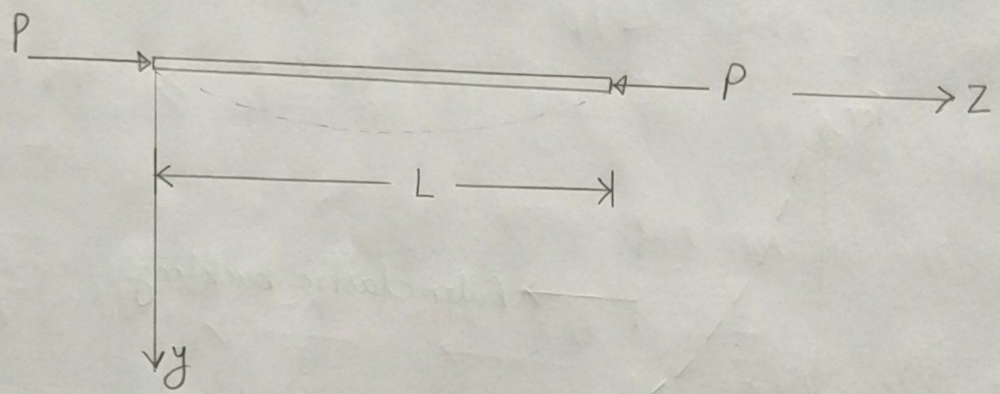
Chapter 6

Compression Members

Euler elastic Buckling



Here, $K^2 = \frac{P}{EI} = \frac{n^2\pi^2}{L^2}$

$$P = \frac{n^2\pi^2 EI}{L^2}$$

The fundamental buckling mode occurs when $\boxed{n = 1}$

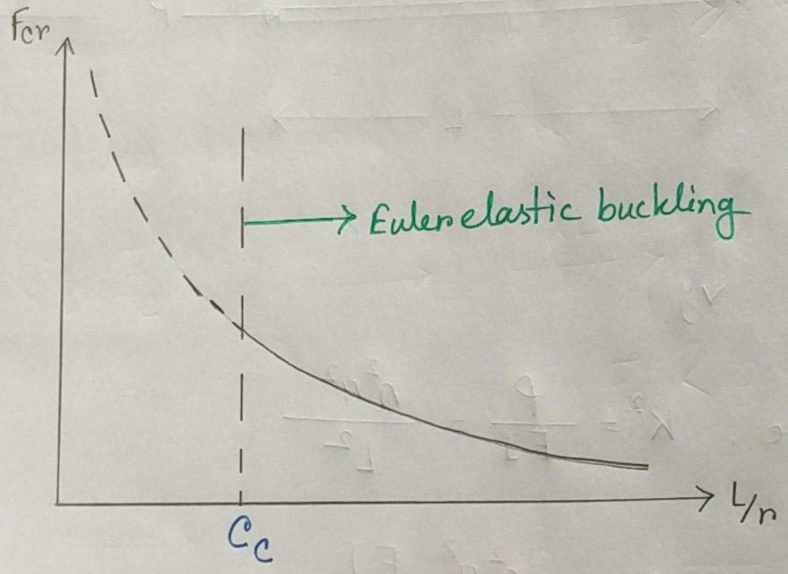
Euler Critical load,

$$\boxed{P_{cr} = \frac{\pi^2 EI}{L^2}}$$

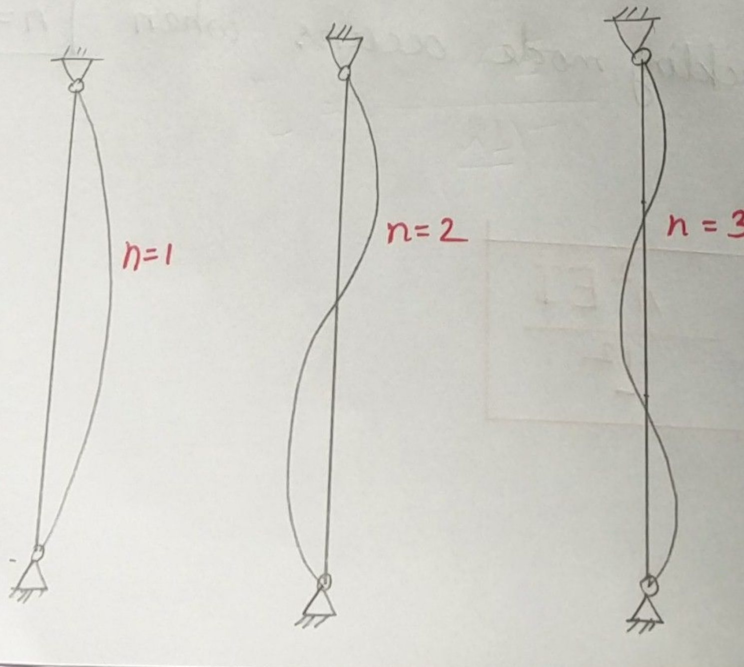
Now,

$$\frac{P_{cr}}{A} = \frac{\pi^2 E \,(I/A)}{L^2} = \frac{\pi^2 E r^2}{L^2}$$

$$\boxed{F_{cr} = \frac{\pi^2 E}{(L/r)^2}}$$ ← Critical buckling stress



C_c = critical slenderness ratio



Basic Column Strength

Strength of column,

$$P_{cr} = F_{cr} A_g = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} A_g$$

E_t = tangent modulus of elasticity at stress $\frac{P_{cr}}{A_g}$

A_g = gross area

$\frac{KL}{r}$ = effective slenderness ratio

K = effective length factor

L = length of member

r = radius of gyration = $\sqrt{\frac{I}{A_g}}$

I = moment of inertia

Column Strength Curve : AISC 2005

Nominal strength,

$$\boxed{P_n = F_{cr} A_g}$$

1) $F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y \longrightarrow \frac{KL}{r} \leq 4.71\sqrt{\frac{E}{F_y}}$

or $F_e \geq 0.44 F_y$ .... (i)

ii) $F_{cr} = 0.877 F_e \longrightarrow \frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$

or $F_e < 0.44 F_y$ ... (ii)

