এটা কিন্তু সাজিদের লেখা না

৭৭/= CE 417

Q.1. Explain the concept of FEM briefly and outline the procedure.

Ans: $\boxed{\text{FEM}}$ The finite element analysis is a numerical technique. In this method all complexities of the problem like varying shape, boundary conditions and loads are maintained as they are but the solutions obtained are approximate. In engineering problem there are some basic unknowns called Field variables. In a continuum, these unknowns are infinite. The finite element procedure reduces such unknowns to a finite number by dividing the solution area into small parts called elements and by expressing the unknown field variables in terms of assumed approximate functions called shape function. The approximate functions are defined in terms of field variables of ~~the~~ specified points called nodes. Once the nodal field variables are found, the field variables at any point can be found by using interpolation/shape function.

Outlines of FEM procedure:

① Select suitable field variables and the elements.

② Discritise the continua.

③ Select interpolation function.

④ Find the element properties.

⑤ Assemble element properties to get global properties.

⑥ Impose the boundary conditions.

⑦ Solve the system equations to get nodal unknowns.

⑧ Make the additional calculations to get the required ~~fed~~ values.

এই চোথাগুলি ফেলে দিবেন না
কারন এই চোথাগুলো M.S.C ও Job এর
সময় লাগে এবং সারা জীবন লাগে।

~~QQQ~~.

** 2. State the advantages and disadvantages of FEM over other classical methods.

Advantages of FEM

① Solutions for a few standard cases ~~are~~ have been solved by classical method, whereas solutions can be obtained for all problems by finite element analysis.

(ii) When material property is not isotropic, solution for the problems become very difficult in classical method but FEM can handle structures with anisotropic properties without any difficulty.

(iii) If structure consists of more than one material, it is difficult to use classical method but finite element can be used without any difficulty.

(iv) Problems with material and geometric non-linearities can not be handled by classical methods but there is no difficulty in FEM.

Disadvantages of FEM

1. In classical methods, exact equations are formed and exact solutions are obtained but in FEM, exact equations are formed but solutions are approximate.
2. For all regular problems, solutions by classical methods are the best solutions.

* 3. When there are several FEM packages available is there any need to study FEM? Discuss.

~~To use~~ The finite element knowledge makes a good engineer better while just user without the knowledge of FEM may produce more dangerous results. To use FEA package properly, the user must know the following points clearly —

1. Which elements are to be used for solving the problem in hand.
2. How to discritise to get good results.
3. How to introduce boundary conditions properly.
4. How the element properties are developed and what are their limitations.
5. How the displays are ~~ds~~ developed in pre and post processor to understand their limitations.

(6) To understand the difficulties involved in the development of FEA programs and hence the need for checking the commercially available packages with the results of standard cases.

Unless user has the background of FEA, he may produce worst results and may go with overconfidence. Hence it is necessary that the users of FEA package should have sound knowledge of FEA.

4 FEM VS. FDM

(1) FDM makes pointwise approximation that ensures continuity only at the node points but FEM makes piece wise approximation that ensures the continuity at nodes as well as sides of the elements.

(2) FDM do not give the values at any point except at node points but FEM gives values at any point.

(3) FEM can handle sloping and curved boundaries while FDM makes stair type approximation for those.

(4) FDM needs large number of nodes to get good results while FEM needs fewer nodes.

(5) FDM can handle fairly complicated problems whereas FEM can handle all complicated problems.

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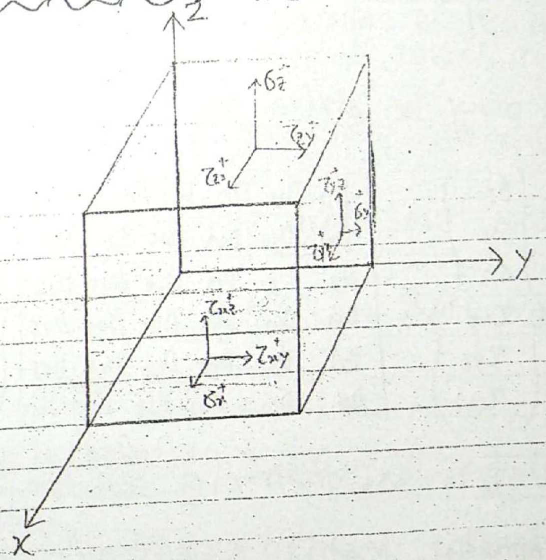
[5] Some popular FEM packages

1. STAAD-PRO
2. NASTRAN
3. ANSYS
4. ABAQUS
5. COSMOS

* [6] Draw a typical 3D element and indicate the state of stress in their positive ~~stress~~ senses.



* [7] State and explain "generalized Hooke's ~~law~~ Law".

In theory of elasticity, the relation between stress and strain is considered linear. ie. stress is proportional to strain. It is very well known as Hooke's law. Cauchy generalized Hooke's law to three dimentional elastic bodies and stated that the 6 components of stress are linearly related to the 6 components of strain. It is called 'Generalized Hooke's Law'.

This may be stated as —

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{21} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{45} & D_{46} \\ D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

where D is 6X6 matrix of ~~stiffness~~ constants of elasticity. As D is a symmetric matrix ($D_{ij} = D_{ji}$), there are 21 material properties for linear elastic Anisotropic materials.

* [8] Explain the terms 'Anisotropic, Orthotropic and Isotropic' as applied to material properties.

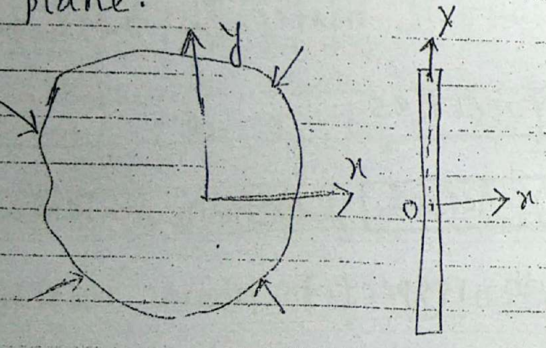
$\boxed{\text{Anisotropic materials}}$ Materials which have different material properties in different directions are called anisotropic materials. For linear elastic anisotropic materials there are 21 material properties.

$\boxed{\text{Orthotropic material}}$ Certain materials exhibit symmetry with respect to planes within the body. Such materials are called orthotropic materials. For orthotropic materials, no. of material constants are 9.

$\boxed{\text{Isotropic materials}}$ An isotropic material is the one that has identical material properties in all direction. For isotropic material, there are 3 [illegible] properties.

9 Explain "plaine stress problem" and "plane strain problem".

Plane stress problem The thin plates subject to forces in their plane only is a typical ~~example~~ plane stress problem. For plane stress problem there is no force in perpendicular to the plane.



