Tension Members

③ controlling limit states :

1) Yielding of the gross cross section away from connection
2) Fracture of the effective net area at connection
3) Block shear failure through bolt holes.

1) Yielding of gross cross section

$$T_n = F_y A_g$$

A_g = gross area

2) Fracture through bolt holes

$$T_n = F_u A_e$$

$A_e = U A_n$ = effective net area

A_n = net area

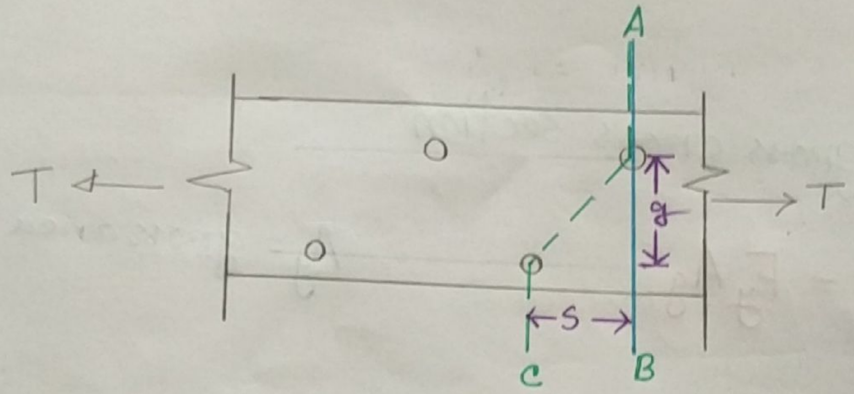
U = Reduction coefficient / shear lag factor

$$\text{Effective hole diameter} = \left(\text{Bolt diameter} + \frac{1}{16''}\right) + \frac{1}{16''}$$

$$= \text{Actual hole diameter} + \frac{1}{16''}$$

$$A_n = A_g - (\text{eff. hole dia})(\text{thickness of plate})$$

Staggered holes



$$\text{Net length of } A\text{-}B = \text{length of } (A\text{-}B) - \left(\text{width of hole} + \frac{1''}{16}\right)$$

$$\text{Net length of } A\text{-}C = \text{length of } (A\text{-}B) - \left(\text{width of hole} + \frac{1''}{16}\right) + \frac{S^2}{4g}$$

$$\frac{S^2}{4g} = \text{length correction}$$

s = spacing of bolt in the dir^n of load

g = " " " " " perpendicular dir^n of load

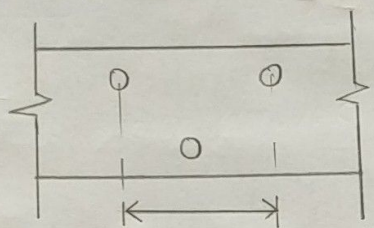
Effective net area

$$A_e = U A_n$$

$$U = 1.0 - \frac{\bar{x}}{L}$$

$\bar{x}$ = distance bet^n loading line and connection plane

L = length of connection



connection length = l

For plates, $U = 1$

LRFD design

$$\phi_t T_n \geq T_u$$

$$\left\{ \begin{array}{l} \phi_t = 0.90 \text{ (yield)} \\ \phi_t = 0.75 \text{ (fracture)} \end{array} \right\}$$

ϕ_t = resistance factor

Yielding on gross section

$$\phi_t T_n = \phi_t F_y A_g = 0.9 F_y A_g$$

Fracture on eff. net section

$$\phi_t T_n = \phi_t F_u A_e = 0.75 F_u A_e$$

ASD design

Yielding on gross section

Nominal strength : $T_n = F_y A_g$

Allowable strength : $\frac{T_n}{\Omega} = \frac{F_y A_g}{\Omega} = \frac{F_y A_g}{1.67}$

Ω = safety factor

Fracture on effective net section

Nominal strength : $T_n = F_u A_e$

Allowable strength : $\frac{T_n}{\Omega} = \frac{F_u A_e}{\Omega} = \frac{F_u A_e}{2.00}$

(3) Block Shear Failure

two types of failure plane

1) Shear failure plane (parallel to dirⁿ of load)
2) Tension failure plane (perpendicular to dirⁿ of load)

