



BANGLADESH UNIVERSITY OF ENGINEERING AND
TECHNOLOGY

Department of Civil Engineering

ASSIGNMENT

CE 311

Structural Analysis and Design I

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➤ Level: 3

➤ Term: 1

➤ Sec: B

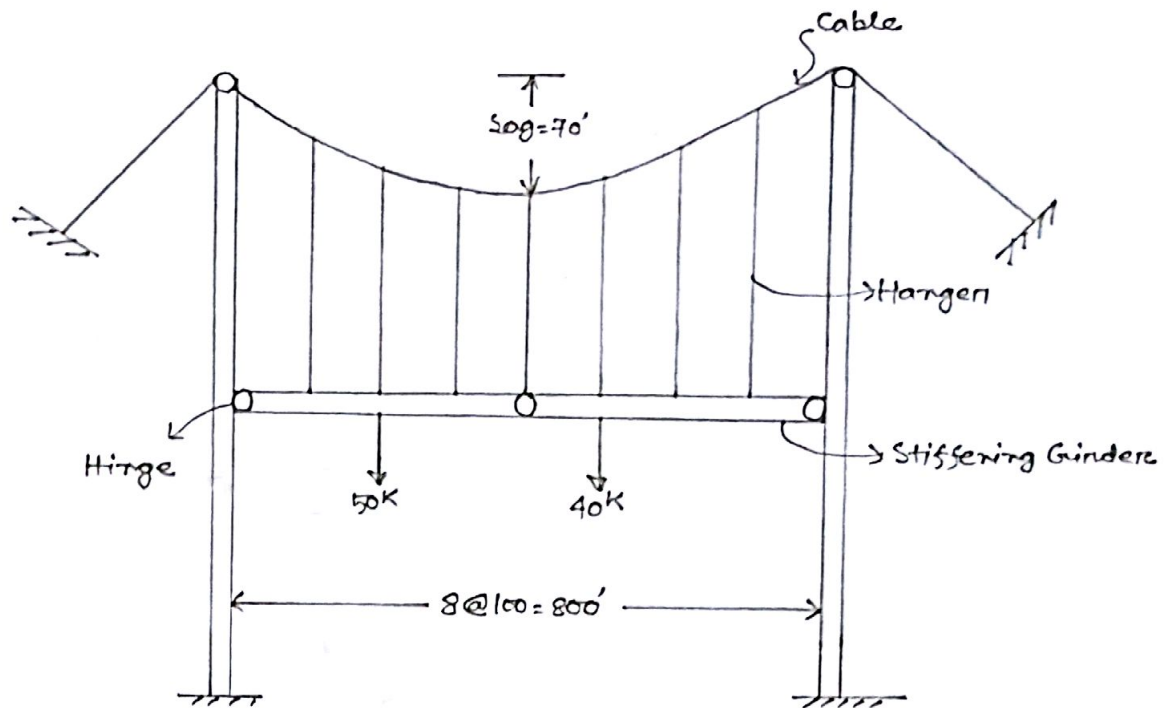
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Submission Date: 21/12/2015

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Assignment # 1 :

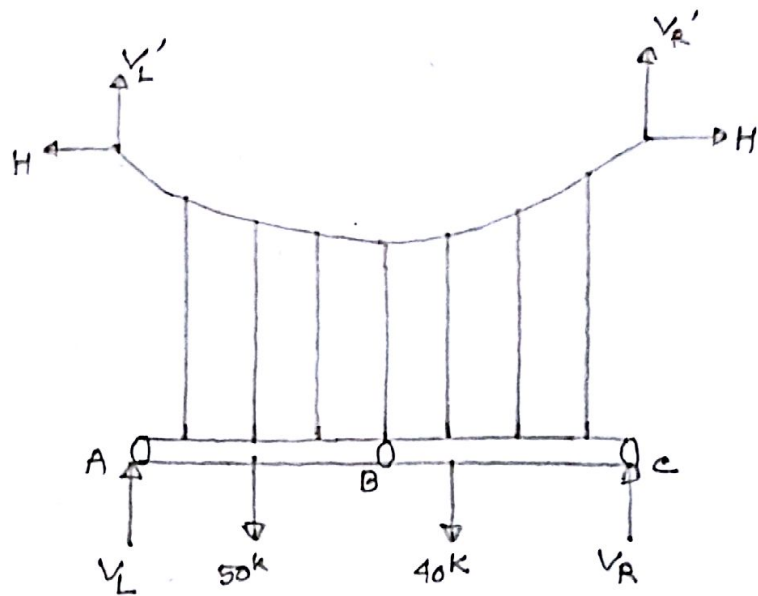


Find

- (1) Hanger force
- (2) Draw SFD for Girder
- (3) Draw BMD for Girder
- (4) T_{max} of cable
- (5) Cable stretch
- (6) Unstressed length of cable

Let, $A = 40 \text{ in}^2$; $E = 27 \times 10^3 \text{ ksi}$

Solution:

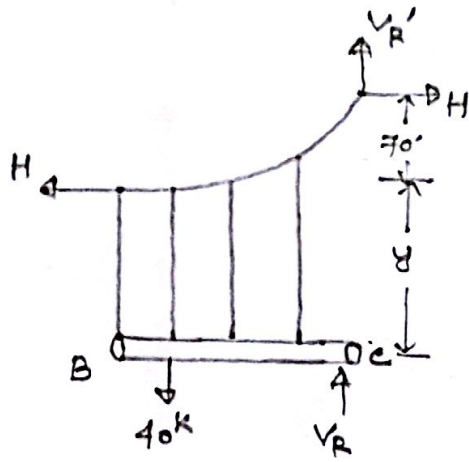


FBD of whole structure

$$\Sigma M @ A = 0 \quad (+ve \downarrow)$$

$$\Rightarrow 50 \times 200 + 40 \times 500 - (V_R + V_R') \times 800 = 0$$

$$\therefore V_R + V_R' = 37.5 \text{ k}$$



FBD for right segment

$$\Sigma M @ B = 0 \quad (+ve \downarrow)$$

$$\Rightarrow 40 \times 100 - H \times y + H(y + 70) - (V_R + V_R') \times 400 = 0$$

$$\Rightarrow 40 \times 100 - Hy + Hy + 70H - 37.5 \times 400 = 0$$

$$\therefore H = 157.14^k$$

Applying General Cable Theorem,

$$\text{Max sag, } h = \frac{wL^2}{8H}$$

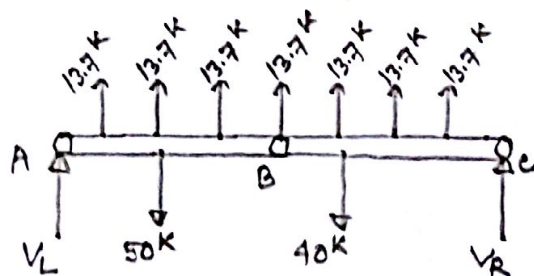
$$\Rightarrow 70 = \frac{w \times 800^2}{8 \times 157.14}$$

$$\therefore w = 0.137 \text{ kip/horizontal foot}$$

① Hanger force = $w \times$ spacing of hanger

$$= 0.137 \times 100$$

$$= 13.7 \text{ kip}$$



$$\Sigma M @ A = 0 \quad (+ve \downarrow)$$

$$\Rightarrow 50 \times 200 + 40 \times 500 - 13.7(100 + 200 + 300 + 400 + 500 + 600 + 700)$$

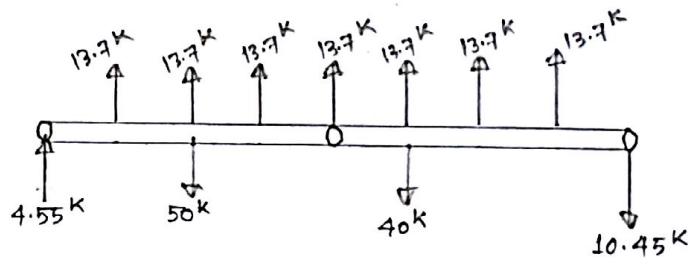
$$- V_R \times 800 = 0$$

$$\therefore V_R = -10.45^k (\downarrow)$$

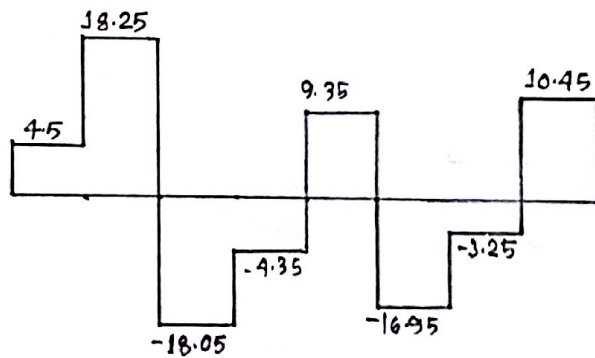
$\Sigma F_y = 0 \quad +ve \uparrow,$

$\Rightarrow V_L + 13.7 \times 7 - 50 - 40 - 10.45 = 0$

$\therefore V_L = +4.55 \text{ K} (\uparrow)$

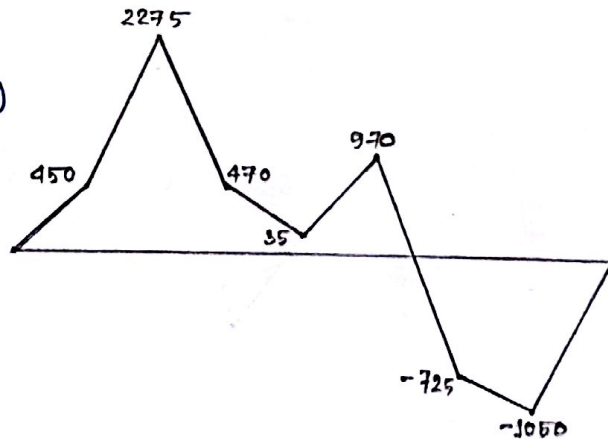


② SFD (Kip)



SFD (Kip)

③ BMD (K-ft)



BMD (Kip-ft)

(5)

T_{max} occurs at ends.

For a horizontal cable chord:

$$\begin{aligned} \textcircled{4} \quad T_{max} &= H (1 + 16\theta^2)^{1/2} \\ &= 157.14 \left[1 + 16 \left(\frac{70}{800} \right)^2 \right]^{1/2} \end{aligned}$$

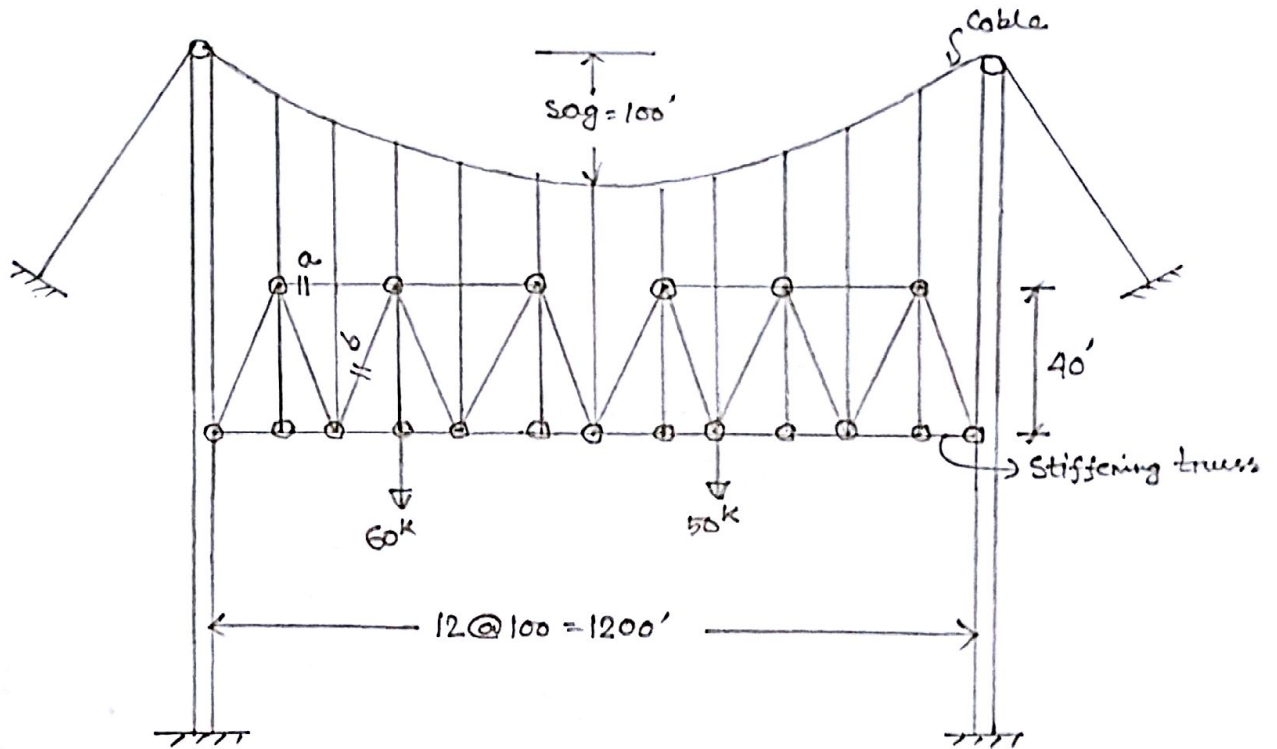
$$= \boxed{166.49 \text{ kip}}$$

$$\begin{aligned} \textcircled{5} \quad \text{Cable stretch, } \Delta s &= \frac{HL}{AE} \left[1 + \frac{16}{3} \theta^2 + \tan^2 \psi \right] \\ &= \frac{157.14 \times 800}{40 \times 27 \times 10^3} \left[1 + \frac{16}{3} \left(\frac{70}{800} \right)^2 + 0 \right] \quad [\because \psi = 0] \\ &= \boxed{0.121 \text{ ft}} \end{aligned}$$

$$\begin{aligned} \text{Stressed length, } s &= \frac{L}{2} (1 + 16\theta^2)^{1/2} + \frac{L}{8\theta} \ln \left[4\theta + (1 + 16\theta^2)^{1/2} \right] \\ &= \frac{800}{2} \left[1 + 16 \times (0.089)^2 \right]^{1/2} + \frac{800}{8 \times 0.089} \ln \left[4 \times 0.089 + (1 + 16(0.089)^2)^{1/2} \right] \\ &= 816.59 \text{ ft} \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad \text{Unstressed length of cable} &= s - \Delta s \\ &= 816.59 - 0.121 \\ &= 816.47 \\ &= \boxed{\approx 816 \text{ ft}} \end{aligned}$$

Assignment # 2:



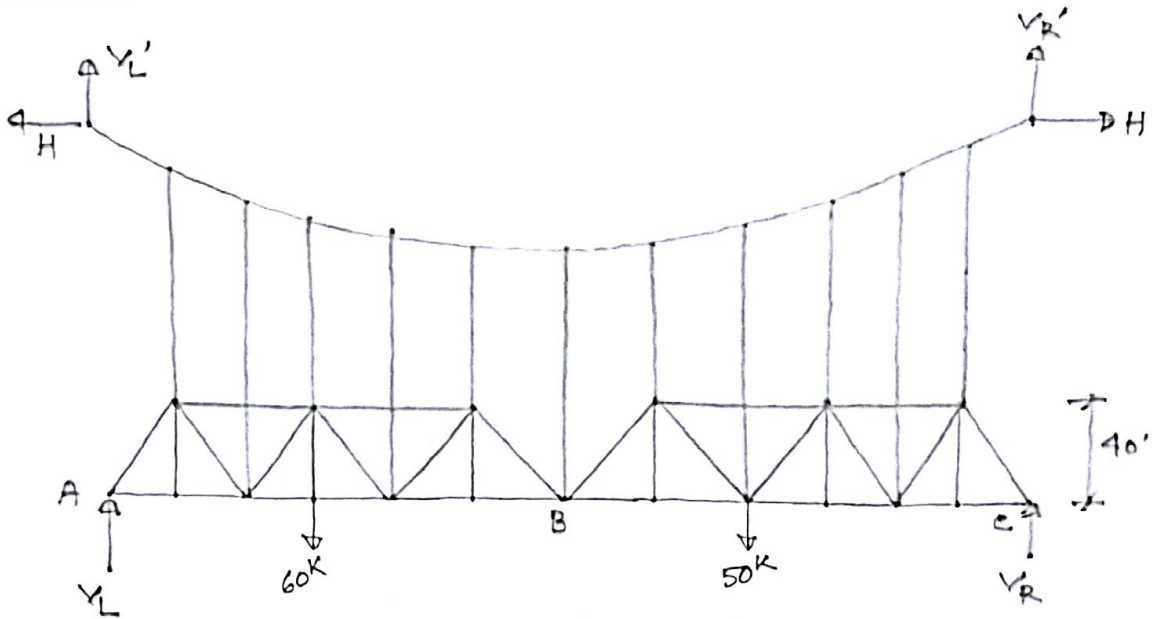
Find

- (1) Horizontal force
- (2) Member force a & b
- (3) T_{max}
- (4) Stretch
- (5) Unstressed length

Let, $A = 40 \text{ in}^2$; $E = 27 \times 10^3 \text{ ksi}$

Solution:

①

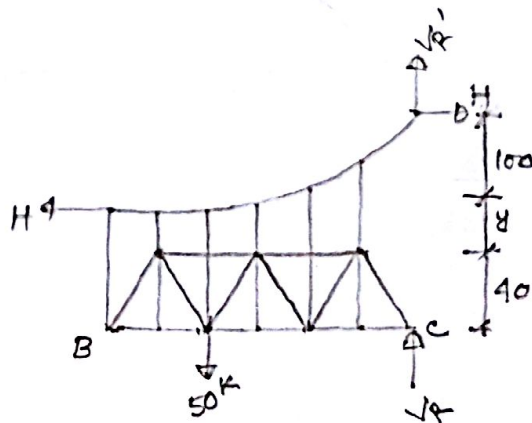


FBD of whole Structure.

$$\Sigma M @ A = 0 \quad +ve \curvearrowright$$

$$\Rightarrow 60 \times 300 + 50 \times 800 - (V_R + V_R') \times 1200 = 0$$

$$\therefore V_R + V_R' = 48.33 \text{ K}$$



$$\Sigma M @ B = 0 \quad +ve \curvearrowright,$$

$$\Rightarrow 50 \times 200 - H(40) + H(100 + 40) - (V_R + V_R') \times 600 = 0$$

$$\Rightarrow 50 \times 200 - 40H + 140H - 48.33 \times 600 = 0$$

$$\therefore \boxed{H = 189.98 \text{ K}}$$

Applying General Cable Theorem,

$$\text{Max sag, } h = \frac{WL^2}{8H}$$

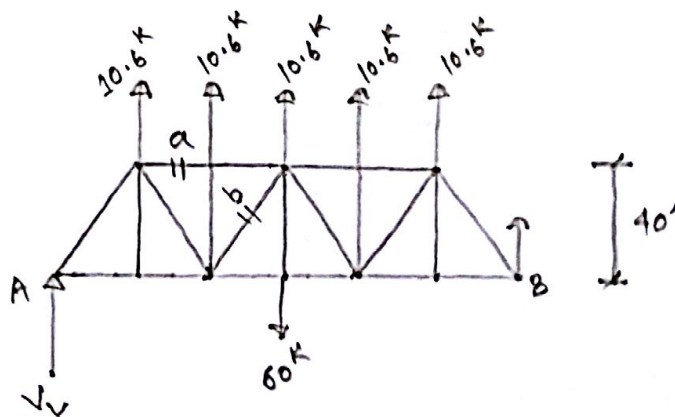
$$\Rightarrow 100 = \frac{W \times 1200^2}{8 \times 189.98}$$

$$\Rightarrow W = 0.1055 \text{ kip/horizontal foot}$$

\therefore Hanger force = $W \times$ spacing of hanger

$$= 0.1055 \times 100$$

$$= 10.6 \text{ kip}$$

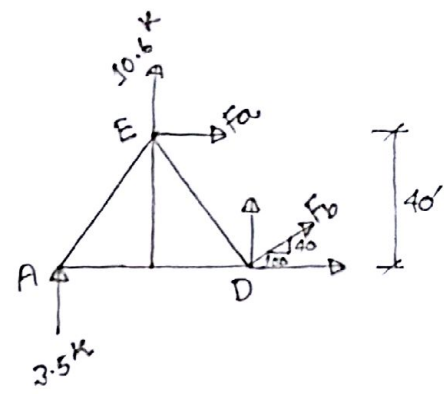


$$\Sigma M @ B = 0 \quad +ve \curvearrowright$$

$$\Rightarrow -60 \times 300 + 10.6 (100 + 200 + 300 + 400 + 500) + V_L \times 600 = 0$$

$$\therefore V_L = +3.5 \text{ k } (\uparrow)$$

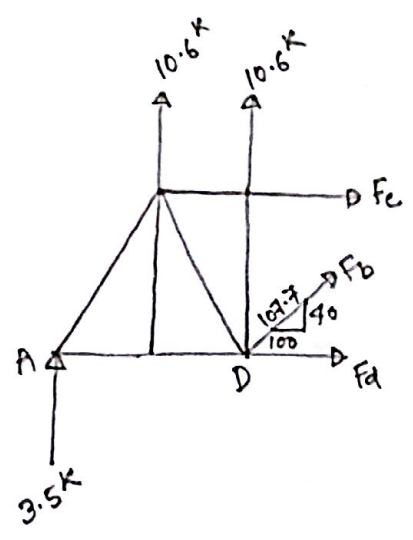
2



$\Sigma M @ D = 0$ (+ve \curvearrowright)

$\Rightarrow 3.5 \times 200 + 10.6 \times 100 + F_a \times 40 = 0$

$\therefore F_a = -44 \text{ kip } (\leftarrow)$



$\Sigma F_y = 0$ (+ve \uparrow),

$\Rightarrow 3.5 + 10.6 + 10.6 + F_b \times \frac{40}{107.7} = 0$

$\therefore F_b = -66.5 \text{ kip } (\leftarrow)$

T_{max} occurs at ends

For a horizontal cable chord:

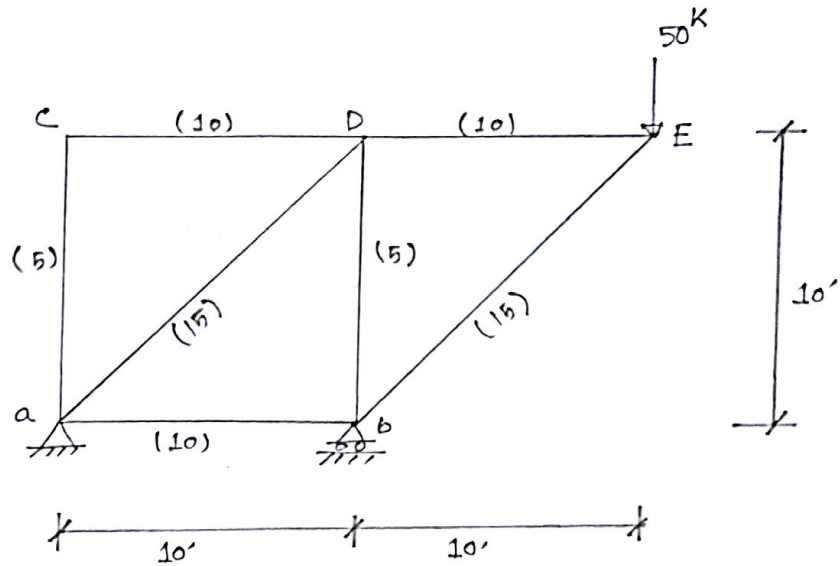
$$\begin{aligned} \textcircled{3} \quad T_{max} &= H (1 + 16\theta^2)^{1/2} \\ &= 189.98 \left[1 + 16 \left(\frac{100}{1200} \right)^2 \right]^{1/2} \\ &= \boxed{200.26 \text{ kip}} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad \text{Cable stretch, } \Delta s &= \frac{HL}{AE} \left[1 + \frac{16}{3}\theta^2 + \tan^2 \gamma \right] \\ &= \frac{189.98 \times 1200}{40 \times 27 \times 10^3} \left[1 + \frac{16}{3} (0.083)^2 + 0 \right] \quad [\because \gamma = 0] \\ &= \boxed{0.219 \text{ ft}} \end{aligned}$$

$$\begin{aligned} \text{Stressed length, } s &= \frac{L}{2} (1 + 16\theta^2)^{1/2} + \frac{L}{8\theta} \ln \left(4\theta + (1 + 16\theta^2)^{1/2} \right) \\ &= \frac{1200}{2} \left[1 + (0.083)^2 \right]^{1/2} + \frac{1200}{8 \times 0.083} \ln \left(4 \times 0.083 + (1 + 16(0.083)^2)^{1/2} \right) \\ &= 1221.69 \text{ ft} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad \text{Unstressed length of cable} &= s - \Delta s \\ &= 1221.69 - 0.219 \\ &= 1221.47 \\ &= \boxed{\approx 1221 \text{ ft}} \end{aligned}$$

