L-4/T-2/CE **Date : 02/08/2017**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-2 B. Sc. Engineering Examinations 2015-2016

Sub : **CE 419** (Introduction to Finite Element Method)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols have their usual meanings. Assume reasonable values for any missing data.

1. (a) What are the basic assumptions in analyzing a structure based on linear static analysis? **(5)**

   (b) Write three statements to describe the advantage of FEM over other methods in solving an engineering problem. **(6)**

   (c) For the spring system with arbitrarily defined nodes and elements as shown in Figure 1, find the global stiffness matrix, Comment on the problem and your derived result. **($12\frac{1}{3}$)**

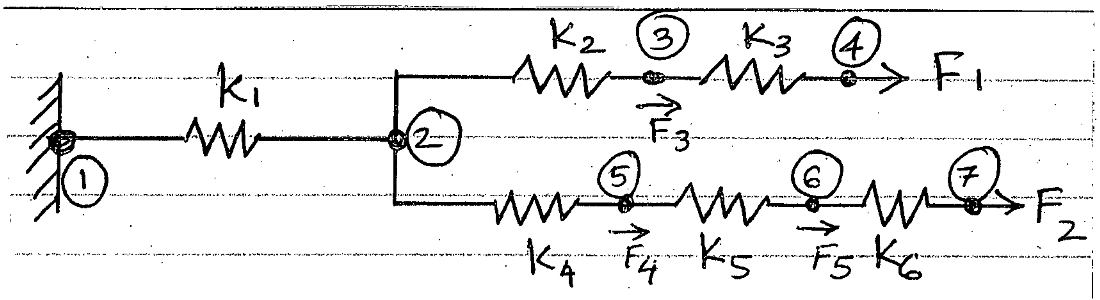

Figure 1

2. (a) Introduce reasonable shape functions for a two nodded beam element (Figure 2) and derive the element stiffness matrix (K),. In this process also consider axial stiffness of a bar element and formulate the stiffness matrix of a 2D general beam element. **(15)**

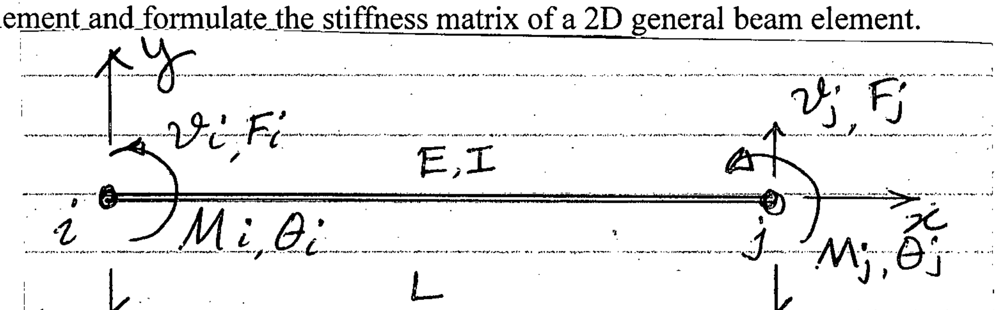

Figure 2

   (b) A cantilever beam shown in Figure 3 is subjected to distributed load. Write down the global FE equation for the beam and find out (i) deflection and rotation at right end (ii) reaction force and moment at the left end. **($8\frac{1}{3}$)**

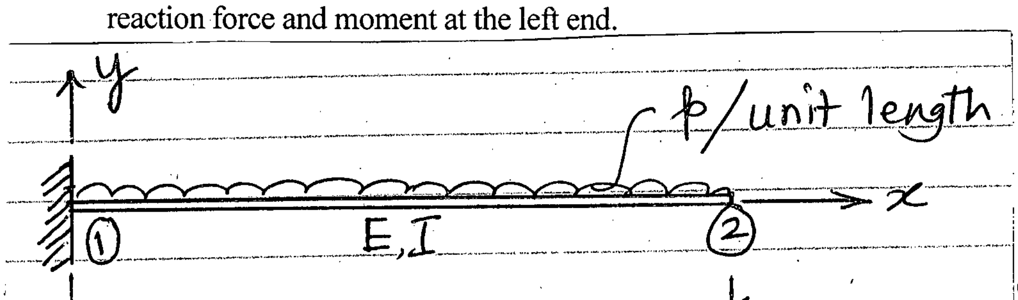

Figure 3

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CE 419

3. (a) Introduce and explain isoparametric concept in finite element analysis. **(5)**

(b) State the three basic theorems on which he isoparametric concept is developed. **(6)**

(c) Discuss the convergence criteria for isoparametric elements. **($6\frac{1}{3}$)**

(d) Explain he following terms: **(6)**

(i) Sub parametric element

(ii) Super parametric element.

4. (a) Explain the term, 'Shape Functions'. Why polynomials are preferred for shape functions? **($11\frac{1}{3}$)**

(b) List four major discontinuities that may exist in a real structure and the necessity of considering those in discretizing a structure. Draw necessary sketches. **(12)**

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) What are the basic components of a general purpose finite element software? **($5\frac{1}{3}$)**

(b) Approximate the area of a circle by dividing it into a number of triangles. In this process show that $S_N = \pi R^2$ when $n \rightarrow \infty$.

where R = radius of the circle, N = number of triangles, S_N = area of the circle. **(12)**

(c) Summarize the basic procedural steps that are followed in FEM for analyzing a structure. **(6)**

6. (a) Define plane stress and plane strain as applied to 2D element. **(8)**

(b) In civil engineering problems, the derived global stiffness matrix is usually 'symmetric" and "banded". Explain the implications of these characteristic features in solving global FE equation by employing band solution technique. **($7\frac{1}{3}$)**

(c) Give an illustrative example to compare frontal solution technique with band solution technique. Explain the aspects where frontal solution technique can offer economic solution. **(8)**

7. (a) Distinguish between Gauss elimation and iterative methods in solving global finite element equations. **(6)**

(b) A displacement based FEM formulation provides lower bound of the exact solution – Explain. **($9\frac{1}{3}$)**

(c) What is the necessity of refining an FEM mesh? Discuss different mesh refining methods. **(8)**

8. (a) What is constitutive relation? What is the purpose of an FEM program? **(5)**

(b) Introduce natural coordinate systems and derive the Jacobian matrix $\underline{\underline{J}}$ for any element. In this process also derive the strain displacement matrix $\underline{\underline{B}}$. **(15)**

(c) What is a numerical error? **($3\frac{1}{3}$)**

L-4/T-2/CE **Date : 18/01/2017**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-2 B. Sc. Engineering Examinations 2014-2015

Sub : **CE 419** (Introduction to Finite Element Method)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols carry their usual meaning.

1. (a) What are the steps involved in Finite Element analysis? **(4)**

(b) What is the definition of strain for beam elements? Derive the strain-displacement matrix of beam elements. **(12)**

(c) What constitutive law will you use for an axially loaded pile as shown in Fig. 1. Will there be any normal stress in the tangential direction? $(7\frac{1}{3})$

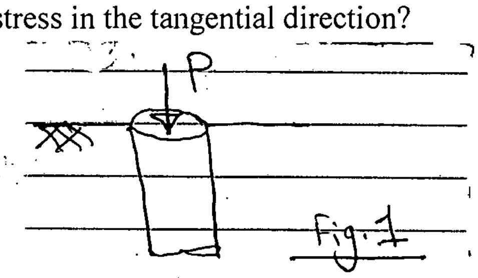

Fig. 1

2. (a) Explain why the two noded bar element yields exact stiffness matrix. **(5)**

(b) For the spring system shown in Fig. 2: **(11)**

$k_1 = 100$ N/mm, $K_2 = 200$ N/mm, $k_3 = 100$ N/mm

$P = 500$ N and $u_1 = 0$

Find:

(i) The global stiffness matrix

(ii) Displacements of nodes 2 and 3

(iii) The reaction force at node 1

(iv) The force in spring 2

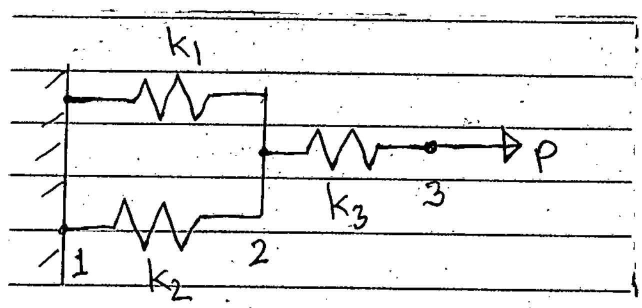

Fig. 2

(c) A simple plane truss is made of two identical bars as shown in Fig. 3. Find the global stiffness matrix. $(7\frac{1}{3})$

