"Atmospheric Water"

Formulas:

1. Specific Humidity, $q_v = \frac{m_v}{m_a} = \frac{\rho_v}{\rho_a}$

   where,

   ρ_v = Density of water vapor

   ρ_a = Density of moist air

2. Vapor Pressure, $e = \rho_v R_v T$ (Ideal Gas Law)

   where,

   e = Vapor pressure of water vapor (pascal)

   ρ_v = Density of water vapor (kg/m^3)

   R_v = Gas constant for water vapor ($J/Kg-°K$)

   T = Absolute temperature (°K)

3. Saturation Vapor Pressure, $e_s = 611 \exp\left(\frac{17.27\,T}{237.3+T}\right)$

   where,

   e_s = Saturation vapor pressure (pascal)

   T = Given air temperature (°c)

4. Relative Humidity, $R_h = \frac{e}{e_s}$

5. Specific Humidity, $q_v = 0.622\,\frac{e}{p}$

   where,

   e = actual vapor pressure (corresponding to dew point temperature, T_d)

   p = total pressure exerted by moist air

6. Air Density, $\rho_a = \frac{P}{R_a T}$

where,

R_a = Gas constant for moist air

7. Gas Constant for Moist air, $R_a = R_d(1+0.608 q_v)$

$= 287(1+0.608 q_v)$ J/Kg-K

where,

R_d = Gas constant for dry air

8. Temperature Variation, $T_2 = T_1 - \alpha(z_2 - z_1)$

where, α = environmental lapse rate = 6.5 °C/km

9. Pressure Variation, $P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{g}{\alpha R_a}}$

10. Precipitable Water, $\Delta m_p = \bar{q}_v \bar{\rho}_a A \Delta z$

where,

$\bar{q}_v = \frac{q_{v1} + q_{v2}}{2}$

$\bar{\rho}_a = \frac{\rho_{a1} + \rho_{a2}}{2}$

A = Cross-sectional area

Δz = interval of height

V.T. Chow Example 3.2.1: At a climate station, air pressure is measured as 100 kPa, air temperature as 20°C, and the wet-bulb or dew-point temperature as 16°C. Calculate the corresponding vapor pressure, relative humidity, specific humidity and air density.

Solution: The saturated vapor pressure at $T = 20°C$ is,

$$e_s = 611 \exp\left(\frac{17.27\,T}{237.3+T}\right)$$

$$= 611 \exp\left(\frac{17.27 \times 20}{237.3+20}\right)$$

$$= 2339.05 \text{ Pa}$$ (Ans.)

The actual vapor pressure is found at dew-point temperature of $T_d = 16°C$.

$$e = 611 \exp\left(\frac{17.27\,T_d}{237.3+T_d}\right)$$

$$= 611 \exp\left(\frac{17.27 \times 16}{237.3+16}\right)$$

$$= 1818.88 \text{ Pa}$$ (Ans.)

The relative humidity,

$$R_h = \frac{e}{e_s} = \frac{1818.88}{2339.05} = 0.7776 = 77.76\%$$

The specific humidity, assuming $P = 101.3\ kPa$

$$q_v = 0.622\ \frac{e}{P}$$

$$= 0.622 \times \frac{1818.88\ Pa}{101.3 \times 10^3\ Pa}$$

$$= 0.0112\ kg\ water / kg\ moist\ air$$

(Ans:)

The gas constant for moist air,

$$R_a = R_d\,(1 + 0.608\ q_v)$$

$$= 287\,(1 + 0.608\ q_v)$$

$$= 287\,(1 + 0.608 \times 0.0112)$$

$$= 288.95\ J/Kg\text{-}K$$

Air Temperature, $T = 20° + 273 = 293\ K$

Air Density, $\rho_a = \frac{P}{R_a T}$

$$= \frac{101.3 \times 10^3\ Pa}{288.95\ J/Kg\text{-}K \times 293\ K}$$

$$= 1.1965\ kg/m^3$$

(Ans:)

V.T. Chow Example 3.2.2: Calculate the precipitable water in a saturated air column 10 km high above 1 m^2 of ground surface. The surface pressure is 101.3 kPa, the surface air temperature is 30°C, and the lapse rate is 6.5°C/km.

Solution: The increment in elevation is taken as, $\Delta z = 2$ km.
= 2000 m.

For the first increment: at the surface,

At $z_1 = 0$ m, $T_1 = 30°C = (273+30)°K = 303$ K.

At $z_2 = 2000$ m, $T_2 = T_1 - \alpha(z_2 - z_1)$ $[\alpha = 6.5°C/km = 0.0065°C/m]$

$= 303 - 0.0065(2000 - 0)$

$= 290$ K or 17°C

Here, dew-point temperature is not given, so we can not calculate actual vapor pressure, e & thus can not determine specific humidity, $q_v = 0.622\frac{e}{p}$.

So, Gas constant, $R_a = R_d = 287$ J/Kg-K

The pressure at 2000 m elevation,

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{g}{\alpha R_a}}$$

$$= 101.3\left(\frac{290}{303}\right)^{\frac{9.81}{0.0065 \times 287}}$$

$$= 80.4 \text{ kPa}$$

The air density at ground ($z_1 = 0\ m$),

$$\rho_{a_1} = \frac{P_1}{R_a T_1} = \frac{101.3 \times 10^3}{287 \times 303} = 1.16\ kg/m^3$$

The air density at 2000m elevation ($z_2 = 2000\ m$),

$$\rho_{a_2} = \frac{P_2}{R_a T_2} = \frac{80.4 \times 10^3}{287 \times 290} = 0.97\ kg/m^3$$

The average density over the 2km increment,

$$\bar{\rho}_a = \frac{\rho_{a_1} + \rho_{a_2}}{2} = \frac{1.16 + 0.97}{2} = 1.07\ kg/m^3$$

The saturated vapor pressure at ground ($z_1 = 0\ m$),

$$e_{s1} = 611 \exp\left(\frac{17.27\ T_1}{237.3 + T_1}\right)$$

$$= 611 \exp\left(\frac{17.27 \times 30}{237.3 + 30}\right) = 4244.45\ Pa \text{ or, } 4.24\ kPa$$

The saturated vapor pressure at 2000m elevation ($z_2 = 2000\ m$),

$$e_{s2} = 611 \exp\left(\frac{17.27\ T_2}{237.3 + T_2}\right)$$

$$= 611 \exp\left(\frac{17.27 \times 17}{237.3 + 17}\right) = 1938.36\ Pa \text{ or, } 1.94\ kPa$$

The specific humidity at ground surface, [$e = e_s$; as saturated air]

$$q_{v_1} = 0.622 \frac{e_1}{P_1} = 0.622 \times \frac{4.24}{101.3} = 0.026\ kg/kg$$

The specific humidity at 2000 m elevation,

$$q_{v_2} = 0.622 \frac{e_2}{P_2} = 0.622 \times \frac{1.94}{80.4} = 0.015\ kg/kg$$

The average value of specific humidity over the 2km increment,

$$\bar{q}_v = \frac{q_{v_1} + q_{v_2}}{2} = \frac{0.026 + 0.015}{2} = 0.0205 \text{ kg/kg}$$

The mass of precipitable water in the first 2km increment,

$$\Delta m_p = \bar{q}_v \bar{\rho}_a A \Delta z = (0.0205 \times 1.07 \times 1 \times 2000 = 43.87 \text{ kg}$$

Elevation Z (km)	Temperature (°C)	Temperature (°K)	Air Pressure p (kPa)	Density ρ_a (kg/m³)	Vapor Pressure e (kPa)	Specific Humidity q_v (kg/kg)	Average $\bar{q}_v$ over 2km increment	Average $\bar{\rho}_a$ over 2km increment	Incremental Mass Δm (kg)	% of Total Mass
0	30	303	101.3	1.16	4.24	0.026				
2	17	290	80.4	0.97	1.94	0.015	0.0205	1.065	43.665	56.8
4	4	277	63.2	0.79	0.81	0.008	0.0115	0.88	20.24	26.3
6	-9	264	49.1	0.65	0.31	0.0039	0.00595	0.72	8.568	11.1
8	-22	251	37.6	0.52	0.10	0.0017	0.0028	0.585	3.276	4.3
10	-35	238	28.5	0.42	0.03	0.0007	0.0012	0.47	1.128	1.5
									76.877	100

Total mass of precipitable water in the column = 76.877 kg

(Ans:)

The equivalent depth of liquid water $= \frac{m_p}{\rho_w A}$

$$= \frac{76.877 \text{ kg}}{1000 \text{ kg/m}^3 \times 1 \text{ m}^2} = 0.076877 \text{ m}$$

$$\approx 76.877 \text{ mm}$$

(Ans:)

2014-15

2.(a) Calculate precipitable water for surface temperature of 20°c in the first km of atmospheric column if the surface pressure and lapse rate are 101.3 kPa and 6.5°c/km respectively. Relative humidity is 90%. and 100% at surface and 1 km elevation respectively. Assume any reasonable value for data if missing.