Two block shear failure modes:

1) Rupture along tensile plane + Yielding along shear planes
2) Rupture along tensile plane + Rupture along shear planes

* Rupture along $\boxed{\text{net shear area}}$
* Yield along $\boxed{\text{gross shear area}}$

* Shear yield stress, $\tau_y = 0.6 F_y$
* Shear strength / rupture, $\tau_u = 0.6 F_u$

i) Shear yielding – tension rupture

$$T_n = 0.6 F_y A_{gv} + F_u U_{bs} A_{nt}$$

ii) Shear fracture – tension rupture

$$T_n = 0.6 F_u A_{nv} + F_u U_{bs} A_{nt}$$

A_{gv} = gross area acted upon by shear

A_n = net area acted upon by tension

A_{nv} = net area acted upon by shear

* For Uniform tension stress, $U_{bs} = 1$

* Non-uniform tension stress, $U_{bs} = 0.5$

LRFD design

Shear yielding – tension rupture

$$\phi_t T_n = \phi_t \left(0.6\, F_y A_{gv} + F_u U_{bs} A_{nt}\right)$$

$$= 0.75 \left(0.6\, F_y A_{gv} + F_u U_{bs} A_{nt}\right)$$

Shear fracture – tension rupture

$$\phi_t T_n = \phi_t \left(0.6\, F_u A_{nv} + F_u U_{bs} A_{nt}\right)$$

$$= 0.75 \left(0.6\, F_u A_{nv} + F_u U_{bs} A_{nt}\right)$$

ASD design

Shear yielding – tension rupture

$$\frac{T_n}{\Omega} = \frac{0.6\, F_y A_{gv} + F_u U_{bs} A_{nt}}{\Omega} = \frac{\left(0.6\, F_y A_{gv} + F_u U_{bs} A_{nt}\right)}{2.00}$$

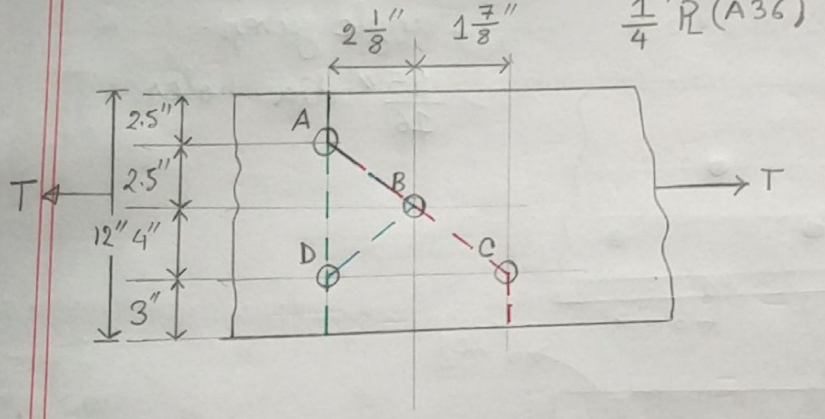
Shear fracture – tension rupture

$$\frac{T_n}{\Omega} = \frac{0.6\, F_u A_{nv} + F_u U_{bs} A_{nt}}{\Omega} = \frac{\left(0.6\, F_u A_{nv} + F_u U_{bs} A_{nt}\right)}{2.00}$$

Example 3.4.1

min^m net area of the plate = ?

$\frac{15}{16}$" dia holes



Path AD:

$$\text{Net Area} = \left\{12'' - 2\left(\frac{15}{16} + \frac{1}{16}\right)\right\} \times 0.25'' = 2.5 \text{ in}^2$$