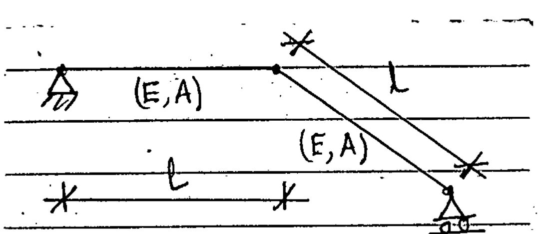

Fig. 3

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CE 419

3\. (a) A propped cantilever beam is subjected to a distributed vertical load p as shown in Fig. 4. Show the work equivalent nodal loads. Ignoring nodal moment at the right end, find the deflection and rotation at node 3. **(12)**

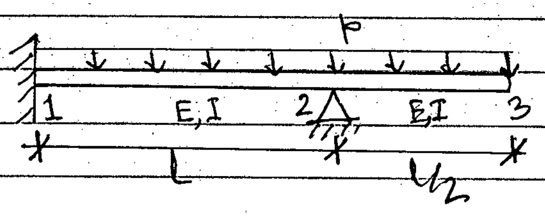

Fig. 4

(b) Determine natural coordinates of the point P of the triangular element ABC shown in Fig. 5. **($11\frac{1}{3}$)**

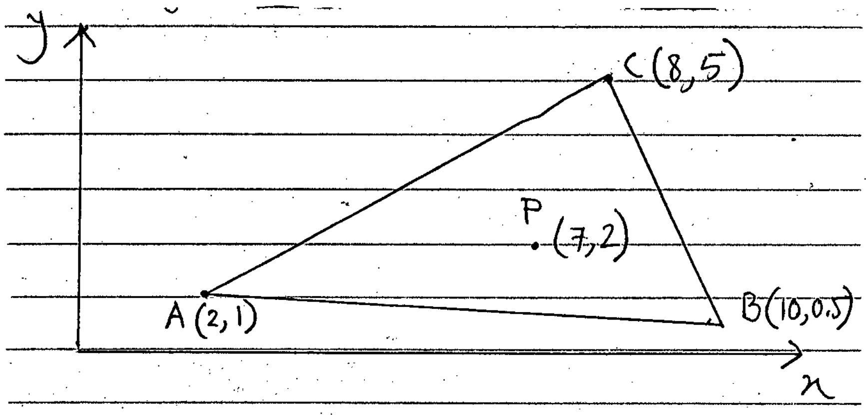

Fig. 5

4\. (a) Determine nodal displacement at node 2 (Fig. 6) due to applied force P = 400 kN and temperature rise 30° C. Given: **($11\frac{1}{3}$)**

Thickness, t = 20 mm, $E_{al} = 0.7 \times 10^5$ N/mm^2,

$E_{st} = 2 \times 10^5$ N/mm^2, $\alpha_{al} = 22 \times 10^{-6}$/°C and $\alpha_{st} = 12 \times 10^{-6}$/°C

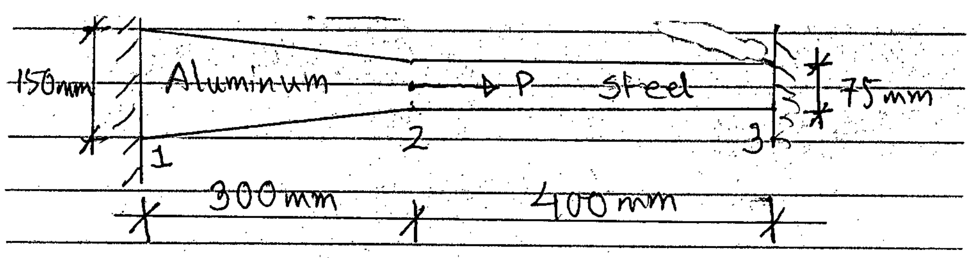

Fig. 6

(b) A jet grout pile as shown in Fig. 7 is subjected to concentrated loads P_1 and P_2 at nodes 1 and 2 and surface tractions T_1 and T_2 in elements 1 and 2. Determine stresses in element 1 and 2. Given: $E = 2.5 \times 10^5$ N/mm^2 and $\rho = 16$ kg/m^3, $P_1 = 400$ kN, $P_2 = 100$ kN, $T_1 = 20$ N/mm^2 and $T_2 = 80$ N/mm^2. **(12)**

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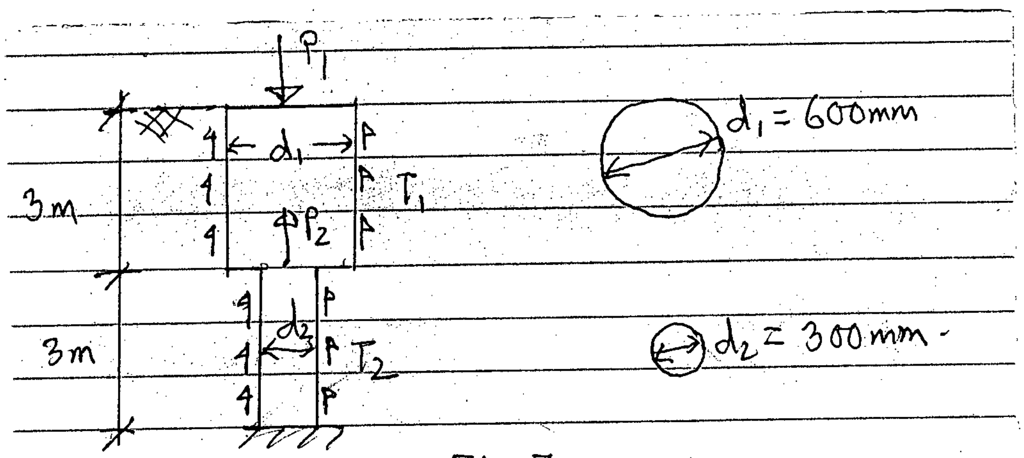

Fig. 7

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Introduce and explain isoparametric concept in finite element analysis. **(5)**

   (b) State three basic theorem on which isoparametric element is developed. **(6)**

   (c) Discuss convergence criteria for isoparametric elements. **($6\frac{1}{3}$)**

   (d) Explain the following terms: **(6)**

   (i) Subparametric element

   (ii) Superparametric element

6. (a) In Civil engineering problem derived global stiffness matrix is usually "banded" and "symmetric" – Explain the implications of these characteristics features in solving global FE equations by employing band solution technique. **($11\frac{1}{3}$)**

   (b) Explain the solution when band solution becomes more expensive than frontal solution technique in terms of memory requirements and storage time. **(12)**

7. (a) Why Gauss quadrature formula is preferred in finite element analysis? Write down the expression that Gauss method used to compute a function at predetermined sampling points. **($11\frac{1}{3}$)**

   (b) What is a numerical error? **(5)**

   (c) Explain Skyline storage technique with an example. **(7)**

8. (a) List four major discontinuities that may exist in a real structure and the necessity of considering those in discretizing a structure. Draw necessary sketches. **(16)**

   (b) Write down a short note on the effect of element aspect ratio on accuracy of a numerical solution. **($7\frac{1}{3}$)**

L-4/T-2/CE Date : 21/01/2016

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-2 B. Sc. Engineering Examinations 2013-2014

Sub : **CE 419** (Introduction to Finite Element Method)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks.

The symbols and abbreviation have their usual meaning.

Assume reasonable values for missing data if any.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Is FEM an exact method of analysis? Explain why? **(5)**

(b) For the spring system shown in Fig. 1, find the global stiffness matrix. **(12)**

(c) A shear wall is subjected to vertical and lateral loads as shown in Fig. 2. What constitutive law will you use for the problem? Will there be any strain in the z-direction? **($6\frac{1}{3}$)**

2. (a) What are the advantages of FEM over other matrix based methods of structural analysis? **(4)**

(b) Determine the support reaction forces at the two ends of the bar shown in Fig. 3, given the following, **(10)**

$P = 6.0 \times 10^4$ N $\quad E = 2.0 \times 10^4$ N/mm^2

$A = 250$ mm^2, $\quad L = 150$ mm, $\Delta = 1.2$ mm

(c) The beam shown in Fig. 4 is clamped at the two ends and acted upon by the distributed force p. Find the deflection and rotation at the center node. **($9\frac{1}{3}$)**

3. (a) What are the assumptions of linear static problems? **(3)**

(b) For the spring system shown in Fig. 5, **(12)**

$k_1 = 100$ N/mm, $k_2 = 200$ N/mm, $k_3 = 100$ N/mm

$P = 500$ N, $u_1 = u_3 = 0$

Find (i) the global stiffness matrix, (ii) displacement of node 2, (iii) the reaction forces at nodes 1 and 3 and (iv) the force in the spring 2.

