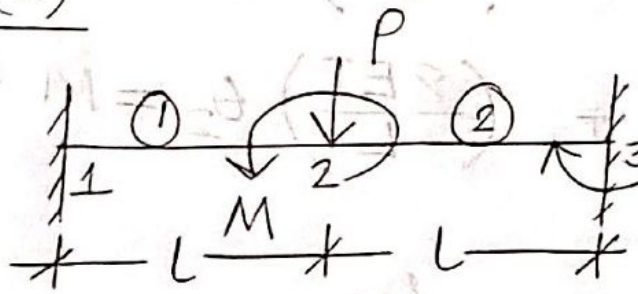
2012 - 2013

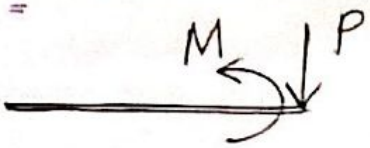
7(b)



$$[K_1] = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \quad [K_2] = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

$$[K] = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & 0 \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & 0 \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{24EI}{L^3} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{8EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ 0 & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

Applying boundary condition the reduced stiffness equation becomes,



$$\left(\frac{24EI}{L^3}\right) v_2 + (0)\,\theta_2 + P = 0$$

$$(0)\, v_2 + \left(\frac{8EI}{L}\right)\theta_2 - M = 0$$

$$\frac{v_2}{-\frac{8EI}{L}P} = \frac{\theta_2}{\frac{24EI}{L^3}M} = \frac{1}{\frac{192E^2I^2}{L^4}}$$

$$v_2 = \frac{-\frac{8EI}{L}P}{\frac{192E^2I^2}{L^4}} = -\frac{8EI}{L}P \times \frac{L^4}{192E^2I^2} = -\frac{PL^3}{24EI}$$

$$\theta_2 = \frac{\frac{24EI}{L^3}M}{\frac{192E^2I^2}{L^4}} = \frac{24EI}{L^3}M \times \frac{L^4}{192E^2I^2} = \frac{ML}{8EI}$$

$$\begin{Bmatrix} F_1 \\ M_1 \\ F_3 \\ M_3 \end{Bmatrix} = \begin{bmatrix} -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} \end{bmatrix} \begin{Bmatrix} -\frac{PL^3}{24EI} \\ \frac{ML}{8EI} \end{Bmatrix} = \begin{Bmatrix} \frac{P}{2} + \frac{3M}{4L} \\ \frac{PL}{4} + \frac{ML}{4} \\ \frac{P}{2} - \frac{3M}{L} \\ -\frac{PL}{4} + \frac{ML}{4} \end{Bmatrix}$$