Flexural Analysis and Design of Beams

Chapter 4

Introduction

- Fundamental Assumptions
- Simple case of axial loading
- Same assumptions and ideal concept apply

- This chapter includes analysis and design for flexure, dimensioning cross section and reinforcement
- Shear design, bond anchorage, serviceability in chapters 4, 5, 6.

Bending of Homogeneous beam

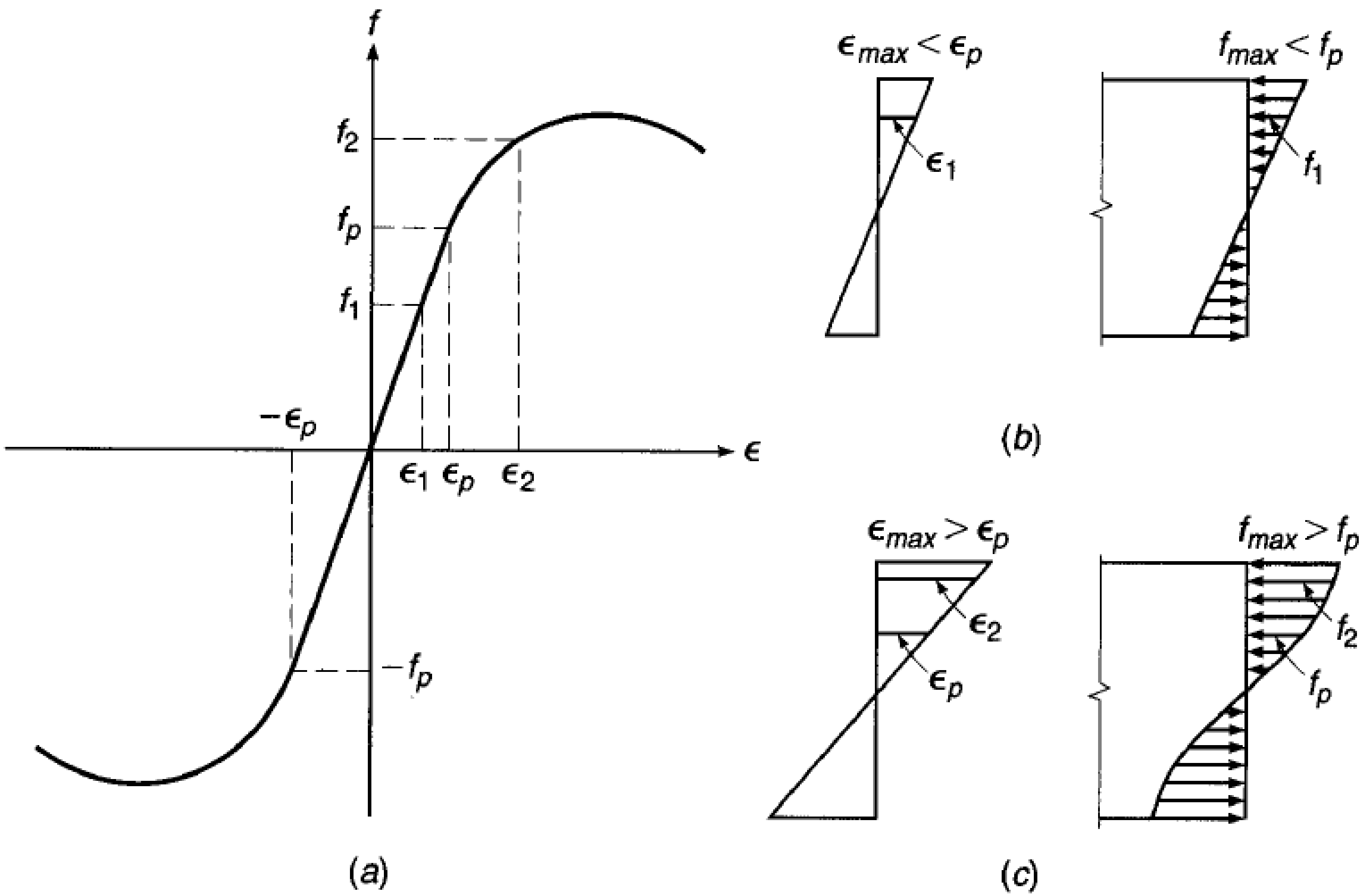
- Steel, timber
- Internal forces-normal and tangential
- Normal-bending/flexural stress-bending moment
- Tangential-shear stress-shear force

Fundamental assumptions relating to flexure and shear

1. Plane cross section remain plane
2. Bending stress *f* at any point depends on the strain at that point
3. Shear stress also depends on cross section and stress-strain diagram. Maximum at neutral axis and zero at extreme fibre. Same horizontal and vertical.
4. The intensity of principal stresses

$$t = \frac{f}{2} \pm \sqrt{\frac{f^2}{4} + \nu^2}$$

where f = intensity of normal fiber stress
ν = intensity of tangential shearing stress


f
f2
fp
f1
−ϵp
ϵ
ϵ1 ϵp ϵ2
−fp
(a)
ϵmax < ϵp
ϵ1
fmax < fp
f1
(b)
ϵmax > ϵp
ϵ2
ϵp
fmax > fp
f2
fp
(c)

5. At neutral axis, only horizontal and vertical shear present-pure shear condition
6. When stress are smaller than proportional limit
   a. Neutral axis = cg
   b. f=My/I
   c. v=VQ/It
   d. Shear distribution parabolic, max at na, zero at outer fibre. For rectangular max=1.5V/bh

Reinforced Concrete Beam Behaviour

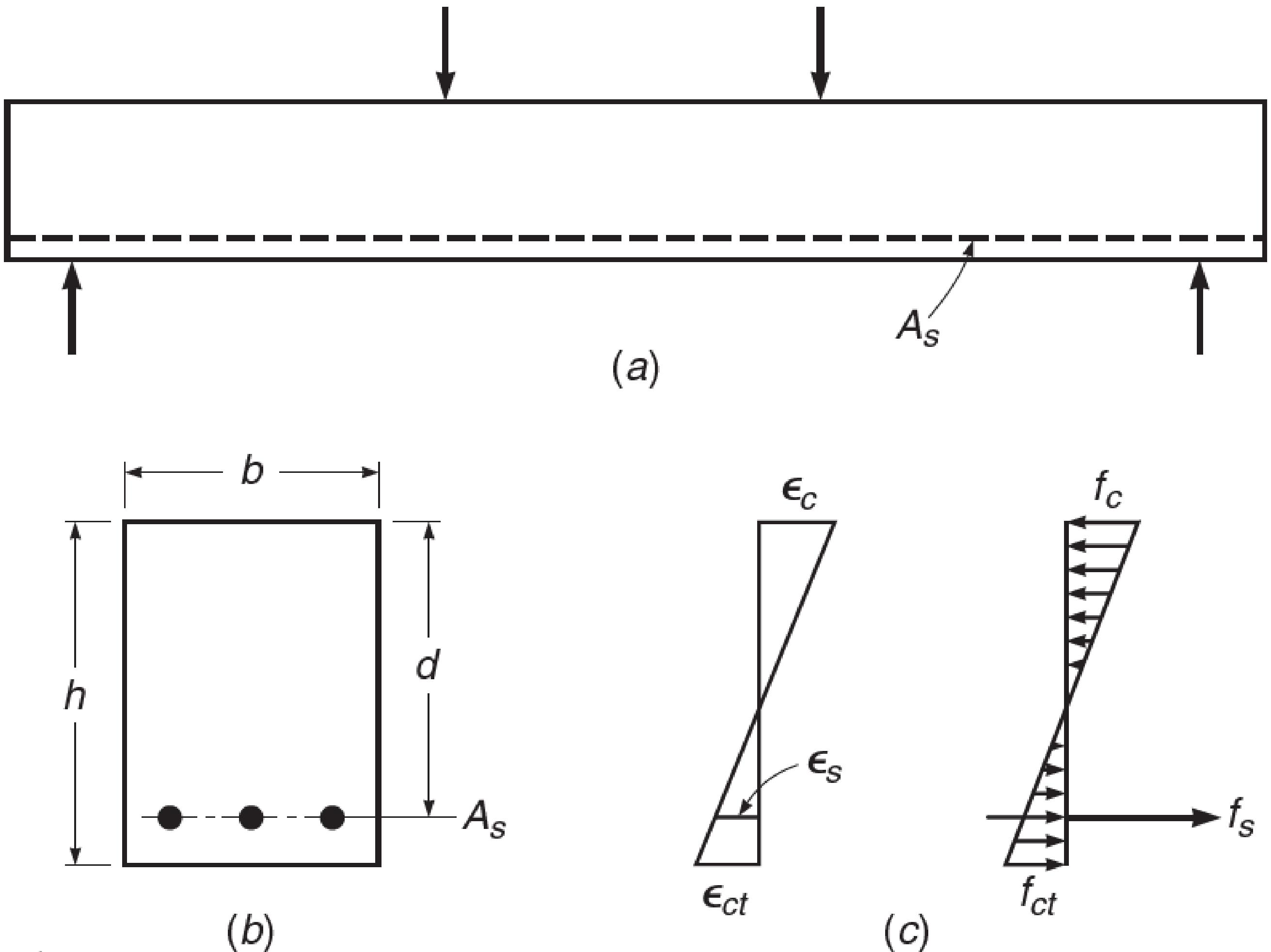


FIGURE 4.1
Behavior of reinforced concrete beam under increasing load.

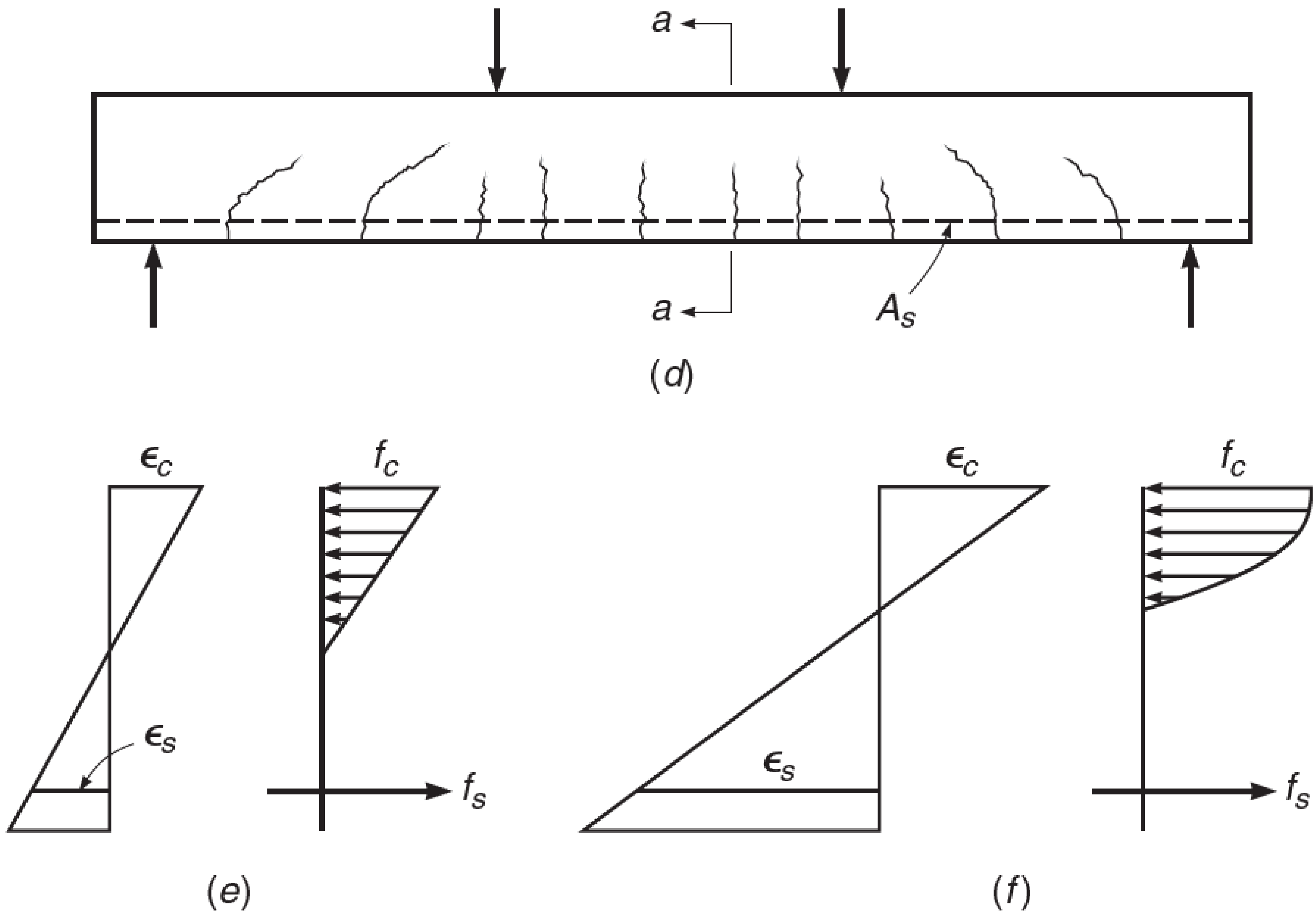


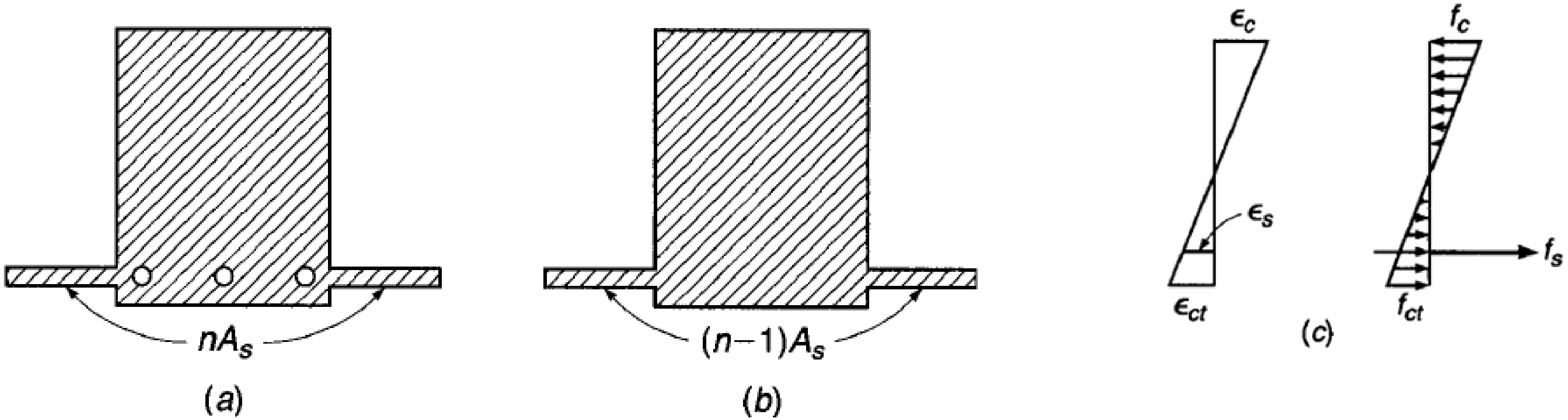
FIGURE 4.1
Behavior of reinforced concrete beam under increasing load.

- Read the article

Video

- See video clips

Stresses elastic, section uncracked



- Tensile stress in concrete is smaller than modulus of rupture

  Transformed section can be used

EXAMPLE 4.1

A rectangular beam has the dimensions (see Fig. 4.3) $b = 10$ in., $h = 25$ in., and $d = 23$ in. and is reinforced with three No. 8 (No. 25) bars so that $A_s = 2.37$ in^2. The concrete compressive strength f'_c is 4000 psi, and the tensile strength in bending (modulus of rupture) is 475 psi. The yield point of the steel f_y is 60,000 psi, the stress-strain curves of the materials being those of Fig. 3.3. Determine the stresses caused by a bending moment $M = 45$ ft-kips.

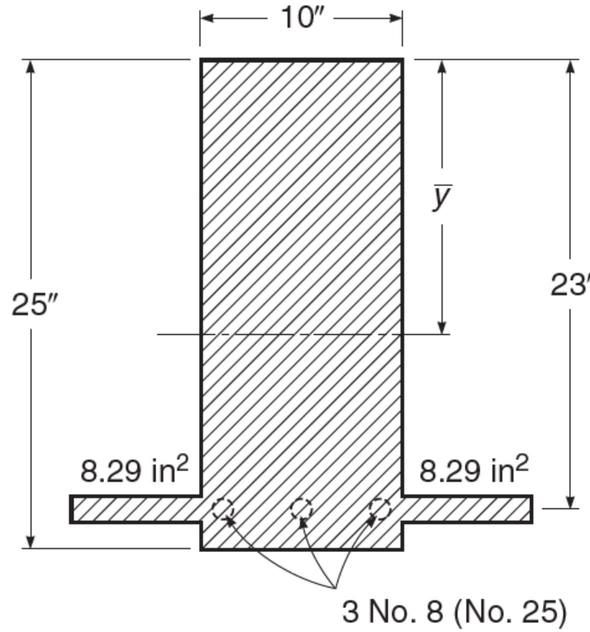
Solution. With a value $n = E_s/E_c = 29{,}000{,}000/3{,}600{,}000 = 8$, one has to add to the rectangular outline an area $(n - 1)A_s = 7 \times 2.37 = 16.59$ in^2, rounded slightly and distributed as shown in Fig. 4.3, to obtain the uncracked, transformed section. Conventional calculations show that the location of the neutral axis of this section is given by $\bar{y} = 13.2$ in. from the top of the section, and its moment of inertia about this axis is 14,740 in^4. For $M = 45$ ft-kips $= 540{,}000$ in-lb, the concrete compression stress at the top fiber is, from Eq. (3.11),

$$f_c = \frac{M\bar{y}}{I} = \frac{540{,}000 \times 13.2}{14{,}740} = 484 \text{ psi}$$

and, similarly, the concrete tension stress at the bottom fiber, 11.8 in. from the neutral axis, is

$$f_{ct} = \frac{540{,}000 \times 11.8}{14{,}740} = 432 \text{ psi}$$

FIGURE 4.3
Transformed beam section of Example 4.1.

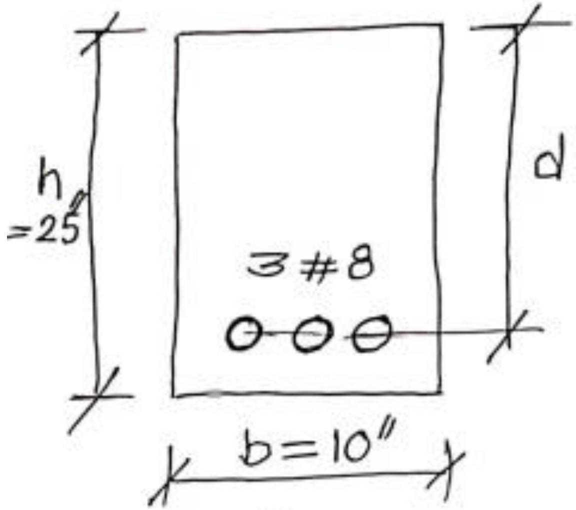


Since this value is below the given tensile bending strength of the concrete, 475 psi, no tension cracks will form, and calculation by the uncracked, transformed section is justified. The stress in the steel, from Eqs. (3.1) and (3.11), is

$$f_s = n\frac{My}{I} = 8\left(\frac{540{,}000 \times 9.8}{14{,}740}\right) = 2870 \text{ psi}$$

By comparing f_c and f_s with the concrete cylinder strength and the yield point, respectively, it is seen that at this stage the actual stresses are quite small compared with the available strengths of the two materials.

Example 4.1



$f_c' = 4\,ksi$, $f_y = 60\,ksi$

Tensile strength of concrete in bending (modulus of rupture)

$= f_r = 7.5\sqrt{f_c'}$ psi

$= 7.5\sqrt{4000} = 475$ psi

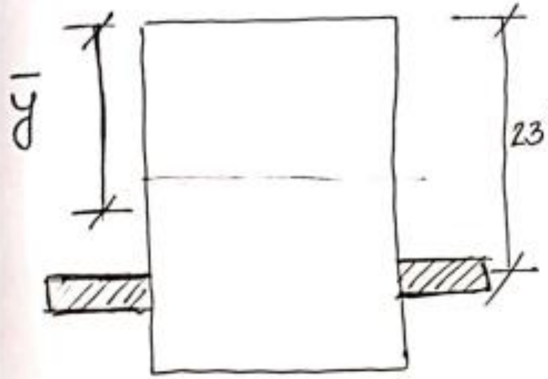
* Determine stresses for $M = 45^{k\text{-}ft}$

Solution

$A_s = 3 * 0.79 = 2.37\ in^2$

$E_s = 29 * 10^6$ psi , $E_c = 57000\sqrt{f_c'}$

$= 3.605 * 10^6$ psi



$n = \frac{E_s}{E_c} = 8$

Equivalent steel area $= (n-1) A_s$
$= 7 * 2.37 = 16.59\ in^2$

Area of transformed section
$= 10 * 25 + 16.59 = 266.59\ in^2$

$\bar{y} = \left(250 * \frac{25}{2} + 16.59 * 23\right) / 266.59$
$= 13.15''$

$I = 10 * \frac{25^3}{12} + 10 * 25 * (13.15 - 12.5)^2 + 16.59 * (23 - 13.15)^2$

$= 14736\ in^4$

Concrete compression stress at top fibre

$$f_c = \frac{My}{I} = \frac{45 * 12000 * 13.15}{14736} = \underline{482 \text{ psi}} \quad (C)$$

Concrete tension stress at bottom fibre

$$f_{ct} = \frac{My}{I} = \frac{45 * 12000 * (25 - 13.15)}{14736} = \underline{434 \text{ psi}} \quad (T)$$

$f_{ct} < f_r \Rightarrow$ Section uncracked.

Tensile stress in steel

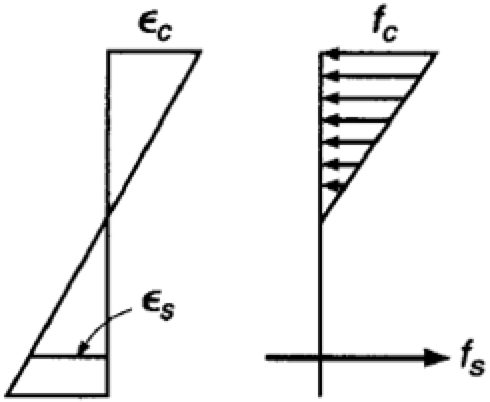
$$= n f_c = n \frac{My}{I}$$

$$= 8 * \frac{45 * 12000 * (23 - 13.15)}{14736} = \underline{2887 \text{ psi}}$$

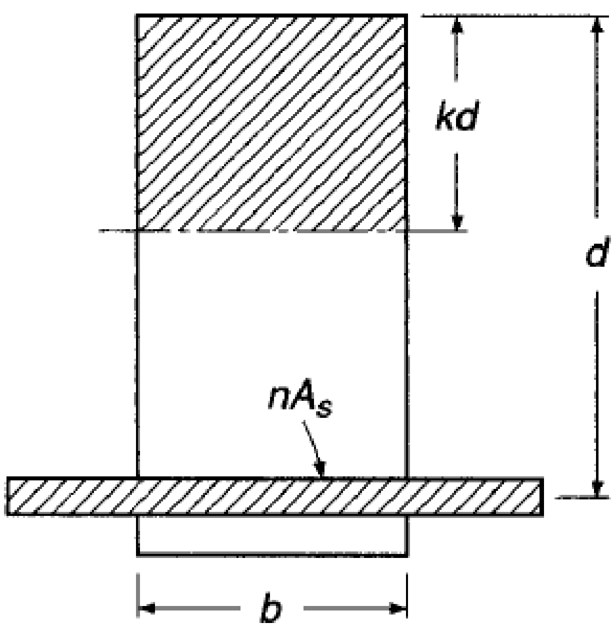
Both concrete and steel stresses are small.

Stresses Elastic, Section cracked

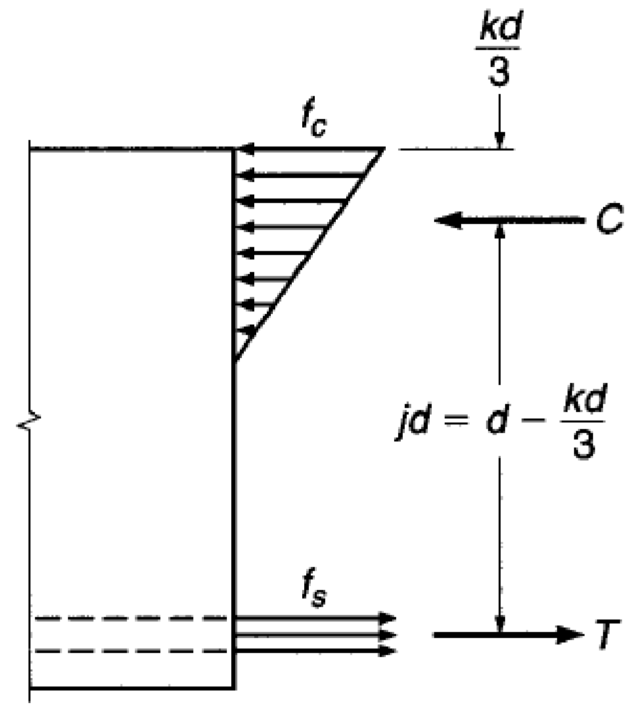
- Concrete tensile stress exceeds mod of rupture
- Concrete compressive stress is less than $f_c'/2$
- Steel stress less than yield
- Assume tension crack up to neutral axis
- Transformed section can still be used


ϵ_c
f_c
ϵ_s
f_s

(e)


kd
d
nA_s
b

(a)


$\frac{kd}{3}$
f_c
C
$jd = d - \frac{kd}{3}$
f_s
T

(b)

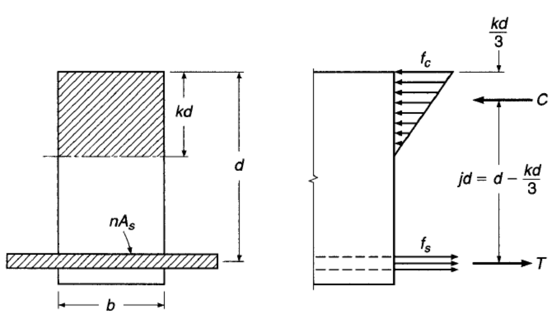
one side of the axis and n times the steel area on the other. The distance to the neutral axis, in this stage, is conventionally expressed as a fraction kd of the effective depth d. (Once the concrete is cracked, any material located below the steel is ineffective, which is why d is the effective depth of the beam.) To determine the location of the neutral axis, the moment of the tension area about the axis is set equal to the moment of the compression area, which gives

Try this differently

$$b\frac{(kd)^2}{2} - nA_s(d - kd) = 0 \tag{4.1}$$

Having obtained kd by solving this quadratic equation, one can determine the moment of inertia and other properties of the transformed section as in the preceding case. Alternatively, one can proceed from basic principles by accounting directly for the forces that act on the cross section. These are shown in Fig. 4.4b. The concrete stress, with maximum value f_c at the outer edge, is distributed linearly as shown. The entire steel area A_s is subject to the stress f_s. Correspondingly, the total compression force C and the total tension force T are

$$C = \frac{f_c}{2}bkd \qquad \text{and} \qquad T = A_s f_s \tag{4.2}$$



The requirement that these two forces be equal numerically has been taken care of by the manner in which the location of the neutral axis has been determined.

Equilibrium requires that the couple constituted by the two forces C and T be equal numerically to the external bending moment M. Hence, taking moments about compression resultant C gives

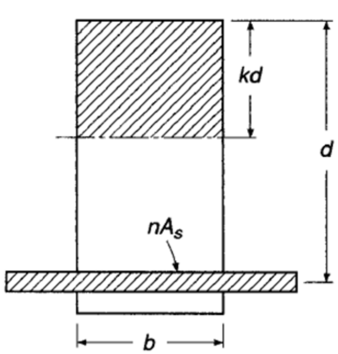
$$M = Tjd = A_s f_s jd \tag{4.3}$$

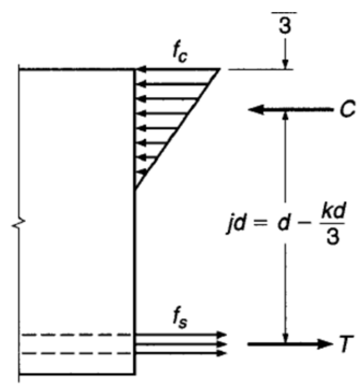
where jd is the internal lever arm between C and T. From Eq. (4.3), the steel stress is

$$f_s = \frac{M}{A_s jd} \tag{4.4}$$

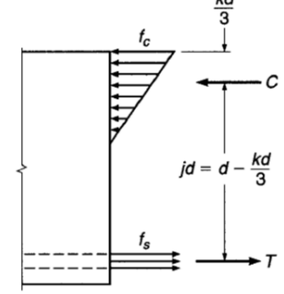
Conversely, taking moments about T gives

$$M = Cjd = \frac{f_c}{2} bkdjd = \frac{f_c}{2} kjbd^2 \tag{4.5}$$





from which the concrete stress is



$$f_c = \frac{2M}{kjbd^2} \tag{4.6}$$

In using Eqs. (4.2) through (4.6), it is convenient to have equations by which k and j may be found directly, to establish the neutral axis distance kd and the internal lever arm jd. First defining the *reinforcement ratio* as

$$\rho = \frac{A_s}{bd} \tag{4.7}$$

then substituting $A_s = \rho bd$ into Eq. (4.1) and solving for k, one obtains

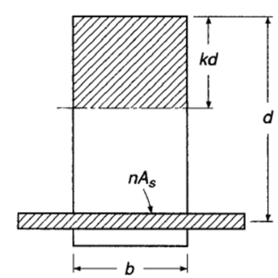
$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n \tag{4.8}$$

Try this

From Fig. 4.4b it is seen that $jd = d - kd/3$, or

$$j = 1 - \frac{k}{3} \tag{4.9}$$

Values of k and j for elastic cracked section analysis, for common reinforcement ratios and modular ratios, are found in Table A.6 of Appendix A.



$$b\frac{(kd)^2}{2} - nA_s(d - kd) = 0 \tag{4.1}$$

EXAMPLE 4.2

The beam of Example 4.1 is subject to a bending moment $M = 90$ ft-kips (rather than 45 ft-kips as previously). Calculate the relevant properties and stresses.

Solution. If the section were to remain uncracked, the tensile stress in the concrete would now be twice its previous value, that is, 864 psi. Since this exceeds by far the modulus of rupture of the given concrete (475 psi), cracks will have formed and the analysis must

be adapted consistent with Fig. 4.4. Equation (4.1), with the known quantities b, n, and A_s inserted, gives the distance to the neutral axis $kd = 7.6$ in., or $k = 7.6/23 = 0.33$. From Eq. (4.9), $j = 1 - 0.33/3 = 0.89$. With these values the steel stress is obtained from Eq. (4.4) as $f_s = 22{,}300$ psi, and the maximum concrete stress from Eq. (4.6) as $f_c = 1390$ psi.

Comparing the results with the pertinent values for the same beam when subject to one-half the moment, as previously calculated, one notices that (1) the neutral axis has migrated upward so that its distance from the top fiber has changed from 13.2 to 7.6 in.; (2) even though the bending moment has only been doubled, the steel stress has increased from 2870 to 22,300 psi, or about 7.8 times, and the concrete compression stress has increased from 484 to 1390 psi, or 2.9 times; (3) the moment of inertia of the cracked transformed section is easily computed to be 5910 in^4, compared with 14,740 in^4 for the uncracked section. This affects the magnitude of the deflection, as discussed in Chapter 7. Thus, it is seen how radical is the influence of the formation of tension cracks on the behavior of reinforced concrete beams.

Example 4.2 Find concrete and steel stresses for $M = 90$ kip-ft.

Since $M > M_{cr}$, Section is now cracked.

Solution

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

$$\rho = \frac{A_s}{bd} = \frac{2.37}{10 * 23} = 0.0103$$

$$\rho n = 0.0824$$

$$k = \sqrt{(.0824)^2 + 2 * .0824} - 0.0824 = 0.332$$

$$kd = 7.63''$$

$$j = 1 - k/3 = 0.889$$

Concrete stress, $f_c = \dfrac{2M}{kjbd^2} = \dfrac{2 * 90 * 12000}{0.332 * 0.889 * 10 * 23^2}$

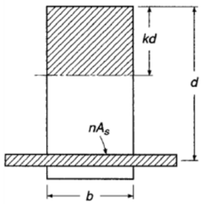
$= 1383$ psi (C)

Steel stress, $f_s = \dfrac{M}{A_s jd} = \dfrac{90 * 12000}{2.37 * 0.889 * 23}$

$= 22{,}286$ psi (T)

$= 22.3$ ksi (T) significantly increased.

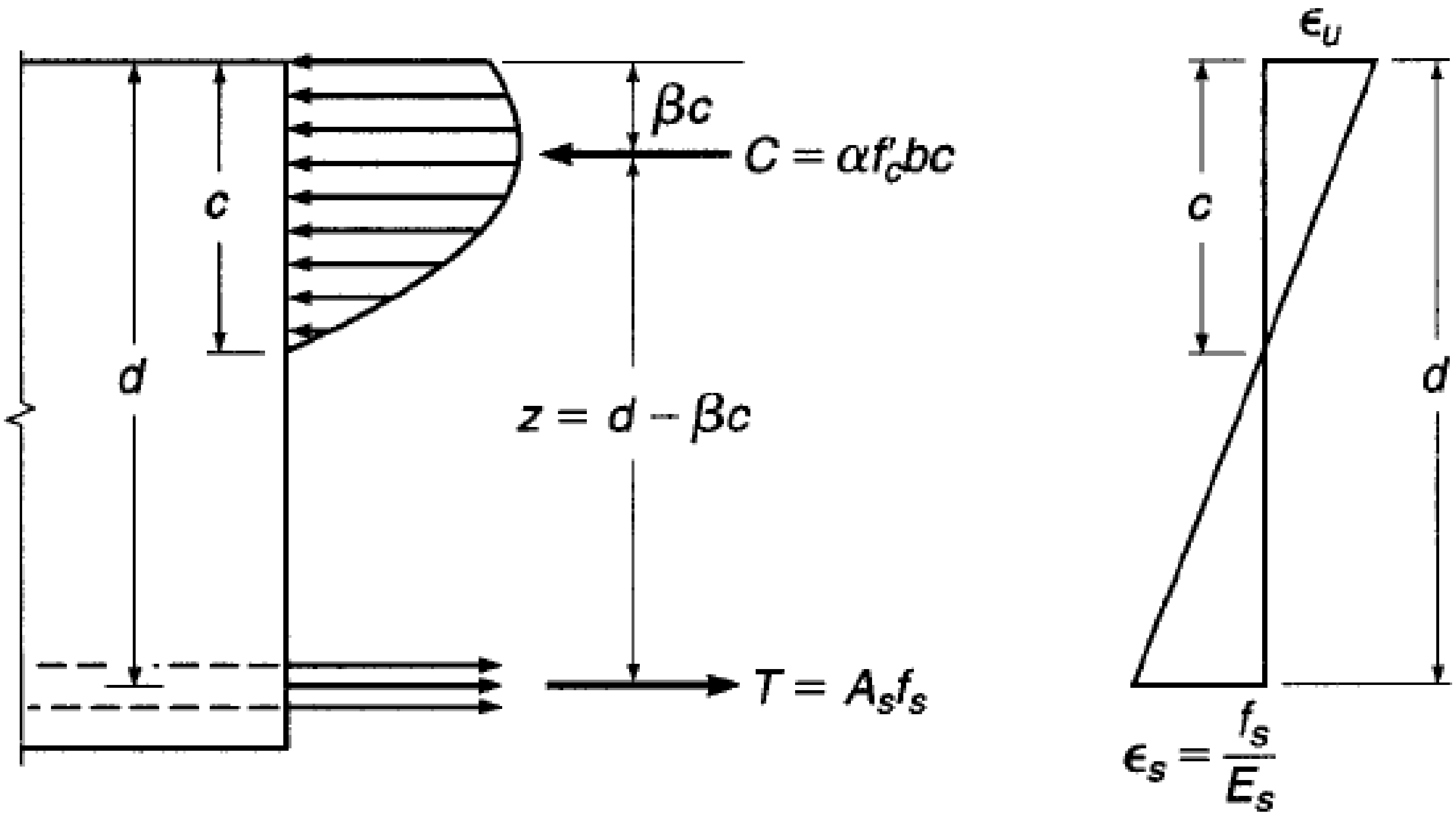
$I = 5910 \text{ in}^4$, check this, greatly reduced.