Assumptions: $\sigma_z = \tau_{xz} = \tau_{yz} = 0$

$\tau_{xz} = \tau_{yz} = 0$ gives $\gamma_{xz} = \gamma_{yz} = 0$

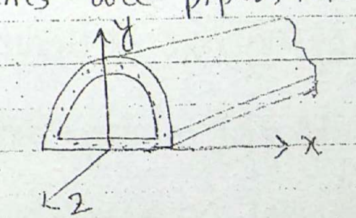
and $\sigma_z = 0$ gives $\sigma_z = \mu\varepsilon_x + \mu\varepsilon_y + (1-\mu)\varepsilon_z = 0$

$\therefore \varepsilon_z = -\frac{\mu}{1-\mu}(\varepsilon_x + \varepsilon_y)$

Hence, the constitutive law reduces to

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\mu^2}\begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Plane strain problem A long body subject to significant lateral forces but very little longitudinal forces falls under this category of problem. Examples of such problems are pipes, retaining wall, tunnel etc.



Assumptions: $\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$

$\gamma_{xz} = \gamma_{yz} = 0$ means $\tau_{xz} = \tau_{yz} = 0$

$\varepsilon_z = 0$ means $\cancel{\sigma_z = \mu\varepsilon_x + \mu\varepsilon_y + (1-\mu)} \times 0$

$\varepsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x + \sigma_y}{E} = 0$ $\cancel{\sigma_z = \mu(\varepsilon}$

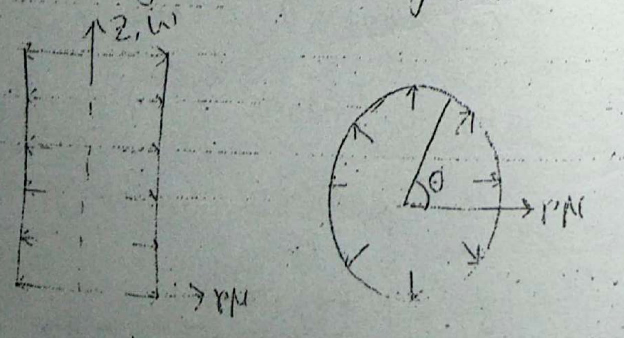
$\sigma_z = \mu(\sigma_x + \sigma_y)$

Hence, the constitutive law reduces to

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & 0 \\ \mu & 1-\mu & 0 \\ 0 & 0 & \frac{1-2\mu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

** $\boxed{10}$ Explain axisymmetric problem

Axi-symmetric structures are those which can be generated by rotating a line or curve about an axis. Cylinders are common examples of axisymmetric structures. If such structures are subjected to ~~axys~~ axisymmetric loading like internal pressures there exist symmetry about any axis.



The constitutive law is

$$\begin{Bmatrix} \sigma_r \\ \sigma_z \\ \sigma_\theta \\ \tau_{rz} \end{Bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & \mu & 0 \\ \mu & 1-\mu & \mu & 0 \\ \mu & \mu & 1-\mu & 0 \\ 0 & 0 & 0 & \frac{1-2\mu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{rz} \end{Bmatrix}$$

** $\boxed{11}$ What is constitutive relation? Write down the relation for a linear isotropic material.

The relationship between stress and strain is called constitutive relation. In one dimensional stress analysis, the linear constitutive law is stress is proportional to strain which is well known as Hooke's law. Constitutive relation can be express as $\{\sigma\} = [D]\{\varepsilon\}$ where $\{\sigma\}$ is stress vector, $\{\varepsilon\}$ is strain vector and $[D]$ is property matrix.

For a linear isotropic material, the constitutive relation is as follows—

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & \mu & 0 & 0 & 0 \\ \mu & 1-\mu & \mu & 0 & 0 & 0 \\ \mu & \mu & 1-\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-\mu \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

* 12 Purpose of constitutive law in FEM program?

Constitutive law establishes a relation between stress and strain. In FEM program, material properties is to be found and assembled. In this purpose the constitutive law is essential to define the material properties ~~and~~ for an element and assembling ~~with~~ the element properties ~~into~~ to get global properties. For a force displacement system, material property is stiffness of the material and it is found from the constitutive law, $\{F\} = [K]\{\delta\}$.

* 13 Define stiffness matrix and explain its special features.

Stiffness matrix Stiffness matrix is the matrix which defines the geometric and material properties of an element ~~and~~ as well as the whole structure. In a matrix displacement equation like $\{F\} = [K]\{\delta\}$, $[K]$ is the stiffness matrix.

Special features of stiffness matrix

1. The stiffness matrix is singular matrix.
2. The matrix is symmetric. It is obvious from Maxwell's reciprocal theorem.
3. The matrix is having diagonal dominance and is positive definite. Hence in the solution process there is no need to rearrange the equation to get diagonal dominance.

(4) The matrix has banded nature i.e. nonzero elements of stiffness matrix are concentrated near the diagonal of the matrix. The elements away from the diagonal are zero.

** **14.** State and explain three approaches to reduce the memory requirement in storing stiffness matrix.

(1) Use of symmetry and banded nature

Since the stiffness matrix is always symmetric and banded in nature, techniques have been developed to store only semiband width of non-zero elements and get the solution. If B is the semiband width of NXN matrix, we need to store only NXB elements. ~~as indicated~~ ~~in the following~~ figure. The diagonal of the given matrix is stored as the first column of the modified matrix. For example, consider a symmetric 6X6 matrix with semi band width 3.

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & 0 \\ & A_{22} & A_{23} & A_{24} & 0 & 0 \\ & & A_{33} & A_{34} & A_{35} & 0 \\ & & & A_{44} & A_{45} & A_{46} \\ & sym & & & A_{55} & A_{56} \\ & & & & & A_{66} \end{bmatrix}$$

This matrix is stored as the 6X3 matrix:-

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{22} & A_{23} & A_{24} \\ A_{33} & A_{34} & A_{35} \\ A_{44} & A_{45} & A_{46} \\ A_{55} & A_{56} & 0 \\ A_{66} & 0 & 0 \end{bmatrix}$$

~~in the following~~ figure. The diagonal of the given matrix is stored as the first column of the modified matrix. For example, consider a symmetric 6X6 matrix with semi band width 3.

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & 0 \\ & A_{22} & A_{23} & A_{24} & 0 & 0 \\ & & A_{33} & A_{34} & A_{35} & 0 \\ & & & A_{44} & A_{45} & A_{46} \\ & sym & & & A_{55} & A_{56} \\ & & & & & A_{66} \end{bmatrix}$$

This matrix is stored as the 6X3 matrix:-

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{22} & A_{23} & A_{24} \\ A_{33} & A_{34} & A_{35} \\ A_{44} & A_{45} & A_{46} \\ A_{55} & A_{56} & 0 \\ A_{66} & A0 & 0 \end{bmatrix}$$

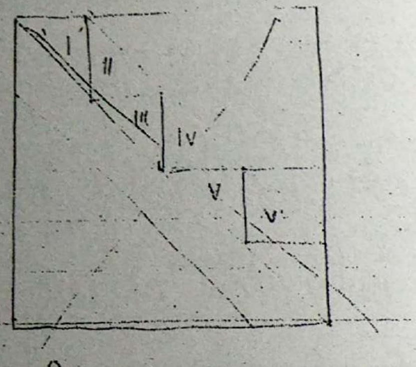
(ii) Partitioning of matrix (Frontal solution)

