Lecture - 17

Flexural Member - 1

1/

• check: shape factor of a rectangular cross-section having width b and depth h.

Shape factor, $\xi = \frac{M_p}{M_y} = \frac{Z}{S}$

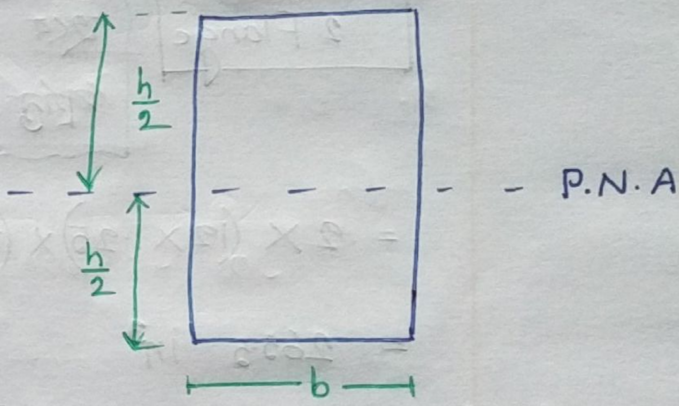
M_p = Plastic Moment, M_y = Yielding Moment

Z = plastic Section Modulus, S = Section Modulus

$$S = \frac{I}{c}$$

$$= \frac{bh^3/12}{\frac{h}{2}}$$

$$= \frac{bh^2}{6}$$



$$Z = \int y\,dA = y\int dA = b\int_{-\frac{h}{2}}^{\frac{h}{2}} y\,dy = 2b\int_0^{\frac{h}{2}} y\,dy$$

$$= 2b\left[\frac{y^2}{2}\right]_0^{\frac{h}{2}} = \frac{bh^2}{4}$$

For Any Rectangular Section, $Z = \frac{bh^2}{4}$

$$\xi = \frac{Z}{S} = \frac{bh^2}{4} \times \frac{6}{bh^2} = 1.5$$

(Ans)

2/ • Compute section factor for W-Section:

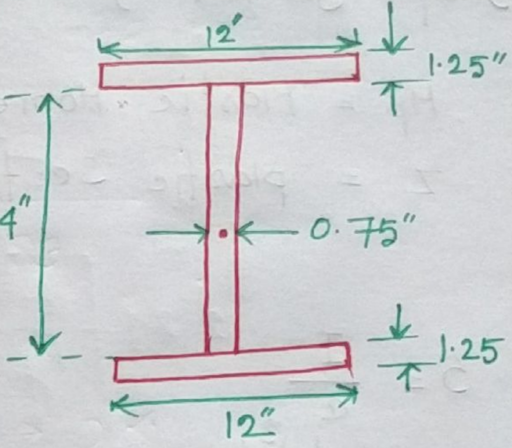
$Z = \int y \, dA$

$= \Sigma A\bar{y}$

সম্পূর্ণ Section এর c.g. থেকে ঐ Section consider করা হবে তার c.g. পর্যন্ত Distance

$\frac{bh^2}{4}$

$\boxed{= \text{Web} + 2\ \text{Flange}}$



$= 2 \times (12 \times 1.25) \times \left(7 + \frac{1.25}{2}\right) + \frac{0.75 \times 14^2}{4}$

$= 265.5 \ in^3$

$I = \frac{bh^3}{12} + \Sigma A d^2$

$\boxed{= \text{Web} + 2\text{Flange}}$

$= \frac{0.75 \times 14^3}{12} + 2\left\{12 \times \frac{1.25^3}{12} + (12 \times 1.25) \times \left(7 + \frac{1.25}{2}\right)^2\right\}$

