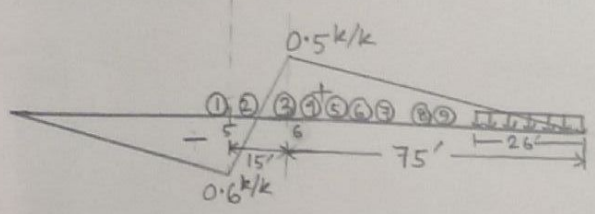
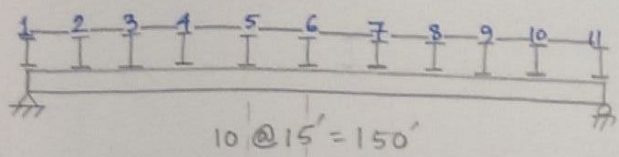
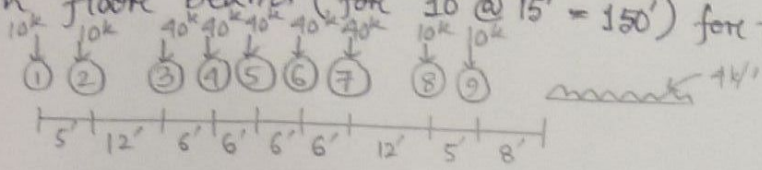


Assignment-32: Find maximum panel shear in 5th panel of the span with floor beams (for 10 @ 15' = 150') for the loading below.



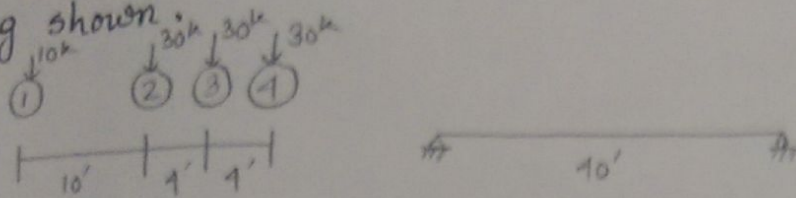
from similar triangles  $\rightarrow$   
 $\frac{.5}{x} = \frac{.6}{15-x}$   
 $\Rightarrow 15 \times .5 = (.6 + .5)x$   
 $\Rightarrow x = 6.82'$   
 $\therefore 15 - x = 8.18$

Trial No.	Position of wheel	$\frac{W}{L}$		$\frac{W_1}{P}$	Remarks	Calculations
1	wheel ③ just to right at 6	$\frac{344}{150}$	$>$	$\frac{10}{15}$	Criterion is Satisfied	$\left\{ \begin{array}{l} W = \text{① to ⑨} + 4 \times 26 = 344k \\ W_1 = \text{wheel ②} = 10k \end{array} \right.$
	wheel ③ just to left at 6	$\frac{344}{150}$	$<$	$\frac{50}{15}$		

$\therefore$  Wheel ③ at 6 gives maximum panel shear.

$\therefore$  Maximum Panel shear =  $\frac{.5}{75} \left[ (75 + 69 + 63 + 57 + 51) \times 40 + (39 + 34) \times 10 \right]$   
 $+ \left( \frac{1}{2} \times 26 \times \frac{.5}{75} \times 26 \right) \times 4 - \frac{.6}{60} \times 58 \times 10 - \frac{.6}{(15 - 6.82)} \times 5.18 \times 10$   
 $= 97.88 - 9.6$   
 $= \underline{\underline{88.28^k}}$   
Ans.

Assignment-31: Find greatest or absolute moment for 40' span for the loading shown



Trial No	Position of wheel	$\frac{W}{L}$		$\frac{W_1}{a}$	Remarks	Calculation
1	wheel ③ at center	$\frac{100}{40}$	>	$\frac{40}{20}$	Criterion Satisfied	$\begin{cases} W = \text{① to ④} = 100^k \\ W_1 = \text{①, ②} = 40^k \end{cases}$
		$\frac{100}{40}$	<	$\frac{70}{20}$		$\begin{cases} W = 100^k \\ W_1 = \text{① to ③} = 70^k \end{cases}$

$\therefore P = \text{wheel ③} = 30^k$

• Locating the cg of loads - from wheel ③  $\rightarrow$

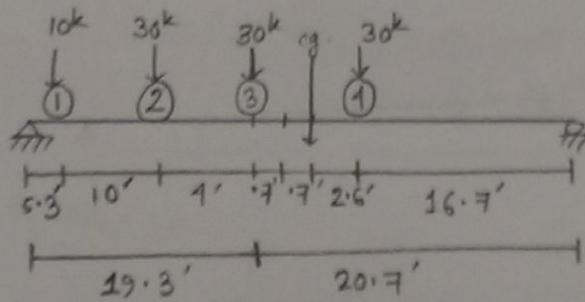
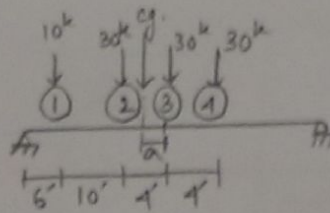
$$10 \times 14 + 30 \times 4 + 100 \times a - 30 \times 4 = 0$$

$$\Rightarrow a = \frac{30 \times 4 - 30 \times 4 - 10 \times 14}{100}$$

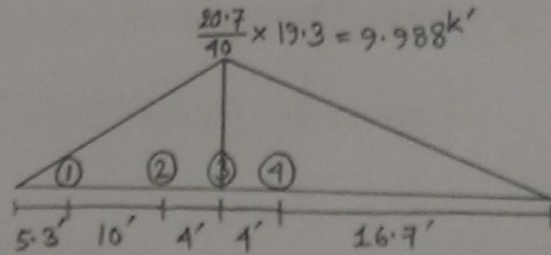
$$\therefore a = -1.4'$$

$$\therefore \frac{a}{2} = 0.7'$$

Here,  $L = 40'$ ,  $W = 100^k$ ,  $a = 19.3'$



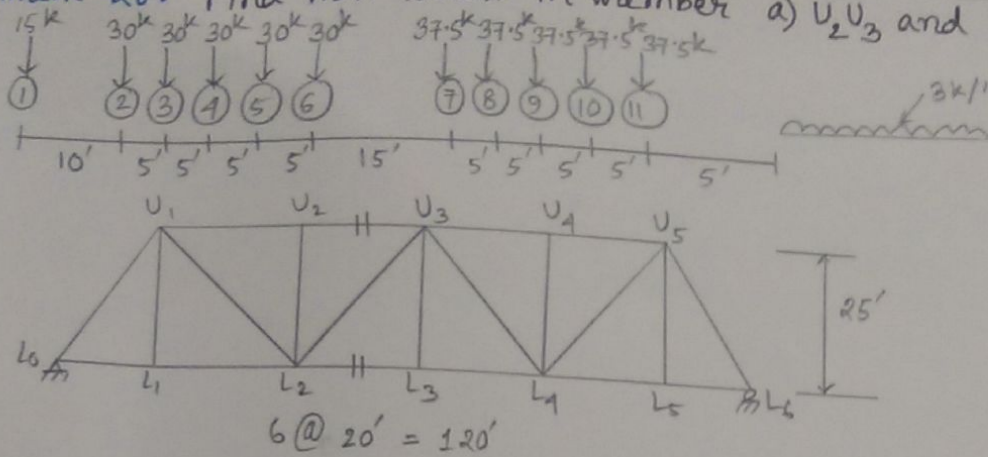
wheel ③ at 19.3' from the left support	just to right	$\frac{100}{40}$	>	$\frac{40}{19.3}$
	just to left	$\frac{100}{40}$	<	$\frac{70}{19.3}$



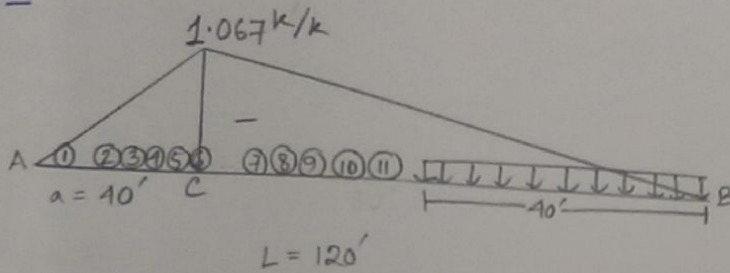
$$\begin{aligned} \text{Greatest moment} &= \frac{9.988}{20.7} [20.7 \times 30 + 16.7 \times 30] + \frac{9.988}{19.3} [15.3 \times 30 + 5.3 \times 10] \\ &= 806.35^k \end{aligned}$$

Ans.

Assignment-28: Find max<sup>m</sup> stress in member a)  $U_2U_3$  and b)  $L_2L_3$



a) for  $U_2U_3$  -



$$\begin{aligned} \sum M_{L_2} = 0 &\Rightarrow \frac{4}{6} \times 40 = -F_{U_2U_3} \times 25 \\ &\Rightarrow F_{U_2U_3} = -1.067 \text{ k} \end{aligned}$$

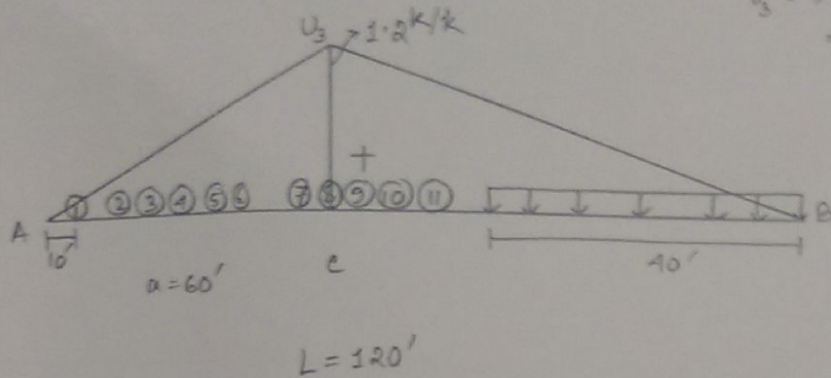
Trial No.	Position of wheel	$\frac{W}{L}$		$\frac{W_1}{a}$	Remarks	Calculations
1	wheel ⑥ at c	just to right	$\frac{472.5}{120}$	$\frac{135}{40}$	Criterion Satisfied	$\begin{cases} W = \text{① to ⑪} + 3 \times 40 = 472.5 \text{ k} \\ W_1 = \text{① to ⑥} = 135 \text{ k} \end{cases}$
		just to left	$\frac{472.5}{120}$	$\frac{165}{40}$		

$\therefore$  wheel ⑥ at C gives max<sup>m</sup> bar force.

$$\begin{aligned} \therefore \text{Max}^m \text{ bar force} &= \frac{1.067}{80} \left[ 80 \times 30 + (65 + 60 + 55 + 50 + 45) \times 37.5 \right] + \left( \frac{1}{2} \times 40 \times \frac{1.067}{80} \times 40 \right) \times 3 \\ &\quad + \frac{1.067}{40} \left[ (35 + 30 + 25 + 20) \times 30 + 10 \times 15 \right] \\ &= \underline{\underline{-293.59 \text{ k (compressive)}}} \end{aligned}$$

Ans.

b) for  $L_2L_3$  -



Trial No.	Position of wheel	$\frac{W}{L}$		$\frac{W_1}{a}$	Remarks	Calculations	
1	wheel ⑦ at c	just to right	$\frac{457.5}{120}$	$>$	$\frac{165}{60}$		$\begin{cases} W = \text{① to ⑪} + 3 \times 35 = 457.5 \text{ k} \\ W_1 = \text{① to ⑥} = 165 \text{ k} \end{cases}$
		just to left	$\frac{457.5}{120}$	$>$	$\frac{202.5}{60}$		$\begin{cases} W = \text{① to ⑪} + 3 \times 35 = 457.5 \text{ k} \\ W_1 = \text{① to ⑦} = 202.5 \text{ k} \end{cases}$
2	wheel ⑧ at c	just to right	$\frac{472.5}{120}$	$>$	$\frac{202.5}{60}$		$\begin{cases} W = \text{① to ⑪} + 3 \times 40 = 472.5 \text{ k} \\ W_1 = \text{① to ⑧} = 202.5 \text{ k} \end{cases}$
		just to left	$\frac{472.5}{120}$	$<$	$\frac{240}{60}$		$\begin{cases} W = 472.5 \text{ k} \\ W_1 = \text{① to ⑧} = 240 \text{ k} \end{cases}$

So wheel ⑧ at c gives maximum stress.

$$\therefore \text{Maximum stress} = \frac{1.2}{60} \left[ (60 + 55 + 50 + 45 + 55) \times 37.5 + (40 + 35 + 30 + 25 + 20) \times 30 + 10 \times 15 \right] + \left( \frac{1}{2} \times 40 \times \frac{1.2}{60} \times 40 \right) \times 3$$

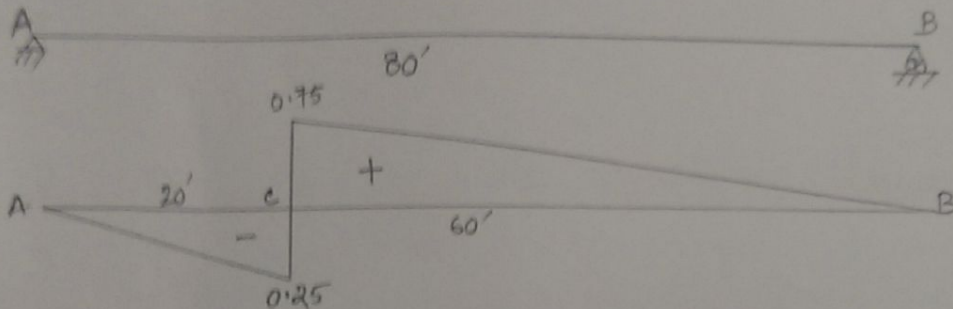
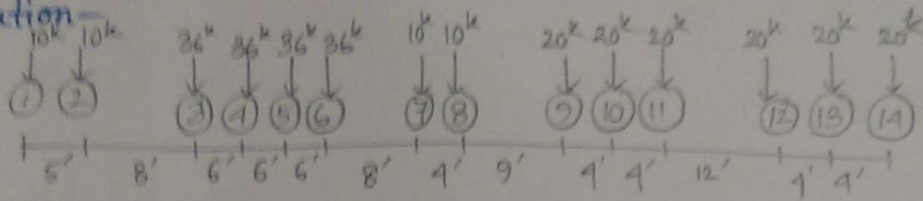
$$= \underline{\underline{339.75 \text{ k (tension)}}}$$

Ans:



Assignment-24: Find maximum shear at quarter point of a 80' simple span.

