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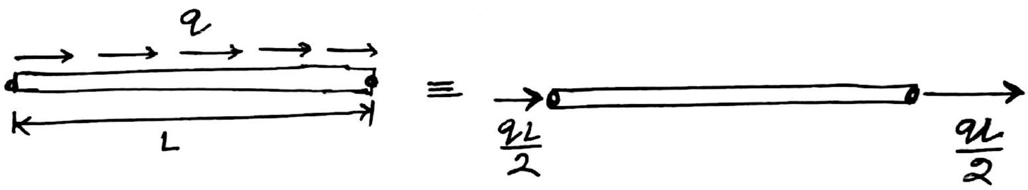
Tirtha

Question solve of 2013-14

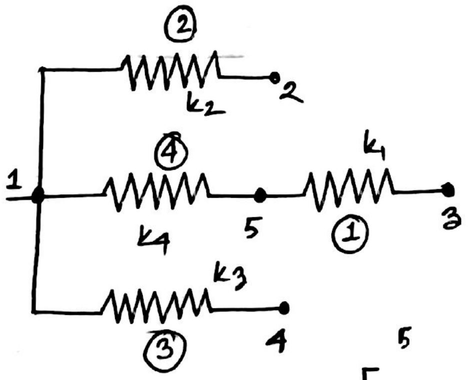
1a

No, FEM is not an exact method of analysis.

In FEM, field variables such as displacement in solid mechanics problems are determined at only nodal points assuming field variable within the elements are function of field variables at nodal points. Moreover, there are also some approximations in loading condition as follows:



1b



For spring 1: $[k_1] = k_1 \begin{array}{c c} \begin{array}{cc} 5 & 3 \end{array} & \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} & \begin{array}{c} 5 \\ 3 \end{array} \end{array}$

$$[k_2] = k_2 \begin{array}{c c} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

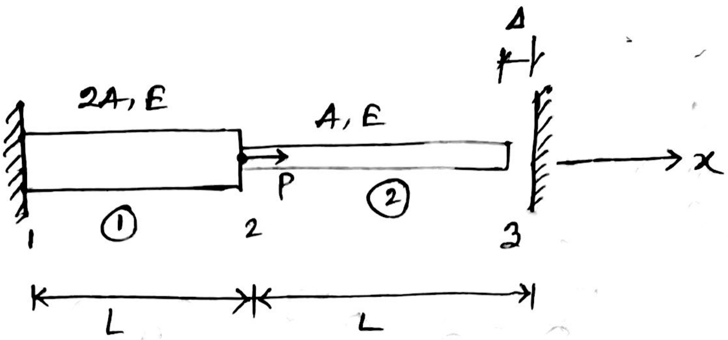
$$[k_4] = k_4 \begin{array}{c c} \begin{array}{cc} 1 & 5 \end{array} & \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} & \begin{array}{c} 1 \\ 5 \end{array} \end{array}$$

$$[k_3] = k_3 \begin{array}{c c} \begin{array}{cc} 1 & 4 \end{array} & \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} & \begin{array}{c} 1 \\ 4 \end{array} \end{array}$$

∴ Global stiffness Matrix:

$$[k] = \begin{bmatrix} k_2+k_3+k_4 & -k_2 & 0 & -k_3 & -k_4 \\ -k_2 & k_2 & 0 & 0 & 0 \\ 0 & 0 & k_1 & 0 & -k_1 \\ -k_3 & 0 & 0 & k_3 & 0 \\ -k_4 & 0 & -k_1 & 0 & k_1+k_4 \end{bmatrix}$$ (Ans.)

2b



Given that, $P = 6.0\times10^4$ N $E = 2.0\times10^4$ N/mm^2

$A = 250$ mm^2 $L = 150$ mm

$\Delta = 1.20$ mm

Check if there is any contact with Right support:

$$\Delta = \frac{PL}{2AE} = \frac{6.0\times10^4 \times 150}{2\times250\times2.0\times10^4}$$

$$= 0.90\text{mm} < 1.20\text{mm}$$

So, no contact with right support.

