

Deflection, Rotation for Beams, Frames

15/09/2015

By Virtual Work
→ (If support movement)

$$\sum Q\delta + W_R = \int \frac{FQ\delta L}{AE} + \int F\alpha\delta tL + \int \frac{M\delta MP}{EI} ds$$

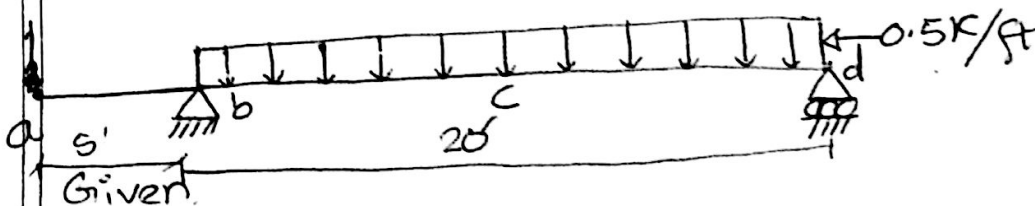
↓
for temp. change

For Beam with No axial force & Temp. change,
support movement

$$\sum Q\delta = \int \frac{M\delta MP}{EI} ds$$

Example 8.1, 8.2, 8.3, 8.4

Example 8.5,



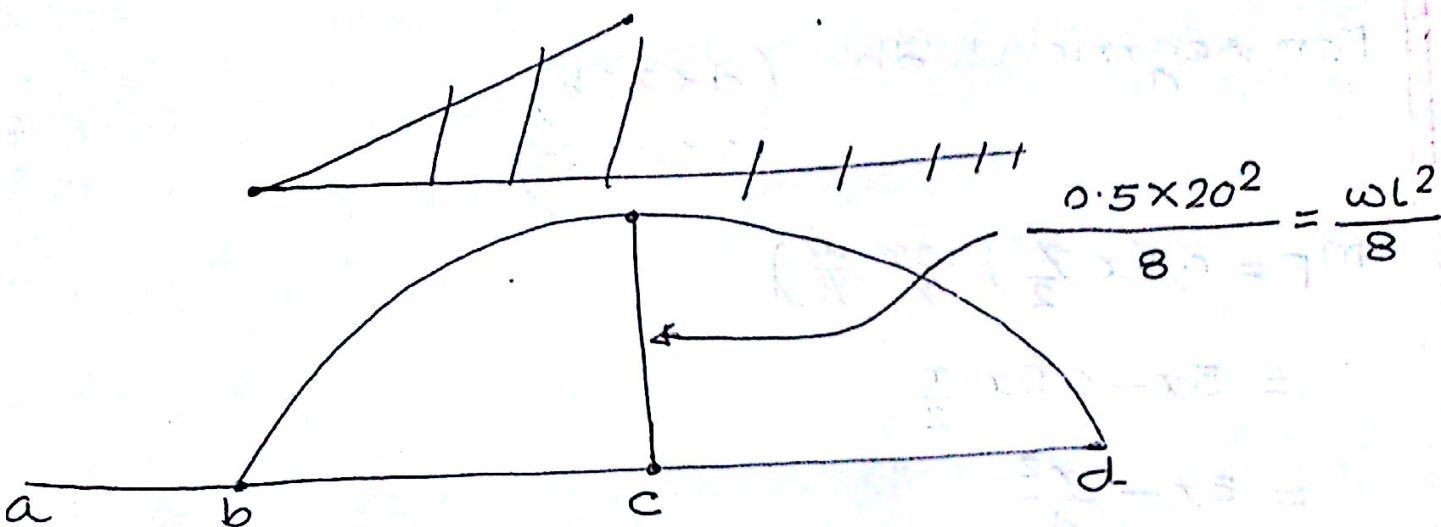
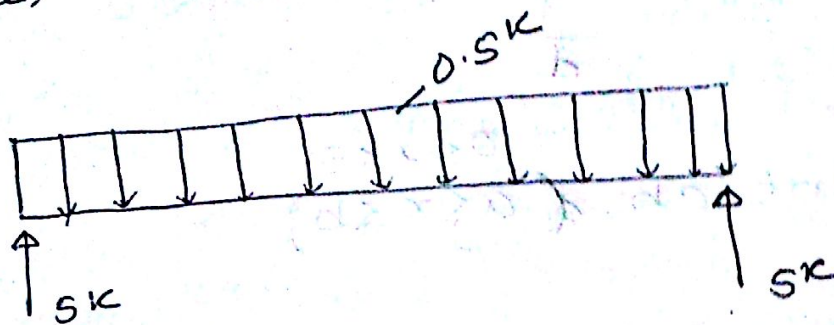
$$E = 30 \times 10^3 \text{ ksi}$$

