \,;\; F_P = 0 \,;\; I = \frac{3}{4} I_1$

Segment DB $(0<x<5)$

$M_Q = -x \,;\; M_P = -100 - 20x \,;\; F_Q = 0 \,;\; F_P = 0 \,;\; I = \frac{3}{4} I_1$

Using principle of virtual work,

$$\sum Q \cdot \delta_{Dv} = \int \frac{M_Q M_P}{EI} dx + \sum \frac{F_Q F_P L}{AE}$$

$$\Rightarrow 1 \times \delta_{Dv} = \int_B^A \frac{(-5)(-200)}{EI_1} + \int_C^D \frac{(0)(-20x)dx}{\frac{3}{4}EI_1} + \int_D^B \frac{(-x)(-100-20x)}{\frac{3}{4}EI_1} dx$$

$$+ \sum_B^A \frac{(-1)(-20)\,20}{15 \times 30 \times 10^3 \times 144} + \sum_C^D \frac{(0)(0)5}{AE} + \sum_D^B \frac{(0)(0)5}{AE}$$

$$\Rightarrow \delta_{Dv} = \int_0^{20} \frac{1000}{EI_1} dx + 0 + \int_0^5 \frac{100x + 20x^2}{\frac{3}{4}EI_1} dx + 8.89 \times 10^{-4}$$

$$\Rightarrow \delta_{Dv} = \frac{1}{EI_1} \left\{ \int_0^{20} 1000\, dx + \frac{4}{3} \int_0^5 (100x + 20x^2) dx \right\} + 8.89 \times 10^{-4}$$

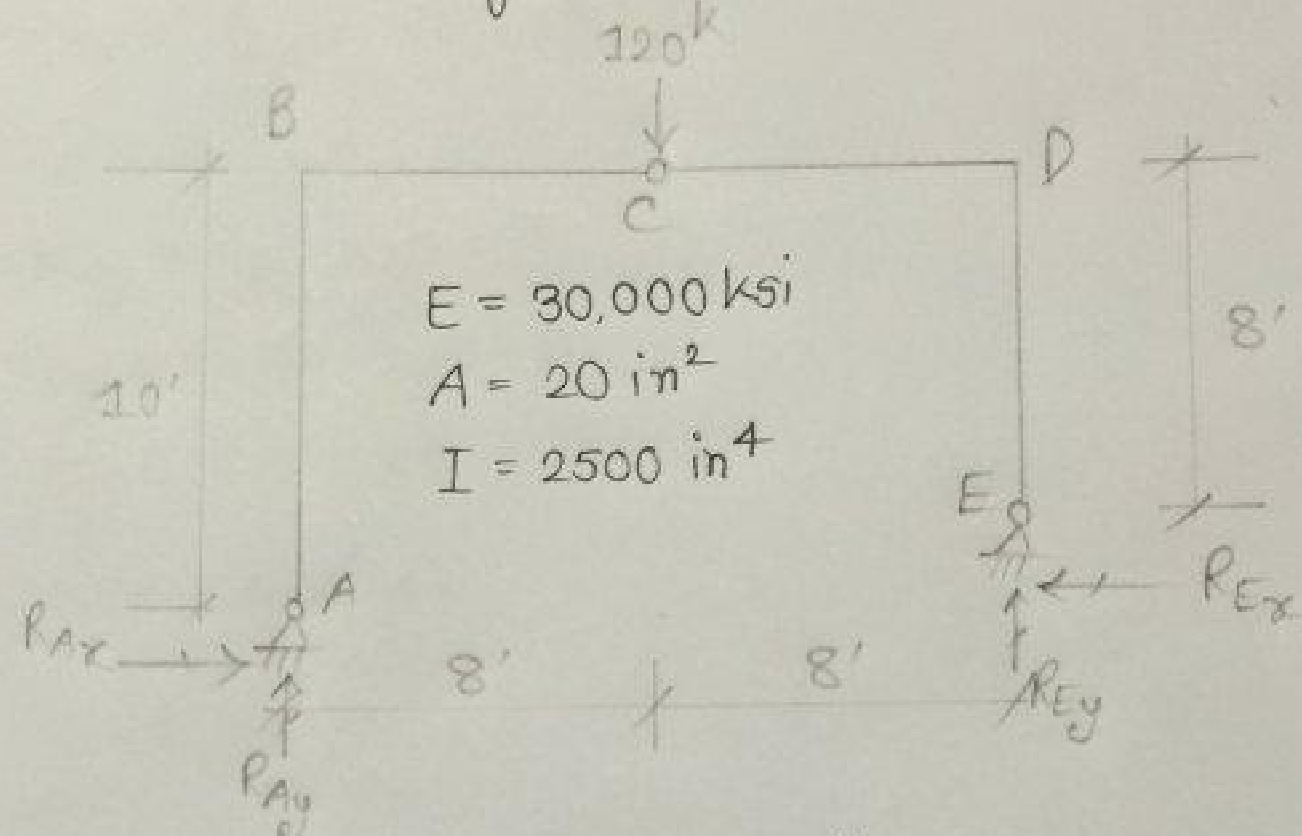
$$\Rightarrow \delta_{Dv} = \frac{1}{EI_1} \left\{ \left[1000x\right]_0^{20} + \frac{4}{3} \left[\frac{100x^2}{2} + \frac{20x^3}{3}\right]_0^5 \right\} + 8.89 \times 10^{-4}$$

$$\Rightarrow \delta_{Dv} = \frac{1}{(30 \times 10^3 \times 144) \times \frac{200}{(144)^2}} \left\{ (1000 \times 20) + \frac{4}{3} \left[\frac{100(5)^2}{2} + \frac{20(5)^3}{3}\right] \right\} + 8.89 \times 10^{-4}$$

$\therefore \delta_{Dv} = +0.5476$ ft (↓ downward)

(Ans:)

Assignment-18 : Compute change in slope of the cross-section on left hinge.



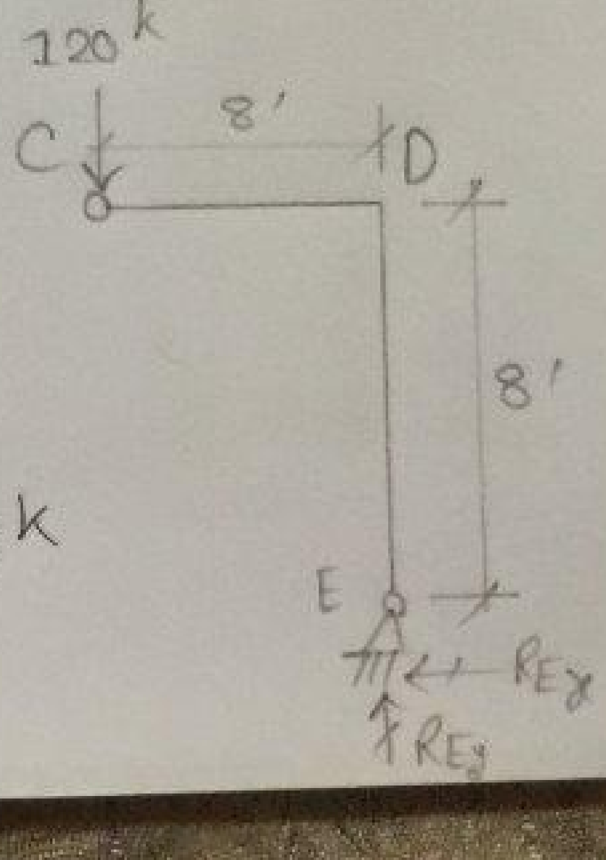

Sol$^n$ : From the whole structure,

$\Sigma M @ A = 0$ ↻+

$\Rightarrow 120 \times 8 - R_{Ex} \times 2 - R_{Ey} \times 16 = 0$

$\Rightarrow 2R_{Ex} + 16\, R_{Ey} = 960$ . . . . . ①

From the right part,




$\Sigma M @ C = 0$ ↻+

$\Rightarrow -8R_{Ey} + 8R_{Ex} = 0$

$\Rightarrow -R_{Ey} + R_{Ex} = 0$ . . . . . ②

By solving eq$^n$ ① & ② we get,

$R_{Ex} = +53.33^k$ & $R_{Ey} = +53.33^k$

$\sum F_x = 0 \longrightarrow +$

$\Rightarrow R_{Ax} - R_{Ex} = 0$

$\Rightarrow R_{Ax} = R_{Ex} = 53.33^k$

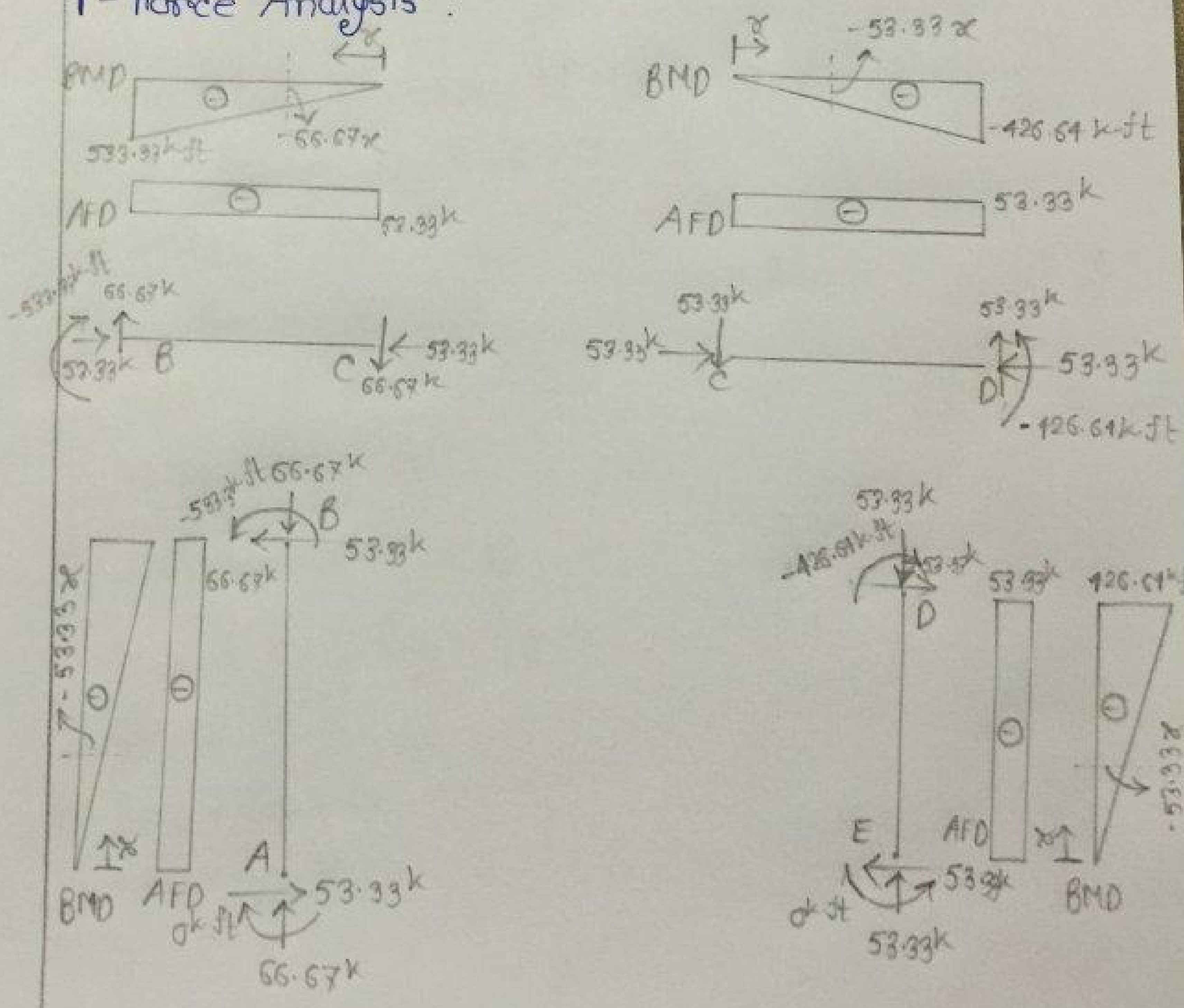
$\sum F_y = 0 \quad \uparrow +$

$\Rightarrow R_{Ay} + R_{Ey} - 120 = 0$

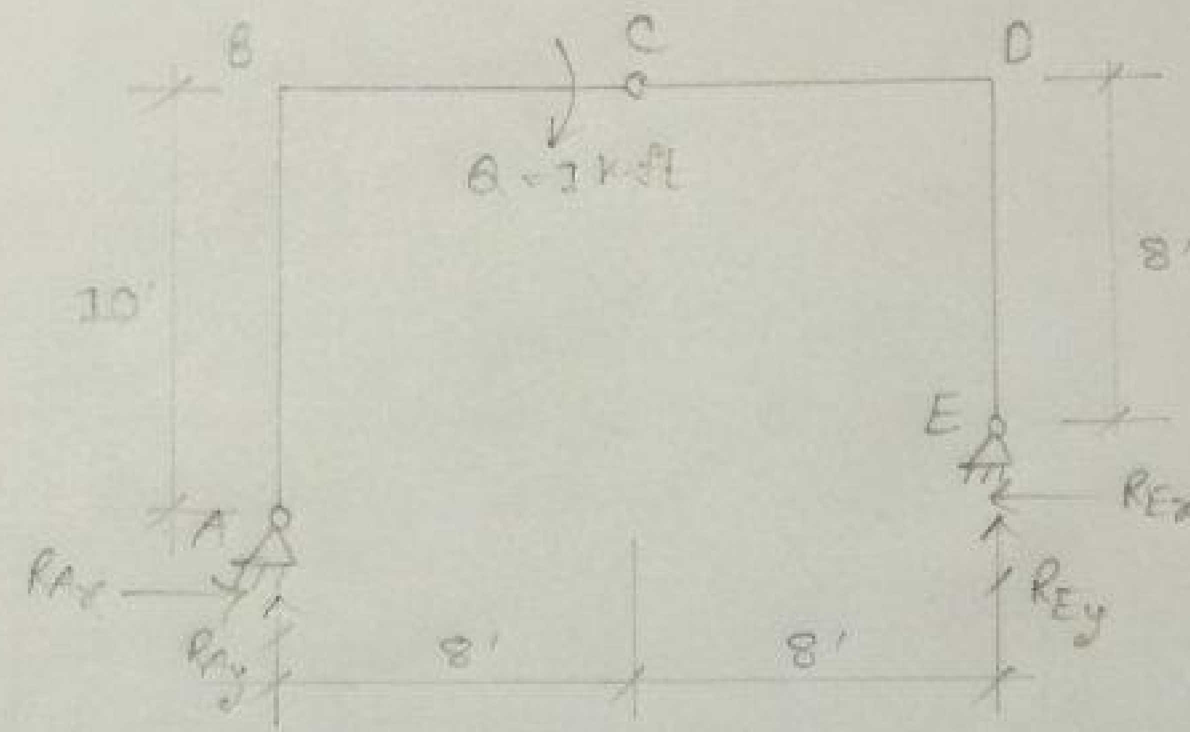
$\Rightarrow R_{Ay} + 53.33 - 120 = 0$

$\therefore R_{Ay} = 66.67^k$

P- Force Analysis :



