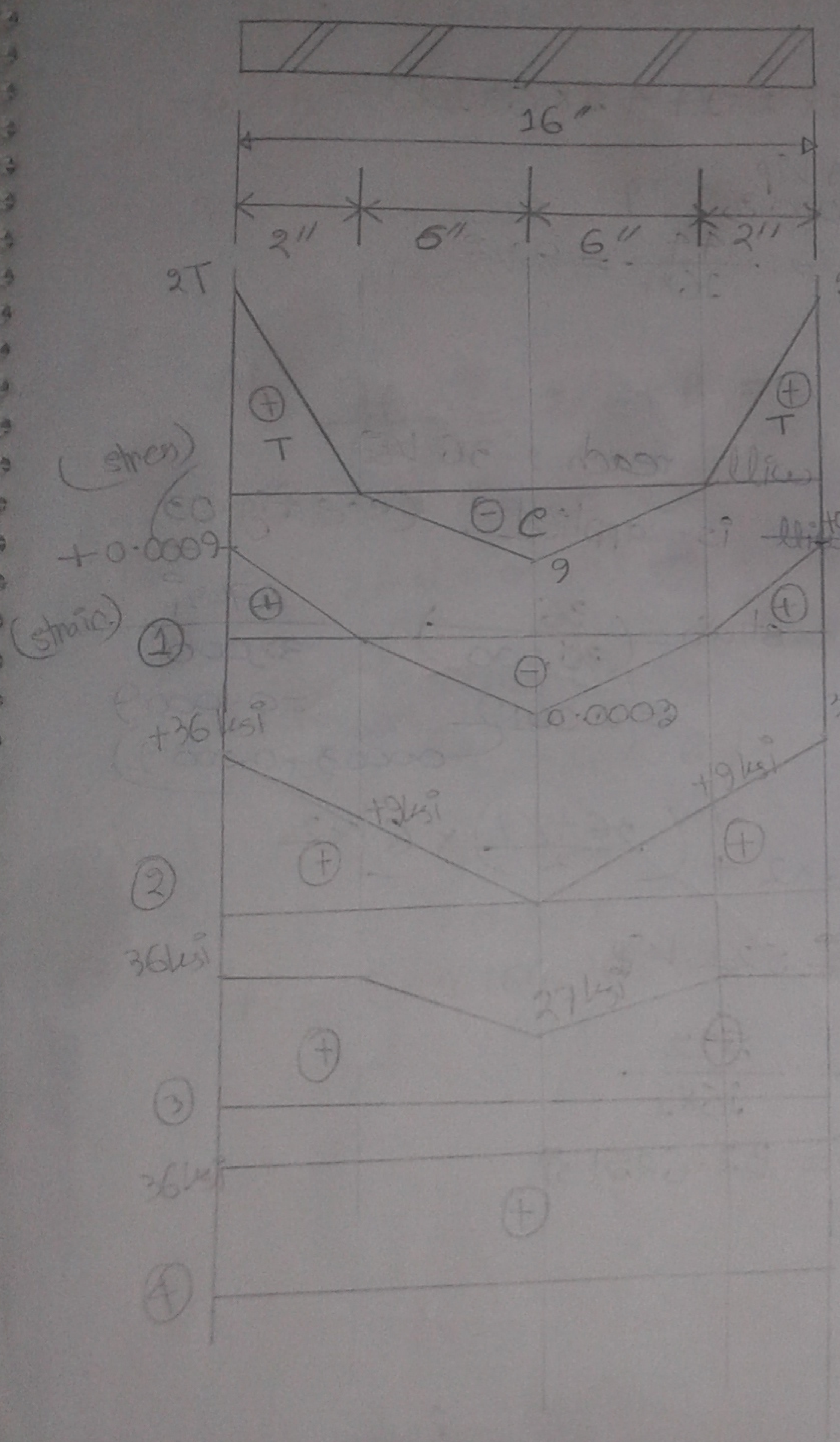
Residual stress

$\boxed{\text{Exp-3-19-1}}$



$\boxed{\text{1rst step}}$

Maximum residual Tensile strain $= \frac{27}{30,000}$

$= +0.0009$ in/in

27 Maximum residual compressive strain

$= \frac{-9}{30,000}$

$= -0.0003$ in/inch

+0.0009

So, $0 \leq \epsilon \leq 0.0003$

(elastic) $f < 36$ ksi

36 ksi $f = E\epsilon$ (Hooke's law is applied.

So, first yield occurs when $\epsilon = 0.0003$ in/inch

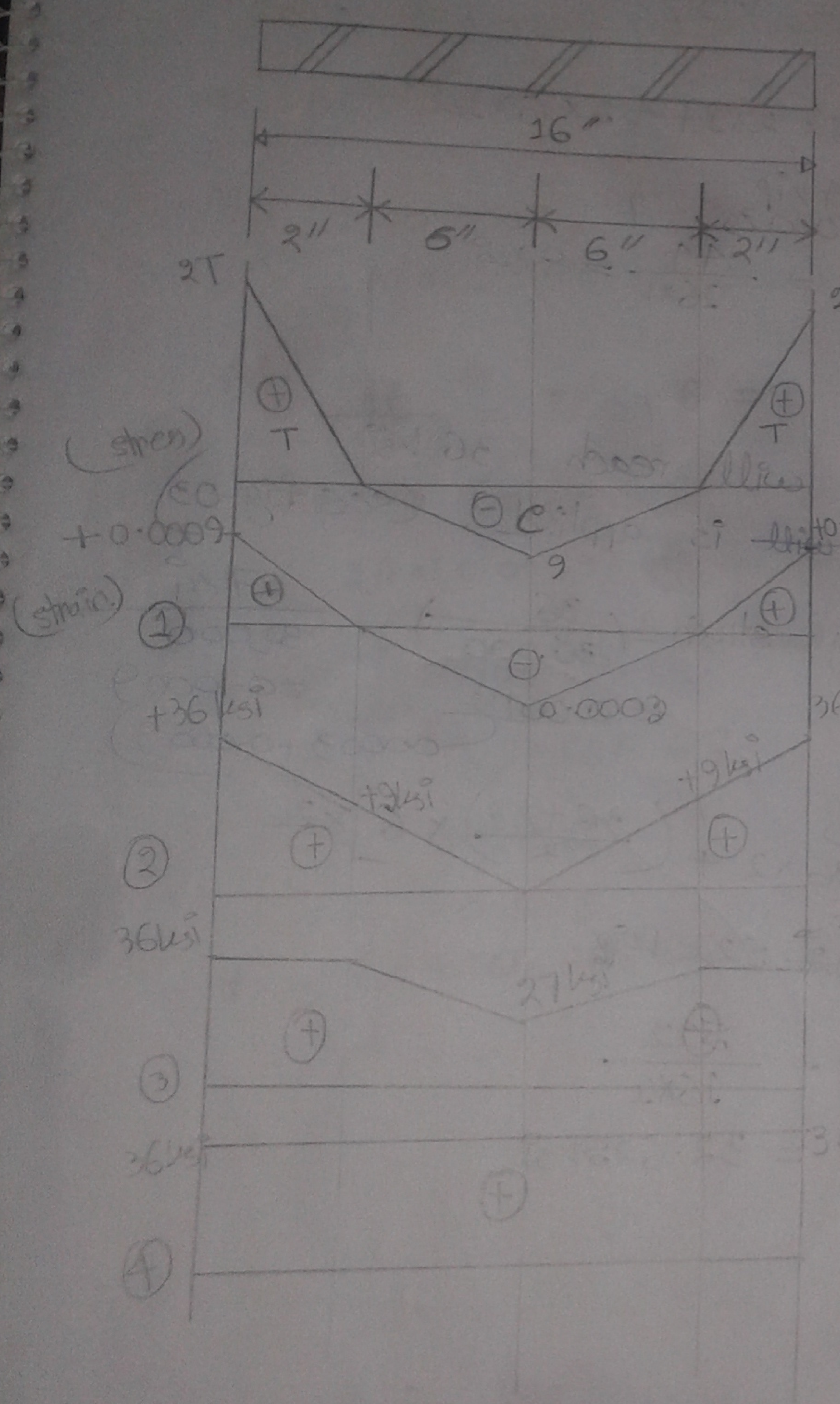
36 ksi

$\boxed{\text{step-02}}$

If 9 ksi imposed on residual stress, then 27 ksi becomes 36 ksi (Fig-02)

Residual stress

Exp-3-19-1



1rst step

Maximum residual Tensile strain $= \frac{27}{30,000}$

$= +0.0009$ in/in

27 Maximum residual compressive strain

$= \frac{-9}{30,000}$

$= -0.0003$ in/inch

So, $0 \leq \epsilon \leq 0.0003$ (elastic) $f < 36$ ksi

36 ksi $f = E\epsilon$ (Hooke's law is applied.

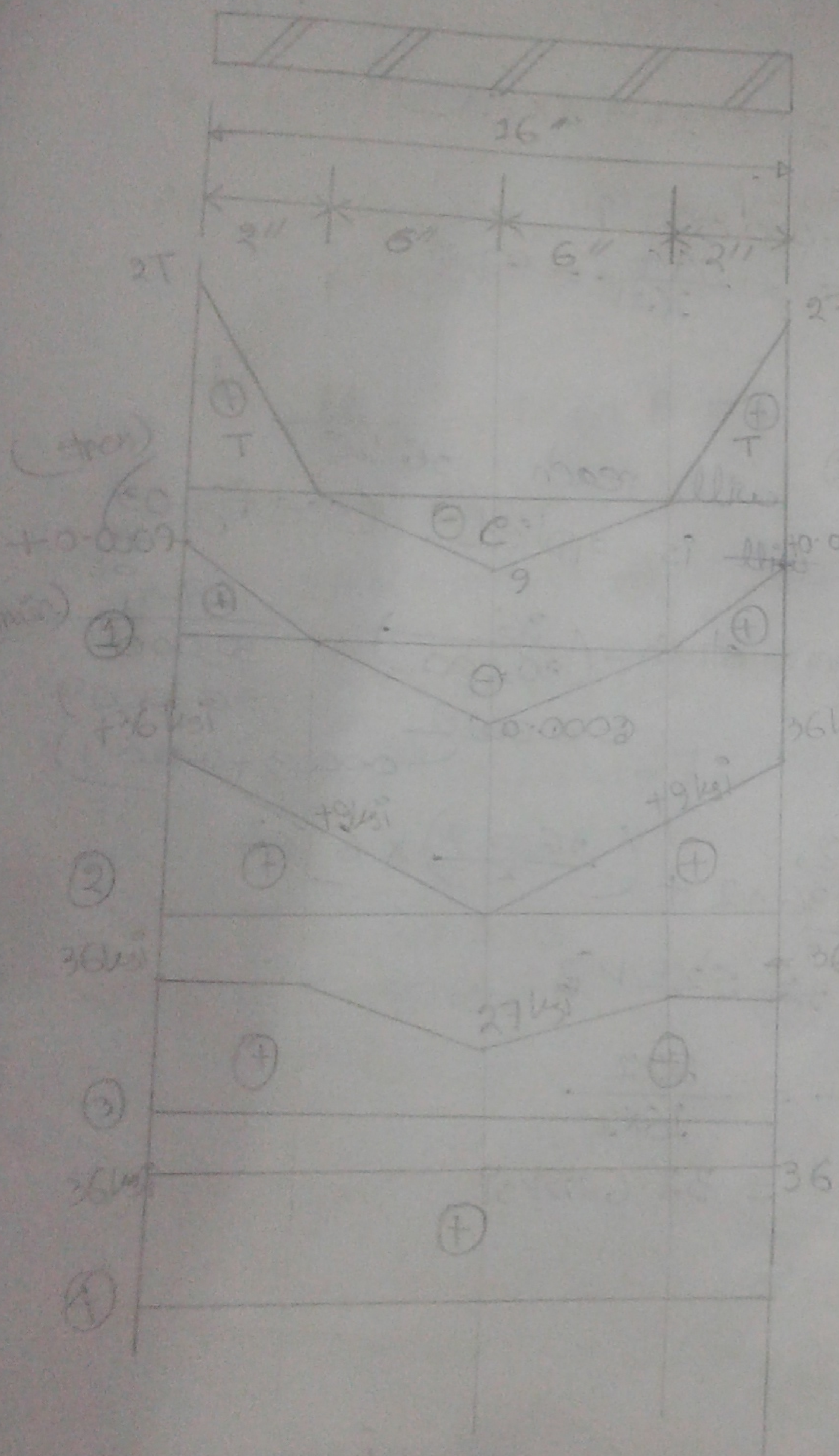
So, first yield occurs when $\epsilon = 0.0003$ in/inch

Step-02

If 9 ksi imposed on residual stress, then 27 ksi becomes 36 ksi (Fig-02)

Residual stress

$\boxed{\text{Exp-3-19-1}}$



$\boxed{\text{first step}}$

Maximum residual Tensile strain $= \dfrac{27}{30{,}000}$

$= +0.0009$ in/in

Maximum residual compressive strain

$= \dfrac{-9}{30{,}000}$

$= -0.0003$ in/inch

So, $0 \le \epsilon \le 0.0003$ (elastic) $f < 36$ ksi

$f = E\epsilon$ (Hooke's law is applied.

