Prof. Saiful Amin Sir

CE 419

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01/03/17 / Wed

Introduction to Finite Element Method

* Finite Element Method – We can't use it as a block box as all computer won't give same result

* This is an 'approximate method' – logical analysis
  – Realistic result

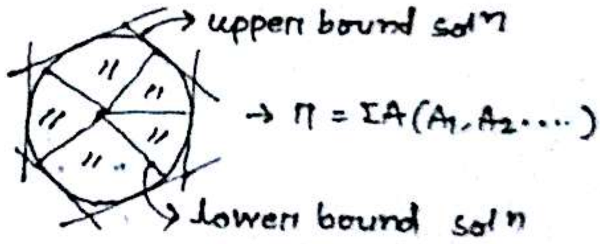
• $\pi = \frac{22}{7}$

$= 3.141$

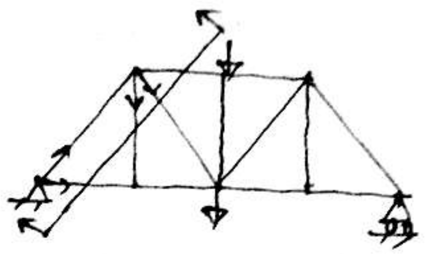
$= \frac{333}{106}$

$= \frac{353}{113}$

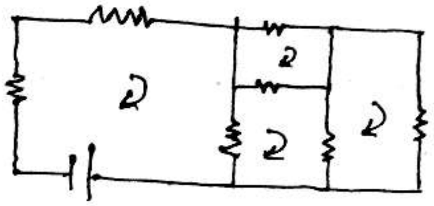
$= \cfrac{4}{1+\cfrac{1^2}{2+\cfrac{3^2}{2+\cfrac{5^2}{2+4^2 \cdots}}}}$



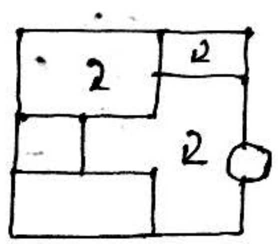
• $F = K\Delta$ → Stiffness Method



- Internal force = External force



- $V = IR$
- $i_{in} = i_{out}$
- $V_{in} = V_{out}$



- $Q_{in} = Q_{out}$
- $Q = AV$

- $F = \underset{\underset{f\left(\frac{EI}{L}\right)}{\downarrow}}{K}\Delta$ · [only MOI (I) design করা লাগবে, বাকি সব known]

- Book (আনা লাগবে)

07/03/17 Tue.

Finite Element Method :

- Pre processor $\longrightarrow$ $F = K\Delta$ $\rightarrow$ Post processor

  Keyboard

  Mouse

  ....

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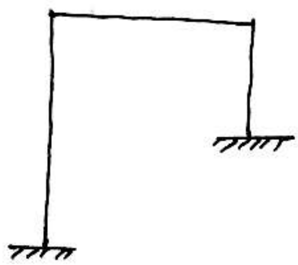
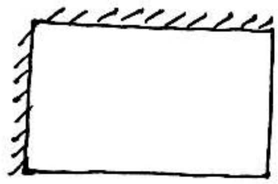
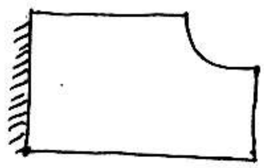
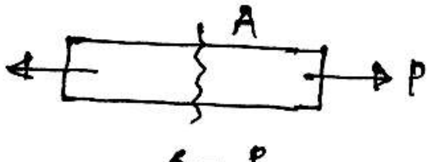
Idealization :



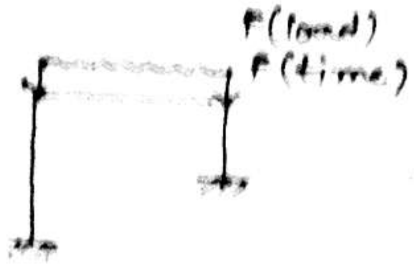




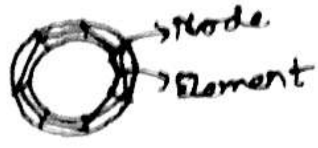


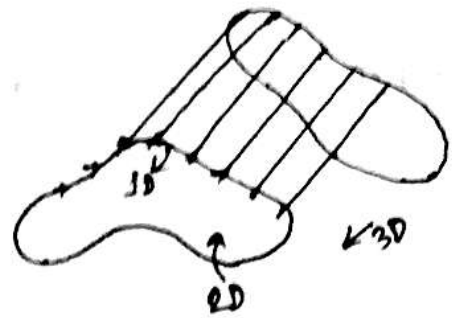
$$\sigma = \frac{P}{A}$$

- → Shape
- → Boundary
- → Loading condition



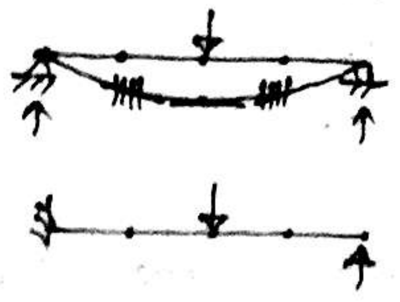
* We divide the structure into finite number of elements.





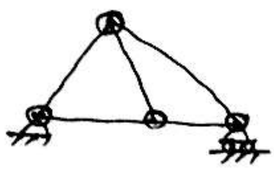
* Molecular form এ structural behaviour থাকে না so finite element এ divide করি।

• Elements are connected by nodes.



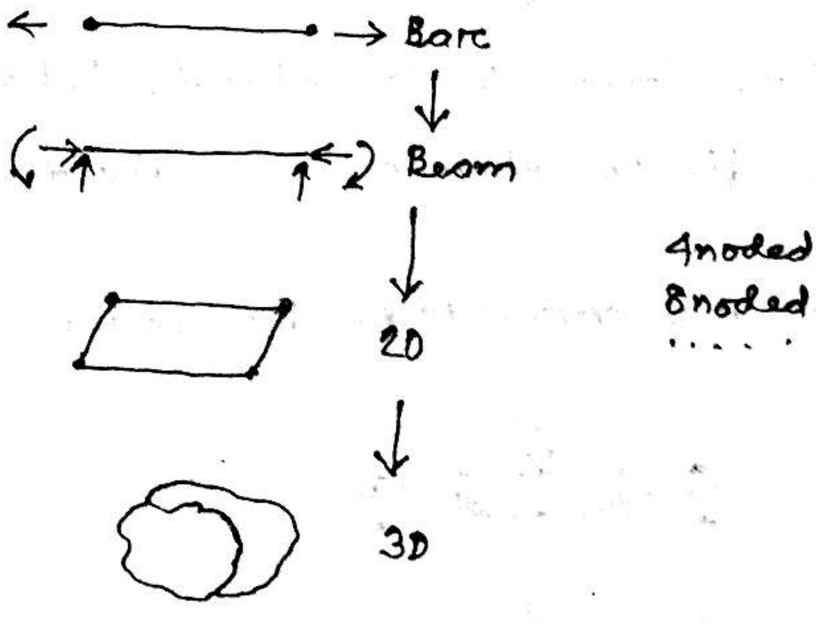
- Node number কম-বেশি হতে পারে।
- Load থাকলে must node দিতে হবে।

* যেখানে output পেতে চাই, সেখানে must node দিতে হবে।



* Shape function এর উপর depend করবে কোন element হবে।

* Shape function ⇒



- Higher order element এর জন্য shape function higher হবে।

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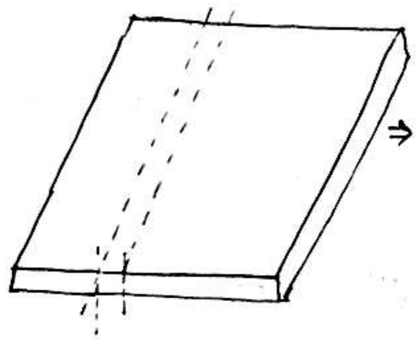
Idealization :

