Lec - 1

Date - 04.02.18.

CE - 419

2hrs/week.

SUNDAY / WEDNESDAY.

+ Class test - 3
+ Assignment
+ Attendance (Collective ...)
+ Continous assessment

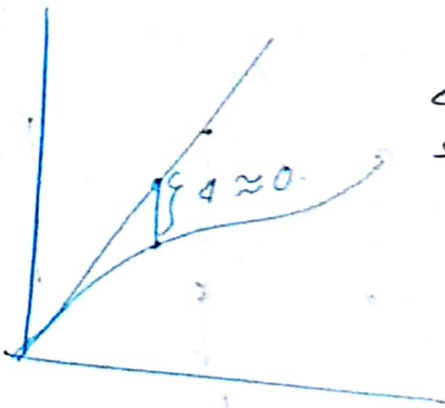
Mechanics

→ Structural Analysis → CE 311, CE 313, CE 411 → CE 419.

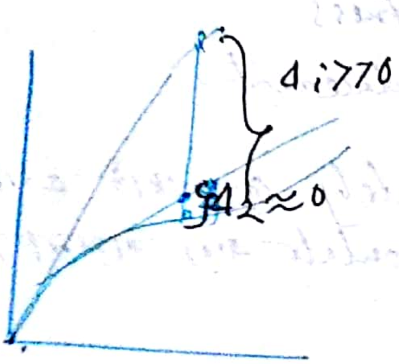
→ Structural Design

1950 — Computer আসাতে

1960 ⟶ stiffness, flexibility.



$\Delta \approx 0$ হলে sol^n-এ reach করা হয়।



$\Delta_2 \approx 0$ হলে sol^n reach করা।

"Art of Approximation"

+ Next sunday ক্লাস হবে।

Lec-2.

Date - 08.04.18

Chapter - 1

Introduction.

→ FEM as general procedure for analysis.

→ Applicable areas.

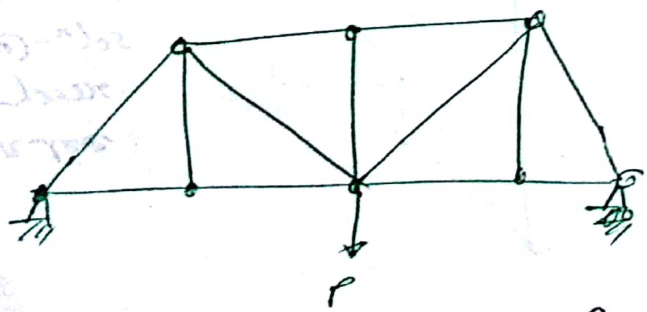
→ FEM vs. classical method

→ Brief history of FEM

→ Caution ~~of~~ about using FEM.

→ Components of FEM

Understanding the system:



যেকোনো structure analysis-এ যেগুলো খেয়াল রাখা লাগে-

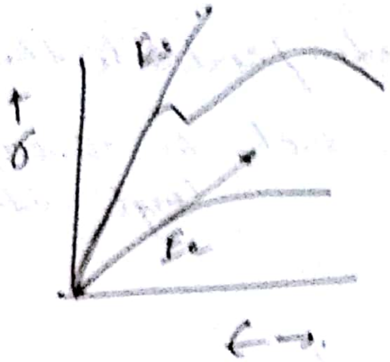
1. Load
2. Boundary condition
3. Stiffness
4. Displacement.

→ roller-এর ক্ষেত্রে consider করি smooth, frictionless pin. But practically হয়না.

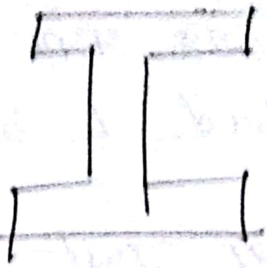
→ unfortunately কোনোটাই সঠিক-ভাবে calculate করা যায়না.

→ Load কখনো fixed হয়না, unpredictable

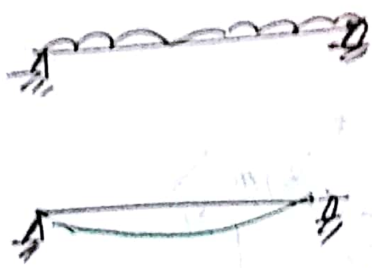
→ Stiffness, $k = f\left(\frac{EI}{L}\right)$



* E const থাকে
+ casting এর সময়
  I change হয়ে যায়।



→ ৮. Displacement এর ক্ষেত্রে.



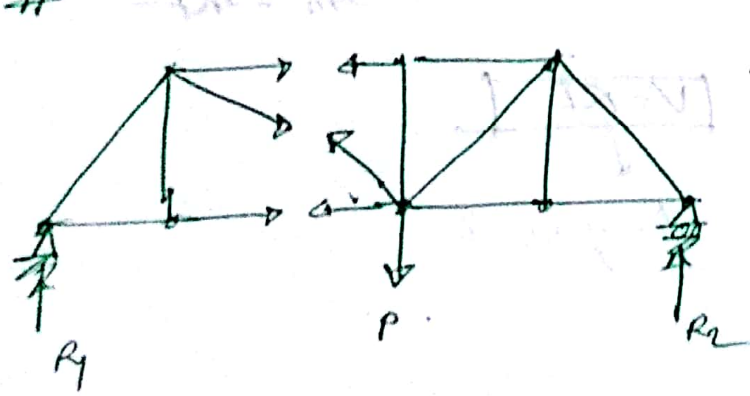
assume করে নিই
* displacement = 0 অবস্থায় — load.
But practically displaced অবস্থায় থাকে.

+ So আমাদের assumptions and approximation বুঝে আমরা করতে হবে।

→ Rational estimation.

→ Interpretation.

#



* Σ Internal forces = Σ External forces
+ Equilibrium conditions.

$$\boxed{F = k\Delta.}$$

code — code

$f\left(\frac{EI}{L}\right)$

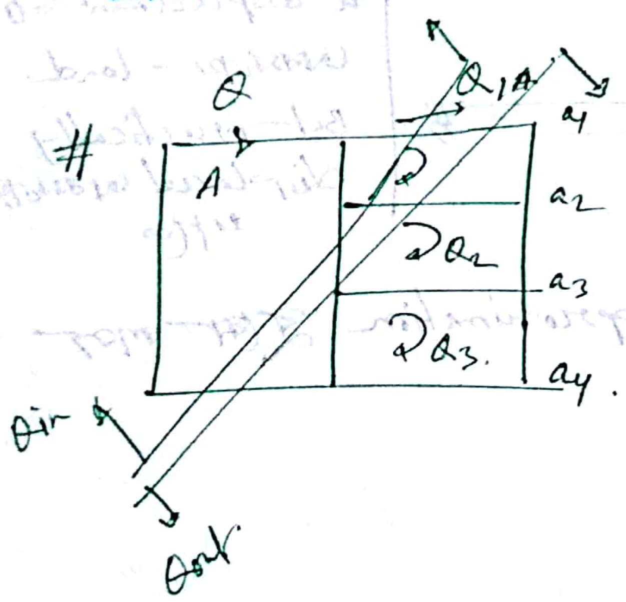
force is always changeable but eqn same

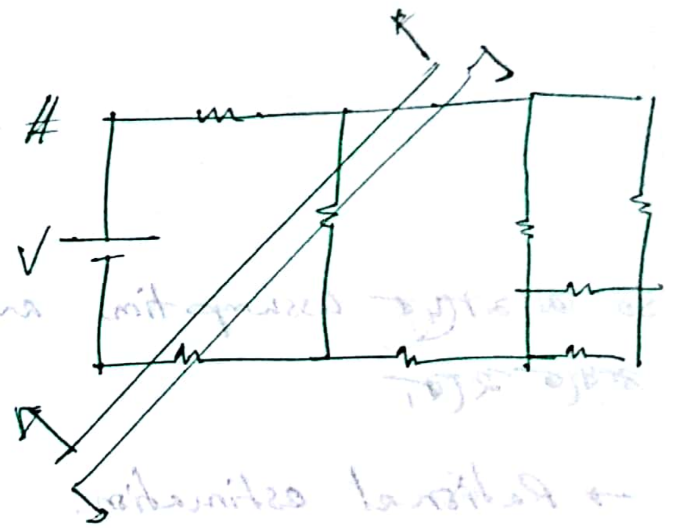
* Design এর সময় যদি বলা হয় wooden/concrete then E fixed. L is fixed by condition such as where length width

* আমরা just I calculate করি.

* force & displacement তো হবে তা-code দেখলে আসে.

Applications





Hardy cross method →

$Q_{in} = Q_{out}$

Equilibrium →

$\boxed{Q = Av}$

Design of the area

Equilibrium → $V_{rise} = V_{drop}$.

$I_{in} = I_{out}$

$\boxed{V = RI}$

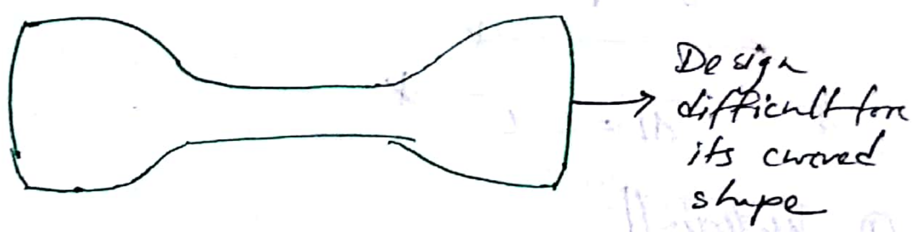
"Design" I.

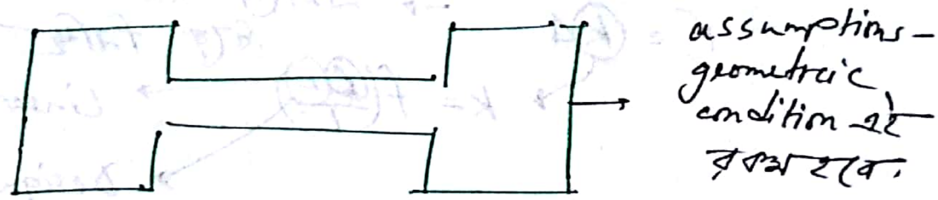
FEM structure -এর জন্য হলেও সব dept -এ applicatn

+ FEM-এর একটি limitations → continuous media.

(যেমন Q flow থাকলে Q_1, Q_2 থাকতে হবে)

Applicable areas:

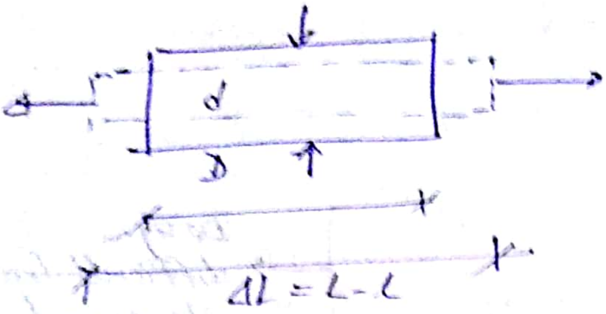




Lec-3.