$$I = 200 \text{ inch}^4$$

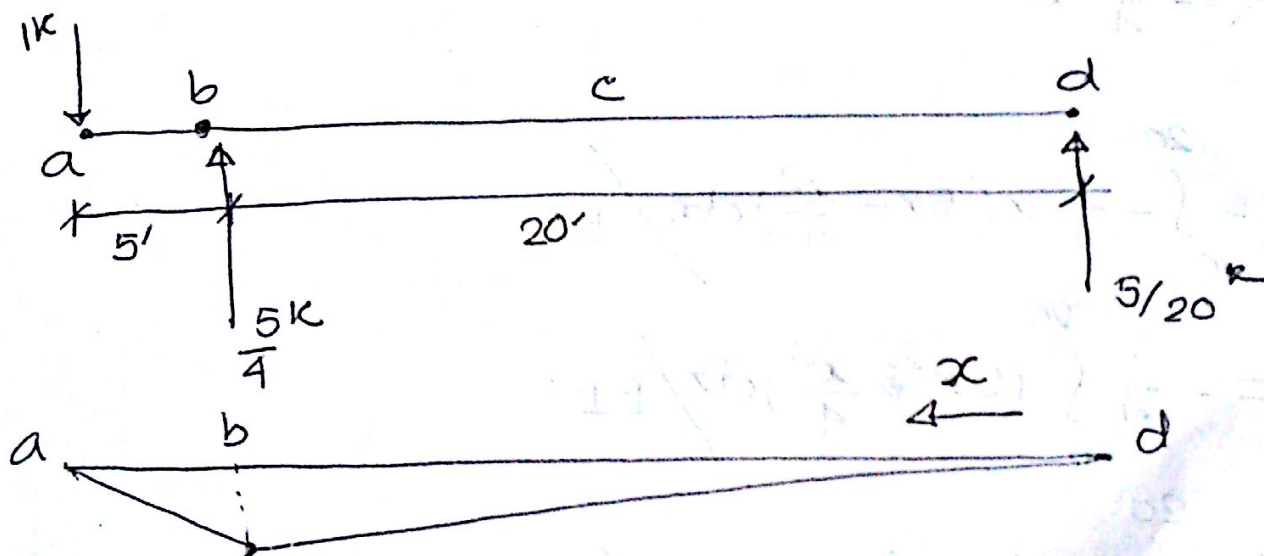
compute vertical deflection at point 'a'.

Solution:

For 'P' force,



For 'Q' force,



By v.v.w [∴ mp = 0]

$$\Delta Q\delta = \int_a^b \frac{M \& mp}{EI} dx + \int_d^b \frac{M \& mp}{EI} dx$$

For segment 'ab' $(0 \leq x \leq 5'$
 $(a \leq x \leq b)$

$$mp = 0$$

$$m\& = 1x$$

For segment 'db' $(d < x \leq b)$
 $(0 < x \leq 20)$

$$mp = 0.5x \cdot \frac{x}{2} \left[\cancel{wx} \cdot \frac{x}{2} \right]$$

$$= 5x - 0.5x \cdot \frac{x}{2}$$

$$= 5x - \frac{x^2}{4}$$

$$MQ = -\frac{1}{4}x^2$$

$$\Delta Q\delta = \int_0^{20} -\frac{1}{4}x \left(5x - \frac{x^2}{4} \right) dx / EI$$