- 3D → 1D



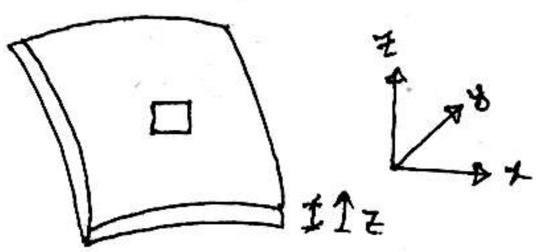
⇒ Length, width, height আছে so 3D

* 3 elements of idealization :

  - Plane stress (problem)
  - Plane strain (")
  - Axy - symmetric (")

Plane stress :

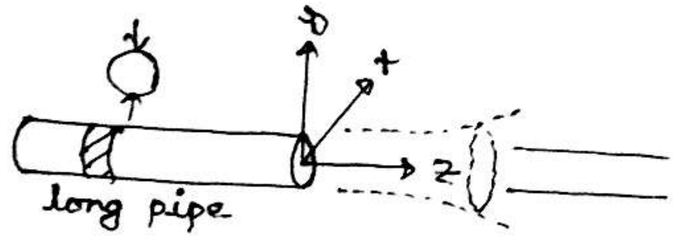
- $\sigma_z = 0$
- $\varepsilon_z \neq 0$



* $\sigma_z = 0$ হলে perpendicular direction এ কোন stress নিতে পারবে না। like balloon.

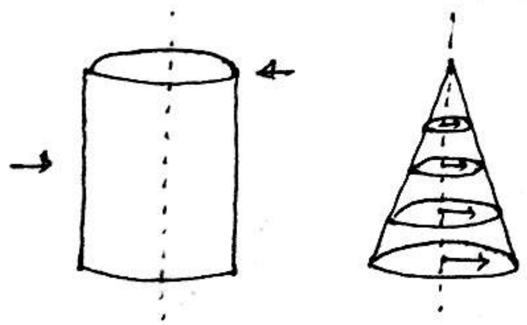
Plane strain :

- $\varepsilon_z = 0$
- $\sigma_z \neq 0$

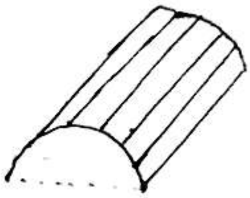


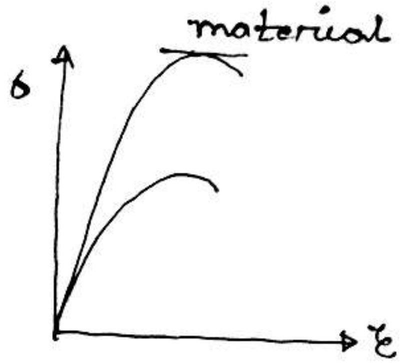
* z direction এ expansion-contraction হয় না, as if $\varepsilon_z \neq 0$, pipe connect করা যাবে না।
pipe এর dia change হলে সেখানে আলাদা section দিতে হবে।

* Chapter 2 : Pg 16 – 19 (Def^n + Example)



* Geometric idealization





⇒ Material idealization is needed as concrete follows Hook's law upto certain limit

Linearization

• $\sigma = f(\varepsilon)$

* $\sigma = E\,\varepsilon$ ⇒ constitutive relation
(can be linear or non-linear)

* Material hook's law follow করলে linear.

Article 2.6

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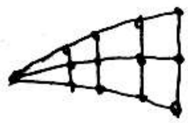
☐ Idealization : 3 idealization in FEM

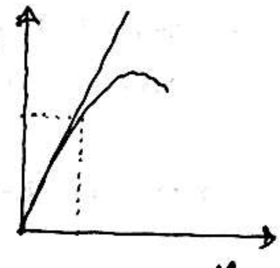
Geometric idealization :

- Nodes / Elements



Material idealization :

- Constitutive relation
(Obeys Hook's law)



Numerical idealization & solution :

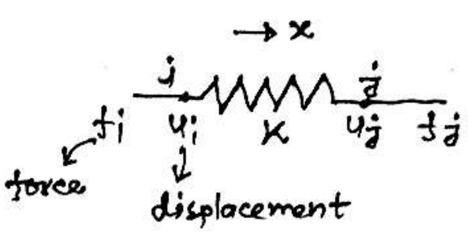
- Shape function
- solⁿ of non-linear equation (if needed)
- Numerical integration

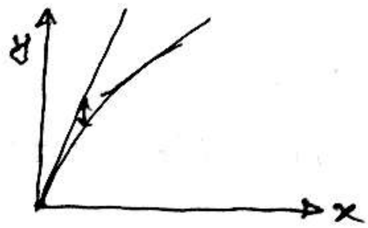
* Chapter 2 (full syllabus) - Bhabikatti's Book

Lecture Note :

III. Spring element :

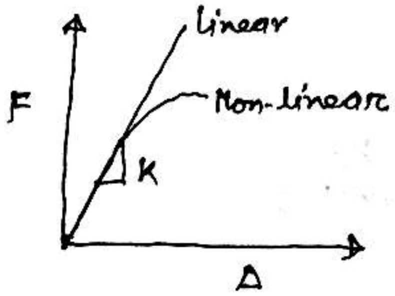
* "Everything important is simple."





* $F = K\Delta$

• $\Delta = u_j - u_i$



$$K = \frac{F}{\Delta}$$

$$f_i = -F = -k(u_j - u_i) = ku_i - ku_j$$

$$f_j = F = k(u_j - u_i) = -ku_i + ku_j$$

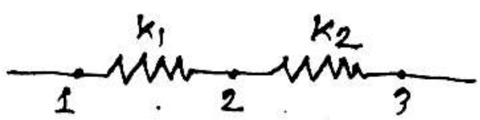
$$* \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} = \begin{Bmatrix} f_i \\ f_j \end{Bmatrix}$$

$$\Rightarrow \underbrace{K}_{\text{Element Stiffness matrix}} \; \underbrace{u}_{\text{displacement vector}} = \underbrace{f}_{\text{force vector}}$$

* Vector এর direction আছে, but matrix এর direction থাকে না।

- Stiffness, $k = f\left(\frac{EI}{L}\right)$

Spring system:



For element 1,

$$\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} f_1^1 \\ f_2^1 \end{Bmatrix}$$

For element 2,

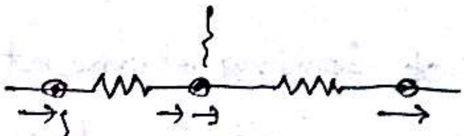
$$\begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} f_1^2 \\ f_2^2 \end{Bmatrix}$$

Idealization,

@ Node 1 $\Rightarrow$ $F_1 = f_1^1$

@ Node 2 $\Rightarrow$ $F_2 = f_2^1 + f_1^2$

@ Node 3 $\Rightarrow$ $F_3 = f_2^2$



Element Topology /
" connectivity

$$F_1 = k_1 u_1 - k_1 u_2$$

$$F_2 = -k_1 u_1 + (k_1 + k_2) u_2 - k_2 u_3$$

$$F_3 = -k_2 u_2 + k_2 u_3$$

* $$\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\Rightarrow KU = F$$

* Page 18 ~ 19 (Assignment)

How to check the results?