Elastic (Euler) buckling stress,

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

AISC LRFD capacity

$$\phi_c P_n \geq P_u$$

ϕ_c = resistance factor = 0.90

P_n = nominal strength = $F_{cr} A_g$

P_u = factored service load

AISC ASD capacity

$$\frac{P_n}{\Omega} \geq P_a$$

Ω = safety factor = 1.67

P_n = nominal strength = $F_{cr} A_g$

P_a = max^m compressive load using ASD load combination

** AISC introduced a reduction factor $Q < 1.0$ in eq^n (i) and (ii) for local buckling effects.

Strength of the member will be $\boxed{QF_y}$ instead of F_y

For the majority cases with AISC standard sections,

$$Q = 1.0$$

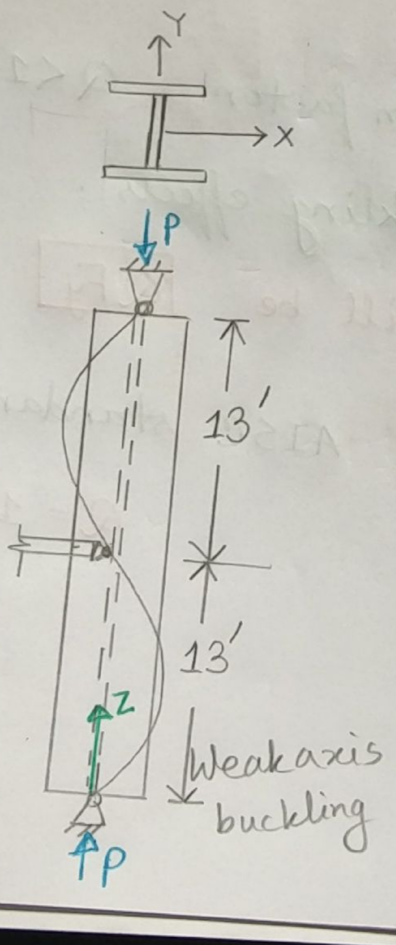
Problem-1

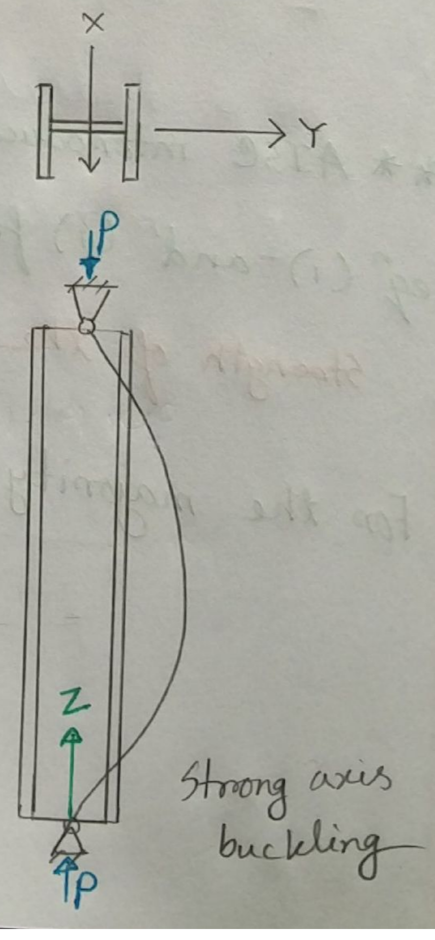
Determine the allowable compressive load carrying capacity of the column shown in figure.

It consists of $\boxed{W10 \times 45}$ section having A992 ($F_y = 50$ ksi) steel.

There are hinge support at top and bottom that allows rotation at any direction.

Also the column has weak direction support (braced at mid-height so the lateral deflection is prevented in x direction. [Use ASD approach]





W10 X 45 section geometry:

$A = 13.3 \ in^2$

$r_x = 4.32$ in

$r_y = 2.01$ in

Now, X axis ⟶ strong axis

Y axis ⟶ weak axis

Column length, $L = 26 \times 12 = 312$ inch

Possibility of buckling in both x any y directions to be checked.

Strong axis buckling

Buckling in Y direction causes bending about X axis or strong axis.

For strong axis buckling (X axis),

$K_x = 1.0$

Now, $\frac{K_x L}{r_x} = \frac{1 \times 312}{4.32} = 72.22$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.43$$

$$\therefore \frac{K_x L}{r_x} < 4.71\sqrt{\frac{E}{F_y}}$$

Now, $F_e = \frac{\pi^2 E}{\left(\frac{K_x L}{r_x}\right)^2} = \frac{\pi^2 \times 29000}{(72.22)^2} = 54.88\ ksi$

Now, $F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y$

$$= \left[0.658^{\frac{50}{54.88}}\right] 50$$

$$\therefore F_{cr} = 34.15\ ksi$$

$$P_{nx} = P_{cr} = F_{cr} A_g = 34.15 \times 13.3 = 454\ kip$$

Weak axis buckling

Buckling in X direction causes bending about Y axis or weak axis.