Solution-



Trial-1: wheel ① at c to wheel ② at c

$$\Sigma P = \text{wheel ① to wheel ⑪} = 244^k$$

$$P_1 = \text{wheel ①} = 10^k$$

$$P' = 0$$

$$e = 0$$

$$P_2 = 0$$

$$e' = 0$$

$$d_1 = 5'$$

$$\Delta V = \frac{\Sigma P d_1}{L} - P_1 + \frac{P' e}{L} + \frac{P_2 e'}{L}$$

$$= \frac{244 \times 5}{80} - 10 + 0 + 0$$

$$= 5.25^k \text{ (tre, increasing)}$$

Trial-2: wheel ② at c to wheel ③ at c

$$\Sigma P = \text{wheel ① to wheel ⑪} = 244^k$$

$$P_1 = \text{wheel ②} = 10^k ; P_2 = 0$$

$$P' = \text{wheel ⑫} = 20^k ; e' = 0$$

$$e = 1' ; d_1 = 8'$$

$$\Delta V = \frac{\Sigma P d_1}{L} - P_1 + \frac{P' e}{L} + \frac{P_2 e'}{L}$$

$$= \frac{244 \times 8}{80} - 10 + \frac{20 \times 1}{80} + 0$$

$$= 14.65 \text{ (tre, increasing)}$$

Trial-3: wheel ③ at C to wheel ④ at C

$$\Sigma P = \text{wheel ① to wheel ⑫} ; \Delta V = \frac{\Sigma P d_1}{L} - P_1 + \frac{P_1' e}{L} + \frac{P_2 e'}{L}$$

$$= 264 \text{ k}$$

$$d_1 = 6'$$

$$P_1 = \text{wheel ③} = 36 \text{ k}$$

$$e = 3'$$

$$P_1' = \text{wheel ⑬} = 20 \text{ k}$$

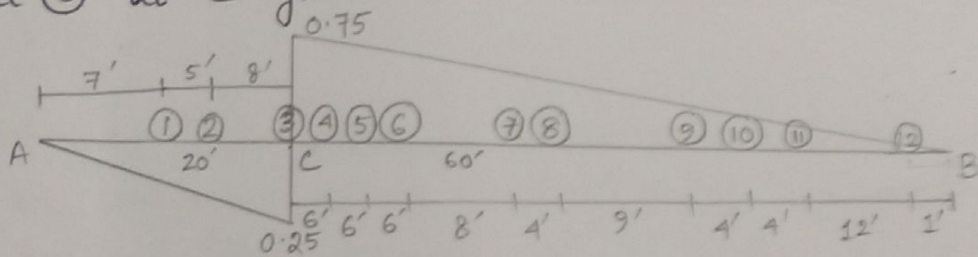
$$P_2 = 0$$

$$e' = 0$$

$$= \frac{264 \times 6}{80} - 36 + \frac{20 \times 3}{80} + 0$$

$$= -15.45 \text{ (-ve, decreasing)}$$

So, wheel ③ at C gives the maximum shear.



$$\therefore \text{Maximum shear} = + \frac{0.75}{80} \left[ 60 \times 36 + 54 \times 36 + 48 \times 36 + 34 \times 10 + 30 \times 10 + 21 \times 20 \right. \\ \left. + 17 \times 20 + 13 \times 20 + 1 \times 20 + 12 \times 36 \right]$$

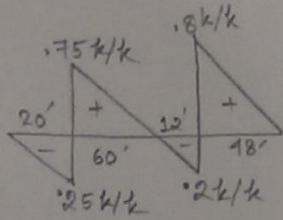
$$- \frac{0.25}{20} \left[ 12 \times 10 + 7 \times 10 \right]$$

$$= +110.425 \text{ k}$$

Ans.

Assignment-21: What is the maximum shear due to 5 k/' UDL and concentrated 100k of moving load?

Solution -

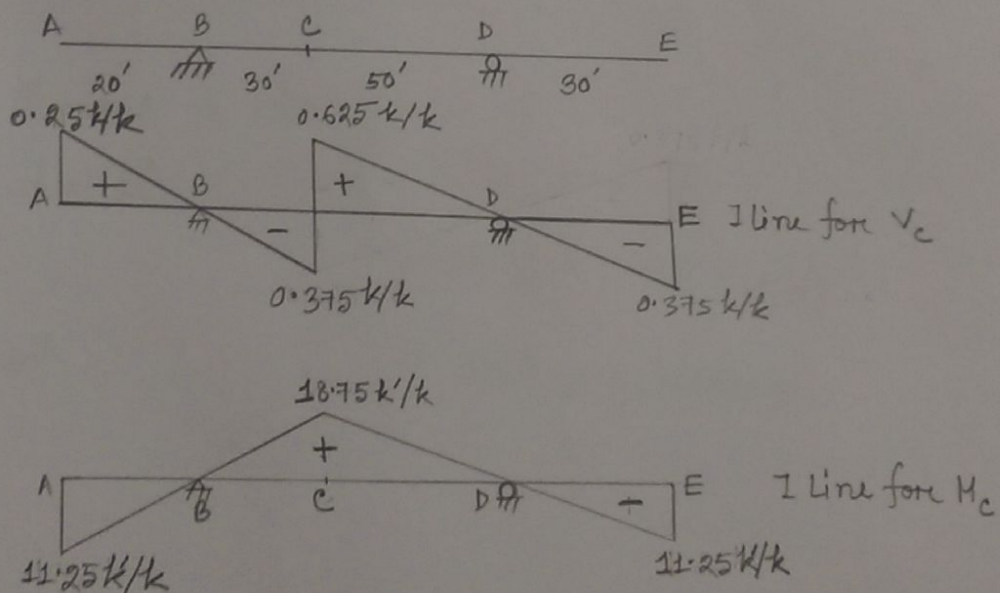


Here, maximum +ve shear =  $\frac{1}{2} \times 0.8 \times 18 \times 5 + 0.8 \times 100 = 176 \text{ k}$   
 and maximum -ve shear =  $\frac{1}{2} \times 0.25 \times 20 \times 5 + 0.25 \times 100 = 37.5 \text{ k}$

Comparing the two, max<sup>m</sup> shear is 176 k (positive) Ans.

Assignment-22: Find maximum shear and moment at section C for an uniform moving load of 7 k/'ft combined with a moving concentration of 90k.

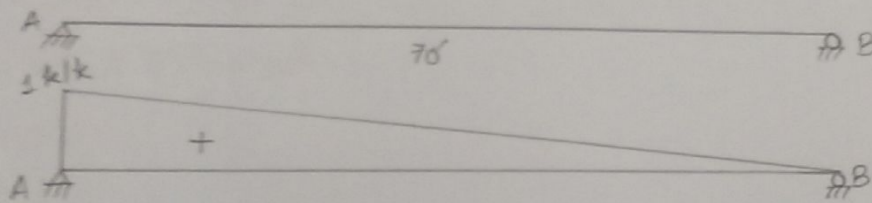
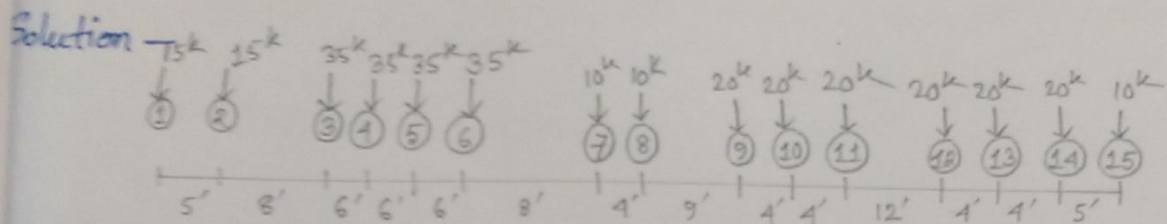
Solution -



$$\text{Maximum shear} = \frac{1}{2} \times 80 \times 0.625 \times 7 + 0.625 \times 90 = \underline{231.25 \text{ k}} \text{ (+ve)} \text{ Ans.}$$

$$\begin{aligned} \text{Maximum moment} &= \frac{1}{2} \times 80 \times 18.75 \times 7 + 18.75 \times 90 \\ &= \underline{6937.5 \text{ k'}} \text{ (+ve)} \text{ Ans.} \end{aligned}$$

Assignment-23: Find maximum reaction of a simple beam of 70' span.



Trial-1: wheel ④ at A to wheel ② at A

$$\begin{aligned} \Sigma P &= \text{wheel ② to wheel ①} \\ &= 235 \text{ k} \end{aligned}$$

$$P_1 = 15 \text{ k}$$

$$P' = \text{wheel ⑩} = 20 \text{ k}$$

$$e = 3'$$

$$d_1 = 5'$$

$$\begin{aligned} \Delta R &= \frac{\Sigma P d_1}{L} - P_1 + \frac{P' e}{L} \\ &= \frac{235 \times 5}{70} - 15 + \frac{20 \times 3}{70} \\ &= 2.64 \text{ (+ve, increase)} \end{aligned}$$

Trial-2: wheel ② at A to wheel ③ at A

$$\Sigma P = \text{wheel ③ to wheel ⑫} = 240^k$$

$$P' = \text{wheel ⑬} = 20^k \text{ and wheel ⑭} = 20^k$$

$$e = 7' \text{ for wheel ⑬ and } 3' \text{ for wheel ⑭}$$

$$P_1 = \text{wheel ②} = 15^k$$

$$d_1 = 8'$$

$$\begin{aligned} \therefore \Delta R &= \frac{\Sigma P d_1}{L} - P_1 + \frac{P' e}{L} = \frac{240 \times 8}{70} - 15 + \frac{20 \times 7 + 20 \times 3}{70} \\ &= 15.29 \text{ (+ve, increase)} \end{aligned}$$

Trial-3: wheel ③ at A to wheel ④ at A

$$\Sigma P = \text{wheel ④ to wheel ⑭} = 245^k$$

$$P' = \text{wheel ⑮} = 10^k$$

$$e = 4'$$

$$d_1 = 6'$$

$$P_1 = \text{wheel ③} = 35^k$$

$$\begin{aligned} \therefore \Delta R &= \frac{\Sigma P d_1}{L} - P_1 + \frac{P' e}{L} = \frac{245 \times 6}{70} - 35 + \frac{10 \times 4}{70} \\ &= 13.42 \text{ (-ve, decrease)} \end{aligned}$$

$\therefore$  wheel ③ at A gives maximum reaction.

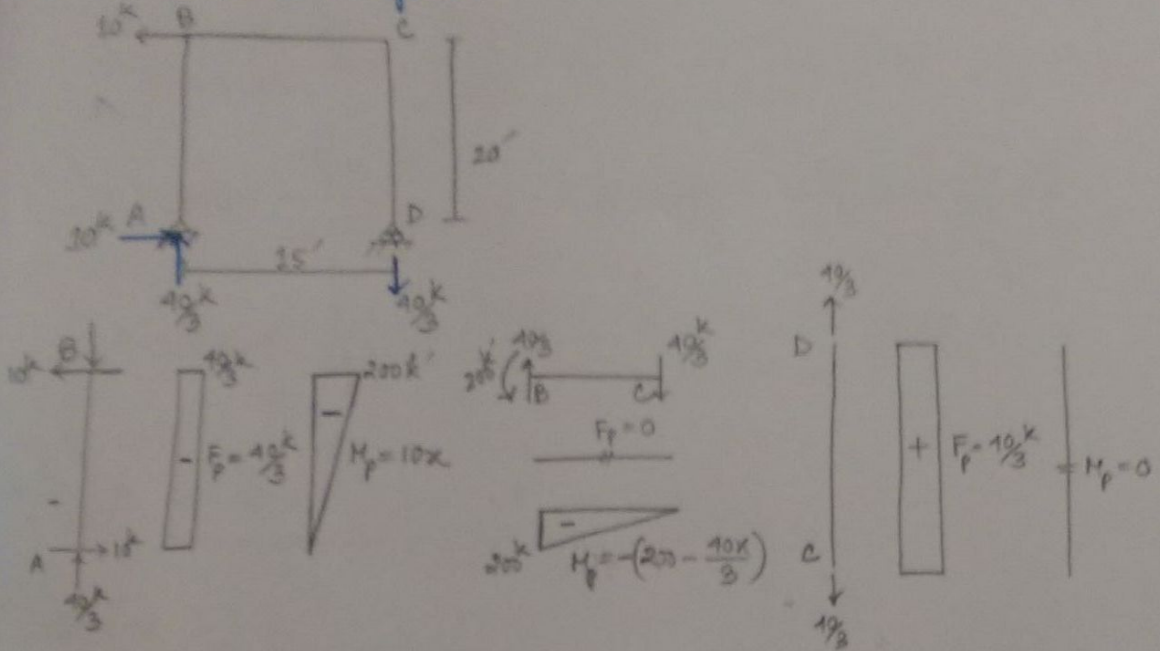
Hence,

$$\begin{aligned} \text{max}^m \text{ reaction} &= \frac{1}{70} \left[ 70 \times 35 + 64 \times 35 + 58 \times 35 + 52 \times 35 + 44 \times 10 + 10 \times 40 \right. \\ &\quad \left. + 20 \times 31 + 20 \times 27 + 20 \times 23 + 20 \times 11 + 20 \times 7 \right. \\ &\quad \left. + 20 \times 3 \right] \end{aligned}$$

$$= \underline{\underline{163.14^k}} \text{ Ans.}$$

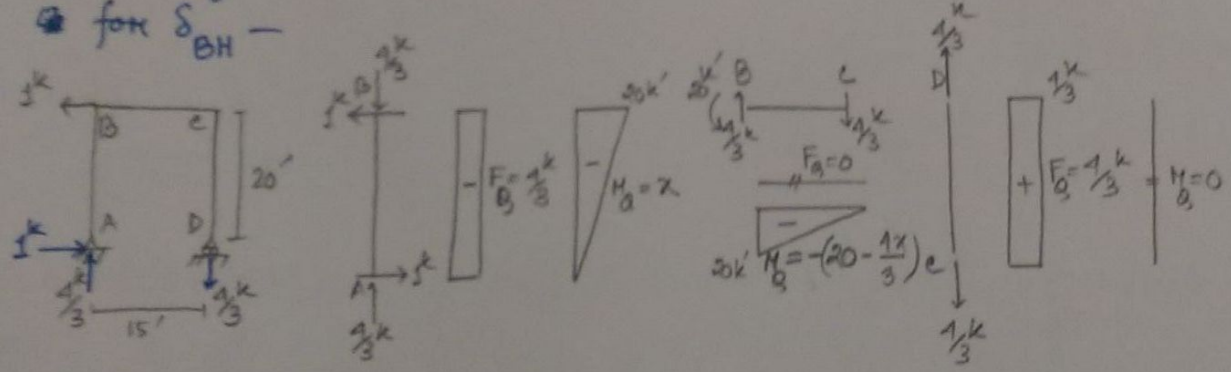
Assignment - 20: Find  $\delta_{BH}$ ,  $\delta_{CH}$ ,  $\theta_B$ ,  $\theta_C$  for the following frame.  
 Given,  $E = 30 \times 10^3$  psi,  $I = 8000$  in<sup>4</sup>,  $A = 20$  in<sup>2</sup>

Solution - P force analysis:



Q forces analysis:

for  $\delta_{BH}$  -



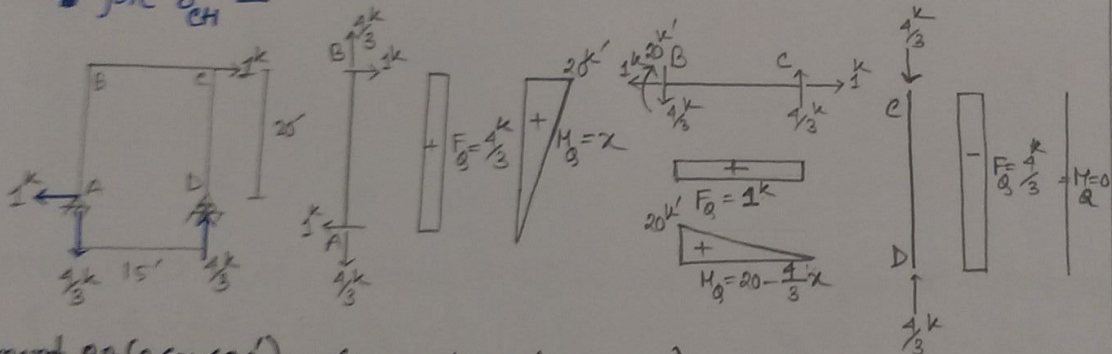
Segment AB ( $0 < x \leq 20'$ )	Segment BC ( $0 < x \leq 15'$ )	Segment CD ( $0 < x \leq 20'$ )
$M_Q = -x$ , $F_Q = -\frac{4}{3}k$	$M_Q = -(20 - \frac{1x}{3})$ , $F_Q = 0$	$M_Q = 0$ , $F_Q = \frac{1}{3}k$
$M_P = -10x$ , $F_P = -\frac{40}{3}$	$M_P = -(200 - \frac{10x}{3})$ , $F_P = 0$	$M_P = 0$ , $F_P = \frac{40}{3}k$

Using principle of virtual work,

$$\begin{aligned}
 EA\delta_{BH} &= \int_0^{20} \frac{10x^2}{EI} dx + \frac{(-\frac{1}{3}) \times (\frac{40}{3}) \times 20}{AE} + \int_0^{15} \frac{(20 - \frac{1}{3}x)(200 - \frac{40x}{3})}{EI} dx + \frac{\frac{1}{3} \times \frac{40}{3} \times 20}{AE} \\
 &= \frac{2 \times 20 \times \frac{1}{3} \times \frac{40}{3}}{20 \times 30000} + \frac{1}{EI} \cdot \frac{10}{3} x^3 \Big|_0^{20} + \frac{1}{EI} \int_0^{15} \left( 4000 - \frac{1600}{3}x + \frac{160}{9}x^2 \right) dx \\
 &= 0.0012 + \frac{\frac{10}{3} \times 20^3 \times 144}{30 \times 10^3 \times 8000} + \frac{1}{EI} \left( 4000x - \frac{800}{3}x^2 + \frac{160}{27}x^3 \right) \Big|_0^{15} \\
 &= 0.0012 + \frac{2}{125} + \frac{(4000 \times 15 - \frac{800}{3} \times 15^2 + \frac{160}{27} \times 15^3) \times 144}{30 \times 10^3 \times 8000}
 \end{aligned}$$

$\therefore \delta_{BH} = \underline{0.0292'}$  (left)

for  $\delta_{CH}$  -



Segment AB ( $0 < x \leq 20'$ ); Segment BC ( $0 < x \leq 15'$ ); Segment CD ( $0 < x \leq 20'$ )

$M_B = x$   
 $V_B = -10x$   
 $F_B = \frac{4}{3} k$   
 $F_P = -\frac{40k}{3}$

$M_B = 20 - \frac{1}{3}x$   
 $F_B = 1 k$   
 $M_P = -200 + \frac{40x}{3}$   
 $F_P = 0$

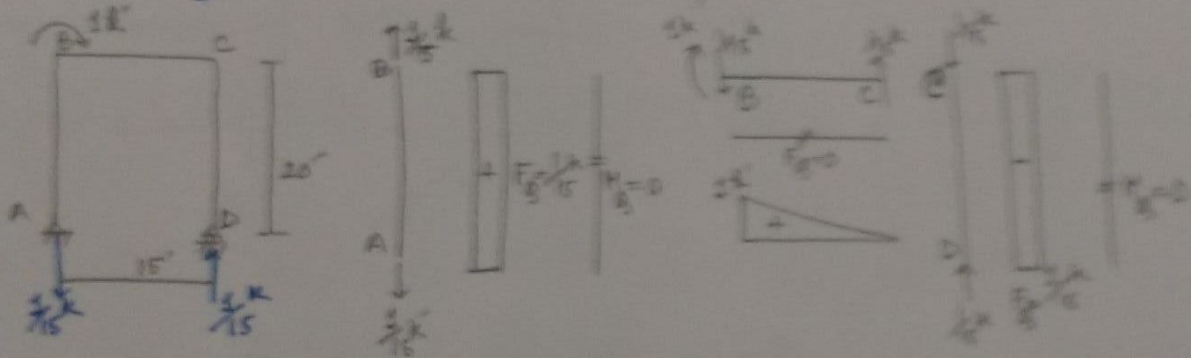
$M_B = 0$   
 $F_B = -\frac{1}{3} k$   
 $M_P = 0$   
 $F_P = \frac{40k}{3}$

Using principle of virtual work -

$$\begin{aligned}
 \Delta S_{CH} &= \int_0^{20} \frac{x(-10x)}{EI} dx + \frac{\frac{10}{3}(-\frac{40}{3})20}{AE} + \int_0^{15} \frac{(20-\frac{2}{3}x)(\frac{1600}{3}-4000-\frac{160}{3}x)}{EI} dx + \frac{(-\frac{1}{3})(\frac{40}{3})20}{AE} \\
 &= \frac{-2 \times \frac{10}{3} \times \frac{40}{3} \times 20}{20 \times 30000} - \frac{10}{9EI} \times \frac{2}{3} \Big|_0^{20} + \frac{1}{EI} \int_0^{15} \left( \frac{1600}{3}x - 4000 - \frac{160}{3}x^2 \right) dx \\
 &= -0.0012 - \frac{2}{125} + \frac{\left( \frac{1600}{300} \times 15^2 - 4000 \times 15 - \frac{160}{81} \times 15^3 \right) \times 144}{30 \times 10^3 \times 3000}
 \end{aligned}$$

$\therefore \Delta_{CH} = -0.0292''$  (leftward)

for  $\theta_B$  -



Segment AB ( $0 < x < 20$ ); Segment CB ( $0 < x < 15$ ); segment CD ( $0 < x < 20$ )

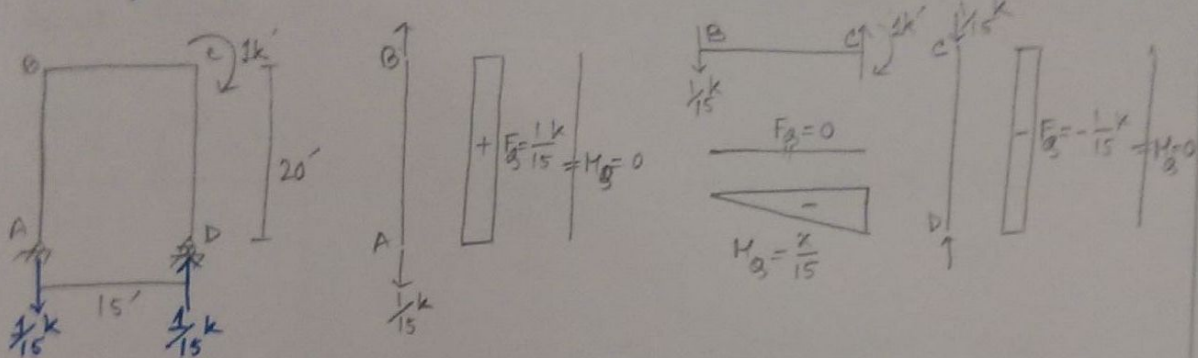
$M_B = 0$	$F_B = \frac{1}{15} k$	$M_B = \frac{x}{15}$	$F_B = 0$	$M_B = 0$	$F_B = -\frac{2}{15} k$
$M_P = -10x$	$F_P = -\frac{40}{3} k$	$M_P = -\frac{40}{3}x$	$F_P = 0$	$M_P = 0$	$F_P = \frac{40}{3} k$

Using principle of virtual work -

$$\begin{aligned}
 \Delta \theta_B &= \frac{(-\frac{40}{3}) \frac{1}{15} \times 20}{AE} + \int_0^{15} \frac{\frac{x}{15} (-\frac{40}{3}x)}{EI} dx + \frac{(-\frac{1}{15})(\frac{40}{3}) \times 20}{AE} \\
 &= -\frac{\frac{40}{3} \times \frac{1}{15} \times 20 \times 2}{20 \times 30000} - \frac{8 \times 144 \times 15^3}{9 \times 30 \times 10^3 \times 3000 \times 3} = -\frac{1}{16875} - \frac{3}{5000}
 \end{aligned}$$

$\therefore \theta_B = -6.59 \times 10^{-4}$  radian (anticlockwise)

for  $\theta_c$  —



Segment AB ( $0 < x < 20$ ) ; Segment BC ( $0 < x < 15$ ) ; Segment CD ( $0 < x < 20$ )

$M_B = 0$ ; $F_B = \frac{1}{15} k$	$M_B = -\frac{x}{15}$ ; $F_B = 0$	$M_B = 0$ ; $F_B = -\frac{1}{15} k$
$M_P = -10x$ ; $F_P = -\frac{40}{3}$	$M_P = -200 + \frac{40x}{3}$ ; $F_P = 0$	$M_P = 0$ ; $F_P = \frac{40}{3} k$

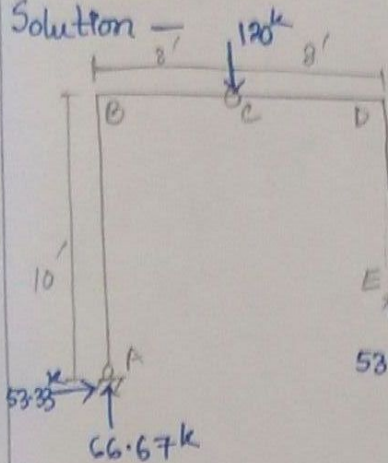
Using principle of virtual work —

$$\begin{aligned} \Sigma \theta \theta_c^v &= \frac{(-\frac{40}{3})(\frac{1}{15})20}{AE} + \int_0^{15} \frac{(-\frac{x}{15})(-200 + \frac{40x}{3})}{EI} dx + \frac{(-\frac{1}{15})(\frac{40}{3})20}{AE} \\ &= -\frac{(\frac{40}{3})(\frac{1}{15}) \times 20 \times 2}{20 \times 30 \times 10^3} - \frac{1}{EI} \int_0^{15} (-200 \frac{x}{15} + \frac{40}{15} x^2) dx \\ &= -\frac{1}{16875} - \frac{1}{EI} \left( -\frac{40}{3} \cdot \frac{x^2}{2} + \frac{8}{9} \cdot \frac{x^3}{3} \right) \Big|_0^{15} \\ &= -\frac{1}{16875} - \frac{(\frac{8}{27} \times 15^3 - \frac{20}{3} \times 15^2) \times 144}{30 \times 10^3 \times 8000} \\ &= -\frac{1}{16875} + \frac{3}{10000} \end{aligned}$$

$\therefore \theta_c^v = \underline{2.4 \times 10^{-4} \text{ radian (clockwise)}}$

18. Compute change in slope for the cross section on left of hinge. Given,  $E = 30 \times 10^3$  ksi,  $A = 20$  in<sup>2</sup>,  $I = 2500$  in<sup>4</sup>.

Solution —



P-force analysis:

$$\sum M_A = 0 \quad \uparrow +$$

$$\Rightarrow R_{Ey} \times 16 + R_{Ex} \times 2 - 120 \times 8 = 0$$

$$\Rightarrow R_{Ex} + 8R_{Ey} = 180 \quad \dots (1)$$

Taking the right part and

$$\sum M_C = 0 \quad \uparrow + \Rightarrow R_{Ey} \times 8 - R_{Ex} \times 8 = 0$$

$$\Rightarrow R_{Ex} = R_{Ey}$$

$$\therefore \text{From eq (1)} \Rightarrow R_{Ey} + 8R_{Ey} = 180$$

$$\Rightarrow R_{Ey} = \frac{180}{9} = 53.33 \text{ k}$$

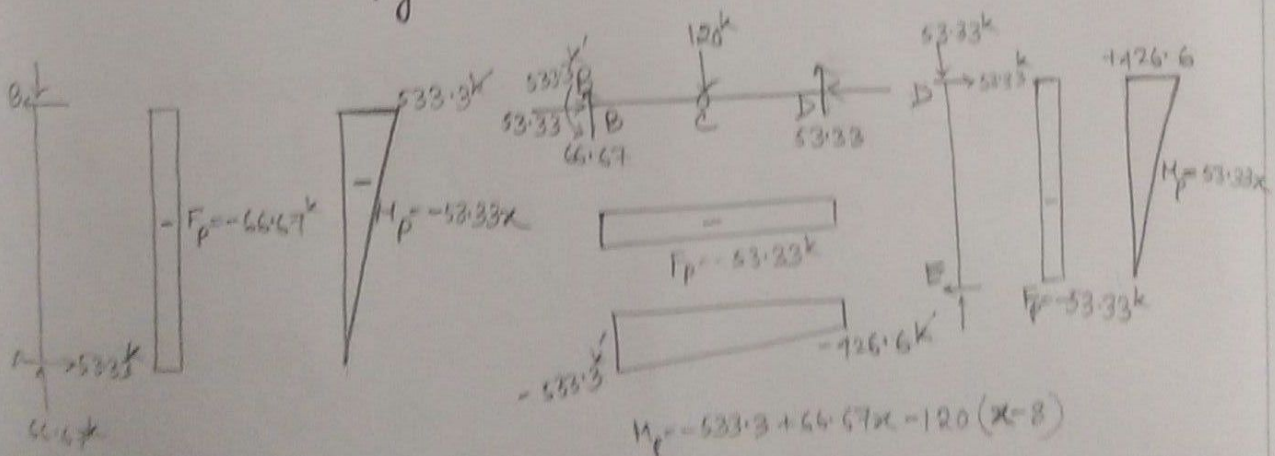
$$\therefore R_{Ex} = 53.33 \text{ k}$$

$$\text{Now, } \sum F_x = 0 \Rightarrow R_{Ex} = R_{Ax} = 53.33 \text{ k}$$

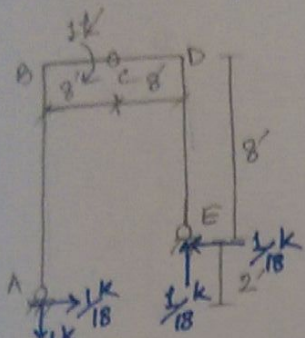
$$\text{and } \sum F_y = 0 \Rightarrow R_{Ay} + R_{Ey} - 120 = 0$$

$$\Rightarrow R_{Ay} = 120 - 53.33$$

$$\therefore R_{Ay} = 66.67 \text{ k}$$



8-force analysis:



$$\sum M_A = 0 \Rightarrow 1 - R_{Ey} \times 16 - R_{Ex} \times 2 = 0$$

$$\Rightarrow 16R_{Ey} + 2R_{Ex} = 1 \quad \dots (1)$$

Considering right part of the hinge and taking

$$\sum M_C = 0 \Rightarrow R_{Ex} \times 8 - R_{Ey} \times 8 = 0 \Rightarrow R_{Ex} = R_{Ey}$$