Q- Force Analysis :

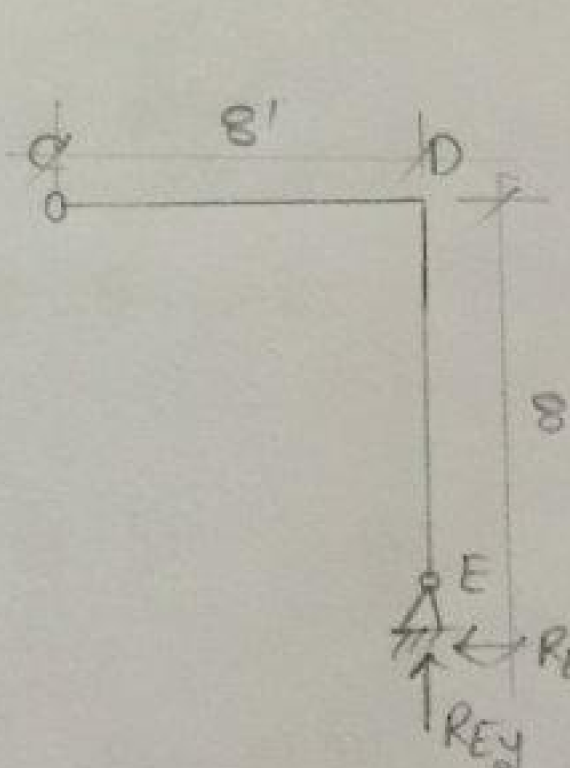



From whole structure, $\Sigma M @ A = 0 \circlearrowright +$

$\Rightarrow 1 - R_{Ex} \times 2 - R_{Ey} \times 16 = 0$

$\Rightarrow 2 R_{Ex} + 16 R_{Ey} = 1 \quad \ldots\ldots (1)$

From the right part,




$\Sigma M @ C = 0 \circlearrowright +$

$\Rightarrow 8 R_{Ex} - 8 R_{Ey} = 0 \quad \ldots\ldots (2)$

By solving eq$^n$ (1) & (2),

$R_{Ex} = \frac{1}{18} k$ , $R_{Ey} = \frac{1}{18} k$

$\Sigma F_x = 0 \rightarrow +$

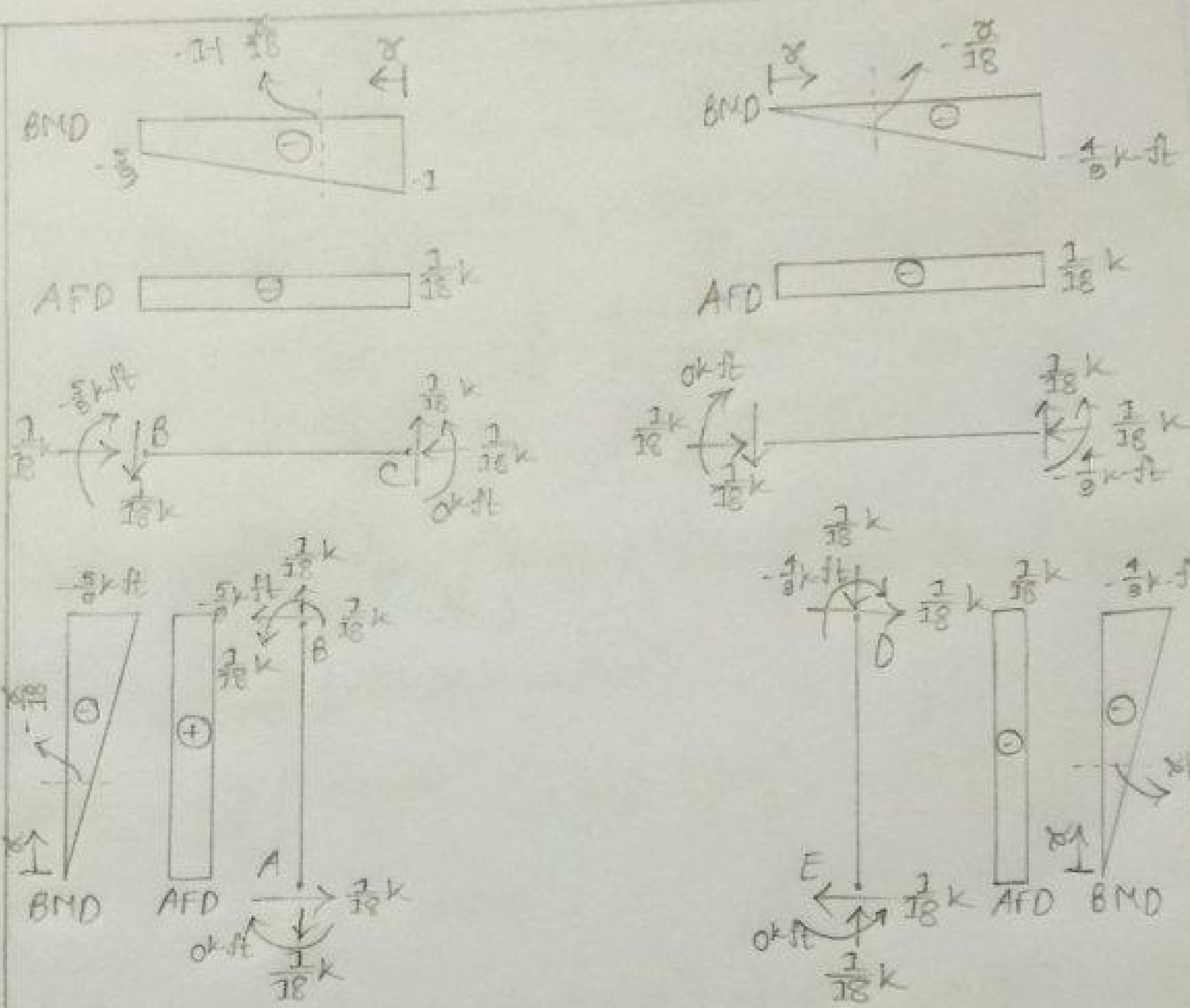
$\Rightarrow -R_{Ex} + R_{Ax} = 0$

$\therefore R_{Ax} = \frac{1}{18} k$

$\Sigma F_y = 0 \uparrow +$

$\Rightarrow R_{Ay} + R_{Ey} = 0$

$\therefore R_{Ay} = -\frac{1}{18} k$




Segment AB $(0<x<10)$

$M_Q = -\frac{x}{18}$ ; $M_P = -53.33x$ ; $F_Q = \frac{1}{18}$ ; $F_P = -66.67$

Segment CB $(0<x<8)$

$M_Q = -1+\frac{x}{18}$ ; $M_P = -66.67x$ ; $F_Q = -\frac{1}{18}$ ; $F_P = -53.33$

Segment ED $(0<x<8)$

$M_Q = -\frac{x}{18}$ ; $M_P = -53.33x$ ; $F_Q = -\frac{1}{18}$ ; $F_P = -53.33$

Segment CD $(0<x<8)$

$M_Q = -\frac{x}{18}$ ; $M_P = -53.33x$ ; $F_Q = -\frac{1}{18}$ ; $F_P = -53.33$

Using principle of virtual work,

$$\sum Q \cdot \theta_{CL} = \int \frac{M_Q M_P}{EI} dx + \sum \frac{F_Q F_P L}{AE}$$

$$\Rightarrow 1 \times \theta_{CL} = \int_A^B \frac{\left(-\frac{x}{18}\right)(-53.33x)}{EI} dx + \frac{\left(\frac{1}{18}\right)(-66.67)10}{AE} + \int_C^B \frac{\left(-1+\frac{x}{18}\right)(-66.67x)}{EI} dx$$

$$+ \frac{\left(-\frac{1}{18}\right)(-53.33)8}{AE} + \int_E^D \frac{\left(-\frac{x}{18}\right)(-53.33x)}{EI} dx + \frac{\left(-\frac{1}{18}\right)(-53.33)8}{AE}$$

$$+ \int_C^D \frac{\left(-\frac{x}{18}\right)(-53.33x)}{EI} dx + \frac{\left(-\frac{1}{18}\right)(-53.33)8}{AE}$$

$$\Rightarrow \theta_{CL} = \int_0^{10} \frac{2.96x^2}{EI} dx + \int_0^8 \frac{(66.67x - 3.7x^2)}{EI} dx + \int_0^8 \frac{2.96x^2}{EI} dx + \int_0^8 \frac{2.96x^2}{EI} dx$$

$$+ \frac{1}{AE}\left[-37.04 + 3 \times 23.7\right]$$

$$\Rightarrow \theta_{CL} = \frac{1}{EI}\left\{\left[\frac{2.96x^3}{3}\right]_0^{10} + \left[\frac{66.67x^2}{2} - \frac{3.7x^3}{3}\right]_0^8 + 2 \times \left[\frac{2.96x^3}{3}\right]_0^8\right\}$$

$$+ \frac{1}{AE} \times 34.06$$

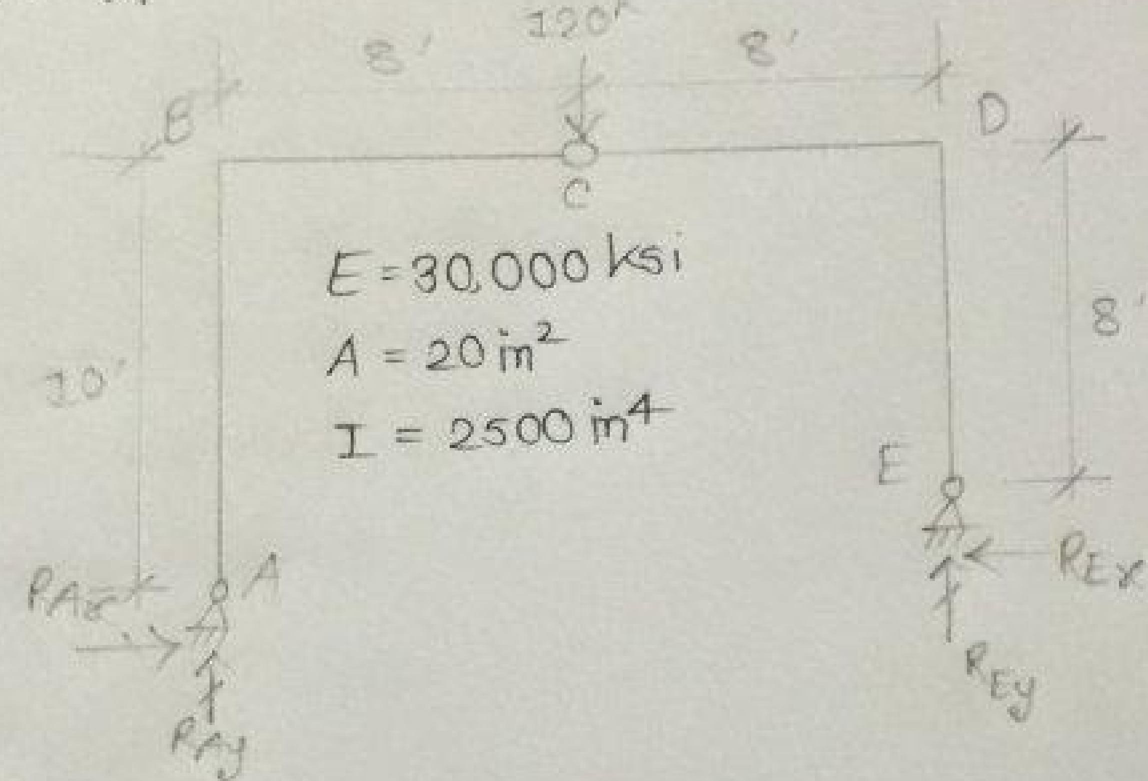
$$\Rightarrow \theta_{CL} = \frac{1}{30 \times 10^3 \times 144 \times \frac{2500}{(144)^2}}\left\{0.99 \times (10)^3 + 33.34(8)^2 - 1.24(8)^3 + 1.97(8)^3\right\}$$

$$+ \frac{34.06}{\frac{20 \times 30 \times 10^3 \times 144}{144}}$$

$$\therefore \theta_{CL} = +0.0068 \text{ radian (clockwise)}$$

(Ans:)

Assignment - 19: Determine the horizontal deflection at point 'B'.