Assignment # 3 :



Given,

$$E = 29000 \text{ ksi}$$

$$A = \text{horizontal } (10 \text{ in}^2)$$

$$\text{Vertical } (5 \text{ in}^2)$$

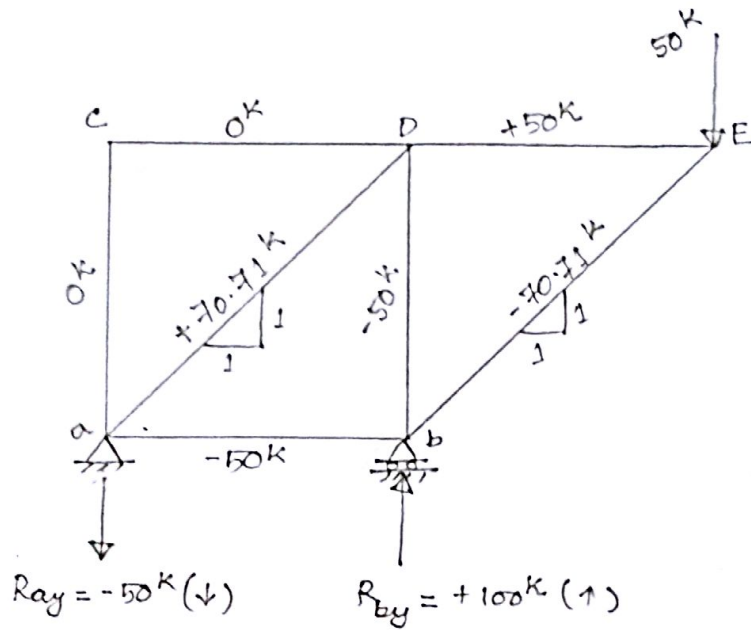
$$\text{Diagonal } (15 \text{ in}^2)$$

Find (a) Vertical Deflection at 'c'

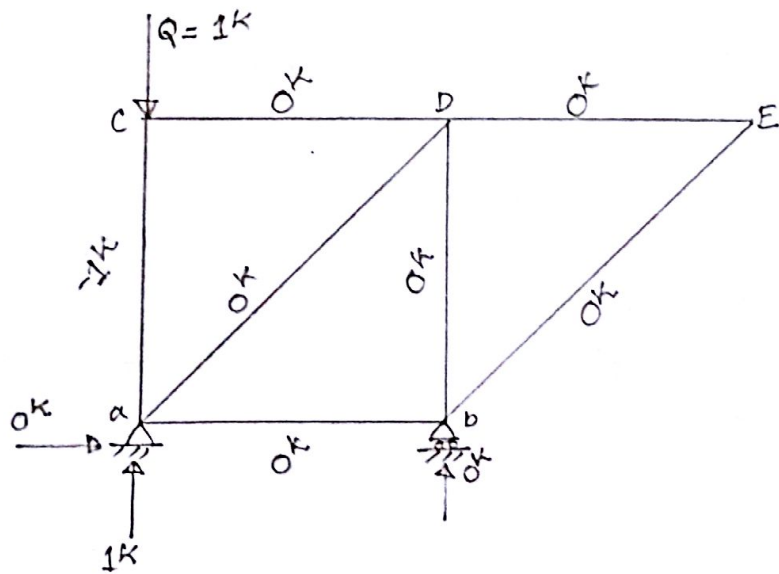
(b) Horizontal deflection at 'E'

Solution:

(a) Analysing for real force.



Analysing for virtual force,



Member	Bar	L (ft)	A (in ²)	$\frac{L}{A}$	F_Q (k)	F_P (k)	$F_Q F_P \frac{L}{A}$
Horizontal	ab	10	10	1	0	-50	0
	cd	10	10	1	0	0	0
	DE	10	10	1	0	+50	0
Vertical	aC	10	5	2	-1	0	0
	bD	10	5	2	0	-50	0
Diagonal	aD	14.14	15	0.943	0	70.71	0
	bE	14.14	15	0.943	0	-70.71	0

$$\sum F_Q F_P \frac{L}{A} = 0$$

If $Q = \text{unit load}$,

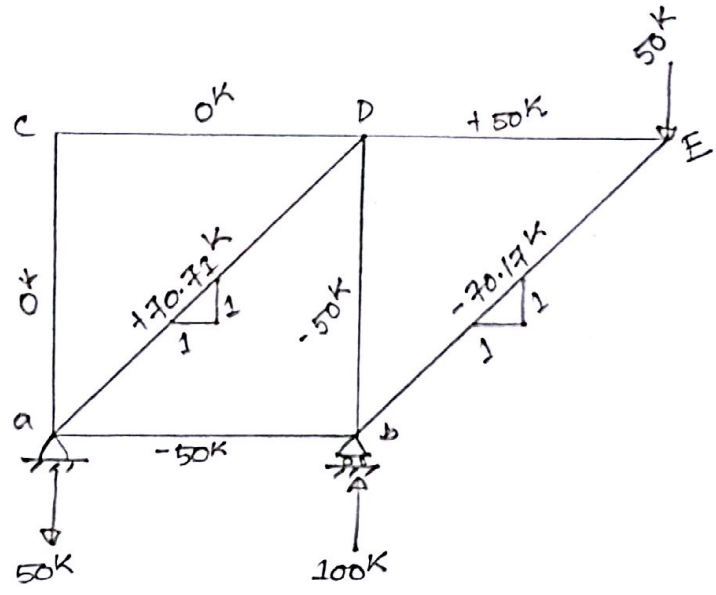
$$1k \cdot \delta = \frac{\sum F_Q F_P L}{AE}$$

$$\Rightarrow 1 \cdot \delta_c = \frac{0}{E}$$

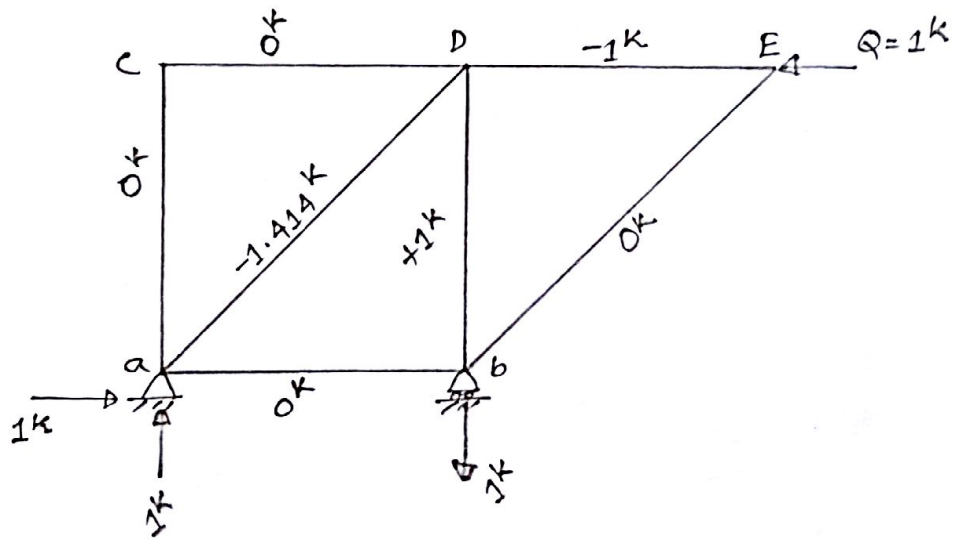
$$\therefore \boxed{\delta_c = 0}$$

(b)

Analysing for real force,



Analysing for virtual force,



Member	Bar	L (ft)	A (in ²)	$\frac{L}{A}$	F _Q (K)	F _P (K)	F _Q F _P $\frac{L}{A}$
Horizontal	ob	10	10	1	0	-50	0
	cd	10	10	1	0	0	0
	DE	10	10	1	-1	+50	-50
Vertical	aC	10	5	2	0	0	0
	bD	10	5	2	+1	-50	-100
Diagonal	aD	14.14	15	0.943	-1.414	70.71	-94.25
	bE	14.14	15	0.943	0	-70.71	0

$$\sum F_Q F_P \frac{L}{A} = -244.25$$

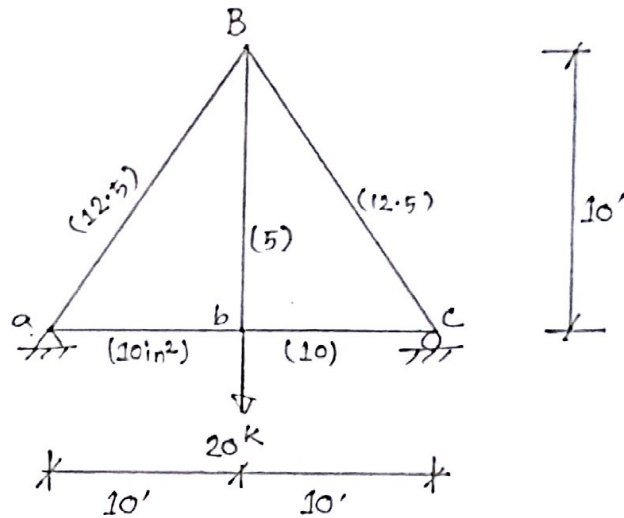
If Q = unit load.

$$1K \cdot \delta = \sum \frac{F_Q F_P L}{AE}$$

$$\Rightarrow 1 \cdot \delta_E = \frac{-244.25}{29000}$$

$$\therefore \delta_E = -8.42 \times 10^{-3} \text{ ft (rightward)}$$

Assignment # 4 :



Compute vertical deflection of joint B if 50°F temp. decreases is observed in bottom chord members only + $P = 20^k$

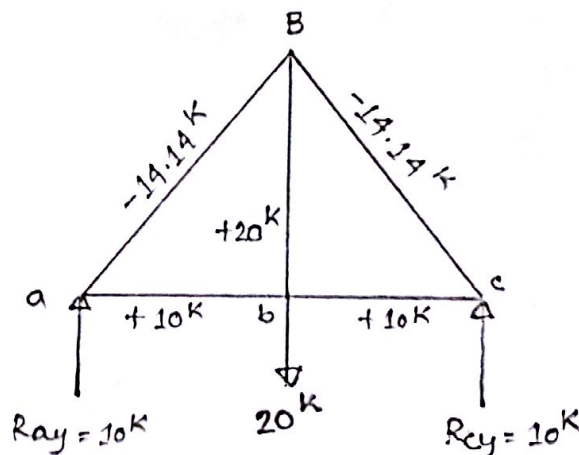
Given, $\alpha_t = \frac{1}{1,50,000}$ per °F

$E = 29000$ ksi

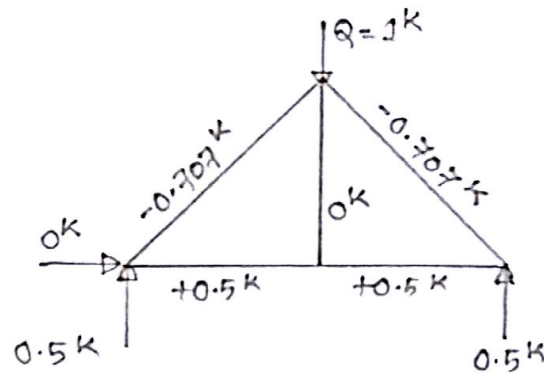
$A = 10$ in² in parentheses

Solution:

Analysis for real force,



Analysing for virtual force,



Members	Bar	L (ft)	A (in ²)	$\frac{L}{A}$	F_Q (K)	F_P (K)	$F_Q F_P \frac{L}{A}$	t	$F_Q t L$
Horizontal	ab	10	10	1	+0.5	+10	5	-50	-250
	bc	10	10	1	+0.5	+10	5	-50	-250
Vertical	bB	10	5	2	0	+20	0	0	0
Diagonal	aB	14.14	12.5	1.13	-0.707	-14.14	11.29	0	0
	cB	14.14	12.5	1.13	-0.707	-14.14	11.29	0	0

$$\sum F_Q F_P \frac{L}{A} = +32.6 \quad \sum F_Q t L = -500$$

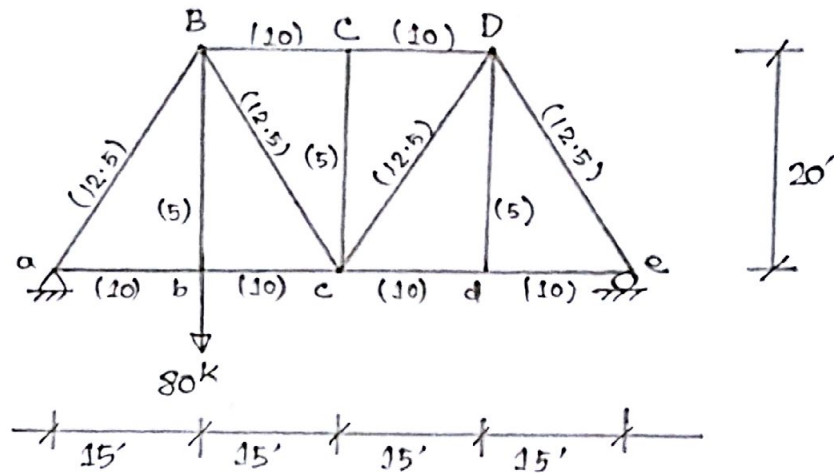
If $Q = \text{unit load}$,

$$1K \cdot \delta = \sum \frac{F_Q F_P L}{AE} + \sum F_Q t L$$

$$\Rightarrow 1 \cdot \delta_B = \frac{32.6}{29000} + \frac{-500}{1,50,000}$$

$$\therefore \delta_B = -2.21 \times 10^{-3} \text{ ft (upward)}$$

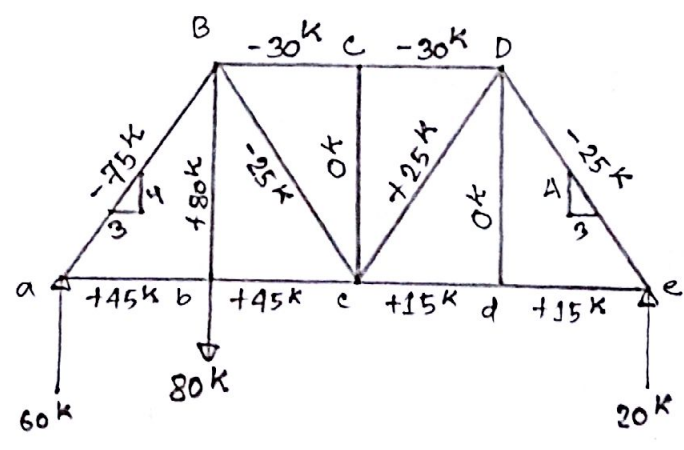
Assignment #5 :



Compute vertical deflection at joint D, due to a decrease of temperature of $50^{\circ}F$ in the bottom chord only. Given, $\alpha_t = \frac{1}{150000}$ per $^{\circ}F$, $E = 30 \times 10^3$ ksi and due to the 80 kip load shown.

Solution:

Analysing for real force,



If $Q = \text{unit load}$,

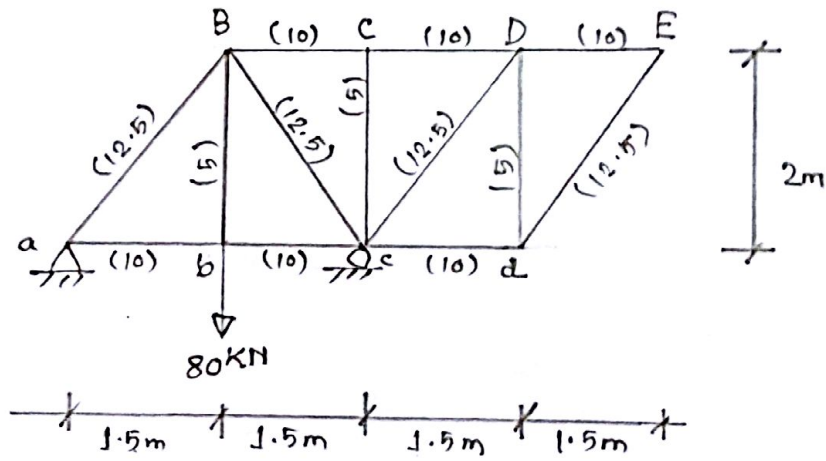
$$1K \cdot \delta = \sum \frac{F_Q F_P L}{AE} + \sum F_Q \alpha_t \pm L$$

$$\Rightarrow 1 \cdot \delta_D = \frac{178 \cdot 15}{30000} + \frac{-1125 \cdot 0.02}{150000}$$

$$\therefore \delta_D = -1.56 \times 10^{-3} \text{ ft (upward)}$$

— 0 —

Assignment # 6:



Calculate vertical component of deflection at E

Given, $P = 80 \text{ kN}$

Temp. Change = $+50^\circ \text{ F}$ in bottom chord member

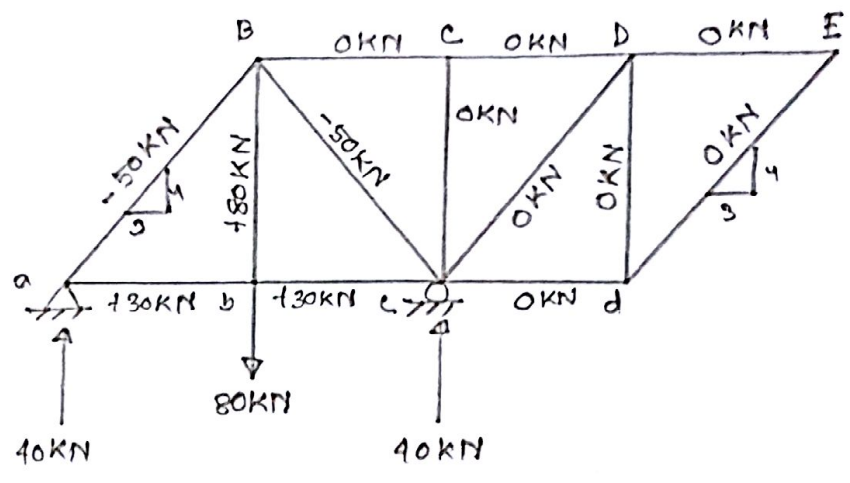
$= -30^\circ \text{ F}$ in vertical member

$$E = 20.7 \times 10^3 \text{ kN/cm}^2$$

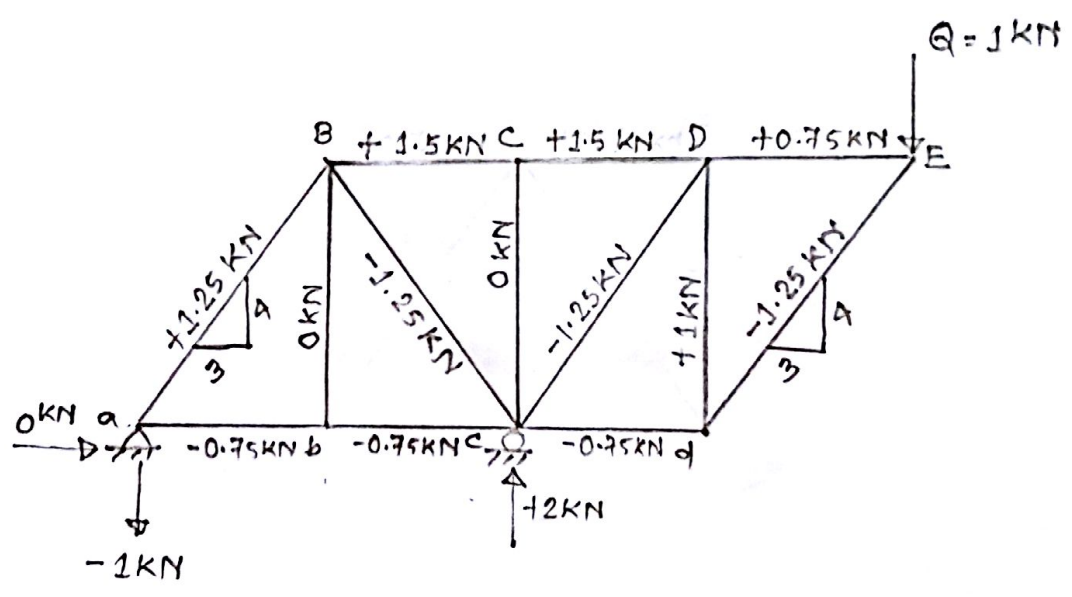
$A =$ in cm^2 in parentheses

$$\alpha_t = \frac{1}{150000} \text{ per } ^\circ \text{ F}$$

Analysis for real force,



Analysis for virtual force,



Member	Bar	L (m)	A (cm ²)	$\frac{L}{A}$	F _a (kN)	F _p (kN)	F _a F _p $\frac{L}{A}$	t (°F)	F _a t L
Horizontal	ab	1.5	10	0.15	-0.75	+30	-3.375	50	-56.25
	bc	1.5	10	0.15	-0.75	+30	-3.375	50	-56.25
	cd	1.5	10	0.15	-0.75	0	0	50	-56.25
	BC	1.5	10	0.15	+1.5	0	0	0	0
	CD	1.5	10	0.15	+1.5	0	0	0	0
	DE	1.5	10	0.15	+0.75	0	0	0	0
Vertical	bB	2	5	0.4	0	+80	0	-30	0
	cC	2	5	0.4	0	0	0	-30	0
	dD	2	5	0.4	+1	0	0	-30	-60
Diagonal	aB	2.5	12.5	0.2	+1.25	-50	-12.5	0	0
	cB	2.5	12.5	0.2	-1.25	-50	12.5	0	0
	cD	2.5	12.5	0.2	-1.25	0	0	0	0
	dE	2.5	12.5	0.2	-1.25	0	0	0	0

$$\sum F_a F_p \frac{L}{A} = -6.75 \quad \sum F_a t L = -228.75$$

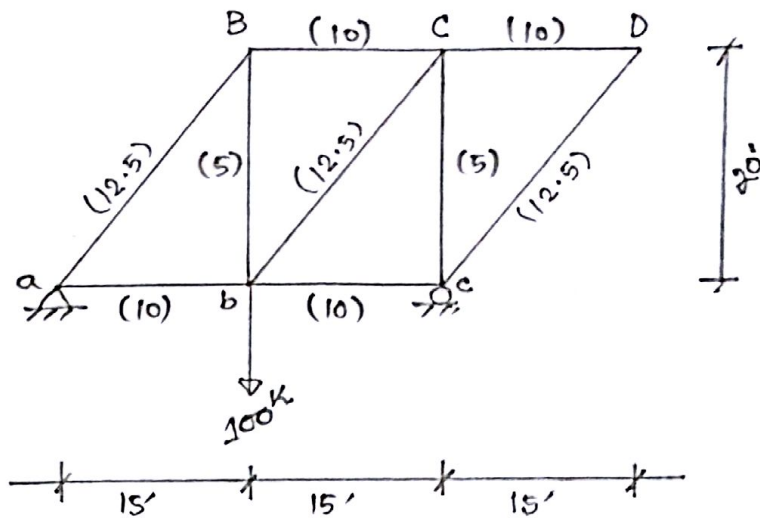
If Q = unit load,

$$1 \text{ kN} \cdot \delta = \sum \frac{F_a F_p L}{AE} + \sum F_a \alpha_t t L$$

$$\Rightarrow 1 \cdot \delta_E = \frac{-6.75}{20.7 \times 10^3} + \frac{-228.75}{150000}$$

$$\therefore \delta_E = -1.85 \times 10^{-3} \text{ m (upward)}$$

Assignment #7:



Calculate horizontal deflection at joint at point D.