(c) Determine the rotation at the right most hinge of the beam shown in Fig. 6. **($8\frac{1}{3}$)**

4. (a) A two noded bar element shown in Fig. 7 is subjected to a triangularly varying axial load with intensity zero at the i-node and maximum intensity q at the j-node. Determine the nodal force vector. **($8\frac{1}{3}$)**

(b) What do you understand by 'Transformation Matrix'? **(3)**

(c) A simple plane truss is made of two bars and loaded as shown in Fig. 8. Find (i) displacement of node 2 and (ii) strain-energy in each bar. **(12)**

Contd P/2

CE 419

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Explain the situation when band solution becomes more expensive than frontal solution technique in terms of memory requirements and storage time. ($7\frac{1}{3}$)

   (b) Name different types of symmetry considerations that an engineer may adopt in modeling a problem using finite element approach. **(8)**

   (c) In modeling a 2D space, a triangular element is often superior than a quadrilateral element while in modeling a 3D solid cube, tetrahedral elements are often better than brick elements. Explain. **(8)**

6. (a) Explain the isoparametric concept in finite element analysis. **(9)**

   (b) Determine shape functions for 4-noded rectangular elements. Use natural coordinate system. In this process show that shape function for i_{th} node can be written generally as: **($14\frac{1}{3}$)**

$$N_i = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i) \qquad \text{for } i = 1, 2, 3 \text{ and } 4.$$

7. (a) Why Gauss quadrate formula is preferred in finite element analysis? Write down also the expression that the Gauss method uses to compute a function at predetermined sampling points. **(10)**

   (b) Write a short note on the effect of element aspect ratio on accuracy. ($5\frac{1}{3}$)

   (c) Discuss convergence criteria for isoparametric elements. **(8)**

8. (a) "An inadequately defined displacement based finite element mesh may provide a lower bound solution" – Explain. **($10\frac{1}{3}$)**

   (b) Describe the sequence of the development of front in terms of letters used for nodes as the front creeps forward one element after another as shown in Figure 9. **(13)**

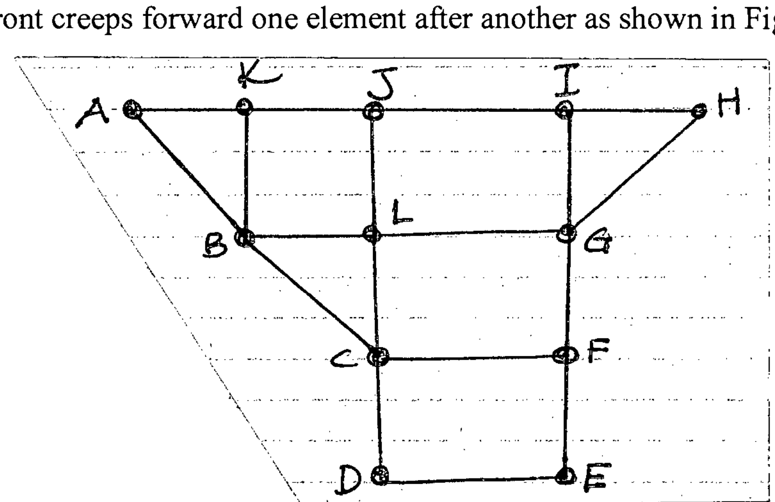

Fig. 9

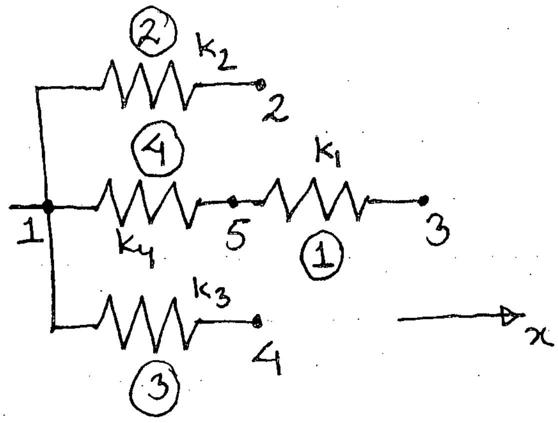

Fig. 1

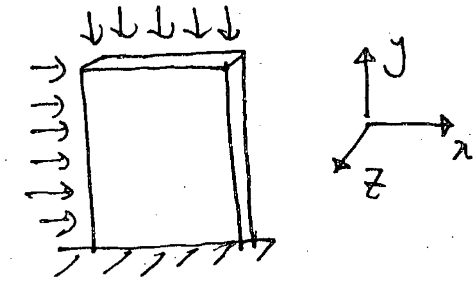

Fig. 2

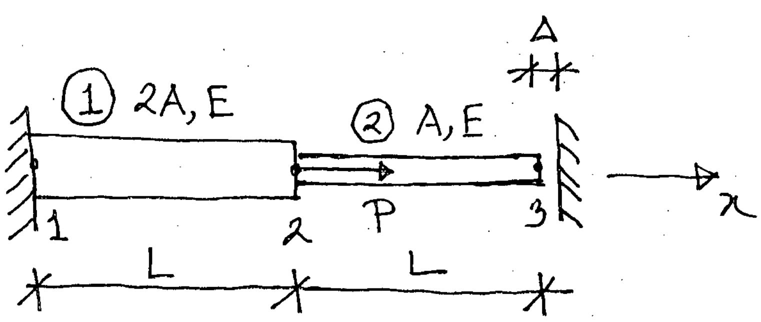

Fig. 3

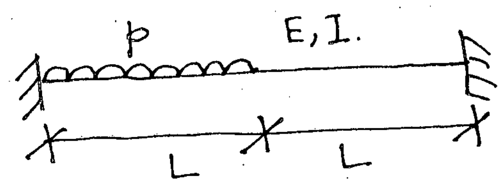

Fig. 4

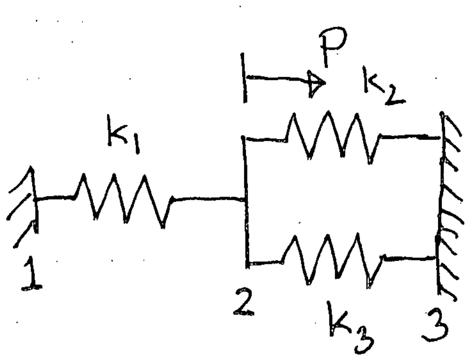

Fig. 5

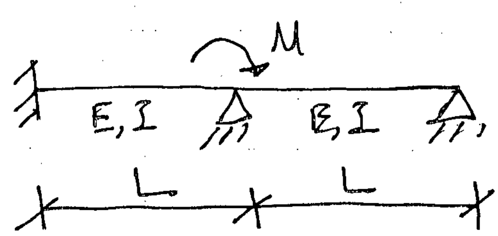

Fig. 6

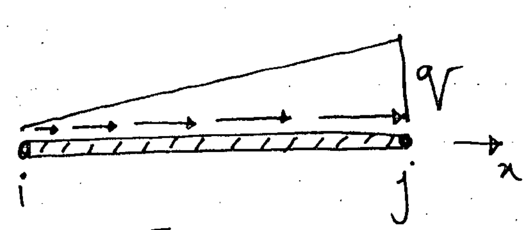

Fig. 7

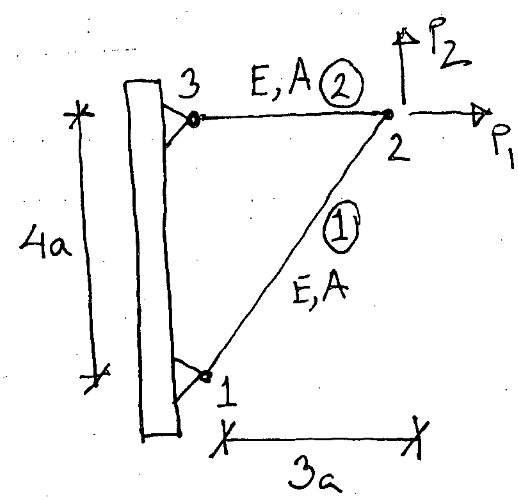

Fig. 8

CE 419

L-4/T-2/CE **Date : 10/08/2015**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-2 B. Sc. Engineering Examinations 2012-2013

Sub : **CE 419** (Introduction to Finite Element Method)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks.