Sol$^n$: From the whole structure,

$\sum M @ A = 0$ ↻+

$\Rightarrow 120 \times 8 - R_{Ex} \times 2 - R_{Ey} \times 16 = 0$

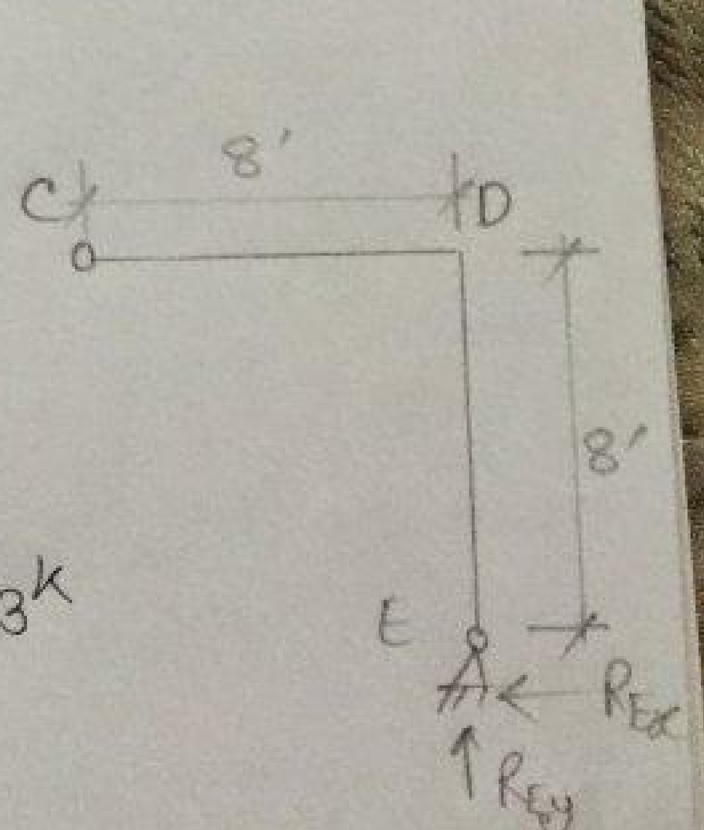
$\Rightarrow 2R_{Ex} + 16R_{Ey} = 960$ . . . . . ①

From the right part,

$\sum M @ C = 0$ ↻+

$\Rightarrow -8R_{Ey} + 8R_{Ex} = 0$

$\Rightarrow R_{Ex} - R_{Ey} = 0$ . . . . . ②




By solving eq$^n$ ① & ②,

$R_{Ex} = +53.33^k$ & $R_{Ey} = +53.33^k$

$$\sum F_x = 0 \rightarrow +$$

$$\Rightarrow R_{Ax} - R_{Ex} = 0$$

$$\therefore R_{Ax} = +53.33^k$$

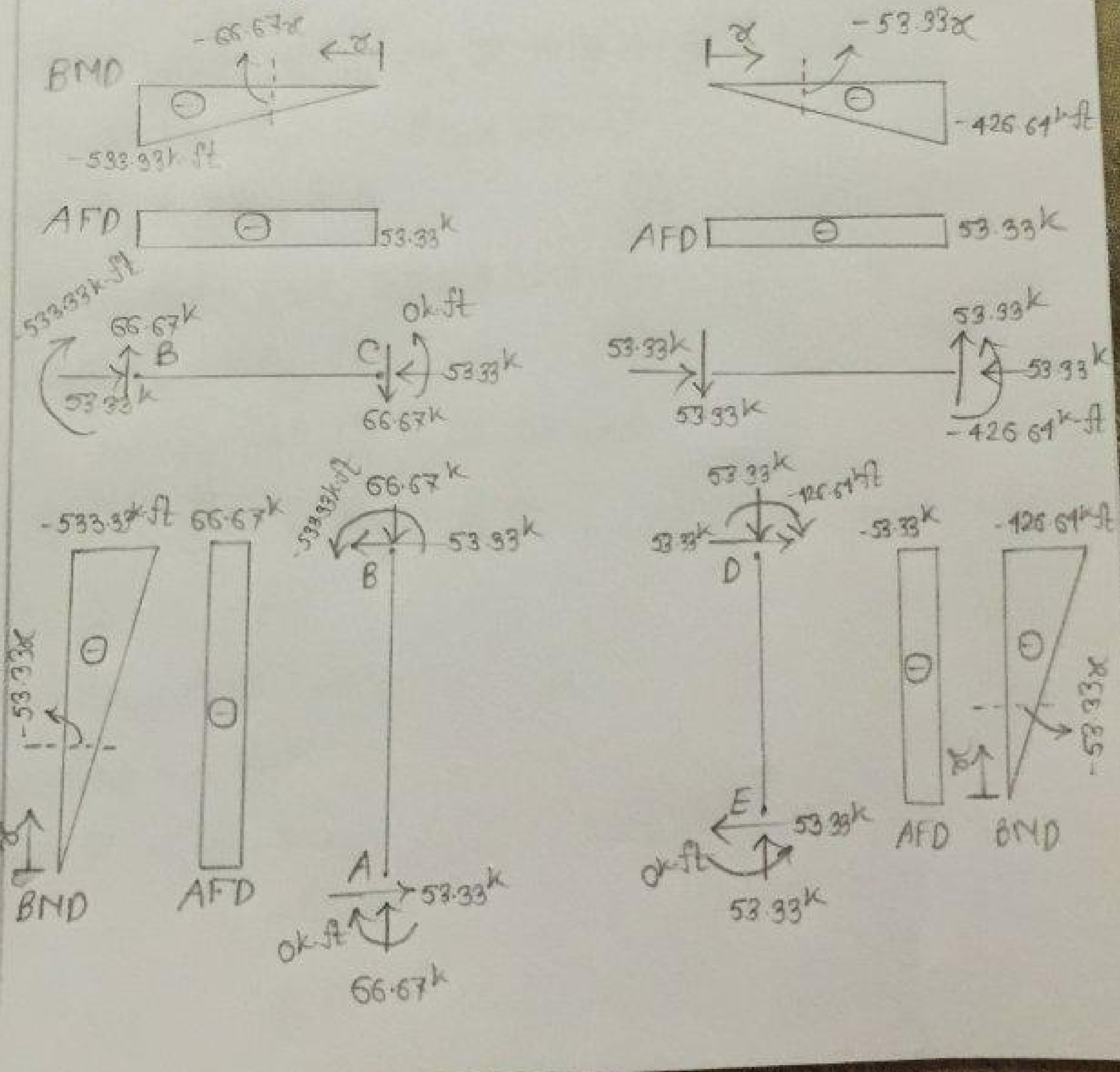
$$\sum F_y = 0 \uparrow +$$

$$\Rightarrow R_{Ay} + R_{Ey} - 120 = 0$$

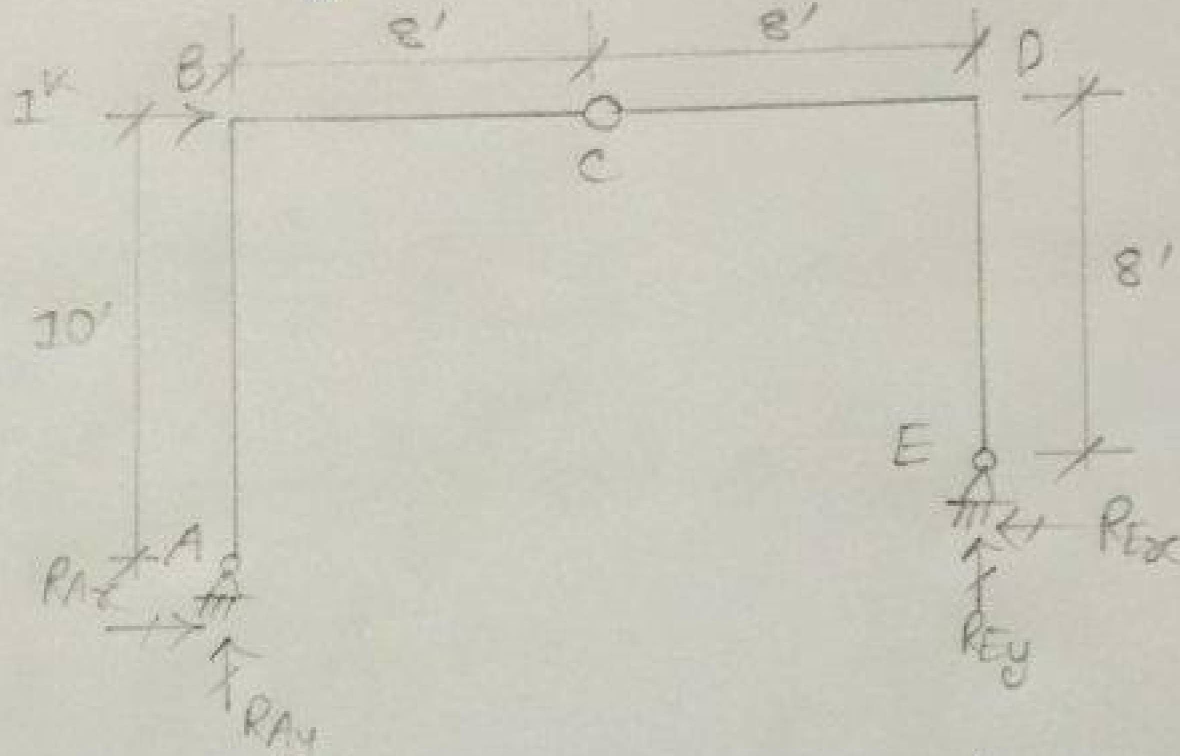
$$\Rightarrow R_{Ay} + 53.33 - 120 = 0$$

$$\therefore R_{Ay} = +66.67^k$$

P- Force Analysis :



Q- Force Analysis :