Temp change = $+50^{\circ}\text{F}$ in bottom chord member

= -30°F in vertical " "

Given, $\alpha_t = \frac{1}{150000}$ per $^{\circ}\text{F}$

at 'a' horizontal movement = $0.75''$ (to right)

at 'a' vertical " = $0.5''$ (down)

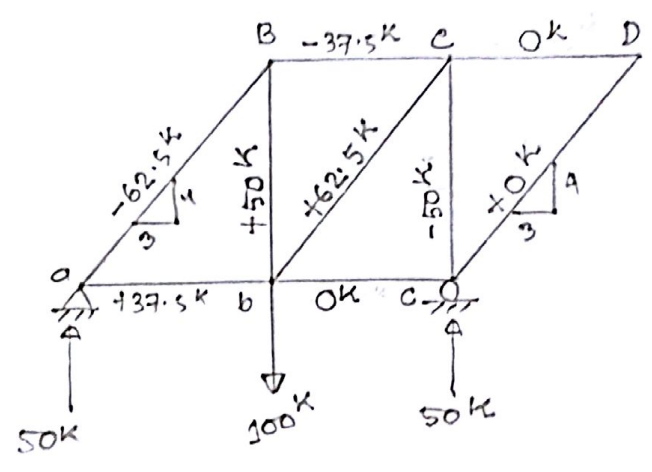
at 'c' vertical " = $0.4''$ (down)

$E = 29000 \text{ ksi}$

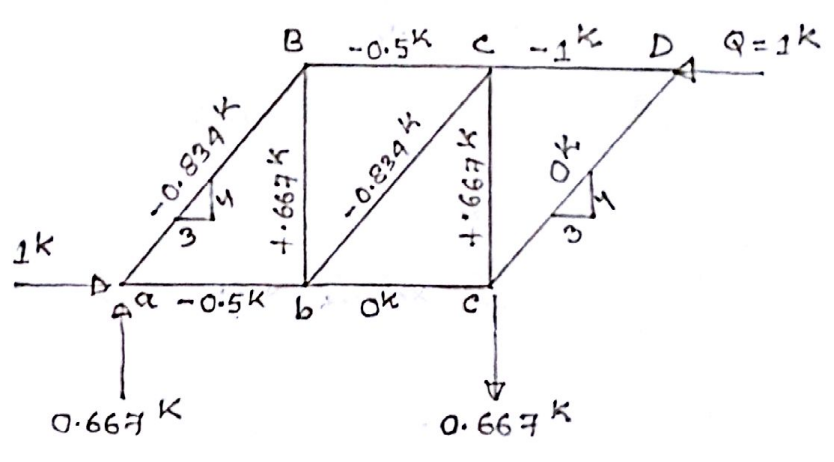
$A = \text{in}^2$ in parentheses

Solution:

Analysing for real force,



Analysing for virtual force,



Member	Bar	L (ft)	A (in ²)	$\frac{L}{A}$	F_a (k)	F_p (k)	$F_a F_p \frac{L}{A}$	t (in)	$F_a t L$
Horizontal	ab	15	10	1.5	-0.5	+37.5	-28.13	50	-375
	bc	15	10	1.5	0	0	0	50	0
	Bc	15	10	1.5	-0.5	-37.5	28.13	0	0
	cd	15	10	1.5	-1	0	0	0	0
Vertical	bB	20	5	4	+0.667	+50	133.4	-30	-400.2
	cC	20	5	4	+0.667	-50	-133.4	-30	-400.2
Diagonal	aB	25	12.5	2	-0.834	-62.5	104.25	0	0
	bC	25	12.5	2	-0.834	+62.5	-104.25	0	0
	cD	25	12.5	2	0	0	0	0	0

$$\sum F_a F_p \frac{L}{A} = 0$$

$$\sum F_a t L = -1175.4$$

Applying principle of virtual work,

$$W_s + W_R = W_d$$

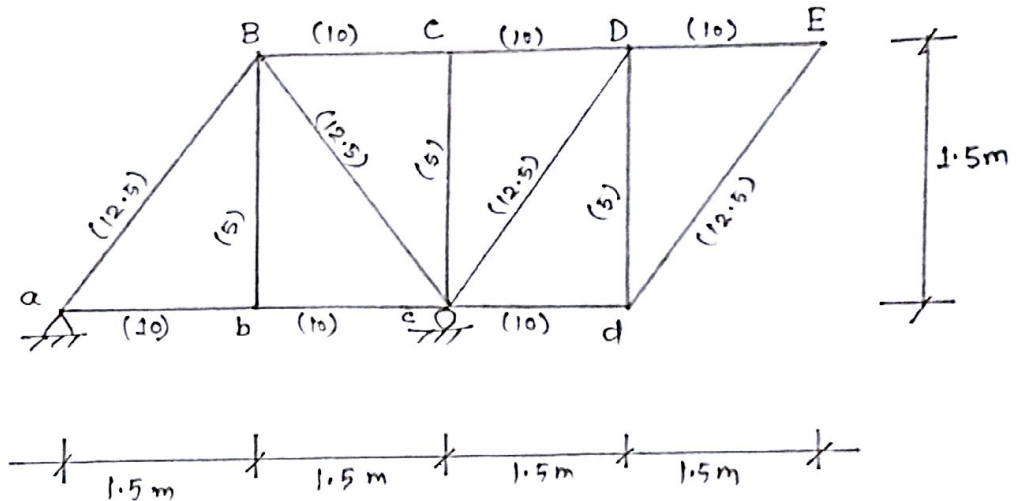
$$\Rightarrow Q \cdot \delta_{Dh} + (R_{ay} \times \Delta_{ay} + R_{ax} \times \Delta_{ax} + R_{cy} \times \Delta_{cy}) = \sum \frac{F_a F_p L}{AE} + \sum F_a t L$$

$$\Rightarrow 1k \cdot \delta_{Dh} + \left(0.667 \times \frac{-0.5}{12} + 1 \times \frac{75}{12} + 0.667 \times \frac{4}{12} \right) = \frac{0}{29000} + \frac{-1175.4}{150000}$$

$$\Rightarrow \delta_{Dh} + 0.0569 = -7.84 \times 10^{-3}$$

$$\therefore \delta_{Dh} = 0.0491 \text{ ft (leftward)}$$

Assignment # 8 :



Compute the horizontal component of the deflection of joint E due to the following movements of the supports :

At a, horizontal = 7.5×10^{-3} m to left

At a, vertical = 11.25×10^{-3} m down

At c, vertical = 3.75×10^{-3} m down

$$E = 20.7 \times 10^3 \text{ KN/cm}^2$$

A = in cm^2 in parentheses

No temperature change

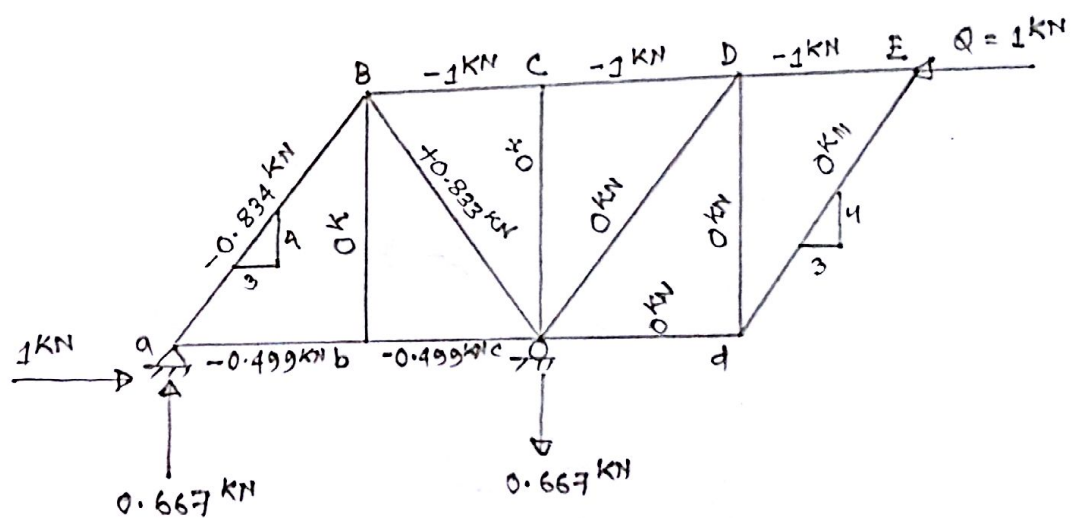
Solution:

Here, there is no real force and no temperature change. The deflection is caused simply by support movements.

There are no changes in length of members.

That is, $\sum \frac{F_a F_p L}{AE} = 0$ and $\sum F_R \alpha_t L = 0$

Analysing for virtual force,



Applying principle of virtual work;

$$W_s + W_R = W_d$$

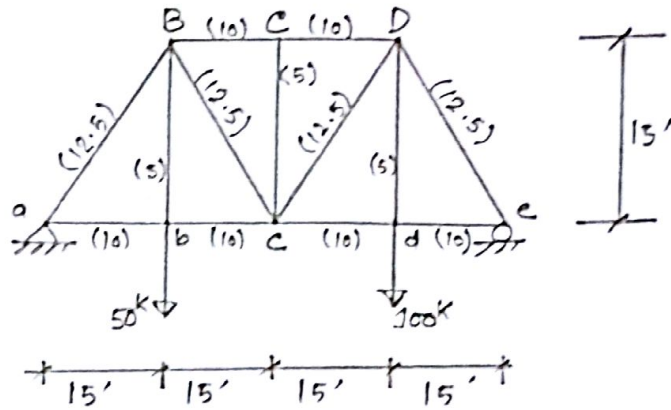
$$\Rightarrow 1 \text{ kN} \cdot \delta_{Eh} + (R_{ay} \times \Delta_{ay} + R_{ax} \times \Delta_{ax} + R_{ey} \times \Delta_{ey}) = \sum \frac{F_Q F_P L}{AE} + \sum F_Q \alpha_i t L$$

$$\Rightarrow 1 \cdot \delta_{Eh} + (.667 \times -11.25 \times 10^{-3} + 1 \times -7.5 \times 10^{-3} + .667 \times 3.75 \times 10^{-3}) = 0$$

$$\therefore \delta_{Eh} = +0.0125 \text{ m (leftward)}$$

— 0 —

Assignment # 9:



Compute the relative deflections of joint d and B along the line joining them

Given, $E = 30 \times 10^3 \text{ ksi}$

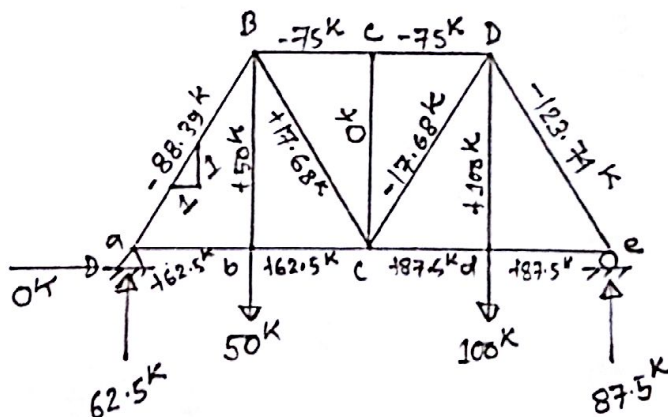
$A = \text{in}^2$ in parentheses

An increase in temperature of 40°C in the top chord; a decrease of 10°C in the bottom chord,

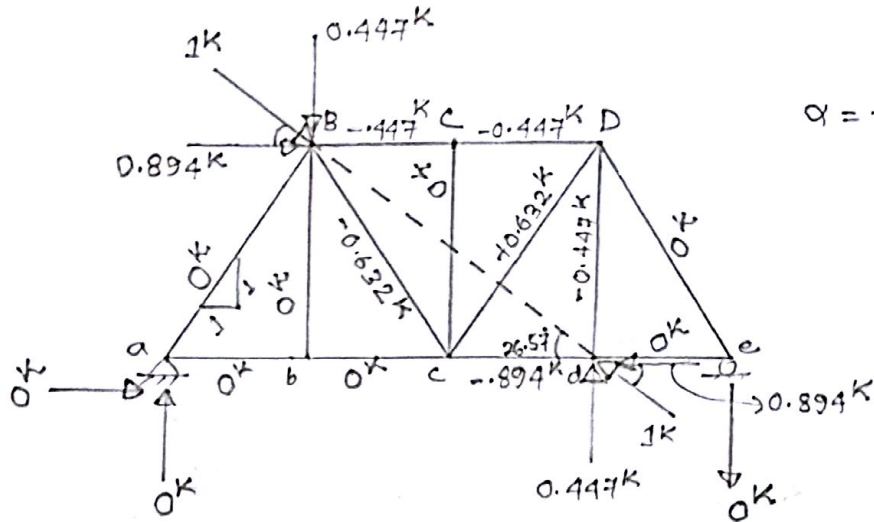
$$\alpha_t = \frac{1}{75000} \text{ per } ^\circ\text{C}$$

Solution:

Analysing for real force,



Analysing for virtual force,



$$\alpha = \tan^{-1} \left(\frac{15}{30} \right) = 26.57^\circ$$

Members	Bar	L (ft)	A (in ²)	$\frac{L}{A}$	F_Q (k)	F_P (k)	$F_Q F_P \frac{L}{A}$	ϵ (°C)	$F_Q \epsilon L$
Horizontal	ab	15	10	1.5	0	+62.5	0	-10	0
	bc	15	10	1.5	0	+62.5	0	-10	0
	cd	15	10	1.5	-0.894	+87.5	-117.34	-10	134.1
	de	15	10	1.5	0	+87.5	0	-10	0
	BC	15	10	1.5	-0.447	-75	50.29	40	-536.4
	CD	15	10	1.5	-0.447	-75	50.29	40	-536.4
Vertical	bB	15	5	3	0	+50	0	0	0
	cC	15	5	3	0	0	0	0	0
	dD	15	5	3	-0.447	+100	-134.1	0	0
	Diagonal	aB	21.2	12.5	1.69	0	-88.39	0	0
cB		21.2	12.5	1.69	-0.632	+17.68	-18.88	0	0
cD		21.2	12.5	1.69	+0.632	-17.68	-18.88	0	0
eD		21.2	12.5	1.69	0	-123.74	0	0	0

$$\sum F_Q F_P \frac{L}{A} = -188.62 \quad \sum F_Q \epsilon L = 998.7$$

If $Q = \text{unit load}$,

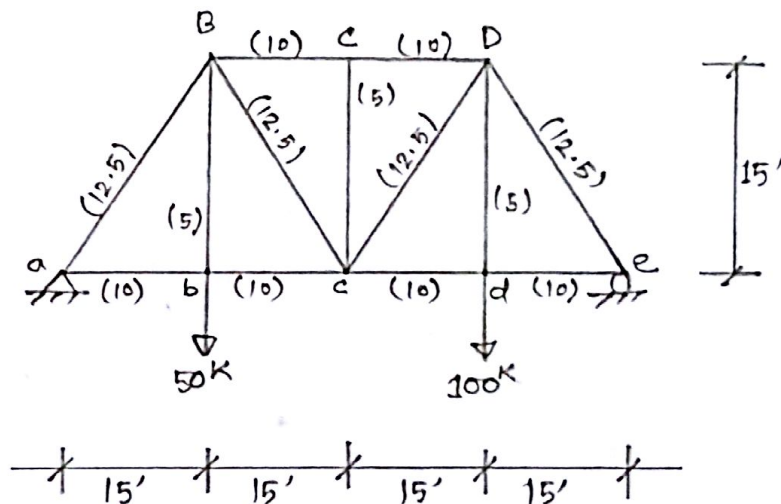
$$1k \cdot \delta_d^x + 1k \cdot \delta_B^y = \sum \frac{F_Q F_P L}{AE} + \sum F_Q \alpha_t t L$$

$$\Rightarrow 1k \cdot \delta_{d-B}^y = \frac{-188.62}{30000} + \frac{-998.7}{75000}$$

$$\therefore \delta_{d-B}^y = -0.0188 \text{ ft (apart)}$$

— o —

Assignment 10:



Find rotation of bar BC.

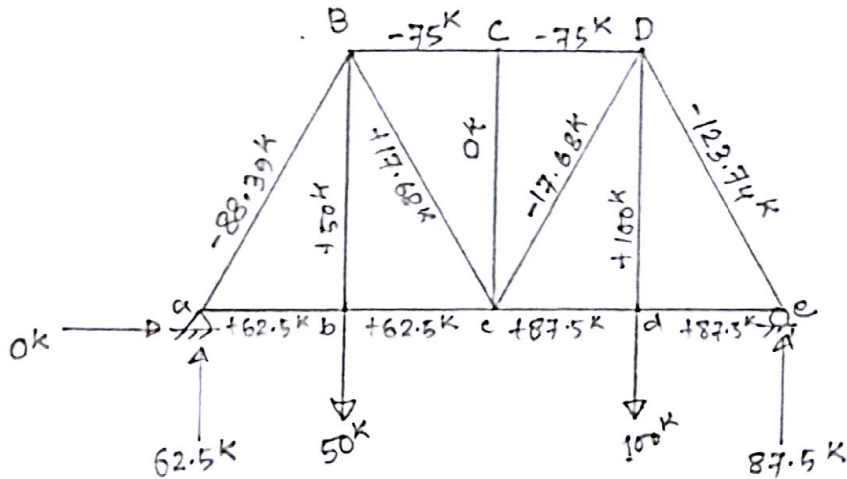
Given, $E = 30 \times 10^3 \text{ ksi}$, $A = \text{in}^2$ in parentheses

An increase of the temperature of 40°C in the top chord, a decrease of 10°C in the bottom

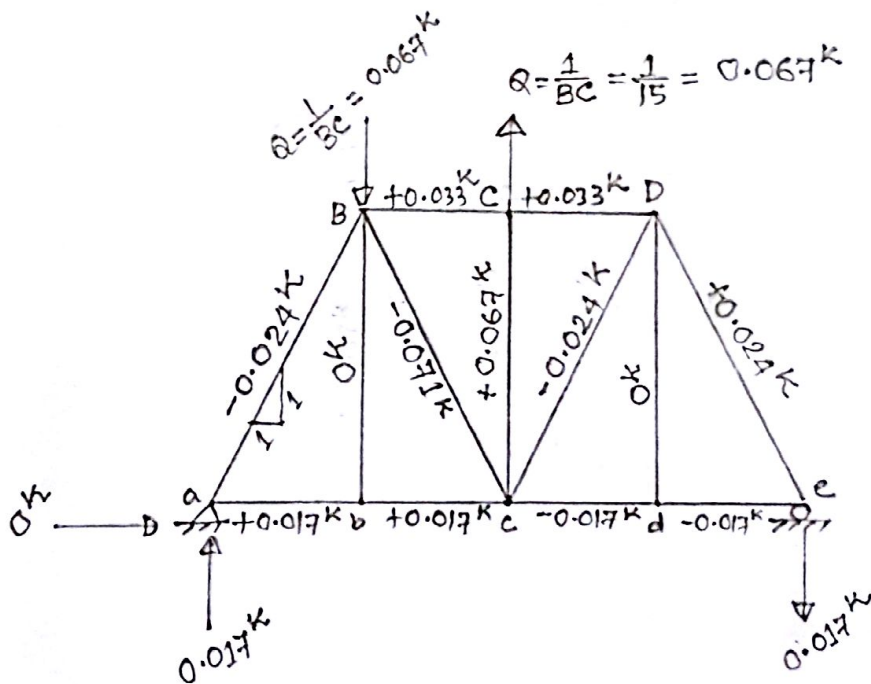
chord, $\alpha_t = \frac{1}{75000} \text{ per } ^\circ\text{C}$.

Solution:

Analysing for real force,



Analysing for virtual force,



Member	Boac	L (ft)	A (in ²)	$\frac{L}{A}$	F _Q (K)	F _P (K)	F _Q F _P $\frac{L}{A}$	t (°C)	F _Q tL
Horizontal	ab	15	10	1.5	+0.017	+62.5	+1.594	-10	-2.55
	bc	15	10	1.5	+0.017	+62.5	+1.594	-10	-2.55
	cd	15	10	1.5	-0.017	+87.5	-2.23	-10	2.55
	de	15	10	1.5	-0.017	+87.5	-2.23	-10	2.55
	BC	15	10	1.5	+0.033	-75	-3.71	40	19.8
	CD	15	10	1.5	+0.033	-75	-3.71	40	19.8
Vertical	bB	15	5	3	0	+50	0	0	0
	cC	15	5	3	+0.067	0	0	0	0
	dD	15	5	3	0	+100	0	0	0
Diagonal	aB	21.2	12.5	1.69	-0.024	-88.39	+3.59	0	0
	cB	21.2	12.5	1.69	-0.071	+17.68	-2.12	0	0
	cD	21.2	12.5	1.69	-0.024	-17.68	0.717	0	0
	eD	21.2	12.5	1.69	+0.024	-123.74	-5.019	0	0

$$\sum F_Q F_P \frac{L}{A} = -11.524 \quad \sum F_Q t L = 39.6$$

If Q = unit load,

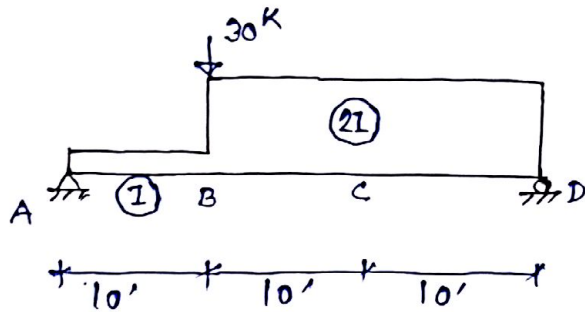
$$\frac{1}{BC} \cdot \delta_B^\downarrow + \frac{1}{BC} \delta_C^\uparrow = \frac{\sum F_Q F_P L}{AE} + \sum F_Q t L$$

$$\Rightarrow \frac{1}{BC} (\delta_B^\downarrow + \delta_C^\uparrow) = \frac{-11.524}{30000} + \frac{39.6}{75000}$$

$$\Rightarrow 1. \theta_{B-C} = 1.44 \times 10^{-4} \text{ radian}$$

$$\therefore \theta_{B-C} = 1.44 \times 10^{-4} \text{ radian (counterclockwise)}$$

Assignment # 11:

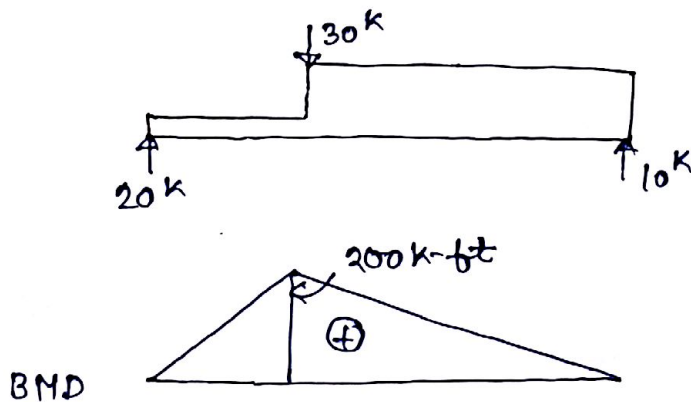


Given, $E = 30000 \text{ ksi}$, $I = 200 \text{ in}^4$

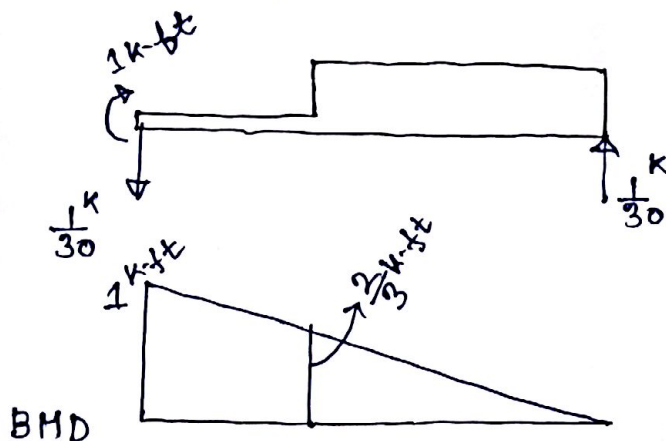
Compute rotation/change in slope at joint A.