So, first yield occurs when $\epsilon = 0.0003$ in/inch

$\boxed{\text{step-02}}$

If 9 ksi imposed on residual stress, then 27 ksi becomes 36 ksi (Fig-02)

(1) Average stress = $\left(\frac{36+9}{2}\right) = 22.5\,ksi$

(2) Average stress = $\left(\frac{9+0}{2}\right) = 4.5\,ksi$

$\therefore P = (22.5 \times 2 \times 1 + 4.5 \times 6 \times 1) \times 2$

$= 144\,kip$

so, at strain 0.0003,

$f = \frac{P}{A} = \frac{144}{16 \times 1} = 9\,ksi$

$\boxed{\text{step-03}}$

Then +9ksi will reach 36 ksi

so, 27 ksi ~~will~~ is applied. (see fig-03)

so, corresponding strain $= \left(\frac{36}{30{,}000}\right)$ $\frac{27ksi}{30{,}000}$

$= 0.0012$ $= 0.0009$

$(0.0003 + 0.0009)$

At $\epsilon = 0.0012$,

$P = \left[36 \times 2 + \left(\frac{36+27}{2}\right) \times 6\right] \times 2$

$= \cancel{32}\ 522\,kip$

$\therefore f = \frac{P}{A} = \frac{522}{16 \times 1}$

$= 32.625\,ksi$

$f = A\epsilon^2 + B\epsilon + C$

At $\epsilon = 0.0003$, $f = 9$ ksi

$\therefore 9 = A(0.0003)^2 + B(0.0003) + C$

$\Rightarrow A(0.0003)^2 + B(0.0003) + C = 9$ —— (i)

At, $\epsilon = 0.0012$, $f = 32.625$ ksi

$\therefore A(0.0012)^2 + B(0.0012) + C = 32.625$ —— (ii)

~~At~~ $\frac{df}{d\epsilon} = 2A\epsilon + B = E$ [For elastic region]

At $\epsilon = 0.0003$ —— (iii)

$\therefore 2A \times (0.0003) + B = 30{,}000$

By solving (i), (ii) and (iii),

$A = -4{,}166{,}666.67$

$B = 32{,}500$

$C = -0.375$

$\therefore f = (-4{,}166{,}666.67)\epsilon^2 + (32{,}500)\epsilon - 0.375$

Step-04

complete yield occurs when +27 ksi reaches +36 ksi.

$\therefore$ 9ksi is applied (see-fig-4)

so, corresponding strain $= \dfrac{9}{30{,}000}$

$= 0.0003$

complete yield occurs when strain reaches $(0.0012+0.0003) = 0.0015$

limit, $0.0012 \le \epsilon \le 0.0015$, $f = 36$ ksi

At $\epsilon = 0.0012$, $f = 32.625$ ksi

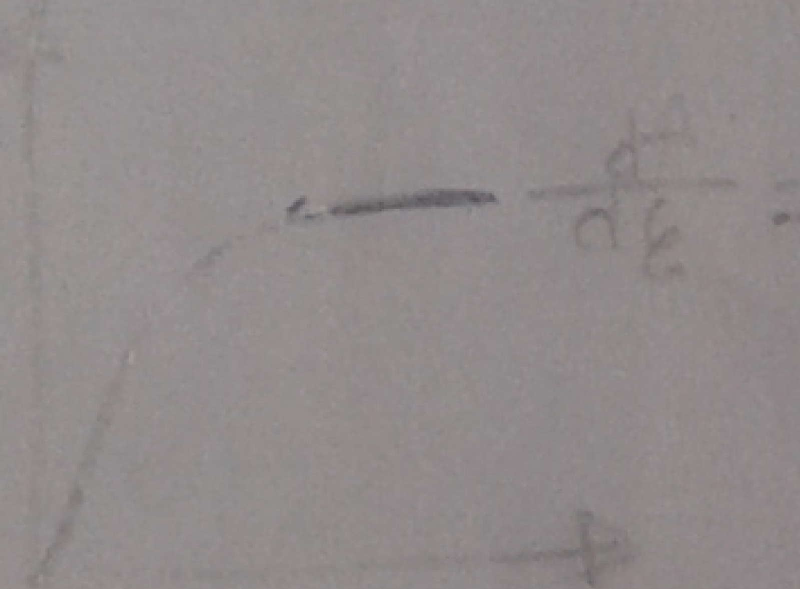
$$32.625 = (0.0012)^2 A + (0.0012) B + C \quad \text{—— (iv)}$$

At $\epsilon = 0.0015$,

$$(0.0015)^2 A + (0.0015) B + C = 36 \quad \text{—— (v)}$$

(*) Beyond 36 ksi, stress won't change with strain and so, horizontal line.

so, $\dfrac{df}{d\epsilon} = 0$



$\epsilon = 0.0015, \frac{df}{d\epsilon} = 0$

$2 \times (0.0015) A + B = 0$ ——— (vi)

By solving (iv), (v) and (vi),

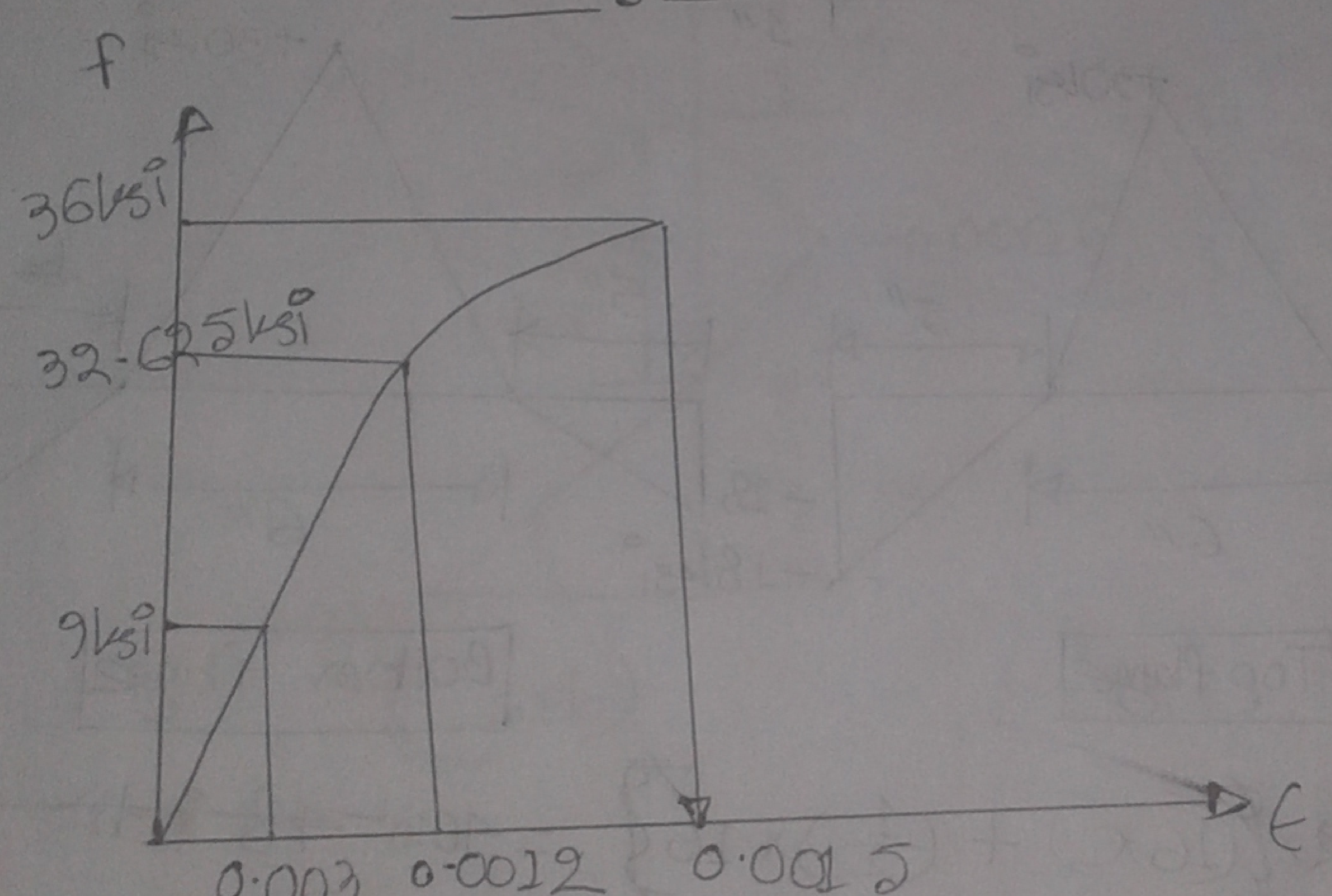
$A = -37500{,}000$

$B = 112500$

$C = -48.375$

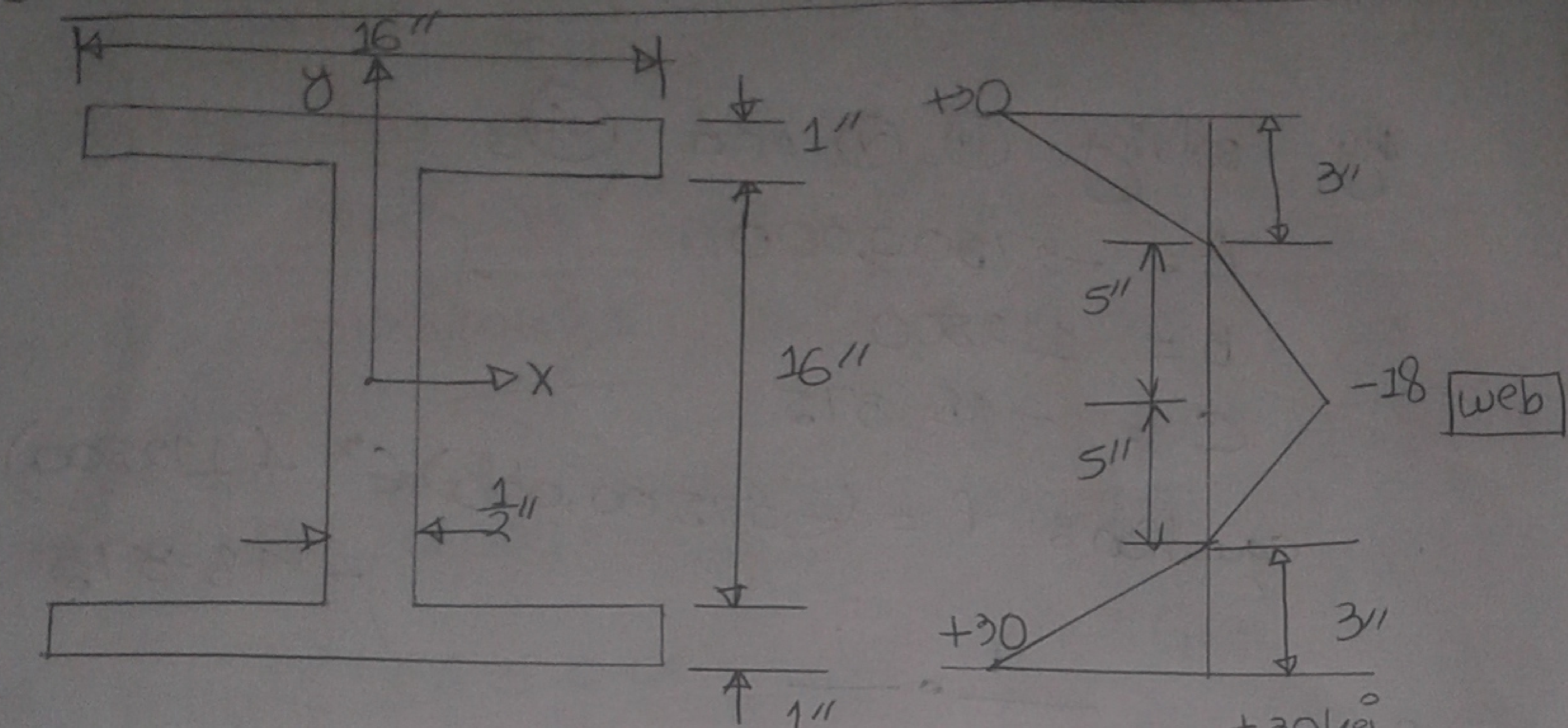
$\therefore$ equation: $f = (-37500{,}000)\epsilon^2 + (112500)\epsilon - 48.375$