The symbols have their usual meaning. Assume reasonable values for missing data if any.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Summarize the basic procedural steps that are followed in analyzing a structure using finite element method. **(6)**

(b) When there are several FEM packages are available, is there any need to study this method? Discuss. **(5)**

(c) State two problems where classical solutions will yield poorer results than FEM solutions. **(4)**

(d) Approximate the area of a circle by dividing it into a number of triangles. In this process show that $S_N = \pi R^2$ when $N \rightarrow \alpha$. Where, R = radius of the circle, N = number of triangles, S_N = Area of the circle. **(8⅓)**

2. (a) Write down the basic assumptions of linear static analysis. **(6)**

(b) Find the stresses in the two bar assembly as shown in Figure 1. The bar is loaded with force P, and constrained at two ends. Use 1-D bar elements. **(12)**

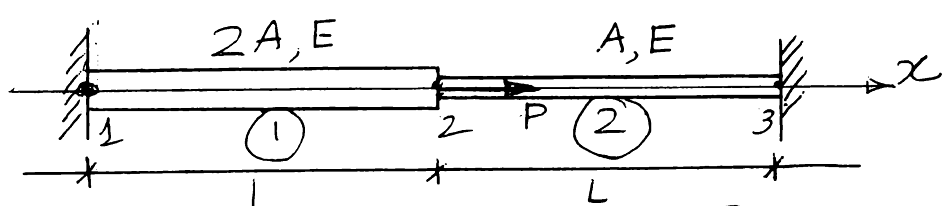

Figure 1

(c) If the two bars in Figure 1 are tappered in cross-sectional area, how will you model the problem in FEM? **(5⅓)**

3. (a) Explain the terms 'Plane stress' and 'Plane strain' problems. Give constitutive laws for these cases. **(4+4)**

(b) What is the difference between (i) Isotropic material, (ii) Orthotropic material in terms of mathematical description in FEM. **(4⅓)**

(c) Explain the terms: **(9)**

(i) Constant Strain triangle (CST)

(ii) Linear strain triangle (LST)

(iii) Quadratic strain triangle (QST)

(d) Define (i) Local coordinate system (ii) Natural Coordinate system with neat sketches. **(2)**

4. (a) Introduce and explain isoparametric concept in finite element analysis. **(5)**

(b) State three basic theorem on which isoparametric concept is developed. **(6)**

(c) Discuss convergence criteria for isoparametric elements. **($6\frac{1}{3}$)**

(d) Explain the following terms: **(6)**

(i) Sub parametric element

(ii) Super parametric element

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Explain the term 'shape functions'. Why polynomial terms are preferred for shape functions? **(6)**

(b) Assume reasonable shape functions and derive the strain displacement matrix (B) and element stiffness matrix (K) for the simple beam shown in Figure 2. The beam has a moment and a shear force at each of the ends. **(10)**

(c) Including axial stiffness of a bar element, write down the stiffness matrix for general 2D beam element. **($7\frac{1}{3}$)**

6. (a) Write short notes on **(8)**

(i) Jacobian matrix

(ii) Gaussian quadrature integration technique

(b) "In a displacement type finite element formulation with inadequately defined mesh, a lower bound solution is expected" Explain. **($7\frac{1}{3}$)**

(c) Assemble Jacobian matrix and strain displacement matrix corresponding to Gauss point (0.57735, 0.57735) for the element shown in Figure 3. Also indicate how do you proceed to assemble the element stiffness matrix. **(8)**

7. (a) Determine shape functions for 4-noded rectangular elements. Use natural coordinate system. In this process show that shape function for i_{th} node can be written generally as: **($13\frac{1}{3}$)**

$$N_i = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i) \text{ for i = 1, 2, 3 and 4.}$$

(b) The beam shown in Figure 4 is clamped at the two ends and acted upon by a force P and moment M in the mid span. Find the deflection and rotation at the central node (Node No. 2) and reaction forces at the two ends (Node No. 1 and 3). **(10)**

8. (a) The spring system shown in Figure 5 contains arbitrarily numbered nodes and elements. Find the global stiffness matrix. **(12)**

(b) Write a short note on the effect of element aspect ratio on the accuracy of the solution. **($5\frac{1}{3}$)**

(c) Discuss the various discontinuities to be considered in discretizing a structure in FEM. **(6)**

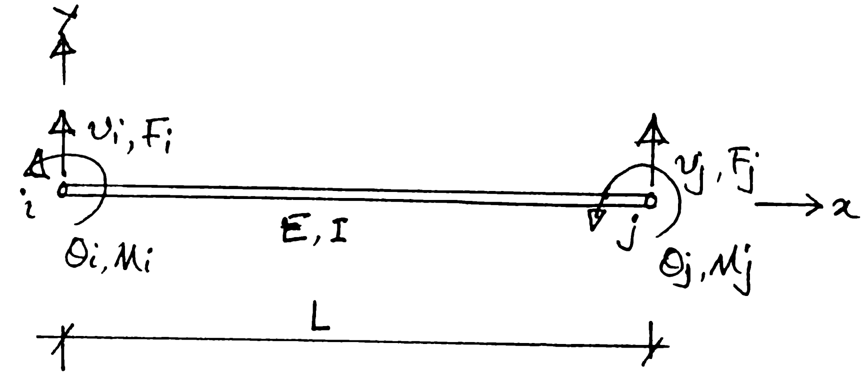

Figure 2

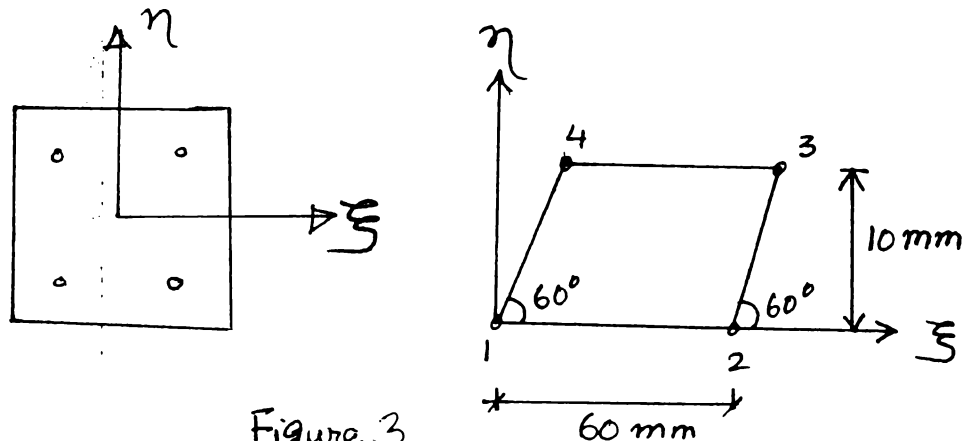

Figure 3

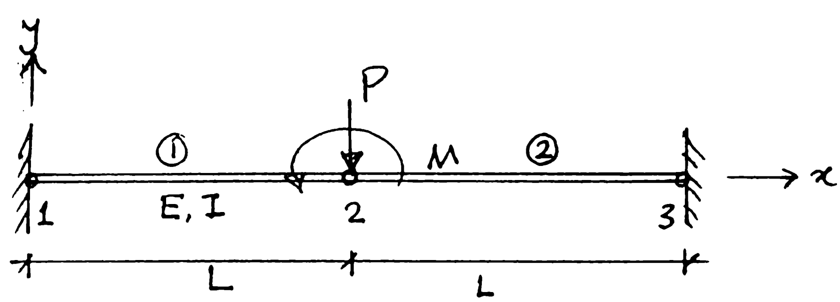

Figure 4

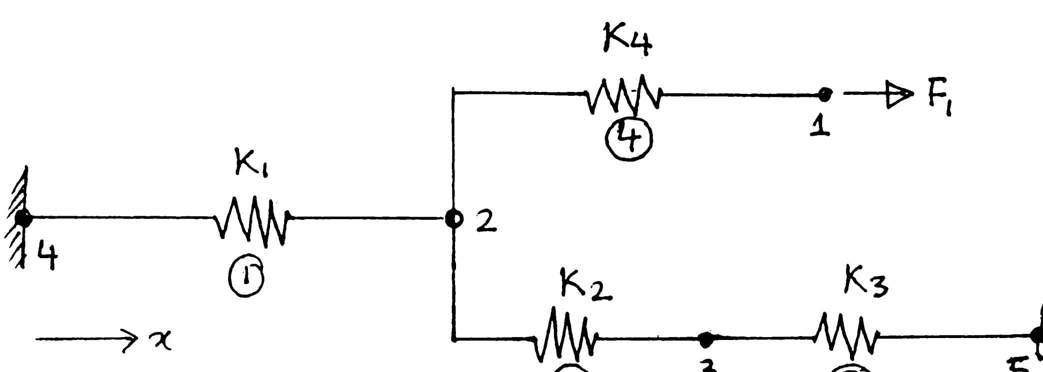

Figure 5

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-2 B. Sc. Engineering Examinations 2011-2012

Sub : **CE 419** (Introduction to Finite Element Method)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks.

The symbols carry their usual meaning.

Assume reasonable values for missing data, if any.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) What are the advantages of finite element method? **(6)**

   (b) Describe the basic steps that are followed in Finite Element formulation. **(11 ⅓)**

   (c) What is isoparametric formulation? Why isoparametric formulation is often known to perform better than super parametric formulation? **(6)**

2. (a) What is a constitutive relation? What is its purpose in a FEM program? **(7)**

   (b) Explain the following terms and write down constitutive laws for each of the cases: **(9)**

   (i) Plane stress problem (ii) Plane strain problem (iii) Axisymmetric problem.