For Element 1:

$$[k_1] = \frac{2EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 66.67\times10^3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 33.33\times10^{+3}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

For element 2:

$$[k_2] = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 33.33\times10^{+3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

∴ Global stiffness matrix:

$$[k] = 33.33\times10^3 \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

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$\frac{2000}{3333}$

$\frac{2000}{3333}$

NOW,

$$\{F\} = \begin{Bmatrix} 0 \\ 6.0\times10^4 \\ 0 \end{Bmatrix}$$

So,

$$33.33\times10^3 \times \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \times \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 6\times10^4 \\ 0 \end{Bmatrix}$$

NOW, $u_1 = 0$

So,

$$33.33\times10^3 \times \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & -1 & 1 \end{bmatrix} \times \begin{Bmatrix} 0 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 6\times10^4 \\ 0 \end{Bmatrix}$$

So,

$$33.33\times10^3 \times \begin{bmatrix} 3 & 0 \\ -1 & 1 \end{bmatrix} \times \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 6\times10^4 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \frac{1}{33.33\times10^3} \times \begin{bmatrix} 3 & 0 \\ -1 & 1 \end{bmatrix}^{-1} \times \begin{Bmatrix} 6\times10^4 \\ 0 \end{Bmatrix}$$

$$\therefore \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0.60 \\ 0.60 \end{Bmatrix} \text{mm}$$

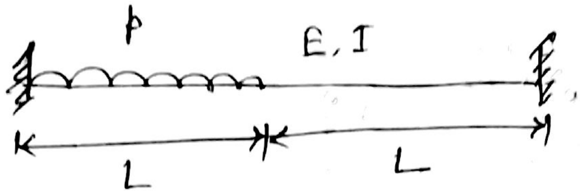
NOW,

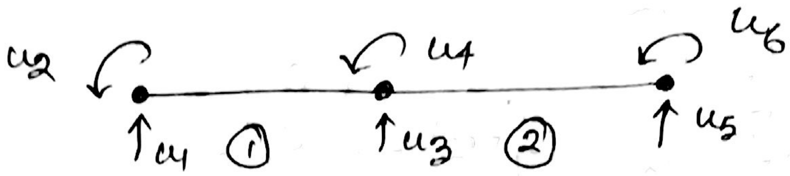
$$\begin{Bmatrix} R_1 \\ R_3 \end{Bmatrix} = 33.33\times10^3 \times \begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix} \times \begin{Bmatrix} 0.60 \\ 0.60 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -40000 \\ 0 \end{Bmatrix} \text{N}$$

$$= \begin{Bmatrix} -40 \\ 0 \end{Bmatrix} \text{kN (Ans).}$$

$\boxed{2c}$





Now, for element 1,

$$\underline{\underline{k_1}} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

for element 2,

$$\underline{\underline{k_2}} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

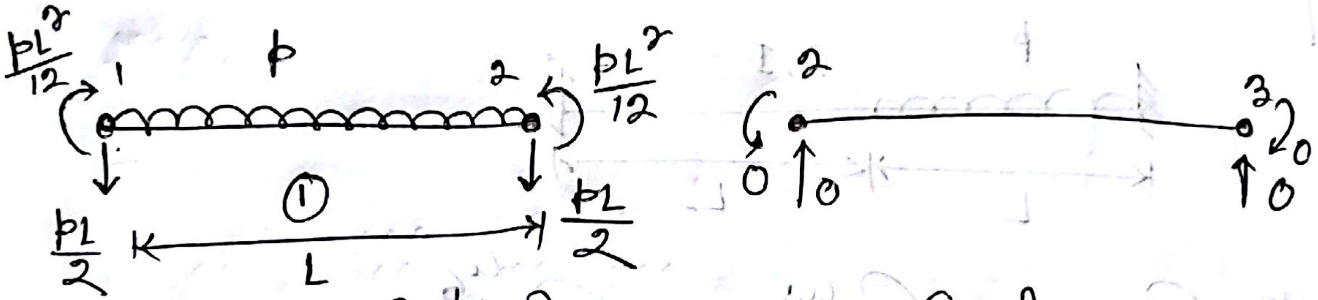
Now,

$$[k] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ +6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Now, $u_1 = u_2 = u_5 = u_6 = 0$