Date-18.04.18

FEM. $f = kd$



- practically এরকমের Deformation হয়না, necking হওয়া শুরু হয়।

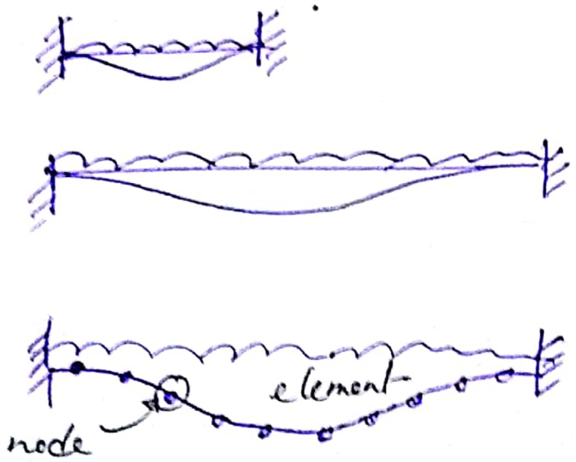
① Material

$F = k d$ → এখানে assume করা হয় material টা linear. হবে নির্দিষ্ট

$k = f\left(\frac{AE}{L}\right)$ → Linear/non linear → Design

② Load

③ Geometry



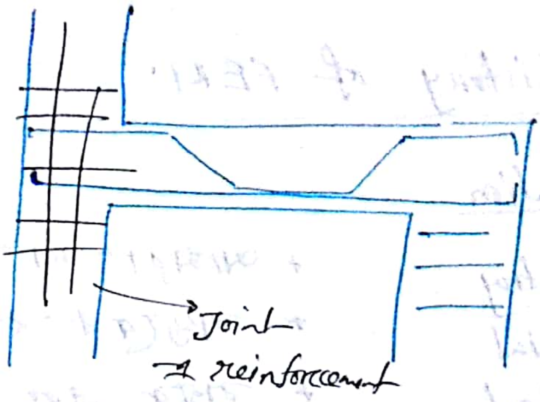
Different consideration

- Beam যেটা হলো self wt এর Deflect হবে, length বাড়লে self wt এ already deflected হয়ে যায়, তাই deflected shape এর উপর load

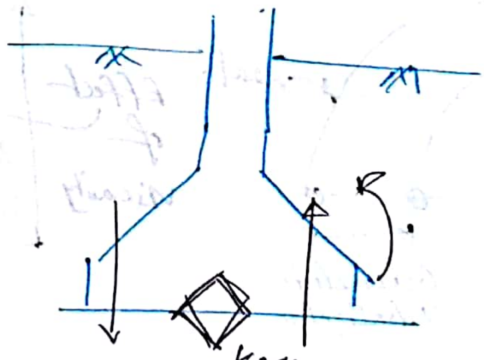
— Divide the structure into finite number of elements connected at nodes.

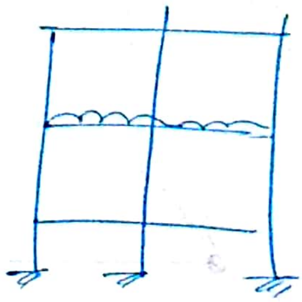
- অসংখ্য Infinite হবেনা, তাই FEM.

(4) Boundary conditions :



* Beam থেকে column-এ load গেলে 1 : 1 ratio অনুযায়ী load share হয়, But beam column-এর Comparison-এ ছোট হলে load share অর্থ 1:3 হতে পারে।





* এই সমস্যাগুলোর জন্য normal procedure-এ এদেরকে solve করা যাবেনা; So,
  - Approximation
  - Esstimation
  - Rationality

  } Important

* FEM
↓
CAD → Computer Aided Design & Drafting
CAM → " " Manufacturing

* mobile tower-এর design-এ stiffness method-এ করা যায়না, FEM
* Bolt-এর জন্য ড্রইং করেছে FEM

Lec-4.

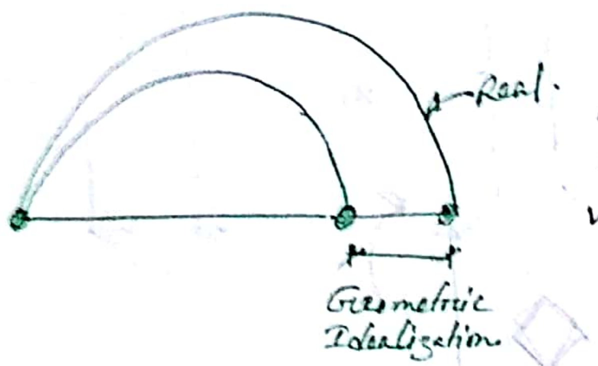
Date – 22.04.18

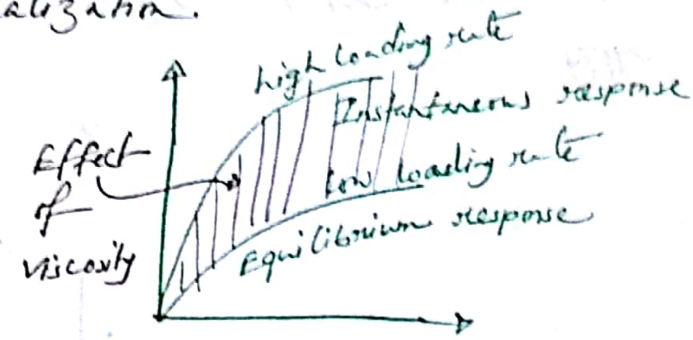
Brief history of FEM:

Idealization:

- Geometry
- Material
- Numerical.

+ আমরা যা দেখব তা বাস্তব.
+ বাস্তবের কাছাকাছি → Analysis এ হয়।
+ বাস্তব এবং বাস্তবের কাছাকাছির মধ্যে difference হল idealization.

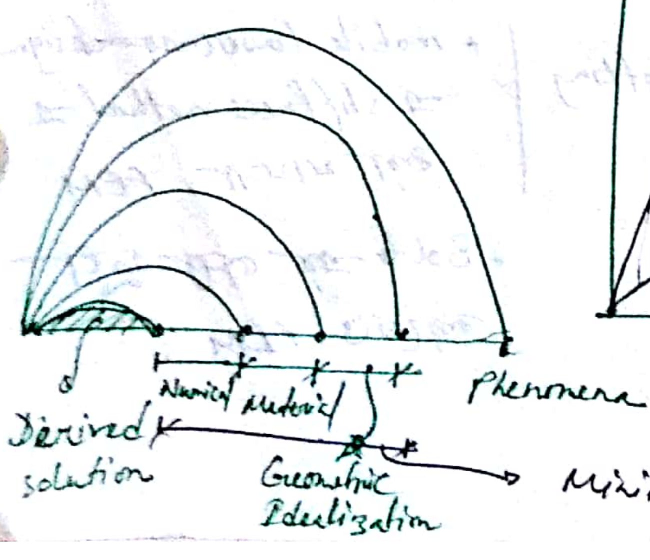


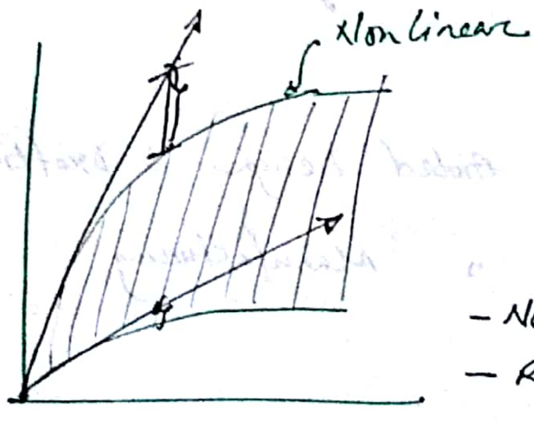


+ যেকোনো material এ দুইটা response থাকে। (Loading rate এর উপর)

① → Instantaneous response
② → Equilibrium response

Design এর সময় lowest loading rate ধরে যাতে lowest possible strength পাওয়া যায়।





- Viscosity.
- Plasticity.

- Newton raphson
- Runge kutta

solution method

What you see depends on what you are looking for → [illegible]

Limits of possible errors :

components of FE software of today :

- Preprocessor → Geometric idealization
- FEM program → Material, Numerical.
- Post-processor

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25.04.18

Lec-5:

Chapter-2.

\# Basic eqn of elasticity.

\# Idealization

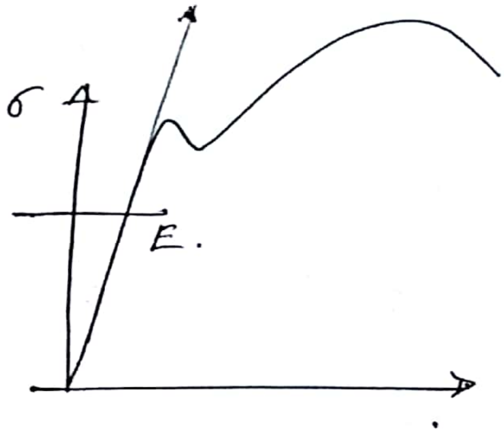
- → Material behaviour
- → Geometry.

+ Nearest/rational দুইটা part নিয়ে আলোচনা করতে হবে.

\# Material behaviour

Time dependent.

- → Viscosity.
- → Creep.



Plasticity.

Nonlinear elasticity.

+ simplicity-এর জন্য Hair-কে 1D বলে assume করা হয়।

Objectives of an Idealized Model.
- ≡ Nearest / Rational
- ≡ simplest possible solution for a purposeful analysis for use in design

* Design এ যে load ধরবে তা sustained load না, তা transient load → Linear FEM is enough

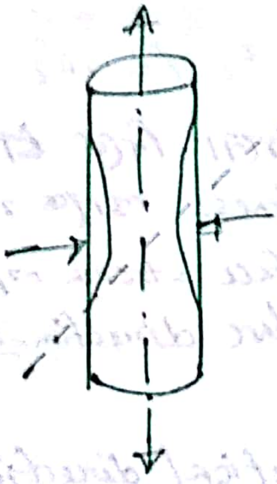
* $\boxed{\text{Art : 2.6}}$

Linear constitutive Relation

$$\{\sigma\} = \{D\}\{\varepsilon\}$$

↑ Elasticity matrix [6X6] (Describes the mat. behaviour)