Solⁿ:

Analysis for P force;



Analysis for Q force;



Principle of V.W. ;

$$Q. \alpha_A = \int_A^B \frac{M_Q M_P}{EI} dx + \int_B^D \frac{M_Q M_P}{EI} dx$$

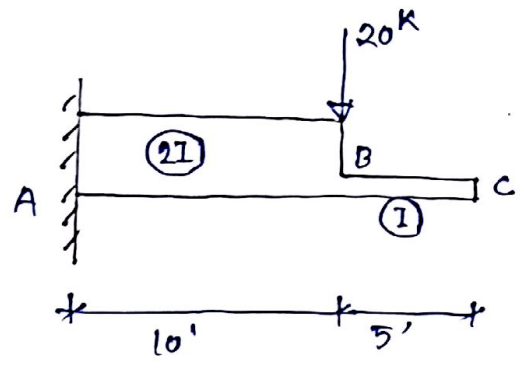
$$\Rightarrow 1k. \alpha_A = \int_A^B \frac{(20x) \left(1 - \frac{x}{30}\right)}{EI_1} dx + \int_B^D \frac{(200 - 10x) \left(\frac{2}{3} - \frac{x}{30}\right)}{EI_2} dx$$

$$\Rightarrow 1k. \alpha_A = \frac{7000}{9EI_1} + \frac{8000}{9EI_2}$$

$$\therefore \alpha_A = 0.04 \text{ radian (clockwise)}$$

— o —

Assignment #12:

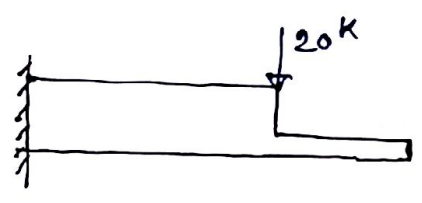


Given, $E = 29000 \text{ ksi}$, $I = 100 \text{ in}^4$

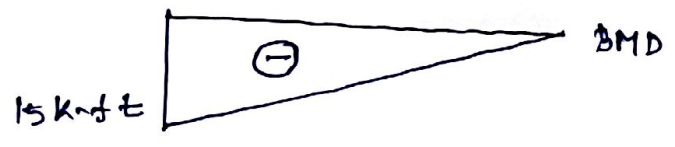
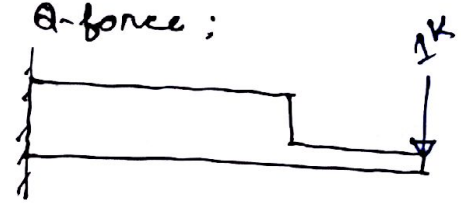
- Compute
- ① Vertical deflection at 'c'.
 - ② Change in slope at 'c'.
 - ③ Change in slope at 'B'.

Soln:

① Analysis for P force;



Analysis for Q-force;



Principle of V.W. ;

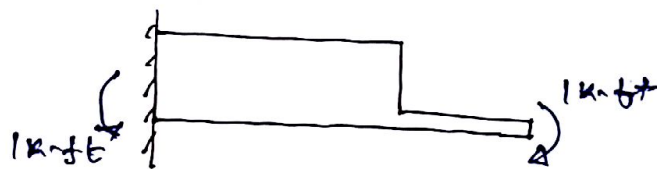
$$Q. \delta_{cv} = \int_A^B \frac{M \alpha M_P}{EI} dx$$

$$\Rightarrow 1K \cdot \delta_{cv} = \frac{1}{3EI} \left[\int_0^{10} (-200 + 20x)(-15 + x) dx \right]$$

$$\Rightarrow \delta_{cv} = \frac{35000/3}{3EI}$$

$$\therefore \delta_{cv} = + 0.1931 \text{ ft (downward)}$$

② Analysis for a force ;



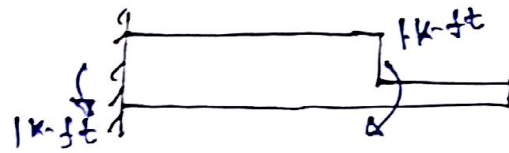
Principle of V.W. ;

$$Q. \alpha_c = \int_A^B \frac{M \alpha M_P}{EI} dx$$

$$\Rightarrow 1K \cdot \alpha_c = \frac{1}{3EI} \int_0^{10} (20x - 200)(-1) dx$$

$$\therefore \alpha_c = 0.01655 \text{ radian (anticlockwise)}$$

③ Analysis for a force;



Principle of V.V. ;

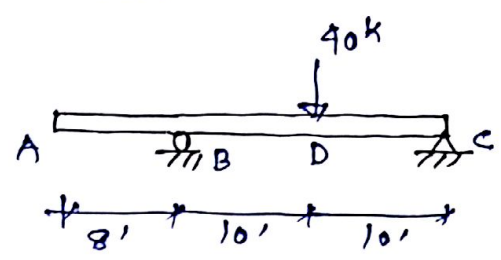
$$\theta_B = \int_A^B \frac{M_A M_P}{EI} dx$$

$$\Rightarrow 1k \cdot \theta_B = \frac{1}{3EI} \int_0^{10} (20x - 200)(-1) dx$$

$$\therefore \theta_B = 0.01655 \text{ radian (clockwise)}$$

— 0 —

Assignment #13:

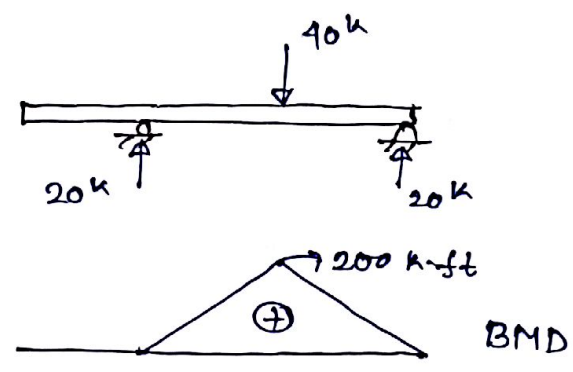


Given, $E = 30000 \text{ ksi}$, $I = 300 \text{ in}^4$

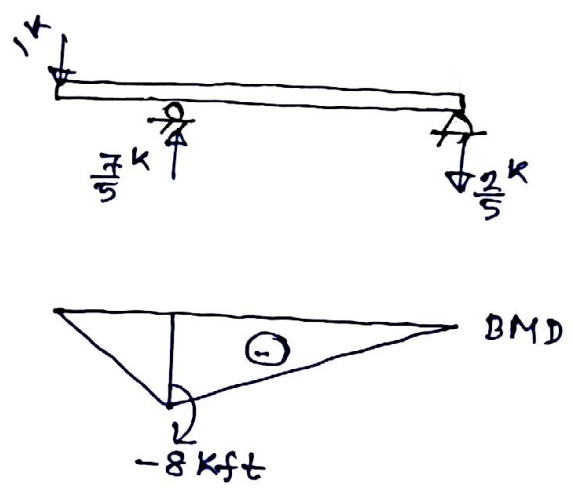
- Find
- ① Deflection at A
 - ② Change in slope at 'A'.
 - ③ Change in slope at 'B'.

Soln:

① Analysis for P force ;



Analysis for Q force ;



Principle of V.W.;

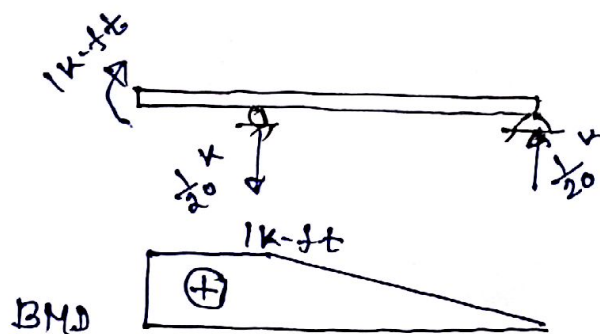
$$Q. \delta_A = \int_B^D \frac{M_A M_P}{EI} dx + \int_D^C \frac{M_A M_P}{EI} dx$$

$$\Rightarrow 1k \cdot \delta_A = \frac{1}{EI} \left[\int_0^{10} (20x) \left(-8 + \frac{2x}{5}\right) dx + \int_0^{10} (200 - 20x) \left(-4 + \frac{2x}{5}\right) dx \right]$$

$$\Rightarrow \delta_A = -\frac{8000}{EI}$$

$$\therefore \delta_A = -0.128 \text{ ft (upward)}$$

② Analysis for a force;



Principle of V.W.;

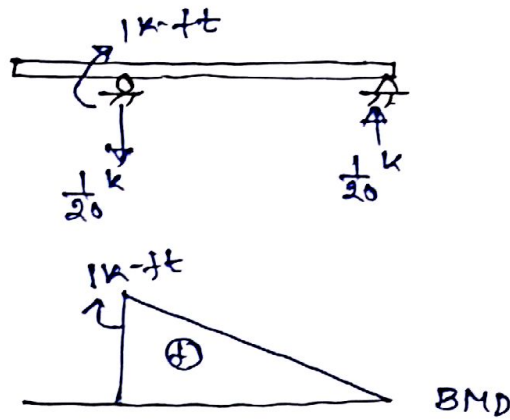
$$Q. \alpha_A = \int_B^D \frac{M_A M_P}{EI} dx + \int_D^C \frac{M_A M_P}{EI} dx$$

$$\Rightarrow 1k \cdot \alpha_A = \frac{1}{EI} \left[\int_0^{10} (20x) \left(1 - \frac{x}{20}\right) dx + \int_0^{10} (20x) \left(\frac{x}{20}\right) dx \right]$$

$$\Rightarrow \alpha_A = \frac{1000}{EI}$$

$$\therefore \alpha_A = 0.016 \text{ radian (clockwise)}$$

③ Analysis for Q-force ;



Principle of V.W. ;

$$Q. \alpha_B = \int_B^D \frac{M_0 M_P}{EI} dx + \int_D^C \frac{M_0 M_P}{EI} dx$$

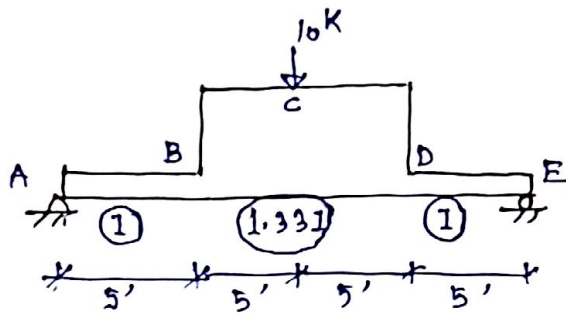
$$\Rightarrow 1k. \alpha_B = \frac{1}{EI} \left[\int_0^{10} (20x) \left(1 - \frac{x}{20}\right) dx + \int_0^{10} (20x) \left(\frac{x}{20}\right) dx \right]$$

$$\Rightarrow \alpha_B = \frac{1000}{EI}$$

$$\therefore \alpha_B = 0.016 \text{ radian (clockwise)}$$

— 0 —

Assignment # 14:



Given,

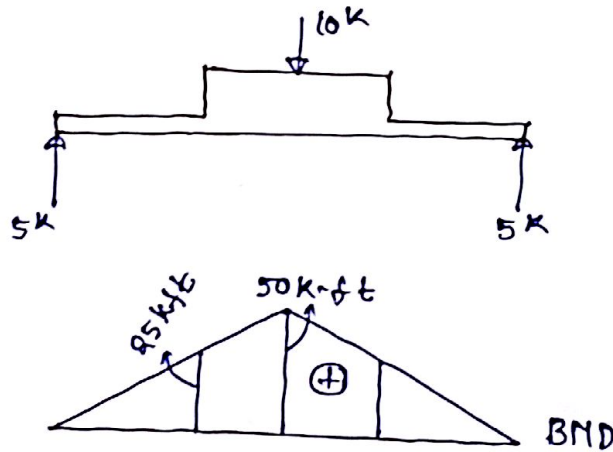
$E = 30 \times 10^3 \text{ kips/in}^2, I = 150 \text{ in}^4$

Compute ① Deflection at 'B'.

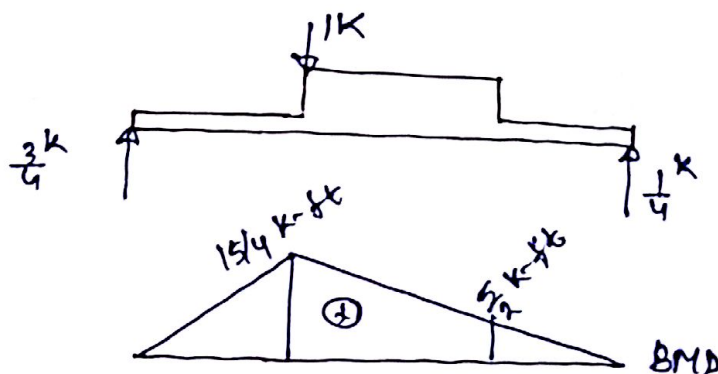
② Change in slope at 'D'.

Soln:

① Analysis for P force:



Analysis for Q force.



Principle of V.W.;

$$Q. \delta_B = \int_A^B \frac{M \Delta M_P}{EI} dx + \int_B^C \frac{M \Delta M_P}{EI} dx + \int_C^D \frac{M \Delta M_P}{EI} dx + \int_D^E \frac{M \Delta M_P}{EI} dx$$

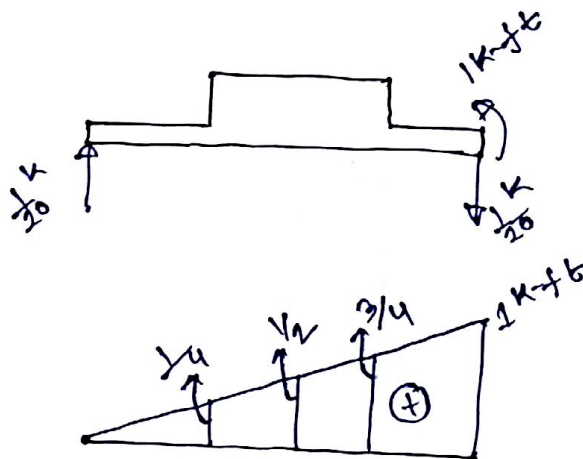
$$\Rightarrow 1k \cdot \delta_B = \frac{1}{EI} \left[\int_0^5 (5x) \left(\frac{3x}{4}\right) dx + \int_0^5 \frac{(25+5x) \left(\frac{15}{4} - \frac{x}{4}\right)}{1.33} dx \right. \\ \left. + \int_0^5 \frac{(50-5x) \left(\frac{5}{2} - \frac{x}{4}\right)}{1.33} dx + \int_0^5 (5x) \left(\frac{7}{4}\right) dx \right]$$

$$\Rightarrow \delta_B = \frac{913.22}{EI}$$

$$\therefore \delta_B = 0.029 \text{ ft (downward)}$$

②

Analysis for a force;



Principle of V.W. ;

$$\theta. \alpha_D = \int_A^B \frac{M \theta M_p}{EI} dx + \int_B^C \frac{M \theta M_p}{EI} dx + \int_C^D \frac{M \theta M_p}{EI} dx + \int_P^E \frac{M \theta M_p}{EI} dx$$

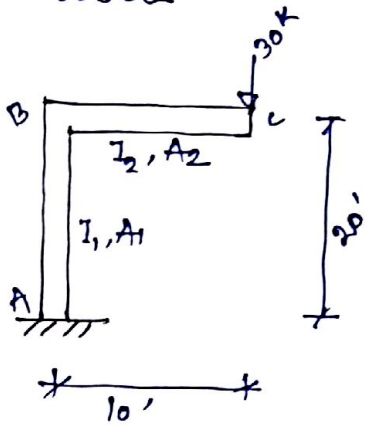
$$\Rightarrow IK. \alpha_D = \frac{1}{E} \left[\frac{1}{150} \left\{ \int_0^5 \frac{x}{20} (5x) dx + \int_0^5 (5x) \left(1 - \frac{x}{20}\right) dx \right\} \right. \\ \left. + \frac{1}{200} \left\{ \int_0^5 (25 + 5x) \left(\frac{1}{4} + \frac{x}{20}\right) dx + \int_0^5 (50 - 5x) \left(\frac{1}{2} + \frac{x}{20}\right) dx \right\} \right]$$

$$\Rightarrow \alpha_D = \frac{5/12 + 15/16}{E}$$

$$\therefore \alpha_D = 0.0065 \text{ radian (anticlockwise)}$$

— • —

Assignment #15:



Compute horizontal deflection at B

Given, $I_1 = 300 \text{ in}^4$

$I_2 = 200 \text{ in}^4$

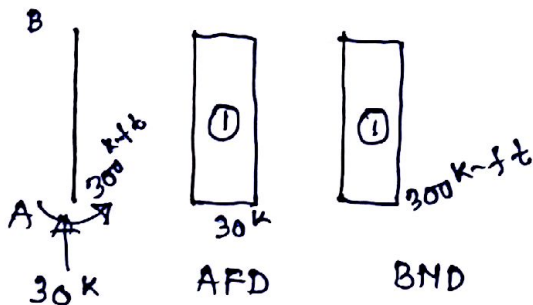
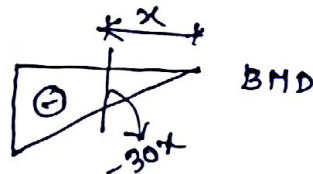
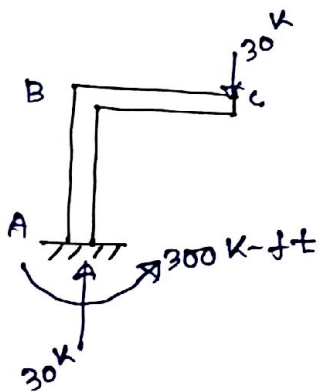
$A_1 = 15 \text{ in}^2$

$A_2 = 10 \text{ in}^2$

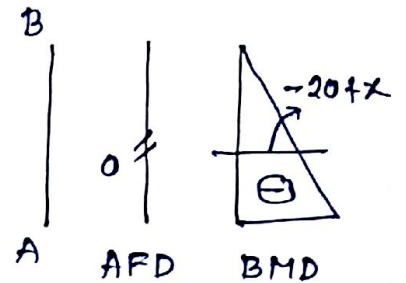
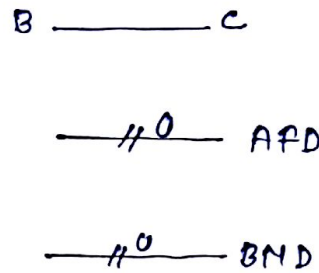
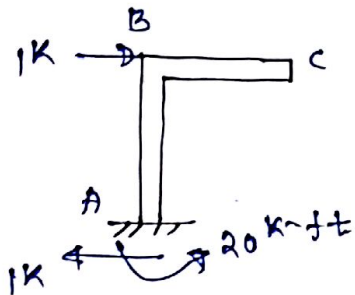
$E = 30000 \text{ ksi}$

Soln:

P-force analysis:



Q-force analysis:



Segment AB

$$M_Q = (x-20)$$

$$M_P = -300$$

$$F_Q = 0$$

$$F_P = -30'$$

Segment BC

$$M_Q = -30x$$

$$M_P = 0$$

$$F_Q = 0$$

$$F_P = 0$$

Principle of V.W.;

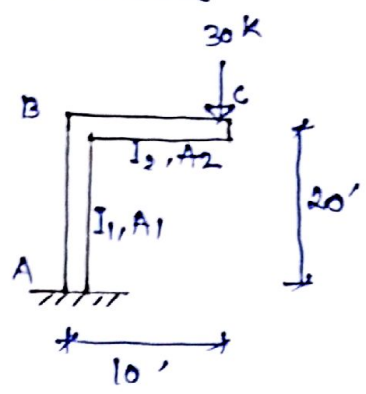
$$Q \cdot \delta_{BH} = \int_A^B \frac{M_Q M_P}{EI} dx + 0$$

$$\Rightarrow 1k \cdot \delta_{BH} = \int_0^{20} \frac{-300(x-20)}{I, E} dx$$

$$\therefore \boxed{\delta_{BH} = 0.96 \text{ ft } (\rightarrow)}$$

_____ o _____

Assignment #16:

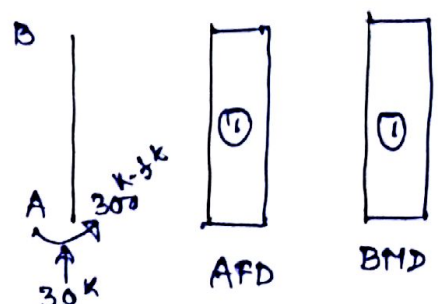
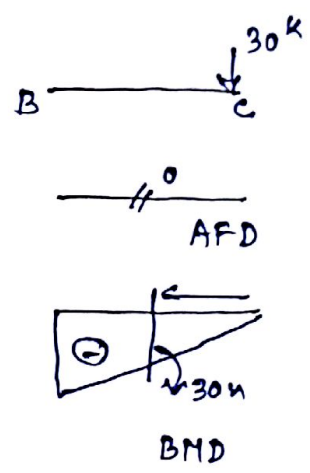
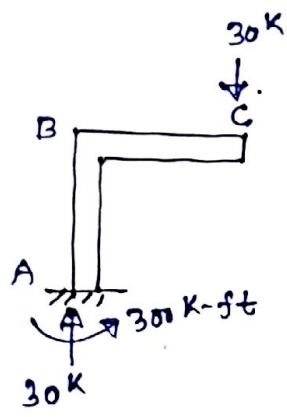


Compute change in slope at C.