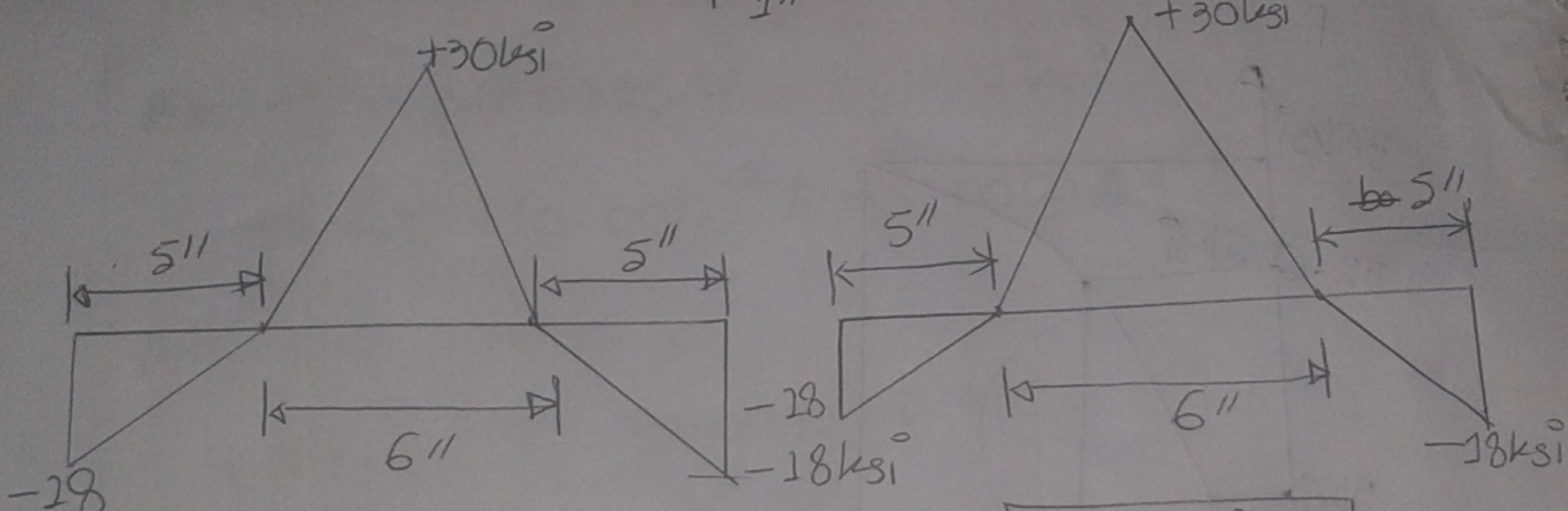
——∘——



~~comp~~

effect of residual stress in compression members





Top flange

Bottom flange

$$A = 2\times\left[(16\times1) + \left(\tfrac{1}{2}\right)\times 16\right] = 40\,in^2 \quad \cancel{48\,inch^2}$$

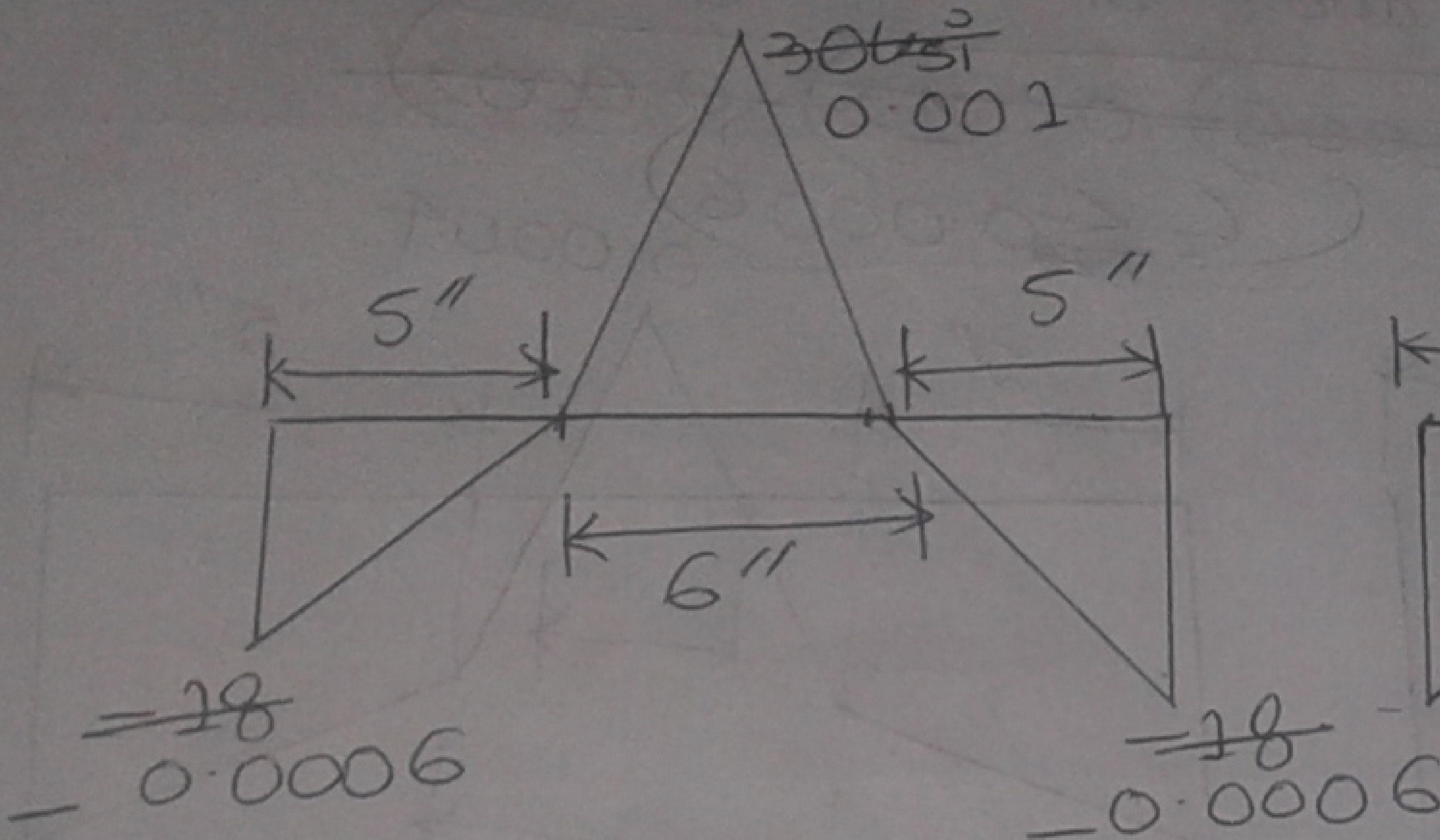
$$I_x = 2\times\frac{16\times1^3}{12} + 2\left[(16\times1)\left(9-\tfrac{1}{2}\right)^2\right] + \frac{\left(\tfrac{1}{2}\right)\times16^3}{12}$$

$$= 2485\ inch^4$$

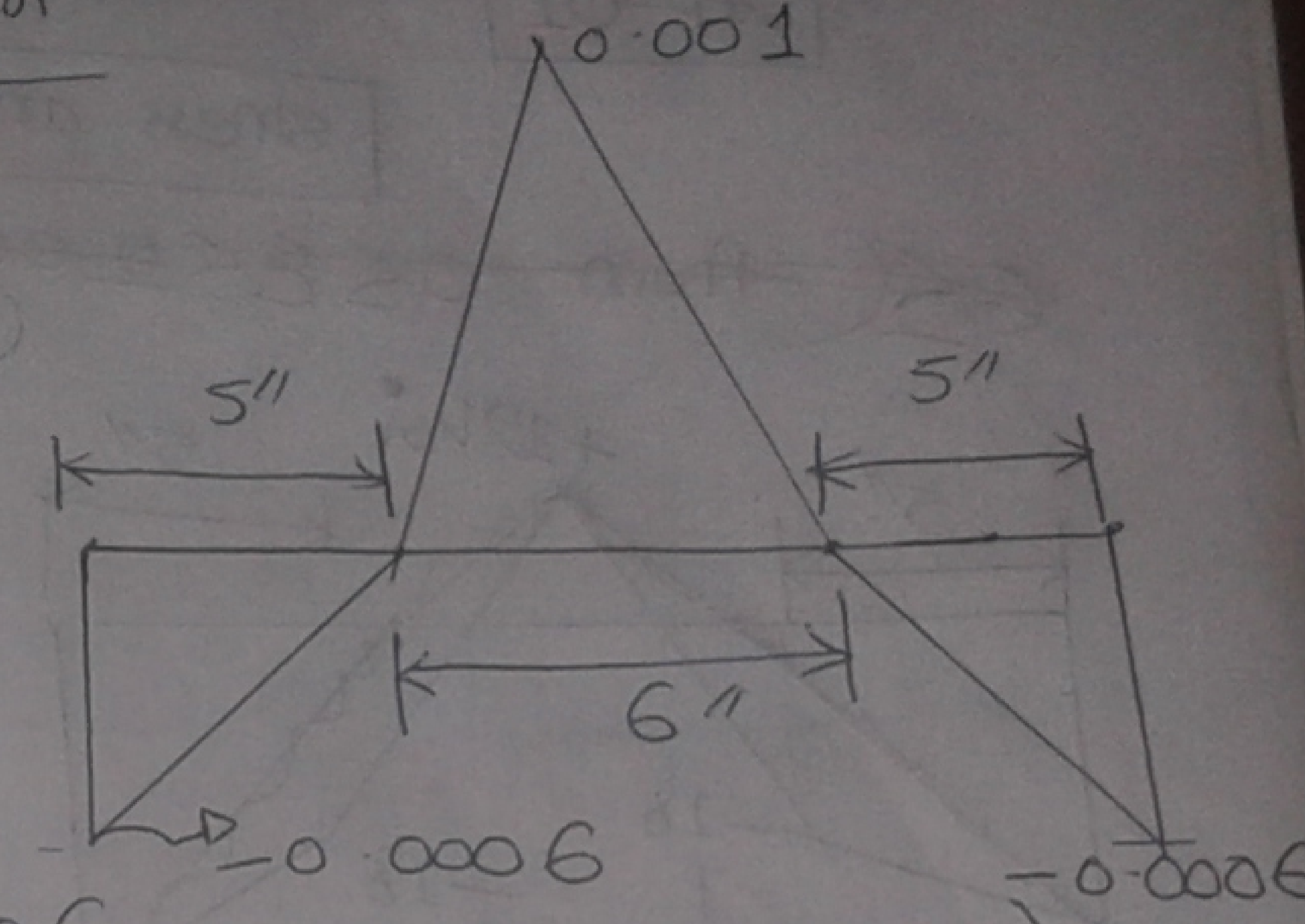
$$I_y = \frac{2\times1\times16^3}{12} + \frac{16\times(0.5)^3}{12}$$

$$= 683\ inch^4$$

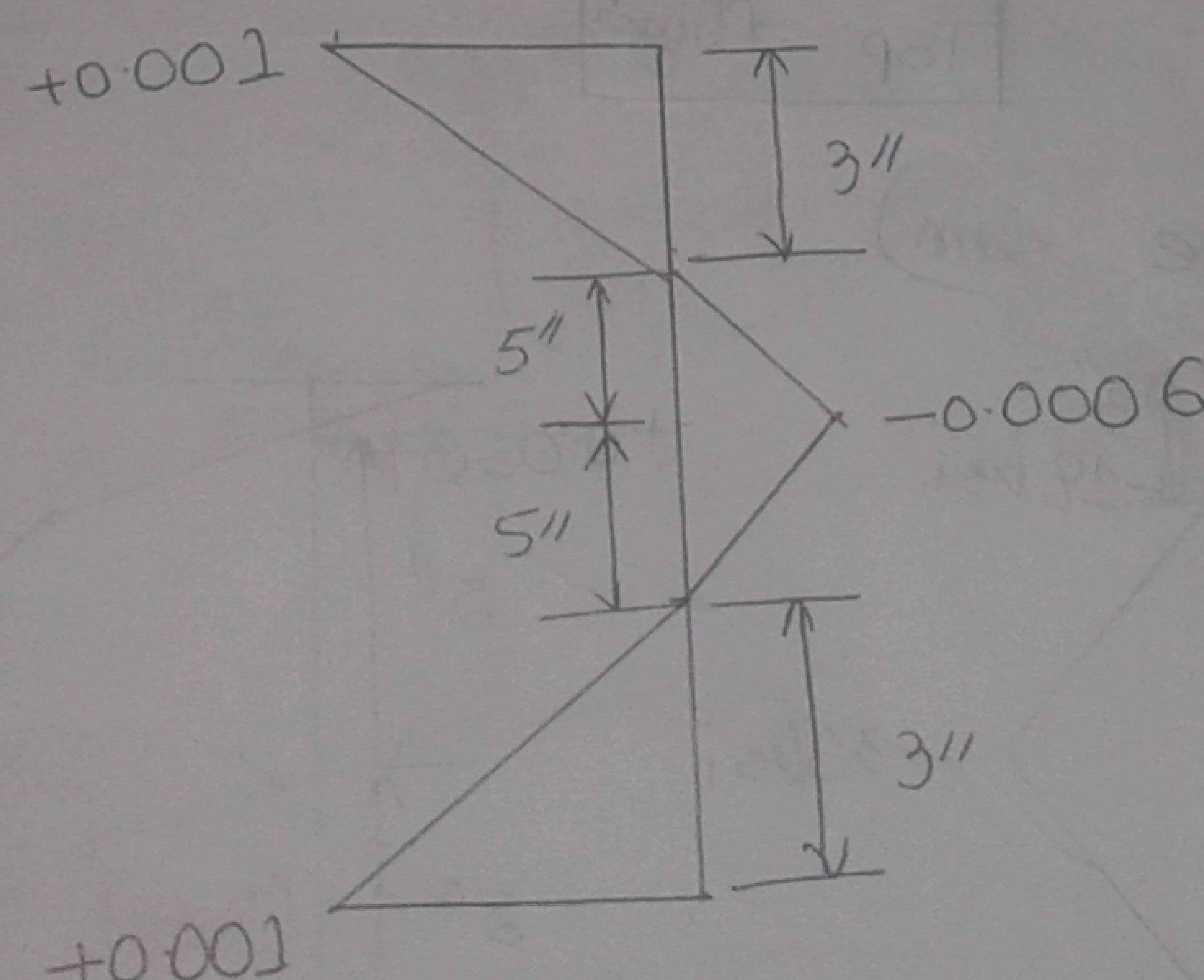
ⓧ corresponding strain diagram



(top flange)



