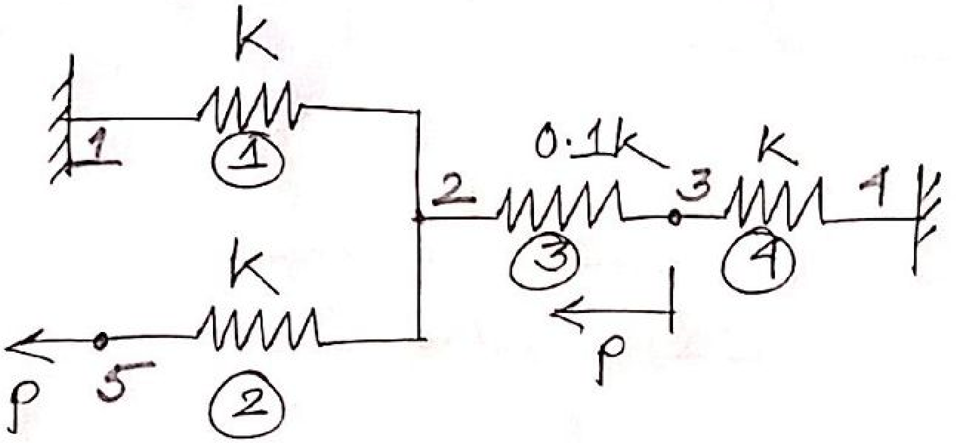
Set A



If $P = 100$ kg & $k = 10$ kg/m find the element with maximum deformation

Solution:

Element connectivity matrix

Element	Node i	Node j
1	1	2
2	5	2
3	2	3
4	3	4

$$\mathbb{K}_1 = \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} \qquad \mathbb{K}_2 = \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix}$$

$$\mathbb{K}_3 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \mathbb{K}_4 = \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix}$$

The stiffness matrix

$$\begin{bmatrix} 10 & -10 & & & \\ -10 & 10+10+1 & -1 & & -10 \\ & -1 & 1+10 & -10 & \\ & & -10 & 10 & \\ & -10 & & & 10 \end{bmatrix}$$

The stiffness equation

$$\begin{bmatrix} 10 & -10 & 0 & 0 & 0 \\ -10 & 21 & -1 & 0 & -10 \\ 0 & -1 & 11 & -10 & 0 \\ 0 & 0 & -10 & 10 & 0 \\ 0 & -10 & 0 & 0 & 10 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix}$$

Applying boundary condition

$$\begin{bmatrix} 10 & -10 & 0 & 0 & 0 \\ -10 & 21 & -1 & 0 & -10 \\ 0 & -1 & 11 & -10 & 0 \\ 0 & 0 & -10 & 10 & 0 \\ 0 & -10 & 0 & 0 & 10 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ 0 \\ u_5 \end{Bmatrix} = \begin{bmatrix} F_1 \\ 0 \\ -100 \\ 0 \\ -100 \end{bmatrix}$$

The reduced FE becomes,

$$(21)u_2 + (-1)u_3 + (-10)u_5 = 0$$
$$(-1)u_2 + (11)u_3 + (0)u_5 = -100$$
$$(-10)u_2 + (0)u_3 + (10)u_5 = -100$$

Now, $u_2 = -10\,m$

$u_3 = -10\,m$

$u_5 = -20\,m$

Displacement in element 1 $= u_2 - u_4$

$= -10 - 0$

$= -10\,m$

Displacement in element 2 $= u_2 - u_5$

$= -10 - (-20)$

$= 10\,m$

Displacement in element 3 $= u_3 - u_2$

$= -10 - (-10)$

$= 0\,m$

Displacement in element 4 $= u_4 - u_3$

$= 0 - (-10)$

$= 10\,m$

So, elements 1, 2 & 4 gets displaced an equal amount.