* $\boxed{\text{Eq}^n : 2.7}$

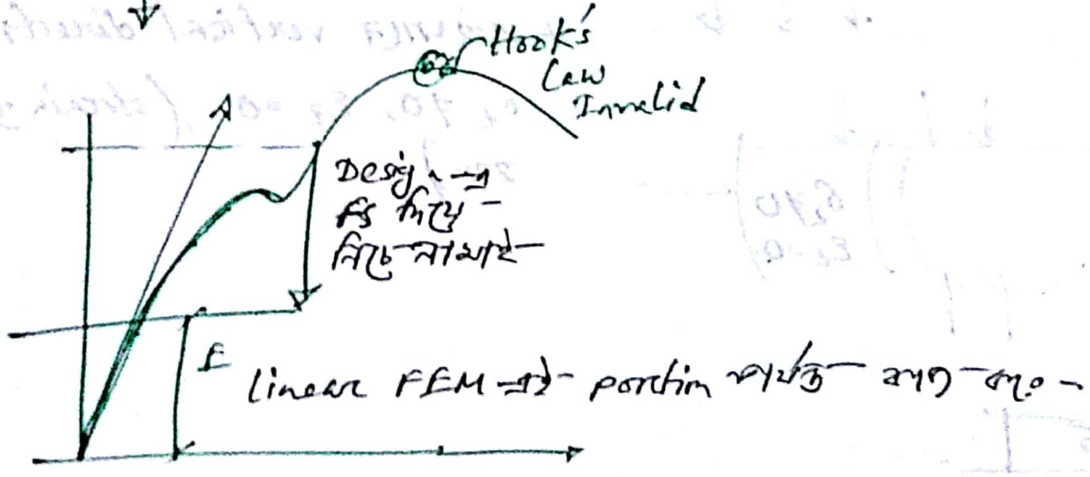


→ Isotropic

→ Anisotropic

* In one direction : $\sigma = E\varepsilon$

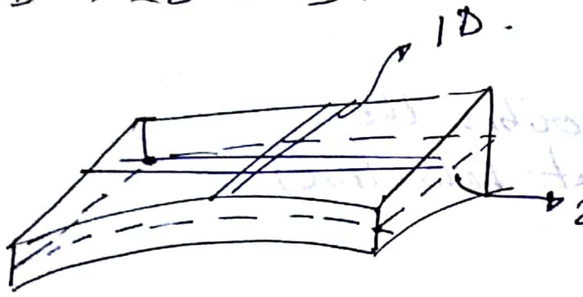
↑ Hook's Law.



Exam-এ আসবে →.

$\boxed{\text{Page-16}}$ — Plane stress.
— plane strain.
— Axisymmetric Prob.

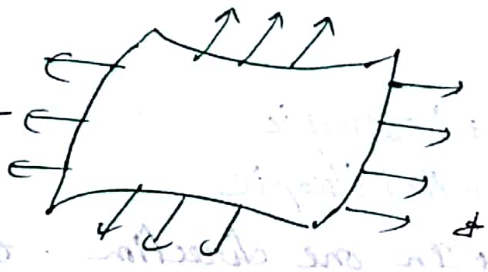
+ 3D → 2D → 1D.



+ slab-এর thickness variable → 3D.

Plane stress Problem

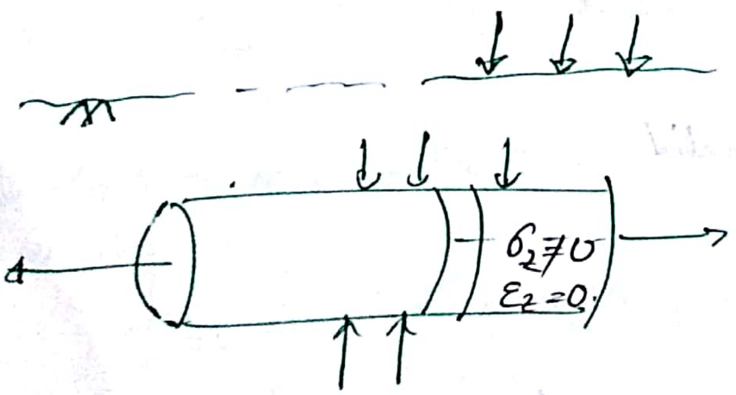
$\boxed{\text{Eq}^n\ 2.14}$



$\varepsilon_z \neq 0$, $\sigma_z = 0$.

+ যেকোনো দিকে টানলে thickness কমতে থাকে,
+ surface strain → perpendicular direction-এ.

plane strain :

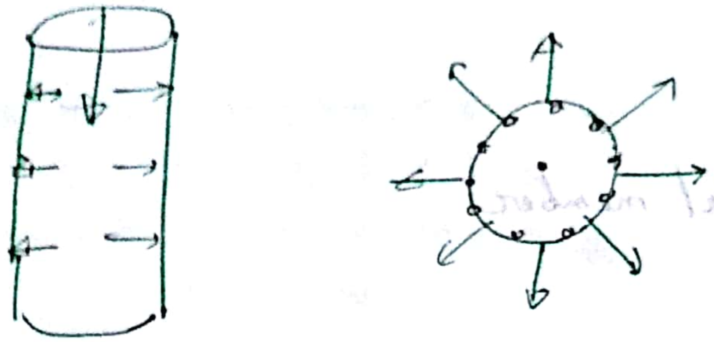


+ এইখানে vertical direction-এ $\sigma_z \neq 0$, $\varepsilon_z = 0$. (strain zero হয়)

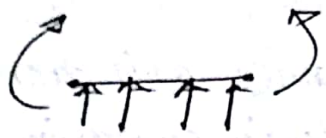
$\boxed{\text{Eq}^n - 2.15}$

Axisymmetric Problem:

+ wind load থাকলে non-axisymmetric



Idealization → 3D → 2D করতে হবে either
- Plane stress
- Plane strain
- Axisymmetric



+ 2D থেকে 1D করলে Beam-এর অনেক node দিয়ে connect করতে হবে।

+ CT → most probably 8 তারিখে.

Assignment → 2.6 থেকে plane stress নিজে নিজে করতে হবে.

Lec-6.

Date – 06.05.18

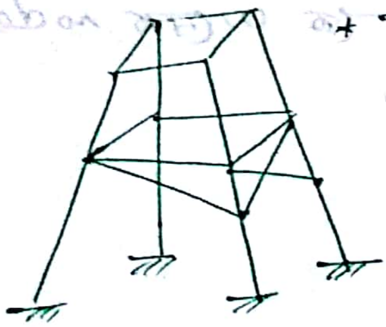
Spring element :

3D → 2D → 1D .

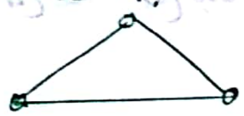
+ This is an uniaxial member

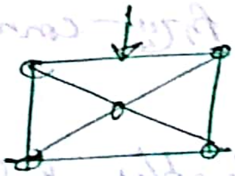
Tension
/
Compression

+ axis বরাবর load দিলে deflection কোথাও হবে না বেঁকে যাবে.



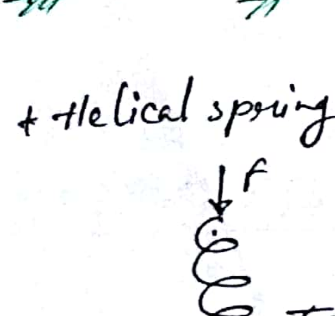
+ Triangulation করা হলে stable হয়।





+ এখানে axis বরাবর tension compression নিতে পারে; moment zero হয়।

Truss.

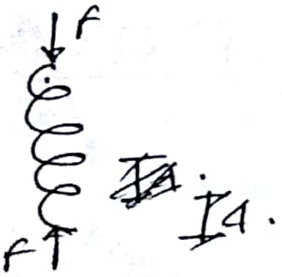


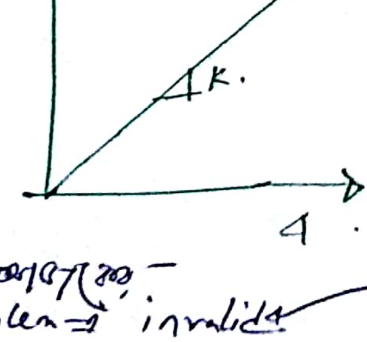
$f = k\Delta$.

k → spring constant.

+ K দেওয়া থাকতে হবে.

+ Helical spring এর ক্ষেত্রে ⇒





$f = k\Delta$.

$\Rightarrow k = \frac{f}{\Delta}$.

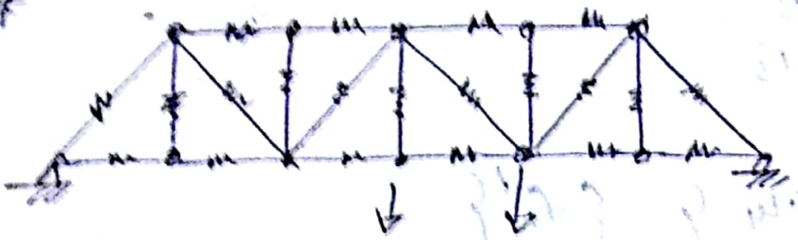
+ May be diff. in tension & compression

অনেকক্ষেত্রে problem ⇒ invalid

+ Euler buckling !!
f (slenderness ratio)

$\boxed{+ f \text{ (torsion in spring element)}}$ ⇒ this is also invalid

#



Anything important is simple.

+ simplest form হলেও আবার হইতেও formal form-এ convert করা।

$\xrightarrow{x}$

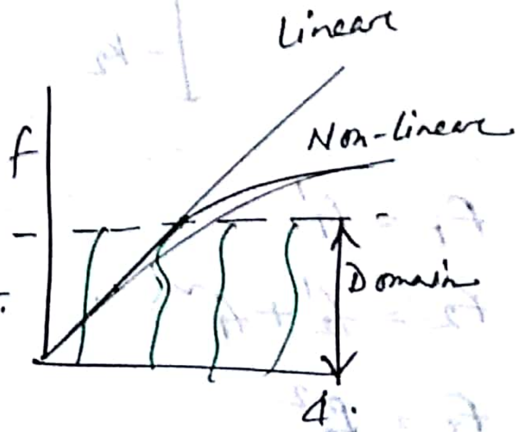
f_s, u_s

f_i, u_i

$ku = f$

$F = k\Delta$.

+ Non linear portion বাদ দেওয়া।



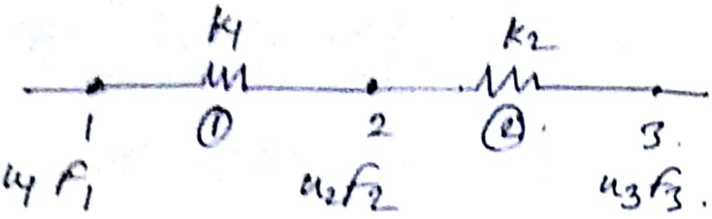
@ i : $f_i = -F = -k(u_i - u_j) = ku_i - ku_j$

@ j : $f_j = +F = k(u_i - u_j) = -ku_i + ku_j$.

In matrix

$$\begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} = \begin{Bmatrix} f_i \\ f_j \end{Bmatrix}$$

Spring system :



Element-1 :
$$\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} f_1^1 \\ f_2^1 \end{Bmatrix}$$

Element-2 :
$$\begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{Bmatrix} f_1^2 \\ f_2^2 \end{Bmatrix}$$

$$f_1 = f_1^1$$
$$f_2 = f_2^1 + f_1^2$$
$$f_3 = f_2^2.$$

$$f_1 = k_1u_1 - k_1u_2$$
$$f_2 = -k_1u_1 + (k_1+k_2)u_2 - k_2u_3.$$
$$f_3 = -k_2u_2 + k_2u_3.$$

$$\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

$$Ku = F.$$

Reading assignment (14-19)

Page-19

check. – Deformed shape
– Balance of ext. force
– Order of magnitude

Notes about the spring element.