From eqn (1)  $\Rightarrow 18R_{Ey} = 1 \Rightarrow R_{Ey} = \frac{1}{18} k$

$$\therefore R_{Ex} = \frac{1}{18} k$$

Now,  $\sum F_x = 0$

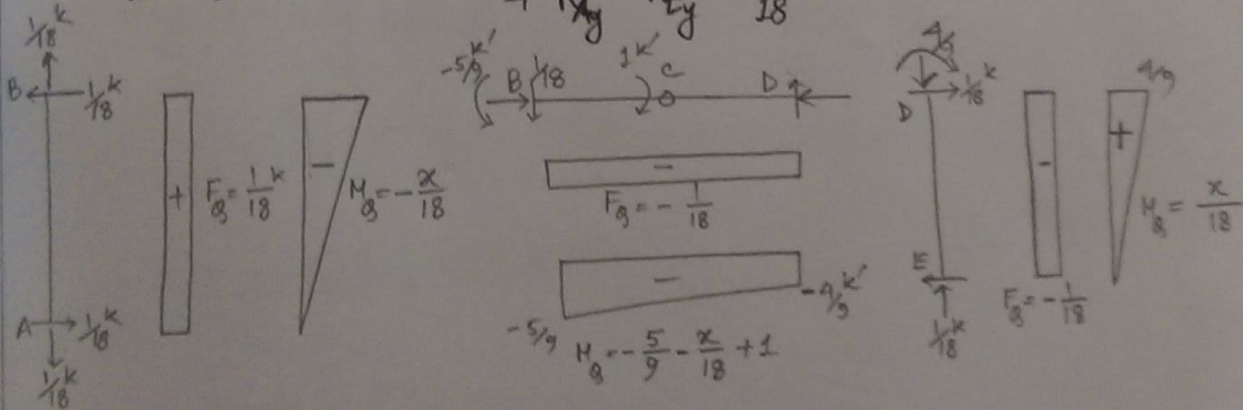
$$\Rightarrow R_{Ex} - R_{Ax} = 0$$

$$\Rightarrow R_{Ax} = R_{Ex} = \frac{1}{18}$$

and  $\sum F_y = 0$

$$\Rightarrow R_{Ey} - R_{Ay} = 0$$

$$\Rightarrow R_{Ay} = R_{Ey} = \frac{1}{18} k$$



Segment AB ( $0 < x \leq 16$ ); Segment BD ( $0 < x \leq 16$ );

$$M_s = -\frac{x}{18} \quad ; \quad F_s = \frac{1}{18} \quad \left| \quad M_s = -\frac{5}{9} - \frac{x}{8} + 1 \quad ; \quad F_s = -\frac{1}{18}$$

$$M_p = -53.33x \quad ; \quad F_p = -66.67 \quad \left| \quad M_p = -533.3 + 66.67x - 12(x-8) \quad ; \quad F_p = -53.33$$

Segment ED ( $0 < x \leq 8$ );

$$M_s = \frac{x}{18} \quad ; \quad F_s = -\frac{1}{18}$$

$$M_p = 53.33x \quad ; \quad F_p = -53.33$$

Using Principle of virtual work -

$$\begin{aligned} \sum \theta_{C_2} &= \int_0^{10} \frac{53.33x \cdot \frac{x}{18}}{EI} dx + \int_0^{16} \frac{\left(-\frac{5}{9} - \frac{x}{18} + 1\right) \{-533.3 + 66.67x - 120(x-8)\}}{EI} dx \\ &+ \int_0^8 \frac{53.33x \cdot \frac{x}{18}}{EI} dx + \frac{\frac{1}{18}(-66.67) \times 10}{AE} + \frac{\left(-\frac{1}{18}\right)(-53.33) \times 16}{AE} + \frac{\left(-\frac{1}{18}\right)(-53.33) \times 8}{AE} \end{aligned}$$

$$\begin{aligned} &= \frac{53.33 \times 10^2 \times 144}{18 \times 2 \times 30 \times 10^3 \times 2500} + \frac{53.33 \times 8^2 \times 144}{18 \times 2 \times 30 \times 10^3 \times 2500} + \frac{\frac{1}{18} \times 53.33 \times 24}{20 \times 30 \times 10^3} - \frac{\frac{10}{18} \times 66.67}{20 \times 30 \times 10^3} \\ &+ \int_0^{16} \frac{\left(\frac{16}{9} - \frac{x}{18}\right)(426.7 - 53.33x)}{EI} dx \end{aligned}$$

$$= 2.84 \times 10^{-4} + 1.82 \times 10^{-4} + 1.19 \times 10^{-4} - 0.62 \times 10^{-4}$$

$$+ \int_0^{16} \left(189.64 - 47.41x + 2.96x^2\right) dx \times \frac{1}{EI}$$

$$= 5.23 \times 10^{-4} + \frac{\left(189.64 \times 16 - \frac{47.41}{2} \times 16^2 + \frac{2.96}{3} \times 16^3\right) \times 144}{30 \times 10^3 \times 2500}$$

$$= 5.23 \times 10^{-4} + 19.34 \times 10^{-4}$$

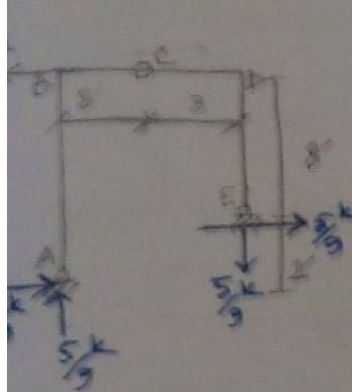
$$= 24.57 \times 10^{-4} \text{ radian}$$

$$\therefore \theta_{C_2} = \underline{\underline{0.0025 \text{ radian (clockwise)}}}$$

Ans.

Assignment-19: Determine horizontal deflection at B.

Solution - A force analysis:



$$\sum M_A = 0 \quad \downarrow +$$

$$\Rightarrow 1 \times 10 + R_{Ex} \times 2 + R_{Ey} \times 16 = 0$$

$$\Rightarrow R_{Ex} + 8R_{Ey} = 5 \quad \dots (1)$$

Considering right part of the hinge and taking,  $\sum M_C = 0 \quad \downarrow$

$$\Rightarrow R_{Ey} \times 8 - R_{Ex} \times 8 = 0$$

$$\Rightarrow R_{Ex} = R_{Ey}$$

From eqn (1)  $\Rightarrow 9R_{Ey} = 5 \Rightarrow R_{Ey} = \frac{5}{9} \text{ k}$

$$\therefore R_{Ex} = \frac{5}{9} \text{ k}$$

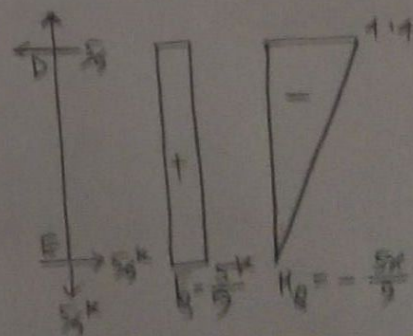
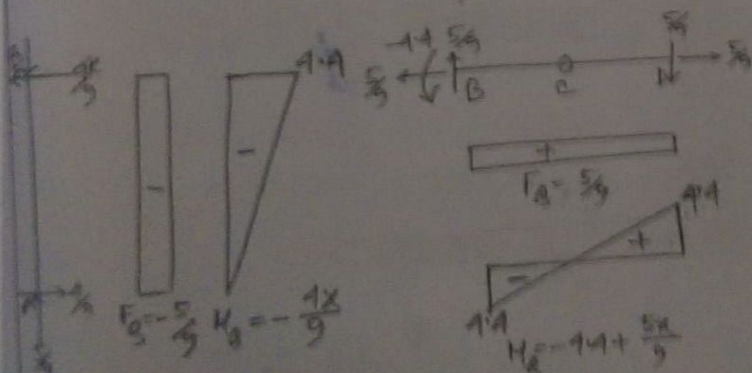
Now,  $\sum F_x = 0 (\rightarrow) \Rightarrow \frac{5}{9} + R_{Ax} - 1 = 0$

$$\Rightarrow R_{Ax} = 1 - \frac{5}{9} = \frac{4}{9} \text{ k}$$

and  $\sum F_y = 0 \quad \uparrow +$

$$\Rightarrow R_{Ay} - R_{By} = 0$$

$$\Rightarrow R_{Ay} = R_{By} = \frac{5}{9} \text{ k}$$



Segment AB ( $0 < x < 10'$ );

$$M_B = -\frac{4x}{9} \quad ; \quad F_B = -\frac{5}{9}$$

$$M_P = -53.33x \quad ; \quad F_P = -66.67$$

Segment BD ( $0 < x < 16'$ );

$$M_B = -4.4 + \frac{5x}{9} \quad ; \quad F_B = \frac{5}{9}$$

$$M_P = -533.3 + 66.67x - 120(x-8) \quad ; \quad F_P = -53.33$$

$$= 426.7 - 53.33x$$

Segment ED ( $0 < x < 8'$ );

$$M_B = -\frac{5x}{9} \quad ; \quad F_B = \frac{5}{9}$$

$$M_P = 53.33x \quad ; \quad F_P = -53.33$$

Using principle of virtual work -

$$\Delta \delta_{BH}^{\leftarrow} = \int_0^{10} \left( \frac{-4x}{9} \right) \left( \frac{-53.33x}{EI} \right) dx + \frac{\left( \frac{-5}{9} \right) (-66.67) 10}{AE} + \frac{\left( \frac{5}{9} \right) (-53.33) 16}{AE}$$

$$+ \frac{\left( \frac{5}{9} \right) (-53.33) 8}{AE} + \int_0^8 \left( \frac{-5x}{9} \right) \left( \frac{53.33x}{EI} \right) dx + \int_0^{16} \left( \frac{-4.4 + \frac{5x}{9}}{EI} \right) (426.7 - 53.33x) dx$$

$$= \frac{53.33 \times 4 \times 10^3 \times 144}{9 \times 3 \times 30 \times 10^3 \times 2500} + \frac{5/9 \times 66.67 \times 10}{20 \times 30 \times 10^3} - \frac{5/9 \times 53.33 \times 24}{20 \times 30 \times 10^3}$$

$$- \frac{5 \times 53.33 \times 8^3 \times 144}{9 \times 3 \times 30 \times 10^3 \times 2500} + \int_0^{16} \frac{(-1877.48 + 471.71x - 29.63x^2)}{EI} dx$$

$$= 5.1 \times 10^{-3} + 0.62 \times 10^{-3} - 1.2 \times 10^{-3} - 9.7 \times 10^{-3}$$

$$+ \frac{(-1877.48 \times 16 + 471.71 \times \frac{16^2}{2} - 29.63 \times \frac{16^3}{3}) \times 144}{30 \times 10^3 \times 2500}$$

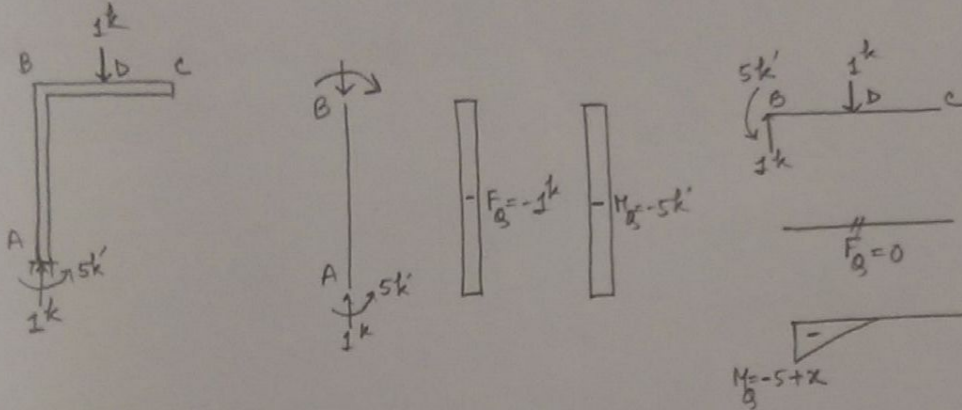
$$= -5.18 \times 10^{-3} - 0.0194$$

$$\therefore \delta_{BH}^{\leftarrow} = -0.0216' \text{ (rightward)}$$

Aus.

Assignment-17: Compute vertical deflection at mid point of BC.

Solution -



For segment AB ( $0 < x \leq 20'$ ): and for segment BD ( $0 < x \leq 5'$ ):

$$F_B = -5k' ; F_D = -1k$$

$$M_B = -200k' ; F_D = -20k'$$

$$M_B = -200 + 20x ; F_D = 0$$

$$M_B = -5 + x ; F_D = 0$$

Using principle of virtual work,

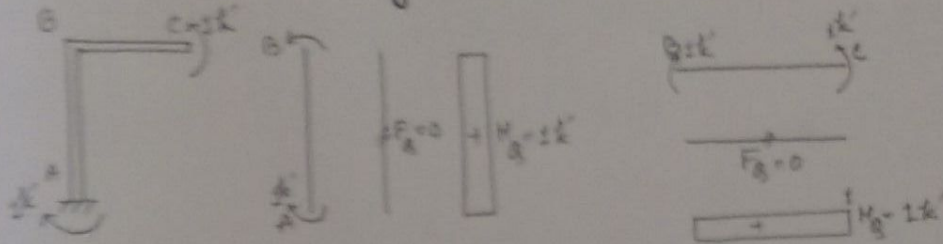
$$\begin{aligned} \Delta_D \downarrow &= \int_0^{20} \frac{(-5)(-200)}{EI_1} dx + \frac{1 \times 20 \times 20}{A_1 E} + \int_0^5 \frac{(20x - 200)(x - 5)}{EI_2} dx \\ &= \frac{1000 \times 20}{30 \times 10^3 \times 144 \times \frac{200}{144^2}} + \frac{400}{15 \times 30000} + \frac{1}{EI_2} \left( \frac{20}{3} x^3 - 150x^2 + 1000x \right) \Big|_0^5 \end{aligned}$$

$$\Rightarrow \Delta_D \downarrow = \frac{12}{25} + \frac{1}{1125} + \frac{1}{15}$$

$$\therefore \Delta_D \downarrow = \underline{\underline{0.55'}} \text{ (down)}$$

Assignment-16: Compute change in slope at "c". Given, same as assignment 15.

