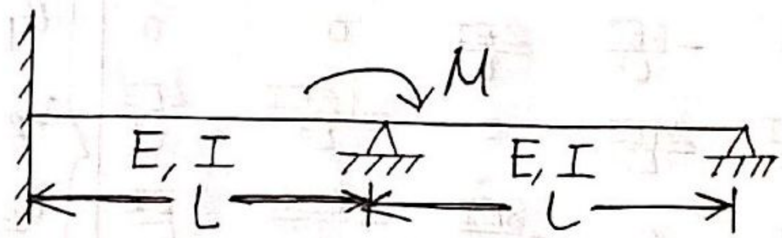
2013 - 2014

3(c)



$$K_1 = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & \frac{-6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

$$K_2 = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & \frac{-6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

The global stiffness equation,

$$\begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & 0 \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & 0 \\ \frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{24EI}{L^3} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{8EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ 0 & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \\ F_3 \\ M_3 \end{Bmatrix}$$

Applying boundary conditions $(v_1 = \theta_1 = v_2 = v_3 = 0)$
the reduced FE equation becomes,

$$\begin{bmatrix} \frac{8EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} -M \\ 0 \end{Bmatrix}$$

$$\left(\frac{8EI}{L}\right)\theta_2 + \left(\frac{2EI}{L}\right)\theta_3 + M = 0$$

$$\left(\frac{2EI}{L}\right)\theta_2 + \left(\frac{4EI}{L}\right)\theta_3 + 0 = 0$$

$$\frac{\theta_2}{-\frac{4EI}{L}M} = \frac{\theta_3}{\frac{2EI}{L}M} = \frac{M}{\frac{32E^2I^2}{L^2} - \frac{4E^2I^2}{L^2}}$$

$$\theta_2 = \frac{-\frac{4EI}{L}M}{\frac{28E^2I^2}{L^2}} = -\frac{4EI}{L} \times M \times \frac{L^2}{28E^2I^2}$$

$$= -\frac{ML}{7EI}$$

$$\theta_3 = \frac{\frac{2EI}{L}M}{\frac{28E^2I^2}{L^2}} = \frac{2EI}{L} \times M \times \frac{L^2}{28E^2I^2}$$

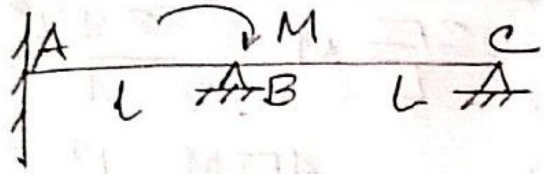
$$= \frac{ML}{14EI}$$

$$F_3 = \left(-\frac{6EI}{L^2}\right)\left(-\frac{ML}{7EI}\right) + \left(-\frac{6EI}{L^2}\right)\left(\frac{ML}{14EI}\right)$$

$$= \frac{6M}{7L} - \frac{3M}{7L}$$

$$= \frac{3M}{7L}$$

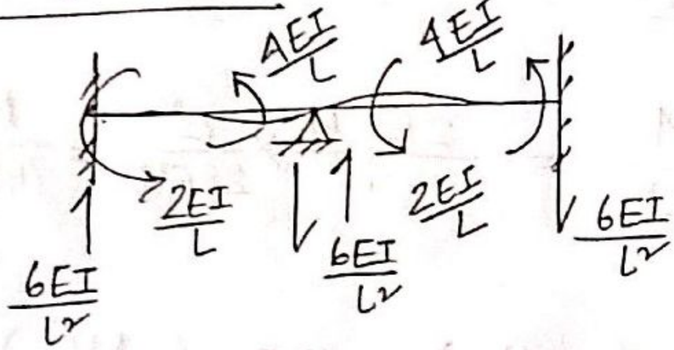
check



$P_j^\circ = \begin{Bmatrix} -M \\ 0 \end{Bmatrix}$

$P_m = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$

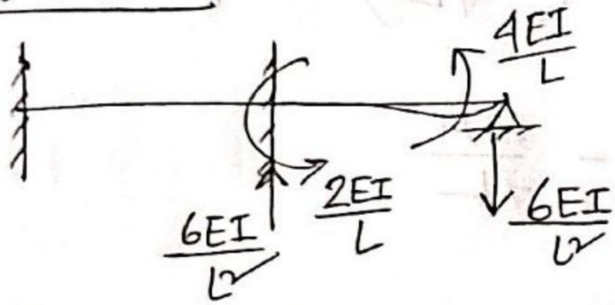
$u_1 = 1 ;\ u_2 = 0$



$K_{11} = \frac{8EI}{L}$

$k_{21} = \frac{2EI}{L}$

$u_1 = 0 ;\ u_2 = 1$



$K_{12} = \frac{2EI}{L}$

$k_{22} = \frac{4EI}{L}$

$$\{P_m\} + [k]\{u\} = \{P_j^\circ\}$$

$$\begin{bmatrix} \frac{8EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} -M \\ 0 \end{Bmatrix}$$

$$\left(\frac{8EI}{L}\right) u_1 + \left(\frac{2EI}{L}\right) u_2 + M = 0$$

$$\left(\frac{2EI}{L}\right) u_1 + \left(\frac{4EI}{L}\right) u_2 + 0 = 0$$

$$\frac{u_1}{-\frac{4EI}{L}M} = \frac{u_2}{\frac{2EI}{L}M} = \frac{1}{\frac{32E^2I^2}{L^2} - \frac{4E^2I^2}{L^2}}$$

$$u_1 = \frac{-\frac{4EI}{L}M}{\frac{28E^2I^2}{L^2}} = -\frac{4EIM}{L} \times \frac{L^2}{28E^2I^2} = -\frac{ML}{7EI}$$

$$u_2 = \frac{\frac{2EI}{L}M}{\frac{28E^2I^2}{L^2}} = \frac{2EIM}{L} \times \frac{L^2}{28E^2I^2} = \frac{ML}{14EI}$$

$$R_c = -\frac{6EI}{L^2}\left(-\frac{ML}{7EI}\right) - \frac{6EI}{L^2}\left(\frac{ML}{14EI}\right)$$

$$= \frac{6M}{7L} - \frac{3M}{7L}$$

$$= \frac{3M}{7L}$$