- Suitable for stiffness analysis.
- Not suitable for stress analysis of spring itself.
- Can have spring elements with stiffness in lateral direction.
spring in tension.

† Next wednesday CT → syllabus: $\boxed{\text{Lec-5. পর্যন্ত}}$

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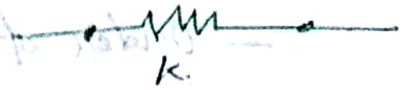
Lec-7

Date - 09.05.18

spring element :

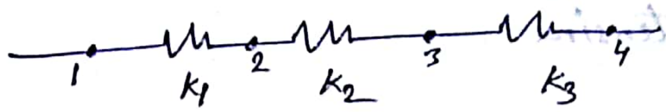
stiffness → $k = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$.



Spring system :

Global stiffness → K.

Example - 1.1 :



$k_1 = 100$ N/m
$k_2 = 200$ N/m
$k_3 = 100$ N/m

$P = 500$ N
$u_1 = u_4 = 0$

Determine
i) Global stiffness matrix.
ii) Displacement @ node 2 & 3
iii) Reaction force @ node 1 & 4
iv) Force in spring 2.

⇒

$$k_1 = \begin{bmatrix} 100 & -100 \\ -100 & 100 \end{bmatrix}$$

$$k_2 = \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix}$$

Diagonal element.
- must be (+ve) & non-zero.
+ যদি (+ve) or non-zero না হয় - তবে ওটা singular হয়ে যায় এবং ওর solve করা যায়না.

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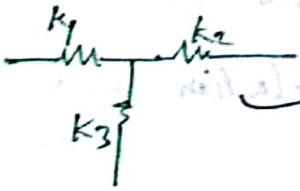
$$k_3 = \begin{bmatrix} 100 & -100 \\ -100 & 100 \end{bmatrix}$$

$$K = \begin{bmatrix} \boxed{k_1} & & \\ & \boxed{k_2} & \\ & & \boxed{k_3} \end{bmatrix}$$

* k_1, k_2, k_3 sequentially বসাবে.
* Member matrix (shape function থেকে)
  - Determines individual behaviour.
  - Connectivity & collective behaviour.



→ Connectivity different

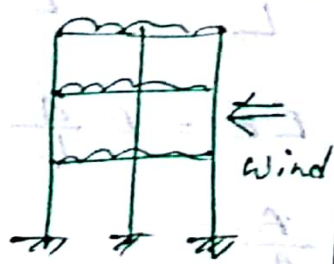
- Element topology (কার সাথে কিভাবে connect হচ্ছে).

Expl – 1.2
⇓
Assignment.

* Exp 1.1 & 1.2 Exam → অনেকক্ষেত্রে হুবহু - আসে।

Chapter – 2

Bar & Beam element:

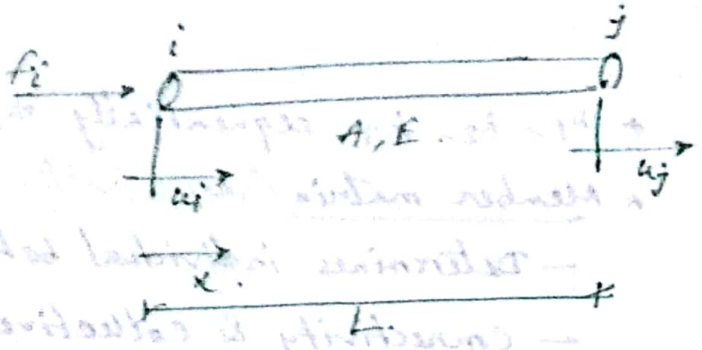


Load এর সাপেক্ষে deflect হয়। wind load deflected structure এ থাকে। But small deformation হলে আমলে কম

1. Linear static analysis.
   - Small deformation
   - ⇒ – Loading pattern has no effect on geometry.
   - Elastic materials.
   - static load

Page – 25.

2. Bar Element : (tension/ comp)



+ এখানে Bar element spring element এর মত কাজ করে।

+ যেকোনো বিন্দু নিতে হলে তার behavior সম্পর্কে জানতে হবে।

+ strain - displacement relation. $\rightarrow$ $\varepsilon = \frac{du}{dx}$ $\Rightarrow$ Relative A

+ stress - strain relation $\rightarrow$ $\sigma = E\varepsilon$ $\Rightarrow$ Relative E

material properties.

$$u(x) = \left(1 - \frac{x}{L}\right) u_i + \frac{x}{L} u_j .$$

[Interpolation is valid between i & j. i এবং j node-এর মধ্যে যেকোনো point-এর জন্য eq^n থেকে u বের হয়

$$\varepsilon = \frac{u_j - u_i}{L} = \frac{\Delta}{L} .$$

$$\sigma = E\varepsilon = E \frac{\Delta}{L} .$$

$$\sigma = \frac{f}{A}$$

$$f = \frac{EA}{L} \Delta = k\Delta .$$

$$k = \frac{EA}{L} , \quad k = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Physical meaning of k.

Represents the forces applied to the bar to maintain a deformed shape with unit - — [Page-28].

lec - 8 Date - 13.05.18

1D | # Bar & Bar element

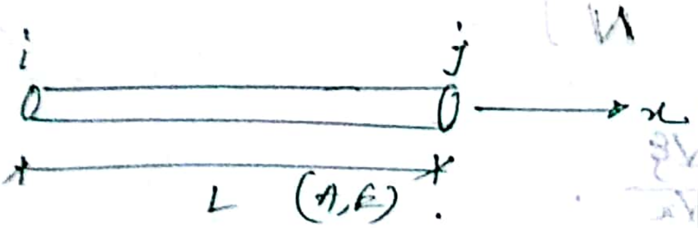
$$K = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}$$

non-zero (+ve)

$k_{ij} = k_{ji}$

* A এবং B দুই point এর আলাদা ভাবে load দিলে deflection same হবে।

* Stiffness matrix — a formal approach.
* Page - 29

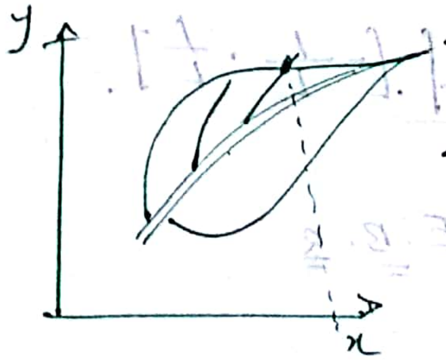


* $N_i(\xi) = 1 - \xi$
* $N_j(\xi) = \xi$

$\xi = \frac{x}{L}$ ইত্যাদি

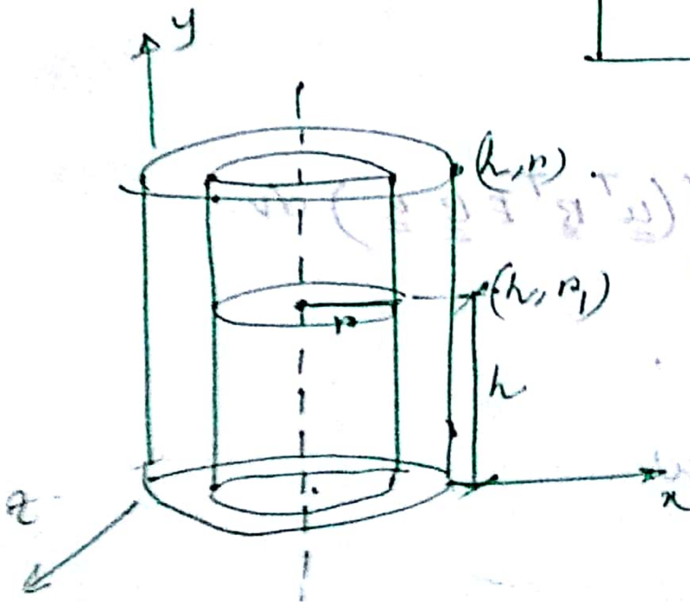
$x \quad y \quad z$

$\xi \quad \varsigma \quad \eta$



Here, Natural co-ordinate

* পাতার শিরা উপশিরা দিয়ে স্থানাঙ্ক বের করা যায়. $(1, \frac{1}{2})$



$\frac{r - r_1}{r}$

* রক্তনালীর দেয়াল (h, r). এবং (h, r_1) স্থানাঙ্ক জানা থাকলে difference থেকে অবস্থান বের করতে পারি।

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$u(x) = u(\xi) = N_i(\xi)\,u_i + N_j(\xi)\,u_j$

$u = [N_i \quad N_j] \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} = \underline{\underline{N}} \cdot \underline{\underline{u}}$

@ $x = 0$, $N_i = 1$

@ $x = L$, $N_j = 1$

$\varepsilon = \frac{du}{dx} = \left[\frac{d}{dx}\,\underline{\underline{N}}\right]\underline{\underline{u}} = \underline{\underline{B}}\,\underline{\underline{u}}$

Here,

$B \Rightarrow$ strain-displacement matrix.

$= \left[\frac{d}{dx}\,\underline{\underline{N}}\right]$

$\left.\begin{matrix} \underline{\underline{N}} \\ N \\ \mathbf{N} \end{matrix}\right\} \Rightarrow$ Matrix/vector. $\left(\underline{V}\quad V\!\!/\!\!/\quad \vec{\omega}\right)$

$\therefore \underline{\underline{B}} = \frac{d}{dx}[N_i(\xi) \cdot N_j(\xi)]$

$= \frac{d}{d\xi}[N_i(\xi)\ N_j(\xi)] \cdot \frac{d\xi}{dx}$.

$\underline{\underline{B}} = \cancel{\left[-\frac{1}{L} \quad \frac{1}{L}\right]} \cdot \left[-\frac{1}{L} \cdot \frac{1}{L}\right]$.

Now, $\sigma = E\varepsilon = E \cdot \underline{\underline{B}} \cdot \underline{\underline{q}}$

strain energy :

$U = \frac{1}{2}\int \sigma^T \varepsilon \, dv = \frac{1}{2}\int_V (\underline{\underline{u}}^T \underline{\underline{B}}^T E\, \underline{\underline{B}}\, \underline{\underline{u}})\, dv$.

$= \frac{1}{2}\underline{\underline{u}}^T \left[\int_V \underline{\underline{B}}^T \cdot E\, \underline{\underline{B}}\, dv\right]$.