Solution -  $\delta$ -force analysis:



For segment AB ( $0 < x \leq 20$ ): and for segment BC ( $0 < x \leq 10$ ):

$$M_B = +1k' ; F_B = 0$$

$$M_P = -200k' ; F_P = -20k'$$

$$M_B = +1k' ; F_B = 0$$

$$M_P = -20x ; F_P = 0$$

Using principle of virtual work,

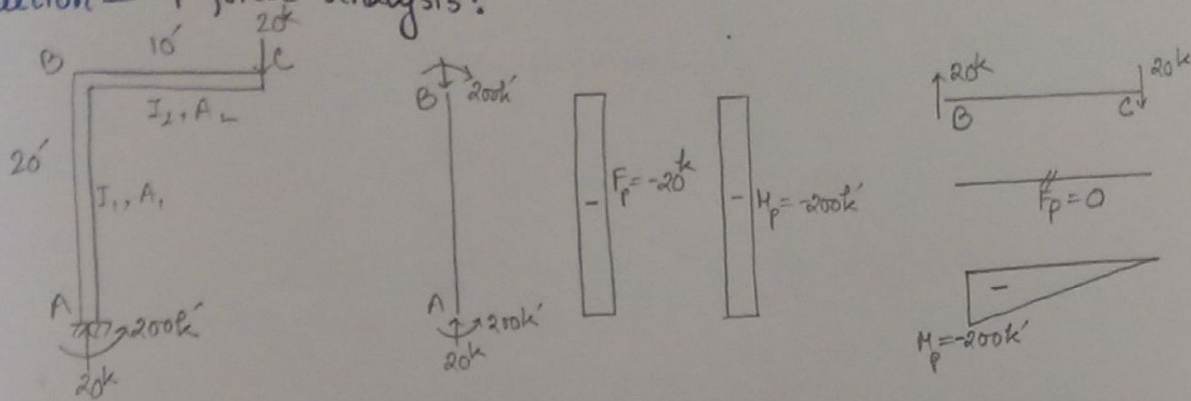
$$\delta \theta \delta_c^{\theta} = \int_0^{20} \frac{(-200)(1)}{EI_1} dx + \int_0^{10} \frac{(-20x)(1)}{EI_2} dx$$

$$= - \frac{200 \times 20}{30 \times 10^3 \times 144 \times \frac{200}{144^2}} - \frac{10 \times 10^2}{30 \times 10^3 \times 144 \times \frac{150}{144^2}}$$

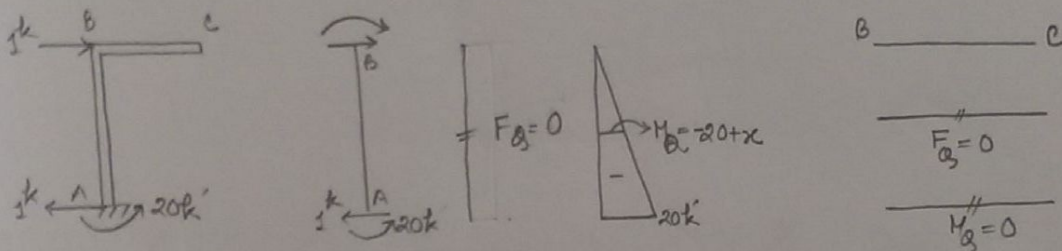
$$\therefore \delta_c^{\theta} = - \underline{0.128 \text{ radian (clockwise)}}$$

Assignment - 15: Compute horizontal deflection at "B". Given,  $I_1 = 200 \text{ in}^4$ ,  $I_2 = 150 \text{ in}^4$ ,  $A_1 = 15 \text{ in}^2$ ,  $A_2 = 10 \text{ in}^2$ ,  $E = 30 \times 10^3 \text{ ksi}$ .

Part (a) - P force analysis:



Part (b) - Q force analysis:



Now, Segment CB ( $0 < x < 10$ ): and Segment AB ( $0 < x < 20$ ):

$$M_Q = 0; F_Q = 0$$

$$M_P = -200k'; F_P = 0$$

$$M_Q = -20 + x; F_Q = 0$$

$$M_P = -200k'; F_P = -20k$$

Using principle of virtual work -

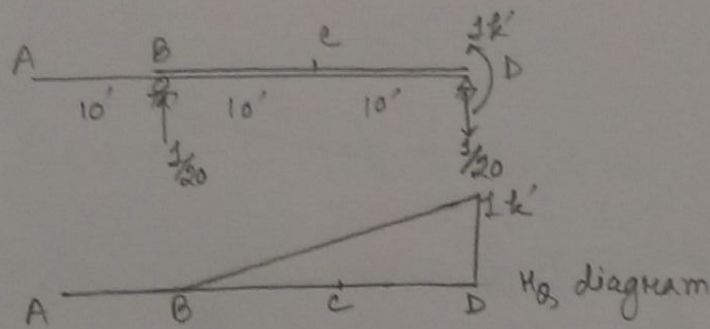
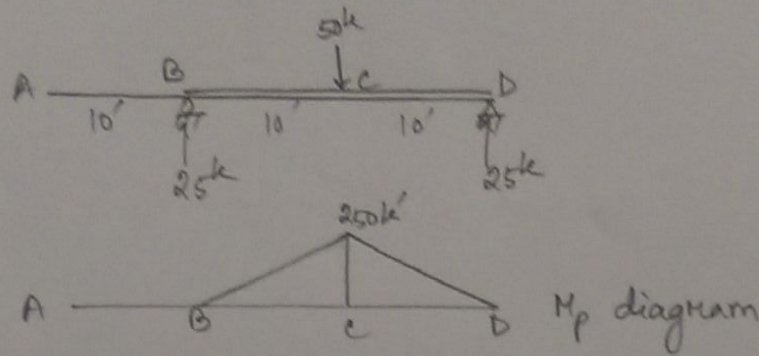
$$\Delta Q \delta_B^{\rightarrow} = \int_0^{20} \frac{4000 - 200x}{EI_1} dx = \frac{1}{EI} (4000 \times 20 - 100 \times 20^2)$$

$$\Rightarrow \delta_B^{\rightarrow} = \frac{40000}{30 \times 10^3 \times 144 \times \frac{200}{1442}}$$

$$\therefore \delta_B^{\rightarrow} = \underline{\underline{0.96'}} \text{ (rightward)}$$

Assignment-14: Find change in slope at "D." Given,  $E=200 \text{ kN/m}^2$

Solution -



Segment AB ( $0 < x < 10$ ); Segment BC ( $0 < x < 10$ ); Segment DC ( $0 < x < 10$ )

$$M_Q = 0$$

$$M_P = 0$$

$$M_Q = \frac{x}{20}$$

$$M_P = 25x$$

$$M_Q = -\frac{x}{20} + 1$$

$$M_P = 25x$$

Using principle of virtual work,

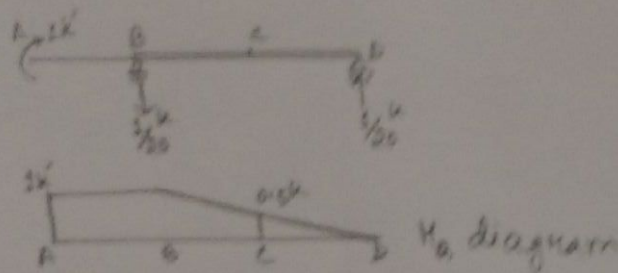
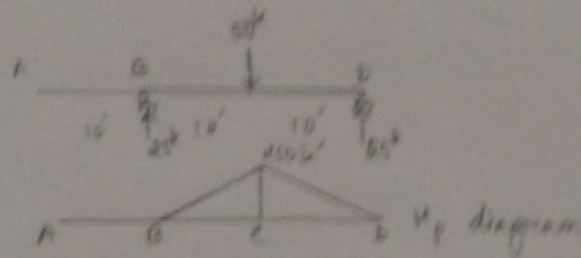
$$\sum Q \delta_D^Q = 0 + \frac{1}{2EI} \left[ \int_0^{10} \frac{5}{4} x^2 dx + \int_0^{10} (25x - \frac{5}{4} x^2) dx \right]$$

$$\Rightarrow \delta_D^Q = \frac{1950 \times 144}{2 \times 30 \times 10^3 \times 200}$$

$$\therefore \delta_D^Q = \underline{0.01 \text{ radian}} \text{ (counter-clockwise)}$$

Assignment-13: Find change in slope at 'A'. Given,  $I = 200 \text{ m}^4$ .

Solution -



Segment AB ( $0 < x < 10$ )	Segment BC ( $0 < x < 10$ )	Segment BC ( $0 < x < 10$ )
$M_B = 1k'$	$M_B = 1 - \frac{x}{20}$	$M_B = \frac{x}{20}$
$M_P = 0$	$M_P = 25x$	$M_P = 25x$

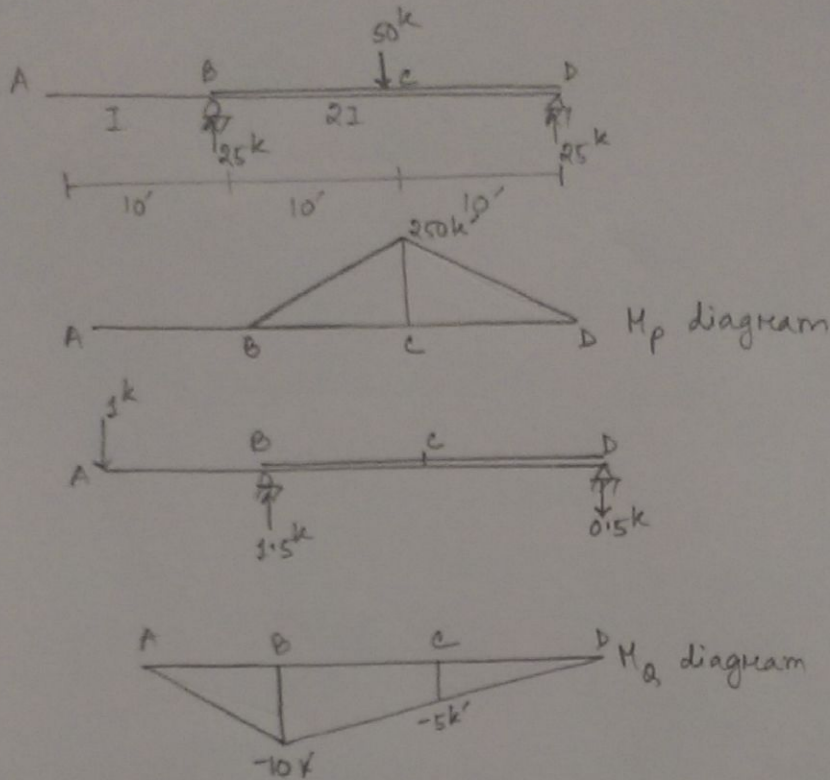
Using the principle of virtual work,

$$\begin{aligned} \Delta \theta_A &= 0 + \frac{1}{2EI} \left[ \int_0^{10} \left( 25x - \frac{5}{4}x^2 \right) dx + \int_0^{10} \frac{5}{4}x^2 dx \right] \\ &= \frac{1250}{2 \times 30 \times 10^9 \times 144 \times \frac{200}{144}} \end{aligned}$$

$$\therefore \Delta \theta_A = \underline{\underline{0.01 \text{ rad (clockwise)}}}$$

Assignment-12: Find vertical deflection at "A". Given,  $I = 200 \text{ in}^4$

Solution -



Segment AB ( $0 < x \leq 10'$ ) ; Segment BC ( $0 < x \leq 10'$ ) ; Segment DC ( $0 < x \leq 10'$ )

$$M_p = 0$$

$$M_q = -x$$

$$M_p = 25x$$

$$M_q = -10 + 0.5x$$

$$M_p = 25x$$

$$M_q = -0.5x$$

Using the principle of virtual work,

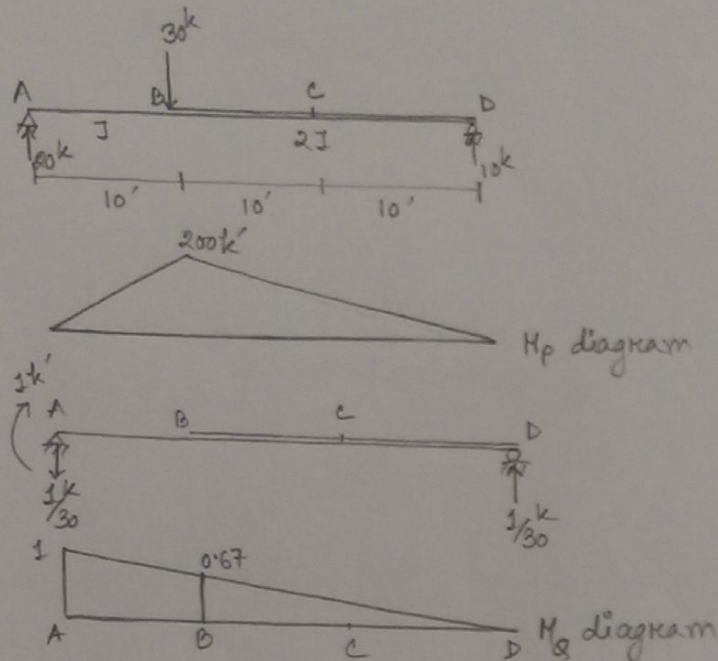
$$\sum Q \delta_A = \int \frac{M_q M_p dx}{EI} = \frac{1}{2EI} \left[ \int_0^{10} (-250x + 12.5x^2) dx + \int_0^{10} -12.5x^2 dx \right]$$

$$\rightarrow \delta_A = -6250 \times \frac{1}{EI} = - \frac{6250}{30 \times 10^9 \times 144 \times \frac{200}{144^2}}$$

$$\therefore \delta_A = \underline{\underline{-0.1' \text{ (up)}}}$$

Assignment-11: Find the change in slope at "A". Given,  $E = 30 \times 10^3 \text{ ksi}$  and  $I = 300 \text{ in}^4$ .

Solution -



Segment AB ( $0 < x \leq 10'$ ):  $M_p = 20x$ ,  $M_B = 1 - \frac{x}{30}$

Segment DB ( $0 < x \leq 20'$ ):  $M_p = 10x$ ,  $M_B = \frac{x}{30}$

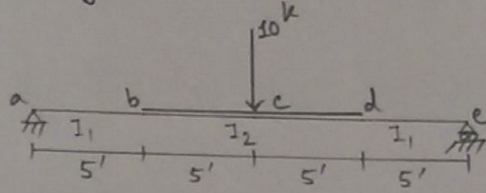
Using principle of virtual work,

$$\begin{aligned} \epsilon \theta \delta_A^v &= \frac{1}{EI} \int_0^{10} M_p M_B dx + \frac{1}{2EI} \int_0^{20} M_p M_B dx \\ &= \frac{1}{EI} \int_0^{10} \left( 20x - \frac{2}{3} x^2 \right) dx + \frac{1}{2EI} \int_0^{20} \frac{1}{3} x^2 dx \\ &= \frac{1}{EI} \left( 10x^2 - \frac{2}{9} x^3 \right) \Big|_0^{10} + \frac{1}{2EI} \left( \frac{1}{9} x^3 \right) \Big|_0^{20} \end{aligned}$$

$$= \frac{11000}{30 \times 10^3 \times 144 \times 200 \div 144^2}$$

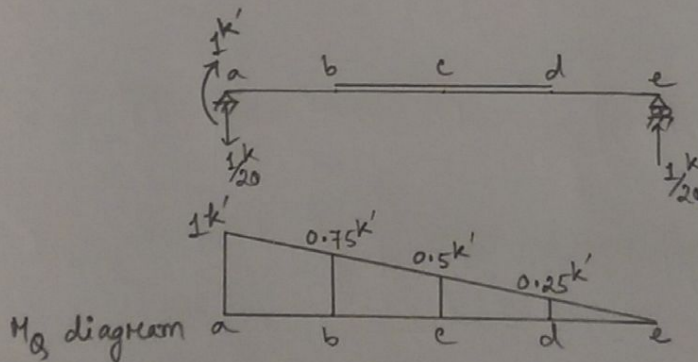
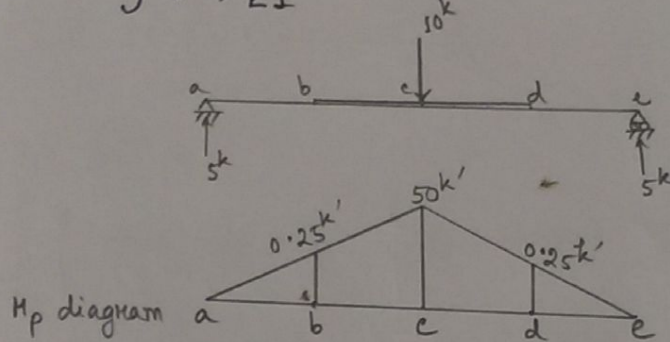
$$\therefore \delta_A^v = \underline{\underline{0.0293 \text{ md (clockwise)}}}$$

Assignment-10: Compute the change in slope of the cross-section at point "a" caused by the load shown.  $E = 30 \times 10^3 \text{ ksi}$ ,  $I_1 = 150 \text{ in}^4$ ,  $I_2 = 200 \text{ in}^4$ .