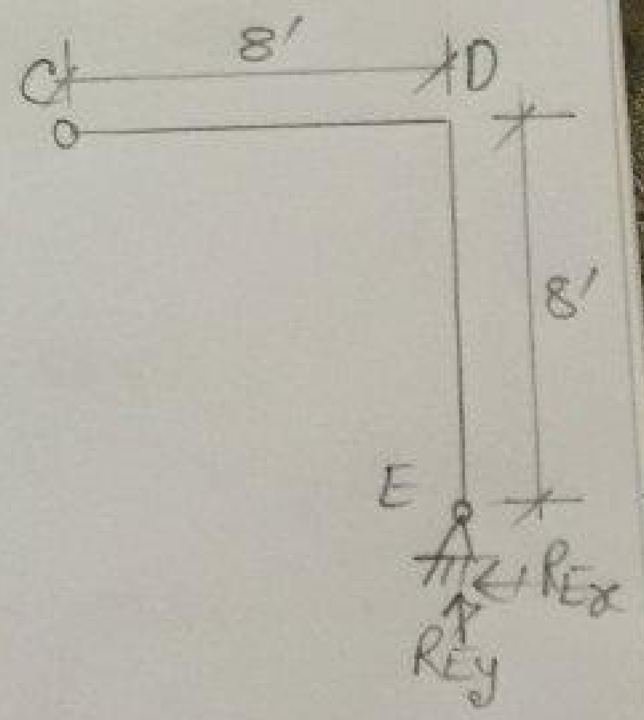



From whole structure, $\Sigma M @ A = 0 \curvearrowright +$

$\Rightarrow 1 \times 10 - R_{Ex} \times 2 - R_{Ey} \times 16 = 0$

$\Rightarrow R_{Ex} + 8R_{Ey} = 5 \quad \ldots\ldots (1)$

From right part, $\Sigma M @ C = 0 \curvearrowright +$




$\Rightarrow 8R_{Ex} - 8R_{Ey} = 0 \quad \ldots\ldots (2)$

By solving eq$^n$ (1) & (2), we get

$R_{Ex} = \frac{5}{9} k$ & $R_{Ey} = \frac{5}{9} k$

$\Sigma F_x = 0 \rightarrow +$

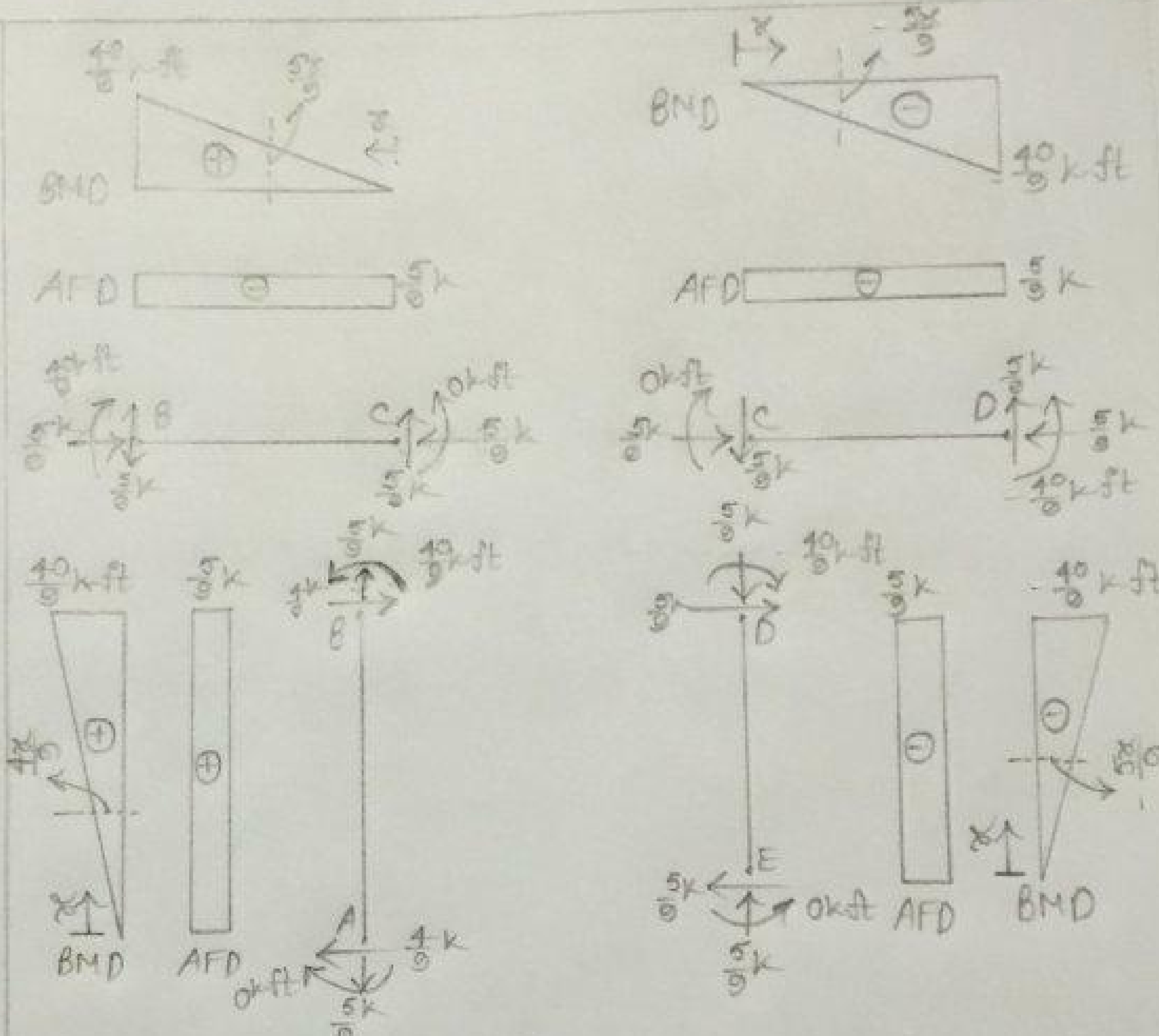
$\Rightarrow R_{Ax} - R_{Ex} + 1 = 0$

$\therefore R_{Ax} = -\frac{4}{9} k$

$\Sigma F_y = 0 \uparrow +$

$\Rightarrow R_{Ay} + R_{Ey} = 0$

$\therefore R_{Ay} = -\frac{5}{9} k$




Segment AB $(0 < x < 10)$

$M_Q = \frac{4x}{9}$ ; $M_P = -53.33x$ , $F_Q = \frac{5}{9}$ ; $F_P = -66.67$

Segment CB $(0 < x < 8)$

$M_P = -66.67x$ ; $M_Q = \frac{5x}{9}$ ; $F_Q = -\frac{5}{9}$ ; $F_P = -53.33$

Segment ED $(0 < x < 8)$

$M_Q = -\frac{5x}{9}$ , $M_P = -53.33x$ ; $F_Q = -\frac{5}{9}$ ; $F_P = -53.33$

Segment CD $(0 < x < 8)$

$M_Q = -\frac{5x}{9}$ ; $M_P = -53.33x$ ; $F_Q = -\frac{5}{9}$ ; $F_P = -53.33$

Using principle of virtual work,

$$\sum Q.\delta_{Bh} = \int \frac{M_Q M_P}{EI} dx + \sum \frac{F_Q F_P L}{AE}$$

$$\Rightarrow 1 \times \delta_{Bh} = \int_A^B \frac{\left(\frac{4x}{3}\right)(-53.33x)}{EI} dx + \sum_A^B \frac{\left(\frac{5}{3}\right)(-66.67)10}{AE} + \int_C^B \frac{(-66.67x)\left(\frac{5x}{3}\right)}{EI} dx$$

$$+ \sum_C^B \frac{\left(-\frac{5}{3}\right)(-53.33)8}{AE} + \int_E^D \frac{\left(-\frac{5x}{3}\right)(-53.33x)}{EI} dx + \sum_E^D \frac{\left(-\frac{5}{3}\right)(-53.33)8}{AE}$$

$$+ \int_C^D \frac{\left(-\frac{5x}{3}\right)(-53.33x)}{EI} dx + \sum_C^D \frac{\left(-\frac{5}{3}\right)(-53.33)8}{AE}$$

$$\Rightarrow \delta_{Bh} = \int_0^{10} \frac{-237x^2}{EI} dx + \int_0^8 \frac{-37.04x^2}{EI} dx + \int_0^8 \frac{29.63x^2}{EI} dx$$

$$+ \int_0^8 \frac{29.63x^2}{EI} dx - \frac{370.39}{AE} + \frac{237.02}{AE} \times 3$$

$$\Rightarrow \delta_{Bh} = \frac{1}{EI} \left\{ \int_0^{10} (-23.7x^2) dx + \int_0^8 (-37.04x^2) dx + 2\int_0^8 (29.63x^2) dx \right\}$$

$$+ \frac{340.67}{AE}$$

$$\Rightarrow \delta_{Bh} = \frac{1}{EI} \left\{ \left[ -\frac{23.7x^3}{3} \right]_0^{10} + \left[ -\frac{37.04x^3}{3} \right]_0^8 + 2\left[ \frac{29.63x^3}{3} \right]_0^8 \right\}$$

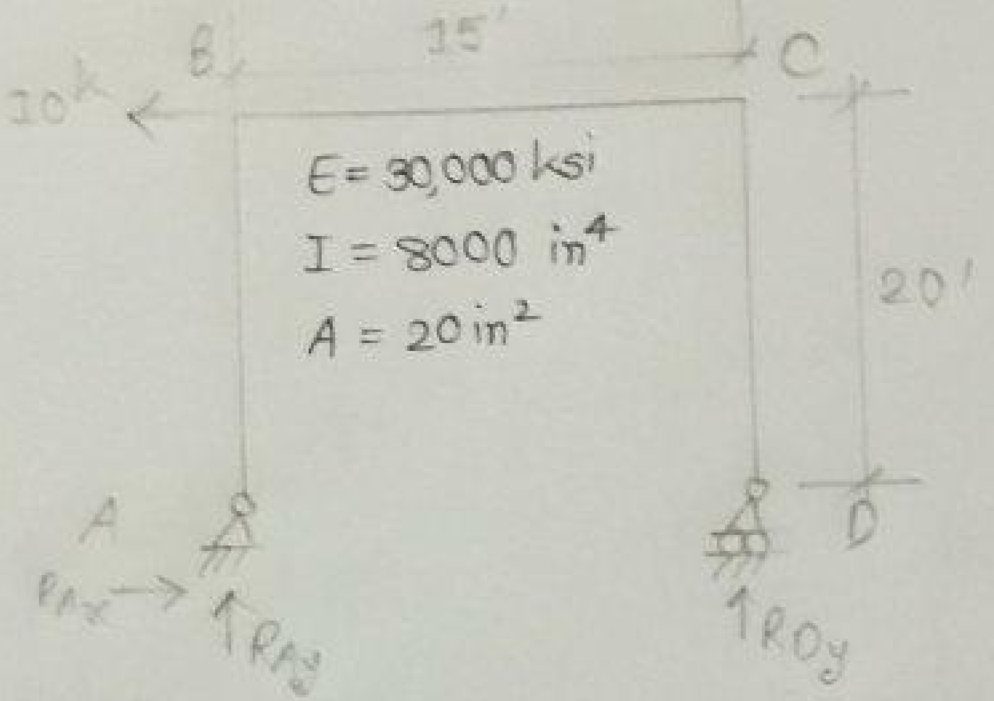
$$+ \frac{340.67}{AE}$$

$$\Rightarrow \delta_{Bh} = \frac{1}{30 \times 10^3 \times 144 \times \frac{2500}{(144)^2}} \left\{ -7.9(10)^3 - 12.35(8)^3 + 2 \times 9.88(8)^3 \right\} + \frac{340.67}{20 \times 30 \times 10^3}$$

$$\therefore \delta_{Bh} = -0.0073 \text{ ft} \quad (\leftarrow \cancel{\text{left}}\text{ward})$$

(Ans:)

Assignment-20: Find $\delta_{Bh}$, $\delta_{Ch}$, $\theta_B$, $\theta_C$.




Sol$^n$: $\Sigma M @ A = 0$ ↻+

$\Rightarrow -10 \times 20 - R_{Dy} \times 15 = 0$

$\therefore R_{Dy} = -\frac{40}{3}$ k

$\Sigma F_x = 0 \rightarrow +$

$\Rightarrow R_{Ax} - 10 = 0$

$\therefore R_{Ax} = 10^k$

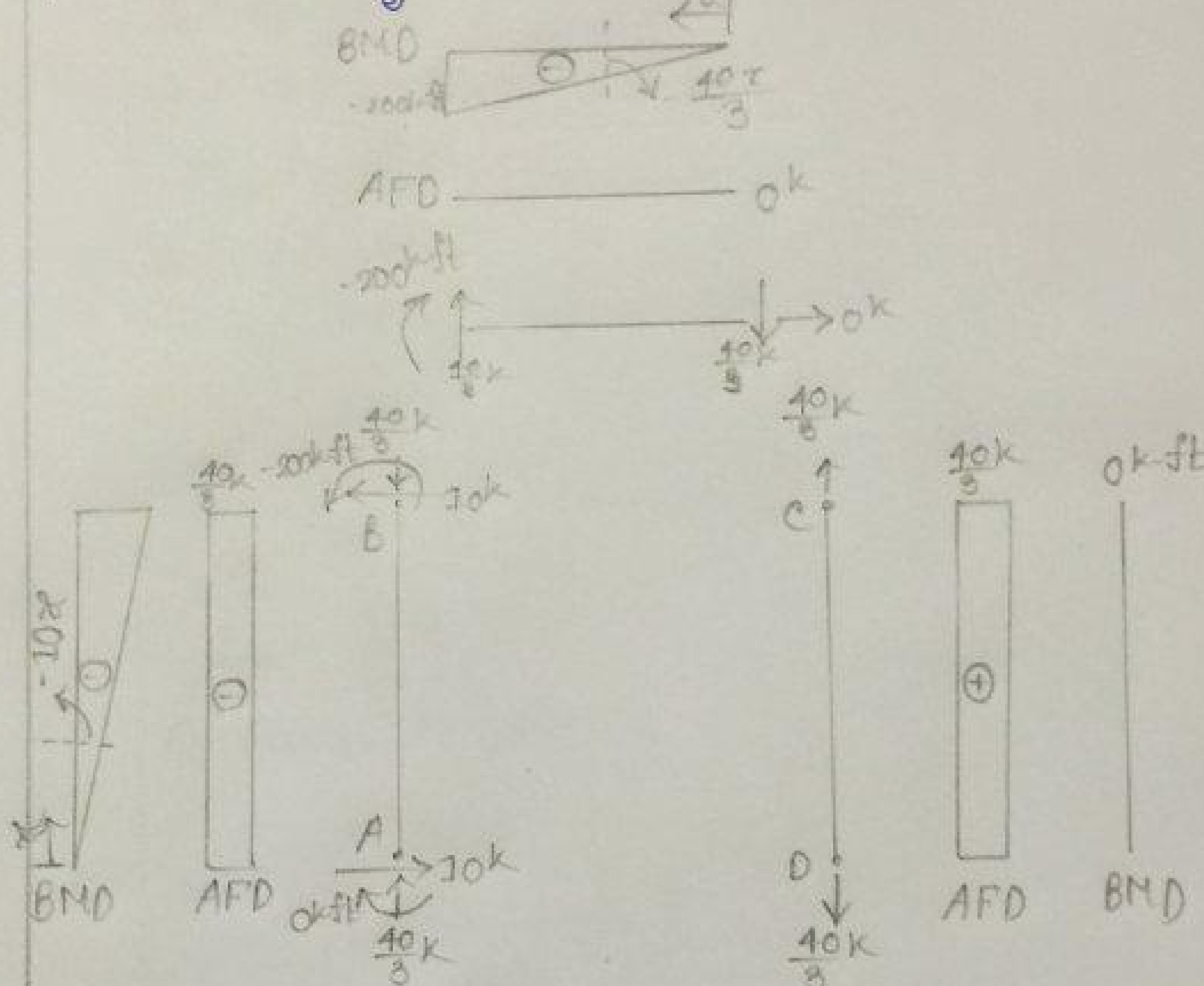
$\Sigma F_y = 0 \uparrow +$

$\Rightarrow R_{Ay} + R_{Dy} = 0$

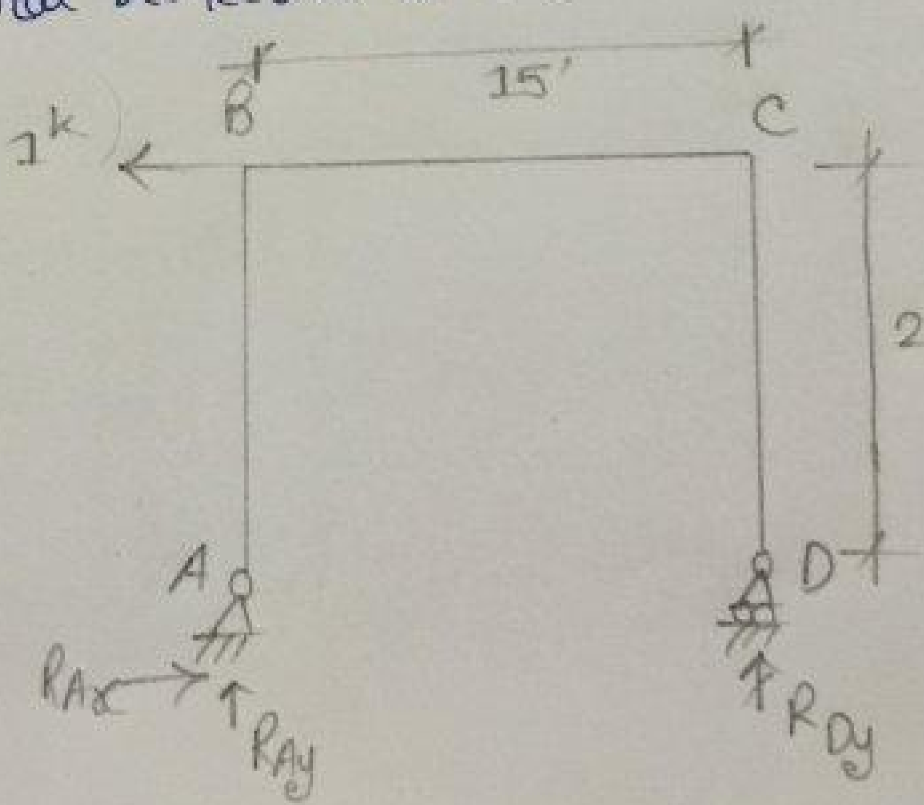
$\Rightarrow R_{Ay} - \frac{40}{3} = 0$

$\therefore R_{Ay} = \frac{40}{3}$ k

P- Force Analysis :




Horizontal Deflection at 'B' :