$$= -\frac{1}{4} \int_0^{20} \left(5x^2 - \frac{x^3}{4} \right) dx / EI$$

$$= \int_0^{20} \left(\frac{x^3}{16} - \frac{5}{4}x^2 \right) dx / EI$$

$$1.8 = \frac{1}{EI} \int_0^{20} \left(\frac{x^3}{16} - \frac{5}{4}x^2 \right) dx$$

$$= -833.3/EI$$

$$= -\frac{833.3}{30000 \times \frac{200}{144}}$$

$$= -0.02 \text{ ft (upward)}$$

$$E = 30000 \text{ ksi } (12^2)$$

$$= 30000 \times 144 \text{ ksf}$$

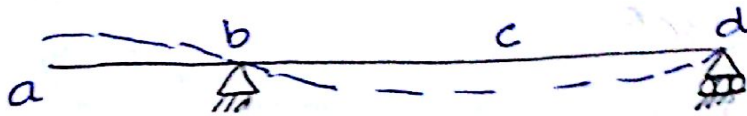
$$I = 200 \text{ inch}^4$$

$$= \frac{200}{(144 \times 144)} \text{ ft}^4$$

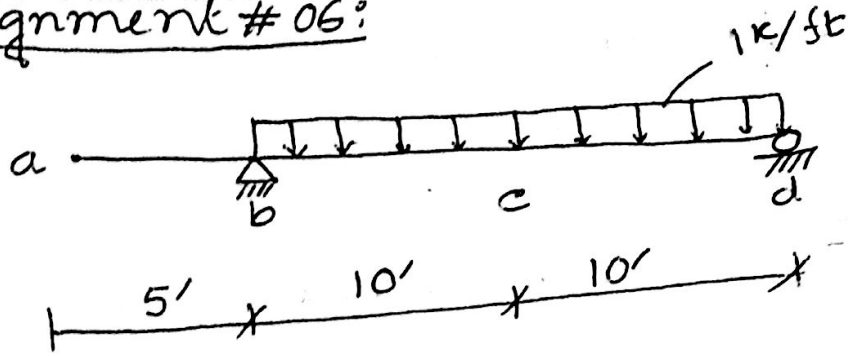
↳ (12)⁴

↳ (exam a upward/downward must

के निशान रख)



Assignment # 06:



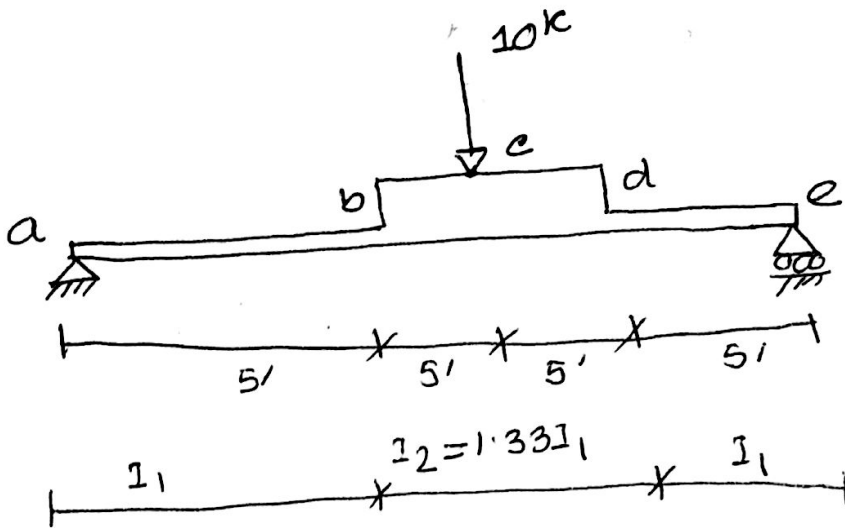
$E = 30,000 \text{ ksi}$

$I = 300 \text{ in}^4$

Compute:

- (a) Vertical deflection at 'c' [b to c & d to c]
- (b) Rotation or slope at point "a"