So,

$$\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ +6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \times \begin{Bmatrix} 0 \\ 0 \\ u_2 \\ u_3 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -PL/2 \\ -PL^2/12 \\ -PL/2 \\ PL^2/12 \\ 0 \\ 0 \end{Bmatrix}$$



$$\therefore \{f_{x1}\} = \begin{Bmatrix} -\frac{pL}{2} \\ -\frac{pL^2}{12} \\ -pL/2 \\ \frac{pL^2}{12} \end{Bmatrix} \qquad \{f_{x2}\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\therefore \{F_x\} = \begin{Bmatrix} -pL/2 \\ -pL^2/12 \\ -pL/2 \\ pL^2/12 \\ 0 \\ 0 \end{Bmatrix}$$

$$-\frac{pL}{2} \times \frac{L^3}{24EI} = -\frac{pL^4}{48EI}$$

$$\frac{pL^2}{12} \times \frac{L^2}{8EI} = \frac{pL^3}{96EI}$$

Now,

$$\frac{EI}{L^3}\begin{bmatrix} 24 & 0 \\ 0 & 8L^2 \end{bmatrix} \times \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} -\frac{pL}{2} \\ \frac{pL^2}{12} \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \frac{L^3}{EI}\begin{bmatrix} 24 & 0 \\ 0 & 8L^2 \end{bmatrix} \times \begin{Bmatrix} -pL/2 \\ \frac{pL^2}{12} \end{Bmatrix}$$

$$\therefore \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} -\frac{pL^4}{48EI} \\ \frac{pL^3}{96EI} \end{Bmatrix}$$ (Ans.)

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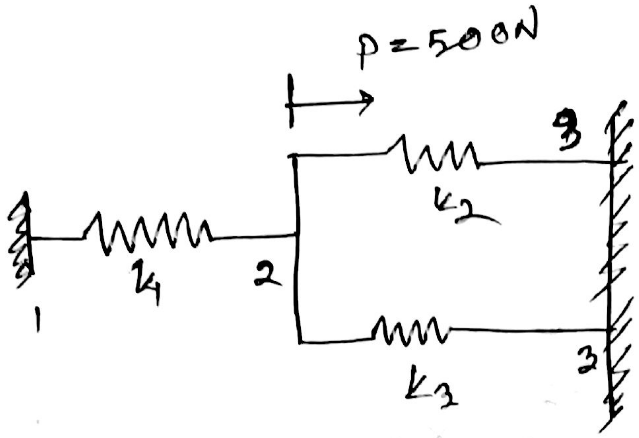
$\boxed{3b}$



Given that, $k_1 = 100\,N/mm$ $\quad k_2 = 200\,N/mm$ $\quad k_3 = 100\,N/mm$

$P = 500\,N$, $\quad u_1 = u_3 = 0$

(i) Now,

$$\underline{\underline{k_1}} = k_1 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{array}{cc} 1 & 2 \\ \left[\begin{matrix} 100 & -100 \\ -100 & 100 \end{matrix}\right] & \begin{matrix} 1 \\ 2 \end{matrix} \end{array}$$

$$\underline{\underline{k_2}} = k_2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{array}{cc} 2 & 3 \\ \left[\begin{matrix} 200 & -200 \\ -200 & 200 \end{matrix}\right] & \begin{matrix} 2 \\ 3 \end{matrix} \end{array}$$

$$\underline{\underline{k_3}} = k_3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{array}{cc} 2 & 3 \\ \left[\begin{matrix} 100 & -100 \\ -100 & 100 \end{matrix}\right] & \begin{matrix} 2 \\ 3 \end{matrix} \end{array}$$

$$\therefore \underline{\underline{k}} = \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & (100+200+100) & -100 & -200 \\ 0 & -100 & 100 & 0 \\ 0 & -200 & 0 & 200 \end{bmatrix}$$

$$= \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 400 & -100 & -200 \\ 0 & -100 & 100 & 0 \\ 0 & -200 & 0 & 200 \end{bmatrix} \quad \text{(Ans)}$$