Path ABD:

$$\text{Net area} = \left\{12'' - 3\left(\frac{15}{16} + \frac{1}{16}\right) + \frac{2.125^2}{4 \times 2.5} + \frac{2.125^2}{4 \times 4}\right\} \times 0.25''$$

$$= 2.43 \text{ in}^2$$

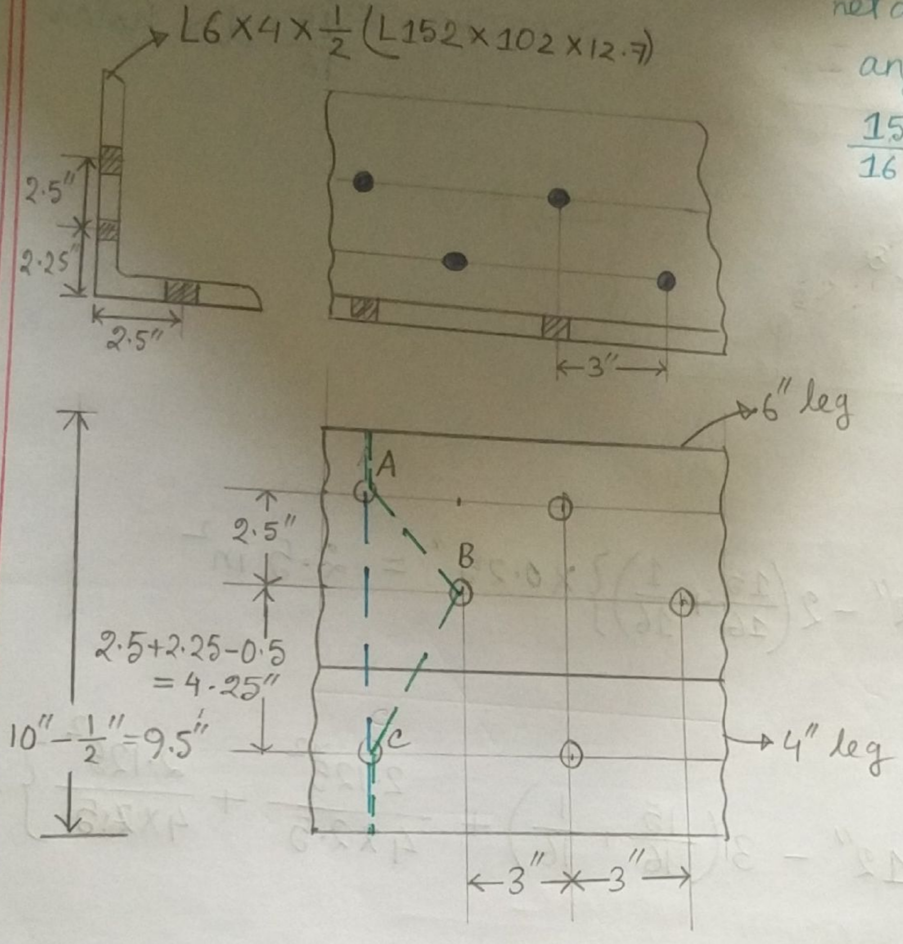
Path ABC:

$$\text{Net area} = \left\{12'' - 3\left(\frac{15}{16} + \frac{1}{16}\right) + \frac{2.125^2}{4 \times 2.5} + \frac{1.875^2}{4 \times 4}\right\} \times 0.25''$$

$$= 2.42 \text{ in}^2$$

$\therefore$ min^m net area of the plate is 2.42 in^2 (along path ABC)

Example 3.4.2



net area A_n for the angle given.
$\frac{15}{16}''$ dia. holes

Path AC:

$$\text{net area} = \left\{9.5 - 2\left(\frac{15}{16}+\frac{1}{16}\right)\right\} \times 0.5 = 3.75 \text{ in}^2$$

Path ABC:

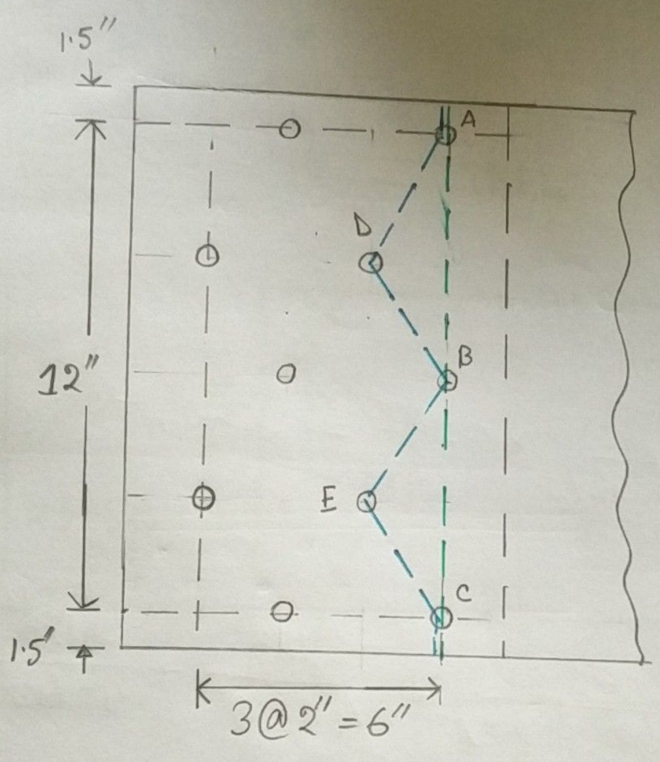
$$\text{net area} = \left\{9.5 - 3\left(\frac{15}{16}+\frac{1}{16}\right) + \frac{3^2}{4\times2.5} + \frac{3^2}{4\times4.25}\right\} \times 0.5$$

$$= 3.96 \text{ in}^2$$

$\therefore$ min^m net area 3.75 in^2 (along path AC)

Example 3.8.1

* Calculate the governing net area for plate A of the single lap joint



$P_L\ \frac{5}{8}$″ thick

$\frac{7}{8}$″ dia. connectors in standard holes

Path ABC:

$$\text{net area} = \left\{15 - 3\left(\frac{7}{8}+\frac{1}{8}\right)\right\}\frac{5}{8} = 7.5 \text{ in}^2$$

Path ADBEC:

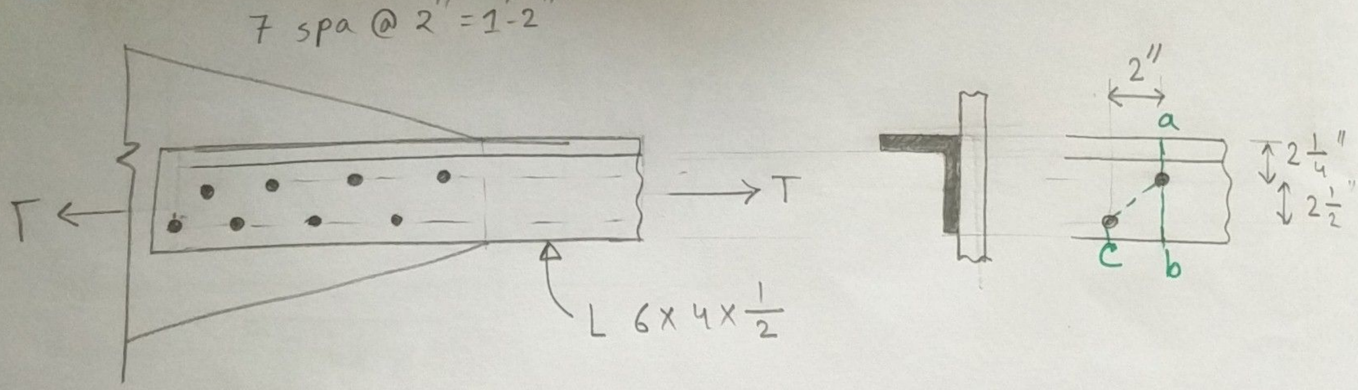
$$\text{net area} = \left\{15 - 5\left(\frac{7}{8}+\frac{1}{8}\right) + \frac{2^2}{4\times 3}\times 4\right\}\times\frac{5}{8} = 7.08 \text{ in}^2$$

Example 3.9.1

Determine the service load capacity in tension for an $L\,6 \times 4 \times \frac{1}{2}$ A572 grade 50 steel connected with $\frac{7}{8}$ in dia bolts in standard holes as shown.