$= 1919.6 \ in^4$

$c = \frac{h}{2} = \left(7 + \frac{1.25}{2}\right) = 8.25$

$S = \frac{I}{c} = \frac{1919.6}{8.25} = 232.7$

$\xi = \frac{M_P}{M_Y} = \frac{Z}{S} = \frac{265.5}{232.7} = 1.14$

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Flexural Member - 2

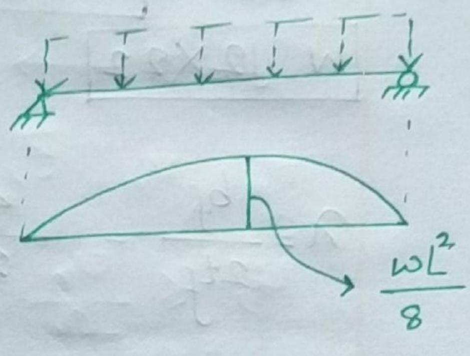
- Select a lighest W - Section:

Dead load = 0.2 kip/ft

Live load = 0.8 kip/ft

span = 20ft

A36 Steel



- কিছু বলা না থাকলে আমরা Compacted Section ব্যবহার করব।

$$M_u = \frac{wL^2}{8} = \frac{1 \times 20^2}{8} = 50 \text{ kip-ft}$$

$$M_u \leq \frac{M_n}{\Omega} \; ; \; M_n = \Omega \times M_u = 1.67 \times 50 = 83.5 \text{ kip ft}$$

For Compacted Section:

$$M_n = M_p = Z_x F_y$$

$$\Rightarrow Z_x = \frac{M_n}{F_y} = \frac{83.5 \times 12}{36} = 27.83 \text{ in}^3$$

choose section $Z_x > 27.83$.

$\boxed{W\ 12 \times 22} \longrightarrow Z_x = 29.30 \text{ in}^3 > 27.83 \text{ in}^3$

$$M_n = Z_x F_y = 29.30 \times 36 = 87.9 \text{ kip-ft}$$

$$M_u \leq \frac{M_n}{\Omega} \; ; \; M_u = \frac{M_n}{\Omega} = \frac{87.9}{1.67} = 52.63 \text{ k-ft}$$

$$M_u = \frac{wL^2}{8} = \frac{(0.2+0.8+0.022) \times 20^2}{8} = 51.1 \text{ k-ft}$$

Scanned by CamScanner

So, $\boxed{M_u < \frac{M_n}{\Omega}} \longrightarrow$ OK

$\boxed{W\ 12 \times 22}$ $\longrightarrow b_f = 4.03$; $t_f = 0.425$

$$\lambda = \frac{b_f}{2t_f} = \frac{4.03}{2 \times 0.425} = 4.74$$

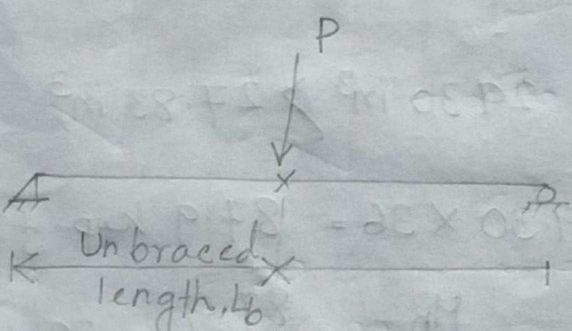
$$\lambda_{pf} = \frac{65}{\sqrt{F_y}} = \frac{65}{\sqrt{36}} = 10.83$$

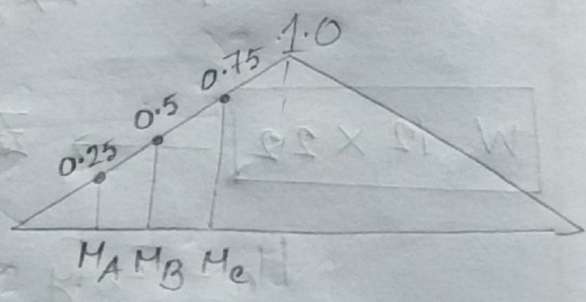
$\boxed{\lambda < \lambda_{pf}} \longrightarrow$ OK

So, Section $\boxed{12 \times 22} \longrightarrow$ Justified.

2/ • Determination of Gradient Factor, C_b.

$$C_b = \frac{12.5 M_{max}}{2.5 M_{Max} + 3M_A + 4M_B + 3M_c} R_m \leq 3.0$$





$$C_b = \frac{12.5 \times 1}{2.5 \times 1 + 3 \times 0.25 + 4 \times 0.5 + 3 \times 0.75} \times 1 = 1.67$$

Flexural Member - 3

- Determine the design bending strength or Moment ϕM_n W14×74, A572 50 grade steel.