Find allowable moment M, if allowable stresses are
$f_c = 0.45f_c'$
$f_s = 24$ ksi

Find I cracked
Try the same problem using My/I

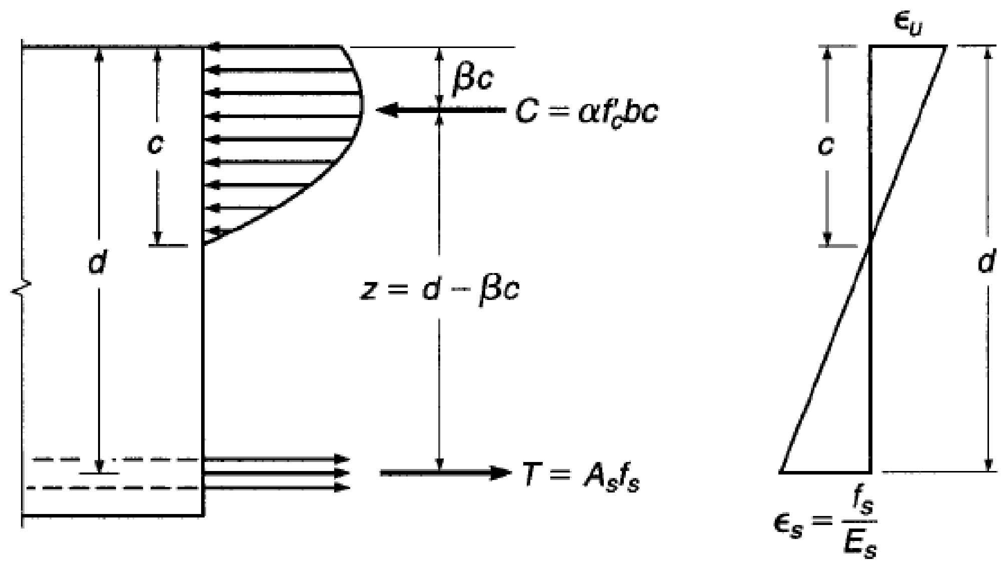
Flexural Strength



- Yielding of steel $f_s=f_y$
- Crushing of concrete ε_u= 0.003-0.004
- Either can reach first
- Exact shape not necessary
- Necessary –Total compressive force and location
- βc- location from comp face

$$\alpha = \frac{f_{av}}{f'_c} \qquad (4.10)$$

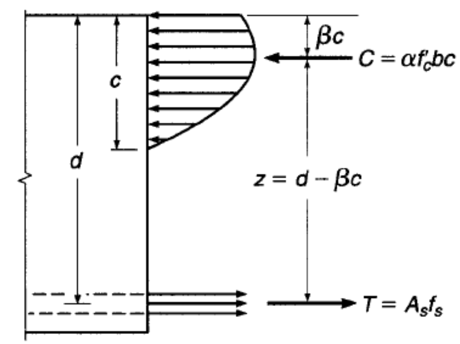
$$C = \alpha f'_c bc \qquad (4.11)$$



α equals 0.72 for $f'_c \leq 4000$ psi and decreases by 0.04 for every 1000 psi above 4000 up to 8000 psi. For $f'_c > 8000$ psi, $\alpha = 0.56$.

β equals 0.425 for $f'_c \leq 4000$ psi and decreases by 0.025 for every 1000 psi above 4000 up to 8000 psi. For $f'_c > 8000$ psi, $\beta = 0.325$.

$$C = T \qquad \text{or} \qquad \alpha f'_c bc = A_s f_s \tag{4.12}$$



Also, the bending moment, being the couple of the forces C and T, can be written as either

$$M = Tz = A_s f_s (d - \beta c) \tag{4.13}$$

or

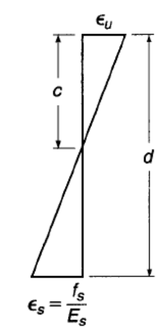
$$M = Cz = \alpha f'_c\, bc\,(d - \beta c) \tag{4.14}$$

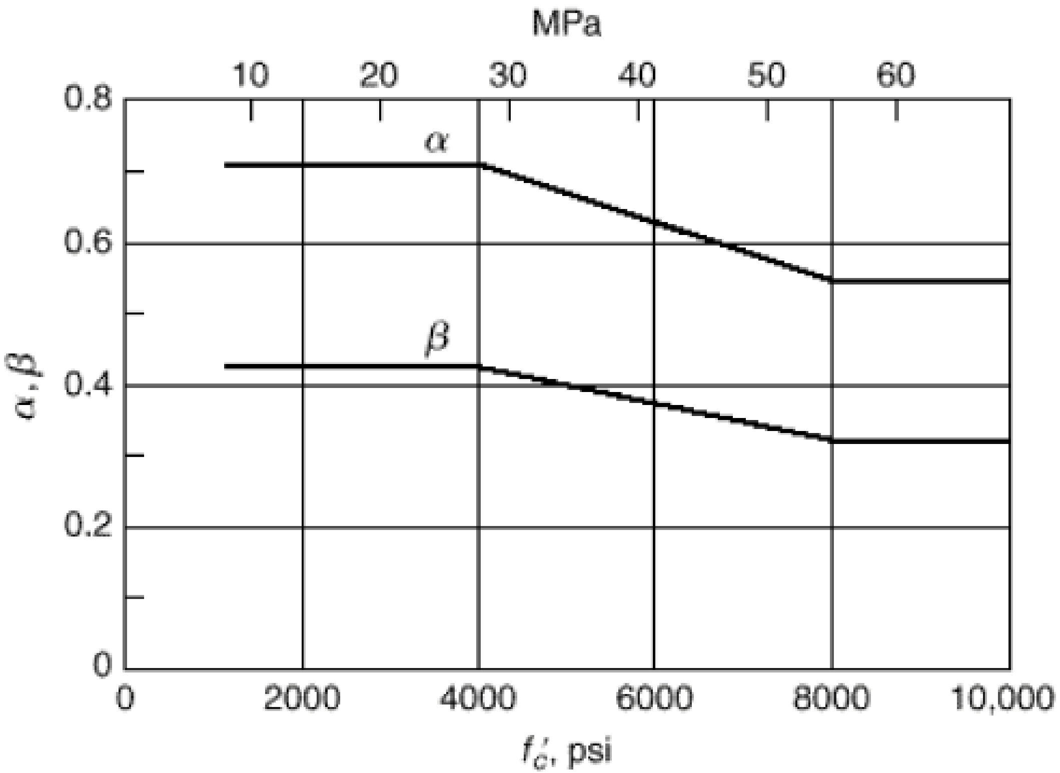
For failure initiated by yielding of the tension steel, $f_s = f_y$. Substituting this value in Eq. (4.12), one obtains the distance to the neutral axis

$$c = \frac{A_s f_y}{\alpha f'_c b} \tag{4.15a}$$

Alternatively, using $A_s = \rho bd$, the neutral axis distance is

$$c = \frac{\rho f_y d}{\alpha f'_c} \tag{4.15b}$$





α equals 0.72 for $f_c' \leq 4000$ psi and decreases by 0.04 for every 1000 psi above 4000 up to 8000 psi. For $f_c' > 8000$ psi, $\alpha = 0.56$.

β equals 0.425 for $f_c' \leq 4000$ psi and decreases by 0.025 for every 1000 psi above 4000 up to 8000 psi. For $f_c' > 8000$ psi, $\beta = 0.325$.

Failure initiated by yielding

For failure initiated by yielding of the tension steel, $f_s = f_y$. Substituting this value in Eq. (4.12), one obtains the distance to the neutral axis

$$c = \frac{A_s f_y}{\alpha f_c' b} \tag{4.15a}$$

Alternatively, using $A_s = \rho bd$, the neutral axis distance is

$$c = \frac{\rho f_y d}{\alpha f_c'} \tag{4.15b}$$

giving the distance to the neutral axis when tension failure occurs. The nominal moment M_n is then obtained from Eq. (4.13) with the value for c just determined, and $f_s = f_y$; that is,

$$M_n = \rho f_y b d^2 \left(1 - \frac{\beta f_y \rho}{\alpha f_c'}\right) \tag{4.16a}$$

With the specific, experimentally obtained values for α and β given previously, this becomes

$$\boxed{M_n = \rho f_y b d^2 \left(1 - 0.59\, \frac{\rho f_y}{f_c'}\right)} \tag{4.16b}$$

Failure by concrete crushing

If, for larger reinforcement ratios, the steel does not reach yield at failure, then the strain in the concrete becomes $\epsilon_u = 0.003$, as previously discussed. The steel stress f_s, not having reached the yield point, is proportional to the steel strain ϵ_s; that is, according to Hooke's law,

$$f_s = \epsilon_s E_s$$

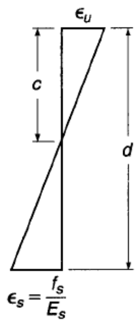
From the strain distribution shown in Fig. 4.5, the steel strain ϵ_s can be expressed in terms of the distance c by evaluating similar triangles, after which it is seen that

$$f_s = \epsilon_u E_s \frac{d-c}{c} \tag{4.17}$$

Then, from Eq. (4.12),

$$\alpha f_c' bc = A_s \epsilon_u E_s \frac{d-c}{c} \tag{4.18}$$

and this quadratic may be solved for c, the only unknown for the given beam. With both c and f_s known, the nominal moment of the beam, so heavily reinforced that failure occurs by crushing of the concrete, may be found from either Eq. (4.13) or Eq. (4.14).

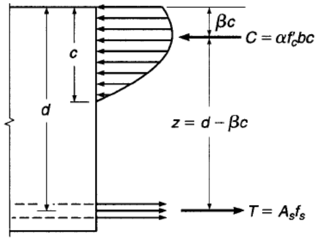


Quadratic equation for c

$$\alpha f_c' bc = A_s f_s$$

$$M = Tz = A_s f_s (d - \beta c) \tag{4.13}$$

$$M = Cz = \alpha f_c' bc (d - \beta c) \tag{4.14}$$

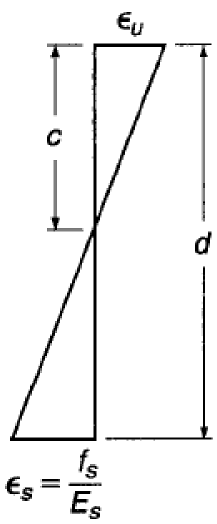


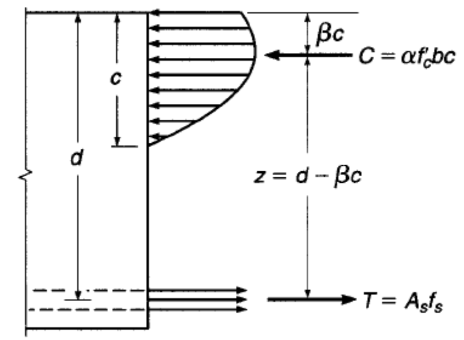
Balanced reinforcement ratio ρ_b

$$c = \frac{\epsilon_u}{\epsilon_u + \epsilon_y} d \tag{4.19}$$

Substituting that value of c into Eq. (4.12), with $A_s f_s = \rho b d f_y$, gives the balanced reinforcement ratio

$$\boxed{\rho_b = \frac{\alpha f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_y}} \tag{4.20}$$





$$C = T \qquad \text{or} \qquad \alpha f_c' bc = A_s f_s \tag{4.12}$$

EXAMPLE 4.3 Determine the nominal moment M_n at which the beam of Examples 4.1 and 4.2 will fail.

Solution. For this beam the reinforcement ratio $\rho = A_s/(bd) = 2.37/(10 \times 23) = 0.0103$. The balanced reinforcement ratio is found from Eq. (4.20) to be 0.0284. Since the amount of steel in the beam is less than that which would cause failure by crushing of the concrete, the beam will fail in tension by yielding of the steel. Its nominal moment, from Eq. (4.16*b*), is

$$M_n = \rho f_y bd^2 \left(1 - 0.59 \frac{\rho f_y}{f'_c}\right) = 0.0103 \times 60{,}000 \times 10 \times 23^2 \left(1 - 0.59 \frac{0.0103 \times 60{,}000}{4000}\right)$$

$$= 2{,}970{,}000 \text{ in-lb} = 248 \text{ ft-kips}$$

When the beam reaches M_n, the distance to its neutral axis, from Eq. (4.15*b*), is

$$c = \frac{\rho f_y d}{\alpha f'_c} = \frac{0.0103 \times 60{,}000 \times 23}{0.72 \times 4000} = 4.94$$

$$\boxed{\rho_b = \frac{\alpha f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_y}} \qquad (4.20)$$

It is informative to compare this result with those of Examples 4.1 and 4.2. In the previous calculations, it was found that at low loads, when the concrete had not yet cracked in tension, the neutral axis was located at a distance of 13.2 in. from the compression edge; at higher loads, when the tension concrete was cracked but stresses were still sufficiently small to be elastic, this distance was 7.6 in. Immediately before the beam fails, as has just been shown, this distance has further decreased to 4.9 in. For these same stages of loading, the stress in the steel increased from 2870 psi in the uncracked section to 22,300 psi in the cracked elastic section and to 60,000 psi at the nominal moment capacity. This migration of the neutral axis toward the compression edge and the increase in steel stress as load is increased is a graphic illustration of the differences between the various stages of behavior through which a reinforced concrete beam passes as its load is increased from zero to the value that causes it to fail. The examples also illustrate the fact that nominal moments cannot be determined accurately by elastic calculations.

Design of Tension-reinforced Rectangular Beams

- Demand < Capacity

$$M_u \leq \phi M_n$$

$$P_u \leq \phi P_n$$

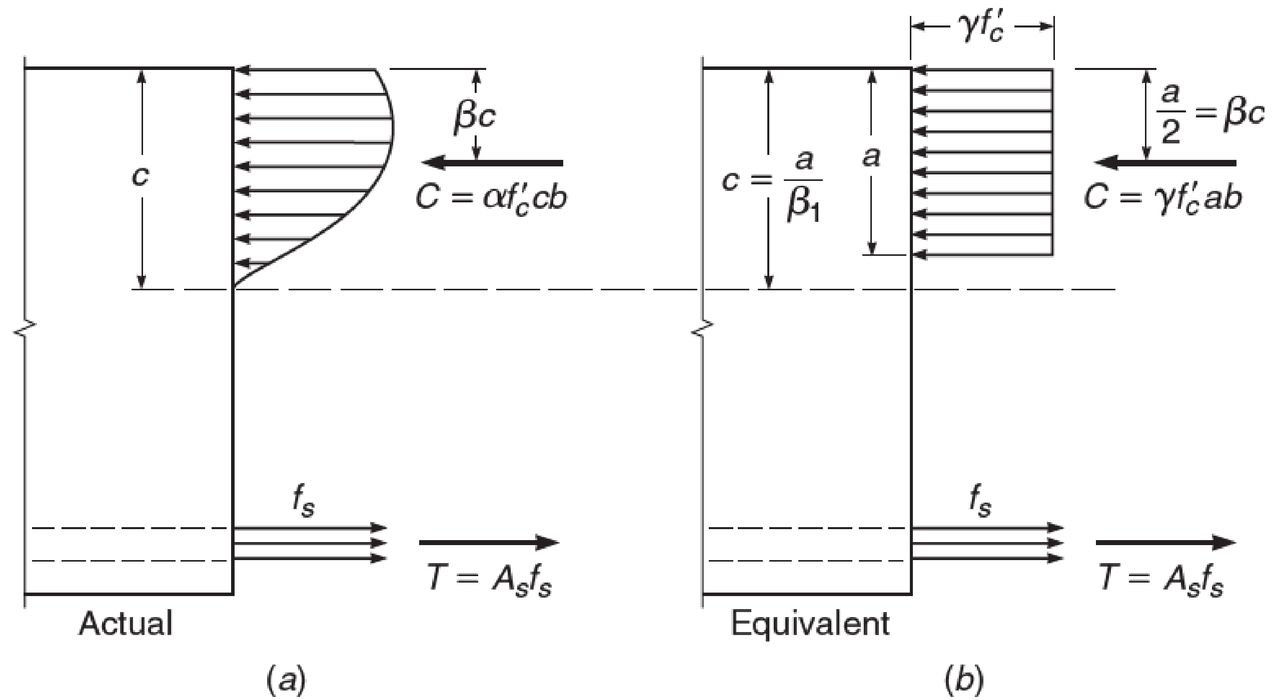
$$V_u \leq \phi V_n$$

- Hypothetical overload stage/demand with load factor
- Reduced capacity with strength reduction factor

- USD method of design- ACI 2008, BNBC 2020
- Limit states Design –Europe
- ULS, SLS limit states

Equivalent Rectangular Stress Distribution

FIGURE 4.7
Actual and equivalent rectangular stress distributions at ultimate load.



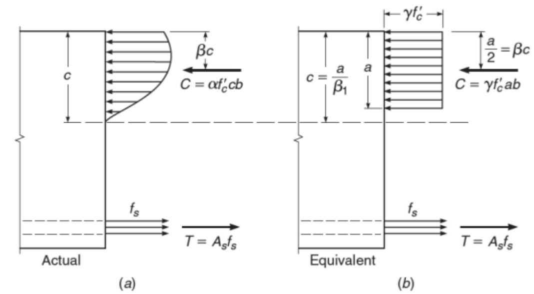
It is seen that the actual stress distribution is replaced by an equivalent one of simple rectangular outline. The intensity $\gamma f'_c$ of this equivalent constant stress and its depth $a = \beta_1 c$ are easily calculated from the two conditions that (1) the total compression force C and (2) its location, that is, distance from the top fiber, must be the same in the equivalent rectangular as in the actual stress distribution. From Fig. 4.7*a* and *b* the first condition gives

$$C = \alpha f'_c cb = \gamma f'_c ab \qquad \text{from which} \qquad \gamma = \alpha \frac{c}{a}$$

C S Whitney

TABLE 4.1
Concrete stress block parameters

	f'_c, psi				
	≤ 4000	5000	6000	7000	≥ 8000
α	0.72	0.68	0.64	0.60	0.56
β	0.425	0.400	0.375	0.350	0.325
$\beta_1 = 2\beta$	0.85	0.80	0.75	0.70	0.65
$\gamma = \alpha/\beta_1$	0.85	0.85	0.85	0.86	0.86



With $a = \beta_1 c$, this gives $\boxed{\gamma = \alpha/\beta_1}$. The second condition simply requires that in the equivalent rectangular stress block, the force C be located at the same distance βc from the top fiber as in the actual distribution. It follows that $\beta_1 = 2\beta$.

To supply the details, the upper two lines of Table 4.1 present the experimental evidence of Fig. 4.6 in tabular form. The lower two lines give the just-derived parameters β_1 and γ for the rectangular stress block. It is seen that the intensity factor for compressive stress γ is essentially independent of f'_c and can be taken as 0.85 throughout. Hence, regardless of f'_c, the concrete compression force at failure in a rectangular beam of width b is

$$\boxed{C = 0.85 f'_c ab} \tag{4.21}$$

$C = \alpha f'_c cb = \gamma f'_c ab$

$\gamma = \alpha \frac{c}{a} = \alpha \frac{c}{\beta_1 c} = \frac{\alpha}{\beta_1}$ $\quad \boxed{\gamma = \frac{\alpha}{\beta_1}}$

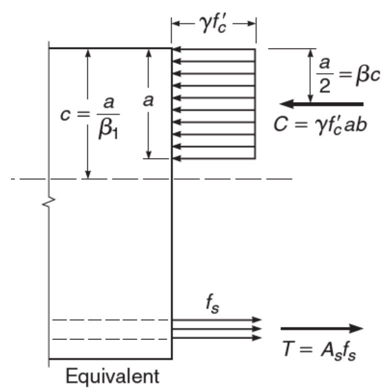
$\frac{a}{2} = \beta c \Rightarrow \frac{\beta_1 c}{2} = \beta c$

$\boxed{\beta_1 = 2\beta}$

Also, for the common concretes with $f_c' \le 4000$ psi, the depth of the rectangular stress block is $a = 0.85c$, with c being the distance to the neutral axis. For higher-strength concretes, this distance is $a = \beta_1 c$, with the β_1 values shown in Table 4.1. This is expressed as follows: For f_c' between 2500 and 4000 psi, β_1 shall be taken as 0.85; for f_c' above 4000 psi, β_1 shall be reduced linearly at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi, but β_1 shall not be taken as less than 0.65. In mathematical terms, the relationship between β_1 and f_c' can be expressed as

$$\boxed{\beta_1 = 0.85 - 0.05\frac{f_c' - 4000}{1000}} \qquad \text{and} \qquad 0.65 \le \beta_1 \le 0.85 \tag{4.22}$$

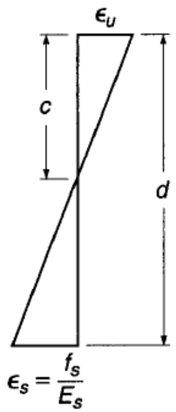
$$a = \beta_1 c$$



(b)

The failure criteria, of course, are the same as before: yielding of the steel at $f_s = f_y$ or crushing of the concrete at $\epsilon_u = 0.003$. Because the rectangular stress block is easily visualized and its geometric properties are extremely simple, many calculations are carried out directly without reference to formally derived equations, as will be seen in the following sections.

Balanced Strain condition



A reinforcement ratio ρ_b producing balanced strain conditions can be established based on the condition that, at balanced failure, the steel strain is exactly equal to ϵ_y when

the strain in the concrete simultaneously reaches the crushing strain of $\epsilon_u = 0.003$. Referring to Fig. 4.5,

$$c = \frac{\epsilon_u}{\epsilon_u + \epsilon_y} d \tag{4.23}$$

which is seen to be identical to Eq. (4.19). Then from the equilibrium requirement that $C = T$

$$\rho_b f_y bd = 0.85 f_c' ab = 0.85 \beta_1 f_c' bc$$

from which

$$\rho_b = 0.85\, \beta_1 \frac{f_c'}{f_y} \frac{c}{d} = \boxed{0.85 \beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_y}} \tag{4.24}$$

This is easily shown to be equivalent to Eq. (4.20).

Underreinforced beam

- Compression failure is abrupt
- Tensile failure gradual
- ρ should be less than ρ_b

A compression failure in flexure, should it occur, gives little if any warning of distress, while a tension failure, initiated by yielding of the steel, typically is gradual. Distress is obvious from observing the large deflections and widening of concrete cracks associated with yielding of the steel reinforcement, and measures can be taken to avoid total collapse. In addition, most beams for which failure initiates by yielding possess substantial strength based on strain-hardening of the reinforcing steel, which is not accounted for in the calculations of M_n.

Because of these differences in behavior, it is prudent to require that beams be designed such that failure, if it occurs, will be by yielding of the steel, not by crushing of the concrete. This can be done, theoretically, by requiring that the reinforcement ratio ρ be less than the balance ratio ρ_b given by Eq. (4.24).

ρ should be less than $\rho_{b,}$ why?

In actual practice, the upper limit on ρ should be below ρ_b for the following reasons: (1) for a beam with ρ exactly equal to ρ_b, the compressive strain limit of the concrete would be reached, theoretically, at precisely the same moment that the steel reaches its yield stress, without significant yielding before failure; (2) material properties are never known precisely; (3) strain-hardening of the reinforcing steel, not accounted for in design, may lead to a brittle concrete compression failure even though ρ may be somewhat less than ρ_b; (4) the actual steel area provided, considering standard reinforcing bar sizes, will always be equal to or larger than required, based on selected reinforcement ratio ρ, tending toward overreinforcement; and (5) the extra ductility provided by beams with lower values of ρ increases the deflection capability substantially and thus provides warning prior to failure.

ACI provisions for underreinforced beam

- ACI establishes some safe limits
- Net tensile strain ϵ_t at farthest from comp face
- Strength reduction factor φ

d. ACI Code Provisions for Underreinforced Beams

While the nominal strength of a member may be computed based on principles of mechanics, the mechanics alone cannot establish safe limits for maximum reinforcement ratios, as discussed in Chapter 3. These limits are defined by the ACI Code. The limitations take two forms. First, the Code addresses the minimum tensile reinforcement strain allowed at nominal strength in the design of beams. Second, the Code defines strength reduction factors that may depend on the tensile strain at nominal strength. Both limitations are based on the *net tensile strain* ϵ_t of the reinforcement farthest from the compression face of the concrete at the depth d_t. The net tensile strain is exclusive of prestress, temperature, and shrinkage effects. For beams with a single layer of reinforcement, the depth to the centroid of the steel d is the same as d_t. For beams with multiple layers of reinforcement, d_t is greater than the depth to the centroid of the reinforcement d. Substituting d_t for d and ϵ_t for ϵ_y in Eq. (4.23), the net tensile strain may be represented as

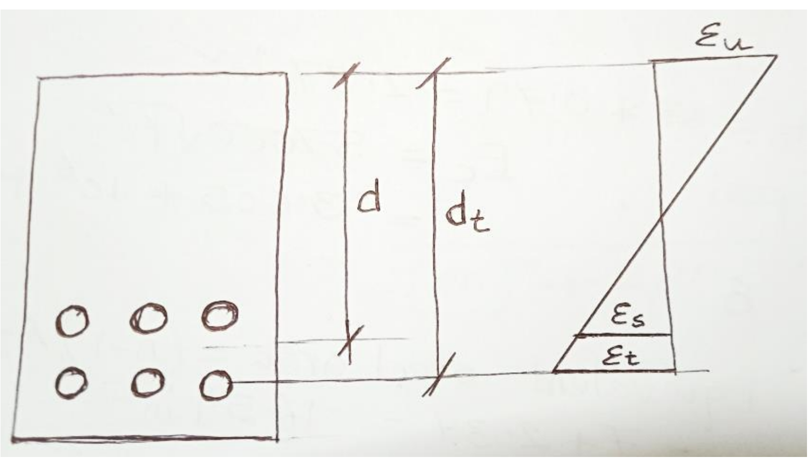
$$\boxed{\epsilon_t = \epsilon_u \frac{d_t - c}{c}} \tag{4.25}$$

Then based on Eq. (4.24), the reinforcement ratio to produce a selected value of net tensile strain is

$$\rho = 0.85\beta_1 \frac{f'_c}{f_y} \frac{d_t}{d} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} \tag{4.26a}$$

or somewhat conservatively

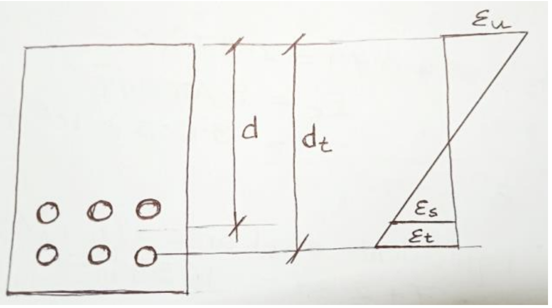
$$\boxed{\rho = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t}} \tag{4.26b}$$



To ensure truly underreinforced behavior, ACI Code Table 21.2.2 establishes a minimum net tensile strain ϵ_t at nominal member strength of 0.005 for beams subjected to axial loads less than $0.10 f'_c A_g$, where A_g is the gross area of the cross section. The ACI Code defines members that meet this requirement, that is, $\epsilon_t \geq 0.005$, as *tension-controlled*. The corresponding strength reduction factor is $\phi = 0.9$. The requirement that $\epsilon_t \geq 0.005$ applies to all grades of reinforcing steel, including prestressing steel. The ACI Code additionally defines *compression-controlled* members as those having a net tensile strain $\epsilon_t \leq \epsilon_y$. The strength reduction factor for compression-controlled members is 0.65; a value of $\phi = 0.75$ may be used if the members are spirally reinforced. Members with net tensile strains between ϵ_y and 0.005 are classified as *transition*, and the ACI Code allows a linear interpolation of ϕ based on ϵ_t, as shown in Fig. 4.8. For the purposes of defining compression-controlled members and calculating ϕ, ACI Code 21.2.2.1 permits a value of $\epsilon_y = 0.002$ to be used for Grade 60 reinforcement, in place of the calculated value 0.00207.

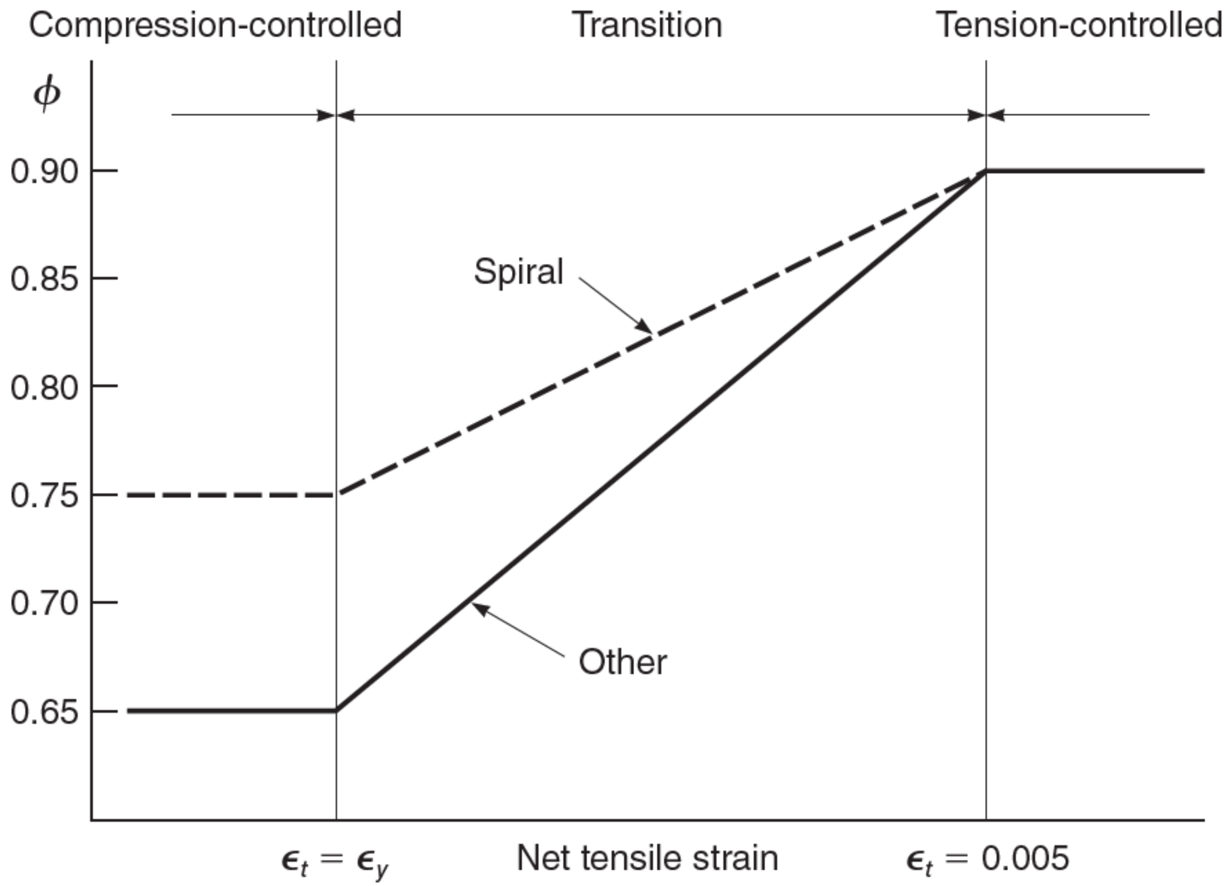
Based on Eq. (4.26*b*), the maximum reinforcement ratio for a tension-controlled beam ($\epsilon_t = 0.005$) is

$$\boxed{\rho_{0.005} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005}} \tag{4.26c}$$



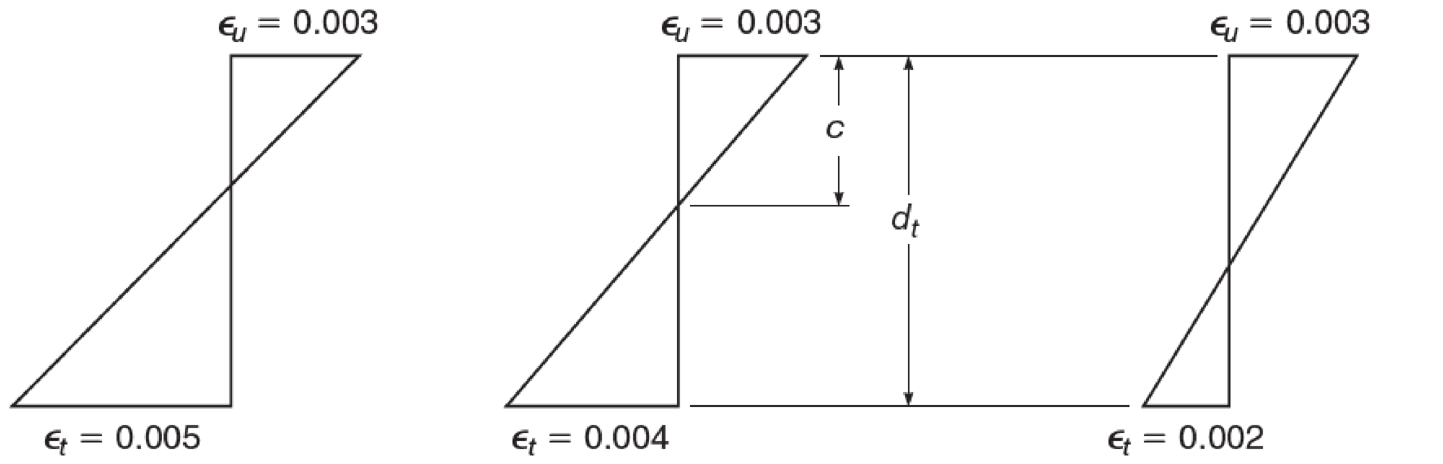
Strength reduction factor ϕ

FIGURE 4.8
Variation of strength reduction factor with net tensile strain in the steel.



$$\phi = 0.483 + 83.3\,\epsilon_t$$

FIGURE 4.9
Net tensile strain and c/d_t ratios.



$$\frac{c}{d_t} = \frac{0.003}{0.003 + 0.005} = 0.375$$

(a)