- Deformed shape of the structure
- Balance of external forces
- Order of magnitude of number

* Spring element
  - Suitable for stiffness analysis
  - Not suitable for spring itself
  - Spring element in lateral direction / torsion

Helical spring

* Example 1.1 & 1.2 (Assignment) [Next class]

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Chapter 2

Bar and Beam Elements

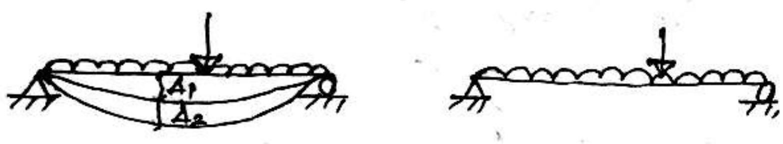
↓ one-dimensional element

(1) Linear static analysis :

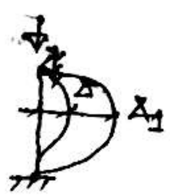
* Assumption : (Imp. for exam, explain করে লিখতে হবে)

1. Small deformation
2. Elastic materials
3. Static loads

(1)

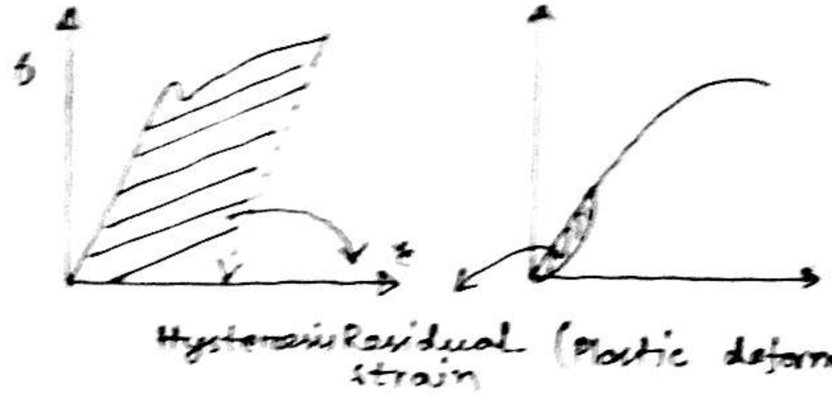


* Self wt. / load এর ফলে deflection হলে beam এর geometric properties change হয় so deformation small হতে হবে।



$\Delta_1 > \Delta$

② যেসব material elongate হওয়ার plastic deformation হয়।



* Linear static analysis এ plastic deformation ignore করা হয়।

③ EQ load (time varying)

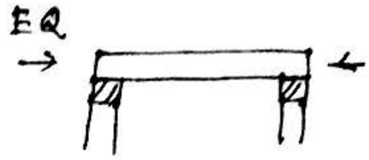
* P is a function of time but we consider it as a constant.

* Superposition theory applicable হবে জিনিসগুলো apply করবো।

$$\Delta = \Delta_1 + \Delta_2$$

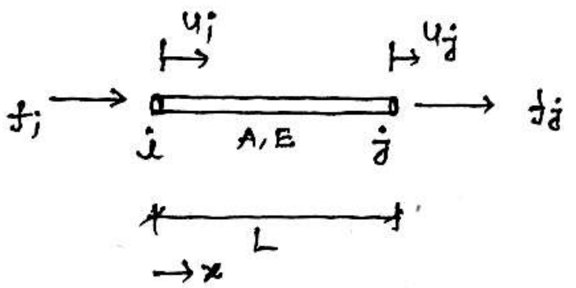
* Non-linear dynamic analysis

- large deformation
- Non-elastic materials
- Dynamic loads



Base isolation

II Bar Element :



$$u = u(x)$$
$$\varepsilon = \varepsilon(x)$$
$$\sigma = \sigma(x)$$

* Strain-displacement relation :

$$\varepsilon = \frac{du}{dx}$$

* Stress-strain relation :

$$\sigma = E\varepsilon$$

* $u(x) = \left(1-\frac{x}{L}\right) u_i + \frac{x}{L} u_j \quad \Rightarrow \quad \begin{array}{l} x = 0,\ u(x) = u_i \\ x = L,\ u(x) = u_j \end{array}$

* $\varepsilon = \dfrac{u_i - u_j}{L} = \dfrac{\Delta}{L}$

* $\sigma = E\varepsilon = \dfrac{EA}{L}$

- $\sigma = \dfrac{F}{A}$

- $F = \dfrac{EA}{L}\Delta = K\Delta$

* $K = \dfrac{EA}{L}$

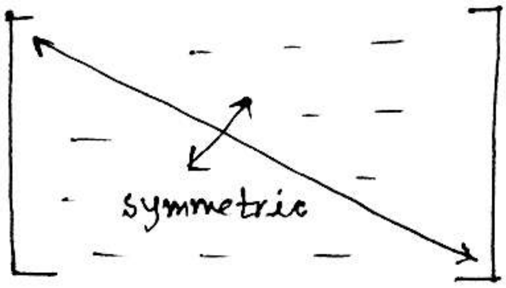
* $\underline{\underline{K}} = \begin{bmatrix} K & -K \\ -K & K \end{bmatrix} = \begin{bmatrix} EA/L & -EA/L \\ -EA/L & EA/L \end{bmatrix}$

means Bold K

* formal approach :

  - Derive from Shape function

- Structure যত বড় হবে, stiffness matrix তত বড় হবে।



* Diagonal element always (+)ve & Non-zero.
বাকি দুই পাশের গুলো symmetric হবে।

- $K_{21} = K_{12}$

- $K_{ji} = K_{ij}$

- $F = K\Delta$ → K: proportionality factor

* Diagonal element 0 হলে, matrix becomes singular. singular matrix solve করা যায় না।

$\begin{bmatrix} -K & 0 \\ 0 & 0 \end{bmatrix} \rightarrow$ not a stiffness matrix

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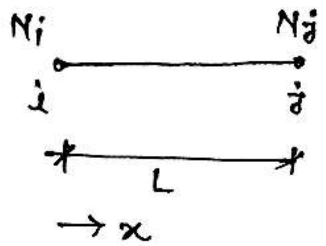
Stiffness Matrix :

Bar Element

$$* \ \underline{\underline{K}} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} = \begin{bmatrix} EA/L & -EA/L \\ -EA/L & EA/L \end{bmatrix}$$

→ Formal approach (pg 29)

* Derivation from shape function :



Two linear shape functions :

$$N_i(\xi) = 1 - \xi$$

$$N_j(\xi) = \xi$$

$$\xi = \frac{x}{L}$$

$$0 \leq \xi \leq 1$$

- ξ → ইকসাই, x
- ρ → ডেট, y
- η → ইটা, z

[* Shape function describes variation of deformation.]

$$u(x) = u(\xi) = N_i(\xi)\, u_i + N_j(\xi)\, u_j$$

$$\Rightarrow \quad u = [N_i \quad N_j] \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} = \underline{\underline{N}}\; \underline{\underline{u}}$$

[* Shape function unitless]

$$\varepsilon = \frac{du}{dx} = \frac{d}{dx}[\underline{\underline{N}}\; \underline{\underline{u}}] = \left[\frac{d}{dx}\underline{\underline{N}}\right] \underline{\underline{u}}$$

$$= \underline{\underline{B}}\; \underline{\underline{u}}$$

- $\underline{\underline{B}} \equiv$ strain-displacement matrix (Shape function 1st derivative)

$$\underline{\underline{B}} = \frac{d}{dx}[N_i(\xi) \quad N_j(\xi)] = \frac{d}{d\xi}[N_i(\xi) \quad N_j(\xi)]\frac{d\xi}{dx}$$

$$\Rightarrow \quad \underline{\underline{B}} = \left[-\frac{1}{L} \quad \frac{1}{L}\right]$$

* Stress – Strain → constitutive relation → material property is represented

* $\sigma = E\varepsilon = E\underline{\underline{B}}\ \underline{\underline{u}}$

$\Rightarrow \underset{\text{Strain energy}}{U} = \frac{1}{2}\int_{\underset{\text{volume}}{V}} \sigma^{\overset{\text{transpose}}{T}} \varepsilon \, dV$

$= \frac{1}{2}\int_V (\underline{\underline{u}}^T \underline{\underline{B}}^T E \underline{\underline{B}}\ \underline{\underline{u}})\, dV$

$= \frac{1}{2}\ \underline{\underline{u}}^T \left[\int_V (\underline{\underline{B}}^T E \underline{\underline{B}})\, dv\right] \underline{\underline{u}}$

