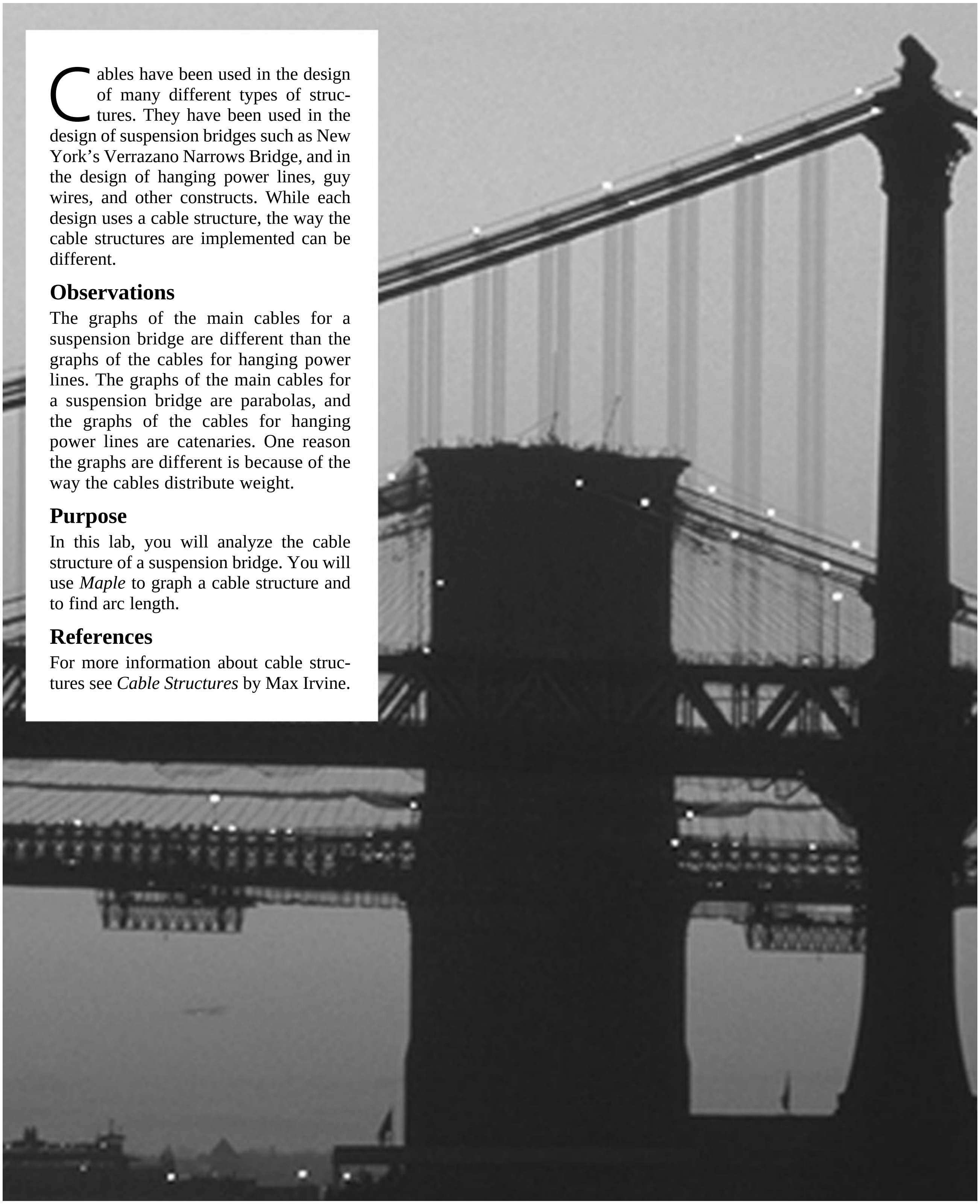
LAB 14

SUSPENSION BRIDGES

Parabolas



Cables have been used in the design of many different types of structures. They have been used in the design of suspension bridges such as New York's Verrazano Narrows Bridge, and in the design of hanging power lines, guy wires, and other constructs. While each design uses a cable structure, the way the cable structures are implemented can be different.

Observations

The graphs of the main cables for a suspension bridge are different than the graphs of the cables for hanging power lines. The graphs of the main cables for a suspension bridge are parabolas, and the graphs of the cables for hanging power lines are catenaries. One reason the graphs are different is because of the way the cables distribute weight.

Purpose

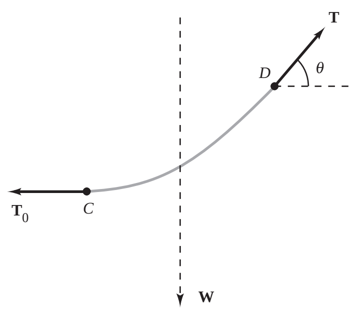
In this lab, you will analyze the cable structure of a suspension bridge. You will use *Maple* to graph a cable structure and to find arc length.