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NOW,

$$\begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 400 & -100 & -200 \\ 0 & -100 & 100 & 0 \\ 0 & -200 & 0 & 200 \end{bmatrix} \times \left\{ \quad \right\}$$

$$\underline{k} = \begin{bmatrix} 100 & -100 & 0 \\ -100 & 400 & -300 \\ 0 & -300 & 300 \end{bmatrix} \quad \text{(Ans).}$$

② NOW,

$$[k] \times \{u\} = \{F\}$$

$$\Rightarrow \begin{bmatrix} 100 & -100 & 0 \\ -100 & 400 & -300 \\ 0 & -300 & 300 \end{bmatrix} \times \begin{Bmatrix} 0 \\ u_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 500 \\ 0 \end{Bmatrix}$$

$$\Rightarrow [400] \times \{u_2\} = \{500\}$$

$$\therefore \{u_2\} = \{1.25\} \text{ mm} \quad \text{(Ans).}$$

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(iii)

$$\begin{Bmatrix} R_1 \\ R_3 \end{Bmatrix} = \begin{bmatrix} -100 \\ -300 \end{bmatrix} \times \{1.25\} - \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -125 \\ -375 \end{Bmatrix} N \quad \text{(Ans).}$$

(iv)

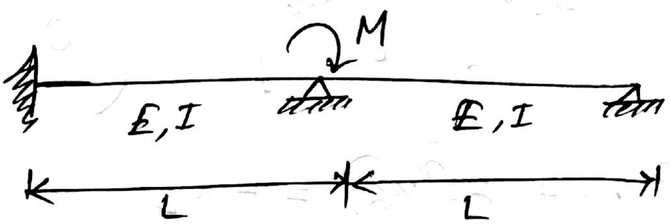
Force for spring (2), $F_2 = k_2(u_3 - u_2)$

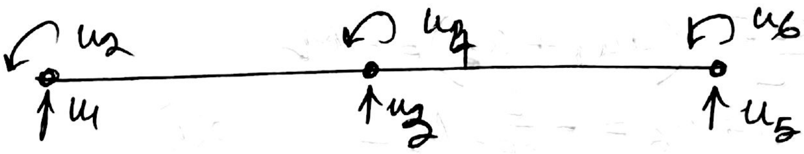
$= 200 \times (0 - 1.25)$

$= -250\,N$ (compression).

(Ans).

3e





$$\underline{\underline{k_1}} = \frac{EI}{L^3} \times \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} = \underline{\underline{k}}$$

$$\therefore \underline{\underline{k}} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ \cancel{12}\,0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Here,

$$u_1 = u_2 = u_3 = u_5 = 0$$

So,

$$\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \times \begin{Bmatrix} 0 \\ 0 \\ 0 \\ u_4 \\ 0 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -M \\ 0 \\ 0 \end{Bmatrix}$$

Now,

$$\frac{EI}{L^3}\begin{bmatrix} 8L^2 & 2L^2 \\ 2L^2 & 4L^2 \end{bmatrix} \times \begin{Bmatrix} u_4 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} -M \\ 0 \end{Bmatrix}$$

