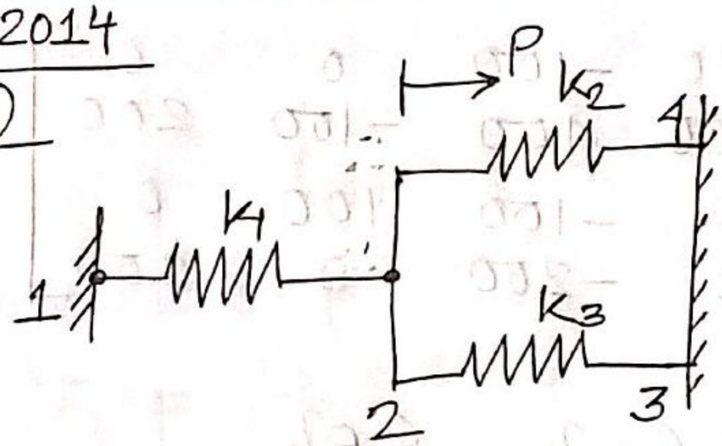
2013-2014

3(b)



The element stiffness matrices are:-

$$k_1 = \begin{array}{cc} u_1 & u_2 \\ \left[\begin{matrix} 100 & -100 \\ -100 & 100 \end{matrix}\right] \end{array}\begin{array}{c} u_1 \\ u_2 \end{array} \qquad k_2 = \begin{array}{cc} u_2 & u_4 \\ \left[\begin{matrix} 200 & -200 \\ -200 & 200 \end{matrix}\right] \end{array}\begin{array}{c} u_2 \\ u_4 \end{array}$$

$$k_3 = \begin{array}{cc} u_2 & u_3 \\ \left[\begin{matrix} 100 & -100 \\ -100 & 100 \end{matrix}\right] \end{array}\begin{array}{c} u_2 \\ u_3 \end{array}$$

The global stiffness matrix becomes,

$$K = \left[\begin{array}{c|c|c|c} 100 & -100 & & \\ \hline -100 & \begin{matrix}100\\200\\100\end{matrix} & -100 & -200 \\ \hline & -100 & 100 & \\ \hline & -200 & & 200 \end{array}\right]$$

$$K = \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 400 & -100 & 200 \\ 0 & -100 & 100 & 0 \\ 0 & -200 & 0 & 200 \end{bmatrix}$$

Now, $[K]\{u\} = \{P\}$

$$\Rightarrow \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 400 & -100 & 200 \\ 0 & -100 & 100 & 0 \\ 0 & -200 & 0 & 200 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

Applying boundary conditions we get,

$$\begin{bmatrix} \cancel{100} & \cancel{-100} & \cancel{0} & \cancel{0} \\ \cancel{-100} & 400 & \cancel{-100} & \cancel{200} \\ \cancel{0} & \cancel{-100} & \cancel{100} & \cancel{0} \\ \cancel{0} & \cancel{-200} & \cancel{0} & \cancel{200} \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} F_1 \\ P \\ F_3 \\ F_4 \end{bmatrix}$$

$$400\, u_2 = P$$

$$\Rightarrow \quad u_2 = \frac{P}{400} = \frac{500}{400} = 1.25$$

Force at Node 1,

$$(100)(u_1) + (-100)(u_2) = F_1$$

$$\Rightarrow 100 \times 0 - 100 \times 1.25 = F_1$$

$$\Rightarrow F_1 = -125 N$$

Force at Node 3,

$$(-100)(u_2) + (100)(u_3) = F_3$$

$$\Rightarrow -100 \times 1.25 + 100 \times 0 = F_3$$

$$\Rightarrow F_3 = -125 N$$

Force in Spring 2,

$$\begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_4 \end{Bmatrix}$$

$$= \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \begin{Bmatrix} 1.25 \\ 0 \end{Bmatrix}$$

$$= \begin{bmatrix} 250 \\ -250 \end{bmatrix}$$

So, $f_j = -f_i = -250 N$