Assume live load to be three times the dead load

Neglect block shear failure



$A_g = 4.75 \text{ in}^2$; $\bar{x} = 0.981''$

Along ab:

$$A_n = A_g - \left(\frac{7}{8} + \frac{1}{8}\right) \times \frac{1}{2} = 4.25 \text{ in}^2$$

$$\therefore A_e = U A_n = \left(1 - \frac{\bar{x}}{L}\right) A_n = \left(1 - \frac{0.981}{14}\right) \times 4.25$$

$$= 5.18 \text{ in}^2$$

along ac:

$$A_n = A_g - \frac{1}{2}\left\{2\left(\frac{7}{8}+\frac{1}{8}\right)\right\} + \left\{\frac{2^2}{4 \times 2.5}\right\} \times \frac{1}{2} = 3.95$$

$$A_e = UA_n = 0.9299 \times 3.95 = 3.67\ in^2 \text{ (governs)}$$

$$\boxed{A_e = 3.67\ in^2}$$

Yielding on gross area

$$T_n = F_y A_g$$

$$T_u = \frac{T_n}{\Omega} = \frac{50 \times 4.75}{1.67} = 142.22 \text{ kips}$$

Fracture on effective net area

$$T_u = \frac{T_n}{\Omega} = \frac{F_u A_e}{2.0} = \frac{65 \times 3.67}{2} = 119.4 \text{ kip}$$

(governs)

Now, $T_u = DL + LL$

or, $119 = DL + 3DL$

$$\therefore \boxed{\begin{aligned} DL &= 29.75 \text{ kip} \\ LL &= 89.25 \text{ kip} \end{aligned}}$$

Ans

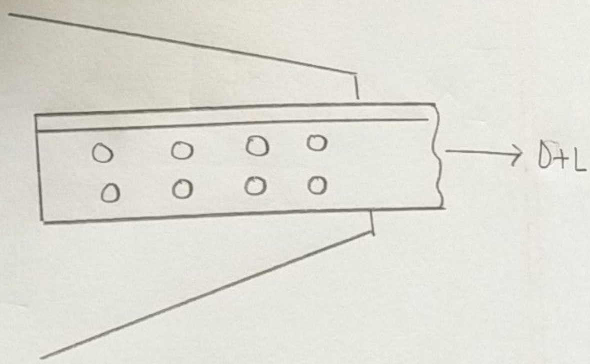
Problem-4 (Tension member design Problem)

Select an unequal-leg angle tension member 15 ft long to resist a service Dead load of 35 kips and a service live load of 70 kips. Use A36 steel ($F_y = 36$ ksi ; $F_u = 58$ ksi)

The connection is shown in figure bellow.

It shall be connected to a gusset plate using 8 nos. $\frac{3}{4}$ in dia bolts in two rows as shown.

[Neglect block shear failure mode and use LRFD principle]



Factored load,

$P_u = 1.2DL + 1.6DL$

$= 154$ kips

On gross area

$P_u \leq \phi P_n = F_y A_g \times 0.90$

$\therefore A_g \geqslant \dfrac{154}{0.90 \times 36}$

$\therefore \boxed{A_g \geqslant 4.75 \text{ in}^2}$

On net area

$P_u \leq \phi P_n = 0.75 F_u A_e$

$A_g \geqslant \dfrac{154}{0.75 \times 58}$

$\therefore \boxed{A_e \geqslant 3.54 \text{ in}^2}$

Now,

to find A_n we need U

Let us assume, $\boxed{U = 0.80}$

Now, $A_e = U A_n$

$$\therefore A_n = \frac{3.54}{0.80} = 4.425 \text{ in}^2$$

Now, $A_g = A_n + (\text{bolt holes} * \text{thickness}) = 4.425 + 2 \times \left(\frac{3}{4} + \frac{1}{8}\right) t$

Let us assume,

thickness of plate, $t = \frac{3}{4}''$

$$\therefore A_g = 4.425 + \frac{7}{4} \times \frac{3}{4} = 5.74 \text{ in}^2$$

$$\therefore \boxed{A_g = 5.74 \text{ in}^2} \text{ (governs)}$$

Now,

Choose a section from AISC manual with $A_g \geq 5.74 \text{ in}^2$ and $t \leq 3/4''$

Let us choose, $\boxed{L\, 6 \times 4 \times \frac{5}{8}}$

$A_g = 5.86\,in^2$; $t = 5/8$ in

$\therefore A_n = 5.86 - 2\left(\frac{3}{4} + \frac{1}{8}\right) \times \frac{5}{8} = 4.77\,in^2$