* Work done, $W = \overset{\text{avg. displacement}}{\frac{1}{2}} f_i u_i + \frac{1}{2} f_j u_j = \frac{1}{2}\underline{\underline{u}}^T \underline{\underline{f}}$

(dot product ~~হবে~~)

$\Rightarrow W = U$

$U = W$

$\Rightarrow \frac{1}{2} u^T \left[\int_V (\underline{\underline{B}}^T E \underline{\underline{B}})\, dv\right] \underline{\underline{u}} = \frac{1}{2}\underline{\underline{u}}^T \underline{\underline{f}}$

$\Rightarrow \left[\int_V (\underline{\underline{B}}^T E \underline{\underline{B}})\, dv\right] \underline{\underline{u}} = \underline{\underline{f}}$

$\Rightarrow \underline{\underline{K}}\ \underline{\underline{u}} = \underline{\underline{f}}$

* $\underline{K} = \int_V (\underline{B}^T E \underline{B}) \, dv$

$\Rightarrow \underline{K} = \dots\dots = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

- $U = \frac{1}{2} \underline{\underline{u}}^T \underline{\underline{K}} \, \underline{\underline{u}}$

* Example 2.1 (Assignment)

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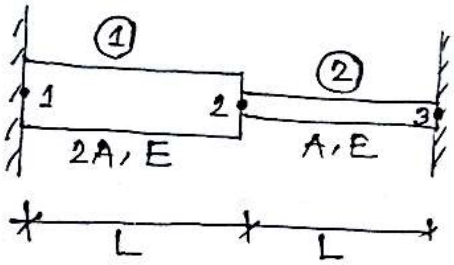
Example 2.1 :



- $A (\uparrow)$ Stiffness $(\uparrow)$
- $L (\uparrow)$ Stiffness $(\downarrow)$

$$K_1 = \frac{2EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K_2 = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\frac{EA}{L} \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$u_1 = u_3 = 0$

$F_2 = P$

FE equation .....

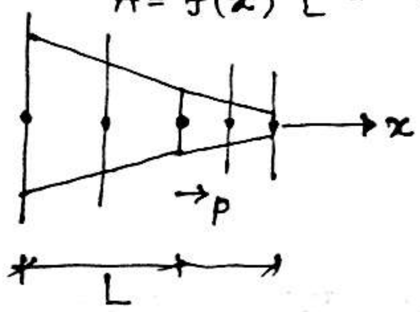
* Pg. 34 Notes (Exam এ short ques আসতে পারে)

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \frac{PL}{3EA} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

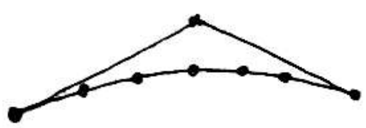
$\Rightarrow \sigma_1 = \frac{P}{3A}$ (Tension)

$\sigma_2 = -\frac{P}{3A}$ (compression)

$A = f(x)$ [cross section length এর সাথে সাথে change হচ্ছে]

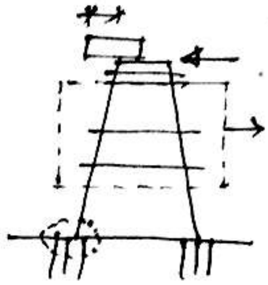


* যেখানে force apply করব এবং যেখানে boundary আছে সেখানে node apply করতে হবে।



• curvature হলে প্রত্যেক point এ node define করতে হবে।

* যেখানে stress / displacement variation বেশি হবে, সেখানে node বেশি হবে।



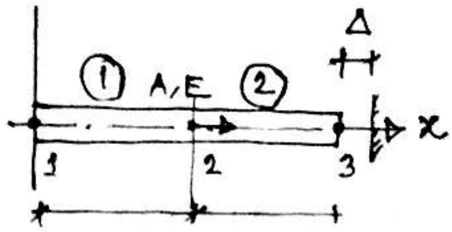
* যত node বেশি হবে, DOF তত বেশি হবে, Stiffness matrix তত বড় হবে।

* Displacement based FEM :

$$\underline{\underline{K}}\ \underline{\underline{u}} = \underline{\underline{F}}$$

⇒ প্রথমে displacement বের করব, then force বের করব।

Example 2.2 : (Assignment)



$P = 6 \times 10^4$ N

$E = 2 \times 10^4$ N/mm^2

$L = 150$ mm

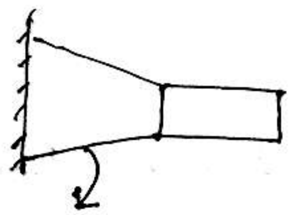
$A = 250$ mm^2

$\Delta = 1.2$ mm

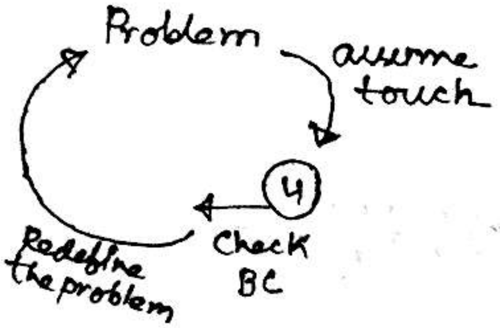
Determine the support reaction forces ?

$\Rightarrow \quad \Delta_0 = 1.8 \text{ mm} > 1.2 \text{ mm}$

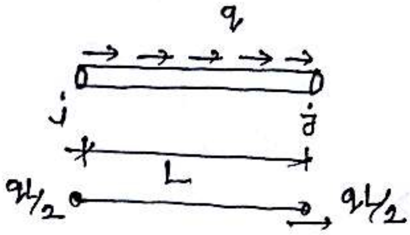
(from $\frac{PL}{EA}$)



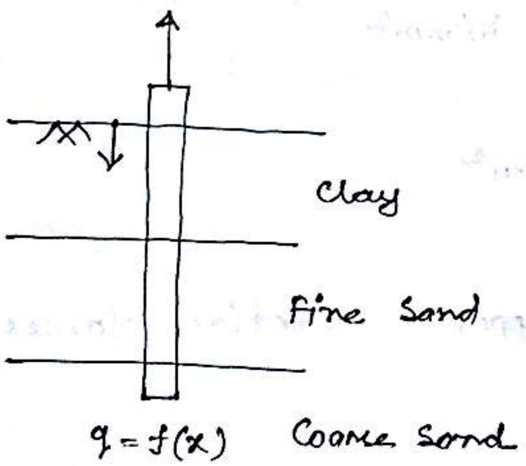
* Cross section varying হলে Δ, $\frac{PL}{EA}$ থেকে বের করা যাবে না।



Distributed load :



Pile का skin friction एक distributed force.



Equivalent Nodal force :

$$\cdot\ W_q = \int_0^L \frac{1}{2} u q \, dx$$

$$= \int_0^1 u(\xi)\, q\, (L d\xi) \frac{L}{2}$$

$$= \frac{qL}{2} \int_0^1 [N_i(\xi) \quad N_j(\xi)] \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} d\xi$$

$$= \frac{qL}{2} \int_0^1 [1-\xi \quad \xi] \, d\xi \begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$$

$$\Rightarrow W_q = \frac{1}{2} [u_i \quad u_j] \begin{Bmatrix} qL/2 \\ qL/2 \end{Bmatrix}$$

$$= \frac{1}{2} u^T f_q$$

* Page 39 → Assignment

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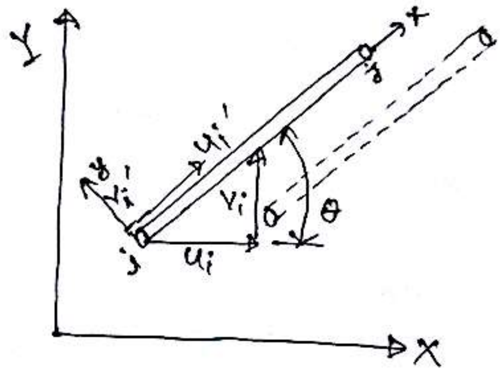
05/04/17/Wed