   (c) In modeling a 2D space, a triangular element is often superior than a quadrilateral element; while modeling a 3D solid cube, tetrahedral elements are often better than brick elements. Explain. **(7 ⅓)**

3. (a) For the spring system with arbitrarily numbered nodes and elements as shown on Fig. 1, derive the global stiffness matrix (k). In this process show that the derived stiffness matrix is symmetric. Indicate the band width. **(11 ⅓)**

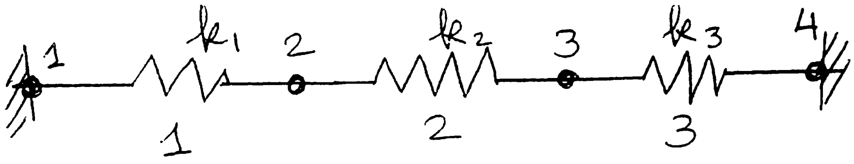

Fig. 1

   (b) What is a numerical error? **(3)**

   (c) Explain the following terms with necessary sketches: **(9)**

   (i) Constant strain triangle (CST) (ii) Linear strain triangle (LST) (iii) Quadratic strain triangle (QST).

Contd P/2

4. (a) Why Gauss quadrate formula is preferred in finite element analysis? Write down also the expression that the Gauss method uses to compute a function at predetermined sampling points. **(7)**

(b) Write a short note on the effect of element aspect ratio on accuracy. **(5)**

(c) List the important discontinuities that need to be considered in discretizing a structure in FEM. Explain each type with examples. **($11\frac{1}{3}$)**

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Summarize the basic procedural steps that are followed in FEM for analyzing a structure. **(6)**

(b) Approximate the area of a circle by dividing it into a number of triangles. In this process show that $S_N = \pi R^2$ when $N \to \infty$. Where R = radius of the circle, N = number of triangles, S_N = area of the circle. **(12)**

(c) Write four statements that describe the necessity and application of FEM in engineering. **($5\frac{1}{3}$)**

6. (a) Discuss with sketches four major discontinuities that may exist in a real structure and the necessity of considering those in discretizing a structure. **(12)**

(b) "In a displacement based finite element formulation with inadequately defined mesh, a lower bound solution is expected" – Explain. **($7\frac{1}{3}$)**

(c) What are the fundamental assumptions in analyzing a structure as a linear elastic problem? **(4)**

7. (a) Explain the term "shape functions". Why polynomial terms are preferred for shape function? **(12)**

(b) Explain the situation when band solution becomes more expensive than frontal solution technique in terms of memory requirements and storage time. **($11\frac{1}{3}$)**

8. (a) Introduce reasonable shape functions for a two nodded beam element (Fig. 2) and derive the element stiffness matrix. **($11\frac{1}{3}$)**

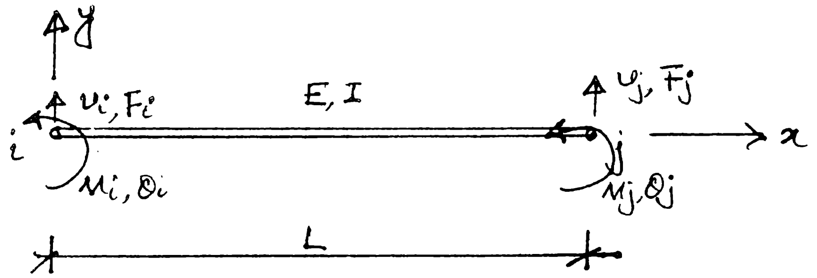

Fig. 2

Contd P/3

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Contd ... Q. No. 8

(b) A cantilever beam with a span length of 10 ft is subjected to an uniform distributed load 5 kip/ft and a point load of 12 kip as shown in Fig. 3. Write down the global FE equation for the beam and determined **(12)**

(i) deflection and rotation at the free end of the beam

(ii) shear force and bending moment at the fixed end of the beam.

Given, E = 3600 ksi and I = 1440 in^4.

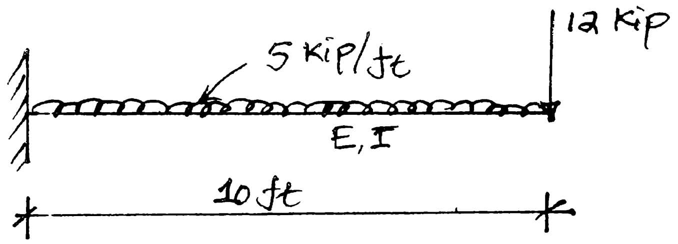


Fig. 3

L-4/T-2/CE **Date : 01/01/2013**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-2 B. Sc. Engineering Examinations 2010-2011

Sub : **CE 419** (Introduction to Finite Element Method)

Full Marks: 140 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

The symbols carry their usual meaning. Assume reasonable values for missing data, if any.

1. (a) What are the basic components of a general purpose finite element software? **($5\frac{1}{3}$)**

(b) Summarize the basic procedural steps that are followed in FEM for analyzing a structure. **(6)**

(c) For the spring system with arbitrarily defined nodes and elements as shown in Figure 1, find the global stiffness matrix. **(12)**

2. (a) Introduce reasonable shape functions for a two nodded beam element (Figure 2) and derive the element stiffness matrix (k). In this process also consider axial stiffness of a bar element and derive the stiffness matrix of a 2D general beam element. **(15)**

CE 419

(b) A cantilever beam shown in Figure 3 is subjected to distribute lateral load. Write down the global FE equation for the beam and find out (i) deflection and rotation at right end (ii) reaction force and moment at left end. **($8\frac{1}{3}$)**

3. (a) Introduce natural coordinate system and derive Jacobbean Matrix $\underline{J}$ for a constant strain triangle element. In this process derive the strain displacement matrix $\underline{B}$. **(15)**

(b) What is a constitutive relation? What is its purpose in a FEM program? **($8\frac{1}{3}$)**

4. (a) Explain the following terms and write down constitutive laws for each of the cases: **(9)**
   (i) Plane stress problem
   (ii) Plane strain problem
   (iii) Axisymmetric problem.

(b) Explain the term 'Shape Functions'. Why polynomials are preferred for shape functions? **(10)**

(c) What are the basic assumptions in analyzing a structure based on linear static analysis? **($4\frac{1}{3}$)**

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.
Symbols carry their usual meaning.

5. (a) Using the Weighted Residual approach, find out an approximate solution to the following differential equation. **($13\frac{1}{3}$)**

$$-\frac{d^2u}{dx^2} - u + x^2 = 0 \qquad 0 < x < 1$$

Given: u(0) = 0 and u(1) = 0.

(b) Using Three Point Gaussian Integration scheme, evaluate, $I = \int_{-1}^{1} e^{-3x/x^2}\, dx$ **(10)**

The Gaussian points and corresponding weights are:

Points	Weights
0	8/9
$\pm\sqrt{0.6}$	5/9

Contd P/3

CE 419

6. (a) How is a Finite Element Formulation classified based on the Gaussian Integration Scheme employed? **(6)**

(b) Express the Weak Formulation to the following differential equation for an element. **(17⅓)**

$$-\frac{d}{dx}\left(u\frac{du}{dx}\right)+f=0$$

Determine the integral expression of a member of the Coefficient Matrix or Stiffness Matrix of an element for the above equation.

7. (a) Member of a stiffness matrix for a problem is given by, **(13⅓)**

$$k_{ij}=\int_{x_A}^{x_B}(1+x)\frac{d\psi_i}{dx}\frac{d\psi_j}{dx}dx$$

Implement a linear element and derive the stiffness matrix.

(b) Suppose that the force-deformation relationship of a structural assemblage is given by, **(10)**

$$\begin{bmatrix} 4 & -3 & 6 & 2 \\ -3 & 2 & 5 & 3 \\ 6 & 5 & 1 & -4 \\ 2 & 3 & -4 & 3 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}=\begin{Bmatrix} 10 \\ R_2 \\ 20 \\ R_4 \end{Bmatrix}$$

The boundary condition is such that $u_2 = 2$ and $u_4 =$. Determine the unknown deformations u_1 and u3 and the unknown reactions R_2 and R_4.

8. (a) Assemble the Finite Element Equations for the structure shown in Fig. 4. **(15⅓)**

(b) Suppose nodal deformations of a quadratic element are $u_1 = 0.5$, $u_2 = 2$ and $u_3 = 0.75$. Determine u at $\bar{x} = 0.75$. **(8)**

L-4/T-2/CE Date : 11/01/2011

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-2 B. Sc. Engineering Examinations 2008-2009

Sub : **CE 419** (Introduction to Finite Element Method)

Full Marks: 140 Time : 3 Hours

The figures in the margin indicate full marks.

Symbols carry their usual meaning.

