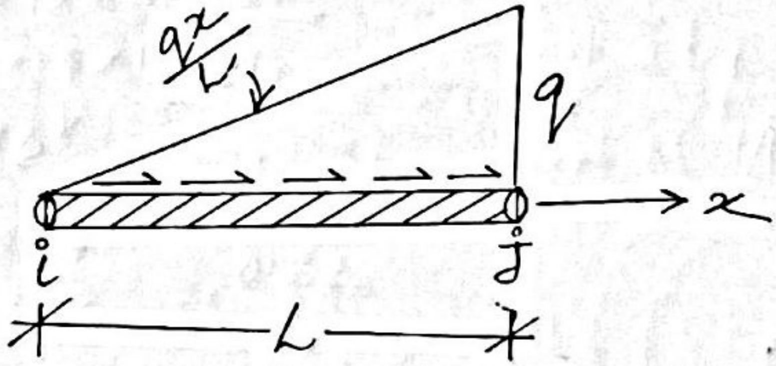
2013-2014

4(a)

$L - q$

$\therefore x - \frac{qx}{L}$



Equivalent nodal forces,

$$\{f_0\} = \begin{Bmatrix} f_{ix} \\ f_{jx} \end{Bmatrix} = \int_0^L [N]^T \left\{\frac{q}{L}\right\} x\,dx$$

$$= \int_0^L \begin{Bmatrix} 1 - \frac{x}{L} \\ \frac{x}{L} \end{Bmatrix} \frac{qx}{L}\,dx$$

$$= \int_0^L \begin{Bmatrix} \frac{qx}{L} - \frac{qx^2}{L^2} \\ \frac{qx^2}{L^2} \end{Bmatrix} dx$$

$$= \begin{bmatrix} \frac{qx^2}{2L} & \frac{qx^3}{3L^2} \\ \frac{qx^3}{3L^2} & \end{bmatrix}_0^L$$

$$= \begin{Bmatrix} \frac{qL^2}{2L} \quad \frac{qL^3}{3L^2} \\ \frac{qL^3}{3L^2} \end{Bmatrix}$$

$$= \begin{Bmatrix} \frac{qL}{2} \quad \frac{qL}{3} \\ \frac{qL}{3} \end{Bmatrix}$$

$$= \begin{Bmatrix} \frac{qL}{6} \\ \frac{qL}{3} \end{Bmatrix}$$

The new nodal force vector,

$$f + f_0 = \begin{Bmatrix} f_i + \frac{qL}{6} \\ f_j + \frac{qL}{3} \end{Bmatrix}$$