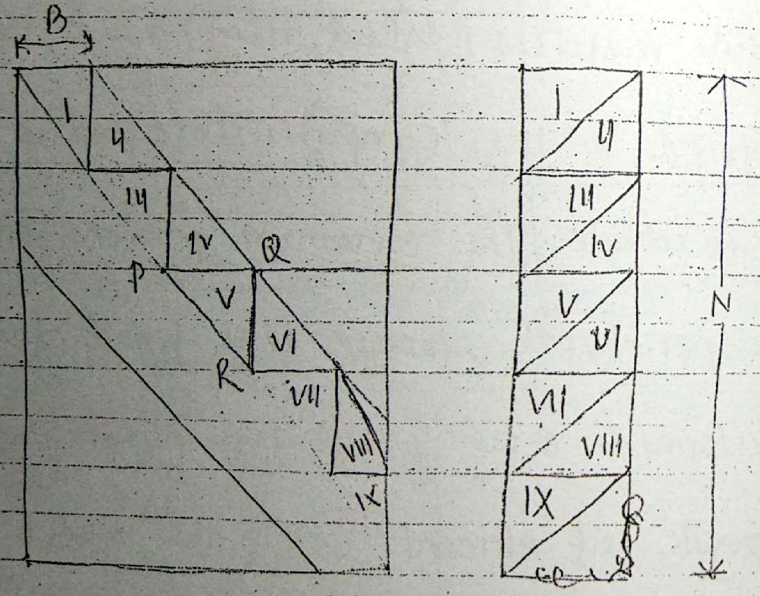
For larger systems, band matrix is storage technique is inadequate. In such cases partitioning of the matrix is made by frontal method.

In this approach only few of the triangular sub matrices (called front) need to be stored in the computer core at a given time while the remaining portions are kept in peripheral storage like hard disk. Elimination performed using one row affects only the triangle of element within the band below that row. For example in the fig, reduction involving row PQ modifies only the triangle PQR.

This permits us to carry out the elimination with only few of the sub matrices in core.

Frontal solution technique is developed on this scheme





(iii) Skyline storage: Further saving in memory requirement is by making use of skyline storage technique. In this system of storage, if there are zeros at the top of a column, only the elements starting from non-zero values need to be stored. The line separating the top zeros from the first non-zero element is called skyline. For the matrix given below the skyline is indicated.

skyline

$$
\begin{bmatrix}
a_{11} & a_{12} & 0 & a_{14} & 0 & 0 & 0 & 0 \\
 & a_{22} & a_{23} & 0 & a_{25} & 0 & 0 & 0 \\
 & & a_{33} & a_{34} & 0 & a_{36} & 0 & 0 \\
 & & & a_{44} & 0 & a_{46} & 0 & a_{48} \\
 & & & & a_{55} & a_{56} & a_{57} & 0 \\
 & & & & & a_{66} & a_{67} & a_{68} \\
 & & & & & & a_{77} & a_{78} \\
 & & & & & & & a_{88}
\end{bmatrix}
$$

*** 15 Give illustrative examples to compare frontal solution techniques with band and skyline technique. Explain the advantages and disadvantages in terms of memory requirements.

Advantages of frontal technique

(1) For larger system, frontal technique is much more better ~~optio~~ option than band technique.

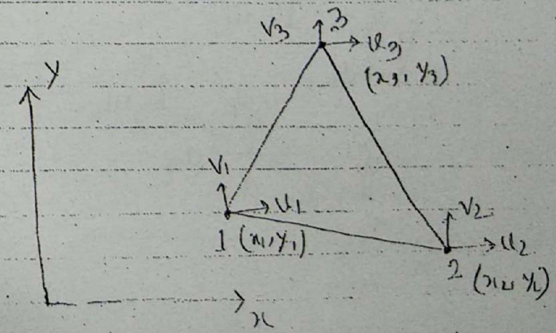
Disadvantages

For very large sparse matrix, skyline approach is better than frontal approach

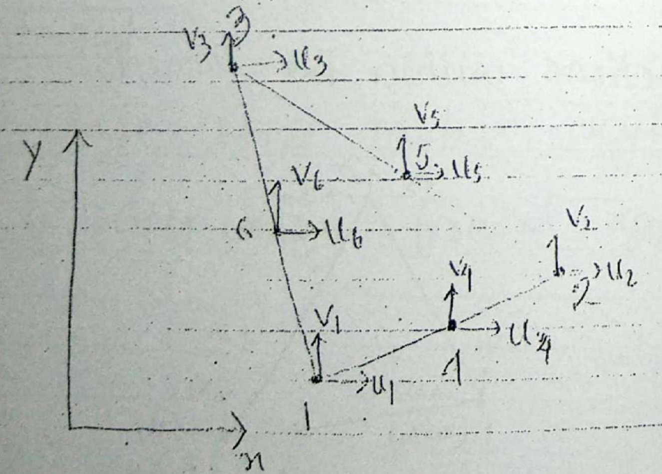
** 16 Explain the following terms with necessary sketches.
(i) CST (ii) LST (iii) QST

*** **(i) Constant strain triangle** This is the simplest 2-D element consisting of three noded triangle.

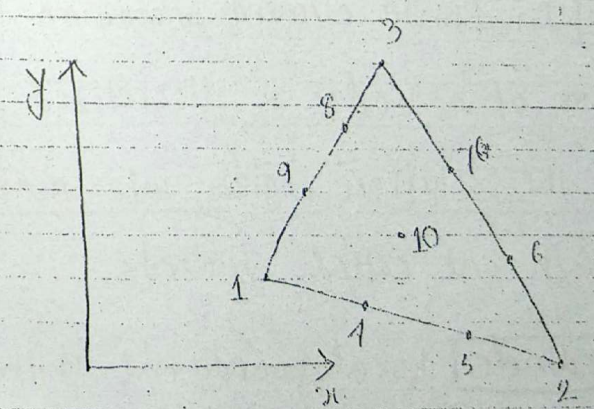
For this element, we have three nodes at the vertices of the triangle which are numbered around the element in the counterclockwise direction. Each node has ~~three~~ two degrees of freedom. The displacements ~~within~~ (u and v) are assumed to be linear functions within the elements.



(ii) Linear strain triangle Six noded triangular 2-D element is called linear strain triangle. ~~The six nodes are~~ Among the six nodes ~~three~~ there are three corner nodes and three mid side nodes. Each node has two degrees of freedom. The displacements (u, v) are assumed to be quadratic functions of (x, y).



(iii) Quadratic strain triangle Ten noded 2-D triangular element is called quadratic strain triangle. Among them 3 nodes at ~~vertexes~~ corners, 6 nodes on 3 sides (2 nodes each) and one node is in within the element. Each node has two degrees of freedom and the displacements (u, v) are assumed to be cubic functions of (x, y)

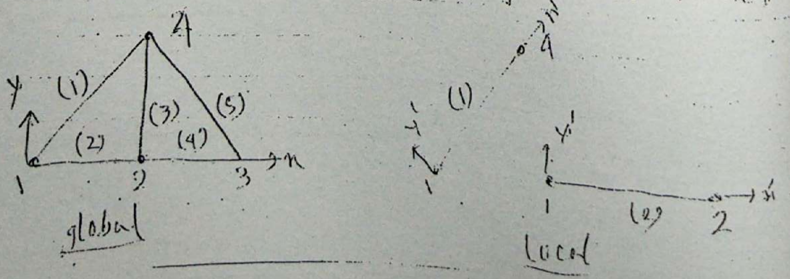


** 17 Explain the following terms :
1. Nodes, elements
2. Local coordinate, global coordinate
3. Transformation matrix

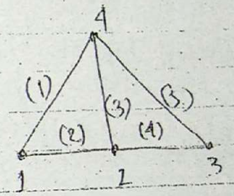
*** **Nodes** Approximating functions are defined in terms of field variables of some specified points of an element. These points are called nodes.