Solution:

We know,  $\Delta \theta = \int M_P M_Q \frac{dx}{EI}$  ----- (1)



• For segment ab ( $0 < x \leq 5'$ ):

$$M_P = 5x ; M_Q = 1 - \frac{x}{20}$$

• For segment cb ( $0 < x \leq 5'$ ):

$$M_P = 5(10-x) = 50 - 5x$$

$$M_Q = 1 - \frac{1}{20}(10-x) = 0.5 + \frac{x}{20}$$

• For segment de ( $0 < x \leq 5'$ ):

$$M_p = 5(15-x) - 10(5-x) = 5x + 25$$

$$M_Q = 1 - \frac{1}{20}(15-x) = 0.25 + \frac{x}{20}$$

• For segment ed ( $0 < x \leq 5'$ ):

$$M_p = 5(20-x) - 10(10-x) = 5x$$

$$M_Q = 1 - \frac{1}{20}(20-x) = \frac{x}{20}$$

Now from eq<sup>n</sup> (1)  $\Rightarrow$

$$\begin{aligned} \Sigma \delta \delta = & \int_a^b \frac{1}{EI_1} 5x \left(1 - \frac{x}{20}\right) dx + \int_c^d \frac{1}{EI_2} (50-5x) \left(0.5 + \frac{x}{20}\right) dx + \int_d^e \frac{1}{EI_2} (5x+25) \left(\frac{25+x}{20}\right) dx \\ & + \int_e^c \frac{1}{EI_1} 5x \left(\frac{x}{20}\right) dx \end{aligned}$$

$$\text{on, } \delta = \frac{1}{EI_1} \int_0^5 (5x - 0.25x^2 + 0.25x^2) dx + \frac{1}{EI_2} \int_0^5 (25 - 0.25x^2 + 2.5x + 6.25 + 0.25x^2) dx$$

$$= \frac{1}{EI_1} \int_0^5 5x dx + \frac{1}{EI_2} \int_0^5 (31.25 + 2.5x) dx$$

$$= \frac{2.5x^2 \Big|_0^5}{EI_1} + \frac{31.25x + 1.25x^2 \Big|_0^5}{EI_2}$$

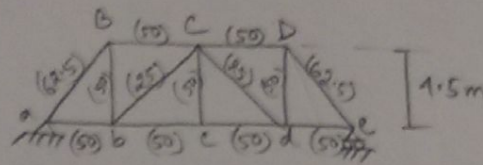
$$= \frac{62.5}{30 \times 10^3 \times 144 \times \frac{150}{144^2}} + \frac{187.5}{30 \times 10^3 \times 144 \times \frac{200}{144^2}}$$

$$= (2 \times 10^{-3} + 4.5 \times 10^{-3}) \text{ radian}$$

$$\therefore \delta = \underline{\underline{+ 0.0065 \text{ radian (clockwise)}}}$$

Ans.

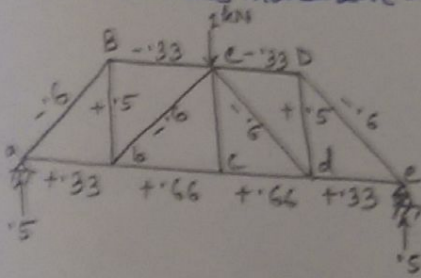
Assignment-9: Determine vertical and horizontal deflection at 'c' (op chord). Given, vertical and horizontal movement of "a" is 0.8" (down) and 0.6" (right) and vertical movement of "c" is 4" (down).



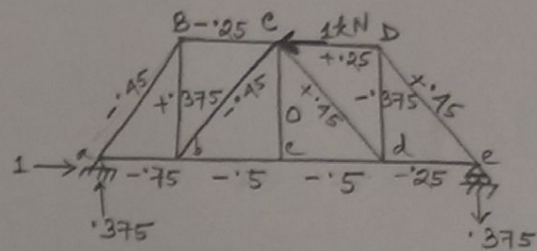
1 @ 3 m = 10m ; area are in cm<sup>2</sup>.

Solution: As no external loading, so only virtual load analysis will be one for horizontal and vertical deflections.

• For vertical movement -



• For horizontal movement -



Bar	L cm	F <sub>0</sub> (kN)		t °F	F <sub>0</sub> tL (kN°Fcm)	
		Vert. Deflect <sup>n</sup>	Hon. Deflect <sup>n</sup>		Vertical	Horizontal
ab	300	+0.33	-0.75	-10	-990	+2250
bc	300	+0.66	-0.5	-10	-1980	+1500
cd	300	+0.66	-0.5	-10	-1980	+1500
de	300	+0.33	-0.25	-10	-990	+750
BC	300	-0.33	-0.25	+40	-3960	-3000
CD	300	-0.33	+0.25	+40	-3960	+3000
					Σ = -13860	Σ = +6000

Here,

$$Q \delta_c \downarrow + 0 - 0.5 \times 8 \times \frac{1}{2.51} - 0.5 \times 4 \times \frac{1}{2.51} = \frac{-13860}{75000}$$

$$\therefore \delta_c \downarrow = \underline{0.0134 \text{ m (up)}}$$

Ans.

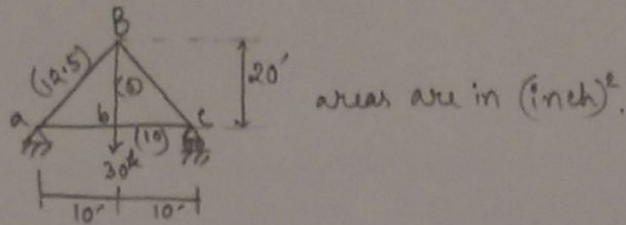
Again,

$$Q \delta_c \leftarrow + 0.6 \times \frac{1}{2.51} - 0.375 \times 8 \times \frac{1}{2.51} + 0.375 \times 4 \times \frac{1}{2.51} = \frac{6000}{75000}$$

$$\therefore \delta_c \leftarrow = \underline{0.0106 \text{ m (left)}}$$

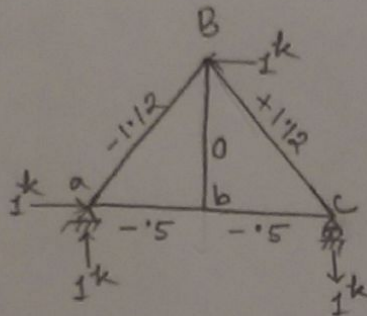
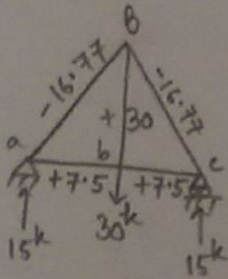
Ans.

Assignment - 8: Compute horizontal deflection of joint B. Given, support movements of 'a' - hor. 0.6" (right), ver. 0.8" (down) and 'c' - ver. 0.5" (down).



Solution:

• For real load -



Bar	L (')	A (in <sup>2</sup> )	L/A '/in <sup>2</sup>	F <sub>P</sub> k	F <sub>Q</sub> k	F <sub>P</sub> F <sub>Q</sub> L/A k <sup>2</sup> '/in <sup>2</sup>	t °F	F <sub>Q</sub> tL k°F'
ab	10	10	1	+7.5	-0.5	-3.75	0	0
bc	10	10	1	+7.5	-0.5	-3.75	0	0
aB	22.36	12.5	1.79	-16.77	-1.12	+33.62	0	0
Bc	22.36	12.5	1.79	-16.77	+1.12	-33.62	0	0
						Σ = -7.5		Σ = 0

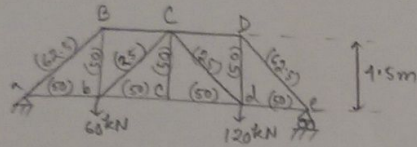
We know,  $W_s + W_R = W_d \Rightarrow Q\delta_B \leftarrow + R_{Ax}\Delta A_x + R_{Ay}\Delta A_y + R_{cy}\Delta c_y = \sum F_P F_Q L/AE$

$$\Rightarrow Q\delta_B \leftarrow + 1 \times \frac{.6}{12} - 1 \times \frac{.8}{12} + 1 \times \frac{.5}{12} = - \frac{7.5}{29 \times 10^3}$$

$$\Rightarrow \delta_B \leftarrow = \underline{\underline{0.0253'}} \text{ (Right)}$$

Ans:

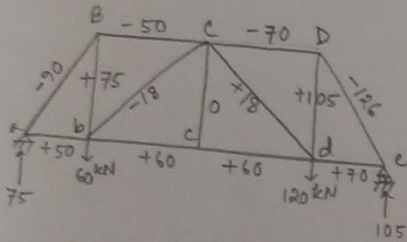
Assignment-7: Find the rotation of the Bar BC of the following truss.



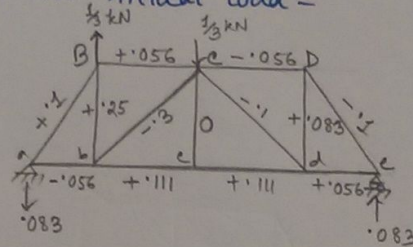
1 @ 3m = 12m ; areas are in cm<sup>2</sup>

Solution:

For real load -



For virtual load -



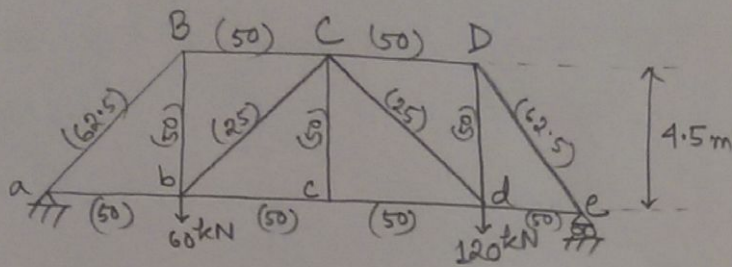
Bar	L cm	A cm <sup>2</sup>	L/A cm/cm <sup>2</sup>	F <sub>p</sub> kN	F <sub>v</sub> kN	F <sub>p</sub> F <sub>v</sub> L/A (kN) <sup>2</sup> /cm	t °F	F <sub>v</sub> tL KN°Fcm
ab	300	50	6	+50	-0.056	-16.8	-10	+168
bc	300	50	6	+60	+0.111	+40	-10	-333
cd	300	50	6	+60	+0.111	+40	-10	-333
de	300	50	6	+70	+0.056	+23.52	-10	-168
BC	300	50	6	-50	+0.056	-16.8	+40	+672
CD	300	50	6	-70	-0.056	+23.52	+40	-672
aB	540	62.5	8.64	-90	+0.1	-77.76	0	0
bC	540	25	21.6	-18	-0.3	+116.84	0	0
cD	540	25	21.6	+18	-0.1	-38.88	0	0
Dd	540	62.5	8.64	-126	-0.1	+108.86	0	0
Bb	450	50	9	+75	+0.25	+168.75	0	0
Dd	450	50	9	+105	+0.083	+78.75	0	0
						Σ = +450		Σ = -666

Rotation of Bar BC,  $\theta_{B-C} = \frac{\sum F_p F_v L}{AE} + \frac{\sum F_v t L}{75000} = \frac{450}{20.7 \times 10^3} - \frac{666}{75000}$

$\therefore \theta_{B-C} = 1.29 \times 10^{-4} \text{ m (clockwise)}$

Ans.

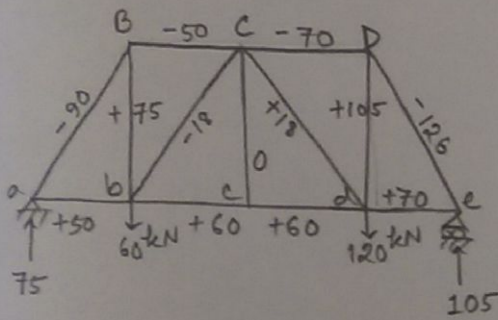
Assignment-6: Find relative deflection of joints Bd of the following truss.



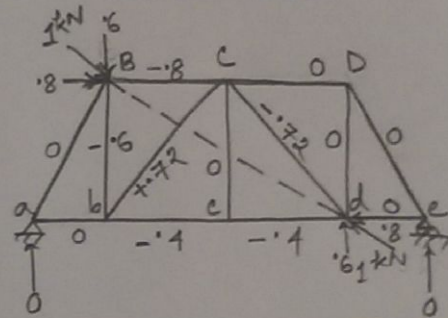
4 @ 3 m = 12m ; areas are in  $\text{cm}^2$

Solution:

• For real load:



For virtual load:

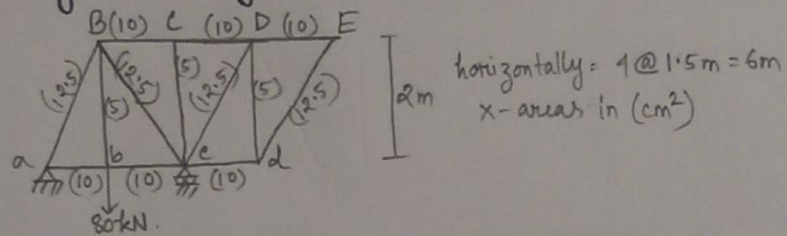


Bar	L cm	A $\text{cm}^2$	L/A $\text{cm}/\text{cm}^2$	$F_p$ kN	$F_B$ kN	$F_p F_B L/A$ $(\text{kN})^2/\text{cm}$	$t$ $^\circ\text{F}$	$F_B t L$ $\text{kN}^\circ\text{Fcm}$
BC	300	50	6	-50	-8	+240	+10	-9600
bc	300	50	6	+60	-1.4	-144	-10	+1200
cd	300	50	6	+60	-1.4	-144	-10	+1200
Bb	450	50	9	+75	-6	-105	0	0
bC	540	25	21.6	-18	+7.2	-279.94	0	0
Cd	540	25	21.6	+18	-7.2	-279.94	0	0
						$\Sigma = -1012.8$		$\Sigma = -7200$

We know,  $\delta_{B-d} = \Sigma F_B F_p L/AE + \Sigma F_B t L = -\frac{1012.8}{20.7 \times 10^3} - \frac{7200}{75000}$

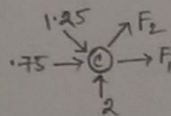
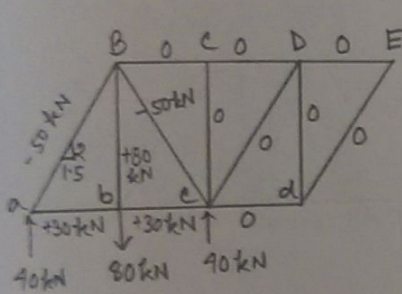
$\therefore \delta_{B-d} = \underline{4.707 \times 10^{-4} \text{ m (apart)}}$   
Ans.