$\therefore A_e = UA_n = 0.8 \times 4.77 = 3.82\,in^2$

Yield on gross area:

$$\phi P_n = 0.9 \times 36 \times 5.86 = 189.86\,k > P_u = 154\,k$$

Fracture on net area:

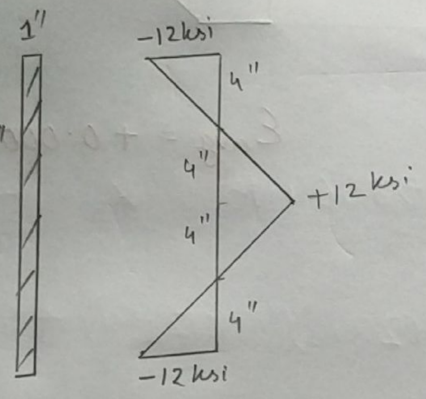
$$\phi P_n = 0.75 \times 58 \times 3.82 = 166.17\,k > P_u$$

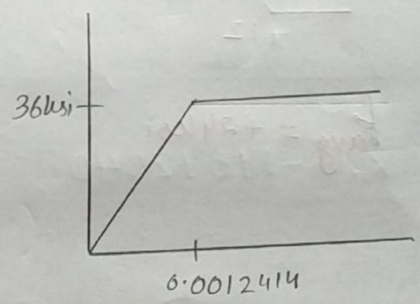
[This is ok]

Problem-6

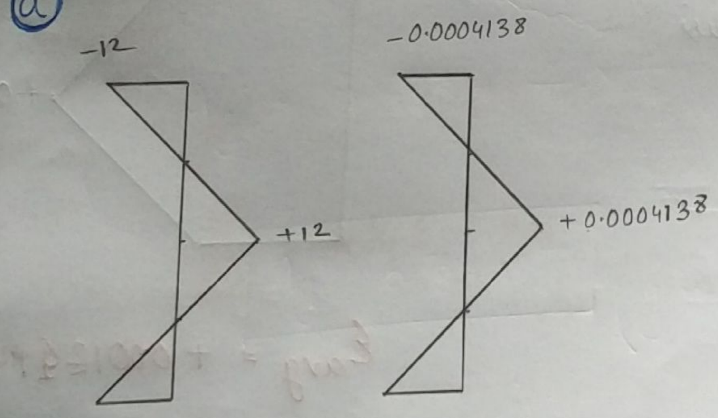
The residual stress for a $16'' \times 1''$ plate to be used as a tension member is shown in the figure.

Derive the eq^n for the stress-strain behavior in tension of the plate. $F_y = 36$ ksi, $E = 29000$ ksi; $\varepsilon = 0.0012414$



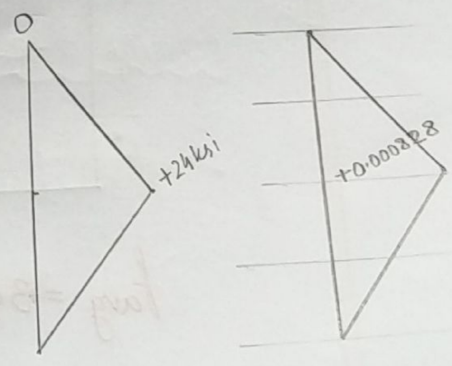


(a)



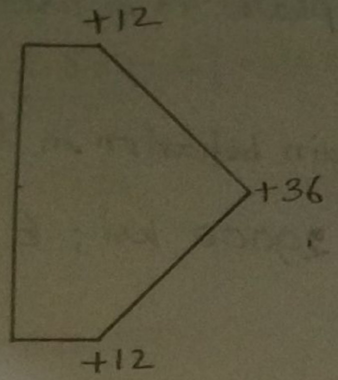
$f_{avg} = 0$ $\varepsilon_{avg} = 0$

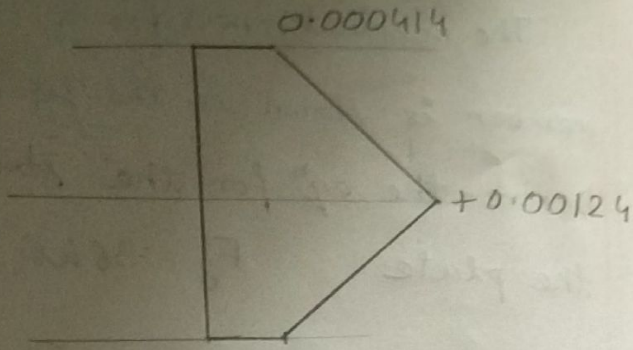
(b)