Answer: The increment in elevation, $\Delta z = 1\ km = 1000\ m$

At, $z_1 = 0\ m$, $T_1 = 20°C$ or $(20+273) = 293°K$

At, $z_2 = 1000\ m$, $T_2 = T_1 - \alpha(z_2 - z_1)$ [$\alpha = 6.5°c/km$ or $0.0065°c/m$]

$$= 20 - 0.0065 \times (1000 - 0)$$

$$= 13.5°C \text{ or } 286.5\ K$$

The air pressure at surface, $P_1 = 101.3\ kPa$

The air pressure at 1000 m elevation, considering, $R_a = R_d = 287\ J/kgK$

$$P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{\frac{g}{\alpha R_a}} = 101.3 \times \left(\frac{286.5}{293}\right)^{\frac{9.81}{0.0065 \times 287}} = 90.03\ kPa$$

The air density at surface,

$$\rho_{a1} = \frac{P_1}{R_a T_1} = \frac{101.3 \times 10^3\ Pa}{287 \times 293} = 1.20\ kg/m^3$$

The air density at 1000 m elevation,

$$\rho_{a2} = \frac{P_2}{R_a T_2} = \frac{90.03 \times 10^3\ Pa}{287 \times 286.5} = 1.09\ kg/m^3$$

The average density, $\bar{\rho}_a = \frac{\rho_{a1}+\rho_{a2}}{2} = \frac{1.20+1.09}{2} = 1.145 kg/m^3$

The saturated vapor pressure at surface,

$$e_{s1} = 611 \exp\left(\frac{17.27 T_1}{237.3+T_1}\right)$$

$$= 611 \exp\left(\frac{17.27\times 20}{237.3+20}\right) = 2339 \text{ Pa or, } 2.34 \text{ kPa}$$

The saturated vapor pressure at 1000 m elevation,

$$e_{s2} = 611 \exp\left(\frac{17.27 T_2}{237.3+T_2}\right)$$

$$= 611 \exp\left(\frac{17.27\times 13.5}{237.3+13.5}\right) = 1547 \text{ Pa or, } 1.55 \text{ kPa}$$

The actual vapor pressure at surface,

$$e_1 = e_{s1}\times R_{h_1} = 2.34\times 0.90 = 2.106 \text{ kPa}$$

The actual vapor pressure at 1000 m elevation,

$$e_2 = e_{s2}\times R_{h_2} = 1.55\times 1.0 = 1.55 \text{ kPa}$$

The specific humidity at surface,

$$q_{v1} = 0.622\frac{e_1}{P_1} = 0.622\times\frac{2.106}{101.3} = 0.0129 \text{ kg/kg}$$

The specific humidity at 1000 m elevation,

$$q_{v2} = 0.622\frac{e_2}{P_2} = 0.622\times\frac{1.55}{90.03} = 0.0107 \text{ kg/kg}$$

The average value of specific humidity,

$$\bar{q}_v = \frac{q_{v1} + q_{v2}}{2} = \frac{0.0129 + 0.0107}{2} = 0.0118 \text{ kg/kg}$$

The mass of the precipitable water in the first km of atmospheric column, assuming $1 m^2$ of area.

$$\Delta m = \bar{q}_v \bar{\rho}_a A \Delta z$$

$$= 0.0118 \times 1.145 \times 1 \times 1000$$

$$= 13.511 \text{ kg} \quad \text{(Ans:)}$$

"2013 - 14"

2.(c) Sol^n: We know,

$$\Delta m = \bar{q}_v \bar{\rho}_a A \Delta z$$

$$\Rightarrow 500 \text{ kg} = \bar{q}_v \times 1 \text{ kg/m}^3 \times 10 \text{ m}^2 \times 2000 \text{ m}$$

$$\therefore \bar{q}_v = 0.025 \text{ kg/kg}$$

$$\text{Now, } \bar{q}_v = \frac{q_{v1} + q_{v2}}{2}$$

$$\Rightarrow 0.025 = \frac{1.5 q_{v2} + q_{v2}}{2}$$

$$\therefore q_{v2} = 0.02 \text{ kg/kg} \text{ @ 2 km elevation.} \quad \text{(Ans:)}$$

Given,

$\Delta m = 500$ kg

$A = 10 m^2$

$\bar{\rho}_a = 1$ kg/m^3

$\Delta z = 2$ km $= 2000$ m

$q_{v1} = 1.5 q_{v2}$

$q_{v_1} = 1.5 q_{v_2}$

$= (1.5 \times 0.02) = 0.03$ kg/kg @ ground surface (Ans:)

Same 2012-13 3(b) 2009-10 2(e)

3.(b) Sol^n: Given, $R_h = 70\%$ & $e_s = 2400$ Pa

(i) We know, $e_s = 611 \exp\left(\frac{17.27T}{237.3+T}\right)$

$\Rightarrow 2400 = 611 \exp\left(\frac{17.27T}{237.3+T}\right)$

$\therefore T = 20.42°C$ (Ans:)

(ii) Again, $R_h = \frac{e}{e_s}$

$\Rightarrow e = e_s \times R_h = 2400 \times 0.70 = 1680$ Pa (Ans:)

(iii) Dew temperature corresponds to actual vapor pressure,

$e = 611 \exp\left(\frac{17.27\, T_d}{237.3+T_d}\right)$

$\Rightarrow 1680 = 611 \exp\left(\frac{17.27 \times T_d}{237.3+T_d}\right)$

$\therefore T_d = 14.76°C$ (Ans:)

Standard Pressure, $P = 101.3$ kPa

(iv) Now, $q_v = 0.622 \frac{e}{P} = 0.622 \times \frac{1680}{101.3 \times 10^3} = 0.0103$ kg/kg

(Ans:)

(v) Again, $R_a = R_d(1 + 0.608 q_v)$

$= 287 \times (1 + 0.608 \times 0.0103)$

$= 288.8$ J/kg-K (Ans:)

(vi) Now, $\rho_a = \frac{P}{R_a T} = \frac{101.3 \times 10^3}{288.8 \times (20.42 + 273)} = 1.1954$ kg/m^3

(Ans:)

"2012-13"

Same 2010-11 1(e)

3.(c) Soln: The increment in height, $\Delta z = 1$ km $= 1000$ m

At $z_1 = 0$ m, $T_1 = 10°C$ or $(10+273) = 283$ K

At $z_2 = 1000$m, $T_2 = T_1 - \alpha(z_2 - z_1)$ [$\alpha = 6.5°C/km$ or $0.0065°C/m$]

$= 10 - 0.0065 \times (1000 - 0)$

$= 3.5°C$ or 276.5 K

The air pressure at surface, $P_1 = 101.3$ kPa

Assume Gas Constant, $R_a = R_d = 287$ J/kg-K

The air pressure at 1000 m elevation,

$$P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{\frac{g}{\alpha R_a}} = 101.3 \times \left(\frac{276.5}{283}\right)^{\frac{9.81}{0.0065 \times 287}} = 89.65 \text{ kPa}$$

The air density at surface,

$$\rho_{a_1} = \frac{P_1}{R_a T_1} = \frac{101.3 \times 10^3}{287 \times 283} = 1.2472 \text{ kg/m}^3$$

The air density at 1000 m elevation,

$$\rho_{a_2} = \frac{P_2}{R_a T_2} = \frac{89.65 \times 10^3}{287 \times 276.5} = 1.1297 \text{ kg/m}^3$$

The average air density,

$$\bar{\rho}_a = \frac{\rho_{a_1} + \rho_{a_2}}{2} = \frac{1.2472 + 1.1297}{2} = 1.18845 \text{ kg/m}^3$$

The saturation vapor pressure at surface,

$$e_{s1} = 611 \exp\left(\frac{17.27\, T_1}{237.3 + T_1}\right)$$

$$= 611 \exp\left(\frac{17.27 \times 10}{237.3 + 10}\right) = 1228.36 \text{ Pa or } 1.2284 \text{ kPa}$$

The saturation vapor pressure at 1000 m elevation,

$$e_{s_2} = 611 \exp\left(\frac{17.27\, T_2}{237.3 + T_2}\right)$$

$$= 611 \exp\left(\frac{17.27 \times 3.5}{237.3 + 3.5}\right) = 785.338 \text{ Pa or, } 0.7853 \text{ kPa}$$

The specific humidity at ground surface, [$e = e_s$; saturated air]

$$q_{v_1} = 0.622 \frac{e_1}{P_1} = 0.622 \times \frac{1.2284}{101.3} = 0.0075 \text{ kg/kg}$$

The specific humidity at 1000 m elevation,

$$q_{v_2} = 0.622 \frac{e_2}{P_2} = 0.622 \times \frac{0.7853}{89.65} = 0.0054 \text{ kg/kg}$$

The average value of specific humidity,

$$\bar{q}_v = \frac{q_{v_1} + q_{v_2}}{2} = \frac{0.0075 + 0.0054}{2} = 0.00645 \text{ kg/kg}$$

The mass of precipitable water, assuming $A = 1 m^2$

$$\Delta m_1 = \bar{q}_v \cdot \bar{\rho}_a A \Delta z = 0.00645 \times 1.18845 \times 1 \times 1000$$

$$= 7.6655 \text{ kg} \quad \text{(Ans:)}$$

For Surface Temperature, $T_1 = 25°C$

At $z_1 = 0$ m, $T_1 = 25°C$ or $(25+273) = 298$ K

At $z_2 = 1000$ m, $T_2 = T_1 - \alpha(z_2 - z_1)$

$$= 25 - 0.0065 \times (1000 - 0)$$

$$= 18.5°C \text{ or } 291.5 \text{ K}$$

The air pressure at 1000 m elevation, for $P_1 = 101.3$ kPa

$$P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{\frac{g}{R_a \alpha}} = 101.3 \times \left(\frac{291.5}{298}\right)^{\frac{9.81}{0.0065 \times 287}} = 90.21 \text{ kPa}$$

The air densities at surface & 1000m elevation are,

$$\rho_{a1} = \frac{P_1}{R_a T_1} = \frac{101.3\times10^3}{287\times298} = 1.1844 \text{ kg/m}^3$$

$$\rho_{a_2} = \frac{P_2}{R_a T_2} = \frac{90.21\times10^3}{287\times291.5} = 1.0783 \text{ kg/m}^3$$

$$\text{So, } \bar{\rho}_a = \frac{\rho_{a1}+\rho_{a2}}{2} = \frac{1.1844+1.0783}{2} = 1.13135 \text{ kg/m}^3$$

The saturation vapor pressures at surface & 1000 m elevation are,

$$e_{s1} = 611\exp\left(\frac{17.27\,T_1}{237.3+T_1}\right) = 611\exp\left(\frac{17.27\times25}{237.3+25}\right) = 3168.8 \text{ Pa}$$

$$\approx 3.17 \text{ kPa}$$

$$e_{s2} = 611\exp\left(\frac{17.27\,T_2}{237.3+T_2}\right) = 611\exp\left(\frac{17.27\times18.5}{237.3+18.5}\right) = 2130.47 \text{ Pa}$$

$$\approx 2.13 \text{ kPa}$$

The specific humidities at surface & 1000 m elevation are,

$$q_{v1} = 0.622\frac{e_1}{P_1} = 0.622\times\frac{3.17}{101.3} = 0.0195 \text{ kg/kg}$$

$$q_{v2} = 0.622\frac{e_2}{P_2} = 0.622\times\frac{2.13}{90.21} = 0.0147 \text{ kg/kg}$$

$$\text{So, } \bar{q}_v = \frac{q_{v1}+q_{v2}}{2} = \frac{0.0195+0.0147}{2} = 0.0171 \text{ kg/kg}$$

Mass of precipitable water, $\Delta m_2 = \bar{q}_v \bar{\rho}_a A \Delta z$

$$= (0.0171\times1.13135\times1\times1000)$$

$$= 19.35 \text{ kg}$$

$$\% \text{ increase} = \frac{\Delta m_2 - \Delta m_1}{\Delta m_1}\times100\% = \frac{19.35-7.6655}{7.6655}\times100\%$$

$$= 152.43\%$$ (Ans.)