Assume reasonable values for any missing data.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) "In a displacement based finite element formulation with inadequately defined mesh, a lower bound solution is expected" – Explain. **(7 1/3)**

(b) List four major discontinuities that may exist in a real structure and the necessity of considering those in discretizing a structure. Draw necessary sketches. **(12)**

(c) Write a short note on the effect of element aspect ratio on accuracy of a numerical solution. **(4)**

2. (a) Explain the following terms with necessary sketches: **(12)**

(i) Constant strain triangle (CST)

(ii) Linear strain triangle (LST)

(iii) Quadratic strain triangle (QST)

(b) The beam element shown in Figure 1 has two degrees of freedom per node, i.e. i and j. Assume necessary shape functions for these degrees of freedom and derive strain displacement matrix $\underline{\underline{B}}$ for the element. **(11 1/3)**

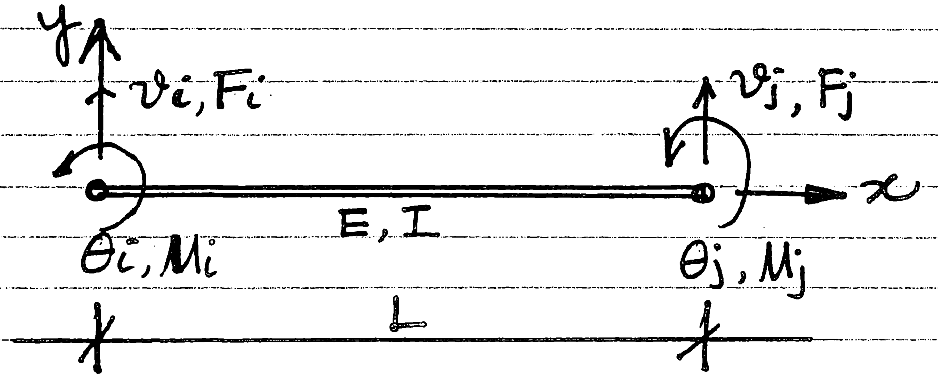


Figure 1

3. (a) The beam shown in Figure 2 is clamped at the two ends and acted upon by a force P and moment M in the mid-span. Find the deflection and rotation at the centre node and reaction forces at the two ends. (12)

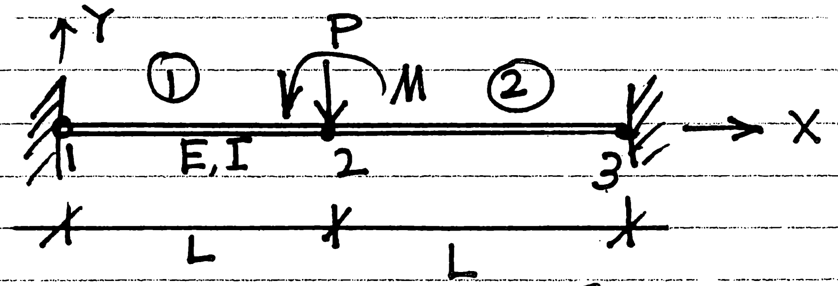


Figure 2

(b) A cantilever beam with distributed load, P as shown in Figure 3 needs to be analyzed. Calculate the work-equivalent nodal load for node 2 and find out deflection and rotation at the right end and the reaction force and moment at the left end. (11 ⅓)

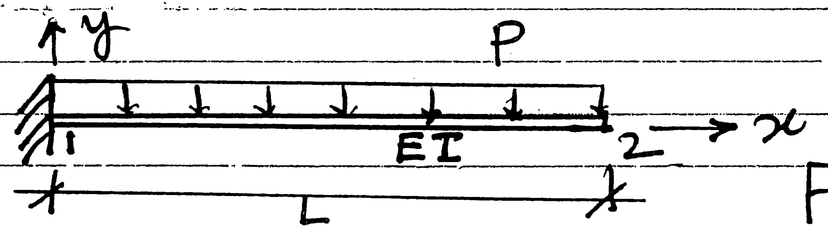
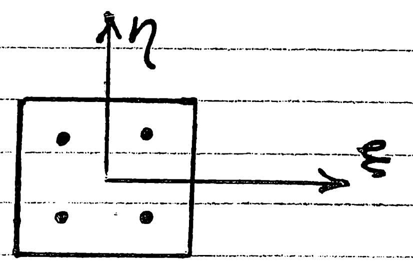


Figure 3

4. (a) Why Gauss quadrate formula is preferred in finite element analysis? Write down also the expression that the Gauss method uses to compute a function at predetermined sampling points. (7)

(b) Assemble Jacobian matrix and strain displacement matrix corresponding to the Gauss point (0.57735, 0.57735) for the element shown in Figure 4. Also indicate how do you proceed to assemble element stiffness matrix. (16 ⅓)



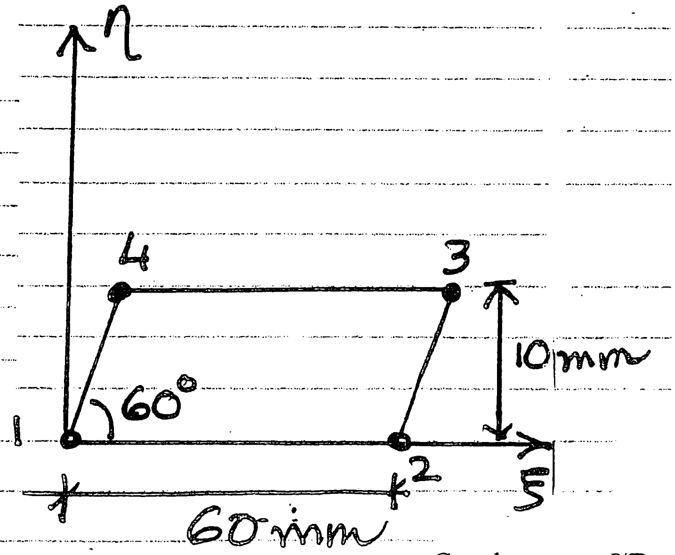


Figure 4

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CE 419

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) What are the basic components a general purpose finite element software? **(7)**

(b) What are the basic assumptions for analyzing a structure based on linear static analysis? **(6)**

(c) For the spring system with arbitrarily numbered nodes and elements shown in Fig. 5, find the global stiffness matrix. **(10 ⅓)**

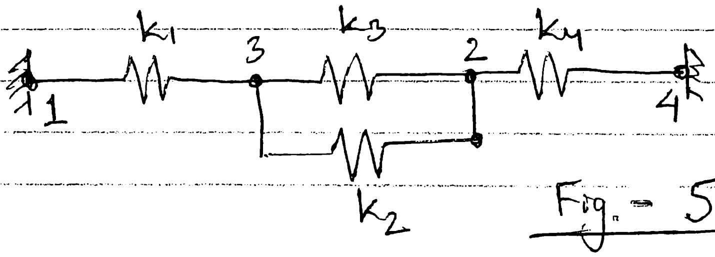

Fig. - 5

6. (a) Explain the following terms : **(6)**

(i) Nodes, Elements

(ii) Local coordinates, Global coordinates

(iii) Transformation matrix

(b) What is a constitutive relation? Write down the relation for a linear isotropic material. **(5 ⅓)**

(c) "A thin plate is subjected to forces in its plane only". Write down the stress strain relation along with necessary assumptions. **(6)**

(d) "A long body is subjected to significant lateral forces, but very little longitudinal forces". Write down stress-strain relations for this range of problem and consider necessary assumptions for the derivation. **(6)**

7. (a) Give illustrative examples to compare frontal solution technique with band and skyline techniques. Explain the advantages and disadvantages in terms of memory requirements. **(11 ⅓)**

(b) Describe the sequence of the development of front in terms of letters used for nodes as the front creeps forward one element after another as shown in Fig. 6. **(12)**

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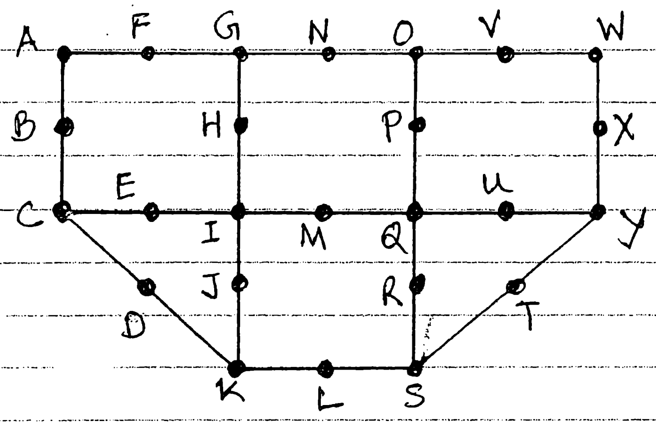

Fig.- 6

8. (a) Introduce and explain isoparametric concept in finite element analysis. (5)

(b) State the three basic theorem on which the isoparametric concept is developed. (6)

(c) Discuss the convergence criteria for isoparametric elements. (6⅓)

(d) Explain the following terms: (6)

(i) Subparametric element

(ii) Superparametric element

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-2 B. Sc. Engineering Examinations, 2007-2008

Sub : **CE 419** (Introduction to Finite Element Method)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks.