For weak axis (Y axis) buckling, $K_y = 0.5$

$$\frac{K_y L}{r_y} = \frac{0.5 \times 312}{2.01} = 77.6$$

$$4.71\sqrt{E/F_y} = 4.71\sqrt{\frac{29000}{50}} = 113.43 > \frac{K_y L}{r_y}$$

$$F_e = \frac{\pi^2 E}{\left(K_y L/r_y\right)^2} = \frac{\pi^2 \times 29 \times 10^3}{(77.6)^2} = 47.53 \text{ ksi}$$

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y = \left[0.658^{\frac{50}{47.53}}\right] \times 50 = 32.2 \text{ ksi}$$

$$P_{ny} = F_{cr} A_g = 32.2 \times 13.3 = 428 \text{ kip}$$

So, P_n = smaller of P_{nx} and P_{ny} = 428 kip

∴ Allowable strength,

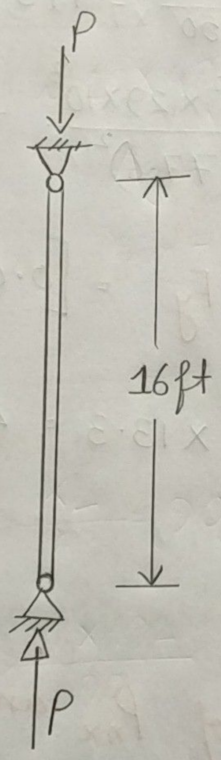
$$P_a = \frac{P_n}{\Omega} = \frac{428}{1.67} = \boxed{256.3 \text{ kip}}$$

Ans

Problem 2

Select the lightest W section of A992 ($F_y = 50 ksi$) steel to serve as a pinned-end main member column 16 ft long to carry an axial compression load of 115 kip DL and 125 kip LL in a braced structure.

[Use ASD approach]



As ASD approach, $P_a = (115+125) = 240 \text{ kip}$

Here, $L = 16 \text{ ft} = 192''$

Both end hinged ; $\therefore K = 1.0$

Nominal strength, $P_n = F_{cr} A_g$

Now, $C_c = 4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.43$

As we prefer "elastic - plastic" behavior,

$\frac{KL}{r} < 113.43$ is preferred.

Trial-1

[start with $\frac{KL}{r} \approx 70-75\%$ of C_c]

Let us assume,

$$\frac{KL}{r} = 80$$

$$\therefore r = \frac{KL}{80} = \frac{1 \times 192}{80} = 2.4 \text{ inch}$$

Now, $F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 \times 29000}{(80)^2} = 44.72$

Now, $F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y = \left[0.658^{\frac{50}{44.72}}\right] 50 = 31.31 \text{ ksi}$

Now, $P_a \leq \frac{P_n}{\Omega}$

$$\therefore P_n = 240 \times 1.67 = 400.8 \text{ kip}$$

Now, $P_n = F_{cr} A_g$

$$\therefore A_g = \frac{400.8}{31.31} = 12.8 \text{ in}^2$$

Now,

From AISC manual choose a W section with

$A_g \geq 12.8 \text{ in}^2$ and $r_{min} \geq 2.4$ in

Let us choose, W 10X49

$A_g = 14.4 \text{ in}^2$

$r_{min} = 2.54$ in

Trial 2

Let us assume, $\frac{KL}{r} = 90$

$$\therefore r = \frac{1 \times 192}{90} = 2.13 \text{ in}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 \times 29000}{(90)^2} = 35.34 \text{ ksi}$$

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y = \left[0.658^{\frac{50}{35.34}}\right] 50 = 27.65 \text{ ksi}$$

Now, $P_a \leq \frac{P_n}{\Omega}$

$\therefore \; P_n \geqslant 240 \times 1.67$

$\therefore \; P_n = 400.8$ kip

Now, $P_n = F_{cr} A_g$

$\therefore \; A_g = \frac{P_n}{F_{cr}} = \frac{400.8}{27.65} = 14.5 \text{ in}^2$

From AISC manual choose a W section with $A_g \geqslant 14.5 \text{ in}^2$

and $r_{min} \geqslant 2.13$ in

Let us choose, W 12 x 53

$A_g = 15.6 \text{ in}^2$

$r_{min} = 2.48$ in

Trial 3

Let us assume, $\frac{kL}{r} = 70$

$$\therefore r = \frac{1 \times 192}{70} = 2.74 \text{ in}$$

Now, $F_e = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} = \frac{\pi^2 \times 29000}{70^2} = 58.4 \text{ ksi}$

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y = \left[0.658^{\frac{50}{58.4}}\right] 50 = 34.95 \text{ ksi}$$

$$P_n = \Omega P_a = 1.67 \times 240 = 400.8 \text{ K}$$

$$P_n = F_{cr} A_g$$

$$\therefore A_g = \frac{P_n}{F_{cr}} = \frac{400.8}{34.95} = 11.47 \text{ in}^2$$

From AISC manual choose a W section with

$$A_g \geq 11.47 \text{ in}^2 \text{ and } r_{min} \geq 2.74 \text{ in}$$

Let us choose, W 12×65

$$A_g = 19.1 \text{ in}^2$$

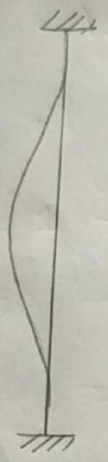
$$r_{min} = 3.02 \text{ in}$$

Among these three trials,

the finally chosen section is $\boxed{W10X49}$

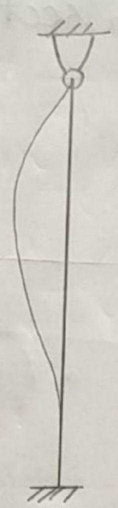
Ans

Effective Length of Compression Members (KL)