"2009-10"

1. (e) Sol^n: The increment in height, $\Delta z = 2\ km = 2000\ m$

Given, at ground level ($z_1 = 0\ m$),

$Rh_1 = 80\%$

$e_{s1} = 2500\ Pa$

$P_1 = 101.3\ kPa$

Let, air temperature $= T_1$

We know, $e_{s1} = 611 \exp\left(\frac{17.27\ T_1}{237.3 + T_1}\right)$

$\Rightarrow 2500 = 611 \exp\left(\frac{17.27 \times T_1}{237.3 + T_1}\right)$

$\therefore T_1 = 21.08\,°C$ or $294.08\ K$

Assuming lapse rate, $\alpha = 6.5\,°C/km$ or $0.0065\,°C/m$

We get, $T_2 = T_1 - \alpha(z_2 - z_1)$

$= 21.08 - 0.0065 \times (2000 - 0) = 8.08\,°C$ or $281.08\ K$

The air pressure at 2000m elevation, assuming $R_a = R_d = 287\ J/kg\text{-}K$

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{g}{\alpha R_a}} = 101.3 \times \left(\frac{281.08}{294.08}\right)^{\frac{9.81}{0.0065 \times 287}} = 79.86\ kPa$$

The saturation vapor pressure at 2000 m elevation,

$$e_{s2} = 611 \exp\left(\frac{17.27\ T_2}{237.3 + T_2}\right)$$

$$= 611 \exp\left(\frac{17.27 \times 8.08}{237.3 + 8.08}\right) = 1078.98\ Pa \text{ or } 1.08\ kPa$$

& $Rh_2 = 90\%$

Now, actual vapor pressures at surface & 2000m elevation are,

$e_1 = e_{s1} \times R_{h1} = 2.5\,kPa \times 0.80 = 2\,kPa$

$e_2 = e_{s2} \times R_{h2} = 1.08\,kPa \times 0.90 = 0.972\,kPa$

The specific humidities at surface & 2000m elevation are,

$q_{v1} = 0.622\,\frac{e_1}{P_1} = 0.622 \times \frac{2}{101.3} = 0.0123\,kg/kg$

$q_{v2} = 0.622\,\frac{e_2}{P_2} = 0.622 \times \frac{0.972}{79.86} = 0.0076\,kg/kg$

So, $\bar{q}_v = \frac{q_{v1} + q_{v2}}{2} = \frac{0.0123 + 0.0076}{2} = 0.00995\,kg/kg$

Mass of precipitable water, for ground surface area, $A = 2m^2$

$\Delta m = \bar{q}_v\,\bar{\rho}_a\,A\,\Delta z$

$= 0.00995 \times 1.095 \times 2 \times 2000$

$= 43.581\,kg$ (Ans:)

The air densities at surface & 2000m elevation are,

$\rho_{a1} = \frac{P_1}{R_a T_1} = \frac{101.3 \times 10^3}{287 \times 294.08}$

$= 1.2\,kg/m^3$

$\rho_{a2} = \frac{P_2}{R_a T_2} = \frac{79.86 \times 10^3}{287 \times 281.08}$

$= 0.99\,kg/m^3$

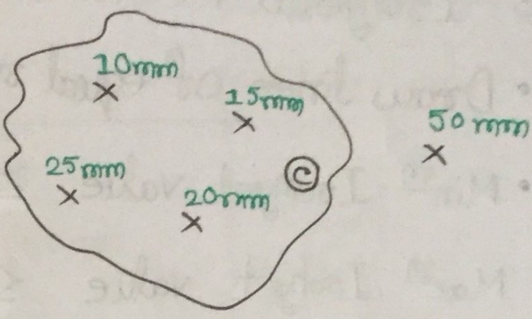
So, $\bar{\rho}_a = \frac{\rho_{a1} + \rho_{a2}}{2}$

$= \frac{1.2 + 0.99}{2} = 1.095\,kg/m^3$

"Computation of Average Rainfall"

1. Arithmetic Mean Method:

$$\bar{P} = \sum_{i=1}^{n} \frac{P_i}{n}$$



Where,

$\bar{P}$ = Average precipitation depth (mm)

P_i = Precipitation depth at gage i (within the topographic basin) (mm)

n = Total number of gages within the topographic basin.

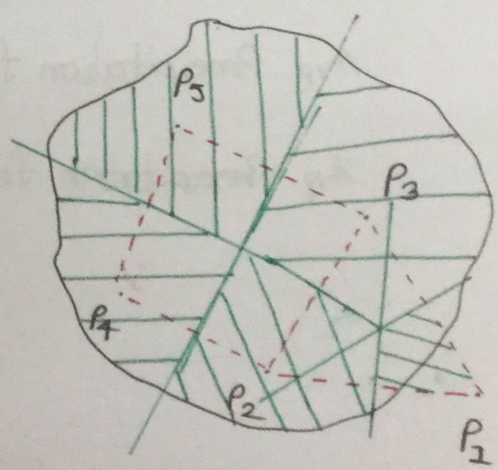
2. Thiessen Polygon Method:

- Divide the area by drawing bisectors
- Three bisectors intersects at a point & this point is equidistant from those three gauges.

Exam Question Pattern:

(i) Area will drawn, one needs need to count the number of grids only. Any partial grid can be considered as 0.5 grids.

(ii) Corner points of a catchment polygon will be given. Rain gauge's co-ordinates will be given.



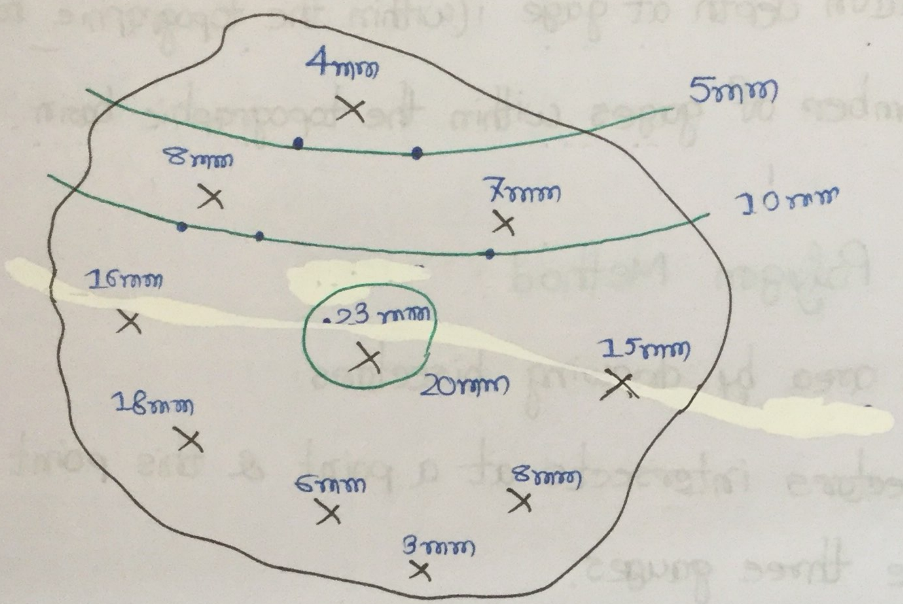
Average Precipation = $\frac{1}{A} \sum_{i=1}^{n} A_i P_i$

where, $A = \sum_{i=1}^{n} A_i$

3. Isohyetal Method :

- Draw lines of equal rainfall
- Min^m Isohyet value > Min^m Precipitation Value

Max^m Isohyet value < Max^m Precipitation Value



- To draw isohyet of 5mm, locate 5mm between rain gauge of 4mm & 8mm, also locate 5mm between rain gauge of 4mm & 7mm. Connect these two points by a random line.

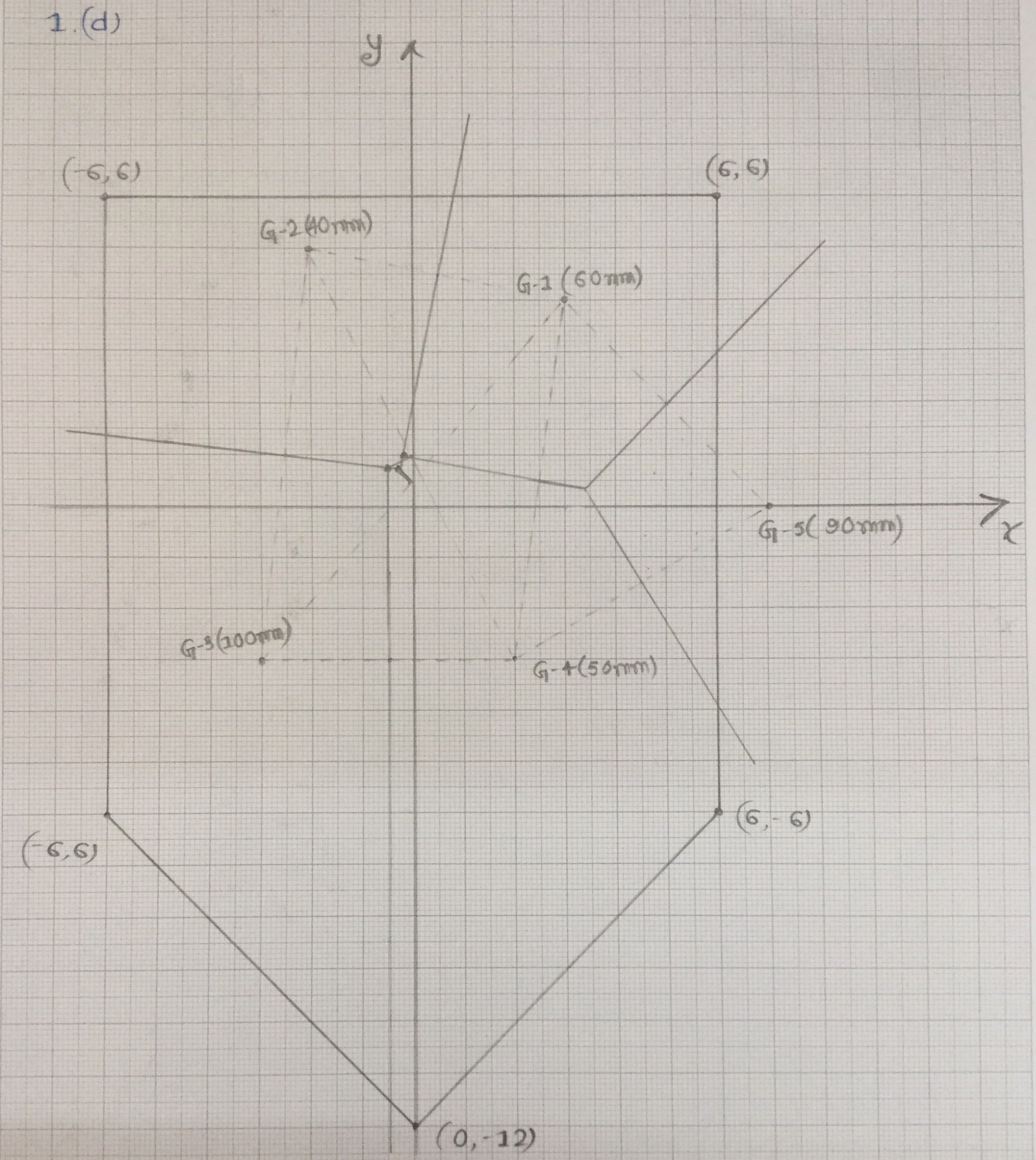
Avg. Precipitation for < 5mm = $\frac{5 + Min^m \text{ value}}{2}$