Also. $W = \frac{1}{2} f_i u_i + \frac{1}{2} f_j u_j$, $U = W$

$= \frac{1}{2}\underline{\underline{u}}^T \underline{\underline{F}}$

strain energy and work load same হবে,

$$\therefore \frac{1}{2} \underline{u}^T \left[\int \underline{B}^T \cdot E \, \underline{B} \, dv \right] \underline{u} = \frac{1}{2} \underline{u}^T \underline{f} .$$

$$\Rightarrow \left[\int_V \underline{B}^T E \, \underline{B} \, dv \right] \underline{u} = \underline{f} .$$

$$\Rightarrow \underline{K} \, \underline{u} = f .$$

$$\underline{k} = \int_V \underline{B}^T E \, \underline{B} \cdot dv \cdot \quad (Eq^n - 20).$$

$$\underline{k} = \int_0^L \begin{Bmatrix} -\frac{1}{L} \\ \frac{1}{L} \end{Bmatrix} E \cdot \left[-\frac{1}{L} \quad \frac{1}{L} \right] A dx$$

$$= \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

* এর member soft থাকে যদি একটু পরিমাণ displacement দিলে তাতে strain energy বেশ stored হবে,

Assignment

✓ Ex - 2.1 .

Date - 20.05.18

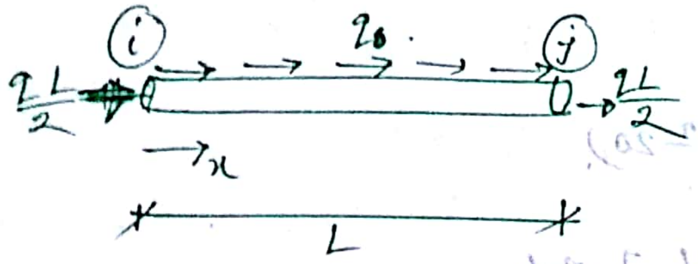
Lec-9.

2nd Lecture note :

[Page 35]

+ Example - 2.2 ⇒ Assignment.

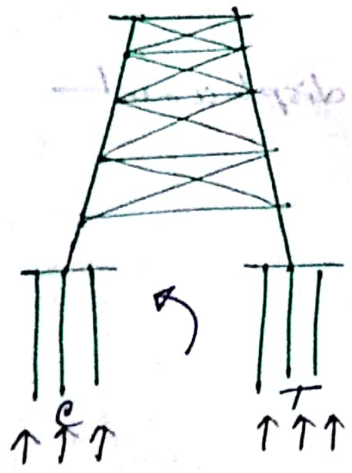
Distributed load :



+ এই Bar element এ একটা uniaxial distributed load আছে।

Q. Derive nodal forces from the shape function.

Telecommunication tower



End bearing
+ skin friction

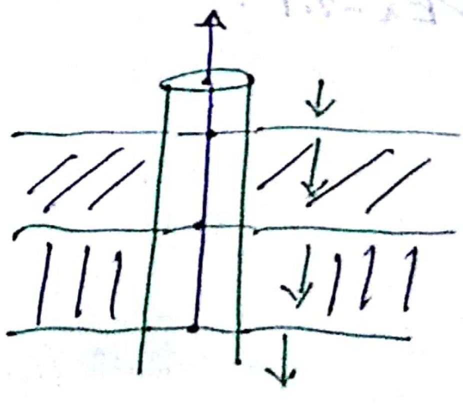
skin friction.

→ End bearing (compression)
+ skin friction

→ skin friction (tension)

+ এখানে skin friction governing. Both end bearing & skin friction কে overcome করতে হবে।

+ Different soil layer এর জন্য skin friction এর কারণে pile ভেঙে যাবে।



Derivation of nodal force :

work done by q.
$$W_q = \int_0^L \frac{1}{2} u q \, dx$$
$$= \frac{1}{2}\int_0^1 u(\xi)\, q\, \{L(d\xi)\}.$$
$$= \frac{qL}{2}\int_0^1 u(\xi)\, d\xi.$$
$$= \frac{qL}{2}\int_0^1 [N_i(\xi)\ N_j(\xi)] \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} d\xi.$$
$$= \frac{qL}{2}\int_0^1 [1-\xi \quad \xi]\, d\xi \begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$$
$$= \frac{1}{2}\left[\frac{qL}{2} \quad \frac{qL}{2}\right] \begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$$
$$= \frac{1}{2}[u_i \quad u_j] \begin{Bmatrix} \frac{qL}{2} \\ \frac{qL}{2} \end{Bmatrix}$$
$$= \frac{1}{2} \underline{u}^T \underline{f}_q. \quad \text{where } \underline{f}_q = \begin{Bmatrix} qL/2 \\ qL/2 \end{Bmatrix}$$

Here,
$$\underline{K}\,\underline{u} = f + f_q.$$

node $\quad$ from distributed load

and
$$f + f_q = \begin{Bmatrix} f_i + qL/2 \\ f_j + qL/2 \end{Bmatrix}$$

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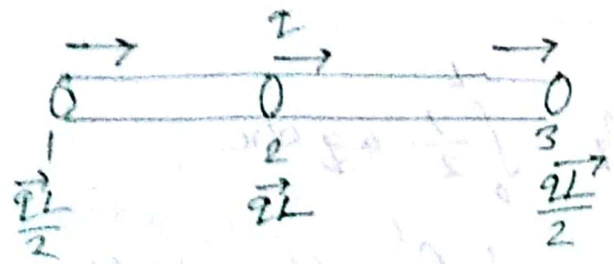
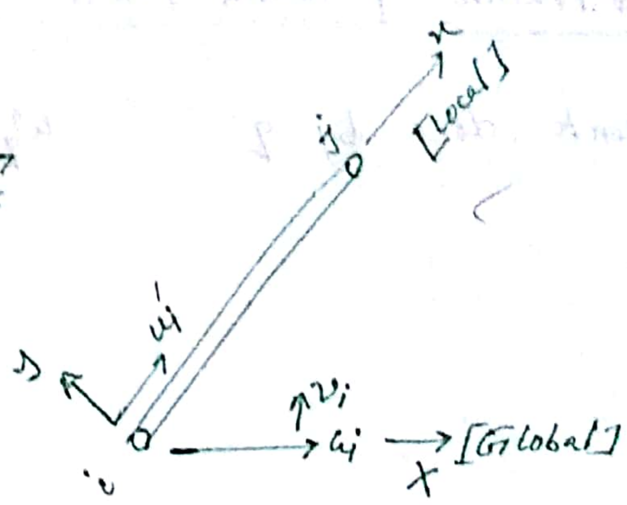




Bar elements:

2D & 3D in space

— Transformation

— Direction cosine

Here, Local	Global
x, y	X, Y
u_i', v_i'	u_i, v_i
1 Dof at a node	2 Dof at a node

~~2 Dof at a node~~

$$u_i' = u_i \cos\theta + v_i \sin\theta = [L \quad m] \begin{Bmatrix} u_i \\ v_i \end{Bmatrix}$$

$$v_i' = -u_i \sin\theta + v_i \cos\theta = [-m \quad L] \begin{Bmatrix} u_i \\ v_i \end{Bmatrix}$$

where, $L = \cos\theta$

$m = \sin\theta$.

Assignment: Eq^n 26–35.

Example — 2.3.

Next class → Beam Element Page 53.

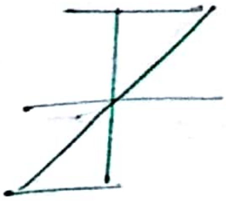
Date - 23.05.18

Lec-10.

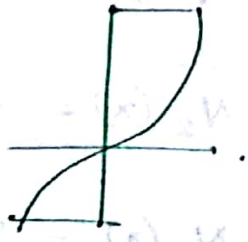
Bar element.

Simple plane beam element:

Page 53

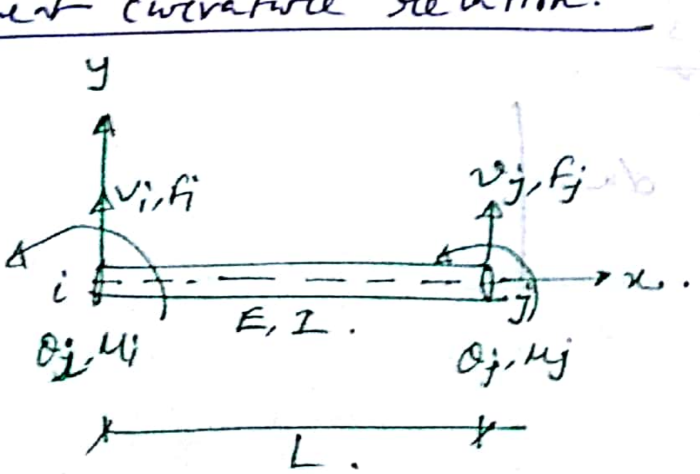


Beam
- Simple beam / Euler's beam ≡ Navier's hypothesis is valid ~ insignificant sheare definition
- Timoshenko beam / Deep beam. ≡ ~ Not valid sheare definition is significant.



$\boxed{\text{Theory}}$ ⇒ Bounded by hypothesis & validity.

Moment curvature relation:



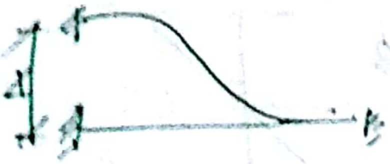
⇒ 2 dof per node (shear, moment)

Elementary beam theory

$$EI \frac{d^2 v}{dx^2} = M(x)$$

$$6 = -\frac{My}{I}.$$

$\Rightarrow K = \begin{bmatrix} & & \\ & & \end{bmatrix}$ Shape function



$$k = \begin{array}{cccc} v_i & \theta_i & v_j & \theta_j \\ \end{array}$$
$$k = \begin{bmatrix} 12 & 6L & -12 & \\ 6L & 4L^2 & -6L & \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

normally eq^n

full moment

calculate [illegible]

But [illegible].

$$N_1(x) = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}.$$

$$N_2(x) = x - \frac{2x^2}{L} + \frac{x^3}{L^2}.$$

$$N_3(x) = 3\frac{x^2}{L^2} - \frac{2x^3}{L^3}.$$

$$N_4(x) = -\frac{x^2}{L} + \frac{x^3}{L^2}.$$

$$\boxed{K = \int_0^L \underline{B}^T E I \underline{B} \, dx}$$

$\cancel{N_0 + N_1 = 1}$

$N_1 + N_3 = 1$

$N_2 + N_3 L + N_4 = x$

$$\frac{d^2 v}{dx^2} = \frac{d^2}{dx^2} \underline{\underline{N}}\, \underline{\underline{u}} = \underline{\underline{B}}\, \underline{\underline{u}}$$

$$\underline{\underline{B}} = \frac{d^2}{dx^2} \underline{\underline{N}} = \left[N_1''(x) \quad N_2''(x) \quad N_3''(x) \quad N_4''(x) \right]$$

$$= \left[\left(\frac{-6}{L^2} + \frac{12x}{L^3} \right) \quad \left(-\frac{4}{L} + \frac{6x}{L^2} \right) \quad \left(\frac{6}{L^2} - \frac{12x}{L^3} \right) \quad \left(\frac{-2}{L} + \frac{6x}{L^2} \right) \right]$$

$$U = \frac{1}{2} \underline{\underline{u}}^T \left[\int_0^L \underline{\underline{B}}^T EI \underline{\underline{B}}\, dx \right]$$

$$\underline{\underline{K}} = \int_0^L \underline{\underline{B}}^T EI \underline{\underline{B}}\, dx = \longrightarrow K \; (?)$$

page $\boxed{55 - 56 \quad 56 - 57}$ $\longrightarrow$ Assignment

- Axial force is ignored in 4 × 4 matrix
- We can include axial force & 3D as well [page 57]

Lec - 11

Date - 01.07.18

Beam element:

formal approach.

