Example 3.12

A bar of length L is subjected to a linearly distributed axial loading that varies from zero at node 1 to a maximum at node 2 (Figure 3–28). Determine the energy equivalent nodal loads.

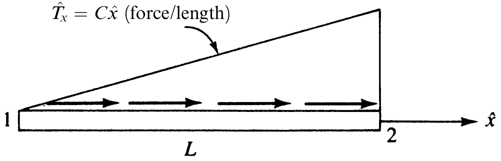

Figure 3–28 Element subjected to linearly varying axial load

Using Eq. (3.10.20a) and shape functions from Eq. (3.10.9), we solve for the energy equivalent nodal forces of the distributed loading as follows:

$$\{\hat{f}_0\} = \begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{2x} \end{Bmatrix} = \iint_{S_1} [N]^T \{\hat{T}_x\}\, dS = \int_0^L \begin{Bmatrix} 1 - \dfrac{\hat{x}}{L} \\ \dfrac{\hat{x}}{L} \end{Bmatrix} \{C\hat{x}\}\, d\hat{x} \tag{3.10.30}$$

$$= \begin{Bmatrix} \dfrac{C\hat{x}^2}{2} - \dfrac{C\hat{x}^3}{3L} \\ \dfrac{C\hat{x}^3}{3L} \end{Bmatrix}_0^L$$

$$= \begin{Bmatrix} \dfrac{CL^2}{6} \\ \dfrac{CL^2}{3} \end{Bmatrix} \tag{3.10.31}$$

where the integration was carried out over the length of the bar, because $\hat{T}_x$ is in units of force/length.

Note that the total load is the area under the load distribution given by

$$F = \frac{1}{2}(L)(CL) = \frac{CL^2}{2} \tag{3.10.32}$$

Therefore, comparing Eq. (3.10.31) with (3.10.32), we find that the equivalent nodal loads for a linearly varying load are

$$\begin{aligned} \hat{f}_{1x} &= \frac{1}{3}F = \text{one-third of the total load} \\ \hat{f}_{2x} &= \frac{2}{3}F = \text{two-thirds of the total load} \end{aligned} \tag{3.10.33}$$

In summary, for the simple two-noded bar element subjected to a linearly varying load (triangular loading), place one-third of the total load at the node where the distributed loading begins (zero end of the load) and two-thirds of the total load at the node where the peak value of the distributed load ends. ■