* Next week

- CT (Wed 1 pm) [upto previous class]

Bar elements in 3D space:



Local	Global
x, y	X, Y
u_i', v_i'	u_i, v_i
2 dof at a node	2 dof at a node

Bar element এর জন্য, $v_i' = 0$

$$u_i' = u_i \cos\theta + v_i \sin\theta = [l \quad m] \begin{Bmatrix} u_i \\ v_i \end{Bmatrix}$$

$$v_i' = -u_i \sin\theta + v_i \cos\theta = [-m \quad l] \begin{Bmatrix} u_i \\ v_i \end{Bmatrix}$$

$l = \cos\theta, \quad m = \sin\theta$

In matrix form,

$$\begin{Bmatrix} u_i' \\ v_i' \end{Bmatrix} = \begin{bmatrix} l & m \\ -m & l \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \end{Bmatrix}$$

$$u_i' = \underset{=}{\tilde{T}}\, u_i \longrightarrow \text{চিহ্নটা}$$

$$\underset{=}{\tilde{T}} = \begin{bmatrix} l & m \\ -m & l \end{bmatrix}$$

$$\underset{=}{\tilde{T}}^{-1} = \underset{=}{\tilde{T}}^{T}$$

Orthogonal Matrix

(Inverse & Transpose সমান)

$$\underset{=}{f}' = \underset{=}{T}\, \underset{=}{f}$$

nodal force

<u>Stiffness Matrix</u>:

$$\underset{=}{k}'\underset{=}{u}' = \underset{=}{f}'$$

$$\underset{=}{k}'\underset{=}{T}\,\underset{=}{u} = \underset{=}{T}\,\underset{=}{f}$$

$$\underset{=}{T}^{T}\underset{=}{k}\,\underset{=}{T}\,\underset{=}{u} = \underset{=}{f}$$

$$\underset{=}{k} = \underset{=}{T}^{T}\underset{=}{k}\,\underset{=}{T}$$

Explicit form:

$$\underline{k} = \frac{EA}{L}\begin{bmatrix} & \end{bmatrix} \quad \cdots\cdots(32)$$

$l = \cos\theta$, $m = \sin\theta$ } Direction cosines

$$\sigma = E\varepsilon = E\underline{\underline{B}}\begin{Bmatrix} u_i' \\ u_j' \end{Bmatrix} = E\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}\begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \end{Bmatrix}$$

$$\sigma = \frac{E}{L}\begin{bmatrix} -l & m & l & m \end{bmatrix}\begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \end{Bmatrix} = \cdots\cdots$$

* Example 2.3 (Assignment)

11/04/17 / Tue

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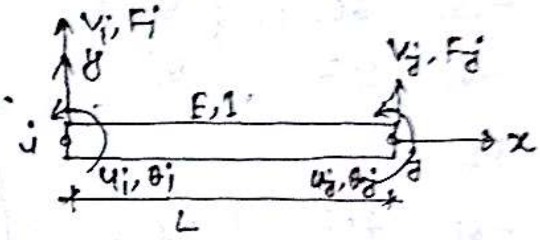
Beam Element :



+ Bar = Beam-Col Action

2D
↓
3D

DOF ⇒ θ , v_i → displacement due to shear force

M, F } per node

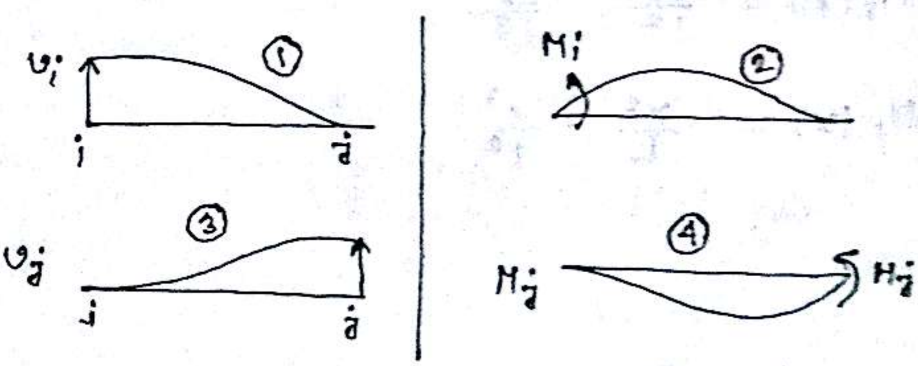
* Elementary Beam theory :

$$EI \frac{d^2 v}{dx^2} = M(x)$$

$$\sigma = (-)\frac{My}{I}$$

for moment sign

* page 54 :



* Element stiffness Equation:

$$\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{Bmatrix} = \begin{Bmatrix} F_i \\ M_i \\ F_j \\ M_j \end{Bmatrix}$$

Formal approach:

$$* \; \underline{\underline{K}} = \int_0^L \underline{\underline{B}}^T EI \, \underline{\underline{B}} \, dx$$

* Shape function:

$$N_i(x) = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}$$

$$N_2(x) = x - \frac{2x^2}{L} + \frac{x^3}{L^2}$$

$$N_3(x) = \frac{3x^2}{L^2} - \frac{2x^3}{L^3}$$

$$N_4(x) = -\frac{x^2}{L} + \frac{x^3}{L^2}$$

$$v(x) = \underline{\underline{N}}\ \underline{\underline{y}}$$

$$= \begin{bmatrix} N_1(x) & N_2(x) & N_3(x) & N_4(x) \end{bmatrix} \begin{Bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{Bmatrix}$$

$$N_1 + N_3 = 1$$

$$N_2 + N_3 L + N_4 = x$$

* From Moment - curvature relation :

$$\frac{d^2 v}{dx^2} = \frac{d}{dx^2} \underline{\underline{N}}\ \underline{\underline{y}} = \underline{\underline{B}}\ \underline{\underline{y}}$$

$$\underline{\underline{B}} = \frac{d^2}{dx^2} \underline{\underline{N}} = \begin{bmatrix} N_1''(x) & N_2''(x) & N_3''(x) & N_4''(x) \end{bmatrix}$$

$$\Rightarrow \underline{\underline{B}} = \begin{bmatrix} -\frac{6}{L^2} + \frac{12x}{L^3} & -\frac{4}{L} + \frac{6x}{L^2} & \frac{6}{L^2} - \frac{12x}{L} & -\frac{2}{L} + \frac{6x}{L^2} \end{bmatrix}$$

- U = strain energy stored in the beam element

$$= \frac{1}{2} \int_V \sigma^T \epsilon \, dV = \frac{1}{2} \iint \left(-\frac{My}{I}\right)^T \frac{1}{E} \left(-\frac{My}{I}\right) dA\, dx$$

$$= \frac{1}{2} \int M^T \frac{1}{EI} M \, dx$$

$$= \frac{1}{2} \int_0^L \left(\frac{d^2 v}{dx^2}\right)^T EI \left(\frac{d^2 v}{dx^2}\right) dx$$

$$= \frac{1}{2} \int (\underline{B}\,\underline{y})^T EI (\underline{B}\,\underline{y})\, dx$$

$$= \frac{1}{2} \underline{y}^T \left(\int_0^L \underline{B}^T EI\, \underline{B}\, dx\right) \underline{y}$$

$$\underline{\underline{K}} = \int_0^L \underline{\underline{B}}^T [EI] \, \underline{\underline{B}} \, dx$$

$\downarrow$ by putting $\underline{\underline{B}}$

$$\frac{EI}{L^3} \begin{bmatrix} 12 & 6L & \cdots \\ \cdots & & \\ \cdots & & \end{bmatrix}$$

* page 57 :