$\hookrightarrow$ from shape function.

$$k = \int \underline{B}^T E I \underline{B}\, dx$$

$I \rightarrow$ Moment of inertia.

$E \rightarrow$ Mod. of elasticity.

$B \rightarrow$ strain displacement matrix

$\rightarrow$ Cartesian co-ordinate

$\rightarrow$ Natural co ordinal

shape function, $\underline{B} = \frac{d^2}{dx^2} \underline{N}$

Now,

$$N_1(x) = 1 - \frac{3x^2}{L^2} + 2\frac{x^3}{L^3}.$$

$$N_2(x) = x - \frac{2x^2}{L} + \frac{x^3}{L^2}.$$

$$N_3(x) = \frac{3x^2}{L^2} - 2\frac{x^3}{L^3}.$$

$$N_4(x) = -\frac{x^2}{L} + \frac{x^3}{L^2}..$$

Let, $x/l_e = s$ [Book, Pg 243]

$$N_1 = 1 - 3s^2 + 2s^2$$

$$N_2 = l_e s (s-1)^2$$

$$N_3 = s(3-2s)$$

$$N_4 = l_e s^2 (s-1).$$

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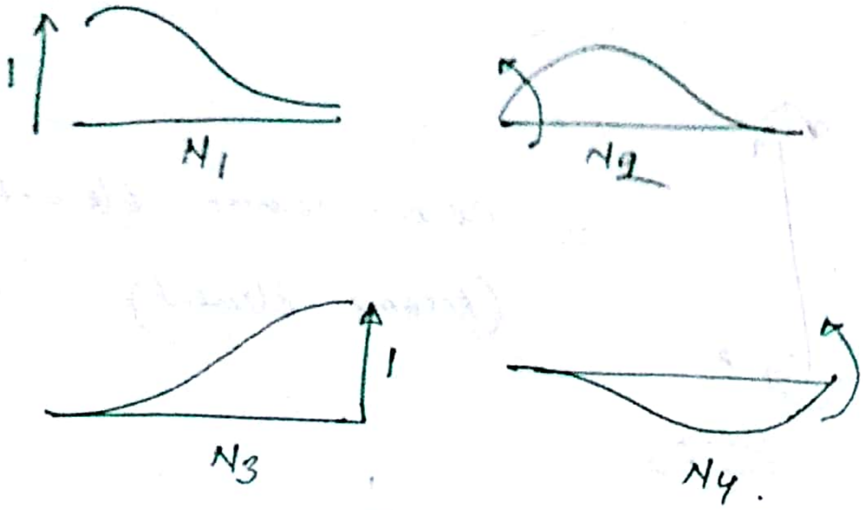


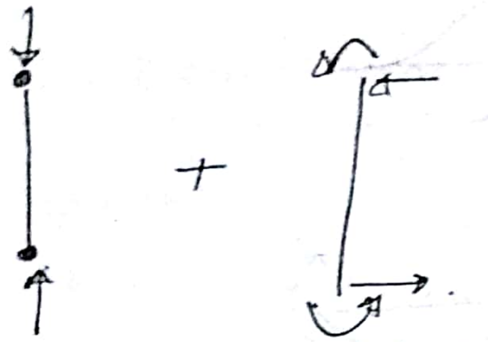
$$\boxed{\begin{aligned} &N_1 + N_3 = 1 \\ &N_2 + N_3 L + N_4 = x \end{aligned}}$$
$\Rightarrow$ properties of shape function for beam

Diagonals +ve moment.

$$K = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

$k_{ij} = k_{ji}$ (symmetric)

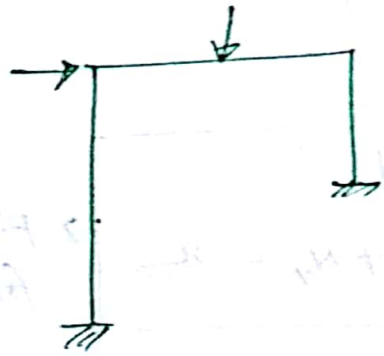
Method of superposition :



$\equiv$ Beam - column element.
(frame element)

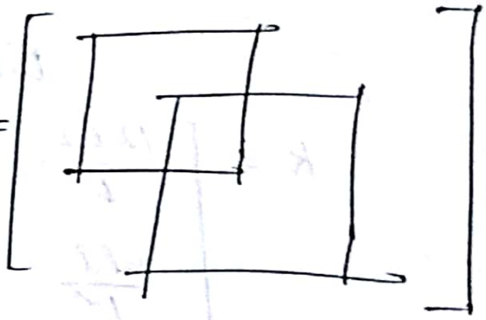
column/truss. Beam

2 DOF. | 2DOF
1 per node | 3 per node



6x6.

$$\underline{\underline{k}} = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix} \quad \bigg\} \quad \underline{\underline{k}} = \_\_\ \_\_$$

$k =$ 

$$k = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & \boxed{-\frac{EA}{L}} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ \boxed{-\frac{EA}{L}} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \rightarrow \text{Diagonals.}$$

Assignment —

Page - 58.

Example 2.5.

$k_{ij} = k_{ji}$. [Note Page - 57

[symmetric]

<u>Class test — 2</u>

আজকের class পর্যন্ত syllabus.

(July - 15)

Lecture-12

04-07-18
Wednesday

Chapter 3

Matrix Displacement formulation

$$\underline{F} = \underline{K} \cdot \underline{\Delta}$$

→ { From Shape Function }

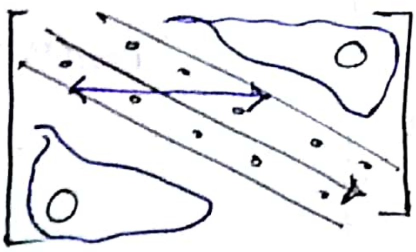
Bar element → Truss element
Beam element

eqn 3.8 → derivation → last class এ করানো হয়েছে

Solution

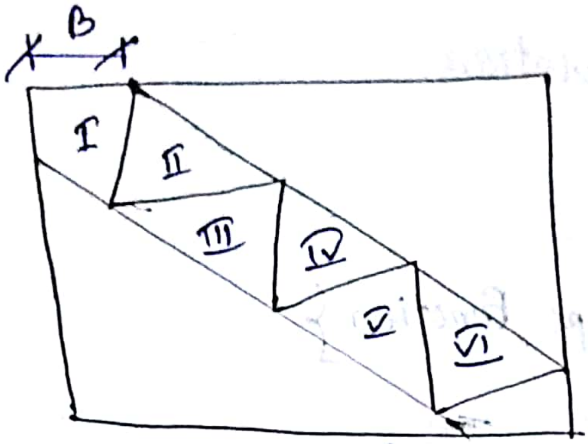
$$[F] = [K]\,[\Delta]$$

element stiffness → Global stiffness



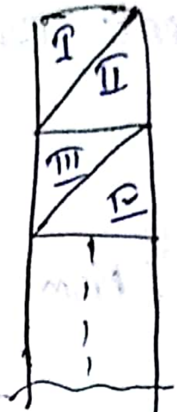
Matrix is banded

Band Solution:



N×N

Array



N×B

Page 28

B = Band width

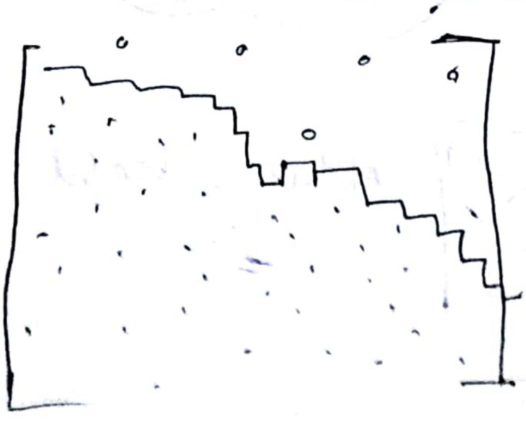
N×N ≥ N×B.

Article: 3.4

i) Use of symmetry & bonded nature

ii) Partitioning of matrix

iii) Skyline Storage:



Next Class:

"Frontal Method"

* Optimum usage of available storage — write a short note

1970

1980

2000

2020

Mandatory

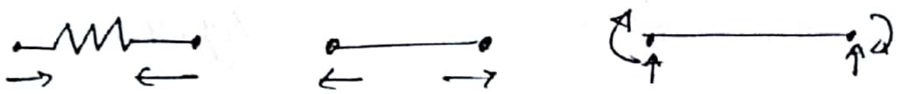
- Lecture Note (up to date)
- Book
- Note sheets

Lec-13

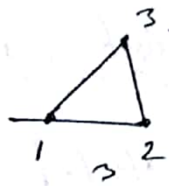
Date – 11.07.18

Chapter – 4

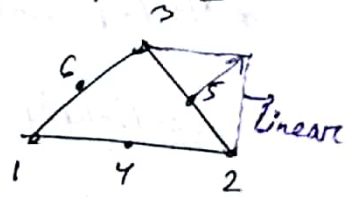
+ **1D element:** spring bare beam



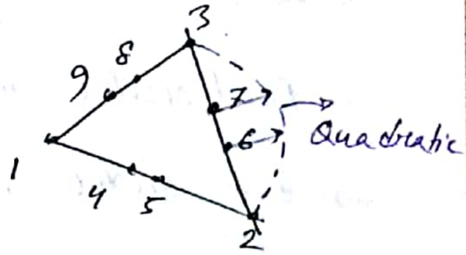
+ **2D element:** i) CST → Constant strain triangle



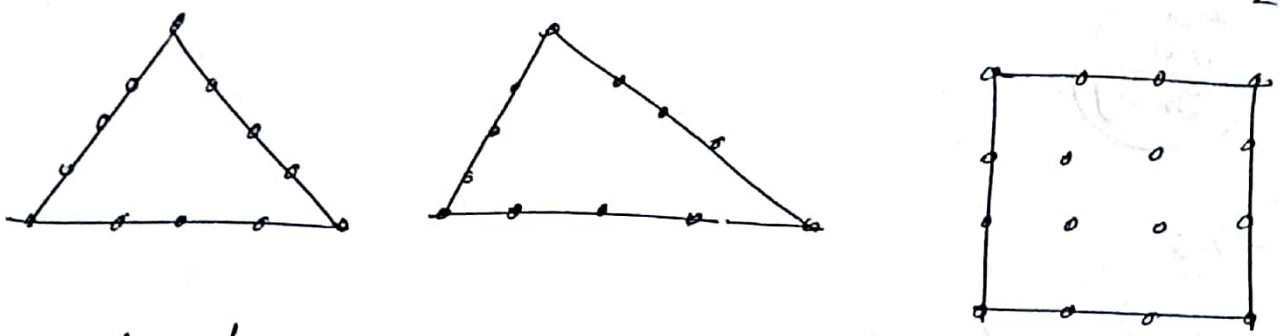
ii) LST → Linear " "



iii) QST → Quadratic " "
(fig- 4.4)

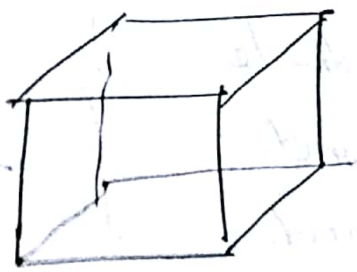


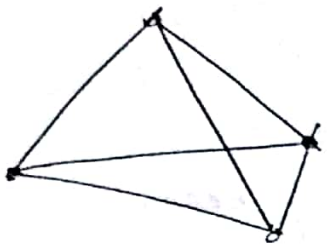
serendipity family. 'serendip'



+ **3D elements:**

Tetrahedron





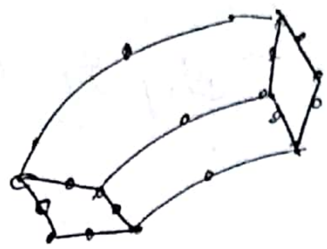


fig-4.2

Art 4.4

* Nodal unknown

continuity.