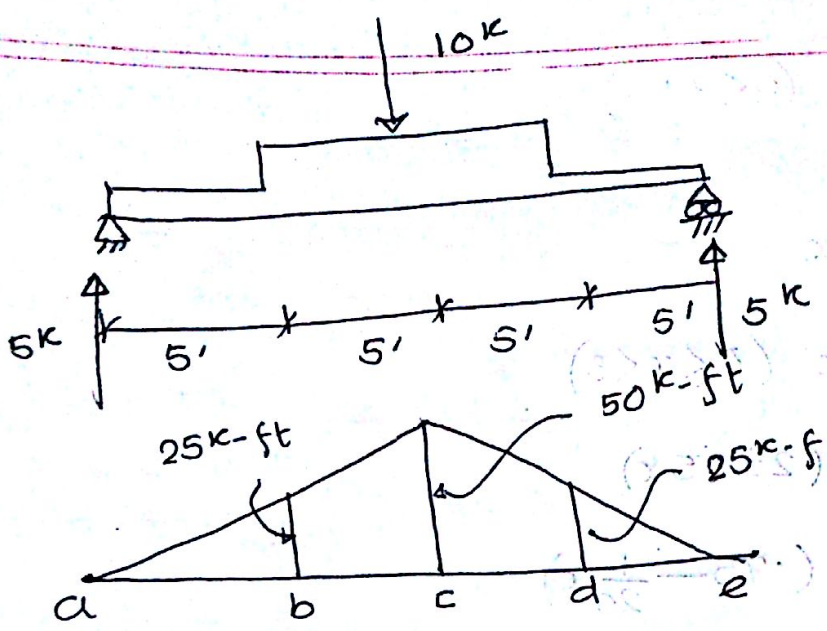
See, Example 8.6



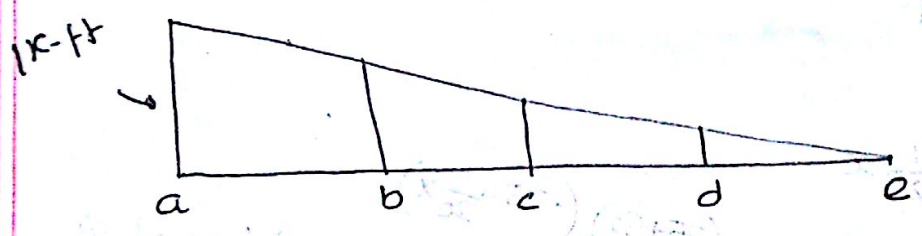
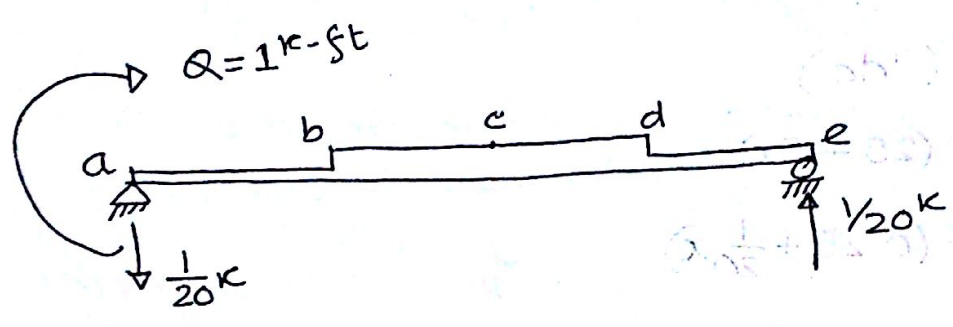
Given, $E = 30,000 \text{ ksi}$

$I_1 = 150 \text{ in}^4$

16/03/2016



Mp diagram



Mv diagram

By virtual work Method:

$\sum Q \theta_a$ change in slope

$$= \int_a^b \frac{d\theta}{EI_1} dx + \int_b^c \frac{d\theta}{EI_2} dx + \int_c^d \frac{d\theta}{EI_2} dx + \int_d^e \frac{d\theta}{EI_1} dx$$

segment 'ab' ($0 < x \leq 5$)

$$M_p = +5x$$

$$M_Q = 1 - \frac{1}{20}x$$

segment 'bc' ($0 < x \leq 5$)

$$M_p = + (25 + 5x)$$

$$M_Q = + \left(0.75 - \frac{1}{20}x\right)$$

segment 'dc'

$$M_p = + (25 + 5x)$$

$$M_Q = + \left(0.25 + \frac{1}{20}x\right)$$

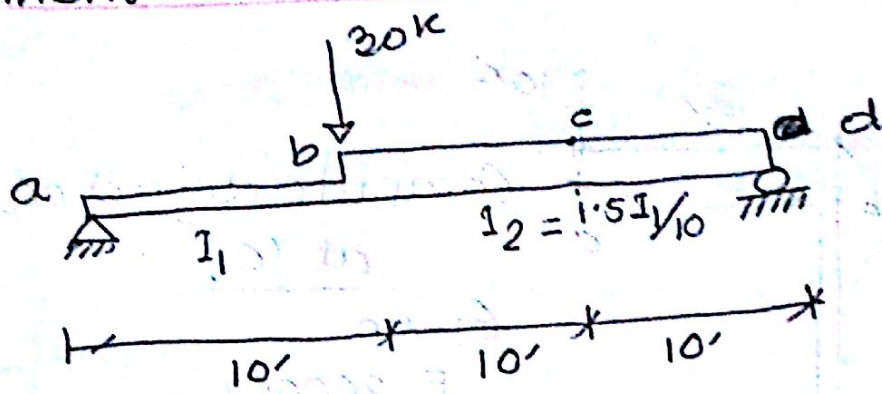
segment 'ed'

$$M_p = +5x$$

$$M_Q = + \frac{1}{20}x$$

$$\begin{aligned} & \int_0^5 \frac{(5x)\left(1 - \frac{x}{20}\right)}{EI_1} dx + \int_0^5 \frac{(25+5x)\left(0.75 - \frac{1}{20}x\right)}{EI_2} dx + \int_0^5 \frac{(25+5x)\left(0.25 + \frac{1}{20}x\right)}{EI_2} dx \\ & + \int_0^5 \frac{(5x)\left(\frac{1}{20}\right)x}{EI_1} dx \\ & = \frac{52.08}{EI_1} + \frac{114.58}{EI_2} + \frac{79.920}{EI_2} + \frac{10.4167}{EI_1} \end{aligned}$$

Assignment 07:

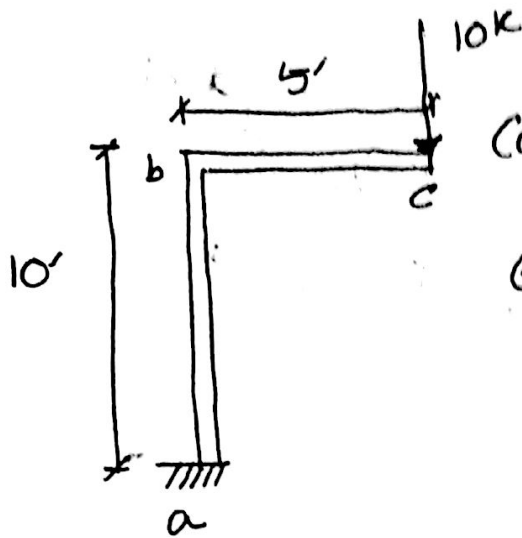


$$E = 30,000 \text{ k/in}^2$$

$$I_1 = 400 \text{ inch}^4$$

- Compute
- i) vertical deflection at 'c'
 - ii) vertical deflection at 'b'
 - iii) change in slope at point 'd'

FRAME



Compute deflection
at 'c'.