1. Continuous lateral Suppor, $L_b = 0$.

প্রথমে check করতে হবে Compacted or Not.

For Flange:

$$\frac{b_f}{2t_f} < \frac{65}{\sqrt{F_y}} \longrightarrow \text{Flange Compacted}$$

For web:

$$\frac{h}{t_w} < 3.76\frac{\sqrt{E}}{\sqrt{F_y}} \longrightarrow \text{web Compacted}$$

Now; W14×74 ⟶ $b_f = 10.10$

$t_f = 0.785$

$h = (d - 2t_f) = 12.63 \rightarrow ??$

$t_w = 0.450$

$$\frac{b_f}{2t_f} = \frac{10.1}{2\times .785} = 6.40$$

$$\frac{65}{\sqrt{F_y}} = \frac{65}{\sqrt{50}} = 19.20$$

$$\boxed{\frac{b_f}{2t_f} < \frac{65}{F_y}} \rightarrow \text{compacted Flange}$$

$$\frac{h}{t_w} = 25.4 \; ; \quad 3.76\frac{\sqrt{E}}{\sqrt{F_y}} = 3.76 \times \sqrt{\frac{29000}{50}} = 90.56$$

$$\boxed{\frac{h}{t_w} < 3.76\sqrt{\frac{E}{F_y}}} \longrightarrow \text{Compacted web.}$$

so, Section is compacted.

$L_b = 0 \longrightarrow$ Cont. laterally support.

$M_p = M_n = Z_x F_y = 126 \times 50 = 525$ kip-ft

$M_u \leq \frac{M_n}{\Omega}$ or, $\boxed{M_u \leq \phi M_n}$

$\phi M_n = 0.9 \times 525 = 473$ kip-ft.

2. $L_b = 15$ ft ; $L_b = 180$ inch

কিছু বলা না থাকলে C_b always 1.0.

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76 \times 2.48 \times \sqrt{\frac{29000}{50}} = 105.12'' = 8.76'$$

$$L_p = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o}} \times \sqrt{1 + \sqrt{1 + 6.76 \left(\frac{0.7 F_y S_x h_o}{EJc}\right)^2}}$$

Here; $F_y = 50$ ksi ; $S_x = 112$; $h_o = 13.4$

$E = 29000$, $J = 3.87$, $c = 1$ (assume)

so; 1.605

Here, $r_{ts} = 2.82$

so; $231.37''$

$L_r = 231.37 \times 1.605 = 370.19$ in
$= 30.85$ ft

$\boxed{L_p < L_b < L_r}$ ⟶ Objective

$$M_n = C_b \times \left[M_p - \{M_p - (0.7 S_x F_y)\} \frac{L_b - L_p}{L_r - L_p} \right]$$

$$= 1.0 \times \left[525 - (525 - 326.67) \frac{6.24}{22.09} \right]$$

$= 469$ kip-ft

$M_u \leq \phi M_n$; $\phi M_n = 0.9 \times 469 = 422.10$ k-ft

1. $L_b = 0$; $\phi M_n = 473$ kip-ft
2. $L_b = 15'$; $\phi M_n = 422.10$ kip-ft.

(Ans.)

2

• Determine the Design Moment W10X12

(1) $L_b = 0$;

$$\frac{b_f}{2t_f} < \frac{65}{\sqrt{F_y}} \quad ; \quad \frac{h}{t_w} < 3.76\sqrt{\frac{E}{F_y}}$$