*** **Elements** In Finite element procedure the continuum of a body is discretized by dividing the solution region into small parts which are called elements.

*** **Local coordinate** Local coordinate is the separate coordinate system for each element.



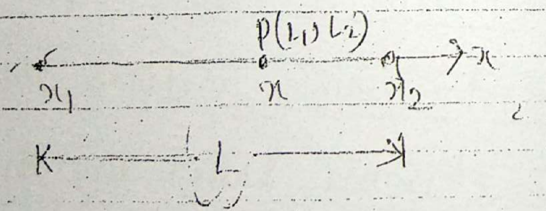
** **Global coordinate** The coordinate system used to define the points in the entire structure is called global coordinate system.



** **Transformation matrix**

** Natural coordinate: A natural coordinate is a system system is a coordinate system which permits the specifications of a point within the element by a set of dimensionless numbers, whose magnitude never exceeds unity. In the following element, ~~let~~ natural coordinate of point P is (L_1, L_2) where

$$L_1 = \frac{x_2 - x}{l}, \quad L_2 = \frac{x - x_1}{l}$$



* Higher order element

*** 18. Explain the term "Shape function". Why polynomials are preferred for shape function?

Shape function: The values of the field variables computed at the nodes are used to approximate the values at non nodal points. The function which relates the field variables at any point within the element to the field variables of nodal points is called shape function. This is also called interpolation function or approximating function.

The interpolation functions are predetermined, known functions of the independent variables and these functions describe the variation of the field variable within the finite element.

For the three node triangle example, the field variable is described by the approximate relation

$$\phi(x,y) = N_1(x,y)\,\phi_1 + N_2(x,y)\,\phi_2 + N_3(x,y)\,\phi_3$$

where ϕ_1, ϕ_2, ϕ_3 are field variables at the nodes, N_1, N_2, N_3 are shape functions.

Why polynomial shape function is preferred

This is because —

1. They are easy to handle mathematically i.e. differentiation and integration of polynomials is easy.
2. Using polynomial any function can be approximated reasonably well. If a function is highly non-linear we may have to approximate with higher order polynomial.

** 19 State and explain the convergence requirements of polynomial shape functions.

Displacements obtained by FEM are lesser than exact values. The more no. of elements, the more close solution approaches to the exact values. In order to ensure this convergence criteria, the shape function should satisfy the following requirement:

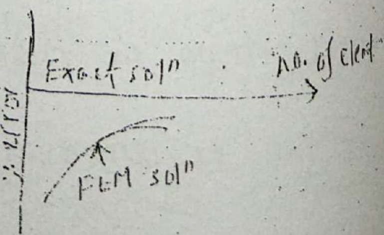
1. The displacement model must be continuous within the elements and the displacements must be compatible between the adjacent elements.

The second part implies that the adjacent elements must deform without causing openings, overlaps or discontinuities between the elements. The requirement is called 'compatibility requirement.

(2) The displacement model should include the rigid body displacements of the elements. It means in displacement model there should be a term which permit all points on the elements to experience the same displacement.

(3) The displacement models must include the constant strain state of the elements. That means there should exist combination of values of polynomials terms that cause all points in the element to experience the same strain.

(4) The pattern of polynomial shape function should be independent of the orientation of the local coordinate system.



** [20] Why shape function is used in finite element formulation? What are its characteristics?

In FEM, field variables of nodal points are determined by stiffness matrix calculation. But field variables other than nodes are not. To approximate the non-nodal point field variables, shape function is required. Shape function describes the variation of the field variables within the finite element. This is why shape function is formulated.

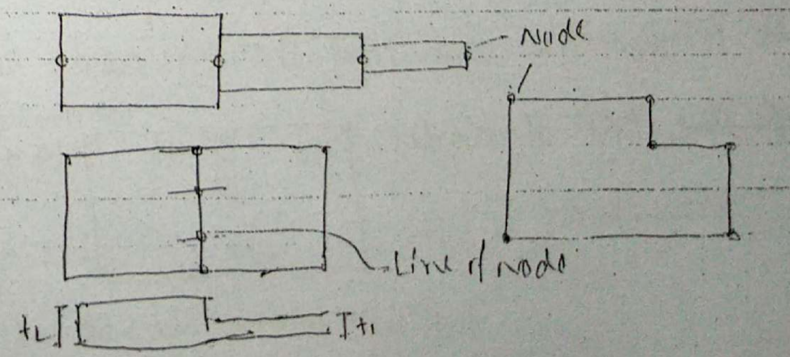
$\boxed{\text{Characteristics of shape function}}$

* 21 What are the requirements for approximating a 2-D shape function?

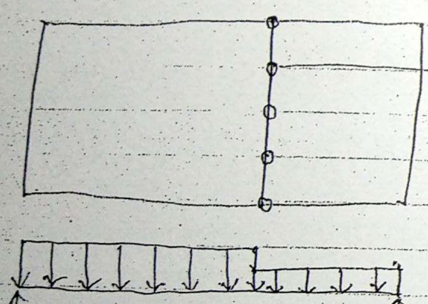
*** 22 Discuss the various discontinuities to be considered in discretizing a structure in FEM.

The process of modeling a structure using suitable number, shape and size of element is called discretization. modeling should be good enough to get the results as close to actual behavior of the structure. There are some discontinuities that need to considered in discretizing a structure, they are —

(1) Geometric discontinuity Whenever there is sudden change in shape and size of the structure there should be a node or line of nodes.

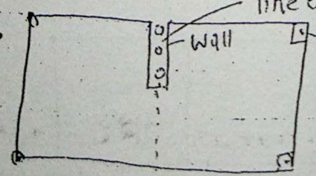


(ii) Discontinuity of loads: Concentrated loads and sudden change in the intensity of uniformly distributed loads are the source of discontinuity of loads. A node or a line of nodes should be there to model the structures.



