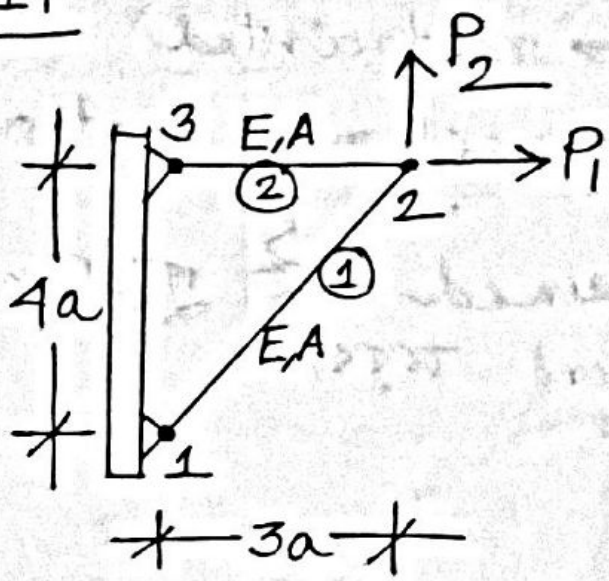
2013-2014

4(c)



Member stiffness matrix,

$$k_K = \frac{A_K E}{l_K} \begin{bmatrix} \cos^2\phi_K & \sin\phi_K\cos\phi_K & -\cos^2\phi_K & -\sin\phi_K\cos\phi_K \\ \sin\phi_K\cos\phi_K & \sin^2\phi_K & -\sin\phi_K\cos\phi_K & -\sin^2\phi_K \\ -\cos^2\phi_K & -\sin\phi_K\cos\phi_K & \cos^2\phi_K & \sin\phi_K\cos\phi_K \\ -\sin\phi_K\cos\phi_K & -\sin^2\phi_K & \sin\phi_K\cos\phi_K & \sin^2\phi_K \end{bmatrix}$$

For member 1,

$l = 5a$, $\therefore \theta = \tan^{-1}\left(\frac{4}{3}\right)$

$$\cos^2\theta = \frac{9}{25}$$

$$\sin^2\theta = \frac{16}{25}$$

$$\sin\theta\cos\theta = \frac{12}{25}$$

For member 2,

$$L = 3a, \quad \theta = \tan^{-1}(0)$$

$$\cos^2\theta = 1$$

$$\sin^2\theta = 0$$

$$\sin\theta\cos\theta = 0$$

$$[K_1] = \frac{AE}{5a}\begin{bmatrix} 9/25 & 12/25 & -9/25 & -12/25 \\ 12/25 & 16/25 & -12/25 & -16/25 \\ -9/25 & -12/25 & 9/25 & 12/25 \\ -12/25 & -16/25 & -12/25 & 16/25 \end{bmatrix}$$

$$= \frac{AE}{a}\begin{bmatrix} \frac{9}{125} & \frac{12}{125} & -\frac{9}{125} & -\frac{12}{125} \\ \frac{12}{125} & \frac{16}{125} & -\frac{12}{125} & -\frac{16}{125} \\ -\frac{9}{125} & -\frac{12}{125} & \frac{9}{125} & \frac{12}{125} \\ -\frac{12}{125} & -\frac{16}{125} & \frac{12}{125} & \frac{16}{125} \end{bmatrix}$$

$$[K_2] = \frac{AE}{3a}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \frac{AE}{a}\begin{bmatrix} 1/3 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & 0 \\ -1/3 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Stiffness matrix,

$$\frac{AE}{a}\begin{bmatrix} \frac{9}{125} & \frac{12}{125} & -\frac{9}{125} & -\frac{12}{125} & 0 & 0 \\ \frac{12}{125} & \frac{16}{125} & -\frac{12}{125} & -\frac{16}{125} & 0 & 0 \\ -\frac{9}{125} & -\frac{12}{125} & \frac{152}{376} & \frac{12}{125} & -\frac{1}{3} & 0 \\ -\frac{12}{125} & -\frac{16}{125} & \frac{12}{125} & \frac{16}{125} & 0 & 0 \\ 0 & 0 & -1/3 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{AE}{a}\begin{bmatrix} \frac{9}{125} & \frac{12}{125} & -\frac{9}{125} & -\frac{12}{125} & 0 & 0 \\ \frac{12}{125} & \frac{16}{125} & -\frac{12}{125} & -\frac{16}{125} & 0 & 0 \\ -\frac{9}{125} & -\frac{12}{125} & \frac{152}{376} & \frac{12}{125} & -\frac{1}{3} & 0 \\ -\frac{12}{125} & -\frac{16}{125} & \frac{12}{125} & \frac{16}{125} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{Bmatrix}$$

$$\frac{AE}{a}\begin{bmatrix} \frac{9}{125} & \frac{12}{125} & -\frac{9}{125} & -\frac{12}{125} & 0 & 0 \\ \frac{12}{125} & \frac{16}{125} & -\frac{12}{125} & -\frac{16}{125} & 0 & 0 \\ -\frac{9}{125} & -\frac{12}{125} & \frac{152}{376} & \frac{12}{125} & -\frac{1}{3} & 0 \\ -\frac{12}{125} & -\frac{16}{125} & \frac{12}{125} & \frac{16}{125} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ u_{2x} \\ u_{2y} \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ F_{1y} \\ P_1 \\ P_2 \\ F_{3x} \\ F_{3y} \end{Bmatrix}$$

$$\frac{AE}{a}\begin{bmatrix} \frac{152}{376} & \frac{12}{125} \\ \frac{12}{125} & \frac{16}{125} \end{bmatrix} \begin{Bmatrix} u_{2x} \\ u_{2y} \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

$$\left(\frac{152}{376}\right)u_{2x} + \left(\frac{12}{125}\right)u_{2y} - \frac{P_1 a}{AE} = 0$$

$$\left(\frac{12}{125}\right)u_{2x} + \left(\frac{16}{125}\right)u_{2y} - \frac{P_2 a}{AE} = 0$$

$$\frac{u_{2x}}{\frac{12P_2a - 16P_1a}{125AE}} = \frac{u_{2y}}{\frac{4512P_1a - 19000P_2a}{47000AE}} = \frac{1}{\frac{31232}{734375}}$$

$$u_{2x} = \frac{12P_2a - 16P_1a}{125AE} \div \frac{31232}{734375}$$

$$= \frac{5875a}{31232AE}(12P_2 - 16P_1)$$

$$u_{2Y} = \frac{4512P_1a - 19000P_2a}{47000AE} \div \frac{31232}{734375}$$

$$= \frac{125a}{249856AE}(4512P_1 - 19000P_2)$$

Displacement of node 2,

$$\begin{Bmatrix} u_{2x} \\ u_{2Y} \end{Bmatrix} = \frac{a}{AE} \begin{Bmatrix} \frac{17625}{7808}P_2 - \frac{5875}{1952}P_1 \\ \frac{17625}{7808}P_1 - \frac{296875}{31232}P_2 \end{Bmatrix}$$