$\boxed{W10X12} \longrightarrow$ $d = 9.87$ in , $S_x = 10.9$

$t_w = 0.190$ in $r_y = 0.785$

$b_f = 3.96$ in $r_{ts} - 0.983$

$t_f = 0.21$ in $J = 0.06$

$h_o = 9.66$ in

$$\frac{b_f}{2t_f} = \frac{3.96}{2 \times 0.21} = 9.43 \; ; \quad \frac{b5}{\sqrt{F_y}} = 9.20$$

$$\frac{b_f}{2t_f} > \frac{65}{\sqrt{F_y}} \longrightarrow \text{Non-Compacted Section.}$$

$$\lambda = \frac{b_f}{2t_f} = 9.43$$

$$\lambda_{pf} = 9.20$$

$$\lambda_{rf} = 1\sqrt{\frac{E}{F_y}} = 24.10$$

$$\boxed{\lambda_{pf} < \lambda < \lambda_{rf}} \rightarrow \boxed{\text{Non-Compact}}$$

$$M_n = \left[M_p - (M_p - 0.7 S_x F_y) \frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}} \right]$$

$$M_p = Z_x S_y = 12.6 \times 50 = 52.5 \text{ kip-ft.}$$

$$M_n = 52.5 - (52.5 - 31.80) \frac{0.23}{14.9}$$

$$= 52.20 \text{ kip-ft}$$

$$\phi M_n = 0.9 \times 52.50 = \boxed{47 \text{ kip-ft}}$$ (Ans)

(2) $L_b = 10 \text{ ft} = 120 \text{ inch}$

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76 \times 0.785 \times \sqrt{\frac{29000}{50}} = 33.3''$$

$$= 2.77 \text{ ft}$$

$$L_r = 1.95 \times 0.983 \times \frac{29000}{0.7 F_y} \sqrt{\frac{0.06 \times 1}{10.9 \times 9.66}} \times$$

$$\sqrt{1 + \sqrt{1 + 6.76 \times \left(\frac{0.7 \times 50 \times 10.9 \times 9.66}{29000 \times 0.06 \times 1}\right)^2}}$$

$$= 97.44 \text{ inch} = 8.12 \text{ ft}$$

3

$L_b > L_r \longrightarrow$ Slender Region

$$M_n = F_{cr} \times S_x$$

So, $$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2}$$

$$= \frac{1 \times \pi^2 \times 29000}{\left(\frac{12 \times 10}{0.983}\right)^2} \sqrt{1 + 0.078 \times \frac{0.06 \times 1}{10.9 \times 9.66} \times \left(\frac{120}{.983}\right)^2}$$

$$= 24.76 \text{ ksi}$$

$$M_n = F_{cr} \times S_x = 24.76 \times 10.9$$

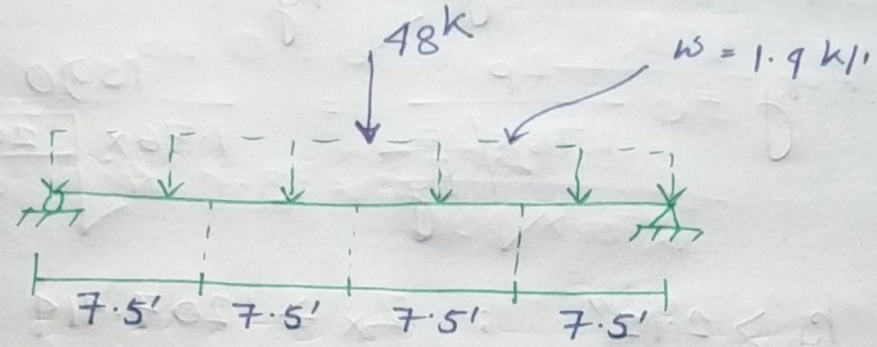
$$= 22.50 \text{ kip-ft}$$

$$\phi M_n = 0.9 \times 22.50 = \boxed{20.25} \text{ kip-ft}$$

(Ans)

3/

• Design lighest W section:



ASD Method

$$M_u \le \frac{M_n}{\Omega}$$

$$M_u = \frac{wL^2}{8} + \frac{PL}{4}$$

$\frac{PL}{4}$ → point load এর জন্য

$\frac{wL^2}{8}$ → UDL এর জন্য

$$= \frac{1.4 \times 30^2}{8} + \frac{48 \times 30}{4} = 517.5 \text{ kip-ft}$$

$$M_n = \Omega \times M_u = 1.67 \times 517.5 = 864.225 \text{ kip-ft}$$

* Consider case - 2 → in compacted Section

$$M_n = Z_x f_y \Rightarrow Z_x = \frac{M_n}{f_y} = 207.414 \text{ in}^3.$$

Here, $L_b = \frac{15'}{2} = \frac{12 \times 15}{2} = \frac{180}{2}$ in $= 90$ inch

$$L_p = 1.76 \times r_y \times \sqrt{\frac{E}{F_y}}$$

$$\therefore r_y = \frac{L_p}{1.76 \times \sqrt{\frac{E}{F_y}}} = \frac{1590}{1.76 \times \sqrt{\frac{29000}{50}}} = 2.12''$$

$r_y > 2.12''$; $Z_x \geq 207.419$

choose $\longrightarrow$ W 18×106

$Z_x = 230 \ in^3$; $r_y = 2.66 \ in$

$M_n = Z_x F_y = 230 \times 50 = 958$ kip-ft

$M_u = \frac{M_n}{\Omega} = 573.9$ kip-ft

$$M_u = \frac{wL^2}{8} + \frac{PL}{4}$$

$$= \frac{1}{8} \times 30^2 \times (1.4 + 0.106) + \frac{48 \times 30}{4}$$

$= 529.43$ kip-ft

$$\boxed{M_u < \frac{M_n}{\Omega}} \longrightarrow \text{ok.}$$

W 18×106

Flexural Member - 4

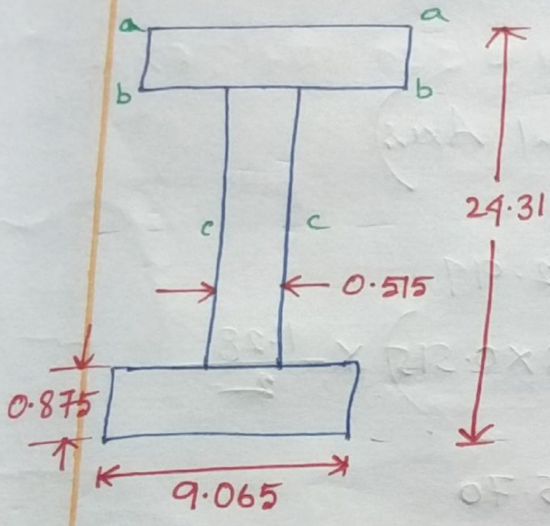
$$v = \frac{VQ}{Ib}$$

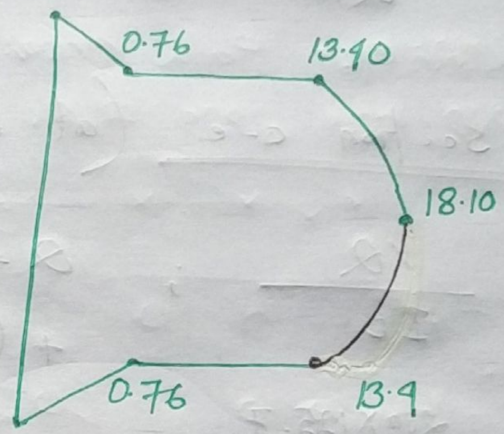
$V \rightarrow$ Shear Force

$Q \rightarrow$ Moment Area (upper Region of Section)

$b \rightarrow$ width of that section

$I \rightarrow$ Inertia





$I = 2700 \text{ in}^4$

At Section a-a :

$$v = \frac{VQ}{Ib} = 0$$

$Q = 0;$

At Section b-b (flange)

$$v = \frac{VQ}{Ib}$$

, $V = 200^k$

$$Q = (9.065 \times 0.875) \times \left(\frac{24.31}{2} - \frac{0.875}{2}\right)$$

$$= 92.94$$

$$I = 2700 \text{ in}^4$$

$$b = 9.065$$

$$= 0.76 \text{ Ksi}$$

At Section b-b (w.b)

$$v = \frac{VQ}{Ib} \quad ; \quad b = 0.515$$

$$= \frac{200 \times 92.94}{2700 \times 0.515} = 13.4 \text{ ksi}$$

At Section c-c (at neutral Axis)

$$v = \frac{VQ}{Ib} \quad ; \quad Q = 92.94 + (11.28 \times 0.515) \times \frac{11.28}{2}$$

$$= \frac{200 \times 125.7}{2700 \times 0.515} \qquad Q = 125.70$$

$$= 18.10 \text{ ksi}$$

Shear carried by flange:

$$\Rightarrow 2\left(\frac{1}{2} \times 0.76 \times 0.875 \times 9.065\right) = 6.03^{k}$$