(Bottom flange)



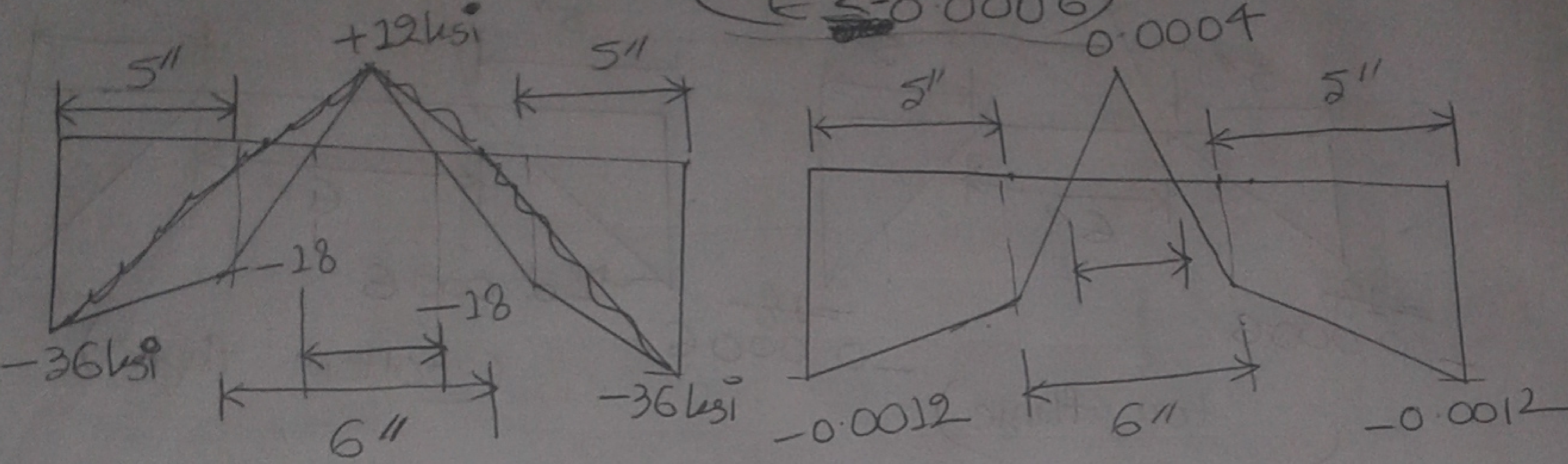
(web)

$\boxed{\text{step-01}}$

$\boxed{\text{stress and strain diagram}}$

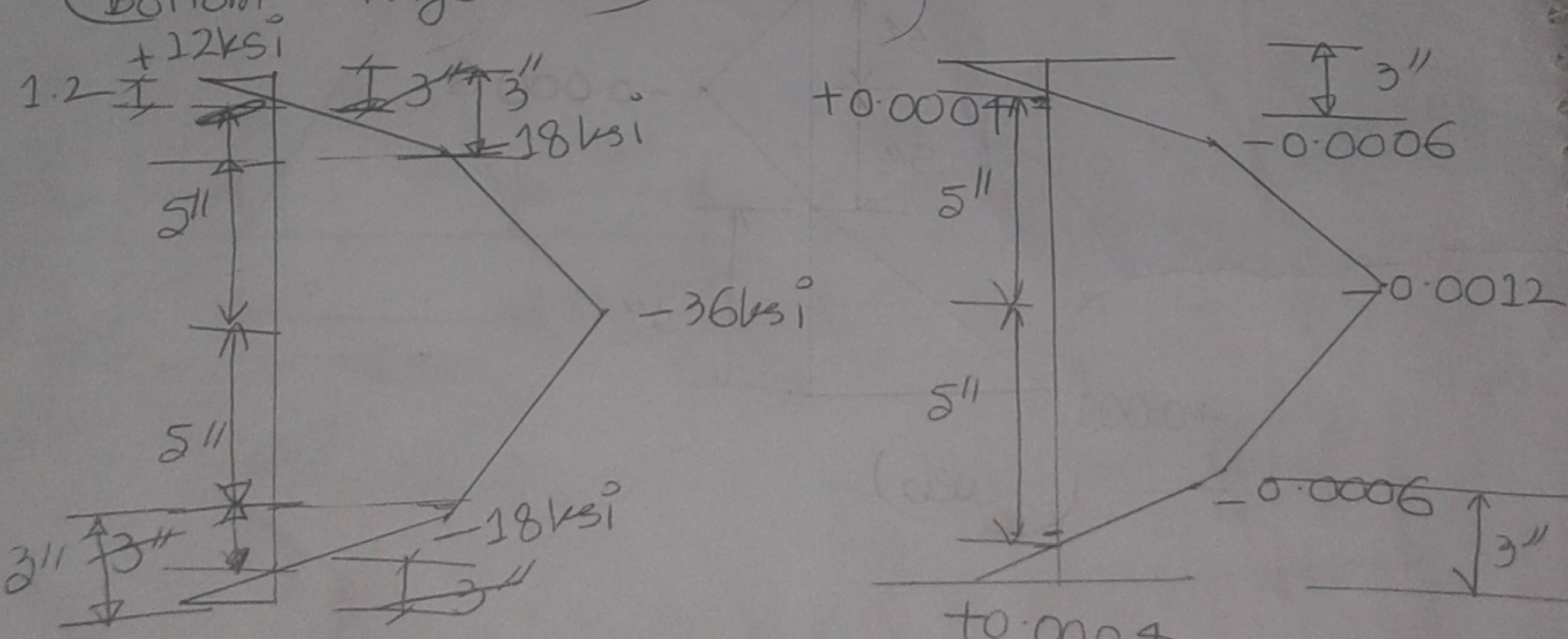
~~$\epsilon \leq$ (from $0 \leq \varepsilon < 0.006$) 0.0012) 0.0002)~~

$(\epsilon \leq -0.0006)$



$\boxed{\text{Top flange}}$

(Bottom flange sam)



$\boxed{\text{web}}$

At $\epsilon = -0.0006$,

$f = \frac{720}{40} = 18\text{ksi}$

$$\frac{df}{d\varepsilon} = 2A\varepsilon + B$$

At $\varepsilon = -0.0006$, $\frac{df}{d\varepsilon} = E$

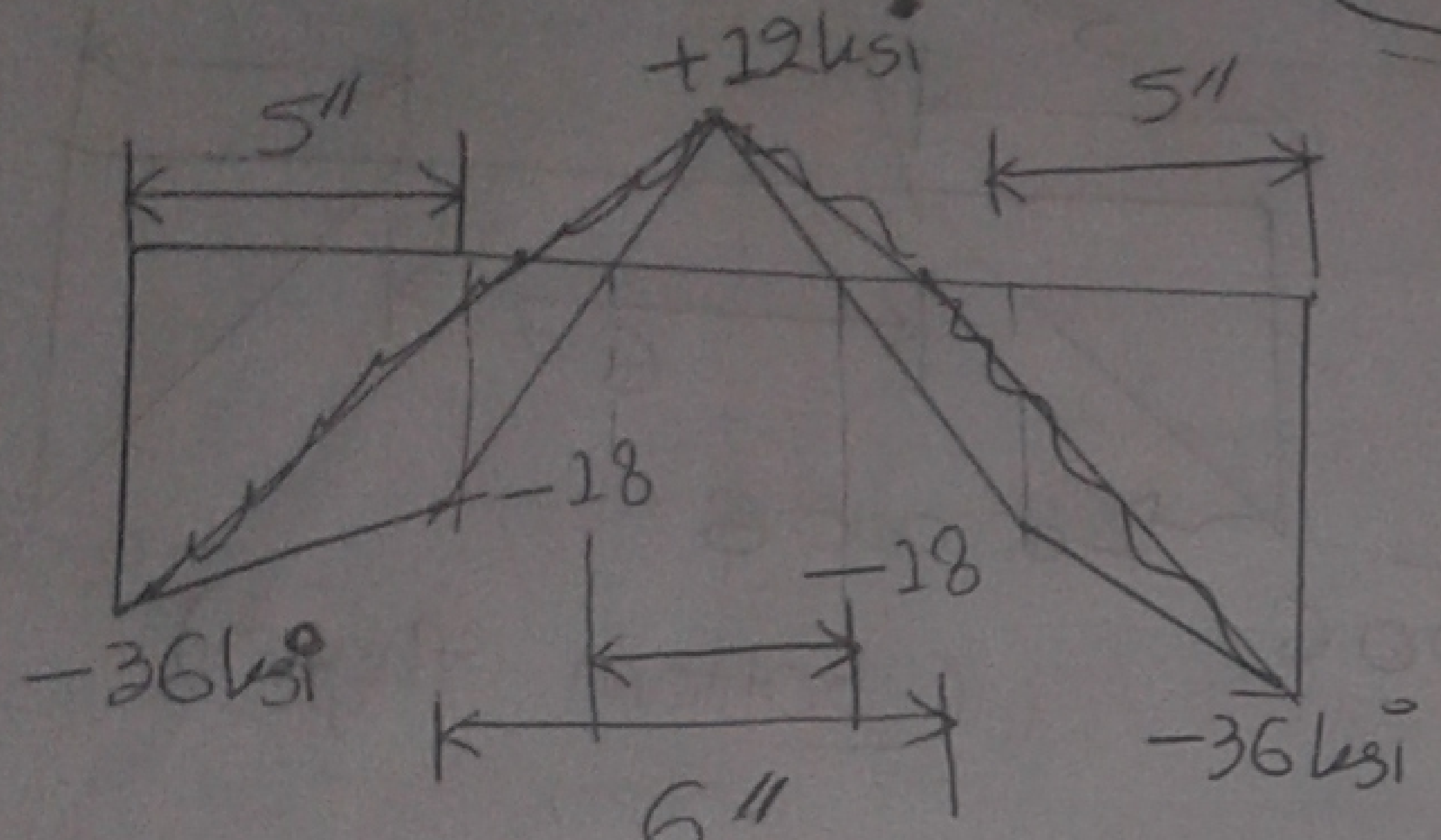
$\therefore 2A \times (+0.0006) + B = 30{,}000$ ——— (1)