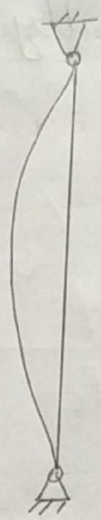
$K_{theo} = 0.5$

$K_{des} = 0.65$



$K_{th} = 0.70$

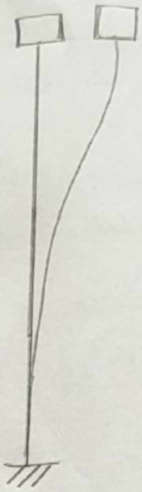
$K_{des} = 0.80$



$K_{th} = 1.0$

$K_{des} = 1.0$

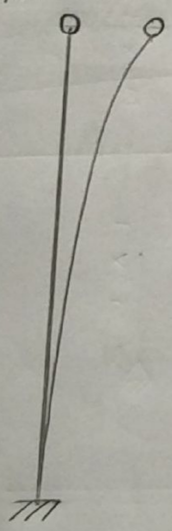
rotation fixed
translation free



$K_{th} = 1.0$

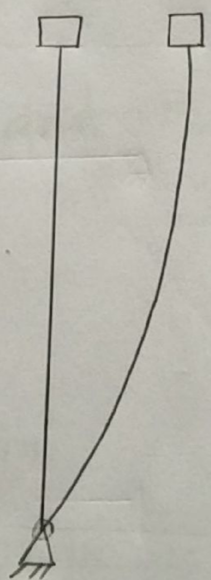
$K_{des} = 1.2$

rotation free
translation free



$K_{th} = 2.0$

$K_{des} = 2.1$



$K_{th} = 2.0$

$K_{des} = 2.0$

But the most commonly used procedure for obtaining effective length is to use

ALIGNMENT CHARTS/NOMOGRAPH

$$\boxed{G = \frac{\sum \left(\frac{EI}{L}\right)_{column}}{\sum \left(\frac{EI}{L}\right)_{girder}}}$$

← Relative Stiffness of Column

Stiffness modification factors for beams:

Condition	Sidesway (Unbraced)	No Sidesway (Braced)
Far end of beam hinged —	$\frac{1}{2}$ 0.5 —	$3/2$ 1.5
Far end of beam fixed —	$2/3$ 0.667 —	2

Problem-3

Determine the effective length co-efficients for the columns AB, FG, GH of the frame.

The moment of inertia for members in in^4 are –

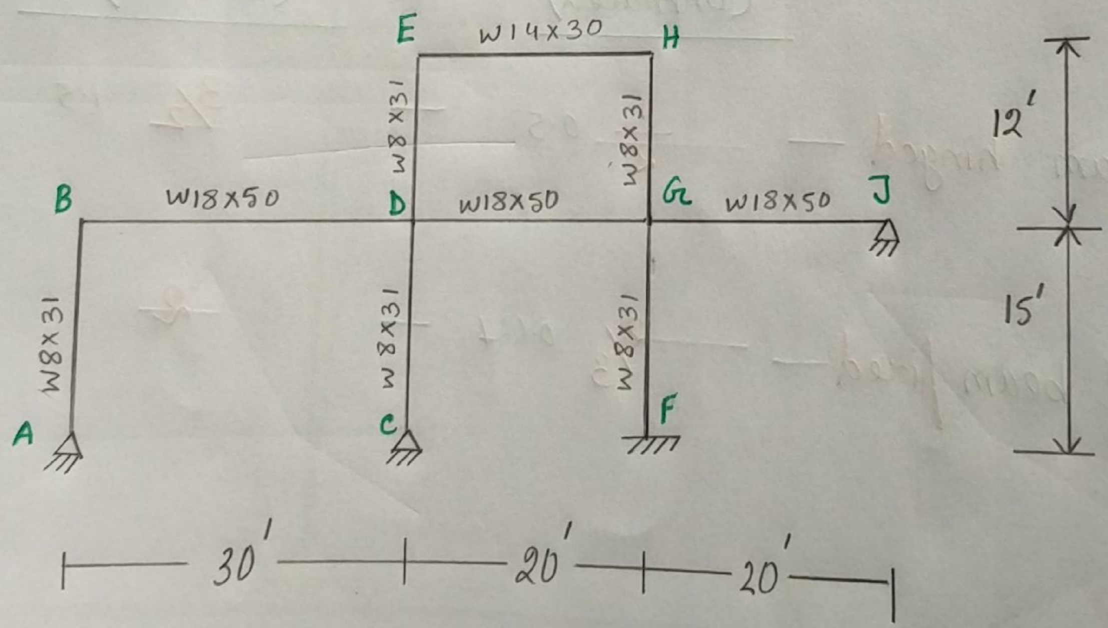
W8 X 31 $\longrightarrow$ 110 in^4

W18 X 50 $\longrightarrow$ 800 in^4

W14 X30 $\longrightarrow$ 291 in^4

The connection factors for beam stiffness with far end of the beam hinged are

$\frac{1}{2}$ with sidesway, $\frac{3}{2}$ without sidesway.



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Column AB:

Column AB is sidesway prevented (braced) frame.

Now, at A, $G_A = 10$ (practical value at hinge instead on infinity)

at B, $G_B = \dfrac{\Sigma (I/L)_{col}}{\Sigma (I/L)_{beam}} = \dfrac{\frac{110}{15}}{\frac{800}{30}} = 0.275$

From alignment chart (braced), $\boxed{K_{AB} = 0.77}$

Column FG:

Column FG is a sidesway prevented (braced) frame.

at F, $G_F = 1$ (practical value at fixed base instead of zero)

at G, $G_G = \dfrac{\Sigma (I/L)_{col}}{\Sigma (I/L)_{beam}} = \dfrac{(I/L)_{GF} + (I/L)_{GH}}{(I/L)_{GD} + (I/L)_{GJ} \times \frac{3}{2}}$

$$= \frac{\frac{110}{15} + \frac{110}{12}}{\frac{800}{20} + \frac{3}{2} \times \frac{800}{20}} = 0.165$$

From alignment chart (braced), $\boxed{K_{FG} = 0.67}$

Column GH:

Column GH is sideway (unbraced) frame.

at G, $G_G = \dfrac{\frac{110}{15} + \frac{110}{12}}{\frac{800}{20} + 0.5\,\frac{800}{20}} = 0.275$

at H, $G_H = \dfrac{\left(I/L\right)_{GH}}{\Sigma \left(I/L\right)_{EH}} = \dfrac{\frac{110}{12}}{\frac{291}{20}}$