$\Sigma M @ A = 0 \circlearrowright +$

$\Rightarrow -1 \times 20 - R_{Dy} \times 15 = 0$

$\therefore R_{Dy} = -\frac{4}{3}k$

$\Sigma F_x = 0 \rightarrow +$

$\Rightarrow R_{Ax} - 1 = 0$

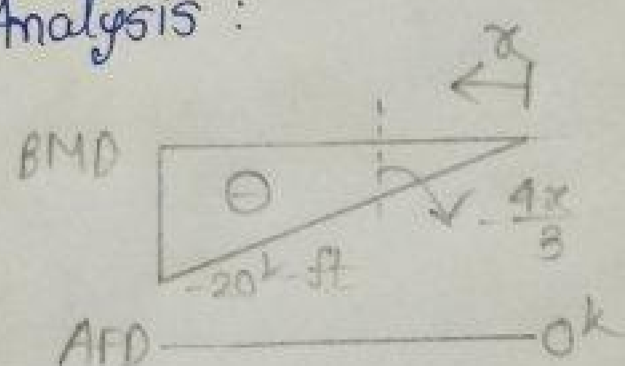
$\therefore R_{Ax} = 1^k$

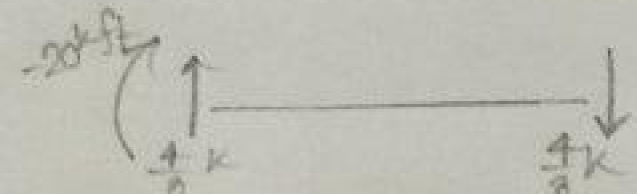
$\Sigma F_y = 0 \uparrow +$

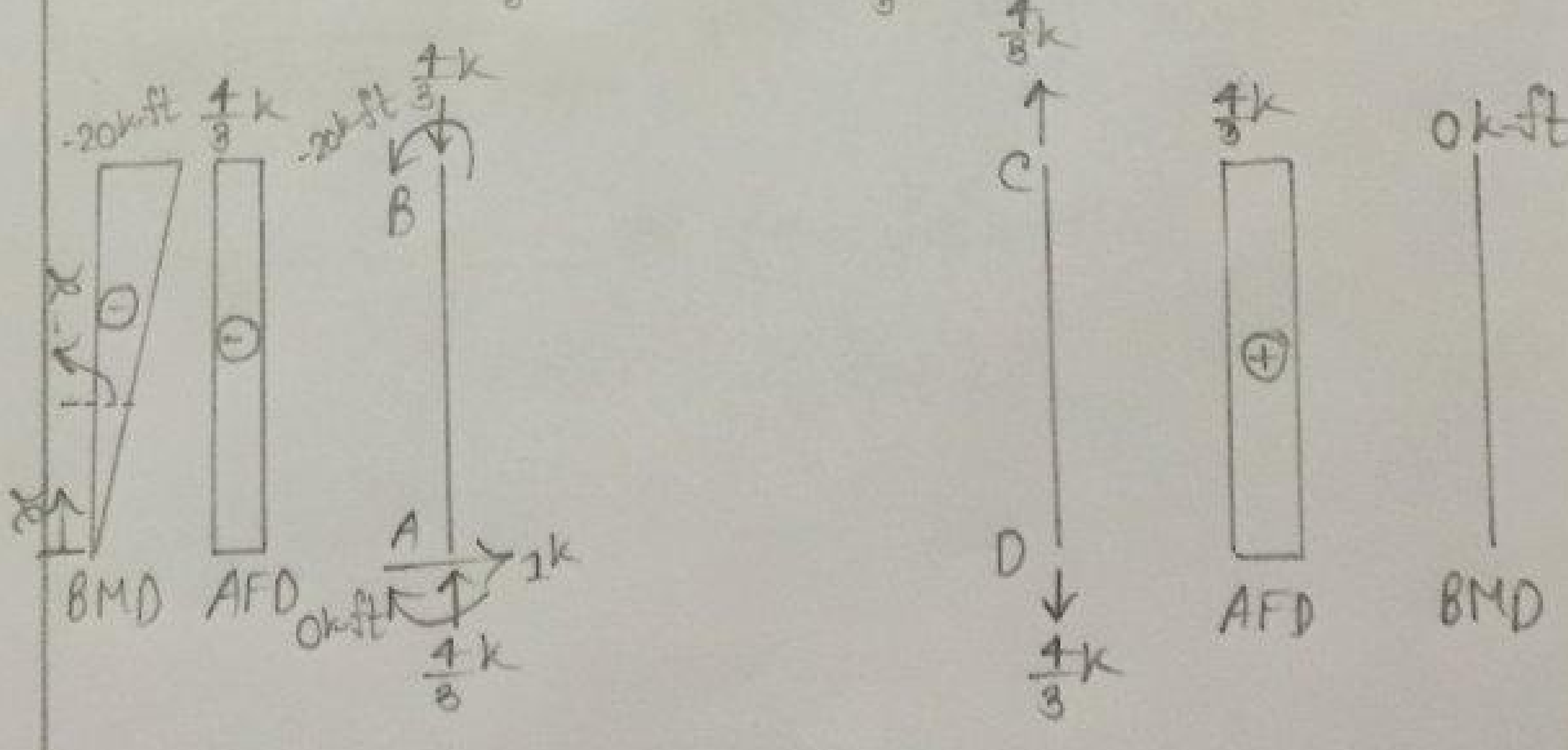
$\Rightarrow R_{Ay} + R_{Dy} = 0$

$\therefore R_{Ay} = \frac{4}{3}k$

Q - Force Analysis :



Segment AB $(0<x<20)$

$M_Q = -x$ ; $M_P = -10x$ ; $F_Q = -\frac{4}{3}$ ; $F_P = -\frac{40}{3}$

Segment DC $(0<x<20)$

$M_Q = 0$ ; $M_P = 0$ ; $F_Q = \frac{4}{3}$ ; $F_P = \frac{40}{3}$

Segment CB $(0<x<15)$

$M_Q = -\frac{4x}{3}$ ; $M_P = -\frac{40x}{3}$ ; $F_Q = 0$ ; $F_P = 0$

Using principle of virtual work,

$$\Sigma Q . \delta_{Bh} = \int \frac{M_Q M_P}{EI} dx + \Sigma \frac{F_Q F_P L}{AE}$$

$$\Rightarrow 1 \times \delta_{Bh} = \int_A^B \frac{(-x)(-10x)}{EI} dx + \sum_A^B \frac{(-\frac{4}{3})(-\frac{40}{3})20}{AE} + \int_D^C \frac{(0)(0)}{EI} dx + \sum_D^C \frac{(\frac{4}{3})(\frac{40}{3})20}{AE}$$

$$+ \int_C^B \frac{(-\frac{4x}{3})(-\frac{40x}{3})}{EI} dx + \sum_C^B \frac{(0)(0)15}{AE}$$

$$\Rightarrow \delta_{Bh} = \frac{1}{EI}\left\{ \int_0^{20} (10x^2) dx + 0 + \int_0^{15} \frac{160x^2}{9} dx \right\} + \frac{1}{AE}\left( \frac{3200}{9} + \frac{3200}{9} \right)$$

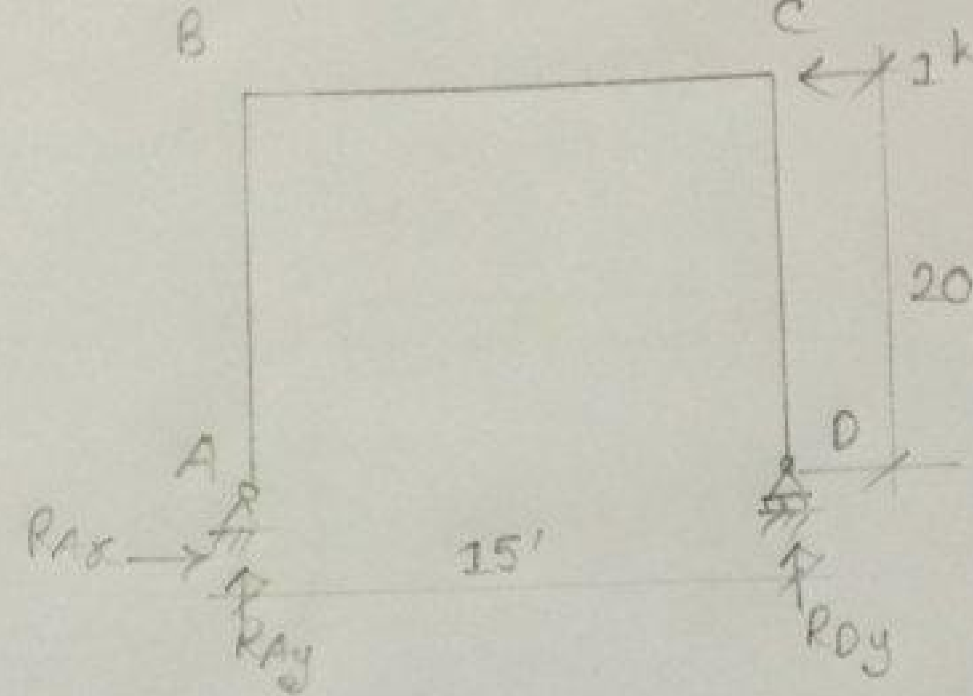
$$\Rightarrow \delta_{Bh} = \frac{1}{EI}\left\{ \left[\frac{10x^3}{3}\right]_0^{20} + \left[\frac{160x^3}{9\times 3}\right]_0^{15} \right\} + \frac{1}{AE} \times \frac{6400}{9}$$

$$\Rightarrow \delta_{Bh} = \frac{1}{30{,}000 \times 144 \times \frac{8000}{(144)^2}} \left\{ \frac{10}{3}(20)^3 + \frac{160}{9\times 3}(15)^3 \right\} + \frac{1}{20 \times 30{,}000} \times \frac{6400}{9}$$

$\therefore \delta_{Bh} = 0.0292$ ft ($\leftarrow$ leftward)

(Ans:)

Horizontal Deflection at 'C' :




$\Sigma M @ A = 0 \curvearrowright +$

$\Rightarrow -1 \times 20 - R_{Dy} \times 15 = 0$

$\therefore R_{Dy} = -\frac{4}{3} k$

$\Sigma F_x = 0 \rightarrow +$

$\Rightarrow R_{Ax} - 1 = 0$

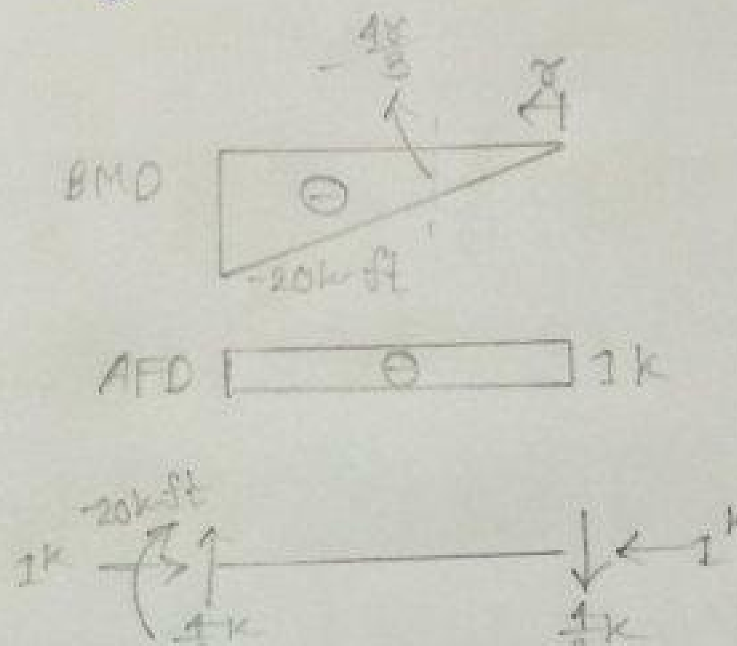
$\therefore R_{Ax} = 1^k$

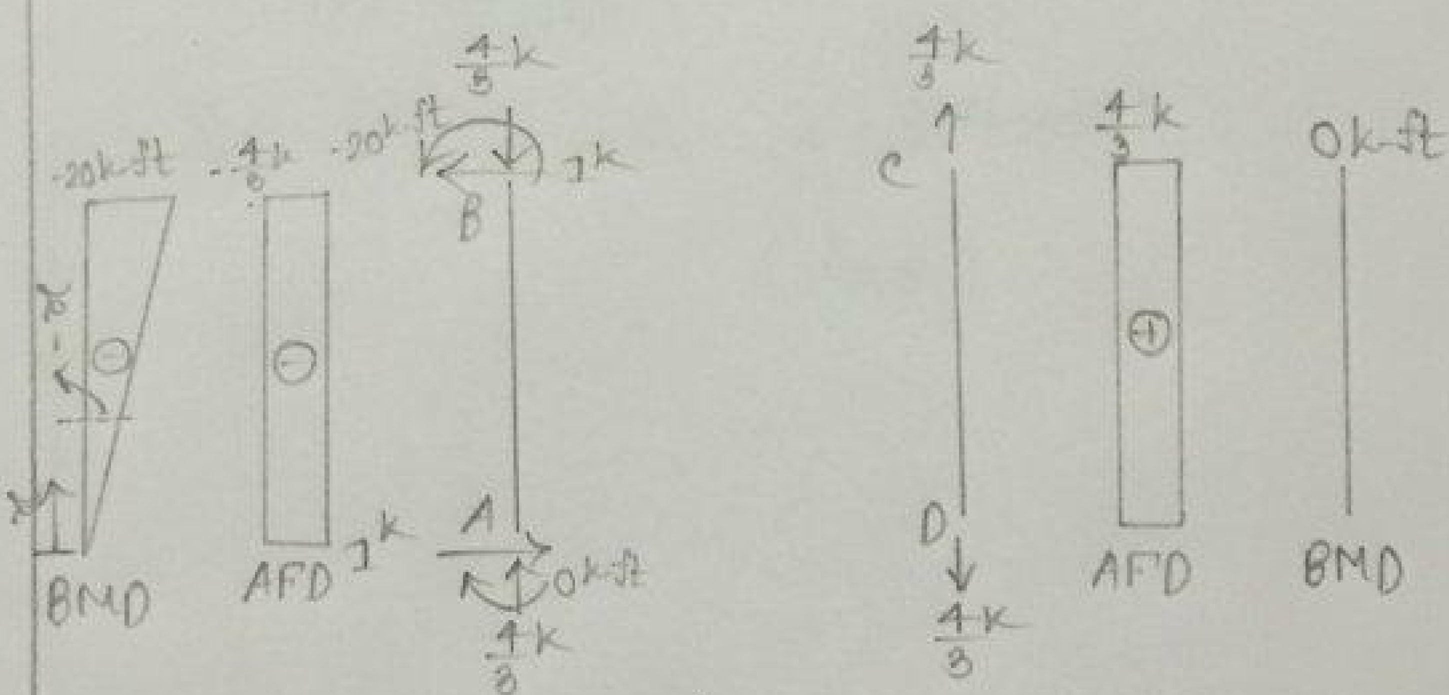
$\Sigma F_y = 0 \uparrow +$

$\Rightarrow R_{Ay} + R_{Dy} = 0$

$\therefore R_{Ay} = \frac{4}{3} k$

Q-Force Analysis :