$f_{avg} = 12$ ksi $\varepsilon_{avg} = 0.000414$

(c)

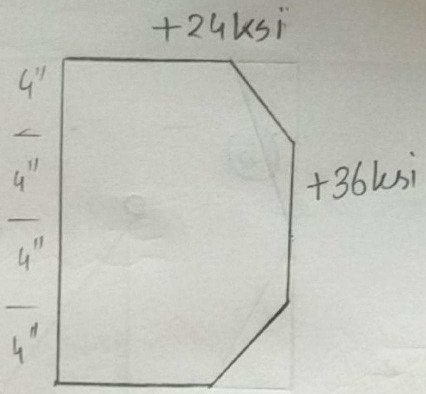


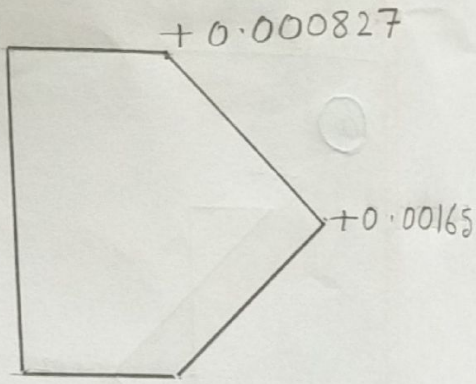


$f_{avg} = +24\,ksi$

$\varepsilon_{avg} = +0.000827$

(d)