Tension-controlled member

$$\frac{c}{d_t} = \frac{0.003}{0.003 + 0.004} = 0.429$$

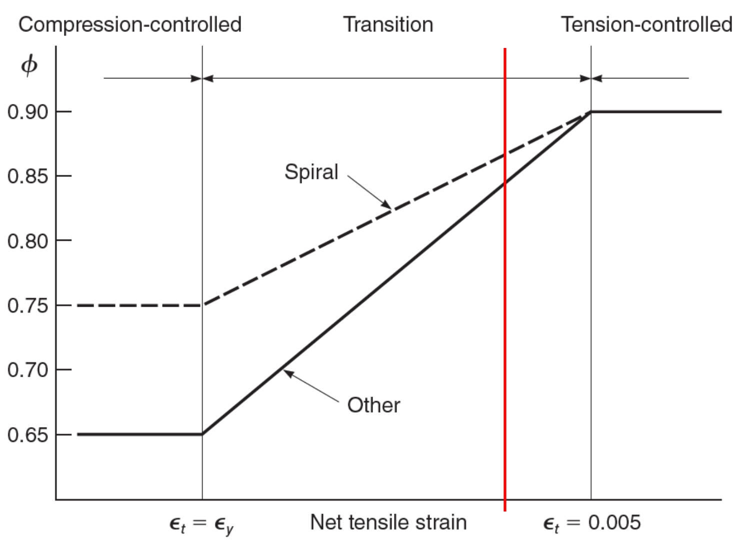
(b)

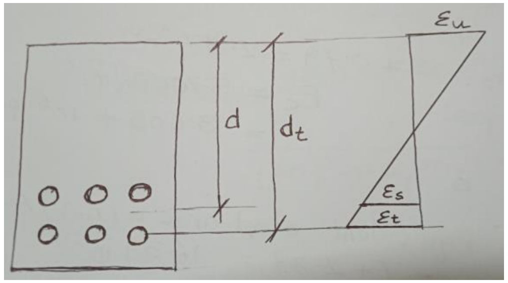
Minimum net tensile strain for flexural member

$$\frac{c}{d_t} = \frac{0.003}{0.003 + 0.002} = 0.600$$

(c)

Compression-controlled member for Grade 60 steel

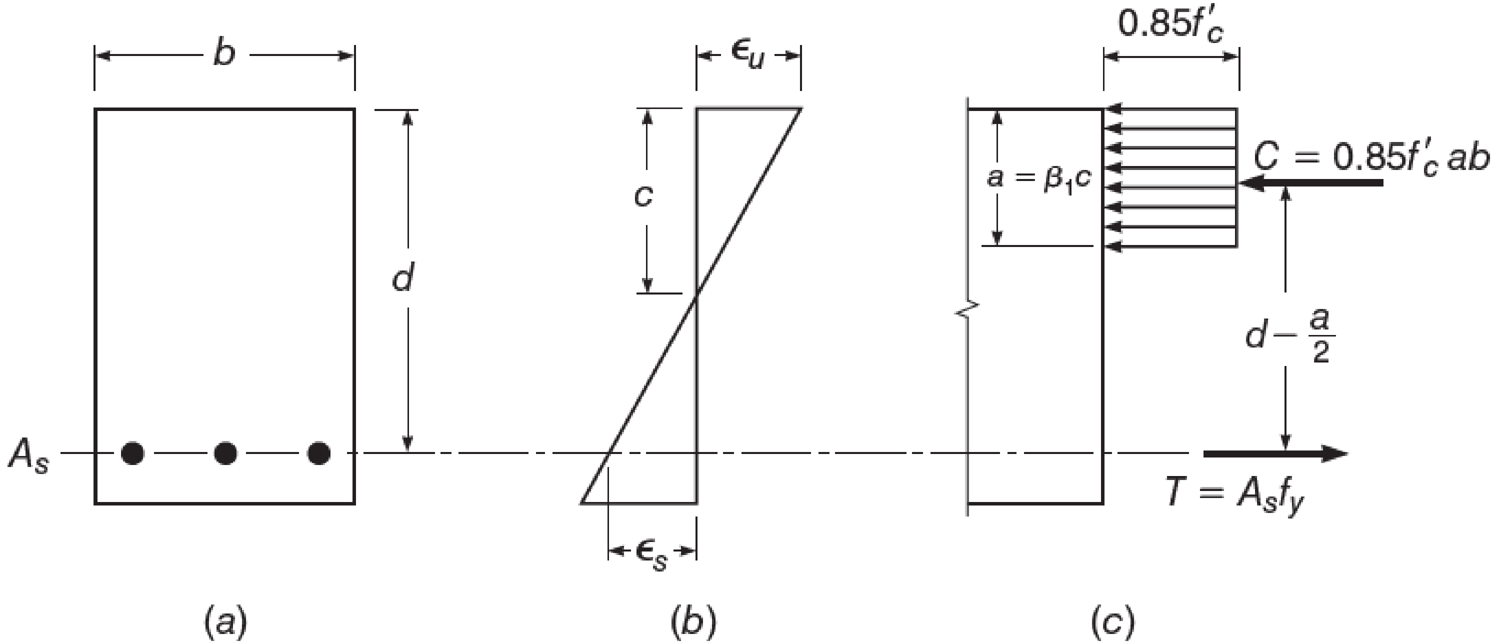




$$\boxed{\epsilon_t = \epsilon_u \frac{d_t - c}{c}}$$

$$\varphi = 0.483 + 83.3\varepsilon_t$$

FIGURE 4.10
Singly reinforced rectangular beam.



reinforcement, where d_t is greater than d. Because $\epsilon_t \geq 0.005$ ensures that steel is yielding in tension, $f_s = f_y$ at failure. Referring to Fig. 4.10, the nominal flexural strength M_n is obtained by summing moments about the centroid of the compression force C.

$$\boxed{M_n = A_s f_y \left(d - \frac{a}{2}\right)} \tag{4.27}$$

The depth of the equivalent stress block a can be found based on equilibrium, $C = T$. Hence, $0.85 f'_c ab = A_s f_y$, giving

$$\boxed{a = \frac{A_s f_y}{0.85 f'_c b}} \tag{4.28}$$

It is convenient for everyday design to combine Eqs. (4.27) and (4.28) as follows. Noting that $A_s = \rho bd$, Eq. (4.28) can be rewritten as

$$a = \frac{\rho f_y d}{0.85 f'_c} \tag{4.29}$$

This is then substituted into Eq. (4.27) to obtain

$$\boxed{M_n = \rho f_y bd^2 \left(1 - 0.59\,\frac{\rho f_y}{f'_c}\right)} \tag{4.30}$$

which is identical to Eq. (4.16*b*) derived in Section 4.2c. This basic equation can be simplified further as follows:

$$M_n = Rbd^2 \tag{4.31}$$

in which

$$R = \rho f_y \left(1 - 0.59\,\frac{\rho f_y}{f'_c}\right) \tag{4.32}$$

The *flexural resistance factor* R depends only on the reinforcement ratio and the strengths of the materials and is easily tabulated. Tables A.5a and A.5b and Graphs A.1*a* and A.1*b* of Appendix A give R values for ordinary combinations of steel and concrete and the full practical range of reinforcement ratios.

In accordance with the safety provisions of the ACI Code, the nominal flexural strength M_n is reduced by imposing the strength reduction factor ϕ to obtain the *design strength* ϕM_n

$$\boxed{\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)} \tag{4.33}$$

or, alternatively,

$$\phi M_n = \phi \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right) \tag{4.34}$$

or

$$\phi M_n = \phi R b d^2 \tag{4.35}$$

EXAMPLE 4.4 Using the equivalent rectangular stress distribution, directly calculate the nominal strength of the beam previously analyzed in Example 4.3. Recall that $b = 10$ in., $d = 23$ in., $A_s = 2.37$ in^2., $f'_c = 4000$ psi, $f_y = 60{,}000$ psi, and $\beta_1 = 0.85$.

Review problem

Solution. The distribution of stresses, internal forces, and strains is shown in Fig. 4.10. The maximum reinforcement ratio is calculated from Eq. (4.26*c*) as

$$\rho_{0.005} = 0.85 \times 0.85 \frac{4000}{60{,}000} \frac{0.003}{0.003 + 0.005} = 0.0181$$

and comparison with the actual reinforcement ratio of 0.0103 confirms that the member is underreinforced and will fail by yielding of the steel. Alternatively, recalling that $c = 4.94$ in.,

$$\frac{c}{d_t} = \frac{c}{d} = \frac{4.94}{23} = 0.215$$

which is less than 0.375, the value of c/d_t corresponding to $\epsilon_t = 0.005$, also confirming that the member is underreinforced. Hence, $0.85 f'_c ab = A_s f_y$, or $a = 2.37 \times 60{,}000/(0.85 \times 4000 \times 10) = 4.18$. The nominal moment is

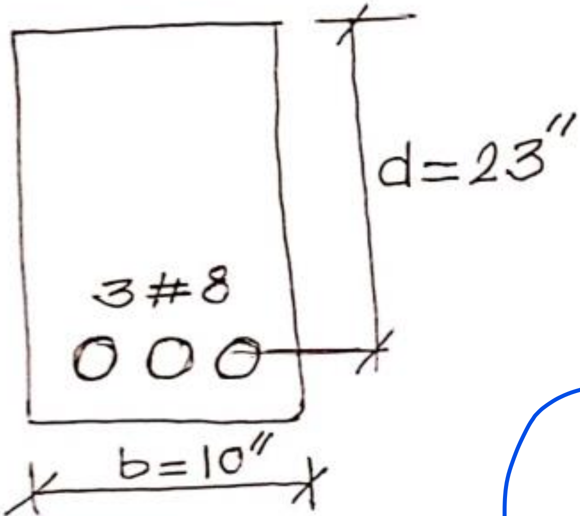
$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 2.37 \times 60{,}000(23 - 2.09) = 2{,}970{,}000 \text{ in-lb} = 248 \text{ ft-kips}$$

EXAMPLE 4.4 (*continued*) Calculate the design moment capacity ϕM_n for the beam analyzed earlier in Example 4.4.

Solution. Comparing ρ with $\rho_{0.005}$ or c/d_t for the beam with the value of c/d_t corresponding to $\epsilon_t = 0.005$ demonstrates that $\epsilon_t > 0.005$. Therefore, $\phi = 0.90$ and the design capacity is

$$\phi M_n = 0.9 \times 248 = 223 \text{ ft-kips}$$

Example 4.4



Find nominal strength

Nominal Moment Capacity, M_n

using equivalent rectangular stress distribution.

$f_c' = 4000$ psi $f_y = 60{,}000$ psi

$$\beta_1 = 0.85 - 0.05\,\frac{f_c' - 4000}{1000}$$

$$0.65 \leq \beta_1 \leq 0.85$$

Solution

$$A_s = 3 * 0.79 = 2.37\ in^2$$

$$\rho = \frac{A_s}{bd} = \frac{2.37}{10 * 23} = 0.0103$$

Let's check if beam is under-reinforced.
(tension-controlled)

① $\rho_{0.005} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.005}$

$\beta_1 = 0.85$

$\rho_{0.005} = 0.85 * 0.85 \frac{4}{60} \frac{0.003}{.003 + .005} = 0.0181$

$\rho < \rho_{0.005} \Rightarrow$ underreinforced (tension- controlled)

② $\frac{c}{d_t} = \frac{c}{d} = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t} = \frac{0.003}{0.003 + .005} = 0.375$ Limit

$C = T \Rightarrow 0.85 f_c' ab = A_s f_y$

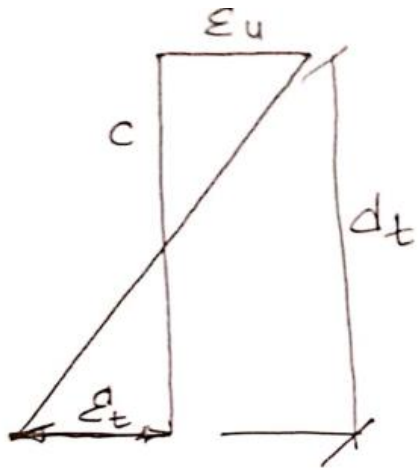
$\Rightarrow 0.85 * 4 * a * 10 = 2.37 * 60$

$\Rightarrow a = 4.18'' = \beta_1 c$

$\Rightarrow c = \frac{4.18}{0.85} = 4.92''$

$\frac{c}{d} = \frac{4.92}{23} = 0.214 < 0.375$ Under-reinforced (tension controlled)

③



$$\frac{\varepsilon_u}{c} = \frac{\varepsilon_t}{d_t - c}$$

$$\varepsilon_t = \frac{\varepsilon_u}{c}(d_t - c)$$

$$= \frac{0.003}{4.92}(23 - 4.92)$$

$$= 0.0110 \gg 0.005$$

So, beam is under-reinforced (tension-controlled)

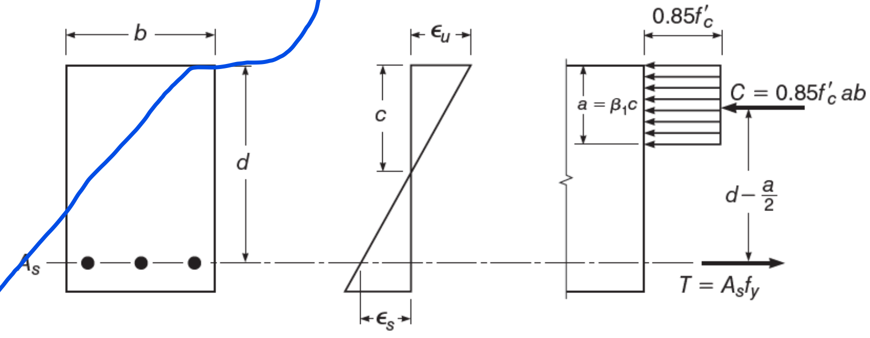
Nominal moment capacity

$$M_n = A_s f_y (d - a/2)$$

$$= 2.37 * 60 \left(23 - \frac{4.18}{2}\right)$$

$$= 2973 \text{ kip inch}$$

$$= 248 \text{ kip-ft.}$$



(a) (b) (c)

Design moment capacity

$$\phi M_n = 0.9 * 248 = 223 \text{ kip-ft}$$

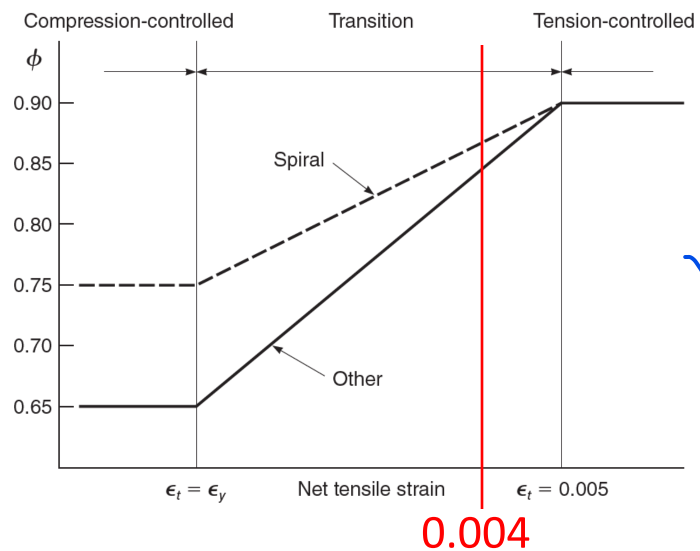
Maximum Reinforcement Ratio

Based on Eq. (4.26*b*), the maximum reinforcement ratio for a tension-controlled beam ($\epsilon_t = 0.005$) is

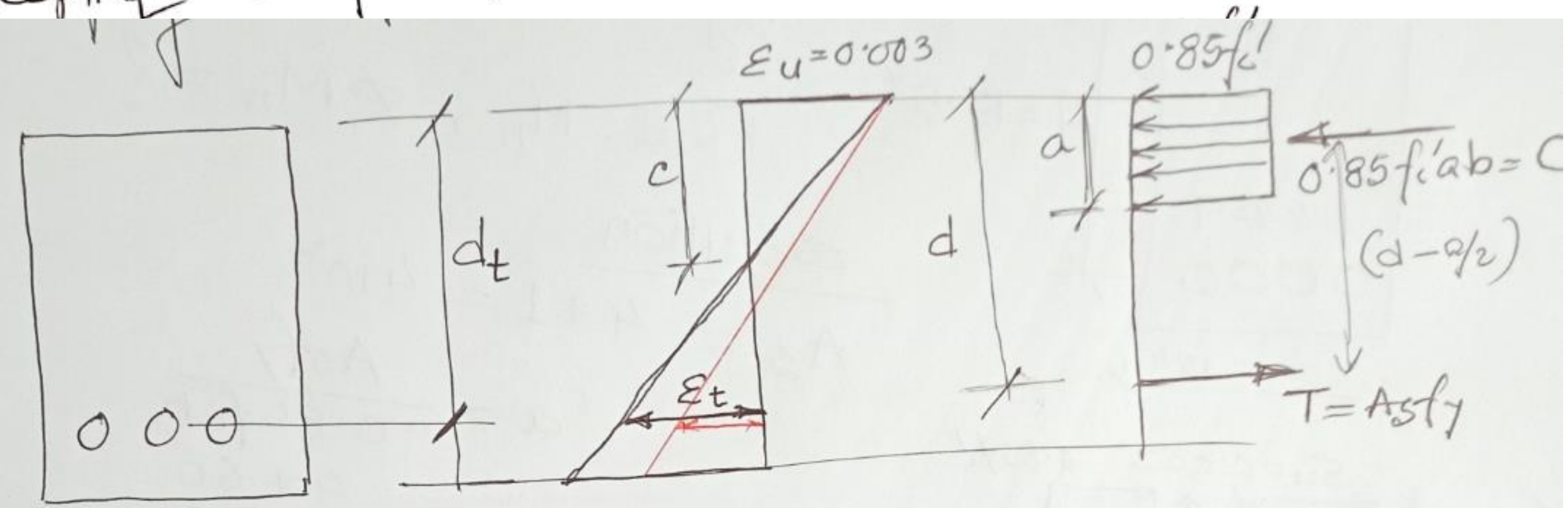
$$\boxed{\rho_{0.005} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005}} \tag{4.26c}$$

For consistency with earlier Codes, ACI Code 7.3.3, 8.3.3, and 9.3.3 allow slabs and beams to have net tensile strains as low as $\epsilon_t = 0.004$, provided the strength reduction factor is adjusted. Based on Eq. (4.26*b*), the reinforcement ratio corresponding to $\epsilon_t = 0.004$ is

$$\boxed{\rho_{0.004} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}} \tag{4.26d}$$



What happens when you increase A_s by keeping d fix?



If A_s increases, T increases

T increases, C increases,

C increases, a increases,

a increases, c increases,

c increases, ε_t decreases.

Minimum Reinforcement Ratio

- If the flexural strength (of cracked section) is less than the moment that produced cracking of the previously uncracked section, the beam fails immediately upon formation of first flexural crack.
- $M_n < M_{cr}$

$$M_u \leq \phi M_n$$

যদি আমাদের অনেক deep section এর Beam লাগতো আমার কোন purpose এ – But যেহেতু Nominal moment (M_n) অনেক বেশি থাকবে, তখন আমরা যদি সেই কম moment এর জন্য কম Reinforcement দিয়ে design করি, then যদি কখনো or overload হয়ে crack করে, then crack

- To ensure against this type of failure, a minimum amount of reinforcement is provided

হওয়ার পরে অনেক moment লাগবে। কিন্তু যদি crack করে, then কম Reinforcement থাকায় কোনো warning না দিয়েই ছিঁড়ে যাবে। কোনো load লাগানোর পরে steel yield হওয়ার কিন্তু deep section crack করলে তখন warning দেয় নাই। এমন failure চাইবো না।!!

* So, cracking moment correspond করে design করবো। $M_n = M_{cr}$ ধরে

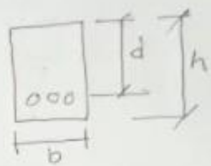
For a rectangular section having width b, total depth h, and effective depth d (see Fig. 4.1b), the section modulus with respect to the tension fiber is $bh^2/6$. For typical cross sections, it is satisfactory to assume that $h/d = 1.1$ and that the internal lever arm at flexural failure is $0.95d$. If the modulus of rupture is taken as $f_r = 7.5\sqrt{f_c'}$, as usual, then an analysis equating the cracking moment to the flexural strength results in

$$\boxed{A_{s,\min} = \frac{1.6\sqrt{f_c'}}{f_y} bd} \qquad (4.36a)$$

not used

minimum reinforcement

Let us find the amount of reinforcement such that nominal capacity M_n equals to cracking moment M_{cr}



$$f = \frac{My}{I} = \frac{M * \frac{h}{2}}{\frac{bh^3}{12}}$$

$$= \frac{M}{bh^2/6}$$

At cracking, $\quad f_t = \frac{M_{cr}}{bh^2/6}$

$$\Rightarrow 7.5\sqrt{f_c'} = \frac{M_{cr}}{bh^2/6} = \frac{M_n}{bh^2/6} \quad [M_u = M_n]$$

$$\Rightarrow 7.5\sqrt{f_c'} = \frac{A_{s,min} * f_y * \text{Lever arm}}{bh^2/6}$$

Assumption, Lever arm $= 0.95d$
$h = 1.1d$ → arbitrary

$$\Rightarrow 7.5\sqrt{f_c'} = \frac{A_{s,min} * f_y * 0.95d}{b/6 * (1.1d)^2}$$

$$= \frac{A_{s,min} * f_y}{0.212\, bd}$$

$$A_{s,min} = \frac{1.59\sqrt{f_c'}\, bd}{f_y} = \frac{1.6\sqrt{f_c'}\, bd}{f_y}$$

ρ_{max} (আগে পড়েছি)

এর মাঝে design করব।

ρ_{min}

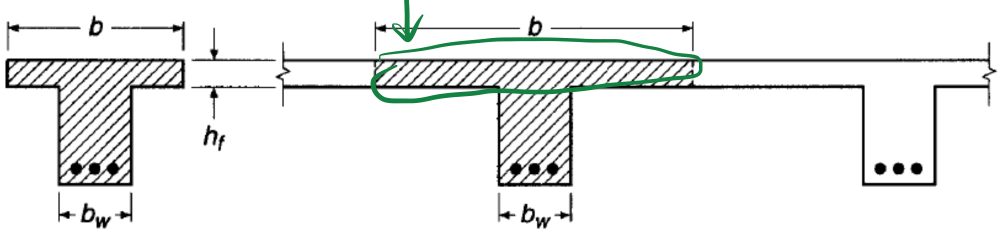
This development can be generalized to apply to beams having a T cross section (see Section 4.7 and Fig. 4.16). The corresponding equations depend on the proportions of the cross section and on whether the beam is bent with the flange (slab) in tension or in compression. For T beams of typical proportions that are bent with the flange in compression, analysis will confirm that the minimum steel area should be

$$A_{s,\min} = \frac{2.7\sqrt{f_c'}}{f_y} b_w d \quad \rightarrow \text{T beam} \quad \text{[Flange compression]} \tag{4.36b}$$

where b_w is the width of the web, or stem, projecting below the slab. For T beams that are bent with the flange in tension, from a similar analysis, the minimum steel area is

$$A_{s,\min} = \frac{6.2\sqrt{f_c'}}{f_y} b_w d \quad \rightarrow \text{T beam} \tag{4.36c}$$

[Flange tension]



(a) (b)

The ACI Code requirements for minimum steel area are based on the results just discussed, but there are some differences. According to ACI Code 9.6.1, at any section where tensile reinforcement is required by analysis, with some exceptions as noted below, the area A_s provided must not be less than

For all beam

$$\boxed{\boxed{A_{s,\min} = \frac{3\sqrt{f_c'}}{f_y} b_w d} \geq \frac{200 b_w d}{f_y}} \tag{4.37a}$$

Old code formula

Note that ACI Code Eq. (4.37*a*) is conveniently expressed in terms of a *minimum tensile reinforcement ratio* $\rho_{\min}$ by dividing both sides by $b_w d$.

$$\boxed{\rho_{\min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y}} \tag{4.37b}$$

$$\boxed{A_{s,min} = 1.33 A_{s,reqd}}$$

According to ACI Code 9.6.1, the requirements of Eq. (4.37a) need not be imposed if, at every section, the area of tensile reinforcement provided is at least one-third greater than that required by analysis. This provides sufficient reinforcement for large members such as grade beams, where the usual equations would require excessive amounts of steel.

Thumb rule

M_n থেকে যে A_s হবে, সেটাকে $A_{s,reqd}$ এ বসালে A_s পাবো।

For structural slabs and footings of uniform thickness, the minimum area of tensile reinforcement in the direction of the span is that required for shrinkage and temperature steel (see Section 12.3 and Table 12.2), and the above minimums need not be imposed. The maximum spacing of such steel is the smaller of 3 times the total slab thickness or 18 in.

Review problem

$\varphi = 0.483 + 83.3\varepsilon_t$

EXAMPLE 4.5 **Flexural strength of a given member.** A rectangular beam has width 12 in. and effective depth 17.5 in. It is reinforced with four No. 9 (No. 29) bars in one row. If $f_y = 60{,}000$ psi and $f'_c = 4000$ psi, what is the nominal flexural strength, and what is the maximum moment that can be utilized in design, according to the ACI Code?

Solution. From Table A.2 of Appendix A, the area of four No. 9 (No. 29) bars is 4.00 in^2. Assuming that the beam is underreinforced and using Eq. (4.28),

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{4.00 \times 60}{0.85 \times 4 \times 12} = 5.88 \text{ in.}$$

The depth of the neutral axis is $c = a/\beta_1 = 5.88/0.85 = 6.92$, giving

$$\frac{c}{d_t} = \frac{6.92}{17.5} = 0.395$$

which is between 0.429 and 0.375, the values corresponding, respectively, to $\epsilon_t = 0.004$ and $\epsilon_t = 0.005$, as shown in Fig. 4.9. Thus, the beam is, as assumed, underreinforced, and from Eq. (4.27)

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 4.00 \times 60 \left(17.5 - \frac{5.88}{2}\right) = 3490 \text{ in-kips}$$

The fact that the beam is unreinforced could also have been established by calculating $\rho = 4.00/(12 \times 17.5) = 0.190$, which just exceeds $\rho_{0.005}$, which is calculated using Eq. (4.26*c*).

$$\rho_{0.005} = 0.85\beta_1 \frac{f_c{}'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005} = 0.85 \times 0.85 \left(\frac{4}{60}\right)\left(\frac{0.003}{0.003 + 0.005}\right) = 0.0181$$

Because the net tensile strain ϵ_t is between 0.004 and 0.005, ϕ must be calculated: $\epsilon_t = \epsilon_u(d - c)/c = 0.003 \times (17.5 - 6.92)/6.92 = 0.00458$. Using linear interpolation from Fig. 4.8, $\phi = 0.87$, and the design strength is taken as

$$\phi M_n = 0.87 \times 3490 = 3040 \text{ in-kips}$$

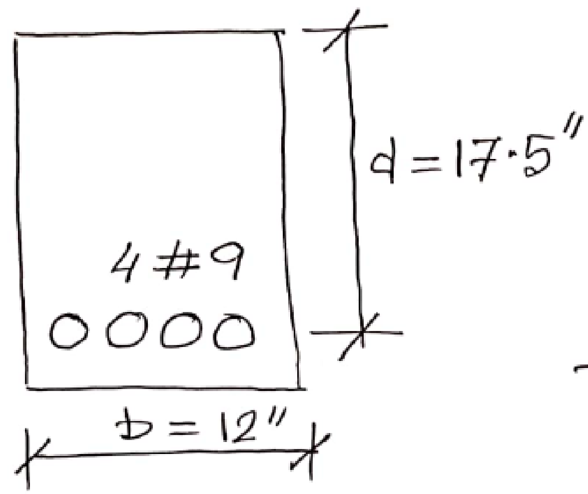
The ACI Code limits on the reinforcement ratio

$$\rho_{0.004} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85 \left(\frac{4}{60}\right)\left(\frac{0.003}{0.003+0.004}\right) = 0.0206$$

$$\rho_{\min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y} = \frac{3\sqrt{4000}}{60{,}000} \geq \frac{200}{60{,}000} = 0.0033$$

are satisfied for this beam.

Example 4·5

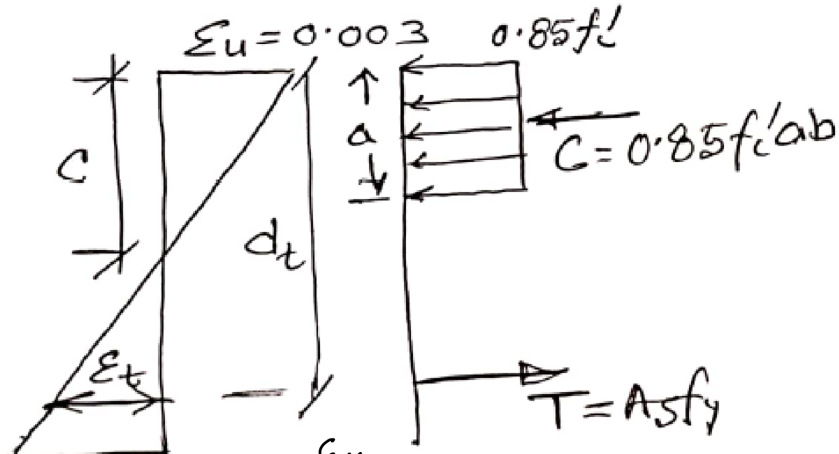


$f_c' = 4000 \text{ psi}$

$f_y = 60{,}000 \text{ psi}$

Find M_n, $\phi M_n = ?$

Solution



$A_s = 4 * 1 = 4 \text{ in}^2$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{4 * 60}{0.85 * 4 * 12} = 5.88''$$

$a = \beta_1 c$

$\Rightarrow c = 5.88/0.85 = 6.92''$

$$\frac{\varepsilon_t}{d_t - c} = \frac{\varepsilon_u}{c}$$

$$\Rightarrow \varepsilon_t = \frac{\varepsilon_u}{c}(d_t - c) = \frac{0.003}{6.92}(17.5 - 6.92)$$

$= 0.00459$, strain is between 0.004 and 0.005

$$\phi = 0.483 + 83.3\,\varepsilon_t$$
$$= 0.865$$

$$M_n = A_s f_y \left(d - \tfrac{a}{2}\right) = 4 * 60 * \left(17.5 - \frac{5.88}{2}\right)$$
$$= 3494^{k''} = \underline{291^{\,k\text{-}ft}}$$

$$\phi M_n = 0.865 * 291 = \underline{252^{\,k\text{-}ft}} \;//$$

old code

$$A_{s,min} = \frac{3\sqrt{f_c'}}{f_y} b_w d \geq \frac{200}{f_y} b_w d$$
$$= \frac{3\sqrt{4000}}{60000} * 12 * 17.5 \geq \frac{200}{60{,}000} * 12 * 17.5$$
$$= 0.664\ in^2 \geq \underline{0.7\ in^2}$$

(ok)

As $A_{s,min} < A_s$ i.e, $0.664 < 4$

So, তাহলে ফেলও না। যদি $\varepsilon_t > 0.005$ হত, the ρ_{max} cross

ঠিক কিনা check করতাম

Design Problem

EXAMPLE 4.6 **Concrete dimensions and steel area to resist a given moment.** Find the concrete cross section and the steel area required for a simply supported rectangular beam with a span of 15 ft that is to carry a computed dead load of 1.27 kips/ft and a service live load of 2.15 kips/ft, as shown in Fig. 4.11. Material strengths are $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

Solution. Load factors are first applied to the given service loads to obtain the factored load for which the beam is to be designed, and the corresponding moment:

$$w_u = 1.2 \times 1.27 + 1.6 \times 2.15 = 4.96 \text{ kips/ft}$$

$$M_u = \frac{1}{8} \times 4.96 \times 15^2 \times 12 = 1670 \text{ in-kips}$$

The concrete dimensions will depend on the designer's choice of reinforcement ratio. To minimize the concrete section, it is desirable to select the maximum permissible reinforcement ratio. To maintain $\phi = 0.9$, the maximum reinforcement ratio corresponding to a net tensile strain of 0.005 will be selected (see Fig. 4.8). Then, from Eq. (4.26*c*)

$$\rho_{0.005} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005} = 0.85 \times 0.85 \left(\frac{4}{60}\right)\left(\frac{0.003}{0.003 + 0.005}\right) = 0.0181$$

Using Eq. (4.26*d*) gives $\rho_{0.004} = 0.0206$, but would require a lower strength reduction factor. Setting the required flexural strength equal to the design strength from Eq. (4.34), and substituting the selected values for ρ and material strengths,

$$M_u = \phi M_n$$

$$1670 = 0.90 \times 0.0181 \times 60bd^2 \left(1 - 0.59\,\frac{0.0181 \times 60}{4}\right)$$

from which

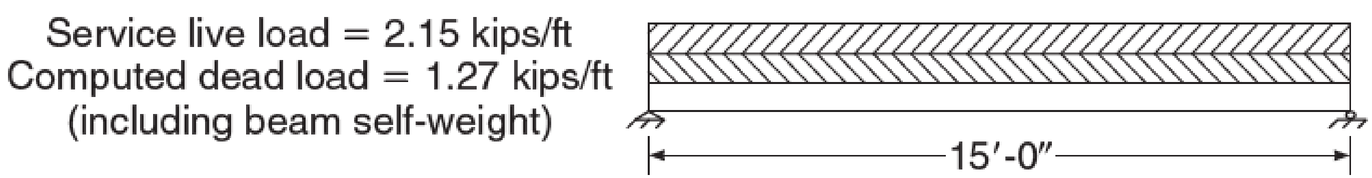
$$bd^2 = 2040 \text{ in}^3$$

A beam with width $b = 10$ in. and $d = 14.3$ in. will satisfy this requirement. The required steel area is found by applying the chosen reinforcement ratio to the required concrete dimensions:

$$A_s = 0.0181 \times 10 \times 14.3 = 2.59 \text{ in}^2$$

Two No. 10 (No. 32) bars provide 2.54 in^2, which is very close to the required area.

Assuming 2.5 in. concrete cover from the centroid of the bars, the required total depth is $h = 16.8$ in. In actual practice, however, the concrete dimensions b and h are always rounded up to the nearest inch, and often to the nearest multiple of 2 in. (see Section 4.4). The actual



d is then found by subtracting the required concrete cover dimension from h. For the present example, $b = 10$ in. and $h = 18$ in. will be selected, resulting in effective depth $d = 15.5$ in. Improved economy then may be possible, refining the steel area based on the actual, larger, effective depth. One can obtain the revised steel requirement directly by solving Eq. (4.34) for ρ, with $\phi M_n = M_u$. A quicker solution can be obtained by iteration. First a reasonable value of a is assumed, and A_s is found from Eq. (4.33). From Eq. (4.28) a revised estimate of a is obtained, and A_s is revised. This method converges very rapidly. For example, assume $a = 5$ in. Then

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)} = \frac{1670}{0.90 \times 60(15.5 - 5/2)} = 2.38 \text{ in}^2$$

Checking the assumed a gives

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2.38 \times 60}{0.85 \times 4 \times 10} = 4.20 \text{ in.}$$

This is close enough to the assumed value that no further calculation is required. The required steel area of 2.38 in^2 could be provided using three No. 8 (No. 25) bars, but for simplicity of construction, two No. 10 (No. 32) bars will be used as before.

A somewhat larger beam cross section using less steel may be more economical and will tend to reduce deflections. As an alternative solution, the beam will be redesigned with a lower reinforcement ratio of $\rho = 0.60\rho_{0.005} = 0.60 \times 0.0181 = 0.0109$. Setting the required strength equal to the design strength [Eq. (4.34)] as before,

$$1670 = 0.90 \times 0.0109 \times 60bd^2 \left(1 - 0.59 \frac{0.0109 \times 60}{4}\right)$$

and

$$bd^2 = 3140 \text{ in}^3$$

A beam with $b = 10$ in. and $d = 17.7$ in. will meet the requirement, for which

$$A_s = 0.0109 \times 10 \times 17.7 = 1.93 \text{ in}^2$$

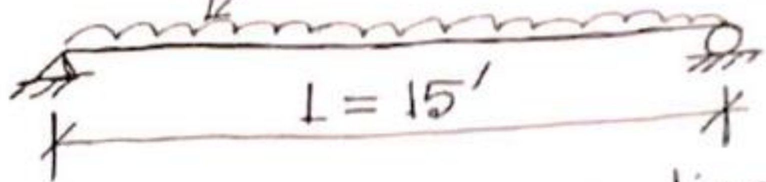
Two No. 9 (No. 29) bars are sufficient, providing an area of 2.00 in^2. If the total concrete height is rounded to 20 in., a 17.5 in. effective depth results, increasing the required steel area to 1.96 in^2. Two No. 9 (No. 29) bars remain the best choice.