Segment AB $(0 < x < 20)$

$M_Q = -x$ ; $M_P = -10x$ ; $F_Q = -\frac{4}{3}$ ; $F_P = -\frac{40}{3}$

Segment DC $(0 < x < 20)$

$M_Q = 0$ ; $M_P = 0$ ; $F_Q = +\frac{4}{3}$ ; $F_P = +\frac{40}{3}$

Segment CB $(0 < x < 15)$

$M_Q = -\frac{4x}{3}$ ; $M_P = -\frac{40x}{3}$ ; $F_Q = -1$ ; $F_P = 0$

Using principle of virtual work,

$$\sum Q.\,\delta_{ch} = \int \frac{M_Q M_P}{EI}dx + \sum \frac{F_Q F_P L}{AE}$$

$$\Rightarrow 1\times\delta_{ch} = \int_A^B \frac{(-x)(-10x)}{EI}dx + \sum_A^B \frac{(-\frac{4}{3})(-\frac{40}{3})20}{AE} + \int_D^C \frac{(0)(0)}{EI}dx + \sum_D^C \frac{(\frac{4}{3})(\frac{40}{3})20}{AE}$$

$$+\int_C^B \frac{(-\frac{4x}{3})(-\frac{40x}{3})}{EI}dx + \sum_C^B \frac{(-1)(0)15}{AE}$$

$$\Rightarrow \delta_{ch} = \frac{1}{EI}\left\{\int_0^{20} 10x^2dx + 0 + \int_0^{15}\frac{160x^2}{9}dx\right\} + \frac{1}{AE}\times 2\times\frac{3200}{9}$$

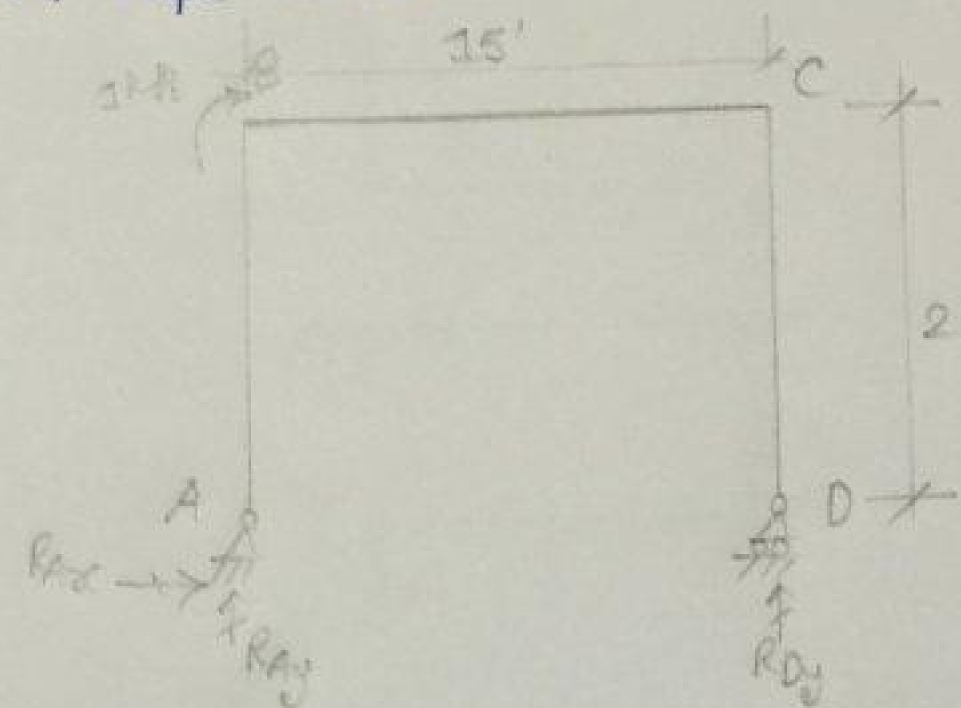
$$\Rightarrow \delta_{ch} = \frac{1}{EI}\left\{\left[\frac{10x^3}{3}\right]_0^{20} + \left[\frac{160x^3}{9\times 3}\right]_0^{15}\right\} + \frac{1}{AE}\times\frac{6400}{9}$$

$$\Rightarrow \delta_{ch} = \frac{1}{30\times10^3\times144\times\frac{8000}{(144)^2}}\left\{\frac{10}{3}(20)^3 + \frac{160}{27}(15)^3\right\} + \frac{1}{25\times30\times10^3}\times\frac{6400}{9}$$

$\therefore \delta_{ch} = +0.0292\,ft$ ($\leftarrow$ leftward)

(Ans:)

Change in slope at 'B':




$\sum M @ A = 0 \curvearrowright +$

$\Rightarrow 1 - R_{Dy} \times 15 = 0$

$\therefore R_{Dy} = \frac{1}{15}\,k$

$\sum F_x = 0 \rightarrow +$

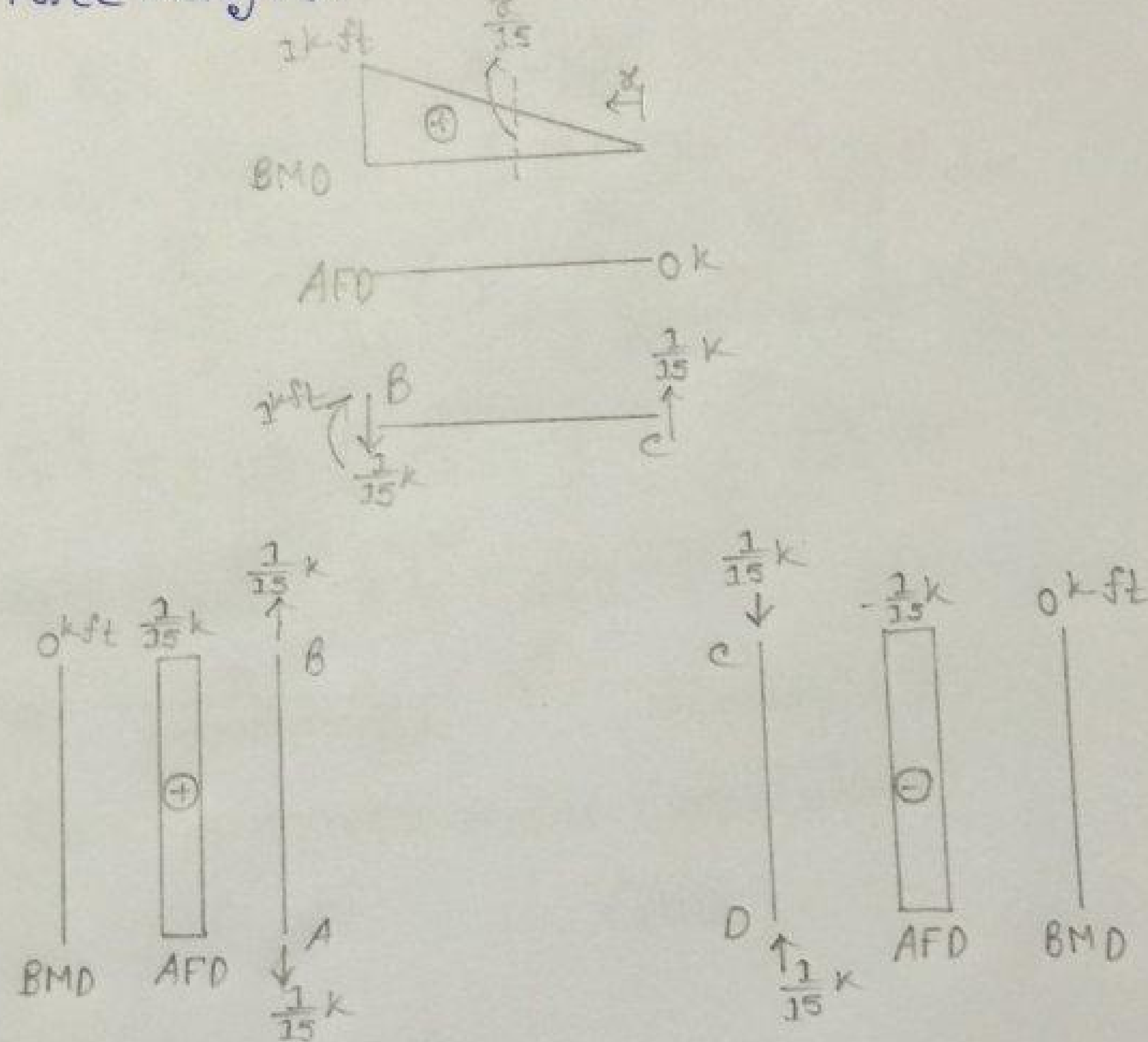
$\Rightarrow R_{Ax} = 0$

$\sum F_y = 0 \uparrow +$

$\Rightarrow R_{Ay} + R_{Dy} = 0$

$\therefore R_{Ay} = -\frac{1}{15}\,k$

Q- Force Analysis:




Segment AB $(0<x<20)$

$M_Q = 0$ ; $M_p = -10x$ ; $F_Q = \frac{1}{15}$ ; $F_p = -\frac{40}{3}$

Segment DC $(0<x<20)$

$M_Q = 0$ ; $M_p = 0$ ; $F_Q = -\frac{1}{15}$ ; $F_p = \frac{40}{3}$

Segment CB $(0<x<15)$

$M_Q = \frac{x}{15}$ ; $M_p = -\frac{40x}{3}$ ; $F_Q = 0$ ; $F_p = 0$

Using principle of virtual work,

$$\Sigma Q \cdot \theta_B = \int \frac{M_Q M_P}{EI} dx + \Sigma \frac{F_Q F_P L}{AE}$$

$$\Rightarrow 1 \times \theta_B = \int_A^B \frac{(0)(-10x)}{EI} dx + \sum_A^B \frac{\left(\frac{1}{15}\right)\left(-\frac{40}{3}\right)20}{AE} + \int_D^C \frac{(0)(0)}{EI} dx$$

$$+ \sum_D^C \frac{\left(-\frac{1}{15}\right)\left(\frac{40}{3}\right)20}{AE} + \int_C^B \frac{\left(\frac{x}{15}\right)\left(-\frac{40x}{3}\right)}{EI} dx + \sum_C^B \frac{(0)(0)15}{AE}$$

$$\Rightarrow \theta_B = 0 + \frac{1}{AE}\left\{-\frac{160}{9} - \frac{160}{9}\right\} + \int_0^{15} -\frac{8x^2}{9EI} dx$$

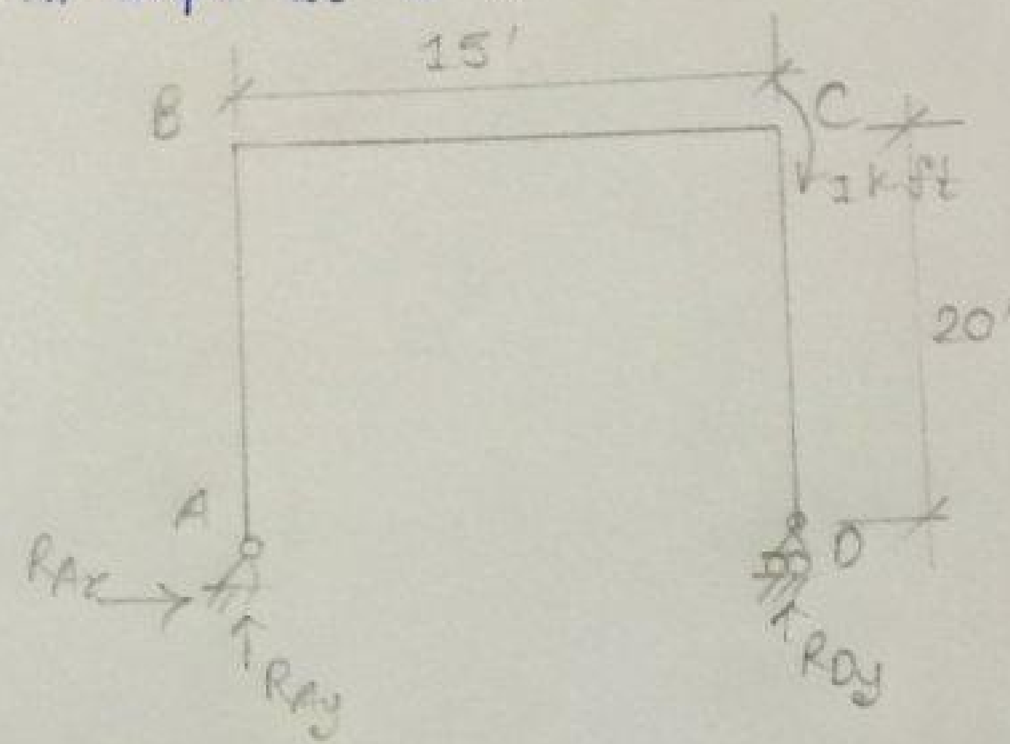
$$\Rightarrow \theta_B = -\frac{8}{9EI}\left[\frac{x^3}{3}\right]_0^{15} - \frac{1}{AE}\left(\frac{320}{9}\right)$$

$$\Rightarrow \theta_B = -\frac{8}{9 \times 30 \times 10^3 \times 144 \times \frac{8000}{(144)^2}}\left(\frac{(15)^3}{3}\right) - \frac{1}{20 \times 30 \times 10^3} \times \left(\frac{320}{9}\right)$$