Given,

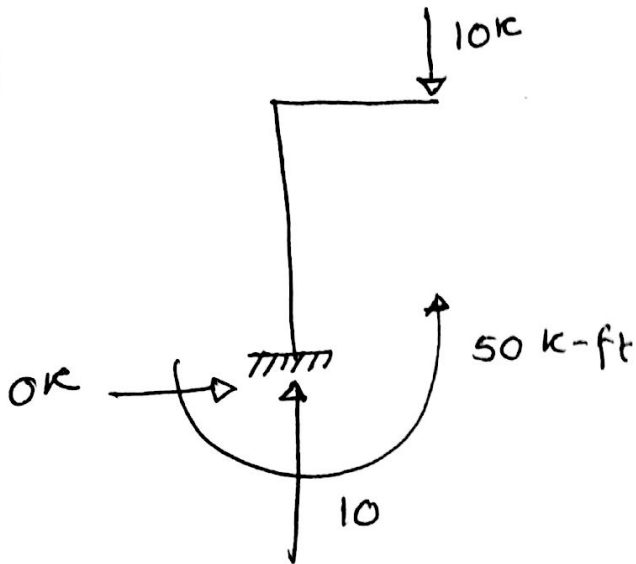
$$E = 20000$$

$$A = 10 \text{ inch}^2$$

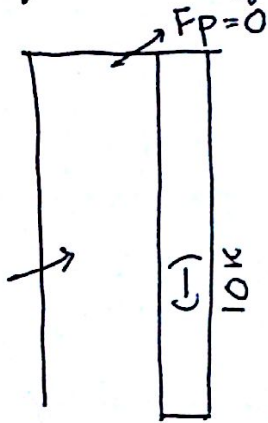
$$I = 200 \text{ inch}^4$$

Frame \rightarrow Axial force, Moment & δ \rightarrow δ

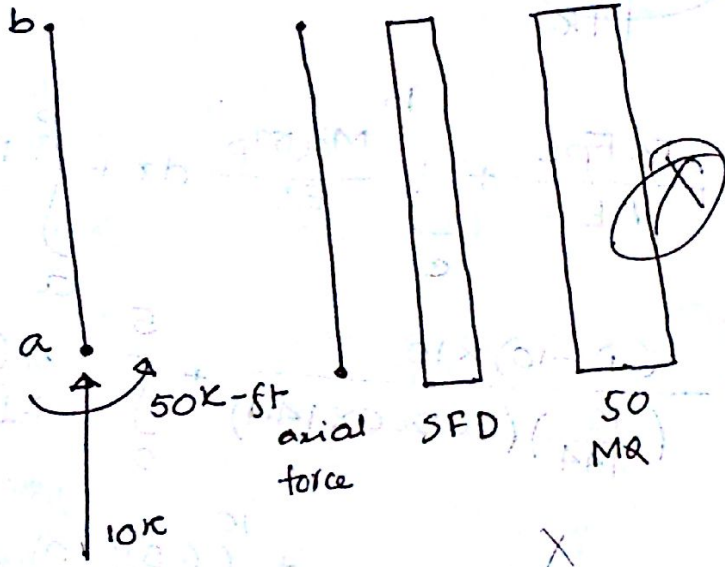
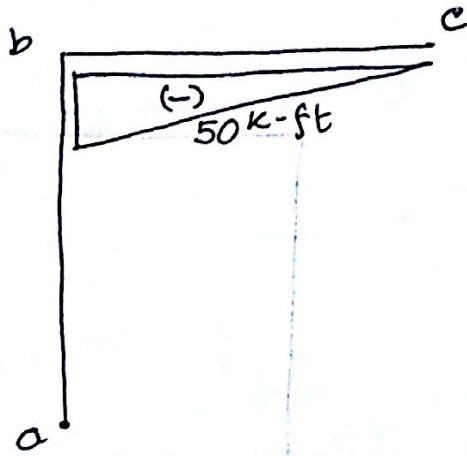
Solⁿ for 'P' force:



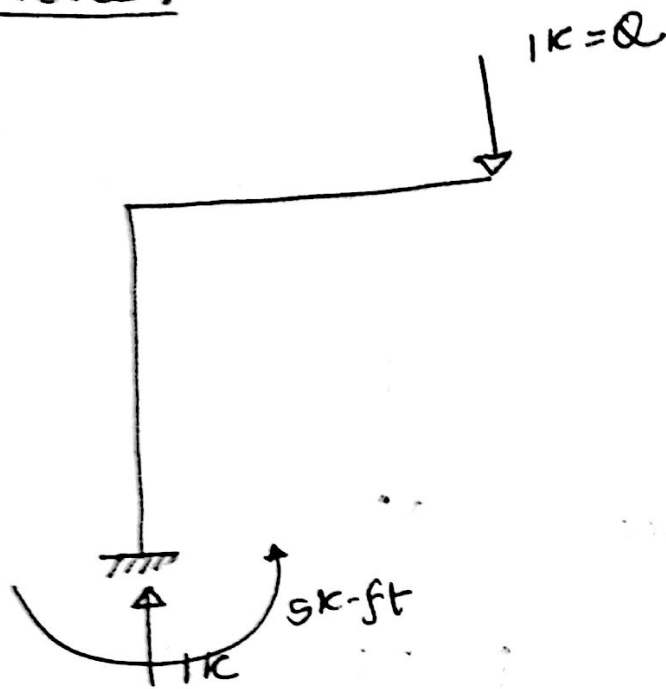
Axial force diagram



Axial force



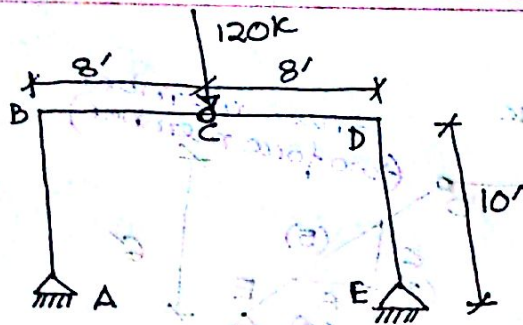
For Q Force :



$$\begin{aligned} \Delta_Q \delta_c &= \frac{\sum F_Q F_{PL}}{AE} + \int_c^b \frac{M_Q M_P}{EI} dx + \int_a^b \frac{M_Q M_P}{EI} dx \\ &= \frac{(-1)(-10) \times 10}{\left(\frac{10}{144}\right)(30,000 \times 144)} + \int_0^5 \frac{(x)(-10x)}{EI} dx \\ &\quad + \int_0^{10} \frac{(-5)(-50)}{EI} dx \end{aligned}$$

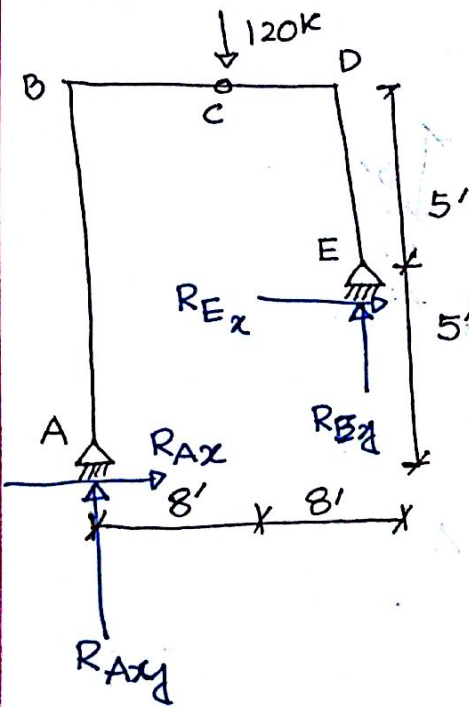
22/03/2015

Ex 86



Given, $E = 30,000 \text{ ksi}$
 $A = 20 \text{ inch}^2$
 $I = 2500 \text{ inch}^4$

Compute the change in slope of cross-section on the left side of the hinge 'c'.