Example 4.6

$f_c' = 4\,ksi$

$f_y = 60\,ksi$

Computed DL = $1.27\ ^{k/'}$
Service LL = $2.15\ ^{k/'}$



Self wt দেয়া থাকলে তখন
আর design এর ক্ষেত্রে

Find beam cross-section and steel.

Solution

Factored load

$$w_u = 1.2\,w_{DL} + 1.6\,w_{LL} = 1.2 * 1.27 + 1.6 * 2.15 = 4.964\ ^{k/'}$$

Factored Moment $M_u = \frac{1}{8} w_u L^2$ → [mid span এ max^m]

$$= \frac{1}{8} * 4.964 * 15^2 = 139.6\ ^{k\text{-}ft}$$

Find b, h, A_s

$$\underbrace{\rho_{0.005}}_{\text{হিসে. নিচ্ছি}} = 0.85\,\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.005} = 0.85 * .85 \frac{4}{60} \frac{0.003}{0.003 + 0.005}$$

$$= 0.0181$$ → 1.81% rod (area)

$\therefore\ d < 0.9$

$\rho_n = 0.0181$

$\boxed{\rho_{0.004} = 0.0206 \rightarrow \text{if use it } \phi < 0.9}$

$M_u \le \phi M_n \Rightarrow \underbrace{M_u = \phi M_n}_{\text{let equal}} = \boxed{\phi \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)}$

(ft → inch)

$\Rightarrow 139.6 \times 12 = 0.9 \times 0.0181 \times 60 \times b d^2 \times \left(1 - 0.59 \frac{0.0181 \times 60}{4}\right)$

$b d^2 = 2040 \text{ in}^3$

$h = d + 2.5 = 16.8''$ clear cover for 1 layer rod

Let $b = 10''$, $d = 14.3''$

10'' এর কম নেওয়া না

$A_s = \rho b d = 0.0181 \times 10 \times 14.3$

$= 2.59 \text{ in}^2$

solution 1 :– $b = 10''$, $d = 14.3''$, $h = 16.8''$, $A_s = 2.59 \text{ in}^2$

solution 2 :– $h = 18''$ (rounded to 18'', not always)

2'' কম।
17'' ও নেওয়া যায়।

$d = 18 - 2.5 = 15.5''$

For sol^n 2,

$$A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{139.6 \times 12}{0.9 \times 60 \left(15.5 - \frac{a}{2}\right)}$$

$$= 2.07,\ 2.27,\ 2.298,\ 2.30$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2.07 \times 60}{0.85 \times 4 \times 10}$$

$$= 3.64,\ 4.00,\ 4.06,\ 4.06$$

এভাবে trial দিতে থাকলে converge হবে।

$A_s = 2.30\ in^2 \rightarrow$ Three No. 8 bar (Can be accommodated in 10″) ২টা

↓ Rod গুলো কাজে লাগে

or, Two No. 10 bar

Solution 2 $\Rightarrow$ $b = 10''$, $d = 15.5''$, $h = 18''$, $A_s \rightarrow 3\#8$ bar

Question :- What will be $\varepsilon_t = ?$

↓ বাড়বে। $A_s \downarrow$ $\varepsilon_t \uparrow$

→ c ছোট হলে ε_t বেড়ে যাবে

$$\frac{\varepsilon_u}{c} = \frac{\varepsilon_t}{d_t - c}$$

Under-reinforced factor

Solution 3

$$\rho = 0.6 * \rho_{0.005} = 0.0109$$

$b = 10''$, $d = 17.7''$, $A_s = 1.93\ in^2$

$h = 20.2''$

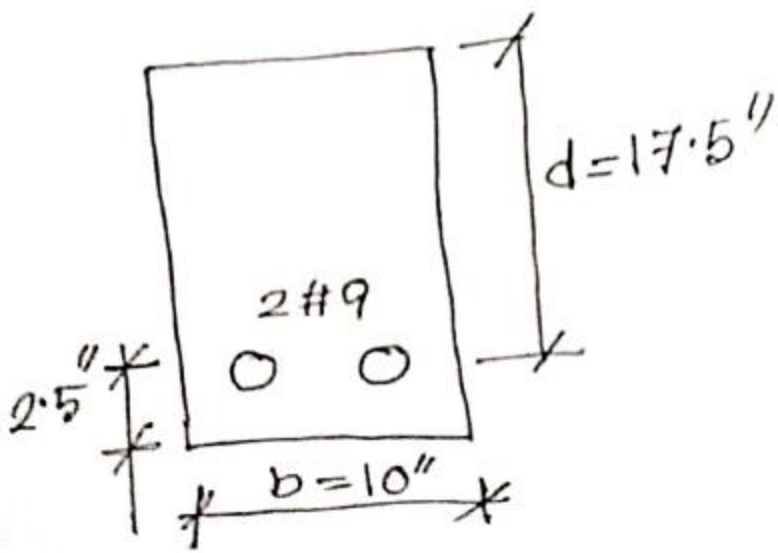
Two No. 9

Solution 4

$b = 10''$ $d = 17.5''$, $A_s = 1.96\ in^2$

$h = 20''$

Two No. 9



- Infinite number of solution is possible

upper limit of $\rho_{0.004}$ to a lower limit of $3\sqrt{f_c'}/f_y \geq 200/f_y$ for beams,

- Economic $0.5\rho_{0.005}$ to $0.75\rho_{0.005}$

There is a type of problem, occurring frequently, that does not fall strictly into either the analysis or the design category. The concrete dimensions are given and are known to be adequate to carry the required moment, and it is necessary only to find the steel area. Typically, this is the situation at critical design sections of continuous beams, in which the concrete dimensions are often kept constant, although the steel reinforcement varies along the span according to the required flexural resistance. Dimensions b, d, and h are determined at the maximum moment section, usually at one of the supports. At other supports, and at midspan locations, where moments

are usually smaller, the concrete dimensions are known to be adequate and only the tensile steel remains to be found. An identical situation was encountered in the design

Determination of steel area

EXAMPLE 4.7 **Determination of steel area.** Using the same concrete dimensions as were used for the second solution of Example 4.6 (b = 10 in., d = 17.5 in., and h = 20 in.) and the same material strengths, find the steel area required to resist a moment M_u of 1300 in-kips.

Solution. Assume $a = 4.0$ in. Then

$$A_s = \frac{1300}{0.90 \times 60(17.5 - 2.0)} = 1.55 \text{ in}^2$$

Checking the assumed a gives

$$a = \frac{1.55 \times 60}{0.85 \times 4 \times 10} = 2.74 \text{ in.}$$

Next assume $a = 2.6$ in. and recalculate A_s:

$$A_s = \frac{1300}{0.90 \times 60(17.5 - 1.3)} = 1.49 \text{ in}^2$$

No further iteration is required. Use $A_s = 1.49$ in^2. Two No. 8 (No. 25) bars, $A_s = 1.58$ in^2, will be used. A check of the reinforcement ratio shows $\rho < \rho_{0.005}$ and $\phi = 0.9$.

* dimension known হলে $A_s \rightleftarrows a$ method এ করব

Example 4.7 Given: $b = 10''$, $d = 17.5''$

$h = 20''$

$f_c' = 4$ ksi $\quad f_y = 60$ ksi

Find steel area required for $M_u = 1300$ k-inch

Solution

$$A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{1300}{0.9 \times 60 \, (17.5 - \frac{1}{2})}$$

ধরে নিয়ে trial করব:

$$= 1.42,\ 1.481,\ \underline{1.49\ in^2} \qquad 2 \# 8$$

trial

$$a = \frac{A_s f_y}{0.85 f_c' \, b} = \frac{1.42 \times 60}{0.85 \times 4 \times 10}$$

$$= 2.50,\ 2.61,\ \underline{2.62}\ in$$

$b = 10''$, $d = 17.5''$, $h = 20''$, $A_s \Rightarrow 2 \# 8 = 1.58\ in^2$.

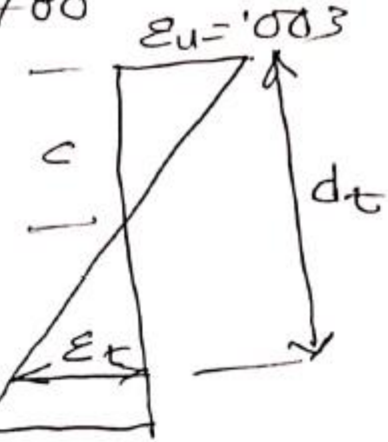
How do you know $\phi = 0.9$?

Check ε_t for actual steel provided.

How do you know $\phi = 0.9$?

Check ε_t for actual steel provided.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{1.58 * 60}{0.85 * 4 * 10} = 2.788''$$

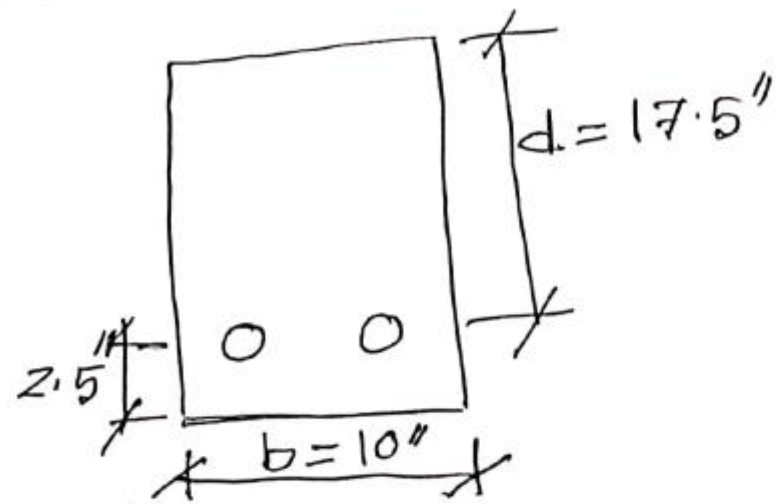


$$c = \frac{a}{\beta_1} = \frac{2.788}{0.85} = 3.28''$$

$$\varepsilon_t = \frac{\varepsilon_u}{c}(d_t - c) = \frac{0.003}{3.28}(17.5 - 3.28)$$

$$= 0.013 >> 0.005 \checkmark$$

$\phi = 0.9$ is ok $\checkmark$



EXAMPLE 4.8 **Determination of steel area and variable strength reduction factor.** Architectural considerations limit the height of a 20 ft long simple span beam to 16 in. and the width to 12 in. The following loads and material properties are given: $w_d = 0.79$ kips/ft, $w_l = 1.65$ kips/ft, $f_c' = 5000$ psi, and $f_y = 60{,}000$ psi. Determine the reinforcement for the beam.

SOLUTION. Calculating the factored loads gives

$$w_u = 1.2 \times 0.79 + 1.6 \times 1.65 = 3.59 \text{ kips/ft}$$

$$M_u = 3.59 \times \frac{20^2}{8} = 179 \text{ ft-kips} = 2150 \text{ in-kips}$$

Assume $a = 4.0$ in. and $\phi = 0.90$. The effective depth is $(16 - 2.5)$ in. $= 13.5$ in. Calculating A_s gives

$$A_s = \frac{M_u/\phi}{f_y(d - a/2)} = \frac{2150/0.90}{60(13.5 - 2.0)} = 3.46 \text{ in}^2$$

Try two No. 10 (No. 32) and one No. 9 (No. 29) bar, $A_s = 3.54$ in^2.

Check $a = 3.54 \times 60/(0.85 \times 5 \times 12) = 4.16$ in. from Eq. (4.28). This is more than assumed; therefore, continue to check the moment capacity.

$$M_n = 3.54 \times 60(13.5 - 4.16/2) = 2426 \text{ in-kips}$$

Using a ϕ of 0.90 gives $\phi M_n = 2183$ in-kips, which is adequate; however, the net tensile strain must be checked to validate the selection of $\phi = 0.9$. In this case $c = a/\beta_1 = 4.16/0.80 = 5.20$ in. The c/d ratio is $0.385 > 0.375$, so $\epsilon_t > 0.005$ is not satisfied. The corresponding net tensile strain is

$$\epsilon_t = \epsilon_u \frac{d - c}{c} = 0.003 \frac{13.5 - 5.2}{5.2} = 0.00479$$

A value of $\epsilon_t = 0.00479$ is allowed by the ACI Code, but only if the strength reduction factor is adjusted. A linear interpolation from Fig. 4.8 gives $\phi = 0.88$ and $M_u = \phi M_n = 2140$ in-kips, which is less than the required capacity. Try increasing the reinforcement to three No. 10 (No. 32) bars, $A_s = 3.81\ \text{in}^2$. Repeating the calculations,

$$a = \frac{3.81 \times 60}{0.85 \times 5 \times 12} = 4.48\ \text{in.}$$

$$c = \frac{4.48}{0.80} = 5.60\ \text{in.}$$

$$M_n = 3.81 \times 60 \left(13.5 - \frac{4.48}{2}\right) = 2574\ \text{in-kips}$$

$$\epsilon_t = \frac{0.003(13.5 - 5.60)}{5.60} = 0.00423$$

$$\varphi = 0.483 + 83.3\varepsilon_t$$

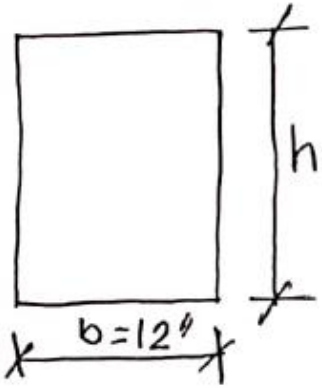
$$\phi = 0.483 + 83.3 \times 0.00423 = 0.835$$

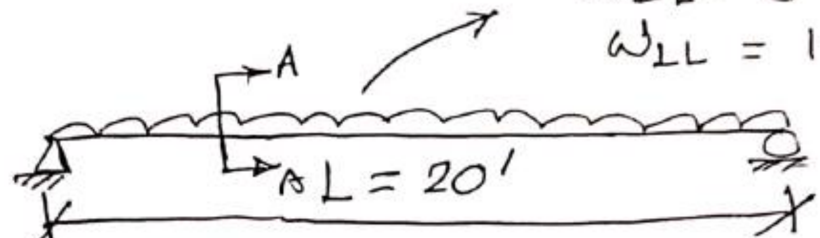
$$M_u = \phi M_n = 0.835 \times 2574 = 2150\ \text{in-kips}$$

which meets the design requirements.

Example 4.8

$w_{DL} = 0.79$ k/ft
$w_{LL} = 1.65$ k/ft



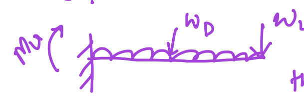


$f_c' = 5{,}000$ psi

$f_y = 60{,}000$ psi

Find A_s

যদি কোন concentrated load থাকে, then moment বের করে factor দিয়ে গুণ করতে হবে।



then $M_u = 1.2 \times M_D + 1.6 \times M_L$

Solution:

$$W_u = 1.2\,W_{DL} + 1.6\,W_{LL}$$
$$= 1.2 * 0.79 + 1.6 * 1.65 = 3.59 \text{ k/ft}$$

$$M_u = \frac{1}{8} W_u L^2 = \frac{1}{8} * 3.59 * 20^2 = 179.4 \text{ k-ft.}$$

clear cover for 1 layer rod.

Let $\phi = 0.9$

$$d = 16 - 2.5 = 13.5''$$

$$A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{179.4 * 12}{0.9 * 60 * (13.5 - \frac{1}{2})} = 3.07,\ 3.41,\ 3.47$$

Trial

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3.07 * 60}{0.85 * 5 * 12} = 3.60,\ 4.01,\ 4.08$$

0.85

* if $f_c' > 4000$ তখন এটা ধরা লাগবে

$0.85 f_c' b$

$$\boxed{\beta_1 = 0.85 - 0.05 \frac{f_c' - 4000}{1000}} \qquad 0.65 \le \beta_1 \le 0.85$$

$$= 0.80$$

Use Actual As used.

$$c = \frac{a}{\beta_1} = \frac{4.08}{0.8} = 5.1''$$

$A_s = 3.47\ in^2$ Provide $2\#10, 1\#9 = 2*1.27 + 1 = 3.54\ in^2$.

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3.54*60}{0.85*5*12} = 4.16''$$

$$c = \frac{4.16}{0.8} = 5.21$$

$$\varepsilon_t = \frac{\varepsilon_u}{c}(d_t - c) = \frac{0.003}{5.21}(13.5 - 5.21)$$

$$= 0.00478 < 0.005 \qquad \phi = 0.9 \text{ NOT ok}$$

$$\phi = 0.483 + 83.3\,\varepsilon_t = 0.88$$

$$\phi M_n = 0.88 * 3.54 * 60 * \left(13.5 - \frac{4.16}{2}\right)$$

$$= 2134^{k''} = 177.9^{k\text{-}ft} < 179.4^{k\text{-}ft}$$

slightly less.

1% diffrence এর হলে চলবে দেখা যায়

Capacity needs to be increased. যেহেতু section fixed, ∴ rod বাড়াতে হবে।

Increase reinforcement $A_s \rightarrow 3\#10$

$A_s = 3 * 1.27 = 3.81 \text{ in}^2$

$$a = \frac{3.81 * 60}{0.85 * 5 * 12} = 4.48''$$

$$c = \frac{4.48}{0.8} = 5.60''$$

$$\varepsilon_t = \frac{0.003}{5.60}(13.5 - 5.60) = 0.00423$$

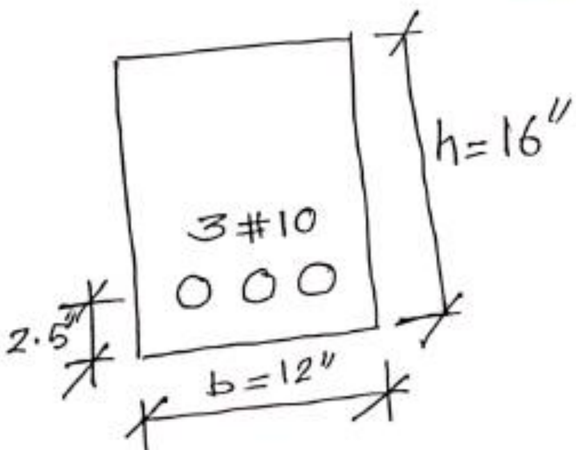
→ code মান্য হচ্ছে না

$$\phi = 0.483 + 83.3\,\varepsilon_t = 0.835$$

$$\phi M_n = 0.835 * 3.81 * 60\left(13.5 - \frac{4.48}{2}\right)$$

$$= 2149^{k''}$$

$$= \underline{179.1}^{\,k\text{-}ft} \simeq 179.4^{\,k\text{-}ft} \quad (OK)$$



← সব problem এর last এমন Pic আঁকতে হবে।

In actuality, the first solution deviates less than 1 percent from the desired value and would likely be acceptable. The remaining portion of the example demonstrates the design implications of requiring a variable strength reduction factor when the net tensile strain falls between 0.005 and 0.004. In this example, the reinforcement increased nearly 8 percent, yet the design moment capacity ϕM_n increased only 0.5 percent due to the decreasing strength reduction factor. For this reason, designs with $\rho < \rho_{0.005}$ are desirable. As will be discussed in Section 4.6, using members with compression as well as tension reinforcement is an option when concrete dimensions are limited.

Overreinforced beam

According to the ACI Code, all beams are to be designed for yielding of the tension steel with ϵ_t not less than 0.004 and thus $\rho \leq \rho_{0.004}$. Occasionally, however, such as when analyzing the capacity of existing construction, it may be necessary to calculate the flexural strength of an overreinforced compression-controlled member, for which f_s is less than f_y at flexural failure.

In this case, the steel strain, in Fig. 4.10*b*, will be less than the yield strain, but can be expressed in terms of the concrete strain ϵ_u and the still-unknown distance c to the neutral axis:

$$\epsilon_s = \epsilon_u \frac{d - c}{c} \tag{4.38}$$

From the equilibrium requirement that $C = T$, one can write

$$0.85\beta_1 f_c' bc = \rho\epsilon_s E_s bd$$

Substituting the steel strain from Eq. (4.38) in the last equation, and defining $k_u = c/d$, one obtains a quadratic equation in k_u as follows:

$$k_u^2 + m\rho k_u - m\rho = 0$$

Here, $\rho = A_s/bd$ as usual, and m is a material parameter given by

$$m = \frac{E_s \epsilon_u}{0.85\beta_1 f_c'} \tag{4.39}$$

Solving the quadratic equation for k_u,

$$k_u = \sqrt{m\rho + \left(\frac{m\rho}{2}\right)^2} - \frac{m\rho}{2} \tag{4.40}$$

The neutral axis depth for the overreinforced beam can then easily be found from $c = k_u d$, after which the stress-block depth $a = \beta_1 c$. With steel strain ϵ_s then computed from Eq. (4.38), and with $f_s = E_s\epsilon_s$, the nominal flexural strength is

$$M_n = A_s f_s\left(d - \frac{a}{2}\right) \tag{4.41}$$

$\rightarrow$ compression controlled

The strength reduction factor ϕ will equal 0.65 for beams in this range or slightly higher if the net tensile strain is in the transition zone shown in Fig. 4.8.

Design Aids: Find M_n

↓ নিচের অংশটা not so important

EXAMPLE 4.9 **Flexural strength of a given member.** Find the nominal flexural strength and design strength of the beam in Example 4.5, which has $b = 12$ in. and $d = 17.5$ in. and is reinforced with four No. 9 (No. 29) bars. Make use of the design aids of Appendix A. Material strengths are $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

Solution. From Table A.2, four No. 9 (No. 29) bars provide $A_s = 4.00$ in^2, and with $b = 12$ in. and $d = 17.5$ in., the reinforcement ratio is $\rho = 4.00/(12 \times 17.5) = 0.0190$. According to Table A.4, this is above $\rho_{0.005} = 0.0181$ but below $\rho_{0.004} = 0.0206$ and above

$\rho_{\min} = 0.0033$. Then from Table A.5b, with $f'_c = 4000$ psi, $f_y = 60{,}000$ psi, and $\rho = 0.019$, the value $R = 949$ psi is found. The nominal and design strengths are (with $\phi = 0.87$ from Example 4.5), respectively,

$$M_n = Rbd^2 = 949 \times 12 \times \frac{17.5^2}{1000} = 3490 \text{ in-kips}$$

$$\phi M_n = 0.87 \times 3490 = 3040 \text{ in-kips}$$

as before.

TABLE A.4

Limiting steel reinforcement ratios for tension-controlled members

ε_y

ρ

f_y, psi	f'_c, psi	β_1	$\rho_{0.005}$[a] $\epsilon_t = 0.005$[b]	ρ_{max}[a] $\epsilon_t = 0.004$[c]	$\rho_{min} = \dfrac{200}{f_y}$	$\rho_{min} = \dfrac{3\sqrt{f'_c}}{f_y}$
40,000	3000	0.85	0.0203	0.0232	0.0050	0.0041
	4000	0.85	0.0271	0.0310	0.0050	0.0047
	5000	0.80	0.0319	0.0364	0.0050	0.0053
	6000	0.75	0.0359	0.0410	0.0050	0.0058
	7000	0.70	0.0390	0.0446	0.0050	0.0063
	8000	0.65	0.0414	0.0474	0.0050	0.0067
	9000	0.65	0.0466	0.0533	0.0050	0.0071
50,000	3000	0.85	0.0163	0.0186	0.0040	0.0033
	4000	0.85	0.0217	0.0248	0.0040	0.0038
	5000	0.80	0.0255	0.0291	0.0040	0.0042
	6000	0.75	0.0287	0.0328	0.0040	0.0046
	7000	0.70	0.0312	0.0357	0.0040	0.0050
	8000	0.65	0.0332	0.0379	0.0040	0.0054
	9000	0.65	0.0373	0.0426	0.0040	0.0057
60,000	3000	0.85	0.0135	0.0155	0.0033	0.0027
	4000	0.85	0.0181	0.0206	0.0033	0.0032
	5000	0.80	0.0213	0.0243	0.0033	0.0035
	6000	0.75	0.0239	0.0273	0.0033	0.0039
	7000	0.70	0.0260	0.0298	0.0033	0.0042
	8000	0.65	0.0276	0.0316	0.0033	0.0045
	9000	0.65	0.0311	0.0355	0.0033	0.0047
75,000	3000	0.85	0.0108	0.0124	0.0027	0.0022
	4000	0.85	0.0145	0.0165	0.0027	0.0025
	5000	0.80	0.0170	0.0194	0.0027	0.0028
	6000	0.75	0.0191	0.0219	0.0027	0.0031
	7000	0.70	0.0208	0.0238	0.0027	0.0033
	8000	0.65	0.0221	0.0253	0.0027	0.0036
	9000	0.65	0.0249	0.0284	0.0027	0.0038

[a] $\rho = 0.85\beta_1 \dfrac{f'_c}{f_y} \dfrac{0.003}{0.003 + \epsilon_t}$

[b] $\dfrac{c}{d_t} = 0.375, \dfrac{a}{d_t} = 0.375\beta_1$

[c] $\dfrac{c}{d_t} = 0.429, \dfrac{a}{d_t} = 0.429\beta_1$

TABLE A.5a
Flexural resistance factor: $R = \rho f_y\left(1 - 0.588\,\dfrac{\rho f_y}{f'_c}\right)$ psi

	f_y = 40,000 psi				f_y = 60,000 psi			
	f'_c, psi				f'_c, psi			
ρ	3000	4000	5000	6000	3000	4000	5000	6000
0.0005	20	20	20	20	30	30	30	30
0.0010	40	40	40	40	59	59	60	60
0.0015	59	59	60	60	88	89	89	89
0.0020	79	79	79	79	117	118	118	119
0.0025	98	99	99	99	146	147	147	148
0.0030	117	118	118	119	174	175	176	177
0.0035	136	137	138	138	201	204	205	206
0.0040	155	156	157	157	229	232	233	234
0.0045	174	175	176	177	256	259	261	263
0.0050	192	194	195	196	282	287	289	291
0.0055	211	213	214	215	309	314	317	319
0.0060	229	232	233	234	335	341	345	347
0.0065	247	250	252	253	360	368	372	375
0.0070	265	268	271	272	385	394	399	403
0.0075	282	287	289	291	410	420	426	430
0.0080	300	305	308	310	435	446	453	457
0.0085	317	323	326	329	459	472	479	485
0.0090	335	341	345	347	483	497	506	511
0.0095	352	359	363	366	506	522	532	538
0.0100	369	376	381	384	529	547	558	565
0.0105	385	394	399	403	552	572	583	591
0.0110	402	412	417	421	575	596	609	617
0.0115	419	429	435	439	597	620	634	643
0.0120	435	446	453	457	618	644	659	669
0.0125	451	463	471	476	640	667	684	695
0.0130	467	480	488	494	661	691	708	720
0.0135	483	497	506	511	681	714	733	746
0.0140	499	514	523	529	702	736	757	771
0.0145	514	531	540	547	722	759	781	796
0.0150	529	547	558	565	741	781	805	821
0.0155	545	563	575	582	760	803	828	845
0.0160	560	580	592	600		825	852	870
0.0165	575	596	609	617		846	875	894
0.0170	589	612	626	635		867	898	918
0.0175	604	628	642	652		888	920	942
0.0180	618	644	659	669		909	943	966
0.0185	633	660	676	686		929	965	989
0.0190	647	675	692	703		949	987	1013
0.0195	661	691	708	720		969	1009	1036
0.0200	675	706	725	737		988	1031	1059

TABLE A.5b

Flexural resistance factor: $R = \rho f_y \left(1 - 0.588 \dfrac{\rho f_y}{f'_c}\right)$ **psi**

	f_y = 40,000 psi				f_y = 60,000 psi			
	f'_c, psi				f'_c, psi			
ρ	3000	4000	5000	6000	3000	4000	5000	6000
0.003	117	118	118	119	174	175	176	177
0.004	155	156	157	157	229	232	233	234
0.005	192	194	195	196	282	287	289	291
0.006	229	232	233	234	335	341	345	347
0.007	265	268	271	272	385	394	399	403
0.008	300	305	308	310	435	446	453	457
0.009	335	341	345	347	483	497	506	511
0.010	369	376	381	384	529	547	558	565
0.011	402	412	417	421	575	596	609	617
0.012	435	446	453	457	618	644	659	669
0.013	467	480	488	494	661	691	708	720
0.014	499	514	523	529	702	736	757	771
0.015	529	547	558	565	741	781	805	821
0.016	560	580	592	600	779	825	852	870
0.017	589	612	626	635		867	898	918
0.018	618	644	659	669		909	943	966
0.019	647	675	692	703		949	987	1013
0.020	675	706	725	737		988	1031	1059
0.021	702	736	757	771			1073	1104
0.022	728	766	789	804			1115	1149
0.023	754	796	820	837			1156	1193
0.024		825	852	870			1196	1237
0.025		853	882	902				1280
0.026		881	913	934				1322
0.027		909	943	966				1363
0.028		936	972	997				
0.029		962	1002	1028				
0.030		988	1031	1059				
0.031		1014	1059	1089				
0.032			1087	1119				
0.033			1115	1149				
0.034			1142	1179				
0.035			1170	1208				
0.036			1196	1237				
0.037				1265				
0.038				1294				
0.039				1322				
0.040				1349				
0.041				1376				

Design Aids: Concrete dimensions and steel

EXAMPLE 4.10 **Concrete dimensions and steel area to resist a given moment.** Find the cross section of concrete and the area of steel required for the beam in Example 4.6, making use of the design aids of Appendix A. $M_u = 1670$ in-kips, $f'_c = 4000$ psi, and $f_y = 60{,}000$ psi. Use a reinforcement ratio of $0.60\rho_{0.005}$.

Solution. From Table A.4, the maximum reinforcement ratio is $\rho_{0.005} = 0.0181$. For economy, a value of $\rho = 0.60\rho_{0.005} = 0.0109$ will be used. For that value, by interpolation from Table A.5a, the required value of R is 596. Then

$$bd^2 = \frac{M_u}{\phi R} = \frac{1670 \times 1000}{0.90 \times 596} = 3113 \text{ in}^3$$

Concrete dimensions $b = 10$ in. and $d = 17.6$ in. will satisfy this, but the depth will be rounded to 17.5 in. to provide a total beam depth of 20.0 in. It follows that

$$R = \frac{M_u}{\phi bd^2} = \frac{1670 \times 1000}{0.90 \times 10 \times 17.5^2} = 606 \text{ psi}$$

and from Table A.5a, by interpolation, $\rho = 0.0112$. This leads to a steel requirement of $A_s = 0.0112 \times 10 \times 17.5 = 1.96 \text{ in}^2$ as before.

Design Aids: find steel area

EXAMPLE 4.11 **Determination of steel area.** Find the steel area required for the beam in Example 4.7, with concrete dimensions $b = 10$ in. and $d = 17.5$ in. known to be adequate to carry the factored load moment of 1300 in-lb. Material strengths are $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

SOLUTION. Note that in cases in which the concrete dimensions are known to be adequate and only the reinforcement must be found, the iterative method used earlier is not required. The necessary flexural resistance factor is

$$R = \frac{M_u}{\phi b d^2} = \frac{1300 \times 1000}{0.90 \times 10 \times 17.5^2} = 472 \text{ psi}$$

According to Table A.5a, with the specified material strengths, this corresponds to a reinforcement ratio of $\rho = 0.0085$, giving a steel area of

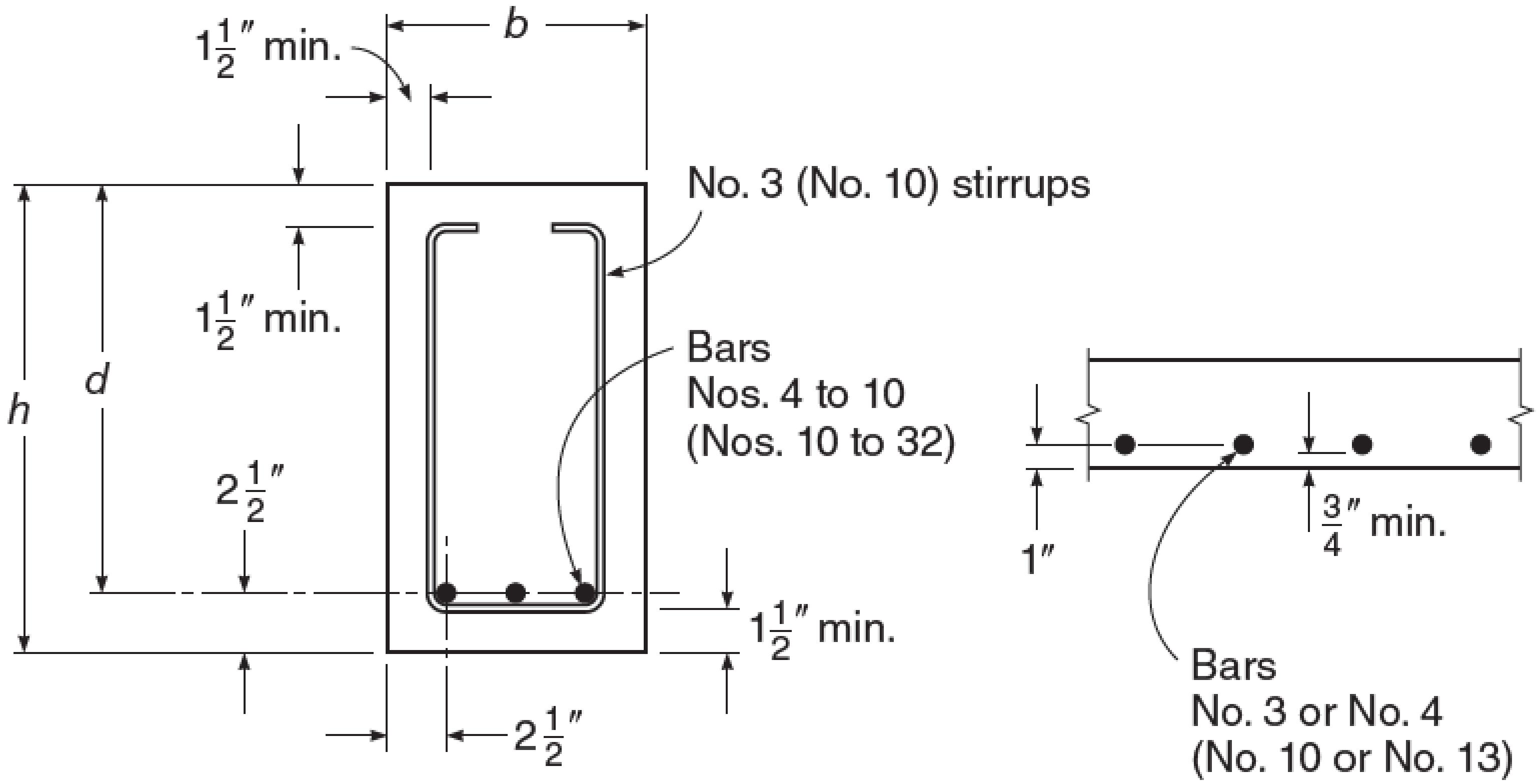
$$A_s = 0.0085 \times 10 \times 17.5 = 1.49 \text{ in}^2$$

as before. Two No. 8 (No. 25) bars will be used.