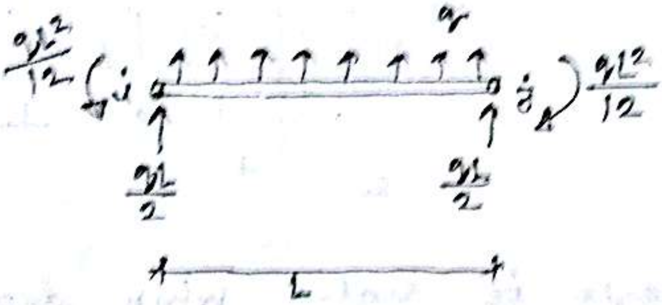
$$\begin{array}{c} u_i \quad v_i \quad \theta_i \quad u_j \quad v_j \quad \theta_j \\ \begin{bmatrix} \\ k_{beam} + k_{bar} \\ \text{Beam-Col. Action} \\ \\ \end{bmatrix} \end{array}$$

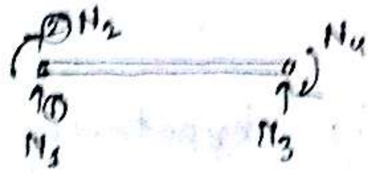
* Example 2.5 (pg 58) [Assignment]

[page 61]

Beam Element

Equivalent Nodal Loads Distributed to transverse loads:





$$N_1(x) = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}$$

$$N_2(x) = x - \frac{2x^2}{L} + \frac{x^3}{L^2}$$

$$N_3(x) = \frac{3x^2}{L^2} - \frac{2x^3}{L^3}$$

Formal approach using shape function

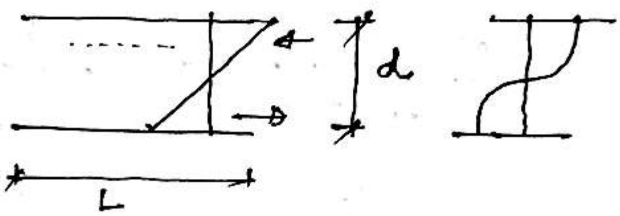
$$N_4(x) = -\frac{x^2}{L} + \frac{x^3}{L^2}$$

$$N_1 + N_3 = 1$$

$$N_2 + N_3 L + N_4 = x$$

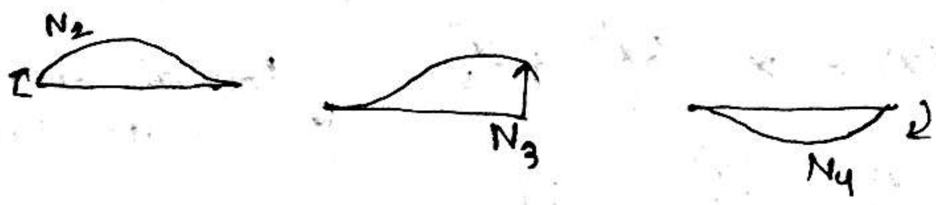
$$\underline{\underline{K}} = \int_0^L \underline{\underline{B}}^T EI \, \underline{\underline{B}} \, dx$$

$$\underline{\underline{B}} = \frac{d^2}{dx^2} \underline{\underline{N}}$$ (Strain-displacement matrix)



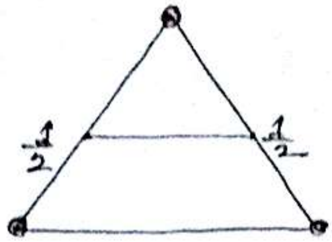
* Navier hypothesis is valid when depth of beam is less $\left(\frac{L}{d} \geq 5\right)$

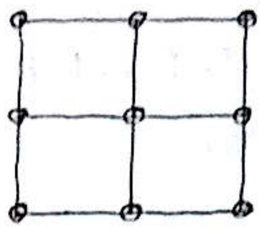
- Euler beam → Navier hypothesis
- Timoshenko beam → Shear def.

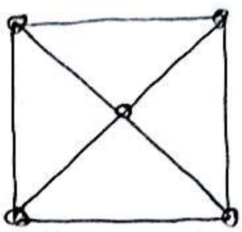


Scanned by CamScanner

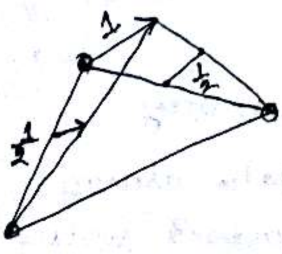
• $EI \frac{d^2v}{dx^2} = M(x)$

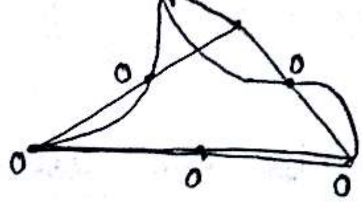






$$2\times2 \xrightarrow{2^3} 3\times3 \xrightarrow{3^3} 4\times4 \xrightarrow{4^3} 5\times5 \xrightarrow{5^3} 6\times6 \rightarrow$$





* Deflected shape এর pattern follow করলে easily shape function লেখা যায়।

* Example 2.6

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Chapter 1

* Article 1.1 – 1.8

Chapter 2

* Full

Chapter 3

* pg. 26 → Beam element

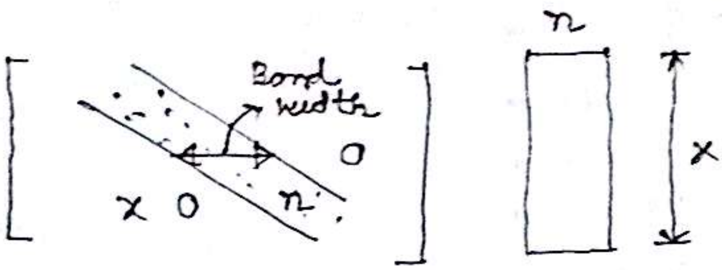
* pg. 28 → Band solution

* Art 3.4 → Techniques of saving computer memory requirement

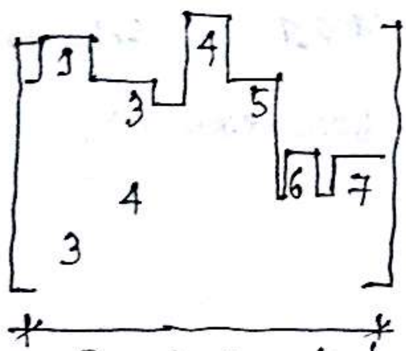
- Solution

→ Optimize the use of memory
(You don't need all data always
Assign a common/redundant value for a missing data)

- Bond solution (pg. 28 & 31)



- Skyline technique

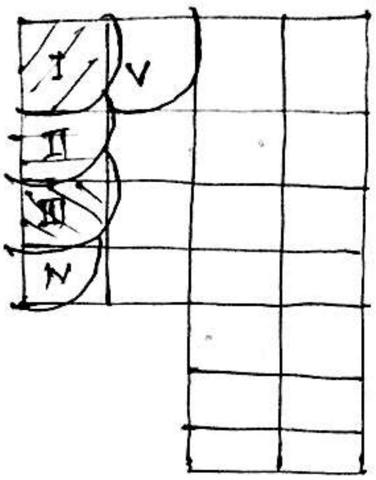


Bond ধরেরাটাই
(you can't reduce the size of the matrix)

* Bond solution efficiency না দিলে Skyline solⁿ এ যেতে পারি।

Frontal solⁿ technique

- Sohraduddin Ahmed – Techniques of Finite element (Irons & Ahmed)
  ↓
  Thickshell Finite Element



** Front কীভাবে move করে - Schemetically দেখাতে বলে ? (Imp. for exam)

Chapter 4

* pg. 40

* pg. 45

* Natural coordinates in 2D pg 45 (Assignment)