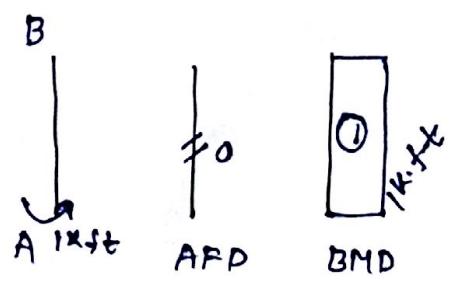
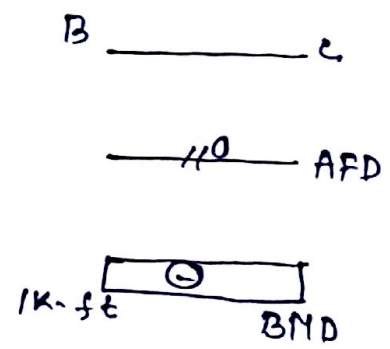
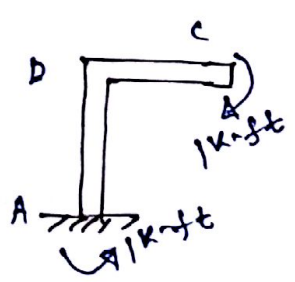
- Given, $I_1 = 300 \text{ in}^4$
- $I_2 = 100 \text{ in}^4$
- $A_1 = 15 \text{ in}^2$
- $A_2 = 10 \text{ in}^2$
- $E = 30000 \text{ ksi}$

H Soln:

P-force analysis:



Q-force analysis:



Principle of V.W.;

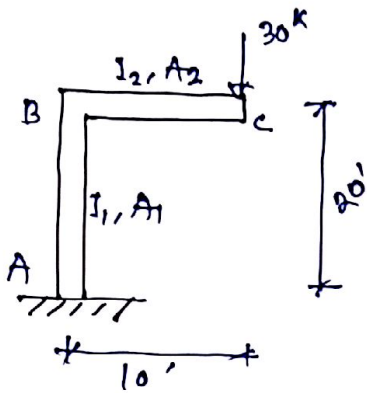
$$\theta_c = \int_A^B \frac{M \Delta y}{EI} dx$$

$$\Rightarrow \theta_c = \int_0^{20} \frac{(-1)(-30x)}{EI} dx$$

$$\therefore \theta_c = 0.096 \text{ radian (clockwise)}$$

— o —

Assignment #17:



Compute vertical deflection at mid-point of BC.

Given, $I_1 = 300 \text{ in}^4$

$I_2 = 200 \text{ in}^4$

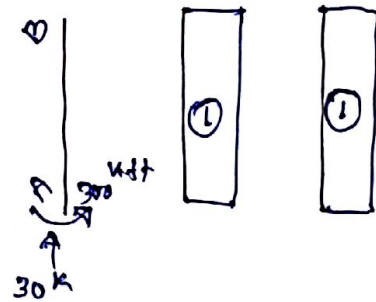
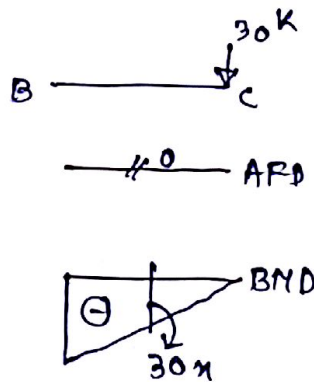
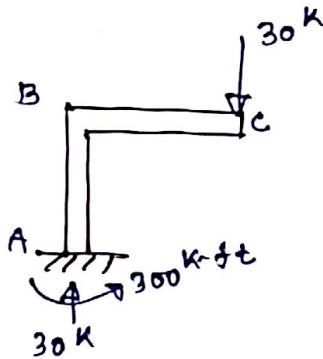
$A_1 = 15 \text{ in}^2$

$A_2 = 10 \text{ in}^2$

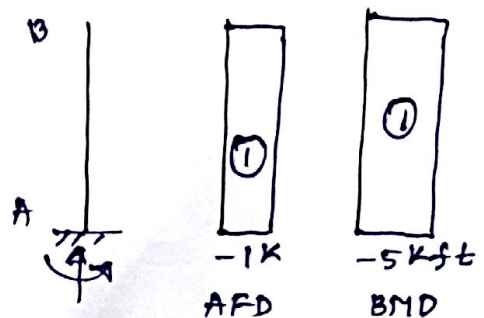
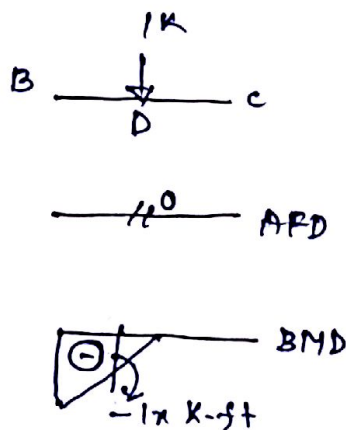
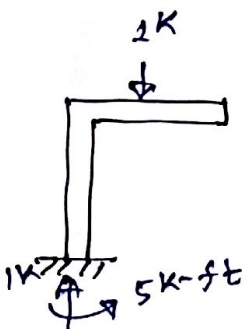
$E = 30000 \text{ ksi}$

Soln:

P-force analysis:



Q-force analysis:



Principle of V.W.;

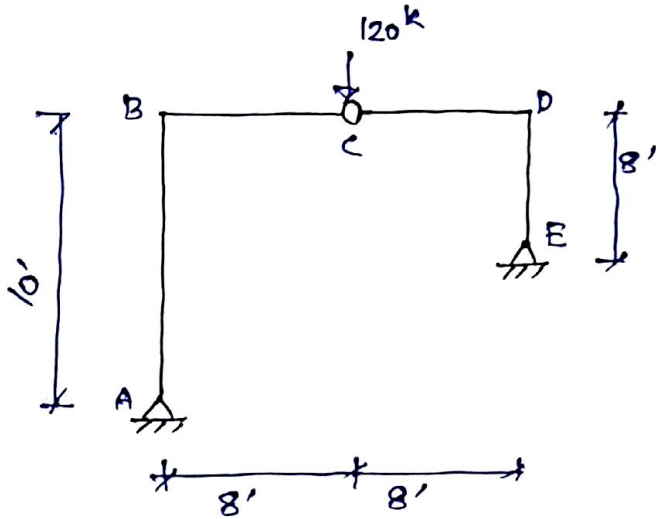
$$Q. \delta_v = \int \frac{M_a M_p}{EI} dx$$

$$\Rightarrow 1k. \delta_v = \int_0^{20} \frac{-900x - 5}{EI} dx + 0 + \int_5^{10} \frac{(-30x)(-x)}{EI} dx$$

$$\therefore \delta_v = 3.52 \times 10^{-3} \text{ ft (downward)}$$

— 0 —

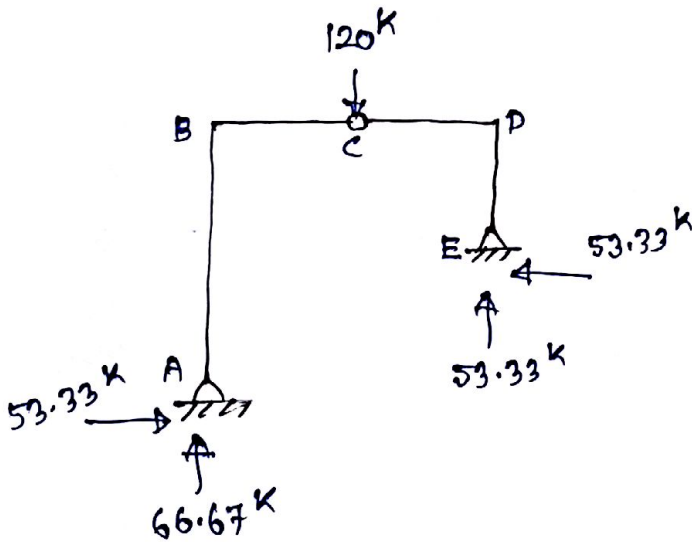
Assignment #18:



Compute change in slope at the cross section on the left of hinge.

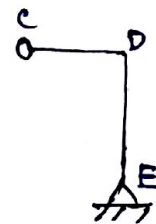
Solⁿ:

P-force analysis:



$$\sum M_A = 0$$

$$\Rightarrow R_{Ex} + 8R_{Ey} = 480 \quad \text{--- (1)}$$



$$\sum M_C = 0$$

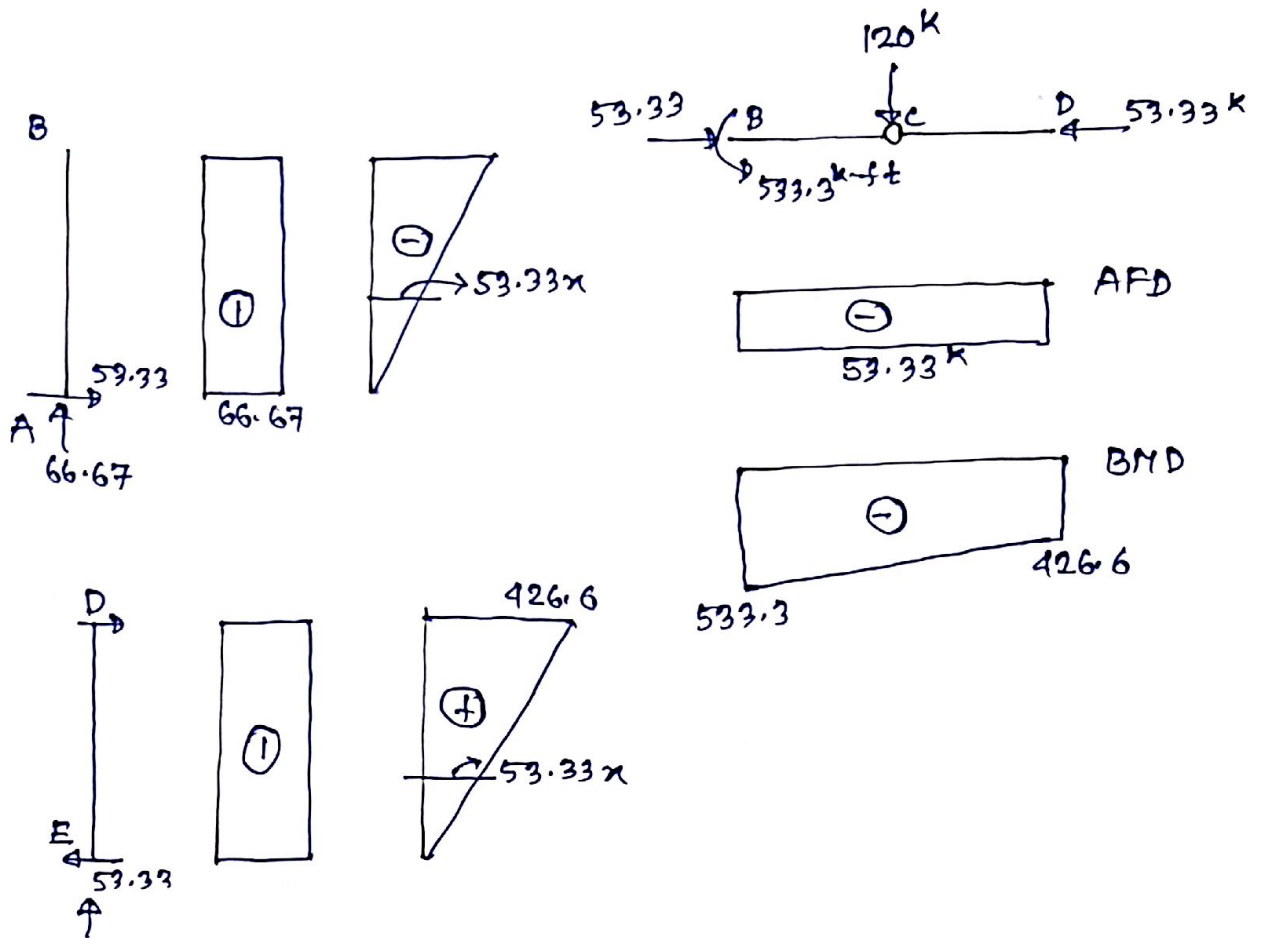
$$\Rightarrow R_{Ex} \cdot 8 = R_{Ey} \cdot 8$$

$$\therefore R_{Ex} = R_{Ey} \quad \text{--- (2)}$$

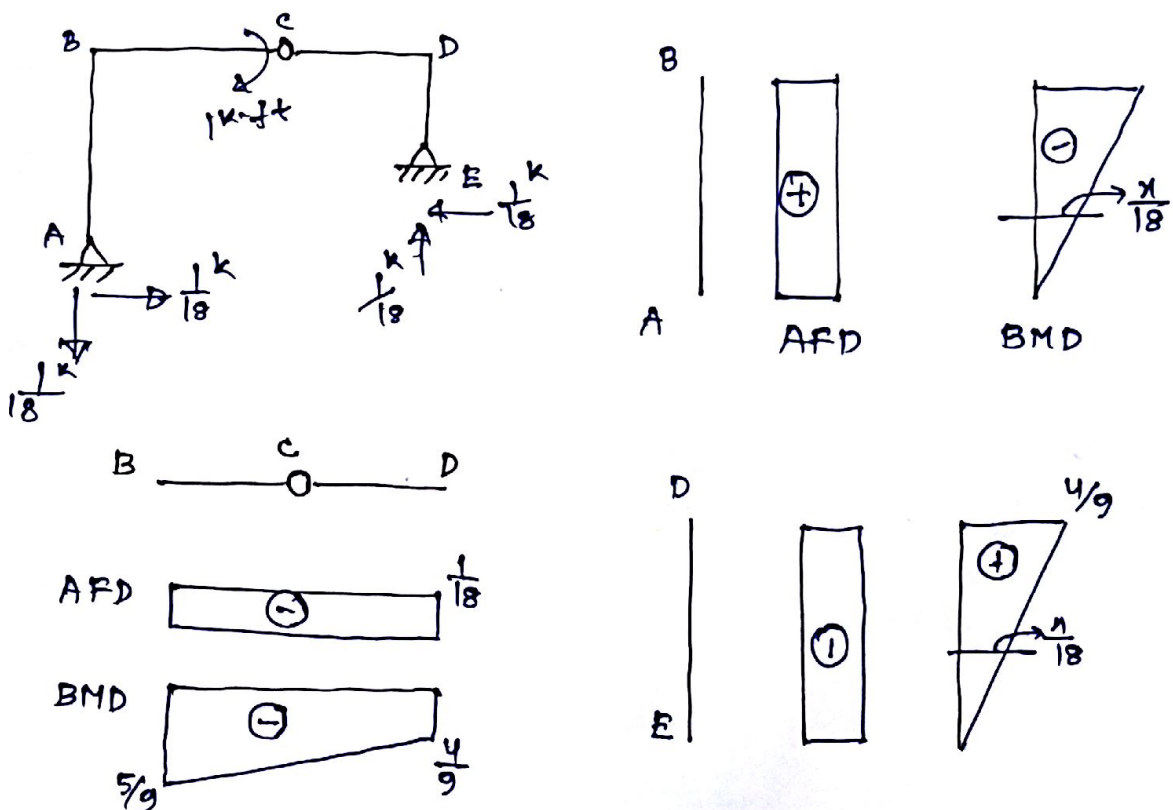
From (1) & (2);

$$R_{Ex} = R_{Ay} = 53.33 \text{ k}$$

$$R_{Ay} = 66.67 \text{ k}$$



Q-force analysis:



Principle of V.W.;

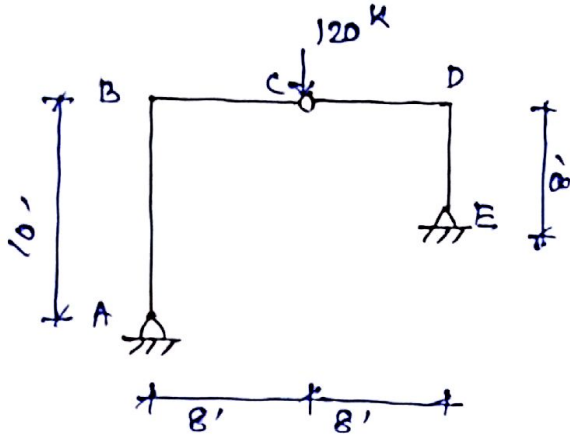
$$\theta_c = \int \frac{M \Delta MP}{EI} dx$$

$$\Rightarrow \theta_c = \frac{1}{EI} \left[\int_0^{10} \left(53.33x + \frac{x}{18} \right) dx + \int_0^{16} \left\{ \left(\frac{-5}{9} + \frac{-x}{18} \right) + 1 \right\} \right. \\ \left. \left[-533.3 + 66.7x - 120(x-8) \right] \right]$$

$$\therefore \theta_c = 24.57 \times 10^{-4} \text{ radian (clockwise)}$$

— 0 —

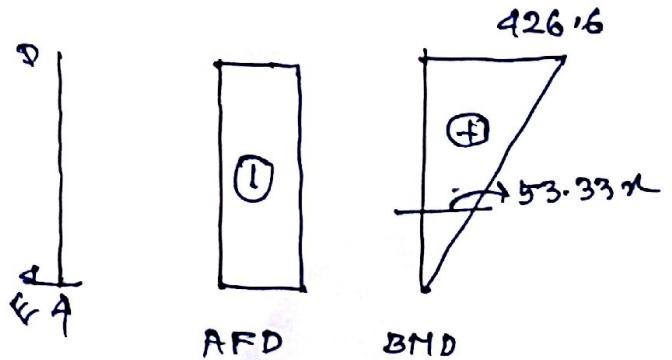
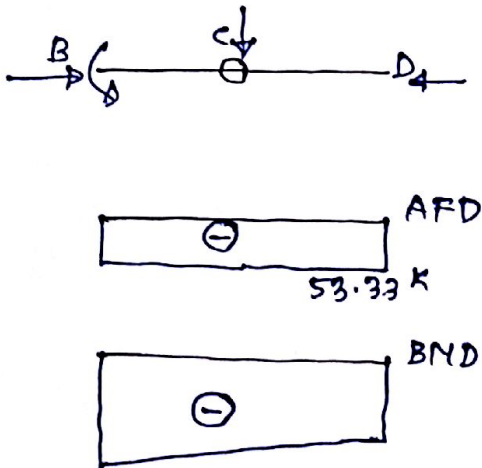
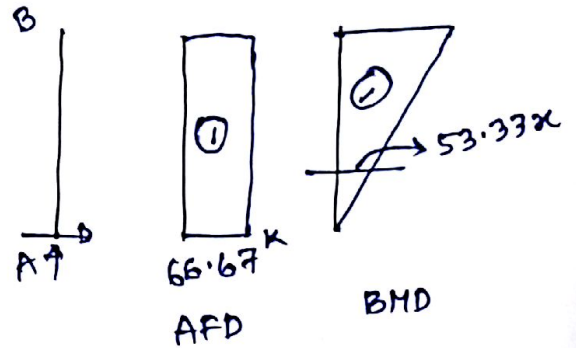
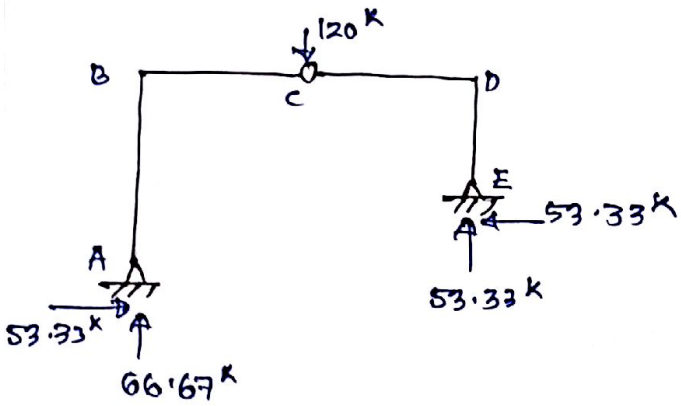
Assignment #19:



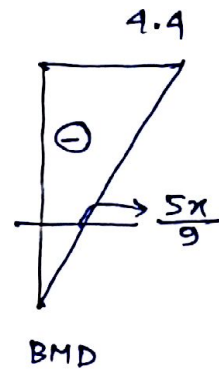
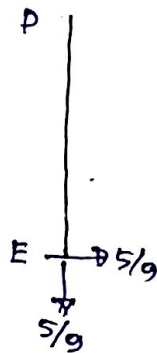
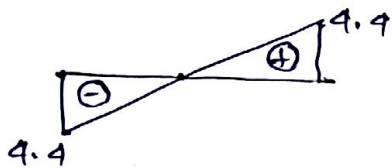
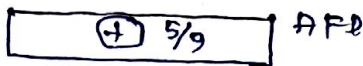
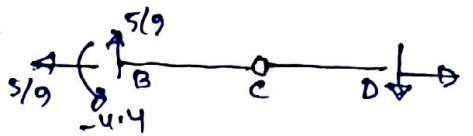
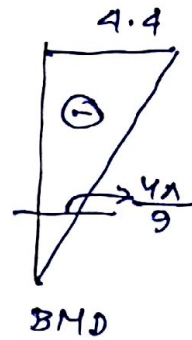
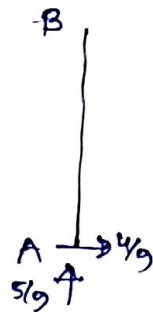
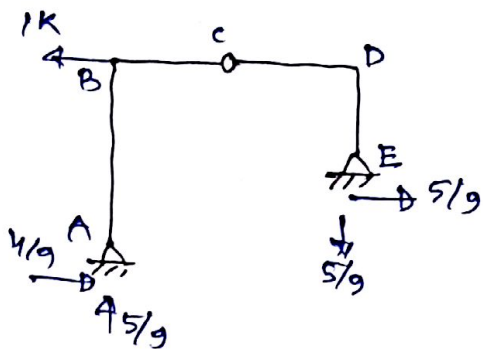
Find horizontal deflection at B.

