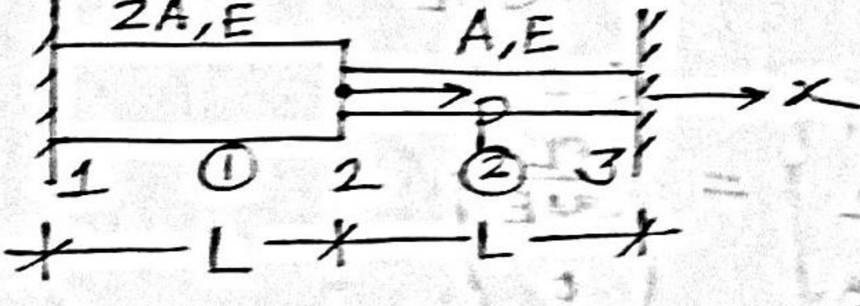
2012-2013

2(b)



$$k_1 = \frac{2AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{AE}{L}\begin{array}{c} \begin{array}{cc} & 2 \end{array} \\ \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{array}{c} \\ 2 \end{array} \end{array}$$

$$k_2 = \frac{AE}{L}\begin{array}{cc} \begin{array}{cc} 2 & 3 \end{array} & \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} & \begin{array}{c} 2 \\ 3 \end{array} \end{array}$$

The stiffness equations

$$\frac{AE}{L}\begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

Applying boundary conditions,

$$\frac{AE}{L}\begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ u_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ P \\ F_3 \end{Bmatrix}$$

Now, $\frac{AE}{L} \times 3u_2 = P$

$$\Rightarrow u_2 = \frac{PL}{3AE}$$

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{PL}{3AE} \\ 0 \end{Bmatrix}$$

stress at element 1,

$$\sigma_1 = E\epsilon_1$$

$$= EB_1u_1$$

$$= E\left[-\frac{1}{L} \quad \frac{1}{L}\right]\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \left[-\frac{E}{L} \quad \frac{E}{L}\right]\begin{Bmatrix} 0 \\ \frac{PL}{3AE} \end{Bmatrix}$$

$$= -\frac{E}{L}\times 0 + \frac{E}{L}\times\frac{PL}{3AE}$$

$$= \frac{P}{3A}$$

Stress at element 2,

$$\sigma_2 = E\,\epsilon_2$$
$$= E\,B_2\,u_2$$
$$= E\begin{bmatrix}-\frac{1}{L} & \frac{1}{L}\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix}$$
$$= \begin{bmatrix}-\frac{E}{L} & \frac{E}{L}\end{bmatrix}\begin{Bmatrix}\frac{PL}{3AE}\\ 0\end{Bmatrix}$$
$$= -\frac{E}{L}\times\frac{PL}{3AE} + \frac{E}{L}\times 0$$
$$= -\frac{P}{3A}$$