* pg. 50

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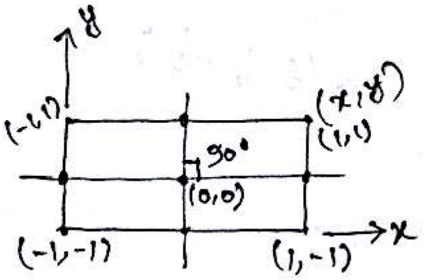
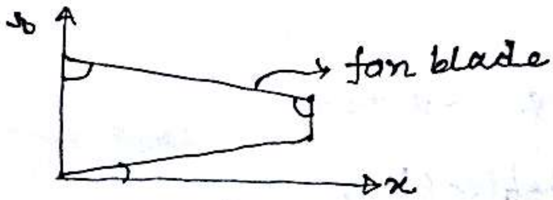
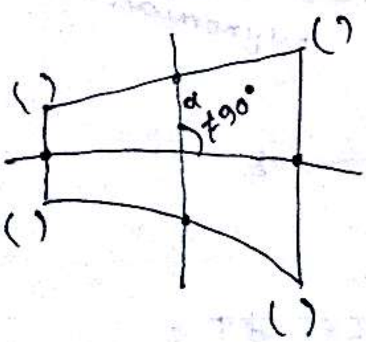
Natural Coordinates for Rectangular Elements (page 49)

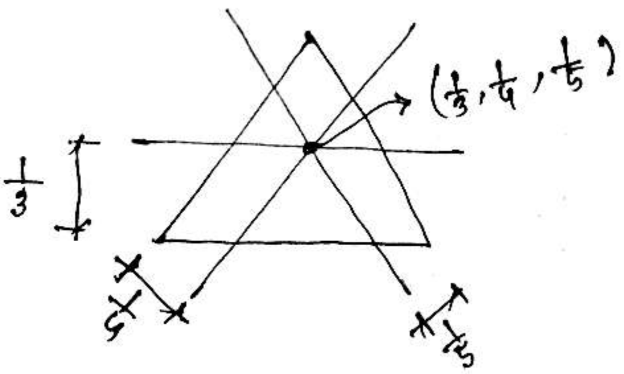






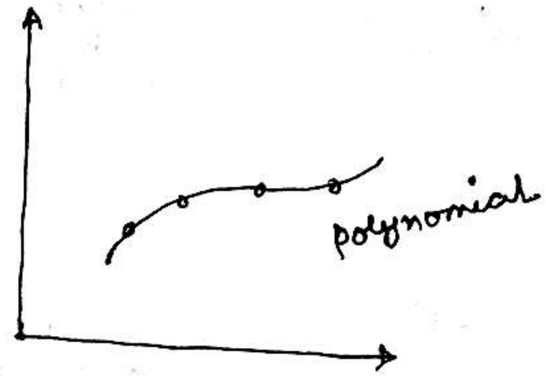
⇒ Iso-parametric element

* Only one parameter α is changing, so the element is called iso-parametric.



* Same Coordinate System
  - predicts deflection
  - " geometry

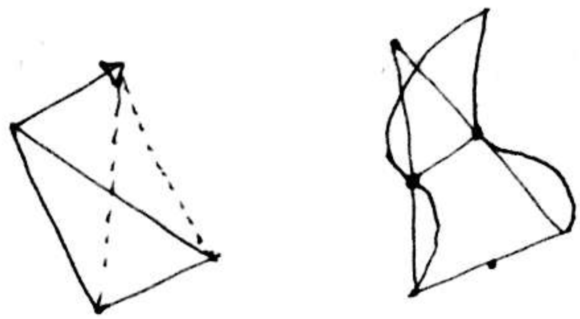
  } Iso-parametric



* Article 5.1 - 5.3 লাগবে না ।

* Example 5.4, 5.5 & 5.7

* Example 5.10, 5.11



* Example 5.12 (Assignment)

- Jacobian

Art. 5.6 (Next class)

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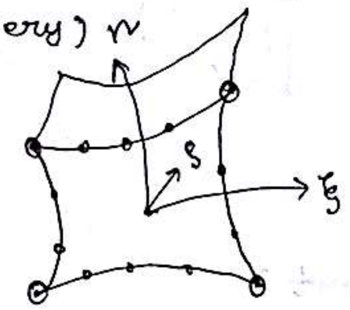
Shape Functions :

Serendipity Family of elements (Pg 89)

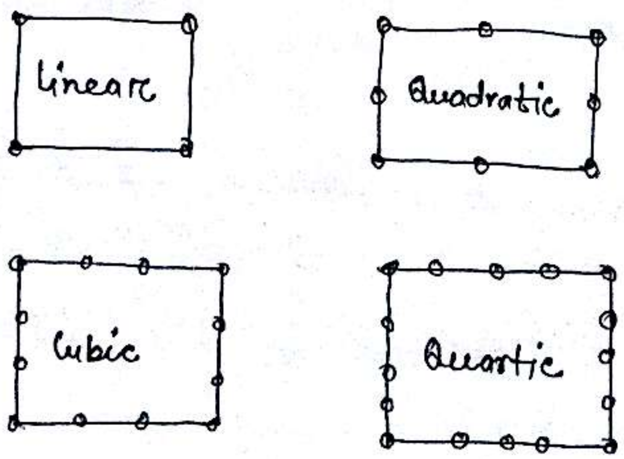
Serendip
(accidental discovery)



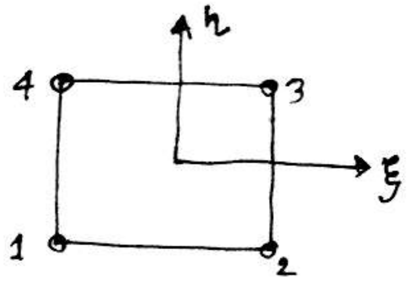
** Eq^n 5.42 মনে রাখতে হবে, derivation লাগবে না।

* যে কোন element এর shape function এই eq^n দ্বারা বের করা যায়।

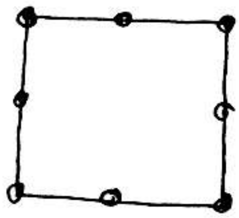
* page 90 :



Example 5.19 :



Example 5.20 (Assignment)



* Chapter - 6 (from lec sheet)
* Chapter - 7 (not in syllabus)
* Chapter - 8 (Article 8.1 & 8.2)
* Chapter - 9 (9.1 & 9.2)
** Chapter - 10 (very imp. for exam)

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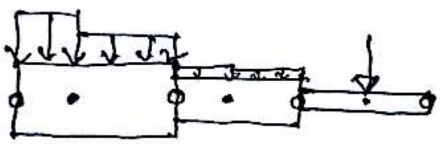
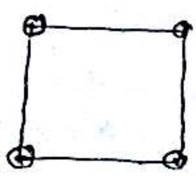
Chapter - 10

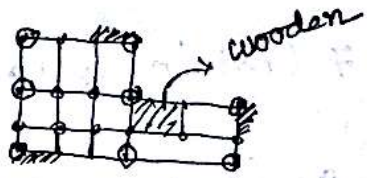
Discretization of Structures

Nodes at discontinuities (Pg. 154)

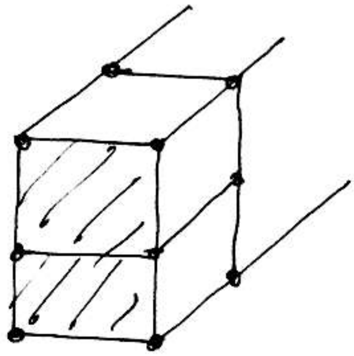
- Geometric
- Load
- Boundary conditions
- Material

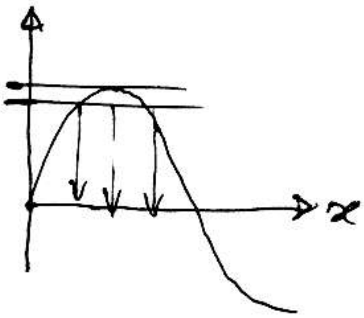




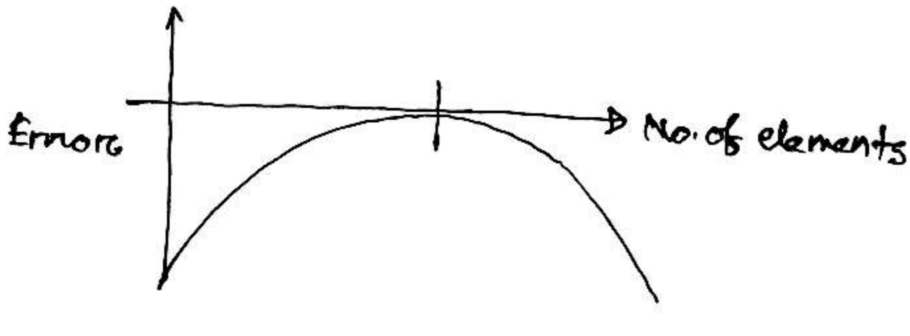


* Discontinuity এর কারণে node সংখ্যা বেড়ে যায়।





* Mesh যত বেশি হবে accuracy তত বাড়বে
upto certain no. of elements.