Solⁿ:

P-force analysis:



Q- force analysis :



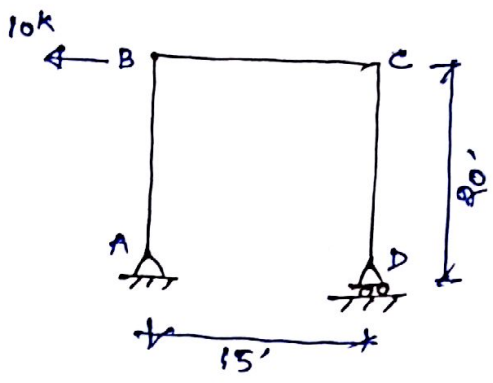
Principle of V.W.;

$$\begin{aligned} \Delta \cdot \delta_{BH} = & \int_0^{10} \frac{(-\frac{4x}{9}) (-53.33x)}{EI} dx + \int_0^8 \frac{-5x(53.33x)}{9EI} dx \\ & + \int_0^{16} \frac{(-4.4 + \frac{5x}{9}) (426.7 - 53.33x)}{EI} dx \\ & + \frac{(-5/9) (-66.67) 10}{AE} + \frac{5/9 (-53.33) 16}{AE} + \frac{5/9 (-53.33) 8}{AE} \end{aligned}$$

$$\Rightarrow 1k \cdot \delta_{BH} = -5.18 \times 10^{-3} - 0.0194$$

$$\therefore \delta_{BH} = -0.0246' \text{ (right)}$$

Assignment # 20 :

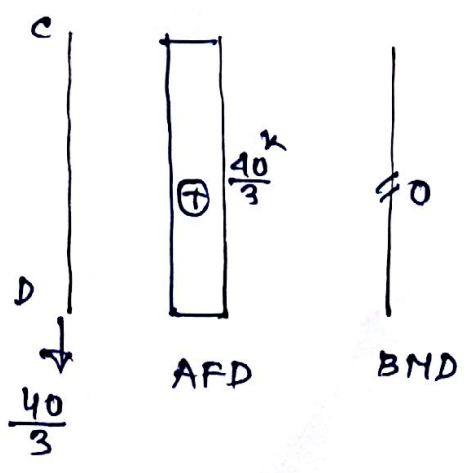
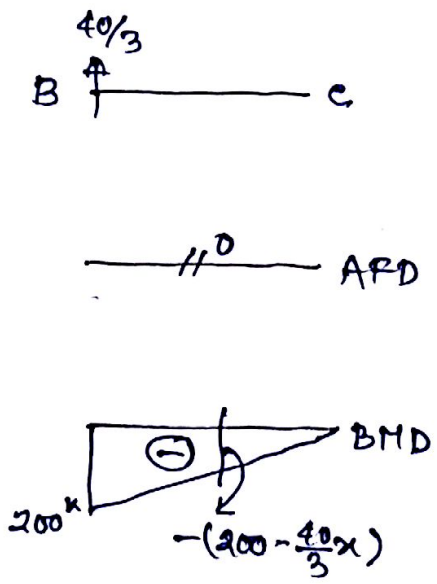
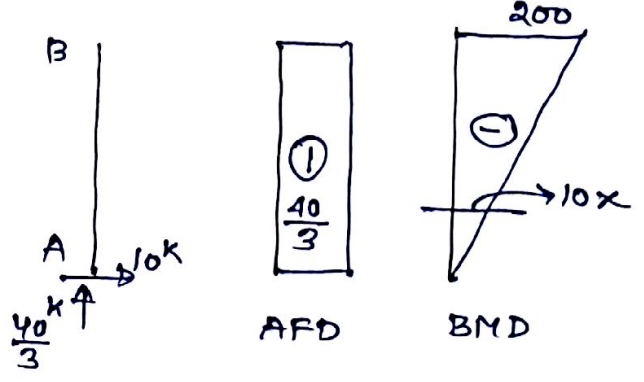
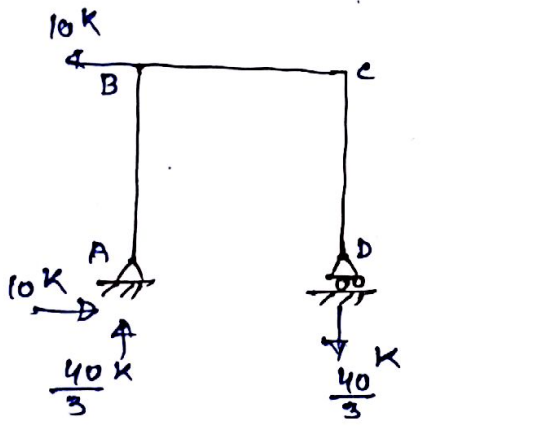


Given, $E = 30000 \text{ ksi}$
 $I = 800 \text{ in}^4$
 $A = 20 \text{ in}^2$

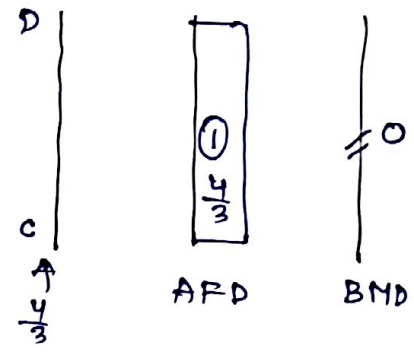
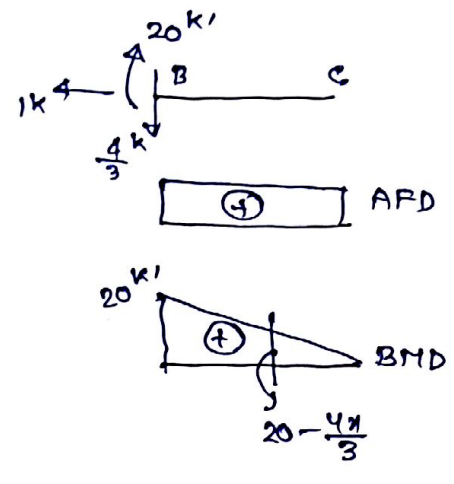
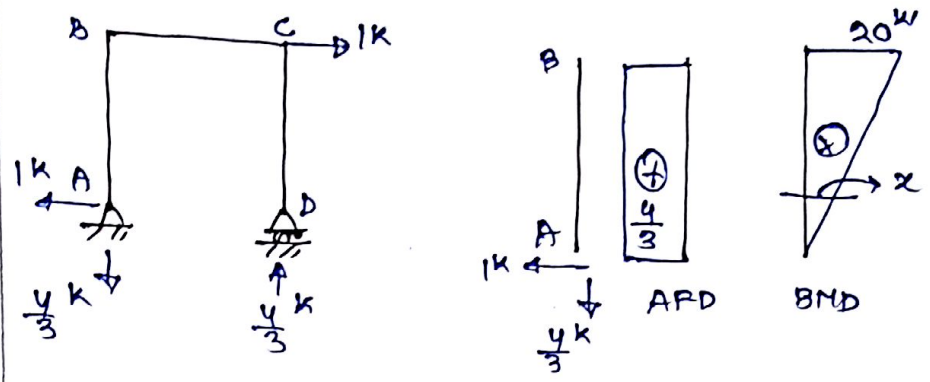
Find $\delta_{ch} = ?$

Soln :

P-force analysis :



Q-force analysis:



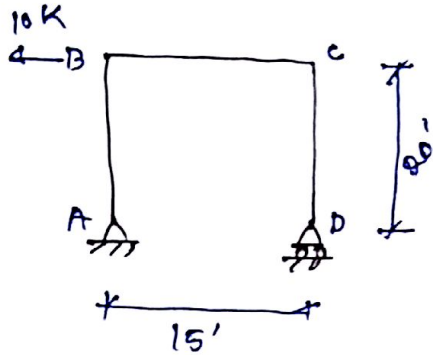
Principle of V.W.'s

$$\begin{aligned} Q. \delta_{ch} = & \int_0^{20} \frac{x(-10x)}{EI} dx + \int_0^{15} \frac{(20 - \frac{4}{3}x)(\frac{40x}{3} - 200)}{EI} dx \\ & + \frac{(-\frac{4}{3})(\frac{40}{3})20}{AE} + \frac{\frac{4}{3}(-\frac{40}{3})20}{AE} \end{aligned}$$

$\therefore \delta_{ch} = -0.0292'$ (leftward)



Assignment # 21!



Given.

$E = 30000 \text{ ksi}$

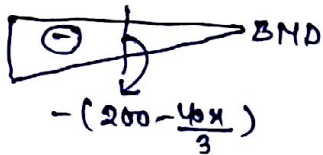
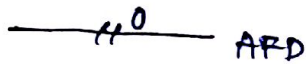
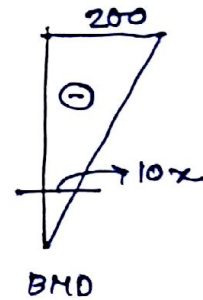
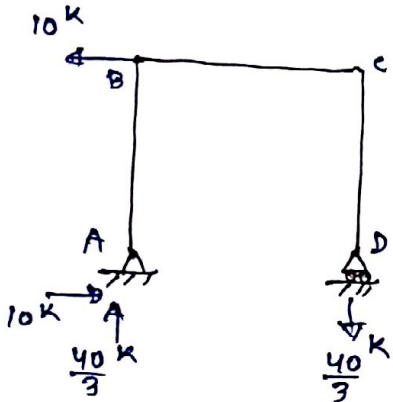
$I = 800 \text{ in}^4$

$A = 20 \text{ in}^2$

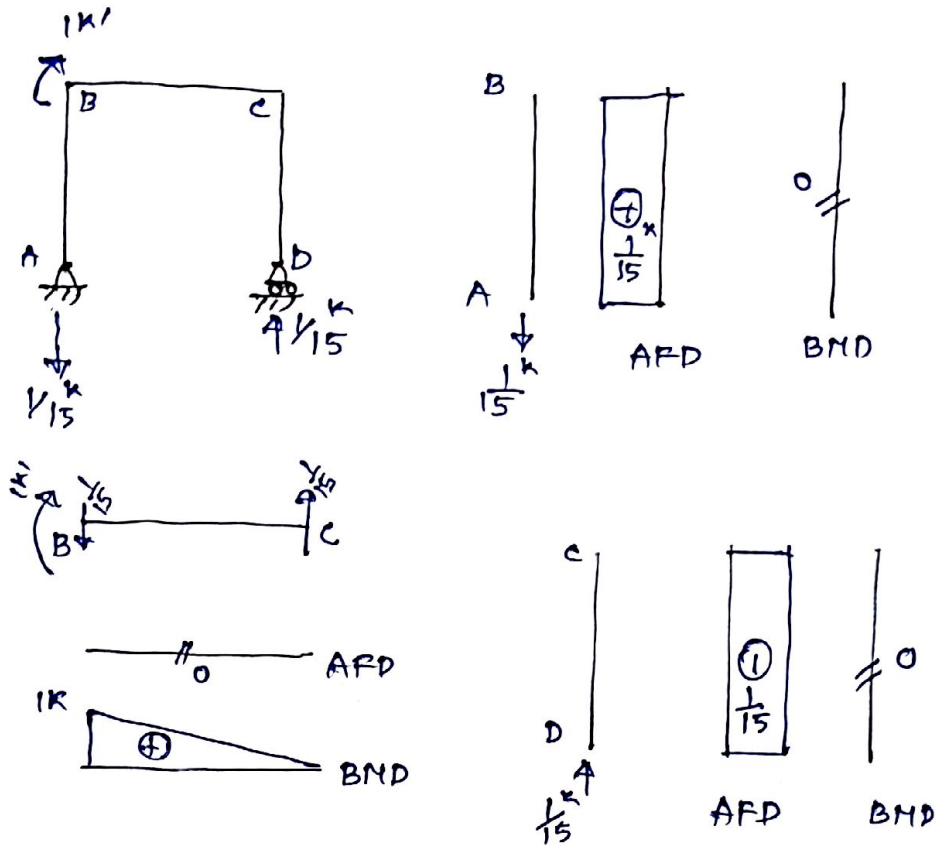
Find θ_B

Soln:

P-force analysis:



Q-force analysis:

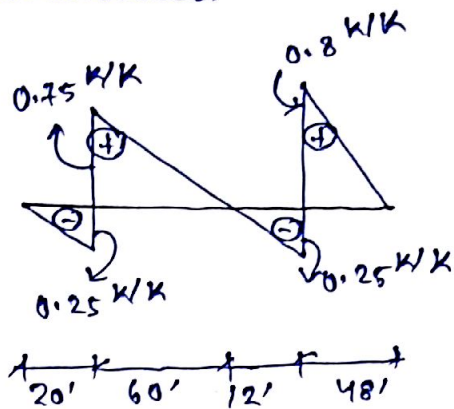


Principle of V.W.;

$$\theta_B = \frac{\left(-\frac{40}{3}\right) \left(\frac{1}{15}\right) \times 20}{AE} + \frac{\left(-\frac{1}{15}\right) \left(\frac{40}{3}\right) \times 20}{AE} + \int_0^{15} \frac{\frac{x}{15} \times \left(-\frac{40x}{3}\right)}{EI} dx$$

$$\therefore \theta_B = -6.59 \times 10^{-4} \text{ radian (anticlockwise)}$$

Assignment # 22:



Find ① Max^m Shear

② Max^m (+)ve shear

③ Max^m (-)ve shear

For uniform load of 6 k/ft
and a concentrated load
of 120 k .

Solⁿ:

$$\text{Max}^m (+)ve \text{ shear} = \frac{1}{2} \times 0.75 \times 60 \times 6 + \frac{1}{2} \times 0.8 \times 48 \times 6 + 0.8 \times 120$$

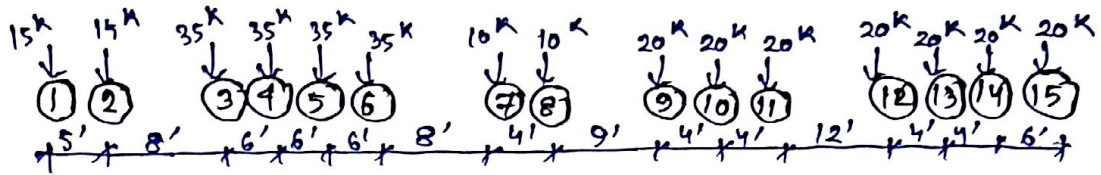
$$= \boxed{346.2 \text{ k}}$$

$$\text{Max}^m (-)ve \text{ shear} = \frac{1}{2} \times 20 \times 0.25 \times 6 + \frac{1}{2} \times 12 \times 0.25 \times 6 + 0.25 \times 120$$

$$= \boxed{54 \text{ k}}$$

$$\text{Max}^m \text{ Shear} = \boxed{346.2 \text{ k (+ve)}}$$

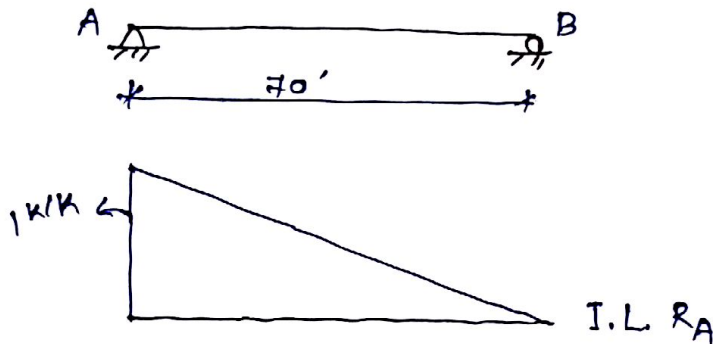
Assignment # 23 :



Span = 70'

Find Maximum R_A .

soln :



$$\Delta R = \frac{\sum Pd}{L} - P_i + \frac{P'e}{L}$$

Trial ① : wheel ① at A to ② at A

Here, $\sum P =$ wheel ② to ⑪ = 235k

$P_i =$ wheel ① = 15k

$P' =$ wheel ⑫ = 20k

$e' = 3'$

$d = 5'$

$\therefore \Delta R = 2.64$ (true, increasing)

Trial ②: wheel ② at A to ③ at A

$$\text{Here, } \Sigma P = \textcircled{2} \text{ to } \textcircled{12} = 240 \text{ k}$$

$$P_1 = \textcircled{1} = 15 \text{ k}$$

$$P' = \textcircled{12} \text{ \& } \textcircled{13} = 20 \text{ k} + 20 \text{ k} = 40 \text{ k}$$

$$e = 11' \text{ for } \textcircled{12} \text{ and } 7' \text{ for } \textcircled{13}$$

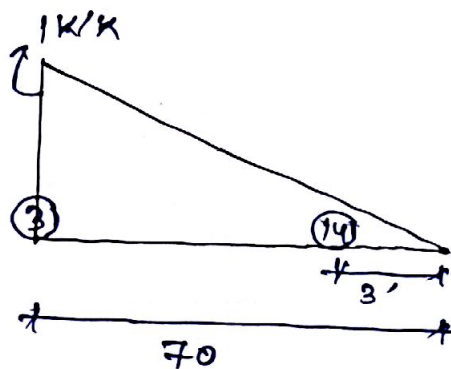
$$d = 8'$$

$$\therefore \Delta R = +17.57 \text{ (increasing)}$$

Trial ③: wheel ③ at A to ④ at A

$$\Delta R = -13.57 \text{ (decreasing)}$$

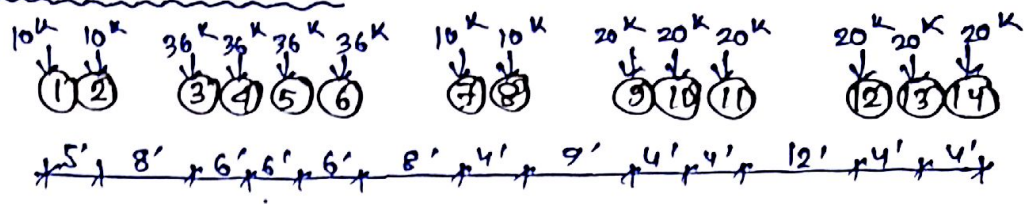
Hence wheel ③ at A will give max^m reaction at A.



$$\therefore \text{Max}^m \text{ Reaction, } R_A = \frac{1}{70} \left[(3 \times 20) + (7 \times 20) + (11 \times 20) + (23 \times 20) \right. \\ \left. + (27 \times 20) + (31 \times 20) + (40 \times 10) \right. \\ \left. + (44 \times 10) + (52 \times 35) + (58 \times 35) \right. \\ \left. + (64 \times 35) + (70 \times 35) \right]$$

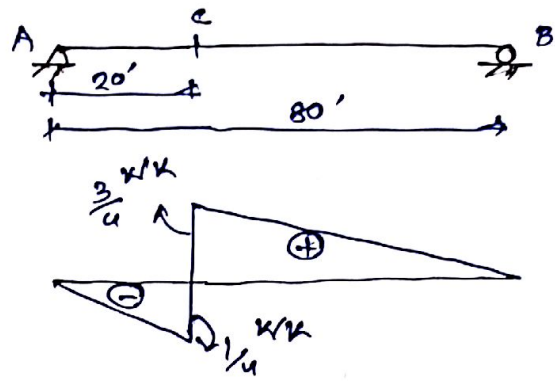
$$\therefore R_{A \text{ max}} = 163.143 \text{ k}$$

Assignment #24:



Find m^m shear at quarter point of a simply supported beam having 80' span for the load shown.

Solⁿ:



$$\Delta V = \frac{\sum P d_i}{L} - P_1 + \frac{P'e}{L} + \frac{P_2 e'}{L}$$

Trial 1: wheel 1 at c to wheel 2 at c

Here,

$$\sum P = \text{wheel 2 to 11} = 234 \text{ k}$$

$$d_1 = 5'$$

$$P_1 = \text{wheel 1} = 10 \text{ k}$$

$$P' = e = P_2 = 0$$

$$\therefore \Delta V = + 4.625 \text{ (increasing)}$$

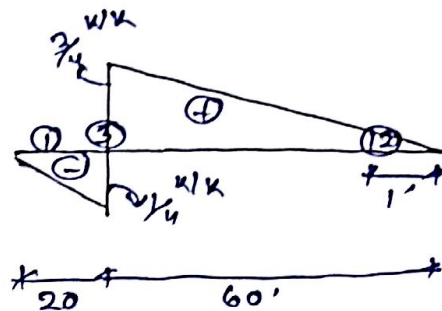
Trial ②: wheel ② at c to ③ at c

$$\Delta V = +12.65 \text{ k (increasing)}$$

Trial ③: wheel ③ at c to ④ at c

$$\Delta V = -21.15 \text{ k (decreasing)}$$

Hence wheel ③ at c will give max^m shear force

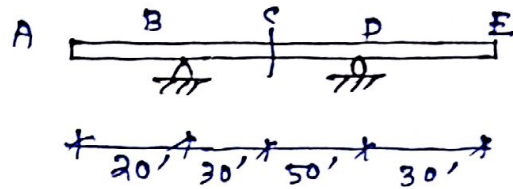


$$\therefore \text{Max}^m \text{ shear at c} = \frac{1}{60} [20 \times 1 + 20(13 + 17 + 21) + 10(30 + 34) + 36(38 + 44 + 50 + 50)]$$

$$- \frac{1}{60} \{10 \times 68 + 10 \times 73\}$$

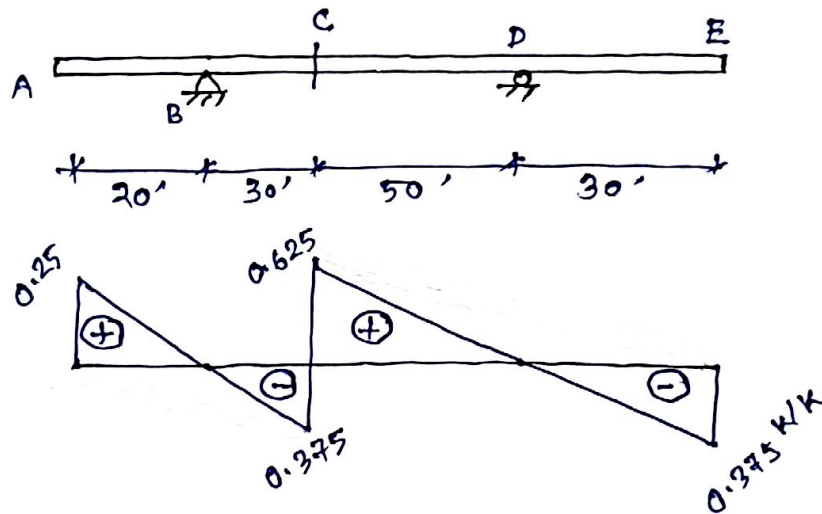
$$\therefore \boxed{V_{\max} = 117.3 \text{ k (tve.)}}$$

— 0 —

Assignment # 25:

Find max^m shear and moment at section C for the beam shown. Consider an uniform moving load of 7 K/ft combined with a moving concentration of 90 K.

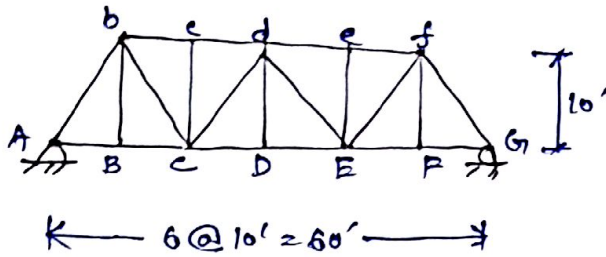
Solⁿ:



$$\therefore \text{Max}^m \text{ shear at C} = 90 \times 0.625 + \frac{1}{2} \times 0.625 \times 50 \times 7 + \frac{1}{2} \times 0.25 \times 20 \times 7$$

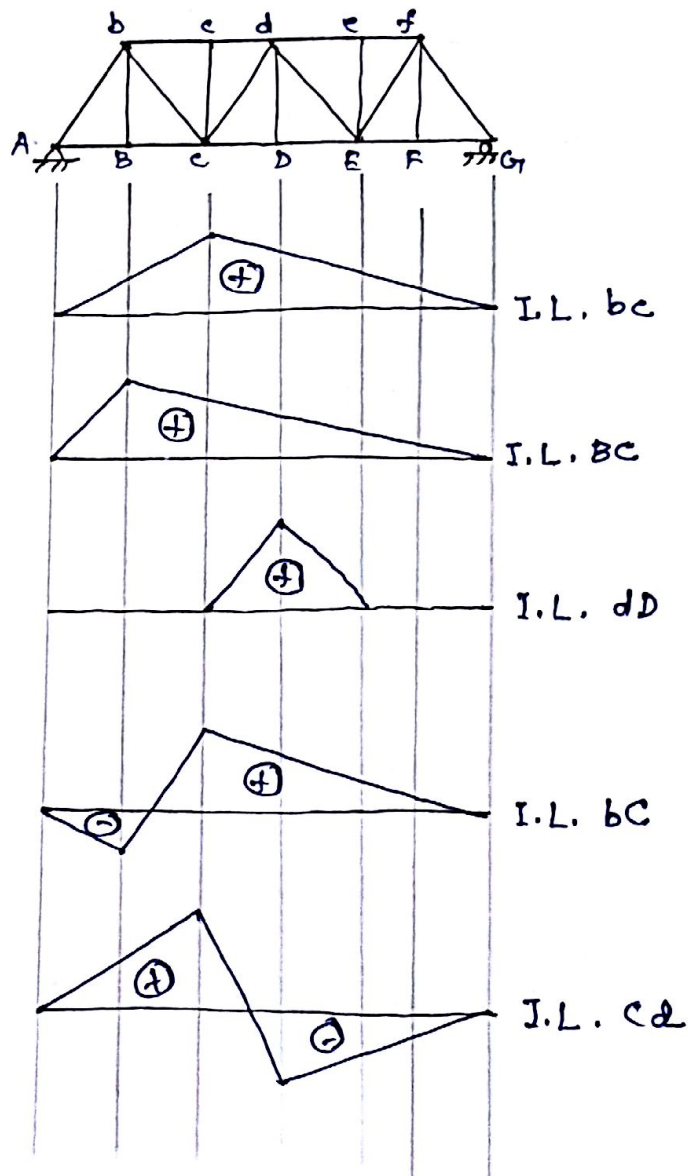
$$\therefore V_{\text{max}} = 183.125 \text{ K}$$

Assignment #26:

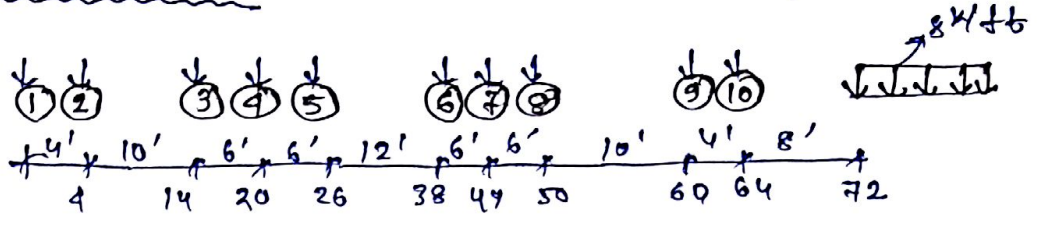


- Draw IL diagrams for
- ① Top chord member
 - ② Bottom " "
 - ③ Diagonal " "
 - ④ Vertical " "

Solⁿ:

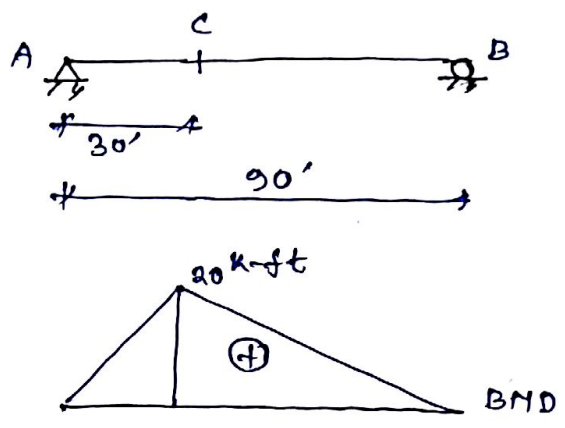


Assignment #27:



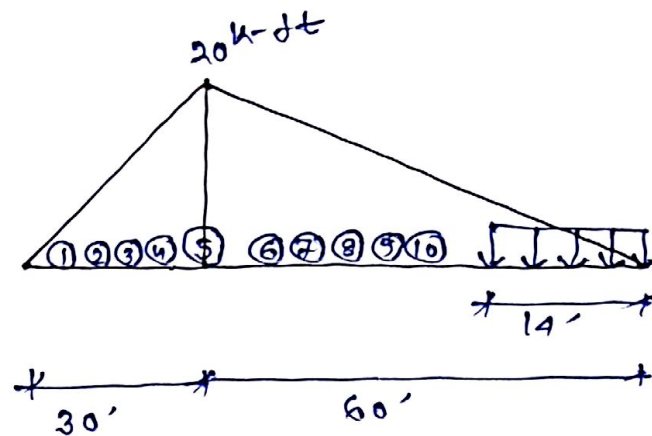
Find max^m moment at one third point of simple span of 90'.