Shear carried by web:

$$\Rightarrow 200 - 6.03 = 193.97^{k}$$

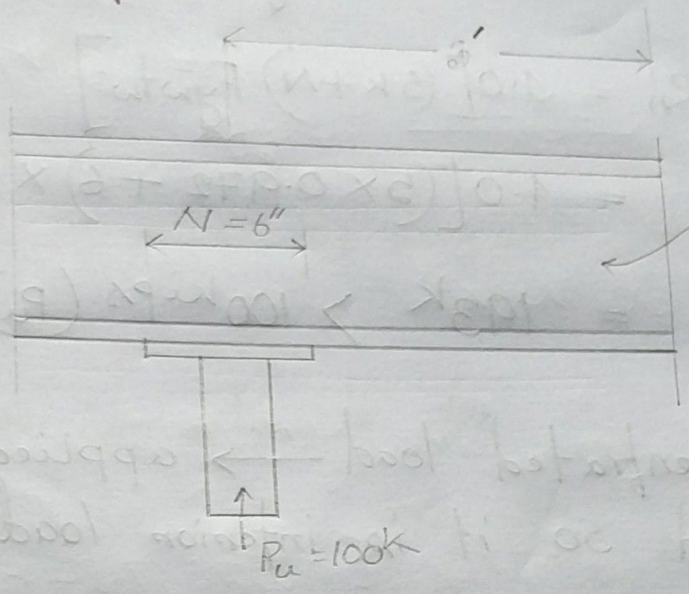
$$v_{avg} = \frac{V}{A_W} = \frac{200}{t_w d} = \frac{200}{24.31 \times .515} = 16 \text{ ksi}$$

(Ans)

Flexural Member - 5

Example :

check Both web yielding and web crippling ASTM A572, grade 50.



For section W 18X50 : (From ASTM)

$K = 0.972$ inch

$t_w = 0.355$ inch

$t_f = 0.57$ inch

$d = 18$ inch

① Web yielding check:

$LRFD \rightarrow \phi = 1.0$

$R_n = (5k + N) F_{yw} t_w$

so, $\phi R_n = 1.0[(5k+N) F_{yw} t_w]$

$= 1.0[(5 \times 0.972 + 6) \times 50 \times 0.355]$

$= 193^k > 100^{kips} (P_u)$

⊛ Concentrated load ⟶ applied not at the end so it is interior load.

② Web - Crippling check:

$LRFD \longrightarrow \phi = 0.75$

⊛ Concentrated load applied at a distance

$x = 3''$;

$d = 18''$; $d/2 = 9''$

so, $x > d/2$

so, $R_n = 0.80 t_w^2 \left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right] \sqrt{\frac{E F_{yw} t_f}{t_w}}$

So, $R_n = 0.80 \times 0.355^2 \times \left[1 + 3 \times \left(\frac{6}{18}\right) \times \left(\frac{0.355}{0.570}\right)^{1.5}\right] \times \sqrt{\frac{29{,}000 \times 50 \times 0.57}{0.355}}$