Avg. Precipitation for > 20 mm = $\frac{20 + Max^m \text{ value}}{2}$

2014-15

Roll No.

1.(d)



MICRO

20 cm x 25 cm

1.(d) Sol^n :

Gauge Number	Recorded Rainfall (mm)	Area (km^2) ≈ No. of Grids	Weighted Rainfall (mm)
1	60	28	1680
2	40	30.5	1220
3	100	56	5600
4	50	56.5	2825
5	90	5.5	495
		176.5	11820

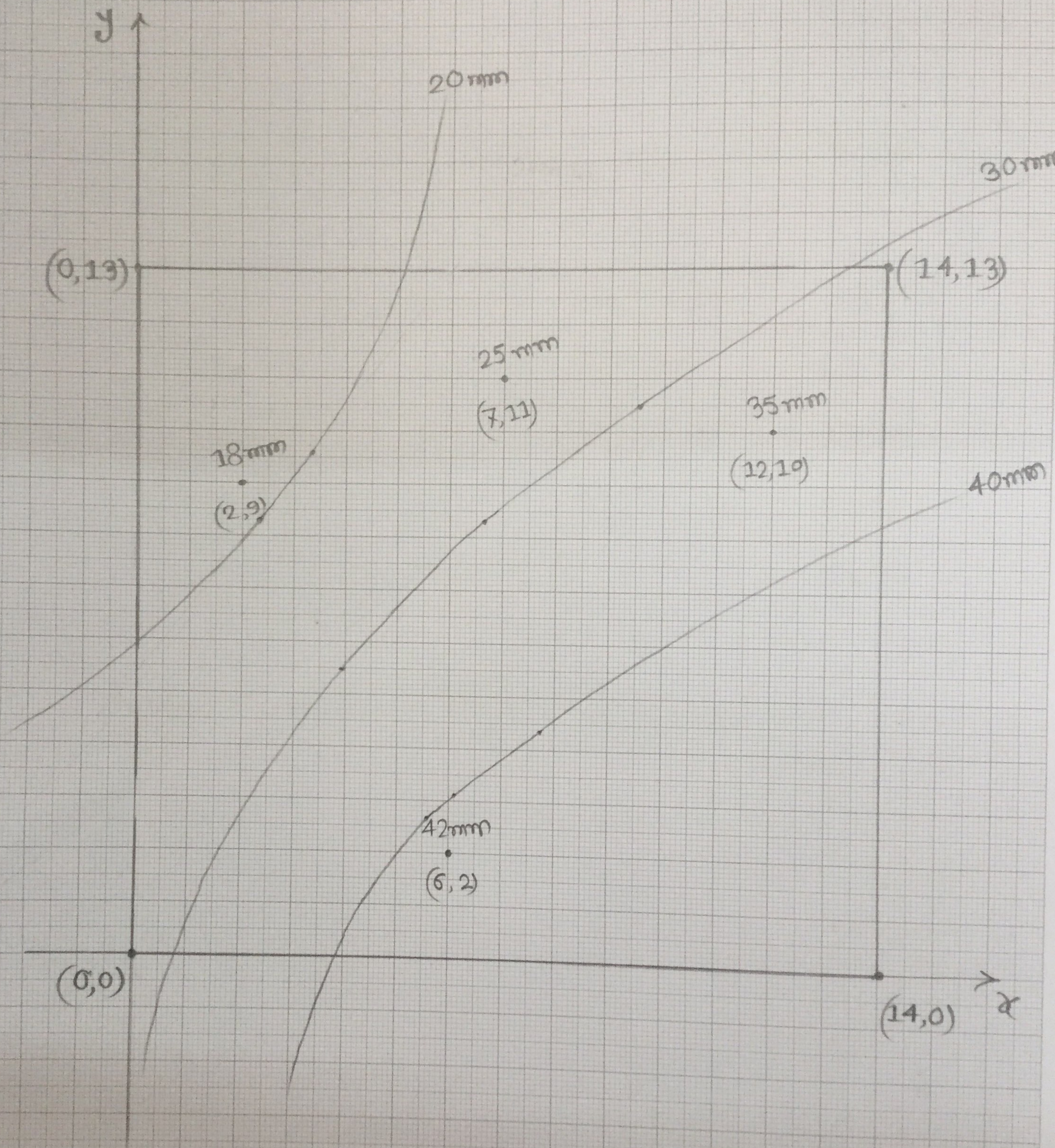
$$\text{Average rainfall} = \frac{11820 \text{ mm}}{176.5} = 66.97 \text{ mm}$$

(Ans:)

2013-14

Roll No.

3.(c)



MICRO

N.P. 20 cm x 25 cm

3.(c) Sol^n:

Isohyets (mm)	Area enclosed ≈ No. of grids (km^2)	Average Rainfall (mm)	Rainfall Volume
<20	20.5	$\frac{18+20}{2} = 19$	389.5
20-30	49.5	$\frac{20+30}{2} = 25$	1237.5
30-40	57	$\frac{30+40}{2} = 35$	1995
>40	50	$\frac{40+42}{2} = 41$	2050
Σ	177		5672

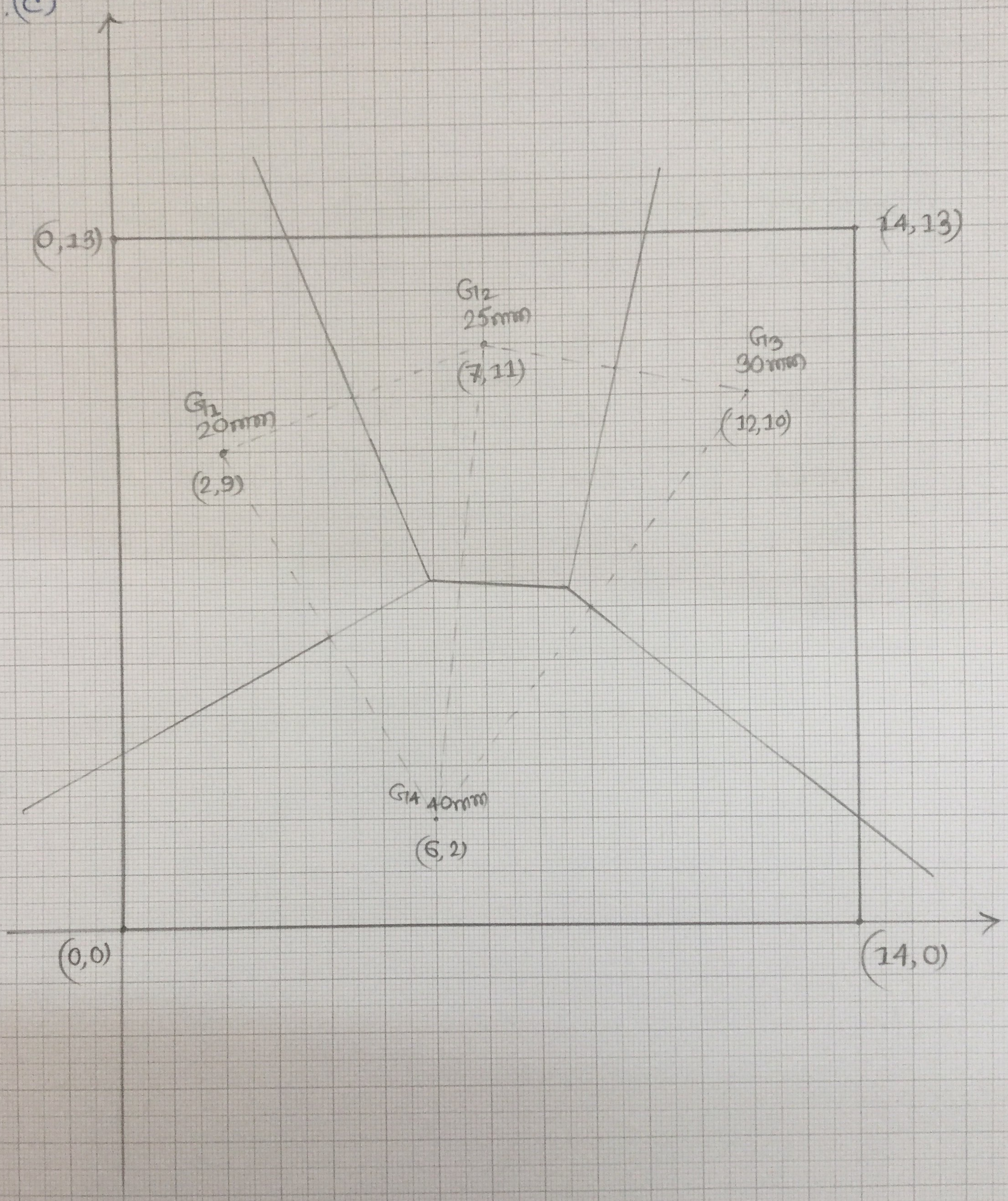
$$\text{Average rainfall} = \frac{5672}{177} = 32.05 \text{ mm}$$

(Ans:)

"2012-13"

Roll No.