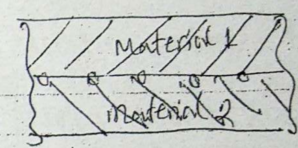
slab with different UDL

(iii) Discontinuity of boundary condition: If the boundary condition of a structure suddenly change we to discretize such that there is node or ~~node~~ ~~of line~~ line of nodes.



slab with intermediate wall and column

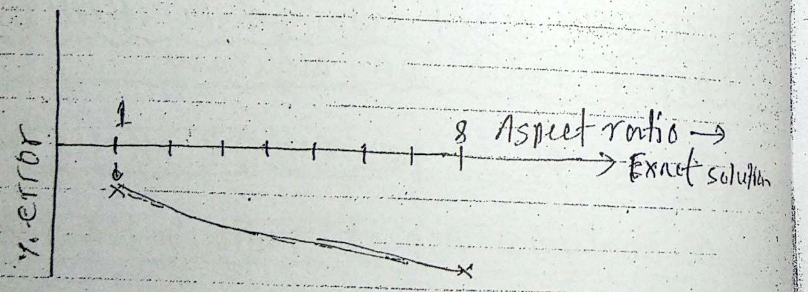
(iv) Material discontinuity: Node or node lines should appear at the places where material discontinuity is seen.



material discontinuity

*** 23. Write short notes on the effect of element aspect ratio on accuracy.

The shape of the element affects the accuracy of analysis. Aspect ratio is the ratio of longest to smallest size in an element. While selecting element size, aspect ratio should be close to unity. Study of aspect ratio shows that when aspect ratio is close to unity, the result is much more accurate. Desai and ~~Abee~~ Abel analysed a beam with 12 element of different aspect ratios and plots the inaccuracy of the displacement ~~vectors~~ versus aspect ratio which shows that aspect ratio close to unity yields better results.



24 Write short notes on:

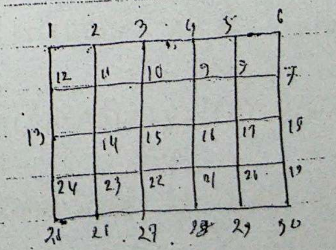
* (i) Numbering nodes for band width minimization

Storing global stiffness matrix in the computer memory imposes a serious limitation on the number of elements/degrees of freedom to be used.

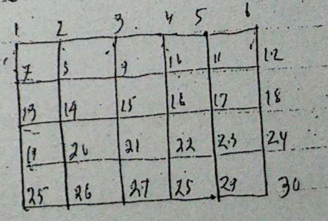
The size of semiband width of stiffness matrix depends upon the numbering system adopted for nodes. The semi band width, B is given by the expression, $B = (D+1)f$

D is maximum difference in node number in an element after considering all elements. f is degrees of freedom per node. The numbering system for which ~~B is Lower~~ B value is Lowest uses less memory.

In many packages numbering is done automatically to keep the semi band width least. Some numbering techniques are illustrated below



$B = (11+1)\times 2 = 24$



$B = (7+1)\times 2 = 16$

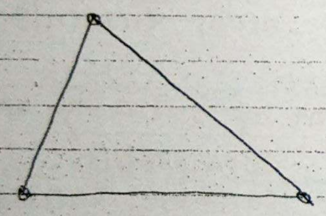
* (11) Mesh refinement vs. order of elements

Accuracy of calculation increases if higher order elements are used. Accuracy can also be increased by using more number of elements. Limitation on use of ~~higher~~ ~~order~~ number of elements comes from the total degrees of freedom the computer can handle. Hence to use higher order elements we have to use less number of such elements, or lower order elements with higher number of elements. ~~Studies show that keeping degree of accuracy~~ ~~as~~ ~~so~~ ~~per unit cost as~~ selection criteria,

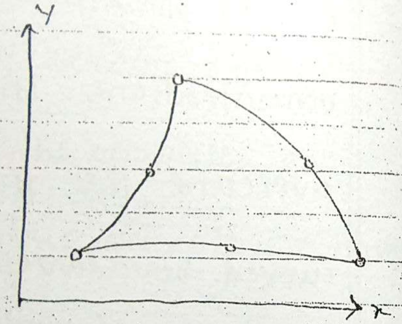
*** 25 Introduce and explain isoparametric concept in finite element analysis.

For curved boundaries refined mesh are used with straight edge elements. Refined meshes sometimes results unnecessary stress concentrations. Higher order elements do not overcome the problem of suitably approximating curved boundarées. The isoparametric ~~isotropic~~ concept revolutionized the finite element analysis and helped properly in mapping such curved boundaries. ~~The concept~~ ~~is~~ of ~~the isoparametric concept~~ is ~~that~~ This ~~is a~~ concept of mapping regular triangular and rectangular elements in natural coordinate system, to arbitrary shapes in global system ~~as~~ is called isoparametric concept.

Using isoparametric concept. Parent element is in natural coordinate system. Concept of mapping in isoparametric elements are shown in the figure below.



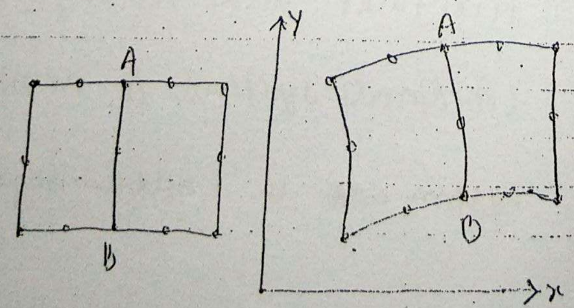
Parent element in natural coordinate system



mapped element in global system.

** [25] State the three basic theorem on which the isoparametric concept is developed.

(1) If two adjacent elements are generated using shape function, then there is continuity at the common edge.



(2) If the shape function used are such that the continuity ~~is~~ of displacement is represented in the parent coordinates, then the continuity requirement will be satisfied in the ~~isotropic~~ isoparametric elements also.

(3) The constant ~~der~~ derivative conditions and conditions for rigid body are satisfied for all isoparametric elements. if $\sum N_i = 1$.

** [26] Discuss the convergence criteria for isoparametric elements.

The constant derivative conditions and condition for rigid body are satisfied for all isoparametric elements if $\sum N_i = 1$. this theorem is known as isoparametric ~~g~~ convergence criteria for isoparametric elements.

(c) the displacement function be

$$u = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 z \text{ ---- (i)}$$

$\therefore$ Nodal displacement at 'i'th node is

$$u_i = \alpha_1 + \alpha_2 x_i + \alpha_3 y_i + \alpha_4 z_i$$

Displacement at any point, $u = \sum N_i u_i$

$$\therefore u = \sum N_i (\alpha_1 + \alpha_2 x_i + \alpha_3 y_i + \alpha_4 z_i)$$

$$= \alpha \sum N_i + \alpha_2 \sum N_i x_i + \alpha_3 \sum N_i y_i + \alpha_4 \sum N_i z_i$$

From isoparametric concept, $\sum N_i x_i = x$

$$\sum N_i y_i = y$$

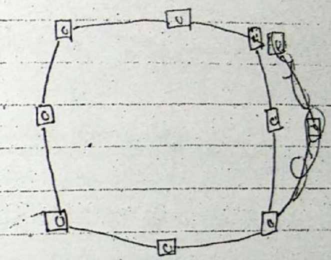
$$\sum N_i z_i = z$$

$$\therefore u = \alpha_1 \sum N_i + \alpha_2 x + \alpha_3 y + \alpha_4 z \text{ ---- (ii)}$$

From eqn (i) and (ii), $\sum N_i = 1$.