$\therefore \theta_B = -0.000659$ radian (anticlockwise)

(Ans:)

Change in slope at 'C':




$\Sigma M @ A = 0$ ↻+

$\Rightarrow 1 - R_{Dy} \times 15 = 0$

$\therefore R_{Dy} = \frac{1}{15}$ k

$\Sigma F_x = 0$ →+

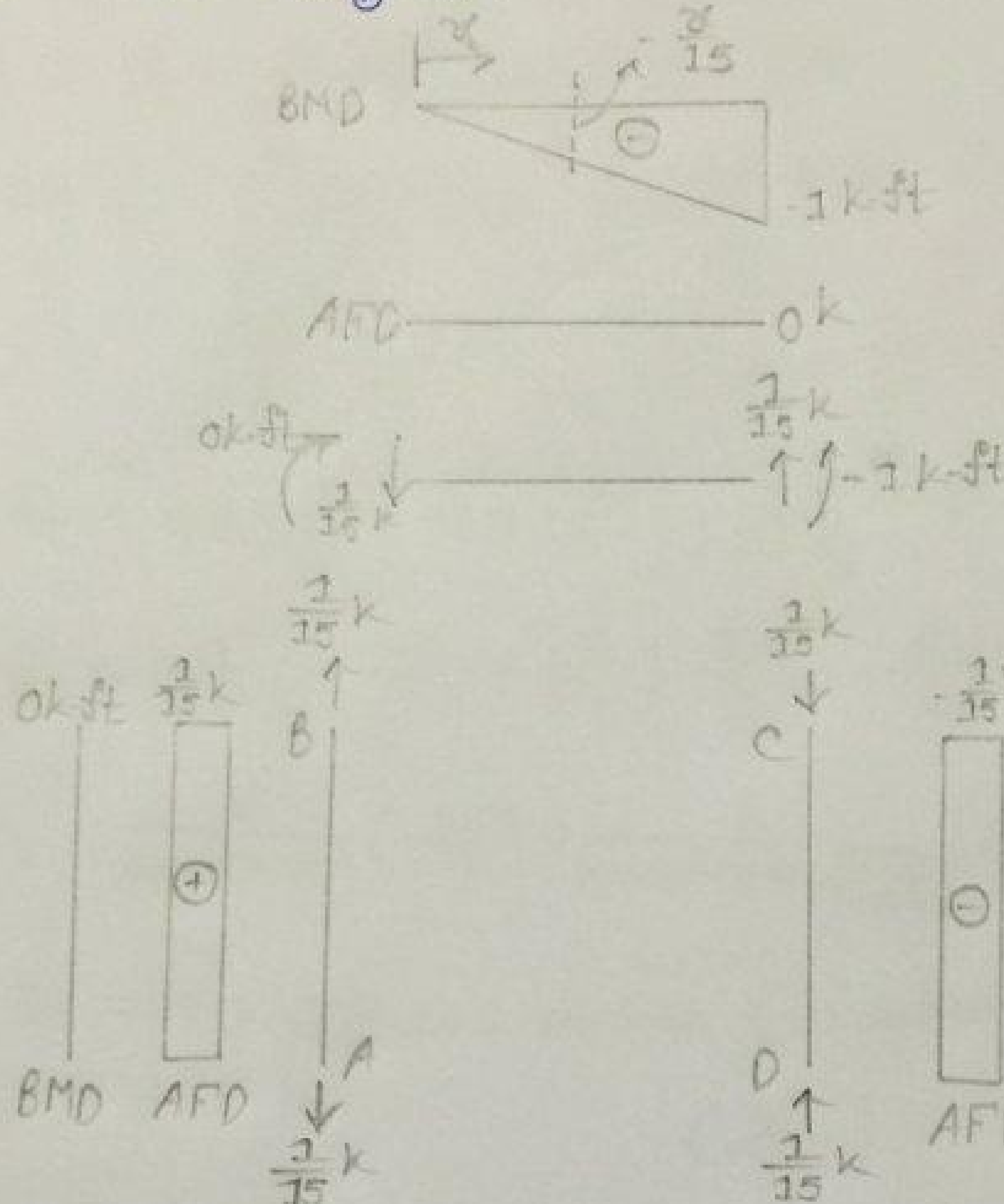
$\therefore R_{Ax} = 0$

$\Sigma F_y = 0$ ↑+

$\Rightarrow R_{Ay} + R_{Dy} = 0$

$\therefore R_{Ay} = -\frac{1}{15}$ k

## Q - Force Analysis :




Segment AB $(0<x<20)$

$M_Q = 0$ ; $M_P = -10x$ ; $F_Q = \frac{1}{15}$ ; $F_P = -\frac{40}{3}$

Segment BC $(0<x<15)$

$M_Q = -\frac{x}{15}$ ; $M_P = -200 + \frac{40x}{3}$ ; $F_Q = 0$ ; $F_P = 0$

Segment DC $(0<x<20)$

$M_Q = 0$ ; $M_P = 0$ ; $F_Q = -\frac{1}{15}$ ; $F_P = \frac{40}{3}$

Using principle of virtual work,

$$\Sigma Q \cdot \theta_C = \int \frac{M_Q M_P}{EI} dx + \Sigma \frac{F_Q F_P L}{AE}$$

$$\Rightarrow 1 \times \theta_C = \int_A^B \frac{(0)(-10x)}{EI} dx + \sum_A^B \frac{\left(\frac{1}{15}\right)\left(-\frac{40}{3}\right)20}{AE} + \int_B^C \frac{\left(-\frac{2x}{15}\right)\left(-200+\frac{40x}{3}\right)}{EI} dx$$

$$+ \sum_B^C \frac{(0)(0)15}{AE} + \int_D^C \frac{(0)(0)}{EI} dx + \sum_D^C \frac{\left(-\frac{1}{15}\right)\left(\frac{40}{3}\right)20}{AE}$$

$$\Rightarrow \theta_C = 0 + \frac{1}{AE}\left\{-2 \times \frac{160}{9}\right\} + \frac{1}{EI}\int_0^{15}\left(\frac{40x}{3} - \frac{8x^2}{9}\right) dx$$

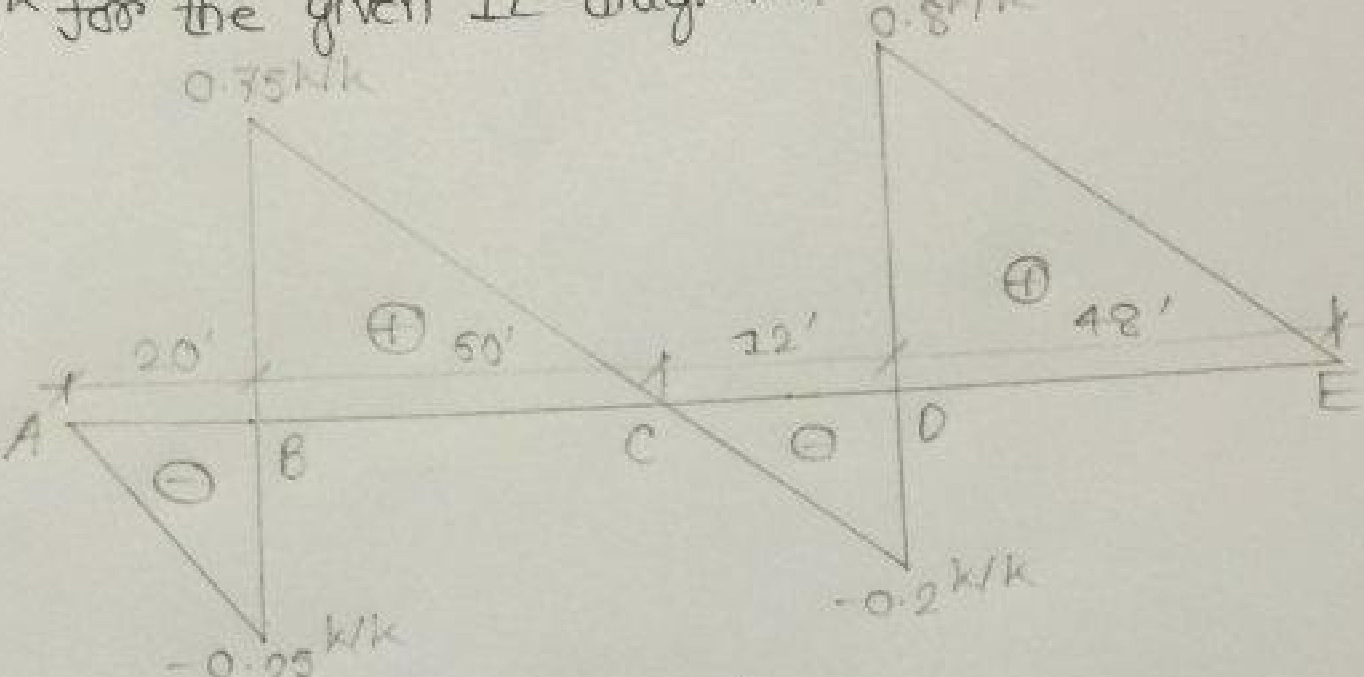
$$\Rightarrow \theta_C = \frac{1}{AE}\left(-\frac{320}{9}\right) + \frac{1}{EI}\left[\frac{20x^2}{3} - \frac{8x^3}{27}\right]_0^{15}$$

$$\Rightarrow \theta_C = \frac{1}{20 \times 30 \times 10^3}\left(-\frac{320}{9}\right) + \frac{1}{30 \times 10^3 \times 144 \times \frac{8000}{(144)^2}}\left[\frac{20}{3}(15)^2 - \frac{8}{27}(15)^3\right]$$

$\therefore \theta_C = +0.000241$ radian (clockwise)

(Ans:)

Assignment - 21 : Find the maximum shear due to an uniform load of $5^{k/ft}$ and a concentrated load of $100^k$ for the given IL-diagram.




Sol$^n$: For max$^m$ positive shear,

BC panel, $= \frac{1}{2} \times 0.75 \times 60 \times 5 = 112.5^k$

DE panel $= \frac{1}{2} \times 0.8 \times 48 \times 5 + 0.8 \times 100 = 176^k$ (max$^m$)

(Ans:)

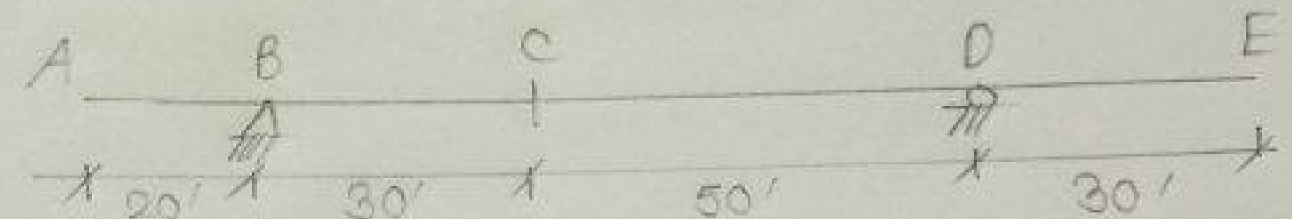
For maximum negative shear,

AB panel $= \frac{1}{2} \times (-0.25) \times 20 \times 5 + (-0.25) \times 100 = -37.5^k$ (max$^m$)

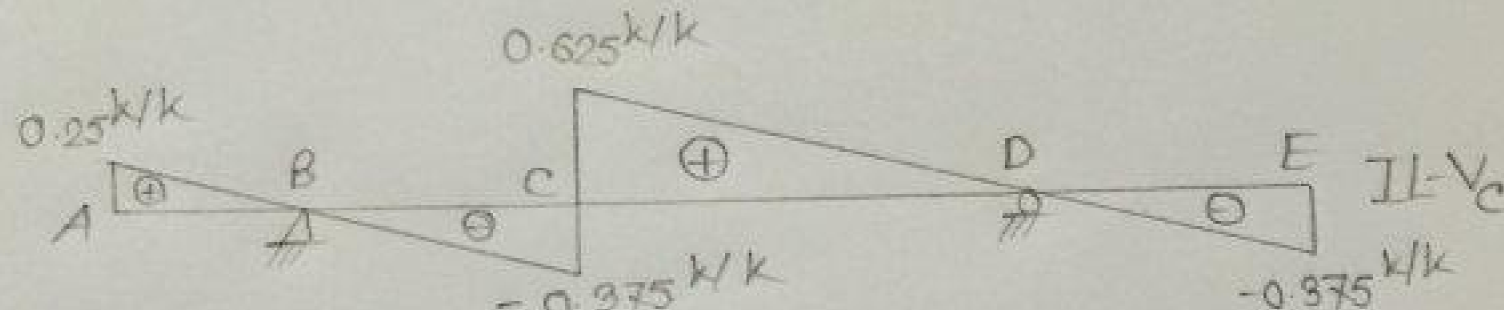
(Ans:)

CD panel $= \frac{1}{2} \times (-0.2) \times 12 \times 5 = -6^k$

Assignment-22 : Find maximum shear and moment at section C for an uniform moving load of $7^{k/ft}$ combined with a moving concentrated load of $90^k$.