So, $\frac{EI}{L^3}\left(8L^2 u_4 + 2L^2 u_6\right) = -M$

$$\Rightarrow \frac{2EI}{L}\left(4u_4 + u_6\right) = -M$$

$$\therefore 4u_4 + u_6 = -\frac{L}{2EI}M \longrightarrow (1)$$

Again, $\frac{EI}{L^3}\left(2L^2 u_4 + 4L^2 u_6\right) = 0$

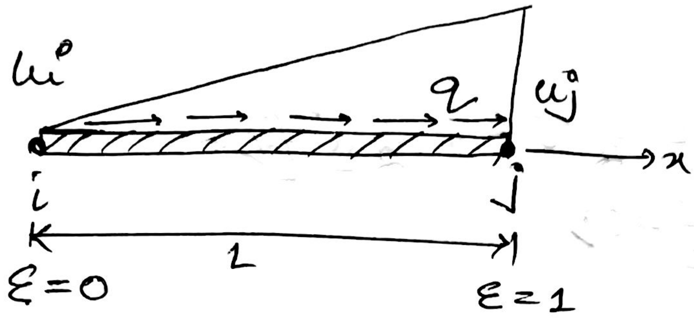
$$\Rightarrow u_4 + 2u_6 = 0$$

$$\therefore u_4 = -2u_6 \longrightarrow (2)$$

From (1), $-8u_6 + u_6 = -\frac{ML}{2EI}$

$$\therefore u_6 = \frac{ML}{14EI} \text{ (Ans)}$$

4a



Here, $N_1 = 1-\xi$

$N_2 = \xi = \frac{x}{L}$

Now, $q(\xi) = \frac{x}{L} q = \xi q$

Now,

Work done by q load,

$$W_q = \frac{1}{2}\int_0^l u\, q(\xi)\, dx$$

$$= \frac{1}{2}\int_0^l \begin{bmatrix} N_i & N_j \end{bmatrix} \times \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} \times \xi q \times dx$$

$$= \frac{q}{2}\int_0^1 \begin{bmatrix} (1-\xi) & \xi \end{bmatrix} \times \xi \times \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} \times l\,d\xi$$

$$= \frac{qL}{2}\int_0^1 \begin{bmatrix} \xi-\xi^2 & \xi^2 \end{bmatrix} \times \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} d\xi$$

$$= \frac{qL}{2} \times \begin{bmatrix} \frac{1}{6} & \frac{1}{3} \end{bmatrix} \times \begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$$

$$= \begin{Bmatrix} u_i & u_j \end{Bmatrix} \times \begin{bmatrix} \frac{qL}{12} \\ \frac{qL}{6} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} u_i & u_j \end{bmatrix} \times \begin{Bmatrix} \frac{qL}{6} \\ qL/3 \end{Bmatrix}$$

$$= \underline{\underline{u}}^T \underline{\underline{f}}_q$$

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Now,

$$U = W$$

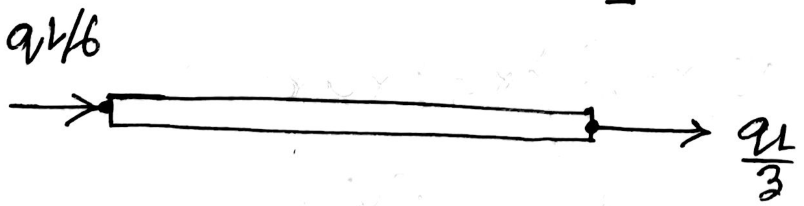
$$\Rightarrow \frac{1}{2}\underline{\underline{u}}^T \underline{\underline{k}}\,\underline{\underline{u}} = \frac{1}{2}u^T \underline{\underline{f}} + \frac{1}{2}\underline{\underline{u}}^T \underline{\underline{fq}}$$

$$\Rightarrow \underline{\underline{k}}\,\underline{\underline{u}} = \underline{\underline{f}} + \underline{\underline{fq}}$$

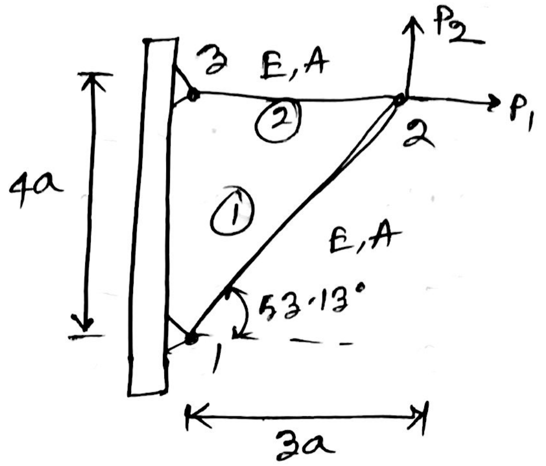
So, Nodal force matrix:

$$[F] = \underline{\underline{f}} + \underline{\underline{fq}} = \begin{bmatrix} f_i \\ f_j \end{bmatrix} + \begin{bmatrix} \frac{qL}{6} \\ \frac{qL}{3} \end{bmatrix}$$

$$= \begin{bmatrix} f_i + \frac{qL}{6} \\ f_j + \frac{qL}{3} \end{bmatrix}$$



4c



For Element ①, $\theta = 53.13°$

$l = \cos\theta = \cos 53.13° = 0.60$

$m = \sin\theta = \sin 53.13° = 0.80$

For element 2: $\theta = 180°$.