*** 27 Explain the following terms:-

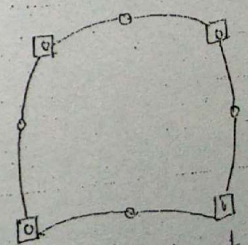
(i) Isoparametric elements: If the shape functions defining the boundary and displacement are the same, the element is called as isoparametric element.



o -- nodes defining geometry
□ -- nodes defining displacement

isoparametric elements

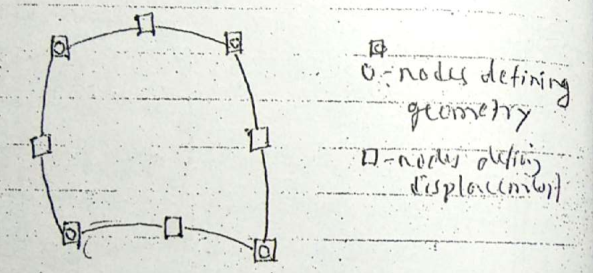
Superparametric elements: The elements in which more number of nodes are used to define geometry compared to the number of nodes defining displacement ~~lower are~~ higher are known as superparametric element.



o - nodes defining geometry
□ - nodes defining displacement

super parametric element.

Subparametric element The elements in which less number of nodes are used to define geometry compared to the ~~no~~ number of nodes used for defining the displacements are called sub-parametric element.



sub parametric elements

* **25** State the differences and similarities between a bare element and a truss element.

Differences :

① Bar elements can be fixed, pinned connected but truss element is always pinned connected.

② Bars elements are considered one dimensional where as truss can be two or three dimensional.

Similarities

① Both the elements have only axial forces.

* 29. State the major steps that one will follow in analysing a bar in finite element approach and there by obtain the unknown reactions.

(1) Selecting suitable field variables and elements.

(2) Discritize the continua

(3) Select interpolation function

(4) Element properties

(5) Global properties

(6) Boundary condition

(7) Solution of simultaneous equation

(8) Additional calculation for reactions.

Q. What are the basic components of a general purpose finite element software? [2010-11, 09-10, 08-09, 07-08]

Ans: Three elements:

- Pre-processor
- Solution algorithm
- Post processor

1) Pre processor:

It creates Finite Element Model & input necessary data for FEA programme. Pre-processor can feed data in the computer. Nodal coordinate & element connectivity can be done with pre-processor.

2) Solution Algorithm:

It is a Global coordinate based system which deals with shape function. Solution algorithm is the heart of FEM.

3) Post - Processor:

post processor will show the final output.

It accepts the results of the analysis and generates tables, diagrams, ~~pies etc~~ ~~pit~~ pictures, graphs, contours etc. for proper interpretation of results. It is a local coordinate based system.

Q. What is a constitutive relation? Write down the relation for a linear isotropic material. [07-02], [08-09]

Ans:

Constitutive relation:

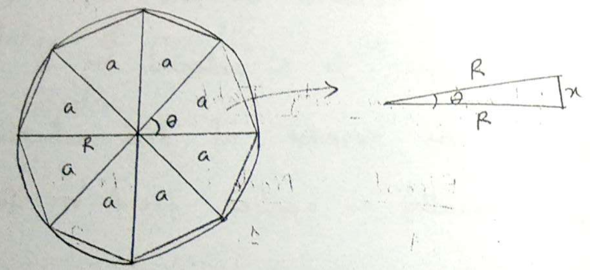
It is a relation between two physical quantities that is specific to a material or a substance, and approximates the response of that material to external stimuli, usually as applied field or forces.

Relation for a linear isotropic material:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{pmatrix} 1-\mu & \mu & \mu & 0 & 0 & 0 \\ & 1-\mu & \mu & 0 & 0 & 0 \\ & & 1-\mu & 0 & 0 & 0 \\ & & & \frac{1-2\mu}{2} & 0 & 0 \\ & & & & \frac{1-2\mu}{2} & 0 \\ & & & & & \frac{1-2\mu}{2} \end{pmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

Q: Approximate the area of circle by dividing it into a number of triangles. In this process, show that $S_N = \pi R^2$, when $N \to \infty$, where R = radius of the circle, N = number of triangles, S_N = area of the circle. [11-12]

Ans:



The area of a circle is to be determined whose radius is R.

Let's divide the circle into n numbers of small triangles.

If $n \to \infty$, then if triangle will be very small.

Let's say, θ is the angle of each triangle which originates in the centre of the circle.

So, for a small size triangle, area, $a = \frac{1}{2} \times \text{base} \times \text{perpendi height}$

$$= \frac{1}{2} \times R \times x$$

$$\therefore a = \frac{1}{2} \times R \times R \sin\theta$$

Now, $\theta = \frac{2\pi}{n}$.

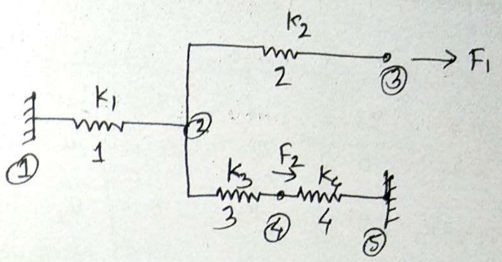
$\therefore$ Area of the circle, $S_N = \underset{n\to\infty}{Lt}\ n a = \underset{n\to\infty}{Lt}\ n \times \frac{1}{2} R^2 \times \sin\theta$

$$\Rightarrow S_n = \underset{n\to\infty}{Lt}\ \frac{1}{2} R^2 \times \frac{\sin\left(\frac{2\pi}{n}\right)}{\frac{2\pi}{n}} \times 2\pi$$

$$= \frac{1}{2} R^2 \times 2\pi = \boxed{\pi R^2}$$

proved.

2010-11	1 (c)



Element Connectivity Table:

Element	Node i	Node j
1	1	2
2	2	3
3	2	4
4	4	5

$$\underline{\underline{k_1}} = \begin{array}{c c} & \begin{array}{cc} u_1 & u_2 \end{array} \\ & \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{array}{c} u_1 \\ u_2 \end{array} \end{array} \qquad \underline{\underline{k_2}} = \begin{array}{c c} & \begin{array}{cc} u_2 & u_3 \end{array} \\ & \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{array}{c} u_2 \\ u_3 \end{array} \end{array}$$

$$\underline{\underline{k_3}} = \begin{array}{c c} & \begin{array}{cc} u_2 & u_{3_4} \end{array} \\ & \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} \begin{array}{c} u_2 \\ u_{3_4} \end{array} \end{array} \qquad \underline{\underline{k_4}} = \begin{array}{c c} & \begin{array}{cc} u_4 & u_5 \end{array} \\ & \begin{bmatrix} k_4 & -k_4 \\ -k_4 & k_4 \end{bmatrix} \begin{array}{c} u_4 \\ u_5 \end{array} \end{array}$$

Applying the superposition method stiffness matrices as follows.