→ zeroth continuity. c^0-continuity

continuity

→ first-order " c^1-continuity

→ 2nd-order " c^n-continuity.

$$\frac{\partial^n w}{\partial x \partial y}.$$

Art-4.5

* coordinate system :

→ Local co-ordinate

→ Global co-ord

→ Natural co-ord

বইতে লেখা কিছু নাই.

Chapter - 5

SHAPE FUNCTION

* Polynomial shape function আসবে না
* Math করতে হবে

Ex-5.4., 5.5, 5.7. — [Page-63]
Next class - Example. 5.10

CT-3.

* Lec-12, 13, 14. ⇒ এর উপর - next week এর পরের - week wednesday. [25.07.18] X

[22.07.18]

⇓

sunday

Lec-14

Date - 18.07.18

Chapter-5

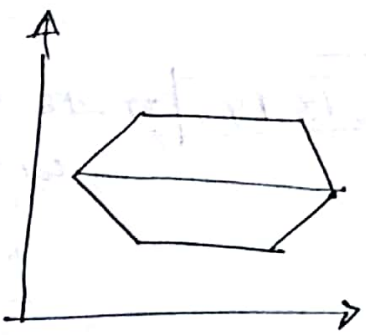
Example-5.10

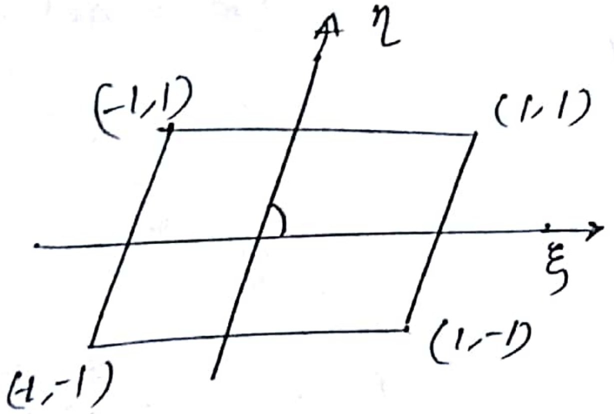
→ Isoparametric Element.

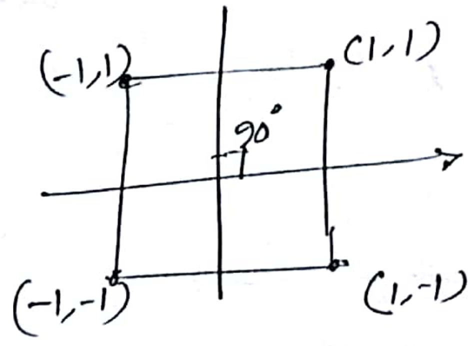
→ "Iaij's Quadrilateral"

→ "Triangular Element"

→ Natural co-ordinate.







Cartesian

iso-parameter (কার্তেসীয়ান এর সাথে এর angle same হতে পারে আবার নাও পারে)

$$J = \frac{d\xi}{dx} \times \frac{d\eta}{dy} \text{ ---}$$

Jacobian

[দুটি parameter হতে 1টি parameter এ নিয়ে আসি → J. তাই এর নাম iso parameter].

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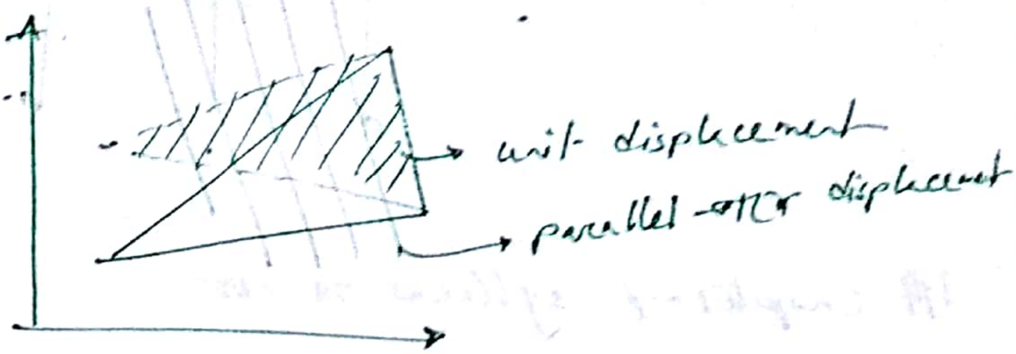
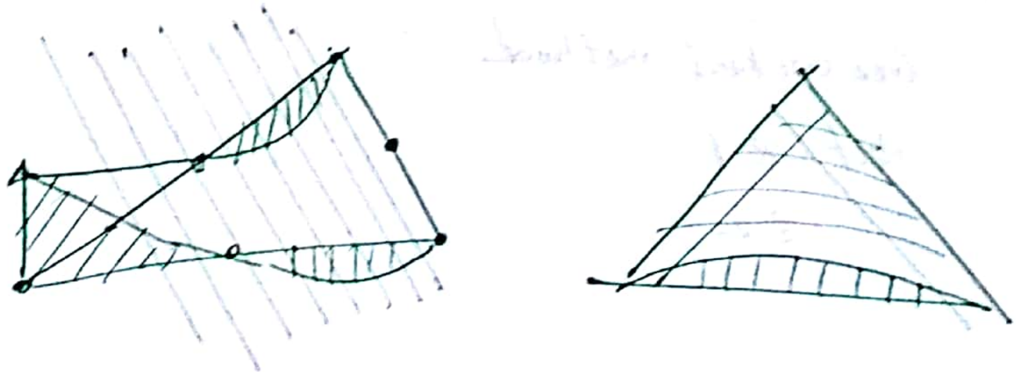
+ Super-parametric (এটা imp. না)

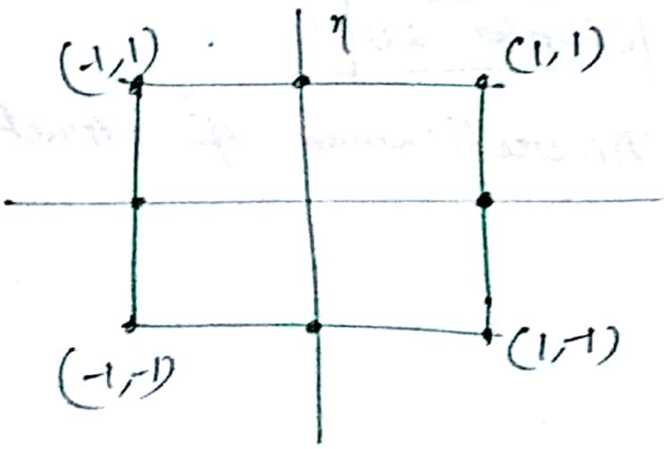
Ex- 5.10 :



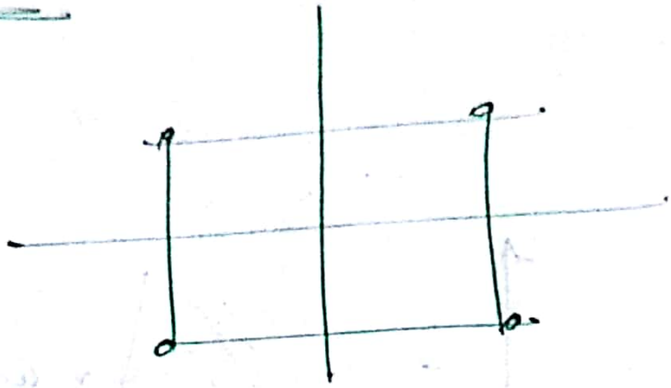
Ex-5.11 :



Ex- 5.12 :



Ex – 5.13 :



- Chapter – 7 syllabus – এ বাদে.

- Chapter – 8 :

  Galerkin's method

  Art. 8.1

  8.2

- Chapter – 9 বাদে.

- Next class ⇒ $\boxed{\text{Chapter – 10}}$

  Discretization of structures.

Chapter 11
12

Chapter 13
14

Lecture - 15

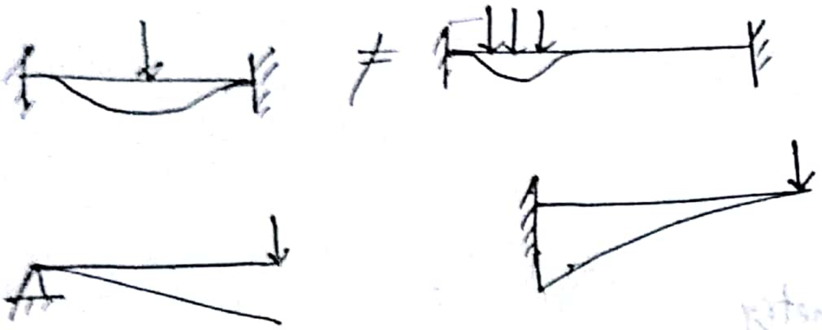
Saiful Amin Sir

25-07-18
~~Monday~~
Wednesday

Discretization of Structures

Art. 102

Geometry
Load
Material
Boundary Condition

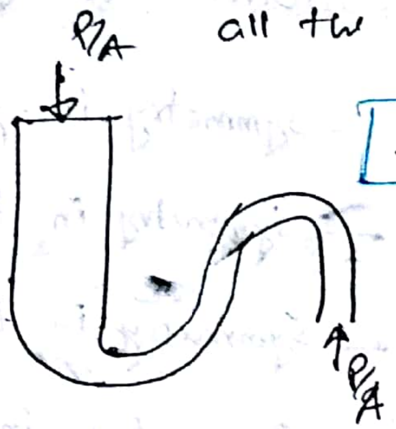


Deformation:

- Deflection
- Rotation

We don't need mesh all the time.

$\boxed{\text{Pascal's law}}$



Boundary condition

- Load
- Material.