4.(c)



MICRO

N.P. 20 cm x 25 cm

"2012-13"

4.(c) Solⁿ:

Gauge Location	Rainfall (mm)	Area ≈ No. of Grids (Km^2)	Weighted Rainfall (mm)
G_1 (2,9)	20	39.5	790
G_2 (7,11)	25	31	775
G_3 (12,10)	30	43	1290
G_4 (6,2)	40	68.5	2740
Σ		182	5595

Average rainfall $= \frac{5595}{182}$ mm $= 30.74$ mm (Ans:)

"Runoff"

'Rational Method'

Rainfall-runoff Correlations:

Regression equation, $R = aP + b$

where, R = runoff ; P = rainfall / precipitation

$$a = \frac{n(\Sigma PR) - (\Sigma P)(\Sigma R)}{n(\Sigma P^2) - (\Sigma P)^2}$$

$$b = \frac{(\Sigma R) - a(\Sigma P)}{n}$$

where, n = number of observations

Correlation coefficient, $r = \dfrac{n(\Sigma PR) - (\Sigma P)(\Sigma R)}{\sqrt{[n(\Sigma P^2) - (\Sigma P)^2] \times [n(\Sigma R^2) - (\Sigma R)^2]}}$

Rational Method:

Peak value of runoff, $Q_p = CiA$ for $t \geq t_c$

Where,

C = coefficient of runoff

A = area of catchment

i = intensity of rainfall

Time of concentration, $t_c\,(min) = 0.01947(L)^{0.77}(S)^{-0.385}$ [Kirpich eqn]

where,

S = average slope of the catchment

L = maximum length of travel of water (m)

Intensity - duration - frequency (IDF) equation:

The rainfall intensity, $i = \frac{KT^x}{(t_c + a)^m}$

where, K, a, x, & m are constants

return period, $T = \frac{1}{P}$

where, p = probability of exceedence

Problem-1 : Given below are the monthly rainfall (P) and the corresponding runoff (R) values covering a period of 18 months for a catchment. Develop a regression equation between R and P

Month	P(cm)	R(cm)	Month	P(cm)	R(cm)
1	5	0.5	10	30	8.0
2	35	10.0	11	10	2.3
3	40	13.8	12	8	1.6
4	30	8.2	13	2	0.0
5	15	3.1	14	22	6.5
6	10	3.2	15	30	9.4
7	5	0.1	16	25	7.6
8	3	12.0	17	8	1.5
9	36	16.0	18	6	0.5

Sol^n:

Month	P(cm)	R(cm)	PR	P^2	R^2
1	5	0.5	2.5	25	0.25
2	35	10.0	350	1225	100
3	40	13.8	552	1600	190.44
4	30	8.2	246	900	67.24
5	15	3.1	46.5	225	9.61
6	10	3.2	32	100	10.24
7	5	0.1	0.5	25	0.01
8	31	12.0	372	961	144
9	36	16.0	576	1296	256
10	30	8.0	240	900	64
11	10	2.3	23	100	5.29
12	8	1.6	12.8	64	2.56
13	2	0.0	0	4	0
14	22	6.5	143	484	42.25
15	30	9.4	282	900	88.36
16	25	7.6	190	625	57.76
17	8	1.5	12	64	2.25
18	6	0.5	3	36	0.25
$n = 18$	$\Sigma P = 348$	$\Sigma R = 104.3$	$\Sigma PR = 3083.3$	$\Sigma P^2 = 9534$	$\Sigma R^2 = 1040.51$

Regression equation, $R = aP + b$

$$a = \frac{n(\Sigma PR) - (\Sigma P)(\Sigma R)}{n(\Sigma P^2) - (\Sigma P)^2}$$

$$= \frac{(18 \times 3083.3) - (348 \times 104.3)}{(18 \times 9534) - (348)^2}$$

$$= 0.38$$

$$b = \frac{(\Sigma R) - a(\Sigma P)}{n}$$

$$= \frac{104.3 - 0.38 \times 348}{18}$$

$$= -1.55$$

& $$r = \frac{n(\Sigma PR) - (\Sigma P)(\Sigma R)}{\sqrt{[n(\Sigma P^2) - (\Sigma P)^2] \times [n(\Sigma R^2) - (\Sigma R)^2]}}$$

$$= \frac{(18 \times 3083.3) - (348 \times 104.3)}{\sqrt{[(18 \times 9534) - (348)^2] \times [(18 \times 1040.51) - (104.3)^2]}}$$

$= 0.964$; correlation is good.

Hence, $R = 0.38P - 1.55$ (Ans:)

when, $R = 0$, $0 = 0.38P - 1.55$

$$\Rightarrow P = 4.0789$$

So, Min^m precipitation of 4.0789 mm is required to initiate runoff.

(Ans:)

K. Subramanya Example 7.1(a): An urban catchment has an area of 85 ha. The slope of the catchment is 0.006 and the maximum length of travel of water is 950 m. The maximum depth of rainfall with a 25 year return period is as below:

Duration (min)	5	10	20	30	40	60
Depth of rainfall (mm)	17	26	40	50	57	62

If a culvert for drainage at the outlet of this area is to be designed for a return period of 25 years, estimate the required peak-flow rate, by assuming the runoff coefficient as 0.3.

Soln: The time of concentration is obtained by Kirpich formula,

$$t_c\,(\text{min}) = 0.01947\, L^{0.77}\, S^{-0.385}$$

$$= 0.01947 \times (950)^{0.77} \times (0.006)^{-0.385}$$

$$= 27.39 \text{ minutes}$$

The time of concentration is taken equal to the duration of rainfall. By interpolation from the given table,

For a rainfall of 27.39 min duration, the maximum depth of rainfall $= \left[40 + \frac{50-40}{30-20} \times (27.39-20)\right] = 47.39$ mm

Hence, Intensity of rainfall = $\dfrac{\text{Rainfall Depth (mm)}}{\text{Rainfall duration (hr)}}$

$$= \frac{47.39 \text{ mm}}{\left(\frac{27.39 \text{ min}}{60}\right) \text{hr}}$$

$$= 103.8116 \text{ mm/hr}$$

Using rational method,

Peak Discharge, $Q_p = CiA$ $\left[1 \text{ ha} = 10^4 \text{ m}^2\right]$

$$= 0.30 \times \frac{103.8116}{1000 \text{ m}} \times \frac{(85 \times 10^4) \text{ m}^2}{(60 \times 60) \text{ sec}}$$

$$= 7.35 \text{ m}^3/\text{s}$$ (Ans:)

Example 7.1(b): If in the urban area of Example 7.1(a), the land use of the area and the corresponding runoff coefficients are as given below, calculate the equivalent runoff coefficient.

Land Use	Area (ha)	Runoff Coefficient
Roads	8	0.70
Lawn	17	0.10
Residential Area	50	0.30
Industrial Area	10	0.80

Sol^{n}: Equivalent runoff coefficient, $C_e = \dfrac{\sum_{i=1}^{N} C_i A_i}{A}$

So, $C_c = \dfrac{[0.70 \times 8 + 0.10 \times 17 + 0.30 \times 50 + 0.80 \times 10]}{(8 + 17 + 50 + 10)}$

$= 0.3565$ (Ans:)

Example 7.2 : A 500 ha watershed has the land use/cover and corresponding runoff coefficient as given below :

Land Use/Cover	Area (ha)	Runoff Coefficient
Forest	250	0.10
Pasture	50	0.11
Cultivated Land	200	0.30

The maximum length of travel of water in the watershed is about 3000 m and the elevation difference between the highest and outlet points of the watershed is 25 m. The maximum intensity duration frequency relationship of the watershed is given by $i = \dfrac{6.311\, T^{0.1523}}{(D + 0.50)^{0.945}}$

where, i = intensity in cm/h, T = Return period in years and D = duration of the rainfall in hours.

Estimate (i) 25 year peak runoff from the watershed and (ii) the 25 year peak runoff if the forest cover has decreased to 50 ha and the cultivated land has encroached upon the pasture and forests lands to have a total coverage of 450 ha.

Solⁿ: (i) Equivalent runoff coefficient,

$$C_e = \frac{\sum_{i=1}^{N} C_i A_i}{A}$$

$$= \frac{[0.10\times250 + 0.11\times50 + 0.30\times200]}{250+50+200}$$

$$= 0.181$$

Now, Slope of catchment $= \frac{\text{Diff}^n \text{ bet}^n \text{ highest \& lowest points}}{\text{Max}^m \text{ length of travel}} = \frac{\Delta H}{L}$

$$= \frac{25\,m}{3000\,m} = \frac{1}{120}$$

Time of concentration, $t_c(\text{min}) = 0.01947\, L^{0.77}\, S^{-0.385}$

$$= 0.01947 \times (3000)^{0.77} \times \left(\frac{1}{120}\right)^{-0.385}$$

$$= 58.51 \text{ min}$$

$$\approx 0.9752 \text{ hours}$$

So, Duration rainfall, D = Time of concentration, t_c

$$= 0.9752 \text{ hours}$$

Now, Intensity of rainfall, $i = \frac{6.311\, T^{0.1523}}{(D+0.50)^{0.945}}$

$$= \frac{6.311 \times (25)^{0.1523}}{(0.9752+0.50)^{0.945}}$$

$$= 7.1358 \text{ cm/hr}$$

$$\approx 0.071358 \text{ m/hr}$$

Area of catchment, $A = 500$ hactres

$$= 500 \times 10^4 \text{ m}^2$$

Peak Discharge, $Q_p = CiA$

$$= 0.181 \times 0.071358 \times 500 \times 10^4 \times \frac{1}{60 \times 60}$$

$$= 17.94 \text{ m}^3/\text{s}$$ (Ans:)

(ii) Here, $C_e = \frac{\sum_{i=1}^{n} C_i A_i}{A} = \frac{[0.10 \times 50 + 0.30 \times 450]}{50+450} = 0.28$

So, $Q_p = CiA$

$$= 0.28 \times \frac{0.071358 \text{ m/hr}}{(60 \times 60)\text{sec}} \times (500\text{ha} \times 10^4)\text{m}^2$$

$$= 27.75 \text{ m}^3/\text{s}$$

(Ans:)

"2014-15"

Same 2012-13 4(b)

3.(a) Sol^n: We know,

Time of concentration,

$t_c(min) = 0.01947\, L^{0.77}\, S^{-0.385}$

$= 0.01947 \times (1500)^{0.77} \times (0.007)^{-0.385}$

$= 36.7$ minutes

Given,

$S = 0.007$

$L = 1500m$

$i = 2.5\ in/hr = \frac{2.5 \times 2.54m}{100 m \times 3600s}$

$= 1.7639 \times 10^{-5}\ m/s$

$A = 3\ km^2$

$C = 0.7$

From IDF curves, for, $i = 2.5$ in/hr

& $t_c = 36.7$ min we get, return period, $T = 5$ years (Ans:)

Now,

Peak Discharge, $Q_p = C\,i\,A$

$= 0.7 \times 1.7639 \times 10^{-5}\ m/s \times 3 \times (1000)^2\ m^2$

$= 37.04\ m^3/s$ (Ans:)

Design Precipitation Volume $= Q_p \times t_c$

$= 37.04\ m^3/s \times (36.7 min \times 60)\ sec$

$= 81562.08\ m^3$

(Ans:)

"2013-14"

Same 2009-10 4(e)

4.(a) Sol^n:

(i) Catchment A has Sandy Soil & High Vegetative Cover, so it will have lower runoff coefficient whereas Catchment B has clay soil & No Vegetative Cover, so it will have higher runoff coefficient.