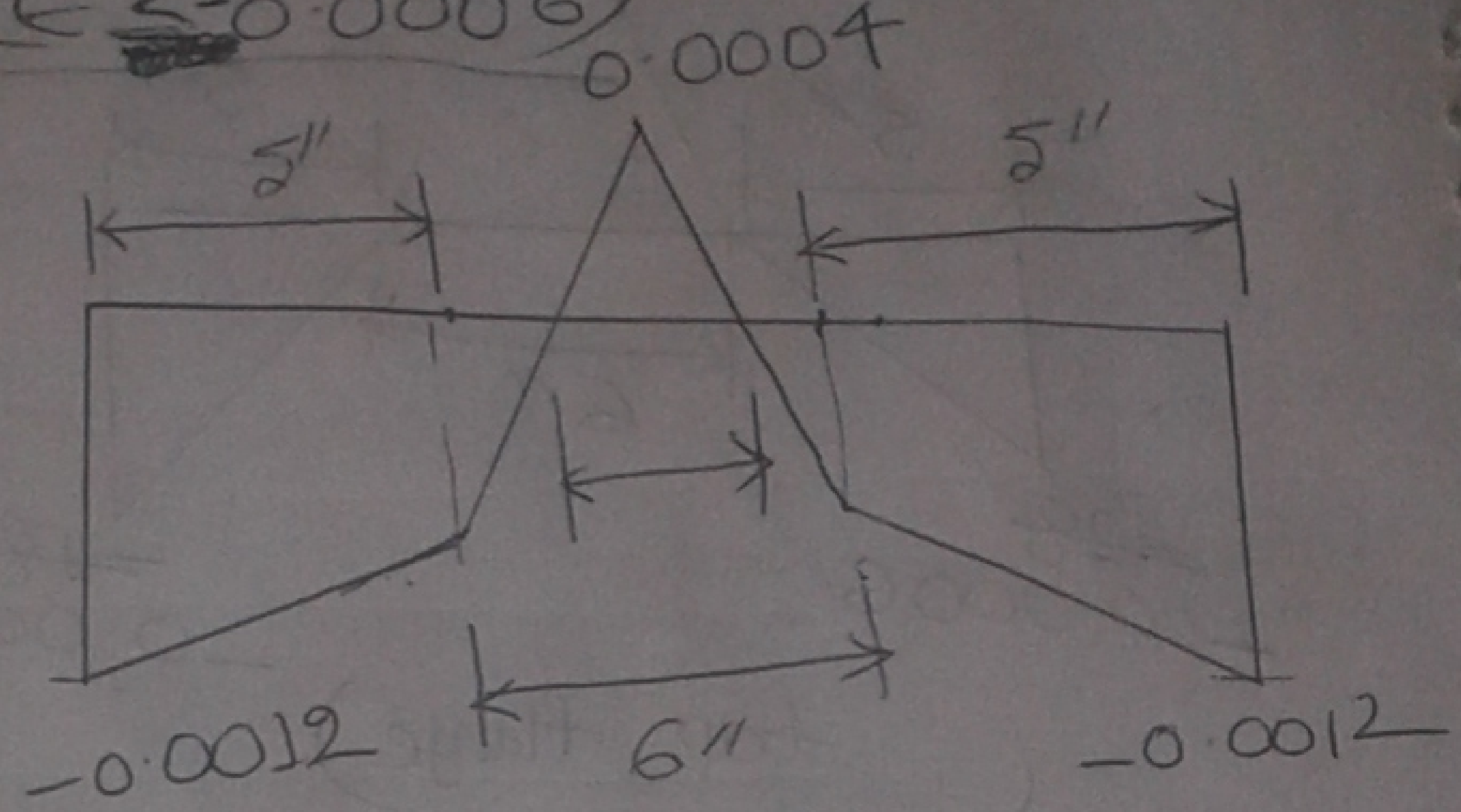
step-01

stress and strain diagram

~~$\epsilon \leq ($ from $0 \leq \varepsilon < 0.0006)$ $0.0012)$ $0.0002)$~~

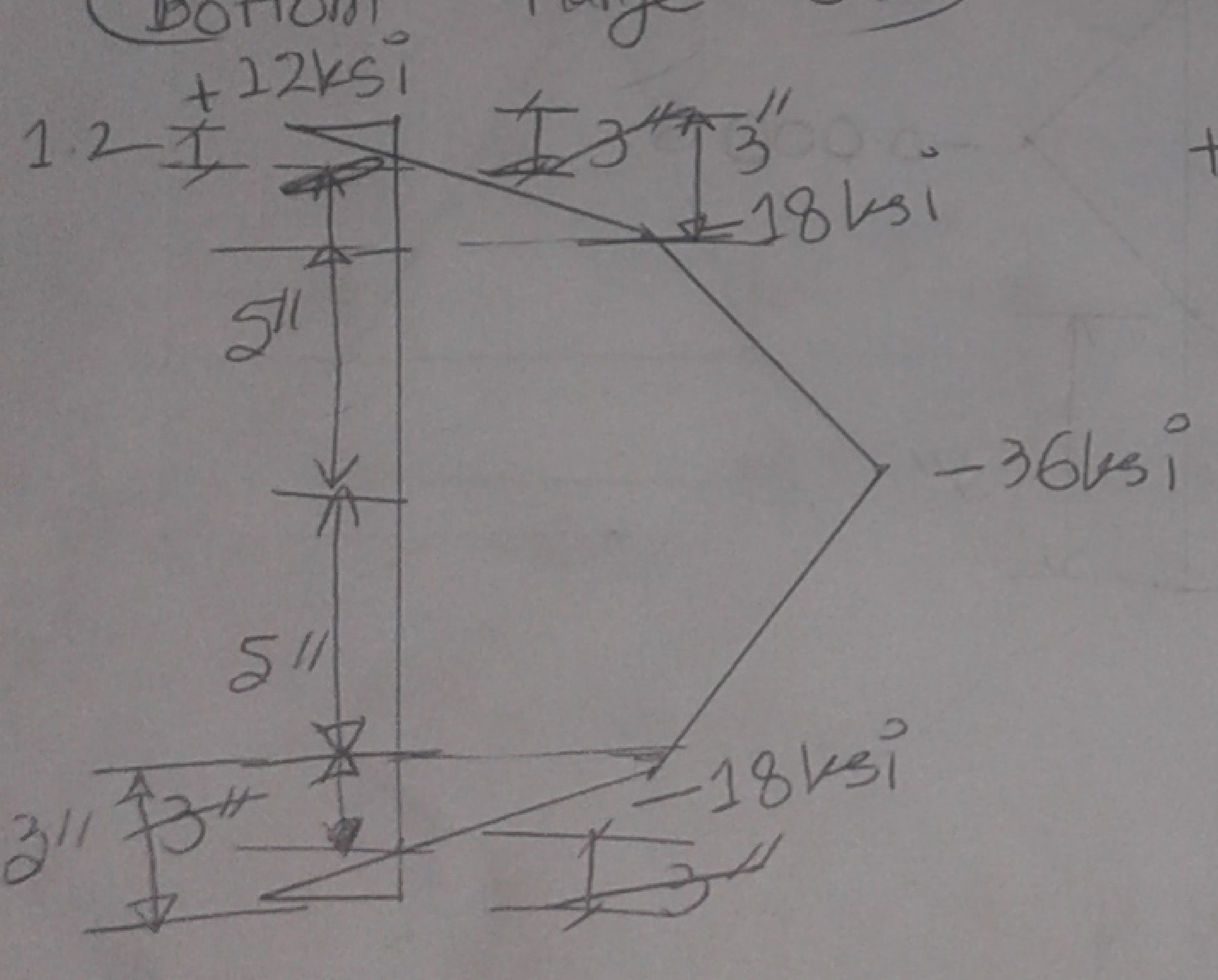
$(\epsilon \leq -0.0006)$

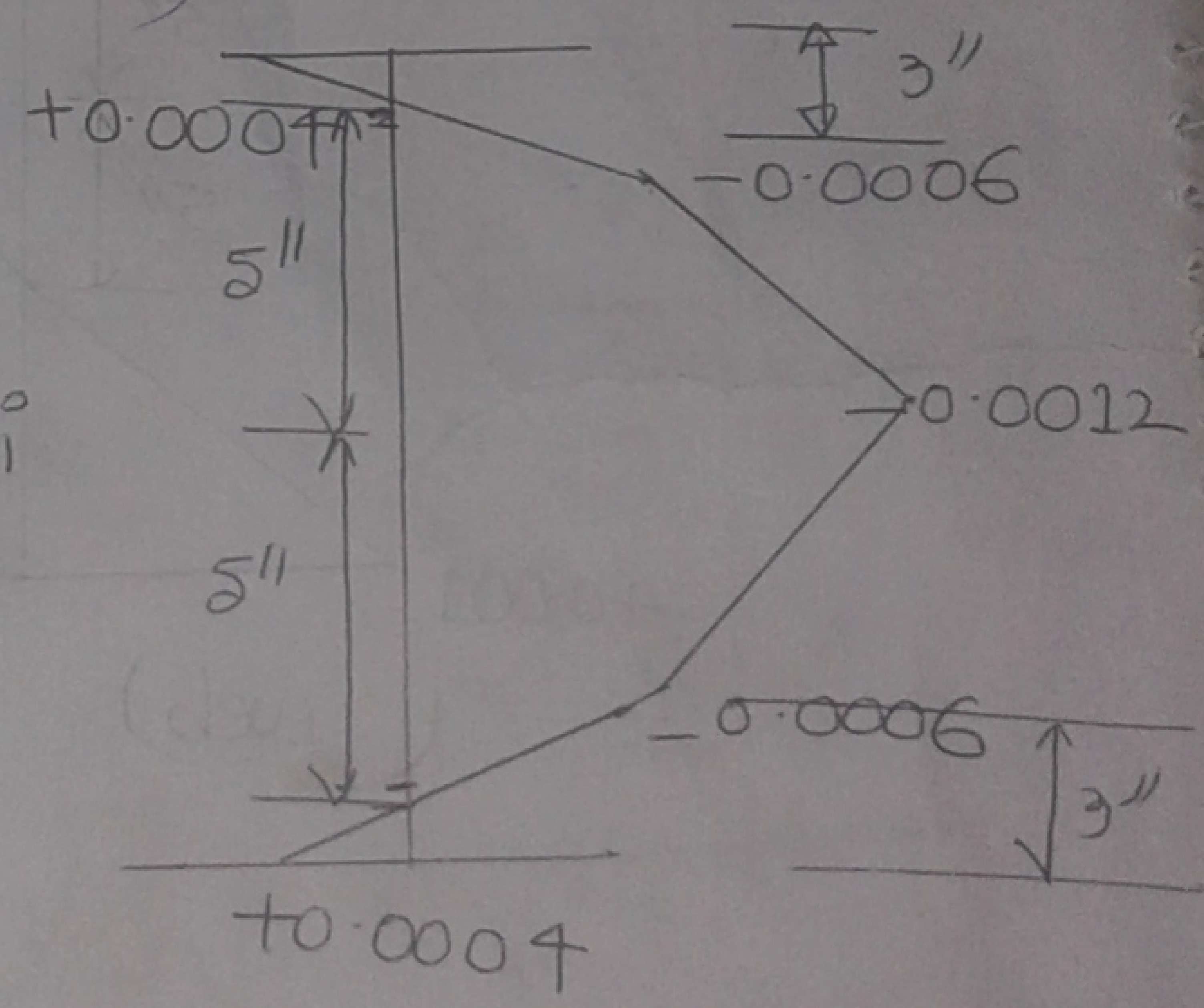




Top flange

(Bottom flange sam)