Soln:



Trial No.	Position of wheel	$\frac{W}{L}$	$\frac{Wl}{a}$	Remarks
1	Wheel ⑤ at c → just right → just left	$\frac{672}{90}$	$> \frac{200}{30}$	Criterion satisfied
		$\frac{672}{90}$	$< \frac{280}{30}$	
2	Wheel ④ at c → just right → just left	$\frac{624}{90}$	$> \frac{120}{30}$	Criterion not satisfied
		$\frac{624}{90}$	$> \frac{200}{30}$	

Hence wheel (5) at C gives max^m moment at C.



$$\begin{aligned} \therefore \text{Max}^m \text{ moment at C} &= \frac{2}{3} [20(4+8) + 30(18+24+30)] \\ &\quad + \frac{1}{3} [20(22+26) + 80(26+42+48)] \\ &\quad + \frac{1}{2} \times 14 \times \frac{1}{3} \times \frac{1}{4} \times 8 \end{aligned}$$

$$\therefore \boxed{M_{\max} = 7941.33 \text{ k}}$$

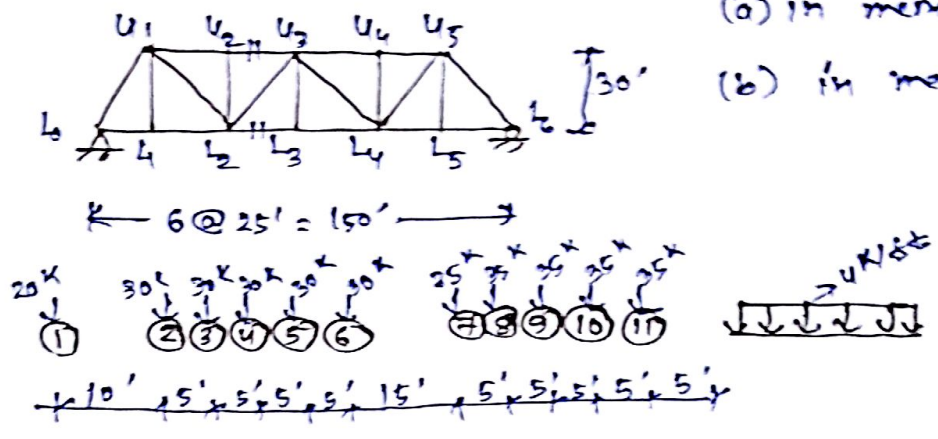
— 0 —

Assignment # 28:

Find max^m stress

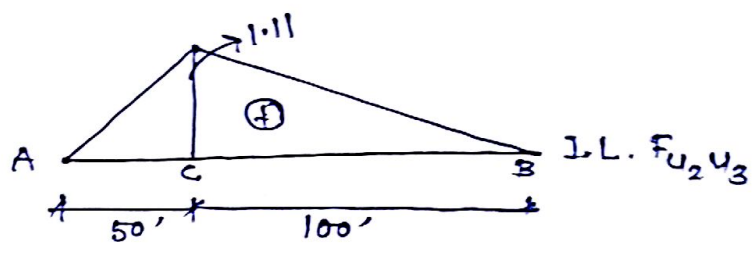
(a) in member U_2U_3

(b) in member L_2L_3



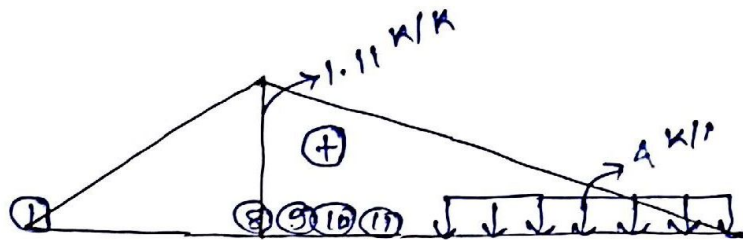
Solⁿ:

(a)



Trial No.	Position of wheel	$\frac{W}{L}$	$\frac{W_1}{a}$	Remarks
1	Wheel ③ → just right at c	3.9 >	2.8	Criterion not satisfied
	← just left	3.9 >	3.4	
2	Wheel ⑦ → just right at c	4.3 >	3.4	Criterion not satisfied
	← just left	4.3 >	4.1	
3	Wheel ⑧ → just right at c	4.43 >	4.1	Criterion satisfied
	← just left	4.3 <	4.4	

Hence wheel (8) will give max^m stress



∴ Max^m stress in member U_2U_3 of truss =

$$\frac{1.11}{50} [30(10+15+20+25+30+35+45+50)]$$

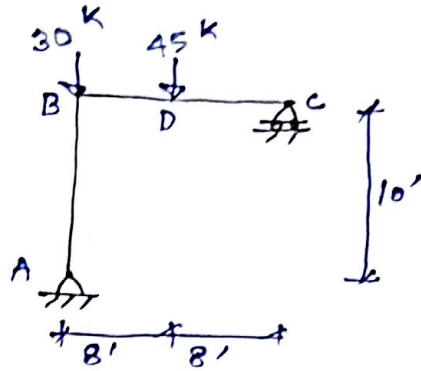
$$+ \frac{1.11}{100} [35(35+40+45)]$$

$$+ \frac{1}{2} \times \frac{1.11}{100} \times 80 \times 80 \times 4$$

$$= 387.49 \text{ k (Compression)}$$

— • —

Assignment # 19:



Compute rotation at C

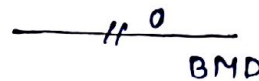
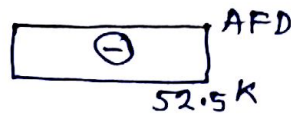
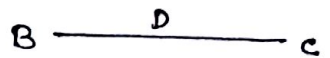
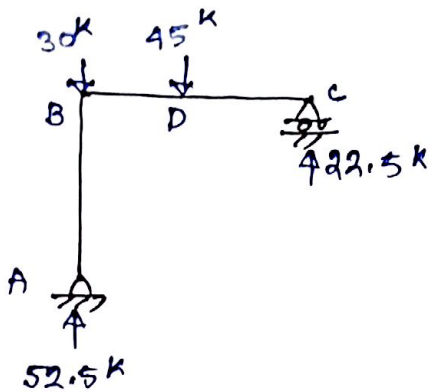
$E = 30000 \text{ ksi}$

$I = 480 \text{ in}^4$

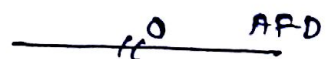
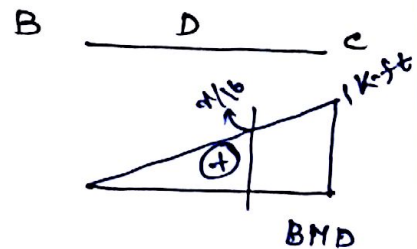
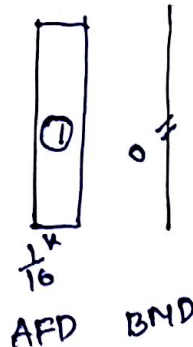
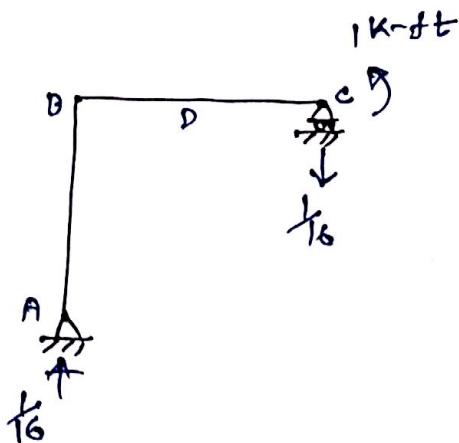
$A = 20 \text{ in}^2$

Soln:

P-force analysis:



Q-force analysis:



Principle of V.W.;

$$\theta_c \alpha_c = \int \frac{M_a M_p}{EI} dx + \sum \frac{F_a F_p L}{AE}$$

$$\Rightarrow 1^k \alpha_c = \int_0^{16} \frac{(22.5x) \cdot \frac{x}{16}}{EI} dx + \int_0^{10} \frac{(1 - \frac{x}{10})(22.5x)}{EI} dx$$

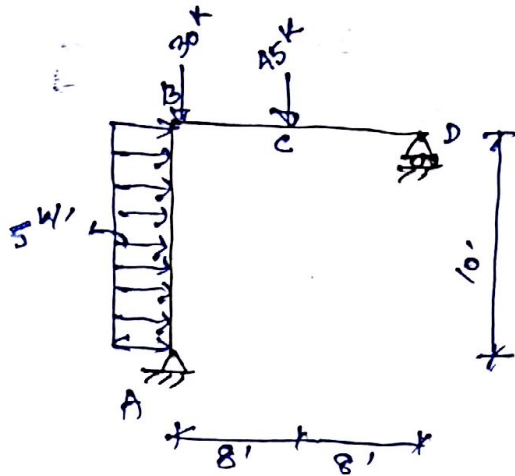
$$+ \int_0^{10} \frac{(\frac{1}{16})(-52.5)}{AE}$$

$$\Rightarrow \alpha_c = 7.2 \times 10^{-3} + 4.375 \times 10^{-5}$$

$$\therefore \alpha_c = 7.2438 \times 10^{-3} \text{ radian (anticlockwise)}$$

— 0 —

Assignment #30:



Compute horizontal deflection at D.

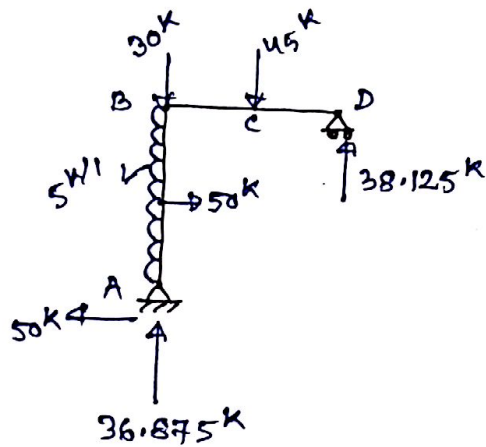
$E = 30000 \text{ ksi}$

$I = 480 \text{ in}^4$

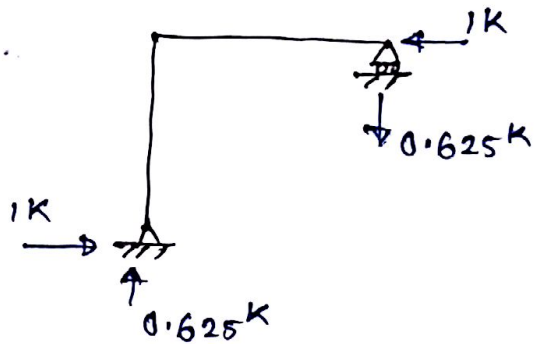
$A = 20 \text{ in}^2$

Soln:

P-force analysis:



Q-force analysis:



AB

$$M_p = 50x - \frac{5x^2}{2}$$

$$M_a = -x$$

$$F_p = -36.875$$

$$F_a = -0.625$$

BC

$$M_p = 6.675x + 250$$

$$M_a = -10 + 0.625x$$

$$F_p = 0$$

$$F_a = -1$$

DC

$$M_p = 38.125x$$

$$M_a = -0.625x$$

$$F_p = 0$$

$$F_a = -1$$

Principle of V.W.;

$$Q. \delta_{DH} = \frac{\sum P_a P_p L}{AE} + \int \frac{M_a M_p}{EI} ds$$

$$\Rightarrow 1k. \delta_{DH} = \frac{-36.875x - 0.625 \times 10}{20 \times 30000} + \int_0^{10} \frac{(50x - \frac{5x^2}{2})(-x)}{EI} dx$$

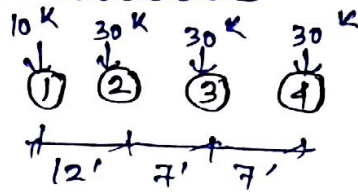
$$+ \int_0^8 \frac{(-10 + 0.625x)(6.875x + 250)}{EI} dx$$

$$+ \int_0^8 \frac{(38.125x)(-0.625x)}{EI} dx$$

$$\therefore \delta_{DH} = -0.1819 \text{ ft } (\rightarrow)$$

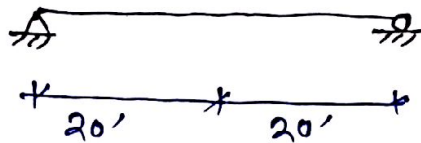
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Assignment #31:



Span = 40'. Find Greatest moment.

Soln:

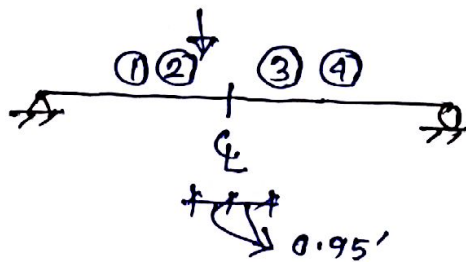


Trial No.	Position of wheel	$\frac{w_1}{L}$	$\frac{w_2}{a}$	Remarks
1	Wheel ③ at center of beam	$\left. \begin{array}{l} \rightarrow \text{just right} \\ \rightarrow \text{just left} \end{array} \right\} \frac{100}{40}$	$\left. \begin{array}{l} > \frac{40}{20} \\ < \frac{70}{20} \end{array} \right\}$	Criterion satisfied

∴ wheel ③ gives max^m moment at center

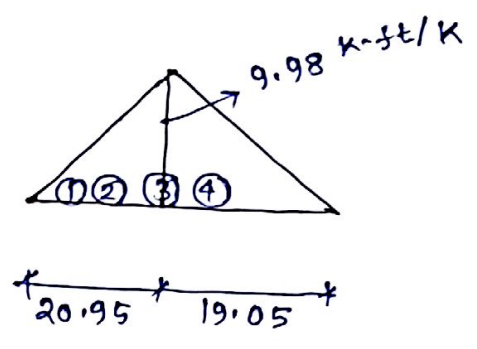
$x = 8.9'$

∴ $a = 1.9'$



∴ Absolute moment may occur at 20.95' from left support.

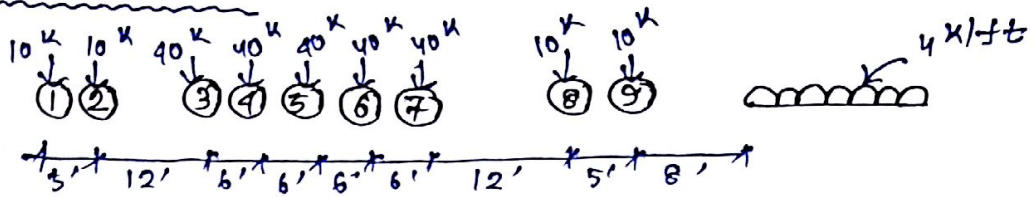
Trial no.	Position of wheel	$\frac{W}{L}$	$\frac{W_1}{a}$	Remarks
1	Wheel ③ at 20.95' from left support	→ just right $\frac{100}{40}$	$>$ $\frac{40}{20.95}$	Criterion satisfied
		→ just left $\frac{100}{40}$	$<$ $\frac{70}{20.95}$	



$$\begin{aligned} \therefore \text{Greatest moment} &= \frac{9.98}{19.05} (19.05 \times 30 + 12.05 \times 30) \\ &+ \frac{9.98}{20.95} (30 \times 13.95 + 1.95 \times 10) \\ &= \boxed{697.44 \text{ k-ft}} \end{aligned}$$

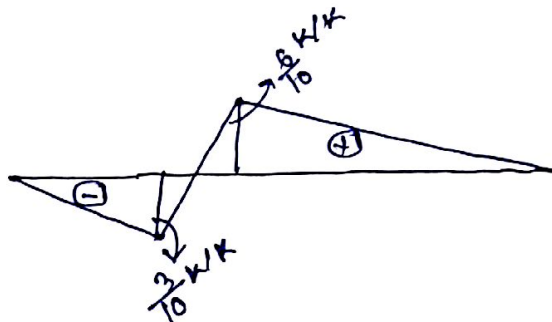
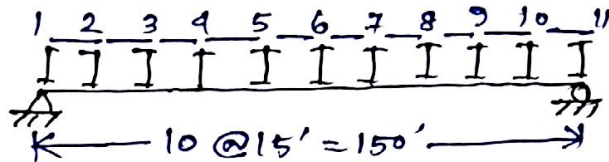
— 0 —

Assignment # 32:



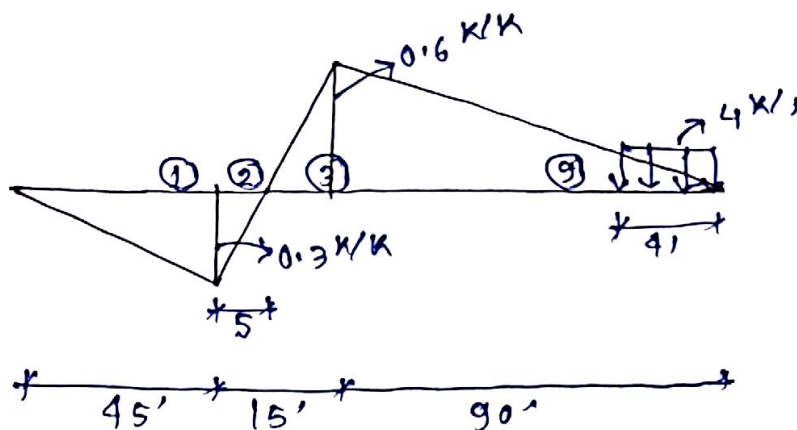
Find max^m shear in 4th panel of 10 panel @ 15'.