$l = \cos 180° = -1$

$m = \sin 180° = 0$

NOW,

$$\underline{\underline{k_1}} = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

$$= \frac{EA}{L}\begin{bmatrix} 0.36 & 0.48 & -.36 & -0.48 \\ 0.48 & 0.64 & -0.48 & -0.64 \\ -0.36 & -0.48 & 0.36 & 0.48 \\ -0.48 & -0.64 & 0.48 & 0.64 \end{bmatrix}$$

$$k_2 = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\therefore [k] = \frac{EA}{L}\begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 & 0 & 0 \\ 0.48 & 0.64 & -0.48 & -0.64 & 0 & 0 \\ -0.36 & -0.48 & 1.36 & 0.48 & -1 & 0 \\ -0.48 & -0.64 & 0.48 & 0.64 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

NOW,

$$[f_0] = \begin{bmatrix} 0 \\ 0 \\ P_1 \\ P_2 \\ 0 \\ 0 \end{bmatrix}$$

$$u_1 = v_1 = u_3 = v_3 = 0$$

So, $[k] \times \{u\} = \{f\}$

$$\Rightarrow \frac{EA}{L}\begin{bmatrix} 1.36 & 0.48 \\ 0.48 & 0.64 \end{bmatrix} \times \begin{Bmatrix} u_2 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

$\cancel{\text{So, } \frac{EA}{L}(1.36u_4 + 0.48v_4) = P_1}$

$\cancel{\Rightarrow}$

$$\Rightarrow \begin{Bmatrix} u_2 \\ v_2 \end{Bmatrix} = \frac{L}{EA} \times \begin{bmatrix} 1.36 & 0.48 \\ 0.48 & 0.64 \end{bmatrix}^{-1} \times \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

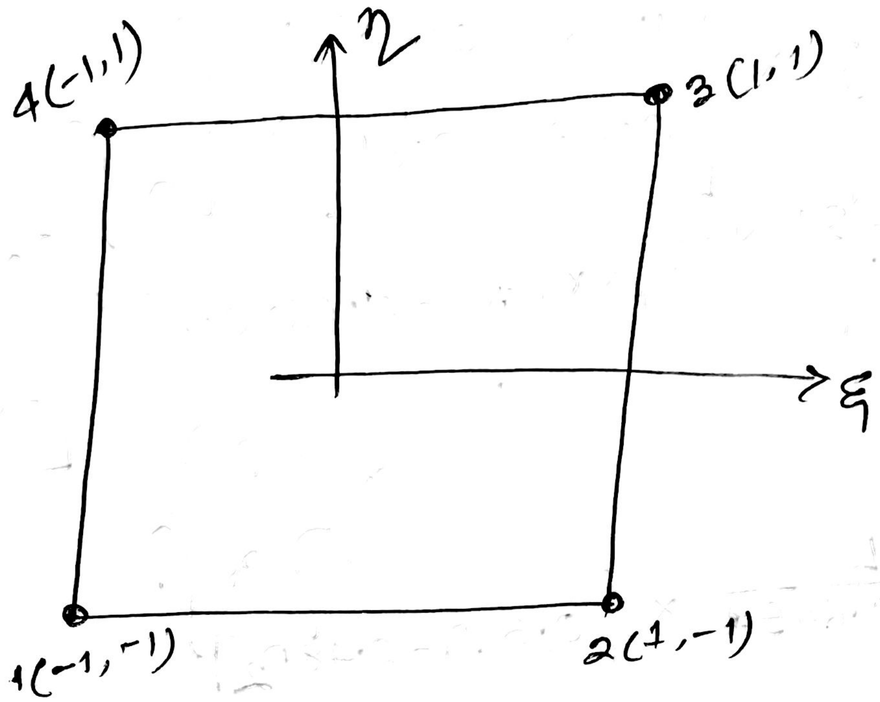
$$= L/EA \times \frac{1}{1.36 * 0.64 - 0.48 \times 0.48} \times \begin{bmatrix} 0.64 & -0.48 \\ -0.48 & 1.36 \end{bmatrix} \times \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