The symbols carry their usual meaning. Assume reasonable values for any missing data.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) What are the basic components of a general purpose finite element software? **(5)**

(b) What is a constitutive relation? Write down the relation for a linear isotropic material. **($6\frac{1}{3}$)**

(c) "A thin plate is subjected to forces in its plane only." Write down the stress-strain relation along with necessary assumptions. **(6)**

(d) "A long body is subjected to significant lateral forces but very little longitudinal forces." Write down stress-strain relations for this range of problem and consider necessary assumptions for the derivation. **(6)**

2. (a) Give illustrative examples to compare frontal solution technique with band and skyline techniques. Explain the advantages and disadvantages in terms of memory requirements. **($11\frac{1}{3}$)**

(b) Describe the sequence of the development of front in terms of the letters used for nodes, as the front creeps forward one element after another as shown in Figure 1. **(6)**

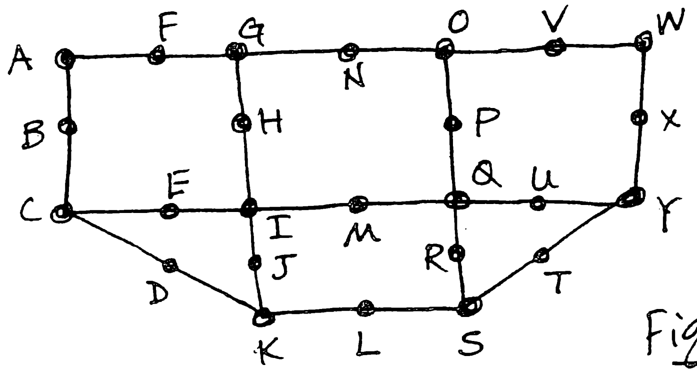

Figure 1

(c) What are the basic assumptions for analyzing a structure based on linear static analysis? **(6)**

Contd P/2

3. (a) The bar element shown in Figure 2 is subjected to axial forces, f_i and f_j at node 1 and node 2. Assume necessary shape functions for node i and j respectively and show that the element stiffness for the element can be expressed as: **(11⅓)**

$$\underline{\underline{k}} = \int_V \underline{\underline{B}}^T E \ \underline{\underline{B}} dV$$

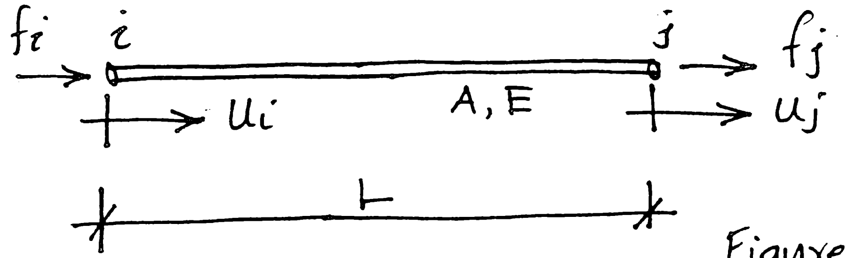

Figure 2

(b) Determine the support reaction forces at the two ends of the bar shown in Figure 3. Given the following: **(12)**

P = 6 × 10^4 N, E = 2 × 10^4 N/mm^2

A = 250 mm^2, L = 150 mm, Δ = 1.2 mm

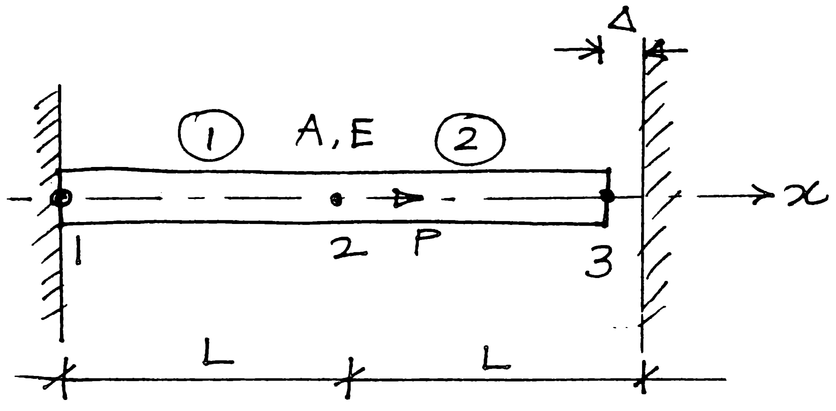

Figure 3

4. (a) Explain the types of discontinuities that may occur in a finite element mesh? What are the ways to minimize such discontinuities? **(5⅓+6=11⅓)**

(b) Write short notes on: **(4+4+4=12)**

(i) Effect of element aspect ratio on accuracy.

(ii) Numbering of nodes for bandwidth minimization.

(iii) Mesh refinement vs. higher order elements.

SECTION–B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) The beam shown in Fig. 4 has two degrees of freedom per node i.e., i and j. Assume necessary shape functions for these degrees of freedom and derive strain displacement matrix B for the element. **(11 ⅓)**

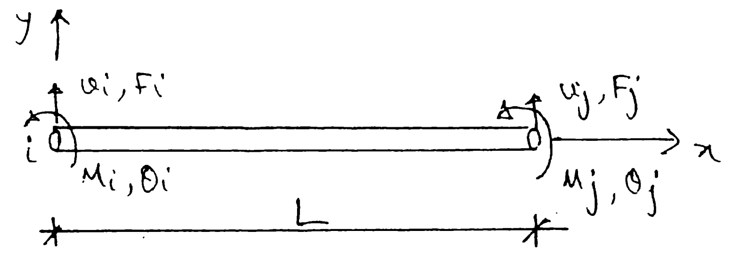

Fig. 4

(b) The bar element shown in Fig. 5 is subjected to axially distributed force, q. Employ proper shape functions to equate the Wq with U and obtain the equivalent nodal forces. **(12)**

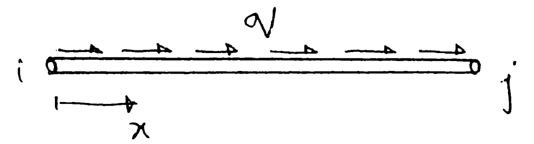

Fig. 5

6. (a) Explain the isoparametric concept in Finite Element analysis. **(8 ⅓)**

(b) State and explain the three basic laws on which the isoparametric concept is developed. **(9)**

(c) Explain the following terms: **(6)**

(i) Isoparametric elements

(ii) Subparametric elements

(iii) Superparametric elements.

7. (a) Explain the following terms: **(6)**

(i) Nodes, Elements

(ii) Local coordinates, Global coordinates

(iii) Transformation matrix

(b) Explain the terms with necessary sketches: **(6)**

(i) Constant strain triangle (CST).

(ii) Linear strain triangle (LST)

(iii) Quadratic strain triangle (QST)

(c) Explain the term "Shape Functions". Why polynomial terms are sometimes preferred for shape functions in finite element method? **(6)**

(d) State and explain the convergence requirements of polynomial shape functions. **($5\frac{1}{3}$)**

8. Analyze the beam shown in Fig. 6 by Finite Element Method and determine:

(a) Element stiffness matrices **($8\frac{1}{3}$)**

(b) Consistent load vectors **(9)**

(c) Deflections at mid spans for each of the two elements. **(6)**

Given: $E = 2 \times 10^5$ N/mm^2 $\quad$ $I = 5 \times 10^6$ mm^4

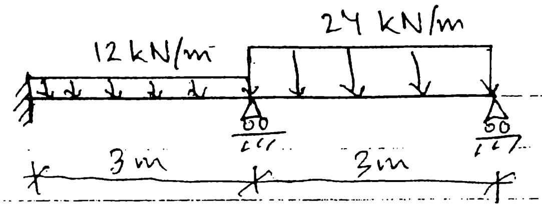

Fig. 6

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L–4/T–2 B. Sc. Engineering Examinations, 2006-2007

Sub : **CE 419** (Introduction to Finite Element Method)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

Assume reasonable value for missing data, if any.

1. (a) Explain the concept of FEM briefly and outline the procedure. (6)

   (b) State the advantages and disadvantages of FEM over classical method. (9)

   (c) When there are several FEM packages are available for analysing a problem, state the major points a user must know clearly to select the best FEM package for his problem and apply the package properly to solve the problem. (8 $\frac{1}{3}$)

2. (a) Draw a typical three dimensional element and indicate the state of stress in their positive senses. (6)

   (b) State and explain the generalized Hook's Law. (4)

   (c) Explain the terms, 'Anisotropic', 'Orthotropic', and 'Isotropic' as applied to material properties. (9)

   (d) Explain (i) Plane stress problem, (ii) Plane strain problem. Give necessary sketches. (4 $\frac{1}{3}$)

3. (a) Define stiffness matrix and explain its special features. (8)

   (b) State and explain three approaches to reduce the memory requirement in storing stiffness matrix. (12)

   (c) Explain the term: "Skyline Storage Technique. (3 $\frac{1}{3}$)

4. (a) Explain the following terms: (6)

   (i) Nodes, elements

   (ii) Local coordinates, global coordinates and natural coordinates

   (iii) Higher order elements and Lower order elements.