Solⁿ:



Trial No.	Position of wheel	$\frac{W}{L}$	$\frac{W}{P}$	Remarks
1	Wheel ② at panel pt. → just right	2.373 >	0.67	Criterion not satisfied
	→ just left	2.373 >	1.33	
2	Wheel ③ at panel pt. → just right	2.7 >	0.67	Criterion satisfied
	→ just left	2.7 <	3.33	

Hence wheel ③ at panel point 5 will give max^m panel shear.



$$\begin{aligned} \therefore \text{Max}^m \text{ panel shear} &= \frac{0.6}{90} \left[\frac{1}{2} \times 4 \times 41 \times 41 + 49 \times 10 + 54 \times 10 \right. \\ &\quad \left. + 66 \times 40 + 72 \times 40 + \right. \\ &\quad \left. 78 \times 40 + 84 \times 40 + 90 \times 40 \right] \\ &\quad - \frac{0.3}{45} (10 \times 43) - \frac{0.3}{15} (2 \times 10) \end{aligned}$$

$$= \boxed{131.03^k}$$

— 0 —



**BANGLADESH UNIVERSITY OF ENGINEERING AND
TECHNOLOGY**

Department of Civil Engineering

ASSIGNMENT

CE 311

Structural Analysis and Design I

➤ **Submitted to: Dr. Md. Zakaria Ahmed**

➤ **Submitted By: Mohammad Irfan Hossain**

➤ **Level: 3**

➤ **Term: 1**

➤ **Sec: B**

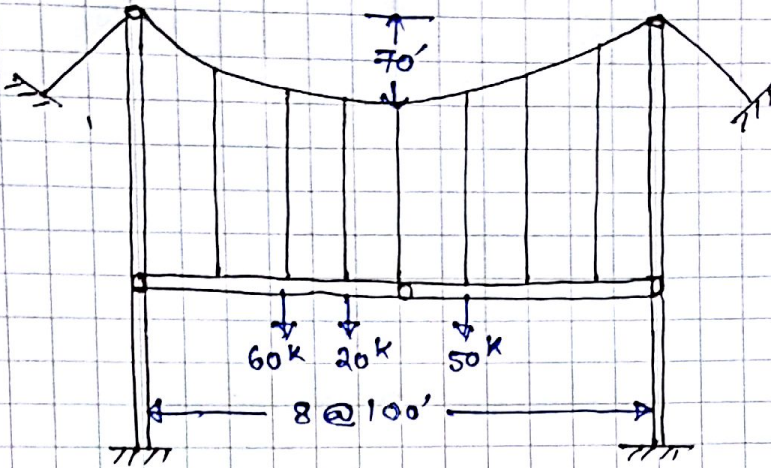
➤ **Roll: 1204092**

Submission Date: 21/12/2015

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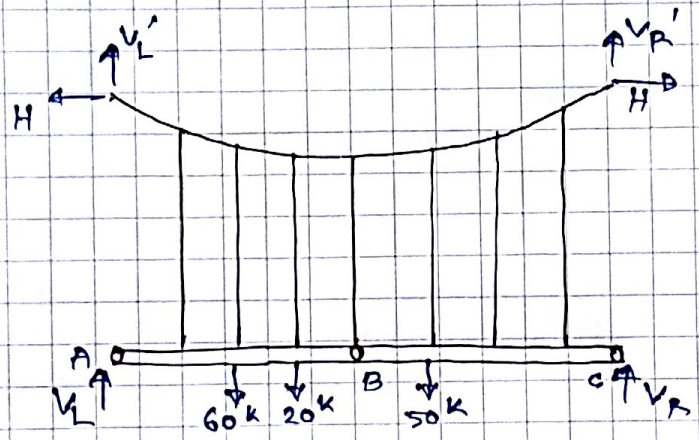
Topic	Subtopic	Assignments	Page
Cable		Assignment number-01	1-3
Virtual Work Method	Frame	Assignment number-02	4-5
Virtual Work Method	Beam	Assignment number-03	6-8
Influence line and application	Locomotive	Assignment number-04	9-13

Assignment # 1:



- ① Find Hanger force
- ② Draw SFD & BMD
- ③ Max^m Tension in Hanger

Solution:



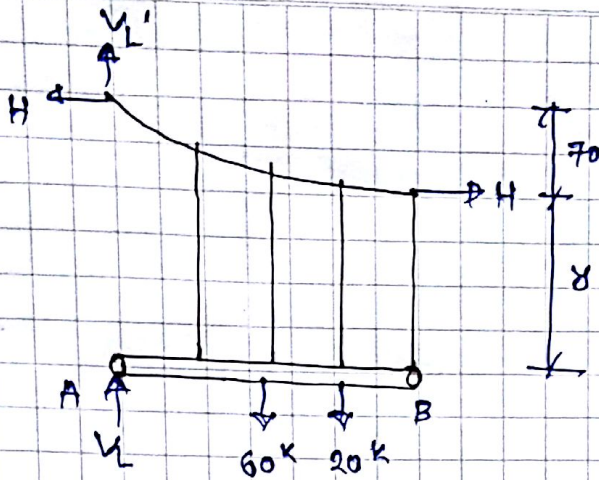
$$\sum M @ C = 0$$

$$\Rightarrow (V_L + V_L') \times 800 - 60 \times 600 - 20 \times 500 - 50 \times 300 = 0$$

$$\therefore V_L + V_L' = 76.25 \text{ k}$$



2



$$\sum M @ B = 0$$

$$\Rightarrow (V_L + V_L') \times 400 - H(y + 70) - 60 \times 200 - 20 \times 100 + Hy = 0$$

$$\therefore H = 235.714 \text{ k}$$

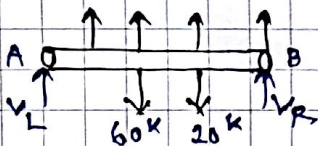
Applying GCT;

$$H = \frac{wL^2}{8h}$$

$$\therefore w = 0.2063 \text{ k/horizontal ft.}$$

$$\text{spacing} = \frac{\text{force}}{w}$$

$$(a) \therefore \text{Hanger force} = w \times \text{spacing} = \boxed{20.625 \text{ k}}$$

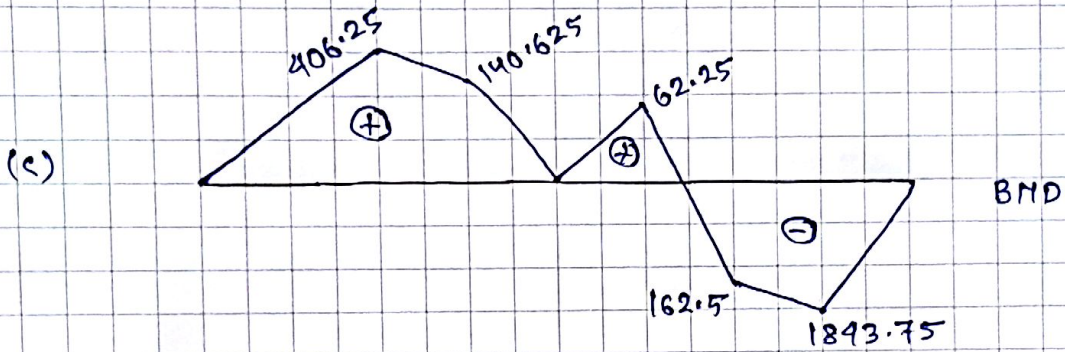
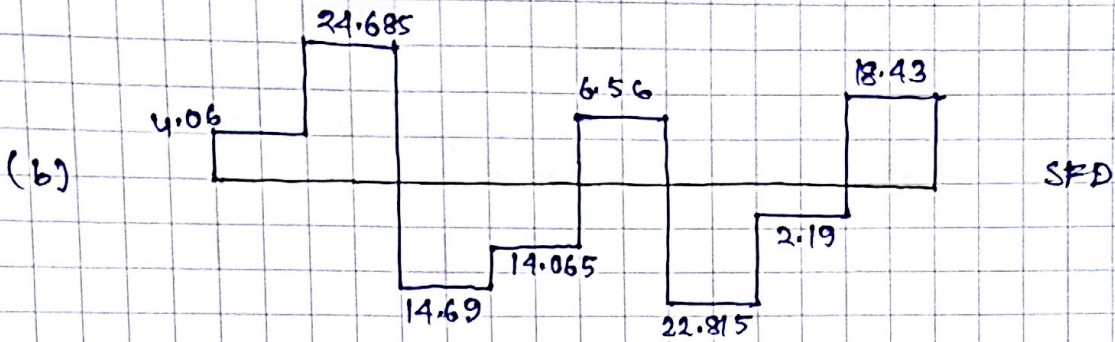
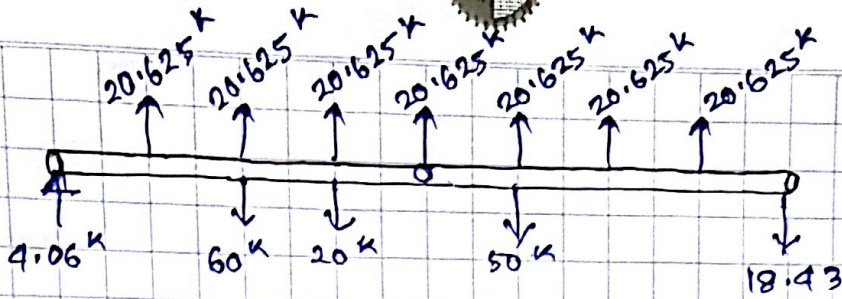


$$\sum M @ B = 0$$

$$\Rightarrow V_L \times 400 + 20.615 (300 + 200 + 100) - 60 \times 200 - 20 \times 100 = 0$$

$$\therefore V_L = 4.06 \text{ k } (\uparrow)$$

$$\therefore V_R = 18.4375 \text{ k } (\downarrow)$$



(d) Here, $\theta = \frac{h}{L} = \frac{70}{800} = 0.0875$

$$T_{max} = H [1 + 16\theta^2]^{1/2}$$

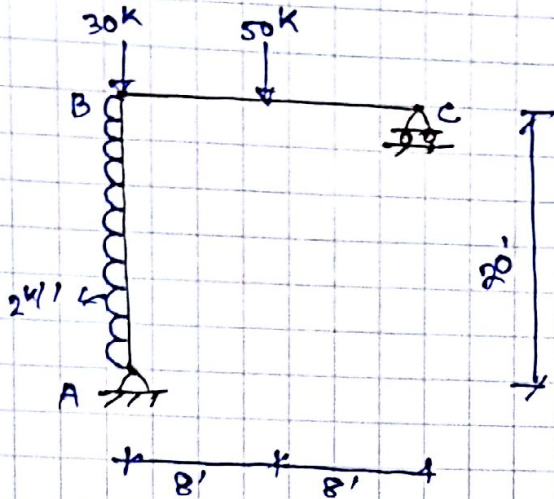
$$= 249.73 \text{ k}$$

— o —



4

Assignment # 2:



Find horizontal deflection at B

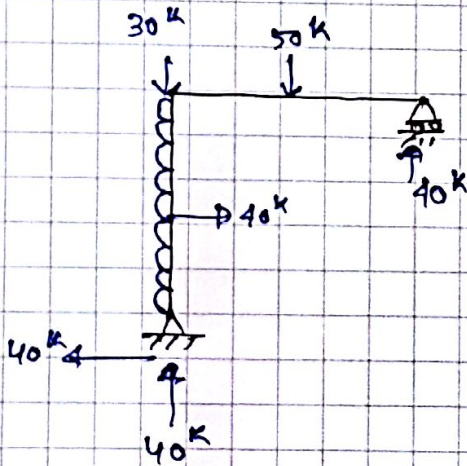
Given, $E = 29000 \text{ ksi}$

$I = 400 \text{ in}^4$

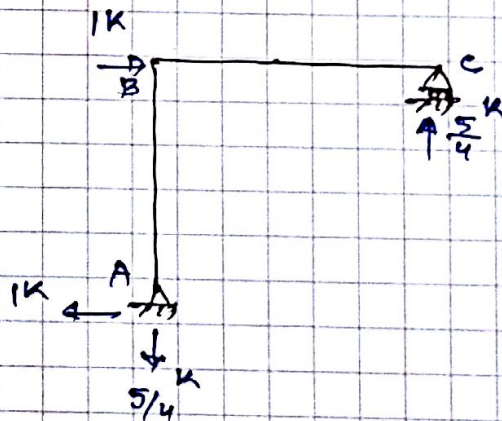
$A = 20 \text{ in}^2$

Solution:

P-force analysis:



Q-force analysis:



AB

$$M_p = 40x - x^2$$

$$M_a = x$$

$$F_p = -40$$

$$F_a = \frac{5}{4}$$

BC

$$M_p = 50(20+x) - 50$$

$$M_a = (20+x) \frac{5}{4}$$

$$F_p = 0$$

$$F_a = 0$$

DC

$$M_p = 50$$

$$M_a = \frac{5}{4}$$

$$F_p = 0$$

$$F_a = 0$$



5

Using principle of virtual work;

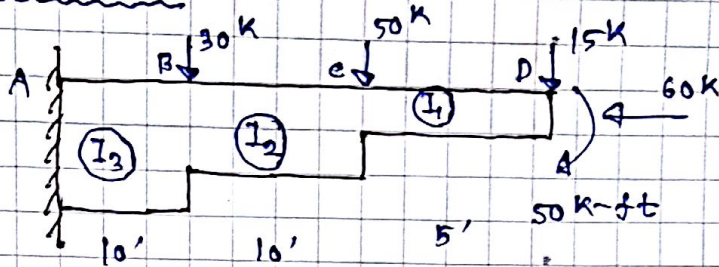
$$Q. \delta_{Bh} = \int \frac{M \delta M_p}{EI} dx + \sum \frac{F \delta F_p L}{AE}$$

$$\Rightarrow 1k, \delta_{Bh} = \frac{125333.33}{29000 \times 12^2 \times \frac{900}{12^4}} + \frac{100}{\frac{20}{12^2} \times 29000}$$

$$\therefore \boxed{\delta_{Bh} = 1.56 \text{ ft } (\rightarrow)}$$



Assignment # 3:



Given,

$$E = 29 \times 10^3 \text{ ksi}$$

$$I_1 = 200 \text{ in}^4, I_2 = 300 \text{ in}^4, I_3 = 400 \text{ in}^4$$

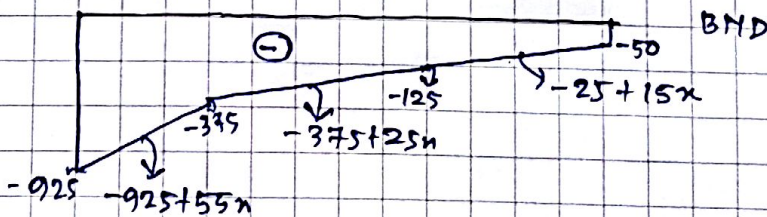
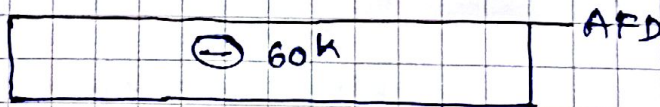
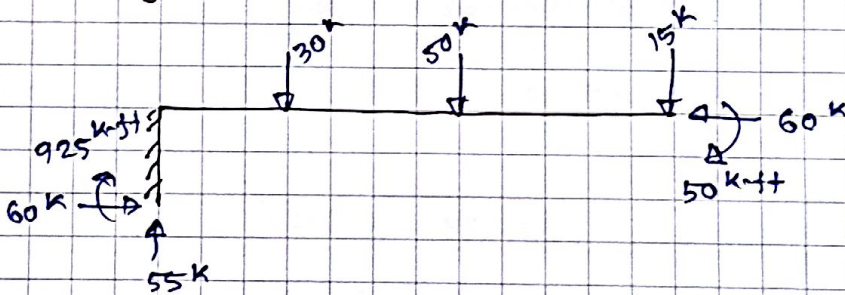
$$A_1 = 10 \text{ in}^2, A_2 = 15 \text{ in}^2, A_3 = 20 \text{ in}^2$$

Find

- (a) Vertical deflection @ c & (b) Rotation at D

Solution:

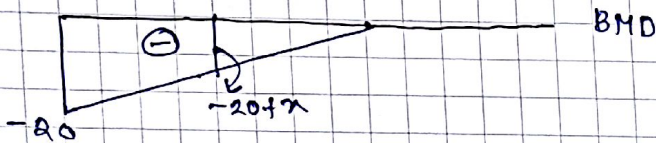
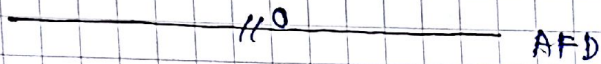
P-force analysis:





7

9) force analysis :



Using principle of V.W.;

$$\delta. \delta_{cv} = \int \frac{M \cdot m}{EI} dx + \frac{\sum P \cdot \delta}{AE}$$

$$\Rightarrow 1k \cdot \delta_{cv} = \int_A^B \frac{(-20+x)(-925+55x)}{EI} dx + \int_B^C \frac{(-10+x)(-375+25x)}{EI} dx$$

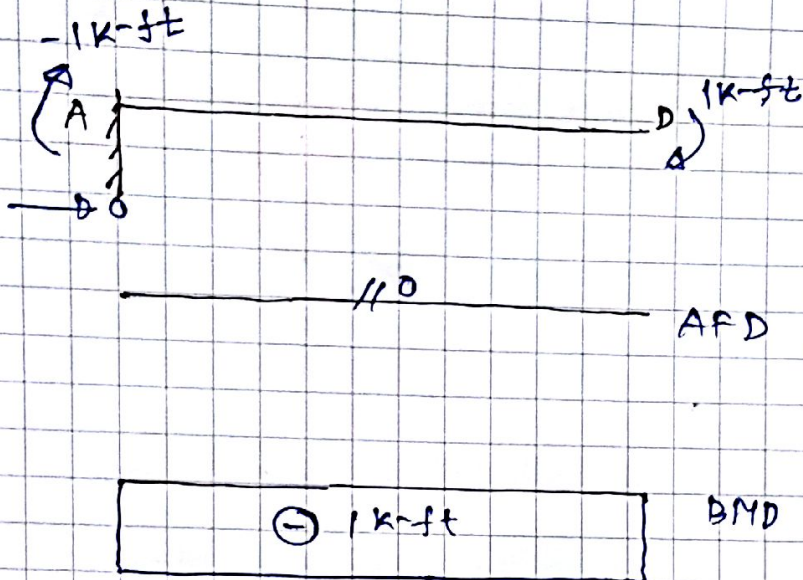
$$\therefore \delta_{cv} = 1.15 \text{ ft (downward)}$$



8

b

Q-force analysis:



Using principle of V.W:

$$\theta_B = \int \frac{M \cdot m}{EI} dx + \frac{\sum F \cdot F_p \cdot L}{AE}$$

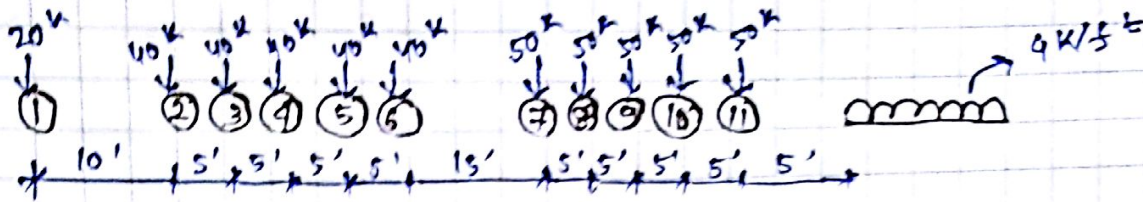
$$\Rightarrow 1k \cdot \theta_B = \int_A^B \frac{(-925 + 55x)(-1)}{4EI} dx + \int_B^C \frac{(-1)(-375 + 25x)}{1.5EI} dx + \int_C^D \frac{(-1)(-125 + 15x)}{EI} dx$$

$$\therefore \theta_B = 17.9 \text{ radian (clockwise)}$$

_____ 0 _____



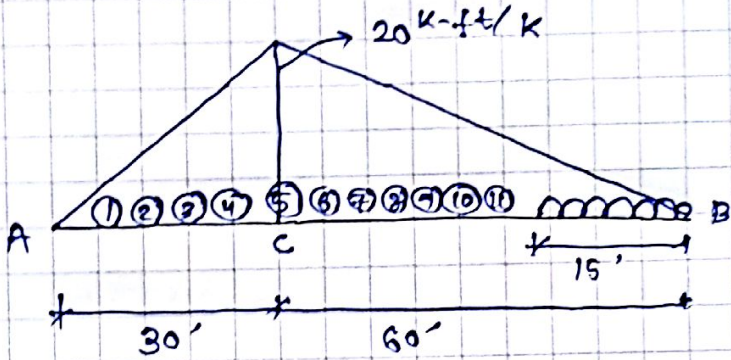
Assignment # 4:



- Find (a) Max^m moment at $\frac{1}{3}$ of 90'
- (b) Max^m shear at quarter point of 100' span
- (c) Max^m reaction for 90' span

Solution:

(a)



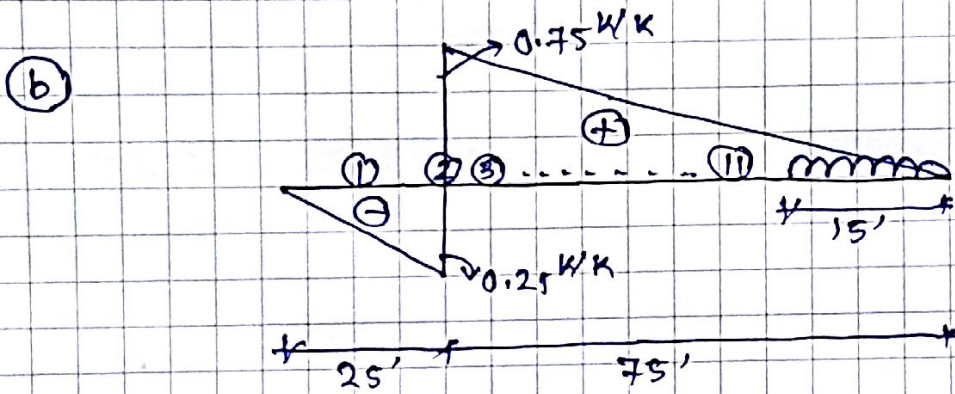
Trial No.	Position of Wheel	$\frac{W}{L}$	$\frac{W_1}{\alpha}$	Remarks
1	Wheel (4) at c → just right ↳ just left	$\frac{510}{90} >$ $\frac{510}{90} >$	$\frac{100}{30}$ $\frac{140}{30}$	Criterion not satisfied
2	Wheel (7) at c → just right ↳ just left	$\frac{530}{90} >$ $\frac{530}{90} <$	$\frac{140}{30}$ $\frac{180}{30}$	Criterion satisfied



(10)

hence wheel ⑤ at C gives max^m moment.

$$\begin{aligned} \therefore M_{\max} &= \frac{20}{60} [40(50+55) + 50(40+35+30+25+20)] + \frac{20}{60} \times 15 \times 0.5 \times 15 \times 4 \\ &\quad + \frac{20}{30} \times [20 \times 5 + 40(15+20+25)] \\ &= 5735.5 \text{ k-ft} \end{aligned}$$



Trial 1: wheel ② at C

Here, $\Sigma p = \text{① to ⑪} + 5' \text{ of UDL} = 490 \text{ k}$

$$d_1 = 10'$$

$$P_1 = \text{①} = 20 \text{ k}$$

$$p' = 10' \text{ of UDL} = 40 \text{ k}$$

$$e = 5'$$

$$P_2 = 0$$

$$\therefore \Delta V = \frac{\Sigma p d_i}{L} = P_1 + \frac{p' e}{L} + \frac{P_2 e'}{L} = +31 \text{ k (increasing)}$$



(11)

Trial 2: wheel (3) at C

$$\text{Here, } \Sigma P = 530 \text{ k}$$

$$d_1 = 5'$$

$$P_1 = 60 \text{ k}$$

$$P' = 20 \text{ k}$$

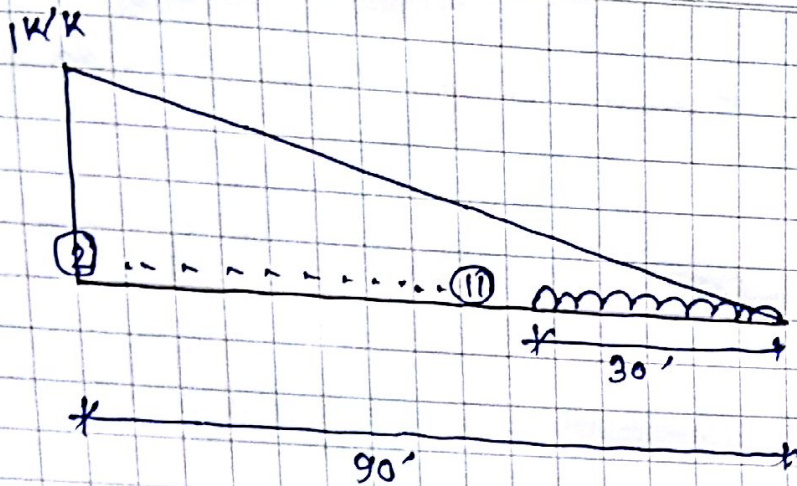
$$e = 2.5'$$

$$\therefore \Delta V = -33 \text{ k (decreasing)}$$

\therefore wheel (2) at C gives max^m shear.

$$\therefore V_{\max} = \frac{0.75}{75} [40(75+70+65+60+55) + 50 \times (40+35+30+25+20) + 7.5 \times 4] - \frac{0.25}{25} (20 \times 15)$$

$$= 202.3 \text{ k}$$



Trial 1: Wheel (2) at c

Here, $\Sigma P = \text{(2) to (1)} + 20' \text{ UDL} = 530 \text{ k}$

$$d_1 = 10'$$

$$P_1 = \text{(1)} = 20 \text{ k}$$

$$e = 5'$$

$$P' = 10' \text{ UDL} = 40 \text{ k}$$

$$\begin{aligned} \therefore \Delta R &= \frac{\Sigma P d_1}{L} - P_1 + \frac{P' e}{L} \\ &= +41.02 \text{ (increasing)} \end{aligned}$$

Trial 2: Wheel (3) at c

Here, $\Sigma P = 410 + 30' \text{ UDL} = 530 \text{ k}$

$$d_1 = 5'$$

$$P_1 = \text{(2)} = 40 \text{ k}$$

$$P' = 5' \text{ UDL} = 20 \text{ k}$$

$$e = 2.5'$$

$$\therefore \Delta R = -10 \text{ k (decreasing)}$$



Hence wheel ② at C gives max^m reaction

$$\therefore \text{Max}^m R_A = \frac{1}{90} [40 \times (90 + 85 + 80 + 75 + 70) + 50(55 + 50 + 45 + 40 + 35) + 4 \times 15]$$

$$= \boxed{303.44 \text{ W/K}}$$

————— 0 —————