$\therefore\ G_H = 0.63$

From Alignment chart (unbraced),

$$\boxed{K_{GH} = 1.15}$$

Effect of Residual stresses in Compression Members

Residual stress diagram must be self balancing

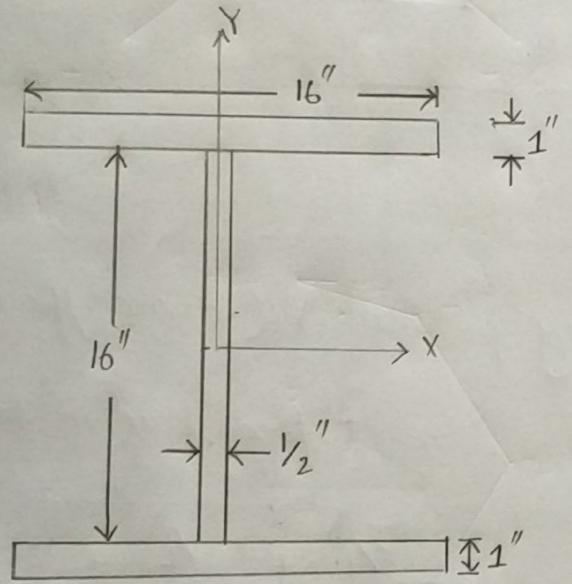
* Residual stress means no external load

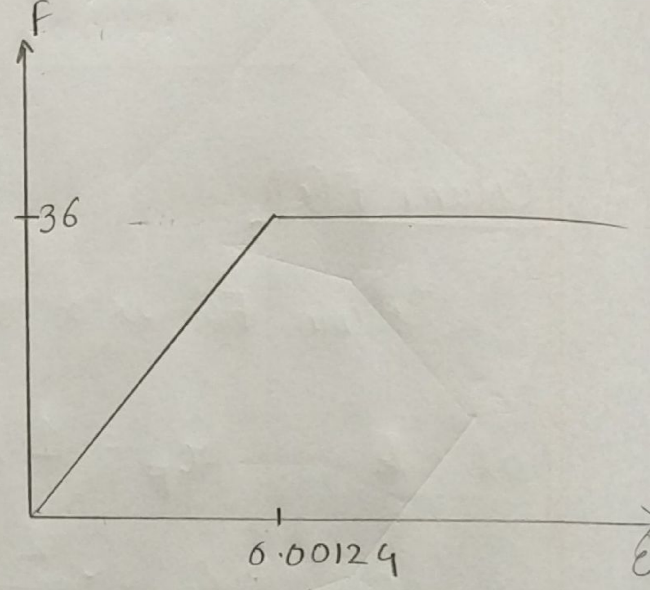
* Self balancing means average stress = 0

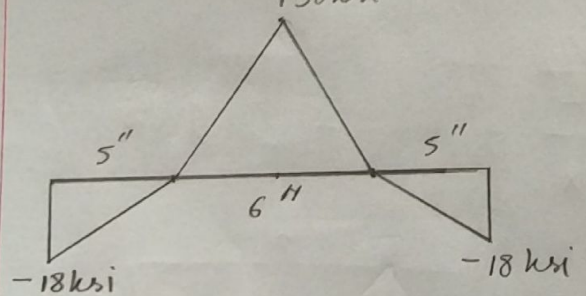
Problem-4

The residual stress for a W section to be used as a compression member is shown in the figures below. Given, $F_y = 36$ ksi, $E = 29000$ ksi

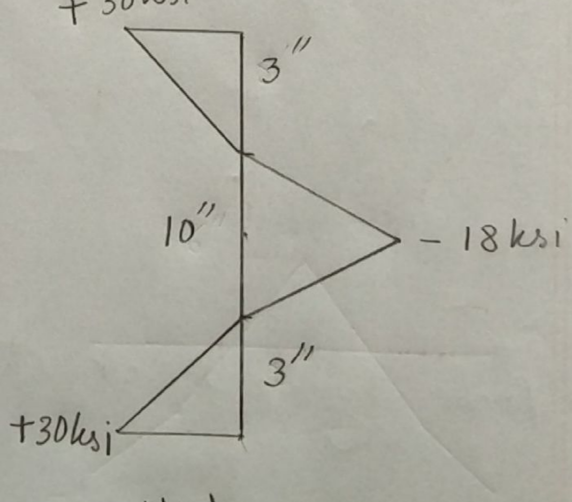
Derive the equation for the stress-strain behavior of the W-section.