$= 229.50 \text{ kips}$

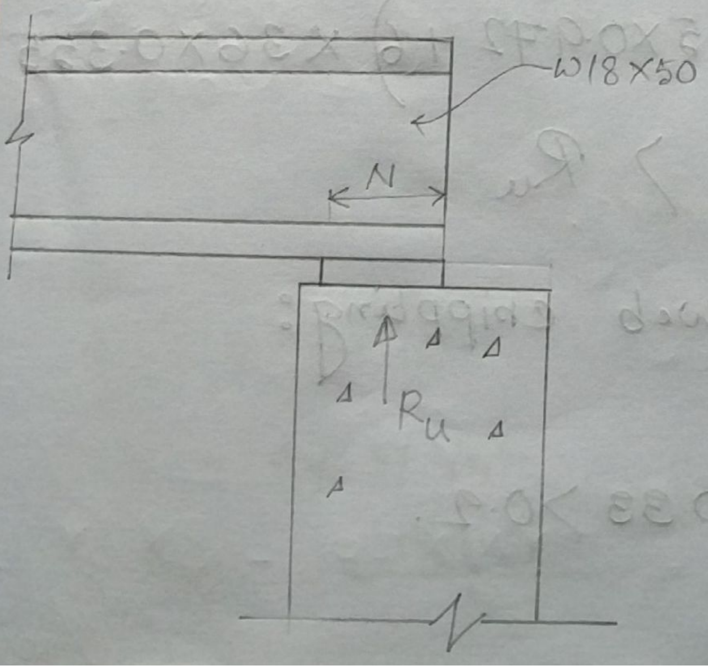
$\phi R_n = 0.75 \times 229.50 = 172^k > 100^k (P_u)$

So, Beam is adequate for
web yielding
web crippling.

(Ans)

• Example:

• Section Properties → আগেরটাই।



Beam Span = 20′

A36 Steel

D = 1.5 k/′

L = 2 k/′

$f'_c = 4000 psi$

Reaction At Support on at the end of the member,

$R_u = \frac{w_u L}{2}$; $w_u = 1.2D + 1.6L$

$= 1.2 \times 1.5 + 1.6 \times 2$

$= 5\ k/'$

$= \frac{5 \times 20 \times 1}{2} = 50^k$

$R_u = 50$ kips

* check for web yielding:

for practical Reason, $N = 6$.

Reaction at end.

$\phi = 1.0$

so, $\phi R_n = 1 \times (2.5k + N) F_{yw} t_w$

$= 1 \times (2.5 \times 0.972 + 6) \times 36 \times 0.355$

$= 107^k > R_u$

* check for web crippling:

$\frac{N}{d} = \frac{6}{18} = 0.33 > 0.2.$

$\phi = 0.75$

$$R_n = 0.40\, t_w^2 \left[1 + \left(\frac{4N}{d} - 0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right] \sqrt{\frac{E F_{yw} t_f}{t_w}}$$

$$= 0.40 \times 0.355^2 \left[1 + \left(\frac{4\times 6}{18} - 0.2\right)\left(\frac{0.355}{0.57}\right)^{1.5}\right] \times \left(\frac{29000 \times 36 \times 0.57}{0.355}\right)^{\frac{1}{2}}$$

$$= 101.6^{k}$$

$$\phi R_n = 0.75 \times 101.6 = 76^{kip} > R_u$$

Beam is adequate.

It is assumed that bearing plate at the end of support. so, $\sqrt{\frac{A_2}{A_1}} = 1$.

$$R_u = \phi R_n = 0.85 f_c' A_1 \sqrt{\frac{A_2}{A_1}} \le 1.7 f_c' A_1$$

$$\text{so, } A_1 = \frac{R_u}{0.85 \times f_c' \times \phi} = \frac{50,000}{0.85 \times 4000 \times 0.6} = 24.51 \text{ in}^2$$

$$A_1 = 24.51 \text{ in}^2$$

$$\text{or, } B \times N = 24.51 \text{ in}^2$$

$$B_{min} = \frac{24.51}{6} = 4.10 \text{ inch}$$

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- B should be equal or greater than width of flange b_f.

$b_f = 7.495$ in

so, $B = 8$ inch

$B \times N = 8 \times 6$

Now,

$2(k_1 + l) = B$

$\Rightarrow l = \frac{B}{2} - k_1 = \frac{8}{2} - \frac{13}{16}$

$= 3.20$ inch

and

Thickness, $t_p \geq \sqrt{\frac{2R_u l^2}{0.9BNF_y}} = \left(\frac{2 \times 50 \times 3.2^2}{0.9 \times 48 \times 36}\right)^{\frac{1}{2}}$

$= 0.81$ in

0.25" upper Rounding.