   (b) Explain the terms with necessary sketches: (6)

   (i) Constant strain triangle (CST)

   (ii) Linear strain triangle (LST)

   (iii) Quadratic strain triangle (QST).

   (c) Explain the term "Shape Functions". Why polynomial terms are sometimes preferred for shape functions in finite element method? (6)

   (d) State and explain the convergence requirements of polynomial shape functions. (5 $\frac{1}{3}$)

Contd P/2

SECTION–B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Selecting interpolation function for a bar element in its natural coordinates, find the strain matrix. **($7\frac{1}{3}$)**

(b) Using polynomial functions (generalized coordinates), determine the shape functions for a two-noded beam element. **(8)**

(c) Differentiate between the terms 'lumped loads' and 'consistent loads'. **(8)**

6. (a) Write short notes on: **(12)**
   (i) Effect of element aspect ratio on accuracy
   (ii) Numbering nodes for band width minimization
   (iii) Mesh refinement vs. order of elements.

(b) State four major discontinuities that may exist in a real structure and the necessity of considering those continuities in discretizing a structure for finite element analysis. Draw necessary sketches. **($11\frac{1}{3}$)**

7. (a) State the differences and the similarities between a bar element and a truss element. **(4)**

(b) State the major steps that one will follow in analysing a bar in finite element approach and thereby obtain the unknown reactions. **($4\frac{1}{3}$)**

(c) Determine the nodal displacements at node 2, stresses at each material and support reactions in the bar shown in Fig. 1 due to the applied force $P = 400 \times 10^3$N and temperature rise of 30°C. Given: **(15)**

$A_1 = 2400$ mm^2, $A_2 = 1200$ mm^2
$l_1 = 300$ mm, $l_2 = 400$ mm
$E_1 = 0.7 \times 10^5$ N/mm^2, $E_2 = 2 \times 10^5$ N/mm^2
$\alpha_1 = 22 \times 10^{-6}$ /°C, $\alpha_2 = 11 \times 10^{-6}$ /°C

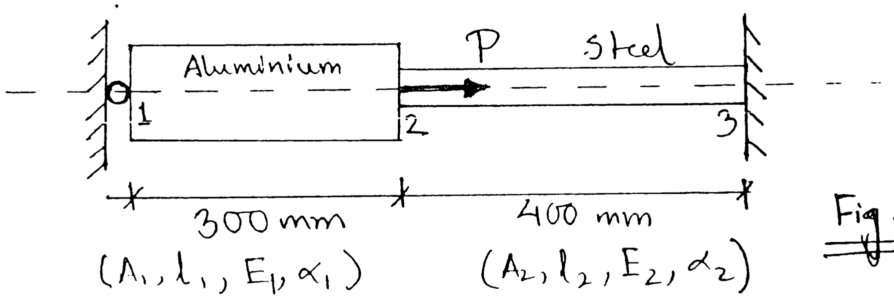

Fig. 1

8. (a) Introduce and explain the isoparametric concept in finite element analysis. **(5)**

(b) State the three basic theorems on which the isoparametric concept is developed. **(6)**

(c) Discuss the convergence criteria for isoparametric elements. **($6\frac{1}{3}$)**

(d) Explain the following terms: **(6)**
   (i) Subparametric and (ii) Superparametric elements.

L-4/T-2/CE Date : 30/4/2007

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-2 B. Sc. Engineering Examinations 2004-2005

Sub : **CE 419** (Introduction of Finite Element Method)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks.

Assume reasonable missing data.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are FOUR questions in this Section. Answer any THREE.

1. (a) Discuss the step by step procedure of deriving stiffness equation in finite element method. **(10)**

(b) Explain how does a bar element act like a spring element when subjected to axial deformation. **(6)**

(c) Describe the advantages of finite element method in solving boundary value problem. **($7\frac{1}{3}$)**

2. (a) Why shape functions (interpolation functions) are used in finite element formulation? What are its characteristics? **($8\frac{1}{3}$)**

(b) Derive the stiffness matrix, k i.e $\int_V B^T E B \, dv$ for one dimensional bar element. Also calculate the Jacobian matrix for the bar element. Assume that the element is 2 node line element. **(9)**

(c) The bar element is subjected to a uniformly distributed load along the length. Using the principle of work done, derive the equivalent nodal forces for the element. **(6)**

3. (a) Consider a 3-rode line element (Fig. 1). Derive the shape functions in natural co-ordinate (ζ) and determine the Jocobian matrix for the same. **(15)**

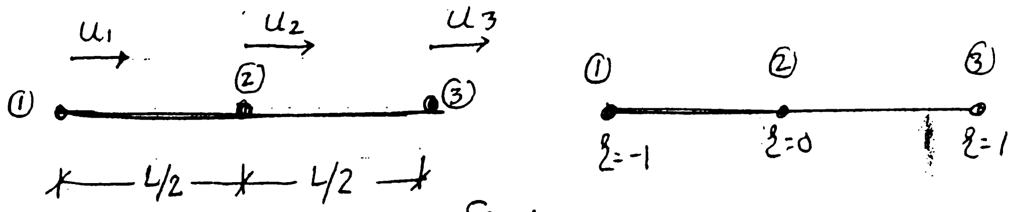

Fig. 1

(b) Calculate the strain displacement transformation matrix, B for the element shown in Fig. 1. **($8\frac{1}{3}$)**

CE 419

4. (a) Write down the expression for strain energy stored in a beam element and mention the meaning of each term. **(5)**

(b) Derive strain displacement transformation matrix i.e. B matrix for the beam element. **(6)**

(c) Write down the expressions for stress-strain-Temperature (constitutive) relations for plane stress and plane strain problems. **(6)**

(d) A cantilever beam of length $10'$ is subjected to a uniform distributed load of 5 kip/ft. The depth and width of the beam are $36''$ and $10''$ respectively. Is it a plane stress or plane strain problem? Justify your answer. **($6\frac{1}{3}$)**

SECTION – B

There are FOUR questions in this Section. Answer any THREE.

5. (a) What are the two basic types of idealization of 2D problems? Write down the elasticity matrix for each of the two cases. **(6)**

(b) For the CST element shown in Fig. 2, determine the stiffness matrix for plane stress. **($17\frac{1}{3}$)**

6. (a) Why do we need an integration scheme in FEM? What is the advantage of Gaussian quadrature? **(5)**

(b) Derive the strain-displacement matrix for a Q4 element. **($18\frac{1}{3}$)**

7. (a) Derive the shape functions of the 9-noded triangular element shown in Fig. 3. **(15)**

(b) Integrate the following using 3-point Gauss rule. **($8\frac{1}{3}$)**

$$I = \int_{-1}^{1}\int_{-1}^{1}\left(xy^2 \cos^2 x \sin y\right)dxdy$$

Given : The integration points are at $-\sqrt{0.6}$, 0, $\sqrt{0.6}$ and the respective weights are $5/9$, $8/9$, $5/9$.

8. (a) What is the basic difference between Banded Matrix Solution and Frontal Solution? **(4)**

(b) Solve the problem shown in Fig. 4 using Frontal Solution technique. Suppose, $W_1 = 0$, $W_4 = 1$. **($19\frac{1}{3}$)**

Given : For a structure shown in Fig. 5, the element stiffness matrix is $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

and the force vector is $\begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$.

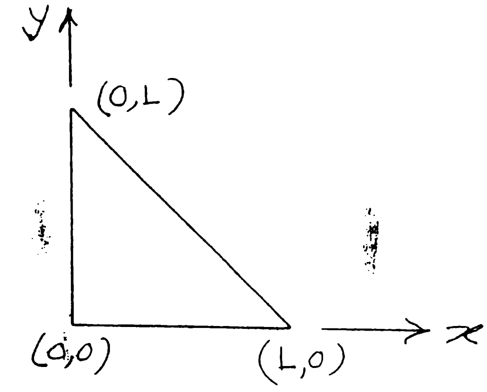

Fig. 2

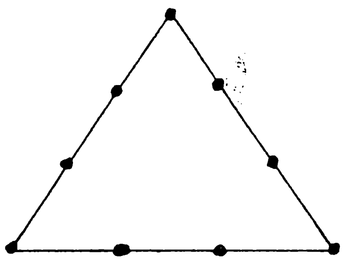

Fig. 3

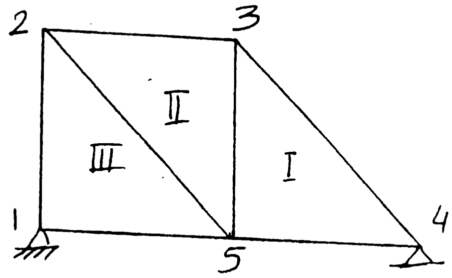

Fig. 4

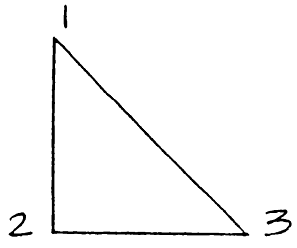

Fig. 5