$$= \frac{L}{0.64EA} \times \begin{bmatrix} 0.64P_1 - 0.48P_2 \\ -0.48P_1 + 1.36P_2 \end{bmatrix}$$

$$= \frac{L}{EA} \begin{bmatrix} \frac{4P_1 - 3P_2}{4} \\ \frac{17P_2 - 6P_1}{8} \end{bmatrix}$$

(Ans).

$\boxed{6b}$



Let, $u = \alpha_1 + \alpha_2 \xi + \alpha_2 \eta + \alpha_3 \xi \eta$

$$\therefore \quad u = \begin{bmatrix} 1 & \xi & \eta & \xi\eta \end{bmatrix} \times \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{Bmatrix} \longrightarrow (1)$$

Now,
$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \times \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{Bmatrix} = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}^{-1} \times \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{Bmatrix} = -\frac{1}{16} \begin{bmatrix} -4 & 4 & 4 & -4 \\ -4 & -4 & 4 & 4 \\ -4 & -4 & -4 & -4 \\ -4 & 4 & -4 & 4 \end{bmatrix}^T \times \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$= -\frac{1}{16} \begin{bmatrix} -4 & -4 & -4 & -4 \\ 4 & -4 & -4 & 4 \\ 4 & 4 & -4 & -4 \\ -4 & 4 & -4 & 4 \end{bmatrix} \times \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$1 \times \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{vmatrix} + 1 \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$\cancel{= -4+4-4+4=0}$

$\cancel{= 1(-1-1)-1(-1+1)-1(1+1)}$

$\cancel{= -2-2}$

$\cancel{= -4}$

$\cancel{= -4-4-4+4}$

$= -4-4-4-4 = -16$

$$= \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{bmatrix} \times \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

From ①,

$$u = \begin{bmatrix} 1 & \xi & \eta & \xi\eta \end{bmatrix} \times \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{bmatrix} \times \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$= \begin{bmatrix} \frac{1-\xi-\eta+\xi\eta}{4} & \frac{-1+\xi+\eta-\xi\eta}{4} & \frac{1+\xi+\eta+\xi\eta}{4} & \frac{1-\xi+\eta-\xi\eta}{4} \end{bmatrix} \times \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

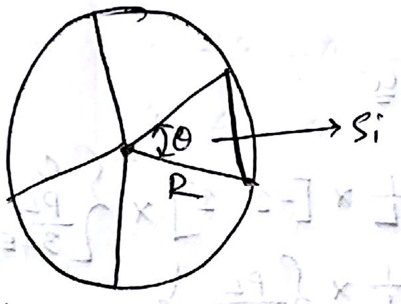
$$= \begin{bmatrix} \frac{(1-\xi)(1-\eta)}{4} & \frac{(1+\xi)(1-\eta)}{4} & \frac{(1+\xi)(1+\eta)}{4} & \frac{(1-\xi)(1+\eta)}{4} \end{bmatrix} \times \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$= \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \times \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

So, $N_i = \frac{(1+\xi\xi_i)(1+\eta\eta_i)}{4}$ where, $i = 1, 2, 3, 4$ etc.