Practical considerations in the design of Beams: Concrete Protection for reinforcement

fire rating

- Protection for steel against fire and corrosion
- Concrete cover depends on member and exposure
- Surfaces not exposed to ground or weather
  - Not less than ¾ in for slab
  - Not less than 1.5 in for beams and columns
- Surfaces exposed to weather or in contact with ground
  - At least 2in
- Cast against ground with no form work
  - Min 3 in cover



(*a*) Beam with stirrups

(*b*) Slab

FIGURE 4.12
Requirements for concrete cover in beams and slabs not exposed to weather or in contact with ground.

1.5+3/8+10/16 =2.5

3/4+4/16 =1

- b and h are rounded to 1 or 2 inch
- Slab rounded to ¼ or ½ inch (greater than 6 inch)

6.125 হলে না
6.5 হবে।

- Proportions- d 2-3 times of b

Selection of bar and spacing

- No 3 to No 11 for beams
- No 14 and No 18 for columns

সেক্ষেত্রে বেশি No. 6 আর 10 use হয়।

- Mixing of sizes allowed with 2 bar sizes

পাশাপাশি size। 10 এর সাথে 3 এর ব্যবহার নয়

Gap between bars

- Clear distance between bars not less than bar dia or 1 inch (for columns 1.5d or 1.5inch)
- Two or more layers- min 1 inch
- Upper bar directly above
- Clear distance and cover not less than 1.33times maximum aggregate size
- Vibrator nozzle → ঢুকানোর gap থাকতে হবে।

Reinforcements –usual sizes

- Slab- No 3, 4, 5 (10mm, 12mm, 16mm)
- Beam- No 5,6, 7, 8 (16 20 ~~22~~ 25mm)
- Stirrup/tie- No, 3 4 (10 12mm)
- Column –No 5, 6 7 8 9 10 11 14 18 (16 20 ~~22~~ 25 28 32 ….)
- Mat- No 4,5,6,8 (12 16 20 25 mm)
- Smaller sizes preferred as long as there is no congestion → ভালো distribution হবে।

TABLE A.7
Maximum number of bars as a single layer in beam stems

দুইটা bar দেওয়া যাবে।

$\frac{3}{4}$ in. Maximum Size Aggregate, No. 4 (No. 13) Stirrups[a]													
Bar No.		**Beam Width b_w, in.**											
Inch-Pound	**SI**	**8**	**10**	**12**	**14**	**16**	**18**	**20**	**22**	**24**	**26**	**28**	**30**
5	16	2	4	5	6	7	8	10	11	12	13	15	16
6	19	2	3	4	6	7	8	9	10	11	12	14	15
7	22	2	3	4	5	6	7	8	9	10	11	12	13
8	25	2	3	4	5	6	7	8	9	10	11	12	13
9	29	1	2	3	4	5	6	7	8	9	9	10	11
10	32	1	2	3	4	5	6	6	7	8	9	10	10
11	36	1	2	3	3	4	5	5	6	7	8	8	9
14	43	1	2	2	3	3	4	5	5	6	6	7	8
18	57	1	1	2	2	3	3	4	4	4	5	5	6

1 in. Maximum Size Aggregate, No. 4 (No. 13) Stirrups[a]													
Bar No.		**Beam Width b_w, in.**											
Inch-Pound	**SI**	**8**	**10**	**12**	**14**	**16**	**18**	**20**	**22**	**24**	**26**	**28**	**30**
5	16	2	3	4	5	6	7	8	9	10	11	12	13
6	19	2	3	4	5	6	7	8	9	9	10	11	12
7	22	1	2	3	4	5	6	7	8	9	10	10	11
8	25	1	2	3	4	5	6	7	7	8	9	10	11
9	29	1	2	3	4	5	6	7	7	8	9	9	10
10	32	1	2	3	4	5	6	6	7	7	8	9	10

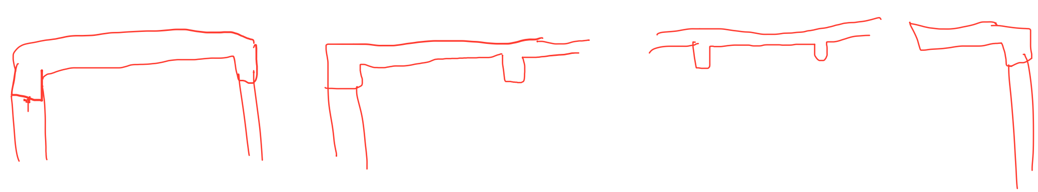
[a]Minimum concrete cover assumed to be $1\frac{1}{2}$ in. to the No. 4 (No. 13) stirrup.

Data Source: Adapted from Ref. 4.8.

Beam minimum depth

TABLE 7.1
Minimum thickness of nonprestressed beams or one-way slabs unless deflections are computed

Member	Minimum Thickness h			
	Simply Supported	**One End Continuous**	**Both Ends Continuous**	**Cantilever**
	Members Not Supporting or Attached to Partitions or Other Construction Likely to Be Damaged by Large Deflections			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$



DOUBLY REINFORCED BEAM

- Beams with tension and compression reinforcement
- Cross section is limited
- Compression steel is used for other reasons-long term deflection, reversal of moment, hanger bar for stirrup

Tension and compression steel both at yields

Tension compression behaviour same

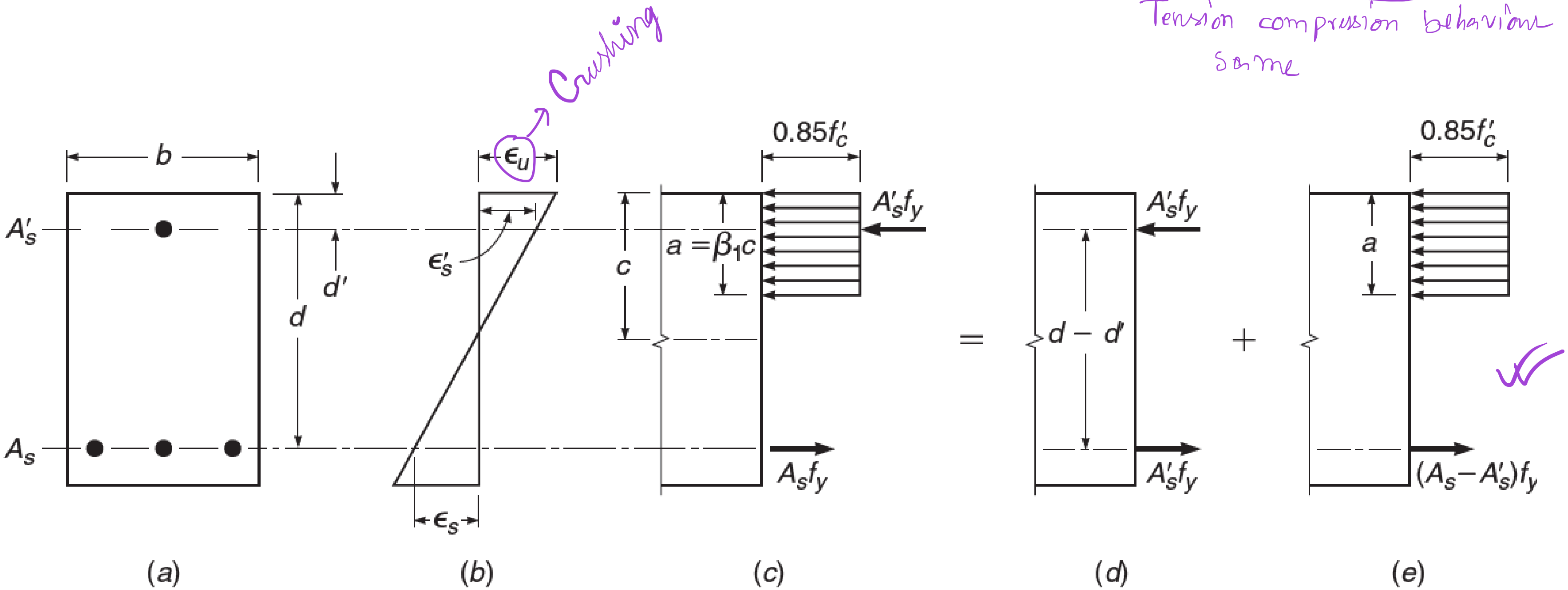


FIGURE 4.13
Doubly reinforced rectangular beam.

* এখানে উভয়ের steel yeild করে থাকবে না, তাই কোন warning দিবে না যা আমাদের লাগবে।

$$M_{n1} = A'_s f_y (d - d') \tag{4.42a}$$

as shown in Fig. 4.13*d*. The second part, M_{n2}, is the contribution of the remaining tension steel $A_s - A'_s$ acting with the compression concrete

$$M_{n2} = (A_s - A'_s) f_y \left(d - \frac{a}{2}\right) \tag{4.42b}$$

as shown in Fig. 4.13*e*, where the depth of the stress block is

$$\boxed{a = \frac{(A_s - A'_s) f_y}{0.85 f'_c\, b}} \tag{4.43a}$$

C × moment arm = T × moment arm হলে এটা আসে

With the definitions $\rho = A_s / bd$ and $\rho' = A'_s / bd$, this can be written

$$a = \frac{(\rho - \rho') f_y d}{0.85 f'_c} \tag{4.43b}$$

The total nominal moment is then

$$\boxed{M_n = M_{n1} + M_{n2} = A'_s f_y (d - d') + (A_s - A'_s) f_y \left(d - \frac{a}{2}\right)} \tag{4.44}$$

কারণ উপরে নিচে steel থাকবে

$$\bar{\rho}_b = \rho_b + \rho' \tag{4.45}$$

where ρ_b is the balanced reinforcement ratio for the corresponding singly reinforced beam and is calculated from Eq. (4.24). The ACI Code establishes the strength reduction factor ϕ based on the net tensile strain, not the reinforcement ratio. The maximum reinforcement ratio for $\phi = 0.90$ is

$$\bar{\rho}_{0.005} = \rho_{0.005} + \rho' \tag{4.46a}$$

Since $\bar{\rho}_{0.005}$ corresponds to $\epsilon_t = 0.005$, no check of ϵ_t is required to determine the strength reduction factor ϕ if $\bar{\rho} \leq \bar{\rho}_{0.005}$. The maximum reinforcement ratio permitted for doubly reinforced beams (producing $\epsilon_t = 0.004$) is

$$\bar{\rho}_{0.004} = \rho_{0.004} + \rho' \tag{4.46b}$$

Compression steel below yield stress

The preceding equations, through which the fundamental analysis of doubly reinforced beams is developed clearly and concisely, are valid *only* if the compression steel has yielded when the beam reached its nominal capacity. In many cases, such as for wide, shallow beams, beams with more than the usual concrete cover over the compression bars, beams with high yield strength steel, or beams with relatively small amounts of tensile reinforcement, the compression bars will be below the yield stress at failure. It is necessary, therefore, to develop more generally applicable equations to account for the possibility that the compression reinforcement has not yielded when the doubly reinforced beam fails in flexure.

Whether or not the compression steel will have yielded at failure can be determined as follows. Referring to Fig. 4.13*b*, and taking as the limiting case $\epsilon_s' = \epsilon_y$, one obtains, from geometry,

নিচের ρ যত কম হবে উপর crush করবে উপরের rod yeild করবে।

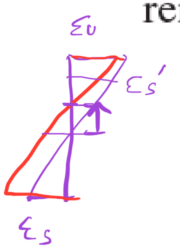
$$\frac{c}{d'} = \frac{\epsilon_u}{\epsilon_u - \epsilon_y} \qquad \text{or} \qquad c = \frac{\epsilon_u}{\epsilon_u - \epsilon_y} d'$$

$T = C = C_c + C_s$

$A_s f_y = 0.85 f_c' ab + A_s' f_y$

এখান β_1, c জানা যায়

Summing forces in the horizontal direction (Fig. 4.13*c*) gives the *minimum* tensile reinforcement ratio $\bar{\rho}_{cy}$ that will ensure yielding of the compression steel at failure:



নিচের Rod

$$\boxed{\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho'} \qquad (4.47)$$

→ বের করব

If the *tensile reinforcement ratio* is *less than* this limiting value, the neutral axis is sufficiently high that the compression steel stress at failure will be less than the

If $\rho < \bar{\rho}_{cy}$, $\rho\downarrow$ $T\downarrow$ $C\downarrow$ $a\downarrow$ $c\downarrow$ $\varepsilon_s'\downarrow$, so, ε_s' doesn't yeild

$\varepsilon_s' < \varepsilon_y$ হবে, তখন Formula use করা যাবে না।

If $\rho > \bar{\rho}_{cy}$, compression steel yields.

Formula use করতে $\varepsilon_s' = \varepsilon_y$ ধরতে

yield stress. In this case, it can easily be shown on the basis of Fig. 4.13*b* and *c* that the balanced reinforcement ratio is

$$\bar{\rho}_b = \rho_b + \rho' \frac{f_s'}{f_y} \tag{4.48}$$

where

$$f_s' = E_s \epsilon_s' = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + \epsilon_y) \right] \le f_y \tag{4.49a}$$

To determine $\rho_{0.005}$, $\epsilon_t = 0.005$ is substituted for ϵ_y in Eq. (4.49*a*), giving

$$f_s' = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.005) \right] \le f_y \tag{4.49b}$$

Likewise, for $\epsilon_t = 0.004$,

$$f_s' = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.004) \right] \le f_y \tag{4.49c}$$

Hence, the maximum reinforcement ratio permitted for $\phi = 0.90$ is

$$\boxed{\bar{\rho}_{0.005} = \rho_{0.005} + \rho' \frac{f_s'}{f_y}} \tag{4.50a}$$

where f_s' is given by Eq. (4.49*b*) and the maximum reinforcement ratio for $\phi < 0.90$ is

$$\boxed{\bar{\rho}_{0.004} = \rho_{0.004} + \rho' \frac{f_s'}{f_y}} \tag{4.50b}$$

It should be emphasized that Eqs. (4.49*a*), (4.49*b*), and (4.49*c*) for compression steel stress apply *only for beams with exact strain values in the extreme tensile reinforcement of* ϵ_y, $\epsilon_t = 0.004$, *or* $\epsilon_t = 0.005$.

If the tensile reinforcement ratio is less than $\overline{\rho}_b$, as given by Eq. (4.48), and less than $\overline{\rho}_{cy}$, as given by Eq. (4.47), then the tensile steel is at the yield stress at failure but the compression steel is not, and new equations must be developed for compression steel stress and flexural strength. The compression steel stress can be expressed in terms of the still-unknown neutral axis depth as

$$\boxed{f'_s = \epsilon_u E_s \frac{c - d'}{c}} \tag{4.51}$$

yeild না হলে

Consideration of horizontal force equilibrium (Fig. 4.13*c* with compression steel stress equal to f'_s) then gives

$$\overbrace{A_s f_y}^{T} = \overbrace{0.85\beta_1 f'_c bc}^{C_c} + \overbrace{A'_s \epsilon_u E_s \frac{c - d'}{c}}^{C_s} \tag{4.52}$$

This is a quadratic equation in c, the only unknown, and is easily solved for c. The nominal flexural strength is found using the value of f'_s from Eq. (4.41), and $a = \beta_1 c$ in the expression

$$\boxed{M_n = 0.85 f'_c ab\left(d - \frac{a}{2}\right) + A'_s f'_s (d - d')} \tag{4.53}$$

If $\varepsilon'_s < \varepsilon_y$

* আমাদের $\rho > \bar{\rho}_{cy}$ হওয়া necessary না।

TABLE 4.2
Minimum beam depths for compression reinforcement to yield

	$\epsilon_t = 0.004$		$\epsilon_t = 0.005$	
f_y, psi	Minimum d'/d	Minimum d for $d' = 2.5$ in., in.	Maximum d'/d	Minimum d for $d' = 2.5$ in., in.
40,000	0.23	10.8	0.20	12.3
60,000	0.13	18.8	0.12	21.5
75,000	0.06	42.7	0.05	48.8
80,000	0.03	72.5	0.03	82.9

Example 4.12

Flexural strength of a given member. A rectangular beam, shown in Fig. 4.14, has a width of 12 in. and an effective depth to the centroid of the tension reinforcement of 24 in. The tension reinforcement consists of six No. 10 (No. 32) bars in two rows. For simplicity in calculating ϵ_t, d_t will be taken as d. Compression reinforcement consisting of two No. 8 (No. 25) bars is placed 2.5 in. from the compression face of the beam. If $f_y = 60{,}000$ psi and $f'_c = 5000$ psi, what is the design moment capacity of the beam?

Solution. The steel areas and ratios are

$$A_s = 7.62 \text{ in}^2 \qquad \rho = \frac{7.62}{12 \times 24} = 0.0265$$

$$A'_s = 1.58 \text{ in}^2 \qquad \rho' = \frac{1.58}{12 \times 24} = 0.0055$$

Check the beam first as a singly reinforced beam to see if the compression bars can be disregarded,

$$\rho_{0.005} = 0.0243 \qquad \text{from Table A.4 or Eq. (4.26}c\text{)}$$

The actual $\rho = 0.0265$ is larger than $\rho_{0.005}$, so the beam must be analyzed as doubly reinforced. From Eq. (4.47), with $\beta_1 = 0.80$,

$$\boxed{\bar{\rho}_{cy}} = 0.85 \times 0.80 \times \frac{5}{60} \times \frac{2.5}{24} \times \frac{0.003}{0.003 - 0.00207} + 0.0055 = 0.0245$$

here, $\rho > \bar{\rho}_{cy}$, ∴ 압축 yeild 한다,

$\rho_{.005}$ [for single reinforced]

The tensile reinforcement ratio is greater than this, so the compression bars will yield when the beam fails. The maximum reinforcement ratio thus can be found from Eq. (4.46*a*),

$$\bar{\rho}_{0.005} = 0.0213 + 0.0055 = 0.0268$$

ρ_s'

The actual tensile reinforcement ratio is below the maximum value, as required. Then, from Eq. (4.43*a*),

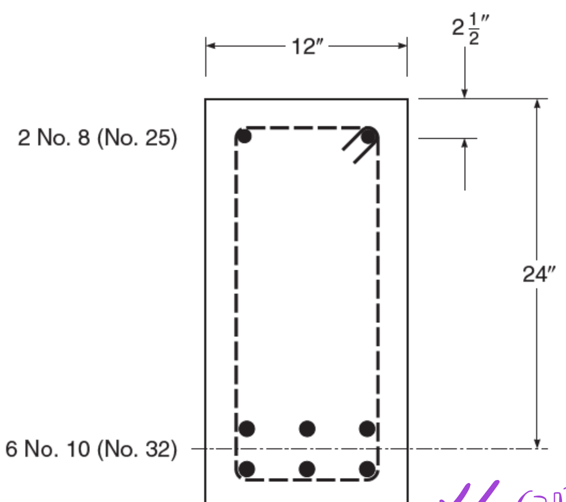
যদি এর চেয়ে নিচে থাকে, then usable, বেশি হলে হবে না।

$$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b} = \frac{(7.62 - 1.58)60}{0.85 \times 5 \times 12} = 7.11 \text{ in.}$$

এই step দরকার নাই।

$$c = a/\beta_1 = \frac{7.11}{0.80} = 8.89 \text{ in.}$$

$$\epsilon_t = \epsilon_u \left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{24 - 8.89}{8.89}\right) = 0.0051 > 0.005$$



বইয়ের logic

If $\rho > \rho_{0.005}$, then we would not consider it as doubly reinforced

But এটা ঠিক না।

significant moment নিতে পারবে না

মোট কথা: steel আর concrete এর যে total compression, তার সমান

যতটুকু পরিমাণ tension Balance করে দিবে।

and thus,

$$\phi = 0.90$$

and from Eq. (4.44),

$$M_n = 1.58 \times 60(24 - 2.5) + 6.04 \times 60\left(24 - \frac{7.11}{2}\right) = 9450 \text{ in-kips}$$

The design strength is

$$\phi M_n = 0.90 \times 9450 = 8500 \text{ in-kips}$$

Nadim Hassoun

Doubly Reinforced beam — Nadim Hassoun

1. Calculate ρ, ρ' and $(\rho - \rho')$

   ρ_{max}, ρ_{min} → জানতে হবে

2. Calculate $$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} + \rho'$$

3. If $\rho \geq \bar{\rho}_{cy}$ compression steel yields, $f_s' = f_y$

   $\rho < \bar{\rho}_{cy}$ " " doesnot yield $f_s' < f_y$

4. If comp steel yields, then ← যদি yeild হলে

   a. Check that $\rho_{max} \geq (\rho - \rho') \geq \rho_{min}$ $\phi = 0.9$

   or check $\varepsilon_t \geq 0.005$

   b. Calculate $$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b}$$

   c. Calculate $$\phi M_n = \phi\left[(A_s - A_s')f_y\left(d - \frac{a}{2}\right) + A_s' f_y (d - d')\right]$$

5. If comp. steel does not yield, then

a. $T = C$

$$\Rightarrow A_s f_y = \underbrace{\boxed{0.85 f_c' * b * \beta_1 c}}_{C_c} + \underbrace{\boxed{A_s' f_s'}}_{C_s}$$

$$A_s f_y = 0.85 \beta_1 b c f_c' + A_s' . E_s \varepsilon_u \frac{c - d'}{c}$$

$\Rightarrow$ solve this quadratic equation to find c ✓

b. Find $f_s' = E_s \boxed{\varepsilon_u \frac{c-d'}{c}}$ ε

স্টিল yeild না করলে

$\phi = 0.9$

$\varepsilon_t =$

c. Check $\rho_{max} \geq \left(\rho - \underbrace{\rho' . \frac{f_s'}{f_y}}\right) \geq \rho_{min}$

নিচের অংশটুকু area টুকু steel এর compression কে nutratize করলে লাগছে।

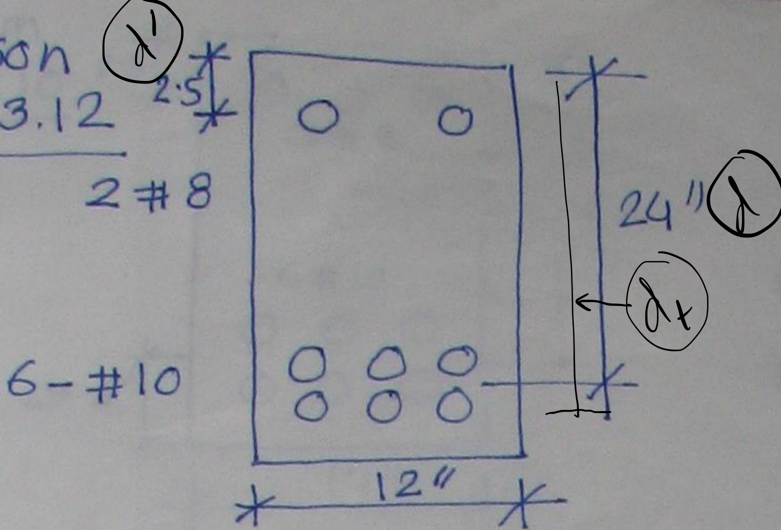
d. Calculate $a = \dfrac{A_s f_y - A_s' f_s'}{0.85 f_c' b}$ or $a = \beta_1 c$ (check)

e. Calculate

$$\phi M_n = \phi \left[(A_s f_y - A_s' f_s')\left(d - \frac{a}{2}\right) + A_s' f_s' (d - d') \right]$$

Find φM_n

Nilson d'
Ex 3.12



$f_c' = 5000$ psi
$f_y = 60{,}000$ psi
Design moment capacity = ?
$\phi M_n = ?$

Sol

$A_s = 6 \times 1.27 = 7.62 \text{ in}^2$ $\qquad \rho = \frac{A_s}{bd} = 0.0265$

$A_s' = 2 * 0.79 = 1.58 \text{ in}^2$ $\qquad \rho' = \frac{A_s'}{bd} = 0.0055$

$$\boxed{\begin{aligned} \rho_{max} &= 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + .004} \\ \rho_{min} &= \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y} \end{aligned}}$$

0.005 হবে 2008, 0.004 ছিল Range এর,

$\rho_{max} = 0.0243$

$\rho_{min} = 0.00354$

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{d'}{d}\frac{\varepsilon_u}{\varepsilon_u-\varepsilon_y} + \rho'$$

$$\varepsilon = \frac{f_y}{E_s} = \frac{60k}{29000k}$$

$$= 0.85*.8\ \frac{5}{60}\ \frac{2.5}{24}\ \frac{.003}{.003-.00207} + .0055 = 0.0245$$

$\rho > \bar{\rho}_{cy} \Rightarrow$ Comp bars yield when beam fails.

$\rho_{max} = .0243$ ok.

$\rho - \rho' = 0.021$

$\rho_{min} = 0.00354$

$$a = \frac{A_s - A_s'}{0.85 f_c' b} f_y = 7.11 \qquad c = \frac{a}{\beta_1} = 8.88$$

$d_t \rightarrow$ outer layer bar

$$\varepsilon_t = 0.003\ \frac{24-8.88}{8.88} = 0.0051 \Rightarrow \phi = 0.9$$

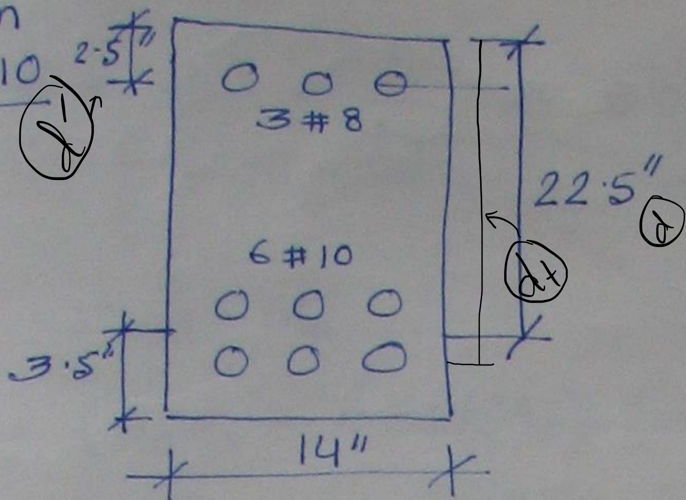
$$M_n = A_s' f_y (d-d') + (A_s - A_s') f_y (d - a/2)$$

$$= 1.58*60(24-2.5) + 6.04*60\left(24 - \frac{7.11}{2}\right) = 9447 \text{ kin}$$

$\phi M_n = 8503$ k-in

Nadim

Ex 3.10



Find ϕM_n

$f_c' = 5$ ksi $f_y = 60$ ksi.

$A_s' = 3 * 0.79 = 2.37 \text{ in}^2$ $\rho' = 0.007524$

$A_s = 6 * 1.27 = 7.62 \text{ in}^2$ $\rho = 0.0242$

$\rho - \rho' = 0.01667$.

$\rho_{max} = 0.0243$ ← $(\rho - \rho')$ is ok.

$\rho_{min} = 0.00354$

2.7% লাগবে yeild করতে

2. $\bar{\rho}_{cy} = 0.85 * 0.8 * \frac{5}{60} \frac{2.5}{22.5} \frac{.003}{.003 - .00207} + 0.007524 = 0.02783$

$\rho < \bar{\rho}_{cy} \Rightarrow$ Comp bar does not yield

to be more correct

$\rho < \rho_{cy} \Rightarrow$ Comp ...

yeild না করলে

লাগবে না

3. $T = C = C_s + C_c$

$\Rightarrow A_s f_y = 0.85 f_c' \beta_1 c . b + A_s' E_s \varepsilon_u \dfrac{c-d'}{c}$ $\boxed{- 0.85 f_c' A_s'}$ ✗

to be more correct

Rod এর ফুটোর বাদ দিয়ে concrete হিসাব করতে

$\Rightarrow 7.62 * 60 = 0.85 * 5 * .8 * c * 14 + 2.37 * 29 * 10^3 * .003 \dfrac{c-2.5}{c} - .85 * 5 * 2.37$

$\Rightarrow 457.2 = 47.6c + 206.19 \dfrac{c-2.5}{c} - 10.07$

$47.6c^2 - 261.08c - 515.475 = 0$

$\Rightarrow c = \dfrac{261.08 \pm \sqrt{261.08^2 + 4 * 47.6 * 515.475}}{2 * 47.6}$

$= 7.026$ in

$a = \beta_1 c = 5.62$

$f_s' = E_s \varepsilon_u \dfrac{c-d'}{c}$

$= 56.04$ ksi.

$$\rho - \rho' \frac{f_s'}{f_y} = 0.0243 - 0.007524 * \frac{56.04}{60}$$

$$= 0.01724 < \rho_{max} \quad \text{ok.}$$

যদি $\rho - \rho' \frac{f'_s}{f'_y} > \rho_{max}$ আসে, then অন্য লাগত যাবেই strain দিতে না

5. Find M_n

$$M_n = (A_s' f_s' - \boxed{0.85 f_c' A_s'})(d - d') + (A_s f_y - A_s' f_s')(d - \tfrac{a}{2})$$

$$M_n = (A_s' f_s' - 0.85 * 5 * 2.37)(22.5 - 2.5)$$

$$= (2.37 * 56 - 0.85 * 5 * 2.37)$$

$$+ (7.62 * 60 - 2.37 * 56.04 + .85 * 5 * 2.37)$$

$$(22.5 - \tfrac{5.62}{2})$$

$$= 122.7 * 20.0 + 334.5 * 19.69.$$

$$= 9040.3 \text{ k.in}.$$

ঠিক দিক

Comp. in concrete $= 0.85 f_c' ab = C_c = 334.5^k$

Comp. in steel $= A_s' f_s'$ − force in displaced concrete

$= C_s = A_s'(f_s' - \boxed{0.85 f_c'}) = 122.7^k.$

Tension in steel $= A_s f_y = 457.2^k$

$T = C$ ok. → Check

6. $$\frac{\varepsilon_t}{d_t - c} = \frac{\varepsilon_u}{c}$$

$22.5 + .5$

$$\varepsilon_t = \frac{.003}{7.026}(23 - 7.026)$$

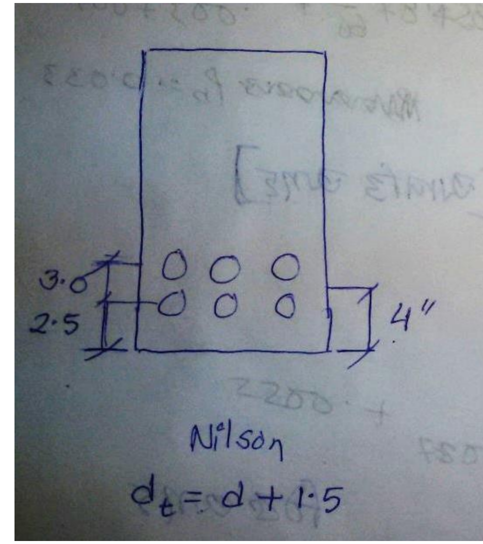
$$= 0.00682 > .005 \quad \phi = 0.9$$

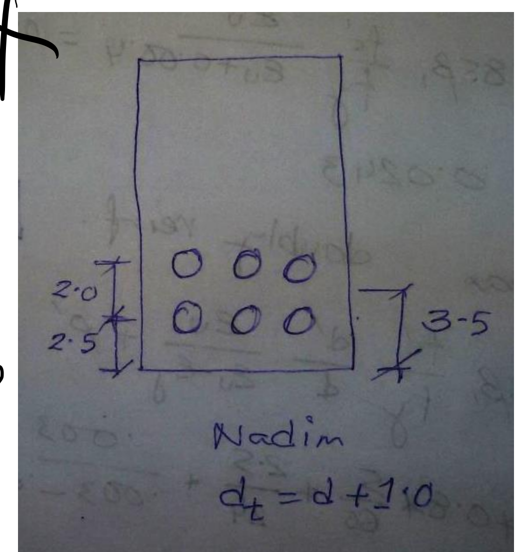
7. $\phi M_n = 8136$ k.in

$d_t = d + 1.0$ if $d = h - 3.5$

$d_t = d + 1.5$ if $d = h - 4.0$

$d_t = h - 2.5$ all case





Nadim ২০ ২ই ৪টা problem assignment

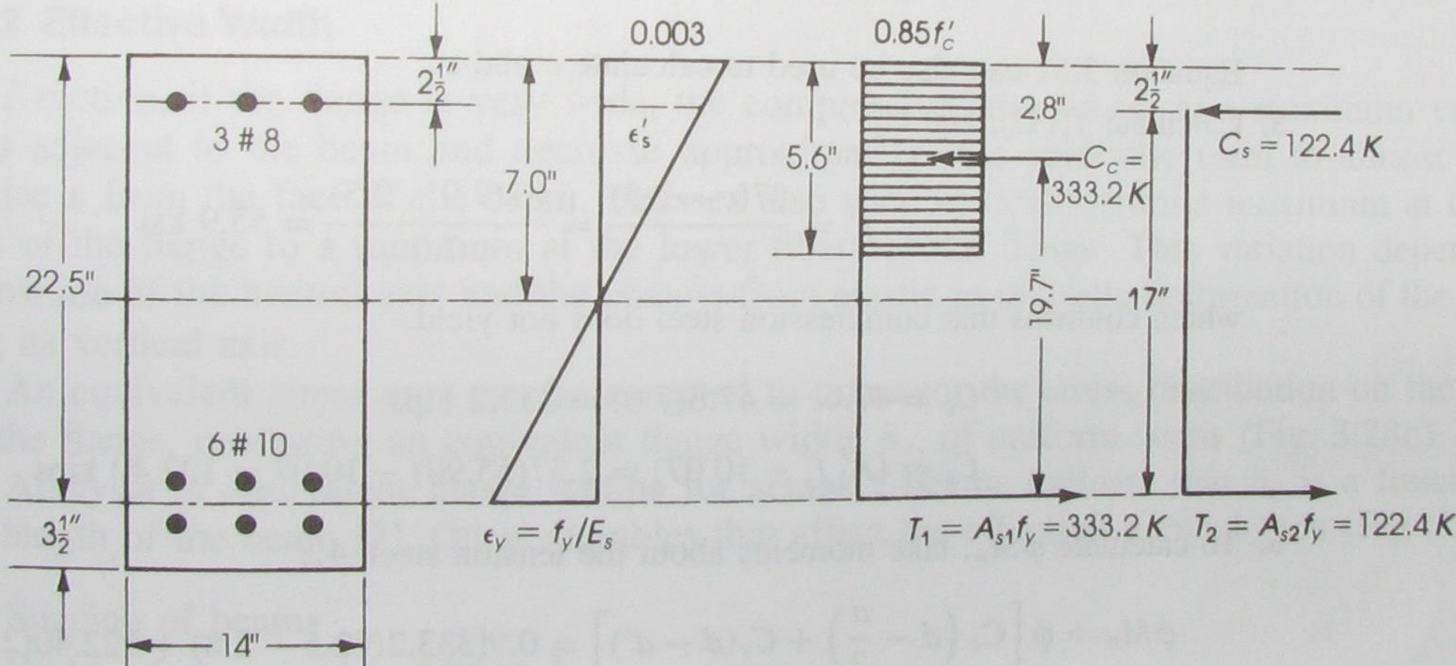


Figure 3.27 Example 3.10 analysis solution.

Design of
Doubly Reinforced Beam

Design problem from Nadim

<u>Ex 4.5</u> A beam section is limited to a width $b = 10\,in$ and total depth of $h = 22\,in$ and has to resist a factored moment of $226.5\,k\text{-}ft$. Calculate the required reinforcement. Given $f_c' = 3\,ksi$ and $f_y = 50\,ksi$.