Sol$^n$ :



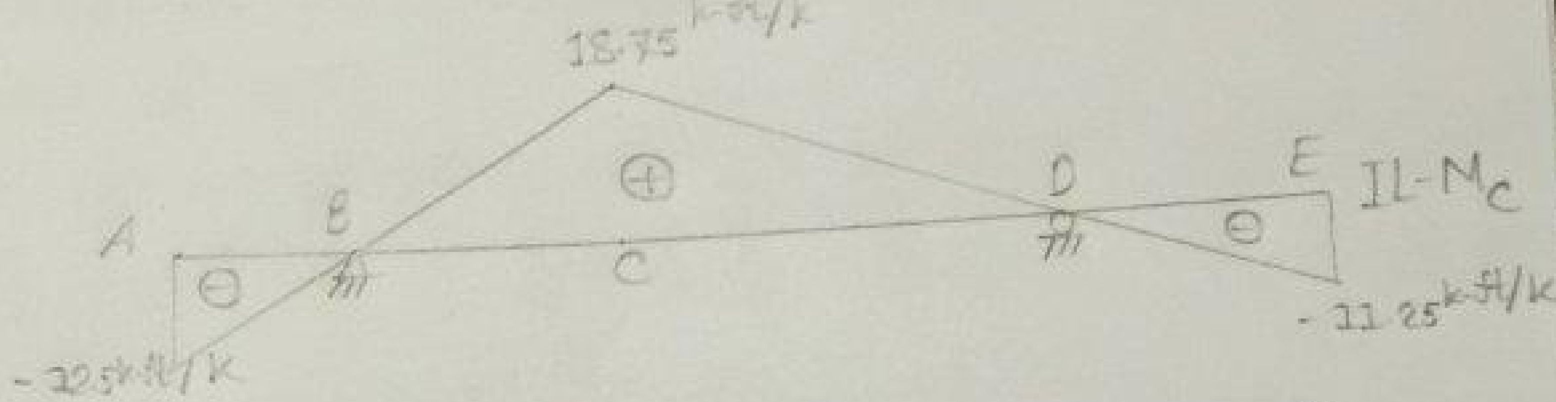

Max$^m$ positive shear at $C = \frac{1}{2} \times 0.625 \times 50 \times 7 + 0.625 \times 90$

$= 165.625^k$ (Ans:)

Max$^m$ Negative shear at $C = \frac{1}{2} \times (-0.375) \times 30 \times 7 + (-0.375) \times 90$

$= -73.125^k$

(Ans:)



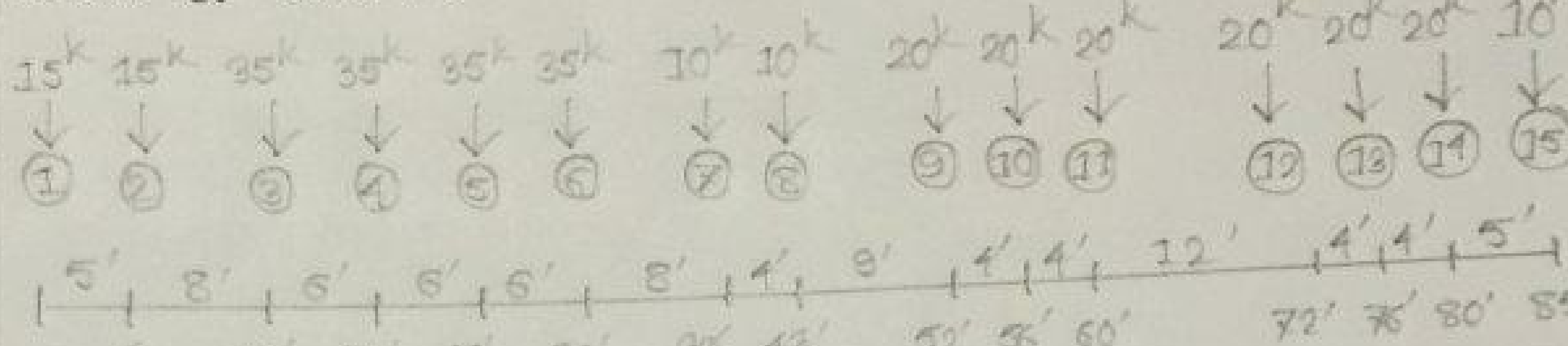

Max$^m$ moment at C (positive) $= \frac{1}{2} \times 18.75 \times 80 \times 7 + 18.75 \times 90$

$= 6937.5$ k-ft (Ans:)

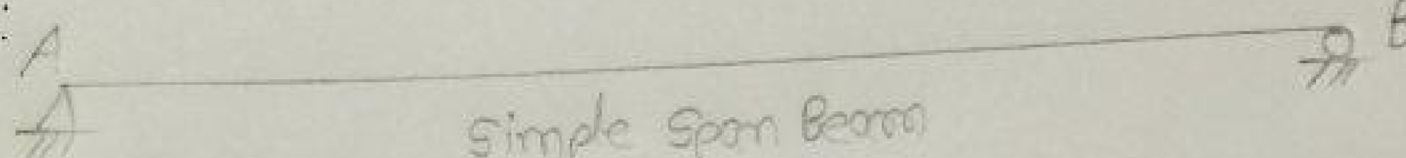
Maximum moment at C (negative) $= \frac{1}{2} \times (-12.5) \times 20 \times 7 + (-12.5) \times 90$

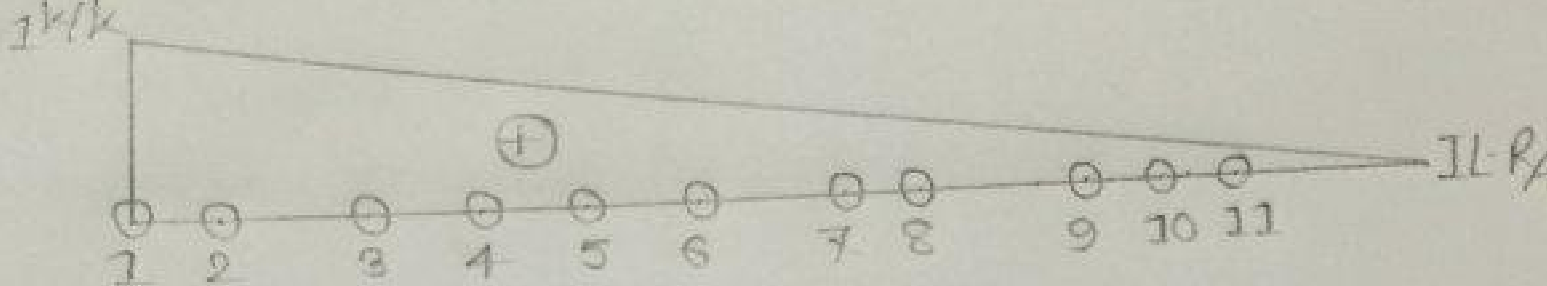
$= -2000$ k-ft (Ans:)

Assignment-23: Find maximum reaction for a simple beam of 70 ft.

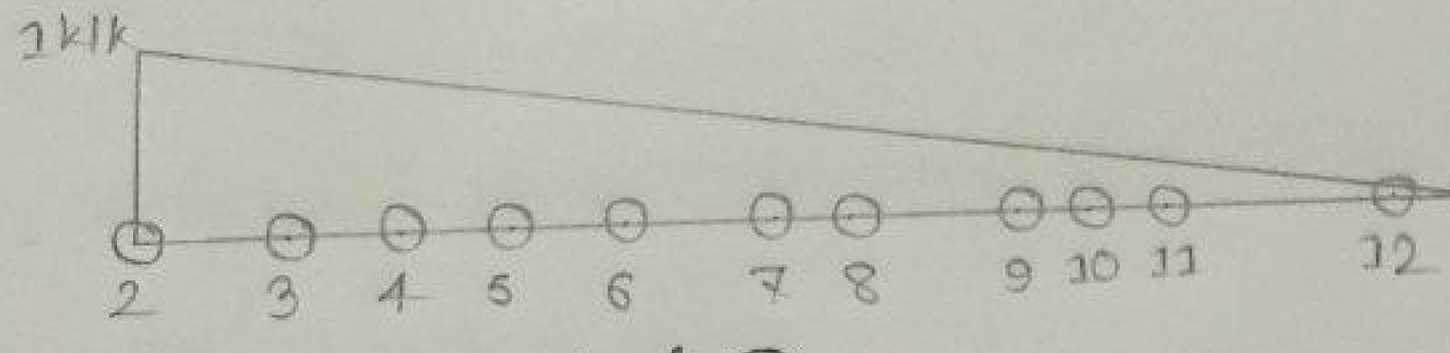



Sol$^n$:




Trial-1: Move wheel 2 to wheel 1 at A.




$\Sigma P$ = wheel 2 to wheel 11

$= (15 + 35 \times 4 + 10 \times 2 + 20 \times 3) = 235^k$

$d_1 = 5'$

$P_1$ = wheel 1 $= 15^k$

$P'$ = wheel 12 $= 20^k$

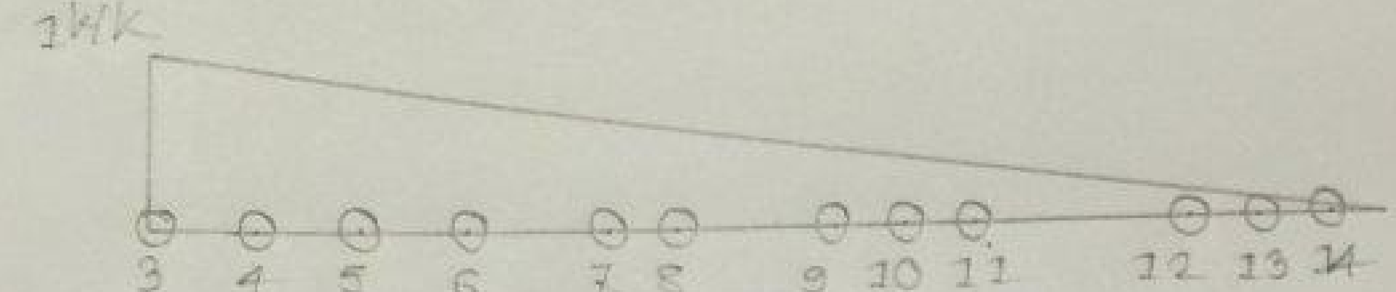
$e = 3'$

$$\Delta R = \frac{\Sigma P d_1}{L} - P_1 + \frac{P' e}{L}$$

$$= \frac{235 \times 5}{70} - 15 + \frac{20 \times 3}{70}$$

$= +2.64$ (+ve, increased)

Trial-2 : move wheel 3 to wheel 2 at A.




$\Sigma P$ = wheel 3 to wheel 12

$= (35 \times 4 + 10 \times 2 + 20 \times 3 + 20) = 240^k$

$d_1 = 8'$

$P_1$ = wheel 2 $= 15^k$

$P'$ = wheel 13 $= 20^k$ & wheel 14 $= 20^k$

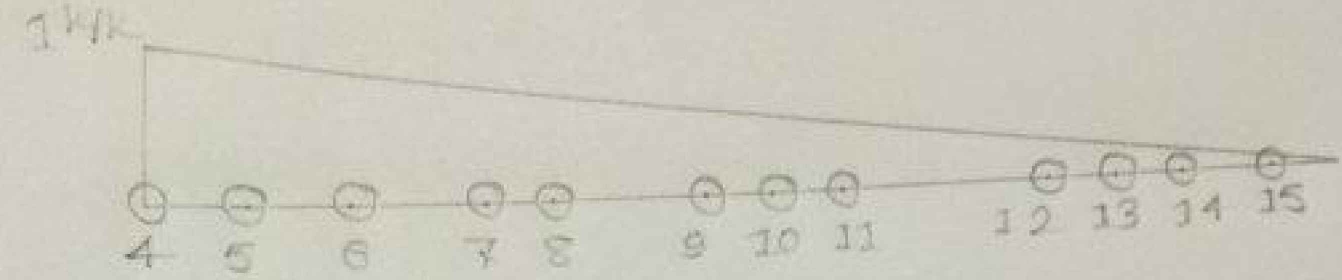
$e = 7'$ by wheel 13 & $3'$ by wheel 14

$$\Delta R = \frac{\Sigma P d_1}{L} - P_1 + \frac{P' e}{L}$$

$$= \frac{240 \times 8}{70} - 15 + \left[\frac{20 \times 7}{70} + \frac{20 \times 3}{70}\right]$$

$= +15.29$ (+ve, increased)

Trial-3 : Move wheel ④ to wheel ③ at A.




$\Sigma P$ = wheel ④ to wheel ⑭

$= (35\times3+10\times2+20\times3+20\times3) = 245^k$

$d_1 = 6'$

$P_1$ = wheel ③ $= +35^k$

$P'$ = wheel ⑮ $= 10^k$

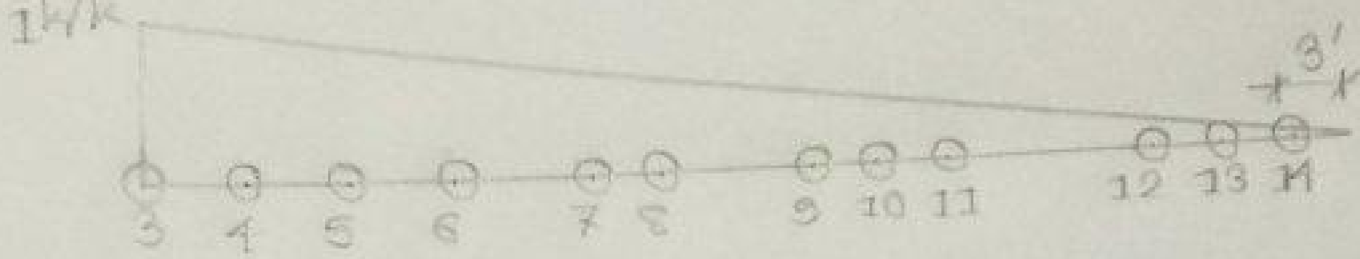
$e = 4'$

$$\Delta R = \frac{\Sigma P d_1}{L} - P_1 + \frac{P'e}{L}$$

$$= \frac{245\times6}{70} - 35 + \frac{10\times4}{70}$$

$= -13.43$ (-ve, decreased)

So, wheel ③ at A gives maximum reaction.




So, Maximum Reaction,

$$R_A = \frac{1}{70}\left[70\times35 + 64\times35 + 58\times35 + 52\times35 + 44\times10 + 40\times10 + 31\times20 + 27\times20 + 23\times20 + 11\times20 + 7\times20 + 3\times20\right]$$

$$= 163.14^k$$

(Ans:)