Hence, $C_A = 0.2$ & $C_B = 0.8$ (Ans:)

(ii) Time of concentration for catchment A,

$$t_{c\,(min)} = 0.01947\, L^{0.77} S^{-0.385}$$

$$= 0.01947 \times (1500)^{0.77} \times (0.002)^{-0.385}$$

$$= 59.44 \text{ minutes}$$ (Ans:)

(iii) from IDF curve, for a return period, T = 100 years and time of concentration, $t_c = 59.44$ min we get, intensity of rainfall, $i = 3.35$ in/hr $= \frac{3.3 \times 2.54 cm}{100 m \times (60\times60) sec} = 2.3283 \times 10^{-5}$ m/s

Peak Discharge, $Q_p = C\,i\,A$

$$= 0.2 \times 2.3283 \times 10^{-5} \text{ m/s} \times \{2 \text{ km}^2 \times (1000)^2\} \text{ m}^2$$

$$= 9.3132 \text{ m}^3/\text{s}$$ (Ans:)

4.(e) Sol^n: We know,

$Q_p = C i A$

$\Rightarrow$ Intensity of rainfall, $i = \frac{Q_p}{CA}$

$\Rightarrow i = \frac{4.2 \, m^3/s}{0.1 \times \{2 \, km^2 \times (1000)^2\} m^2}$

$\therefore i = 2.1 \times 10^{-5} \, m/s$

$= \frac{2.1 \times 10^{-5} \times 100 \, cm}{2.54 \, in} \times (60 \times 60) \, hr$

$= 2.9764$ in/hr (Ans:)

Given,

$S = 0.005$

$A = 2 \, km^2$

$C = 0.1$

$Q_p = 4.2 \, m^3/s$

$T = 100$ years

From IDF curves, for $i = 2.9764$ in/hr & $T = 100$ years, we get, $t_c = 70$ min (Ans:)

From Kirpich formula,

$t_c (min) = 0.01947 \, L^{0.77} \, S^{-0.385}$

$\Rightarrow 70 = 0.01947 \times (L)^{0.77} \times (0.005)^{-0.385}$

$\therefore L = 2933.05$ m (Ans:)

"2009-10"

4.(e) Sol^n: (i) Catchment A has heavy suspended solid in infiltrating water & during Rainy season, so it will have higher runoff coefficient, whereas

Catchment B has little suspended solid in infiltrating water & during Dry season, so it will have lower runoff coefficient.

Hence, $C_A = 0.7$ & $C_B = 0.4$ (Ans:)

(ii) Time of concentration, for "A"

Given,

$L = 1000 m$

$A = 1 km^2$

$S = 0.005$

$T = 50$ years

$t_c(min) = 0.01947 L^{0.77} S^{-0.385}$

$= 0.01947 \times (1000)^{0.77} \times (0.005)^{-0.385}$

$= 30.57$ min

From IDF curves, for $t_c = 30.57$ min & $T = 50$ years we get,

$i = 4.25$ in/hr $= \frac{4.25 \times 2.54 cm}{100 m \times 3600 sec} = 2.9986 \times 10^{-5}$ m/s

Peak Discharge, $Q_p = C i A$

$= 0.7 \times 2.9986 \times 10^{-5} m/s \times \{1 km^2 \times (1000)^2\} m^2$

$= 20.9902 m^3/s$

(Ans:)

5. (d) Sol^n: Equivalent runoff coefficient, $C_e = \frac{\sum_{i=1}^{N} C_i A_i}{A}$

$$\Rightarrow C_e = \frac{[(0.45\times0.85)+(0.25\times0.60)+(0.20\times0.40)+(0.10\times0.70)]}{(0.45+0.25+0.20+0.10)}$$

$\therefore C_e = 0.6825$

Given,

$A = 295\times10^5\ m^2$

$S = 0.0036$

$L = 1.65\ km = 1650\ m$

$T = 20$ years

$K = 6.16$

$x = 0.694$

$a = 0.50$

$m = 0.972$

Now, Time of concentration,

$t_c(min) = 0.01947(L)^{0.77}(S)^{-0.385}$

$= 0.01947\times(1650)^{0.77}\times(0.0036)^{-0.385}$

$= 51$ min

≈ 0.85 hours

Let, Duration of rainfall, $D = t_c$

$= 0.85$ hours

We know,

Intensity of rainfall, $i_{(cm/hr)} = \frac{KT^x}{(D+a)^m} = \frac{6.16\times(20)^{0.694}}{(0.85+0.50)^{0.972}}$

$= 36.8$ cm/hr

Peak Discharge, $Q_P = C\,i\,A$

$= 0.6825\times\frac{36.8\times295\times10^5}{3600\times100}$

$= 2058.12\ m^3/s$ (Ans:)

"2009-10"

5.(d) Sol^n: Equivalent runoff coefficient, $C_e = \frac{\sum_{i=1}^{N} C_i A_i}{A}$

$$\Rightarrow C_e = \frac{0.50\times0.65+0.20\times0.95+0.30\times0.40}{0.50+0.20+0.30} = 0.635$$

Time of concentration, $t_c = 0.01947\, L^{0.77} S^{-0.385}$

$$\Rightarrow t_c = 0.01947\times(2200)^{0.77}\times(0.0025)^{-0.385}$$

$\therefore t_c = 73.25$ min

From given table, for $t_c = 73.25$ min we get maximum depth of rainfall $= \left[85 + \frac{96-85}{75-60}\times(73.25-60)\right] = 94.7167$ mm

$$\text{Rainfall intensity} = \frac{\text{Depth of Rainfall}}{\text{Rainfall Duration}} = \frac{94.7167\times\frac{1}{1000}\text{ m}}{73.25\times60\text{sec}}$$

$$\therefore i = 2.1551\times10^{-5}\text{ m/s}$$

Now, Peak Flow Rate, $Q_p = C\,i\,A$

$$= 0.635\times\underbrace{2.1551\times10^{-5}}_{\text{m/s}}\times(550\times10^4\text{ m}^2)$$

$$= 75.27\text{ m}^3/\text{s}$$ (Ans:)

"Infiltration"

Infiltration Index :

$$\phi\text{-index} = \frac{P-R}{t_e}$$

Where,

P = total storm precipitation

R = total storm runoff

t_e = duration of rainfall excess, i.e. the total time in which the rainfall intensity is greater than ϕ-index

Now,

if $i < \phi$-index, the infiltration rate $(f) = i$ (no runoff case)

if $i > \phi$-index, then infiltration rate $(f) = \phi$-index, and the difference between rainfall & infiltration in an interval of time represents the runoff volume in that time.

Example 3.5: A storm with 10.0 cm precipitation produced a direct run off of 5.8 cm. Given the time distribution of the storm as below, estimate the ϕ index of the storm.

Time from start (h)	0	1	2	3	4	5	6	7	8
Incremental Rainfall in each hour (cm)	0	0.4	0.9	1.5	2.3	1.8	1.6	1.0	0.5

Sol^n: Total infiltration = Rainfall - Direct Runoff

$= (10 - 5.8)$ or, 4.2 cm

Assume, t_e = time of rainfall excess = 8h for first trial.

Then, $\phi = \frac{P-R}{t_e} = \frac{(10-5.8)\text{cm}}{8h} = 0.525$ cm/h

But this value of ϕ makes the rainfalls of the first & eighth hour ineffective as their magnitude is less than 0.525 cm/hr. The value of t_e is therefore modified.

Assume, $t_e = 6$ h for the second trial.

Now, $\phi = \frac{P-R}{t_e} = \frac{[(10-0.4-0.5)-5.8]\text{cm}}{6h} = 0.55$ cm/hr (Ans:)

This magnitude is less than the rainfall values for all other instances, so its ok.