Assignment-5: Find vertical deflection at point "E" of the following truss due to the given 80 kN load and due to a decrease of temperature of 40°F in diagonals only.  $E = 20.7 \times 10^3 \text{ kN/cm}^2$  and  $\alpha = 1/150,000 \text{ per } ^\circ\text{F}$ .



Solution:

• For real load -



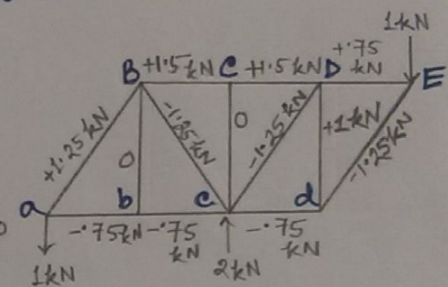
$$\sum F_y = -0.8 \times 1.25 + 2 + 0.8 \times F_2 = 0$$

$$\Rightarrow F_2 = -\frac{1}{8} = -0.125 \text{ kN}$$

$$\sum F_x = -0.8 \times 1.25 + 0.8 \times 1.25 + 0.75 + F_1 = 0$$

$$\Rightarrow F_1 = -0.75 \text{ kN}$$

• For virtual load -



Bar	L cm	A cm <sup>2</sup>	L/A cm/cm <sup>2</sup>	F <sub>p</sub> kN	F <sub>Q</sub> kN	F <sub>p</sub> F <sub>Q</sub> L/A kN <sup>2</sup> cm/cm <sup>2</sup>	t °F	F <sub>Q</sub> tL kN <sup>2</sup> Fem
ab	150	10	15	30	-0.75	-337.5	0	
bc	150	10	15	30	-0.75	-337.5	0	
cd	150	10	15	0	-0.75	0	0	
Bc	150	10	15	0	1.5	0	0	
Cd	150	10	15	0	1.5	0	0	
DE	150	10	15	0	0.75	0	0	
aB	250	12.5	20	-50	1.25	-1250	-40	-12500
Bc	250	12.5	20	-50	-1.25	1250	-40	12500
cd	250	12.5	20	0	-1.25	0	-40	12500
dE	250	12.5	20	0	-1.25	0	0	
Bb	200	5	40	80	0	0	0	
Cc	200	5	40	0	0	0	0	
Dd	200	5	40	0	1	0	0	
$\Sigma = -675$								$\Sigma = 25000$

We know,  $\delta = \sum F_p F_Q \frac{L}{AE} + \sum F_Q \alpha t L$

$$= \left[ (-675) \times \frac{1}{20.7 \times 10^3} + \frac{25000}{150,000} \right] \text{ kNem}$$

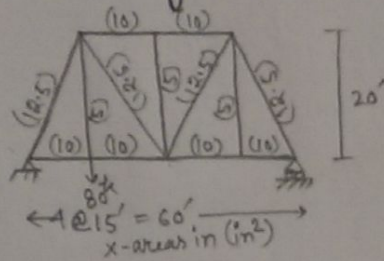
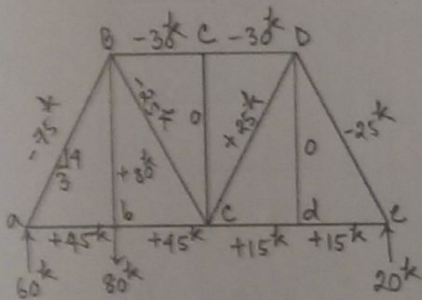
$$\delta = 0.134 \text{ cm (down)}$$

Ans.

Assignment-4: Find horizontal deflection at point "D" of following truss due to  $80^k$  load and due to a decrease of temperature of  $50^{\circ}F$  in the bottom chord only.  $E = 30 \times 10^3 \text{ kip/in}^2$  and  $\alpha_t = 1/150,000 \text{ per } ^{\circ}F$ .

Solution:

• For true load -

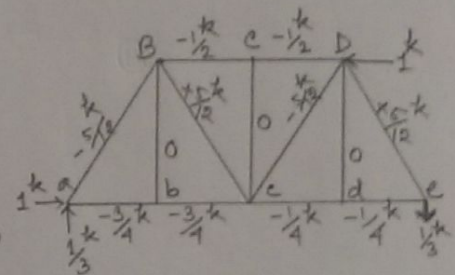


• For virtual load -

Free body diagram of the truss under a virtual load of 1 k applied at point D. Reaction forces are  $F_1 = 25$  k at the left support and  $F_2 = -30$  k at the right support.

$$\sum F_y = 8 \times 25 - 8 \times F_1 = 0 \Rightarrow F_1 = +25^k$$

$$\sum F_x = -6 \times 25 - 6 \times 25 - F_2 = 0 \Rightarrow F_2 = -30^k$$



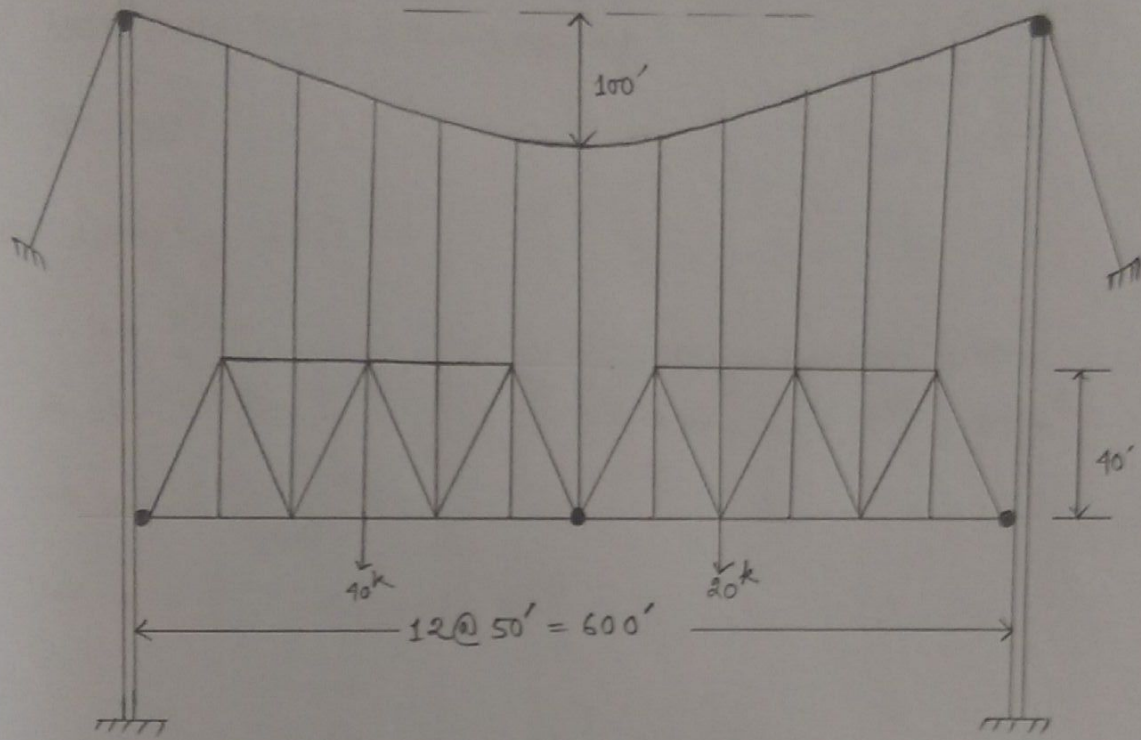
Bar	L ft	A in <sup>2</sup>	L/A ft/in <sup>2</sup>	F <sub>p</sub> kip	F <sub>q</sub> kip	F <sub>p</sub> F <sub>q</sub> L/A k <sup>2</sup> ft/in <sup>2</sup>	t °F	F <sub>q</sub> tL k°Fft
ab	15	10	1.5	45	-75	-50.625	-50	562.5
bc	15	10	1.5	45	-75	-50.625	-50	562.5
cd	15	10	1.5	15	-25	-5.625	-50	187.5
de	15	10	1.5	15	-25	-5.625	-50	187.5
Bc	15	10	1.5	-30	-5	22.5	0	0
Cd	15	10	1.5	-30	-5	22.5	0	0
aB	25	12.5	2	-75	-4167	62.505	0	0
bC	25	12.5	2	-25	4167	-20.835	0	0
cD	25	12.5	2	25	-4167	-20.835	0	0
Dc	25	12.5	2	-25	4167	-20.835	0	0
Bb	20	5	4	80	0	0	0	0
Cc	20	5	4	0	0	0	0	0
Dd	20	5	4	0	0	0	0	0
$\Sigma = -67.5$								$\Sigma = 1500$

We know,  $\delta = \sum F_p F_q \frac{L}{AE} + \sum F_q \alpha_t t L$

$$= \left[ -(67.5) \times \frac{1}{30 \times 10^3} + 1500 \times \frac{1}{150,000} \right] \text{ ft-k}$$

$\therefore \delta = 7.75 \times 10^{-3} \text{ ft (left)}$  Ans.

Assignment-3: Determine the hanger force and the two member forces 'a' and 'b' of the following suspension bridge.

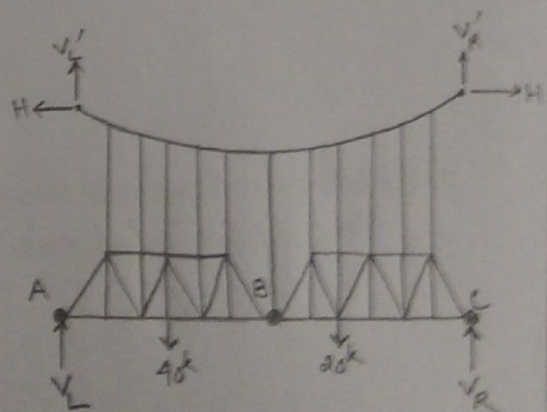


Solution:

$$\sum M_A = 0$$

$$\Rightarrow (V_R + V'_R) \times 600 = 40 \times 150 + 20 \times 400$$

$$\Rightarrow V_R + V'_R = 23.333 \dots (1)$$



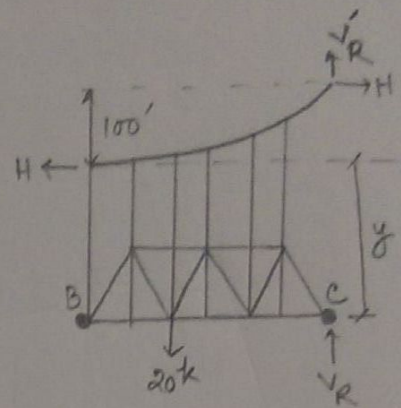
Here,

$$\Sigma M_B = 0$$

$$\Rightarrow H(100+y) - Hy - (V_R + V_R') \times 300 + 20 \times 100 = 0$$

$$\Rightarrow 100H - 23.333 \times 300 + 2000 = 0$$

$$\Rightarrow H = 50^k$$



Now applying G.E.T  $\rightarrow$

$$H = \frac{wL^2}{8h}$$

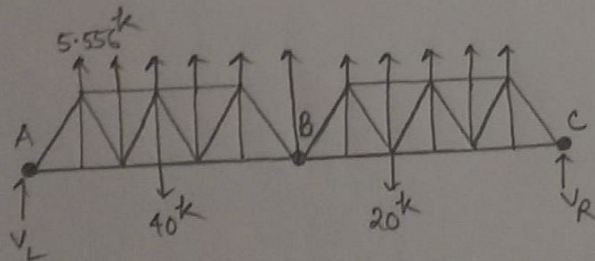
$$\Rightarrow w = \frac{8hH}{L^2}$$

$$\Rightarrow w = \frac{8 \times 100 \times 50}{600^2}$$

$$\Rightarrow w = \frac{1}{9}^k / \text{horizontal foot}$$

$\therefore$  Hanger tension force,

$$HF = \frac{1}{9} \times 50^k = 5.556^k$$



For this figure  $\rightarrow$

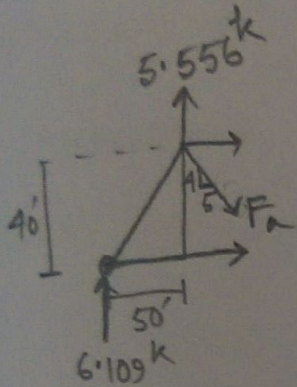
$$\Sigma M_A = 0$$

$$\Rightarrow 40 \times 150 + 20 \times 400 - V_R \times 600 - 5.556 [50 + 100 + 150 + 200 + 250 + 300 + 350 + 400 + 450 + 500 + 550] = 0$$

$$\Rightarrow V_R = -7.225^k (\downarrow)$$

$$\text{And } \Sigma F_y = 0 \uparrow \Rightarrow V_L + V_R - 40 - 20 + 5.556 \times 11 = 0$$

$$\Rightarrow V_L = 6.109^k (\uparrow)$$

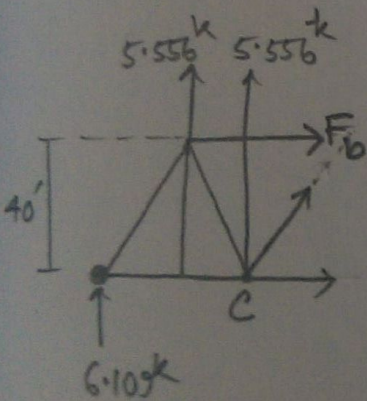


Here,  $\Sigma F_y = 0 \uparrow$

$$\Rightarrow 5.556 + 6.109 - \frac{4}{\sqrt{41}} F_a = 0$$

$$\Rightarrow F_a = 18.673^k$$

Aus.



Here,  $\Sigma M_c = 0 \downarrow$

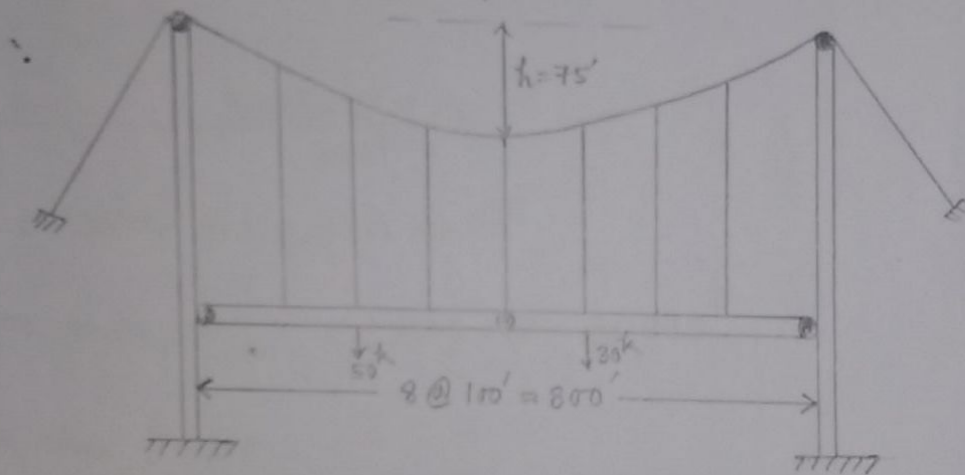
$$\Rightarrow F_b \times 40 + 5.556 \times 50 + 6.109 \times 100 = 0$$

$$\Rightarrow F_b = -22.2175^k$$

Aus.