Web

At $\epsilon = -0.0006$,

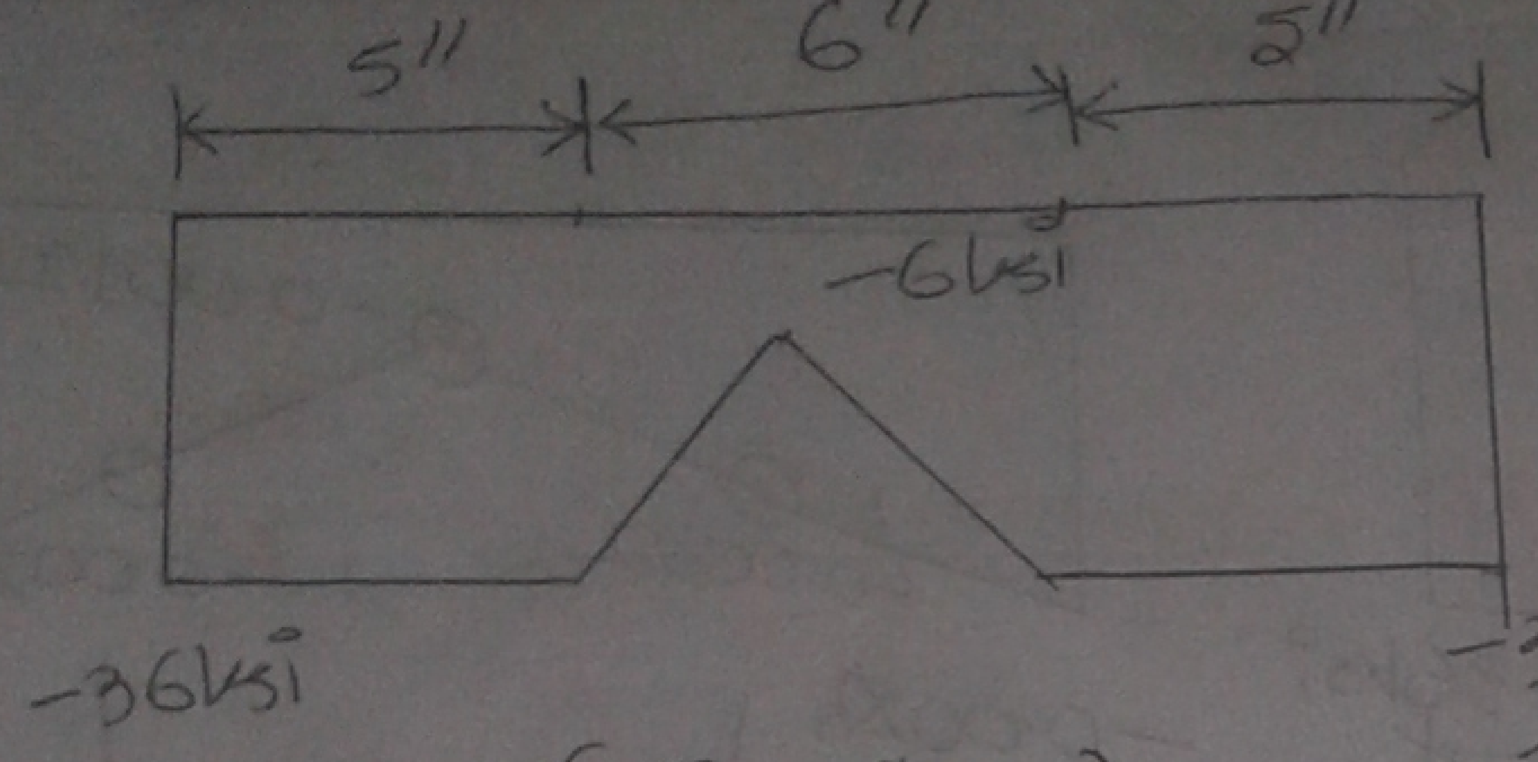
$f = \frac{720}{40} = 18$ ksi

$\frac{df}{d\varepsilon} = 2A\varepsilon + B$

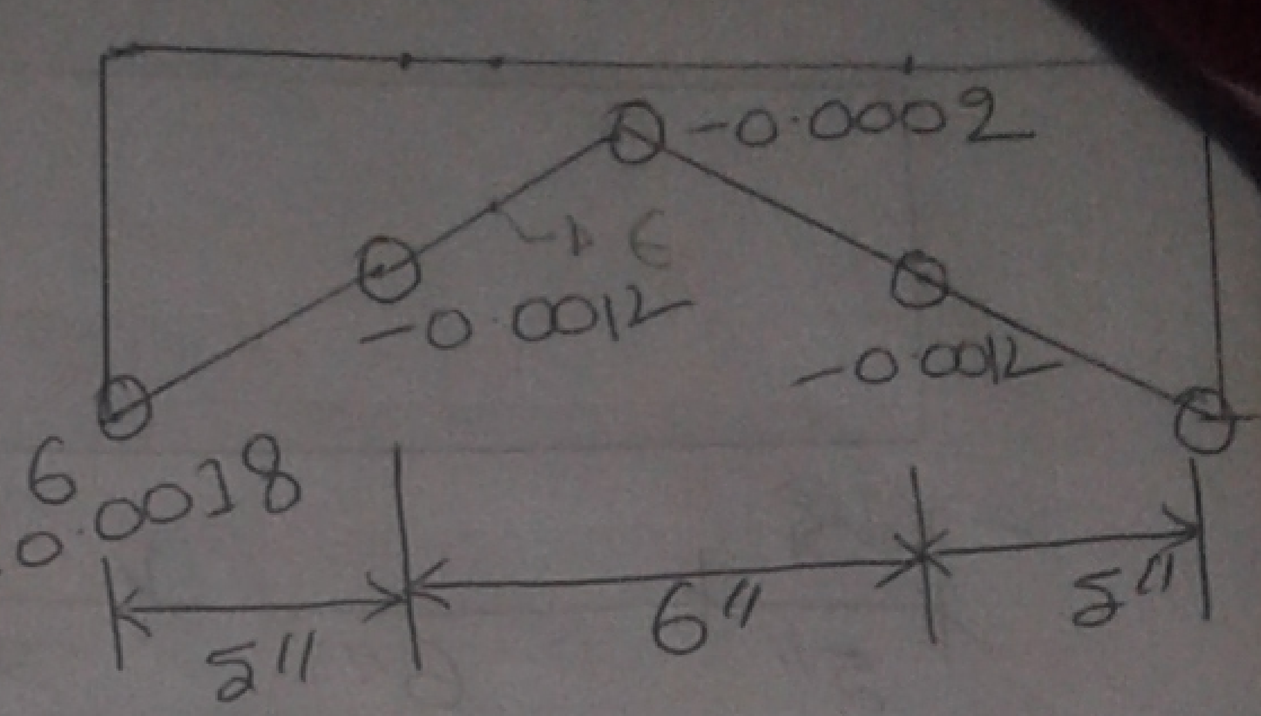
At $\varepsilon = -0.0006$, $\frac{df}{d\varepsilon} = E$

$\therefore 2A \times (+0.0006) + B = 30{,}000$ ——— (i)

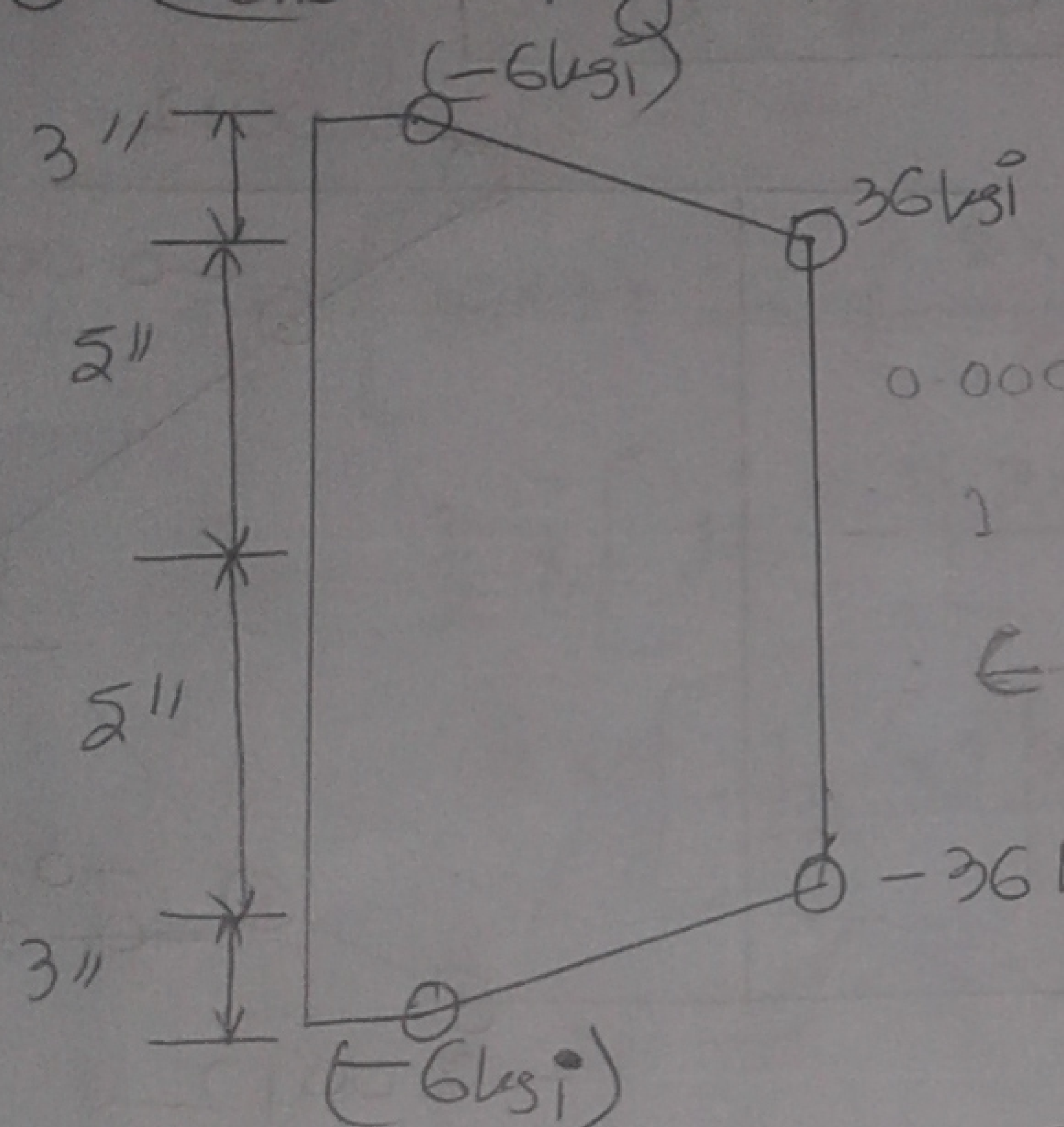
$\boxed{\text{step-02}}$ ($-0.0002 \le \epsilon \le 0.0008$) ($-0.0006 \le$



(Top flange)



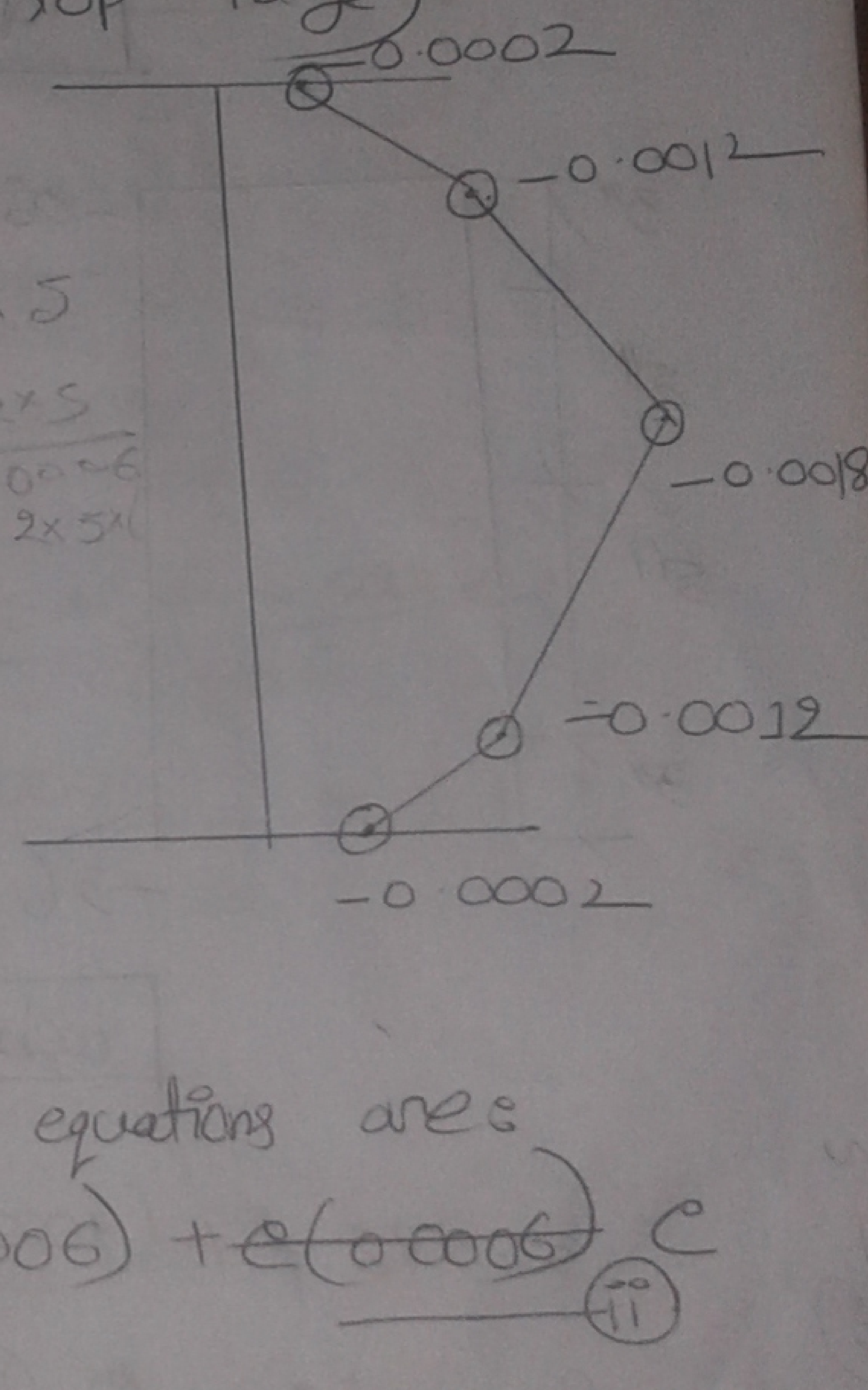
(*) (Bottom flange is same as top flange)



$0.0006 \rightarrow 2\times 5$

$1 \rightarrow \frac{2\times 5}{0.0006}$

$\epsilon\ 0.0006 = 2\times 5 \times$



$\boxed{\text{web}}$

$-0.006 \le \epsilon \le -0.0012$, The equations are:

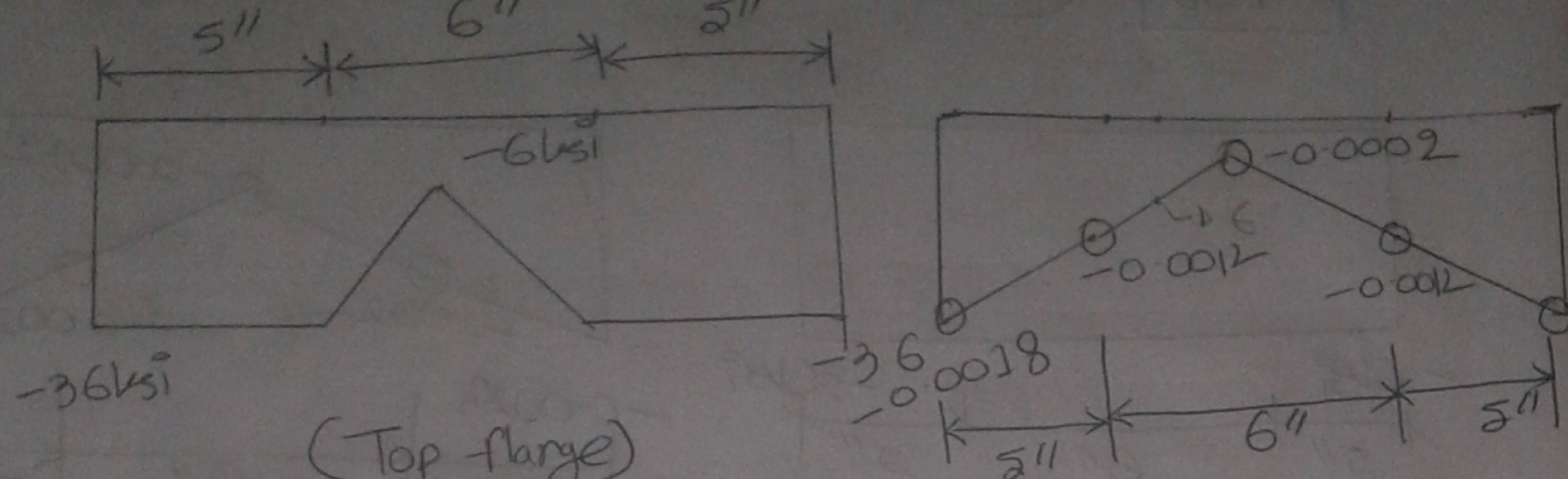
$$18 = A(0.0006)^2 + B(0.0006) + \cancel{C(0.0006)}\ C \quad \text{(ii)}$$