Governing eqⁿ এটা হবে mesh simple হলে, multiple হলে complex হবে।

Question কেমন আসবে? এমন আসবে। Describe করতে হবে।

Larger discontinuity

Coarser Mesh

Refined Mesh

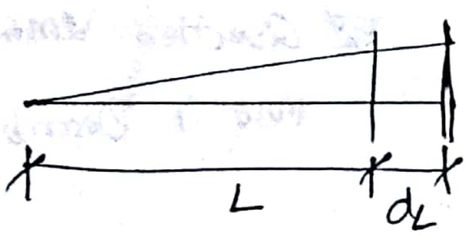
Art 10.4

Use of Symmetry

একটা structure এ symmetry থাকে ৪ ধরনের যেমন—

- Symmetry in geometry
- Symmetry in boundary condition
- Symmetry in load
- Symmetry in material

Defining Infinity



dL will be parallel to L if L tends to infinity.

$dL \approx ||$

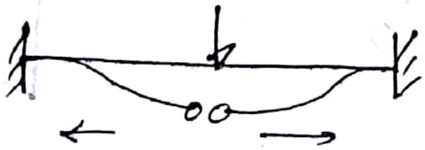
$L \to \infty$

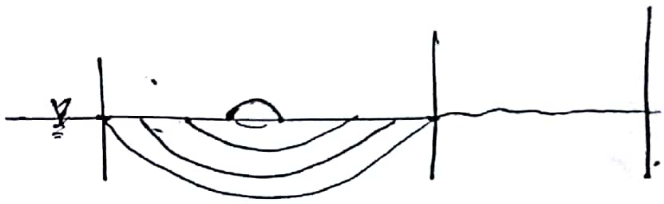
$\theta \to 0^\circ / 180^\circ$

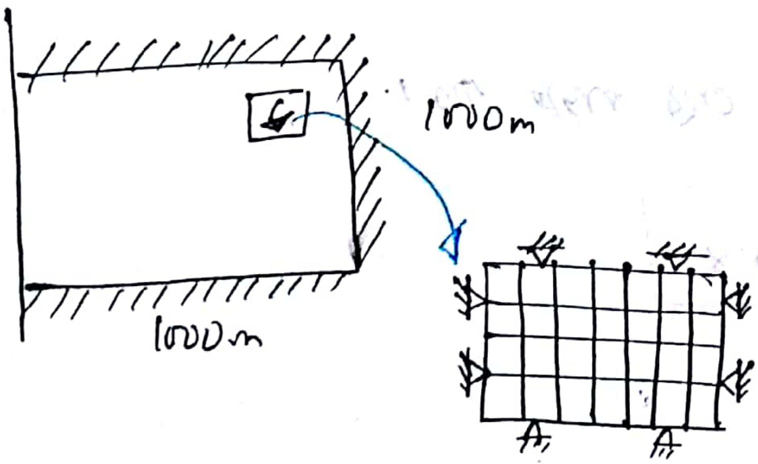
Art 10.4

Finite Representation of Infinite Bodies

Definition of Infinite Boundary Condition (Art 10.6)

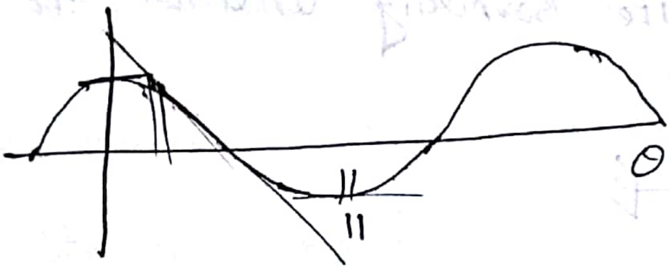


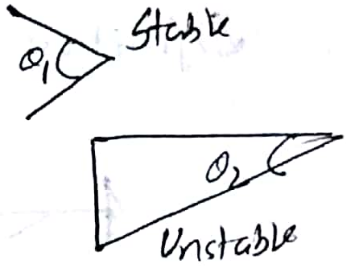




Art 10·6

Element Aspect Ratio





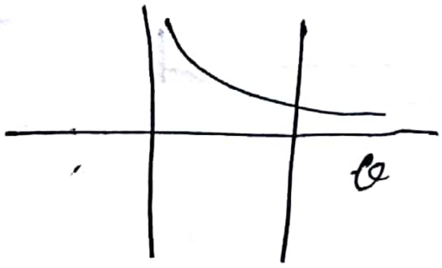


Fig 10:8 এর পরে আসবে না।

Art 10·7, 10·8

Chapter 11

Example 11·1 to 11·4

Chapter 12

Upto Art ~~10~~ 12·2

Lec — 16

Date - 29.07.18

Chapter - 13

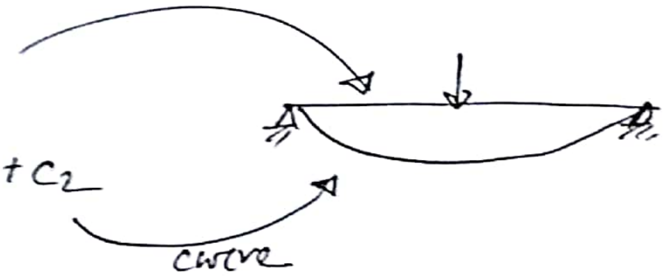
Iso-parametric formulation

↓

one

A single function to describe geometry and deformed shape

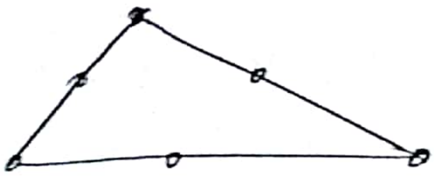
Same function
$$\begin{cases} y = mx + c \\ y = mx^2 + C_1 + C_2 \end{cases}$$

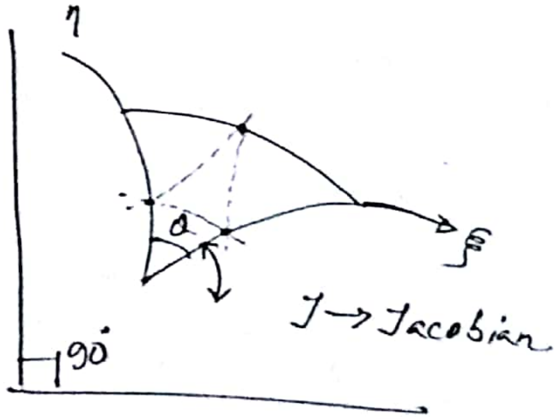


Co-ordinate Transformation :

Fig - 13.1

Page 219—220 .





Art - 13.2

$$\{x\} = [N]\{x\}_e .$$

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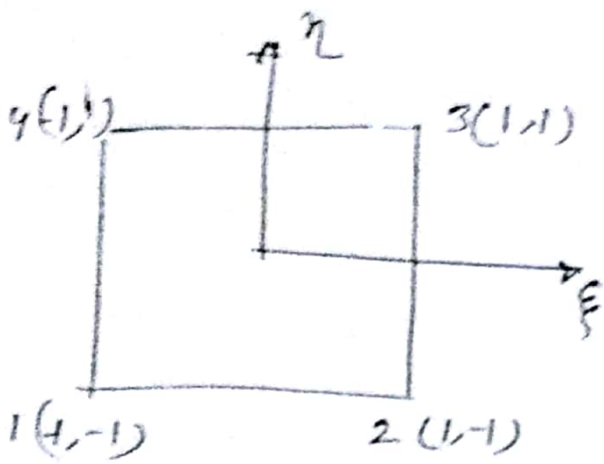
for Quadrilateral [Trig. Quadrilateral]



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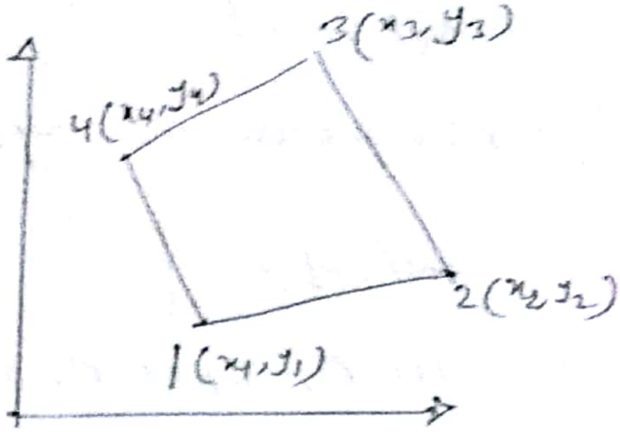
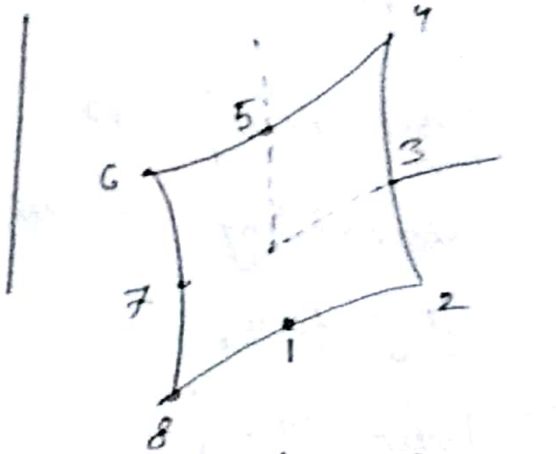


$$N_1 = \frac{(1-\xi)(1-\eta)}{4}$$

$$N_2 = \frac{(1+\xi)(1-\eta)}{4}$$

$$N_3 = \frac{(1+\xi)(1+\eta)}{4}$$

$$N_4 = \frac{(1-\xi)(1+\eta)}{4}$$



shape function for?

Chapter-5 (Page 74)

Ex-5.12

$$N_i = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)(1+\zeta\zeta_i)$$

$N_1 =$

$N_2 =$

$N_3 =$

$N_4 =$

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Triangular . (Ex-5.11)

Art 13.4

Uniqueness in mapping

Art-13.5.

Iso-P, Super-P, Sub-P.

+ Iso-P → function ও SRতT same

+ Super-P → " " " same ঘT.

Art 13.6

Assembling of stiffness matrix

Fig-13.6

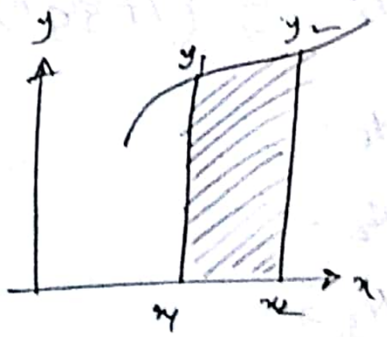
Art-13.7

Numerical Integration

$$k = \int \underline{\underline{B}}^T \underline{\underline{D}}\, \underline{\underline{B}}\, dV$$

Trapezoidal rule

Gauss Integration coeff.



$$\int f(\xi)\, \partial\xi = \sum_{i=1}^{n} w_i f(\xi_i) = + + +$$ n = no. of Gauss

1st ...

$\boxed{\text{Table 13.1}}$

Page - 230

$\boxed{\text{Art - 13.8}}$ → Numeric Example

Ex - 13.1 - 13.8

Last Assignment: (Example - 14.1 - 14.3)