Assignment-1: For the following figure, determine -

- Hanger force
- Draw shear force and bending moment diagram of stiffening girder
- Maximum tension of cable
- Stressed and unstressed length
- Stretch of the cable



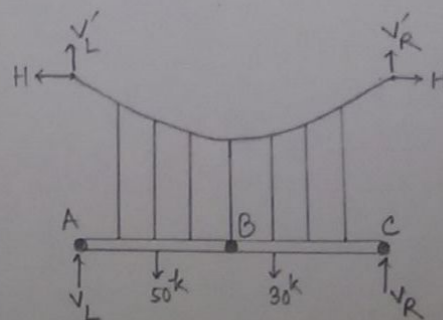
Solution:

From the free body diagram of the whole structure, we can find by taking moment about point A.

$$\sum M_A = 0 \quad \uparrow +ve$$

$$\Rightarrow (V_R + V'_R) \times 800 = 50 \times 200 + 30 \times 500$$

$$\Rightarrow V_R + V'_R = 31.25^k \quad \text{----- (1)}$$



Now from the free body diagram of the right stiffening girder we get-

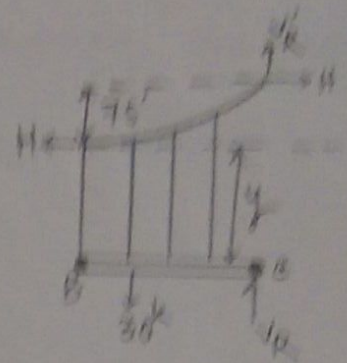
$$\sum M_B = 0 \quad \uparrow +ve$$

$$\Rightarrow H(75+y) - Hy + 30 \times 100 = (V_R + V'_R) 400 = 0$$

$$\Rightarrow 75H = (V_R + V'_R) 400 - 30 \times 100$$

$$\Rightarrow 75H = 31.25 \times 400 - 30 \times 100 \quad ; \text{ [from (1)]}$$

$$\therefore H = 126.67 \text{ k}$$



Applying general cable theorem (GCT),

$$h = \frac{wL^2}{8H}$$

$$\Rightarrow w = \frac{8Hh}{L^2} = \frac{8 \times 126.67 \times 75}{800^2}$$

$$\Rightarrow w = 0.11875 \text{ k/horizontal foot}$$

$$\therefore \text{Hanger force, H.F} = 0.11875 \times 100 = 11.875 \text{ k}$$

Now from this figure,

$$H_A = 0 \quad \uparrow +ve$$

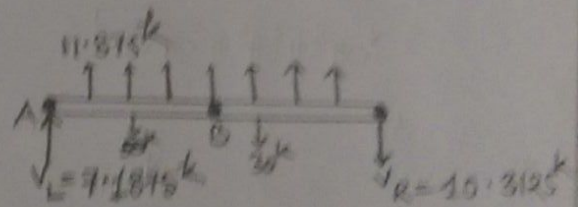
$$V_R \times 800 = 11.9(100+200+300+400+500 + 600+700) - 30 \times 500 - 50 \times 200$$

$$V_R = 10.3125 \text{ k} (\downarrow)$$

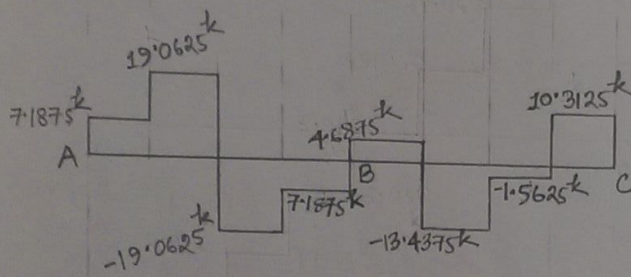
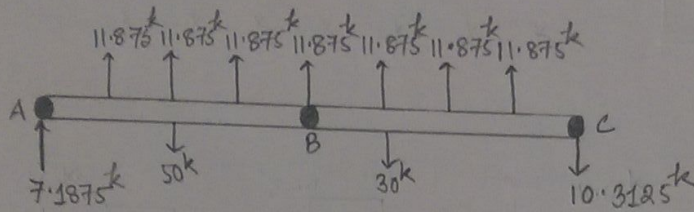
$$\text{Now, } \sum F_y = 0 \quad \uparrow +ve$$

$$\Rightarrow V_L - V_R + 11.9 \times 7 = 80$$

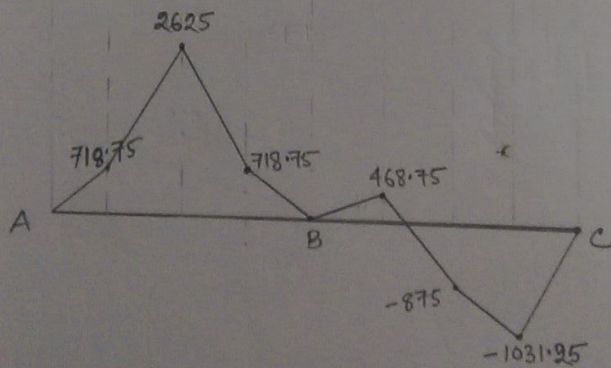
$$\Rightarrow V_L = -7.1875 \text{ k} (\uparrow)$$



Here, for the shear force and bending moment diagram of the stiffening girder -



SFD



BMD (kip-ft)

(c) Here,  $\gamma = 0$  and  $\theta = \frac{h}{L} = \frac{75}{800} = 0.09375$

$$T_{\max} = H(1 + 16\theta^2)^{1/2}$$

$$= 126.67 (1 + 16 \times (0.09375)^2)^{1/2}$$

$$= 135.28^k$$

(d) Cable length,

$$S = \frac{L}{2} (1 + 16\theta^2)^{1/2} + \frac{L}{8\theta} \ln [40 + (1 + 16\theta^2)^{1/2}]$$

$$= \frac{800}{2} [1 + 16 \times (0.09375)^2]^{1/2} + \frac{800}{8 \times 0.09375} \ln [4 \times 0.09375 + \{1 + 16 \times (0.09375)^2\}^{1/2}]$$

$$= 818.373'$$

Stretch,  $\Delta S = \frac{HL}{AE} [1 + \frac{16}{3} \theta^2]$

$$= \frac{126.67 \times 800}{50 \times 27 \times 10^3} [1 + \frac{16}{3} (0.09375)^2] ; \left[ \begin{array}{l} \text{Assuming} \\ A = 50 \text{ in} \\ \text{and } E = 27 \times 10^6 \text{ psi} \end{array} \right]$$

$$= 0.0786'$$

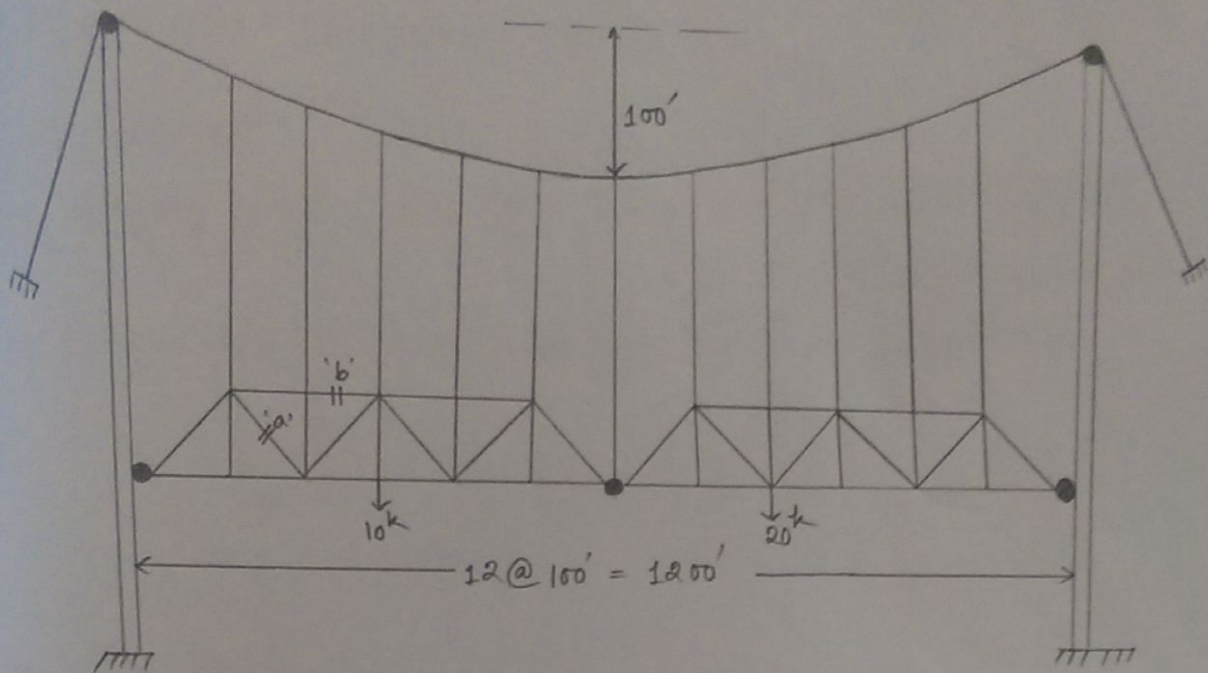
$\therefore$  Unstressed length =  $S - \Delta S$

$$= (818.373 - 0.0786)'$$

$$= 818.29'$$

Assignment-2:

Determine the hanger force and the two member forces 'a' and 'b' of the following suspension bridge.



Solution :

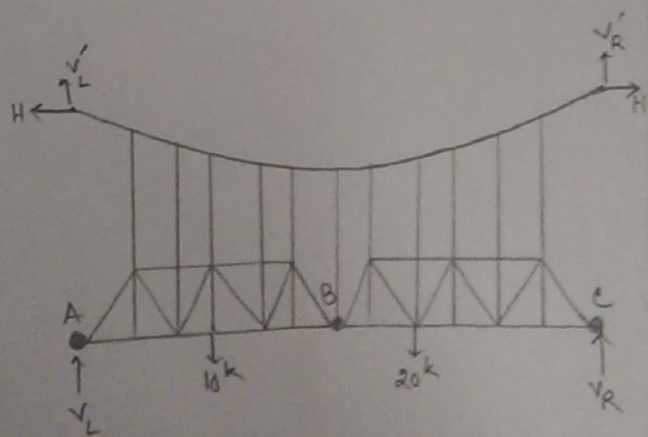
$$\sum M_A = 0 \curvearrowright$$

$$\Rightarrow (V_R + V'_R) \times 1200 = 10 \times 300 + 20 \times 800$$

$$\Rightarrow V_R + V'_R = (3000 + 16000) \times \frac{1}{1200}$$

$$\Rightarrow V_R + V'_R = 19000 \times \frac{1}{1200}$$

$$\therefore V_R + V'_R = 15.833 \dots \dots (1)$$



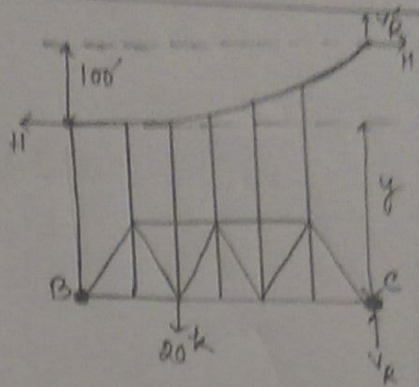
Here,

$$\sum M_B = 0 \quad \uparrow +ve$$

$$\Rightarrow H(100+y) - Hy + 20 \times 200 - (V_R + V'_R) \times 600 = 0$$

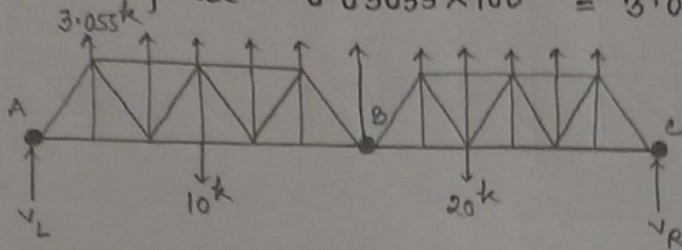
$$\Rightarrow 100H + 4000 - 15.833 \times 600 = 0$$

$$\Rightarrow H = 54.998 \text{ k}$$



Now applying G.C.T,  $h = \frac{WL^2}{8H} \Rightarrow W = \frac{8Hh}{L^2} = \frac{8 \times 54.998 \times 100}{1200^2}$   
 $\Rightarrow W = 0.03055 \text{ k/horizontal foot}$

$\therefore$  Hanger tension force =  $0.03055 \times 100 \text{ k} = 3.055 \text{ k}$



For this figure,

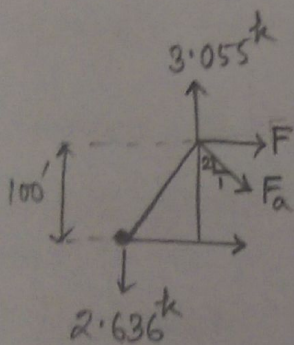
$$\sum M_A = 0 \quad \uparrow +ve \Rightarrow 10 \times 300 + 20 \times 800 - V_R \times 1200 - 3.055 (100 + 200 + 300 + 400 + 500 + 600 + 700 + 800 + 900 + 1000) = 0$$

$$\Rightarrow V_R = -0.969 \text{ k} (\downarrow)$$

And,  $\sum F_y = 0 \quad \uparrow +ve$

$$\Rightarrow V_L + V_R + 3.055 \times 11 - 30 = 0$$

$$\Rightarrow V_L = -2.636 \text{ k} (\downarrow)$$

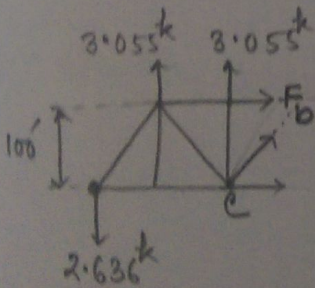


Here,

$$\sum F_y = 0 \quad \uparrow +$$

$$\Rightarrow 3.055 - 2.636 - \frac{2}{\sqrt{5}} F_a = 0$$

$$\Rightarrow F_a = 0.468 \text{ k}$$



Here,  $\sum M_c = 0 \quad \curvearrowright +ve$

$$\Rightarrow F_b \times 100 + 3.055 \times 100 - 2.636 \times 200 = 0$$

$$\Rightarrow F_b = 2.217 \text{ k}$$