~~At $\epsilon = -0.0006$,~~

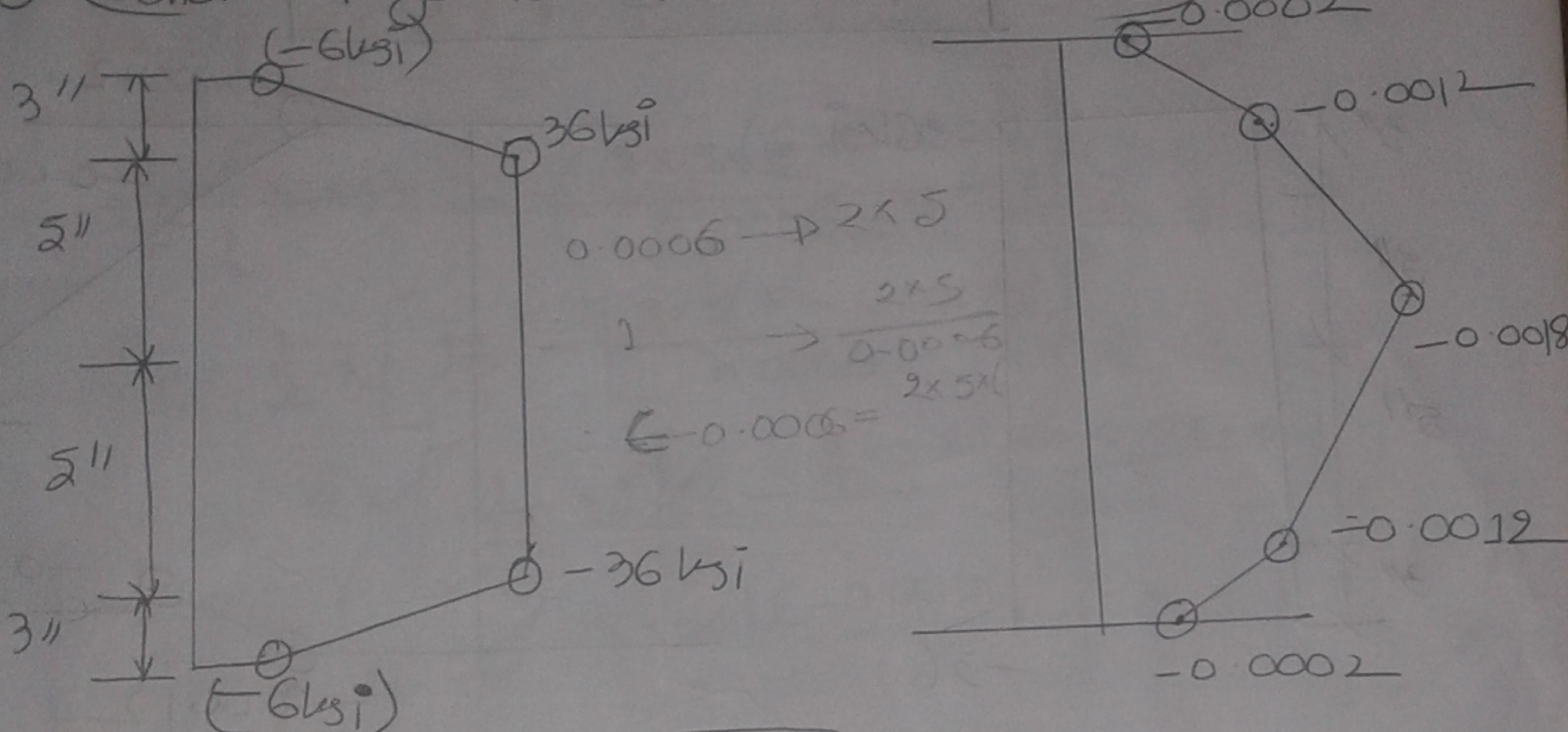
At $\epsilon = -0.0012$,

$$f = \frac{36\times 10\times 0.5 + 2\times 3\times\left(\frac{36+6}{2}\right)\times 0.5 + 2\times\left[\cancel{10}\ 2\times 5\times 36\times 1 + 2\times 3\times(21)\times 1\right]}{40} = \frac{1215}{40} = 30.375\ \text{ksi}$$

$\boxed{\text{step - 02}}$ $\cancel{(-0.0002 \le \epsilon \le 0.0008)}$ $(-0.0006 \le \epsilon \le 0.0012)$



(*) (Bottom flange is same as top flange)



$\boxed{\text{web}}$

$-0.006 \le \epsilon \le 0.0012$, The equations are:

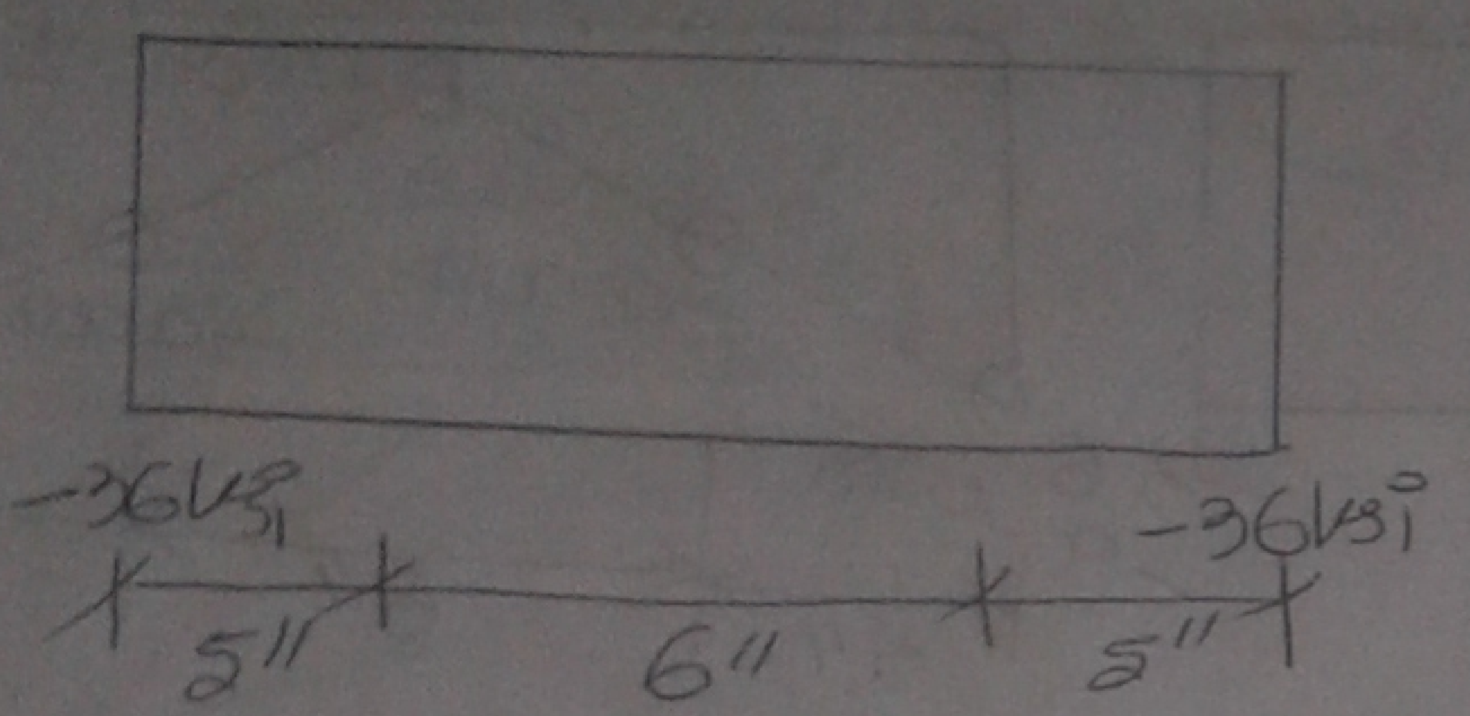
$$18 = A(0.0006)^2 + B(0.0006) + \cancel{C(0.0006)}\ C \quad (ii)$$