Mathematical Problems of 2012-13

$\boxed{1d}$



Number of triangles = N

Now, area of a triangle $= \frac{1}{2}R^2\sin\theta = \frac{1}{2}R^2\sin\frac{2\pi}{N}$

$\therefore$ Total area of ~~trian~~ circle $= \sum_{n=i}^{N} \frac{1}{2}R^2\sin\frac{2\pi}{N}$

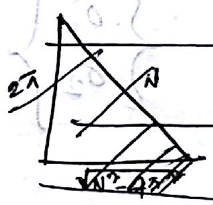
$= \frac{1}{2}NR^2\sin\frac{2\pi}{N}$

$= \frac{1}{2}R^2\left(N\sin\frac{2\pi}{N}\right)$

When, $N\rightarrow\alpha$, $N\sin\frac{2\pi}{N} \longrightarrow 2\pi$

$\therefore$ Area of circle $= \frac{1}{2}R^2\times 2\pi$

$= \pi R^2$



$\lim_{N\to\alpha} N\times\sin\frac{2\pi}{N}$

$= N\sin\frac{\pi}{N}+\frac{\pi}{N}$

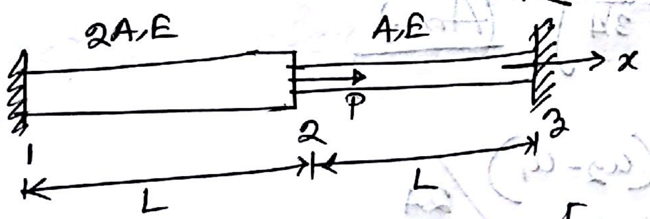
$= N(\sin\frac{\pi}{N}\cos\frac{\pi}{4} + \cos\frac{N}{2}\sin\frac{\pi}{N})$

$= 2N\sin\frac{\pi}{N}\cos\frac{\pi}{N}$

$N\to\alpha$ $\theta\to\alpha$

~~$\lim_{N\to\alpha} N\sin\theta = \lim_{\theta\to\alpha}$~~

$\boxed{2b}$



$$\underline{k_1} = \frac{AE}{L}\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix} \quad \underline{k_2} = \frac{AE}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$\therefore [k] = \frac{AE}{L}\begin{bmatrix}2 & -2 & 0\\ -2 & 3 & -2\\ 0 & -2 & 2\end{bmatrix}$$

Now, $u_1 = u_3 = 0$

So, $\frac{AE}{L}[3]\times\{u_2\} = \{P\}$

$\therefore \{u_2\} = \frac{L}{AE}\{P/3\}$

NOW,

Stress at bar 1:

$$\sigma_1 = E\underline{\underline{B}}\,\underline{\underline{u}}$$

$$= E \times \frac{1}{L} \times \begin{bmatrix} -1 & 1 \end{bmatrix} \times \begin{Bmatrix} 0 \\ \frac{PL}{3AE} \end{Bmatrix}$$

$$= E \times \frac{1}{L} \times \left\{ \frac{PL}{3AE} \right\}$$

$$= \left\{ \frac{P}{3A} \right\}$$

Stress at bar 2:

$$\sigma_2 = E\underline{\underline{B}}\,\underline{\underline{u}}$$

$$= E \times \frac{1}{L} \times \begin{bmatrix} -1 & 1 \end{bmatrix} \times \begin{Bmatrix} \frac{PL}{3AE} \\ 0 \end{Bmatrix}$$

$$= E \times \frac{1}{L} \times \left\{ -\frac{PL}{3AE} \right\}$$

$$= \left\{ -\frac{P}{3A} \right\}$$

$$\therefore \begin{Bmatrix} \sigma_1 \\ \sigma_2 \end{Bmatrix} = \begin{Bmatrix} \frac{P}{3A} \\ -\frac{P}{3A} \end{Bmatrix} \quad \text{(Ans)}$$

Check:

$$\sigma_1 = \frac{2EA}{L}(u_2 - u_1) / 2A$$

$$= \frac{2EA}{L} \times \left(\frac{PL}{3EA}\right) / 2A$$

$$= \frac{2P}{3} \times \frac{1}{2A}$$

$$= \frac{P}{3A}$$

$$\sigma_2 = \frac{EA}{L}(u_3 - u_2) \times \frac{1}{A}$$

$$= \frac{EA}{L}\left(0 - \frac{PL}{3EA}\right) \times \frac{1}{A}$$

$$= -\frac{EA}{L} \times \frac{PL}{3AE} \times \frac{1}{A}$$

$$= \frac{-P}{3A}$$

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