"2013-14"

Same
2014-15
2012-13

2.(d) Sol^n: Given, ϕ-index = 2.0 cm/hr ≈ 1.0 cm/30 min

Time from start (min)	30	60	90	120	150	180
Rainfall in 30 min (cm)	1.65	1.8	4.5	3.3	0.3	0.45
Infiltration in 30 min (cm)	1.0	1.0	1.0	1.0	0.3	0.45
Runoff in 30 min (cm)	0.65	0.8	3.5	2.3	0	0

(i) Total Runoff = $0.65 + 0.8 + 3.5 + 2.3 + 0 + 0 = 7.25$ cm

$\therefore$ Total volume of runoff = Runoff × catchment area

$$= \frac{7.25\text{cm}}{100} \times \{3\text{ km}^2 \times (1000)^2\}$$

$$= 217500 \text{ m}^3$$ (Ans:)

(ii) Total Volume of infiltration $= \frac{(1+1+1+1+0.3+0.45)\text{cm}}{100} \times \{3\text{km}^2 \times (1000)^2\}$

$$= 142500 \text{ m}^3$$ (Ans:)

(iii) Duration of rainfall excess = $(30 + 30 + 30 + 30)$ min

= 120 mins or 2 hours

(Ans:)

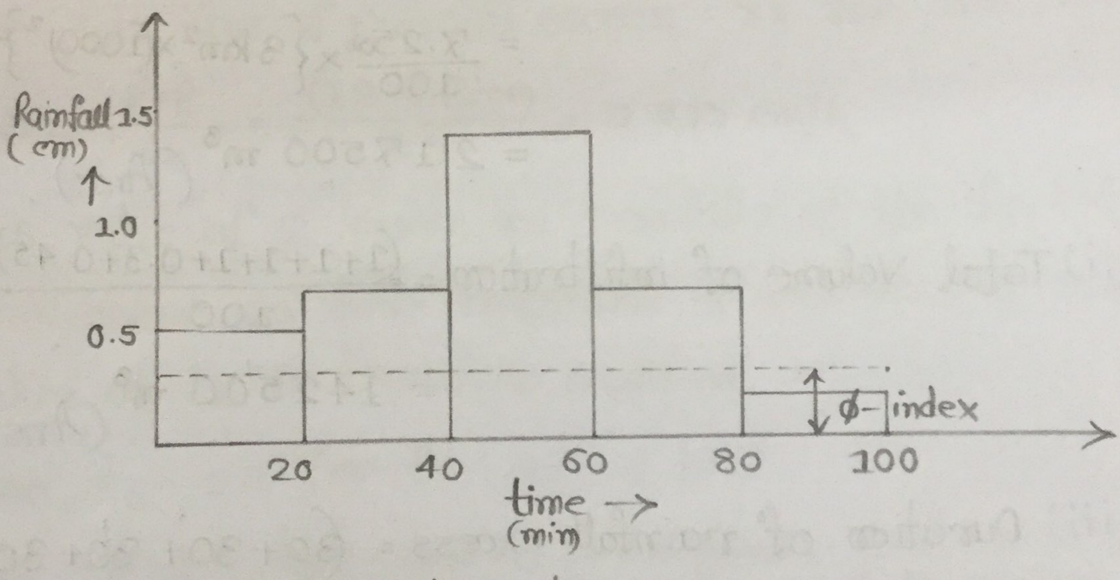
"2009-10"

3.(e) Sol^n: Given, ϕ-index = 0.9cm/hr ≈ 0.3cm/20min

Catchment area, $A = 0.5 km^2 = 5\times10^5 m^2$

Time from start (h)	20	40	60	80	100
Rainfall-Cumulative (cm)	0.5	1.2	2.6	3.3	3.5
Incremental-Rainfall in 20min (cm)	0.5	0.7	1.4	0.7	0.2
Infiltration in 20 min (cm)	0.3	0.3	0.3	0.3	0.2
Runoff in 20 min (cm)	0.2	0.4	1.1	0.4	0

(i)



Hyetograph

(ii) Total surface Runoff = R × A

$$= \frac{(0.2+0.4+1.1+0.4+0)\,cm}{100} \times 5\times10^5 m^2$$

$$= 10500 m^3$$ (Ans:)

"2009-10"

1.(e) Sol^n: (i) Given,

Total amount of rainfall, $P = 12cm$

Runoff coefficient, $= 0.5$

So, total amount of runoff, $R = (12 \times 0.5) = 6\ cm$

Rainfall remaining for the missing portion $= 12 - (0.6 + 3.2 + 2.4 + 0.5 + 1.0 + 1.2)$
$= 3.1\ cm$

(i) Now, equally divide this rainfall $= \frac{3.1cm}{2} = 1.55\ cm$ (Ans:)

Time from start (hours)	1	2	3	4	5	6	7	8
Incremental Rainfall (cm)	0.6	3.2	1.55	2.4	0.5	1.55	1.0	1.2

(ii) Assume, $t_e = 8h$ for first trial.

So, $\phi = \frac{P-R}{t_e} = \frac{(12-6)cm}{8hr} = 0.75\ cm/hr$

But the rainfall value in the first & fifth hour is less than $\phi = 0.75\ cm/hr$.

Now, for second trial, assume, $t_e = 6\ hr$

So, $\phi = \frac{P-R}{t_e} = \frac{(12-6-0.6-0.5)cm}{6\ hr} = 0.8167\ cm/hr$

(Ans:)

"Hydrographs"

Effective Rainfall

K. Subramanya Same 2014-15 4(b)

Example 6.2: Rainfall of magnitude 3.8 cm and 2.8 cm occurring on two consecutive 4-h durations on a catchment of area 27 km^2 produced the following hydrograph of flow at the outlet of the catchment. Estimate the rainfall excess and ϕ-index.

Time from start of rainfall (h)	-6	0	6	12	18	24	30	36	42	48	54	60	66
Observed flow (m^3/s)	6	5	13	26	21	16	12	9	7	5	5	4.5	4.5

Sol^n: By inspection, direct runoff starts at $t = 0$, has the peak at $t = 12$ h and ends at $t = 48$ h (as the flow becomes constant afterwards)

A straight line base flow separation gives a constant value of 5 m^3/s for the base flow.

excluded from DRH   excluded from DRH

Time from start of rainfall (h)	-6	0	6	12	18	24	30	36	42	48	54	60	66
Observed Flow (m^3/s)	6	5	13	26	21	16	12	9	7	5	5	4.5	4.5
Base Flow (m^3/s)	5	5	5	5	5	5	5	5	5	5	5	5	5
Direct Runoff Hydrograph (m^3/s)	1	0	8	21	16	11	7	4	2	0	0	-0.5	-0.5

$$\text{Area of DRH} = (6\,hr \times 60 \times 60)\,sec \times \left[\frac{1}{2}\times 8 + \frac{8+21}{2} + \frac{21+16}{2} + \frac{16+11}{2} + \frac{11+7}{2} + \frac{7+4}{2} + \frac{4+2}{2} + \frac{1}{2}\times 2\right] m^3/s$$

$$= 1.4904 \times 10^6\ m^3$$

= Total direct runoff due to storm

$$\text{Runoff depth} = \frac{\text{runoff volume}}{\text{catchment area}} = \frac{1.4904 \times 10^6 m^3}{27 \times (1000)^2 m^2} = 0.0552\ m$$

$\approx 5.52\ cm$ (Ans:)

rainfall excess

Total rainfall $= (3.8 + 2.8) = 6.6\ cm$

Duration, $t_e = 8\ h$

$$\text{Now, } \phi\text{-index} = \frac{\text{Rainfall} - \text{Runoff}}{\text{Duration}} = \frac{6.6 - 5.52}{8} = 0.135\ cm/hr$$

Total loss in 4-h rainfall $= (0.135 \times 4) = 0.54\ cm <$ Rainfall.

So, $\phi = 0.135\ cm/hr$ is OK. (Ans:)

Example 6.3: A storm over a catchment of area $5.0\ km^2$ had a duration of 14 hours. The mass curve of rainfall of the storm is as:

Time from start of storm (h)	0	2	4	6	8	10	12	14
Accumulated rainfall (cm)	0	0.6	2.8	5.2	6.6	7.5	9.2	9.6

If the ϕ-index for the catchment is 0.4 cm/hr, determine the effective rainfall hyetograph and the volume of direct runoff from the catchment due to the storm.

Soln: We know, Rainfall − Infiltration = Effective Rainfall

$0.4 \times 2 = 0.8$ cm

Time from start of storm (h)	Time interval Δt (h)	Accumulated rainfall in time t (cm)	Depth of rainfall in Δt (cm)	Infiltration in Δt time $= \phi \Delta t$ (cm)	Effective Rainfall (ER) (cm)	Intensity of ER (cm/h)
0	–	0	–	–	–	–
2	2	0.6	0.6	0.8	0	0
4	2	2.8	2.2	0.8	1.4	0.7
6	2	5.2	2.4	0.8	1.6	0.8
8	2	6.6	1.4	0.8	0.6	0.3
10	2	7.5	0.9	0.8	0.1	0.05
12	2	9.2	1.7	0.8	0.9	0.45
14	2	9.6	0.4	0.8	0	0

Total Effective Rainfall $\Sigma = 4.6$ cm (Ans:)

Volume of direct runoff = ER × Catchment Area

$$= \frac{4.6}{100}\,m \times \{5 \times (1000)^2\}\,m^2$$

$$= 230000\ m^3$$

(Ans:)

"Unit Hydrograph"

Example 6.5 (Example 6.4 is Same) : Two storms of 6-h duration and having rainfall excess values of 3.0 and 2.0 cm respectively occur successively. The 2 cm ER rain follows the 3-cm ER rain. The 6-h unit hydrograph for the catchment is given. Calculate the resulting DRH.

Time (h)	Ordinate of 6-h unit hydrograph (m^3/s)	
0	0	
3	25	
6	50	
9	85	
12	125	
15	160	
18	185	
24	160	
30	110	
36	60	
42	36	
48	25	
54	16	
60	8	
69	0	

Sol^n:

Time (h)	Ordinate of 6-h UH (m^3/s)	Ordinate of 3-cm DRH × 3	Ordinate of 2-cm DRH (lagged by 6 hr) × 2	Ordinate of 5-cm DRH (m^3/s)	Remarks
0	0	0	0	0	
3	25	75	0	75	
6	50	150	0	150	
9	85	255	50	305	
12	125	375	100	475	
15	160	480	170	650	
18	185	555	250	805	
(21)	(172.5)	(517.5)	(320)	(837.5)	Interpolation
24	160	480	370	850	
30	110	330	320	650	
36	60	180	220	400	
42	36	108	120	228	
48	25	75	72	147	
54	16	48	50	98	
60	8	24	32	56	
(66)	(2.7)	(8.1)	(16)	(24.1)	Interpolation
69	0	0	(10.6)	(10.6)	Interpolation
75	0	0	0		

Same 2013-14 1(b) 2012-13 1(b)

Example 6.6: The ordinates of a 6-hour unit hydrograph of a catchment is given below:

Time(h)	0	3	6	9	12	15	18	24	30	36	42	48	54	60	69
Ordinate of 6-h UH	0	25	50	85	125	160	185	160	110	60	36	25	16	8	0

Derive the flood hydrograph in the catchment due to the storm given below:

Time from start of storm(h)	0	6	12	18
Accumulated Rainfall(cm)	0	3.5	11.0	16.5

The storm loss rate(ϕ-index) for the catchment is estimated as 0.25 cm/h. The base flow can be assumed to be 15 m^3/s at the beginning and increasing by 2.0 m^3/s for every 12 hours till the end of the direct-runoff hydrograph.