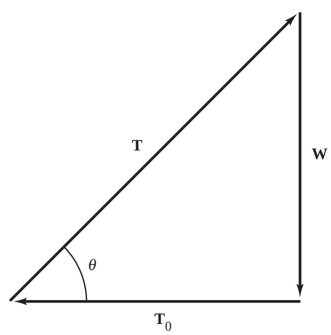
References

For more information about cable structures see *Cable Structures* by Max Irvine.

Data

Suspension bridges are stable when the horizontal and vertical forces acting on its main cables are in equilibrium. This equilibrium occurs when the main cables are in the shape of parabolas.





The forces acting on the main cable are shown in the diagram above as directed line segments, indicating both the magnitude and the direction of the forces. At the center of the cable, the tension force $\mathbf{T}_0$ is horizontal. **T** is the tension at point *D*, and is directed along the tangent at point *D*. The uniformly distributed load supported by the section *CD* of the cable is represented by **W**.

Exercises

Name ______________________________

Date ____________________ Class ____________

Instructor ______________________________

1. ***Finding a Parabolic Equation.*** When the Verrazano Narrows Bridge opened in 1964, it was the world's largest suspension bridge. The bridge's main cable is suspended from towers 693 feet above the roadway at either end of a 4260-foot span. The low point in the center of the cable is 303 feet above the roadway. Given that the cable hangs in the shape of a parabola, find a rectangular equation of the form

$$(x - h)^2 = 4p(y - k)$$

describing the shape of the cable. To determine the equation, where did you place the origin of the coordinate axes?

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2. ***Another Parabolic Equation.*** The equation that describes the shape of a suspension bridge's cable is given by

$$y = \frac{wx^2}{2\|\mathbf{T}_0\|}$$

where w is the load per unit length measured horizontally and $\|\mathbf{T}_0\|$ is the minimum tension. For a suspension bridge such as the Verrazano Narrows Bridge, the equation describes a parabola centered at the origin with the origin occurring at the lowest point of the cable midway between the two main supports. Use this equation to describe the shape of the main cable if the cable supports a load of $w = 10{,}800$ pounds per foot and the cable's sag (the vertical distance from the supports to the cable's lowest point) is 390 feet.

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3. ***Graph the Equations.*** Use *Maple* to graph the parabolic equations you found in Exercises 1 and 2 on the same set of axes. (If necessary, rewrite the equation found in Exercise 1 so that the origin is the same as the one used in Exercise 2.) Are the graphs the same? Why or why not?

4. ***Arc Length.*** Determine the length of Verrazano Narrows Bridge's main cable by evaluating the following integral by hand, where dy/dx is the derivative of either equation found in Exercises 1 and 2.

$$s = 2\int_0^{2130} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Evaluate the integral using *Maple* and compare the result to the one you obtained by hand. Compared to the span of the bridge and the lowest point of the cable, does the length of the cable seem reasonable? Why or why not? What is the length of the cable from the cable's lowest point to either support?

5. ***Arc Length at Extreme Temperatures.*** The extreme temperatures of summer and winter can effect the sag of the main cable. The Verrazano Narrows Bridge was designed with extreme temperatures in mind, and the sag height can vary from 386 feet to 394 feet. Use the equation from Exercise 2 to describe the shape of the cable at each height. Then use *Maple* to find the length of the cable for the minimum and maximum sag heights. During what time of year would you expect the sag height to be 386 feet? During what time of year would you expect the sag height to be 394 feet? Explain your reasoning.

6. ***Magnitude of Tension.*** The tension at a point (x, y) along the main cable is given by

$$\|\mathbf{T}\| = \sqrt{\|\mathbf{T}_0\| + wx^2}.$$

Substitute the values for $\|\mathbf{T}_0\|$ and w from Exercise 2. Determine the maximum and minimum magnitudes of **T** with respect to x. Where on the Verrazano Narrows Bridge does the maximum tension occur? Where on the bridge does the minimum tension occur? Determine the maximum and minimum magnitudes of **T** with respect to x when the sag height is 386 feet and 394 feet.

7. ***Tension and Tangent Lines.*** Find the tangents to the cable at the maximum and minimum values found in Exercise 6. Graph the tangents and the tension **T** at these values on the same graph. What do you notice about the tangent and the tension **T** at each value? Can you use the tangent line to determine the direction of **T**? If so, determine the direction of **T** at the maximum and minimum values with respect to the horizontal.

8. ***Direction of Tension.*** The direction of the tension **T** at *D* is given by

$$\tan\theta = \frac{wx}{\|\mathbf{T}_0\|}.$$

Use this formula to verify the directions you found in Exercise 7 for **T**.

9. ***Supports at Different Elevations.*** Because the suspension towers for the main cable of the Verrazano Narrows Bridge are the same height, the cable's lowest point is midway between the towers. When the heights of the supports for a cable structure are different, the position of the cable's lowest point is *not* midway between the supports. Determine the following for a cable that hangs between a 75-foot high tower and a 125-foot high tower, and supports a uniformly distributed load of 800 pounds per foot. Assume the towers are 1000 feet apart and the lowest point of the cable just touches the roadway.

a. Determine an equation that describes the shape of the cable.

b. Where is the lowest point of the cable? Where is the highest point?

c. How long is the cable? How long is the cable from the lowest point to the highest point?