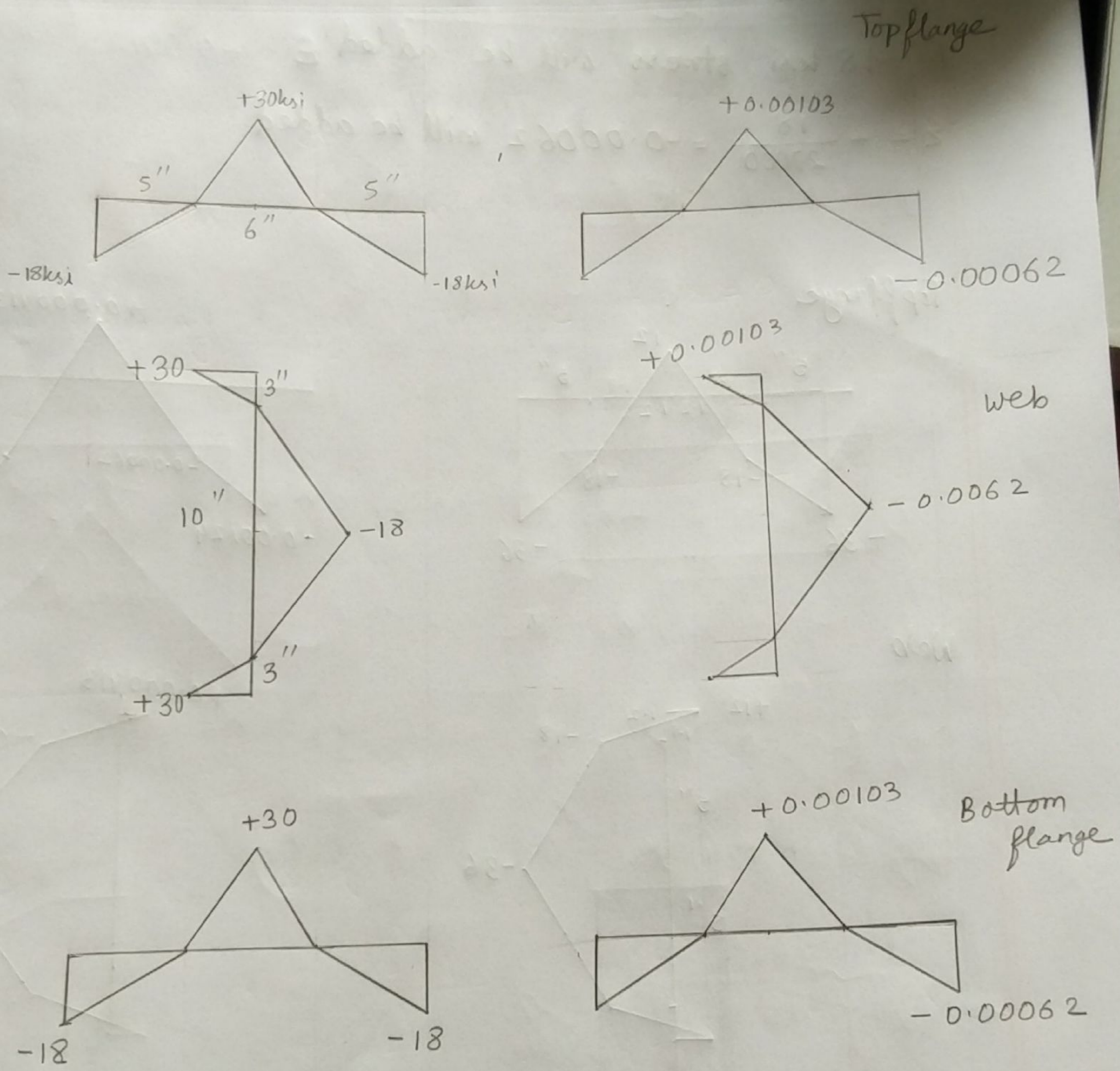


Top flange, Bottom flange



Web

(a)



$$F_{avg} = \frac{\Sigma \text{ area}}{\text{Total length}} = \frac{(-0.5 \times 5 \times 18 + 0.5 \times 3 \times 30) \times 2}{16} = 0$$

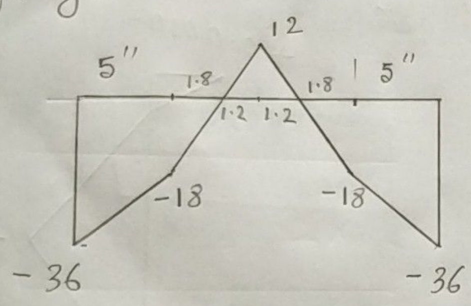
$$\varepsilon_{avg} = \frac{F_{avg}}{E} = 0$$

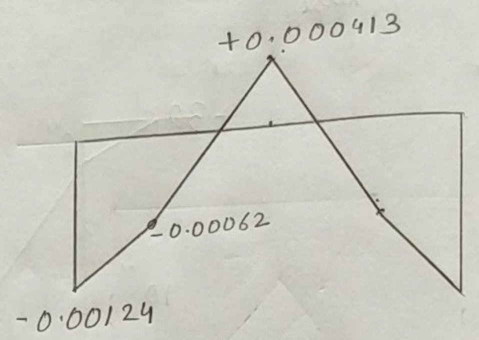
(b) Elastic limit

F= −18 ksi stress will be added

$$\varepsilon = -\frac{18}{29000} = -0.00062 \text{ will be added}$$

Top flange





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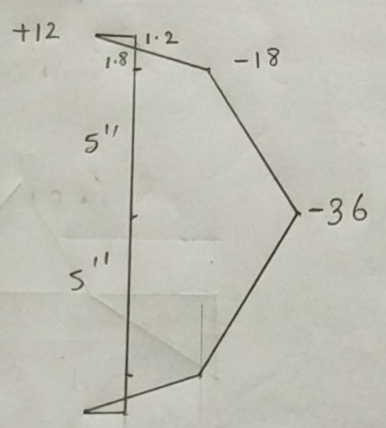
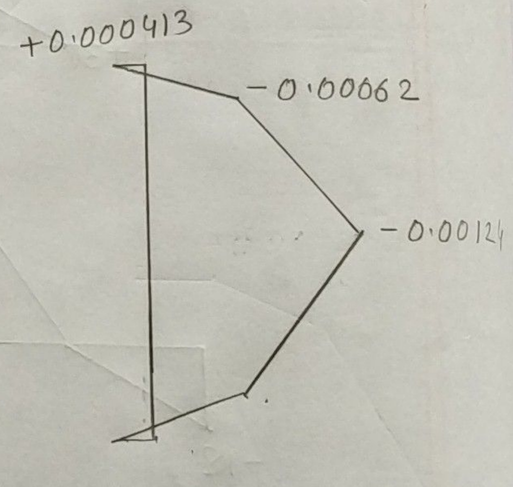
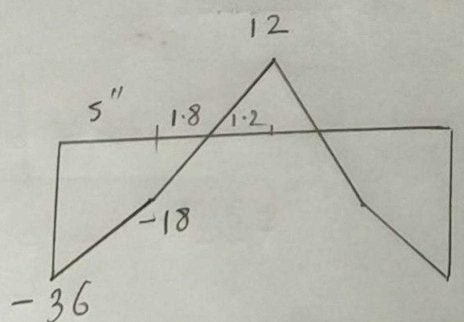
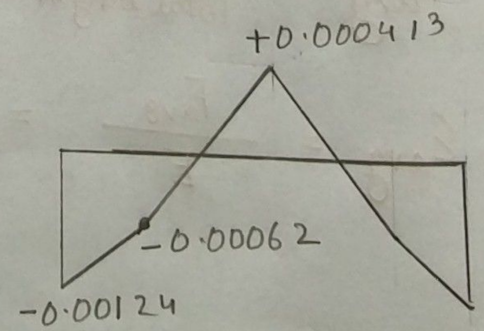