Soln: Firstly, calculate Effective Rainfall.

Interval	1st 6 hours	2nd 6 hours	3rd 6 hours
Rainfall depth(cm)	3.5	(11-3.5) = 7.5	(16.5-11) = 5.5
Loss @ 0.25 cm/h for 6 h	0.25×6 = 1.50	0.25×6 = 1.50	0.25×6 = 1.50
Effective Rainfall(cm)	2.0	6.0	4.0

We know, ER = Rainfall − Infiltration Loss

Time (h)	Ordinate of 6-h UH (m^3/s)	DRH due to 2cm ER ×2	DRH due to 6cm ER ×6	DRH due to 4cm ER ×4	Ordinates of Final DRH	Base flow (m^3/s)	Ordinates of Flood Hydrograph (m^3/s)
0	0	0	0	0	0	15	15
3	25	50	0	0	50	15	65
6	50	100	0	0	100	15	115
9	85	170	150	0	320	15	335
12	125	250	300	0	550	17	567
15	160	320	510	100	930	17	947
18	185	370	750	200	1320	17	1337
(21)	(172.5)	(345)	960	340	1645	(17)	1662
24	160	320	1110	500	1930	19	1949
(27)	(135)	270	(1035)	640	1945	(19)	1964
30	110	220	960	740	1920	19	1939
36	60	120	660	640	1420	21	1441
42	36	72	360	440	872	21	893
48	25	50	216	240	506	23	529
54	16	32	150	144	326	23	349
60	8	16	96	100	212	25	237
66	(2.7)	(5.4)	48	64	117	25	142
69	0	0	—	—	—	—	—
72		0	16	32	48	27	75
75		0	0	—	—	—	—
78		0	0	10.8	(11)	27	49
81				0	0	27	27
84						27	27

Same 2014-15 4(a)
2013-14 1(a)
2012-13 1(c)

Example 6.7: Following are the ordinates of a storm hydrograph of a river draining a catchment area of 423 km^2 due to a 6-h isolated storm. Derive the ordinates of a 6-h unit hydrograph for the catchment.

Time from start of storm (h)	-6	0	6	12	18	24	30	36	42	48
Discharge (m^3/s)	10	10	30	87.5	115.5	102.5	85.0	71.0	59.0	47.5
Time from start of storm (h)	54	60	66	72	78	84	90	96	102	
Discharge (m^3/s)	39.0	31.5	26.0	21.5	17.5	15.0	12.5	12.0	12.0	

Soln: Beginning of DRH at $t = 0$ } By inspection
End of DRH at $t = 90h$

Base flow increases by $\left(\frac{12.5-10}{90} \times 6\right) = 0.17\ m^3/s$ every 6 hr. starting from $10\ m^3/s$ at $t=0$ & ends to $12.5\ m^3/s$ at $t = 90h$

Volume of DRH $= (6hr \times 60 \times 60)sec \times \Sigma$ DRH Ordinates

$= (21600 \times 587)\ m^3 = 12.68 \times 10^6\ m^3$

Drainage Area $= 423\ km^2 = 423 \times 10^6\ m^2$

Runoff Depth = ER Depth $= \frac{\text{Volume of DRH}}{\text{Drainage Area}} = \frac{12.68 \times 10^6 m^3}{423 \times 10^6 m^2} = 0.03m$

$= 3cm$

By dividing DRH values by 3cm we get 6-h UH

Time from beginning of storm (h)	Ordinate of flood hydrograph (m^3/s)	Base Flow (m^3/s)	Ordinate of DRH (m^3/s)	Ordinate of 6-h unit hydrograph ÷ 3 (m^3/s)
-6	10.0	10	0	0
0	10.0	10	0	0
6	30.0	10.17	19.83	6.61
12	87.5	10.33	77.17	25.723
18	111.5	10.5	101.0	33.67
24	102.5	10.67	91.83	30.61
30	85.0	10.83	74.17	24.723
36	71.0	11.0	60.0	20.0
42	59.0	11.17	47.83	15.943
48	47.5	11.33	36.17	12.0567
54	39.0	11.5	27.5	9.167
60	31.5	11.67	19.83	6.61
66	26.0	11.83	14.17	4.723
72	21.5	12.0	9.5	3.167
78	17.5	12.17	5.33	1.7767
84	15.0	12.33	2.67	0.89
90	12.5	12.50	0	0
96	12.0	12.0	0	0
102	12.0	12.0	0	0

$\Sigma = 587$

Example 6.8 (a) The peak of flood hydrograph due to a 3-h duration isolated storm in a catchment is 270 m^3/s. The total depth of rainfall is 5.9 cm. Assuming an average infiltration loss of 0.3 cm/h and a constant base flow of 20 m^3/s estimate the peak of the 3-h unit hydrograph (UH) of this catchment.

(b) If the area of the catchment is 567 km^2 determine the base width of the 3-h unit hydrograph by assuming it to be triangular in shape.

Solⁿ: (a) Duration of rainfall excess = 3h

Total depth of rainfall = 5.9 cm

Loss @ 0.3 cm/h for 3 h = (0.3×3) = 0.9 cm

So, Rainfall excess = Rainfall − Loss = (5.9 − 0.9) = 5.0 cm

Now,

Peak of flood hydrograph = 270 m^3/s

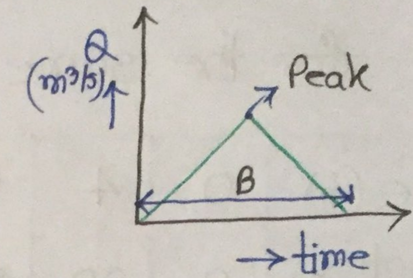
Base flow = 20 m^3/s

∴ Peak of Direct Runoff Hydrograph = 250 m^3/s

$$\text{Peak of 3-h UH} = \frac{\text{Peak of DRH}}{\text{Rainfall excess}} = \frac{250\ m^3/s}{5} = 50\ m^3/s$$

(Ans:)

(b) Let, B = base width of the 3-h UH



Volume represented by the area of UH = volume of 1 cm depth over the catchment

$\Rightarrow$ Area of UH = Area of Catchment × 1 cm

$$\Rightarrow \frac{1}{2} \times B \times (50\ m^3/s \times 60 \times 60)\ m^3/h = \{567\ km^2 \times (1000)^2\}\ m^2 \times \left(\frac{1\ cm}{100}\right) m$$

$\therefore$ B = 63 hours

(Ans:)

Example 6.9: Given the ordinates of a 4-h unit hydrograph as below derive the ordinates of a 12-h unit hydrograph for the same catchment.

Time (h)	0	4	8	12	16	20	24	28	32	36	40	44
Ordinate of 4-h UH	0	20	80	130	150	130	90	52	27	15	5	0

Sol^n:

Time (h)	Ordinates of 4-h UH (m^3/s) A	B Lagged by 4h	C Lagged by 8 h	DRH of 3cm in 12-h (1+1+1 = 3 cm 4+4+4 = 12h) (m^3/s)	Ordinate of 12h - UH (m^3/s)
0	0	0	0	0	0
4	20	0	0	20	6.7
8	80	20	0	100	33.3
12	130	80	20	230	76.7
16	150	130	80	360	120.0
20	130	150	130	410	136.7
24	90	130	150	370	123.3
28	52	90	130	272	90.7
32	27	52	90	169	56.3
36	15	27	52	94	31.3
40	5	15	27	47	15.7
44	0	5	15	20	6.7
48		0	5	5	1.7
52			0	0	0

Example 6.11: Same Problem using S-Curve

Time (h) (1)	Ordinate of 4-h UH (m^3/s) (2)	S-Curve Ordinate (Cumulative) (3)	S-curve lagged by 12h (m^3/s) (4)	Col 3 - Col 4 (5)	Col. 5 / $\left(\frac{12h}{4h}\right)$ = 12-h UH (m^3/s)
0	0	0	0	0	0
4	20	20	0	20	6.7
8	80	100	0	100	33.3
12	130	230	0	230	76.7
16	150	380	20	360	120.0
20	130	510	100	410	136.7
24	90	600	230	370	123.3
28	52	652	380	272	90.7
32	27	679	510	169	56.3
36	15	694	600	94	31.3
40	5	699	652	47	15.7
44	0	699	679	20	6.7
48		699	694	5	1.7
52		699	699	0	0

Same 2014-15 2(b)

Example 6.12: For the values stated in Example 6.9, derive the ordinates of a 2-h unit hydrograph for same catchment.

Sol^n: Common multiple of 4 & 2 is 2, so ordinates at 2-h intervals need to be determined.

Time (h) (1)	Ordinate of 4-h UH (m^3/s) (2)	S-Curve Ordinate (Cumulative) (m^3/s) (3)	S-Curve lagged by 2 h (4)	Col.(3) − Col.(4) DRH of $\left(\frac{2}{4}\right)$ = 0.5 cm (5)	2-h UH ordinates $\frac{\text{Col. (5)}}{\left(\frac{2h}{4h}\right)}$ (m^3/s) (6)
0	0	0	–	0	0
2	10	10	0	10	20
4	20	20	10	10	20
6	50	60	20	40	80
8	80	100	60	40	80
10	105	165	100	65	130
12	130	230	165	65	130
14	140	305	230	75	150
16	150	380	305	75	150
18	140	445	380	65	130
20	130	510	445	65	130
22	110	555	510	45	90
24	90	600	555	45	90
26	71	626	600	26	52
28	52	652	626	26	52
30	39.5	665.5	652	13.5	27

Time (h)	Ordinate of 4-h UH (m^3/s)	S-Curve Ordinate (Cumulative) (m^3/s)	S-Curve lagged by 2h	Col.(3) - Col.(4) DRH of $\frac{2}{4}$ = 0.5 cm	2-h UH Ordinate $\frac{\text{Col.(5)}}{\left(\frac{2h}{4h}\right)}$ (m^3/s)
32	27	679	665.5	13.5	27
34	21	686.5	679	7.5	15
36	15	694	686.5	7.5	15
38	10	696.5	694	2.5	5
40	5	699	696.5	2.5	5
42	2.5	699	699	0	0
44	0	699	699	0	0

"2010-11"

7(c)