<u>Sol</u>

$$\rho_{0.005} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.005} = 0.01625$$

$$d = 22 - 3.5 = 18.5$$ two layer

ধরি। But 1 layer দিয়ে করা যেত হত।

$$A_s = 0.01625 * 10 * 18.5 = 3\,in^2$$

$$M_u = 226.5 \times 12 = 2718\,k\text{-}''$$

ঠিক।

$$a = \frac{A_s f_y}{0.85 f_c' b} = 5.88\,in.$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 2333.8\,kin.$$

$$\phi M_n = 0.9 * 2333.8 = 2100.4\,k\text{-}in.$$

Doubly Reinforced beam reqd.

যখন h কম থাকবে, But moment অনেক, তখন doubly R.B use করি।

$\phi M_{n1} = 2100.4$

$\phi M_{n2} = 2718 - 2100.4 = 617.6 \text{ k}''$

Assuming comp steel yield

$$A_{s2} = \frac{617.6}{\phi f_y (d-d')} = \frac{617.6}{0.9 * 50\,(18.5-2.5)} = 0.86 \text{ in}^2$$

Total tension steel $= A_s = 3.0 + 0.86 = 3.86 \text{ in}^2$

Comp. steel $A_s' = 0.86 \text{ in}^2$.

$\rho' = 0.00465$

$\rho = 0.02086$

$\rho - \rho' = 0.01621$

যেই steel দিলে সেটাকে balance করতে যত দিতে হবে steel সেটা দেওয়া হলো.

Use actual area provided

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} + \rho'$$

$$= 0.85 * 0.85 \frac{3}{50} \frac{2.5}{18.5} \frac{.003}{.003 - 0.001724} + 0.00465$$

$$= 0.01842$$

$\rho > \bar{\rho}_{cy}$ comp steel yields.

exact rod দিয়ে করা হয়েছে। Required দিয়ে করা না।

$$a = \frac{A_s - A_s'}{0.85 f_c' b} f_y = \frac{3.86 - 0.86}{0.85 * 3 * 10} \times 50 = 5.88 \text{ in.}$$

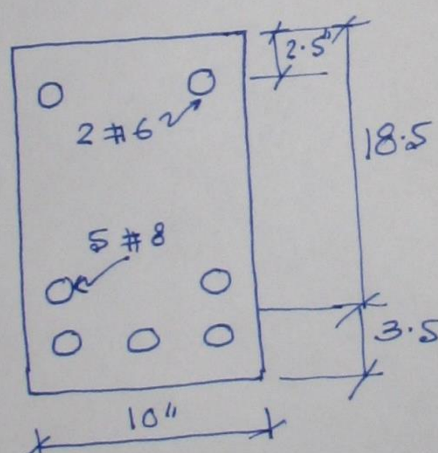
$$c = \frac{a}{\beta_1} = 6.92 \text{ in.}$$

outer layer

$$\varepsilon_t = \varepsilon_u \frac{d_t - c}{c} = 0.003 \frac{19.5 - 6.92}{6.92} = 0.00545$$

+ exact formula দিয়ে

$\phi = 0.9$ ok.



Nilson

Ex 3.13

Design

$LL = 2.47^{k/'}$ simply supported span $= 18'$

$DL = 1.05^{k/'}$ Beam section $\Rightarrow 10'' \times 20''$

$f_c' = 4000$ psi $f_y = 60{,}000$ psi

find reinforcement.

Sol

$w_u = 1.2 * 1.05 + 1.6 * 2.47 = 5.212^{k/'}$

$M_u = \frac{1}{8} w_u l^2 = \frac{1}{8} \times 5.212 * 18^2 = 211.09^{k'} = 2533^{k''}$

$d' = 2.5''$ (If needed)

$d = 20 - 4 = 16''$ (two layer)

এখানে 2.5" দিয়ে check করে দেখা উচিত। হয়ে যেতে পারে।

First check if possible to design singly reinforced.

$\varepsilon_t = 0.005$ $\rho_{0.005} = 0.0181$

$A_s = \rho bd = 0.0181 * 10 * 16 = 2.89 \text{ in}^2$

$a = 6''$

$A_s = \rho b d = 0.0181 * 10 * 16 = \ldots$

$a = \dfrac{A_s f_y}{.85 f'_c b} = 5.1$ in — এখানে exact rod নেই তাই। তোমরা check করতেছি দেখো

$c = \dfrac{a}{\beta_1} = 6''$

$M_n = A_s f_y \left(d - \dfrac{a}{2}\right) = 2.89 * 60 \left(16 - \dfrac{5.1}{2}\right) = 2332^{k''}$

$\phi M_n = 0.9 * 2332 = 2099^{k''} < M_u$

Doubly Reinf reqd.

Remaining moment $\phi M_n = 2533 - 2099 = 434^{k''}$ → short আছে।

$M_n = 482.4^{k''}$

Assuming comp bars yield

$A_{s2} = \dfrac{434}{0.9 * 60 * (16 - 2.5)} = 0.6 \text{ in}^2.$

Total tension steel $= 2.89 + 0.6 = 3.49 \text{ in}^2$

Comp steel $= 0.6 \text{ in}^2$

4 # 9
4 in^2.

2 # 6
0.88 in^2.

$\rho = \frac{4}{10\times16} = 0.025 \rightarrow$ actual $\qquad \rho' = \frac{0.88}{10\times16} = 0.0055$

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{d'}{d}\frac{\varepsilon_u}{\varepsilon_u-\varepsilon_y} + \rho'$$

$$= 0.85*0.85\frac{4}{60}\frac{2.5}{16}\frac{.003}{.003-.00207} + 0.0055$$

$$= 0.0298$$

$\rho < \bar{\rho}_{cy} \Rightarrow$ Comp steel does not yield

$$A_sf_y = 0.85f_c'b\beta_1c + A_s'f_s' - A_s'*0.85f_c'$$ X

$$\Rightarrow 4*60 = 0.85*4*10*.85c + A_s'*\left[E_s\varepsilon_u\frac{c-d'}{c} - 0.85f_c'\right]$$ X

$$\Rightarrow 240c = 28.9c^2 - 2.992c + (76.56c - 191.4)$$ X

$$\Rightarrow 28.9c^2 - 166.43c - 191.4 = 0$$ X

$$c = 6.74 \text{ in.} \qquad a = 5.73 \text{ in.}$$

$$f_s' = E_s \varepsilon_u \frac{c-d'}{c} = 54.7^{\text{ksi}}.$$

$$C_c = 0.85 f_c' . b . \beta_1 c = 194.8^{k}$$

$$C_s = A_s' \left[f_s' - 0.85 f_c' \right] = 45.14^{k}$$

checked.

$$T = A_s f_y = 4 * 60 = 240^{k}$$

$$M_n = C_c \left(d - \frac{a}{2}\right) + C_s (d - d')$$

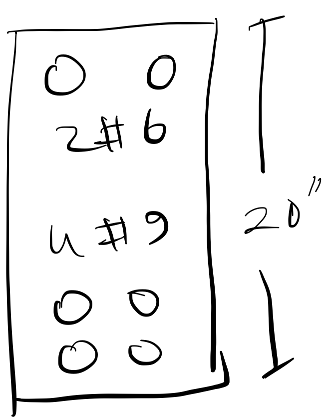
$$= 194.8 \left(16 - \frac{5.73}{2}\right) + 45.14 (16 - 2.5)$$

$$= 2558.5 + 609.44 = 3167.9^{k''}$$

$$\varepsilon_t = \varepsilon_u \frac{d_t - c}{c} = 0.003 \frac{17.5 - 6.74}{6.74} = 0.00479$$

$$\phi = 0.483 + 83.3\,\varepsilon_t = 0.882 \rightarrow \text{interpolate}$$

$$\phi M_n = 2793.9^{k} > M_u = 2533^{k''} \quad \underline{\text{ok}}.$$



$A_s = 4''$ $\quad A_s' = .88''$

$$\rho = \frac{4}{10 \times 16} = .025 \qquad \rho' = \frac{.88}{10 \times 16} = .0055$$

$$\rho_{max} \geq \left(\rho - \rho' \frac{f_s'}{f_y}\right) \geq \rho_{min}$$

$$\rho_{max} = \rho_{.004} = .85\beta_1 \frac{f_c'}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + .004} = .0206$$

$$\rho_{.005} = .0181$$

$$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y} = 0.0033$$

$$\rho - \rho' \frac{f_s'}{f_y} = 0.025 - .0055 \times \frac{54.7}{60} = 0.01999$$

$$\phi < 0.9$$

T-beam

- RC beam and slab are monolithically cast
- Beam stirrups and bent bars extend into the slab
- A part of slab act along with beam top to take longitudinal compression
- Slab forms the beam flange
- Part of beam below slab is called web/stem

Effective flange width

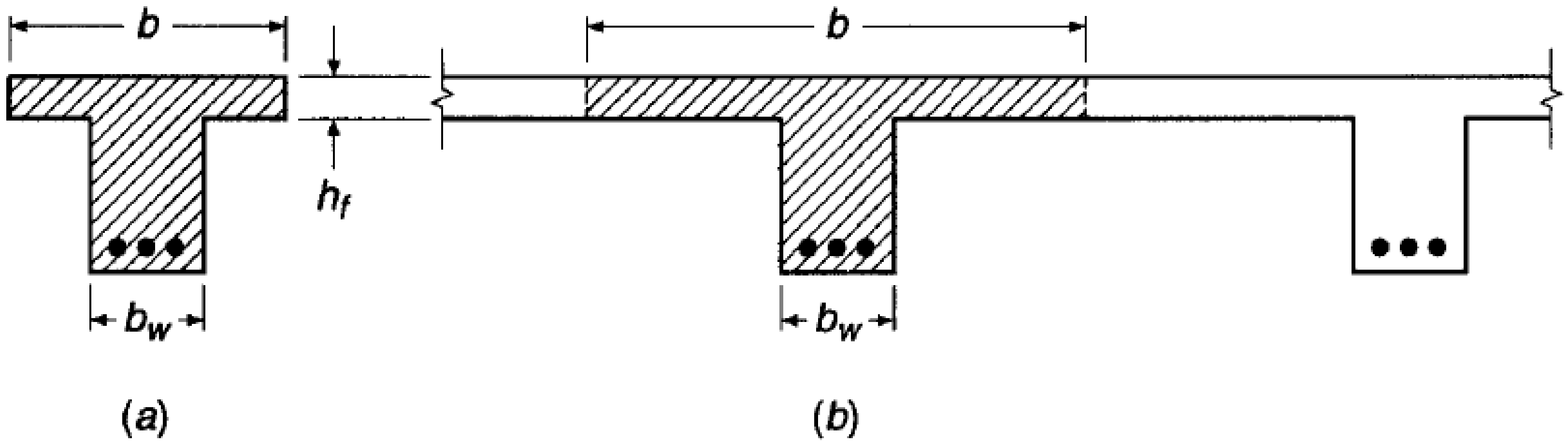


FIGURE 3.17
Effective flange width of T beams.

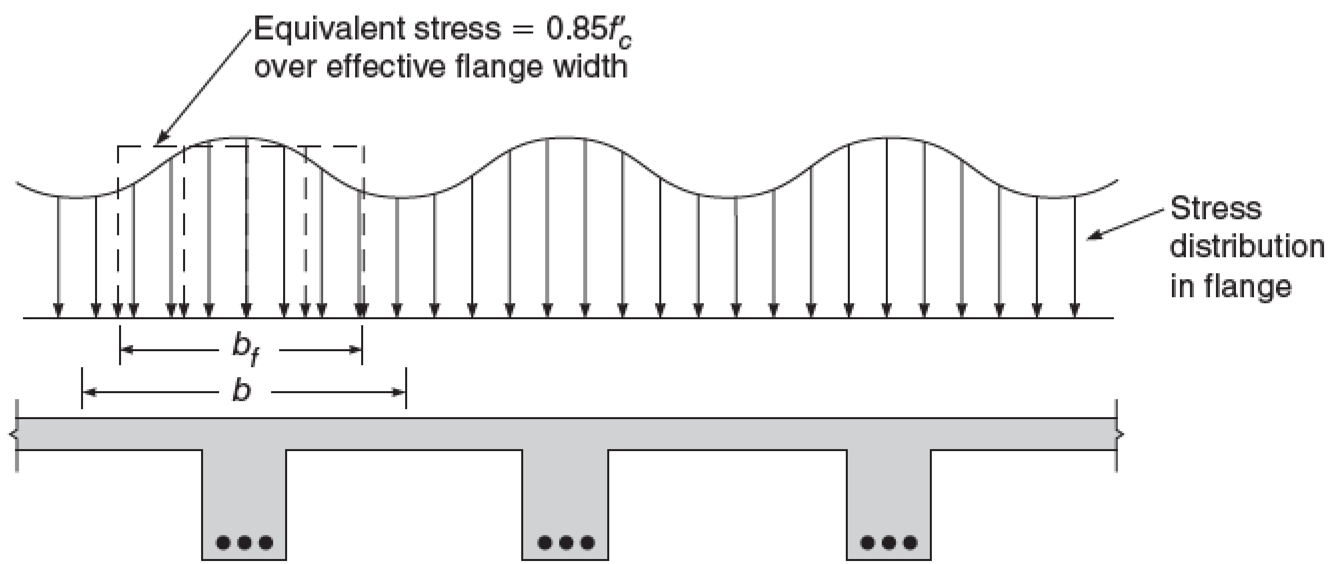


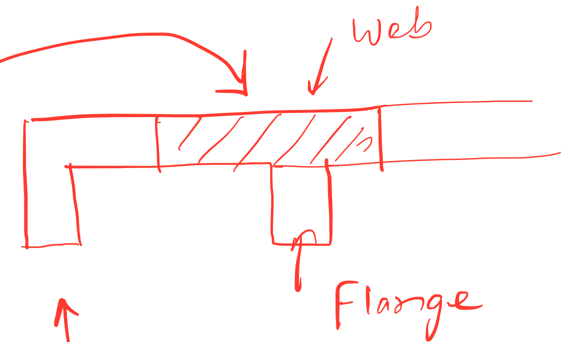
FIGURE 4.17
Stress distribution in T beam.

Effective flange width

The criteria for effective width given in ACI Code 8.12 are as follows:

1. For symmetric T beams, the effective width b shall not exceed one-fourth the span length of the beam. The overhanging slab width on either side of the beam web shall not exceed 8 times the thickness of the slab or go beyond one-half the clear distance to the next beam.
2. For beams having a slab on one side only, the effective overhanging slab width shall not exceed one-twelfth the span length of the beam, 6 times the slab thickness, or one-half the clear distance to the next beam.
3. For isolated beams in which the flange is used only for the purpose of providing additional compressive area, the flange thickness shall not be less than one-half the width of the web, and the total flange width shall not be more than 4 times the web width.

Effective flange width



1. Symmetrical T beams (T beam)

   $b < 16h_f + b_w$

   $b < Span/4 + b_w$

   $b <$ c/c beam spacing

2. Beam having slab on one side (L beam)

   $b < span/12 + b_w$

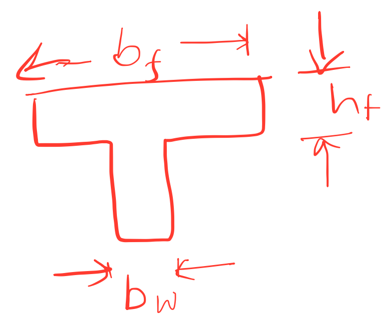
   $b < 6h_f + b_w$

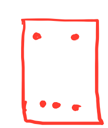
   $b <$ Half the clear span $+ b_w$

3. Isolated T beam

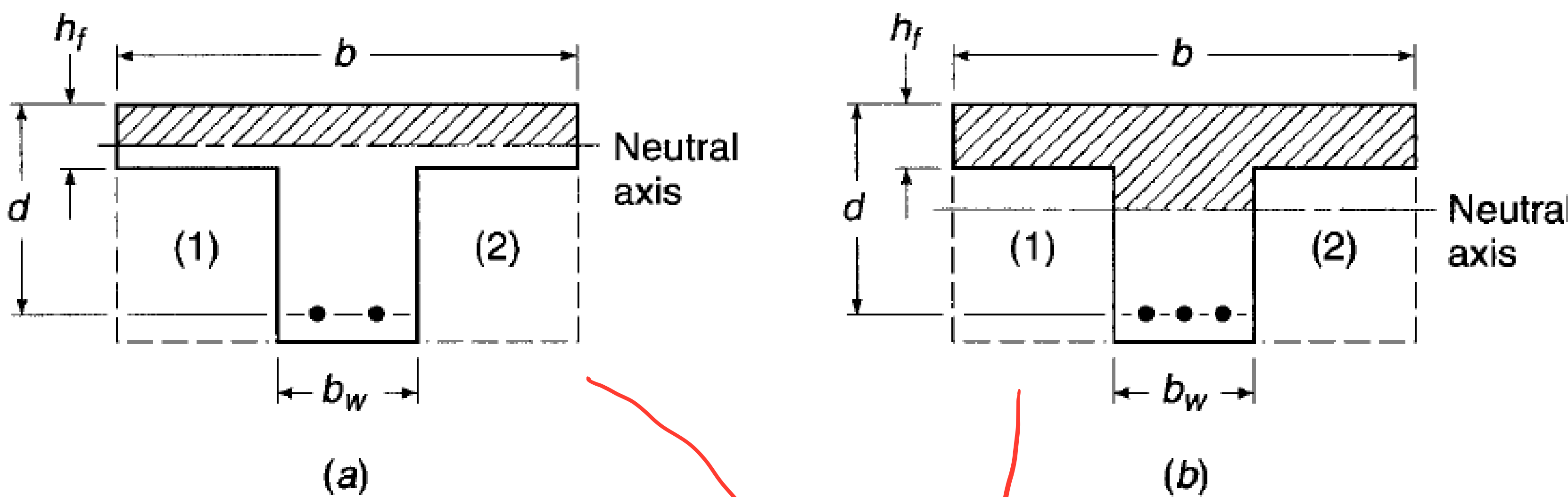
   $h_f > b_w/2$

   $b < 4b_w$



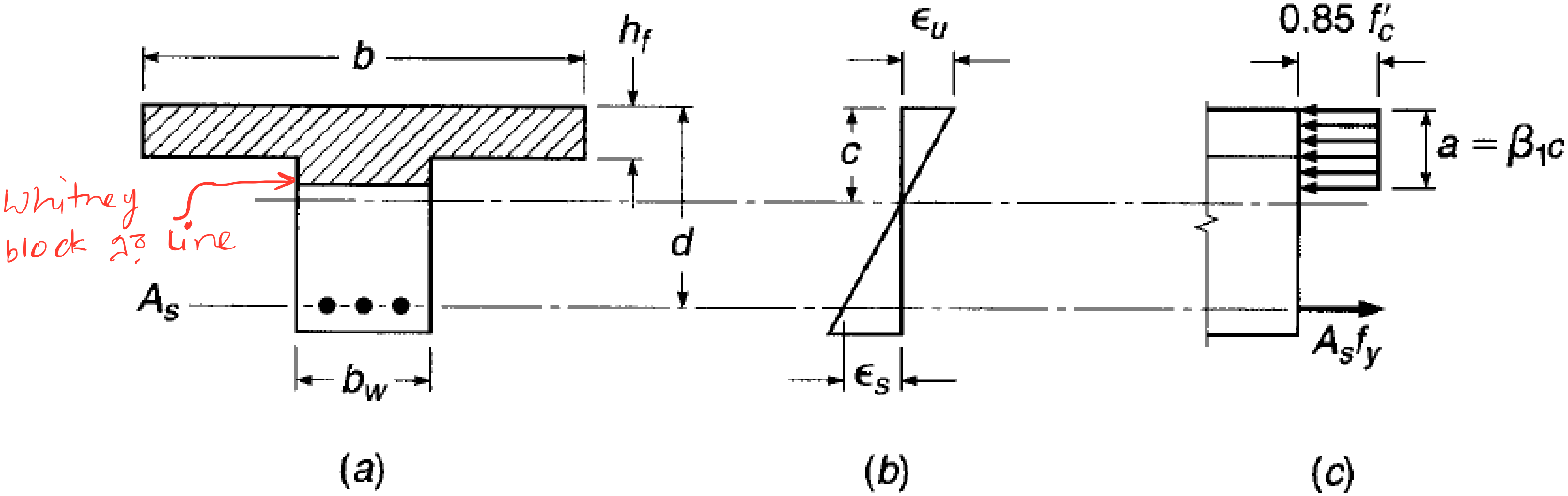
এই contd. গুলো না মিললে অন্য structure হবে, তাকে  box Beam হিসাবে ধরবো।

Strength Analysis



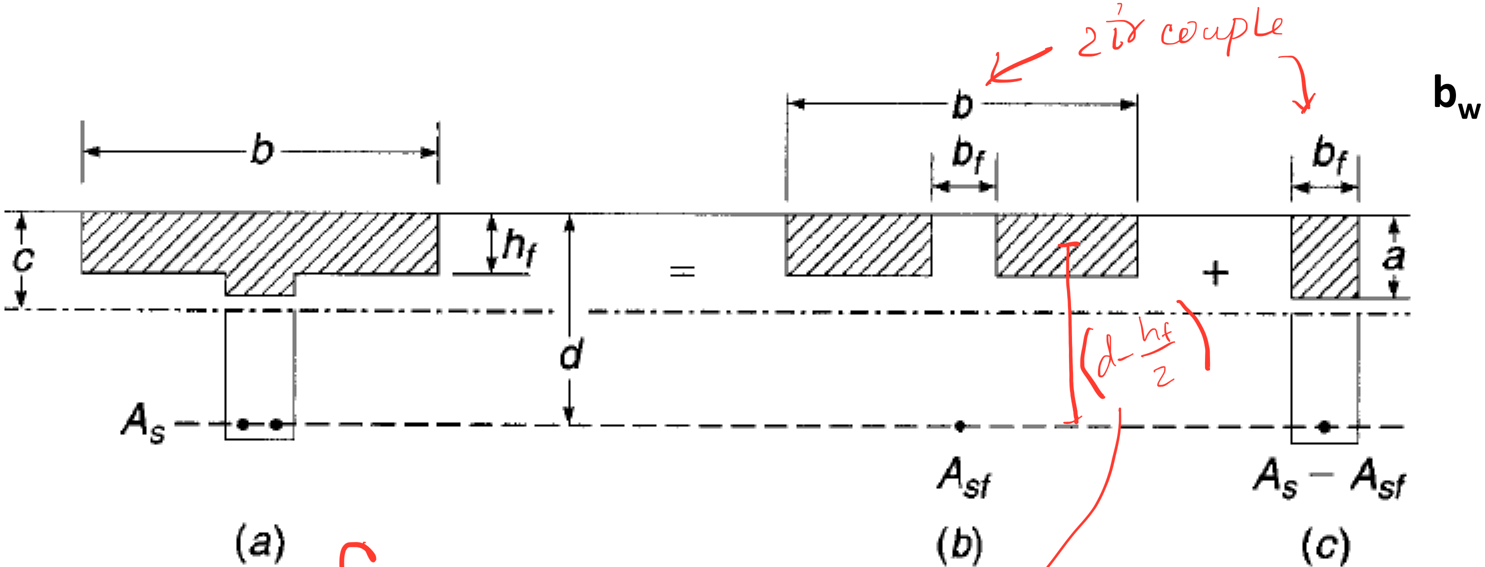
Nutral axis এর নিচে আমরা দুটা মনে করি।

Two possibilities
- Just like rectangular beam
- T-beam analysis required



$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{\rho f_y d}{0.85 f'_c} \tag{3.58}$$

If $a > h_f$ T-beam



T C

$$A_{sf} = \frac{0.85 f_c' (b - b_w) h_f}{f_y} \qquad (3.59)$$

$$M_{n1} = A_{sf} f_y \left(d - \frac{h_f}{2} \right) \qquad (3.60)$$

The remaining steel area $A_s - A_{sf}$:

C T

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w} \qquad (3.61)$$

$$M_{n2} = (A_s - A_{sf}) f_y \left(d - \frac{a}{2} \right) \qquad (3.62)$$

and the total nominal resisting moment is the sum of the parts:

$$M_n = M_{n1} + M_{n2} = A_{sf} f_y \left(d - \frac{h_f}{2} \right) + (A_s - A_{sf}) f_y \left(d - \frac{a}{2} \right) \tag{3.63}$$

As for rectangular beams, the tensile steel should yield prior to sudden crushing of the compression concrete, as assumed in the preceding development. Yielding of the tensile reinforcement and Code compliance are ensured if the net tensile strain ϵ_t is greater than 0.004. If $\epsilon_t \geq 0.005$, a strength reduction factor $\phi = 0.90$ may be used. From the geometry of the section,

$$\frac{c}{d_t} \leq \frac{\epsilon_u}{\epsilon_u + \epsilon_t} \tag{4.60}$$

Setting $\epsilon_u = 0.003$ and $\epsilon_t = 0.005$ provides a maximum c/d_t ratio of 0.375, as shown in Fig. 4.9. Thus, as long as the depth to the neutral axis is less than $0.375d_t$, the net tensile strain requirements are satisfied, as they are for rectangular beam sections. This will occur if $\rho_w = A_s/b_w d$ is less than

$$\rho_{w,\,0.005} = \rho_{0.005} + \rho_f \tag{4.61}$$

where $\rho_f = A_{sf}/b_w d$ and $\rho_{0.005}$ is as previously defined for a rectangular cross section [Eq. (4.26c)]. For c/d_t ratios between 0.375 and 0.429, equivalent to ρ_w between the

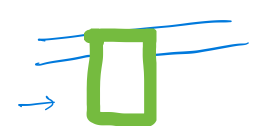
$\rho_{w,0.005}$ from Eq. (4.61) and $\rho_{w,0.004}$, calculated by substituting $\rho_{0.004}$ from Eq. (4.26d) for $\rho_{0.005}$ in Eq. (4.61), the strength reduction factor ϕ must be adjusted for ϵ_t, as shown in Fig. 4.8. For $\rho_w \le \rho_{w,0.005}$ or $c/d_t \le 0.375$, $\phi = 0.90$.

The practical result of applying Eq. (4.61) is that the stress block of T beams will almost always be within the flange, except for unusual geometry or combinations of material strength. Consequently, rectangular beam equations may be applied in most cases.

বেশিরভাগ সময় Rectangular নিলেই হয়, আসলে কম লাগে না

The ACI Code restriction that the tensile reinforcement ratio for beams not be less than $\rho_{min} = 3\sqrt{f'_c}/f_y$ and $\ge 200/f_y$ (see Section 4.3d) applies to T beams as well as rectangular beams. For T beams, the ratio ρ should be computed for this purpose based on the web width b_w.

এই Math গুলো support নয় কারণ কোনো কাজ করে না, তখন tension zone হয়। ∴ compression area আসলে কোন Meaning নাই। তখন Rectangular ধরে হিসাব করবো।



d. Examples of Analysis and Design of T Beams

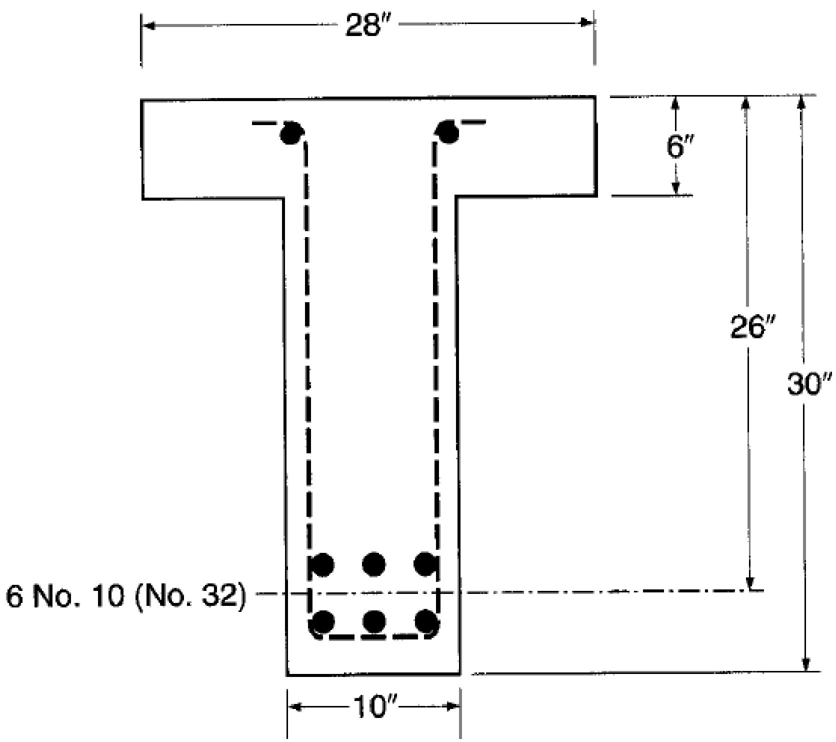
For *analyzing* the capacity of a T beam with known concrete dimensions and tensile steel area, it is reasonable to start with the assumption that the stress block depth a does not exceed the flange thickness h_f. In that case, all ordinary rectangular beam equations (see Section 4.3) apply, with beam width taken equal to the effective width of the flange. If, upon checking that assumption, a proves to exceed h_f, then T beam analysis must be applied. Equations (4.55) through (4.59) can be used, in sequence, to obtain the nominal flexural strength, after which the design strength is easily calculated.

For *design*, the following sequence of calculations may be followed:

1. Establish flange thickness h_f based on flexural requirements of the slab, which normally spans transversely between parallel T beams.
2. Determine the effective flange width b_f according to ACI limits.
3. Choose web dimensions b_w and d based on either of the following:
   (*a*) Negative bending requirements at the supports, if a continuous T beam
   (*b*) Shear requirements, setting a reasonable upper limit on the nominal unit shear stress v_u in the beam web (see Chapter 5)
4. With all concrete dimensions thus established, calculate a trial value of A_s, assuming that a does not exceed h_f, with beam width equal to flange width b_f. Use ordinary rectangular beam design methods.
5. For the trial A_s, check the depth of stress block a to confirm that it does not exceed h_f. If it should exceed that value, revise A_s, using the T beam equations.
6. Check to ensure that $\epsilon_t \geq 0.005$ or $c/d \leq 0.375$ to ensure that $\phi = 0.90$. (This will almost invariably be the case.)
7. Check to ensure that $\rho_w \geq \rho_{w,\min}$.

EXAMPLE 3.14

Moment capacity of a given section. The isolated T beam shown in Fig. 3.21 is composed of a flange 28 in. wide and 6 in. deep cast monolithically with a web of 10 in. width that extends 24 in. below the bottom surface of the flange to produce a beam of 30 in. total depth. Tensile reinforcement consists of six No. 10 (No. 32) bars placed in two horizontal rows separated by 1 in. clear spacing. The centroid of the bar group is 26 in. from the top of the beam. The concrete has a strength of 3000 psi, and the yield strength of the steel is 60,000 psi. What is the design moment capacity of the beam? see new slide



Practice এর কাজ করিবে
chapter 4 এর maths

Solution. It is easily confirmed that the flange dimensions are satisfactory according to the ACI Code for an isolated beam. The entire flange can be considered effective. For six No. 10 (No. 32) bars, $A_s = 7.62$ in^2. First check the location of the neutral axis, on the assumption that rectangular beam equations may be applied. Using Eq. (3.32)

$$a = \frac{7.62 \times 60}{0.85 \times 3 \times 28} = 6.40 \text{ in.}$$

This exceeds the flange thickness, and so a T beam analysis is required. From Eq. (3.59) and Fig. 3.19*b*,

$$A_{sf} = 0.85 \times \frac{3}{60}(28 - 10) \times 6 = 4.59 \text{ in}^2$$

Then, from Eq. (3.60),

$$M_{n1} = 4.59 \times 60(26 - 3) = 6330 \text{ in-kips}$$

Then, from Fig. 3.19*c*,

$$A_s - A_{sf} = 7.62 - 4.59 = 3.03 \text{ in}^2$$

and from Eqs. (3.58) and (3.59)

$$a = \frac{3.03 \times 60}{0.85 \times 3 \times 10} = 7.13 \text{ in.}$$

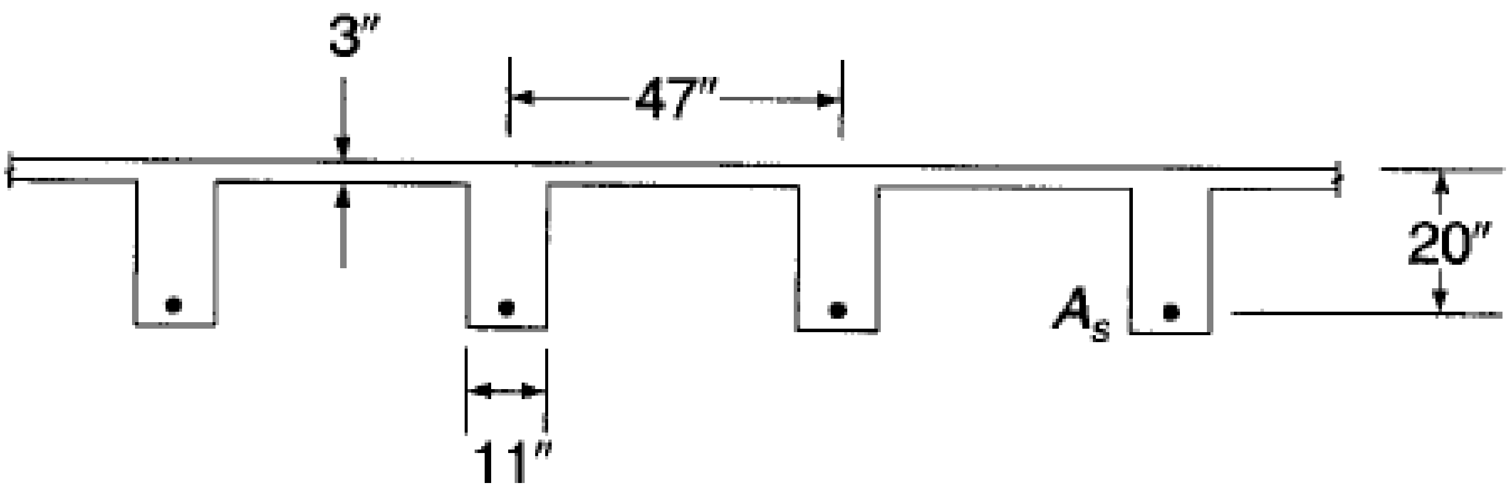
$$M_{n2} = 3.03 \times 60(26 - 3.56) = 4080 \text{ in-kips}$$

The depth to the neutral axis is $c = a/\beta_1 = 7.13/0.85 = 8.39$ and $d_t = 27.5$ in. to the lowest bar. The c/d_t ratio is $8.39/27.5 = 0.305 < 0.375$, so the $\epsilon_t > 0.005$ requirement is met and $\phi = 0.90$. When the ACI strength reduction factor is incorporated, the design strength is

$$\phi M_n = 0.90(6330 + 4080) = 9370 \text{ in-kips}$$

EXAMPLE 3.15

Determination of steel area for a given moment. A floor system, shown in Fig. 3.22, consists of a 3 in. concrete slab supported by continuous T beams with a 24 ft span, 47 in. on centers. Web dimensions, as determined by negative-moment requirements at the supports, are $b_w = 11$ in. and $d = 20$ in. What tensile steel area is required at midspan to resist a factored moment of 6400 in-kips if $f_y = 60{,}000$ psi and $f'_c = 3000$ psi?