Bottom flange





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$$F_{avg} = \frac{\Sigma \text{ area}}{\text{Total length}} = \frac{2\left(-\frac{1}{2} \times 5 \times (18+36) - \frac{1}{2} \times 1.8 \times 18 + \frac{1}{2} \times 1.2 \times 12\right)}{16}$$

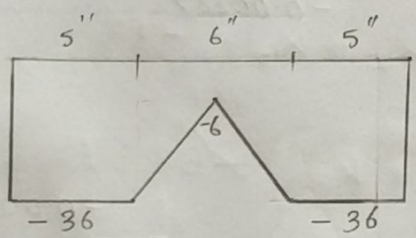
$$= -18 \text{ ksi}$$

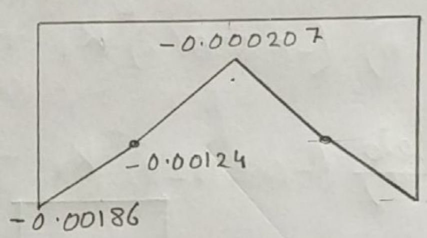
$$\varepsilon_{avg} = \frac{F_{avg}}{E} = -0.00062$$

(c) Inelastic limit - 1

F = -18 ksi will be added i.e. $\varepsilon = -.00062$ will be added

Top flange, Bottom flange





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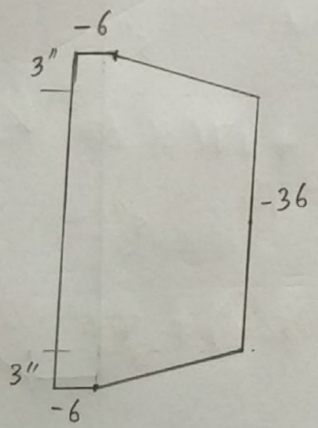

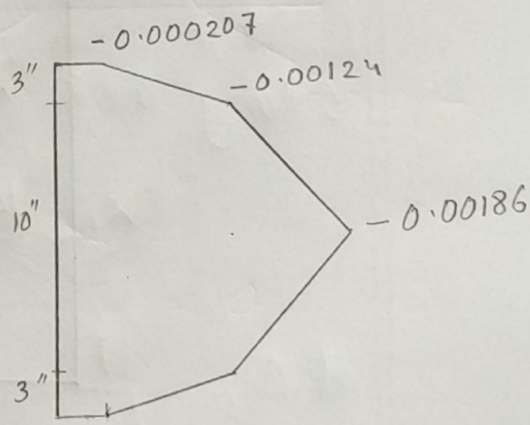

$$F_{avg} = \frac{-\{2(0.5 \times 3 \times (36+6)) + 36 \times 10\}}{16} = -30.375$$

$$\varepsilon_{avg} = \frac{\Sigma \text{ area}}{\text{Total length}} = -0.00124$$

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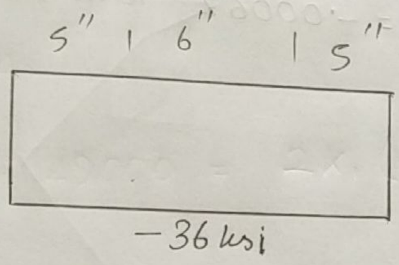
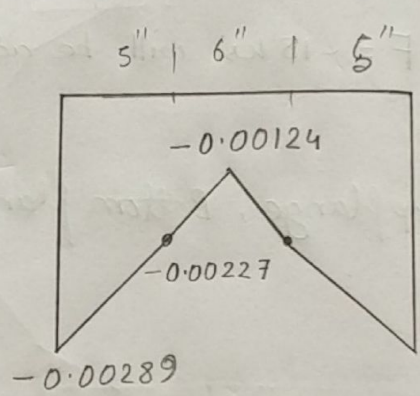
(d) Inelastic limit, fully yield