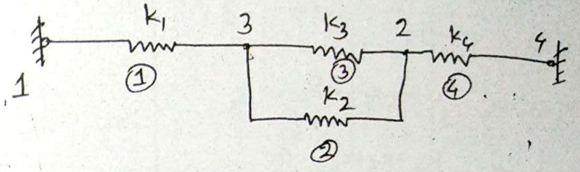
$$
\underline{\underline{K}} = \begin{array}{c} \\ \end{array}
\begin{array}{cccccc}
u_1 & u_2 & u_3 & u_4 & u_5 & \\
\end{array}
$$

$$
\underline{\underline{K}} = \begin{array}{cc}
\begin{bmatrix}
k_1 & -k_1 & 0 & 0 & 0 \\
-k_1 & k_1+k_2+k_3 & -k_2 & -k_3 & 0 \\
0 & -k_2 & k_2 & 0 & 0 \\
0 & -k_3 & 0 & k_3+k_4 & -k_4 \\
0 & 0 & 0 & -k_4 & k_4
\end{bmatrix} & \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array}
\end{array}
$$

confirm

checked with partho

2078-09 | 5 (c)



Element connectivity table:

element	Node i	Node j
1	1	3
2	3	2
3	3	2
4	2	4

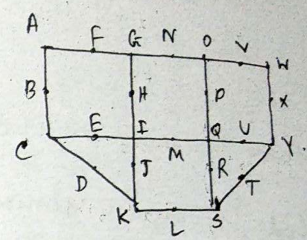
$$
\underline{\underline{k_1}} = \begin{array}{cc} \begin{array}{cc} u_1 & u_3 \end{array} & \\ \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} & \begin{array}{c} u_1 \\ u_3 \end{array} \end{array}
\qquad
\underline{\underline{k_2}} = \begin{array}{cc} \begin{array}{cc} u_3 & u_2 \end{array} & \\ \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} & \begin{array}{c} u_3 \\ u_2 \end{array} \end{array}
$$

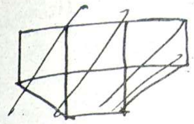
$$
\underline{\underline{k_3}} = \begin{array}{cc} \begin{array}{cc} u_3 & u_2 \end{array} & \\ \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} & \begin{array}{c} u_3 \\ u_2 \end{array} \end{array}
\qquad
\underline{\underline{k_4}} = \begin{array}{cc} \begin{array}{cc} u_2 & u_4 \end{array} & \\ \begin{bmatrix} k_4 & -k_4 \\ -k_4 & k_4 \end{bmatrix} & \begin{array}{c} u_2 \\ u_4 \end{array} \end{array}
$$

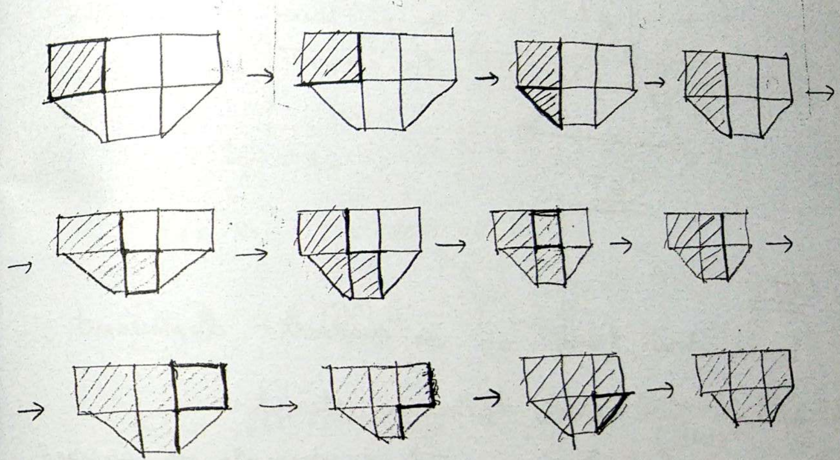
$$
\begin{array}{c|cccc|c}
 & u_1 & u_2 & u_3 & u_4 & \\
\hline
K = & k_1 & 0 & -k_1 & 0 & u_1 \\
 & 0 & k_2+k_3+k_4 & -k_2-k_3 & -k_4 & u_2 \\
 & -k_1 & -k_2-k_3 & k_2+k_3+k_1 & 0 & u_3 \\
 & 0 & -k_4 & 0 & k_4 & u_4
\end{array}
$$

checked with partho

2018-09 $\boxed{7(b)}$







ABC EI HGF ⟶ CEIHJ ⟶ CEIHGJKD ⟶ GHIJK ⟶

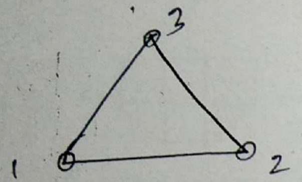
GHIJK ~~B~~LSRQM ⟶ GHIMQRS ⟶ GHI~~Q~~MQPONRS ⟶ OPQRS ⟶

OPQRS ~~T~~UY XWV ⟶ SRQUY ⟶ QRSTYU ⟶ Finish

2011 - 12 . 3(c)

(i) CST:

constant strain Triangle or Linear Displacement Triangle element. 3 noded triangular element.

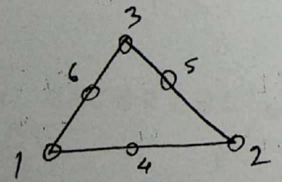


The displacement are assumed to be linear functions within the element.

CST

(ii) ~~QST~~: LST:

Linear strain triangle or ~~six~~ quadratic displacement triangle element six noded triangular element.

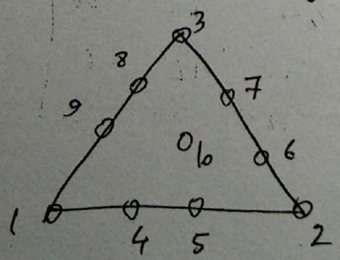


The displacements are assumed to be quadratic function

LST

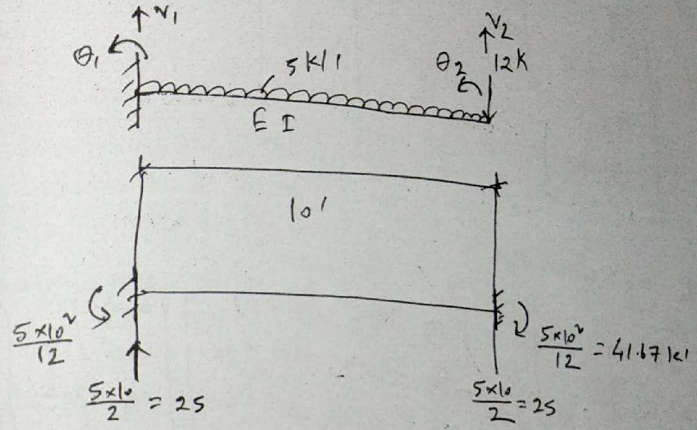
(iii) QST:

Quadratic strain triangle or ~~cubid~~ cubic Displacement triangle. 10 ~~noddes~~ noded triangular element.



The displacement are assumed to be cubic function.

2011-12 | 2(b)



$E = 3600$ ksi, $I = \cancel{1440}\ 1440$ in^4.

$[P_m] + [K]u = [\Sigma P_j]$.

