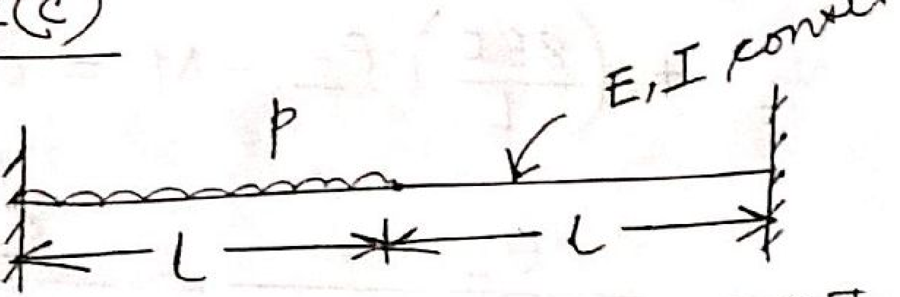
2013-2014

2(c)



$$\mathbb{K}_1 = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

$$\mathbb{K}_2 = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

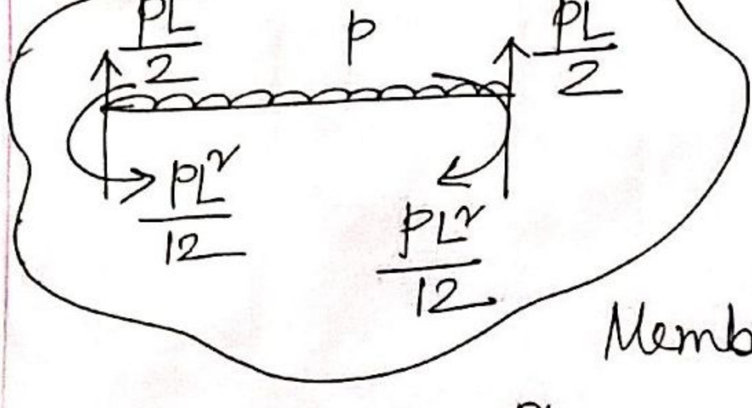
The global stiffness equation,

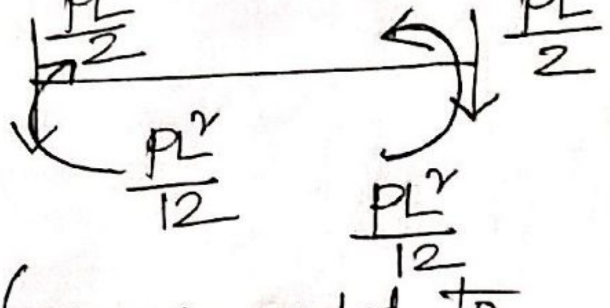
$$
\begin{bmatrix}
\frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & 0 \\
\frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & 0 \\
-\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{24EI}{L^3} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{8EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\
0 & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\
0 & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L}
\end{bmatrix}
\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{Bmatrix}
=
\begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \\ F_3 \\ M_3 \end{Bmatrix}
$$

Applying boundary condition $(v_1 = \theta_1 = v_3 = \theta_3 = 0)$ the reduced FE equation becomes,

$$\left(\frac{24EI}{L^3}\right) v_2 + (0)\,\theta_2 = F_2$$

$$(0)\, v_2 + \left(\frac{8EI}{L}\right)\theta_2 = M_2$$





Member forces converted to

So, $F_2 = -\frac{PL}{2}$ & $M_2 = -\frac{PL^2}{12}$

$$\left(\frac{24EI}{L^3}\right) v_2 + (0)\,\theta_2 + \frac{PL}{2} = 0$$

$$(0)\, v_2 + \left(\frac{8EI}{L}\right) \theta_2 + \frac{PL^2}{12} = 0$$

$$\frac{v_2}{-4PEI} = \frac{\theta_2}{-\frac{2\,PEI}{L}} = \frac{1}{\frac{192E^2I^2}{L^4}}$$

$$v_2 = -\frac{4PEI}{\frac{192E^2I^2}{L^4}} = -\frac{4PEI \times L^4}{192E^2I^2}$$

$$= \frac{-PL^4}{48EI}$$

$$\theta_2 = \frac{-\frac{2PEI}{L}}{\frac{192E^2I^2}{L^4}} = -\frac{2PEI \times L^4}{L \times 192E^2I^2}$$

$$= \frac{-PL^3}{96EI}$$