SOLUTION. First determining the effective flange width,

$$16h_f + b_w = 16 \times 3 + 11 = 59 \text{ in.}$$

$$\frac{\text{Span}}{4} = 24 \times \frac{12}{4} = 72 \text{ in.}$$

$$\text{Centerline beam spacing} = 47 \text{ in.}$$

The centerline T beam spacing controls in this case, and $b = 47$ in. The concrete dimensions b_w and d are known to be adequate in this case, since they have been selected for the larger negative support moment applied to the effective rectangular section $b_w d$. The tensile steel at midspan is most conveniently found by trial. Assuming the stress-block depth a is equal to the flange thickness of $h_f = 3$ in., one gets

$$d - \frac{a}{2} = 20 - 1.50 = 18.50 \text{ in.}$$

Trial:

$$A_s = \frac{M_u}{\phi f_y(d - a/2)} = \frac{6400}{0.90 \times 60 \times 18.50} = 6.41 \text{ in}^2$$

Checking the assumed value for a,

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{6.41 \times 60}{0.85 \times 3 \times 47} = 3.21 \text{ in.}$$

Since a is greater than h_f, a T beam design is required and $\phi = 0.90$ is assumed.

$$A_{sf} = \frac{0.85 f'_c (b - b_w) h_f}{f_y} = \frac{0.85 \times 3 \times 36 \times 3}{60} = 4.59 \text{ in}^2$$

$$\phi M_{n1} = \phi A_{sf} f_y\left(d - \frac{h_f}{2}\right) = 0.90 \times 4.59 \times 60 \times 18.50 = 4590 \text{ in-kips}$$

$$\phi M_{n2} = M_u - \phi M_{n1} = 6400 - 4590 = 1810 \text{ in-kips}$$

Assume $a = 4.00$ in.:

$$A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y(d - a/2)} = \frac{1810}{0.90 \times 60 \times (20 - 4.0/2)} = 1.86 \text{ in}^2$$

Check:

$$a = \frac{(A_s - A_{sf})f_y}{0.85 f'_c b_w} = \frac{1.86 \times 60}{0.85 \times 3 \times 11} = 3.98 \text{ in.}$$

This is satisfactorily close to the assumed value of 4 in. Then

$$A_s = A_{sf} + A_s - A_{sf} = 4.59 + 1.86 = 6.45 \text{ in}^2$$

Checking to ensure that the net tensile strain of 0.005 is met to allow $\phi = 0.90$,

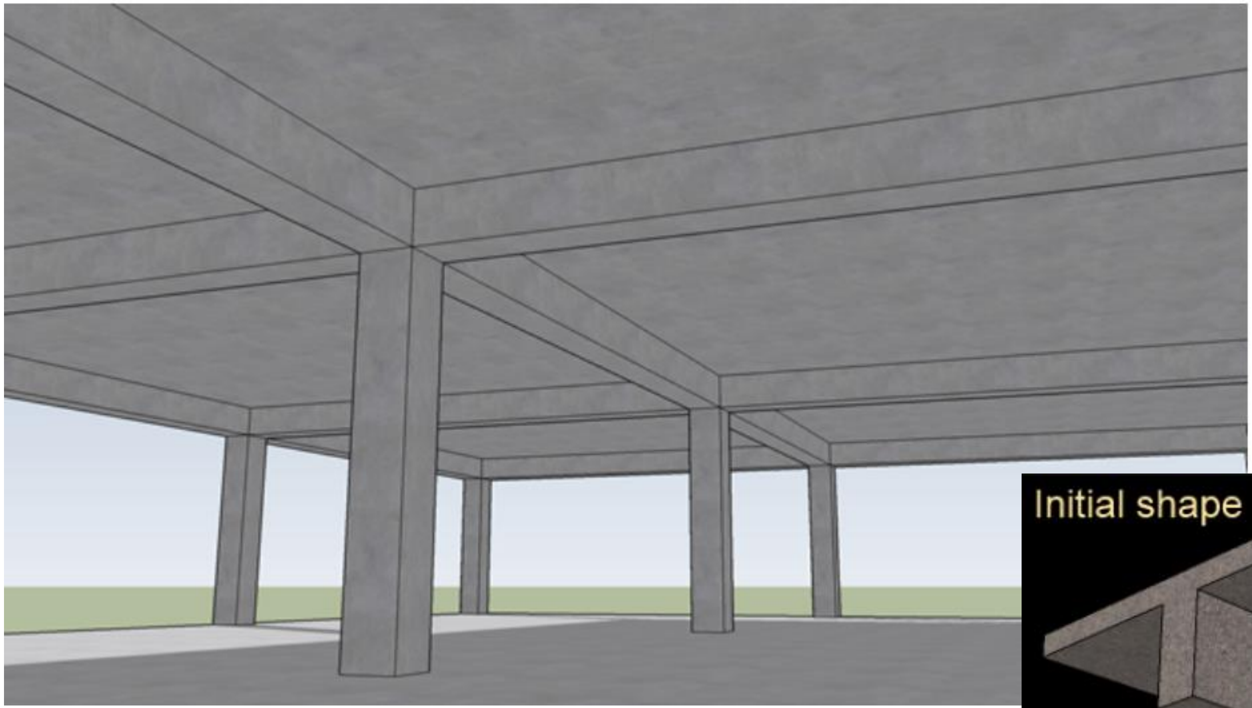
$$c = \frac{a}{\beta_1} = \frac{3.98}{0.85} = 4.68$$

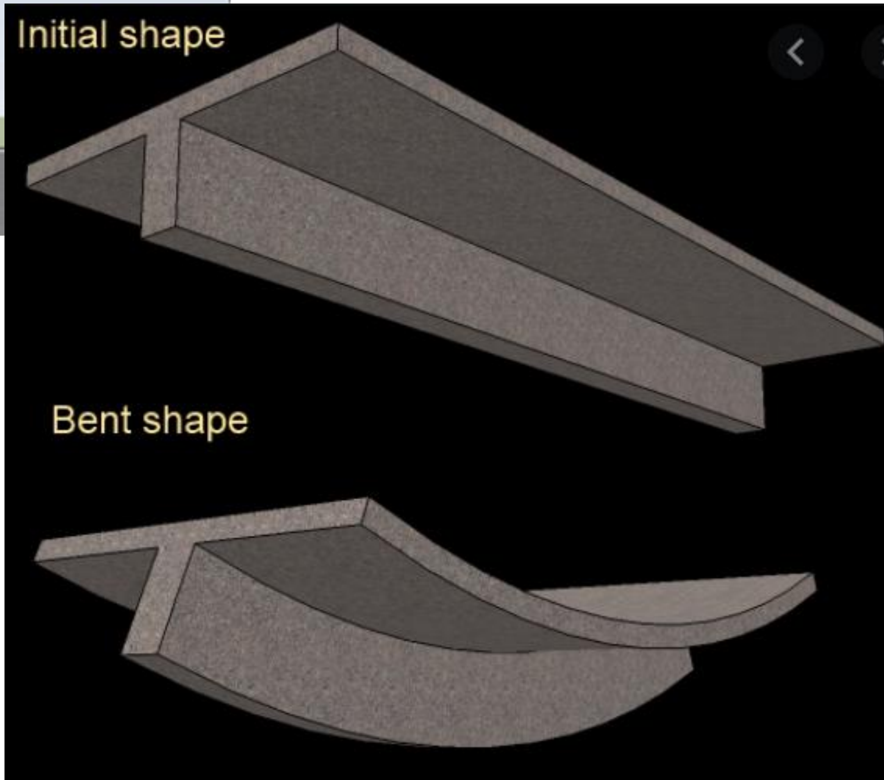
$$\frac{c}{d_t} = \frac{4.68}{20} = 0.23 < 0.325$$

indicating that the design is satisfactory.

The close agreement should be noted between the approximate tensile steel area of 6.41 in^2 found by assuming the stress-block depth equal to the flange thickness and the more exact value of 6.45 in^2 found by T beam analysis. The approximate solution would be satisfactory in most cases.

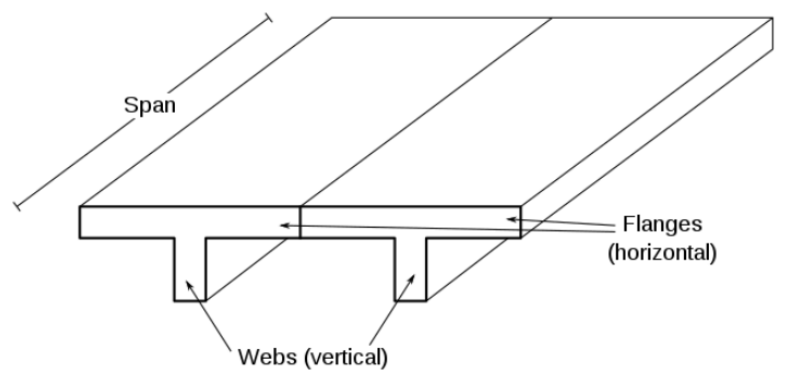
T-beam





T-beam

- RC beam and slab are monolithically cast
- Beam stirrups and bent bars extend into the slab
- A part of slab act along with beam top to take longitudinal compression
- Slab forms the beam flange
- Part of beam below slab is called web/stem



Effective flange width

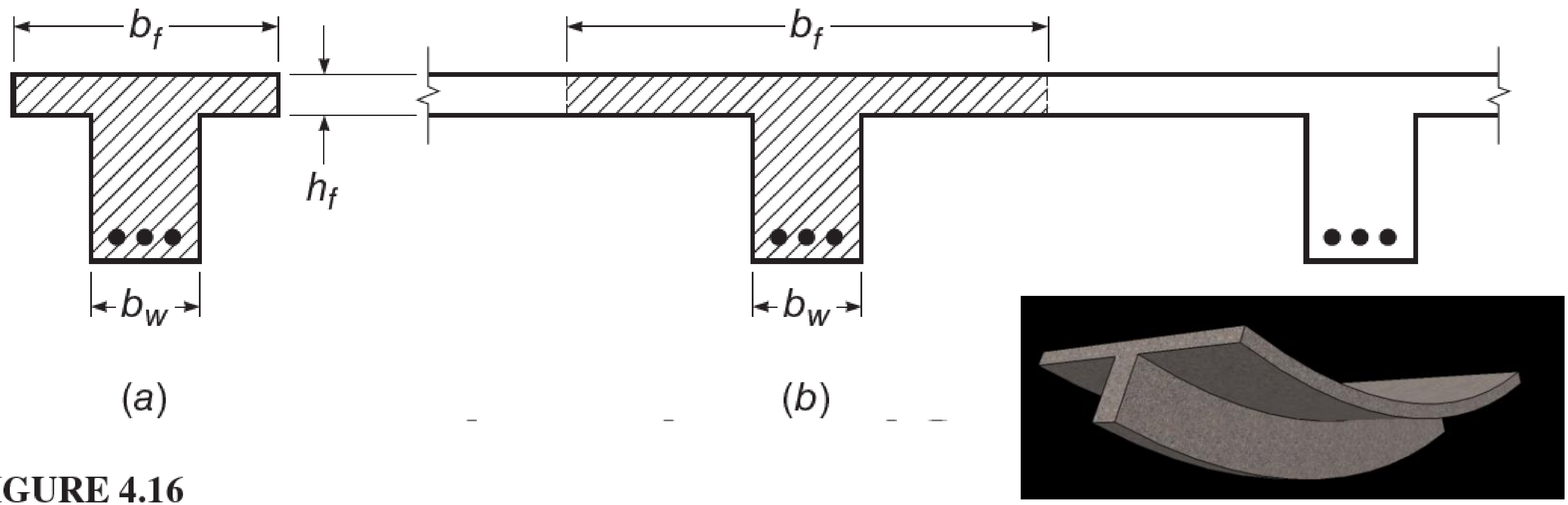


FIGURE 4.16
Effective flange width of T beams.

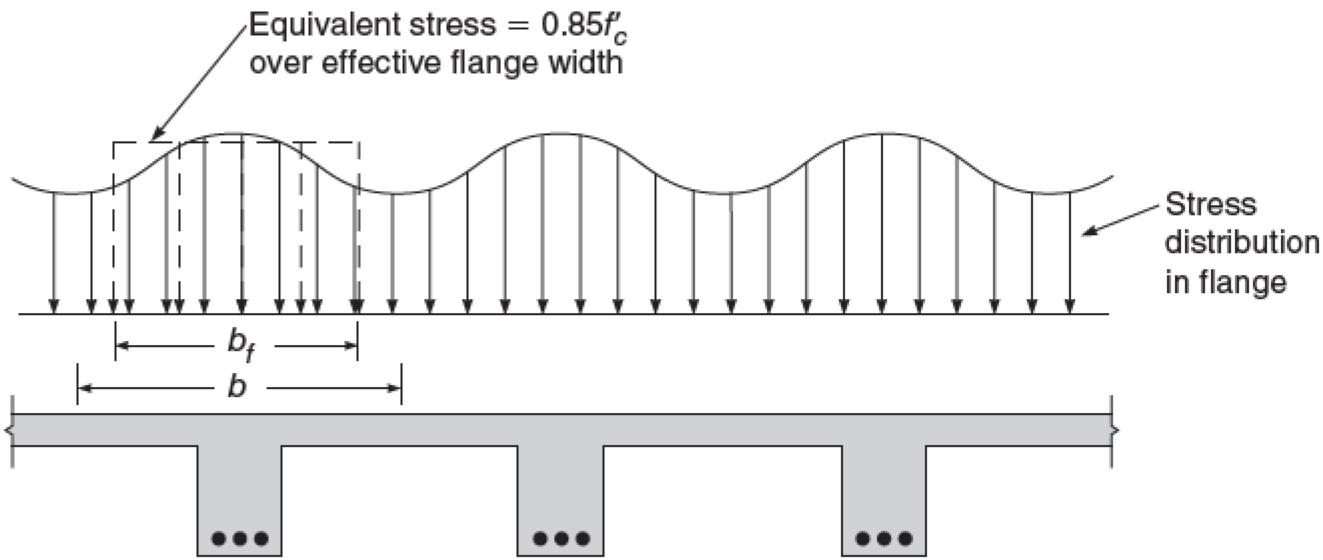


FIGURE 4.17
Stress distribution in T beam.

Effective flange width

The criteria for effective width b_f given in ACI Code 6.3.2 can be summarized as follows:

1. For T beams with flanges on both sides of the web, the overhanging slab width on either side of the beam web shall not exceed one-eighth of the beam clear span ℓ_n, 8 times the thickness of the slab h, or go beyond one-half the clear distance to the next beam s_w.
2. For beams having a slab on one side only, the effective overhanging slab width shall not exceed one-twelfth the beam clear span ℓ_n, 6 times the thickness of the slab h, or go beyond one-half the clear distance to the next beam s_w.
3. For isolated beams in which the flange is used only for the purpose of providing additional compressive area, the flange thickness shall not be less than one-half the width of the web b_w, and the total flange width shall not be more than 4 times the web width b_w.

Effective flange width

1. Symmetrical T beams

$b_f < 16h_f + b_w$

$b_f < \text{Span}/4 + b_w$ Changed

$b_f <$ c/c beam spacing

2. Beam having slab on one side

$b_f < \text{span}/12 + b_w$

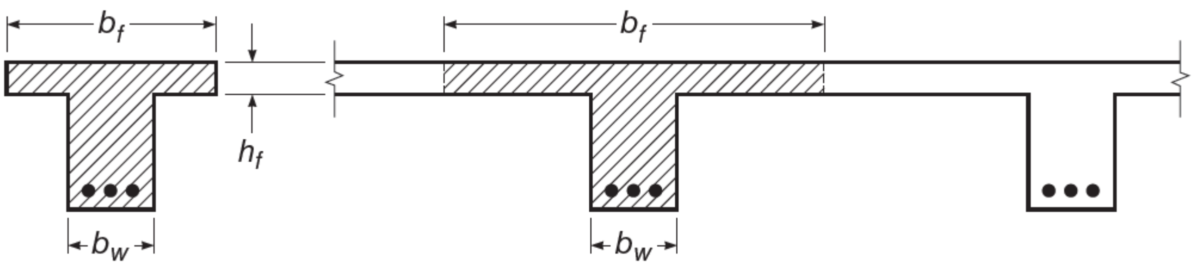
$b_f < 6h_f + b_w$

$b_f <$ Half the clear span $+ b_w$

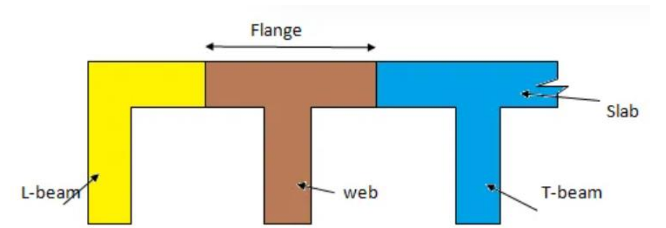
3. Isolated T beam

$h_f > b_w/2$

$b_f < 4b_w$



(a) (b)



Strength Analysis

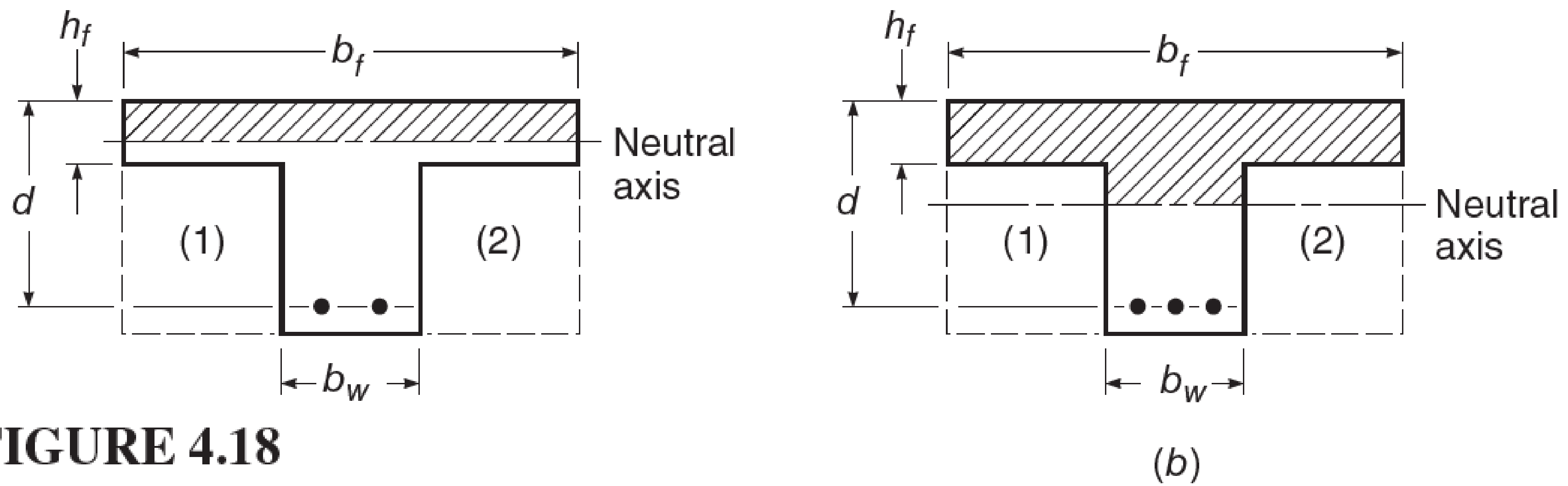


FIGURE 4.18
Effective cross sections of T beams.

Two possibilities

- Just like rectangular beam
- T-beam analysis required

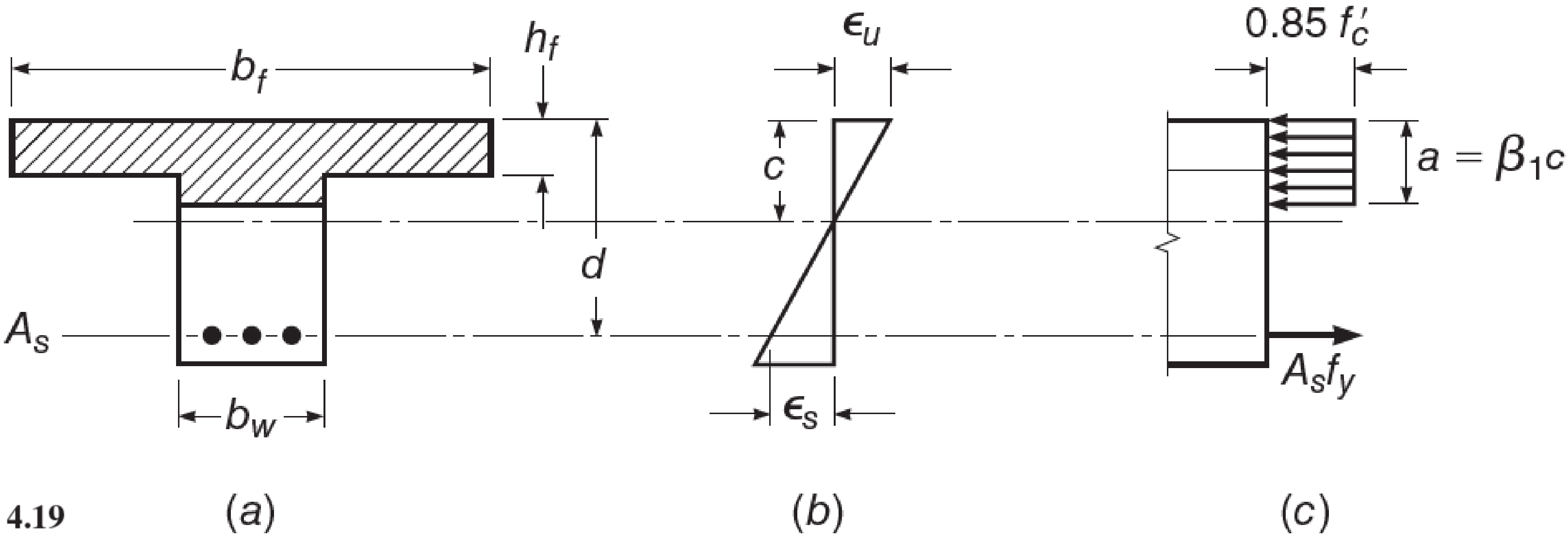


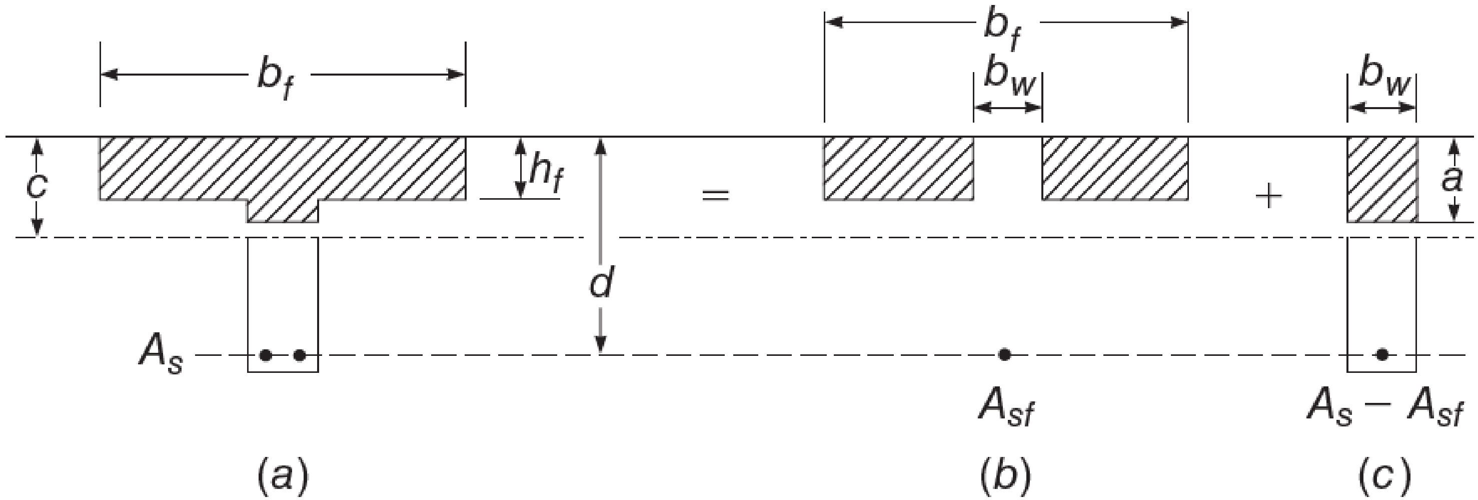
FIGURE 4.19
Strain and equivalent stress distributions for T beams.

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{\rho f_y d}{0.85 f_c'} \tag{4.54}$$

b_f

If $a > h_f$ T-beam

else Rectangular beam with $b = b_f$



$$\boxed{A_{sf} = \frac{0.85 f'_c (b_f - b_w) h_f}{f_y}} \tag{4.55}$$

The force $A_{sf} f_y$ and the equal and opposite force $0.85\, f'_c\, (b_f - b_w) h_f$ act with a lever arm $d - h_f/2$ to provide the nominal resisting moment

$$\boxed{M_{n1} = A_{sf} f_y \left(d - \frac{h_f}{2} \right)} \tag{4.56}$$

The remaining steel area $A_s - A_{sf}$, at a stress f_y, is balanced by the compression in the rectangular portion of the beam (Fig. 4.20*c*). The depth of the equivalent rectangular stress block in this zone is found from horizontal equilibrium.

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f'_c b_w} \tag{4.57}$$

An additional moment M_{n2} is thus provided by the forces $(A_s - A_{sf}) f_y$ and $0.85 f'_c\, a b_w$ acting at the lever arm $d - a/2$.

$$\boxed{M_{n2} = (A_s - A_{sf}) f_y \left(d - \frac{a}{2} \right)} \tag{4.58}$$

and the total nominal resisting moment is the sum of the parts:

$$\boxed{M_n = M_{n1} + M_{n2} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) + (A_s - A_{sf}) f_y \left(d - \frac{a}{2}\right)} \tag{4.59}$$

This moment is reduced by the strength reduction factor ϕ in accordance with the safety provisions of the ACI Code to obtain the design strength.

As for rectangular beams, the tensile steel should yield prior to sudden crushing of the compression concrete, as assumed in the preceding development. Yielding of the tensile reinforcement and Code compliance are ensured if the net tensile strain ϵ_t is greater than 0.004. If $\epsilon_t \geq 0.005$, a strength reduction factor $\phi = 0.90$ may be used. From the geometry of the section,

$$\frac{c}{d_t} \leq \frac{\epsilon_u}{\epsilon_u + \epsilon_t} \tag{4.60}$$

Setting $\epsilon_u = 0.003$ and $\epsilon_t = 0.005$ provides a maximum c/d_t ratio of 0.375, as shown in Fig. 4.9. Thus, as long as the depth to the neutral axis is less than $0.375d_t$, the net tensile strain requirements are satisfied, as they are for rectangular beam sections. This will occur if $\rho_w = A_s/b_w d$ is less than

$$\rho_{w,\,0.005} = \rho_{0.005} + \rho_f \tag{4.61}$$

where $\rho_f = A_{sf}/b_w d$ and $\rho_{0.005}$ is as previously defined for a rectangular cross section [Eq. (4.26c)]. For c/d_t ratios between 0.375 and 0.429, equivalent to ρ_w between the

$\rho_{w,0.005}$ from Eq. (4.61) and $\rho_{w,0.004}$, calculated by substituting $\rho_{0.004}$ from Eq. (4.26*d*) for $\rho_{0.005}$ in Eq. (4.61), the strength reduction factor ϕ must be adjusted for ϵ_t, as shown in Fig. 4.8. For $\rho_w \leq \rho_{w,0.005}$ or $c/d_t \leq 0.375$, $\phi = 0.90$.

The practical result of applying Eq. (4.61) is that the stress block of T beams will almost always be within the flange, except for unusual geometry or combinations of material strength. Consequently, rectangular beam equations may be applied in most cases.

The ACI Code restriction that the tensile reinforcement ratio for beams not be less than $\rho_{\min} = 3\sqrt{f'_c}/f_y$ and $\geq 200/f_y$ (see Section 4.3d) applies to T beams as well as rectangular beams. For T beams, the ratio ρ should be computed for this purpose based on the web width b_w.

d. Examples of Analysis and Design of T Beams

For *analyzing* the capacity of a T beam with known concrete dimensions and tensile steel area, it is reasonable to start with the assumption that the stress block depth a does not exceed the flange thickness h_f. In that case, all ordinary rectangular beam equations (see Section 4.3) apply, with beam width taken equal to the effective width of the flange. If, upon checking that assumption, a proves to exceed h_f, then T beam analysis must be applied. Equations (4.55) through (4.59) can be used, in sequence, to obtain the nominal flexural strength, after which the design strength is easily calculated.

For *design*, the following sequence of calculations may be followed:

1. Establish flange thickness h_f based on flexural requirements of the slab, which normally spans transversely between parallel T beams.
2. Determine the effective flange width b_f according to ACI limits.
3. Choose web dimensions b_w and d based on either of the following:
   (*a*) Negative bending requirements at the supports, if a continuous T beam
   (*b*) Shear requirements, setting a reasonable upper limit on the nominal unit shear stress v_u in the beam web (see Chapter 5)
4. With all concrete dimensions thus established, calculate a trial value of A_s, assuming that a does not exceed h_f, with beam width equal to flange width b_f. Use ordinary rectangular beam design methods.
5. For the trial A_s, check the depth of stress block a to confirm that it does not exceed h_f. If it should exceed that value, revise A_s, using the T beam equations.
6. Check to ensure that $\epsilon_t \geq 0.005$ or $c/d \leq 0.375$ to ensure that $\phi = 0.90$. (This will almost invariably be the case.)
7. Check to ensure that $\rho_w \geq \rho_{w,\min}$.

EXAMPLE 4.14

Moment capacity of a given section. The isolated T beam shown in Fig. 4.21 is composed of a flange 28 in. wide and 6 in. deep cast monolithically with a web of 10 in. width that extends 24 in. below the bottom surface of the flange to produce a beam of 30 in. total depth. Tensile reinforcement consists of six No. 10 (No. 32) bars placed in two horizontal rows separated by 1 in. clear spacing. The centroid of the bar group is 26 in. from the top of the beam. The concrete has a strength of 3000 psi, and the yield strength of the steel is 60,000 psi. What is the design moment capacity of the beam?

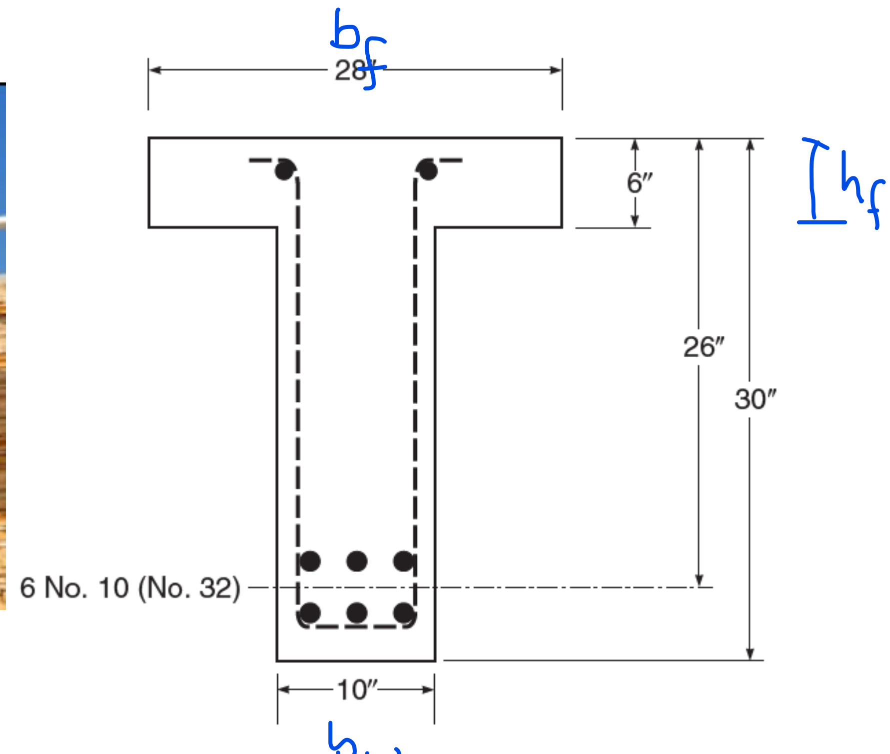


FIGURE 4.21
T beam of Example 4.14.

Solution. It is easily confirmed that the flange dimensions are satisfactory according to the ACI Code for an isolated beam. The entire flange can be considered effective. For six No. 10 (No. 32) bars, $A_s = 7.62$ in^2. First check the location of the neutral axis, on the assumption that rectangular beam equations may be applied. Using Eq. (4.28) with $b_f = b$

$$a = \frac{A_s f_y}{0.85\, f_c' b_f} = \frac{7.62 \times 60}{0.85 \times 3 \times 28} = 6.40 \text{ in.}$$

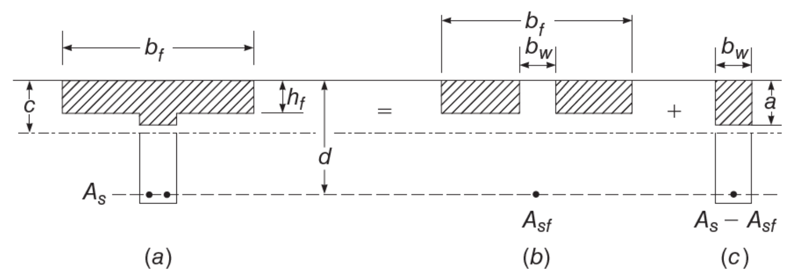
This exceeds the flange thickness, and so a T beam analysis is required. From Eq. (4.55) and Fig. 4.19b,

$$A_{sf} = 0.85 \frac{f_c'}{f_y}(b_f - b_w) h_f = 0.85 \times \frac{3}{60}(28 - 10) \times 6 = 4.59 \text{ in}^2$$

Then, from Eq. (4.56),

$$M_{n1} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) = 4.59 \times 60\,(26 - 3) = 6330 \text{ in-kips}$$

Then, from Fig. 4.19c,



$$A_s - A_{sf} = 7.62 - 4.59 = 3.03 \text{ in}^2$$

and from Eqs. (4.54) and (4.55)

$$a = \frac{A_s f_y}{0.85 f_c' b_w} = \frac{3.03 \times 60}{0.85 \times 3 \times 10} = 7.13 \text{ in.}$$

$$M_{n2} = (A_s - A_{sf})\, f_y \left(d - \frac{a}{2}\right) = 3.03 \times 60(26 - 3.56) = 4080 \text{ in-kips}$$

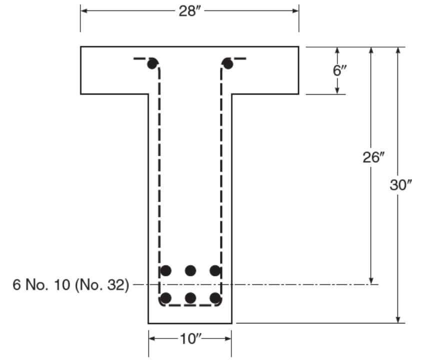
The depth to the neutral axis is $c = a/\beta_1 = 7.13/0.85 = 8.39$ and $d_t = 27.5$ in. to the lowest bar. The c/d_t ratio is $8.39/27.5 = 0.305 < 0.375$, so the $\epsilon_t > 0.005$ requirement is met and $\phi = 0.90$. When the ACI strength reduction factor is incorporated, the design strength is

$$\phi M_n = \phi(M_{n1} + M_{n2}) = 0.90(6330 + 4080) = 9370 \text{ in-kips}$$

Example 4.14

Isolated T-beam.

Review problem. Find ϕM_n.



Solution

First check if flange dimensions are satisfactory otherwise overhang will be disregarded.

1. $h_f > 0.5 b_w$, $6'' \text{ is } > 0.5 * 10''$ (ok)

2. $b_f < 4 b_w$, $28'' < 4 * 10''$ (ok)

$A_s = 6 * 1.27 = 7.62 \text{ in}^2$

Let us check if it is T-beam or Rect-beam.