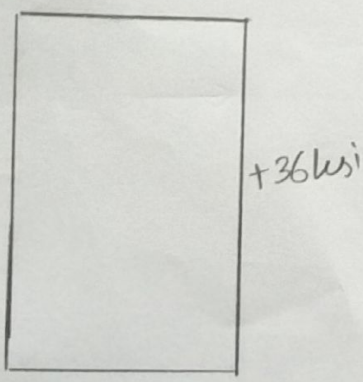


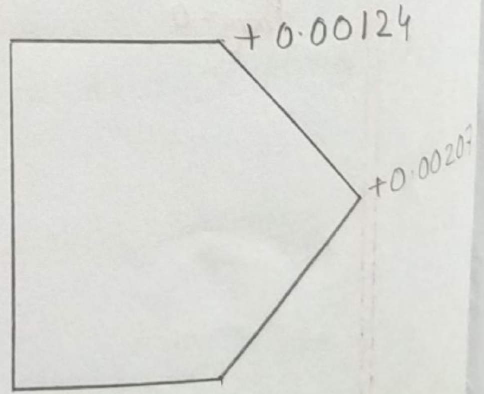


$f_{avg} = +33\,ksi$

$\varepsilon_{avg} = +0.0012414$

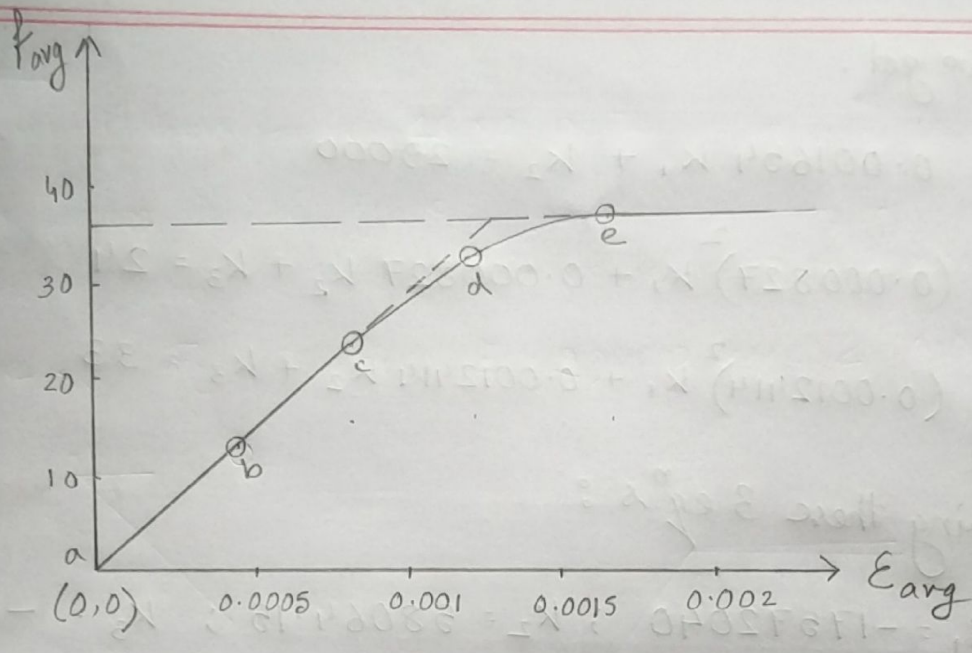
(e)





$f_{avg} = 36\,ksi$

$\varepsilon_{avg} = +0.001655$



Portion a-b-c : elastic (stress - strain relationship linear)

Portion c-d-e : inelastic

after d : curve is flat

Now, $f = k_1 \varepsilon^2 + k_2 \varepsilon + k_3$

$$\frac{dF}{d\varepsilon} = 2k_1 \varepsilon + k_2$$

Portion c-d

Boundary conditions -

i) at c $\longrightarrow$ $\frac{dF}{d\varepsilon} = 29000$; $\varepsilon = +0.000827$

ii) at c $\longrightarrow$ $f = +24$; $\varepsilon = +0.000827$

iii) at d $\longrightarrow$ $f = +33$; $\varepsilon = +0.0012414$

So, we get,

i) $0.001654\, k_1 + k_2 = 29000$

ii) $(0.000827)^2 k_1 + 0.000827\, k_2 + k_3 = 24$

iii) $(0.0012414)^2 k_1 + 0.0012414\, k_2 + k_3 = 33$

Solving these 3 $eq^n s$:

$k_1 = -17572040$; $k_2 = 58064.15$; $k_3 = -12$

$\therefore$ at d : $\frac{dF}{d\varepsilon} = 2(-17572040)(0.0012414) + 58064.15$

$= 14436.3$

Portion d-e

Boundary conditions —

i) at d $\longrightarrow$ $\frac{dF}{d\varepsilon} = 14436.3$; $\varepsilon = +0.0012414$

ii) at d $\longrightarrow$ $f = +33$; $\varepsilon = +0.0012414$

iii) at e $\longrightarrow$ $f = +36$; $\varepsilon = +0.001655$

So, we get,

i) $0.00248 k_1 + k_2 = 14436.3$

ii) $(0.0012414)^2 k_1 + 0.0012414 k_2 + k_3 = +33$

iii) $(0.001655)^2 k_1 + 0.001655 k_2 + k_3 = +36$

Solving these 3 eq^{n}s,

$K_1 = -17250036.23$; $K_2 = 57216.4$; $K_3 = -11.44$

Therefore,

i) $f = 29000\,\varepsilon$; $\frac{dF}{d\varepsilon} = 29000$ $\quad 0 \leq \varepsilon \leq 0.000827$ (portion a-c)

ii) $f = -17572040\,\varepsilon^2 + 58064.15\,\varepsilon - 12$

$\frac{dF}{d\varepsilon} = -35144080\,\varepsilon + 58064.15$; $\quad 0.000827 < \varepsilon \leq 0.0012414$

(c-d)

iii) $f = -17250036.23\,\varepsilon^2 + 57216.4\,\varepsilon - 11.44$

$\frac{dF}{d\varepsilon} = -34500072\,\varepsilon + 57216.4$ $\quad 0.0012414 \leq \varepsilon \leq 0.001655$

(d-e)

iv) $f = 36$; $\frac{dF}{d\varepsilon} = 0$ $\quad \varepsilon \geq 0.001655$

(after e)