$F = -30$ ksi will be added i.e. $\varepsilon = -0.00103$ will be added

Top flange, bottom flange





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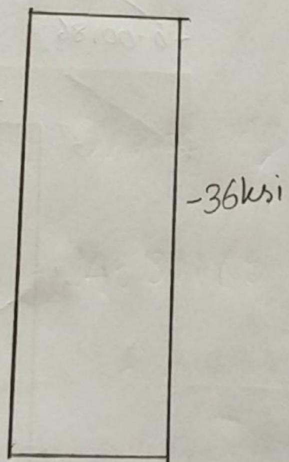
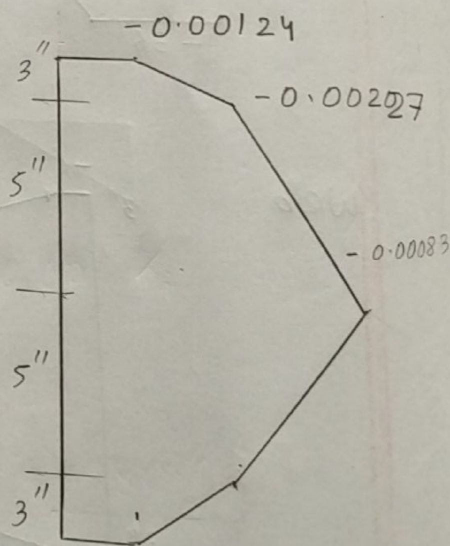
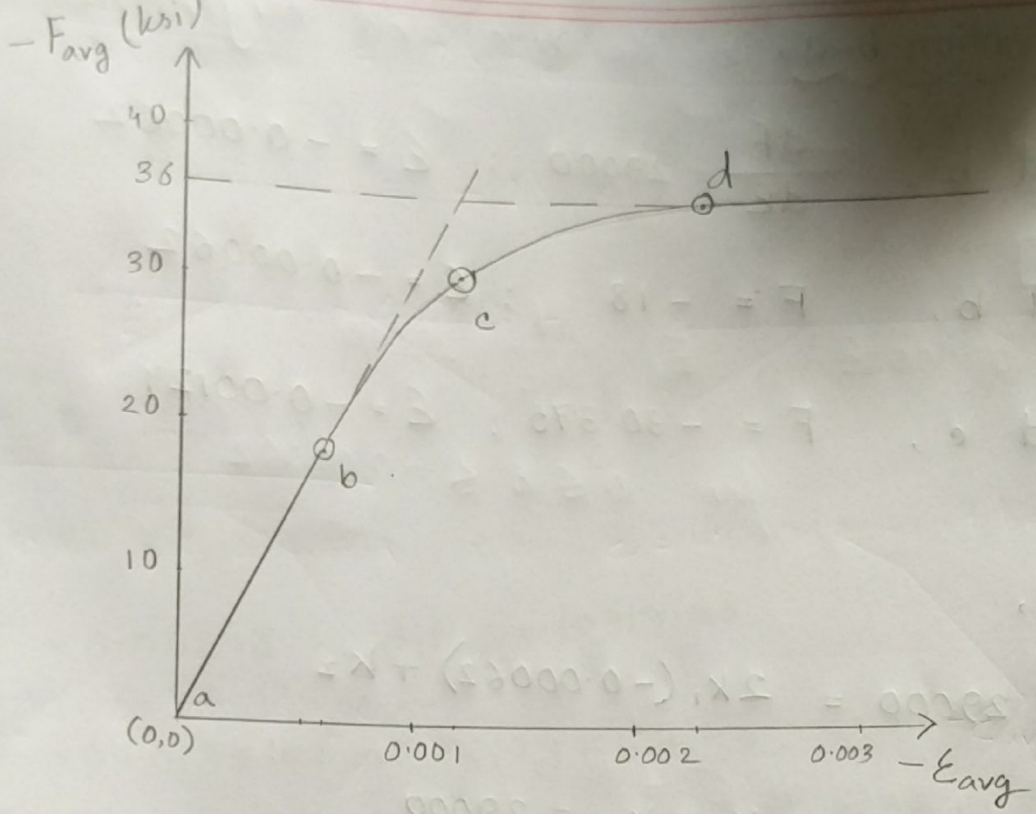




$F_{avg} = 36$ ksi

$\varepsilon_{avg} = -0.00227$



Upto point b, strain-strain relationship is linear

After point d, the curve is flat.

Portion b-c and c-d are in inelastic limit.

Now,

$$F = k_1 \varepsilon^2 + k_2 \varepsilon + k_3$$

$$\frac{dF}{d\varepsilon} = E_t = 2k_1 \varepsilon + k_2$$

At portion b-c:

i) at b, $\frac{dF}{d\varepsilon} = 29000$; $\varepsilon = -0.00062$

ii) at b, $F = -18$; $\varepsilon = -0.00062$

iii) at c, $F = -30.375$; $\varepsilon = -0.00124$

Now,

i) $29000 = 2k_1(-0.00062) + k_2$

$\therefore -0.00124\,k_1 + k_2 = 29000$

ii) $(-0.00062)^2 k_1 + (-0.00062) k_2 + k_3 = -18$

iii) $(-0.00124)^2 k_1 - 0.00124\,k_2 + k_3 = -30.375$

Solving these three eq^{n}s,

$k_1 = 14581165.45$; $k_2 = +47080.65$; $k_3 = 5.585$

at c, $\frac{dF}{d\varepsilon} = 2(14581165.45)(-0.00124) + 47080.65$

$= 10919.36$ ksi

At Portion cd :

i) at c, $\frac{dF}{d\varepsilon} = 10919.36$; $\varepsilon = -0.00124$

ii) at c, $F = -30.375$; $\varepsilon = -.00124$

iii) at d, $F = -36$; $\varepsilon = -0.00227$

Now,

i) $-0.00248\, k_1 + k_2 = 10919.36$

ii) $+0.00154\, k_1 - 0.00124\, k_2 + k_3 = -30.375$

iii) $+0.00515\, k_1 - 0.00227\, k_2 + k_3 = -36$

solving these three eq^{n}s,

$k_1 = 5299218.4$; $k_2 = 24061.4$; $k_3 = 8.67$

So, at point d, $\frac{dF}{d\varepsilon} = 2(5299218.4)(-0.00227) + 24061.4$

$= 2.95$ very small compared to $E = 29000$ ksi

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The stress strain relation and tangent modulus eq^{n}s are —

$\boxed{a-b}$

$F = 29000\,\varepsilon$; $E_t = \frac{dF}{d\varepsilon} = 29000$

for $0 \le \varepsilon \le -0.00062$

$\boxed{b-c}$

$F = 14581165.45\,\varepsilon^2 + 47080.65\,\varepsilon + 5.585$

$E_t = \frac{dF}{d\varepsilon} = 29162330.9\,\varepsilon + 47080.65$

for $-0.00062 < \varepsilon \le -0.00124$

$\boxed{c-d}$

$F = 5299218.4\,\varepsilon^2 + 24061.4\,\varepsilon + 8.67$

$E_t = \frac{dF}{d\varepsilon} = 10598436.8\,\varepsilon + 24061.4$

for $-0.00124 < \varepsilon \le -0.00227$

$\boxed{\text{after } d}$

$F = -36$; $E_t = \frac{dF}{d\varepsilon} = 0$

for $\varepsilon \geqslant -0.00227$