$EI = 3600 \times 1440$ $\frac{k\text{-}in^2}{}$ $\frac{EI}{L^3} = \frac{3600\times1440}{(10\times12)^3} = 3$ k/in.
$= 36$ k/ft.

$$K = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$= 36\begin{bmatrix} 12 & 60 & -12 & 60 \\ 60 & 400 & -60 & 200 \\ -12 & -60 & 12 & -60 \\ 60 & 200 & -60 & 400 \end{bmatrix}$$

$$P_m = \begin{bmatrix} 25 \\ 41.67 \\ 25 \\ -41.67 \end{bmatrix} \qquad P_j = \begin{bmatrix} 0 \\ 0 \\ -12 \\ 0 \end{bmatrix}$$

$$\therefore P_m + Ku = P_j$$

$$\Rightarrow \begin{bmatrix} 25 \\ 41.67 \\ 25 \\ -41.67 \end{bmatrix} + 36 \times \begin{bmatrix} 12 & 60 & -12 & 60 \\ 60 & 400 & -60 & 200 \\ -12 & -60 & 12 & -60 \\ 60 & 200 & -60 & 400 \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -12 \\ 0 \end{Bmatrix}$$

Applying boundary condition, $v_1 = \theta_1 = 0$.

$$\therefore \begin{Bmatrix} 25 \\ -41.67 \end{Bmatrix} + 36 \begin{Bmatrix} 12 & -60 \\ -60 & 400 \end{Bmatrix} \begin{Bmatrix} v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} -12 \\ 0 \end{Bmatrix}$$

$$\therefore \begin{Bmatrix} v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} -0.2847' \\ -0.0398 \text{ rad} \end{Bmatrix}. \quad \underline{\underline{\text{Ans}}}$$

$$\begin{Bmatrix} 25 \\ 41.67 \end{Bmatrix} + 36 \times \begin{bmatrix} 12 & 60 & -12 & 60 \\ 60 & 400 & -60 & 200 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -0.2847 \\ -0.0398 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} F_{1y} \\ M_1 \end{Bmatrix}$$

$$\therefore \begin{Bmatrix} F_{1y} \\ M_1 \end{Bmatrix} = \begin{Bmatrix} 62^{K} \\ 370^{K'} \end{Bmatrix} \quad \underline{\underline{\text{Ans}}}.$$

checked with Nafis

Pq Processor:

2010-11	5(b)

Gaussian Integration:

Given, $I = \int_{-1}^{1} e^{\frac{-3x}{x^2}} \, dx$.

by Gaussian integration, $I = \sum_{i=1}^{n} W_i f(x_i)$

$$= \frac{8}{9} \times e^{-\frac{1}{0}} + \frac{5}{9} \times \left[e^{-\frac{3\times\sqrt{0.6}}{0.6}} + e^{\frac{3\sqrt{0.6}}{0.6}} \right]$$

$$= \frac{8}{9} \times \frac{1}{e^{\infty}} + \frac{5}{9} \left[e^{\frac{-3}{\sqrt{0.6}}} + e^{3/\sqrt{0.6}} \right]$$

$$= \boxed{26.72579} \ (5.d.p)$$

2010-11 | 7(b)

$u_2 = 2, \quad u_4 = 0$

$$\begin{bmatrix} 4 & -3 & 6 & 2 \\ -3 & 2 & 5 & 3 \\ 6 & 5 & 1 & -4 \\ 2 & 3 & -4 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ 2 \\ u_3 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 10 \\ R_2 \\ 20 \\ R_4 \end{Bmatrix}$$

$$\therefore \; 4u_1 - 6 + 6u_3 = 10 \quad \text{——— (1)}$$

$$6u_1 + 10 + u_3 = 20 \quad \text{——— (2)}$$

$$\therefore \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 1.375 \\ 1.75 \end{Bmatrix}. \quad \underline{\text{Ans}}.$$

$$R_2 = -3 \times 1.375 + 2 \times 2 + 5 \times 1.75.$$

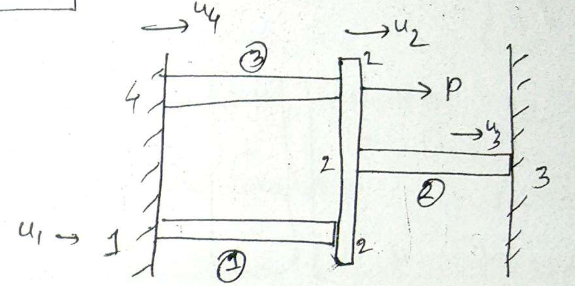
$$= \boxed{8.625}$$

$$R_4 = 2 \times 1.375 + 3 \times 2 - 4 \times 1.75$$

$$= \boxed{1.75}$$

<u>Ans</u>.

2010-11 | 7(b)



As the element containing node 2 is rigid, so it only has displacement in direction of applied force.

Assume, A; E & L are same for all members.

$$K_1 = \frac{AE}{L} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array}$$

$$K_2 = \frac{AE}{L} \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} 2 \\ 3 \end{array}$$

$$K_3 = \frac{AE}{L} \begin{array}{c} \begin{array}{cc} 2 & 4 \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} 2 \\ 4 \end{array}$$

$$\therefore K = \frac{AE}{L} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}$$

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Now, $\{P_m\} + [k]\,u = \{P_i\}$

$$\Rightarrow \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \frac{AE}{L}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$$

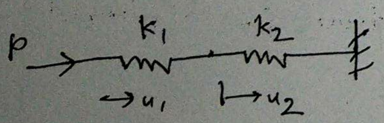
Applying boundary conditions, $u_1 = u_4 = u_3 = 0$.

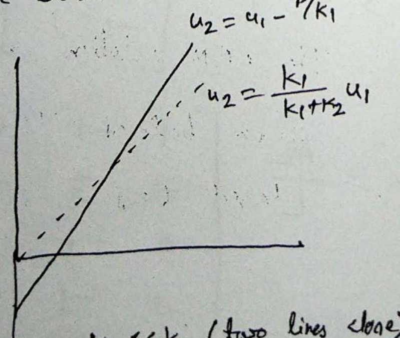
$$\therefore \frac{AE}{L}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ u_2 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ P \\ F_3 \\ F_4 \end{Bmatrix}$$ Ans.

2011-12	3(b)

Numerical errors:

Large difference in stiffness ~~in~~ of different parts in FE model may cause ill-conditioning of FE equations. Hence giving results with larger error.





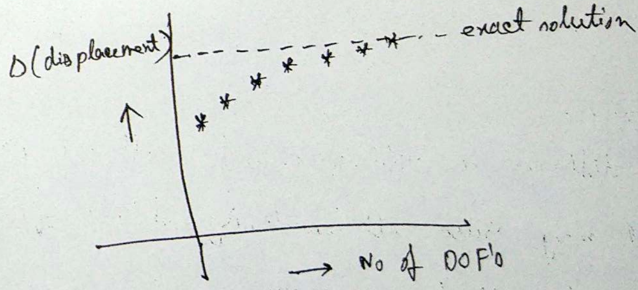
$k_2 << k_1$ (two lines close)
system is ill-conditiones.

2011-12	6(b)

Displacement field is controlled by values at a limited number of nodes.

Because of stiffening effect, FE model is stiffer than real structures. In general, displacement results in smaller in magnitude than real values.

Hence FEM solution of displacement provides a lower bound ~~solution of~~ of exact solution.



The FEM solution approaches the exact solution from below. This is ~~too~~ true for displacement based FEA.

2009-10, 3(a)

Types of Symmetry:

① Reflective (mirror, bilateral) symmetry

② Rotational (cyclic) symmetry.

③ Anisymmetry

④ Translational symmetry.