10.3

10.4

* Chapter - 10 (full in syllabus)
* CT (Wed - 1 pm) (upto today's class)

20|5|17| Sat

Chapter 11: (from lec sheet)

- # Exa 11.1 (pg. 169)
- # " 11.2
- # " 11.3
- # " 11.4 (pg. 178)

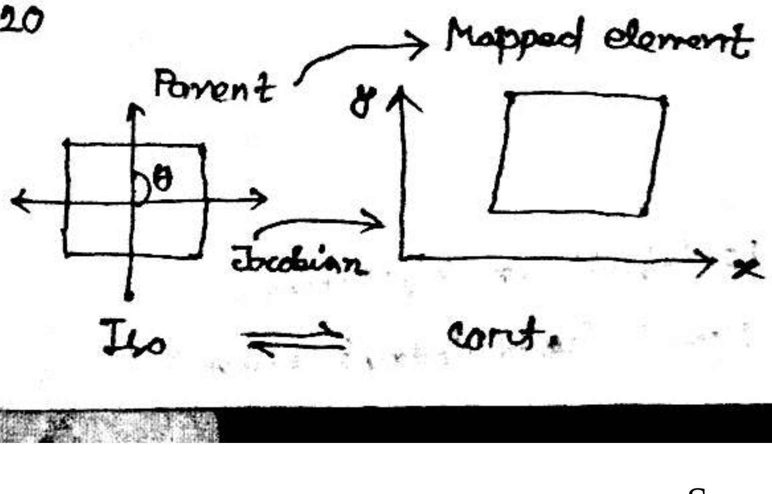
Chapter 12 : Plane stress and Plane strain

* Exa 12.1 এর আগে পর্যন্ত

Chapter 13 : Iso-parametric formulation

13.1

* Pg. 220



* Fig 13.1

Art 13.2 : Coordinate transformation

* $\{x\} = [N]\{x\}_e$

Pg. 227

* $J = \ldots\ldots$ Eq^n 13.8

* Jacobian is one kind of transformation matrix in isoparametric element.

Art 13.6

* $N_i = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)$

Art 13.5

$$\left.\begin{array}{l}\rightarrow \text{Iso - P} \\ \rightarrow \text{Super - P} \\ \rightarrow \text{Sub - P}\end{array}\right\} \text{Fig } 13.5$$

* Isoparametric element কাজ করবে for small deformation, যেখানে warping হবে না।

Fig 11.4, Art. 11.4

Chapter 13 (Full in syllabus)

08/07/17/Sat

CE 419

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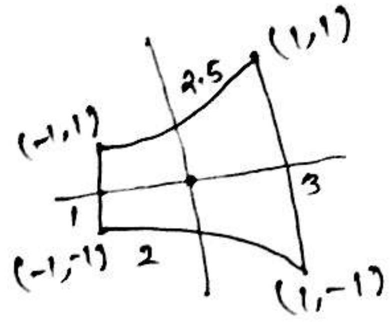
13.7 Numerical Integration:

* $\underline{\underline{K}} = \int B^T D B \, dV$



* Finite element এর final result গুলো numerical হবে।

- Trapezoidal rule
- Simpson's $\frac{1}{3}$ rule
- " $\frac{3}{8}$th rule
- . . . .
- Gauss Quadrature

* $$\int_{-1}^{+1} f(\xi)\, d\xi = \sum_{i=1}^{n} w_i f(\xi_i)$$

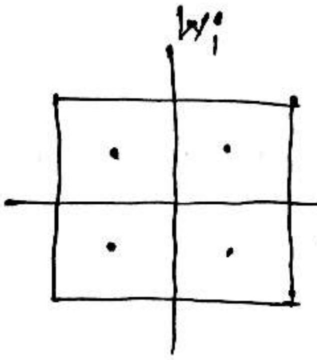
Weight function

- Table 13.1 :

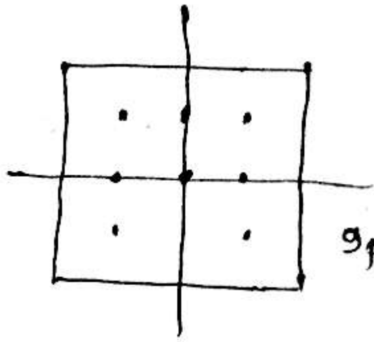
$$* \int_{-1}^{1} f(\xi)\, d\xi = \sum_{i=1}^{n} W_i f(\xi_i)$$

$$= W_1 f_1 + W_2 f_2 + W_3 f_3 + W_4 f_4$$

n	ξ
1	.
2	.
3	.
4	.



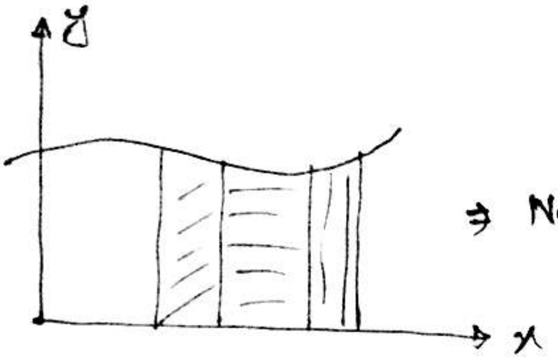
4point



9point

* Gauss point always element এর ভিতরে থাকবে।

* 9 point Gauss Quadrature use করলে আরও accurate হবে।



⇒ Numerical approximation

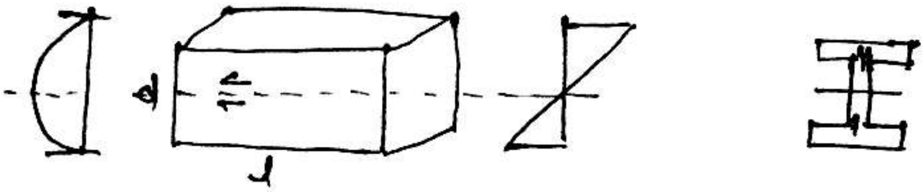
Exa 13.1, 13.2, 13.3 (assignment)

Chapter 14 :

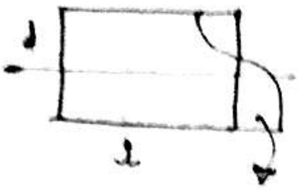
* Exa 14.1 to 14.3

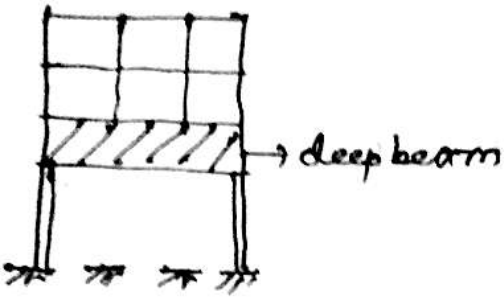
14.5 :

* Beams
  - Euler beam
  - Timoshenko beam } Navier's hypothesis
    Accounting the shear deformation



* $\frac{d}{l} < \frac{1}{5}$ হলে shear deformation কম হবে।





CT-2 (Tue 8am)

* Beam Element (Lec. sheet + Ch-14 3 Example)

CT-3 (Wed 9am)

* Chapter 10 & 13