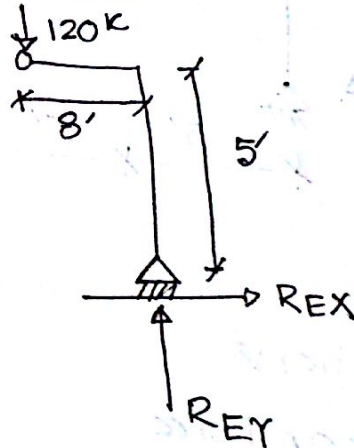


whole structure

$$\sum M @ A = 0 = 120 \times 8 + R_{EX} \times 5 - R_{EY} \times 16$$

$$5 R_{EX} - 16 R_{EY} = -960 \quad (1)$$

Right bar



$$\sum M @ C = 0$$

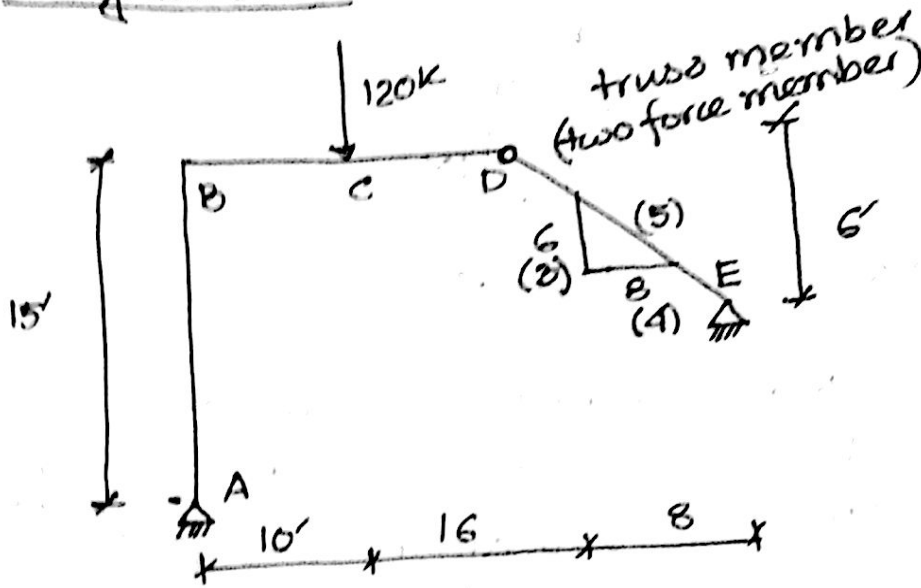
$$= -R_{EY} \times 8 - R_{EX} \times 5$$

$$\text{or, } 8 R_{EY} + 5 R_{EX} = 0$$

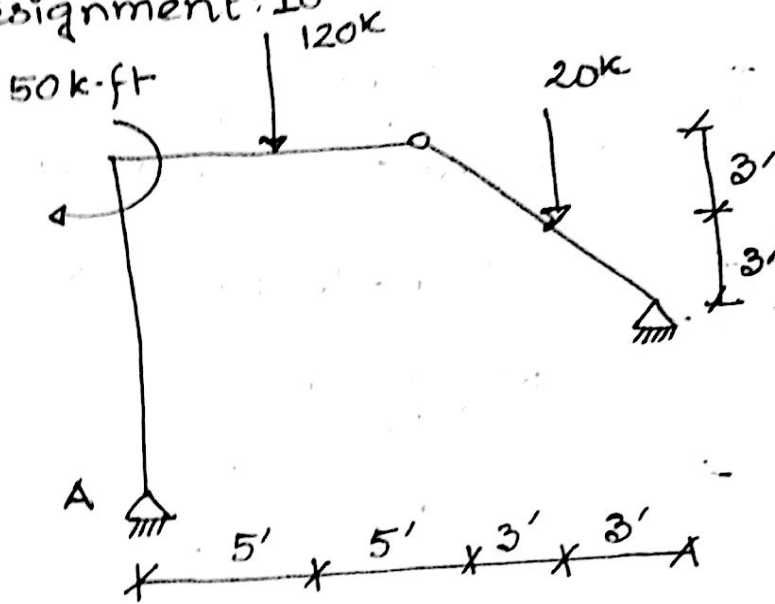
$$R_{EY} = 40 \text{ k} (\uparrow)$$

$$R_{EX} = -64 \text{ k} (\leftarrow)$$

Assignment 09



Assignment: 10



Find i

- 1) Reactions
- 2) Shear Force
- 3) Bending moment