$b_f = 28''$

Assume, rectangular beam.

$$a = \frac{A_s f_y}{0.85 f_c' b_f} = \frac{7.62 * 60}{0.85 * 3 * 28} = 6.40 \text{ in}$$

$a > h_f \Rightarrow$ T-beam analysis is reqd.

$$A_{sf} = \frac{0.85 f_c' (b_f - b_w) h_f}{f_y} = \frac{0.85 * 3 (28-10) * 6}{60}$$

$$= 4.59 \text{ in}^2$$

$$M_{n1} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) = 4.59 * 60 \left(26 - \frac{6}{2}\right)$$

$$= \underline{6334 \text{ k-in}}$$

$$A_s - A_f = 7.62 - 4.59 = 3.03 \text{ in}^2$$

$$a = \frac{A_s f_y}{0.85 f_c' b_w} = \frac{3.03 * 60}{0.85 * 3 * 10} = 7.13 \text{ in}$$

$$M_{n2} = (A_s - A_{sf}) f_y (d - a/2) = 3.03 * 60 \left(26 - \frac{7.13}{2}\right)$$

$$= \underline{4078 \text{ k-in}}$$

$$M_n = M_{n_1} + M_{n_2} = 10413 \text{ k-in}$$

$$c = \frac{a}{\beta_1} = \frac{7.13}{0.85} = 8.39''$$

$$d_t = 26 + 1.5 = 27.5''$$

$$c/d_t = 0.305 < 0.375 \implies \underline{\phi = 0.9}$$

$$\text{or } \varepsilon_t = \frac{\varepsilon_u}{c}(d_t - c) = \frac{0.003}{8.39}(27.5 - 8.39)$$

$$= 0.00683 > 0.005 \implies \underline{\phi = 0.9}$$

$$\therefore \text{ Design strength} = \phi M_n$$

$$= 0.9 * 10413$$

$$= 9372 \text{ k-}''$$

$$= \underline{781 \text{ k-ft}}$$

EXAMPLE 4.15

Determination of steel area for a given moment. A floor system, shown in Fig. 4.22, consists of a 3 in. concrete slab supported by continuous T beams with a clear span $\ell_n = 24$ ft, 47 in. on centers. Web dimensions, as determined by negative-moment requirements at the supports, are $b_w = 11$ in. and $d = 20$ in. What tensile steel area is required at midspan to resist a factored moment of 6400 in-kips if $f_y = 60{,}000$ psi and $f'_c = 3000$ psi?

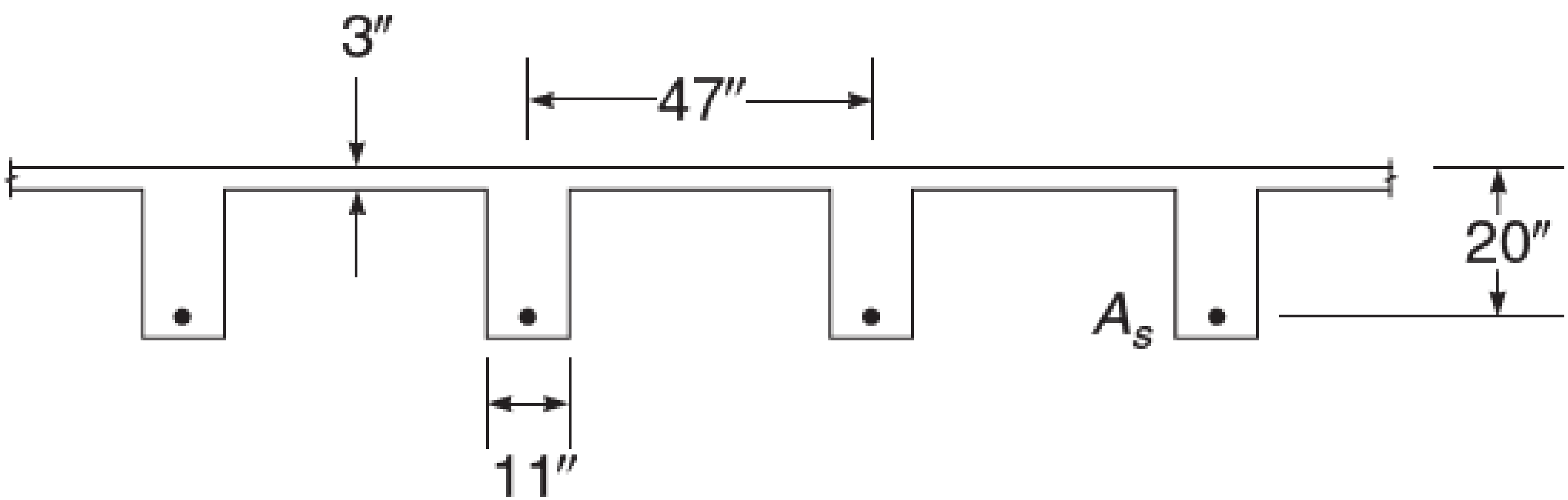
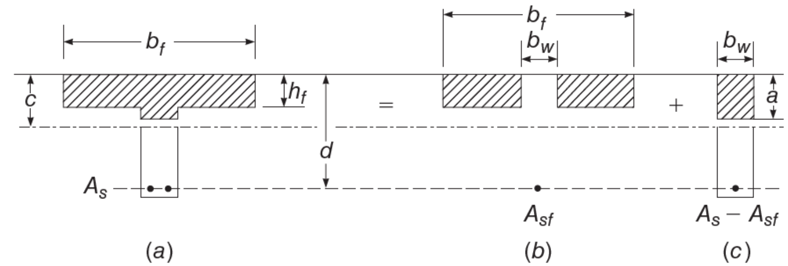


FIGURE 4.22
T beam of Example 4.15.

Solution. First determining the effective flange width b_f,



$$16h_f + b_w = 16 \times 3 + 11 = 59 \text{ in.}$$

$$2\frac{\ell_n}{8} + b_w = 2 \times \frac{24 \times 12}{8} + 11 = 83 \text{ in.}$$

$$\text{Centerline beam spacing} = 47 \text{ in.}$$

The centerline T beam spacing controls in this case, and $b_f = 47$ in. The concrete dimensions b_w and d are known to be adequate in this case, since they have been selected for the larger negative support moment applied to the effective rectangular section $b_w d$. The tensile steel at midspan is most conveniently found by trial. Assuming the stress-block depth a is equal to the flange thickness of $h_f = 3$ in., one gets

$$d - \frac{a}{2} = 20 - \frac{3}{2} = 18.50 \text{ in.}$$

Trial:

$$A_s = \frac{M_u}{\phi f_y(d - a/2)} = \frac{6400}{0.90 \times 60 \times 18.50} = 6.41 \text{ in}^2$$

Checking the assumed value for a,

$$a = \frac{A_s f_y}{0.85 f_c' b_f} = \frac{6.41 \times 60}{0.85 \times 3 \times 47} = 3.21 \text{ in.}$$

Since a is greater than h_f, a T beam design is required and $\phi = 0.90$ is assumed.

$$A_{sf} = \frac{0.85f'_c(b_f - b_w)h_f}{f_y} = \frac{0.85 \times 3 \times (47 - 11) \times 3}{60} = 4.59 \text{ in}^2$$

$$\phi M_{n1} = \phi A_{sf} f_y \left(d - \frac{h_f}{2}\right) = 0.90 \times 4.59 \times 60 \times \left(20 - \frac{3}{2}\right) = 4590 \text{ in-kips}$$

$$\phi M_{n2} = M_u - \phi M_{n1} = 6400 - 4590 = 1810 \text{ in-kips}$$

Assume $a = 4.0$ in.:

$$A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y(d - a/2)} = \frac{1810}{0.90 \times 60 \times (20 - 4/2)} = 1.86 \text{ in}^2$$

Check:

$$a = \frac{(A_s - A_{sf})f_y}{0.85f'_c b_w} = \frac{1.86 \times 60}{0.85 \times 3 \times 11} = 3.98 \text{ in.}$$

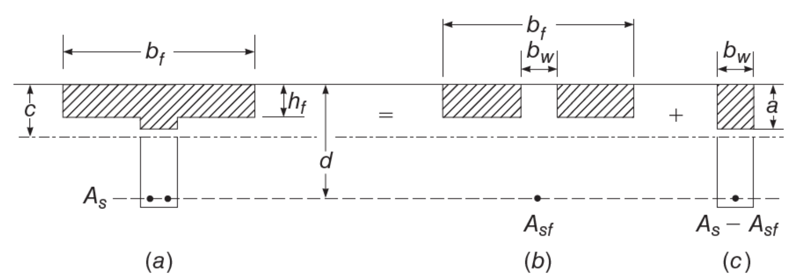
This is satisfactorily close to the assumed value of 4 in. Then

$$A_s = A_{sf} + A_s - A_{sf} = 4.59 + 1.86 = 6.45 \text{ in}^2$$

Checking to ensure that the net tensile strain of 0.005 is met to allow $\phi = 0.90$,

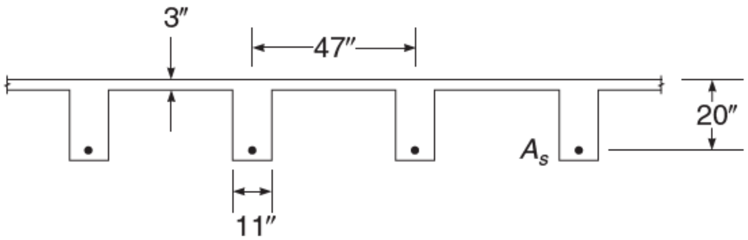
$$c = \frac{a}{\beta_1} = \frac{3.98}{0.85} = 4.68$$

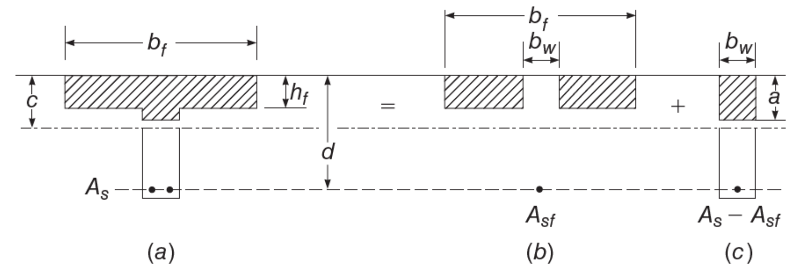
$$\frac{c}{d_t} = \frac{4.68}{20} = 0.23 < 0.325$$



indicating that the design is satisfactory.

The close agreement should be noted between the approximate tensile steel area of 6.41 in^2 found by assuming the stress-block depth equal to the flange thickness and the more exact value of 6.45 in^2 found by T beam analysis. The approximate solution would be satisfactory in most cases.





Example 4.15 — Design Problem

Find $A_s = ?$ for $M_u = 6400^{k\text{-}in} = 534^{k\text{-}ft}$.

Solution

Find Effective flange width, b_f

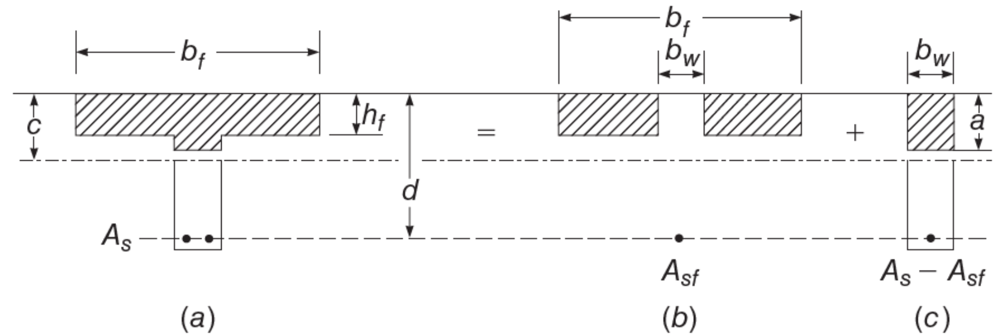
1. $16 h_f + b_w = 16 * 3 + 11 = 59''$
2. $\frac{l_n}{4} + b_w = \frac{24*12}{4} + 11 = 83''$
3. c/c of beam spacing $= 47''$ smallest

$\Longrightarrow b_f = 47''$

Let us first assume $a = h_f = 3''$

$$\therefore A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{6400}{0.9*60*(20-\frac{3}{2})}$$

$$= 6.41 \text{ in}^2$$



$$a = \frac{A_{sf} f_y}{0.85 f_c' b_f} = \frac{6.41 * 60}{0.85 * 3 * 47} = 3.21 > h_f = 3''$$

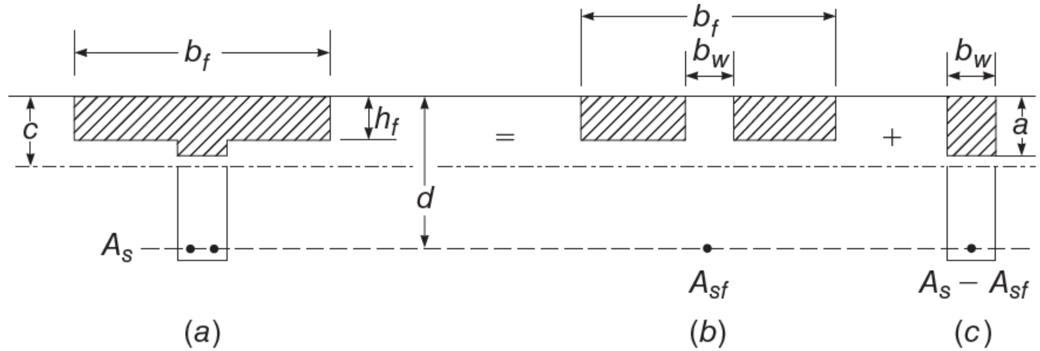
T-beam analysis is required.

$$A_{sf} = \frac{0.85 f_c' (b_f - b_w) h_f}{f_y}$$

$$= \frac{0.85 * 3 * (47 - 11) * 3}{60} = \underline{4.59 \text{ in}^2}$$

$$\phi M_{n1} = \phi A_{sf} f_y \left(d - \frac{h_f}{2}\right)$$

$$= 0.9 * 4.59 * 60 \left(20 - \frac{3}{2}\right) = 4585^{\text{ k-''}}$$



$$\phi M_{n2} = M_u - \phi M_{n1} = 6400 - 4585$$

$$= 1814 \text{ k-in}$$

$$A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y (d - a/2)} = \frac{1814}{0.9 * 60 \left(20 - \frac{1}{2}\right)}$$

$$= 1.72, 1.85, 1.86, 1.87 \text{ in}^2$$

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w}$$

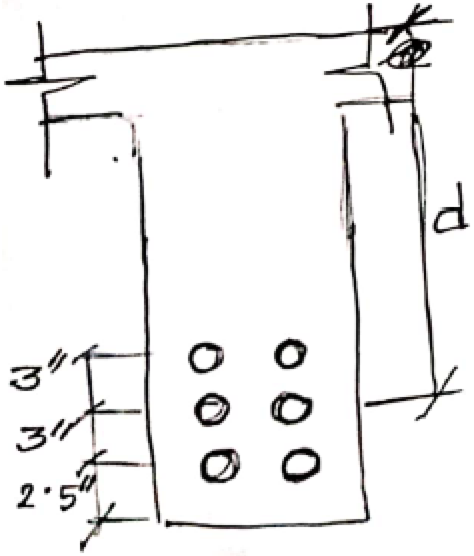
$$= \frac{1.72 * 60}{0.85 * 3 * 11} = 3.68, 3.96, 3.98, 3.99''$$

$$\therefore A_s = A_{sf} + (A_s - A_{sf}) = 4.59 + 1.87$$

$$= \underline{6.46 \text{ in}^2} \leftrightarrow \text{very close to initial value} = 6.41 \text{ in}^2$$

A_s, provided $\Rightarrow$ 4 # 10, 2 # 8 = $\underline{6.66 \text{ in}^2}$.

$$a = \frac{(6.66 - 4.59) * 60}{0.85 * 3 * 11} = 4.427''$$



$$c = \frac{a}{\beta_1} = \frac{4.427}{0.85} = 5.21''$$

$d_t = d + 3''$ (when there is 3 layers of reinf.)

$$= 20 + 3 = 23''$$

$$\varepsilon_t = \frac{\varepsilon_u}{c}(d_t - c)$$

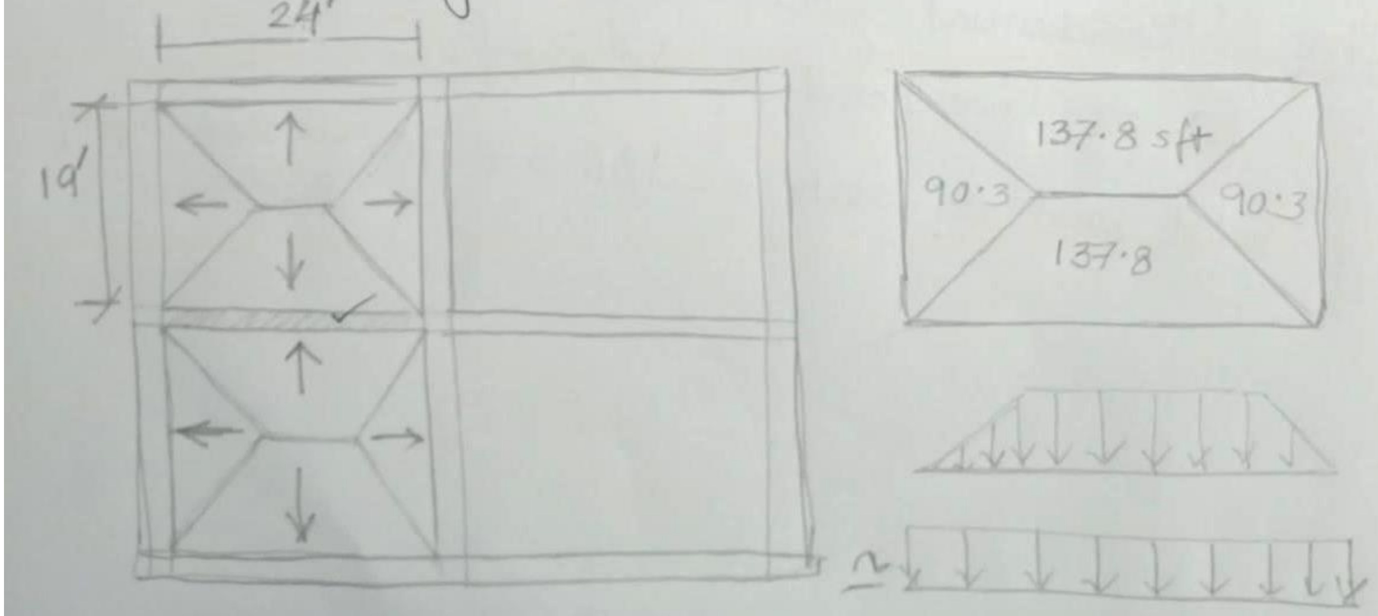
$$= \frac{0.003}{5.21}(23 - 5.21)$$

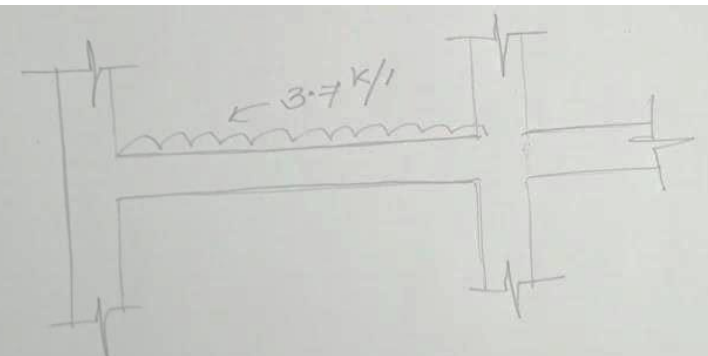
$$= 0.010 >> 0.005$$

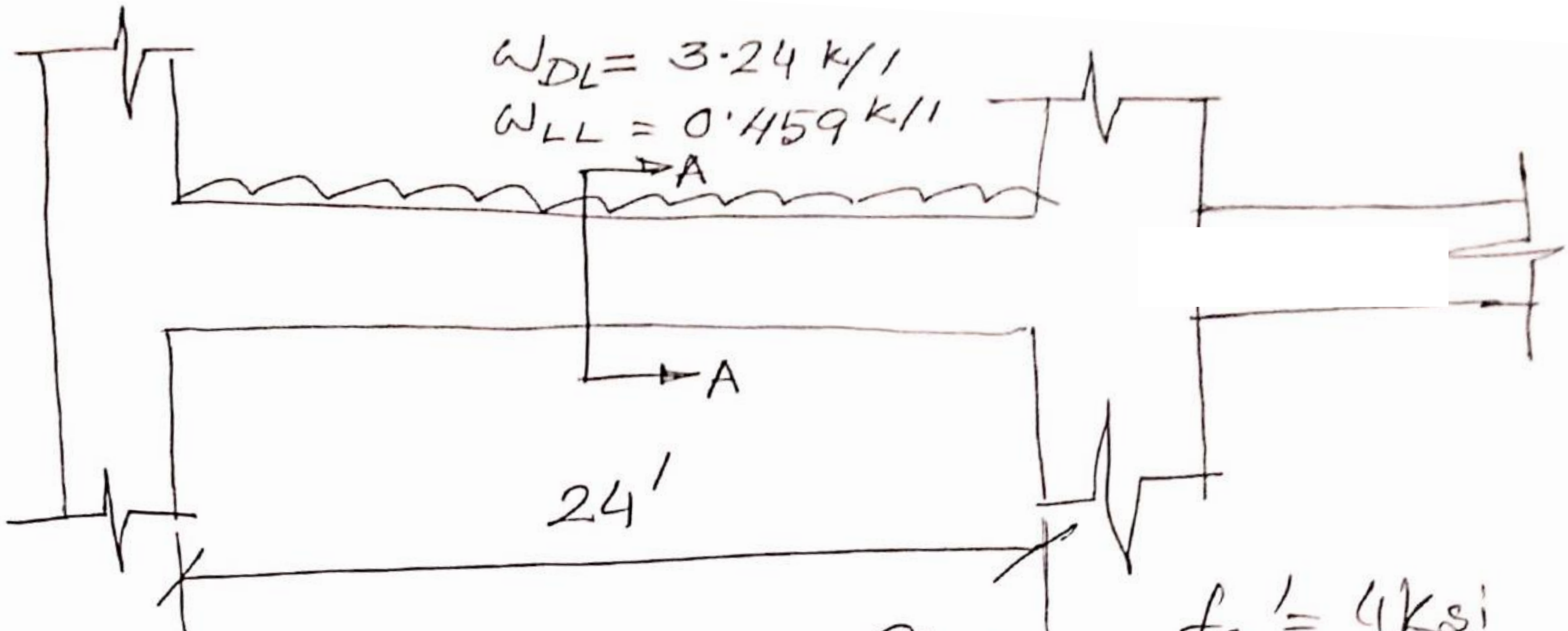
Why strain is calculated corresponding to $(A_s - A_{sf})$?

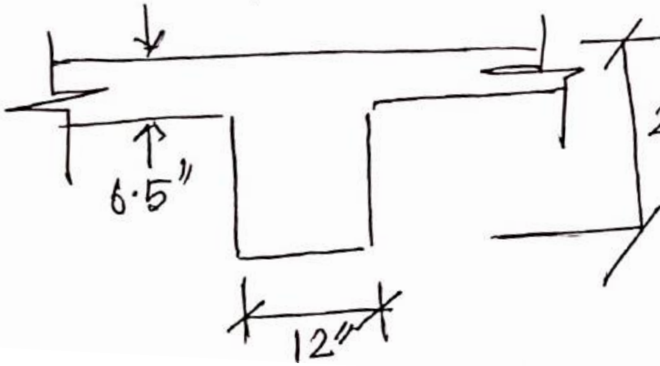
The close agreement should be noted between the approximate tensile steel area of 6.41 in^2 found by assuming the stress-block depth equal to the flange thickness and the more exact value of 6.45 in^2 found by T beam analysis. The approximate solution would be satisfactory in most cases.

(4) Load coming from slab







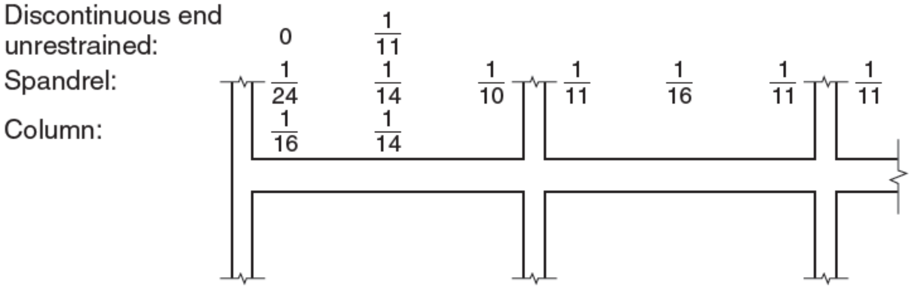


Given: $f_c' = 4\ ksi$

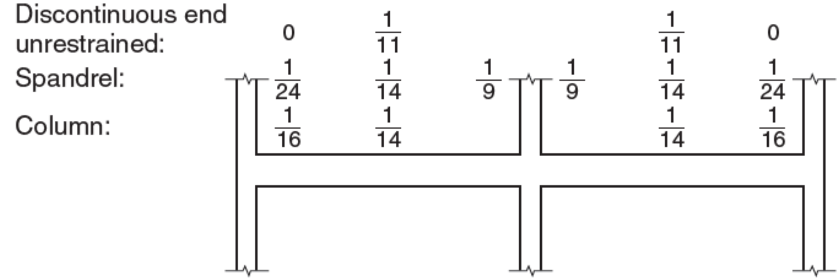
$f_y = 60\ ksi$

Find A_s at midspan and support

(a)



(a)

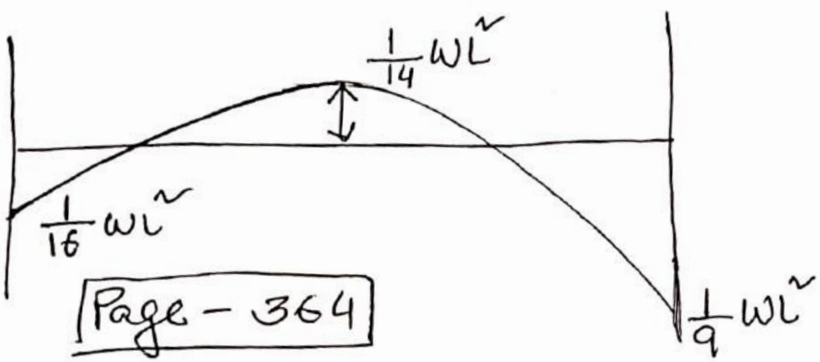


(b)

Solution

$W_u = 1.2\, W_{DL} + 1.6\, W_{LL} = 1.2 * 3.24 + 1.6 * 0.459$

$= 4.62\ ^{k/\prime}$



Design at Midspan

Max^m positive moment $= \frac{1}{14} W_u L^2 = \frac{1}{14} * 4.62 * 24^2$

$= 190$ k-ft.

Effective flange width, b_f

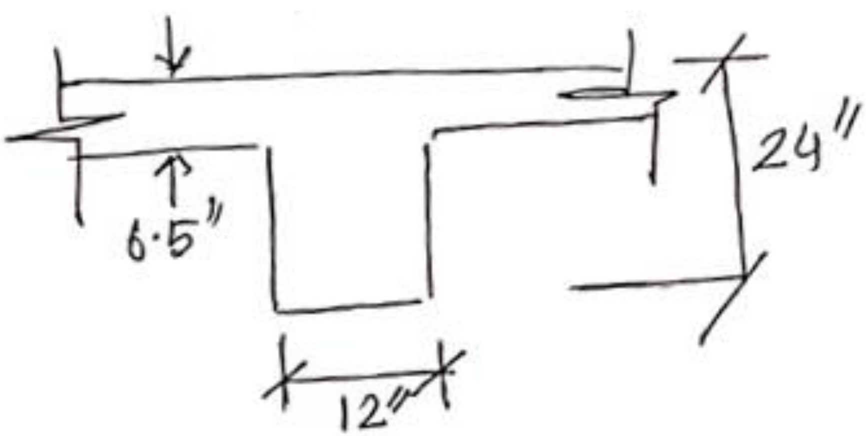
$$= 16\,\overset{h_f}{b_f} + b_w = 16 \times 6.5 + 12 = 116''$$

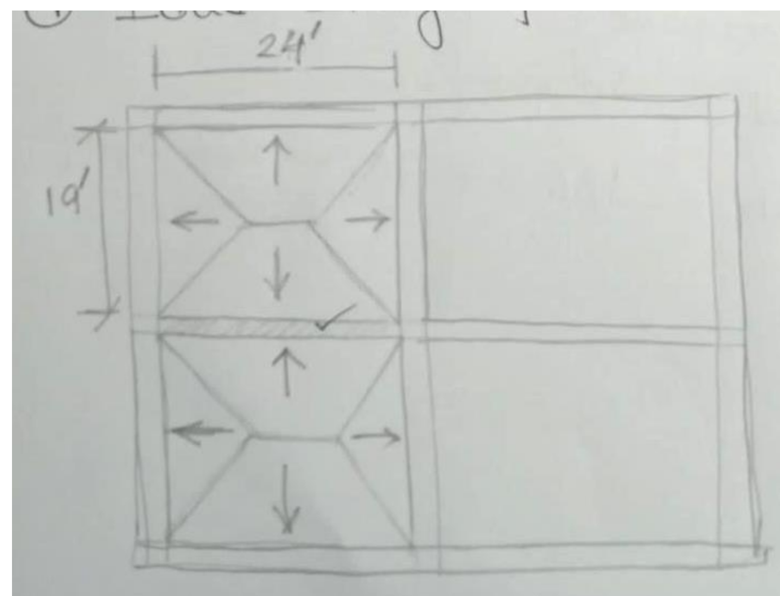
$$= \frac{\text{span}}{4} + b_w = \frac{24 \times 12}{4} + 12 = 84''$$

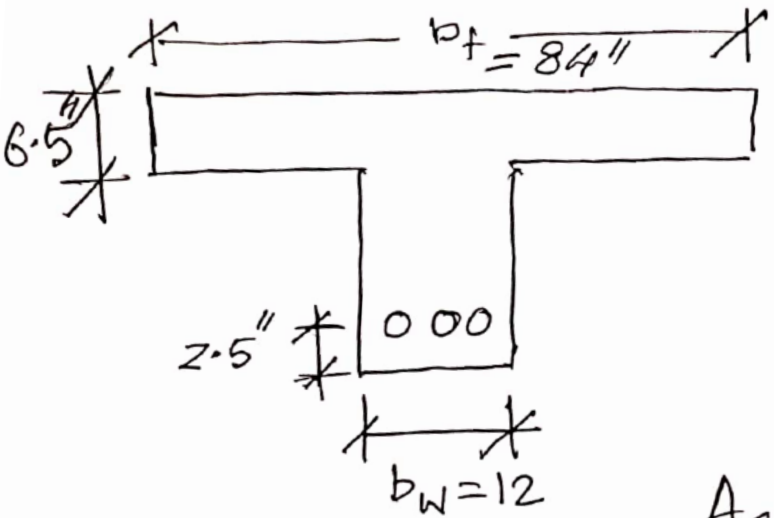
$$= c/c \text{ of beam} = \cancel{10 \times 12 = 120''}$$

20'X12=240"

$$\underline{b_f = 84''}$$







beam section

Let us, $d = 24'' - 4''$

$= 20''$ (Two Layer)

Check if T-beam

$a = h_f = 6.5''$

$$A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{190 * 12}{0.9 * 60 \left(20 - \frac{6.5}{2}\right)}$$

$= 2.52 \text{ in}^2$.

$$a = \frac{A_s f_y}{0.85 f_c' b_f} = \frac{2.52 * 60}{0.85 * 4 * 84} = 0.53'' < h_f$$

$\therefore$ This is a Rectangular beam with $b_f = 84''$

$$A_s = \frac{190 * 12}{0.9 * 60\left(20 - \frac{0.53}{2}\right)} = 2.14, 2.14$$

* Try this ignoring flange

$$a = \frac{2.14 * 60}{0.85 * 4 * 84} = 0.449'', 0.449''$$

$$A_s, \text{provided} \Longrightarrow 3 \# 8 = 2.37 \text{ in}^2.$$

(1 Layer reinf)

$$a = 0.498, \quad c = \frac{a}{\beta_1} = \frac{0.498}{0.85} = 0.586''$$

$$\varepsilon_t = \frac{\varepsilon_u}{c}(d_t - c) = \frac{0.003}{0.586}(20 - 0.586)$$

$$= 0.099 >>> 0.005 \quad \phi = 0.9 \text{ (OK)}$$

Design at Interior support

M_u = Negative moment = tension at top

$= \frac{1}{9} w_u L^2 = \frac{1}{9} * 4.62 * 24^2 = 296$ k-ft.

The flange is in tension, so this is a Rectangular beam with $b_w = 12''$

no b_f for −ve moment

$$A_s = \frac{296 * 12}{0.9 * 60 * (20 - \frac{a}{2})} = 3.37, 3.75, 3.81, 3.83 \text{ in}$$

$$a = \frac{3.37 * 60}{0.85 * 4 * 12} = 4.96, 5.52, 5.61, 5.63 \text{ in}$$

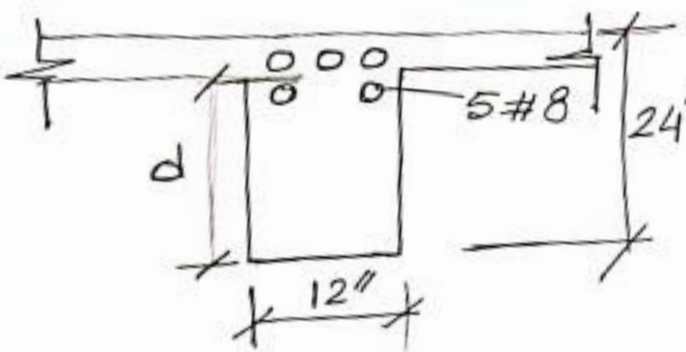
Let $A_{s,provided}$ = 5 # 8 = 3.95 in^2

$$a = \frac{3.95 * 60}{0.85 * 4 * 12} = 5.81''$$

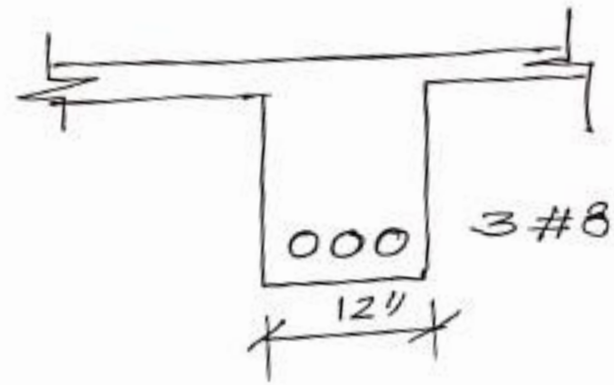
$$c = \frac{a}{\beta_1} = \frac{5.81}{0.85} = 6.83''$$

$$\varepsilon_t = \frac{\varepsilon_u}{c}(d_t - c) = \frac{0.003}{6.83}(21.5 - 6.83)$$

$$= 0.00644 > 0.005 \quad \phi = 0.9 \text{ (ok)}$$



At support



At midspan

At interior support

End of

Chapter 4: Flexural Analysis and Design of Beams