$t_p = 1$ inch

Bearing Plate $= B \times N \times t_p = 8 \times 6 \times 1. = 48 \text{ in}^3$

Ans.

Example :

For Section W 14X30 ;

$t_f = 0.385'$

$\frac{h}{t_w} = 45.4$

$t_w = 0.27''$

$b_f = 6.73$

Here, $\left(\frac{h}{t_w}\right) / \left(\frac{L_b}{b_f}\right)$

$$= \frac{45.40}{(20 \times 12)/6.73} = 1.273 < 2.30$$

$\phi = 0.85$

$$R_n = \frac{C_r t_w^3 t_f}{h^2}\left[1 + 0.4\left(\frac{h/t_w}{L_b/L_f}\right)^3\right]$$

So, $$\phi R_n = 0.85 \times \frac{480000 \times 0.27^3 \times 0.385}{12.26^2}\left[1 + 0.40 \times (1.273)^3\right]$$

$$= 37.5^K$$

(Ans)

Flexural Member - 6

1 **Example:**

Dead load $= 0.5$ kip/ft

Live load $= 1$ kip/ft

span, $L = 42$ ft

$\Delta_{max} < \frac{L}{360}$

$F_y = 50$

⊛ Uniform distributed load so, Moment $= \frac{wL^2}{8}$

ASD Method

choose self wt of the section $= 100$ lb/ft $= 0.1$ k/ft

$w = 1 + 0.5 + 0.1 = 1.6$ kip/ft

$$M_u = \frac{wL^2}{8} = \frac{1.6 \times 42^2}{8} = 352.8 \text{ kip-ft}$$

$$M_u = \frac{M_n}{\Omega}$$

so, $M_n = Z_x F_y = \Omega M_u = 1.67 \times 352.8$

$$Z_x = \frac{1.67 \times 352.8}{50} = 11.8 \text{ ft} \cdot \text{in}^2 = 141.6 \text{ in}^3$$

$$\boxed{Z_x = 141.6 \text{ in}^3}$$

Here, $\frac{L}{360} = \frac{42 \times 12}{360} = 1.4$ inch

$\Delta_{max} = 1.40 \text{ inch} = \frac{5}{384} \times \frac{wL^4}{EI} \longrightarrow \boxed{LL \text{ consider}}$

$$\text{so, } I = \frac{5 \times wL^4}{384 \times E \times 1.4}$$

$$= \frac{5 \times 1.\text{[illegible]} \times (42 \times 12)^4}{12 \times 384 \times 29000 \times 14}$$

$$= 1724.50 \text{ in}^4$$

$$\boxed{I = 1724.50}$$

For this I and $Z_x \longrightarrow$ Section choose.

$$\boxed{W\ 21 \times 93}$$

self wt = 93 lb/ft < assumed 100 lb/ft.

(*) Section (এর জন্যে)

(a) check $\frac{d}{t_w} \le 2.24 \sqrt{\frac{E}{F_y}}$

$\boxed{\text{Compacted}}$

(b) shear check,

$$V_n = (0.6F_y)(t_w d)$$

Scanned by CamScanner

So,

From properties;

$d = 21.6$ in

$t_w = 0.580$ inch

$$\frac{d}{t_w} = \frac{21.6}{0.580} = 37.24$$

$$2.24\sqrt{\frac{E}{F_y}} = 2.24 \times \sqrt{\frac{29000}{50}} = 53.94$$

$$\frac{d}{t_w} < 2.24\sqrt{\frac{E}{F_y}} \longrightarrow OK$$

$$V_n = (0.6F_y)(d\,t_w) = 0.6 \times 50 \times 21.6 \times .58 = 375.84$$

$$V_u = \frac{V_n}{\Omega} = \frac{375.84}{1.5} = 250.56 \text{ kip}$$

$$V_u = \frac{wL}{2} = \frac{(1+0.5+0.93)}{12 \times 2} \times 42 \times 12$$

$$= 33.50 \text{ kip} < 250.56^{k} \rightarrow OK$$

So, Section $\boxed{W\ 21 \times 93}$ — OK