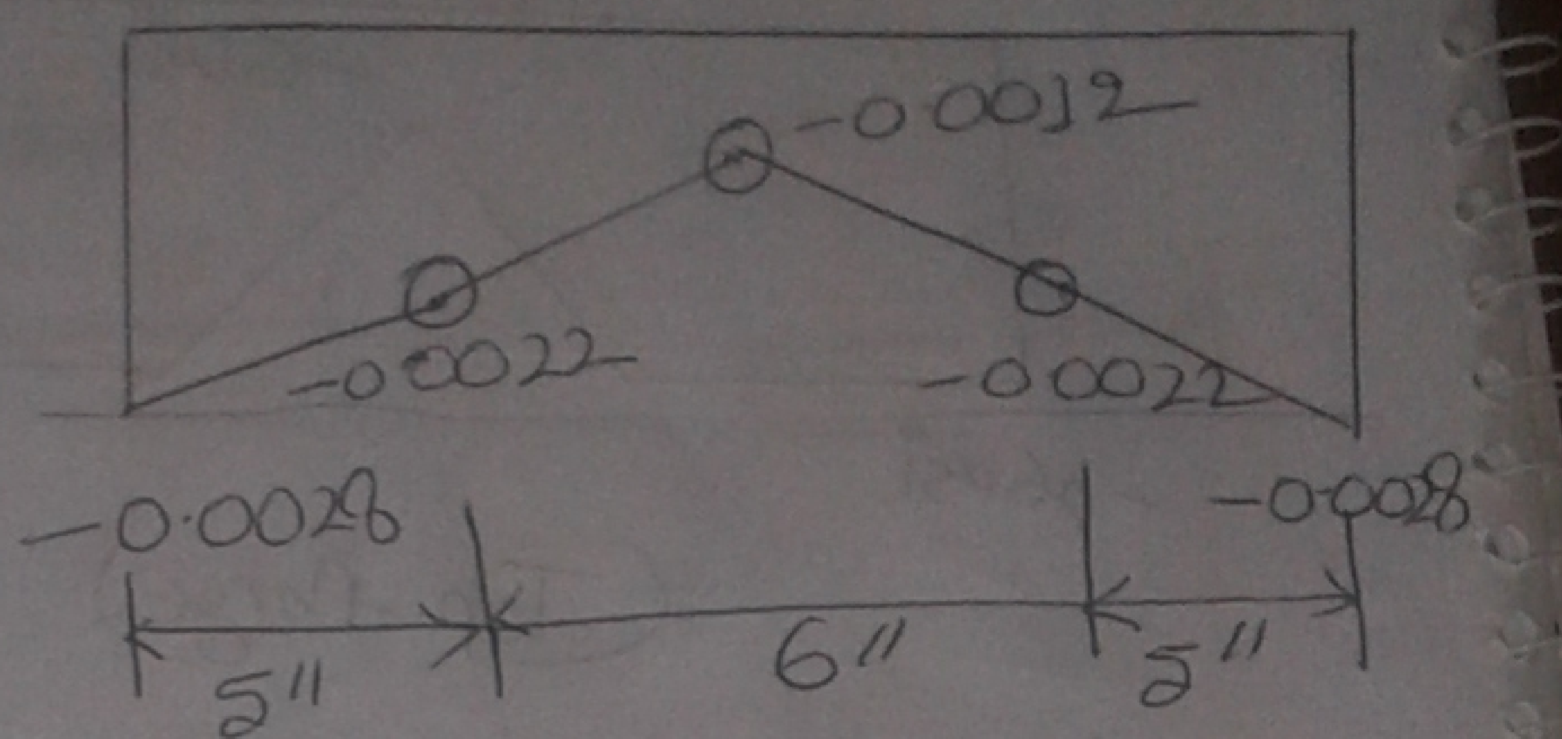
~~At $\epsilon = -0.0006$,~~

At $\epsilon = -0.0012$,

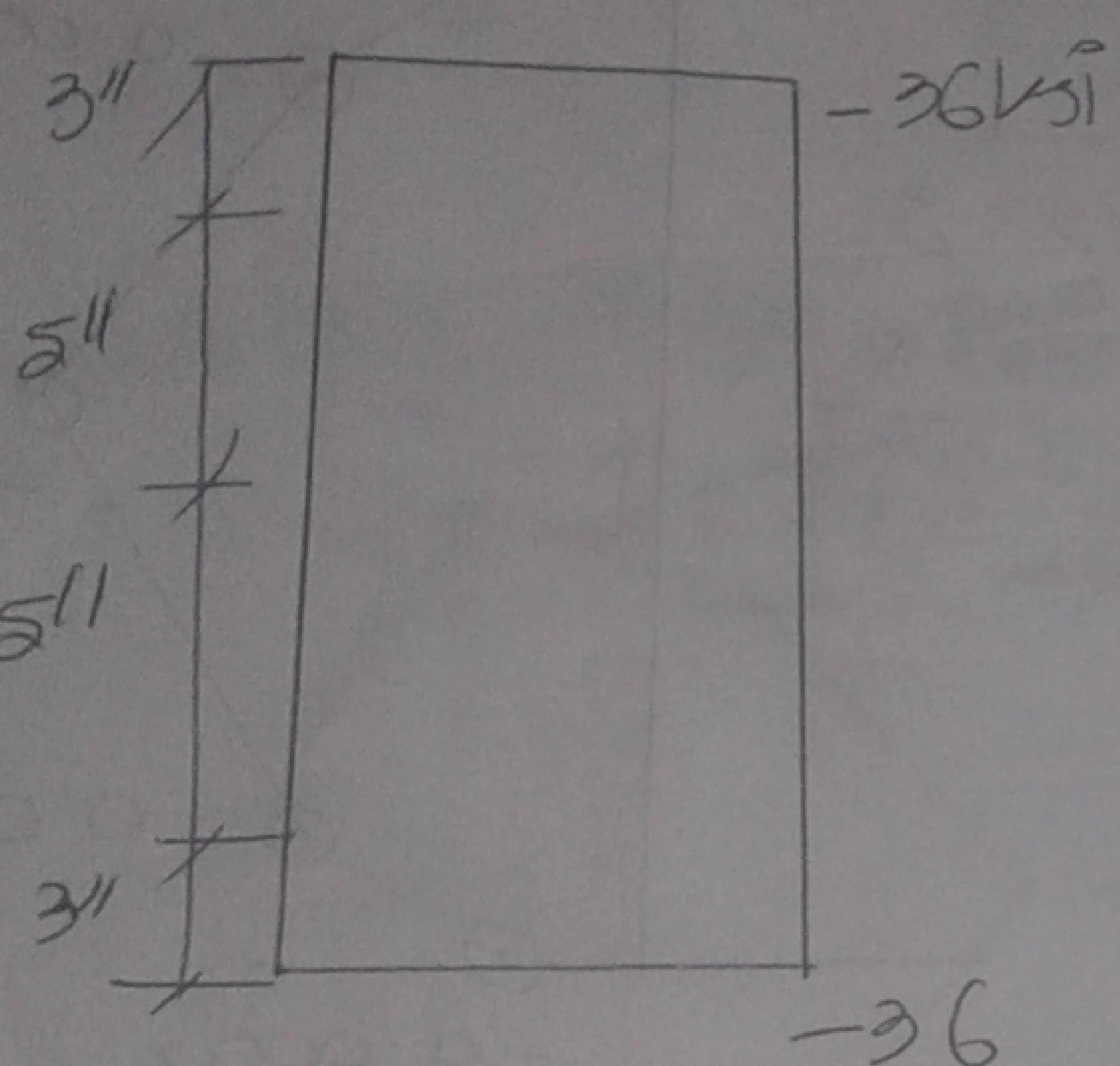
$$f = \frac{36 \times 10 \times 0.5 + 2 \times 3 \times \left(\frac{36+6}{2}\right) \times 0.5 + 2 \times \left[\cancel{10}\ 2 \times 5 \times 36 \times 1 + 2 \times 3 \times (21) \times 1\right]}{40} = \frac{1215}{40} = 30.375 \text{ ksi}$$

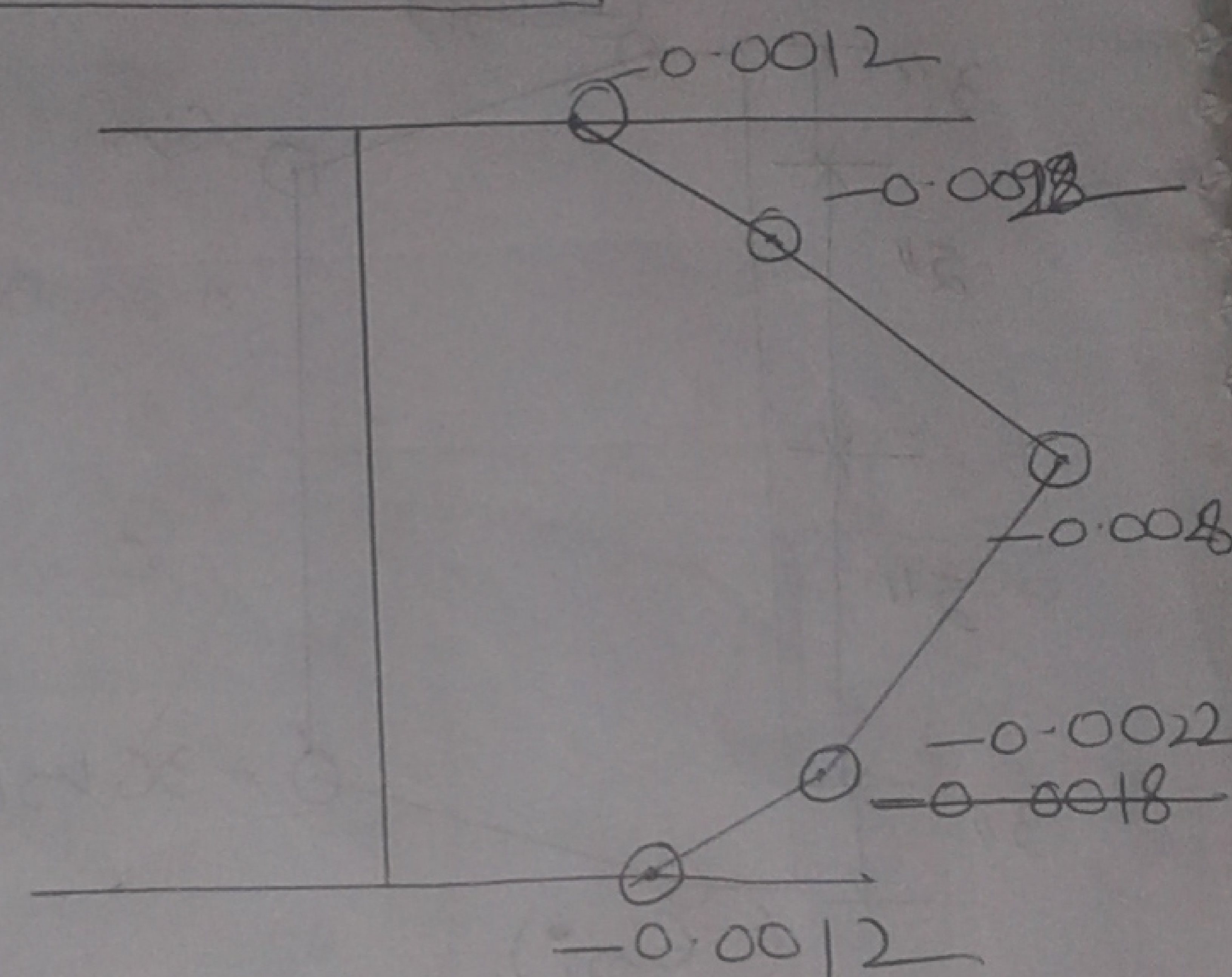
$\boxed{\text{step 02}}$ $(-0.0012 \leq \epsilon \leq 0.0022)$





$\boxed{\text{Top and bottom flange}}$





$\boxed{\text{web}}$

$$\therefore 30.375 = A(+0.0012)^2 + B(+0.0012) + C \quad \text{(iii)}$$

By solving (i), (ii) and (iii),

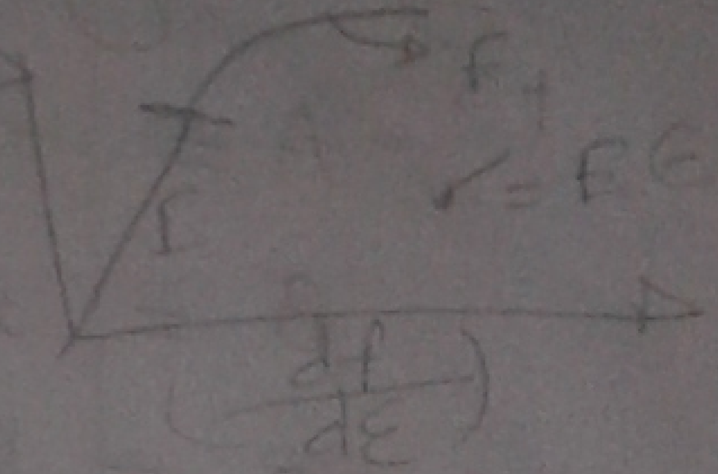
$A = -15{,}625{,}000$

$B = 48{,}750$

$C = -5.625$

solution yields :

$$f = -15{,}625{,}000\,\varepsilon^2 + 48{,}750\,\varepsilon - 5.625$$



Now,

$$E_t = -31{,}250{,}000\,\varepsilon + 48{,}750$$

In this region, $-0.0006 \le \epsilon \le -0.0012$,

$$I_{x,eff} = \left\{ \frac{2\times}{12}\left[16 - \frac{2\times5(\epsilon - 0.0006)}{0.0006}\right]\times 1^3 \right\}$$

$$+\ 2\times\left[\cancel{(16\times1)}\ 16 - \left(\frac{2\times5\times(\epsilon - 0.0006)}{0.0006}\right)\right]\times\left(8\tfrac{1}{2}\right)^2$$

$$+\ \frac{1}{12}\times\frac{1}{2}\times\left[16^3 - \left\{\frac{2\times5(\varepsilon - 0.0006)}{0.0006}\right\}^3\right]$$

$$I_{y,eff} = 2\times\left\{\frac{1}{12}\times1\left[16 - \frac{2\times5\times(\epsilon - 0.0006)}{0.0006}\right]^3\right\}$$

$$+\ \frac{1}{12}\times\left[16 - \frac{2\times5\times(\varepsilon - 0.0006)}{0.0006}\right]\times\left(\frac{1}{2}\right)^3$$

For $-0.0012 \le \varepsilon \le -0.0022$

The equations can be solved as:

At $\varepsilon = -0.0012$,

$$30.375 = A(0.0012)^2 + B(0.0012) + C$$

At $\varepsilon = -0.0022$

$$f = \frac{1440}{40} = 36\,\text{ksi}$$

$\therefore 36 = A(0.0022)^2 + B(0.0022) + C$

$0 = A \times (2 \times 0.0022) + B$

By solving 3 equations:

$A = -5{,}625{,}000$

$B = 24{,}750$

$C = 8.775$

$\therefore f = (-5{,}625{,}000)\varepsilon^2 + (24{,}750)\varepsilon + 8.775$

$\therefore E_t = (-11250000)\varepsilon + (24750)$

In this region,

~~$I_{x,eff} = 2 \times \left[\frac{1}{12} \times 16 - \right]$~~

$$I_{x,eff} = 2 \times \left[\frac{1}{12} \left\{ 6 - \frac{2 \times 3(\epsilon - 0.0012)}{0.0010} \right\} \times 1^3 \right]$$

$$+ 2 \times \left[\left[\left\{ 6 - \frac{2 \times 3(\epsilon - 0.0012)}{0.0010} \right\} \right] \times \left(8\frac{1}{2}\right)^2 \right]$$

$$+ \frac{1}{12} \times \frac{1}{2} \times \left[16^3 - \left\{ 10 + \frac{2 \times 3(\epsilon - 0.0012)}{0.0010} \right\}^3 \right]$$

$$I_{y,eff} = 2 \times \left\{ \frac{1}{12} \times 1 \left[6 - \frac{2 \times 3(\epsilon - 0.0012)}{0.0010} \right]^3 \right\}$$

$$+ \frac{1}{12} \left\{ 16 - \left[10 + \frac{2 \times 3(\epsilon - 0.0012)}{0.0010} \right] \right\} \times \left(\frac{2}{2}\right)^3$$

stress - strain value for column cross section

ε	σ	E_t	$I_{x\,eff}$	$I_{y\,eff}$
0	0	30,000		
0.0002	6	30,000		
0.0004	12	30,000		
0.0006	18	30,000		
0.0008	219(?)	23,750		
0.0010		17,500		
0.0012		11,250		
0.0014		9000		
0.0016		6,750		
0.0018		4,500		
0.0020		2250		
0.0022		0		

$$\cancel{\varepsilon =} \frac{\sigma}{\varepsilon} = E_t$$

$$\therefore (\sigma = E_t \times \varepsilon)$$